Induction Machines

1 Introduction

The induction machine was invented by NIKOLA TESLA in 1888. Right from its inception its ease of manufacture and its robustness have made it a very strong candidate for electromechanical energy conversion. It is available from fractional horsepower ratings to megawatt levels. It finds very wide usage in all various application areas. The induction machine is an AC electromechanical energy conversion device. The machine interfaces with the external world through two connections (ports) one mechanical and one electrical. The mechanical port is in the form of a rotating shaft and the electrical port is in the form of terminals where AC supply is connected. There are machines available to operate from three phase or single phase electrical input. In this module we will be discussing the three phase induction machine. Single phase machines are restricted to small power levels.

2 The Rotating Magnetic Field

The principle of operation of the induction machine is based on the generation of a rotating magnetic field. Let us understand this idea better.

Click on the following steps in sequence to get a graphical picture. It is suggested that the reader read the text before clicking the link.

- Consider a cosine wave from 0 to 360°. This sine wave is plotted with unit amplitude.
- Now allow the amplitude of the sine wave to vary with respect to time in a simisoidal fashion with a frequency of 50Hz.Let the maximum value of the amplitude is, say, 10 units. This waveform is a pulsating sine wave.

$$i_{apk} = I_m \cos 2\pi.50.t \tag{1}$$

- Now consider a second sine wave, which is displaced by 120° from the first (lagging)...
- and allow its amplitude to vary in a similar manner, but with a 120°time lag.

$$i_{bpk} = I_m \cos(2\pi.50.t - 120°) \tag{2}$$

- Similarly consider a third sine wave, which is at 240° lag...
- and allow its amplitude to change as well with a 240° time lag. Now we have three pulsating sine waves.

$$i_{cpk} = I_m \cos(2\pi.50.t - 240°) \tag{3}$$

Let us see what happens if we sum up the values of these three sine waves at every angle. The result really speaks about Tesla's genius. What we get is a constant amplitude travelling sine wave!

In a three phase induction machine, there are three sets of windings — phase A winding, phase B and phase C windings. These are excited by a balanced three-phase voltage supply. This would result in a balanced three phase current. Equations 1 — 3 represent the currents that flow in the three phase windings. Note that they have a 120° *time* lag between them.

Further, in an induction machine, the windings are not all located in the same place. They are distributed in the machine 120° away from each other (more about this in the section on alternators). The correct terminology would be to say that the windings have their axes separated in space by 120°. This is the reason for using the phase A, B and C since waves separated in *space* as well by 120°.

When currents flow through the coils, they generate mmfs. Since mmf is proportional to current, these waveforms also represent the mmf generated by the coils and the total mmf. Further, due to magnetic material in the machine (iron), these mmfs generate magnetic flux, which is proportional to the mmf (we may assume that iron is infinitely permeable and non-linear effects such as hysterisis are neglected). Thus the waveforms seen above would also represent the flux generated within the machine. The net result as we have seen is a travelling flux wave. The x-axis would represent the space angle in the machine as one travels around the air gap. The first pulsating waveform seen earlier would then represent the a-phase flux, the second represents the b-phase flux and the third represents the c-phase.

This may be better visualized in a polar plot. The angles of the polar plot represent the *space* angle in the machine, i.e., angle as one travels around the stator bore of the machine. Click on the links below to see the development on a polar axes.

- This plot shows the pulsating wave at the zero degree axes. The amplitude is maximum at zero degree axes and is zero at 90° axis. Positive parts of the waveform are shown in red while negative in blue. Note that the waveform is pulsating at the $0 - 180°$ axis and red and blue alternate in any given side. This corresponds to the sinewave current changing polarity. Note that the maximum amplitude of the sinewave is reached only along the $0 - 180°$ axis. At all other angles, the amplitude does not reach a maximum of this value. It however reaches *a* maximum value which is less than that of the peak occuring at the $0 - 180°$ axis. More exactly, the maximum reached at any space angle θ would be equal to $\cos\theta$ times the peak at the $0 - 180°$ axis. Further, at any space angle θ, the time variation is sinusoidal with the frequency and phase lag being that of the excitation, and amplitude being that corresponding to the *space* angle.
- This plot shows the pulsating waveforms of all three cosines. Note that the first is pulsating about the $0 - 180°$ axis, the second about the$120° - 300°$axis and the third at $240° - 360°$axis.
- This plot shows the travelling wave in a circular trajectory. Note that while individual pulsating waves have maximum amplitude of 10, the resultant has amplitude of 15.

If f_1 is the amplitude of the flux waveform in each phase, the travelling wave can then be represented as

$$\begin{aligned} f(t) &= f_1 \cos \omega t \cos \theta + f_1 \cos(\omega t - \frac{2\pi}{3}) \cos(\theta - \frac{2\pi}{3}) + f_1 \cos(\omega t - \frac{4\pi}{3}) \cos(\theta - \frac{4\pi}{3}) \\ &= \frac{3}{2} f_1 \cos(\omega t - \theta) \qquad (4) \end{aligned}$$

It is worthwhile pondering over the following points.

1. what is the interpretation of the pulsating plots of the animation? If one wants to know the 'a' phase flux at a particular angle for all instants of time, how can it be obtained?
2. What will this time variation look like? It is obviously periodic. What will be the amplitude and frequency?
3. Why does eqn. 4 represent a travelling wave?

3 Principles of Torque Production

In the earlier section, we saw how a rotating flux is produced. Now let us consider a rotor, which is placed in this field. Let the rotor have a coil such that the coil sides are placed diametrically opposite each other. This is shown in the fig. 1. Since the flux generated by the stator rotates flux linked by this rotor coil also changes.

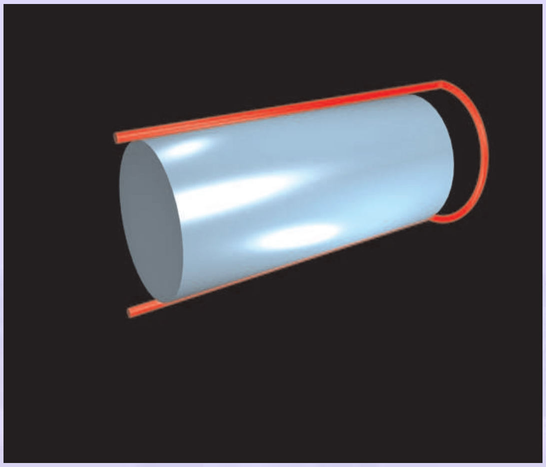


Figure 1: A Coil on the rotor

Since the flux pattern is varying *sinusoidally* in space, as the flux waveform rotates, the flux linkage varies sinusoidally. The rate of variation of this flux linkage will then be equal to the speed of rotation of the air gap flux produced. This sinusoidal variation of the flux linkage produces a sinusoidal induced emf in the rotor coil. If the coil is short circuited, this induced emf will cause a current flow in the coil as per Lenz's law.

Now imagine a second coil on the rotor whose axis is 120° away from the first. This is shown in fig. 2. The flux linkage in this coil will also vary sinusoidally with respect to time and therefore cause an induced voltage varying sinusoidally with time. However the flux linkages in these two coils will have a phase difference of 120° (the rotating flux wave will have to travel 120° in order to cause a similar flux linkage variation as in the first coil), and hence the time varying voltages induced in the coils will also have a 120° phase difference.

A third coil placed a further 120° away is shown in fig. 3. This will have a time varying induced emf lagging 240° in time with respect to the first.

When these three coils are shorted upon themselves currents flow in them as per Lenz's law. The mechanism by which torque is produced may now be understood as follows. The diagram in fig. 4 shows a view of the rotor seen from one end. Positive current is said to

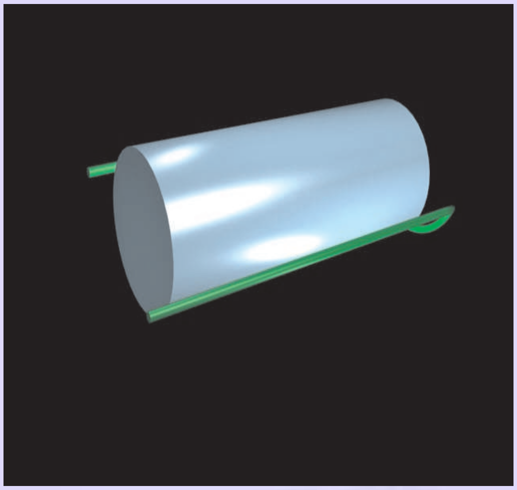


Figure 2: A coil displaced 120° from the first

flow in these coils when current flows out of the page in a, b, c conductors and into a', b' and c' respectively.

If we look at the voltage induced in these coils as phasors, the diagram looks as shown in fig. 5. The main flux is taken as the reference phasor. Considering that the induced emf is $-d\psi/dt$ where ψ is the flux linkage, the diagram is drawn as shown.

As usual, the horizontal component of these phasors gives the instantaneous values of the induced emf in these coils.

Let these coils be purely resistive. Then these emf phasors also represent the currents flowing in these coils. If we consider the instant $t = 0$, it can be seen that

1. The field flux is along 0° axis.
2. The current in a phase coil is zero.
3. The current in b phase coil is $-\frac{\sqrt{3}}{2}$ units.
4. The current in c phase coil is $+\frac{\sqrt{3}}{2}$ units.

These currents act to produce mmf and flux along the axes of the respective coils. Let us consider the space around b' and c coil sides. The situation is shown in fig. 6.

The resulting flux pattern causes a tendency to move in the anticlockwise direction. This is easy to see through the so called whiplash rule. Alternatively, since the force on a current

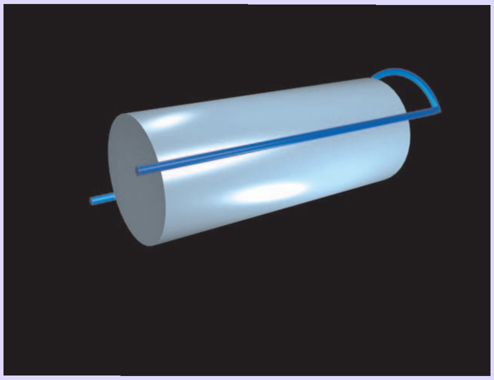


Figure 3: A coil displaced 240° from the first

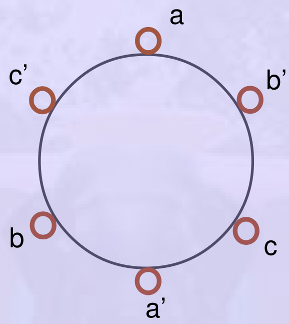


Figure 4: Coils on the rotor

carrying conductor is $\overline{F} = q(\overline{v}\ X\ \overline{B})$, it can be seen that the torque produced tends to rotate the rotor counter-clockwise. The magnitude of the torque would increase with the current magnitude in the coils. This current is in turn dependent on the magnitude of the main field flux and its speed of rotation. Therefore one may say that motion of the main field tends to drag the rotor along with it.

When the rotor is free to move and begins moving, the motion reduces the relative speed between the main field and the rotor coils. Less emf would therefore be induced and the torque would come down. Depending on the torque requirement for the load, the difference in speed between the rotor and the main field settles down at some particular value.

From the foregoing, the following may be noted.

1. The torque produced depends on a non-zero relative speed between the field and the rotor.

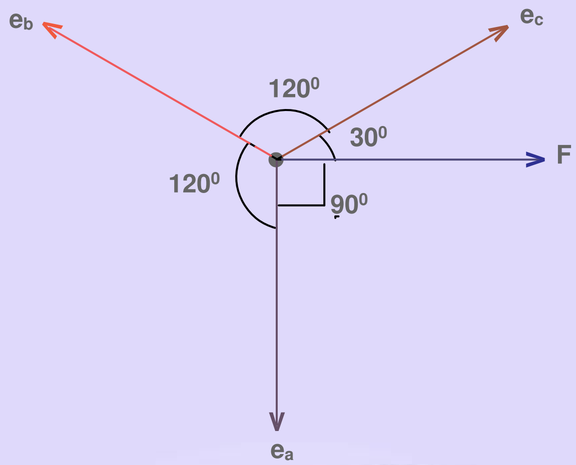


Figure 5: EMF induced in the coils : Resistive rotor

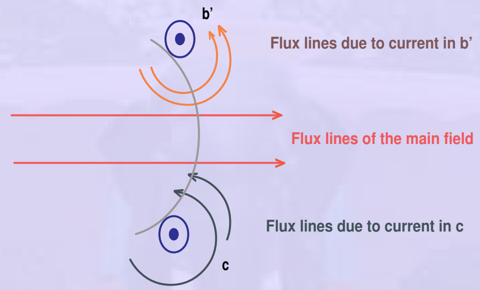


Figure 6: Flux around conductors : Resistive rotor

2. It is therefore not possible for the rotor to run continuously at the same speed of the field. This is so because in such a condition, no emf would be induced in the rotor and hence no rotor current, no torque.
3. The frequency of currents induced in the rotor coils and their magnitude depends on this difference in speed.

These are important conclusions. The speed of the main field is known as the synchronous speed, n_s. If the actual speed of the rotor is n_r then the ratio

$$s = \frac{n_s - n_r}{n_s} \quad (5)$$

is known as slip and is frequently expressed as a percentage. Typically induction machines are designed to operate at about less than 4 percent slip at full load.

It is instructive to see the situation if the rotor resistance is neglected and is considered to be purely inductive. The phasor diagram of voltages and the currents would then look as shown in fig. 7.

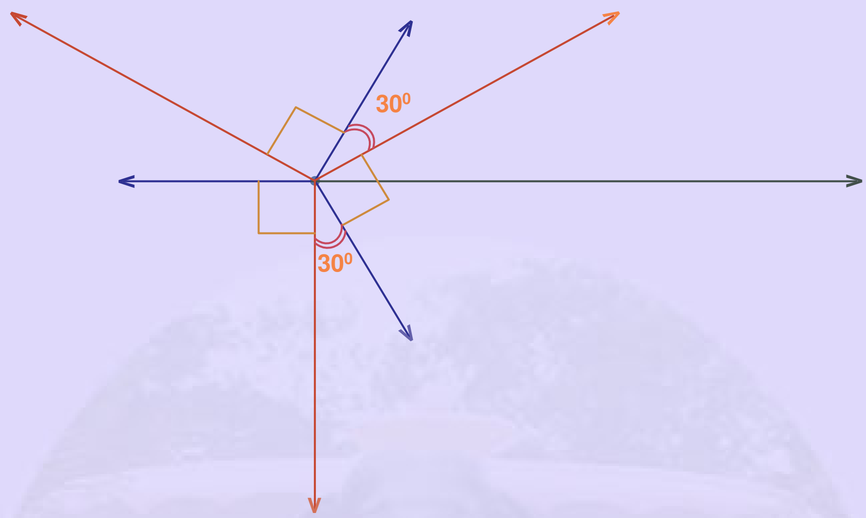


Figure 7: EMF induced in coils : Inductive rotor

At $t = 0$, one can see that current in a phase coil is at negative maximum, while b and c phases have positive current of 0.5 units. Now if we consider the current flux profiles at coil sides $a, b^{'}, c$, the picture that emerges is shown in fig. 8.

Since main flux at the a coil side is close to zero, there is very little torque produced from there. There is a tendency to move due to the $b^{'}$ and c coil sides, but they are in opposite directions however. Hence there is no net torque on the rotor. This brings up another important conclusion — the resistance of the rotor is an important part of torque production in the induction machine. While a high resistance rotor is better suited for torque production, it would also be lossy.

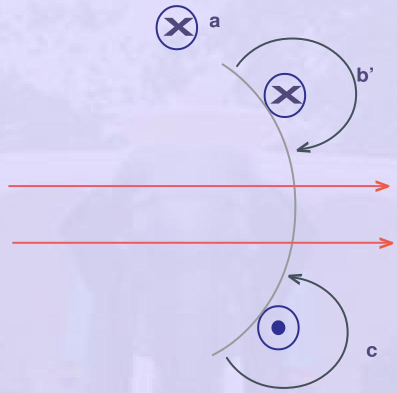


Figure 8: Flux around conductors : Inductive rotor

4 Construction

In actual practice, the three coils form three windings distributed over several slots. These windings may be connected in star or delta and three terminations are brought out. These are conventional three phase windings which are discussed in greater detail in the chapters on alternators. Such windings are present n the stator as well as rotor. A photograph of

Figure 9: stator of an induction machine

the stator of an induction machine is shown in fig. 9. A close up of the windings is shown in fig. 10.the several turns that makeup a coil are seen in this picture. The three terminations are connected to rings on which three brushes make a sliding contact. As the rotor rotates the brushes slip over the rings and provide means of connecting stationary external circuit elements to the rotating windings. A schematic of these arrangements is shown in fig. 13. A photograph of a wound rotor for an induction machine is shown in fig. 11. Fig. 12 shows a close up of the slip ring portion. Brushes are not shown in this picture.

Induction machines, which have these kinds of windings and terminals that are brought out, are called slip ring machines. The reader may note that in order that torque is produced current must flow in the rotor. To achieve that, the stationary brush terminals must either be shorted, or connected to a circuit allowing current flow. Sometimes a star connected resistor bank is connected so that the developed starting torque is higher. There are also other forms of power electronic circuitry that may be connected to the rotor terminals to achieve various functions.

The popularity of the induction machine however, stems from another variety of rotor

Figure 10: Coils in the stator

Figure 11: A wound rotor with slip rings

Figure 12: slip rings

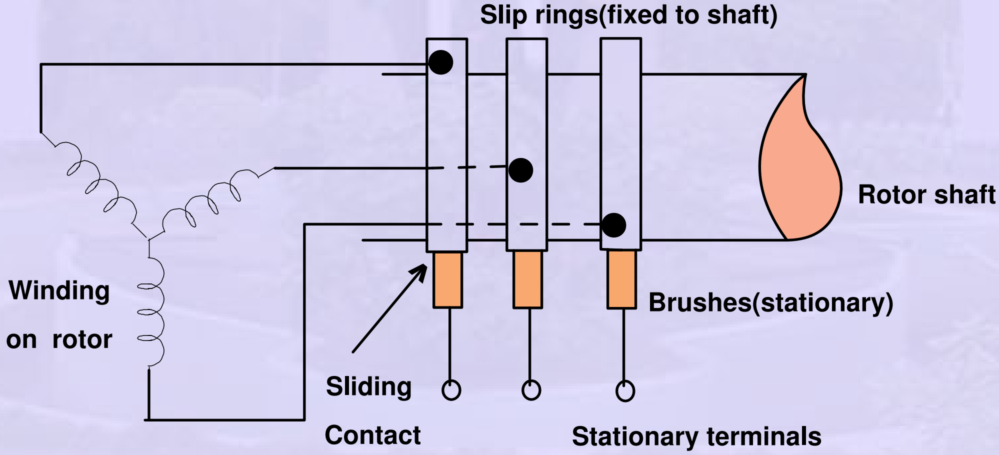


Figure 13: Slip rings and brushes in induction machines — a schematic

that is used. This rotor has slots into which copper or aluminium bars are inserted. These bars are then shorted by rings that are brazed on to each of the rotor ends. Figure 14 shows a simple schematic.

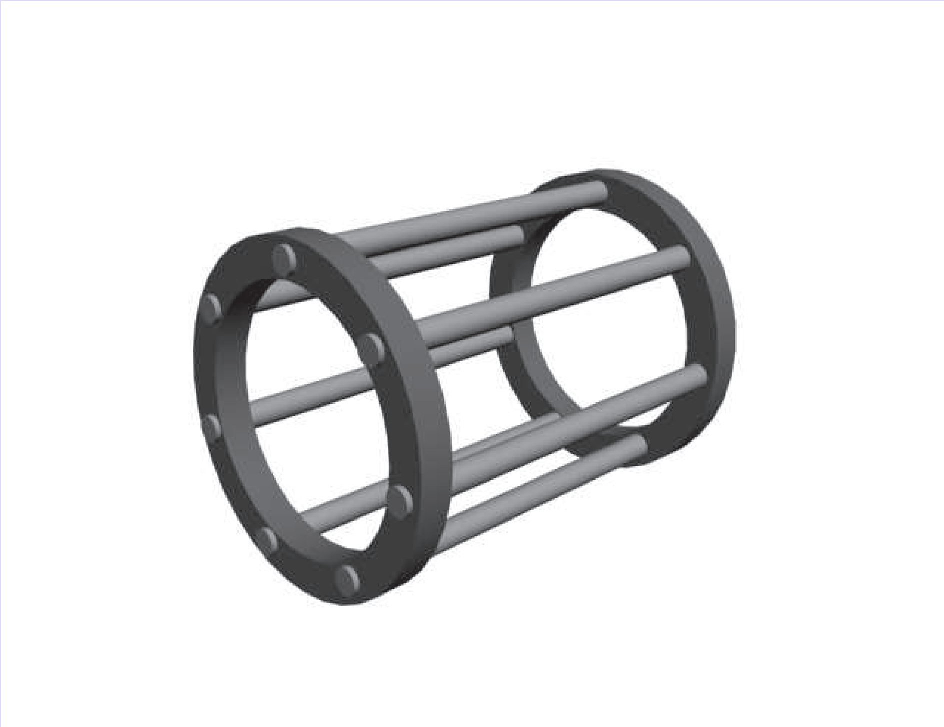


Figure 14: Squirrel cage rotor — a schematic

Such a rotor is called squirrel cage rotor. This rotor behaves like a short-circuited winding and hence the machine is able to perform electromechanical energy conversion. This type of rotor is easy to manufacture, has no sliding contacts and is very robust. It is this feature that makes induction machine suitable for use even in hazardous environments and reliable operation is achieved. The disadvantage of this type of rotor is that the motor behavior cannot be altered by connecting anything to the rotor — there are no rotor terminals.

Fig. 15 shows a photograph of a squirrel cage rotor. The rotor also has a fan attached to it. This is for cooling purposes. The bars (white lines on the surface) are embedded in the rotor iron which forms the magnetic circuit. The white lines correspond to the visible portion of the rotor bar.

Sometimes two rotor bars are used per slot to achieve some degree of variability in the starting and running performances. It is to make use of the fact that while high rotor

Figure 15: squirrel cage rotor

resistance is desirable from the point of view of starting torque, low rotor resistance is desirable from efficiency considerations while the machine is running. Such rotors are called double cage rotors or deep-bar rotors.

To summarize the salient features discussed so far,

1. The stator of the 3 - phase induction machine consists of normal distributed AC windings.
2. Balanced three phase voltages impressed on the stator, cause balanced three phase currents to flow in the stator.
3. These stator currents cause a rotating flux pattern (the pattern is a flux distribution which is sinusoidal with respect to the space angle) in the air gap.
4. The rotating flux pattern causes three phase induced e.m.f.s in rotor windings (again normal ac windings). These windings, if shorted, carry three phase-balanced currents. Torque is produced as a result of interaction of the currents and the air gap flux.
5. The rotor may also take the form of a squirrel cage arrangement, which behaves in a manner similar to the short-circuited three phase windings.

5 Equivalent Circuit

It is often required to make quantitative predictions about the behavior of the induction machine, under various operating conditions. For this purpose, it is convenient to represent the machine as an equivalent circuit under sinusoidal steady state operating conditions. Since the operation is balanced, a single-phase equivalent circuit is sufficient for most purposes.

In order to derive the equivalent circuit, let us consider a machine with an open circuited rotor. Since no current can flow and as a consequence no torque can be produced, the situation is like a transformer open-circuited on the secondary (rotor). The equivalent circuit under this condition can be drawn as shown in fig. 16.

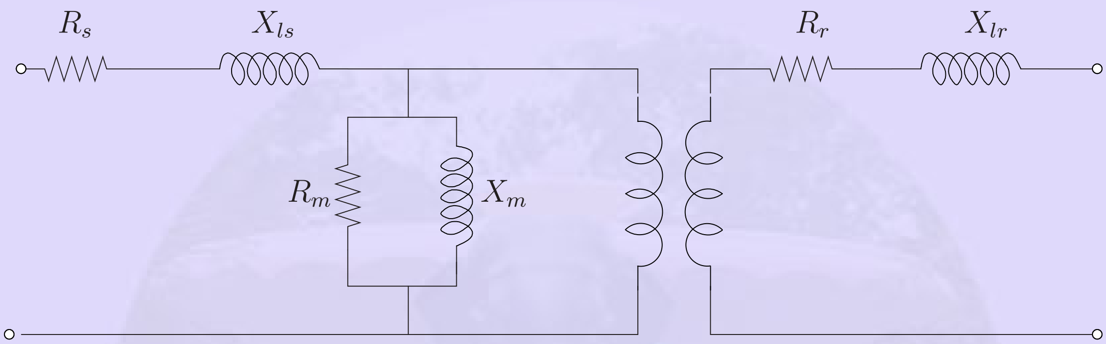


Figure 16: Induction machine with the rotor open

This is just the normal transformer equivalent circuit (*why?*). Measurements aregenerally made on the stator side and the rotor, in most circumstances, is shorted (if required, through some external circuitry). Since most of the electrical interaction is from the stator, it makes sense to refer all parameters to the stator.

Let us consider the rotor to be shorted. Let the steady speed attained by the rotor be ω_r and the synchronous speed be ω_s. The induced voltage on the rotor is now proportional to the slip i.e., slip times the induced voltage under open circuit (*why?*). Further, the voltage induced and the current that flows in the rotor is at a frequency equal to slip times the stator excitation frequency (*why?*). The equivalent circuit can be made to represent this by shorting the secondary side and is shown in fig. 17.

$R_r^{'}$ and $X_{lr}^{'}$ refer to the rotor resistance and leakage resistance referred to the stator side (using the square of the turns ratio, as is done in transformer). The secondary side loop is excited by a voltage sE_1, which is also at a frequency sf_1. This is the reason why the rotor

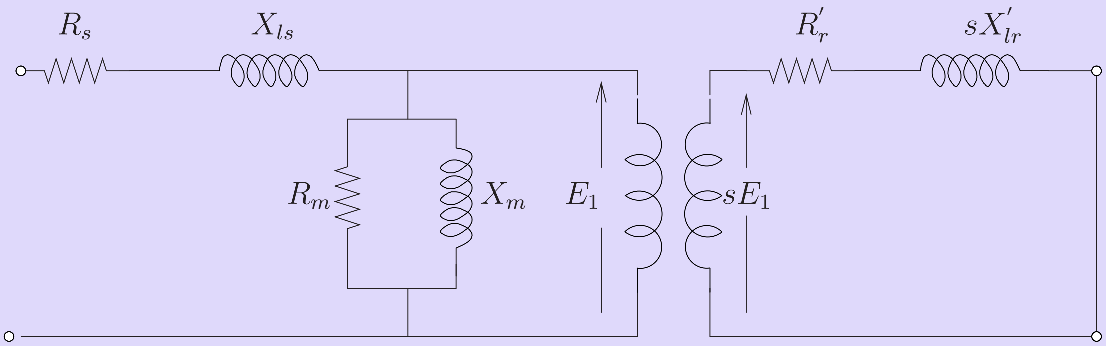


Figure 17: Equivalent circuit : rotor at its own frequency

leakage is $sX_{lr}^{'}$ now . The current amplitude in the rotor side would therefore be

$$I_r^{'} = \frac{sE_1}{\sqrt{R_r^{'2} + (sX_{lr}^{'2})}} \tag{6}$$

This expression can be modified as follows (dividing numerator and denominator by s)

$$I_r^{'} = \frac{E_1}{\sqrt{\frac{R_r^{'2}}{s^2} + (X_{lr}^{'2})}} \tag{7}$$

Equation 7 tells us that the rotor current is the same as the current flowing in a circuit with a load impedance consisting of a resistance $R_r^{'}/s$ and inductive reactance $X_{lr}^{'}$. This current would also now be at the frequency of E_1 (stator frequency). Note that the slip no longer multiplies the leakage reactance. Further this current is now caused by a voltage of E_1 itself (no multiplying factor of s). Hence the transformer in fig. 17 can also be removed.

Since, with this, the conversion to slip frequency is no longer there, the equivalent circuit can be represented as in fig. 18.

This is then the per-phase equivalent circuit of the induction machine, also called as *exact* equivalent circuit. Note that the voltage coming across the magnetizing branch is the applied stator voltage, reduced by the stator impedance drop. Generally the stator impedance drop is only a small fraction of the applied voltage. This fact is taken to advantage and the magnetizing branch is shifted to be directly across the input terminals and is shown in fig. 19.

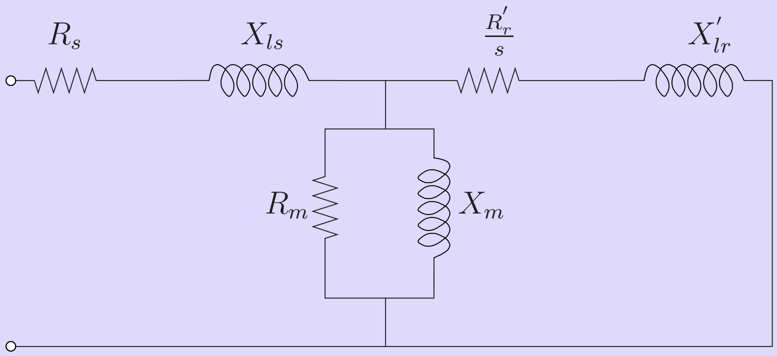


Figure 18: The Exact equivalent circuit

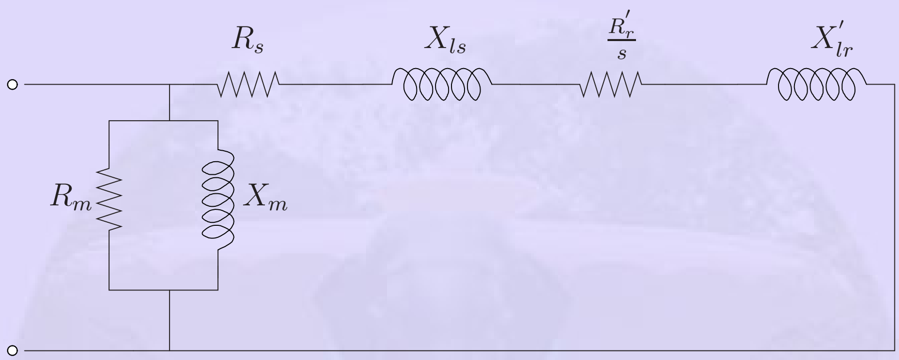


Figure 19: The approximate equivalent circuit

This circuit, called the *approximate* equivalent circuit, is simple to use for quick calculations.

The resistance term $\frac{R_r'}{s}$ could be split into two parts.

$$\frac{R_r'}{s} = R_r' + \frac{R_r'(1-s)}{s} \tag{8}$$

With this equation the equivalent circuit can be modified as shown in fig. 20.

Dividing the equation for the rotor current by s and merging the two sides of the transformer is not just a mathematical jugglery. The power dissipated in the rotor resistance (per phase) is obviously $I_2'^2 R_r'$. From the equivalent circuit of fig. 20 one can see that the rotor current (referred to stator of course) flows through a resistance R_r'/s which has a component $R_r'(1-s)/s$ in addition to R_r', which also dissipates power. What does this represent?

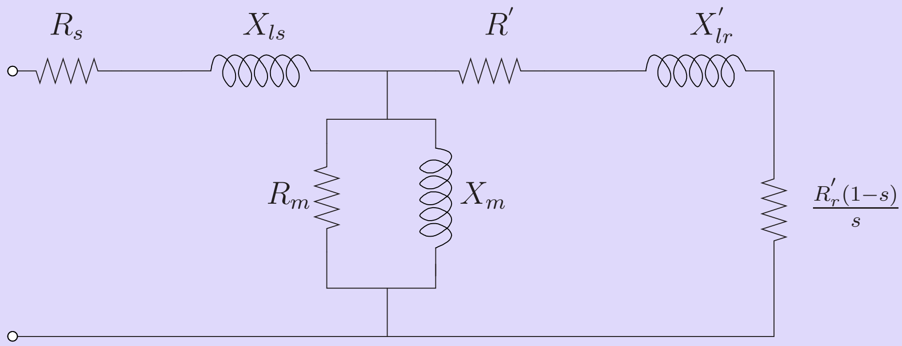


Figure 20: The exact equivalent circuit - separation of rotor resistance

From the equivalent circuit, one can see that the dissipation in R_s represents the stator loss, and dissipation in R_m represents the iron loss. Therefore, the power absorption indicated by the rotor part of the circuit must represent all other means of power consumption - the actual mechanical output, friction and windage loss components and the rotor copper loss components. Since the dissipation in R'_r is rotor copper loss, the power dissipation in $R'_r(1-s)/s$ is the sum total of the remaining. In standard terminology, dissipation in

- R'_r/s is called the air gap power.
- R'_r is the rotor copper loss.
- $R'_r(1-s)/s$ is the mechanical output.

In an ideal case where there are no mechanical losses, the last term would represent the actual output available at the shaft. Out of the power P_g Transferred at the air gap, a fraction s is dissipated in the rotor and $(1-s)$ is delivered as output at the shaft. If there are no mechanical losses like friction and windage, this represents the power available to the load.

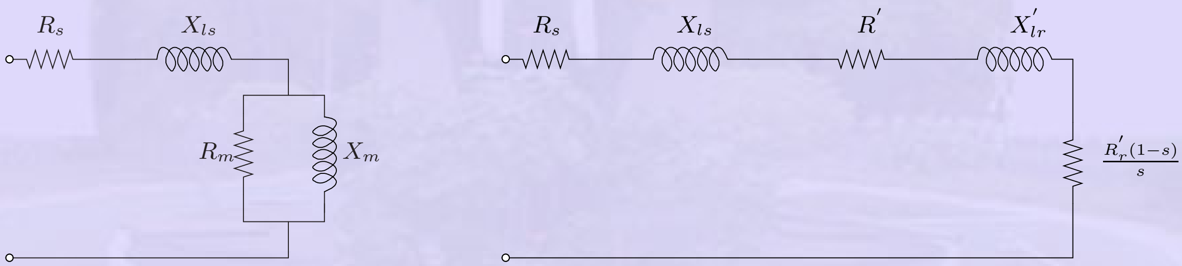
6 Determination of Circuit Parameters

In order to find values for the various elements of the equivalent circuit, tests must be conducted on a particular machine, which is to be represented by the equivalent circuit. In order to do this, we note the following.

1. When the machine is run on no-load, there is very little torque developed by it. In an ideal case where there is no mechanical losses, there is no mechanical power deveoped at no-load. Recalling the explanations in the section on torque production, the flow of current in the rotor is indicative of the torque that is produced. If no torque is produced, one may conclude that no current would be flowing in the rotor either. The rotor branch acts like an open circuit. This conclusion may also be reached by reasoning that when there is no load, an ideal machine will run up to its synchronous speed where the slip is zero resulting in an infinite impedance in the rotor branch.

2. When the machine is prevented from rotation, and supply is given, the slip remains at unity. The elements representing the magnetizing branch R_m&X_m are high impedances much larger than $R_r^{'}$ & $X_{lr}^{'}$ in series. Thus, in the exact equivalent circuit of the induction machine, the magnetizing branch may be neglected.

From these considerations, we may reduce the induction machine exact equivalent circuit of fig.18 to those shown in fig. 21.



(a) No-load equivalent (b) Blocked rotor equivalent

Figure 21: Reduced equivalent circuits

These two observations and the reduced equivalent circuits are used as the basis for the two most commonly used tests to find out the equivalent circuit parameters — the blocked rotor test and no load test. They are also referred to as the short circuit test and open circuit test respectively in conceptual analogy to the transformer.

6.1 The no-load test

The behaviour of the machine may be judged from the equivalent circuit of fig. 21(a). The current drawn by the machine causes a stator-impedance drop and the balance voltage is applied across the magnetizing branch. However, since the magnetizing branch impedance is large, the current drawn is small and hence the stator impedance drop is small compared to the applied voltage (rated value). This drop and the power dissipated in the stator resistance are therefore neglected and the total power drawn is assumed to be consumed entirely as core loss. This can also be seen from the approximate equivalent circuit, the use of which is justified by the foregoing arguments. This test therefore enables us to compute the resistance and inductance of the magnetizing branch in the following manner.

Let applied voltage $= V_s$. Then current drawn is given by

$$I_s = \frac{V_s}{R_m} + \frac{V_s}{jX_m} \tag{9}$$

The power drawn is given by

$$P_s = \frac{V_s^2}{R_m} \Rightarrow R_m = \frac{V_s^2}{P_s} \tag{10}$$

V_s, I_s and P_s are measured with appropriate meters. With R_m known from eqn. 10, X_m can be found from eqn. 9. The current drawn is at low power factor and hence a suitable wattmeter should be used.

6.2 Blocked-rotor Test

In this test the rotor is prevented from rotation by mechanical means and hence the name. Since there is no rotation, slip of operation is unity, $s = 1$. The equivalent circuit valid under these conditions is shown in fig. 21(b). Since the current drawn is decided by the resistance and leakage impedances alone, the magnitude can be very high when rated voltage is applied. Therefore in this test, only small voltages are applied — just enough to cause rated current to flow. While the current magnitude depends on the resistance and the reactance, the power drawn depends on the resistances.

The parameters may then be determined as follows. The source current and power drawn may be written as

$$I_s = \frac{V_s}{(R_s + R_r') + j(X_s + X_r')} \tag{11}$$

$$P_s = |I_s|^2(R_s + R_r') \tag{12}$$

In the test V_s, I_s and P_s are measured with appropriate meters. Equation 12 enables us to compute$(R_s + R_r^{'})$. Once this is known, $(X_s + X_r^{'})$ may be computed from the eqn. 11.

Note that this test only enables us to determine the series combination of the resistance and the reactance only and not the individual values. Generally, the individual values are assumed to be equal; the assumption $R_s = R_r^{'}$, and$X_s = X_r^{'}$suffices for most purposes. In practice, there are differences. If more accurate estimates are required IEEE guidelines may be followed which depend on the size of the machine.

Note that these two tests determine the equivalent circuit parameters in a 'Stator-referred' sense, i.e., the rotor resistance and leakage inductance are not the actual values but what they 'appear to be' when looked at from the stator. This is sufficient for most purposes as interconnections to the external world are generally done at the stator terminals.

