ELEC330, ELEC7302
Electrical Energy Conversion and Utilisation

Lecture 3 - Transformers

Transformers

- Ideal Transformers
- Non-ideal Transformers
  - Losses
- Transformer Equivalent Circuits
- Voltage Regulation
- Efficiency

Transformers

- Makes possible:
  1. Power generation at the most economical level
  2. Transmission and distribution at the most economical level
  3. Power utilisation at the most suitable level
  4. Measurement of high voltages (potential transformer) and high current (current transformer)
  5. Impedance matching, insulating one circuit from another or insulating DC circuits from AC circuits

Single Phase Transformer

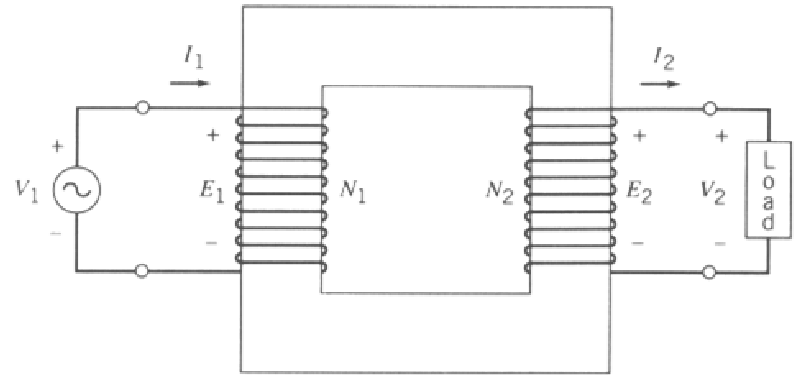


FIGURE 4.8 A transformer circuit.

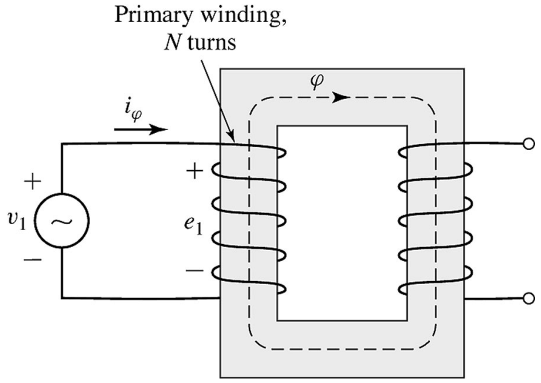
- A single phase transformer
  - Two or more winding, coupled by a common magnetic core

Single Phase Transformer

- One winding connected to an AC voltage source,
    - A time varying flux is produced in the core
- Flux is maintained within the magnetic core & it links the second winding
- A voltage is induced in the secondary winding.
- If a load is connected to the secondary winding, a secondary current flows
    - Allows energy transfer from primary to secondary side of transformer

A closer look…

- Current i_φ flows in the first coil



- Causes flux φ in core (NI_φ/R)
  - Links second coil and induces voltage e_1 in first coil
  - But e_1 must be equal to v_1
- Let the flux be described as:

$$\phi = \phi_{max} \sin\omega t$$

A closer look…

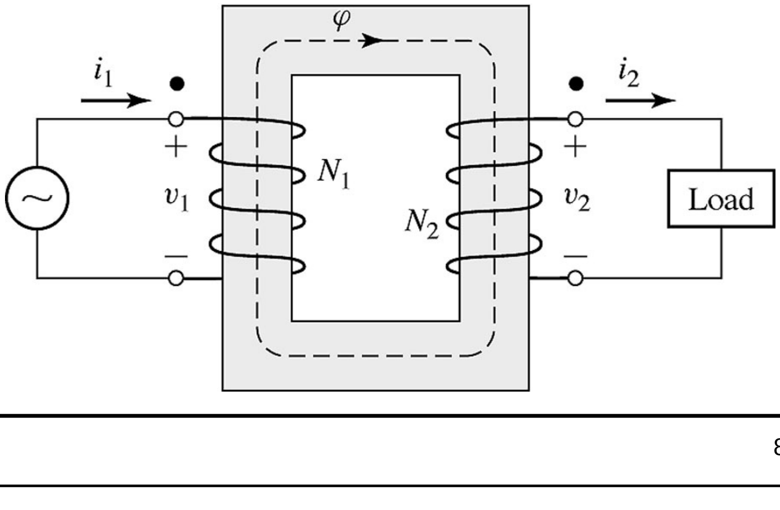
- Induced voltage on coil one from flux φ is:

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi_m}{dt} = \omega\phi_{\max} N_1 \cos\omega t = v_1$$

  - Where *φ* = Φ_m – often referred to as *magnetising* flux
  - But induced voltage e_1 equal to v_1
  - Therefore, v_1 “forces” a certain magnitude of flux in the core - Φ_m

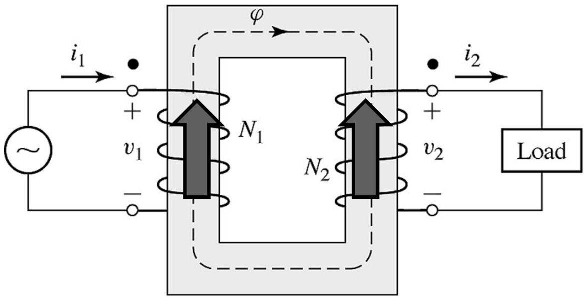
A closer look…

- What happens when a load is connected to other winding?



Flux direction

- Can use right hand rule to determine flux direction produced by coil



- wrap right hand around coil in direction of current
- thumb gives direction of generated flux

A closer look…

- The induced voltage on the second coil will be of polarity to oppose flux that created it (Faraday's law) – will be equal to:

$$e_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\phi_m}{dt} = \omega\phi_{\max} N_2 \cos \omega t$$

- A current i_2 will flow through the winding due to e_2 and load

A closer look…

- However this current will act to decrease the total flux in the core (see slide 10)
- This can't occur however, as v_1 forces flux to be a certain value, i.e.:

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi_m}{dt} = \omega\phi_{\max} N_1 \cos\omega t = v_1 \longrightarrow \phi_{\max} = \frac{v_1}{\omega N_1 \cos\omega t}$$

  - Therefore current in coil one must compensate – total MMF/flux must cancel (only flux remaining is magnetising flux)
  - Therefore: $N_1 I_1 - N_2 I_2 = 0$ (neglecting magnetising MMF/flux – ideal transformer)

Ideal Transformer

- Zero leakage flux
  - Fluxes produced by the primary and secondary currents are confined within the core.
- The windings have no resistance
  - Applied voltage v_1 equals the induced primary voltage e_1
  - Similarly $v_2 = e_2$
- The core has infinite permeability
  - Reluctance of the core is zero.
  - negligible current is required to set up the magnetic flux
- The magnetic core is loss-less.

Voltage relationships of ideal transformer

- Summing up then…
- Emf (voltage) produced in windings then given by:

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi_m}{dt} = \omega\phi_p N_1 \cos\omega t$$

$$e_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\phi_m}{dt} = \omega\phi_p N_2 \cos\omega t$$

Voltage relationships of ideal transformer

$$E_1 = \frac{1}{\sqrt{2}} \omega \phi_{\max} N_1 = 4.44 f \phi_{\max} N_1$$

$$E_2 = \frac{1}{\sqrt{2}} \omega \phi_{\max} N_2 = 4.44 f \phi_{\max} N_2$$

- RMS values of voltage produced

Turns ratio of transformer

- Turns ratio

  $[E_1/E_2] = [N_1/N_2] = a$
- Polarities of induced voltages given by Lenz's law
  - EMF's induce currents that tend to oppose flux change (that created them)
- For ideal transformer

  $[E_1/E_2] = [V_1/V_2] = [N_1/N_2] = a$
  - Induced voltages equal to corresponding terminal voltages

Current ratio of Transformer

- For ideal transformer magnetic field intensity required in core negligible (due to high relative permeability)
- From Ampere's law

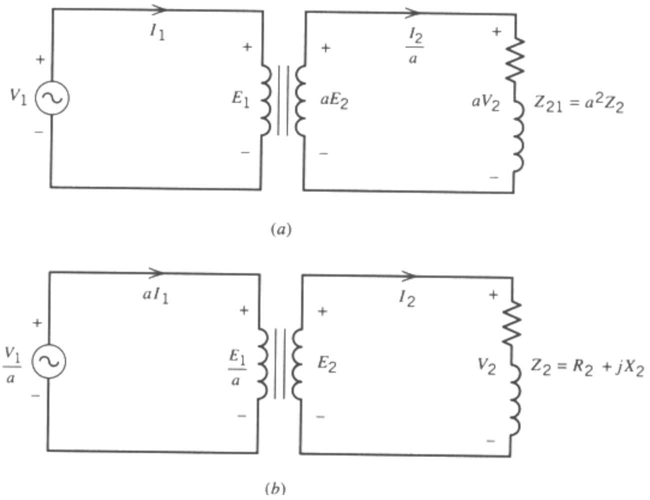
  $H \bullet Lc = N_1 I_1 - N_2 I_2 = 0$

  Thus

  $[I_1/I_2] = [N_2/N_1] = [1/a]$
- Because $V_1 = a V_2$ and $I_1 = I_2/a$, can then be shown that

  $V_1 I_1 = V_2 I_2$ - Power invariance of ideal transformer
- Power input is equal to the power output.

