1

Circuit Variables

Assessment Problems

AP 1.1 To solve this problem we use a product of ratios to change units from dollars/year to dollars/millisecond. We begin by expressing \$10 billion in scientific notation:

$$\$100 \text{ billion } = \$100 \times 10^9$$

Now we determine the number of milliseconds in one year, again using a product of ratios:

$$\frac{1 \text{ year}}{365.25 \text{ days}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ mins}} \cdot \frac{1 \text{ min}}{60 \text{ secs}} \cdot \frac{1 \text{ sec}}{1000 \text{ ms}} = \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}}$$

Now we can convert from dollars/year to dollars/millisecond, again with a product of ratios:

$$\frac{\$100 \times 10^9}{1 \text{ year}} \cdot \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}} = \frac{100}{31.5576} = \$3.17/\text{ms}$$

AP 1.2 First, we recognize that $1 \text{ ns} = 10^{-9}$ s. The question then asks how far a signal will travel in 10^{-9} s if it is traveling at 80% of the speed of light. Remember that the speed of light $c = 3 \times 10^8$ m/s. Therefore, 80% of c is $(0.8)(3 \times 10^8) = 2.4 \times 10^8$ m/s. Now, we use a product of ratios to convert from meters/second to inches/nanosecond:

$$\frac{2.4 \times 10^8 \text{ m}}{1\text{s}} \cdot \frac{1 \text{ s}}{10^9 \text{ ns}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \cdot \frac{1 \text{ in}}{2.54 \text{ cm}} = \frac{(2.4 \times 10^8)(100)}{(10^9)(2.54)} = \frac{9.45 \text{ in}}{1 \text{ ns}}$$

Thus, a signal traveling at 80% of the speed of light will travel 9.45″ in a nanosecond.

AP 1.3 Remember from Eq. (1.2), current is the time rate of change of charge, or $i = \frac{dq}{dt}$ In this problem, we are given the current and asked to find the total charge. To do this, we must integrate Eq. (1.2) to find an expression for charge in terms of current:

$$q(t) = \int_0^t i(x)\,dx$$

We are given the expression for current, i, which can be substituted into the above expression. To find the total charge, we let $t \to \infty$ in the integral. Thus we have

$$q_{\text{total}} = \int_0^\infty 20e^{-5000x}\,dx = \frac{20}{-5000}e^{-5000x}\Big|_0^\infty = \frac{20}{-5000}(e^{-\infty} - e^0)$$

$$= \frac{20}{-5000}(0-1) = \frac{20}{5000} = 0.004\ \text{C} = 4000\,\mu\text{C}$$

AP 1.4 Recall from Eq. (1.2) that current is the time rate of change of charge, or $i = \frac{dq}{dt}$. In this problem we are given an expression for the charge, and asked to find the maximum current. First we will find an expression for the current using Eq. (1.2):

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[\frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-\alpha t}\right]$$

$$= \frac{d}{dt}\left(\frac{1}{\alpha^2}\right) - \frac{d}{dt}\left(\frac{t}{\alpha}e^{-\alpha t}\right) - \frac{d}{dt}\left(\frac{1}{\alpha^2}e^{-\alpha t}\right)$$

$$= 0 - \left(\frac{1}{\alpha}e^{-\alpha t} - \alpha\frac{t}{\alpha}e^{-\alpha t}\right) - \left(-\alpha\frac{1}{\alpha^2}e^{-\alpha t}\right)$$

$$= \left(-\frac{1}{\alpha} + t + \frac{1}{\alpha}\right)e^{-\alpha t}$$

$$= te^{-\alpha t}$$

Now that we have an expression for the current, we can find the maximum value of the current by setting the first derivative of the current to zero and solving for t:

$$\frac{di}{dt} = \frac{d}{dt}(te^{-\alpha t}) = e^{-\alpha t} + t(-\alpha)e^{\alpha t} = (1-\alpha t)e^{-\alpha t} = 0$$

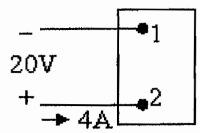
Since $e^{-\alpha t}$ never equals 0 for a finite value of t, the expression equals 0 only when $(1 - \alpha t) = 0$. Thus, $t = 1/\alpha$ will cause the current to be maximum. For this value of t, the current is

$$i = \frac{1}{\alpha}e^{-\alpha/\alpha} = \frac{1}{\alpha}e^{-1}$$

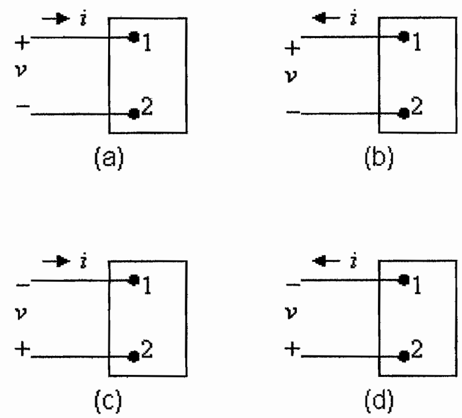
Remember in the problem statement, $\alpha = 0.03679$. Using this value for α,

$$i = \frac{1}{0.03679}e^{-1} \cong 10 \text{ A}$$

AP 1.5 Start by drawing a picture of the circuit described in the problem statement:



Also sketch the four figures from Fig. 1.6:



[a] Now we have to match the voltage and current shown in the first figure with the polarities shown in Fig. 1.6. Remember that 4A of current entering Terminal 2 is the same as 4A of current leaving Terminal 1. We get

$$\text{(a) } v = -20\,\text{V}, \quad i = -4\,\text{A}; \quad \text{(b) } v = -20\,\text{V}, \quad i = 4\,\text{A}$$

$$\text{(c) } v = 20\,\text{V}, \quad i = -4\,\text{A}; \quad \text{(d) } v = 20\,\text{V}, \quad i = 4\,\text{A}$$

[b] Using the reference system in Fig. 1.6(a) and the passive sign convention, $p = vi = (-20)(-4) = 80\,\text{W}$. Since the power is greater than 0, the box is absorbing power.

[c] From the calculation in part (b), the box is absorbing 80 W.

AP 1.6 Applying the passive sign convention to the power equation using the voltage and current polarities shown in Fig. 1.5, $p = vi$. From Eq. (1.3), we know that power is the time rate of change of energy, or $p = \frac{dw}{dt}$. If we know the power, we can find the energy by integrating Eq. (1.3). To begin, find the expression for power:

$$p = vi = (10{,}000e^{-5000t})(20e^{-5000t}) = 200{,}000e^{-10{,}000t} = 2 \times 10^5 e^{-10{,}000t}\text{ W}$$

Now find the expression for energy by integrating Eq. (1.3):

$$w(t) = \int_0^t p(x)\,dx$$

Substitute the expression for power, p, above. Note that to find the total energy, we let $t \to \infty$ in the integral. Thus we have

$$w = \int_0^\infty 2 \times 10^5 e^{-10{,}000x}\,dx = \frac{2 \times 10^5}{-10{,}000} e^{-10{,}000x}\Big|_0^\infty$$

$$= \frac{2 \times 10^5}{-10{,}000}(e^{-\infty} - e^0) = \frac{2 \times 10^5}{-10{,}000}(0 - 1) = \frac{2 \times 10^5}{10{,}000} = 20 \text{ J}$$

AP 1.7 At the Oregon end of the line the current is leaving the upper terminal, and thus entering the lower terminal where the polarity marking of the voltage is negative. Thus, using the passive sign convention, $p = -vi$. Substituting the values of voltage and current given in the figure,

$$p = -(800 \times 10^3)(1.8 \times 10^3) = -1440 \times 10^6 = -1440 \text{ MW}$$

Thus, because the power associated with the Oregon end of the line is negative, power is being generated at the Oregon end of the line and transmitted by the line to be delivered to the California end of the line.

Chapter Problems

P 1.1 $\dfrac{(250 \times 10^6)(440)}{10^9} = 110$ giga-watt hours

P 1.2 $(4 \text{ cond.}) \cdot (845 \text{ mi}) \cdot \dfrac{5280 \text{ ft}}{1 \text{ mi}} \cdot \dfrac{2526 \text{ lb}}{1000 \text{ ft}} \cdot \dfrac{1 \text{ kg}}{2.2 \text{ lb}} = 20.5 \times 10^6 \text{ kg}$

P 1.3 [a] $\dfrac{1000 \text{ songs}}{(32)(24)(2.1) \text{ mm}^3} = \dfrac{x \text{ songs}}{1 \text{ mm}^3}$

$$x = \frac{(1000)(1)}{(32)(24)(2.1)} = 0.62 \text{ 3-minute songs, or about 111.6 seconds of music}$$

[b] $\dfrac{4 \times 10^9 \text{ bytes}}{(32)(24)(2.1)\text{ mm}^3} = \dfrac{x \times 10^6 \text{ MB}}{(0.1)^3 \text{ mm}^3}$

$$x = \frac{(4 \times 10^9)(0.001)}{(32)(24)(2.1)} = 2480 \text{ bytes}$$

P 1.4 $\dfrac{(320)(240)\text{ pixels}}{1 \text{ frame}} \cdot \dfrac{2 \text{ bytes}}{1 \text{ pixel}} \cdot \dfrac{30 \text{ frames}}{1 \text{ sec}} = 4.608 \times 10^6 \text{ bytes/sec}$

$(4.608 \times 10^6 \text{ bytes/sec})(x \text{ secs}) = 30 \times 10^9 \text{ bytes}$

$$x = \frac{30 \times 10^9}{4.608 \times 10^6} = 6510 \text{ sec} = 108.5 \text{ min of video}$$

P 1.5 [a] We can set up a ratio to determine how long it takes the bamboo to grow $10\,\mu$m First, recall that 1 mm $= 10^3\mu$m. Let's also express the rate of growth of bamboo using the units mm/s instead of mm/day. Use a product of ratios to perform this conversion:

$$\frac{250 \text{ mm}}{1 \text{ day}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{250}{(24)(60)(60)} = \frac{10}{3456} \text{ mm/s}$$

Use a ratio to determine the time it takes for the bamboo to grow $10\,\mu$m:

$$\frac{10/3456 \times 10^{-3} \text{ m}}{1 \text{ s}} = \frac{10 \times 10^{-6} \text{ m}}{x \text{ s}} \quad \text{so} \quad x = \frac{10 \times 10^{-6}}{10/3456 \times 10^{-3}} = 3.456 \text{ s}$$

[b] $\dfrac{1 \text{ cell}}{3.456 \text{ s}} \cdot \dfrac{3600 \text{ s}}{1 \text{ hr}} \cdot \dfrac{(24)(7) \text{ hr}}{1 \text{ week}} = 175{,}000 \text{ cells/week}$

P 1.6 Volume = area $\times$ thickness

Convert values to millimeters, noting that $10 \text{ m}^2 = 10^6 \text{ mm}^2$

$10^6 = (10 \times 10^6)(\text{thickness})$

$$\Rightarrow \text{thickness} = \frac{10^6}{10 \times 10^6} = 0.10 \text{ mm}$$

P 1.7 $\text{C/m}^3 = \dfrac{1.6022 \times 10^{-19} \text{ C}}{1 \text{ electron}} \times \dfrac{10^{29} \text{ electrons}}{1 \text{ m}^3} = 1.6022 \times 10^{10} \text{ C/m}^3$

Cross-sectional area of wire $= \pi r^2 = \pi(1.5 \times 10^{-3} \text{ m})^2 = 7.07 \times 10^{-6} \text{ m}^2$

$\text{C/m} = (1.6022 \times 10^{10}\text{C/m}^3)(7.07 \times 10^{-6}\text{m}^2) = 113.253 \times 10^3 \text{ C/m}$

Therefore, $i\left(\dfrac{\text{C}}{\text{sec}}\right) = (113.253 \times 10^3)\left(\dfrac{\text{C}}{\text{m}}\right) \times \text{avg vel}\left(\dfrac{\text{m}}{\text{s}}\right)$

Thus, average velocity $= \dfrac{i}{113.253 \times 10^3} = \dfrac{1200}{113.253 \times 10^3} = 0.0106 \text{ m/s}$

P 1.8 $$n = \frac{35 \times 10^{-6}\ \text{C/s}}{1.6022 \times 10^{-19}\ \text{C/elec}} = 2.18 \times 10^{14}\ \text{elec/s}$$

P 1.9 First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 24\cos 4000t$$

Therefore, $dq = 24\cos 4000t\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 24\int_0^t \cos 4000y\,dy$$

We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 24\frac{\sin 4000y}{4000}\bigg|_0^t = \frac{24}{4000}\sin 4000t - \frac{24}{4000}\sin 4000(0) = \frac{24}{4000}\sin 4000t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 6 \times 10^{-3}\sin 4000t\,\text{C} = 6\sin 4000t\,\text{mC}$

P 1.10 $w = qV = (1.6022 \times 10^{-19})(6) = 9.61 \times 10^{-19} = 0.961$ aJ

P 1.11 $p = (9)(100 \times 10^{-3}) = 0.9$ W; $\quad 5\ \text{hr} \cdot \dfrac{3600\ \text{s}}{1\ \text{hr}} = 18{,}000\ \text{s}$

$$w(t) = \int_0^t p\,dt \qquad w(18{,}000) = \int_0^{18{,}000} 0.9\,dt = 0.9(18{,}000) = 16.2\ \text{kJ}$$

P 1.12 Assume we are standing at box A looking toward box B. Then, using the passive sign convention $p = vi$, since the current i is flowing into the + terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

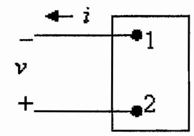
[a] $p = (120)(5) = 600$ W 600 W from A to B

[b] $p = (250)(-8) = -2000$ W 2000 W from B to A

[c] $p = (-150)(16) = -2400$ W 2400 W from B to A

[d] $p = (-480)(-10) = 4800$ W 4800 W from A to B

P 1.13 [a]



$p = vi = (40)(-10) = -400$ W
Power is being delivered by the box.

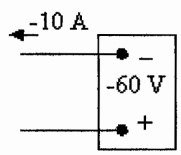
[b] Leaving

[c] Gaining

P 1.14 [a] $p = vi = (-60)(-10) = 600$ W, so power is being absorbed by the box.

[b] Entering

[c] Losing



P 1.15 [a] In Car A, the current i is in the direction of the voltage drop across the 12 V battery(the current i flows into the + terminal of the battery of Car A). Therefore using the passive sign convention,
$p = vi = (30)(12) = 360$ W.
Since the power is positive, the battery in Car A is absorbing power, so Car A must have the "dead" battery.

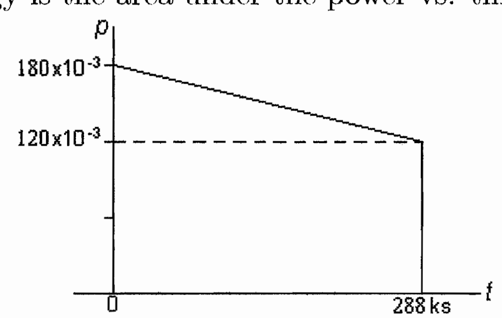
[b] $w(t) = \int_0^t p\,dx; \qquad 1\text{ min} = 60\text{ s}$

$$w(60) = \int_0^{60} 360\,dx$$

$$w = 360(60 - 0) = 360(60) = 21{,}600\text{ J} = 21.6\text{ kJ}$$

P 1.16 $p = vi; \qquad w = \int_0^t p\,dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.



Note that in constructing the plot above, we used the fact that 80 hr = 288,000 s = 288 ks

$p(0) = (9)(20 \times 10^{-3}) = 180 \times 10^{-3}$ W

$$p(288\text{ ks}) = (6)(20 \times 10^{-3}) = 120 \times 10^{-3}\text{ W}$$

$$w = (120 \times 10^{-3})(288 \times 10^3) + \frac{1}{2}(180 \times 10^{-3} - 120 \times 10^{-3})(288 \times 10^3) = 43.2\text{ kJ}$$

P 1.17 [a] $p = vi = 30e^{-500t} - 30e^{-1500t} - 40e^{-1000t} + 50e^{-2000t} - 10e^{-3000t}$

$p(1\text{ ms}) = 3.1\text{ mW}$

[b]

$$\begin{aligned} w(t) &= \int_0^t (30e^{-500x} - 30e^{-1500x} - 40e^{-1000x} + \\ &\quad 50e^{-2000x} - 10e^{-3000x})dx \\ &= 21.67 - 60e^{-500t} + 20e^{-1500t} + 40e^{-1000t} - \\ &\quad 25e^{-2000t} + 3.33e^{-3000t}\mu\text{J} \\ w(1\text{ ms}) &= 1.24\mu\text{J} \end{aligned}$$

[c] $w_{\text{total}} = 21.67\mu\text{J}$

P 1.18 [a] $v(10\text{ ms}) = 400e^{-1}\sin 2 = 133.8\text{ V}$

$i(10\text{ ms}) = 5e^{-1}\sin 2 = 1.67\text{ A}$

$p(10\text{ ms}) = vi = 223.79\text{ W}$

[b]

$$\begin{aligned} p &= vi = 2000e^{-200t}\sin^2 200t \\ &= 2000e^{-200t}\left[\frac{1}{2} - \frac{1}{2}\cos 400t\right] \\ &= 1000e^{-200t} - 1000e^{-200t}\cos 400t \\ w &= \int_0^\infty 1000e^{-200t}\,dt - \int_0^\infty 1000e^{-200t}\cos 400t\,dt \\ &= 1000\left.\frac{e^{-200t}}{-200}\right|_0^\infty \\ &\quad -1000\left.\left\{\frac{e^{-200t}}{(200)^2 + (400)^2}[-200\cos 400t + 400\sin 400t]\right\}\right|_0^\infty \\ &= 5 - 1000\left[\frac{200}{4 \times 10^4 + 16 \times 10^4}\right] = 5 - 1 \\ \text{w} &= 4\text{ J} \end{aligned}$$

P 1.19 [a] $0\text{ s} \le t < 4\text{ s}$:

$v = 2.5t$ V; $i = 1\,\mu$A; $p = 2.5t\,\mu$W

$4\text{ s} < t \le 8\text{ s}$:

$v = 10$ V; $i = 0$ A; $p = 0$ W

$8\text{ s} \le t < 16\text{ s}$:

$v = -2.5t + 30$ V; $i = -1\,\mu$A; $p = 2.5t - 30\,\mu$W

$16\text{ s} < t \le 20\text{ s}$:

$v = -10$ V; $i = 0$ A; $p = 0$ W

$20\text{ s} \le t < 36\text{ s}$:

$v = t - 30$ V; $i = 0.4\,\mu$A; $p = 0.4t - 12\,\mu$W

$36\text{ s} < t \le 46\text{ s}$:

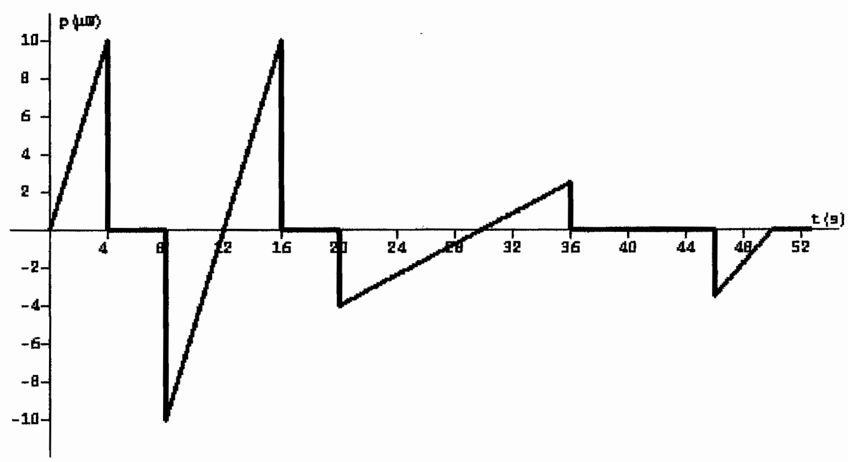
$v = 6$ V; $i = 0$ A; $p = 0$ W

$46\text{ s} \le t < 50\text{ s}$:

$v = -1.5t + 75$ V; $i = -0.6\,\mu$A; $p = 0.9t - 45\,\mu$W

$t > 50$ s:

$v = 0$ V; $i = 0$ A; $p = 0$ W



[b] Calculate the area under the curve from zero up to the desired time:

$$w(4) = \tfrac{1}{2}(4)(10) = 20\,\mu\text{J}$$

$$w(12) = w(4) - \tfrac{1}{2}(4)(10) = 0\text{ J}$$

$$w(36) = w(12) + \tfrac{1}{2}(4)(10) - \tfrac{1}{2}(10)(4) + \tfrac{1}{2}(6)(2.4) = 7.2\,\mu\text{J}$$

$$w(50) = w(36) - \tfrac{1}{2}(4)(3.6) = 0\text{ J}$$

P 1.20 [a] $p = vi = (0.05e^{-1000t})(75 - 75e^{-1000t}) = (3.75e^{-1000t} - 3.75e^{-2000t})$ W

$$\frac{dp}{dt} = -3750e^{-1000t} + 7500e^{-2000t} = 0 \qquad \text{so} \qquad 2e^{-2000t} = e^{-1000t}$$

$2 = e^{1000t}$ so $\ln 2 = 1000t$ thus p is maximum at $t = 693.15\,\mu$s

$p_{\text{max}} = p(693.15\,\mu\text{s}) = 937.5$ mW

[b] $$w = \int_0^{\infty} [3.75e^{-1000t} - 3.75e^{-2000t}]\,dt = \left[\frac{3.75}{-1000}e^{-1000t} - \frac{3.75}{-2000}e^{-2000t}\Big|_0^{\infty}\right]$$

$$= \frac{3.75}{1000} - \frac{3.75}{2000} = 1.875 \text{ mJ}$$

P 1.21 [a] $p = vi = 900\sin(200\pi t)\cos(200\pi t) = 450\sin(400\pi t)$ W

Therefore, $p_{\text{max}} = 450$ W

[b] $p_{\text{max}}(\text{extracting}) = 450$ W

[c] $$p_{\text{avg}} = 200\int_0^{5\times10^{-3}} 450\sin(400\pi t)\,dt$$

$$= 9\times10^4\left[\frac{-\cos 400\pi t}{400\pi}\Big|_0^{5\times10^{-3}}\right] = \frac{225}{\pi}[1 - \cos 2\pi] = 0$$

[d] $$p_{\text{avg}} = \frac{180}{\pi}[1 - \cos 2.5\pi] = \frac{180}{\pi} = 57.3 \text{ W}$$

P 1.22 [a] q = area under i vs. t plot

$$= \left[\tfrac{1}{2}(5)(4) + (10)(4) + \tfrac{1}{2}(8)(4) + (8)(6) + \tfrac{1}{2}(3)(6)\right]\times 10^3$$

$$= [10 + 40 + 16 + 48 + 9]10^3 = 123{,}000 \text{ C}$$

[b] $$w = \int p\,dt = \int vi\,dt$$

$$v = 0.2\times10^{-3}t + 9 \qquad 0 \le t \le 15 \text{ ks}$$

$0 \le t \le 4000s$

$$i = 15 - 1.25\times10^{-3}t$$

$$p = 135 - 8.25\times10^{-3}t - 0.25\times10^{-6}t^2$$

$$w_1 = \int_0^{4000}(135 - 8.25\times10^{-3}t - 0.25\times10^{-6}t^2)\,dt$$

$$= (540 - 66 - 5.3333)10^3 = 468.667 \text{ kJ}$$

$4000 \le t \le 12{,}000$

$$i = 12 - 0.5\times10^{-3}t$$

$$p = 108 - 2.1\times10^{-3}t - 0.1\times10^{-6}t^2$$

$$w_2 = \int_{4000}^{12{,}000}(108 - 2.1\times10^{-3}t - 0.1\times10^{-6}t^2)\,dt$$

$$= (864 - 134.4 - 55.467)10^3 = 674.133 \text{ kJ}$$

$12{,}000 \leq t \leq 15{,}000$

$$\begin{aligned}
i &= 30 - 2\times 10^{-3}t \\
p &= 270 - 12\times 10^{-3}t - 0.4\times 10^{-6}t^2 \\
w_3 &= \int_{12{,}000}^{15{,}000} (270 - 12\times 10^{-3}t - 0.4\times 10^{-6}t^2)\,dt \\
&= (810 - 486 - 219.6)10^3 = 104.4 \text{ kJ} \\
w_T &= w_1 + w_2 + w_3 = 468.667 + 674.133 + 104.4 = 1247.2 \text{ kJ}
\end{aligned}$$

P 1.23 [a]

$$\begin{aligned}
p &= vi = [16{,}000t + 20)e^{-800t}][(128t + 0.16)e^{-800t}] \\
&= 2048\times 10^3 t^2 e^{-1600t} + 5120te^{-1600t} + 3.2e^{-1600t} \\
&= 3.2e^{-1600t}[640{,}000t^2 + 1600t + 1] \\
\frac{dp}{dt} &= 3.2\{e^{-1600t}[1280\times 10^3 t + 1600] - 1600e^{-1600t}[640{,}000t^2 + 1600t + 1]\} \\
&= -3.2e^{-1600t}[128\times 10^4(800t^2 + t)] = -409.6\times 10^4 e^{-1600t}t(800t + 1)
\end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $t = 0$
so $p_{\max}$ occurs at $t = 0$.

[b]

$$\begin{aligned}
p_{\max} &= 3.2e^{-0}[0 + 0 + 1] \\
&= 3.2 \text{ W}
\end{aligned}$$

[c]

$$\begin{aligned}
w &= \int_0^t p dx \\
\frac{w}{3.2} &= \int_0^t 640{,}000x^2 e^{-1600x}\,dx + \int_0^t 1600xe^{-1600x}\,dx + \int_0^t e^{-1600x}\,dx \\
&= \frac{640{,}000e^{-1600x}}{-4096\times 10^6}[256\times 10^4 x^2 + 3200x + 2]\Big|_0^t + \\
&\quad \frac{1600e^{-1600x}}{256\times 10^4}(-1600x - 1)\Big|_0^t + \frac{e^{-1600x}}{-1600}\Big|_0^t
\end{aligned}$$

When $t \to \infty$ all the upper limits evaluate to zero, hence

$$\frac{w}{3.2} = \frac{(640{,}000)(2)}{4096\times 10^6} + \frac{1600}{256\times 10^4} + \frac{1}{1600}$$

$$w = 10^{-3} + 2\times 10^{-3} + 2\times 10^{-3} = 5 \text{ mJ}.$$

P 1.24 [a] We can find the time at which the power is a maximum by writing an expression for $p(t) = v(t)i(t)$, taking the first derivative of $p(t)$ and setting it to zero, then solving for t. The calculations are shown below:

$$p = 0 \quad t < 0, \qquad p = 0 \quad t > 3\text{ s}$$

$$p = vi = t(3-t)(6-4t) = 18t - 18t^2 + 4t^3 \text{ mW} \qquad 0 \le t \le 3\text{ s}$$

$$\frac{dp}{dt} = 18 - 36t + 12t^2 = 12(t^2 - 3t + 1.5)$$

$$\frac{dp}{dt} = 0 \qquad \text{when } t^2 - 3t + 1.5 = 0$$

$$t = \frac{3 \pm \sqrt{9-6}}{2} = \frac{3 \pm \sqrt{3}}{2}$$

$$t_1 = 3/2 - \sqrt{3}/2 = 0.634\text{ s}; \qquad t_2 = 3/2 + \sqrt{3}/2 = 2.366\text{ s}$$

$$p(t_1) = 18(0.634) - 18(0.634)^2 + 4(0.634)^3 = 5.196\text{ mW}$$

$$p(t_2) = 18(2.366) - 18(2.366)^2 + 4(2.366)^3 = -5.196\text{ mW}$$

Therefore, maximum power is being delivered at $t = 0.634$ s.

[b] The maximum power was calculated in part (a) to determine the time at which the power is maximum: $p_{\text{max}} = 5.196$ mW (delivered)

[c] As we saw in part (a), the other "maximum" power is actually a minimum, or the maximum negative power. As we calculated in part (a), maximum power is being extracted at $t = 2.366$ s.

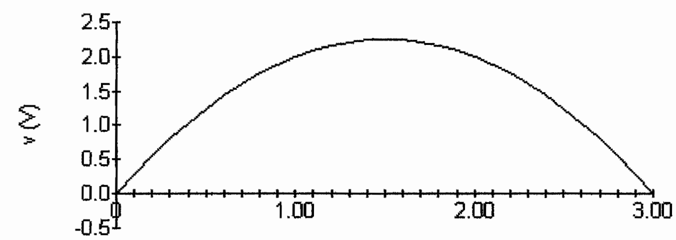
[d] This maximum extracted power was calculated in part (a) to determine the time at which power is maximum: $p_{\text{max}} = 5.196$ mW (extracted)

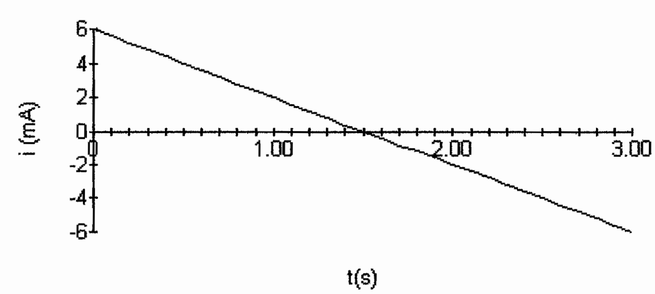
[e] $w = \int_0^t p\,dx = \int_0^t (18x - 18x^2 + 4x^3)dx = 9t^2 - 6t^3 + t^4$

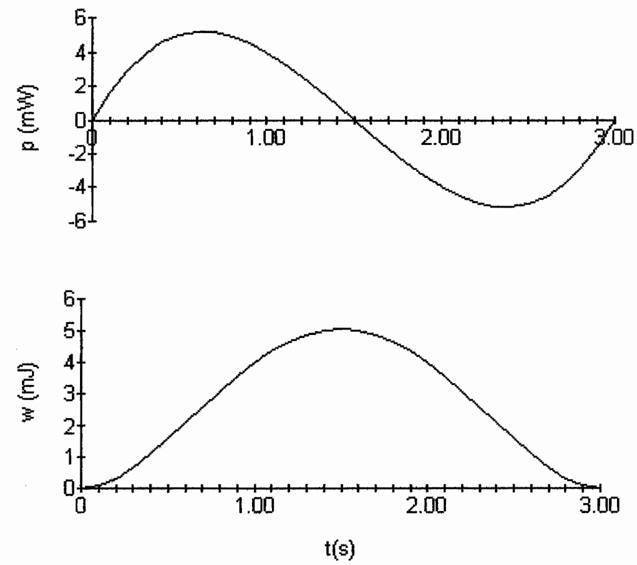
$$w(0) = 0\text{ mJ} \qquad w(2) = 4\text{ mJ}$$

$$w(1) = 4\text{ mJ} \qquad w(3) = 0\text{ mJ}$$

To give you a feel for the quantities of voltage, current, power, and energy and their relationships among one another, they are plotted below:







P 1.25 [a]

$$\begin{aligned} p &= vi \\ &= 400 \times 10^3 t^2 e^{-800t} + 700te^{-800t} + 0.25e^{-800t} \\ &= e^{-800t}[400{,}000t^2 + 700t + 0.25] \\ \frac{dp}{dt} &= \{e^{-800t}[800 \times 10^3 t + 700] - 800e^{-800t}[400{,}000t^2 + 700t + 0.25]\} \\ &= [-3{,}200{,}000t^2 + 2400t + 5]100e^{-800t} \end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $3{,}200{,}000t^2 - 2400t - 5 = 0$
so $p_{\max}$ occurs at $t = 1.68$ ms.

[b]

$$\begin{aligned} p_{\max} &= [400{,}000(.00168)^2 + 700(.00168) + 0.25]e^{-800(.00168)} \\ &= 666 \text{ mW} \end{aligned}$$

[c]

$$\begin{aligned} w &= \int_0^t p dx \\ w &= \int_0^t 400{,}000x^2 e^{-800x}\,dx + \int_0^t 700xe^{-800x}\,dx + \int_0^t 0.25e^{-800x}\,dx \\ &= \frac{400{,}000e^{-800x}}{-512 \times 10^6}[64 \times 10^4 x^2 + 1600x + 2]\Big|_0^t + \\ &\quad \frac{700e^{-800x}}{64 \times 10^4}(-800x - 1)\Big|_0^t + 0.25\frac{e^{-800x}}{-800}\Big|_0^t \end{aligned}$$

When $t = \infty$ all the upper limits evaluate to zero, hence

$$w = \frac{(400{,}000)(2)}{512 \times 10^6} + \frac{700}{64 \times 10^4} + \frac{0.25}{800} = 2.97 \text{ mJ}.$$

P 1.26 We use the passive sign convention to determine whether the power equation is $p = vi$ or $p = -vi$ and substitute into the power equation the values for v and i, as shown below:

$$p_a = v_a i_a = (0.150)(0.6) = 90 \text{ mW}$$
$$p_b = v_b i_b = (0.150)(-1.4) = -210 \text{ mW}$$
$$p_c = -v_c i_c = -(0.100)(-0.8) = 80 \text{ mW}$$
$$p_d = v_d i_d = (0.250)(-0.8) = -200 \text{ mW}$$
$$p_e = -v_e i_e = -(0.300)(-2) = 600 \text{ mW}$$
$$p_f = v_f i_f = (-0.300)(1.2) = -360 \text{ mW}$$

Remember that if the power is positive, the circuit element is absorbing power, whereas is the power is negative, the circuit element is developing power. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal:
$\sum P_{\text{dev}} = 210 + 200 + 360 = 770$ mW;
$\sum P_{\text{abs}} = 90 + 80 + 600 = 770$ mW
Thus, the power balances and the total power developed in the circuit is 770 mW.

P 1.27 [a] From the diagram and the table we have

$$p_a = -v_a i_a = -(5000)(-0.150) = 750 \text{ W}$$
$$p_b = v_b i_b = (2000)(0.250) = 500 \text{ W}$$
$$p_c = -v_c i_c = -(3000)(0.200) = -600 \text{ W}$$
$$p_d = v_d i_d = (-5000)(0.400) = -2000 \text{ W}$$
$$p_e = -v_e i_e = -(1000)(-0.050) = 50 \text{ W}$$
$$p_f = v_f i_f = (4000)(0.350) = 1400 \text{ W}$$
$$p_g = -v_g i_g = -(-2000)(0.400) = 800 \text{ W}$$
$$p_h = -v_h i_h = -(-6000)(-0.350) = -2100 \text{ W}$$
$$\sum P_{\text{del}} = 600 + 2000 + 2100 = 4700 \text{ W}$$
$$\sum P_{\text{abs}} = 750 + 500 + 50 + 1400 + 800 = 3500 \text{ W}$$

Therefore, $\sum P_{\text{del}} \neq \sum P_{\text{abs}}$ and the subordinate engineer is correct.

[b] The difference between the power delivered to the circuit and the power absorbed by the circuit is

$-4700 + 3500 = 1200$ W

One-half of this difference is 600 W, so it is likely that p_c is in error. Either the voltage or the current probably has the wrong sign. (In Chapter 2, we will discover that using KCL at the top node, the current v_c should be -3.0 kV, not 3.0 kV!) If the sign of p_c is changed from negative to positive, we can recalculate the power delivered and the power absorbed as follows:

$$\sum P_{\text{del}} = 2000 + 2100 = 4100 \text{ W}$$
$$\sum P_{\text{abs}} = 750 + 500 + 600 + 50 + 1400 + 800 = 4100 \text{ W}$$

Now the power delivered equals the power absorbed and the power balances for the circuit.

P 1.28
$$p_a = -v_a i_a = -(36)(250 \times 10^{-6}) = -9 \text{ mW}$$
$$p_b = v_b i_b = (44)(-250 \times 10^{-6}) = -11 \text{ mW}$$
$$p_c = v_c i_c = (28)(-250 \times 10^{-6}) = -7 \text{ mW}$$
$$p_d = v_d i_d = (-108)(100 \times 10^{-6}) = -10.8 \text{ mW}$$
$$p_e = v_e i_e = (-32)(150 \times 10^{-6}) = -4.8 \text{ mW}$$
$$p_f = -v_f i_f = -(60)(-350 \times 10^{-6}) = 21 \text{ mW}$$
$$p_g = v_g i_g = (-48)(-200 \times 10^{-6}) = 9.6 \text{ mW}$$
$$p_h = v_h i_h = (80)(-150 \times 10^{-6}) = -12 \text{ mW}$$
$$p_j = -v_j i_j = -(80)(-300 \times 10^{-6}) = 24 \text{ mW}$$

Therefore,

$$\sum P_{\text{abs}} = 21 + 9.6 + 24 = 54.6 \text{ mW}$$

$$\sum P_{\text{del}} = 9 + 11 + 7 + 10.8 + 4.8 + 12 = 54.6 \text{ W}$$

$$\sum P_{\text{abs}} = \sum P_{\text{del}}$$

Thus, the interconnection satisfies the power check

P 1.29
$$p_a = -v_a i_a = -(1.6)(0.080) = -128 \text{ mW}$$
$$p_b = -v_b i_b = -(2.6)(0.060) = -156 \text{ mW}$$
$$p_c = v_c i_c = (-4.2)(-0.050) = 210 \text{ mW}$$
$$p_d = -v_d i_d = -(1.2)(0.020) = -24 \text{ mW}$$
$$p_e = v_e i_e = (1.8)(0.030) = 54 \text{ mW}$$
$$p_f = -v_f i_f = -(-1.8)(-0.040) = -72 \text{ mW}$$
$$p_g = v_g i_g = (-3.6)(-0.030) = 108 \text{ mW}$$
$$p_h = v_h i_h = (3.2)(-0.020) = -64 \text{ mW}$$
$$p_j = -v_j i_j = -(-2.4)(0.030) = 72 \text{ mW}$$
$$\sum P_{\text{del}} = 128 + 156 + 24 + 72 + 64 = 444 \text{ mW}$$
$$\sum P_{\text{abs}} = 210 + 54 + 108 + 72 = 444 \text{ mW}$$

Therefore, $\sum P_{\text{del}} = \sum P_{\text{abs}} = 444\ \text{mW}$

Thus, the interconnection satisfies the power check

P 1.30 **[a]** From an examination of reference polarities, elements a, b, e, and f absorb power, while elements c, d, g, and h supply power.

[b]
$$\begin{aligned}
p_{\text{a}} &= v_{\text{a}}i_{\text{a}} = (0.300)(25 \times 10^{-6}) = 7.5\,\mu\text{W} \\
p_{\text{b}} &= -v_{\text{b}}i_{\text{b}} = -(-0.100)(10 \times 10^{-6}) = 1\,\mu\text{W} \\
p_{\text{c}} &= v_{\text{c}}i_{\text{c}} = (-0.200)(15 \times 10^{-6}) = -3\,\mu\text{W} \\
p_{\text{d}} &= -v_{\text{d}}i_{\text{d}} = -(-0.200)(-35 \times 10^{-6}) = -7\,\mu\text{W} \\
p_{\text{e}} &= -v_{\text{e}}i_{\text{e}} = -(0.350)(-25 \times 10^{-6}) = 8.75\,\mu\text{W} \\
p_{\text{f}} &= v_{\text{f}}i_{\text{f}} = (0.200)(10 \times 10^{-6}) = 2\,\mu\text{W} \\
p_{\text{g}} &= v_{\text{g}}i_{\text{g}} = (-0.250)(35 \times 10^{-6}) = -8.75\,\mu\text{W} \\
p_{\text{h}} &= v_{\text{h}}i_{\text{h}} = (0.050)(-10 \times 10^{-6}) = -0.5\,\mu\text{W}
\end{aligned}$$

$\sum P_{\text{abs}} = 7.5 + 1 + 8.75 + 2 = 19.25\,\mu\text{W}$

$\sum P_{\text{del}} = 3 + 7 + 8.75 + 0.5 = 19.25\,\mu\text{W}$

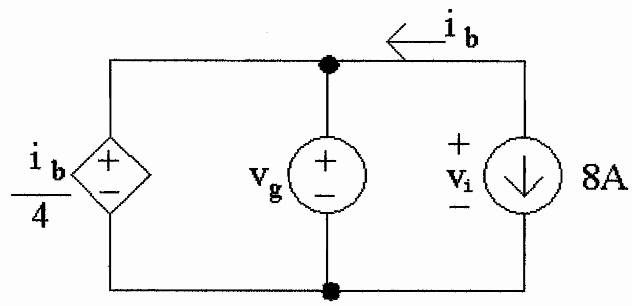
Thus, $19.25\,\mu$W of power is delivered and $19.25\,\mu$W of power is absorbed, and the power balances

2

Circuit Elements

Assessment Problems

AP 2.1



[a] Note that the current i_b is in the same circuit branch as the 8 A current source; however, i_b is defined in the opposite direction of the current source. Therefore,

$$i_b = -8\,\text{A}$$

Next, note that the dependent current source and the independent current source are in parallel with the same polarity. Therefore, their voltages are equal, and

$$v_g = \frac{i_b}{4} = \frac{-8}{4} = -2\,\text{V}$$

[b] To find the power associated with the 8 A source, we need to find the voltage drop across the source, v_i. Note that the two independent sources are in parallel, and that the voltages v_g and v_1 have the same polarities, so these voltages are equal:

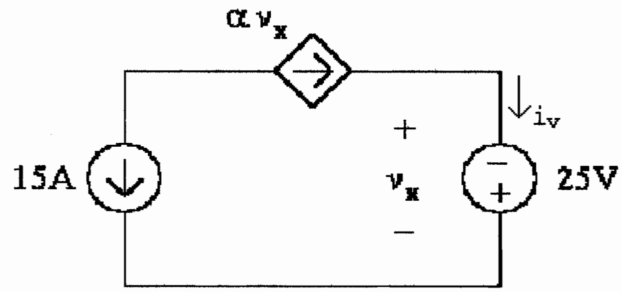
$$v_i = v_g = -2\,\text{V}$$

Using the passive sign convention,

$$p_s = (8\,\text{A})(v_i) = (8\,\text{A})(-2\,\text{V}) = -16\,\text{W}$$

Thus the current source generated 16 W of power.

AP 2.2



[a] Note from the circuit that $v_x = -25$ V. To find α note that the two current sources are in the same branch of the circuit but their currents flow in opposite directions. Therefore

$$\alpha v_x = -15\,\text{A}$$

Solve the above equation for α and substitute for v_x,

$$\alpha = \frac{-15\,\text{A}}{v_x} = \frac{-15\,\text{A}}{-25\,\text{V}} = 0.6\ \text{A/V}$$

[b] To find the power associated with the voltage source we need to know the current, i_v. Note that this current is in the same branch of the circuit as the dependent current source and these two currents flow in the same direction. Therefore, the current i_v is the same as the current of the dependent source:

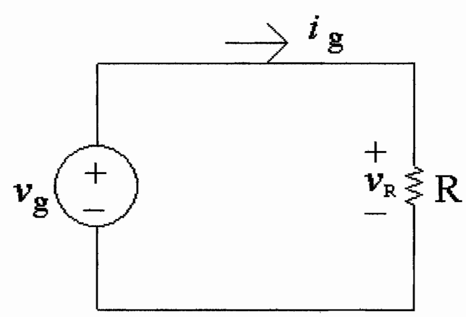
$$i_v = \alpha v_x = (0.6)(-25) = -15\,\text{A}$$

Using the passive sign convention,

$$p_s = -(i_v)(25\,\text{V}) = -(-15\,\text{A})(25\,\text{V}) = 375\,\text{W}.$$

Thus the voltage source dissipates 375 W.

AP 2.3



[a] The resistor and the voltage source are in parallel and the resistor voltage and the voltage source have the same polarities. Therefore these two voltages are the same:

$$v_R = v_g = 1\,\text{kV}$$

Note from the circuit that the current through the resistor is $i_g = 5$ mA. Use Ohm's law to calculate the value of the resistor:

$$R = \frac{v_R}{i_g} = \frac{1\,\text{kV}}{5\,\text{mA}} = 200\,\text{k}\Omega$$

Using the passive sign convention to calculate the power in the resistor,

$$p_R = (v_R)(i_g) = (1\,\text{kV})(5\,\text{mA}) = 5\,\text{W}$$

The resistor is dissipating 5 W of power.

[b] Note from part (a) the $v_R = v_g$ and $i_R = i_g$. The power delivered by the source is thus

$$p_{\text{source}} = -v_g i_g \qquad \text{so} \qquad v_g = -\frac{p_{\text{source}}}{i_g} = -\frac{-3\,\text{W}}{75\,\text{mA}} = 40\,\text{V}$$

Since we now have the value of both the voltage and the current for the resistor, we can use Ohm's law to calculate the resistor value:

$$R = \frac{v_g}{i_g} = \frac{40\,\text{V}}{75\,\text{mA}} = 533.33\,\Omega$$

The power absorbed by the resistor must equal the power generated by the source. Thus,

$$p_R = -p_{\text{source}} = -(-3\,\text{W}) = 3\,\text{W}$$

[c] Again, note the $i_R = i_g$. The power dissipated by the resistor can be determined from the resistor's current:

$$p_R = R(i_R)^2 = R(i_g)^2$$

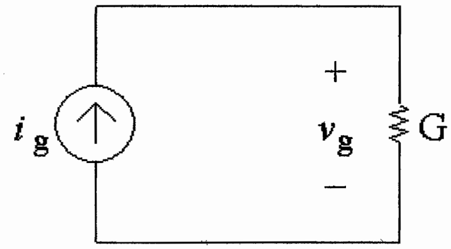
Solving for i_g,

$$i_g^2 = \frac{p_r}{R} = \frac{480\,\text{mW}}{300\,\Omega} = 0.0016 \qquad \text{so} \qquad i_g = \sqrt{0.0016} = 0.04\,\text{A} = 40\,\text{mA}$$

Then, since $v_R = v_g$

$$v_R = Ri_R = Ri_g = (300\,\Omega)(40\,\text{mA}) = 12\,\text{V} \qquad \text{so} \qquad v_g = 12\,\text{V}$$

AP 2.4



[a] Note from the circuit that the current through the conductance G is i_g, flowing from top to bottom, because the current source and the conductance are in the same branch of the circuit so must have the same current. The voltage drop across the current source is v_g, positive at the top, because the current source and the conductance are also in parallel so must have the same voltage. From a version of Ohm's law,

$$v_g = \frac{i_g}{G} = \frac{0.5\,\text{A}}{50\,\text{mS}} = 10\,\text{V}$$

Now that we know the voltage drop across the current source, we can find the power delivered by this source:

$$p_{\text{source}} = -v_g i_g = -(10)(0.5) = -5\,\text{W}$$

Thus the current source delivers 5 W to the circuit.

[b] We can find the value of the conductance using the power, and the value of the current using Ohm's law and the conductance value:

$$p_g = Gv_g^2 \qquad \text{so} \qquad G = \frac{p_g}{v_g^2} = \frac{9}{15^2} = 0.04\,\text{S} = 40\,\text{mS}$$

$$i_g = Gv_g = (40\,\text{mS})(15\,\text{V}) = 0.6\,\text{A}$$

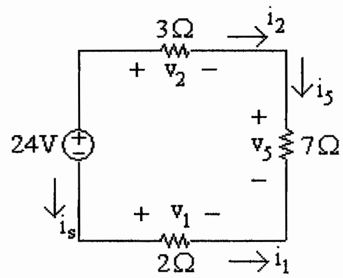
[c] We can find the voltage from the power and the conductance, and then use the voltage value in Ohm's law to find the current:

$$p_g = Gv_g^2 \qquad \text{so} \qquad v_g^2 = \frac{p_g}{G} = \frac{8\,\text{W}}{200\,\mu\text{S}} = 40{,}000$$

Thus $\quad v_g = \sqrt{40{,}000} = 200\,\text{V}$

$$i_g = Gv_g = (200\,\mu\text{S})(200\,\text{V}) = 0.04\,\text{A} = 40\,\text{mA}$$

AP 2.5 [a] Redraw the circuit with all of the voltages and currents labeled for every circuit element.



Write a KVL equation clockwise around the circuit, starting below the voltage source:

$$-24\,\text{V} + v_2 + v_5 - v_1 = 0$$

Next, use Ohm's law to calculate the three unknown voltages from the three currents:

$$v_2 = 3i_2; \qquad v_5 = 7i_5; \qquad v_1 = 2i_1$$

A KCL equation at the upper right node gives $i_2 = i_5$; a KCL equation at the bottom right node gives $i_5 = -i_1$; a KCL equation at the upper left node gives $i_s = -i_2$. Now replace the currents i_1 and i_2 in the Ohm's law equations with i_5:

$$v_2 = 3i_2 = 3i_5; \qquad v_5 = 7i_5; \qquad v_1 = 2i_1 = -2i_5$$

Now substitute these expressions for the three voltages into the first equation:

$$24 = v_2 + v_5 - v_1 = 3i_5 + 7i_5 - (-2i_5) = 12i_5$$

Therefore $i_5 = 24/12 = 2$ A

[b] $v_1 = -2i_5 = -2(2) = -4\,\text{V}$

[c] $v_2 = 3i_5 = 3(2) = 6\,\text{V}$

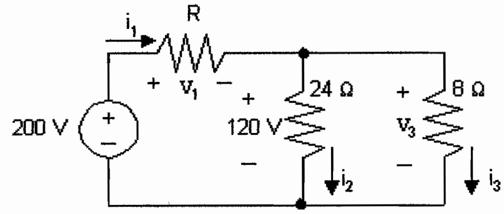
[d] $v_5 = 7i_5 = 7(2) = 14\,\text{V}$

[e] A KCL equation at the lower left node gives $i_s = i_1$. Since $i_1 = -i_5$, $i_s = -2\,\text{A}$. We can now compute the power associated with the voltage source:

$$p_{24} = (24)i_s = (24)(-2) = -48\,\text{W}$$

Therefore 24 V source is delivering 48 W.

AP 2.6 Redraw the circuit labeling all voltages and currents:



We can find the value of the unknown resistor if we can find the value of its voltage and its current. To start, write a KVL equation clockwise around the right loop, starting below the 24 Ω resistor:

$$-120\,\text{V} + v_3 = 0$$

Use Ohm's law to calculate the voltage across the 8 Ω resistor in terms of its current:

$$v_3 = 8i_3$$

Substitute the expression for v_3 into the first equation:

$$-120\,\text{V} + 8i_3 = 0 \qquad \text{so} \qquad i_3 = \frac{120}{8} = 15\,\text{A}$$

Also use Ohm's law to calculate the value of the current through the 24 Ω resistor:

$$i_2 = \frac{120\,\text{V}}{24\,\Omega} = 5\,\text{A}$$

Now write a KCL equation at the top middle node, summing the currents leaving:

$$-i_1 + i_2 + i_3 = 0 \qquad \text{so} \qquad i_1 = i_2 + i_3 = 5 + 15 = 20\,\text{A}$$

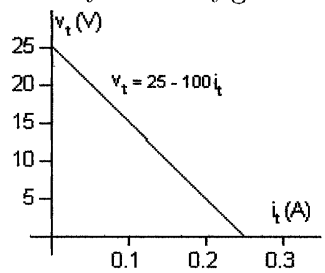
Write a KVL equation clockwise around the left loop, starting below the voltage source:

$$-200\,\text{V} + v_1 + 120\,\text{V} = 0 \qquad \text{so} \qquad v_1 = 200 - 120 = 80\,\text{V}$$

Now that we know the values of both the voltage and the current for the unknown resistor, we can use Ohm's law to calculate the resistance:

$$\text{R} = \frac{v_1}{i_1} = \frac{80}{20} = 4\,\Omega$$

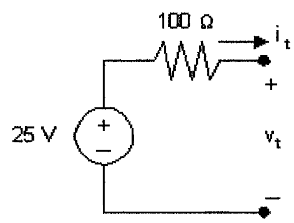
AP 2.7 [a] Plotting a graph of v_t versus i_t gives



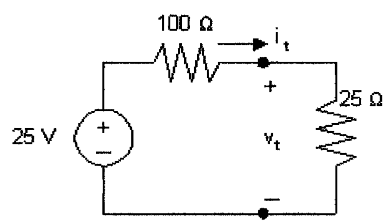
Note that when $i_t = 0$, $v_t = 25$ V; therefore the voltage source must be 25 V. Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25-0}{0.25-0} = \frac{25}{0.25} = 100\,\Omega$$

A circuit model having the same $v - i$ characteristic is a 25 V source in series with a 100Ω resistor, as shown below:



[b] Draw the circuit model from part (a) and attach a 25 Ω resistor:



To find the power delivered to the 25 Ω resistor we must calculate the current through the 25 Ω resistor. Do this by first using KCL to recognize that the current in each of the components is i_t, flowing in a clockwise direction. Write a KVL equation in the clockwise direction, starting below the voltage source, and using Ohm's law to express the voltage drop across the resistors in the direction of the current i_t flowing through the resistors:

$$-25\,\text{V} + 100i_t + 25i_t = 0 \qquad \text{so} \qquad 125i_t = 25 \qquad \text{so} \qquad i_t = \frac{25}{125} = 0.2\,\text{A}$$

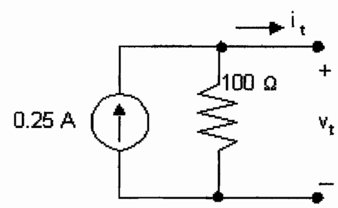
Thus, the power delivered to the 25 Ω resistor is

$p_{25} = (25)i_t^2 = (25)(0.2)^2 = 1\,\text{W}$.

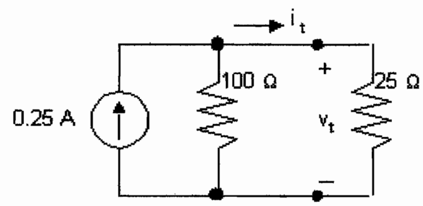
AP 2.8 [a] From the graph in Assessment Problem 2.7(a), we see that when $v_t = 0$, $i_t = 0.25\,\text{A}$. Therefore the current source must be 0.25 A. Since the plot is a straight line, its slope can be used to calculate the value of resistance:

$$R = \frac{\Delta v}{\Delta i} = \frac{25 - 0}{0.25 - 0} = \frac{25}{0.25} = 100\,\Omega$$

A circuit model having the same $v - i$ characteristic is a 0.25 A current source in parallel with a 100Ω resistor, as shown below:



[b] Draw the circuit model from part (a) and attach a 25 Ω resistor:



Note that by writing a KVL equation around the right loop we see that the voltage drop across both resistors is v_t. Write a KCL equation at the top center node, summing the currents leaving the node. Use Ohm's law to specify the currents through the resistors in terms of the voltage drop across the resistors and the value of the resistors.

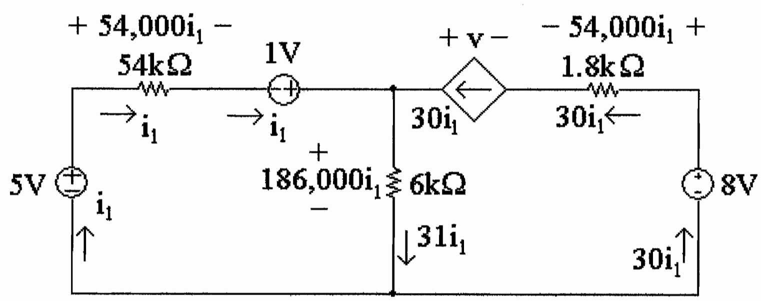
$$-0.25 + \frac{v_t}{100} + \frac{v_t}{25} = 0, \qquad \text{so} \qquad 5v_t = 25, \qquad \text{thus} \qquad v_t = 5\,\text{V}$$

$$p_{25} = \frac{v_t^2}{25} = 1\,\text{W}.$$

AP 2.9 First note that we know the current through all elements in the circuit except the 6 kΩ resistor (the current in the three elements to the left of the 6 kΩ resistor is i_1; the current in the three elements to the right of the 6 kΩ resistor is $30i_1$). To find the current in the 6 kΩ resistor, write a KCL equation at the top node:

$$i_1 + 30i_1 = i_{6\text{k}} = 31i_1$$

We can then use Ohm's law to find the voltages across each resistor in terms of i_1. The results are shown in the figure below:



[a] To find i_1, write a KVL equation around the left-hand loop, summing voltages in a clockwise direction starting below the 5V source:

$$-5\,\text{V} + 54{,}000i_1 - 1\,\text{V} + 186{,}000i_1 = 0$$

Solving for i_1

$$54{,}000i_1 + 186{,}000i_1 = 6\,\text{V} \qquad \text{so} \qquad 240{,}000i_1 = 6\,\text{V}$$

Thus,

$$i_1 = \frac{6}{240{,}000} = 25\,\mu\text{A}$$

[b] Now that we have the value of i_1, we can calculate the voltage for each component except the dependent source. Then we can write a KVL equation for the right-hand loop to find the voltage v of the dependent source. Sum the voltages in the clockwise direction, starting to the left of the dependent source:

$$+v - 54{,}000i_1 + 8\,\text{V} - 186{,}000i_1 = 0$$

Thus,

$$v = 240{,}000i_1 - 8\,\text{V} = 240{,}000(25 \times 10^{-6}) - 8\,\text{V} = 6\,\text{V} - 8\,\text{V} = -2\,\text{V}$$

We now know the values of voltage and current for every circuit element.

Let's construct a power table:

Element	**Current** (μ**A**)	**Voltage** (**V**)	**Power Equation**	**Power** (μ**W**)
5 V	25	5	$p = -vi$	−125
54 kΩ	25	1.35	$p = Ri^2$	33.75
1 V	25	1	$p = -vi$	−25
6 kΩ	775	4.65	$p = Ri^2$	3603.75
Dep. source	750	−2	$p = -vi$	1500
1.8 kΩ	750	1.35	$p = Ri^2$	1012.5
8 V	750	8	$p = -vi$	−6000

[c] The total power generated in the circuit is the sum of the negative power values in the power table:

$$-125\,\mu\text{W} + -25\,\mu\text{W} + -6000\,\mu\text{W} = -6150\,\mu\text{W}$$

Thus, the total power generated in the circuit is 6150 μW.

[d] The total power absorbed in the circuit is the sum of the positive power values in the power table:

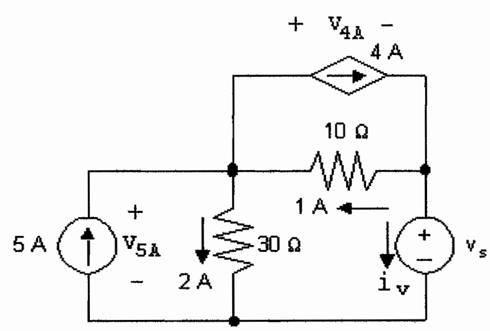
$$33.75\,\mu\text{W} + 3603.75\,\mu\text{W} + 1500\,\mu\text{W} + 1012.5\,\mu\text{W} = 6150\,\mu\text{W}$$

Thus, the total power absorbed in the circuit is 6150 μW.

AP 2.10 Given that $i_\phi = 2$ A, we know the current in the dependent source is $2i_\phi = 4$ A. We can write a KCL equation at the left node to find the current in the 10 Ω resistor. Summing the currents leaving the node,

$$-5\,\text{A} + 2\,\text{A} + 4\,\text{A} + i_{10\Omega} = 0 \qquad \text{so} \qquad i_{10\Omega} = 5\,\text{A} - 2\,\text{A} - 4\,\text{A} = -1\,\text{A}$$

Thus, the current in the 10 Ω resistor is 1 A, flowing right to left, as seen in the circuit below.



[a] To find v_s, write a KVL equation, summing the voltages counter-clockwise around the lower right loop. Start below the voltage source.

$$-v_s + (1\,\text{A})(10\,\Omega) + (2\,\text{A})(30\,\Omega) = 0 \qquad \text{so} \qquad v_s = 10\,\text{V} + 60\,\text{V} = 70\,\text{V}$$

[b] The current in the voltage source can be found by writing a KCL equation at the right-hand node. Sum the currents leaving the node

$$-4\,\text{A} + 1\,\text{A} + i_v = 0 \qquad \text{so} \qquad i_v = 4\,\text{A} - 1\,\text{A} = 3\,\text{A}$$

The current in the voltage source is 3 A, flowing top to bottom. The power associated with this source is

$$p = vi = (70\,\text{V})(3\,\text{A}) = 210\,\text{W}$$

Thus, 210 W are absorbed by the voltage source.

[c] The voltage drop across the independent current source can be found by writing a KVL equation around the left loop in a clockwise direction:

$$-v_{5A} + (2\,\text{A})(30\,\Omega) = 0 \qquad \text{so} \qquad v_{5A} = 60\,\text{V}$$

The power associated with this source is

$$p = -v_{5A}i = -(60\,\text{V})(5\,\text{A}) = -300\,\text{W}$$

This source thus delivers 300 W of power to the circuit.

[d] The voltage across the controlled current source can be found by writing a KVL equation around the upper right loop in a clockwise direction:

$$+v_{4A} + (10\,\Omega)(1\,\text{A}) = 0 \qquad \text{so} \qquad v_{4A} = -10\,\text{V}$$

The power associated with this source is

$$p = v_{4A}i = (-10\,\text{V})(4\,\text{A}) = -40\,\text{W}$$

This source thus delivers 40 W of power to the circuit.

[e] The total power dissipated by the resistors is given by

$$(i_{30\Omega})^2(30\,\Omega) + (i_{10\Omega})^2(10\,\Omega) = (2)^2(30\,\Omega) + (1)^2(10\,\Omega) = 120 + 10 = 130\,\text{W}$$

Problems

P 2.1 [a] Yes, independent voltage sources can carry the 8 A current required by the connection; independent current source can support any voltage required by the connection, in this case 20 V, positive at the top.

[b] 30 V source: absorbing

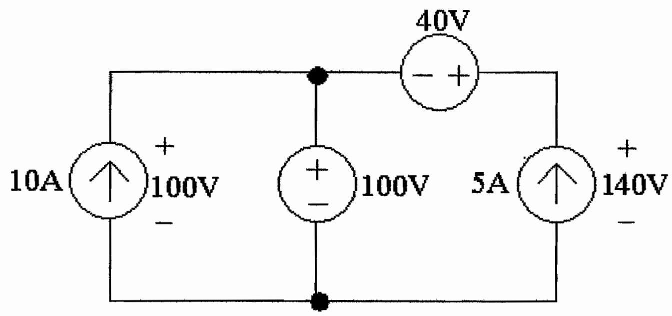
10 V source: delivering

8 A source: delivering

[c]
$$P_{30\text{V}} = (30)(8) = 240 \text{ W} \quad \text{(abs)}$$
$$P_{10\text{V}} = -(10)(8) = -80 \text{ W} \quad \text{(del)}$$
$$P_{8\text{A}} = -(20)(8) = -160 \text{ W} \quad \text{(del)}$$
$$\sum P_{\text{abs}} = \sum P_{\text{del}} = 240 \text{ W}$$

[d] The interconnection is valid, but in this circuit the voltage drop across the 8 A current source is 40 V, positive at the top; 30 V source is absorbing, the 10 V source is absorbing, and the 8 A source is delivering
$$P_{30\text{V}} = (30)(8) = 240 \text{ W} \quad \text{(abs)}$$
$$P_{10\text{V}} = (10)(8) = 80 \text{ W} \quad \text{(abs)}$$
$$P_{8\text{A}} = -(40)(8) = -320 \text{ W} \quad \text{(del)}$$
$$\sum P_{\text{abs}} = \sum P_{\text{del}} = 320 \text{ W}$$

P 2.2 The interconnection is valid. The 10 A current source has a voltage drop of 100 V, positive at the top, because the 100 V source supplies its voltage drop across a pair of terminals shared by the 10 A current source. The right hand branch of the circuit must also have a voltage drop of 100 V from the left terminal of the 40 V source to the bottom terminal of the 5 A current source, because this branch shares the same terminals as the 100 V source. This means that the voltage drop across the 5 A current source is 140 V, positive at the top. Also, the two voltage sources can carry the current required of the interconnection. This is summarized in the figure below:



From the values of voltage and current in the figure, the power supplied by the currents sources is calculated as follows:

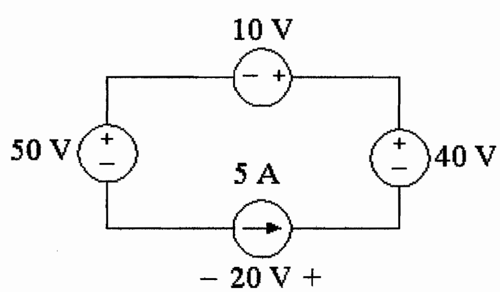
$$P_{10\text{A}} = -(100)(10) = -1000 \text{ W} \quad \text{(dev)}$$

$$P_{5\text{A}} = -(140)(5) = -700 \text{ W} \quad \text{(dev)}$$

$$\sum P_{\text{dev}} = 1700 \text{ W}$$

P 2.3 The interconnection is not valid. Note that both current sources in the right hand branch supply current through the 100 V source. If the interconnection was valid, these two current sources would supply the same current in the same direction, which they do not.

P 2.4 The interconnect is valid since the voltage sources can all carry 5 A of current supplied by the current source, and the current source can carry the voltage drop required by the interconnection. Note that the branch containing the 10 V, 40 V, and 5 A sources must have the same voltage drop as the branch containing the 50 V source, so the 5 A current source must have a voltage drop of 20 V, positive at the right. The voltages and currents are summarize in the circuit below:



$$P_{50\text{V}} = (50)(5) = 250 \text{ W} \quad \text{(abs)}$$

$$P_{10\text{V}} = (10)(5) = 50 \text{ W} \quad \text{(abs)}$$

$$P_{40\text{V}} = -(40)(5) = -200 \text{ W} \quad \text{(dev)}$$

$$P_{5\text{A}} = -(20)(5) = -100 \text{ W} \quad \text{(dev)}$$

$\sum P_{\text{dev}} = 300$ W

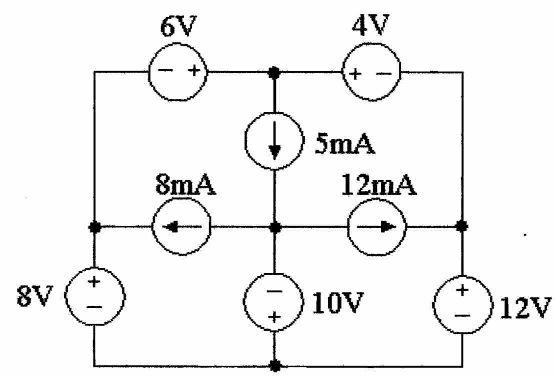
P 2.5 The interconnection is valid, since the voltage sources can carry the 10 A current supplied by the current source, and the current sources can carry whatever voltage drop is required by the interconnection. In particular, note the the voltage drop across the three sources in the right hand branch must be the same as the voltage drop across the 20 A current source in the middle branch, since the middle and right hand branch are connected between the same two terminals. In particular, this means that

v_1(the voltage drop across the middle branch)

$= 100\text{V} - 50\text{V} - v_2$(the voltage drop across the right hand branch)

Hence any combination of v_1 and v_2 such that $v_1 + v_2 = 50\,\text{V}$ is a valid solution.

P 2.6

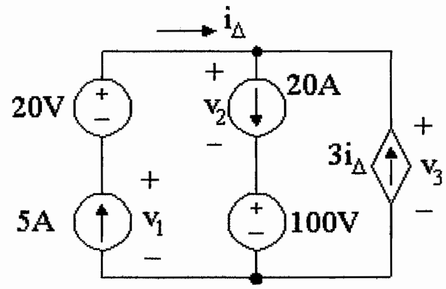


The interconnection is invalid. The voltage drop between the top terminal and the bottom terminal on the left hand side is due to the 6 V and 8 V sources, giving a total voltage drop between these terminals of 14 V. But the voltage drop between the top terminal and the bottom terminal on the right hand side is due to the 4 V and 12 V sources, giving a total voltage drop between these two terminals of 16 V. The voltage drop between any two terminals in a valid circuit must be the same, so the interconnection is invalid.

P 2.7 [a] Yes, each of the voltage sources can carry the current required by the interconnection, and each of the current sources can carry the voltage drop required by the interconnection. (Note that $i_\Delta = 5$ A.)

[b] No, because the voltage drop between the top terminal and the bottom terminal cannot be determined. For example, define v_1, v_2, and v_3 as

shown:



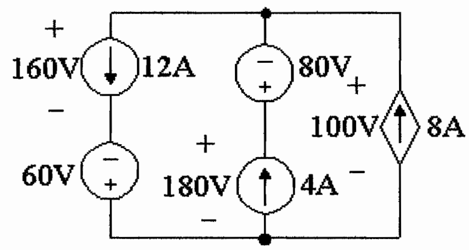
The voltage drop across the left branch, the center branch, and the right branch must be the same, since these branches are connected at the same two terminals. This requires that

$$20 + v_1 = v_2 + 100 = v_3$$

But this equation has three unknown voltages, so the individual voltages cannot be determined, and thus the power of the sources cannot be determined.

P 2.8 The interconnection is invalid. In the middle branch, the value of the current i_Δ must be -25 A, since the 25 A current source supplies current in this branch in the direction opposite the direction of the current i_Δ. Therefore, the voltage supplied by the dependent voltage source in the left hand branch is $6(-25) = -150$ V. This gives a voltage drop from the top terminal to the bottom terminal in the left hand branch of $50 - (-150) = 200$ V. But the voltage drop between these same terminals in the right hand branch is 250 V, due to the voltage source in that branch. Therefore, the interconnection is invalid.

P 2.9 The middle branch has a 4 A current source, so the current i_Δ in that branch must also be 4 A, since the two currents are in the same direction. This means that the current supplied by the dependent source is $2(4) = 8$ A. Next, $v_o = 100$ V, and this must be the voltage drop across all three branches in the circuit, since all three branches connect at the same two terminals. Therefore, the voltage drop across the current source in the left hand branch must be 160 V, positive at the top and the voltage drop across the current source in the middle branch must be 180 V, positive at the top. The voltages and currents for all sources are summarized in the figure below:



From the values of voltage and current in the figure, the power supplied by the currents sources is calculated as follows:

$$\begin{aligned} P_{12\text{A}} &= (160)(12) = 1920 \text{ W} \quad \text{(abs)} \\ P_{60\text{V}} &= -(60)(12) = -720 \text{ W} \quad \text{(dev)} \\ P_{80\text{V}} &= (80)(4) = 320 \text{ W} \quad \text{(abs)} \\ P_{4\text{A}} &= -(180)(4) = -720 \text{ W} \quad \text{(dev)} \\ P_{\text{depsource}} &= -(100)(8) = -800 \text{ W} \quad \text{(dev)} \end{aligned}$$

$$\sum P_{\text{dev}} = 720 + 720 + 800 = 2240 \text{ W}$$

P 2.10 Since we know the device is a resistor, we can use Ohm's law to calculate the resistance. From Fig. P2.10(a),

$$v = Ri \qquad \text{so} \qquad R = \frac{v}{i}$$

Using the values in the table of Fig. P2.10(b),

$$R = \frac{-160}{-0.02} = \frac{-80}{-0.01} = \frac{80}{0.01} = \frac{160}{0.02} = \frac{240}{0.03} = 8\text{k}\Omega$$

P 2.11 Since we know the device is a resistor, we can use the power equation. From Fig. P2.11(a),

$$p = vi = \frac{v^2}{R} \qquad \text{so} \qquad R = \frac{v^2}{p}$$

Using the values in the table of Fig. P2.11(b)

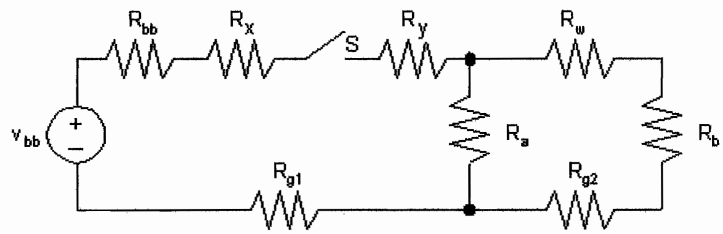
$$R = \frac{(-10)^2}{25 \times 10^{-3}} = \frac{(-5)^2}{6.25 \times 10^{-3}} = \frac{(5)^2}{6.25 \times 10^{-3}} = \frac{(10)^2}{25 \times 10^{-3}}$$

$$= \frac{(15)^2}{56.25 \times 10^{-3}} = \frac{(20)^2}{100 \times 10^{-3}} = 4\,\text{k}\Omega$$

P 2.12 The resistor value is the ratio of the power to the square of the current: $R = \frac{p}{i^2}$. Using the values for power and current in Fig. P2.12(b),

$$\frac{100}{2^2} = \frac{400}{4^2} = \frac{900}{6^2} = \frac{1600}{8^2} = \frac{2500}{10^2} = \frac{3600}{12^2} = 25\,\Omega$$

P 2.13



V_{bb} = no-load voltage of battery

R_{bb} = internal resistance of battery

R_x = resistance of wire between battery and switch

R_y = resistance of wire between switch and lamp A

R_a = resistance of lamp A

R_b = resistance of lamp B

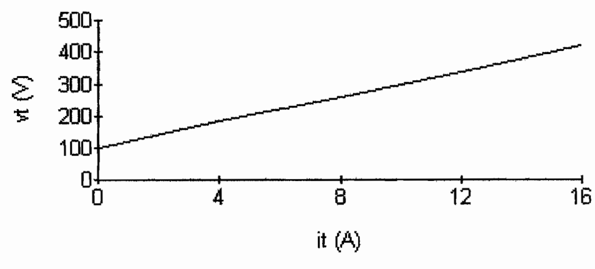
R_w = resistance of wire between lamp A and lamp B

R_{g1} = resistance of frame between battery and lamp A

R_{g2} = resistance of frame between lamp A and lamp B

S = switch

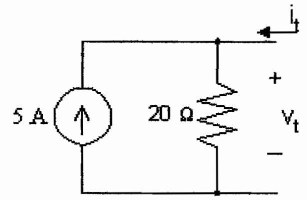
P 2.14 [a] Plot the v—i characteristic:



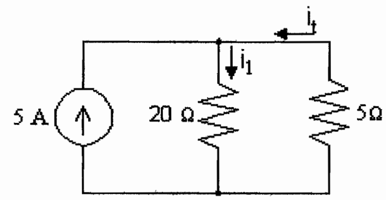
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(420 - 100)}{(16 - 0)} = 20\,\Omega$$

When $i_t = 0$, $v_t = 100$ V; therefore the ideal current source must have a current of $100/20 = 5$ A



[b] We attach a 5 Ω resistor to the device model developed in part (a):



Write a KCL equation at the top node:

$5 + i_t = i_1$

Write a KVL equation for the right loop, in the direction of the two currents, using Ohm's law:

$20i_1 + 5i_t = 0$

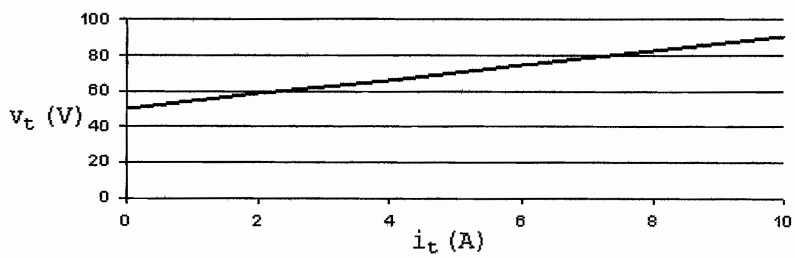
Combining the two equations and solving,

$20(5 + i_t) + 5i_t = 0$ so $25i_t = -100;$ thus $i_t = -4\,\text{A}$

Now calculate the power dissipated by the resistor:

$p_{5\,\Omega} = 5i_t^2 = 5(-4)^2 = 80\,\text{W}$

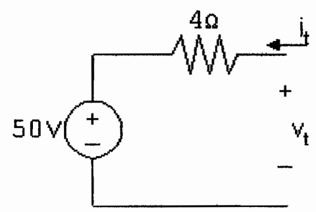
P 2.15 [a] Plot the $v - i$ characteristic



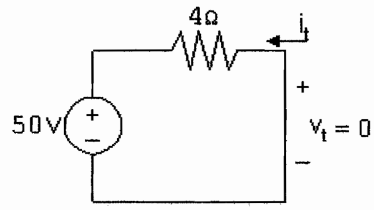
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(90 - 50)}{(10 - 0)} = 4\,\Omega$$

When $i_t = 0$, $v_t = 50$ V; therefore the ideal voltage source has a voltage of 50 V.



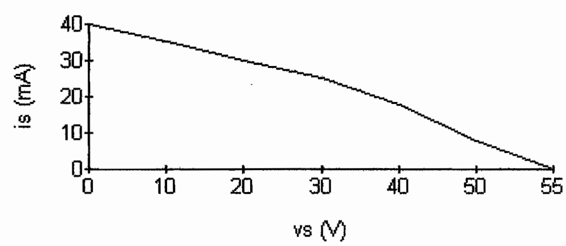
[b]



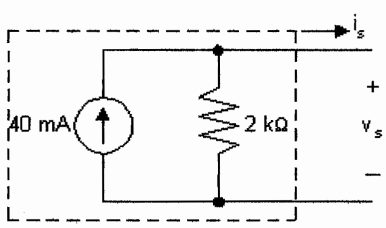
When $v_t = 0$, $$i_t = \frac{-50}{4} = -12.5\text{A}$$

Note that this result can also be obtained by extrapolating the $v - i$ characteristic to $v_t = 0$.

P 2.16 [a]

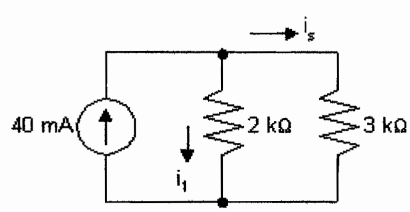


[b] $\Delta v = 20\text{V}$; $\Delta i = 10$ mA; $R = \dfrac{\Delta v}{\Delta i} = 2\ \text{k}\Omega$



[c] $2i_1 = 3i_s$, $i_1 = 1.5i_s$

$40 = i_1 + i_s = 2.5i_s$, $i_s = 16$ mA

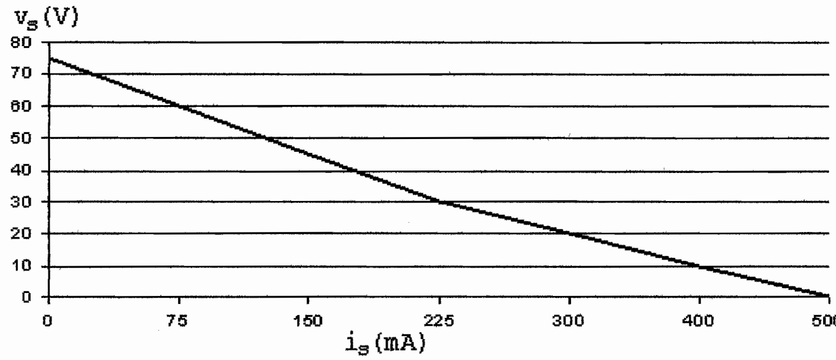


[d] $v_s(\text{open circuit}) = (40 \times 10^{-3})(2 \times 10^3) = 80$ V

[e] The open circuit voltage can be found in the table of values (or from the plot) as the value of the voltage v_s when the current $i_s = 0$. Thus, $v_s(\text{open circuit}) = 55$ V (from the table)

[f] Linear model cannot predict the nonlinear behavior of the practical current source.

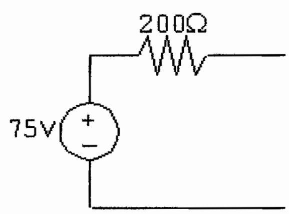
P 2.17 [a] Begin



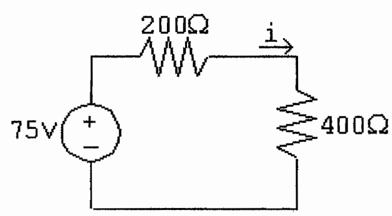
[b] Since the plot is linear for $0 \leq i_s \leq 225$ mA amd since $R = \Delta v/\Delta i$, we can calculate R from the plotted values as follows:

$$R = \frac{\Delta v}{\Delta i} = \frac{75 - 30}{0.225 - 0} = \frac{45}{0.225} = 200\,\Omega$$

We can determine the value of the ideal voltage source by considering the value of v_s when $i_s = 0$. When there is no current, there is no voltage drop across the resistor, so all of the voltage drop at the output is due to the voltage source. Thus the value of the voltage source must be 75 V. The model, valid for $0 \leq i_s \leq 225$ mA, is shown below:



[c] The circuit is shown below:

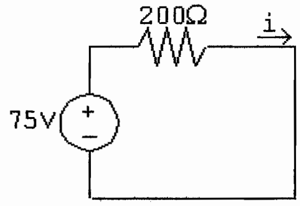


Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i:

$$-75\,\text{V} + 200i + 400i = 0 \qquad \text{so} \qquad 600i = 75\,\text{V}$$

Thus, $i = \dfrac{75\,\text{V}}{600\,\Omega} = 125\,\text{mA}$

[d] The circuit is shown below:



Write a KVL equation in the clockwise direction, starting below the voltage source. Use Ohm's law to express the voltage drop across the resistors in terms of the current i:

$-75\,\text{V} + 200i = 0 \qquad \text{so} \qquad 200i = 75\,\text{V}$

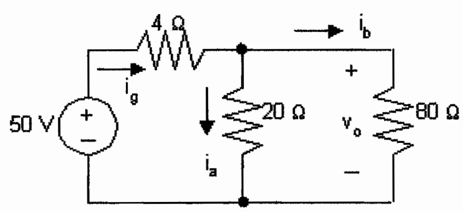
Thus, $i = \dfrac{75\,\text{V}}{200\,\Omega} = 375\,\text{mA}$

[e] The short circuit current can be found in the table of values (or from the plot) as the value of the current i_s when the voltage $v_s = 0$. Thus,

$i_{sc} = 500\,\text{mA} \qquad \text{(from table)}$

[f] The plot of voltage versus current constructed in part (a) is not linear (it is piecewise linear, but not linear for all values of i_s). Since the proposed circuit model is a linear model, it cannot be used to predict the nonlinear behavior exhibited by the plotted data.

P 2.18 [a]



$$\begin{aligned} 20i_a &= 80i_b \qquad i_g = i_a + i_b = 5i_b \\ i_a &= 4i_b \\ 50 &= 4i_g + 80i_b = 20i_b + 80i_b = 100i_b \\ i_b &= 0.5\text{ A, therefore, } i_a = 2\text{ A} \quad \text{and} \quad i_g = 2.5\text{ A} \end{aligned}$$

[b] $i_b = 0.5$ A

[c] $v_o = 80i_b = 40$ V

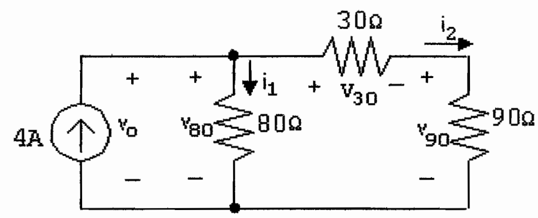
[d]
$$\begin{aligned} p_{4\Omega} &= i_g^2(4) = 6.25(4) = 25\text{ W} \\ p_{20\Omega} &= i_a^2(20) = (4)(20) = 80\text{ W} \\ p_{80\Omega} &= i_b^2(80) = 0.25(80) = 20\text{ W} \end{aligned}$$

[e] $p_{50\text{V}}$ (delivered) $= 50i_g = 125$ W
Check:

$\sum P_{\text{dis}} = 25 + 80 + 20 = 125\text{W}$

$\sum P_{\text{del}} = 125\text{W}$

P 2.19



[a] Write a KCL equation at the top node:

$-4 + i_1 + i_2 = 0$ so $i_1 + i_2 = 4$

Write a KVL equation around the right loop:

$-v_{80} + v_{30} + v_{90} = 0$

From Ohm's law,

$v_{80} = 80i_1, \qquad v_{30} = 30i_2, \qquad v_{90} = 90i_2$

Substituting,

$-80i_1 + 30i_2 + 90i_2 = 0$ so $-80i_1 + 120i_2 = 0$

Solving the two equations for i_1 and i_2 simultaneously,

$i_1 = 2.4\,\text{A}$ and $i_2 = 1.6\,\text{A}$

[b] Write a KVL equation clockwise around the left loop:

$-v_o + v_{80} = 0$ but $v_{80} = 80i_1 = 80(2.4) = 192\,\text{A}$

So $v_o = v_{80} = 192\,\text{V}$

[c] Calculate power using $p = vi$ for the source and $p = Ri^2$ for the resistors:

$p_{\text{source}} = -v_o(4) = -(192)(4) = -768\,\text{W}$

$p_{80\Omega} = 2.4^2(80) = 460.8\,\text{W}$

$p_{30\Omega} = 1.6^2(30) = 76.8\,\text{W}$

$p_{90\Omega} = 1.6^2(90) = 230.4\,\text{W}$

$\sum P_{\text{dev}} = 768\,\text{W} \qquad \sum P_{\text{abs}} = 460.8 + 76.8 + 230.4 = 768\,\text{W}$

P 2.20 [a] Use KVL for the right loop to calculate the voltage drop across the right-hand branch v_o. This is also the voltage drop across the middle branch, so once v_o is known, use Ohm's law to calculate i_o:

$$v_o = 1000i_a + 4000i_a + 3000i_a = 8000i_a = 8000(0.002) = 16 \text{ V}$$

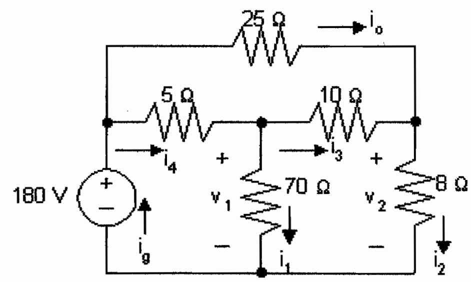
$$16 = 2000i_o$$

$$i_o = \frac{16}{2000} = 8 \text{ mA}$$

[b] KCL at the top node: $i_g = i_a + i_o = 0.002 + 0.008 = 0.010 \text{ A} = 10 \text{ mA}.$

[c] The voltage drop across the source is v_0, seen by writing a KVL equation for the left loop. Thus,
$p_g = -v_o i_g = -(16)(0.01) = -0.160 \text{ W} = -160 \text{ mW}.$
Thus the source delivers 160 mW.

P 2.21 [a]



$$v_2 = 180 - 100 = 80\text{V}$$

$$i_2 = \frac{v_2}{8} = 10\text{A}$$

$$i_3 + 4 = i_2, \qquad i_3 = 10 - 4 = 6\text{A}$$

$$v_1 = 10i_3 + 8i_2 = 10(6) + 8(10) = 140\text{V}$$

$$i_1 = \frac{v_1}{70} = \frac{140}{70} = 2\text{A}$$

Note also that

$$i_4 = i_1 + i_3 = 2 + 6 = 8\text{ A}$$

$$i_g = i_4 + i_o = 8 + 4 = 12\text{ A}$$

[b]

$$p_{5\Omega} = 8^2(5) = 320 \text{ W}$$

$$p_{25\Omega} = (4)^2(25) = 400 \text{ W}$$

$$p_{70\Omega} = 2^2(70) = 280 \text{ W}$$

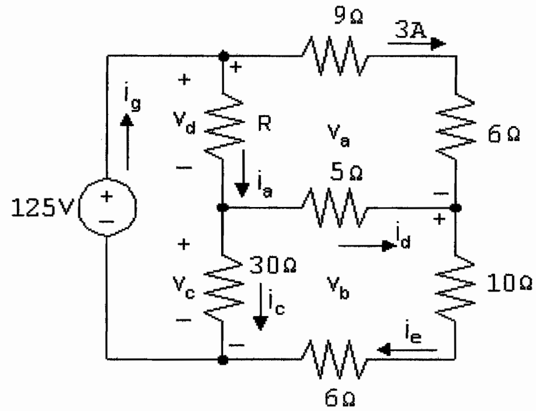
$$p_{10\Omega} = 6^2(10) = 360 \text{ W}$$

$$p_{8\Omega} = 10^2(8) = 800 \text{ W}$$

[c] $\sum P_{\text{dis}} = 320 + 400 + 280 + 360 + 800 = 2160\text{W}$

$P_{\text{dev}} = 180i_g = 180(12) = 2160\text{W}$

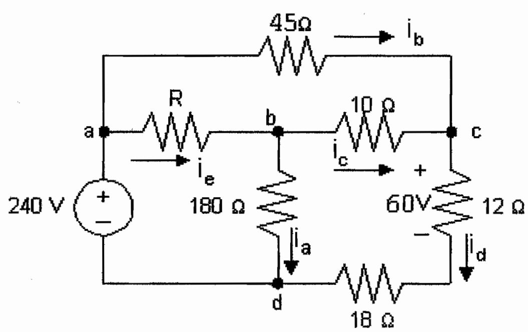
P 2.22 **[a]**



$v_a = (9+6)(3) = 45\,\text{V}$
$-125 + v_a + v_b = 0 \qquad \text{so} \qquad v_b = 125 - v_a = 125 - 45 = 80\,\text{V}$
$i_e = v_b/(10+6) = 80/16 = 5\,\text{A}$
$i_d = i_e - 3 = 5 - 3 = 2\,\text{A}$
$v_c = 5i_d + v_b = 5(2) + 80 = 90\,\text{V}$
$i_c = v_c/30 = 90/30 = 3\,\text{A}$
$v_d = 125 - v_c = 125 - 90 = 35\,\text{V}$
$i_a = i_d + i_c = 2 + 3 = 5\,\text{A}$
$R = v_d/i_a = 35/5 = 7\,\Omega$

[b] $i_g = i_a + 3 = 5 + 3 = 8$ A
p_g (supplied) $= (125)(8) = 1000$ W

P 2.23



$i_d = 60/12 = 5\,\text{A}; \quad$ therefore, $v_{cd} = 60 + 18(5) = 150\,\text{V}$
$-240 + v_{ac} + v_{cd} = 0; \quad$ therefore, $v_{ac} = 240 - 150 = 90\,\text{V}$
$i_b = v_{ac}/45 = 90/45 = 2\,\text{A}; \quad$ therefore, $i_c = i_d - i_b = 5 - 2 = 3\,\text{A}$
$v_{bd} = 10i_c + v_{cd} = 10(3) + 150 = 180\,\text{V};$
therefore, $i_a = v_{bd}/180 = 180/180 = 1\,\text{A}$
$i_e = i_a + i_c = 1 + 3 = 4\,\text{A}$
$-240 + v_{ab} + v_{bd} = 0 \quad$ therefore, $v_{ab} = 240 - 180 = 60\,\text{V}$
$R = v_{ab}/i_e = 60/4 = 15\,\Omega$

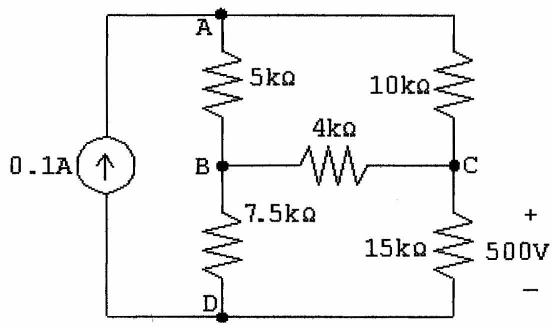
CHECK: $i_g = i_b + i_e = 2 + 4 = 6$ A

$$p_{\text{dev}} = (240)(6) = 1440 \text{ W}$$

$$\sum P_{\text{dis}} = 1^2(180) + 4^2(15) + 3^2(10) + 5^2(12) + 5^2(18) + 2^2(45)$$

$$= 1440 \text{ W (CHECKS)}$$

P 2.24 [a]



$$i_{\text{cd}} = 500/15{,}000 = 33.33\,\text{mA}$$

$$i_{\text{bd}} + i_{\text{cd}} = 0.1 \quad \text{so} \quad i_{\text{bd}} = 0.1 - 0.033 = 66.67\,\text{mA}$$

$$4000 i_{\text{bc}} + 500 - 7500 i_{\text{bd}} = 0 \quad \text{so} \quad i_{\text{bc}} = (500 - 500)/4000 = 0$$

$$i_{\text{ac}} = i_{\text{cd}} - i_{\text{bc}} = 33.33 - 0 = 33.33\,\text{mA}$$

$$0.1 = i_{\text{ab}} + i_{\text{ac}} \quad \text{so} \quad i_{\text{ab}} = 0.1 - 33.33 = 66.67\,\text{mA}$$

Calculate the power dissipated by the resistors using the equation $p_R = Ri_R^2$:

$$p_{5\text{k}\Omega} = (5000)(0.0667)^2 = 22.22\,\text{W} \qquad p_{7.5\text{k}\Omega} = (7500)(0.0667)^2 = 33.33\,\text{W}$$

$$p_{10\text{k}\Omega} = (10{,}000)(0.03333)^2 = 11.11\,\text{W} \qquad p_{15\text{k}\Omega} = (15{,}000)(0.0333)^2 = 16.67\,\text{W}$$

$$p_{4\text{k}\Omega} = (4000)(0)^2 = 0\,\text{W}$$

[b] Calculate the voltage drop across the current source:

$$v_{\text{ad}} = 5000 i_{\text{ab}} + 7500 i_{\text{bd}} = 5000(0.0667) + 7500(0.0667) = 833.33\,\text{V}$$

Now that we have both the voltage and the current for the source, we can calculate the power supplied by the source:

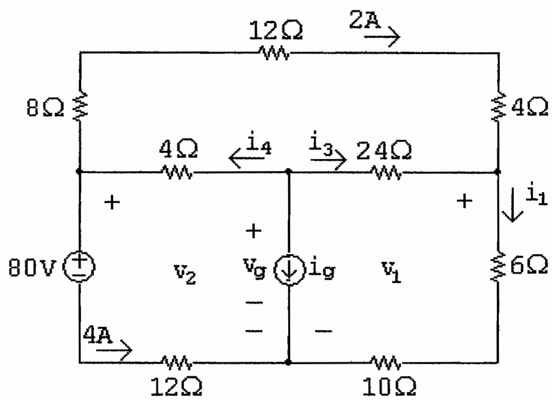
$$p_g = -833.33(0.1) = -83.33\,\text{W} \qquad \text{thus} \qquad p_g \text{ (supplied) } = 83.33\,\text{W}$$

[c] $\sum P_{\text{dis}} = 22.22 + 33.33 + 11.11 + 16.67 + 0 = 83.33\,\text{W}$

Therefore,

$$\sum P_{\text{supp}} = \sum P_{\text{dis}}$$

P 2.25 [a]



$v_2 = 80 + 4(12) = 128\,\text{V}; \qquad v_1 = 128 - (8 + 12 + 4)(2) = 80\,\text{V}$

$i_1 = \frac{v_1}{6+10} = \frac{80}{16} = 5\,\text{A}; \qquad i_3 = i_1 - 2 = 5 - 2 = 3\,\text{A}$

$v_g = v_1 + 24i_3 = 80 + 24(3) = 152\,\text{V}$

$i_4 = 2 + 4 = 6\,\text{A}$

$i_g = -i_4 - i_3 = -6 - 3 = -9\,\text{A}$

[b] Calculate power using the formula $p = Ri^2$:

$p_{8\,\Omega} = (8)(2)^2 = 32\,\text{W}; \qquad p_{12\,\Omega} = (12)(2)^2 = 48\,\text{W}$

$p_{4\,\Omega} = (4)(2)^2 = 16\,\text{W}; \qquad p_{4\,\Omega} = (4)(6)^2 = 144\,\text{W}$

$p_{24\,\Omega} = (24)(3)^2 = 216\,\text{W}; \qquad p_{6\,\Omega} = (6)(5)^2 = 150\,\text{W}$

$p_{10\,\Omega} = (10)(5)^2 = 250\,\text{W}; \qquad p_{12\,\Omega} = (12)(4)^2 = 192\,\text{W}$

[c] $v_g = 152\,\text{V}$

[d] Sum the power dissipated by the resistors:

$\sum p_{\text{diss}} = 32 + 48 + 16 + 144 + 216 + 150 + 250 + 192 = 1048\,\text{W}$

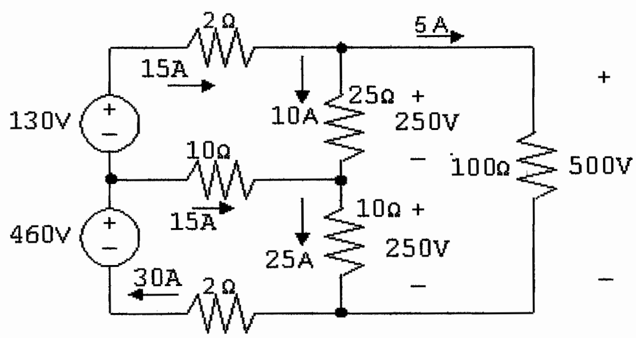
The power associated with the sources is

$p_{\text{volt–source}} = (80)(4) = 320\,\text{W}$

$p_{\text{curr–source}} = -v_g i_g = -(152)(9) = -1368\,\text{W}$

Thus the total power dissipated is $1048 + 320 = 1368$ W and the total power developed is 1368 W, so the power balances.

P 2.26 [a] Start by calculating the voltage drops due to the currents i_1 and i_2. Then use KVL to calculate the voltage drop across and $100\,\Omega$ resistor, and Ohm's law to find the current in the $100\,\Omega$ resistor. Finally, KCL at each of the middle three nodes yields the currents in the two sources and the current in the middle $10\,\Omega$ resistor. These calculations are summarized in the figure below:



$$p_{130} = -(130)(15) = -1950 \text{ W}$$

$$p_{460} = -(460)(30) = -13{,}800 \text{ W}$$

[b]

$$\sum P_{\text{dis}} = (15)^2(2) + (15)^2(10) + (30)^2(2) + (10)^2(25) + (25)^2(10) + (5)^2(100)$$
$$= 450 + 2250 + 1800 + 2500 + 6250 + 2500 = 15{,}750 \text{ W}$$

$$\sum P_{\text{sup}} = 1950 + 13{,}800 = 15{,}750 \text{ W}$$

Therefore, $\sum P_{\text{dis}} = \sum P_{\text{sup}} = 15{,}750$ W

P 2.27 $i_E - i_B - i_C = 0$

$i_C = \beta i_B$ therefore $i_E = (1+\beta)i_B$

$i_2 = -i_B + i_1$

$$V_o + i_E R_E - (i_1 - i_B)R_2 = 0$$

$$-i_1 R_1 + V_{CC} - (i_1 - i_B)R_2 = 0 \quad \text{or} \quad i_1 = \frac{V_{CC} + i_B R_2}{R_1 + R_2}$$

$$V_o + i_E R_E + i_B R_2 - \frac{V_{CC} + i_B R_2}{R_1 + R_2} R_2 = 0$$

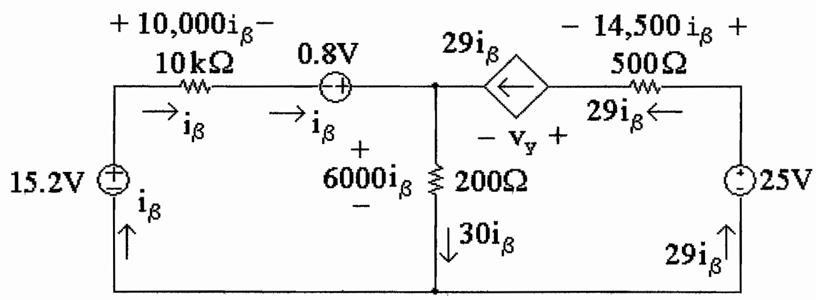
Now replace i_E by $(1+\beta)i_B$ and solve for i_B. Thus

$$i_B = \frac{[V_{CC}R_2/(R_1 + R_2)] - V_o}{(1+\beta)R_E + R_1R_2/(R_1 + R_2)}$$

P 2.28 First note that we know the current through all elements in the circuit except the 200 Ω resistor (the current in the three elements to the left of the 200 Ω resistor is i_β; the current in the three elements to the right of the 200 Ω resistor is $29i_\beta$). To find the current in the 200 Ω resistor, write a KCL equation at the top node:

$$i_\beta + 29i_\beta = i_{200\,\Omega} = 30i_\beta$$

We can then use Ohm's law to find the voltages across each resistor in terms of i_β. The results are shown in the figure below:



[a] To find i_β, write a KVL equation around the left-hand loop, summing voltages in a clockwise direction starting below the 15.2V source:

$$-15.2\,\text{V} + 10{,}000i_1 - 0.8\,\text{V} + 6000i_\beta = 0$$

Solving for i_β

$$10{,}000i_\beta + 6000i_\beta = 16\,\text{V} \qquad \text{so} \qquad 16{,}000i_\beta = 16\,\text{V}$$

Thus,

$$i_\beta = \frac{16}{16{,}000} = 1\,\text{mA}$$

Now that we have the value of i_β, we can calculate the voltage for each component except the dependent source. Then we can write a KVL equation for the right-hand loop to find the voltage v_y of the dependent source. Sum the voltages in the clockwise direction, starting to the left of the dependent source:

$$-v_y - 14{,}500i_\beta + 25\,\text{V} - 6000i_\beta = 0$$

Thus,

$$v_y = 25\,\text{V} - 20{,}500i_\beta = 25\,\text{V} - 20{,}500(10^{-3}) = 25\,\text{V} - 20.5\,\text{V} = 4.5\,\text{V}$$

[b] We now know the values of voltage and current for every circuit element. Let's construct a power table:

Element	**Current (mA)**	**Voltage (V)**	**Power Equation**	**Power (mW)**
15.2 V	1	15.2	$p = -vi$	−15.2
10 kΩ	1	10	$p = Ri^2$	10
0.8 V	1	0.8	$p = -vi$	−0.8
200 Ω	30	6	$p = Ri^2$	180
Dep. source	29	4.5	$p = vi$	130.5
500 Ω	29	14.5	$p = Ri^2$	420.5
25 V	29	25	$p = -vi$	−725

The total power generated in the circuit is the sum of the negative power values in the power table:

$$-15.2\,\text{mW} + -0.8\,\text{mW} + -725\,\text{mW} = -741\,\text{mW}$$

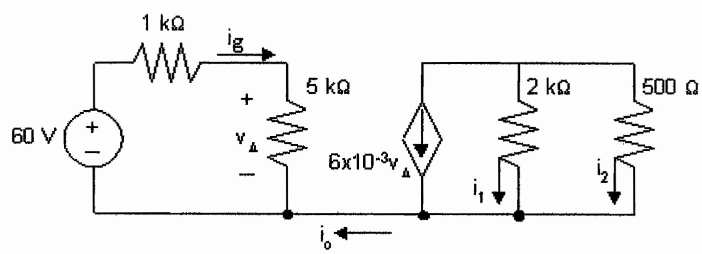
Thus, the total power generated in the circuit is 741 mW. The total power absorbed in the circuit is the sum of the positive power values in the power table:

$$10\,\text{mW} + 180\,\text{mW} + 130.5\,\text{mW} + 420.5\,\text{mW} = 741\,\text{mW}$$

Thus, the total power absorbed in the circuit is 741 mW and the power in the circuit balances.

P 2.29 [a] $i_o = 0$ because no current can exist in a single conductor connecting two parts of a circuit.

[b]



$$60 = 6000i_g \qquad i_g = 10 \text{ mA}$$

$$v_\Delta = 5000i_g = 50\text{V} \qquad 6 \times 10^{-3}v_\Delta = 300 \text{ mA}$$

$$2000i_1 = 500i_2, \text{ so } i_1 + 4i_1 = -300 \text{ mA; therefore, } i_1 = -60 \text{ mA}$$

[c] $300 - 60 + i_2 = 0$, so $i_2 = -240$ mA.

P 2.30 $50i_2 + \frac{0.250}{50} + \frac{0.250}{12.5} = 0; \quad i_2 = -0.5 \text{ mA}$

$$v_1 = 100i_2 = -50 \text{ mV}$$

$$20i_1 + \frac{(-0.050)}{25} + (-0.0005) = 0; \quad i_1 = 125\,\mu\text{A}$$

$$v_g = 10i_1 + 40i_1 = 50i_1$$

Therefore, $v_g = 6.25$ mV.

P 2.31 [a] $-50 - 20i_\sigma + 18i_\Delta = 0$

$-18i_\Delta + 5i_\sigma + 40i_\sigma = 0$ so $18i_\Delta = 45i_\sigma$

Therefore, $-50 - 20i_\sigma + 45i_\sigma = 0,$ so $i_\sigma = 2\,\text{A}$

$18i_\Delta = 45i_\sigma = 90;$ so $i_\Delta = 5\,\text{A}$

$v_o = 40i_\sigma = 80\,\text{V}$

[b] i_g = current out of the positive terminal of the 50 V source
v_{d} = voltage drop across the $8i_\Delta$ source

$i_g = i_\Delta + i_\sigma + 8i_\Delta = 9i_\Delta + i_\sigma = 47\,\text{A}$

$v_d = 80 - 20 = 60\,\text{V}$

$$\begin{aligned} \sum P_{\text{gen}} &= 50i_g + 20i_\sigma i_g = 50(47) + 20(2)(47) = 4230 \text{ W} \\ \sum P_{\text{diss}} &= 18i_\Delta^2 + 5i_\sigma(i_g - i_\Delta) + 40i_\sigma^2 + 8i_\Delta v_d + 8i_\Delta(20) \\ &= (18)(25) + 10(47 - 5) + 4(40) + 40(60) + 40(20) \\ &= 4230 \text{ W; Therefore,} \\ \sum P_{\text{gen}} &= \sum P_{\text{diss}} = 4230 \text{ W} \end{aligned}$$

P 2.32 Here is Equation 2.25:

$$i_{\text{B}} = \frac{(V_{\text{CC}}R_2)/(R_1 + R_2) - V_0}{(R_1R_2)/(R_1 + R_2) + (1 + \beta)R_{\text{E}}}$$

$$\frac{V_{CC}R_2}{R_1 + R_2} = \frac{(15)(80)}{100} = 12\text{V}$$

$$\frac{R_1R_2}{R_1 + R_2} = \frac{(20)(80)}{100} = 16 \text{ k}\Omega$$

$$i_B = \frac{12 - 0.2}{16 + 40(0.1)} = \frac{11.8}{20} = 0.59 \text{ mA}$$

$$i_C = \beta i_B = (39)(0.59) = 23.01 \text{ mA}$$

$$i_E = i_C + i_B = 23 + 0.59 = 23.6 \text{ mA}$$

$$v_{3d} = (23.6)(0.1) = 2.36\text{V}$$

$$v_{bd} = V_o + v_{3d} = 2.56\text{V}$$

$$i_2 = \frac{v_{bd}}{R_2} = \frac{2.56}{80} \times 10^{-3} = 32\,\mu\text{A}$$

$$i_1 = i_2 + i_B = 32 + 590 = 622\,\mu\text{A}$$

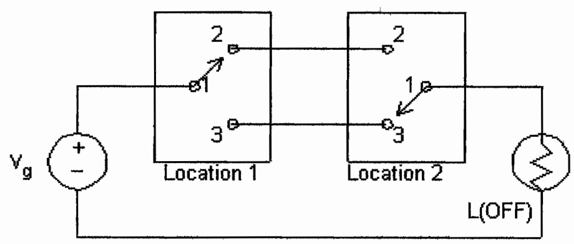
$$v_{\text{ab}} = 20(0.622) = 12.44\text{V}$$

$$i_{CC} = i_C + i_1 = 23.01 + 0.622 = 23.632 \text{ mA}$$

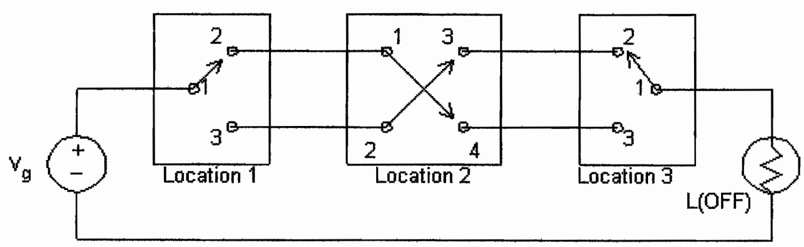
$$v_{13} + 23.01(0.5) + 2.36 = 15$$

$$v_{13} = 1.135\text{V}$$

P 2.33 [a]



[b]

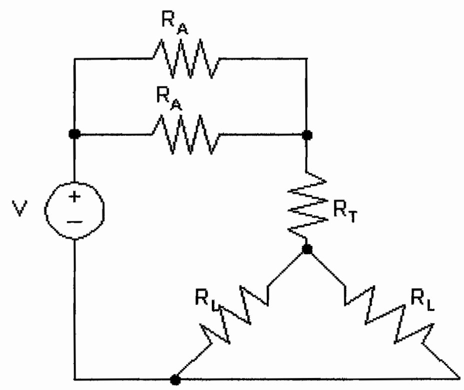


P 2.34 From the simplified circuit model, using Ohm's law and KVL:

$400i + 50i + 200i - 250 = 0$ so $i = 250/650 = 385$ mA

This current is nearly enough to stop the heart, according to Table 2.1, so a warning sign should be posted at the 250 V source.

P 2.35



P 2.36 [a] $p = i^2R$

$$p_{\text{arm}} = \left(\frac{250}{650}\right)^2 (400) = 59.17\,\text{W}$$

$$p_{\text{leg}} = \left(\frac{250}{650}\right)^2 (200) = 29.59\,\text{W}$$

$$p_{\text{trunk}} = \left(\frac{250}{650}\right)^2 (50) = 7.40\,\text{W}$$

[b] $$\left(\frac{dT}{dt}\right)_{\text{arm}} = \frac{2.39 \times 10^{-4} p_{\text{arm}}}{4} = 35.36 \times 10^{-4}\,{}^\circ\text{C/s}$$

$$t_{\text{arm}} = \frac{5}{35.36} \times 10^4 = 1414.23 \text{ s or } 23.57 \text{ min}$$

$$\left(\frac{dT}{dt}\right)_{\text{leg}} = \frac{2.39 \times 10^{-4}}{10} P_{\text{leg}} = 7.07 \times 10^{-4\circ}\,\text{C/s}$$

$$t_{\text{leg}} = \frac{5 \times 10^4}{7.07} = 7{,}071.13 \text{ s or } 117.85 \text{ min}$$

$$\left(\frac{dT}{dt}\right)_{\text{trunk}} = \frac{2.39 \times 10^{-4}(7.4)}{25} = 0.71 \times 10^{-4}\,{}^\circ\text{C/s}$$

$$t_{\text{trunk}} = \frac{5 \times 10^4}{0.71} = 70{,}422.54 \text{ s or } 1{,}173.71 \text{ min}$$

[c] They are all much greater than a few minutes.

P 2.37 [a] $R_{\text{arms}} = 400 + 400 = 800\Omega$

$$i_{\text{letgo}} = 50 \text{ mA (minimum)}$$

$$v_{\text{min}} = (800)(50) \times 10^{-3} = 40\,\text{V}$$

[b] No, $12/800 = 15$ mA. Note this current is sufficient to give a perceptible shock.

P 2.38 $R_{\text{space}} = 1\ \text{M}\Omega$

$i_{\text{space}} = 3$ mA

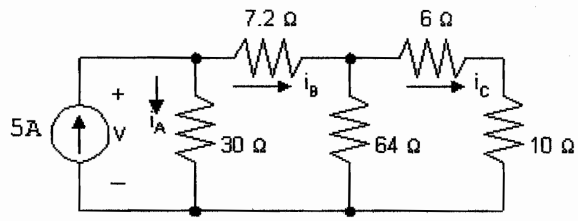
$v = i_{\text{space}} R_{\text{space}} = 3000\,\text{V}.$

3

Simple Resistive Circuits

Assessment Problems

AP 3.1



Start from the right hand side of the circuit and make series and parallel combinations of the resistors until one equivalent resistor remains. Begin by combining the $6\,\Omega$ resistor and the $10\,\Omega$ resistor in series:

$6\,\Omega + 10\,\Omega = 16\,\Omega$

Now combine this $16\,\Omega$ resistor in parallel with the $64\,\Omega$ resistor:

$$16\,\Omega \| 64\,\Omega = \frac{(16)(64)}{16+64} = \frac{1024}{80} = 12.8\,\Omega$$

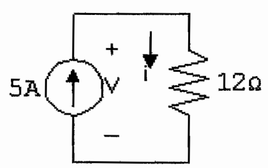
This equivalent $12.8\,\Omega$ resistor is in series with the $7.2\,\Omega$ resistor:

$12.8\,\Omega + 7.2\,\Omega = 20\,\Omega$

Finally, this equivalent $20\,\Omega$ resistor is in parallel with the $30\,\Omega$ resistor:

$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = \frac{600}{50} = 12\,\Omega$$

Thus, the simplified circuit is as shown:



[a] With the simplified circuit we can use Ohm's law to find the voltage across both the current source and the $12\,\Omega$ equivalent resistor:

$$v = (12\,\Omega)(5\text{ A}) = 60\text{ V}$$

[b] Now that we know the value of the voltage drop across the current source, we can use the formula $p = -vi$ to find the power associated with the source:

$$p = -(60\text{ V})(5\text{ A}) = -300\text{ W}$$

Thus, the source delivers 300 W of power to the circuit.

[c] We now can return to the original circuit, shown in the first figure. In this circuit, $v = 60$ V, as calculated in part (a). This is also the voltage drop across the $30\,\Omega$ resistor, so we can use Ohm's law to calculate the current through this resistor:

$$i_A = \frac{60\text{ V}}{30\,\Omega} = 2\text{ A}$$

Now write a KCL equation at the upper left node to find the current i_B:

$$-5\text{ A} + i_A + i_B = 0 \qquad \text{so} \qquad i_B = 5\text{ A} - i_A = 5\text{ A} - 2\text{ A} = 3\text{ A}$$

Next, write a KVL equation around the outer loop of the circuit, using Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:

$$-v + 7.2i_B + 6i_C + 10i_C = 0$$

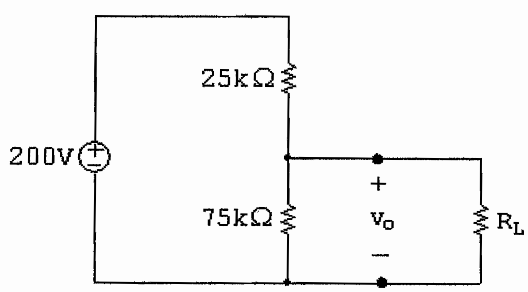
So $\quad 16i_C = v - 7.2i_B = 60\text{ V} - (7.2)(3) = 38.4\text{ V}$

Thus $\quad i_C = \dfrac{38.4}{16} = 2.4\text{ A}$

Now that we have the current through the $10\,\Omega$ resistor we can use the formula $p = Ri^2$ to find the power:

$$p_{10\,\Omega} = (10)(2.4)^2 = 57.6\text{ W}$$

AP 3.2



[a] We can use voltage division to calculate the voltage v_o across the 75 kΩ resistor:

$$v_o(\text{no load}) = \frac{75{,}000}{75{,}000 + 25{,}000}(200\text{ V}) = 150\text{ V}$$

[b] When we have a load resistance of 150 kΩ then the voltage v_o is across the parallel combination of the 75 kΩ resistor and the 150 kΩ resistor. First, calculate the equivalent resistance of the parallel combination:

$$75\text{ k}\Omega \| 150\text{ k}\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50{,}000\,\Omega = 50\text{ k}\Omega$$

Now use voltage division to find v_o across this equivalent resistance:

$$v_o = \frac{50{,}000}{50{,}000 + 25{,}000}(200\text{ V}) = 133.3\text{ V}$$

[c] If the load terminals are short-circuited, the 75 kΩ resistor is effectively removed from the circuit, leaving only the voltage source and the 25 kΩ resistor. We can calculate the current in the resistor using Ohm's law:

$$i = \frac{200\text{ V}}{25\text{ k}\Omega} = 8\text{ mA}$$

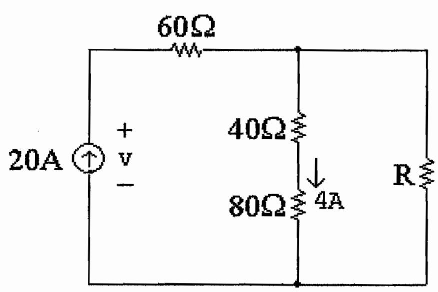
Now we can use the formula $p = Ri^2$ to find the power dissipated in the 25 kΩ resistor:

$$p_{25k} = (25{,}000)(0.008)^2 = 1.6\text{ W}$$

[d] The power dissipated in the 75 kΩ resistor will be maximum at no load since v_o is maximum. In part (a) we determined that the no-load voltage is 150 V, so be can use the formula $p = v^2/R$ to calculate the power:

$$p_{75k}(\text{max}) = \frac{(150)^2}{75{,}000} = 0.3\text{ W}$$

AP 3.3



[a] We will write a current division equation for the current throught the 80Ω resistor and use this equation to solve for R:

$$i_{80\Omega} = \frac{R}{R + 40\,\Omega + 80\,\Omega}(20\text{ A}) = 4\text{ A} \qquad \text{so} \qquad 20R = 4(R + 120)$$

$$\text{Thus} \qquad 16R = 480 \qquad \text{and} \qquad R = \frac{480}{16} = 30\,\Omega$$

[b] With $R = 30\,\Omega$ we can calculate the current through R using current division, and then use this current to find the power dissipated by R, using the formula $p = Ri^2$:

$$i_R = \frac{40+80}{40+80+30}(20 \text{ A}) = 16 \text{ A} \qquad \text{so} \qquad p_R = (30)(16)^2 = 7680 \text{ W}$$

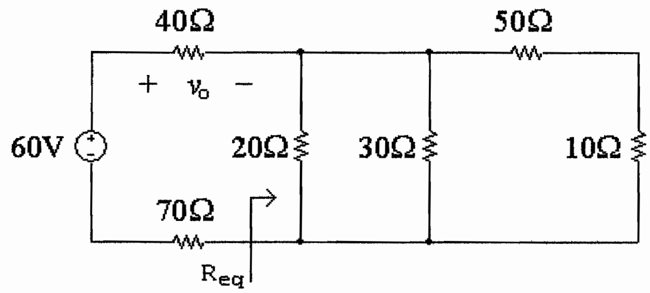
[c] Write a KVL equation around the outer loop to solve for the voltage v, and then use the formula $p = -vi$ to calculate the power delivered by the current source:

$$-v + (60\,\Omega)(20 \text{ A}) + (30\,\Omega)(16 \text{ A}) = 0 \qquad \text{so} \qquad v = 1200 + 480 = 1680 \text{ V}$$

Thus, $\quad p_{\text{source}} = -(1680 \text{ V})(20 \text{ A}) = -33{,}600 \text{ W}$

Thus, the current source generates 33,600 W of power.

AP 3.4



[a] First we need to determine the equivalent resistance to the right of the $40\,\Omega$ and $70\,\Omega$ resistors:

$$R_{\text{eq}} = 20\,\Omega \| 30\,\Omega \| (50\,\Omega + 10\,\Omega) \qquad \text{so} \qquad \frac{1}{R_{\text{eq}}} = \frac{1}{20\,\Omega} + \frac{1}{30\,\Omega} + \frac{1}{60\,\Omega} = \frac{1}{10\,\Omega}$$

Thus, $\quad R_{\text{eq}} = 10\,\Omega$

Now we can use voltage division to find the voltage v_o:

$$v_o = \frac{40}{40+10+70}(60 \text{ V}) = 20 \text{ V}$$

[b] The current through the $40\,\Omega$ resistor can be found using Ohm's law:

$$i_{40\Omega} = \frac{v_o}{40} = \frac{20 \text{ V}}{40\,\Omega} = 0.5 \text{ A}$$

This current flows from left to right through the $40\,\Omega$ resistor. To use current division, we need to find the equivalent resistance of the two parallel branches containing the $20\,\Omega$ resistor and the $50\,\Omega$ and $10\,\Omega$ resistors:

$$20\,\Omega \| (50\,\Omega + 10\,\Omega) = \frac{(20)(60)}{20+60} = 15\,\Omega$$

Now we use current division to find the current in the $30\,\Omega$ branch:

$$i_{30\Omega} = \frac{15}{15+30}(0.5 \text{ A}) = 0.16667 \text{ A} = 166.67 \text{ mA}$$

[c] We can find the power dissipated by the 50 Ω resistor if we can find the current in this resistor. We can use current division to find this current from the current in the 40 Ω resistor, but first we need to calculate the equivalent resistance of the 20 Ω branch and the 30 Ω branch:

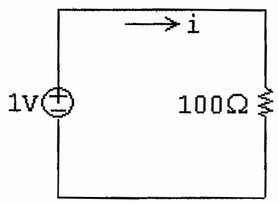
$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = 12\,\Omega$$

Current division gives:

$$i_{50\Omega} = \frac{12}{12+50+10}(0.5 \text{ A}) = 0.08333 \text{ A}$$

Thus, $\quad p_{50\Omega} = (50)(0.08333)^2 = 0.34722 \text{ W} = 347.22 \text{ mW}$

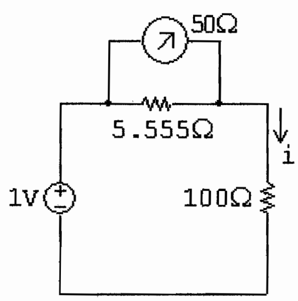
AP 3.5 [a]



We can find the current i using Ohm's law:

$$i = \frac{1 \text{ V}}{100\,\Omega} = 0.01 \text{ A} = 10 \text{ mA}$$

[b]

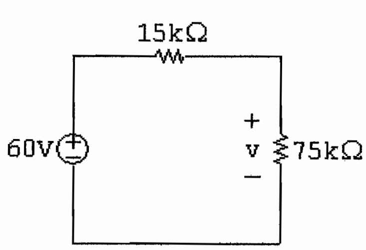


$$R_m = 50\,\Omega \| 5.555\,\Omega = 5\,\Omega$$

We can use the meter resistance to find the current using Ohm's law:

$$i_{\text{meas}} = \frac{1 \text{ V}}{100\,\Omega + 5\,\Omega} = 0.009524 = 9.524 \text{ mA}$$

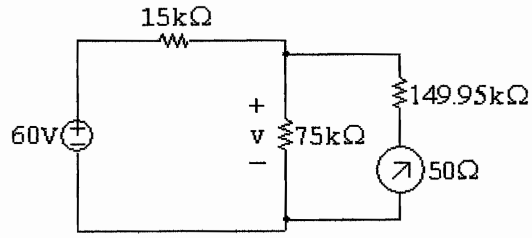
AP 3.6 [a]



Use voltage division to find the voltage v:

$$v = \frac{75{,}000}{75{,}000 + 15{,}000}(60\ \text{V}) = 50\ \text{V}$$

[b]



The meter resistance is a series combination of resistances:

$$R_m = 149{,}950 + 50 = 150{,}000\,\Omega$$

We can use voltage division to find v, but first we must calculate the equivalent resistance of the parallel combination of the 75 kΩ resistor and the voltmeter:

$$75{,}000\,\Omega \| 150{,}000\,\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50\ \text{k}\Omega$$

Thus, $$v_{\text{meas}} = \frac{50{,}000}{50{,}000 + 15{,}000}(60\ \text{V}) = 46.15\ \text{V}$$

AP 3.7 [a] Using the condition for a balanced bridge, the products of the opposite resistors must be equal. Therefore,

$$100R_x = (1000)(150) \qquad \text{so} \qquad R_x = \frac{(1000)(150)}{100} = 1500\,\Omega = 1.5\ \text{k}\Omega$$

[b] When the bridge is balanced, there is no current flowing through the meter, so the meter acts like an open circuit. This places the following branches in parallel: The branch with the voltage source, the branch with the series combination R_1 and R_3 and the branch with the series combination of R_2 and R_x. We can find the current in the latter two branches using Ohm's law:

$$i_{R_1,R_3} = \frac{5\ \text{V}}{100\,\Omega + 150\,\Omega} = 20\ \text{mA}; \qquad i_{R_2,R_x} = \frac{5\ \text{V}}{1000 + 1500} = 2\ \text{mA}$$

We can calculate the power dissipated by each resistor using the formula $p = Ri^2$:

$$p_{100\Omega} = (100\,\Omega)(0.02\ \text{A})^2 = 40\ \text{mW}$$

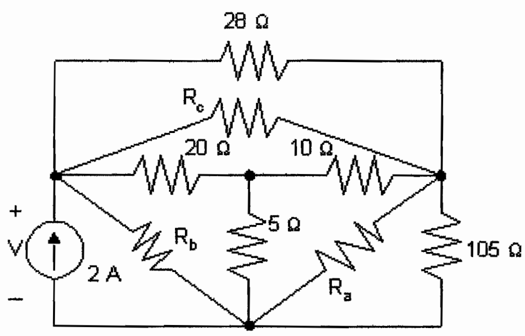
$$p_{150\Omega} = (150\,\Omega)(0.02\ \text{A})^2 = 60\ \text{mW}$$

$$p_{1000\Omega} = (1000\,\Omega)(0.002\ \text{A})^2 = 4\ \text{mW}$$

$$p_{1500\Omega} = (1500\,\Omega)(0.002\ \text{A})^2 = 6\ \text{mW}$$

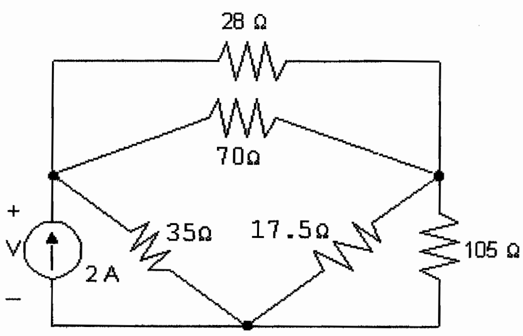
Since none of the power dissipation values exceeds 250 mW, the bridge can be left in the balanced state without exceeding the power-dissipating capacity of the resistors.

AP 3.8 Convert the three Y-connected resistors, 20 Ω, 10 Ω, and 5 Ω to three Δ-connected resistors R_a, R_b, and R_c. To assist you the figure below has both the Y-connected resistors and the Δ-connected resistors



$$R_a = \frac{(5)(10) + (5)(20) + (10)(20)}{20} = 17.5\,\Omega$$
$$R_b = \frac{(5)(10) + (5)(20) + (10)(20)}{10} = 35\,\Omega$$
$$R_c = \frac{(5)(10) + (5)(20) + (10)(20)}{5} = 70\,\Omega$$

The circuit with these new Δ-connected resistors is shown below:



From this circuit we see that the 70 Ω resistor is parallel to the 28 Ω resistor:

$$70\,\Omega \| 28\,\Omega = \frac{(70)(28)}{70 + 28} = 20\,\Omega$$

Also, the 17.5 Ω resistor is parallel to the 105 Ω resistor:

$$17.5\,\Omega \| 105\,\Omega = \frac{(17.5)(105)}{17.5 + 105} = 15\,\Omega$$

Once the parallel combinations are made, we can see that the equivalent $20\,\Omega$ resistor is in series with the equivalent $15\,\Omega$ resistor, giving an equivalent resistance of $20\,\Omega + 15\,\Omega = 35\,\Omega$. Finally, this equivalent $35\,\Omega$ resistor is in parallel with the other $35\,\Omega$ resistor:

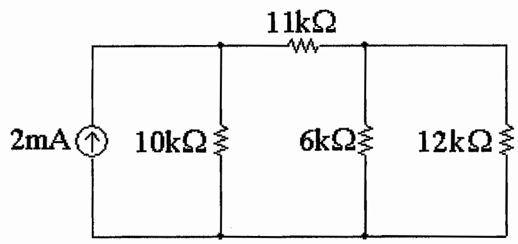
$$35\,\Omega \| 35\,\Omega = \frac{(35)(35)}{35+35} = 17.5\,\Omega$$

Thus, the resistance seen by the 2 A source is $17.5\,\Omega$, and the voltage can be calculated using Ohm's law:

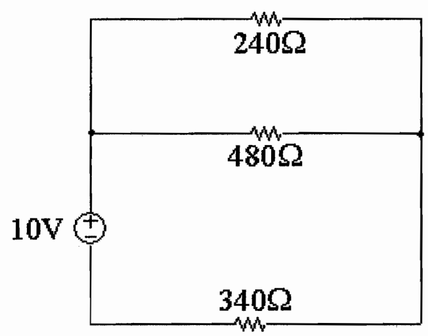
$$v = (17.5\,\Omega)(2\text{ A}) = 35\text{ V}$$

Problems

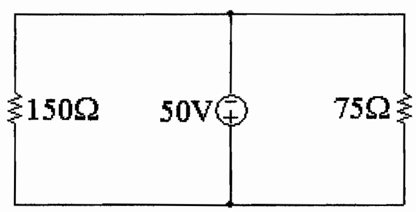
P 3.1 [a] The 3 kΩ and 8 kΩ resistors are in series, as are the 5 kΩ and 7 kΩ resistors. The simplified circuit is shown below:



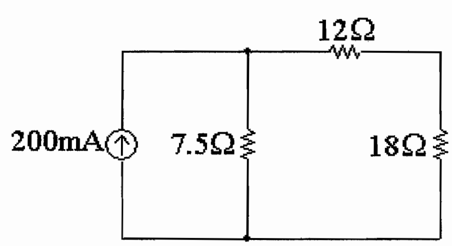
[b] The 180 Ω and 300 Ω resistors are in series, as are the 140 Ω and 200 Ω resistors. The simplified circuit is shown below:



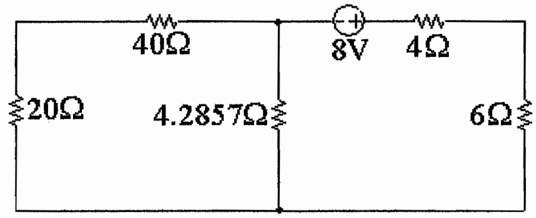
[c] The 40 Ω, 50 Ω, and 60 Ω resistors are in series, as are the 45 Ω and 30 Ω resistors. The simplified circuit is shown below:



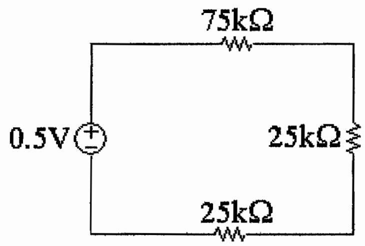
P 3.2 [a] The 12 Ω and 20 Ω resistors are in parallel, as are the 28 Ω and 21 Ω resistors. The simplified circuit is shown below:



[b] The 30 Ω and 5 Ω resistors are in parallel, as are the 9 Ω and 18 Ω resistors. The simplified circuit is shown below:



[c] The 100 kΩ and 300 kΩ resistors are in parallel, as are the 75 kΩ, 50 kΩ, and 150 kΩ resistors. The simplified circuit is shown below:



P 3.3 [a] $p_{4\Omega} = i_s^2 4 = (12)^2 4 = 576\text{ W}$ $\qquad p_{18\Omega} = (4)^2 18 = 288\text{ W}$

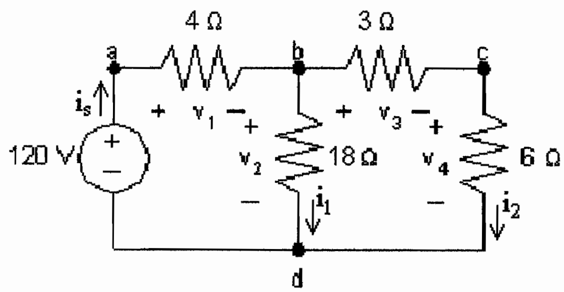
$p_{3\Omega} = (8)^2 3 = 192\text{ W}$ $\qquad p_{6\Omega} = (8)^2 6 = 384\text{ W}$

[b] $p_{120\text{V}}(\text{delivered}) = 120 i_s = 120(12) = 1440\text{ W}$

[c] $p_{\text{diss}} = 576 + 288 + 192 + 384 = 1440\text{ W}$

P 3.4 [a] From Ex. 3-1: $i_1 = 4$ A, $\quad i_2 = 8$ A, $\quad i_s = 12$ A

at node b: $-12 + 4 + 8 = 0$, $\quad$ at node d: $12 - 4 - 8 = 0$



[b] $v_1 = 4 i_s = 48\text{ V}$ $\qquad v_3 = 3 i_2 = 24\text{ V}$

$v_2 = 18 i_1 = 72\text{ V}$ $\qquad v_4 = 6 i_2 = 48\text{ V}$

loop abda: $-120 + 48 + 72 = 0$,
loop bcdb: $-72 + 24 + 48 = 0$,
loop abcda: $-120 + 48 + 24 + 48 = 0$

P 3.5 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[a] $R_{eq} = \{[(5\text{ k} + 7\text{ k})\|6\text{ k}] + 3\text{ k} + 8\text{ k}\}\|10\text{ k} = [(12\text{ k}\|6\text{ k}) + 11\text{ k}]\|10\text{ k}$

$$= (4\text{ k} + 11\text{ k})\|10\text{ k} = 15\text{ k}\|10\text{ k} = 6\text{ k}\Omega$$

[b] $R_{eq} = [240\|(180 + 300)] + 140 + 200 = (240\|480) + 340 = 160 + 340 = 500\,\Omega$

[c] $R_{eq} = (40 + 50 + 60)\|(30 + 45) = 150\|75 = 50\,\Omega$

P 3.6 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[a] $R_{eq} = 12\|20\|[18 + (28\|21)] = 12\|20\|(18 + 12) = 12\|20\|30 = 6\,\Omega$

[b] $R_{eq} = 4 + (9\|18) + [5\|30\|(20 + 40)] = 4 + 6 + (5\|30\|60) = 4 + 6 + 4 = 14\,\Omega$

[c] $R_{eq} = (100\text{ k}\|300\text{ k}) + (75\text{ k}\|50\text{ k}\|150\text{ k}) + 25\text{ k} = 75\text{ k} + 25\text{ k} + 25\text{ k} = 125\text{ k}\Omega$

P 3.7 [a] $12\,\Omega\|24\,\Omega = 8\,\Omega$ Therefore, $R_{ab} = 8 + 2 + 6 = 16\,\Omega$

[b] $$\frac{1}{R_{eq}} = \frac{1}{24\text{ k}\Omega} + \frac{1}{30\text{ k}\Omega} + \frac{1}{20\text{ k}\Omega} = \frac{15}{120\text{ k}\Omega} = \frac{1}{8\text{ k}\Omega}$$

$$R_{eq} = 8\text{ k}\Omega; \qquad R_{eq} + 7 = 15\text{ k}\Omega$$

$$\frac{1}{R_{ab}} = \frac{1}{15\text{ k}\Omega} + \frac{1}{30\text{ k}\Omega} + \frac{1}{15\text{ k}\Omega} = \frac{5}{30\text{ k}\Omega} = \frac{1}{6\text{ k}\Omega}$$

$$R_{ab} = 6\text{ k}\Omega$$

P 3.8 [a] $60\|20 = 1200/80 = 15\,\Omega$ $\qquad 12\|24 = 288/36 = 8\,\Omega$

$15 + 8 + 7 = 30\,\Omega$ $\qquad 30\|120 = 3600/150 = 24\,\Omega$

$R_{ab} = 15 + 24 + 25 = 64\,\Omega$

[b] $35 + 40 = 75\,\Omega$ $\qquad 75\|50 = 3750/125 = 30\,\Omega$

$30 + 20 = 50\,\Omega$ $\qquad 50\|75 = 3750/125 = 30\,\Omega$

$30 + 10 = 40\,\Omega$ $\qquad 40\|60 + 9\|18 = 24 + 6 = 30\,\Omega$

$30\|30 = 15\,\Omega$ $\qquad R_{ab} = 10 + 15 + 5 = 30\,\Omega$

[c] $50 + 30 = 80\,\Omega$ $\qquad 80\|20 = 16\,\Omega$

$16 + 14 = 30\,\Omega$ $\qquad 30 + 24 = 54\,\Omega$

$54\|27 = 18\,\Omega$ $\qquad 18 + 12 = 30\,\Omega$

$30\|30 = 15\,\Omega$ $\qquad R_{ab} = 3 + 15 + 2 = 20\,\Omega$

P 3.9 [a] For circuit (a)

$$R_{ab} = 15\|(18 + 48\|16) = 10\,\Omega$$

For circuit (b)

$$5\|10\|15\|10\|(12 + 18) = 2\,\Omega$$

$$16\|(14 + 2) = 8\,\Omega$$

$$R_{ab} = 4 + 8 + 12 = 24\,\Omega$$

For circuit (c)

$$144\|(4 + 12) = 14.4\,\Omega$$

$$14.4 + 5.6 = 20\,\Omega$$

$$20\|12 = 7.5\,\Omega$$

$$7.5 + 2.5 = 10\,\Omega$$

$$10\|15 = 6\,\Omega$$

$$14 + 6 + 10 = 30\,\Omega$$

$$R_{ab} = 30\|60 = 20\,\Omega$$

[b] $P_a = \dfrac{20^2}{10} = 40 \text{ W}$

$$P_b = \frac{144^2}{27} = 768 \text{ W}$$

$$P_c = 5^2(20) = 500 \text{ W}$$

P 3.10 $R_{eq} = 6\|30\|20 = 4\,\Omega$

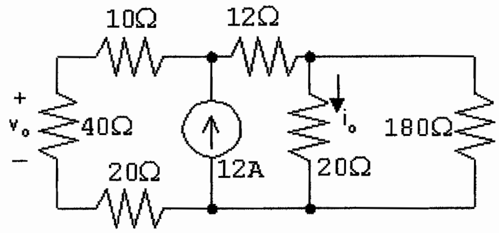
$$v_{30\text{A}} = v_{4\Omega} = (30 \text{ A})(4\,\Omega) = 120 \text{ V}$$

Therefore, since the three original resistors are in parallel with the current source:

$$v_{30\Omega} = v_{30\text{A}} = 120 \text{ V}$$

Thus, $p_{30\Omega} = \dfrac{v_{30\Omega}^2}{30} = \dfrac{120^2}{30} = 480 \text{ W}$

P 3.11 [a]



$$R_{eq} = (10 + 40 + 20)\|[12 + (20\|180)] = 70\|30 = 21\,\Omega$$

$$v_{12A} = 12(21) = 252 \text{ V}$$

$$v_o = v_{40\Omega} = \frac{40}{10 + 40 + 20}(252) = 144 \text{ V}$$

$$v_{20\Omega} = \frac{20\|180}{12 + (20\|180)}(252) = \frac{18}{30}(252) = 151.2 \text{ V}$$

$$i_o = \frac{151.2}{20} = 7.56 \text{ A}$$

[b] $p_{12\Omega} = (252/30)^2(12) = 846.72$ W

[c] $p_{12A} = -(252)(12) = -3024$ W

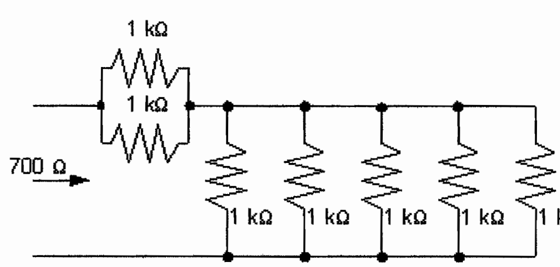
Thus the power developed by the current source is 3024 W.

P 3.12 [a] $R_{eq} = R\|R = \dfrac{R^2}{2R} = \dfrac{R}{2}$

[b]
$$\begin{aligned} R_{eq} &= R\|R\|R\|\cdots\|R \qquad (n\ R\text{'s}) \\ &= R\|\frac{R}{n-1} \\ &= \frac{R^2/(n-1)}{R + R/(n-1)} = \frac{R^2}{nR} = \frac{R}{n} \end{aligned}$$

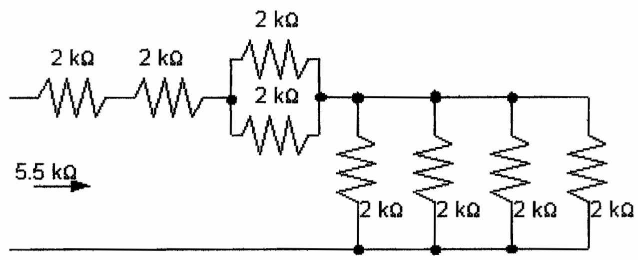
[c] One solution:

$$\begin{aligned} 700\,\Omega &= 200\,\Omega + 500\,\Omega \\ &= 1000/5 + 1000/2 \\ &= 1\text{ k}\Omega\|1\text{ k}\Omega\|1\text{ k}\Omega\|1\text{ k}\Omega\|1\text{ k}\Omega + 1\text{ k}\Omega\|1\text{ k}\Omega \end{aligned}$$



[d] One solution:

$$\begin{aligned} 5.5\ \text{k}\Omega &= 5\ \text{k}\Omega + 0.5\ \text{k}\Omega \\ &= 2\ \text{k}\Omega + 2\ \text{k}\Omega + 1\ \text{k}\Omega + 0.5\ \text{k}\Omega \\ &= 2\ \text{k}\Omega + 2\ \text{k}\Omega + \frac{2\ \text{k}\Omega}{2} + \frac{2\ \text{k}\Omega}{4} \\ &= 2\ \text{k}\Omega + 2\ \text{k}\Omega + 2\ \text{k}\Omega \| 2\ \text{k}\Omega + 2\ \text{k}\Omega \| 2\ \text{k}\Omega \| 2\ \text{k}\Omega \| 2\ \text{k}\Omega \end{aligned}$$



P 3.13 [a] $v_o = \dfrac{160(3300)}{(4700 + 3300)} = 66\ \text{V}$

[b] $i = 160/8000 = 20\ \text{mA}$

$$P_{R_1} = (400 \times 10^{-6})(4.7 \times 10^3) = 1.88\ \text{W}$$

$$P_{R_2} = (400 \times 10^{-6})(3.3 \times 10^3) = 1.32\ \text{W}$$

[c] Since R_1 and R_2 carry the same current and $R_1 > R_2$ to satisfy the voltage requirement, first pick R_1 to meet the 0.5 W specification

$$i_{R_1} = \frac{160 - 66}{R_1}, \qquad \text{Therefore,} \left(\frac{94}{R_1}\right)^2 R_1 \leq 0.5$$

$$\text{Thus, } R_1 \geq \frac{94^2}{0.5} \qquad \text{or} \qquad R_1 \geq 17{,}672\,\Omega$$

Now use the voltage specification:

$$\frac{R_2}{R_2 + 17{,}672}(160) = 66$$

Thus, $R_2 = 12{,}408\,\Omega$

P 3.14 $4 = \dfrac{20R_2}{R_2 + 40} \qquad \text{so} \qquad R_2 = 10\,\Omega$

$$3 = \frac{20R_e}{40 + R_e} \qquad \text{so} \qquad R_e = \frac{120}{17}\,\Omega$$

$$\text{Thus,} \quad \frac{120}{17} = \frac{10R_L}{10 + R_L} \qquad \text{so} \qquad R_L = 24\,\Omega$$

P 3.15 [**a**] $v_o = \frac{100R_2}{R_1 + R_2} = 20 \quad \text{so} \quad R_1 = 4R_2$

Let $R_e = R_2 \| R_L = \frac{R_2 R_L}{R_2 + R_L}$

$$v_o = \frac{100R_e}{R_1 + R_e} = 16 \quad \text{so} \quad R_1 = 5.25R_e$$

Then, $4R_2 = 5.25R_e = \frac{5.25(48R_2)}{48 + R_2}$

Thus, $R_2 = 15\ \text{k}\Omega$ and $R_1 = 4(15\ \text{k}) = 60\ \text{k}\Omega$

[**b**] The resistor that must dissipate the most power is R_1, as it has the largest resistance and carries the same current as the parallel combination of R_2 and the load resistor. The power dissipated in R_1 will be maximum when the voltage across R_1 is maximum. This will occur when the voltage divider has a resistive load. Thus,

$$v_{R_1} = 100 - 16 = 84\ \text{V}$$

$$p_{R_1} = \frac{84^2}{60\ \text{k}} = 117.6\ \text{m W}$$

Thus the minimum power rating for all resistors should be 1/8 W.

P 3.16 Refer to the solution to Problem 3.15. The voltage divider will reach the maximum power it can safely dissipate when the power dissipated in R_1 equals 0.15 W. Thus,

$$\frac{v_{R_1}^2}{60\ \text{k}} = 0.15 \quad \text{so} \quad v_{R_1} = 94.87\ \text{V}$$

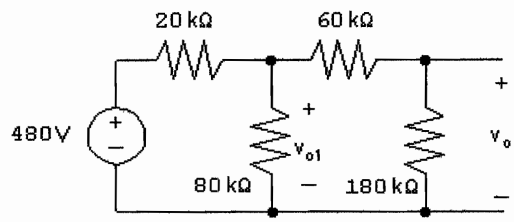
$$v_o = 100 - 94.87 = 5.13\ \text{V}$$

So, $\frac{100R_e}{60\ \text{k} + R_e} = 5.13$ and $R_e = 3.25\ \text{k}\Omega$

Thus, $\frac{(15\ \text{k})R_L}{15\ \text{k} + R_L} = 3250$ and $R_L = 4.14\ \text{k}\Omega$

The minimum value for R_L is thus 4.14 kΩ.

P 3.17 [a]



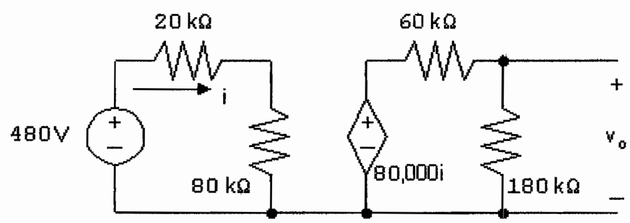
$180\ \text{k}\Omega + 60\ \text{k}\Omega = 240\ \text{k}\Omega$

$80\ \text{k}\Omega \| 240\ \text{k}\Omega = 60\ \text{k}\Omega$

$$v_{o1} = \frac{60{,}000}{(20{,}000 + 60{,}000)}(480) = 360\ \text{V}$$

$$v_o = \frac{180{,}000}{(240{,}000)}(v_{o1}) = 270\ \text{V}$$

[b]



$$i = \frac{480}{100{,}000} = 4.8\ \text{mA}$$

$80{,}000i = 384\ \text{V}$

$$v_o = \frac{180{,}000}{240{,}000}(384) = 288\ \text{V}$$

[c] It removes loading effect of second voltage divider on the first voltage divider. Observe that the open circuit voltage of the first divider is

$$v_{o1}' = \frac{80{,}000}{(100{,}000)}(480) = 384\ \text{V}$$

Now note this is the input voltage to the second voltage divider when the current controlled voltage source is used.