7 Deducing the machine performance.

From the equivalent circuit, many aspects of the steady state behavior of the machine can be deduced. We will begin by looking at the speed-torque characteristic of the machine. We will consider the approximate equivalent circuit of the machine. We have reasoned earlier that the power consumed by the 'rotor-portion' of the equivalent circuit is the power transferred across the air-gap. Out of that quantity the amount dissipated in R_r' is the rotor copper loss and the quantity consumed by $R_r'(1-s)/s$ is the mechanical power developed. Neglecting mechanical losses, this is the power available at the shaft. The torque available can be obtained by dividing this number by the shaft speed.

7.1 The complete torque-speed characteristic

In order to estimate the speed torque characteristic let us suppose that a sinusoidal voltage is impressed on the machine. Recalling that the equivalent circuit is the per-phase representation of the machine, the current drawn by the circuit is given by

$$I_s = \frac{V_s}{(R_s + \frac{R_r'}{s}) + j(X_{ls} + X_{lr}')} \tag{14}$$

where V_s is the phase voltage phasor and I_s is the current phasor. The magnetizing current is neglected. Since this current is flowing through $\frac{R_r'}{s}$, the air-gap power is given by

$$\begin{aligned} P_g &= |I_s|^2 \frac{R_r'}{s} \\ &= \frac{V_s}{(R_s + \frac{R_r'}{s})^2 + (X_{ls} + X_{lr}')^2} \frac{R_r'}{s} \end{aligned} \tag{15}$$

The mechanical power output was shown to be $(1-s)P_g$ (power dissipated in R_r'/s). The torque is obtained by dividing this by the shaft speed ω_m.Thus we have,

$$\frac{P_g(1-s)}{\omega_m} = \frac{P_g(1-s)}{\omega_s(1-s)} = |I_s|^2 \frac{R_r'}{s\omega_s} \tag{16}$$

where ω_s is the synchronous speed in radians per second and s is the slip. Further, this is the torque produced per phase. Hence the overall torque is given by

$$T_e = \frac{3}{\omega_s} . \frac{V_s^2}{(R_s + \frac{R_r'}{s})^2 + (X_{ls} + X_{lr}')} . \frac{R_r'}{s} \tag{17}$$

The torque may be plotted as a function of 's' and is called the torque-slip (or torque-speed, since slip indicates speed) characteristic — a very important characteristic of the induction machine. Eqn. 17 is valid for a two-pole (one pole pair) machine. In general, this expression should be multiplied by p, the number of pole-pairs. A typical torque-speed characteristic is shown in fig. 22. This plot corresponds to a 3 kW, 4 pole, 60 Hz machine. The rated operating speed is 1780 rpm.

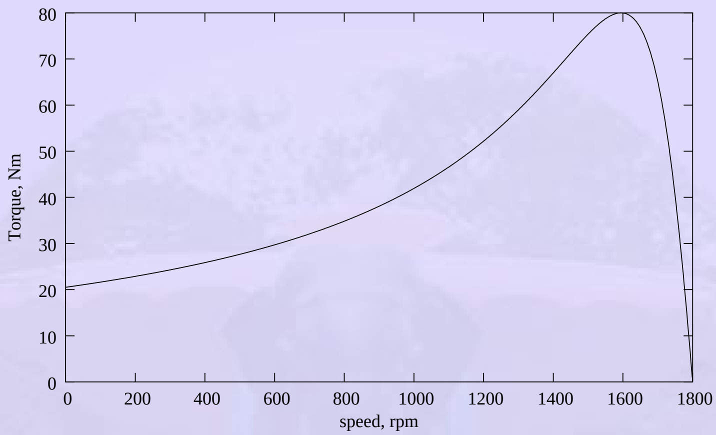


Figure 22: Induction machine speed-torque characteristic

We must note that the approximate equivalent circuit was used in deriving this relation. Readers with access to MATLAB or suitable equivalents (octave, scilab available free under GNU at the time of this writing) may find out the difference caused by using the 'exact' equivalent circuit by using the script found **here**. A comparison between the two is found in the plot of fig. 23. The plots correspond to a 3 kW, 4 pole, 50 Hz machine, with a rated speed of 1440 rpm. It can be seen that the approximate equivalent circuit is a good approximation in the operating speed range of the machine. Comparing fig. 22 with fig. 23, we can see that the slope and shape of the characteristics are dependent intimately on the machine parameters.

Further, this curve is obtained by varying slip with the applied voltage being held constant. Coupled with the fact that this is an equivalent circuit valid under steady state, it implies that if this characteristic is to be measured experimentally, we need to look at the torque for a given speed after all transients have died down. One cannot, for example, try to obtain this curve by directly starting the motor with full voltage applied to the terminals

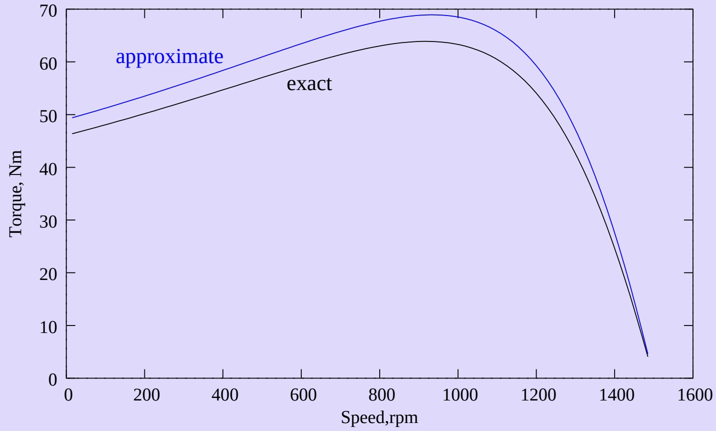


Figure 23: Comparison of exact and approximate circuit predictions

and measuring the torque and speed dynamically as it runs up to steady speed.

Another point to note is that the equivalent circuit and the values of torque predicted is valid when the applied voltage waveform is sinusoidal. With non-sinusoidal voltage waveforms, the procedure is not as straightforward.

With respect to the direction of rotation of the air-gap flux, the rotor maybe driven to higher speeds by a prime mover or may also be rotated in the reverse direction. The torque-speed relation for the machine under the entire speed range is called the complete speed-torque characteristic. A typical curve is shown in fig. 7.1 for a four-pole machine, the synchronous speed being 1500 rpm. Note that negative speeds correspond to slip values greater than 1, and speeds greater than 1500 rpm correspond to negative slip. The plot also shows the operating modes of the induction machine in various regions. The slip axis is also shown for convenience.

Restricting ourselves to positive values of slip, we see that the curve has a peak point. This is the maximum torque that the machine can produce, and is called as stalling torque. If the load torque is more than this value, the machine stops rotating or *stalls*. It occurs at a slip $\hat{s}$, which for the machine of fig. 7.1 is 0.38. At values of slip lower than $\hat{s}$, the curve falls steeply down to zero at $s = 0$. The torque at synchronous speed is therefore zero. At values of slip higher than $s = \hat{s}$, the curve falls slowly to a minimum value at $s = 1$. The torque at $s = 1$ (speed = 0) is called the starting torque.

The value of the stalling torque may be obtained by differentiating the expression for torque with respect to zero and setting it to zero to find the value of $\hat{s}$. Using this method,

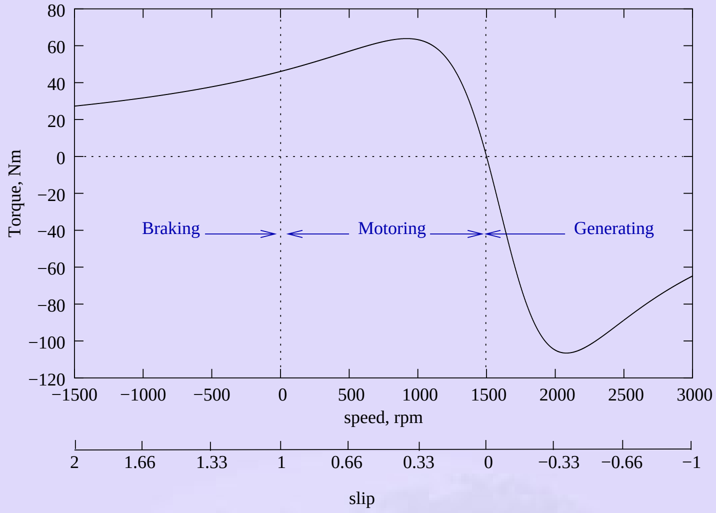


Figure 24: Complete speed-torque characteristic

$$\hat{s} = \frac{\pm R_r'}{\sqrt{R_r'^2 + (X_{ls} + X_{lr}')^2}} \tag{18}$$

Substituting $\hat{s}$ into the expression for torque gives us the value of the stalling torque $\hat{T}_e$.

$$\hat{T}_e = \frac{3V_s^2}{2\omega_s} . \frac{1}{R_s \pm \sqrt{R_s^2 + (X_{ls} + X_{lr}')^2}} \tag{19}$$

the negative sign being valid for negative slip.

The expression shows that $\hat{T}_e$ is the independent of R_r', while $\hat{s}$ is directly proportional to R_r'. This fact can be made use of conveniently to alter $\hat{s}$. If it is possible to change R_r', then we can get a whole series of torque-speed characteristics, the maximum torque remaining constant all the while. But this is a subject to be discussed later.

We may note that if R_r' is chosen equal to $\sqrt{R_s^2 + (X_{ls} + X_{lr}')^2}$, $\hat{s}$, becomes unity, which means that the maximum torque occurs at starting. Thus changing of R_r', wherever possible can serve as a means to control the starting torque.

While considering the negative slip range, (generator mode) we note that the maximum torque is higher than in the positive slip region (motoring mode).

7.2 Operating Point

Consider a speed torque characteristic shown in fig. 25 for an induction machine, having the load characteristic also superimposed on it. The load is a constant torque load i.e.,the torque required for operation is fixed irrespective of speed.

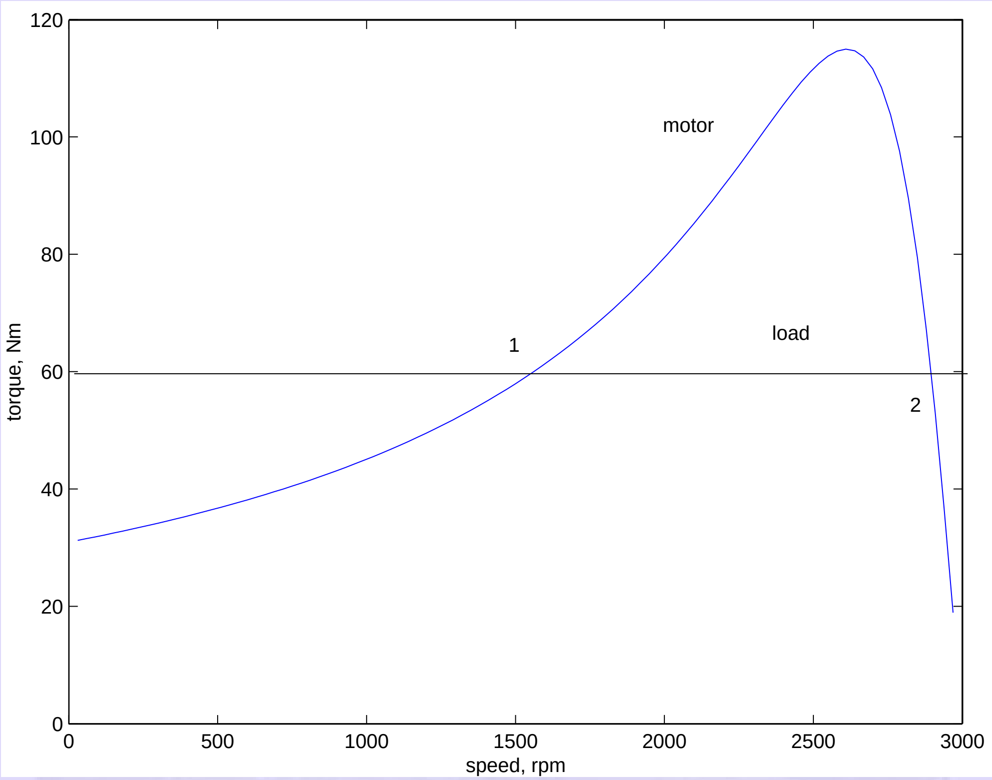


Figure 25: Machine and load characteristics

The system consisting of the motor and load will operate at a point where the two characteristics meet. From the above plot, we note that there are two such points. We therefore need to find out which of these is the actual operating point.

To answer this we must note that, in practice, the characteristics are never fixed; they change slightly with time. It would be appropriate to consider a small band around the curve drawn where the actual points of the characteristic will lie. This being the case let us considers that the system is operating at point 1, and the load torque demand increases slightly. This is shown in fig. 26, where the change is exaggerated for clarity. This would shift the point of operation to a point $1'$ at which the slip would be less and the developed torque higher.

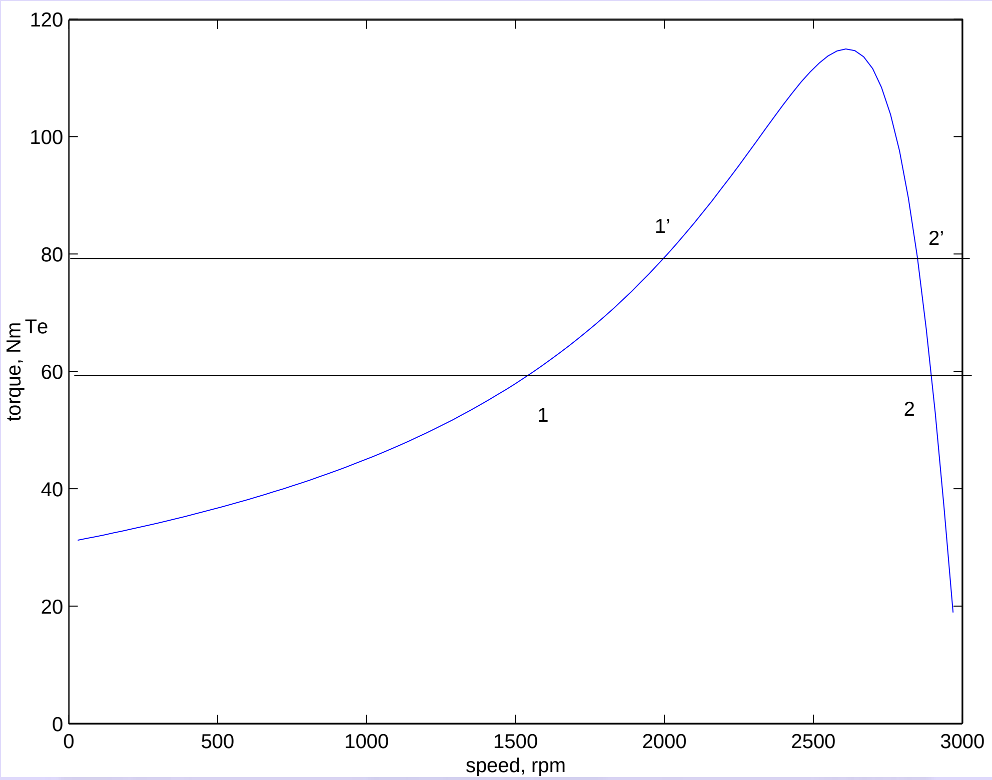


Figure 26: Stability of operating point

The difference in torque developed $\triangle T_e$, being positive will accelerate the machine. Any overshoot in speed as it approaches the point $1'$ will cause it to further accelerate since the developed torque is increasing. Similar arguments may be used to show that if for some reason the developed torque becomes smaller the speed would drop and the effect is cumulative. Therefore we may conclude that 1 is not a stable operating point.

Let us consider the point 2. If this point shifts to $2'$, the slip is now higher (speed is lower) and the positive difference in torque will accelerate the machine. This behavior will tend to bring the operating point towards 2 once again. In other words, disturbances at point 2 will not cause a runaway effect. Similar arguments may be given for the case where the load characteristic shifts down. Therefore we conclude that point 2 is a stable operating point.

From the foregoing discussions, we can say that the entire region of the speed-torque characteristic from $s = 0$ to $s = \hat{s}$ is an unstable region, while the region from $s = \hat{s}$ to $s = 0$ is a stable region. Therefore the machine will always operate between $s = 0$ and $s = \hat{s}$.

7.3 Modes of Operation

The reader is referred to fig. 7.1 which shows the complete speed-torque characteristic of the induction machine along with the various regions of operation.

Let us consider a situation where the machine has just been excited with three phase supply and the rotor has not yet started moving. A little reflection on the definition of the slip indicates that we are at the point $s = 1$. When the rotating magnetic field is set up due to stator currents, it is the induced emf that causes current in the rotor, and the interaction between the two causes torque. It has already been pointed out that it is the presence of the non-zero slip that causes a torque to be developed. Thus the region of the curve between $s = 0$ and $s = 1$ is the region where the machine produces torque to rotate a passive load and hence is called the motoring region. Note further that the direction of rotation of the rotor is the same as that of the air gap flux.

Suppose when the rotor is rotating, we change the phase sequence of excitation to the machine. This would cause the rotating stator field to reverse its direction — the rotating stator mmf and the rotor are now moving in opposite directions. If we adopt the convention that positive direction is the direction of the air gap flux, the rotor speed would then be a negative quantity. The slip would be a number greater than unity. Further, the rotor as we know should be "dragged along" by the stator field. Since the rotor is rotating in the opposite direction to that of the field, it would now tend to slow down, and reach zero speed. Therefore this region ($s > 1$) is called the braking region. *(What would happen if the supply is not cut-off when the speed reaches zero?)*

There is yet another situation. Consider a situation where the induction machine is operating from mains and is driving an active load (a load capable of producing rotation by itself). A typical example is that of a windmill, where the fan like blades of the wind mill are connected to the shaft of the induction machine. Rotation of the blades may be caused by the motoring action of the machine, or by wind blowing. Further suppose that both acting independently cause rotation in the same direction. Now when both grid and wind act, a strong wind may cause the rotor to rotate faster than the mmf produced by the stator excitation. A little reflection shows that slip is then negative. Further, the wind is rotating the rotor to a speed higher than what the electrical supply alone would cause. In order to do this it has to contend with an opposing torque generated by the machine preventing the speed build up. The torque generated is therefore negative. It is this action of the wind against the torque of the machine that enables wind-energy generation. The region of slip $s > 1$ is the generating mode of operation. Indeed this is at present the most commonly used approach in wind-energy generation. It may be noted from the torque expression of eqn. 17 that torque is negative for negative values of slip.

8 Speed control of Induction Machines

We have seen the speed torque characteristic of the machine. In the stable region of operation in the motoring mode, the curve is rather steep and goes from zero torque at synchronous speed to the stall torque at a value of slip $s = \hat{s}$. Normally $\hat{s}$ may be such that stall torque is about three times that of the rated operating torque of the machine, and hence may be about 0.3 or less. This means that in the entire loading range of the machine, the speed change is quite small. The machine speed is quite stiff with respect to load changes. The entire speed variation is only in the range n_s to $(1 - \hat{s})n_s$, n_s being dependent on supply frequency and number of poles.

The foregoing discussion shows that the induction machine, when operating from mains is essentially a constant speed machine. Many industrial drives, typically for fan or pump applications, have typically constant speed requirements and hence the induction machine is ideally suited for these. However,the induction machine, especially the squirrel cage type, is quite rugged and has a simple construction. Therefore it is good candidate for variable speed applications if it can be achieved.

8.1 Speed control by changing applied voltage

From the torque equation of the induction machine given in eqn.17, we can see that the torque depends on the square of the applied voltage. The variation of speed torque curves with respect to the applied voltage is shown in fig. 27. These curves show that the slip at maximum torque $\hat{s}$ remains same, while the value of stall torque comes down with decrease in applied voltage. The speed range for stable operation remains the same.

Further, we also note that the starting torque is also lower at lower voltages. Thus, even if a given voltage level is sufficient for achieving the running torque, the machine may not start. This method of trying to control the speed is best suited for loads that require very little starting torque, but their torque requirement may increase with speed.

Figure 27 also shows a load torque characteristic — one that is typical of a fan type of load. In a fan (blower) type of load,the variation of torque with speed is such that $T \propto \omega^2$. **Here** one can see that it may be possible to run the motor to lower speeds within the range n_s to $(1 - \hat{s})n_s$. Further, since the load torque at zero speed is zero, the machine can start even at reduced voltages. This will not be possible with constant torque type of loads.

One may note that if the applied voltage is reduced, the voltage across the magnetising branch also comes down. This in turn means that the magnetizing current and hence flux level are reduced. Reduction in the flux level in the machine impairs torque production

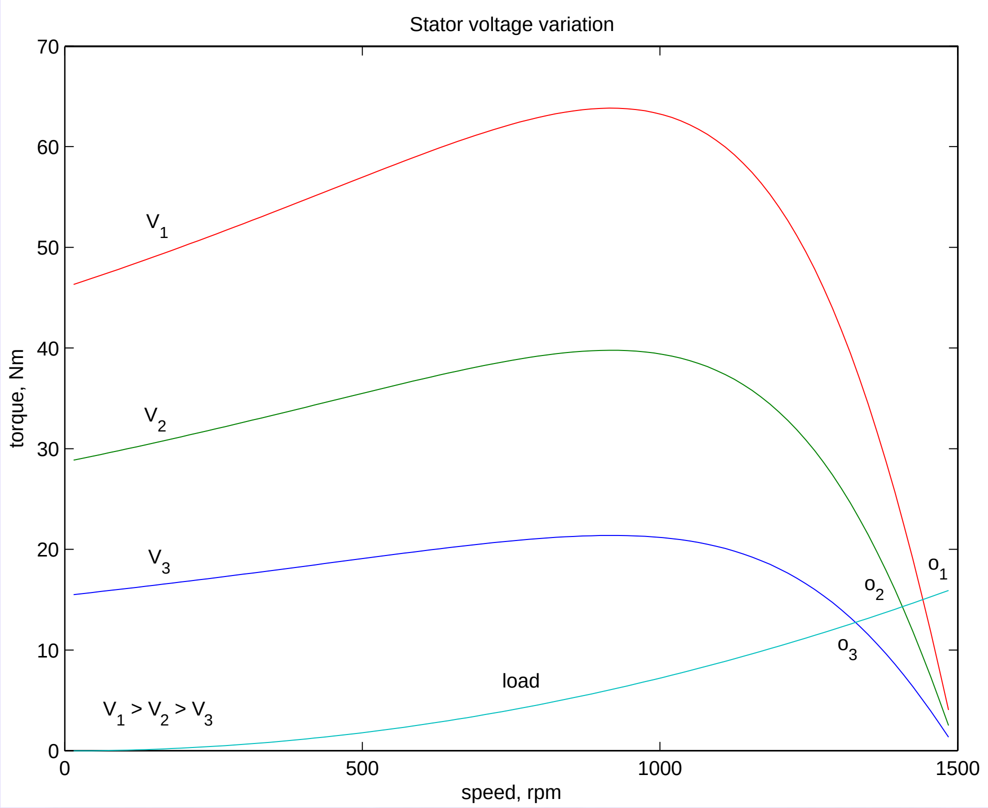


Figure 27: Speed-torque curves: voltage variation

(recall explantions on torque production), which is primarily the explanation for fig. 27. If, however, the machine is running under lightly loaded conditions, then operating under rated flux levels is not required. Under such conditions, reduction in magnetizing current improves the power factor of operation. Some amount of energy saving may also be achieved.

Voltage control may be achieved by adding series resistors (a lossy, inefficient proposition), or a series inductor / autotransformer (a bulky solution) or a more modern solution using semiconductor devices. A typical solid state circuit used for this purpose is the AC voltage controller or AC chopper. Another use of voltage control is in the so-called 'soft-start' of the machine. This is discussed in the section on starting methods.

8.2 Rotor resistance control

The reader may recall from eqn.17 the expression for the torque of the induction machine. Clearly, it is dependent on the rotor resistance. Further, eqn.19 shows that the maximum value is independent of the rotor resistance. The slip at maximum torque eqn.18 is dependent on the rotor resistance. Therefore, we may expect that if the rotor resistance is changed, the maximum torque point shifts to higher slip values, while retaining a constant torque. Figure 28 shows a family of torque-speed characteristic obtained by changing the rotor resistance.

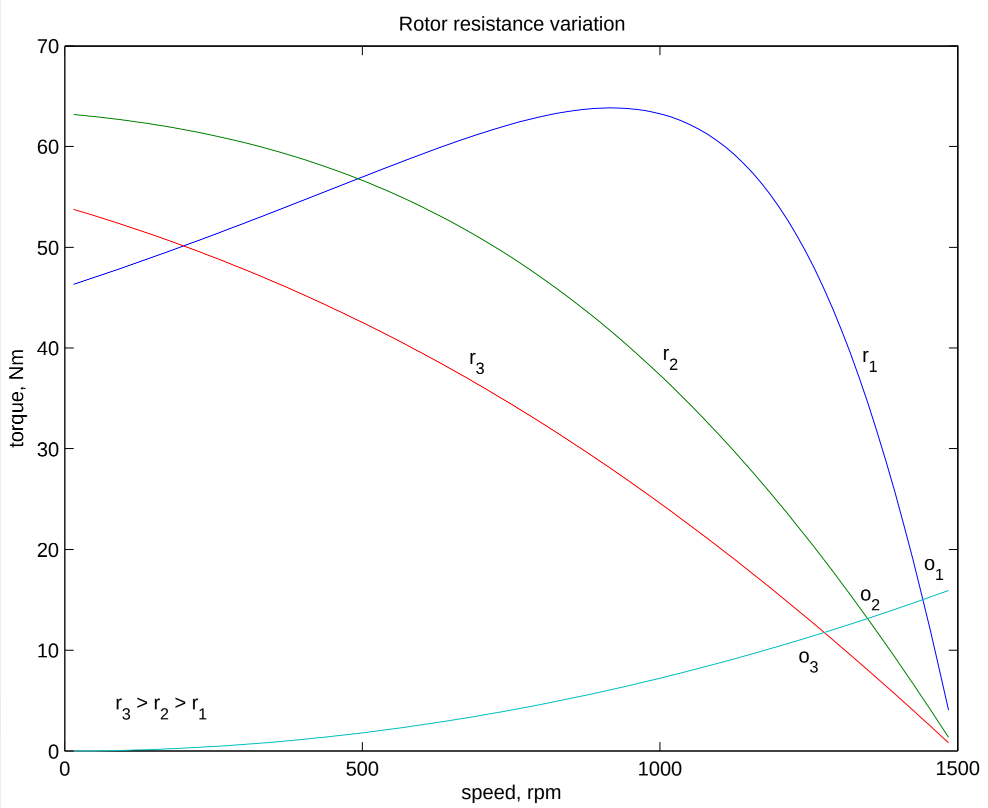


Figure 28: Speed-torque curves : rotor resistance variation

Note that while the maximum torque and synchronous speed remain constant, the slip at which maximum torque occurs increases with increase in rotor resistance, and so does the starting torque. whether the load is of constant torque type or fan-type, it is evident that the speed control range is more with this method. Further, rotor resistance control could also be used as a means of generating high starting torque.

For all its advantages, the scheme has two serious drawbacks. Firstly, in order to vary the rotor resistance, it is necessary to connect external variable resistors (winding resistance itself cannot be changed). This, therefore necessitates a slip-ring machine, since only in that case rotor terminals are available outside. For cage rotor machines, there are no rotor terminals. Secondly, the method is not very efficient since the additional resistance and operation at high slips entails dissipation.

The resistors connected to the slip-ring brushes should have good power dissipation capability. Water based rheostats may be used for this. A 'solid-state' alternative to a rheostat is a chopper controlled resistance where the duty ratio control of of the chopper presents a variable resistance load to the rotor of the induction machine.

8.3 Cascade control

The power drawn from the rotor terminals could be spent more usefully. Apart from using the heat generated in meaning full ways, the slip ring output could be connected to another induction machine. The stator of the second machine would carry slip frequency currents of the first machine which would generate some useful mechanical power. A still better option would be to mechanically couple the shafts of the two machines together. This sort of a connection is called cascade connection and it gives some measure of speed control as shown below.

Let the frequency of supply given to the first machine be f_1 , its number poles be p_1, and its slip of operation be s_1 . Let f_2, p_2 and s_2 be the corresponding quantities for the second machine. The frequency of currents flowing in the rotor of the first machine and hence in the stator of the second machine is $s_1 f_1$. Therefore $f_2 = s_1 f_1$. Since the machines are coupled at the shaft, the speed of the rotor is common for both. Hence, if n is the speed of the rotor in radians,

$$n = \frac{f_1}{p_1}(1 - s_1) = \pm\frac{s_1 f_1}{p_2}(1 - s_2). \tag{20}$$

Note that while giving the rotor output of the first machine to the stator of the second, the resultant stator mmf of the second machine may set up an air-gap flux which rotates in the same direction as that of the rotor, or opposes it. this results in values for speed as

$$n = \frac{f_1}{p_1 + p_2} \quad or \quad n = \frac{f_1}{p_1 - p_2} \quad (s_2\ negligible) \tag{21}$$

The latter expression is for the case where the second machine is connected in opposite phase sequence to the first. The cascade connected system can therefore run at two possible

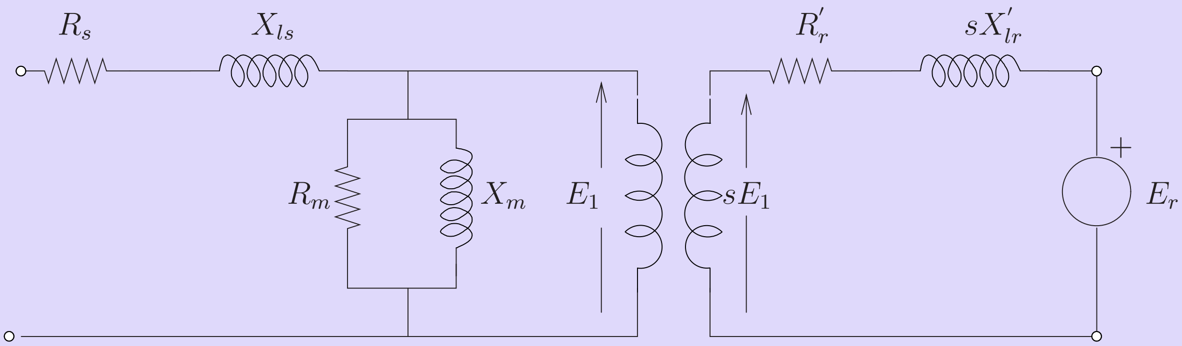


Figure 29: Generalized rotor control

speeds.

Speed control through rotor terminals can be considered in a much more general way. Consider the induction machine equivalent circuit of fig. 29, where the rotor circuit has been terminated with a voltage source E_r.

If the rotor terminals are shorted, it behaves like a normal induction machine. This is equivalent to saying that across the rotor terminals a voltage source of zero magnitude is connected. Different situations could then be considered if this voltage source E_r had a non-zero magnitude. Let the power consumed by that source be P_r. Then considering the rotor side circuit power dissipation per phase

$$sE_1 I_2' \cos\phi_2 = I_2' R_2' + P_r. \tag{22}$$

Clearly now, the value of s can be changed by the value of P_r. For $P_r = 0$, the machine is like a normal machine with a short circuited rotor. As P_r becomes positive, for all other circuit conditions remaining constant, s increases or in the other words, speed reduces. As P_r becomes negative,the right hand side of the equation and hence the slip decreases. The physical interpretation is that we now have an active source connected on the rotor side which is able to supply part of the rotor copper losses. When $P_r = -I_2'^2 R_2$ the entire copper loss is supplied by the external source. The RHS and hence the slip is zero. This corresponds to operation at synchronous speed. In general the circuitry connected to the rotor may not be a simple resistor or a machine but a power electronic circuit which can process this power requirement. This circuit may drive a machine or recover power back to the mains. Such circuits are called static kramer drives.

8.4 Pole changing schemes

Sometimes induction machines have a special stator winding capable of being externally connected to form two different number of pole numbers. Since the synchronous speed of the induction machine is given by $n_s = f_s/p$ (in rev./s) where p is the number of pole pairs, this would correspond to changing the synchronous speed. With the slip now corresponding to the new synchronous speed, the operating speed is changed. This method of speed control is a stepped variation and generally restricted to two steps.

If the changes in stator winding connections are made so that the air gap flux remains constant, then at any winding connection, the same maximum torque is achievable. Such winding arrangements are therefore referred to as constant-torque connections. If however such connection changes result in air gap flux changes that are inversely proportional to the synchronous speeds, then such connections are called constant-horsepower type.

The following figure serves to illustrate the basic principle. Consider a magnetic pole structure consisting of four pole faces A, B, C, D as shown in fig. 30.

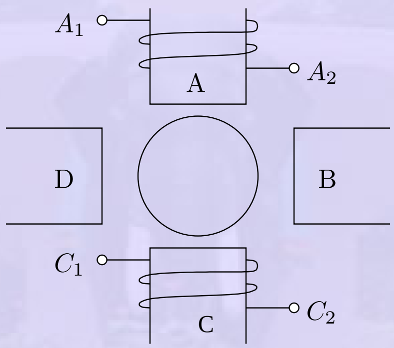


Figure 30: Pole arrangement

Coils are wound on A & C in the directions shown. The two coils on A & C may be connected in series in two different ways — A2 may be connected to C1 or C2. A1 with the other terminal at C then form the terminals of the overall combination. Thus two connections result as shown in fig. 31 (a) & (b).

Now, for a given direction of current flow at terminal A1, say into terminal A1, the flux directions within the poles are shown in the figures. In case (a), the flux lines are out of the pole A (seen from the rotor) for and into pole C, thus establishing a two-pole structure. In case (b) however, the flux lines are out of the poles in A & C. The flux lines will be then have to complete the circuit by flowing into the pole structures on the sides. If, when seen from the rotor, the pole emanating flux lines is considered as north pole and the pole into which

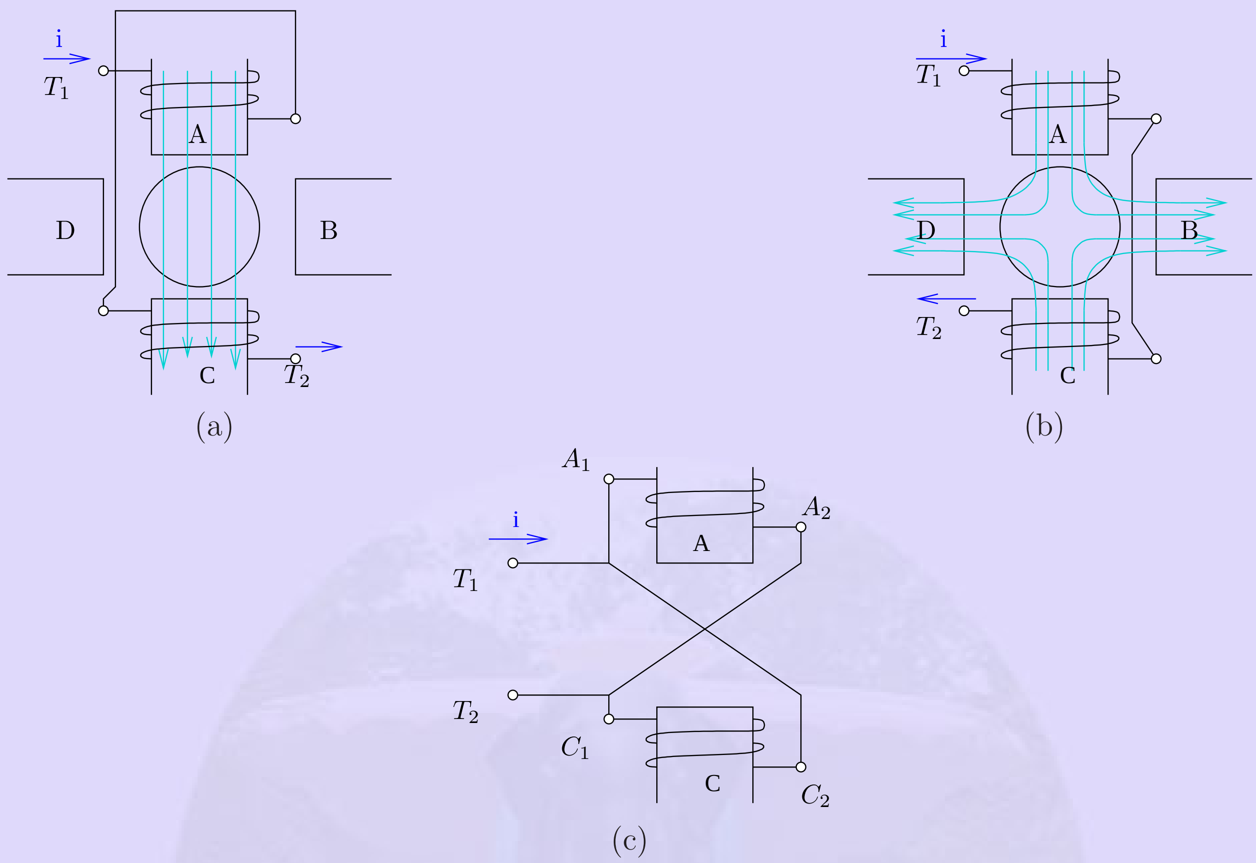


Figure 31: Pole Changing: Various connections

they enter is termed as south, then the pole configurations produced by these connections is a two-pole arrangement in fig. 31(a) and a four-pole arrangement in fig. 31(b).

Thus by changing the terminal connections we get either a two pole air-gap field or a four-pole field. In an induction machine this would correspond to a synchronous speed reduction in half from case (a) to case (b). Further note that irrespective of the connection, the applied voltage is balanced by the series addition of induced emfs in two coils. Therefore the air-gap flux in both cases is the same. Cases (a) and (b) therefore form a pair of constant torque connections.

Consider, on the other hand a connection as shown in the fig. 31(c). The terminals T1 and T2 are where the input excitation is given. Note that current direction in the coils now resembles that of case (b), and hence this would result in a four-pole structure. However, in fig. 31(c), there is only one coil induced emf to balance the applied voltage. Therefore flux in case (c) would therefore be halved compared to that of case (b) (or case (a), for that matter). Cases (a) and (c) therefore form a pair of constant horse-power connections.

It is important to note that in generating a different pole numbers, the current through one coil (out of two, coil C in this case) is reversed. In the case of a three phase machine, the following example serves to explain this. Let the machine have coils connected as shown $[C_1 - C_6]$ as shown in fig. 32.