Equivalent Circuit of Ideal Transformer



- Can refer or "move" parameters on primary side to secondary side
E.g.

$$\frac{V_1}{I_1}(= Z_1) \div \frac{V_2}{I_2}(= Z_2)$$

$$= \frac{V_1}{V_2} * \frac{I_2}{I_1}\left(= \frac{Z_1}{Z_2}\right) = a^2$$

$$Z_1 = a^2 Z_2$$

Actual Transformer

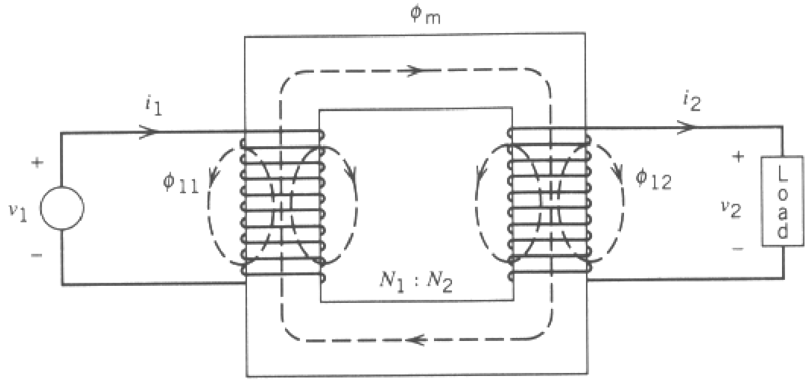
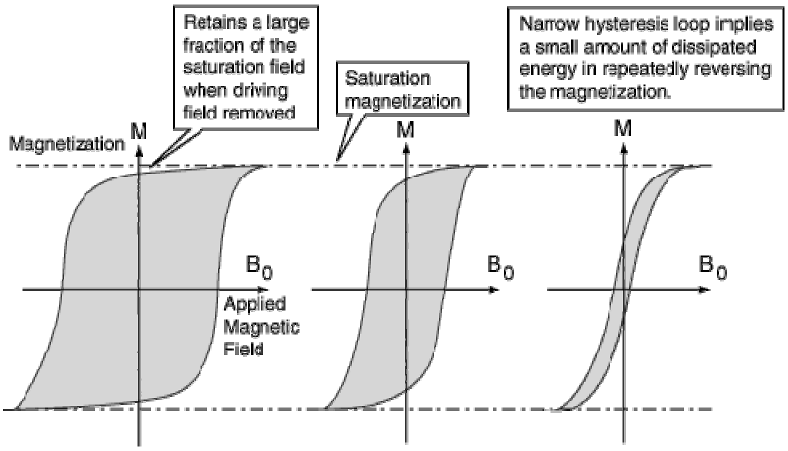


FIGURE 4.10 An actual transformer.

Actual Transformer

- Has resistances in the windings.
- Not all of the flux produced by one winding will link the other winding
  - Flux leakage.
- Core of the actual transformer has finite permeability
- There will be core losses (iron losses)
  - Hysteresis losses
  - Eddy current losses

Hysteresis Losses



Retains a large fraction of the saturation field when driving field removed

Saturation magnetization

Narrow hysteresis loop implies a small amount of dissipated energy in repeatedly reversing the magnetization.

Desirable for permanent magnets and magnetic recording and memory devices.

The area of the hysteresis loop is related to the amount of energy dissipation upon reversal of the field.

Desirable for transformer and motor cores to minimize the energy dissipation with the alternating fields associated with AC electrical applications.

Hysteresis Losses

- Energy dissipated in process of moving “domain” (small magnetic sections within ferromagnetic material) walls past impurities and strains in crystal structure
- Proportional to area enclosed by B-H loop of magnetisation curve
- Loss per cycle a non-linear function of maximum flux density within material
- Total hysteresis loss proportional to frequency of excitation

Eddy Current Losses

- Occurs in conducting magnetic material as a result of rate of change of flux density with time
    - Sections of core experience varying flux density, which produce circulating currents
        - Induces power dissipation within core causing heating and losses
    - Need to ensure uniform flux density across core using either laminations in the core or ferrite material
- Eddy current losses proportional to SQUARE of frequency of excitation

Behaviour of Actual Transformer

- Primary winding flux $\phi_1 = \phi_m + \phi_{11}$
- Secondary winding flux $\phi_2 = \phi_m - \phi_{12}$

$$v_1 = R_1 i_1 + N_1 \frac{d\phi_{11}}{dt} + N_1 \frac{d\phi_m}{dt} = R_1 i_1 + L_1 \frac{di_1}{dt} + N_1 \frac{d\phi_m}{dt}$$

$$v_2 = -R_2 i_2 + \frac{d\lambda_2}{dt} = -R_2 i_2 - N_2 \frac{d\phi_{12}}{dt} + N_2 \frac{d\phi_m}{dt}$$

$$= -R_2 i_2 - L_2 \frac{di_2}{dt} + N_2 \frac{d\phi_m}{dt}$$

Turns ratio: *a*

$$e_1 = N_1 \frac{d\phi_m}{dt}$$

$$e_2 = N_2 \frac{d\phi_m}{dt}$$

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} = a$$

Equivalent circuit

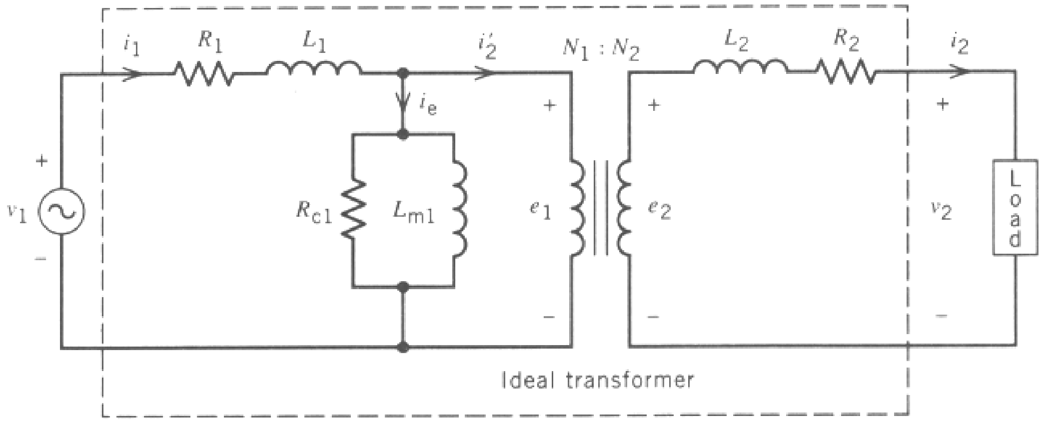


FIGURE 4.11 Equivalent circuit of a transformer.

Core modelling

- Core magnetisation and core losses can be either in primary or in secondary.
- The inductor L_{m1} represents core magnetisation (finite permeability)
- The resistor R_{c1} represents the core losses (hysteresis and eddy current losses combined)
- The core related elements are usually determined at rated voltage and are referred to the primary side

Transformer equivalent circuit: Phasor form

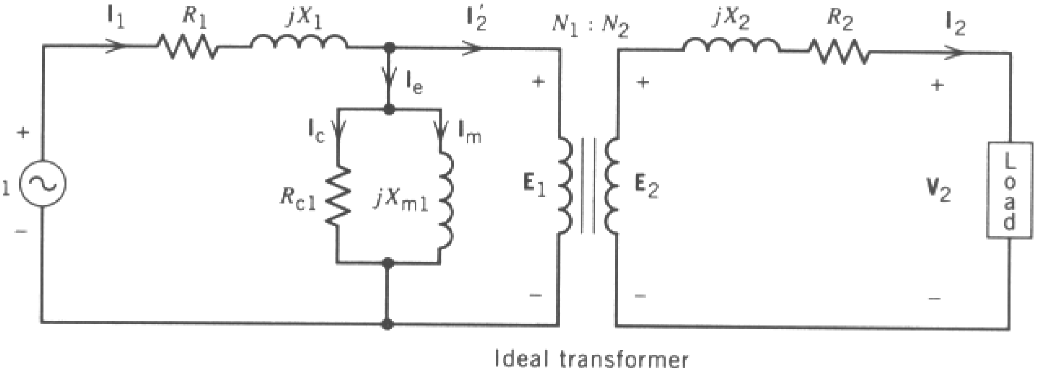


FIGURE 4.12 Transformer equivalent circuit in phasor form.

