Chapter 2

Electrical Laws and Circuits

ELECTRIC AND MAGNETIC FIELDS

When something occurs at one point in space because something else happened at another point, with no visible means by which the "cause" can be related to the "effect," we say the two events are connected by a **field.** In radio work, the fields with which we are concerned are the **electric** and **magnetic,** and the combination of the two called the **electromagnetic** field.

A field has two important properties, intensity (magnitude) and direction. The field exerts a *force* on an object immersed in it; this force represents potential (ready-to-be-used) energy, so the **potential** of the field is a measure of the **field intensity.** The **direction** of the field is the direction in which the object on which the force is exerted will tend to move.

An electrically charged object in an electric field will be acted on by a force that will tend to move it in a direction determined by the direction of the field. Similarly, a magnet in a magnetic field will be subject to a force. Everyone has seen demonstrations of magnetic fields with pocket magnets, so intensity and direction are not hard to grasp.

A "static" field is one that neither moves nor changes in intensity. Such a field can be set up by a stationary electric charge **(electrostatic field)** or by a stationary magnet **(magnetostatic field).** But if either an electric or magnetic field is moving in space or changing in intensity, the motion or change sets up the other kind of field. That is, a changing electric field sets up a magnetic field, and a changing magnetic field generates an electric field. This interrelationship between magnetic and electric fields makes possible such things as the electromagnet and the electric motor. It also makes possible the **electromagnetic waves** by which radio communication is carried on, for such waves are simply traveling fields in which the energy is alternately handed back and forth between the electric and magnetic fields.

Lines of Force

Although no one knows what it is that composes the field itself, it is useful to invent a picture of it that will help in visualizing the forces and the way in which they act.

A field can be pictured as being made up of **lines of force,** or **flux lines.** These are purely imaginary threads that show, by the direction in which they lie, the direction the object on which the force is exerted will move. The *number* of lines in a chosen cross section of the field is a measure of the *intensity* of the force. The number of lines per unit of area (square inch or square centimeter) is called the **flux density.**

ELECTRICITY AND THE ELECTRIC CURRENT

Everything physical is built up of atoms, particles so small that they cannot be seen even through the most powerful microscope. But the atom in turn consists of several different kinds of still smaller particles. One is the **electron,** essentially a small particle of electricity. The quantity or **charge** of electricity represented by the electron is, in fact, the smallest quantity of electricity that can exist. The kind of electricity associated with the electron is called **negative.**

An ordinary atom consists of a central core called the **nucleus,** around which one or more electrons circulate somewhat as the earth and other planets circulate around the sun. The nucleus has an electric charge of the kind of electricity called **positive,** the amount of its charge being just exactly equal to the sum of the negative charges on all the electrons associated with that nucleus.

The important fact about these two "opposite" kinds of electricity is that they are strongly attracted to each other. Also, there is a strong force of repulsion between two charges of the *same* kind. The positive nucleus and the negative electrons are attracted to each other, but two electrons will be repelled from each other and so will two nuclei.

In a normal atom the positive charge on the nucleus is exactly balanced by the negative charges on the electrons. However, it is possible for an atom to lose one of its electrons. When that happens the atom has a little less negative charge than it should — that is, it has a net positive charge. Such an atom is said to be **ionized,** and in this case the atom is a **positive ion.** If an atom picks up an extra electron, as it sometimes does, it has a net negative charge and is called a **negative ion.** A positive ion will attract any stray electron in the vicinity, including the extra one that may be attached to a nearby negative ion. In this way it is possible for electrons to travel from atom to atom. The movement of ions or electrons constitutes the **electric current.**

The **amplitude** of the current (its intensity or magnitude) is determined by the rate at which electric charge — an accumulation of electrons or ions of the same kind — moves past a point in a circuit. Since the charge on a single electron or ion is extremely small, the number that must move as a group to form even a tiny current is almost inconceivably large.

Conductors and Insulators

Atoms of some materials, notably metals and acids, will give up an electron readily, but atoms of other materials will not part with any of their electrons even when the electric force is extremely strong. Materials in which electrons or ions can be moved with relative ease are called **conductors,** while those that refuse to permit such movement are called **nonconductors** or **insulators**. The following list shows how some common materials are classified:

Conductors	*Insulators*	
Metals	Dry Air	Glass
Carbon	Wood	Rubber
Acids	Porcelain	Resins
	Textiles	

Electromotive Force

The electric force or potential (called **electromotive force,** and abbreviated **e.m.f.**) that causes current flow may be developed in several ways. The action of certain chemical solutions on dissimilar metals sets up an e.m.f.; such a combination is called a **cell,** and a group of cells forms an electric **battery.** The amount of current that such cells can carry is limited, and in the course of current flow one of the metals is eaten away. The amount of electrical energy that can be taken from a battery consequently is rather small. Where a large amount of energy is needed it is usually furnished by an electric **generator,** which develops its e.m.f. by a combination of magnetic and mechanical means.

Direct and Alternating Currents

In picturing current flow it is natural to think of a single, constant force causing the electrons to move. When this is so, the electrons always move in the same direction through a path or **circuit** made up of conductors connected together in a continuous chain. Such a current is called a **direct current,** abbreviated **d.c.** It is the type of current furnished by batteries and by certain types of generators.

It is also possible to have an e.m.f. that periodically reverses. With this kind of e.m.f. the current flows first in one direction through the circuit and then in the other. Such an e.m.f. is called an alternating e.m.f., and the current is called an **alternating current** (abbreviated **a.c.**). The reversals (alternations) may occur at any rate from a few per second up to several billion per second. Two reversals make a **cycle**; in one cycle the force acts first in one direction, then in the other, and then returns to the first direction to begin the next cycle. The number of cycles in one second is called the **frequency** of the alternating current.

The difference between direct current and alternating current is shown in Fig. 2-1. In these graphs the horizontal axis measures time, increasing toward the right away from the vertical axis. The vertical axis represents the amplitude or strength of the current, increasing in either the up or down direction away from the horizontal axis. If the graph is *above* the horizontal axis the current is flowing in one direction through the circuit (indicated by the + sign) and if it is *below* the horizontal axis the current is flowing in the reverse direction through the circuit (indicated by the – sign). Fig. 2-1A shows that, if we close the circuit — that is, make the path for the current complete — at the time indicated by X, the current instantly takes the amplitude indicated by the height A. After that, the current continues at the same amplitude as time goes on. This is an ordinary *direct* current.

In Fig. 2-1B, the current starts flowing with the amplitude A at time X, continues at that amplitude until time Y and then instantly ceases. After an interval YZ the current again begins to flow and the same sort of start-and-stop performance is repeated. This is an *intermittent* direct current. We could get it by alternately closing and opening a switch in the circuit. It is a *direct* current because the *direction* of current flow does not change; the graph is always on the + side of the horizontal axis.

In Fig. 2-1C the current starts at zero, increases in amplitude as time goes on until it reaches the amplitude A_1 while flowing in the + direction, then decreases until it drops to zero amplitude once more. At that time (X) the *direction* of the current flow reverses; this is indicated by the fact that the next part of the graph is below the axis. As time goes on the amplitude increases, with the current now flowing in the – direction, until it reaches amplitude A_2. Then

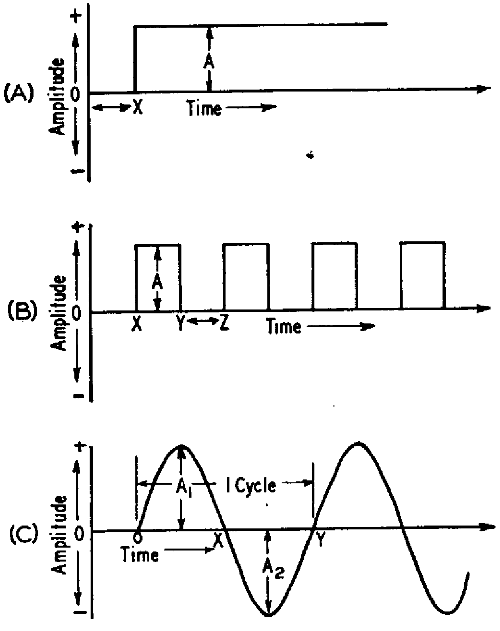


Fig. 2-1—Three types of current flow. A—direct current; B—intermittent direct current; C—alternating current.

the amplitude decreases until finally it drops to zero (Y) and the direction reverses once more. This is an *alternating* current.

Waveforms

The type of alternating current shown in Fig. 2-1C is known as a **sine wave.** The variations in many a.c. waves are not so smooth, nor is one half-cycle necessarily just like the preceding one in shape. However, these **complex waves** can be shown to be the sum of two or more sine waves of frequencies that are exact integral (whole-number) multiples of some lower frequency. The lowest frequency is called the **fundamental,** and the higher frequencies are called **harmonics.**

Fig. 2-2 shows how a fundamental and a second harmonic (twice the fundamental) might add to form a complex wave. Simply by changing the relative amplitudes of the two waves, as well as the times at which they pass through zero amplitude, an infinite number of waveshapes can be constructed from just a fundamental and second harmonic. More complex waveforms can be constructed if more harmonics are used.

Frequency multiplication, the generation of second, third and higher-order harmonics, takes place whenever a fundamental sine wave is passed through a nonlinear device. The distorted output is made up of the fundamental frequency plus harmonics; a desired harmonic can be selected through the use of tuned circuits. Typical nonlinear devices used for frequency multiplication include rectifiers of any kind and amplifiers that distort an applied signal.

Electrical Units

The unit of electromotive force is called the **volt.** An ordinary flashlight cell generates an e.m.f. of about 1.5 volts. The e.m.f. commonly supplied for domestic lighting and power is 115 volts a.c. at a frequency of 60 cycles per second.

The flow of electric current is measured in **amperes.** One ampere is equivalent to the movement of many billions of electrons past a point in the circuit in one second. The *direct* currents used in amateur radio equipment usually are not large, and it is customary to measure such currents in **milliamperes.** One milliampere is equal to one one-thousandth of an ampere.

A "d.c. ampere" is a measure of a *steady* current, but the "a.c. ampere" must measure a current that is continually varying in amplitude and periodically reversing direction. To put the two on the same basis, an a.c. ampere is defined as the current that will cause the same heating effect as one ampere of steady direct current. For sine-wave a.c., this **effective** (or **r.m.s.**, for *root mean square,* the mathematical derivation) **value** is equal to the *maximum* (or **peak**) amplitude (A_1 or A_2 in Fig. 2-1C) multiplied by 0.707. The **instantaneous value** is the value that the current (or voltage) has at any selected instant in the cycle. If all the instantaneous values in a sine wave are averaged over a *half*-cycle, the resulting figure is the **average** value. It is equal to 0.636 times the maximum amplitude.

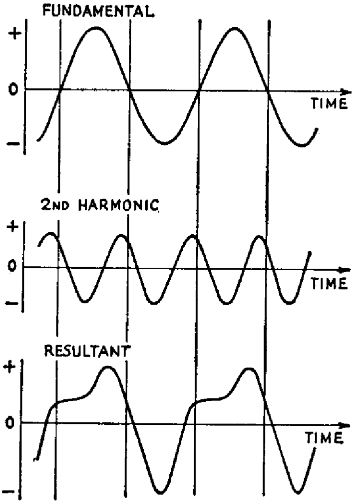


Fig. 2-2—A complex waveform. A fundamental (top) and second harmonic (center) added together, point by point at each instant, result in the waveform shown at the bottom. When the two components have the same polarity at a selected instant, the resultant is the simple sum of the two. When they have opposite polarities, the resultant is the *difference;* if the negative-polarity component is larger, the resultant is negative at that instant.

FREQUENCY AND WAVELENGTH

Frequency Spectrum

Frequencies ranging from about 15 to 15,000 cycles per second (c.p.s.) are called **audio** frequencies, because the vibrations of air particles that our ears recognize as sounds occur at a similar rate. Audio frequencies (abbreviated **a.f.**) are used to actuate loudspeakers and thus create sound waves.

Frequencies above about 15,000 c.p.s. are called **radio** frequencies **(r.f.)** because they are useful in radio transmission. Frequencies all the way up to and beyond 10,000,000,000 c.p.s. have been used for radio purposes. At radio frequencies the numbers become so large that it becomes convenient to use a larger unit than the cycle. Two such units are the **kilocycle,** which is equal to 1000 cycles and is abbreviated **kc.**, and the **megacycle,** which is equal to 1,000,000 cycles or 1000 kilocycles and is abbreviated **Mc.**

The various radio frequencies are divided off into classifications for ready identification. These classifications, listed below, constitute the **frequency spectrum** so far as it extends for radio purposes at the present time.

Frequency	*Classification*	*Abbreviation*
10 to 30 kc.	Very-low frequencies	v.l.f.
30 to 300 kc.	Low frequencies	l.f.
300 to 3000 kc.	Medium frequencies	m.f.
3 to 30 Mc.	High frequencies	h.f.
30 to 300 Mc.	Very-high frequencies	v.h.f.
300 to 3000 Mc.	Ultrahigh frequencies	u.h.f.
3000 to 30,000 Mc.	Superhigh frequencies	s.h.f.

Wavelength

Radio waves travel at the same speed as light —300,000,000 meters or about 186,000 miles a second in space. They can be set up by a radio-frequency current flowing in a circuit, because the rapidly changing current sets up a magnetic field that changes in the same way, and the varying magnetic field in turn sets up a varying electric field. And whenever this happens, the two fields move outward at the speed of light.

Suppose an r.f. current has a frequency of 3,000,000 cycles per second. The fields will go through complete reversals (one cycle) in 1/3,000,000 second. In that same period of time the fields — that is, the wave — will move 300,000,000/3,000,000 meters, or 100 meters. By the time the wave has moved that distance the next cycle has begun and a new wave has started out. The first wave, in other words, covers a distance of 100 meters before the beginning of the next, and so on. This distance is the **wavelength.**

The longer the time of one cycle—that is, the lower the frequency—the greater the distance occupied by each wave and hence the longer the wavelength. The relationship between wavelength and frequency is shown by the formula

$$\lambda = \frac{300{,}000}{f}$$

where λ = Wavelength in meters
f = Frequency in kilocycles

or
$$\lambda = \frac{300}{f}$$

where λ = Wavelength in meters
f = Frequency in megacycles

Example: The wavelength corresponding to a frequency of 3650 kilocycles is

$$\lambda = \frac{300{,}000}{3650} = 82.2 \text{ meters}$$

RESISTANCE

Given two conductors of the same size and shape, but of different materials, the amount of current that will flow when a given e.m.f. is applied will be found to vary with what is called the **resistance** of the material. The lower the resistance, the greater the current for a given value of e.m.f.

Resistance is measured in **ohms.** A circuit has a resistance of one ohm when an applied e.m.f. of one volt causes a current of one ampere to flow. The **resistivity** of a material is the resistance, in ohms, of a cube of the material measuring one centimeter on each edge. One of the best conductors is copper, and it is frequently convenient, in making resistance calculations, to compare the resistance of the material under consideration with that of a copper conductor of the same size and shape. Table 2-I gives the ratio of the resistivity of various conductors to that of copper.

The longer the path through which the current flows the higher the resistance of that conductor. For direct current and low-frequency alternating currents (up to a few thousand cycles per second) the resistance is *inversely* proportional to the cross-sectional area of the path the current must travel; that is, given two conductors of the same material and having the same length, but differing in cross-sectional area, the one with the larger area will have the lower resistance.

TABLE 2-I

Relative Resistivity of Metals

Material	*Resistivity Compared to Copper*
Aluminum (pure)	1.6
Brass	3.7–4.9
Cadmium	4.4
Chromium	1.8
Copper (hard-drawn)	1.03
Copper (annealed)	1.00
Gold	1.4
Iron (pure)	5.68
Lead	12.8
Nickel	5.1
Phosphor Bronze	2.8–5.4
Silver	0.94
Steel	7.6–12.7
Tin	6.7
Zinc	3.4

Resistance of Wires

The problem of determining the resistance of a round wire of given diameter and length—or its opposite, finding a suitable size and length of wire to supply a desired amount of resistance—can be easily solved with the help of the copper-wire table given in a later chapter. This table gives the resistance, in ohms per thousand feet, of each standard wire size.

Example: Suppose a resistance of 3.5 ohms is needed and some No. 28 wire is on hand. The wire table in Chapter 20 shows that No. 28 has a resistance of 66.17 ohms per thousand feet. Since the desired resistance is 3.5 ohms, the length of wire required will be

$$\frac{3.5}{66.17} \times 1000 = 52.89 \text{ feet.}$$

Or, suppose that the resistance of the wire in the circuit must not exceed 0.05 ohm and that the length of wire required for making the connections totals 14 feet. Then

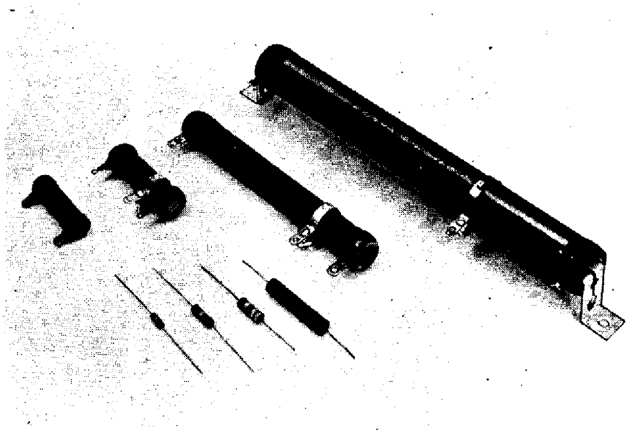
$$\frac{14}{1000} \times R = 0.05 \text{ ohm}$$

where R is the maximum allowable resistance in ohms per thousand feet. Rearranging the formula gives

$$R = \frac{0.05 \times 1000}{14} = 3.57 \text{ ohms/1000 ft.}$$

Reference to the wire table shows that No. 15 is the smallest size having a resistance less than this value.

When the wire is not copper, the resistance values given in the wire table should be multiplied by the ratios given in Table 2-I to obtain the resistance.



Types of resistors used in radio equipment. Those in the foreground with wire leads are carbon types, ranging in size from ½ watt at the left to 2 watts at the right. The larger resistors use resistance wire wound on ceramic tubes; sizes shown range from 5 watts to 100 watts. Three are of the adjustable type, having a sliding contact on an exposed section of the resistance winding.

Example: If the wire in the first example were nickel instead of copper the length required for 3.5 ohms would be

$$\frac{3.5}{66.17 \times 5.1} \times 1000 = 10.37 \text{ feet.}$$

Temperature Effects

The resistance of a conductor changes with its temperature. Although it is seldom necessary to consider temperature in making resistance calculations for amateur work, it is well to know that the resistance of practically all metallic conductors increases with increasing temperature. Carbon, however, acts in the opposite way; its resistance *decreases* when its temperature rises. The temperature effect is important when it is necessary to maintain a constant resistance under all conditions. Special materials that have little or no change in resistance over a wide temperature range are used in that case.

Resistors

A "package" of resistance made up into a single unit is called a **resistor.** Resistors having the same resistance value may be considerably different in size and construction. The flow of current through resistance causes the conductor to become heated; the higher the resistance and the larger the current, the greater the amount of heat developed. Resistors intended for carrying large currents must be physically large so the heat can be radiated quickly to the surrounding air. If the resistor does not get rid of the heat quickly it may reach a temperature that will cause it to melt or burn.

Skin Effect

The resistance of a conductor is not the same for alternating current as it is for direct current. When the current is alternating there are internal effects that tend to force the current to flow mostly in the outer parts of the conductor. This decreases the effective cross-sectional area of the conductor, with the result that the resistance increases.

For low audio frequencies the increase in resistance is unimportant, but at radio frequencies this **skin effect** is so great that practically all the current flow is confined within a few thousandths of an inch of the conductor surface. The r.f. resistance is consequently many times the d.c. resistance, and increases with increasing frequency. In the r.f. range a conductor of thin tubing will have just as low resistance as a solid conductor of the same diameter, because material not close to the surface carries practically no current.

Conductance

The reciprocal of resistance (that is, $1/R$) is called **conductance.** It is usually represented by the symbol G. A circuit having large conductance has low resistance, and vice versa. In radio work the term is used chiefly in connection with vacuum-tube characteristics. The unit of conductance is the **mho.** A resistance of one ohm has a conductance of one mho, a resistance of 1000 ohms has a conductance of 0.001 mho, and so on. A unit frequently used in connection with vacuum tubes is the **micromho,** or one-millionth of a mho. It is the conductance of a resistance of one megohm.

OHM'S LAW

The simplest form of electric circuit is a battery with a resistance connected to its terminals, as shown by the symbols in Fig. 2-3. A complete circuit must have an unbroken path so current can flow out of the battery, through the apparatus connected to it, and back into the battery. The circuit is **broken,** or **open,** if a connection is removed at any point. A **switch** is a device for making and breaking connections and thereby closing or opening the circuit, either allowing current to flow or preventing it from flowing.

Fig. 2-3—A simple circuit consisting of a battery and resistor.

The values of current, voltage and resistance in a circuit are by no means independent of each other. The relationship between them is known as **Ohm's Law.** It can be stated as follows: The

TABLE 2-II

Conversion Factors for Fractional and Multiple Units

To change from	*To*	*Divide by*	*Multiply by*
Units	Micro-units		1,000,000
	Milli-units		1000
	Kilo-units	1000	
	Mega-units	1,000,000	
Micro-units	Milli-units	1000	
	Units	1,000,000	
Milli-units	Micro-units		1000
	Units	1000	
Kilo-units	Units		1000
	Mega-units	1000	
Mega-units	Units		1,000,000
	Kilo-units		1000

current flowing in a circuit is directly proportional to the applied e.m.f. and inversely proportional to the resistance. Expressed as an equation, it is

$$I \text{ (amperes)} = \frac{E \text{ (volts)}}{R \text{ (ohms)}}$$

The equation above gives the value of current when the voltage and resistance are known. It may be transposed so that each of the three quantities may be found when the other two are known:

$$E = IR$$

(that is, the voltage acting is equal to the current in amperes multiplied by the resistance in ohms) and

$$R = \frac{E}{I}$$

(or, the resistance of the circuit is equal to the applied voltage divided by the current).

All three forms of the equation are used almost constantly in radio work. It must be remembered that the quantities are in *volts, ohms* and *amperes;* other units cannot be used in the equations without first being converted. For example, if the current is in milliamperes it must be changed to the equivalent fraction of an ampere before the value can be substituted in the equations.

Table 2-II shows how to convert between the various units in common use. The prefixes attached to the basic-unit name indicate the nature of the unit. These prefixes are:

micro — one-millionth (abbreviated μ)
milli — one-thousandth (abbreviated *m*)
kilo — one thousand (abbreviated *k*)
mega — one million (abbreviated *M*)

For example, one microvolt is one-millionth of a volt, and one megohm is 1,000,000 ohms. There are therefore 1,000,000 microvolts in one volt, and 0.000001 megohm in one ohm.

The following examples illustrate the use of Ohm's Law:

The current flowing in a resistance of 20,000 ohms is 150 milliamperes. What is the voltage? Since the voltage is to be found, the equation to use is $E = IR$. The current must first be converted from milliamperes to amperes, and reference to the table shows that to do so it is necessary to divide by 1000. Therefore,

$$E = \frac{150}{1000} \times 20{,}000 = 3000 \text{ volts}$$

When a voltage of 150 is applied to a circuit the current is measured at 2.5 amperes. What is the resistance of the circuit? In this case R is the unknown, so

$$R = \frac{E}{I} = \frac{150}{2.5} = 60 \text{ ohms}$$

No conversion was necessary because the voltage and current were given in volts and amperes.

How much current will flow if 250 volts is applied to a 5000-ohm resistor? Since I is unknown

$$I = \frac{E}{R} = \frac{250}{5000} = 0.05 \text{ ampere}$$

Milliampere units would be more convenient for the current, and 0.05 amp. × 1000 = 50 milliamperes.

SERIES AND PARALLEL RESISTANCES

Very few actual electric circuits are as simple as the illustration in the preceding section. Commonly, resistances are found connected in a variety of ways. The two fundamental methods of connecting resistances are shown in Fig. 2-4. In the upper drawing, the current flows from the source of e.m.f. (in the direction shown by the arrow, let us say) down through the first resistance, R_1, then through the second, R_2, and then back to the source. These resistors are connected in **series**. The current everywhere in the circuit has the same value.

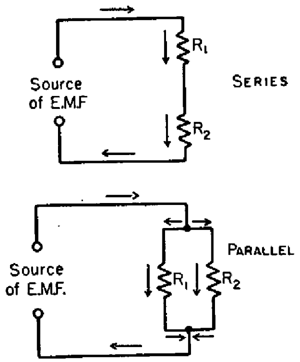


Fig. 2-4—Resistors connected in series and in parallel.

In the lower drawing the current flows to the common connection point at the top of the two resistors and then divides, one part of it flowing through R_1 and the other through R_2. At the lower connection point these two currents again combine; the total is the same as the current that flowed into the upper common connection. In this case the two resistors are connected in **parallel.**

Resistors in Series

When a circuit has a number of resistances connected in series, the total resistance of the circuit is the sum of the individual resistances. If these are numbered R_1, R_2, R_3, etc., then

$$R \text{ (total)} = R_1 + R_2 + R_3 + R_4 + \cdots$$

where the dots indicate that as many resistors as necessary may be added.

Example: Suppose that three resistors are connected to a source of e.m.f. as shown in Fig. 2-5. The e.m.f. is 250 volts, R_1 is 5000 ohms, R_2 is 20,000 ohms, and R_3 is 8000 ohms. The total resistance is then

$$R = R_1 + R_2 + R_3 = 5000 + 20{,}000 + 8000 = 33{,}000 \text{ ohms}$$

The current flowing in the circuit is then

$$I = \frac{E}{R} = \frac{250}{33{,}000} = 0.00757 \text{ amp.} = 7.57 \text{ ma.}$$

(We need not carry calculations beyond three significant figures, and often two will suffice because the accuracy of measurements is seldom better than a few per cent.)

Voltage Drop

Ohm's Law applies to *any part* of a circuit as well as to the whole circuit. Although the current is the same in all three of the resistances in the example, the total voltage divides among them. The voltage appearing across each resistor (the **voltage drop**) can be found from Ohm's Law.

Example: If the voltage across R_1 (Fig. 2-5) is called E_1, that across R_2 is called E_2, and that across R_3 is called E_3, then

$$E_1 = IR_1 = 0.00757 \times 5000 = 37.9 \text{ volts}$$
$$E_2 = IR_2 = 0.00757 \times 20{,}000 = 151.4 \text{ volts}$$
$$E_3 = IR_3 = 0.00757 \times 8000 = 60.6 \text{ volts}$$

The applied voltage must equal the sum of the individual voltage drops:

$$E = E_1 + E_2 + E_3 = 37.9 + 151.4 + 60.6 = 249.9 \text{ volts}$$

The answer would have been more nearly exact if the current had been calculated to more decimal places, but as explained above a very high order of accuracy is not necessary.

In problems such as this considerable time and trouble can be saved, when the current is small enough to be expressed in milliamperes, if the resistance is expressed in kilohms rather than ohms. When resistance in kilohms is substituted directly in Ohm's Law the current will be in milliamperes if the e.m.f. is in volts.

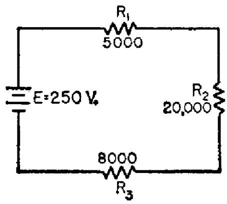


Fig. 2-5—An example of resistors in series. The solution of the circuit is worked out in the text.

Resistors in Parallel

In a circuit with resistances in parallel, the total resistance is *less* than that of the *lowest* value of resistance present. This is because the total current is always greater than the current in any individual resistor. The formula for finding the total resistance of resistances in parallel is

$$R = \cfrac{1}{\cfrac{1}{R_1} + \cfrac{1}{R_2} + \cfrac{1}{R_3} + \cfrac{1}{R_4} + \cdots}$$

where the dots again indicate that any number of resistors can be combined by the same method. For only two resistances in parallel (a very common case) the formula becomes

$$R = \frac{R_1R_2}{R_1 + R_2}$$

Example: If a 500-ohm resistor is paralleled with one of 1200 ohms, the total resistance is

$$R = \frac{R_1R_2}{R_1 + R_2} = \frac{500 \times 1200}{500 + 1200} = \frac{600{,}000}{1700} = 353 \text{ ohms}$$

It is probably easier to solve practical problems by a different method than the "reciprocal of reciprocals" formula. Suppose the three resistors of the previous example are connected in parallel as shown in Fig. 2-6. The same e.m.f., 250 volts, is applied to all three of the resistors. The current in each can be found from Ohm's Law as shown below, I_1 being the current through R_1, I_2 the current through R_2 and I_3 the current through R_3.

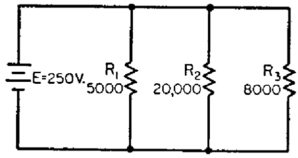


Fig. 2-6—An example of resistors in parallel. The solution is worked out in the text.

For convenience, the resistance will be expressed in kilohms so the current will be in milliamperes.

$$I_1 = \frac{E}{R_1} = \frac{250}{5} = 50 \text{ ma.}$$

$$I_2 = \frac{E}{R_2} = \frac{250}{20} = 12.5 \text{ ma.}$$

$$I_3 = \frac{E}{R_3} = \frac{250}{8} = 31.25 \text{ ma.}$$

The total current is

$$I = I_1 + I_2 + I_3 = 50 + 12.5 + 31.25 = 93.75 \text{ ma.}$$

The total resistance of the circuit is therefore

$$R = \frac{E}{I} = \frac{250}{93.75} = 2.66 \text{ kilohms } (= 2660 \text{ ohms})$$

Resistors in Series-Parallel

An actual circuit may have resistances both in parallel and in series. To illustrate, we use the same three resistances again, but now connected as in Fig. 2-7. The method of solving a circuit such as Fig. 2-7 is as follows: Consider R_2 and R_3 in parallel as though they formed a single resistor. Find their equivalent resistance. Then this resistance in series with R_1 forms a simple series circuit, as shown at the right in Fig. 2-7. An example of the arithmetic is given under the illustration.

Using the same principles, and staying within the practical limits, a value for R_2 can be computed that will provide a given voltage drop across R_3 or a given current through R_1. Simple algebra is required.

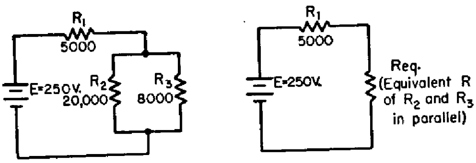


Fig. 2-7—An example of resistors in series-parallel. The equivalent circuit is at the right. The solution is worked out in the text.