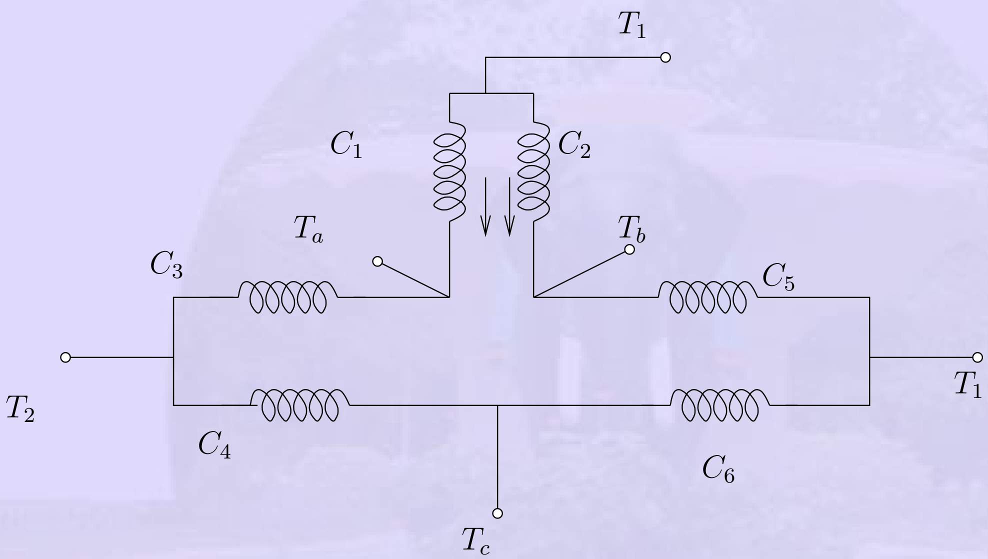


Figure 32: Pole change example: three phase

The current directions shown in C_1 & C_2 correspond to the case where T_1, T_2, T_3 are supplied with three phase excitation and T_a, T_b & T_c are shorted to each other (STAR point). The applied voltage must be balanced by induced emf in one coil only (C_1 & C_2 are parallel). If however the excitation is given to T_a, T_b& $T_c$with T_1, T_2, T_3 open, then current through one of the coils (C_1 & C_2) would reverse. Thus the effective number of poles would increase, thereby bringing down the speed. The other coils also face similar conditions.

8.5 Stator frequency control

The expression for the synchronous speed indicates that by changing the stator frequency also it can be changed. This can be achieved by using power electronic circuits called inverters which convert dc to ac of desired frequency. Depending on the type of control scheme of the inverter, the ac generated may be variable-frequency-fixed-amplitude or variable-frequency-variable-amplitude type. Power electronic control achieves smooth variation of voltage and frequency of the ac output. This when fed to the machine is capable of running at a controlled speed. However, consider the equation for the induced emf in the induction machine.

$$V = 4.44N\phi_m f \tag{23}$$

where N is the number of the turns per phase, ϕ_m is the peak flux in the air gap and f is the frequency. Note that in order to reduce the speed, frequency has to be reduced. If the frequency is reduced while the voltage is kept constant, thereby requiring the amplitude of induced emf to remain the same, flux has to increase. This is not advisable since the machine likely to enter deep saturation. If this is to be avoided, then flux level must be maintained constant which implies that voltage must be reduced along with frequency. The ratio is held constant in order to maintain the flux level for maximum torque capability.

Actually, it is the voltage across the magnetizing branch of the exact equivalent circuit that must be maintained constant, for it is that which determines the induced emf. Under conditions where the stator voltage drop is negligible compared the applied voltage, eqn. 23 is valid.

In this mode of operation, the voltage across the magnetizing inductance in the 'exact' equivalent circuit reduces in amplitude with reduction in frequency and so does the inductive reactance. This implies that the current through the inductance and the flux in the machine remains constant. The speed torque characteristics at any frequency may be estimated as before. There is one curve for every excitation frequency considered corresponding to every value of synchronous speed. The curves are shown below. It may be seen that the maximum torque remains constant.

This may be seen mathematically as follows. If E is the voltage across the magnetizing branch and f is the frequency of excitation, then $E = kf$, where k is the constant of proportionality. If $\omega = 2\pi f$, the developed torque is given by

$$T_{E/f} = \frac{k^2 f^2}{\left(\frac{R_r'}{s}\right)^2 + (\omega L_{lr}')^2} \frac{R_r'}{s\omega} \tag{24}$$

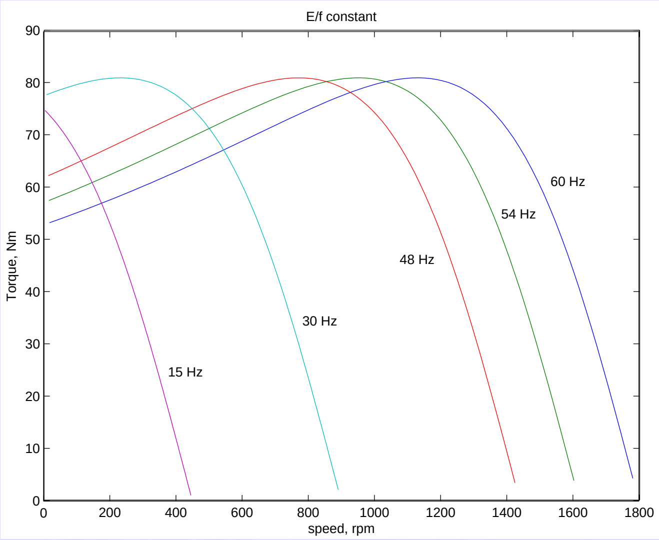


Figure 33: Torque-speed curves with E/f held constant

If this equation is differentiated with repsect to s and equated to zero to find the slip at maximum torque $\hat{s}$, we get $\hat{s} = \pm R_r'/(\omega L_{lr}')$. The maximum torque is obtained by substituting this value into eqn. 24.

$$\hat{T}_{E/f} = \frac{k^2}{8\pi^2 L_{lr}'} \tag{25}$$

Equation 25 shows that this maximum value is indepedent of the frequency. Further $\hat{s}\omega$ is independent of frequency. This means that the maximum torque always occurs at a speed lower than synchronous speed by a fixed difference, independent of frequency. The overall effect is an apparent shift of the torque-speed characteristic as shown in fig. 33.

Though this is the aim, E is an internal voltage which is not accessible. It is only the terminal voltage V which we have access to and can control. For a fixed V, E changes with operating slip (rotor branch impedance changes) and further due to the stator impedance drop. Thus if we approximate E/f as V/f, the resulting torque-speed characteristic shown in fig. 34 is far from desirable.

At low frequencies and hence low voltages the curves show a considerable reduction in peak torque. At low frequencies (and hence at low voltages) the drop across the stator impedance prevents sufficient voltage availability. Therefore, in order to maintain sufficient torque at low frequencies, a voltage more than proportional needs to be given at low speeds.

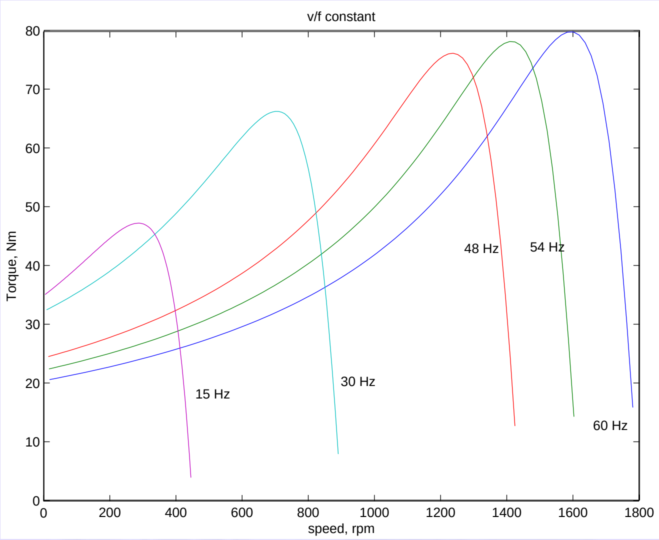


Figure 34: Torque-speed curves with V/f constant

Another component of compensation that needs to be given is due to operating slip. With these two components, therefore, the ratio of applied voltage to frequency is not a constant but is a curve such as that shown in fig. 35

With this kind of control, it is possible to get a good starting torque and steady state performance. However, under dynamic conditions, this control is insufficient. Advanced control techniques such as field- oriented control (vector control) or direct torque control (DTC) are necessary.

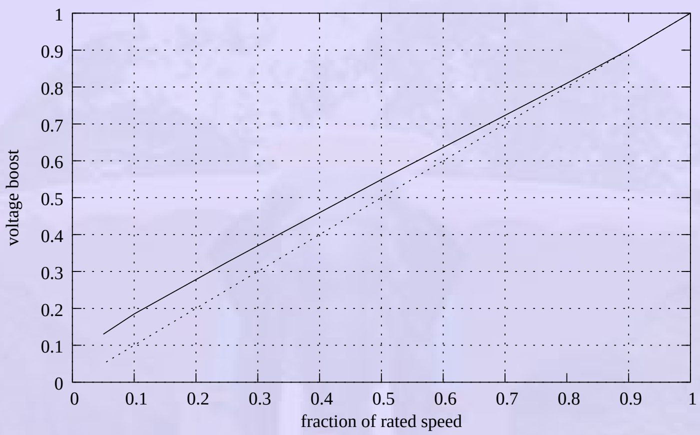


Figure 35: Voltage boost required for V/f control

9 Harmonics in Induction Machines

In attempting to understand the performance of an induction machine, we consider that the air-gap flux wave is purely sinusoidal. It is from that assumption the analysis of induced emf, sinusoidal currents, the expressions for generated torque etc. proceed. In practice, there are deviations from this idealistic picture.

9.1 Time Harmonics

The first non-ideality is the presence of harmonics in the input supply given to the three phase machine. The source may contain $3^{rd}, 5^{th}, 7^{th}\ldots$ harmonics. Note that due to the symmetry of the waveform ($f(t) = -f(t + T/2)$, where T is the period of the supply sine waveform, even ordered harmonics cannot exist. Let the R phase supply voltage be given by the expression

$$\begin{aligned} v_R &= V_{1m}\sin(\omega_1 t + \phi_1) + V_{3m}\sin(3\omega_1 t + \phi_3) \\ &\quad + V_{5m}\sin(5\omega_1 t + \phi_5) + V_{7m}\sin(7\omega_1 t + \phi_7) + - - - \end{aligned} \tag{25}$$

Being a balanced three phase supply, we know that the waveforms of v_Y and v_B are 120° and 240° shifted from v_R respectively. It is further well known that if a waveform is shifted by ϕ degrees, its harmonics are shifted by $n\phi$ degrees, where n is the order of the harmonic. Thus the expressions for v_Y and v_B would be

$$\begin{aligned} v_Y &= V_{1m}\sin(\omega_1 t + \phi_1 - \frac{2\pi}{3}) + V_{3m}\sin(3\omega_1 t + \phi_3 - 3.\frac{2\pi}{3}) \\ &\quad + V_{5m}\sin(5\omega_1 t + \phi_5 - 5.\frac{2\pi}{3}) + V_{7m}\sin(7\omega_1 t + \phi_7 - 7.\frac{2\pi}{3}) + - - - \end{aligned} \tag{26}$$

$$\begin{aligned} v_B &= V_{1m}\sin(\omega_1 t + \phi_1 - \frac{4\pi}{3}) + V_{3m}\sin(3\omega_1 t + \phi_3 - 3.\frac{4\pi}{3}) \\ &\quad V_{5m}\sin(5\omega_1 t + \phi_5 - 5.\frac{4\pi}{3}) + V_{7m}\sin(7\omega_1 t + \phi_7 - 7.\frac{4\pi}{3}) + - - - \end{aligned} \tag{27}$$

If we consider the third harmonic components of the three phase waveforms, and if $v_{x3}(t)$ is the third harmonic of phase x, we can see that

$$\begin{aligned} v_{R3} &= V_{3m}\sin(3\omega_1 t + \phi_3) \\ v_{Y3} &= V_{3m}\sin(3\omega_1 t + \phi_3) \\ v_{B3} &= V_{3m}\sin(3\omega_1 t + \phi_3) \end{aligned} \tag{28}$$

Therefore, all the three third harmonics are in phase. In a STAR connected system with isolated neutral, these voltages cannot cause any current flow since all three terminals are equal in potential. If the neutral point is connected to some point, then then current can flow through the neutral connection. Such a connection is however rare in induction machines. The machine is therefore an open circuit to third harmonics. In fact, one can see that any harmonic whose order is a multiple of three, i.e., the triplen harmonics, as they are called, will face an identical situation. Since the machine is an open circuit to triplen harmonics in the excitation voltage, these do not have effect on the machine.

Let us now consider the fifth harmonic. From the equations above, one can see that

$$\begin{aligned} v_{RS} &= V_{5m}\sin(5\omega_1 t + \phi_5) \\ v_{YS} &= V_{5m}\sin(5\omega_1 t + \phi_5 - 5.\frac{2\pi}{3}) \\ &= V_{5m}\sin(5\omega_1 t + \phi_5 - \frac{4\pi}{3}) \\ v_{BS} &= V_{5m}\sin(5\omega_1 t + \phi_5 - 5.\frac{4\pi}{3}) \\ &= V_{5m}\sin(5\omega_1 t + \phi_5 - \frac{2\pi}{3}) \end{aligned} \tag{29}$$

From eqns. 29 we see that the fifth harmonic of the excitation forms a negative sequence system — B phase lags R by 120° and Y phase lags R by 120°.

The MMF caused by a negative sequence excitation causes backward revolving flux pattern (compared to the direction of the fundamental). The torque which it generates will act as an opposing torque to that generated by the fundamental.

Looking at the seventh harmonic, we can see that

$$\begin{aligned} v_{R7} &= V_{7m}\sin(7\omega_1 t + \phi_7) \\ v_{Y7} &= V_{7m}\sin(7\omega_1 t + \phi_7 - 7.\frac{2\pi}{3}) \\ &= V_{7m}\sin(7\omega_1 t + \phi_7 - \frac{2\pi}{3}) \\ v_{B7} &= V_{7m}\sin(7\omega_1 t + \phi_7 - 7.\frac{4\pi}{3}) \\ &= V_{7m}\sin(7\omega_1 t + \phi_7 - \frac{4\pi}{3}) \end{aligned} \tag{30}$$

From eqns. 30, it is evident that the seventh harmonic components of the excitation form a positive sequence system. The torque produced by these currents will therefore be additive with respect to the fundamental component's torque.

The actual effect of these harmonics on the induction machine would depend on the reactance of the machine since at high frequencies, it is the reactance component that dominates the inductance. Excitation voltage waveforms with considerable harmonic content may result when induction machines are controlled through inverters. Apart from the effects on torque, these harmonics cause considerable heating in the machine and are hence a cause for concern. These harmonics are called *time* harmonics since they are generated by a source that varies non-sinusoidally in time.

9.2 Space Harmonics

Apart from this, there is another kind of harmonic generated in machines called *space* harmonic. To understand that this behaviour, it is necessary to consider MMF/flux production in the machine. It may not be out of place to recall once again that in all our foregoing analysis we have assumed that both air-gap mmf and flux are sinusoidally distributed in space.

Let us consider a single full-pitched coil excited by a sinusoidal voltage. The current flowing through it is sinusoidal and hence the time variation of the mmf produced by it is sinusoidal. But if we travel around the span of the coil, the MMF variation that we would encounter is square. It is the amplitude of this square wave that varies sinusoidally in time. The behaviour is depicted diagrammatically in fig. 36.

Let one more coil be connected in series to this, which is spatially displaced by the slot angle. This is shown in fig. 37. The same current passes through both and hence the mmf pattern generated by both would vary in tandem. However, they will be displaced by a slot angle as far as spatial distribution is concerned. The resulting situation is also shown in fig. 37 at a particular time instant. It can be seen that the resultant mmf waveform a non-sinusoidal function of the space angle θ. The harmonics are functions of the space angle. These are called space harmonics. Let us consider the effects of these.

Let the net flux waveform as a function of angle at an instant of time when unit current flows in the coils be described by $f(\theta)$. Clearly $f(\theta)$ is a periodic function of θ with a period equal to 2π. Therefore one may express this as a fourier series. If $f_A(\theta)$ is the distribution function for phase A,

$$f_A(\theta) = A_1 \sin(\theta + \phi_1) + A_3 \sin(3\theta + \phi_3) + A_5 \sin(5\theta + \phi_5) + - - - \quad (31)$$

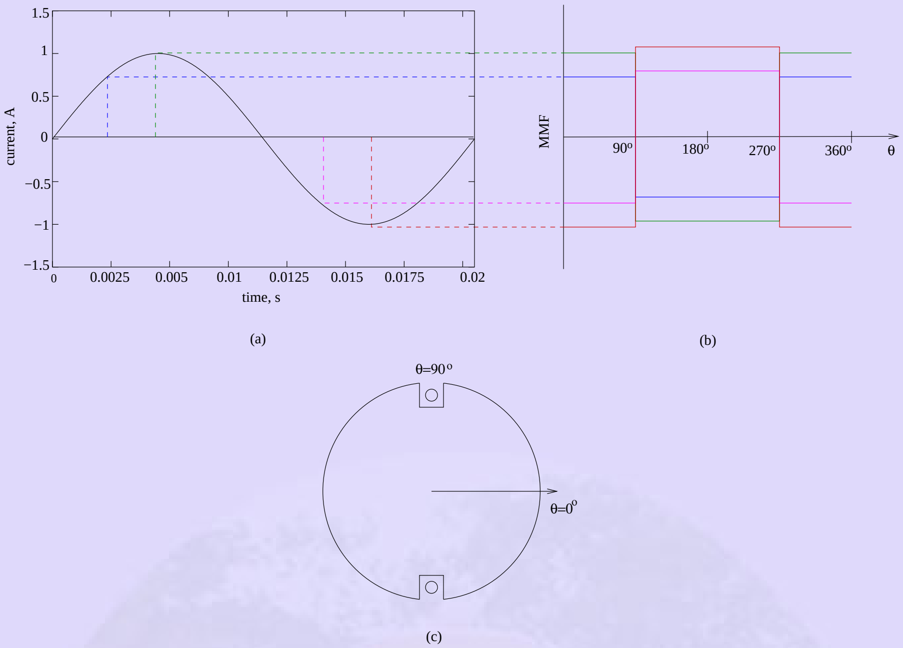


Figure 36: Coil MMF with sinusoidal excitation

The distribution functions for phases B & C will be displaced from that of A phase by 120° and 240° respectively and hence are given by

$$f_B(\theta) = A_1 \sin(\theta + \phi_1 - \frac{2\pi}{3}) + A_3 \sin(3\theta + \phi_3) + A_5 \sin(5\theta + \phi_5 - \frac{4\pi}{3}) + - - - \quad (32)$$

$$f_C(\theta) = A_1 \sin(\theta + \phi_1 - \frac{4\pi}{3}) + A_3 \sin(3\theta + \phi_3) + A_5 \sin(5\theta + \phi_5 - \frac{2\pi}{3}) + - - - \quad (33)$$

Note that we have written these at a given instant of time when unit current flows. We know that this current variation is sinusoidal in time. Considering the fifth harmonic, let the resultant fifth harmonic variation is given by,

$$\begin{aligned} f_5(t, \theta) = {} & A_{5m} \sin \omega t \sin(5\theta + \phi_5) + \\ & A_{5m} \sin(\omega t - \frac{2\pi}{3}) \sin(5\theta + \phi_5 - \frac{4\pi}{3}) + \\ & A_{5m} \sin(\omega t - \frac{4\pi}{3}) \sin(5\theta + \phi_5 - \frac{2\pi}{3}) + - - - \end{aligned} \quad (34)$$

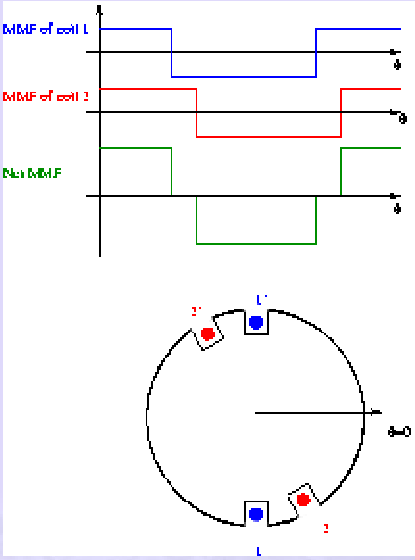


Figure 37: MMF in a distributed winding in two slots

Upon simplification, we get

$$f_5(t,\theta) = -\frac{3}{2}A_{5m}\cos(\omega t + 5\theta) \tag{35}$$

Consider the behaviour of this function. At t=0, the function a value of $-\frac{3}{2}A_{5m}$ at θ =0. Now let $\omega t = \frac{\pi}{3}$. At this instant, we find that the function reaches a value $-\frac{3}{2}A_{5m}$ at $\theta = -\frac{\pi}{3*5}$. In other words the function $f_5(t,\theta)$ has shifted by an angle which is a fifth of the value of ωt, in the negative direction. The fifth harmonic therefore rotates opposite to the direction of the fundamental at a speed which is one-fifth of the fundamental.

Similarly, if we consider the seventh harmonic, it can be shown that the resultant distribution is

$$f_7(t,\theta) = -\frac{3}{2}A_{7m}\cos(\omega t - 7\theta) \tag{36}$$

By similar arguments as above we conclude that the seventh space harmonic rotates in the same direction as that of the fundamental at one seventh the speed. In general , we may have harmonics of the order $h = 6n{\pm}1$, where n is an integer greater than or equal to 1. Harmonics orders generated by the addition operation move in the same direction as the fundamental and those generated by the subtraction operation move in the opposite direction. The speed of rotation is ω_1/h, where ω_1, is the synchronous speed of the fundamental.

The space harmonics, it may be noticed are a result of non-sinusoidal distribution of the coils in the machine and slotting. These have their effects on the speed torque current of the machine. An example speed-torque characteristic of an induction machine is compared with its ideal characteristic in fig. 38. The effect of 5^{th}, 7^{th}, 11^{th} and 13^{th} harmonics have been considered. It can be seen that these harmonics result in kinks in the speed-torque characteristic near starting region.

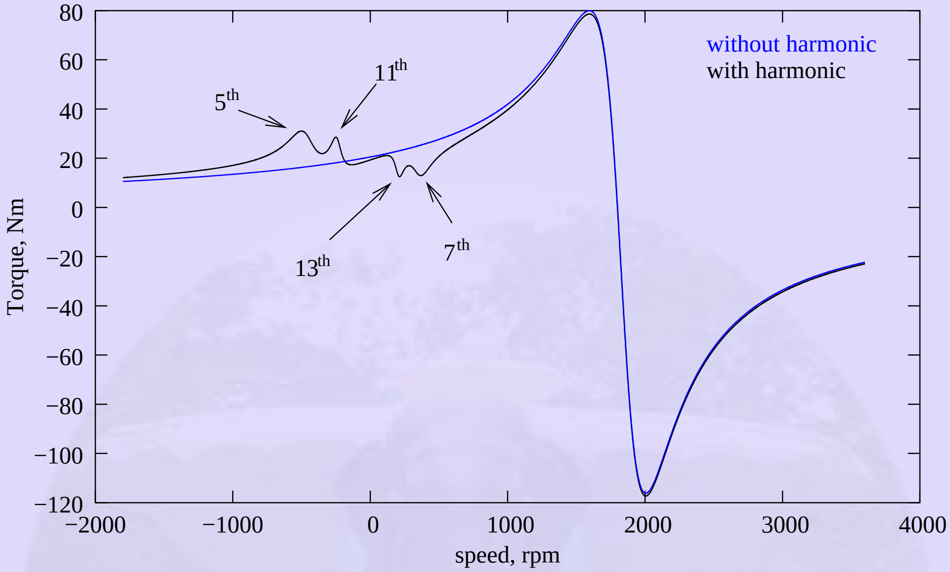


Figure 38: Ideal and actual speed-torque curves

To understand the effect of these kinks, consider fig. 39, which shows the same speed torque characteristic in the motoring region. A load characteristic is also shown, which intersects the machine characteristic at various points. Note that point 1 is stable and hence the machine would have a tendency to operate there, though the intended operating point might be point 5. This tendency is referred to as *crawling*. A momentary reduction in load torque conditions might accelerate the machine to print 2, which is unstable. The operating point would then settle down at 3. The intended operating point may be reached if favourable torque variations are there.

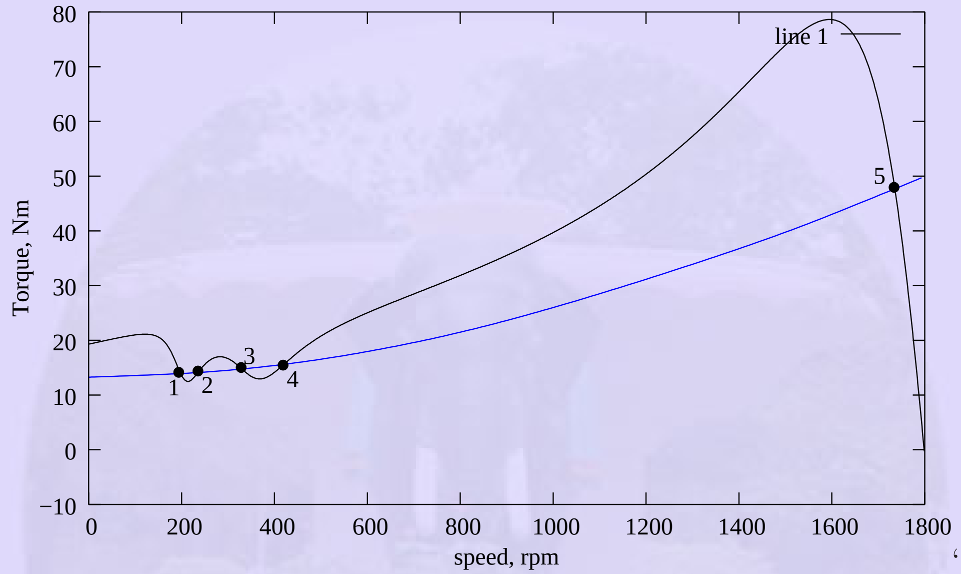


Figure 39: Effect of harmonics on loaded machine

Synchronous Machines

1 Introduction

With the development of the technology and the way in which human labour is getting minimized and the comforts increasing tremendously the use of electrical energy is ever increasing. Basically electric power is the main source of energy for carrying out many functions, as it is a clean and efficient energy source, which can be easily transmitted over long distances. With the availability of Transformer for changing the voltage levels to a very high value (of say 132kV to 400kV) the use of AC power has increased rapidly and the DC power is used only at remote places where AC power cannot be supplied through power lines or cables or for a few social purposes.

A synchronous generator is an electrical machine producing alternating emf (Electromotive force or voltage) of constant frequency. In our country the standard commercial frequency of AC supply is 50 Hz. In U.S.A. and a few other countries the frequency is 60 Hz. The AC voltages generated may be single phase or 3-phase depending on the power supplied. For low power applications single phase generators are preferable. The basic principles involved in the production of emf and the constructional details of the generators are discussed below.

1.1 Generation of emf

In 1831 Faraday discovered that an emf can be induced (or generated) due to relative motion between a magnetic field and a conductor of electricity. This voltage was termed as the induced emf since the emf is produced only due to motion between the conductor and the magnetic field without actual physical contact between them. The principle of electromagnetic induction is best understood by referring to Fig. 1. The magnetic field is produced by the two fixed poles one being the north pole from which the magnetic flux lines emerge and enter into the other pole known as the south pole. It was found that the magnitude of the voltage induced in the conductor is proportional to the rate of change of flux lines linking the conductor.
Mathematically it is given as

$$e = \frac{d\phi}{dt} \approx \frac{\phi}{t} \quad volts \tag{1}$$

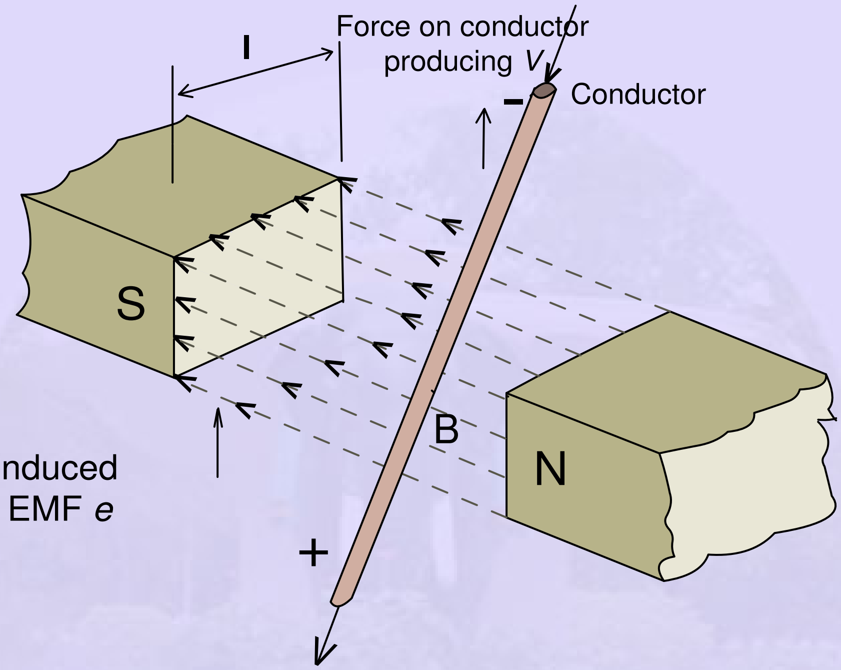


Figure 1: Conductor of length 'l' moving through a magnetic field B generate an EMF

where
ϕ = flux in Webers
t = time in seconds
e = average induced emf in volts.

The above Eqn. 1 holds good only when the magnetic circuit is physically the same at the end as at the beginning and also during the period of change of flux linkages as well. In practical rotating machinery, however the change of flux linking each individual conductor during rotation (of either the conductors or the poles) is not clearly defined or cannot be easily measured. It is therefore more convenient to express this rate of change of flux in terms of an average flux density (assumed constant) and the relative velocity between this field and a single conductor moving through it. For the conductor of active length l moving with a velocity of v in a magnetic field of flux density B, as shown in Fig. 1, the instantaneous induced emf is expressed as,

$$e = Blv \quad Volts \tag{2}$$

where
B= flux density in Tesla (Wb/m^2)
l = active conductor length (m)
v = relative linear velocity between the conductor and the field (m/s).
This animation would help to understand the concept for a coil rotating in a magnetic field.

Thus the instantaneous voltage e and the average value E of the induced emf are

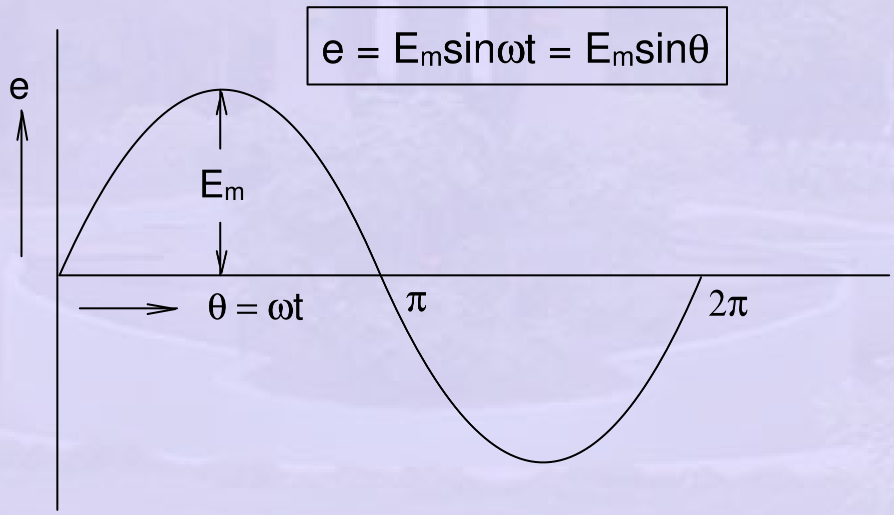
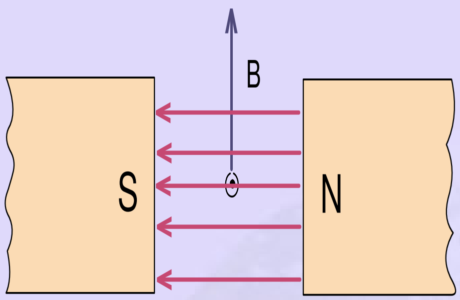


Figure 2: Sinusoidal voltage waveform

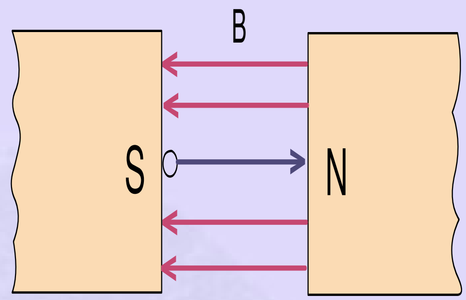
the same if the flux density B and the relative velocity v are both uniform and constant. In an alternator we want the instantaneous emf to be varying in a sinusoidal manner as shown in Fig. 2. Hence we should have a field system which will produce a sinusoidal distribution of flux density in the plane perpendicular to the plane of motion of the conductor.Then,

$$e = E_m \sin \omega t = E_m \sin \theta \tag{3}$$

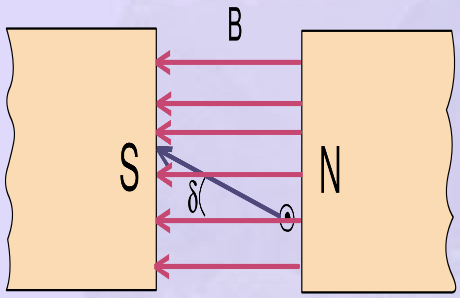
We have assumed that the conductor is moved in a direction perpendicular to the



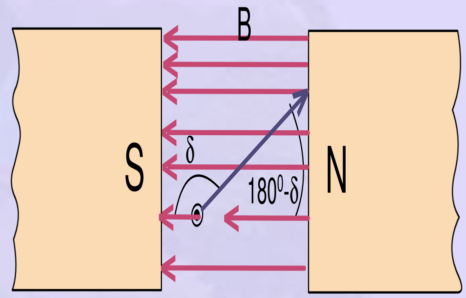
(a) Conductor moving at right angles to magnetic field



(b) Conductor moving parallel to magnetic field



(c) Conductor moving at any angle across magnetic field



(d) Conductor moving at any angle across magnetic field

Figure 3: Effect of change of flux linkages on induced EMF in a conductor

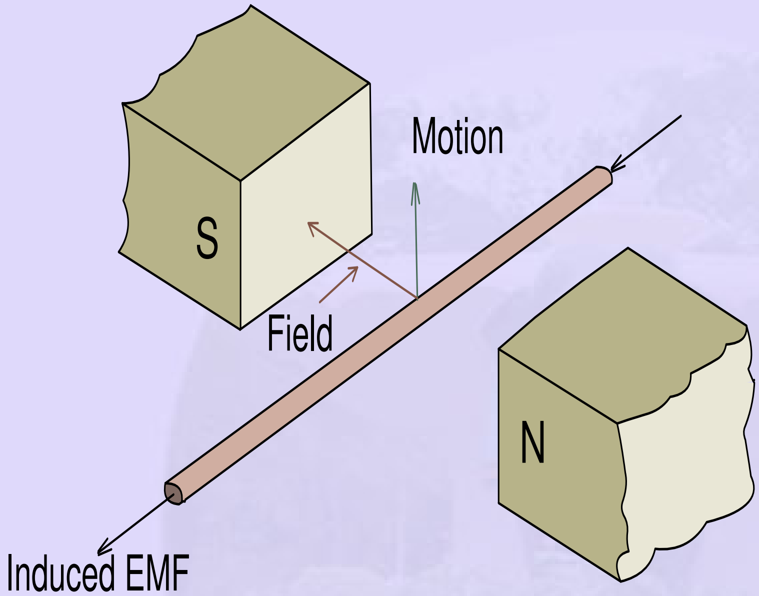
magnetic field as shown in Fig. 1. Eqn. 1 or Eqn. 2 are valid only for this mutually orthogonal condition for B and v. The other possible cases of motion of conductor with respect to B are shown in Fig.3 in addition to the mutually orthogonal condition of Fig. 1. When the conductor moves parallel to B, the induced emf will be zero because the rate of change of flux linkage is zero as the conductor does not link any new flux line/lines. To account for this condition of operation, Eqn. 2 must be multiplied by some factor, that takes into account the direction of motion of conductor so as to make 'e' zero for this condition of operation although B, l and v are finite quantities. Intuitively we may infer that this factor must be a sine function as it has a zero value at $0°$ and also at $180°$ and a maximum value at $90°$.

Indeed the emf equation for the general case of a conductor moving in any direction with respect to the field as shown in Fig. 3 is given by

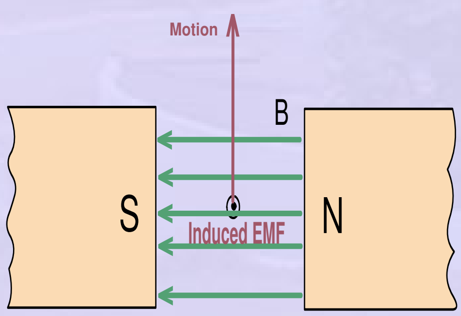
$$e = Blv \sin \delta \tag{4}$$

where δ is the angle formed between B and v always taking B as the reference. All other quantities are the same as in Eqn. 2.

1.2 Direction of induced e.m.f



(a) Right-hand rule



(b) orthogonal relations

Figure 4: Fleming's right hand rule for direction of induced EMF

The direction of the induced emf is given by Fleming's Right Hand Rule which states: If the thuMb, First finger and the seCond finger of the right hand are stretched out and held in three mutually perpendicular directions such that the First finger is held pointing in the direction of the magnetic field and the thuMb pointing in the direction of motion, then the seCond finger will be pointing in the direction of the induced emf such that the current flows in that direction. As shown in Fig. 4 the induced emf is in a direction so as to circulate current in the direction shown by the middle finger. Schematically we indicate the direction of the emf by a dot as shown in Fig. 5(a) to represent an emf so as to send current in a direction perpendicular to the plane of the paper and out of it. A cross will indicate the emf of opposite polarity, see Fig. 5(b). Although the Right Hand Rule assumes the magnetic filed to be stationary, we can also apply this rule to the case of a stationary conductor and moving magnetic field, by assuming that the conductor is moving in the opposite direction. For example, as shown in Fig. 4 the direction of the induced emf will be the same if the poles producing the field had been moved upwards.