P 3.18 $\frac{(24)^2}{R_1 + R_2 + R_3} = 80,$ Therefore, $R_1 + R_2 + R_3 = 7.2\,\Omega$

$$\frac{(R_1 + R_2)24}{(R_1 + R_2 + R_3)} = 12$$

Therefore, $2(R_1 + R_2) = R_1 + R_2 + R_3$

Thus, $R_1 + R_2 = R_3$; $2R_3 = 7.2$; $R_3 = 3.6\,\Omega$

$$\frac{R_2(24)}{R_1 + R_2 + R_3} = 5$$

$$4.8R_2 = R_1 + R_2 + 3.6 = 7.2$$

Thus, $R_2 = 1.5\,\Omega$; $R_1 = 7.2 - R_2 - R_3 = 2.1\,\Omega$

P 3.19 [a] At no load: $v_o = kv_s = \dfrac{R_2}{R_1 + R_2}v_s.$

At full load: $v_o = \alpha v_s = \dfrac{R_e}{R_1 + R_e}v_s$, where $R_e = \dfrac{R_o R_2}{R_o + R_2}$

$$\text{Therefore } k = \frac{R_2}{R_1 + R_2} \text{ and } R_1 = \frac{(1-k)}{k}R_2$$

$$\alpha = \frac{R_e}{R_1 + R_e} \text{ and } R_1 = \frac{(1-\alpha)}{\alpha}R_e$$

Thus $\left(\dfrac{1-\alpha}{\alpha}\right)\left[\dfrac{R_2 R_o}{R_o + R_2}\right] = \dfrac{(1-k)}{k}R_2$

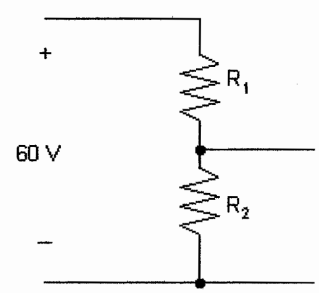
Solving for R_2 yields $R_2 = \dfrac{(k-\alpha)}{\alpha(1-k)}R_o$

Also, $R_1 = \dfrac{(1-k)}{k}R_2 \quad \therefore \quad R_1 = \dfrac{(k-\alpha)}{\alpha k}R_o$

[b] $R_1 = \left(\dfrac{0.05}{0.68}\right)R_o = 2.5\text{ k}\Omega$

$R_2 = \left(\dfrac{0.05}{0.12}\right)R_o = 14.167\text{ k}\Omega$

[c]



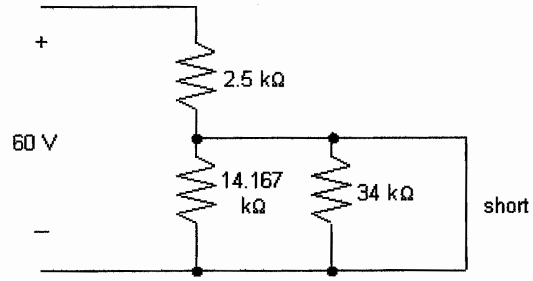
Maximum dissipation in R_2 occurs at no load, therefore,

$$P_{R_2(\text{max})} = \frac{[(60)(0.85)]^2}{14{,}167} = 183.6\text{ mW}$$

Maximum dissipation in R_1 occurs at full load.

$$P_{R_1(\text{max})} = \frac{[60 - 0.80(60)]^2}{2500} = 57.60\text{ mW}$$

[d]



$$P_{R_1} = \frac{(60)^2}{2500} = 1.44 \text{ W} = 1440 \text{ mW}$$

$$P_{R_2} = \frac{(0)^2}{14{,}167} = 0 \text{ W}$$

P 3.20 [a] Let v_o be the voltage across the parallel branches, positive at the upper terminal, then

$$i_g = v_oG_1 + v_oG_2 + \cdots + v_oG_N = v_o(G_1 + G_2 + \cdots + G_N)$$

It follows that $\quad v_o = \dfrac{i_g}{(G_1 + G_2 + \cdots + G_N)}$

The current in the k^{th} branch is $\quad i_k = v_oG_k;\qquad$ Thus,

$$i_k = \frac{i_gG_k}{[G_1 + G_2 + \cdot + G_N]}$$

[b] $i_{6.25} = \dfrac{1142(0.16)}{[4 + 0.4 + 1 + 0.16 + 0.1 + 0.05]} = 32 \text{ mA}$

P 3.21 Begin by using the relationships among the branch currents to express all branch currents in terms of i_4:

$$i_1 = 4i_2 = 4(8i_3) = 5(32i_4)$$

$$i_2 = 8i_3 = 5(8i_4)$$

$$i_3 = 5i_4$$

Now use KCL at the top node to relate the branch currents to the current supplied by the source.

$$i_1 + i_2 + i_3 + i_4 = 5 \text{ mA}$$

Express the branch currents in terms of i_4 and solve for i_4:

$$5 \text{ mA} = 160i_4 + 40i_4 + 5i_4 + i_4 = 206i_4 \qquad \text{so} \qquad i_4 = \frac{0.005}{206} \text{ A}$$

Since the resistors are in parallel, the same voltage, 1 V appears across each of them. We know the current and the voltage for R_4 so we can use Ohm's law to calculate R_4:

$$R_4 = \frac{v_g}{i_4} = \frac{1\text{ V}}{(5/206)\text{ mA}} = 41.2\text{ k}\Omega$$

Calculate i_3 from i_4 and use Ohm's law as above to find R_3:

$$i_3 = 5i_4 = \frac{25}{206}\text{ A} \qquad \therefore\ R_3 = \frac{v_g}{i_3} = \frac{1\text{ V}}{(25/206)\text{ mA}} = 8240\,\Omega$$

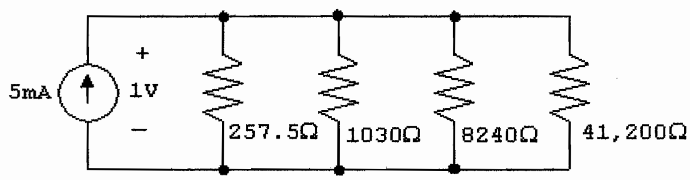
Calculate i_2 from i_4 and use Ohm's law as above to find R_2:

$$i_2 = 40i_4 = \frac{0.2}{206}\text{ A} \qquad \therefore\ R_2 = \frac{v_g}{i_2} = \frac{1\text{ V}}{(200/206)\text{ mA}} = 1030\,\Omega$$

Calculate i_1 from i_4 and use Ohm's law as above to find R_1:

$$i_1 = 160i_4 = \frac{0.8}{206}\text{ A} \qquad \therefore\ R_1 = \frac{v_g}{i_1} = \frac{1\text{ V}}{(800/206)\text{ mA}} = 257.5\,\Omega$$

The resulting circuit is shown below:



P 3.22 [a] The equivalent resistance to the right of the 10 kΩ resistor is $3\text{ k} + 8\text{ k} + [6\text{ k}\|(5\text{ k} + 7\text{ k})] = 15\text{ k}\Omega$. Therefore,

$$i_{10\text{k}} = \frac{15\text{ k}\|10\text{ k}}{10\text{ k}}(0.002) = \frac{6\text{ k}}{10\text{ k}}(0.002) = 1.2\text{ mA}$$

[b] The voltage drop across the 10 kΩ resistor can be found using Ohm's law:

$$v_{10\text{k}} = (10{,}000)i_{10\text{k}} = (10{,}000)(0.0012) = 12\text{ V}$$

[c] The voltage $v_{10\text{k}}$ drops across the 3 kΩ resistor, the 8 kΩ resistor and the equivalent resistance of the 6 kΩ and the parallel branch containing the 5 kΩ and 7 kΩ resistors. Thus, using voltage division,

$$v_{6\text{k}} = \frac{6\text{ k}\|(5\text{ k} + 7\text{ k})}{3\text{ k} + 8\text{ k} + [6\text{ k}\|(5\text{ k} + 7\text{ k})]}(12) = \frac{4}{15}(12) = 3.2\text{ V}$$

[d] The voltage $v_{6\text{k}}$ drops across the branch containing the 5 kΩ and 7 kΩ resistors. Thus, using voltage division,

$$v_{5\text{k}} = \frac{5\text{ k}}{5\text{ k} + 7\text{ k}}(3.2) = 1.33\text{ V}$$

P 3.23 [a] The voltage drop across the 240 Ω resistor is the same as the voltage drop across the parallel combination of the branch containing the 240 Ω resistor and the branch containing the 180 Ω and 300, Ω resistors. Thus by voltage division,

$$v_{240} = \frac{240\|(180+300)}{[240\|(180+300)] + 140 + 200}(10) = \frac{160}{500}(10) = 3.2 \text{ V}$$

[b] The current in the 240 Ω resistor can be found from its voltage using Ohm's law:

$$i_{240} = \frac{v_{240}}{240} = \frac{3.2}{240} = 13.33 \text{ mA}$$

[c] The current in the 140 Ω resistor divides between two branches – one containing the 180 Ω and 300 Ω resistors and the other containing the 240 Ω resistor. Using current division,

$$i_{240} = \frac{240\|(180+300)}{240}(i_{140}) = 0.01333 \quad \text{so} \quad i_{140} = \frac{240(0.01333)}{160} = 20 \text{ mA}$$

P 3.24 [a] $v_{1\text{k}} = \frac{1}{1+5}(30) = 5 \text{ V}$

$$v_{15\text{k}} = \frac{15}{15+60}(30) = 6 \text{ V}$$

$$v_x = v_{15\text{k}} - v_{1\text{k}} = 6 - 5 = 1 \text{ V}$$

[b] $v_{1\text{k}} = \frac{v_s}{6}(1) = v_s/6$

$$v_{15\text{k}} = \frac{v_s}{75}(15) = v_s/5$$

$$v_x = (v_s/5) - (v_s/6) = v_s/30$$

P 3.25 $60\|30 = 20\,\Omega$

$$i_{30\Omega} = \frac{(25)(75)}{125} = 15 \text{ A}$$

$$v_2 = (15)(20) = 300 \text{ V}$$

$$v_2 + 30i_{30} = 750 \text{ V}$$

$$v_1 - 12(25) = 750$$

$$v_1 = 1050 \text{ V}$$

P 3.26 $i_{10k} = \dfrac{(18)(15\text{ k})}{40\text{ k}} = 6.75\text{ mA}$

$v_{15k} = -(6.75\text{ m})(15\text{ k}) = -101.25\text{ V}$

$i_{3k} = 18\text{ m} - 6.75\text{ m} = 11.25\text{ mA}$

$v_{12k} = -(12\text{ k})(11.25\text{ m}) = -135\text{ V}$

$v_o = -101.25 - (-135) = 33.75\text{ V}$

P 3.27 $54\,\Omega \| 27\,\Omega = 18\,\Omega$; $18\,\Omega + 2\,\Omega = 20\,\Omega$; $20 \| (10 + 15 + 35) = 15\,\Omega$;

Therefore, $i_g = \dfrac{675}{30 + 15} = 15\text{ A}$

$i_{2\Omega} = \dfrac{20\|60}{20}(15) = 11.25\text{ A}$; $i_o = \dfrac{27\|54}{27}(11.25) = 7.5\text{ A}$

P 3.28 [a] $40\|10 = 8\,\Omega$

$8 + 2 = 10\,\Omega$

$15\|10 = 6\,\Omega$

$6 + 4 = 10\,\Omega$

$30\|10 = 7.5\,\Omega$

$i_{120V} = \dfrac{120}{7.5} = 16\text{ A}$

$i_{4\Omega} = \dfrac{7.5}{4+6}(16) = 12\text{ A}$

$i_{2\Omega} = \dfrac{6}{2+8}(12) = 7.2\text{ A}$

$i_o = \dfrac{8}{40}(7.2) = 1.44\text{ A}$

[b] $i_{15\Omega} = i_{4\Omega} - i_{2\Omega} = 12 - 7.2 = 4.8\text{ A}$

$P_{15\Omega} = (4.8)^2(15) = 345.6\text{ W}$

P 3.29 [a] The voltage across the $9\,\Omega$ resistor is $1(12 + 6) = 18$ V.
The current in the $9\,\Omega$ resistor is $18/9 = 2$ A. The current in the $2\,\Omega$ resistor is $1 + 2 = 3$ A. Therefore, the voltage across the $24\,\Omega$ resistor is $(2)(3) + 18 = 24$ V.
The current in the $24\,\Omega$ resistor is 1 A. The current in the $3\,\Omega$ resistor is $1 + 2 + 1 = 4$ A. Therefore, the voltage across the $72\,\Omega$ resistor is $24 + 3(4) = 36$ V.
The current in the $72\,\Omega$ resistor is $36/72 = 0.5$ A.
The $20\,\Omega\|5\,\Omega$ resistors are equivalent to a $4\,\Omega$ resistor. The current in this equivalent resistor is $0.5 + 1 + 3 = 4.5$ A. Therefore the voltage across the $108\,\Omega$ resistor is $36 + 4.5(4) = 54$ V.
The current in the $108\,\Omega$ resistor is $54/108 = 0.5$ A. The current in the $1.2\,\Omega$ resistor is $4.5 + 0.5 = 5$ A. Therefore,

$$v_g = (1.2)(5) + 54 = 60\text{ V}$$

[b] The current in the $20\,\Omega$ resistor is

$$i_{20} = \frac{(4.5)(4)}{20} = \frac{18}{20} = 0.9 \text{ A}$$

Thus, the power dissipated by the $20\,\Omega$ resistor is

$$p_{20} = (0.9)^2(20) = 16.2 \text{ W}$$

P 3.30 [a] The model of the ammeter is an ideal ammeter in parallel with a resistor whose resistance is given by

$$R_m = \frac{100\,\text{mV}}{2\,\text{mA}} = 50\,\Omega.$$

We can calculate the current through the real meter using current division:

$$i_m = \frac{(25/12)}{50 + (25/12)}(i_{\text{meas}}) = \frac{25}{625}(i_{\text{meas}}) = \frac{1}{25}i_{\text{meas}}$$

[b] At full scale, $i_{\text{meas}} = 5$ A and $i_{\text{m}} = 2$ mA so $5 - 0.002 = 4998$ mA flows throught the resistor R_{A}:

$$R_{\text{A}} = \frac{100 \text{ mV}}{4998 \text{ m A}} = \frac{100}{4998}\,\Omega$$

$$i_m = \frac{(100/4998)}{50 + (100/4998)}(i_{\text{meas}}) = \frac{1}{2500}(i_{\text{meas}})$$

[c] Yes

P 3.31 The measured value is $60\|30.5 = 20.22\,\Omega$.

$$i_g = \frac{180}{(20.22 + 10)} = 5.96 \text{ A}; \qquad i_{\text{meas}} = \frac{60}{90.5}(5.96) = 3.95 \text{ A}$$

The true value is $60\|30 = 20\,\Omega$.

$$i_g = \frac{180}{(20 + 10)} = 6 \text{ A}; \qquad i_{\text{true}} = \frac{60}{90}(6) = 4 \text{ A}$$

$$\%\text{error} = \left[\frac{3.95}{4} - 1\right] \times 100 = -1.28\%$$

P 3.32 Begin by using current division to find the actual value of the current i_o:

$$i_{\text{true}} = \frac{24}{24 + 5.5}(20 \text{ mA}) = 16.27 \text{ mA}$$

$$i_{\text{meas}} = \frac{24}{24 + 5.5 + 0.5}(20 \text{ mA}) = 16 \text{ mA}$$

$$\% \text{ error} = \left[\frac{16}{16.27} - 1\right] 100 = -1.66\%$$

P 3.33 For all full-scale readings the total resistance is

$$R_V + R_{\text{movement}} = \frac{\text{full-scale reading}}{10^{-3}}$$

$$\therefore \quad R_V = 1000\ (\text{full-scale reading}) \ - 50$$

[a] $R_V = 1000(100) - 50 = 99,950\,\Omega$

[b] $R_V = 1000(5) - 50 = 4950\,\Omega$

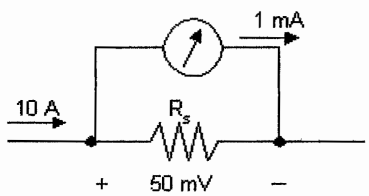
[c] $R_V = 100 - 50 = 50\,\Omega$

P 3.34 [a] $v_{\text{meas}} = (20 \times 10^{-3})(24\|5.5\|4950) = 0.089411$ V

[b] $v_{\text{true}} = (20 \times 10^{-3})(24\|5.5) = 0.089492$ V

$$\%\ \text{error} \ = \left(\frac{0.089411}{0.089492} - 1\right) \times 100 = -0.08998\%$$

P 3.35



Original meter: $R_e = \dfrac{50 \times 10^{-3}}{10} = 0.005\,\Omega$

Modified meter: $R_e = \dfrac{(0.015)(0.005)}{0.02} = 0.00375\,\Omega$

$$\therefore \quad (I_{\text{fs}})(0.00375) = 50 \times 10^{-3}$$

$$\therefore \quad I_{\text{fs}} = 13.33 \text{ A}$$

P 3.36 At full scale the voltage across the shunt resistor will be 50 mV; therefore the power dissipated will be

$$P_{\text{A}} = \frac{(50 \times 10^{-3})^2}{R_{\text{A}}}$$

Therefore $R_{\text{A}} \geq \dfrac{(50 \times 10^{-3})^2}{0.5} = 5\ \text{m}\Omega$

Otherwise the power dissipated in R_{A} will exceed its power rating of 0.5 W

When $R_A = 5\ \text{m}\Omega$, the shunt current will be

$$i_A = \frac{50 \times 10^{-3}}{5 \times 10^{-3}} = 10\ \text{A}$$

The measured current will be $i_{meas} = 10 + 0.001 = 10.001$ A

$\therefore$ Full-scale reading is for practical purposes is 10 A

P 3.37 The current in the shunt resistor at full-scale deflection is $i_A = i_{fullscale} - 2 \times 10^{-3}$ A. The voltage across R_A at full-scale deflection is always 100 mV; therefore,

$$R_A = \frac{100 \times 10^{-3}}{i_{fullscale} - 2 \times 10^{-3}} = \frac{100}{1000 i_{fullscale} - 2}$$

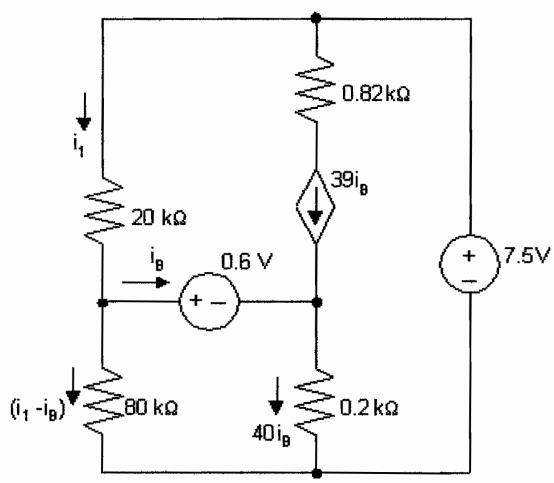
[a] $R_A = \dfrac{100}{5000 - 2} = 20{,}008\ \text{m}\Omega$

[b] $R_A = \dfrac{100}{2000 - 2} = 50.05\ \text{m}\Omega$

[c] $R_A = \dfrac{100}{1000 - 2} = 100.20\ \text{m}\Omega$

[d] $R_A = \dfrac{100}{50 - 2} = 2.083\ \Omega$

P 3.38 [a]



$$20 \times 10^3 i_1 + 80 \times 10^3 (i_1 - i_B) = 7.5$$

$$80 \times 10^3 (i_1 - i_B) = 0.6 + 40 i_B (0.2 \times 10^3)$$

$$\therefore \quad 100 i_1 - 80 i_B = 7.5 \times 10^{-3}$$

$$80 i_1 - 88 i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 225\ \mu\text{A}$

[b] With the insertion of the ammeter the equations become

$$100i_1 - 80i_B = 7.5 \times 10^{-3} \quad \text{(no change)}$$

$$80 \times 10^3(i_1 - i_B) = 10^3 i_B + 0.6 + 40i_B(200)$$

$$80i_1 - 89i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 216\,\mu\text{A}$

[c] $\%\text{ error } = \left(\frac{216}{225} - 1\right)100 = -4\%$

P 3.39 [a] $v_{\text{meter}} = 180$ V

[b] $R_{\text{meter}} = (100)(200) = 20$ kΩ

$$20\|70 = 15.56 \text{ k}\Omega$$

$$v_{\text{meter}} = \frac{180}{35.56} \times 15.56 = 78.76 \text{ V}$$

[c] $20\|20 = 10$ kΩ

$$v_{\text{meter}} = \frac{180}{80}(10) = 22.5 \text{ V}$$

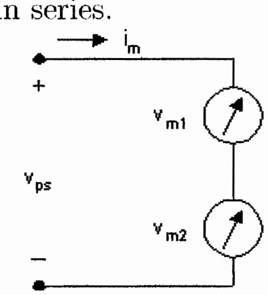
[d] $v_{\text{meter a}} = 180$ V

$$v_{\text{meter b}} + v_{\text{meter c}} = 101.26 \text{ V}$$

No, because of the loading effect.

P 3.40 [a] Since the unknown voltage is greater than either voltmeter's maximum reading, the only possible way to use the voltmeters would be to connect them in series.

[b]



$$R_{m1} = (400)(1000) = 400 \text{ k}\Omega = R_{m2}$$

$$\therefore \quad R_{m1} + R_{m2} = 800 \text{ k}\Omega$$

$$i_{1\text{ max}} = \frac{400}{400} \times 10^{-3} = 1 \text{ mA} = i_{2\text{ max}}$$

$$\therefore \quad i_{\text{max}} = 1 \text{ mA since meters are in series}$$

$$v_{\text{max}} = 10^{-3}(400 + 400)10^3 = 800 \text{ V}$$

Thus the meters can be used to measure the voltage

[c] $i_m = \dfrac{504}{800 \times 10^3} = 0.63$ mA

$$v_{m1} = (0.63)(400) = 252 \text{ V} = v_{m2}$$

P 3.41 The current in the series-connected voltmeters is

$$i_m = \frac{328}{400} = 0.82 \text{ mA}$$

$$v_{50\text{ k}\Omega} = (0.82)(50) = 41 \text{ V}$$

$$V_{\text{power supply}} = 328 + 328 + 41 = 697 \text{ V}$$

P 3.42 $R_{\text{meter}} = R_m + R_{\text{movement}} = \dfrac{800 \text{ V}}{1 \text{ mA}} = 800 \text{ k}\Omega$

$$v_{\text{meas}} = (300 \text{ k}\Omega \| 600 \text{ k}\Omega \| 800 \text{ k}\Omega)(3.5 \text{ mA}) = (160 \text{ k}\Omega)(3.5 \text{ mA}) = 560 \text{ V}$$

$$v_{\text{true}} = (300 \text{ k}\Omega \| 600 \text{ k}\Omega)(3.5 \text{ mA}) = (200 \text{ k}\Omega)(3.5 \text{ mA}) = 700 \text{ V}$$

$$\% \text{ error} = \left(\frac{560}{700} - 1\right) 100 = -20\%$$

P 3.43 [a] $R_{\text{meter}} = 300 \text{ k}\Omega + 600 \text{ k}\Omega \| 200 \text{ k}\Omega = 450 \text{ k}\Omega$

$$450 \| 360 = 200 \text{ k}\Omega$$

$$V_{\text{meter}} = \frac{200}{240}(600) = 500 \text{ V}$$

[b] What is the percent error in the measured voltage?

$$\text{True value} = \frac{360}{400}(600) = 540 \text{ V}$$

$$\% \text{ error} = \left(\frac{500}{540} - 1\right) 100 = -7.41\%$$

P 3.44 [a] $R_1 = (50)10^3 = 50 \text{ k}\Omega$

$$R_2 = (20)10^3 = 20 \text{ k}\Omega$$

$$R_3 = (2)10^3 = 2 \text{ k}\Omega$$

[b] Let i_a = actual current in the movement

i_d = design current in the movement

$$\text{Then \% error} = \left(\frac{i_\text{a}}{i_\text{d}} - 1\right)100$$

For the 50 V scale:

$$i_\text{a} = \frac{50}{50{,}000 + 100} = \frac{50}{50{,}100}, \qquad i_\text{d} = \frac{50}{50{,}000}$$

$$\frac{i_\text{a}}{i_\text{d}} = \frac{50{,}000}{50{,}100} = 0.9980 \qquad \text{\% error} = (0.9980 - 1)100 = -0.20\%$$

For the 20 V scale:

$$\frac{i_\text{a}}{i_\text{d}} = \frac{20{,}000}{20{,}100} = 0.995 \qquad \text{\% error} = (0.995 - 1.0)100 = -0.4975\%$$

For the 2 V scale:

$$\frac{i_\text{a}}{i_\text{d}} = \frac{2000}{2100} = 0.9524 \qquad \text{\% error} = (0.9524 - 1.0)100 = -4.76\%$$

P 3.45 [a] $R_\text{movement} = 5\,\Omega$

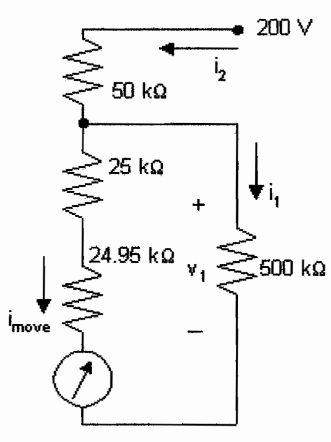
$$R_1 + R_\text{movement} = \frac{50}{2 \times 10^{-3}} = 25\text{ k}\Omega \quad \therefore \quad R_1 = 24{,}995\,\Omega$$

$$R_2 + R_1 + R_\text{movement} = \frac{100}{2 \times 10^{-3}} = 50\text{ k}\Omega \quad \therefore \quad R_2 = 25\text{ k}\Omega$$

$$R_3 + R_2 + R_1 + R_\text{movement} = \frac{200}{2 \times 10^{-3}} = 100\text{ k}\Omega$$

$$\therefore \quad R_3 = 50\text{ k}\Omega$$

[b]



$$i_\text{move} = \frac{188}{200}(2) = 1.88\text{ mA}$$

$$v_1 = (1.88)(50) = 94 \text{ V}$$

$$i_1 = \frac{94}{500} = 0.188 \text{ mA}$$

$$i_2 = i_{\text{move}} + i_1 = 1.88 + 0.188 = 2.068 \text{ mA}$$

$$v_{\text{meas}} = v_x = 94 + 50i_2 = 197.4 \text{ V}$$

[c] $v_1 = 100$ V $\qquad i_2 = 2 + 0.20 = 2.20$ mA

$i_1 = 100/500 = 0.20$ mA $\qquad v_{\text{meas}} = v_x = 100 + 50(2.20) = 210$ V

P 3.46 From the problem statement we have

$$80 = \frac{V_s(10)}{10 + R_s} \quad (1) \quad V_s \text{ in mV; } R_s \text{ in M}\Omega$$

$$72 = \frac{V_s(5)}{5 + R_s} \quad (2)$$

[a] From Eq (1) $\quad 10 + R_s = 0.125V_s$

$\therefore \; R_s = 0.125V_s - 10$

Substituting into Eq (2) yields

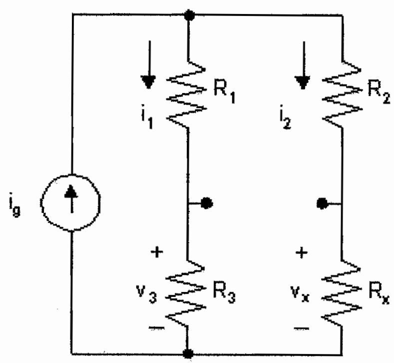
$$72 = \frac{5V_s}{0.125V_s - 5} \quad \text{or} \quad V_s = 90 \text{ mV}$$

[b] From Eq (1)

$$80 = \frac{900}{10 + R_s} \quad \text{or} \quad 80R_s = 100$$

So $R_s = 1250$ kΩ

P 3.47 Since the bridge is balanced, we can remove the detector without disturbing the voltages and currents in the circuit.



It follows that

$$i_1 = \frac{i_g(R_2 + R_x)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_2 + R_x)}{\sum R}$$

$$i_2 = \frac{i_g(R_1 + R_3)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_1 + R_3)}{\sum R}$$

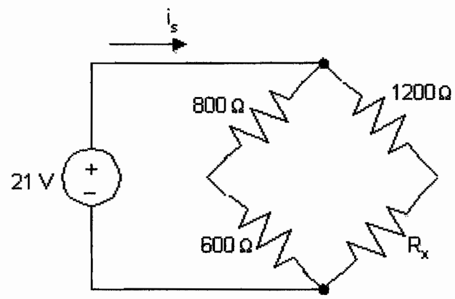
$$v_3 = R_3 i_1 = v_x = i_2 R_x$$

$$\therefore \quad \frac{R_3 i_g(R_2 + R_x)}{\sum R} = \frac{R_x i_g(R_1 + R_3)}{\sum R}$$

$$\therefore \quad R_3(R_2 + R_x) = R_x(R_1 + R_3)$$

From which $R_x = \dfrac{R_2 R_3}{R_1}$

P 3.48 [a]



The condition for a balanced bridge is that the product of the opposite resistors must be equal:

$$(800)(R_x) = (1200)(600) \qquad \text{so} \qquad R_x = \frac{(1200)(600)}{800} = 900\,\Omega$$

[b] The source current is the sum of the two branch currents. Each branch current can be determined using Ohm's law, since the resistors in each branch are in series and the voltage drop across each branch is 21 V:

$$i_s = \frac{21 \text{ V}}{800\,\Omega + 600\,\Omega} + \frac{21 \text{ V}}{1200\,\Omega + 900\,\Omega} = 25 \text{ mA}$$

[c] We can use current division to find the current in each branch:

$$i_{\text{left}} = \frac{1200 + 900}{1200 + 900 + 800 + 600}(25 \text{ mA}) = 15 \text{ mA}$$

$$i_{\text{right}} = 25 \text{ mA} - 15 \text{ mA} = 10 \text{ mA}$$

Now we can use the formula $p = Ri^2$ to find the power dissipated by each resistor:

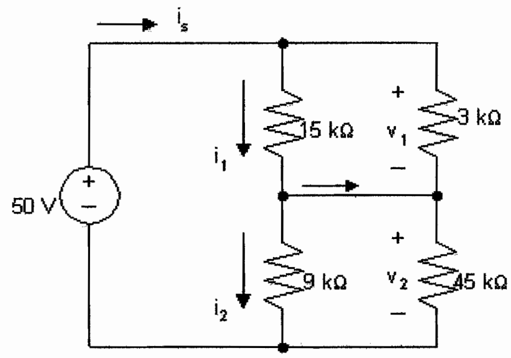
$$p_{800} = (800)(0.015)^2 = 180 \text{ mW} \qquad p_{600} = (600)(0.015)^2 = 135 \text{ mW}$$

$$p_{1200} = (1200)(0.010)^2 = 120 \text{ mW} \qquad p_{900} = (900)(0.010)^2 = 90 \text{ mW}$$

Thus, the $800\,\Omega$ resistor absorbs the most power; it absorbs 180 mW of power.

[d] From the analysis in part (c), the $900\,\Omega$ resistor absorbs the least power; it absorbs 90 mW of power.

P 3.49 Redraw the circuit, replacing the detector branch with a short circuit.



$$15\text{ k}\Omega \| 3\text{ k}\Omega = 2.5\text{ k}\Omega$$

$$9\text{ k}\Omega \| 45\text{ k}\Omega = 7.5\text{ k}\Omega$$

$$i_g = \frac{50}{10} = 5\text{ mA}$$

$$v_1 = 5(2.5) = 12.5\text{ V}$$

$$v_2 = 5(7.5) = 37.5\text{ V}$$

$$i_1 = \frac{12.5}{15} = 833.3\,\mu\text{A}$$

$$i_2 = \frac{37.5}{9} = 4166.7\,\mu\text{A}$$

$$i_{\text{d}} = i_1 - i_2 = -3333.4\,\mu\text{A}$$

P 3.50 Note the bridge structure is balanced, that is $10 \times 18 = 30 \times 6$, hence there is no current in the 50Ω resistor. It follows that the equivalent resistance of the circuit is

$$R_{\text{eq}} = 3 + (10 + 6)\|(30 + 18) = 3 + 12 = 15\,\Omega$$

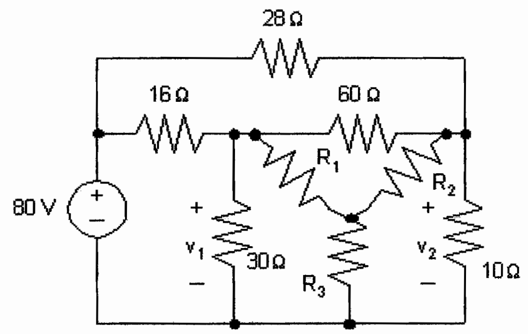
The source current is $300/15 = 20$ A.
The current down through the branch containing the $30\,\Omega$ and $18\,\Omega$ resistors is

$$i_{18} = \frac{12}{30 + 18}(20) = 5\text{ A}$$

$$\therefore\ p_{18} = 18(5)^2 = 450\text{ W}$$

P 3.51 In order that all four decades (1, 10, 100, 1000) that are used to set R_3 contribute to the balance of the bridge, the ratio R_2/R_1 should be set to 0.001.

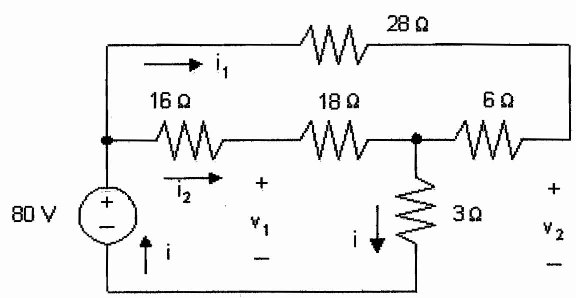
P 3.52 Begin by transforming the Δ-connected resistors $(10\,\Omega, 30\,\Omega, 60\,\Omega)$ to Y-connected resistors. Both the Y-connected and Δ-connected resistors are shown below to assist in using Eqs. 3.44 – 3.46:



Now use Eqs. 3.44 – 3.46 to calculate the values of the Y-connected resistors:

$$R_1 = \frac{(30)(60)}{10+30+60} = 18\,\Omega; \quad R_2 = \frac{(60)(10)}{10+30+60} = 6\,\Omega; \quad R_3 = \frac{(30)(10)}{10+30+60} = 3\,\Omega$$

The transformed circuit is shown below:



The equivalent resistance seen by the 80 V source can be calculated by making series and parallel combinations of the resistors to the right of the 24 V source:

$$R_{eq} = (28+6)\|(16+18) + 3 = 34\|34 + 3 = 17 + 3 = 20\,\Omega$$

Therefore, the current i in the 80 V source is given by

$$i = \frac{80\text{ V}}{20\,\Omega} = 4\text{ A}$$

Use current division to calculate the currents i_1 and i_2. Note that the current i_1 flows in the branch containing the $28\,\Omega$ and $6\,\Omega$ series connected resistors, while the current i_2 flows in the parallel branch that contains the series connection of the $16\,\Omega$ and $18\,\Omega$ resistors:

$$i_1 = \frac{16+18}{16+18+28+6}(i) = \frac{34}{68}(4\text{ A}) = 2\text{ A}, \qquad \text{and} \qquad i_2 = 4\text{ A} - 2\text{ A} = 2\text{ A}$$

Now use KVL and Ohm's law to calculate v_1. Note that v_1 is the sum of the voltage drop across the $18\,\Omega$ resistor, $18i_2$, and the voltage drop across the $3\,\Omega$ resistor, $3i$:

$$v_1 = 18i_2 + 3i = 18(2\text{ A}) + 3(4\text{ A}) = 36 + 12 = 48\text{ V}$$

Finally, use KVL and Ohm's law to calculate v_2. Note that v_2 is the sum of the voltage drop across the $6\,\Omega$ resistor, $6i_1$, and the voltage drop across the $3\,\Omega$ resistor, $3i$:

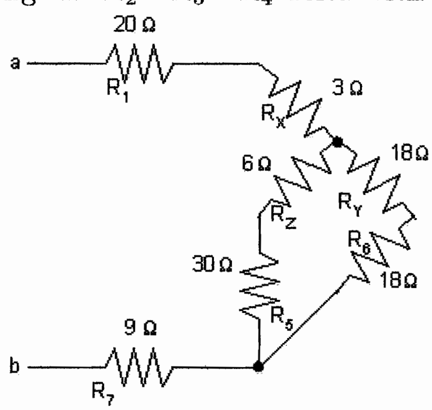
$$v_2 = 6i_1 + 3i = 6(2\text{ A}) + 3(4\text{ A}) = 12 + 12 = 24\text{ V}$$

P 3.53 [a] Calculate the values of the Y-connected resistors that are equivalent to the $10\,\Omega, 30\,\Omega$, and 60Ω Δ-connected resistors:

$$R_X = \frac{(10)(30)}{10+30+60} = 3\,\Omega; \qquad R_Y = \frac{(30)(60)}{10+30+60} = 18\,\Omega;$$

$$R_Z = \frac{(10)(60)}{10+30+60} = 6\,\Omega$$

Replacing the R_2—R_3—R_4 delta with its equivalent Y gives



Now calculate the equivalent resistance R_{ab} by making series and parallel combinations of the resistors:

$$R_{ab} = 20 + 3 + [(30+6)\|(18+18)] + 9 = 50\,\Omega$$

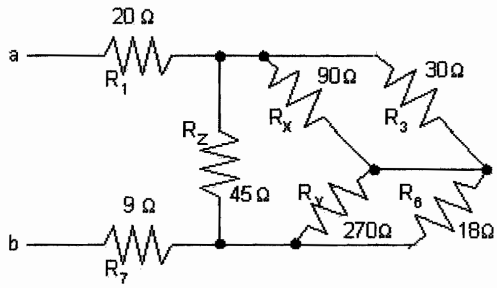
[b] Calculate the values of the Δ-connected resistors that are equivalent to the $10\,\Omega, 30\,\Omega$, and $60\,\Omega$ Y-connected resistors:

$$R_X = \frac{(10)(30) + (30)(60) + (10)(60)}{30} = \frac{2700}{30} = 90\,\Omega$$

$$R_Y = \frac{(10)(30) + (30)(60) + (10)(60)}{10} = \frac{2700}{10} = 270\,\Omega$$

$$R_Z = \frac{(10)(30) + (30)(60) + (10)(60)}{60} = \frac{2700}{60} = 45\,\Omega$$

Replacing the R_2, R_4, R_5 wye with its equivalent Δ gives



Make series and parallel combinations of the resistors to find the equivalent resistance R_{ab}:

$90\,\Omega \| 30\,\Omega = 22.5\,\Omega$; $270\,\Omega \| 18\,\Omega = 16.875\,\Omega$

$\therefore \quad 45 \| (22.5 + 16.875) = 21\,\Omega$

$\therefore \quad R_{ab} = 20 + 21 + 9 = 50\,\Omega$

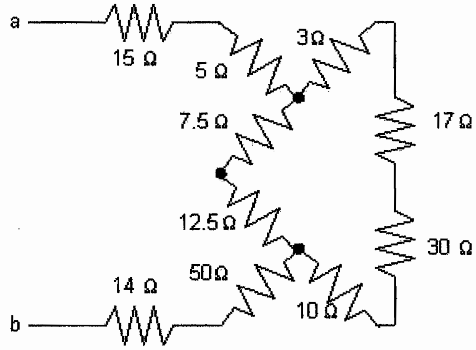
[c] Convert the delta connection R_4—R_5—R_6 to its equivalent wye. Convert the wye connection R_3—R_4—R_6 to its equivalent delta.

P 3.54 Replace the upper and lower deltas with the equivalent wyes:

$$R_{1U} = \frac{(25)(10)}{50} = 5\,\Omega;\ R_{2U} = \frac{(10)(15)}{50} = 3\,\Omega;\ R_{3U} = \frac{(25)(15)}{50} = 7.5\,\Omega$$

$$R_{1L} = \frac{(125)(25)}{250} = 12.5\,\Omega;\ R_{2L} = \frac{(25)(100)}{250} = 10\,\Omega;\ R_{3L} = \frac{(125)(100)}{250} = 50\,\Omega$$

The resulting circuit is shown below:

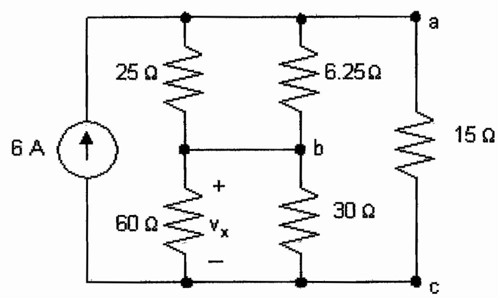


Now make series and parallel combinations of the resistors:

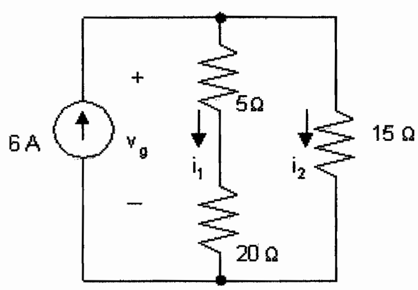
$(7.5 + 12.5)||(3 + 17 + 30 + 10) = 20||60 = 15\,\Omega$

$R_{\text{ab}} = 15 + 5 + 15 + 50 + 14 = 99\,\Omega$

P 3.55



$25||6.25 = 5\,\Omega$ $\qquad$ $60||30 = 20\,\Omega$



$$i_1 = \frac{(6)(15)}{(40)} = 2.25 \text{ A}; \qquad v_x = 20i_1 = 45 \text{ V}$$

$$v_g = 25i_1 = 56.25 \text{ V}$$

$v_{6.25} = v_g - v_x = 11.25$ V

$$P_{\text{device}} = \frac{11.25^2}{6.25} + \frac{45^2}{30} + \frac{56.25^2}{15} = 298.6875 \text{ W}$$

P 3.56 $8 + 12 = 20\,\Omega$

$20\|60 = 15\,\Omega$

$15 + 20 = 35\,\Omega$

$35\|140 = 28\,\Omega$

$28 + 22 = 50\,\Omega$

$50\|75 = 30\,\Omega$

$30 + 10 = 40\,\Omega$

$i_g = 240/40 = 6$ A

$i_o = (6)(50)/125 = 2.4$ A

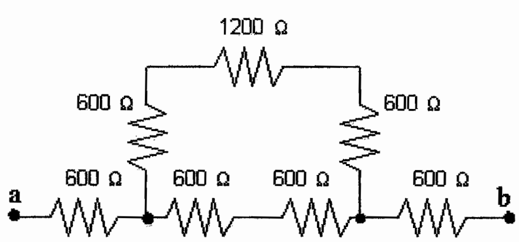
$i_{140\Omega} = (6 - 2.4)(35)/175 = 0.72$ A

$p_{140\Omega} = (0.72)^2(140) = 72.576$ W

P 3.57 The top of the pyramid can be replaced by a resistor equal to

$$R_1 = \frac{(3.6)(1.8)}{5.4} = 1.2 \text{ k}\Omega$$

The lower left and right deltas can be replaced by wyes. Each resistance in the wye equals $600\,\Omega$. Thus our circuit can be reduced to



Now the $2400\,\Omega$ in parallel with $1200\,\Omega$ reduces to $800\,\Omega$.

$\therefore\ R_{\text{ab}} = 600 + 800 + 600 = 2000 = 2$ kΩ

P 3.58 [a] Convert the upper delta to a wye.

$$R_1 = \frac{(80)(200)}{400} = 40\,\Omega$$

$$R_2 = \frac{(80)(120)}{400} = 24\,\Omega$$

$$R_3 = \frac{(120)(200)}{400} = 60\,\Omega$$

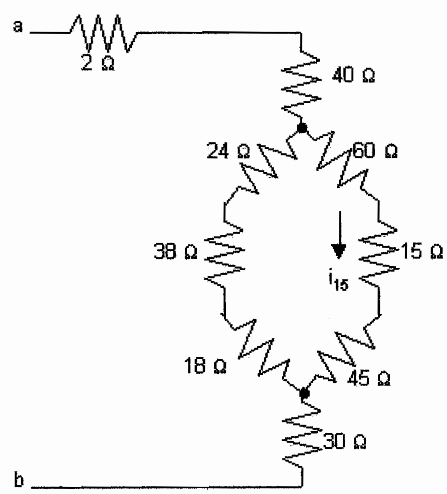
Convert the lower delta to a wye.

$$R_4 = \frac{(60)(90)}{300} = 18\,\Omega$$

$$R_5 = \frac{(60)(150)}{300} = 30\,\Omega$$

$$R_6 = \frac{(90)(150)}{300} = 45\,\Omega$$

Now redraw the circuit using the wye equivalents.



$$R_{ab} = 2 + 40 + \frac{(80)(120)}{200} + 30 = 42 + 48 + 30 = 120\,\Omega$$

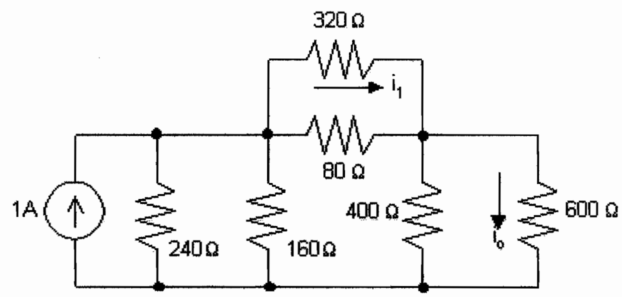
[b] When $v_{ab} = 600$ V

$$i_g = \frac{600}{120} = 5 \text{ A}$$

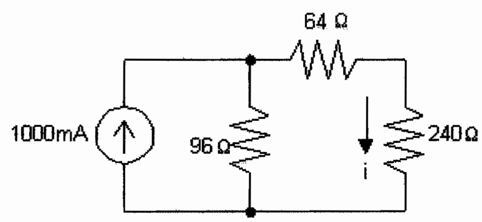
$$i_{15} = \frac{(5)(80)}{200} = 2 \text{ A}$$

$$p_{15\Omega} = (4)(15) = 60 \text{ W}$$

P 3.59 [a] After the 20 Ω—100 Ω—50 Ω wye is replaced by its equivalent delta, the circuit reduces to



Now the circuit can be reduced to

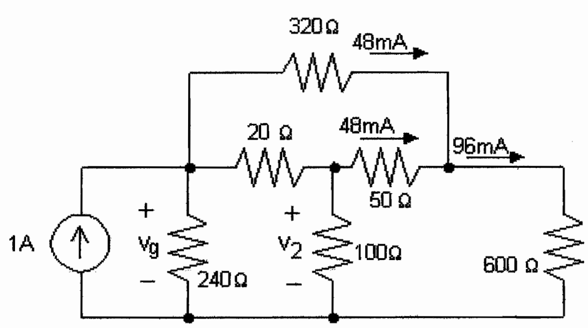


$$i = \frac{96}{400}(1000) = 240 \text{ mA}$$

$$i_o = \frac{400}{1000}(240) = 96 \text{ mA}$$

[b] $i_1 = \frac{80}{400}(240) = 48 \text{ mA}$

[c] Now that i_o and i_1 are known return to the original circuit



$$v_2 = (50)(0.048) + (600)(0.096) = 60 \text{ V}$$

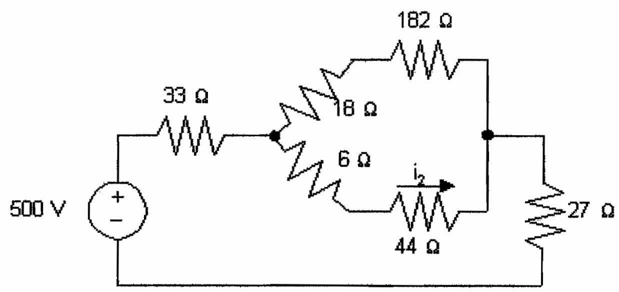
$$i_2 = \frac{v_2}{100} = \frac{60}{100} = 600 \text{ mA}$$

[d] $v_g = v_2 + 20(0.6 + 0.048) = 60 + 12.96 = 72.96 \text{ V}$

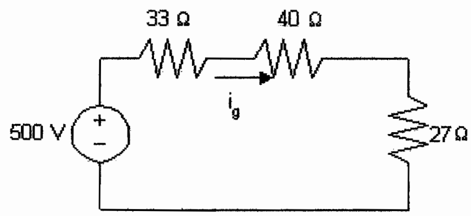
$$p_g = -(v_g)(1) = -72.96 \text{ W}$$

Thus the current source delivers 72.96 W.

P 3.60 [a] Replace the 30—60—10 Ω delta with a wye equivalent to get



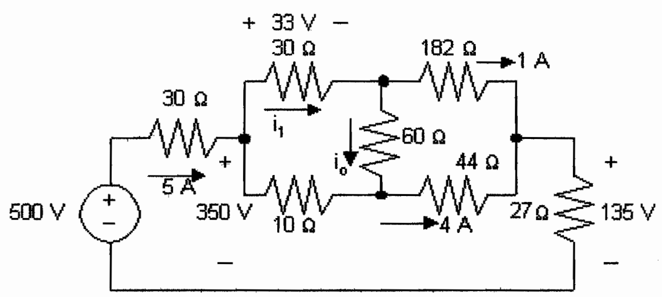
Using series/parallel reductions the circuit reduces to



$$i_g = \frac{500}{100} = 5 \text{ A}$$

$$i_2 = \frac{200}{250}(5) = 4 \text{ A}$$

[b] $i_1 = 33/30 = 1.1$ A
Returning to the original circuit we have



$$i_o = 1.1 - 1.0 = 0.1 \text{ A}$$

[c] $v = 60i_o = 6$ V

[d] $P_{\text{supplied}} = (500)(5.0) = 2500$ W

P 3.61 Subtracting Eq. 3.42 from Eq. 3.43 gives

$$R_1 - R_2 = (R_cR_b - R_cR_a)/(R_a + R_b + R_c).$$

Adding this expression to Eq. 3.41 and solving for R_1 gives

$$R_1 = R_cR_b/(R_a + R_b + R_c).$$

To find R_2, subtract Eq. 3.43 from Eq. 3.41 and add this result to Eq. 3.42. To find R_3, subtract Eq. 3.41 from Eq. 3.42 and add this result to Eq. 3.43. Using the hint, Eq. 3.43 becomes

$$R_1 + R_3 = \frac{R_b[(R_2/R_3)R_b + (R_2/R_1)R_b]}{(R_2/R_1)R_b + R_b + (R_2/R_3)R_b} = \frac{R_b(R_1 + R_3)R_2}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

Solving for R_b gives $R_b = (R_1R_2 + R_2R_3 + R_3R_1)/R_2$. To find R_a: First use Eqs. 3.44–3.46 to obtain the ratios $(R_1/R_3) = (R_c/R_a)$ or $R_c = (R_1/R_3)R_a$ and $(R_1/R_2) = (R_b/R_a)$ or $R_b = (R_1/R_2)R_a$. Now use these relationships to eliminate R_b and R_c from Eq. 3.42. To find R_c, use Eqs. 3.44–3.46 to obtain the ratios $R_b = (R_3/R_2)R_c$ and $R_a = (R_3/R_1)R_c$. Now use the relationships to eliminate R_b and R_a from Eq. 3.41.

P 3.62

$$\begin{aligned} G_a &= \frac{1}{R_a} = \frac{R_1}{R_1R_2 + R_2R_3 + R_3R_1} \\ &= \frac{1/G_1}{(1/G_1)(1/G_2) + (1/G_2)(1/G_3) + (1/G_3)(1/G_1)} \\ &= \frac{(1/G_1)(G_1G_2G_3)}{G_1 + G_2 + G_3} = \frac{G_2G_3}{G_1 + G_2 + G_3} \end{aligned}$$

Similar manipulations generate the expressions for G_b and G_c.

P 3.63 [a] $R_{ab} = 2R_1 + \dfrac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = R_L$

Therefore $\quad 2R_1 - R_L + \dfrac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = 0$

Thus $\quad R_L^2 = 4R_1^2 + 4R_1R_2 = 4R_1(R_1 + R_2)$

When $R_{ab} = R_L$, the current into terminal a of the attenuator will be v_i/R_L

Using current division, the current in the R_L branch will be

$$\frac{v_i}{R_L} \cdot \frac{R_2}{2R_1 + R_2 + R_L}$$

Therefore $\quad v_o = \dfrac{v_i}{R_L} \cdot \dfrac{R_2}{2R_1 + R_2 + R_L} R_L$

and $\quad \dfrac{v_o}{v_i} = \dfrac{R_2}{2R_1 + R_2 + R_L}$

[b] $(600)^2 = 4(R_1 + R_2)R_1$

$$9 \times 10^4 = R_1^2 + R_1R_2$$

$$\frac{v_o}{v_i} = 0.6 = \frac{R_2}{2R_1 + R_2 + 600}$$

$$\therefore \quad 1.2R_1 + 0.6R_2 + 360 = R_2$$

$$0.4R_2 = 1.2R_1 + 360$$

$$R_2 = 3R_1 + 900$$

$$\therefore \quad 9 \times 10^4 = R_1^2 + R_1(3R_1 + 900) = 4R_1^2 + 900R_1$$

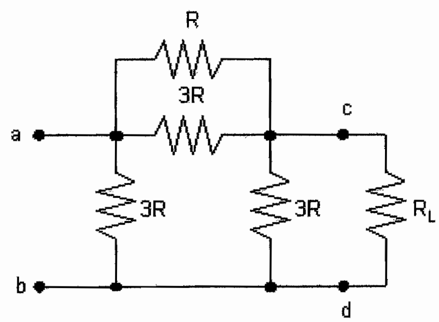
$$\therefore \quad R_1^2 + 225R_1 - 22{,}500 = 0$$

$$R_1 = -112.5 \pm \sqrt{(112.5)^2 + 22{,}500} = -112.5 \pm 187.5$$

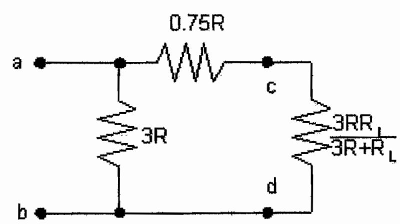
$$\therefore \quad R_1 = 75\,\Omega$$

$$\therefore \quad R_2 = 3(75) + 900 = 1125\,\Omega$$

P 3.64 [a] After making the Y-to-Δ transformation, the circuit reduces to



Combining the parallel resistors reduces the circuit to



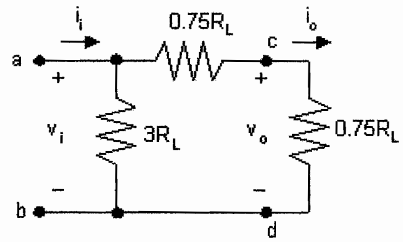
Now note: $$0.75R + \frac{3RR_L}{3R + R_L} = \frac{2.25R^2 + 3.75RR_L}{3R + R_L}$$

Therefore $$R_{ab} = \frac{3R\left(\dfrac{2.25R^2 + 3.75RR_L}{3R + R_L}\right)}{3R + \left(\dfrac{2.25R^2 + 3.75RR_L}{3R + R_L}\right)} = \frac{3R(3R + 5R_L)}{15R + 9R_L}$$

When $R_{ab} = R_L$, we have $15RR_L + 9R_L^2 = 9R^2 + 15RR_L$

Therefore $R_L^2 = R^2$ or $R_L = R$

[b] When $R = R_L$, the circuit reduces to



$$i_o = \frac{i_i(3R_L)}{4.5R_L} = \frac{1}{1.5}i_i = \frac{1}{1.5}\frac{v_i}{R_L}, \qquad v_o = 0.75R_L i_o = \frac{1}{2}v_i,$$

Therefore $\frac{v_o}{v_i} = 0.5$

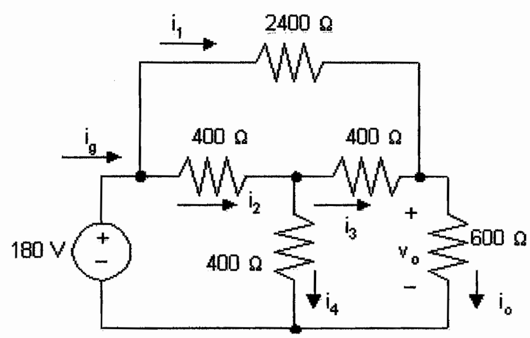
P 3.65 [a] $3(3R - R_L) = 3R + R_L$

$$9R - 1800 = 3R + 600$$

$$6R = 2400, \qquad R = 400\,\Omega$$

$$R_2 = \frac{2(400)(600)^2}{3(400)^2 - (600)^2} = 2400\,\Omega$$

[b]



$$v_o = \frac{v_i}{3} = \frac{180}{3} = 60\text{ V}$$

$$i_o = \frac{60}{600} = 100\text{ mA}$$

$$i_1 = \frac{180 - 60}{2400} = \frac{120}{2400} = 50\text{ mA}$$

$$i_g = \frac{180}{600} = 300\text{ mA}$$

$$i_2 = 300 - 50 = 250\text{ mA}$$

$$i_3 = 100 - 50 = 50\text{ mA}$$

$$i_4 = 250 - 50 = 200\text{ mA}$$

$$p_{2400\ \text{top}} = (50 \times 10^{-3})^2(2400) = 6\ \text{W}$$

$$p_{400\ \text{left}} = (250 \times 10^{-3})^2(400) = 25\ \text{W}$$

$$p_{400\ \text{right}} = (50 \times 10^{-3})^2(400) = 1\ \text{W}$$

$$p_{400\ \text{vertical}} = (200 \times 10^{-3})^2(400) = 16\ \text{W}$$

$$p_{600\ \text{load}} = (100 \times 10^{-3})^2(600) = 6\ \text{W}$$

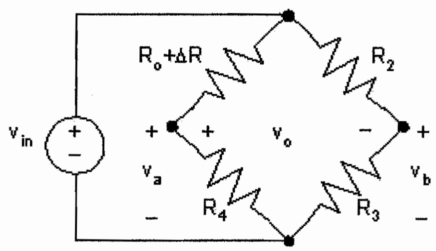
The 400 Ω resistor carrying i_2

[c] $p_{400\ \text{left}} = 25$ W

[d] The 400 Ω resistor carrying i_3

[e] $p_{400\ \text{right}} = 1$ W

P 3.66 [a]



$$v_a = \frac{v_{in}R_4}{R_o + R_4 + \Delta R}$$

$$v_b = \frac{R_3}{R_2 + R_3}v_{in}$$

$$v_o = v_a - v_b = \frac{R_4 v_{in}}{R_o + R_4 + \Delta R} - \frac{R_3}{R_2 + R_3}v_{in}$$

When the bridge is balanced,

$$\frac{R_4}{R_o + R_4}v_{in} = \frac{R_3}{R_2 + R_3}v_{in}$$

$$\therefore \quad \frac{R_4}{R_o + R_4} = \frac{R_3}{R_2 + R_3}$$

Thus,

$$\begin{aligned} v_o &= \frac{R_4 v_{in}}{R_o + R_4 + \Delta R} - \frac{R_4 v_{in}}{R_o + R_4} \\ &= R_4 v_{in}\left[\frac{1}{R_o + R_4 + \Delta R} - \frac{1}{R_o + R_4}\right] \\ &= \frac{R_4 v_{in}(-\Delta R)}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &\approx \frac{-(\Delta R)R_4 v_{in}}{(R_o + R_4)^2}, \qquad \text{since } \Delta R << R_4 \end{aligned}$$

[b] $\Delta R = 0.03R_o$

$$R_o = \frac{R_2R_4}{R_3} = \frac{(1000)(5000)}{500} = 10{,}000\,\Omega$$

$$\Delta R = (0.03)(10^4) = 300\,\Omega$$

$$\therefore \quad v_o \approx \frac{-300(5000)(6)}{(15{,}000)^2} = -40\text{ mV}$$

[c]
$$\begin{aligned} v_o &= \frac{-(\Delta R)R_4v_{\text{in}}}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &= \frac{-300(5000)(6)}{(15{,}300)(15{,}000)} \\ &= -39.2157\text{ mV} \end{aligned}$$

P 3.67 [a]
$$\text{approx value} = \frac{-(\Delta R)R_4v_{\text{in}}}{(R_o + R_4)^2}$$

$$\text{true value} = \frac{-(\Delta R)R_4v_{\text{in}}}{(R_o + R_4 + \Delta R)(R_o + R_4)}$$

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{(R_o + R_4 + \Delta R)}{(R_o + R_4)}$$

$$\therefore \quad \%\text{ error} = \left[\frac{R_o + R_4 + \Delta R}{R_o + R_4} - 1\right] \times 100 = \frac{\Delta R}{R_o + R_4} \times 100$$

But $R_o = \dfrac{R_2R_4}{R_3}$

$$\therefore \quad \%\text{ error} = \frac{R_3\Delta R}{R_4(R_2 + R_3)}$$

[b]
$$\%\text{ error} = \frac{(500)(300)}{(5000)(1500)} \times 100 = 2\%$$

P 3.68
$$\frac{\Delta R(R_3)(100)}{(R_2 + R_3)R_4} = 0.5$$

$$\frac{\Delta R(500)(100)}{(1500)(5000)} = 0.5$$

$$\therefore \quad \Delta R = 75\,\Omega$$

$$\%\text{ change} = \frac{75}{10{,}000} \times 100 = 0.75\%$$

P 3.69 [a] From Eq 3.64 we have

$$\left(\frac{i_1}{i_2}\right)^2 = \frac{R_2^2}{R_1^2(1+2\sigma)^2}$$

Substituting into Eq 3.63 yields

$$R_2 = \frac{R_2^2}{R_1^2(1+2\sigma)^2} R_1$$

Solving for R_2 yields

$$R_2 = (1+2\sigma)^2 R_1$$

[b] From Eq 3.67 we have

$$\frac{i_1}{i_b} = \frac{R_2}{R_1 + R_2 + 2R_a}$$

But $R_2 = (1+2\sigma)^2 R_1$ and $R_a = \sigma R_1$ therefore

$$\frac{i_1}{i_b} = \frac{(1+2\sigma)^2 R_1}{R_1 + (1+2\sigma)^2 R_1 + 2\sigma R_1} = \frac{(1+2\sigma)^2}{(1+2\sigma) + (1+2\sigma)^2}$$

$$= \frac{1+2\sigma}{2(1+\sigma)}$$

It follows that

$$\left(\frac{i_1}{i_b}\right)^2 = \frac{(1+2\sigma)^2}{4(1+\sigma)^2}$$

Substituting into Eq 3.66 gives

$$R_b = \frac{(1+2\sigma)^2 R_a}{4(1+\sigma)^2} = \frac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2}$$

P 3.70 From Eq 3.69

$$\frac{i_1}{i_3} = \frac{R_2 R_3}{D}$$

But $D = (R_1 + 2R_a)(R_2 + 2R_b) + 2R_b R_2$

where $R_a = \sigma R_1$; $R_2 = (1+2\sigma)^2 R_1$ and $R_b = \dfrac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2}$

Therefore D can be written as

$$\begin{aligned}
D &= (R_1 + 2\sigma R_1)\left[(1+2\sigma)^2 R_1 + \frac{2(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2}\right] + \\
&\quad 2(1+2\sigma)^2 R_1\left[\frac{(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2}\right] \\
&= (1+2\sigma)^3 R_1^2\left[1 + \frac{\sigma}{2(1+\sigma)^2} + \frac{(1+2\sigma)\sigma}{2(1+\sigma)^2}\right] \\
&= \frac{(1+2\sigma)^3 R_1^2}{2(1+\sigma)^2}\{2(1+\sigma)^2 + \sigma + (1+2\sigma)\sigma\} \\
&= \frac{(1+2\sigma)^3 R_1^2}{(1+\sigma)^2}\{1 + 3\sigma + 2\sigma^2\}
\end{aligned}$$

$$D = \frac{(1+2\sigma)^4 R_1^2}{(1+\sigma)}$$

$$\begin{aligned}
\therefore\ \frac{i_1}{i_3} &= \frac{R_2 R_3(1+\sigma)}{(1+2\sigma)^4 R_1^2} \\
&= \frac{(1+2\sigma)^2 R_1 R_3(1+\sigma)}{(1+2\sigma)^4 R_1^2} \\
&= \frac{(1+\sigma)R_3}{(1+2\sigma)^2 R_1}
\end{aligned}$$

When this result is substituted into Eq 3.69 we get

$$R_3 = \frac{(1+\sigma)^2 R_3^2 R_1}{(1+2\sigma)^4 R_1^2}$$

Solving for R_3 gives

$$R_3 = \frac{(1+2\sigma)^4 R_1}{(1+\sigma)^2}$$

P 3.71 From the dimensional specifications, calculate σ and R_3:

$$\sigma = \frac{y}{x} = \frac{0.025}{1} = 0.025; \qquad R_3 = \frac{V_{\text{dc}}^2}{p} = \frac{12^2}{120} = 1.2\,\Omega$$

Calculate R_1 from R_3 and σ:

$$R_1 = \frac{(1+\sigma)^2}{(1+2\sigma)^4}R_3 = 1.0372\,\Omega$$

Calculate R_a, R_b, and R_2:

$$R_a = \sigma R_1 = 0.0259\,\Omega \qquad R_b = \frac{(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2} = 0.0068\,\Omega$$

$$R_2 = (1 + 2\sigma)^2 R_1 = 1.1435\,\Omega$$

Using symmetry,

$$R_4 = R_2 = 1.1435\,\Omega \qquad R_5 = R_1 = 1.0372\,\Omega$$

$$R_c = R_b = 0.0068\,\Omega \qquad R_d = R_a = 0.0259\,\Omega$$

Test the calculations by checking the power dissipated, which should be 120 W/m. Calculate D, then use Eqs. (3.58)-(3.60) to calculate i_b, i_1, and i_2:

$$D = (R_1 + 2R_a)(R_2 + 2R_b) + 2R_2R_b = 1.2758$$

$$i_b = \frac{V_{\text{dc}}(R_1 + R_2 + 2R_a)}{D} = 21 \text{ A}$$

$$i_1 = \frac{V_{\text{dc}}R_2}{D} = 10.7561 \text{ A} \qquad i_2 = \frac{V_{\text{dc}}(R_1 + 2R_a)}{D} = 10.2439 \text{ A}$$

It follows that $i_b^2R_b = 3$ W and the power dissipation per meter is $3/0.025 = 120$ W/m. The value of $i_1^2R_1 = 120$ W/m. The value of $i_2^2R_2 = 120$ W/m. Finally, $i_1^2R_a = 3$ W/m.

P 3.72 From the solution to Problem 3.71 we have $i_{\text{b}} = 21$ A and $i_3 = 10$ A. By symmetry $i_{\text{c}} = 21$ A thus the total current supplied by the 12 V source is $21 + 21 + 10$ or 52 A. Therefore the total power delivered by the source is $p_{12\text{V}}$ (del) $= (12)(52) = 624$ W. We also have from the solution that $p_{\text{a}} = p_{\text{b}} = p_{\text{c}} = p_{\text{d}} = 3$ W. Therefore the total power delivered to the vertical resistors is $p_{\text{V}} = (8)(3) = 24$ W. The total power delivered to the five horizontal resistors is $p_{\text{H}} = 5(120) = 600$ W.

$$\therefore \sum p_{\text{diss}} = p_{\text{H}} + p_{\text{V}} = 624 \text{ W} = \sum p_{\text{del}}$$

P 3.73 [a] $\sigma = 0.05/1.25 = 0.04$

Since the power dissipation is 150 W/m the power dissipated in R_3 must be 150(1.25) or 187.5 W. Therefore

$$R_3 = \frac{12^2}{187.5} = 0.768\,\Omega$$

From Table 3.1 we have

$$R_1 = \frac{(1 + \sigma)^2 R_3}{(1 + 2\sigma)^4} = 0.6106\,\Omega$$

$$R_{\text{a}} = \sigma R_1 = 0.0244\,\Omega$$

$$R_2 = (1 + 2\sigma)^2 R_1 = 0.7122\,\Omega$$

$$R_b = \frac{(1 + 2\sigma)^2 \sigma R_1}{4(1 + \sigma)^2} = 0.0066\,\Omega$$

Therefore

$$R_4 = R_2 = 0.7122\,\Omega \qquad R_5 = R_1 = 0.6106\,\Omega$$

$$R_c = R_b = 0.0066\,\Omega \qquad R_d = R_a = 0.0244\,\Omega$$

[b] $D = 0.4877$

$$i_1 = \frac{V_{dc} R_2}{D} = 17.52 \text{ A}$$

$$i_1^2 R_1 = 187.5 \text{ W or } 150 \text{ W/m}$$

$$i_2 = \frac{R_1 + 2R_a}{D} V_{dc} = 16.23 \text{ A}$$

$$i_2^2 R_2 = 187.5 \text{ W or } 150 \text{ W/m}$$

$$i_1^2 R_a = 7.5 \text{ W or } 150 \text{ W/m}$$

$$i_b = \frac{R_1 + R_2 + 2R_a}{D} V_{dc} = 33.75 \text{ A}$$

$$i_b^2 R_b = 7.5 \text{ W or } 150 \text{ W/m}$$

$$i_{source} = 33.75 + 33.75 + \frac{12}{0.768} = 83.125 \text{ A}$$

$$p_{del} = 12(83.125) = 997.50 \text{ W}$$

$$p_H = 5(187.5) = 937.5 \text{ W}$$

$$p_V = 8(7.5) = 60 \text{ W}$$

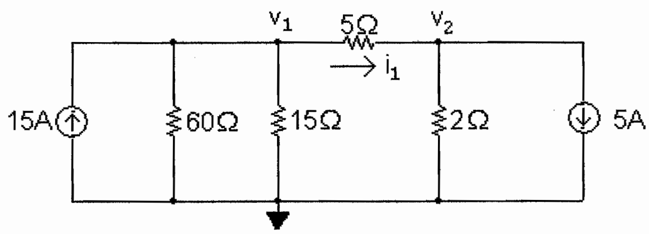
$$\sum p_{del} = \sum p_{diss} = 997.50 \text{ W}$$

4

Techniques of Circuit Analysis

Assessment Problems

AP 4.1 [a] Redraw the circuit, labeling the reference node and the two node voltages:



The two node voltage equations are

$$-15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0$$

$$5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{60} + \frac{1}{15} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) = 15$$

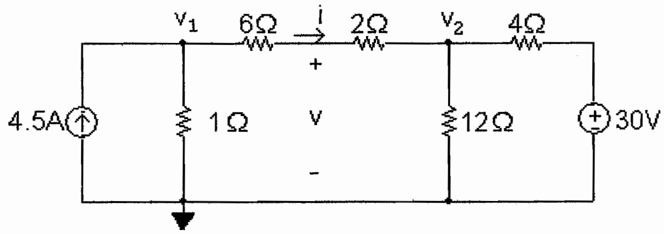
$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{2} + \frac{1}{5}\right) = -5$$

Solving, $v_1 = 60$ V and $v_2 = 10$ V;
Therefore, $i_1 = (v_1 - v_2)/5 = 10$ A

[b] $p_{15\text{A}} = -(15\text{ A})v_1 = -(15\text{ A})(60\text{ V}) = -900\text{ W} = 900\text{ W(delivered)}$

[c] $p_{5\text{A}} = (5\text{ A})v_2 = (5\text{ A})(10\text{ V}) = 50\text{ W}= -50\text{ W(delivered)}$

AP 4.2 Redraw the circuit, choosing the node voltages and reference node as shown:



The two node voltage equations are:

$$-4.5 + \frac{v_1}{1} + \frac{v_1 - v_2}{6+2} = 0$$

$$\frac{v_2}{12} + \frac{v_2 - v_1}{6+2} + \frac{v_2 - 30}{4} = 0$$

Place these equations in standard form:

$$v_1\left(1 + \frac{1}{8}\right) + v_2\left(-\frac{1}{8}\right) = 4.5$$

$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) = 7.5$$

Solving, $v_1 = 6$ V $\qquad v_2 = 18$ V

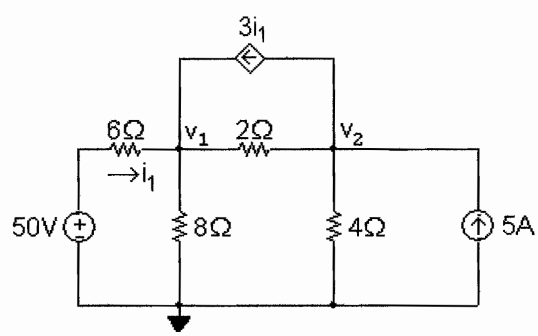
To find the voltage v, first find the current i through the series-connected $6\,\Omega$ and $2\,\Omega$ resistors:

$$i = \frac{v_1 - v_2}{6+2} = \frac{6-18}{8} = -1.5 \text{ A}$$

Using a KVL equation, calculate v:

$$v = 2i + v_2 = 2(-1.5) + 18 = 15 \text{ V}$$

AP 4.3 [a] Redraw the circuit, choosing the node voltages and reference node as shown:



The node voltage equations are:

$$\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0$$

$$-5 + \frac{v_2}{4} + \frac{v_2 - v_1}{2} + 3i_1 = 0$$

The dependent source requires the following constraint equation:

$$i_1 = \frac{50 - v_1}{6}$$

Place these equations in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{6}+\frac{1}{8}+\frac{1}{2}\right) &+ v_2\left(-\frac{1}{2}\right) &+ i_1(-3) &= \frac{50}{6}\\
v_1\left(-\frac{1}{2}\right) &+ v_2\left(\frac{1}{4}+\frac{1}{2}\right) &+ i_1(3) &= 5\\
v_1\left(\frac{1}{6}\right) &+ v_2(0) &+ i_1(1) &= \frac{50}{6}
\end{aligned}$$

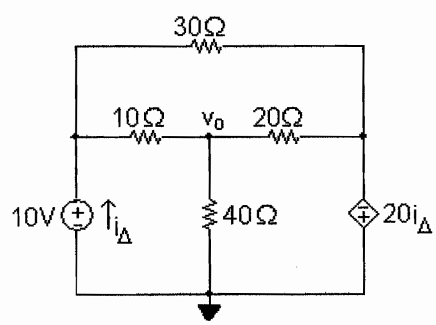
Solving, $v_1 = 32$ V; $v_2 = 16$ V; $i_1 = 3$ A

Using these values to calculate the power associated with each source:

$$\begin{aligned}
p_{50\text{V}} = -50i_1 &= -150\ \text{W}\\
p_{5\text{A}} = -5(v_2) &= -80\ \text{W}\\
p_{3i_1} = 3i_1(v_2 - v_1) &= -144\ \text{W}
\end{aligned}$$

[b] All three sources are delivering power to the circuit because the power computed in (a) for each of the sources is negative.

AP 4.4 Redraw the circuit and label the reference node and the node at which the node voltage equation will be written:



The node voltage equation is

$$\frac{v_o}{40} + \frac{v_o - 10}{10} + \frac{v_o + 20i_\Delta}{20} = 0$$

The constraint equation required by the dependent source is

$$i_\Delta = i_{10\,\Omega} + i_{30\,\Omega} = \frac{10 - v_o}{10} + \frac{10 + 20i_\Delta}{30}$$

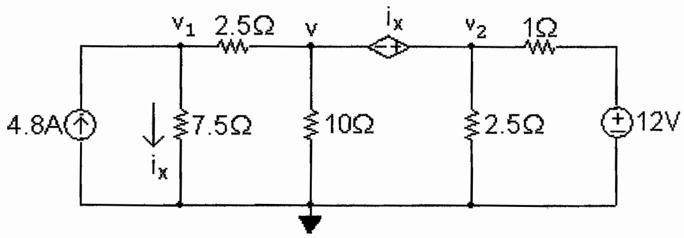
Place these equations in standard form:

$$v_o\left(\frac{1}{40}+\frac{1}{10}+\frac{1}{20}\right) + i_\Delta(1) = 1$$

$$v_o\left(\frac{1}{10}\right) + i_\Delta\left(1-\frac{20}{30}\right) = 1+\frac{10}{30}$$

Solving, $i_\Delta = -3.2$ A and $v_o = 24$ V

AP 4.5 Redraw the circuit identifying the three node voltages and the reference node:



Note that the dependent voltage source and the node voltages v and v_2 form a supernode. The v_1 node voltage equation is

$$\frac{v_1}{7.5}+\frac{v_1-v}{2.5}-4.8=0$$

The supernode equation is

$$\frac{v-v_1}{2.5}+\frac{v}{10}+\frac{v_2}{2.5}+\frac{v_2-12}{1}=0$$

The constraint equation due to the dependent source is

$$i_x=\frac{v_1}{7.5}$$

The constraint equation due to the supernode is

$$v+i_x=v_2$$

Place this set of equations in standard form:

$$v_1\left(\frac{1}{7.5}+\frac{1}{2.5}\right) + v\left(-\frac{1}{2.5}\right) + v_2(0) + i_x(0) = 4.8$$

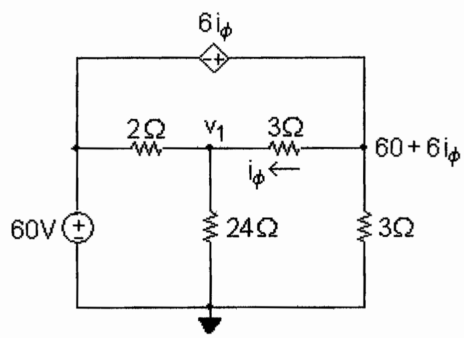
$$v_1\left(-\frac{1}{2.5}\right) + v\left(\frac{1}{2.5}+\frac{1}{10}\right) + v_2\left(\frac{1}{2.5}+1\right) + i_x(0) = 12$$

$$v_1\left(-\frac{1}{7.5}\right) + v(0) + v_2(0) + i_x(1) = 0$$

$$v_1(0) + v(1) + v_2(-1) + i_x(1) = 0$$

Solving this set of equations gives $v_1 = 15$ V, $v_2 = 10$ V, $i_x = 2$ A, and $v = 8$ V.

AP 4.6 Redraw the circuit identifying the reference node and the two unknown node voltages. Note that the right-most node voltage is the sum of the 60 V source and the dependent source voltage.



The node voltage equation at v_1 is

$$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - (60 + 6i_\phi)}{3} = 0$$

The constraint equation due to the dependent source is

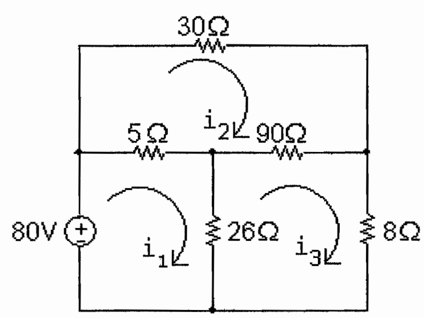
$$i_\phi = \frac{60 + 6i_\phi - v_1}{3}$$

Place these two equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{2} + \frac{1}{24} + \frac{1}{3}\right) &+ i_\phi(-2) &= 30 + 20 \\ v_1\left(\frac{1}{3}\right) &+ i_\phi(1 - 2) &= 20 \end{aligned}$$

Solving, $i_\phi = -4$ A and $v_1 = 48$ V

AP 4.7 [a] Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$\begin{aligned} -80 + 5(i_1 - i_2) + 26(i_1 - i_3) &= 0 \\ 30i_2 + 90(i_2 - i_3) + 5(i_2 - i_1) &= 0 \\ 8i_3 + 26(i_3 - i_1) + 90(i_3 - i_2) &= 0 \end{aligned}$$

Place these equations in standard form:

$$\begin{aligned} 31i_1 - 5i_2 - 26i_3 &= 80 \\ -5i_1 + 125i_2 - 90i_3 &= 0 \\ -26i_1 - 90i_2 + 124i_3 &= 0 \end{aligned}$$

Solving,

$i_1 = 5$ A; $\quad i_2 = 2$ A; $\quad i_3 = 2.5$ A

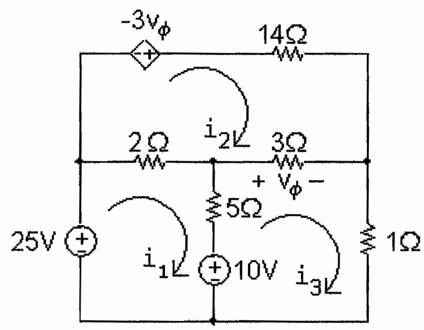
$p_{80V} = -(80)i_1 = -(80)(5) = -400$ W

Therefore the 80 V source is delivering 400 W to the circuit.

[b] $p_{8\Omega} = (8)i_3^2 = 8(2.5)^2 = 50$ W, so the 8 Ω resistor dissipates 50 W.

AP 4.8 **[a]** $b = 8, \quad n = 6, \quad b - n + 1 = 3$

[b] Redraw the circuit identifying the three mesh currents:



The three mesh-current equations are

$$\begin{aligned} -25 + 2(i_1 - i_2) + 5(i_1 - i_3) + 10 &= 0 \\ -(-3v_\phi) + 14i_2 + 3(i_2 - i_3) + 2(i_2 - i_1) &= 0 \\ 1i_3 - 10 + 5(i_3 - i_1) + 3(i_3 - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is

$v_\phi = 3(i_3 - i_2)$

Place these four equations in standard form:

$$\begin{aligned} 7i_1 - 2i_2 - 5i_3 + 0v_\phi &= 15 \\ -2i_1 + 19i_2 - 3i_3 + 3v_\phi &= 0 \\ -5i_1 - 3i_2 + 9i_3 + 0v_\phi &= 10 \\ 0i_1 + 3i_2 - 3i_3 + 1v_\phi &= 0 \end{aligned}$$

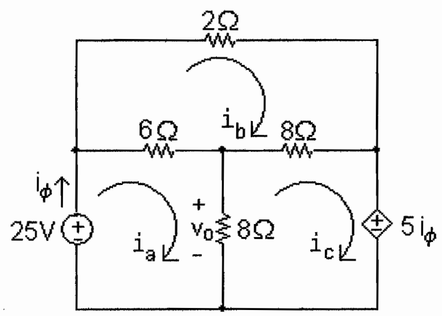
Solving

$i_1 = 4$ A; $\quad i_2 = -1$ A; $\quad i_3 = 3$ A; $\quad v_\phi = 12$ V

$$p_{ds} = -(-3v_\phi)i_2 = 3(12)(-1) = -36 \text{ W}$$

Thus, the dependent source is delivering 36 W, or absorbing −36 W.

AP 4.9 Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$\begin{aligned}
-25 + 6(i_a - i_b) + 8(i_a - i_c) &= 0 \\
2i_b + 8(i_b - i_c) + 6(i_b - i_a) &= 0 \\
5i_\phi + 8(i_c - i_a) + 8(i_c - i_b) &= 0
\end{aligned}$$

The dependent source constraint equation is $i_\phi = i_a$. We can substitute this simple expression for i_ϕ into the third mesh equation and place the equations in standard form:

$$\begin{aligned}
14i_a - 6i_b - 8i_c &= 25 \\
-6i_a + 16i_b - 8i_c &= 0 \\
-3i_a - 8i_b + 16i_c &= 0
\end{aligned}$$

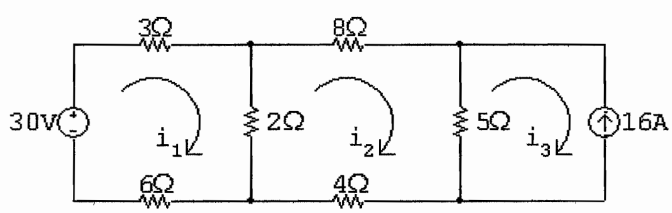
Solving,

$i_a = 4$ A; $\quad i_b = 2.5$ A; $\quad i_c = 2$ A

Thus,

$$v_o = 8(i_a - i_c) = 8(4 - 2) = 16 \text{ V}$$

AP 4.10 Redraw the circuit identifying the mesh currents:



Since there is a current source on the perimeter of the i_3 mesh, we know that $i_3 = -16$ A. The remaining two mesh equations are

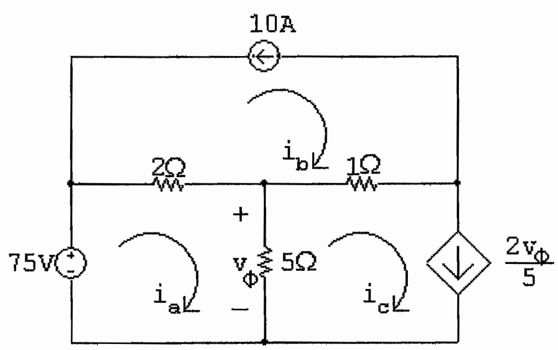
$$-30 + 3i_1 + 2(i_1 - i_2) + 6i_1 = 0$$
$$8i_2 + 5(i_2 + 16) + 4i_2 + 2(i_2 - i_1) = 0$$

Place these equations in standard form:

$$11i_1 - 2i_2 = 30$$
$$-2i_1 + 19i_2 = -80$$

Solving: $i_1 = 2$ A, $i_2 = -4$ A, $i_3 = -16$ A
The current in the $2\,\Omega$ resistor is $i_1 - i_2 = 6$ A $\therefore$ $p_{2\Omega} = (6)^2(2) = 72$ W
Thus, the $2\,\Omega$ resistors dissipates 72 W.

AP 4.11 Redraw the circuit and identify the mesh currents:



There are current sources on the perimeters of both the i_b mesh and the i_c mesh, so we know that

$$i_b = -10 \text{ A}; \qquad i_c = \frac{2v_\phi}{5}$$

The remaining mesh current equation is

$$-75 + 2(i_a + 10) + 5(i_a - 0.4v_\phi) = 0$$

The dependent source requires the following constraint equation:

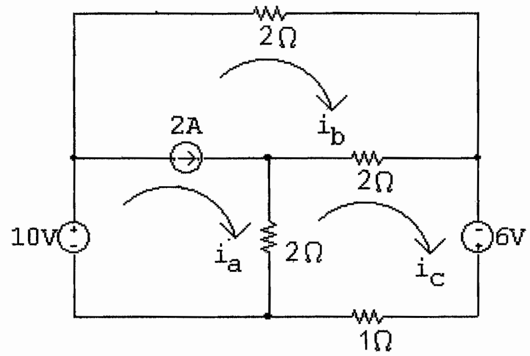
$$v_\phi = 5(i_a - i_c) = 5(i_a - 0.4v_\phi)$$

Place the mesh current equation and the dependent source equation is standard form:

$$7i_a - 2v_\phi = 55$$
$$5i_a - 3v_\phi = 0$$

Solving: $i_a = 15$ A; $i_b = -10$ A; $i_c = 10$ A; $v_\phi = 25$ V
Thus, $i_a = 15$ A.

AP 4.12 Redraw the circuit and identify the mesh currents:



The 2 A current source is shared by the meshes i_a and i_b. Thus we combine these meshes to form a supermesh and write the following equation:

$$-10 + 2i_b + 2(i_b - i_c) + 2(i_a - i_c) = 0$$

The other mesh current equation is

$$-6 + 1i_c + 2(i_c - i_a) + 2(i_c - i_b) = 0$$

The supermesh constraint equation is

$$i_a - i_b = 2$$

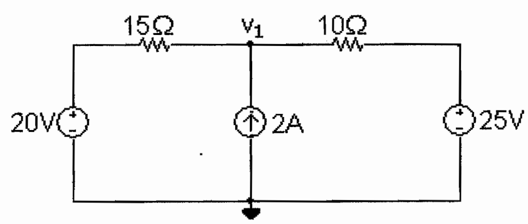
Place these three equations in standard form:

$$\begin{aligned} 2i_a + 4i_b - 4i_c &= 10 \\ -2i_a - 2i_b + 5i_c &= 6 \\ i_a - i_b + 0i_c &= 2 \end{aligned}$$

Solving, $i_a = 7$ A; $i_b = 5$ A; $i_c = 6$ A

Thus, $p_{1\,\Omega} = i_c^2(1) = (6)^2(1) = 36$ W

AP 4.13 Redraw the circuit and identify the reference node and the node voltage v_1:



The node voltage equation is

$$\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0$$

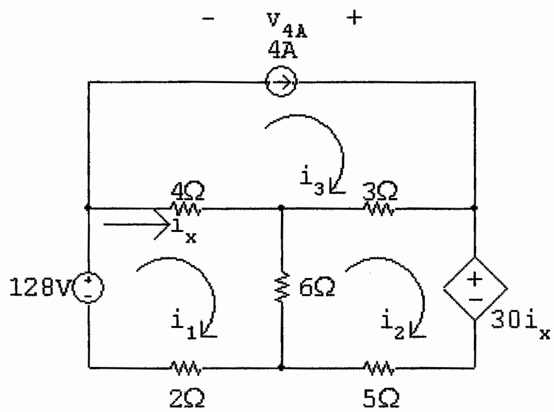
Rearranging and solving,

$$v_1\left(\frac{1}{15}+\frac{1}{10}\right)=2+\frac{20}{15}+\frac{25}{10} \qquad \therefore \quad v_1=35 \text{ V}$$

$$p_{2A}=-35(2)=-70 \text{ W}$$

Thus the 2 A current source delivers 70 W.

AP 4.14 Redraw the circuit and identify the mesh currents:



There is a current source on the perimeter of the i_3 mesh, so $i_3 = 4$ A. The other two mesh current equations are

$$-128+4(i_1-4)+6(i_1-i_2)+2i_1 = 0$$
$$30i_x+5i_2+6(i_2-i_1)+3(i_2-4) = 0$$

The constraint equation due to the dependent source is

$$i_x=i_1-i_3=i_1-4$$

Substitute the constraint equation into the second mesh equation and place the resulting two mesh equations in standard form:

$$12i_1-6i_2 = 144$$
$$24i_1+14i_2 = 132$$

Solving,

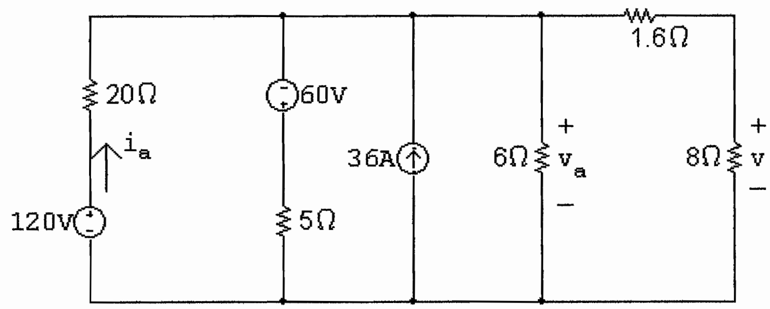
$$i_1=9 \text{ A}; \qquad i_2=-6 \text{ A}; \qquad i_3=4 \text{ A}; \qquad i_x=9-4=5 \text{ A}$$

$$\therefore \quad v_{4A}=3(i_3-i_2)-4i_x=10 \text{ V}$$

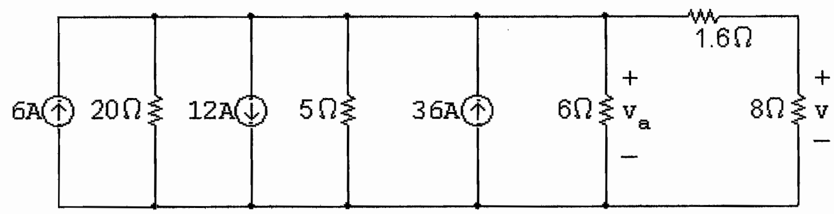
$$p_{4A}=-v_{4A}(4)=-(10)(4)=-40 \text{ W}$$

Thus, the 2 A current source delivers 40 W.

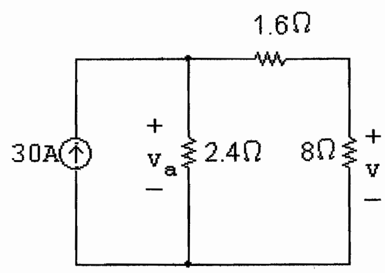
AP 4.15 [a] Redraw the circuit with a helpful voltage and current labeled:

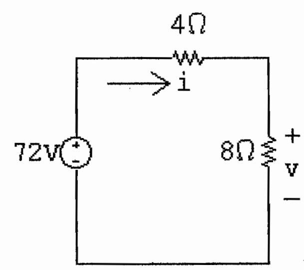


Transform the 120 V source in series with the 20 Ω resistor into a 6 A source in parallel with the 20 Ω resistor. Also transform the −60 V source in series with the 5 Ω resistor into a −12 A source in parallel with the 5 Ω resistor. The result is the following circuit:



Combine the three current sources into a single current source, using KCL, and combine the 20 Ω, 5 Ω, and 6 Ω resistors in parallel. The resulting circuit is shown on the left. To simplify the circuit further, transform the resulting 30 A source in parallel with the 2.4 Ω resistor into a 72 V source in series with the 2.4 Ω resistor. Combine the 2.4 Ω resistor in series with the 1.6 Ω resisor to get a very simple circuit that still maintains the voltage v. The resulting circuit is on the right.





Use voltage division in the circuit on the right to calculate v as follows:

$$v = \frac{8}{12}(72) = 48 \text{ V}$$

[b] Calculate i in the circuit on the right using Ohm's law:

$$i = \frac{v}{8} = \frac{48}{8} = 6 \text{ A}$$

Now use i to calculate v_{a} in the circuit on the left:

$v_{\text{a}} = 6(1.6 + 8) = 57.6$ V

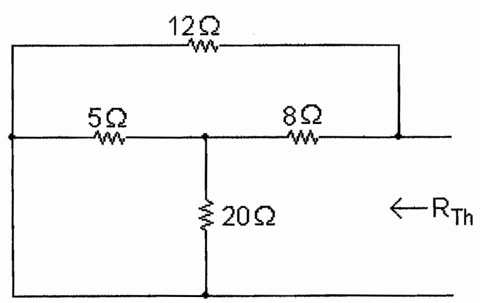
Returning back to the original circuit, note that the voltage v_{a} is also the voltage drop across the series combination of the 120 V source and 20 Ω resistor. Use this fact to calculate the current in the 120 V source, i_{a}:

$$i_{\text{a}} = \frac{120 - v_{\text{a}}}{20} = \frac{120 - 57.6}{20} = 3.12 \text{ A}$$

$$p_{120V} = -(120)i_{\text{a}} = -(120)(3.12) = -374.40 \text{ W}$$

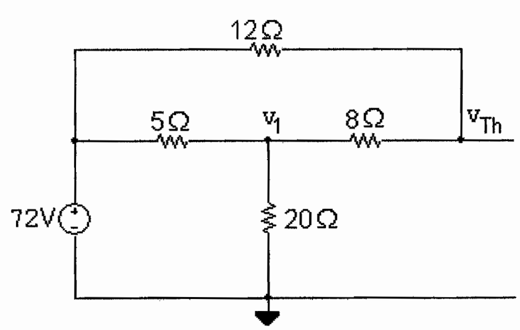
Thus, the 120 V source delivers 374.4 W.

AP 4.16 To find R_{Th}, replace the 72 V source with a short circuit:



Note that the 5 Ω and 20 Ω resistors are in parallel, with an equivalent resistance of $5\|20 = 4\,\Omega$. The equivalent 4 Ω resistance is in series with the 8 Ω resistor for an equivalent resistance of $4 + 8 = 12\,\Omega$. Finally, the 12 Ω equivalent resistance is in parallel with the 12 Ω resistor, so $R_{\text{Th}} = 12\|12 = 6\,\Omega$.

Use node voltage analysis to find v_{Th}. Begin by redrawing the circuit and labeling the node voltages:



The node voltage equations are

$$\frac{v_1 - 72}{5} + \frac{v_1}{20} + \frac{v_1 - v_{\text{Th}}}{8} = 0$$

$$\frac{v_{\text{Th}} - v_1}{8} + \frac{v_{\text{Th}} - 72}{12} = 0$$

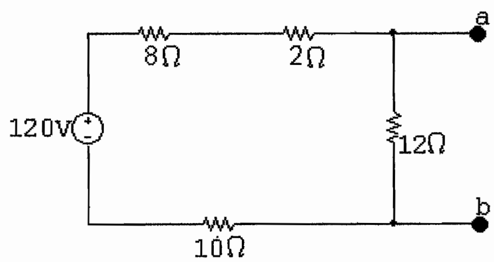
Place these equations in standard form:

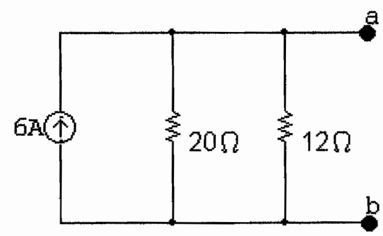
$$v_1\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{8}\right) + v_{\text{Th}}\left(-\frac{1}{8}\right) = \frac{72}{5}$$

$$v_1\left(-\frac{1}{8}\right) + v_{\text{Th}}\left(\frac{1}{8}+\frac{1}{12}\right) = 6$$

Solving, $v_1 = 60$ V and $v_{\text{Th}} = 64.8$ V. Therefore, the Thévenin equivalent circuit is a 64.8 V source in series with a $6\,\Omega$ resistor.

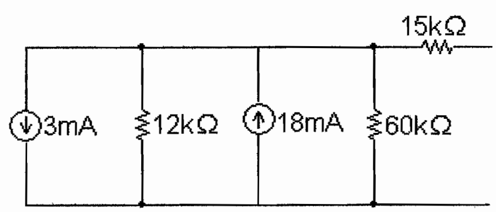
AP 4.17 We begin by performing a source transformation, turning the parallel combination of the 15 A source and $8\,\Omega$ resistor into a series combination of a 120 V source and an $8\,\Omega$ resistor, as shown in the figure on the left. Next, combine the $2\,\Omega$, $8\,\Omega$ and $10\,\Omega$ resistors in series to give an equivalent $20\,\Omega$ resistance. Then transform the series combination of the 120 V source and the $20\,\Omega$ equivalent resistance into a parallel combination of a 6 A source and a $20\,\Omega$ resistor, as shown in the figure on the right.



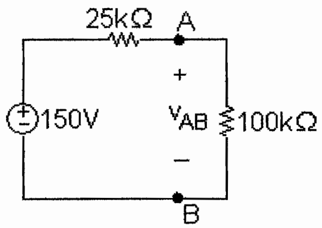


Finally, combine the $20\,\Omega$ and $12\,\Omega$ parallel resistors to give $R_{\text{N}} = 20\|12 = 7.5\,\Omega$. Thus, the Norton equivalent circuit is the parallel combination of a 6 A source and a $7.5\,\Omega$ resistor.

AP 4.18 Find the Thévenin equivalent with respect to A, B using source transformations. To begin, convert the series combination of the -36 V source and 12 kΩ resistor into a parallel combination of a -3 mA source and 12 kΩ resistor. The resulting circuit is shown below:



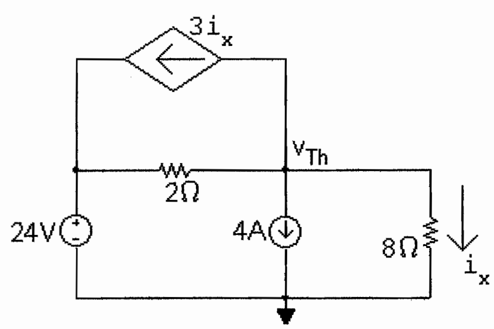
Now combine the two parallel current sources and the two parallel resistors to give a $-3+18=15$ mA source in parallel with a 12 k$\|$60 k$=$ 10 kΩ resistor. Then transform the 15 mA source in parallel with the 10 kΩ resistor into a 150 V source in series with a 10 kΩ resistor, and combine this 10 kΩ resistor in series with the 15 kΩ resistor. The Thévenin equivalent is thus a 150 V source in series with a 25 kΩ resistor, as seen to the left of the terminals A,B in the circuit below.



Now attach the voltmeter, modeled as a 100 kΩ resistor, to the Thévenin equivalent and use voltage division to calculate the meter reading v_{AB}:

$$v_{AB} = \frac{100{,}000}{125{,}000}(150) = 120 \text{ V}$$

AP 4.19 Begin by calculating the open circuit voltage, which is also v_{Th}, from the circuit below:



Summing the currents away from the node labeled v_{Th} We have

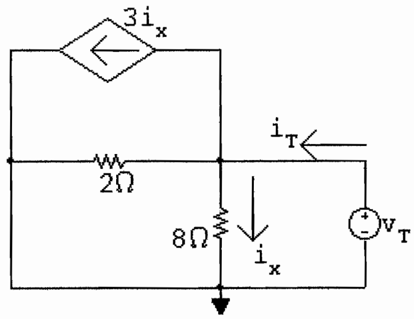
$$\frac{v_{Th}}{8} + 4 + 3i_x + \frac{v_{Th} - 24}{2} = 0$$

Also, using Ohm's law for the 8 Ω resistor,

$$i_x = \frac{v_{Th}}{8}$$

Substituting the second equation into the first and solving for v_{Th} yields $v_{Th} = 8$ V.

Now calculate R_{Th}. To do this, we use the test source method. Replace the voltage source with a short circuit, the current source with an open circuit, and apply the test voltage v_T, as shown in the circuit below:



Write a KCL equation at the middle node:

$$i_T = i_x + 3i_x + v_T/2 = 4i_x + v_T/2$$

Use Ohm's law to determine i_x as a function of v_T:

$$i_x = v_T/8$$

Substitute the second equation into the first equation:

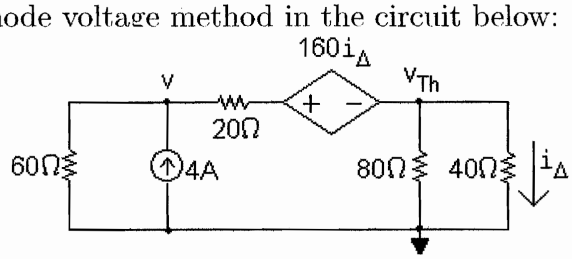
$$i_T = 4(v_T/8) + v_T/2 = v_T$$

Thus,

$$R_{Th} = v_T/i_T = 1\,\Omega$$

The Thévenin equivalent is an 8 V source in series with a $1\,\Omega$ resistor.

AP 4.20 Begin by calculating the open circuit voltage, which is also v_{Th}, using the node voltage method in the circuit below:



The node voltage equations are

$$\frac{v}{60} + \frac{v - (v_{Th} + 160i_\Delta)}{20} - 4 = 0,$$
$$\frac{v_{Th}}{40} + \frac{v_{Th}}{80} + \frac{v_{Th} + 160i_\Delta - v}{20} = 0$$

The dependent source constraint equation is

$$i_\Delta = \frac{v_{Th}}{40}$$

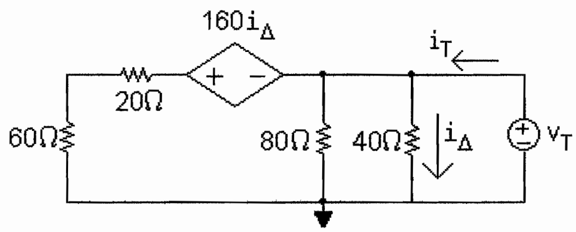
Substitute the constraint equation into the node voltage equations and put the two equations in standard form:

$$v\left(\frac{1}{60}+\frac{1}{20}\right) + v_{\text{Th}}\left(-\frac{5}{20}\right) = 4$$

$$v\left(-\frac{1}{20}\right) + v_{\text{Th}}\left(\frac{1}{40}+\frac{1}{80}+\frac{5}{20}\right) = 0$$

Solving, $v = 172.5$ V and $v_{\text{Th}} = 30$ V.

Now use the test source method to calculate the test current and thus R_{Th}. Replace the current source with a short circuit and apply the test source to get the following circuit:



Write a KCL equation at the rightmost node:

$$i_{\text{T}} = \frac{v_{\text{T}}}{80} + \frac{v_{\text{T}}}{40} + \frac{v_{\text{T}} + 160i_{\Delta}}{80}$$

The dependent source constraint equation is

$$i_{\Delta} = \frac{v_{\text{T}}}{40}$$

Substitute the constraint equation into the KCL equation and simplify the right-hand side:

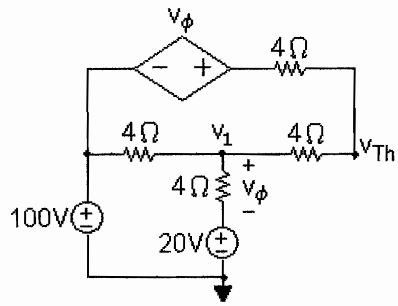
$$i_{\text{T}} = \frac{v_{\text{T}}}{10}$$

Therefore,

$$R_{\text{Th}} = \frac{v_{\text{T}}}{i_{\text{T}}} = 10\,\Omega$$

Thus, the Thévenin equivalent is a 30 V source in series with a 10 Ω resistor.

AP 4.21 First find the Thévenin equivalent circuit. To find v_{Th}, create an open circuit between nodes a and b and use the node voltage method with the circuit below:



The node voltage equations are:

$$\frac{v_{\text{Th}} - (100 + v_\phi)}{4} + \frac{v_{\text{Th}} - v_1}{4} = 0$$

$$\frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1 - v_{\text{Th}}}{4} = 0$$

The dependent source constraint equation is

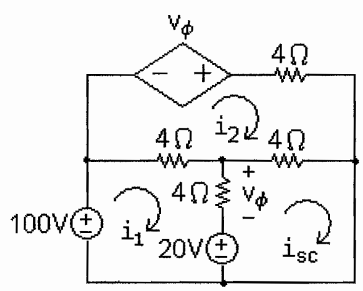
$$v_\phi = v_1 - 20$$

Place these three equations in standard form:

$$\begin{aligned}
v_{\text{Th}}\left(\frac{1}{4}+\frac{1}{4}\right) &+ v_1\left(-\frac{1}{4}\right) &+ v_\phi\left(-\frac{1}{4}\right) &= 25\\
v_{\text{Th}}\left(-\frac{1}{4}\right) &+ v_1\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right) &+ v_\phi(0) &= 30\\
v_{\text{Th}}(0) &+ v_1(1) &+ v_\phi(-1) &= 20
\end{aligned}$$

Solving, $v_{\text{Th}} = 120$ V, $v_1 = 80$ V, and $v_\phi = 60$ V.

Now create a short circuit between nodes a and b and use the mesh current method with the circuit below:



The mesh current equations are

$$\begin{aligned}
-100 + 4(i_1 - i_2) + v_\phi + 20 &= 0\\
-v_\phi + 4i_2 + 4(i_2 - i_{\text{sc}}) + 4(i_2 - i_1) &= 0\\
-20 - v_\phi + 4(i_{\text{sc}} - i_2) &= 0
\end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_{sc})$$

Place these four equations in standard form:

$$\begin{aligned} 4i_1 - 4i_2 + 0i_{sc} + v_\phi &= 80 \\ -4i_1 + 12i_2 - 4i_{sc} - v_\phi &= 0 \\ 0i_1 - 4i_2 + 4i_{sc} - v_\phi &= 20 \\ 4i_1 + 0i_2 - 4i_{sc} - v_\phi &= 0 \end{aligned}$$

Solving, $i_1 = 45$ A, $i_2 = 30$ A, $i_{sc} = 40$ A, and $v_\phi = 20$ V. Thus,

$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{120}{40} = 3\,\Omega$$

[a] For maximum power transfer, $R = R_{Th} = 3\,\Omega$

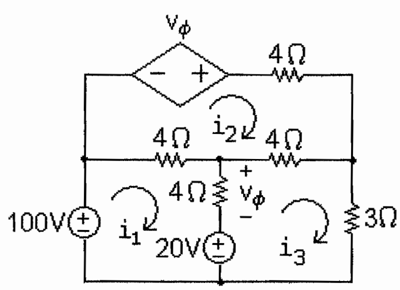
[b] The Thévenin voltage, $v_{Th} = 120$ V, splits equally between the Thévenin resistance and the load resistance, so

$$v_{load} = \frac{120}{2} = 60 \text{ V}$$

Therefore,

$$p_{max} = \frac{v_{load}^2}{R_{load}} = \frac{60^2}{3} = 1200 \text{ W}$$

AP 4.22 Sustituting the value $R = 3\,\Omega$ into the circuit and identifying three mesh currents we have the circuit below:



The mesh current equations are:

$$\begin{aligned} -100 + 4(i_1 - i_2) + v_\phi + 20 &= 0 \\ -v_\phi + 4i_2 + 4(i_2 - i_3) + 4(i_2 - i_1) &= 0 \\ -20 - v_\phi + 4(i_3 - i_2) + 3i_3 &= 0 \end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_3)$$

Place these four equations in standard form:

$$\begin{aligned} 4i_1 - 4i_2 + 0i_3 + v_\phi &= 80 \\ -4i_1 + 12i_2 - 4i_3 - v_\phi &= 0 \\ 0i_1 - 4i_2 + 7i_3 - v_\phi &= 20 \\ 4i_1 + 0i_2 - 4i_3 - v_\phi &= 0 \end{aligned}$$

Solving, $i_1 = 30$ A, $i_2 = 20$ A, $i_3 = 20$ A, and $v_\phi = 40$ V.

[**a**] $p_{100\text{V}} = -(100)i_1 = -(100)(30) = -3000$ W. Thus, the 100 V source is delivering 3000 W.

[**b**] $p_{\text{depsource}} = -v_\phi i_2 = -(40)(20) = -800$ W. Thus, the dependent source is delivering 800 W.

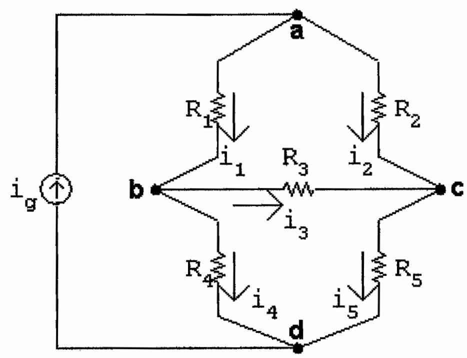
[**c**] From Assessment Problem 4.21(b), the power delivered to the load resistor is 1200 W, so the load power is $(1200/3800)100 = 31.58\%$ of the combined power generated by the 100 V source and the dependent source.

Problems

P 4.1 [**a**] There are six circuit components, five resistors and the current source. Since the current is known only in the current source, it is unknown in the five resistors. Therefore there are **five** unknown currents.

[**b**] There are four essential nodes in this circuit, identified by the dark black dots in Fig. P4.4. At three of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the fourth node would be dependent on the first three. Therefore there are **three** independent KCL equations.

[**c**]



Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

$$-i_g + i_1 + i_2 = 0$$

$$-i_1 + i_4 + i_3 = 0$$

$$i_5 - i_2 - i_3 = 0$$

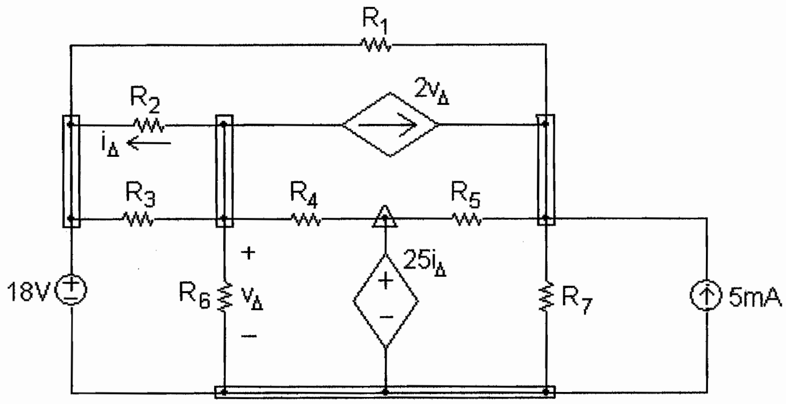
[**d**] There are three meshes in this circuit: one on the left with the components i_g, R_1, and R_4; one on the top right with components R_1, R_2, and R_3; and one on the bottom right with components R_3, R_4, and R_5. We cannot write a KVL equation for the left mesh because we don't know the voltage drop across the current source. Therefore, we can write KVL equations for the two meshes on the right, giving a total of **two** independent KVL equations.

[**e**] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

$$R_1i_1 + R_3i_3 - R_2i_2 = 0$$

$$R_3i_3 + R_5i_5 - R_4i_4 = 0$$

P 4.2



[a] 11 branches, 7 branches with resistors, 2 branches with independent sources, 2 branches with dependent sources

[b] The current is unknown in every branch except the one containing the 5 mA current source, so the current is unknown in 10 branches.

[c] 11 essential branches each containing a single element.

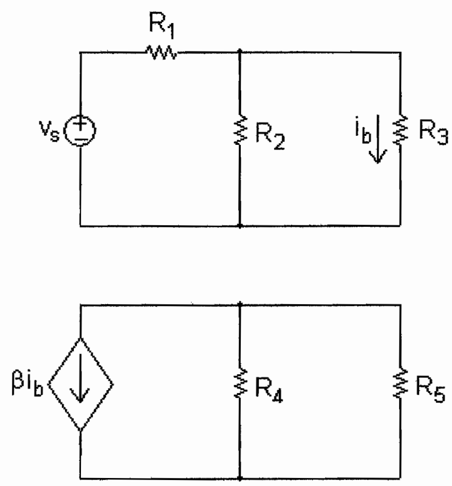
[d] The current is known only in the essential branch containing the current source, and is unknown in the remaining 10 essential branches

[e] From the figure there are 5 nodes – four identified by rectangular boxes and one identified by a triangle.

[f] There are 5 essential nodes, four identified with rectangular boxes and one identified with a triangle

[g] A mesh is like a window pane, and as can be seen from the figure there are 7 window panes or meshes.

P 4.3



[a] As can be seen from the figure, the circuit has 2 separate parts.

[b] There are 5 nodes – the four black dots and the node betweem the voltage source and the resistor R_1.

[c] There are 7 branches, each containing one of the seven circuit components.

[d] When a conductor joins the lower nodes of the two separate parts, there is now only a single part in the circuit. There would now be 4 nodes, because the two lower nodes are now joined as a single node. The number of branches remains at 7, where each branch contains one of the seven individual circuit components.

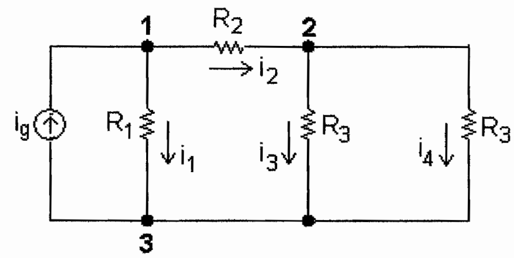
P 4.4 [a] From Problem 4.2(d) there are 10 essential branches were the current is unknown, so we need 10 simultaneous equations to describe the circuit.

[b] From Problem 4.2(f), there are 5 essential nodes, so we can apply KCL at $(5-1) = 4$ of these essential nodes. There would also be two dependent source constraint equations.

[c] The remaining 4 equations needed to describe the circuit will be derived from KVL equations.

[d] We must avoid using the meshes containing current sources, as we have no way of determining the voltage drop across a current source.

P 4.5



[a] At node 1: $\quad -i_g + i_1 + i_2 = 0$

At node 2: $\quad -i_2 + i_3 + i_4 = 0$

At node 3: $\quad i_g - i_1 - i_3 - i_4 = 0$

[b] There are many possible solutions. For example, solve the equation at node 1 for i_g:

$i_g = i_1 + i_2$

Substitute this expression for i_g into the equation at node 3:

$(i_1 + i_2) - i_1 - i_3 - i_4 = 0 \qquad \text{so} \qquad i_2 - i_3 - i_4 = 0$

Multiply this last equation by -1 to get the equation at node 2:

$-(i_2 - i_3 - i_4) = -0 \qquad \text{so} \qquad -i_2 + i_3 + i_4 = 0$

P 4.6 Use the lower terminal of the 5 Ω resistor as the reference node.

$$\frac{v_o - 60}{10} + \frac{v_o}{5} + 3 = 0$$

Solving, $\quad v_o = 10\ \text{V}$

P 4.7 [a] From the solution to Problem 4.5 we know $v_o = 10$ V, therefore

$$p_{3\text{A}} = 3v_o = 30 \text{ W}$$

$$\therefore \quad p_{3\text{A}} \text{ (developed)} \ = -30 \text{ W}$$

[b] The current into the negative terminal of the 60 V source is

$$i_g = \frac{60 - 10}{10} = 5 \text{ A}$$

$$p_{60\text{V}} = -60(5) = -300 \text{ W}$$

$$\therefore \quad p_{60\text{V}} \text{ (developed)} \ = 300 \text{ W}$$

[c] $p_{10\Omega} = (5)^2(10) = 250$ W

$$p_{5\Omega} = (10)^2/5 = 20 \text{ W}$$

$$\sum p_{\text{dev}} = 300 \text{ W}$$

$$\sum p_{\text{dis}} = 250 + 20 + 30 = 300 \text{ W}$$

P 4.8 [a] $\dfrac{v_0 - 60}{10} + \dfrac{v_o}{5} + 3 = 0; \qquad v_o = 10$ V

[b] Let $v_x =$ voltage drop across 3 A source

$$v_x = v_o - (10)(3) = -20 \text{ V}$$

$$p_{3\text{A}} \text{ (developed)} \ = (3)(20) = 60 \text{ W}$$

[c] Let $i_g =$ be the current into the positive terminal of the 60 V source

$$i_g = (10 - 60)/10 = -5 \text{ A}$$

$$p_{60\text{V}} \text{ (developed)} \ = (5)(60) = 300 \text{ W}$$

[d] $\sum p_{\text{dis}} = (5)^2(10) + (3)^2(10) + (10)^2/5 = 360$ W

$$\sum p_{\text{dis}} = 300 + 60 = 360 \text{ W}$$

[e] v_o is independent of any finite resistance connected in series with the 3 A current source

P 4.9 $2.4 + \dfrac{v_1}{125} + \dfrac{v_1 - v_2}{25} = 0$

$$\frac{v_2 - v_1}{25} + \frac{v_2}{250} + \frac{v_2}{375} - 3.2 = 0$$

Solving, $v_1 = 25$ V; $\qquad v_2 = 90$ V

CHECK:

$$p_{125\Omega} = \frac{(25)^2}{125} = 5\ \text{W}$$

$$p_{25\Omega} = \frac{(90-25)^2}{25} = 169\ \text{W}$$

$$p_{250\Omega} = \frac{(90)^2}{250} = 32.4\ \text{W}$$

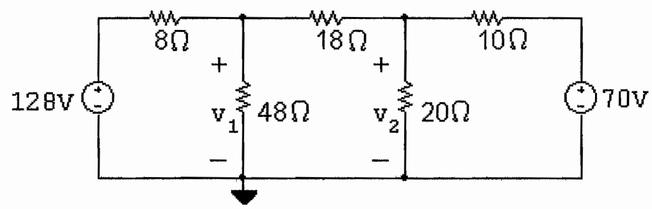
$$p_{375\Omega} = \frac{(90)^2}{375} = 21.6\ \text{W}$$

$$p_{2.4\text{A}} = (25)(2.4) = 60\ \text{W}$$

$$\sum p_{\text{abs}} = 5 + 169 + 32.4 + 21.6 + 60 = 288\ \text{W}$$

$$\sum p_{\text{dev}} = (90)(3.2) = 288\ \text{W} \quad \text{(CHECKS)}$$

P 4.10 [a]



$$\frac{v_1 - 128}{8} + \frac{v_1}{48} + \frac{v_1 - v_2}{18} = 0$$

$$\frac{v_2 - v_1}{18} + \frac{v_2}{20} + \frac{v_2 - 70}{10} = 0$$

In standard form,

$$v_1\left(\frac{1}{8} + \frac{1}{48} + \frac{1}{18}\right) + v_2\left(-\frac{1}{18}\right) = \frac{128}{8}$$

$$v_1\left(-\frac{1}{18}\right) + v_2\left(\frac{1}{18} + \frac{1}{20} + \frac{1}{10}\right) = \frac{70}{10}$$

Solving, $v_1 = 96$ V; $v_2 = 60$ V

$$i_a = \frac{128 - 96}{8} = 4\ \text{A}$$

$$i_b = \frac{96}{48} = 2\ \text{A}$$

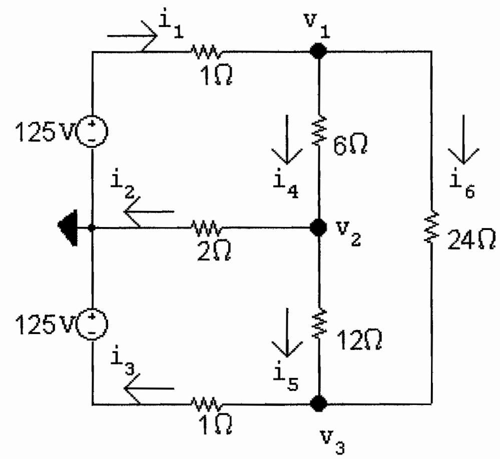
$$i_c = \frac{96 - 60}{18} = 2\ \text{A}$$

$$i_d = \frac{60}{20} = 3\text{ A}$$

$$i_e = \frac{60-70}{10} = -1\text{ A}$$

[b] $p_{dev} = 128(4) + 70(1) = 582$ W

P 4.11 [a]



$$\begin{aligned}
\frac{v_1-125}{1} + \frac{v_1-v_2}{6} + \frac{v_1-v_3}{24} &= 0\\
\frac{v_2-v_1}{6} + \frac{v_2}{2} + \frac{v_2-v_3}{12} &= 0\\
\frac{v_3+125}{1} + \frac{v_3-v_2}{12} + \frac{v_3-v_1}{24} &= 0
\end{aligned}$$

In standard form:

$$\begin{aligned}
v_1\left(\frac{1}{1}+\frac{1}{6}+\frac{1}{24}\right) + v_2\left(-\frac{1}{6}\right) + v_3\left(-\frac{1}{24}\right) &= 125\\
v_1\left(-\frac{1}{6}\right) + v_2\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{12}\right) + v_3\left(-\frac{1}{12}\right) &= 0\\
v_1\left(-\frac{1}{24}\right) + v_2\left(-\frac{1}{12}\right) + v_3\left(\frac{1}{1}+\frac{1}{12}+\frac{1}{24}\right) &= -125
\end{aligned}$$

Solving, $v_1 = 101.24$ V; $v_2 = 10.66$ V; $v_3 = -106.57$ V

Thus, $i_1 = \frac{125-v_1}{1} = 23.76$ A $\quad i_4 = \frac{v_1-v_2}{6} = 15$ A

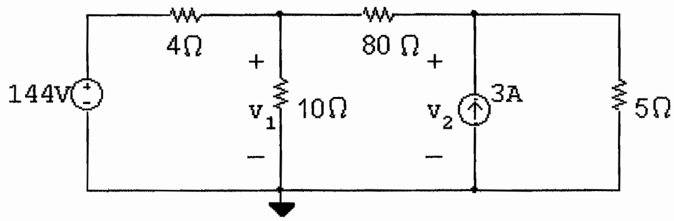
$i_2 = \frac{v_2}{2} = 5.33$ A $\quad i_5 = \frac{v_2-v_3}{12} = 9.77$ A

$i_3 = \frac{v_3+125}{1} = 18.43$ A $\quad i_6 = \frac{v_1-v_3}{24} = 8.66$ A

[b] $\sum P_{dev} = 125 i_1 + 125 i_3 = 5273.09$ W

$$\sum P_{dis} = i_1^2(1) + i_2^2(2) + i_3^2(1) + i_4^2(6) + i_5^2(12) + i_6^2(24) = 5273.09\text{ W}$$

P 4.12

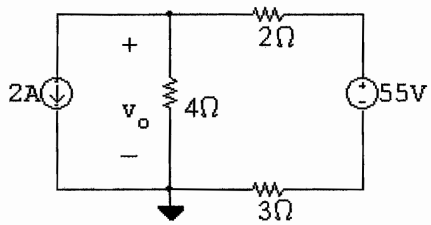


$$\frac{v_1-144}{4}+\frac{v_1}{10}+\frac{v_1-v_2}{80}=0 \qquad \text{so} \qquad 29v_1-v_2=2880$$

$$-3+\frac{v_2-v_1}{80}+\frac{v_2}{5}=0 \qquad \text{so} \qquad -v_1+17v_2=240$$

Solving, $v_1 = 100$ V; $v_2 = 20$ V

P 4.13

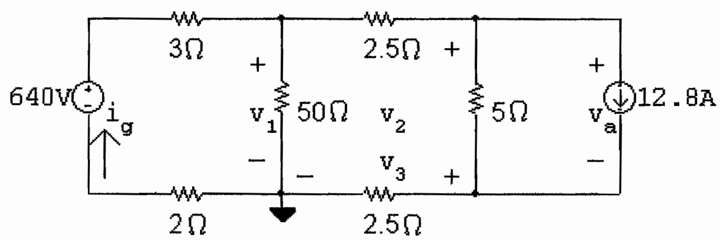


$$2+\frac{v_o}{4}+\frac{v_o-55}{5}=0$$

$v_o = 20$ V

$p_{2\mathrm{A}} = (20)(2) = 40$ W (absorbing)

P 4.14 [a]



$$\frac{v_1}{50}+\frac{v_1-640}{5}+\frac{v_1-v_2}{2.5}=0 \qquad \text{so} \qquad 31v_1-20v_2+0v_3=6400$$

$$\frac{v_2-v_1}{2.5}+\frac{v_2-v_3}{5}+12.8=0 \qquad \text{so} \qquad -2v_1+3v_2-v_3=-64$$

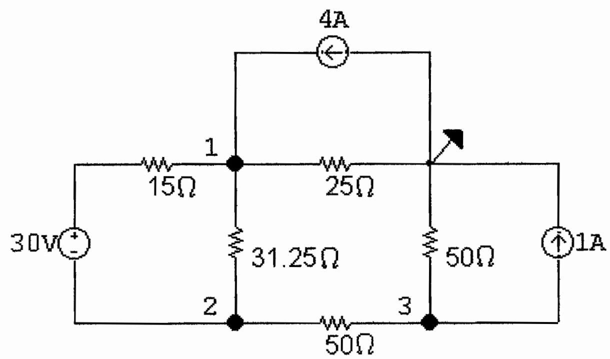
$$\frac{v_3}{2.5}+\frac{v_3-v_2}{5}-12.8=0 \qquad \text{so} \qquad 0v_1-v_2+3v_3=64$$

Solving, $v_1 = 380\ V$; $v_2 = 269$ V; $v_3 = 111$ V,

[b] $i_g = \dfrac{640-380}{5} = 52$ A

$p_g(\text{del}) = (640)(52) = 33{,}280$ W

P 4.15



$$\frac{v_1 - (v_2 + 30)}{15} + \frac{v_1 - v_2}{31.25} + \frac{v_1}{25} - 4 = 0$$

$$-\left[\frac{v_1 - (v_2 + 30)}{15}\right] + \frac{v_2 - v_3}{50} + \frac{v_2 - v_1}{31.25} = 0$$

$$\frac{v_3 - v_2}{50} + \frac{v_3}{50} + 1 = 0$$

Solving, $v_1 = 76$ V; $v_2 = 46$ V; $v_3 = -2$ V; $i_{30\text{V}} = 0$ A

$$p_{4\text{A}} = -4v_1 = -4(76) = -304 \text{ W} \quad \text{(del)}$$

$$p_{1\text{A}} = (1)(-2) = -2 \text{ W} \quad \text{(del)}$$

$$p_{30\text{V}} = (30)(0) = 0 \text{ W}$$

$$p_{15\Omega} = (0)^2(15) = 0 \text{ W}$$

$$p_{25\Omega} = \frac{v_1^2}{25} = \frac{76^2}{25} = 231.04 \text{ W}$$

$$p_{31.25\Omega} = \frac{(v_1 - v_2)^2}{31.25} = \frac{30^2}{31.25} = 28.8 \text{ W}$$

$$p_{50\Omega}(\text{lower}) = \frac{(v_2 - v_3)^2}{50} = \frac{48^2}{50} = 46.08 \text{ W}$$

$$p_{50\Omega}(\text{right}) = \frac{v_3^2}{50} = \frac{4}{50} = 0.08 \text{ W}$$

$$\sum p_{\text{diss}} = 0 + 231.04 + 28.8 + 46.08 + 0.08 = 306 \text{ W}$$

$$\sum p_{\text{dev}} = 304 + 2 = 306 \text{ W} \qquad \text{(CHECKS)}$$

P 4.16 [a] $\frac{v_o - v_1}{R} + \frac{v_o - v_2}{R} + \frac{v_o - v_3}{R} + \cdots + \frac{v_o - v_n}{R} = 0$

$$\therefore \quad nv_o = v_1 + v_2 + v_3 + \cdots + v_n$$

$$\therefore \quad v_o = \frac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \frac{1}{n}\sum_{k=1}^{n} v_k$$

[b] $v_o = \frac{1}{3}(150 + 200 - 50) = 100$ V

P 4.17 $-3 + \frac{v_o}{200} + \frac{v_o + 5i_\Delta}{10} + \frac{v_o - 80}{20} = 0; \quad i_\Delta = \frac{v_o - 80}{20}$

[a] Solving, $v_o = 50$ V

[b] $i_{ds} = \frac{v_o + 5i_\Delta}{10}$

$$i_\Delta = (50 - 80)/20 = -1.5 \text{ A}$$

$$\therefore \quad i_{ds} = 4.25 \text{ A}; \quad 5i_\Delta = -7.5 \text{ V}: \quad p_{ds} = (-5i_\Delta)(i_{ds}) = 31.875 \text{ W}$$

[c] $p_{3A} = -3v_o = -3(50) = -150$ W (del)

$p_{80V} = 80i_\Delta = 80(-1.5) = -120$ W (del)

$\sum p_{del} = 150 + 120 = 270$ W

CHECK:

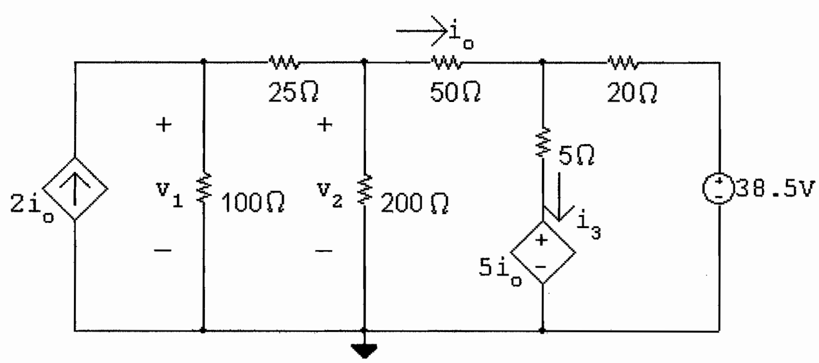
$p_{200\Omega} = 2500/200 = 12.5$ W

$p_{20\Omega} = (80 - 50)^2/20 = 900/20 = 45$ W

$p_{10\Omega} = (4.25)^2(10) = 180.625$ W

$\sum p_{diss} = 31.875 + 180.625 + 12.5 + 45 = 270$ W

P 4.18 [a]



$$i_o = \frac{v_2 - v_3}{50}$$

$$-2i_o + \frac{v_1}{100} + \frac{v_1 - v_2}{25} = 0 \qquad \text{so} \qquad 5v_1 - 8v_2 + 4v_3 = 0$$

$$\frac{v_2 - v_1}{25} + \frac{v_2}{200} + \frac{v_2 - v_3}{50} \qquad \text{so} \qquad -8v_1 + 13v_2 - 4v_3 = 0$$

$$\frac{v_3 - v_2}{50} + \frac{v_3 - 5i_o}{5} + \frac{v_3 - 38.5}{20} = 0 \qquad \text{so} \qquad 0v_1 - 4v_2 + 29v_3 = 192.5$$

Solving, $v_1 = -50$ V; $v_2 = -30$ V; $v_3 = 2.5$ V

[b] $i_o = \frac{v_2 - v_3}{50} = \frac{-30 - 2.5}{50} = -0.65$ A

$$i_3 = \frac{v_3 - 5i_o}{5} = \frac{2.5 - 5(-0.65)}{5} = 1.15 \text{ A}$$

$$i_g = \frac{38.5 - 2.5}{20} = 1.8 \text{ A}$$

$$\sum p_{\text{dis}} = \sum p_{\text{dev}}$$

Calculate $\sum p_{\text{dev}}$ because we don't know if the dependent sources are developing or absorbing power. Likewise for the independent source.

$$p_{2i_o} = -2i_o v_1 = -2(-0.65)(-50) = -65 \text{ W(dev)}$$

$$p_{5i_o} = 5i_o i_3 = 5(-0.65)(1.15) = -3.7375 \text{ W(dev)}$$

$$p_g = -38.5(1.8) = -69.30 \text{ W(dev)}$$

$$\sum p_{\text{dev}} = 69.3 + 65 + 3.7375 = 138.0375 \text{ W}$$

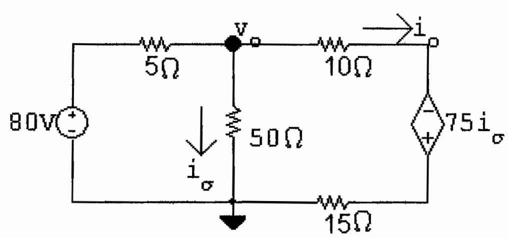
CHECK

$$\sum p_{\text{dis}} = \frac{2500}{100} + \frac{900}{200} + \frac{400}{25} + (0.65)^2(50) + (1.15)^2 5 + (1.8)^2(20)$$

$$= 138.0375 \text{ W}$$

$$\therefore \sum p_{\text{dev}} = \sum p_{\text{dis}} = 138.0375 \text{ W}$$

P 4.19



$$\frac{v_o - 80}{5} + \frac{v_o}{50} + \frac{v_o + 75i_\sigma}{25} = 0; \quad i_\sigma = \frac{v_o}{50}$$

Solving, $v_o = 50$ V; $i_\sigma = 1$ A

$$i_o = \frac{50 - (-75)(1)}{25} = 5 \text{ A}$$

$$p_{75i_\sigma} = 75 i_\sigma i_o = -375 \text{ W}$$

$\therefore$ The dependent voltage source delivers 375 W to the circuit.

P 4.20 **[a]** $-5 + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0 \quad \text{so} \quad 4v_1 - 3v_2 + 0i_\Delta = 75$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{30} + \frac{v_2}{10} + \frac{v_2 + 5i_\Delta}{30} = 0 \quad \text{so} \quad -6v_1 + 11v_2 + 5i_\Delta = 0$$

$$i_\Delta = \frac{v_1 - v_2}{5} \quad \text{so} \quad v_1 - v_2 - 5i_\Delta = 0$$

Solving, $v_1 = 30$ V; $v_2 = 15$ V; $i_\Delta = 3$ A; $i_o = \frac{15 + 15}{30} = 1$ A

$$p_{5i_\Delta} = (-15)(1) = -15 \text{ W(del)}$$

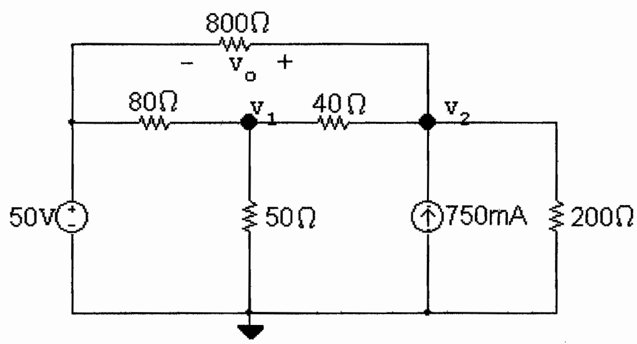
$$p_{5\text{A}} = -5(30) = -150 \text{ W(del)}$$

$$\therefore \quad p_{\text{dev}} = 165 \text{ W}$$

[b] $\sum p_{\text{abs}} = \frac{(30)^2}{15} + \frac{(15)^2}{30} + \frac{(15)^2}{10} + (3)^2(5) + (1)^2(30) = 165 \text{ W}$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{abs}} = 165 \text{ W}$$

P 4.21



The two node voltage equations are:

$$\frac{v_1 - 50}{80} + \frac{v_1}{50} + \frac{v_1 - v_2}{40} = 0$$

$$\frac{v_2 - v_1}{40} - 0.75 + \frac{v_2}{200} + \frac{v_2 - 50}{800} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{80} + \frac{1}{50} + \frac{1}{40}\right) + v_2\left(-\frac{1}{40}\right) = \frac{50}{80}$$

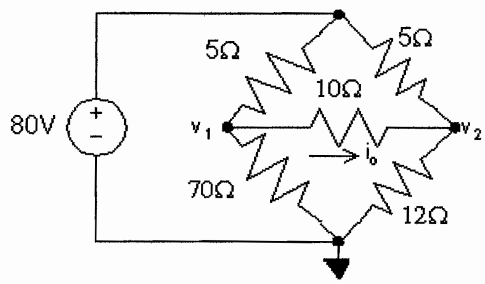
$$v_1\left(-\frac{1}{40}\right) + v_2\left(\frac{1}{40} + \frac{1}{200} + \frac{1}{800}\right) = 0.75 + \frac{50}{800}$$

Solving, $v_1 = 34$ V; $v_2 = 53.2$ V.
Thus, $v_o = v_2 - 50 = 53.2 - 50 = 3.2$ V.
POWER CHECK:

$$
\begin{aligned}
i_g &= (50-34)/80 + (50-53.2)/800 = 196 \text{ m A}\\
p_{50\text{V}} &= -(50)(0.196) = -9.8 \text{ W}\\
p_{80\Omega} &= (50-34)^2/80 = 3.2 \text{ W}\\
p_{800\Omega} &= (50-53.2)^2/800 = 12.8 \text{ m W}\\
p_{40\Omega} &= (53.2-34)^2/40 = 9.216 \text{ W}\\
p_{50\Omega} &= 34^2/50 = 23.12 \text{ W}\\
p_{200\Omega} &= 53.2^2/200 = 14.1512 \text{ W}\\
p_{0.75\text{A}} &= -(53.2)(0.75) = -39.9 \text{ W}
\end{aligned}
$$

$\sum p_{\text{abs}} = 3.2 + .0128 + 9.216 + 23.12 + 14.1512 = 49.7 \text{ W} = \sum p_{\text{del}} =$
$9.8 + 39.9 = 49.7$

P 4.22

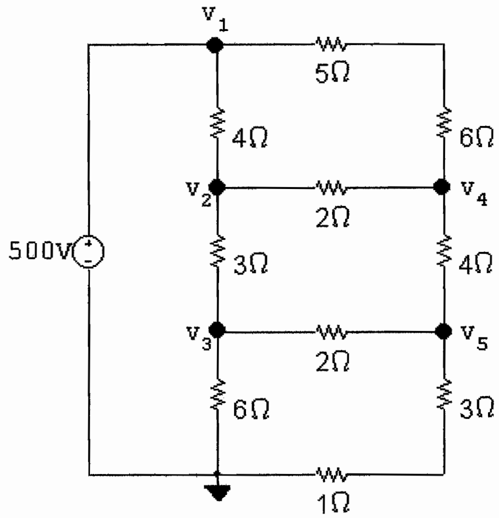


$$\frac{v_1}{70} + \frac{v_1 - v_2}{10} + \frac{v_1 - 80}{5} = 0 \qquad \text{so} \qquad 22v_1 - 7v_2 = 1120$$
$$\frac{v_2}{12} + \frac{v_2 - v_1}{10} + \frac{v_2 - 80}{5} = 0 \qquad \text{so} \qquad -6v_1 + 23v_2 = 960$$

Solving, $v_1 = 70$ V; $v_2 = 60$ V

Thus, $i_o = \dfrac{v_1 - v_2}{10} = 1$ A

P 4.23 [a]



$$\frac{v_2-500}{4}+\frac{v_2-v_4}{2}+\frac{v_2-v_3}{3}=0 \quad \text{so} \quad 13v_2-4v_3-6v_4+0v_5=1500$$

$$\frac{v_3-v_2}{3}+\frac{v_3}{6}+\frac{v_3-v_5}{2}=0 \quad \text{so} \quad -2v_2+6v_3+0v_4-3v_5=0$$

$$\frac{v_4-v_2}{2}+\frac{v_4-500}{11}+\frac{v_4-v_5}{4}=0 \quad \text{so} \quad -22v_2+0v_3+37v_4-11v_5=2000$$

$$\frac{v_5-v_3}{2}+\frac{v_5}{4}+\frac{v_5-v_4}{4}=0 \quad \text{so} \quad 0v_2-2v_3-v_4+4v_5=0$$

Solving, $v_2 = 300$ V; $v_3 = 180$ V; $v_4 = 280$ V; $v_5 = 160$ V

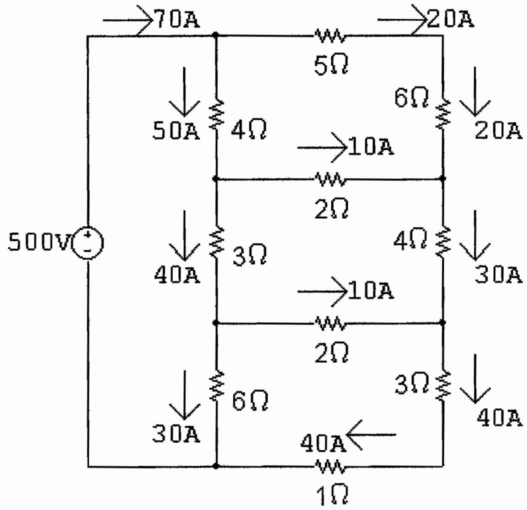
$$i_{5\Omega}=\frac{500-v_4}{11}=\frac{500-280}{11}=20 \text{ A}$$

$$p_{5\Omega}=(20)^2(5)=2000 \text{ W}$$

[b]
$$i_{500\text{V}} = \frac{v_1-v_2}{4}+\frac{v_1-v_4}{11}$$
$$= \frac{500-300}{4}+\frac{500-280}{11}=50+20=70 \text{ A}$$

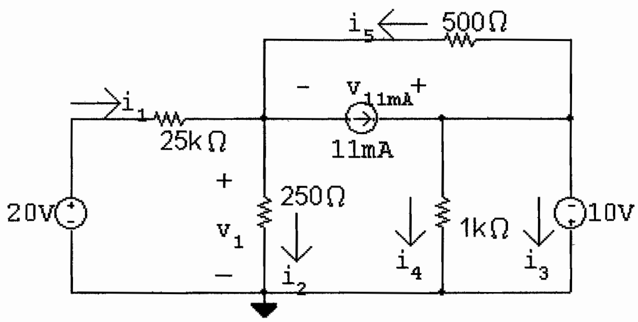
$$p_{500\text{V}} = 35{,}000 \text{ W}$$

Check:



$$\sum P_{\rm dis} = (50)^2(4) + (40)^2(3) + (30)^2(6) + (20)^2(11) + (10)^2(2)$$
$$+(30)^2(4) + (10)^2(2) + (40)^2(4) = 35{,}000 \text{ W}$$

P 4.24 [a]



$$\frac{v_1 - 20}{25 \times 10^3} + \frac{v_1}{0.25 \times 10^3} + 11 \times 10^{-3} + \frac{v_1 + 10}{0.5 \times 10^3} = 0$$

$$v_1 = -5 \text{ V}$$

$$i_1 = \frac{20 + 5}{25{,}000} = 1 \text{ mA}$$

$$i_2 = \frac{v_1}{250} = \frac{-5}{250} = -20 \text{ mA}$$

$$i_5 = \frac{-10 + 5}{500} = -10 \text{ mA}$$

$$i_4 = \frac{-10}{1000} = -10 \text{ mA}$$

$$i_4 + i_3 - 11 + i_5 = 0$$

$$\therefore \quad i_3 = 11 - i_4 - i_5 = 11 + 10 + 10 = 31 \text{ mA}$$

[b] $p_{20V} = 20i_1 = 20(1 \times 10^{-3}) = 20$ mW

$p_{10V} = 10i_3 = 10(31 \times 10^{-3}) = 310$ mW

$v_{11mA} + v_1 = -10, \quad v_{11mA} = -10 + 5 = -5$ V

$p_{11mA} = -11v_{11mA} = -55$ mW (del)

$\sum p_{dev} = 20 + 310 = 330$ mW

$p_{25k} = 25 \times 10^3 i_1^2 = 25$ mW

$p_{0.25k} = 0.25 \times 10^3 i_2^2 = 100$ mW

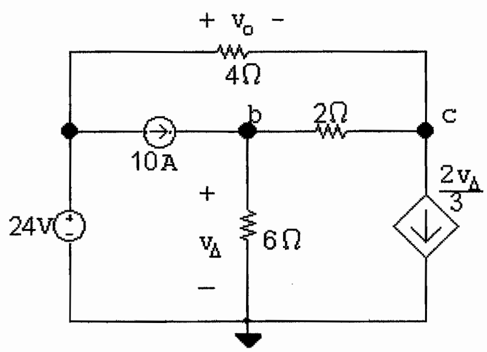
$p_{0.5k} = 0.5 \times 10^3 i_5^2 = 50$ mW

$p_{1k} = 1 \times 10^3 i_4^2 = 100$ mW

$\sum p_{diss} = 25 + 100 + 50 + 100 + 55 = 330$ mW

$\sum p_{diss} = \sum p_{dev} = 330$ mW

P 4.25



The two node voltage equations are:

$$-10 + \frac{v_b}{6} + \frac{v_b - v_c}{2} = 0$$
$$\frac{2v_\Delta}{3} + \frac{v_c - v_b}{2} + \frac{v_c - 24}{4} = 0$$

The constraint equation for the dependent source is:

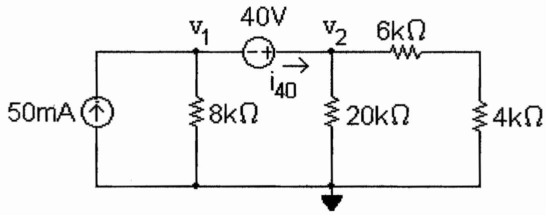
$v_\Delta = v_b$

Place these equations in standard form:

$$v_b\left(\frac{1}{6} + \frac{1}{2}\right) + v_c(-\frac{1}{2}) + v_\Delta(0) = 10$$
$$v_b(-\frac{1}{2}) + v_c\left(\frac{1}{2} + \frac{1}{4}\right) + v_\Delta\left(\frac{2}{3}\right) = \frac{24}{4}$$
$$v_b(1) + v_c(0) + v_\Delta(-1) = 0$$

Solving, $v_b = 18$ V, $v_c = 4$ V, $v_\Delta = 18$ V, and $v_o = 24 - v_c = 20$ V

P 4.26



This circuit has a supernode includes the nodes v_1, v_2 and the 40 V source. The supernode equation is

$$-0.05 + \frac{v_1}{8000} + \frac{v_2}{20{,}000} + \frac{v_2}{10{,}000} = 0$$

The supernode constraint equation is

$$v_2 + -v_1 = 40$$

Place these two equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{8000}\right) \quad &+ \quad v_2\left(\frac{1}{20{,}000} + \frac{1}{10{,}000}\right) &= \quad 0.05 \\ v_1(-1) \quad &+ \quad v_2(1) &= \quad 40 \end{aligned}$$

Solving, $v_1 = 160$ V and $v_2 = 200$ V, so $v_o = v_2 = 200$ V.

$$i_{40} = 0.05 - \frac{v_1}{8000} = 30 \text{ m A}$$

$$p_{40\text{V}} = -(40)i_{40} = -(40)(0.03) = -1.2 \text{ W}$$

The 40 V source delivers 1.2 W.

P 4.27 Place $v_\Delta/5$ inside a supernode and use the lower node as a reference. Then

$$\frac{v_1 - 50}{10} + \frac{v_1}{30} + \frac{v_1 - v_\Delta/5}{39} + \frac{v_1 - v_\Delta/5}{78} = 0$$

$$134v_1 - 6v_\Delta = 3900; \qquad v_\Delta = 50 - v_1$$

Solving, $v_1 = 30$ V; $\quad v_\Delta = 20$ V; $\quad v_o = 30 - v_\Delta/5 = 30 - 4 = 26$ V

P 4.28 $i_\phi = \dfrac{v_3 - v_4}{4} = \dfrac{235 - 222}{4} = 3.25$ A

$$30i_\phi = 30(3.25) = 97.5 \text{ V}$$

$$v_1 + 30i_\phi = v_4$$

$$v_1 = v_4 - 30i_\phi = 222 - 97.5 = 124.5 \text{ V}$$

$$v_3 + v_\Delta = 250$$

$$\therefore \quad v_\Delta = 250 - 235 = 15 \text{ V}$$

$$3.2v_\Delta = (3.2)(15) = 48 \text{ A}$$

$$i_g = \frac{250 - 124.5}{2} + \frac{250 - 235}{1} = 77.75 \text{ A}$$

$$p_{250\text{V}} = -250i_g = -250(77.75) = -19{,}437.5 \text{ W(del)}$$

$$i_{30i_\phi} - i_\phi + v_4/40 + 48 = 0$$

$$i_{30i_\phi} = i_\phi - 222/40 - 48 = 3.25 - 5.55 - 48 = -50.3 \text{ A}$$

$$p_{30i_\phi} = (30i_\phi)i_{30i_\phi} = (97.5)(-50.3) = -4904.25 \text{ W(dev)}$$

$$p_{3.2v_\Delta} = (3.2v_\Delta)(v_4) = (48)(222) = 10{,}656 \text{ W(abs)}$$

$$\therefore \quad \sum p_{\text{dev}} = 19{,}437.5 + 4904.25 = 24{,}341.75 \text{ W}$$

$$p_{10\Omega} = \frac{v_1^2}{10} = \frac{(124.5)^2}{10} = 1550.025 \text{ W}$$

$$p_{2\Omega} = \frac{(250 - 124.5)^2}{2} = 7875.125 \text{ W}$$

$$p_{1\Omega} = \frac{(250 - 235)^2}{1} = 225 \text{ W}$$

$$p_{20\Omega} = \frac{(235)^2}{20} = 2761.25 \text{ W}$$

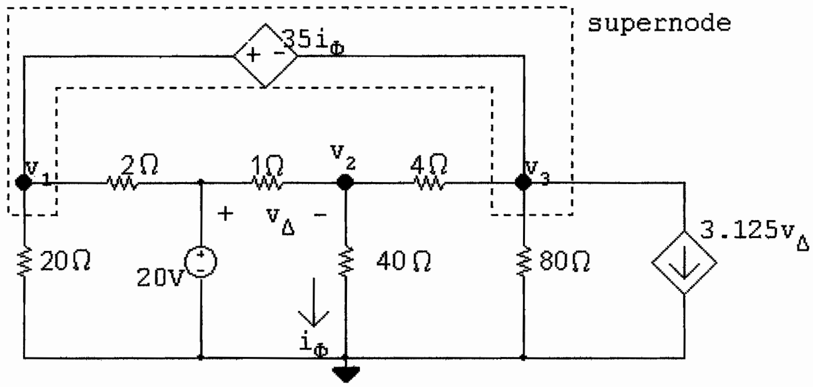
$$p_{4\Omega} = (3.25)^2(4) = 42.25 \text{ W}$$

$$p_{40\Omega} = \frac{(222)^2}{40} = 1232.10 \text{ W}$$

$$\therefore \quad \sum p_{\text{diss}} = 10{,}656 + 1550.025 + 7875.125 + 225 + 2761.250 + 42.25 + 1232.1 = 24{,}341.75 \text{ W}$$

Thus, $\sum p_{\text{dev}} = \sum p_{\text{diss}}$; Agree with analyst

P 4.29



Node equations:

$$\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_3 - v_2}{4} + \frac{v_3}{80} + 3.125v_\Delta = 0$$

$$\frac{v_2}{40} + \frac{v_2 - v_3}{4} + \frac{v_2 - 20}{1} = 0$$

Constraint equations:

$$v_\Delta = 20 - v_2$$

$$v_1 - 35i_\phi = v_3$$

$$i_\phi = v_2/40$$

Solving, $v_1 = -20.25$ V; $v_2 = 10$ V; $v_3 = -29$ V

Let i_g be the current delivered by the 20 V source, then

$$i_g = \frac{20 - (20.25)}{2} + \frac{20 - 10}{1} = 30.125 \text{ A}$$

$$p_g \text{ (delivered) } = 20(30.125) = 602.5 \text{ W}$$

P 4.30 From Eq. 4.16, $i_B = v_c/(1+\beta)R_E$

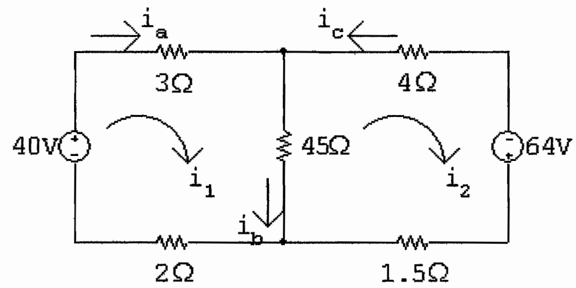
From Eq. 4.17, $i_B = (v_b - V_o)/(1+\beta)R_E$

From Eq. 4.19,

$$i_B = \frac{1}{(1+\beta)R_E}\left[\frac{V_{CC}(1+\beta)R_ER_2 + V_oR_1R_2}{R_1R_2 + (1+\beta)R_E(R_1+R_2)} - V_o\right]$$

$$= \frac{V_{CC}R_2 - V_o(R_1+R_2)}{R_1R_2 + (1+\beta)R_E(R_1+R_2)} = \frac{[V_{CC}R_2/(R_1+R_2)] - V_o}{[R_1R_2/(R_1+R_2)] + (1+\beta)R_E}$$

P 4.31 [a]



$40 = 50i_1 - 45i_2$

$64 = -45i_1 + 50.5i_2$

Solving, $i_1 = 9.8$ A; $i_2 = 10$ A

$i_a = i_1 = 9.8$ A; $i_b = i_1 - i_2 = -0.2$ A; $i_c = -i_2 = -10$ A

[b] If the polarity of the 64 V source is reversed, we have

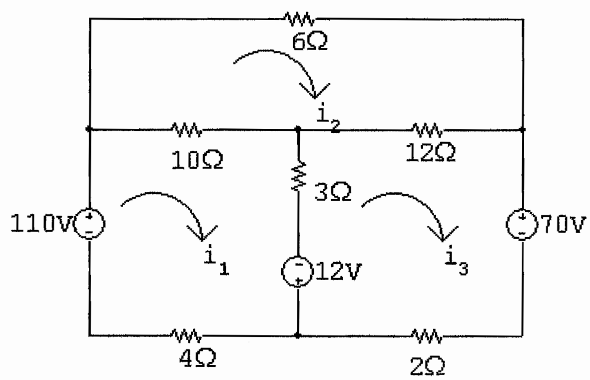
$40 = 50i_1 - 45i_2$

$-64 = -45i_1 + 50.5i_2$

$i_1 = -1.72$ A and $i_2 = -2.8$ A

$i_a = i_1 = -1.72$ A; $i_b = i_1 - i_2 = 1.08$ A; $i_c = -i_2 = 2.8$ A

P 4.32 [a]



$110 + 12 = 17i_1 - 10i_2 - 3i_3$

$0 = -10i_1 + 28i_2 - 12i_3$

$-12 - 70 = -3i_1 - 12i_2 + 17i_3$

Solving, $i_1 = 8$ A; $i_2 = 2$ A; $i_3 = -2$ A

$p_{110} = -110i_1 = -880$ W(del)

$p_{12} = -12(i_1 - i_3) = -120$ W(del)

$p_{70} = 70i_3 = -140$ W(del)

$\therefore \sum p_{\text{dev}} = 1140$ W

[b] $p_{4\Omega} = (8)^2(4) = 256$ W

$p_{10\Omega} = (6)^2(10) = 360$ W

$p_{12\Omega} = (-4)^2(12) = 192$ W

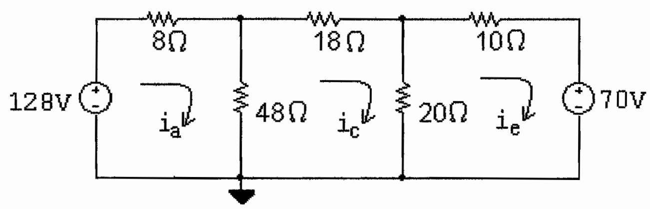
$p_{2\Omega} = (-2)^2(2) = 8$ W

$p_{6\Omega} = (2)^2(6) = 24$ W

$p_{3\Omega} = (10)^2(3) = 300$ W

$\therefore\ \sum p_{\text{abs}} = 1140$ W

P 4.33 [a]



The three mesh current equations are:

$$-128 + 8i_a + 48(i_a - i_c) = 0$$

$$18i_c + 20(i_c - i_e) + 48(i_c - i_a) = 0$$

$$70 + 20(i_e - i_c) + 10i_e = 0$$

Place these equations in standard form:

$$i_a(8 + 48) + i_c(-48) + i_e(0) = 128$$

$$i_a(-48) + i_c(18 + 20 + 48) + i_e(-20) = 0$$

$$i_a(0) + i_c(-20) + i_e(20 + 10) = -70$$

Solving, $i_a = 4$ A; $\quad i_c = 2$ A; $\quad i_e = -1$ A
Now calculate the remaining branch currents:

$$i_b = i_a - i_c = 2 \text{ A}$$

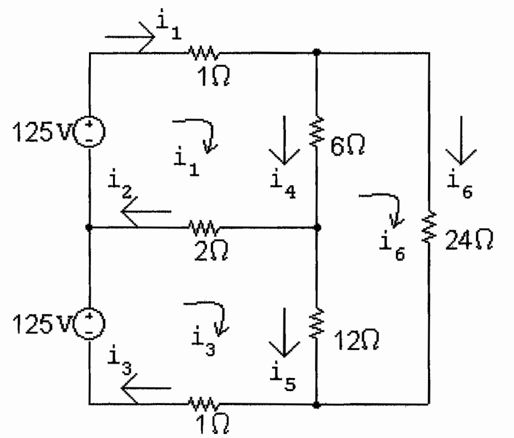
$$i_d = i_c - i_e = 3 \text{ A}$$

[b] $p_{\text{sources}} = p_{128\text{V}} + p_{70\text{V}} = -(128)i_a + (70)i_e$

$$= -(128)(4) + (70)(-1) = -512 - 70 = -582 \text{ W}$$

Thus, the power developed in the circuit is 582 W. Note that the resistors cannot develop power!