Vector Diagram – lagging load

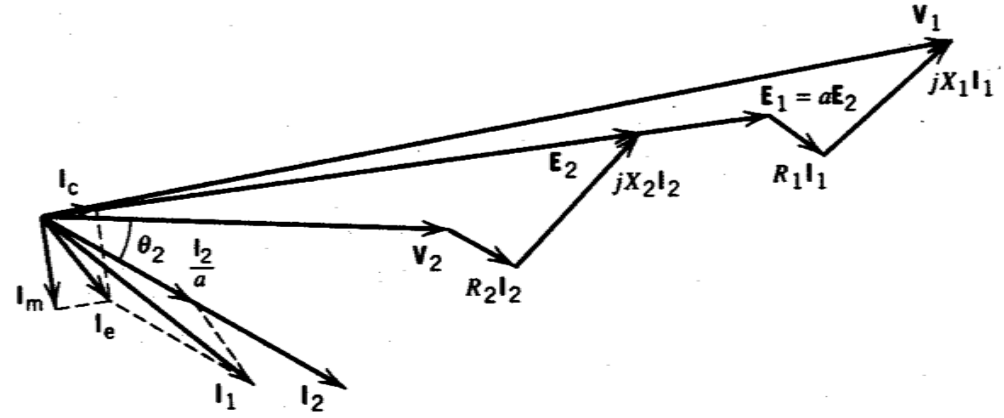
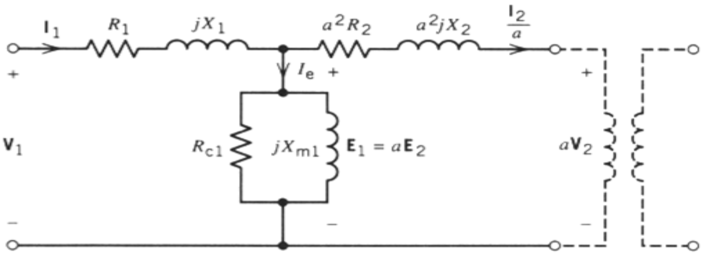


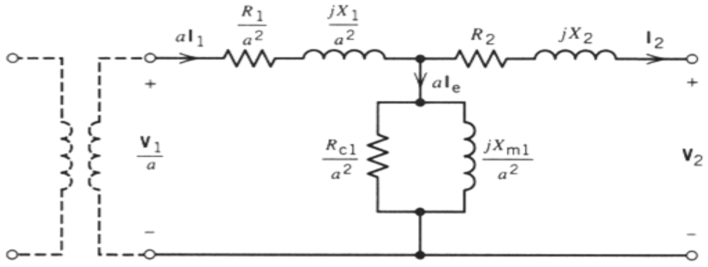
FIGURE 4.13 Phasor diagram for Fig. 4.12.

Transformer Equivalent circuits: Referred to primary or Secondary



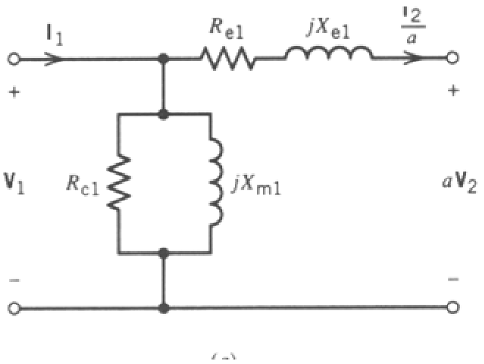
(*a*)

Referred to primary



Referred to secondary

Approximate equivalent circuit - referred to primary



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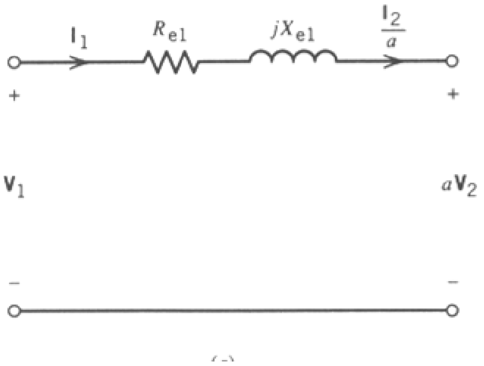
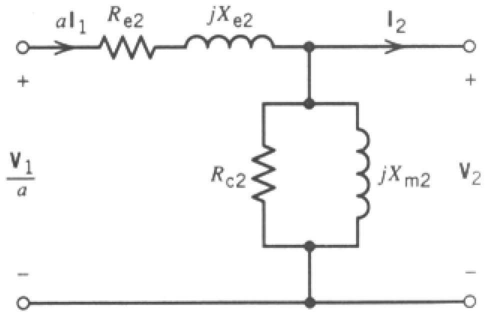


- All impedances “moved” to primary side (above is a cantilever circuit – magnetising branch directly connected across V_1)
  $R_{e1} = R_1 + a^2 R_2$ and $X_{e1} = X_1 + a^2 X_2$
- Core losses/magnetisation losses assumed dependent on primary voltage not magnetising flux
- If core losses and magnetisation losses neglected, equivalent circuit reduces to simple series equivalent

Approximate equivalent circuit - referred to secondary



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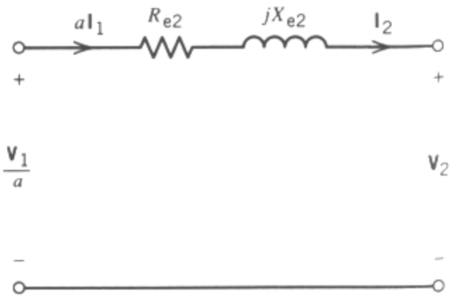


- All impedances “moved” to secondary side
  $R_{e2}= R_1/a^2 + R_2$ and $X_{e2}= X_1/a^2 + X_2$
- Core losses/magnetisation losses assumed dependent on load voltage not magnetising flux
- If core losses and magnetisation losses neglected, equivalent circuit reduces to simple series equivalent

Voltage Regulation

- The voltage regulation of a transformer is defined as the change in the magnitude of the secondary voltage as the current changes from full load to no load with the primary voltage held fixed
  E.g.

$$regulation = \frac{|V_{2,nl}| - |V_{2,fl}|}{|V_{2,fl}|} \times 100\%$$

Voltage regulation

$$regulation = \frac{|V_1| - |aV_2|}{|aV_2|} \times 100\% \quad = \frac{|V_1/a| - |V_2|}{|V_2|} \times 100\%$$

- Approximate equivalent circuits for which core losses and magnetization losses assumed dependent upon terminal voltage allows use of these simpler formulae
- Regulation provides a measure of transformer's ability to maintain voltage under load conditions
    - Indicator of size of copper losses and leakage reactance of transformer windings

Importance of Voltage Regulation

- Most electrical apparatus designed to operate at specified or rated voltage ±5%
  - If voltage too high
    - Space heater over-heat
    - Magnetic loads saturate
  - If voltage too low
    - Constant power loads draw excessive current, over-heating load and possibly also transformer

Efficiency

- The efficiency of a transformer is defined as the ratio of the power output (P_{OUTPUT}) to the power input (P_{INPUT}).
- Generally, the output power will be governed by the requirements of the loads, making it a more easily specified term than input power

$$\eta = \frac{P_{output}}{P_{input}} 100\%$$

$$= \frac{P_{output}}{P_{output} + \Sigma(losses)} 100\%$$

Losses

- $\sum$ Losses= core losses + copper losses
  - Copper losses depend upon load current
  - Iron losses are constant for constant flux (constant voltage) conditions
- $\sum$ Losses= core losses + ($I_1^2R_1$+ $I_2^2R_2$)
- $\sum$ Losses= core losses + ($I_1^2R_{e1}$)
- $\sum$ Losses= core losses + ($I_2^2R_{e2}$)

Transformer Efficiency – practical considerations

- Measure of efficiency only valid for specific load at which calculation performed
  - For constant power factor load however can be shown that maximum efficiency achieved if:
    Total copper losses = core losses
- Need to consider how transformer operated
  - High voltage transformers operating continuously near rated capacity
    - Design for maximum efficiency at or near RATED load
  - Distribution class transformers connected 24 hrs/day but experience significant load variations
    - Design for maximum efficiency at or near AVERAGE load

Transformers – Part B

- Determination of Equivalent Circuit Parameters
  - Open Circuit Test
  - Short Circuit Test
  - Example
  - Test of Three Phase Transformers
- Transformer Rating

Determination of Equivalent Circuit Parameters

- Two simple tests used to determine transformer equivalent circuit parameters
  - Open circuit test
  - Short circuit test
- Tests used by manufacturers to confirm design values or to obtain value where no data available
- If complete equivalent circuit used customary to assume equal distribution of losses (real or reactive) between primary and secondary

  E.g. $R_1 = a^2R_2$, $X_1 = a^2X_2$,
  - This step unnecessary if using approximate equivalent circuits