Example: The first step is to find the equivalent resistance of R_2 and R_3. From the formula for two resistances in parallel,

$$R_{eq.} = \frac{R_2R_3}{R_2 + R_3} = \frac{20 \times 8}{20 + 8} = \frac{160}{28}$$
$$= 5.71 \text{ kilohms}$$

The total resistance in the circuit is then

$$R = R_1 + R_{eq.} = 5 + 5.71 \text{ kilohms}$$
$$= 10.71 \text{ kilohms}$$

The current is

$$I = \frac{E}{R} = \frac{250}{10.71} = 23.3 \text{ ma.}$$

The voltage drops across R_1 and $R_{eq.}$ are

$$E_1 = IR_1 = 23.3 \times 5 = 117 \text{ volts}$$
$$E_2 = IR_{eq.} = 23.3 \times 5.71 = 133 \text{ volts}$$

with sufficient accuracy. These total 250 volts, thus checking the calculations so far, because the sum of the voltage drops must equal the applied voltage. Since E_2 appears across both R_2 and R_3,

$$I_2 = \frac{E_2}{R_2} = \frac{133}{20} = 6.65 \text{ ma.}$$
$$I_3 = \frac{E_2}{R_3} = \frac{133}{8} = 16.6 \text{ ma.}$$

where I_2 = Current through R_2
I_3 = Current through R_3

The total is 23.25 ma., which checks closely enough with 23.3 ma., the current through the whole circuit.

POWER AND ENERGY

Power—the rate of doing work—is equal to voltage multiplied by current. The unit of electrical power, called the **watt,** is equal to one volt multiplied by one ampere. The equation for power therefore is

$$P = EI$$

where P = Power in watts
E = E.m.f. in volts
I = Current in amperes

Common fractional and multiple units for power are the **milliwatt,** one one-thousandth of a watt, and the **kilowatt,** or one thousand watts.

Example: The plate voltage on a transmitting vacuum tube is 2000 volts and the plate current is 350 milliamperes. (The current must be changed to amperes before substitution in the formula, and so is 0.35 amp.) Then

$$P = EI = 2000 \times 0.35 = 700 \text{ watts}$$

By substituting the Ohm's Law equivalents for E and I, the following formulas are obtained for power:

$$P = \frac{E^2}{R}$$

$$P = I^2R$$

These formulas are useful in power calculations when the resistance and either the current or voltage (but not both) are known.

Example: How much power will be used up in a 4000-ohm resistor if the voltage applied to it is 200 volts? From the equation

$$P = \frac{E^2}{R} = \frac{(200)^2}{4000} = \frac{40,000}{4000} = 10 \text{ watts}$$

Or, suppose a current of 20 milliamperes flows through a 300-ohm resistor. Then

$$P = I^2R = (0.02)^2 \times 300 = 0.0004 \times 300$$
$$= 0.12 \text{ watt}$$

Note that the current was changed from milliamperes to amperes before substitution in the formula.

Electrical power in a resistance is turned into heat. The greater the power the more rapidly the heat is generated. Resistors for radio work are made in many sizes, the smallest being rated to "dissipate" (or carry safely) about ¼ watt. The largest resistors used in amateur equipment will dissipate about 100 watts.

Generalized Definition of Resistance

Electrical power is not always turned into heat. The power used in running a motor, for example, is converted to mechanical motion. The power supplied to a radio transmitter is largely converted into radio waves. Power applied to a loudspeaker is changed into sound waves. But in every case of this kind the power is completely "used up"—it cannot be recovered. Also, for proper operation of the device the power must be supplied at a definite ratio of voltage to current. Both these features are characteristics of resistance, so it can be said that any device that dissipates power has a definite value of "resistance." This concept of resistance as something that absorbs power at a definite voltage/current ratio is very useful, since it permits substituting a simple resistance for the **load** or power-consuming part of the device receiving power, often with considerable simplification of calculations. Of course, every electrical device has some resistance of its own in the more narrow sense, so a part of the power supplied to it is dissipated in that resistance and hence appears as heat even though the major part of the power may be converted to another form.

Efficiency

In devices such as motors and vacuum tubes, the object is to obtain power in some other form than heat. Therefore power used in heating is considered to be a loss, because it is not the *useful* power. The **efficiency** of a device is the useful power output (in its converted form) divided by the power input to the device. In a vacuum-tube transmitter, for example, the object is to convert power from a d.c. source into a.c. power at some radio frequency. The ratio of the r.f. power output to the d.c. input is the efficiency of the tube. That is,

$$Eff. = \frac{P_o}{P_i}$$

where $Eff.$ = Efficiency (as a decimal)
P_o = Power output (watts)
P_I = Power input (watts)

Example: If the d.c. input to the tube is 100 watts and the r.f. power output is 60 watts, the efficiency is

$$Eff. = \frac{P_O}{P_I} = \frac{60}{100} = 0.6$$

Efficiency is usually expressed as a percentage; that is, it tells what per cent of the input power will be available as useful output. The efficiency in the above example is 60 per cent.

Energy

In residences, the power company's bill is for electric **energy,** not for power. What you pay for is the *work* that electricity does for you, not the *rate* at which that work is done. Electrical work is equal to power multiplied by time; the common unit is the **watt-hour,** which means that a power of one watt has been used for one hour. That is,

$$W = PT$$

where W = Energy in watt-hours
P = Power in watts
T = Time in hours

Other energy units are the **kilowatt-hour** and the **watt-second.** These units should be self-explanatory.

Energy units are seldom used in amateur practice, but it is obvious that a small amount of power used for a long time can eventually result in a "power" bill that is just as large as though a large amount of power had been used for a very short time.

CAPACITANCE

Suppose two flat metal plates are placed close to each other (but not touching) and are connected to a battery through a switch, as shown in Fig. 2-8. At the instant the switch is closed, electrons will be attracted from the upper plate to the positive terminal of the battery, and the same number will be repelled into the lower plate from the negative battery terminal. Enough electrons move into one plate and out of the other to make the e.m.f. between them the same as the e.m.f. of the battery.

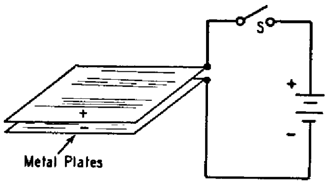


Fig. 2-8—A simple capacitor.

If the switch is opened after the plates have been **charged** in this way, the top plate is left with a deficiency of electrons and the bottom plate with an excess. The plates remain charged despite the fact that the battery no longer is connected. However, if a wire is touched between the two plates (**short-circuiting** them) the excess electrons on the bottom plate will flow through the wire to the upper plate, thus restoring electrical neutrality. The plates have then been **discharged.**

The two plates constitute an electrical **capacitor**; a capacitor possesses the property of storing electricity. (The energy actually is stored in the electric field between the plates.) During the time the electrons are moving—that is, while the capacitor is being charged or discharged—a current is flowing in the circuit even though the circuit is "broken" by the gap between the capacitor plates. However, the current flows only during the time of charge and discharge, and this time is usually very short. There can be no continuous flow of direct current "through" a capacitor, but an alternating current can pass through easily if the frequency is high enough.

The **charge** or quantity of electricity that can be placed on a capacitor is proportional to the applied voltage and to the **capacitance** of the capacitor. The larger the plate area and the smaller the spacing between the plate the greater the capacitance. The capacitance also depends upon the kind of insulating material between the plates; it is smallest with air insulation, but substitution of other insulating materials for air may increase the capacitance many times. The ratio of the capacitance with some material other than air between the plates, to the capacitance of the same capacitor with air insulation, is called the **dielectric constant** of that particular insulating material. The material itself is called a **dielectric.** The dielectric constants of a number of materials commonly used as dielectrics in capacitors are

Table 2-III

Dielectric Constants and Breakdown Voltages

Material	*Dielectric Constant* *	*Puncture Voltage* **
Air	1.0	
Alsimag 196	5.7	240
Bakelite	4.4–5.4	300
Bakelite, mica-filled	4.7	325–375
Cellulose acetate	3.3–3.9	250–600
Fiber	5–7.5	150–180
Formica	4.6–4.9	450
Glass, window	7.6–8	200–250
Glass, Pyrex	4.8	335
Mica, ruby	5.4	3800–5600
Mycalex	7.4	250
Paper, Royalgrey	3.0	200
Plexiglass	2.8	990
Polyethylene	2.3	1200
Polystyrene	2.6	500–700
Porcelain	5.1–5.9	40–100
Quartz, fused	3.8	1000
Steatite, low-loss	5.8	150–315
Teflon	2.1	1000–2000

* At 1 Mc. ** In volts per mil (0.001 inch)

given in Table 2-III. If a sheet of polystyrene is substituted for air between the plates of a capacitor, for example, the capacitance will be increased 2.6 times.

Units

The fundamental unit of capacitance is the **farad,** but this unit is much too large for practical work. Capacitance is usually measured in **microfarads** (abbreviated μf.) or **picofarads** (pf.). The microfarad is one-millionth of a farad, and the picofarad (formerly micromicrofarad) is one-millionth of a microfarad. Capacitors nearly always have more than two plates, the alternate plates being connected together to form two sets as shown in Fig. 2-9. This makes it possible to attain a fairly large capacitance in a small space, since several plates of smaller individual area can be stacked to form the equivalent of a single large plate of the same total area. Also, all plates, except the two on the ends, are exposed to plates of the other group on *both sides,* and so are twice as effective in increasing the capacitance.

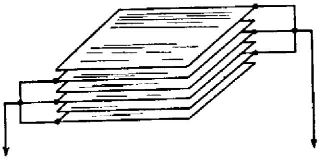


Fig. 2-9—A multiple-plate capacitor. Alternate plates are connected together.

The formula for calculating capacitance is:

$$C = 0.224 \frac{KA}{d} (n - 1)$$

where C = Capacitance in pf.
K = Dielectric constant of material between plates
A = Area of one side of *one* plate in square inches
d = Separation of plate surfaces in inches
n = Number of plates

If the plates in one group do not have the same area as the plates in the other, use the area of the *smaller* plates.

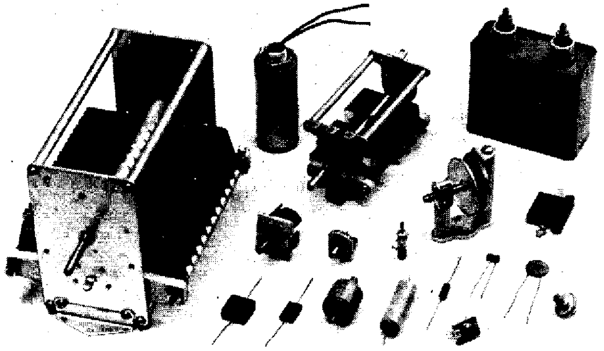
Capacitors in Radio

The types of capacitors used in radio work differ considerably in physical size, construction, and capacitance. Some representative types are shown in the photograph. In **variable** capacitors (almost always constructed with air for the dielectric) one set of plates is made movable with respect to the other set so that the capacitance can be varied. **Fixed** capacitors—that is, assemblies having a single, non-adjustable value of capacitance—also can be made with metal plates and with air as the dielectric, but usually are constructed from plates of metal foil with a thin solid or liquid dielectric sandwiched in between, so that a relatively large capacitance can be secured in a small unit. The solid dielectrics commonly used are mica, paper and special ceramics. An example of a liquid dielectric is mineral oil. The **electrolytic** capacitor uses aluminum-foil plates with a semiliquid conducting chemical compound between them; the actual dielectric is a very thin film of insulating material that forms on one set of plates through electrochemical action when a d.c. voltage is applied to the capacitor. The capacitance obtained with a given plate area in an electrolytic capacitor is very large, compared with capacitors having other dielectrics, because the film is so thin—much less than any thickness that is practicable with a solid dielectric.

The use of electrolytic and oil-filled capacitors is confined to power-supply filtering and audio bypass applications. Mica and ceramic capacitors are used throughout the frequency range from audio to several hundred megacycles.

Voltage Breakdown

When a high voltage is applied to the plates of a capacitor, a considerable force is exerted on the electrons and nuclei of the dielectric. Because the dielectric is an insulator the electrons do not become detached from atoms the way they do in conductors. However, if the force is great enough the dielectric will "break down"; usually it will puncture and may char (if it is solid) and permit current to flow. The **breakdown voltage** depends upon the kind and thickness of the dielectric, as shown in Table 2-III. It is not directly proportional to the thickness; that is, doubling



Fixed and variable capacitors. The large unit at the left is a transmitting-type variable capacitor for r.f. tank circuits. To its right are other air-dielectric variables of different sizes ranging from the midget "air padder" to the medium-power tank capacitor at the top center. The cased capacitors in the top row are for power-supply filters, the cylindrical-can unit being an electrolytic and the rectangular one a paper-dielectric capacitor. Various types of mica, ceramic, and paper-dielectric capacitors are in the foreground.

the thickness does not quite double the breakdown voltage. If the dielectric is air or any other gas, breakdown is evidenced by a spark or arc between the plates, but if the voltage is removed the arc ceases and the capacitor is ready for use again. Breakdown will occur at a lower voltage between pointed or sharp-edged surfaces than between rounded and polished surfaces; consequently, the breakdown voltage between metal plates of given spacing in air can be increased by buffing the edges of the plates.

Since the dielectric must be thick to withstand high voltages, and since the thicker the dielectric the smaller the capacitance for a given plate area, a high-voltage capacitor must have more plate area than a low-voltage one of the same capacitance. High-voltage high-capacitance capacitors are physically large.

CAPACITORS IN SERIES AND PARALLEL

The terms "parallel" and "series" when used with reference to capacitors have the same circuit meaning as with resistances. When a number of capacitors are connected in parallel, as in Fig. 2-10, the total capacitance of the group is equal to the sum of the individual capacitances, so

$$C\ (\text{total}) = C_1 + C_2 + C_3 + C_4 + \ldots\ldots\ldots\ldots$$

However, if two or more capacitors are connected in series, as in the second drawing, the total capacitance is less than that of the smallest capacitor in the group. The rule for finding the capacitance of a number of series-connected capacitors is the same as that for finding the resistance of a number of *parallel*-connected resistors. That is,

$$C\ (\text{total}) = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \cdots\cdots\cdots}$$

and, for only two capacitors in series,

$$C\ (\text{total}) = \frac{C_1 C_2}{C_1 + C_2}$$

The same units must be used throughout; that is, all capacitances must be expressed in either µf. or pf.; both kinds of units cannot be used in the same equation.

Capacitors are connected in parallel to obtain a larger total capacitance than is available in one unit. The largest voltage that can be applied safely to a group of capacitors in parallel is the voltage that can be applied safely to the one having the *lowest* voltage rating.

When capacitors are connected in series, the applied voltage is divided up among them; the situation is much the same as when resistors are in series and there is a voltage drop across each. However, the voltage that appears across each capacitor of a group connected in series is in *inverse* proportion to its capacitance, as compared with the capacitance of the whole group.

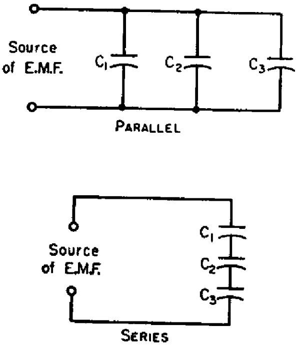


Fig. 2-10—Capacitors in parallel and in series.

Example: Three capacitors having capacitances of 1, 2, and 4 µf., respectively, are connected in series as shown in Fig. 2-11. The total capacitance is

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{1}{\frac{1}{1} + \frac{1}{2} + \frac{1}{4}} = \frac{1}{\frac{7}{4}} = \frac{4}{7}$$

$$= 0.571\ \mu\text{f.}$$

The voltage across each capacitor is proportional to the *total* capacitance divided by the capacitance of the capacitor in question, so the voltage across C_1 is

$$E_1 = \frac{0.571}{1} \times 2000 = 1142 \text{ volts}$$

Similarly, the voltages across C_2 and C_3 are

$$E_2 = \frac{0.571}{2} \times 2000 = 571 \text{ volts}$$

$$E_3 = \frac{0.571}{4} \times 2000 = 286 \text{ volts}$$

totaling approximately 2000 volts, the applied voltage.

Capacitors are frequently connected in series to enable the group to withstand a larger voltage (at the expense of decreased total capacitance) than any individual capacitor is rated to stand. However, as shown by the previous example, the applied voltage does not divide equally among the capacitors (except when all the capacitances are the same) so care must be taken to see that the voltage rating of no capacitor in the group is exceeded.

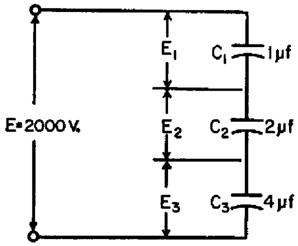


Fig. 2-11—An example of capacitors connected in series. The solution to this arrangement is worked out in the text.

INDUCTANCE

It is possible to show that the flow of current through a conductor is accompanied by magnetic effects; a compass needle brought near the conductor, for example, will be deflected from its normal north-south position. The current, in other words, sets up a magnetic field.

The transfer of energy to the magnetic field represents work done by the source of e.m.f. Power is required for doing work, and since power is equal to current multiplied by voltage, there must be a voltage drop in the circuit during the time in which energy is being stored in the field. This voltage "drop" (which has nothing to do with the voltage drop in any resistance in the circuit) is the result of an opposing voltage "induced" in the circuit while the field is building up to its final value. When the field becomes constant the **induced e.m.f.** or **back e.m.f.** disappears, since no further energy is being stored.

Since the induced e.m.f. opposes the e.m.f. of the source, it tends to prevent the current from rising rapidly when the circuit is closed. The amplitude of the induced e.m.f. is proportional to the rate at which the current is changing and to a constant associated with the circuit itself, called the **inductance** of the circuit.

Inductance depends on the physical characteristics of the conductor. If the conductor is formed into a coil, for example, its inductance is increased. A coil of many turns will have more inductance than one of few turns, if both coils are otherwise physically similar. Also, if a coil is placed on an iron core its inductance will be greater than it was without the magnetic core.

The polarity of an induced e.m.f. is always such as to oppose any change in the current in the circuit. This means that when the current in the circuit is increasing, work is being done against the induced e.m.f. by storing energy in the magnetic field. If the current in the circuit tends to decrease, the stored energy of the field returns to the circuit, and thus adds to the energy being supplied by the source of e.m.f. This tends to keep the current flowing even though the applied e.m.f. may be decreasing or be removed entirely.

The unit of inductance is the **henry.** Values of inductance used in radio equipment vary over a wide range. Inductance of several henrys is required in power-supply circuits (see chapter on Power Supplies) and to obtain such values of inductance it is necessary to use coils of many turns wound on iron cores. In radio-frequency circuits, the inductance values used will be measured in **millihenrys** (a **mh.**, one one-thousandth of a henry) at low frequencies, and in **microhenrys** (**μh.**, one one-millionth of a henry) at medium frequencies and higher. Although coils for radio frequencies may be wound on special iron cores (ordinary iron is not suitable) most r.f. coils made and used by amateurs are of the "air-core" type; that is, wound on an insulating support consisting of nonmagnetic material.

Every conductor has inductance, even though the conductor is not formed into a coil. The inductance of a short length of straight wire is small, but it may not be negligible because if the current through it changes its intensity rapidly enough the induced voltage may be appreciable. This will be the case in even a few inches of wire when an alternating current having a frequency of the order of 100 Mc. or higher is flowing. However, at much lower frequencies the inductance of the same wire could be ignored because the induced voltage would be negligibly small.

Calculating Inductance

The approximate inductance of single-layer air-core coils may be calculated from the simplified formula

$$L\ (\mu\text{h.}) = \frac{a^2n^2}{9a + 10b}$$

where L = Inductance in microhenrys
a = Coil radius in inches
b = Coil length in inches
n = Number of turns

The notation is explained in Fig. 2-12. This

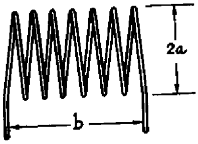
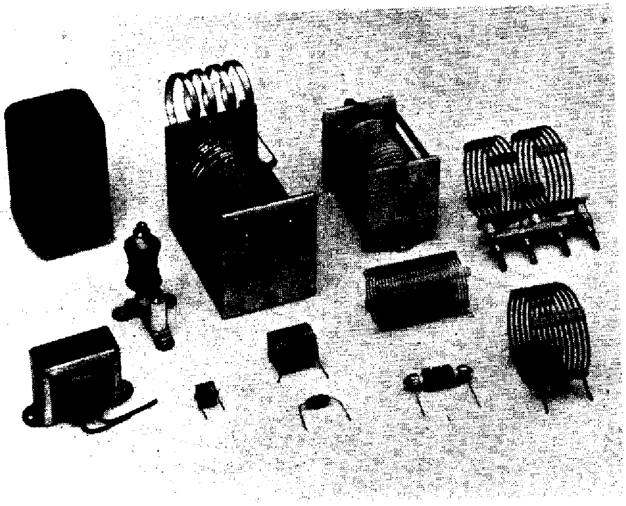


Fig. 2-12—Coil dimensions used in the inductance formula. The wire diameter does not enter into the formula.



Inductors for power and radio frequencies. The two iron-core coils at the left are "chokes" for power-supply filters. The mounted air-core coils at the top center are adjustable inductors for transmitting tank circuits. The "pie-wound" coils at the left and in the foreground are radio-frequency choke coils. The remaining coils are typical of inductors used in r.f. tuned circuits, the larger sizes being used principally for transmitters.

formula is a close approximation for coils having a length equal to or greater than $0.8a$.

Example: Assume a coil having 48 turns wound 32 turns per inch and a diameter of ¾ inch. Thus $a = 0.75 \div 2 = 0.375$, $b = 48 \div 32 = 1.5$, and $n = 48$. Substituting,

$$L = \frac{.375 \times .375 \times 48 \times 48}{(9 \times .375) + (10 \times 1.5)} = 17.6\ \mu\text{h}.$$

To calculate the number of turns of a single-layer coil for a required value of inductance,

$$n = \sqrt{\frac{L\,(9a + 10b)}{a^2}}$$

Example: Suppose an inductance of 10 µh. is required. The form on which the coil is to be wound has a diameter of one inch and is long enough to accommodate a coil of 1¼ inches. Then $a = 0.5$, $b = 1.25$, and $L = 10$. Substituting,

$$n = \sqrt{\frac{10\,(4.5 + 12.5)}{.5 \times .5}} = \sqrt{680} = 26.1 \text{ turns}$$

A 26-turn coil would be close enough in practical work. Since the coil will be 1.25 inches long, the number of turns per inch will be $26.1 \div 1.25 = 20.8$. Consulting the wire table, we find that No. 17 enameled wire (or anything smaller) can be used. The proper inductance is obtained by winding the required number of turns on the form and then adjusting the spacing between the turns to make a uniformly-spaced coil 1.25 inches long.

Inductance Charts

Most inductance formulas lose accuracy when applied to small coils (such as are used in v.h.f. work and in low-pass filters built for reducing harmonic interference to television) because the conductor thickness is no longer negligible in comparison with the size of the coil. Fig. 2-13 shows the measured inductance of v.h.f. coils, and may be used as a basis for circuit design. Two curves are given: curve *A* is for coils wound to an inside diameter of ½ inch; curve *B* is for coils of ¾-inch inside diameter. In both curves the wire size is No. 12, winding pitch 8 turns to the inch (⅛ inch center-to-center turn spacing). The inductance values given include leads ½ inch long.

The charts of Figs. 2-14 and 2-15 are useful for rapid determination of the inductance of coils of the type commonly used in radio-frequency circuits in the range 3-30 Mc. They are of sufficient accuracy for most practical work. Given the coil length in inches, the curves show the multiplying factor to be applied to the inductance value given in the table below the curve for a coil of the same diameter and number of turns per inch.

Example: A coil 1 inch in diameter is 1¼ inches long and has 20 turns. Therefore it has 16 turns per inch, and from the table under Fig. 2-15 it is found that the reference inductance for a coil of this diameter and number of turns per inch is 16.8 µh. From curve *B* in the figure the multiplying factor is 0.35, so the inductance is

$$16.8 \times 0.35 = 5.9\ \mu\text{h}.$$

The charts also can be used for finding suitable dimensions for a coil having a required value of inductance.

Example: A coil having an inductance of 12 µh. is required. It is to be wound on a form having a diameter of 1 inch, the length available for the winding being not more than 1¼ inches. From Fig. 2-15, the multiplying factor for a 1-inch diameter coil (curve *B*) having the maximum possible length of 1¼ inches is 0.35. Hence the number of turns per inch must be chosen for a reference inductance of at least 12/0.35, or 34 µh. From the Table under Fig. 2-15 it is seen that 16 turns per inch (reference inductance 16.8 µh.) is too small. Using 32 turns per inch, the multiplying factor is 12/68, or 0.177, and from curve *B* this corresponds to a coil length of ¾ inch. There will be 24 turns in this length, since the winding "pitch" is 32 turns per inch.

Machine-wound coils with the diameters and turns per inch given in the tables are available in many radio stores, under the trade names of "B&W Miniductor" and "Illumitronic Air Dux."

IRON-CORE COILS

Permeability

Suppose that the coil in Fig. 2-16 is wound on an iron core having a cross-sectional area of 2 square inches. When a certain current is sent through the coil it is found that there are 80,000 lines of force in the core. Since the area is 2 square inches, the flux density is 40,000 lines per square inch. Now suppose that the iron core is removed and the same current is maintained in the coil, and that the flux density without the iron core is found to be 50 lines per square inch. The ratio of the flux density with the given core material to the flux density (with the same coil and same current) with an air core is called the **permeability** of the material. In this case the permeability of the iron is 40,000/50 = 800. The inductance of the coil is increased 800 times by inserting the iron core since, other things being equal, the inductance will be proportional to the magnetic flux through the coil.

The permeability of a magnetic material varies with the flux density. At low flux densities (or with an air core) increasing the current through

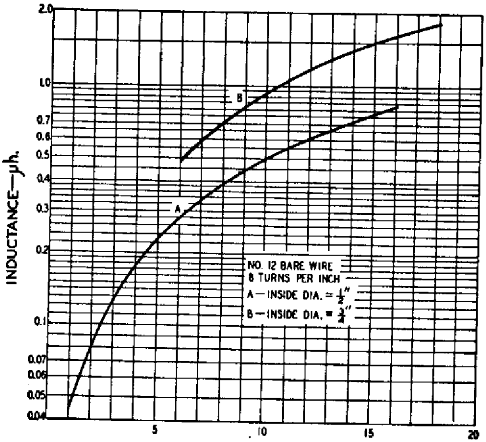


Fig. 2-13—Measured inductance of coils wound with No. 12 bare wire, 8 turns to the inch. The values include half-inch leads.

the coil will cause a proportionate increase in flux, but at very high flux densities, increasing the current may cause no appreciable change in the flux. When this is so, the iron is said to be **saturated.** Saturation causes a rapid decrease in permeability, because it decreases the ratio of flux lines to those obtainable with the same current and an air core. Obviously, the inductance of an iron-core inductor is highly dependent upon the current flowing in the coil. In an air-core coil, the inductance is independent of current because air does not saturate.

Iron core coils such as the one sketched in Fig. 2-16 are used chiefly in power-supply equipment. They usually have direct current flowing through the winding, and the variation in inductance with current is usually undesirable. It may be overcome by keeping the flux density below the saturation point of the iron. This is done by opening the core so that there is a small "air gap," as indicated by the dashed lines. The magnetic "resistance" introduced by such a gap is so large—even though the gap is only a small fraction of an inch—compared with that of the iron that the gap, rather than the iron, controls the flux density. This reduces the inductance, but makes it practically constant regardless of the value of the current.

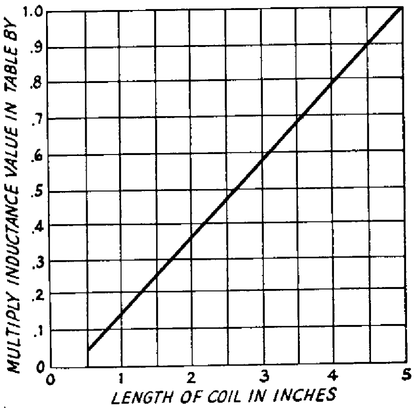


Fig. 2-14—Factor to be applied to the inductance of coils listed in the table below, for coil lengths up to 5 inches.

Coil diameter, Inches	*No. of turns per inch*	*Inductance in μh.*
1¼	4	2.75
	6	6.3
	8	11.2
	10	17.5
	16	42.5
1½	4	3.9
	6	8.8
	8	15.6
	10	24.5
	16	63
1¾	4	5.2
	6	11.8
	8	21
	10	33
	16	85
2	4	6.6
	6	15
	8	26.5
	10	42
	16	108
2½	4	10.2
	6	23
	8	41
	10	64
3	4	14
	6	31.5
	8	56
	10	89

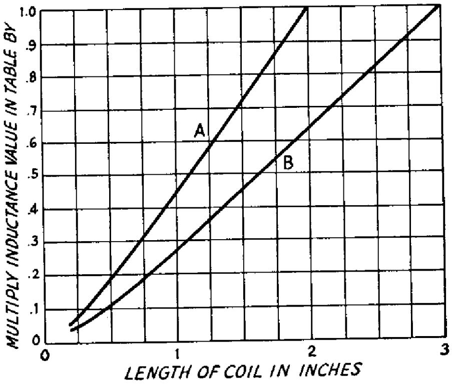


Fig. 2-15—Factor to be applied to the inductance of coils listed in the table below, as a function of coil length. Use curve A for coils marked A, curve B for coils marked B.

Coil diameter, Inches	*No. of turns per inch*	*Inductance in μh.*
½ (A)	4	0.18
	6	0.40
	8	0.72
	10	1.12
	16	2.9
	32	12
⅝ (A)	4	0.28
	6	0.62
	8	1.1
	10	1.7
	16	4.4
	32	18
¾ (B)	4	0.6
	6	1.35
	8	2.4
	10	3.8
	16	9.9
	32	40.
1 (B)	4	1.0
	6	2.3
	8	4.2
	10	6.6
	16	16.8
	32	68

Eddy Currents and Hysteresis

When alternating current flows through a coil wound on an iron core an e.m.f. will be induced, as previously explained, and since iron is a conductor a current will flow in the core. Such currents (called **eddy currents**) represent a waste

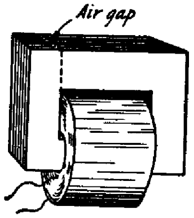


Fig. 2-16—Typical construction of an iron-core inductor. The small air gap prevents magnetic saturation of the iron and thus maintains the inductance at high currents.

