DC MACHINES

Just as with any other type of rotating machine, the DC machine can be used as either a motor or a generator. However, applications requiring operation of the DC machine as a generator are limited, while applications requiring the DC machine as a motor are commonplace. DC motors are the obvious choice in applications where DC sources are all that is available (e.g., automotive systems). One capability that DC machines possess that induction and synchronous machines do not is precise speed and/or torque control. Many industrial applications require motors with a high degree of flexibility in the control of speed and torque making the DC machine the machine of choice.

DC Machine Characteristics

1.   Adjustable motor speed over wide ranges
2.   Constant mechanical output (torque)
3.   Rapid acceleration or deceleration
4.   Responsive to feedback signals

The rotor of the DC machine is the armature winding with field windings located in the stator. The poles on a DC machine are salient poles that are, in the case of a motor, excited by a DC current in the field windings. The armature windings are interconnected by way of the *commutator* which is made up of conducting bars (*commutator segments*) which are electrically insulated from one another. The rotating commutator makes contact with stationary contacts by way of *brushes*. In the case of a DC motor, the role of the commutator is to ensure that the forces on the armature windings point in the same rotation direction at all positions of the armature. This requires that the direction of the armature currents be changed depending on their alignment relative to the stator field. In the case of a DC generator, the commutator ensures that the armature output voltage (which is oscillatory) is rectified to approximate a DC value.

The operation of the DC machine can be illustrated by using a simplified 2-pole DC machine (see Figure 4.13, p. 131). Note that the magnetic field of the poles in this example is pointing left to right. For the case of a DC motor, assume that armature current labeled as "*a*" is flowing inward while the current labeled "*b*" is flowing outward. The directions of the forces on the two armature currents are found according to

$$F = \int_L (I_a \times B_f)\, dl$$

where I_a is the vector armature current and B_f is the vector magnetic flux density produced by the field windings. At the armature location shown in the figure, the vector force on the *a* conductor is downward, while the vector force on the *b* conductor is upward, giving a net counterclockwise rotational force. However, if the armature current directions remain unchanged, when the conductors rotate to the opposite side of the machine, the directions of the forces on the armature conductors would remain unchanged (*a* downward and *b* upward). These forces would now present a net clockwise rotational force and prevent rotation.

If, on the other hand, the armature currents can be made to change directions as these conductors approach either side of the machine, the rotational force will be unidirectional and the machine will rotate. This current reversal is accomplished via the commutator/brush combination. The commutator acts as a mechanical rectifier. As the armature rotates, the brush contacts alternate from one commutator segment to the other. Thus, for this example, when the armature conductors are rotating on the left side of the machine, their currents are always outward while when the conductors are rotating on the right side of the machine, their current are always inward. This current reversal keeps the rotational force unidirectional.

A DC motor can have more than 2 poles (see Figure 4.16, p.133). The number of windings on the armature should equal to number of poles on the stator. In addition, the number of commutator segments will be equal to the number of machine poles.

DC GENERATOR
(INDUCED ARMATURE VOLTAGE)

If the operation of the DC machine as a generator is considered, an expression for the voltage induced at the armature terminals can be developed. As the armature rotates in the magnetic field produced by the field windings, and emf is induced in the conductors. The induced emf in each armature turn V_t (shown is defined according to the concept of *motional induction* as

$$V_t = \int \boldsymbol{E}_a \cdot d\boldsymbol{l} = \int (\boldsymbol{u}_a \times \boldsymbol{B}_f) \cdot d\boldsymbol{l} \qquad \left(\begin{array}{c} \text{motional induction} \\ \text{"flux cutting emf"} \end{array} \right)$$

where $\boldsymbol{u}_a$ is the vector velocity of the armature conductor, $\boldsymbol{B}_f$ is the vector magnetic flux density produced by the field windings and $\boldsymbol{E}_a$ is the vector electric field associated with the induced emf in the armature conductor.
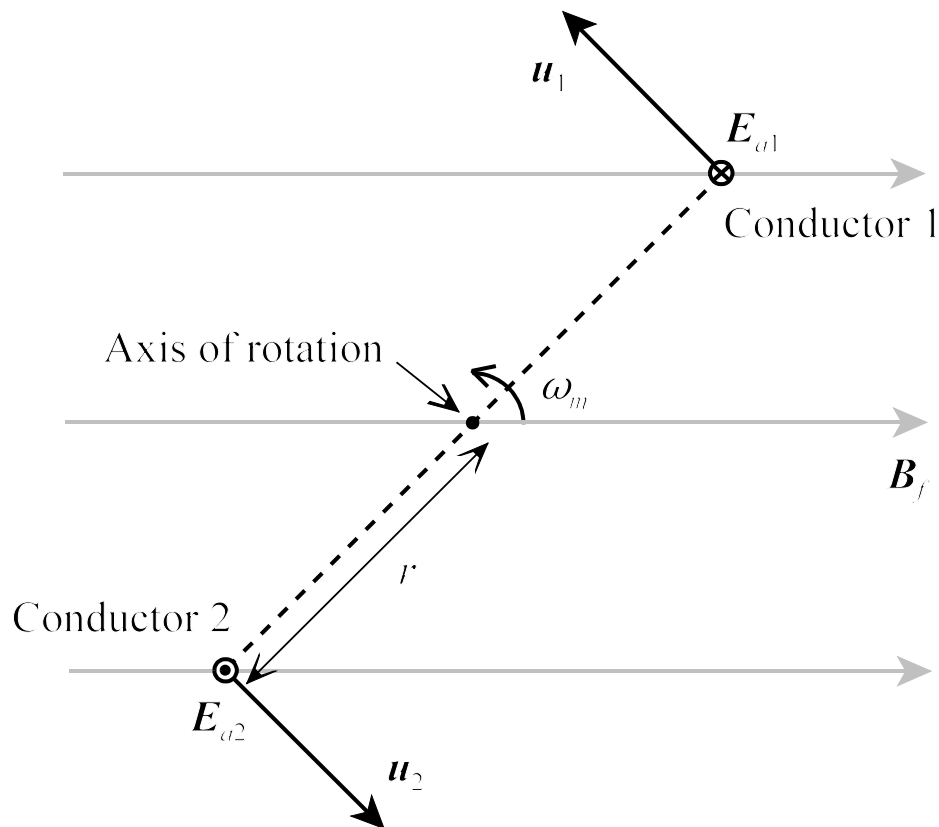
The magnitude of the velocity of each conductor is given by