Open Circuit Test

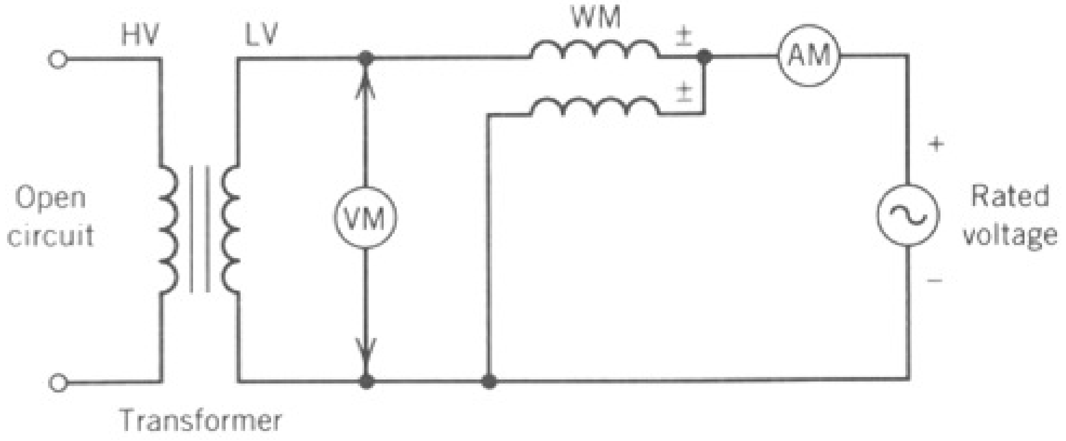
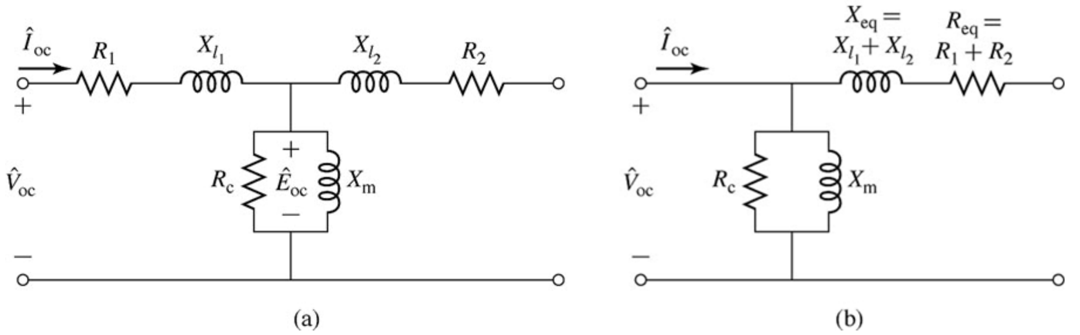


FIGURE 4.16 Connections for open-circuit test.

Open Circuit Test

- Transformer rated voltage applied to one winding while other winding open circuited
  - Choice of energized winding depends upon availability of suitable voltage source
- Allows measurement of magnetizing inductance L_{m1}, core loss resistance R_{c1}
- Also allows verification of turns ratio

Open Circuit Test



Open Circuit Test

- Since transformer unloaded, current I_{OC} represents excitation current through shunt branch
  - Current ~5% rated current
    - Voltage drop across leakage reactance, winding resistance of energized winding can be ignored
- When rated voltage and rated frequency applied during open circuit test, measured power practically equal to core loss
  - Core loss assumed to remain constant for different load levels

Open Circuit Test

- Equivalent impedance of open circuit test:

$$Z_{OC} = Z_m = \frac{R_C\left(jX_m\right)}{R_c + jX_m}$$

- Determining equivalent magnetising impedance

$$\left|Z_m\right| = \frac{V_{OC}}{I_{OC}}$$

(error in Fitzgerald with this formula, denominator should be I_{OC} as above)

- Determining equivalent core resistance

$$R_C = \frac{V_{OC}{}^2}{P_{OC}}$$

- Determining equivalent magnetising reactance

$$X_m = \frac{1}{\sqrt{\left(1/\left|Z_m\right|\right)^2 - \left(1/R_c\right)^2}}$$

Open Circuit test

- Another way of determining X_m from open circuit measurements:

$$\cos\theta = \frac{P_{OC}}{S_{OC}} = \frac{P_{OC}}{I_{OC}V_{OC}} \qquad \theta = \cos^{-1}\left(\frac{P_{OC}}{I_{OC}V_{OC}}\right)$$

$$Q_{OC} = S_{OC}\sin\theta = V_{OC}I_{OC}\sin\theta$$

$$Q_{OC} = \frac{V_{OC}^{\ 2}}{X_m} \quad \Longrightarrow \quad X_m = \frac{V_{OC}^{\ 2}}{Q_{OC}}$$

Open Circuit Test

- If necessary parameters may be referred to primary or secondary side as required

$$R_{C1} = a^2 R_{C2}$$

$$X_{M1} = a^2 X_{M2}$$

Example – Open Circuit Test

- 50 kVA, 2400/240 V, 50 Hz single phase transformer
  - Open circuit test performed on low voltage side
  - Results

V_{OC} = 240 V
I_{OC} = 5.4 A
P_{OC} = 186 W

Example Contd – Open Circuit Test

- $|Z_{oc}| = V_{oc}/I_{oc}$
  $= 44.44\ \Omega$
- $R_{C2} = V_{oc}^2/P_{oc}$
  $= 309.6\ \Omega$
- $X_{m2} = \dfrac{1}{\sqrt{\left(1/|Z_m|\right)^2 - \left(1/R_c\right)^2}}$
  $= 44.9\ \Omega$

- Referring these quantities to HV side
  - a = 2400/240 = 10
- $R_{c1} = a^2R_{c2} = 30.96\ k\Omega$
  $X_{m1} = a^2X_{m2} = 4.49\ k\Omega$
  $L_{m1} = X_{m1} / 2\pi f$
  $=14.26\ H$

Short Circuit Test

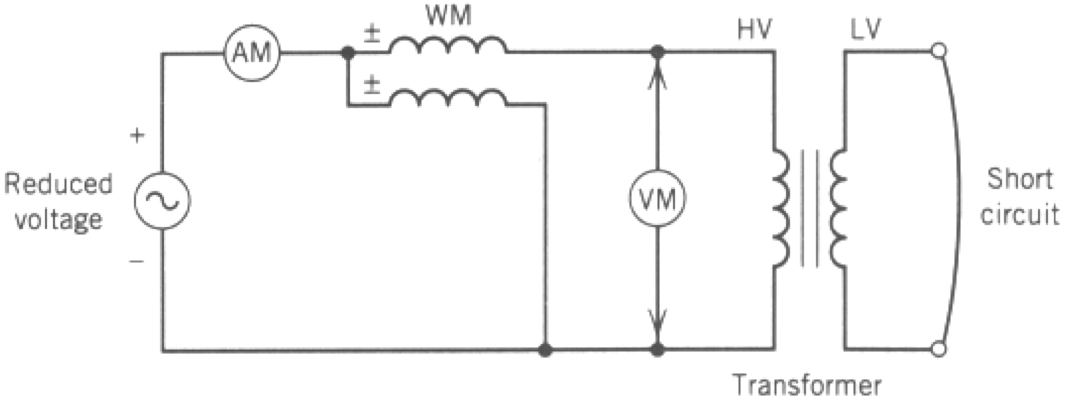
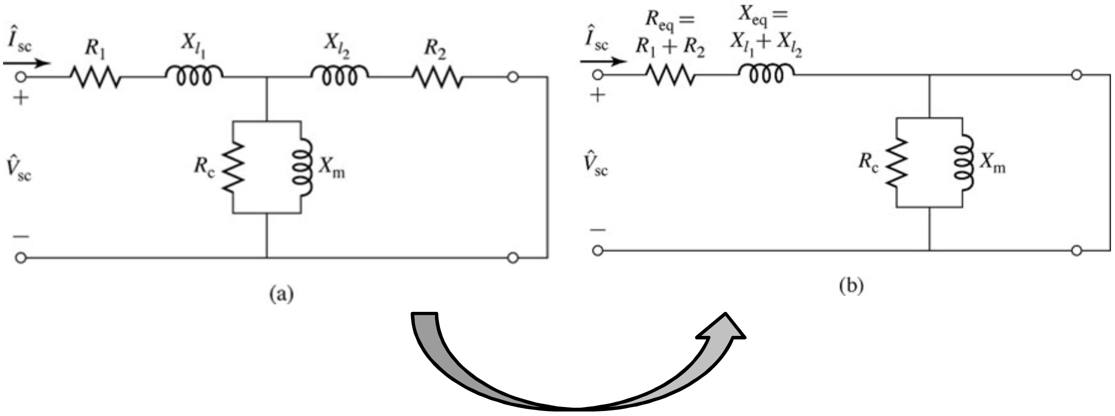


FIGURE 4.18 Connections for short-circuit test.