P 4.34 [a]



The three mesh current equations are:

$$-125 + 1i_1 + 6(i_1 - i_6) + 2(i_1 - i_3) = 0$$

$$24i_6 + 12(i_6 - i_3) + 6(i_6 - i_1) = 0$$

$$-125 + 2(i_3 - i_1) + 12(i_3 - i_6) + 1i_3 = 0$$

Place these equations in standard form:

$$i_1(1 + 6 + 2) + i_3(-2) + i_6(-6) = 125$$

$$i_1(-6) + i_3(-12) + i_6(24 + 12 + 6) = 0$$

$$i_1(-2) + i_3(2 + 12 + 1) + i_6(-12) = 125$$

Solving, $i_1 = 23.76$ A; $i_3 = 18.43$ A; $i_6 = 8.66$ A
Now calculate the remaining branch currents:

$$i_2 = i_1 - i_3 = 5.33 \text{ A}$$

$$i_4 = i_1 - i_6 = 15.10 \text{ A}$$

$$i_5 = i_3 - i_6 = 9.77 \text{ A}$$

[b] $p_{\text{sources}} = p_{\text{top}} + p_{\text{bottom}} = -(125)(23.76) - (125)(18.43)$

$$= -2970 - 2304 = -5274 \text{ W}$$

Thus, the power developed in the circuit is 5274 W.
Now calculate the power absorbed by the resistors:

$$p_{1\text{top}} = (23.76)^2(1) = 564.54 \text{ W}$$

$$p_2 = (5.33)^2(2) = 56.82 \text{ W}$$

$$p_{1\text{bot}} = (18.43)^2(1) = 339.66 \text{ W}$$

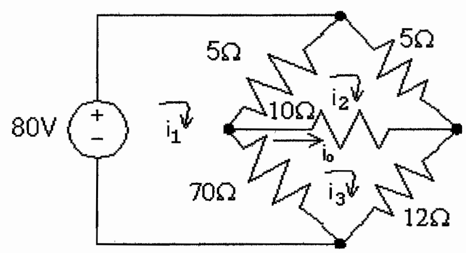
$$p_6 = (15.10)^2(6) = 1368.06 \text{ W}$$

$$p_{12} = (9.77)^2(12) = 1145.43 \text{ W}$$

$$p_{24} = (8.66)^2(24) = 1799.89 \text{ W}$$

The power absorbed by the resistors is
$564.54 + 56.82 + 339.66 + 1368.06 + 1145.43 + 1799.89 = 5274$ W so the power balances.

P 4.35



The three mesh current equations are:

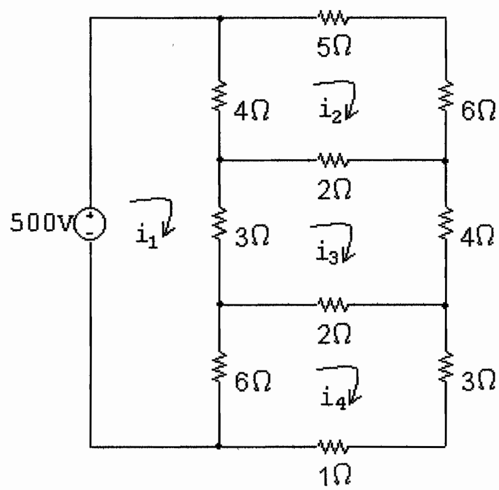
$$\begin{aligned} -80 + 5(i_1 - i_2) + 70(i_1 - i_3) &= 0 \\ 5i_2 + 10(i_2 - i_3) + 5(i_2 - i_1) &= 0 \\ 12i_3 + 70(i_3 - i_1) + 10(i_3 - i_2) &= 0 \end{aligned}$$

Place these equations in standard form:

$$\begin{aligned} i_1(5 + 70) + i_2(-5) + i_3(-70) &= 80 \\ i_1(-5) + i_2(5 + 10 + 5) + i_3(-10) &= 0 \\ i_1(-70) + i_2(-10) + i_3(12 + 70 + 10) &= 0 \end{aligned}$$

Solving, $i_1 = 6$ A; $\quad i_2 = 4$ A; $\quad i_3 = 5$ A
Thus, $i_o = i_3 - i_2 = 1$ A.

P 4.36 [a]



The four mesh current equations are:

$$-500 + 4(i_1 - i_2) + 3(i_1 - i_3) + 6(i_1 - i_4) = 0$$

$$5i_2 + 6i_2 + 2(i_2 - i_3) + 4(i_2 - i_1) = 0$$

$$4i_3 + 2(i_3 - i_4) + 3(i_3 - i_1) + 2(i_3 - i_2) = 0$$

$$3i_4 + 1i_4 + 6(i_4 - i_1) + 2(i_4 - i_3) = 0$$

Place these equations in standard form:

$$i_1(4+3+6) + i_2(-4) + i_3(-3) + i_4(-6) = 500$$

$$i_1(-4) + i_2(5+6+2+4) + i_3(-2) + i_4(0) = 0$$

$$i_1(-3) + i_2(-2) + i_3(2+4+2+3) + i_4(-2) = 0$$

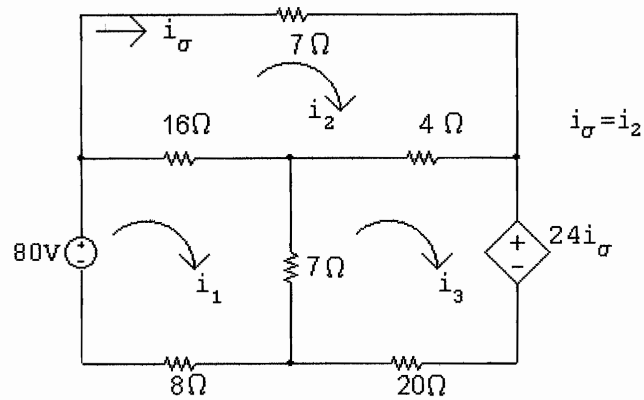
$$i_1(-6) + i_2(0) + i_3(-2) + i_4(2+3+1+6) = 0$$

Solving, $i_1 = 70$ A; $i_2 = 20$ A; $i_3 = 30$ A; $i_4 = 40$ A
The power absorbed by the 5 Ω resistor is

$$p_5 = i_2^2(5) = (20)^2(5) = 2000 \text{ W}$$

[b] $p_{500} = -(500)i_1 = -(500)(70) = -35 \text{ kW}$

P 4.37



$$-80 + 31i_1 - 16i_2 - 7i_3 = 0$$

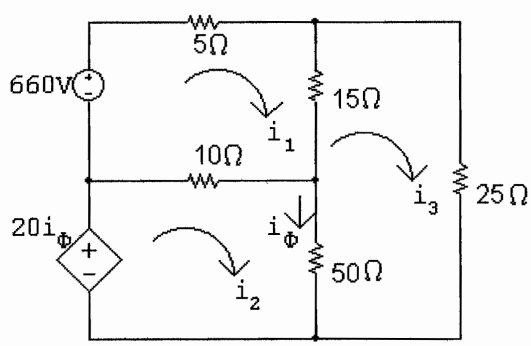
$$-16i_1 + 27i_2 - 4i_3 = 0$$

$$-7i_1 - 4i_2 + 31i_3 + 24i_2 = 0$$

Solving, $i_1 = 3.5$ A

$$p_{8\Omega} = (3.5)^2(8) = 98 \text{ W}$$

P 4.38



$$660 = 30i_1 - 10i_2 - 15i_3$$

$$20i_\phi = -10i_1 + 60i_2 - 50i_3$$

$$0 = -15i_1 - 50i_2 + 90i_3$$

$$i_\phi = i_2 - i_3$$

Solving, $i_1 = 42$ A; $i_2 = 27$ A; $i_3 = 22$ A; $i_\phi = 5$ A

$$20i_\phi = 100 \text{ V}$$

$$p_{20i_\phi} = -100i_2 = -100(27) = -2700 \text{ W}$$

$$\therefore \; p_{20i_\phi} \text{ (developed)} = 2700 \text{ W}$$

CHECK:

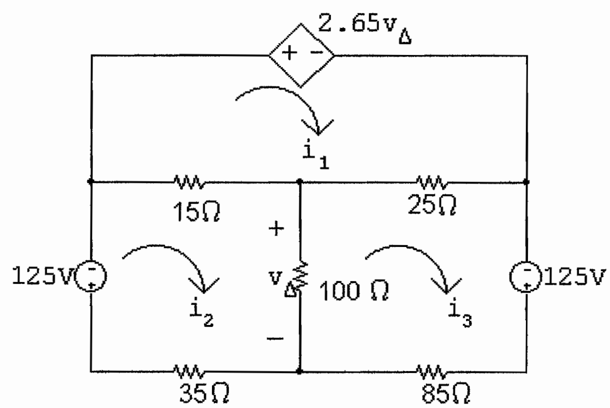
$$p_{660\text{V}} = -660(42) = -27{,}720 \text{ W (dev)}$$

$$\therefore \; \sum P_{\text{dev}} = 27{,}720 + 2700 = 30{,}420 \text{ W}$$

$$\sum P_{\text{dis}} = (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) + (15)^2(10)$$

$$= 30{,}420 \text{ W}$$

P 4.39



Mesh equations:

$$2.65v_\Delta + 40i_1 - 15i_2 - 25i_3 = 0$$

$$-15i_1 + 150i_2 - 100i_3 = -125$$

$$-25i_1 - 100i_2 + 210i_3 = 125$$

Constraint equations:

$$v_\Delta = 100(i_2 - i_3)$$

Solving, $i_1 = 7$ A; $\quad i_2 = 1.2$ A; $\quad i_3 = 2$ A

$$v_\Delta = 100(i_2 - i_3) = 100(1.2 - 2) = -80 \text{ V}$$

$$p_{2.65v_\Delta} = 2.65v_\Delta i_1 = -1484 \text{ W}$$

Therefore, the dependent source is developing 1484 W.
CHECK:

$p_{125\text{V}} = 125i_2 = 150$ W (left source)

$p_{125\text{V}} = -125i_3 = -250$ W (right source)

$\sum p_{\text{dev}} = 1484 + 250 = 1734$ W

$p_{35\Omega} = (1.2)^2(35) = 50.4$ W

$p_{85\Omega} = (2)^2(85) = 340$ W

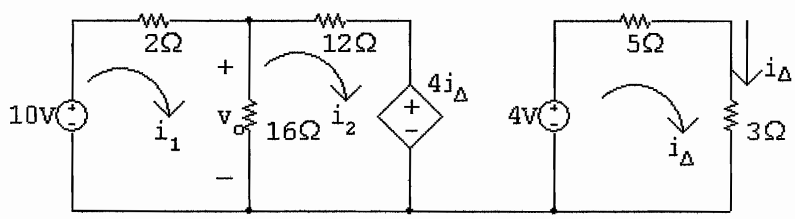
$p_{15\Omega} = (7 - 1.2)^2(15) = 504.6$ W

$p_{25\Omega} = (7 - 2)^2(25) = 625$ W

$p_{100\Omega} = (1.2 - 2)^2(100) = 64$ W

$\sum p_{\text{diss}} = 50.4 + 340 + 504.6 + 625 + 64 + 150 = 1734$ W

P 4.40 [a]



$10 = 18i_1 - 16i_2$

$0 = -16i_1 + 28i_2 + 4i_\Delta$

$4 = 8i_\Delta$

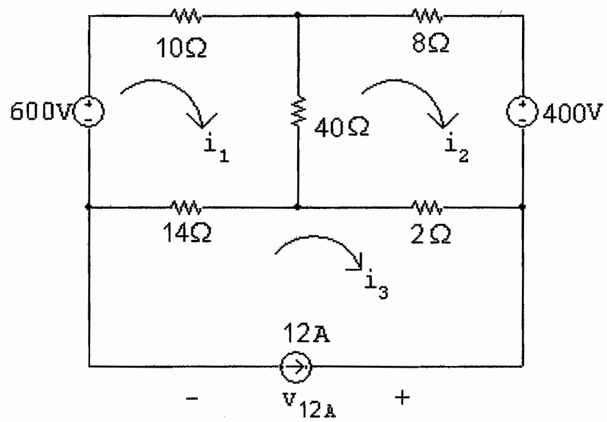
Solving, $i_1 = 1$ A; $\quad i_2 = 0.5$ A; $\quad i_\Delta = 0.5$ A

$v_0 = 16(i_1 - i_2) = 16(0.5) = 8$ V

[b] $p_{4i_\Delta} = 4i_\Delta i_2 = (4)(0.5)(0.5) = 1$ W (abs)

$\therefore\ p_{4i_\Delta}$ (deliver) $= -1$ W

P 4.41



$600 = 64i_1 - 40i_2 - 14i_3$

$-400 = -40i_1 + 50i_2 - 2i_3$

$-12 = i_3$

Solving, $i_1 = 2.9$ A; $i_2 = -6.16$ A; $i_3 = -12$ A

[a] $v_{12\text{A}} = 2(12 - 6.16) + 14(12 + 2.9)$

$= 220.28$ V

$p_{12\text{A}} = -12v_{12\text{A}} = -12(220.28) = -2643.36$ W

Therefore, the 12 A source delivers 2643.36 W.

[b] $p_{400\text{V}} = 400(-6.16) = -2464$ W

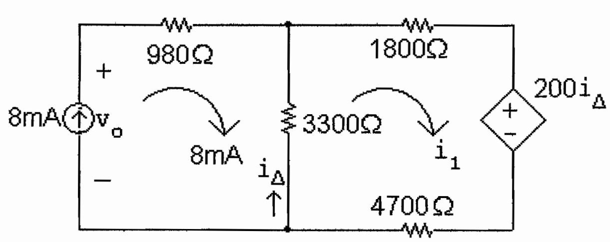
$p_{600\text{V}} = -600i_1 = -600(2.9) = -1740$ W

Therefore, the total power delivered is $2643.36 + 2464 + 1740 = 6847.36$ W

[c] $\sum p_{\text{resistors}} = (2.9)^2(10) + (6.16)^2(8) + (9.06)^2(40) + (14.9)^2(14) + (5.84)^2(2)$

$\sum p_{\text{abs}} = 6847.36\text{ W} = \sum p_{\text{del}}$ (CHECKS)

P 4.42 [a]

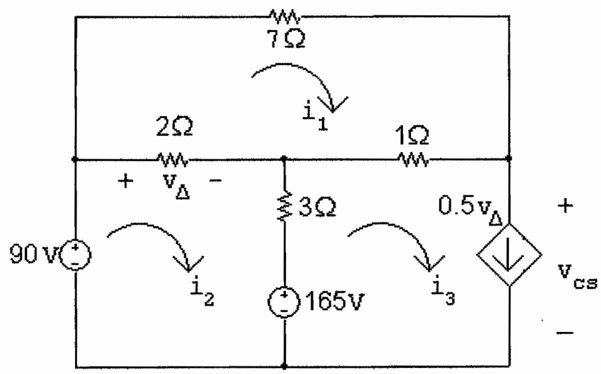


The mesh current equation for the right mesh is:
$3300(i_1 - 0.008) + 6500i_1 + 200(i_1 - 0.008) = 0$
Solving, $10{,}000i_1 = 28$ $\quad\therefore\quad i_1 = 2.8$ mA
Then, $i_\Delta = i_1 - 0.008 = -5.2$ mA

[b] $v_o = (0.008)(980) - (-0.0052)(3300) = 25$ V
$p_{8\text{mA}} = -(25)(0.008) = -200$ mW
Thus, the 8 mA source delivers 200 mW

[c] $200i_\Delta = 200(-0.0052) = -1.04$ V
$p_{\text{dep source}} = 200i_\Delta i_1 = (-1.04)(0.0028) = -2.912$ mW
The dependent source delivers 2.912 mW.

P 4.43



Mesh equations:

$$7i_1 + 1(i_1 - i_3) + 2(i_1 - i_2) = 0$$
$$-90 + 2(i_2 - i_1) + 3(i_2 - i_3) + 165 = 0$$

Constraint equations:
$i_3 = 0.5v_\Delta$; $\quad v_\Delta = 2(i_2 - i_1)$
Place these equations in standard form:

$$i_1(7+1+2) + i_2(-2) + i_3(-1) + v_\Delta(0) = 0$$
$$i_1(-2) + i_2(2+3) + i_3(-3) + v_\Delta(0) = -75$$
$$i_1(0) + i_2(0) + i_3(1) + v_\Delta(-0.5) = 0$$
$$i_1(-2) + i_2(2) + i_3(0) + v_\Delta(-1) = 0$$

Solving, $i_1 = -9$ A; $\quad i_2 = -33$ A; $\quad i_3 = -24$ A; $\quad v_\Delta = -48$ V
Solve the outer loop KVL equation to find v_{cs}:
$-90 + 7i_1 + v_{cs} = 0$; $\quad \therefore \quad v_{cs} = 90 - 7(-9) = 153$ V
Calculate the power for the sources:

$$p_{90V} = -(90)(-33) = 2970 \text{ W}$$
$$p_{165V} = (165)(-33 + 24) = -1485 \text{ W}$$
$$p_{\text{dep source}} = (153)[0.5(-48)] = -3672 \text{ W}$$

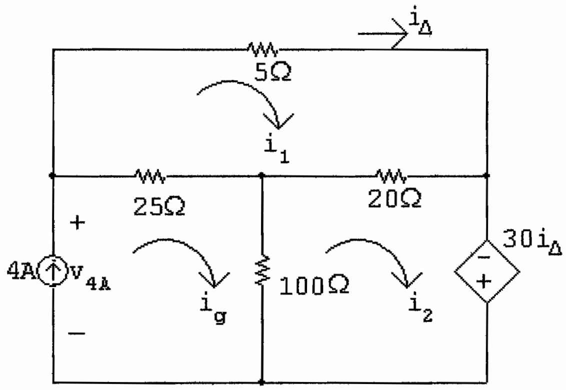
Thus, the total power developed is $1485 + 3672 = 5157$ W.

CHECK:

$$p_{7\Omega} = (9)^2(7) = 567 \text{ W}$$
$$p_{2\Omega} = (24)^2(2) = 1152 \text{ W}$$
$$p_{3\Omega} = (9)^2(3) = 243 \text{ W}$$
$$p_{1\Omega} = (15)^2(1) = 225 \text{ W}$$

$\therefore \sum p_{\text{abs}} = 567 + 1152 + 243 + 225 + 2970 = 5157$ W (checks!)

P 4.44



Mesh equations:

$$50i_1 - 20i_2 - 25i_g = 0$$

$$-20i_1 + 120i_2 - 30i_\Delta - 100i_g = 0$$

Constraint equations:

$i_g = 4;$ $\qquad i_\Delta = i_1$

Solving, $i_1 = 4$ A; $\qquad i_2 = 5$ A

$$i_{25\Omega} = 4 - i_1 = 0 \text{ A}$$

$$i_{20\Omega} = i_2 - i_1 = 1 \text{ A}$$

$$i_{100\Omega} = 4 - i_2 = -1 \text{ A}$$

$$i_{5\Omega} = i_1 = 4 \text{ A}$$

$$v_{4A} = 100(4 - i_2) = -100 \text{ V}$$

$$p_{4A} = -v_{4A}i_g = -(-100)(4) = 400 \text{ W (abs)}$$

$v_{30i_\Delta} = 30i_\Delta = 30i_1 = 120$ V

$p_{30i_\Delta} = -30i_\Delta i_2 = -120(5) = -600$ W

Therefore, the dependent source is developing 600 W, all other elements are absorbing power, and the total power developed is thus 600 W.
CHECK:

$p_{5\Omega} = 16(5) = 80$ W

$p_{25\Omega} = 0$ W

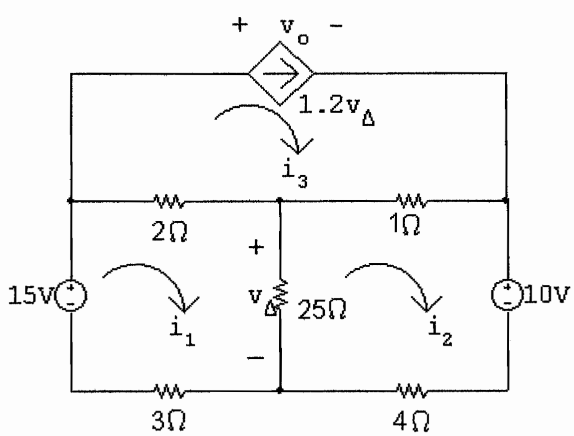
$p_{20\Omega} = 1(20) = 20$ W

$p_{100\Omega} = 1(100) = 100$ W

$p_{4\text{A}} = 400$ W

$\sum p_{\text{abs}} = 80 + 0 + 20 + 100 + 400 = 600$ W (CHECKS)

P 4.45 [a]



Mesh equations:

$15 = 30i_1 - 25i_2 - 2i_3$

$-10 = -25i_1 + 30i_2 - i_3$

Constraint equations:

$i_3 = 1.2v_\Delta; \qquad v_\Delta = 25(i_1 - i_2)$

Solving, $i_1 = 10$ A; $\quad i_2 = 9$ A; $\quad i_3 = 30$ A; $\quad v_\Delta = 25$ V

$i_{2\Omega} = i_1 - i_3 = 9 - 30 = -20$ A

$p_{2\Omega} = (-20)^2(2) = 800$ W

[b] $p_{15\text{V}} = -15(10) = -150 \text{ W(dev)}$

$p_{10\text{V}} = 10i_2 = 10(9) = 90 \text{ W (abs)}$

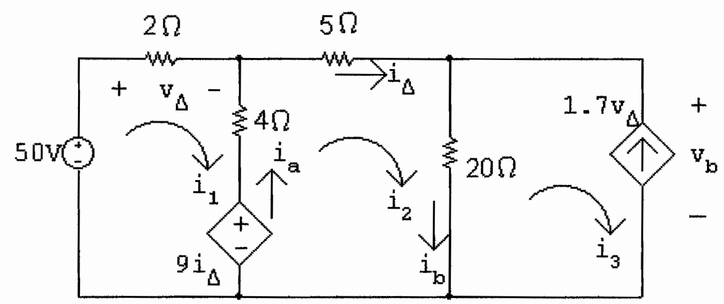
$v_o = (i_1 - i_3)2 + (i_2 - i_3)1 = -40 - 21 = -61 \text{ V}$

$p_{1.2v_\Delta} = i_3 v_o = (30)(-61) = -1830 \text{ W (dev)}$

$\sum P_{\text{dev}} = 1830 + 150 = 1980 \text{ W}$

$$\% \text{ delivered to } 2\Omega = \frac{800}{1980} \times 100 = 40.4\%$$

P 4.46 [a]



Mesh equations:

$-50 + 6i_1 - 4i_2 + 9i_\Delta = 0$

$-9i_\Delta - 4i_1 + 29i_2 - 20i_3 = 0$

Constraint equations:

$i_\Delta = i_2; \qquad i_3 = -1.7v_\Delta; \qquad v_\Delta = 2i_1$

Solving, $i_1 = -5$ A; $\quad i_2 = 16$ A; $\quad i_3 = 17$ A; $\quad v_\Delta = -10$ V

$9i_\Delta = 9(16) = 144 \text{ V}$

$i_a = i_2 - i_1 = 21 \text{ A}$

$i_b = i_2 - i_3 = -1 \text{ A}$

$v_b = 20i_b = -20 \text{ V}$

$p_{50\text{V}} = -50i_1 = 250 \text{ W (absorbing)}$

$p_{9i_\Delta} = -i_a(9i_\Delta) = -(21)(144) = -3024 \text{ W (delivering)}$

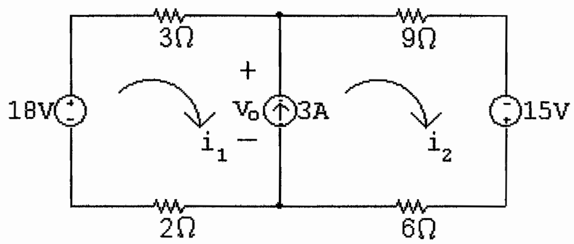
$p_{1.7\text{V}} = -1.7v_\Delta v_b = i_3 v_b = (17)(-20) = -340 \text{ W (delivering)}$

[b] $\sum P_{\text{dev}} = 3024 + 340 = 3364 \text{ W}$

$\sum P_{\text{dis}} = 250 + (-5)^2(2) + (21)^2(4) + (16)^2(5) + (-1)^2(20)$

$= 3364 \text{ W}$

P 4.47



$$-18 + 3i_1 + 9i_2 - 15 + 6i_2 + 2i_1 = 0; \quad i_2 - i_1 = 3$$

Solving, $i_1 = -0.6$ A; $i_2 = 2.4$ A

$$p_{18V} = -18i_1 = 10.8 \text{ W (diss)}$$

$$p_{3\,\Omega} = (-0.6)^2(3) = 1.08 \text{ W}$$

$$p_{2\,\Omega} = (-0.6)^2(2) = 0.72 \text{ W}$$

$$p_{9\,\Omega} = (2.4)^2(9) = 51.84 \text{ W}$$

$$p_{6\,\Omega} = (2.4)^2(6) = 34.56 \text{ W}$$

$$\sum p_{\text{diss}} = 99 \text{ W}$$

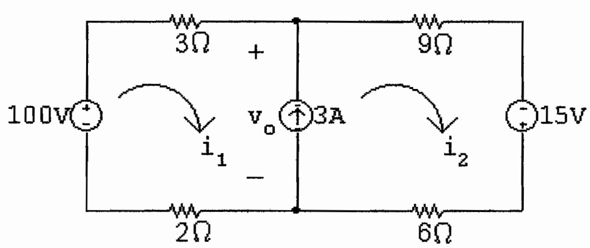
$$v_o = 15i_2 - 15 = 36 - 15 = 21 \text{ V}$$

$$p_{3A} = -3v_o = -63 \text{ W (dev)}$$

$$p_{15V} = -15i_2 = -36 \text{ W (dev)}$$

$$\sum p_{\text{dev}} = 99 \text{ W} = \sum p_{\text{diss}}$$

P 4.48



$$-100 + 5i_1 + 15i_2 - 15 = 0$$

$$5i_1 + 15i_2 = 115$$

$i_2 - i_1 = 3; \qquad i_2 = i_1 + 3; \qquad 15i_2 = 15i_1 + 45$

$\therefore \quad 20i_1 = 70$

$i_1 = 3.5 \text{ A}; \qquad i_2 = 6.5 \text{ A}$

$v_o = 15i_2 - 15 = 97.5 - 15 = 82.5 \text{ V}$

$p_{100\text{V}} = -100i_1 = -350 \text{ W(dev)}$

$p_{3\text{A}} = -3v_o = -247.5 \text{ W(dev)}$

$p_{15\text{V}} = -15i_2 = -97.5 \text{ W(dev)}$

$\sum p_{\text{dev}} = \sum p_{\text{dis}} = 695 \text{ W}$

Check: $\sum p_{\text{dis}} = (3.5)^2(5) + (6.5)^2(15) = 695 \text{ W}$

P 4.49 [a] Summing around the supermesh used in the solution to Problem 3.27 gives

$-(-10) + 5i_1 + 15i_2 - 15 = 0$

$i_2 = i_1 + 3$

$\therefore \quad i_1 = -2 \text{ A}; \qquad i_2 = 1 \text{ A}$

$p_{10\text{V}} = 10(-2) = -20 \text{ W (del)}$

$v_o = 15i_2 - 15 = 0 \text{ V}$

$p_{3\text{A}} = 3v_o = 0 \text{ W}$

$p_{15\text{V}} = -15i_2 = -15 \text{ W (del)}$

$\sum p_{\text{diss}} = (-2)^2(5) + (1)^2(15) = 35 \text{ W}$

$\sum p_{\text{dev}} = 35 \text{ W} = \sum p_{\text{diss}}$

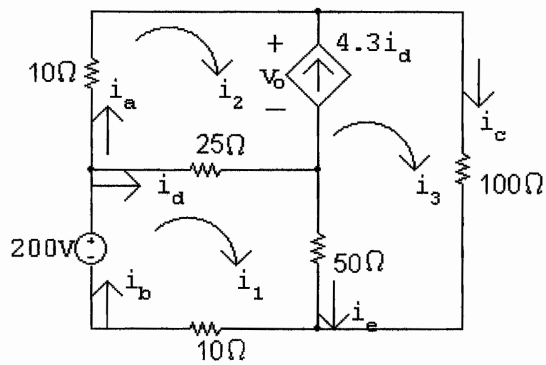
[b] With 3 A current source replaced with a short circuit

$i_1 = -2 \text{ A}, \qquad i_2 = 1 \text{ A}$

$\therefore \quad \sum P_{\text{diss}} = (-2)^2(5) + (1)^2(15) = 35 \text{ W}$

[c] A 3 A source with zero terminal voltage is equivalent to a short circuit carrying 3 A.

P 4.50 [a]



$$200 = 85i_1 - 25i_2 - 50i_3$$

$$0 = -75i_1 + 35i_2 + 150i_3 \qquad \text{(supermesh)}$$

$$i_3 - i_2 = 4.3(i_1 - i_2)$$

Solving, $i_1 = 4.6$ A; $\quad i_2 = 5.7$ A; $\quad i_3 = 0.97$ A

$i_a = i_2 = 5.7$ A; $\quad i_b = i_1 = 4.6$ A

$i_c = i_3 = 0.97$ A; $\quad i_d = i_1 - i_2 = -1.1$ A

$i_e = i_1 - i_3 = 3.63$ A

[b] $10i_2 + v_o + 25(i_2 - i_1) = 0$

$\therefore \quad v_o = -57 - 27.5 = -84.5$ V

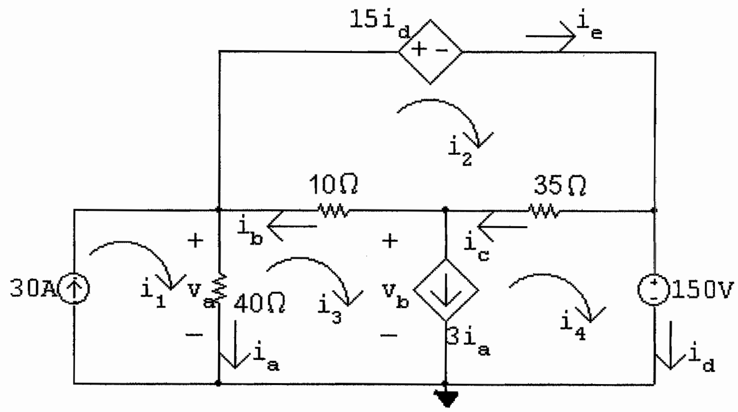
$p_{4.3i_d} = -v_o(4.3i_d) = -(-84.5)(4.3)(-1.1) = -399.685$ W(dev)

$p_{200V} = -200(4.6) = -920$ W(dev)

$$\begin{aligned} \sum P_{dev} &= 1319.685 \text{ W} \\ \sum P_{dis} &= (5.7)^2 10 + (1.1)^2(25) + (0.97)^2 100 + (4.6)^2(10) + \\ &\quad (3.63)^2(50) \\ &= 1319.685 \text{ W} \end{aligned}$$

$\therefore \quad \sum P_{dev} = \sum P_{dis} = 1319.685$ W

P 4.51 [a]



$$40(i_3 - i_1) + 10(i_3 - i_2) + 35(i_4 - i_2) + 150 = 0$$

$$35(i_2 - i_4) + 10(i_2 - i_3) + 15i_d = 0$$

$$3i_a = i_3 - i_4; \qquad i_a = i_1 - i_3$$

$$i_d = i_4; \qquad i_1 = 30 \text{ A}$$

Solving, $i_1 = 30$ A; $\quad i_2 = 8$ A; $\quad i_3 = 24$ A; $\quad i_4 = 6$ A

$i_a = 30 - 24 = 6$ A; $\quad i_b = 8 - 24 = -16$ A; $\quad i_c = 8 - 6 = 2$ A;

$i_d = 6$ A; $\quad i_e = i_c + i_d = 6 + 2 = 8$ A

[b] $v_{\text{a}} = 40i_{\text{a}} = 240$ V; $\quad v_{\text{b}} = 150 - 35i_c = 80$ V

$$p_{30\text{A}} = -30v_a = -30(240) = -7200 \text{ W (gen)}$$

$$p_{15i_d} = 15i_di_e = 15(6)(8) = 720 \text{ W (diss)}$$

$$p_{3i_a} = 3i_av_b = 3(6)(80) = 1440 \text{ W (diss)}$$

$$p_{150\text{V}} = 150i_d = 150(6) = 900 \text{ W (diss)}$$

$$p_{40\Omega} = (6)^2(40) = 1440 \text{ W (diss)}$$

$$p_{10\Omega} = (-16)^2(10) = 2560 \text{ W (diss)}$$

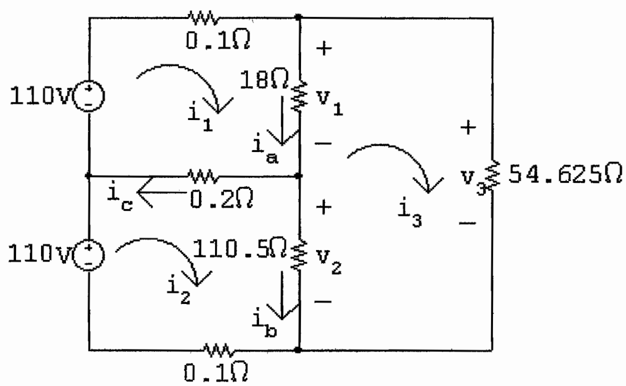
$$p_{35\Omega} = (2)^2(35) = 140 \text{ W (diss)}$$

$$\sum P_{\text{gen}} = 7200 \text{ W}$$

$$\sum P_{\text{diss}} = 720 + 1440 + 900 + 1440 + 2560 + 140 = 7200 \text{ W}$$

P 4.52 [a] Both the mesh-current method and the node-voltage method require three equations. The mesh-current method is a bit more intuitive due to the presence of the voltage sources. We choose the mesh-current method, although technically it is a toss-up.

[b]



$$110 = 18.3i_1 - 0.2i_2 - 18i_3$$

$$110 = -0.2i_1 + 110.8i_2 - 110.5i_3$$

$$0 = -18i_1 - 110.5i_2 + 183.125i_3$$

Solving, $i_1 = 10$ A; $i_2 = 5$ A; $i_3 = 4$ A

$$v_1 = 18(i_1 - i_3) = 108 \text{ V}$$

$$v_2 = 110.5(i_2 - i_3) = 110.5 \text{ V}$$

$$v_3 = 54.625i_3 = 218.5 \text{ V}$$

[c]

$$p_{R1} = (i_1 - i_3)^2(18) = 648 \text{ W}$$

$$p_{R2} = (i_2 - i_3)^2(110.5) = 110.5 \text{ W}$$

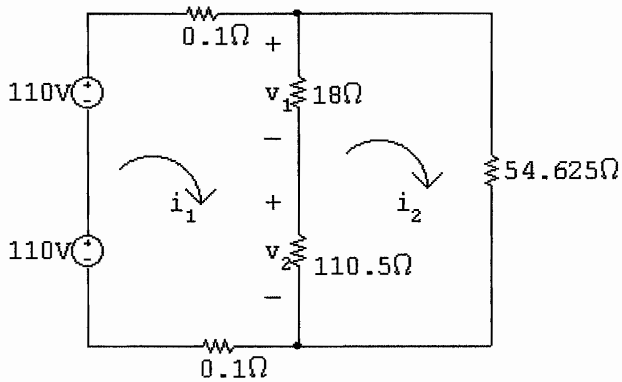
$$p_{R3} = i_3^2(54.625) = 874 \text{ W}$$

[d] $\sum p_{\text{dev}} = 110(i_1 + i_2) = 1650$ W

$\sum p_{\text{load}} = 1632.5$ W

$$\% \text{ delivered} = \frac{1632.5}{1650} \times 100 = 98.94\%$$

[e]



$$220 = 128.7i_1 - 128.5i_2$$

$$0 = -128.5i_1 + 183.125i_2$$

Solving, $i_1 = 5.71$ A; $i_2 = 4.01$ A

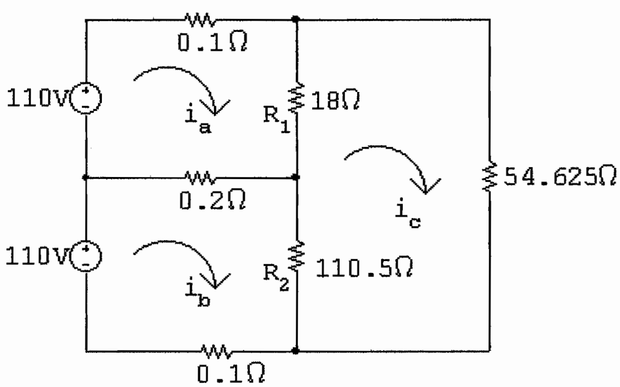
$$i_1 - i_2 = 1.7 \text{ A}$$

$$v_1 = (1.7)(18) = 30.6 \text{ V}$$

$$v_2 = (1.7)(110.5) = 187.85 \text{ V}$$

Note v_1 is low and v_2 is high. Therefore, loads designed for 110 V would not function properly, and could be damaged.

P 4.53



$$110 = (R + 0.3)i_a - 0.2i_b - Ri_c$$

$$110 = -0.2i_a + (R + 0.3)i_b - Ri_c$$

$$\therefore \quad (R + 0.3)i_a - 0.2i_b - Ri_c = -0.2i_a + (R + 0.3)i_b - Ri_c$$

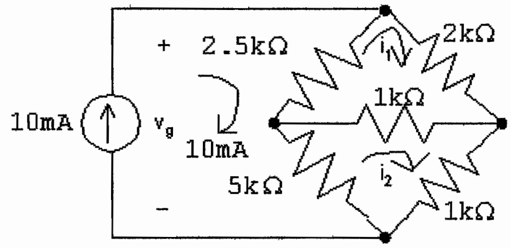
$$\therefore \quad (R + 0.3)i_a - 0.2i_b = -0.2i_a + (R + 0.3)i_b$$

$$\therefore \quad (R + 0.5)i_a = (R + 0.5)i_b$$

Thus, $i_a = i_b$ so $i_o = i_b - i_a = 0$

P 4.54 [a] There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b]



The mesh current equations:

$$2500(i_1 - 0.01) + 2000i_1 + 1000(i_1 - i_2) = 0$$

$$5000(i_2 - 0.01) + 1000(i_2 - i_1) + 1000i_2 = 0$$

Place the equations in standard form:

$$i_1(2500 + 2000 + 1000) + i_2(-1000) = 25$$

$$i_1(-1000) + i_2(5000 + 1000 + 1000) = 50$$

Solving, $i_1 = 6$ mA; $\quad i_2 = 8$ mA

Find the power in the 1 kΩ resistor:

$i_{1k} = i_1 - i_2 = -2$ m A

$p_{1k} = (-0.002)^2(1000) = 4$ mW

[c] No, the voltage across the 10 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

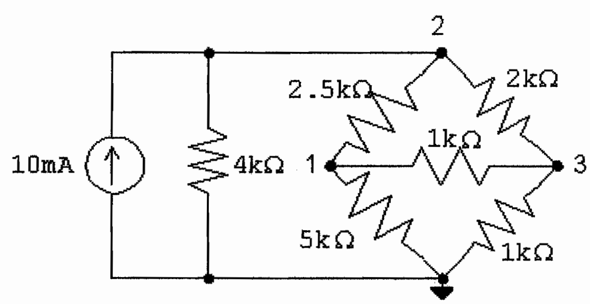
[d] $v_g = 2000i_1 + 1000i_2 = 12 + 8 = 20$ V

$p_{10\text{mA}} = -(20)(0.01) = -200$ m W

Thus the 10 mA source develops 200 mW.

P 4.55 [a] There are three unknown node voltages and three unknown mesh currents, so the number of simultaneous equations required is the same for both methods. The node voltage method has the advantage of having to solve the three simultaneous equations for one unknown voltage provided the connection at either the top or bottom of the circuit is used as the reference node. Therefore recommend the node voltage method.

[b]



The node voltage equations are:

$$\frac{v_1}{5000} + \frac{v_1 - v_2}{2500} + \frac{v_1 - v_3}{1000} = 0$$

$$-0.01 + \frac{v_2}{4000} + \frac{v_2 - v_1}{2500} + \frac{v_2 - v_3}{2000} = 0$$

$$\frac{v_3 - v_1}{1000} + \frac{v_3 - v_2}{2000} + \frac{v_3}{1000} = 0$$

Put the equations in standard form:

$$v_1\left(\frac{1}{5000} + \frac{1}{2500} + \frac{1}{1000}\right) + v_2\left(-\frac{1}{2500}\right) + v_3\left(-\frac{1}{1000}\right) = 0$$

$$v_1\left(-\frac{1}{2500}\right) + v_2\left(\frac{1}{4000} + \frac{1}{2500} + \frac{1}{2000}\right) + v_3\left(-\frac{1}{2000}\right) = 0.01$$

$$v_1\left(-\frac{1}{1000}\right) + v_2\left(-\frac{1}{2000}\right) + v_3\left(\frac{1}{2000} + \frac{1}{1000} + \frac{1}{1000}\right) = 0$$

Solving, $v_1 = 6.67$ V; $v_2 = 13.33$ V; $v_3 = 5.33$ V

$p_{10m} = -(13.33)(0.01) = -133.33$ m W

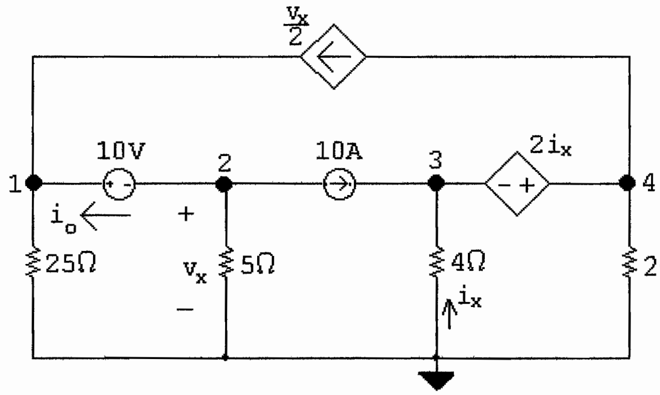
Therefore, the 10 mA source is developing 133.33 mW

P 4.56 [a] The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 10 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 10 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node voltage method, it is the preferred approach.

[b]



Node voltage equations:

$$\frac{v_1}{25} - \frac{v_x}{2} + \frac{v_2}{5} + 10 = 0$$

$$-10 + \frac{v_3}{4} + \frac{v_4}{2} + \frac{v_x}{2} = 0$$

Constraints:

$$v_2 = v_x; \qquad -\frac{v_3}{4} = i_x; \qquad v_4 - v_3 = 2i_x; \qquad v_1 - v_2 = 10$$

Solving,

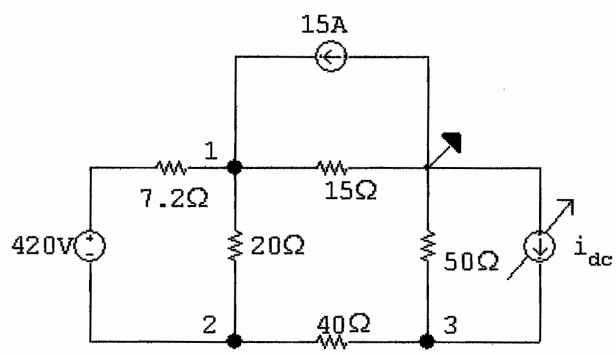
$v_1 = 50$ V; $v_2 = 40$ V; $v_3 = -20$ V; $v_4 = -10$ V; $i_x = 5$ A.

$$i_o = \frac{v_1}{25} - \frac{v_x}{2} = -18 \text{ A}$$

$$p_{10V} = -10i_o = 180 \text{ W}$$

Thus, the 10 V source absorbs 180 W.

P 4.57 Choose the reference node so that a node voltage is identical to the voltage across the 15 A source; thus:



Since the 15 A source is developing 3750 W, v_1 must be 250 V.

Since v_1 is known, we can sum the currents away from node 1 to find v_2; thus:

$$\frac{250 - (420 + v_2)}{7.2} + \frac{250 - v_2}{20} + \frac{250}{15} - 15 = 0$$

$\therefore \quad v_2 = -50$ V

Now that we know v_2 we sum the currents away from node 2 to find v_3; thus:

$$\frac{v_2 + 420 - 250}{7.2} + \frac{v_2 - 250}{20} + \frac{v_2 - v_3}{40} = 0$$

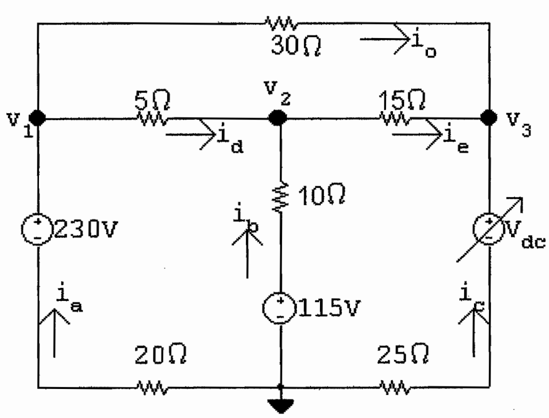
$\therefore \quad v_3 = 50/3$ V

Now that we know v_3 we sum the currents away from node 3 to find i_{dc}; thus:

$$\frac{v_3}{50} + \frac{v_3 + 50}{40} = i_{dc}$$

$\therefore \quad i_{dc} = 2$ A

P 4.58 [a]



If $i_o = 0$ then $v_1 = v_3$; therefore,

$$\frac{v_1 - v_2}{5} + \frac{v_1 - 230}{20} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2 - v_3}{15} + \frac{v_2 - 115}{10} = 0$$

Solving, $v_1 = 170$ V $= v_3$; $\quad v_2 = 155$ V

$$\therefore \quad \frac{170 - 155}{15} + \frac{170 - v_{dc}}{25} = 0$$

Solving, $v_{dc} = 195$ V

[b] $i_a = \dfrac{230 - 170}{20} = 3$ A

$$i_b = \frac{115 - 155}{10} = -4 \text{ A}$$

$$i_c = \frac{195 - 170}{25} = 1 \text{ A}$$

$$i_d = \frac{170 - 155}{5} = 3 \text{ A}$$

$$i_e = \frac{155 - 170}{15} = -1 \text{ A}$$

$$p_{230\text{V}} = -230 i_a = -690 \text{ W (dev)}$$

$$p_{115\text{V}} = -115 i_b = 460 \text{ W (abs)}$$

$$p_{v_{\text{dc}}} = -v_{\text{dc}} i_c = -195 \text{ W (dev)}$$

$$p_{20\Omega} = i_a^2(20) = 180 \text{ W}$$

$$p_{5\Omega} = i_d^2(5) = 45 \text{ W}$$

$$p_{10\Omega} = i_b^2(10) = 160 \text{ W}$$

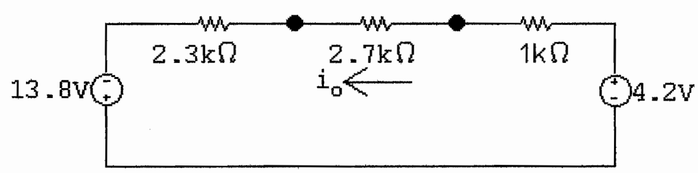
$$p_{15\Omega} = i_e^2(15) = 15 \text{ W}$$

$$p_{25\Omega} = i_c^2(25) = 25 \text{ W}$$

$$\sum p_{\text{diss}} = 460 + 180 + 45 + 160 + 15 + 25 = 885 \text{ W}$$

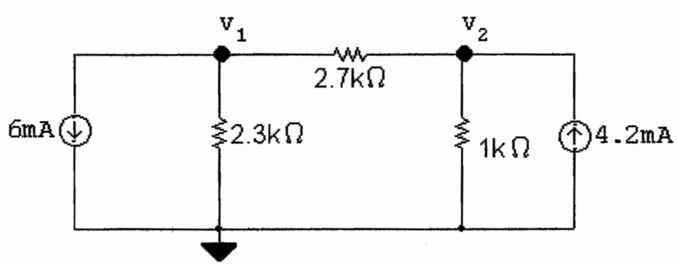
$$\sum p_{\text{dev}} = 690 + 195 = 885 \text{ W (CHECKS)}$$

P 4.59 [a] Apply source transformations to both current sources to get



$$i_o = \frac{13.8 + 4.2}{2700 + 2300 + 1000} = 3 \text{ mA}$$

[b]



The node voltage equations:

$$6 \times 10^{-3} + \frac{v_1}{2300} + \frac{v_1 - v_2}{2700} = 0$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{2700} - 4.2 \times 10^{-3} = 0$$

Place these equations in standard form:

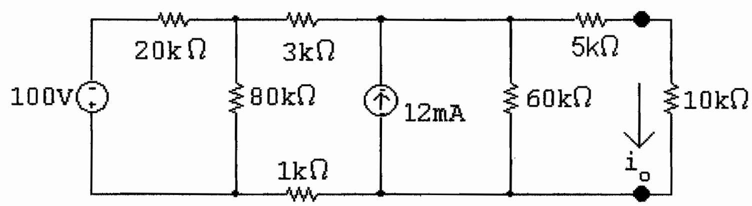
$$v_1\left(\frac{1}{2700}+\frac{1}{2300}\right)+v_2\left(-\frac{1}{2700}\right) = -6\times10^{-3}$$

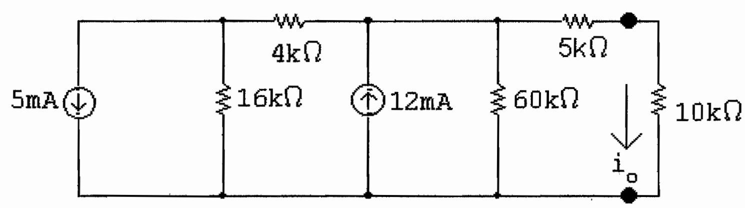
$$v_1\left(-\frac{1}{2700}\right)+v_2\left(\frac{1}{1000}+\frac{1}{2700}\right) = 4.2\times10^{-3}$$

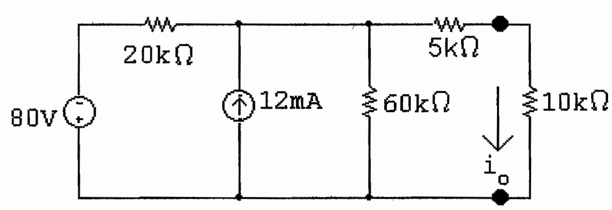
Solving, $v_1 = -6.9$ V; $\quad v_2 = 1.2$ V

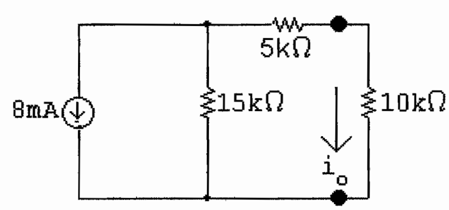
$$\therefore\ i_o = \frac{v_2 - v_1}{2700} = 3 \text{ mA}$$

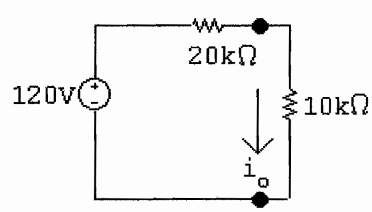
P 4.60 [a]





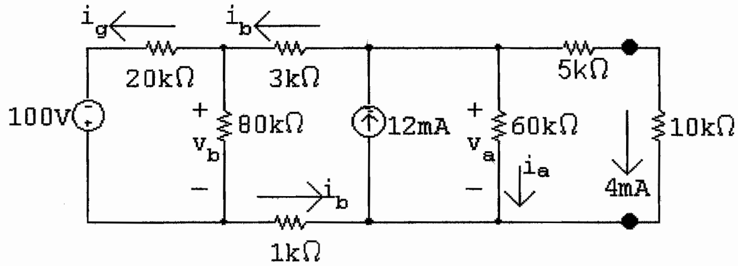






$$i_o = \frac{120}{30{,}000} = 4 \text{ mA}$$

[b]



$$v_a = (15{,}000)(0.004) = 60 \text{ V}$$

$$i_a = \frac{v_a}{60{,}000} = 1 \text{ mA}$$

$$i_b = 12 - 1 - 4 = 7 \text{ mA}$$

$$v_b = 60 - (0.007)(4000) = 32 \text{ V}$$

$$i_g = 0.007 - \frac{32}{80{,}000} = 6.6 \text{ mA}$$

$$p_{100\text{V}} = -(100)(6.6 \times 10^{-3}) = -660 \text{ mW}$$

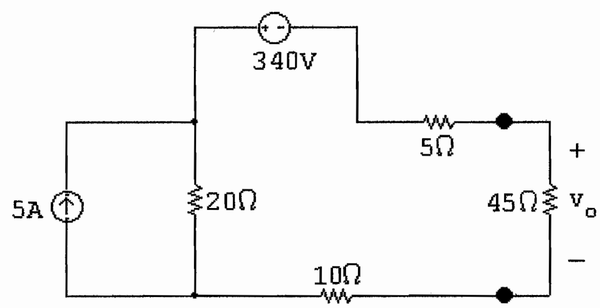
Check:

$$p_{12\text{mA}} = -(60)(12 \times 10^{-3}) = -720 \text{ mW}$$

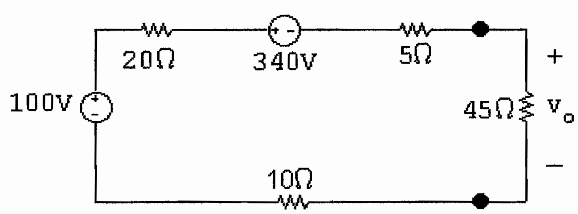
$$\sum P_{\text{dev}} = 660 + 720 = 1380 \text{ mW}$$

$$\sum P_{\text{dis}} = (20{,}000)(6.6 \times 10^{-3})^2 + (80{,}000)(0.4 \times 10^{-3})^2 + (4000)(7 \times 10^{-3})^2$$
$$+(60{,}000)(1 \times 10^{-3})^2 + (15{,}000)(4 \times 10^{-3})^2$$
$$= 1380 \text{ mW}$$

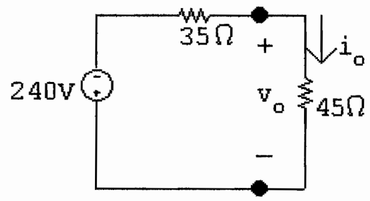
P 4.61 [a] First remove the 8 Ω and 80 Ω resistors:



Next use a source transformation to convert the 5 A current source and 20 Ω resistor:

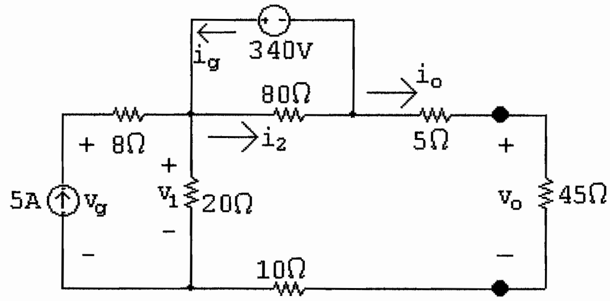


which simplifies to



$$\therefore \quad v_o = \frac{45}{80}(-240) = -135 \text{ V}; \qquad i_o = \frac{-135}{45} = -3 \text{ A}$$

[b] Return to the original circuit with $v_o = -135$ V and $i_o = -3$ A:



$$i_g = \frac{340}{80} - (-3) = 7.25 \text{ A}$$

$$p_{340\text{V}} = -(340)(7.25) = -2465 \text{ W}$$

Therefore, the 340 V source is developing 2465 W.

[c] $v_1 = 340 + 60i_o = 340 - 180 = 160$ V

$$v_g = v_1 + 5(8) = 160 + 40 = 200 \text{ V}$$

$$p_{5\text{A}} = -(5)(200) = -1000 \text{ W}$$

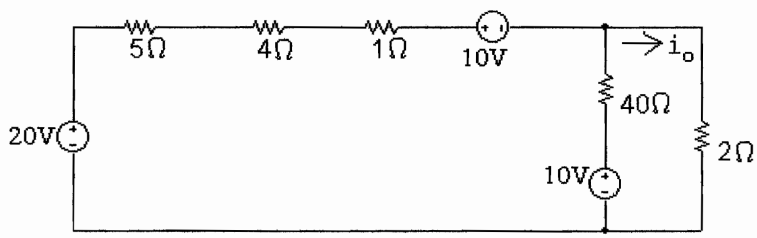
Therefore the 5 A source is developing 1000 W.

[d] $\sum p_{\text{dev}} = 2465 + 1000 = 3465$ W

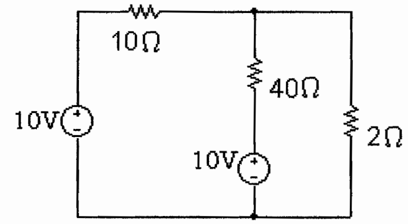
$$\sum p_{\text{diss}} = (5)^2(8) + (8)^2(20) + (4.25)^2(80) + (3)^2(60) = 3465 \text{ W}$$

$$\therefore \quad \sum p_{\text{diss}} = \sum p_{\text{dev}}$$

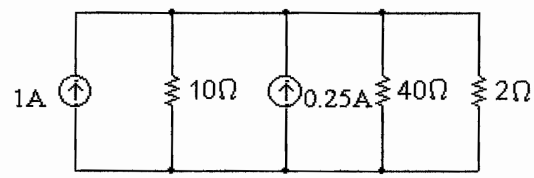
P 4.62 [a] Applying a source transformation to each current source yields



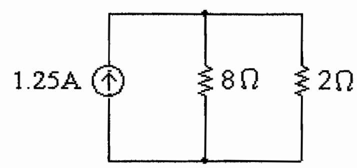
Now combine the 20 V and 10 V sources into a single voltage source and the 5 Ω, 4 Ω and 1 Ω resistors into a single resistor to get



Now use a source transformation on each voltage source, thus

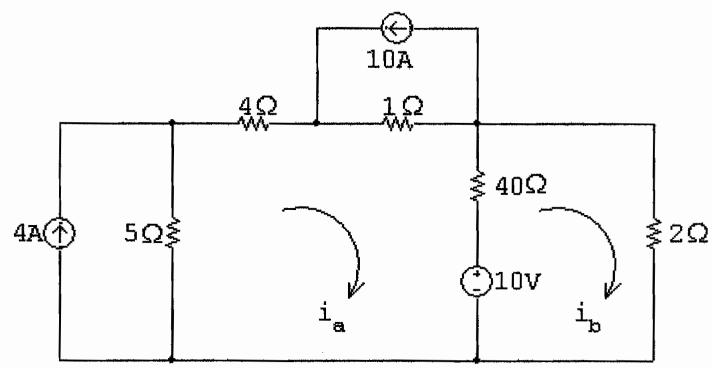


which can be reduced to



$$\therefore \quad i_o = \frac{(1.25)(8)}{10} = 1 \text{ A}$$

[b]



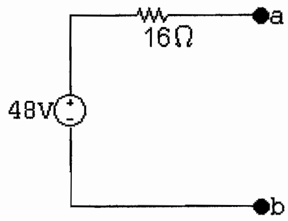
$$50i_a - 40i_b = 20 - 10 - 10 = 0$$

$$-40i_a + 42_b = 10$$

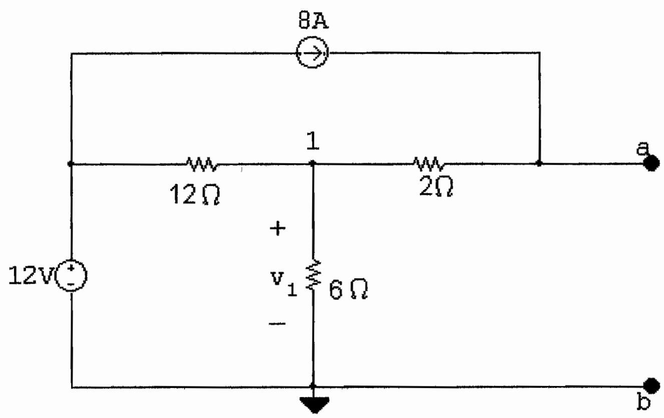
$$\text{Solving, } i_b = \frac{N_b}{\Delta} = 1 \text{ A} = i_o$$

P 4.63 $v_{Th} = \frac{60}{50}(40) = 48$ V

$$R_{Th} = 8 + \frac{(40)(10)}{50} = 16\,\Omega$$



P 4.64

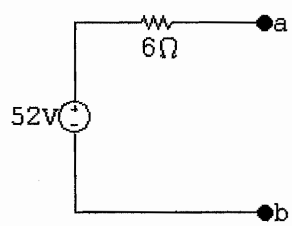


$$\frac{v_1 - 12}{12} + \frac{v_1}{6} - 8 = 0$$

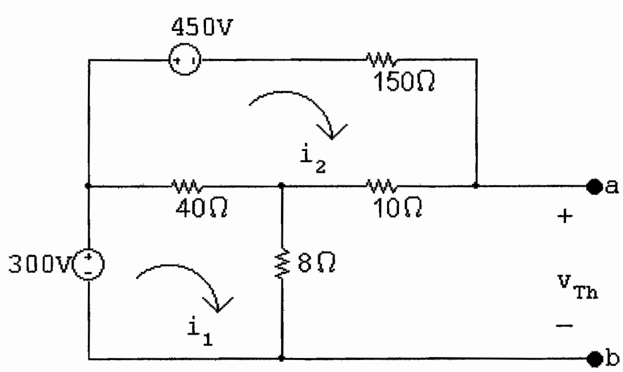
$$v_1 = 36 \text{ V}$$

$$v_{\text{Th}} = v_1 + (2)(8) = 52 \text{ V}$$

$$R_{\text{Th}} = 2 + \frac{(12)(6)}{18} = 6\,\Omega$$



P 4.65 After making a source transformation the circuit becomes



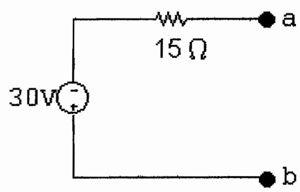
$$300 = 48i_1 - 40i_2$$

$-450 = -40i_1 + 200i_2$

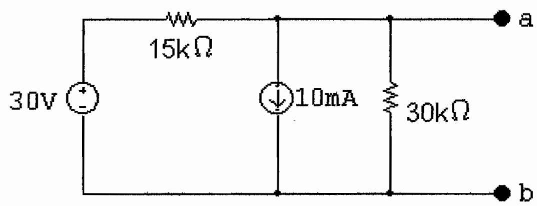
$\therefore \quad i_1 = 5.25 \text{ A and } i_2 = -1.2 \text{ A}$

$v_{\text{Th}} = 8i_1 + 10i_2 = 30 \text{ V}$

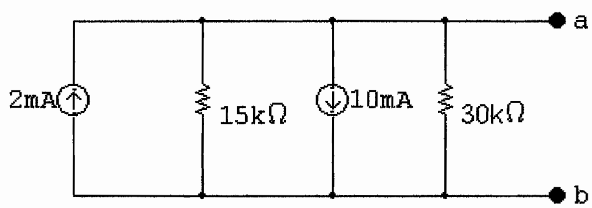
$R_{\text{Th}} = (40\|8 + 10)\|150 = 15\,\Omega$



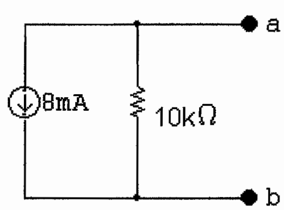
P 4.66 First we make the observation that the 8-mA current source and the 20 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



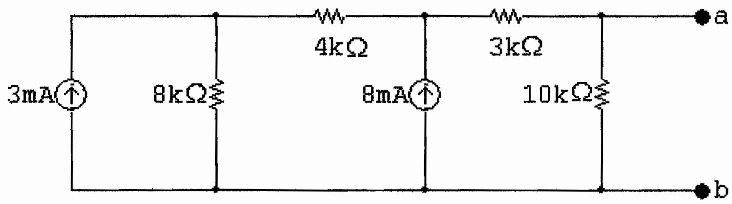
or

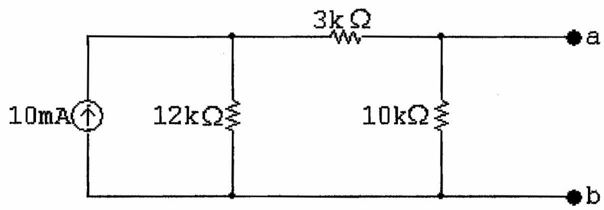


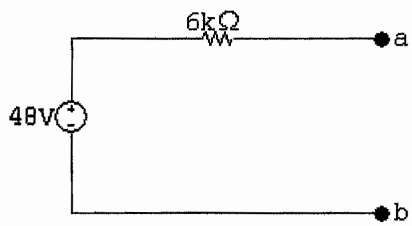
Therefore the Norton equivalent is



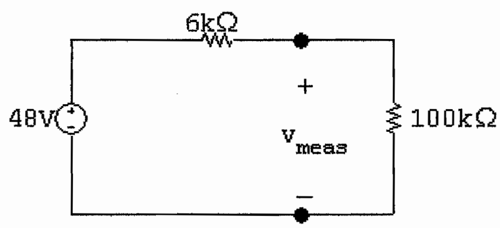
P 4.67 [a] First, find the Thévenin equivalent with respect to a,b using a succession of source transformations.







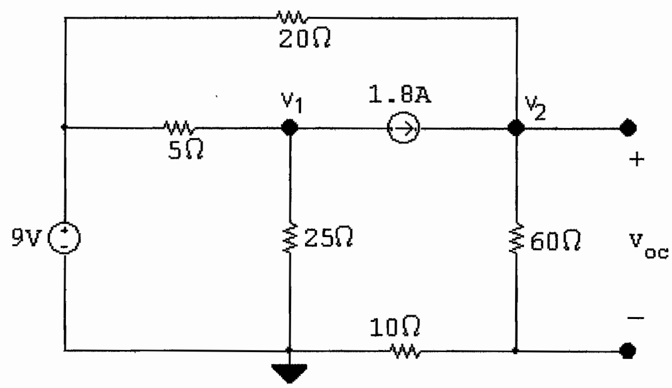
$$\therefore \quad v_{\text{Th}} = 48 \text{ V} \qquad R_{\text{Th}} = 6 \text{ k}\Omega$$



$$v_{\text{meas}} = \frac{100}{106}(48) = 45.28 \text{ V}$$

[b] $\%\text{error} = \left(\frac{45.28 - 48}{48}\right) \times 100 = -5.67\%$

P 4.68 [a] Open circuit:

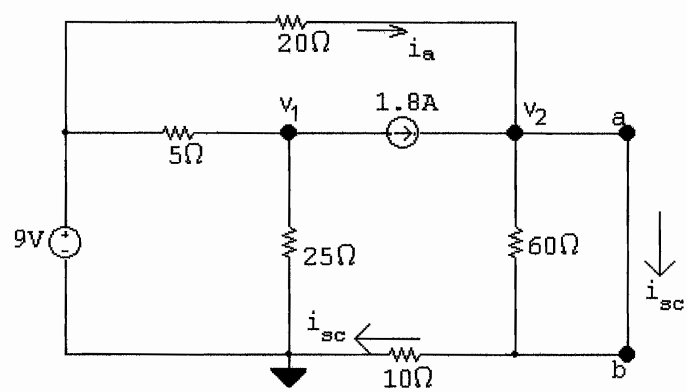


$$\frac{v_2 - 9}{20} + \frac{v_2}{70} - 1.8 = 0$$

$v_2 = 35$ V

$$v_{\text{Th}} = \frac{60}{70}v_2 = 30 \text{ V}$$

Short circuit:



$$\frac{v_2 - 9}{20} + \frac{v_2}{10} - 1.8 = 0$$

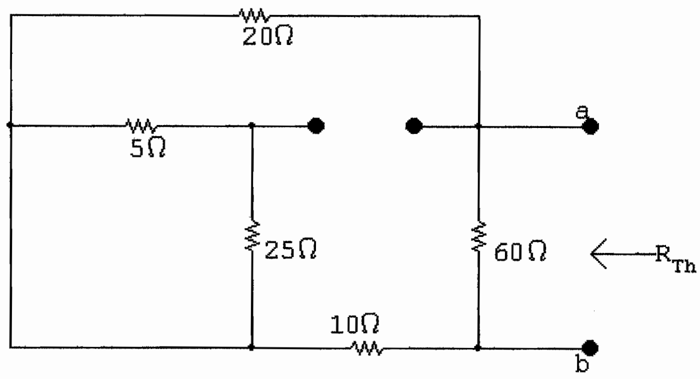
$\therefore \quad v_2 = 15$ V

$$i_a = \frac{9 - 15}{20} = -0.3 \text{ A}$$

$i_{sc} = 1.8 - 0.3 = 1.5$ A

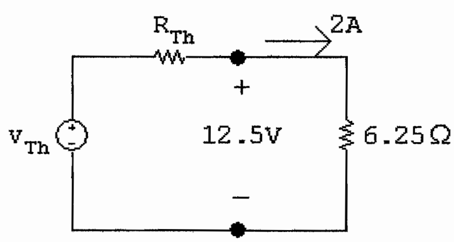
$$R_{\text{Th}} = \frac{30}{1.5} = 20\,\Omega$$

[b]

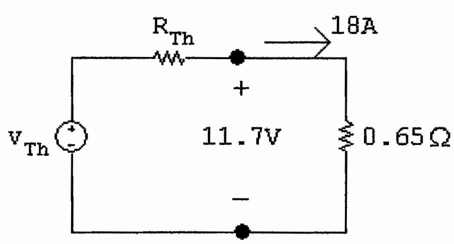


$R_{\text{Th}} = (20 + 10\|60 = 20\,\Omega$ (CHECKS)

P 4.69



$12.5 = v_{\text{Th}} - 2R_{\text{Th}}$



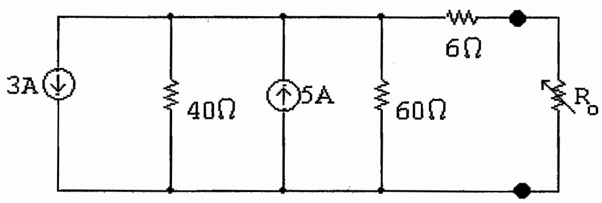
$11.7 = v_{\text{Th}} - 18R_{\text{Th}}$

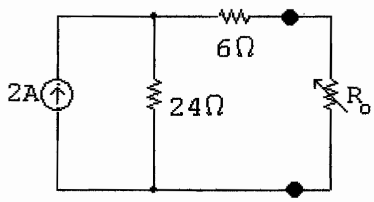
Solving the above equations for V_{Th} and R_{Th} yields

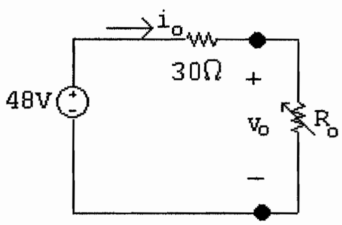
$v_{\text{Th}} = 12.6v, \qquad R_{\text{Th}} = 50\ \text{m}\Omega$

$\therefore\ \ I_N = 252\ \text{A}, \qquad R_N = 50\ \text{m}\Omega$

P 4.70 First, find the Thévenin equivalent with respect to R_o.

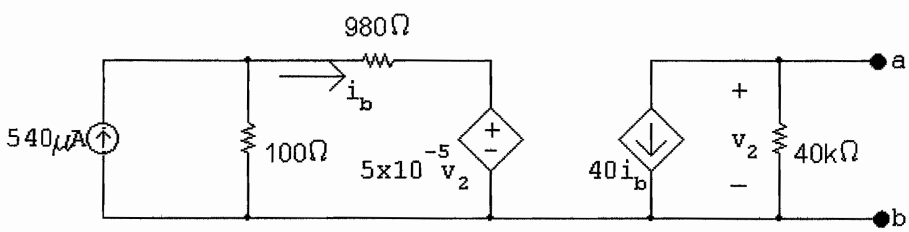






R_o	i_o	v_o	R_o	i_o	v_o
0	1.6	0	20	0.96	19.2
2	1.5	3	30	0.8	24
6	1.33	8	50	0.6	30
10	1.2	12	60	0.533	32
15	1.067	16	70	0.48	33.6

P 4.71



OPEN CIRCUIT

$v_2 = -40i_b\ 40 \times 10^3 = -16 \times 10^5 i_b$

$5 \times 10^{-5} v_2 = -80i_b$

$980i_b + 5 \times 10^{-5} v_2 = 980i_b - 80i_b = 900i_b$

So $900i_b$ is the voltage across the $100\,\Omega$ resistor.

From KCL at the top left node, $540\,\mu\text{A} = \frac{900i_b}{100} + i_b = 10i_b$

$$\therefore \quad i_b = \frac{540 \times 10^{-6}}{10} = 54\,\mu\text{A}$$

$$v_{\text{Th}} = -16 \times 10^5 (54 \times 10^{-6}) = -86.40\ \text{V}$$

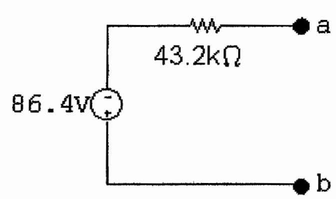
SHORT CIRCUIT

$$v_2 = 0; \qquad i_{\text{sc}} = -40i_b$$

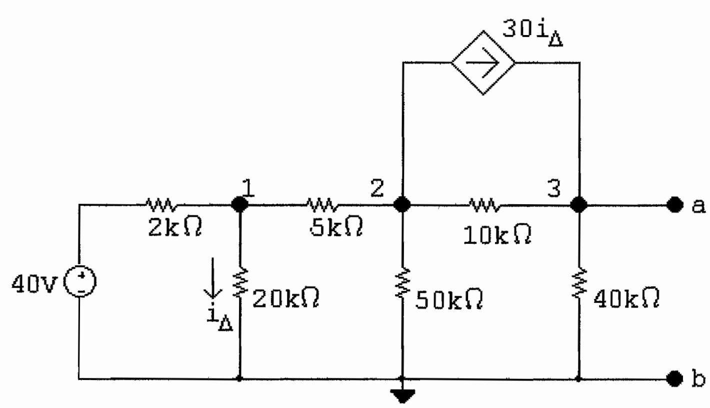
$$i_b = \frac{54 \times 10^{-3}}{1080} = \frac{54}{1.08} \times 10^{-6} = 50\,\mu\text{A}$$

$$i_{\text{sc}} = -40(50) = -2000\,\mu\text{A} = -2\ \text{mA}$$

$$R_{\text{Th}} = \frac{-86.4}{-2} \times 10^3 = 43.2\ \text{k}\Omega$$



P 4.72



The node voltage equations are:

$$\frac{v_1 - 40}{2000} + \frac{v_1}{20{,}000} + \frac{v_1 - v_2}{5000} = 0$$
$$\frac{v_2 - v_1}{5000} + \frac{v_2}{50{,}000} + \frac{v_2 - v_3}{10{,}000} + 30\frac{v_1}{20{,}000} = 0$$
$$\frac{v_3 - v_2}{10{,}000} + \frac{v_3}{40{,}000} - 30\frac{v_1}{20{,}000} = 0$$

In standard form:

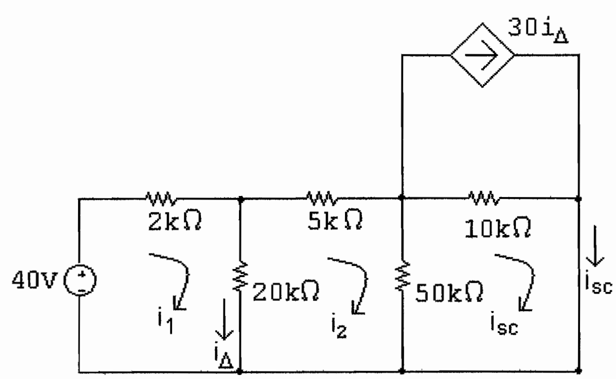
$$v_1\left(\frac{1}{2000} + \frac{1}{20{,}000} + \frac{1}{5000}\right) + v_2\left(-\frac{1}{5000}\right) + v_3(0) = \frac{40}{2000}$$

$$v_1\left(-\frac{1}{5000} + \frac{30}{20{,}000}\right) + v_2\left(\frac{1}{5000} + \frac{1}{50{,}000} + \frac{1}{10{,}000}\right) + v_3\left(-\frac{1}{10{,}000}\right) = 0$$

$$v_1\left(-\frac{30}{20{,}000}\right) + v_2\left(-\frac{1}{10{,}000}\right) + v_3\left(\frac{1}{10{,}000} + \frac{1}{40{,}000}\right) = 0$$

Solving, $v_1 = 24$ V; $v_2 = -10$ V; $v_3 = 280$ V

$V_{\text{Th}} = v_3 = 280$ V



The mesh current equations are:

$$-40 + 2000i_1 + 20{,}000(i_1 - i_2) = 0$$
$$5000i_2 + 50{,}000(i_2 - i_{sc}) + 20{,}000(i_2 - i_1) = 0$$
$$50{,}000(i_{sc} - i_2) + 10{,}000(i_{sc} - 30i_\Delta) = 0$$

The constraint equation is:

$$i_\Delta = i_1 - i_2$$

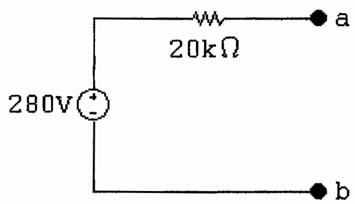
Put these equations in standard form:

$$\begin{aligned}
i_1(22{,}000) + i_2(-20{,}000) + i_{sc}(0) + i_\Delta(0) &= 40\\
i_1(-20{,}000) + i_2(75{,}000) + i_{sc}(-50{,}000) + i_\Delta(0) &= 0\\
i_1(0) + i_2(-50{,}000) + i_{sc}(60{,}000) + i_\Delta(-300{,}000) &= 0\\
i_1(-1) + i_2(1) + i_{sc}(0) + i_\Delta(1) &= 0
\end{aligned}$$

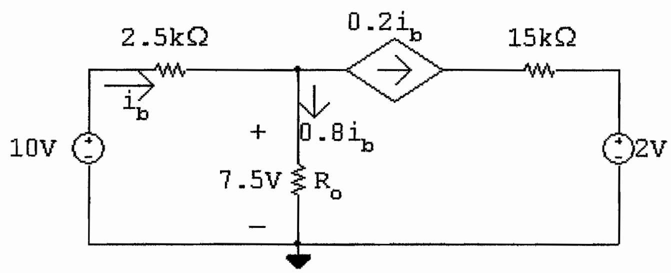
Solving,

$i_1 = 13.6$ m A; $i_2 = 12.96$ m A; $i_{sc} = 14$ m A; $i_\Delta = 640\,\mu$ A

$$R_{Th} = \frac{280}{0.014} = 20\text{ k}\Omega$$



P 4.73 [a] Use source transformations to simplify the left side of the circuit.



$$i_b = \frac{10 - 7.5}{2.5} = 1\text{ mA}$$

Let $R_o = R_{meter}\|10\text{ k}\Omega = 7.5/0.8 = 9.375\text{ k}\Omega$

$$\therefore \quad \frac{(R_{meter})(10)}{R_{metere} + 10} = 9.375; \qquad R_{meter} = \frac{(9.375)(10)}{0.625} = 150\text{ k}\Omega$$

[b] Actual value of v_e:

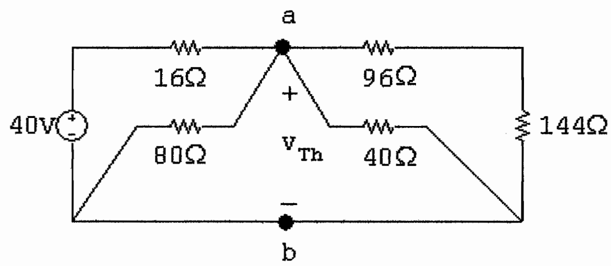
$$i_b = \frac{10}{2.5 + (0.8)(10)} = 0.9524\text{ mA}$$

$$v_e = 0.8 i_b(10) = 7.62\text{ V}$$

$$\%\text{ error} = \left(\frac{7.5 - 7.62}{7.62}\right) \times 100 = -1.57\%$$

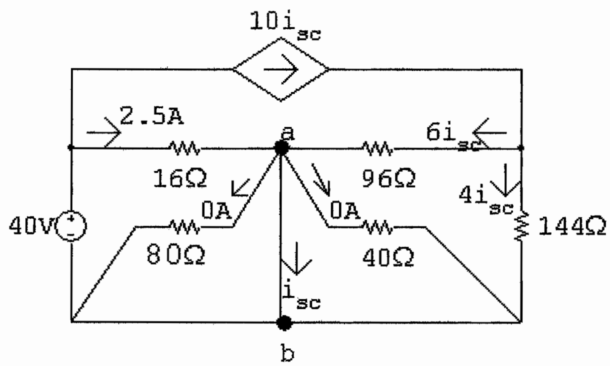
P 4.74 [a] Find the Thévenin equivalent with respect to the terminals of the ammeter. This is most easily done by first finding the Thévenin with respect to the terminals of the $50\,\Omega$ resistor.

Thévenin voltage: note i_ϕ is zero.



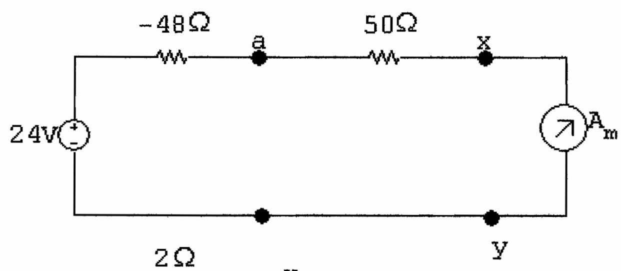
$$\frac{v_{\rm Th}}{80} + \frac{v_{\rm Th}}{40} + \frac{v_{\rm Th}}{240} + \frac{v_{\rm Th} - 40}{16} = 0$$

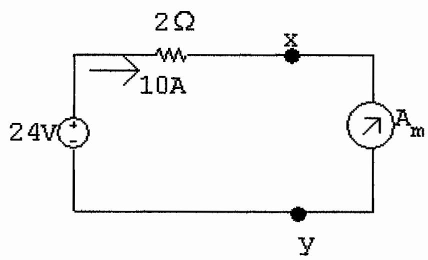
Solving, $v_{\rm Th} = 24$ V.
Short-circuit current:



$$i_{\rm sc} = 2.5 + 6i_{\rm sc}, \qquad \therefore \quad i_{\rm sc} = -0.5 \text{ A}$$

$$R_{\rm Th} = \frac{24}{-0.5} = -48\,\Omega$$

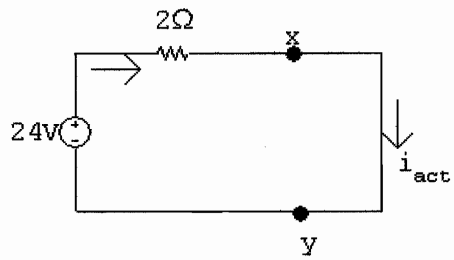




$$R_{\rm total} = \frac{24}{10} = 2.4\,\Omega$$

$$R_{\text{meter}} = 2.4 - 2 = 0.40\,\Omega$$

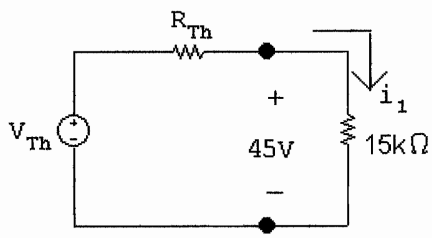
[b] Actual current:



$$i_{\text{actual}} = \frac{24}{2} = 12 \text{ A}$$

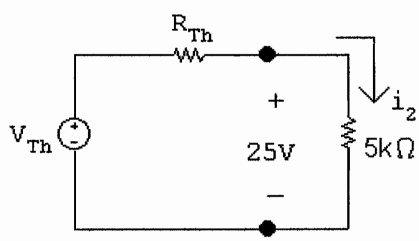
$$\% \text{ error } = \frac{10 - 12}{12} \times 100 = -16.67\%$$

P 4.75



$i_1 = 45/15{,}000 = 3$ mA

$45 = v_{\text{Th}} - 0.003R_{\text{Th}}, \qquad v_{\text{Th}} = 45 + 0.003R_{\text{Th}}$

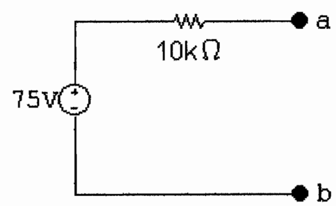


$i_2 = 25/5000 = 5$ mA

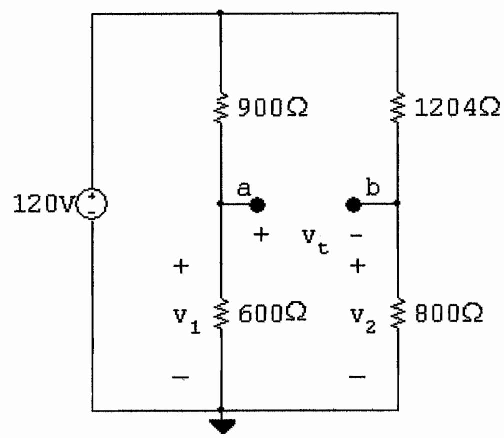
$25 = v_{\text{Th}} - 0.005R_{\text{Th}}, \qquad v_{\text{Th}} = 25 + 0.005R_{\text{Th}}$

$\therefore \quad 45 + 0.003R_{\text{Th}} = 25 + 0.005R_{\text{Th}} \qquad$ so $\qquad R_{\text{Th}} = 10 \text{ k}\Omega$

$v_{\text{Th}} = 45 + 30 = 75$ V



P 4.76

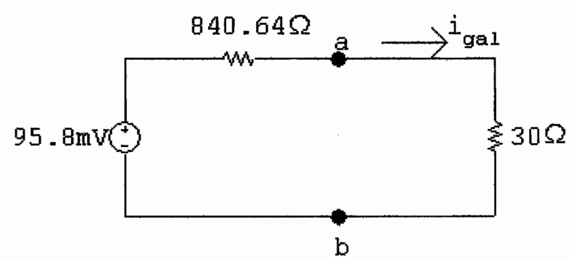


$$v_1 = \frac{600}{1500}(120) = 48 \text{ V}$$

$$v_2 = \frac{800}{2004}(120) = 47.9042 \text{ V}$$

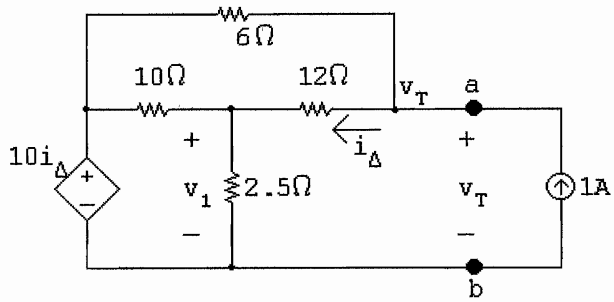
$$v_{\text{Th}} = v_1 - v_2 = 48 - 47.9042 = 95.8 \text{ mV}$$

$$R_{\text{Th}} = \frac{(900)(600)}{1500} + \frac{(1204)(800)}{2004} = \frac{2{,}105{,}800}{2505} = 840.64\,\Omega$$



$$i_{\text{gal}} = \frac{95.8 \times 10^{-3}}{0.87064 \times 10^3} = 110.03\,\mu\text{A}$$

P 4.77 $V_{\text{Th}} = 0$, since circuit contains no independent sources.



$$\frac{v_1 - 10i_\Delta}{10} + \frac{v_1}{2.5} + \frac{v_1 - v_T}{12} = 0$$

$$\frac{v_T - v_1}{12} + \frac{v_T - 10i_\Delta}{6} - 1 = 0$$

$$i_\Delta = \frac{v_T - v_1}{12}$$

In standard form:

$$v_1\left(\frac{1}{10} + \frac{1}{2.5} + \frac{1}{12}\right) + v_T\left(-\frac{1}{12}\right) + i_\Delta(-1) = 0$$

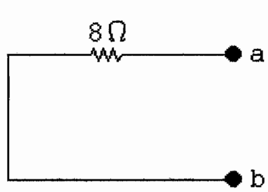
$$v_1\left(-\frac{1}{12}\right) + v_T\left(\frac{1}{12} + \frac{1}{6}\right) + i_\Delta\left(-\frac{10}{6}\right) = 1$$

$$v_1(1) + v_T(-1) + i_\Delta(12) = 0$$

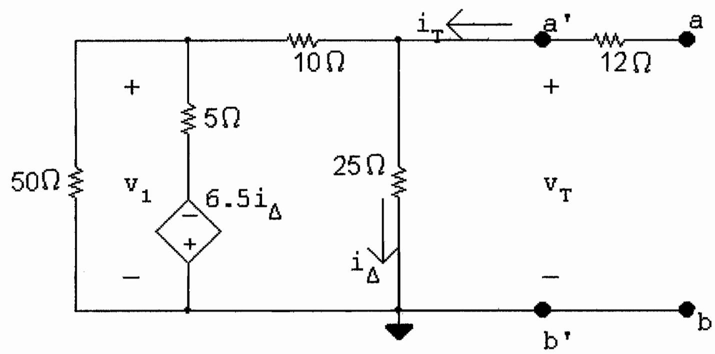
Solving,

$$v_1 = 2 \text{ V}; \qquad v_T = 8 \text{ V}; \qquad i_\Delta = 0.5 \text{ A}$$

$$\therefore \quad R_{\text{Th}} = \frac{v_T}{1 \text{ A}} = 8\,\Omega$$



P 4.78 $V_{\text{Th}} = 0$ since there are no independent sources in the circuit. To find R_{Th} we first find $R_{a'b'}$.



$$i_{\text{T}} = \frac{v_{\text{T}}}{25} + \frac{v_{\text{T}} - v_1}{10}$$

$$\frac{v_1}{50} + \frac{v_1 + 6.5i_\Delta}{5} + \frac{v_1 - v_{\text{T}}}{10} = 0 \text{ so } 16v_1 + 65i_\Delta = 5v_{\text{T}}$$

$$i_\Delta = \frac{v_{\text{T}}}{25}, \qquad 65i_\Delta = 2.6v_{\text{T}}$$

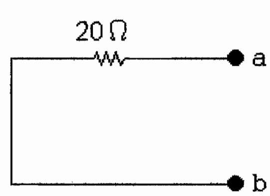
$$16v_1 + 2.6v_{\text{T}} = 5v_{\text{T}}$$

$$\therefore \quad v_1 = 0.15v_{\text{T}}$$

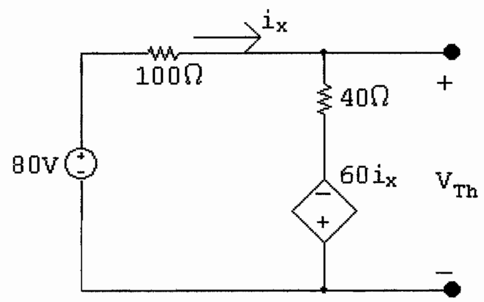
$$i_{\text{T}} = \frac{v_{\text{T}}}{25} + \frac{v_{\text{T}} - 0.15v_{\text{T}}}{10} = \frac{6.25}{50}v_{\text{T}}$$

$$\frac{v_{\text{T}}}{i_{\text{T}}} = 50/6.25 = 8\,\Omega = R_{a'b'}$$

$$\therefore \quad R_{\text{Th}} = 12 + 8 = 20\,\Omega$$



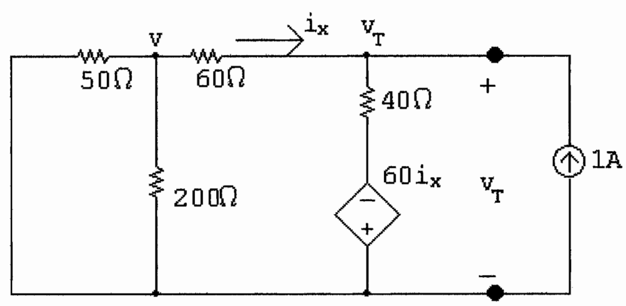
P 4.79 We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_x = \frac{80 + 60i_x}{140}; \qquad i_x = 1 \text{ A}$$

$$V_{\text{Th}} = 40i_x - 60i_x = -20i_x = -20 \text{ V}$$

Using the test-source method to find the Thévenin resistance gives



Use the node voltage method:

$$\frac{v}{50} + \frac{v - v_{\text{T}}}{60} + \frac{v}{200} = 0$$

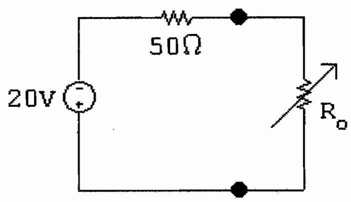
$$\frac{v_{\text{T}} - v}{60} + \frac{v_{\text{T}} + 60i_x}{40} - 1 = 0$$

$$i_x = \frac{v - v_{\text{T}}}{60}$$

Solving, $\quad v_{\text{T}} = 50$ V.

$$R_{\text{Th}} = \frac{v_{\text{T}}}{1 \text{ A}} = 50\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



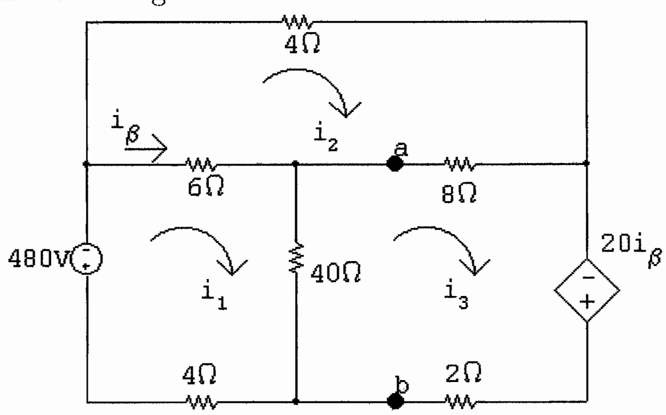
$$\left(\frac{-20}{50+R_o}\right)^2 R_o = 1.5$$

$$\frac{400R_o}{R_o^2+100R_o+2500} = 1.5$$

$$1.5R_o^2 - 250R_o + 3750 = 0$$

$$\therefore \quad R_o = 16.67\,\Omega; \quad R_o = 150\,\Omega$$

P 4.80 [a] Find the Thévenin equivalent with respect to the terminals of R_L.
Open circuit voltage:



The mesh current equations are:

$$\begin{aligned}
480 + 6(i_1 - i_2) + 40(i_1 - i_3) + 4i_1 &= 0\\
4i_2 + 8(i_2 - i_3) + 6(i_2 - i_1) &= 0\\
-20i_\beta + 2i_3 + 40(i_3 - i_1) + 8(i_3 - i_2) &= 0
\end{aligned}$$

The dependent source constraint equation is:
$i_\beta = i_1 - i_2$

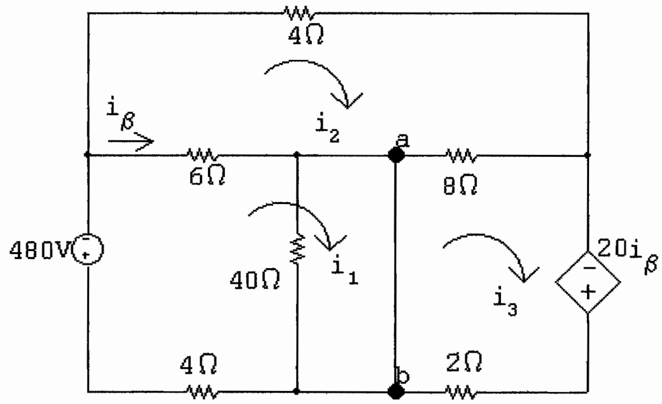
Place these equations in standard form:

$$\begin{aligned}
i_1(6+40+4) + i_2(-6) + i_3(-40) + i_\beta(0) &= -480\\
i_1(-6) + i_2(4+8+6) + i_3(-8) + i_\beta(0) &= 0\\
i_1(-40) + i_2(-8) + i_3(8+2+40) + i_\beta(-20) &= 0\\
i_1(-1) + i_2(1) + i_3(0) + i_\beta(1) &= 0
\end{aligned}$$

Solving, $i_1 = -99.6$ A; $i_2 = -78$ A; $i_3 = -100.8$ A; $i_\beta = -21.6$ A

$V_{\text{Th}} = 40(i_1 - i_3) = 48$ V

Short-circuit current:



The mesh current equations are:

$$\begin{aligned} 480 + 6(i_1 - i_2) + 4i_1 &= 0 \\ 4i_2 + 8(i_2 - i_3) + 6(i_2 - i_1) &= 0 \\ -20i_\beta + 2i_3 + 8(i_3 - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is:

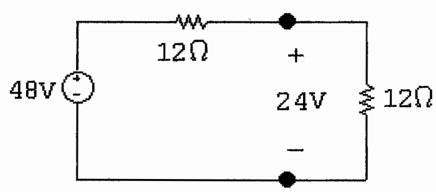
$i_\beta = i_1 - i_2$

Place these equations in standard form:

$$\begin{aligned} i_1(6+4) + i_2(-6) + i_3(0) + i_\beta(0) &= -480 \\ i_1(-6) + i_2(4+8+6) + i_3(-8) + i_\beta(0) &= 0 \\ i_1(0) + i_2(-8) + i_3(8+2) + i_\beta(-20) &= 0 \\ i_1(-1) + i_2(1) + i_3(0) + i_\beta(1) &= 0 \end{aligned}$$

Solving, $i_1 = -92$ A; $i_2 = -73.33$ A; $i_3 = -96$ A; $i_\beta = -18.67$ A

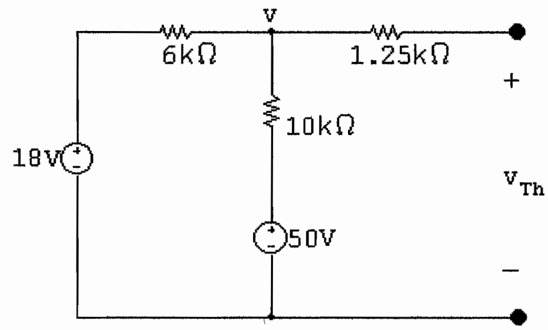
$i_{\text{sc}} = i_1 - i_3 = 4$ A; $\quad R_{\text{Th}} = \dfrac{V_{\text{Th}}}{i_{\text{sc}}} = \dfrac{48}{4} = 12\,\Omega$



$R_{\text{L}} = R_{\text{Th}} = 12\,\Omega$

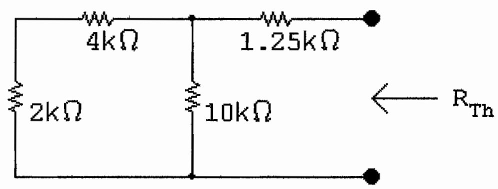
[b] $p_{\text{max}} = \dfrac{24^2}{12} = 48$ W

P 4.81 [a]



$$\frac{v_{\mathrm{Th}} - 18}{6000} + \frac{v_{\mathrm{Th}} - 50}{10{,}000} = 0$$

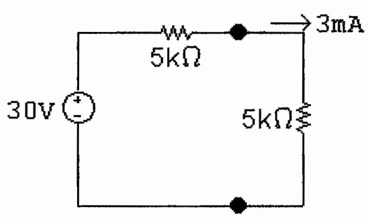
Solving, $\quad v_{\mathrm{Th}} = 30$ V



$$R_{\mathrm{Th}} = 1250 + 10{,}000\|(2000 + 4000) = 5\ \mathrm{k}\Omega$$

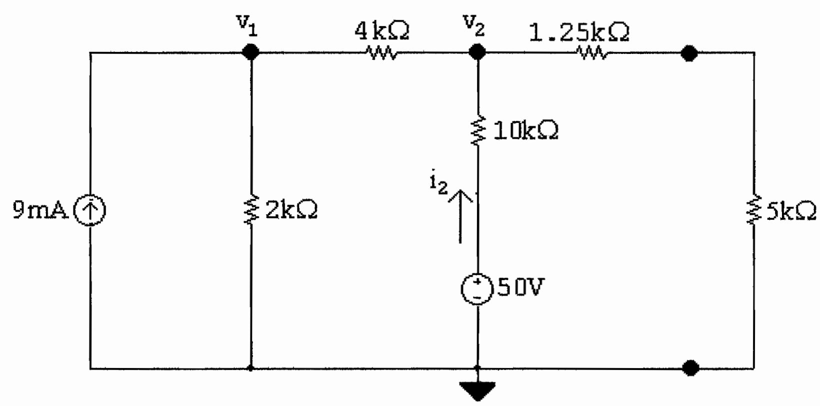
$$R_o = R_{\mathrm{Th}} = 5\ \mathrm{k}\Omega$$

[b]



$$p_{\max} = (3 \times 10^{-3})^2(5000) = 45\ \text{m W}$$

P 4.82 Write KCL equations at each of the labeled nodes, place them in standard form, and solve:



At v_1: $$-9\times10^{-3}+\frac{v_1}{2000}+\frac{v_1-v_2}{4000}=0$$

At v_2: $$\frac{v_2-v_1}{4000}+\frac{v_2-50}{10{,}000}+\frac{v_2}{6250}=0$$

Standard form:

$$v_1\left(\frac{1}{2000}+\frac{1}{4000}\right)+v_2\left(-\frac{1}{4000}\right)=0.009$$

$$v_1\left(-\frac{1}{4000}\right)+v_2\left(\frac{1}{4000}+\frac{1}{10{,}000}+\frac{1}{6250}\right)=\frac{50}{10{,}000}$$

Calculator solution:

$v_1 = 18.25\ \text{V}$ $\qquad$ $v_2 = 18.75\ \text{V}$

Calculate currents:

$$i_2=\frac{50-v_2}{10{,}000}=3.125\ \text{m A}$$

Calculate power delivered by the sources:

$$p_{9\text{mA}}=(9\times10^{-3})v_1=(9\times10^{-3})(18.25)=164.25\ \text{mW}$$

$$p_{50\text{V}}=i_2(50)=(3.125\times10^{-3})(50)=156.25\ \text{mW}$$

$p_{\text{delivered}} = 164.25 + 156.25 = 320.5$ mW

From Problem 4.81,

$p_{5\text{k}} = 45$ mW

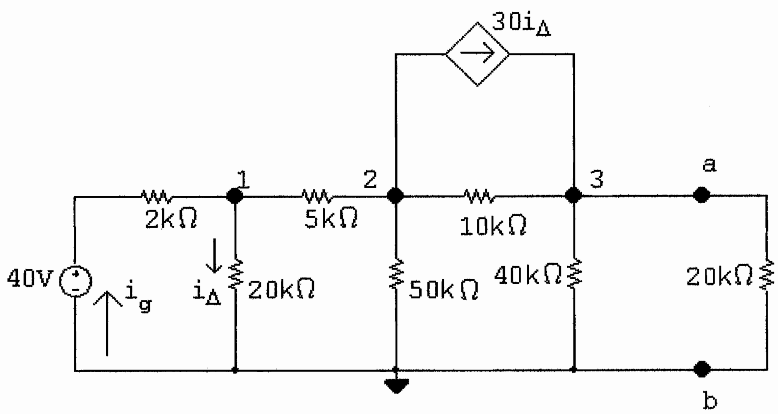
% delivered to R_o: $\quad \frac{45}{320.5}(100) = 14.04\%$

P 4.83 [a] From the solution of Problem 4.72 we have $R_{\text{Th}} = 20$ kΩ and $V_{\text{Th}} = 280$ V. Therefore

$$R_o = R_{\text{Th}} = 20 \text{ k}\Omega$$

[b] $p = \frac{(140)^2}{20{,}000} = 980$ mW

[c]



The node voltage equations are:

$$\frac{v_1 - 40}{2000} + \frac{v_1}{20{,}000} + \frac{v_1 - v_2}{5000} = 0$$

$$\frac{v_2 - v_1}{5000} + \frac{v_2}{50{,}000} + \frac{v_2 - v_3}{10{,}000} + 30i_\Delta = 0$$

$$\frac{v_3 - v_2}{10{,}000} + \frac{v_3}{40{,}000} - 30i_\Delta + \frac{v_3}{20{,}000} = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{v_1}{20{,}000}$$

Place these equations in standard form:

$$v_1\left(\frac{1}{2000} + \frac{1}{20{,}000} + \frac{1}{5000}\right) + v_2\left(-\frac{1}{5000}\right) + v_3(0) + i_\Delta(0) = \frac{40}{2000}$$

$$v_1\left(-\frac{1}{4000}\right) + v_2\left(\frac{1}{4000} + \frac{1}{50{,}000} + \frac{1}{10{,}000}\right) + v_3\left(-\frac{1}{10{,}000}\right) + i_\Delta(30) = 0$$

$$v_1(0) + v_2\left(-\frac{1}{10{,}000}\right) + v_3\left(\frac{1}{10{,}000} + \frac{1}{40{,}000} + \frac{1}{20{,}000}\right) + i_\Delta(-30) = 0$$

$$v_1\left(\frac{-1}{20{,}000}\right) + v_2(0) + v_3(0) + i_\Delta(1) = 0$$

Solving, $v_1 = 18.4$ V; $v_2 = -31$ V; $v_3 = 140$ V; $i_\Delta = 920\,\mu$A

Calculate the power:

$$i_g = \frac{40 - 18.4}{2000} = 10.8 \text{ mA}$$

$$p_{40\text{V}} = -(40)(10.8 \times 10^{-3}) = -432 \text{ mW}$$

$$p_{\text{dep source}} = (v_2 - v_3)(30 i_\Delta) = -4719.6 \text{ mW}$$

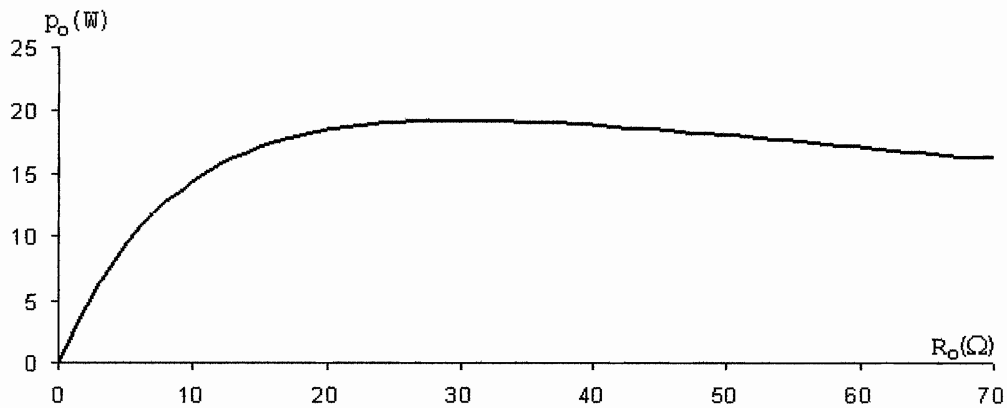
$$\sum p_{\text{dev}} = 432 + 4719.6 = 5151.6 \text{ mW}$$

$$\%\text{ delivered} = \frac{980 \times 10^{-3}}{5151.6 \times 10^{-3}} \times 100 = 19.02\%$$

P 4.84 [a] From the solution to Problem 4.70 we have

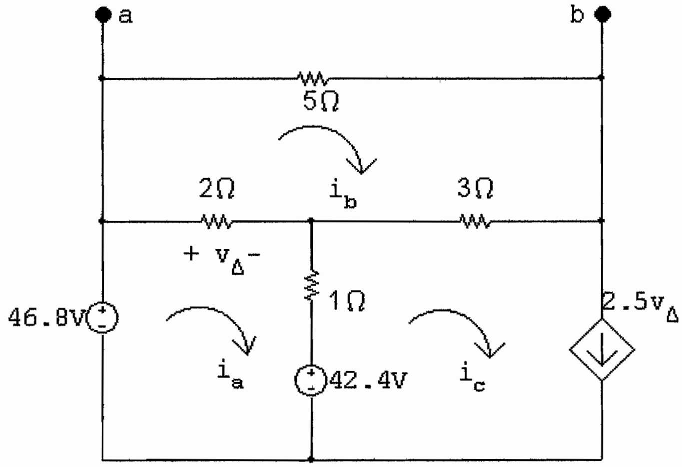
$R_o(\Omega)$	P_o(W)	$R_o(\Omega)$	P_o(W)
0	0	20	18.432
2	4.5	30	19.2
6	10.67	50	18
10	14.4	60	17.067
15	17.067	70	16.128

[b]



[c] $R_o = 30\,\Omega$, P_o (max) $= 19.2$ W

P 4.85 Find the Thévenin equivalent with respect to the terminals of R_o.
Open circuit voltage:



$$(46.8 - 42.4) = 3i_a - 2i_b - i_c$$

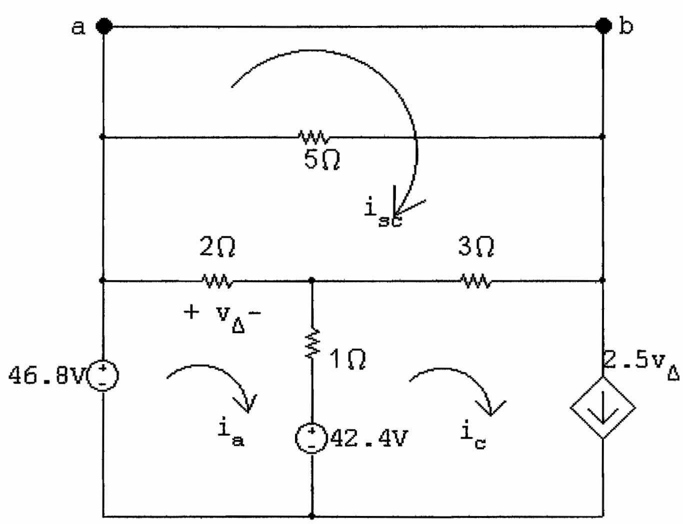
$$0 = -2i_a + 10i_b - 3i_c$$

$$i_c = 2.5v_\Delta; \qquad v_\Delta = 2(i_a - i_b)$$

Solving, $i_b = 74.8$ A

$\therefore \quad v_{\text{Th}} = 5i_b = 374$ V

Short circuit current:



$$46.8 - 42.4 = 3i_a - 2i_{sc} - i_c$$

$0 = -2i_a + 5i_{sc} - 3i_c$

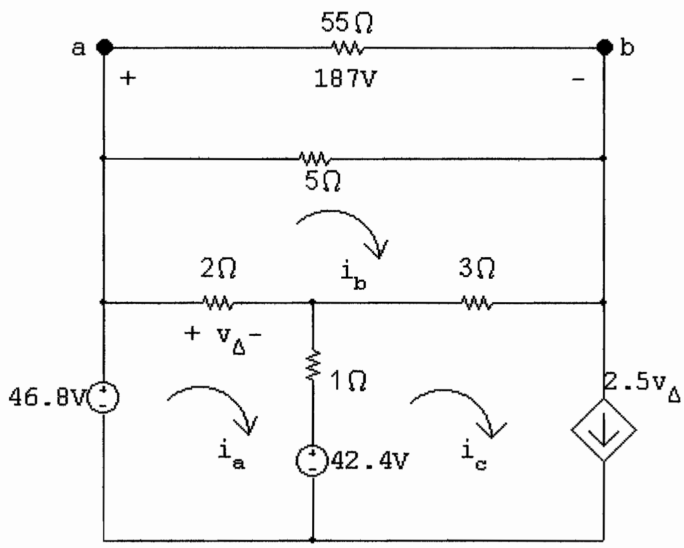
$i_c = 2.5v_\Delta; \qquad v_\Delta = 2(i_a - i_{sc})$

Solving, $i_{sc} = 6.8$ A; $\quad i_a = 8$ A; $\quad i_c = 6$ A; $\quad v_\Delta = 2.4$ V

$R_{\text{Th}} = v_{\text{Th}}/i_{sc} = 374/6.8 = 55\,\Omega$

$R_o = 55\,\Omega$

With R_o equal to $55\,\Omega$ the circuit becomes



$46.8 - 42.4 = 3i_a - 2i_b - 2.5(2)(i_a - i_b)$

$i_c = 2.5v_\Delta; \qquad v_\Delta = 2(i_a - i_b)$

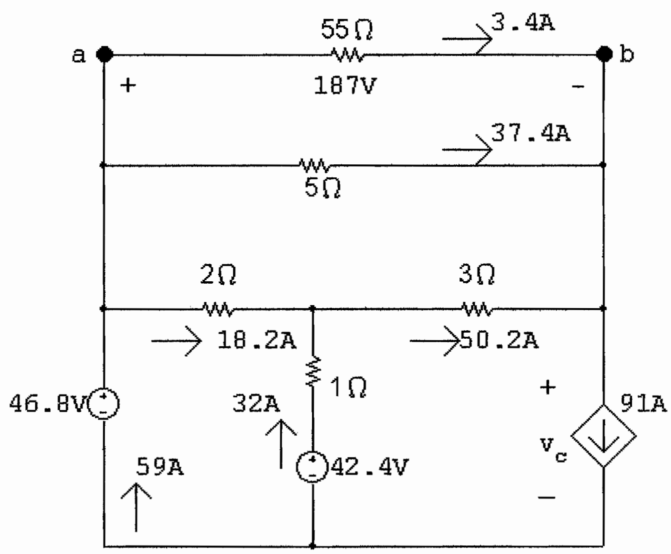
$187 + 3i_b - 3(2.5)(2)(i_a - i_b) + 2i_b - 2i_a = 0$

Solving, $i_a = 59$ A; $\qquad i_b = 40.8$ A

$v_\Delta = 2(59 - 40.80) = 36.4$ V

$i_c = 91$ A

Thus we have



$$v_c = 42.4 - 32 - 150.6 = -140.20 \text{ V}$$

$$\sum P_{\text{dev}} = 46.8(59) + 42.4(32) + 140.20(91) = 16{,}876.20 \text{ W}$$

CHECK:

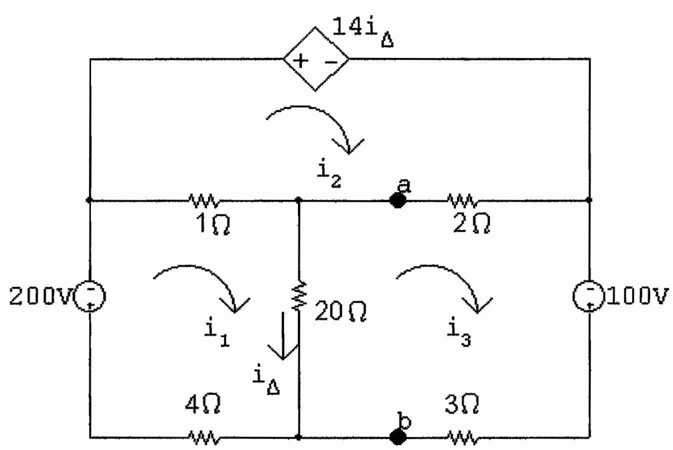
$$\sum P_{\text{dis}} = (18.2)^2(2) + (50.2)^2(3) + (32)^2(1)$$

$$+187(3.4) + 187(37.4) = 16{,}876.20 \text{ W}$$

$$\% \text{ delivered } = \frac{(55)(3.4)^2(100)}{16{,}876.2} = 3.77\%$$

P 4.86 [a] We begin by finding the Thévenin equivalent with respect to the terminals of R_o.

Open circuit voltage



$$-200 = 25i_1 - 1i_2 - 20i_3$$

$0 = -i_1 + 3i_2 - 2i_3 + 14i_\Delta$

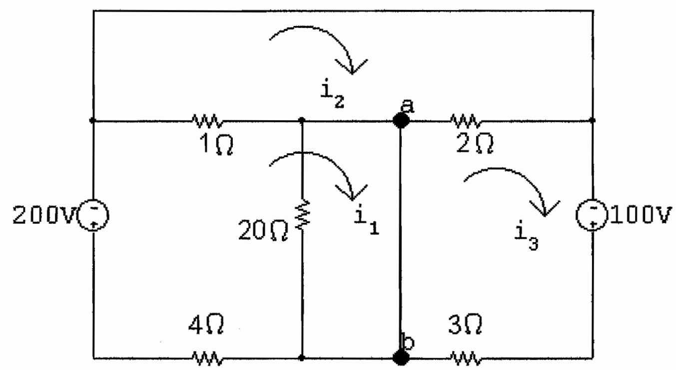
$100 = -20i_1 - 2i_2 + 25i_3$

$i_\Delta = i_1 - i_3$

Solving, $i_1 = -2.5$ A; $i_2 = 37.5$ A; $i_3 = 5$ A; $i_\Delta = -7.5$ A

$v_{\text{Th}} = 20(i_1 - i_3) = 20(-7.5) = -150$ V

Now find the short-circuit current.



Note with the short circuit from a to b that i_Δ is zero, hence $14i_\Delta$ is also zero.

$-200 = 5i_1 - 1i_2 + 0i_3$

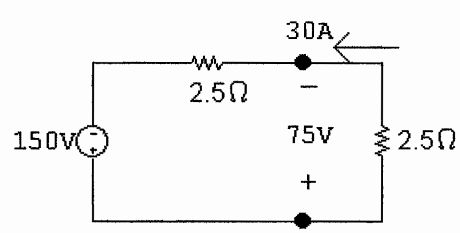
$0 = -1i_1 + 3i_2 - 2i_3$

$100 = 0i_1 - 2i_2 + 5i_3$

Solving, $i_1 = -40$ A; $i_2 = 0$ A; $i_3 = 20$ A

$i_{\text{sc}} = i_1 - i_3 = -60$ A

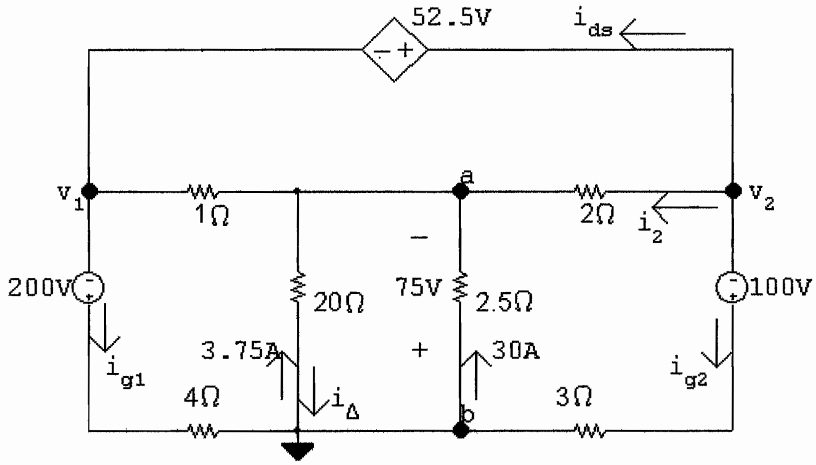
$R_{\text{Th}} = (-150)/(-60) = 2.5\,\Omega$



For maximum power transfer $R_o = R_{\text{Th}} = 2.5\,\Omega$

[b] $p_{\max} = \dfrac{75^2}{2.5} = 2250$ W

P 4.87 From the solution of Problem 4.86 we know that when R_o is 2.5 Ω, the voltage across R_o is 75 V, positive at the lower terminal. Therefore our problem reduces to the analysis of the following circuit. In constructing the circuit we have used the fact that i_Δ is −3.75 A, and hence $14i_\Delta$ is −52.5 V.



Using the node voltage method to find v_1 and v_2 yields

$$-33.75 + \frac{-75 - v_1}{1} + \frac{-75 - v_2}{2} = 0$$

$$v_1 + 52.5 = v_2$$

Solving, $v_1 = -115$ V; $\qquad v_2 = -62.5$ V.It follows that

$$i_{g_1} = \frac{-115 + 200}{4} = 21.25 \text{ A}$$

$$i_{g_2} = \frac{-62.5 + 100}{3} = 12.5 \text{ A}$$

$$i_2 = \frac{-62.5 + 75}{2} = 6.25 \text{ A}$$

$$i_{\text{ds}} = -6.25 - 12.5 = -18.75 \text{ A}$$

$$p_{200\text{V}} = -200i_{g_1} = -4250 \text{ W(dev)}$$

$$p_{100\text{V}} = -100i_{g_2} = -1250 \text{ W(dev)}$$

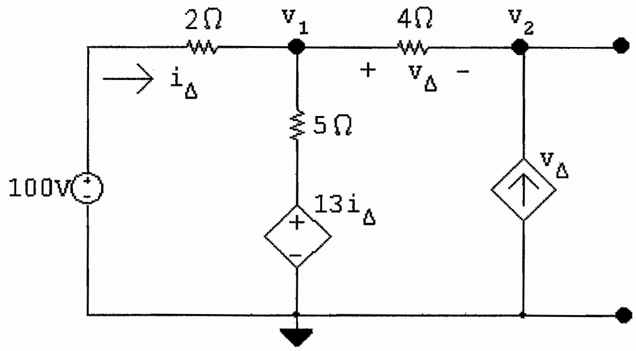
$$p_{\text{ds}} = 52.5i_{\text{ds}} = -984.375 \text{ W(dev)}$$

$$\therefore \sum p_{\text{dev}} = 4250 + 1250 + 984.375 = 6484.375 \text{ W}$$

$$\therefore \ \% \text{ delivered} = \frac{2250}{6484.375}(100) = 34.7\%$$

∴ 34.7% of developed power is delivered to load

P 4.88 [**a**] Open circuit voltage



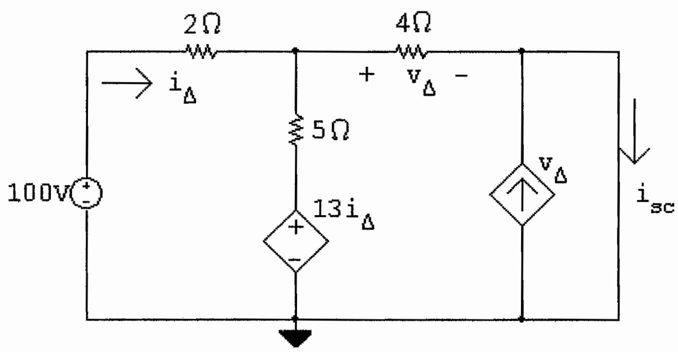
Node voltage equation:

$$\frac{v_1 - 100}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1 - v_2}{4} = 0$$

Constraint equations:

$$i_\Delta = \frac{100 - v_1}{2}; \qquad \frac{v_2 - v_1}{4} - v_\Delta = 0; \qquad v_\Delta = v_1 - v_2$$

Solving, $v_2 = 90 \text{ V} = v_{\text{Th}}$; $v_1 = 90 \text{ V}$; $v_\Delta = 0 \text{ V}$; $i_\Delta = 5 \text{ A}$
Short circuit current:



$$\frac{v_1 - 100}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1}{4} = 0$$

$$i_\Delta = \frac{100 - v_1}{2}$$

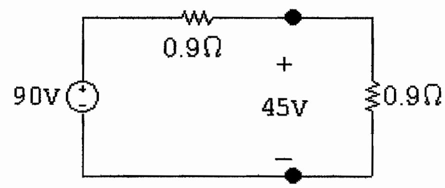
Solving, $v_1 = 80 \text{ V} = v_\Delta$; $i_\Delta = 10 \text{ A}$

$$i_{\text{sc}} = \frac{v_1}{4} + v_\Delta = 20 + 80 = 100 \text{ A}$$

$$R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{\text{sc}}} = \frac{90}{100} = 0.9\,\Omega$$

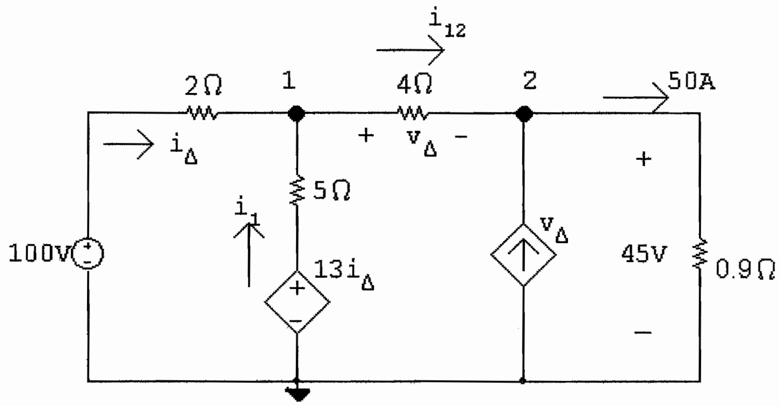
$$\therefore \quad R_o = R_{\text{Th}} = 0.9\,\Omega$$

[b]



$$p_{\max} = \frac{(45)^2}{0.9} = 2250 \text{ W}$$

[c]



$$\frac{v_1 - 100}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1 - 45}{4} = 0$$

$$i_\Delta = \frac{100 - v_1}{2}$$

Solving, $v_1 = 85$ V; $i_\Delta = 7.5$ A; $v_\Delta = v_1 - v_2 = 85 - 45 = 40$ V

$i_{100\text{V}} = i_\Delta = 7.5$ A

$p_{100\text{V}}$ (dev) $= 100(7.5) = 750$ W

$i_{12} = v_\Delta/4 = 40/4 = 10$ A

$i_1 = i_{12} - i_\Delta - 10 - 7.5 = 2.5$ A

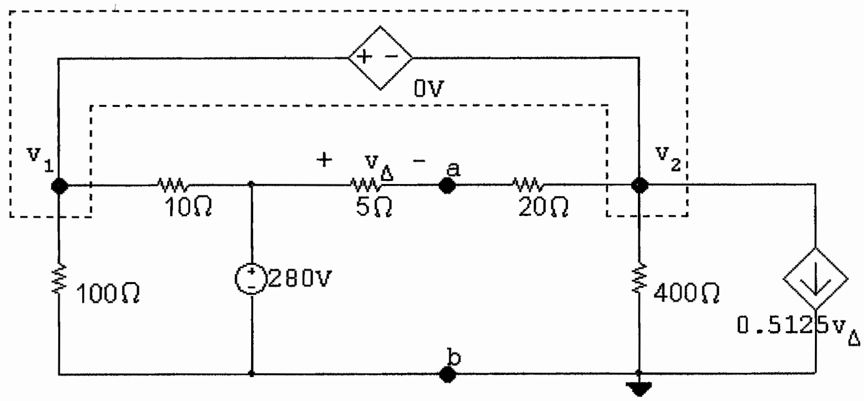
p_{13i_Δ} (dev) $= (97.5)(2.5) = 243.75$ W

p_{v_Δ} (dev) $= (45)(40) = 1800$ W

$\sum p_{\text{dev}} = 750 + 243.75 + 1800 = 2793.75$ W

$$\% \text{ delivered } = \frac{2250}{2793.75} \times 100 = 80.54\%$$

P 4.89 [a] First find the Thévenin equivalent with respect to R_o.
Open circuit voltage: $i_\phi = 0$; $50i_\phi = 0$



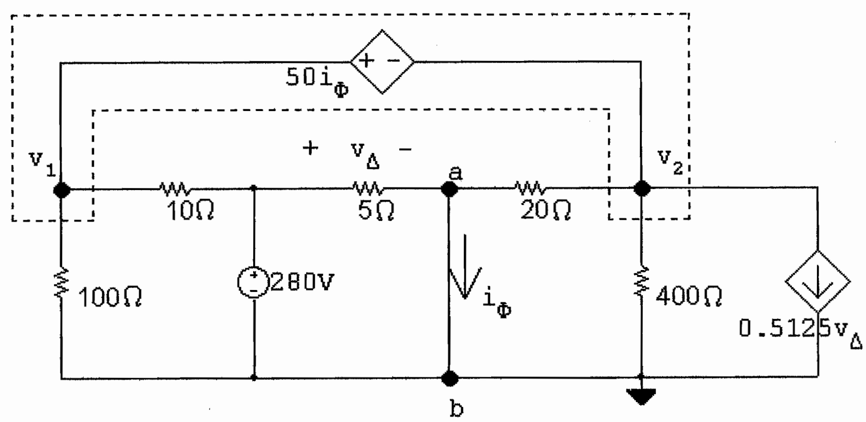
$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_1 - 280}{25} + \frac{v_1}{400} + 0.5125v_\Delta = 0$$

$$v_\Delta = \frac{(280 - v_1)}{25}5 = 56 - 0.2v_1$$

$$v_1 = 210 \text{ V}; \qquad v_\Delta = 14 \text{ V}$$

$$V_{\text{Th}} = 280 - v_\Delta = 280 - 14 = 266 \text{ V}$$

Short circuit current



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2}{20} + \frac{v_2}{400} + 0.5125(280) = 0$$

$$v_\Delta = 280 \text{ V}$$

$$v_2 + 50i_\phi = v_1$$

$$i_\phi = \frac{280}{5} + \frac{v_2}{20} = 56 + 0.05v_2$$

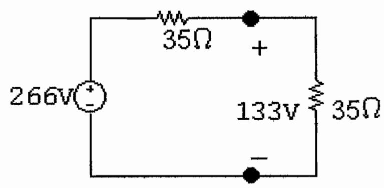
$v_2 = -968$ V; $\quad v_1 = -588$ V

$i_\phi = i_{sc} = 56 + 0.05(-968) = 7.6$ A

$R_{Th} = V_{Th}/i_{sc} = 266/7.6 = 35\,\Omega$

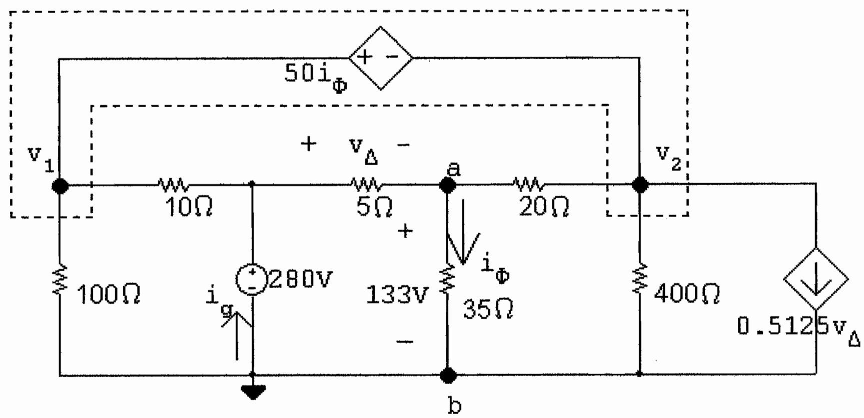
$\therefore \quad R_o = 35\,\Omega$

[b]



$p_{max} = (133)^2/35 = 505.4$ W

[c]



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2 - 133}{20} + \frac{v_2}{400} + 0.5125(280 - 133) = 0$$

$v_2 + 50i_\phi = v_1; \qquad i_\phi = 133/35 = 3.8$ A

Therefore, $v_1 = -189$ V and $v_2 = -379$ V; thus,

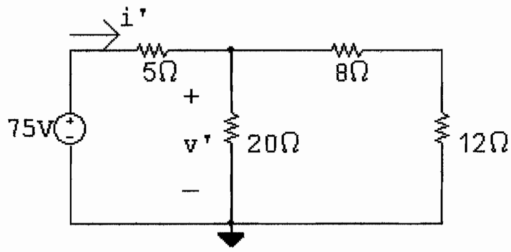
$$i_g = \frac{280 - 133}{5} + \frac{280 + 189}{10} = 76.30 \text{ A}$$

p_{280V} (dev) $= (280)(76.3) = 21{,}364$ W

P 4.90 [a] Since $0 \le R_o \le \infty$ maximum power will be delivered to the 6 Ω resistor when $R_o = 0$.

[b] $P = \dfrac{30^2}{6} = 150$ W

P 4.91 [a] 75 V source acting alone:

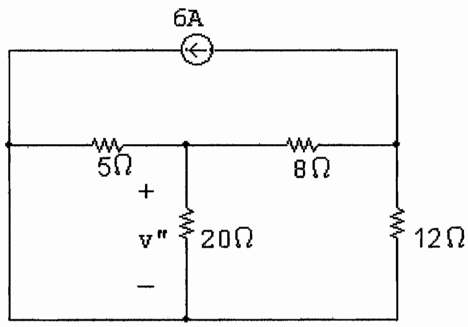


$R_e = 20\|20 = 10\,\Omega$

$$i' = \frac{75}{5+10} = 5 \text{ A}$$

$v' = (5)(10) = 50$ V

6 A source acting alone:

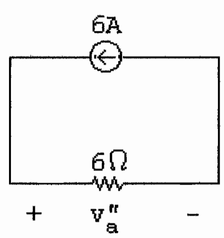


$5\|20 = 4\,\Omega$

$4 + 8 = 12\,\Omega$

$12\|12 = 6\,\Omega$

Hence our circuit reduces to:



It follows that

$v''_a = 6(6) = 36$ V

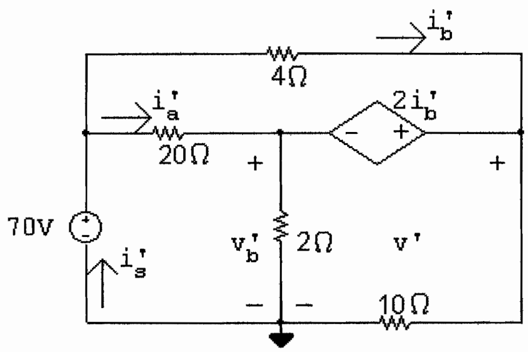
and

$$v'' = \frac{4}{4+8}(-36) = -12 \text{ V}$$

$$\therefore \quad v = v' + v'' = 50 - 12 = 38 \text{ V}$$

[b] $p = \dfrac{v^2}{20} = 72.2$ W

P 4.92 70-V source acting alone:



$$v' = 70 - 4i'_b$$

$$i'_s = \frac{v'_b}{2} + \frac{v'}{10} = i'_a + i'_b$$

$$70 = 20i'_a + v'_b$$

$$i'_a = \frac{70 - v'_b}{20}$$

$$\therefore \quad i'_b = \frac{v'_b}{2} + \frac{v'}{10} - \frac{70 - v'_b}{20} = \frac{11}{20}v'_b + \frac{v'}{10} - 3.5$$

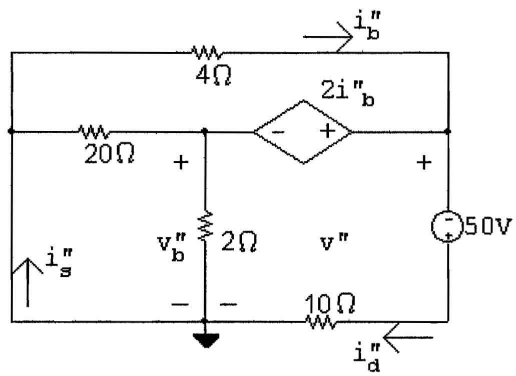
$$v' = v'_b + 2i'_b$$

$$\therefore \quad v'_b = v' - 2i'_b$$

$$\therefore \quad i'_b = \frac{11}{20}(v' - 2i'_b) + \frac{v'}{10} - 3.5 \quad \text{or} \quad i'_b = \frac{13}{42}v' - \frac{70}{42}$$

$$\therefore \quad v' = 70 - 4\left(\frac{13}{42}v' - \frac{70}{42}\right) \quad \text{or} \quad v' = \frac{3220}{94} = \frac{1610}{47} \text{ V} = 34.255 \text{ V}$$

50-V source acting alone:



$$v'' = -4i_b''$$

$$v'' = v_b'' + 2i_b''$$

$$v'' = -50 + 10i_d''$$

$$\therefore \quad i_d'' = \frac{v'' + 50}{10}$$

$$i_s'' = \frac{v_b''}{2} + \frac{v'' + 50}{10}$$

$$i_b'' = \frac{v_b''}{20} + i_s'' = \frac{v_b''}{20} + \frac{v_b''}{2} + \frac{v'' + 50}{10} = \frac{11}{20}v_b'' + \frac{v'' + 50}{10}$$

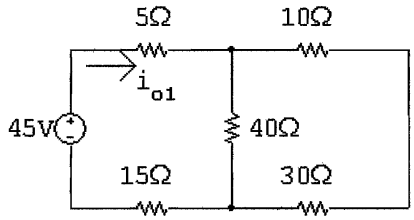
$$v_b'' = v'' - 2i_b''$$

$$\therefore \quad i_b'' = \frac{11}{20}(v'' - 2i_b'') + \frac{v'' + 50}{10} \qquad \text{or} \qquad i_b'' = \frac{13}{42}v'' + \frac{100}{42}$$

Thus, $\quad v'' = -4\left(\frac{13}{42}v'' + \frac{100}{42}\right) \qquad$ or $\qquad v'' = -\frac{200}{47}\text{ V} = -4.255\text{ V}$

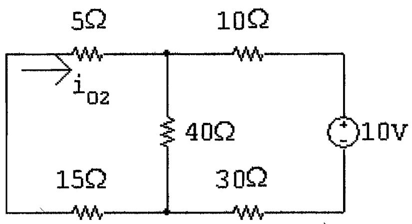
Hence, $\quad v = v' + v'' = \frac{1610}{47} - \frac{200}{47} = \frac{1410}{47} = 30\text{ V}$

P 4.93 45 V source acting alone:



$i_{o1} = 45/40 = 1.125$ A

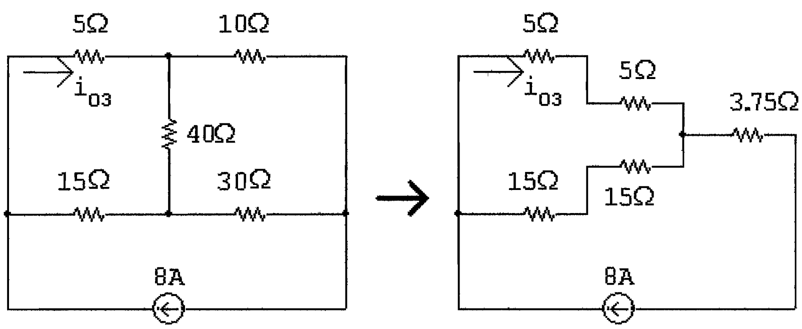
10 V source acting alone:



$$i_s = \frac{10}{40 + 40/3} = \frac{30}{160} \text{ A}$$

$$i_{o2} = -\frac{30}{160} \cdot \frac{40}{60} = -0.125 \text{ A}$$

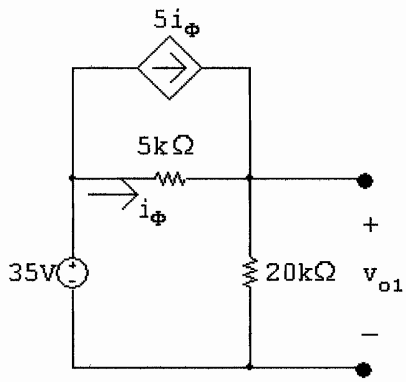
8 A current source acting alone:



$$i_{o3} = \frac{(8)(30)}{40} = 6 \text{ A}$$

$i_o = i_{o1} + i_{o2} + i_{o3} = 1.125 - 0.125 + 6 = 7$ A

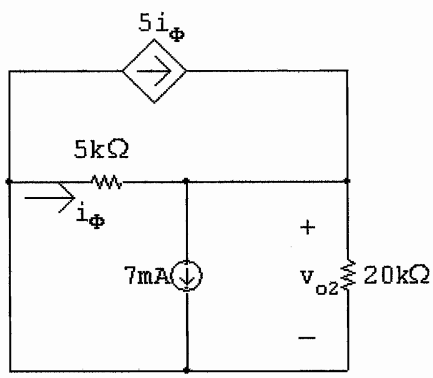
P 4.94 Voltage source acting alone:



$$\frac{v_{o1}}{20}+\frac{v_{o1}-35}{5}-5\left(\frac{35-v_{o1}}{5}\right)=0$$

$$\therefore\quad v_{o1}=33.6\text{ V}$$

Current source acting alone:

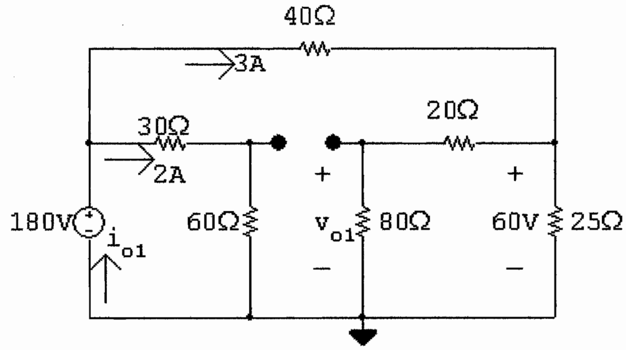


$$\frac{v_{o2}}{20}+7+\frac{v_{o2}}{5}-5\left(\frac{-v_{o2}}{5}\right)=0$$

$$\therefore\quad v_{o2}=-5.6\text{ V}$$

$$v_o=v_{o1}+v_{o2}=33.6-5.6=28\text{ V}$$

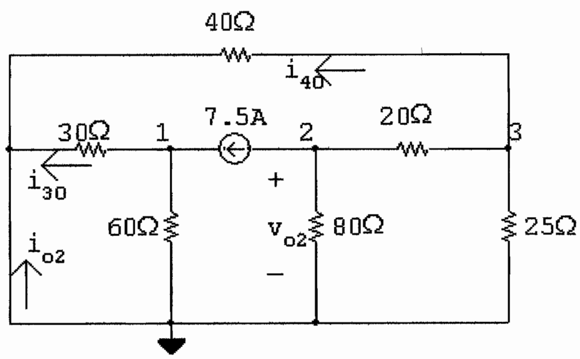
P 4.95 Voltage source acting alone:



$$i_{o1} = \frac{180}{90} + \frac{180}{40 + 100\|25} = 2 + 3 = 5 \text{ A}$$

$$v_{o1} = (3)(20)\left(\frac{80}{100}\right) = 48 \text{ V}$$

Current source acting alone:



$$\frac{v_2}{80} + 7.5 + \frac{v_2 - v_3}{20} = 0$$

$$\frac{v_3}{25} + \frac{v_3 - v_2}{20} + \frac{v_3}{40} = 0$$

Solving, $v_2 = -184 \text{ V} = v_{o2}$; $v_3 = -80$ V

$$i_{40} = \frac{v_3}{40} = -2 \text{ A}$$

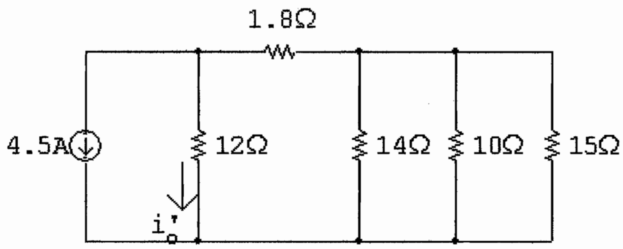
$$i_{30} = \frac{7.5(60)}{90} = 5 \text{ A}$$

$$i_{o2} = -i_{30} - i_{40} = -5 + 2 = -3 \text{ A}$$

$$\therefore \quad v_o = v_{o1} + v_{o2} = 48 - 184 = -136 \text{ V}$$

$$i_o = i_{o1} + i_{o2} = 5 - 3 = 2 \text{ A}$$

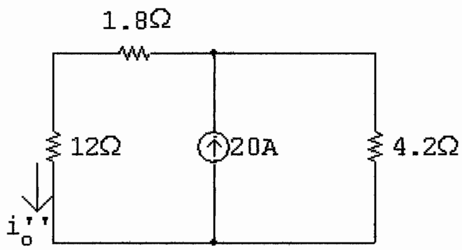
P 4.96 4.5 A source:



$$14\,\Omega \| 10\,\Omega \| 15\,\Omega = 4.2\,\Omega$$

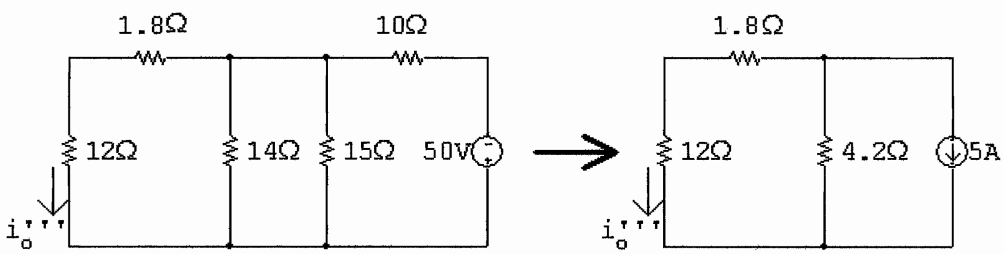
$$\therefore \quad i_o' = \frac{-4.5(6)}{18} = -1.5 \text{ A}$$

20 A source:



$$i_o'' = \frac{4.2(20)}{18} = 4.67 \text{ A}$$

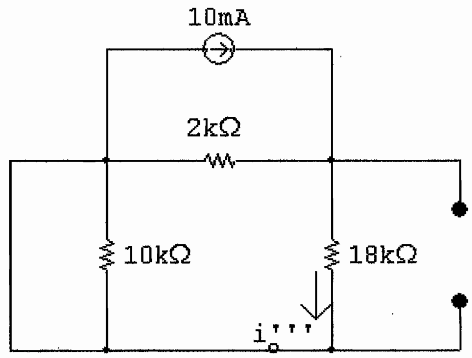
50 V source:



$$i_o''' = \frac{4.2(-5)}{18} = -1.167 \text{ A}$$

$$i_o = i_o' + i_o'' + i_o''' = -1.5 + 4.67 - 1.167 = 2 \text{ A}$$

P 4.97 [a] By hypothesis $i_o' + i_o'' = 1.5$ mA.



$$i_o''' = 10\frac{(2)}{(20)} = 1 \text{ mA}; \qquad \therefore \quad i_o = 1.5 + 1 = 2.5 \text{ mA}$$

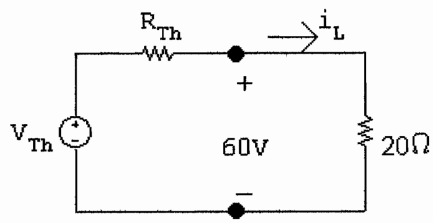
[b] With all three sources in the circuit write a single node voltage equation.

$$\frac{v_b}{18} + \frac{v_b - 20}{2} - 5 - 10 = 0$$

$$\therefore \quad v_b = 45 \text{ V}$$

$$i_o = \frac{v_b}{18} = 2.5 \text{ mA}$$

P 4.98 [a]



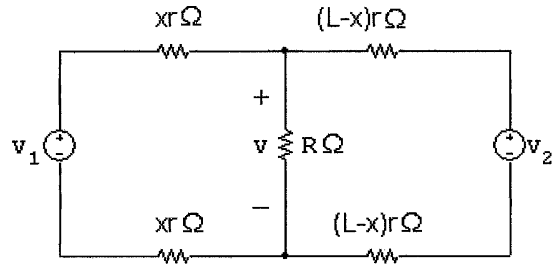
$$v_{\text{oc}} = V_{\text{Th}} = 75 \text{ V}; \qquad i_L = \frac{60}{20} = 3 \text{ A}; \qquad i_L = \frac{75 - 60}{R_{\text{Th}}} = \frac{15}{R_{\text{Th}}}$$

$$\text{Therefore} \quad R_{\text{Th}} = \frac{15}{3} = 5\,\Omega$$

[b] $i_L = \dfrac{v_o}{R_L} = \dfrac{V_{\text{Th}} - v_o}{R_{\text{Th}}}$

$$\text{Therefore} \quad R_{\text{Th}} = \frac{V_{\text{Th}} - v_o}{v_o/R_L} = \left(\frac{V_{\text{Th}}}{v_o} - 1\right) R_L$$

P 4.99 **[a]**



$$\frac{v-v_1}{2xr}+\frac{v}{R}+\frac{v-v_2}{2r(L-x)}=0$$

$$v\left[\frac{1}{2xr}+\frac{1}{R}+\frac{1}{2r(L-x)}\right]=\frac{v_1}{2xr}+\frac{v_2}{2r(L-x)}$$

$$v=\frac{v_1RL+xR(v_2-v_1)}{RL+2rLx-2rx^2}$$

[b] Let $D = RL + 2rLx - 2rx^2$

$$\frac{dv}{dx}=\frac{(RL+2rLx-2rx^2)R(v_2-v_1)-[v_1RL+xR(v_2-v_1)]2r(L-2x)}{D^2}$$

$\frac{dv}{dx}=0$ when numerator is zero.