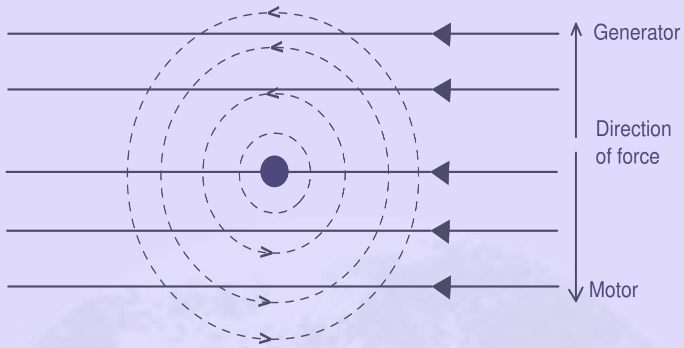
1.3 Electromagnetic Force

The motion of the conductor in a magnetic field can be imparted by the application of an external mechanical force to the conductor. In such a case the mechanical work done in moving the conductor is converted to an electric energy in agreement with the law of conservation of energy. The electric energy is not produced by the magnetic field since the field is not changed or destroyed in the process. The name electro mechanical energy conversion is given to the process of converting energy from mechanical form obtained from a prime mover, such as an IC engine, water/steam turbine etc, into electric energy.

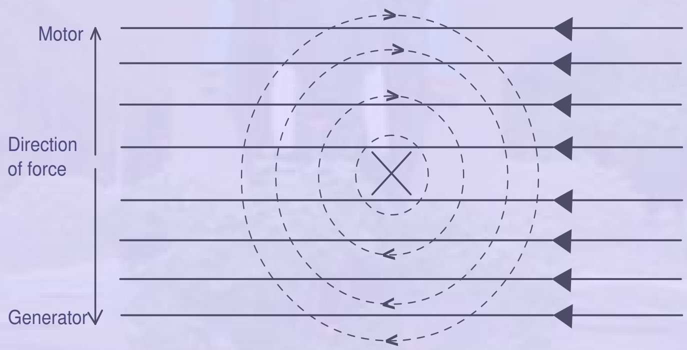
The emf induced in the conductor will circulate a current through it if a closed circuit is formed by an external connection. The direction of the current flowing in the conductor will be such as to oppose the cause of it as stated by Lenz's Law. A current carrying conductor located in a magnetic field will experience a force given by Biot-savart's law:

$$f = Bli \tag{5}$$

In other words, whenever a change in flux linkages occur, an emf is induced that tends to set up a current in such a direction as to produce a magnetic flux that opposes the cause of it. Thus if a current carrying conductor is placed in a magnetic field as shown in Fig. 5 the current tends to produce a magnetic field in the direction shown by the dotted circles.



(a)Current, coming out of the plane of paper



(b)Current entering the plane of paper

Figure 5: Force on a current carrying conductor in a magnetic field

The direction of the flux lines around the current carrying conductor can be easily determined by Corkscrew Rule - which states that the flux lines will be in the same direction as the rotation of a right threaded screw so as to advance in the direction of flow of current. As a result the magnetic field, for the case shown in Fig. 5(a), is strengthened at the top and weakened at the bottom of the conductor, thereby setting up a force to move the conductor downwards. For the case of a Generator, the conductor must be moved up against this counter force or the opposing force. Similarly the current is to be supplied to the conductor against the emf generated (known as the counter emf or back emf) in the conductor as it moves due to the motor action. Thus, the same machine can be operated as a generator or a motor, depending on whether we supply mechanical power or electrical power to it, respectively.

1.4 Elementary AC. Generators

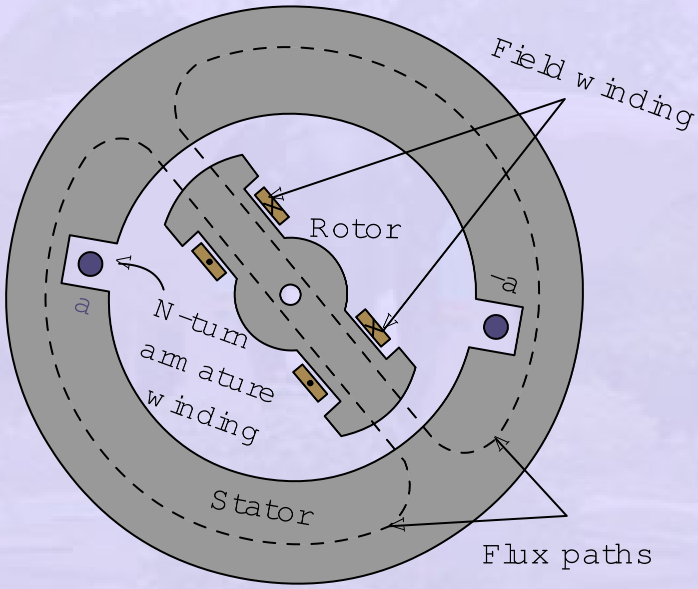


Figure 6: Elementary synchronous generator

The generators shown in Fig. 1 and Fig. 4 and discussed in the earlier sections are clearly impractical for a number of reasons. The main reason is that such generators require a prime mover that imparts linear or reciprocating motion to the conductor. Most of the commercial prime movers provide rotary motion in the commercial generators. The conductors of most commercial generators are rotated about a central axis of a shaft. The conductors are housed in slots cut in a cylindrical structure (made of magnetic material) known as the armature. The armature is supported at both ends by means of bearings attached to the shaft that goes through the center of the armature. The armature is rotated inside the field structure by providing a small gap between these two members. This gap is known as the air gap and is usually of the order of 1 to 1.5 cms. If the air gap is maintained constant throughout the spread of the pole arc, we have a fairly constant flux density under it in a plane perpendicular to the plane of the conductor's motion. i.e. in a radial direction with respect to the field and armature structure. Since the emf is also proportional to B, the flux density in the air gap of AC generators is arranged to be distributed as closely to a sine wave as possible by suitable shaping (chamfering as it is technically known) of the pole shoe. Since the relative motion between the conductors and the magnetic flux lines is responsible for the production of emf, it is immaterial whether the conductors are rotated or the magnetic flux producing poles are rotated. In most of the alternators it is the field that is rotated rather than the conductors. In an alternator the external connection to the load can be taken directly from the conductors since there is no need for any rectification as in a DC generator. In a DC generator the rectification of the emf is achieved through a mechanical rectifier—- the commutator and brush arrangement. Moreover the load current supplied by the alternator can be easily supplied from stationary coils without any difficulty as there will be no sparking and wear and tear of the brushes and slip rings. Where as the low values of D.C excitation current to the field coils can be easily sent through the slip rings and brush arrangement. Thus the usual arrangement in an elementary synchronous generator is as shown in Fig. 6. The conductors are housed in slots cut in the armature structure. Only a single coil of N turns, indicated in its cross-section by the two coil sides a and -a placed in diametrically opposite slots on the inner periphery of the stator (i.e. the armature, as it is a stationary member here) is shown in Fig. 6.

The conductors forming these coil sides are parallel to the shaft of the machine and are connected in series by end connections (not shown in the figure). The coils are actually formed by taking a continuous copper wire of suitable cross section round a diamond shaped bobbin. The completed coil is shown in Fig. 7. The copper wire is usually of fine linen covered, cotton covered or enamel covered so as to have sufficient insulation between the conductors of the same coil. The actual layout and interconnection of various coils so as to obtain the required voltage from the synchronous machine (alternator) is presented in the following section.

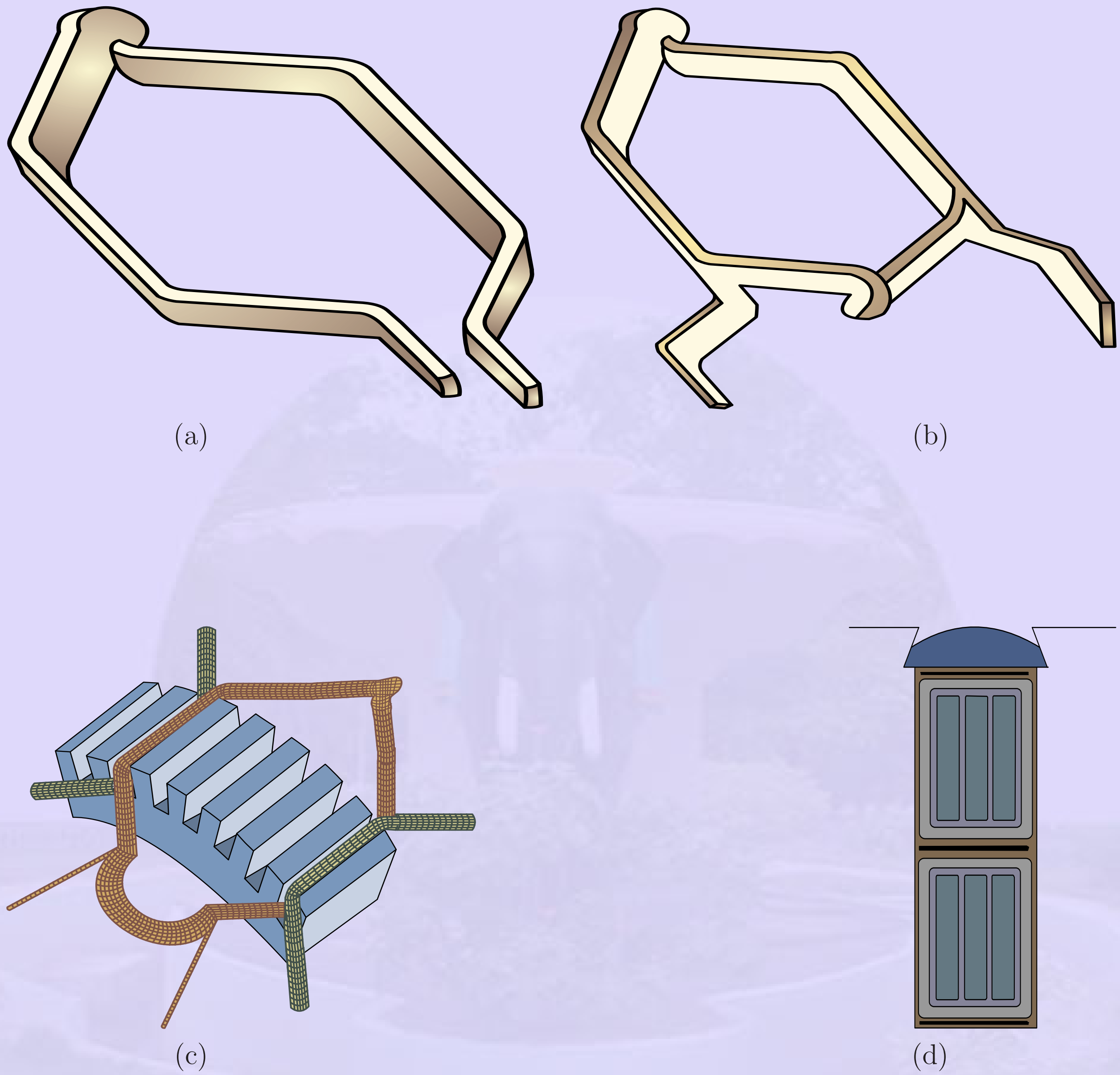


Figure 7: The completed coil

2 Synchronous Machine Armature Windings

2.1 Winding Types

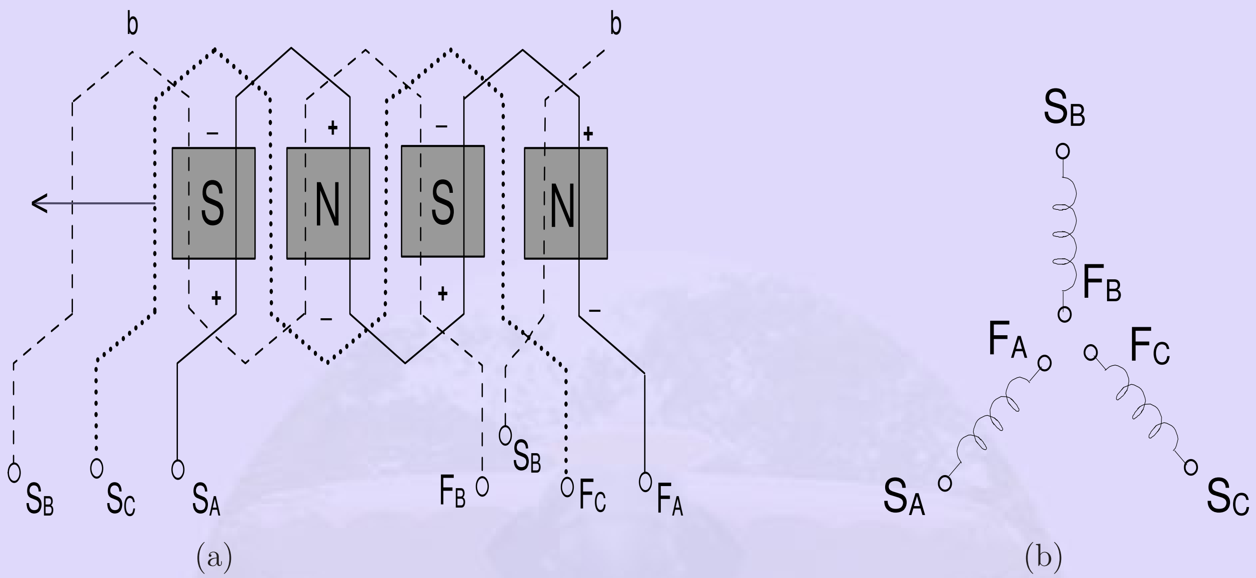


Figure 8: Concentrated three-phase,half-coil wave winding with one slot per phase(one coil side per slot and instantaneous polarity and phase relation of coils)

A three phase winding, in extremely simplified form, is shown in Fig. 8. The start and finish of all the coils in phase A are designated, respectively, as S_A and F_A. Phase A is shown as a solid line in the figure, phase B as a dashed line, and phase C as a dotted line. Note that each winding does not start and finish under the same pole. Further, note that the two coil sides of a given coil lie in identical magnetic conditions of opposite polarity. This implies that when seen from the coil terminals, the emfs produced in the two coil sides add up. If we assume that the poles on the rotor are moving to the left as shown, then the relative motion of the armature conductors is to the right. This implies that identical magnetic conditions will be seen by conductors of phase A, followed by phase C, followed by phase B. The induced emfs in phases A,C and B may be said to produce a phase sequence of ACBACBA.The time interval between two phases to achieve identical magnetic conditions would depend on the relative speed of motion, and on the spatial seperation of the phases. In Fig 8, the phases are so laid out that each phase is seperated from another by 120 electrical degrees (360° being defined by the distance to achieve identical magnetic conditions).

As the distance between two adjacent corresponding points on the poles is 180 electrical degrees, we can see that the distance between the coil side at the start of A and that at the start of C must be 120 electrical degrees. Thus, the leading pole tip of a unit north pole moving to the left in Fig. 8 will induce identical voltages in corresponding coil sides A, C, and B, respectively, 120 electrical degrees apart. Note that phase B lags phase A by 240 electrical degrees or leads phase A by 120 electrical degrees.Fig. 8(b) is a representation that is frequently used to depict the windings of the three phases and the phase relationship between them.

The winding depicted in Fig. 8 is an open winding since both ends of the windings have been brought out for suitable connections. It is a wave winding since it progresses from pole to pole. It is a concentrated winding because all the coils of one phase are concentrated in the same slot under one pole. It is a half-coil winding because there is only one-half of a coil (one coil side) in each slot. It is a full-pitch winding because the coil sides of one coil are 180° electrical degrees apart i.e., they lie under identical magnetic conditions, but of opposite polarity under adjacent poles.

Fig. 9, on the other hand shows the coils of a single phase,(A, in this case) distributed winding distributed over two slots under each pole.

2.1.1 Half-coil and whole-coil windings

Half-coil (also called single-layer) windings are sometimes used in small induction motor stators and in the rotors of small wound-rotor induction motors. A cross section of a half-coil, single-layer winding is shown in Fig. 9(c)(i). Like the dc dynamo armature windings, most commercial armatures for ac synchronous generators are of the full or whole-coil two-layer type, shown in cross section at the right in Fig. 9(c)(ii). The whole-coil, two-layer winding gets its name from the fact that there are two coil sides (one coil) per slot. Fig. 9(a) shows a single-layer, half-coil lap windings;Fig. 9(b) shows a double-layer, full-coil lap winding. A cross section of a single layer (half-coil) winding is shown in Fig. 9(c)(i).

2.1.2 Chorded or fractional -pitch windings

Whereas most single-layer windings are full-pitch windings, the two-layer, whole-coil windings are generally designed on an armature as a chorded or fractional-pitch windings. This common practice stems from the fact that the primary advantage of the whole-coil windings is that it permits the use of fractional-pitch coils in order to save copper. As will

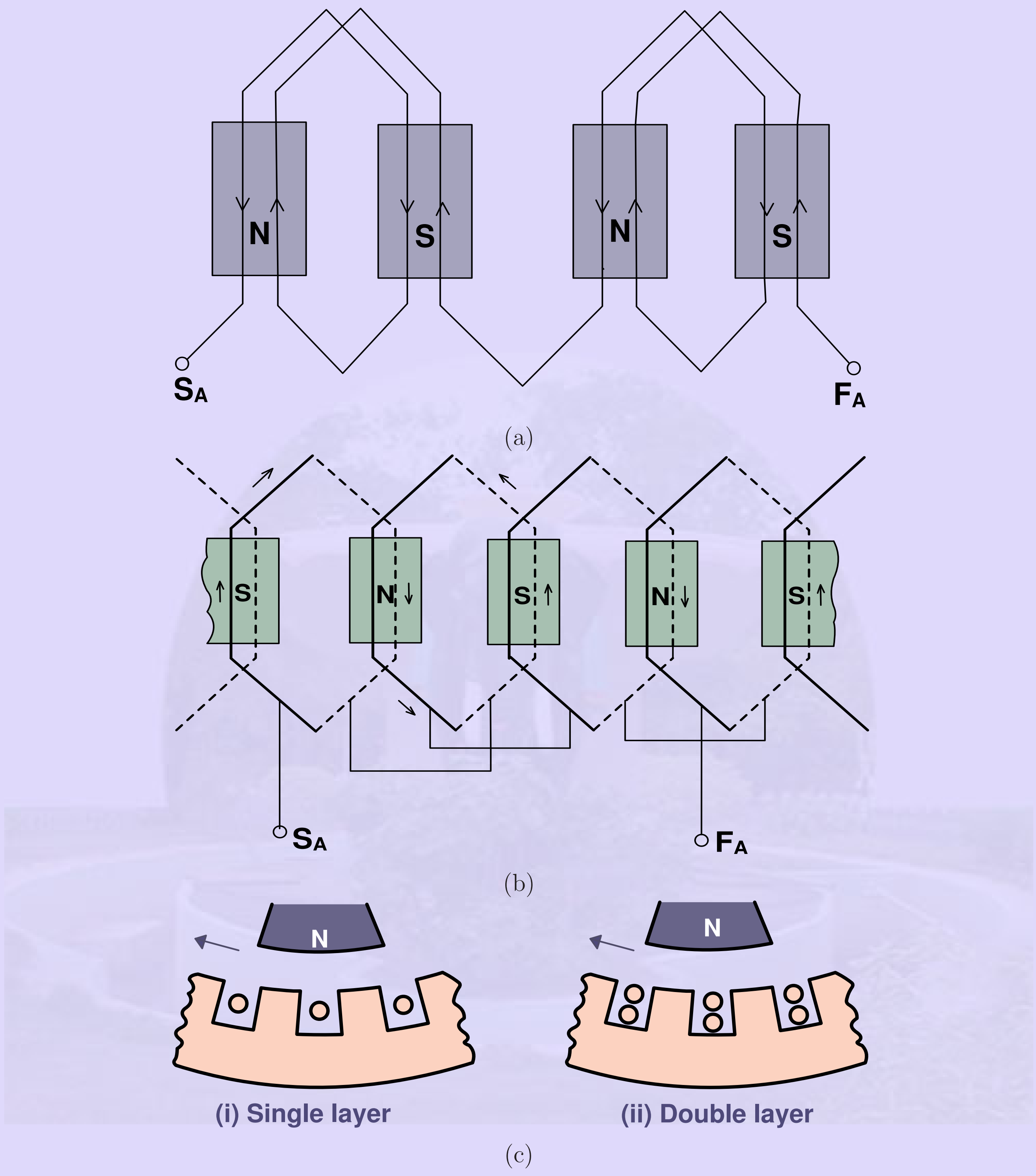


Figure 9: Distributed and concentrated half-coil and whole-coil windings

be shown later, fractional-pitch windings, when used in ac synchronous and asynchronous generator armatures, in addition to saving copper, (1) reduce the MMF harmonics produced by the armature winding and (2) reduce the EMF harmonics induced in the windings, without reducing the magnitude of the fundamental EMF wave to a great extent. For the three reasons cited, fractional-pitch windings are almost universally used in ac synchronous generator armatures.

2.1.3 EMF of Fractional Pitch Windings

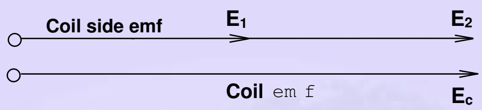


Figure 10: Full pitch coil

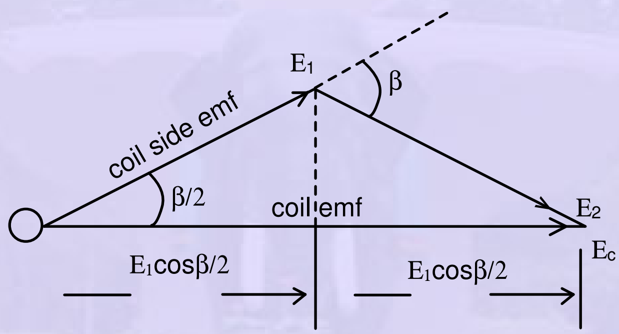


Figure 11: Fractional-pitch coil - Coil EMF in terms of coil side EMFs for fractional-pitch coil

In the case of an ac generator using a full-pitch coil, such as that shown in Fig. 8, the two coil sides span a distance exactly equal to the pole pitch of 180 electrical degrees. As a result, the EMFs induced in a full-pitch coil are such that the coil side EMFs are in phase, as shown in Fig. 10. The total coil voltage E_c is $2E_1$, if E_1 is the emf induced in a coil-side.

In the case of the two-layer winding shown in Fig. 9(b), note that the coil span of single coil is less than the pole span of 180 electrical degrees. The EMF induced in each coil side is not in phase, and the resultant coil voltage E_c would be less than the arithmetic sum of the EMF of each coil side, or less than $2E_1$. It is obvious that $2E_1$ must be multiplied by a factor,k_p, that is less than unity, to get the proper value for coil voltage E_c (or $E_c = 2E_1k_p$). The pitch factor k_p is given by

$$k_p = \frac{E_c}{2E_1} = \frac{phasor\ sum\ of\ the\ EMF\ of\ the\ two\ coil\ sides}{arithmetic\ sum\ of\ the\ EMF's\ of\ the\ two\ coil\ sides} \quad (6)$$

The pitch factor may be quantified in terms of angles as follows. If we assume that the induced EMFs of two coils, E_1 and E_2, are out of phase with respect to each other by some angle β as shown in Fig. 11, then the angle between E_1 and the resultant coil voltage E_c is $\frac{\beta}{2}$ .The resultant coil voltage E_c is from Eqn. 6 and Fig. 11.

$$E_c = 2E_1 \cos\frac{\beta}{2} = 2E_1k_p. \quad (7)$$

and, therefore,

$$k_p = \cos\frac{\beta}{2} \quad (8)$$

The angle β is 180^0 minus the number of electrical degrees spanned by the coil, for a short-pitched coil. For a full pitched coil, therefore, $k_p = 1$ as $\beta = 0$.

Since β is the supplementary of the coil span, the pitch factor k_p may also be expressed as

$$k_p = \sin\frac{p^0}{2} \quad (9)$$

where p^0 is the span of the coil in electrical degrees.

It is sometimes convenient to speak of an armature coil span as having a fractional pitch expressed as a fraction e.g., a $\frac{5}{6}$ pitch, or an $\frac{11}{12}$ pitch, etc. This fraction is infact the ratio of the number of slots spanned by a coil to the number of slots in a full pitch. In such a case, the electrical degrees spanned, p^0 is $\frac{5}{6} * 180^0$, or 150^0; or $\frac{11}{12} * 180^0$ or 165^0; etc. The pitch factor k_p is still computed as in Eqn. 9. Over pitched coils are not normaly used in practice as there is an increased requirement of copper wire without any additional advantage.

2.1.4 Relation between Electrical and Mechanical Degrees of Rotation

As stated earlier there are 180 electrical degrees between the centres of two adjacent north and south poles. Since 360 electrical degrees represents a full cycle of sinusoidal EMF, we are interested in determining how many sinusoidal cycles are generated in one complete mechanical rotation, i.e., 360 mechanical degrees for a machine having P poles. The number of electrical degrees as a function of degrees of mechanical rotation is

$$\alpha = \frac{P\theta}{2} = p\theta. \tag{10}$$

where P is the number of poles (always an even integer), p is the number of pole-pairs, and θ is the number of mechanical degrees of rotation.

Thus, a two-pole machine generates one cycle of sinusoid; a four-pole machine generates two cycles and so on, in one full revolution of the armature.

2.1.5 Distributed windings and distribution (or Belt) factor

The windings shown in Fig. 8 and Fig. 9(b) are called concentrated windings because all the coil sides of a given phase are concentrated in a single slot under a given pole. For Fig. 8., in determining the induced ac voltage per phase, it would be necessary only to multiply the voltage induced in any given coil by the number of series-connected coils in each phase. This is true for the winding shown in Fig. 8 because the conductors of each coil, respectively, lie in the same position with respect to the N and S poles as other series coils in the same phase. Since these individual coil voltages are induced in phase with each other, they may be added arithmetically. In otherwords, the induced emf per phase is the product of the emf in one coil and the number of series connected coils in that phase.

Concentrated windings in which all conductors of a given phase per pole are concentrated in a single slot, are not commercially used because they have the following disadvantages,

1. They fail to use the entire inner periphery of the stator iron efficiently.
2. They make it necessary to use extremely deep slots where the windings are concentrated. This causes an increase in the mmf required to setup the airgap flux.
3. The effect of the second disadvantage is to also increase the armature leakage flux and the armature reactance.
4. They result in low copper-to-iron ratios by not using the armature iron completely.
5. They fail to reduce harmonics as effectively as distributed windings.

For the five reasons just given, it is more advantageous to distribute the armature winding, using more slots and a uniform spacing between slots, than to concentrate the windings in a few deep slots.

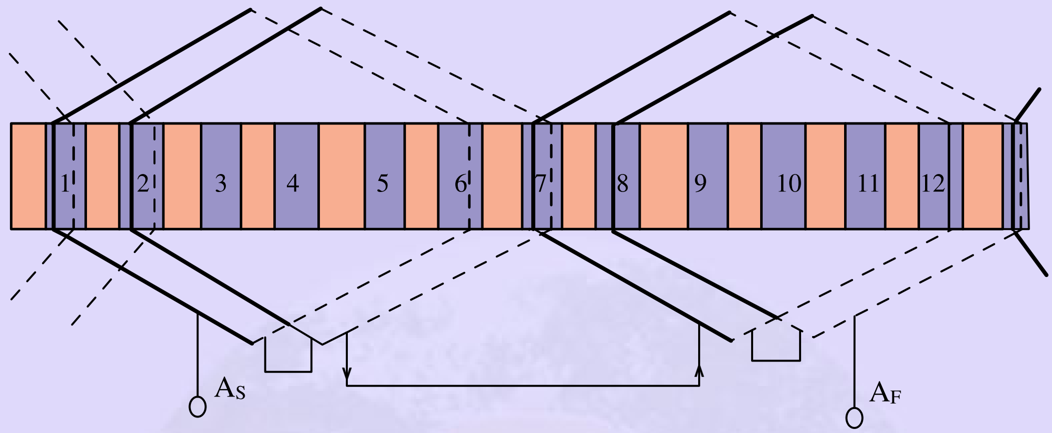


Figure 12: Lap winding

When the slots are distributed around the armature uniformly, the winding that is inserted is called a distributed winding. A distributed lap winding is shown in Fig. 12. Note that two coils in phase belt A are displaced by one slot angle (the angular displacement between two successive slots) with respect to each other. The induced voltages of each of these coils will be displaced by the same degree to which the slots have been distributed, and the total voltage induced in any phase will be the phasor sum of the individual coil voltages. For an armature winding having four coils distributed over say, 2/3 rd of a pole-pitch, in four slots, the four individual coil side voltages are represented by phasors in Fig. 13 as displaced by some angle α, the number of electrical degrees between adjacent slots, known as slot angle. It is 30^0 for the case of 4 slots per phase belt. Voltages E_{c1}, E_{c2}, etc., are the individual coil voltages, and n is the number of coils in a given phase belt, in general.

For a machine using n slots for a phase belt, the belt or distribution factor k_d by which the arithmetic sum of the individual coil voltages must be multiplied in order to yield the phasor sum is determined by the following method,

$$k_d = \frac{E_\phi}{nE_c} \tag{11}$$

where all terms are previously defined

As in the case of Eqn. 12., the computation of k_d in terms of voltages (either theo-

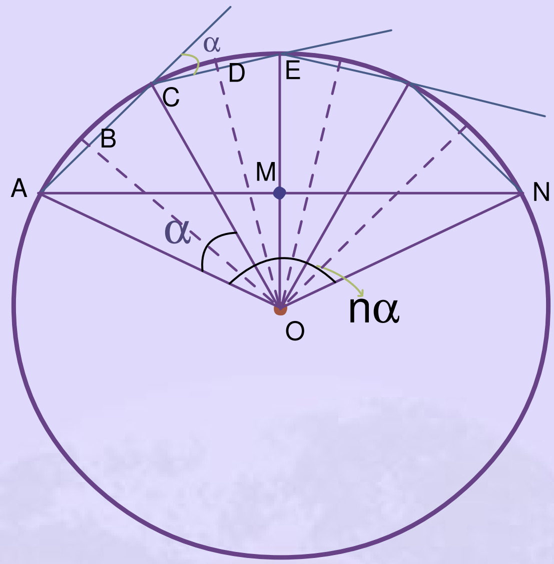


Figure 13: Determination of distribution factor

retical or actual) is impractical. The construction of Fig. 13 in which perpendiculars have been drawn to the center of each of the individual coil voltage phasors to a common center of radius 'r' (using dashed lines) serves to indicate that $\alpha/2$ is the angle BOA. Coil side voltage AB equals $OA \sin\alpha/2$, and coil voltage represented by chord AC equals $2OA\sin\alpha/2$. For n coils in series per phase, chord AN, is also $2OA\sin n\alpha/2$, and the distribution or belt factor k_d is

$$k_d = \frac{E_\phi}{nE_c} = \frac{2OA\sin(n\alpha/2)}{n.2OA(\sin(\alpha/2))} = \frac{AN}{nE_c} = \frac{2AM}{n*AC}$$

$$= \frac{2AM}{n*2AB} = \frac{2AM}{n*2OA\sin\frac{\alpha}{2}} = \frac{2OA\sin(n\alpha/2)}{n.2OA(\sin(\alpha/2))} = \frac{\sin n\alpha/2}{n\sin\alpha/2}$$

where
n is the number of slots per pole per phase (s.p.p)
α is the number of electrical degrees between adjacent slots i.e. slot angle

It should be noted from Eqn. 12. that the distribution factor k_d for any fixed or given number of phases is a sole function of the number of distributed slots under a given pole. As the distribution of coils (slots/pole) increases, the distribution factor k_d decreases. It is not affected by the type of winding, lap or wave, or by the number of turns per coil, etc.

2.2 Generated EMF in a Synchronous Generator

It is now possible to derive the computed or expected EMF per phase generated in a synchronous generator. Let us assume that this generator has an armature winding consisting of a total number of full pitched concentrated coils C, each coil having a given number of turns N_c. Then the total number of turns in any given phase of an m-phase generator armature is

$$N_p = \frac{CN_c}{m} \tag{12}$$

But Faraday's law Sec. **??** states that the average voltage induced in a single turn of two coil sides is

$$E_{av} = \frac{\phi}{t} \tag{13}$$

The voltage induced in one conductor is $2\phi/(1/s) = 2\phi s$, where s=speed of rotation in r.p.s, for a 2 pole generator. Furthermore, when a coil consisting of N_c turns rotates in a uniform magnetic field, at a uniform speed, the average voltage induced in an armature coil is

$$E_{\frac{av}{coil}} = 4\phi N_c s \quad Volts \tag{14}$$

where ϕ is the number of lines of flux (in Webers) per pole, N_c is number of turns per coil, s is the relative speed in revolutions/second (rps) between the coil of N_c turns and the magnetic field ϕ.

A speed s of 1 rps will produce a frequency f of 1 Hz. Since f is directly proportional and equivalent to s, (for a 2-pole generator) replacing the latter in Eqn. 14, for all the series turns in any phase,

$$E_{\frac{av}{phase}} = 4\phi N_p f \quad Volts \tag{15}$$

However, in the preceding section we discovered that the voltage per phase is made more completely sinusoidal by intentional distribution of the armature winding. The effective rms value of a sinusoidal ac voltage is 1.11 times the average value. The effective ac voltage per phase is

$$E_{eff} = 4.44\phi N_p f \quad Volts \tag{16}$$

But Eqn. 16 is still not representative of the effective value of the phase voltage generated in an armature in which fractional-pitch coils and a distributed winding are employed. Taking the pitch factor k_p and the distribution factor k_d into account, we may now write the equation for the effective value of the voltage generated in each phase of an AC synchronous generator as

$$E_{gp} = 4.44\phi N_p f k_p k_d \quad Volts \tag{17}$$

2.3 Frequency of an A.C. Synchronous Generator

Commercial ac synchronous generators have many poles and may rotate at various speeds, either as alternators or as synchronous or induction motors.Eqn. 13 was derived for a two-pole device in which the generated EMF in the stationary armature winding changes direction every half-revolution of the two-pole rotor. One complete revolution will produce one complete positive and negative pulse each cycle. The frequency in cycles per second (Hz) will, as stated previously, depend directly on the speed or number of revolutions per second (rpm/60) of the rotating field.

If the ac synchronous generator has multiple poles (having, say, two, four, six, or eight poles...), then for a speed of one revolution per second (1 rpm/60), the frequency per revolution will be one, two, three, or four ..., cycles per revolution, respectively. The frequency per revolution, is therefore, equal to the number of pairs of poles. Since the frequency depends directly on the speed (rpm/60) and also on the number of pairs of poles (P/2), we may combine these into a single equation in which

$$f = \frac{P}{2} * \frac{rpm}{60} = \frac{PN}{120} = \frac{P}{120} * \frac{\omega_m * 60}{2\pi} = \frac{P}{2} * \frac{\omega_m}{2\pi} = \frac{\omega_e}{2\pi} \tag{18}$$

where
P is the number of poles
N is the speed in rpm (rev/min)
f is. the frequency in hertz
ω_m is the speed in radians per second (rad/s)
ω_e is the speed electrical radians per second.

2.4 Constructional Details of Rotor

As stated earlier the field windings are provided in the rotor or the rotating member of the synchronous machine. Basically there are two general classifications for large 3 phase synchronous generators ——cylindrical rotor and salient-pole rotor - .

The cylindrical-rotor construction is peculiar to synchronous generators driven by steam turbines and which are also known as turbo alternators or turbine generators. Steam turbines operate at relatively high speeds, 1500 and 3000 rpm being common for 50 Hz, accounting for the cylindrical-rotor construction, which because of its compactness readily withstands the centrifugal forces developed in the large sizes at those speeds. In addition, the smoothness of the rotor contour makes for reduced windage losses and for quiet operation.

Salient-pole rotors are used in low-speed synchronous generators such as those driven by water wheels. They are also used in synchronous motors. Because of their low speeds salient-pole generators require a large number of poles as, for example, 60 poles for a 100-rpm 50 Hz generator.

Fig. 14 illustrates two and four pole cylindrical rotors along with a developed view of the field winding for one pair of poles. One pole and its associated field coil of a salient-pole rotor is shown in fig. 14.The stator slots in which the armature winding is embedded are not shown for reasons of simplicity. The approximate path taken by the field flux, not including leakage flux, is indicated by the dashed lines in Fig. 14. The field coils in Fig. 14 are represented by filaments but actually (except for the insulation between turns and between the coil sides and the slot) practically fill the slot more nearly in keeping with fig. 15.

The stepped curve in fig. 15. represents the waveform of the mmf produced by the distributed field winding if the slots are assumed to be completely filled by the copper in the coil sides instead of containing current filaments. The sinusoid indicated by the dashed line in fig. 15 represents approximately the fundamental component of the mmf wave.

The air gap in cylindrical-rotor machines is practically of uniform length except for the slots in the rotor and in the stator, and when the effect of the slots and the tangential component of H, which is quite small for the low ratio of air-gap length to the arc subtended by one pole in conventional machines, are neglected, the stepped mmf wave in fig. 15 produces a flux-density space wave in which the corners of the steps are rounded due to fringing. The flux density wave form is therefore more nearly sinusoidal than the mmf waveform when the effect of the slots is neglected. However, saturation of the iron in the region of maximum mmf tends to flatten the top of the flux-density wave.

2.5 Excitation Systems for Synchronous Machines

A number of arrangements for supplying direct current to the fields of synchronous machines have come into use. Adjustments in the field current may be automatic or manual depending upon the complexity and the requirements of the power system to which the generator is connected.