Short Circuit Test

- One winding short circuited while just enough voltage applied to other winding to ensure rated current flows in both windings
- Allows measurement of equivalent resistance, R_E, and leakage reactance X_E, of windings as seen from supply side
  - Equivalent resistance can be compared with measurement of DC resistance to determine impact of frequency of windings resistance
- Also allows check of turns ratio

Short Circuit Test



(a) (b)

Short Circuit Test

- With transformer short circuited voltage required to produce rated current very low
  - Voltage ~5 - 10% rated voltage
    - Current through magnetising branch is negligible
- Applied voltage may be assumed to occur wholly as voltage drop across transformer equivalent series impedance
- Also when rated current flows through windings during short circuit test, measured power equal to rated copper loss

Short Circuit Test

- Magnitude of series impedance of transformer, referred to energized (HV) side

  $|Z_{E1}| = |V_{SC}|/|I_{SC}|$

- Equivalent series resistance (referred to HV side)

  $R_{E1} = P_{SC}/I^2_{SC} = R_1 + a^2R_2$

- Equivalent series reactance (referred to HV side)

  $$X_{E1} = \sqrt{(|Z_{E1}|^2 - R_{E1}{}^2)}$$
  $$= X_1 + a^2X_2$$

Short Circuit Test

- If using complete equivalent circuit parameters for secondary can be determined according to:

$$R_1 = a^2R_2 = R_{E1}/2$$

$$X_1 = a^2X_2 = X_{E1}/2$$

Example – Short Circuit Test

- 50 kVA, 2400/240 – V, 50 Hz single phase transformer
  - Short circuit test performed with low voltage side shorted
  - Results

V_{SC} = 48 V
I_{SC} = 20.8 A
P_{SC} = 620 W

Example - Short Circuit Test

- Magnitude of series impedance of transformer, referred to energized (HV) side

  $|Z_{E1}| = 48/20.8 = 2.3\ \Omega$

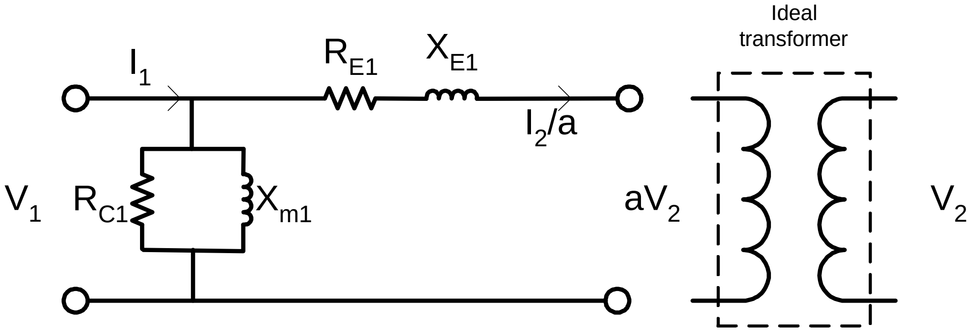
- Equivalent series resistance (referred to HV side)

  $R_{E1} = P_{SC}/I^2_{SC} = R_1 + a^2R_2$
  $= 620/(20.8)^2 = 1.43\ \Omega$

- Equivalent series reactance (referred to HV side)

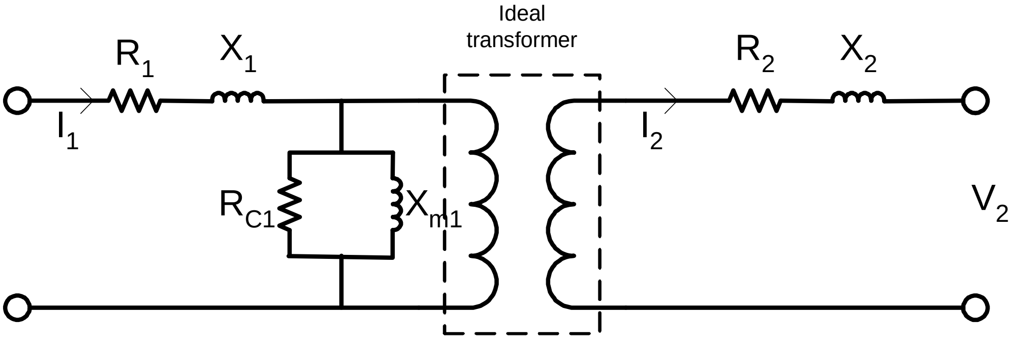
  $X_{E1} = \sqrt{(|Z_{E1}|^2 - R_{E1}{}^2)}$
  $= X_1 + a^2X_2$
  $= \sqrt{(|2.3|^2 - 1.43^2)} = 1.8\Omega$

Example
Transformer Equivalent parameters



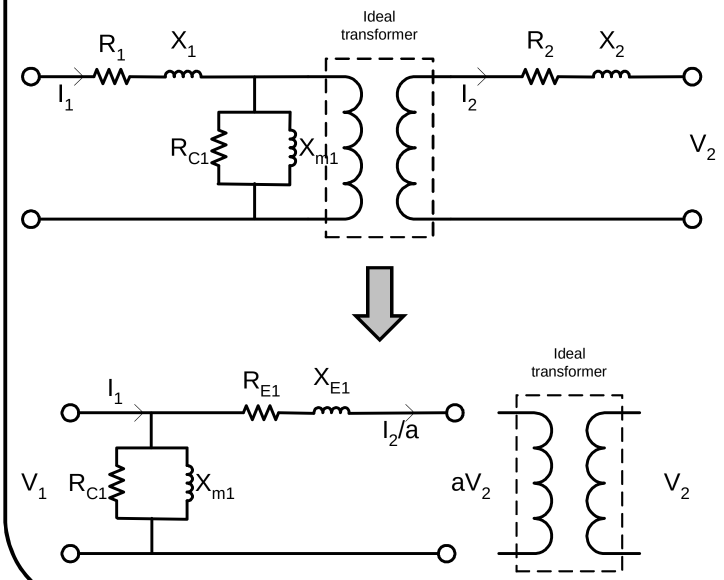
- From open circuit and short circuit test can define needed parameters of equivalent circuit
  for 50 kVA, 2400/240 V transformer:
  R_{C1}= 30.96 kΩ, X_{M1}= 4.48 kΩ,
  R_{E1} = 1.43 Ω, X_{E1} = 1.8Ω

Example
Transformer Equivalent parameters



- By assuming $R_1 = a^2R_2 = (R_{E1})/2$ and
  $X_1 = a^2X_2 = (X_{E1})/2$
  can derive necessary parameters for full equivalent circuit of 50 kVA 2400/240 V 50 Hz transformer:
  R1 = 0.715 Ω, R2 = 0.00715 Ω
  X1 = 0.9 Ω, X2 = 0.009 Ω

Cantilever circuit



- Notice in second diagram that magnetising/core branch is placed directly across V_1 i.e. before R_{E1} and X_{E1}

- This is called a cantilever equivalent circuit

- Not as accurate as standard equivalent circuit
  - But commonly used as it speeds up computation and inaccuracy is minor

Example
Transformer Voltage Regulation/Efficiency

- Determine regulation/efficiency of 50 kVA transformer with parameters as determined previously if operated at rated load, 0.8 power factor lagging, at rated secondary voltage

- Rated load current (rated secondary current)
  $= 50\,000 \text{ VA} / 240 \text{ V} = 208.3 \text{ A}$
  $I_2 = 208.3 \angle -\cos^{-1}(0.8) = 208.3 \angle -36.87°$
  if secondary voltage selected as reference phasor

- Required parameter for approximate equivalent circuit referred to primary
  $aV_2 = 2400 \angle 0°$, $(I_2/a) = 20.83 \angle -36.87°$ A