The numerator simplifies to

$$x^2+\frac{2L-v_1}{(v_2-v_1)}x+\frac{RL(v_2-v_1)-2rv_1L^2}{2r(v_2-v_1)}=0$$

Solving for the roots of the quadratic yields

$$x=\frac{L}{v_2-v_1}\left\{-v_1\pm\sqrt{v_1v_2-\frac{R}{2rL}(v_2-v_1)^2}\right\}$$

[c] $$x=\frac{L}{v_2-v_1}\left\{-v_1\pm\sqrt{v_1v_2-\frac{R}{2rL}(v_1-v_2)^2}\right\}$$

$v_2 = 1200$ V, $\quad v_1 = 1000$ V, $\quad L = 16$ km

$r = 5\times 10^{-5}\,\Omega/m;\quad R = 3.9\,\Omega$

$$\frac{L}{v_2-v_1}=\frac{16{,}000}{1200-1000}=80;\qquad v_1v_2=1.2\times 10^6$$

$$\frac{R}{2rL}(v_1-v_2)^2=\frac{3.9(-200)^2}{(10\times 10^{-5})(16\times 10^3)}=0.975\times 10^5$$

$$x=80\{-1000\pm\sqrt{1.2\times 10^6-0.0975\times 10^6}\}$$

$$=80\{-1000\pm 1050\}=80(50)=4000\text{ m}$$

[d]

$$v_{\min} = \frac{v_1 RL + R(v_2 - v_1)x}{RL + 2rLx - 2rx^2}$$

$$= \frac{(1000)(3.9)(16 \times 10^3) + 3.9(200)(4000)}{(3.9)(16{,}000) + 10 \times 10^{-5}(16{,}000)(4000) - 10 \times 10^{-5}(16 \times 10^6)}$$

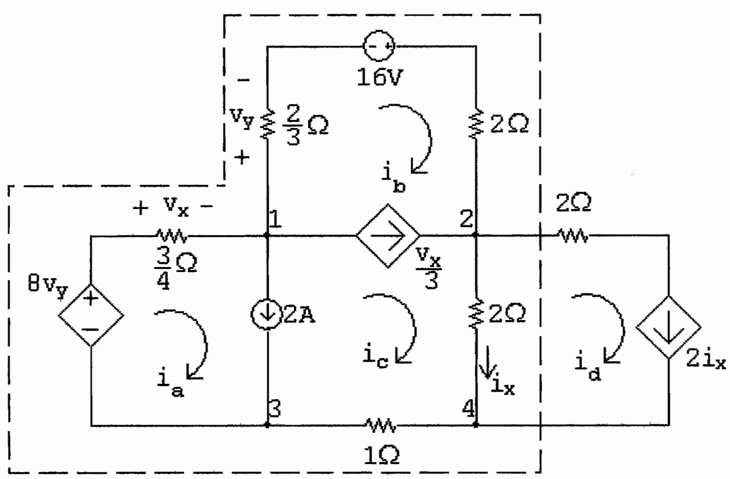
$$= 975 \text{ V}$$

P 4.100 [a] In studying the circuit in Fig. P4.100 we note it contains six meshes and six essential nodes. Further study shows that by replacing the parallel resistors with their equivalent values the circuit reduces to four meshes and four essential nodes as shown in the following diagram.

The node Voltage approach will require solving three node Voltage equations along with equations involving v_Δ and i_β.

The mesh-current approach will require writing one supermesh equation plus three constraint equations involving the three current sources. Thus at the outset we know the supermesh equation can be reduced to a single unknown current. Since we are interested in the power developed by the 16 V source, we will retain the mesh current i_b and eliminate the mesh currents i_a, i_c And i_d.

The supermesh is denoted by the dashed line in the following figure.



[b] Summing the voltages around the supermesh yields

$$-8v_y + \frac{3}{4}i_a + \frac{2}{3}i_b - 16 + 2i_b + 2(i_c - i_d) + 1i_c = 0$$

Note that $v_y = 2i_b/3$; make that substitution and multiply the equation by 12:

$$-96\left(\frac{2}{3}i_b\right) + 9i_a + 8i_b - 192 + 24i_b + 24(i_c - i_d) + 12i_c = 0$$

or

$$9i_a - 32i_b + 36i_c - 24i_d = 192$$

Now note:

$i_d = 2i_x;$ and $i_x = i_c - i_d$

so

$i_d = 2(i_c - i_d)$ $\therefore$ $3i_d = 2i_c$

Now use the following constraints:

$\frac{v_x}{3} = i_c - i_b$ and $v_x = \frac{3}{4}i_a$

Therefore

$i_a = 4i_c - 4i_b$

Finally,

$i_a - i_c = 2$

In standard form:

$9i_a - 32i_b + 36i_c - 24i_d = 192$

$0i_a + 0i_b + 2i_c - 3i_d = 0$

$1i_a + 4i_b - 4i_c + 0i_d = 0$

$1i_a + 0i_b - 1i_c + 0i_d = 2$

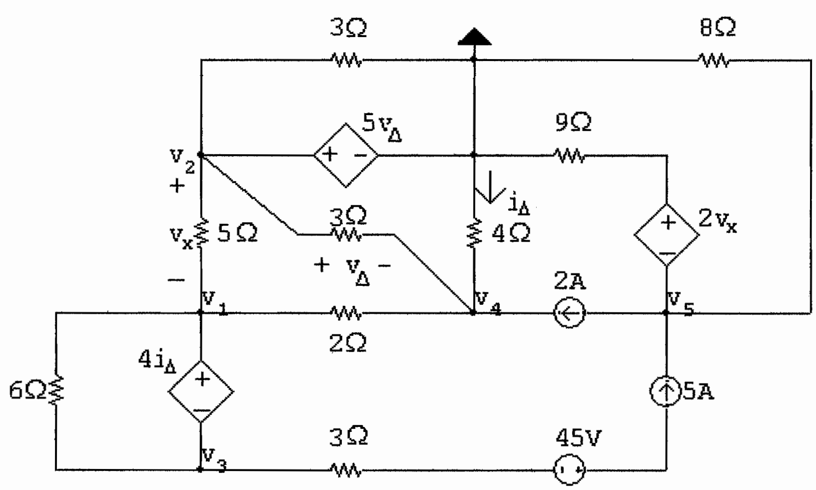
Solving,

$i_a = 33.6$ V; $i_b = 23.2$ V; $i_c = 31.6$ V; $i_a = 21.067$ V

$p_{16V} = -16i_b = -16(23.2) = -371.2$ W

Therefore, the 16 V source deliveres 371.2 W of power.

P 4.101



At v_1 : $\frac{v_1 - v_2}{5} + \frac{v_1 - v_4}{2} + 5 = 0$

At v_4: $\quad \dfrac{v_4 - v_1}{2} + \dfrac{v_4 - v_2}{3} + \dfrac{v_4}{4} - 2 = 0$

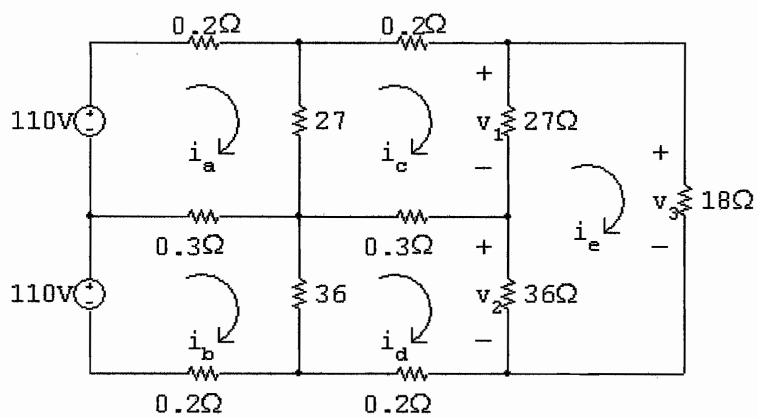
Also, $v_2 = 5v_\Delta = 5(v_2 - v_4)$
Solving, $v_1 = -20$ V; $\quad v_2 = -15$ V; $\quad v_4 = -12$ V

At v_5: $\quad 2 - 5 + \dfrac{v_5}{8} + \dfrac{v_5 + 2v_x}{9} = 0$

Also, $v_x = v_2 - v_1 = 5$ V Solving, $v_5 = 8$ V

$\therefore \quad p_{2\text{A}} = 2(v_5 - v_4) = 40$ W

P 4.102



$$110 = 27.5i_a - 0.3i_b - 27i_c$$
$$110 = -0.3i_a + 36.5i_b - 36i_d$$
$$0 = -27i_a + 54.5i_c - 0.3i_d - 27i_e$$
$$0 = -36i_b - 0.3i_c + 72.5i_d - 36i_e$$
$$0 = -27i_c - 36i_d + 81i_e$$

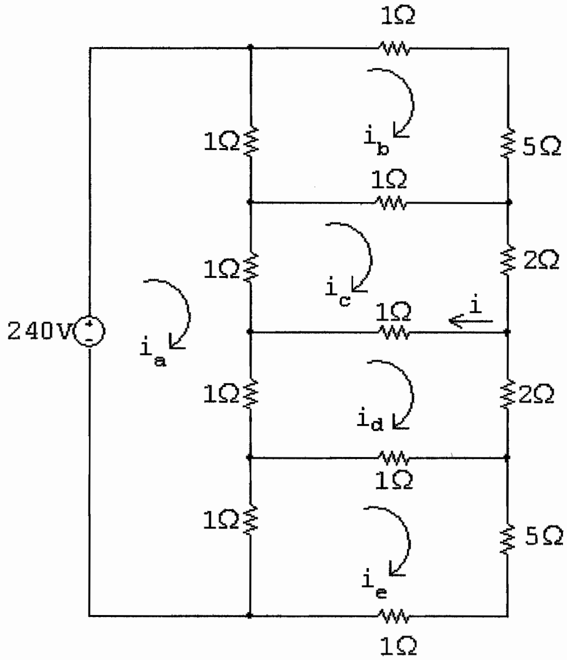
Solving,

$i_a = 19.19$ A; $\quad i_b = 17.36$ A; $\quad i_c = 15.275$ A;
$i_d = 14.39$ A; $\quad i_e = 11.49$ A

So,

$v_1 = 27(15.275 - 11.49) = 102.2$ V;
$v_2 = 36(14.49 - 11.49) = 104.4$ V;
$v_3 = 18(11.49) = 206.8$ V.

P 4.103



$$240 = 4i_a - 1i_b - 1i_c - 1i_d - 1i_e$$
$$0 = -1i_a + 8i_b - 1i_c + 0i_d + 0i_e$$
$$0 = -1i_a - 1i_b + 5i_c - 1i_d + 0i_e$$
$$0 = -1i_a + 0i_b - 1i_c + 5i_d - 1i_e$$
$$0 = -1i_a + 0i_b + 0i_c - 1i_d + 8i_e$$

A calculator solution yields

$i_a = 77.5$ A; $\quad i_d = 22.5$ A;

$i_b = 12.5$ A; $\quad i_e = 12.5$ A;

$i_c = 22.5$ A

$$\therefore \quad i = i_c - i_d = 0 \text{ A}$$

CHECK:

$$\begin{aligned}\sum p_{\text{abs}} &= 1(77.5 - 12.5)^2 + 1(77.5 - 22.5)^2 + 1(77.5 - 22.5)^2 + 1(77.5 - 12.5)^2 \\ &\quad +1(12.5)^2 + 5(12.5)^2 + 1(12.5 - 22.5)^2 + 2(22.5)^2 + 1(22.5 - 22.5)^2 \\ &\quad +2(22.5)^2 + 1(22.5 - 12.5)^2 + 5(12.5)^2 + 1(12.5)^2 \\ &= 4225 + 3025 + 3025 + 4225 + 156.25 + 781.25 \\ &\quad +100 + 1012.5 + 1012.5 + 100 + 781.25 + 156.25 = 18{,}000 \text{ W} \\ \sum p_{\text{gen}} &= 240i_a = 240(77.5) = 18{,}000 \text{ W (CHECKS)}\end{aligned}$$

P 4.104 $$\frac{dv_1}{dI_{g1}} = \frac{-R_1[R_2(R_3 + R_4) + R_3R_4]}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_1}{dI_{g2}} = \frac{R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g1}} + \frac{-R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g2}} = \frac{R_3R_4(R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

P 4.105 From the solution to Problem 4.104 we have

$$\frac{dv_1}{dI_{g1}} = \frac{-25[5(125) + 3750]}{30(125) + 3750} = -\frac{175}{12} \text{ V/A} = -14.5833 \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g1}} = \frac{-(25)(50)(75)}{30(125) + 3750} = -12.5 \text{ V/A}$$

By hypothesis, $\Delta I_{g1} = 11 - 12 = -1$ A

$$\therefore \quad \Delta v_1 = (-\frac{175}{12})(-1) = \frac{175}{12} = 14.5833 \text{ V}$$

Thus, $v_1 = 25 + 14.5833 = 39.5833$ V
Also,

$$\Delta v_2 = (-12.5)(-1) = 12.5 \text{ V}$$

Thus, $v_2 = 90 + 12.5 = 102.5$ V
The PSpice solution is

$$v_1 = 39.5830 \text{ V}$$

and

$$v_2 = 102.5000 \text{ V}$$

These values are in agreement with our predicted values.

P 4.106 From the solution to Problem 4.104 we have

$$\frac{dv_1}{dI_{g2}} = \frac{(25)(50)(75)}{30(125) + 3750} = 12.5 \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g2}} = \frac{(50)(75)(30)}{30(125) + 3750} = 15 \text{ V/A}$$

By hypothesis, $\Delta I_{g2} = 17 - 16 = 1$ A

$$\therefore \quad \Delta v_1 = (12.5)(1) = 12.5 \text{ V}$$

Thus, $v_1 = 25 + 12.5 = 37.5$ V
Also,

$$\Delta v_2 = (15)(1) = 15 \text{ V}$$

Thus, $v_2 = 90 + 15 = 105$ V
The PSpice solution is

$$v_1 = 37.5 \text{ V}$$

and

$$v_2 = 105 \text{ V}$$

These values are in agreement with our predicted values.

P 4.107 From the solutions to Problems 4.104 — 4.106 we have

$$\frac{dv_1}{dI_{g1}} = -\frac{175}{12} \text{ V/A}; \qquad \frac{dv_1}{dI_{g2}} = 12.5 \text{ V/A}$$

$$\frac{dv_2}{dI_{g1}} = -12.5 \text{ V/A}; \qquad \frac{dv_2}{dI_{g2}} = 15 \text{ V/A}$$

By hypothesis,

$$\Delta I_{g1} = 11 - 12 = -1 \text{ A}$$

$$\Delta I_{g2} = 17 - 16 = 1 \text{ A}$$

Therefore,

$$\Delta v_1 = \frac{175}{12} + 12.5 = 27.0833 \text{ V}$$

$\Delta v_2 = 12.5 + 15 = 27.5$ V

Hence

$v_1 = 25 + 27.0833 = 52.0833$ V

$v_2 = 90 + 27.5 = 117.5$ V

The PSpice solution is

$v_1 = 52.0830$ V

and

$v_2 = 117.5$ V

These values are in agreement with our predicted values.

P 4.108 By hypothesis,

$\Delta R_1 = 27.5 - 25 = 2.5\,\Omega$

$\Delta R_2 = 4.5 - 5 = -0.5\,\Omega$

$\Delta R_3 = 55 - 50 = 5\,\Omega$

$\Delta R_4 = 67.5 - 75 = -7.5\,\Omega$

So

$\Delta v_1 = 0.5833(2.5) - 5.417(-0.5) + 0.45(5) + 0.2(-7.5) = 4.9168$ V

$\therefore \quad v_1 = 25 + 4.9168 = 29.9168$ V

$\Delta v_2 = 0.5(2.5) + 6.5(-0.5) + 0.54(5) + 0.24(-7.5) = -1.1$ V

$\therefore \quad v_2 = 90 - 1.1 = 88.9$ V

The PSpice solution is

$v_1 = 29.6710$ V

and

$v_2 = 88.5260$ V

Note our predicted values are within a fraction of a volt of the actual values.

5

The Operational Amplifier

Assessment Problems

AP 5.1 [a] This is an inverting amplifier, so

$$v_o = (-R_f/R_i)v_s = (-80/16)v_s, \qquad \text{so} \qquad v_o = -5v_s$$

v_s(V)	0.4	2.0	3.5	−0.6	−1.6	−2.4
v_o(V)	−2.0	−10.0	−15.0	3.0	8.0	10.0

Two of the values, 3.5 V and −2.4 V, cause the op amp to saturate.

[b] Use the negative power supply value to determine the largest input voltage:

$$-15 = -5v_s, \quad v_s = 3 \text{ V}$$

Use the positive power supply value to determine the smallest input voltage:

$$10 = -5v_s, \qquad v_s = -2 \text{ V}$$

Therefore $\; -2 \leq v_s \leq 3$ V

AP 5.2 From Assessment Problem 5.1

$$v_o = (-R_f/R_i)v_s = (-R_x/16{,}000)v_s = (-R_x/16{,}000)(-0.640)$$

$$= 0.64R_x/16{,}000 = 4 \times 10^{-5}R_x$$

Use the negative power supply value to determine one limit on the value of R_x:

$$4 \times 10^{-5}R_x = -15 \qquad \text{so} \qquad R_x = -15/4 \times 10^{-5} = -375\,\text{k}\Omega$$

Since we cannot have negative resistor values, the lower limit for R_x is 0. Now use the positive power supply value to determine the upper limit on the value of R_x:

$$4 \times 10^{-5} R_x = 10 \qquad \text{so} \qquad R_x = 10/4 \times 10^{-5} = 250\,\text{k}\Omega$$

Therefore,

$$0 \leq R_x \leq 250\,\text{k}\Omega$$

AP 5.3 [a] This is an inverting summing amplifier so

$$v_o = (-R_f/R_\text{a})v_\text{a} + (-R_f/R_\text{b})v_\text{b} = -(250/5)v_\text{a} - (250/25)v_\text{b} = -50v_\text{a} - 10v_\text{b}$$

Substituting the values for v_a and v_b:

$$v_o = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$$

[b] Substitute the value for v_b into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50v_\text{a} - 10(0.25) = -50v_\text{a} - 2.5 = -10 \text{ V}$$

$$\text{Therefore} \quad 50v_\text{a} = 7.5, \quad \text{so} \quad v_\text{a} = 0.15 \text{ V}$$

[c] Substitute the value for v_a into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50(0.10) - 10v_\text{b} = -5 - 10v_\text{b} = -10 \text{ V};$$

$$\text{Therefore} \quad 10v_\text{b} = 5, \quad \text{so} \quad v_\text{b} = 0.5 \text{ V}$$

[d] The effect of reversing polarity is to change the sign on the v_b term in each equation from negative to positive.
Repeat part (a):

$$v_o = -50v_\text{a} + 10v_\text{b} = -5 + 2.5 = -2.5 \text{ V}$$

Repeat part (b):

$$v_o = -50v_\text{a} + 2.5 = -10 \text{ V}; \qquad 50v_\text{a} = 12.5, \quad v_\text{a} = 0.25 \text{ V}$$

Repeat part (c), using the value of the positive power supply:

$$v_o = -5 + 10v_\text{b} = 15 \text{ V}; \qquad 10v_\text{b} = 20; \qquad v_\text{b} = 2.0 \text{ V}$$

AP 5.4 [a] Write a node voltage equation at v_n; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v_n}{4500} + \frac{v_n - v_o}{63{,}000} = 0$$

Solve for v_o in terms of v_n by multiplying both sides by 63,000 and collecting terms:

$$14v_n + v_n - v_o = 0 \qquad \text{so} \qquad v_o = 15v_n$$

Now use voltage division to calculate v_p. We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 kΩ resistor and the R_x resistor:

$$v_p = \frac{R_x}{15{,}000 + R_x}(0.400)$$

Now substitute the value $R_x = 60$ kΩ:

$$v_p = \frac{60{,}000}{15{,}000 + 60{,}000}(0.400) = 0.32 \text{ V}$$

Finally, remember that for an ideal op amp, $v_n = v_p$, so substitute the value of v_p into the equation for v_0

$$v_o = 15v_n = 15v_p = 15(0.32) = 4.8 \text{ V}$$

[b] Substitute the expression for v_p into the equation for v_o and set the resulting equation equal to the positive power supply value:

$$v_o = 15\left(\frac{0.4R_x}{15{,}000 + R_x}\right) = 5$$

$$15(0.4R_x) = 5(15{,}000 + R_x) \quad \text{so} \quad R_x = 75\,\text{k}\Omega$$

AP 5.5 [a] Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. 5.22:

$$v_o = \frac{20(60)}{10(24)}v_\text{b} - \frac{50}{10}v_\text{a}$$

Simplify this expression and subsitute in the value for v_b:

$$v_o = 5(v_\text{b} - v_\text{a}) = 20 - 5v_\text{a}$$

Set this expression for v_o to the positive power supply value:

$$20 - 5v_\text{a} = 10 \text{ V} \quad \text{so} \quad v_\text{a} = 2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$20 - 5v_\text{a} = -10 \text{ V} \quad \text{so} \quad v_\text{a} = 6 \text{ V}$$

Therefore $\;2 \le v_\text{a} \le 6$ V

[b] Begin as before by substituting the appropriate values into Eq. 5.22:

$$v_o = \frac{8(60)}{10(12)}v_b - 5v_a = 4v_b - 5v_a$$

Now substitute the value for v_b:

$$v_o = 4(4) - 5v_a = 16 - 5v_a$$

Set this expression for v_o to the positive power supply value:

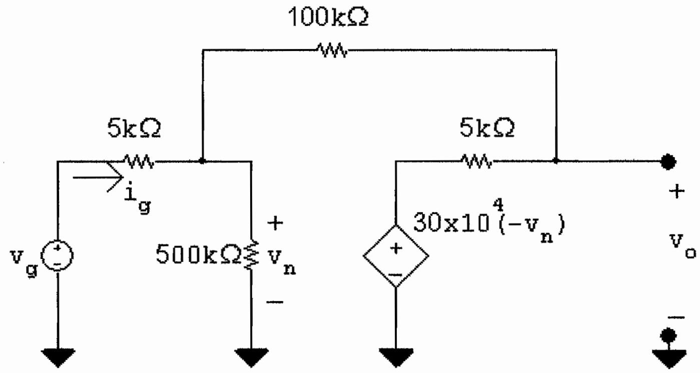
$$16 - 5v_a = 10 \text{ V} \quad \text{so} \quad v_a = 1.2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$16 - 5v_a = -10 \text{ V} \quad \text{so} \quad v_a = 5.2 \text{ V}$$

Therefore $1.2 \leq v_a \leq 5.2$ V

AP 5.6 [a] Replace the op amp with the more realistic model of the op amp from Fig. 5.15:



Write the node voltage equation at the left hand node:

$$\frac{v_n}{500{,}000} + \frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} = 0$$

Multiply both sides by 500,000 and simplify:

$$v_n + 100v_n - 100v_g + 5v_n - 5v_0 = 0 \quad \text{so} \quad 21.2v_n - v_o = 20v_g$$

Write the node voltage equation at the right hand node:

$$\frac{v_o - 300{,}000(-v_n)}{5000} + \frac{v_o - v_n}{100{,}000} = 0$$

Multiply through by 100,000 and simplify:

$$20v_o + 6 \times 10^6 v_n + v_o - v_n = 0 \quad \text{so} \quad 6 \times 10^6 v_n + 21v_o = 0$$

Use Cramer's method to solve for v_o:

$$\Delta = \begin{vmatrix} 21.2 & -1 \\ 6 \times 10^6 & 21 \end{vmatrix} = 6{,}000{,}445.2$$

$$N_o = \begin{vmatrix} 21.2 & 20v_g \\ 6 \times 10^6 & 0 \end{vmatrix} = -120 \times 10^6 v_g$$

$$v_o = \frac{N_o}{\Delta} = -19.9985 v_g; \qquad \text{so } \frac{v_o}{v_g} = -19.9985$$

[b] Use Cramer's method again to solve for v_n:

$$N_1 = \begin{vmatrix} 20v_g & -1 \\ 0 & 21 \end{vmatrix} = 420 v_g$$

$$v_n = \frac{N_1}{\Delta} = 6.9995 \times 10^{-5} v_g$$

$$v_g = 1 \text{ V}, \qquad v_n = 69.995\,\mu\text{ V}$$

[c] The resistance seen at the input to the op amp is the ratio of the input voltage to the input current, so calculate the input current as a function of the input voltage:

$$i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 6.9995 \times 10^{-5} v_g}{5000}$$

Solve for the ratio of v_g to i_g to get the input resistance:

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 6.9995 \times 10^{-5}} = 5000.35\,\Omega$$

[d] This is a simple inverting amplifier configuration, so the voltage gain is the ratio of the feedback resistance to the input resistance:

$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

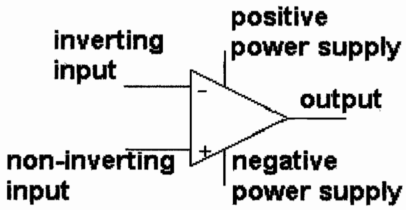
Since this is now an ideal op amp, the voltage difference between the two input terminals is zero; since $v_p = 0$, $v_n = 0$

Since there is no current into the inputs of an ideal op amp, the resistance seen by the input voltage source is the input resistance:

$$R_g = 5000\,\Omega$$

Problems

P 5.1 [a] The five terminals of the op amp are identified as follows:



[b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i_n = 0$ A.

[c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v_p - v_n) = 0$.

[d] Write a node voltage equation at v_n:

$$\frac{v_n - 1}{2000} + \frac{v_n - v_o}{8000} = 0$$

But $v_p = 0$ and $v_n = v_p = 0$. Thus,

$$\frac{-1}{2000} - \frac{v_o}{8000} = 0 \quad \text{so} \quad v_o = -4 \text{ V}$$

P 5.2 $$\frac{v_b - v_a}{20} + \frac{v_b - v_o}{160} = 0, \qquad \text{therefore} \quad v_o = 9v_b - 8v_a$$

[a] $v_a = 1.5$ V, $v_b = 0$ V, $v_o = -12$ V

[b] $v_a = 3.0$ V, $v_b = 0$ V, $v_o = -18$ V (sat)

[c] $v_a = 1.0$ V, $v_b = 2$ V, $v_o = 10$ V

[d] $v_a = 4.0$ V, $v_b = 2$ V, $v_o = -14$ V

[e] $v_a = 6.0$ V, $v_b = 8$ V, $v_o = 18$ V (sat)

[f] If $v_b = 4.5$ V, $v_o = 40.5 - 8v_a = \pm 18$

$$\therefore \quad 2.8125 \leq v_a \leq 7.3125 \text{ V}$$

P 5.3 $$v_o = (1)(9) = 9 \text{ V}; \qquad i_{15k\Omega} = \frac{9}{15} = 0.6\,\text{mA};$$

$$i_{6k\Omega} = \frac{9}{6} = 1.5\,\text{mA}; \qquad i_{9k\Omega} = \frac{9}{9} = 1\,\text{mA}$$

$$\therefore \quad i_o = -0.6 - 1.5 - 1 = -3.1\,\text{mA}$$

P 5.4 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the 3.3 MΩ resistor is $(3.3 \times 10^6)(2.5 \times 10^{-6})$ or 8.25 V. Therefore the voltmeter reads 8.25 V.

P 5.5 [a] $i_a = \frac{120}{6} \times 10^{-6} = 20\,\mu\text{A}$

$$v_a = -20 \times 10^3 i_a = -400\,\text{mV}$$

[b] $\frac{v_a}{60{,}000} + \frac{v_a}{20{,}000} + \frac{v_a - v_o}{240{,}000} = 0$

$$\therefore \quad v_o = 17v_a = -6.8 \text{ V}$$

[c] $i_a = 20\,\mu\text{A}$

[d] $i_o = \frac{-v_o}{80{,}000} + \frac{v_a - v_o}{240{,}000} = 111.67\,\mu\text{ A}$

P 5.6 $v_p = \frac{3000}{3000 + 6000}(3) = 1 \text{ V} = v_n$

$$\frac{v_n - 5}{10{,}000} + \frac{v_n - v_o}{5000} = 0$$

$$(1 - 5) + 2(1 - v_o) = 0$$

$$v_o = -1.0 \text{ V}$$

$$i_L = \frac{v_o}{4000} = -\frac{1}{4000} = -250 \times 10^{-6}$$

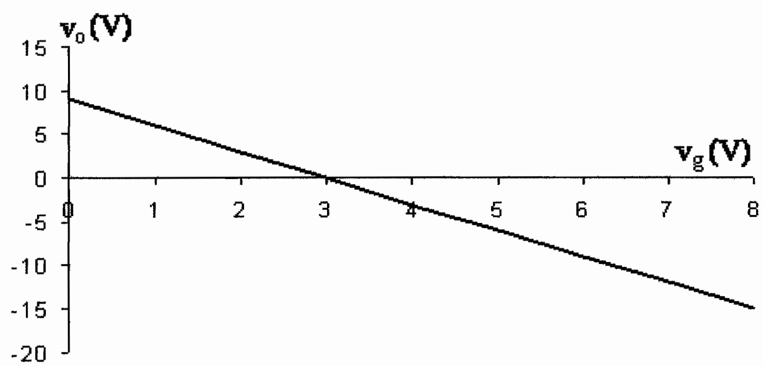
$$i_L = -250\,\mu\text{A}$$

P 5.7 [a] First, note that $v_n = v_p = 3$ V

Let v_{o1} equal the voltage output of the op-amp. Then

$$\frac{3 - v_g}{5000} + \frac{3 - v_{o1}}{15{,}000} = 0, \qquad \therefore \quad v_{o1} = 12 - 3v_g$$

Also note that $v_{o1} - 3 = v_o$, $\quad \therefore \quad v_o = 9 - 3v_g$



[b] Yes, the circuit designer is correct!

P 5.8 [a] The circuit shown is a non-inverting amplifier.

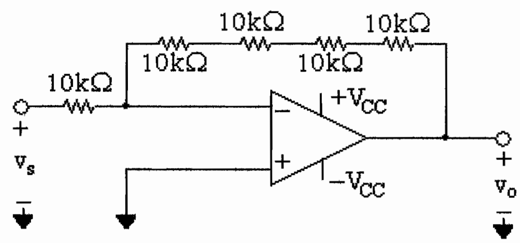
[b] We assume the op amp to be ideal, so $v_n = v_p = 750$ mV. Write a KCL equation at v_n:

$$\frac{0.75}{20{,}000} + \frac{0.75 - v_o}{100{,}000} = 0$$

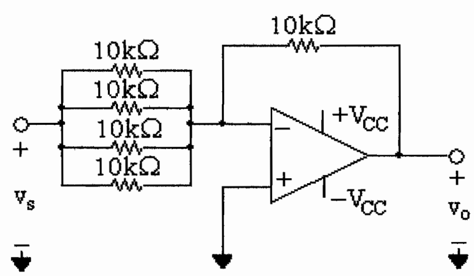
Solving,

$$v_o = 4.5 \text{ V}.$$

P 5.9 [a] The gain of an inverting amplifier is the ratio of the feedback resistor to the input resistor. If the gain of the inverting amplifier is to be 4, the feedback resistor must be 4 times as large as the input resistor. There are many possible designs that use only 10 kΩ resistors. We present two here. Use a single 10 kΩ resistor as the input resistor, and use four 10 kΩ resistors in series as the feedback resistor to give a total of 40 kΩ.



Alternately, Use a single 10 kΩ resistor as the feedback resistor and use four 10 kΩ resistors in parallel as the input resistor to give a total of 2.5 kΩ.



[b] To amplify a 2.5 V signal without saturating the op amp, the power supply voltages must be greater than or equal to the product of the input voltage and the amplifier gain. Thus, the power supplies should have a magnitude of $(2.5)(4) = 10$ V.

P 5.10 [a] Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{4000} + \frac{0 - v_\Delta}{20{,}000} = 0$$

$$-5v_g - v_\Delta = 0, \qquad \therefore \quad v_\Delta = -5(40 \times 10^{-3}) = -0.2 \text{ V}$$

$$\frac{v_\Delta}{\alpha R_\Delta} + \frac{v_\Delta - 0}{20{,}000} + \frac{v_\Delta - v_o}{(1-\alpha)R_\Delta} = 0$$

$$\frac{v_\Delta}{\alpha} + 6v_\Delta + \frac{v_\Delta - v_o}{1-\alpha} = 0$$

$$v_\Delta\left(\frac{1}{\alpha} + 6 + \frac{1}{1-\alpha}\right) = \frac{v_o}{1-\alpha}$$

$$\therefore \quad v_o = -0.2\left[1 + 6(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right]$$

When $\alpha = 0.25, \qquad v_o = -0.2(1 + 4.5 + 3) = -1.7$ V

When $\alpha = 0.8, \qquad v_o = -0.2(1 + 1.2 + 0.25) = -0.49$ V

$\therefore \quad -1.7 \text{ V} \le v_o \le -0.49$ V

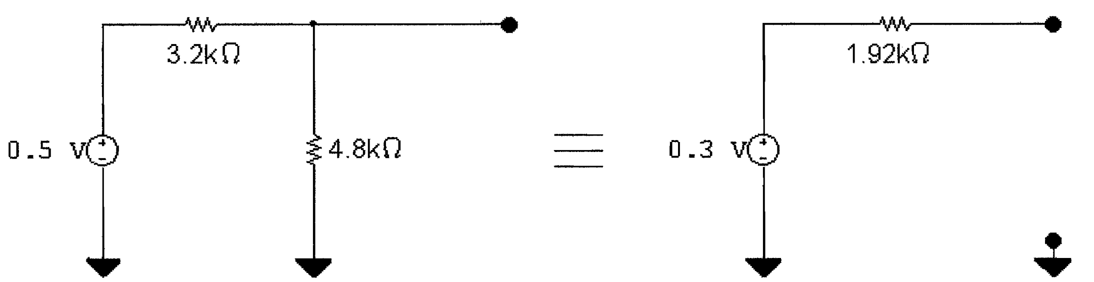
[b] $-0.2\left[1 + 6(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right] = -12$

$$\alpha + 6\alpha(1-\alpha) + (1-\alpha) = 60\alpha$$

$$\alpha + 6\alpha - 6\alpha^2 + 1 - \alpha = 60\alpha$$

$$\therefore \quad 6\alpha^2 + 54\alpha - 1 = 0 \quad \text{so} \quad \alpha \cong 0.0185$$

P 5.11 [a] Replace the combination of v_g, 3.2 kΩ, and the 4.8 kΩ resistors with its Thévenin equivalent.



Then $v_o = \frac{-[30 + \sigma 170]}{1.92}(0.30)$

At saturation $v_o = -10$ V; therefore

$$-\left(\frac{30 + \sigma 170}{1.92}\right)(0.3) = -10, \quad \text{or} \quad \sigma = 0.2$$

Thus for $0 \le \sigma < 0.20$ the operational amplifier will not saturate.

[b] When $\sigma = 0.12$, $\qquad v_o = \dfrac{-(30+20.4)}{1.92}(0.30) = -7.875\ \text{V}$

Also $\dfrac{v_o}{180} + \dfrac{v_o}{50.4} + i_o = 0$

$$\therefore\quad i_o = -\frac{v_o}{180} - \frac{v_o}{50.4} = \frac{7.875}{180} + \frac{7.875}{50.4}\,\text{mA} = 200\,\mu\text{A}$$

P 5.12 [a] This circuit is an example of an inverting summing amplifier.

[b] $v_o = -\dfrac{180}{20}v_\text{a} - \dfrac{180}{30}v_\text{b} - \dfrac{180}{60}v_\text{c} = -4.5 - 9 + 7.5 = -6\ \text{V}$

[c] $v_o = -13.5 - 3v_\text{c} = \pm 9$

$$\therefore\quad v_\text{c} = -7.5\ \text{V} \quad \text{when} \quad v_o = 9\ \text{V};$$

$$v_\text{c} = 1.5\ \text{V} \quad \text{when} \quad v_o = -9\ \text{V}$$

$$\therefore\quad -7.5\ \text{V} \le v_\text{c} \le 1.5\ \text{V}$$

P 5.13 [a] Write a KCL equation at the inverting input to the op amp:

$$\frac{v_\text{d} - v_\text{a}}{55{,}000} + \frac{v_\text{d} - v_\text{b}}{66{,}000} + \frac{v_\text{d} - v_\text{c}}{220{,}000} + \frac{v_\text{d}}{550{,}000} + \frac{v_\text{d} - v_o}{330{,}000} = 0$$

$$v_o = 14.1v_\text{d} - 6v_\text{a} - 5v_\text{b} - 1.5v_\text{c} = 141 - 96 - 60 + 9 = -6\ \text{V}$$

[b] $v_o = 141 - 96 - 5v_\text{b} + 9 = 54 - 5i_\text{b}$

$$54 - 5v_\text{b} = -12 \qquad \text{so} \qquad v_\text{b} = 13.2\ \text{V}$$

$$54 - 5v_\text{b} = 12 \qquad \text{so} \qquad v_\text{b} = 8.4\ \text{V}$$

$$\therefore\quad 8.4\ \text{V} \le v_\text{b} \le 13.2\ \text{V}$$

P 5.14 [a] $\dfrac{v_\text{d} - v_\text{a}}{55{,}000} + \dfrac{v_\text{d} - v_\text{b}}{66{,}000} + \dfrac{v_\text{d} - v_\text{c}}{220{,}000} + \dfrac{v_\text{d}}{550{,}000} + \dfrac{v_\text{d} - v_o}{R_\text{f}} = 0$

$$12v_\text{d} - 12v_\text{a} + 10v_\text{d} - 10v_\text{b} + 3v_\text{d} - 3v_\text{c} + 1.2v_\text{d} + \frac{660}{R_\text{f}}v_\text{d} = \frac{660}{R_\text{f}}v_o \ (R_\text{f} \text{ in k}\Omega)$$

$$26.2v_\text{d} + \frac{660}{R_\text{f}}v_\text{d} - 12v_\text{a} - 10v_\text{b} - 3v_\text{c} = \frac{660}{R_\text{f}}v_o$$

$$262 + \frac{6600}{R_\text{f}} - 192 - 120 + 18 = \frac{660}{R_\text{f}}v_o$$

$$6600 - 32R_\text{f} = 660v_o \qquad \text{so} \qquad 32R_\text{f} = 6600 - 660v_o$$

$$v_o = \pm 12 \qquad \text{but} \qquad R_\text{f} > 0$$

$$\therefore\quad 32R_\text{f} = 6600 - 660(-12) \qquad \text{so} \qquad R_\text{f} = 453.75\,\text{k}\Omega$$

[b] $v_o = -12$ V; A KCL equation at the output node gives

$$i_o + \frac{-12}{33{,}000} + \frac{-12-10}{453{,}750} = 0$$

$$\therefore \quad i_o = 412.12\,\mu\text{ A}$$

P 5.15 We want the following expression for the output voltage:

$$v_o = -(3v_a + 5v_b + 4v_c + 2v_d)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage:

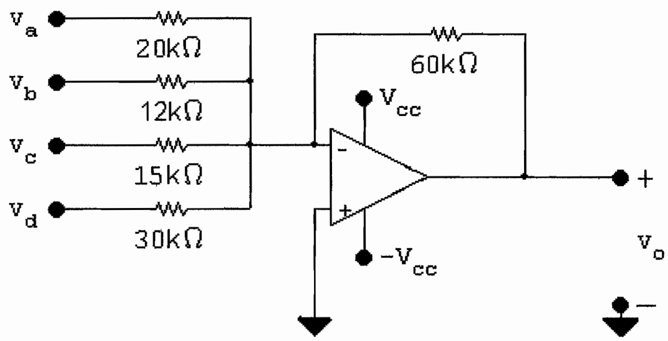
$$v_o = -\left[\frac{60\text{k}}{R_\text{a}}v_\text{a} + \frac{60\text{k}}{R_\text{b}}v_\text{b} + \frac{60\text{k}}{R_\text{c}}v_\text{c} + \frac{60\text{k}}{R_\text{d}}v_\text{d}\right]$$

Solve for each input resistance value to yield the desired gain:

$$\therefore \quad R_\text{a} = 60{,}000/3 = 20\,\text{k}\Omega \qquad R_\text{c} = 60{,}000/4 = 15\,\text{k}\Omega$$

$$R_\text{b} = 60{,}000/5 = 12\,\text{k}\Omega \qquad R_\text{d} = 60{,}000/2 = 30\,\text{k}\Omega$$

The final circuit is shown here:



P 5.16 $v_o = -\left[\frac{R_\text{f}}{4000}(0.2) + \frac{R_\text{f}}{5000}(0.15) + \frac{R_\text{f}}{20{,}000}(0.4)\right]$

$$-6 = -0.1 \times 10^{-3} R_\text{f}; \qquad R_\text{f} = 60\,\text{k}\Omega; \qquad \therefore \quad 0 \le R_\text{f} \le 60\,\text{k}\Omega$$

P 5.17 [a] This circuit is an example of the non-inverting amplifier.

[b] Use voltage division to calculate v_p:

$$v_p = \frac{75{,}000}{25{,}000 + 75{,}000}v_s = \frac{3v_s}{4}$$

Write a KCL equation at $v_n = v_p = 3v_s/4$:

$$\frac{3v_s/4}{8000} + \frac{3v_s/4 - v_o}{32{,}000} = 0$$

Solving,

$$v_o = 12v_s/4 + 3v_s/4 = 3.75v_s$$

[c] $3.75v_s = 15 \qquad \text{so} \qquad v_s = 4\ \text{V}$

$3.75v_s = -9 \qquad \text{so} \qquad v_s = -2.4\ \text{V}$

Thus, $-2.4\ \text{V} \leq v_s \leq 4\ \text{V}$.

P 5.18 [a] $v_p = v_n = \frac{45}{75}v_g = 0.6v_g$

$$\therefore \quad \frac{0.6v_g}{15} + \frac{0.6v_g - v_o}{48} = 0;$$

$$\therefore \quad v_o = 2.52v_g = 2.52(3), \qquad v_o = 7.56\ \text{V}$$

[b] $v_o = 2.52v_g = \pm 10$

$v_g = \pm 3.97\ \text{V}, \qquad -3.97 \leq v_g \leq 3.97\ \text{V}$

[c] $\frac{0.6v_g}{15} + \frac{0.6v_g - v_o}{R_f} = 0$

$$\left(\frac{0.6R_f}{15} + 0.6\right) v_g = v_o = \pm 10$$

$$\therefore \quad 3R_f + 45 = \pm 150; \qquad 3R_f = 150 - 45; \qquad R_f = 35\,\text{k}\Omega$$

P 5.19 [a] This circuit is an example of a non-inverting summing amplifier.

[b] Write a KCL equation at v_p and solve for v_p in terms of v_s:

$$\frac{v_p - v_s}{12{,}000} + \frac{v_p + 4}{48{,}000} = 0$$

$4v_p - 4v_s + v_p + 4 = 0 \qquad \text{so} \qquad v_p = 4v_s/5 - 4/5$

Now write a KCL equation at v_n and solve for v_o:

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0 \qquad \text{so} \qquad v_o = 5v_n$$

Since we assume the op amp is ideal, $v_n = v_p$. Thus,

$$v_o = 5(4v_s/5 - 4/5) = 4v_s - 4$$

[c] $4v_s - 4 = 10 \qquad \text{so} \qquad v_s = 3.5\ \text{V}$

$4v_s - 4 = -10 \qquad \text{so} \qquad v_s = -1.5\ \text{V}$

Thus, $-1.5\ \text{V} \leq v_s \leq 3.5\ \text{V}$.

P 5.20 [a] This is a non-inverting summing amplifier.

[**b**] $$\frac{v_p - v_a}{80 \times 10^3} + \frac{v_p - v_b}{64 \times 10^3} = 0$$

$$\therefore \quad 9v_p = 4v_a + 5v_b$$

$$\frac{v_n}{18{,}000} + \frac{v_n - v_o}{72{,}000} = 0$$

$$\therefore \quad v_o = 5v_n = 5v_p = (20/9)v_a + (25/9)v_b = 4.44 \text{ V}$$

[**c**] $$v_p = v_n = \frac{v_o}{5} = 0.889 \text{ V}$$

$$i_a = \frac{v_a - v_p}{80 \times 10^3} = -4.86\,\mu\text{A}$$

$$i_b = \frac{v_b - v_p}{64 \times 10^3} = 4.86\,\mu\text{A}$$

[**d**] (20/9) for v_a

(25/9) for v_b

P 5.21 [**a**] $$\frac{v_p - v_a}{R_a} + \frac{v_p - v_b}{R_b} + \frac{v_p - v_c}{R_c} + \frac{v_p}{R_g} = 0$$

$$\therefore \quad v_p = \frac{R_bR_cR_g}{D}v_a + \frac{R_aR_cR_g}{D}v_b + \frac{R_aR_bR_g}{D}v_c$$

where $D = R_bR_cR_g + R_aR_cR_g + R_aR_bR_g + R_aR_bR_c$

$$\frac{v_n}{R_s} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n\left(\frac{1}{R_s} + \frac{1}{R_f}\right) = \frac{v_o}{R_f}$$

$$\therefore \quad v_o = \left(1 + \frac{R_f}{R_s}\right)v_n = kv_n$$

where $k = \left(1 + \frac{R_f}{R_s}\right)$

$$v_p = v_n$$

$$\therefore \quad v_o = kv_p$$

or

$$v_o = \frac{kR_gR_bR_c}{D}v_a + \frac{kR_gR_aR_c}{D}v_b + \frac{kR_gR_aR_b}{D}v_c$$

$$\frac{kR_gR_bR_c}{D} = 6 \qquad \frac{kR_gR_aR_c}{D} = 3 \qquad \frac{kR_gR_aR_b}{D} = 4$$

$$\therefore \quad \frac{R_b}{R_a} = \frac{6}{3} = 2 \qquad \frac{R_c}{R_b} = \frac{3}{4} = 0.75 \qquad \frac{R_c}{R_a} = \frac{6}{4} = 1.5$$

Since $R_a = 1\,\text{k}\Omega \qquad R_b = 2\,\text{k}\Omega \qquad R_c = 1.5\,\text{k}\Omega$

$$\therefore \quad D = [(2)(1.5)(3) + (1)(1.5)(3) + (1)(2)(3) + (1)(2)(1.5)] \times 10^9 = 22.5 \times 10^9$$

$$\frac{k(3)(2)(1.5) \times 10^9}{22.5 \times 10^9} = 6$$

$$k = \frac{135 \times 10^9}{9 \times 10^9} = 15$$

$$\therefore \quad 15 = 1 + \frac{R_f}{R_s}$$

$$\frac{R_f}{R_s} = 14$$

$$R_f = (14)(15{,}000) = 210\,\text{k}\Omega$$

[b] $v_o = 6(0.5) + 3(2.5) + 4(1) = 14\ \text{V}$

$$v_n = v_p = \frac{14.5}{15} = 0.967\ \text{V}$$

$$i_a = \frac{0.5 - 0.967}{1000} = -466.67\,\mu\text{A}$$

$$i_b = \frac{2.5 - 0.967}{2000} = 766.67\,\mu\text{A}$$

$$i_c = \frac{1 - 0.967}{1500} = 22.22\,\mu\text{A}$$

$$i_g = \frac{0.967}{3000} = 322.22\,\mu\text{A}$$

$$i_s = \frac{v_n}{15{,}000} = \frac{0.967}{15{,}000} = 64.44\,\mu\text{A}$$

P 5.22 **[a]** $\dfrac{v_p - v_a}{R_a} + \dfrac{v_p - v_b}{R_b} + \dfrac{v_p - v_c}{R_c} = 0$

$$\therefore \quad v_p = \frac{R_bR_c}{D}v_a + \frac{R_aR_c}{D}v_b + \frac{R_aR_b}{D}v_c$$

where $D = R_bR_c + R_aR_c + R_aR_b$

$$\frac{v_n}{20{,}000} + \frac{v_n - v_o}{R_f} = 0$$

$$\left(\frac{R_f}{20{,}000} + 1\right) v_n = v_o$$

Let $\dfrac{R_f}{20{,}000} + 1 = k$

$v_o = kv_n = kv_p$

$\therefore \quad v_o = \dfrac{kR_bR_c}{D}v_a + \dfrac{kR_aR_c}{D}v_b + \dfrac{kR_aR_b}{D}v_c$

$\therefore \quad \dfrac{kR_bR_c}{D} = 4 \qquad\qquad \therefore \quad \dfrac{R_b}{R_a} = 4$

$\dfrac{kR_aR_c}{D} = 1$

$\dfrac{kR_aR_b}{D} = 2 \qquad\qquad \therefore \quad \dfrac{R_c}{R_a} = 2$

$\therefore \quad R_b = 4R_a = 4\,\text{k}\Omega$

$R_c = 2R_a = 2\,\text{k}\Omega$

$\therefore \quad D = (4)(2) + (1)(2) + (1)(4) = 14 \times 10^6$

$\therefore \quad k = \dfrac{4D}{R_bR_c} = \dfrac{(4)(14) \times 10^6}{(4)(2) \times 10^6} = 7$

$\therefore \quad \dfrac{R_f}{20{,}000} + 1 = 7, \qquad R_f = 120\,\text{k}\Omega$

[b] $v_o = 4(0.75) + 1.0 + 2(1.5) = 7$ V

$v_n = v_o/7 = 1$ V $= v_p$

$i_a = \dfrac{v_a - v_p}{1000} = \dfrac{0.75 - 1}{1000} = -250\,\mu\text{A}$

$i_b = \dfrac{v_b - v_p}{4000} = \dfrac{1 - 1}{4000} = 0\,\mu\text{A}$

$i_c = \dfrac{v_c - v_p}{2000} = \dfrac{1.5 - 1}{2000} = 250\,\mu\text{A}$

P 5.23 [a] Assume v_a is acting alone. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_a}{R_a} + \frac{0 - v_o'}{R_b} + i_n = 0, \qquad i_n = 0$$

Therefore

$$\frac{v_o'}{R_b} = -\frac{v_a}{R_a}, \qquad v_o' = -\frac{R_b}{R_a}v_a$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_b R_d}{R_c + R_d}$$

$$\frac{v_n}{R_a} + \frac{v_n - v_o''}{R_b} + i_n = 0, \qquad i_n = 0$$

$$\left(\frac{1}{R_a} + \frac{1}{R_b}\right)\left(\frac{R_d}{R_c + R_d}\right) v_b - \frac{v_o''}{R_b} = 0$$

$$v_o'' = \left(\frac{R_b}{R_a} + 1\right)\left(\frac{R_d}{R_c + R_d}\right) v_b = \frac{R_d}{R_a}\left(\frac{R_a + R_b}{R_c + R_d}\right) v_b$$

$$v_o = v_o' + v_o'' = \frac{R_d}{R_a}\left(\frac{R_a + R_b}{R_c + R_d}\right) v_b - \frac{R_b}{R_a} v_a$$

[b] $\dfrac{R_d}{R_a}\left(\dfrac{R_a + R_b}{R_c + R_d}\right) = \dfrac{R_b}{R_a}$, therefore $R_d(R_a + R_b) = R_b(R_c + R_d)$

$R_d R_a = R_b R_c$, therefore $\dfrac{R_a}{R_b} = \dfrac{R_c}{R_d}$

When $\dfrac{R_d}{R_a}\left(\dfrac{R_a + R_b}{R_c + R_d}\right) = \dfrac{R_b}{R_a}$

Eq. (5.22) reduces to $v_o = \dfrac{R_b}{R_a} v_b - \dfrac{R_b}{R_a} v_a = \dfrac{R_b}{R_a}(v_b - v_a)$.

P 5.24 Use voltage division to find v_p:

$$v_p = \frac{25{,}000}{25{,}000 + 15{,}000}(8) = 5 \text{ V}$$

Write a KCL equation at v_n and solve it for v_o:

$$\frac{v_n - v_a}{10{,}000} + \frac{v_n - v_o}{R_f} = 0 \qquad \text{so} \qquad \left(\frac{R_f}{10{,}000} + 1\right) v_n - \frac{R_f}{10{,}000} v_a = v_o$$

Since the op amp is ideal, $v_n = v_p = 5\text{V}$, so

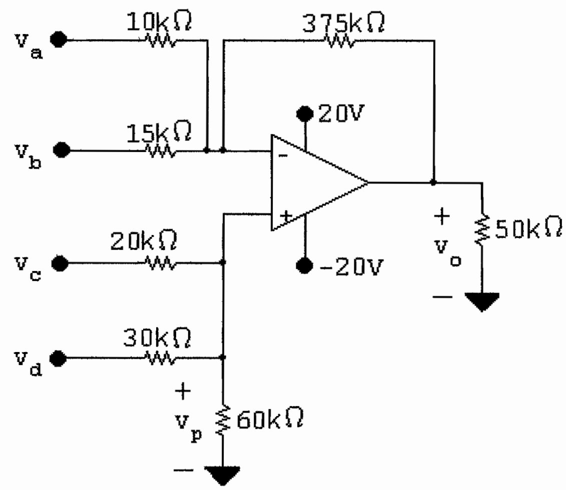
$$v_o = \left(\frac{R_f}{2000} + 5\right) - \frac{R_f}{10{,}000} v_a$$

To satisfy the equation,

$$\left(\frac{R_f}{2000} + 5\right) = 15 \qquad \text{and} \qquad \frac{R_f}{10{,}000} = 2$$

Thus, $R_f = 20\ \text{k}\Omega$.

P 5.25 [a]



$$\frac{v_p}{60{,}000} + \frac{v_p - v_c}{20{,}000} + \frac{v_p - v_d}{30{,}000} = 0$$

$$\therefore \quad 6v_p = 3v_c + 2v_d = 6v_n$$

$$\frac{v_n - v_a}{10{,}000} + \frac{v_n - v_b}{15{,}000} + \frac{v_n - v_o}{375{,}000} = 0$$

$$\begin{aligned} \therefore \quad v_o &= 63.5v_n - 37.5v_a - 25v_b \\ &= 63.5[(1/2)v_c + (1/3)v_d] - 37.5v_a - 25v_b \\ &= 63.5(0.1 + 0.2) - 37.5(0.4) - 25(0.8) = -15.95 \text{ V} \end{aligned}$$

[b] $v_o = 63.5(0.3) - 37.5(0.4) - 25v_b$

$$\pm 20 = 4.05 - 25v_b$$

$$\therefore \quad v_b = -0.638 \text{ V} \qquad \text{and} \qquad v_b = 0.962 \text{ V}$$

$$\therefore \quad -638 \leq v_b \leq 962 \,\text{mV}$$

P 5.26 [a] $v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a = \frac{33(100)}{20(80)}(0.90) - 4(0.45)$

$$v_o = 1.8563 - 1.8 = 56.25 \text{ mV}$$

[b] $v_n = v_p = \frac{(0.90)(33)}{80} = 371.25\,\text{mV}$

$$i_a = \frac{(450 - 371.25)10^{-3}}{20 \times 10^3} = 3.9375\,\mu\text{A}$$

$$R_a = \frac{v_a}{i_a} = \frac{450 \times 10^{-3}}{3.9375 \times 10^{-6}} = 114.3\,\text{k}\Omega$$

[c] $R_{\text{in b}} = R_c + R_d = 80\,\text{k}\Omega$

P 5.27 $v_p = \dfrac{v_b R_b}{R_a + R_b} = v_n$

$$\frac{v_n - v_a}{4000} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n\left(\frac{R_f}{4000} + 1\right) - \frac{v_a R_f}{4000} = v_o$$

$$\therefore \quad \left(\frac{R_f}{4000} + 1\right)\frac{R_b}{R_a + R_b}v_b - \frac{R_f}{4000}v_a = v_o$$

$$\therefore \quad \frac{R_f}{4000} = 7.5; \qquad R_f = 30\,\text{k}\Omega$$

$$\therefore \quad \frac{R_f}{4000} + 1 = 8.5$$

$$\therefore \quad 8.5\left(\frac{R_b}{R_a + R_b}\right) = 7.5$$

$$8.5R_b = 7.5R_b + 7.5R_a \qquad R_b = 7.5R_a$$

$$R_a + R_b = 170\,\text{k}\Omega$$

$$8.5R_a = 170\,\text{k}\Omega$$

$$R_a = 20\,\text{k}\Omega$$

$$R_b = 170 - 20 = 150\,\text{k}\Omega$$

P 5.28 $v_p = v_n = R_b i_b$

$$\frac{R_b i_b - 1000 i_a}{1000} + \frac{R_b i_b - v_o}{R_f} = 0$$

$$\left(\frac{R_b}{1000} + \frac{R_b}{R_f}\right) i_b - i_a = \frac{v_o}{R_f}$$

$$v_o = \left[\frac{R_b R_f}{1000} + R_b\right] i_b - R_f i_a$$

$$\therefore \quad R_f = 4000\,\Omega$$

$$4R_b + R_b = 4000$$

$$\therefore \quad R_b = 800\,\Omega$$

P 5.29 $v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a$

By hypothesis: $R_b/R_a = 5; \quad R_c + R_d = 600\,\text{k}\Omega; \quad \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} = 2$

$$\therefore \quad \frac{R_d}{R_a}\frac{(R_a + 5R_a)}{600{,}000} = 2 \quad \text{so} \quad R_d = 200\,\text{k}\Omega; \quad R_c = 400\,\text{k}\Omega$$

Also, when $v_o = 0$ we have

$$\frac{v_n - v_a}{R_a} + \frac{v_n}{R_b} = 0$$

$$\therefore \quad v_n\left(1 + \frac{R_a}{R_b}\right) = v_a; \qquad v_n = (5/6)v_a$$

$$i_a = \frac{v_a - (5/6)v_a}{R_a} = \frac{1}{6}\frac{v_a}{R_a}; \qquad R_{in} = \frac{v_a}{i_a} = 6R_a = 18\,\text{k}\Omega$$

$$\therefore \quad R_a = 3\,\text{k}\Omega; \qquad R_b = 15\,\text{k}\Omega$$

P 5.30 [a] $v_p = \frac{\alpha R_g}{\alpha R_g + (R_g - \alpha R_g)}v_g$

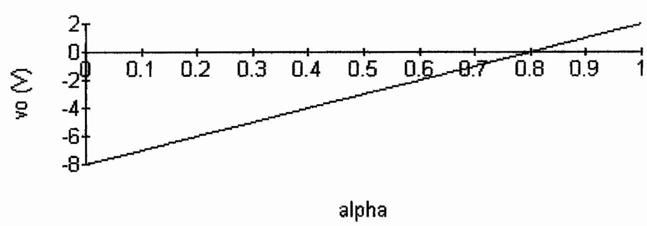
$$v_n = v_p = \alpha v_g$$

$$\frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_f} = 0$$

$$(v_n - v_g)\frac{R_f}{R_1} + v_n - v_o = 0$$

$$\begin{aligned} v_o &= \left(1 + \frac{R_f}{R_g}\right)\alpha v_g - \frac{R_f}{R_1}v_g \\ &= (\alpha v_g - v_g)4 + \alpha v_g \\ &= [(\alpha - 1)4 + \alpha]v_g \\ &= (5\alpha - 4)v_g \\ &= (5\alpha - 4)(2) = 10\alpha - 8 \end{aligned}$$

α	v_o	α	v_o	α	v_o
0.0	−8 V	0.4	−4 V	0.8	0 V
0.1	−7 V	0.5	−3 V	0.9	1 V
0.2	−6 V	0.6	−2 V	1.0	2 V
0.3	−5 V	0.7	−1 V		



[b] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1\right) v_g\alpha + -\left(\frac{R_f}{R_1}\right) v_g$$

Therefore,

$$\text{slope } = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad \text{intercept } = -\left(\frac{R_f}{R_1}\right) v_g$$

[c] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad 4 = -\left(\frac{R_f}{R_1}\right) v_g$$

Solving,

$$v_g = -2 \text{ V}; \qquad \frac{R_f}{R_1} = 2$$

P 5.31 $v_p = \dfrac{20{,}00}{100{,}000}(-4) = -0.8 \text{ V} = v_n$

$$\frac{-0.8 + 4}{2000} + \frac{-0.8 - v_o}{R_\text{f}} = 0$$

$$\therefore \quad v_o = 0.0016R_\text{f} - 0.8$$

$$v_o = 20 \text{ V}; \qquad R_\text{f} = 13\,\text{k}\Omega$$

$$v_o = -10 \text{ V}; \qquad R_\text{f} = -5.75\,\text{k}\Omega$$

But $R_\text{f} \geq 0, \qquad \therefore \; R_\text{f} = 13\,\text{k}\Omega$

P 5.32 [a] $A_\text{dm} = \dfrac{95(100+5) + 100(5+95)}{2(5)(5+95)} = 19.975$

[b] $A_\text{cm} = \dfrac{(5)(95) - 5(100)}{(5)(5+95)} = -0.05$

[c] $\text{CMRR} = \left|\dfrac{19.975}{0.05}\right| = 399.5$

P 5.33 $A_\text{cm} = \dfrac{(33)(47) - (47)R_x}{33(47 + R_x)}$

$$A_\text{dm} = \frac{47(33+47) + 47(47+R_x)}{2(33)(47+R_x)}$$

$$\frac{A_\text{dm}}{A_\text{cm}} = \frac{R_x + 127}{2(33 - R_x)}$$

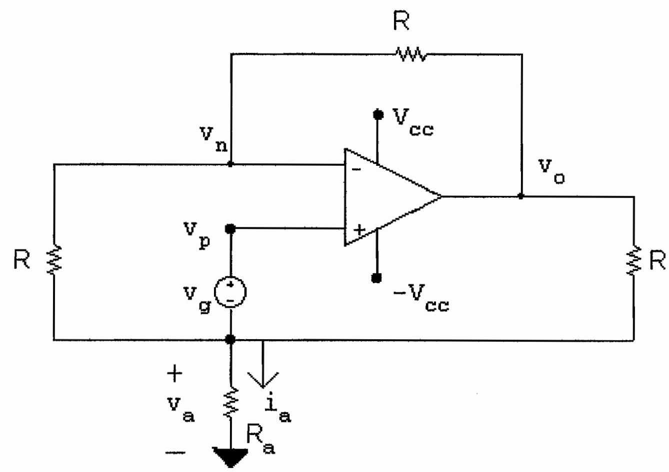
$$\therefore \quad \frac{R_x + 127}{2(33 - R_x)} = \pm 750 \quad \text{for the limits on the value of } R_x$$

If we use +750 $\quad R_x = 32.89\,\text{k}\Omega$

If we use −750 $\quad R_x = 33.11\,\text{k}\Omega$

$$32.89\,\text{k}\Omega \le R_x \le 33.11\,\text{k}\Omega$$

P 5.34 [a]



$$\frac{v_n - v_a}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_a = v_o$$

$$\frac{v_a}{R_a} + \frac{v_a - v_n}{R} + \frac{v_a - v_o}{R} = 0$$

$$v_a\left[\frac{1}{R_a} + \frac{2}{R}\right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_a\left(2 + \frac{R}{R_a}\right) - v_n = v_o$$

$$v_n = v_p = v_a + v_g$$

$$\therefore \quad 2v_n - v_a = 2v_a + 2v_g - v_a = v_a + 2v_g$$

$$\therefore \quad v_a - v_o = -2v_g \qquad (1)$$

$$2v_a + v_a\left(\frac{R}{R_a}\right) - v_a - v_g = v_o$$

$$\therefore \quad v_a\left(1 + \frac{R}{R_a}\right) - v_o = v_g \qquad (2)$$

Now combining equations (1) and (2) yields

$$-v_a\frac{R}{R_a} = -3v_g$$

$$\text{or}\quad v_a = 3v_g\frac{R_a}{R}$$

$$\text{Hence}\quad i_a = \frac{v_a}{R_a} = \frac{3v_g}{R} \qquad \text{Q.E.D.}$$

[b] At saturation $\quad V_o = \pm\, V_{cc}$

$$\therefore\quad v_a = \pm\, V_{cc} - 2v_g \qquad (3)$$

and

$$\therefore\quad v_a\left(1+\frac{R}{R_a}\right) = \pm\, V_{cc} + v_g \qquad (4)$$

Dividing Eq (4) by Eq (3) gives

$$1+\frac{R}{R_a} = \frac{\pm\, V_{cc}+v_g}{\pm\, V_{cc}-2v_g}$$

$$\therefore\quad \frac{R}{R_a} = \frac{\pm\, V_{cc}+v_g}{\pm\, V_{cc}-2v_g} - 1 = \frac{3v_g}{\pm\, V_{cc}-2v_g}$$

$$\text{or}\quad R_a = \frac{(\pm\, V_{cc}-2v_g)}{3v_g}R \qquad \text{Q.E.D.}$$

P 5.35 [a] Assume the op-amp is operating within its linear range, then

$$i_L = \frac{3}{1.5} = 2\,\text{mA}$$

For $R_L = 2.5\,\text{k}\Omega \qquad v_o = (2.5+1.5)(2) = 8\ \text{V}$

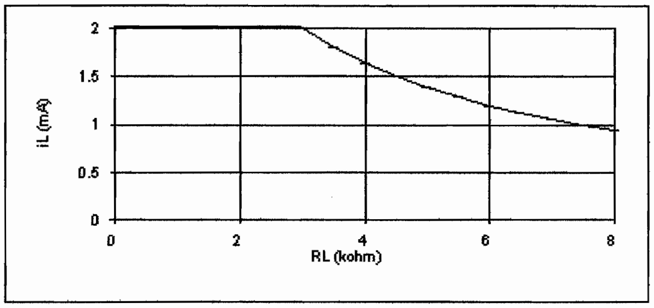
Now since $v_o < 9$ V our assumption of linear operation is correct, therefore

$$i_L = 2\,\text{mA}$$

[b] $9 = 2(1.5+R_L); \qquad R_L = 3\,\text{k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L. From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 3\,\text{k}\Omega$. Therefore when $R_L = 6.5\,\text{k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 9/(1.5+6.5) = 1.125\,\text{mA}$. To justify neglecting the current into the op-amp assume the drop across the 47 kΩ resistor is negligible, and the input resistance to the op-amp is at least $500\,\text{k}\Omega$. Then $i_p = i_n = (3-1.5)/500\times 10^{-3} = 3\,\mu\text{A}$. But $3\,\mu\text{A} \ll 1.125\,\text{mA}$, hence our assumption is reasonable.

[d]



P 5.36 [a] Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-90}{15}(-0.5) = 3 \text{ V}; \qquad v_{o2} = \frac{-120}{30}(0.4) = -1.6 \text{ V}$$

$$i_a = \frac{v_{o2} - v_{o1}}{1000} = -4.6\,\text{mA}$$

[b] $i_a = 0$ when $v_{o1} = v_{02}$ so from (a) $v_{o2} = 3$ V

Thus

$$\frac{-120}{30}(v_L) = 3$$

$$v_L = -\frac{90}{120} = -750 \text{ mV}$$

P 5.37 [a] $p_{16\,\text{k}\Omega} = \dfrac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4\,\mu\text{W}$

[b] $v_{16\,\text{k}\Omega} = \left(\dfrac{16}{64}\right)(320) = 80\,\text{mV}$

$$p_{16\,\text{k}\Omega} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4\,\mu\text{W}$$

[c] $\dfrac{p_a}{p_b} = \dfrac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification.*
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

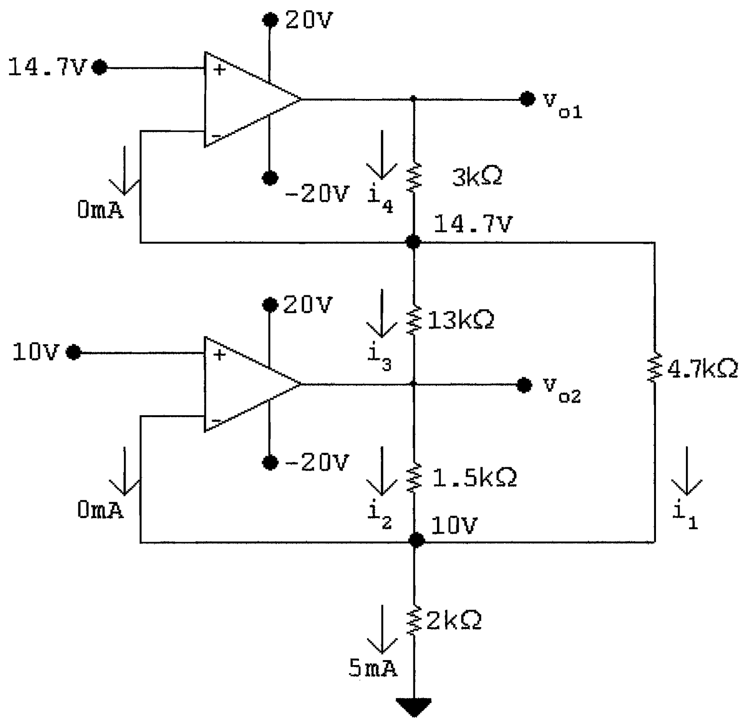
P 5.38 [a] $v_p = v_s, \qquad v_n = \frac{R_1 v_o}{R_1 + R_2}, \qquad v_n = v_p$

Therefore $v_o = \left(\frac{R_1 + R_2}{R_1}\right) v_s = \left(1 + \frac{R_2}{R_1}\right) v_s$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

P 5.39



$$i_1 = \frac{14.7 - 10}{4700} = 1\,\text{mA}$$

$$i_2 + i_1 + 0 = 5\,\text{mA}; \qquad i_2 = 4\,\text{mA}$$

$$v_{o2} = 10 + (1500)(0.004) = 16 \text{ V}$$

$$i_3 = \frac{14.7 - 16}{13{,}000} = -0.1\,\text{mA}$$

$$i_4 = i_3 + i_1 = 0.9\,\text{mA}$$

$$v_{o1} = 14.7 + 3000(0.0009) = 17.4 \text{ V}$$

P 5.40 $v_p = \dfrac{5.6}{8.0}v_g = 0.7v_g = 2.8\sin(5\pi/3)t\ \text{V}$

$$\frac{v_n}{2000} + \frac{v_n - v_o}{18{,}000} = 0$$

$$10v_n = v_o; \qquad v_n = v_p$$

$$\therefore\quad v_o = 28\sin(5\pi/3)t\ \text{V} \qquad 0 \le t \le \infty$$

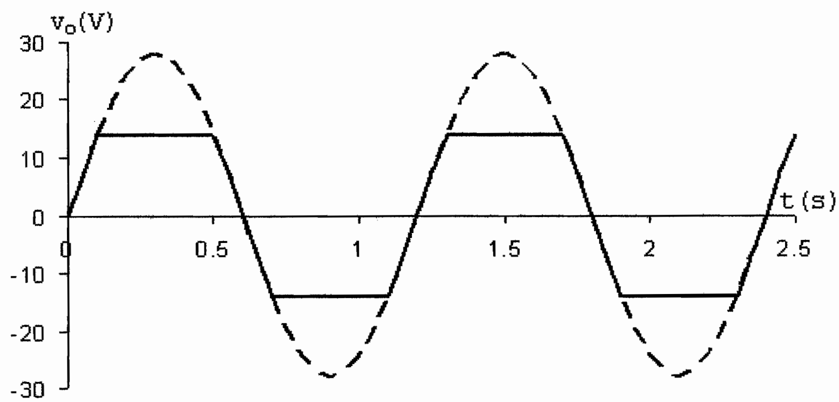
$$v_o = 0 \qquad t \le 0$$

At saturation

$$28\sin\left(\frac{5\pi}{3}\right)t = \pm 14; \qquad \sin\frac{5\pi}{3}t = \pm 0.5$$

$$\therefore\quad \frac{5\pi}{3}t = \frac{\pi}{6},\quad \frac{5\pi}{6},\quad \frac{7\pi}{6},\quad \frac{11\pi}{6},\quad \text{etc.}$$

$t = 0.10\,\text{s}, \quad 0.50\,\text{s}, \quad 0.70\,\text{s}, \quad$ etc.



P 5.41 It follows directly from the circuit that $v_o = -5v_g$
From the plot of v_g we have $\quad v_g = 0, \quad t < 0$

$$
\begin{aligned}
v_g &= 4t & 0 \le t \le 0.5\\
v_g &= 4 - 4t & 0.5 \le t \le 1.5\\
v_g &= 4t - 8 & 1.5 \le t \le 2.5\\
v_g &= 12 - 4t & 2.5 \le t \le 3.5\\
v_g &= 4t - 16 & 3.5 \le t \le 4.5, \quad \text{etc.}
\end{aligned}
$$

Therefore

$$v_o = -20t \qquad 0 \le t \le 0.5$$

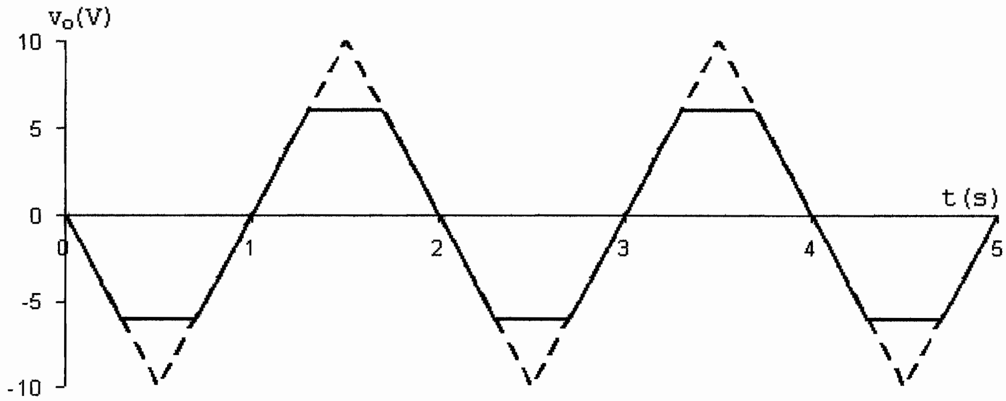
$$v_o = 20t - 20 \qquad 0.5 \le t \le 1.5$$

$$v_o = 40 - 20t \qquad 1.5 \le t \le 2.5$$

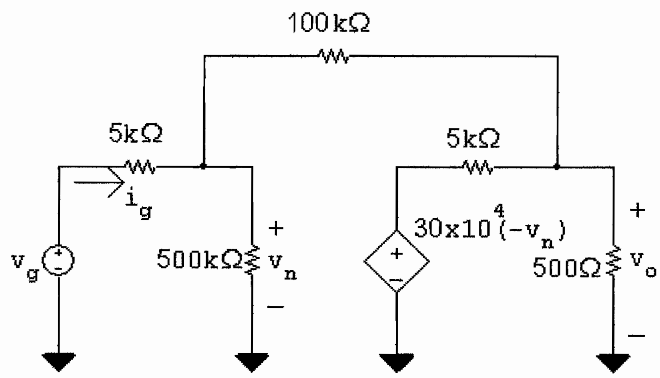
$$v_o = 20t - 60 \qquad 2.5 \le t \le 3.5$$

$$v_o = 80 - 20t \qquad 3.5 \le t \le 4.5, \quad \text{etc.}$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ± 6, the output is clipped at ± 6. The plot is shown below.



P 5.42 [a] Replace the op amp with the model from Fig. 5.15:



Write two node voltage equations, one at the left node, the other at the right node:

$$\frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} + \frac{v_n}{500{,}000} = 0$$

$$\frac{v_o + 3 \times 10^5 v_n}{5000} + \frac{v_o - v_n}{100{,}000} + \frac{v_o}{500} = 0$$

Simplify and place in standard form:

$106v_n - 5v_o = 100v_g$

$(6 \times 10^6 - 1)v_n + 221v_o = 0$

Let $v_g = 1$ V and solve the two simultaneous equations:

$v_o = -19.9844$ V; $\quad v_n = 736.1\,\mu$V

Thus the voltage gain is $v_o/v_g = -19.9844$.

[b] From the solution in part (a), $v_n = 736.1\,\mu$V.

[c] $i_g = \dfrac{v_g - v_n}{5000} = \dfrac{v_g - 736.1 \times 10^{-6}v_g}{5000}$

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 736.1 \times 10^{-6}} = 5003.68\,\Omega$$

[d] For an ideal op amp, the voltage gain is the ratio between the feedback resistor and the input resistor:

$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

For an ideal op amp, the difference between the voltages at the input terminals is zero, and the input resistance of the op amp is infinite. Therefore,

$v_n = v_p = 0$ V; $\quad R_g = 5000\,\Omega$

P 5.43 [a] $\dfrac{v_n}{8000} + \dfrac{v_n - v_g}{600{,}000} + \dfrac{v_n - v_o}{240{,}000} = 0 \quad$ or $\quad 78.5v_n - 2.5v_o = v_g$

$$\frac{v_o}{30{,}000} + \frac{v_o - v_n}{240{,}000} + \frac{v_o - 100{,}000(v_p - v_n)}{5000} = 0$$

$57v_o - v_n - 48 \times 10^5(v_p - v_n) = 0$

$$v_p = v_g + \frac{(v_n - v_g)(160)}{600} = (11/15)v_g + (4/15)v_n$$

$57v_o - v_n - 48 \times 10^5[(11/15)v_g - (11/15)v_n] = 0$

$57v_o + 3{,}520{,}000v_n = 3{,}520{,}000v_g$

$$\Delta = \begin{vmatrix} 78.5 & -2.5 \\ 3.52 \times 10^6 & 57 \end{vmatrix} = 8{,}804{,}474.5$$

$$N_o = \begin{vmatrix} 78.5 & v_g \\ 3.52 \times 10^6 & 3.52 \times 10^6 v_g \end{vmatrix} = 272.8 \times 10^6 v_g$$

$v_o = \dfrac{N_o}{\Delta} = 30.98v_g; \qquad \dfrac{v_o}{v_g} = 30.98$

[b] $N_1 = \begin{vmatrix} v_g & -2.5 \\ 3.52 \times 10^6 v_g & 57 \end{vmatrix} = 8{,}800{,}057 v_g$

$$v_n = \frac{N_1}{\Delta} = 0.9995 v_g; \qquad v_n = 999.5\,\text{mV}$$

$$v_p = (11/15)(1000) + (4/15)(999.5) = 999.87\,\text{mV}$$

[c] $v_p - v_n = 367.94\,\mu$ V

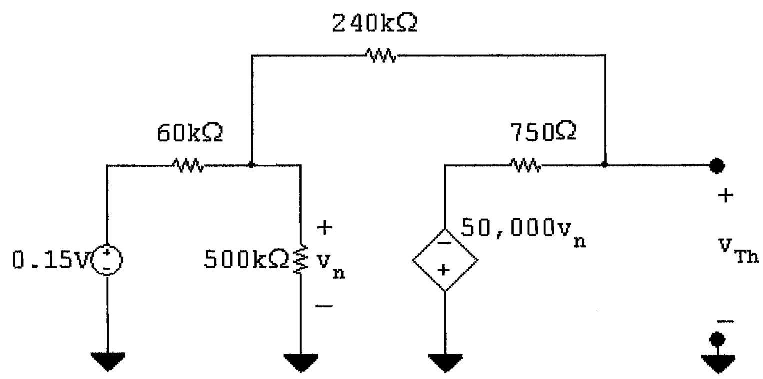
[d] $i_g = \dfrac{(1000 - 999.87)10^{-3}}{160 \times 10^3} = 836.22\,\text{pA}$

[e] $\dfrac{v_g}{8} + \dfrac{v_g - v_o}{240} = 0, \qquad \text{since } v_n = v_p = v_g$

$$\therefore\ v_o = 31 v_g, \qquad \frac{v_o}{v_g} = 31$$

$$v_n = v_p = 1\ \text{V}; \qquad v_p - v_n = 0\ \text{V}; \qquad i_g = 0\ \text{A}$$

P 5.44 [a]

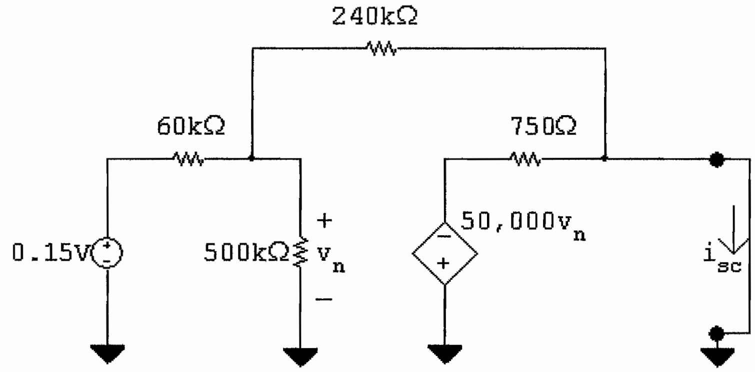


$$\frac{v_n - 0.15}{60{,}000} + \frac{v_n}{500{,}000} + \frac{v_n - v_{\text{Th}}}{240{,}000} = 0$$

$$\frac{v_{\text{Th}} + 5 \times 10^4 v_n}{750} + \frac{v_{\text{Th}} - v_n}{240{,}000} = 0$$

Solving, $v_{\text{Th}} = -0.6$ V

Short-circuit current calculation:

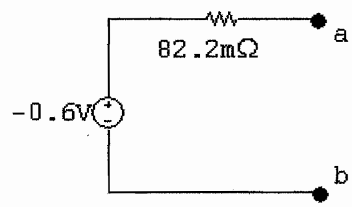


$$\frac{v_n}{500{,}000} + \frac{v_n - 0.15}{60{,}000} + \frac{v_n - 0}{240{,}000} = 0$$

$$\therefore \quad v_n = 0.1095 \text{ V}$$

$$i_{sc} = \frac{v_n}{240{,}000} - \frac{5 \times 10^4}{750} v_n = -7.3 \text{ A}$$

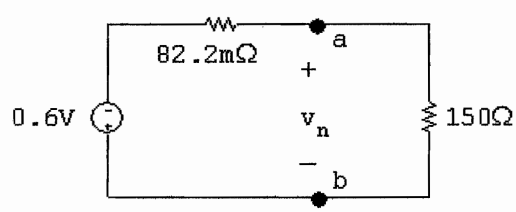
$$R_{\rm Th} = \frac{v_{\rm Th}}{i_{sc}} = 82.2\,\text{m}\Omega$$



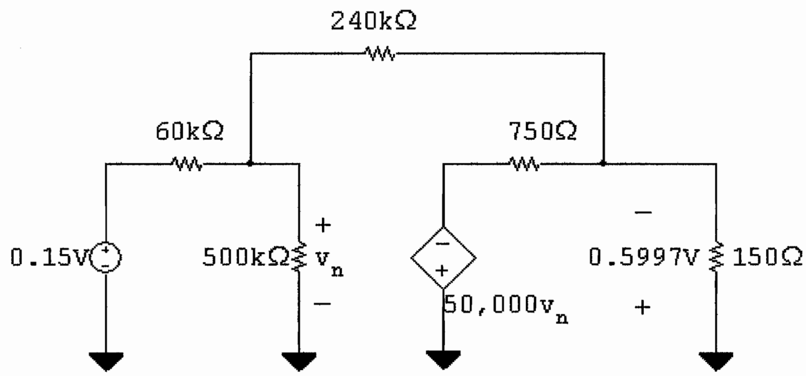
[b] The output resistance of the inverting amplifier is the same as the Thévenin resistance, i.e.,

$$R_o = R_{\rm Th} = 82.2\,\text{m}\Omega$$

[c]



$$v_o = \left(\frac{150}{150.082}\right)(-0.6) = -0.5997 \text{ V}$$



$$\frac{v_n - 0.15}{60{,}000} + \frac{v_n}{500{,}000} + \frac{v_n + 0.5997}{240{,}000} = 0$$

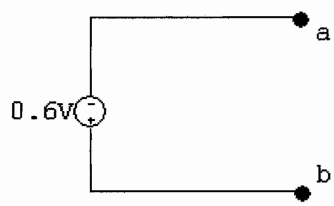
$$\therefore \quad v_n = 54.7445\,\mu\text{V}$$

$$i_g = \frac{0.15 - 54.7445 \times 10^{-6}}{60{,}000} = 2.4991\,\mu\text{A}$$

$$R_g = \frac{0.15}{i_g} = 60{,}021.6\,\Omega$$

P 5.45 [a] $v_{\text{Th}} = \frac{-240}{60}(0.15) = -0.6 \text{ V}$

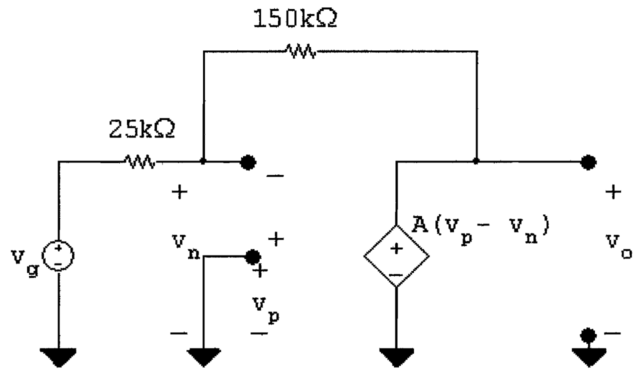
$R_{\text{Th}} = 0$, since op-amp is ideal



[b] $R_o = R_{\text{Th}} = 0\,\Omega$

[c] $R_g = 60\,\text{k}\Omega$ since $v_n = 0$

P 5.46 [a]



$$\frac{v_n - v_g}{25{,}000} + \frac{v_n - v_o}{150{,}000} = 0$$

$$\therefore \quad v_o = 7v_n - 6v_g$$

Also $\quad v_o = A(v_p - v_n) = -Av_n$

$$\therefore \quad v_n = \frac{-v_o}{A}$$

$$\therefore \quad v_o\left(1 + \frac{7}{A}\right) = -6v_g$$

$$v_o = \frac{-6A}{(7 + A)}v_g$$

[b] $v_o = \dfrac{-6(150)(0.5)}{(7 + 150)} = -2.866 \text{ V}$

[c] $v_o = -6(0.5) = -3 \text{ V}$

[d] $-2.94 = \dfrac{-6(0.5)A}{7 + A}$

$$\therefore \quad A = 343$$

P 5.47 From Eq. 5.57,

$$\frac{v_{\text{ref}}}{R + \Delta R} = v_n\left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f}\right) - \frac{v_o}{R_f}$$

Substituting Eq. 5.59 for $v_p = v_n$:

$$\frac{v_{\text{ref}}}{R + \Delta R} = \frac{v_{\text{ref}}\left(\frac{1}{R+\Delta R} + \frac{1}{R-\Delta R} + \frac{1}{R_f}\right)}{(R - \Delta R)\left(\frac{1}{R+\Delta R} + \frac{1}{R-\Delta R} + \frac{1}{R_f}\right)} - \frac{v_o}{R_f}$$

Rearranging,

$$\frac{v_o}{R_f} = v_{\text{ref}}\left(\frac{1}{R-\Delta R} - \frac{1}{R+\Delta R}\right)$$

Thus,

$$v_o = v_{\text{ref}}\left(\frac{2\Delta R}{R^2-\Delta R^2}\right)R_f$$

P 5.48 [a] Use Eq. 5.61 to solve for R_f; note that since we are using 1% strain gages, $\Delta = 0.01$:

$$R_f = \frac{v_o R}{2\Delta v_{\text{ref}}} = \frac{(5)(120)}{(2)(0.01)(15)} = 2\,\text{k}\Omega$$

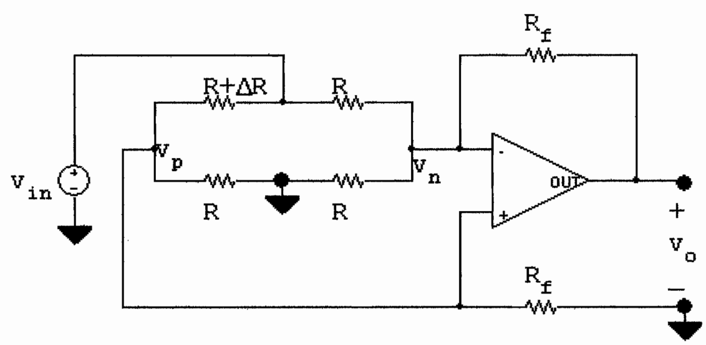
[b] Now solve for Δ given $v_o = 50$ mV:

$$\Delta = \frac{v_o R}{2R_f v_{\text{ref}}} = \frac{(0.05)(120)}{2(2000)(15)} = 100\times10^{-6}$$

The change in strain gage resistance that corresponds to a 50 mV change in output voltage is thus

$$\Delta R = \Delta R = (100\times10^{-6})(120) = 12\ \text{m}\Omega$$

P 5.49 [a]



Let $R_1 = R + \Delta R$

$$\frac{v_p}{R_f} + \frac{v_p}{R} + \frac{v_p - v_{\text{in}}}{R_1} = 0$$

$$\therefore\quad v_p\left[\frac{1}{R_f} + \frac{1}{R} + \frac{1}{R_1}\right] = \frac{v_{\text{in}}}{R_1}$$

$$\therefore\quad v_p = \frac{RR_f v_{\text{in}}}{RR_1 + R_f R_1 + R_f R} = v_n$$

$$\frac{v_n}{R} + \frac{v_n - v_{\text{in}}}{R} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n\left[\frac{1}{R}+\frac{1}{R}+\frac{1}{R_f}\right]-\frac{v_o}{R_f}=\frac{v_{\text{in}}}{R}$$

$$\therefore\quad v_n\left[\frac{R+2R_f}{RR_f}\right]-\frac{v_{\text{in}}}{R}=\frac{v_o}{R_f}$$

$$\therefore\quad \frac{v_o}{R_f}=\left[\frac{R+2R_f}{RR_f}\right]\left[\frac{RR_fv_{\text{in}}}{[RR_1+R_fR_1+R_fR]}\right]-\frac{v_{\text{in}}}{R}$$

$$\therefore\quad \frac{v_o}{R_f}=\left[\frac{R+2R_f}{RR_1+R_fR_1+R_fR}-\frac{1}{R}\right]v_{\text{in}}$$

$$\therefore\quad v_o=\frac{[R^2+2RR_f-R_1(R+R_f)-RR_f]R_f}{R[R_1(R+R_f)+RR_f]}v_{\text{in}}$$

Now substitute $R_1=R+\Delta R$ and get

$$v_o=\frac{-\Delta R(R+R_f)R_fv_{\text{in}}}{R[(R+\Delta R)(R+R_f)+RR_f]}$$

If $\Delta R\ll R$

$$v_o\approx\frac{(R+R_f)R_f(-\Delta R)v_{\text{in}}}{R^2(R+2R_f)}$$

[b] $v_o\approx\dfrac{47\times10^4(48\times10^4)(-95)15}{10^8(95\times10^4)}\approx-3.384\text{ V}$

[c] $v_o=\dfrac{-95(48\times10^4)(47\times10^4)15}{10^4[(1.0095)10^4(48\times10^4)+47\times10^8]}=-3.368\text{ V}$

P 5.50 [a] $v_o\approx\dfrac{(R+R_f)R_f(-\Delta R)v_{\text{in}}}{R^2(R+2R_f)}$

$$v_o=\frac{(R+R_f)(-\Delta R)R_fv_{\text{in}}}{R[(R+\Delta R)(R+R_f)+RR_f]}$$

$$\therefore\quad \frac{\text{approx value}}{\text{true value}}=\frac{R[(R+\Delta R)(R+R_f)+RR_f]}{R^2(R+2R_f)}$$

$$\therefore\quad \text{Error}=\frac{R[(R+\Delta R)(R+R_f)+RR_f]-R^2(R+2R_f)}{R^2(R+2R_f)}$$

$$=\frac{\Delta R}{R}\frac{(R+R_f)}{(R+2R_f)}$$

$$\therefore\quad \%\text{ error}=\frac{\Delta R(R+R_f)}{R(R+2R_f)}\times100$$

[b] $\%\text{ error}=\dfrac{95(48\times10^4)\times100}{10^4(95\times10^4)}=0.48\%$

P 5.51 $$1 = \frac{\Delta R(48 \times 10^4)}{10^4(95 \times 10^4)} \times 100$$

$$\therefore \quad \Delta R = \frac{9500}{48} = 197.91667\,\Omega$$

$$\therefore \quad \%\text{ change in } R = \frac{197.19667}{10^4} \times 100 \approx 1.98\%$$

P 5.52 [a] It follows directly from the solution to Problem 5.49 that

$$v_o = \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f v_{\text{in}}}{R[R_1(R + R_f) + RR_f]}$$

Now $R_1 = R - \Delta R$. Substituting into the expression gives

$$v_o = \frac{(R + R_f)R_f(\Delta R)v_{\text{in}}}{R[(R - \Delta R)(R + R_f) + RR_f]}$$

Now let $\Delta R \ll R$ and get

$$v_o \approx \frac{(R + R_f)R_f \Delta R v_{\text{in}}}{R^2(R + 2R_f)}$$

[b] It follows directly from the solution to Problem 5.49 that

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{R[(R - \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore \quad \text{Error} = \frac{(R - \Delta R)(R + R_f) + RR_f - R(R + 2R_f)}{R(R + 2R_f)}$$

$$= \frac{-\Delta R(R + R_f)}{R(R + 2R_f)}$$

$$\therefore \quad \%\text{ error} = \frac{-\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

[c] $R - \Delta R = 9810\,\Omega \qquad \therefore \quad \Delta R = 10{,}000 - 9810 = 190\,\Omega$

$$\therefore \quad v_o \approx \frac{(48 \times 10^4)(47 \times 10^4)(190)(15)}{10^8(95 \times 10^4)} \approx 6.768 \text{ V}$$

[d] $$\%\text{ error} = \frac{-190(48 \times 10^4)(100)}{10^4(95 \times 10^4)} = -0.96\%$$

6

Inductance, Capacitance, and Mutual Inductance

Assessment Problems

AP 6.1 [a] $i_g = 8e^{-300t} - 8e^{-1200t}\,\text{A}$

$$v = L\frac{di_g}{dt} = -9.6e^{-300t} + 38.4e^{-1200t}\,\text{V}, \qquad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8\,\text{V}$$

[b] $v = 0$ when $38.4e^{-1200t} = 9.6e^{-300t}$ or $t = (\ln 4)/900 = 1.54\,\text{ms}$

[c] $p = vi = 384e^{-1500t} - 76.8e^{-600t} - 307.2e^{-2400t}\,\text{W}$

[d] $\dfrac{dp}{dt} = 0$ when $e^{1800t} - 12.5e^{900t} + 16 = 0$

Let $x = e^{900t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.44766, \qquad t = \frac{\ln 1.45}{900} = 411.05\,\mu\text{s}$$

$$x = 11.0523, \qquad t = \frac{\ln 11.05}{900} = 2.67\,\text{ms}$$

p is maximum at $t = 411.05\,\mu\text{s}$

[e] $p_{\text{max}} = 384e^{-1.5(0.41105)} - 76.8e^{-0.6(0.41105)} - 307.2e^{-2.4(0.41105)} = 32.72\,\text{W}$

[f] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54\,\text{ms}$.

[g] $i_{\text{max}} = 8[e^{-0.3(1.54)} - e^{-1.2(1.54)}] = 3.78\,\text{A}$

$$w_{\text{max}} = (1/2)(4 \times 10^{-3})(3.78)^2 = 28.6\,\text{mJ}$$

AP 6.2 [a] $i = C\frac{dv}{dt} = 24 \times 10^{-6}\frac{d}{dt}[e^{-15{,}000t}\sin 30{,}000t]$

$= [0.72\cos 30{,}000t - 0.36\sin 30{,}000t]e^{-15{,}000t}\,\text{A}, \qquad i(0^+) = 0.72\,\text{A}$

[b] $i\left(\frac{\pi}{80}\,\text{ms}\right) = -31.66\,\text{mA}, \quad v\left(\frac{\pi}{80}\,\text{ms}\right) = 20.505\,\text{V},$

$p = vi = -649.23\,\text{mW}$

[c] $w = \left(\frac{1}{2}\right)Cv^2 = 126.13\,\mu\text{J}$

AP 6.3 [a] $v = \left(\frac{1}{C}\right)\int_{0^-}^{t} i\,dx + v(0^-)$

$= \frac{1}{0.6 \times 10^{-6}}\int_{0^-}^{t} 3\cos 50{,}000x\,dx = 100\sin 50{,}000t\,\text{V}$

[b] $p(t) = vi = [300\cos 50{,}000t]\sin 50{,}000t$

$= 150\sin 100{,}000t\,\text{W}, \qquad p_{(\text{max})} = 150\,\text{W}$

[c] $w_{(\text{max})} = \left(\frac{1}{2}\right)Cv_{\text{max}}^2 = 0.30(100)^2 = 3000\,\mu\text{J} = 3\,\text{mJ}$

AP 6.4 [a] $L_{\text{eq}} = \frac{60(240)}{300} = 48\,\text{mH}$

[b] $i(0^+) = 3 + -5 = -2\,\text{A}$

[c] $i = \frac{125}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 2 = 0.125e^{-5t} - 2.125\,\text{A}$

[d] $i_1 = \frac{50}{3}\int_{0^+}^{t}(-0.03e^{-5x})\,dx + 3 = 0.1e^{-5t} + 2.9\,\text{A}$

$i_2 = \frac{25}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 5 = 0.025e^{-5t} - 5.025\,\text{A}$

$i_1 + i_2 = i$

AP 6.5 $v_1 = 0.5 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 10 = -12e^{-10t} + 2\,\text{V}$

$v_2 = 0.125 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 5 = -3e^{-10t} - 2\,\text{V}$

$v_1(\infty) = 2\,\text{V}, \qquad v_2(\infty) = -2\,\text{V}$

$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right] \times 10^{-6} = 20\,\mu\text{J}$

AP 6.6 [a] Summing the voltages around mesh 1 yields

$$4\frac{di_1}{dt} + 8\frac{d(i_2 + i_g)}{dt} + 20(i_1 - i_2) + 5(i_1 + i_g) = 0$$

or

$$4\frac{di_1}{dt} + 25i_1 + 8\frac{di_2}{dt} - 20i_2 = -\left(5i_g + 8\frac{di_g}{dt}\right)$$

Summing the voltages around mesh 2 yields

$$16\frac{d(i_2 + i_g)}{dt} + 8\frac{di_1}{dt} + 20(i_2 - i_1) + 780i_2 = 0$$

or

$$8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 800i_2 = -16\frac{di_g}{dt}$$

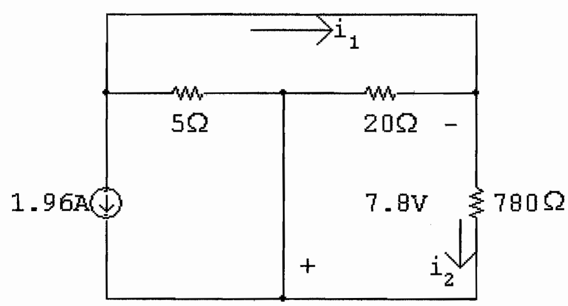
[b] From the solutions given in part (b)

$$i_1(0) = -0.4 - 11.6 + 12 = 0; \qquad i_2(0) = -0.01 - 0.99 + 1 = 0$$

These values agree with zero initial energy in the circuit. At infinity,

$$i_1(\infty) = -0.4\text{A}; \qquad i_2(\infty) = -0.01\text{A}$$

When $t = \infty$ the circuit reduces to



$$\therefore\ i_1(\infty) = -\left(\frac{7.8}{20} + \frac{7.8}{780}\right) = -0.4\text{A}; \quad i_2(\infty) = -\frac{7.8}{780} = -0.01\text{A}$$

From the solutions for i_1 and i_2 we have

$$\frac{di_1}{dt} = 46.40e^{-4t} - 60e^{-5t}$$

$$\frac{di_2}{dt} = 3.96e^{-4t} - 5e^{-5t}$$

Also, $\frac{di_g}{dt} = 7.84e^{-4t}$

Thus

$$4\frac{di_1}{dt} = 185.60e^{-4t} - 240e^{-5t}$$

$$25i_1 = -10 - 290e^{-4t} + 300e^{-5t}$$

$$8\frac{di_2}{dt} = 31.68e^{-4t} - 40e^{-5t}$$

$$20i_2 = -0.20 - 19.80e^{-4t} + 20e^{-5t}$$

$$5i_g = 9.8 - 9.8e^{-4t}$$

$$8\frac{di_g}{dt} = 62.72e^{-4t}$$

Test:

$$185.60e^{-4t} - 240e^{-5t} - 10 - 290e^{-4t} + 300e^{-5t} + 31.68e^{-4t} - 40e^{-5t}$$

$$+0.20 + 19.80e^{-4t} - 20e^{-5t} \stackrel{?}{=} -[9.8 - 9.8e^{-4t} + 62.72e^{-4t}]$$

$$-9.8 + (300 - 240 - 40 - 20)e^{-5t}$$

$$+(185.60 - 290 + 31.68 + 19.80)e^{-4t} \stackrel{?}{=} -(9.8 + 52.92e^{-4t})$$

$$-9.8 + 0e^{-5t} + (237.08 - 290)e^{-4t} \stackrel{?}{=} -9.8 - 52.92e^{-4t}$$

$$-9.8 - 52.92e^{-4t} = -9.8 - 52.92e^{-4t} \quad \text{(OK)}$$

Also,

$$8\frac{di_1}{dt} = 371.20e^{-4t} - 480e^{-5t}$$

$$20i_1 = -8 - 232e^{-4t} + 240e^{-5t}$$

$$16\frac{di_2}{dt} = 63.36e^{-4t} - 80e^{-5t}$$

$$800i_2 = -8 - 792e^{-4t} + 800e^{-5t}$$

$$16\frac{di_g}{dt} = 125.44e^{-4t}$$

Test:

$$371.20e^{-4t} - 480e^{-5t} + 8 + 232e^{-4t} - 240e^{-5t} + 63.36e^{-4t} - 80e^{-5t}$$

$$-8 - 792e^{-4t} + 800e^{-5t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(8 - 8) + (800 - 480 - 240 - 80)e^{-5t}$$

$$+(371.20 + 232 + 63.36 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(800 - 800)e^{-5t} + (666.56 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$-125.44e^{-4t} = -125.44e^{-4t} \quad \text{(OK)}$$

Problems

P 6.1 [a]

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 16t\,\text{A} && 0 \le t \le 25\,\text{ms} \\ i &= 0.8 - 16t\,\text{A} && 25 \le t \le 50\,\text{ms} \\ i &= 0 && 50\,\text{ms} < t \end{aligned}$$

[b] $v = L\dfrac{di}{dt} = 375 \times 10^{-3}(16) = 6\,\text{V} \qquad 0 \le t \le 25\,\text{ms}$

$v = 375 \times 10^{-3}(-16) = -6\,\text{V} \qquad 25 \le t \le 50\,\text{ms}$

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 6\,\text{V} && 0 < t < 25\,\text{ms} \\ v &= -6\,\text{V} && 25 < t < 50\,\text{ms} \\ v &= 0 && 50\,\text{ms} < t \end{aligned}$$

$p = vi$

$$\begin{aligned} p &= 0 && t < 0 \\ p &= 96t\,\text{W} && 0 < t < 25\,\text{ms} \\ p &= 96t - 4.8\,\text{W} && 25 < t < 50\,\text{ms} \\ p &= 0 && 50\,\text{ms} < t \end{aligned}$$

$w = \dfrac{1}{2}Li^2$

$$\begin{aligned} w &= 0 && t < 0 \\ w &= 48t^2\,\text{J} && 0 < t < 25\,\text{ms} \\ w &= 48t^2 - 4.8t + 0.12\,\text{J} && 25 < t < 50\,\text{ms} \\ w &= 0 && 50\,\text{ms} < t \end{aligned}$$

P 6.2 [a] $0 \le t \le 1\,\text{ms}$:

$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{10^6}{300}\int_0^t 6 \times 10^{-3}\,dx + 0$$

$$= 20x \Big|_0^t = 20t\,\text{A}$$

$1\,\text{ms} \le t \le 2\,\text{ms}:$

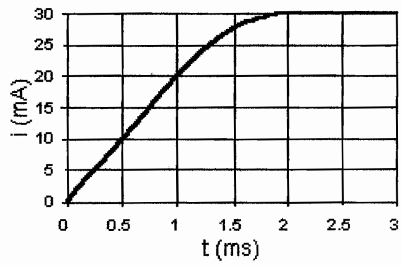
$$i = \frac{10^6}{300}\int_{10^{-3}}^{t}(12\times 10^{-3} - 6x)\,dx + 20\times 10^{-3}$$

$$\therefore\quad i = 40t - 10{,}000t^2 - 10\times 10^{-3}\,\text{A}$$

$2\,\text{ms} \le t \le \infty:$

$$i = \frac{10^6}{300}\int_{2\times 10^{-3}}^{t}(0)\,dx + 30\times 10^{-3} = 30\,\text{mA}$$

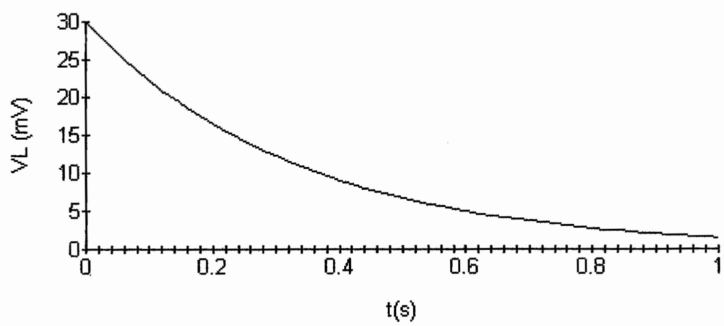
[b]

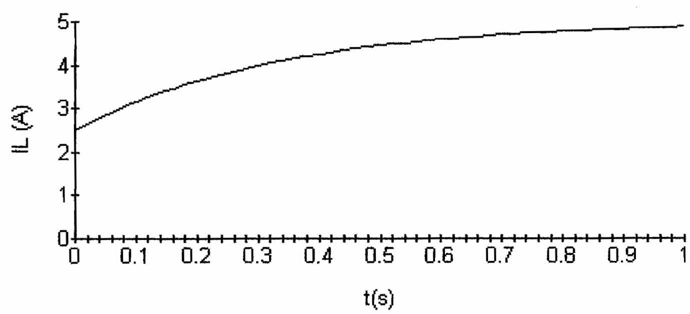


P 6.3 $\quad 0 \le t < \infty$

$$i_L = \frac{10^3}{4}\int_0^t 30\times 10^{-3}e^{-3x}\,dx + 2.5 = 7.5\frac{e^{-3x}}{-3}\Big|_0^t + 2.5$$

$$= 5 - 2.5e^{-3t}\,\text{A}, \qquad 0 \le t \le \infty$$





P 6.4 [a] $v = L\dfrac{di}{dt}$

$$\frac{di}{dt} = 20[e^{-5t} - 5te^{-5t}] = 20e^{-5t}(1 - 5t)$$

$$v = (100 \times 10^{-6})(20)e^{-5t}(1 - 5t)$$

$$= 2e^{-5t}(1 - 5t)\,\text{mV}, \quad t > 0$$

[b] $p = vi = 0.04te^{-10t}(1 - 5t)$

$$p(100\,\text{ms}) = 0.04(0.1)e^{-1}(1 - 0.5) = 735.76\,\mu\text{W}$$

[c] absorbing

[d] $i(100\,\text{ms}) = 20(0.1)e^{-0.5} = 2e^{-0.5}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(100 \times 10^{-6})(2e^{-0.5})^2 = 73.58\,\mu\text{J}$$

[e] The energy is a maximum where the current is a maximum:

$$\frac{di_L}{dt} = 0 \quad \text{when} \quad 1 - 5t = 0 \quad \text{or} \quad t = 0.2\,\text{s}$$

$$i_{\max} = 20(0.2)e^{-1} = 4e^{-1}\,\text{A}$$

$$w_{\max} = \frac{1}{2}(100 \times 10^{-6})(4e^{-1})^2 = 108.27\,\mu\text{J}$$

P 6.5 [a] $0 \le t \le 2\,\text{s}:$

$$v = -25t$$

$$i = \frac{1}{2.5}\int_0^t -25x\,dx + 0 = -10\frac{x^2}{2}\Big|_0^t$$

$$i = -5t^2\,\text{A}$$

$2\,\text{s} \le t \le 6\,\text{s}:$

$$v = -100 + 25t$$

$$i(2) = -20\,\text{A}$$

$$\begin{aligned} \therefore\ i &= \frac{1}{2.5}\int_2^t (25x - 100)\,dx - 20 \\ &= 10\int_2^t x\,dx - 40\int_2^t dx - 20 \\ &= 5(t^2 - 4) - 40(t - 2) - 20 \\ &= 5t^2 - 40t + 40\,\text{A} \end{aligned}$$

$6\,\text{s} \le t \le 10\,\text{s}:$

$v = 200 - 25t$

$i(6) = 5(36) - 240 + 40 = -20\,\text{A}$

$$\begin{aligned} i &= \frac{1}{2.5}\int_6^t (200 - 25x)\,dx - 20 \\ &= 80\int_6^t dx - 10\int_6^t x\,dx - 20 \\ &= 80(t-6) - 10(t^2 - 36)/2 - 20 = 80t - 5t^2 - 320\,\text{A} \end{aligned}$$

$10\,\text{s} \le t \le 12\,\text{s}:$

$v = 25t - 300$

$i(10) = 800 - 500 - 320 = -20\,\text{A}$

$$\begin{aligned} i &= \frac{1}{2.5}\int_{10}^t (25x - 300)\,dx - 20 \qquad t \ge 12\,\text{s}: \\ &= 10\int_{10}^t x\,dx - 120\int_{10}^t dx - 20 \\ &= 5(t^2 - 100) - 120(t - 10) - 20 \\ &= 5t^2 - 120t + 680\,\text{A} \end{aligned}$$

$v = 0$

$i(12) = 5(12)^2 - 120(12) + 680 = -40\,\text{A}$

$$\begin{aligned} i &= \frac{1}{2.5}\int_{12}^t 0\,dx - 40 \\ &= -40\,\text{A} \end{aligned}$$

[**b**] For $0 \le t \le 2\,\text{s}$, $\quad v = -25t\,\text{V}; \qquad i = -5t^2\,\text{A}$

$v = 0 \quad \text{when} \quad t = 0 \quad \text{so} \quad i = 0\,\text{A}$

For $2 \le t \le 6\,\text{s}$, $\quad v = -100 + 25t\,\text{V}; \qquad i = 5t^2 - 40t + 40\,\text{A}$

$v = 0 \quad \text{when} \quad t = 4\,\text{s} \quad \text{so} \quad i = 5(4)^2 - 40(4) + 40 = -40\,\text{A}$

For $6 \le t \le 10\,\text{s}$, $\quad v = 200 - 25t\,\text{V}; \qquad i = -5t^2 + 80t - 320\,\text{A}$

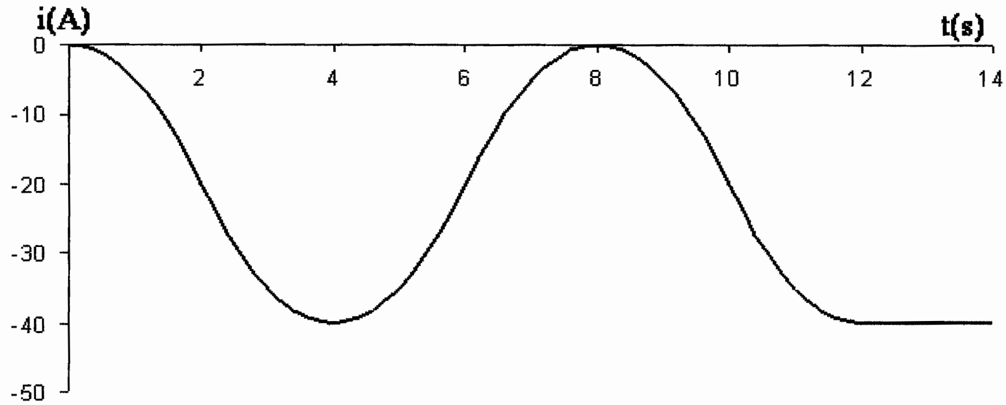
$v = 0 \quad \text{when} \quad t = 8\,\text{s} \quad \text{so} \quad i = -5(8)^2 + 80(8) - 320 = 0\,\text{A}$

For $10 \le t \le 12\,\text{s}$, $\quad v = 25t - 300\,\text{V}; \qquad i = 5t^2 - 120t + 680\,\text{A}$

$v = 0 \quad \text{when} \quad t = 12\,\text{s} \quad \text{so} \quad i = 5(12)^2 - 120(12) + 680 = -40\,\text{A}$

For $t \ge 12\,\text{s}$, $\quad v = 0; \qquad i = -40\,\text{A}$

[c]



P 6.6 [a] $v_L = L\frac{di}{dt} = [56\cos 140t + 92\sin 140t]e^{-20t}\,\text{mV}$

$$\therefore \quad \frac{dv_L}{dt} = [11{,}760\cos 140t - 9680\sin 140t]e^{-20t}\,\text{mV/s}$$

$$\frac{dv_L}{dt} = 0 \quad \text{when} \quad \tan 140t = \frac{11{,}760}{9680} = 1.21$$

$$\therefore \quad t = 6.30\,\text{ms}$$

Also $140t = 0.8821 + \pi$ etc.

Because of the decaying exponential v_L will be maximum the first time the derivative is zero.

[b] $v_L(\text{max}) = [56\cos 0.8821 + 92\sin 0.8821]e^{-0.12602} = 93.997\,\text{mV}$

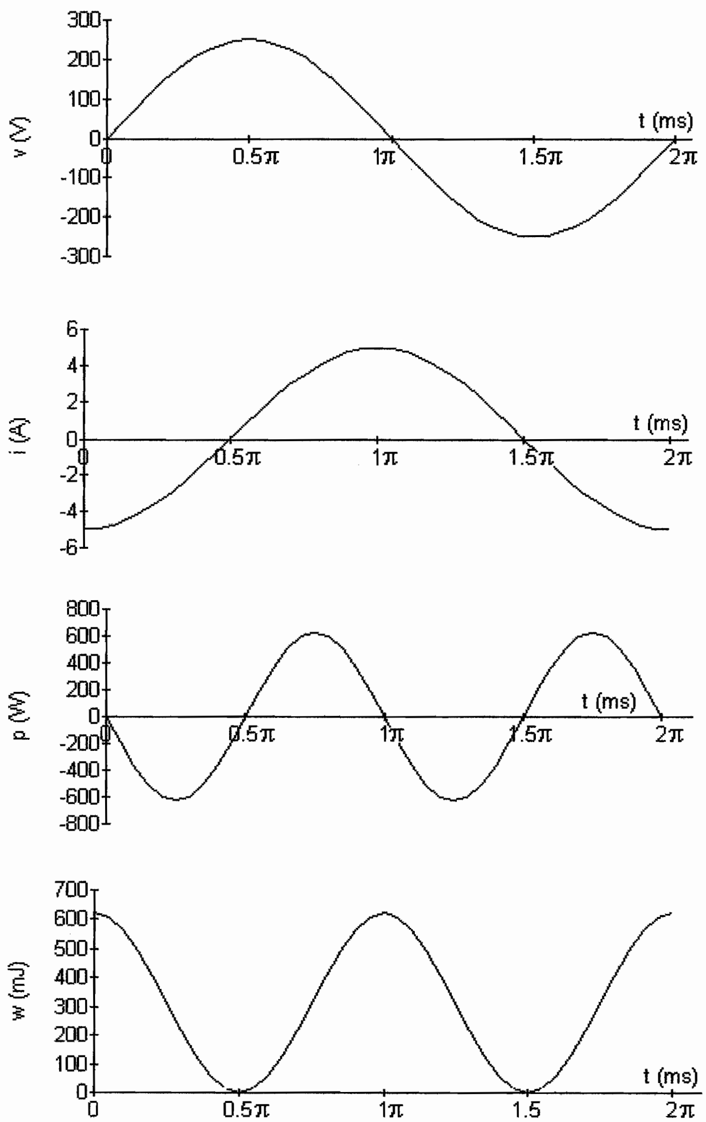
$v_L\,\text{max} \approx 94\,\text{mV}$

Note: When $t = \frac{0.8821 + \pi}{140}$; $\quad v_L = -60\,\text{mV}$

P 6.7 [a]

$$\begin{aligned} i &= \frac{1000}{50}\int_0^t 250\sin 1000x\,dx - 5 \\ &= 5000\int_0^t \sin 1000x\,dx - 5 \\ &= 5000\left[\frac{-\cos 1000x}{1000}\Bigg|_0^t\right] - 5 \\ &= 5(1 - \cos 1000t) - 5 \\ i &= -5\cos 1000t\,\text{A} \end{aligned}$$

[b]
$$p = vi = (250\sin 1000t)(-5\cos 1000t)$$
$$= -1250\sin 1000t\cos 1000t$$
$$p = -625\sin 2000t\ \mathrm{W}$$
$$w = \frac{1}{2}Li^2$$
$$= \frac{1}{2}(50\times 10^{-3})25\cos^2 1000t$$
$$= 625\cos^2 1000t\ \mathrm{mJ}$$
$$w = [312.5 + 312.5\cos 2000t]\ \mathrm{mJ}.$$



[c] Absorbing power: Delivering power:

$0.5\pi \leq t \leq \pi$ ms $\quad$ $0 \leq t \leq 0.5\pi$ ms

$1.5\pi \leq t \leq 2\pi$ ms $\quad$ $\pi \leq t \leq 1.5\pi$ ms

P 6.8 [a] $i(0) = A_1 + A_2 = 1$

$$\frac{di}{dt} = -2000A_1e^{-2000t} - 8000A_2e^{-8000t}$$

$$v = -30A_1e^{-2000t} - 120A_2e^{-8000t}\,\text{V}$$

$$v(0) = -30A_1 - 120A_2 = 60$$

Solving, $A_1 = 2$ and $A_2 = -1$

Thus,

$$i_1 = (2e^{-2000t} - e^{-8000t})\,\text{A} \qquad t \geq 0$$

$$v = -60e^{-2000t} + 120e^{-8000t}\,\text{V}, \qquad t \geq 0$$

[b] $p = vi = 300e^{-10{,}000t} - 120e^{-4000t} - 120e^{-16{,}000t}$

$p = 0$ when $300e^{6000t} - 120e^{12{,}000t} - 120 = 0$

Let $x = e^{6000t}$; then $300x - 120x^2 - 120 = 0$

Thus $x^2 - 2.5x + 1 = 0$ so $x = 0.5$ and $x = 2$

If $x = e^{6000t} = 0.5$, t will be negative. Hence, the solution for $t > 0$ must be $x = 2$:

$e^{6000t} = 2$ so $6000t = \ln 2$

Thus, $t = \dfrac{\ln 2}{6000} = 115.52\,\mu\text{s}$

P 6.9 [a] From Problem 6.8 we have

$$i = A_1e^{-2000t} + A_2e^{-8000t}\,\text{A}$$

$$v = -30A_1e^{-2000t} - 120A_2e^{-8000t}\,\text{V}$$

$$i(0) = A_1 + A_2 = 1$$

$$v(0) = -30A_1 - 120A_2 = -300$$

Solving, $A_1 = -2$; $A_2 = 3$

Thus,

$$i = -2e^{-2000t} + 3e^{-8000t}\,\text{A} \quad t \geq 0$$

$$v = 60e^{-2000t} - 360e^{-8000t}\,\text{V} \quad t \geq 0$$

[b] $i = 0$ when $3e^{-8000t} = 2e^{-2000t}$

$\therefore\ e^{6000t} = 1.5$ so $t = (\ln 1.5)/6000 = 67.58\,\mu\text{s}$

Thus,

$i > 0$ for $0 \leq t \leq 67.58\,\mu\text{s}$ and $i < 0$ for $67.58\,\mu\text{s} \leq t < \infty$

$v = 0$ when $60e^{-2000t} = 3600e^{-8000t}$

$\therefore\ t = (\ln 6)/6000 = 298.63\,\mu\text{s}$

Thus,

$v < 0$ for $0 \leq t \leq 298.63\,\mu\text{s}$ and $v > 0$ for $298.63\,\mu\text{s} \leq t < \infty$

Therefore,

$p < 0$ for $0 \leq t \leq 67.58\,\mu\text{s}$ and $298.63\,\mu\text{s} \leq t < \infty$

(inductor delivers energy)

$p > 0$ for $67.58\,\mu\text{s} \leq t \leq 298.63\,\mu\text{s}$ (inductor stores energy)

[c] $p = vi = 900e^{-10{,}000t} - 120e^{-4000t} - 1080e^{-16{,}000t}$ W

$$\therefore\ w_{\text{stored}} = \int_{t_2}^{t_1} p\,dx + w(0)$$

$$w_{\text{stored}} = 10^{-3}\left[-90e^{-10{,}000x}\Big|_{t_1}^{t_2} + 30e^{-4000x}\Big|_{t_1}^{t_2} + 67.5e^{-16{,}000x}\Big|_{t_1}^{t_2}\right] + 7.5\times 10^{-3}$$

$$= 30e^{-4000t_2} + 67.5e^{-16{,}000t_2} - 90e^{-10{,}000t_2} + 90e^{-10{,}000t_1} - 30e^{-4000t_1}$$

$$- 67.5e^{-16{,}000t_1} + 7.5\,\text{mJ}$$

where $t_1 = 67.58\,\mu\text{s}$ and $t_2 = 298.63\,\mu\text{s}$

$$\therefore\ w_{\text{stored}} = 5.11 + 7.5 = 12.61\,\text{mJ}$$

$$w_{\text{extracted}} = \int_0^{t_1} p\,dt + \int_{t_2}^{\infty} p\,dt$$

$$= \int_0^{t_1} [900e^{-10{,}000x} - 120e^{-4000x} - 1080e^{-16{,}000x}]\,dx$$

$$+ \int_{t_2}^{\infty} [900e^{-10{,}000x} - 120e^{-4000x} - 1080e^{-16{,}000x}]\,dx$$

$$= 10^{-3}\left(-90e^{-10{,}000x}\Big|_0^{t_1} + 30e^{-4000x}\Big|_0^{t_1} + 67.5e^{-16{,}000x}\Big|_0^{t_1}\right)$$

$$-10^{-3}\left(90e^{-10{,}000x}\Big|_{t_2}^{\infty} + 30e^{-4000x}\Big|_{t_2}^{\infty} + 67.5e^{-16{,}000x}\Big|_{t_2}^{\infty}\right)$$

$$= 90e^{-10{,}000t_2} - 30e^{-4000t_2} - 67.5e^{-16{,}000t_2} + 30e^{-4000t_1}$$

$$+67.5e^{-16{,}000t_1} - 90e^{-10{,}000t_1} - 7.5\,\text{mJ}$$

$$\text{where } t_1 = 67.58\,\mu\text{s} \quad \text{and} \quad t_2 = 298.63\,\mu\text{s}$$

$$\therefore \quad w_{\text{extracted}} = -12.61\,\text{mJ}$$

Thus, the energy stored equals the energy extracted.

P 6.10 $i = (B_1 \cos 5t + B_2 \sin 5t)e^{-t}$

$$i(0) = B_1 = 25\,\text{A}$$

$$\frac{di}{dt} = (B_1 \cos 5t + B_2 \sin 5t)(-e^{-t}) + e^{-t}(-5B_1 \sin 5t + 5B_2 \cos 5t)$$

$$= [(5B_2 - B_1)\cos 5t - (5B_1 + B_2)\sin 5t]e^{-t}$$

$$v = 2\frac{di}{dt} = [(10B_2 - 2B_1)\cos 5t - (10B_1 + 2B_2)\sin 5t]e^{-t}$$

$$v(0) = 100 = 10B_2 - 2B_1 = 10B_2 - 50 \quad \therefore \quad B_2 = 150/10 = 15\,\text{A}$$

Thus,

$$i = (25\cos 5t + 15\sin 5t)e^{-t}\,\text{A}, \qquad t \ge 0$$

$$v = (100\cos 5t - 280\sin 5t)e^{-t}\,\text{V}, \qquad t \ge 0$$

$$i(0.5) = -6.70\,\text{A}; \qquad v(0.5) = -150.23\,\text{V}$$

$$p(0.5) = (-6.70)(-150.23) = 1007.00\,\text{W absorbing}$$

P 6.11 For $0 \le t \le 1.2\,\text{s}$:

$$i_L = \frac{1}{20}\int_0^t 14 \times 10^{-3}\,dx + 0 = 0.7 \times 10^{-3}t$$

$$i_L(1.2\,\text{s}) = (0.7 \times 10^{-3})(1.2) = 0.84\,\text{mA}$$

$$R_m = (25)(1000) = 25\,\text{k}\Omega$$

$$v_m(1.2\,\text{s}) = (0.84 \times 10^{-3})(25 \times 10^3) = 21\,\text{V}$$

P 6.12 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

$$W = \int_0^{\infty} p\,dx = \int_0^{\infty} 40x[e^{-10x} - 10xe^{-20x} - e^{-20x}]\,dx = 0.2\,\text{J}$$

This is energy stored in the inductor at $t = \infty$.

P 6.13 [a]
$$\begin{aligned} v(20\,\mu\text{s}) &= 12.5 \times 10^9(20 \times 10^{-6})^2 = 5\,\text{V} \quad \text{(end of first interval)} \\ v(20\,\mu\text{s}) &= 10^6(20 \times 10^{-6}) - (12.5)(400) \times 10^{-3} - 10 \\ &= 5\,\text{V (start of second interval)} \\ v(40\,\mu\text{s}) &= 10^6(40 \times 10^{-6}) - (12.5)(1600) \times 10^{-3} - 10 \\ &= 10\,\text{V (end of second interval)} \end{aligned}$$

[b] $p(10\mu\text{s}) = 62.5 \times 10^{12}(10^{-5})^3 = 62.5\,\text{mW}$, $v(10\,\mu\text{s}) = 1.25\,\text{V}$,

$i(10\mu\text{s}) = 50\,\text{mA}$, $p(10\,\mu\text{s}) = vi = (1.25)(50\,\text{m}) = 62.5\,\text{mW}$ (checks)

$p(30\,\mu\text{s}) = 437.50\,\text{mW}$, $v(30\,\mu\text{s}) = 8.75\,\text{V}$, $i(30\,\mu\text{s}) = 0.05\,\text{A}$

$p(30\,\mu\text{s}) = vi = (8.75)(0.05) = 62.5\,\text{mW}$ (checks)

[c] $w(10\,\mu\text{s}) = 15.625 \times 10^{12}(10 \times 10^{-6})^4 = 0.15625\,\mu\text{J}$

$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625\,\mu\text{J}$

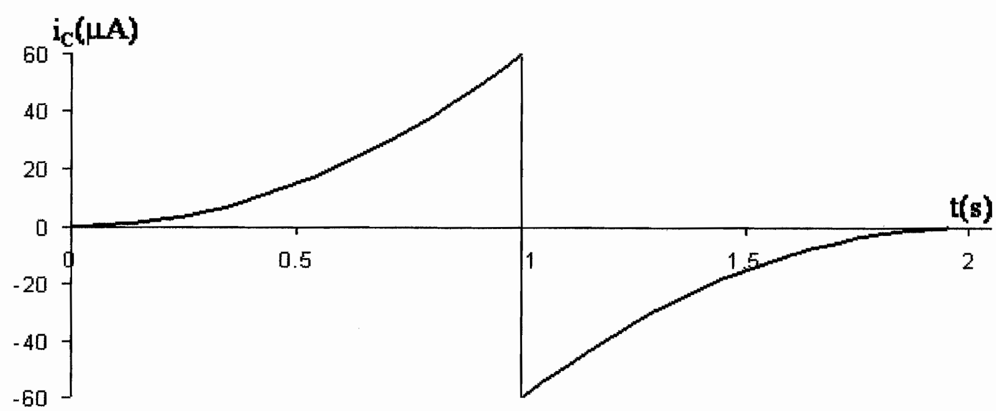
$w(30\,\mu\text{s}) = 7.65625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625\,\mu\text{J}$

P 6.14 $i_C = C(dv/dt)$

$0 < t < 1:$ $\quad i_C = 0.5 \times 10^{-6}(120)t^2 = 60t^2\,\mu\text{A}$

$1 < t < 2:$ $\quad i_C = 0.5 \times 10^{-6}(120)(2-t)^2(-1) = -60(2-t)^2\,\mu\text{A}$



P 6.15 **[a]** $0 \le t \le 100\,\mu$s

$$C = 0.2\,\mu\text{F} \qquad \frac{1}{C} = 5 \times 10^6$$

$$v = 5 \times 10^6 \int_0^t -0.04\,dx + 40$$

$$v = -200 \times 10^3 t + 40\,\text{V} \qquad 0 \le t \le 100\,\mu\text{s}$$

$$v(100\,\mu\text{s}) = -20 + 40 = 20\,\text{V}$$

[b] $100\,\mu\text{s} \le t \le 300\,\mu$s

$$v = 5 \times 10^6 \int_{100\times 10^{-6}}^t 0.08\,dx + 20 = 4 \times 10^5 t - 40 + 20$$

$$v = 4 \times 10^5 t - 20\text{V} \qquad 100 \le t \le 300\,\mu\text{s}$$

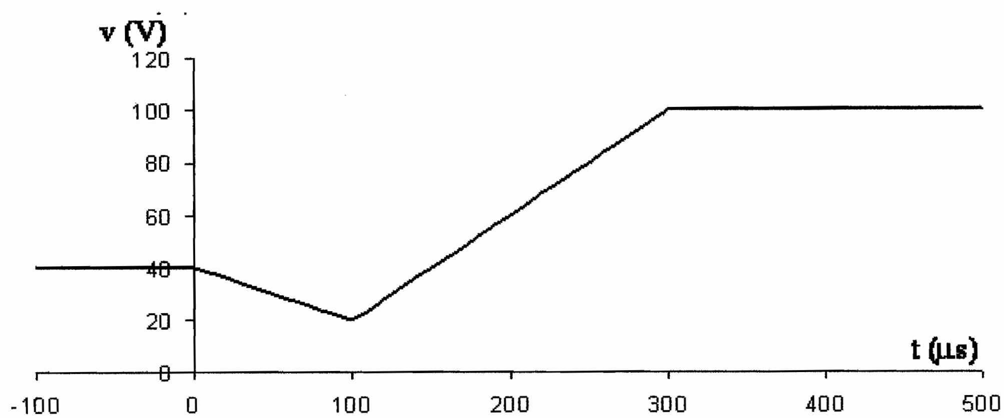
$$v(300\,\mu\text{s}) = 4 \times 10^5(300 \times 10^{-6}) - 20 = 100\,\text{V}$$

[c] $300\,\mu\text{s} \le t < \infty$

$$v = 5 \times 10^6 \int_{300\times 10^{-6}}^t 0\,dx + 100 = 100$$

$$v = 100\,\text{V}, \qquad 300\,\mu\text{s} \le t < \infty$$

[d]



P 6.16 **[a]** $i = C\dfrac{dv}{dt} = 0, \quad t < 0$

[b] $i = C\dfrac{dv}{dt} = 5e^{-1000t}[\cos 3000t + 13 \sin 3000t]\,\text{mA}, \quad t \ge 0$

[c] no, $v(0^-) = -30$ V
$v(0^+) = 10 - 40 = -30$ V

[d] yes, $i(0^-) = 0$ A
$i(0^+) = 5$ mA

[e] $v(\infty) = 10\,\text{V}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.5 \times 10^{-6})(10)^2 = 25\,\mu\text{J}$$

P 6.17 [a] $i = \dfrac{50 \times 10^{-3}}{10 \times 10^{-6}}t = 5 \times 10^3 t \qquad 0 \le t \le 10\,\mu\text{s}$

$i = 50 \times 10^{-3} \qquad 10 \le t \le 30\,\mu\text{s}$

$$\begin{aligned} q &= \int_0^{10\times10^{-6}} 5 \times 10^3 t\,dt + \int_{10\times10^{-6}}^{30\times10^{-6}} 50 \times 10^{-3}\,dt \\ &= 5 \times 10^3 \frac{t^2}{2}\bigg|_0^{10\times10^{-6}} + 50 \times 10^{-3}(20 \times 10^{-6}) \\ &= 5 \times 10^3(\tfrac{1}{2})(100 \times 10^{-12}) + 1000 \times 10^{-3} \times 10^{-6} \\ &= 1.25\,\mu\text{C} \end{aligned}$$

[b] $i = 200 \times 10^{-3} - 5 \times 10^{-3}t \qquad 30\,\mu\text{s} \le t \le 50\,\mu\text{s}$

$$\begin{aligned} q &= 1.25 \times 10^{-6} + \int_{30\times10^{-6}}^{50\times10^{-6}} [200 \times 10^{-3} - 5 \times 10^3 t]\,dt \\ &= 1.25 \times 10^{-6} + 200 \times 10^{-3}(20 \times 10^{-6}) - 5 \times 10^3 \frac{t^2}{2}\bigg|_{30\times10^{-6}}^{50\times10^{-6}} \\ &= 1.25 \times 10^{-6} + 4000 \times 10^{-9} - 5 \times 10^3\left[\frac{2500 - 900}{2}\right]10^{-12} \\ &= 1.25\,\mu\text{C} \end{aligned}$$

Since $q = vC$, $\quad \therefore \quad v = 1.25/0.25 = 5\,\text{V}.$

[c] $i = -300 \times 10^{-3} + 5 \times 10^{-3}t \qquad 50\,\mu\text{s} \le t \le 60\,\mu\text{s}$

$$\begin{aligned} q &= 1.25 \times 10^{-6} + \int_{50\times10^{-6}}^{60\times10^{-6}} [-300 \times 10^{-3} + 5 \times 10^3 t]\,dt \\ &= 1.25 \times 10^{-6} - 300 \times 10^{-3}(10 \times 10^{-6}) \\ &\quad + 5 \times 10^3\left[\frac{3600 - 2500}{2}\right]10^{-12} \\ &= 1\,\mu\text{C} \end{aligned}$$

$$v = \frac{1 \times 10^{-6}}{0.25 \times 10^{-6}} = 4\,\text{V}$$

$$w = \frac{C}{2}v^2 = \frac{1}{2}(0.25) \times 10^{-6}(16) = 2\,\mu\text{J}$$

P 6.18 [a]

$$v = 5\times 10^6\int_0^{250\times10^{-6}} 100\times10^{-3}e^{-1000t}\,dt - 60.6$$

$$= 500\times10^3\frac{e^{-1000t}}{-1000}\Big|_0^{250\times10^{-6}} - 60.6$$

$$= 500(1-e^{-0.25}) - 60.6 = 50\,\text{V}$$

$$w = \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.2)(10^{-6})(50)^2 = 250\,\mu\text{J}$$

[b] $v = 500 - 60.6 = 439.40\,\text{V}$

$$w = \frac{1}{2}(0.2)\times10^{-6}(439.40)^2 = 19.31\,\text{mJ} = 19{,}307.24\,\mu\text{J}$$

P 6.19 [a] $w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(0.40)\times10^{-6}(25)^2 = 125\,\mu\text{J}$

[b] $v = (A_1t + A_2)e^{-1500t}$

$v(0) = A_2 = 25\,\text{V}$

$$\frac{dv}{dt} = -1500e^{-1500t}(A_1t + A_2) + e^{-1500t}(A_1)$$

$$= (-1500A_1t - 1500A_2 + A_1)e^{-1500t}$$

$$\frac{dv}{dt}(0) = A_1 - 1500A_2$$

$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore\ \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{90\times10^{-3}}{0.40\times10^{-6}} = 225\times10^3$$

$$\therefore\ 225\times10^3 = A_1 - 1500(25)$$

Thus, $A_1 = 2.25\times10^5 + 3.75\times10^4 = 262{,}500\,\frac{\text{V}}{\text{s}}$

[c] $v = (262{,}500t + 25)e^{-1500t}$

$$i = C\frac{dv}{dt} = 0.40\times10^{-6}\frac{d}{dt}(262{,}500t + 25)e^{-1500t}$$

$$i = \frac{d}{dt}[(0.105t + 10\times10^{-6})e^{-1500t}]$$

$$= (0.105t + 10\times10^{-6})(-1500)e^{-1500t} + e^{-1500t}(0.105)$$

$$= (-157.5t - 15\times10^{-3} + 0.105)e^{-1500t}$$

$$= (0.09 - 157.5t)e^{-1500t}\,\text{A}, \qquad t\ge 0$$

$$= (90 - 157{,}500t)e^{-1500t}\,\text{mA}, \qquad t\ge 0$$

P 6.20 $10\|(15+25) = 8\,\text{H}$

$8\|12 = 4.8\,\text{H}$

$44\|(1.2+4.8) = 5.28\,\text{H}$

$21\|4 = 3.36\,\text{H}$

$5.28 + 3.36 = 8.64\,\text{H}$

P 6.21 $6\|14 = 4.2\,\text{H}$

$15.8 + 4.2 = 20\,\text{H}$

$20\|60 = 15\,\text{H}$

$15 + 5 = 20\,\text{H}$

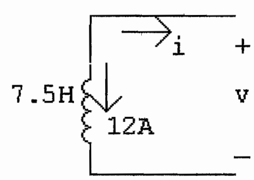
$20\|80 = 16\,\text{H}$

$16 + 24 = 40\,\text{H}$

$40\|10 = 8\,\text{H}$

$L_{\text{ab}} = 12 + 8 = 20\,\text{H}$

P 6.22 [a]



$$\begin{aligned} i(t) &= -\frac{1}{7.5}\int_0^t -1800e^{-20x}\,dx - 12 \\ &= 240\frac{e^{-20x}}{-20}\Big|_0^t -12 \\ &= -12(e^{-20t}-1) - 12 \\ i(t) &= -12e^{-20t}\,\text{A} \end{aligned}$$

[b] $$i_1(t) = -\frac{1}{10}\int_0^t -1800e^{-20x}\,dx + 4$$

$$= 180\frac{e^{-20x}}{-20}\Big|_0^t + 4$$

$$= -9(e^{-20t} - 1) + 4$$

$$i_1(t) = -9e^{-20t} + 13\,\text{A}$$

[c] $$i_2(t) = -\frac{1}{30}\int_0^t -1800e^{-20x}\,dx - 16$$

$$= 60\frac{e^{-20x}}{-20}\Big|_0^t - 16$$

$$= -3(e^{-20t} - 1) - 16$$

$$i_2(t) = -3e^{-20t} - 13\,\text{A}$$

[d] $p = vi = (-1800e^{-20t})(-12e^{-20t}) = 21{,}600e^{-40t}\,\text{W}$

$$w = \int_0^\infty p\,dt = \int_0^\infty 21{,}600e^{-40t}\,dt$$

$$= 21{,}600\frac{e^{-40t}}{-40}\Big|_0^\infty$$

$$= 540\,\text{J}$$

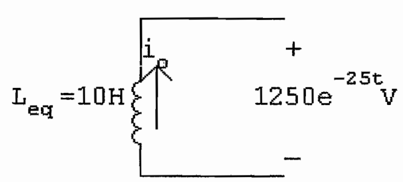
[e] $w = \frac{1}{2}(10)(16) + \frac{1}{2}(30)(256) = 3920\,\text{J}$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 3920 - 540 = 3380\,\text{J}$

[g] $w_{\text{trapped}} = \frac{1}{2}(10)(13)^2 + \frac{1}{2}(30)(13)^2 = 3380\,\text{J}$ checks

P 6.23 [a] $i_o(0) = i_1(0) + i_2(0) = 5\,\text{A}$

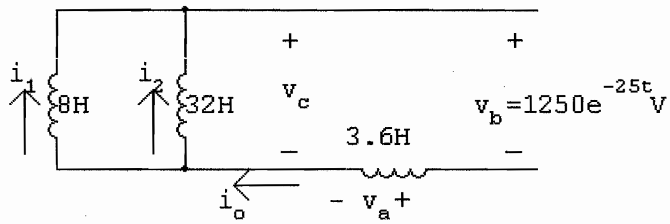
[b]



$$i_o = -\frac{1}{10}\int_0^t 1250e^{-25x}\,dx + 5 = -125\left[\frac{e^{-25x}}{-25}\right|_0^t + 5$$

$$= 5(e^{-25t} - 1) + 5 = 5e^{-25t}\,\text{A}, \qquad t \geq 0$$

[c]



$$v_a = 3.6\frac{d}{dt}(5e^{-25t}) = -450e^{-25t}\,\text{V}$$

$$\begin{aligned} v_c &= v_a + v_b = -450e^{-25t} + 1250e^{-25t} \\ &= 800e^{-25t}\,\text{V} \\ i_1 &= -\frac{1}{8}\int_0^t 800e^{-25x}\,dx + 10 \\ &= 4e^{-25t} - 4 + 10 \\ i_1 &= 4e^{-25t} + 6\,\text{A} \qquad t \geq 0 \end{aligned}$$

[d] $$\begin{aligned} i_2 &= -\frac{1}{32}\int_0^t 800e^{-25x}\,dx - 5 \\ &= e^{-25t} - 1 - 5 \\ i_2 &= e^{-25t} - 6\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[e] $w(0) = \frac{1}{2}(8)(100) + \frac{1}{2}(32)(25) + \frac{1}{2}(3.6)(25) = 845\,\text{J}$

[f] $w_{\text{del}} = \frac{1}{2}(10)(25) = 125\,\text{J}$

[g] $w_{\text{trapped}} = 845 - 125 = 720\,\text{J}$

P 6.24 $v_b = 1250e^{-25t}\,\text{V}$

$i_o = 5e^{-25t}\,\text{A}$

$p = 6250e^{-50t}\,\text{W}$

$$w = \int_0^t 6250e^{-50x}\,dx = 6250\frac{e^{-50x}}{-50}\Big|_0^t = 125(1 - e^{-50t})\,\text{W}$$

$w_{\text{total}} = 125\,\text{J}$

$80\%w_{\text{total}} = 100\,\text{J}$

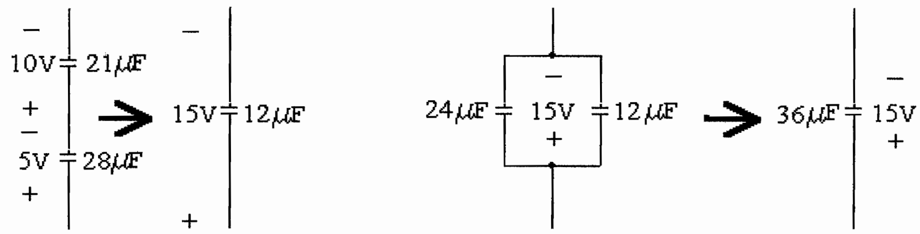
Thus,

$125 - 125e^{-50t} = 100; \qquad e^{50t} = 5; \qquad \therefore\ t = 32.19\,\text{ms}$

P 6.25 $\frac{1}{21} + \frac{1}{28} = \frac{7}{84} \quad \therefore \quad C_{eq} = 12\,\mu\text{F}$

$-10\,\text{V} - 5\,\text{V} = -15\,\text{V}$

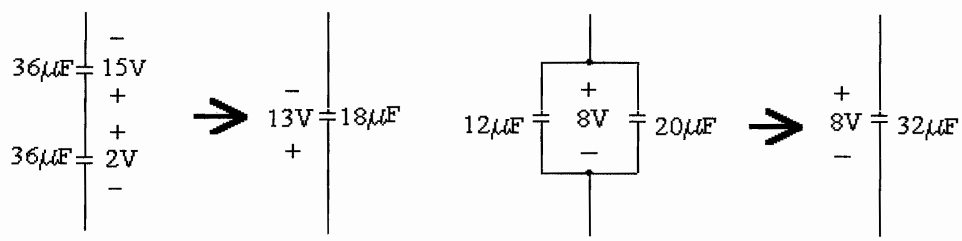
$24 + 12 = 36\,\mu\text{F}$



$\frac{1}{36} + \frac{1}{36} = \frac{2}{36} \quad \therefore \quad C_{eq} = 18\,\mu\text{F}$

$-15\,\text{V} + 2\,\text{V} = -13\,\text{V}$

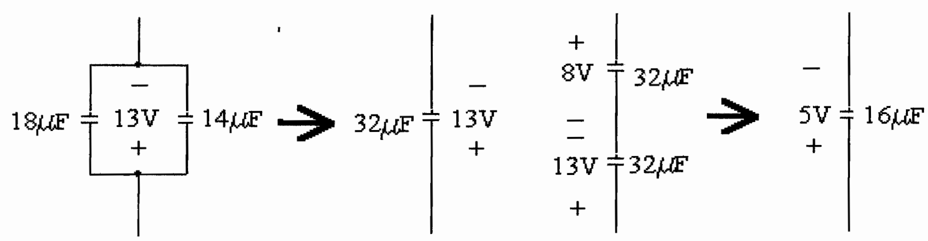
$12 + 20 = 32\,\mu\text{F}$



$18 + 14 = 32\,\mu\text{F}$

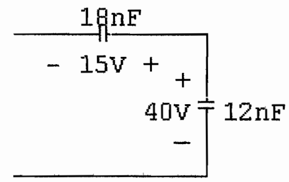
$\frac{1}{32} + \frac{1}{32} = \frac{2}{32} \quad \therefore \quad C_{eq} = 16\,\mu\text{F}$

$8\,\text{V} - 13\,\text{V} = -5\,\text{V}$



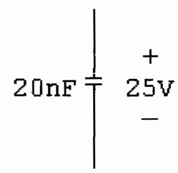
P 6.26 $\frac{1}{C_1} = \frac{1}{8} + \frac{1}{32} = \frac{5}{32}; \qquad C_1 = 6.4\,\text{nF}$

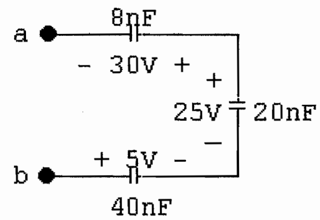
$C_2 = 5.6 + 6.4 = 12\,\text{nF}$



$\frac{1}{C_3} = \frac{1}{18} + \frac{1}{12} = \frac{10}{72}; \qquad C_3 = 7.2\,\text{nF}$

$C_4 = 12.8 + 7.2 = 20\,\text{nF}$

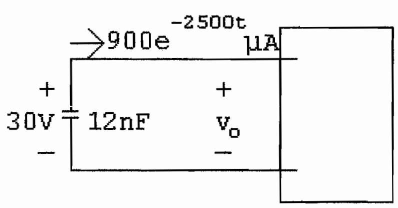




$\frac{1}{C_5} = \frac{1}{8} + \frac{1}{20} + \frac{1}{40} = \frac{1}{5}; \qquad C_5 = 5\,\text{nF}$

Equivalent capacitance is 5 nF with an initial voltage drop of −10 V.