$$u = \omega_m r$$

where r is the radius of the armature and ω_m is the mechanical angular velocity of the armature in rad/s. The magnitude of the resulting emf electric field along each conductor is given by

$$E_{a1} = E_{a2} = u B_f = \omega_m r B_f$$

Note that the direction of the emf electric fields are opposite on the armature turn conductors. The induced emf in each conductor is found by integrating the emf electric field along the length of the conductor. Assuming the emf electric field is uniform along the conductor, this gives

$$V_{t1} = V_{t2} = u B_f l = \omega_m r B_f l$$

where l is the length of the armature conductor in the magnetic flux of the field winding. These induced voltages have opposite polarities so that they add to give the overall induced voltage in each armature turn

$$V_t = 2 \omega_m r B_f l$$

The magnetic flux density in a DC machine will be non-uniform. If the magnetic flux density is interpreted as an average value, the average magnetic flux density can be written as

$$B_f = \frac{\psi_m}{A} = \frac{\psi_m}{\dfrac{2\pi r l}{p}} = \frac{\psi_m p}{2\pi r l}$$

where ψ_m is the total magnetic flux per pole and A is the area per pole. Inserting this expression for the average magnetic flux density into the induced armature turn voltage gives

$$V_t = \frac{\psi_m p \, \omega_m}{\pi}$$

The overall armature winding voltage (V_a) in all of the turns (N) connected in series is equal to the single turn voltage times the number of turns. If there several of these series combinations of turns connected in parallel, then the overall voltage is divided by the number of parallel paths which gives

$$V_a = \frac{N}{a} V_t = \frac{N}{a} \frac{\Psi_m p \, \omega_m}{\pi}$$

where a is the number of parallel paths. The armature voltage can be written as

$$V_a = K_a \Psi_m \omega_m$$

where the constant K_a is known as the *machine* or *armature* constant given by

$$K_a = \frac{Np}{a\pi}$$

The armature constant contains the information about how the armature winding is configured. The two most common configurations of DC machine windings are the *lap winding* and the *wave winding*.

Lap winding - the number of parallel paths is equal to the number of machine poles ($a = p$).

Wave winding - two parallel paths ($a = 2$).

The lap winding is well-suited for DC machines with high currents and low voltages while the wave winding is suitable for DC machines with high voltages and low currents.

DC MACHINE DEVELOPED TORQUE

The torque developed by a DC machine can be determined by summing the Lorentz forces on all of the armature conductors. The force (F_c) on an individual armature conductor is given by

$$F_c = \int_L (I_c \times B_f) dl$$

where I_c is the conductor current, B_f is the average flux density seen by the conductor as it rotates and the integration is along the length of the conductor located in the magnetic flux (l) produced by the field windings. Assuming a uniform flux density along the length of the conductor, the magnitude of the vector force on the armature conductor is

$$F_c = I_c B_f l$$

The current in an individual armature conductor will depend on the number of parallel paths in the armature (a) and the total armature current (I_a). The force on an individual armature conductor in terms of the total armature current is

$$F_c = \frac{I_a}{a} B_f l$$

The developed torque associated with a single armature conductor (T_c) is given by the product of the force on the conductor and the radius of the armature (r).

$$T_c = F_c r = \frac{I_a}{a} B_f r l$$

The total developed torque (T) is found by multiplying T_c by the total number of conductors in the armature ($2N$ = two conductors per turn $\times$ N total turns).

$$T = 2NT_c = \frac{2NI_a r l}{a} B_f$$

Inserting the previously determined expression for the average flux density into the total developed torque equation gives

$$B_f = \frac{\psi_m p}{2\pi r l}$$

$$T = \frac{2NI_a r l}{a} B_f$$

$$= \frac{2NI_a r l}{a} \frac{\psi_m p}{2\pi r l}$$

$$= \frac{Np}{\pi a} \psi_m I_a$$

$$= K_a \psi_m I_a$$

where K_a is the armature constant. For the case of a DC motor, the armature output power (P_a) is the product of the developed torque and the angular velocity of the machine (ω_m).