Example
Transformer Voltage Regulation/Efficiency

- Primary voltage required to supply load

  $V_1 = aV_2 + (I_2/a)(R_{E1} + j^*X_{E1})$

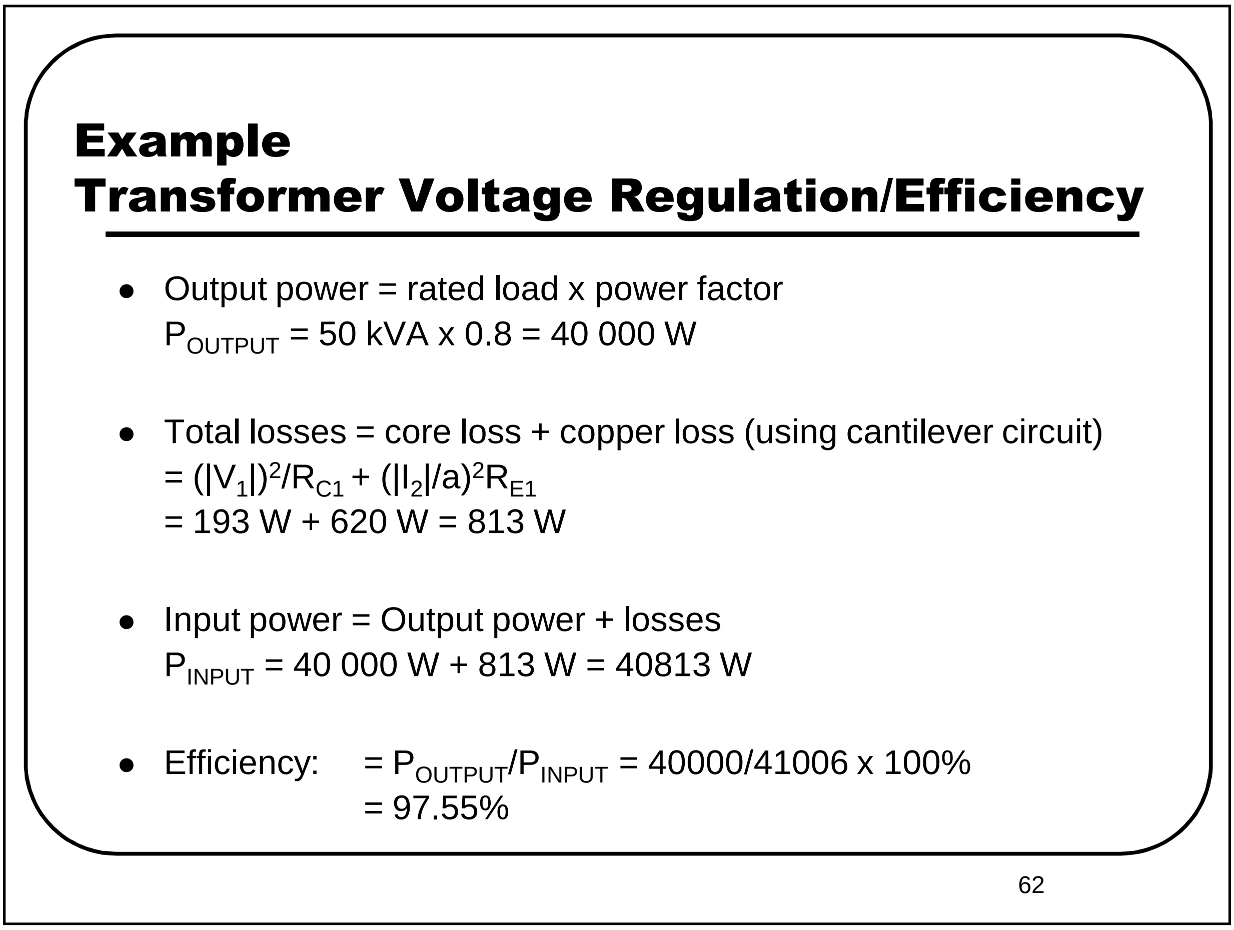
  $= 2400 \angle 0° + 20.83 \angle -36.87°(1.43 + j1.80)$

  $= 2446.4 \angle 0.28°$

- Voltage regulation

$$= \frac{|V_{2,no\ load}| - |V_{2,\ full\ load}|}{|V_{2,\ full\ load}|} \times 100\% = \frac{|V_1| - |aV_2|}{|aV_2|} \times 100\%$$

$$= \left(\frac{2446.4 - 2400}{2400}\right) \times 100\% = 1.93\%$$



Example
Transformer Voltage Regulation/Efficiency

- Output power = rated load x power factor
  P_{OUTPUT} = 50 kVA x 0.8 = 40 000 W

- Total losses = core loss + copper loss (using cantilever circuit)
  $= (|V_1|)^2/R_{C1} + (|I_2|/a)^2 R_{E1}$
  = 193 W + 620 W = 813 W

- Input power = Output power + losses
  P_{INPUT} = 40 000 W + 813 W = 40813 W

- Efficiency: $= P_{OUTPUT}/P_{INPUT}$ = 40000/41006 x 100%
  = 97.55%

Test on Three Phase Transformers

- For tests on three phase transformers
  - Power being measured is total three phase power
  - Measured voltage is line-to-line voltage
  - Measured current is line current
- Previous formulae are valid for single phase transformer
  - Three phase measurements must be converted to per-phase values

Transformer Rating

- Transformer rating determines conditions under which transformer designed to operate
- Defined by:
  - frequency
  - voltage
  - current
  - apparent power (volt-ampere product)

Voltage Rating

$$V_{1rated} = \frac{\omega N_1 A B_{\max}}{\sqrt{2}} = \frac{\omega N_1 \phi_{\max}}{\sqrt{2}}$$

- Rated voltage controlled by maximum flux density permissible within core
  - Problems with high peak flux density
    - high magnetising current due to core saturation
    - Increase in cores losses with both hysteresis and eddy current losses controlled by maximum flux density
- Size of transformer (cross sectional area of core) affected by maximum flux density that can be tolerated by transformer

Current rating

- Rated current of transformer is maximum rms current that will not produce excessive heating in transformer insulation
  - For oil-impregnated paper insulation maximum temperature ~100°C

$$I_{1,rated} = \left(\frac{P_L - P_C}{R_{E1}} \right)^{1/2}$$

P_L – power that can be dissipated as heat

P_C – core losses of transformer at rated voltage

Current Rating

- Method of cooling, surface area of transformer and even ambient temperature will control amount of heat that can be dissipated
  - If transformer temperature considerably below maximum permissible level can increase current above rated current until insulation reaches design limit
    - Operation at temperatures above design limit can reduce life of transformer appreciably

Apparent Power Rating

- Transformer has ***volt-ampere*** rating, not a (real) power rating
  - $S_{rated} = V_{1,rated}I_{1,rated}$ or $S_{rated} = V_{2,rated}I_{2,rated}$
  - Voltage rating and current rating essentially independent
- Rating independent of power factor or load
  - Transformer can become fully loading supplying capacitive or inductive loads even if load requires little real power

Transformers – Part C

- Polyphase Transformer
  - Winding connection types
  - Phase shift
  - Harmonics
  - Per – unit impedance
- Transformer Construction
  - Core
  - Winding
  - Tank
- Special Transformers
  - Instrument transformers

Polyphase Transformers

- Formed as either
  - Three single phase transformer connected together
    - Easy to replace failed units
  - Three phase transformer bank constructed with all three phase on a common core
    - Lower weight and cost for given transformer rating than 3 individual units
    - 6 rather than 12 external connections
      (large saving for HV windings with complicated structure)
    - Whole transformer must be replaced if single winding fails
- In both case, analysis procedure identical

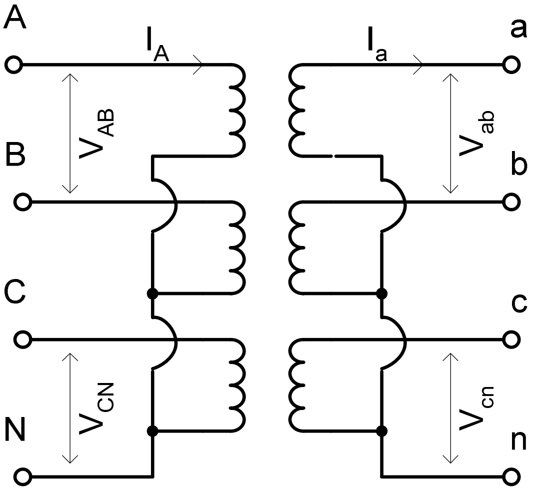
Polyphase Transformer

- Winding connections
  - Wye – wye
  - Delta – delta
  - Wye – delta
  - Delta – wye

Wye – wye connection

- Each single phase transformer winding controls ratio of phase – neutral voltages and phase currents
  E.g
  $|V_{AN}|/|V_{an}| = N_1/N_2$
  $|I_{AN}|/|I_{an}| = N_2/N_1$

- Ratio of line-line voltages
  $|V_{AB}|/|V_{ab}| = \sqrt{3}|V_{AN}|/\sqrt{3}|V_{an}|$
  $= N_1/N_2$
- Ratio of line – currents
  $|I_A|/|I_a| = |I_{AN}|/|I_{an}|$
  $= N_2/N_1$



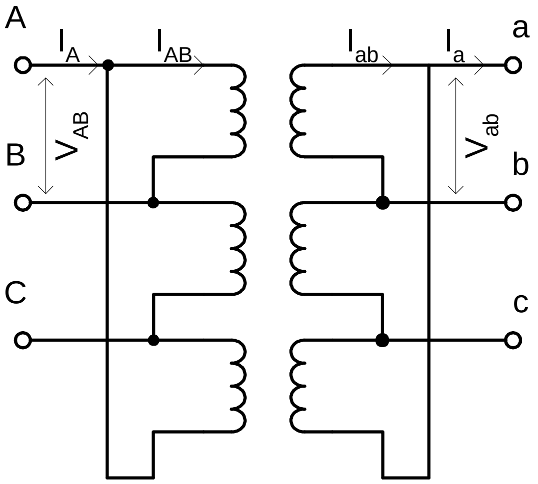
Wye – wye connection

- Seldom used in industrial applications
  - Easy to develop voltage unbalances
  - Allows propagation of harmonics, especially “triplen” harmonics (3rd, 6th, 9th, 12th harmonic, etc) through transformer