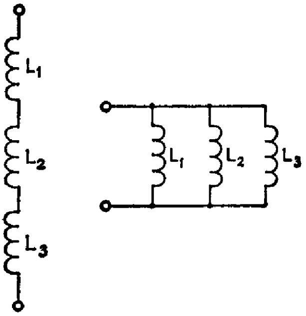


Fig. 2-17—Inductances in series and parallel.

of power because they flow through the resistance of the iron and thus cause heating. Eddy-current losses can be reduced by **laminating** the core; that is, by cutting it into thin strips. These strips or **laminations** must be insulated from each other by painting them with some insulating material such as varnish or shellac.

There is also another type of energy loss: the iron tends to resist any change in its magnetic state, so a rapidly-changing current such as a.c. is forced continually to supply energy to the iron to overcome this "inertia." Losses of this sort are called **hysteresis** losses.

Eddy-current and hysteresis losses in iron increase rapidly as the frequency of the alternating current is increased. For this reason, ordinary iron cores can be used only at power and audio frequencies—up to, say, 15,000 cycles. Even so, a very good grade of iron or steel is necessary if the core is to perform well at the higher audio frequencies. Iron cores of this type are completely useless at radio frequencies.

For radio-frequency work, the losses in iron cores can be reduced to a satisfactory figure by grinding the iron into a powder and then mixing it with a "binder" of insulating material in such a way that the individual iron particles are insulated from each other. By this means cores can be made that will function satisfactorily even through the v.h.f. range—that is, at frequencies up to perhaps 100 Mc. Because a large part of the magnetic path is through a nonmagnetic material, the permeability of the iron is low compared with the values obtained at power-supply frequencies. The core is usually in the form of a "slug" or cylinder which fits inside the insulating form on which the coil is wound. Despite the fact that, with this construction, the major portion of the magnetic path for the flux is in air, the slug is quite effective in increasing the coil inductance. By pushing the slug in and out of the coil the inductance can be varied over a considerable range.

INDUCTANCES IN SERIES AND PARALLEL

When two or more inductors are connected in series (Fig. 2-17, left) the total inductance is equal to the sum of the individual inductances, *provided the coils are sufficiently separated so that no coil is in the magnetic field of another.* That is,

$$L_{total} = L_1 + L_2 + L_3 + L_4 + \dots\dots\dots$$

If inductors are connected in parallel (Fig. 2-17, right)—and the coils are separated sufficiently, the total inductance is given by

$$L_{total} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4} + \dots\dots}$$

and for two inductances in parallel,

$$L = \frac{L_1 L_2}{L_1 + L_2}$$

Thus the rules for combining inductances in series and parallel are the same as for resistances, *if* the coils are far enough apart so that each is unaffected by another's magnetic field. When this is not so the formulas given above cannot be used.

MUTUAL INDUCTANCE

If two coils are arranged with their axes on the same line, as shown in Fig. 2-18, a current sent through Coil 1 will cause a magnetic field which "cuts" Coil 2. Consequently, an e.m.f. will be induced in Coil 2 whenever the field strength is changing. This induced e.m.f. is similar to the e.m.f. of self-induction, but since it appears in the *second* coil because of current flowing in the *first,* it is a "mutual" effect and results from the **mutual inductance** between the two coils.

If all the flux set up by one coil cuts all the turns of the other coil the mutual inductance has its maximum possible value. If only a small part of the flux set up by one coil cuts the turns of the other the mutual inductance is relatively small. Two coils having mutual inductance are said to be **coupled.**

The ratio of actual mutual inductance to the maximum possible value that could theoretically be obtained with two given coils is called the **coefficient of coupling** between the coils. It is frequently expressed as a percentage. Coils that

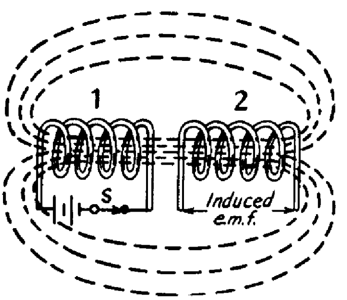


Fig. 2-18—Mutual inductance. When the switch, S, is closed current flows through coil No. 1, setting up a magnetic field that induces an e.m.f. in the turns of coil No. 2.

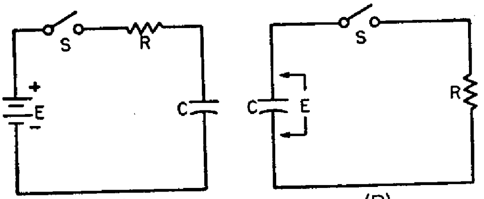
have nearly the maximum possible (coefficient = 1 or 100%) mutual inductance are said to be **closely**, or **tightly**, coupled, but if the mutual inductance is relatively small the coils are said to be **loosely** coupled. The degree of coupling depends upon the physical spacing between the coils and how they are placed with respect to each other. Maximum coupling exists when they have a common axis and are as close together as possible (one wound over the other). The coupling is least when the coils are far apart or are placed so their axes are at right angles.

The maximum possible coefficient of coupling is closely approached only when the two coils are wound on a closed iron core. The coefficient with air-core coils may run as high as 0.6 or 0.7 if one coil is wound over the other, but will be much less if the two coils are separated.

TIME CONSTANT

Capacitance and Resistance

Connecting a source of e.m.f. to a capacitor causes the capacitor to become charged to the full e.m.f. practically instantaneously, if there is no resistance in the circuit. However, if the circuit contains resistance, as in Fig. 2-19A, the resistance limits the current flow and an appreciable length of time is required for the e.m.f. between the capacitor plates to build up to the same value as the e.m.f. of the source. During this "building-up" period the current gradually decreases from its initial value, because the increasing e.m.f. stored on the capacitor offers increasing opposition to the steady e.m.f. of the source.



(A) (B)

Fig. 2-19—Illustrating the time constant of an RC circuit.

Theoretically, the charging process is never really finished, but eventually the charging current drops to a value that is smaller than anything that can be measured. The **time constant** of such a circuit is the length of time, in seconds, required for the voltage across the capacitor to reach 63 per cent of the applied e.m.f. (this figure is chosen for mathematical reasons). The voltage across the capacitor rises with time as shown by Fig. 2-20.

The formula for time constant is

$$T = RC$$

where T = Time constant in seconds
C = Capacitance in farads
R = Resistance in ohms

If C is in microfarads and R in megohms, the time constant also is in seconds. These units usually are more convenient.

Example: The time constant of a 2-μf. capacitor and a 250,000-ohm (0.25 megohm) resistor is

$$T = RC = 0.25 \times 2 = 0.5 \text{ second}$$

If the applied e.m.f. is 1000 volts, the voltage between the capacitor plates will be 630 volts at the end of ½ second.

If a charged capacitor is *discharged* through a resistor, as indicated in Fig. 2-19B, the same time constant applies. If there were no resistance, the capacitor would discharge instantly when S was closed. However, since R limits the current flow the capacitor voltage cannot instantly go to zero, but it will decrease just as rapidly as the capacitor can rid itself of its charge through R. When the capacitor is discharging through a resistance, the time constant (calculated in the same way as above) is the time, in seconds, that it takes for the capacitor to *lose* 63 per cent of its voltage; that is, for the voltage to drop to 37 per cent of its initial value.

Example: If the capacitor of the example above is charged to 1000 volts, it will discharge to 370 volts in ½ second through the 250,000-ohm resistor.

Inductance and Resistance

A comparable situation exists when resistance and inductance are in series. In Fig. 2-21, first consider L to have no resistance and also assume that R is zero. Then closing S would tend to

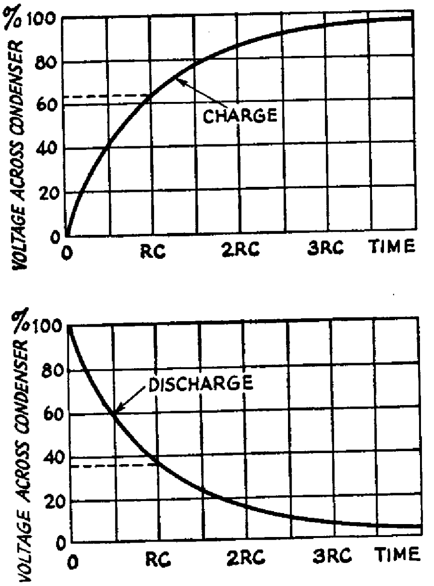


Fig. 2-20—How the voltage across a capacitor rises, with time, when charged through a resistor. The lower curve shows the way in which the voltage decreases across the capacitor terminals on discharging through the same resistor.

send a current through the circuit. However, the instantaneous transition from no current to a finite value, however small, represents a very rapid *change* in current, and a *back e.m.f.* is developed by the self-inductance of L that is practically equal and opposite to the applied e.m.f. The result is that the initial current is very small.

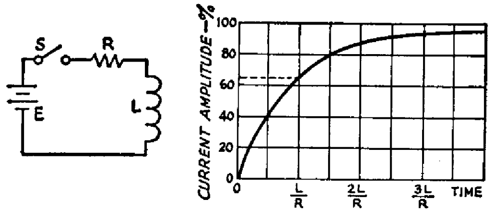


Fig. 2-21—Time constant of an *LR* circuit.

The back e.m.f. depends upon the *change* in current and would cease to offer opposition if the current did not continue to increase. With no resistance in the circuit (which would lead to an infinitely large current, by Ohm's Law) the current would increase forever, always growing just fast enough to keep the e.m.f. of self-induction equal to the applied e.m.f.

When resistance is in series, Ohm's Law sets a limit to the value that the current can reach. The back e.m.f. generated in L has only to equal the *difference* between E and the drop across R, because that difference is the voltage actually applied to L. This difference becomes smaller as the current approaches the final Ohm's Law value. Theoretically, the back e.m.f. never quite disappears and so the current never quite reaches the Ohm's Law value, but practically the difference becomes unmeasurable after a time. The time constant of an inductive circuit is the time in seconds required for the current to reach 63 per cent of its final value. The formula is

$$T=\frac{L}{R}$$

where T = Time constant in seconds

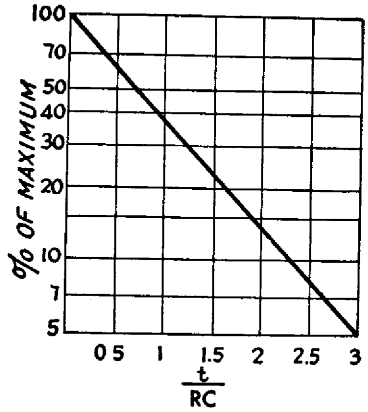


Fig. 2-22—Voltage across capacitor terminals in a discharging *RC* circuit, in terms of the initial charged voltage. To obtain time in seconds, multiply the factor *t/RC* by the time constant of the circuit.

L = Inductance in henrys
R = Resistance in ohms

The resistance of the wire in a coil acts as if it were in series with the inductance.

Example: A coil having an inductance of 20 henrys and a resistance of 100 ohms has a time constant of

$$T=\frac{L}{R}=\frac{20}{100}=0.2\text{ second}$$

if there is no other resistance in the circuit. If a d.c. e.m.f. of 10 volts is applied to such a coil, the final current, by Ohm's Law, is

$$I=\frac{E}{R}=\frac{10}{100}=0.1\text{ amp. or }100\text{ ma.}$$

The current would rise from zero to 63 milliamperes in 0.2 second after closing the switch.

An inductor cannot be "discharged" in the same way as a capacitor, because the magnetic field disappears as soon as current flow ceases. Opening S does not leave the inductor "charged." The energy stored in the magnetic field instantly returns to the circuit when S is opened. The rapid disappearance of the field causes a very large voltage to be induced in the coil—ordinarily many times larger than the voltage applied, because the induced voltage is proportional to the *speed* with which the field changes. The common result of opening the switch in a circuit such as the one shown is that a spark or arc forms at the switch contacts at the instant of opening. If the inductance is large and the current in the circuit is high, a great deal of energy is released in a very short period of time. It is not at all unusual for the switch contacts to burn or melt under such circumstances. The spark or arc at the opened switch can be reduced or suppressed by connecting a suitable capacitor and resistor in series across the contacts.

Time constants play an important part in numerous devices, such as electronic keys, timing and control circuits, and shaping of keying characteristics by vacuum tubes. The time constants of circuits are also important in such applications as automatic gain control and noise limiters. In nearly all such applications a resistance-capacitance (RC) time constant is involved, and it is usually necessary to know the voltage across the capacitor at some time interval larger or smaller than the actual time constant of the circuit as given by the formula above. Fig. 2-22 can be used for the solution of such problems, since the curve gives the voltage across the capacitor, in terms of percentage of the initial charge, for percentages between 5 and 100, at any time after discharge begins.

Example: A 0.01-μf. capacitor is charged to 150 volts and then allowed to discharge through a 0.1-megohm resistor. How long will it take the voltage to fall to 10 volts? In percentage, 10/150 = 6.7%. From the chart, the factor corresponding to 6.7% is 2.7. The time constant of the circuit is equal to $RC = 0.1 \times 0.01 = 0.001$. The time is therefore $2.7 \times 0.001 = 0.0027$ second, or 2.7 milliseconds.

ALTERNATING CURRENTS

PHASE

The term **phase** essentially means "time," or the *time interval* between the instant when one thing occurs and the instant when a second related thing takes place. The later event is said to **lag** the earlier, while the one that occurs first is said to **lead.** In a.c. circuits the current amplitude changes continuously, so the concept of phase or time becomes important. Phase can be measured in the ordinary time units, such as the second, but there is a more convenient method: Since each a.c. cycle occupies exactly the same amount of time as every other cycle of the same frequency, we can use the cycle itself as the time unit. Using the cycle as the time unit makes the specification or measurement of phase independent of the frequency of the current, so long as only one frequency is under consideration at a time. When two or more frequencies are to be considered, as in the case where harmonics are present, the phase measurements are made with respect to the lowest, or fundamental, frequency.

The time interval or "phase difference" under consideration usually will be less than one cycle. Phase difference could be measured in decimal parts of a cycle, but it is more convenient to divide the cycle into 360 parts or **degrees.** A phase degree is therefore 1/360 of a cycle. The reason for this choice is that with sine-wave alternating current the value of the current at any instant is proportional to the sine of the angle that corresponds to the number of degrees—that is, length of time—from the instant the cycle began. There is no actual "angle" associated with an alternating current. Fig. 2-23 should help make this method of measurement clear.

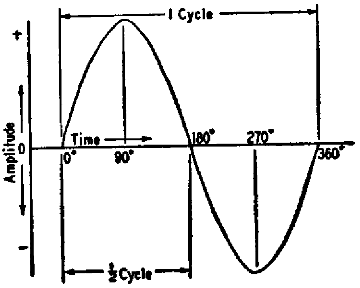


Fig. 2-23—An a.c. cycle is divided off into 360 degrees that are used as a measure of time or phase.

Measuring Phase

The phase difference between two currents of the same frequency is the time or angle difference between corresponding parts of cycles of the two currents. This is shown in Fig. 2-24. The current labeled *A* leads the one marked *B* by 45 degrees, since *A*'s cycles begin 45 degrees earlier in time. It is equally correct to say that *B lags A* by 45 degrees.

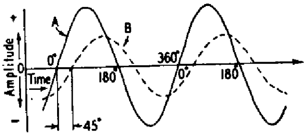


Fig. 2-24—When two waves of the same frequency start their cycles at slightly different times, the time difference or phase difference is measured in degrees. In this drawing wave B starts 45 degrees (one-eighth cycle) later than wave A, and so lags 45 degrees behind A.

Two important special cases are shown in Fig. 2-25. In the upper drawing *B* lags 90 degrees behind *A*; that is, its cycle begins just one-quarter cycle later than that of *A*. When one wave is passing through zero, the other is just at its maximum point.

In the lower drawing *A* and *B* are 180 degrees out of phase. In this case it does not matter which one is considered to lead or lag. *B* is always positive while *A* is negative, and vice versa. The two waves are thus *completely* out of phase.

The waves shown in Figs. 2-24 and 2-25 could represent current, voltage, or both. *A* and *B* might be two currents in separate circuits, or *A* might represent voltage and *B* current in the same circuit. If *A* and *B* represent two currents in the *same* circuit (or two voltages in the same circuit) the total or **resultant** current (or voltage) also is a sine wave, because adding any number of sine waves of the same frequency always gives a sine wave also of the same frequency.

Phase in Resistive Circuits

When an alternating voltage is applied to a resistance, the current flows exactly in step with the voltage. In other words, the voltage and current are **in phase.** This is true at any frequency if the resistance is "pure"—that is, is free from the reactive effects discussed in the next section. Practically, it is often difficult to obtain a purely

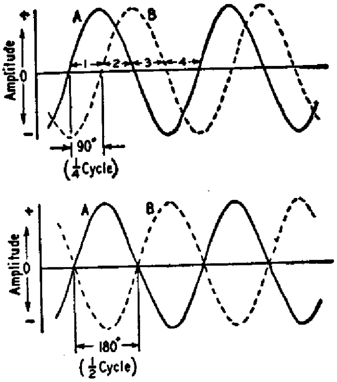


Fig. 2-25—Two important special cases of phase difference. In the upper drawing, the phase difference between A and B *is* 90 degrees; in the lower drawing the phase difference is 180 degrees.

resistive circuit at radio frequencies, because the reactive effects become more pronounced as the frequency is increased.

In a purely resistive circuit, or for purely resistive parts of circuits, Ohm's Law is just as valid for a.c. of any frequency as it is for d.c.

REACTANCE

Alternating Current in Capacitance

In Fig. 2-26 a sine-wave a.c. voltage having a maximum value of 100 volts is applied to a capacitor. In the period *OA*, the applied voltage increases from zero to 38 volts; at the end of this period the capacitor is charged to that voltage. In interval *AB* the voltage increases to 71 volts; that is, 33 volts additional. In this interval a *smaller* quantity of charge has been added than in *OA*, because the voltage rise during interval *AB* is smaller. Consequently the average current during *AB* is smaller than during *OA*. In the third interval, *BC*, the voltage rises from 71 to 92 volts, an increase of 21 volts. This is less than the voltage increase during *AB*, so the quantity of electricity added is less; in other words, the average current during interval *BC* is still smaller. In the fourth interval, *CD*, the voltage increases only 8 volts; the charge added is smaller than in any preceding interval and therefore the current also is smaller.

By dividing the first quarter cycle into a very large number of intervals it could be shown that the current charging the capacitor has the shape of a sine wave, just as the applied voltage does. The current is largest at the beginning of the cycle and becomes zero at the maximum value of the voltage, so there is a phase difference of 90 degrees between the voltage and current. During the first quarter cycle the current is flowing in the normal direction through the circuit, since the capacitor is being charged. Hence the current is positive, as indicated by the dashed line in Fig. 2-26.

In the second quarter cycle—that is, in the time from *D* to *H*, the voltage applied to the capacitor decreases. During this time the capacitor *loses* its charge. Applying the same reasoning, it is plain that the current is small in interval *DE* and continues to increase during each succeeding interval. However, the current is flowing *against* the applied voltage because the capacitor is discharging into the circuit. The current flows in the *negative* direction during this quarter cycle.

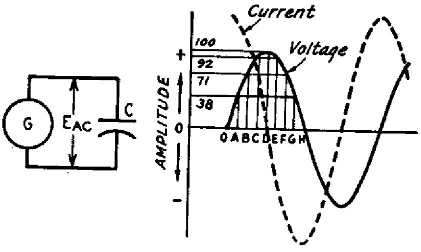


Fig. 2-26—Voltage and current phase relationships when an alternating voltage is applied to a capacitor.

The third and fourth quarter cycles repeat the events of the first and second, respectively, with this difference—the polarity of the applied voltage has reversed, and the current changes to correspond. In other words, an alternating current flows in the circuit because of the alternate charging and discharging of the capacitance. As shown by Fig. 2-26, the current starts its cycle 90 degrees before the voltage, so the current in a capacitor leads the applied voltage by 90 degrees.

Capacitive Reactance

The quantity of electric charge that can be placed on a capacitor is proportional to the applied e.m.f. and the capacitance. This amount of charge moves back and forth in the circuit once each cycle, and so the *rate* of movement of charge—that is, the current—is proportional to voltage, capacitance and frequency. If the effects of capacitance and frequency are lumped together, they form a quantity that plays a part similar to that of resistance in Ohm's Law. This quantity is called **reactance,** and the unit for it is the ohm, just as in the case of resistance. The formula for it is

$$X_C = \frac{1}{2\pi fC}$$

where X_C = Capacitive reactance in ohms
f = Frequency in cycles per second
C = Capacitance in farads
π = 3.14

Although the unit of reactance is the ohm, there is no power dissipation in reactance. The energy stored in the capacitor in one quarter of the cycle is simply returned to the circuit in the next.

The fundamental units (cycles per second, farads) are too large for practical use in radio circuits. However, if the capacitance is in microfarads and the frequency is in megacycles, the reactance will come out in ohms in the formula.

Example: The reactance of a capacitor of 470 pf. (0.00047 μf.) at a frequency of 7150 kc. (7.15 Mc.) is

$$X = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 7.15 \times 0.00047} = 47.4 \text{ ohms}$$

Inductive Reactance

When an alternating voltage is applied to a *pure* inductance (one with no resistance—all *practical* inductors have resistance) the current is again 90 degrees out of phase with the applied voltage. However, in this case the current *lags* 90 degrees behind the voltage—the opposite of the capacitor current-voltage relationship.

The primary cause for this is the *back e.m.f.* generated in the inductance, and since the amplitude of the back e.m.f. is proportional to the rate at which the current changes, and this in turn is proportional to the frequency, the amplitude of the current is inversely proportional to the applied frequency. Also, since the back e.m.f. is proportional to inductance for a given rate of current change, the current flow is inversely proportional to inductance for a given applied voltage and frequency. (Another way of saying this is that just enough current flows to generate an induced e.m.f. that equals and opposes the applied voltage.)

The combined effect of inductance and frequency is called **inductive reactance**, also expressed in ohms, and the formula for it is

$$X_L = 2\pi f L$$

where X_L = Inductive reactance in ohms
f = Frequency in cycles per second
L = Inductance in henrys
π = 3.14

Example: The reactance of a coil having an inductance of 8 henrys, at a frequency of 120 cycles, is

$$X_L = 2\pi f L = 6.28 \times 120 \times 8 = 6029 \text{ ohms}$$

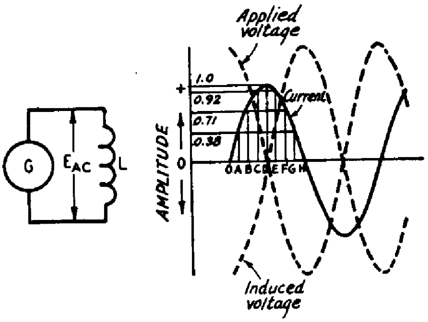


Fig. 2-27—Phase relationships between voltage and current when an alternating voltage is applied to an inductance.

In radio-frequency circuits the inductance values usually are small and the frequencies are large. If the inductance is expressed in millihenrys and the frequency in kilocycles, the conversion factors for the two units cancel, and the formula for reactance may be used without first converting to fundamental units. Similarly, no conversion is necessary if the inductance is in microhenrys and the frequency is in megacycles.

Example: The reactance of a 15-microhenry coil at a frequency of 14 Mc. is

$$X_L = 2\pi f L = 6.28 \times 14 \times 15 = 1319 \text{ ohms}$$

The resistance of the wire of which the coil is wound has no effect on the reactance, but simply acts as though it were a separate resistor connected in series with the coil.

Ohm's Law for Reactance

Ohn's Law for an a.c. circuit containing *only* reactance is

$$I = \frac{E}{X}$$

$$E = IX$$

$$X = \frac{E}{I}$$

where E = E.m.f. in volts
I = Current in amperes
X = Reactance in ohms

The reactance in the circuit may, of course, be either inductive or capacitive.

Example: If a current of 2 amperes is flowing through the capacitor of the earlier example (reactance = 47.4 ohms) at 7150 kc., the voltage drop across the capacitor is

$$E = IX = 2 \times 47.4 = 94.8 \text{ volts}$$

If 400 volts at 120 cycles is applied to the 8-henry inductor of the earlier example, the current through the coil will be

$$I = \frac{E}{X} = \frac{400}{6029} = 0.0663 \text{ amp. (66.3 ma.)}$$

Reactance Chart

The accompanying chart, Fig. 2-28, shows the reactance of capacitances from 1 pf. to 100 µf., and the reactance of inductances from 0.1 µh. to 10 henrys, for frequencies between 100 c.p.s. and 100 megacycles per second. The approximate value of reactance can be read from the chart or, where more exact values are needed, the chart will serve as a check on the order of magnitude of reactances calculated from the formulas given above, and thus avoid "decimal-point errors".

Reactances in Series and Parallel

When reactances of the same kind are connected in series or parallel the resultant reactance is that of the resultant inductance or capacitance. This leads to the same rules that are used when determining the resultant resistance when resistors are combined. That is, for series reactances of the same kind the resultant reactance is

$$X = X_1 + X_2 + X_3 + X_4$$

and for reactances of the same kind in parallel the resultant is

$$X = \frac{1}{\frac{1}{X_1} + \frac{1}{X_2} + \frac{1}{X_3} + \frac{1}{X_4}}$$

or for two in parallel,

$$X = \frac{X_1 X_2}{X_1 + X_2}$$

The situation is different when reactances of opposite kinds are combined. Since the current in a capacitance leads the applied voltage by 90 degrees and the current in an inductance lags the applied voltage by 90 degrees, the voltages at the terminals of opposite types of reactance are 180 degrees out of phase in a series circuit (in which the current has to be the same through all elements), and the currents in reactances of opposite types are 180 degrees out of phase in a parallel circuit (in which the same voltage is applied to all elements). The 180-degree phase relationship means that the currents or voltages are of opposite polarity, so in the series circuit of Fig. 2-29A the voltage E_L across the inductive reactance X_L is of opposite polarity to the voltage E_C across the capacitive reactance X_C. Thus if we call X_L "positive" and X_C "negative" (a common convention) the applied voltage E_{AC} is $E_L - E_C$. In the parallel circuit at B the total current, I, is equal to $I_L - I_C$, since the currents are 180 degrees out of phase.

In the series case, therefore, the resultant re-

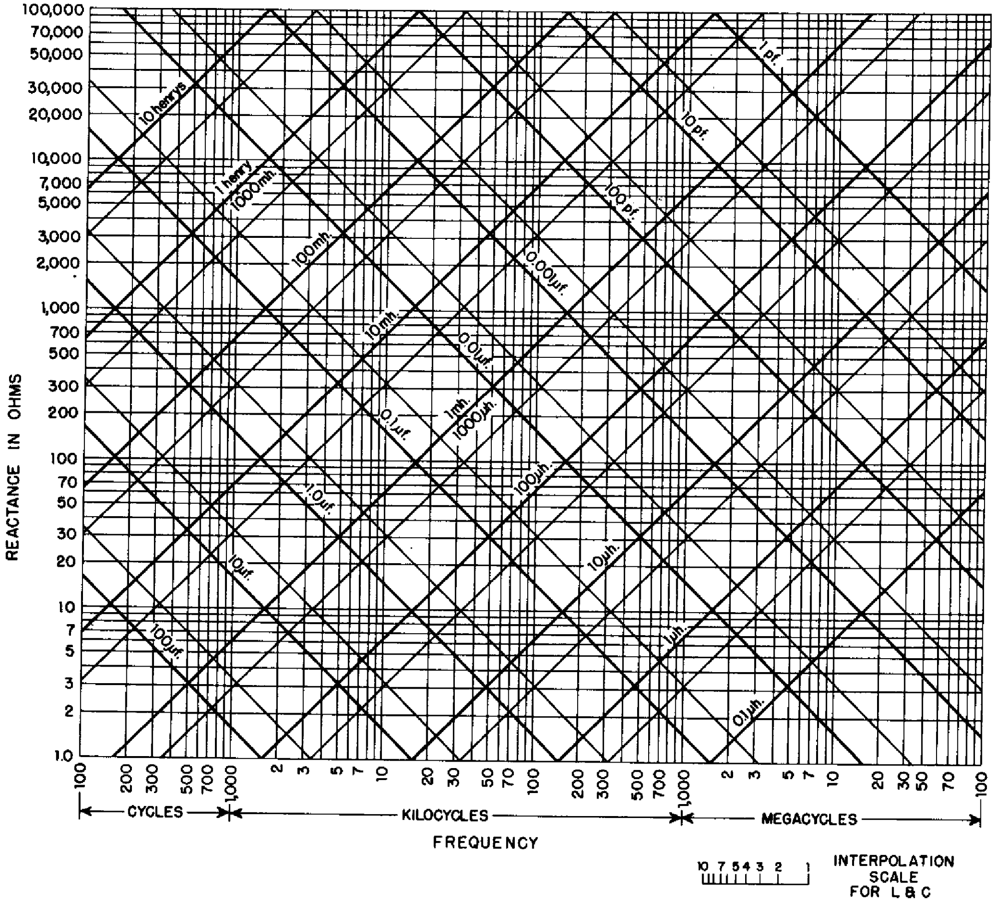


Fig. 2-28—Inductive and capacitive reactance vs. frequency. Heavy lines represent multiples of 10, intermediate light lines multiples of 5; e.g., the light line between 10 μh. and 100 μh. represents 50 μh., the light line between 0.1 μf. and 1 μf. represents 0.5 μf., etc. Intermediate values can be estimated with the help of the interpolation scale.

Reactances outside the range of the chart may be found by applying appropriate factors to values within the chart range. For example, the reactance of 10 henrys at 60 cycles can be found by taking the reactance to 10 henrys at 600 cycles and dividing by 10 for the 10-times decrease in frequency.

actance of X_L and X_C is

$$X = X_L - X_C$$

and in the parallel case

$$X = \frac{-X_L X_C}{X_L - X_C}$$

Note that in the series circuit the total reactance is negative if X_C is larger than X_L; this indicates that the total reactance is capacitive in such a case. The resultant reactance in a series circuit is always smaller than the larger of the two individual reactances.

In the parallel circuit, the resultant reactance is negative (i.e., capacitive) if X_L is larger than X_C, and positive (inductive) if X_L is smaller than X_C, but in every case is always larger than the smaller of the two individual reactances.