$$P_a = T\omega_m = K_a \psi_m I_a \omega_m$$

For the case of a DC generator, the armature output power is simply the product of the armature voltage and the armature current ($V_a I_a$). Given the previously determined expression for the armature voltage,

$$V_a = K_a \psi_m \omega_m$$

the armature output power is

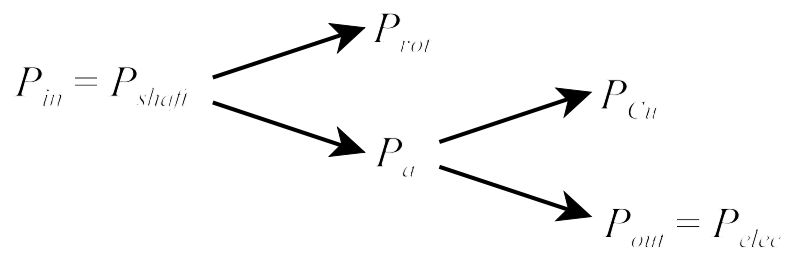$$P_a = V_a I_a = K_a \psi_m I_a \omega_m$$

Note that the armature output power is equivalent for a DC motor and a DC generator. The armature ouput power is an intermediate power quantity in the overall operation of a DC motor or generator. If we designate the power loss components as:

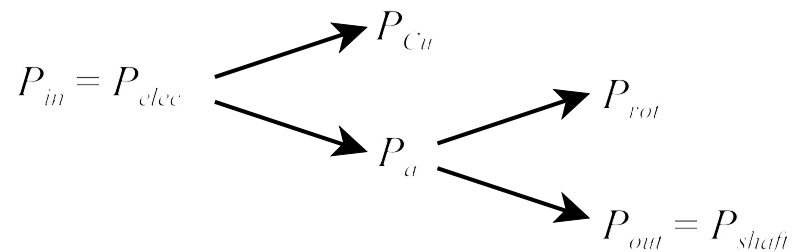P_{Cu} - DC machine copper losses (armature, field winding)

P_{rot} - DC machine rotational losses (windage, friction)

then the power flow relationships for the DC generator and the DC motor are shown below.

DC Generator

$$P_{in} = P_{shaft}$$

$$P_{rot}$$

$$P_a$$

$$P_{Cu}$$

$$P_{out} = P_{elec}$$

DC Motor

$$P_{in} = P_{elec}$$

$$P_{Cu}$$

$$P_a$$

$$P_{rot}$$

$$P_{out} = P_{shaft}$$

<u>Example</u> (DC Machine, armature characteristics)

The armature winding of a four pole DC machine (armature radius r = 12.5cm, armature length l = 25cm) has 33 coils with 7 turns per coil. The poles cover 75% of the armature while the average magnetic flux density per pole is 0.75T. For lap-wound and wave-wound machines, determine (a.) the armature constant (b.) the induced armature voltage when the armature rotates at 1000rpm (c.) the coil current (d.) the developed torque in the machine when the armature current is 400A (e.) the power developed by the armature.

<u>Lap-Wound machine</u> ($a = p = 4$)

(a.) $N = 33 \times 7 = 231$ total turns

$$K_a = \frac{Np}{a\pi} = \frac{N}{\pi} = \frac{231}{\pi} = 73.53$$

(b.) $V_a = K_a \psi_m \omega_m$

$$\psi_m = B_f A = B_f \left[0.75 \frac{2\pi rl}{p} \right]$$

$$= 0.75 \left[0.75 \frac{2\pi(0.125)(0.25)}{4} \right]$$

$$= 0.0276 \text{ T}$$

$$\omega_m = 1000 \frac{\text{rev}}{\text{min}} \left[2\pi \frac{\text{rad}}{\text{rev}} \right] \left[\frac{1 \text{ min}}{60 \text{ sec}} \right] = 104.72 \text{ rad/s}$$

$$V_a = (73.53)(0.0276)(104.72) = 212.52 \text{ V}$$

(c.) $$I_{coil} = \frac{I_a}{a} = \frac{400}{4} = 100 \text{ A}$$

(d.) $T = K_a \psi_m I_a = (73.53)(0.0276)(400) = 811.8 \text{ T-m}$

(e.) $P_a = T\omega_m = = (811.8)(104.72) = 85.0 \text{ kW}$

      or

$$P_a = V_a I_a = (212.52)(400) = 85.0 \text{ kW}$$

Wave-Wound machine ($a = 2$)

(a.) $N = 33 \times 7 = 231$ total turns

$$K_a = \frac{Np}{a\pi} = \frac{(231)(4)}{2\pi} = 147.06$$

(b.) $V_a = K_a \psi_m \omega_m$

$$= (147.06)(0.0276)(104.72) = 425.04 \text{ V}$$

(c.) $I_{coil} = \frac{I_a}{a} = \frac{400}{2} = 200 \text{ A}$

(d.) $T = K_a \psi_m I_a = (147.06)(0.0276)(200) = 811.8 \text{ T-m}$

(e.) $P_a = T\omega_m = = (811.8)(104.72) = 85.0 \text{ kW}$

      or

$$P_a = V_a I_a = (425.04)(200) = 85.0 \text{ kW}$$

DC MACHINE MAGNETIZATION CURVES

The equivalent magnetic circuit of the DC machine consists of mmf sources associated with the field windings and the reluctances of the various machine components in the direction of the magnetic flux [poles, air gaps, rotor teeth, rotor core and the outer portion of the stator (yoke)] (see Figure 4.21, p. 142). At low values of magnetic flux density, the reluctances of the ferromagnetic components in the magnetic circuit are negligible in comparison to the reluctances of the air gaps. However, at higher values of magnetic flux density, the certain components of the magnetic circuit may saturate (particularly the rotor teeth), and the increased reluctance of these components begins to affect the overall magnetic circuit.