Excitation systems are usually 125 V up to ratings of 50kW with higher voltages for the larger ratings. The usual source of power is a direct-connected exciter, motor- generator

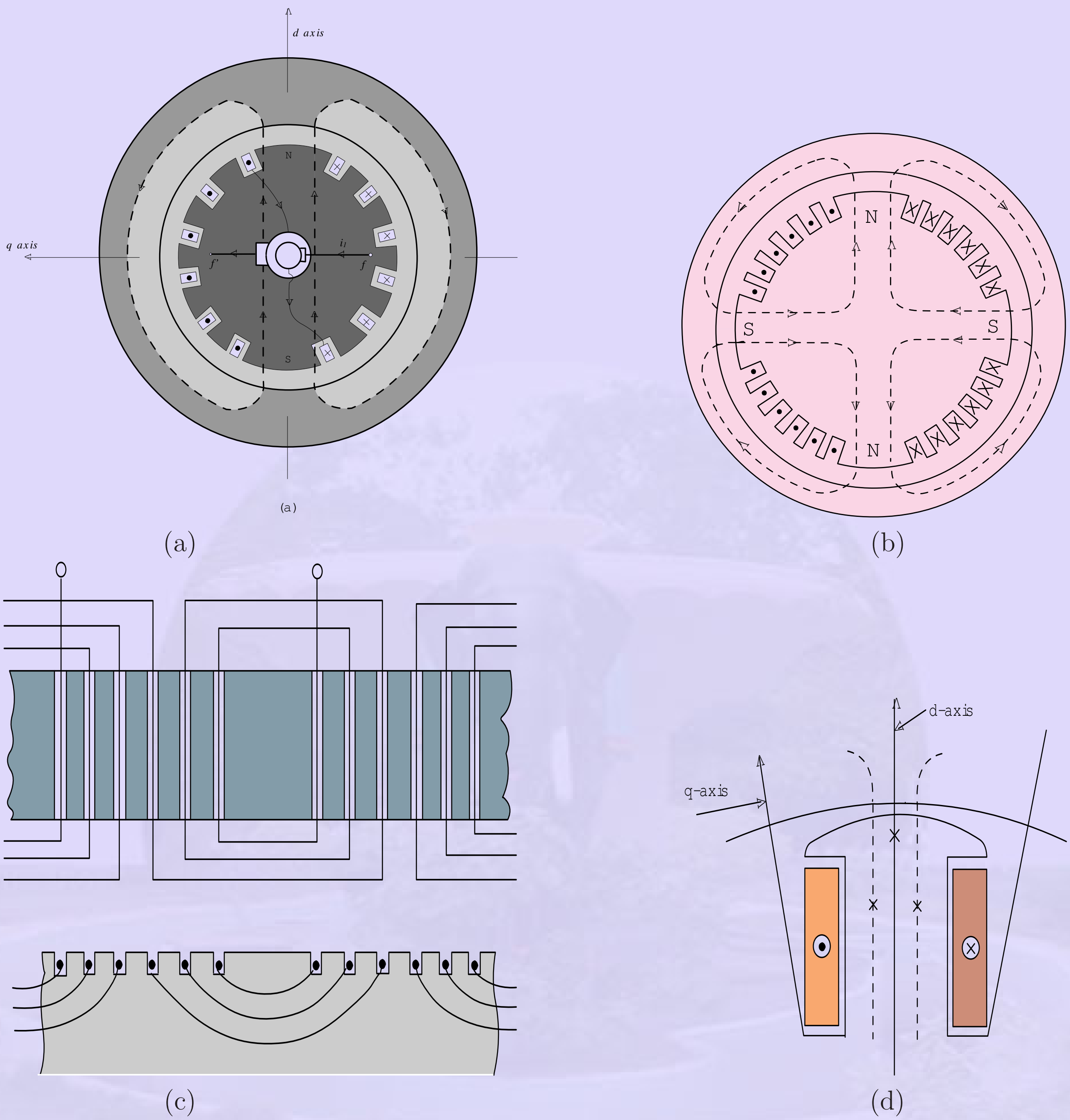


Figure 14: Synchronous machines with stator slots and armature windings omitted (a)Two-pole cylindrical rotor, (b) Four-pole cylindrical rotor, (c) Developed view of two pole cylindrical rotor field structure, (d) Salient pole and field coil

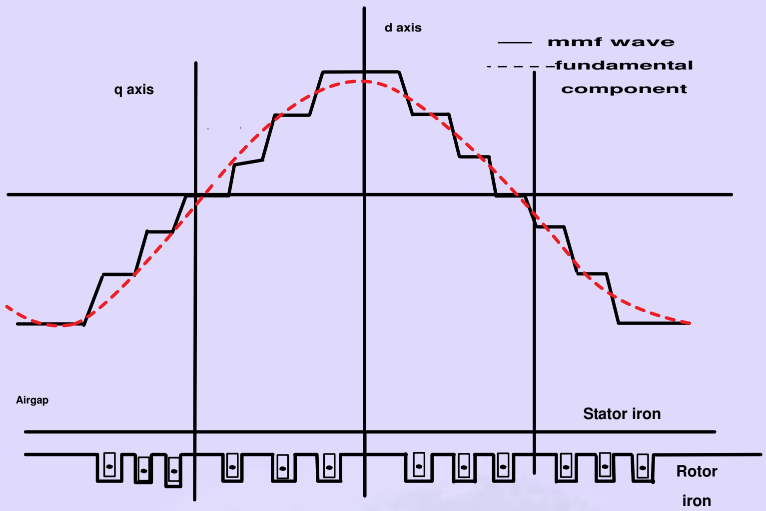


Figure 15: Cylindrical rotor mmf wave and its fundamental of a synchronous machine

set, rectifier, or battery. A common excitation system in which a conventional dc shunt generator mounted on the shaft of the synchronous machine furnishes the field excitation is shown in Fig. 16. The output of the exciter (i.e., the field current of the synchronous machine) is varied by adjusting the exciter field rheostat. A somewhat more complex system that makes use of a pilot exciter—- a compound dc generator—- also mounted on the generator shaft, which in turn excites the field of the main exciter, is shown in Fig. 16. This arrangement makes for greater rapidity of response, a feature that is important in the case of synchronous generators when there are disturbances on the system to which the generator is connected. In some installations a separate motor-driven exciter furnishes the excitation. An induction motor is used instead of a synchronous motor because in a severe system disturbance a synchronous motor may pullout of synchronism with the system. In addition, a large flywheel is used to carry the exciter through short periods of severely reduced system voltage.

2.5.1 Brushless Excitation System

The brushless excitation system eliminates the usual commutator, collector rings, and brushes. One arrangement in which a permanent magnet pilot exciter, an ac main exciter, and a rotating rectifier are mounted on the same shaft as the field of the ac turbogenerator is shown in Fig. 17. The permanent magnet pilot excitor has a stationary armature and a rotating permanent magnetic field. It feeds 400 Hz, three-phase power to a regulator, which in turn supplies regulated dc power to the stationary field of a rotating-armature ac exciter, The

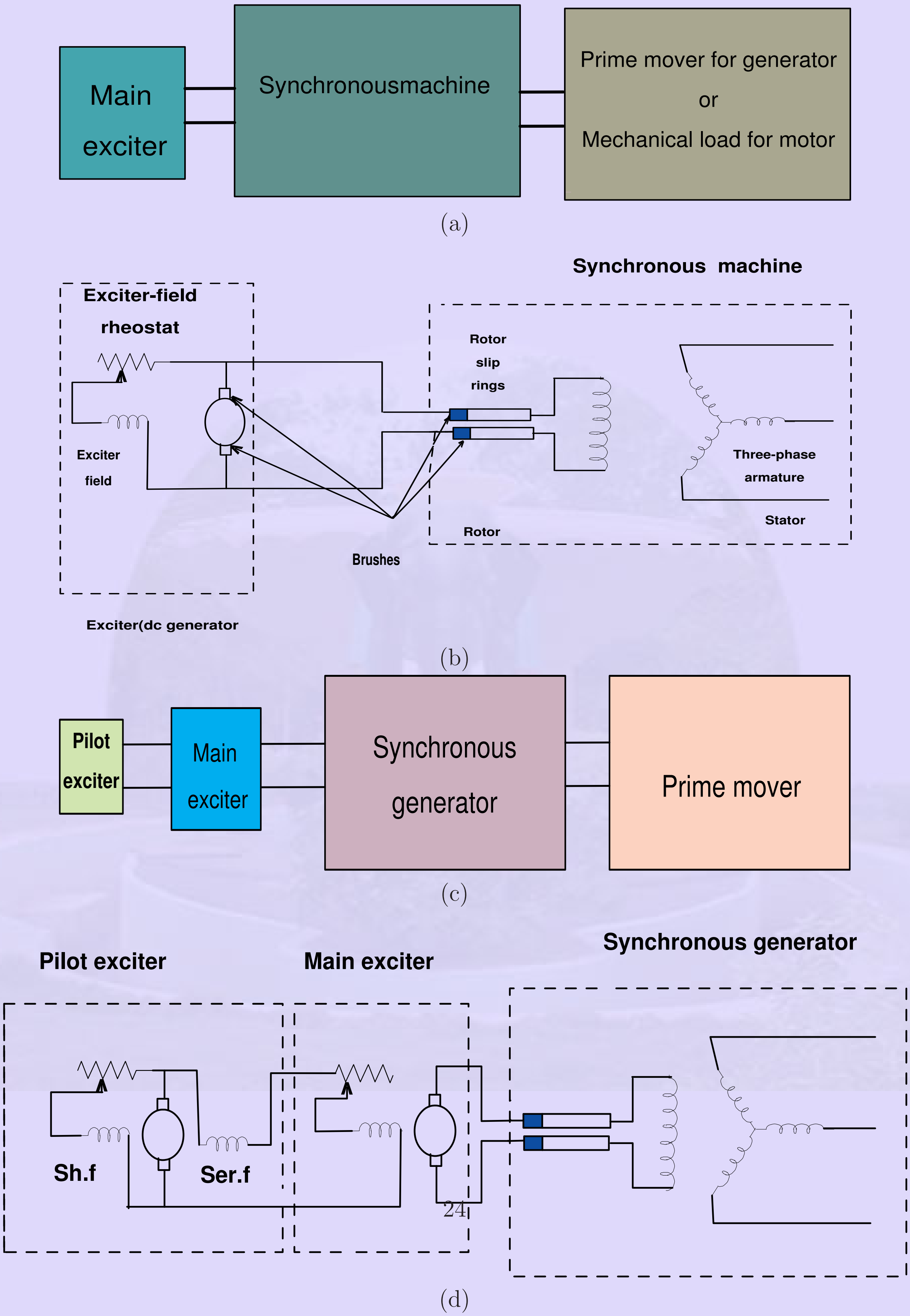


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Figure 16: Conventional excitation systems for synchronous machines

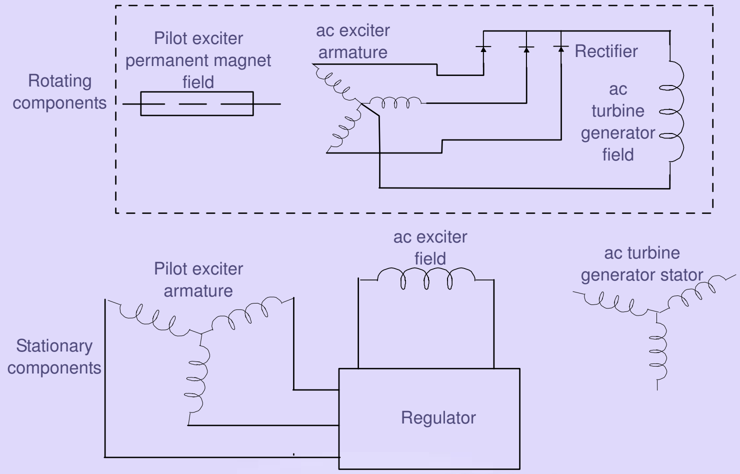


Figure 17: Brushless excitation system

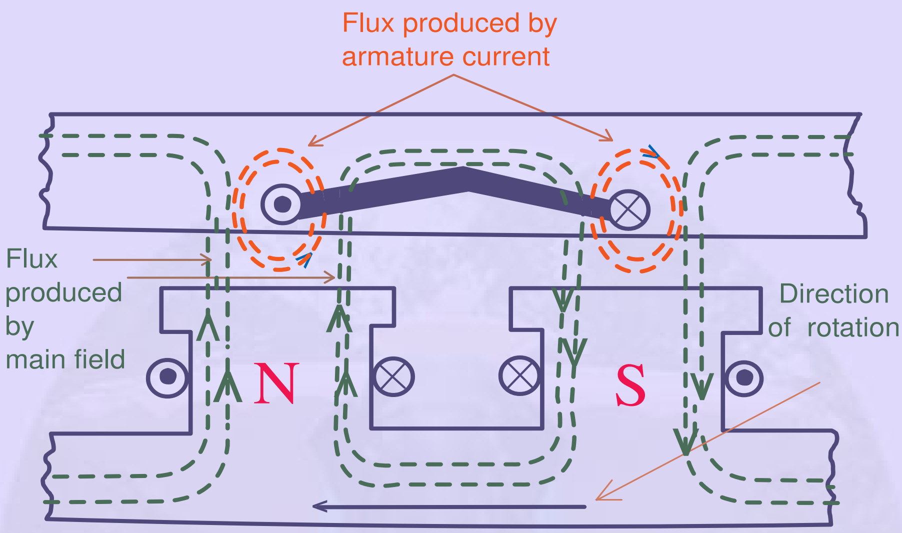
output of the ac exciter is rectified by diodes and delivered to the field of the turbo generator.

Brush less excitation systems have been also used extensively in the much smaller generators employed in aircraft applications where reduced atmospheric pressure intensifies problems of brush deterioration. Because of their mechanical simplicity, such systems lend themselves to military and other applications that involve moderate amounts of power.

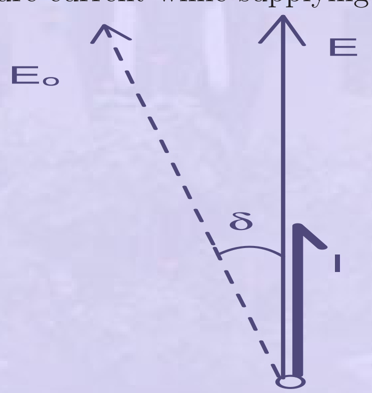
2.6 The Action of the Synchronous Machine

Just like the DC generator, the behaviour of a Synchronous generator connected to an external load is not the same as at no-load. In order to understand the action of the Synchronous machine when it is loaded, let us take a look at the flux distributions in the machine when the armature also carries a current. Unlike in the DC machine here the current peak and the emf peak will not occur in the same coil due to the effect of the power factor (pf) of the load. In other words the current and the induced emf will be at their peaks in the same coil only for upf loads. For zero power factor (zpf)(lagging) loads, the current reaches its peak in a coil which falls behind that coil wherein the induced emf is at its peak by nearly 90 electrical degrees or half a pole-pitch. Likewise for zero power factor (zpf)(leading) loads, the current reaches its peak in a coil which is ahead of that coil wherein the induced emf is at its peak by nearly 90 electrical degrees or half a pole-pitch. For simplicity, let us assume the resistance and leakage reactance of the stator windings to be negligible. Let us also assume the magnetic circuit to be linear i.e. the flux in the magnetic circuit is deemed to be proportional to the resultant ampere-turns - in other words we assume that there is no saturation of the magnetic core. Thus the e.m.f. induced is the same as the terminal voltage, and the phase-angle between current and e.m.f. is determined only by the power factor (pf) of the external load connected to the synchronous generator.

2.6.1 Armature Reaction



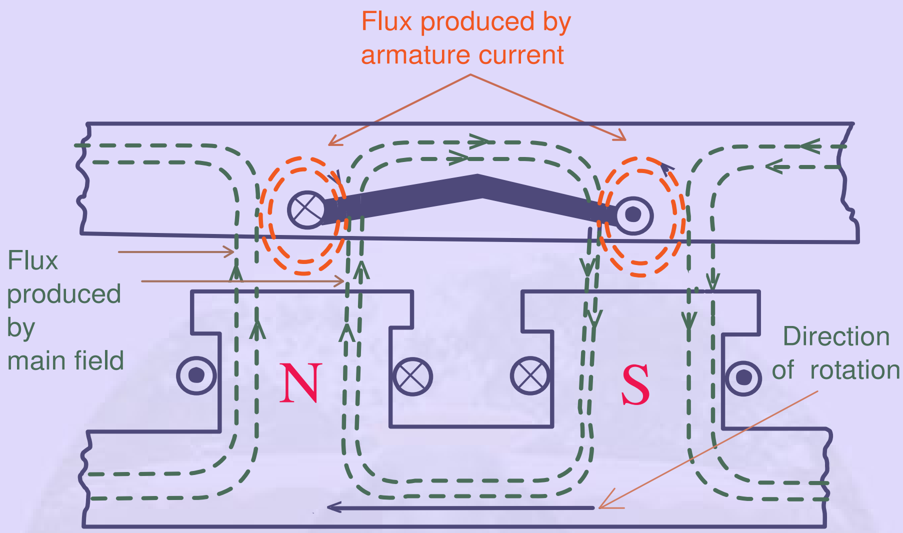
(a)The effect of armature current while supplying a pure resistance load



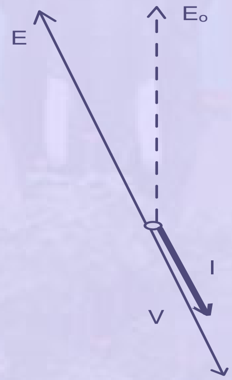
(b)Phasor diagram

Figure 18: Stretched out synchronous generator

In order to understand more clearly let us consider a sketch of a stretched-out synchronous machine shown in Fig. 18(a) which shows the development of a fixed stator car-

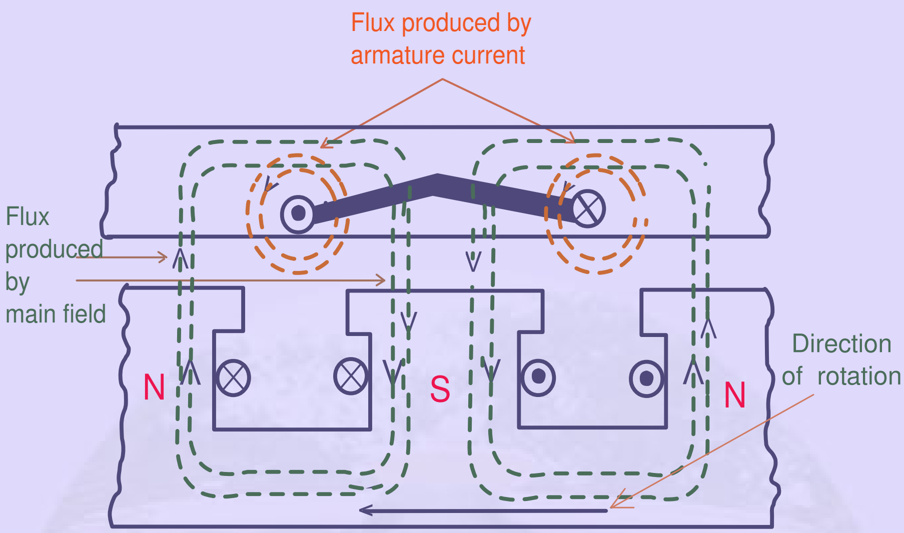


(a)The effect of armature current when the machine operates as a motor at u.p.f

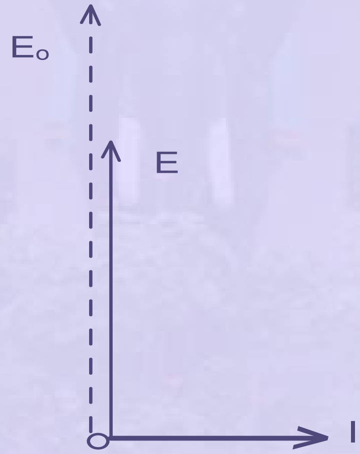


(b)Phasor diagram

Figure 19: Stretched out synchronous motor

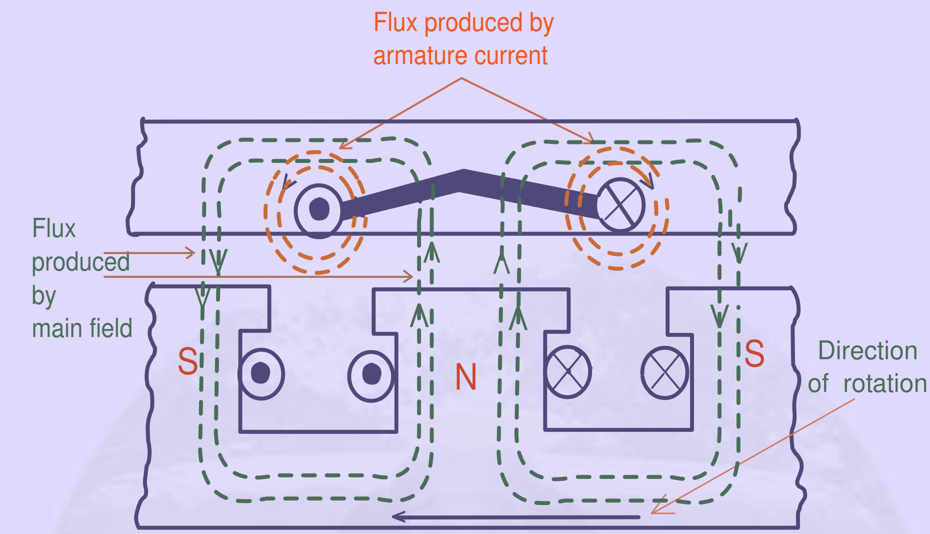


(a)The effect of armature current when it supplies a zpf(lagging) load



(b)Phasor diagram

Figure 20: Stretched out synchronous generator



(a)The effect of armature current when it supplies a zpf(leading) load

(b)Phasor diagram

Figure 21: Stretched out synchronous generator

rying armature windings, and a rotor carrying field windings and capable of rotation within it. The directions of the currents and the flux distribution are as shown in Fig. 18(a), when the emf induced in the stator coils is the maximum. The coil links no resultant flux but is in the position of greatest rate of change of flux. The coil position shown is also that for maximum current when the current is in phase with the voltage: i.e for a pure resistive load. The current in the coil has no effect on the total flux per pole, but causes a strengthening on one side and a weakening on the other side of the pole shoes. Thus the armature conductors find themselves in the circumstances illustrated in Fig. 19, and a torque is produced by the interaction of the main flux ϕ_m with the current in the conductors. The torque thus produced is seen to be opposed to the direction of motion of the rotor - the force on the conductors is such as to push them to the left and by reaction to push the rotor to the right (as the armature coils are stationary). The rotor is rotated by a prime mover against this reaction, so that the electrical power, the product EI, is produced by virtue of the supply of a corresponding mechanical power. Thus it is evident from the distortion of the main flux distribution that electrical energy is converted from mechanical energy and the machine operates as a generator. An unidirectional torque is maintained as the stator conductors cut N-Pole and S-Pole fluxes alternately resulting in alternating emfs at a frequency equal to the number of pole-pairs passed per second and the currents also alternate with the emf. The assumption that the conditions shown in Fig. 18(a) represent co-phasal emf and current is not quite true. The strengthening of the resultant flux on the right of the poles and an equivalent amount of weakening on the left effectively shift the main field flux axis against the direction of rotation, so that the actual e.m.f. E induced in the armature winding is an angle δ behind the position E_0 that it would occupy if the flux were undistorted as shown in the adjacent phasor diagram Fig. 18(b) pertaining to this condition of operation. Thus the effect of a resistive (unit power factor (upf)) load connected to a synchronous generator is to shift the main field flux axis due to what is known as cross-magnetization.

The action of a synchronous machine operating as a motor at unit power factor (upf) is shown in Fig. 19(a). Just like a DC motor, a synchronous motor also requires an externally-applied voltage V in order to circulate in it a current in opposition to the induced e.m.f. E. The coil is shown in the position of maximum induced emf and current, but the current is oppositely directed to that shown in Fig. 18(a). Again the m.m.f. of the coil does not affect the total flux in the common magnetic circuit, but distorts the distribution in such a way as to produce a torque in the same direction as the motion. The machine is a motor by virtue of the electrical input VI causing a torque in the direction of motion. The flux distortion causes a shift of the flux axis across the poles, so that the actual e.m.f. E is an angle δ ahead of the position E_0 that it would occupy if the flux were undistorted as shown in the adjacent phasor diagram Fig. 19(b), pertaining to this condition of operation.

Next let us consider this generator to be connected to a purely inductive load so that the current I in the coils lags behind the e.m.f. E by 90 electrical degrees i.e. corresponding to a quarter-period, in time scale. Since the coil-position in Fig. 18(a) or Fig. 19(a) represents that for maximum e.m.f., the poles would have moved through half a pole-pitch before the current in the coil has reached a maximum as shown in Fig. 20(a). As seen from this figure it is obvious that the ampere-turns of the stator coils are now in direct opposition to those on the pole, thereby reducing the total flux and e.m.f. Since the stator and rotor ampere-turns act in the same direction, there is no flux-distortion, no torque, and hence no additional mechanical power. This circumstance is in accordance with the fact that there is also no electrical power output as E and I are in phase quadrature, as shown in Fig. 20(b). The phasor E_o represents the e,m.f. with no demagnetizing armature current, emphasizing the reduction in e.m.f. due to the reduced flux.

Likewise, when this generator is connected to a purely capacitive load i.e the current I in the coil leads the emf E by 90 electrical degrees, the conditions are such that the armature AT and the field AT will be assisting each other as shown in Fig. 21.

When the generator supplies a load at any other power factor intermediate between unity and zero, a combination of cross- and direct-magnetization is produced on the magnetic circuit by the armature current. The cross-magnetization is distorting and torque-producing as in Fig. 18; the direct-magnetization decreases (for lagging currents) or increases (for leading currents) the ampere-turns acting on the magnetic circuit as in Fig. 20 and Fig. 21, affecting the main flux and the e.m.f. accordingly.

For a motor the torque is reversed on account of the current reversal, and the direct-magnetizing effect is assisting the field ampere-turns for lagging currents. The action of the armature ampere-turns as described above is called armature-reaction. The effect of the armature reaction has a far-reaching influence on the performance of the synchronous motor, particularly as regards the power factor at which it operates and the amount of field excitation that it requires.

2.6.2 Behaviour of a loaded synchronous generator

The simple working of the synchronous machine can be summed up as follows:
A synchronous machine driven as a generator produces e.m.f.'s in its armature windings at a frequency $f = np$. These e.m.f.'s when applied to normal circuits produce currents of the same frequency. Depending on the p.f of the load, field distortion is produced, generating a mechanical torque and demanding an input of mechanical energy to satisfy the electrical

output. As the stator currents change direction in the same time as they come from one magnetic polarity to the next, the torque is unidirectional. The torque of individual phases is pulsating just like in a single-phase induction machine - but the torque of a three-phase machine is constant for balanced loads.

For the cylindrical rotor machine the fundamental armature reaction can be more

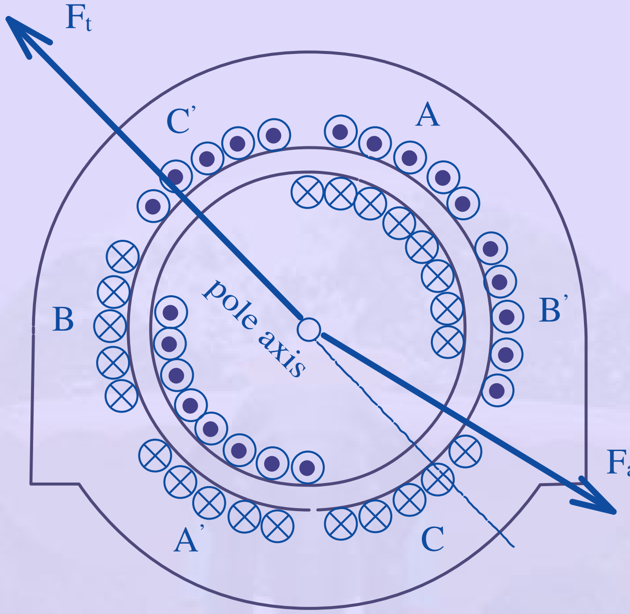


Figure 22: Synchronous generator supplying a lagging pf load

convincingly divided into cross-magnetizing and direct-magnetizing components, since the uniform air-gap permits sinusoidal m.m.f s to produce more or less sinusoidal fluxes. Fig. 22 shows a machine with two poles and the currents in the three-phase armature winding produce a reaction field having a sinusoidally-distributed fundamental component and an axis coincident, for the instant considered, with that of one phase such as $A - A'$. The rotor windings, energized by direct current, give also an approximately sinusoidal rotor m.m.f. distribution. The machine is shown in operation as a generator supplying a lagging current. The relation of the armature reaction m.m.f. F_a to the field m.m.f. F_t is shown in Fig. 23. The F_a sine wave is resolved into the components F_{aq} corresponding to the cross-component and F_{ad} corresponding to the direct-component, which in this case demagnetizes in accordance with Fig. 20. F_{ad} acts in direct opposition to F_t and reduces the effective m.m.f. acting round the normal magnetic circuit. F_{aq} shifts the axis of the resultant m.m.f. (and flux) backward against the direction of rotation of the field system.

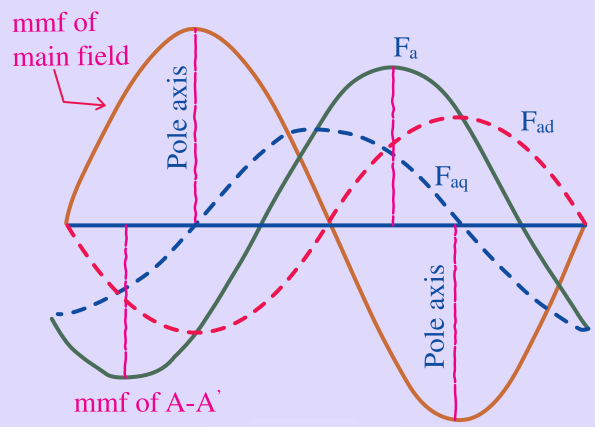


Figure 23: Sinusoidal distribution of the components of armature reaction in a synchronous generator

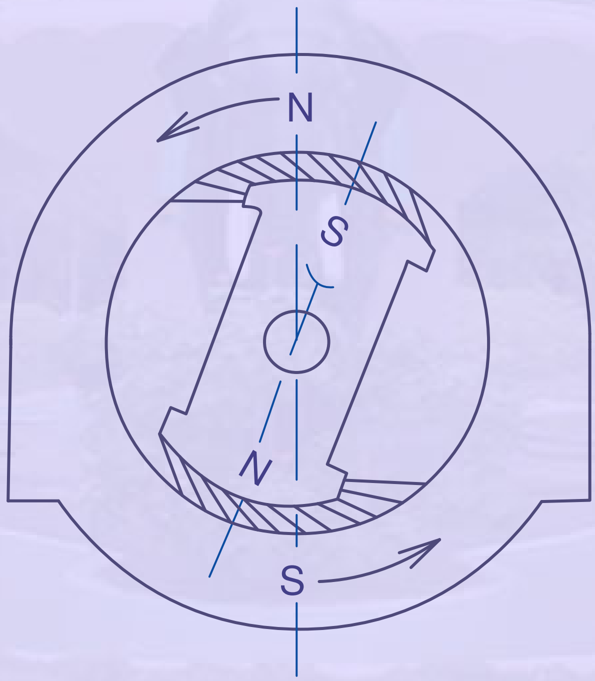


Figure 24: Elementary synchronous motor action - Attraction of the unlike poles keep the rotor locked to the rotating field produced in the stator

2.6.3 Behaviour of a loaded synchronous motor

Likewise when a synchronous machine operates as a motor with a mechanical load on its shaft, it draws an alternating current which interacts with the main flux to produce a driving torque. The torque remains unidirectional only if the rotor moves one pole-pitch per half-cycle; i.e. it can run only at the synchronous speed. In a balanced three-phase machine, the armature reaction due to the fundamental component of the current is a steady mmf revolving synchronously with the rotor - its constant cross-component producing a constant torque by interaction with the main flux, while its direct-component affects the amount of the main flux. A very simple way of regarding a synchronous motor is illustrated in Fig. 24. The stator, like that of the induction motor produces a magnetic field rotating at synchronous speed. The poles on the rotor (salient-pole is shown in Fig. 24 only for clarity), excited by direct current in their field windings, undergo magnetic attraction by the stator poles, and are dragged round to align themselves and locked up with with the stator poles (of opposite polarity- obviously). On no load the axes of the stator and rotor poles are practically coincident. When a retarding torque is applied to the shaft, the rotor tends to fall behind. In doing so the attraction of the stator on the rotor becomes tangential to an extent sufficient to develop a counter torque - however the rotor continues to rotate only at synchronous speed. The angular shift between the stator and rotor magnetic axes represents the torque (or load) angle (as shown later, in the phasor diagram). This angle naturally increases with the mechanical load on the shaft. The maximum possible load is that which retards the rotor so that the tangential attraction is a maximum. (It will be shown later that the maximum possible value for the torque angle is 90 electrical degrees - corresponding to a retardation of the rotor pole by one half of a pole pitch). If the load be increased above this amount, the rotor poles come under the influence of a like pole and the attraction between the stator and rotor poles ceases and the rotor comes to a stop. At this point we say that the synchronous motor pulled out of step. This situation arises much above the rated loads in any practical machine.

It is to be noted that the magnetic field shown in Fig. 24 is only diagrammatic and for better understanding of the action of the synchronous machine - the flux lines may be considered as elastic bands which will be stretched by application of the mechanical load on the shaft. Actually the flux lines will enter or leave the stator and rotor surfaces nearly normally, on account of the high permeability of these members. In a salient-pole machine the torque is developed chiefly on the sides of the poles and on the sides of the teeth in a non-salient-pole machine.

2.7 Concept of Synchronous Reactance

The operation of the synchronous machine can be reduced to comparatively simple expression by the convenient concept of synchronous reactance. The resultant linkage of flux with any phase of the armature of a synchronous machine is due, as has been seen, to the combined action of the field and armature currents. For a simple treatment it is convenient to separate the resultant flux into components: (a) the field flux due to the field current alone; and (b) the armature flux due to the armature current alone. This separation does not affect qualitative matters, but its quantitative validity rests on the assumption that the magnetic circuit has a constant permeability. In brief the simplifying assumptions are:

1. The permeability of all parts of the magnetic circuit of the synchronous machine is constant - in other words the field and armature fluxes can be treated separately as proportional to their respective currents so that their effects can be superposed.

2. The air gap is uniform, so that the armature flux is not affected by its position relative to the poles - in other words we assume the rotor to be cylindrical

3. The distribution of the field flux in the air gap is sinusoidal.

4. The armature winding is uniformly distributed and carries balanced sinusoidal currents. In other words, the harmonics are neglected so that the armature flux is directly proportional to the fundamental component of the armature reaction mmf implying that the armature reaction mmf is distributed sinusoidally and rotates at synchronous speed with constant magnitude.

Assumption (1) is roughly fulfilled when the machine works at low saturation; (2) and (3) are obviously inaccurate with salient-pole machines and assumption (4) is commonly made and introduces negligible error in most cases. The behaviour of an "ideal" synchronous machine can be indicated qualitatively when the above assumptions (1) to (4) are made.

The phasor diagrams Fig. 25 for the several conditions contain the phasors of two emfs viz. E_o and E . The latter is the e.m.f actually existing, while the former is that which would be induced under no-load conditions, i.e. with no armature current (or armature reaction).

Thus E_o is the e.m.f. corresponding to the flux produced by the field winding only, while E is that actually produced by the resultant flux due to the combined effect of stator and rotor ampere-turns. The actual e.m.f. E can be considered as E_o plus a fictitious e.m.f. proportional to the armature current.

Fig. 25 is drawn in this manner with E_r such that the following phasor relationship is satisfied:

$$\mathbf{E} = \mathbf{E}_o + \mathbf{E}_r \tag{19}$$

It can be seen from Fig. 25, that E_r, is always in phase-quadrature with armature current and proportional to it (as per the four assumptions (1) to (4) above). The emf E_r is thus similar to an emf induced in an inductive reactance, so that the effect of armature reaction is exactly the same as if the armature windings had a reactance $x_a = E_r/I_a$. This fictitious reactance x_a can added to the armature leakage reactance x_l and the combined reactance ($x_a + x_l$) is known as the synchronous reactance x_s. The armature winding apart from these reactance effects, presents a resistive behaviour also. Synchronous impedance is a tern used to denote the net impedance presented by each phase of the alternator winding, consisting of both resistive and reactive components. The behavior of a synchronous machine can be easily predicted from the equivalent circuit developed using this synchronous reactance x_s, as explained in the following section.