In the special case where $X_L = X_C$ the total reactance is zero in the series circuit and infinitely large in the parallel circuit.

Reactive Power

In Fig. 2-29A the voltage drop across the inductor is larger than the voltage applied to the circuit. This might seem to be an impossible condition, but it is not; the explanation is that while energy is being stored in the inductor's magnetic field, energy is being returned to the circuit from the capacitor's electric field, and

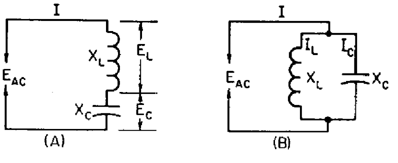


Fig. 2-29—Series and parallel circuits containing opposite kinds of reactance.

vice versa. This stored energy is responsible for the fact that the voltages across reactances in series can be larger than the voltage applied to them.

In a resistance the flow of current causes heating and a power loss equal to I^2R. The power in a reactance is equal to I^2X, but is not a "loss"; it is simply power that is transferred back and forth between the field and the circuit but not used up in heating anything. To distinguish this "nondissipated" power from the power which is actually consumed, the unit of reactive power is called the **volt-ampere-reactive,** or **var,** instead of the watt. Reactive power is sometimes called "wattless" power.

IMPEDANCE

When a circuit contains both resistance and reactance the combined effect of the two is called **impedance,** symbolized by the letter Z. (Impedance is thus a more general term than either resistance or reactance, and is frequently used even for circuits that have only resistance or reactance, although usually with a qualification—such as "resistive impedance" to indicate that the circuit has only resistance, for example.)

The reactance and resistance comprising an impedance may be connected either in series or in parallel, as shown in Fig. 2-30. In these circuits the reactance is shown as a box to indicate that it may be either inductive or capacitive. In the series circuit the current is the same in both elements, with (generally) different voltages appearing across the resistance and reactance. In the parallel circuit the same voltage is applied to both elements, but different currents flow in the two branches.

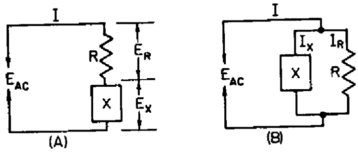


Fig. 2-30—Series and parallel circuits containing resistance and reactance.

Since in a resistance the current is in phase with the applied voltage while in a reactance it is 90 degrees out of phase with the voltage, the phase relationship between current and voltage in the circuit as a whole may be anything between zero and 90 degrees, depending on the relative amounts of resistance and reactance.

Series Circuits

When resistance and reactance are in series, the impedance of the circuit is

$$Z = \sqrt{R^2 + X^2}$$

where Z = impedance in ohms
R = resistance in ohms
X = reactance in ohms.

The reactance may be either capacitive or inductive. If there are two or more reactances in the circuit they may be combined into a resultant by the rules previously given, before substitution into the formula above; similarly for resistances.

The "square root of the sum of the squares" rule for finding impedance in a series circuit arises from the fact that the voltage drops across the resistance and reactance are 90 degrees out of phase, and so combine by the same rule that applies in finding the hypothenuse of a right-angled triangle when the base and altitude are known.

Parallel Circuits

With resistance and reactance in parallel, as in Fig. 2-30B, the impedance is

$$Z = \frac{RX}{\sqrt{R^2 + X^2}}$$

where the symbols have the same meaning as for series circuits.

Just as in the case of series circuits, a number of reactances in parallel should be combined to find the resultant reactance before substitution into the formula above; similarly for a number of resistances in parallel.

Equivalent Series and Parallel Circuits

The two circuits shown in Fig. 2-30 are equivalent if the same current flows when a given voltage of the same frequency is applied, and if the phase angle between voltage and current is the same in both cases. It is in fact possible to "transform" any given series circuit into an equivalent parallel circuit, and vice versa.

Transformations of this type often lead to simplification in the solution of complicated circuits. However, from the standpoint of practical work the usefulness of such transformations lies in the fact that the impedance of a circuit may be modified by the addition of *either* series or parallel elements, depending on which happens to be most convenient in the particular case. Typical applications are considered later in connection with tuned circuits and transmission lines.

Ohm's Law for Impedance

Ohm's Law can be applied to circuits containing impedance just as readily as to circuits having resistance or reactance only. The formulas are

$$I = \frac{E}{Z}$$

$$E = IZ$$

$$Z = \frac{E}{I}$$

where E = E.m.f. in volts
I = Current in amperes
Z = Impedance in ohms

Fig. 2-31 shows a simple circuit consisting of a resistance of 75 ohms and a reactance of 100 ohms in series. From the formula previously given, the impedance is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(75)^2 + (100)^2} = 125$$

ohms.

If the applied voltage is 250 volts, then

$$I = \frac{E}{Z} = \frac{250}{125} = 2 \text{ amperes.}$$

This current flows though both the resistance and reactance, so the voltage drops are

$$E_R = IR = 2 \times 75 = 150 \text{ volts}$$
$$E_{XL} = IX_L = 2 \times 100 = 200 \text{ volts}$$

The simple arithmetical sum of these two drops, 350 volts, is greater than the applied voltage because the two voltages are 90 degrees out of phase. Their actual resultant, when phase is taken into account, is

$$\sqrt{(150)^2 + (200)^2} = 250 \text{ volts.}$$

Power Factor

In the circuit of Fig. 2-31 an applied e.m.f. of 250 volts results in a current of 2 amperes, giving an apparent power of 250 × 2 = 500 watts. However, only the resistance actually consumes power. The power in the resistance is

$$P = I^2R = (2)^2 \times 75 = 300 \text{ watts}$$

The ratio of the power consumed to the apparent power is called the **power factor** of the circuit, and in this example the power factor would be 300/500 = 0.6. Power factor is frequently expressed as a percentage; in this case, it would be 60 per cent.

"Real" or dissipated power is measured in watts; apparent power, to distinguish it from real power, is measured in volt-amperes. It is simply the product of volts and amperes and has no direct relationship to the power actually used up or dissipated unless the power factor of the circuit is known. The power factor of a purely resistive circuit is 100 per cent or 1, while the power factor of a pure reactance is zero. In this illustration, the reactive power is $VAR = I^2X = (2)^2 \times 100 = 400$ volt-amperes.

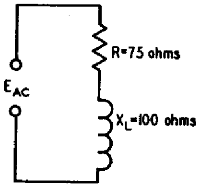


Fig. 2-31—Circuit used as an example for impedance calculations.

Reactance and Complex Waves

It was pointed out earlier in this chapter that a complex wave (a "nonsinusoidal" wave) can be resolved into a fundamental frequency and a series of harmonic frequencies. When such a complex voltage wave is applied to a circuit containing reactance, the current through the circuit will not have the same wave shape as the applied voltage. This is because the reactance of an inductor and capacitor depend upon the applied frequency. For the second-harmonic component of a complex wave, the reactance of the inductor is twice and the reactance of the capacitor one-half their respective values at the fundamental frequency; for the third harmonic the inductor reactance is three times and the capacitor reactance one-third, and so on. Thus the circuit impedance is different for each harmonic component.

Just what happens to the current wave shape depends upon the values of resistance and reactance involved and how the circuit is arranged. In a simple circuit with resistance and inductive reactance in series, the amplitudes of the harmonic currents will be reduced because the inductive reactance increases in proportion to frequency. When capacitance and resistance are in series, the harmonic current is likely to be accentuated because the capacitive reactance becomes lower as the frequency is raised. When both inductive and capacitive reactance are present the shape of the current wave can be altered in a variety of ways, depending upon the circuit and the "constants," or the relative values of *L*, *C*, and *R*, selected.

This property of nonuniform behavior with respect to fundamental and harmonics is an extremely useful one. It is the basis of "filtering," or the suppression of undesired frequencies in favor of a single desired frequency or group of such frequencies.

TRANSFORMERS FOR AUDIO FREQUENCIES

Two coils having mutual inductance constitute a **transformer.** The coil connected to the source of energy is called the **primary** coil, and the other is called the **secondary** coil.

The usefulness of the transformer lies in the fact that electrical energy can be transferred from one circuit to another without direct connection, and in the process can be readily changed from one voltage level to another. Thus, if a device to be operated requires, for example, 115 volts a.c. and only a 440-volt source is available, a transformer can be used to change the source voltage to that required. A transformer can be used only with a.c., since no voltage will be induced in the secondary if the magnetic field is not changing. If d.c. is applied to the primary of a transformer, a voltage will be induced in the secondary only at the instant of closing or opening the primary circuit, since it is only at these times that the field is changing.

THE IRON-CORE TRANSFORMER

As shown in Fig. 2-32, the primary and secondary coils of a transformer may be wound on a core of magnetic material. This increases the inductance of the coils so that a relatively small number of turns may be used to induce a given value of voltage with a small current. A **closed core** (one having a continuous magnetic path) such as that shown in Fig. 2-32 also tends to insure that practically all of the field set up by the current in the primary coil will cut the turns of the secondary coil. However, the core introduces a power loss because of hysteresis and eddy currents so this type of construction is normally practicable only at power and audio frequencies.

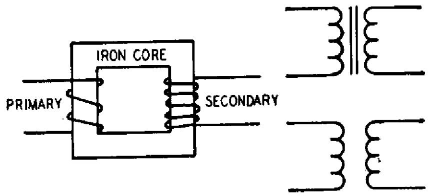


Fig. 2-32—The transformer. Power is transferred from the primary coil to the secondary by means of the magnetic field. The upper symbol at right indicates an iron-core transformer, the lower one an air-core transformer.

The discussion in this section is confined to transformers operating at such frequencies.

Voltage and Turns Ratio

For a given varying magnetic field, the voltage induced in a coil in the field will be proportional to the number of turns in the coil. If the two coils of a transformer are in the same field (which is the case when both are wound on the same closed core) it follows that the induced voltages will be proportional to the number of turns in each coil. In the primary the induced voltage is practically equal to, and opposes, the applied voltage, as described earlier. Hence,

$$E_S = \frac{n_S}{n_P} E_P$$

where E_s = Secondary voltage
E_p = Primary applied voltage
n_s = Number of turns on secondary
n_p = Number of turns on primary

The ratio n_s/n_p is called the secondary-to-primary **turns ratio** of the transformer.

Example: A transformer has a primary of 400 turns and a secondary of 2800 turns, and an e.m.f. of 115 volts is applied to the primary. The secondary voltage will be

$$E_s = \frac{n_s}{n_p} E_p = \frac{2800}{400} \times 115 = 7 \times 115$$
$$= 805 \text{ volts}$$

Also, if an e.m.f. of 805 volts is applied to the 2800-turn winding (which then becomes the primary) the output voltage from the 400-turn winding will be 115 volts.

Either winding of a transformer can be used as the primary, providing the winding has enough turns (enough inductance) to induce a voltage equal to the applied voltage without requiring an excessive current flow.

Effect of Secondary Current

The current that flows in the primary when no current is taken from the secondary is called the **magnetizing current** of the transformer. In any properly-designed transformer the primary inductance will be so large that the magnetizing current will be quite small. The power consumed by the transformer when the secondary is "open" —that is, not delivering power—is only the amount necessary to supply the losses in the iron core and in the resistance of the wire with which the primary is wound.

When power is taken from the secondary winding, the secondary current sets up a magnetic field that opposes the field set up by the primary current. But if the induced voltage in the primary is to equal the applied voltage, the original field must be maintained. Consequently, the primary must draw enough additional current to set up a field exactly equal and opposite to the field set up by the secondary current.

In practical calculations on transformers it may be assumed that the entire primary current is caused by the secondary "load." This is justifiable because the magnetizing current should be very small in comparison with the primary "load" current at rated power output.

If the magnetic fields set up by the primary and secondary currents are to be equal, the primary current multiplied by the primary turns must equal the secondary current multiplied by the secondary turns. From this it follows that

$$I_P = \frac{n_S}{n_P} I_S$$

where I_p = Primary current
I_s = Secondary current
n_p = Number of turns on primary
n_s = Number of turns on secondary

Example: Suppose that the secondary of the transformer in the previous example is delivering a current of 0.2 ampere to a load. Then the primary current will be

$$I_p = \frac{n_s}{n_p} I_s = \frac{2800}{400} \times 0.2 = 7 \times 0.2 = 1.4 \text{ amp.}$$

Although the secondary voltage is higher than the primary voltage, the secondary *current* is *lower* than the primary current, and by the same ratio.

Power Relationships; Efficiency

A transformer cannot create power; it can only transfer it and change the e.m.f. Hence, the power taken from the secondary cannot exceed that taken by the primary from the source of applied e.m.f. There is always some power loss in the resistance of the coils and in the iron core, so in all practical cases the power taken from the source will exceed that taken from the secondary. Thus,

$$P_o = nP_i$$

where P_o = Power output from secondary
P_i = Power input to primary
n = Efficiency factor

The efficiency, n, always is less than 1. It is usually expressed as a percentage; if n is 0.65, for instance, the efficiency is 65 per cent.

Example: A transformer has an efficiency of 85% at its full-load output of 150 watts. The power input to the primary at full secondary, load will be

$$P_i = \frac{P_o}{n} = \frac{150}{0.85} = 176.5 \text{ watts}$$

A transformer is usually designed to have its highest efficiency at the power output for which it is rated. The efficiency decreases with either lower or higher outputs. On the other hand, the *losses* in the transformer are relatively small at low output but increase as more power is taken.

The amount of power that the transformer can handle is determined by its own losses, because these heat the wire and core. There is a limit to the temperature rise that can be tolerated, because too-high temperature either will melt the wire or cause the insulation to break down. A transformer can be operated at reduced output, even though the efficiency is low, because the actual loss also will be low under such conditions.

The full-load efficiency of small power transformers such as are used in radio receivers and transmitters usually lies between about 60 and 90 per cent, depending upon the size and design.

Leakage Reactance

In a practical transformer not all of the magnetic flux is common to both windings, although in well-designed transformers the amount of flux that "cuts" one coil and not the other is only a small percentage of the total flux. This **leakage flux** causes an e.m.f. of self-induction; consequently, there are small amounts of **leakage inductance** associated with both windings of the transformer. Leakage inductance acts in exactly the same way as an equivalent amount of ordinary inductance inserted in series with the circuit.

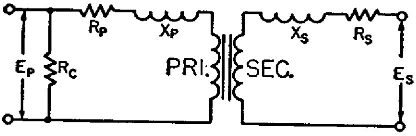


Fig. 2-33—The equivalent circuit of a transformer includes the effects of leakage inductance and resistance of both primary and secondary windings. The resistance R_C is an equivalent resistance representing the core losses, which are essentially constant for any given applied voltage and frequency. Since these are comparatively small, their effect may be neglected in many approximate calculations.

It has, therefore, a certain reactance, depending upon the amount of leakage inductance and the frequency. This reactance is called **leakage reactance.**

Current flowing through the leakage reactance causes a voltage drop. This voltage drop increases with increasing current, hence it increases as more power is taken from the secondary. Thus, the greater the secondary current, the smaller the secondary terminal voltage becomes. The resistances of the transformer windings also cause voltage drops when current is flowing; although these voltage drops are not in phase with those caused by leakage reactance, together they result in a lower secondary voltage under load than is indicated by the turns ratio of the transformer.

At power frequencies (60 cycles) the voltage at the secondary, with a reasonably well-designed transformer, should not drop more than about 10 per cent from open-circuit conditions to full load. The drop in voltage may be considerably more than this in a transformer operating at audio frequencies because the leakage reactance increases directly with the frequency.

Impedance Ratio

In an ideal transformer—one without losses or leakage reactance—the following relationship is true:

$$Z_p = Z_s N^2$$

where Z_p = Impedance looking into primary terminals from source of power

Z_s = Impedance of load connected to secondary

N = Turns ratio, primary to secondary

That is, a load of any given impedance connected to the secondary of the transformer will be transformed to a different value "looking into" the primary from the source of power. The impedance transformation is proportional to the square of the primary-to-secondary turns ratio.

Example: A transformer has a primary-to-secondary turns ratio of 0.6 (primary has 6/10 as many turns as the secondary) and a load of 3000 ohms is connected to the secondary. The impedance looking into the primary then will be

$$Z_p = Z_s N^2 = 3000 \times (0.6)^2 = 3000 \times 0.36$$
$$= 1080 \text{ ohms}$$

By choosing the proper turns ratio, the impedance of a fixed load can be transformed to any desired value, within practical limits. The transformed or "reflected" impedance has the same phase angle as the actual load impedance; thus if the load is a pure resistance the load presented by the primary to the source of power also will be a pure resistance.

The above relationship may be used in practical work even though it is based on an "ideal" transformer. Aside from the normal design requirements of reasonably low internal losses and low leakage reactance, the only requirement is that the primary have enough inductance to operate with low magnetizing current at the voltage applied to the primary.

The primary impedance of a transformer—*as it appears to the source of power*—is determined wholly by the load connected to the secondary and by the turns ratio. If the characteristics of the transformer have an appreciable effect on the impedance presented to the power source, the transformer is either poorly designed or is not suited to the voltage and frequency at which it is being used. Most transformers will operate quite well at voltages from slightly above to well below the design figure.

Impedance Matching

Many devices require a specific value of load resistance (or impedance) for optimum operation. The impedance of the actual load that is to dissipate the power may differ widely from this value, so a transformer is used to change the actual load into an impedance of the desired value. This is called **impedance matching.** From the preceding,

$$N = \sqrt{\frac{Z_p}{Z_s}}$$

where N = Required turns ratio, primary to secondary
Z_p = Primary impedance required
Z_s = Impedance of load connected to secondary

Example: A vacuum-tube a.f. amplifier requires a load of 5000 ohms for optimum performance, and is to be connected to a loudspeaker having an impedance of 10 ohms. The turns ratio, primary to secondary, required in the coupling transformer is

$$N = \sqrt{\frac{Z_p}{Z_s}} = \sqrt{\frac{5000}{10}} = \sqrt{500} = 22.4$$

The primary therefore must have 22.4 times as many turns as the secondary.

Impedance matching means, in general, adjusting the load impedance—by means of a transformer or otherwise—to a desired value. However, there is also another meaning. It is possible to show that any source of power will deliver its maximum possible output when the impedance of the load is equal to the internal impedance of the source. The impedance of the source is said to be "matched" under this condition. The efficiency is only 50 per cent in such a case; just as much power is used up in the source as is delivered to the load. Because of the poor efficiency, this type of impedance matching is limited to cases where only a small amount of power is available and heating from power loss in the source is not important.

Transformer Construction

Transformers usually are designed so that the magnetic path around the core is as short as possible. A short magnetic path means that the transformer will operate with fewer turns, for a given applied voltage, than if the path were long. A short path also helps to reduce flux leakage and therefore minimizes leakage reactance.

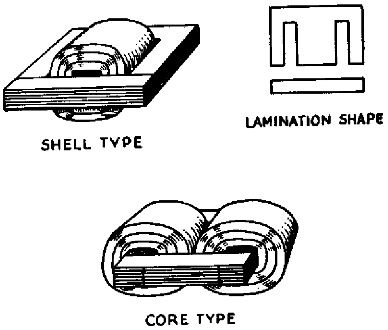


Fig. 2-34—Two common types of transformer construction. Core pieces are interleaved to provide a continuous magnetic path.

Two core shapes are in common use, as shown in Fig. 2-34. In the shell type both windings are placed on the inner leg, while in the core type the primary and secondary windings may be placed on separate legs, if desired. This is sometimes done when it is necessary to minimize capacitive effects between the primary and secondary, or when one of the windings must operate at very high voltage.

Core material for small transformers is usually silicon steel, called "transformer iron." The core is built up of laminations, insulated from each other (by a thin coating of shellac, for example) to prevent the flow of eddy currents. The laminations are interleaved at the ends to make the magnetic path as continuous as possible and thus reduce flux leakage.

The number of turns required in the primary for a given applied e.m.f. is determined by the size, shape and type of core material used, and the frequency. The number of turns required is inversely proportional to the cross-sectional area of the core. As a rough indication, windings of small power transformers frequently have about six to eight turns per volt on a core of 1-square-inch cross section and have a magnetic path 10 or 12 inches in length. A longer path or smaller cross section requires more turns per volt, and vice versa.

In most transformers the coils are wound in layers, with a thin sheet of treated-paper insulation between each layer. Thicker insulation is used between coils and between coils and core.

Autotransformers

The transformer principle can be utilized with only one winding instead of two, as shown in Fig. 2-35; the principles just discussed apply equally well. A one-winding transformer is called an **autotransformer.** The current in the common section (A) of the winding is the difference between the line (primary) and the load (secondary) currents, since these currents are out of phase. Hence if the line and load currents are nearly equal the common section of the winding may be wound with comparatively small wire. This will be the case only when the primary (line) and secondary (load) voltages are not very different. The autotransformer is used chiefly for boosting or reducing the power-line voltage by relatively small amounts. Continuously-variable autotransformers are commercially available under a variety of trade names; "Variac" and "Powerstat" are typical examples.

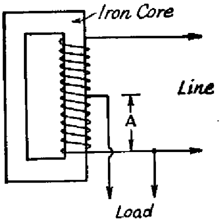


Fig. 2-35—The autotransformer is based on the transformer principle, but uses only one winding. The line and load currents in the common winding (A) flow in opposite directions, so that the resultant current is the difference between them. The voltage across A is proportional to the turns ratio.

THE DECIBEL

In most radio communication the received signal is converted into sound. This being the case, it is useful to appraise signal strengths in terms of relative loudness as registered by the ear. A peculiarity of the ear is that an increase or decrease in loudness is responsive to the *ratio* of the amounts of power involved, and is practically independent of absolute value of the power. For example, if a person estimates that the signal is "twice as loud" when the transmitter power is increased from 10 watts to 40 watts, he will also estimate that a 400-watt signal is twice as loud as a 100-watt signal. In other words, the human ear has a *logarithmic* response.

This fact is the basis for the use of the relative-power unit called the **decibel** (abbreviated **db.**) A change of one decibel in the power level is just detectable as a change in loudness under ideal conditions. The number of decibels corresponding to a given power ratio is given by the following formula:

$$Db. = 10 \log \frac{P_2}{P_1}$$

Common logarithms (base 10) are used.

Voltage and Current Ratios

Note that the decibel is based on *power* ratios. Voltage or current ratios can be used, but only when the impedance is the same for both values of voltage, or current. The gain of an amplifier cannot be expressed correctly in db. if it is based on the ratio of the output voltage to the input voltage unless both voltages are measured across the same value of impedance. When the impedance at both points of measurement is the same, the following formula may be used for voltage or current ratios:

$$Db. = 20 \log \frac{V_2}{V_1}$$

$$\text{or } 20 \log \frac{I_2}{I_1}$$

Decibel Chart

The two formulas are shown graphically in Fig. 2-36 for ratios from 1 to 10. Gains (increases) expressed in decibels may be added arithmetically; losses (decreases) may be subtracted. A power decrease is indicated by prefixing the decibel figure with a minus sign. Thus +6 db. means that the power has been multiplied by 4, while —6 db. means that the power has been divided by 4.

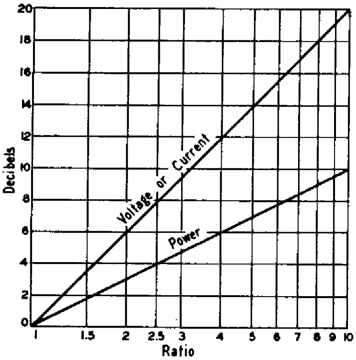


Fig. 2-36—Decibel chart for power, voltage and current ratios for power ratios of 1:1 to 10:1. In determining decibels for current or voltage ratios the currents (or voltages) being compared must be referred to the same value of impedance.

The chart may be used for other ratios by adding (or subtracting, if a loss) 10 db. each time the ratio scale is multiplied by 10, for power ratios; or by adding (or subtracting) 20 db. each time the scale is multiplied by 10 for voltage or current ratios. For example, a power ratio of 2.5 is 4 db. (from the chart). A power ratio of 10 times 2.5, or 25, is 14 db. (10 + 4), and a power ratio of 100 times 2.5, or 250, is 24 db. (20 + 4). A voltage or current ratio of 4 is 12 db., a voltage or current ratio of 40 is 32 db. (20 + 12), and one of 400 is 52 db. (40 + 12).

RADIO-FREQUENCY CIRCUITS

RESONANCE IN SERIES CIRCUITS

Fig. 2-37 shows a resistor, capacitor and inductor connected in series with a source of alternating current, the frequency of which can be varied over a wide range. At some *low* frequency the capacitive reactance will be much larger than the resistance of *R*, and the inductive reactance will be small compared with either the reactance of *C* or the resistance of *R*. (*R* is assumed to be the same at all frequencies.) On the other hand, at some very *high* frequency the reactance of *C* will be very small and the reactance of *L* will be very large. In either case the current will be small, because the net reactance is large.

At some intermediate frequency, the reactances of *C* and *L* will be equal and the voltage drops across the coil and capacitor will be equal and

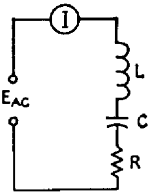


Fig. 2-37.—A series circuit containing *L*, *C* and *R* is "resonant" at the applied frequency when the reactance of *C* is equal to the reactance of *L*.

180 degrees out of phase. Therefore they cancel each other completely and the current flow is determined wholly by the resistance, R. At that frequency the current has its largest possible value, assuming the source voltage to be constant regardless of frequency. A series circuit in which the inductive and capacitive reactances are equal is said to be **resonant.**

The principle of resonance finds its most extensive application in radio-frequency circuits. The reactive effects associated with even small inductances and capacitances would place drastic limitations on r.f. circuit operation if it were not possible to "cancel them out" by supplying the right amount of reactance of the opposite kind—in other words, "tuning the circuit to resonance."

Resonant Frequency

The frequency at which a series circuit is resonant is that for which $X_L = X_C$. Substituting the formulas for inductive and capacitive reactance gives

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where f = Frequency in cycles per second
L = Inductance in henrys
C = Capacitance in farads
$\pi = 3.14$

These units are inconveniently large for radio-frequency circuits. A formula using more appropriate units is

$$f = \frac{10^6}{2\pi\sqrt{LC}}$$

where f = Frequency in kilocycles (kc.)
L = Inductance in microhenrys (μh.)
C = Capacitance in picofarads (pf.)
$\pi = 3.14$

Example: The resonant frequency of a series circuit containing a 5-μh. inductor and a 35-pf. capacitor is

$$f = \frac{10^6}{2\pi\sqrt{LC}} = \frac{10^6}{6.28 \times \sqrt{5 \times 35}}$$

$$= \frac{10^6}{6.28 \times 13.2} = \frac{10^6}{83} = 12{,}050 \text{ kc.}$$

The formula for resonant frequency is not affected by resistance in the circuit.

Resonance Curves

If a plot is drawn of the current flowing in the circuit of Fig. 2-37 as the frequency is varied (the applied voltage being constant) it would look like one of the curves in Fig. 2-38. The shape of the **resonance curve** at frequencies near resonance is determined by the ratio of reactance to resistance.

If the reactance of either the coil or capacitor is of the same order of magnitude as the resistance, the current decreases rather slowly as the frequency is moved in either direction away from resonance. Such a curve is said to be **broad.** On the other hand, if the reactance is considerably larger than the resistance the current decreases rapidly as the frequency moves away from resonance and the circuit is said to be **sharp.** A sharp circuit will respond a great deal more readily to the resonant frequency than to frequencies quite close to resonance; a broad circuit will respond almost equally well to a group or band of frequencies centering around the resonant frequency.

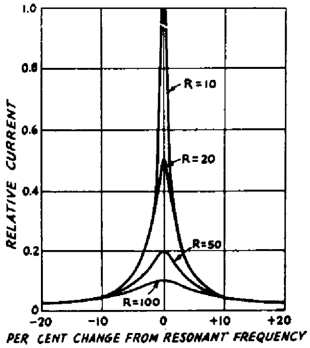


Fig. 2-38—Current in a series-resonant circuit with various values of series resistance. The values are arbitrary and would not apply to all circuits, but represent a typical case. It is assumed that the reactances (at the resonant frequency) are 1000 ohms. Note that at frequencies more than plus or minus ten per cent away from the resonant frequency the current is substantially unaffected by the resistance in the circuit.

Both types of resonance curves are useful. A sharp circuit gives good **selectivity**—the ability to respond strongly (in terms of current amplitude) at one desired frequency and discriminate against others. A broad circuit is used when the apparatus must give about the same response over a band of frequencies rather than to a single frequency alone.

Q

Most diagrams of resonant circuits show only inductance and capacitance; no resistance is indicated. Nevertheless, resistance is always present. At frequencies up to perhaps 30 Mc. this resist-

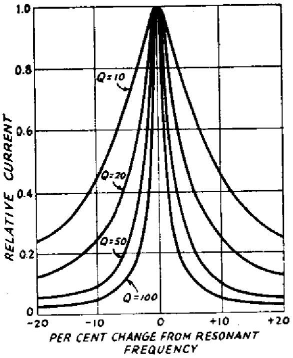


Fig. 2-39—Current in series-resonant circuits having different Qs. In this graph the current at resonance is assumed to be the same in all cases. The lower the Q, the more slowly the current decreases as the applied frequency is moved away from resonance.

ance is mostly in the wire of the coil. Above this frequency energy loss in the capacitor (principally in the solid dielectric which must be used to form an insulating support for the capacitor plates) also becomes a factor. This energy loss is equivalent to resistance. When maximum sharpness or selectivity is needed the object of design is to reduce the inherent resistance to the lowest possible value.

The value of the reactance of either the inductor or capacitor at the resonant frequency of a series-resonant circuit, divided by the *series* resistance in the circuit, is called the ***Q*** (quality factor) of the circuit, or

$$Q = \frac{X}{r}$$

where Q = Quality factor
X = Reactance of either coil or capacitor in ohms
r = Series resistance in ohms

Example: The inductor and capacitor in a series circuit each have a reactance of 350 ohms at the resonant frequency. The resistance is 5 ohms. Then the Q is

$$Q = \frac{X}{r} = \frac{350}{5} = 70$$

The effect of Q on the sharpness of resonance of a circuit is shown by the curves of Fig. 2-39. In these curves the frequency change is shown in percentage above and below the resonant frequency. Qs of 10, 20, 50 and 100 are shown; these values cover much of the range commonly used in radio work. The **unloaded Q** of a circuit is determined by the inherent resistances associated with the components.