The armature voltage is a critical parameter in the determination of performance characteristics for the DC machine. The armature voltage has been shown to be proportional to the magnetic flux density per pole and the armature speed. Thus, the armature voltage will also exhibit nonlinear behavior when saturation occurs. The analysis of DC machine performance typically requires that the characteristics of the armature voltage saturation be known. This information is typically provided in a plot of the armature voltage vs. field current (magnetization curve) for a given operating speed.

DC MACHINE CLASSIFICATIONS

One significant advantage of a DC machine is the variety of performance characteristics that can be achieved by interconnecting the field and armature windings in various ways.

The field windings can be excited by a DC source which is connected only to the field windings and not connected to the armature. Such a configuration is known as a *separately excited DC machine*. The field excitation can also be provided by permanent magnets. This type of machine is also classified as a separately excited machine since the armature current is independent of the field excitation.

If the field winding current is all or part of the armature current, the machine is known as a *self-excited DC machine*. There are three different configurations of self-excited DC machines. In a *series self-excited DC machine*, the field windings are connected in series with the armature. A *shunt self-excited DC machine* results when the field windings are connected in parallel with the armature. If a combination of series and parallel connections are made between the field and armature windings, this machine is classified as a *compound self-excited DC machine*.

If a shunt connection of the field windings is implemented, a variable resistor is typically inserted in series with the field winding in order to easily control the field winding current. This variable resistor is known as the *field rheostat*. The field winding current in shunt-connected winding is typically low (approximately 1 to 2% of the rated armature current). Thus, copper losses in the shunt-connected winding are typically low.

A series connected field winding must carry the much larger armature current. Thus, the resistance of a series field winding must be kept low in order that the copper losses of the field winding are not too severe. A series field winding will typically have a resistance on the order of tens of $m\Omega$'s.

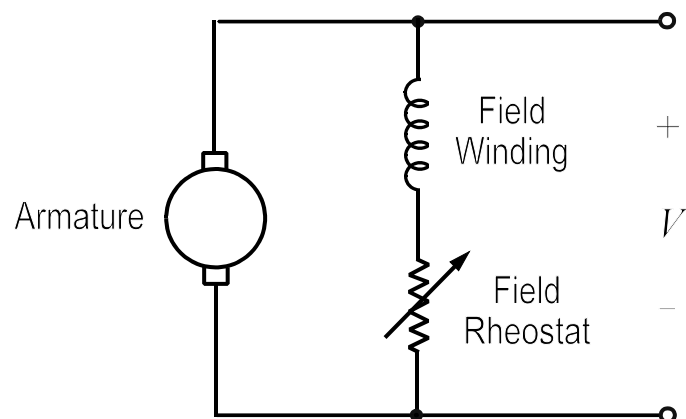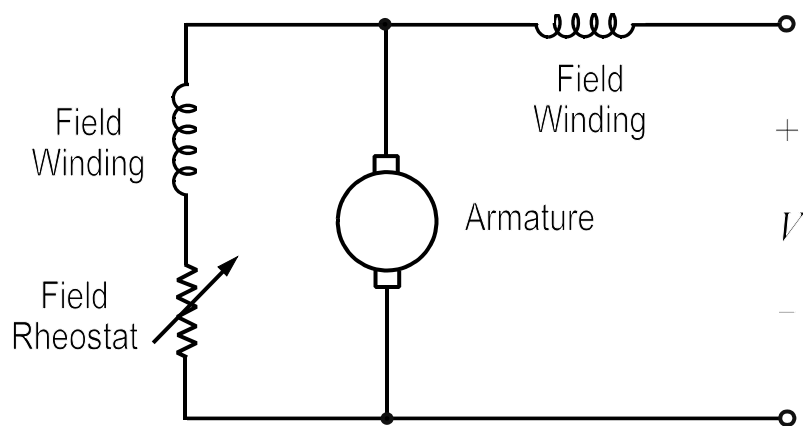## Separately excited DC machine