P 6.27 [a]



$$v_o = -\frac{10^9}{12}\int_0^t 900 \times 10^{-6} e^{-2500x}\,dx + 30$$

$$= -75{,}000 \frac{e^{-2500x}}{-2500}\Big|_0^t + 30$$

$$= 30e^{-2500t}\,\text{V}, \qquad t \ge 0$$

[b] $$v_1 = -\frac{10^9}{20}(900 \times 10^{-6})\frac{e^{-2500x}}{-2500}\Big|_0^t + 45$$

$$= 18e^{-2500t} + 27\,\text{V}, \qquad t \ge 0$$

[c] $$v_2 = -\frac{10^9}{30}(900 \times 10^{-6})\frac{e^{-2500x}}{-2500}\Big|_0^t - 15$$

$$= 12e^{-2500t} - 27\,\text{V}, \qquad t \ge 0$$

[d] $p = vi = (30e^{-2500t})(900 \times 10^{-6})e^{-2500t}$

$= 27 \times 10^{-3}e^{-5000t}$

$$w = \int_0^\infty 27 \times 10^{-3}e^{-5000t}\,dt$$

$$= 27 \times 10^{-3}\frac{e^{-5000t}}{-5000}\Big|_0^\infty$$

$$= -5.4 \times 10^{-6}(0-1) = 5.4\,\mu\text{J}$$

[e] $w = \frac{1}{2}(20 \times 10^{-9})(45)^2 + \frac{1}{2}(30 \times 10^{-9})(15)^2$

$= 20.25 \times 10^{-6} + 3.375 \times 10^{-6}$

$= 23.625\,\mu\text{J}$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 23.625 - 5.4 = 18.225\,\mu\text{J}$

[g] $w_{\text{trapped}} = \frac{1}{2}(20 \times 10^{-9})(27)^2 + \frac{1}{2}(30 \times 10^{-9})(27)^2$

$= (10 + 15)(27)^2 \times 10^{-9}$

$= 18.225\,\mu\text{J}$

CHECK: $18.225 + 5.4 = 23.625\,\mu\text{J}$

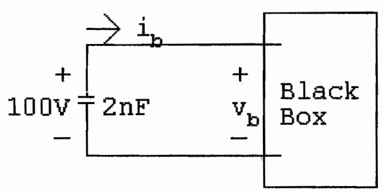
P 6.28 $C_1 = 1 + 1.5 = 2.5\,\text{nF}$

$$\frac{1}{C_2} = \frac{1}{2.5} + \frac{1}{12.5} + \frac{1}{50} = \frac{1}{2}$$

$\therefore\ C_2 = 2\,\text{nF}$

$v_d(0) + v_a(0) - v_c(0) = 40 + 15 + 45 = 100\,\text{V}$

[a]



$$v_b = -\frac{10^9}{2}\int_0^t 50 \times 10^{-6}e^{-250x}\,dx + 100$$

$$= -25{,}000\frac{e^{-250x}}{-250}\Big|_0^t + 100$$

$$= 100(e^{-250t} - 1) + 100$$

$$= 100e^{-250t}\,\text{V}$$

[b]
$$v_a = -\frac{10^9}{12.5}\int_0^t 50\times 10^{-6}e^{-250x}\,dx + 15$$
$$= -4000\frac{e^{-250x}}{-250}\Big|_0^t + 15$$
$$= 16(e^{-250t}-1)+15$$
$$= 16e^{-250t} - 1\,\text{V}$$

[c]
$$v_c = \frac{10^9}{50}\int_0^t 50\times 10^{-6}e^{-250x}\,dx - 45$$
$$= 1000\frac{e^{-250x}}{-250}\Big|_0^t - 45$$
$$= -4(e^{-250t}-1)-45$$
$$= -4e^{-250t} - 41\,\text{V}$$

[d]
$$v_d = -\frac{10^9}{2.5}\int_0^t 50\times 10^{-6}e^{-250x}\,dx + 40$$
$$= -20{,}000\frac{e^{-250x}}{-250}\Big|_0^t + 40$$
$$= 80(e^{-250t}-1)+40$$
$$= 80e^{-250t} - 40\,\text{V}$$

CHECK:
$$v_b = v_d + v_a - v_c$$
$$= 80e^{-250t} - 40 + 16e^{-250t} - 1 + 4e^{-250t} + 41$$
$$= 100e^{-250t}\,\text{V} \quad \text{(checks)}$$

[e]
$$i_1 = -10^{-9}\frac{d}{dt}[80e^{-250t} - 40]$$
$$= -10^{-9}(-20{,}000e^{-250t})$$
$$= 20e^{-250t}\,\mu\text{A}$$

[f]
$$i_2 = -1.5\times 10^{-9}\frac{d}{dt}[80e^{-250t} - 40]$$
$$= -1.5\times 10^{-9}(-20{,}000e^{-250t})$$
$$= 30e^{-250t}\,\mu\text{A}$$

CHECK: $i_1 + i_2 = 50e^{-250t}\,\mu\text{A} = i_b$

P 6.29 [a]
$$\begin{aligned} w(0) &= [\tfrac{1}{2}(2.5)(40)^2 + \tfrac{1}{2}(12.5)(15)^2 + \tfrac{1}{2}(50)(45)^2] \times 10^{-9} \\ &= 54{,}031.25\,\text{nJ} \end{aligned}$$

[b]
$$\begin{aligned} v_{\text{a}}(\infty) &= -1\,\text{V} \\ v_{\text{c}}(\infty) &= -41\,\text{V} \\ v_{\text{d}}(\infty) &= -40\,\text{V} \\ w(\infty) &= [\tfrac{1}{2}(2.5)(40)^2 + \tfrac{1}{2}(12.5)(1)^2 + \tfrac{1}{2}(50)(41)^2] \times 10^{-9} \\ &= 44{,}031.25\,\text{nJ} \end{aligned}$$

[c] $w = \int_0^{\infty} (100e^{-250t})(50e^{-250t}) \times 10^{-6}\,dt = 10{,}000\,\text{nJ}$

CHECK: $54{,}031.25 - 44{,}031.25 = 10{,}000$

[d] $\%\ \text{delivered} = \dfrac{10{,}000}{54{,}031.25} \times 100 = 18.51\%$

[e]
$$\begin{aligned} w &= 5 \times 10^{-3} \int_0^t e^{-500x}\,dx \\ &= 10^4(1 - e^{-500t})\,\text{nJ} \end{aligned}$$

$\therefore\ 10^4(1 - e^{-500t}) = 5000; \qquad e^{-500t} = 0.5$

Thus, $t = \dfrac{\ln 2}{500} = 1.39\,\text{ms}.$

P 6.30 From Figure 6.17(a) we have

$$v = \frac{1}{C_1}\int_0^t i + v_1(0) + \frac{1}{C_2}\int_0^t i\,dx + v_2(0) + \cdots$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]\int_0^t i\,dx + v_1(0) + v_2(0) + \cdots$$

Therefore $\dfrac{1}{C_{\text{eq}}} = \left[\dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots\right], \qquad v_{\text{eq}}(0) = v_1(0) + v_2(0) + \cdots$

P 6.31 From Fig. 6.18(a)

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots = [C_1 + C_2 + \cdots]\frac{dv}{dt}$$

Therefore $C_{\text{eq}} = C_1 + C_2 + \cdots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on C_{eq}.

P 6.32

$$\begin{aligned} v_2(t) &= 20 \times 10^{-3}\frac{di_o}{dt} \\ &= (20 \times 10^{-3})(50 \times 10^{-3})\{e^{-8000t}[-6000 \sin 6000t + 12{,}000 \cos 6000t] \\ &\quad +(-8000e^{-8000t})[\cos 6000t + 2 \sin 6000t]\} \\ &= e^{-8000t}\{4 \cos 6000t - 22 \sin 6000t\}\text{ V} \end{aligned}$$

$$\therefore\ v_2(0) = 4\text{ V}$$

$$i_0(0) = 50\text{ mA}$$

$$v_R(0) = 320(50 \times 10^{-3}) = 16\text{ V}$$

$$v_1(0) = 16 + 4 = 20\text{ V}$$

P 6.33

$$\begin{aligned} v_c &= \frac{-10^6}{20}\int_0^t e^{-80x} \sin 60x\, dx - 300 \\ &= 5e^{-80t}[80 \sin 60t + 60 \cos 60t] + 300 - 300 \\ &= 400e^{-80t} \sin 60t + 300e^{-80t} \cos 60t\ \text{ V} \\ v_L &= 5\frac{di_o}{dt} \\ &= 5[-80e^{-80t} \sin 60t + 60e^{-80t} \cos 60t] \\ &= -400e^{-80t} \sin 60t + 300e^{-80t} \cos 60t\ \text{ V} \\ v_o &= v_c - v_L \\ &= (300e^{-80t} \cos 60t - 300e^{-80t} \cos 60t + 400e^{-80t} \sin 60t+ \\ &\quad 400e^{-80t} \sin 60t) \\ &= 800e^{-80t} \sin 60t\ \text{ V} \end{aligned}$$

P 6.34 [a] $5\frac{di_g}{dt} + 40\frac{di_2}{dt} + 90i_2 = 0$

$$40\frac{di_2}{dt} + 90i_2 = -5\frac{di_g}{dt}$$

[b] $i_2 = e^{-t} - 5e^{-2.25t}$ A

$$\frac{di_2}{dt} = -e^{-t} + 11.25e^{-2.25t}\text{ A/s}$$

$$i_g = 10e^{-t} - 10\text{ A}$$

[c]
$$\begin{aligned} p_{\text{dev}} &= v_g i_g \\ &= 960 + 92{,}480e^{-4t} - 94{,}400e^{-5t} - 92{,}480e^{-9t} + \\ &\quad 93{,}440e^{-10t}\,\text{W} \end{aligned}$$

[d] $p_{\text{dev}}(\infty) = 960\,\text{W}$

[e] $i_1(\infty) = 4\,\text{A}; \quad i_2(\infty) = 1\,\text{A}; \quad i_g(\infty) = 16\,\text{A};$

$$p_{5\Omega} = (16-4)^2(5) = 720\,\text{W}$$

$$p_{20\Omega} = 3^2(20) = 180\,\text{W}$$

$$p_{60\Omega} = 1^2(60) = 60\,\text{W}$$

$$\sum p_{\text{abs}} = 720 + 180 + 60 = 960\,\text{W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{abs}} = 960\,\text{W}$$

P 6.37 [a] Rearrange by organizing the equations by di_1/dt, i_1, di_2/dt, i_2 and transfer the i_g terms to the right hand side of the equations. We get

$$4\frac{di_1}{dt} + 25i_1 - 8\frac{di_2}{dt} - 20i_2 = 5i_g - 8\frac{di_g}{dt}$$

$$-8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 80i_2 = 16\frac{di_g}{dt}$$

[b] From the given solutions we have

$$\frac{di_1}{dt} = -320e^{-5t} + 272e^{-4t}$$

$$\frac{di_2}{dt} = 260e^{-5t} - 204e^{-4t}$$

Thus,

$$4\frac{di_1}{dt} = -1280e^{-5t} + 1088e^{-4t}$$

$$25i_1 = 100 + 1600e^{-5t} - 1700e^{-4t}$$

$$8\frac{di_2}{dt} = 2080e^{-5t} - 1632e^{-4t}$$

$$20i_2 = 20 - 1040e^{-5t} + 1020e^{-4t}$$

$$5i_g = 80 - 80e^{-5t}$$

$$8\frac{di_g}{dt} = 640e^{-5t}$$

Thus,

$$-1280e^{-5t} + 1088e^{-4t} + 100 + 1600e^{-5t} - 1700e^{-4t} - 2080e^{-5t}$$

$$+1632e^{-4t} - 20 + 1040e^{-5t} - 1020e^{-4t} \stackrel{?}{=} 80 - 80e^{-5t} - 640e^{-5t}$$

$$80 + (1088 - 1700 + 1632 - 1020)e^{-4t}$$

$$+(1600 - 1280 - 2080 + 1040)e^{-5t} \stackrel{?}{=} 80 - 720e^{-5t}$$

$$80 + (2720 - 2720)e^{-4t} + (2640 - 3360)e^{-5t} = 80 - 720e^{-5t} \quad \text{(OK)}$$

$$8\frac{di_1}{dt} = -2560e^{-5t} + 2176e^{-4t}$$

$$20i_1 = 80 + 1280e^{-5t} - 1360e^{-4t}$$

$$16\frac{di_2}{dt} = 4160e^{-5t} - 3264e^{-4t}$$

$$80i_2 = 80 - 4160e^{-5t} + 4080e^{-4t}$$

$$16\frac{di_g}{dt} = 1280e^{-5t}$$

$$2560e^{-5t} - 2176e^{-4t} - 80 - 1280e^{-5t} + 1360e^{-4t} + 4160e^{-5t} - 3264e^{-4t}$$

$$+80 - 4160e^{-5t} + 4080e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$(-80 + 80) + (2560 - 1280 + 4160 - 4160)e^{-5t}$$

$$+(1360 - 2176 - 3264 + 4080)e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$0 + 1280e^{-5t} + 0e^{-4t} = 1280e^{-5t} \quad \text{(OK)}$$

P 6.38 [a] Dot terminal 2; with current entering terminal 2, the flux is right-to-left coil 1-2. Assign the current into terminal 4; the flux is left-to-right in coil 3-4. The flux is in the same direction, due to the topology of the core, so dot terminal 4. Hence, 2 and 4 or 1 and 3.

[b] Dot terminal 1; with current entering terminal 1 the flux is down in coil 1-2. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore the flux is in the same direction, due to the topology of the core, so dot terminal 4. Hence, 1 and 4 or 2 and 3.

[c] Dot terminal 1; with current entering terminal 1 the flux is up in coil 1-2. Assign the current into terminal 4; the flux is left-to-right in coil 3-4. Therefore the flux is in the same direction, due to the topology of the core, so dot terminal 4. Hence, 1 and 4 or 2 and 3.

[d] Dot terminal 2; with current entering terminal 2, the flux is down in coil 1-2. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, the flux is in the same direction, so dot terminal 4. Hence, 2 and 4 or 1 and 3.

P 6.39 When the switch is closed, the induced voltage in the coil connected to the source is negative at the dotted terminal. Since the dc voltmeter kicks up-scale, the induced voltage in the coil connected to the voltmeter is positive at the lower terminal. Therefore, dot the upper terminal of the coil connected to the voltmeter.

P 6.40 [a] $v_{ab} = L_1\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$

It follows that $L_{ab} = (L_1 + L_2 + 2M)$

[b] $v_{ab} = L_1\frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt} = (L_1 + L_2 - 2M)\frac{di}{dt}$

Therefore $L_{ab} = (L_1 + L_2 - 2M)$

P 6.41 [a] $v_{ab} = L_1\frac{d(i_1 - i_2)}{dt} + M\frac{di_2}{dt}$

$$0 = L_1\frac{d(i_2 - i_1)}{dt} - M\frac{di_2}{dt} + M\frac{d(i_1 - i_2)}{dt} + L_2\frac{di_2}{dt}$$

Collecting coefficients of $[di_1/dt]$ and $[di_2/dt]$, the two mesh-current equations become

$$v_{ab} = L_1\frac{di_1}{dt} + (M - L_1)\frac{di_2}{dt}$$

and

$$0 = (M - L_1)\frac{di_1}{dt} + (L_1 + L_2 - 2M)\frac{di_2}{dt}$$

Solving for $[di_1/dt]$ gives

$$\frac{di_1}{dt} = \frac{L_1 + L_2 - 2M}{L_1L_2 - M^2}v_{ab}$$

from which we have

$$v_{ab} = \left(\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right)\left(\frac{di_1}{dt}\right)$$

$$\therefore \quad L_{ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

[b] If the magnetic polarity of coil 2 is reversed, the sign of M reverses, therefore

$$L_{ab} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

P 6.42 [a] $L_2 = \left(\frac{M^2}{k^2 L_1}\right) = \frac{(0.1)^2}{(0.5)^2(0.250)} = 160\,\text{mH}$

$$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{250}{160}} = 1.25$$

[b] $\mathcal{P}_1 = \frac{L_1}{N_1^2} = \frac{0.250}{(1000)^2} = 0.25 \times 10^{-6}$ Wb/A

$$\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{0.16}{(800)^2} = 0.25 \times 10^{-6} \text{ Wb/A}$$

P 6.43 $\mathcal{P}_1 = \frac{L_1}{N_1^2} = \frac{400 \times 10^{-6}}{250^2} = 6.4$ nWb/A

$$\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{900 \times 10^{-6}}{500^2} = 3.6 \text{ nWb/A}; \quad M = k\sqrt{L_1 L_2} = 450\,\mu\text{H}$$

$$\mathcal{P}_{12} = \mathcal{P}_{21} = \frac{M}{N_1 N_2} = \frac{450 \times 10^{-6}}{(250)(500)} = 3.6 \text{ nWb/A}$$

$$\mathcal{P}_{11} = \mathcal{P}_1 - \mathcal{P}_{21} = 6.4 - 3.6 = 2.8 \text{ nWb/A}$$

P 6.44 [a] $k = \frac{M}{\sqrt{L_1 L_2}} = \frac{19.5}{\sqrt{676}} = 0.75$

[b] $M_{\text{max}} = \sqrt{676} = 26\,\text{mH}$

[c] $\frac{L_1}{L_2} = \frac{N_1^2 \mathcal{P}_1}{N_2^2 \mathcal{P}_2} = \left(\frac{N_1}{N_2}\right)^2$

$$\therefore \quad \left(\frac{N_1}{N_2}\right)^2 = \frac{52}{13} = 4$$

$$\frac{N_1}{N_2} = \sqrt{4} = 2$$

P 6.45 [a] $L_1 = N_1^2 \mathcal{P}_1; \qquad \mathcal{P}_1 = \frac{288 \times 10^{-3}}{10^6} = 288$ nWb/A

$$\frac{d\phi_{11}}{d\phi_{21}} = \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}} = 0.5; \qquad \mathcal{P}_{21} = 2\mathcal{P}_{11}$$

$$\therefore \quad 288 \times 10^{-9} = \mathcal{P}_{11} + \mathcal{P}_{21} = 3\mathcal{P}_{11}$$

$$\mathcal{P}_{11} = 96 \text{ nWb/A}; \qquad \mathcal{P}_{21} = 192 \text{ nWb/A}$$

$$M = k\sqrt{L_1 L_2} = (1/3)\sqrt{(0.288)(0.162)} = 72\,\text{mH}$$

$$N_2 = \frac{M}{N_1 \mathcal{P}_{21}} = \frac{72 \times 10^{-3}}{(1000)(192 \times 10^{-9})} = 375 \text{ turns}$$

[b] $\mathcal{P}_2 = \dfrac{L_2}{N_2^2} = \dfrac{162 \times 10^{-3}}{(375)^2} = 1152 \text{ nWb/A}$

[c] $\mathcal{P}_{11} = 96 \text{ nWb/A}$ [see part (a)]

[d] $\dfrac{\phi_{22}}{\phi_{12}} = \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2 - \mathcal{P}_{12}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2}{\mathcal{P}_{12}} - 1$

$$\mathcal{P}_{21} = \mathcal{P}_{21} = 192 \text{ nWb/A}; \qquad \mathcal{P}_2 = 1152 \text{ nWb/A}$$

$$\frac{\phi_{22}}{\phi_{12}} = \frac{1152}{192} - 1 = 5$$

P 6.46 [a] $\dfrac{1}{k^2} = \left(1 + \dfrac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right) = \left(1 + \dfrac{\mathcal{P}_{11}}{\mathcal{P}_{21}}\right)\left(1 + \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)$

Therefore

$$k^2 = \frac{\mathcal{P}_{12}\mathcal{P}_{21}}{(\mathcal{P}_{21} + \mathcal{P}_{11})(\mathcal{P}_{12} + \mathcal{P}_{22})}$$

Now note that

$$\phi_1 = \phi_{11} + \phi_{21} = \mathcal{P}_{11}N_1i_1 + \mathcal{P}_{21}N_1i_1 = N_1i_1(\mathcal{P}_{11} + \mathcal{P}_{21})$$

and similarly

$$\phi_2 = N_2i_2(\mathcal{P}_{22} + \mathcal{P}_{12})$$

It follows that

$$(\mathcal{P}_{11} + \mathcal{P}_{21}) = \frac{\phi_1}{N_1i_1}$$

and

$$(\mathcal{P}_{22} + \mathcal{P}_{12}) = \left(\frac{\phi_2}{N_2i_2}\right)$$

Therefore

$$k^2 = \frac{(\phi_{12}/N_2i_2)(\phi_{21}/N_1i_1)}{(\phi_1/N_1i_1)(\phi_2/N_2i_2)} = \frac{\phi_{12}\phi_{21}}{\phi_1\phi_2}$$

or

$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right)\left(\frac{\phi_{12}}{\phi_2}\right)}$$

[b] The fractions (ϕ_{21}/ϕ_1) and (ϕ_{12}/ϕ_2) are by definition less than 1.0, therefore $k < 1$.

P 6.47 [a] $W = (0.5)L_1i_1^2 + (0.5)L_2i_2^2 + Mi_1i_2$

$$M = 0.8\sqrt{(0.025)(0.1)} = 40 \,\text{mH}$$

$$W = (0.5)(0.025)(10)^2 + (0.5)(0.1)(15)^2 + (0.04)(10)(15) = 18.5 \,\text{J}$$

[b] $W = (0.5)(0.025)(-10)^2 + (0.5)(0.1)(-15)^2 + (0.04)(-10)(-15) = 18.5\,\text{J}$

[c] $W = (0.5)(0.025)(-10)^2 + (0.5)(0.1)(15)^2 + (0.04)(-10)(15) = 6.5\,\text{J}$

[d] $W = (0.5)(0.025)(10)^2 + (0.5)(0.1)(-15)^2 + (0.04)(10)(-15) = 6.5\,\text{J}$

P 6.48 [a] $M = 1.0\sqrt{(0.025)(0.1)} = 50\,\text{mH}, \qquad i_1 = 10\,\text{A}$

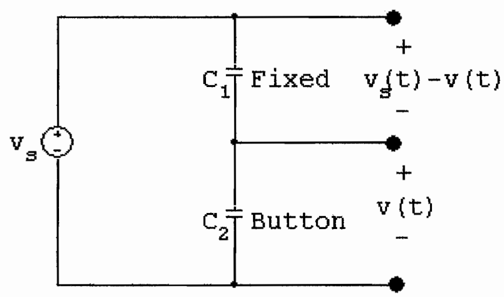
Therefore $50i_2^2 + 500i_2 + 1250 = 0, \qquad i_2^2 + 10i_2 + 25 = 0$

Therefore $i_2 = -\left(\frac{10}{2}\right) \pm \sqrt{\left(\frac{10}{2}\right)^2 - 25} = -5 \pm \sqrt{0}$

Therefore $i_2 = -5\,\text{A}$

[b] No, setting W equal to a negative value will make the quantity under the square root sign negative.

P 6.49 When the button is not pressed we have



$$C_2\frac{dv}{dt} = C_1\frac{d}{dt}(v_s - v)$$

or

$$(C_1 + C_2)\frac{dv}{dt} = C_1\frac{dv_s}{dt}$$

$$\frac{dv}{dt} = \frac{C_1}{(C_1 + C_2)}\frac{dv_s}{dt}$$

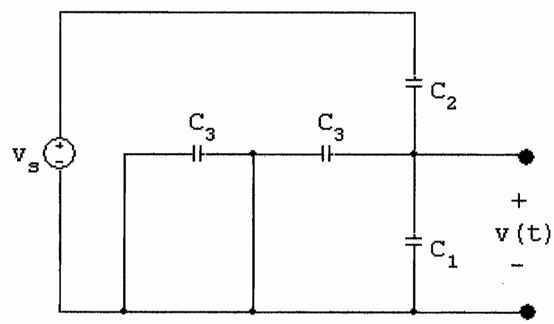
Assuming $C_1 = C_2 = C$

$$\frac{dv}{dt} = 0.5\frac{dv_s}{dt}$$

or

$$v = 0.5v_s(t) + v(0)$$

When the button is pressed we have



$$C_1\frac{dv}{dt} + C_3\frac{dv}{dt} + C_2\frac{d(v - v_s)}{dt} = 0$$

$$\therefore \quad \frac{dv}{dt} = \frac{C_2}{C_1 + C_2 + C_3}\frac{dv_s}{dt}$$

Assuming $C_1 = C_2 = C_3 = C$

$$\frac{dv}{dt} = \frac{1}{3}\frac{dv_s}{dt}$$

$$v = \frac{1}{3}v_s(t) + v(0)$$

Therefore interchanging the fixed capacitor and the button has no effect on the change in $v(t)$.

P 6.50 With no finger touching and equal 10 pF capacitors

$$v(t) = \frac{10}{20}(v_s(t)) + 0 = 0.5v_s(t)$$

With a finger touching

Let C_e = equivalent capacitance of person touching lamp

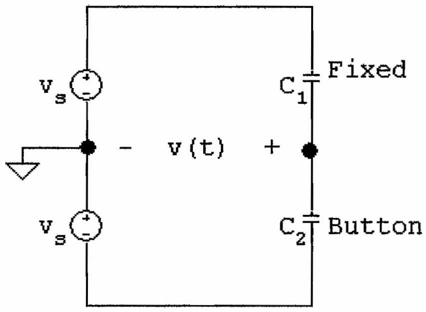
$$C_e = \frac{(10)(100)}{110} = 9.091 \text{ pF}$$

Then $\quad C + C_e = 10 + 9.091 = 19.091$ pF

$$\therefore \quad v(t) = \frac{10}{29.091}v_s = 0.344v_s$$

$$\therefore \quad \Delta v(t) = (0.5 - 0.344)v_s = 0.156v_s$$

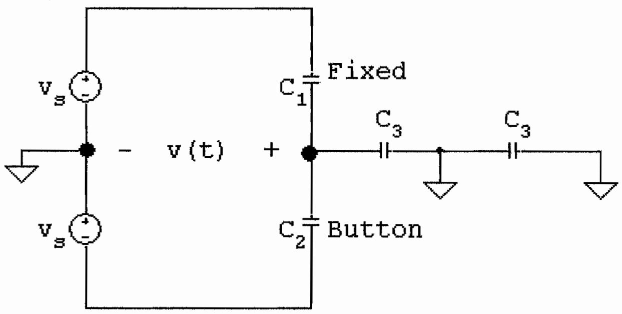
P 6.51 With no finger on the button the circuit is



$$C_1 \frac{d}{dt}(v - v_s) + C_2 \frac{d}{dt}(v + v_s) = 0$$

when $\quad C_1 = C_2 = C \qquad (2C)\frac{dv}{dt} = 0$

With a finger on the button



$$C_1 \frac{d(v - v_s)}{dt} + C_2 \frac{d(v + v_s)}{dt} + C_3 \frac{dv}{dt} = 0$$

$$(C_1 + C_2 + C_3)\frac{dv}{dt} + C_2 \frac{dv_s}{dt} - C_1 \frac{dv_s}{dt} = 0$$

when $\quad C_1 = C_2 = C_3 = C \qquad (3C)\frac{dv}{dt} = 0$

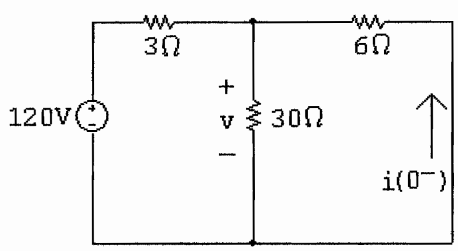
$\therefore$ there is no change in the output voltage of this circuit.

7

Response of First-Order *RL* and *RC* Circuits

Assessment Problems

AP 7.1 [a] The circuit for $t < 0$ is shown below. Note that the inductor behaves like a short circuit, effectively eliminating the $2\,\Omega$ resistor from the circuit.



First combine the $30\,\Omega$ and $6\,\Omega$ resistors in parallel:

$30\|6 = 5\,\Omega$

Use voltage division to find the voltage drop across the parallel resistors:

$$v = \frac{5}{5+3}(120) = 75\,\text{V}$$

Now find the current using Ohm's law:

$$i(0^-) = -\frac{v}{6} = -\frac{75}{6} = -12.5\,\text{A}$$

[b] $w(0) = \frac{1}{2}Li^2(0) = \frac{1}{2}(8 \times 10^{-3})(12.5)^2 = 625\,\text{mJ}$

[c] To find the time constant, we need to find the equivalent resistance seen by the inductor for $t > 0$. When the switch opens, only the $2\,\Omega$ resistor remains connected to the inductor. Thus,

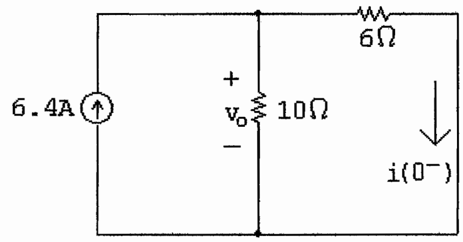
$$\tau = \frac{L}{R} = \frac{8 \times 10^{-3}}{2} = 4\,\text{ms}$$

[d] $i(t) = i(0^-)e^{t/\tau} = -12.5e^{-t/0.004} = -12.5e^{-250t}\,\text{A}, \qquad t \geq 0$

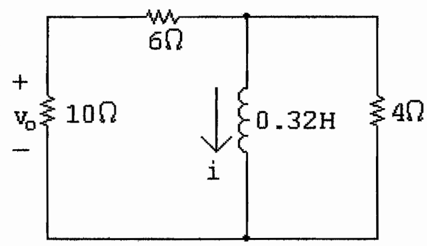
[e] $i(5\,\text{ms}) = -12.5e^{-250(0.005)} = -12.5e^{-1.25} = -3.58\,\text{A}$

So $w\,(5\,\text{ms}) = \frac{1}{2}Li^2(5\,\text{ms}) = \frac{1}{2}(8)\times 10^{-3}(3.58)^2 = 51.3\,\text{mJ}$
$w\,(\text{dis}) = 625 - 51.3 = 573.7\,\text{mJ}$
$\%\ \text{dissipated} = \left(\frac{573.7}{625}\right)100 = 91.8\%$

AP 7.2 [a] First, use the circuit for $t < 0$ to find the initial current in the inductor:



Using current division,
$i(0^-) = \frac{10}{10+6}(6.4) = 4\,\text{A}$
Now use the circuit for $t > 0$ to find the equivalent resistance seen by the inductor, and use this value to find the time constant:



$R_{eq} = 4\|(6+10) = 3.2\,\Omega, \quad \therefore \quad \tau = \frac{L}{R_{eq}} = \frac{0.32}{3.2} = 0.1\,\text{s}$
Use the initial inductor current and the time constant to find the current in the inductor:
$i(t) = i(0^-)e^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t}\,\text{A}, \quad t \geq 0$
Use current division to find the current in the $10\,\Omega$ resistor:

$$i_o(t) = \frac{4}{4+10+6}(-i) = \frac{4}{20}(-4e^{-10t}) = -0.8e^{-10t}\,\text{A}, \quad t \geq 0^+$$

Finally, use Ohm's law to find the voltage drop across the $10\,\Omega$ resistor:
$v_o(t) = 10i_o = 10(-0.8e^{-10t}) = -8e^{-10t}\,\text{V}, \quad t \geq 0^+$

[b] The initial energy stored in the inductor is

$$w(0) = \frac{1}{2}Li^2(0^-) = \frac{1}{2}(0.32)(4)^2 = 2.56\,\text{J}$$

Find the energy dissipated in the $4\,\Omega$ resistor by integrating the power over all time:

$$v_{4\Omega}(t) = L\frac{di}{dt} = 0.32(-10)(4e^{-10t}) = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

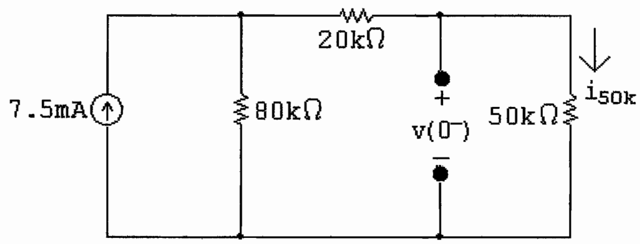
$$p_{4\Omega}(t) = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0^+$$

$$w_{4\Omega}(t) = \int_0^\infty 40.96e^{-20t}dt = 2.048\,\text{J}$$

Find the percentage of the initial energy in the inductor dissipated in the $4\,\Omega$ resistor:

$$\%\ \text{dissipated} = \left(\frac{2.048}{2.56}\right)100 = 80\%$$

AP 7.3 [a] The circuit for $t < 0$ is shown below. Note that the capacitor behaves like an open circuit.



Find the voltage drop across the open circuit by finding the voltage drop across the $50\,\text{k}\Omega$ resistor. First use current division to find the current through the $50\,\text{k}\Omega$ resistor:

$$i_{50\text{k}} = \frac{80 \times 10^3}{80 \times 10^3 + 20 \times 10^3 + 50 \times 10^3}(7.5 \times 10^{-3}) = 4\,\text{mA}$$

Use Ohm's law to find the voltage drop:
$v(0^-) = (50 \times 10^3)i_{50\text{k}} = (50 \times 10^3)(0.004) = 200\,\text{V}$

[b] To find the time constant, we need to find the equivalent resistance seen by the capacitor for $t > 0$. When the switch opens, only the $50\,\text{k}\Omega$ resistor remains connected to the capacitor. Thus,
$\tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20\,\text{ms}$

[c] $v(t) = v(0^-)e^{-t/\tau} = 200e^{-t/0.02} = 200e^{-50t}\,\text{V}, \quad t \geq 0$

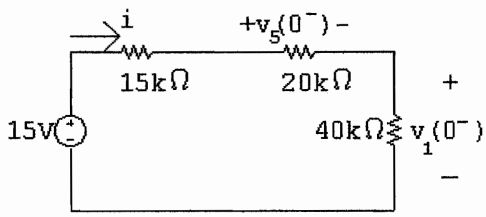
[d] $w(0) = \frac{1}{2}Cv^2 = \frac{1}{2}(0.4 \times 10^{-6})(200)^2 = 8\,\text{mJ}$

[e] $w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}(0.4 \times 10^{-6})(200e^{-50t})^2 = 8e^{-100t}\,\text{mJ}$

The initial energy is 8 mJ, so when 75% is dissipated, 2 mJ remains:

$$8 \times 10^{-3}e^{-100t} = 2 \times 10^{-3}, \qquad e^{100t} = 4, \qquad t = (\ln 4)/100 = 13.86\,\text{ms}$$

AP 7.4 [a] This circuit is actually two RC circuits in series, and the requested voltage, v_o, is the sum of the voltage drops for the two RC circuits. The circuit for $t < 0$ is shown below:



Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:

$$i = \frac{15}{75{,}000} = 0.2\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$$

There are two time constants in the circuit, one for each RC subcircuit. τ_5 is the time constant for the $5\,\mu$F – $20\,$kΩ subcircuit, and τ_1 is the time constant for the $1\,\mu$F – $40\,$kΩ subcircuit:

$$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}; \qquad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}$$

Therefore,

$$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t}\,\text{V}, \quad t \geq 0$$

$$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t}\,\text{V}, \quad t \geq 0$$

Finally,

$$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$$

[b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:

$$v_1(60\,\text{ms}) = 8e^{-25(0.06)} \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) = 4e^{-10(0.06)} \cong 2.20\,\text{V}$$

$$w_1(60\,\text{ms}) = \tfrac{1}{2}Cv_1^2(60\,\text{ms}) = \tfrac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59\,\mu\text{J}$$

$$w_5(60\,\text{ms}) = \tfrac{1}{2}Cv_5^2(60\,\text{ms}) = \tfrac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05\,\mu\text{J}$$

$$w(60\,\text{ms}) = 1.59 + 12.05 = 13.64\,\mu\text{J}$$

Find the initial energy from the initial voltage:

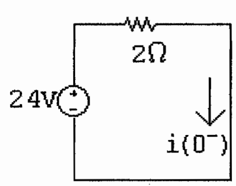
$$w(0) = w_1(0) + w_2(0) = \tfrac{1}{2}(1 \times 10^{-6})(8)^2 + \tfrac{1}{2}(5 \times 10^{-6})(4)^2 = 72\,\mu\text{J}$$

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:

$$w_{\text{diss}} = w(0) - w(60\,\text{ms}) = 72 - 13.64 = 58.36\,\mu\text{J}$$

$$\%\ \text{dissipated} = (58.36 \times 10^{-6}/72 \times 10^{-6})(100) = 81.05\,\%$$

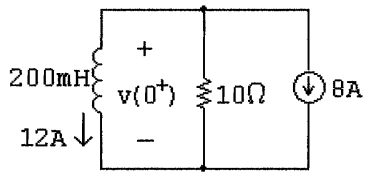
AP 7.5 [a] Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:



$i(0^-) = 24/2 = 12\,\text{A} = i(0^+)$

Note that $i(0^-) = i(0^+)$ because the current in an inductor is continuous.

[b] Use the circuit at $t = 0^+$, shown below, to calculate the voltage drop across the inductor at 0^+. Note that this is the same as the voltage drop across the $10\,\Omega$ resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.

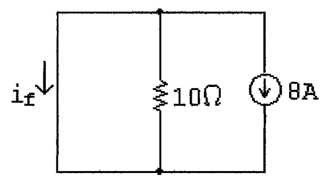


$v(0^+) = -10(8 + 12) = -200\,\text{V}$

[c] To calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the $10\,\Omega$ resistor is connected to the inductor for $t > 0$. Thus,

$\tau = L/R = (200 \times 10^{-3}/10) = 20\,\text{ms}$

[d] To find $i(t)$, we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:



Note that the inductor behaves as a short circuit and all of the current from the 8 A source flows through the short circuit. Thus,

$i_f = -8\,\text{A}$

Now,

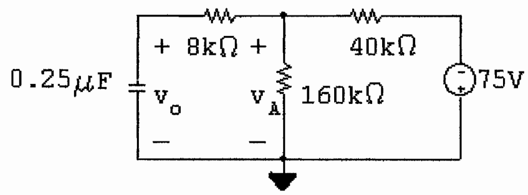
$i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02}$

$\quad = -8 + 20e^{-50t}\,\text{A}, \quad t \geq 0$

[e] To find $v(t)$, use the relationship between voltage and current for an inductor:

$$v(t) = L\frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$$

AP 7.6 [a]



From Example 7.6,

$$v_o(t) = -60 + 90e^{-100t}\,\text{V}$$

Write a KVL equation at the top node and use it to find the relationship between v_o and v_A:

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160{,}000} + \frac{v_A + 75}{40{,}000} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

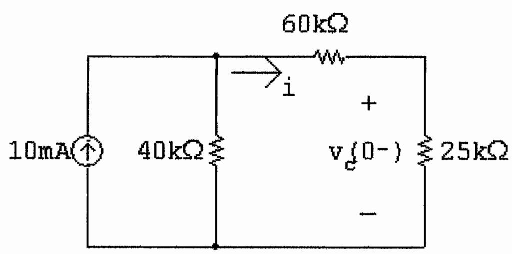
$$v_A = 0.8v_o - 12$$

Use the above equation for v_A in terms of v_o to find the expression for v_A:

$$v_A(t) = 0.8(-60 + 90e^{-100t}) - 12 = -60 + 72e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

[b] $t \geq 0^+$, since there is no requirement that the voltage be continuous in a resistor.

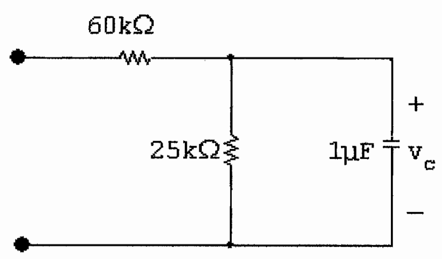
AP 7.7 [a] Use the circuit shown below, for $t < 0$, to calculate the initial voltage drop across the capacitor:



$$i = \left(\frac{40 \times 10^3}{125 \times 10^3}\right)(10 \times 10^{-3}) = 3.2\,\text{mA}$$

$$v_c(0^-) = (3.2 \times 10^{-3})(25 \times 10^3) = 80\,\text{V} \quad \text{so} \quad v_c(0^+) = 80\,\text{V}$$

Now use the next circuit, valid for $0 \leq t \leq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:

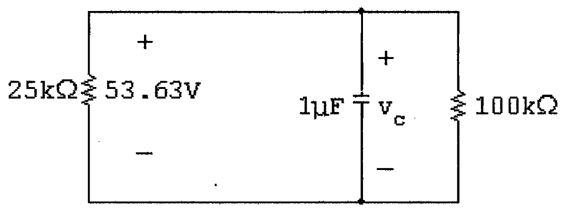


For $0 \le t \le 100\,\text{ms}$:

$$\tau = RC = (25 \times 10^3)(1 \times 10^{-6}) = 25\,\text{ms}$$

$$v_c(t) = v_c(0^-)e^{t/\tau} = 80e^{-40t}\,\text{V} \quad 0 \le t \le 10\,\text{ms}$$

[b] Calculate the starting capacitor voltage in the interval $t \ge 10\,\text{ms}$, using the capacitor voltage from the previous interval:
$v_c(0.01) = 80e^{-40(0.01)} = 53.63\,\text{V}$
Now use the next circuit, valid for $t \ge 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:



For $t \ge 10\,\text{ms}$:

$$R_{\text{eq}} = 25\,\text{k}\Omega \| 100\,\text{k}\Omega = 20\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (20 \times 10^3)(1 \times 10^{-6}) = 0.02\,\text{s}$$

Therefore $v_c(t) = v_c(0.01^+)e^{-(t-0.01)/\tau} = 53.63e^{-50(t-0.01)}\,\text{V}, \qquad t \ge 0.01\,\text{s}$

[c] To calculate the energy dissipated in the 25 kΩ resistor, integrate the power absorbed by the resistor over all time. Use the expression $p = v^2/R$ to calculate the power absorbed by the resistor.

$$w_{25\text{k}} = \int_0^{0.01} \frac{[80e^{-40t}]^2}{25{,}000}dt + \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{25{,}000}dt = 2.91\,\text{mJ}$$

[d] Repeat the process in part (c), but recognize that the voltage across this resistor is non-zero only for the second interval:

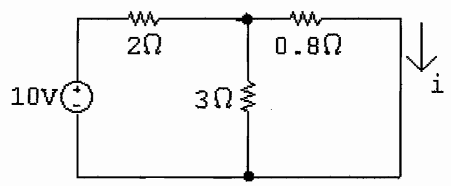
$$w_{100\,\text{k}\Omega} = \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{100{,}000}dt = 0.29\,\text{mJ}$$

We can check our answers by calculating the initial energy stored in the capacitor. All of this energy must eventually be dissipated by the 25 kΩ resistor and the 100 kΩ resistor.

Check: $w_{\text{stored}} = (1/2)(1 \times 10^{-6})(80)^2 = 3.2\,\text{mJ}$

$$w_{\text{diss}} = 2.91 + 0.29 = 3.2\,\text{mJ}$$

AP 7.8 [a] Prior to switch a closing at $t = 0$, there are no sources connected to the inductor; thus, $i(0^-) = 0$.
At the instant A is closed, $i(0^+) = 0$.
For $0 \le t \le 1\,\text{s}$,



The equivalent resistance seen by the 10 V source is $2 + (3\|0.8)$. The current leaving the 10 V source is

$$\frac{10}{2 + (3\|0.8)} = 3.8\,\text{A}$$

The final current in the inductor, which is equal to the current in the $0.8\,\Omega$ resistor is

$$I_{\text{F}} = \frac{3}{3 + 0.8}(3.8) = 3\,\text{A}$$

The resistance seen by the inductor is calculated to find the time constant:

$$[(2\|3) + 0.8]\|3\|6 = 1\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{1} = 2\,\text{s}$$

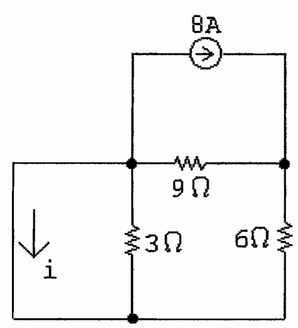
Therefore,

$$i = i_{\text{F}} + [i(0^+) - i_{\text{F}}]e^{-t/\tau} = 3 - 3e^{-0.5t}\,\text{A}, \quad 0 \le t \le 1\,\text{s}$$

For part (b) we need the value of $i(t)$ at $t = 1$ s:

$$i(1) = 3 - 3e^{-0.5} = 1.18\,\text{A}$$

[b] For $t > 1$ s



Use current division to find the final value of the current:

$$i = \frac{9}{9 + 6}(-8) = -4.8\,\text{A}$$

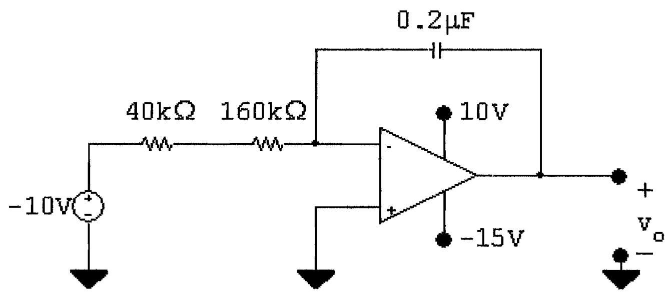
The equivalent resistance seen by the inductor is used to calculate the time constant:

$$3\|(9+6) = 2.5\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2.5} = 0.8\,\text{s}$$

Therefore,

$$i = i_F + [i(1^+) - i_F]e^{-(t-1)/\tau}$$
$$= -4.8 + 5.98e^{-1.25(t-1)}\,\text{A}, \quad t \geq 1\,\text{s}$$

AP 7.9 $0 \leq t \leq 32$ ms:

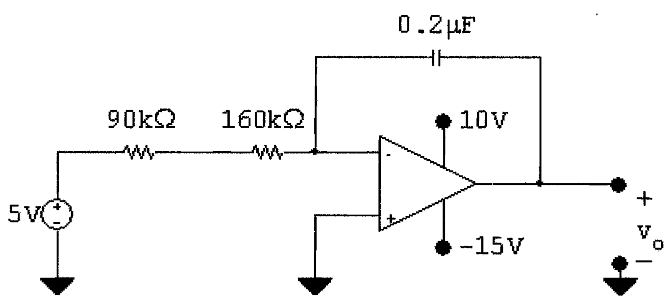


$$v_o = -\frac{1}{RC_f}\int_0^{32\times10^{-3}} -10\,dt + 0 = -\frac{1}{RC_f}(-10t)\Big|_0^{32\times10^{-3}} = -\frac{1}{RC_f}(-320\times10^{-3})$$

$$RC_f = (200\times10^3)(0.2\times10^{-6}) = 40\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 25$$

$$v_o = -25(-320\times10^{-3}) = 8\,\text{V}$$

$t \geq 32$ ms:



$$v_o = -\frac{1}{RC_f}\int_{32\times10^{-3}}^{t} 5\,dy + 8 = -\frac{1}{RC_f}(5y)\Big|_{32\times10^{-3}}^{t} + 8 = -\frac{1}{RC_f}5(t-32\times10^{-3}) + 8$$

$$RC_f = (250\times10^3)(0.2\times10^{-6}) = 50\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 20$$

$$v_o = -20(5)(t - 32 \times 10^{-3}) + 8 = -100t + 11.2$$

The output will saturate at the negative power supply value:

$$-15 = -100t + 11.2 \quad \therefore \quad t = 262\,\text{ms}$$

AP 7.10 [a] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (0 + 2)e^{-t/\tau}$$

$$\tau = (160 \times 10^3)(10 \times 10^{-9}) = 10^{-3}; \qquad 1/\tau = 625$$

$$v_p = -2 + 2e^{-625t}\,\text{V}; \qquad v_n = v_p$$

Write a KVL equation at the inverting input, and use it to determine v_o:

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0$$

$$\therefore \quad v_o = 5v_n = 5v_p = -10 + 10e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 10e^{-625t} = -5; \quad e^{-625t} = 1/2; \quad t = \ln 2/625 = 1.11\,\text{ms}$$

[b] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (1 + 2)e^{-625t} = -2 + 3e^{-625t}\,\text{V}$$

The analysis for v_o is the same as in part (a):

$$v_o = 5v_p = -10 + 15e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 15e^{-625t} = -5; \qquad e^{-625t} = 1/3; \quad t = \ln 3/625 = 1.76\,\text{ms}$$

Problems

P 7.1 [a] $i(0) = 125/25 = 5\,\text{A}$

[b] $\tau = \dfrac{L}{R} = \dfrac{4}{100} = 40\,\text{ms}$

[c] $i = 5e^{-25t}\,\text{A}, \qquad t \geq 0$

$$v_1 = -80i = -400e^{-25t}\,\text{V} \qquad t \geq 0$$

$$v_2 = L\frac{di_1}{dt} = 4(-125e^{-25t}) = -500e^{-25t}\,\text{V} \qquad t \geq 0^+$$

[d] $p_{\text{diss}} = i^2(20) = 25e^{-50t}(20) = 500e^{-50t}\,\text{W}$

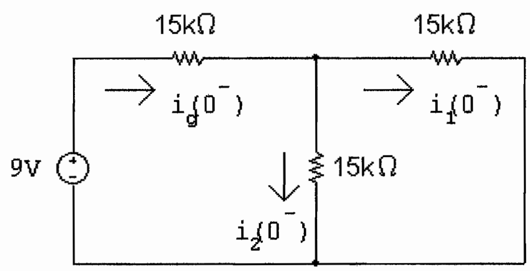
$$w_{\text{diss}} = \int_0^t 500e^{-50x}\,dx = 500\frac{e^{-50x}}{-50}\Big|_0^t = 10 - 10e^{-50t}\,\text{J}$$

$$w_{\text{diss}}(12\,\text{ms}) = 10 - 10e^{-0.6} = 4.51\,\text{J}$$

$$w(0) = \frac{1}{2}(4)(25) = 50\,\text{J}$$

$$\%\ \text{dissipated} = \frac{4.51}{50}(100) = 9.02\%$$

P 7.2 [a] $t < 0$



$$15\,\text{k}\Omega \| 15\,\text{k}\Omega = 7.5k\Omega$$

$$i_g(0^-) = \frac{9}{(15 + 7.5) \times 10^3} = 0.4\,\text{mA}$$

$$i_1(0^-) = i_2(0^-) = (0.4) \times 10^{-3}\frac{(15)}{(30)} = 0.2\,\text{mA}$$

[b] $i_1(0^+) = i_1(0^-) = 0.2\,\text{mA}$

$i_2(0^+) = -i_1(0^+) = -0.2\,\text{mA}$ (when switch is open)

[c] $\tau = \dfrac{L}{R} = \dfrac{30 \times 10^{-3}}{30 \times 10^3} = 10^{-6}; \qquad \dfrac{1}{\tau} = 10^6$

$$i_1(t) = i_1(0^+)e^{-t/\tau}$$

$$i_1(t) = 0.2e^{-10^6 t}\,\text{mA}, \qquad t \geq 0$$

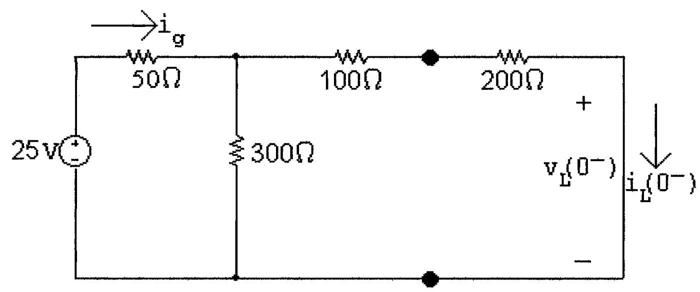
[d] $i_2(t) = -i_1(t)$ when $t \geq 0^+$

$$\therefore \quad i_2(t) = -0.2e^{-10^6 t}\,\text{mA}, \qquad t \geq 0^+$$

[e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal 0.2 mA and $i_2(0^+) = -0.2$ mA.

P 7.3 [a] $i_o(0^-) = 0$ since the switch is open for $t < 0$.

[b] For $t = 0^-$ the circuit is:

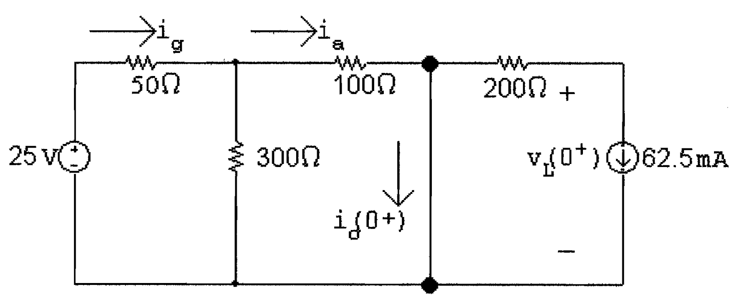


$$300\,\Omega \| 300\,\Omega = 150\,\Omega$$

$$\therefore \quad i_g = \frac{25}{50 + 150} = 125\,\text{mA}$$

$$i_L(0^-) = \left(\frac{300}{600}\right) i_g = 62.5\,\text{mA}$$

[c] For $t = 0^+$ the circuit is:



$$300\,\Omega \| 100\,\Omega = 75\,\Omega$$

$$\therefore \quad i_g = \frac{25}{50 + 75} = 200\,\text{mA}$$

$$i_a = \left(\frac{300}{400}\right) 200 = 150\,\text{mA}$$

$$\therefore \quad i_o(0^+) = 150 - 62.5 = 87.5\,\text{mA}$$

[d] $i_L(0^+) = i_L(0^-) = 62.5\,\text{mA}$

[e] $i_o(\infty) = i_a = 150\,\text{mA}$

[f] $i_L(\infty) = 0$, since the switch short circuits the branch containing the $200\,\Omega$ resistor and the 50 mH inductor.

[g] $\tau = \dfrac{L}{R} = \dfrac{50 \times 10^{-3}}{200} = 0.25\,\text{ms}; \qquad \dfrac{1}{\tau} = 4000$

$$\therefore \quad i_L = 0 + (62.5 - 0)e^{-4000t} = 62.5e^{-4000t}\,\text{mA}, \qquad t \geq 0$$

[h] $v_L(0^-) = 0$ since for $t < 0$ the current in the inductor is constant

[i] Refer to the circuit at $t = 0^+$ and note:

$$200(0.0625) + v_L(0^+) = 0; \qquad \therefore \quad v_L(0^+) = -12.5\,\text{V}$$

[j] $v_L(\infty) = 0$, since the current in the inductor is a constant at $t = \infty$.

[k] $v_L(t) = 0 + (-12.5 - 0)e^{-4000t} = -12.5e^{-4000t}\,\text{V}, \qquad t \geq 0^+$

[l] $i_o = i_a - i_L = 150 - 62.5e^{-4000t}\,\text{mA}, \qquad t \geq 0^+$

P 7.4 [a] $\dfrac{v}{i} = R = \dfrac{100e^{-80t}}{4e^{-80t}} = 25\,\Omega$

[b] $\tau = \dfrac{1}{80} = 12.5\,\text{ms}$

[c] $\tau = \dfrac{L}{R} = 12.5 \times 10^{-3}$

$$L = (12.5)(25) \times 10^{-3} = 312.5\,\text{mH}$$

[d] $w(0) = \dfrac{1}{2}L[i(0)]^2 = \dfrac{1}{2}(0.3125)(16) = 2.5\,\text{J}$

[e] $w_{\text{diss}} = \displaystyle\int_0^t 400e^{-160x}\,dx = 2.5 - 2.5e^{-160t}$

$$0.8w(0) = (0.8)(2.5) = 2\,\text{J}$$

$$2.5 - 2.5e^{-160t} = 2 \qquad \therefore \quad e^{160t} = 5$$

Solving, $t = 10.06$ ms.

P 7.5 $w(0) = \dfrac{1}{2}(20 \times 10^{-3})(10^2) = 1\,\text{J}$

$$0.5w(0) = 0.5\,\text{J}$$

$$i_R = 10e^{-t/\tau}$$

$$p_{\text{diss}} = i_R^2 R = 100Re^{-2t/\tau}$$

$$w_{\text{diss}} = \int_0^t R(100)e^{-2x/\tau}\,dx$$

$$w_{\text{diss}} = 100R\frac{e^{-2x/\tau}}{-2/\tau}\Big|_0^{t_o} = -50\tau R(e^{-2t_o/\tau} - 1) = 50L(1 - e^{-2t_o/\tau})$$

$$50L = (50)(20)\times 10^{-3} = 1; \qquad t_o = 10\,\mu\text{s}$$

$$1 - e^{-2t_o/\tau} = 0.5$$

$$e^{2t_o/\tau} = 2; \qquad \frac{2t_o}{\tau} = \frac{2t_oR}{L} = \ln 2$$

$$R = \frac{L\ln 2}{2t_o} = \frac{20\times 10^{-3}\ln 2}{20\times 10^{-6}} = 693.15\,\Omega$$

P 7.6 [a] $w(0) = \frac{1}{2}LI_g^2$

$$w_{\text{diss}} = \int_0^{t_o} I_g^2Re^{-2t/\tau}\,dt = I_g^2R\frac{e^{-2t/\tau}}{(-2/\tau)}\Big|_0^{t_o}$$

$$= \frac{1}{2}I_g^2R\tau(1 - e^{-2t_o/\tau}) = \frac{1}{2}I_g^2L(1 - e^{-2t_o/\tau})$$

$$w_{\text{diss}} = \sigma w(0)$$

$$\therefore\ \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \sigma\left(\frac{1}{2}LI_g^2\right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \qquad e^{2t_o/\tau} = \frac{1}{(1-\sigma)}$$

$$\frac{2t_o}{\tau} = \ln\left[\frac{1}{(1-\sigma)}\right]; \qquad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L\ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \dfrac{(20\times 10^{-3})\ln[1/0.5]}{20\times 10^{-6}}$

$$R = 693.15\,\Omega$$

P 7.7 [a] $i_L(0) = \dfrac{80}{40} = 2\text{ A}$

$$i_o(0^+) = \frac{80}{20} - 2 = 4 - 2 = 2\text{ A}$$

$$i_o(\infty) = \frac{80}{20} = 4\text{ A}$$

[b] $i_L = 2e^{-t/\tau}; \qquad \tau = \dfrac{L}{R} = \dfrac{20}{20} \times 10^{-3} = 1\,\text{ms}$

$$i_L = 2e^{-1000t}\,\text{A}$$

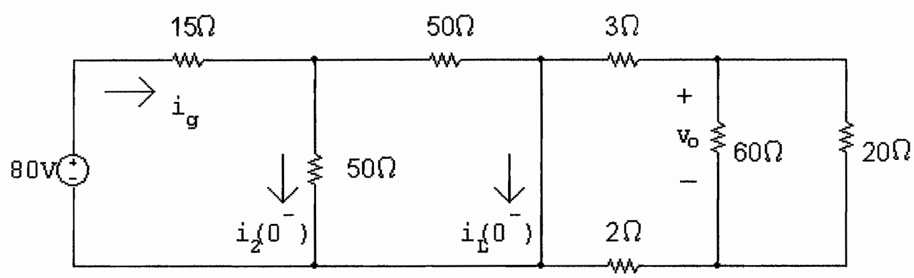
$$i_o = 4 - i_L = 4 - 2e^{-1000t}\,\text{A}, \qquad t \geq 0^+$$

[c] $4 - 2e^{-1000t} = 3.8$

$$0.2 = 2e^{-1000t}$$

$$e^{1000t} = 10 \qquad \therefore \quad t = 2.30\,\text{ms}$$

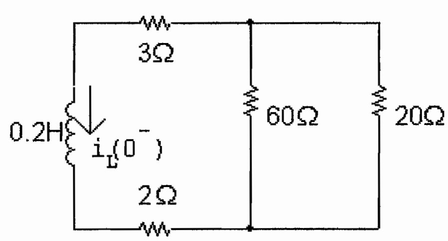
P 7.8 [a] For $t < 0$



$$i_g = \frac{80}{40} = 2\,\text{A}$$

$$i_L(0^-) = \frac{2(50)}{(100)} = 1\,\text{A} = i_L(0^+)$$

For $t > 0$



$$i_L(t) = i_L(0^+)e^{-t/\tau}\,\text{A}, \qquad t \geq 0$$

$$\tau = \frac{L}{R} = \frac{0.20}{5+15} = \frac{1}{100} = 0.01\,\text{s}$$

$$i_L(0^+) = 1\,\text{A}$$

$$i_L(t) = e^{-100t}\,\text{A}, \qquad t \geq 0$$

$$v_o(t) = -15i_L(t)$$

$$v_o(t) = -15e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

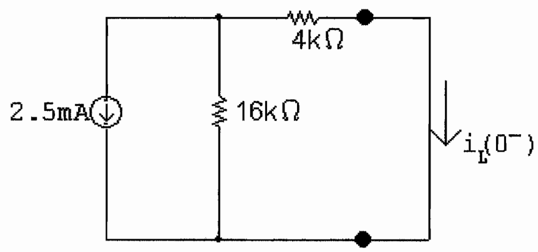
P 7.9 $P_{20\Omega} = \frac{v_o^2}{20} = 11.25e^{-200t}\text{W}$

$$w_{\text{diss}} = \int_0^{0.01} 11.25e^{-200t}\,dt$$
$$= \frac{11.25}{-200}e^{-200t}\Big|_0^{0.01}$$
$$= 56.25 \times 10^{-3}(1 - e^{-2}) = 48.64\,\text{mJ}$$

$$w_{\text{stored}} = \frac{1}{2}(0.2)(1)^2 = 100\,\text{mJ.}$$

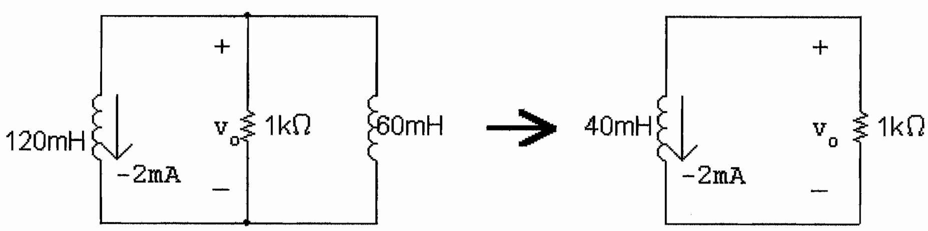
$$\%\ \text{diss} = \frac{48.64}{100} \times 100 = 48.64\%$$

P 7.10 [a] $t < 0$



$$i_L(0^-) = \frac{-2.5(16)}{(20)} = -2\,\text{mA}$$

$t \geq 0$



$$\tau = \frac{40 \times 10^{-3}}{10^3} = 40 \times 10^{-6}; \qquad 1/\tau = 25{,}000$$

$$v_o = -1000(-2 \times 10^{-3})e^{-25{,}000t} = 2e^{-25{,}000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $w_{\text{del}} = \frac{1}{2}(40 \times 10^{-3})(4 \times 10^{-6}) = 80\,\text{nJ}$

[c] $0.95w_{\text{del}} = 76\,\text{nJ}$

$$\therefore\ 76 \times 10^{-9} = \int_0^{t_o} \frac{4e^{-50{,}000t}}{1000}\,dt$$

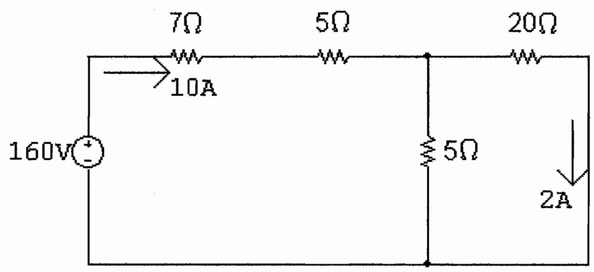
$$\therefore \quad 76 \times 10^{-9} = 80 \times 10^{-9} e^{-50{,}000t} \Big|_0^{t_o} = 80 \times 10^{-9}(1 - e^{-50{,}000t_o})$$

$$\therefore \quad e^{-50{,}000t_o} = 0.05$$

$$50{,}000t_o = \ln 20 \quad \text{so} \quad t_o = 59.9\,\mu\text{s}$$

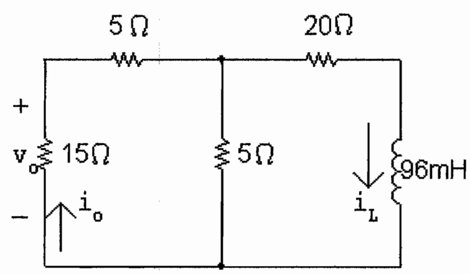
$$\therefore \quad \frac{t_o}{\tau} = \frac{59.9}{40} = 1.498 \quad \text{so} \quad t_o \approx 1.5\tau$$

P 7.11 $t < 0$:



$i_L(0^+) = 2\,\text{A}$

$t > 0$:



$$R_e = \frac{(20)(5)}{25} + 20 = 24\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{96}{24} \times 10^{-3} = 4\,\text{ms}; \qquad \frac{1}{\tau} = 250$$

$$\therefore \quad i_L = 2e^{-250t}\,\text{A}$$

$$\therefore \quad i_o = \frac{5}{25} i_L = 0.4e^{-250t}\,\text{A}$$

$$v_o = -15i_o = -6e^{-250t}\,\text{V}, \quad t \geq 0^+$$

P 7.12 $p_{20\Omega} = 20i_L^2 = 20(4)(e^{-250t})^2 = 80e^{-500t}\,\text{W}$

$$w_{20\Omega} = \int_0^\infty 80e^{-500t}\,dt = 80\frac{e^{-500t}}{-500}\Big|_0^\infty = 160\,\text{mJ}$$

$$w(0) = \frac{1}{2}(96)(10^{-3})(4) = 192\,\text{mJ}$$

$$\%\ \text{diss} = \frac{160}{192}(100) = 83.33\%$$

P 7.13 [a] $v_o(t) = v_o(0^+)e^{-t/\tau}$

$$\therefore \quad v_o(0^+)e^{-5\times10^{-3}/\tau} = 0.25v_o(0^+)$$

$$\therefore \quad e^{5\times10^{-3}/\tau} = 4$$

$$\therefore \quad \tau = \frac{L}{R} = \frac{5\times10^{-3}}{\ln 4}$$

$$\therefore \quad L = \frac{250\times10^{-3}}{\ln 4} = 180.34\,\text{mH}$$

[b] $i_L(0^-) = 60\left(\frac{1}{6}\right) = 10\,\text{mA} = i_L(0^+)$

$$w_{\text{stored}} = \frac{1}{2}Li_L(0^+)^2 = \frac{1}{2}(R\tau)(100\times10^{-6}) = 2500\tau\,\mu\text{J}.$$

$$i_L(t) = 10e^{-t/\tau}\,\text{mA}$$

$$p_{50\Omega} = i_L^2(50) = 5000\times10^{-6}e^{-2t/\tau}$$

$$w_{\text{diss}} = \int_0^{5\times10^{-3}} 5000\times10^{-6}e^{-2t/\tau}\,dt$$

$$= 5000\times10^{-6}\frac{e^{-2t/\tau}}{(-2/\tau)}\Big|_0^{5\times10^{-3}}$$

$$= 2500\times10^{-6}\tau\left[1 - e^{\frac{-10\times10^{-3}}{\tau}}\right]$$

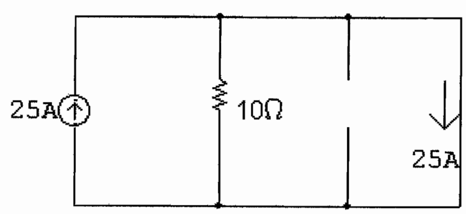
$$e^{\frac{-10\times10^{-3}}{\tau}} = e^{-2\ln 4} = 0.0625$$

$$w_{\text{diss}} = 2500\times10^{-6}\tau(0.9375)$$

$$\%\ \text{diss} = \frac{2500\times10^{-6}\tau(0.9375)}{2500\times10^{-6}\tau}\times 100$$

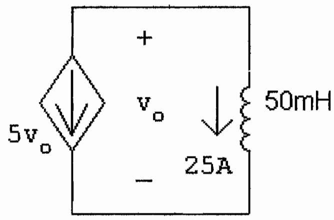
$$w_{\text{diss}} = 93.75\%$$

P 7.14 $t < 0$

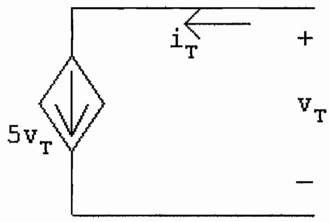


$$i_L(0^-) = i_L(0^+) = 25\,\text{A}$$

$t > 0$

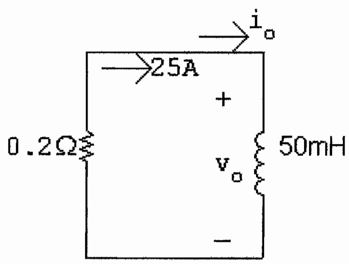


Find Thévenin resistance seen by inductor



$$i_T = 5v_T; \qquad \frac{v_T}{i_T} = R_{\text{Th}} = \frac{1}{5} = 0.2\,\Omega$$

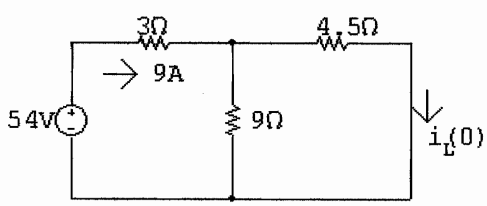
$$\tau = \frac{L}{R} = \frac{50 \times 10^{-3}}{0.2} = 250\,\text{ms}; \qquad 1/\tau = 4$$



$$i_o = 25e^{-4t}\,\text{A}, \qquad t \geq 0$$

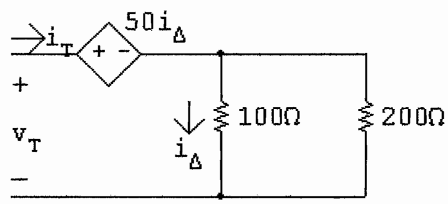
$$v_o = L\frac{di_o}{dt} = (50 \times 10^{-3})(-100e^{-4t}) = -5e^{-4t}\,\text{V}, \quad t \geq 0^+$$

P 7.15 [a] $t < 0$:



$$\frac{(9)(4.5)}{13.5} = 3\,\Omega; \qquad i_L(0) = 9\frac{9}{13.5} = 6\,\text{A}$$

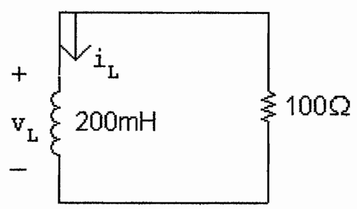
$t > 0$:



$$i_\Delta = \frac{i_T(200)}{300} = \frac{2}{3}i_T$$

$$v_T = 50i_\Delta + i_T\frac{(100)(200)}{300} = 50i_T\frac{2}{3} + \frac{200}{3}i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = \frac{100}{3} + \frac{200}{3} = 100\,\Omega$$

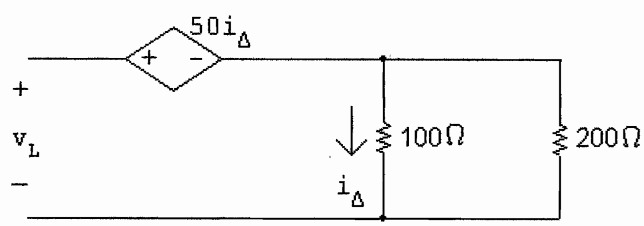


$$\tau = \frac{L}{R} = \frac{200}{100} \times 10^{-3} \qquad \frac{1}{\tau} = 500$$

$$i_L = 6e^{-500t}\,\text{A}, \qquad t \geq 0$$

[b] $v_L = 200 \times 10^{-3}(-3000e^{-500t}) = -600e^{-500t}\,\text{V}, \quad t \geq 0^+$

[c]



$$v_L = 50i_\Delta + 100i_\Delta = 150i_\Delta$$

$$i_\Delta = \frac{v_L}{150} = -4e^{-500t}\,\text{A} \qquad t \geq 0^+$$

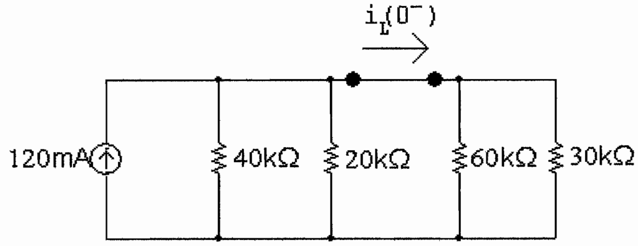
P 7.16 $w(0) = \frac{1}{2}(200 \times 10^{-3})(36) = 3.6\,\text{J}$

$$p_{50i_\Delta} = -50i_\Delta i_L = -50(-4e^{-500t})(6e^{-500t}) = 1200e^{-1000t}\,\text{W}$$

$$w_{50i_\Delta} = \int_0^\infty 1200e^{-1000t}\,dt = 1200\frac{e^{-1000t}}{-1000}\Big|_0^\infty = 1.2\,\text{J}$$

$$\%\ \text{dissipated} = \frac{1.2}{3.6}(100) = 33.33\%$$

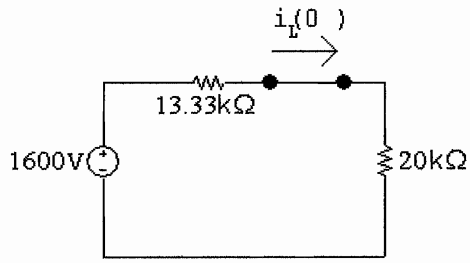
P 7.17 [a] $t < 0$



$$40\,\text{k}\Omega \| 20\,\text{k}\Omega = 13.33\,\text{k}\Omega$$

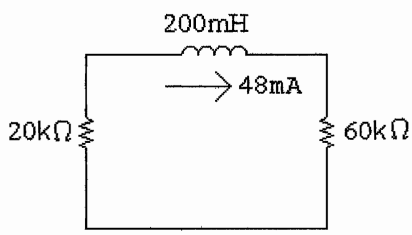
$$60\,\text{k}\Omega \| 30\,\text{k}\Omega = 20\,\text{k}\Omega$$

$$(120 \times 10^{-3})(13.33 \times 10^{3}) = 1600\,\text{V}$$



$$i_L(0^-) = \frac{1600}{33{,}333.33} = 48\,\text{mA}$$

$t > 0$



$$\tau = \frac{L}{R} = \frac{0.2}{80{,}000} = 2.5\,\mu\text{s}; \qquad \frac{1}{\tau} = 400{,}000$$

$$i_L(t) = 48e^{-400{,}000t}\,\text{mA}, \qquad t \geq 0$$

$$p_{60k} = (0.048e^{-400{,}000t})^2(60{,}000) = 138.24e^{-800{,}000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^t 138.24e^{-800{,}000x}\,dx = 172.8 \times 10^{-6}[1 - e^{-800{,}000t}]\,\text{J}$$

$$w(0) = \frac{1}{2}(.2)(48 \times 10^{-3})^2 = 230.4\,\mu\text{J}$$

$$0.25w(0) = 57.6\,\mu\text{J}$$

$$172.8(1 - e^{-800{,}000t}) = 57.6; \qquad \therefore \quad e^{800{,}000t} = 1.5$$

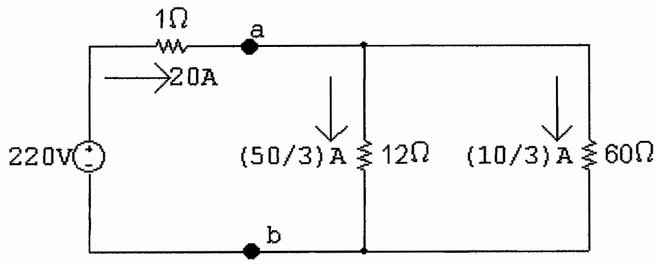
$$t = \frac{\ln 1.5}{800{,}000} = 0.507\,\mu\text{s}$$

[b] $w_{\text{diss}}(\text{total}) = 230.4(1 - e^{-800{,}000t})\ \mu\text{J}$

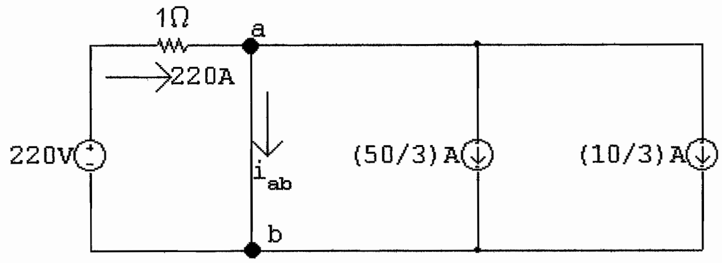
$w_{\text{diss}}(0.507\ \mu\text{s}) = 76.82\ \mu\text{J}$

$\% = (76.82/230.4)(100) = 33.3\%$

P 7.18 [a] $t < 0$:

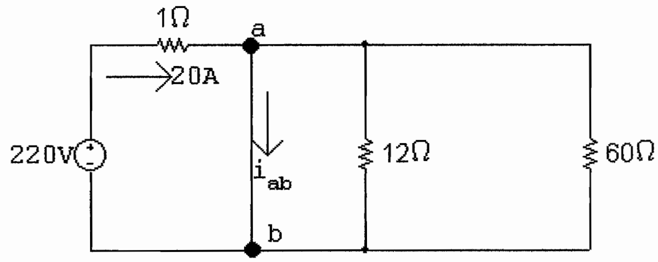


$t = 0^+$:

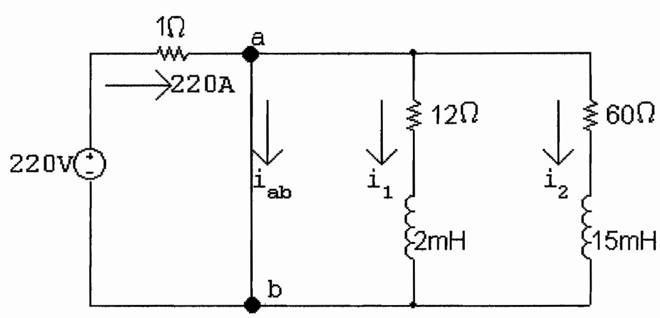


$$220 = i_{\text{ab}} + (50/3) + (10/3), \qquad i_{\text{ab}} = 200\,\text{A}, \quad t = 0^+$$

[b] At $t = \infty$:



$$i_{\text{ab}} = 220/1 = 220\,\text{A}, \quad t = \infty$$



[c] $i_1(0) = 50/3, \qquad \tau_1 = \frac{2}{12} \times 10^{-3} = 0.167\,\text{ms}$

$i_2(0) = 10/3, \qquad \tau_2 = \frac{15}{60} \times 10^{-3} = 0.25\,\text{ms}$

$i_1(t) = (50/3)e^{-6000t}\ \text{A}, \quad t \geq 0$

$i_2(t) = (10/3)e^{-4000t}\,\text{A}, \quad t \geq 0$

$i_{ab} = 220 - (50/3)e^{-6000t} - (10/3)e^{-4000t}\,\text{A}, \quad t \geq 0$

$220 - (50/3)e^{-6000t} - (10/3)e^{-4000t} = 210$

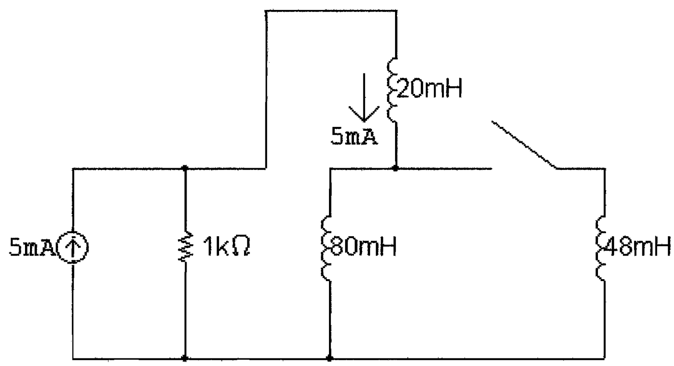
$30 = 50e^{-6000t} + 10e^{-4000t}$

$3 = 5e^{-6000t} + e^{-4000t}$

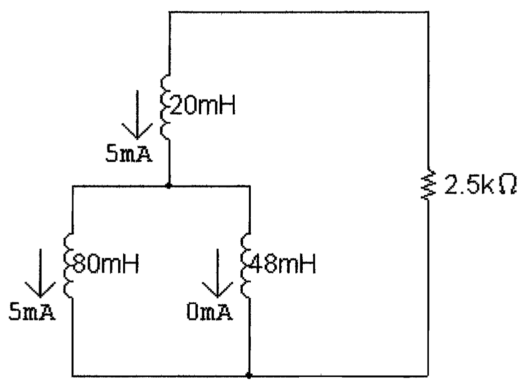
By trial and error

$t = 123.1\,\mu\text{s}$

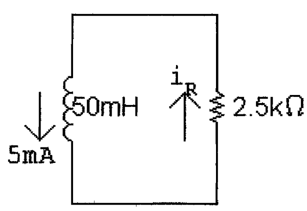
P 7.19 [a] $t < 0$:



$t = 0^+$:

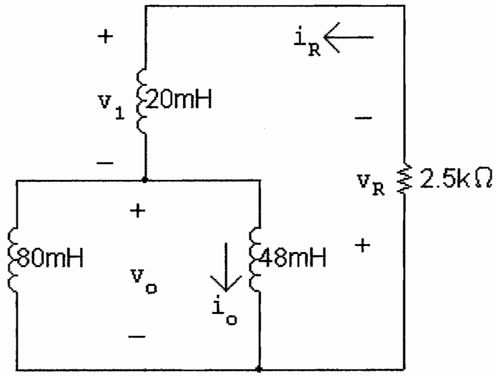


$t > 0$:



$i_R = 5e^{t/\tau}\,\text{mA}; \qquad \tau = \dfrac{L}{R} = 20 \times 10^{-6}$

$i_R = 5e^{-50{,}000t}\,\text{mA}$



$$v_R = (2.5\times10^3)(5\times10^{-3})e^{-50{,}000t} = 12.5e^{-50{,}000t}\,\text{V}$$

$$v_1 = 20\times10^{-3}[5\times10^{-3}(-50{,}000)e^{-50{,}000t}] = -5e^{-50{,}000t}\,\text{V}$$

$$v_o = -v_1 - v_R = -7.5e^{-50{,}000t}\,\text{V}$$

[b] $$i_o = \frac{10^3}{48}\int_0^t -7.5e^{-50{,}000x}\,dx + 0 = 3.125e^{-50{,}000t} - 3.125\,\text{mA}$$

P 7.20 [a] From the solution to Problem 7.19,

$$i_R = 5\times10^{-3}e^{-50{,}000t}\,\text{A}$$

$$p_R = (25\times10^{-6}e^{-100{,}000t})(2.5\times10^3) = 62.5\times10^{-3}e^{-100{,}000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^\infty 62.5\times10^{-3}e^{-100{,}000t}\,dt$$

$$= 62.5\times10^{-3}\frac{e^{-100{,}000t}}{-10^5}\Bigg|_0^\infty = 625\,\text{nJ}$$

[b] $$w_{\text{trapped}} = \frac{1}{2}L_{\text{eq}}i_R^2(0) = \frac{1}{2}(50\times10^{-3})(5\times10^{-3})^2 = 625\,\text{nJ}$$

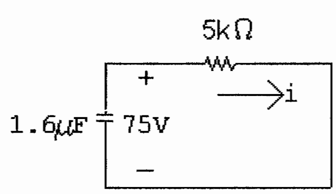
CHECK:

$$w(0) = \tfrac{1}{2}(20)(25\times10^{-6})\times10^{-3} + \tfrac{1}{2}(80)(25\times10^{-6})\times10^{-3} = 1250\,\text{nJ}$$

$$\therefore\quad w(0) = w_{\text{diss}} + w_{\text{trapped}}$$

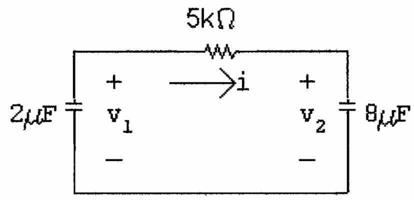
P 7.21 [a] $v_1(0^-) = v_1(0^+) = 75\,\text{V}$ $\qquad v_2(0^+) = 0$

$$C_{\text{eq}} = 2\times8/10 = 1.6\,\mu\text{F}$$



$$\tau = (5)(1.6)\times10^{-3} = 8\text{ms};\qquad \frac{1}{\tau} = 125$$

$$i = \frac{75}{5} \times 10^{-3}e^{-125t} = 15e^{-125t}\,\text{mA}, \qquad t \geq 0^+$$



$$v_1 = \frac{-10^6}{2}\int_0^t 15 \times 10^{-3}e^{-125x}\,dx + 75 = 60e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{10^6}{8}\int_0^t 15 \times 10^{-3}e^{-125x}\,dx + 0 = -15e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(2 \times 10^{-6})(5625) = 5625\,\mu\text{J}$

[c] $w_{\text{trapped}} = \frac{1}{2}(2 \times 10^{-6})(225) + \frac{1}{2}(8 \times 10^{-6})225 = 1125\,\mu\text{J}.$

$$w_{\text{diss}} = \frac{1}{2}(1.6 \times 10^{-6})(5625) = 4500\,\mu\text{J}.$$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 1125 + 4500 = 5625\,\mu\text{J}; \qquad w(0) = 5625\,\mu\text{J}.$

P 7.22 [a] $R = \frac{v}{i} = 20\,\text{k}\Omega$

[b] $\frac{1}{\tau} = \frac{1}{RC} = 1000; \qquad C = \frac{1}{(10^3)(20 \times 10^3)} = 0.05\,\mu\text{F}$

[c] $\tau = \frac{1}{1000} = 1\,\text{ms}$

[d] $w(0) = \frac{1}{2}(0.05 \times 10^{-6})(10^4) = 250\,\mu\text{J}$

[e]

$$W_{\text{diss}} = \int_0^{t_o} \frac{v^2}{R}dt = \int_0^{t_o} \frac{(10^4)e^{-2000t}}{(20 \times 10^3)}dt$$

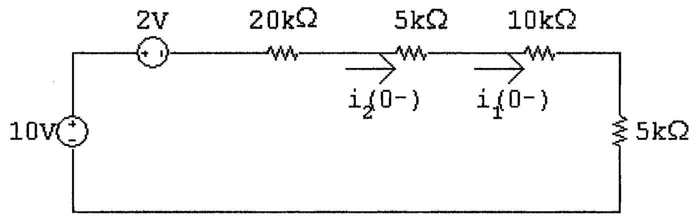
$$= 0.5\frac{e^{-2000t}}{-2000}\Big|_0^{t_o} = 250(1 - e^{-2000t_o})\,\mu\text{J}$$

$$200 = 250(1 - e^{-2000t_o})$$

$$\therefore\ e^{-2000t_o} = 0.2; \qquad e^{2000t_o} = 5$$

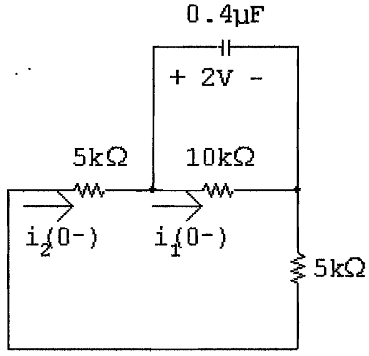
$$t_o = \frac{1}{2000}\ln 5; \qquad t_o \cong 804.72\,\mu\text{s}$$

P 7.23 [a] $t < 0$:



$$i_1(0^-) = i_2(0^-) = \left(\frac{8}{40} \times 10^{-3}\right) = 0.2\,\text{mA}$$

[b] $t > 0$:



$$i_1(0^+) = \frac{2}{10} \times 10^{-3} = 0.2\,\text{mA}$$

$$i_2(0^+) = \frac{-2}{10} \times 10^{-3} = -0.2\,\text{mA}$$

[c] Capacitor voltage cannot change instantaneously, therefore,

$$i_1(0^-) = i_1(0^+) = 0.2\,\text{mA}$$

[d] Switching can cause an instantaneous change in the current in a resistive branch. In this circuit

$$i_2(0^-) = 0.2\,\text{mA} \quad \text{and} \quad i_2(0^+) = -0.2\,\text{mA}$$

[e] $v_c = 2e^{-t/\tau}\,\text{V}, \qquad t \geq 0$

$$\tau = R_eC = 5000(0.4) \times 10^{-6} = 2 \times 10^{-3}$$

$$v_c = 2e^{-500t}\,\text{V}, \qquad t \geq 0$$

$$i_1 = \frac{v_c}{10{,}000} = 0.2e^{-500t}\,\text{mA}, \qquad t \geq 0$$

[f] $i_2 = \dfrac{-v_c}{10{,}000} = -0.2e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

P 7.24 [a] $v(0) = \dfrac{(8)(27)(33)}{60} = 118.80\,\text{V}$

$$R_e = \frac{(3)(6)}{9} = 2\,\text{k}\Omega$$

$$\tau = R_eC = (2000)(0.25) \times 10^{-6} = 500\,\mu\text{s}; \qquad \frac{1}{\tau} = 2000$$

$$v = 118.80e^{-2000t}\,\text{V} \qquad t \geq 0$$

$$i_o = \frac{v}{3000} = 39.6e^{-2000t}\,\text{mA}$$

[b] $w(0) = \frac{1}{2}(0.25)(118.80)^2 = 1764.18\,\mu\text{J}$

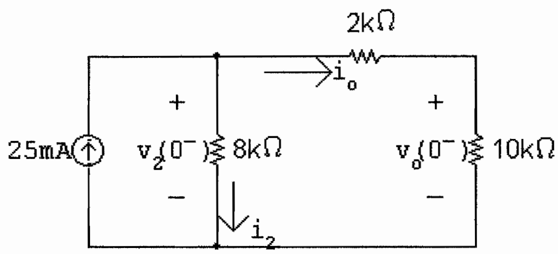
$$i_{4k} = \frac{118.80e^{-2000t}}{6} = 19.8e^{-2000t}\,\text{mA}$$

$$p_{4k} = [(19.8)e^{-2000t}]^2(4000) \times 10^{-6} = 1568.16 \times 10^{-3}e^{-4000t}$$

$$w_{4k} = 1568.16 \times 10^{-3}\frac{e^{-4000x}}{-4000}\Big|_0^{250\times10^{-6}} = 392.04(1 - e^{-1})\,\mu\text{J}$$

$$\% = \frac{392.04}{1764.18}(1 - e^{-1}) \times 100 = 14.05\%$$

P 7.25 [a] $t < 0$:



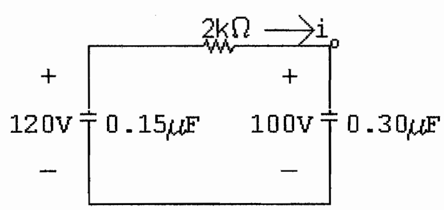
$$i_o(0^-) = \frac{(25)(8)}{(20)} = 10\,\text{mA}$$

$$v_o(0^-) = (10)(10) = 100\,\text{V}$$

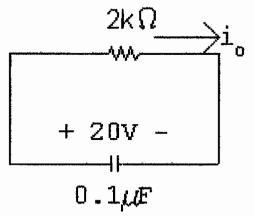
$$i_2(0^-) = 25 - 10 = 15\,\text{mA}$$

$$v_2(0^-) = 15(8) = 120\,\text{V}$$

$t > 0$

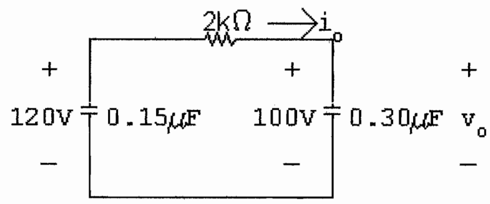


$$\tau = RC = 0.2\,\text{ms} = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$



$$i_o(t) = \frac{20}{2 \times 10^3} e^{-t/\tau} = 10e^{-5000t}\,\text{mA}, \qquad t \geq 0^+$$

[b]



$$\begin{aligned} v_o &= \frac{10^6}{0.3}\int_0^t 10 \times 10^{-3} e^{-5000x}\,dx + 100 \\ &= \frac{10^5}{3}\frac{e^{-5000x}}{-5000}\Big|_0^t + 100 \\ &= -(20/3)e^{-5000t} + (20/3) + 100 \\ v_o &= [-(20/3)e^{-5000t} + (320/3)]\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[c] $w_{\text{trapped}} = (1/2)(0.15) \times 10^{-6}(320/3)^2 + (1/2)(0.3) \times 10^{-6}(320/3)^2$

$w_{\text{trapped}} = 2560\,\mu\text{J}.$

Check by combining the capacitors into a single equivalent capacitance of 0.1 μF with a 20 V initial voltage:

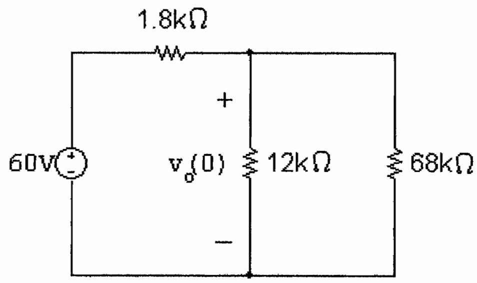
$$w_{\text{diss}} = \frac{1}{2}C_{\text{eq}}(V_o)^2 = \frac{1}{2}(0.1 \times 10^{-6})(20)^2 = 20\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.15) \times 10^{-6}(120)^2 + \frac{1}{2}(0.3 \times 10^{-6})(100)^2 = 2580\,\mu\text{J}.$$

$$w_{\text{trapped}} + w_{\text{diss}} = w(0)$$

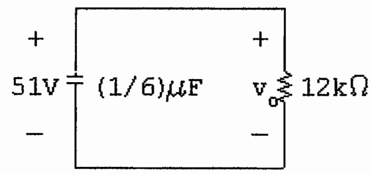
$2560 + 20 = 2580$ OK.

P 7.26 [a] $t < 0$:



$$v_o(0) = \frac{(60)(10.2)}{12} = 51\,\text{V}$$

$t > 0$:



$$\tau = \frac{1}{6}(12) \times 10^{-3} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 51e^{-500t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12} \times 10^{-3} = 216.75 \times 10^{-3}e^{-1000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{2\times 10^{-3}} 216.75 \times 10^{-3}e^{-1000t}\,dt$$
$$= 216.75 \times 10^{-6}(1 - e^{-2}) = 187.42\,\mu\text{J}$$

[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)(51)^2 \times 10^{-6} = 216.75\,\mu\text{J}$

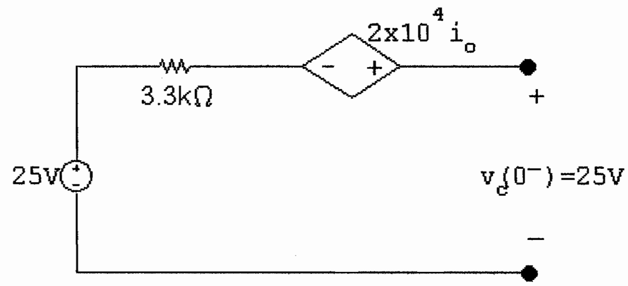
$$0.95w(0) = 205.9125\,\mu\text{J}$$

$$\int_0^{t_o} 216.75 \times 10^{-3}e^{-1000x}\,dx = 205.9125 \times 10^{-6}$$

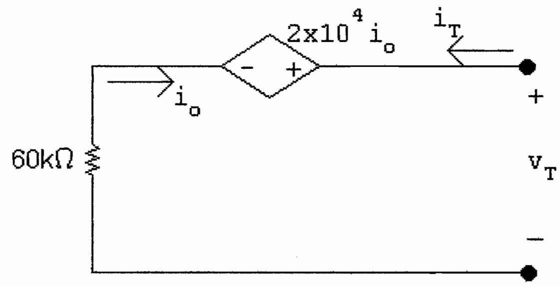
$$\int_0^{t_o} e^{-1000x}\,dx = 0.95 \times 10^{-3}$$

$$\therefore \quad 1 - e^{-1000t_o} = 0.95; \qquad e^{1000t_o} = 20; \quad \text{so} \quad t_o = 3\,ms$$

P 7.27 $t < 0$

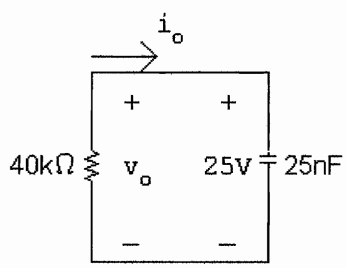


$t > 0$



$$
\begin{aligned}
v_T &= 2 \times 10^4 i_o + 60{,}000 i_T \\
&= 20{,}000(-i_T) + 60{,}000 i_T = 40{,}000 i_T
\end{aligned}
$$

$$\therefore \frac{v_T}{i_T} = R_{\rm Th} = 40\,\text{k}\Omega$$

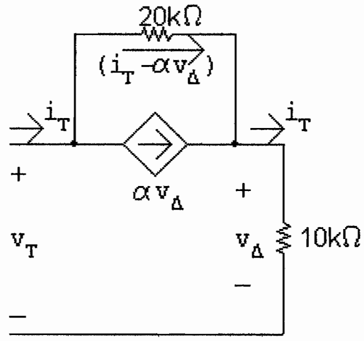


$$\tau = RC = 1\,\text{ms}; \qquad \frac{1}{\tau} = 1000$$

$$v_o = 25e^{-1000t}\,\text{V}, \qquad t \geq 0$$

$$i_o = 25 \times 10^{-9} \frac{d}{dt}[25e^{-1000t}] = -625e^{-1000t}\,\mu\text{A}, \qquad t \geq 0^+$$

P 7.28 [a] $\tau = RC = R_{\text{Th}}(0.2) \times 10^{-6} = 10^{-3}; \qquad \therefore \quad R_{\text{Th}} = \dfrac{1000}{0.2} = 5\,\text{k}\Omega$



$$v_T = 20 \times 10^3(i_T - \alpha v_\Delta) + 10 \times 10^3 i_T$$

$$v_\Delta = 10 \times 10^3 i_T$$

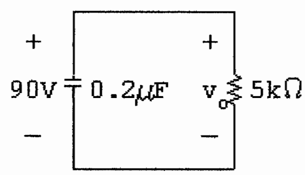
$$v_T = 30 \times 10^3 i_T - 20 \times 10^3 \alpha 10 \times 10^3 i_T$$

$$\frac{v_T}{i_T} = 30 \times 10^3 - 200 \times 10^6 \alpha = 5 \times 10^3$$

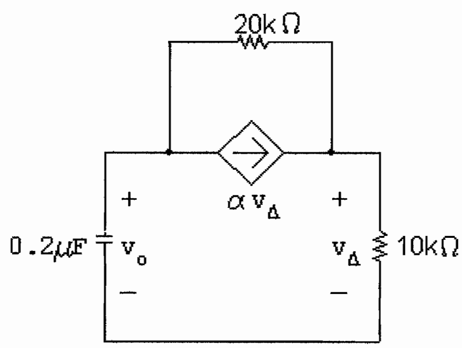
$$\therefore \quad 30 - 200{,}000\alpha = 5; \qquad \alpha = 125 \times 10^{-6}\ \text{A/V}$$

[b] $v_o(0) = (0.018)(5000) = 90\,\text{V} \qquad t < 0$

$t > 0$:



$$v_o = 90e^{-1000t}\,\text{V}, \quad t \geq 0$$

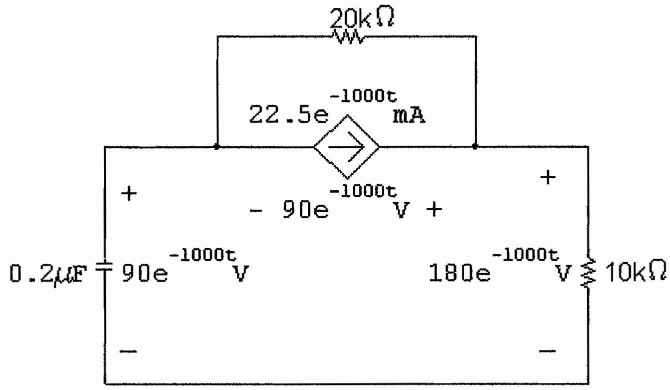


$$\frac{v_\Delta}{10 \times 10^3} + \frac{v_\Delta - v_o}{20{,}000} - 125 \times 10^{-6} v_\Delta = 0$$

$$2v_\Delta + v_\Delta - v_o - 2500 \times 10^{-3} v_\Delta = 0$$

$$\therefore \quad v_\Delta = 2v_o = 180e^{-1000t}\,\text{V}$$

P 7.29 [a]



$$p_{ds} = (-90e^{-1000t})(22.5 \times 10^{-3}e^{-1000t}) = -2025 \times 10^{-3}e^{-2000t}\,\text{W}$$

$$w_{ds} = \int_0^\infty p_{ds}\,dt = -1012.5\,\mu\text{J}.$$

$\therefore$ dependent source is delivering 1012.5 μJ

[b] $$p_{10k} = \frac{(180)^2e^{-2000t}}{10 \times 10^3}$$

$$w_{10k} = \int_0^\infty p_{10k}\,dt = 1620\,\mu\text{J}$$

$$p_{20k} = \frac{(90)^2e^{-2000t}}{20 \times 10^3}$$

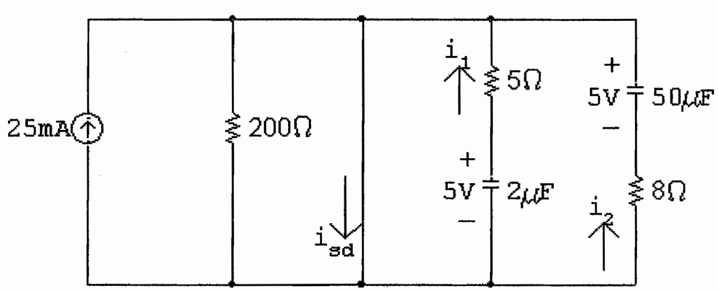
$$w_{20k} = \int_0^\infty p_{20k}\,dt = 202.5\,\mu\text{J}$$

$$w_c(0) = \frac{1}{2}(0.2) \times 10^{-6}(90)^2 = 810\,\mu\text{J}$$

$$\sum w_{\text{dev}} = 810 + 1012.5 = 1822.5\,\mu\text{J}$$

$$\sum w_{\text{diss}} = 202.5 + 1620 = 1822.5\,\mu\text{J}.$$

P 7.30 [a] At $t = 0^-$ the voltage on each capacitor will be $25 \times 10^{-3} \times 200 = 5$ V, positive at the upper terminal. Hence at $t \geq 0^+$ we have



$$\therefore\quad i_{sd}(0^+) = 0.025 + \frac{5}{5} + \frac{5}{8} = 1.65\,\text{A}$$

At $t = \infty$, both capacitors will have completely discharged.

$\therefore \quad i_{sd}(\infty) = 25\,\text{mA}$

[b] $i_{sd}(t) = 0.025 + i_1(t) + i_2(t)$

$\tau_1 = (5)(2) \times 10^{-6} = 10\,\mu\text{s}$

$\tau_2 = (8)(50 \times 10^{-6}) = 400\,\mu\text{s}$

$\therefore \quad i_1(t) = e^{-10^5 t}\,\text{A}, \qquad t \geq 0^+$

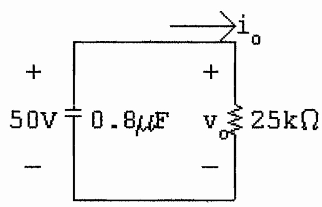
$i_2(t) = 0.625e^{-2500t}\,\text{A}, \qquad t \geq 0$

$\therefore \quad i_{sd} = 25 + 1000e^{-100{,}000t} + 625e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

P 7.31 [a] $\dfrac{1}{C_e} = 1 + \dfrac{1}{4} = 1.25$

$\therefore \quad C_e = 0.8\,\mu\text{F}; \qquad v_o(0) = 60 - 10 = 50\,\text{V}$

$\tau = (0.8)(25) \times 10^{-3} = 20\,\text{ms}; \qquad \dfrac{1}{\tau} = 50$



$v_o = 50e^{-50t}\,\text{V}, \qquad t > 0^+$

[b] $w_o = \dfrac{1}{2}(1 \times 10^{-6})(3600) + \dfrac{1}{2}(4 \times 10^{-6})(100) = 2\,\text{mJ}$

$w_{\text{diss}} = \dfrac{1}{2}(0.8 \times 10^{-6})(2500) = 1\,\text{mJ}$

$\%\ \text{diss} = \dfrac{1}{2} \times 100 = 50\%$

[c] $i_o = \dfrac{v_o}{25} \times 10^{-3} = 2e^{-50t}\,\text{mA}$

$$v_1 = -\frac{10^6}{4}\int_0^t 2 \times 10^{-3} e^{-50x}\,dx - 10 = -500\int_0^t e^{-50x}\,dx - 10$$

$$= -500\frac{e^{-50x}}{-50}\bigg|_0^t - 10 = 10e^{-50t} - 20\,\text{V} \qquad t \geq 0$$

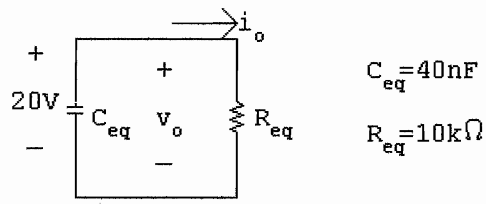
[d] $v_1 + v_2 = v_o$

$v_2 = v_o - v_1 = 50e^{-50t} - 10e^{-50t} + 20 = 40e^{-50t} + 20\,\text{V} \quad t \geq 0$

[e] $w_{\text{trapped}} = \frac{1}{2}(4 \times 10^{-6})(400) + \frac{1}{2}(1 \times 10^{-6})(400) = 1\,\text{mJ}$

$w_{\text{diss}} + w_{\text{trapped}} = 2\,\text{mJ}$ (check)

P 7.32 [a] The equivalent circuit for $t > 0$:



$\tau = 0.4\,\text{ms}; \qquad 1/\tau = 2500$

$v_o = 20e^{-2500t}\,\text{V}, \qquad t \geq 0$

$i_o = 2e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$i_{25k\Omega} = 2e^{-2500t}\left(\frac{15}{40}\right) = 0.75e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$p_{25k\Omega} = (0.5625 \times 10^{-6}e^{-5000t})(25{,}000) = 14{,}062.5 \times 10^{-6}e^{-5000t}\,\text{W}$

$w_{25k\Omega} = \int_0^\infty 14{,}062.5 \times 10^{-6}e^{-5000t}\,dt = -2.8125 \times 10^{-6}(0-1) = 2.8125\,\mu\text{J}$

$w(0) = \frac{1}{2}(0.2 \times 10^{-6})(100) + \frac{1}{2}(0.05 \times 10^{-6})(900) = 32.5\,\mu\text{J}$

$\%\,\text{diss}\,(25\,\text{k}\Omega) = \frac{2.8125}{32.5} \times 100 = 8.65\%$

[b] $p_{625\Omega} = 625(2 \times 10^{-3}e^{-2500t})^2 = 2.5 \times 10^{-3}e^{-5000t}$

$w_{625\Omega} = \int_0^\infty p_{625}\,dt = 0.50\,\mu\text{J}$

$\%\,\text{diss}\,(625\Omega) = \frac{0.5}{32.5} \times 100 = 1.54\%$

$i_{15k\Omega} = 2e^{-2500t}\left(\frac{25}{40}\right) = 1.25e^{-2500t}\,\text{mA}, \quad t \geq 0^+$

$p_{15k\Omega} = (1.25 \times 10^{-3}e^{-2500t})^2(15{,}000) = 23.4375 \times 10^{-3}e^{-5000t}\,\text{W}$

$w_{15k\Omega} = \int_0^\infty 23.4375 \times 10^{-3}e^{-5000t}\,dt = 4.6875\,\mu\text{J}$

$\%\,\text{diss}\,(15\text{k}\Omega) = 14.42\%$

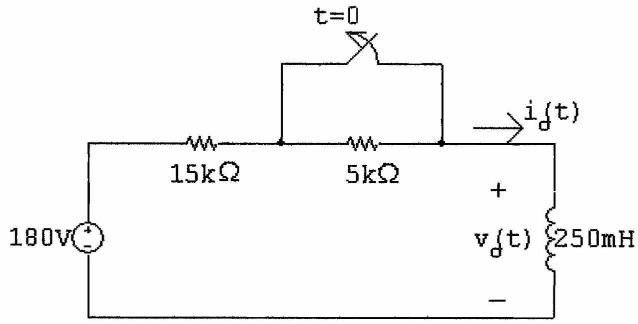
[c] $\sum w_{\text{diss}} = 2.8125 + 0.50 + 4.6875 = 8\,\mu\text{J}$

$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 32.5 - 8 = 24.5\,\mu\text{J}$

$\%\,\text{trapped} = \frac{24.5}{32.5} \times 100 = 75.38\%$

Check: $8.65 + 1.54 + 14.42 + 75.38 = 99.99 \approx 100\%$

P 7.33 After making a Thévenin equivalent we have



$$I_o = 180/15 = 12\,\text{mA}$$

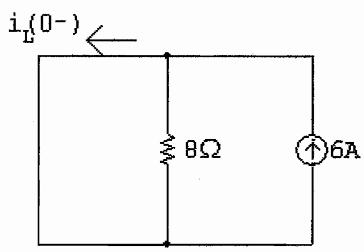
$$\tau = (0.25/20) \times 10^{-3} = 0.125 \times 10^{-4}; \qquad \frac{1}{\tau} = 80{,}000$$

$$I_{\text{f}} = \frac{V_s}{R} = \frac{180}{20} = 9\,\text{mA}$$

$$i_o = 9 + (12 - 9)e^{-80{,}000t} = 9 + 3e^{-80{,}000t}\,\text{mA}$$

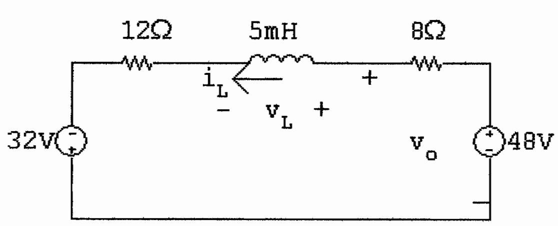
$$v_o = [180 - 12(20]e^{-80{,}000t} = -60e^{-80{,}000t}\,\text{V}$$

P 7.34 $t < 0$



$$i_L(0^-) = 6\,\text{A}$$

$t > 0$



$$i_L(\infty) = \frac{32 + 48}{20} = 4\,\text{A}$$

$$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{20} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L = 4 + (6-4)e^{-4000t} = 4 + 2e^{-4000t}\,\text{A}, \qquad t \geq 0$$

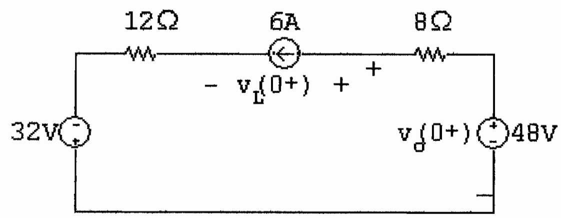
$$v_o = -8i_L + 48 = -8(4 + 2e^{-4000t}) + 48 = 16 - 16e^{-4000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_L = 5 \times 10^{-3}\dfrac{di_L}{dt} = 5 \times 10^{-3}[-8000e^{-4000t}] = -40e^{-4000t}\,\text{V}, \qquad t \geq 0^+$

$v_L(0^+) = -40\,\text{V}$

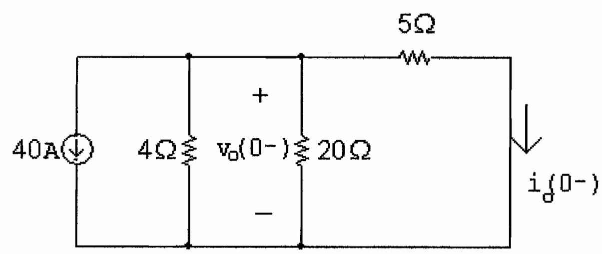
$v_o(0^+) = 0\,\text{V}$

Check: at $t = 0^+$ the circuit is:



$v_L(0^+) = 32 - 72 + 0 = -40\,\text{V}, \qquad v_o(0^+) = 48 - 48 = 0\,\text{V}$

P 7.35 [a] $t < 0$



KVL equation at the top node:

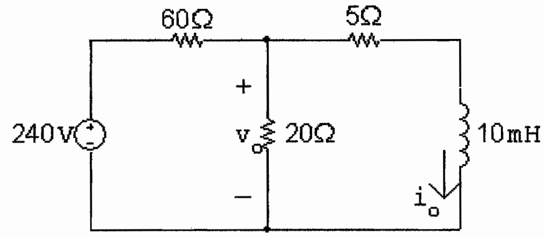
$$-40 = \frac{v_o(0^-)}{4} + \frac{v_o(0^-)}{20} + \frac{v_o(0^-)}{5}$$

Multiply by 20 and solve:

$$-800 = (5 + 1 + 4)v_o; \qquad v_o = -80\,\text{V}$$

$$\therefore \quad i_o(0^-) = \frac{v_o}{5} = -80/5 = -16\,\text{A}$$

$t > 0$



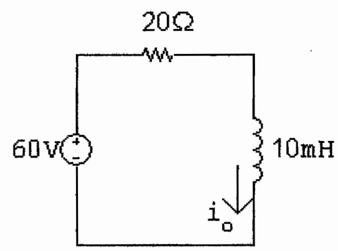
Use voltage division to find the Thévenin voltage:

$$V_{\text{Th}} = v_o = \frac{20}{20+60}(240) = 60\,\text{V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{\text{Th}} = 5 + 20\|60 = 5 + 15 = 20\,\Omega$$

The simplified circuit is:



$$\tau = \frac{L}{R} = \frac{10\times 10^{-3}}{20} = 0.5\,\text{ms}; \qquad \frac{1}{\tau} = 2000$$

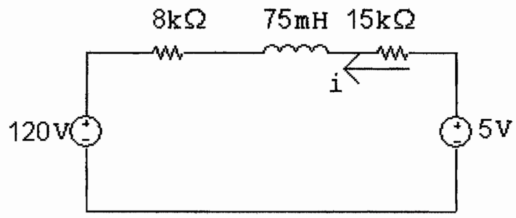
$$i_o(\infty) = \frac{60}{20} = 3\,\text{A}$$

$$\therefore\ i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 3 + (-16-3)e^{-2000t} = 3 - 19e^{-2000t}\,\text{A}, \qquad t \geq 0$$

[b]
$$\begin{aligned} v_o &= 5i_o + (0.01)\frac{di_o}{dt} \\ &= 15 - 95e^{-2000t}) + 0.01(38{,}000)(e^{-2000t}) \\ &= 15 - 95e^{-2000t} + 380e^{-2000t} \\ v_o &= 15 + 285e^{-2000t}\,\text{V}, \qquad t \geq 0^+ \end{aligned}$$

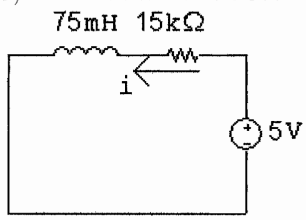
P 7.36 [a] For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 75 mH inductor. We get



$$i(0^-) = \frac{5 - 120}{15\,\text{k} + 8\,\text{k}} = -5\,\text{mA}$$

$$i(0^-) = i(0^+) = -5\,\text{mA}$$

[b] For $t > 0$, the circuit reduces to



Therefore $i(\infty) = 5/15{,}000 = 0.333\,\text{mA}$

[c] $\tau = \frac{L}{R} = \frac{75 \times 10^{-3}}{15{,}000} = 5\,\mu\text{s}$

[d] $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

$$= 0.333 + [-5 - 0.333]e^{-200{,}500t} = 0.333 - 5.333e^{-200{,}500t}\,\text{mA}, \qquad t \geq 0$$

P 7.37 [a] From Eqs. (7.35) and (7.42)

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-(R/L)t}$$

$$v = (V_s - I_oR)e^{-(R/L)t}$$

$$\therefore \quad \frac{V_s}{R} = 10; \qquad I_o - \frac{V_s}{R} = -10$$

$$V_s - I_oR = 200; \qquad \frac{R}{L} = 500$$

$$\therefore \quad I_o = -10 + \frac{V_s}{R} = 0\,\text{A}$$

Therefore, $V_s = 200\text{V}$.

$$i(\infty) = 10 = \frac{200}{R} \quad \text{so} \quad R = 20\,\Omega$$

$$L = \frac{R}{500} = 40\,\text{mH}$$

[b] $i = 10 - 10e^{-500t}; \qquad i^2 = 100 - 200e^{-500t} + 100e^{-1000t}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.04)[100 - 200e^{-500t} + 100e^{-1000t}] = 2 - 4e^{-500t} + 2e^{-1000t}$$

$$w(\infty) = 2\,\text{J}$$

$$w(t_o) = 2 - 4e^{-500t_o} + 2e^{-1000t_o} = 0.25(2)$$

$$\therefore \quad 1 - 2x + x^2 = 0.25 \quad \text{and thus} \quad x^2 - 2x + 0.75 = 0$$

Solving, $x = 1.5$ and $x = 0.5$ but only the second solution is possible

$$\therefore \quad 0.5 = e^{-500t_o} \quad \text{so} \quad t_o = \frac{\ln 2}{500} = 1.386\,\text{ms}$$

P 7.38 [a] $v_o(0^+) = -I_gR_2; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_gR_2e^{-[(R_1+R_2)/L]t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_o(0^+) \to \infty$, and the duration of $v_o(t) \to$ zero

[c] $v_{sw} = R_2i_o; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g\frac{R_1}{R_1 + R_2}$$

Therefore $\quad i_o(t) = \frac{I_gR_1}{R_1+R_2} + \left[I_g - \frac{I_gR_1}{R_1+R_2}\right]e^{-[(R_1+R_2)/L]t}$

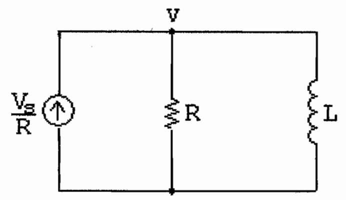
$$i_o(t) = \frac{R_1I_g}{(R_1+R_2)} + \frac{R_2I_g}{(R_1+R_2)}e^{-[(R_1+R_2)/L]t}$$

Therefore $\quad v_{sw} = \frac{R_1I_g}{(1+R_1/R_2)} + \frac{R_2I_g}{(1+R_1/R_2)}e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0^+$

[d] $|v_{sw}(0^+)| \to \infty; \qquad$ duration $\to 0$

P 7.39 Opening the inductive circuit causes a very large voltage to be induced across the inductor L. This voltage also appears across the switch (part [e] of Problem 7.38) causing the switch to arc over. At the same time, the large voltage across L damages the meter movement.

P 7.40 [a]



$$-\frac{V_s}{R} + \frac{v}{R} + \frac{1}{L}\int_0^t v\,dt + I_o = 0$$

Differentiating both sides,

$$\frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

$$\therefore \quad \frac{dv}{dt} + \frac{R}{L}v = 0$$

[b] $\dfrac{dv}{dt} = -\dfrac{R}{L}v$

$$\frac{dv}{dt}dt = -\frac{R}{L}v\,dt \qquad \text{so} \qquad dv = -\frac{R}{L}v\,dt$$

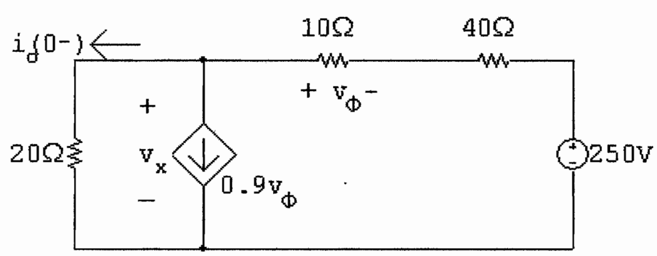
$$\frac{dv}{v} = -\frac{R}{L}dt$$

$$\int_{V_o}^{v(t)} \frac{dx}{x} = -\frac{R}{L}\int_0^t dy$$

$$\ln\frac{v(t)}{V_o} = -\frac{R}{L}t$$

$$\therefore \quad v(t) = V_o e^{-(R/L)t} = (V_s - RI_o)e^{-(R/L)t}$$

P 7.41 For $t < 0$



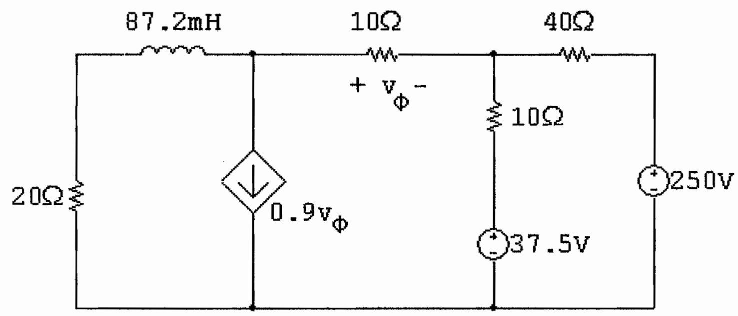
$$\frac{v_x}{20} + 9\left[\frac{v_x - 250}{50}\right] + \left[\frac{v_x - 250}{50}\right] = 0$$

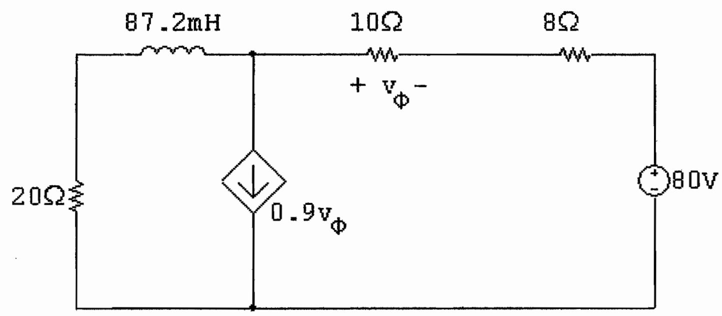
$$\frac{v_x}{20} + 10\frac{(v_x - 250)}{50} = 0$$

$$5v_x - 5000 + 20v_x = 0; \qquad v_x = 200\,\text{V}$$

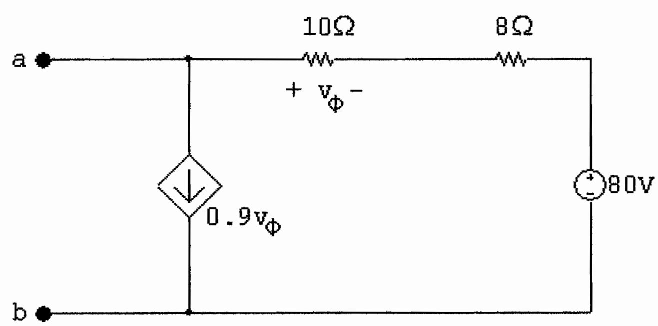
$i_o(0^-) = 200/20 = 10\,\text{A}$

$t > 0$

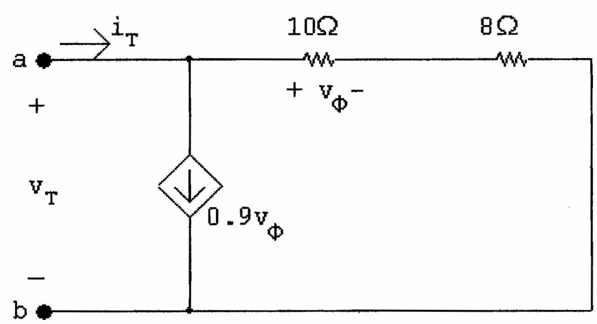




Find Thévenin equivalent with respect to a, b



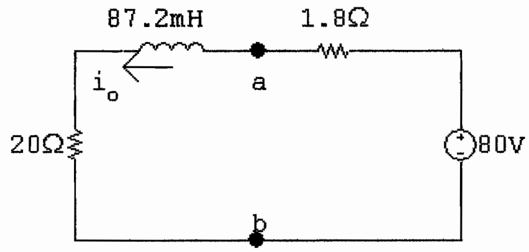
$$\frac{V_{\text{Th}} - 80}{18} + 9\frac{(V_{\text{Th}} - 80)}{18} = 0 \qquad V_{\text{Th}} = 80\,\text{V}$$



$$v_T = (i_T - 0.9v_\phi)18 = \left[i_T - 0.9\left(\frac{10v_T}{18}\right)\right]18$$

$$v_T = 18i_T - 9v_T \qquad \therefore \quad 10v_T = 18i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = 1.8\,\Omega$$

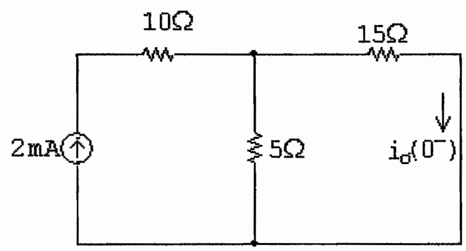


$$i_o(\infty) = 80/21.8 = 3.67\,\text{A}$$

$$\tau = \frac{87.2}{21.8} \times 10^{-3} = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$i_o = 3.67 + (10 - 3.67)e^{-250t} = 3.67 + 6.33e^{-250t}\,\text{A}, \qquad t \geq 0$$

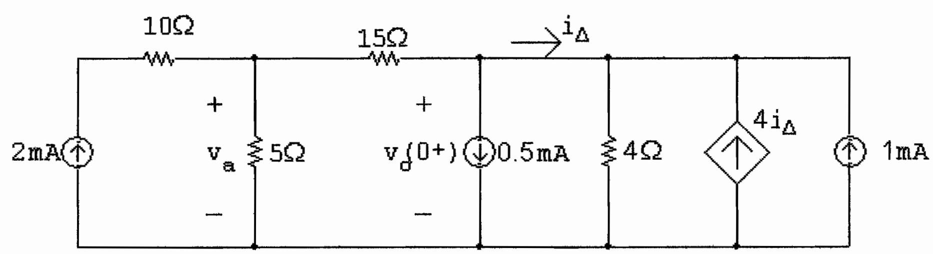
P 7.42 $t < 0$;



$$i_o(0^-) = \frac{5}{5 + 15}(0.002) = 0.5\,\text{mA}$$

$$i_o(0^+) = i_o(0^-) = 0.5\,\text{mA}$$

$t > 0$;



$$-0.002 + \frac{v_a}{5} + \frac{v_a - v_o}{15} = 0$$

$$\frac{v_o - v_a}{15} + 5 \times 10^{-4} + \frac{v_o}{4} - 4i_\Delta - 0.001 = 0$$

$$i_\Delta = \frac{v_o}{4} - 4i_\Delta - 0.001$$

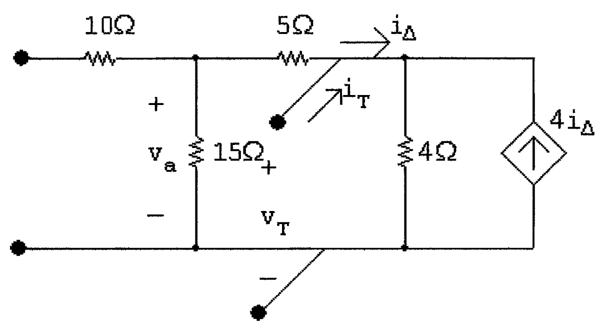
Solving,

$$v_o(0^+) = 2\,\text{mV}$$

We also know that

$$v_o(\infty) = 0$$

Find the Thévenin resistance seen by the 2 mH inductor:



$$i_T = \frac{v_T}{20} + \frac{v_T}{4} - 4i_\Delta$$

$$i_\Delta = \frac{v_T}{4} - 4i_\Delta \qquad \therefore\ 5i_\Delta = \frac{v_T}{4}; \qquad i_\Delta = \frac{v_T}{20}$$

$$i_T = \frac{v_T}{20} + \frac{v_T}{4} - \frac{4v_T}{20}$$

$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{4} - \frac{1}{5} = \frac{2}{20} = 0.1\,\text{S}$$

$$\therefore\ R_{\text{Th}} = 10\Omega$$

$$\tau = \frac{2\times 10^{-3}}{10} = 0.2\,\text{ms}; \qquad 1/\tau = 5000$$

$$\therefore\ v_o = 0 + (2-0)e^{-5000t} = 2e^{-5000t}\,\text{mV}, \qquad t \geq 0^+$$

P 7.43 [a] Let v be the voltage drop across the parallel branches, positive at the top node, then

$$-I_g + \frac{v}{R_g} + \frac{1}{L_1}\int_0^t v\,dx + \frac{1}{L_2}\int_0^t v\,dx = 0$$

$$\frac{v}{R_g} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_0^t v\,dx = I_g$$

$$\frac{v}{R_g} + \frac{1}{L_e}\int_0^t v\,dx = I_g$$

$$\frac{1}{R_g}\frac{dv}{dt} + \frac{v}{L_e} = 0$$

$$\frac{dv}{dt} + \frac{R_g}{L_e}v = 0$$

Therefore $v = I_gR_ge^{-t/\tau}; \qquad \tau = L_e/R_g$

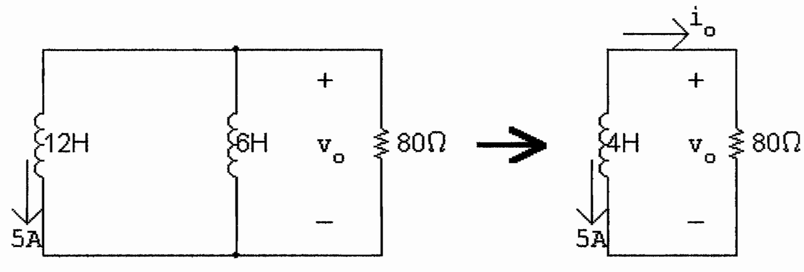
Thus

$$i_1 = \frac{1}{L_1}\int_0^t I_gR_ge^{-x/\tau}\,dx = \frac{I_gR_g}{L_1}\frac{e^{-x/\tau}}{(-1/\tau)}\Big|_0^t = \frac{I_gL_e}{L_1}(1 - e^{-t/\tau})$$

$$i_1 = \frac{I_gL_2}{L_1 + L_2}(1 - e^{-t/\tau}) \quad \text{and} \quad i_2 = \frac{I_gL_1}{L_1 + L_2}(1 - e^{-t/\tau})$$

[b] $i_1(\infty) = \dfrac{L_2}{L_1 + L_2}I_g; \qquad i_2(\infty) = \dfrac{L_1}{L_1 + L_2}I_g$

P 7.44 $t > 0$



$$\tau = \frac{4}{80} = \frac{1}{20}$$

$$i_o = -5e^{-20t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 80i_o = -400e^{-20t}\,\text{V}, \qquad t > 0^+$$

$$-400e^{-20t} = -80; \qquad e^{20t} = 5$$

$$\therefore\ t = \frac{1}{20}\ln 5 = 80.47\,\text{ms}$$

P 7.45 **[a]** $w_{\text{diss}} = \frac{1}{2}L_e i^2(0) = \frac{1}{2}(4)(25) = 50\,\text{J}$

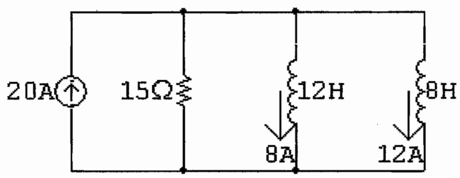
[b]

$$i_{12H} = \frac{1}{12}\int_0^t (-400)e^{-20x}\,dx + 5$$
$$= \frac{-100}{3}\frac{e^{-20x}}{-20}\Big|_0^t + 5 = \frac{5}{3}e^{-20t} + \frac{10}{3}\,\text{A}$$
$$i_{6H} = \frac{1}{6}\int_0^t (-400)e^{-20x}\,dx + 0$$
$$= \frac{-200}{3}\frac{e^{-20x}}{-20}\Big|_0^t + 0 = \frac{10}{3}e^{-20t} - \frac{10}{3}\,\text{A}$$

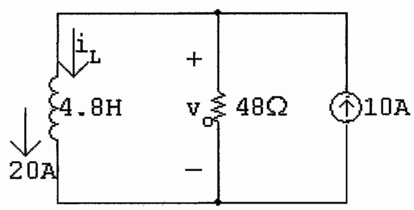
$$w_{\text{trapped}} = \frac{1}{2}(18)(100/9) = 100\,\text{J}$$

[c] $w(0) = \frac{1}{2}(12)(25) = 150\,\text{J}$

P 7.46 **[a]** $t < 0$



$t > 0$



$$i_L(0^-) = i_L(0^+) = 20\,\text{A}; \qquad \tau = \frac{4.8}{48} = 0.1\,\text{s}; \qquad \frac{1}{\tau} = 10$$

$$i_L(\infty) = 10\,\text{A}$$

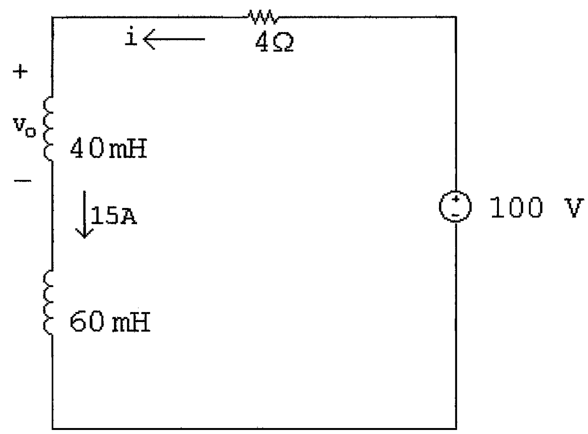
$$i_L = 10 + [20 - 10]e^{-10t} = 10 + 10e^{-10t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 4.8[-100e^{-10t}] = -480e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_1 = \frac{1}{12}\int_0^t -480e^{-10x}\,dx + 8 = 4e^{-10t} + 4\,\text{A}, \qquad t \geq 0$

[c] $i_2 = \frac{1}{8}\int_0^t -480e^{-10x}\,dx + 12 = 6e^{-10t} + 6\,\text{A}, \qquad t \geq 0$

P 7.47 For $t < 0$, $\quad i_{40\text{mH}}(0) = 75/5 = 15\,\text{A}$
For $t > 0$, after making a Thévenin equivalent we have



$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-t/\tau}$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{4}{100} \times 10^3 = 40$$

$$I_o = 15\,\text{A}; \qquad \frac{V_s}{R} = \frac{100}{4} = 25\,\text{A}$$

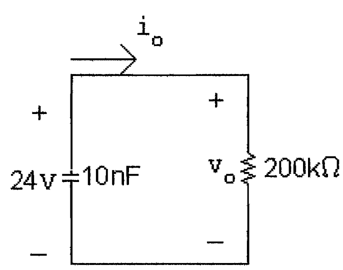
$$i = 25 + (15 - 25)e^{-40t} = 25 - 10e^{-40t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 0.04\frac{di}{dt} = 0.04(400e^{-40t}) = 16e^{-40t}\,\text{V}, \qquad t > 0^+$$

P 7.48 [a] $v_c(0^-) = \dfrac{16}{20}(30) = 24\,\text{V}$

$$C_{\text{eq}} = \left(\frac{1}{30} + \frac{1}{15}\right)^{-1} = 10\,\text{nF}$$

For $t > 0$:



$$\tau = RC = 200 \times 10^3 \times 10 \times 10^{-9} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 24e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_o = \dfrac{v_o}{200{,}000} = \dfrac{24e^{-500t}}{200{,}000} = 120e^{-500t}\,\mu\text{A}$

$$v_1 = \frac{1}{15 \times 10^{-9}} \times 120 \times 10^{-6} \int_0^t e^{-500x}\,dx + 0 = 16 - 16e^{-500t}\,\text{V}, \quad t \geq 0$$

P 7.49 [a] The energy delivered to the 200 kΩ resistor is equal to the energy stored in the equivalent capcitor. From the solution to Problem 7.48 we have

$$w = \frac{1}{2}C_{\text{eq}}v_o^2 = \frac{1}{2}(10 \times 10^{-9})(24)^2 = 2.88\,\mu\text{J}$$

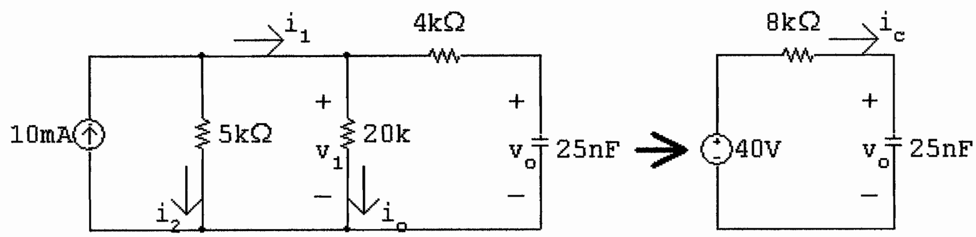
[b] From the solution to Problem 7.48 we know the voltage on the 15 nF capacitor at $t = \infty$ is 16 V. THerefore, the voltage across the 30 nF capacitor at $t = \infty$ is −16 V. It follows that the total energy trapped is

$$w_{\text{trapped}} = \frac{1}{2}(30 \times 10^{-9})(-16)^2 + \frac{1}{2}(15 \times 10^{-9})(16)^2 = 5.76\,\mu\text{J}$$

[c] $w(0) = \dfrac{1}{2}(30 \times 10^{-9})(24^2) = 8.64\,\mu\text{J}$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 5.76 + 2.88 = 8.64 = w(0)$

P 7.50 [a] $t > 0$



$$v_o(0^-) = v_o(0^+) = 0\,\text{V}$$

$$v_o(\infty) = 40\,\text{V}$$

$$\tau = (8 \times 10^3)(25) \times 10^{-9} = 0.2\,\text{ms} \qquad 1/\tau = 5000$$

$$v_o = (40 - 40e^{-5000t})\,\text{V}, \qquad t \geq 0$$

[b] $i_c = 25 \times 10^{-9}\dfrac{dv_o}{dt}$

$$i_c = 25 \times 10^{-9}(200{,}000e^{-5000t}) = 5e^{-5000t}\,\text{mA}$$

$$v_1 = 4(5e^{-5000t}) + 40 - 40e^{-5000t} = 40 - 20e^{-5000t}$$

$$i_o = \frac{v_1}{20 \times 10^3} = 2 - e^{-5000t}\,\text{mA}$$

[c] $i_1(t) = i_o + i_c = 2 + 4e^{-5000t}\,\text{mA}$

[d] $i_2(t) = \dfrac{v_1}{5\times10^3} = 8 - 4e^{-5000t}\,\text{mA}$

[e] $i_1(0^+) = 2 + 4 = 6\,\text{mA}$

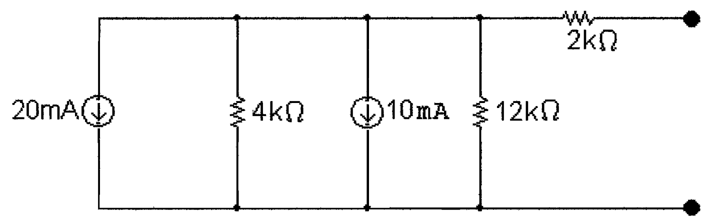
Checks: $i_1 + i_2 = 10\,\text{mA}$

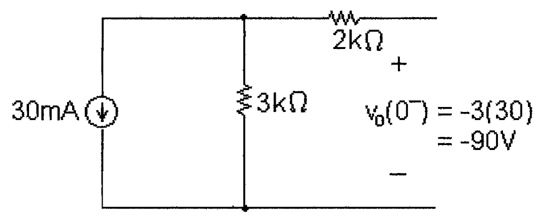
$$i_c(0^+) = \frac{10\left(\frac{1}{4}\right)}{\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{4}\right)} = 5\,\text{mA}$$

$$i_o(0^+) = \frac{10\left(\frac{1}{20}\right)}{\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{4}\right)} = 1\,\text{mA}$$

$$i_1(0^+) = 5 + 1 = 6\,\text{mA}$$

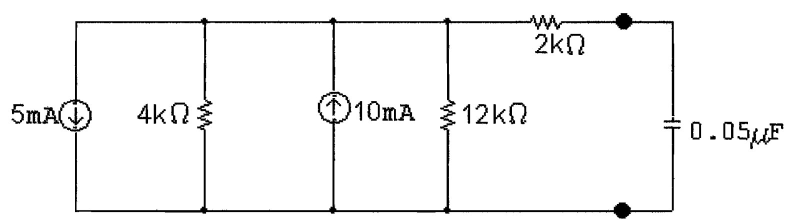
P 7.51 For $t < 0$

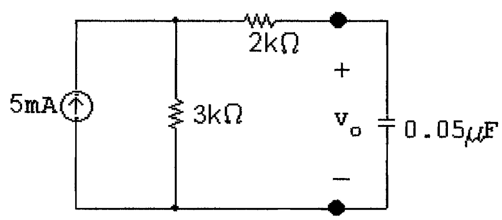


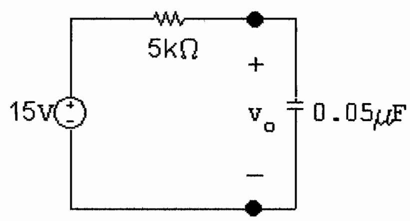


$\therefore\ v_o(0^-) = v_o(0^+) = -90\,\text{V}$

$t > 0$







$$v_o(\infty) = 15\,\text{V}; \qquad \tau = RC = (5\,\text{k})(0.05\,\mu) = 0.25\,\text{ms}; \qquad \frac{1}{\tau} = 4000$$

$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = 15 + [-90 - 15]e^{-4000t}$$

$$= 15 - 105e^{-4000t}\,\text{V} \qquad t \geq 0$$

P 7.52 [a] $I_s = i(0^+) = 50\,\text{mA}; \qquad V_o = 0\,\text{V}$

$$I_sR = v(\infty) = 80$$

$$\therefore \quad R = \frac{80}{0.05} = 1.6\,\text{k}\Omega$$

$$RC = \frac{1}{2500}; \qquad C = \frac{1}{2500(1600)} = 250\ \text{nF}$$

[b] $w(t) = \frac{1}{2}(250 \times 10^{-9})[80 - 80e^{-2500t}]^2$

$$= 125 \times 10^{-9}(6400)[1 - e^{-2500t}]^2$$

$$= 800[1 - 2e^{-2500t} + e^{-5000t}]\ \mu\text{J}$$

Let $x = e^{-2500t}$; then $800[1 - 2x + x^2] = 0.64(800)$

$$\therefore \quad x^2 - 2x + 0.36 = 0$$

The two solutions are $x = 1.8$, $x = 0.2$

Only the second solution is valid $\therefore \; e^{+2500t} = 5$

$2500t = \ln 5$ so $t = 400 \ln 5\,\mu\text{s} = 643.787\,\mu\text{s}$

P 7.53 [a] $v_c(0^+) = 120\,\text{V}$

[b] Use voltage division to find the final value of voltage:

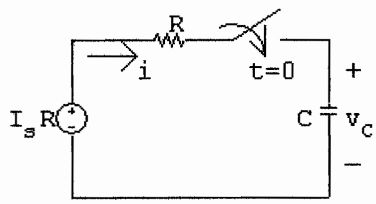
$$v_c(\infty) = \frac{150}{150 + 50}(-200) = -150\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = -150\,\text{V}, \qquad R_{\text{Th}} = 12.5\,\text{k} + 150\,\text{k}\|50\,\text{k} = 50\,\text{k}\Omega,$$

Therefore $\tau = R_{\text{eq}}C = (50{,}000)(40 \times 10^{-9}) = 2\,\text{ms}$

The simplified circuit for $t > 0$ is:



[d] $i(0^+) = \dfrac{-150 - 120}{50{,}000} = -5.4\,\text{mA}$

[e] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -150 + [120 - (-150)]e^{-t/\tau} = -150 + 270e^{-500t}\,\text{V}, \qquad t \geq 0$$

[f] $i = C\dfrac{dv_c}{dt} = (40 \times 10^{-9})(-500)(270e^{-500t}) = 5.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

P 7.54 [a] Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{10k} = \frac{10\,\text{k}}{10\,\text{k} + 15\,\text{k}}(-75) = -30\,\text{V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{5k} = (5 \times 10^{-3})(5000) = 25\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = 25\,\text{V}, \qquad R_{\text{Th}} = 5\,\text{k} + 20\,\text{k} = 25\,\text{k}\Omega$$

$$\tau = R_{\text{Th}}C = 2.5\,\text{ms}$$

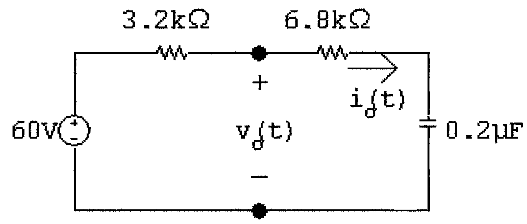
[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= 25 + (-30 - 25)e^{-400t} = 25 - 55e^{-400t}\,\text{V}, \quad t \geq 0$$

We want $v_c = 25 - 55e^{-400t} = 0$:

Therefore $t = \dfrac{\ln(55/25)}{400} = 1.97\,\text{ms}$

P 7.55 [a]



$$i_o(0^+) = \frac{60}{10} \times 10^{-3} = 6\,\text{mA}$$

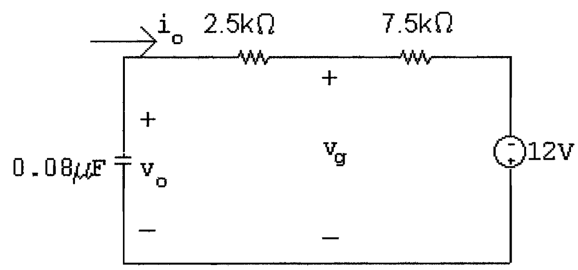
[b] $i_o(\infty) = 0$

[c] $\tau = RC = (10 \times 10^3)(0.2 \times 10^{-6}) = 2\,\text{ms}$

[d] $i_o = 0 + (6 - 0)e^{-500t} = 6e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

[e] $v_o = 60 - 3.2 \times 10^3 i_o = 60 - 19.2e^{-500t}\,\text{V}, \qquad t \geq 0^+$

P 7.56 [a] $v_o(0^-) = v_o(0^+) = 48\,\text{V}$



$$v_o(\infty) = -12\,\text{V}; \qquad \tau = 0.8\,\text{ms}; \qquad \frac{1}{\tau} = 1250$$

$$v_o = -12 + (48 - (-12))e^{-1250t}$$

$$v_o = -12 + 60e^{-1250t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o = -0.08 \times 10^{-6}[-75{,}000e^{-1250t}]$

$$i_o = 6e^{-1250t}\,\text{mA}, \qquad t \geq 0^+$$

[c] $v_g = v_o - 2.5 \times 10^3 i_o$

$$v_g = -12 + 45e^{-1250t}\,\text{V}$$

[d] $v_g(0^+) = -12 + 45 = 33\,\text{V}$

Checks:

$$v_g(0^+) = i_o(0^+)7.5 \times 10^3 - 12 = 45 - 12 = 33\,\text{V}$$

$$i_{10\text{k}} = \frac{v_g}{10\text{k}} = -1.2 + 4.5e^{-1250t}\,\text{mA}$$

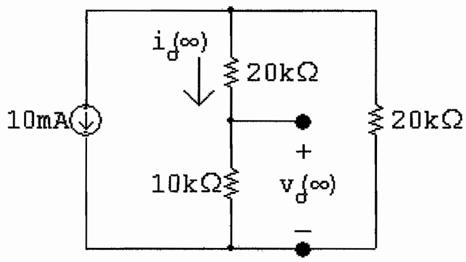
$$i_{30k} = \frac{v_g}{30k} = -0.4 + 1.5e^{-1250t}\,\text{mA}$$

$$-i_o + i_{10} + i_{30} + 1.6 = 0 \qquad \text{(ok)}$$

P 7.57 $t < 0$;

$$i_o(0^-) = (15)\frac{20}{50} = 6\,\text{mA}; \qquad v_o(0^-) = (6)(10) = 60\,\text{V}$$

$t = \infty$:

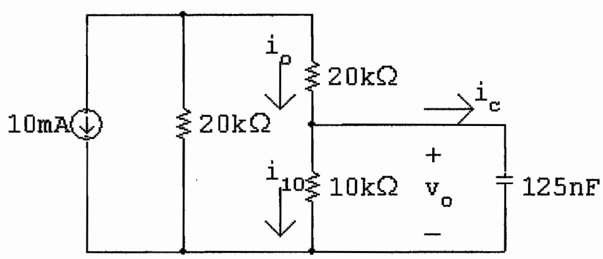


$$i_o(\infty) = -10\left(\frac{20}{50}\right) = -4\,\text{mA}; \qquad v_o(\infty) = i_o(\infty)(10) = -40\,\text{V}$$

$$R_{\text{Th}} = 10\,\text{k}\Omega \| 40\,\text{k}\Omega = 8\,\text{k}\Omega; \qquad C = 125\,\text{nF}$$

$$\tau = (8)(0.125) = 1\,\text{ms}; \qquad \frac{1}{\tau} = 1000$$

$$\therefore\quad v_o(t) = -40 + 100e^{-1000t}\,\text{V}, \qquad t \geq 0^+$$



$$i_c = C\frac{dv_o}{dt} = -12.5e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{10} = \frac{v_o}{10} = -4 + 10e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_o = i_c + i_{10} = -(4 + 2.5e^{-1000t})\,\text{mA}, \qquad t \geq 0^+$$

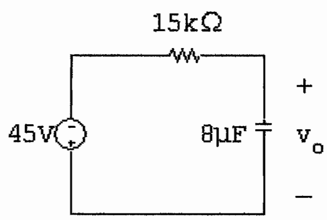
P 7.58 For $t > 0$

$$V_{\mathrm{Th}} = (-15)(30)i_{\mathrm{b}} = -450 \times 10^3 i_{\mathrm{b}}$$

$$i_{\mathrm{b}} = \frac{400(12)}{48} = 100\,\mu\mathrm{A}$$

$$V_{\mathrm{Th}} = -450 \times 10^3(100 \times 10^{-6}) = -45\,\mathrm{V}$$

$$R_{\mathrm{Th}} = 15\,\mathrm{k}\Omega$$



$$v_o(\infty) = -45\,\mathrm{V}; \qquad v_o(0^+) = 0$$

$$\tau = (15,000)(8)10^{-6} = 120\,\mathrm{ms}; \qquad 1/\tau = 8.33$$

$$v_o = -45 + 45e^{-8.33t}\,\mathrm{V}, \qquad t \geq 0$$

$$w(t) = \frac{1}{2}(8 \times 10^{-6})v_o^2 = 8100(1 - 2e^{-8.33t} + e^{-16.67t})\,\mu\mathrm{J}$$

$$w(\infty) = 8100\,\mu\mathrm{J}$$

$$\therefore\ \ 8100(1 - 2e^{-8.33t_o} + e^{-16.67t_o}) = 0.90(8100)$$

$$\therefore\ \ 1 - 2x + x^2 = 0.90; \qquad x = e^{-8.33t_o}$$

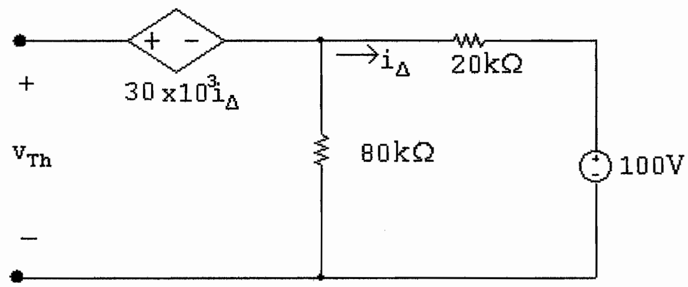
$$\therefore\ \ x^2 - 2x + 0.10 = 0$$

$$x_{1,2} = 1.9487, \qquad 0.0513$$

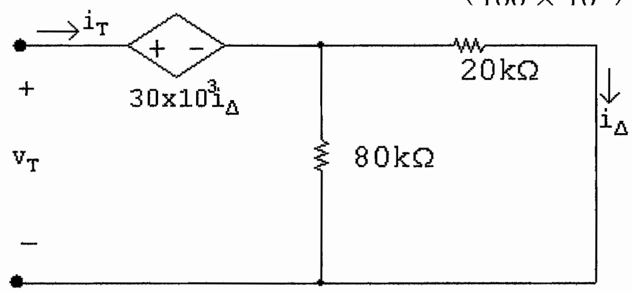
$$e^{-(25/3)t_o} = 0.0513; \qquad (25/3)t_o = \ln 19.4868; \qquad t_o = 356.4\,\mathrm{ms}$$

P 7.59 For $t < 0$, $v_o(0) = 80\,\text{V}$

$t > 0$:



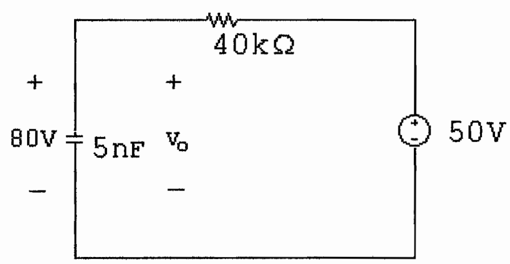
$$v_{\text{Th}} = 30 \times 10^3 i_\Delta + 0.8(100) = 30 \times 10^3 \left(\frac{-100}{100 \times 10^3}\right) + 80 = 50\,\text{V}$$



$$v_T = 30 \times 10^3 i_\Delta + 16 \times 10^3 i_T = 30 \times 10^3(0.8)i_T + 16 \times 10^3 i_T = 40 \times 10^3 i_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = 40\,\text{k}\Omega$$

$t > 0$



$$v_o = 50 + (80 - 50)e^{-t/\tau}$$

$$\tau = RC = (40 \times 10^3)(5 \times 10^{-9}) = 200 \times 10^{-6}; \qquad \frac{1}{\tau} = 5000$$

$$v_o = 50 + 30e^{-5000t}\,\text{V}$$

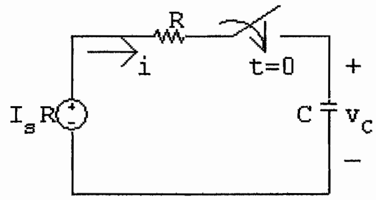
P 7.60 $v_o(0) = 50\,\text{V}$; $\quad v_o(\infty) = 80\,\text{V}$

$$R_{\text{Th}} = 16\,\text{k}\Omega$$

$$\tau = (16)(5 \times 10^{-6}) = 80 \times 10^{-6}; \qquad \frac{1}{\tau} = 12{,}500$$

$$v = 80 + (50 - 80)e^{-12{,}500t} = 80 - 30e^{-12{,}500t}\,\text{V}$$

P 7.61 [a]



$$I_sR = Ri + \frac{1}{C}\int_{0^+}^{t} i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \quad \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $\displaystyle \frac{di}{dt} = -\frac{i}{RC}; \qquad \frac{di}{i} = -\frac{dt}{RC}$

$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0^+}^{t} dx$$

$$\ln\frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_sR - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore \quad i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$$

P 7.62 [a] Let i be the current in the clockwise direction around the circuit. Then

$$\begin{aligned} V_g &= iR_g + \frac{1}{C_1}\int_0^t i\,dx + \frac{1}{C_2}\int_0^t i\,dx \\ &= iR_g + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i\,dx = iR_g + \frac{1}{C_e}\int_0^t i\,dx \end{aligned}$$

Now differentiate the equation

$$0 = R_g\frac{di}{dt} + \frac{i}{C_e} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{R_gC_e}i = 0$$

Therefore $\displaystyle i = \frac{V_g}{R_g}e^{-t/R_gC_e} = \frac{V_g}{R_g}e^{-t/\tau}; \qquad \tau = R_gC_e$

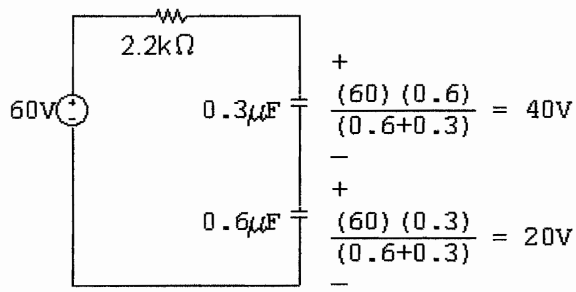
$$v_1(t) = \frac{1}{C_1}\int_0^t \frac{V_g}{R_g}e^{-x/\tau}\,dx = \frac{V_g}{R_gC_1}\left.\frac{e^{-x/\tau}}{-1/\tau}\right|_0^t = -\frac{V_gC_e}{C_1}(e^{-t/\tau} - 1)$$

$$v_1(t) = \frac{V_gC_2}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

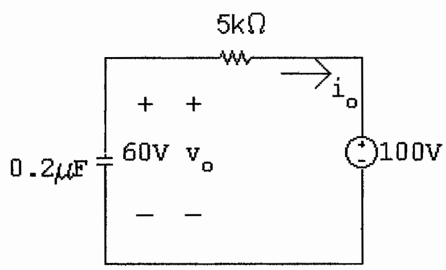
$$v_2(t) = \frac{V_g C_1}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_g C_e$$

[b] $v_1(\infty) = \frac{C_2}{C_1 + C_2} V_g; \qquad v_2(\infty) = \frac{C_1}{C_1 + C_2} V_g$

P 7.63 [a] $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 60\,\text{V}$$

$$v_o(\infty) = 100\,\text{V}$$

$$\tau = (0.2)(5) \times 10^{-3} = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 100 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o = -C\frac{dv_o}{dt} = -0.2 \times 10^{-6}[40{,}000e^{-1000t}]$

$$= -8e^{-1000t}\,\text{mA}; \qquad t \geq 0^+$$

[c] $v_1 = \frac{-10^6}{0.3}\int_0^t -8 \times 10^{-3} e^{-1000x}\,dx + 40$

$$= 66.67 - 26.67e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[d] $v_2 = \frac{-10^6}{0.6}\int_0^t -8 \times 10^{-3} e^{-1000x}\,dx + 20$

$$= 33.33 - 13.33e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[e] $w_{\text{trapped}} = \frac{1}{2}(0.3)10^{-6}(66.67)^2 + \frac{1}{2}(0.6)10^{-6}(33.33)^2$

$= 666.67 + 333.33 = 1000\,\mu\text{J}.$

P 7.64 $v_o(0) = \frac{120}{120}(80) = 80\,\text{V}$

$v_o(\infty) = -6(25) = -150\,\text{V}$

$\tau = (25 \times 10^3)(40 \times 10^{-9}) = 10^{-3}\,\text{s}; \qquad \frac{1}{\tau} = 1000$

$v_o = -150 + (80 + 150)e^{-1000t} = -150 + 230e^{-1000t}\,\text{V}, \qquad t \geq 0$

P 7.65 [a] From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{(8\,\text{m})(20\,\text{m}) - (10\,\text{m})^2}{8\,\text{m} + 20\,\text{m} - 2(10\,\text{m})} = 7.5\,\text{mH}$$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{(7.5\,\text{m})}{75} = \frac{1}{10{,}000}$$

$$i_o = \frac{15}{75} - \frac{15}{75}e^{-10{,}000t} = 0.2 - 0.2e^{-10{,}000t}\,\text{A} \quad t \geq 0$$

[b] $v_o = 15 - 75i_o = 15 - 75(0.2 - 0.2e^{-10{,}000t}) = 15e^{-10{,}000t}\,\text{V} \quad t \geq 0^+$

[c] $v_o = 0.008\frac{di_1}{dt} + 0.01\frac{di_2}{dt}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{di_2}{dt} = \frac{di_o}{dt} - \frac{di_1}{dt} = 2000e^{-10{,}000t} - \frac{di_1}{dt}$$

$$\therefore \quad 15e^{-10{,}000t} = 0.008\frac{di_1}{dt} + 0.01\left(2000e^{-10{,}000t} - \frac{di_1}{dt}\right)$$

$$\therefore \quad \frac{di_1}{dt} = 2500e^{-10{,}000t}$$

$$di_1 = 2500e^{-10{,}000t}\,dt$$

$$\int_0^{i_1} dx = 2500\int_0^t e^{-10{,}000y}\,dy$$

$$\therefore \quad i_1 = 2500\frac{e^{-10{,}000y}}{-10{,}000}\Big|_0^t = 0.25 - 0.25e^{-10{,}000t}\,\text{A}, \quad t \geq 0$$

[d]
$$\begin{aligned} i_2 &= i_o - i_1 \\ &= 0.2 - 0.2e^{-10{,}000t} - 0.25 + 0.25e^{-10{,}000t} \\ &= -50 + 50e^{-10{,}000t}\,\text{mA}, \quad t \geq 0 \end{aligned}$$

[e]
$$\begin{aligned} v_o &= L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \\ &= 0.02(-500e^{-10{,}000t}) + 0.01(2500e^{-10{,}000t}) \\ &= 15e^{-10{,}000t}\,\text{V}, \quad t \geq 0^+ \qquad \text{(checks)} \end{aligned}$$

$i_1(0) = 0.25 - 0.25 = 0$; agrees with initial conditions;
$i_2(0) = -0.05 + 0.05 = 0$; agrees with initial conditions;
The final values of i_o, i_1, and i_2 can be checked via the conservation of Wb-turns:

$$i_o(\infty)L_{eq} = 0.2 \times (7.5\,\text{m}) = 1.5\ \text{mWb-turns}$$

$$i_1(\infty)L_1 + i_2(\infty)M = 0.25(8\,\text{m}) - 0.05(10\,\text{m}) = 15\ \text{mWb-turns}$$

$$i_2(\infty)L_2 + i_1(\infty)M = -0.05(0.02) + 0.25(0.01) = 15\ \text{mWb-turns}$$

Thus our solutions make sense in terms of known circuit behavior.