Voltage Rise at Resonance

When a voltage of the resonant frequency is inserted in series in a resonant circuit, the voltage that appears across either the inductor or capacitor is considerably higher than the applied voltage. The current in the circuit is limited only by the resistance and may have a relatively high value; however, the same current flows through the high reactances of the inductor and capacitor and causes large voltage drops. The ratio of the reactive voltage to the applied voltage is equal to the ratio of reactance to resistance. This ratio is also the Q of the circuit. Therefore, the voltage across either the inductor or capacitor is equal to QE, where E is the voltage inserted in series. This fact accounts for the high voltages developed across the components of series-tuned antenna couplers (see chapter on "Transmission Lines").

RESONANCE IN PARALLEL CIRCUITS

When a variable-frequency source of constant voltage is applied to a parallel circuit of the type shown in Fig. 2-40 there is a resonance effect similar to that in a series circuit. However, in this case the "line" current (measured at the point indicated) is *smallest* at the frequency for which the inductive and capacitive reactances are equal. At that frequency the current through L is exactly canceled by the out-of-phase current through C, so that only the current taken by R flows in the line. At frequencies *below* resonance the current through L is larger than that through C, because the reactance of L is smaller and that of C higher at low frequencies; there is only partial cancellation of the two reactive currents and the line current therefore is larger than the current taken by R alone. At frequencies *above* resonance the situation is reversed and more current flows through C than through L, so the line current again increases. The current at resonance, being determined wholly by R, will be small if R is large and large if R is small.

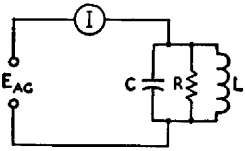
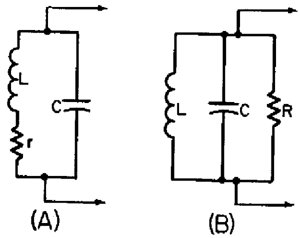


Fig. 2-40—Circuit illustrating parallel resonance.

The resistance R shown in Fig. 2-40 is not necessarily an actual resistor. In many cases it will be the series resistance of the coil "transformed" to an equivalent parallel resistance (see later). It may be antenna or other load resistance coupled into the tuned circuit. In all cases it represents the total effective resistance in the circuit.

Parallel and series resonant circuits are quite alike in some respects. For instance, the circuits given at A and B in Fig. 2-41 will behave identically, when an external voltage is applied, if (1) L and C are the same in both cases; and (2) R multiplied by r equals the square of the reactance (at resonance) of either L or C. When these conditions are met the two circuits will have the same Q. (These statements are approximate, but are quite accurate if the Q is 10 or more.) The circuit at A is a *series* circuit if it is viewed from the "inside"—that is, going around the loop formed by L, C and r—so its Q can be found from the ratio of X to r.

Thus a circuit like that of Fig. 2-41A has an equivalent **parallel impedance** (at resonance) of $R = \frac{X^2}{r}$; X is the reactance of either the inductor or the capacitor. Although R is not an actual resistor, to the source of voltage the



(A) (B)

Fig. 2-41—Series and parallel equivalents when the two circuits are resonant. The series resistance, r, in A is replaced in B by the equivalent parallel resistance ($R = X_C^2/r = X_L^2/r$) and vice versa.

parallel-resonant circuit "looks like" a pure resistance of that value. It is "pure" resistance because the inductive and capacitive currents are 180 degrees out of phase and are equal; thus there is no reactive current in the line. In a practical circuit with a high-Q capacitor, at the resonant frequency the parallel impedance is

$$Z_r = QX$$

where Z_r = Resistive impedance at resonance
Q = Quality factor of inductor
X = Reactance (in ohms) of either the inductor or capacitor

Example: The parallel impedance of a circuit with a coil Q of 50 and having inductive and capacitive reactances of 300 ohms will be

$$Z_r = QX = 50 \times 300 = 15{,}000 \text{ ohms.}$$

At frequencies off resonance the impedance is no longer purely resistive because the inductive and capacitive currents are not equal. The off-resonant impedance therefore is complex, and is lower than the resonant impedance for the reasons previously outlined.

The higher the Q of the circuit, the higher the parallel impedance. Curves showing the variation of impedance (with frequency) of a parallel circuit have just the same shape as the curves showing the variation of current with frequency in a series circuit. Fig. 2-42 is a set of such curves. A set of curves showing the relative response as a function of the departure from the resonant frequency would be similar to Fig. 2-39. The —3 db. bandwidth (bandwidth at 0.707 relative response) is given by

$$\text{Bandwidth}_{-3\text{ db.}} = f_o/Q$$

where f_o is the resonant frequency and Q the circuit Q. It is also called the "half-power" bandwidth, for ease of recollection.

Parallel Resonance in Low-Q Circuits

The preceding discussion is accurate only for Qs of 10 or more. When the Q is below 10, resonance in a parallel circuit having resistance in series with the coil, as in Fig. 2-41A, is not so easily defined. There is a set of values for L and C that will make the parallel impedance a pure resistance, but with these values the impedance does not have its maximum possible value. Another set of values for L and C will make the parallel impedance a maximum, but this maximum value is not a pure resistance. Either condition could be called "resonance," so with low-Q circuits it is necessary to distinguish between **maximum impedance** and **resistive impedance** parallel resonance. The difference between these L and C values and the equal reactances of a series-resonant circuit is appreciable when the Q is in the vicinity of 5, and becomes more marked with still lower Q values.

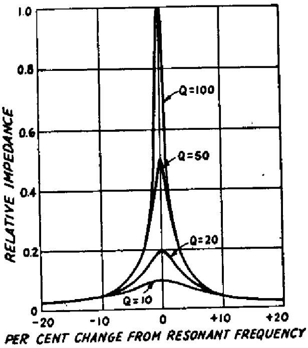


Fig. 2-42.—Relative impedance of parallel-resonant circuits with different Qs. These curves are similar to those in Fig. 2-39 for current in a series-resonant circuit. The effect of Q on impedance is most marked near the resonant frequency.

Q of Loaded Circuits

In many applications of resonant circuits the only power lost is that dissipated in the resistance of the circuit itself. At frequencies below 30 Mc. most of this resistance is in the coil. Within limits, increasing the number of turns in the coil increases the reactance faster than it raises the resistance, so coils for circuits in which the Q must be high are made with relatively large inductance for the frequency.

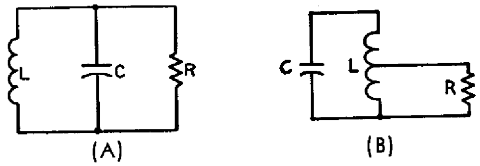


Fig. 2-43—The equivalent circuit of a resonant circuit delivering power to a load. The resistor *R* represents the load resistance. At B the load is tapped across part of *L*, which by transformer action is equivalent to using a higher load resistance across the whole circuit.

However, when the circuit delivers energy to a load (as in the case of the resonant circuits used in transmitters) the energy consumed in the circuit itself is usually negligible compared with that consumed by the load. The equivalent of such a circuit is shown in Fig. 2-43A, where the parallel resistor represents the load to which power is delivered. If the power dissipated in the load is at least ten times as great as the power lost in the inductor and capacitor, the parallel impedance of the resonant circuit itself will be so high compared with the resistance of the load that for all practical purposes the impedance of the combined circuit is equal to the load resistance. Under these conditions the Q of a parallel-resonant circuit loaded by a resistive impedance is

$$Q = \frac{R}{X}$$

where R = Parallel load resistance (ohms)
X = Reactance (ohms)

Example: A resistive load of 3000 ohms is connected across a resonant circuit in which the inductive and capacitive reactances are each 250 ohms. The circuit Q is then

$$Q = \frac{R}{X} = \frac{3000}{250} = 12$$

The "effective" Q of a circuit loaded by a parallel resistance becomes higher when the reactances are decreased. A circuit loaded with a relatively low resistance (a few thousand ohms) must have low-reactance elements (large capacitance and small inductance) to have reasonably high Q.

Impedance Transformation

An important application of the parallel-resonant circuit is as an impedance-matching device in the output circuit of a vacuum-tube r.f. power amplifier. As described in the chapter on vacuum tubes, there is an optimum value of load resistance for each type of tube and set of operating conditions. However, the resistance of the load to which the tube is to deliver power usually is considerably lower than the value required for proper tube operation. To transform the actual load resistance to the desired value the load may be tapped across part of the coil, as shown in Fig. 2-43B. This is equivalent to connecting a higher value of load resistance across the whole circuit, and is similar in principle to impedance transformation with an iron-core transformer. In high-frequency resonant circuits the impedance ratio does not vary exactly as the square of the turns ratio, because all the magnetic flux lines do not cut every turn of the coil. A desired reflected impedance usually must be obtained by experimental adjustment.

When the load resistance has a very low value (say below 100 ohms) it may be connected in series in the resonant circuit (as in Fig. 2-41A, for example), in which case it is transformed to an equivalent parallel impedance as previously described. If the Q is at least 10, the equivalent parallel impedance is

$$Z_r = \frac{X^2}{r}$$

where Z_r = Resistive parallel impedance at resonance
X = Reactance (in ohms) of either the coil or capacitor
r = Load resistance inserted in series

If the Q is lower than 10 the reactance will have to be adjusted somewhat, for the reasons given in the discussion of low-Q circuits, to obtain a resistive impedance of the desired value.

Reactance Values

The charts of Figs. 2-44 and 2-45 show reactance values of inductances and capacitances in the range commonly used in r.f. tuned circuits for the amateur bands. With the exception of the 3.5-4 Mc. band, limiting values for which are shown on the charts, the change in reactance over a band, for either inductors or capacitors, is small enough so that a single curve gives the reactance with sufficient accuracy for most practical purposes.

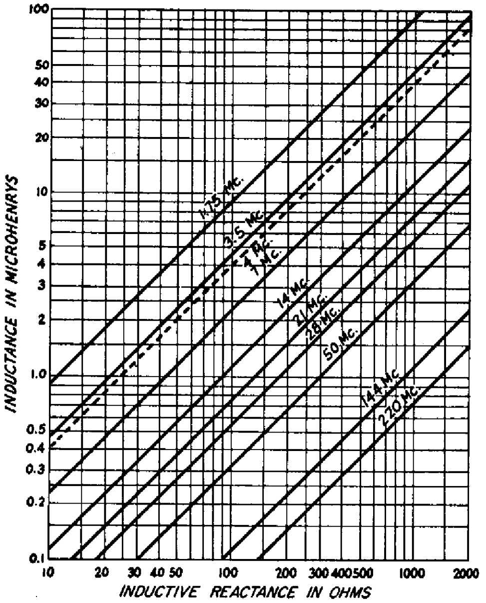


Fig. 2-44—Reactance chart for inductance values commonly used in amateur bands from 1.75 to 220 Mc.

L/C Ratio

The formula for resonant frequency of a circuit shows that the same frequency always will be obtained so long as the *product* of L and C is constant. Within this limitation, it is evident that L can be large and C small, L small and C large, etc. The relation between the two for a fixed frequency is called the ***L/C* ratio.** A **high-*C*** circuit is one that has more capacitance than "normal" for the frequency; a **low-*C*** circuit one that has less than normal capacitance. These terms depend to a considerable extent upon the particular ap-

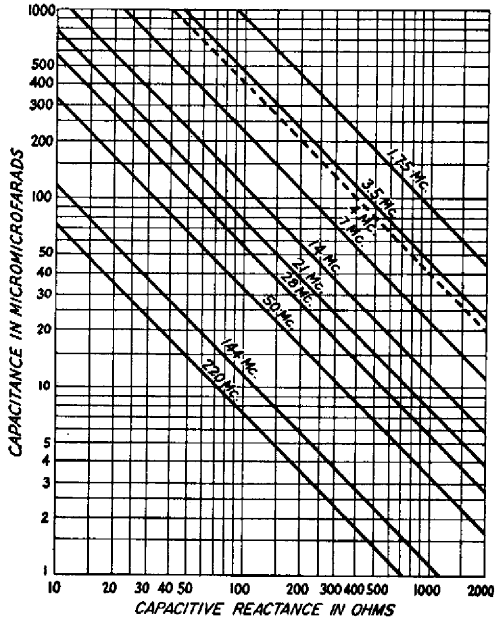


Fig. 2-45—Reactance chart for capacitance values commonly used in amateur bands from 1.75 to 220 Mc.

plication considered, and have no exact numerical meaning.

LC Constants

It is frequently convenient to use the numerical value of the LC **constant** when a number of calculations have to be made involving different L/C ratios for the same frequency. The constant for any frequency is given by the following equation:

$$LC = \frac{25{,}330}{f^2}$$

where L = Inductance in microhenrys (μh.)
C = Capacitance in micromicrofarads ($\mu\mu$f.)
f = Frequency in megacycles

Example: Find the inductance required to resonate at 3650 kc. (3.65 Mc.) with capacitances of 25, 50, 100, and 500 $\mu\mu$f. The LC constant is

$$LC = \frac{25{,}330}{(3.65)^2} = \frac{25{,}330}{13.35} = 1900$$

With 25 $\mu\mu$f. $L = 1900/C = 1900/25$
$= 76$ μh.
50 $\mu\mu$f. $L = 1900/C = 1900/50$
$= 38$ μh.
100 $\mu\mu$f. $L = 1900/C = 1900/100$
$= 19$ μh.
500 $\mu\mu$f. $L = 1900/C = 1900/500$
$= 3.8$ μh.

COUPLED CIRCUITS

Energy Transfer and Loading

Two circuits are **coupled** when energy can be transferred from one to the other. The circuit delivering power is called the **primary** circuit; the one receiving power is called the **secondary** circuit. The power may be practically all dissipated in the secondary circuit itself (this is usually the case in receiver circuits) or the secondary may simply act as a medium through which the power is transferred to a load. In the latter case, the coupled circuits may act as a radio-frequency impedance-matching device. The matching can be accomplished by adjusting the loading on the secondary and by varying the amount of coupling between the primary and secondary.

Coupling by a Common Circuit Element

One method of coupling between two resonant circuits is through a circuit element common to both. The three common variations of this type of coupling are shown in Fig. 2-46; the circuit element common to both circuits carries the subscript M. At A and B current circulating in L_1C_1 flows through the common element, and the voltage developed across this element causes current to flow in L_2C_2. At C,C_M and C_2 form a capacitive voltage divider across L_1C_1, and some of the voltage developed across L_1C_1 is applied across L_2C_2.

If both circuits are resonant to the same frequency, as is usually the case, the value of coupling reactance required for maximum energy transfer can be approximated by the following, based on $L_1 = L_2$, $C_1 = C_2$ and $Q_1 = Q_2$:

(A) $L_M \approx L_1/Q_1$; (B) $C_M \approx Q_1C_1$; (C) $C_M \approx C_1/Q_1$.

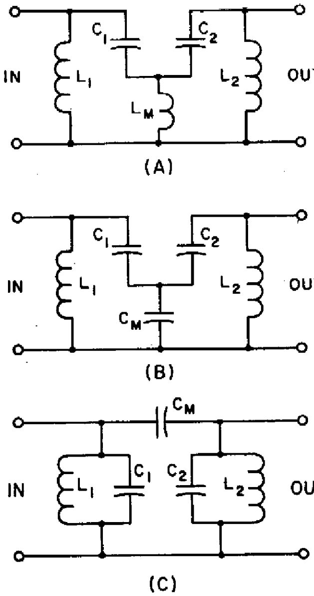


Fig. 2-46—Three methods of circuit coupling.

The coupling can be increased by increasing the above coupling elements in A and C and decreasing the value in B. When the coupling is increased, the resultant bandwidth of the combination is increased, and this principle is sometimes applied to "broad-band" the circuits in a transmitter or receiver. When the coupling elements in A and C are decreased, or when the coupling element in B is increased, the coupling between the circuits is decreased below the *critical coupling* value on which the above approximations are based. Less than critical coupling will decrease the bandwidth and the energy transfer; the principle is often used in receivers to improve the selectivity.

Inductive Coupling

Figs. 2-47 and 2-48 show inductive coupling, or coupling by means of the mutual inductance between two coils. Circuits of this type resemble the iron-core transformer, but because only a part of the magnetic flux lines set up by one coil cut the turns of the other coil, the simple relationships between turns ratio, voltage ratio and impedance

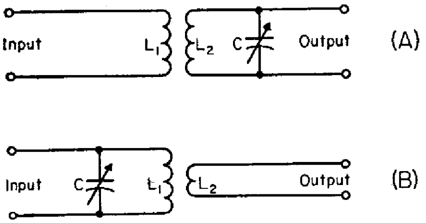


Fig. 2-47—Single-tuned inductively coupled circuits.

ratio in the iron-core transformer do not hold.

Two types of inductively-coupled circuits are shown in Fig. 2-47. Only one circuit is resonant. The circuit at A is frequently used in receivers for coupling between amplifier tubes when the tuning of the circuit must be varied to respond to signals of different frequencies. Circuit B is used principally in transmitters, for coupling a radio-frequency amplifier to a resistive load.

In these circuits the coupling between the primary and secondary coils usually is "tight"—that is, the coefficient of coupling between the coils is large. With very tight coupling either circuit operates nearly as though the device to which the untuned coil is connected were simply tapped across a corresponding number of turns on the tuned-circuit coil, thus either circuit is approximately equivalent to Fig. 2-43B.

By proper choice of the number of turns on the untuned coil, and by adjustment of the coupling, the parallel impedance of the tuned circuit may be adjusted to the value required for the proper operation of the device to which it is connected. In any case, the maximum energy transfer possible for a given coefficient of coupling is obtained when the reactance of the untuned coil is equal to the resistance of its load.

The Q and parallel impedance of the tuned circuit are reduced by coupling through an untuned coil in much the same way as by the tapping arrangement shown in Fig. 2-43B.

Coupled Resonant Circuits

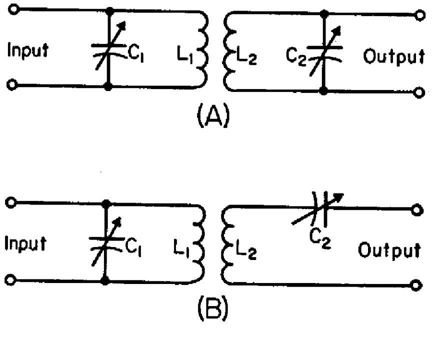


Fig. 2-48—Inductively-coupled resonant circuits. Circuit A is used for high-resistance loads (load resistance much higher than the reactance of either L_2 or C_2 at the resonant frequency). Circuit B is suitable for low resistance loads (load resistance much lower than the reactance of either L_2 or C_2 at the resonant frequency).

When the primary and secondary circuits are both tuned, as in Fig. 2-48, the resonance effects in both circuits make the operation somewhat more complicated than in the simpler circuits just considered. Imagine first that the two circuits are not coupled and that each is independently tuned to the resonant frequency. The impedance of each will be purely resistive. If the primary circuit is connected to a source of r.f. energy of the resonant frequency and the secondary is then loosely coupled to the primary, a current will flow in the secondary circuit. In flowing through the resistance of the secondary circuit and any load that may be connected to it, the current causes a power loss. This power must come from the energy source through the primary circuit, and manifests itself in the primary as an increase in the equivalent resistance in series with the primary coil. Hence the Q and parallel impedance of the primary circuit are decreased by the coupled secondary. As the coupling is made greater (without changing the tuning of either circuit) the coupled resistance becomes larger and the parallel impedance of the primary continues to decrease. Also, as the coupling is made tighter the amount of power transferred from the primary to the secondary will increase to a maximum at one value of coupling, called **critical coupling**, but then decreases if the coupling is tightened still more (still without changing the tuning).

Critical coupling is a function of the Qs of the two circuits. A higher coefficient of coupling is required to reach critical coupling when the Qs are low; if the Qs are high, as in receiving applications, a coupling coefficient of a few per cent may give critical coupling.

With loaded circuits such as are used in transmitters the Q may be too low to give the desired power transfer even when the coils are coupled as tightly as the physical construction permits. In such case, increasing the Q of either circuit will be helpful, although it is generally better to increase the Q of the lower-Q circuit rather than the reverse. The Q of the parallel-tuned primary (input) circuit can be increased by decreasing the L/C ratio because, as shown in connection with Fig. 2-43, this circuit is in effect loaded by a parallel resistance (effect of coupled-in resistance). In the parallel-tuned secondary circuit, Fig. 2-48A, the Q can be increased, for a fixed value of load resistance, either by decreasing the L/C ratio or by tapping the load down (see Fig. 2-43). In the series-tuned secondary circuit, Fig. 2-48B, the Q may be increased by *increasing* the L/C ratio. There will generally be no difficulty in securing sufficient coupling, with practicable coils, if the product of the Qs of the two tuned circuits is 10 or more. A smaller product will suffice if the coil construction permits tight coupling.

Selectivity

In Fig. 2-47 only one circuit is tuned and the selectivity curve will be essentially that of a single resonant circuit. As stated, the effective Q depends upon the resistance connected to the untuned coil.

In Fig. 2-48, the selectivity is increased. It approaches that of a single tuned circuit having a Q equalling the sum of the individual circuit Qs—*if* the coupling is well below critical (this is not the condition for optimum power transfer discussed immediately above) and both circuits are tuned to resonance. The Qs of the individual circuits are affected by the degree of coupling, because each couples resistance into the other; the

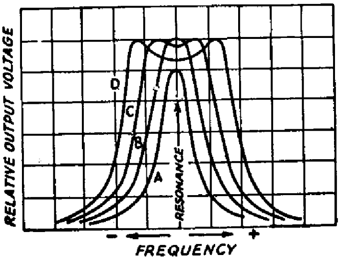


Fig. 2-49—Showing the effect on the output voltage from the secondary circuit of changing the coefficient of coupling between two resonant circuits independently tuned to the same frequency. The voltage applied to the primary is held constant in amplitude while the frequency is varied, and the output voltage is measured across the secondary.

tighter the coupling, the lower the individual *Q*s and therefore the lower the over-all selectivity.

If both circuits are independently tuned to resonance, the over-all selectivity will vary about as shown in Fig. 2-49 as the coupling is varied. With loose coupling, *A*, the output voltage (across the secondary circuit) is small and the selectivity is high. As the coupling is increased the secondary voltage also increases until critical coupling, *B*, is reached. At this point the output voltage at the resonant frequency is maximum but the selectivity is lower than with looser coupling. At still tighter coupling, *C*, the output voltage at the resonant frequency decreases, but as the frequency is varied either side of resonance it is found that there are two "humps" to the curve, one on either side of resonance. With very tight coupling, *D*, there is a further decrease in the output voltage at resonance and the "humps" are farther away from the resonant frequency. Curves such as those at *C* and *D* are called **flat-topped** because the output voltage does not change much over an appreciable band of frequencies.

Note that the off-resonance humps have the same maximum value as the resonant output voltage at critical coupling. These humps are caused by the fact that at frequencies off resonance the secondary circuit is reactive and couples reactance as well as resistance into the primary. The coupled resistance decreases off resonance, and each hump represents a new condition of critical coupling at a frequency to which the primary is tuned by the additional coupled-in reactance from the secondary.

Fig. 2-50 shows the response curves for various degrees of coupling between two circuits tuned to a frequency f_0. Equals *Q*s are assumed in both circuits, although the curves are representative if the *Q*s differ by ratios up to 1.5 or even 2 to 1. In these cases, a value of $Q = \sqrt{Q_1 Q_2}$ should be used.

Band-Pass Coupling

Over-coupled resonant circuits are useful where substantially uniform output is desired over a continuous band of frequencies, without readjustment of tuning. The width of the flat top of the resonance curve depends on the *Q*s of the two circuits as well as the tightness of coupling; the frequency separation between the humps will increase, and the curve become more flat-topped, as the *Q*s are lowered.

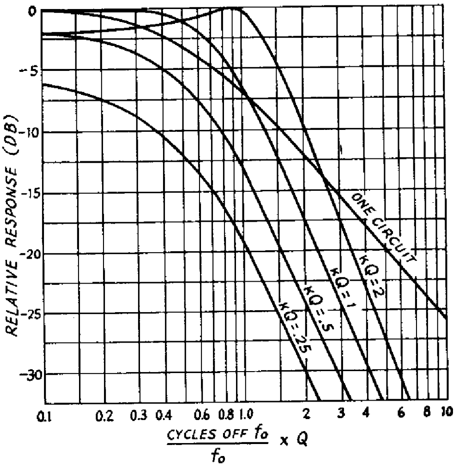


Fig. 2-50—Relative response for a single tuned circuit and for coupled circuits. For inductively-coupled circuits (Figs. 2-46A and 2-48A), $k = \frac{M}{\sqrt{L_1 L_2}}$ where *M* is the mutual inductance. For capacitance-coupled circuits (Figs. 2-46B and 2-46C), $k \cong \frac{\sqrt{C_1 C_2}}{C_M}$ and $k \cong \frac{C_M}{\sqrt{C_1 C_2}}$ respectively.

Band-pass operation also is secured by tuning the two circuits to slightly different frequencies, which gives a double-humped resonance curve even with loose coupling. This is called **stagger tuning.** To secure adequate power transfer over the frequency band it is usually necessary to use tight coupling and experimentally adjust the circuits for the desired performance.

Link Coupling

A modification of inductive coupling, called **link coupling,** is shown in Fig. 2-51. This gives the effect of inductive coupling between two coils that have no mutual inductance; the link is simply a means for providing the mutual inductance. The total mutual inductance between two coils coupled by a link cannot be made as great as if the coils themselves were coupled. This is because the coefficient of coupling between air-

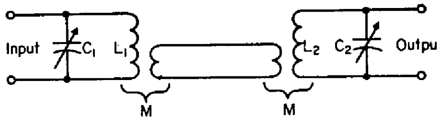


Fig. 2-51—Link coupling. The mutual inductances at both ends of the link are equivalent to mutual inductance between the tuned circuits, and serve the same purpose.

core coils is considerably less than 1, and since there are two coupling points the over-all coupling coefficient is less than for any *pair* of coils. In practice this need not be disadvantageous because the power transfer can be made great enough by making the tuned circuits sufficiently high-Q. Link coupling is convenient when ordinary inductive coupling would be impracticable for constructional reasons.

The link coils usually have a small number of turns compared with the resonant-circuit coils. The number of turns is not greatly important, because the coefficient of coupling is relatively independent of the number of turns on either coil; it is more important that both link coils should have about the *same* inductance. The length of the link between the coils is not critical if it is very small compared with the wavelength, but if the length is more than about one-twentieth of a wavelength the link operates more as a transmission line than as a means for providing mutual inductance. In such case it should be treated by the methods described in the chapter on Transmission Lines.

IMPEDANCE-MATCHING CIRCUITS

The coupling circuits discussed in the preceding section have been based either on inductive coupling or on coupling through a common circuit element between two resonant circuits. These are not the only circuits that may be used for transferring power from one device to another.

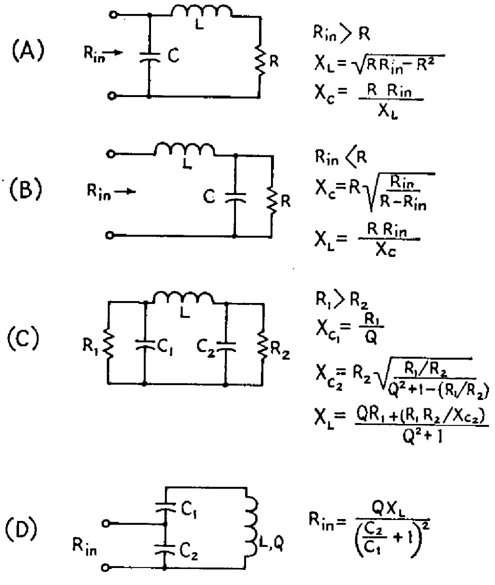


Fig. 2-52—Impedance-matching networks adaptable to amateur work. (A) L network for transforming to a lower value of resistance. (B) L network for transforming to a higher resistance value. (C) Pi network. R_1 is the larger of the two resistors; Q is defined as R_1/X_{C1}. (D) Tapped tuned circuit used in some receiver applications. The impedance of the tuned circuit is transformed to a lower value, R_{in}, by the capacitive divider.

There is, in fact, a wide variety of such circuits available, all of them being classified generally as **impedance-matching networks.** Several networks frequently used in amateur equipment are shown in Fig. 2-52.

The L Network

The L network is the simplest possible impedance-matching circuit. It closely resembles an ordinary resonant circuit with the load resistance, R, Fig. 2-52, either in series or parallel. The arrangement shown in Fig. 2-52A is used when the desired impedance, R_{IN}, is larger than the actual load resistance, R, while Fig. 2-52B is used in the opposite case. The design equations for each case are given in the figure, in terms of the circuit reactances. The reactances may be converted to inductance and capacitance by means of the formulas previously given or taken directly from the charts of Figs. 2-44 and 2-45.

When the impedance transformation ratio is large—that is, one of the two impedances is of the order of 100 times (or more) larger than the other—the operation of the circuit is exactly the same as previously discussed in connection with impedance transformation with a simple LC resonant circuit.

The Q of an L network is found in the same way as for simple resonant circuits. That is, it is equal to X_L/R or R_{IN}/X_C in Fig. 2-52A, and to X_L/R_{IN} or R/X_C in Fig. 2-52B. The value of Q is determined by the ratio of the impedances to be matched, and cannot be selected independently. In the equations of Fig. 2-52 it is assumed that both R and R_{IN} are pure resistances.

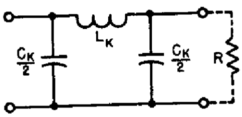
The Pi Network

The pi network, shown in Fig. 2-52C, offers more flexibility than the L since the operating Q may be chosen practically at will. The only limitation on the circuit values that may be used is that the reactance of the series arm, the inductor L in the figure, must not be greater than the square root of the product of the two values of resistive impedance to be matched. As the circuit is applied in amateur equipment, this limiting value of reactance would represent a network with an undesirably low operating Q, and the circuit values ordinarily used are well on the safe side of the limiting values.

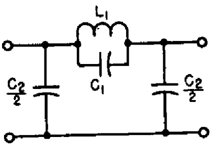
In its principal application as a "tank" circuit matching a transmission line to a power amplifier tube, the load R_2 will generally have a fairly low value of resistance (up to a few hundred ohms) while R_1, the required load for the tube, will be of the order of a few thousand ohms. In such a case the Q of the circuit is defined as R_1/X_{C1}, so the choice of a value for the operating Q immediately sets the value of X_{C1} and hence of C_1. The values of X_{C2} and X_L are then found from the equations given in the figure.