Field Winding

External DC Source
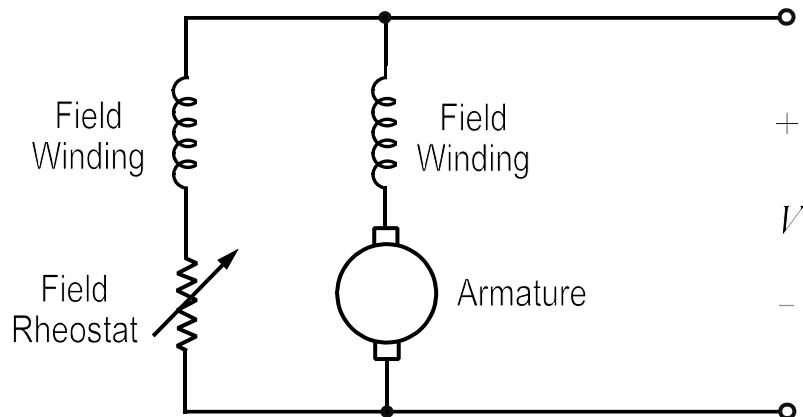
Field Rheostat

Armature

$+$

V

$-$
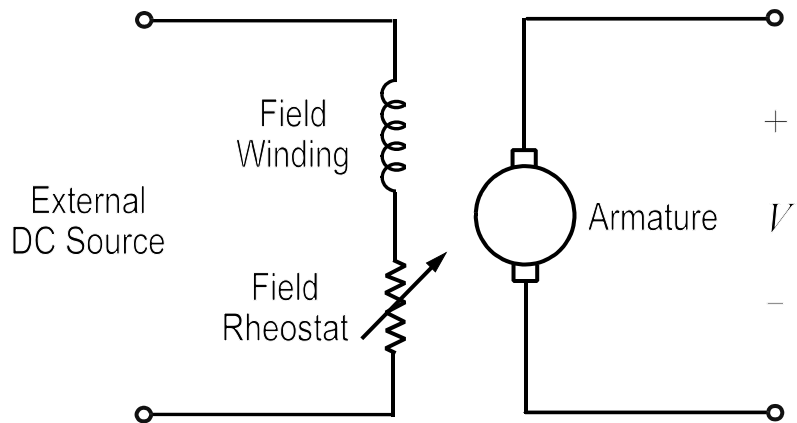
Self-excited series DC machine

Field Winding

Armature

$+$

V

$-$
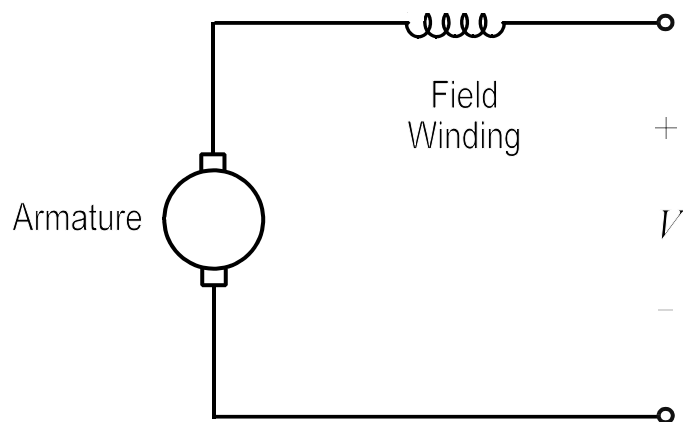
Self-excited shunt DC machine

Field Winding

Field Rheostat

Armature

$+$

V

$-$

Self-excited compound DC machine (short shunt)

Field Winding

Field Winding

Field Rheostat

Armature

$+$

V

$-$

Self-excited compound DC machine (long shunt)

Field Winding

Field Winding

Field Rheostat

Armature

$+$

V

$-$

The various DC machine configurations can be analyzed by including the resistances of the field windings, the armature circuit (armature winding resistance and brush resistances) along with the load resistance and the field rheostat, if present. The analysis at DC steady-state does not require inclusion of the various winding inductances. For transient analysis, these inductances are important.

SEPARATELY EXCITED DC GENERATOR

The complete equivalent circuit of the separately excited DC generator is shown below. The individual components of the equivalent circuit are:

R_a - armature resistance (including the resistance of the armature winding along with brush contact resistance)
R_{fw} - field winding resistance
R_{fc} - field winding resistance
R_L - load resistance
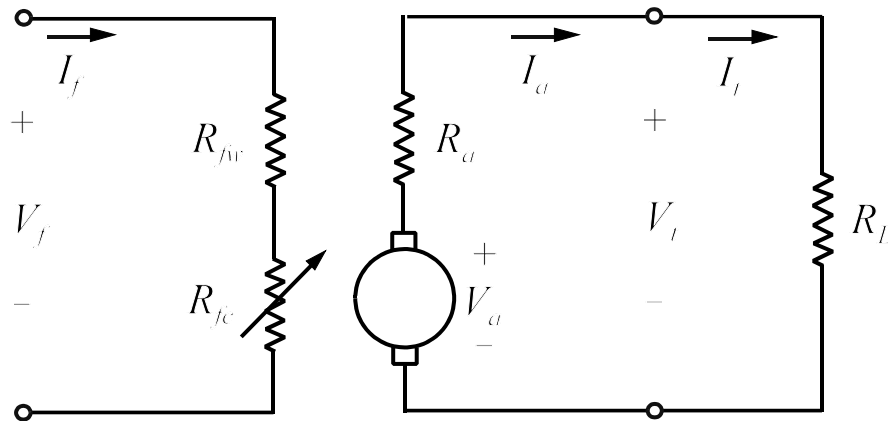
Analysis of the resistive network is straightforward assuming the armature voltage is known. Note that the terminal voltage is dependent on the armature voltage and the voltage drop across the armature resistance (dependent on the terminal current). The terminal voltage is given by

$$V_t = V_a - I_a R_a$$

The circuit model predicts that the terminal voltage decreases linearly with an increase in the terminal (armature) current. However, due to an effect known as the *armature reaction* or the *demagnetization effect*, the actual terminal voltage is slightly lower than that predicted by the equivalent circuit. The armature reaction is more pronounced at higher armature currents and is usually neglected at currents below the rated armature current.