Delta – delta connection

- Each single phase transformer winding controls ratio of line – line voltages and phase currents
  E.g
  $|V_{AB}|/|V_{ab}| = N_1/N_2$
  $|I_{AB}|/|I_{ab}| = N_2/N_1$

- Ratio of line – currents
  $|I_A|/|I_a| = \sqrt{3}|I_{AB}|/\sqrt{3}|I_{ab}|$
  $= N_2/N_1$



Open delta or V – V connection

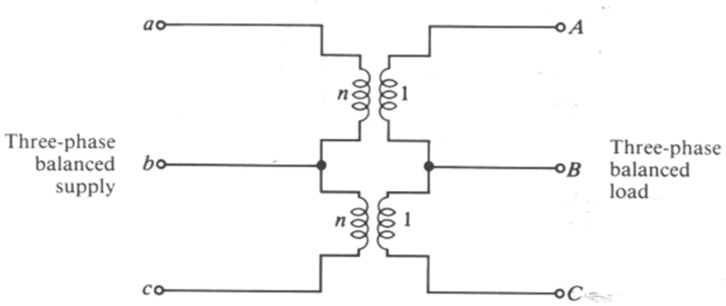


Figure 7.33 Open-delta transformer connection for Problem 7.20.

- One single-phase transformer can be removed and remaining two continue to operate as a three-phase bank
- Apparent power rating (kVA) of bank reduced to 58% or 1/√3 of original rating
- Sometimes used to supply small load that is expected to grow
  - 2 transformers used for 3 phase supply, with 3rd serving as spare

Wye – delta connection

- Each single phase transformer winding controls ratio of phase – neutral voltage (HV) to line – line voltage (LV) and ratios of currents through each single phase winding E.g

  $|V_{AN}|/|V_{ab}| = N_1/N_2$

  $|I_{AN}|/|I_{ab}| = N_2/N_1$

- Ratio of line-line voltages

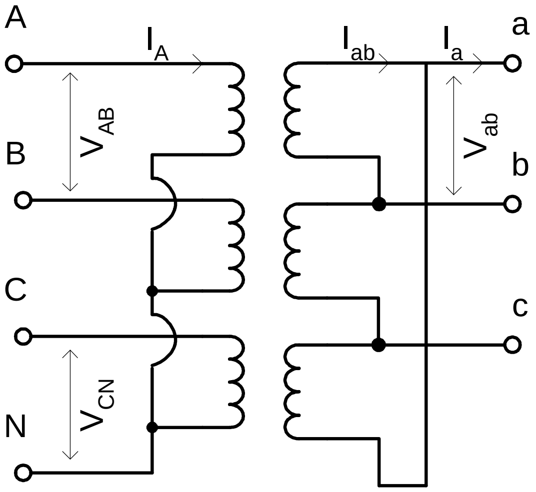
  $|V_{AB}|/|V_{ab}| = \sqrt{3}|V_{AN}|/|V_{ab}|$

  $= \sqrt{3}N_1/N_2$

- Ratio of line – currents

  $|I_A|/|I_a| = |I_{AN}|/\sqrt{3}|I_{ab}|$

  $= N_2/\sqrt{3}N_1$



Delta – wye connection

- Each single phase transformer winding controls ratio of line – line voltage (HV) to phase – neutral voltage (LV) and ratios of currents through each single phase winding E.g

  $|V_{AB}|/|V_{an}| = N_1/N_2$

  $|I_{AB}|/|I_{an}| = N_2/N_1$

- Ratio of line-line voltages

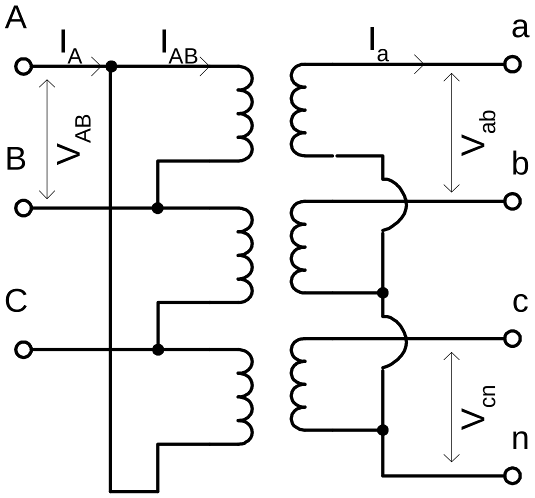
  $|V_{AB}|/|V_{ab}| = |V_{AB}|/\sqrt{3}|V_{an}|$

  $= N_1/\sqrt{3}N_2$

- Ratio of line – currents

  $|I_A|/|I_a| = \sqrt{3}|I_{AB}|/|I_{an}|$

  $= \sqrt{3}N_2/N_1$



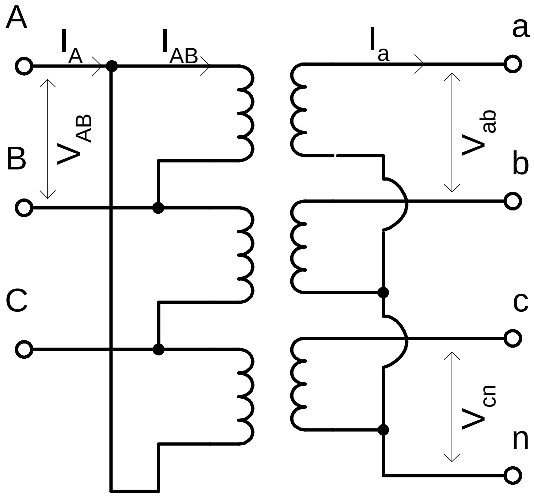
Wye – delta connections

- Wye (HV) – delta (LV) connections
  - Utilises insulation more efficiently
    - Turns ratio effectively increased
    - Commonly applied to generator step-up transformers
  - Grounding point desirable because it limits stress on line-ground impedance of high voltage winding under some fault conditions
  - Delta connection allows circulating path for harmonics (especially triplen harmonics)
    - Allows maintenance of system balance and “good” shape of voltage waveform even in presence of load unbalances and magnetizing current harmonics

Delta – wye connections

- Delta (HV) – wye (LV) connections
  - Less commonly used as turns ratio on transformers not used as effectively
  - Often applied for step-down transformers for connection to distribution or LV network where single phase loads connected that require neutral point

Transformer Phase Shift



- Delta-wye (or wye-delta) connections introduce phase shift into voltage and current waveforms
  E.g. Delta – wye connection
  $|V_{AB}| / |V_{an}| = N_1/N_2$
  $|V_{an}| = |V_{AB}|*(N_2/N_1)$

  $V_{ab} = \sqrt{3}|V_{an}|\angle{+30^\circ}$
  $= \sqrt{3}\,|V_{AB}|*(N_2/N_1)\angle{+30^\circ}$
- Turns ratio of this transformer then contains magnitude change and phase shift change
  $V_{ab}/V_{AB} = \sqrt{3}\,|V_{AB}|*(N_2/N_1)\angle{+30^\circ}$

Transformer Phase Shift

- Can determine similar relationship for current ratio
- Ratio of line – currents of delta – wye connection

  $|I_A|/|I_a| = \sqrt{3}|I_{AB}|/|I_{an}|$

  $= \sqrt{3}N_2/N_1$
- For line current feeding delta winding I_A

  $I_A = \sqrt{3}I_{AB}\angle -30^\circ$

  $I_A/I_a = \sqrt{3}(N_2/N_{1)}\ \angle -30^\circ$

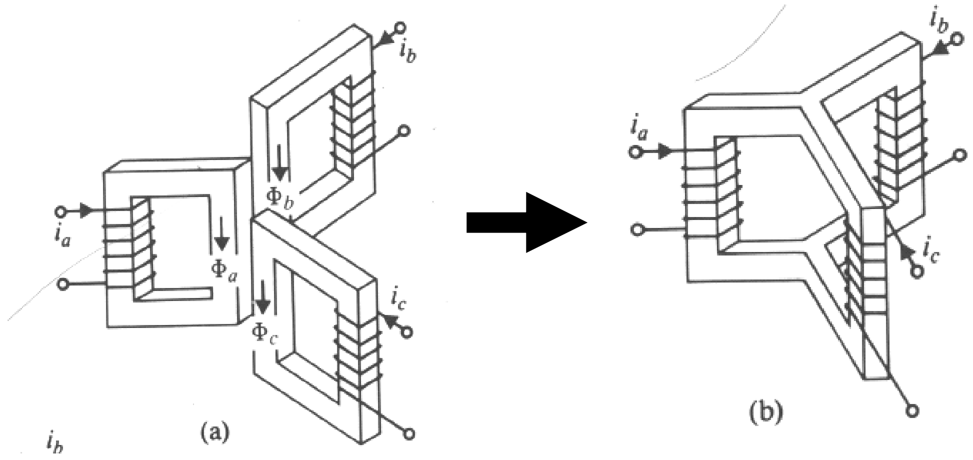
- Possible to achieve phase shifts of
  - ±30°
  - ±150°
  - ±90°

  depending upon manner by which windings are connected
- Need to standardize the relationship