Graphical solutions for practical cases are given in the chapter on transmitter design in the discussion of plate tank circuits. The L and C values may be calculated from the reactances or read from the charts of Figs. 2-44 and 2-45.

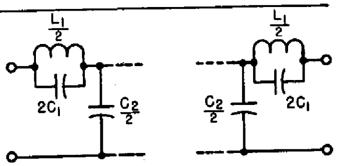
LOW-PASS FILTERS



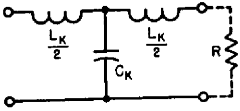
Constant-k π section



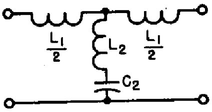
m-derived π section



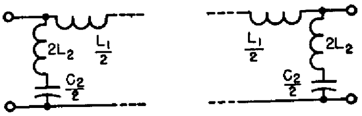
m-derived end sections for use with intermediate π section



Constant-k T section



m-derived T section



m-derived end sections for use with intermediate T section

$L_K = \frac{R}{\pi f_c}$ $C_K = \frac{1}{\pi f_c R}$

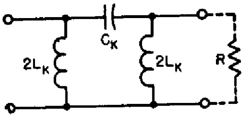
$L_1 = mL_K$ $C_1 = \frac{1-m^2}{4m} C_K$

$L_2 = \frac{1-m^2}{4m} L_K$ $C_2 = m C_K$

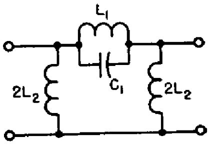
$L_1 = mL_K$ $C_1 = \frac{1-m^2}{4m} C_K$

$L_2 = \frac{1-m^2}{4m} L_K$ $C_2 = m C_K$

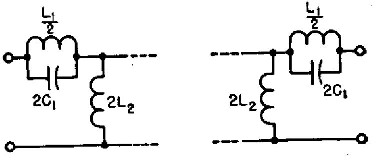
HIGH-PASS FILTERS



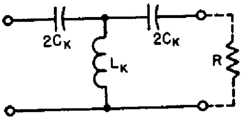
Constant-k π section



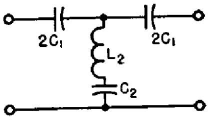
m-derived π section



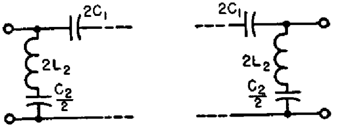
m-derived end sections for use with intermediate π section



Constant-k T section



m-derived T section



m-derived end section for use with intermediate T section

$L_K = \frac{R}{4\pi f_c}$ $C_K = \frac{1}{4\pi f_c R}$

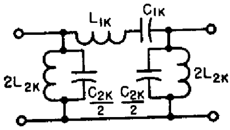
$L_1 = \frac{4m}{1-m^2} L_K$ $C_1 = \frac{C_K}{m}$

$L_2 = \frac{L_K}{m}$ $C_2 = \frac{4m}{1-m^2} C_K$

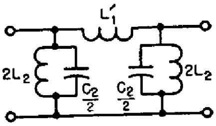
$L_1 = \frac{4m}{1-m^2} L_K$ $C_1 = \frac{C_K}{m}$

$L_2 = \frac{L_K}{m}$ $C_2 = \frac{4m}{1-m^2} C_K$

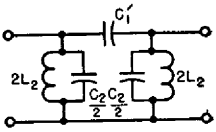
BANDPASS FILTERS



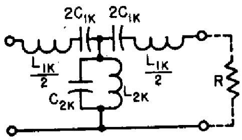
Constant-k π section



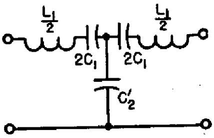
Three-element π section



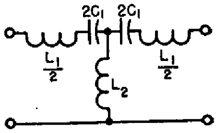
Three-element π section



Constant-k T section



Three-element T section



Three-element T section

$L_{1K} = \frac{R}{\pi(f_2-f_1)}$ $C_{1K} = \frac{f_2-f_1}{4\pi f_1 f_2 R}$

$L_{2K} = \frac{(f_2-f_1)R}{4\pi f_1 f_2}$ $C_{2K} = \frac{1}{\pi(f_2-f_1)R}$

$L_1 = L_{1K}$ $L_1' = \frac{R}{\pi(f_1+f_2)}$

$C_1 = \frac{f_2-f_1}{4\pi f_1^2 R}$ $L_2 = \frac{(f_2-f_1)R}{4\pi f_1^2}$

$C_2 = C_{2K}$ $C_2' = \frac{1}{\pi(f_1+f_2)R}$

$L_1 = \frac{f_1 R}{\pi f_2(f_2-f_1)}$ $C_1 = C_{1K}$

$C_1' = \frac{f_1+f_2}{4\pi f_1 f_2 R}$ $L_2 = L_{2K}$

$L_2 = \frac{(f_1+f_2)R}{4\pi f_1 f_2}$ $C_2 = \frac{f_1}{\pi f_2(f_2-f_1)R}$

Fig. 2-53—Basic filter sections and design formulas. In the above formulas *R* is in ohms, C in farads, *L* in henrys, and *f* in cycles per second.

Tapped Tuned Circuit

The tapped tuned circuit of Fig. 2-52D is useful in some receiver applications, where it is desirable to use a high-impedance tuned circuit as a lower-impedance load. When the Q of the inductor has been determined, the capacitors can be selected to give the desired impedance transformation and the necessary resultant capacitance to tune the circuit to resonance.

FILTERS

A **filter** is an electrical circuit configuration **(network)** designed to have specific characteristics with respect to the transmission or attenuation of various frequencies that may be applied to it. There are three general types of filters: **low-pass, high-pass,** and **band-pass.**

A low-pass filter is one that will permit all frequencies below a specified one called the **cut-off frequency** to be transmitted with little or no loss, but that will attenuate all frequencies above the cut-off frequency.

A high-pass filter similarly has a cut-off frequency, above which there is little or no loss in transmission, but below which there is considerable attenuation. Its behavior is the opposite of that of the low-pass filter.

A band-pass filter is one that will transmit a selected band of frequencies with substantially no loss, but that will attenuate all frequencies either higher or lower than the desired band.

The **pass band** of a filter is the frequency spectrum that is transmitted with little or no loss. The transmission characteristic is not necessarily perfectly uniform in the pass band, but the variations usually are small.

The **stop band** is the frequency region in which attenuation is desired. The attenuation may vary in the stop band, and in a simple filter usually is least near the cut-off frequency, rising to high values at frequencies considerably removed from the cut-off frequency.

Filters are designed for a specific value of purely resistive impedance (the **terminating impedance** of the filter). When such an impedance is connected to the output terminals of the filter, the impedance looking into the input terminals has essentially the same value, throughout most of the pass band. Simple filters do not give perfectly uniform performance in this respect, but the input impedance of a properly-terminated filter can be made fairly constant, as well as closer to the design value, over the pass band by using **m-derived** filter sections.

A discussion of filter design principles is beyond the scope of this *Handbook,* but it is not difficult to build satisfactory filters from the circuits and formulas given in Fig. 2-53. Filter circuits are built up from elementary sections as shown in the figure. These sections can be used alone or, if greater attenuation and sharper cut-off (that is, a more rapid rate of rise of attenuation with frequency beyond the cut-off frequency) are required, several sections can be connected in series. In the low- and high-pass filters, f_c represents the cut-off frequency, the highest (for the low-pass) or the lowest (for the high-pass) frequency transmitted without attenuation. In the band-pass filter designs, f_1 is the low-frequency cut-off and f_2 the high-frequency cut-off. The units for L, C, R and f are henrys, farads, ohms and cycles per second, respectively.

All of the types shown are "unbalanced" (one side grounded). For use in balanced circuits (e.g., 300-ohm transmission line, or push-pull audio circuits), the series reactances should be equally divided between the two legs. Thus the balanced constant-k π-section low-pass filter would use two inductors of a value equal to $L_k/2$, while the balanced constant-k π-section high-pass filter would use two capacitors each equal to $2C_k$.

If several low- (or high-) pass sections are to be used, it is advisable to use m-derived end sections on either side of a constant-k center section, although an m-derived center section can be used. The factor m determines the ratio of the cut-off frequency, f_c, to a frequency of high attenuation, f_∞. Where only one m-derived section is used, a value of 0.6 is generally used for m, although a deviation of 10 or 15 per cent from this value is not too serious in amateur work. For a value of $m = 0.6$, f_∞ will be $1.25f_c$ for the low-pass filter and $0.8f_c$ for the high-pass filter. Other values can be found from

$$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2} \text{ for the low-pass filter and}$$

$$m = \sqrt{1 - \left(\frac{f_\infty}{f_c}\right)^2} \text{ for the high-pass filter.}$$

The output sides of the filters shown should be terminated in a resistance equal to R, and there should be little or no reactive component in the termination.

PIEZOELECTRIC CRYSTALS

A number of crystalline substances found in nature have the ability to transform mechanical strain into an electrical charge, and *vice versa.* This property is known as the **piezoelectric effect.** A small plate or bar cut in the proper way from a quartz crystal and placed between two conducting electrodes will be mechanically strained when the electrodes are connected to a source of voltage. Conversely, if the crystal is squeezed between two electrodes a voltage will be developed between the electrodes.

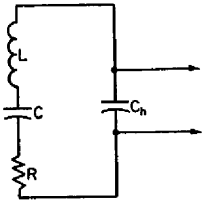
Piezoelectric crystals can be used to transform mechanical energy into electrical energy, and vice versa. They are used in microphones and phonograph pick-ups, where mechanical vibrations are transformed into alternating voltages of corresponding frequency. They are also used in headsets and loudspeakers, transforming electrical energy into mechanical vibration. Crystals of Rochelle salts are used for these purposes.

Crystal Resonators

Crystalline plates also are mechanical resonators that have natural frequencies of vibration ranging from a few thousand cycles to tens of megacycles per second. The vibration frequency depends on the kind of crystal, the way the plate is cut from the natural crystal, and on the dimensions of the plate. The thing that makes the **crystal resonator** valuable is that it has extremely high Q, ranging from a minimum of about 20,000 to as high as 1,000,000.

Analogies can be drawn between various mechanical properties of the crystal and the electrical characteristics of a tuned circuit. This leads to an "equivalent circuit" for the crystal. The electrical coupling to the crystal is through the holder plates between which it is sandwiched; these plates form, with the crystal as the dielectric, a small capacitator like any other capacitor constructed of two plates with a dielectric between. The crystal itself is equivalent to a series-resonant circuit, and together with the capacitance of the holder forms the equivalent circuit shown in Fig. 2-54. At frequencies of the order of 450 kc., where crystals are widely used as resonators, the equivalent L may be several henry's and the equivalent C only a few hundredths of a micromicrofarad. Although the equivalent R is of the order of a few thousand ohms, the reactance at resonance is so high that the Q of the crystal likewise is high.

Fig. 2-54—Equivalent circuit of a crystal resonator. L, C and R are the electrical equivalents of mechanical properties of the crystal; C_h is the capacitance of the holder plates with the crystal plate between them.



A circuit of the type shown in Fig. 2-54 has a series-resonant frequency, when viewed from the circuit terminals indicated by the arrowheads, determined by L and C only. At this frequency the circuit impedance is simply equal to R, providing the reactance of C_h is large compared with R (this is generally the case). The circuit also has a parallel-resonant frequency determined by L and the equivalent capacitance of C and C_h in series. Since this equivalent capacitance is smaller than C alone, the parallel-resonant frequency is higher than the series-resonant frequency. The separation between the two resonant frequencies depends on the ratio of C_h to C, and when this ratio is large (as in the case of a crystal resonator, where C_h will be a few μμf. in the average case) the two frequencies will be quite close together. A separation of a kilocycle or less at 455 kc. is typical of a quartz crystal.

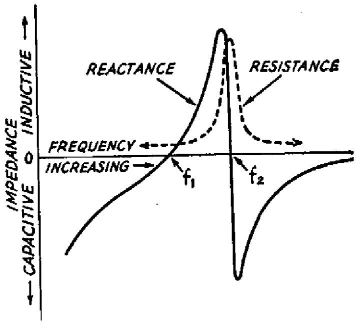


Fig. 2-55—Reactance and resistance vs. frequency of a circuit of the type shown in Fig. 2-54. Actual values of reactance, resistance and the separation between the series- and parallel-resonant frequencies, f_1, and f_2, respectively, depend on the circuit constants.

Fig. 2-55 shows how the resistance and reactance of such a circuit vary as the applied frequency is varied. The reactance passes through zero at both resonant frequencies, but the resistance rises to a large value at parallel resonance, just as in any tuned circuit.

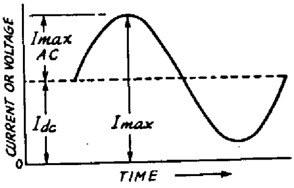
Quartz crystals may be used either as simple resonators for their selective properties or as the frequency-controlling elements in oscillators as described in later chapters. The series-resonant frequency is the one principally used in the former case, while the more common forms of oscillator circuit use the parallel-resonant frequency.

PRACTICAL CIRCUIT DETAILS

COMBINED A.C. AND D.C.

Most radio circuits are built around vacuum tubes, and it is the nature of these tubes to require direct current (usually at a fairly high voltage) for their operation. They convert the direct current into an alternating current (and sometimes the reverse) at frequencies varying from well down in the audio range to well up in the superhigh range. The conversion process almost invariably requires that the direct and alternating currents meet somewhere in the circuit.

Fig. 2-56—Pulsating d.c., composed of an alternating current or voltage superimposed on a steady direct current or voltage.



In this meeting, the a.c. and d.c. are actually combined into a single current that "pulsates" (at the a.c. frequency) about an average value equal to the direct current. This is shown in Fig. 2-56. It is convenient to consider that the alternating current is **superimposed** on the direct current, so we may look upon the actual current as having two components, one d.c. and the other a.c.

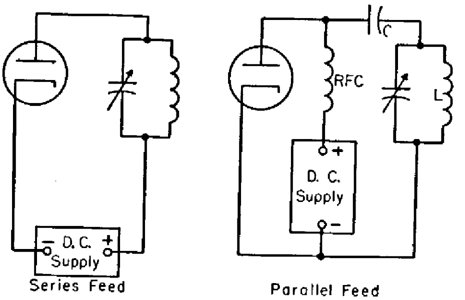
In an alternating current the positive and negative alternations have the same average amplitude, so when the wave is superimposed on a direct current the latter is alternately increased and decreased by the same amount. There is thus no *average* change in the direct current. If a d.c. instrument is being used to read the current, the reading will be exactly the same whether or not the a.c. is superimposed.

However, there is actually more power in such a combination current than there is in the direct current alone. This is because power varies as the square of the instantaneous value of the current, and when all the instantaneous squared values are averaged over a cycle the total power is greater than the d.c. power alone. If the a.c. is a sine wave having a peak value just equal to the d.c., the power in the circuit is 1.5 times the d.c. power. An instrument whose readings are proportional to power will show such an increase.

Series and Parallel Feed

Fig. 2-57 shows in simplified form how d.c. and a.c. may be combined in a vacuum-tube circuit. In this case, it is assumed that the a.c. is at radio frequency, as suggested by the coil-and-capacitor tuned circuit. It is also assumed that r.f. current can easily flow through the d.c. supply; that is, the impedance of the supply at radio frequencies is so small as to be negligible.

In the circuit at the left, the tube, tuned circuit, and d.c. supply all are connected in series. The direct current flows through the r.f. coil to get to the tube; the r.f. current generated by the tube flows through the d.c. supply to get to the tuned circuit. This is **series feed.** It works because the impedance of the d.c. supply at radio frequencies is so low that it does not affect the flow of *r.f.* current, and because the d.c. resistance of the coil is so low that it does not affect the flow of *direct* current.



Series Feed

Parallel Feed

Fig. 2-57—Illustrating series and parallel feed.

In the circuit at the right the direct current does not flow through the r.f. tuned circuit, but instead goes to the tube through a second coil, *RFC* **(radio-frequency choke).** Direct current cannot flow through *L* because a **blocking capacitance,** *C,* is placed in the circuit to prevent it. (Without *C,* the d.c. supply would be short-circuited by the low resistance of *L.*) On the other hand, the r.f. current generated by the tube can easily flow through *C* to the tuned circuit because the capacitance of *C* is intentionally chosen to have low reactance (compared with the impedance of the tuned circuit) at the radio frequency. The r.f. current cannot flow through the d.c. supply because the inductance of *RFC* is intentionally made so large that it has a very high reactance at the radio frequency. The resistance of *RFC,* however, is too low to have an appreciable effect on the flow of direct current. The two currents are thus in *parallel,* hence the name **parallel feed.**

Either type of feed may be used for both a.f. and r.f. circuits. In parallel feed there is no d.c. voltage on the a.c. circuit, a desirable feature from the viewpoint of safety to the operator, because the voltages applied to tubes—particularly transmitting tubes—are dangerous. On the other hand, it is somewhat difficult to make an r.f. choke work well over a wide range of frequencies. Series feed is often preferred, therefore, because it is relatively easy to keep the impedance between the a.c. circuit and the tube low.

Bypassing

In the series-feed circuit just discussed, it was assumed that the d.c. supply had very low impedance at radio frequencies. This is not likely to be true in a practical power supply, partly because the normal physical separation between the supply and the r.f. circuit would make it necessary to use rather long connecting wires or leads. At radio frequencies, even a few feet of wire can have fairly large reactance—too large to be considered a really "low-impedance" connection.

An actual circuit would be provided with a **bypass capacitor,** as shown in Fig. 2-58. Capacitor *C* is chosen to have low reactance at the operating frequency, and is installed right in the circuit where it can be wired to the other parts with quite short connecting wires. Hence the r.f. current will tend to flow through it rather than through the d.c. supply.

To be effective, the reactance of the bypass capacitor should not be more than one-tenth of the impedance of the bypassed part of the circuit. Very often the latter impedance is not known, in which case it is desirable to use the largest capacitance in the bypass that circumstances permit. To make doubly sure that r.f. current will not flow through a non-r.f. circuit such as a power supply, an r.f. choke may be connected in the lead to the latter, as shown in Fig. 2-58.

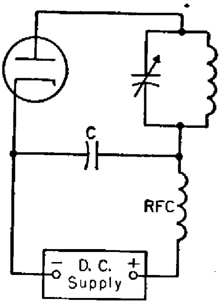


Fig. 2-58—Typical use of a bypass capacitor and r.f. choke in a series-feed circuit.

The same type of bypassing is used when audio frequencies are present in addition to r.f. Because the reactance of a capacitor changes with frequency, it is readily possible to choose a capacitance that will represent a very low reactance at radio frequencies but that will have such high reactance at audio frequencies that it is practically an open circuit. A capacitance of 0.001 μf. is practically a short circuit for r.f., for example, but is almost an open circuit at audio frequencies. (The actual value of capacitance that is usable will be modified by the impedances concerned.) Capacitors also are used in audio circuits to carry the audio frequencies around a d.c. supply.

Distributed Capacitance and Inductance

In the discussions earlier in this chapter it was assumed that a capacitor has only capacitance and that an inductor has only inductance. Unfortunately, this is not strictly true. There is always a certain amount of inductance in a conductor of any length, and a capacitor is bound to have a little inductance in addition to its intended capacitance. Also, there is always capacitance between two conductors or between parts of the same conductor, and thus there is appreciable capacitance between the turns of an inductance coil.

This **distributed inductance** in a capacitor and the **distributed capacitance** in an inductor have important practical effects. Actually, every capacitor is in effect a series-tuned circuit, resonant at the frequency where its capacitance and inductance have the same reactance. Similarly, every inductor is in effect a parallel-tuned circuit, resonant at the frequency where its inductance and distributed capacitance have the same reactance. At frequencies well below these **natural resonances**, the capacitor will act like a capacitance and the coil will act like an inductor. Near the natural resonance points, the inductor will have its highest impedance and the capacitor will have its lowest impedance. At frequencies above resonance, the capacitor acts like an inductor and the inductor acts like a capacitor. Thus there is a limit to the amount of capacitance that can be used at a given frequency. There is a similar limit to the inductance that can be used. At audio frequencies, capacitances measured in microfarads and inductances measured in henrys are practicable. At low and medium radio frequencies, inductances of a few mh. and capacitances of a few thousand pf. are the largest practicable. At high radio frequencies, usable inductance values drop to a few μh. and capacitances to a few hundred pf.

Distributed capacitance and inductance are important not only in r.f. tuned circuits, but in bypassing and choking as well. It will be appreciated that a bypass capacitor that actually acts like an inductance, or an r.f. choke that acts like a low-reactance capacitor, cannot work as it is intended they should.

Grounds

Throughout this book there are frequent references to **ground** and **ground potential.** When a connection is said to be "grounded" it does not necessarily mean that it actually goes to earth. What it means is that an actual earth connection to that point in the circuit should not disturb the operation of the circuit in any way. The term also is used to indicate a "common" point in the circuit where power supplies and metallic supports (such as a metal chassis) are electrically tied together. It is general practice, for example, to "ground" the negative terminal of a d.c. power supply, and to "ground" the filament or heater power supplies for vacuum tubes. Since the cathode of a vacuum tube is a junction point for grid and plate voltage supplies, and since the various circuits connected to the tube elements have at least one point connected to cathode, these points also are "returned to ground." Ground is therefore a common reference point in the radio circuit. "Ground potential" means that there is no "difference of potential"—no voltage—between the circuit point and the earth.

Single-Ended and Balanced Circuits

With reference to ground, a circuit may be either **single-ended** (unbalanced) or **balanced.** In a single-ended circuit, one side of the circuit (the **cold** side) is connected to ground. In a balanced circuit, the electrical midpoint is connected to ground, so that the circuit has two "hot" ends each at the same voltage "above" ground.

Typical single-ended and balanced circuits are shown in Fig. 2-59. R.f. circuits are shown in the upper row, while iron-core transformers (such as are used in power-supply and audio circuits) are shown in the lower row. The r.f. circuits may be balanced either by connecting the center of the coil to ground or by using a "balanced" or "split-stator" capacitor and connecting its rotor to r.f. ground. In the iron-core transformer, one or both windings may be tapped at the center of the winding to provide the ground connection.

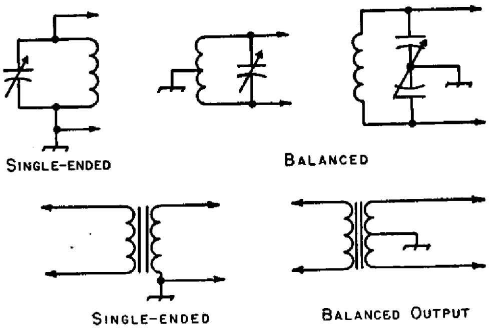


Fig. 2-59—Single-ended and balanced circuits.

Shielding

Two circuits that are physically near each other usually will be coupled to each other in some degree even though no coupling is intended. The metallic parts of the two circuits form a small capacitance through which energy can be transferred by means of the electric field. Also, the magnetic field about the coil or wiring of one circuit can couple that circuit to a second through the latter's coil and wiring. In many cases these unwanted couplings must be prevented if the circuits are to work properly.

Capacitive coupling may readily be prevented by enclosing one or both of the circuits in grounded low-resistance metallic containers, called **shields.** The electric field from the circuit components does not penetrate the shield. A metallic plate, called a **baffle shield,** inserted between two components also may suffice to prevent electrostatic coupling between them. It should be large enough to make the components invisible to each other.

Similar metallic shielding is used at radio frequencies to prevent magnetic coupling. The shielding effect for magnetic fields increases with frequency and with the conductivity and thickness of the shielding material.

A closed shield is required for good magnetic shielding; in some cases separate shields, one about each coil, may be required. The baffle shield is rather ineffective for magnetic shielding, although it will give partial shielding if placed at right angles to the axes of, and between, the coils to be shielded from each other.

Shielding a coil reduces its inductance, because part of its field is canceled by the shield. Also, there is always a small amount of resistance in the shield, and there is therefore an energy loss. This loss raises the effective resistance of the coil. The decrease in inductance and increase in resistance lower the Q of the coil, but the reduction in inductance and Q will be small if the spacing between the sides of the coil and the shield is at least half the coil diameter, and if the spacing at the ends of the coil is at least equal to the coil diameter. The higher the conductivity of the shield material, the less the effect on the inductance and Q. Copper is the best material, but aluminum is quite satisfactory.

For good magnetic shielding at audio frequencies it is necessary to enclose the coil in a container of high-permeability iron or steel. In this case the shield can be quite close to the coil without harming its performance.

U.H.F. CIRCUITS

RESONANT LINES

In resonant circuits as employed at the lower frequencies it is possible to consider each of the reactance components as a separate entity. The fact that an inductor has a certain amount of self-capacitance, as well as some resistance, while a capacitor also possesses a small self-inductance, can usually be disregarded.

At the very-high and ultrahigh frequencies it is not readily possible to separate these components. Also, the connecting leads, which at lower frequencies would serve merely to join the capacitor and coil, now may have more inductance than the coil itself. The required inductance coil may be no more than a single turn of wire, yet even this single turn may have dimensions comparable to a wavelength at the operating frequency. Thus the energy in the field surrounding the "coil" may in part be radiated. At a sufficiently high frequency the loss by radiation may represent a major portion of the total energy in the circuit.

For these reasons it is common practice to utilize resonant sections of transmission line as tuned circuits at frequencies above 100 Mc. or so. A quarter-wavelength line, or any odd multiple thereof, shorted at one end and open at the other exhibits large standing waves, as described in the section on transmission lines. When a voltage of the frequency at which such a line is resonant is applied to the open end, the response is very similar to that of a parallel resonant circuit. The equivalent relationships are shown in Fig. 2-60. At frequencies off resonance the line displays qualities comparable with the inductive and capacitive reactances of a conventional tuned circuit, so sections of transmission line can be used in much the same manner as inductors and capacitors.

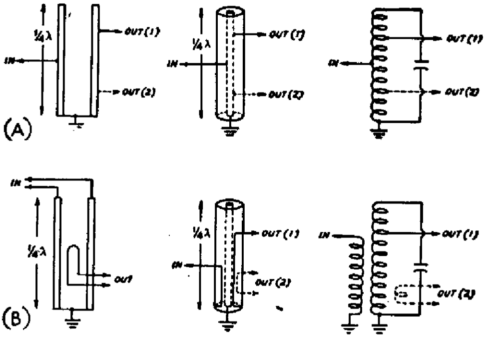


Fig. 2-60—Equivalent coupling circuits for parallel-line, coaxial-line and conventional resonant circuits.

To minimize radiation loss the two conductors of a parallel-conductor line should not be more than about one-tenth wavelength apart, the spacing being measured between the conductor axes. On the other hand, the spacing should not be less than about twice the conductor diameter because of "proximity effect," which causes eddy currents and an increase in loss. Above 300 Mc. it is difficult to satisfy both these requirements simultaneously, and the radiation from an open line tends to become excessive, reducing the Q. In such case the coaxial type of line is to be preferred, since it is inherently shielded.

Representative methods for adjusting coaxial lines to resonance are shown in Fig. 2-61. At the left, a sliding shorting disk is used to reduce the effective length of the line by altering the position of the short-circuit. In the center, the same effect is accomplished by using a telescoping tube in the end of the inner conductor to vary its length and thereby the effective length of the line. At the right, two possible methods of using

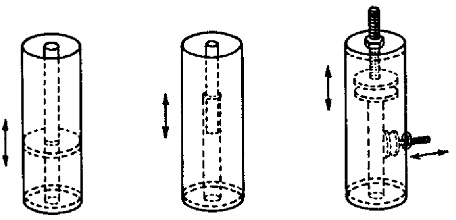


Fig. 2-61—Methods of tuning coaxial resonant lines.

parallel-plate capacitors are illustrated. The arrangement with the loading capacitor at the open end of the line has the greatest tuning effect per unit of capacitance; the alternative method, which is equivalent to tapping the capacitor down on the line, has less effect on the Q of the circuit. Lines with capacitive "loading" of the sort illustrated will be shorter, physically, than unloaded lines resonant at the same frequency.

Two methods of tuning parallel-conductor lines are shown in Fig. 2-62. The sliding short-circuiting strap can be tightened by means of screws and nuts to make good electrical contact. The parallel-plate capacitor in the second drawing may be placed anywhere along the line, the tuning effect becoming less as the capacitor is located nearer the shorted end of the line. Although a low-capacitance variable capacitor of ordinary construction can be used, the circular-plate type shown is symmetrical and thus does not unbalance the line. It also has the further advantage that no insulating material is required.

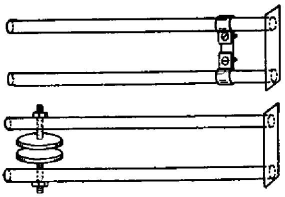


Fig. 2-62—Methods of tuning parallel-type resonant lines.

WAVEGUIDES

A waveguide is a conducting tube through which energy is transmitted in the form of electromagnetic waves. The tube is not considered as carrying a current in the same sense that the wires of a two-conductor line do, but rather as a *boundary* which confines the waves to the enclosed space. Skin effect prevents any electromagnetic effects from being evident outside the guide. The energy is injected at one end, either through capacitive or inductive coupling or by radiation, and is received at the other end. The waveguide then merely confines the energy of the fields, which are propagated through it to the receiving end by means of reflections against its inner walls.

Analysis of waveguide operation is based on the assumption that the guide material is a perfect conductor of electricity. Typical distributions of electric and magnetic fields in a rectangular guide are shown in Fig. 2-63. It will be observed that the intensity of the electric field is greatest (as indicated by closer spacing of the lines of force) at the center along the x dimension, Fig. 2-63(B), diminishing to zero at the end walls. The latter is a necessary condition, since the existence of any electric field parallel to the walls at the surface would cause an infinite current to flow in a perfect conductor. This represents an impossible situation.

Modes of Propagation

Fig. 2-63 represents a relatively simple distribution of the electric and magnetic fields.

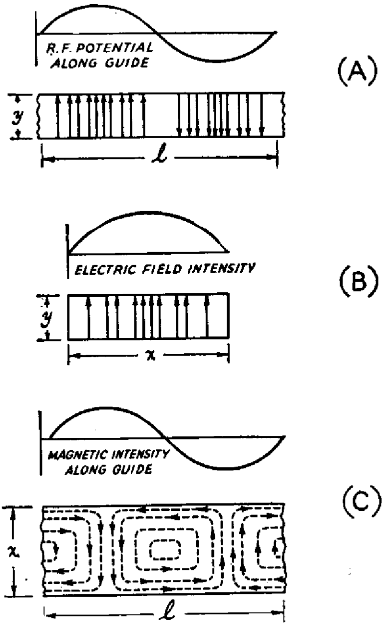


Fig. 2-63—Field distribution in a rectangular waveguide. The $TE_{1,0}$ mode of propagation is depicted.