The armature reaction is caused by the fact that the currents induced in the armature by motional induction create secondary components of magnetic field that tend to oppose the magnetic field of the field circuit over a portion of the armature while enhancing the magnetic field of the field circuit over the remainder of the armature. If the enhanced portion of the armature field causes magnetic saturation, the net effect is a reduction in the flux per pole which decreases the armature voltage, and thus, decreases the terminal voltage.

The armature reaction can be modeled as an effective reduction in the field current. The effective armature current ($I_{f,eff}$) is related to the actual field current by

$$I_{f,eff} = I_f - I_{f,AR}$$

where $I_{f,AR}$ is the amount by which the field current must be reduced to model the armature reaction.

Example (Separately excited DC generator)

A 12 kW, 100V, 1000rpm DC shunt generator has an armature resistance of $R_a = 0.1$ Ω, shunt winding resistance of $R_{fw} = 80$ Ω and $N_f = 1200$ turns per pole. The rated field current is 1 A. The magnetization characteristic at 1000 rpm is given in Figure 4.24 (p. 143). The machine is operated as a separately excited DC generator at 1000 rpm with rated field current.

(a.) Determine the terminal voltage at full load neglecting the armature reaction.

(b.) Determine the terminal voltage at full load accounting for the armature reaction. Assume the armature reaction at full load is equivalent to 0.06 field Amperes.

(c.) Determine the field current necessary to make the terminal voltage equal to 100 V at full load.

The machine description provides the following data:

> Rated armature power (at full load) = 12 kW
> Rated armature voltage (at full load) = 100 V
> Rated speed = 1000 rpm

The rated armature current is found from the rated power and voltage.

> Rated armature current = 12000/100 = 120 A

(a.) The terminal voltage at full-load is given by

$$V_t = V_a - I_a R_a = 100 - (120)(0.1) = 88 \text{ V}$$

(b.) The effective field current accounting for the armature reaction is

$$I_{f,eff} = I_f - I_{f,AR} = 1.0 - 0.06 = 0.94 \text{ A}$$

At the effective field current, the armature voltage is approximately 98 V. The resulting terminal voltage is

$$V_t = V_a - I_a R_a = 98 - (120)(0.1) = 86 \text{ V}$$

(c.) If the terminal voltage is 100 V, the corresponding armature voltage is

$$V_a = V_t + I_a R_a = 100 + (120)(0.1) = 112 \text{ V}$$

From Figure 4.24, the effective field current is approximately 1.4 A. The actual field current is then

$$I_f = I_{f,eff} + I_{f,AR} = 1.4 + 0.06 = 1.46 \text{ A}$$

SHUNT DC MOTOR

The equivalent circuit of the shunt DC motor is shown below. Note that for motor operation, a DC voltage source is applied at the input terminal and the terminal current is assumed to flow into the motor.
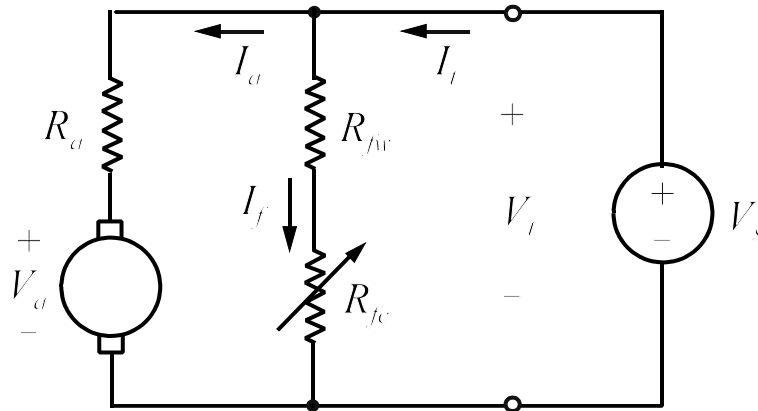
The operation of the field circuit is no different from the separately excited machine since a constant DC voltage is applied to the field winding (just as the separately excited case). In this regard, the function of the field circuit is independent of the of the armature circuit. However, the input DC source for the shunt motor must supply a current equal to the sum of the armature and the field currents.

Example (Shunt DC motor)

The DC machine used in the separately excited DC generator example (12 kW, 100V, 1000 rpm, R_a = 0.1 Ω, R_{fw} = 80 Ω) is used as DC shunt motor. The motor operates at 1000 rpm with an armature current of 6 A at no-load. Determine
    (a.)   the resistance value of the field rheostat (R_{fc}).
    (b.)   the rotational losses of the DC motor at 1000 rpm.

(c.) the speed, developed torque and efficiency given rated armature current neglecting armature reaction (assume the air gap flux at rated load equals that at no load).

(d.) the speed, developed torque and efficiency given rated armature current assuming the air gap flux is reduced by 5% from the no-load value due to armature reaction.

(e.) the starting torque if the starting armature current is limited to 150% of its rated value (neglect armature reaction).

(f.) the starting torque if the starting armature current is limited to 150% of its rated value accounting for armature reaction with $I_{f,AR} = 0.16$ A.