Transformer Phase Shift

- IEEE C57.12.70 – 1978 *American National Standard Terminal Marking and Connections for Distribution and Power Transformer*
  - For both delta-wye and wye-delta connections the HV terminal voltage will lead the corresponding LV terminal voltage by 30°
  - Currents in the transformers are displace by 30° in the direction of the voltages since the phase angles of the currents are determined by the load impedances

Polyphase transformers



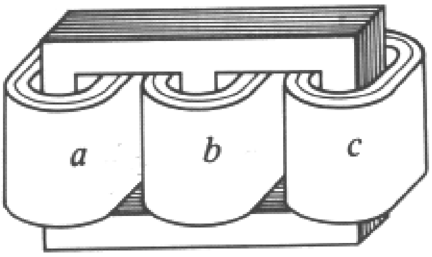
(a) (b)

Polyphase transformers

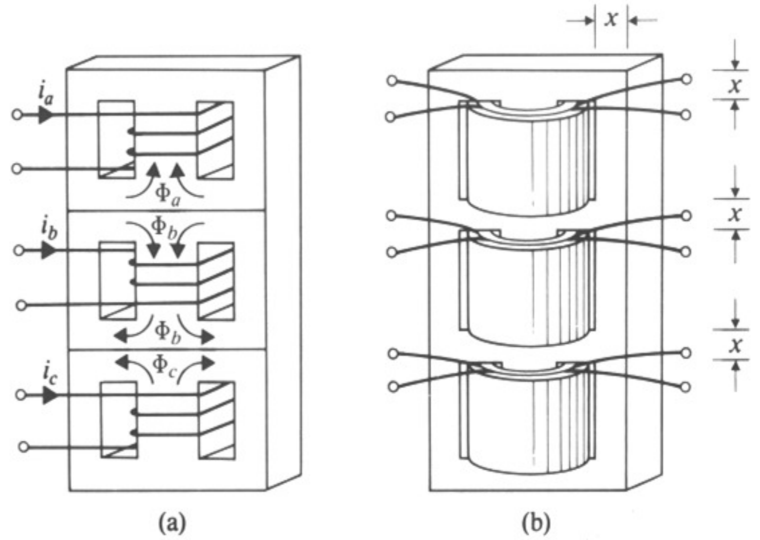
- Consist of 3 sets of primary and secondary windings on a common magnetic structure
- For balanced excitation flux produced in each winding of Φ_1, Φ_2 and Φ_3 balanced
  - $\Phi_1 + \Phi_2 + \Phi_3 = 0$
    – no flux in central magnetic path allowing it to be removed, simplifying construction, reducing mass and cost of transformer
- Two main transformer bank construction types
  - Core
    - Most common three phase transformer construction
  - Shell

Core type transformer

- Magnetic circuit consists of three core sections in parallel
  - Similar to delta connected bank of single phase transformer
  - Removal of return path for flux ensures that both the flux and voltage per phase must sum to zero even for unbalanced loading conditions
  - Limits production of triplen harmonics under un-balanced loading conditions



Shell type transformer



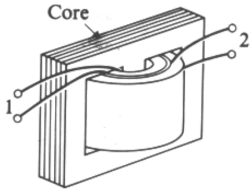
(a) (b)

Shell type transformer

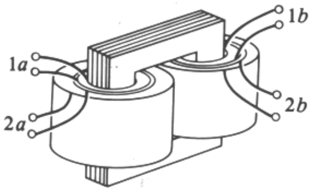
- Can be consider as a stack of 3 single phase units
- Phase B coil wound in opposite direction to Phase A or Phase C coils
    - Ensures that magnitude of combined fluxes such as $0.5*\Phi_a + 0.5*\Phi_b$ or $0.5*\Phi_c + 0.5*\Phi_a$ will have same magnitude as flux in outer section of core of $0.5*\Phi_a$
    - Allows significant reduction in core size (with respective to stack of 3 single phase units)

Core construction – Laminated steel core

- Transformer core built from layers of steel laminate
- Positions of joints between layers alternated to give mechanical strength
- Carefully constructed to leave no air-gaps in corner where laminates overlap
  - Air-gaps lead to increased losses within core

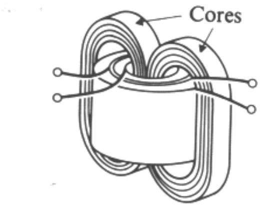


(a)

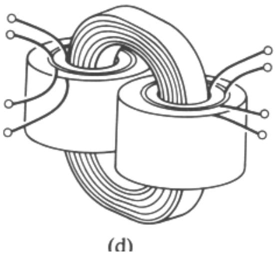


(b)

Core construction – wound steel core



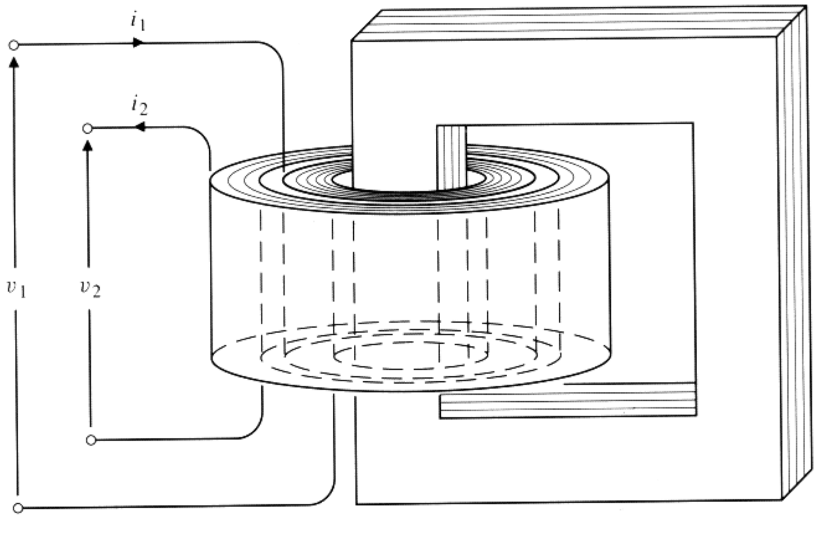
(c)



(d)

- Core wound from continuous strip of grain-oriented steel fed through core

Winding construction



Winding construction

- Windings made of copper or aluminium
- While resistive losses may be significant in distribution class transformers, leakage losses
  most important for HV transformers
  - Windings construction to maximize coupling between primary and secondary coils
- Diagram shows common arrangement used to limit leakage flux.
- Performance enhanced further by
  - Minimize area in which flux leakage occurs
  - Increasing windings length to maximize path length for leakage flux

Instrument Transformers
- Current Transformer

- Toroidal core wound from continuous strip of low-loss, low-field intensity magnetic material
  - Secondary winding wound around toroid transformers high current to 1-5A for measurement
  - Insulation of secondary must be adequate for voltage of current carrying conductor
- Rated conservatively and may be considered as ideal with little error. Sources of error include
  - Excitation currents
    - Core designed to operate at low flux densities
    - Load currents on secondary kept low

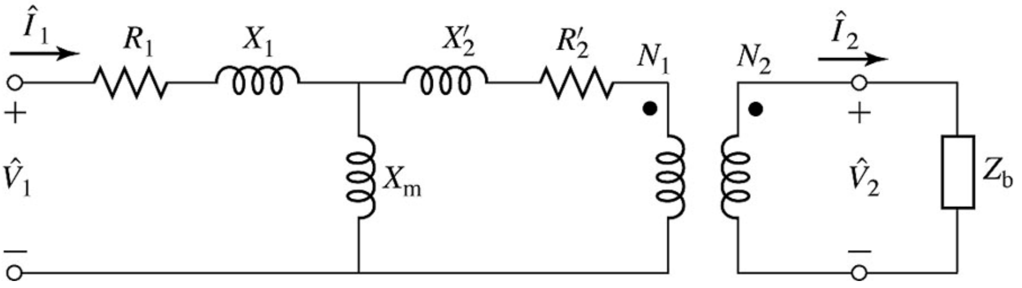
Instrument Transformers
- Current Transformer

- Operating considerations
  - Secondary should never be open-circuited
    - All primary current would become magnetizing current driving core alternatively between positive and negative saturation producing high voltage pulses in secondary windings

Instrument Transformers
- Potential Transformer

- On most systems, line-voltages cannot be measured directly
  - Voltage measured using potential transformer
    - Allows low current metering
    - Performs isolation from high voltage system
  - VA rating of potential transformer often very small
    - Transformer however may be physically large due to need for insulation from line voltage

Instrument Transformers
- Potential Transformer



Questions?