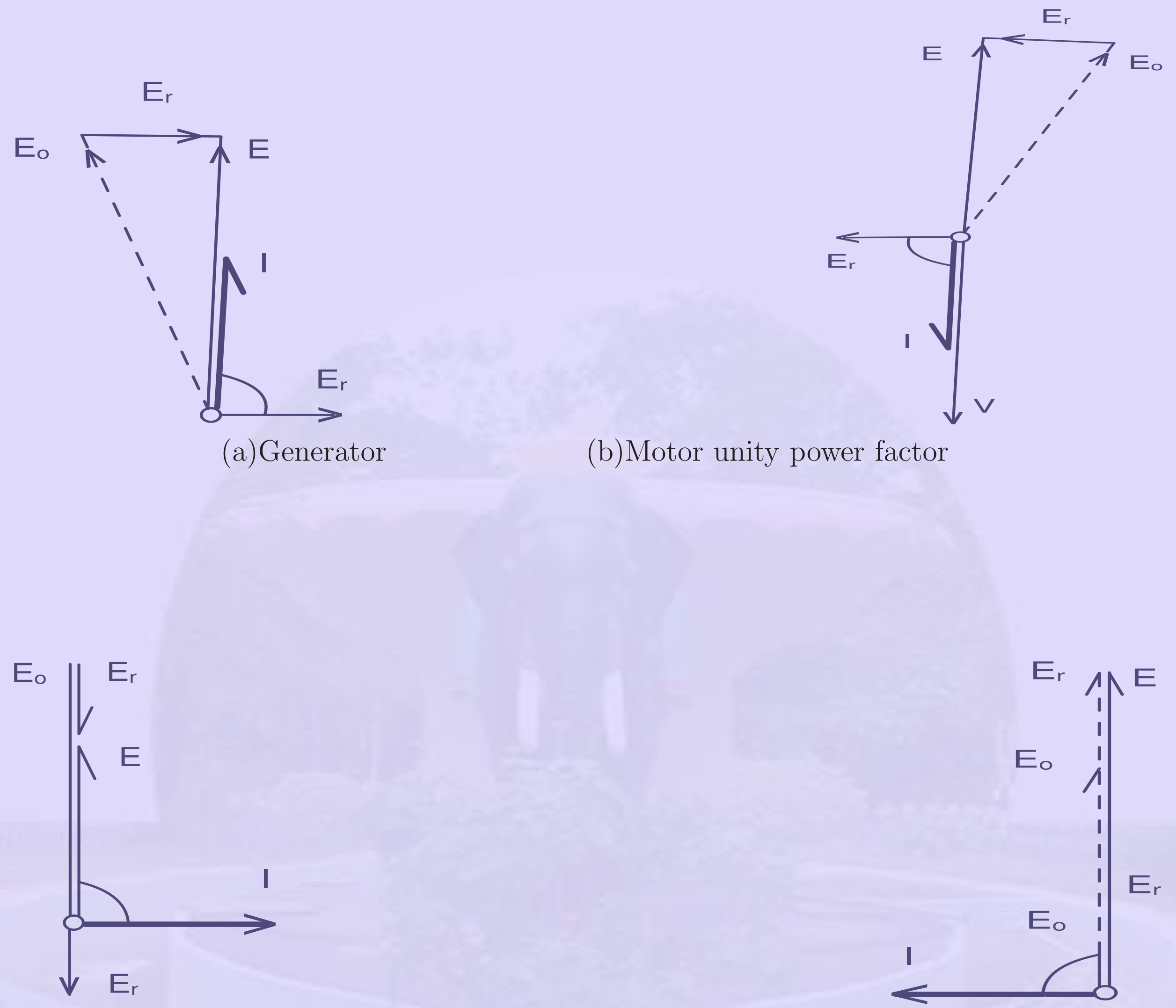
2.8 Approximation of the Saturated Synchronous Reactance

Economical size requires the magnetic circuit to be somewhat saturated under normal operating conditions. However, the machine is unsaturated in the short-circuit test, and the synchronous reactance based on short-circuits and open-circuit test data is only an approximation at best. Nevertheless, there are many studies in which a value based on rated open-circuit voltage and the short circuit current suffices. Hence, in Fig. 29, if oc is rated voltage, ob is the required no-load field current, which also produces the armature current $o'e$ on short circuit. The synchronous impedance assuming the armature winding is star-connected is, accordingly,

$$Z_s = \frac{oc}{\sqrt{3} * o'e} \tag{20}$$

Except in very small machines, the synchronous reactance is much greater than the resistance (r_a) of the armature and the saturated value as well as the unsaturated value of the synchronous reactance and therefore is considered equal to the magnitude of the synchronous impedance

$$X_d = (Z_s^2 - r_a^2)^{\frac{1}{2}} \approx Z_s \tag{21}$$



(a)Generator (b)Motor unity power factor

(c) Generator (d)Generator zero power factor

Figure 25: Phasor diagrams for different operating conditions

The line *of* in Fig. 29 is more nearly representative of the saturated machine than is the air-gap line. On the basis of this line, an estimate of the field current can be obtained for a given terminal voltage, load current, and power factor. This is done by calculating E_{af} and making use of the saturated synchronous reactance as follows.

$$\mathbf{E_{af}} = \mathbf{V} + \mathbf{Z_s I} \tag{22}$$

The field current is that required to produce E_{af} on the line *of*.

2.8.1 Open-circuit and Short-circuit Tests

The effect of saturation on the performance of synchronous machines is taken into account by means of the magnetization curve and other data obtained by tests on an existing machine. Only some basic test methods are considered. The unsaturated synchronous impedance and approximate value of the saturated synchronous impedance can be obtained form the open-circuit and short-circuit tests.

In the case of a constant voltage source having constant impedance, the impedance can be found by dividing the open-circuit terminal voltage by the short circuit current. However, when the impedance is a function of the open-circuit voltage, as it is when the machine is saturated, the open-circuit characteristic or magnetization curve in addition to the short-circuit characteristic is required.

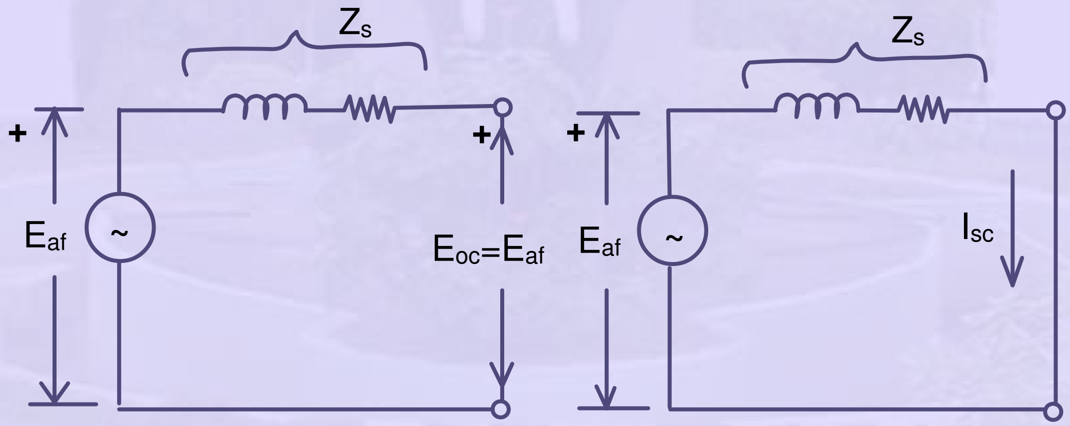


Figure 26: Synchronous generator(a) Open circuit (b) Short circuit

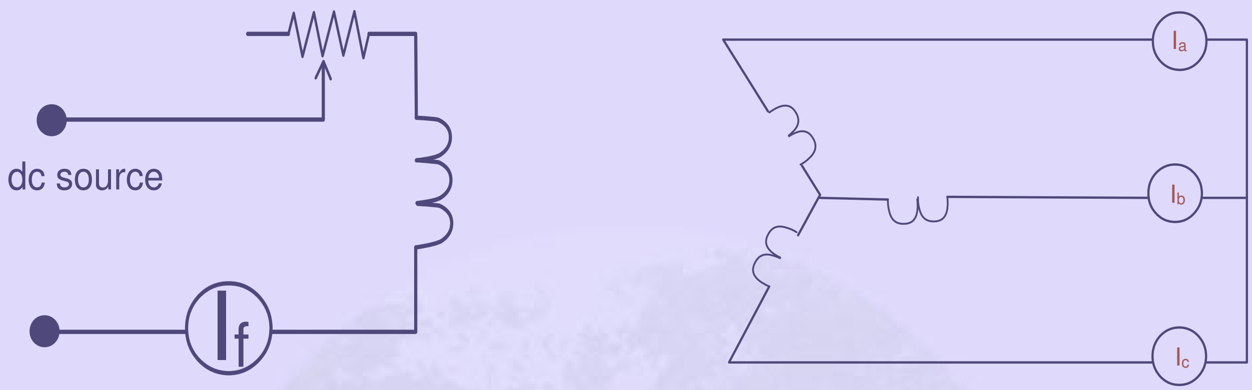
The unsaturated synchronous reactance is constant because the reluctance of the unsaturated iron is negligible. The equivalent circuit of one phase of a polyphase synchronous machine is shown in Fig. 26 for the open-circuit condition and for the short circuit condition. Now E_{af} is the same in both cases when the impedance Z_s. Where E_{af} is the open-circuit volts per phase and I_{sc} is the short-circuit current per phase.

2.8.2 Open-circuit Characteristic

Figure 27: (a) Open circuit characteristic and (b) Short-circuit characteristic

To obtain the open-circuit characteristic the machine is driven at its rated speed without load. Readings of line-to-line voltage are taken for various values of field current. The voltage except in very low-voltage machines is stepped down by means of instrument potential transformers. Fig. 27 shows the open-circuit characteristic or no-load saturation curve. Two sets of scales are shown; one, line to-line volts versus field current in amperes and the other per-unit open-circuit voltage versus per-unit field current. If it were not for the magnetic saturation of the iron, the open-circuit characteristic would be linear as represented by the air-gap line in Fig. 27. It is important to note that 1.0 per unit field current corresponds to the value of the field current that would produce rated voltage if there were no saturation. On the basis of this convention, the per-unit representation is such as to make the air-gap lines of all synchronous machines identical.

2.8.3 Short circuit Test



(a)Field circuit (b)Armature circuit

Figure 28: Connections for short-circuit test

The three terminals of the armature are short -circuited each through a current-measuring circuit, which except for small machines is an instrument current transformer with an ammeter in its secondary. A diagram of connections in which the current transformers are omitted is shown in Fig. 28.

The machine is driven at approximately synchronous (rated) speed and measurements of armature short-circuit current are made for various values of field current, usually up to and somewhat above rated armature current. The short-circuit characteristic (i.e. armature short circuit current versus field current) is shown in Fig. 27. In conventional synchronous machines the short-circuit characteristic is practically linear because the iron is unsaturated up to rated armature current and somewhat beyond, because the magnetic axes of the armature and the field practically coincide (if the armature had zero resistance the magnetic axes would be in exact alignment), and the field and armature mmfs oppose each other.

2.8.4 Unsaturated Synchronous Impedance

The open circuit and short-circuit characteristics are represented on the same graph in Fig. 29. The field current *oa* produces a line-to line voltage *oc* on the air- gap line, which

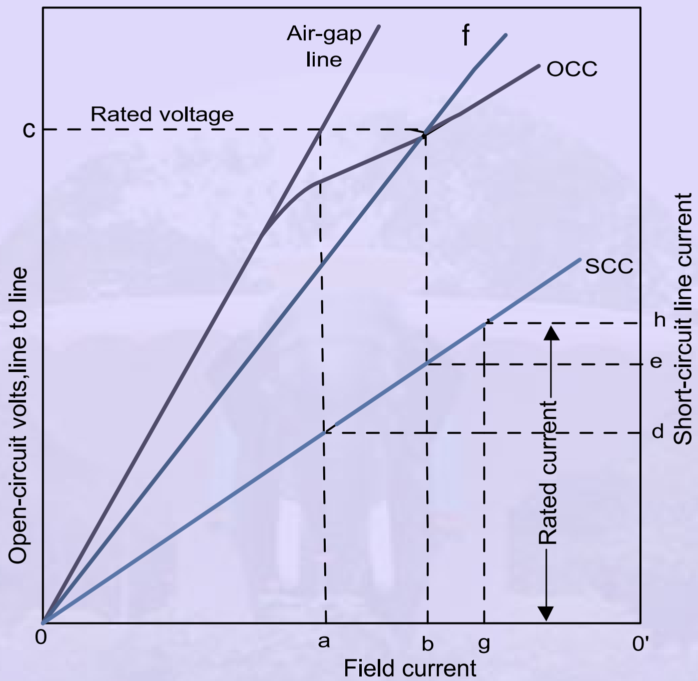


Figure 29: Open-circuit and short circuit characteristic

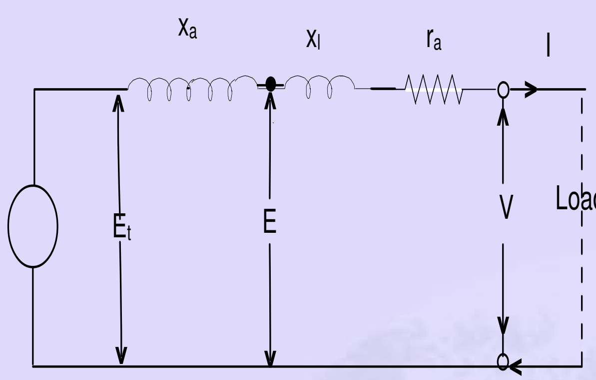
would be the open-circuit voltage if there were no saturation. The same value of field current produces the armature current o'd and the unsaturated synchronous reactance is given by:

$$X_d = \frac{oc}{\sqrt{3}\ o'd}\ \Omega \text{ phase, for a star connected armature} \tag{23}$$

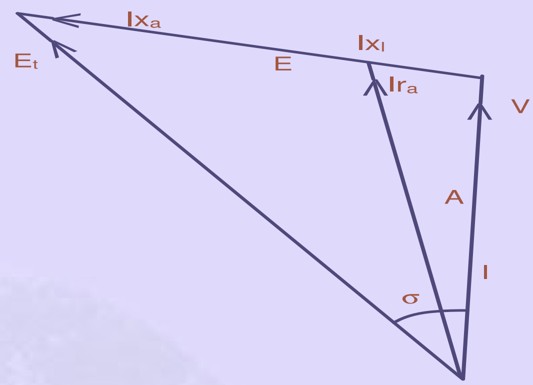
When the open-circuit characteristic, air-gap line, and the short-circuit characteristic are plotted in per-unit, then the per unit value of unsaturated synchronous reactance equals the per-unit voltage on the air-gap line which results from the same value of field current as that which produces rated short-circuit (one-per unit) armature current. In Fig. 29 this would be the per-unit value on the air gap line corresponding to the field current og.

3 Synchronous Generator Operation

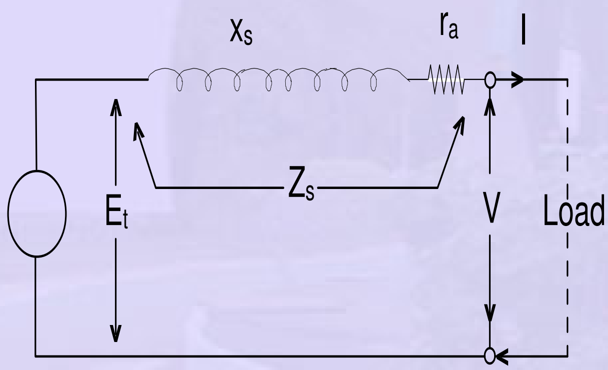
3.1 Cylindrical Rotor Machine



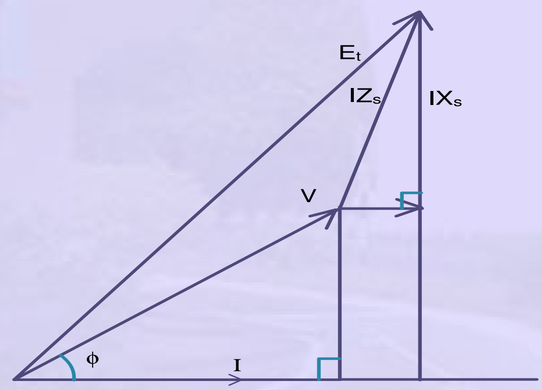
(a)



(b)phasor diagram for R load



(c)



(d)phasor diagram for R-L load

Figure 30: Equivalent circuits

The synchronous generator, under the assumption of constant synchronous reactance, may be considered as representable by an equivalent circuit comprising an ideal winding in which an e.m.f. E_t proportional to the field excitation is developed, the winding being connected to the terminals of the machine through a resistance r_a and reactance

$(X_l + X_a) = X_s$ all per phase. This is shown in Fig. 30. The principal characteristics of the synchronous generator will be obtained qualitatively from this circuit.

3.1.1 Generator Load Characteristics

Consider a synchronous generator driven at constant speed and with constant excitation. On open circuit the terminal voltage V is the same as the open circuit **e.m.f.** E_t. Suppose a unity-power-factor load be connected to the machine. The flow of load current produces a voltage drop IZ_s in the synchronous impedance, and terminal voltage V is reduced. Fig. 31 shows the complexor diagram for three types of load. It will be seen that the angle σ between E_t and V increases with load, indicating a shift of the flux across the pole faces due to cross- magnetization. The terminal voltage is obtained from the complex summation

$$V + Z_s = E_t$$
$$or\ V = E_t - IZ_s \tag{24}$$

Algebraically this can be written

$$V = \sqrt{(E_t^2 - I^2X_s^2)} - I_r \tag{25}$$

for non-reactive loads. Since normally r is small compared with X_s

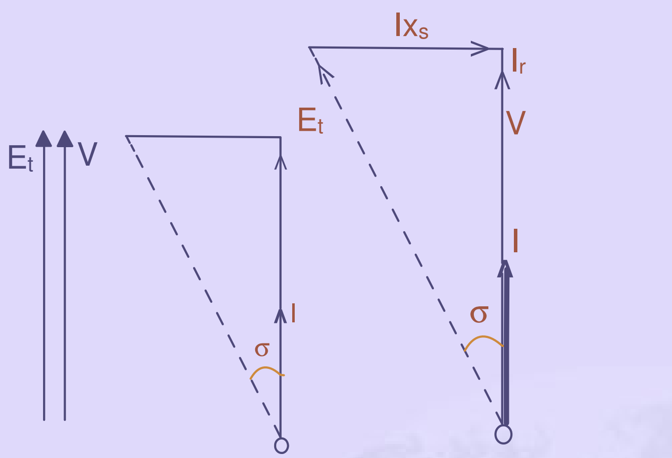
$$V^2 + I^2X_s^2 \approx E_t^2 = constant \tag{26}$$

so that the V/I curve, Fig. 32, is nearly an ellipse with semi-axes E_t and I_{sc}. The current I_{sc} is that which flows when the load resistance is reduced to zero. The voltage V falls to zero also and the machine is on short-circuit with V = 0 and

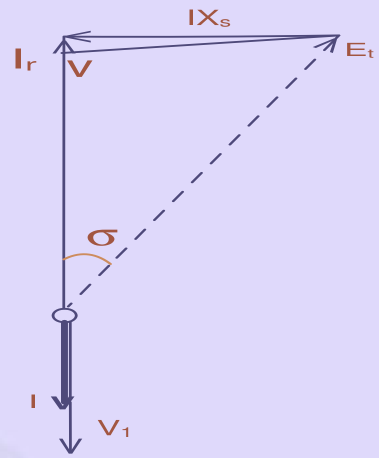
$$I = I_{sc} = E_t/Z_s \approx E_t/X_s \tag{27}$$

For a lagging load of zero power-factor, diagram is given in Fig. 31 The voltage is given as before and since the resistance in normal machines is small compared with the synchronous reactance, the voltage is given approximately by

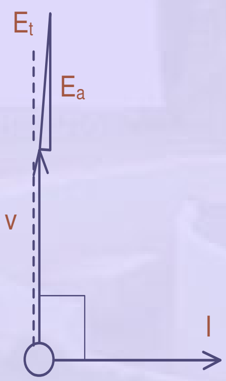
$$V \approx E_t - IX_s \tag{28}$$



(a)phasor diagram for different R loads



(b)



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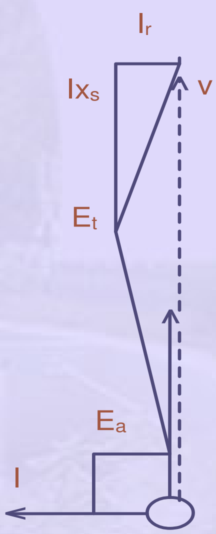


(d)

Figure 31: Variation of voltage with load at constant Excitation

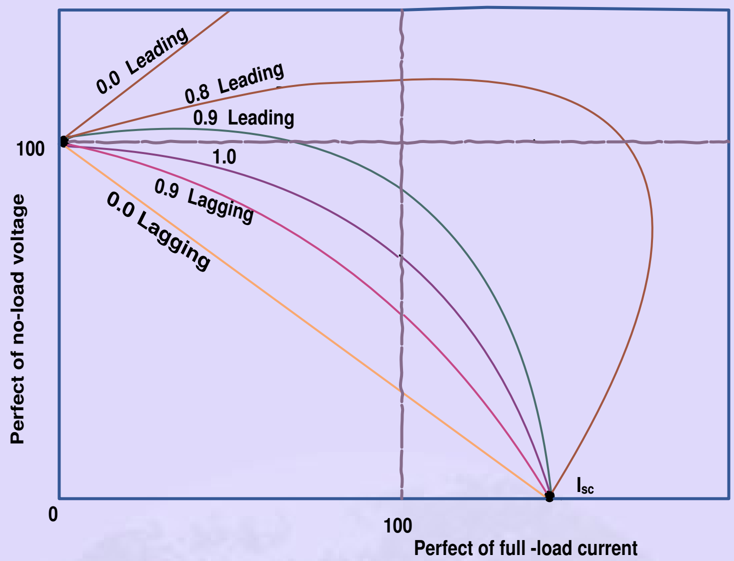


Figure 32: Generator Load characteristics

which is the straight line marked for $\cos\phi = 0$ lagging in Fig. 32. A leading load of zero power factor Fig. 31. will have the voltage

$$V \approx E_t + IX_s \tag{29}$$

another straight line for which, by reason of the direct magnetizing effect of leading currents, the voltage increases with load.

Intermediate load power factors produce voltage/current characteristics resembling those in Fig. 32. The voltage-drop with load (i.e. the regulation) is clearly dependent upon the power factor of the load. The short-circuit current I_{sc} at which the load terminal voltage falls to zero may be about 150 per cent (1.5 per unit) of normal current in large modern machines.

3.1.2 Generator Voltage-Regulation

The voltage-regulation of a synchronous generator is the voltage rise at the terminals when a given load is thrown off, the excitation and speed remaining constant. The voltage-rise is clearly the numerical difference between E_t and V, where V is the terminal voltage for a given load and E_t is the open-circuit voltage for the same field excitation. Expressed as a fraction, the regulation is

$$\varepsilon = (E_t - V)/V \; perunit \tag{30}$$

Comparing the voltages on full load (1.0 per unit normal current) in Fig. 32, it will be seen that much depends on the power factor of the load. For unity and lagging power factors there is always a voltage drop with increase of load, but for a certain leading power factor the full-load regulation is zero, i.e. the terminal voltage is the same for both full and no-load conditions. At lower leading power factors the voltage rises with increase of load, and the regulation is negative. From Fig. 30, the regulation for a load current I at power factor $\cos\phi$ is obtained from the equality

$$E_t^2 = (V\cos\phi + Ir)^2 + (V\sin\phi + IX_s)^2 \tag{31}$$

from which the regulation is calculated, when both E_t and V are known or found.

3.1.3 Generator excitation for constant voltage

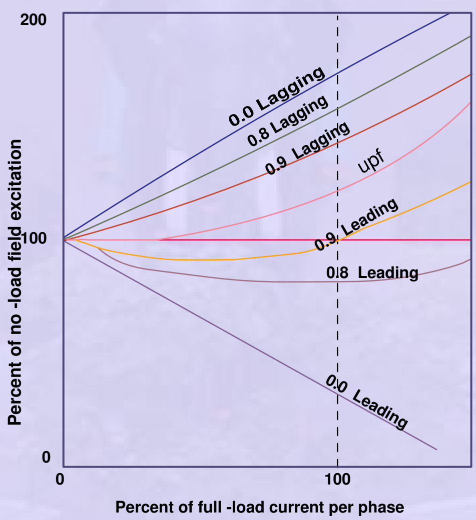


Figure 33: Generator Excitation for constant Voltage

Since the e.m.f. E_t is proportional to the excitation when the synchronous reactance is constant, the Eqn. 31 can be applied directly to obtain the excitation necessary to maintain constant output voltage for all loads. All unity-and lagging power-factor loads will require an increase of excitation with increase of load current, as a corollary of Fig. 32.

Low-leading-power-factor loads, on the other hand, will require the excitation to be reduced on account of the direct magnetizing effect of the zero- power-factor component. Fig. 33 shows typical e.m.f./current curves for a constant output voltage. The ordinates of Fig. 33 are marked in percentage of no-load field excitation, to which the e.m.f E_t exactly corresponds when saturation is neglected.

3.1.4 Generator input and output

For any load conditions as represented by Fig. 30, the output per phase is $P = VI\cos\phi$. The electrical power converted from mechanical power input is per phase

$$P_1 = E_t I \cos(\phi + \sigma) \tag{32}$$

Resolving E_t along I

$$P_1 = E_t I \cos(\phi + \sigma) = (V\cos\phi + Ir).I = VI\cos\phi + I^2 R \tag{33}$$

The electrical input is thus the output plus the I^2R loss, as might be expected. The prime mover must naturally supply also the friction, windage and core losses, which do not appear in the phasor diagram.

In large machines the resistance is small compared with the synchronous reactance so that $\theta = arc\tan(x_s/r) \approx 90°$, it can be shown that

$$\frac{V}{\sin(90 - \overline{\theta + \sigma})^2} = \frac{Z_s}{\sin\sigma} \tag{34}$$

and hence,

$$P = P_1 = E_t I \cos(\phi + \sigma) \approx (E_t/X_s).V\sin\sigma \tag{35}$$

Thus the power developed by a synchronous machine with given values of E_t V and Z_s is proportional to $sin\sigma$: or, for small angles, to σ, and the displacement angle σ representing the change in relative position between the rotor and resultant pole- axes is proportional to the load power. The term load-, power- or torque-angle may be applied to σ.

An obvious deduction from the above Eqn. 35is that the greater the field excitation (corresponding to E_t) the greater is the output per unit angle σ: that is, the more stable will be the operation.

3.2 Salient Pole Rotor Machine

As discussed earlier in Sec. 3.1 the behaviour of a synchronous machine on load can be determined by the use of synchronous reactance x_s which is nothing but the sum of x_a and x_l , where x_a is a fictitious reactance representing the effect of armature reaction while x_l is the leakage reactance. It was also mentioned that this method of representing the effect of armature reaction by a fictitious reactance x_a was applicable more aptly only for a cylindrical rotor (non-salient pole) machine. This was so as the procedure followed therein was valid only when both the armature and main field m.m.f.'s act upon the same magnetic circuit and saturation effects are absent.

3.2.1 Theory of Salient-pole machines (Blondel's Two-reaction Theory)

It was shown in Sec. **??** that the effect of armature reaction in the case of a salient pole synchronous machine can be taken as two components - one acting along the direct axis (coinciding with the main field pole axis) and the other acting along the quadrature axis (inter-polar region or magnetic neutral axis) - and as such the mmf components of armature-reaction in a salient-pole machine cannot be considered as acting on the same magnetic circuit. Hence the effect of the armature reaction cannot be taken into account by considering only the synchronous reactance, in the case of a salient pole synchronous machine.

In fact, the direct-axis component F_{ad} acts over a magnetic circuit identical with that of the main field system and produces a comparable effect while the quadrature-axis component F_{aq} acts along the interpolar space, resulting in an altogether smaller effect and, in addition, a flux distribution totally different from that of F_{ad} or the main field m.m.f. This explains why the application of cylindrical-rotor theory to salient-pole machines for predicting the performance gives results not conforming to the performance obtained from an actual test.

Blondel's two-reaction theory considers the effects of the quadrature and direct-axis components of the armature reaction separately. Neglecting saturation, their different effects are considered by assigning to each an appropriate value of armature-reaction "reactance," respectively x_{ad} and x_{aq} . The effects of armature resistance and true leakage reactance (x_l) may be treated separately, or may be added to the armature reaction coefficients on the assumption that they are the same, for either the direct-axis or quadrature-axis components of the armature current (which is almost true). Thus the combined reactance values can be expressed as :

$$x_{sd} = x_{ad} + x_l \text{ and } x_{sq} = x_{aq} + x_l \tag{36}$$

for the direct- and cross-reaction axes respectively. These values can be determined experimentally as described in Sec. 3.2.3

In a salient-pole machine, x_{aq}, the cross- or quadrature-axis reactance is smaller than x_{ad}, the direct-axis reactance, since the flux produced by a given current component in that axis is smaller as the reluctance of the magnetic path consists mostly of the interpolar spaces.

It is essential to clearly note the difference between the quadrature- and direct-axis components I_{aq}, and I_{ad} of the armature current I_a, and the reactive and active components I_{aa} and I_{ar}. Although both pairs are represented by phasors in phase quadrature, the former are related to the induced emf E_t while the latter are referred to the terminal voltage V. These phasors are clearly indicated with reference to the phasor diagram of a (salient pole) synchronous generator supplying a lagging power factor (pf) load, shown in Fig. **??**(a). We have

$$I_{aq} = I_a \cos(\delta + \phi); I_{ad} = I_a \sin(\delta + \phi); \text{and } I_a = \sqrt{(I_{aq}^2 + I_{ad}^2)} \tag{37}$$

$$I_{aa} = I_a \cos(\phi);\ I_{ar} = I_a \sin(\phi);\ \text{and } I_a = \sqrt{(I_{aa}^2 + I_{ar}^2)} \tag{38}$$

where σ = torque or power angle and ϕ = the p.f. angle of the load.

The phasor diagram Fig. 34 shows the two reactance voltage components $I_{aq} * x_{sq}$ and $I_{ad} * x_{sd}$ which are in quadrature with their respective components of the armature current. The resistance drop $\mathbf{I}_a * R_a$ is added in phase with I_a although we could take it as $I_{aq} * R_a$ and $I_{ad} * R_a$ separately, which is unnecessary as

$$\mathbf{I}_a = \mathbf{I}_{ad} + j\mathbf{I}_{aq}$$

Actually it is not possible to straight-away draw this phasor diagram as the power angle σ is unknown until the two reactance voltage components $I_{aq} * x_{sq}$ and $I_{ad} * x_{sd}$ are known. However this difficulty can be easily overcome by following the simple geometrical construction shown in Fig. 34(d), assuming that the values for terminal voltage V, the load power factor (pf) angle ϕ and the two synchronous reactances x_{sd} and x_{sq} are known to us.

The resistance drop $I_a * R_a$ (length AB) is added to the tip of the voltage phasor (OA) in phase with the current phasor (i.e. in a direction parallel to OQ). Then we draw line BC (of length equal to $I_a * x_{sq}$) and extend it to D such that BD will be (of length equal to $I_a * x_{sd}$) at the extremity B of $I_a * R_a$ and at right-angles to I_a . Draw OC and extend it (to F). From D draw the perpendicular DF on OC extended. Then OF represents the induced voltage E_t. The proof for this can be given as follows:. If DF is extended to G such that this line is perpendicular to BG drawn parallel to OF, we have :

$$BG = BD * \cos(90 - (\sigma + \phi)) = I_a * x_{sd} * \sin(\sigma + \phi) = I_{ad} * x_{sd} \text{ and} \quad (39)$$
$$GF = CH = BC * \sin(90 - (\sigma + \phi)) = I_a * x_{sq} * \cos(\sigma + \phi) = I_{aq} * x_{sq} \quad (40)$$

3.2.2 Power relations in a Salient Pole Synchronous Machine:

Neglecting the armature winding resistance, the power output of the generator is given by:

$$P = V * I_a * \cos\phi \quad (41)$$

This can be expressed in terms of σ, by noting from Fig. 34 that :

$$\begin{aligned} I_a * \cos\phi &= I_{aq} * \cos\sigma + I_{ad} * \sin\sigma \quad (42)\\ V * \cos\sigma &= E_o - I_{ad} * x_{sd} \\ \text{and } V * \sin\sigma &= I_{aq} * x_{sd} \end{aligned}$$

Substituting these in the expression for power, we have.

$$\begin{aligned} P &= V[(V * \sin\sigma/x_{sd}) * \cos\sigma + (E_o - V * \cos\sigma)/x_{sd} * \sin\sigma] \quad (43)\\ &= (V * E_o/x_{sd}) * \sin\sigma + V^2 * (x_{sd} - x_{sq})/(2 * x_{sq} * x_{sq}) * \sin 2\sigma \end{aligned}$$

It is clear from the above expression that the power is a little more than that for a cylindrical rotor synchronous machine, as the first term alone represents the power for a cylindrical rotor synchronous machine. A term in ($\sin 2\sigma$) is added into the power - angle characteristic of a non-salient pole synchronous machine. This also shows that it is possible to generate an emf even if the excitation E_0 is zero. However this magnitude is quite less compared with that obtained with a finite E_0. Likewise we can show that the machine develops a torque - called the reluctance torque - as this torque is developed due to the variation of the reluctance in the magnetic circuit even if the excitation E_0 is zero.

3.2.3 Experimental Determination of x_d and x_q

The unsaturated values of x_d and x_q of a 3-Phase synchronous machine can be easily determined experimentally by conducting the following test known as slip test. The rotor of

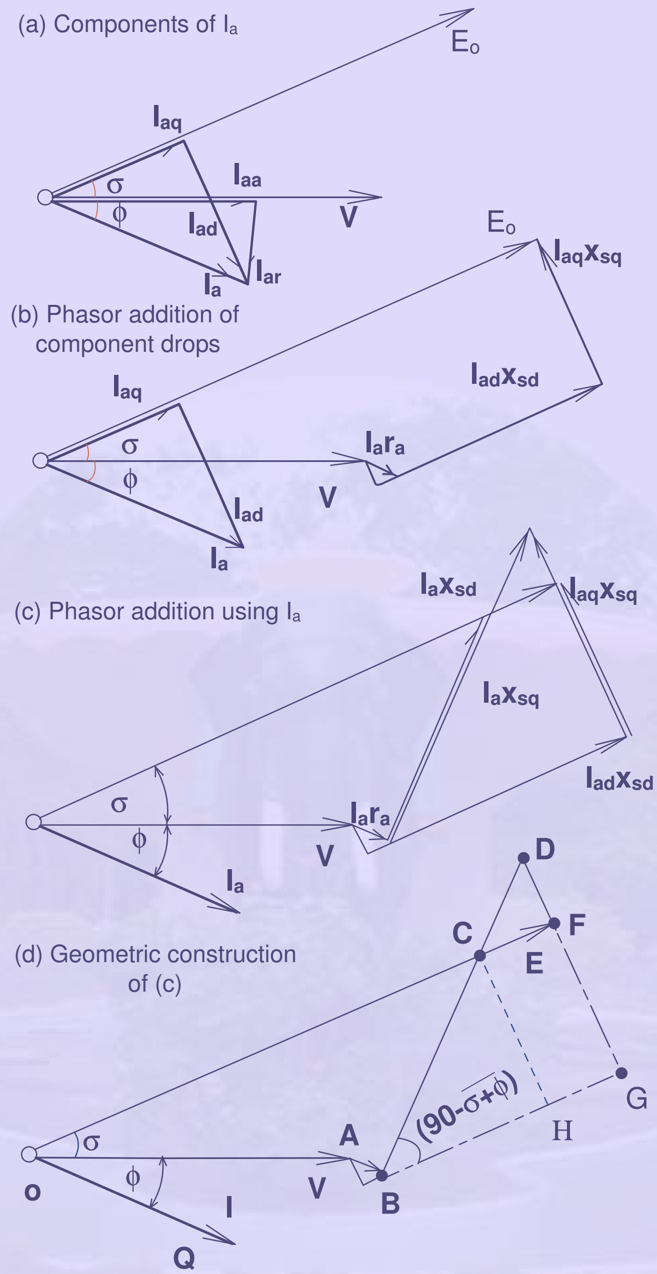


Figure 34: Phasor diagram of a generator-Two reaction theory

the synchronous machine is driven by means of a prime mover (usually a DC motor in the laboratory) at a speed close to the synchronous speed in the proper direction but not equal to it. The armature is supplied with a low voltage 3-Phase balanced supply through a variac, while the field circuit is kept open. The armature current varies between two limits since it moves through, since the synchronously rotating armature mmf acts through the varying magnetic reluctance paths as it goes from inter-polar axis to pole axis region. The values of x_{sd} and x_{sq} are determined based on the applied voltage and the armature current values. The ratio of applied voltage to the minimum value of the armature current gives the direct axis synchronous reactance x_{sd} which is usually the same as the synchronous reactance x_s that we usually determine from normal no-load and short-circuit tests as explained in Sec. **??** The ratio of applied voltage to the maximum value of the armature current gives the the quadrature-axis reactance x_{sq}. For more accurate determination of these values the oscillogram of the armature current and voltage can be recorded.

3.3 Losses and Efficiency

To calculate the efficiency of a synchronous generator, a procedure is to be followed for establishing the total losses when operating under load. For generators these losses are,

1. Rotational losses such as friction and windage.
2. Eddy current and hysteresis losses in the magnetic circuit
3. Copper losses in the armature winding and in the field coils
4. Load loss due to armature leakage flux causing eddy current and hysteresis losses in the armature-surrounding iron.

With regard to the losses, the following comments may be made,

1. The rotational losses, which include friction and windage losses, are constant, since the speed of a synchronous generator is constant. It may be determined from a no-load test.
2. The core loss includes eddy current and hysteresis losses as a result of normal flux density changes. It can be determined by measuring the power input to an auxiliary motor used to drive the generator at no load, with and without the field excited. The difference in power measured constitutes this loss.

3. The armature and field copper losses are obtained as $I_a^2 R_a$ and $V_f I_f$ Since per phase quantities are dealt with, the armature copper loss for the generator must be multiplied by the number of phases. The field winding loss is as a result of the excitation current flowing through the resistance of the field winding.

4. Load loss or stray losses result from eddy currents in the armature conductors and increased core losses due to distorted magnetic fields. Although it is possible to separate this loss by tests, in calculating the efficiency, it may be accounted for by taking the effective armature resistance rather than the dc resistance.

After all the foregoing losses have been determined, the efficiency η is calculated as,

$$\eta = \frac{kVA * PF}{kVA * PF + (total\ losses)} * 100\% \tag{44}$$

where η = efficiency,

kvA = load on the generator (output)

PF = power factor of the load

The quantity (kVA*PF) is, of course, the real power delivered to the load (in kW) by the synchronous generator. Thus, it could in general be stated as

$$\eta = \frac{P_{out}}{P_{in}} * 100 = \frac{P_{out}}{P_{out} + P_{losses}} * 100 \tag{45}$$

The input power $P_{in} = P_{out} + P_{losses}$ is the power required from the prime mover to drive the loaded generator.

4 Parallel Operation of two Generators

When two synchronous generators are connected in parallel, they have an inherent tendency to remain in step, on account of the changes produced in their armature currents by a divergence of phase. Consider identical machines 1 and 2, Fig. 35 in parallel and working on to the same load. With respect to the load, their e.m.fs are normally in phase: with respect to the local circuit formed by the two armature windings, however, their e.m.fs are in phase-opposition.