There is in general an infinite number of ways in which the fields can arrange themselves in a guide so long as there is no upper limit to the frequency to be transmitted. Each field configuration is called a **mode.** All modes may be separated into two general groups. One group, designated *TM* **(transverse magnetic),** has the magnetic field entirely transverse to the direction of propagation, but has a component of electric field in that direction. The other type, designated *TE* **(transverse electric)** has the electric field entirely transverse, but has a component of magnetic field in the direction of propagation. *TM* waves are sometimes called *E* waves, and *TE* waves are sometimes called *H* waves, but the *TM* and *TE* designations are preferred.

The particular mode of transmission is identified by the group letters followed by two subscript numerals; for example, $TE_{1,0}$, $TM_{1,1}$, etc. The number of possible modes increases with frequency for a given size of guide. There is only one possible mode (called the **dominant mode**) for the lowest frequency that can be transmitted. The dominant mode is the one generally used in practical work.

Waveguide Dimensions

In the rectangular guide the critical dimension is x in Fig. 2-63; this dimension must be more than one-half wavelength at the lowest frequency to be transmitted. In practice, the y dimension usually is made about equal to $\frac{1}{2}x$ to avoid the possibility of operation at other than the dominant mode.

Other cross-sectional shapes than the rectangle can be used, the most important being the circular pipe. Much the same considerations apply as in the rectangular case.

Wavelength formulas for rectangular and circular guides are given in the following table, where x is the width of a rectangular guide and r is the radius of a circular guide. All figures are in terms of the dominant mode.

	Rectangular	*Circular*
Cut-off wavelength	$2x$	$3.41r$
Longest wavelength transmitted with little attenuation	$1.6x$	$3.2r$
Shortest wavelength before next mode becomes possible	$1.1x$	$2.8r$

Cavity Resonators

Another kind of circuit particularly applicable at wavelengths of the order of centimeters is the **cavity resonator,** which may be looked upon as a section of a waveguide with the dimensions chosen so that waves of a given length can be maintained inside.

Typical shapes used for resonators are the cylinder, the rectangular box and the sphere, as shown in Fig. 2-64. The resonant frequency depends upon the dimensions of the cavity and the mode of oscillation of the waves (comparable to the transmission modes in a waveguide). For the lowest modes the resonant wavelengths are* as follows:

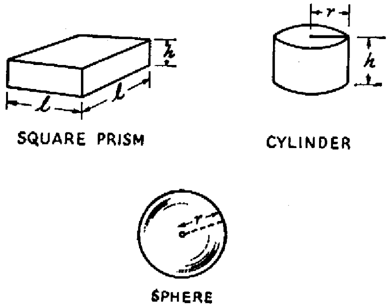


Fig. 2-64—Forms of cavity resonators.

Cylinder	$2.61r$
Square box	$1.41l$
Sphere	$2.28r$

The resonant wavelengths of the cylinder and square box are independent of the height when the height is less than a half wavelength. In other modes of oscillation the height must be a multiple of a half wavelength as measured inside the cavity. A cylindrical cavity can be tuned by a sliding shorting disk when operating in such a mode. Other tuning methods include placing adjustable tuning paddles or "slugs" inside the cavity so that the standing-wave pattern of the electric and magnetic fields can be varied.

A form of cavity resonator in practical use is the re-entrant cylindrical type shown in Fig. 2-65. In construction it resembles a concentric line closed at both ends with capacitive loading at the top, but the actual mode of oscillation may differ considerably from that occurring in coaxial lines. The resonant frequency of such a cavity depends upon the diameters of the two cylinders and the distance d between the cylinder ends.

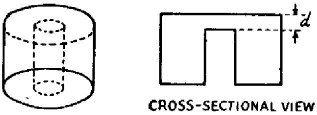


Fig. 2-65—Re-entrant cylindrical cavity resonator.

Compared with ordinary resonant circuits, cavity resonators have extremely high Q. A value of Q of the order of 1000 or more is readily obtainable, and Q values of several thousand can be secured with good design and construction.

Coupling to Waveguides and Cavity Resonators

Energy may be introduced into or abstracted from a waveguide or resonator by means of either the electric or magnetic field. The energy transfer frequently is through a coaxial line, two methods for coupling to which are shown in Fig. 2-66. The probe shown at A is simply a short extension of the inner conductor of the coaxial line, so oriented that it is parallel to the electric lines of force. The loop shown at B is arranged so that it encloses some of the magnetic lines of force. The point at which maximum coupling will be secured depends upon the particular mode of propagation in the guide or cavity; the coupling will be maximum when the coupling device is in the most intense field.

Coupling can be varied by turning the probe or loop through a 90-degree angle. When the probe is perpendicular to the electric lines the coupling will be minimum; similarly, when the plane of the loop is parallel to the magnetic lines the coupling will have its minimum value.

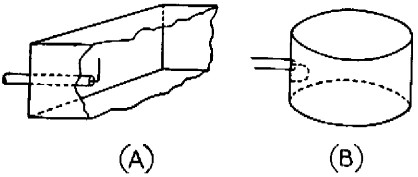


Fig. 2-66—Coupling to waveguides and resonators.

MODULATION, HETERODYNING AND BEATS

Since one of the most widespread uses of radio frequencies is the transmission of speech and music, it would be very convenient if the audio spectrum to be transmitted could simply be shifted up to some radio frequency, transmitted as radio waves, and shifted back down to audio at the receiving point. Suppose the audio signal to be transmitted by radio is a pure 1000-cycle tone, and we wish to transmit it at 1 Mc. (1,000,000 cycles per second). One possible way might be to add 1.000 Mc. and 1 kc. together, thereby obtaining a radio frequency of 1.001 Mc. No simple method for doing this directly has been devised, although the *effect* is obtained and used in "single-sideband transmission."

When two different frequencies are present simultaneously in an ordinary circuit (specifically, one in which Ohm's Law holds) each behaves as though the other were not there. The total or resultant voltage (or current) in the circuit will be the sum of the instantaneous values of the two at every instant. This is because there can be only one value of current or voltage at any single point in a circuit at any instant. Figs. 2-67A and B show two such frequencies, and C shows the resultant. The amplitude of the 1-Mc. current is not affected by the presence of the 1-kc. current, but the axis is shifted back and forth at the 1-kc. rate. An attempt to transmit such a combination as a radio wave would result in only the radiation of the 1-Mc. frequency, since the 1-kc. frequency retains its identity as an audio frequency and will not radiate.

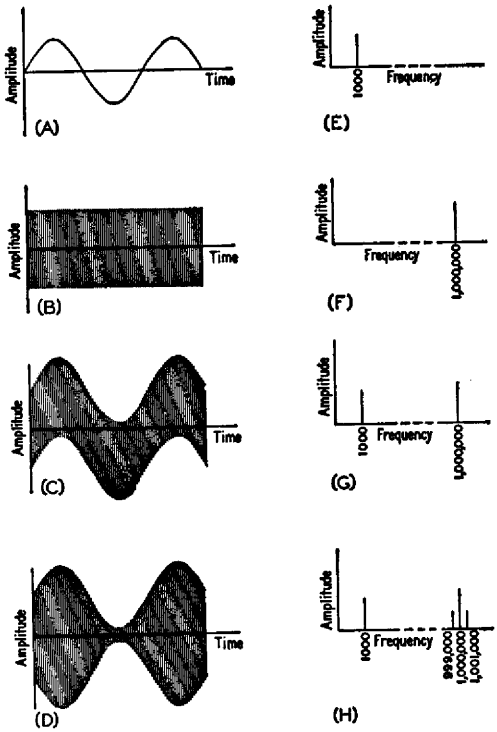


Fig. 2-67—Amplitude-vs.-time and amplitude-vs.-frequency plots of various signals. (A) 1½ cycles of an audio signal, assumed to be 1000 c.p.s. in this example. (B) A radio-frequency signal, assumed to be 1 Mc.; 1500 cycles are completed during the same time as the 1½ cycles in A, so they cannot be shown accurately. (C) The signals of A and B in the same circuit; each maintains its own identity. (D) The signals of A and B in a circuit where the amplitude of A can control the amplitude of B. The 1-Mc. signal is modulated by the 1000-cycle signal.

E, F, G and H show the spectrums for the signals in A, B, C and D, respectively. Note the new frequencies in H, resulting from the modulation process.

There are devices, however, which make it possible for one frequency to control the amplitude of the other. If, for example, a 1-kc. tone is used to control a 1-Mc. signal, the maximum r.f. output will be obtained when the 1-kc. signal is at the peak of one alternation and the minimum will occur at the peak of the next alternation. The process is called **amplitude modulation,** and the effect is shown in Fig. 2-67D. The resultant signal is now entirely at radio frequency, but with its amplitude varying at the modulation rate (1 kc.). Receiving equipment adjusted to receive the 1-Mc. r.f. signal can reproduce these changes in amplitude, and reveal what the audio signal is, through a process called **detection.**

It might be assumed that the only radio frequency present in such a signal is the original 1.000 Mc., but such is not the case. Two new frequencies have appeared. These are the sum (1.000 + .001) and the difference (1.000 − .001) of the two, and thus the radio frequencies appearing after modulation are 1.001, 1.000 and .999 Mc.

When an audio frequency is used to control the amplitude of a radio frequency, the process is generally called "amplitude modulation," as mentioned, but when a radio frequency modulates another radio frequency it is called **heterodyning.** The processes are identical. A general term for the sum and difference frequencies generated during heterodyning or amplitude modulation is **"beat frequencies,"** and a more specific one is **upper side frequency,** for the sum, and **lower side frequency** for the difference.

In the simple example, the modulating signal was assumed to be a pure tone, but the modulating signal can just as well be a *band* of frequencies making up speech or music. In this case, the side frequencies are grouped into the **upper sideband** and the **lower sideband.** Fig. 2-67H shows the side frequencies appearing as a result of the modulation process.

Amplitude modulation **(a.m.)** is not the only possible type nor is it the only one in use. Such signal properties as phase and frequency can also be modulated. In every case the modulation process leads to the generation of a new set (or sets) of radio frequencies symmetrically disposed about the original radio **(carrier)** frequency.

Chapter 4

Semiconductor Devices

Materials whose conductivity falls approximately midway between that of good conductors (e.g., copper) and good insulators (e.g., quartz) are called **semi-conductors.** Some of these materials (primarily germanium and silicon) can, by careful processing, be used in **solid-state** electronic devices that perform many or all of the functions of thermionic tubes. In many applications their small size, long life and low power requirements make them superior to tubes.

The conductivity of a material is proportional to the number of free electrons in the material. Pure germanium and pure silicon crystals have relatively few free electrons. If, however, carefully controlled amounts of "impurities" (materials having a different atomic structure, such as arsenic or antimony) are added, the number of free electrons, and consequently the conductivity, is increased. When certain other impurities are introduced (such as aluminum, gallium or indium) are introduced, an electron deficiency, or **hole,** is produced. As in the case of free electrons, the presence of holes encourages the flow of electrons in the semiconductor material, and the conductivity is increased. Semiconductor material that conducts by virtue of the free electrons is called **n-type** material; material that conducts by virtue of an electron deficiency is called **p-type.**

Electron and Hole Conduction

If a piece of p-type material is joined to a piece of n-type material as at A in Fig. 4-1 and a voltage is applied to the pair as at B, current will flow across the boundary or junction between the two (and also in the external circuit) when the battery has the polarity indicated. Electrons, indicated by the minus symbol, are attracted across the junction from the n material through the p material to the positive terminal of the battery, and holes, indicated by the plus symbol, are attracted in the opposite direction across the junction by the negative potential of the battery. Thus current flows through the circuit by means of electrons moving one way and holes the other.

If the battery polarity is reversed, as at C, the excess electrons in the n material are attracted away from the junction and the holes in the p material are attracted by the negative potential of the battery away from the junction. This leaves the junction region without any current carriers, consequently there is no conduction.

In other words, a junction of p- and n-type materials constitutes a rectifier. It differs from the tube diode rectifier in that there is a measurable, although comparatively very small, reverse current. The reverse current results fom the presence of some carriers of the type opposite to those which principally characterize the material.

With the two plates separated by practically zero spacing, the junction forms a capacitor of relatively high capacitance. This places a limit on the upper frequency at which semiconductor devices of this construction will operate, as compared with vacuum tubes. Also, the number of excess electrons and holes in the material depends upon temperature, and since the conductivity in turn depends on the number of excess holes and electrons, the device is more temperature sensitive than is a vacuum tube.

Capacitance may be reduced by making the contact area very small. This is done by means of a **point contact,** a tiny p-type region being formed under the contact point during manufacture when n-type material is used for the main body of the device.

SEMICONDUCTOR DIODES

Point-contact type diodes are used for many of the same purposes for which tube diodes are used. The construction of such a diode is shown in Fig. 4-2. Germanium and silicon are the most widely used materials; silicon finds much application as a microwave mixer diode. As compared with the tube diode for r.f. applications, the semiconductor point-contact diode has the advantages of very low interelectrode capacitance (on the order of 1 pf. or less) and not requiring any heater or filament power.

The germanium diode is characterized by relatively large current flow with small applied voltages in the "forward" direction, and small, although finite, current flow in the reverse or "back" direction for much larger applied voltages. A typical characteristic curve is shown in Fig. 4-3.

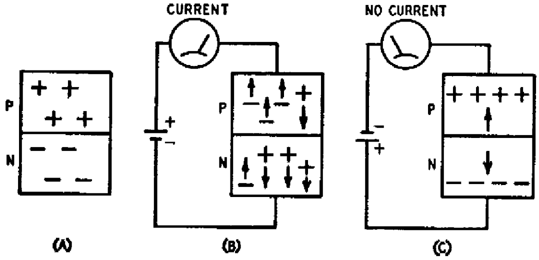


Fig. 4-1—A p-n junction (A) and its behavior when conducting (B) and non-conducting (C).

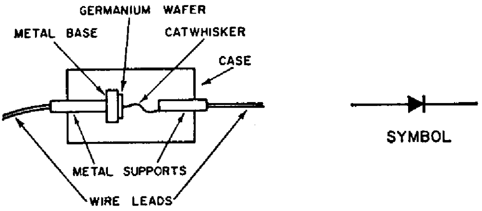


Fig. 4-2—Construction of a germanium-point-contact diode. In the circuit symbol for a contact rectifier the arrow points in the direction of minimum resistance measured by the conventional method—that is, going from the positive terminal of the voltage source through the rectifier to the negative terminal of the source. The arrow thus corresponds to the plate and the bar to the cathode of a tube diode.

The dynamic resistance in either the forward or back direction is determined by the change in current that occurs, at any given point on the curve, when the applied voltage is changed by a small amount. The forward resistance shows some variation in the region of very small applied voltages, but the curve is for the most part quite straight, indicating fairly constant dynamic resistance. For small applied voltages, the forward resistance is of the order of 200 ohms in most such diodes. The back resistance shows considerable variation, depending on the particular voltage chosen for the measurement. It may run from a few thousand ohms to over a megohm. In applications such as meter rectifiers for r.f. indicating instruments (r.f. voltmeters, wavemeter indicators, and so on) where the load resistance may be small and the applied voltage of the order of several volts, the resistances vary with the value of the applied voltage and are considerably lower.

Junction Diodes

Junction-type diodes made of silicon are employed widely as power rectifiers. Depending upon the design of the diode, they are capable of rectifying currents up to 40 or 50 amperes, and up to reverse peak voltages of 1000. They can be connected in series or in parallel, with suitable circuitry, to provide higher capabilities than those given above. A big advantage over thermionic rectifiers is their large surge-to-average-current ratio, which makes them suitable for use with capacitor-only filter circuits. This in turn leads to improved no-load-to-full-load voltage characteristics. Some consideration must be given to the operating temperature of silicon diodes, although many carry ratings to 150° C or so. A silicon junction diode requires a forward voltage of from 0.4 to 0.7 volts to overcome the junction potential barrier.

Ratings

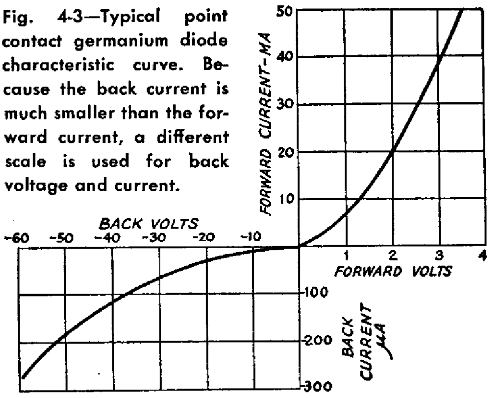
Semiconductor diodes are rated primarily in terms of **maximum safe inverse voltage** and **maximum average rectified current.** Inverse voltage is a voltage applied in the direction opposite to that which would be read by a d.c. meter connected in the current path.

It is also customary with some types to specify standards of performance with respect to forward and back current. A minimum value of forward current is usually specified for one volt applied. The voltage at which the maximum tolerable back current is specified varies with the type of diode.

Zener Diodes

The "Zener diode" is a special type of silicon junction diode that has a characteristic similar to that shown in Fig. 4-4. The sharp break from non-conductance to conductance is called the Zener Knee; at applied voltages greater than this breakdown point, the voltage drop across the diode is essentially constant over a wide range of currents. The substantially constant voltage drop over a wide range of currents allows this semiconductor device to be used as a constant

Fig. 4-3—Typical point contact germanium diode characteristic curve. Because the back current is much smaller than the forward current, a different scale is used for back voltage and current.



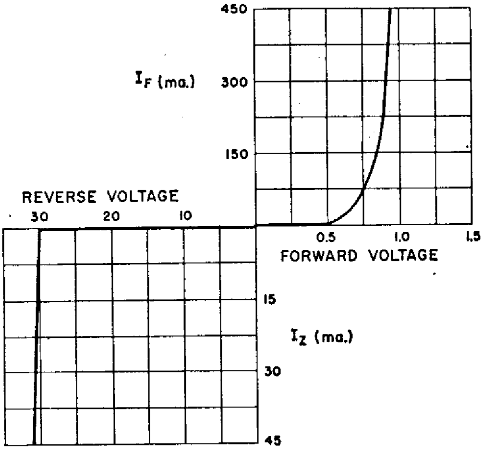


Fig. 4-4—Typical characteristic of a zener diode. In this example, the voltage drop is substantially constant at 30 volts in the (normally) reverse direction. Compare with Fig. 4-3. A diode with this characteristic would be called a "30-volt zener diode."

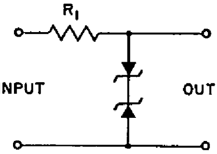


Fig. 4-5—Full-wave clipping action with two Zener diodes in opposition. The output level would be at a peak-to-peak voltage of twice the zener rating of a single diode. R_1 should have a resistance value sufficient to limit the current to the zener diode rating.

voltage reference or control element, in a manner somewhat similar to the gaseous voltage-regulator tube. Voltages for Zener diode action range from a few volts to several hundred and power ratings run from a fraction of a watt to 50 watts.

Zener diodes can be connected in series to advantage; the temperature coefficient is improved over that of a single diode of equivalent rating and the power-handling capability is increased.

Two Zener diodes connected in opposition, Fig. 4-5, form a simple and highly effective clipper.

Voltage-Dependent Capacitors

Voltage-dependent capacitors, or **varactors,** are p-n junction diodes that behave as capacitors of reasonable *Q* when biased in the reverse direction. They are useful in many applications because the actual capacitance value is dependent upon the d.c. bias voltage that is applied. In a typical capacitor the capacitance can be varied over a 10-to-1 range with a bias change from 0 to — 100 volts. The current demand on the bias supply is on the order of a few microamperes.

Typical applications include remote control of tuned circuits, automatic frequency control of receiver local oscillators, and simple frequency modulators for communications and for sweep-tuning applications.

An important transmitter application of the varactor is as a high-efficiency frequency multiplier. The basic circuits for varactor doublers and triplers is shown in Fig. 4-6. In these circuits, the fundamental frequency flows around the input loop. Harmonics generated by the varactor are passed to the load through a filter tuned to the desired harmonic. In the case of the tripler circuit at B, an **idler** circuit, tuned to the second harmonic, is required. Tripling, efficiencies of 75 per cent are not too difficult to come by, at power levels of 10 to 20 watts.

An important receiver application of the varactor is as a **parametric** amplifier. The diode is modulated by r.f. several times higher in frequency than the signal. This **pump** r.f. adds energy to the stored signal charge. To provide the necessary phase relationship between the signal and the pump, an idler circuit is included.

Tunnel Diode

Much hope is held for the future use of the "tunnel diode," a junction semiconductor of special construction that has a "negative resistance" characteristic at low voltages. This characteristic (*decrease* of current with increase of voltage) permits the diode to be used as an oscillator and as an amplifier. Since electrical charges move through the diode with the speed of light, in contrast to the relatively slow motion of electrical charge carriers in other semiconductors, it has been possible to obtain oscillations at 2000 Mc. and higher.

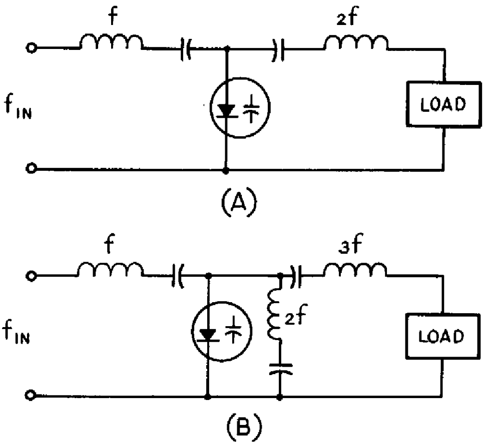


Fig. 4-6—Varactor frequency multipliers. A— Doubler circuit requires filters tuned to fundamental and second-harmonic frequencies. B— Tripler circuit shunts varactor with "idler" circuit tuned to second harmonic.

Efficiencies on the order of 75 per cent or higher can be obtained with these devices.

TRANSISTORS

Fig. 4-7 shows a "sandwich" made from two layers of p-type semiconductor material with a thin layer of n-type between. There are in effect two p-n junction diodes back to back. If a positive bias is applied to the p-type material at the left, current will flow through the left-hand junction, the holes moving to the right and the electrons from the n-type material moving to the left. Some of the holes moving into the n-type material will combine with the electrons there and be neutralized, but some of them also will travel to the region of the right-hand junction.

If the p-n combination at the right is biased negatively, as shown, there would normally be no current flow in this circuit (see Fig. 4-1C). However, there are now additional holes available at the junction to travel to point *B* and electrons can travel toward point *A*, so a current can flow even though this section of the sandwich

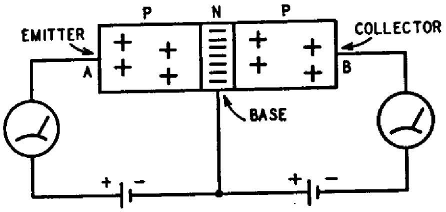


Fig. 4-7—The basic arrangement of a transistor. This represents a junction-type p-n-p unit.

considered alone is biased to prevent conduction. Most of the current is between *A* and *B* and does not flow out through the common connection to the n-type material in the sandwich.

A semiconductor combination of this type is called a **transistor,** and the three sections are known as the **emitter, base** and **collector,** respectively. The amplitude of the collector current depends principally upon the amplitude of the emitter current; that is, the collector current is controlled by the emitter current.

Power Amplification

Because the collector is biased in the back direction the collector-to-base resistance is high. On the other hand, the emitter and collector currents are substantially equal, so the power in the collector circuit is larger than the power in the emitter circuit ($P = I^2 R$, so the powers are proportional to the respective resistances, if the currents are the same). In practical transistors emitter resistance is of the order of a few hundred ohms while the collector resistance is hundreds or thousands of times higher, so power gains of 20 to 40 db. or even more are possible.

Types

The transistor may be one of the several types shown in Fig. 4-8. The assembly of p- and n-types materials may be reversed, so that p-n-p and n-p-n transistors are both possible.

The first two letters of the n-p-n and p-n-p designations indicate the respective polarities of the voltages applied to the emitter and collector in normal operation. In a p-n-p transistor, for example, the emitter is made positive with respect to both the collector and the base, and the collector is made negative with respect to both the emitter and the base.

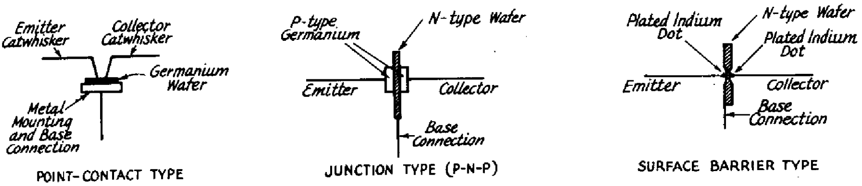
Point-Contact Transistors

The point-contact transistor, shown at the left in Fig. 4-8, has two "cat whiskers" placed very close together on the surface of a germanium wafer. It is principally of historical interest and is now superseded by the junction type. It was difficult to manufacture, since the two contact points must be extremely close together if good high-frequency characteristics are to be secured.

Junction Transistors

The majority of transistors being manufactured are one or another version of junction transistors. These may be grown junctions, alloyed or fused junctions, diffused junctions, epitaxial junctions and electroetched and/or electroplated junctions. The diffused-junction transistor, in widespread use because the product of this type of manufacture is generally consistent, involves applying the doping agent to a semiconductor wafer by electroplating, painting, or exposing the surface to a gaseous form of the dopant. A carefully-controlled temperature cycling causes the dopant to diffuse into the surface of the solid. The diffused layer is then a different type than the base material. Epitaxial junctions refers to growth of new layers on the original base in such a manner that the new (epitaxial) layer perpetuates the crystalline structure of the original.

The surface-barrier transistor is still another type of junction transistor. High-frequency operation requires that the base be physically thin. This is obtained by a process known as "jet etching." After the base wafer has been ground as thin as possible, it is placed between two opposed streams of jet solution, and the electrical etching current is turned on. The wafer center may be etched down to a thickness of only 0.0002 inch, at which time the current is reversed. At this point electroplating from the solution begins. An n-type wafer would be plated with a solution containing a p-type impurity. When leads are soldered to the plated surfaces, the heat causes the junctions to form between the base material and the plating. Surface-barrier transistors will op-



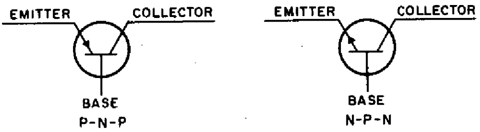
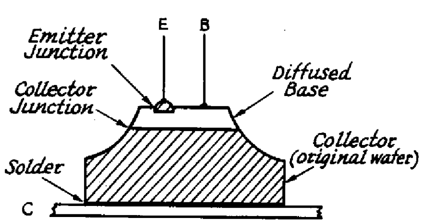
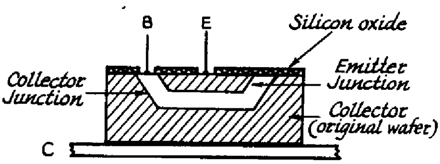


Fig. 4-8—Point-contact, junction-type and surface-barrier types of transistors.

Circuit symbols for n-p-n and p-n-p transistors.



MESA TRANSISTOR



PLANAR TRANSISTOR

Fig. 4-9—Two basic types of transistor construction.

erate as amplifiers and oscillators at frequencies measured in hundreds of megacycles.

Transistor Structures

There are two general terms used to describe the general physical structure of many transistors. As shown in Fig. 4-9, the **mesa** transistor is formed by etching away the metal around the emitter and base connections, leaving the junctions exposed and with very small cross sections. This construction makes for good high-frequency response.

In the **planar** construction shown in Fig. 4-9, the junctions are protected at the upper surface by an impervious layer of silicon oxide. This reduces leakage and increases current gain at low signal levels.

Note that in either type of construction, the collector lead also serves as a heat sink to cool the transistor.

TRANSISTOR CHARACTERISTICS

An important characteristic of a transistor is its **current amplification factor,** usually designated by the symbol α. This is the ratio of the change in collector current to a small change in emitter current, measured in the common-base circuit described later, and is comparable with the voltage amplification factor (μ) of a vacuum tube. The current amplification factor is almost, but not quite, 1 in a junction transistor. It is larger than 1 in the point-contact type, values in the neighborhood of 2 being typical.

The **α cut-off frequency** is the frequency at which the current amplification drops 3 db. below its low-frequency value. Cut-off frequencies range from 500 kc. to frequencies in the u.h.f. region. The cut-off frequency indicates in a general way the frequency spread over which the transistor is useful.

Each of the three elements in the transistor has a resistance associated with it. The emitter and collector resistances were discussed earlier. There is also a certain amount of resistance associated with the base, a value of a few hundred to 1000 ohms being typical of the base resistance.

The values of all three resistances vary with the type of transistor and the operating voltages. The collector resistance, in particular, is sensitive to operating conditions.

Characteristic Curves

The operating characteristics of transistors can be shown by a series of characteristic curves. One such set of curves is shown in Fig. 4-10. It shows the collector current *vs.* collector voltage for a number of fixed values of emitter current. Practically, the collector current depends almost entirely on the emitter current and is independent of the collector voltage. The separation between curves representing equal steps of emitter current is quite uniform, indicating that almost distortionless output can be obtained over the useful operating range of the transistor.

Another type of curve is shown in Fig. 4-11, together with the circuit used for obtaining it. This also shows collector current *vs.* collector voltage, but for a number of different values of base current. In this case the emitter element is used as the common point in the circuit. The collector current is not independent of collector voltage with this type of connection, indicating that the output resistance of the device is fairly low. The base current also is quite low, which means that the resistance of the base-emitter

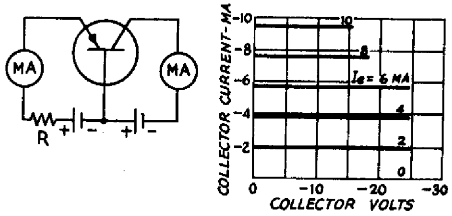


Fig. 4-10—A typical collector-current vs. collector-voltage characteristic of a junction-type transistor, for various emitter-current values. The circuit shows the setup for taking such measurements. Since the emitter resistance is low, a current-limiting resistor, R, is connected in series with the source of current. The emitter current can be set at a desired value by adjustment of this resistance.