(a.) The armature voltage at no-load ($I_a = 6$ A) is given by

$$V_a = V_t - I_a R_a = 100 - (6)(0.1) = 99.4 \text{ V}$$

From the magnetization curve for the machine, an armature voltage of 99.4 V at 1000 rpm requires a field current of approximately 0.99 A. Thus,

$$V_t = I_f(R_{fw} + R_{fc})$$

$$R_{fc} = \frac{V_t}{I_f} - R_{fw} = \frac{100}{0.99} - 80 = 21 \text{ }\Omega$$

(b.) At no-load, all of the armature power is dissipated in the form of rotational loss.

$$P_{rot} = V_a I_a = (99.4)(6) = 596.4 \text{ W}$$

(c.) The motor is loaded (full load) such that the armature carries the rated current of I_a = 120 A. The corresponding full load armature voltage is

$$V_a = V_t - I_a R_a = 100 - (120)(0.1) = 88 \text{ V}$$

According to the armature voltage definition,

$$V_a = K_a \psi_m \omega_m$$

the ratio of the full-load armature voltage to the no-load armature voltage is

$$\frac{V_{a,FL}}{V_{a,NL}} = \frac{K_a \psi_{m,FL} \omega_{m,FL}}{K_a \psi_{m,NL} \omega_{m,NL}} = \frac{\omega_{m,FL}}{\omega_{m,NL}}$$

assuming that the full-load magnetic flux is equal to that of the no-load case. The full load angular velocity is then

$$\omega_{m,FL} = \frac{V_{a,FL}}{V_{a,NL}} \omega_{m,NL} = \frac{88}{99.4} 1000 = 885.31 \text{ rpm}$$

$$= 885.31 \frac{2\pi}{60} = 92.71 \text{ rad/s}$$

The resulting full-load torque is

$$T = \frac{P_a}{\omega_m} = \frac{V_a I_a}{\omega_m} = \frac{(88)(120)}{92.71} = \frac{10560}{92.71} = 113.9 \text{ N-m}$$

The motor output power is equal to the armature power minus the rotational losses.

$$P_{out} = P_a - P_{rot} = 10.560 - 0.596 = 9.964 \text{ kW}$$

The motor input power is equal to the product of the terminal voltage and current.

$$P_{in} = V_t I_t = V_a (I_a + I_f)$$

$$= 100(120 + 0.99) = 12.099 \text{ kW}$$

The motor efficiency is defined by the ratio of output power to input power.

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{9.964}{12.099} \times 100\% = 82.35\%$$

(d.) If the air gap flux is reduced by 5% from no-load to full-load, the ratio of full-load to no-load armature voltages gives

$$\frac{V_{a,FL}}{V_{a,NL}} = \frac{K_a \Psi_{m,FL} \, \omega_{m,FL}}{K_a \Psi_{m,NL} \, \omega_{m,NL}} = 0.95 \, \frac{\omega_{m,FL}}{\omega_{m,NL}}$$

The full load angular velocity is then

$$\omega_{m,FL} = \frac{V_{a,FL}}{V_{a,NL}} \frac{\omega_{m,NL}}{0.95} = \frac{88}{99.4} \frac{1000}{0.95} = 931.91 \text{ rpm}$$

$$= 931.91 \frac{2\pi}{60} = 97.59 \text{ rad/s}$$

The resulting full-load torque is

$$T = \frac{P_a}{\omega_m} = \frac{V_a I_a}{\omega_m} = \frac{(88)(120)}{97.59} = \frac{10560}{97.59} = 108.21 \text{ N-m}$$

The calculations for the output power, the input power, and the motor efficiency are identical to those in part (c.). The actual motor efficiency decrease from that of part (c.) since the motor rotational losses would be larger at a higher speed.

(e.) The starting torque can be found according to

$$T = K_a \psi_m I_a \qquad I_a = (1.5)(120) = 180 \text{ A}$$

The product of $K_a \psi_m$ can be found from the armature emf equation evaluated at no-load or full-load.

$$V_a = K_a \psi_m \omega_m \qquad \Rightarrow \qquad K_a \psi_m = \frac{V_a}{\omega_m} = \frac{88}{92.71} = 0.949 \text{ V-s}$$

The starting torque is given by

$$T_{st} = (0.949)(180) = 170.82 \text{ N-m}$$

(f.) The effective field current is

$$I_{f,eff} = I_f - I_{f,AR} = 0.99 - 0.16 = 0.83 \text{ A}$$

From the magnetization curve, the armature voltage for this field current is 93.5 V at a speed of 1000 rpm. The flux condition for this case is

$$K_a \psi_m = \frac{V_a}{\omega_m} = \frac{93.5}{1000 \frac{2\pi}{60}} = 0.893 \text{ V-s}$$

which gives the following starting torque.