P 7.66 [a] $L_{eq} = \dfrac{(3)(15)}{3+15} = 2.5\,\text{H}$

$$\tau = \frac{L_{eq}}{R} = \frac{2.5}{7.5} = \frac{1}{3}\,\text{s}$$

$$i_o(0) = 0; \qquad i_o(\infty) = \frac{120}{7.5} = 16\,\text{A}$$

$$\therefore \quad i_o = 16 - 16e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 120 - 7.5i_o = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

$$i_1 = \frac{1}{3}\int_0^t 120e^{-3x}\,dx = \frac{40}{3} - \frac{40}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$i_2 = i_o - i_1 = \frac{8}{3} - \frac{8}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

[b] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with initial conditions.
$v_o(0^+) = 120$ V, consistent with $i_o(0) = 0$.

$$v_o = 3\frac{di_1}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

or

$$v_o = 15\frac{di_2}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

The voltage solution is consistent with the current solutions.

$\lambda_1 = 3i_1 = 40 - 40e^{-3t}$ Wb-turns

$\lambda_2 = 15i_2 = 40 - 40e^{-3t}$ Wb-turns

$\therefore \quad \lambda_1 = \lambda_2$ as it must, since

$$v_o = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt}$$

$\lambda_1(\infty) = \lambda_2(\infty) = 40$ Wb-turns

$\lambda_1(\infty) = 3i_1(\infty) = 3(40/3) = 40$ Wb-turns

$\lambda_2(\infty) = 15i_2(\infty) = 15(8/3) = 40$ Wb-turns

$\therefore \quad i_1(\infty)$ and $i_2(\infty)$ are consistent with $\lambda_1(\infty)$ and $\lambda_2(\infty)$.

P 7.67 [a] From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{0.125 - 0.0625}{0.75 + 0.5} = 50\,\text{mH}$$

$$\tau = \frac{L}{R} = \frac{1}{5000}; \qquad \frac{1}{\tau} = 5000$$

$$\therefore \quad i_o(t) = 40 - 40e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[b] $v_o = 10 - 250i_o = 10 - 250(0.04 + 0.04e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 0.5\dfrac{di_1}{dt} - 0.25\dfrac{di_2}{dt} = 10e^{-5000t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 200e^{-5000t}\ \text{A/s}$$

$$\therefore \quad \frac{di_2}{dt} = 200e^{-5000t} - \frac{di_1}{dt}$$

$$\therefore \quad 10e^{-5000t} = 0.5\frac{di_1}{dt} - 50e^{-5000t} + 0.25\frac{di_1}{dt}$$

$$\therefore \quad 0.75\frac{di_1}{dt} = 60e^{-5000t}; \qquad di_1 = 80e^{-5000t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 80e^{-5000y}\,dy$$

$$i_1 = \frac{80}{-5000}e^{-5000y}\bigg|_0^t = 16 - 16e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 40 - 40e^{-5000t} - 16 + 16e^{-5000t}$

$$= 24 - 24e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = (0.05)(200)e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 0.5\frac{di_1}{dt} - 0.25\frac{di_2}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 0.25\frac{di_2}{dt} - 0.25\frac{di_1}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 10\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

$i_o(\infty) = 40\,\text{mA}; \qquad i_o(\infty)L_{\text{eq}} = (0.04)(0.05) = 2$ mWb-turns

$i_1(\infty)L_1 + i_2(\infty)M = (16\,\text{m})(500) + (24\,\text{m})(-250) = 2$ mWb-turns (ok)

$i_2(\infty)L_2 + i_1(\infty)M = (24\,\text{m})(250) + (16\,\text{m})(-250) = 2$ mWb-turns (ok)

Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.68 [a] $L_{\text{eq}} = 4 + 8 - 2(5) = 2\,\text{H}$

$$\tau = \frac{L}{R} = \frac{2}{50} = \frac{1}{25}; \qquad \frac{1}{\tau} = 25$$

$$i = 4 - 4e^{-25t}\,\text{A}, \quad t \geq 0$$

[b] $v_1(t) = 4\frac{di}{dt} - 5\frac{di}{dt} = -\frac{di}{dt} = -(100e^{-25t}) = -100e^{-25t}\,\text{V}, \quad t \geq 0^+$

[c] $v_2(t) = 8\frac{di}{dt} - 5\frac{di}{dt} = 3\frac{di}{dt} = 3(100e^{-25t}) = 300e^{-25t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 4 - 4 = 0$, which agrees with initial conditions.

$$200 = 50i_1 + v_1 + v_2 = 50(4 - 4e^{-25t}) - 100e^{-25t} + 300e^{-25t} = 200\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.69 [a] $L_{\text{eq}} = 4 + 8 + 2(5) = 22\,\text{H}$

$$\tau = \frac{L}{R} = \frac{22}{50}; \qquad \frac{1}{\tau} = 2.273$$

$$i = 4 - 4e^{-2.273t}\,\text{A}, \quad t \geq 0$$

[b] $v_1(t) = 4\frac{di}{dt} + 5\frac{di}{dt} = 9\frac{di}{dt} = 9(9.09e^{-2.273t}) = 81.81e^{-2.273t}\,\text{V}, \quad t \geq 0^+$

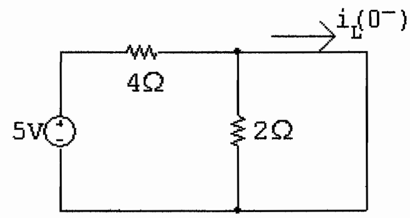
[c] $v_2(t) = 8\frac{di}{dt} + 5\frac{di}{dt} = 13\frac{di}{dt} = 13(9.09e^{-2.273t}) = 118.18e^{-2.273t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 0$, which agrees with initial conditions.

$$200 = 50i_1 + v_1 + v_2 = 50(4 - 4e^{-2.273t}) + 81.81e^{-2.273t} + 118.18e^{-2.273t} = 200\,\text{V}$$

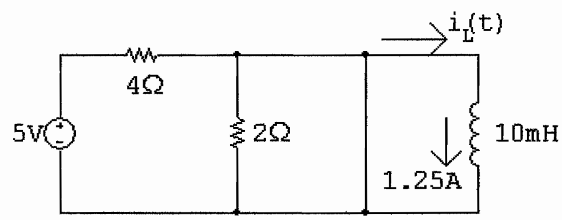
Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.70 $t < 0$:



$i_L(0^-) = 5/4 = 1.25\,\text{A} = i_L(0^+)$

$0 \leq t \leq 1$:

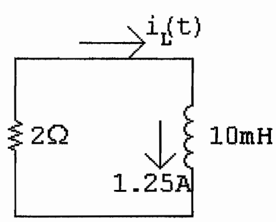


$\tau = 5/0 = \infty$

$i_L(t) = 1.25e^{-t/\infty} = 1.25e^{-0} = 1.25$

$i_L(t) = 1.25\,\text{A}$

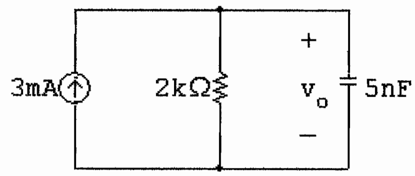
$1 \leq t < \infty$:



$$\tau = \frac{10 \times 10^{-3}}{2} = 5\,\text{ms}; \qquad 1/\tau = 200$$

$i_L(t) = 1.25e^{-200(t-1)}\,\text{A}$

P 7.71 $0 \le t \le 3\,\mu$s:

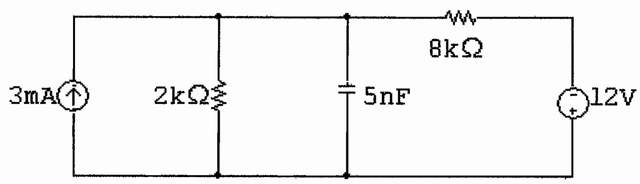


$$\tau = RC = (2\times10^3)(5\times10^{-9}) = 10\,\mu\text{s}; \qquad 1/\tau = 100{,}000$$

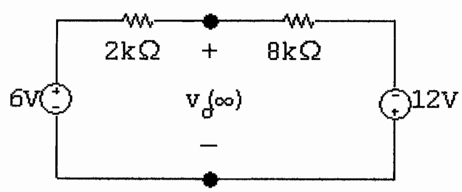
$$v_o(0) = 0\,\text{V}; \qquad v_o(\infty) = 6\,\text{V}$$

$$v_o = 6 - 6e^{-100{,}000t}\,\text{V} \qquad 0 \le t \le 3\,\mu\text{s}$$

$3\,\mu\text{s} \le t < \infty$:



$t = \infty$:



$$i = \frac{6-(-12)}{10} = 1.8\,\text{mA}$$

$$v_o(\infty) = 6 - 2i = 2.4\,\text{V}$$

$$v_o(3\,\mu\text{s}) = 6 - 6e^{-0.3} = 1.555\,\text{V}$$

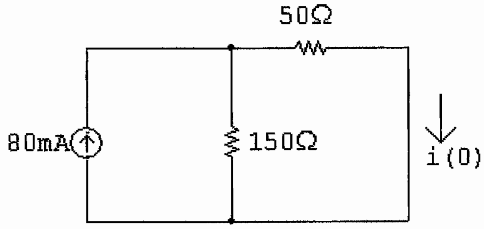
$$v_o = 2.4 + (1.555 - 2.4)e^{-(t-3\,\mu\text{s})/\tau}$$

$$R_{\text{Th}} = 2\,\text{k}\Omega \| 8\,\text{k}\Omega = 1.6\,\text{k}\Omega$$

$$\tau = (1.6)(5) = 8\,\mu\text{s}; \qquad 1/\tau = 125{,}000$$

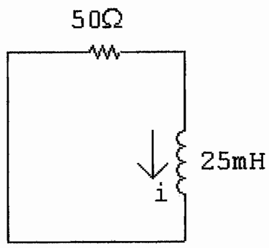
$$v_o = 2.4 - 0.845e^{-125{,}000(t-3\,\mu\text{s})} \qquad 3\,\mu\text{s} \le t < \infty$$

P 7.72 For $t < 0$:



$$i(0) = \frac{80(150)}{200} = 60\,\text{mA}$$

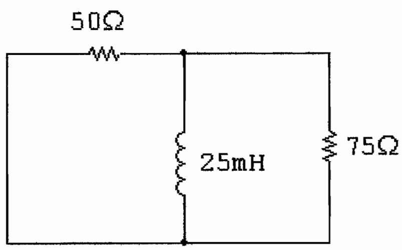
$0 \leq t \leq 250\,\mu\text{s}$:



$$i = 60e^{-2000t}\,\text{mA}$$

$$i(250\mu\text{s}) = 60e^{-0.5} = 36.39\,\text{mA}$$

$250\,\mu\text{s} \leq t \leq 650\,\mu\text{s}$:



$$R_{\text{eq}} = \frac{(50)(75)}{125} = 30\,\Omega$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{30}{25} \times 10^3 = 1200$$

$$i = 36.39e^{-1200(t-250\times10^{-6})}\,\text{mA}$$

$650\,\mu\text{s} \leq t < \infty$:

$$i(650\mu\text{s}) = 36.39e^{-0.48} = 22.52\,\text{mA}$$

$$i = 22.52e^{-2000(t-650\times10^{-6})}\,\text{mA}$$

$$v = L\frac{di}{dt}; \qquad L = 25\,\text{mH}$$

$$\frac{di}{dt} = 22.52(-2000)\times10^{-3}e^{-2000(t-650\times10^{-6})} = -45.04e^{-2000(t-650\times10^{-6})}$$

$$v = (25\times10^{-3})(-45.04)e^{-2000(t-650\times10^{-6})}$$

$$= -1.13e^{-2000(t-650\times10^{-6})}\,\text{V}, \qquad t > 650^{+}\,\mu\text{s}$$

$$v(1\text{ms}) = -1.13e^{-2000(350)\times10^{-6}} = -559.12\,\text{mV}$$

P 7.73 From the solution to Problem 7.72, the initial energy is

$$w(0) = \frac{1}{2}(25\text{ mH})(60\text{ mA})^2 = 45\,\mu\text{J}$$

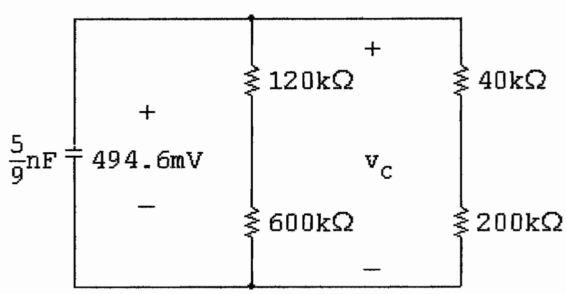
For $650\,\mu\text{s} \le t < \infty$:

$$w(t) = \frac{1}{2}(25\text{ mH})(22.52e^{-2000(t-650\times10^{-6})}\,\text{mA})^2 = (0.04)(45\,\mu\text{J})$$

Solving,

$$t = 964.72\,\mu\text{s}$$

P 7.74 $0 \le t \le 50\,\mu\text{s}$;

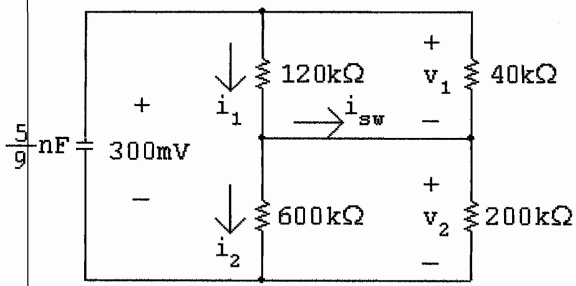


$$R_e = 720\|240 = 180\,\text{k}\Omega; \qquad \tau = \left(\frac{5}{9}\right)(180) = 100\,\mu\text{s}$$

$$v_c = 494.6e^{-10{,}000t}\,\text{mV}$$

$$v_c(50\,\mu\text{s}) = 494.6e^{-0.5} = 300\,\text{mV}$$

$50\,\mu s \le t < \infty$:



$$R_e = 120\|40 + 600\|200 = 30 + 150 = 180\,\text{k}\Omega$$

$$\tau = \left(\frac{5}{9}\right)(180) = 100\,\mu s; \qquad \frac{1}{\tau} = 10{,}000$$

$$v_c = 300e^{-10{,}000(t-50\,\mu s)}\ \text{mV}$$

$$v_1 = \frac{30}{180}v_c = 50e^{-10{,}000(t-50\,\mu s)}\ \text{mV}$$

$$v_2 = \frac{150}{180}v_c = 250e^{-10{,}000(t-50\,\mu s)}\ \text{mV}$$

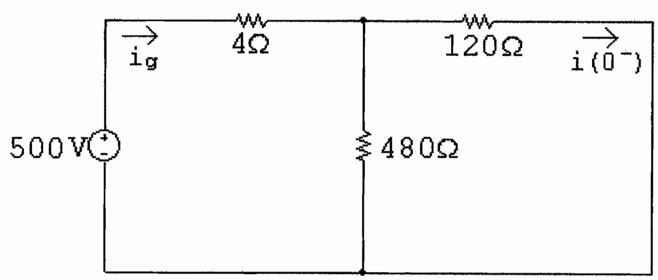
$$i_1 = \frac{v_1}{120\times 10^3} = 416.7e^{-10{,}000(t-50\,\mu s)}\ \text{nA}$$

$$i_2 = \frac{v_2}{600\times 10^3} = 416.7e^{-10{,}000(t-50\,\mu s)}\ \text{nA}$$

$$i_{sw} = i_1 - i_2 = 0\,\text{A}$$

$$i_{sw}(100\,\mu s) = 0\,\text{A}$$

P 7.75 [a] $t < 0$:



$$i_g = \frac{500}{4+96} = 5\,\text{A}$$

$$i(0^-) = \frac{5(480)}{600} = 4\,\text{A} = i(0^+)$$

[b] $0 \le t \le 100\,\mu$s:

$$i = 4e^{-t/\tau}$$

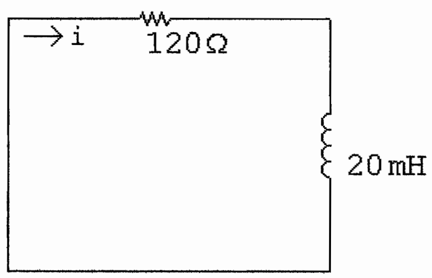
$$\frac{1}{\tau} = \frac{R}{L} = \frac{120 + 96\|480}{20 \times 10^{-3}} = 10{,}000$$

$$i = 4e^{-10{,}000t}$$

$$i(25\mu\text{s}) = 4e^{-10^4(25)\times 10^{-6}} = 4e^{-0.25} = 3.12\,\text{A}$$

[c] $i(100\mu\text{s}) = 4e^{-1} = 1.47\,\text{A}$

$100\,\mu\text{s} \le t < \infty$:



$$\frac{1}{\tau} = \frac{R}{L} = \frac{120}{20} \times 10^3 = 6000$$

$$i = 1.47e^{-6000(t-100\times 10^{-6})}\,\text{A}$$

$$i(200\mu\text{s}) = 1.47e^{-6000(100)\times 10^{-6}} = 1.47e^{-0.6} = 807.59\,\text{mA}$$

[d] $0 \le t \le 100\,\mu$s:

$$i = 4e^{-10{,}000t}$$

$$v = L\frac{di}{dt} = (20 \times 10^{-3})(4)(-10^4)e^{-10^4 t} = -800e^{-10^4 t}\,\text{V}$$

$$v(100^-\,\mu\text{s}) = -800e^{-10^4(100\times 10^{-6})} = -800e^{-1} = -294.30\,\text{V}$$

[e] $100\,\mu\text{s} \le t < \infty$:

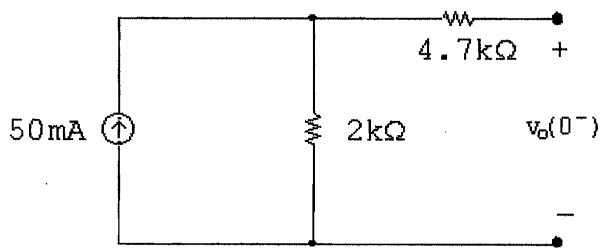
$$i = 1.47e^{-6000(t-100\times 10^{-6})}$$

$$v = (20 \times 10^{-3})(1.47)(-6000)e^{-6000(t-100\times 10^{-6})}$$

$$= -176.58e^{-6000(t-100\times 10^{-6})}\,\text{V}$$

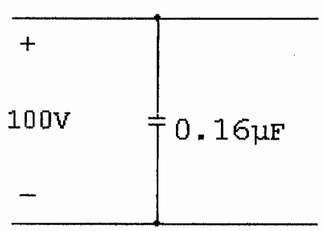
$$v(100^+\,\mu\text{s}) = -176.58\,\text{V}$$

P 7.76 $t < 0$:



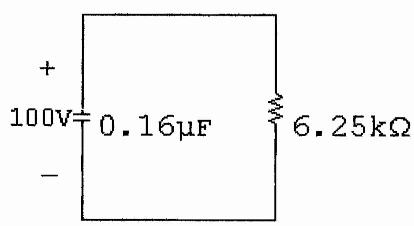
$$v_c(0^-) = (50)(2000) \times 10^{-3} = 100\,\text{V} = v_c(0^+)$$

$0 \le t \le 250\,\text{ms}$:



$\tau = \infty; \qquad 1/\tau = 0; \qquad v_o = 100e^{-0} = 100\,\text{V}$

$250\,\text{ms} \le t < \infty$:



$\tau = (6.25)(0.16)10^{-3} = 1\,\text{ms}; \qquad 1/\tau = 1000; \qquad v_o = 100e^{-1000(t-0.25)}\,\text{V}$

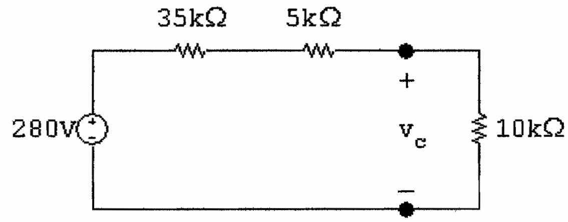
Summary:

$v_o = 100\,\text{V}, \qquad 0 \le t \le 250\,\text{ms}$

$v_o = 100e^{-1000(t-0.25)}\,\text{V}, \qquad 250\,\text{ms} \le t < \infty$

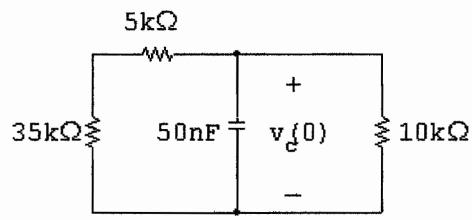
P 7.77 Note that for $t > 0$, $v_o = (35/40)v_c$, where v_c is the voltage across the 50 nF capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{280}{50}(10) = 56\,\text{V}$$

$0 \leq t \leq 400\,\mu$s:



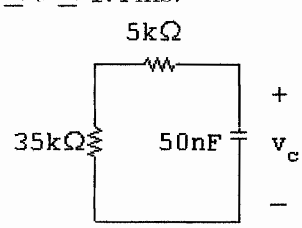
$$\tau = R_eC, \qquad R_e = \frac{(10)(40)}{50} = 8\,\text{k}\Omega$$

$$\tau = (8 \times 10^3)(50 \times 10^{-9}) = 400\,\mu\text{s}, \qquad \frac{1}{\tau} = 2500$$

$$v_c = 56e^{-2500t}\,\text{V}, \qquad t \geq 0$$

$$v_c(400\,\mu\text{s}) = 56e^{-1} = 20.60\,\text{V}$$

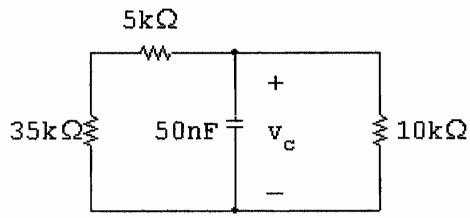
$400\,\mu\text{s} \leq t \leq 1.4\,\text{ms}$:



$$\tau = (40 \times 10^3)(50 \times 10^{-9}) = 2\,\text{ms}, \qquad \frac{1}{\tau} = 500$$

$$v_c = 20.60e^{-500(t-400\times10^{-6})}\,\text{V}$$

$1.4\,\text{ms} \leq t < \infty$:



$$\tau = 400\,\mu\text{s}, \qquad \frac{1}{\tau} = 2500$$

$$v_c(1.4\text{ms}) = 20.60e^{-500(1400-400)10^{-6}} = 20.60e^{-0.5} = 12.50\,\text{V}$$

$$v_c = 12.50e^{-2500(t-1.4\times10^{-3})}\,\text{V}$$

$$v_c(1.6\text{ms}) = 12.50e^{-2500(1.6-1.4)10^{-3}} = 12.50e^{-0.5} = 7.58\,\text{V}$$

$$v_o = (35/40)(7.58) = 6.63\,\text{V}$$

P 7.78 $w(0) = \frac{1}{2}(50 \times 10^{-9})(56)^2 = 78.4\,\mu\text{J}$

$0 \leq t \leq 400\,\mu\text{s}$:

$$v_c = 56e^{-2500t}: \qquad v_c^2 = 3136e^{-5000t}$$

$$p_{10k} = 3136 \times 10^{-4}e^{-5000t}$$

$$\begin{aligned} w_{10k} &= \int_0^{400\times10^{-6}} 3136 \times 10^{-4}e^{-5000t}\,dt \\ &= 3136 \times 10^{-4}\frac{e^{-5000t}}{-5000}\Bigg|_0^{400\times10^{-6}} \\ &= -6272 \times 10^{-8}(e^{-2} - 1) = 54.23\,\mu\text{J} \end{aligned}$$

$1.4\,\text{ms} \leq t < \infty$:

$$v_c = 12.50e^{-2500(t-1.4\times10^{-3})}\,\text{V}; \qquad v_c^2 = 156.13e^{-5000(t-1.4\times10^{-3})}$$

$$p_{10k} = 156.13 \times 10^{-4}e^{-5000(t-1.4\times10^{-3})}$$

$$\begin{aligned} w_{10k} &= \int_{1.4\times10^{-3}}^{\infty} 156.13 \times 10^{-4}e^{-5000(t-1.4\times10^{-3})}\,dt \\ &= 156.13 \times 10^{-4}\frac{e^{-5000(t-1.4\times10^{-3})}}{-5000}\Bigg|_{1.4\times10^{-3}}^{\infty} \\ &= -311.83 \times 10^{-8}(0 - 1) = 3.12\,\mu\text{J} \end{aligned}$$

$$w_{10k} = 54.23 + 3.12 = 57.35\,\mu\text{J}$$

$$\% = \frac{57.35}{78.4}(100) = 73.15\%$$

To check, find the energy dissipated in the 40 kΩ resistance:
$0 \leq t \leq 400\,\mu$s:

$$v_c = 56e^{-2500t}; \qquad v_c^2 = 3136e^{-5000t}$$

$$p_{40k} = \frac{3136}{40} \times 10^{-3}e^{-5000t}$$

$$w_{40k} = 784 \times 10^{-4}\frac{e^{-5000t}}{-5000}\Big|_0^{400\times10^{-6}}$$
$$= -156.8(10^{-7})(e^{-2} - 1) = 13.56\,\text{mJ}$$

$400\,\mu\text{s} \leq t \leq 1\,\text{ms}$:

$$v_c = 20.60e^{-500(t-400\times10^{-6})}; \qquad v_c^2 = 424.41e^{-1000(t-400\times10^{-6})}$$

$$w_{40k} = 106.10 \times 10^{-4}\int_{400\times10^{-6}}^{10^{-3}}$$
$$= 106.10 \times 10^{-4}\frac{e^{-1000(t-400\times10^{-6})}}{-1000}\Big|_{400\times10^{-6}}^{10^{-3}}$$
$$= -106.10(10^{-7})(e^{-0.6} - 1) = 4.79\,\text{mJ}$$

$1.4\,\text{ms} \leq t < \infty$:

$$v_c = 12.49e^{-2500(t-1.4\times10^{-3})}; \qquad v_c^2 = 156.13e^{-5000(t-1.4\times10^{-3})}$$

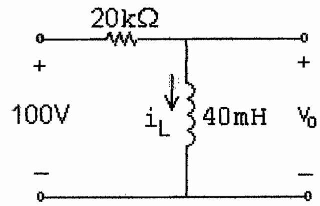
Note in this interval the energy dissipated in the 40k resistor will be 1/4th that dissipated in the 10k resistor.

$$w_{40k} = \frac{1}{4}(3.12) = 0.78\,\mu\text{J}$$

$$w_{40k} = 13.56 + 6.71 + 0.78 = 21.05\,\mu\text{J}$$

$$w_{40k} + w_{10k} = 57.35 + 21.05 = 78.40\,\mu\text{J}$$

P 7.79 [a] $0 \le t \le 2\,\mu$s



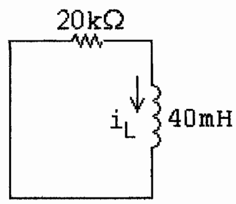
$i_L(0) = 0; \qquad i_L(\infty) = 5\,$mA

$\tau = \dfrac{L}{R} = \dfrac{0.04}{20{,}000} = 2\,\mu$s

$i_L = 5 - 5e^{-500{,}000t}\,$mA, $\qquad 0 \le t \le 2\,\mu$s

$v_o = (0.04)[(500{,}000)(0.005)e^{-500{,}000t}] = 100e^{-500{,}000t}\,$V, $\qquad 0^+ \le t < 2\,\mu$s

$2\,\mu\text{s} \le t < \infty$



$i_L(2\,\mu\text{s}) = 5 - 5e^{-1} \approx 3.16\,$mA

$i_L(\infty) = 0; \qquad \tau = 2\,\mu$s; $\qquad 1/\tau = 500{,}000$

$i_L = 0 + (3.16 - 0)e^{-500{,}000(t-2\,\mu\text{S})}\,$mA

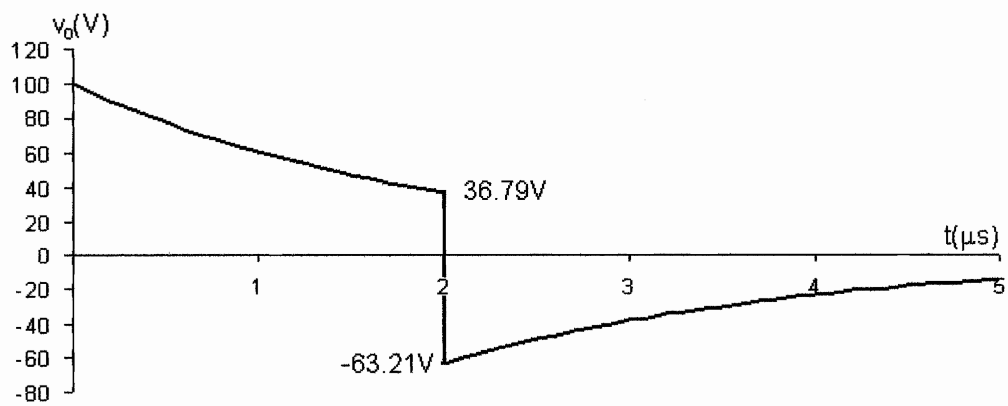
$\quad = 3.16e^{-500{,}000(t-2\,\mu\text{S})}\,$mA, $\qquad 2\,\mu\text{s} \le t < \infty$

$v_o = L\dfrac{di_L}{dt} = (0.04)(3.16 \times 10^{-3})[-500{,}000e^{-500{,}000(t-2\,\mu\text{S})}]$

$\quad = (-5)(4)(3.16)e^{-500{,}000(t-2\,\mu\text{S})}$

$\quad = -63.21e^{-500{,}000(t-2\,\mu\text{S})}\,$V, $\qquad 2\,\mu\text{s} \le t < \infty$

[b]



[c] $v_o(4\,\mu s) = -23.25\,V$

$$i_o = \frac{+23.25}{20{,}000} = 1.16\,mA$$

P 7.80 [a] $i_o(0) = 0; \qquad i_o(\infty) = 25\,mA$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{8000}{250} \times 10^3 = 32{,}000$$

$$i_o = (25 - 25e^{-32{,}000t})\,mA, \qquad 0 \le t \le 50\,\mu s$$

$$v_o = 0.25\frac{di_o}{dt} = 200e^{-32{,}000t}\,V, \qquad 0 \le t \le 50\,\mu s$$

$50\,\mu s \le t < \infty$:

$$i_o(50\mu s) = 25 - 25e^{-1.6} = 19.95\,mA; \qquad i_o(\infty) = 0$$

$$i_o = 19.95e^{-32{,}000(t-50\times10^{-6})}\,mA$$

$$v_o = (0.25)\frac{di_o}{dt} = -159.62e^{-32{,}000(t-50\mu s)}$$

$$\begin{aligned} \therefore\quad t < 0: \qquad v_o &= 0 \\ 0 \le t \le 50\,\mu s: \qquad v_o &= 200e^{-32{,}000t}\,V \\ 50\,\mu s \le t < \infty: \qquad v_o &= -159.62e^{-32{,}000(t-50\mu S)} \end{aligned}$$

[b] $v_o(50^-\mu s) = 200e^{-1.6} = 40.38\,V$

$v_o(50^+\mu s) = -159.62\,V$

[c] $i_o(50^-\mu s) = i_o(50^+\mu s) = 19.95\,mA$

P 7.81 [a] $0 \le t \le 6\,ms$:

$v_c(0^+) = 0; \qquad v_c(\infty) = 40\,V;$

$RC = 500 \times 10^3(0.02 \times 10^{-6}) = 10\,ms; \qquad 1/RC = 100$

$v_c = 40 - 40e^{-100t}$

$v_o = 40 - 40 + 40e^{-100t} = 40e^{-100t}\,V, \qquad 0 \le t \le 6\,ms$

$6\,ms \le t < \infty$:

$v_c(6\,ms) = 40 - 40e^{-0.6} = 18.05\,V$

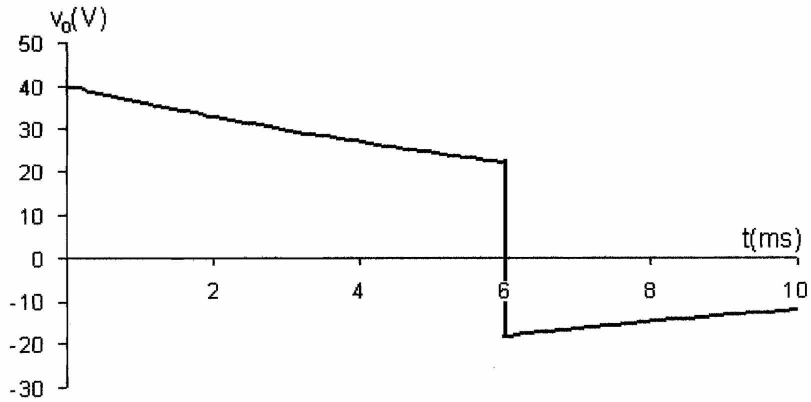
$v_c(\infty) = 0\,V$

$\tau = 10\,ms; \qquad 1/\tau = 100$

$v_c = 18.05e^{-100(t-0.006)}\,V$

$v_o = -v_c = -18.05e^{-100(t-0.006)}\,V, \qquad t \ge 6\,ms$

[b]



P 7.82 [a] $t < 0; \qquad v_o = 0$

$0 \le t \le 10\,\text{ms}$:

$$\tau = (50)(0.4) \times 10^{-3} = 20\,\text{ms}; \qquad 1/\tau = 50$$

$$v_o = 40 - 40e^{-50t}\,\text{V}, \qquad 0 \le t \le 10\,\text{ms}$$

$$v_o(10\,\text{ms}) = 40(1 - e^{-0.5}) = 15.74\,\text{V}$$

$10\,\text{ms} \le t \le 20\,\text{ms}$:

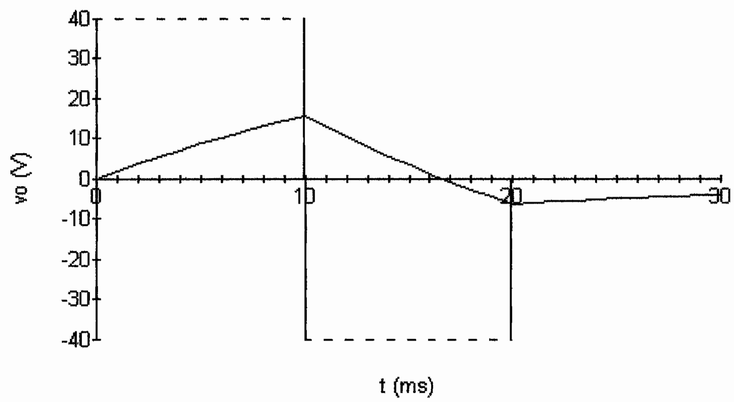
$$v_o = -40 + 55.74e^{-50(t-0.01)}\,\text{V}$$

$$v_o(20\,\text{ms}) = -40 + 55.74e^{-0.5} = -6.19\,\text{V}$$

$20\,\text{ms} \le t \le \infty$:

$$v_o = -6.19e^{-50(t-0.02)}\,\text{V}$$

[b]



[c] $t \le 0: \qquad v_o = 0$

$0 \le t \le 10\,\text{ms}$:

$$\tau = 10(0.4 \times 10^{-3}) = 4\,\text{ms}$$

$v_o = 40 - 40e^{-250t}\,\text{V}, \qquad 0 \le t \le 10\,\text{ms}$

$v_o(10\,\text{ms}) = 40 - 40e^{-2.5} = 36.72\,\text{V}$

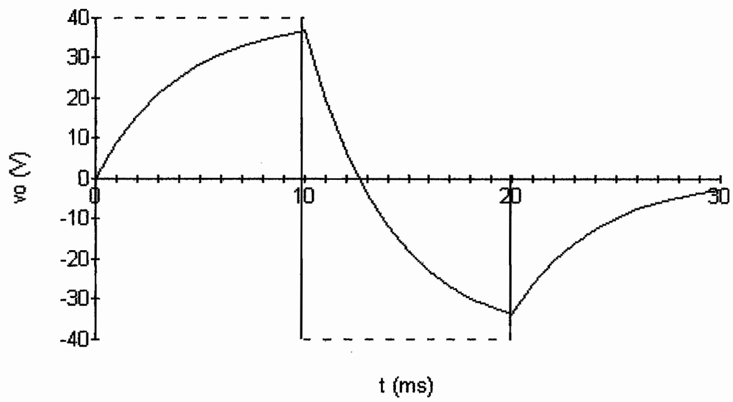
$10\,\text{ms} \le t \le 20\,\text{ms}$:

$v_o = -40 + 76.72e^{-250(t-0.01)}\,\text{V}, \qquad 10\,\text{ms} \le t \le 20\,\text{ms}$

$v_o(20\,\text{ms}) = -40 + 76.72e^{-2.5} = -33.7\,\text{V}$

$20\,\text{ms} \le t \le \infty$:

$v_o = -33.7e^{-250(t-0.02)}\,\text{V}, \qquad 20\,\text{ms} \le t \le \infty$



P 7.83 [a] $\tau = RC = (8000)(100) \times 10^{-9} = 800\,\mu\text{s}; \qquad 1/\tau = 1250$

$i_o = v_o = 0 \qquad t < 0$

$i_o(0^+) = 20\left(\frac{6}{8}\right) = 15\,\text{mA}, \qquad i_o(\infty) = 0$

$\therefore \quad i_o = 15e^{-1250t}\,\text{mA} \qquad 0^+ \le t \le 0.5^-\,\text{ms}$

$i_{6k\Omega} = 20 - 15e^{-1250t}\,\text{mA}$

$\therefore \quad v_o = 120 - 90e^{-1250t}\,\text{V} \qquad 0^+ \le t \le 05^-\,\text{ms}$

$v_c = v_o - 2 \times 10^3 i_o = 120 - 120e^{-1250t}\,\text{V} \qquad 0 \le t \le 0.5\,\text{ms}$

$v_c(0.5\,\text{ms}) = 120 - 120e^{-0.625} = 55.77\,\text{V}$

$\therefore \quad i_o(0.5^+\,\text{ms}) = \frac{-55.77}{8} = -6.97\,\text{mA}$

$i_o(\infty) = 0$

$i_o = -6.97e^{-1250(t-500\mu s)}\,\text{mA}, \qquad 0.5^+\,\text{ms} \le t < \infty$

$v_o = -6000 i_o = 41.83e^{-1250(t-500\mu s)}\,\text{V} \qquad 0.5^+\,\text{ms} \le t < \infty$

Summary part (a)

$i_o = 0 \qquad t < 0$

$i_o = 15e^{-1250t}\,\text{mA} \qquad (0^+ \le t \le 0.5^-\,\text{ms})$

$i_o = -6.97e^{-1250(t-500\mu s)}\,\text{mA} \qquad 0.5^+\,\text{ms} \le t < \infty$

$v_o = 0 \qquad t < 0$

$v_o = 120 - 90e^{-1250t}\,\text{V}, \qquad 0 \le t \le 0.5^-\,\text{ms}$

$v_o = 41.83e^{-1250(t-500\mu s)}\,\text{V}, \qquad 0.5^+\,\text{ms} \le t < \infty$

[b] $i_o(0^-) = 0$

$i_o(0^+) = 15\,\text{mA}$

$i_o(0.5^-\,\text{ms}) = 15e^{-0.625} = 8.03\,\text{mA}$

$i_o(0.5^+\,\text{ms}) = -6.97\,\text{mA}$

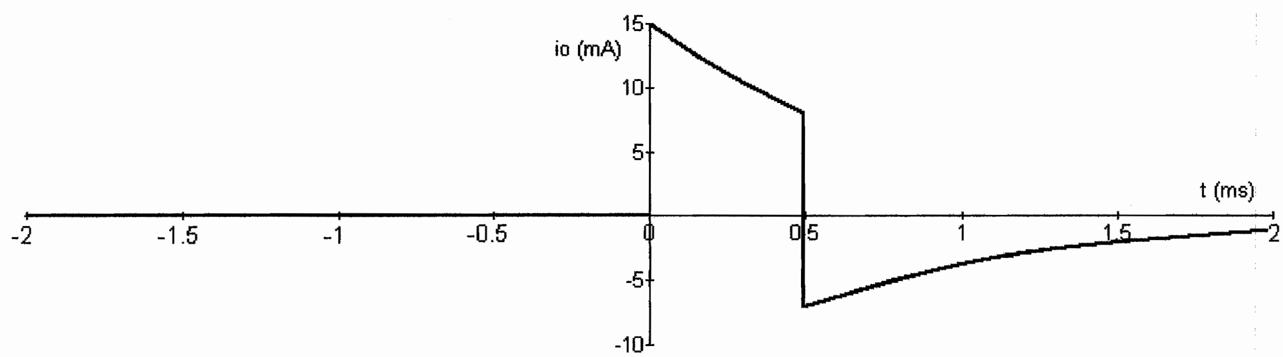
[c] $v_o(0^-) = 0$

$v_o(0^+) = 30\,\text{V}$

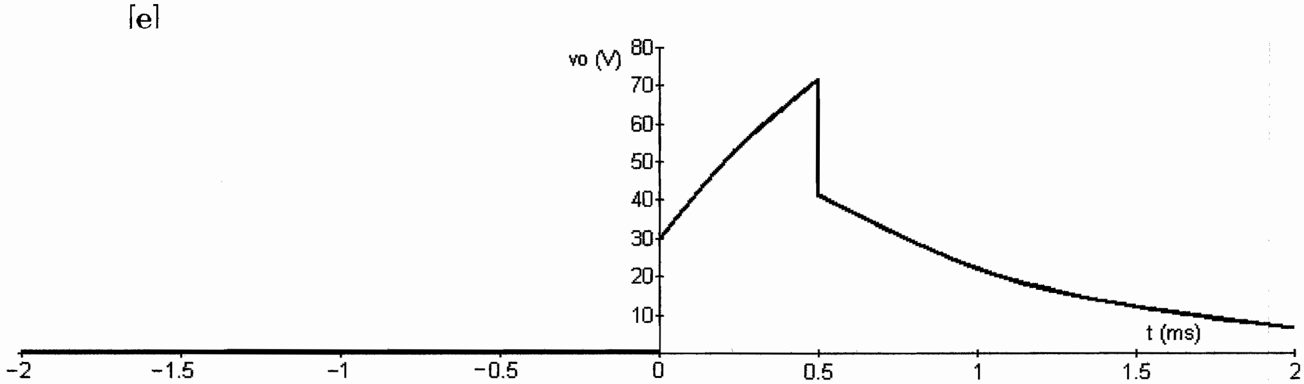
$v_o(0.5^-\,\text{ms}) = 120 - 90e^{-0.625} = 71.83\,\text{V}$

$v_o(0.5^+\,\text{ms}) = 41.83$

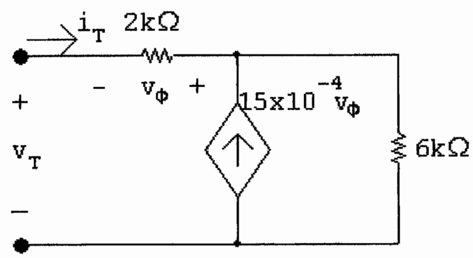
[d]



[e]



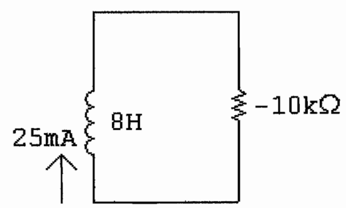
P 7.84



$$v_T = 2000i_T + 6000(i_T + 15 \times 10^{-4}v_\phi) = 8000i_T + 9v_\phi$$

$$= 8000i_T + 9(-2000i_T)$$

$$\frac{v_T}{i_T} = -10{,}000$$

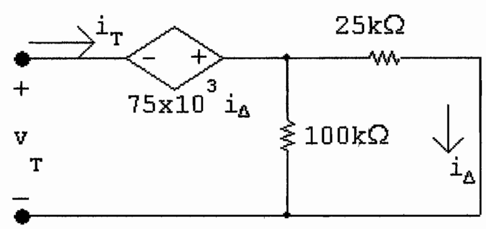


$$\tau = \frac{8}{-10} \times 10^{-3} = -0.8\,\text{ms}; \qquad 1/\tau = -1250$$

$$i = 25e^{1250t}\,\text{mA}$$

$$\therefore \quad 25e^{1250t} \times 10^{-3} = 12; \qquad t = \frac{\ln 480}{1250} = 4.94\,\text{ms}$$

P 7.85 $t > 0$:



$$v_T = -75 \times 10^3 i_\Delta + 20 \times 10^3 i_T$$

$$i_\Delta = \frac{100}{125} i_T = 0.8i_T$$

$$\therefore \quad v_T = -60 \times 10^3 i_T + 20 \times 10^3 i_T$$

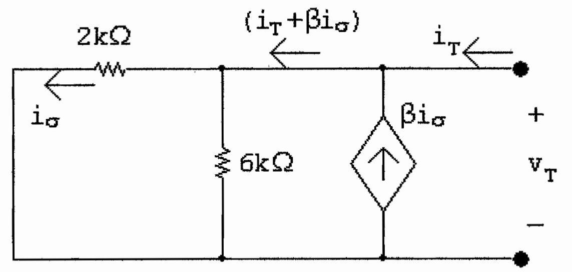
$$R_{\text{Th}} = \frac{v_T}{i_T} = -40\,\text{k}\Omega$$

$$\tau = RC = -40 \times 10^3(0.025) \times 10^{-6} = -10^{-3}$$

$$v_c = 25e^{1000t}\,\text{V}; \qquad 25e^{1000t} = 50{,}000$$

$$1000t = \ln 2000 \quad \therefore \quad t = 7.6\,\text{ms}$$

P 7.86 [a]



$$v_T = 2000 i_\sigma$$

$$i_\sigma = \frac{6}{8}(i_T + \beta i_\sigma) = 0.75 i_T + 0.75\beta i_\sigma$$

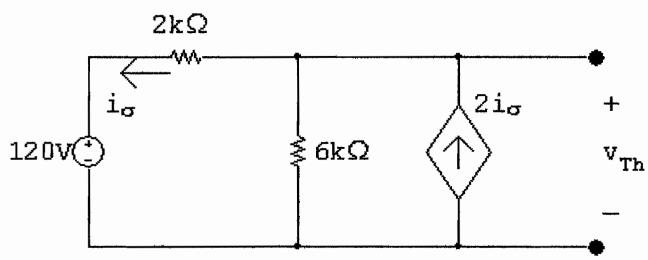
$$i_\sigma(1 - 0.75\beta) = 0.75 i_T$$

$$i_\sigma = \frac{0.75 i_T}{1 - 0.75\beta}; \qquad 2000 i_\sigma = \frac{1500 i_T}{(1 - 0.75\beta)}$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = \frac{1500}{1 - 0.75\beta} = -3000$$

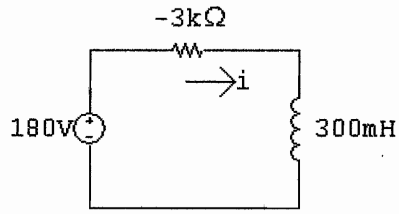
$$1 - 0.75\beta = -0.5 \qquad \therefore \quad \beta = 2$$

[b] Find V_{Th};



$$\frac{V_{\text{Th}} - 120}{2000} + \frac{V_{\text{Th}}}{6000} - 2\frac{(V_{\text{Th}} - 120)}{2000} = 0$$

$$V_{\text{Th}} = 180\,\text{V}$$



$$180 = -3000i + 0.3\frac{di}{dt}$$

$$\frac{di}{dt} = 600 + 10{,}000i = 10{,}000(i + 0.06)$$

$$\frac{di}{i + 0.06} = 10{,}000\,dt$$

$$\int_0^i \frac{dx}{x + 0.06} = \int_0^t 10{,}000\,dx$$

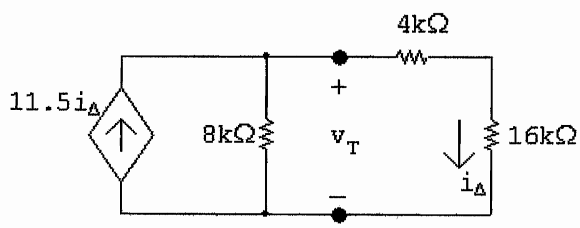
$$\therefore \quad i = -60 + 60e^{10{,}000t}\,\text{mA}$$

$$\frac{di}{dt} = (60 \times 10^{-3})(10{,}000)e^{10{,}000t} = 600e^{10{,}000t}$$

$$v = 0.3\frac{di}{dt} = 180e^{10{,}000t} = 36{,}000; \qquad e^{10{,}000t} = 200$$

$$\therefore \quad t = \frac{\ln 200}{10{,}000} = 529.83\,\mu\text{s}$$

P 7.87 Find the Thévenin equivalent with respect to the terminals of the capacitor. R_{Th} calculation:

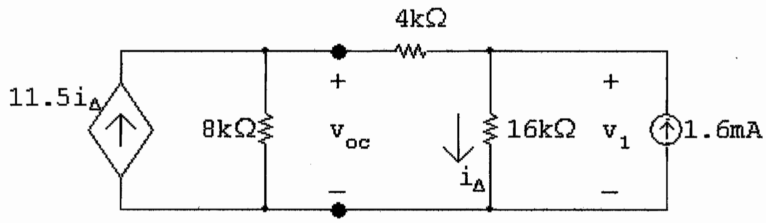


$$i_T = \frac{v_T}{8000} + \frac{v_T}{20{,}000} - 11.5\frac{v_T}{20{,}000}$$

$$\frac{i_T}{v_T} = \frac{2.5 + 1 - 11.5}{20{,}000} = \frac{-8}{20{,}000}$$

$$\therefore \quad \frac{v_T}{i_T} = \frac{-20{,}000}{8} = -2500\,\Omega$$

Open circuit voltage calculation:

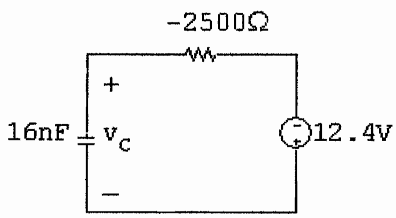


$$\frac{v_{oc}}{8000} + \frac{v_{oc} - v_1}{4000} - 11.5i_\Delta = 0$$

$$\frac{v_1 - v_{oc}}{4000} + \frac{v_1}{16{,}000} - 1.6 \times 10^{-3} = 0$$

$$i_\Delta = \frac{v_1}{16{,}000}$$

Solving, $v_{oc} = -12.4\,\text{V}$



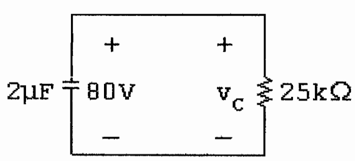
$v_c(0) = 0; \qquad v_c(\infty) = -12.4\,\text{V}$

$\tau = RC = (-2500)(16 \times 10^{-9}) = -40 \times 10^{-6}; \qquad \frac{1}{\tau} = -25{,}000$

$v_c = -12.4 + 12.4e^{25{,}000t} = 930$

$e^{25{,}000t} = 76; \qquad 25{,}000t = \ln 76; \qquad t = 173.23\,\mu\text{s}$

P 7.88 [a]



$\tau = (25)(2) \times 10^{-3} = 50\,\text{ms}; \qquad 1/\tau = 20$

$$v_c(0^+) = 80\,\text{V}; \qquad v_c(\infty) = 0$$

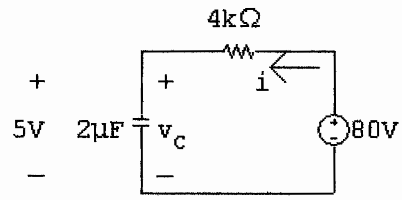
$$v_c = 80e^{-20t}\,\text{V}$$

$$\therefore \quad 80e^{-20t} = 5; \qquad e^{20t} = 16; \qquad t = \frac{\ln 16}{20} = 138.63\,\text{ms}$$

[b] $0^+ < t < 138.63\,\text{ms}$:

$$i = (2 \times 10^{-6})(-1600e^{-20t}) = -3.2e^{-20t}\,\text{mA}$$

$t \geq 138.63^+\,\text{ms}$:



$$\tau = (2)(4) \times 10^{-3} = 8\,\text{ms}; \qquad 1/\tau = 125$$

$$v_c(138.63^+\,\text{ms}) = 5\,\text{V}; \qquad v_c(\infty) = 80\,\text{V}$$

$$v_c = 80 - 75e^{-125(t-0.13863)}\,\text{V}, \qquad t \geq 138.63\,\text{ms}$$

$$i = 2 \times 10^{-6}(9375)e^{-125(t-0.13863)}$$
$$= 18.75e^{-125(t-0.13863)}\,\text{mA}, \qquad t \geq 138.63^+\,\text{ms}$$

[c] $80 - 75e^{-125\Delta t} = 0.85(80) = 68$

$$80 - 68 = 75e^{-125\Delta t} = 12$$

$$e^{125\Delta t} = 6.25; \qquad \Delta t = \frac{\ln 6.25}{125} \cong 14.66\,\text{ms}$$

P 7.89 $\displaystyle\frac{0 - 15}{R} - 60 \times 10^{-9}\frac{dv_o}{dt} = 0$

$$\therefore \quad v_o = \frac{-250 \times 10^6 t}{R}$$

$$\therefore \quad R = \frac{(-250 \times 10^6)(3 \times 10^{-3})}{-15} = 50 \times 10^3 = 50\,\text{k}\Omega$$

P 7.90 $\displaystyle\frac{0 - 15}{R} - C\frac{dv_o}{dt} = 0; \qquad dv_o = \frac{-15}{RC}\,dt$

$$v_o - v_o(0) = \frac{-15}{RC}t$$

$$v_o = \frac{-15}{RC}t + v_o(0) = \frac{-250 \times 10^6 t}{R} + 5 = -15$$

$$\therefore \quad R = \frac{250 \times 10^6(8 \times 10^{-3})}{20} = 100\,\text{k}\Omega$$

P 7.91 [a] $\frac{Cdv_p}{dt} + \frac{v_p - v_b}{R} = 0;$ therefore $\frac{dv_p}{dt} + \frac{1}{RC}v_p = \frac{v_b}{RC}$

$$\frac{v_n - v_a}{R} + C\frac{d(v_n - v_o)}{dt} = 0;$$

therefore $\frac{dv_o}{dt} = \frac{dv_n}{dt} + \frac{v_n}{RC} - \frac{v_a}{RC}$

But $v_n = v_p$

Therefore $\frac{dv_n}{dt} + \frac{v_n}{RC} = \frac{dv_p}{dt} + \frac{v_p}{RC} = \frac{v_b}{RC}$

Therefore $\frac{dv_o}{dt} = \frac{1}{RC}(v_b - v_a);$ $v_o = \frac{1}{RC}\int_0^t (v_b - v_a)\,dy$

[b] The output is the integral of the difference between v_b and v_a and then scaled by a factor of $1/RC$.

[c] $v_o = \frac{1}{RC}\int_0^t (v_b - v_a)\,dx$

$$RC = (40) \times 10^3(25) \times 10^{-9} = 1\,\text{ms}$$

$$v_b - v_a = 50\,\text{mV}$$

$$v_o = 50\int_0^t dx = 50t; \qquad 50t_{sat} = 12; \qquad t_{sat} = 240\,\text{ms}$$

P 7.92 $v_2 = \frac{15(20)}{(50)} = 6\,\text{V}$

$$\frac{6+4}{50{,}000} + C\frac{d}{dt}(6 - v_o) = 0$$

$$\therefore \quad \frac{dv_o}{dt} = \frac{10 \times 10^6}{50{,}000(0.5)} = 400$$

$$dv_o = 400\,dt; \qquad v_o = 400t + v_o(0)$$

$$v_o(0) = 6 - 16 = -10\,\text{V}$$

$$\therefore \quad v_o = 400t - 10\,\text{V}$$

$$0 = 400t_o - 10$$

$$t_o = \frac{10}{400} = 25\,\text{ms}$$

P 7.93 $v_o = \frac{1}{RC}\int_0^t (v_b - v_a)\,dy + v_o(0)$

$$RC = (40 \times 10^3)(12.5 \times 10^{-9}) = 500 \times 10^{-6} = 0.5\,\text{ms}$$

$$\frac{1}{RC} = 2000; \qquad v_b - v_a = 10 - (-5) = 15\,\text{mV}$$

$$v_o(0) = 15 - 45 = -30\,\text{mV}$$

$$v_o = (2000)(15) \times 10^{-3}t - 30 \times 10^{-3} = (30{,}000t - 30)\,\text{mV}$$

$$v_2 = 10 + (15 - 10)e^{-2000t}\,\text{mV} = [10 + 5e^{-2000t}]\,\text{mV}$$

$$v_f = v_o - v_p = (30{,}000t - 40 - 5e^{-2000t})\,\text{mV}$$

P 7.94 [a] $RC = 40(50) \times 10^{-6} = 2\,\text{ms}; \qquad \frac{1}{RC} = 500; \qquad v_o = 0, \quad t < 0$

[b] $0 \le t \le 50\,\text{ms}:$

$$v_o = -500\int_0^t -0.50\,dx = 250t\,\text{V}$$

[c] $50\,\text{ms} \le t \le 100\,\text{ms};$

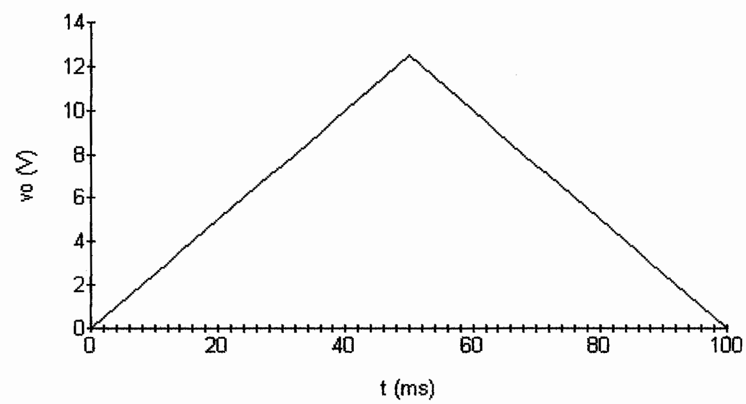
$$v_o(0.05) = 250(0.05) = 12.5\,\text{V}$$

$$v_o(t) = -500\int_{0.05}^t 0.50\,dx + 12.5 = -250(t - 0.05) + 12.5 = -250t + 25\,\text{V}$$

[d] $100\,\text{ms} \le t \le \infty:$

$$v_o(0.1) = -25 + 25 = 0\,\text{V}$$

$$v_o(t) = 0\,\text{V}$$



P 7.95 Write a KCL equation at the inverting input to the op amp, where the voltage is 0:

$$\frac{0-v_g}{R_i} + \frac{0-v_o}{R_f} + C_f\frac{d}{dt}(0-v_o) = 0$$

$$\therefore \qquad \frac{dv_o}{dt} + \frac{1}{R_fC_f}v_o = -\frac{v_g}{R_i}$$

Note that this first-order differential equation is in the same form as Eq. 7.50 if $I_s = -v_g/R_i$. Therefore, its solution is the same as Eq. 7.51:

$$v_o = \frac{-v_gR_f}{R_i} + \left(V_o - \frac{-v_gR_f}{R_i}\right)e^{-t/R_fC_f}$$

[a] $v_o = 0, \qquad t < 0$

[b] $R_fC_f = (4\times10^6)(50\times10^{-9}) = 0.2; \qquad \frac{1}{R_fC_f} = 5$

$$\frac{-v_gR_f}{R_i} = \frac{-(-0.5)(4\times10^6)}{40{,}000} = 50$$

$$V_o = v_o(0) = 0$$

$$\therefore \qquad v_o = 50 + (0-50)e^{-5t} = 50(1-e^{-5t})\,\text{V}, \qquad 0 \le t \le 50\,\text{ms}$$

[c] $\frac{1}{R_fC_f} = 5$

$$\frac{-v_gR_f}{R_i} = \frac{-(0.5)(4\times10^6)}{40{,}000} = -50$$

$$V_o = v_o(0.05) = 50(1-e^{-0.25}) \cong 11.06\,\text{V}$$

$$\therefore \qquad v_o = -50 + [11.06 - (-50)]e^{-5(t-0.05)}$$

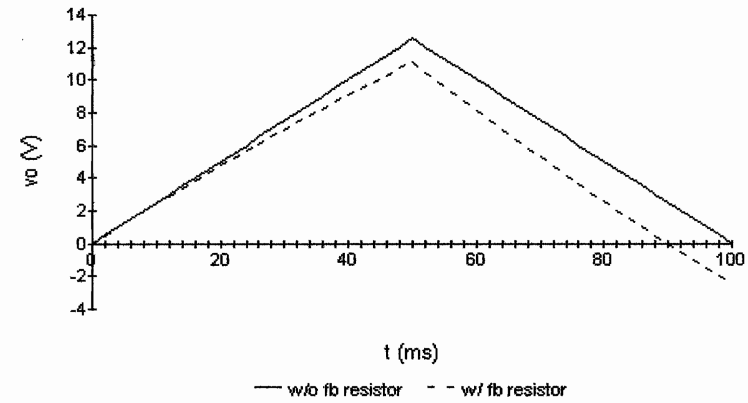
$$= 61.06e^{-5(t-0.05)} - 50\,\text{V}, \quad 50\,\text{ms} \le t \le 100\,\text{ms}$$

[d] $\frac{1}{R_fC_f} = 5$

$$\frac{-v_gR_f}{R_i} = 0$$

$$V_o = v_o(0.10) = 61.06e^{-0.25} - 50 \cong -2.45\,\text{V}$$

$$v_o = 0 + (-2.45-0)e^{-5(t-0.1)} = -2.45e^{-5(t-0.1)}\,\text{V}, \qquad 100\,\text{ms} \le t \le \infty$$



P 7.96 [a] $RC = (200 \times 10^3)(25 \times 10^{-9}) = 5 \times 10^{-3}; \qquad \frac{1}{RC} = 200$

$0 \le t \le 5\,\mu s$:

$$v_g = 0.6 \times 10^6 t$$

$$v_o = -200 \int_0^t 0.6 \times 10^6 x\,dx + 0$$

$$= -12 \times 10^7 \frac{x^2}{2} \Big|_0^t = -6 \times 10^7 t^2$$

$$v_o(5\,\mu s) = -6 \times 10^7 (5 \times 10^{-6})^2 = -1.5 \times 10^{-3}\,\text{V}$$

$5\,\mu s \le t \le 15\,\mu s$:

$$v_g = 6 - 0.6 \times 10^6 t$$

$$v_o = -200 \int_{5\times10^{-6}}^t (6 - 0.6 \times 10^6 x)\,dx - 1.5 \times 10^{-3}$$

$$= -\left[1200x \Big|_{5\times10^{-6}}^t + 12 \times 10^7 \frac{x^2}{2} \Big|_{5\times10^{-6}}^t\right] - 1.5 \times 10^{-3}$$

$$= -1200t + 6 \times 10^{-3} + 6 \times 10^7 t^2 - 1.5 \times 10^{-3} - 1.5 \times 10^{-3}$$

$$= 6 \times 10^7 t^2 - 1200t + 3 \times 10^{-3}$$

$$v_o(15\,\mu s) = 6 \times 10^7 (15 \times 10^{-6})^2 - 1200(15 \times 10^{-6}) + 3 \times 10^{-3}$$

$$= -1.5 \times 10^{-3}$$

$15\,\mu s \le t \le 20\,\mu s$:

$$v_g = -12 + 0.6 \times 10^6 t$$

$$v_o = -200 \int_{15\times10^{-6}}^t (-12 + 0.6 \times 10^6 x)\,dx - 1.5 \times 10^{-3}$$

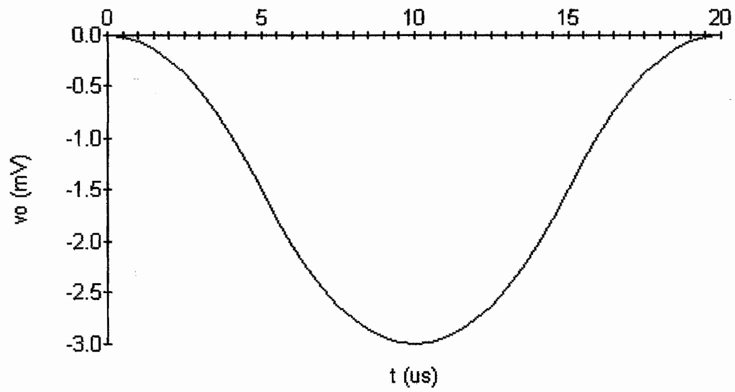
$$= -\left[2400x \Big|_{15\times10^{-6}}^t - 12 \times 10^7 \frac{x^2}{2} \Big|_{15\times10^{-6}}^t\right] - 1.5 \times 10^{-3}$$

$$= 2400t - 36 \times 10^{-3} - 6 \times 10^7 t^2 + 13.5 \times 10^{-3} - 1.5 \times 10^{-3}$$

$$= -6 \times 10^7 t^2 + 2400t - 24 \times 10^{-3}$$

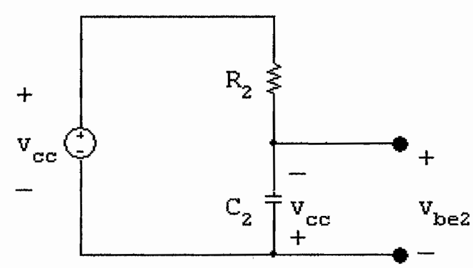
$$v_o(20\,\mu\text{s}) = -6 \times 10^7(20 \times 10^{-6})^2 + 2400(20 \times 10^{-6}) - 24 \times 10^{-3} = 0$$

[b]



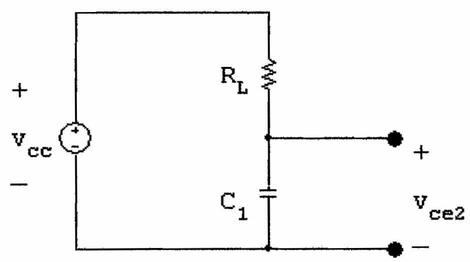
[c] The output voltage will also repeat. This follows from the observation that at $t = 20\,\mu$s the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 20\,\mu$s as it was at $t = 0$, thus as v_g repeats itself, so will v_o.

P 7.97 [a] While T_2 has been ON, C_2 is charged to V_{CC}, positive on the left terminal. At the instant T_1 turns ON the capacitor C_2 is connected across $b_2 - e_2$, thus $v_{be2} = -V_{CC}$. This negative voltage snaps T_2 OFF. Now the polarity of the voltage on C_2 starts to reverse, that is, the right-hand terminal of C_2 starts to charge toward $+V_{CC}$. At the same time, C_1 is charging toward V_{CC}, positive on the right. At the instant the charge on C_2 reaches zero, v_{be2} is zero, T_2 turns ON. This makes $v_{be1} = -V_{CC}$ and T_1 snaps OFF. Now the capacitors C_1 and C_2 start to charge with the polarities to turn T_1 ON and T_2 OFF. This switching action repeats itself over and over as long as the circuit is energized. At the instant T_1 turns ON, the voltage controlling the state of T_2 is governed by the following circuit:



It follows that $v_{be2} = V_{CC} - 2V_{CC}e^{-t/R_2C_2}$.

[b] While T_2 is OFF and T_1 is ON, the output voltage v_{ce2} is the same as the voltage across C_1, thus



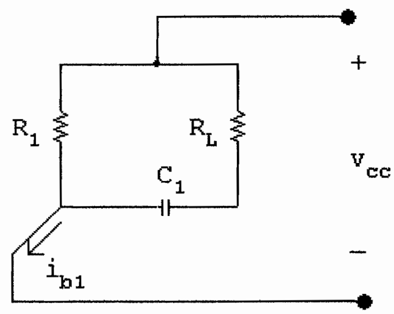
It follows that $v_{ce2} = V_{CC} - V_{CC}e^{-t/R_L C_1}$.

[c] T_2 will be OFF until v_{be2} reaches zero. As soon as v_{be2} is zero, i_{b2} will become positive and turn T_2 ON. $v_{be2} = 0$ when $V_{CC} - 2V_{CC}e^{-t/R_2C_2} = 0$, or when $t = R_2C_2 \ln 2$.

[d] When $t = R_2C_2 \ln 2$, we have

$$v_{ce2} = V_{CC} - V_{CC}e^{-[(R_2C_2 \ln 2)/(R_L C_1)]} = V_{CC} - V_{CC}e^{-10 \ln 2} \cong V_{CC}$$

[e] Before T_1 turns ON, i_{b1} is zero. At the instant T_1 turns ON, we have



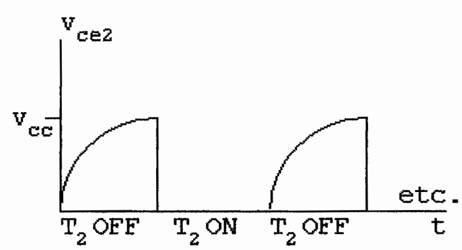
$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_L}e^{-t/R_L C_1}$$

[f] At the instant T_2 turns back ON, $t = R_2C_2 \ln 2$; therefore

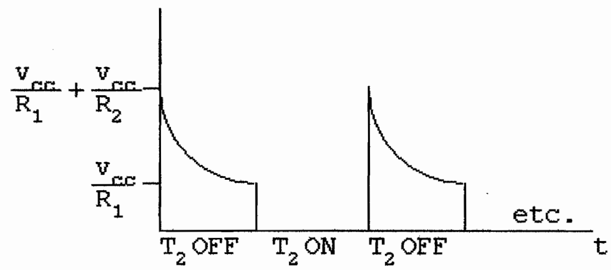
$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_L}e^{-10 \ln 2} \cong \frac{V_{CC}}{R_1}$$

When T_2 turns OFF, i_{b1} drops to zero instantaneously.

[g]



[h]



P 7.98 [a] $t_{\text{OFF2}} = R_2C_2 \ln 2 = 18 \times 10^3(2 \times 10^{-9}) \ln 2 \cong 25\,\mu\text{s}$

[b] $t_{\text{ON2}} = R_1C_1 \ln 2 \cong 25\,\mu\text{s}$

[c] $t_{\text{OFF1}} = R_1C_1 \ln 2 \cong 25\,\mu\text{s}$

[d] $t_{\text{ON1}} = R_2C_2 \ln 2 \cong 25\,\mu\text{s}$

[e] $i_{\text{b1}} = \frac{9}{3} + \frac{9}{18} = 3.5\,\text{mA}$

[f] $i_{\text{b1}} = \frac{9}{18} + \frac{9}{3}e^{-25/6} \cong 0.5465\,\text{mA}$

[g] $v_{\text{ce2}} = 9 - 9e^{-25/6} \cong 8.86\,\text{V}$

P 7.99 [a] $t_{\text{OFF2}} = R_2C_2 \ln 2 = (18 \times 10^3)(2.8 \times 10^{-9}) \ln 2 \cong 35\,\mu\text{s}$

[b] $t_{\text{ON2}} = R_1C_1 \ln 2 \cong 37.4\,\mu\text{s}$

[c] $t_{\text{OFF1}} = R_1C_1 \ln 2 \cong 37.4\,\mu\text{s}$

[d] $t_{\text{ON1}} = R_2C_2 \ln 2 = 35\,\mu\text{s}$

[e] $i_{\text{b1}} = 3.5\,\text{mA}$

[f] $i_{\text{b1}} = \frac{9}{18} + 3e^{-35/9} \cong 0.561\,\text{mA}$

[g] $v_{\text{ce2}} = 9 - 9e^{-35/9} \cong 8.81\,\text{V}$

Note in this circuit T_2 is OFF 35 μs and ON 37.4 μs of every cycle, whereas T_1 is ON 35 μs and OFF 37.4 μs every cycle.

P 7.100 If $R_1 = R_2 = 50R_L = 100\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{100 \times 10^3 \ln 2} = 692.49\,\text{pF}; \qquad C_2 = \frac{36 \times 10^{-6}}{100 \times 10^3 \ln 2} = 519.37\,\text{pF}$$

If $R_1 = R_2 = 6R_L = 12\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{12 \times 10^3 \ln 2} = 5.77\,\text{nF}; \qquad C_2 = \frac{36 \times 10^{-6}}{12 \times 10^3 \ln 2} = 4.33\,\text{nF}$$

Therefore $692.49\,\text{pF} \leq C_1 \leq 5.77\,\text{nF}$ and $519.37\,\text{pF} \leq C_2 \leq 4.33\,\text{nF}$

P 7.101 [a] T_2 is normally ON since its base current i_{b2} is greater than zero, i.e., $i_{b2} = V_{CC}/R$ when T_2 is ON. When T_2 is ON, $v_{ce2} = 0$, therefore $i_{b1} = 0$. When $i_{b1} = 0$, T_1 is OFF. When T_1 is OFF and T_2 is ON, the capacitor C is charged to V_{CC}, positive at the left terminal. This is a stable state; there is nothing to disturb this condition if the circuit is left to itself.

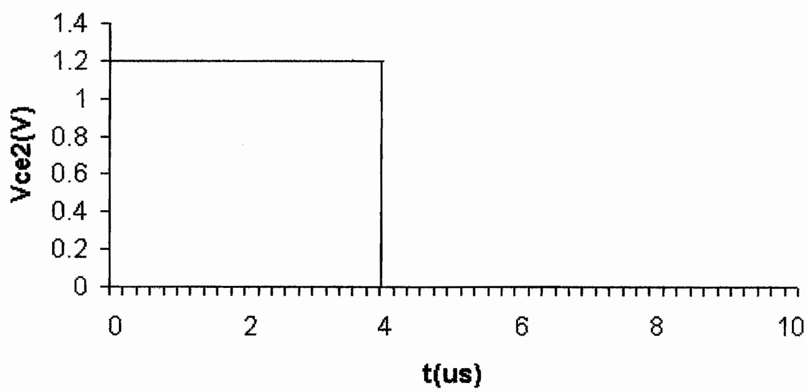
[b] When S is closed momentarily, v_{be2} is changed to $-V_{CC}$ and T_2 snaps OFF. The instant T_2 turns OFF, v_{ce2} jumps to $V_{CC}R_1/(R_1 + R_L)$ and i_{b1} jumps to $V_{CC}/(R_1 + R_L)$, which turns T_1 ON.

[c] As soon as T_1 turns ON, the charge on C starts to reverse polarity. Since v_{be2} is the same as the voltage across C, it starts to increase from $-V_{CC}$ toward $+V_{CC}$. However, T_2 turns ON as soon as $v_{be2} = 0$. The equation for v_{be2} is $v_{be2} = V_{CC} - 2V_{CC}e^{-t/RC}$. $v_{be2} = 0$ when $t = RC\ln 2$, therefore T_2 stays OFF for $RC\ln 2$ seconds.

P 7.102 [a] For $t < 0$, $v_{ce2} = 0$. When the switch is momentarily closed, v_{ce2} jumps to

$$v_{ce2} = \left(\frac{V_{CC}}{R_1 + R_L}\right) R_1 = \frac{6(5)}{25} = 1.2\,\text{V}$$

T_2 remains open for $(23{,}083)(250) \times 10^{-12} \ln 2 \cong 4\,\mu$s.

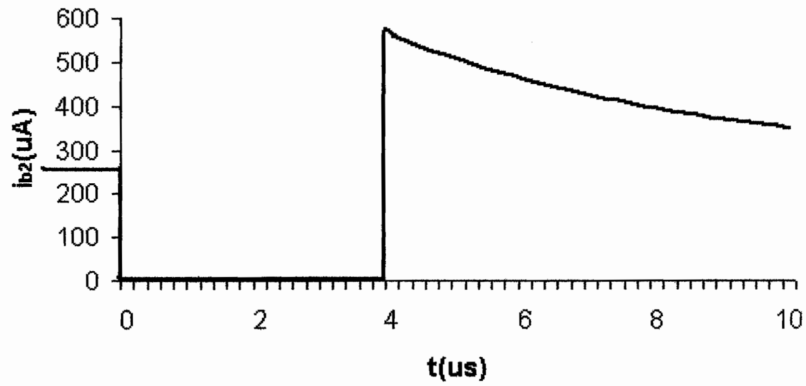


[b] $i_{b2} = \dfrac{V_{CC}}{R} = 259.93\,\mu\text{A}, \qquad -5 \le t \le 0\,\mu\text{s}$

$i_{b2} = 0, \qquad 0 < t < RC\ln 2$

$$i_{b2} = \frac{V_{CC}}{R} + \frac{V_{CC}}{R_L} e^{-(t-RC\ln 2)/R_L C}$$

$$= 259.93 + 300e^{-0.2\times10^6(t-4\times10^{-6})}\ \mu\text{A}, \qquad RC\ln 2 < t$$



P 7.103 [a] We want the lamp to be in its nonconducting state for no more than 10 s, the value of t_o:

$$10 = R(10\times10^{-6})\ln\frac{1-6}{4-6} \quad \text{and} \quad R = 1.091\,\text{M}\Omega$$

[b] When the lamp is conducting

$$V_{\text{Th}} = \frac{20\times10^3}{20\times10^3 + 1.091\times10^6}(6) = 0.108\,\text{V}$$

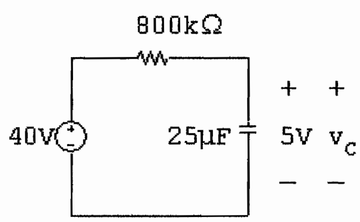
$$R_{\text{Th}} = 20\,\text{k}\|1.091\ \text{M} = 19{,}640\,\Omega$$

So,

$$(t_c - t_o) = (19{,}640)(10\times10^{-6})\ln\frac{4-0.108}{1-0.108} = 0.289\,\text{s}$$

The flash lasts for 0.289 s.

P 7.104 [a] At $t = 0$ we have



$$\tau = (800)(25)\times10^{-3} = 20\ \text{sec}; \qquad 1/\tau = 0.05$$

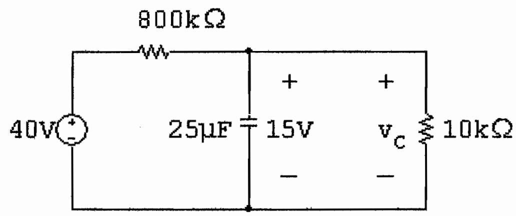
$$v_c(\infty) = 40\,\text{V}; \qquad v_c(0) = 5\,\text{V}$$

$$v_c = 40 - 35e^{-0.05t}\,\text{V}, \qquad 0 \le t \le t_o$$

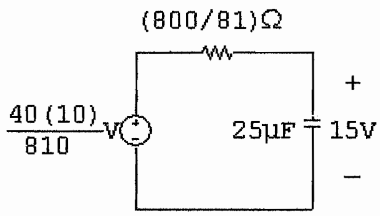
$$40 - 35e^{-0.05t_o} = 15; \qquad \therefore \quad e^{0.05t_o} = 1.4$$

$$t_o = 20 \ln 1.4\,\text{s} = 6.73\,\text{s}$$

At $t = t_o$ we have



The Thévenin equivalent with respect to the capacitor is



$$\tau = \left(\frac{800}{81}\right)(25) \times 10^{-3} = \frac{20}{81}\,\text{s}; \qquad \frac{1}{\tau} = \frac{81}{20} = 4.05$$

$$v_c(t_o) = 15\,\text{V}; \qquad v_c(\infty) = \frac{40}{81}\,\text{V}$$

$$v_c(t) = \frac{40}{81} + \left(15 - \frac{40}{81}\right)e^{-4.05(t-t_o)}\,\text{V} = \frac{40}{81} + \frac{1175}{81}e^{-4.05(t-t_o)}$$

$$\therefore \quad \frac{40}{81} + \frac{1175}{81}e^{-4.05(t-t_o)} = 5$$

$$\frac{1175}{81}e^{-4.05(t-t_o)} = \frac{365}{81}$$

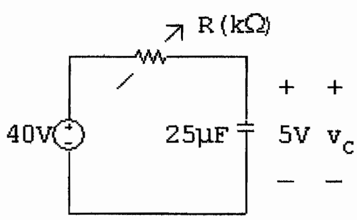
$$e^{4.05(t-t_o)} = \frac{1175}{365} = 3.22$$

$$t - t_o = \frac{1}{4.05}\ln 3.22 \cong 0.29\,\text{s}$$

One cycle = 7.02 seconds.

$N = 60/7.02 = 8.55$ flashes per minute

[b] At $t = 0$ we have



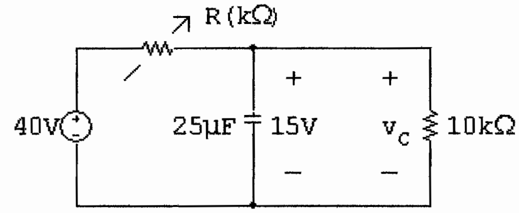
$$\tau = 25R \times 10^{-3}; \qquad 1/\tau = 40/R$$

$v_c = 40 - 35e^{-(40/R)t}$

$40 - 35e^{-(40/R)t_o} = 15$

$\therefore \quad t_o = \dfrac{R}{40}\ln 1.4, \qquad R \quad \text{in} \quad \text{k}\Omega$

At $t = t_o$:



$$v_{\text{Th}} = \frac{10}{R+10}(40) = \frac{400}{R+10}; \qquad R_{\text{Th}} = \frac{10R}{R+10}\,\text{k}\Omega$$

$$\tau = \frac{(25)(10R)\times 10^{-3}}{R+10} = \frac{0.25R}{R+10}; \qquad \frac{1}{\tau} = \frac{4(R+10)}{R}$$

$$v_c = \frac{400}{R+10} + \left(15 - \frac{400}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)}$$

$$\therefore \quad \frac{400}{R+10} + \left[\frac{15R-250}{R+10}\right]e^{-\frac{4(R+10)}{R}(t-t_o)} = 5$$

$$\text{or} \quad \left(\frac{15R-250}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)} = \frac{5R-350}{(R+10)}$$

$$\therefore \quad e^{\frac{4(R+10)}{R}(t-t_o)} = \frac{3R-50}{R-70}$$

$$\therefore \quad t - t_o = \frac{R}{4(R+10)}\ln\left(\frac{3R-50}{R-70}\right)$$

At 12 flashes per minute $\quad t_o + (t - t_o) = 5\,\text{s}$

$$\therefore \quad \underbrace{\frac{R}{40}\ln 1.4}_{\text{dominant term}} + \frac{R}{4(R+10)}\ln\left(\frac{3R-50}{R-70}\right) = 5$$

Start the trial-and-error procedure by setting $(R/40)\ln 1.4 = 5$, then $R = 200/(\ln 1.4)$ or $594.40\,\text{k}\Omega$. If $R = 594.40\,\text{k}\Omega$ then $t - t_o \cong 0.29\,\text{s}$. Second trial set $(R/40)\ln 1.4 = 4.7\,\text{s}$ or $R = 558.74\,\text{k}\Omega$.

With $\quad R = 558.74\,\text{k}\Omega, \quad t - t_o \cong 0.30\,\text{s}$

This procedure converges to $R = 559.3\,\text{k}\Omega$.

P 7.105 [a] $t_o = RC \ln\left(\frac{V_{\text{min}} - V_s}{V_{\text{max}} - V_s}\right) = (3700)(250 \times 10^{-6}) \ln\left(\frac{-700}{-100}\right)$

$= 1.80\,\text{s}$

$$t_c - t_o = \frac{RCR_L}{R + R_L} \ln\left(\frac{V_{\text{max}} - V_{\text{Th}}}{V_{\text{min}} - V_{\text{Th}}}\right)$$

$$\frac{R_L}{R + R_L} = \frac{1.3}{1.3 + 3.7} = 0.26; \qquad RC = (3700)(250 10^{-6}) = 0.925\,\text{s}$$

$$V_{\text{Th}} = \frac{1000(1.3)}{1.3 + 3.7} = 260\,\text{V}; \qquad R_{\text{Th}} = 3.7\,\text{k}\|1.3\,\text{k} = 962\,\Omega$$

$$\therefore \quad t_c - t_o = (0.925)(0.26)\ln(640/40) = 0.67\,\text{s}$$

$$\therefore \quad t_c = 1.8 + 0.67 = 2.47\,\text{s}$$

$$\text{flashes/min} = \frac{60}{2.47} = 24.32$$

[b] $0 \leq t \leq t_o$:

$$v_L = 1000 - 700e^{-t/\tau_1}$$

$$\tau_1 = RC = 0.925\,\text{s}$$

$t_o \leq t \leq t_c$:

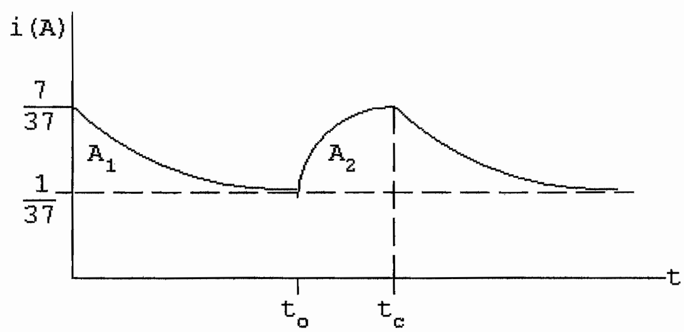
$$v_L = 260 + 640e^{-(t-t_o)/\tau_2}$$

$$\tau_2 = R_{\text{Th}}C = 962(250) \times 10^{-6} = 0.2405\,\text{s}$$

$$0 \leq t \leq t_o: \qquad i = \frac{1000 - v_L}{3700} = \frac{7}{37}e^{-t/0.925}\,\text{A}$$

$$t_o \leq t \leq t_c: \qquad i = \frac{1000 - v_L}{3700} = \frac{74}{370} - \frac{64}{370}e^{-(t-t_o)/0.2405}$$

Graphically, i versus t is



The average value of i will equal the areas $(A_1 + A_2)$ divided by t_c.

$$\therefore \quad i_{\text{avg}} = \frac{A_1 + A_2}{t_c}$$

$$A_1 = \frac{7}{37}\int_0^{t_o} e^{-t/0.925}\,dt$$
$$= \frac{6.475}{37}(1 - e^{-\ln 7}) = 0.15 \text{ A–s}$$

$$A_2 = \int_{t_o}^{t_c} \frac{74 - 64e^{-(t-t_o)/0.2405}}{370}\,dt$$
$$= \frac{74}{370}(t_c - t_o) + \frac{15.392}{370}(e^{-\ln 16} - 1)$$
$$= \frac{17.797}{370}\ln 16 - \frac{15.392}{370}(1 - e^{-\ln 16})$$
$$= 0.09436 \text{ A–s}$$

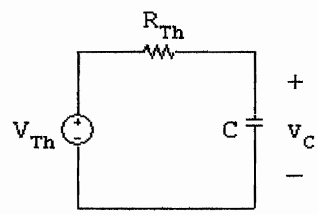
$$i_{\text{avg}} = \frac{(0.15 + 0.09436)}{0.925\ln 7 + 0.2405\ln 16}(1000) = 99.06\,\text{mA}$$

[c] $P_{\text{avg}} = (1000)(99.06 \times 10^{-3}) = 99.06\,\text{W}$

$$\text{No. of kw hrs/yr} = \frac{(99.06)(24)(365)}{1000} = 867.77$$

$$\text{Cost/year} = (867.77)(0.05) = 43.39 \text{ dollars/year}$$

P 7.106 [a] Replace the circuit attached to the capacitor with its Thévenin equivalent, where the equivalent resistance is the parallel combination of the two resistors, and the open-circuit voltage is obtained by voltage division across the lamp resistance. The resulting circuit is



$$R_{\text{Th}} = R\|R_{\text{L}} = \frac{RR_{\text{L}}}{R + R_{\text{L}}}; \qquad V_{\text{Th}} = \frac{R_{\text{L}}}{R + R_{\text{L}}}V_s$$

From this circuit,

$$v_{\text{C}}(\infty) = V_{\text{Th}}; \qquad v_{\text{C}}(0) = V_{\text{max}}; \qquad \tau = R_{\text{Th}}C$$

Thus,

$$v_{\text{C}}(t) = V_{\text{Th}} + (V_{\text{max}} - V_{\text{Th}})e^{-(t-t_o)/\tau}$$

where

$$\tau = \frac{RR_{\text{L}}C}{R + R_{\text{L}}}$$

[b] Now, set $v_C(t_c) = V_{\text{min}}$ and solve for $(t_c - t_o)$:

$$V_{\text{Th}} + (V_{\text{max}} - V_{\text{Th}})e^{-(t_c - t_o)/\tau} = V_{\text{min}}$$

$$e^{-(t_c - t_o)/\tau} = \frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}}$$

$$\frac{-(t_c - t_o)}{\tau} = \ln \frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}}$$

$$(t_c - t_o) = -\frac{RR_{\text{L}}C}{R + R_{\text{L}}} \ln \frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}} = \frac{RR_{\text{L}}C}{R + R_{\text{L}}} \ln \frac{V_{\text{max}} - V_{\text{Th}}}{V_{\text{min}} - V_{\text{Th}}}$$

P 7.107 [a] $0 \le t \le 0.5$:

$$i = \frac{21}{60} + \left(\frac{30}{60} - \frac{21}{60}\right) e^{-t/\tau} \qquad \text{where } \tau = L/R.$$

$$i = 0.35 + 0.15e^{-60t/L}$$

$$i(0.5) = 0.35 + 0.15e^{-30/L} = 0.40$$

$$\therefore \quad e^{30/L} = 3; \qquad L = \frac{30}{\ln 3} = 27.31\,\text{H}$$

[b] $0 \le t \le t_r$, where t_r is the time the relay releases:

$$i = 0 + \left(\frac{30}{60} - 0\right) e^{-60t/L} = 0.5e^{-60t/L}$$

$$\therefore \quad 0.4 = 0.5e^{-60t_r/L}; \qquad e^{60t_r/L} = 1.25$$

$$t_r = \frac{27.31 \ln 1.25}{60} \cong 0.10\,\text{s}$$

8

Natural and Step Responses of RLC Circuits

Assessment Problems

AP 8.1 [a] $\dfrac{1}{(2RC)^2} = \dfrac{1}{LC}$, therefore $C = 500\,\text{nF}$

[b] $\alpha = 5000 = \dfrac{1}{2RC}$, therefore $C = 1\,\mu\text{F}$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - \frac{(10^3)(10^6)}{20}} = (-5000 \pm j5000)\ \text{rad/s}$$

[c] $\dfrac{1}{\sqrt{LC}} = 20{,}000$, therefore $C = 125\,\text{nF}$

$$s_{1,2} = \left[-40 \pm \sqrt{(40)^2 - 20^2}\right] 10^3,$$

$$s_1 = -5.36\,\text{krad/s}, \qquad s_2 = -74.64\,\text{krad/s}$$

AP 8.2
$$\begin{aligned}
i_L &= \frac{1}{50 \times 10^{-3}} \int_0^t [-14e^{-5000x} + 26e^{-20{,}000x}]\,dx + 30 \times 10^{-3} \\
&= 20 \left\{ \frac{-14e^{-5000x}}{-5000}\Bigg|_0^t + \frac{26e^{-20{,}000t}}{-20{,}000}\Bigg|_0^t \right\} + 30 \times 10^{-3} \\
&= 56 \times 10^{-3}(e^{-5000t} - 1) - 26 \times 10^{-3}(e^{-20{,}000t} - 1) + 30 \times 10^{-3} \\
&= [56e^{-5000t} - 56 - 26e^{-20{,}000t} + 26 + 30]\,\text{mA} \\
&= 56e^{-5000t} - 26e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0
\end{aligned}$$

AP 8.3 From the given values of R, L, and C, $s_1 = -10\,\text{krad/s}$ and $s_2 = -40$ krad/s.

[a] $v(0^-) = v(0^+) = 0$, therefore $i_R(0^+) = 0$

[b] $i_C(0^+) = -(i_L(0^+) + i_R(0^+)) = -(-4+0) = 4\,\text{A}$

[c] $C\frac{dv_c(0^+)}{dt} = i_c(0^+) = 4, \qquad \text{therefore} \quad \frac{dv_c(0^+)}{dt} = \frac{4}{C} = 4\times 10^8\,\text{V/s}$

[d] $v = [A_1e^{-10{,}000t} + A_2e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0^+$

$$v(0^+) = A_1 + A_2, \qquad \frac{dv(0^+)}{dt} = -10{,}000A_1 - 40{,}000A_2$$

Therefore $A_1 + A_2 = 0, \qquad -A_1 - 4A_2 = 40{,}000; \qquad A_1 = 40{,}000/3\,\text{V}$

[e] $A_2 = -40{,}000/3\,\text{V}$

[f] $v = [40{,}000/3][e^{-10{,}000t} - e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0$

AP 8.4 [a] $\frac{1}{2RC} = 8000, \qquad \text{therefore} \quad R = 62.5\,\Omega$

[b] $i_R(0^+) = \frac{10\,\text{V}}{62.5\,\Omega} = 160\,\text{mA}$

$$i_C(0^+) = -(i_L(0^+) + i_R(0^+)) = -80 - 160 = -240\,\text{mA} = C\frac{dv(0^+)}{dt}$$

Therefore $\frac{dv(0^+)}{dt} = \frac{-240\,\text{m}}{C} = -240\,\text{kV/s}$

[c] $B_1 = v(0^+) = 10\,\text{V}, \qquad \frac{dv_c(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

Therefore $6000B_2 - 8000B_1 = -240{,}000, \qquad B_2 = (-80/3)\,\text{V}$

[d] $i_L = -(i_R + i_C); \qquad i_R = v/R; \qquad i_C = C\frac{dv}{dt}$

$$v = e^{-8000t}[10\cos 6000t - \frac{80}{3}\sin 6000t]\,\text{V}$$

Therefore $i_R = e^{-8000t}[160\cos 6000t - \frac{1280}{3}\sin 6000t]\,\text{mA}$

$$i_C = e^{-8000t}[-240\cos 6000t + \frac{460}{3}\sin 6000t]\,\text{mA}$$

$$i_L = 10e^{-8000t}[8\cos 6000t + \frac{82}{3}\sin 6000t]\,\text{mA}, \qquad t \geq 0$$

AP 8.5 [a] $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = \frac{10^6}{4}, \qquad \text{therefore} \quad \frac{1}{2RC} = 500, \quad R = 100\,\Omega$

[b] $0.5CV_0^2 = 12.5\times 10^{-3}, \qquad \text{therefore} \quad V_0 = 50\,\text{V}$

[c] $0.5LI_0^2 = 12.5\times 10^{-3}, \qquad I_0 = 250\,\text{mA}$

[d] $D_2 = v(0^+) = 50, \qquad \frac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$i_R(0^+) = \frac{50}{100} = 500\,\text{mA}$

Therefore $i_C(0^+) = -(500 + 250) = -750\,\text{mA}$

Therefore $\frac{dv(0^+)}{dt} = -750 \times \frac{10^{-3}}{C} = -75{,}000\,\text{V/s}$

Therefore $D_1 - \alpha D_2 = -75{,}000; \qquad \alpha = \frac{1}{2RC} = 500, \quad D_1 = -50{,}000\ \text{V/s}$

[e] $v = [50e^{-500t} - 50{,}000te^{-500t}]\,\text{V}$

$i_R = \frac{v}{R} = [0.5e^{-500t} - 500te^{-500t}]\,\text{A}, \qquad t \geq 0^+$

AP 8.6 [a] $i_R(0^+) = \frac{V_0}{R} = \frac{40}{500} = 0.08\,\text{A}$

[b] $i_C(0^+) = I - i_R(0^+) - i_L(0^+) = -1 - 0.08 - 0.5 = -1.58\,\text{A}$

[c] $\frac{di_L(0^+)}{dt} = \frac{V_o}{L} = \frac{40}{0.64} = 62.5\,\text{A/s}$

[d] $\alpha = \frac{1}{2RC} = 1000; \qquad \frac{1}{LC} = 1{,}562{,}500; \qquad s_{1,2} = -1000 \pm j750\ \text{rad/s}$

[e] $i_L = i_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t, \qquad i_f = I = -1\,\text{A}$

$i_L(0^+) = 0.5 = i_f + B_1', \qquad$ therefore $B_1' = 1.5\,\text{A}$

$\frac{di_L(0^+)}{dt} = 62.5 = -\alpha B_1' + \omega_d B_2', \qquad$ therefore $B_2' = (25/12)\,\text{A}$

Therefore $i_L(t) = -1 + e^{-1000t}[1.5\cos 750t + (25/12)\sin 750t]\,\text{A}, \qquad t \geq 0$

[f] $v(t) = \frac{L\,di_L}{dt} = 40e^{-1000t}[\cos 750t - (154/3)\sin 750t]V \qquad t \geq 0$

AP 8.7 [a] $i(0^+) = 0$, since there is no source connected to L for $t < 0$.

[b] $v_c(0^+) = v_C(0^-) = \left(\frac{15\,\text{k}}{15\,\text{k} + 9\,\text{k}}\right)(80) = 50\,\text{V}$

[c] $50 + 80i(0^+) + L\frac{di(0^+)}{dt} = 100, \qquad \frac{di(0^+)}{dt} = 10{,}000\,\text{A/s}$

[d] $\alpha = 8000; \qquad \frac{1}{LC} = 100 \times 10^6; \qquad s_{1,2} = -8000 \pm j6000\ \text{rad/s}$

[e] $i = i_f + e^{-\alpha t}[B_1'\cos\omega_d t + B_2'\sin\omega_d t]; \qquad i_f = 0, \quad i(0^+) = 0$

Therefore $B_1' = 0; \qquad \frac{di(0^+)}{dt} = 10{,}000 = -\alpha B_1' + \omega_d B_2'$

Therefore $B_2' = 1.67\,\text{A}; \qquad i = 1.67e^{-8000t}\sin 6000t\,\text{A}, \qquad t \geq 0$

AP 8.8 $v_c(t) = v_f + e^{-\alpha t}[B_1' \cos\omega_d t + B_2' \sin\omega_d t], \qquad v_f = 100\,\text{V}$

$$v_c(0^+) = 50\,\text{V}; \qquad \frac{dv_c(0^+)}{dt} = 0; \qquad \text{therefore} \quad 50 = 100 + B_1'$$

$$B_1' = -50\,\text{V}; \qquad 0 = -\alpha B_1' + \omega_d B_2'$$

$$\text{Therefore} \quad B_2' = \frac{\alpha}{\omega_d} B_1' = \left(\frac{8000}{6000}\right)(-50) = -66.67\,\text{V}$$

$$\text{Therefore} \quad v_c(t) = 100 - e^{-8000t}[50\cos 6000t + 66.67\sin 6000t]\,\text{V}, \qquad t \geq 0$$

Problems

P 8.1 [a] $\alpha = \dfrac{1}{2RC} = \dfrac{10^9}{(10{,}000)(8)} = 12{,}500$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(1.25)(8)} = 10^8$$

$$s_{1,2} = -12{,}500 \pm \sqrt{(1.5625 - 1)10^8} = -12{,}500 \pm 7500$$

$$s_1 = -5000 \text{ rad/s}$$

$$s_2 = -20{,}000 \text{ rad/s}$$

[b] overdamped

[c] $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 10^8 - 36 \times 10^6 = 0.64 \times 10^8$$

$$\alpha = 0.8 \times 10^4 = 8000$$

$$\frac{1}{2RC} = 8000; \qquad \therefore \quad R = \frac{10^9}{(16{,}000)(8)} = 7812.5\,\Omega$$

[d] $s_1 = -8000 + j6000$ rad/s; $\qquad s_2 = -8000 - j6000$ rad/s

[e] $\alpha = 10^4 = \dfrac{1}{2RC}; \qquad \therefore \quad R = \dfrac{1}{2C(10^4)} = 6250\,\Omega$

P 8.2 [a] $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -5000$

$$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -20{,}000$$

$$\therefore \quad -2\alpha = -25{,}000$$

$$\alpha = 12{,}500\,\text{rad/s}$$

$$\frac{1}{2RC} = \frac{10^6}{2R(0.05)} = 12{,}500$$

$$R = 800\,\Omega$$

$$2\sqrt{\alpha^2 - \omega_o^2} = 15{,}000$$

$$4(\alpha^2 - \omega_o^2) = 225 \times 10^6$$

$$\therefore \quad \omega_o = 10{,}000\,\text{rad/s}$$

$$\omega_o^2 = 10^8 = \frac{1}{LC}$$

$$\therefore \quad L = \frac{1}{10^8 C} = 200\,\text{mH}$$

[b] $i_\text{R} = \dfrac{v(t)}{R} = -6.25e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$

$$i_\text{C} = C\frac{dv(t)}{dt} = 1.25e^{-5000t} - 20e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_\text{L} = -(i_\text{R} + i_\text{C}) = 5e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

P 8.3 [a] $\alpha = 4000; \qquad \omega_d = 3000$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 9 \times 10^6 + 16 \times 10^6 = 25 \times 10^6$$

$$\frac{1}{LC} = 25 \times 10^6$$

$$L = \frac{1}{(25 \times 10^6)(50 \times 10^{-9})} = 0.8\,\text{H} = 800\,\text{mH}$$

[b] $\alpha = \dfrac{1}{2RC}$

$$\therefore \quad R = \frac{1}{2\alpha C} = \frac{10^9}{(8000)(50)} = 2500\,\Omega$$

[c] $V_o = v(0) = 125\,\text{V}$

[d] $I_o = i_L(0) = -i_R(0) - i_C(0)$

$$i_R(0) = \frac{V_o}{R} = \frac{125}{2.5} \times 10^{-3} = 50\,\text{mA}$$

$$i_C(0) = C\frac{dv}{dt}(0)$$

$$\frac{dv}{dt} = 125\{e^{-4000t}[-3000\sin 3000t - 6000\cos 3000t] -$$

$$4000e^{-4000t}[\cos 3000t - 2\sin 3000t]$$

$$\frac{dv}{dt}(0) = 125\{1(-6000) - 4000\} = -125 \times 10^4$$

$$C\frac{dv}{dt}(0) = -125 \times 10^4(50 \times 10^{-9}) = -6250 \times 10^{-5} = -62.5\,\text{mA}$$

$$\therefore\quad I_o = -50 + 62.5 = 12.5\,\text{mA}$$

[e]
$$\frac{dv}{dt} = 125e^{-4000t}[5000\sin 3000t - 10{,}000\cos 3000t]$$

$$= 625 \times 10^3 e^{-4000t}[\sin 3000t - 2\cos 3000t]$$

$$C\frac{dv}{dt} = 31{,}250 \times 10^{-6}e^{-4000t}(\sin 3000t - 2\cos 3000t)$$

$$i_C(t) = 31.25e^{-4000t}(\sin 3000t - 2\cos 3000t)\,\text{mA}$$

$$i_R(t) = 50e^{-4000t}(\cos 3000t - 2\sin 3000t)\,\text{mA}$$

$$i_L(t) = -i_R(t) - i_C(t)$$

$$= e^{-4000t}(12.5\cos 3000t + 68.75\sin 3000t)\,\text{mA}, \quad t \geq 0$$

CHECK:

$$\frac{di_L}{dt} = \{-4000e^{-4000t}[12.5\cos 3000t + 68.75\sin 3000t]$$

$$+e^{-4000t}[-37.5 \times 10^3 \sin 3000t$$

$$+206.25 \times 10^3 \cos 3000t] \times 10^{-3}$$

$$= e^{-4000t}[156.25\cos 3000t - 312.5\sin 3000t]$$

$$L\frac{di_L}{dt} = e^{-4000t}[125\cos 3000t - 250\sin 3000t]$$

$$= 125e^{-4000t}[\cos 3000t - 2\sin 3000t]\,\text{V}$$

P 8.4 [a] $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = (4000)^2$

$$\therefore \quad C = \frac{1}{(16 \times 10^6)(5)} = 12.5\,\text{nF}$$

$$\frac{1}{2RC} = 4000$$

$$\therefore \quad R = \frac{10^9}{(8000)(12.5)} = 10\,\text{k}\Omega$$

$$v(0) = D_2 = 25\,\text{V}$$

$$i_{\text{R}}(0) = \frac{25}{10} = 2.5\text{mA}$$

$$i_{\text{C}}(0) = -2.5 - 5 = -7.5\,\text{mA}$$

$$\frac{dv}{dt}(0) = D_1 - 4000D_2 = \frac{-7.5 \times 10^{-3}}{12.5 \times 10^{-9}} = -6 \times 10^5$$

$$\therefore \quad D_1 = -6 \times 10^5 + 4000(25) = -5 \times 10^5\ \text{V/s}$$

[b] $v = -5 \times 10^5 te^{-4000t} + 25e^{-4000t}$

$$\frac{dv}{dt} = [20 \times 10^8 t - 6 \times 10^5]e^{-4000t}$$

$$i_{\text{C}} = C\frac{dv}{dt} = 12.5 \times 10^{-9}[20 \times 10^8 t - 6 \times 10^5]e^{-4000t}$$

$$= (25{,}000t - 7.5)e^{-4000t}\,\text{mA}, \qquad t > 0$$

P 8.5 [a] $2\alpha = 200; \qquad \alpha = 100\,\text{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 120; \qquad \omega_o = 80\,\text{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{200(200)} = 25\,\mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(80)^2(25)} = 6.25\,\text{H}$$

$$i_{\text{C}}(0^+) = A_1 + A_2 = 15\,\text{mA}$$

$$\frac{di_{\text{C}}}{dt} + \frac{di_{\text{L}}}{dt} + \frac{di_{\text{R}}}{dt} = 0$$

$$\frac{di_{\text{C}}(0)}{dt} = -\frac{di_{\text{L}}(0)}{dt} - \frac{di_{\text{R}}(0)}{dt}$$

$$\frac{di_L(0)}{dt} = \frac{0}{6.25} = 0\,\text{A/s}$$

$$\frac{di_R(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_C(0)}{C} = \frac{15 \times 10^{-3}}{(200)(25 \times 10^{-6})} = 3\,\text{A/s}$$

$$\therefore \quad \frac{di_C(0)}{dt} = -3\,\text{A/s}$$

$$\therefore \quad 160A_1 + 40A_2 = 3$$

$$4A_1 + A_2 + = 75 \times 10^{-3}; \qquad \therefore \quad A_1 = 20\,\text{mA}; \qquad A_2 = -5\,\text{mA}$$

$$\therefore \quad i_C = 20e^{-160t} - 5e^{-40t}\,\text{mA}, \qquad t \geq 0$$

[b] By hypothesis

$$v = A_3e^{-160t} + A_4e^{-40t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 0$$

$$\frac{dv(0)}{dt} = \frac{15 \times 10^{-3}}{25 \times 10^{-6}} = 600\,\text{V/s}$$

$$-160A_3 - 40A_4 = 600; \qquad \therefore \quad A_3 = -5\,\text{V}; \qquad A_4 = 5\,\text{V}$$

$$v = -5e^{-160t} + 5e^{-40t}\,\text{V}, \qquad t \geq 0$$

[c] $i_R(t) = \dfrac{v}{200} = -25e^{-160t} + 25e^{-40t}\,\text{mA}, \qquad t \geq 0^+$

[d] $i_L = -i_R - i_C$

$$i_L = 5e^{-160t} - 20e^{-40t}\,\text{mA}, \qquad t \geq 0$$

P 8.6 [a] $i_R(0) = \dfrac{90}{2000} = 45\text{mA}$

$$i_L(0) = -30\text{mA}$$

$$i_C(0) = -i_L(0) - i_R(0) = 30 - 45 = -15\,\text{mA}$$

[b] $\alpha = \dfrac{1}{2RC} = \dfrac{10^9}{(4000)(10)} = 25{,}000$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^9)}{(250)(10)} = 4 \times 10^8$$

$$s_{1,2} = -25{,}000 \pm \sqrt{6.25 \times 10^8 - 10^8(4)} = -25{,}000 \pm 15{,}000$$

$$s_1 = -10{,}000\ \text{rad/s}; \qquad s_2 = -40{,}000\ \text{rad/s}$$

$$v = A_1e^{-10{,}000t} + A_2e^{-40{,}000t}$$

$$v(0) = A_1 + A_2 = 90$$

$$\frac{dv}{dt}(0) = -10^4 A_1 - 4A_2 \times 10^4 = \frac{-15 \times 10^{-3}}{10 \times 10^{-9}} = -1.5 \times 10^6 \text{V/s}$$

$$-A_1 - 4A_2 = -150$$

$$\therefore \quad -3A_2 = -60; \qquad A_2 = 20; \qquad A_1 = 70$$

$$v = 70e^{-10{,}000t} + 20e^{-40{,}000t}\,\text{V}, \qquad t \geq 0$$

[c] $$i_C = C\frac{dv}{dt}$$

$$= 10 \times 10^{-9}[-70 \times 10^4 e^{-10{,}000t} - 80 \times 10^4 e^{-40{,}000t}]$$

$$= -7e^{-10{,}000t} - 8e^{-40{,}000t}\,\text{mA}$$

$$i_R = 35e^{-10{,}000t} + 10e^{-40{,}000t}\,\text{mA}$$

$$i_L = -i_C - i_R = -28e^{-10{,}000t} - 2e^{-40{,}000t}\,\text{mA}, \quad t \geq 0$$

P 8.7 $$\alpha = \frac{1}{2RC} = \frac{10^9}{(5000)(10)} = 2 \times 10^4$$

$$\alpha^2 = 4 \times 10^8; \qquad \therefore \quad \alpha^2 = \omega_o^2$$

Critical damping:

$$v = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

$$i_R(0^+) = \frac{90}{2500} = 36\,\text{mA}$$

$$i_C(0^+) = -[i_L(0^+) + i_R(0^+)] = -[-30 + 36] = -6\,\text{mA}$$

$$v(0) = D_2 = 90$$

$$\frac{dv}{dt} = D_1[t(-\alpha e^{-\alpha t}) + e^{-\alpha t}] - \alpha D_2 e^{-\alpha t}$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{i_C(0)}{C} = \frac{-6 \times 10^{-3}}{10 \times 10^{-9}} = -6 \times 10^5$$

$$D_1 = \alpha D_2 - 6 \times 10^5 = (2 \times 10^4)(90) - 6 \times 10^5 = 120 \times 10^4$$

$$v = (120 \times 10^4 t + 90)e^{-20{,}000t}\,\text{V}, \qquad t \geq 0$$

P 8.8 $$\frac{1}{2RC} = \frac{3 \times 10^9}{(25{,}000)(10)} = 12{,}000$$

$$\frac{1}{LC} = 4 \times 10^8$$

$$s_{1,2} = -12{,}000 \pm j16{,}000\,\text{rad/s}$$

$\therefore$ response is underdamped

$$v(t) = B_1 e^{-12{,}000t}\cos 16{,}000t + B_2 e^{-12{,}000t}\sin 16{,}000t$$

$$v(0^+) = 90\,\text{V} = B_1; \qquad i_\text{R}(0^+) = \frac{90}{(12{,}500/3)} = 21.6\,\text{mA}$$

$$i_\text{C}(0^+) = [-i_\text{L}(0^+) + i_\text{R}(0^+)] = -[-30 + 21.6] = 8.4\,\text{mA}$$

$$\frac{dv(0^+)}{dt} = \frac{8.4 \times 10^{-3}}{10 \times 10^{-9}} = 840{,}000\,\text{V/s}$$

$$\frac{dv(0)}{dt} = -12{,}000B_1 + 16{,}000B_2 = 840{,}000$$

$$\text{or } -3B_1 + 4B_2 = 210; \qquad \therefore \quad B_2 = 120\,\text{V}$$

$$v(t) = 90e^{-12{,}000t}\cos 16{,}000t + 120e^{-12{,}000t}\sin 16{,}000t\,\text{V}, \qquad t \geq 0$$

P 8.9 $$\alpha = 2000/2 = 1000$$

$$R = \frac{1}{2\alpha C} = \frac{10^6}{(2000)(18)} = 27.78\,\Omega$$

$$v(0^+) = -24\,\text{V}$$

$$i_\text{R}(0^+) = \frac{-24}{27.78} = -864\,\text{mA}$$

$$\frac{dv}{dt} = 2400e^{-200t} + 21{,}600e^{-1800t}$$

$$\frac{dv(0^+)}{dt} = 2400 + 21{,}600 = 24{,}000\,\text{V/s}$$

$$i_\text{C}(0^+) = 18 \times 10^{-6}(24{,}000) = 432\,\text{mA}$$

$$i_\text{L}(0^+) = -[i_\text{R}(0^+) + i_\text{C}(0^+)] = -[-864 + 432] = 432\,\text{mA}$$

P 8.10 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{40} = 25 \times 10^6$

$\omega_o = 5000$ rad/s

$\dfrac{1}{2RC} = 5000; \qquad R = \dfrac{1}{10{,}000C}$

$R = \dfrac{10^9}{8 \times 10^4} = 12.5\,\text{k}\Omega$

[b] $v(t) = D_1te^{-5000t} + D_2e^{-5000t}$

$v(0) = -25\,\text{V} = D_2$

$\dfrac{dv}{dt} = (D_1t - 25)(-5000e^{-5000t}) + D_1e^{-5000t}$

$\dfrac{dv}{dt}(0) = 125 \times 10^3 + D_1 = \dfrac{i_\text{C}(0)}{C}$

$i_\text{C}(0) = -i_\text{R}(0) - i_\text{L}(0)$

$i_\text{R}(0) = \dfrac{-25}{12.5} = -2\,\text{mA}$

$\therefore \quad i_\text{C}(0) = 2 - (-1) = 3\,\text{mA}$

$\therefore \quad \dfrac{dv}{dt}(0) = \dfrac{3 \times 10^{-3}}{8 \times 10^{-9}} = 0.375 \times 10^6 = 3.75 \times 10^5$

$\therefore \quad 1.25 \times 10^5 + D_1 = 3.75 \times 10^5$

$D_1 = 2.5 \times 10^5 = 25 \times 10^4\text{V/s}$

$\therefore \quad v(t) = (25 \times 10^4t - 25)e^{-5000t}\,\text{V}, \qquad t \geq 0$

[c] $i_\text{C}(t) = 0$ when $\dfrac{dv}{dt}(t) = 0$

$$\frac{dv}{dt} = (25 \times 10^4t - 25)(-5000)e^{-5000t} + e^{-5000t}(25 \times 10^4)$$

$$= (375{,}000 - 125 \times 10^7t)e^{-5000t}$$

$\dfrac{dv}{dt} = 0$ when $125 \times 10^7t_1 = 375{,}000; \qquad \therefore \quad t_1 = 300\,\mu\text{s}$

$v(300\mu\text{s}) = 50e^{-1.5} = 11.16\,\text{V}$

[d] $i_{\mathrm{L}}(300\mu\mathrm{s}) = -i_{\mathrm{R}}(300\mu\mathrm{s}) = \dfrac{11.16}{12.5} = 0.89\,\mathrm{mA}$

$$\omega_{\mathrm{C}}(300\mu\mathrm{s}) = 4\times 10^{-9}(11.16)^2 = 497.87\,\mathrm{nJ}$$

$$\omega_{\mathrm{L}}(300\mu\mathrm{s}) = (2.5)(0.89)^2\times 10^{-6} = 1991.48\,\mathrm{nJ}$$

$$\omega(300\mu\mathrm{s}) = \omega_{\mathrm{C}} + \omega_{\mathrm{L}} = 2489.35\,\mathrm{nJ}$$

$$\omega(0) = 4\times 10^{-9}(625) + 2.5(10^{-6}) = 5000\,\mathrm{nJ}$$

$$\%\text{ remaining } = \frac{2489.35}{5000}(100) = 49.79\%$$

P 8.11 [a] $\alpha = \dfrac{1}{2RC} = 1\text{ rad/s}$

$$\omega_o^2 = \frac{1}{LC} = 10$$

$$\omega_d = \sqrt{10-1} = 3\text{ rad/s}$$

$$\therefore\quad v = B_1e^{-t}\cos 3t + B_2e^{-t}\sin 3t$$

$$v(0) = B_1 = 0; \qquad v = B_2e^{-t}\sin 3t$$

$$i_R(0^+) = 0\,\mathrm{A}; \qquad i_C(0^+) = 3\,\mathrm{A}; \qquad \frac{dv}{dt}(0^+) = \frac{3}{0.25} = 12\text{ V/s}$$

$$12 = -\alpha B_1 + \omega_d B_2 = -1(0) + 3B_2$$

$$\therefore\quad B_2 = 4$$

$$\therefore\quad v = 4e^{-t}\sin 3t\,\mathrm{V}, \qquad t \geq 0$$

[b] $\dfrac{dv}{dt} = 4e^{-t}(3\cos 3t - \sin 3t)$

$$\frac{dv}{dt} = 0 \quad\text{when}\quad 3\cos 3t = \sin 3t \quad\text{or}\quad \tan 3t = 3$$

$$\therefore\quad 3t_1 = 1.25, \qquad t_1 = 416.35\,\mathrm{ms}$$

$$3t_2 = 1.25 + \pi, \qquad t_2 = 1463.55\,\mathrm{ms}$$

$$3t_3 = 1.25 + 2\pi, \qquad t_3 = 2510.74\,\mathrm{ms}$$

[c] $t_3 - t_1 = 2094.40\,\mathrm{ms}; \qquad T_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{3} = 2094.40\,\mathrm{ms}$

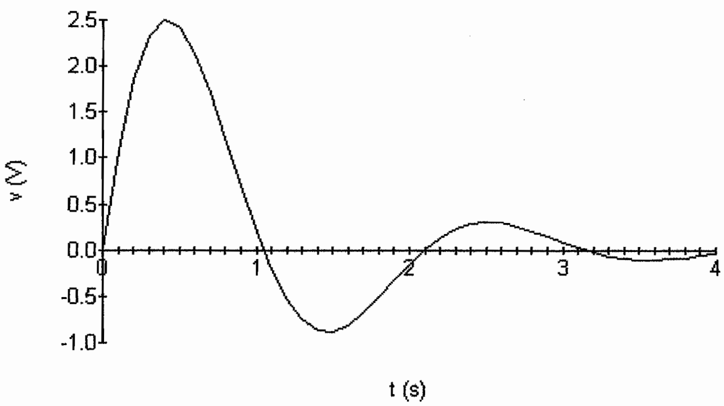
[d] $t_2 - t_1 = 1047.20\,\mathrm{ms}; \qquad \dfrac{T_d}{2} = \dfrac{2094.40}{2} = 1047.20\,\mathrm{ms}$

[e] $v(t_1) = 4e^{-(0.41635)} \sin 3(0.41635) = 2.50\,\text{V}$

$$v(t_2) = 4e^{-(1.46355)} \sin 3(1.46355) = -0.88\,\text{V}$$

$$v(t_3) = 4e^{-(2.51074)} \sin 3(2.51074) = 0.31\,\text{V}$$

[f]



P 8.12 [a] $\alpha = 0; \qquad \omega_d = \omega_o = \sqrt{10} = 3.16\,\text{rad/s}$

$$v = B_1 \cos\omega_o t + B_2 \sin\omega_o t; \qquad v(0) = B_1 = 0; \qquad v = B_2 \sin\omega_o t$$

$$C\frac{dv}{dt}(0) = -i_L(0) = 3$$

$$12 = -\alpha B_1 + \omega_d B_2 = -0 + \sqrt{10}B_2$$

$$\therefore \quad B_2 = 12/\sqrt{10} = 3.79\,\text{V}$$

$$v = 3.79 \sin 3.16t\,\text{V}, \qquad t \geq 0$$

[b] $2\pi f = 3.16; \qquad f = \dfrac{3.16}{2\pi} \cong 0.50\,\text{Hz}$

[c] 3.79 V

P 8.13 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2, respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$

The characteristic determinate is

$$\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$$

The numerator determinates are

$$N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$$

and $N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$

It follows that $A_1 = \dfrac{N_1}{\Delta} = \dfrac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$

and $A_2 = \dfrac{N_2}{\Delta} = \dfrac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$

We see from these expressions that $A_1 = A_2^*$

P 8.14 By definition, $B_1 = A_1 + A_2$. From the solution to Problem 8.13 we have

$$A_1 + A_2 = \frac{2\omega_d K_1}{2\omega_d} = K_1$$

But K_1 is $v(0)$, therefore, $B_1 = v(0)$, which is identical to Eq. (8.30).
By definition, $B_2 = j(A_1 - A_2)$. From Problem 8.13 we have

$$B_2 = j(A_1 - A_2) = \frac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \frac{\alpha K_1 + K_2}{\omega_d}$$

It follows that

$K_2 = -\alpha K_1 + \omega_d B_2$, but $K_2 = \dfrac{dv(0^+)}{dt}$ and $K_1 = B_1$

Thus we have

$$\frac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2,$$

which is identical to Eq. (8.31).

P 8.15 [a] $\alpha = \dfrac{1}{2RC} = 1000\sqrt{2}, \qquad \omega_o = 10^3,$ therefore overdamped

$s_1 = -414.21, \qquad s_2 = -2414.21$

therefore $v = A_1e^{-414.21t} + A_2e^{-2414.21t}$

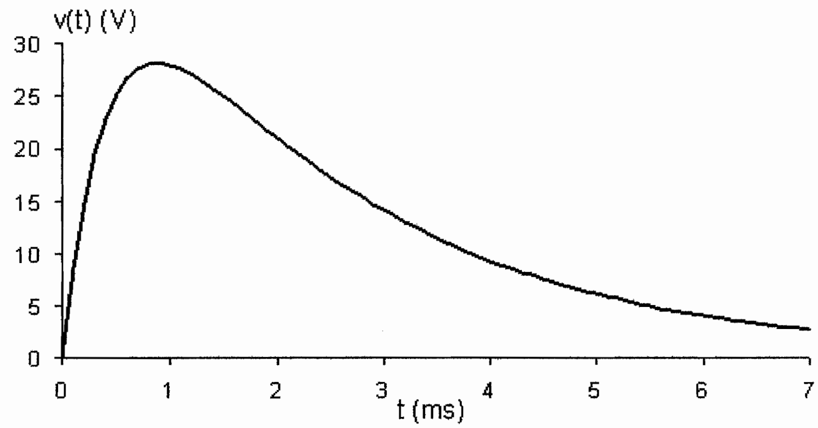
$v(0^+) = 0 = A_1 + A_2; \qquad \left[\dfrac{dv(0^+)}{dt}\right] = \dfrac{i_C(0^+)}{C} = 98{,}000\,\text{V/s}$

Therefore $-414.21A_1 - 2414.21A_2 = 98{,}000$

$A_1 = 49, \quad A_2 = -49$

$v(t) = 49[e^{-414.21t} - e^{-2414.21t}]\,\text{V}, \qquad t \geq 0$

[b]

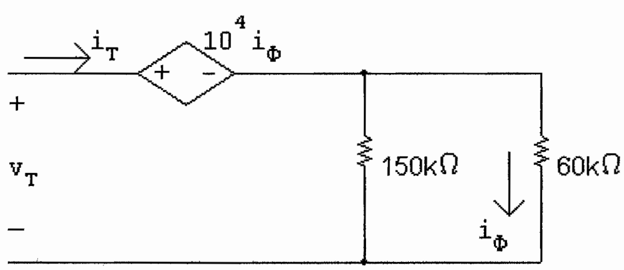


Example 8.4: $v_{\text{max}} \cong 74.1\,\text{V}$ at 1.4 ms

Example 8.5: $v_{\text{max}} \cong 36.1\,\text{V}$ at 1.0 ms

Problem 8.15: $v_{\text{max}} \cong 28.2\,\text{V}$ at 0.9 ms

P 8.16



$$v_T = 10^4\frac{i_T(150\times 10^3)}{210\times 10^3} + \frac{(150)(60)10^6}{210\times 10^3}i_T$$

$$\frac{v_T}{i_T} = \frac{1500\times 10^3}{210} + \frac{9000\times 10^3}{210} = \frac{10{,}500}{210}\times 10^3 = 50\,\text{k}\Omega$$

$$V_o = \frac{75}{10}(6) = 45\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{45}{50{,}000} = -0.9\,\text{mA}$$

$$\frac{i_C(0)}{C} = \frac{-0.9}{1.25} \times 10^6 = -720 \times 10^3$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(8)(1.25)} = 10^8; \qquad \omega_o = 10^4\ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(50)(1.25)\times 10^3} = 8000\ \text{rad/s}$$

$$\omega_d = \sqrt{(100-64)\times 10^6} = 6000\ \text{rad/s}$$

$$v_o = B_1 e^{-8000t}\cos 6000t + B_2 e^{-8000t}\sin 6000t$$

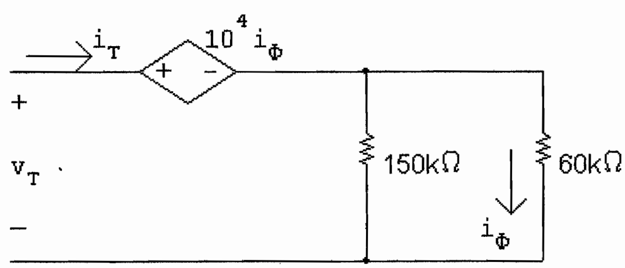
$$v_o(0) = B_1 = 45\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 6000B_2 - 8000B_1 = -720\times 10^3$$

$$\therefore\ \ 6000B_2 = 8000(45) - 720\times 10^3; \qquad \therefore\ \ B_2 = -60\,\text{V}$$

$$v_o = 45e^{-8000t}\cos 6000t - 60e^{-8000t}\sin 6000t\,\text{V}, \qquad t \geq 0$$

P 8.17



$$v_T = 10^4 \frac{i_T(150\times 10^3)}{210\times 10^3} + \frac{(150)(60)10^6}{210\times 10^3} i_T$$

$$\frac{v_T}{i_T} = \frac{1500\times 10^3}{210} + \frac{9000\times 10^3}{210} = \frac{10{,}500}{210}\times 10^3 = 50\,\text{k}\Omega$$

$$V_o = \frac{75}{10}(6) = 45\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{45}{50{,}000} = -0.9\,\text{mA}$$

$$\frac{i_C(0)}{C} = \frac{-0.9\text{m}}{10^{-9}} = -900 \times 10^3$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(10)(10^{-9})} = 10^8; \qquad \omega_o = 10{,}000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(50{,}000)(10^{-9})} = 10{,}000 \text{ rad/s}$$

$\alpha^2 = \omega_o^2$ so the response is critically damped

$$v_o = D_1te^{-10{,}000t} + D_2e^{-10{,}000t}$$

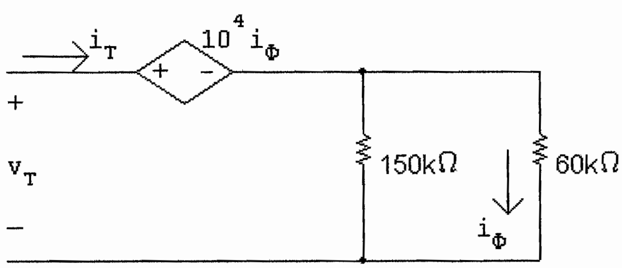
$$v_o(0) = D_2 = 45\,\text{V}$$

$$\frac{dv_o}{dt}(0) = D_1 - \alpha D_2 = -900 \times 10^3$$

$$\therefore \; D_1 = -900 \times 10^3 + (10{,}000)(45); \qquad \therefore \; D_1 = -450{,}000\,\text{V/s}$$

$$v_o = -450{,}000te^{-10{,}000t} + 45e^{-10{,}000t}\,\text{V}, \qquad t \geq 0$$

P 8.18



$$v_T = 10^4\frac{i_T(150 \times 10^3)}{210 \times 10^3} + \frac{(150)(60)10^6}{210 \times 10^3}i_T$$

$$\frac{v_T}{i_T} = \frac{1500 \times 10^3}{210} + \frac{9000 \times 10^3}{210} = \frac{10{,}500}{210} \times 10^3 = 50\,\text{k}\Omega$$

$$V_o = \frac{75}{10}(6) = 45\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{45}{50{,}000} = -0.9\,\text{mA}$$

$$\frac{i_C(0)}{C} = \frac{-0.9}{800 \times 10^{-12}} = -1125 \times 10^3$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(12.5)(800 \times 10^{-12})} = 10^8; \qquad \omega_o = 10{,}000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(50{,}000)(800 \times 10^{-12})} = 12{,}500 \text{ rad/s}$$

$\alpha^2 > \omega_o^2$ so the response is overdamped

$$v_o = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -12{,}500 \pm \sqrt{(12{,}500)^2 - 10^8} = -12{,}500 \pm 7500$$

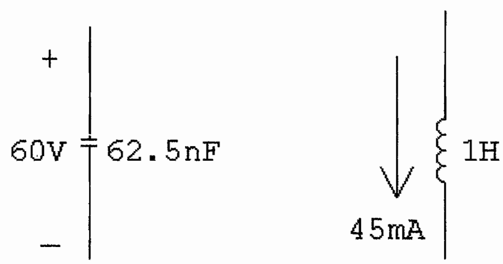
$$\therefore \quad s_{1,2} = -5000 \text{ r/s}, -20{,}000 \text{ r/s}$$

$$A_1 + A_2 = V_o = 45 \quad \text{and} \quad -5000A_1 - 20{,}000A_2 = -1125 \times 10^3$$

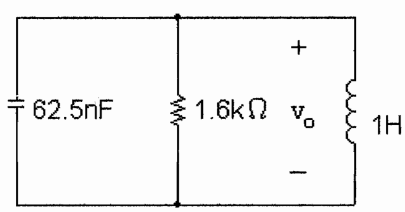
$$\therefore \quad A_1 = -15, \quad A_2 = 60$$

$$v_o = -15e^{-5000t} + 60e^{-20{,}000t} \text{ V}, \qquad t \geq 0$$

P 8.19 $t < 0: \qquad V_o = 60\,\text{V}, \qquad I_o = 45\,\text{mA}$



$t > 0$:



$$i_R(0) = \frac{60}{1600} = 37.5\,\text{mA}; \qquad i_L(0) = 45\,\text{mA}$$

$$i_C(0) = -37.5 - 45 = -82.5\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{3200(62.5)} = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{62.5} = 16 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 16 \times 10^6} = -5000 \pm 3000$$

$$s_1 = -2000 \text{ rad/s}; \qquad s_2 = -8000 \text{ rad/s}$$

$$\therefore \quad v_o = A_1 e^{-2000t} + A_2 e^{-8000t}$$

$$A_1 + A_2 = v_o(0) = 60$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = \frac{-82.5 \times 10^{-3}}{62.5 \times 10^{-9}} = -1320 \times 10^3$$

Solving, $\quad A_1 = -140\,\text{V}, \qquad A_2 = 200\,\text{V}$

$$\therefore \quad v_o = -140e^{-2000t} + 200e^{-8000t}\,\text{V}, \qquad t \geq 0$$

P 8.20 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{16 \times 10^6}{0.64} = 25 \times 10^6$

$$\alpha = \frac{1}{2RC} = \frac{16 \times 10^6}{4000} = 4000 \text{ rad/s}; \qquad \alpha^2 = 16 \times 10^3$$

$$\omega_d = \sqrt{(25 - 16) \times 10^6} = 3000 \text{ rad/s}$$

$$s_{1,2} = -4000 \pm j3000\text{rad/s}$$

$$v_o(t) = B_1 e^{-4000t} \cos 3000t + B_2 e^{-4000t} \sin 3000t$$

$$v_o(0) = B_1 = 60\,\text{V}$$

$$i_R(0) = \frac{60}{2000} = 30\,\text{mA}$$

$$i_L(0) = 45\,\text{mA}$$

$$i_C(0) = -i_R(0) - i_L(0) = -75\,\text{mA}$$

$$\frac{i_C(0)}{C} = (-75 \times 10^{-3})(16 \times 10^6) = -12 \times 10^5$$

$$\frac{dv_o}{dt}(0) = -4000B_1 + 3000B_2 = -12 \times 10^5$$

$$\therefore\ \ 3B_2 = 4B_1 - 1200 = 240 - 1200 = -960; \qquad \therefore\ \ B_2 = -320\,\text{V}$$

$$v_o(t) = 60e^{-4000t}\cos 3000t - 320e^{-4000t}\sin 3000t\,\text{V}, \qquad t \geq 0$$

P 8.21 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{16 \times 10^6}{0.16} = 10^8; \qquad \omega_o = 10^4$

$$\alpha = \frac{1}{2RC} = \frac{16 \times 10^6}{1600} = 10^4$$

$$\therefore\ \ \alpha^2 = \omega_o^2 \text{ (critical damping)}$$

$$v_o(t) = D_1te^{-10{,}000t} + D_2e^{-10{,}000t}$$

$$v_o(0) = D_2 = 60\,\text{V}$$

$$i_R(0) = \frac{60}{800} = 75\,\text{mA}$$

$$i_L(0) = 45\,\text{mA}$$

$$i_C(0) = -120\,\text{mA}$$

$$\frac{dv_o}{dt}(0) = -10{,}000D_2 + D_1$$

$$\frac{i_C(0)}{C} = (-120 \times 10^{-3})(16 \times 10^6) = -1920 \times 10^3$$

$$D_1 - 10{,}000D_2 = -1920 \times 10^3; \qquad D_1 = -1320 \times 10^3\text{V/s}$$

$$v_o(t) = (60 - 132 \times 10^4 t)e^{-10{,}000t}\,\text{V}, \qquad t > 0$$

P 8.22 [a] $v = L\left(\dfrac{di_L}{dt}\right) = 16[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{V}, \qquad t \geq 0$

[b] $i_R = \dfrac{v}{R} = 40[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

[c] $i_C = I - i_L - i_R = [-8e^{-20{,}000t} + 32e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

P 8.23 [a] $v = L\left(\dfrac{di_L}{dt}\right) = 40e^{-32,000t}\sin 24{,}000t\,\text{V}, \qquad t \geq 0$

[b] $i_C(t) = I - i_R - i_L = 24 \times 10^{-3} - \dfrac{v}{625} - i_L$

$$= [24e^{-32,000t}\cos 24{,}000t - 32e^{-32,000t}\sin 24{,}000t]\,\text{mA}, \qquad t \geq 0^+$$

P 8.24 $v = L\left(\dfrac{di_L}{dt}\right) = 960{,}000te^{-40,000t}\,\text{V}, \qquad t \geq 0$

P 8.25 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^6}{(20)(5)} = 10^4; \qquad \omega_o = 100 \text{ rad/s}$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{(1600)(5)} = \frac{10^4}{80} = 125 \text{ rad/s}$$

$$s_{1,2} = -125 \pm \sqrt{(125)^2 - 10^4} = -125 \pm 75$$

$$s_1 = -50 \text{ rad/s}; \qquad s_2 = -200 \text{ rad/s}$$

$$I_f = 15\,\text{mA}$$

$$i_L = 15 + A_1'e^{-50t} + A_2'e^{-200t}$$

$$\therefore \quad -30 = 15 + A_1' + A_2'; \qquad A_1' + A_2' = -45 \times 10^{-3}$$

$$\frac{di_L}{dt} = -50A_1' - 200A_2' = \frac{60}{20} = 3$$

Solving, $\quad A_1' = -40\,\text{mA}; \qquad A_2' = -5\,\text{mA}$

$$i_L = 15 - 40e^{-50t} - 5e^{-200t}\,\text{mA}, \quad t \geq 0$$

P 8.26 $\alpha = \dfrac{1}{2RC} = \dfrac{10^6}{(2500)(5)} = 80; \qquad \alpha^2 = 6400$

$$\omega_o^2 = 10^4; \qquad \omega_d = \sqrt{10^4 - 6400} = 60 \text{ rad/s}$$

$$s_{1,2} = -\alpha \pm j\omega_d = -80 \pm j60 \text{ rad/s}$$

$$i_L = 15 + B_1'e^{-80t}\cos 60t + B_2'e^{-80t}\sin 60t$$

$$-30 = 15 + B_1' \qquad \therefore \quad B_1' = -45\,\text{mA}$$

$$\frac{di_L}{dt}(0) = -80B_1' + 60B_2' = 3$$

$$\therefore \quad B_2' = -10\,\text{mA}$$

$$i_L = 15 - 45e^{-80t}\cos 60t - 10e^{-80t}\sin 60t\,\text{mA}, \quad t \geq 0$$

P 8.27 $\alpha = \dfrac{1}{2RC} = \dfrac{10^6}{(2000)(5)} = 100$

$\alpha^2 = 10^4 = \omega_o^2 \qquad$ critical damping

$i_L = I_f + D_1' te^{-100t} + D_2' e^{-100t} = 15 + D_1' te^{-100t} + D_2' e^{-100t}$

$i_L(0) = -30 = 15 + D_2'; \qquad \therefore \;\; D_2' = -45\,\text{mA}$

$\dfrac{di_L}{dt}(0) = -100D_2' + D_1' = 3000 \times 10^{-3}$

$\therefore \;\; D_1' = 3000 \times 10^{-3} + 100(-45 \times 10^{-3}) = -1500 \times 10^{-3}$

$i_L = 15 - 1500te^{-100t} - 45e^{-100t}\,\text{mA}, \quad t \geq 0$

P 8.28 $\alpha = \dfrac{1}{2RC} = \dfrac{10^6}{(1600)(6.25)} = 100; \qquad \alpha^2 = 10^4$

$\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^6}{(25)(6.25)} = 6400$

$s_{1,2} = -100 \pm \sqrt{10^4 - 6400} = -100 \pm 60$

$s_1 = -40\text{ rad/s}; \qquad s_2 = -160\text{ rad/s}$

$v_o(\infty) = 0 = V_f$

$\therefore \;\; v_o = A_1' e^{-40t} + A_2' e^{-160t}$

$v_o(0) = 30 = A_1' + A_2'$

Note: $\qquad i_C(0^+) = 0$

$\therefore \;\; \dfrac{dv_o}{dt}(0) = 0 = -40A_1' - 160A_2'$

Solving, $\qquad A_1' = 40\,\text{V}, \qquad A_2' = -10\,\text{V}$

$v_o(t) = 40e^{-40t} - 10e^{-160t}\,\text{V}, \qquad t > 0^+$

P 8.29 [a] $i_o = I_f + A_1' e^{-40t} + A_2' e^{-160t}$

$$I_f = \frac{30}{800} = 37.5\text{mA}; \qquad i_o(0) = 0$$

$$0 = 37.5 \times 10^{-3} + A_1' + A_2', \qquad \therefore \quad A_1' + A_2' = -37.5 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = \frac{30}{25} = -40A_1' - 160A_2'$$

Solving, $\quad A_1' = -40\,\text{mA}; \qquad A_2' = 2.5\,\text{mA}$

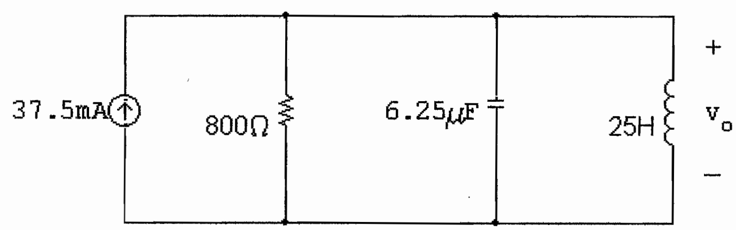
$$i_o = 37.5 - 40e^{-40t} + 2.5e^{-160t}\,\text{mA}, \quad t \geq 0$$

[b] $\displaystyle\frac{di_o}{dt} = [1600e^{-40t} - 400e^{-160t}] \times 10^{-3}$

$$L\frac{di_o}{dt} = 25(1.6)e^{-40t} - 25(0.4)e^{-160t}$$

$$\therefore \quad v_o = 40e^{-40t} - 10e^{-160t}\,\text{V}, \quad t \geq 0$$

P 8.30 For $\quad t > 0$



$$\alpha = \frac{1}{2RC} = 100; \qquad \frac{1}{LC} = 6400$$

$$s_{1,2} = -100 \pm 60$$

$$s_1 = -40\,\text{rad/s}; \qquad s_2 = -160\,\text{rad/s}$$

$$v_o = V_f + A_1' e^{-40t} + A_2' e^{-160t}$$

$$V_f = 0; \qquad v_o(0^+) = 0; \qquad i_C(0^+) = 37.5\,\text{mA}$$

$$\therefore \quad A_1' + A_2' = 0$$

$$\frac{dv_o(0^+)}{dt} = \frac{i_C(0^+)}{6.25 \times 10^{-6}} = 6000\,\text{V/s}$$

$$\frac{dv_o(0^+)}{dt} = -40A_1' - 160A_2'$$

$$-40A_1' - 160A_2' = 6000$$

$$A_1' + 4A_2' = -150$$

$$A_1' + A_2' = 0$$

$$\therefore \quad A_1' = 50\,\text{V}; \qquad A_2' = -50\,\text{V}$$

$$v_o = 50e^{-40t} - 50e^{-160t}\,\text{V}, \qquad t \geq 0$$

P 8.31 [a] From the solution to Prob. 8.30 $s_1 = -40\,\text{rad/s}$ and $s_2 = -160\,\text{rad/s}$, therefore

$$i_o = I_f + A_1'e^{-40t} + A_2'e^{-160t}$$

$$I_f = 37.5\,\text{mA}; \qquad i_o(0^+) = 0; \qquad \frac{di_o(0^+)}{dt} = 0$$

$$\therefore \quad 0 = 37.5 + A_1' + A_2'; \qquad -40A_1' - 160A_2' = 0$$

It follows that

$$A_1' = -50\,\text{mA}; \qquad A_2' = 12.5\,\text{mA}$$

$$\therefore \quad i_o = 37.5 - 50e^{-40t} + 12.5e^{-160t}\,\text{mA}, \qquad t \geq 0$$

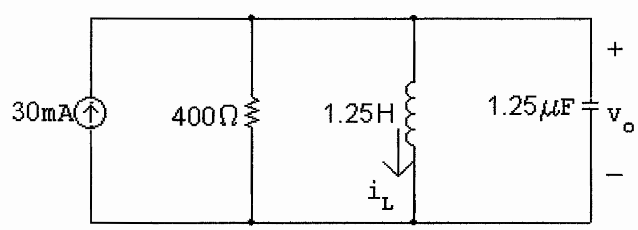
[b] $$\frac{di_o}{dt} = 2e^{-40t} - 2e^{-160t}$$

$$v_o = L\frac{di_o}{dt} = 25[2e^{-40t} - 2e^{-160t}]$$

$$v_o = 50e^{-40t} - 50e^{-160t}\,\text{V}, \qquad t \geq 0$$

P 8.32 $i_L(0^-) = i_L(0^+) = 30\,\text{mA}$

For $t > 0$



$$i_L(0^-) = i_L(0^+) = 30\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = 1000\,\text{rad/s}; \qquad \omega_o^2 = \frac{1}{LC} = 64 \times 10^4$$

$$s_1 = -400\,\text{rad/s} \qquad s_2 = -1600\,\text{rad/s}$$

$$v_o(\infty) = 0 = V_f$$

$$v_o = A_1' e^{-400t} + A_2' e^{-1600t}$$

$$i_C(0^+) = -30 + 30 + 0 = 0$$

$$\therefore \quad \frac{dv_o}{dt} = 0$$

$$\frac{dv_o}{dt}(0) = -400A_1' - 1600A_2'$$

$$\therefore \quad A_1' + 400A_2' = 0; \qquad A_1' + A_2' = 0$$

$$\therefore \quad A_1' = 0; \qquad A_2' = 0$$

$$\therefore \quad v_o = 0 \text{ for } t \geq 0$$

Note: $\quad v_o(0) = 0; \qquad v_o(\infty) = 0; \qquad \dfrac{dv_o(0)}{dt} = 0$

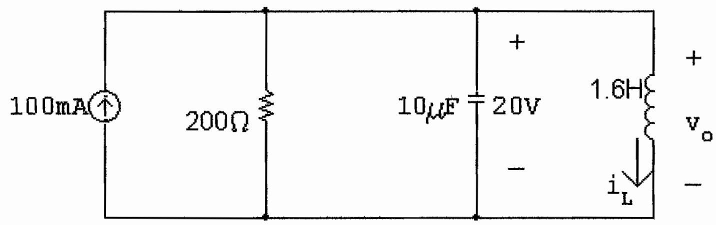
Hence the 30 mA current circulates between the current source and the ideal inductor in the equivalent circuit. In the original circuit the 12 V source sustains a current of 30 mA in the inductor. This is an example of a circuit going directly into steady state when the switch is closed. There is no transient period, or interval.