Suppose there to be no external load. If machine 1 for some reason accelerates, its e.m.f. will draw ahead of that of machine 2. The resulting phase difference 2δ causes e.m.fs to lose phase-opposition in the local circuit so that there is in effect a local e.m.f E_s which will circulate a current I_s in the local circuit of the two armatures. The current I_s flows in the synchronous impedance of the two machines together, so that it lags by $\theta = arc\tan(x_s/r) \approx 90°$ on E_s on account of the preponderance of reactance in$Z_s I_s$ therefore flows out of machine 1 nearly in phase with the e.m.f., and enters 2 in opposition to the e.m.f. Consequently machine 1 produces a power $P_s \approx E_1 I_s$ as a generator, and supplies it (I^2R losses excepted) to 2 as a synchronous motor. The synchronizing power P_s tends to retard the faster machine 1 and accelerate the slower, 2, pulling the two back into step. Within the limits of maximum power, therefore, it is not possible to destroy the synchronous running of two synchronous generators in parallel, for a divergence of their angular positions results in the production of synchronizing power, which loads the forward machine and accelerates the backward machine to return the two to synchronous running.

The development of synchronizing power depends on the fact that the armature impedance is preponderating reactive. If it were not, the machines could not operate stably in parallel: for the circulating current I_s would be almost in phase- quandrature with the generated e.m.f.'s, and would not contribute any power to slow the faster or speed up the slower machine.

When both machines are equally loaded pn to an external circuit, the synchronizing power is developed in the same way as on no load, the effect being to reduce the load of the slower machine at the same time as that of the faster machine is increased. The conditions are shown in Fig. 35, where I_1, I_2 are the equal load currents of the two machines before the occurrence of phase displacement, and $I_1^{'}, I_2^{'}$ are the currents as changed by the circulation of the synchronizing current I_s.

The argument above has been applied to identical machines. Actually, it is not essential for them to be identical, nor to have equal excitations nor power supplies. In

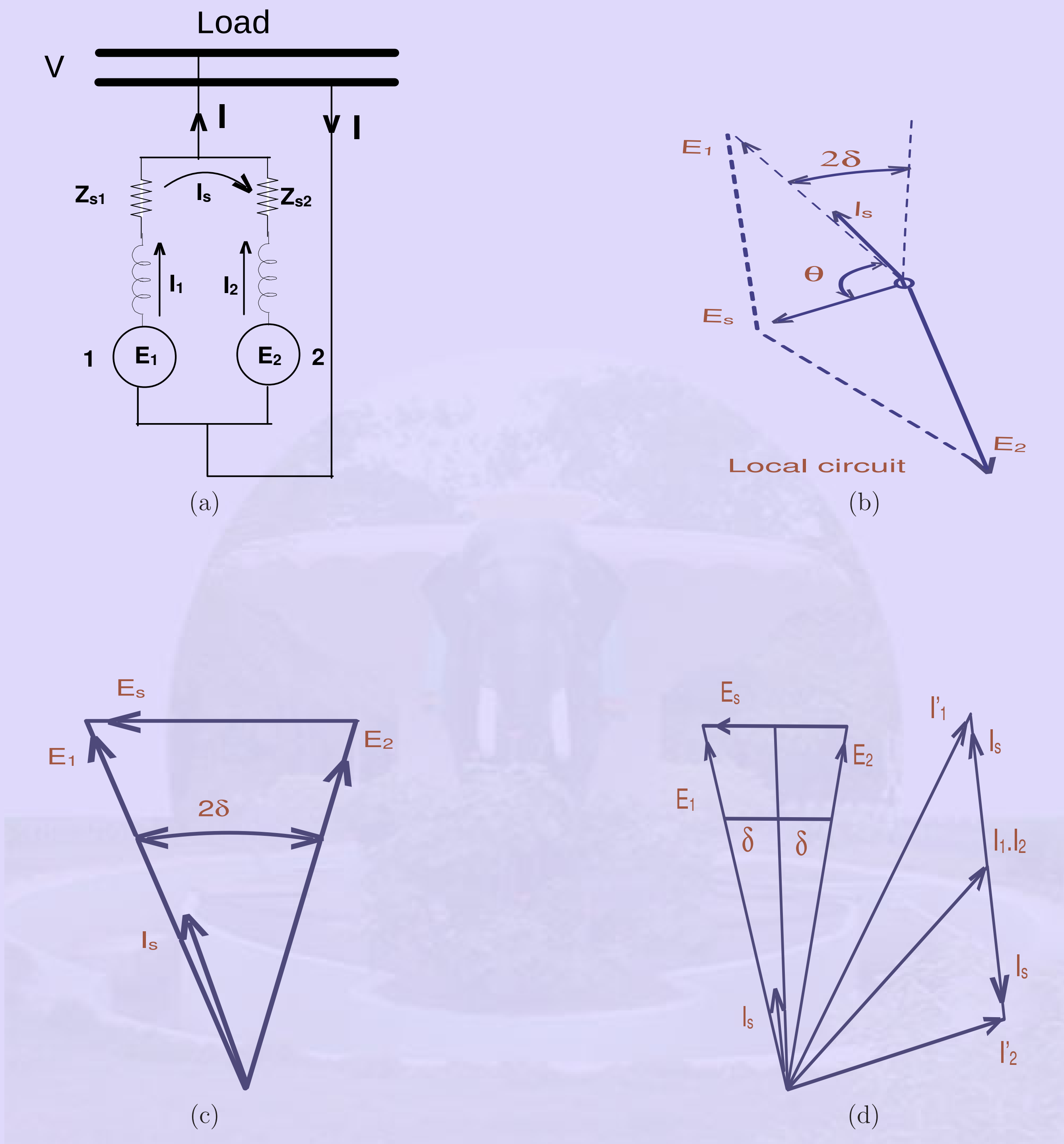


Figure 35: Parallel operation

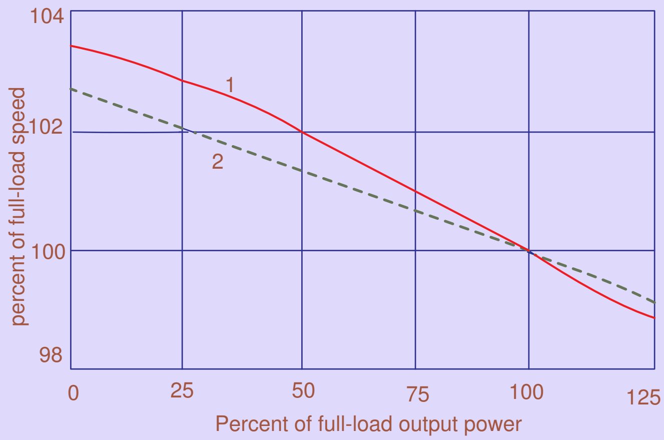


Figure 36: Governer Characteristic

general, the machines will have different synchronous impedance Z_{s1}, Z_{s2}; different e.m.f.'s E_1 and E_2 and different speed regulations. The governors of prime movers are usually arranged so that a reduction of the speed of the prime mover is necessary for the increase of the power developed. Unless the governor speed/load characteristics are identical the machines can never share the total load in accordance with their ratings. The governor characteristics take the form shown in Fig. 36. If the two are not the same, the load will be shared in accordance with the relative load values at the running speed, for synchronous machines must necessarily run at identical speeds.

5 Interconnected Synchronous Generators

The study of interconnection of several synchronous generators is important because of the following main reasons:

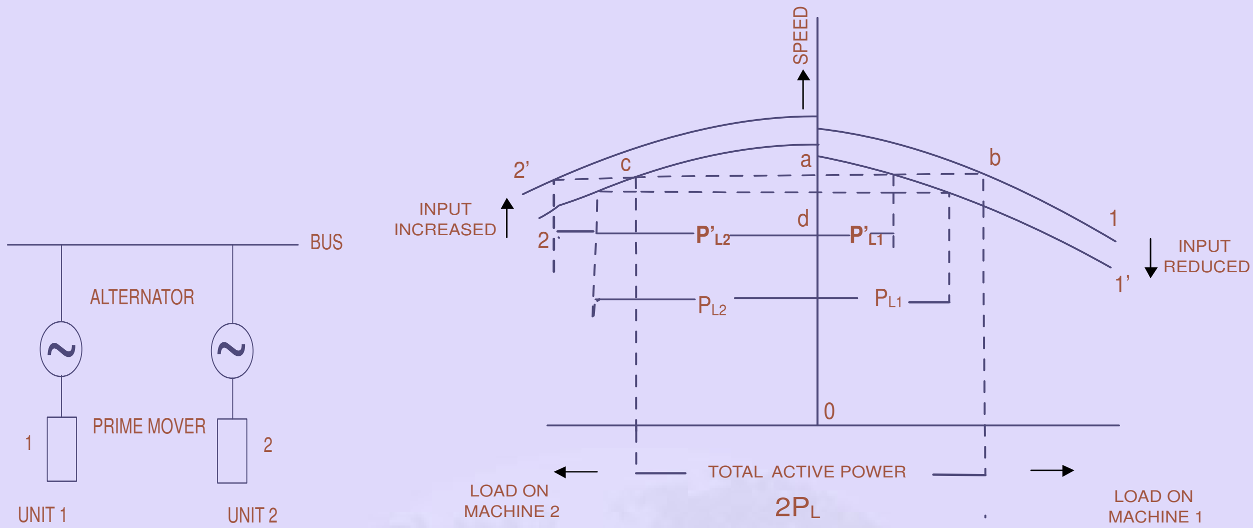
1. Since the demand of electricity varies during a day, also during the various seasons in a year, a modern power station employs two or more units so that one or more alternators can supply power efficiently according to the need. Installation of a single generator of capacity equal to the installed capacity of a station will be uneconomic, as such a generator will have to be run at a reduced load for certain periods of the day, and also building of such a generator is difficult proposition. Further, routine maintenance requires a unit to be shut down for a certain period of time and as such the capacity requirement of the stand by unit in a power station with several alternators is less.

2. Connections of several stations by a grid is economic and advantageous. This reduces the installed capacity of the stand by unit considerably, and enables economic distributions of load between several stations. Also, in a country like India, where considerable amount of power is generated by harnessing waterpower,parallel operation of steam and hydro-stations is essential to maintain continuity of supply throughout the year and also to ensure the maximum utilization of water power resources.

5.1 Load Sharing

For alternators in parallel, change in field excitation will mainly change the operating power factor of the generator and has primarily no effect on the active power delivered by the generators (change in power factor will change the total current of an alternator thereby changing copper loss. The output active power will alter through a very small amount). The control of active power shared between alternators is affected by changing the input power to the prime mover. For example, in a thermal power station: having alternators driven by steam turbines, an increase of throttle opening and thus allowing more steam into the turbine will increase the power input; in a hydro station, the power input is controlled by water inlet into the turbine. The prime-mover, speed-load characteristics thus determine the load sharing between the alternators.

Consider for simplicity, a two machine case, consisting of two non-salient pole synchronous machines (generators) 1 and 2 respectively coupled to prime-movers 1 and 2 Fig. 37 shows the speed-load characteristics of the prime- movers. Assume that initially the two generators share equal active power and it is now required to transfer a certain amount of power from unit 1 to unit 2, the total power remaining constant.



(a) (b)

Figure 37: Interconnection and load sharing

The initial operating points are indicated on the characteristic by points b and c, the busbar speed (or frequency) being given by the point a. The load on each machine is P_L. the total load being $2P_L$. To reduce the load on unit 1, its input is decreased (by reducing the throttle opening) so that the prime-mover characteristic is now given by $1'$. The total load being constant, the loads shared by the machines are

$$\text{machine } 1 \rightarrow P_{L1},$$
$$\text{machine } 2 \rightarrow P_{L2},$$

the total load being $P_{L1} + P_{L2} = 2P_L$, and the bus frequency given by the point d is reduced. To maintain the bus frequency constant at its original value (given by point a) the input to unit 2 must be suitably increased so that its speed-load characteristic is given by $2'$. The final load sharing is thus given by

$$\text{machine } 1 \rightarrow P'_{L1}, \text{ machine } 2 \rightarrow P'_{L2}$$

and

$$P'_{L1} + P'_{L2} = 2P_L \tag{46}$$

5.2 Generator input and output

For any load conditions represented e.g. by Fig. 38 the output per phase is $P = VI\cos\phi$. The electrical power converted from mechanical power input is per phase

$$P_1 = EI\cos(\phi + \sigma) \tag{47}$$

Resolving E along I,

$$P_1 = EI\cos(\phi + \delta) = VI\cos\phi + Ir.I = VI\cos\phi + I^2 r \tag{48}$$

The electrical input is thus the output plus the I^2R loss, as might be expected. The prime mover must naturally supply also the friction, windage and core losses, which do not appear in the complexor diagram.
For a given load current I at external phase-angle ϕ to V, the magnitude and phase of E are determined by Z_s The impedance angle θ is arc $\tan(x_3/r)$, and using Fig. 38.

$$\begin{aligned} I &= (E - V)\angle z_s = (E\angle\delta - V/0)/z_s/\angle\theta \\ &= (E/z_s)\angle(\delta - \theta) - (V/z_s)\angle\theta \end{aligned} \tag{49}$$

when referred to the datum direction $V = V\angle\theta$ Converting to the rectangular form:

$$\begin{aligned} I &= (E/z_s)[\cos(\theta - \delta) - j\sin(\theta - \delta)] - (V/Z_s)[\cos\theta - j\sin\theta] \\ &= [\frac{E}{Z_s}\cos(\theta - \delta) - \frac{V}{Z_s}\cos\theta] + j[\frac{E}{Z_s}\sin(\theta - \delta) - \frac{V}{Z_s}\sin\theta]. \end{aligned} \tag{50}$$

These components represent $I\cos\phi$ and $I\sin\phi$. The power converted internally is the sum of the corresponding components of the current with $E\cos\delta$ and $E\sin\delta$, to give $P_1 = E\cos(\phi + \delta)$:

$$\begin{aligned} P_1 &= E\cos\delta[(E/Z_s)\cos(\theta - \delta) - (V/Z_s)\cos\theta] \\ &+ E\sin\delta[(E/Z_s)\sin(\theta - \delta)(V/Z_s)\sin\theta] \\ &= E[(E/Z_s)\cos\theta] - E[(V/Z_s)\cos(\theta + \delta)] \\ &= (E/Z_s)[E\cos\theta - V\cos(\theta + \delta)]\ perphase \end{aligned} \tag{51}$$

The output power is $VI\cos\phi$, which is given similarly by

$$P = (V/Z_s)[E\cos(\theta - \delta) - V\cos\theta]\ perphase \tag{52}$$

In large machines the resistance is small compared with the synchronous reactance so that $\theta = arc\tan(x_s/r) \simeq 90°$.Eqn. 50 and Eqn. 52 the simplify to $P_1 = P$, where

$$P = P_1 = E\cos(\theta + \delta) \simeq (E/X_s)V\sin\delta \tag{53}$$

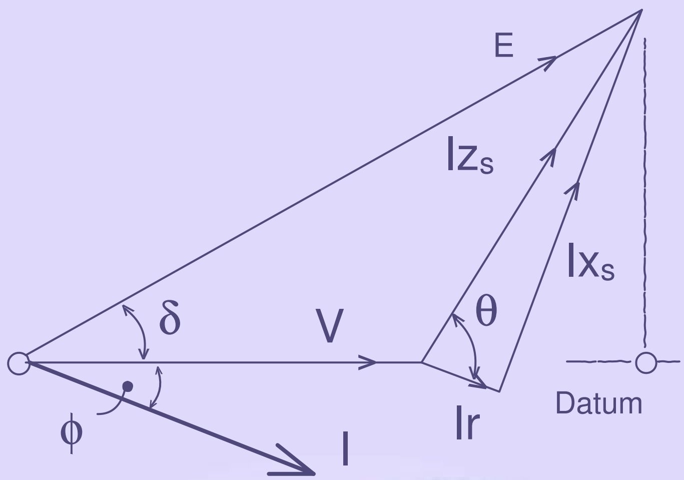


Figure 38: Power conditions

Thus the power developed by a synchronous machine with given values of E, V and Z_s, is proportional to $\sin\delta$ or, for small angles, to δ itself. The displacement angle δ represents the change in relative position between the rotor and resultant pole-axes and sis proportional to the load power. The term load-, power- or torque-angle may be applied to δ.
An obvious deduction from Eqn. 53 is that the greater the field excitation (corresponding to E), the greater is the output per unit angle δ; that is, the more stable will be the operation.

5.3 Synchronous Machine on Infinite Bus-bars.

So far we have discussed the behavior of a synchronous generator or a pair of synchronous generator supplying a single concentrated load. In view of the tremendous increase in the size of interconnected transmission and distribution systems in the last few decades, and the power generation is contracted at a few large power stations. The generating plant capacity is of a few hundred or thousand MVAs. In such a plant several generators (of say a few hundred 100 kvAs each) will be operated in parallel. Not all of them will be operating simultaneously as we may not have the demand for the total cdapacity of the plant all the time. Assume the behaviour of a single machine connected to this type of a large generating plant is not likely to disturb the voltage and frequency provided the rating of the machine is only a fraction of the total capacity of the generating plant. In the limit, we may presume that the generating plant maintains an invariable voltage and frequency at all points.In other words a network has zero impedance and infinite rotational inertia. A synchronous machine connected to such a network is said to be operating on infinite bus-bars.

As such, we can expect that, characteristics of a synchronous generator on infinite bus-bars are going to be quite different from those when it operates on its own concentrated load. As already described, a change in the excitation changes the terminal voltage, while the power factor is determined by the load, supplied by the stand alone synchronous generator. On the other hand, no alteration of the excitation can change the terminal voltage, (which is fixed by the network) when it is connected to bus bars, the power factor, however, is affected. In both cases the power developed by a generator depends on the mechanical power supplied. Likewise the electrical power received by a motor depends on the mechanical load applied at its shaft.

Practically all synchronous motors and generators in normal industrial use on large power supply systems can be considered as connected to infinite bus-bars, the former because they are relatively small, the latter on account of the modern automatic voltage regulators for keeping the voltage practical, constant at all loads. The behaviour of the synchronous machine connected to infinite bus bars can be easily described from the electrical load diagram of a synchronous generator.

5.3.1 Basis for drawing the general load diagram.

Consider a synchronous machine connected to infinite bus bars (of constant-voltage, constant-frequency) of phase voltage V, Fig. 39. Let the machine run on no load with mechanical and core losses only supplied. If the e.m.f. E be adjusted to equality with V, no current will flow into or out of the armature on account of the exact balance between the e.m.f. and the bus-bar voltage. This will be the case when a synchronous generator is just parallel to infinite bus bar. If the excitation,I_f is reduced, machine E will tend to be less than V, so that a leading current I_r will flow which will add to the field ampere-turns due to direct magnetizing effect of armature reaction. Under the assumption of constant synchronous impedance, this is taken into account by I_rZ_s as the difference between E and V. The current I_r must be completely reactive because the machine is on no-load and no electrical power is being supplied to or by the machine, as it is on no-load. If now the excitation be increased, E will tend to be greater than V. A current will therefore be circulated in the armature circuit, this time a lagging current which will reduce the net excitation due to the demagnetizing effect of armature reaction so that the machine will again generates a voltage equal to that of the constant bus-bar voltage. The synchronous impedance drop $I_r Z_s$ is, as before, the difference between E and V, and there should be only a zero-power-factor lagging current, as the machine is running on no-load.

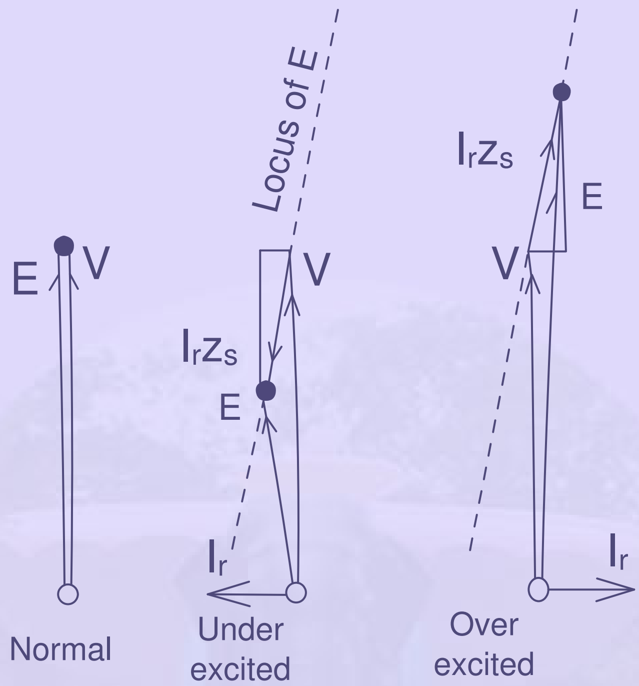


Figure 39: Generator on infinite Bus-Bar -(No load)

Suppose the machine to be supplied with full-load mechanical power. Then as a generator it must produce the equivalent in electrical power: i.e. the output current must have an active component I_{aa} corresponding to full-load electrical power. For an output at exactly unity power factor, the excitation must be adjusted so that the voltage triangle $E, V, I_a Z_s$, satisfies the conditions required, Fig. 40. If the excitation be reduced, a magnetizing reactive component is supplied in addition, i.e. a leading current I_{ar}, which assists the field winding to produce the necessary flux. If the machine is over excited, a lagging reactive demagnetizing current component is supplied, in addition to the constant power component.

In Fig. 40 the IZ_s drop has been added in components corresponding to the current components I_{aa} and I_{ar}. For all three diagrams of Fig. 40, I_{aa} and $I_{aa}Z_s$ are constant, since the electrical power supplied is constant. Only the component $I_{aa}Z_s$ (and therefore I_{ar}) varies with the excitation. Thus the excitation controls only the power factor of the current

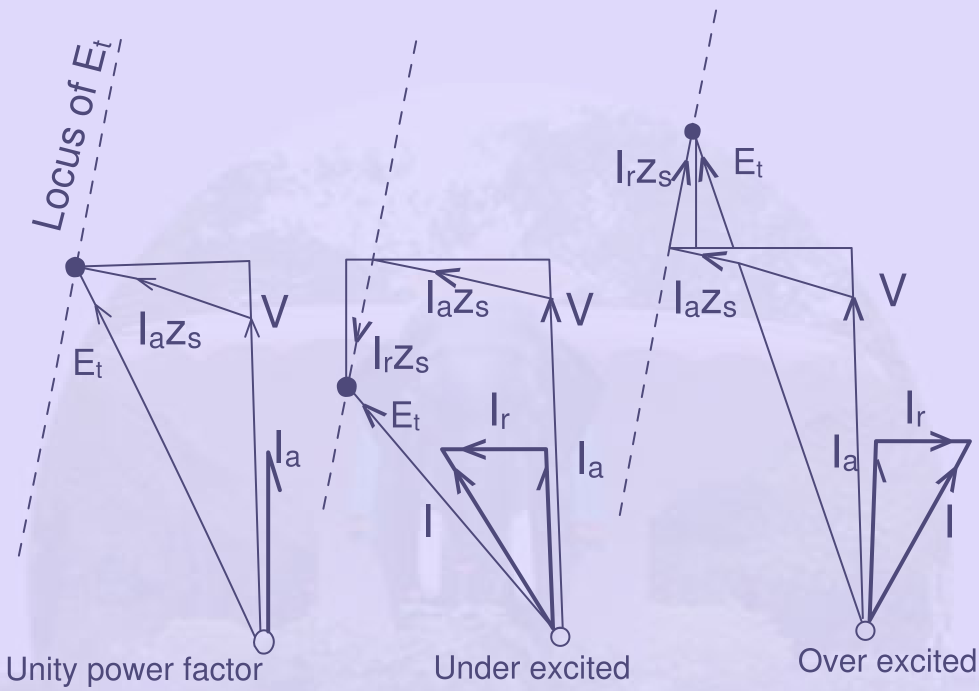


Figure 40: Generator on infinite Bus-Bar - (Full load power)

supplied by the generator to the infinite bus-bars and not the active power.

From this diagram for different excitations, we can see that the extremities of the phasor of E (indicated by dots) are seen to lie on the straight line shown dashed. Since all three diagrams refer to full-load power, the dotted line becomes the locus of E and of the excitation, to scale for constant power output. This is the basis of the electrical load diagram, Fig. 42.

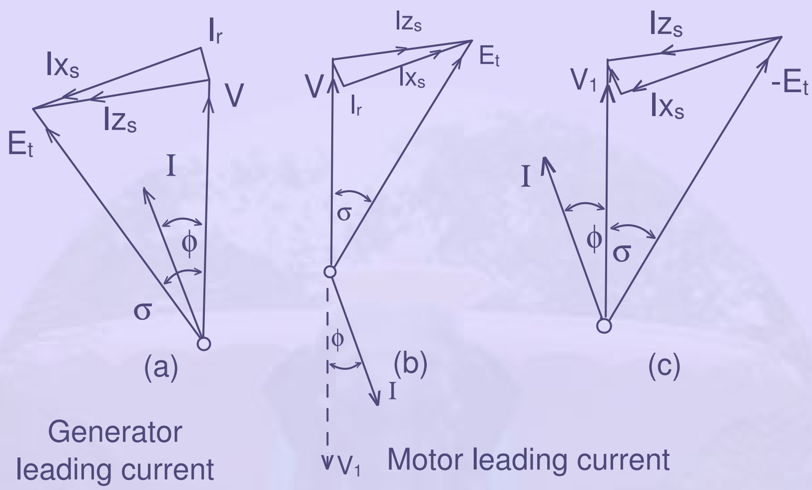


Figure 41: Generator and motor on infinite Bus-bars

A generator working-on infinite bus-bars will become a motor if its excitation is maintained and the prime mover replaced by a mechanical load. The change in the phasor of I_a is shown in the phasor diagrams Fig. 41(a and b). V is the output voltage of the machine, furnished by the e.m.f generated. For the motor, the current is in phase-opposition to V, since it is forced into the machine against the output voltage. For convenience, the supply voltage V_1 (equal and opposite to V) may be used when the motor is considered, and the diagram then becomes that of Fig. 41(c). The retarded angle δ of E or $-E$ is descriptive of the fact that when the shaft of the machine is loaded, it falls slightly relative to the stator rotating field in order to develop the torque, required by the load.

Thus, the power-angle δ, Fig. 41, plays an important role in the operation of a synchronous machine. Changes in load or excitation change its magnitude. When a machine alters from generator to motor action, δ reverses; and when δ is caused to increase excessively, the machine becomes unstable.

5.3.2 Electrical load diagram.

The electrical load diagram is shown in Fig. 42. The phasor V represents the constant voltage of the infinite bus-bars. At the extremity of V is drawn an axis showing the direction of the I_aZ_s drops—i.e. the voltage drops for unity-power-factor output currents. This axis must be drawn at the angle $\theta = arc\tan(X_s/r)$ to V, to scale along the axis is a distance corresponding to, say, full load at unity power factor. At this point a line is drawn at right angles to the axis. It is the locus of the E values for constant power, or constant-electrical-power line. Other parallel lines are drawn for other loads, one through the extremity of V itself corresponding to zero power output, others on the right-hand side of V corresponding to negative power output, i.e. input to the machine as a motor.

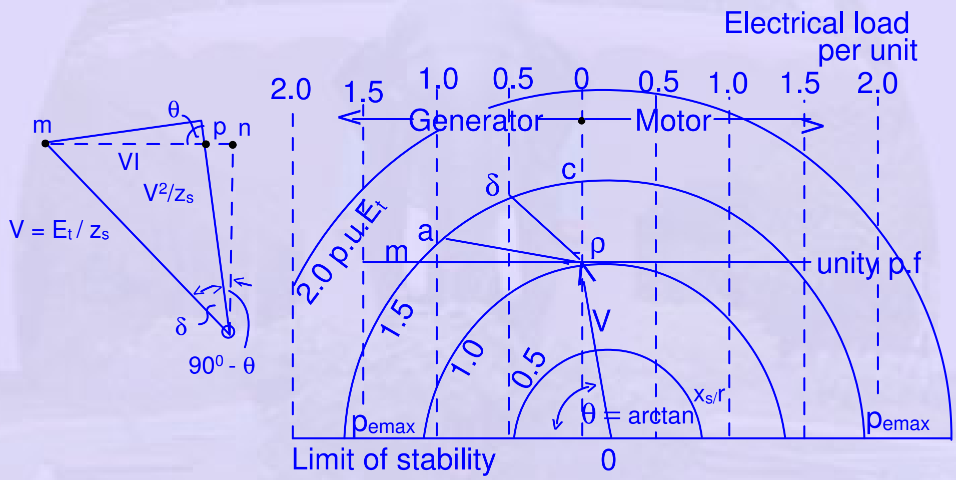


Figure 42: Electrical load diagram

The diagram solves Eqn. 52. Consider the full-load unity-power- factor case in Fig. 40, and multiply each complex voltage by the constant (V/Z_s). This gives the inset in Fig. 42, from which $VI = P = mp = mn - np$. Now $mn = (EV/Z_s)\sin(90^\circ - \theta + \delta)$ and $np = (V^2/Z_s)\cos\theta$, so that P is given directly by Eqn. 52.

If the excitation be fixed, the extremity of the e.m.f. vector E, will have a circular locus as indicated by the circular arcs struck with O as centre. Taking 1.0 per unit E as that for which $E = V$ on no load and no current, the per-unit excitation for any other loading condition can be found from the diagram. Thus with 1.5 per unit excitation, the machine will work on full-load power as a generator with a power factor of $\cos 8°$ lagging; on half-fun-load power with a power factor of $\cos 42°$ lagging; and on zero power output with a power-factor of zero lagging, as shown by the lines pa, pb and pc. The variation of the power output (controlled by the input from the prime mover in the case of a generator and by the load applied to the shaft for a motor) with constant excitation is thl1S accompanied by changes in the load power factor.

If the generator be provided with greater mechanical power with say, 150 per cent (or 1.5 per unit) excitation, then the output power increases with reducing power factor from lagging values until, with an output (for this case) of 1.2 per unit power (see Fig. 42), the power factor becomes unity. Thereafter the power increases with a reducing power factor-now leading. Finally the excitation will not include any more constant-power lines, for the circle of its locus becomes tangential to these. If more power is supplied by the prime mover, the generator will be forced to rise out of step, and synchronous running will be lost. The maximum power that can be generated is indicated by intercepts on the limit of stability. The typical point P_{emax} on the left of the load diagram is for an excitation of 1.5 per unit.

Similarly, if a motor is mechanically overloaded it will fall out of step, because of its limited electrical power intake. The point P_{emax} in the motor region again corresponds to 1.5 per unit excitation, and all such points again lie on the limiting-stability line. This maximum power input includes I^2R loss, and the remainder-the mechanical power output-in fact becomes itself limited before maximum electrical input can be attained.

5.3.3 Mechanical load diagram

The mechanical load, or electromagnetically-converted power P_1 of Eqn. 52, is for a generator the net mechanical input. For a motor it is the gross mechanical output including core friction and windage loss. A diagram resembling that of Fig. 42 could be devised* by. resolving the current along E to give $P_1 = EI\cos(\theta + \phi)$. But as the terminal voltage V is taken to be constant, a new circle with another centre is needed for each value of E selected. The following method obtains the mechanical loading from the difference I^2r between P and P_1.

The input to a motor is $P = V_1 I \cos\phi$. The electro-magnetic or converted or developed power, which includes the losses due to rotation, is $P_1 = V_1 I \cos\phi$. From the latter,

$$I^2 - V_1 I \cos\phi/r + P_1/r = 0 \tag{54}$$

giving

$$I = \frac{V_1 \cos\phi}{2r} \pm \sqrt{[(\frac{V_1 \cos\phi}{2r})^2 - \frac{P_1}{r}]} \tag{55}$$

For each power factor $\cos\phi$, and given voltage V_1 and electro-magnetic power P_1, there are two values of current, one leading and one lagging. The complexor diagrams, Fig. 43 and Fig. 45, show that there will be two corresponding values of excitation E one large and one small, associated respectively with leading and lagging reactive current components $I_r = I \sin\phi$. At the same time the increased I^2R loss for power factors less than unity requires the active component $I_a = I \cos\phi$ to be larger. The locus of I then forms an O-curve, while the plot of the current magnitude to a base of excitation E gives a V -curve, Fig. 46.

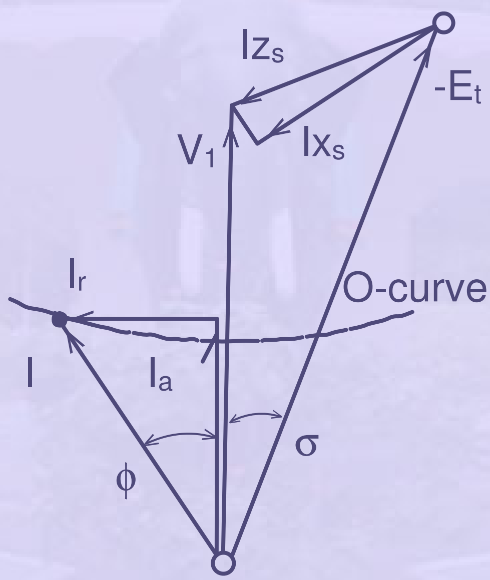


Figure 43: Synchronous motor with constant output and variable excitation -Leading current

The O-curves are circular arcs, because Eqn. 55 represents the equation to a circle. Writing

$$(I\cos\phi)^2 + (I\sin\phi)^2 - (V_1/r)(I\cos\phi) + P_1/r = 0 \tag{56}$$

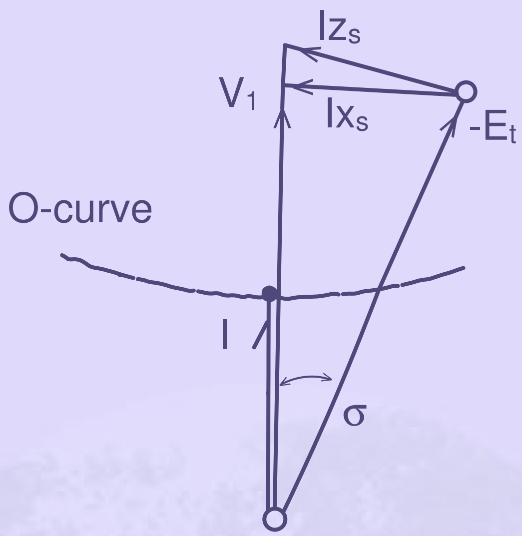


Figure 44: Synchronous motor with constant output and variable excitation- Unity p.f

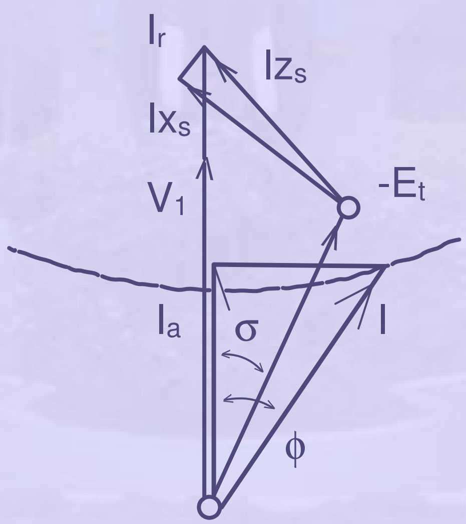


Figure 45: Synchronous motor with constant output and variable excitation-Lagging current

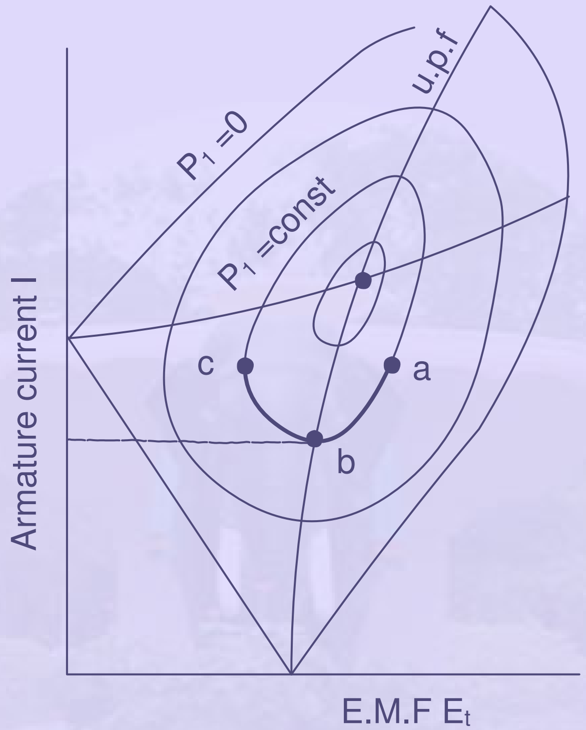


Figure 46: Synchronous motor with constant output and variable excitation-V-curves

it is seen that I must lie on a circle centred at a point distant $V_1/2r$

from the origin the axis of $I\cos\phi$, the radius of the circle being $\sqrt{[(V_1^2/4r^2)(P_1/r)]}$.

The construction of the mechanical load diagram is given in Fig. 47. Let $OM = V_1/2r$ to scale: draw with M as centre a circle of radius OM. This circle, from Eqn. 55, corresponds to $PI = 0$, a condition for M which the circle radius is $V_1/2r$. The circle thus represents the current locus for zero mechanical power. Any smaller circle on centre M represents the current locus for some constant , mechanical power output P_1.

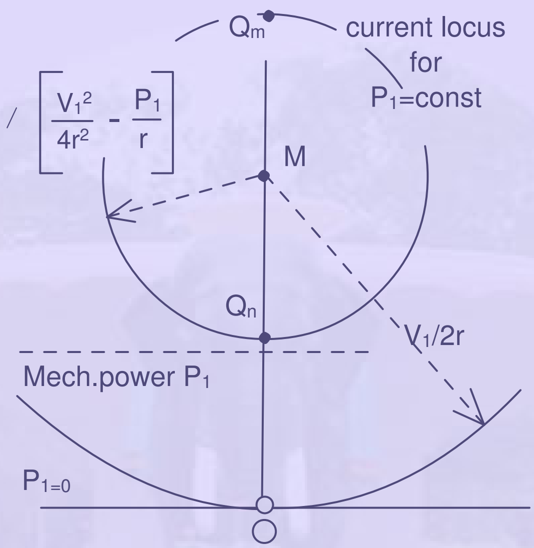


Figure 47: Pertaining to O-curves

For unity power factor

$$I = (V_1/2r) \pm \sqrt{[(V_1/2r)^2 - (P_1/r)]} \tag{57}$$

Again there are in general two values O-CURVES of current for each power output P_1, the smaller OQ_n in the working range, the greater OQ_m above the limit of stability. If $P_1/r = V_1^2/4r^2$, there is a single value of current $I = V_1/2r$ corresponding to the maximum power $P_{1m} = V_1^2/4r$. The power circle has shrunk to zero radius and becomes in fact the point M. The efficiency is 50 per cent, the I^2R loss being equal to the mechanical output. Such a condition is well outside the normal working range, not only because of heating but also because the stability is critical. The case corresponds to the requirement of the maximum-power-transfer theorem, commonly employed to determine maximum-power-output conditions in telecommunication circuits.

