STAT 100 Lecture 17: Testing Hypotheses: Examples

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Last Time...

1. Formulating the hypotheses.
2. Test criterion and rejection region.
3. Two types of error and their probabilities.

Today's Agenda

1. Performing a test.
2. P-value: how strong is a rejection of H_0?
3. Examples.

Two types of error and their probabilities

Definition

Type I error: *Rejection of* H_0 *when* H_0 *is true.*
Type II error: *Nonrejection of* H_0 *when* H_1 *is true.*
$\alpha =$ *Probability of making type I error (also called* **the level of significance***)*
$\beta =$ *Probability of making type II error*

Two types of error and their probabilities

$\alpha =$ Probability of making type I error, $\beta =$ Probability of making type II error.

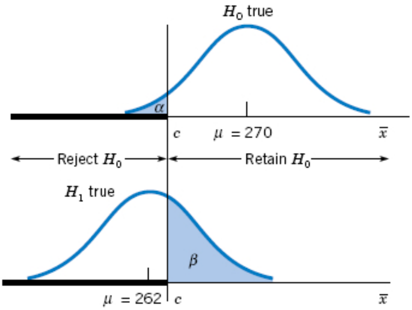


Figure 8 The error probabilities α and β.

Performing a Large Sample Normal Test or Z Test

Example

A sample of 40 sales receipts from a grocery store has $\overline{x} = \$141$ *and* $s = \$30.2$*. Test the null hypothesis* $H_0 : \mu = 150$ *versus* $H_1 : \mu < 150$ *using a 5% level of significance and state whether or not the claim* $\mu < 150$ *is substantiated.*

Solution

Because $n = 40$ *is large, the* $\overline{X}$ *has the approximately normal distribution by CLT. If* σ *is unknown we can estimate it by the sample standard deviation* S*. The null hypothesis* H_0 *specifies that* μ *has the value* $\mu_0 = 150$*. The test statistics*

$$Z = \frac{\overline{X} - 150}{30.2/\sqrt{40}}$$

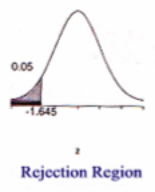
. *With* $\alpha = 0.05$*, the rejection region is* $R : Z \leq -1.645$.

Example

The observed value of the Z test statistic is

$$z = \frac{141 - 150}{30.2/\sqrt{40}} = -1.88,$$

which is in R. Therefore, we reject H_0 at the $\alpha = 0.05$ level.



P-value: How Strong is a rejection of H_0?

Question

How small an α could we use and still arrive at the conclusion of rejecting H_0?

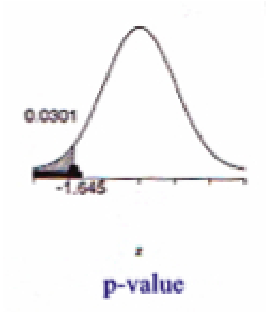
Answer

The observed $z = -1.88$. Calculate the rejection probability

$$P[Z \leq -1.88] = 0.0301$$

P–value: How Strong is a rejection of H_0?

The smallest possible α that would permit rejection of H_0, on the basis of the observed $z = -1.88$, is therefore .0301.



.0301 is called the **significance probability** or **P–value** of the observed z. This very small P–value, .0301, signifies a strong rejection of H_0 or that the result is highly statistically significant.

P–value

Definition

The **P–value** *is the probability, calculated under H_0, that the test statistic takes a value equal to or more extreme than the value actually observed.*

The P–value serves as a measure of the strength of evidence against H_0.

A small P–value *means that the null hypothesis is* **strongly rejected** *or the result is* **highly statistically significant.**

The Steps for Testing Hypotheses

Definition

1. *Formulate the null hypothesis H_0 and the alternative hypothesis H_1.*
2. *Test criterion: State the test statistic and the form of the rejection region.*
3. *With a specified α, determine the rejection region.*
4. *Calculate the test statistic from the data.*
5. *Draw a conclusion: State whether or not H_0 is rejected at the specified α and interpret the conclusion in the context of the problem. Also, it is a good statistical practice to calculate the P–value and strengthen the conclusion.*

Example Evaluating a Weight Loss Diet–Calculation of a P–Value

Example

A brochure inviting subscriptions for a new diet program states that the participants are expected to lose over 22 pounds in five weeks. Suppose that, from the data of the five-week weight losses of 56 participants, the sample mean and standard deviation are found to be 23.5 and 10.2 pounds, respectively. Could the statement in the brochure be substantiated on the basis of these findings? Test with $\alpha = .05$. *Also calculate the P–value and interpret the result.*

Two types of alternative hypothesis.

So far, we have only seen **one-sided hypotheses**:

$$H_0 : \mu = \mu_0 \text{ versus } H_1 : \mu > \mu_0$$

or

$$H_0 : \mu = \mu_0 \text{ versus } H_1 : \mu < \mu_0$$

The corresponding tests are called **one-sided tests** or **one-tailed tests.**

We can also have **two-sided alternative** or **two-sided hypothesis**:

$$H_0 : \mu = \mu_0 \text{ versus } H_1 : \mu \neq \mu_0$$

Two types of alternative hypothesis.

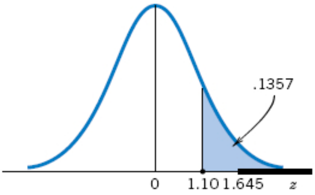


Figure: Rejection region for one-sided alternative.

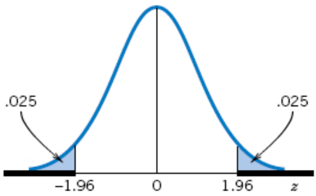


Figure: Rejection region for two-sided alternative.

Example. Testing Hypotheses about the Mean Height of Seedlings

Example

The height measurements of 40 pine seedlings were performed. We find $\overline{x} = 1.715$ *and* $s = .475$. *Do these data indicate that the population mean height is different from 1.9 centimeters?*

Summary

Large Sample Test for μ

When the sample size is large, a Z test concerning μ is based on the normal test statistic

$$Z = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}.$$

The rejection region is one- or two-sided depending on the alternative hypothesis. Specifically,

$$H_1 : \mu > \mu_0 \text{ requires } R : Z \geq z_\alpha.$$

$$H_1 : \mu < \mu_0 \text{ requires } R : Z \leq -z_\alpha.$$

$$H_1 : \mu \neq \mu_0 \text{ requires } R : |Z| \geq z_{\alpha/2}.$$

For Next Time

- Read Section 8.5 from Johnson and Bhattacharyya
- Online homework 8.4: 8.37, 8.41, 8.49, 8.51, 8.53