P 8.33 $\quad t < 0$:

$$v_o(0^-) = v_o(0^+) = \frac{1000}{1250}(25) = 20\,\text{V}$$

$$i_L(0^-) = i_L(0^+) = 0$$

$t > 0$



$$-100 + \frac{20}{0.2} + i_C(0^+) + 0 = 0; \qquad \therefore \quad i_C(0^+) = 0$$

$$\frac{1}{2RC} = \frac{10^6}{(400)(10)} = 250\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{10(1.6)} = 62{,}500$$

$\therefore \quad \alpha^2 = \omega_o^2 \quad$ critically damped

[a] $v_o = V_f + D_1' t e^{-250t} + D_2' e^{-250t}$

$V_f = 0$

$$\frac{dv_o(0)}{dt} = -250D_2' + D_1' = 0$$

$v_o(0^+) = 20 = D_2'$

$D_1' = 250D_2' = 5000\,\text{V/s}$

$\therefore \quad v_o = 5000te^{-250t} + 20e^{-250t}\,\text{V}, \quad t \geq 0^+$

[b] $i_L = I_f + D_3' t e^{-250t} + D_4' e^{-250t}$

$$i_L(0^+) = 0; \qquad I_f = 100\,\text{mA}; \qquad \frac{di_L(0^+)}{dt} = \frac{20}{1.6} = 12.5\text{ A/s}$$

$\therefore \quad 0 = 100 + D_4'; \qquad D_4' = -100\,\text{mA};$

$-250D_4' + D_3' = 12.5; \qquad D_3' = -12.5\,\text{A/s}$

$\therefore \quad i_L = 100 - 12{,}500te^{-250t} - 100e^{-250t}\,\text{mA} \qquad t \geq 0$

P 8.34 [a] $w_L = \int_0^\infty p\,dt = \int_0^\infty v_o i_L\,dt$

$v_o = 5000te^{-250t} + 20e^{-250t}\,\text{V}$

$i_L = 0.1 - 12.5te^{-250t} - 0.1e^{-250t}\,\text{A}$

$$p = 2e^{-250t} + 500te^{-250t} - 750te^{-500t} - 62{,}500t^2e^{-500t} - 2e^{-500t}\,\text{W}$$

$$\frac{w_L}{2} = \int_0^\infty e^{-250t}\,dt + 250\int_0^\infty te^{-250t}\,dt - 375\int_0^\infty te^{-500t} -$$

$$31{,}250\int_0^\infty t^2e^{-500t}\,dt - \int_0^\infty e^{-500t}\,dt$$

$$= \frac{e^{-250t}}{-250}\Big|_0^\infty + \frac{250}{(250)^2}e^{-250t}(-250t-1)\Big|_0^\infty -$$

$$\frac{375}{(500)^2}e^{-500t}(-500t-1)\Big|_0^\infty -$$

$$\frac{31{,}250}{(-500)^3}e^{-500t}(500^2t^2+1000t+2)\Big|_0^\infty -$$

$$\frac{e^{-500t}}{(-500)}\Big|_0^\infty$$

All the upper limits evaluate to zero hence

$$\frac{w_L}{2} = \frac{1}{250} + \frac{250}{62{,}500} - \frac{375}{25\times10^4} - \frac{(31{,}250)(2)}{(5)^3 10^6} - \frac{1}{500}$$

$$w_L = 8 + 8 - 3 - 1 - 4 = 8\,\text{mJ}$$

Note this value corresponds to the final energy stored in the inductor, i.e.

$$w_L(\infty) = \frac{1}{2}(1.6)(0.1)^2 = 8\,\text{mJ}.$$

[b] $v = 5000te^{-250t} + 20e^{-250t}\,\text{V}$

$$i_R = \frac{v}{200} = 25te^{-250t} + 0.1e^{-250t}\,\text{A}$$

$$p_R = vi_R = 2e^{-500t}[62{,}500t^2 + 500t + 1]$$

$$w_R = \int_0^\infty p_R\,dt$$

$$\frac{w_R}{2} = 62{,}500\int_0^\infty t^2e^{-500t}\,dt + 500\int_0^\infty te^{-500t}\,dt + \int_0^\infty e^{-500t}\,dt$$

$$= \frac{62{,}500e^{-500t}}{-125\times10^6}[25\times10^4t^2 + 1000t + 2]\Big|_0^\infty +$$

$$\frac{500e^{-500t}}{25\times10^4}(-500t-1)\Big|_0^\infty + \frac{e^{-500t}}{(-500)}\Big|_0^\infty$$

Since all the upper limits evaluate to zero we have

$$\frac{w_R}{2} = \frac{62{,}500(2)}{125\times10^6} + \frac{500}{25\times10^4} + \frac{1}{500}$$

$w_R = 2 + 4 + 4 = 10\,\text{mJ}$

[c] $100 = i_R + i_C + i_L \quad (\text{mA})$

$$\begin{aligned} i_R + i_L &= 25{,}000te^{-250t} + 100e^{-250t} + 100 - 12{,}500te^{-250t} - 100e^{-250t}\,\text{mA} \\ &= 100 + 12{,}500te^{-250t}\,\text{mA} \end{aligned}$$

$$\therefore \quad i_C = 100 - (i_R + i_L) = -12{,}500te^{-250t}\,\text{mA} = -12.5te^{-250t}\,\text{A}$$

$$\begin{aligned} p_C = vi_C &= [5000te^{-250t} + 20e^{-250t}][-12.5te^{-250t}] \\ &= -250[250t^2e^{-500t} + te^{-500t}] \end{aligned}$$

$$\frac{w_C}{-250} = 250\int_0^\infty t^2e^{-500t}\,dt + \int_0^\infty te^{-500t}\,dt$$

$$\frac{w_C}{-250} = \frac{250e^{-500t}}{-125\times 10^6}[25\times 10^4t^2 + 1000t + 2]\Big|_0^\infty + \frac{e^{-500t}}{25\times 10^4}(-500t - 1)\Big|_0^\infty$$

Since all upper limits evaluate to zero we have

$$w_C = \frac{-250(250)(2)}{125\times 10^6} - \frac{250(1)}{25\times 10^4} = -1000\times 10^{-6} - 10\times 10^{-4} = -2\,\text{mJ}$$

Note this 2 mJ corresponds to the initial energy stored in the capacitor, i.e.,

$$w_C(0) = \frac{1}{2}(10\times 10^{-6})(20)^2 = 2\,\text{mJ}.$$

Thus $w_C(\infty) = 0\,\text{mJ}$ which agrees with the final value of $v = 0$.

[d] $i_s = 100\,\text{mA}$

$$\begin{aligned} p_s(\text{del}) &= 100v\,\text{mW} \\ &= 0.1[5000te^{-250t} + 20e^{-250t}] \\ &= 2e^{-250t} + 500te^{-250t}\,\text{W} \\ \frac{w_s}{2} &= \int_0^\infty e^{-250t}\,dt + \int_0^\infty 250te^{-250t}\,dt \\ &= \frac{e^{-250t}}{-250}\Big|_0^\infty + \frac{250e^{-250t}}{62{,}500}(-250t - 1)\Big|_0^\infty \\ &= \frac{1}{250} + \frac{1}{250} \\ w_s &= \frac{2(2)}{250} = \frac{4}{250} = 16\,\text{mJ} \end{aligned}$$

[e] $w_L = 8\,\text{mJ} \quad (\text{absorbed})$

$w_R = 10\,\text{mJ} \quad (\text{absorbed})$

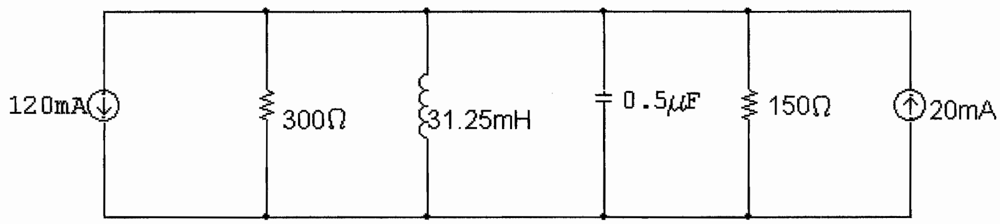
$$w_C = 2\,\text{mJ} \quad \text{(delivered)}$$

$$w_S = 16\,\text{mJ} \quad \text{(delivered)}$$

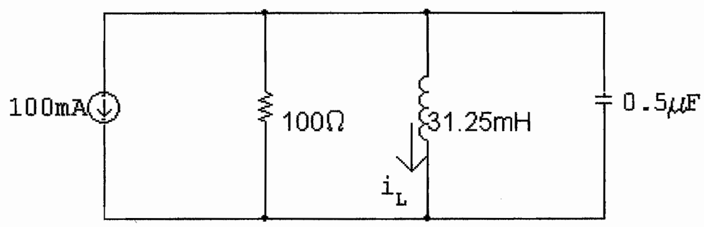
$$\sum w_{\text{del}} = w_{\text{abs}} = 18\,\text{mJ}.$$

P 8.35 $t < 0:$ $\quad i_L = 3/150 = 20\,\text{mA}$

$t > 0$:



$300\|150 = 100\,\Omega$



$$i_L(0) = 20\,\text{mA}, \qquad i_L(\infty) = -100\,\text{mA}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(31.25)(0.5)} = 64 \times 10^6; \qquad \omega_o = 8000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{(200)(0.5)} = 10^4; \qquad \alpha^2 = 100 \times 10^6$$

$$\alpha^2 - \omega_o^2 = (100 - 64)10^6 = 36 \times 10^6$$

$$s_{1,2} = -10{,}000 \pm 6000$$

$$s_1 = -4000 \text{ rad/s}; \qquad s_2 = -16{,}000 \text{ rad/s}$$

$$i_L = I_f + A_1' e^{-4000t} + A_2' e^{-16{,}000t}$$

$$i_L(\infty) = I_f = -100\text{mA}$$

$$i_L(0) = A_1' + A_2' + I_f = 20\,\text{mA}$$

$\therefore\quad A_1' + A_2' - 100 = 20 \quad \text{so} \quad A_1' + A_2' = 120\,\text{mA}$

$\frac{di_L}{dt}(0) = 0 = -4000A_1 - 16{,}000A_2'$

Solving, $\quad A_1' = 160\,\text{mA}, \qquad A_2' = -40\,\text{mA}$

$i_L = -100 + 160e^{-4000t} - 40e^{-16{,}000t}\,\text{mA}, \qquad t \geq 0$

P 8.36 $v_C(0^+) = \frac{1}{2}(240) = 120\,\text{V}$

$i_L(0^+) = 60\,\text{mA}; \qquad i_L(\infty) = \frac{240}{5} \times 10^{-3} = 48\,\text{mA}$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{2(2500)(5)} = 40$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{400} = 2500$$

$\alpha^2 = 1600; \qquad \alpha^2 < \omega_o^2; \qquad \therefore \quad \text{underdamped}$

$s_{1,2} = -40 \pm j\sqrt{2500 - 1600} = -40 \pm j30\,\text{rad/s}$

$$\begin{aligned} i_L &= I_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t \\ &= 48 + B_1'e^{-40t}\cos 30t + B_2'e^{-40t}\sin 30t \end{aligned}$$

$i_L(0) = 48 + B_1'; \qquad B_1' = 60 - 48 = 12\,\text{mA}$

$\frac{di_L}{dt}(0) = 30B_2' - 40B_1' = \frac{120}{80} = 1.5 = 1500 \times 10^{-3}$

$\therefore\quad 30B_2' = 40(12) \times 10^{-3} + 1500 \times 10^{-3}; \qquad B_2' = 66\,\text{mA}$

$\therefore\quad i_L = 48 + 12e^{-40t}\cos 30t + 66e^{-40t}\sin 30t\,\text{mA}, \qquad t \geq 0$

P 8.37 [a] $2\alpha = 5000; \quad \alpha = 2500\,\text{rad/s}$

$\sqrt{\alpha^2 - \omega_o^2} = 1500; \qquad \omega_o^2 = 4 \times 10^6; \qquad \omega_o = 2000\,\text{rad/s}$

$\alpha = \frac{R}{2L} = 2500; \qquad R = 5000L$

$\omega_o^2 = \frac{1}{LC} = 4 \times 10^6; \qquad L = \frac{10^9}{4 \times 10^6(50)} = 5\text{H}$

$R = 25{,}000\,\Omega$

[b] $i(0) = 0$

$$L\frac{di(0)}{dt} = v_c(0); \qquad \frac{1}{2}(50) \times 10^{-9}v_c^2(0) = 90 \times 10^{-6}$$

$$\therefore \quad v_c^2(0) = 3600; \qquad v_c(0) = 60\,\text{V}$$

$$\frac{di(0)}{dt} = \frac{60}{5} = 12\,\text{A/s}$$

[c] $i(t) = A_1e^{-1000t} + A_2e^{-4000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -1000A_1 - 4000A_2 = 12$$

Solving,

$$\therefore \quad A_1 = 4\,\text{mA}; \qquad A_2 = -4\,\text{mA}$$

$$i(t) = 4e^{-1000t} - 4e^{-4000t}\,\text{mA} \qquad t \geq 0$$

[d] $\dfrac{di(t)}{dt} = -4e^{-1000t} + 16e^{-4000t}$

$$\frac{di}{dt} = 0 \text{ when } 16e^{-4000t} = 4e^{-1000t}$$

$$\text{or } e^{3000t} = 4$$

$$\therefore \quad t = \frac{\ln 4}{3000}\mu\text{s} = 462.10\,\mu\text{s}$$

[e] $i_{\text{max}} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89\,\text{mA}$

[f] $v_L(t) = 5\dfrac{di}{dt} = [-20e^{-1000t} + 80e^{-4000t}]\,\text{V}, \quad t \geq 0^+$

P 8.38 $\alpha = 800\,\text{rad/s}; \qquad \omega_d = 600\,\text{rad/s}$

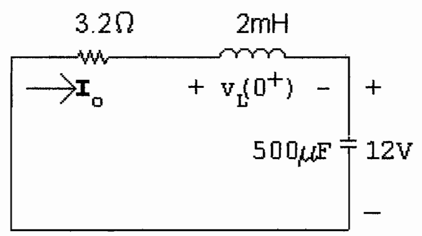
$$\omega_o^2 - \alpha^2 = 36 \times 10^4; \qquad \omega_o^2 = 100 \times 10^4; \quad w_o = 1000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 800; \qquad R = 1600L$$

$$\frac{1}{LC} = 100 \times 10^4; \qquad L = \frac{10^6}{(100 \times 10^4)(500)} = 2\,\text{mH}$$

$$\therefore \quad R = 3.2\,\Omega$$

$$i(0^+) = B_1 = 0\,\text{A}; \qquad \text{at } t = 0^+$$



$$12 + 0 + v_L(0^+) = 0; \qquad v_L(0^+) = -12\,\text{V}$$

$$\frac{di(0^+)}{dt} = \frac{-12}{0.002} = -6000\,\text{A/s}$$

$$\therefore \quad \frac{di(0^+)}{dt} = 600B_2 - 800B_1 = -6000$$

$$\therefore \quad 600B_2 = 800B_1 - 6000; \qquad \therefore \quad B_2 = -10\,\text{A}$$

$$\therefore \quad i = -10e^{-800t}\sin 600t\,\text{A}, \quad t \geq 0$$

P 8.39 From Prob. 8.38 we know v_c will be of the form

$$v_c = B_3e^{-800t}\cos 600t + B_4e^{-800t}\sin 600t$$

From Prob. 8.38 we have

$$v_c(0) = 12\,\text{V} = B_3$$

and

$$\frac{dv_c(0)}{dt} = \frac{i_C(0)}{C} = 0$$

$$\frac{dv_c(0)}{dt} = 600B_4 - 800B_3$$

$$\therefore \quad 600B_4 = 800B_3 + 0; \qquad B_4 = 16\,\text{V}$$

$$v_c(t) = 12e^{-800t}\cos 600t + 16e^{-800t}\sin 600t\,\text{V} \qquad t \geq 0$$

P 8.40 [a] $t < 0$:

$$i_o = \frac{120}{8000} = 15\,\text{mA}; \qquad v_o = (5000)(0.015) = 75\,\text{V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{5000}{2(1)} = 2500\ \text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(1)(250)} = 4 \times 10^6 = 400 \times 10^4$$

$$\alpha^2 - \omega_o^2 = 625 \times 10^4 - 400 \times 10^4 = 225 \times 10^4$$

$$\therefore \quad s_{1,2} = -2500 \pm 1500$$

$$s_1 = -1000\ \text{rad/s} \qquad s_2 = -4000\ \text{rad/s}$$

$$\therefore \quad i_o(t) = A_1 e^{-1000t} + A_2 e^{-4000t}$$

$$i_o(0) = A_1 + A_2 = 15 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = -1000A_1 - 4000A_2 = 0$$

Solving, $\qquad A_1 = 20\,\text{mA}; \qquad A_2 = -5\,\text{mA}$

$$i_o(t) = 20e^{-1000t} - 5e^{-4000t}\,\text{mA}, \qquad t \geq 0^+$$

[b] $v_o(t) = A_1 e^{-1000t} + A_2 e^{-4000t}$

$$v_o(0) = A_1 + A_2 = 75$$

$$\frac{dv_o}{dt}(0) = -1000A_1 - 4000A_2 = \frac{-15 \times 10^{-3}}{250 \times 10^{-9}}$$

Solving, $\qquad A_1 = 80\,\text{V}; \qquad A_2 = -5\,\text{V}$

$$v_o(t) = 80e^{-1000t} - 5e^{-4000t}\,\text{V}, \qquad t \geq 0^+$$

Check:

$$5000i_o + 1\frac{di_o}{dt} = v_o$$

$$5000i_o = 100e^{-1000t} - 25e^{-4000t}$$

$$\frac{di_o}{dt} = -20e^{-1000t} + 20e^{-4000t}$$

$$\therefore \quad 5000i_o + \frac{di_o}{dt} = 80e^{-1000t} - 5e^{-4000t}\,\text{V} \qquad \text{(checks)}$$

P 8.41 [a] $\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(0.25)(160)} = \frac{10^8}{4} = 25 \times 10^6$

$$\alpha = \frac{R}{2L} = \omega_o = 5000\,\text{rad/s}$$

$$\therefore \quad R = (5000)(2)L = 2500\,\Omega$$

[b] $i(0) = i_L(0) = 24\,\text{mA}$

$$v_L(0) = 90 - (0.024)(2500) = 30\,\text{V}$$

$$\frac{di}{dt}(0) = \frac{30}{0.25} = 120\ \text{A/s}$$

[c] $v_C = D_1 te^{-5000t} + D_2 e^{-5000t}$

$$v_C(0) = D_2 = 90\,\text{V}$$

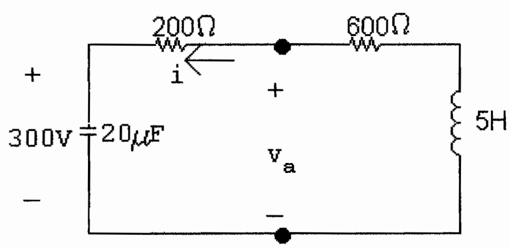
$$\frac{dv_C}{dt}(0) = D_1 - 5000D_2 = \frac{i_C(0)}{C} = \frac{-i_L(0)}{C}$$

$$D_1 - 450{,}000 = -\frac{24 \times 10^{-3}}{160 \times 10^{-9}} = -150{,}000$$

$$\therefore \quad D_1 = 300{,}000\ \text{V/s}$$

$$v_C = 300{,}000te^{-5000t} + 90e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

P 8.42 [a] For $t > 0$:



Since $i(0^-) = i(0^+) = 0$

$$v_a(0^+) = 300\,\text{V}$$

[b] $v_a = 200i + 5 \times 10^4 \int_0^t i\,dx + 300$

$$\frac{dv_a}{dt} = 200\frac{di}{dt} + 5 \times 10^4 i$$

$$\frac{dv_a(0^+)}{dt} = 200\frac{di(0^+)}{dt} + 5 \times 10^4 i(0^+) = 200\frac{di(0^+)}{dt}$$

$$-L\frac{di(0^+)}{dt} = 300$$

$$\frac{di(0^+)}{dt} = -0.2(300) = -60\,\text{A/s}$$

$$\therefore \quad \frac{dv_a(0^+)}{dt} = -12{,}000\,\text{V/s}$$

[c] $$\alpha = \frac{R}{2L} = \frac{800}{10} = 80\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(5)(20)} = 10^4$$

$$s_{1,2} = -80 \pm \sqrt{6400 - 10^4} = -80 \pm j60\,\text{rad/s}$$

Underdamped:

$$v_a = B_1 e^{-80t}\cos 60t + B_2 e^{-80t}\sin 60t$$

$$v_a(0) = B_1 = 300\,\text{V}$$

$$\frac{dv_a(0)}{dt} = -80B_1 + 60B_2 = -12{,}000; \qquad \therefore \quad B_2 = 200\,\text{V}$$

$$v_a = 300e^{-80t}\cos 60t + 200e^{-80t}\sin 60t\,\text{V}, \quad t \geq 0^+$$

P 8.43 $$i_L(0^-) = i_L(0^+) = \frac{70}{50 + 200} = 280\,\text{mA}$$

$$v_c(0^-) = v_c(0^+) = 200(0.280) = 56\,\text{V}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.100)(200 \times 10^{-9})} = 50 \times 10^6$$

$$\alpha = \frac{R}{2L} = \frac{200}{2(0.100)} = 1000; \qquad \alpha^2 = 10^6$$

$$\alpha^2 < \omega_o^2 \quad \therefore \quad \text{underdamped}$$

$$s_{1,2} = -1000 \pm j7000\ \text{rad/s}$$

$$i = B_1 e^{-1000t}\cos 7000t + B_2 e^{-1000t}\sin 7000t$$

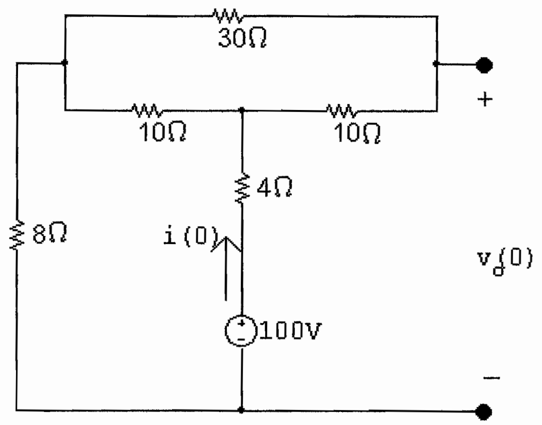
$$i(0) = B_1 = 280\,\text{mA}$$

$$\frac{di}{dt}(0) = 7000B_2 - 1000B_1 = 0$$

$$\therefore \quad B_2 = \frac{1}{7}B_1 = 40\,\text{mA}$$

$$i = 280e^{-1000t}\cos 7000t + 40e^{-1000t}\sin 7000t\,\text{mA}, \qquad t \geq 0^+$$

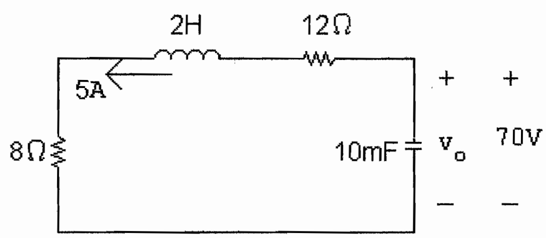
P 8.44 $t < 0$:



$$i(0) = \frac{100}{4+8+8} = \frac{100}{20} = 5\,\text{A}$$

$$v_o(0) = 100 - 5(4) - 10(5)\left(\frac{10}{50}\right) = 70\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{20}{4} = 5, \qquad \alpha^2 = 25$$

$$\omega_o^2 = \frac{1}{LC} = \frac{100}{2} = 50$$

$\omega_o^2 > \alpha^2$ underdamped

$$v_o = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t; \qquad \omega_d = \sqrt{50-25} = 5$$

$$v_o = B_1 e^{-5t}\cos 5t + B_2 e^{-5t}\sin 5t$$

$$v_o(0) = B_1 = 70\,\text{V}$$

$$C\frac{dv_o}{dt}(0) = -5, \qquad \frac{dv_o}{dt} = \frac{-5}{10} \times 10^3 = -500\,\text{V/s}$$

$$\frac{dv_o}{dt}(0) = -5B_1 + 5B_2 = -500$$

$$5B_2 = -500 + 5B_1 = -500 + 350; \qquad B_2 = -150/5 = -30\,\text{V}$$

$$\therefore\quad v_o = 70e^{-5t}\cos 5t - 30e^{-5t}\sin 5t\,\text{V}, \qquad t \geq 0$$

P 8.45 $\alpha = \dfrac{R}{2L} = 5000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{20} = 50 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 50 \times 10^6} = -5000 \pm j5000\,\text{rad/s}$$

$$v_o = V_f + B_1'e^{-5000t}\cos 5000t + B_2'e^{-5000t}\sin 5000t$$

$$v_o(0) = 0 = V_f + B_1'$$

$$v_o(\infty) = 40\,\text{V}; \qquad \therefore\ B_1' = -40\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = 5000B_2' - 5000B_1'$$

$$\therefore\ B_2' = B_1' = -40\,\text{V}$$

$$v_o = 40 - 40e^{-5000t}\cos 5000t - 40e^{-5000t}\sin 5000t\,\text{V}, \quad t \geq 0$$

P 8.46 $\alpha = \dfrac{R}{2L} = 5000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.4)(100 \times 10^{-9})} = 25 \times 10^6 \qquad \therefore\ \omega_o = 5000\ \text{rad/s}$$

The response is therefore critically damped

$$v_o = V_f + D_1'te^{-5000t} + D_2'e^{-5000t}$$

$$v_o(0) = 0 = V_f + D_2'$$

$$v_o(\infty) = 40\,\text{V}; \qquad \therefore\ D_2' = -40\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = D_1' - \alpha D_2'$$

$$\therefore\ D_1' = (5000)(-40) = -200{,}000\ \text{V/s}$$

$$v_o = 40 - 200{,}000te^{-5000t} - 40e^{-5000t}\,\text{V}, \quad t \geq 0$$

P 8.47 $\alpha = \dfrac{R}{2L} = 5000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.4)(156.25\times 10^{-9})} = 16\times 10^6 \qquad \therefore\ \ \omega_o = 4000\ \text{rad/s}$$

The response is therefore overdamped

$$s_{1,2} = -5000 \pm \sqrt{5000^2 - 4000^2} = -5000 \pm 3000 = -2000\,\text{rad/s},\ \ -8000\,\text{rad/s},$$

$$v_o = V_f + A_1'e^{-2000t} + A_2'e^{-8000t}$$

$$v_o(0) = 0 = V_f + A_1' + A_2'$$

$$v_o(\infty) = 40\,\text{V}; \qquad \therefore\ \ A_1' + A_2' = -40\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = s_1A_1' + S_2A_2' = -2000A_1' - 8000A_2'$$

$$\therefore\ \ A_1' = -53.33\,\text{V}, \quad A_2' = 13.33\,\text{V}$$

$$v_o = 40 - 53.33e^{-2000t} + 13.33e^{-8000t}\,\text{V}, \quad t \geq 0$$

P 8.48 [a] Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $\quad i = B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$

$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]\,e^{-\alpha t}$$

Therefore $\quad \omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \dfrac{\alpha}{\omega_d}B_1' = \dfrac{\alpha}{\omega_d}\dfrac{V_g}{R}$

Therefore

$$v_o = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\frac{V_g L}{R}\left(\frac{\alpha^2 + \omega_d^2}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R}\left(\frac{\omega_o^2}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R\omega_d}\left(\frac{1}{LC}\right)e^{-\alpha t}\sin\omega_d t$$

$$v_o = -\frac{V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t\,\text{V}, \quad t \geq 0^+$$

[b] $$\frac{dv_o}{dt} = -\frac{V_g}{\omega_d RC}\{\omega_d\cos\omega_d t - \alpha\sin\omega_d t\}e^{-\alpha t}$$

$$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}$$

Therefore $\omega_d t = \tan^{-1}(\omega_d/\alpha)$ (smallest t)

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

P 8.49 [a] From Problem 8.48 we have

$$v_o = \frac{-V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t$$

$$\alpha = \frac{R}{2L} = \frac{120}{0.01} = 12{,}000\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^{12}}{2500} = 400 \times 10^6$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 16\,\text{krad/s}$$

$$\frac{-V_g}{RC\omega_d} = \frac{-(-600)10^9}{(120)(500)(16)\times 10^3} = 625$$

$$\therefore \quad v_o = 625e^{-12{,}000t}\sin 16{,}000t\,\text{V}$$

[b] From Problem 8.48

$$t_d = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = \frac{1}{16{,}000}\tan^{-1}\left(\frac{16{,}000}{12{,}000}\right)$$

$$t_d = 57.96\,\mu\text{s}$$

[c] $v_{\text{max}} = 625e^{-0.012(57.96)}\sin[(0.016)(57.96)] = 249.42\,\text{V}$

[d] $R = 12\,\Omega;\qquad \alpha = 1200\,\text{rad/s}$

$$\omega_d = 19{,}963.97\,\text{rad/s}$$

$$v_o = 5009.02e^{-1200t}\sin 19{,}963.97t\,\text{V},\quad t \geq 0$$

$$t_d = 75.67\,\mu\text{s}$$

$$v_{\text{max}} = 4565.96\,\text{V}$$

P 8.50 $i_C(0) = 0;\qquad v_o(0) = 200\,\text{V}$

$$\alpha = \frac{R}{2L} = \frac{4}{2(0.04)} = 50\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^3}{0.4} = 2500$$

$$\therefore\ \alpha^2 = \omega_o^2;\qquad \text{critical damping}$$

$$v_o(t) = V_f + D_1' te^{-50t} + D_2' e^{-50t}$$

$$V_f = 100\,\text{V}$$

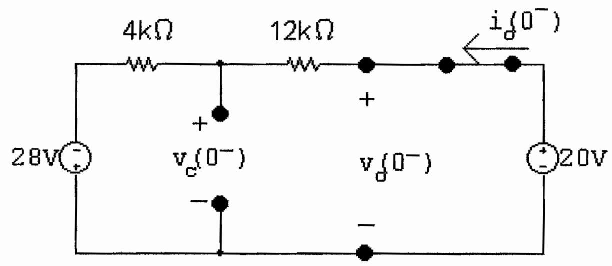
$$v_o(0) = 100 + D_2' = 200;\qquad D_2' = 100\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -50D_2' + D_1' = 0$$

$$D_1' = 50D_2' = 5000\ \text{V/s}$$

$$v_o = 100 + 5000te^{-50t} + 100e^{-50t}\,\text{V},\quad t \geq 0$$

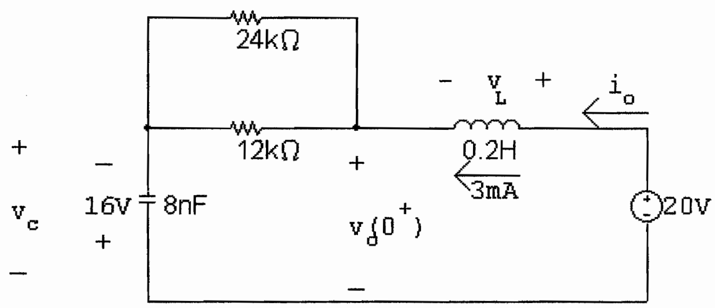
P 8.51 **[a]** $t < 0$:



$$i_o(0^-) = \frac{48}{16{,}000} = 3\,\text{mA}$$

$$v_C(0^-) = 20 - (12{,}000)(0.003) = -16\text{ V}$$

$t = 0^+$:



$$12\,\text{k}\Omega \| 24\,\text{k}\Omega = 8\,\text{k}\Omega$$

$$\therefore \quad v_o(0^+) = (0.003)(8000) - 16 = 24 - 16 = 8\text{ V}$$

$$\text{and } v_L(0^+) = 20 - 8 = 12\text{ V}$$

[b] $v_o(t) = 8000 i_o + v_C$

$$\frac{dv_o}{dt}(t) = 8000\frac{di_o}{dt} + \frac{dv_C}{dt}$$

$$\frac{dv_o}{dt}(0^+) = 8000\frac{di_o}{dt}(0^+) + \frac{dv_C}{dt}(0^+)$$

$$v_L(0^+) = L\frac{di_o}{dt}(0^+)$$

$$\frac{di_o}{dt}(0^+) = \frac{v_L(0^+)}{L} = \frac{12}{0.2} = 60\text{ A/s}$$

$$C\frac{dv_c}{dt}(0^+) = i_o(0^+)$$

$$\therefore \quad \frac{dv_c}{dt}(0^+) = \frac{3\times 10^{-3}}{8\times 10^{-9}} = 375{,}000$$

$$\therefore \quad \frac{dv_o}{dt}(0^+) = 8000(60) + 375{,}000 = 855{,}000\text{ V/s}$$

[c] $\omega_o^2 = \frac{1}{LC} = \frac{10^9}{1.6} = 625 \times 10^6; \qquad \omega_o = 25{,}000 \text{ rad/s}$

$$\alpha = \frac{R}{2L} = \frac{8000}{0.4} = 20{,}000 \text{ rad/s}; \qquad \alpha^2 = 400 \times 10^6$$

$\alpha^2 < \omega_o^2$ underdamped

$$s_{1,2} = -20{,}000 \pm j15{,}000 \text{ rad/s}$$

$$v_o(t) = V_f + B_1'e^{-20{,}000t}\cos 15{,}000t + B_2'e^{-20{,}000t}\sin 15{,}000t$$

$$V_f = v_o(\infty) = 20\,\text{V}$$

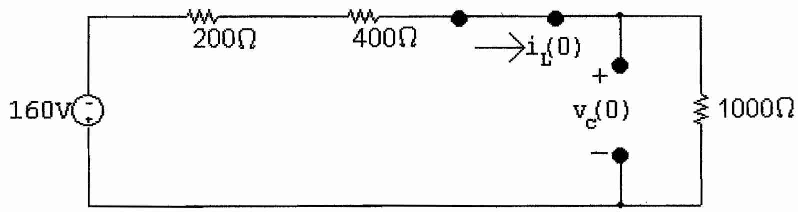
$$8 = 20 + B_1'; \qquad B_1' = -12\,\text{V}$$

$$-20{,}000B_1' + 15{,}000B_2' = 855{,}000$$

Solving, $B_2' = 41$ V

$$\therefore \quad v_o(t) = 20 - 12e^{-20{,}000t}\cos 15{,}000t + 41e^{-20{,}000t}\sin 15{,}000t\,\text{V}, \qquad t \geq 0^+$$

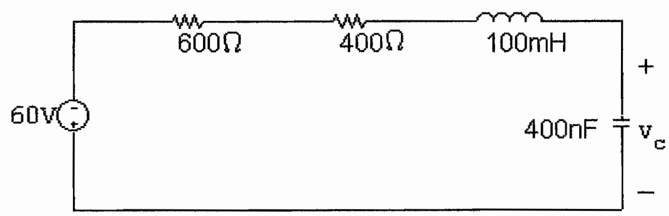
P 8.52 $t < 0$:



$$i_L(0) = \frac{-160}{1600} = -100\,\text{mA}$$

$$v_C(0) = 1000 i_L(0) = -100\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{1000}{200} \times 10^3 = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^9)(10^3)}{(100)(400)} = \frac{10^8}{4} = 25 \times 10^6$$

$\omega_o = 5000$ rad/s $\qquad \therefore \quad$ critical damping

$$v_C(t) = V_f + D_1'te^{-5000t} + D_2'e^{-5000t}$$

$$v_C(0) = -100\,\text{V}; \qquad V_f = -60\,\text{V}$$

$$\therefore \quad -100 = -60 + D_2'; \qquad D_2' = -40\,\text{V}$$

$$C\frac{dv_C}{dt}(0) = i_L(0) = -100 \times 10^{-3}$$

$$\frac{dv_C}{dt}(0) = \frac{-100 \times 10^{-3}}{400 \times 10^{-9}} = -250{,}000 \text{ V/s}$$

$$\therefore \quad D_1' = 5000(-40) - 250{,}000 = -450{,}000$$

$$v_C(t) = -60 - 450{,}000te^{-5000t} - 40e^{-5000t}\,\text{V}, \qquad t \geq 0$$

P 8.53 [a] $v_c = V_f + [B_1' \cos\omega_d t + B_2' \sin\omega_d t]\, e^{-\alpha t}$

$$\frac{dv_c}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]e^{-\alpha t}$$

Since the initial stored energy is zero,

$$v_c(0^+) = 0 \quad \text{and} \quad \frac{dv_c(0^+)}{dt} = 0$$

It follows that $B_1' = -V_f$ and $B_2' = \dfrac{\alpha B_1'}{\omega_d}$

When these values are substituted into the expression for $[dv_c/dt]$, we get

$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{\omega_d} + \omega_d\right) V_f e^{-\alpha t}\sin\omega_d t$$

But $V_f = V$ and $\dfrac{\alpha^2}{\omega_d} + \omega_d = \dfrac{\alpha^2 + \omega_d^2}{\omega_d} = \dfrac{\omega_o^2}{\omega_d}$

Therefore $\dfrac{dv_c}{dt} = \left(\dfrac{\omega_o^2}{\omega_d}\right) Ve^{-\alpha t}\sin\omega_d t$

[b] $\dfrac{dv_c}{dt} = 0$ when $\sin\omega_d t = 0$, or $\omega_d t = n\pi$

where $n = 1, 2, 3, \ldots$

Therefore $t = \dfrac{n\pi}{\omega_d}$

[c] When $t_n = \dfrac{n\pi}{\omega_d}$, $\cos\omega_d t_n = \cos n\pi = (-1)^n$

and $\sin\omega_d t = \sin n\pi = 0$

Therefore $v_c(t_n) = V[1 - (-1)^n e^{-\alpha n\pi/\omega_d}]$

[d] It follows from [c] that

$$v_(t_1) = V + Ve^{-(\alpha\pi/\omega_d)} \quad \text{and} \quad v_c(t_3) = V + Ve^{-(3\alpha\pi/\omega_d)}$$

Therefore $\dfrac{v_c(t_1) - V}{v_c(t_3) - V} = \dfrac{e^{-(\alpha\pi/\omega_d)}}{e^{-(3\alpha\pi/\omega_d)}} = e^{(2\alpha\pi/\omega_d)}$

But $\dfrac{2\pi}{\omega_d} = t_3 - t_1 = T_d$, thus $\alpha = \dfrac{1}{T_d}\ln\dfrac{[v_c(t_1) - V]}{[v_c(t_3) - V]}$

P 8.54 $\alpha = \dfrac{1}{T_d}\ln\left\{\dfrac{v_c(t_1) - V}{v_c(t_3) - V}\right\}$; $\quad T_d = t_3 - t_1 = \dfrac{3\pi}{12} - \dfrac{\pi}{12} = \dfrac{2\pi}{12}$ ms

$$\alpha = \frac{12{,}000}{2\pi}\ln\left[\frac{13.505}{0.985}\right] = 5000; \qquad \omega_d = \frac{2\pi}{T_d} = 12{,}000\,\text{rad/s}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = 144\times10^6 + 25\times10^6 = 169\times10^6$$

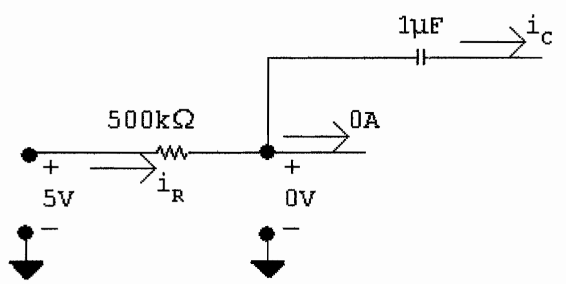
$$L = \frac{1}{(169)(0.2)} = 29.6\,\text{mH}; \qquad R = 2\alpha L = 295.86\,\Omega$$

P 8.55 At $t = 0$ the voltage across each capacitor is zero. It follows that since the operational amplifiers are ideal, the current in the 500 kΩ is zero. Therefore there cannot be an instantaneous change in the current in the 1 μF capacitor. Since the capacitor current equals $C(dv_o/dt)$, the derivative must be zero.

P 8.56 [a] From Example 8.13 $\dfrac{d^2v_o}{dt^2} = 2$

therefore $\dfrac{dg(t)}{dt} = 2$, $\quad g(t) = \dfrac{dv_o}{dt}$

$$g(t) - g(0) = 2t; \qquad g(t) = 2t + g(0); \qquad g(0) = \frac{dv_o(0)}{dt}$$



$$i_R = \frac{5}{500}\times10^{-3} = 10\,\mu\text{A} = i_C = -C\frac{dv_o(0)}{dt}$$

$$\frac{dv_o(0)}{dt} = \frac{-10 \times 10^{-6}}{1 \times 10^{-6}} = -10 = g(0)$$

$$\frac{dv_o}{dt} = 2t - 10$$

$$dv_o = 2t\,dt - 10\,dt$$

$$v_o - v_o(0) = t^2 - 10t; \qquad v_o(0) = 8\,\text{V}$$

$$v_o = t^2 - 10t + 8, \qquad 0 \le t \le t_{\text{sat}}$$

[b] $t^2 - 10t + 8 = -9$

$$t^2 - 10t + 17 = 0$$

$$t \cong 2.17\,\text{s}$$

P 8.57 Part (1) — Example 8.14, with R_1 and R_2 removed:

[a] $R_a = 100\,\text{k}\Omega; \qquad C_1 = 0.1\,\mu\text{F}; \qquad R_b = 25\,\text{k}\Omega; \qquad C_2 = 1\,\mu\text{F}$

$$\frac{d^2v_o}{dt^2} = \left(\frac{1}{R_aC_1}\right)\left(\frac{1}{R_bC_2}\right)v_g; \qquad \frac{1}{R_aC_1} = 100 \quad \frac{1}{R_bC_2} = 40$$

$$v_g = 250 \times 10^{-3}; \qquad \text{therefore} \quad \frac{d^2v_o}{dt^2} = 1000$$

[b] Since $v_o(0) = 0 = \dfrac{dv_o(0)}{dt}$, our solution is $v_o = 500t^2$

The second op-amp will saturate when

$$v_o = 6\,\text{V}, \quad \text{or} \quad t_{\text{sat}} = \sqrt{6/500} \cong 0.1095\,\text{s}$$

[c] $\dfrac{dv_{o1}}{dt} = -\dfrac{1}{R_aC_1}v_g = -25$

[d] Since $v_{o1}(0) = 0, \quad v_{o1} = -25t\,\text{V}$

At $t = 0.1095\,\text{s}, \quad v_{o1} \cong -2.74\,\text{V}$

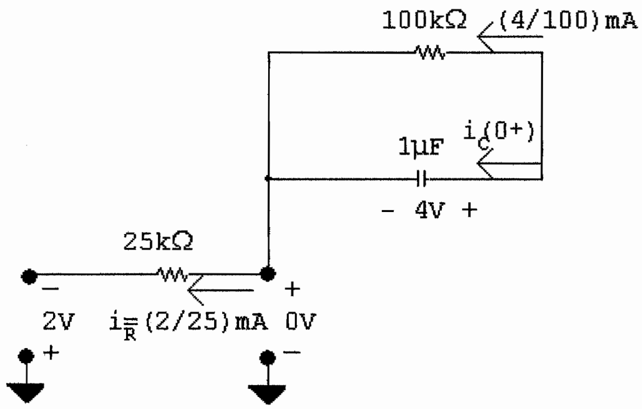
Therefore the second amplifier saturates before the first amplifier saturates. Our expressions are valid for $0 \le t \le 0.1095\,\text{s}$. Once the second op-amp saturates, our linear model is no longer valid.

Part (2) — Example 8.14 with $v_{o1}(0) = -2\,\text{V}$ and $v_o(0) = 4\,\text{V}$:

[a] Initial conditions will not change the differential equation; hence the equation is the same as Example 8.14.

[b] $v_o = 5 + A_1'e^{-10t} + A_2'e^{-20t}$ (from Example 8.14)

$v_o(0) = 4 = 5 + A_1' + A_2'$



$$\frac{4}{100} + i_C(0^+) - \frac{2}{25} = 0$$

$$i_C(0^+) = \frac{4}{100}\,\text{mA} = C\frac{dv_o(0^+)}{dt}$$

$$\frac{dv_o(0^+)}{dt} = \frac{0.04 \times 10^{-3}}{10^{-6}} = 40\,\text{V/s}$$

$$\frac{dv_o}{dt} = -10A_1'e^{-10t} - 20A_2'e^{-20t}$$

$$\frac{dv_o}{dt}(0^+) = -10A_1' - 20A_2' = 40$$

Therefore $-A_1' - 2A_2' = 4$ and $A_1' + A_2' = -1$
Thus, $A_1' = 2$ and $A_2' = -3$

$v_o = 5 + 2e^{-10t} - 3e^{-20t}$ V

[c] Same as Example 8.14:

$$\frac{dv_{o1}}{dt} + 20v_{o1} = -25$$

[d] From Example 8.14:

$v_{o1}(\infty) = -1.25\,\text{V};$ $v_1(0) = -2\,\text{V}$ (given)

Therefore

$v_{o1} = -1.25 + (-2 + 1.25)e^{-20t} = -1.25 - 0.75e^{-20t}$ V

P 8.58 [a] $\dfrac{d^2v_o}{dt^2} = \dfrac{1}{R_1C_1R_2C_2}v_g$

$$\frac{1}{R_1C_1R_2C_2} = \frac{10^{-6}}{(50)(20)(2)(4) \times 10^{-6} \times 10^{-6}} = 125$$

$$\therefore \quad \frac{d^2v_o}{dt^2} = 125v_g$$

$0 \le t \le 0.2^-$:

$v_g = 400\,\text{mV}$

$$\frac{d^2v_o}{dt^2} = 50$$

Let $g(t) = \dfrac{dv_o}{dt}$, then $\dfrac{dg}{dt} = 50$ or $dg = 50\,dt$

$$\int_{g(0)}^{g(t)} dx = 50\int_0^t dy$$

$$g(t) - g(0) = 50t, \quad g(0) = \frac{dv_o}{dt}(0) = 0$$

$$g(t) = \frac{dv_o}{dt} = 50t$$

$dv_o = 50t\,dt$

$$\int_{v_o(0)}^{v_o(t)} dx = 50\int_0^t x\,dx; \qquad v_o(t) - v_o(0) = 25t^2, \quad v_o(0) = 0$$

$v_o(t) = 25t^2\,\text{V}, \quad 0 \le t \le 0.2^-$

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1C_1}v_g = -10v_g = -4$$

$dv_{o1} = -4\,dt$

$$\int_{v_{o1}(0)}^{v_{o1}(t)} dx = -4\int_0^t dy$$

$v_{o1}(t) - v_{o1}(0) = -4t, \qquad v_{o1}(0) = 0$

$v_{o1}(t) = -4t\,\text{V}, \qquad 0 \le t \le 0.2^-$

$0.2^+ \le t \le t_{\text{sat}}$:

$$\frac{d^2v_o}{dt^2} = -12.5, \qquad \text{let} \quad g(t) = \frac{dv_o}{dt}$$

$$\frac{dg(t)}{dt} = -12.5; \qquad dg(t) = -12.5\,dt$$

$$\int_{g(0.2^+)}^{g(t)} dx = -12.5\int_{0.2}^t dy$$

$$g(t) - g(0.2^+) = -12.5(t - 0.2) = -12.5t + 2.5$$

$$g(0.2^+) = \frac{dv_o(0.2^+)}{dt}$$

$$C\frac{dv_o}{dt}(0.2^+) = \frac{0 - v_{o1}(0.2^+)}{20\times10^3}$$

$$v_{o1}(0.2^+) = v_o(0.2^-) = -4(0.2) = -0.80\,\text{V}$$

$$\therefore\quad C\frac{dv_{o1}(0.2^+)}{dt} = \frac{0.80}{20\times10^3} = 40\,\mu\text{A}$$

$$\frac{dv_{o1}}{dt}(0.2^+) = \frac{40\times10^{-6}}{4\times10^{-6}} = 10\,\text{V/s}$$

$$\therefore\quad g(t) = -12.5t + 2.5 + 10 = -12.5t + 12.5 = \frac{dv_o}{dt}$$

$$\therefore\quad dv_o = -12.5t\,dt + 12.5\,dt$$

$$\int_{v_o(0.2^+)}^{v_o(t)} dx = \int_{0.2^+}^{t} -12.5y\,dy + \int_{0.2^+}^{t} 12.5\,dy$$

$$v_o(t) - v_o(0.2^+) = -6.25y^2\Big|_{0.2}^{t} + 12.5y\Big|_{0.2}^{t}$$

$$v_o(t) = v_o(0.2^+) - 6.25t^2 + 0.25 + 12.5t - 2.5$$

$$v_o(0.2^+) = v_o(0.2^-) = 1\,\text{V}$$

$$\therefore\quad v_o(t) = -6.25t^2 + 12.5t - 1.25\,\text{V}, \qquad 0.2^+ \le t \le t_{\text{sat}}$$

$$\frac{dv_{o1}}{dt} = -10(-0.1) = 1, \qquad 0.2^+ \le t \le t_{\text{sat}}$$

$$dv_{o1} = dt; \qquad \int_{v_{o1}(0.2^+)}^{v_{o1}(t)} dx = \int_{0.2^+}^{t} dy$$

$$v_{o1}(t) - v_{o1}(0.2^+) = t - 0.2; \qquad v_{o1}(0.2^+) = v_{o1}(0.2^-) = -0.8\,\text{V}$$

$$\therefore\quad v_{o1}(t) = t - 1\,\text{V}, \qquad 0.2^+ \le t \le t_{\text{sat}}$$

Summary:

$0 \le t \le 0.2^-\text{s}:\qquad v_{o1} = -4t\,\text{V},\quad v_o = 25t^2\,\text{V}$

$0.2^+\text{s} \le t \le t_{\text{sat}}:\qquad v_{o1} = t - 1\,\text{V},\quad v_o = -6.25t^2 + 12.5t - 1.25\,\text{V}$

[b] $-10 = -6.25t_{\text{sat}}^2 + 12.5t_{\text{sat}} - 1.25$

$$\therefore\quad 6.25t_{\text{sat}}^2 - 12.5t_{\text{sat}} - 8.75 = 0$$

$$t_{\text{sat}}^2 - 2t_{\text{sat}} - 1.4 = 0$$

$$t_{\text{sat}} = 1 \pm \sqrt{2 + 1.4} = 1 \pm 1.844$$

$$\therefore\quad t_{\text{sat}} = 2.844\,\text{sec}$$

$$v_{o1}(t_{\text{sat}}) = 1.844 - 1 = 0.844\,\text{V}$$

P 8.59 $\tau_1 = (0.25 \times 10^6)(2 \times 10^{-6}) = 0.50\,\text{s}$

$$\frac{1}{\tau_1} = 2; \qquad \tau_2 = (0.25 \times 10^6)(4 \times 10^{-6}) = 1\,\text{s}; \qquad \therefore\ \frac{1}{\tau_2} = 1$$

$$\therefore\ \frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = 50$$

$$s^2 + 3s + 2 = 0$$

$$(s+1)(s+2) = 0; \qquad s_1 = -1, \quad s_2 = -2$$

$$v_o = V_f + A_1'e^{-t} + A_2'e^{-2t}; \qquad V_f = \frac{50}{2} = 25\,\text{V}$$

$$v_o = 25 + A_1'e^{-t} + A_2'e^{-2t}$$

$$v_o(0) = 0 = 25 + A_1' + A_2'; \qquad \frac{dv_o}{dt}(0) = 0 = -A_1' - 2A_2'$$

$$\therefore\ A_1' = -50, \qquad A_2' = 25\,\text{V}$$

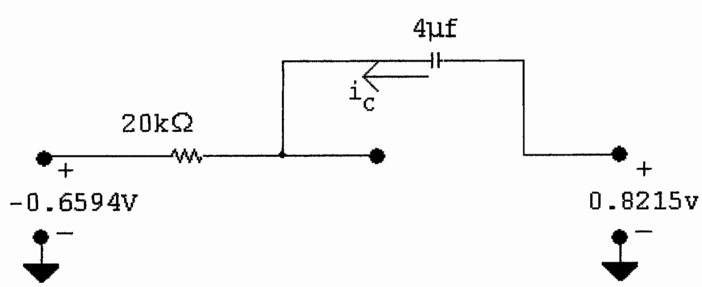
$$v_o(t) = 25 - 50e^{-t} + 25e^{-2t}\,\text{V}, \qquad 0 \le t \le 0.2\,\text{s}$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = -4; \qquad \therefore\ v_{o1} = -2 + 2e^{-2t}\,\text{V}, \quad 0 \le t \le 0.2\,\text{s}$$

$$v_o(0.2) = 25 - 50e^{-0.2} + 25e^{-0.4} = 0.8215\,\text{V}$$

$$v_{o1}(0.2) = -2 + 2e^{-0.4} = -0.6594\,\text{V}$$

At $t = 0.2\,\text{s}$



$$i_C = \frac{0 + 0.6594}{20 \times 10^3} = 32.97\,\mu\text{A}$$

$$C\frac{dv_o}{dt} = 32.97\,\mu\text{A}; \qquad \frac{dv_o}{dt} = \frac{32.97}{4} = 8.24\,\text{V/s}$$

$0.2\,\text{s} \leq t < \infty$:

$$\frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2 = -12.5$$

$$v_o(\infty) = -6.25$$

$$\therefore \quad v_o = -6.25 + A_1'e^{-(t-0.2)} + A_2'e^{-2(t-0.2)}$$

$$0.8215 = -6.25 + A_1' + A_2'$$

$$\frac{dv_o}{dt}(0.2) = 8.24 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' + A_2' = 7.07; \qquad -A_1' - 2A_2' = 8.24$$

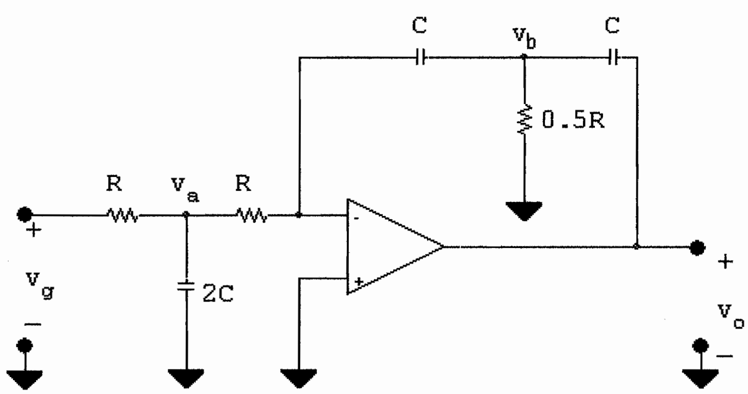
$$A_1' = 22.38; \qquad A_2' = -15.31$$

$$\therefore \quad v_o = -6.25 + 22.38e^{-(t-0.2)} - 15.31e^{-2(t-0.2)}\,\text{V}, \qquad 0.2 \leq t < \infty$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = 1$$

$$\therefore \quad v_{o1} = 0.5 + (-0.6594 - 1)e^{-2(t-0.2)} = 0.5 - 1.66e^{-2(t-0.2)}\,\text{V}, \qquad 0.2 \leq t < \infty$$

P 8.60 [a]



$$2C\frac{dv_a}{dt} + \frac{v_a - v_g}{R} + \frac{v_a}{R} = 0$$

(1) Therefore $\dfrac{dv_a}{dt} + \dfrac{v_a}{RC} = \dfrac{v_g}{2RC}$; $\qquad \dfrac{0 - v_a}{R} + C\dfrac{d(0 - v_b)}{dt} = 0$

(2) Therefore $\frac{dv_b}{dt} + \frac{v_a}{RC} = 0, \quad v_a = -RC\frac{dv_b}{dt}$

$$\frac{2v_b}{R} + C\frac{dv_b}{dt} + C\frac{d(v_b - v_o)}{dt} = 0$$

(3) Therefore $\frac{dv_b}{dt} + \frac{v_b}{RC} = \frac{1}{2}\frac{dv_o}{dt}$

From (2) we have $\frac{dv_a}{dt} = -RC\frac{d^2v_b}{dt^2}$ and $v_a = -RC\frac{dv_b}{dt}$

When these are substituted into (1) we get

(4) $-RC\frac{d^2v_b}{dt^2} - \frac{dv_b}{dt} = \frac{v_g}{2RC}$

Now differentiate (3) to get

(5) $\frac{d^2v_b}{dt^2} + \frac{1}{RC}\frac{dv_b}{dt} = \frac{1}{2}\frac{d^2v_o}{dt^2}$

But from (4) we have

(6) $\frac{d^2v_b}{dt^2} + \frac{1}{RC}\frac{dv_b}{dt} = -\frac{v_g}{2R^2C^2}$

Now substitute (6) into (5)

$$\frac{d^2v_o}{dt^2} = -\frac{v_g}{R^2C^2}$$

[b] When $R_1C_1 = R_2C_2 = RC: \quad \frac{d^2v_o}{dt^2} = \frac{v_g}{R^2C^2}$

The two equations are the same except for a reversal in algebraic sign.

[c] Two integrations of the input signal with one operational amplifier.

P 8.61 [a] $f(t) = \text{inertial force} + \text{frictional force} + \text{spring force}$

$$= M[d^2x/dt^2] + D[dx/dt] + Kx$$

[b] $\frac{d^2x}{dt^2} = \frac{f}{M} - \left(\frac{D}{M}\right)\left(\frac{dx}{dt}\right) - \left(\frac{K}{M}\right)x$

Given $v_A = \frac{d^2x}{dt^2}$, then

$$v_B = -\frac{1}{R_1C_1}\int_0^t \left(\frac{d^2x}{dy^2}\right)dy = -\frac{1}{R_1C_1}\frac{dx}{dt}$$

$$v_C = -\frac{1}{R_2C_2}\int_0^t v_B\,dy = \frac{1}{R_1R_2C_1C_2}x$$

$$v_D = -\frac{R_3}{R_4} \cdot v_B = \frac{R_3}{R_4 R_1 C_1} \frac{dx}{dt}$$

$$v_E = \left[\frac{R_5 + R_6}{R_6}\right] v_C = \left[\frac{R_5 + R_6}{R_6}\right] \cdot \frac{1}{R_1 R_2 C_1 C_2} \cdot x$$

$$v_F = \left[\frac{-R_8}{R_7}\right] f(t), \qquad v_A = -(v_D + v_E + v_F)$$

Therefore $$\frac{d^2x}{dt^2} = \left[\frac{R_8}{R_7}\right] f(t) - \left[\frac{R_3}{R_4 R_1 C_1}\right] \frac{dx}{dt} - \left[\frac{R_5 + R_6}{R_6 R_1 R_2 C_1 C_2}\right] x$$

Therefore $M = \frac{R_7}{R_8}$, $\quad D = \frac{R_3 R_7}{R_8 R_4 R_1 C_1}$ and $K = \frac{R_7(R_5 + R_6)}{R_8 R_6 R_1 R_2 C_1 C_2}$

Box Number	Function
1	inverting and scaling
2	inverting and scaling
3	integrating and scaling
4	integrating and scaling
5	inverting and scaling
6	noninverting and scaling

P 8.62 [a] Given that the current response is underdamped we know i will be of the form

$$i = I_f + [B_1' \cos \omega_d t + B_2' \sin \omega_d t]e^{-\alpha t}$$

where $$\alpha = \frac{R}{2L}$$

and $$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \alpha^2}$$

The capacitor will force the final value of i to be zero, therefore $I_f = 0$.
By hypothesis $i(0^+) = V_{dc}/R$ therefore $B_1' = V_{dc}/R$.
At $t = 0^+$ the voltage across the primary winding is zero hence $di(0^+)/dt = 0$.
From our equation for i we have

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1') \cos \omega_d t - (\omega_d B_1' + \alpha B_2') \sin \omega_d t]e^{-\alpha t}$$

Hence

$$\frac{di(0^+)}{dt} = \omega_d B_2' - \alpha B_1' = 0$$

Thus

$$B_2' = \frac{\alpha}{\omega_d}B_1' = \frac{\alpha V_{dc}}{\omega_d R}$$

It follows directly that

$$i = \frac{V_{dc}}{R}\left[\cos\omega_d t + \frac{\alpha}{\omega_d}\,\sin\omega_d t\right]e^{-\alpha t}$$

[b] Since $\omega_d B_1' - \alpha B_1' = 0$ it follows that

$$\frac{di}{dt} = -(\omega_d B_1' + \alpha B_2')e^{-\alpha t}\,\sin\omega_d t$$

But $\quad \alpha B_2' = \dfrac{\alpha^2 V_{dc}}{\omega_d R} \quad$ and $\quad \omega_d B_1' = \dfrac{\omega_d V_{dc}}{R}$

Therefore

$$\omega_d B_1' + \alpha B_2' = \frac{\omega_d V_{dc}}{R} + \frac{\alpha^2 V_{dc}}{\omega_d R} = \frac{V_{dc}}{R}\left[\frac{\omega_d^2 + \alpha^2}{\omega_d}\right]$$

But $\quad \omega_d^2 + \alpha^2 = \omega_o^2 = \dfrac{1}{LC}$

Hence

$$\omega_d B_1' + \alpha B_2' = \frac{V_{dc}}{\omega_d RLC}$$

Now since $\quad v_1 = L\dfrac{di}{dt} \quad$ we get

$$v_1 = -L\frac{V_{dc}}{\omega_d RLC}e^{-\alpha t}\,\sin\omega_d t = -\frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\,\sin\omega_d t$$

[c] $v_c = V_{dc} - iR - L\dfrac{di}{dt}$

$$iR = V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\,\sin\omega_d t\right)e^{-\alpha t}$$

$$v_c = V_{dc} - V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\,\sin\omega_d t\right)e^{-\alpha t} + \frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\sin\omega_d t$$

$$= V_{dc} - V_{dc}e^{-\alpha t}\cos\omega_d t + \left(\frac{V_{dc}}{\omega_d RC} - \frac{\alpha V_{dc}}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right)e^{-\alpha t}\sin\omega_d t\right]$$

$$= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t\right]$$

P 8.63 $v_{sp} = V_{dc}\left[1 - \frac{a}{\omega_d RC}e^{-\alpha t}\sin\omega_d t\right]$

$$\frac{dv_{sp}}{dt} = \frac{-aV_{dc}}{\omega_d RC}\frac{d}{dt}[e^{-\alpha t}\sin\omega_d t]$$

$$= \frac{-aV_{dc}}{\omega_d RC}[-\alpha e^{-\alpha t}\sin\omega_d t + \omega_d\cos\omega_d t e^{-\alpha t}]$$

$$= \frac{aV_{dc}e^{-\alpha t}}{\omega_d RC}[\alpha\sin\omega_d t - \omega_d\cos\omega_d t]$$

$$\frac{dv_{sp}}{dt} = 0 \quad \text{when} \quad \alpha\sin\omega_d t = \omega_d\cos\omega_d t$$

$$\text{or} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}; \qquad \omega_d t = \tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

$$\therefore\ t_{\max} = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

Note that because $\tan\theta$ is periodic, i.e., $\tan\theta = \tan(\theta \pm n\pi)$, where n is an integer, there are an infinite number of solutions for t where $dv_{sp}/dt = 0$, that is

$$t = \frac{\tan^{-1}(\omega_d/\alpha) \pm n\pi}{\omega_d}$$

Because of $e^{-\alpha t}$ in the expression for v_{sp} and knowing $t \geq 0$ we know v_{sp} will be maximum when t has its smallest positive value. Hence

$$t_{\max} = \frac{\tan^{-1}(\omega_d/\alpha)}{\omega_d}.$$

P 8.64 [a] $v_c = V_{dc}[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t]$

$$\frac{dv_c}{dt} = V_{dc}\frac{d}{dt}[1 + e^{-\alpha t}(K\sin\omega_d t - \cos\omega_d t)]$$

$$= V_{dc}\{(-\alpha e^{-\alpha t})(K\sin\omega_d t - \cos\omega_d t) +$$
$$e^{-\alpha t}[\omega_d K\cos\omega_d t + \omega_d\sin\omega_d t]\}$$
$$= V_{dc}e^{-\alpha t}[(\omega_d - \alpha K)\sin\omega_d t + (\alpha + \omega_d K)\cos\omega_d t]$$

$$\frac{dv_c}{dt} = 0 \quad \text{when} \quad (\omega_d - \alpha K)\sin\omega_d t = -(\alpha + \omega_d K)\cos\omega_d t$$

or $\tan\omega_d t = \left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$

$$\therefore \quad \omega_d t \pm n\pi = \tan^{-1}\left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$t_c = \frac{1}{\omega_d}\left\{tan^{-1}\left(\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right) \pm n\pi\right\}$$

$$\alpha = \frac{R}{2L} = \frac{4 \times 10^3}{6} = 666.67\,\text{rad/s}$$

$$\omega_d = \sqrt{\frac{10^9}{1.2} - (666.67)^2} = 28{,}859.81\,\text{rad/s}$$

$$K = \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right) = 21.63$$

$$t_c = \frac{1}{\omega_d}\left\{\tan^{-1}(-43.29) + n\pi\right\} = \frac{1}{\omega_d}\{-1.55 + n\pi\}$$

The smallest positive value of t occurs when $n = 1$, therefore

$$t_{c\,\text{max}} = 55.23\,\mu\text{s}$$

[b] $v_c(t_{c\,\text{max}}) = 12[1 - e^{-\alpha t_{c\,\text{max}}}\cos\omega_d t_{c\,\text{max}} + Ke^{-\alpha t_{c\,\text{max}}}\sin\omega_d t_{c\,\text{max}}]$

$$= 262.42\,\text{V}$$

[c] From the text example the voltage across the spark plug reaches its maximum value in 53.63 μs. If the spark plug does not fire the capacitor voltage peaks in 55.23 μs. When v_{sp} is maximum the voltage across the capacitor is 262.15 V. If the spark plug does not fire the capacitor voltage reaches 262.42 V.

P 8.65 [a] $w = \frac{1}{2}L[i(0^+)]^2 = \frac{1}{2}(5)(16) \times 10^{-3} = 40\,\text{mJ}$

[b] $\alpha = \frac{R}{2L} = \frac{3 \times 10^3}{10} = 300\,\text{rad/s}$

$$\omega_d = \sqrt{\frac{10^9}{1.25} - (300)^2} = 28{,}282.68\,\text{rad/s}$$

$$\frac{1}{Rc} = \frac{10^6}{0.75} = \frac{4 \times 10^6}{3}$$

$$t_{\text{max}} = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = 55.16\,\mu\text{s}$$

$$v_{sp}\,(t_{\text{max}}) = 12 - \frac{12(50)(4 \times 10^6)}{3(28{,}282.68)}e^{-\alpha t_{\text{max}}}\sin\omega_d t_{\text{max}} = -27{,}808.04\,\text{V}$$

[c] $v_c\,(t_{\text{max}}) = 12[1 - e^{-\alpha t_{\text{max}}} \cos \omega_d t_{\text{max}} + K e^{-\alpha t_{\text{max}}} \sin \omega_d t_{\text{max}}]$

$$K = \frac{1}{\omega_d}\left[\frac{1}{RC} - \alpha\right] = 47.13$$

$$v_c\,(t_{\text{max}}) = 568.15\,\text{V}$$

9

Sinusoidal Steady State Analysis

Assessment Problems

AP 9.1 [a] $\mathbf{V} = 170\underline{/-40^\circ}\,\mathrm{V}$

[b] $10\sin(1000t + 20^\circ) = 10\cos(1000t - 70^\circ)$

$\therefore \quad \mathbf{I} = 10\underline{/-70^\circ}\,\mathrm{A}$

[c] $\mathbf{I} = 5\underline{/36.87^\circ} + 10\underline{/-53.13^\circ}$

$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\underline{/-26.57^\circ}\,\mathrm{A}$

[d] $\sin(20{,}000\pi t + 30^\circ) = \cos(20{,}000\pi t - 60^\circ)$

Thus,

$$\mathbf{V} = 300\underline{/45^\circ} - 100\underline{/-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$$

$$= 162.13 + j298.73 = 339.90\underline{/61.51^\circ}\,\mathrm{mV}$$

AP 9.2 [a] $v = 18.6\cos(\omega t - 54^\circ)\,\mathrm{V}$

[b] $\mathbf{I} = 20\underline{/45^\circ} - 50\underline{/-30^\circ} = 14.14 + j14.14 - 43.3 + j25$

$= -29.16 + j39.14 = 48.81\underline{/126.68^\circ}$

Therefore $\quad i = 48.81\cos(\omega t + 126.68^\circ)\,\mathrm{mA}$

[c] $\mathbf{V} = 20 + j80 - 30\underline{/15^\circ} = 20 + j80 - 28.98 - j7.76$

$= -8.98 + j72.24 = 72.79\underline{/97.08^\circ}$

$v = 72.79\cos(\omega t + 97.08^\circ)\,\mathrm{V}$

AP 9.3 [a] $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

[b] $Z_L = j\omega L = j200\,\Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\underline{/30^\circ})(200\underline{/90^\circ}) \times 10^{-3} = 2\underline{/120^\circ}\,\text{V}$

[d] $v_L = 2\cos(10{,}000t + 120^\circ)\,\text{V}$

AP 9.4 [a] $X_C = \dfrac{-1}{\omega C} = \dfrac{-1}{4000(5 \times 10^{-6})} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{30\underline{/25^\circ}}{50\underline{/-90^\circ}} = 0.6\underline{/115^\circ}\,\text{A}$

[d] $i = 0.6\cos(4000t + 115^\circ)\,\text{A}$

AP 9.5 $\mathbf{I}_1 = 100\underline{/25^\circ} = 90.63 + j42.26$

$\mathbf{I}_2 = 100\underline{/145^\circ} = -81.92 + j57.36$

$\mathbf{I}_3 = 100\underline{/-95^\circ} = -8.71 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A}, \qquad \text{therefore} \quad i_4 = 0\,\text{A}$

AP 9.6 [a] $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{|Z|\underline{/\theta_z}} = \dfrac{125}{|Z|}\underline{/(-60 - \theta_Z)^\circ}$

But $-60 - \theta_Z = -105^\circ \qquad \therefore\ \theta_Z = 45^\circ$

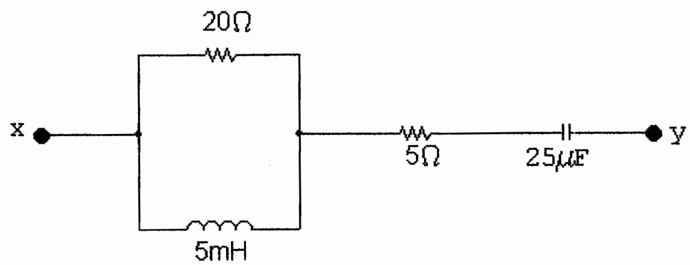
$Z = 90 + j160 + jX_C$

$\therefore\ X_C = -70\,\Omega; \qquad X_C = -\dfrac{1}{\omega C} = -70$

$\therefore\ C = \dfrac{1}{(70)(5000)} = 2.86\,\mu\text{F}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}_s}{Z} = \dfrac{125\underline{/-60^\circ}}{(90 + j90)} = 0.982\underline{/-105^\circ}A; \qquad \therefore\ |\mathbf{I}| = 0.982\,\text{A}$

AP 9.7 [a]



$\omega = 2000\,\text{rad/s}$

$\omega L = 10\,\Omega, \qquad \dfrac{-1}{\omega C} = -20\,\Omega$

$Z_{xy} = 20\|j10 + 5 + j20 = \dfrac{20(j10)}{(20 + j10)} + 5 - j20$

$= 4 + j8 + 5 - j20 = (9 - j12)\,\Omega$

[b] $\omega L = 40\,\Omega, \qquad \frac{-1}{\omega C} = -5\,\Omega$

$$Z_{xy} = 5 - j5 + 20\|j40 = 5 - j5 + \left[\frac{(20)(j40)}{20 + j40}\right]$$

$$= 5 - j5 + 16 + j8 = (21 + j3)\,\Omega$$

[c] $Z_{xy} = \left[\frac{20(j\omega L)}{20 + j\omega L}\right] + \left(5 - \frac{j10^6}{25\omega}\right)$

$$= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for ω yields $\omega = 4000\,\text{rad/s}$.

[d] $Z_{xy} = \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15\,\Omega$

AP 9.8 The frequency 4000 rad/s was found to give $Z_{xy} = 15\,\Omega$ in Assessment Problem 9.7. Thus,

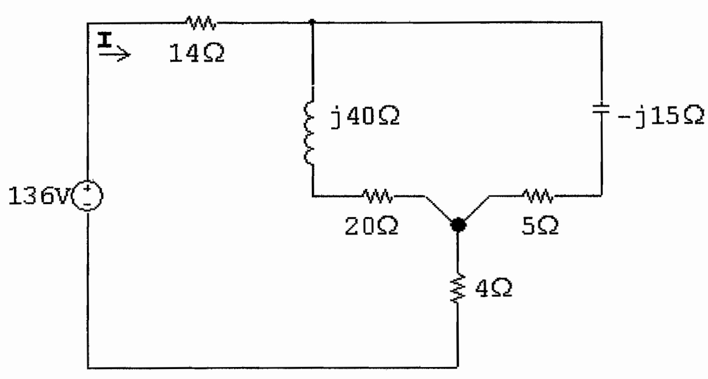
$$\mathbf{V} = 150\angle 0^\circ, \qquad \mathbf{I}_s = \frac{\mathbf{V}}{Z_{xy}} = \frac{150\angle 0^\circ}{15} = 10\angle 0^\circ\,\text{A}$$

Using current division,

$$\mathbf{I}_L = \frac{20}{20 + j20}(10) = 5 - j5 = 7.07\angle -45^\circ\,\text{A}$$

$$i_L = 7.07\cos(4000t - 45^\circ)\,\text{A}, \qquad I_m = 7.07\,\text{A}$$

AP 9.9 After replacing the delta made up of the $50\,\Omega$, $40\,\Omega$, and $10\,\Omega$ resistors with its equivalent wye, the circuit becomes



The circuit is further simplified by combining the parallel branches,

$$(20 + j40)\|(5 - j15) = (12 - j16)\,\Omega$$

$$\text{Therefore}\quad \mathbf{I} = \frac{136\angle 0^\circ}{14 + 12 - j16 + 4} = 4\angle 28.07^\circ\ \text{A}$$

AP 9.10

$$\mathbf{V}_1 = 240\angle 53.13^\circ = 144 + j192\,\text{V}$$

$$\mathbf{V}_2 = 96\angle -90^\circ = -j96\,\text{V}$$

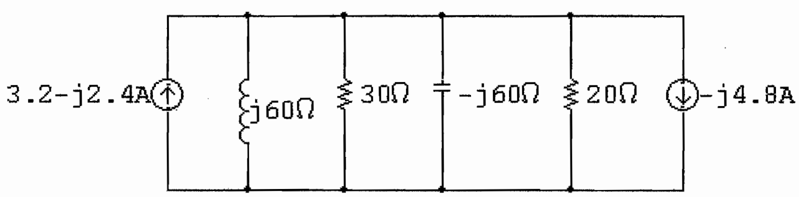
$$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$$

$$\frac{1}{j\omega C} = -j\frac{6 \times 10^6}{(4000)(25)} = -j60\,\Omega$$

Perform a source transformation:

$$\frac{\mathbf{V}_1}{j60} = \frac{144 + j192}{j60} = 3.2 - j2.4\,\text{A}$$

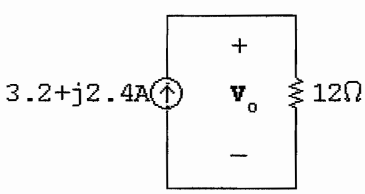
$$\frac{\mathbf{V}_2}{20} = -j\frac{96}{20} = -j4.8\,\text{A}$$



Combine the parallel impedances:

$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

$$Z = \frac{1}{Y} = 12\,\Omega$$



$$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8\,\text{V} = 48\angle 36.87^\circ\,\text{V}$$

$$v_o = 48\cos(4000t + 36.87^\circ)\,\text{V}$$

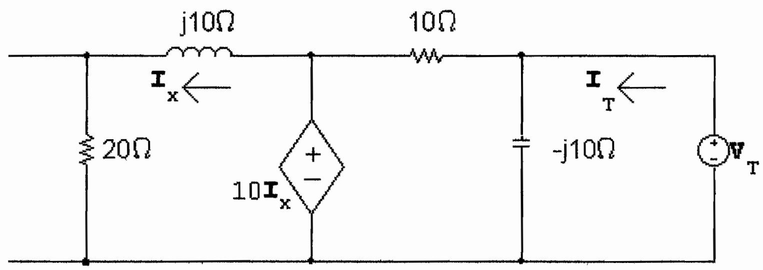
AP 9.11 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the $20\,\Omega$ resistor and $\mathbf{V}_{\text{Th}}$ = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2\underline{/45^\circ} + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\text{Th}} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for $\mathbf{V}_{\text{Th}}$ gives $\mathbf{V}_{\text{Th}} = 10\underline{/45^\circ}V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

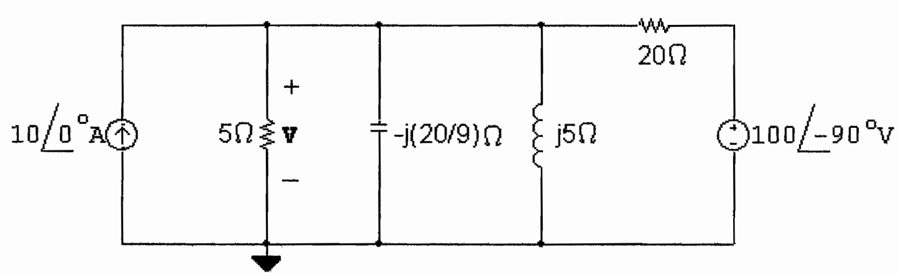
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\text{Th}} = (5 - j5)\,\Omega$$

AP 9.12 The phasor domain circuit is as shown in the following diagram:



The node voltage equation is

$$-10+\frac{\mathbf{V}}{5}+\frac{\mathbf{V}}{-j(20/9)}+\frac{\mathbf{V}}{j5}+\frac{\mathbf{V}-100\underline{/-90^\circ}}{20}=0$$

Therefore $\mathbf{V}=10-j30=31.62\underline{/-71.57^\circ}$

Therefore $v=31.62\cos(50{,}000t-71.57^\circ)\,\text{V}$

AP 9.13 Let $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8=(1+j2)\mathbf{I}_a+(3-j5)(\mathbf{I}_a-\mathbf{I}_b)$$

and

$$0=(3-j5)(\mathbf{I}_b-\mathbf{I}_a)+2(\mathbf{I}_b-\mathbf{I}_c).$$

But

$$\mathbf{V}_x=-j5(\mathbf{I}_a-\mathbf{I}_b),$$

therefore

$$\mathbf{I}_c=-0.75[-j5(\mathbf{I}_a-\mathbf{I}_b)].$$

Solving for $\mathbf{I}=\mathbf{I}_a=29+j2=29.07\underline{/3.95^\circ}\,\text{A}$.

AP 9.14 [a] $M=0.4\sqrt{0.0625}=0.1\,\text{H}, \qquad \omega M=80\,\Omega$

$$Z_{22}=40+j800(0.125)+360+j800(0.25)=(400+j300)\,\Omega$$

Therefore $|Z_{22}|=500\,\Omega, \qquad Z_{22}^*=(400-j300)\,\Omega$

$$Z_r=\left(\frac{80}{500}\right)^2(400-j300)=(10.24-j7.68)\,\Omega$$

[b] $\mathbf{I}_1=\dfrac{245.20}{184+100+j400+Z_r}=0.50\underline{/-53.13^\circ}\,\text{A}$

$$i_1=0.5\cos(800t-53.13^\circ)\,\text{A}$$

[c] $\mathbf{I}_2=\left(\dfrac{j\omega M}{Z_{22}}\right)\mathbf{I}_1=\dfrac{j80}{500\underline{/36.87^\circ}}(0.5\underline{/-53.13^\circ})=0.08\underline{/0^\circ}\,\text{A}$

$$i_2=80\cos 800t\,\text{mA}$$

AP 9.15

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_1 + 2s^2 Z_2} = \frac{25 \times 10^3 \underline{/0^\circ}}{1500 + j6000 + (25)^2(4 - j14.4)}$$

$$= 4 + j3 = 5\underline{/36.87^\circ}\text{ A}$$

$$\mathbf{V}_1 = \mathbf{V}_s - Z_1\mathbf{I}_1 = 25{,}000\underline{/0^\circ} - (4 + j3)(1500 + j6000)$$

$$= 37{,}000 - j28{,}500$$

$$\mathbf{V}_2 = -\frac{1}{25}\mathbf{V}_1 = -1480 + j1140 = 1868.15\underline{/142.39^\circ}\text{ V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{1868.15\underline{/142.39^\circ}}{4 - j14.4} = 125\underline{/216.87^\circ}\text{ A}$$

Problems

P 9.1 [a] $\omega = 2\pi f = 240\pi\,\text{rad/s}, \qquad f = \frac{\omega}{2\pi} = 120\,\text{Hz}$

[b] $T = 1/f = 8.33\,\text{ms}$

[c] $V_m = 100\,\text{V}$

[d] $v(0) = 100\cos(45^\circ) = 70.71\,\text{V}$

[e] $\phi = 45^\circ; \qquad \phi = \frac{45^\circ(2\pi)}{360^\circ} = \frac{\pi}{4} = 0.7854\text{ rad}$

[f] $V = 0$ when $240\pi t + 45^\circ = 90^\circ$. Now resolve the units:

$$(240\pi\text{ rad/s})t = \frac{45^\circ}{57.3^\circ/\text{rad}} = \frac{\pi}{4}\text{ rad}, \qquad t = 1.042\,\text{ms}$$

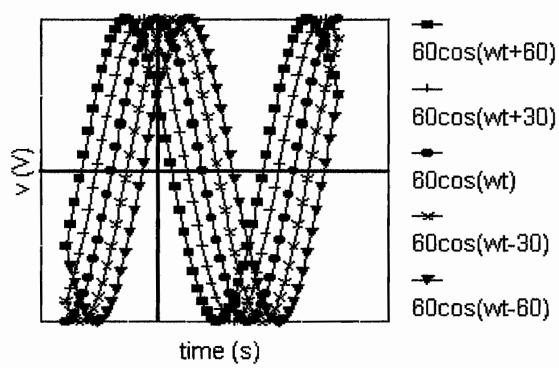
[g] $(dv/dt) = (-100)240\pi\sin(240\pi t + 45^\circ)$

$$(dv/dt) = 0 \quad \text{when} \quad 240\pi t + 45^\circ = 180^\circ$$

$$\text{or} \qquad 240\pi t = \frac{135^\circ}{57.3^\circ/\text{rad}} = \frac{3\pi}{4}\text{ rad}$$

Therefore $\quad t = 3.125\,\text{ms}$

P 9.2



[a] Left as ϕ becomes more positive

[b] Right

P 9.3 [a] $\frac{T}{2} = \frac{1250}{6} + \frac{250}{6} = 250\,\mu\text{s}; \qquad T = 500\,\mu\text{s}$

$$f = \frac{1}{T} = \frac{10^6}{500} = 2000\text{Hz}$$

[b] $v = V_m \sin(\omega t + \theta)$

$$\omega = 2\pi f = 4000\pi \text{ rad/s}$$

$$4000\pi\left(\frac{-250}{6} \times 10^{-6}\right) + \theta = 0; \qquad \therefore \quad \theta = \frac{\pi}{6} \text{ rad} = 30^\circ$$

$$v = V_m \sin[4000\pi t + 30^\circ]$$

$$75 = V_m \sin 30^\circ; \qquad V_m = 150\,\text{V}$$

$$v = 150 \sin[4000\pi t + 30^\circ] = 150 \cos[4000\pi t - 60^\circ]\,\text{V}$$

P 9.4 [a] By hypothesis

$$i = 10\cos(\omega t + \theta)$$

$$\frac{di}{dt} = -10\omega \sin(\omega t + \theta)$$

$$\therefore \quad 10\omega = 20{,}000\pi; \qquad \omega = 2000\pi\,\text{rad/s}$$

[b] $f = \dfrac{\omega}{2\pi} = 1000 \text{ Hz}; \qquad T = \dfrac{1}{f} = 1 \text{ ms} = 1000\,\mu\text{s}$

$$\frac{150}{1000} = \frac{3}{20}, \qquad \therefore \quad \theta = -90 - \frac{3}{20}(360) = -144^\circ$$

$$\therefore \quad i = 10\cos(2000\pi t - 144^\circ)\,\text{A}$$

P 9.5 [a] 170 V

[b] $2\pi f = 120\pi; \qquad f = 60\text{Hz}$

[c] $\omega = 120\pi = 376.99 \text{ rad/s}$

[d] $\theta(\text{rad}) = \dfrac{-\pi}{180}(60) = \dfrac{-\pi}{3} = -1.05 \text{ rad}$

[e] $\theta = -60^\circ$

[f] $T = \dfrac{1}{f} = \dfrac{1}{60} = 16.67\,\text{ms}$

[g] $120\pi t - \dfrac{\pi}{3} = 0; \qquad \therefore \quad t = \dfrac{1}{360} = 2.78\,\text{ms}$

[h] $v = 170\cos\left[120\pi\left(t + \dfrac{0.125}{18}\right) - \dfrac{\pi}{3}\right]$

$$= 170\cos[120\pi t + (15\pi/18) - (\pi/3)]$$

$$= 170\cos[120\pi t + (\pi/2)]$$

$$= -170 \sin 120\pi t\,\text{V}$$

[i] $120\pi(t - t_o) - (\pi/3) = 120\pi t - (\pi/2)$

$$\therefore \quad 120\pi t_o = \frac{\pi}{6}; \qquad t_o = \frac{25}{18}\,\text{ms}$$

[j] $120\pi(t - t_o) - (\pi/3) = 120\pi t$

$$\therefore \quad 120\pi t_o = \frac{\pi}{3}; \qquad t_o = \frac{25}{9}\,\text{ms}$$

P 9.6
$$\begin{aligned} u &= \int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi)\,dt \\ &= V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\,dt \\ &= \frac{V_m^2}{2}\left\{\int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi)\,dt\right\} \\ &= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t + 2\phi)\,\Big|_{t_o}^{t_o+T}\right]\right\} \\ &= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi)\right]\right\} \\ &= V_m^2\left(\frac{T}{2}\right) + \frac{1}{2\omega}(0) = V_m^2\left(\frac{T}{2}\right) \end{aligned}$$

P 9.7 $V_m = \sqrt{2}V_{\text{rms}} = \sqrt{2}(120) = 169.71\,\text{V}$

P 9.8
$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2 \sin^2\frac{2\pi}{T}t\,dt}$$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}t\right)\,dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right)\,dt = \frac{V_m^2 T}{4}$$

Therefore $\quad V_{\text{rms}} = \sqrt{\frac{1}{T}\frac{V_m^2 T}{4}} = \frac{V_m}{2}$

P 9.9 [a] The numerical values of the terms in Eq. 9.8 are

$$V_m = 100, \qquad R/L = 533.33, \qquad \omega L = 30$$

$$\sqrt{R^2 + \omega^2 L^2} = 50$$

$$\phi = 60^\circ, \qquad \theta = \tan^{-1} 30/40, \qquad \theta = 36.87^\circ$$

$$i = \left[-1.84e^{-533.33t} + 2\cos(400t + 23.13^\circ)\right]\,\text{A}, \qquad t \geq 0$$

[b] Transient component $= -1.84e^{-533.33t}$ A
Steady-state component $= 2\cos(400t + 23.13^\circ)$ A

[c] By direct substitution into Eq 9.9, $\quad i(1.875\,\text{ms}) = 133.61\,\text{mA}$

[d] 2 A, 400 rad/s, 23.13°

[e] The current lags the voltage by 36.87°.

P 9.10 [a] From Eq. 9.9 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi-\theta)e^{-(R/L)t}}{\sqrt{R^2+\omega^2L^2}} + \frac{V_m R\cos(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$L\frac{di}{dt} + Ri = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$

But

$$\frac{R}{\sqrt{R^2+\omega^2L^2}} = \cos\theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2+\omega^2L^2}} = \sin\theta$$

Therefore the right-hand side reduces to

$V_m\cos(\omega t+\phi)$

At $t = 0$, Eq. 9.9 reduces to

$$i(0) = \frac{-V_m\cos(\phi-\theta)}{\sqrt{R^2-\omega^2L^2}} + \frac{V_m\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}} = 0$$

[b] $i_{ss} = \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$

Therefore

$$L\frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t+\phi-\theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$$

$$L\frac{di_{ss}}{dt} + Ri_{ss} = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$

$$= V_m\cos(\omega t+\phi)$$

P 9.11 [a] $\mathbf{Y} = 100\underline{/45^\circ} + 500\underline{/-60^\circ} = 483.86\underline{/-48.48^\circ}$

$y = 483.86\cos(300t - 48.48^\circ)$

[b] $\mathbf{Y} = 250\underline{/30^\circ} - 150\underline{/50^\circ} = 120.51\underline{/4.8^\circ}$

$y = 120.51\cos(377t + 4.8^\circ)$

[c] $\mathbf{Y} = 60\underline{/60^\circ} - 120\underline{/-215^\circ} + 100\underline{/90^\circ} = 152.88\underline{/32.94^\circ}$

$$y = 152.88\cos(100t + 32.94^\circ)$$

[d] $\mathbf{Y} = 100\underline{/40^\circ} + 100\underline{/160^\circ} + 100\underline{/-80^\circ} = 0$

$$y = 0$$

P 9.12 [a] 50Hz

[b] $\theta_v = 0^\circ$

$$\mathbf{I} = \frac{340\underline{/0^\circ}}{j\omega L} = \frac{340}{\omega L}\underline{/-90^\circ} = 8.5\underline{/-90^\circ}; \qquad \theta_i = -90^\circ$$

[c] $\dfrac{340}{\omega L} = 8.5; \qquad \omega L = 40\,\Omega$

[d] $L = \dfrac{40}{100\pi} = \dfrac{400}{\pi}\,\text{mH} = 127.32\,\text{mH}$

[e] $Z_L = j\omega L = j40\,\Omega$

P 9.13 [a] $\omega = 2\pi f = 80\pi \times 10^3 = 251.33\,\text{krad/s} = 251{,}327.41\,\text{rad/s}$

[b] $\mathbf{I} = \dfrac{2.5 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(2.5 \times 10^{-3})\underline{/0^\circ} = 2.5 \times 10^{-3}\omega C\underline{/90^\circ}$

$$\therefore \quad \theta_i = 90^\circ$$

[c] $125.66 \times 10^{-6} = 2.5 \times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{2.5 \times 10^{-3}}{125.66 \times 10^{-6}} = 19.89\,\Omega, \quad \therefore \quad X_C = -19.89\,\Omega$$

[d] $C = \dfrac{1}{19.89(\omega)} = \dfrac{1}{(19.89)(80\pi \times 10^3)}$

$$C = 0.2 \times 10^{-6} = 0.2\,\mu\text{F}$$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j19.89\,\Omega$

P 9.14 [a] $\mathbf{V}_g = 150\underline{/20^\circ}; \qquad \mathbf{I}_g = 30\underline{/-52^\circ}$

$$\therefore \quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 5\underline{/72^\circ}\,\Omega$$

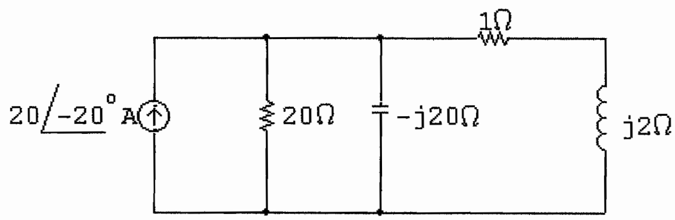
[b] i_g lags v_g by 72°:

$$2\pi f = 8000\pi; \qquad f = 4000\,\text{Hz}; \qquad T = 1/f = 250\,\mu\text{s}$$

$$\therefore \quad i_g \text{ lags } v_g \text{ by } \frac{72}{360}(250) = 50\,\mu\text{s}$$

P 9.15 [a] $j\omega L = j(5\times 10^4)(40\times 10^{-6}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{5\times 10^4} = -j20\,\Omega; \qquad \mathbf{I}_g = 20\underline{/-20^\circ}\text{ A}$$



[b] $\mathbf{V}_o = 20\underline{/-20^\circ} Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{20} + j\frac{1}{20} + \frac{1}{1+j2}$$

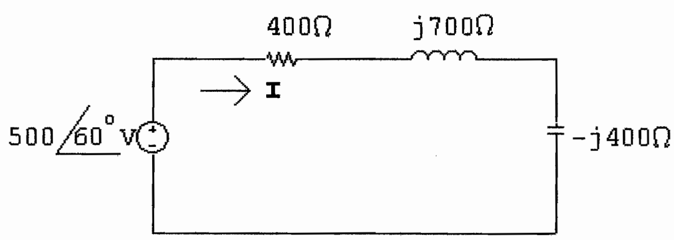
$$Y_e = 0.05 + j0.05 + 0.20 - j0.40 = 0.25 - j0.35\,\text{S}$$

$$Z_e = \frac{1}{0.25 - j0.35} = 2.32\underline{/54.46^\circ}\,\Omega$$

$$\mathbf{V}_o = (20\underline{/-20^\circ})(2.32\underline{/54.46^\circ}) = 46.4\underline{/34.46^\circ}\text{ V}$$

[c] $v_o = 46.4\cos(5\times 10^4 t + 34.46^\circ)$ V

P 9.16 [a]



[b] $\mathbf{I} = \dfrac{500\underline{/60^\circ}}{400 + j700 - j400} = 1\underline{/23.13^\circ}\text{ A}$

[c] $i = 1\cos(8000t + 23.13^\circ)$ A

P 9.17 [a] $Z_1 = R_1 - j\dfrac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and}$$

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

[b] $R_1 = \dfrac{500}{1 + (64 \times 10^8)(25 \times 10^4)(625 \times 10^{-18})} = 250\,\Omega$

$$C_1 = \frac{2}{(64 \times 10^8)(25 \times 10^4)(25 \times 10^{-9})} = 50\,\text{nF}$$

P 9.18 [a] $Y_2 = \dfrac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $\quad Y_1 = Y_2 \quad$ when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[b] $R_2 = \dfrac{1 + (4 \times 10^8)(4 \times 10^6)(2500 \times 10^{-18})}{(4 \times 10^8)(2 \times 10^3)(2500 \times 10^{-18})} = 2500 = 2.5\text{k}\Omega$

$$C_2 = \frac{50 \times 10^{-9}}{5} = 10\,\text{nF}$$

P 9.19 [a] $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[b] $R_1 = \dfrac{(4 \times 10^8)(6.25)(5 \times 10^4)}{25 \times 10^8 + (4 \times 10^8)(6.25)} = 2.5 \times 10^4$

$\therefore \quad R_1 = 25\,\text{k}\Omega$

$$L_1 = \frac{(25 \times 10^8)2.5}{50 \times 10^8} = 1.25\,\text{H}$$

P 9.20 [a] $Y_2 = \dfrac{1}{R_2} - \dfrac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $\quad Y_2 = Y_1 \quad$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $R_2 = \dfrac{25\times 10^6 + 10^8(0.25)}{5\times 10^3} = 10\times 10^3$

$\therefore\quad R_2 = 10\,\text{k}\Omega$

$L_2 = \dfrac{50\times 10^6}{10^8(0.5)} = 1\,\text{H}$

P 9.21 [a] $Y = \dfrac{1}{4-j3} + \dfrac{1}{16+j12} + \dfrac{1}{-j100}$

$= 0.16 + j0.12 + 0.04 - j0.03 + j0.01$

$= 0.2 + j0.1 = 223.6\underline{/26.57^\circ}\,\text{mS}$

[b] $G = 200\,\text{mS}$

[c] $B = 100\,\text{mS}$

[d] $\mathbf{I} = 50\underline{/0^\circ}\,\text{A},\qquad \mathbf{V} = \dfrac{\mathbf{I}}{Y} = \dfrac{50}{0.223\underline{/26.57^\circ}} = 223.61\underline{/-26.57^\circ}\,\text{V}$

$\mathbf{I}_C = \dfrac{\mathbf{V}}{Z_C} = \dfrac{223.6\underline{/-26.57^\circ}}{100\underline{/-90^\circ}} = 2.24\underline{/63.43^\circ}\,\text{A}$

$i_C = 2.24\cos(\omega t + 63.43^\circ)\,\text{A},\qquad I_m = 2.24\,\text{A}$

P 9.22 [a] $Z_{\text{ab}} = j5\omega + \dfrac{(4000)(10^9/j\omega 625)}{4000 + (10^9/j625\omega)}$

$= j5\omega + \dfrac{4\times 10^{12}}{25\times 10^5 j\omega + 10^9}$

$= j5\omega + \dfrac{4\times 10^7}{10^4 + j25\omega}$

$= j5\omega + \dfrac{4\times 10^{11}}{10^8 + 625\omega^2} - j\dfrac{100\times 10^7\omega}{10^8 + 625\omega^2}$

$\therefore\quad 5 = \dfrac{10^9}{10^8 + 625\omega^2}$

$5\times 10^8 + 3125\omega^2 = 10^9$

$\omega = 4\times 10^2 = 400\,\text{rad/s}$

[b] $Z_{\text{ab}}(400) = j2000 + \dfrac{(4000)(-j4000)}{4000 - j4000} = 2\,\text{k}\Omega$

P 9.23 $Z_1 = 10 - j40\,\Omega$

$Z_2 = \dfrac{(5-j10)(10+j30)}{15+j20} = 10 - j10\,\Omega$

$$Z_3 = \frac{20(j20)}{20 + j20} = 10 + j10\,\Omega$$

$$\therefore \quad Z_{\text{ab}} = Z_1 + Z_2 + Z_3 = 30 - j40\,\Omega = 50\underline{/-53.13^\circ}\,\Omega$$

P 9.24 First find the admittance of the parallel branches

$$Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125\,\text{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8\,\Omega$$

$$Z_{\text{ab}} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12\,\Omega$$

$$Y_{\text{ab}} = \frac{1}{Z_{\text{ab}}} = \frac{1}{16 - j12} = 0.04 + j0.03\,\text{S}$$

$$= 40 + j30\,\text{mS} = 50\underline{/36.87^\circ}\,\text{mS}$$

P 9.25 $Z = 400 + j(5)(40) - j\dfrac{1000}{(5)(0.4)} = 500\underline{/-36.87^\circ}\,\Omega$

$$\mathbf{I}_o = \frac{750\underline{/0^\circ} \times 10^{-3}}{500\underline{/-36.87^\circ}} = 1.5\underline{/36.87^\circ}\,\text{mA}$$

$$i_o(t) = 1.5\cos(5000t + 36.87^\circ)\,\text{mA}$$

P 9.26 $\mathbf{V}_g = 50\underline{/-45^\circ}\,\text{V}; \qquad \mathbf{I}_g = 100\underline{/-8.13^\circ}\,\text{mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 500\underline{/-36.87^\circ}\,\Omega = 400 - j300\,\Omega$$

$$Z = 400 + j\left(0.04\omega - \frac{2.5 \times 10^6}{\omega}\right)$$

$$\therefore \quad 0.04\omega - \frac{2.5 \times 10^6}{\omega} = -300$$

$$\therefore \quad \omega^2 + 7500\omega - 62.5 \times 10^6 = 0$$

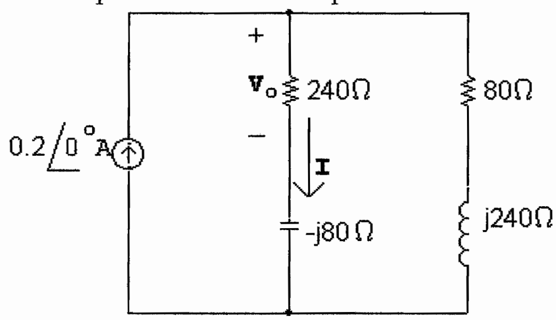
$$\therefore \quad \omega = -3750 \pm \sqrt{(3750)^2 + 62.5 \times 10^6} = -3750 \pm 8750$$

$$\omega > 0, \qquad \therefore \quad \omega = 5000\,\text{rad/s}$$

P 9.27 $Z_L = j(5000)(48 \times 10^{-3}) = j240\,\Omega$

$$Z_C = \frac{-j}{(5000)(2.5 \times 10^{-6})} = -j80\,\Omega$$

Construct the phasor domain equivalent circuit:



Using current division:

$$\mathbf{I} = \frac{(80 + j240)}{240 - j80 + 80 + j240}(0.2) = 0.1 + j0.1\,\text{A}$$

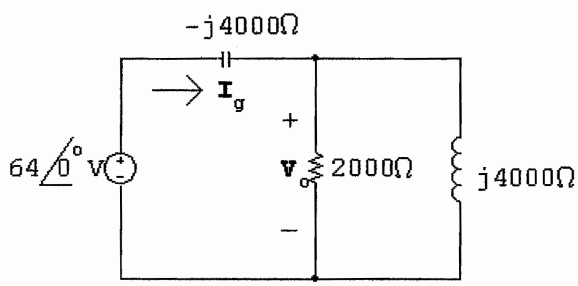
$$\mathbf{V}_o = 240\mathbf{I} = 24 + j24 = 33.94\underline{/45^\circ}$$

$$v_o = 33.94\cos(5000t + 45^\circ)\,\text{V}$$

P 9.28 $\dfrac{1}{j\omega C} = \dfrac{10^9}{(31.25)(8000)} = -j4000\,\Omega$

$$j\omega L = j8000(500)10^{-3} = j4000\,\Omega$$

$$\mathbf{V}_g = 64\underline{/0^\circ}\,\text{V}$$



$$Z_e = \frac{(2000)(j4000)}{2000 + j4000} = 1600 + j800\,\Omega$$

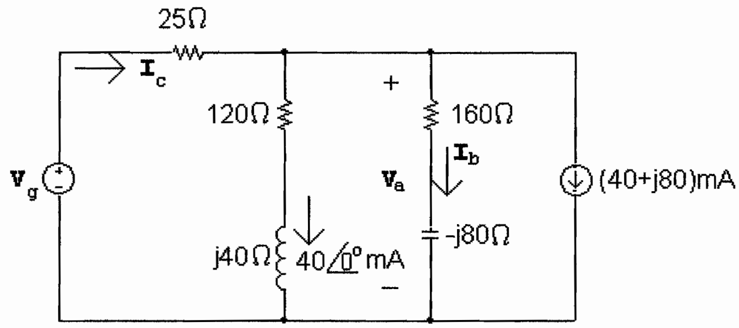
$$Z_T = 1600 + j800 - j4000 = 1600 - j3200\,\Omega$$

$$\mathbf{I}_g = \frac{64\underline{/0^\circ}}{1600 - j3200} = 8 + j16\,\text{mA}$$

$$\mathbf{V}_o = Z_e\mathbf{I}_g = (1600 + j800)(0.008 + j0.016) = j32 = 32\underline{/90^\circ}\,\text{V}$$

$$v_o = 32\cos(8000t + 90^\circ)\,\text{V}$$

P 9.29 [a]



$$\mathbf{V}_a = (120 + j40)(0.04\underline{/0^\circ}) = 4.8 + j1.6\,\text{V}$$

$$\mathbf{I}_b = \frac{4.8 + j1.6}{160 - j80} = 20 + j20\,\text{mA}$$

$$\mathbf{I}_c = 40\underline{/0^\circ} + (20 + j20) + (40 + j80)\,\text{mA} = 100 + j100\,\text{mA}$$

$$\mathbf{V}_g = 25\mathbf{I}_c + \mathbf{V}_a = 25(0.100 + j0.100) + 4.8 + j1.6 = 7.3 + j4.1\,\text{V}$$

[b] $i_b = 28.28\cos(800t + 45^\circ)\,\text{mA}$

$$i_c = 141.42\cos(800t + 45^\circ)\,\text{mA}$$

$$v_g = 8.37\cos(800t + 29.32^\circ)\,\text{V}$$

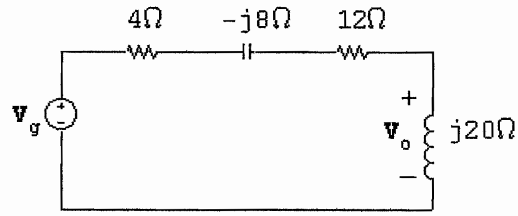
P 9.30 [a] $\dfrac{1}{j\omega C} = \dfrac{10^9}{j8 \times 10^5(125)} = -j10\,\Omega$

$$j\omega L = j8 \times 10^5(25 \times 10^{-6}) = j20\,\Omega$$

$$Z_e = \frac{(-j10)(20)}{20 - j10} = 4 - j8\,\Omega$$

$$\mathbf{I}_g = 5\underline{/0^\circ}$$

$$\mathbf{V}_g = \mathbf{I}_g Z_e = 5(4 - j8) = 20 - j40\,\text{V}$$



$$\mathbf{V}_o = \frac{(20-j40)(j20)}{(16+j12)} = 44 - j8 = 44.72\underline{/-10.30^\circ}\text{ V}$$

$$v_o = 44.72\cos(8\times 10^5 t - 10.30^\circ)\text{ V}$$

[b] $\omega = 2\pi f = 8\times 10^5; \qquad f = \dfrac{4\times 10^5}{\pi}$

$$T = \frac{1}{f} = \frac{\pi}{4\times 10^5} = 2.5\pi\ \mu\text{s}$$

$$\therefore \quad \frac{10.30}{360}(2.5\pi) = 224.82\text{ ns}$$

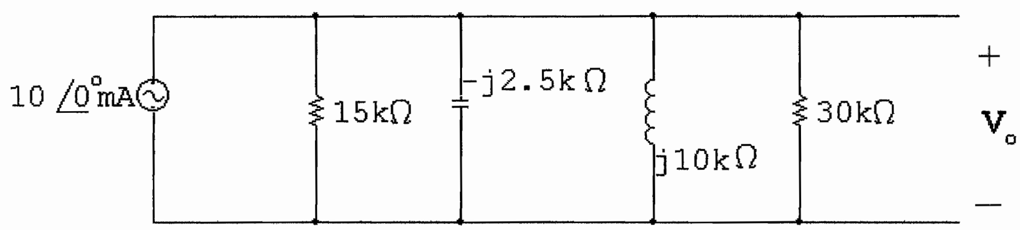
$\therefore \quad v_o$ lags i_g by 224.82 ns

P 9.31 $\mathbf{I}_s = 15\underline{/0^\circ}\text{ mA}$

$$\frac{1}{j\omega C} = \frac{10^6}{j0.05(8000)} = -j2500\,\Omega$$

$$j\omega L = j8000(1.25) = j10{,}000\,\Omega$$

After two source transformations we have



$$15\text{ k}\Omega \| 30\text{ k}\Omega = 10\text{ k}\Omega$$

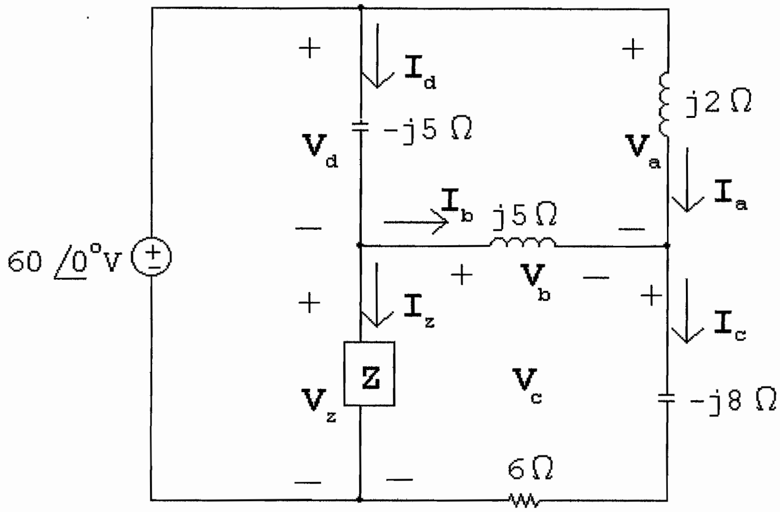
$$Y_o = \frac{10^{-3}}{10} + \frac{1}{-j2500} + \frac{1}{j10^4} = 10^{-4}(1+j3)$$

$$Z_o = \frac{10^4}{1+j3} = (1-j3)\text{ k}\Omega$$

$$\mathbf{V}_o = \mathbf{I}_g Z_o = (10)(1-j3) = 10 - j30 = 31.62\underline{/-71.57^\circ}\text{ V}$$

$$v_o = 31.62\cos(8000t - 71.57^\circ)\text{ V}$$

P 9.32



$$\mathbf{V}_\mathrm{a} = j2\mathbf{I}_\mathrm{a} = j2(-j5) = 10\underline{/0^\circ}\,\mathrm{V}$$

$$\mathbf{V}_\mathrm{c} = 60\underline{/0^\circ} - \mathbf{V}_\mathrm{a} = 50\underline{/0^\circ}\,\mathrm{V}$$

$$\mathbf{I}_\mathrm{c} = \frac{\mathbf{V}_\mathrm{c}}{6 - j8} = \frac{50\underline{/0^\circ}}{10\underline{/-53.13^\circ}} = 5\underline{/53.13^\circ} = 3 + j4\,\mathrm{A}$$

$$\mathbf{I}_\mathrm{b} = \mathbf{I}_\mathrm{c} - \mathbf{I}_\mathrm{a} = 3 + j4 - (-j5) = 3 + j9\,\mathrm{A} = 9.49\underline{/71.57^\circ}\,\mathrm{A}$$

$$\mathbf{V}_\mathrm{b} = \mathbf{I}_\mathrm{b}(j5) = (3 + j9)(j5) = -45 + j15\,\mathrm{V}$$

$$\mathbf{V}_\mathrm{z} = \mathbf{V}_\mathrm{b} + \mathbf{V}_\mathrm{c} = -45 + j15 + 50 + j0 = 5 + j15\,\mathrm{V}$$

$$\mathbf{V}_\mathrm{d} + \mathbf{V}_\mathrm{z} = 60\underline{/0^\circ}; \qquad \therefore\ \mathbf{V}_\mathrm{d} = 60 - 5 - j15 = 55 - j15\,\mathrm{V}$$

$$\mathbf{I}_\mathrm{d} = \frac{\mathbf{V}_\mathrm{d}}{-j5} = 3 + j11\,\mathrm{A}$$

$$\mathbf{I}_\mathrm{z} = \mathbf{I}_\mathrm{d} - \mathbf{I}_\mathrm{b} = 3 + j11 - 3 - j9 = j2\,\mathrm{A}$$

$$Z = \frac{\mathbf{V}_\mathrm{z}}{\mathbf{I}_\mathrm{z}} = \frac{5 + j15}{j2} = 7.5 - j2.5\,\Omega$$

P 9.33 $\mathbf{V}_2$ is the voltage across the $-j10\,\Omega$ impedance.

$$\frac{\mathbf{V}_1 - \mathbf{V}_g}{20} + \frac{\mathbf{V}_1}{j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{Z} = 0$$

$$\frac{(40 + j30) - (100 - j50)}{20} + \frac{40 + j30}{j5} + \frac{(40 + j30) - \mathbf{V}_2}{Z} = 0$$

$\therefore \quad \mathbf{V}_2 = 40 + j30 + (3 - j4)Z$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{Z} + \frac{\mathbf{V}_L}{-j10} - \mathbf{I}_g + \frac{\mathbf{V}_2 - \mathbf{V}_g}{3 + j1} = 0$$

$$\frac{\mathbf{V}_2 - (40 + j30)}{Z} + \frac{\mathbf{V}_2}{-j10} - (20 + j30) + \frac{\mathbf{V}_2 - (100 - j50)}{3 + j1} = 0$$

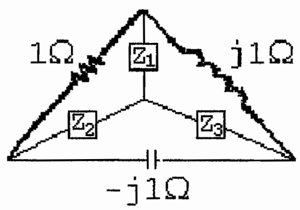
Substituting the expression for $\mathbf{V}_2$ found at the start and simplifying yields

$Z = 12 + j16\,\Omega$

P 9.34 Simplify the top triangle using series and parallel combinations:

$(1 + j1)\|(1 - j1) = 1\,\Omega$

Convert the lower left delta to a wye:

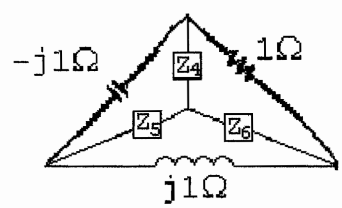


$$Z_1 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

$$Z_2 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_3 = \frac{(j1)(-j1)}{1 + j1 - j1} = 1\,\Omega$$

Convert the lower right delta to a wye:

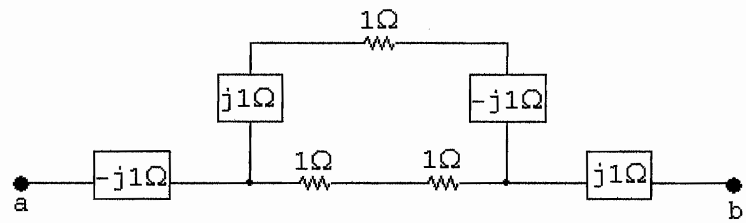


$$Z_4 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_5 = \frac{(-j1)(j1)}{1 + j1 - j1} = 1\,\Omega$$

$$Z_6 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

The resulting circuit is shown below:



Simplify the middle portion of the circuit by making series and parallel combinations:

$$(1 + j1 - j1)\|(1 + 1) = 1\|2 = 2/3\,\Omega$$

$$Z_{ab} = -j1 + 2/3 + j1 = 2/3\,\Omega$$

P 9.35 [a] $Y_p = \dfrac{1}{10 + j2\omega} + j4 \times 10^{-3}\omega$

$$= \frac{10 - j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

$$= \frac{10}{100 + 4\omega^2} - \frac{j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

Y_p is real when

$$4 \times 10^{-3}\omega = \frac{2\omega}{100 + 4\omega^2}$$

or $\omega^2 = 100;$ $\omega = 10$ rad/s; $f = 5/\pi = 1.59$Hz

[b] $Y_p(10\,\text{rad/s}) = \dfrac{10}{500} = 20\,\text{mS}$

$$Z_p(10\,\text{rad/s}) = \frac{10^3}{20} = 50\,\Omega$$

$$Z(10\,\text{rad/s}) = 50 + 150 = 200\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_g}{200}\,\text{A} = \frac{10\underline{/0^\circ}}{200} = 50\underline{/0^\circ}\,\text{mA}$$

$$i_o = 50\cos 10t\,\text{mA}$$

P 9.36 [a] $Z_g = 4000 - j\frac{10^9}{25\omega} + \frac{10^4(j2\omega)}{10^4 + j2\omega}$

$$= 4000 - j\frac{10^9}{25\omega} + \frac{2\times 10^4 j\omega(10^4 - j2\omega)}{10^8 + 4\omega^2}$$

$$= 4000 - j\frac{10^9}{25\omega} + \frac{4\times 10^4\omega^2}{10^8 + 4\omega^2} + j\frac{2\times 10^8\omega}{10^8 + 4\omega^2}$$

$$\therefore \quad \frac{10^9}{25\omega} = \frac{0.2\times 10^9\omega}{10^8 + 4\omega^2}$$

$$10^8 + 4\omega^2 = 5\omega^2$$

$$\omega^2 = 10^8; \qquad \omega = 10{,}000\,\text{rad/s}$$

[b] When $\omega = 10{,}000$ rad/s

$$Z_g = 4000 + \frac{4\times 10^4(10^4)^2}{10^8 + 4(10^4)^2} = 12{,}000\,\Omega$$

$$\therefore \quad \mathbf{I}_g = \frac{45\underline{/0^\circ}}{12{,}000} = 3.75\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{V}_o = \mathbf{V}_g - \mathbf{I}_g Z_1$$

$$Z_1 = 4000 - j\frac{10^9}{25\times 10^4} = 4000 - j4000\,\Omega$$

$$\mathbf{V}_o = 45\underline{/0^\circ} - (3.75\times 10^{-3})(4000 - j4000) = 45 - (15 - j15)$$

$$= 30 + j15 = 33.54\underline{/26.57^\circ}\,\text{V}$$

$$v_o = 33.54\cos(10{,}000t + 26.57^\circ)\,\text{V}$$

P 9.37 [a] $Y_1 = \frac{1}{5000} = 0.2\times 10^{-3}\,\text{S}$

$$Y_2 = \frac{1}{1200 + j0.2\omega}$$

$$= \frac{1200}{1.44\times 10^6 + 0.04\omega^2} - j\frac{0.2\omega}{1.44\times 10^6 + 0.04\omega^2}$$

$$Y_3 = j\omega 50\times 10^{-9}$$

$$Y_T = Y_1 + Y_2 + Y_3$$

For i_g and v_o to be in phase the j component of Y_T must be zero; thus,

$$\omega 50\times 10^{-9} = \frac{0.2\omega}{1.44\times 10^6 + 0.04\omega^2}$$

or

$$0.04\omega^2 + 1.44 \times 10^6 = \frac{0.2 \times 10^9}{50} = 4 \times 10^6$$

$$\therefore \quad 0.04\omega^2 = 2.56 \times 10^6 \qquad \therefore \quad \omega = 8000\,\text{rad/s} = 8\,\text{krad/s}$$

[b] $Y_T = 0.2 \times 10^{-3} + \dfrac{1200}{1.44 \times 10^6 + 0.04(64) \times 10^6} = 0.5 \times 10^{-3}\,\text{S}$

$$\therefore \quad Z_T = 2000\,\Omega$$

$$\mathbf{V}_o = (2.5 \times 10^{-3}\underline{/0^\circ})(2000) = 5\underline{/0^\circ}$$

$$v_o = 5\cos 8000t\,\text{V}$$

P 9.38 [a] $Z_p = \dfrac{\dfrac{R}{j\omega C}}{R + (1/j\omega C)} = \dfrac{R}{1 + j\omega RC}$

$$= \frac{12{,}500}{1 + j(1000)(12{,}500)C} = \frac{12{,}500}{1 + j12.5 \times 10^6 C}$$

$$= \frac{12{,}500(1 - j12.5 \times 10^6 C)}{1 + 156.25 \times 10^{12} C^2}$$

$$= \frac{12{,}500}{1 + 156.25 \times 10^{12} C^2} - j\frac{156.25 \times 10^9 C}{1 + 156.25 \times 10^{12} C^2}$$

$$j\omega L = j1000(5) = j5000$$

$$\therefore \quad 5000 = \frac{156.25 \times 10^9 C}{1 + 156.25 \times 10^{12} C^2}$$

$$\therefore \quad 781.25 \times 10^{15} C^2 - 156.25 \times 10^9 C + 5000 = 0$$

$$\therefore \quad C^2 - 20 \times 10^{-8} C + 64 \times 10^{-16} = 0$$

$$\therefore \quad C_{1,2} = 10 \times 10^{-8} \pm \sqrt{100 \times 10^{-16} - 64 \times 10^{-16}}$$

$$C_1 = 10 \times 10^{-8} + 6 \times 10^{-8} = 16 \times 10^{-8} = 0.16\,\mu\text{F}$$

$$C_2 = 10 \times 10^{-8} - 6 \times 10^{-8} = 4 \times 10^{-8} = 0.04\,\mu\text{F}$$

[b] $R_e = \dfrac{12{,}500}{1 + 156.25 \times 10^{12} C^2}$

When $C = 160\,\text{nF}$ $\qquad R_e = 2500\,\Omega$;

$$\mathbf{I}_g = \frac{250\underline{/0^\circ}}{2500} = 0.1\underline{/0^\circ}\,\text{A}; \qquad i_g = 100\cos 1000t\,\text{mA}$$

When $C = 40\,\text{nF}$ $\qquad R_e = 10{,}000\,\Omega$;

$$\mathbf{I}_g = \frac{250\underline{/0^\circ}}{10{,}000} = 0.025\underline{/0^\circ}\,\text{A}; \qquad i_g = 25\cos 1000t\,\text{mA}$$

P 9.39 [a] $Z_1 = 1600 - j\dfrac{10^9}{10^4(62.5)} = 1600 - j1600\,\Omega$

$$Z_1 = \frac{4000(j10^4L)}{4000 + j10^4L} = \frac{4 \times 10^5L^2 + j16 \times 10^4L}{16 + 100L^2}$$

$$Z_T = Z_1 + Z_2 = 1600 + \frac{4 \times 10^5L^2}{16 + 100L^2} - j1600 + j\frac{16 \times 10^4L}{16 + 100L^2}$$

Z_T is resistive when

$$\frac{16 \times 10^4L}{16 + 100L^2} = 1600 \qquad \text{or}$$

$$L^2 - L + 0.16 = 0$$

Solving, $L_1 = 0.8$ H and $L_2 = 0.2$ H.

[b] When $L = 0.8$ H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.64)}{16 + 64} = 4800\,\Omega$$

$$\mathbf{I}_g = \frac{96\underline{/0^\circ}}{4.8} \times 10^{-3} = 20\underline{/0^\circ}\,\text{mA}$$

$$i_g = 20\cos 10{,}000t\,\text{mA}$$

When $L = 0.2$ H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.04)}{16 + 4} = 2400\,\Omega$$

$$i_g = 40\cos 10{,}000t\,\text{mA}$$

P 9.40 Step 1 to Step 2:

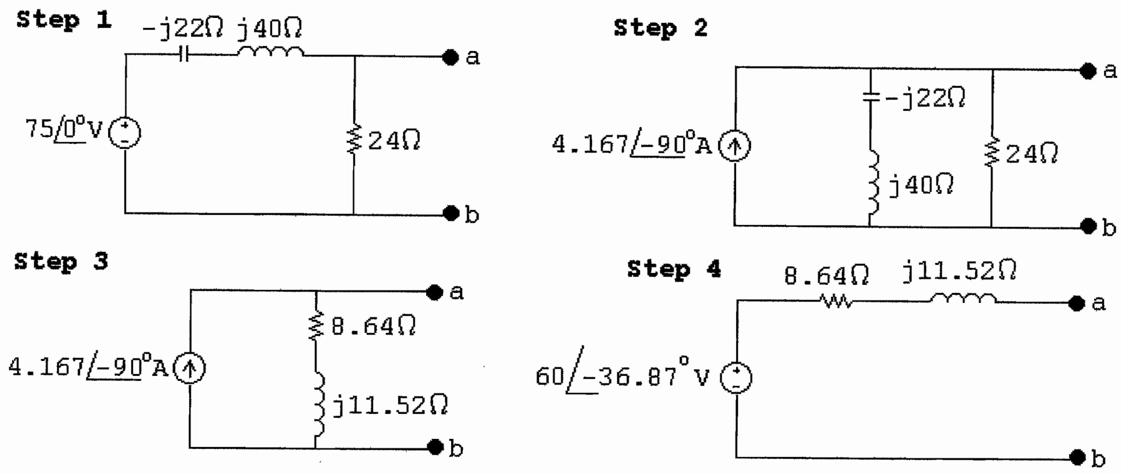
$$\frac{75\underline{/0^\circ}}{j18} = -j4.167 = 4.167\underline{/-90^\circ}\,\text{A}$$

Step 2 to Step 3:

$$(j18)\|24 = \frac{(j18)(24)}{24 + j18} = 8.64 + j11.52\,\Omega$$

Step 3 to Step 4:

$$(4.167\underline{/-90^\circ})(8.64 + j11.52) = 60\underline{/-36.87^\circ}\,\text{V}$$



P 9.41 Step 1 to Step 2:

$$(16\underline{/0^\circ})(25) = 400\underline{/0^\circ}\text{ V}$$

Step 2 to Step 3:

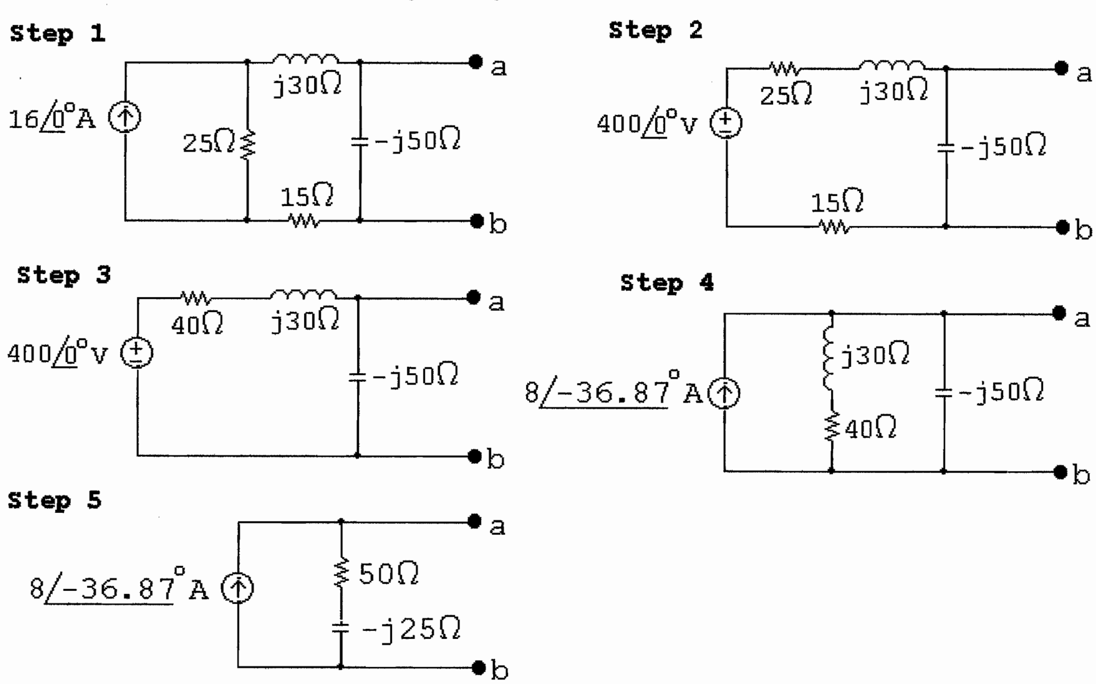
$$25 + 15 + j30 = (40 + j30)\,\Omega$$

Step 3 to Step 4:

$$\frac{400\underline{/0^\circ}}{(40 + j30)} = 8\underline{/-36.87^\circ}\text{ A}$$

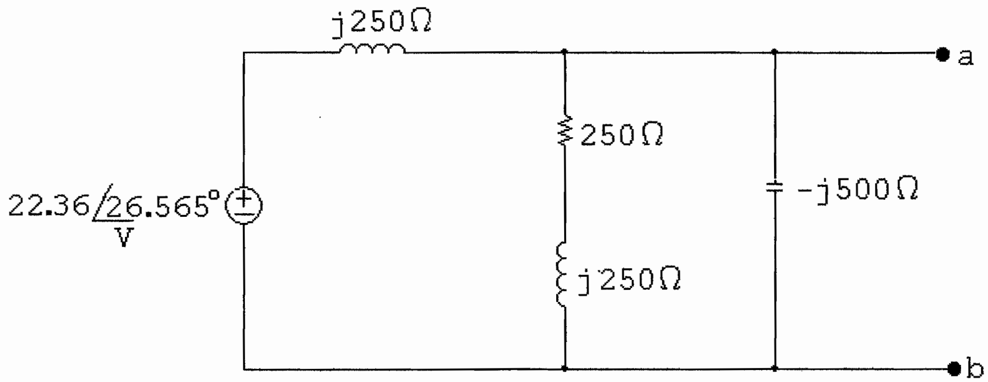
Step 4 to Step 5:

$$(40 + j30\| - j50 = \frac{(-j50)(40 + j30)}{40 + j30 - j50} = 50 - j25\,\Omega$$



P 9.42 [a] $j\omega L = j(5000)(50)\times 10^{-3} = j250\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{1}{(5000)(400\times 10^{-9})} = -j500\,\Omega$$



Using voltage division,

$$\mathbf{V}_{\rm ab} = \frac{(250+j250)\|(-j500)}{j250+(250+j250)\|(-j500)}(23.36\underline{/26.565^\circ}) = 20\underline{/0^\circ}$$

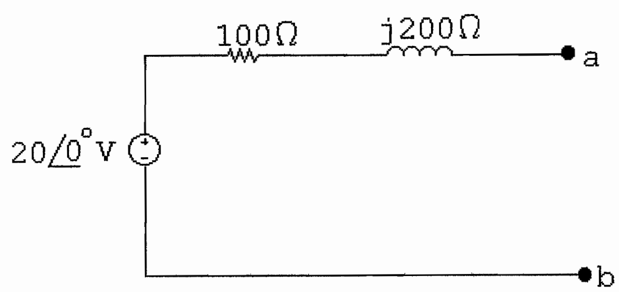
$$\mathbf{V}_{\rm Th} = \mathbf{V}_{\rm ab} = 20\underline{/0^\circ}\text{ V}$$

[b] Remove the voltage source and combine impedances in parallel to find $Z_{\rm Th} = Z_{\rm ab}$:

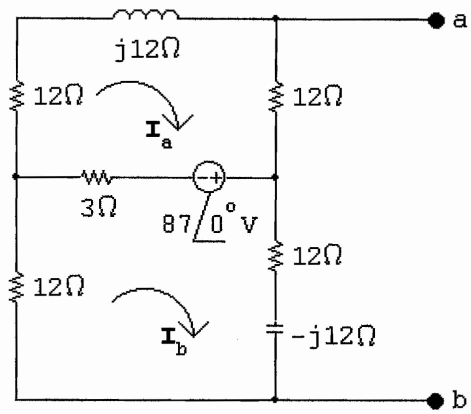
$$Y_{\rm ab} = \frac{1}{j250} + \frac{1}{250+j250} + \frac{1}{-j500} = 2 - j4\text{ mS}$$

$$Z_{\rm Th} = Z_{\rm ab} = \frac{1}{Y_{\rm ab}} = 100 + j200\,\Omega$$

[c]



P 9.43



$(27 + j12)\mathbf{I}_a - 3\mathbf{I}_b = -87\underline{/0^\circ}$

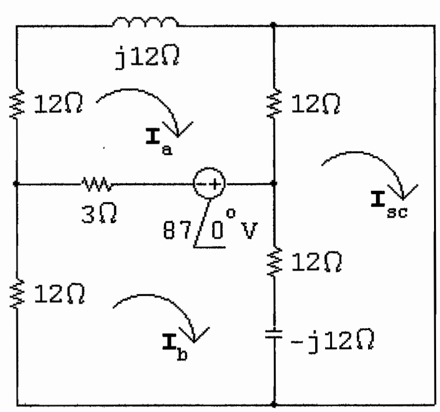
$-3\mathbf{I}_a + (27 - j12)\mathbf{I}_b = 87\underline{/0^\circ}$

Solving,

$\mathbf{I}_a = -2.4167 + j1.21; \qquad \mathbf{I}_b = 2.4167 + j1.21$

$\mathbf{V}_{\mathrm{Th}} = 12\mathbf{I}_a + (12 - j12)\mathbf{I}_b = 14.5\underline{/0^\circ}\,\mathrm{V}$

Short Circuit Test:



$(27 + j12)\mathbf{I}_a - 3\mathbf{I}_b - 12\mathbf{I}_{sc} = -87$

$-3\mathbf{I}_a + (27 - j12)\mathbf{I}_b - (12 - j12)\mathbf{I}_{sc} = 87$

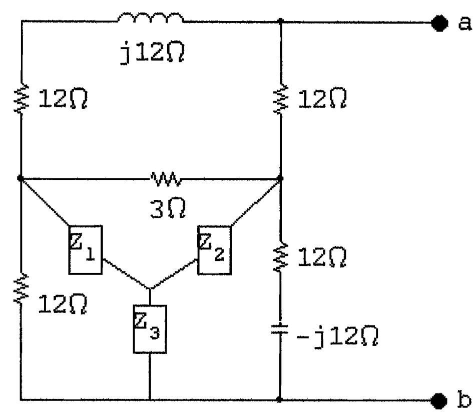
$-12\mathbf{I}_a - (12 - j12)\mathbf{I}_b + (24 - j12)\mathbf{I}_{sc} = 0$

Solving,

$\mathbf{I}_{sc} = 1\underline{/0^\circ}$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{14.5/0^\circ}{1/0^\circ} = 14.5\,\Omega$$

Alternate calculation for Z_{Th}:

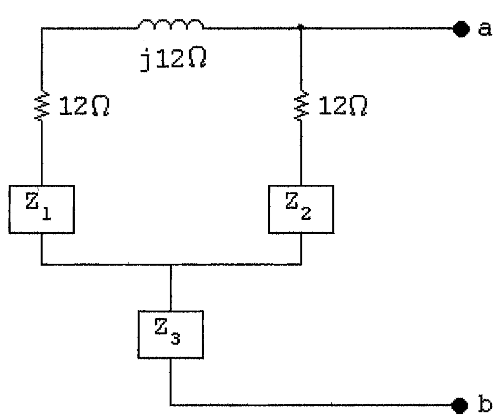


$$\sum Z = 12 + 3 + 12 - j12 = 27 - j12$$

$$Z_1 = \frac{36}{27 - j12} = \frac{12}{9 - j4}$$

$$Z_2 = \frac{36 - j36}{27 - j12} = \frac{12 - j12}{9 - j4}$$

$$Z_3 = \frac{12(12 - j12)}{27 - j12} = \frac{48 - j48}{9 - j4}$$



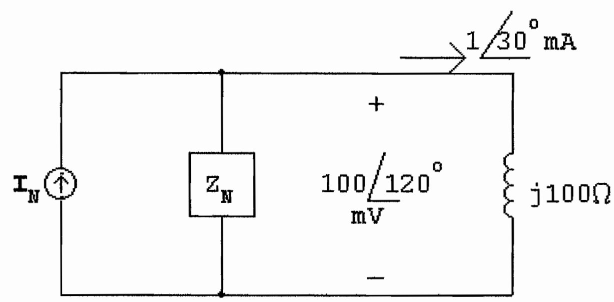
$$Z_{\text{a}} = 12 + j12 + \frac{12}{9 - j4} = \frac{12(14 + j5)}{9 - j4}$$

$$Z_{\text{b}} = 12 + \frac{12 - j12}{9 - j4} = \frac{12(10 - j5)}{9 - j4}$$

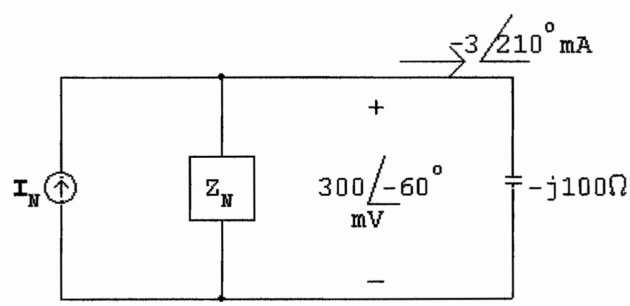
$$Z_a \| Z_b = \frac{165 - j20}{18 - j8}$$

$$Z_3 + Z_a \| Z_b = \frac{48 - j48}{9 - j4} + \frac{165 - j20}{18 - j8} = 14.5\,\Omega$$

P 9.44



$$\mathbf{I}_N = \frac{0.1\angle 120^\circ}{Z_N} + 1\angle 30^\circ\,\text{mA}, \quad Z_N \text{ in k}\Omega$$



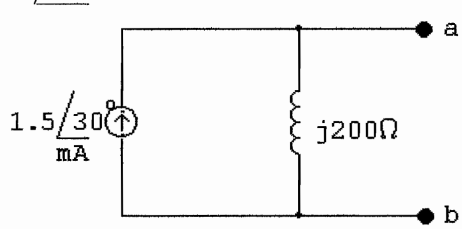
$$\mathbf{I}_N = \frac{0.3\angle -60^\circ}{Z_N} + (-3\angle 210^\circ)\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

$$\frac{0.1\angle 120^\circ}{Z_N} + 1\angle 30^\circ = \frac{0.3\angle -60^\circ}{Z_N} + (-3\angle 210^\circ)$$

$$\frac{0.3\angle -60^\circ - 0.1\angle 120^\circ}{Z_N} = 1\angle 30^\circ + 3\angle 210^\circ$$

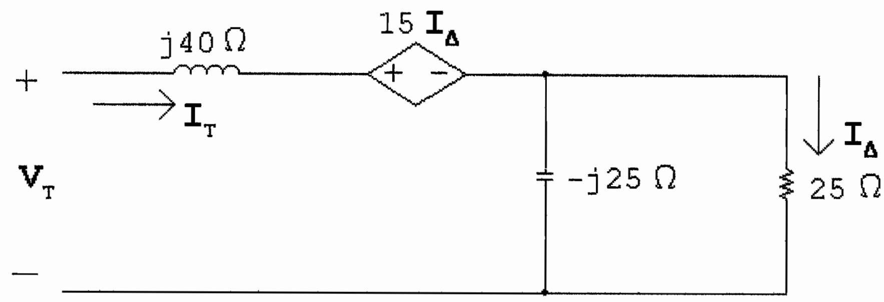
$$Z_N = \frac{0.3\angle -60^\circ - 0.1\angle 120^\circ}{1\angle 30^\circ + 3\angle 210^\circ} = 0.2\angle 90^\circ = j0.2\,\text{k}\Omega$$

$$\mathbf{I}_N = \frac{0.1\angle 120^\circ}{0.2\angle 90^\circ} + 1\angle 30^\circ = 1.5\angle 30^\circ\,\text{mA}$$



P 9.45 $j\omega L = j1.6 \times 10^6(25 \times 10^{-6}) = j40\,\Omega$

$$\frac{1}{j\omega C} = \frac{10^{-6} \times 10^9}{j1.6(25)} = -j25\,\Omega$$



$$\mathbf{V}_T = j40\mathbf{I}_T + 15\mathbf{I}_\Delta + 25\mathbf{I}_\Delta$$

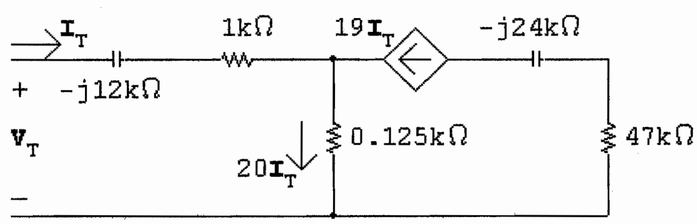
$$\mathbf{I}_\Delta = \frac{\mathbf{I}_T(-j25)}{25 - j25} = \frac{-j\mathbf{I}_T}{1 - j1}$$

$$\mathbf{V}_T = j40\mathbf{I}_T + 40\frac{(-j\mathbf{I}_T)}{1 - j1}$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{\text{ab}} = j40 + 20(-j)(1 + j) = 20 + j20\,\Omega = 28.28\underline{/45^\circ}\,\Omega$$

P 9.46 $\dfrac{1}{\omega C_1} = \dfrac{(10^{-3})(10^9)}{25(10/3)} = 12\,\text{k}\Omega$

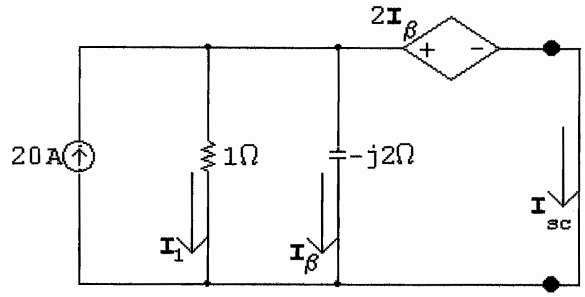
$$\frac{1}{\omega C_2} = \frac{(10^{-3})(10^9)}{25(5/3)} = 24\,\text{k}\Omega$$



$$\mathbf{V}_T = (1 - j12)\mathbf{I}_T + 20\mathbf{I}_T(0.125)$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 3.5 - j12\,\text{k}\Omega$$

P 9.47 Short circuit current

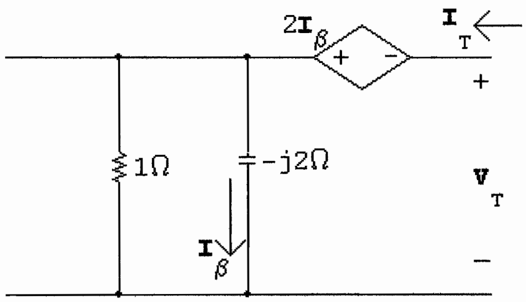


$$\mathbf{I}_\beta = \frac{2\mathbf{I}_\beta}{-j2}$$

$$-j2\mathbf{I}_\beta = 2\mathbf{I}_\beta; \qquad \therefore\ \mathbf{I}_\beta = 0$$

$$\mathbf{I}_1 = 0; \qquad \therefore\ \mathbf{I}_{sc} = 20\,\text{A} = \mathbf{I}_N$$

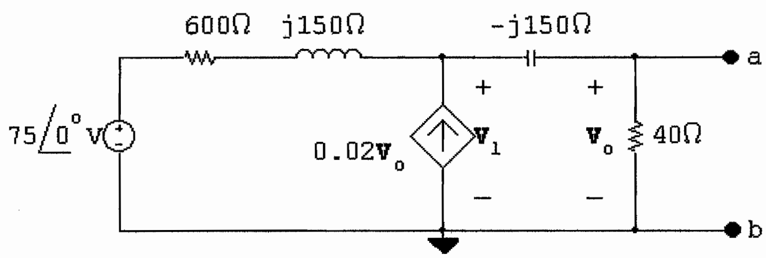
The Norton impedance is the same as the Thévenin impedance. Find it using a test source



$$\mathbf{V}_T = -2\mathbf{I}_\beta - j2\mathbf{I}_\beta = (-2 - j2)\mathbf{I}_\beta, \qquad \mathbf{I}_\beta = \frac{1}{1 - j2}\mathbf{I}_T$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{(-2 - j2)\mathbf{I}_\beta}{[(1 - j2)/1]\mathbf{I}_\beta} = \frac{-2 - j2}{1 - j2} = 0.4 - j1.2\,\Omega$$

P 9.48



$$\frac{\mathbf{V}_1 - 75}{150(4 + j1)} - \frac{0.02\mathbf{V}_1(40)}{40 - j150} + \frac{\mathbf{V}_1}{40 - j150} = 0$$

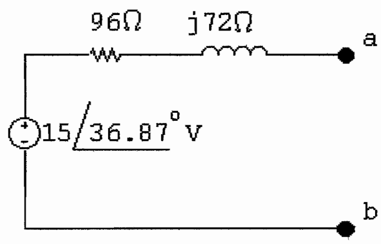
$$\therefore \quad \mathbf{V}_1 = \frac{75(4 - j15)}{16 - j12}$$

$$\mathbf{V}_{\text{Th}} = \frac{40\mathbf{V}_1}{40 - j150} = \frac{4}{4 - j15} \cdot \frac{75(4 - j15)}{16 - j12}$$

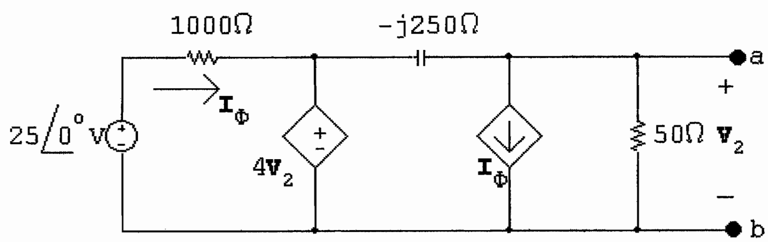
$$= \frac{75}{4 - j3} = 15\underline{/36.87^\circ}\,\text{V}$$

$$\mathbf{I}_{\text{sc}} = \frac{75}{600} = \frac{1}{8}\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = 120\underline{/36.87^\circ} = 96 + j72\,\Omega$$



P 9.49



$$\frac{\mathbf{V}_2}{50} + \frac{25 - 4\mathbf{V}_2}{1000} + \frac{\mathbf{V}_2 - 4\mathbf{V}_2}{-j250} = 0$$

Solving,

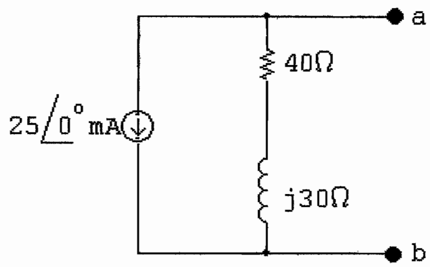
$$\mathbf{V}_2 = -1 - j0.75\,\text{V} = 1.25\underline{/216.87^\circ}\,\text{V}$$

$$\mathbf{I}_{sc} = -\mathbf{I}_\phi = \frac{-25\underline{/0^\circ}}{1000} = -25\underline{/0^\circ}\,\text{mA}$$

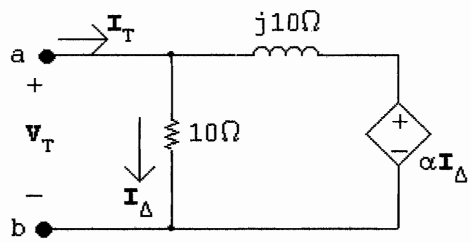
$$Z_{\text{Th}} = \frac{1.25\underline{/216.87^\circ}}{-25 \times 10^{-3}\underline{/0^\circ}} = 50\underline{/36.87^\circ}\,\Omega = 40 + j30\,\Omega$$

$$\mathbf{I}_N = \mathbf{I}_{sc} = -25\underline{/0^\circ}\,\text{mA}$$

$$Z_N = Z_{\text{Th}} = 50\underline{/36.87^\circ} = 40 + j30\,\Omega$$



P 9.50 [a]



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + \frac{\mathbf{V}_T - \alpha\mathbf{V}_T/10}{j10}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{10} + \frac{(1-\alpha/10)}{j10} = \frac{(10-\alpha)+j10}{j100}$$

$$\therefore\quad Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{1000 + j100(10-\alpha)}{(10-\alpha)^2 + 100}$$

Z_{Th} is real when $\alpha = 10$.