$$T_{st} = (0.893)(180) = 160.74 \text{ N-m}$$

SERIES DC MOTOR

The equivalent circuit of the series DC motor is shown below. Note that in addition to the field winding resistance (R_{fw}) and the armature resistance (R_a), a series resistance R_{sc} (*speed control resistance*) has been included in the series motor equivalent circuit. The speed control resistance can be adjusted to control the speed of the series DC motor.
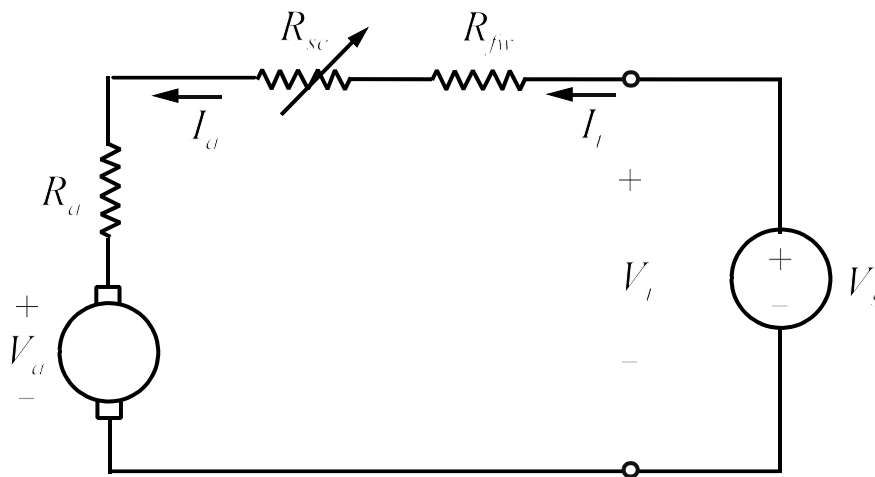
The armature voltage and developed torque for any DC machine can be written as

$$V_a = K_a \Psi_m \omega_m$$

$$T = K_a \Psi_m I_a$$

If magnetic linearity is assumed, the magnetic flux in the DC machine will be proportional to the field current. Thus, we may write

$$K_a \Psi_m = K_1 I_a$$

Given this relationship, the armature voltage and the torque can be written as

$$V_a = K_1 I_a \omega_m$$

$$T = K_1 I_a^2$$

The armature voltage is the series DC motor is

$$V_a = V_t - I_a \left(R_a + R_{fw} + R_{sc} \right)$$

Combining the equations for the armature voltage gives

$$\omega_m = \frac{V_t}{K_1 I_a} - \frac{R_a + R_{fw} + R_{sc}}{K_1}$$

This equation can be rewritten in terms of torque by substituting the following relationship:

$$I_a = \sqrt{\frac{T}{K_1}}$$

which yields

$$\omega_m = \frac{V_t}{\sqrt{K_1 T}} - \frac{R_a + R_{fw} + R_{sc}}{K_1}$$

As shown in the previous equation, the series DC motor is characterized by high torque at low speeds and low torque at high speeds.

Example (Series DC motor)

A fan is driven by a 220 V, 7 hp series DC motor which draws 25 A and runs at 300 rpm when connected to a 220 V supply with $R_{sc} = 0$. The armature resistance of the DC motor is $R_a = 0.6\ \Omega$ and the resistance of the field winding is $R_{fw} = 0.4\ \Omega$. The torque required by the fan is proportional to the square of the motor speed. Neglect the rotational losses and armature reaction. Determine

    (a.)   the power delivered to the fan and the torque developed by the motor.

    (b.)   the value of R_{sc} necessary to reduce the motor speed to 200 rpm and the power delivered to the fan at this speed.

    (a.)   The armature voltage of the series DC motor is given by

$$V_a = V_t - I_a(R_a + R_{fw} + R_{sc})$$
$$= 220 - 25\,(0.6 + 0.4 + 0) = 195\ \text{V}$$

The armature power is equal to the output power when rotational losses are neglected. Thus, the power delivered to the fan is

$$P = P_a = V_a I_a = (195)(25) = 4.875\ \text{kW}$$
$$= \frac{4875}{746} = 6.53\ \text{hp}$$

The developed torque is equal to the armature power divided by the motor mechanical speed.

$$\omega_m = 300\,\frac{2\,\pi}{60} = 31.42\ \text{rad/s}$$

$$T = \frac{P_a}{\omega_m} = \frac{4875}{31.42} = 155.2\ \text{N-m}$$

(b.) The constant K_1 can be determined from the torque found in part (a.) at $\omega_m = 300$ rpm.

$$T = K_1 I_a^2 \qquad \Rightarrow \qquad 155.2 = K_1\, 25^2$$

$$K_1 = 0.248$$

If the required torque is proportional to the square of the speed, then

$$\frac{T(200\,\text{rpm})}{T(300\,\text{rpm})} = \frac{200^2}{300^2}$$

$$T(200\,\text{rpm}) = \left(\frac{200}{300}\right)^2 (155.2) = 68.98\ \text{N-m}$$

The power delivered to the fan at 200 rpm is

$$\omega_m = 200\,\frac{2\,\pi}{60} = 20.94\ \text{rad/s}$$

$$P = P_a = \omega_m T = (68.98)(20.94) = 1.444\ \text{kW}$$

The value of speed control resistance required is found from

$$\omega_m = \frac{V_t}{\sqrt{K_1 T}} - \frac{R_a + R_{fw} + R_{sc}}{K_1}$$

$$20.94 = \frac{220}{\sqrt{(0.248)(68.98)}} - \frac{0.6 + 0.4 + R_{sc}}{0.248}$$

The resulting speed control resistance is

$$R_{sc} = 7.0\ \Omega$$