The completed mechanical load diagram is shown in Fig. 48, with the addition of $OR = V/Z_s$ drawn at angle arc $\cos(r/Z_s)$ to OM. Circles drawn with R as centre represent constant values of E_1/Z_s, or E, or the field excitation.

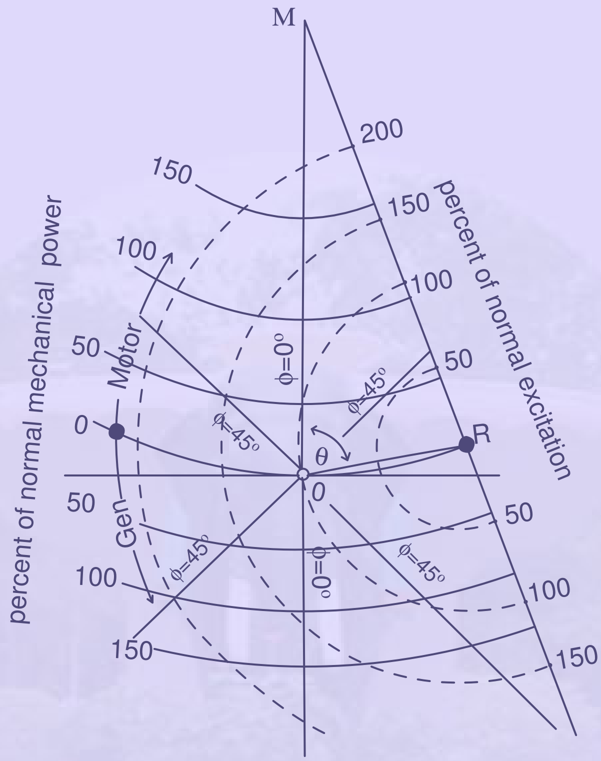


Figure 48: Load diagram-O-curves

5.3.4 O-Curves and V -Curves.

The current loci in Fig. 48 are continued below the base line for generator operation. The horizontal lines of constant mechanical power are now constant input (from the prime mover) and a departure from unity-power-factor working, giving increased currents, increases the I^2R loss and lowers the available electrical output. The whole system of lines depends, of course, on constant bus-bar voltage. The circular current loci are called the O - curves for

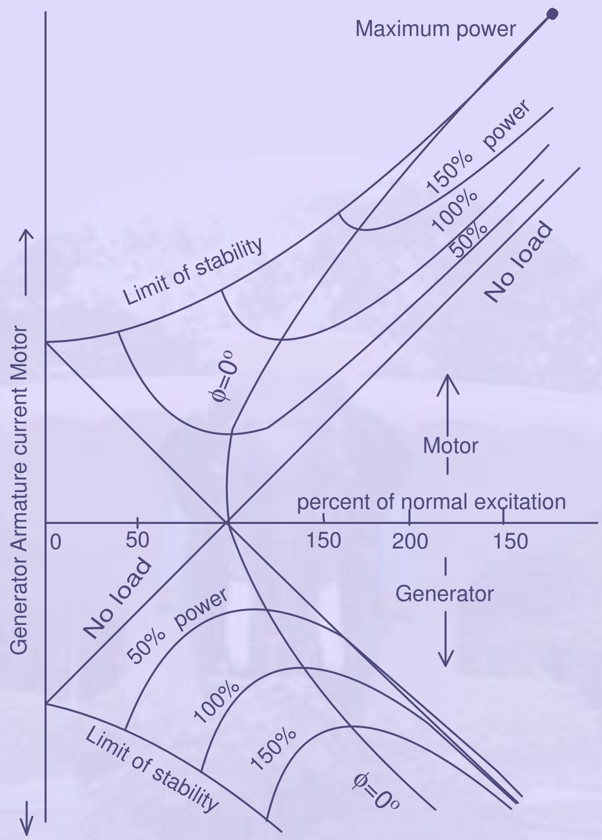


Figure 49: Load diagram-V-curves

constant mechanical power. Any point P on the diagram, fixed by the percentage excitation and load, gives by the line OP the current to scale in magnitude and phase. Directly from the O-curves, Fig. 48, the V -curves, relating armature current and excitation for various constant mechanical loads can be derived. These are shown in Fig. 49.

6 Synchronous motor

6.1 Principle of operation

In order to understand the principle of operation of a synchronous motor, let us examine what happens if we connect the armature winding (laid out in the stator) of a 3-phase synchronous machine to a suitable balanced 3-phase source and the field winding to a D.C source of appropriate voltage. The current flowing through the field coils will set up stationary magnetic poles of alternate North and South. (for convenience let us assume a salient pole rotor, as shown in Fig. 50). On the other hand, the 3-phase currents flowing in the armature winding produce a rotating magnetic field rotating at synchronous speed. In other words there will be moving North and South poles established in the stator due to the 3-phase currents i.e at any location in the stator there will be a North pole at some instant of time and it will become a South pole after a time period corresponding to half a cycle. (after a time = $\frac{1}{2f}$, where f = frequency of the supply). Let us assume that the stationary South pole in the rotor is aligned with the North pole in the stator moving in clockwise direction at a particular instant of time, as shown in Fig. 50. These two poles get attracted and

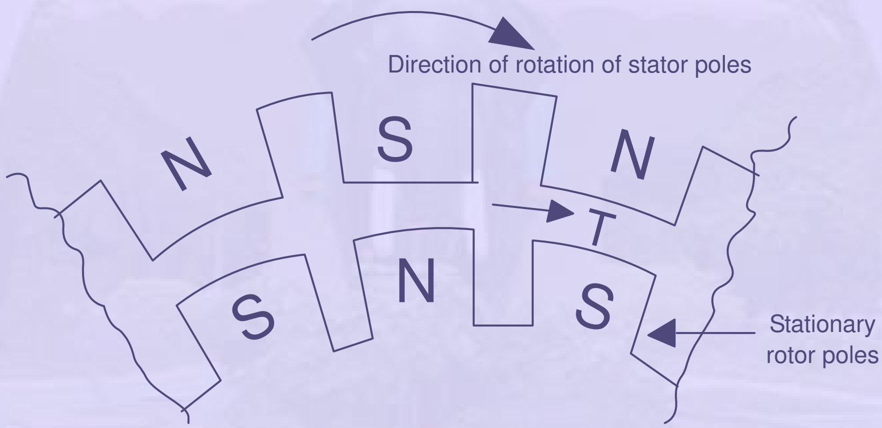


Figure 50: Force of attraction between stator poles and rotor poles - resulting in production of torque in clockwise direction

try to maintain this alignment (as per lenz's law) and hence the rotor pole tries to follow the stator pole as the conditions are suitable for the production of torque in the clockwise direction. However the rotor cannot move instantaneously due to its mechanical inertia, and so it needs sometime to move. In the mean time, the stator pole would quickly (a time duration corresponding to half a cycle) change its polarity and becomes a South pole. So the force of attraction will no longer be present and instead the like poles experience a force of repulsion as shown in Fig. 51. In other words, the conditions are now suitable for the

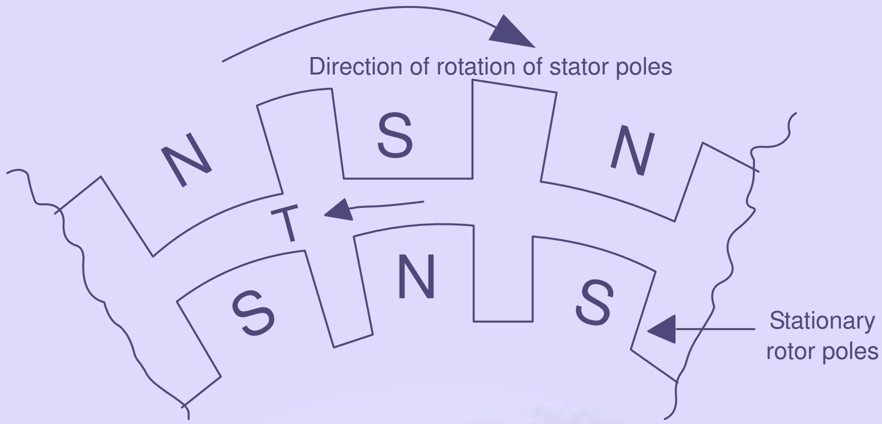


Figure 51: Force of repulsion between stator poles and rotor poles - resulting in production of torque in anticlockwise direction

production of torque in the anticlockwise direction. Even this condition will not last longer as the stator pole would again change to North pole after a time of $\frac{1}{2f}$. Thus the rotor will experience an alternating force which tries to move it clockwise and anticlockwise at twice the frequency of the supply, i.e. at intervals corresponding to $\frac{1}{2f}$ seconds. As this duration is quite small compared to the mechanical time constant of the rotor, the rotor cannot respond and move in any direction. The rotor continues to be stationary only.

On the contrary if the rotor is brought to near synchronous speed by some external means say a small motor (known as pony motor-which could be a D.C or AC induction rotor) mounted on the same shaft as that of the rotor, the rotor poles get locked to the unlike poles in the stator and the rotor continues to run at the synchronous speed even if the supply to the pony motor is disconnected.

Thus the synchronous rotor cannot start rotating on its own or usually we say that the synchronous rotor has no starting torque. So, some special provision has to be made either inside the machine or outside of the machine so that the rotor is brought to near about its synchronous speed. At that time, if the armature is supplied with electrical power, the rotor can pull into step and continue to operate at its synchronous speed. Some of the commonly used methods for starting synchronous rotor are described in the following section.

6.2 Methods of starting synchronous motor

Basically there are three methods that are used to start a synchronous motor:

- To reduce the speed of the rotating magnetic field of the stator to a low enough value that the rotor can easily accelerate and lock in with it during one half-cycle of the rotating magnetic field's rotation. This is done by reducing the frequency of the applied electric power. This method is usually followed in the case of inverter-fed synchronous motor operating under variable speed drive applications.
- To use an external prime mover to accelerate the rotor of synchronous motor near to its synchronous speed and then supply the rotor as well as stator. Ofcourse care should be taken to ensure that the direction of rotation of the rotor as well as that of the rotating magnetic field of the stator are the same. This method is usually followed in the laboratory- the synchronous machine is started as a generator and is then connected to the supply mains by following the synchronization or paralleling procedure. Then the power supply to the prime mover is disconnected so that the synchronous machine will continue to operate as a motor.
- To use damper windings or amortisseur windings if these are provided in the machine. The damper windings or amortisseur windings are provided in most of the large synchronous motors in order to nullify the oscillations of the rotor whenever the synchronous machine is subjected to a periodically varying load.

Each of these methods of starting a synchronous motor are described below in detail.

6.2.1 Motor Starting by Reducing the supply Frequency

If the rotating magnetic field of the stator in a synchronous motor rotates at a low enough speed, there will be no problem for the rotor to accelerate and to lock in with the stator's magnetic field. The speed of the stator magnetic field can then be increased to its rated operating speed by gradually increasing the supply frequency f up to its normal 50- or 60-Hz value.
This approach to starting of synchronous motors makes a lot of sense, but there is a big problem: Where from can we get the variable frequency supply? The usual power supply systems generally regulate the frequency to be 50 or 60 Hz as the case may be. However, variable-frequency voltage source can be obtained from a dedicated generator only in the

olden days and such a situation was obviously impractical except for very unusual or special drive applications.
But the present day solid state power converters offer an easy solution to this. We now have the rectifier- inverter and cycloconverters, which can be used to convert a constant frequency AC supply to a variable frequency AC supply. With the development of such modern solid-state variable-frequency drive packages, it is thus possible to continuously control the frequency of the supply connected to the synchronous motor all the way from a fraction of a hertz up to and even above the normal rated frequency. If such a variable-frequency drive unit is included in a motor-control circuit to achieve speed control, then starting the synchronous motor is very easy-simply adjust the frequency to a very low value for starting, and then raise it up to the desired operating frequency for normal running.
When a synchronous motor is operated at a speed lower than the rated speed, its internal generated voltage (usually called the counter EMF) $E_A = K\phi\omega$ will be smaller than normal. As such the terminal voltage applied to the motor must be reduced proportionally with the frequency in order to keep the stator current within the rated value. Generally, the voltage in any variable-frequency power supply varies roughly linearly with the output frequency.

6.2.2 Motor Starting with an External Motor

The second method of starting a synchronous motor is to attach an external starting motor (pony motor) to it and bring the synchronous machine to near about its rated speed (but not exactly equal to it, as the synchronization process may fail to indicate the point of closure of the main switch connecting the synchronous machine to the supply system) with the pony motor. Then the output of the synchronous machine can be synchronised or paralleled with its power supply system as a generator, and the pony motor can be detached from the shaft of the machine or the supply to the pony motor can be disconnected. Once the pony motor is turned OFF, the shaft of the machine slows down, the speed of the rotor magnetic field B_R falls behind B_{net}, momentarily and the synchronous machine continues to operate as a motor. As soon as it begins to operates as a motor the synchronous motor can be loaded in the usual manner just like any motor.

This whole procedure is not as cumbersome as it sounds, since many synchronous motors are parts of motor-generator sets, and the synchronous machine in the motor-generator set may be started with the other machine serving as the starting motor. More over, the starting motor is required to overcome only the mechanical inertia of the synchronous machine without any mechanical load (load is attached only after the synchronous machine is paralleled to the power supply system). Since only the motor's inertia must be overcome, the starting motor can have a much smaller rating than the synchronous motor it is going to start. Generally most of the large synchronous motors have brushless excitation systems mounted on their shafts. It is then possible to use these exciters as the starting motors. For many medium-size to large synchronous motors, an external starting motor or starting by using the exciter may be the only possible solution, because the power systems they are tied to may not be able to handle the starting currents needed to use the damper (amortisseur) winding approach described next.

6.2.3 Motor Starting by Using damper (Amortisseur) Winding

As already mentioned earlier most of the large synchronous motors are provided with damper windings, in order to nullify the oscillations of the rotor whenever the synchronous machine is subjected to a periodically varying load. Damper windings are special bars laid into slots cut in the pole face of a synchronous machine and then shorted out on each end by a large shorting ring, similar to the squirrel cage rotor bars. A pole face with a set of damper windings is shown in Figure..

When the stator of such a synchronous machine is connected to the 3-Phase AC supply, the machine starts as a 3-Phase induction machine due to the presence of the damper bars, just like a squirrel cage induction motor. Just as in the case of a 3-Phase squirrel cage induction motor, the applied voltage must be suitably reduced so as to limit the starting current to the safe rated value. Once the motor picks up to a speed near about its synchronous speed, the DC supply to its field winding is connected and the synchronous motor pulls into step i.e. it continues to operate as a Synchronous motor running at its synchronous speed.

6.3 Behavior of a synchronous motor

The behavior of a synchronous motor can be predicted by considering its equivalent circuit on similar lines to that of a synchronous generator as described below.

6.3.1 Equivalent circuit model and phasor diagram of a synchronous motor

The equivalent-circuit model for one armature phase of a cylindrical rotor three phase synchronous motor is shown in Fig. 52 exactly similar to that of a synchronous generator except that the current flows in to the armature from the supply. All values are given per phase. Applying Kirchhoff's voltage law to Fig. 52,

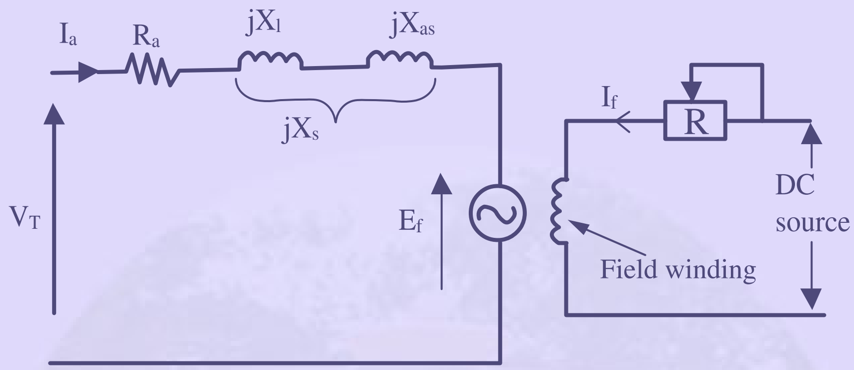


Figure 52: Equivalent-circuit model for one phase of a synchronous motor armature

$$\mathbf{V}_T = \mathbf{I}_a R_a + j\mathbf{I}_a X_l + j\mathbf{I}_a X_{as} + \mathbf{E}_f \quad (58)$$

Combining reactances, we have

$$X_s = X_l + X_{as} \quad (59)$$

Substituting Eqn. 59 in Eqn. 58

$$\mathbf{V}_T = \mathbf{E}_f + \mathbf{I}_a(R_a + jX_s) \quad (60)$$

$$or \quad \mathbf{V}_T = \mathbf{E}_f + \mathbf{I}_a\mathbf{Z}_s \quad (61)$$

where:
R_a = armature resistance (Ω/phase)
X_l = armature leakage reactance (Ω/phase)
X_s = synchronous reactance (Ω/phase)
Z_s = synchronous impedance (Ω/phase)
V_T = applied voltage/phase (V)
I_a = armature current/phase(A)

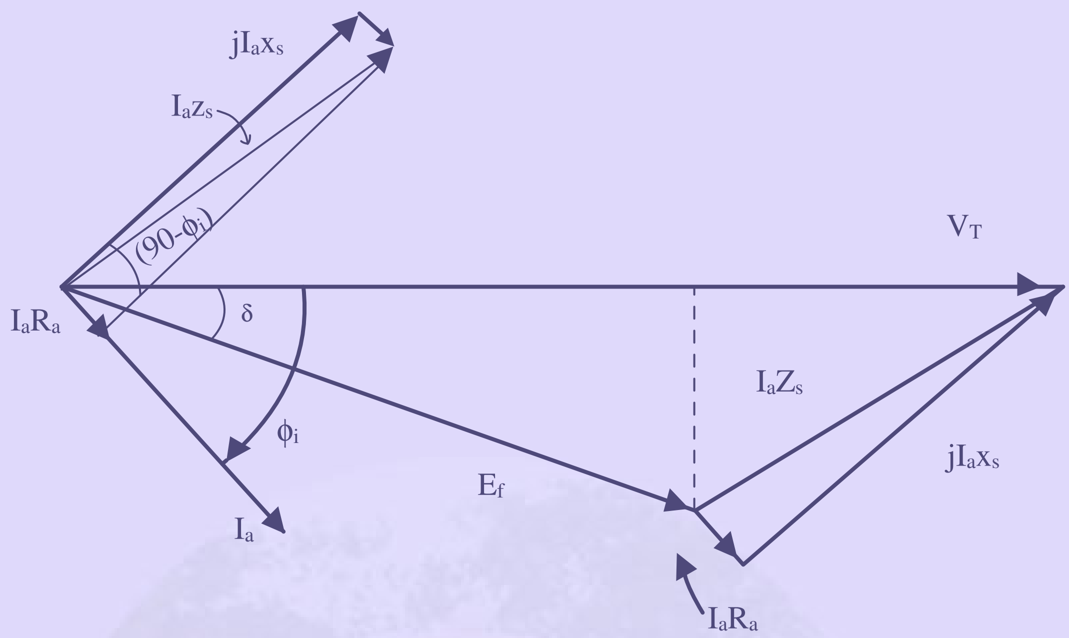


Figure 53: Phasor diagram corresponding to the equivalent-circuit model

A phasor diagram shown in Fig. 53, illustrates the method of determining the counter EMF which is obtained from the phasor equation;

$$\mathbf{E}_f = \mathbf{V}_T - \mathbf{I}_a\mathbf{Z}_s$$

The phase angle δ between the terminal voltage V_T and the excitation voltage E_f in Fig. 53 is usually termed the torque angle. The torque angle is also called the load angle or power angle.

6.3.2 Synchronous-motor power equation

Except for very small machines, the armature resistance of a synchronous motor is relatively insignificant compared to its synchronous reactance, so that Eqn. 61 to be approximated to

$$\mathbf{V}_T = \mathbf{E}_f + j\mathbf{I}_a X_s \tag{62}$$

The equivalent-circuit and phasor diagram corresponding to this relation are shown in Fig. 54 and Fig. 55. These are normally used for analyzing the behavior of a synchronous

motor, due to changes in load and/or changes in field excitation.
From this phasor diagram, we have,

$$I_a X_s \cos\theta_i = -E_f \sin\delta \tag{63}$$

Multiplying through by V_T and rearranging terms we have,

$$V_T I_a \cos\phi_i = \frac{-V_T E_f}{X_s}\sin\delta \tag{64}$$

Since the left side of Eqn. 64 is an expression for active power input and as the winding resistance is assumed to be negligible this power input will also represent the electromagnetic power developed, per phase, by the synchronous motor.
Thus,

$$P_{in,ph} = V_T I_a \cos\phi_i \tag{65}$$

or

$$P_{in,ph} = \frac{-V_T E_f}{X_s}\sin\delta \tag{66}$$

Thus, for a three-phase synchronous motor,

$$P_{in} = 3 * V_T I_a \cos\phi_i \tag{67}$$

or

$$P_{in} = 3 * \frac{-V_T E_f}{X_s}\sin\delta \tag{68}$$

Eqn. 66, called the synchronous-machine power equation, expresses the electro magnetic power developed per phase by a cylindrical-rotor motor, in terms of its excitation voltage and power angle. Assuming a constant source voltage and constant supply frequency, Eqn. 65 and Eqn. 66 may be expressed as proportionalities that are very useful for analyzing the behavior of a synchronous-motor:

$$P \propto I_a \cos\theta \tag{69}$$

$$P \propto E_f \sin\delta \tag{70}$$

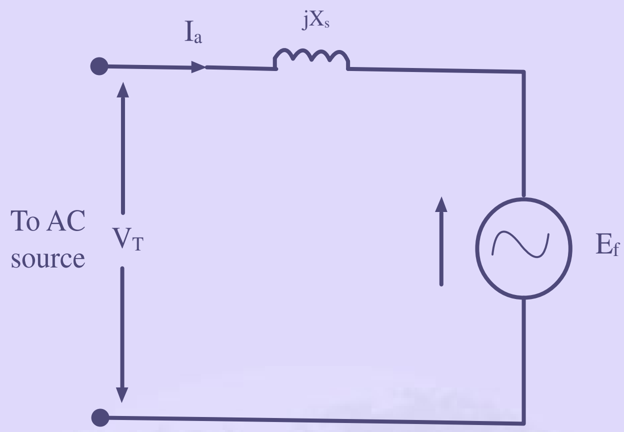


Figure 54: Equivalent-circuit of a synchronous-motor, assuming armature resistance is negligible

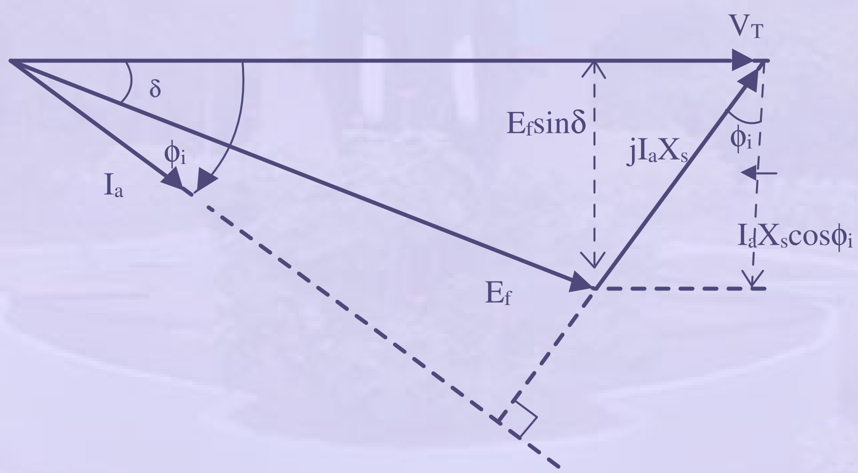


Figure 55: Phasor diagram model for a synchronous-motor, assuming armature resistance is negligible

6.3.3 Effect of changes in load on armature current, power angle, and power factor of synchronous motor

The effects of changes in mechanical or shaft load on armature current, power angle, and power factor can be seen from the phasor diagram shown in Fig. 56; As already stated, the applied stator voltage, frequency, and field excitation are assumed, constant. The initial load conditions, are represented by the thick lines. The effect of increasing the shaft load to twice its initial value are represented by the light lines indicating the new steady state conditions. These are drawn in accordance with Eqn. 69 and Eqn. 70, when the shaft load is doubled both $I_a \cos\phi_i$ and $E_f \sin\delta$ are doubled. While redrawing the phasor diagrams to show new steady-state conditions, the line of action of the new jI_aX_s phasor must be perpendicular to the new I_a phasor. Furthermore, as shown in Fig. 56, if the excitation is not changed, increasing the shaft load causes the locus of the E_f phasor to follow a circular arc, thereby increasing its phase angle with increasing shaft load. Note also that an increase in shaft load is also accompanied by a decrease in ϕ_i; resulting in an increase in power factor.

As additional load is placed on the machine, the rotor continues to increase its angle

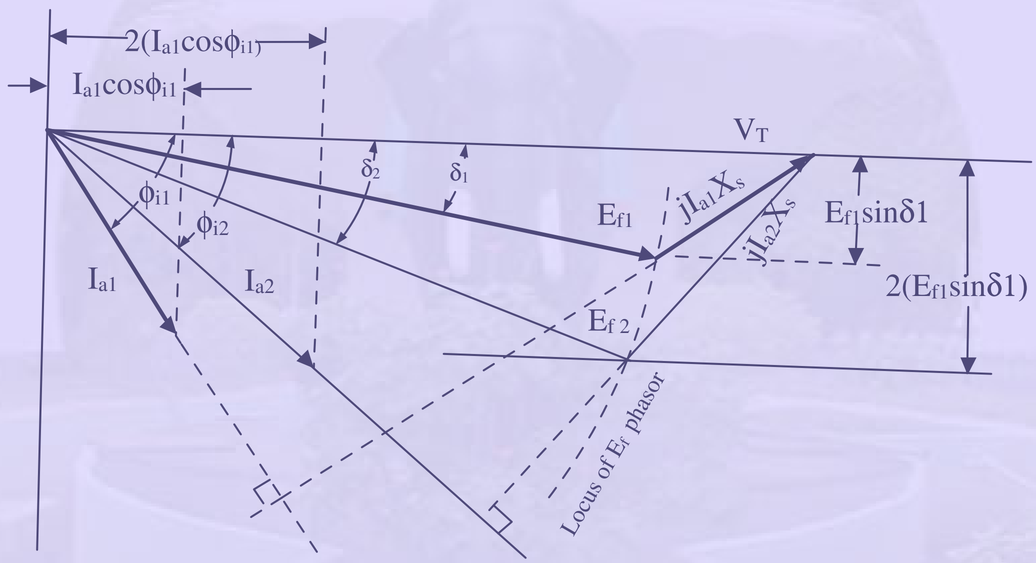


Figure 56: Phasor diagram showing effect of changes in shaft load on armature current, power angle and power factor of a synchronous motor

of lag relative to the rotating magnetic field, thereby increasing both the angle of lag of the counter EMF phasor and the magnitude of the stator current. It is interesting to note that during all this load variation, however, except for the duration of transient conditions whereby the rotor assumes a new position in relation to the rotating magnetic field, the average speed of the machine does not change. As the load is being increased, a final point is reached at which a further increase in δ fails to cause a corresponding increase in motor torque, and the rotor pulls out of synchronism. In fact as stated earlier, the rotor poles at this point, will fall behind the stator poles such that they now come under the influence of like poles and the force of attraction no longer exists. Thus, the point of maximum torque occurs at a power angle of approximately 90° for a cylindrical-rotor machine, as is indicated by Eqn. 68. This maximum value of torque that causes a synchronous motor to pull out of synchronism is called the pull-out torque. In actual practice, the motor will never be operated at power angles close to 90° as armature current will be many times its rated value at this load.

6.3.4 Effect of changes in field excitation on synchronous motor performance

Intuitively we can expect that increasing the strength of the magnets will increase the magnetic attraction, and thereby cause the rotor magnets to have a closer alignment with the corresponding opposite poles of the rotating magnetic poles of the stator. This will obviously result in a smaller power angle. This fact can also be seen in Eqn. 68. When the shaft load is assumed to be constant, the steady-state value of $E_f \sin\delta$ must also be constant. An increase in E_f will cause a transient increase in $E_f \sin\delta$, and the rotor will accelerate. As the rotor changes its angular position, δ decreases until $E_f \sin\delta$ has the same steady-state value as before, at which time the rotor is again operating at synchronous speed, as it should run only at the synchronous speed. This change in angular position of the rotor magnets relative to the poles of rotating magnetic field of the stator occurs in a fraction of a second.

The effect of changes in field excitation on armature current, power angle, and power factor of a synchronous motor operating with a constant shaft load, from a constant voltage, constant frequency supply, is illustrated in Fig. 57. From Eqn. 69, we have for a constant shaft load,

$$E_{f1}\sin\delta_1 = E_{f2}\sin\delta_2 = E_{f3}\sin\delta_3 = E_f\sin\delta \tag{71}$$

This is shown in Fig. 57, where the locus of the tip of the E_f phasor is a straight line parallel to the V_T phasor. Similarly, from Eqn. 69, for a constant shaft load,

$$I_{a1}\cos\phi_{i1} = I_{a2}\cos\phi_{i2} = I_{a3}\cos\phi_{i3} = I_a\cos\phi_i \tag{72}$$

This is also shown in Fig. 57, where the locus of the tip of the I_a phasor is a line

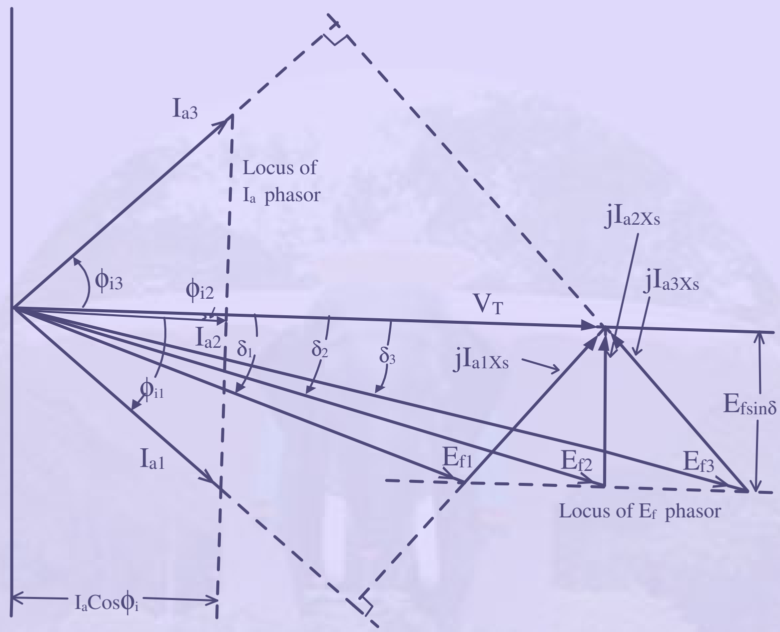


Figure 57: Phasor diagram showing effect of changes in field excitation on armature current, power angle and power factor of a synchronous motor

perpendicular to the V_T phasor.

Note that increasing the excitation from E_{f1} to E_{f3} in Fig. 57 caused the phase angle of the current phasor with respect to the terminal voltage V_T (and hence the power factor) to go from lagging to leading. The value of field excitation that results in unity power factor is called normal excitation. Excitation greater than normal is called over excitation, and excitation less than normal is called under excitation. Furthermore, as indicated in Fig. 57, when operating in the overexcited mode, $|E_f| > |V_T|$. In fact a synchronous motor operating under over excitation condition is sometimes called a synchronous condenser.

6.3.5 V curves

Curves of armature current vs. field current (or excitation voltage to a different scale) are called V curves, and are shown in Fig. 58 for typical values of synchronous motor loads. The curves are related to the phasor diagram in Fig. 57, and illustrate the effect of the variation of field excitation on armature current and power factor for typical shaft loads. It can be easily noted from these curves that an increase in shaft loads require an increase in field excitation in order to maintain the power factor at unity.

The locus of the left most point of the V curves in Fig. 58 represents the stability limit ($\delta = -90°$). Any reduction in excitation below the stability limit for a particular load will cause the rotor to pullout of synchronism.

The V curves shown in Fig. 58 can be determined experimentally in the laboratory by varying I_f at a constant shaft load and noting I_a as I_f is varied. Alternatively the V curves shown in Fig. 58 can be determined graphically by plotting $|I_a| vs. |E_f|$ from a family of phasor diagrams as shown in Fig. 57, or from the following mathematical expression for the V curves

$$
\begin{aligned}
(I_a X_s)^2 &= V_T^2 + E_f^2 - 2V_T E_f \cos\delta \qquad (73)\\
&= V_T^2 + E_f^2 - 2V_T E_f \sqrt{1-\sin^2\delta}\\
&= V_T^2 + E_f^2 - 2\sqrt{V_T^2 E_f^2 - V_T^2 E_f^2 \sin^2\delta}
\end{aligned}
$$

$$I_a = \frac{1}{X_s}.\sqrt{V_T^2 + E_f^2 - 2\sqrt{V_T^2 E_f^2 - X_s^2.P_{in,ph}^2}} \qquad (74)$$

Eqn. 74 is based on the phasor diagram and the assumption that R_a is negligible. It is to be noted that instability will occur, if the developed torque is less than the shaft load plus friction and windage losses, and the expression under the square root sign will be negative.

The family of V curves shown in Fig. 58 represent computer plots of Eqn. 74, by taking the data pertaining to a three-phase 10 hp synchronous motor i.e $V_{ph} = 230V$ and $X_s = 1.2\Omega/phase$.

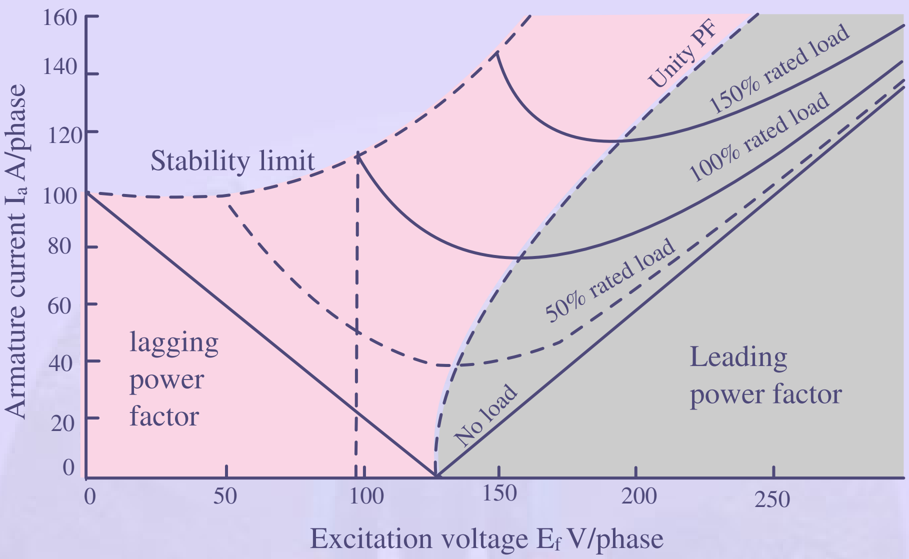


Figure 58: Family of representative V curves for a synchronous motor

6.3.6 Synchronous-motor losses and efficiency

The flow of power through a synchronous motor, from stator to rotor and then to shaft output, is shown in Fig. 59. As indicated in the power-flow diagram, the total power loss for the motor is given by

$$P_{loss} = P_{scl} + P_{core} + P_{fcl} + P_{f,w} + P_{stray}\ W \tag{75}$$

where:
P_{scl}= stator-copper loss
P_{fcl} = fie1d-copper.loss
P_{core} = core loss

$P_{f,w}$ = friction and windage loss
P_{stray} = stray load loss
Except for the transient conditions that occur when the field current is increased or decreased (magnetic energy stored or released), the total energy supplied to the field coils is constant and all of it is consumed as I^2R losses in the field winding. Just as in the case of the synchronous generator, the overall efficiency of a synchronous motor is given by

$$\eta = \frac{P_{shaft}}{P_{in} + P_{field}} = \frac{P_{shaft}}{P_{shaft} + P_{loss}} \tag{76}$$

Generally, the nameplates of synchronous motors and manufacturers' specification sheets customarily provide the overall efficiency for rated load and few load conditions only. Hence, only the total losses at these loads can be determined. The separation of losses into the components listed in Eqn. 75 needs a very involved test procedure in the laboratory. However, a closer approximation of the mechanical power developed can be calculated by subtracting the copper losses of the armature and field winding if these losses can be calculated. The shaft power can then be calculated subtracting the mechanical losses from the mechanical power developed.

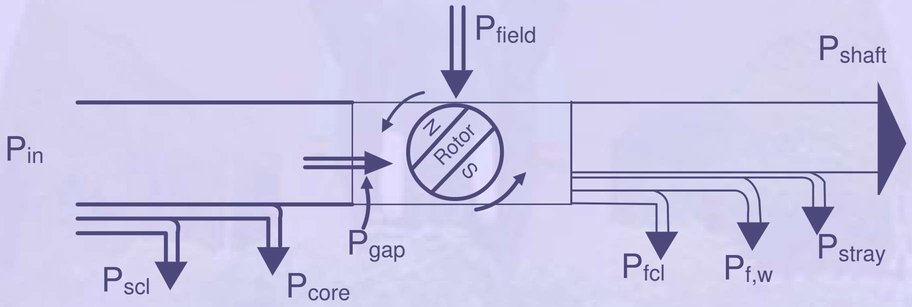


Figure 59: Power flow diagram for a synchronous motor