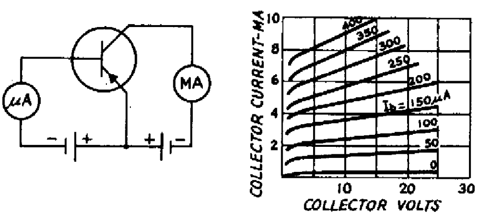


Fig. 4-11—Collector current vs. collector voltage for various values of base current, for a junction-type transistor. The values are determined by means of the circuit shown.

circuit is moderately high with this method of connection. This may be contrasted with the high values of emitter current shown in Fig. 4-10.

Ratings

The principal ratings applied to transistors are maximum collector dissipation, maximum collector voltage, maximum collector current, and maximum emitter current. The voltage and current ratings are self-explanatory.

The collector dissipation is the power, usually expressed in milliwatts, that can safely be dissipated by the transistor as heat. With some types of transistors provision is made for transferring heat rapidly through the container, and such units usually require installation on a heat "sink," or mounting that can absorb heat.

The amount of undistorted output power that can be obtained depends on the collector voltage, the collector current being practically independent of the voltage in a given transistor. Increasing the collector voltage extends the range of linear operation, but must not be carried beyond the point where either the voltage or dissipation ratings are exceeded.

TRANSISTOR AMPLIFIERS

Amplifier circuits used with transistors fall into one of three types, known as the **grounded-base, grounded-emitter,** and **grounded-collector** circuits. These are shown in Fig. 4-12 in elementary form. The three circuits correspond approximately to the grounded-grid, grounded-cathode and cathode-follower circuits, respectively, used with vacuum tubes.

The important transistor **parameters** in these circuits are the **short-circuit current transfer ratio,** the **cut-off frequency,** and the **input** and **output impedances.** The short-circuit current transfer ratio is the ratio of a small change in output current to the change in input current that causes it, the output circuit being short-circuited. The cut-off frequency is the frequency at which the amplification decreases by 3 db. from its value at some frequency well below that at which frequency effects begin to assume importance. The input and output impedances are, respectively, the impedance which a signal source working into the transistor would see, and the internal output impedance of the transistor (corresponding to the plate resistance of a vacuum tube, for example).

Grounded-Base Circuit

The input circuit of a grounded-base amplifier must be designed for low impedance, since the emitter-to-base resistance is of the order of $25/I_e$ ohms, where I_e is the emitter current in milliamperes. The optimum output load impedance, R_L, may range from a few thousand ohms to 100,000, depending upon the requirements.

The current transfer ratio is α and the cut-off frequency is as defined previously.

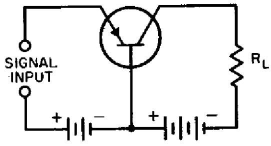
In this circuit the phase of the output (collector) current is the same as that of the input (emitter) current. The parts of these currents that flow through the base resistance are likewise in phase, so the circuit tends to be regenerative and will oscillate if the current amplification factor is greater than 1. A junction transistor is stable in this circuit since α is less than 1.

Grounded-Emitter Circuit

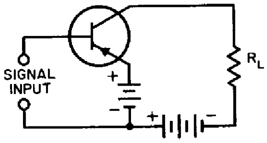
The grounded-emitter circuit shown in Fig. 4-12 corresponds to the ordinary grounded-cathode vacuum-tube amplifier. As indicated by the curves of Fig. 4-11, the base current is small and the input impedance is therefore fairly high — several thousand ohms in the average case. The collector resistance is some tens of thousands of ohms, depending on the signal source impedance. The current transfer ratio in the common-emitter circuit is equal to

$$\frac{\alpha}{1-\alpha}$$

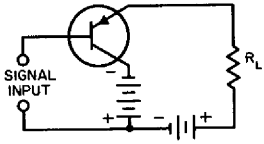
Since α is close to 1 (0.98 or higher being representative), the short-circuit current gain in the grounded-emitter circuit may be 50 or more. The cut-off frequency is equal to the α cut-off frequency multiplied by $(1 - \alpha)$, and therefore is relatively low. (For example a transistor with an α cut-off of 1000 kc. and $\alpha = 0.98$ would have a cut-off frequency of $1000 \times 0.02 = 20$ kc. in the grounded-emitter circuit.)



COMMON BASE



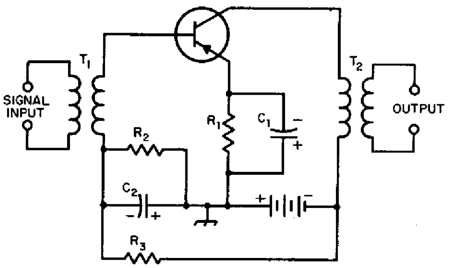
COMMON EMITTER



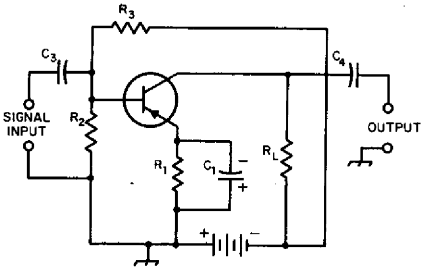
COMMON COLLECTOR

Fig. 4-12—Basic transistor amplifier circuits. R_L, the load resistance, may be an actual resistor or the primary of a transformer. The input signal may be supplied from a transformer secondary or by resistance-capacitance coupling. In any case it is to be understood that a d.c. path must exist between the base and emitter.

P-n-p transistors are shown in these circuits. If n-p-n types are used the battery polarities must be reversed.



TRANSFORMER COUPLING



RESISTANCE COUPLING

Fig. 4-13—Practical grounded-emitter circuits using transformer and resistance coupling. A combination of either also can be used—e.g., resistance-coupled input and transformer-coupled output. Tuned transformers may be used for r.f. and i.f. circuits.

With small transistors used for low-level amplification the input impedance will be of the order of 1000 ohms and the input circuit should be designed for an impedance step-down, if necessary. This can be done by appropriate choice of turns ratio for T_1 or, in the case of tuned circuits, by tapping the base down on the tuned secondary circuit. In the resistance-coupled circuit R_2 should be large compared with the input impedance, values of the order of 10,000 ohms being used.

In low-level circuits R_1 will be of the order of 1000 ohms. R_3 should be chosen to bias the transistor to the desired no-signal collector current; its value depends on R_1 and R_2 (see text).

Within its frequency limitations, the grounded-emitter circuit gives the highest power gain of the three.

In this circuit the phase of the output (collector) current is opposite to that of the input (base) current so such feedback as occurs through the small emitter resistance is negative and the amplifier is stable with either junction or point-contact transistors.

Grounded-Collector Circuit

Like the vacuum-tube cathode follower, the grounded-collector transistor amplifier has high input impedance and low output impedance. The latter is approximately equal to the impedance of the signal input source multiplied by $(1 - \alpha)$. The input resistance depends on the load resistance, being approximately equal to the load resistance divided by $(1 - \alpha)$. The fact that input resistance is directly related to the load resistance is a disadvantage of this type of amplifier if the load is one whose resistance or impedance varies with frequency.

The current transfer ratio with this circuit is

$$\frac{1}{1 - \alpha}$$

and the cut-off frequency is the same as in the grounded-emitter circuit. The output and input currents are in phase.

Practical Circuit Details

The transistor is essentially a low-voltage device, so the use of a battery power supply rather than a rectified-a.c. supply is quite common. Usually, it is more convenient to employ a single battery as a power source in preference to the two-battery arrangements shown in Fig. 4-12, so most circuits are designed for single-battery operation. Provision must be included, therefore, for obtaining proper biasing voltage for the emitter-base circuit from the battery that supplies the power in the collector circuit.

Coupling arrangements for introducing the input signal into the circuit and for taking out the amplified signal are similar to those used with vacuum tubes. However, the actual component values will in general be quite different from those used with tubes. This is because the impedances associated with the input and output circuits of transistors may differ widely from the comparable impedances in tube circuits. Also, d.c. voltage drops in resistances may require more careful attention with transistors because of the much lower voltage available from the ordinary battery power source. Battery economy becomes an important factor in circuit design, both with respect to voltage required and to overall current drain. A bias voltage divider, for example, easily may use more power than the transistor with which it is associated.

Typical single-battery grounded-emitter circuits are shown in Fig. 4-13. R_1, in series with the emitter, is for the purpose of "swamping out" the resistance of the emitter-base diode; this swamping helps to stabilize the emitter current. The resistance of R_1 should be large compared with that of the emitter-base diode, which, as stated earlier, is approximately equal to 25 divided by the emitter current in ma.

Since the current in R_1 flows in such a direction as to bias the emitter negatively with respect to the base (a p-n-p transistor is assumed), a base-emitter bias slightly greater than the drop in R_1 must be supplied. The proper operating point is achieved through adjustment of voltage divider R_2R_3, which is proportioned to give the desired value of no-signal collector current.

In the transformer-coupled circuit, input signal currents flow through R_1 and R_2, and there would be a loss of signal power at the base-emitter diode if these resistors were not bypassed by C_1 and C_2. The capacitors should have low reactance compared with the resistances across which they are connected. In the resistance-coupled circuit R_2 serves as part of the bias voltage divider and also as part of the load for the signal-input source. As seen by the signal source, R_3 is in parallel with R_2 and thus becomes part of the input load resistance. C_3 must have low reactance compared with the parallel combination of R_2, R_3 and the base-to-emitter resistance of the transistor. The load impedance will determine the reactance of C_4.

The output load resistance in the transformer-coupled case will be the actual load as reflected at the primary of the transformer, and its proper value will be determined by the transistor characteristics and the type of operation (Class A, B, etc.). The value of R_L in the resistance-coupled case is usually such as to permit the maximum a.c. voltage swing in the collector circuit without undue distortion, since Class A operation is usual with this type of amplifier.

Bias Stabilization

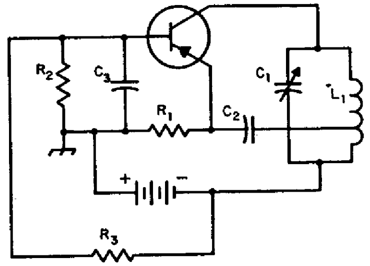
Transistor currents are sensitive to temperature variations, and so the operating point tends to shift as the transistor heats. The shift in operating point is in such a direction as to increase the heating, leading to "thermal runaway" and possible destruction of the transistor. The heat developed depends on the amount of power dissipated in the transistor, so it is obviously advantageous in this respect to operate with as little internal dissipation as possible: i.e., the d.c. input should be kept to the lowest value that will permit the type of operation desired and should never exceed the rated value for the particular transistor used.

A contributing factor to the shift in operating point is the collector-to-base leakage current (usually designated I_{co}) — that is, the current that flows from collector to base with the emitter connection open. This current, which is highly temperature sensitive, has the effect of increasing the emitter current by an amount much larger than I_{co} itself, thus shifting the operating point in such a way as to increase the collector current. This effect is reduced to the extent that I_{co} can be made to flow out of the base terminal rather than through the base-emitter diode. In the circuits of Fig. 4-13, bias stabilization is improved by making the resistance of R_1 as large as possible and both R_2 and R_3 as small as possible, consistent with gain and battery economy.

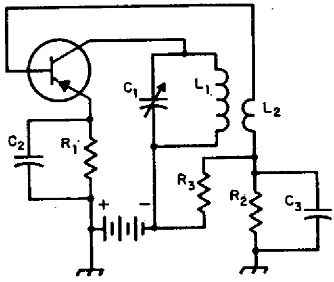
TRANSISTOR OSCILLATORS

Since more power is available from the output circuit than is necessary for its generation in the input circuit, it is possible to use some of the output power to supply the input circuit with a signal and thus sustain self-oscillation. Representative self-controlled oscillator circuits, based on vacuum-tube circuits of the same names, are shown in Fig. 4-14.

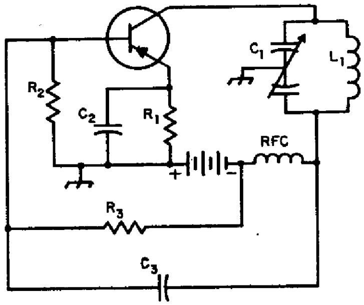
The upper frequency limit for oscillation is principally a function of the cut-off frequency of the transistor used, and oscillation will cease at the frequency at which there is insufficient amplification to supply the energy required to overcome circuit losses. Transistor oscillators usually will operate up to, and sometimes well beyond, the α cut-off frequency of the particular transistor used.



HARTLEY



TICKLER



COLPITTS

Fig. 4-14—Typical transistor oscillator circuits. Component values are discussed in the text.

The approximate oscillation frequency is that of the tuned circuit, L_1C_1. R_1, R_2 and R_3 have the same functions as in the amplifier circuits given in Fig. 4-13. Bypass capacitors C_2 and C_3 should have low reactances compared with the resistances with which they are associated.

Feedback in these circuits is adjusted in the same way as with tube oscillators: position of the tap on L_1 in the Hartley, turns and coupling of L_2 in the tickler circuit, and ratio of the sections of C_1 in the Colpitts.

FIELD-EFFECT TRANSISTORS

A relatively new semiconductor device, the field-effect transistor, may turn out to be superior to older transistor types in many applications. Because it has a high input impedance, its characteristics more nearly approach those of a vacuum tube.

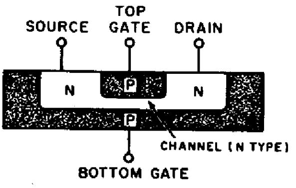


Fig. 4-15—The junction field-effect transistor.

When a p-n junction is reverse biased, the holes and free electrons in the vicinity of the junction are moved away, and there are no current carriers available. This region is called the "depletion region" and its thickness depends on the magnitude of the applied reverse voltage. No current can flow in the depletion region because there are no current carriers in that region.

The Junction FET

Field-effect transistors are divided into two main groups: junction FETS, and insulated-gate FETS. The basic JFET is shown in Fig. 4-15.

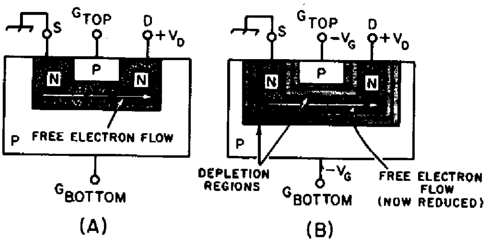


Fig. 4-16—Operation of the JFET under applied bias. A depletion region (light shading) is formed, compressing the channel and increasing its resistance to current flow.

The reason for the terminal names will become clear later. A d.c. operating condition is set up by starting a current flow between source and drain. This current flow is made up of free electrons since the semiconductor is n-type in the channel, so a positive voltage is applied at the drain. This positive voltage attracts the negatively-charged free electrons and the current flows (Fig. 4-16A). The next step is to apply a gate voltage of the polarity shown in Fig. 4-16B. Note that this reverse-biases the gates with respect to the source, channel, and drain. This reverse-bias gate voltage causes a depletion layer to be formed which takes up part of the channel, and since the electrons now have less volume in which to move the resistance is greater and the current between source and drain is reduced. If a large gate voltage is applied the depletion regions meet, and consequently the source-drain current is reduced nearly to zero. Since the large source-drain current changed with a relatively small gate voltage, the device acts as an amplifier. In the operation of the JFET, the gate terminal is never forward biased, because if it were the source-drain current would all be diverted through the forward-biased gate junction diode.

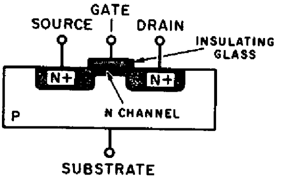


Fig. 4-17—The insulated-gate field-effect transistor.

The resistance between the gate terminal and the rest of the device is very high, since the gate terminal is always reverse biased, so the JFET has a very high input resistance. The source terminal is the *source* of current carriers, and they are *drained* out of the circuit at the drain. The gate *opens* and *closes* the amount of channel current which flows. Thus the operation of a FET closely resembles the operation of the vacuum tube with its high grid input impedance. Comparing the JFET to a vacuum tube, the source corresponds to the cathode, the gate to the grid, and the drain to the plate.

Insulated-Gate FET

The other large family which makes up field-effect transistors is the insulated-gate field-effect transistor, or IGFET, which is pictured schematically in Fig. 4-17. In order to set up a d.c. operating condition, a positive polarity is applied to the drain terminal. The substrate is connected to the source, and both are at ground potential, so the channel electrons are attracted to the positive drain. In order to regulate this source-drain current, voltage is applied to the gate contact. The gate is insulated from the rest of the device by a piece of insulating glass so this is not a p-n junction between the gate and the device—

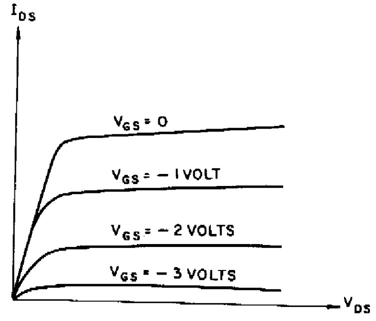


Fig. 4-18—Typical JFET characteristic curves.

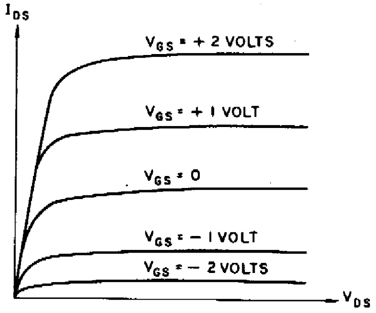


Fig. 4-19—Typical IGFET characteristic curves.

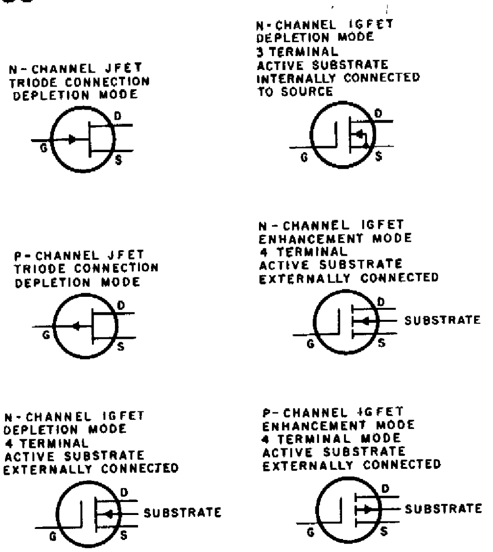


Fig. 4-20—Symbols for most-commonly available field-effect transistors.

thus the name insulated gate. When a negative gate polarity is applied, positively-charged holes from the p-type substrate are attracted towards the gate and the conducting channel is made more narrow; thus the source-drain current is reduced. When a positive gate voltage is connected, the holes in the substrate are repelled away, the conducting channel is made larger, and the source-drain current is increased. The IGFET is more flexible since either a positive or negative voltage can be applied to the gate. The resistance between the gate and the rest of the device is extremely high because they are separated by a layer of glass—not as clear as window glass, but it conducts just as poorly. Thus the IGFET has an extremely high input impedance. In fact, since the leakage through the insulating glass is generally much smaller than through the reverse-biased p-n gate junction in the JFET, the IGFET has a much higher input impedance. Typical values of R_{in} for the IGFET are over a million megohms, while R_{in} for the JFET ranges from megohms to over a thousand megohms.

Characteristic Curves

The characteristic curves for the FETs described above are shown in Figs. 4-18 and 4-19, where drain-source current is plotted against drain-source voltage for given gate & voltages.

The discussion of the JFET so far has left both gates separate so the device can be used as a tetrode in mixer applications. However, the gates can be internally connected for triode applications. When using the IGFET the substrate is always a.c.-shorted to the source, and only the insulated gate is used to control the current flow. This is done so that both positive and negative polarities can be applied to the device, as opposed to JFET operation where only one polarity can be used, because if the gate itself becomes forward biased the unit is no longer useful.

Classifications

Field-effect transistors are classed into two main groupings for application in circuits, enhancement mode and depletion mode. The enhancement-mode devices are those specifically constructed so that they have *no* channel. They become useful only when a gate voltage is applied that causes a channel to be formed. IGFETs can be used as enhancement-mode devices since both polarities can be applied to the gate without the gate becoming forward based and conducting.

A depletion-mode unit corresponds to Figs. 4-15 and 4-17 shown earlier, where a channel exists with no gate voltage applied. For the JFET we can apply a gate voltage and deplete the channel, causing the current to decrease. With the IGFET we can apply a gate voltage of either polarity so the device can be depleted (current decreased) or enhanced (current increased).

To sum up, a depletion-mode FET is one which has a channel constructed; thus it has a current flow for zero gate voltage. Enhancement-mode FETs are those which have no channel, so no current flows with zero gate voltage. The latter type devices are useful in logic applications.

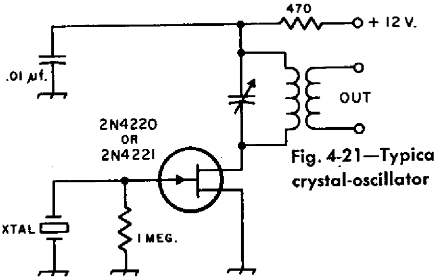


Fig. 4-21—Typical JFET crystal-oscillator cir-

Circuit symbols approved for FETs are shown in Fig. 4-20. Both depletion-mode and enhancement-mode devices are illustrated.

A typical application is the crystal oscillator circuit of Fig. 4-21. Note the resemblance to a vacuum-tube oscillator circuit.

SILICON CONTROLLED RECTIFIERS

The silicon controlled rectifier is a four-layer (p-n-p-n or n-p-n-p) three-electrode semiconductor rectifier. The three terminals are called anode, cathode and gate. The SCR differs from the diode silicon rectifier in that it will not conduct until the voltage exceeds a value called the *forward breakover* voltage. The value of this voltage can be controlled by the gate current. As the gate current is increased, the value of the forward breakover voltage is decreased. Once the rectifier conducts in the forward direction, the gate current no longer has any control, and the rectifier behaves as a low-forward-resistance diode. The gate regains control when the current through the rectifier is cut off, as during the other half cycle.

The SCR finds wide use in power-control applications, but not much in amateur radio, despite the fact that it is a highly-efficient means for controlling power from an a.c. supply.

TESTING UNKNOWN RECTIFIERS

There are many "bargain" rectifiers advertised; many of these are indeed bargains if they live up to their claimed characteristics. Checking them is not too difficult; a few meters and a couple of voltage sources are required.

Two basic checks can be made on any unknown silicon rectifier; a p.i.v. (peak inverse voltage) test and a (forward) current rating test.

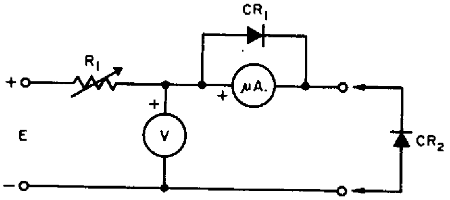


Fig. 4-22—Test circuit for determining p.i.v. rating of unknown rectifiers.
CR_1—400 p.i.v. silicon, to protect meter.
CR_2—Diode under test.
E—Voltage source, low current.
R_1—About 1000 ohms per inverse volt. See text.

Referring to Fig. 4-22, the p.i.v. test requires a source of adjustable high voltage, a high-sensitivity voltmeter and a microammeter. The maximum of the high-voltage source should be about 2½ times the expected p.i.v. Typical values for R_1, the limiting resistor, are 50,000 ohms for a 50 p.i.v. rectifier and 0.5 megohm for a 400 p.i.v. diode.

To test an unknown rectifier, the voltage E is increased slowly while the two meters are monitored. A good silicon diode will show very little reverse current until a value of about 10 μa. is reached; then the reverse current will increase rapidly as the voltage is increased. The diode should be given a p.i.v. rating of about 80 per cent of the voltage at which the current started to increase rapidly.

> Example: A diode was tested and found to run 9 μa. reverse current at 500 volts, after which the current increased rapidly as the voltage was increased. The diode was rated at 400 p.i.v. (0.8 × 500 = 400)

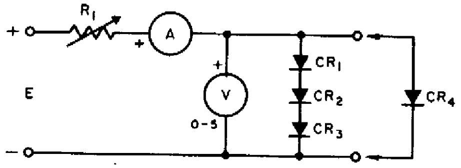


Fig. 4-23—Test circuit for checking semiconductor diode current rating.
A—Ammeter or milliammeter, 2 to 5 times expected current rating.
CR_1-CR_3—400 p.i.v. silicon diode
CR_4—Diode under test.
E—10 to 25 volts
R_1—Sufficient to limit current to maximum expected rating of CR_4.

The current rating of a diode is checked by using the test circuit of Fig. 4-23. It is essentially a measurement of the voltage drop across the rectifier; a v.t.v.m. can be used.

The test consists of setting R_1 for the rated current through the diode, as indicated by A. If the voltage drop across the diode is greater than 3 volts, throw away the diode. If the drop with 0.75 ampere through the diode is 1.4 volts, rate the diode at 400 ma. A diode goods for 3 amperes will show less than 1.5 volts drop at that current; a diode good for 2 amperes will show 2.5 volts or less drop at 2 amperes forward current. If an alleged 3-ampere diode shows 2 volts drop, reduce its rating to 2 amperes.

A Simple Transistor Tester

The transistor test circuit shown in Fig. 4-24 is useful to the experimenter or inveterate purchaser of "bargain" transistors. It can be built on a piece of Vectorboard; the two flashlight cells can be plugged into a battery holder. The contacts marked C, B and E can be a transistor socket or three leads terminated in miniature clips, or both.

After connecting the transistor to be tested and with S_1 at LEAK, S_3 should be tried in both positions if the transistor type is unknown. In the correct position, only a small reading should appear on the meter. This is the collector-emitter leakage current.

With S_1 closed to GAIN, a current of 30μa. (LO) or slightly more than 1 ma. (HI) is fed to the base. In the LO position the meter maximum is less than 1 ma.; in the HI position the maximum is about 200 ma.

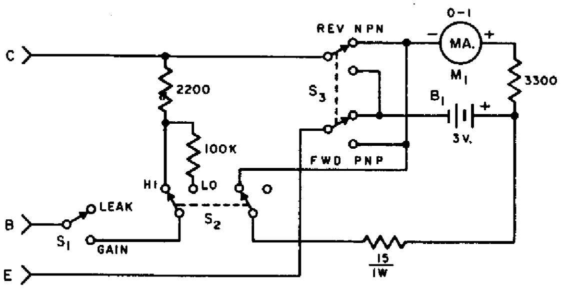


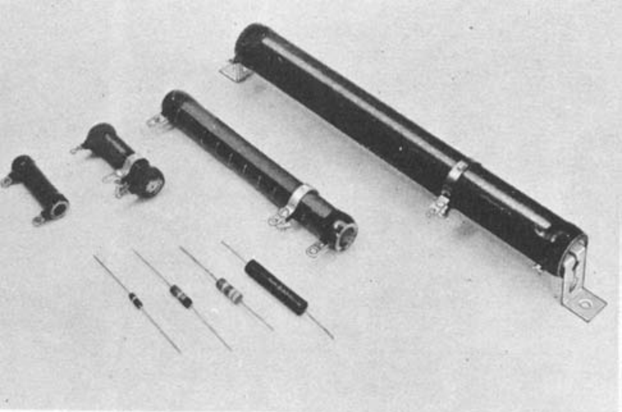
Fig. 4-24—Circuit diagrams of the transistor tester. Resistors are ½ watt.
B_1—Two C cells connected in series
M_1—0-1 milliameter (Lafayette 99 C 5052)
S_1, S_2, S_3—D.p.d.t. miniature slide switch

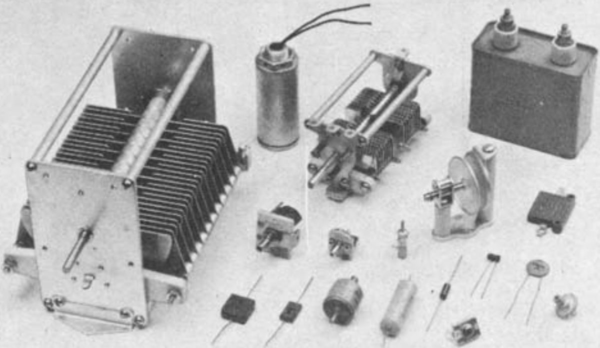
Semiconductor Bibliography

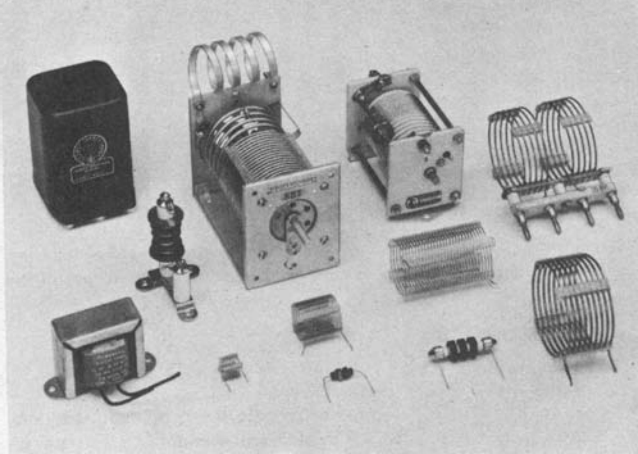
Many books are available on semiconductor theory and application. A few that are written at about the level of this handbook and can be found in most radio stores are listed below:

G. E. Controlled Rectifier Manual
G. E. Rectifier Guide Manual
G. E. Transistor Manual
G. E. Tunnel Diode Manual
*RCA Transistor Manual**
RCA Tunnel Diode Manual

* Including rectifiers, silicon controlled rectifiers, varactor diodes, and tunnel diodes.







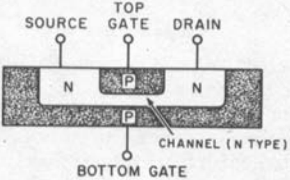
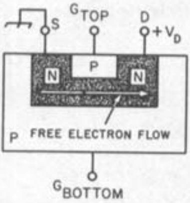


Fig. 4-15—The junction field-effect transistor.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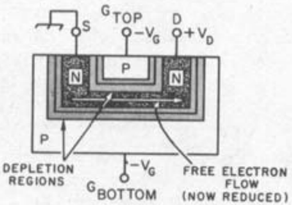
(A)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(B)

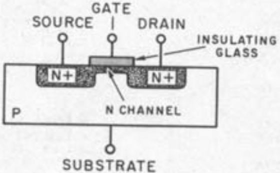

Fig. 4-17—The insulated-gate field-effect transistor.