[b] $Z_{\text{Th}} = \dfrac{1000}{100} = 10\,\Omega$

[c] $Z_{\text{Th}} = 5 + j5$

$$\frac{1000}{(10-\alpha)^2 + 100} = 5; \qquad (10-\alpha)^2 = 100$$

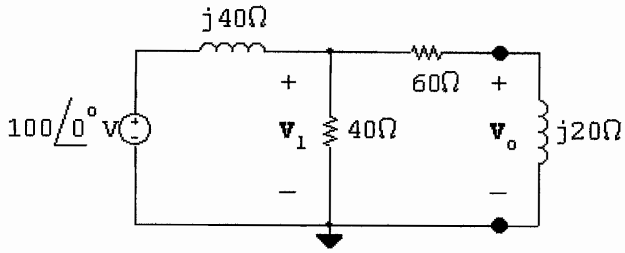
$$\therefore\quad 10 - \alpha = \pm 10; \qquad \alpha = 10 \mp 10$$

$$\alpha = 0; \qquad \alpha = 20$$

But the j term can only equal the real term with $\alpha = 0$. Thus, $\alpha = 0$.

[d] Z_{Th} will be inductive when $\alpha < 10$.

P 9.51



$$\frac{\mathbf{V}_1 - 100}{j40} + \frac{\mathbf{V}_1}{40} + \frac{\mathbf{V}_1}{60 + j20} = 0$$

Solving for $\mathbf{V}_1$ yields

$$\mathbf{V}_1 = 30 - j40\,\text{V}$$

$$\mathbf{V}_o = \frac{\mathbf{V}_1}{60 + j20}(j20) = \left(\frac{j}{3 + j}\right)\mathbf{V}_1$$

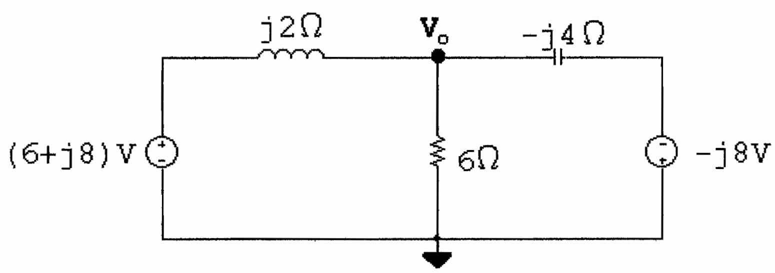
$$\mathbf{V}_o = 15 + j5\,\text{V} = 15.81\underline{/18.43^\circ}\,\text{V}$$

P 9.52 $j\omega L = j(5000)(0.4 \times 10^{-3}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(5000)(50)} = -j4\,\Omega$$

$$\mathbf{V}_{g1} = 10\underline{/53.13^\circ} = 6 + j8\,\text{V}$$

$$\mathbf{V}_{g2} = 8\underline{/-90^\circ} = -j8\,\text{V}$$



$$\frac{\mathbf{V}_o - 6 - j8}{j2} + \frac{\mathbf{V}_o}{6} + \frac{\mathbf{V}_o + (-j8)}{-j4} = 0$$

Solving,

$$\mathbf{V}_o = 12\underline{/0^\circ}$$

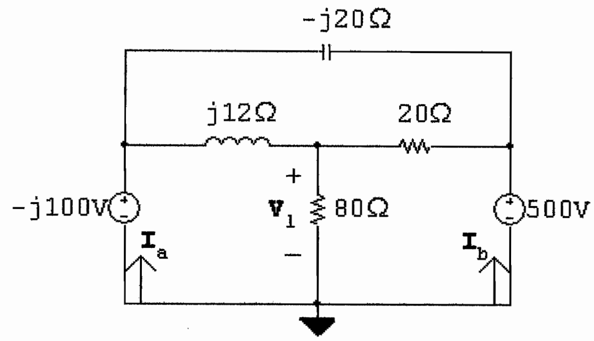
$$v_o(t) = 12\cos 5000t\,\text{V}$$

P 9.53 $j\omega L = j10^4(1.2 \times 10^{-3}) = j12\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j10^6}{5 \times 10^4} = -j20\,\Omega$$

$$\mathbf{V}_a = 100\angle -90^\circ = -j100\,\text{V}$$

$$\mathbf{V}_b = 500\angle 0^\circ = 500\,\text{V}$$



$$\frac{\mathbf{V}_1}{80} + \frac{\mathbf{V}_1 - 500}{20} + \frac{\mathbf{V}_1 + j100}{j12} = 0$$

Solving,

$$\mathbf{V}_1 = 160\angle 53.13^\circ\,\text{V} = 96 + j128\,\text{V}$$

$$\mathbf{I}_a = \frac{-j100 - 96 - j128}{j12} + \frac{-j100 - 500}{-j20}$$

$$= -14 - j17 = 22.02\angle -129.47^\circ\,\text{A}$$

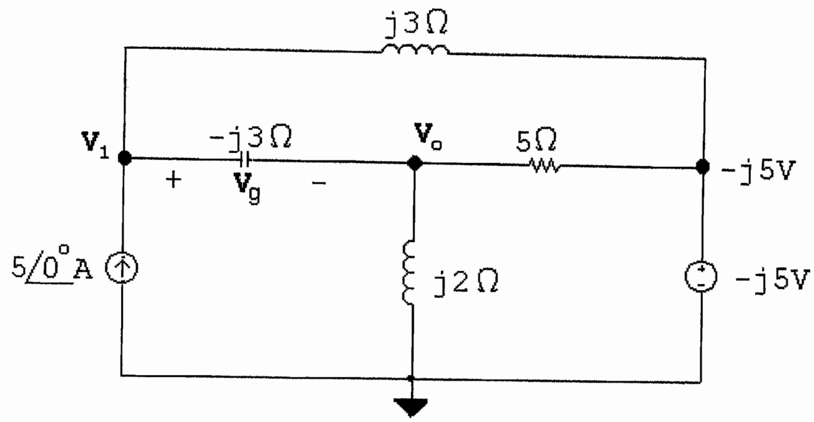
$$i_a = 22.02\cos(10{,}000t - 129.47^\circ)\,\text{A}$$

$$\mathbf{I}_b = \frac{500 - 96 - j128}{20} + \frac{500 + j100}{-j20}$$

$$= 15.2 + j18.6 = 24.02\angle 50.74^\circ\,\text{A}$$

$$i_b = 24.02\cos(10{,}000t + 50.74^\circ)\,\text{A}$$

P 9.54



$$\frac{\mathbf{V}_o}{j2} + \frac{\mathbf{V}_o + j5}{5} + \frac{\mathbf{V}_o - \mathbf{V}_1}{-j3} = 0$$

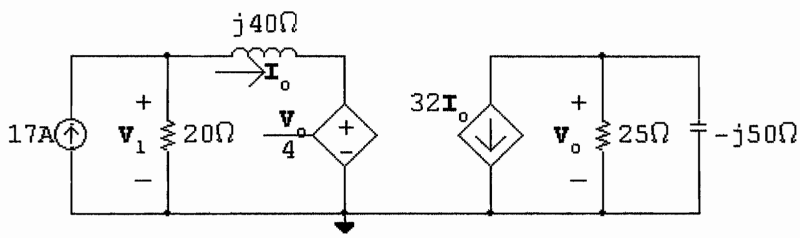
$$(5 + j6)\mathbf{V}_o + 10\mathbf{V}_1 = 30$$

$$-5 + \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j3} + \frac{\mathbf{V}_1 + j5}{j3} = 0$$

$$\mathbf{V}_o = j10; \qquad \mathbf{V}_1 = 9 - j5$$

$$\mathbf{V}_g = \mathbf{V}_1 - \mathbf{V}_o = 9 - j5 - j10 = 9 - j15 = 17.49\underline{/-59.04^\circ}\text{ V}$$

P 9.55



$$\frac{\mathbf{V}_o}{25} + \frac{\mathbf{V}_o}{-j50} + 32\mathbf{I}_o = 0$$

$$(2 + j)\mathbf{V}_o = -1600\mathbf{I}_o$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/4)}{j40}$$

$$\therefore \quad \mathbf{V}_1 = (-160 + j120)\mathbf{I}_o$$

$$17 = \frac{\mathbf{V}_1}{20} + \mathbf{I}_o = (-8 + j6)\mathbf{I}_o + \mathbf{I}_o = (-7 + j6)\mathbf{I}_o$$

$$\therefore \quad \mathbf{I}_o = \frac{17}{(-7 + j6)} = -1.4 - j1.2\,\text{A} = 1.84\underline{/-139.40^\circ}\,\text{A}$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o = 1280 + j320 = 1319.39\underline{/14.04^\circ}\,\text{V}$$

P 9.56 $$-15\underline{/0^\circ} + \frac{\mathbf{V}_o}{8} + \frac{\mathbf{V}_o - 2.5\mathbf{I}_\Delta}{j5} + \frac{\mathbf{V}_o}{-j10} = 0$$

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j10}$$

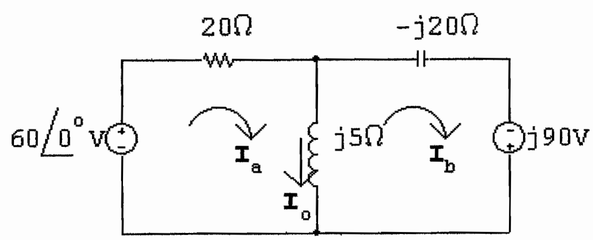
Solving,

$$\mathbf{V}_o = 72 + j96 = 120\underline{/53.13^\circ}\,\text{V}$$

P 9.57 $\mathbf{V}_\text{a} = 60\underline{/0^\circ}\,\text{V}$; $\quad\mathbf{V}_\text{b} = 90\underline{/90^\circ}\,\text{V}$

$$j\omega L = j(4 \times 10^4)(125 \times 10^{-6}) = j5\Omega$$

$$\frac{-j}{\omega C} = \frac{-j10^6}{40{,}000(1.25)} = -j20\,\Omega$$



$$60 = (20 + j5)\mathbf{I}_\text{a} - j5\mathbf{I}_\text{b}$$

$$j90 = -j5\mathbf{I}_\text{a} - j15\mathbf{I}_\text{b}$$

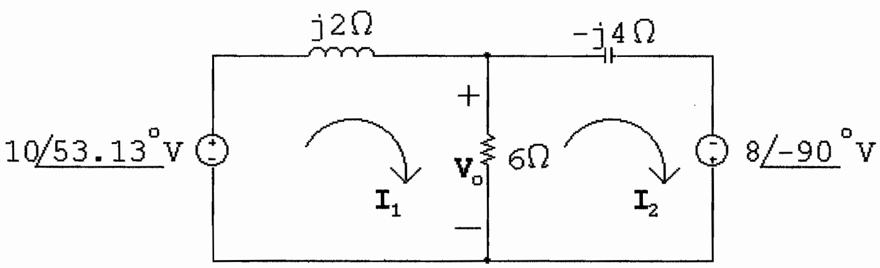
Solving,

$\mathbf{I}_\text{a} = 2.25 - j2.25\,\text{A}$; $\qquad \mathbf{I}_\text{b} = -6.75 + j0.75\,\text{A}$

$$\mathbf{I}_o = \mathbf{I}_\text{a} - \mathbf{I}_\text{b} = 9 - j3 = 9.49\underline{/-18.43^\circ}\,\text{A}$$

$$i_o(t) = 9.49\cos(40{,}000t - 18.43^\circ)\,\text{A}$$

P 9.58 From the solution to Problem 9.52 the phasor-domain circuit is



$$10\angle 53.13^\circ = (6 + j2)\mathbf{I}_1 - 6\mathbf{I}_2$$

$$8\angle -90^\circ = -6\mathbf{I}_1 + (6 - j4)\mathbf{I}_2$$

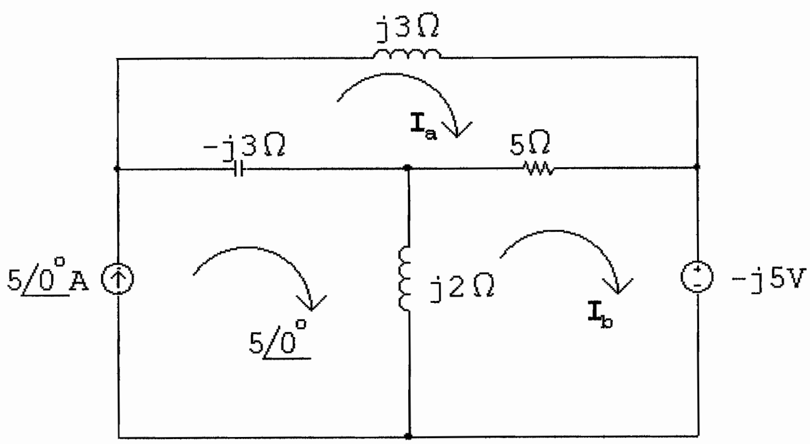
$$\mathbf{V}_o = (\mathbf{I}_1 - \mathbf{I}_2)6$$

Solving,

$$\mathbf{V}_o = 12\angle 0^\circ\text{ V}$$

$$v_o(t) = 12\cos 5000t\text{ V}$$

P 9.59



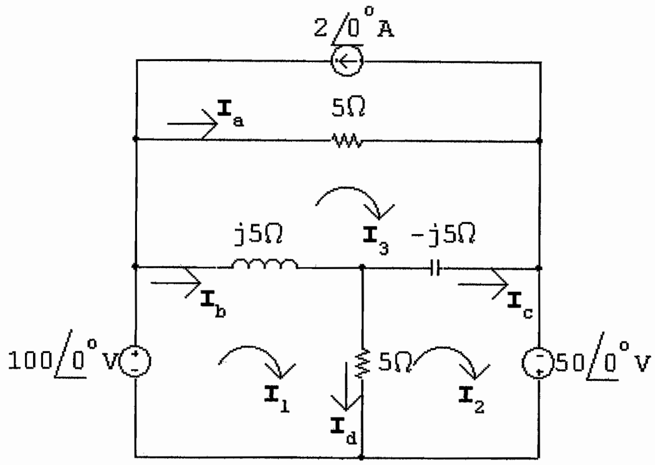
$$j3\mathbf{I}_a + 5(\mathbf{I}_a - \mathbf{I}_b) - j3(\mathbf{I}_a - 5) = 0$$

$$j2(\mathbf{I}_b - 5) + 5(\mathbf{I}_b - \mathbf{I}_a) - j5 = 0$$

Solving,

$$\mathbf{I}_a = -j3; \qquad \mathbf{I}_g = -j3 = 3\angle -90^\circ\text{ A}$$

P 9.60



$$100\underline{/0^\circ} = (5 + j5)\mathbf{I}_1 - 5\mathbf{I}_2 - j5\mathbf{I}_3$$

$$50\underline{/0^\circ} = -5\mathbf{I}_1 + (5 - j5)\mathbf{I}_2 + j5\mathbf{I}_3$$

$$-10\underline{/0^\circ} = -j5\mathbf{I}_1 + j5\mathbf{I}_2 + 5\mathbf{I}_3$$

Solving,

$\mathbf{I}_1 = 58 - j20\,\text{A}$; $\quad \mathbf{I}_2 = 58 + j10\,\text{A}$; $\quad \mathbf{I}_3 = 28 + j0\,\text{A}$

$$\mathbf{I}_a = \mathbf{I}_3 + 2 = 30 + j0\,\text{A}$$

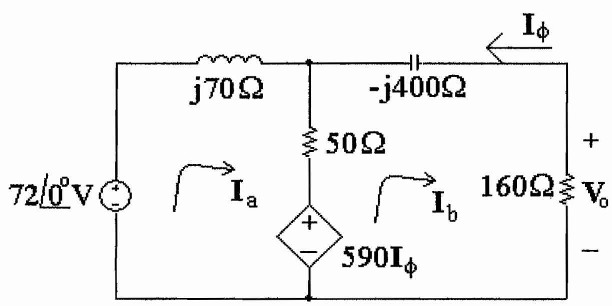
$$\mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_3 = 58 - j20 - 28 = 30 - j20\,\text{A}$$

$$\mathbf{I}_c = \mathbf{I}_2 - \mathbf{I}_3 = 58 + j10 - 28 = 30 + j10\,\text{A}$$

$$\mathbf{I}_d = \mathbf{I}_1 - \mathbf{I}_2 = 58 - j20 - 58 - j10 = -j30\,\text{A}$$

P 9.61 $j\omega L = j5000(14 \times 10^{-3}) = j70\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(5000)(0.5 \times 10^{-6})} = -j400\,\Omega$$



$$72\underline{/0^\circ} = (50 + j70)\mathbf{I}_a - 50\mathbf{I}_b + 590(-\mathbf{I}_b)$$

$$0 = -50\mathbf{I}_a - 590(-\mathbf{I}_b) + (210 - j400)\mathbf{I}_b$$

Solving,

$$\mathbf{I}_b = (50 - j50)\,\text{mA}$$

$$\mathbf{V}_o = 160\mathbf{I}_b = 8 - j8 = 11.31\underline{/-45^\circ}$$

$$v_o = 11.31\cos(5000t - 45^\circ)\,\text{V}$$

P 9.62 $$Z_o = 600 - j\frac{10^6}{(5000)(0.25)} = 600 - j800\,\Omega$$

$$Z_T = 300 + j2000 + 600 - j800 = 900 + j1200\,\Omega = 1500\underline{/53.13^\circ}\,\Omega$$

$$\mathbf{V}_o = \mathbf{V}_g\frac{Z_o}{Z_T} = \frac{(75\underline{/0^\circ})(1000\underline{/-53.13^\circ})}{1500\underline{/53.13^\circ}} = 50\underline{/-106.26^\circ}\,\text{V}$$

$$v_o = 50\cos(5000t - 106.26^\circ)\,\text{V}$$

P 9.63 $$\frac{1}{j\omega C} = -j\frac{10^6}{10^4} = -j100\,\Omega$$

$$j\omega L = j(500)(1) = j500\,\Omega$$

Let $Z_1 = 50 - j100\,\Omega$; $\qquad Z_2 = 250 + j500\,\Omega$

$$\mathbf{I}_g = 125\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{I}_o = \frac{\mathbf{I}_g Z_1}{Z_1 + Z_2} = \frac{125\underline{/0^\circ}(50 - j100)}{(300 + j400)}$$

$$= -12.5 - j25\,\text{mA} = 27.95\underline{/-116.57^\circ}\,\text{mA}$$

$$i_o = 27.95\cos(500t - 116.57^\circ)\,\text{mA}$$

P 9.64 $\mathbf{V}_g = 1.2\underline{/0^\circ}\,\text{V}$; $\qquad \dfrac{1}{j\omega C} = \dfrac{10^6}{j100} = -j10\,\text{k}\Omega$

Let $\mathbf{V}_a$ = voltage across $1\,\mu$F capacitor, positive at upper terminal
Then:

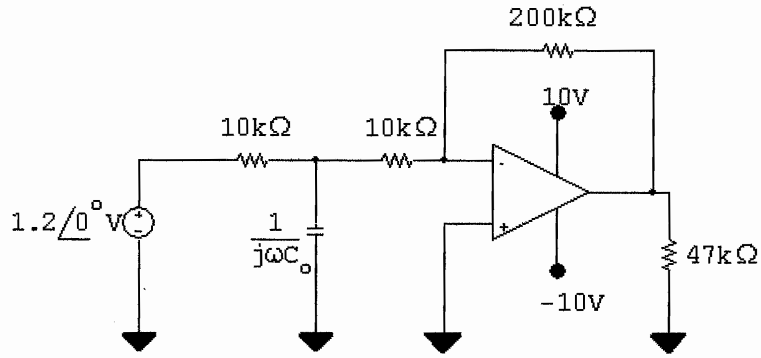
$$\frac{\mathbf{V}_a - 1.2\underline{/0^\circ}}{10} + \frac{\mathbf{V}_a}{-j10} + \frac{\mathbf{V}_a}{10} = 0; \qquad \therefore\ \mathbf{V}_a = (0.48 - j0.24)\,\text{V}$$

$$\frac{0 - \mathbf{V}_a}{10} + \frac{0 - \mathbf{V}_o}{200} = 0; \qquad \mathbf{V}_o = -20\mathbf{V}_a$$

$$\therefore\ \mathbf{V}_o = -9.6 + j4.8 = 10.73\underline{/153.43^\circ}\,\text{V}$$

$$v_o = 10.73\cos(100t + 153.43^\circ)\,\text{V}$$

P 9.65 [a]



$$\frac{\mathbf{V}_a - 1.2\underline{/0^\circ}}{10{,}000} + j\omega C_o \mathbf{V}_a + \frac{\mathbf{V}_a}{10{,}000} = 0$$

$$\mathbf{V}_a = \frac{1.2}{2 + j10^4\omega C_o}$$

$\mathbf{V}_o = -20\mathbf{V}_a$ (see solution to Prob. 9.73)

$$\mathbf{V}_o = \frac{-24}{2 + j10^6 C_o} = \frac{24\underline{/180^\circ}}{2 + j10^6 C_o}$$

$\therefore$ denominator angle $= 60^\circ$

$\tan 60^\circ = \sqrt{3}$

$$\frac{10^6 C_o}{2} = \sqrt{3}$$

$$\text{or } C_o = \frac{2\sqrt{3}}{10^6} = 2\sqrt{3}\,\mu\text{F} = 3.46\,\mu\text{F}$$

[b] $\mathbf{V}_o = \dfrac{24\underline{/180^\circ}}{2 + j2\sqrt{3}} = 6\underline{/120^\circ}\text{ V}$

$v_o = 6\cos(100t + 120^\circ)\text{ V}$

P 9.66 [a] $\mathbf{V}_g = 2\underline{/0^\circ}\text{ V}$

$$\mathbf{V}_p = \frac{80}{100}\mathbf{V}_g = 1.6\underline{/0^\circ}; \qquad \mathbf{V}_n = \mathbf{V}_p = 1.6\underline{/0^\circ}\text{ V}$$

$$\frac{1.6}{160} + \frac{1.6 - \mathbf{V}_o}{Z_p} = 0$$

$$Z_p = \frac{(200)(1/j\omega C)}{200 + (1/j\omega C)}$$

$$\frac{1}{j\omega C} = \frac{10^9}{j10^5(0.1)} = -j10^5 = -j100\text{ k}\Omega$$

$$Z_p = \frac{200(-j100)}{200 - j100} = 40 - j80\,\text{k}\Omega$$

$$\mathbf{V}_o = 1.6 + \frac{Z_p}{100} = 2 - j0.8 = 2.15\underline{/-21.80^\circ}$$

$$v_o = 2.15\cos(10^5 t - 21.80^\circ)\,\text{V}$$

[b] $\mathbf{V}_p = 0.8V_m\underline{/0^\circ}$; $\quad \mathbf{V}_n = \mathbf{V}_p = 0.8V_m\underline{/0^\circ}$

$$\frac{0.8V_m}{160} + \frac{0.8V_m - \mathbf{V}_o}{40 - j80} = 0$$

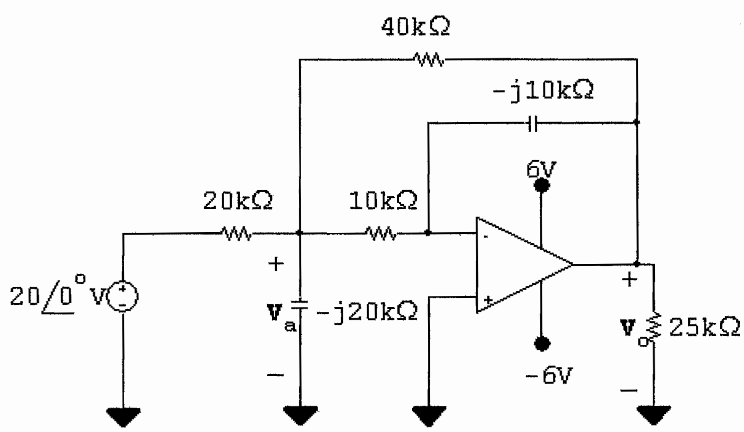
$$\therefore \quad \mathbf{V}_o = 0.8V_m + \frac{40 - j80}{160}V_m(0.8) = 0.8V_m(1.25 - j0.5)$$

$$\therefore \quad |0.8V_m(1.25 - j0.5)| \leq 5$$

$$\therefore \quad V_m \leq 4.64\,\text{V}$$

P 9.67 $\quad \dfrac{1}{j\omega C_1} = \dfrac{10^{12}}{j10^6(100)} = -j10\,\text{k}\Omega$

$$\frac{1}{j\omega C_2} = \frac{10^{12}}{j(10^6)(50)} = -j20\,\text{k}\Omega$$



$$\frac{\mathbf{V}_a}{-j20} + \frac{\mathbf{V}_a - 20}{20} + \frac{\mathbf{V}_a - \mathbf{V}_o}{40} + \frac{\mathbf{V}_a}{10} = 0$$

$$\therefore \quad (-2 + j7)\mathbf{V}_a - j\mathbf{V}_o = j40$$

$$\frac{0 - \mathbf{V}_a}{10} + \frac{0 - \mathbf{V}_o}{-j10} = 0; \qquad \therefore \quad \mathbf{V}_a = -j\mathbf{V}_o$$

$$\therefore \quad (7 + j)\mathbf{V}_o = j40$$

$$\mathbf{V}_o = \frac{j40}{7+j} = 0.8 + j5.6 = 5.657\underline{/81.87^\circ}\,\mathrm{V}$$

$$v_o(t) = 5.657\cos(10^6 t + 81.87^\circ)\,\mathrm{V}$$

P 9.68 [a] $\dfrac{1}{j\omega C} = \dfrac{-j10^9}{(2\times 10^5)(12.5)} = -j400\,\Omega$

$$\frac{\mathbf{V}_\mathrm{n}}{200} + \frac{\mathbf{V}_\mathrm{n} - \mathbf{V}_o}{-j400} = 0$$

$$\frac{\mathbf{V}_o}{-j400} = \frac{\mathbf{V}_\mathrm{n}}{200} + \frac{\mathbf{V}_\mathrm{n}}{-j400}$$

$$\mathbf{V}_o = \mathbf{V}_\mathrm{n} - j2\mathbf{V}_\mathrm{n} = (1-j2)\mathbf{V}_\mathrm{n}$$

$$\mathbf{V}_\mathrm{p} = \frac{\mathbf{V}_g(1/j\omega C_o)}{500 + (1/j\omega C_o)} = \frac{\mathbf{V}_g}{1 + j(500)(2\times 10^5)C_o}$$

$$\mathbf{V}_g = 10\underline{/0^\circ}\,\mathrm{V}$$

$$\mathbf{V}_\mathrm{p} = \frac{10\underline{/0^\circ}}{1 + j10^8 C_o} = \mathbf{V}_\mathrm{n}$$

$$\therefore\quad \mathbf{V}_o = \frac{(1-j2)10\underline{/0^\circ}}{1 + j10^8 C_o}$$

$$|\mathbf{V}_o| = \frac{\sqrt{5}(10)}{\sqrt{1 + 10^{16} C_o^2}} = 10$$

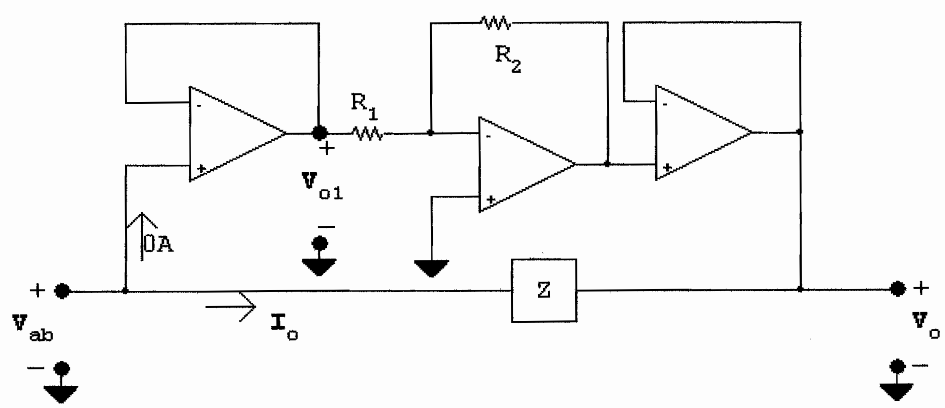
Solving,

$C_o = 20\,\mathrm{nF}$

[b] $\mathbf{V}_o = \dfrac{10(1-j2)}{1+j2} = 10\underline{/-126.87^\circ}$

$$v_o = 10\cos(2\times 10^5 t - 126.87^\circ)\,\mathrm{V}$$

P 9.69 [a]



Because the op-amps are ideal $\mathbf{I}_{\text{in}} = \mathbf{I}_o$, thus

$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_{\text{in}}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_o}; \qquad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - \mathbf{V}_o}{Z}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{\text{ab}}; \qquad \mathbf{V}_{o2} = -\left(\frac{R_2}{R_1}\right)\mathbf{V}_{o1} = -K\mathbf{V}_{o1} = -K\mathbf{V}_{\text{ab}}$$

$$\mathbf{V}_o = \mathbf{V}_{o2} = -K\mathbf{V}_{\text{ab}}$$

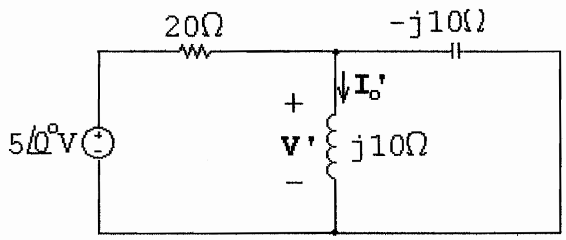
$$\therefore \quad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - (-K\mathbf{V}_{\text{ab}})}{Z} = \frac{(1+K)\mathbf{V}_{\text{ab}}}{Z}$$

$$\therefore \quad Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{(1+K)\mathbf{V}_{\text{ab}}}Z = \frac{Z}{(1+K)}$$

[b] $Z = \dfrac{1}{j\omega C}; \qquad Z_{\text{ab}} = \dfrac{1}{j\omega C(1+K)}; \qquad \therefore \; C_{\text{ab}} = C(1+K)$

P 9.70 [a] Superposition must be used because the frequencies of the two sources are different.

[b] For $\omega = 80{,}000$ rad/s:



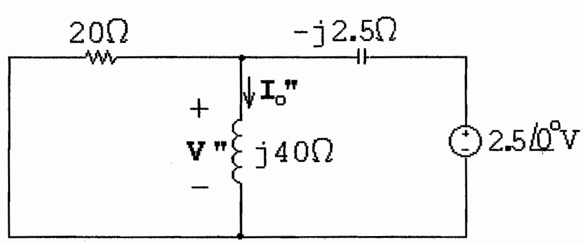
$$\frac{\mathbf{V}_o' - 5}{20} + \frac{\mathbf{V}_o'}{j10} + \frac{\mathbf{V}_o'}{-j10} = 0$$

$$\mathbf{V}_o'\left(\frac{1}{20} + \frac{1}{j10} + \frac{1}{-j10}\right) = \frac{5}{20}$$

$$\therefore \quad \mathbf{V}_o' = 5\underline{/0^\circ}\text{ V}$$

$$\mathbf{I}_o' = \frac{\mathbf{V}_o'}{j10} = -j0.5 = 500\underline{/-90^\circ}\text{ mA}$$

For $\omega = 320{,}000$ rad/s:



$$20\|j40 = 16 + j8\,\Omega$$

$$\mathbf{V}'' = \frac{16 + j8}{16 + j8 - j2.5}(2.5\underline{/0^\circ}) = 2.643\underline{/7.59^\circ}\,\text{V}$$

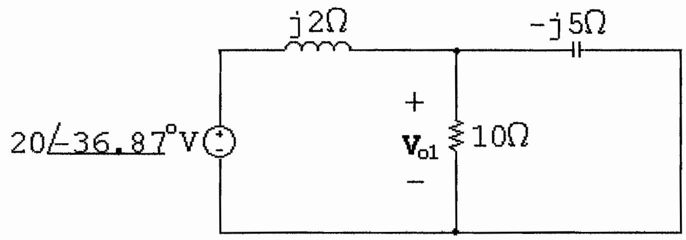
$$\therefore\;\; \mathbf{I}_o'' = \frac{\mathbf{V}''}{j40} = 66.08\underline{/-82.4^\circ}\,\text{mA}$$

Thus,

$$i_o(t) = [500 \sin 80{,}000t + 66.08 \cos(320{,}000t - 82.4^\circ)]\,\text{mA}, \quad t \geq 0$$

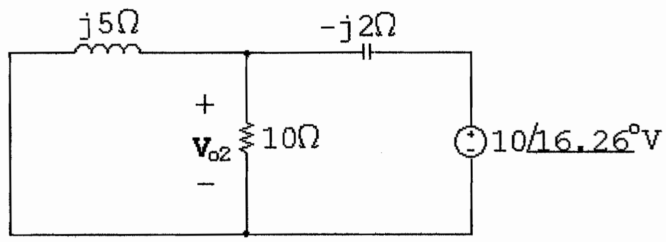
P 9.71 **[a]** Superposition must be used because the frequencies of the two sources are different.

[b] For $\omega = 2000$ rad/s:



$10 \| -j5 = 2 - j4\,\Omega$ so $\mathbf{V}_{o1} = \dfrac{2 - j4}{2 - j4 + j2}(20\underline{/-36.87^\circ} = 31.62\underline{/-55.3^\circ}\,\text{V}$

For $\omega = 5000$ rad/s:



$j5 \| 10 = 2 + j4\,\Omega$

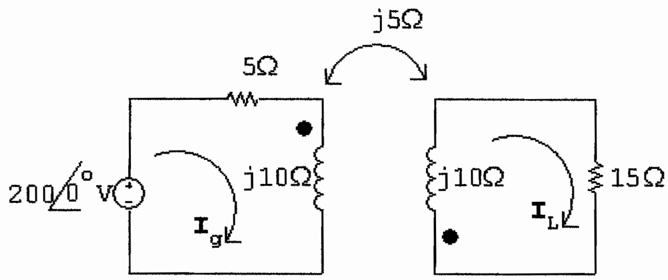
$$\mathbf{V}_{o2} = \frac{2 + j4}{2 + j4 - j2}(10\underline{/16.26^\circ}) = 15.81\underline{/34.69^\circ}\,\text{V}$$

Thus,

$$v_o(t) = [31.62 \cos(2000t - 55.3^\circ) + 15.81 \cos(5000t + 34.69^\circ)]\,\text{V}, \quad t \geq 0$$

P 9.72 **[a]** $j\omega L_1 = j\omega L_2 = j(10{,}000)(1 \times 10^{-3}) = j10\,\Omega$

$j\omega M = j(10{,}000)(0.5 \times 10^{-3}) = j5\,\Omega$



$$200 = (5 + j10)\mathbf{I}_g + j5\mathbf{I}_L$$

$$0 = j5\mathbf{I}_g + (15 + j10)\mathbf{I}_L$$

Solving,

$$\mathbf{I}_g = 10 - j15\,\text{A}; \qquad \mathbf{I}_L = -5\,\text{A}$$

$$i_g = 18.03\cos(10{,}000t - 56.31^\circ)\,\text{A}$$

$$i_L = 5\cos(10{,}000t - 180^\circ)\,\text{A}$$

[b] $k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{0.5}{\sqrt{1}} = 0.5$

[c] When $t = 50\pi\,\mu$s,

$$10{,}000t = (10{,}000)(50\pi)\times 10^{-6} = 0.5\pi = \pi/2\,\text{rad} = 90^\circ$$

$$i_g(50\pi\mu s) = 18.03\cos(90 - 56.31^\circ) = 15\,\text{A}$$

$$i_L(50\pi\mu s) = 5\cos(90 + 180^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2 = \frac{1}{2}(1\times 10^{-3})(15)^2 + 0 + 0 = 112.5\,\text{mJ}$$

When $t = 100\pi\,\mu$s,

$$10{,}000t = \pi\,\text{rad} = 180^\circ$$

$$i_g(100\pi\mu s) = -10\,\text{A}$$

$$i_L(100\pi\mu s) = 5\,\text{A}$$

$$w = \frac{1}{2}(1\times 10^{-3})(10)^2 + \frac{1}{2}(1\times 10^{-3})(5)^2 + 0.5\times 10^{-3}(-10)(5) = 37.5\,\text{mJ}$$

P 9.73 [a] $j\omega L_1 = j(50)(5) = j250\,\Omega$

$$j\omega L_2 = j(50)(20) = j1000\,\Omega$$

$$\frac{1}{j\omega C} = \frac{10^9}{j(50\times 10^3)(40)} = -j500\,\Omega$$

$$\therefore \quad Z_{22} = 75 + 300 + j1000 - j500 = 375 + j500\,\Omega$$

$$\therefore \quad Z_{22}^* = 375 - j500\,\Omega$$

$$M = k\sqrt{L_1 L_2} = 10k \times 10^{-3}$$

$$\omega M = (50)(10k) = 500k$$

$$Z_r = \left[\frac{500k}{625}\right]^2 (375 - j500) = k^2(240 - j320)\,\Omega$$

$$Z_{\rm in} = 120 + j250 + 240k^2 - j320k^2$$

$$|Z_{\rm in}| = [(120 + 240k^2)^2 + (250 - 320k^2)^2]^{\frac{1}{2}}$$

$$\frac{d|Z_{\rm in}|}{dk} = \frac{1}{2}[(120 + 240k^2)^2 + (250 - 320k^2)^2]^{-\frac{1}{2}} \times$$

$$[2(120 + 240k^2)480k + 2(250 - 320k^2)(-640k)]$$

$$\frac{d|Z_{\rm in}|}{dk} = 0 \text{ when}$$

$$960k(120 + 240k^2) - 1280k(250 - 320k^2) = 0$$

$$\therefore \quad k^2 = 0.32; \qquad \therefore \quad k = \sqrt{0.32} = 0.5657$$

[b] $Z_{\rm in}\ (\text{min}) = 120 + 240(0.32) + j[250 - 0.32(320)]$

$$= 196.8 + j147.6 = 246\underline{/36.87^\circ}\,\Omega$$

$$\mathbf{I}_1\ (\text{max}) = \frac{369\underline{/0^\circ}}{246\underline{/36.87^\circ}} = 1.5\underline{/-36.87^\circ}\,\text{A}$$

$$\therefore \quad i_1\ (\text{peak}) = 1.5\,\text{A}$$

Note — You can test that the k value obtained from setting $d|Z_{\rm in}|/dt = 0$ leads to a minimum by noting $0 \le k \le 1$. If $k = 1$,

$$Z_{\rm in} = 360 - j70 = 366.74\underline{/-11^\circ}\,\Omega$$

Thus,

$$|Z_{\rm in}|_{k=1} > |Z_{\rm in}|_{k=\sqrt{0.32}}$$

If $k = 0$,

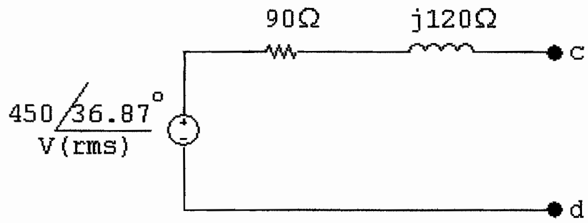
$$Z_{\rm in} = 120 + j250 = 277.31\underline{/64.36^\circ}\,\Omega$$

Thus,

$$|Z_{\rm in}|_{k=0} > |Z_{\rm in}|_{k=\sqrt{0.32}}$$

P 9.74 $Z_{\text{Th}} = 30 + j200 + (50/25)^2(15 - j20) = 90 + j120\,\Omega$

$$\mathbf{V}_{\text{Th}} = \frac{225\underline{/0^\circ}}{15 + j20}(j50) = 450\underline{/36.87^\circ}\,\text{V}$$



P 9.75 $j\omega L_1 = j(25 \times 10^3)(3.2 \times 10^{-3}) = j80\,\Omega$

$$j\omega L_2 = j(25 \times 10^3)(12.8 \times 10^{-3}) = j320\,\Omega$$

$$\frac{1}{j\omega C} = \frac{10^9}{j(25 \times 10^3)(250)} = -j160\,\Omega$$

$$j\omega M = j(25 \times 10^3)k\sqrt{(3.2)(12.8)} \times 10^{-3} = j160k\,\Omega$$

$$Z_{22} = 40 + j320 - j160 = 40 + j160\,\Omega$$

$$Z_{22}^* = 40 - j160\,\Omega$$

$$Z_r = \left[\frac{160k}{|40 + j160|}\right]^2 (40 - j160) = 37.647k^2 - j150.588k^2$$

$$Z_{\text{ab}} = 10 + j80 + 37.647k^2 - j150.588k^2 = (10 + 37.647k^2) + j(80 - 150.588k^2)$$

Z_{ab} is resistive when

$$80 - 150.588k^2 = 0 \quad \text{or} \quad k^2 = 0.53125$$

$$\therefore \quad Z_{\text{ab}} = 10 + (37.647)(0.53125) = 30\,\Omega$$

P 9.76 [a] $j\omega L_2 = j(500)10^3(500)10^{-6} = j250\,\Omega$

$$\frac{1}{j\omega C} = \frac{10^9}{j(500 \times 10^3)(20)} = -j100\,\Omega$$

$$Z_{22} = 150 + 50 + j250 - j100 = 200 + j150\,\Omega$$

$$Z_{22}^* = 200 - j150\,\Omega$$

$$\omega M = (500 \times 10^3)(100 \times 10^{-6}) = 50\,\Omega$$

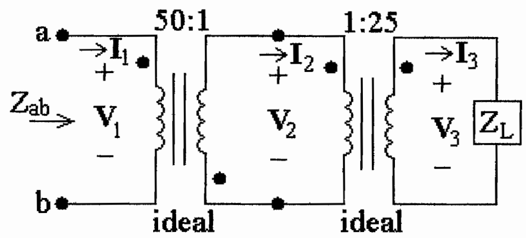
$$Z_r = \left(\frac{50}{250}\right)^2 [200 - j150] = 8 - j6\,\Omega$$

[b] $Z_{ab} = R_1 + j\omega L_1 + 8 - j6$

$$j\omega L_1 = j(500 \times 10^3)(80 \times 10^{-6}) = j40\,\Omega$$

$$Z_{ab} = 20 + j34\,\Omega$$

P 9.77



$$Z_{ab} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{50} = -\frac{\mathbf{V}_2}{1}; \qquad 50\mathbf{I}_1 = -\mathbf{I}_2$$

$$\therefore \quad Z_{ab} = \frac{-50\mathbf{V}_2}{-\mathbf{I}_2/50} = 2500\frac{\mathbf{V}_2}{\mathbf{I}_2}$$

$$\frac{\mathbf{V}_2}{1} = \frac{\mathbf{V}_3}{25}; \qquad \mathbf{I}_2 = 25\mathbf{I}_3$$

$$\therefore \quad Z_{ab} = 2500\frac{\mathbf{V}_3/25}{25\mathbf{I}_2} = \frac{2500}{625}\frac{\mathbf{V}_3}{\mathbf{I}_3}$$

$$= 4Z_L = 4(200 + j150) = (800 + j600)\,\Omega$$

P 9.78 In Eq. 9.69 replace $\omega^2 M^2$ with $k^2\omega^2 L_1 L_2$ and then write X_{ab} as

$$X_{ab} = \omega L_1 - \frac{k^2\omega^2 L_1 L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}$$

$$= \omega L_1 \left\{1 - \frac{k^2 \omega L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}\right\}$$

For X_{ab} to be negative requires

$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 < k^2\omega L_2(\omega L_2 + \omega L_L)$$

or

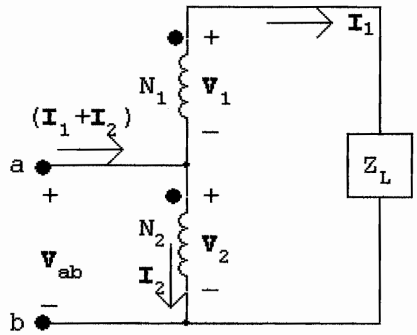
$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 - k^2\omega L_2(\omega L_2 + \omega L_L) < 0$$

which reduces to

$$R_{22}^2 + \omega^2 L_2^2(1-k^2) + \omega L_2 \omega L_L(2-k^2) + \omega^2 L_L^2 < 0$$

But $k \leq 1$ hence it is impossible to satisfy the inequality. Therefore X_{ab} can never be negative if X_L is an inductive reactance.

P 9.79 [a]



$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{(1 + N_1/N_2)\mathbf{I}_1}$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_2}, \qquad \mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2$$

$$\mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2$$

$$Z_{\text{ab}} = \frac{\mathbf{I}_1 Z_L}{(N_1/N_2 + 1)(1 + N_1/N_2)\mathbf{I}_1}$$

$$\therefore \quad Z_{\text{ab}} = \frac{Z_L}{[1 + (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

[b] Assume dot on the N_2 coil is moved to the lower terminal. Then

$$\mathbf{V}_1 = -\frac{N_1}{N_2}\mathbf{V}_2 \quad \text{and} \quad \mathbf{I}_2 = -\frac{N_1}{N_2}\mathbf{I}_1$$

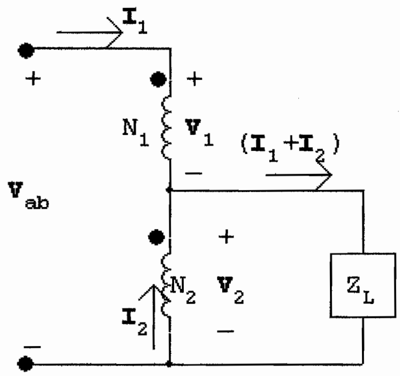
As before

$$Z_{\text{ab}} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} \quad \text{and} \quad \mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\text{ab}} = \frac{\mathbf{V}_2}{(1 - N_1/N_2)\mathbf{I}_1} = \frac{Z_L\mathbf{I}_1}{[1 - (N_1/N_2)]^2\mathbf{I}_1}$$

$$Z_{\text{ab}} = \frac{Z_L}{[1 - (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

P 9.80 [a]



$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}_1} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_2 = \frac{N_2}{N_1}\mathbf{V}_1$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\mathbf{V}_2 = (\mathbf{I}_1 + \mathbf{I}_2)Z_L = \mathbf{I}_1\left(1 + \frac{N_1}{N_2}\right)Z_L$$

$$\mathbf{V}_1 + \mathbf{V}_2 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2 = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{ab} = \frac{(1 + N_1/N_2)^2 Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{ab} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L \quad \text{Q.E.D.}$$

[b] Assume dot on N_2 is moved to the lower terminal, then

$$\frac{\mathbf{V}_1}{N_1} = \frac{-\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_1 = \frac{-N_1}{N_2}\mathbf{V}_2$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{-N_1}{N_2}\mathbf{I}_1$$

As in part [a]

$$\mathbf{V}_2 = (\mathbf{I}_2 + \mathbf{I}_1)Z_L \quad \text{and} \quad Z_{ab} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$Z_{ab} = \frac{(1 - N_1/N_2)\mathbf{V}_2}{\mathbf{I}_1} = \frac{(1 - N_1/N_2)(1 - N_1/N_2)Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{ab} = [1 - (N_1/N_2)]^2 Z_L \quad \text{Q.E.D.}$$

P 9.81 [a] $\mathbf{I} = \frac{240}{24} + \frac{240}{j32} = (10 - j7.5)\text{ A}$

$$\mathbf{V}_s = 240\underline{/0^\circ} + (0.1 + j0.8)(10 - j7.5) = 247 + j7.25 = 247.11\underline{/1.68^\circ}\text{ V}$$

[b] Use the capacitor to eliminate the j component of $\mathbf{I}$, therefore

$$\mathbf{I}_c = j7.5\text{ A}, \qquad Z_c = \frac{240}{j7.5} = -j32\,\Omega$$

$$\mathbf{V}_s = 240 + (0.1 + j0.8)10 = 241 + j8 = 241.13\underline{/1.90^\circ}\text{ V}$$

[c] Let I_c denote the magnitude of the current in the capacitor branch. Then

$$\mathbf{I} = (10 - j7.5 + jI_c) = 10 + j(I_c - 7.5)\text{ A}$$

$$\mathbf{V}_s = 240\underline{/\alpha} = 240 + (0.1 + j0.8)[10 + j(I_c - 7.5)]$$
$$= (247 - 0.8I_c) + j(7.25 + 0.1I_c)$$

It follows that

$$240\cos\alpha = (247 - 0.8I_c) \quad\text{and}\quad 240\sin\alpha = (7.25 + 0.1I_c)$$

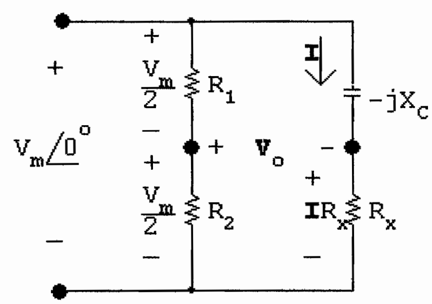
Now square each term and then add to generate the quadratic equation

$$I_c^2 - 605.77I_c + 5325.48 = 0; \qquad I_c = 302.88 \pm 293.96$$

Therefore

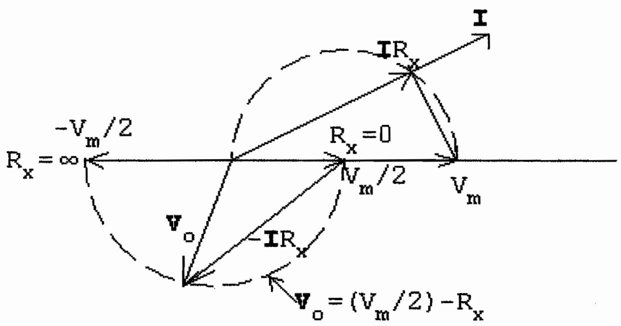
$I_c = 8.92\text{ A}$ (smallest value) and $Z_c = 240/j8.92 = -j26.90\,\Omega$.

P 9.82 The phasor domain equivalent circuit is

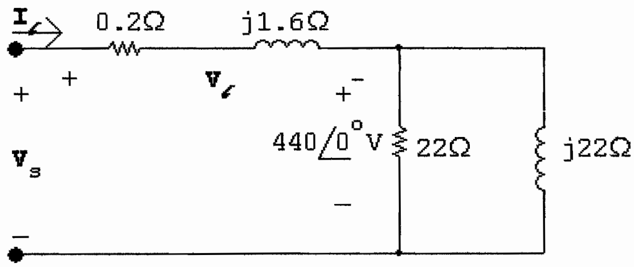


$$V_o = \frac{V_m}{2} - \mathbf{I}R_x; \qquad \mathbf{I} = \frac{V_m}{R_x - jX_C}$$

As R_x varies from 0 to ∞, the amplitude of v_o remains constant and its phase angle increases from 0° to −180°, as shown in the following phasor diagram:



P 9.83 [a]

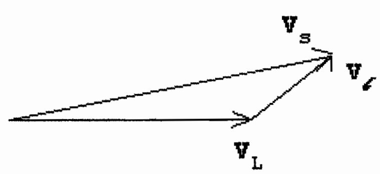


$$\mathbf{I}_\ell = \frac{440}{22} + \frac{440}{j22} = 20 - j20\,\text{A}$$

$$\mathbf{V}_\ell = (0.2 + j1.6)(20 - j20) = 36 + j28 = 45.61\underline{/37.87^\circ}\,\text{V(rms)}$$

$$\text{V}_s = 440\underline{/0^\circ} + \mathbf{V}_\ell = 476 + j28 = 476.82\underline{/3.37^\circ}\,\text{V}$$

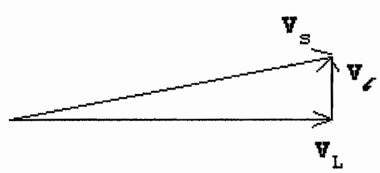
[b]



[c] $\mathbf{I}_\ell = \dfrac{440}{22} + \dfrac{440}{j22} + \dfrac{440}{-j22} = 20 + j0\,\text{A}$

$$\mathbf{V}_\ell = (0.2 + j1.6)(20 + j0) = 4 + j32 = 32.25\underline{/82.87^\circ}$$

$$\mathbf{V}_s = 440 + \mathbf{V}_\ell = 444 + j32 = 445.15\underline{/4.12^\circ}$$



P 9.84 [a] $\mathbf{I}_1 = \frac{120}{24} + \frac{240}{8.4 + j6.3} = 23.29 - j13.71 = 27.02\underline{/-30.5^\circ}\text{ A}$

$$\mathbf{I}_2 = \frac{120}{12} - \frac{120}{24} = 5\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_3 = \frac{120}{12} + \frac{240}{8.4 + j6.3} = 28.29 - j13.71 = 31.44\underline{/-25.87^\circ}\text{ A}$$

$$\mathbf{I}_4 = \frac{120}{24} = 5\underline{/0^\circ}\text{ A}; \qquad \mathbf{I}_5 = \frac{120}{12} = 10\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_6 = \frac{240}{8.4 + j6.3} = 18.29 - j13.71 = 22.86\underline{/-36.87^\circ}\text{ A}$$

[b] When fuse A is interrupted,

$\mathbf{I}_1 = 0$ $\qquad \mathbf{I}_3 = 15\text{ A}$ $\qquad \mathbf{I}_5 = 10\text{ A}$

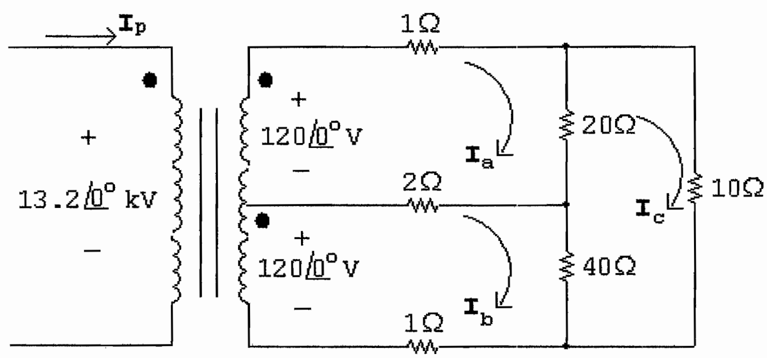
$\mathbf{I}_2 = 10 + 5 = 15\text{ A}$ $\qquad \mathbf{I}_4 = -5\text{ A}$ $\qquad \mathbf{I}_6 = 5\text{ A}$

[c] The clock and television set were fed from the uninterrupted side of the circuit, that is, the 12 Ω load includes the clock and the TV set.

[d] No, the motor current drops to 5 A, well below its normal running value of 22.86 A.

[e] After fuse A opens, the current in fuse B is only 15 A.

P 9.85 [a] The circuit is redrawn, with mesh currents identified:



The mesh current equations are:

$$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120\underline{/0^\circ} = -2\mathbf{I}_a + 43\mathbf{I}_b - 40\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 40\mathbf{I}_b + 70\mathbf{I}_c$$

Solving,

$\mathbf{I}_a = 24\underline{/0^\circ}\text{ A}$ $\qquad \mathbf{I}_b = 21.96\underline{/0^\circ}\text{ A}$ $\qquad \mathbf{I}_c = 19.40\underline{/0^\circ}\text{ A}$

The branch currents are:

$\mathbf{I}_1 = \mathbf{I}_a = 24\underline{/0^\circ}$ A

$\mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 2.04\underline{/0^\circ}$ A

$\mathbf{I}_3 = \mathbf{I}_b = 21.96\underline{/0^\circ}$ A

$\mathbf{I}_4 = \mathbf{I}_c = 19.40\underline{/0^\circ}$ A

$\mathbf{I}_5 = \mathbf{I}_a - \mathbf{I}_c = 4.6\underline{/0^\circ}$ A

$\mathbf{I}_6 = \mathbf{I}_b - \mathbf{I}_c = 2.55\underline{/0^\circ}$ A

[b] Let N_1 be the number of turns on the primary winding; because the secondary winding is center-tapped, let $2N_2$ be the total turns on the secondary. From Fig. 9.58,

$$\frac{13{,}200}{N_1} = \frac{240}{2N_2} \qquad \text{or} \qquad \frac{N_2}{N_1} = \frac{1}{110}$$

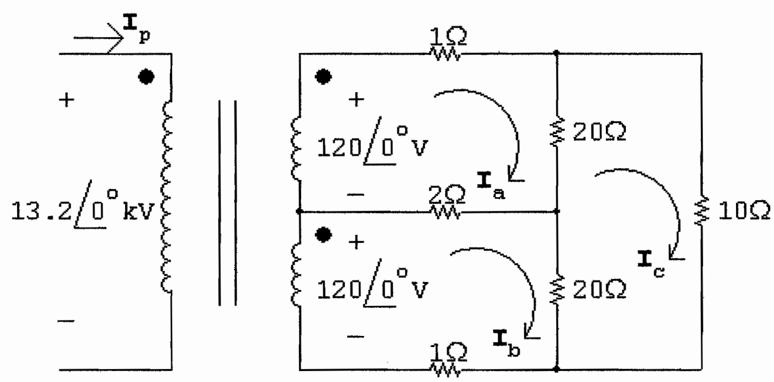
The ampere turn balance requires

$N_1\mathbf{I}_p = N_2\mathbf{I}_1 + N_2\mathbf{I}_3$

Therefore,

$$\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{1}{110}(24 + 21.96) = 0.42\underline{/0^\circ}\text{ A}$$

P 9.86 [a]



The three mesh current equations are

$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$

$120\underline{/0^\circ} = -2\mathbf{I}_a + 23\mathbf{I}_b - 20\mathbf{I}_c$

$0 = -20\mathbf{I}_a - 20\mathbf{I}_b + 50\mathbf{I}_c$

Solving,

$\mathbf{I}_a = 24\underline{/0^\circ}$ A; $\qquad \mathbf{I}_b = 24\underline{/0^\circ}$ A; $\qquad \mathbf{I}_c = 19.2\underline{/0^\circ}$ A

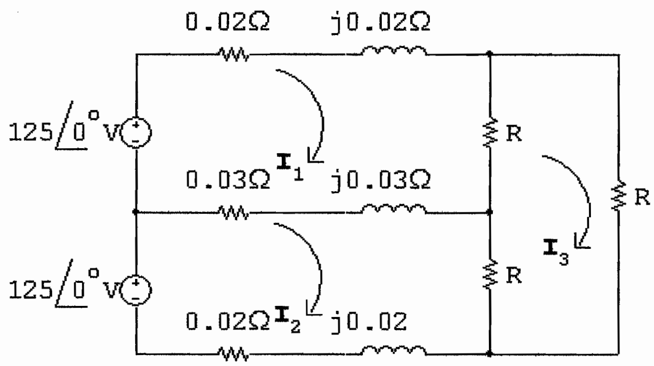
$\therefore\ \mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 0$ A

[b] $\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{N_2}{N_1}(\mathbf{I}_a + \mathbf{I}_b$

$= \frac{1}{110}(24 + 24) = 0.436\text{ A}$

[c] When the two loads are equal, more current is drawn from the primary.

P 9.87 [a]



$$125 = (R + 0.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - R\mathbf{I}_3$$

$$125 = -(0.03 + j0.03)\mathbf{I}_1 + (R + 0.05 + j0.05)\mathbf{I}_2 - R\mathbf{I}_3$$

Subtracting the above two equations gives

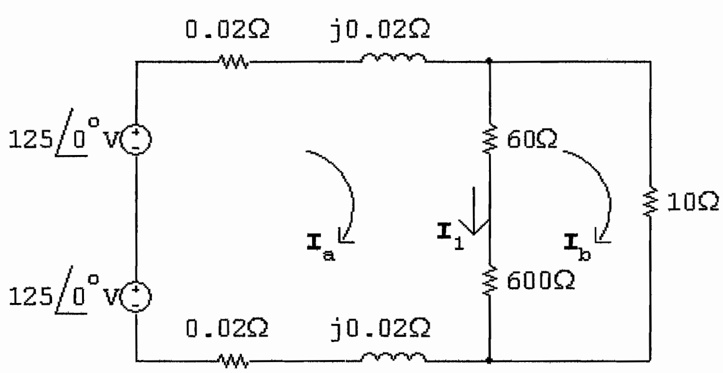
$$0 = (R + 0.08 + j0.08)\mathbf{I}_1 - (R + 0.08 + j0.08)\mathbf{I}_2$$

$\therefore\ \mathbf{I}_1 = \mathbf{I}_2$ so $\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = 0\text{ A}$

[b] $\mathbf{V}_1 = R(\mathbf{I}_1 - \mathbf{I}_3);\qquad \mathbf{V}_2 = R(\mathbf{I}_2 - \mathbf{I}_3)$

Since $\mathbf{I}_1 = \mathbf{I}_2$ (from part [a]) $\mathbf{V}_1 = \mathbf{V}_2$

[c]



$$250 = (660.04 + j0.04)\mathbf{I}_a - 660\mathbf{I}_b$$

$$0 = -660\mathbf{I}_a + 670\mathbf{I}_b$$

Solving,

$\mathbf{I}_a = 25.275945\underline{/-0.231714^\circ} = 25.275738 - j0.10222\,\text{A}$

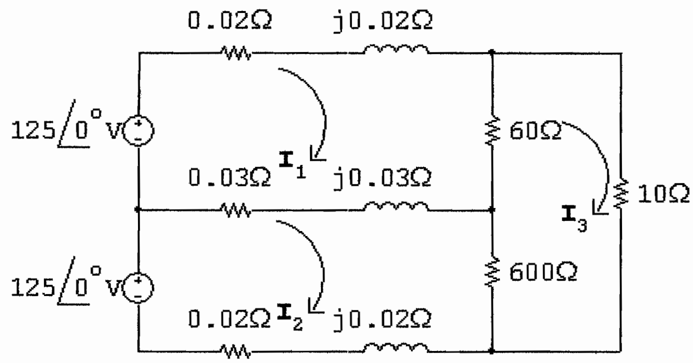
$\mathbf{I}_b = 24.898692\underline{/-0.231713^\circ} = 24.898488 - j0.100694\,\text{A}$

$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_b = 0.37725 - j0.001526\,\text{A}$

$\mathbf{V}_1 = 60\mathbf{I}_1 = 22.635 - j0.09156 = 22.635185\underline{/-0.231764^\circ}\,\text{V}$

$\mathbf{V}_2 = 600\mathbf{I}_1 = 226.35 - j0.9156 = 226.35185\underline{/-0.231764^\circ}\,\text{V}$

[d]



$125 = (60.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - 60\mathbf{I}_3$

$125 = -(0.03 + j0.03)\mathbf{I}_1 + (600.05 + j0.05)\mathbf{I}_2 - 600\mathbf{I}_3$

$0 = -60\mathbf{I}_1 - 600\mathbf{I}_2 + 670\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 26.97\underline{/-0.24^\circ} = 26.97 - j0.113\,\text{A}$

$\mathbf{I}_2 = 25.10\underline{/-0.24^\circ} = 25.10 - j0.104\,\text{A}$

$\mathbf{I}_3 = 24.90\underline{/-0.24^\circ} = 24.90 - j0.104\,\text{A}$

$\mathbf{V}_1 = 60(\mathbf{I}_1 - \mathbf{I}_3) = 124.4\underline{/-0.27^\circ}\,\text{V}$

$\mathbf{V}_2 = 600(\mathbf{I}_2 - \mathbf{I}_3) = 124.6\underline{/-0.20^\circ}\,\text{V}$

[e] Because an open neutral can result in severely unbalanced voltages across the 125 V loads.

P 9.88 [a] Let N_1 = primary winding turns and $2N_2$ = secondary winding turns. Then

$$\frac{14{,}000}{N_1} = \frac{250}{2N_2}; \qquad \therefore \quad \frac{N_2}{N_1} = \frac{1}{112} = a$$

In part c),

$\mathbf{I}_\mathrm{p} = 2a\mathbf{I}_\mathrm{a}$

$$\therefore \quad \mathbf{I}_\mathrm{p} = \frac{2N_2\mathbf{I}_\mathrm{a}}{N_1} = \tfrac{1}{56}\mathbf{I}_\mathrm{a}$$

$$= \frac{1}{56}(25.28 - j0.10)$$

$$\mathbf{I}_\mathrm{p} = 451.4 - j1.8\,\mathrm{mA}$$

In part d),

$\mathbf{I}_\mathrm{p}N_1 = \mathbf{I}_1N_2 + \mathbf{I}_2N_2$

$$\therefore \quad \mathbf{I}_\mathrm{p} = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_2)$$

$$= \frac{1}{112}(26.97 - j0.11 + 25.10 - j0.10)$$

$$= \frac{1}{112}(52.07 - j0.22)$$

$$\mathbf{I}_\mathrm{p} = 464.9 - j1.9\,\mathrm{mA}$$

[b] Yes, because the neutral conductor carries non-zero current whenever the load is not balanced.

10

Sinusoidal Steady State Power Calculations

Assessment Problems

AP 10.1 [a] $\mathbf{V} = 100\underline{/-45^\circ}\,\text{V}, \qquad \mathbf{I} = 20\underline{/15^\circ}\,\text{A}$

Therefore

$$P = \frac{1}{2}(100)(20)\cos[-45 - (15)] = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin -60^\circ = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/165^\circ}$

$$P = 1000\cos(-210^\circ) = -866.03\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-210^\circ) = 500\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/-105^\circ}$

$$P = 1000\cos(60^\circ) = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin(60^\circ) = 866.03\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[d] $\mathbf{V} = 100\underline{/0^\circ}, \qquad \mathbf{I} = 20\underline{/120^\circ}$

$$P = 1000\cos(-120^\circ) = -500\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-120^\circ) = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

AP 10.2

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos[15 - (75)] = \cos(-60^\circ) = 0.5 \text{ leading}$$

$$\text{rf} = \sin(\theta_v - \theta_i) = \sin(-60^\circ) = -0.866$$

AP 10.3

From Ex. 9.4 $I_{\text{eff}} = \dfrac{I_\rho}{\sqrt{3}} = \dfrac{0.18}{\sqrt{3}}$ A

$$P = I_{\text{eff}}^2 R = \left(\frac{0.0324}{3}\right)(5000) = 54\,\text{W}$$

AP 10.4 [a] $Z = (39 + j26)\|(-j52) = 48 - j20 = 52\underline{/-22.62^\circ}\,\Omega$

$$\text{Therefore}\quad \mathbf{I}_\ell = \frac{250\underline{/0^\circ}}{48 - j20 + 1 + j4} = 4.85\underline{/18.08^\circ}\,\text{A(rms)}$$

$$\mathbf{V}_\text{L} = Z\mathbf{I}_\ell = (52\underline{/-22.62^\circ})(4.85\underline{/18.08^\circ}) = 252.20\underline{/-4.54^\circ}\,\text{V(rms)}$$

$$\mathbf{I}_\text{L} = \frac{\mathbf{V}_\text{L}}{39 + j26} = 5.38\underline{/-38.23^\circ}\,\text{A(rms)}$$

[b] $S_\text{L} = \mathbf{V}_L\mathbf{I}_L^* = (252.20\underline{/-4.54^\circ})(5.38\underline{/+38.23^\circ}) = 1357\underline{/33.69^\circ}$

$= (1129.09 + j752.73)$ VA

$P_\text{L} = 1129.09$ W; $\quad Q_\text{L} = 752.73$ VAR

[c] $P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52$ W; $\quad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09$ VAR

[d] $S_g(\text{delivering}) = 250\mathbf{I}_\ell^* = (1152.62 - j376.36)$ VA

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

[e] $Q_\text{cap} = \dfrac{|\mathbf{V}_\text{L}|^2}{-52} = \dfrac{(252.20)^2}{-52} = -1223.18$ VAR

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

Check: $\quad 94.09 + 752.73 + 376.36 = 1223.18$ VAR $\quad$ and

$1129.09 + 23.52 = 1152.62$ W

AP 10.5 Series circuit derivation:

$$S = 250\mathbf{I}^* = (40{,}000 - j30{,}000)$$

$$\text{Therefore}\quad \mathbf{I}^* = 160 - j120 = 200\underline{/-36.87^\circ}\,\text{A(rms)}$$

$$\mathbf{I} = 200\underline{/36.87^\circ}\,\text{A(rms)}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{250}{200\underline{/36.87^\circ}} = 1.25\underline{/-36.87^\circ} = (1 - j0.75)\,\Omega$$

Therefore $\quad R = 1\,\Omega, \qquad X_\text{C} = -0.75\,\Omega$

Parallel circuit derivation

$$P = \frac{(250)^2}{R}; \qquad \text{therefore} \quad R = \frac{(250)^2}{40{,}000} = 1.5625\,\Omega$$

$$Q = \frac{(250)^2}{X_C}; \qquad \text{therefore} \quad X_C = \frac{(250)^2}{-30{,}000} = -2.083\,\Omega$$

AP 10.6

$$S_1 = 15{,}000(0.6) + j15{,}000(0.8) = 9000 + j12{,}000\,\text{VA}$$

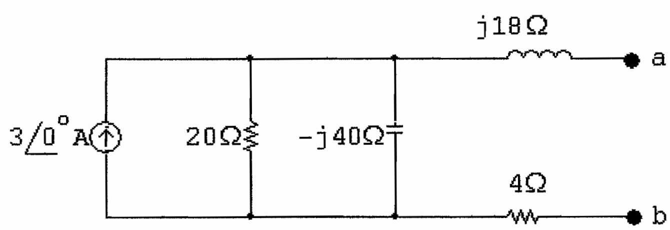
$$S_2 = 6000(0.8) - j6000(0.6) = 4800 - j3600\,\text{VA}$$

$$S_T = S_1 + S_2 = 13{,}800 + j8400\,\text{VA}$$

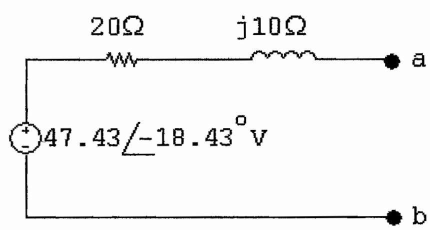
$$S_T = 200\mathbf{I}^*; \qquad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \qquad \mathbf{I} = 69 - j42\,\text{A}$$

$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64\underline{/15.91^\circ}\,\text{V(rms)}$$

AP 10.7 [a] The phasor domain equivalent circuit and the Thévenin equivalent are shown below:
Phasor domain equivalent circuit:



Thévenin equivalent:



$$\mathbf{V}_{\text{Th}} = 3\frac{-j800}{20 - j40} = 48 - j24 = 53.67\underline{/-26.57^\circ}\,\text{V}$$

$$Z_{\text{Th}} = 4 + j18 + \frac{-j800}{20 - j40} = 20 + j10 = 22.36\underline{/26.57^\circ}\,\Omega$$

For maximum power transfer, $Z_L = (20 - j10)\,\Omega$

[b] $\mathbf{I} = \dfrac{53.67\underline{/-26.57^\circ}}{40} = 1.34\underline{/-26.57^\circ}\,\text{A}$

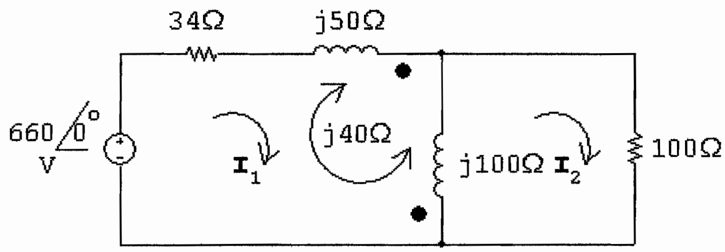
$$\text{Therefore}\quad P = \left(\frac{1.34}{\sqrt{2}}\right)^2 20 = 17.96\,\text{W}$$

[c] $R_L = |Z_{Th}| = 22.36\,\Omega$

[d] $\mathbf{I} = \dfrac{53.67\underline{/-26.57^\circ}}{42.36 + j10} = 1.23\underline{/-39.85^\circ}\,\text{A}$

$$\text{Therefore}\quad P = \left(\frac{1.23}{\sqrt{2}}\right)^2 (22.36) = 17\,\text{W}$$

AP 10.8



Mesh current equations:

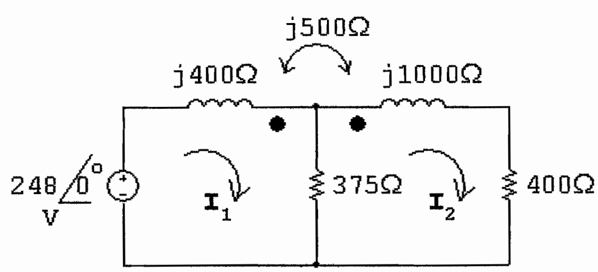
$$660 = (34 + j50)\mathbf{I}_1 + j100(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = j100(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 + 100\mathbf{I}_2$$

Solving,

$$\mathbf{I}_2 = 3.5\underline{/0^\circ}\,\text{A}; \qquad \therefore\quad P = \frac{1}{2}(3.5)^2(100) = 612.50\,\text{W}$$

AP 10.9 [a]



$$248 = j400\mathbf{I}_1 - j500\mathbf{I}_2 + 375(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = 375(\mathbf{I}_2 - \mathbf{I}_1) + j1000\mathbf{I}_2 - j500\mathbf{I}_1 + 400\mathbf{I}_2$$

Solving,

$$\mathbf{I}_1 = 0.80 - j0.62\,\text{A}; \qquad \mathbf{I}_2 = 0.4 - j0.3 = 0.5\underline{/-36.87^\circ}$$

$$\therefore\quad P = \frac{1}{2}(0.25)(400) = 50\,\text{W}$$

[b] $\mathbf{I}_1 - \mathbf{I}_2 = 0.4 - j0.32\,\mathrm{A}$

$$P_{375} = \frac{1}{2}|\mathbf{I}_1 - \mathbf{I}_2|^2(375) = 49.20\,\mathrm{W}$$

[c] $P_g = \frac{1}{2}(248)(0.8) = 99.20\,\mathrm{W}$

$$\sum P_{\mathrm{abs}} = 50 + 49.2 = 99.20\,\mathrm{W} \quad \text{(checks)}$$

AP 10.10 [a] $V_{\mathrm{Th}} = 210\,\mathrm{V}; \qquad \mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_1 = \frac{1}{4}\mathbf{I}_2$

Short circuit equations:

$$840 = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = 14\,\mathrm{A}; \qquad R_{\mathrm{Th}} = \frac{210}{14} = 15\,\Omega$$

[b] $P_{\max} = \left(\frac{210}{30}\right)^2 15 = 735\,\mathrm{W}$

AP 10.11 [a] $\mathbf{V}_{\mathrm{Th}} = -4(146\underline{/0^\circ}) = -584\underline{/0^\circ}\,\mathrm{V(rms)}$

$$\mathbf{V}_2 = 4\mathbf{V}_1; \qquad \mathbf{I}_1 = -4\mathbf{I}_2$$

Short circuit equations:

$$146\underline{/0^\circ} = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

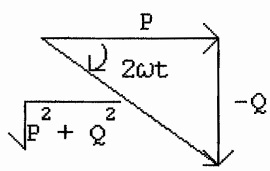
$$\therefore \quad \mathbf{I}_2 = -146/365 = -0.40\,\mathrm{A}; \qquad R_{\mathrm{Th}} = \frac{-584}{-0.4} = 1460\,\Omega$$

[b] $P = \left(\frac{-584}{2920}\right)^2 1460 = 58.40\,\mathrm{W}$

Problems

P 10.1 $p = P + P\cos 2\omega t - Q\sin 2\omega t;\qquad \frac{dp}{dt} = -2\omega P\sin 2\omega t - 2\omega Q\cos 2\omega t$

$$\frac{dp}{dt} = 0 \quad \text{when} \quad -2\omega P\sin 2\omega t = 2\omega Q\cos 2\omega t \quad \text{or} \quad \tan 2\omega t = -\frac{Q}{P}$$



$$\cos 2\omega t = \frac{P}{\sqrt{P^2+Q^2}};\qquad \sin 2\omega t = -\frac{Q}{\sqrt{P^2+Q^2}}$$

Let $\theta = \tan^{-1}(-Q/P)$, then p is maximum when $2\omega t = \theta$ and p is minimum when $2\omega t = (\theta + \pi)$.

$$\text{Therefore}\quad p_{\text{max}} = P + P\cdot\frac{P}{\sqrt{P^2+Q^2}} - \frac{Q(-Q)}{\sqrt{P^2+Q^2}} = P + \sqrt{P^2+Q^2}$$

$$\text{and}\quad p_{\text{min}} = P - P\cdot\frac{P}{\sqrt{P^2+Q^2}} - Q\cdot\frac{Q}{\sqrt{P^2+Q^2}} = P - \sqrt{P^2+Q^2}$$

P 10.2 [a] $P = \frac{1}{2}(340)(20)\cos(60-15) = 3400\cos 45^\circ = 2404.16\ \text{W}\quad \text{(abs)}$

$Q = 3400\sin 45^\circ = 2404.16\ \text{VAR}\quad \text{(abs)}$

[b] $P = \frac{1}{2}(16)(75)\cos(-15-60) = 600\cos(-75^\circ) = 155.29\ \text{W}\quad \text{(abs)}$

$Q = 600\sin(-75^\circ) = -579.56\ \text{VAR}\quad \text{(del)}$

[c] $P = \frac{1}{2}(625)(4)\cos(40-150) = 1250\cos(-110^\circ) = -427.53\ \text{W}\quad \text{(del)}$

$Q = 1250\sin(-110^\circ) = -1174.62\ \text{VAR}\quad \text{(del)}$

[d] $P = \frac{1}{2}(180)(10)\cos(130-20) = 900\cos(110^\circ) = -307.82\ \text{W}\quad \text{(del)}$

$Q = 900\sin(110^\circ) = 845.72\ \text{VAR}\quad \text{(abs)}$

P 10.3 [a] coffee maker = 1200 W radio = 71 W

television = 145 W portable heater = 1322 W

$\sum P = 2738$ W

Therefore $I_{\text{eff}} = \dfrac{2738}{120} = 22.82$ A

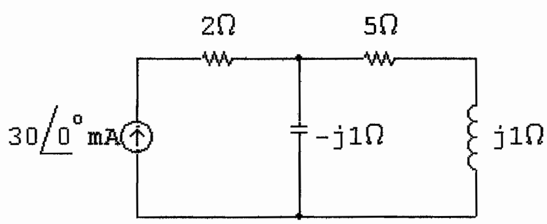
Yes, the breaker will trip.

[b] $\sum P = 2738 - 1200 = 1538$ W; $I_{\text{eff}} = \dfrac{1538}{120} = 12.82$ A

Yes, the breaker will not trip if the current is reduced to 12.82 A.

P 10.4 $\mathbf{I}_g = 30\underline{/0^\circ}$ mA; $\dfrac{1}{j\omega C} = \dfrac{10^6}{j(25 \times 10^3)(40)} = -j1\,\Omega$

$$j\omega L = j(25 \times 10^3)(40) \times 10^{-6} = j1\,\Omega$$



$$Z_1 = -j1\|(5 + j1) = 0.2 - j1\,\Omega$$

$$Z_{\text{eq}} = 2 + Z_1 = 2.2 - j1\,\Omega$$

$$P_g = |I_{\text{rms}}|^2\text{Re}\{Z_{\text{eq}}\} = \left(\frac{30}{\sqrt{2}} \times 10^{-3}\right)^2 (2.2) = 990\,\mu\text{W}$$

P 10.5 $\dfrac{1}{\omega C} = \dfrac{10^9}{(5000)(80)} = 2500\,\Omega$

$$Z_{\text{f}} = \frac{-j2500(7500)}{7500 - j2500} = 750 - j2250\,\Omega$$

$$Z_{\text{i}} = 1500\,\Omega$$

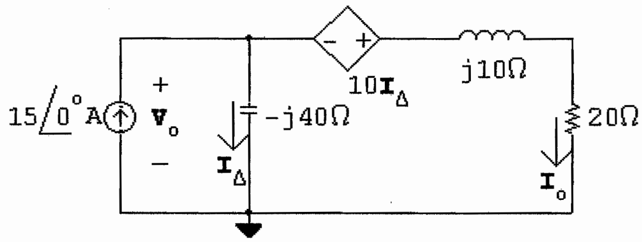
$$\therefore \quad \frac{Z_{\text{f}}}{Z_{\text{i}}} = \frac{750 - j2250}{1500} = 0.5 - j1.5$$

$$\mathbf{V}_o = -\frac{Z_{\text{f}}}{Z_{\text{i}}}\mathbf{V}_g; \qquad \mathbf{V}_g = 4\underline{/0^\circ}\text{ V}$$

$$\mathbf{V}_o = (-0.5 + j1.5)(4) = -2 + j6 = 6.32\underline{/108.43^\circ}\,\text{V}$$

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\frac{(4)(10)}{1000} = 20 \times 10^{-3} = 20\,\text{mW}$$

P 10.6 $j\omega L = j10{,}000(10^{-3}) = j10\,\Omega;\qquad \frac{1}{j\omega C} = \frac{10^6}{j10{,}000(2.5)} = -j40\,\Omega$



$$-15 + \frac{\mathbf{V}_o}{-j40} + \frac{\mathbf{V}_o + 10(\mathbf{V}_o/-j40)}{20 + j10} = 0$$

$$\therefore\quad \mathbf{V}_o\left[\frac{1}{-j40} + \frac{1 + j0.25}{20 + j10}\right] = 15$$

$$\therefore\quad \mathbf{V}_o = 300 - j100\,\text{V}$$

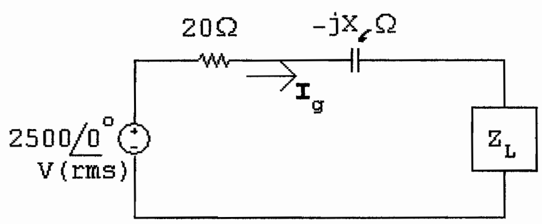
$$\therefore\quad \mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j40} = 2.5 + j7.5\,\text{A}$$

$$\mathbf{I}_o = 15\underline{/0^\circ} - \mathbf{I}_\Delta = 15 - 2.5 - j7.5 = 12.5 - j7.5 = 14.58\underline{/-30.9^\circ}\,\text{A}$$

$$P_{20\Omega} = \frac{1}{2}|\mathbf{I}_o|^2 20 = 2125\,\text{W}$$

P 10.7 [a] line loss $= 50{,}000 - 40{,}000 = 10\,\text{kW}$

line loss $= |\mathbf{I}_g|^2 20 \qquad \therefore\quad |\mathbf{I}_g|^2 = 500$

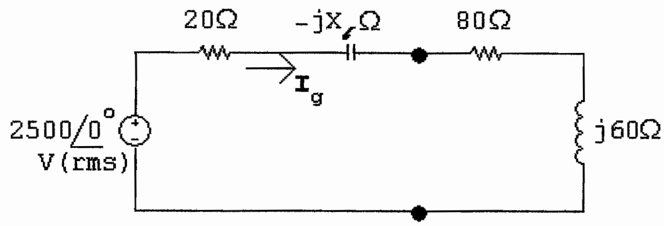


$$|\mathbf{I}_g| = \sqrt{500}\,\text{A}$$

$$|\mathbf{I}_g|^2 R_\text{L} = 40{,}000 \qquad \therefore\quad R_\text{L} = 80\,\Omega$$

$|\mathbf{I}_g|^2 X_L = 30{,}000 \qquad \therefore \quad X_L = 60\,\Omega$

Thus,



$$|Z| = \sqrt{(100)^2 + (60 - X_\ell)^2} \qquad |\mathbf{I}_g| = \frac{2500}{\sqrt{10{,}000 + (60 - X_\ell)^2}}$$

$$\therefore \quad 10{,}000 + (60 - X_\ell)^2 = \frac{625 \times 10^4}{500} = 12{,}500$$

Solving, $\qquad (60 - X_\ell) = \pm 50.$

Thus, $\quad X_\ell = 10\,\Omega \qquad$ or $\qquad X_\ell = 110\,\Omega$

[b] If $X_\ell = 10\,\Omega$:

$$\mathbf{I}_g = \frac{2500}{100 + j50} = 20 - j10\,\text{A}$$

$$S_g = -2500\mathbf{I}_g^* = -50 - j25\,\text{kVA}$$

Thus, the voltage source is delivering 50 KW and 25 magnetizing Kvars.

$$Q_{-j10} = |\mathbf{I}_g|^2 X_\ell = 500(-10) = -5000\,\text{VAR}$$

Therefore the line reactance is generating 5 magnetizing kvars.

$$Q_{j60} = |\mathbf{I}_g|^2 X_L = 500(60) = 30{,}000\,\text{VAR}$$

Therefore the load reactance is absorbing 30 magnetizing kvars.

$$\sum Q_{\text{gen}} = 25{,}000\,\text{kVAR} = \sum Q_{\text{abs}}$$

If $X_\ell = 110\,\Omega$:

$$\mathbf{I}_g = \frac{2500}{100 - j50} = 20 + j10\,\text{A}$$

$$S_g = -2500\mathbf{I}_g^* = -50 + j25\,\text{kVA}$$

Thus, the voltage source is delivering 50 kW and absorbing 25 magnetizing kvars.

$$Q_{-j110} = |\mathbf{I}_g|^2(-110) = 500(-110) = -55\,\text{kVAR}$$

Therefore the line reactance is generating 55 magnetizing kvars. The load continues to absorb 30 magnetizing kvars.

$$\sum Q_{\text{gen}} = 55\,\text{kVAR} = \sum Q_{\text{abs}}$$

P 10.8 [a] $P = \frac{1}{2}\frac{(90)^2}{1350} = 3\,\text{W}$

$$Q = \frac{1}{2}\frac{(90)^2}{(1012.5)} = 4\,\text{VAR}$$

$$p_{\max} = P + \sqrt{P^2 + Q^2} = 3 + \sqrt{(3)^2 + (4)^2} = 8\,\text{W(del)}$$

[b] $p_{\min} = 3 - 5 = -2\,\text{W(abs)}$

[c] $P = 4\,\text{W}$ from (a)

[d] $Q = 4\,\text{VAR}$ from (a)

[e] absorb, because $Q > 0$

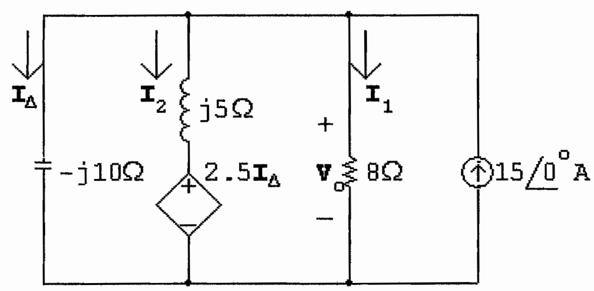
[f] pf $= \cos(\theta_v - \theta_i)$

$$\mathbf{I} = \frac{90}{1350} + \frac{90}{j1012.5} = 0.0667 - j0.08889 = 111.11\underline{/-53.13^\circ}\,\text{mA}$$

$$\therefore \quad \text{pf} = \cos(0 + 53.13^\circ) = 0.6 \text{ lagging}$$

[g] rf $= \sin(53.13^\circ) = 0.8$

P 10.9 [a] From the solution to Problem 9.56 we have:



$$\mathbf{V}_o = 72 + j96 = 120\underline{/53.13^\circ}\,\text{V}$$

$$S_g = -\frac{1}{2}\mathbf{V}_o\mathbf{I}_g^* = -\frac{1}{2}(72 + j96)(15) = -540 - j720\,\text{VA}$$

Therefore, the independent current source is delivering 540 W and 720 magnetizing vars.

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{8} = 15\underline{/53.13^\circ}\,\text{A}$$

$$P_{8\Omega} = \frac{1}{2}(15)^2(8) = 900\,\text{W}$$

Therefore, the $8\,\Omega$ resistor is absorbing 900 W.

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j10} = -9.6 + j7.2 = 12\underline{/143.13^\circ}\,\text{A}$$

$$Q_{\text{cap}} = \frac{1}{2}(12)^2(-10) = -720\,\text{VAR}$$

Therefore, the $-j10\,\Omega$ capacitor is delivering 720 magnetizing vars.

$$2.5\mathbf{I}_\Delta = -24 + j18\,\text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o - 2.5\mathbf{I}_\Delta}{j5} = \frac{72 + j96 + 24 - j18}{j5}$$

$$= 15.6 - j19.2\,\text{A} = 24.72\underline{/-50.91^\circ}\,\text{A}$$

$$Q_{j5} = \frac{1}{2}|\mathbf{I}_2|^2(5) = 1530\,\text{VAR}$$

Therefore, the $j5\,\Omega$ inductor is absorbing 1530 magnetizing vars.

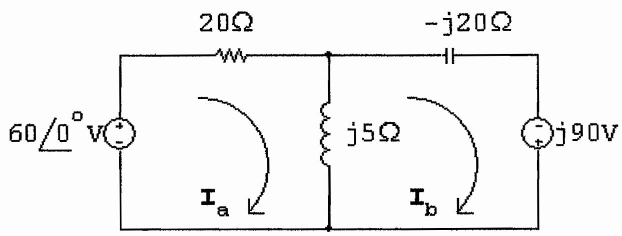
$$S_{2.5\mathbf{I}_\Delta} = \tfrac{1}{2}(2.5\mathbf{I}_\Delta)\mathbf{I}_2^* = \tfrac{1}{2}(-24 + j18)(15.6 + j19.2)$$
$$= -360 - j90\,\text{VA}$$

Thus the dependent source is delivering 360 W and 90 magnetizing vars.

[b] $\sum P_{\text{gen}} = 360 + 540 = 900\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{gen}} = 720 + 90 + 720 = 1530\,\text{VAR} = \sum Q_{\text{abs}}$

P 10.10 [a] From the solution to Problem 9.57 we have



$\mathbf{I}_a = 2.25 - j2.25\,\text{A}$; $\mathbf{I}_b = -6.75 + j0.75\,\text{A}$; $\mathbf{I}_o = 9 - j3\,\text{A}$

$$S_{60V} = -\frac{1}{2}(60)\mathbf{I}_a^* = -30(2.25 + j2.25) = -67.5 - j67.5\,\text{VA}$$

Thus, the 60 V source is developing 67.5 W and 67.5 magnetizing vars.

$$S_{90V} = -\tfrac{1}{2}(j90)\mathbf{I}_b^* = -j45(-6.75 - j0.75)$$
$$= -33.75 + j303.75\,\text{VA}$$

Thus, the 90 V source is delivering 33.75 W and absorbing 303.75 magnetizing vars.

$$P_{20\Omega} = \frac{1}{2}|\mathbf{I}_a|^2(20) = 101.25\,\text{W}$$

Thus the $20\,\Omega$ resistor is absorbing 101.25 W.

$$Q_{-j20\Omega} = \frac{1}{2}|\mathbf{I}_b|^2(-20) = -461.25\,\text{VAR}$$

Thus the $-j20\,\Omega$ capacitor is developing 461.25 magnetizing vars.

$$Q_{j5\Omega} = \frac{1}{2}|\mathbf{I}_o|^2(5) = 225\,\text{VAR}$$

Thus the $j5\,\Omega$ inductor is absorbing 225 magnetizing vars.

[b] $\sum P_{\text{dev}} = 67.5 + 33.75 = 101.25\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{dev}} = 67.5 + 461.25 = 528.75\,\text{VAR}$

$$\sum Q_{\text{abs}} = 225 + 303.75 = 528.75\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.11 $W_{\text{dc}} = \dfrac{V_{\text{dc}}^2}{R}T; \qquad W_s = \displaystyle\int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$

$$\therefore \quad \frac{V_{\text{dc}}^2}{R}T = \int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$$

$$V_{\text{dc}}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt$$

$$V_{\text{dc}} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt} = V_{\text{rms}} = V_{\text{eff}}$$

P 10.12 [a] $I_{\text{eff}} = 60/110 \cong 0.545\,\text{A}$; [b] $I_{\text{eff}} = (60+80)/110 \cong 1.273\,\text{A}$

P 10.13 [a] Area under one cycle of v_g^2:

$$A = (400)(4)(20 \times 10^{-6}) + 10{,}000(2)(20 \times 10^{-6})$$
$$= 21{,}600(20 \times 10^{-6})$$

Mean value of v_g^2:

$$\text{M.V.} = \frac{A}{120 \times 10^{-6}} = \frac{21{,}600(20 \times 10^{-6})}{120 \times 10^{-6}} = 3600$$

$$\therefore \quad V_{\text{rms}} = \sqrt{3600} = 60\,\text{V(rms)}$$

[b] $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{3600}{12} = 300\,\text{W}$

P 10.14 $i(t) = \frac{30}{40} \times 10^3 t = 750t \qquad 0 \le t \le 40\,\text{ms}$

$$i(t) = M - \frac{30}{10} \times 10^3 t \qquad 40\,\text{ms} \le t \le 50\,\text{ms}$$

$$i(t) = 0 \text{ when } t = 50\,\text{ms}$$

$$\therefore \quad M = 3000(50 \times 10^{-3}) = 150$$

$$i(t) = 150 - 3000t \qquad 40\,\text{ms} \le t \le 50\,\text{ms}$$

$$\therefore \quad I_{\text{rms}} = \sqrt{\frac{1000}{50}\left\{\int_0^{0.04} (750)^2 t^2\,dt + \int_{0.04}^{0.05} (150 - 3000t)^2\,dt\right\}}$$

$$\int_0^{0.04} (750)^2 t^2\,dt = (750)^2 \frac{t^3}{3}\Big|_0^{0.04} = 12$$

$$(150 - 3000t)^2 = 22{,}500 - 9 \times 10^5 t + 9 \times 10^6 t^2$$

$$\int_{0.04}^{0.05} 22{,}500\,dt = 225$$

$$\int_{0.04}^{0.05} 9 \times 10^5 t\,dt = 45 \times 10^4 t^2 \Big|_{0.04}^{0.05} = 405$$

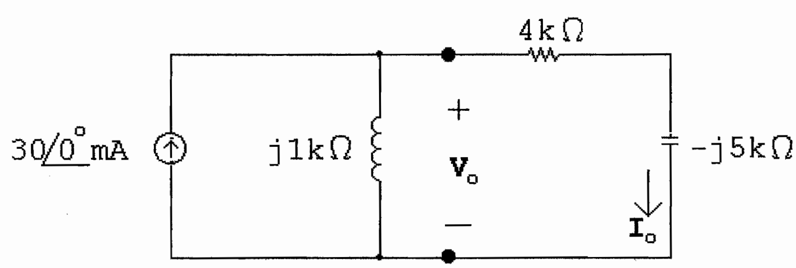
$$9 \times 10^6 \int_{0.04}^{0.05} t^2\,dt = 3 \times 10^6 t^3 \Big|_{0.04}^{0.05} = 183$$

$$\therefore \quad I_{\text{rms}} = \sqrt{20\{12 + (225 - 405 + 183)\}} = \sqrt{300} = 17.32\,\text{A}$$

P 10.15 $P = I_{\text{rms}}^2 R \qquad \therefore \quad R = \frac{24 \times 10^3}{300} = 80\,\Omega$

P 10.16 $\mathbf{I}_g = 30\underline{/0^\circ}\,\text{mA}$

$$j\omega L = j(100)(10) = j1000\,\Omega; \qquad \frac{1}{j\omega C} = \frac{10^6}{j(100)(2)} = -j5000\,\Omega$$



$$\mathbf{I}_o = \frac{30\underline{/0^\circ}(j1000)}{4000 - j4000} = 3.75\sqrt{2}\underline{/135^\circ}\,\text{mA}$$

$$P = |\mathbf{I}_o|^2_{\text{rms}}(4000) = (3.75)^2(4000) = 56.25\,\text{mW}$$

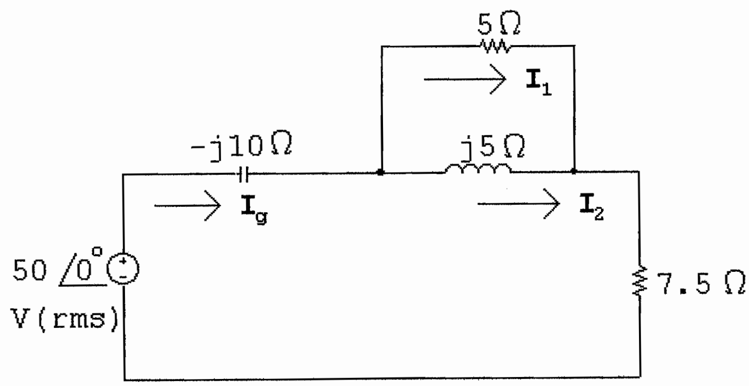
$$Q = |\mathbf{I}_o|^2_{\text{rms}}(-5000) = -70.3125\,\text{mVAR}$$

$$S = P + jQ = 56.25 - j70.3125\,\text{mVA}$$

$$|S| = 90.044\,\text{mVA}$$

P 10.17 [a] $\dfrac{1}{j\omega C} = \dfrac{10^6}{j10^5} = -j10\,\Omega$

$$j\omega L = j10^5(50\times10^{-6}) = j5\,\Omega$$



$$Z = -j10 + \frac{(5)(j5)}{5+j5} + 7.5 = 10 - j7.5\,\Omega$$

$$\mathbf{I}_g = \frac{50/0^\circ}{10 - j7.5} = 3.2 + j2.4\,\text{A}$$

$$S_g = -\frac{1}{2}\mathbf{V}_g\mathbf{I}_g^* = -25(3.2 - j2.4) = -80 + j60\,\text{VA}$$

$P = 80\,\text{W(abs)}$; $\qquad Q = 60\,\text{VAR(del)}$

$$|S| = |S_g| = 100\,\text{VA}$$

[b] $\mathbf{I}_1 = \dfrac{\mathbf{I}_g(j5)}{5+j5} = \dfrac{1}{2}(3.2 + j2.4)(1 + j1) = 0.4 + j2.8\,\text{A}$

$$P_{5\Omega} = \frac{1}{2}|\mathbf{I}_1|^2(5) = 20\,\text{W}$$

$$P_{7.5\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(7.5) = 60\,\text{W}$$

$$\sum P_{\text{diss}} = 20 + 60 = 80\,\text{W} = \sum P_{\text{dev}}$$

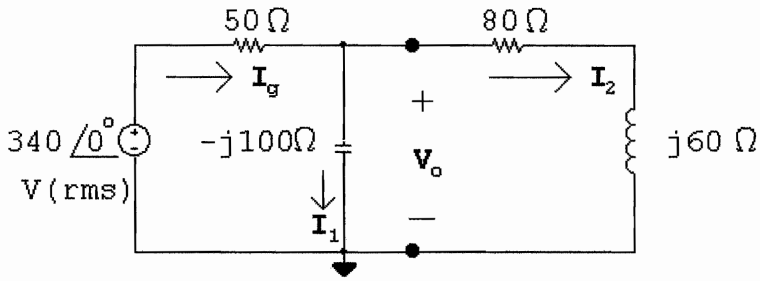
[c] $\mathbf{I}_{j5} = \dfrac{\mathbf{I}_g 5}{5 + j5} = \dfrac{1}{2}(3.2 + j2.4)(1 - j1) = 2.8 - j0.4\,\text{A}$

$$Q_{j5\Omega} = \frac{1}{2}|\mathbf{I}_{j5}|^2(5) = 20\,\text{VAR(abs)}$$

$$Q_{-j10\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(-10) = -80\,\text{VAR(dev)}$$

$$\sum Q_{\text{abs}} = 20 + 60 = 80\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.18 [a]



$$\frac{\mathbf{V}_o}{-j100} + \frac{\mathbf{V}_o - 340}{50} + \frac{\mathbf{V}_o}{80 + j60} = 0$$

$$\therefore\ \mathbf{V}_o = 238 - j34\,\text{V}$$

$$\mathbf{I}_g = \frac{340 - 238 + j34}{50} = 2.04 + j0.68\,\text{A}$$

$$S_g = \mathbf{V}_g\mathbf{I}_g^* = (340)(2.04 - j0.68)$$
$$= 693.6 - j231.2\,\text{VA}$$

[b] Source is delivering 693.6 W.

[c] Source is absorbing 231.2 magnetizing VAR.

[d] $\mathbf{I}_1 = \dfrac{\mathbf{V}_o}{-j100} = 0.34 + j2.38\,\text{A}$

$$S_1 = \mathbf{V}_o\mathbf{I}_1^* = (238 - j34)(0.34 - j2.38)$$
$$= 0 - j578\,\text{VA}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o}{80 + j60} = \frac{238 - j34}{80 + j60} = 1.7 - j1.7\,\text{A}$$

$$S_2 = \mathbf{V}_o\mathbf{I}_2^* = (238 - j34)(1.7 + j1.7)$$
$$= 462.4 + j346.8\,\text{VA}$$

$$S_{50\Omega} = |\mathbf{I}_g|^2(50) + j0 = (2.15)^2(50) = 231.2\,\text{W}$$

[e] $\sum P_{\text{del}} = 693.6\,\text{W}$

$\sum P_{\text{diss}} = 462.4 + 231.2 = 693.6\,\text{W}$

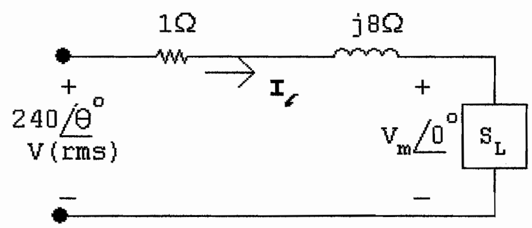
$\therefore \quad \sum P_{\text{del}} = \sum P_{\text{diss}} = 693.6\,\text{W}$

[f] $\sum Q_{\text{abs}} = 231.2 + 346.8 = 578\,\text{VAR}$

$\sum Q_{\text{dev}} = 578\,\text{VAR}$

$\therefore \quad \sum \text{ mag VAR dev } = \sum \text{ mag VAR abs } = 578$

P 10.19 [a] Let $\mathbf{V}_{\text{L}} = V_m\underline{/0^\circ}$:



$S_{\text{L}} = 250(0.6 + j0.8) = 150 + j200\,\text{VA}$

$\mathbf{I}_\ell^* = \dfrac{150}{V_m} + j\dfrac{200}{V_m}; \qquad \mathbf{I}_\ell = \dfrac{150}{V_m} - j\dfrac{200}{V_m}$

$240\underline{/\theta} = V_m + \left(\dfrac{150}{V_m} - j\dfrac{200}{V_m}\right)(1 + j8)$

$240V_m\underline{/\theta} = V_m^2 + (150 - j200)(1 + j8) = V_m^2 + 1750 + j1000$

$240V_m \cos\theta = V_m^2 + 1750; \qquad 240V_m \sin\theta = 1000$

$(240)^2V_m^2 = (V_m^2 + 1750)^2 + 1000^2$

$57{,}600V_m^2 = V_m^4 + 3500V_m^2 + (3.0625 + 1) \times 10^6$

or

$V_m^4 - 54{,}100V_m^2 + 4{,}062{,}500 = 0$

Solving,

$V_m^2 = 27{,}050 \pm 26{,}974.8; \qquad V_m = 232.43\,\text{V}$ and $V_m = 8.67\,\text{V}$

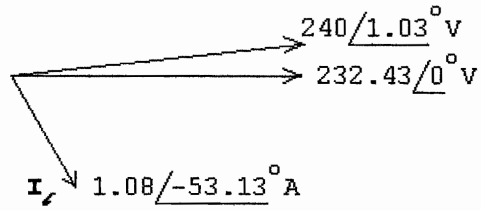
If $V_m = 232.43$ V:

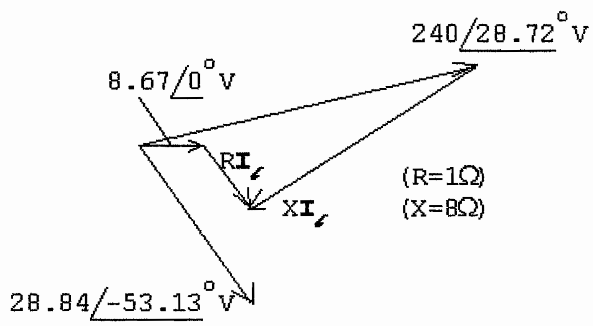
$\sin\theta = \dfrac{1000}{(232.43)(240)} = 0.0179; \qquad \therefore \quad \theta = 1.03^\circ$

If $V_m = 8.67$ V:

$\sin\theta = \dfrac{1000}{(8.67)(240)} = 0.4805; \qquad \therefore \quad \theta = 28.72^\circ$

[b]





P 10.20 $S_T = 52{,}800 - j\frac{52{,}800}{0.8}(0.6) = 52{,}800 - j39{,}600\,\text{VA}$

$S_1 = 40{,}000(0.96 + j0.28) = 38{,}400 + j11{,}200\,\text{VA}$

$S_2 = S_T - S_1 = 14{,}400 - j50{,}800 = 52{,}801.52\underline{/-74.17^\circ}\,\text{VA}$

$\text{rf} = \sin(-74.17^\circ) = -0.9621$

$\text{pf} = \cos(-74.17^\circ) = 0.2727$ leading

P 10.21 [a] $Z_1 = 12 + j(2\pi)(60)(15 \times 10^{-3}) = 13.27\underline{/25.23^\circ}\,\Omega$

$\text{pf} = \cos(25.23^\circ) = 0.9$ lagging

$\text{rf} = \sin(25.23^\circ) = 0.43$

$$Z_2 = 80 - \frac{j}{2\pi(60)(16 \times 10^{-6})} = 184.08\underline{/-64.24^\circ}\,\Omega$$

$\text{pf} = \cos(-64.24^\circ) = 0.43$ leading

$\text{rf} = \sin(-64.24^\circ) = -0.9$

$Z_3 = 400 + Z_p$

$$Z_p = \frac{j\omega L(1/j\omega C)}{j\omega L + 1/j\omega C} = \frac{j\omega L}{1 - \omega^2 LC}$$

$$= \frac{j(120\pi)(20)}{1 - (120\pi)^2(20)(5 \times 10^{-6})} = -j570.67\,\Omega$$

$$\therefore \quad Z_3 = 400 - j570.67 = 696.90\underline{/-54.97^\circ}\,\Omega$$

$$\text{pf} = \cos(-54.97^\circ) = 0.57 \text{ leading}$$

$$\text{rf} = \sin(-54.97^\circ) = -0.82$$

[b] $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{13.27\underline{/25.23^\circ}}; \qquad Y_2 = \frac{1}{184.08\underline{/-64.24^\circ}}; \qquad Y_3 = \frac{1}{696.90\underline{/-54.97^\circ}}$$

$$Y = 71.35 - j26.05\,\text{mS}$$

$$Z = \frac{1}{Y} = 13.16\underline{/20.06^\circ}\,\Omega$$

$$\text{pf} = \cos(20.06^\circ) = 0.94 \text{ lagging}$$

$$\text{rf} = \sin(20.06^\circ) = 0.343$$

P 10.22 [a] $S_1 = 18 + j24\,\text{kVA}; \qquad S_2 = 36 - j48\,\text{kVA}; \qquad S_3 = 18 + j0\,\text{kVA}$

$$S_T = S_1 + S_2 + S_3 = 72 - j24\,\text{kVA}$$

$$2400\mathbf{I}^* = (72 - j24) \times 10^3; \qquad \therefore \quad \mathbf{I} = 30 + j10\,\text{A}$$

$$Z = \frac{2400}{30 + j10} = 72 - j24\,\Omega = 75.89\underline{/-18.43^\circ}\,\Omega$$

[b] $\text{pf} = \cos(-18.43^\circ) = 0.9487$ leading

P 10.23 [a] From the solution to Problem 10.22 we have

$$\mathbf{I}_L = 30 + j10\,\text{A(rms)}$$

$$\therefore \quad \mathbf{V}_s = 2400\underline{/0^\circ} + (30 + j10)(0.2 + j1.6) = 2390 + j50$$

$$= 2390.52\underline{/1.20^\circ}\,\text{V(rms)}$$

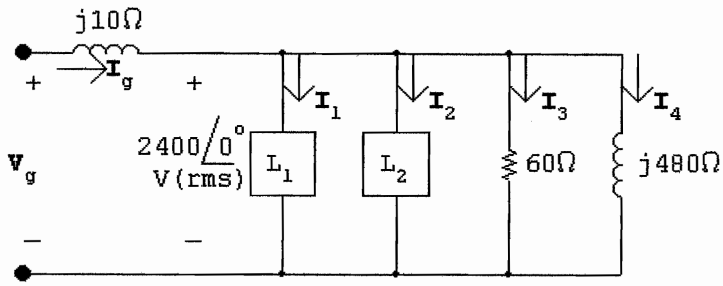
[b] $|\mathbf{I}_L| = \sqrt{1000}$

$$P_\ell = (1000)(0.2) = 200\,\text{W} \qquad Q_\ell = (1000)(1.6) = 1600\,\text{VAR}$$

[c] $P_s = 72{,}000 + 200 = 72.2\,\text{kW} \qquad Q_s = -24{,}000 + 1600 = -22.4\,\text{kVAR}$

[d] $\eta = \dfrac{72}{72.2}(100) = 99.72\%$

P 10.24



$$2400\mathbf{I}_1^* = 24{,}000 + j18{,}000$$

$$\mathbf{I}_1^* = 10 + j7.5; \qquad \therefore \quad \mathbf{I}_1 = 10 - j7.5\,\text{A(rms)}$$

$$2400\mathbf{I}_2^* = 48{,}000 - j30{,}000$$

$$\mathbf{I}_2^* = 20 - j12.5; \qquad \therefore \quad \mathbf{I}_2 = 20 + j12.5\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{2400\underline{/0^\circ}}{60} = 40 + j0\,\text{A}; \qquad \mathbf{I}_4 = \frac{2400\underline{/0^\circ}}{j480} = 0 - j5\,\text{A}$$

$$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 70\,\text{A}$$

$$\mathbf{V}_g = 2400 + (70)(j10) = 2400 + j700 = 2500\underline{/16.26^\circ}\,\text{V(rms)}$$

P 10.25 [a] $S_1 = 24{,}960 + j47{,}040\,\text{VA}$

$$S_2 = \frac{|\mathbf{V}_\text{L}|^2}{Z_2^*} = \frac{(480)^2}{5 + j5} = 23{,}040 - j23{,}040\,\text{VA}$$

$$S_1 + S_2 = 48{,}000 + j24{,}000\,\text{VA}$$

$$480\mathbf{I}_\text{L}^* = 48{,}000 + j24{,}000; \qquad \therefore \quad \mathbf{I}_\text{L} = 100 - j50\,\text{A(rms)}$$

$$\mathbf{V}_g = \mathbf{V}_\text{L} + \mathbf{I}_\text{L}(0.02 + j0.20) = 480 + (100 - j50)(0.02 + j0.20)$$

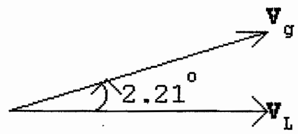
$$= 492 + j19 = 492.37\underline{/2.21^\circ}\,\text{Vrms}$$

$$|\mathbf{V}_g| = 492.37\,\text{Vrms}$$

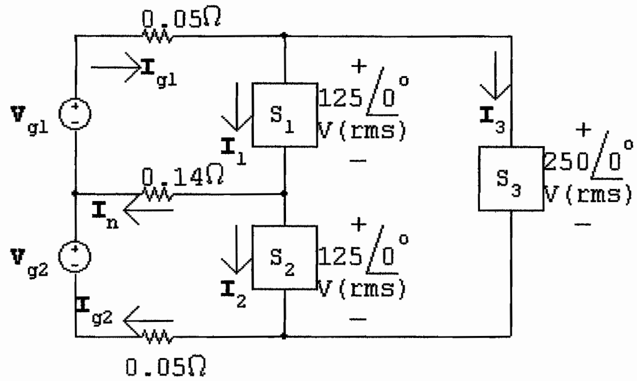
[b] $T = \dfrac{1}{f} = \dfrac{1}{60} = 16.67\,\text{ms}$

$$\frac{2.21^\circ}{360^\circ} = \frac{t}{16.67\ \text{ms}}; \qquad \therefore \quad t = 102.39\,\mu\text{s}$$

[c] $\mathbf{V}_L$ lags $\mathbf{V}_g$ by 2.21° or 102.31 μs



P 10.26 [a]



$$\mathbf{I}_1 = \frac{5000 - j2000}{125} = 40 - j16\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{3750 - j1500}{125} = 30 - j12\,\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{8000 + j0}{250} = 32 + j0\,\text{A (rms)}$$

$$\therefore\quad \mathbf{I}_{g1} = 72 - j16\,\text{A (rms)}$$

$$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = 10 - j4\,\text{A (rms)}$$

$$\mathbf{I}_{g2} = 62 - j12\,\text{A}$$

$$\mathbf{V}_{g1} = 0.05\mathbf{I}_{g1} + 125 + j0 + 0.14\mathbf{I}_n = 130 - j1.36\,\text{V(rms)}$$

$$\mathbf{V}_{g2} = -0.14\mathbf{I}_n + 125 + j0 + 0.05\mathbf{I}_{g2} = 126.7 - j0.04\,\text{V(rms)}$$

$$S_{g1} = [(130 - j1.36)(72 + j16)] = [9381.76 + j1982.08]\,\text{VA}$$

$$S_{g2} = [(126.7 - j0.04)(62 + j12)] = [7855.88 + j1517.92]\,\text{VA}$$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

[b] $P_{0.05} = |\mathbf{I}_{g1}|^2(0.05) = 272\,\text{W}$

$$P_{0.15} = |\mathbf{I}_n|^2(0.14) = 16.24\,\text{W}$$

$$P_{0.05} = |\mathbf{I}_{g2}|^2(0.05) = 199.4\,\text{W}$$

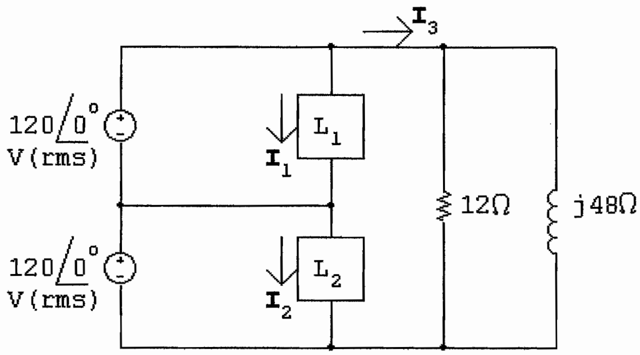
$$\sum P_{\text{dis}} = 272 + 16.24 + 199.4 + 5000 + 3750 + 8000 = 17{,}237.64\,\text{W}$$

$$\sum P_{\text{dev}} = 9381.76 + 7855.88 = 17{,}237.64\,\text{W} = \sum P_{\text{dis}}$$

$$\sum Q_{\text{abs}} = 2000 + 1500 = 3500\,\text{VAR}$$

$$\sum Q_{\text{del}} = 1982.08 + 1517.92 = 3500\,\text{VAR} = \sum Q_{\text{abs}}$$

P 10.27 [a]



$$120\mathbf{I}_1^* = 1800 + j600; \qquad \therefore \quad \mathbf{I}_1 = 15 - j5\,\text{A(rms)}$$

$$120\mathbf{I}_2^* = 1200 - j900; \qquad \therefore \quad \mathbf{I}_2 = 10 + j7.5\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{240}{12} + \frac{240}{j48} = 20 - j5\,\text{A(rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 35 - j10\,\text{A}$$

$$S_{g1} = 120(35 + j10) = 4200 + j1200\,\text{VA}$$

Thus the $\mathbf{V}_{g1}$ source is delivering 4200 W and 1200 magnetizing vars.

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 30 + j2.5\,\text{A(rms)}$$

$$S_{g2} = 120(30 - j2.5) = 3600 - j300\,\text{VA}$$

Thus the $\mathbf{V}_{g2}$ source is delivering 3600 W and absorbing 300 magnetizing vars.

[b] $\sum P_{\text{gen}} = 4200 + 3600 = 7800\,\text{W}$

$$\sum P_{\text{abs}} = 1800 + 1200 + \frac{(240)^2}{12} = 7800\,\text{W} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 1200 + 900 = 2100\,\text{VAR}$$

$$\sum Q_{\text{abs}} = 300 + 600 + \frac{(240)^2}{48} = 2100\,\text{VAR} = \sum Q_{\text{del}}$$

P 10.28 $S_1 = 1200 + 1196 + 516 + j0 = 2912 + j0\,\text{VA}$

$$\therefore \quad \mathbf{I}_1 = \frac{2912}{120} + j0 = 24.27 + j0\,\text{A}$$

$$S_2 = 600 + 279 + 88 + 512 + j0 = 1479 + j0\,\text{VA}$$

$$\therefore \quad \mathbf{I}_2 = \frac{1479}{120} + j0 = 12.33 + j0\,\text{A}$$

$$S_3 = 4474 + 12{,}200 + j0 = 16{,}674 + j0\,\text{VA}$$

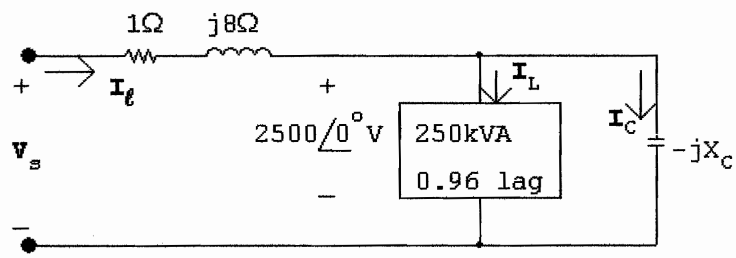
$$\therefore \quad \mathbf{I}_3 = \frac{16{,}674}{240} + j0 = 69.48 + j0\,\text{A}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 93.75 + j0\,\text{A}$$

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 81.81 + j0\,\text{A}$$

Breakers will not trip since both feeder currents are less than 100 A.

P 10.29



$$\mathbf{I}_\text{L} = \frac{240{,}000 - j70{,}000}{2500} = 96 - j28\,\text{A(rms)}$$

$$\mathbf{I}_\text{C} = \frac{2500}{-jX_\text{C}} = j\frac{2500}{X_\text{C}} = jI_\text{C}$$

$$\mathbf{I}_\ell = 96 - j28 + jI_\text{C} = 96 + j(I_\text{C} - 28)$$

$$\mathbf{V}_s = 2500 + (1 + j8)[96 + j(I_\text{C} - 28)]$$
$$= (2820 - 8I_\text{C}) + j(740 + I_\text{C})$$

$$|\mathbf{V}_s|^2 = (2820 - 8I_\text{C})^2 + (740 + I_\text{C})^2 = (2500)^2$$

$$\therefore \quad 65I_\text{C}^2 - 43{,}640I_\text{C} + 2{,}250{,}000 = 0$$

$$I_C = \frac{43{,}640 \pm \sqrt{(43{,}640)^2 - 4(65)(2{,}250{,}000)}}{2(65)}$$
$$= 335.69 \pm 279.42 = 56.27\,\text{A(rms)}^*$$

*Select the smaller value of I_C to minimize the magnitude of I_ℓ.

$$\therefore \quad X_C = -\frac{2500}{56.27} = -44.43$$

$$\therefore \quad C = \frac{1}{(44.43)(120\pi)} = 59.7\,\mu\text{F}$$

P 10.30 [a] $\mathbf{I} = \dfrac{7200\underline{/0^\circ}}{140 + j480} = 14.4\underline{/-73.74^\circ}\,\text{A(rms)}$

$$P = (14.4)^2(2) = 414.72\,\text{W}$$

[b] $Y_L = \dfrac{1}{138 + j460} = \dfrac{138 - j460}{230{,}644}$

$$\therefore \quad -j\omega C = -j\frac{460}{230{,}644} \qquad \therefore \quad X_C = \frac{-230{,}644}{460} = -501.40\,\Omega$$

[c] $Z_L = \dfrac{230{,}644}{138} = 1671.33\,\Omega$

[d] $\mathbf{I} = \dfrac{7200}{1673.33 + j20} = 4.30\underline{/-0.68^\circ}\,\text{A}$

$$P = (4.30)^2(2) = 37.02\,\text{W}$$

[e] $\% = \dfrac{37.02}{414.72}(100) = 8.93\%$

Thus the power loss after the capacitor is added is 8.93% of the power loss before the capacitor is added.

P 10.31 [a] $S_L = 24 + j7\,\text{kVA}$

$$125\mathbf{I}_L^* = (24 + j7) \times 10^3; \quad \mathbf{I}_L^* = 192 + j56\,\text{A(rms)}$$

$$\therefore \quad \mathbf{I}_L = 192 - j56\,\text{A(rms)}$$

$$\mathbf{V}_s = 125 + (192 - j56)(0.006 + j0.048) = 128.84 + j8.88$$
$$= 129.15\underline{/3.94^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 129.15\,\text{V(rms)}$$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.006) = (200)^2(0.006) = 240\,\text{W}$

[c] $\frac{(125)^2}{X_C} = -7000; \qquad X_C = -2.23\,\Omega$

$$-\frac{1}{\omega C} = -2.23; \qquad C = \frac{1}{(2.23)(120\pi)} = 1188.36\,\mu\text{F}$$

[d] $\mathbf{I}_\ell = 192 + j0\,\text{A(rms)}$

$$\mathbf{V}_s = 125 + 192(0.006 + j0.048) = 126.152 + j9.216$$
$$= 126.49\underline{/4.18^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 126.49\,\text{V(rms)}$$

[e] $P_\ell = (192)^2(0.006) = 221.184\,\text{W}$

P 10.32 [a] $S_o = \text{ original load } = 1800 + j\frac{1800}{0.6}(0.8) = 1800 + j2400\,\text{kVA}$

$$S_f = \text{ final load } = 2400 + j\frac{2400}{0.96}(0.28) = 2400 + j700\,\text{kVA}$$

$$\therefore \quad Q_{\text{added}} = 700 - 2400 = -1700\,\text{kVAR}$$

[b] deliver

[c] $S_a = \text{ added load } = 600 - j1700 = 1802.78\underline{/-70.56^\circ}\,\text{kVA}$

$$\text{pf } = \cos(-70.56) = 0.3328 \text{ leading}$$

[d] $\mathbf{I}_L^* = \frac{(1800 + j2400) \times 10^3}{4800} = 375 + j500\,\text{A}$

$$\mathbf{I}_L = 375 - j500 = 625\underline{/53.13^\circ}\,\text{A(rms)}$$

$$|\mathbf{I}_L| = 625\,\text{A(rms)}$$

[e] $\mathbf{I}_L^* = \frac{(2400 + j700) \times 10^3}{4800} = 500 + j145.83$

$$\mathbf{I}_L = 500 - j145.83 = 520.83\underline{/-16.26^\circ}\,\text{A(rms)}$$

$$|\mathbf{I}_L| = 520.83\,\text{A(rms)}$$

P 10.33 [a] $P_{\text{before}} = (625)^2(0.02) = 7812.50\,\text{W}$

$$P_{\text{after}} = (520.83)^2(0.02) = 5425.35\,\text{W}$$

[b] $\mathbf{V}_s(\text{before}) = 4800 + (375 - j500)(0.02 + j0.16) = 4887.5 + j50$

$= 4887.5\underline{/0.59^\circ}\,\text{V(rms)}$

$|\mathbf{V}_s(\text{before})| = 4887.76\,\text{V(rms)}$

$\mathbf{V}_s(\text{after}) = 4800 + (500 - j145.83)(0.02 + j0.16)$

$= 4833.33 + j77.08 = 4833.95\underline{/0.91^\circ}\,\text{V(rms)}$

$|\mathbf{V}_s(\text{after})| = 4833.95\,\text{V(rms)}$

P 10.34 **[a]** $\mathbf{I}_1 = \dfrac{125\underline{/0^\circ}}{20 + j34 + 5 + j16} = \dfrac{125}{25 + j50} = 1 - j2\,\text{A}$

$\mathbf{I}_2 = \dfrac{j\omega M}{Z_{22}}\mathbf{I}_1 = \dfrac{j50}{200 + j150}(1 - j2)$

$= 0.44 - j0.08 = 0.45\underline{/-10.30^\circ}\,\text{A}$

$\mathbf{V}_\text{L} = (150 - j100)(0.44 - j0.08) = 58 - j56$

$= 80.62\underline{/-43.99^\circ}\,\text{V}$

$|\mathbf{V}_\text{L}| = 80.62\,\text{V}$

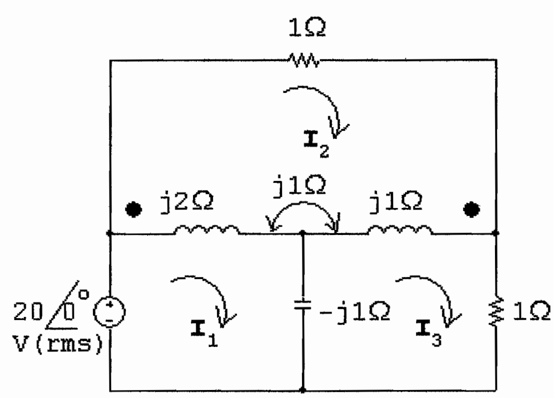
[b] $P_g(\text{ideal}) = 125(1) = 125\,\text{W}$

$P_g(\text{practical}) = 125 - |\mathbf{I}_1|^2(5) = 125 - 25 = 100\,\text{W}$

$P_\text{L} = |\mathbf{I}_2|^2(150) = 30\,\text{W}$

$\%\ \text{delivered} = \dfrac{30}{100}(100) = 30\%$

P 10.35 **[a]**



$20 = j2(\mathbf{I}_1 - \mathbf{I}_2) + j1(\mathbf{I}_2 - \mathbf{I}_3) - j1(\mathbf{I}_1 - \mathbf{I}_3)$

$0 = 1\mathbf{I}_2 + j1(\mathbf{I}_2 - \mathbf{I}_3) + j1(\mathbf{I}_1 - \mathbf{I}_2) + j2(\mathbf{I}_2 - \mathbf{I}_1) - j1(\mathbf{I}_2 - \mathbf{I}_3)$

$$0 = -j1(\mathbf{I}_3 - \mathbf{I}_1) + j1(\mathbf{I}_3 - \mathbf{I}_2) - j1(\mathbf{I}_1 - \mathbf{I}_2) + 1\mathbf{I}_3$$

Solving,

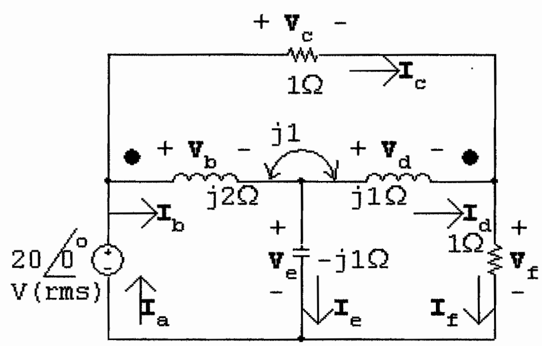
$\mathbf{I}_1 = 20 - j20\,\text{A(rms)}$; $\mathbf{I}_2 = 20 + j0\,\text{A(rms)}$; $\mathbf{I}_3 = 0\,\text{A(rms)}$

$\mathbf{I}_a = \mathbf{I}_1 = 20 - j20\,\text{A}$ $\quad$ $\mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_2 = -j20\,\text{A}$

$\mathbf{I}_c = \mathbf{I}_2 = 20\,\text{A}$ $\quad$ $\mathbf{I}_d = \mathbf{I}_3 - \mathbf{I}_2 = -20\,\text{A}$

$\mathbf{I}_e = \mathbf{I}_1 - \mathbf{I}_3 = 20 - j20\,\text{A}$ $\quad$ $\mathbf{I}_f = \mathbf{I}_3 = 0\,\text{A}$

[b]



$\mathbf{V}_a = 20 + j0\,\text{V}$ $\quad$ $\mathbf{V}_b = j2\mathbf{I}_b - j1\mathbf{I}_d = 40 + j20\,\text{V}$

$\mathbf{V}_c = 1\mathbf{I}_c = 20 + j0\,\text{V}$ $\quad$ $\mathbf{V}_d = j1\mathbf{I}_d - j1\mathbf{I}_b = -20 - j20\,\text{V}$

$\mathbf{V}_e = -j1\mathbf{I}_e = -20 - j20\,\text{V}$ $\quad$ $\mathbf{V}_f = 1\mathbf{I}_f = 0\,\text{V}$

$$S_a = -20\mathbf{I}_a^* = -400 - j400\,\text{VA}$$

$$S_b = \mathbf{V}_b\mathbf{I}_b^* = -400 + j800\,\text{VA}$$

$$S_c = \mathbf{V}_c\mathbf{I}_c^* = 400 + j0\,\text{VA}$$

$$S_d = \mathbf{V}_d\mathbf{I}_d^* = 400 + j400\,\text{VA}$$

$$S_e = \mathbf{V}_e\mathbf{I}_e^* = 0 - j800\,\text{VA}$$

$$S_f = \mathbf{V}_f\mathbf{I}_f^* = 0 + j0\,\text{VA}$$

[c] $\sum P_{\text{dev}} = 400\,\text{W}$

$$\sum P_{\text{abs}} = -400 + 400 + 400 = 400\,\text{W}$$

Note that the total power absorbed by the coupled coils is zero: $-400 + 400 = 0 = P_b + P_d$

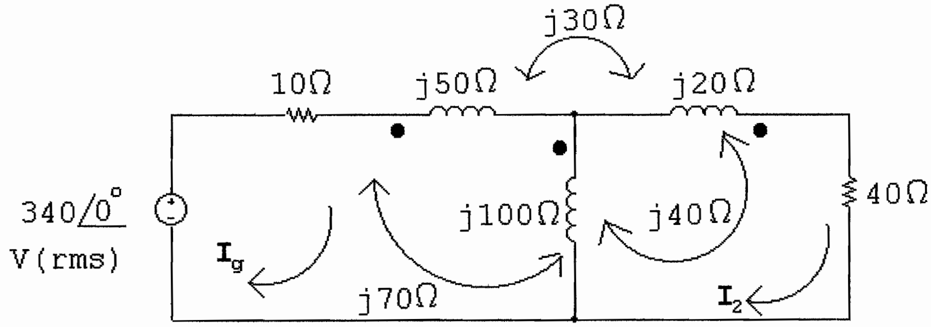
[d] $\sum Q_{\text{dev}} = 400 + 800 = 1200\,\text{VAR}$

Both the source and the capacitor are developing magnetizing vars.

$\sum Q_{\text{abs}} = 400 + 800 = 1200\,\text{VAR}$

$\sum Q$ absorbed by the coupled coils is $Q_b + Q_d$

P 10.36 [a]



$$340\underline{/0^\circ} = 10\mathbf{I}_g + j50\mathbf{I}_g + j70(\mathbf{I}_g - \mathbf{I}_2) - j30\mathbf{I}_2$$
$$+j70\mathbf{I}_g - j40\mathbf{I}_2 + j100(\mathbf{I}_g - \mathbf{I}_2)$$
$$0 = j100(\mathbf{I}_2 - \mathbf{I}_g) - j70\mathbf{I}_g + j40\mathbf{I}_2 + j20\mathbf{I}_2$$
$$+j40(\mathbf{I}_2 - \mathbf{I}_g) - j30\mathbf{I}_g + 40\mathbf{I}_2$$

Solving,

$I_g = 5 - j1\,\text{A(rms)}; \qquad \mathbf{I}_2 = 6\underline{/0^\circ}\,\text{A(rms)}$

$P_{40\Omega} = (6)^2(40) = 1440\,\text{W}$

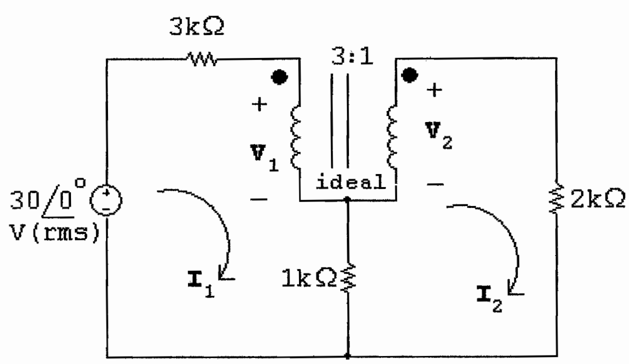
[b] $P_g(\text{developed}) = (340)(5) = 1700\,\text{W}$

[c] $Z_{\text{ab}} = \dfrac{\mathbf{V}_g}{\mathbf{I}_g} - 10 = \dfrac{340}{5-j} - 10 = 55.38 + j13.08 = 56.91\underline{/13.28^\circ}\,\Omega$

[d] $P_{10\Omega} = |I_g|^2(10) = 260\,\text{W}$

$\sum P_{\text{diss}} = 1440 + 260 = 1700\,\text{W} = \sum P_{\text{dev}}$

P 10.37 [a]



$$30 = 3000\mathbf{I}_1 + \mathbf{V}_1 + 1000(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = 1000(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2 + 2000\mathbf{I}_2$$

$$\mathbf{V}_2 = \frac{1}{3}\mathbf{V}_1; \qquad \mathbf{I}_2 = 3\mathbf{I}_1$$

Solving,

$$\mathbf{V}_1 = 28.8\,\text{V(rms)}; \qquad \mathbf{V}_2 = 9.6\,\text{V(rms)}$$

$$\mathbf{I}_1 = 1.2\,\text{mA(rms)}; \qquad \mathbf{I}_2 = 3.6\,\text{mA(rms)}$$

$$\mathbf{V}_{10\text{mA}} = \mathbf{V}_1 + 1000(\mathbf{I}_1 - \mathbf{I}_2) = 26.4\,\text{V(rms)}$$

$$\therefore \quad P = -(26.4)(10 \times 10^{-3}) = -264\,\text{mW}$$

Thus 264 mW is delivered by the current source to the circuit.

[b] $\mathbf{I}_{1k\Omega} = \mathbf{I}_1 - \mathbf{I}_2 = -2.4\,\text{mA(rms)}$

$$\therefore \quad P_{1k\Omega} = (-0.0024)^2(1000) = 5.76\,\text{mW}$$

P 10.38 [a] $Z_{\text{ab}} = \left(1 + \frac{N_1}{N_2}\right)^2 (4 - j8) = 36 - j72\,\Omega$

$$\therefore \quad \mathbf{I}_1 = \frac{250\underline{/0^\circ}}{4 + j42 + 36 - j72} = 5\underline{/36.87^\circ}\,\text{A}$$

$$P_{4(\text{left})} = |\mathbf{I}_1|^2(4) = (5)^2(4) = 100\,\text{W}$$

$$\mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1 = 10\underline{/36.87^\circ}\,\text{A}$$

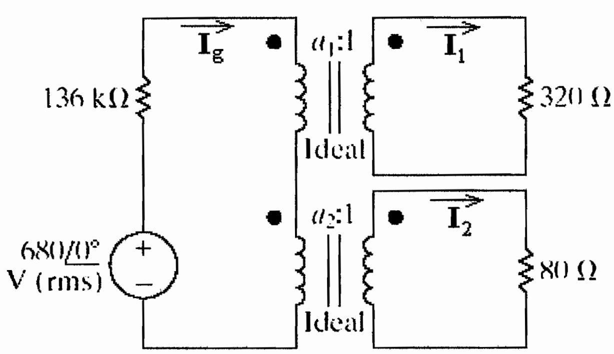
$$\therefore \quad \mathbf{I}_\text{L} = 15\underline{/36.87^\circ}\,\text{A(rms)}$$

$$P_{4(\text{right})} = (225)(4) = 900\,\text{W}$$

[b] $P_g = (250)(5)\cos(36.87^\circ) = 1000\,\text{W(developed)}$

$$\sum P_{\text{abs}} = (5)^2(4) + 900 = 1000\,\text{W} = \sum P_{\text{dev}}$$

P 10.39 [a]



$$a_1\mathbf{I}_g = \mathbf{I}_1; \qquad a_2\mathbf{I}_g = \mathbf{I}_2; \qquad \text{so} \qquad \frac{a_1}{a_2} = \frac{\mathbf{I}_1}{\mathbf{I}_2}$$

$$P_{320} = |\mathbf{I}_1|^2(320); \qquad P_{80} = |\mathbf{I}_2|^2(80); \qquad P_{80} = 16P_{320}$$

$$\therefore \quad |\mathbf{I}_2|^2(80) = 16[|\mathbf{I}_1|^2(320)] \qquad \text{thus} \qquad \frac{a_1}{a_2} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{1}{8}$$

The load impedances are matched to the source impedance:

$$a_1^2(320) + a_2^2(80) = 136{,}000 \qquad \text{so} \qquad a_1^2(320) + (8a_1)^2(80) = 136{,}000$$

$$\therefore \quad a_1^2 = 25 \qquad \text{so} \qquad a_1 = 5 \quad \text{and} \quad a_2 = 8a_1 = 40$$

[b] $\mathbf{I}_g = \dfrac{680\angle 0^\circ}{(136+136)10^3} = 2.5\angle 0^\circ\,\text{mA(rms)}$

$\mathbf{I}_2 = 40\mathbf{I}_g = 100\,\text{mA(rms)}$

$\therefore \quad P_{80\Omega} = (0.1)^2(80) = 800\,\text{mW}$

[c] $\mathbf{I}_1 = 5\mathbf{I}_g = 12.5\angle 0^\circ\,\text{mA(rms)}$

$\mathbf{V}_{320} = (12.5\times 10^{-3})(320) = 4\,\text{V(rms)}$

P 10.40 $Z_\text{L} = |Z_\text{L}|\angle\theta^\circ = |Z_\text{L}|\cos\theta^\circ + j|Z_\text{L}|\sin\theta^\circ$

$$\text{Thus} \quad |\mathbf{I}| = \frac{|\mathbf{V}_\text{Th}|}{\sqrt{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}}$$

$$\text{Therefore} \quad P = \frac{0.5|\mathbf{V}_\text{Th}|^2|Z_\text{L}|\cos\theta}{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}$$

Let D = demoninator in the expression for P, then

$$\frac{dP}{d|Z_\text{L}|} = \frac{(0.5|\mathbf{V}_\text{Th}|^2\cos\theta)(D\cdot 1 - |Z_\text{L}|dD/d|Z_\text{L}|)}{D^2}$$

$$\frac{dD}{d|Z_\text{L}|} = 2(R_\text{Th} + |Z_\text{L}|\cos\theta)\cos\theta + 2(X_\text{Th} + |Z_\text{L}|\sin\theta)\sin\theta$$

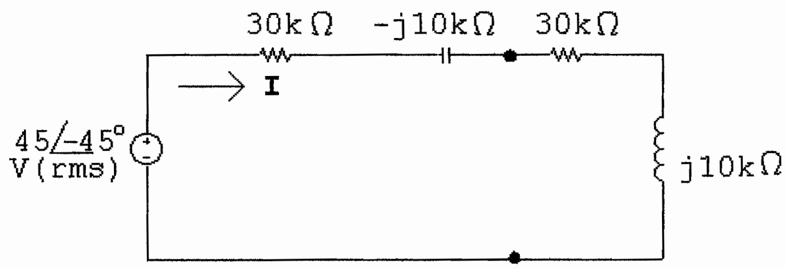
$$\frac{dP}{d|Z_\text{L}|} = 0 \quad \text{when} \quad D = |Z_\text{L}|\left(\frac{dD}{d|Z_\text{L}|}\right)$$

Substituting the expressions for D and $(dD/d|Z_\text{L}|)$ into this equation gives us the relationship $R_\text{Th}^2 + X_\text{Th}^2 = |Z_\text{L}|^2$ or $|Z_\text{Th}| = |Z_\text{L}|$.

P 10.41 [a] $Z_\text{Th} = \dfrac{1}{j\omega C} + \dfrac{(60)(j60)}{60+j60} = -j40 + 30 + j30 = 30 - j10\,\text{k}\Omega$

$\therefore \quad Z_\text{L} = Z_\text{Th}^* = 30 + j10\,\text{k}\Omega$

[b] $\mathbf{V}_{\text{Th}} = \dfrac{90\underline{/0^\circ}(60)}{60 + j60} = 45(1 - j1) = 45\sqrt{2}\underline{/-45^\circ}\,\text{V}$



$\mathbf{I} = \dfrac{45\sqrt{2}\underline{/-45^\circ}}{60 \times 10^3} = 0.75\sqrt{2}\underline{/-45^\circ}\,\text{mA}$

$|\mathbf{I}_{\text{rms}}| = 0.75\,\text{mA}$

$P_{\text{load}} = (0.75)^2 \times 10^{-6}(30 \times 10^3) = 16.875\,\text{mW}$

P 10.42 [a] $\dfrac{240 - j80 - 480}{Z_{\text{Th}}} + \dfrac{240 - j80}{100} = 0$

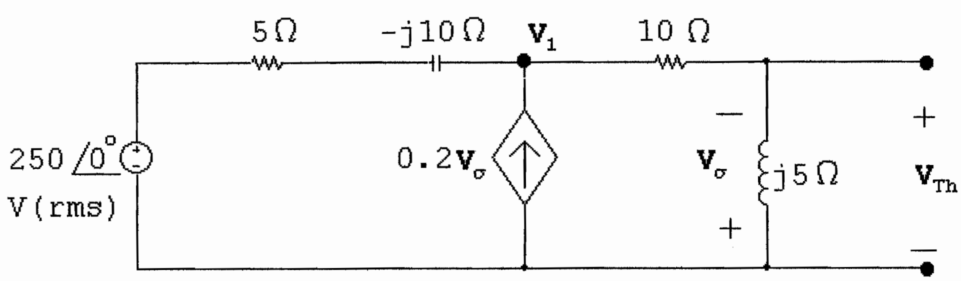
$\therefore\quad Z_{\text{Th}} = \dfrac{-100(240 + j80)}{-(240 - j80)} = 80 + j60\,\Omega$

$\therefore\quad Z_{\text{L}} = 80 - j60\,\Omega$

[b] $\mathbf{I} = \dfrac{480\underline{/0^\circ}}{160\underline{/0^\circ}} = 3\underline{/0^\circ}\,\text{A(rms)}$

$P = (9)(80) = 720\,\text{W}$

P 10.43 [a]



$\dfrac{\mathbf{V}_1 - 250}{5 - j10} - 0.2\mathbf{V}_\sigma + \dfrac{\mathbf{V}_1}{10 + j5} = 0$

$\mathbf{V}_\sigma = \dfrac{-j5\mathbf{V}_1}{10 + j5} = \dfrac{-j\mathbf{V}_1}{2 + j1}$

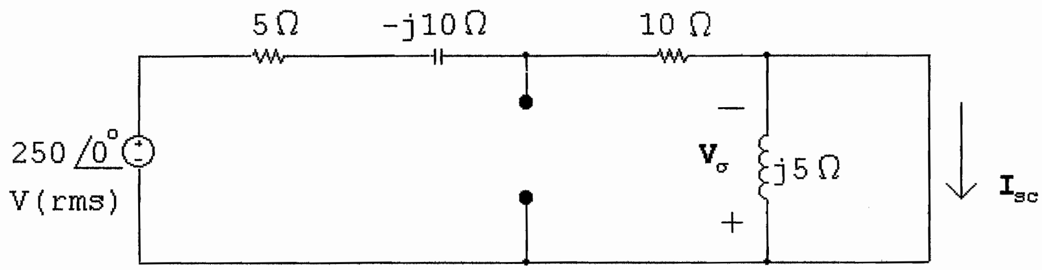
$-0.2\mathbf{V}_\sigma = \dfrac{j0.2\mathbf{V}_1}{2 + j1}$

$\therefore\quad \mathbf{V}_1\left[\dfrac{1}{5 - j10} + \dfrac{j0.2}{2 + j1} + \dfrac{1}{10 + j5}\right] = \dfrac{250}{5 - j10}$

Thus, $\mathbf{V}_1 = 10(10 + j5)$

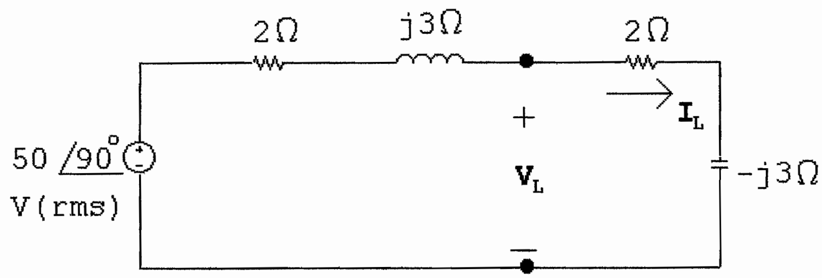
$$\mathbf{V}_{\text{Th}} = \frac{j5}{10 + j5}\mathbf{V}_1 = j50 = 50\underline{/90^\circ}\,\text{V(rms)}$$

Short circuit current:



$$\mathbf{I}_{\text{sc}} = \frac{250\underline{/0^\circ}}{15 - j10} = \frac{50}{3 - j2}\,\text{A(rms)}$$

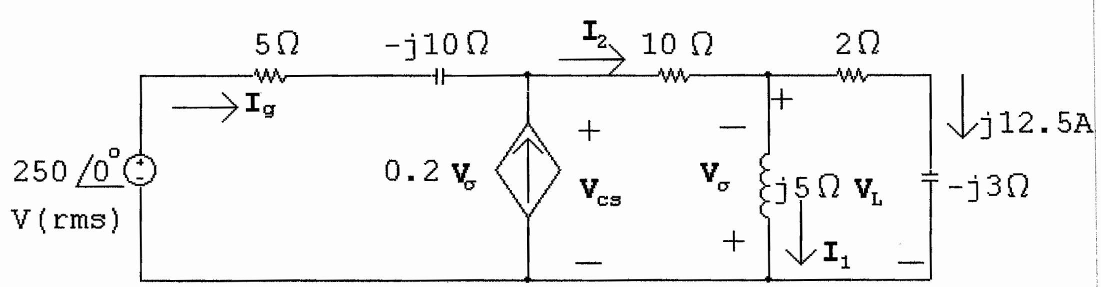
$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{j50}{50}(3 - j2) = 2 + j3\,\Omega$$



$$\mathbf{I}_{\text{L}} = \frac{50\underline{/90^\circ}}{4} = 12.5\underline{/90^\circ}\,\text{A(rms)}$$

$$P = (12.5)^2(2) = 312.50\,\text{W}$$

[b] $\mathbf{V}_{\text{L}} = (2 - j3)(j12.5) = 37.5 + j25\,\text{V(rms)}$



$$\mathbf{I}_1 = \frac{\mathbf{V}_{\text{L}}}{j5} = \frac{37.5 + j25}{j5} = 5 - j7.5\,\text{A(rms)}$$

$$\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_{\text{L}} = 5 - j7.5 + j12.5 = 5 + j5\,\text{A(rms)}$$

$$\mathbf{V}_{cs} = \mathbf{V}_L + 10\mathbf{I}_2 = 37.5 + j25 + 50 + j50 = 87.5 + j75\,\text{V(rms)}$$

$$\mathbf{V}_\sigma = -\mathbf{V}_L = -37.5 - j25$$

$$0.2\mathbf{V}_\sigma = -7.5 - j5$$

$$S_{cs} = -\mathbf{V}_{cs}\mathbf{I}_{cs}^* = -(87.5 + j75)(-7.5 + j5) = 1031.25 + j125\,\text{VA}$$

Therefore, the dependent source is absorbing 1031.25 W and 125 magnetizing vars. Only the independent voltage source is developing power.

$$\mathbf{I}_g = -0.2\mathbf{V}_\sigma + \mathbf{I}_2 = 7.5 + j5 + 5 + j5 = 12.5 + j10\,\text{A}$$

$$S_g = -250\mathbf{I}_g^* = -3125 + j2500\,\text{VA}$$

$$\therefore \quad P_{\text{dev}} = 3125\,\text{W}$$

$$\%\ \text{delivered} = \frac{312.5}{3125}(100) = 10\%$$

Thus, 10% of the developed power is delivered to the load.
Checks:

$$P_{10\Omega} = (5\sqrt{2})^2 10 = 500\,\text{W}$$

$$P_{2\Omega} = 312.5\,\text{W}$$

$$P_{5\Omega} = (\sqrt{256.25})^2 5 = 1281.25\,\text{W}$$

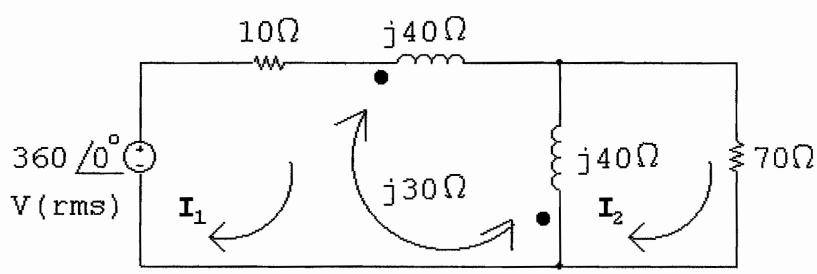
$$\therefore \quad \sum P_{\text{dev}} = \sum P_{\text{abs}} = 500 + 312.5 + 1281.25 + 1031.25 = 3125\,\text{W}$$

VAR Check:
The 250 V source is absorbing 2500 vars; the dependent current source is absorbing 125 vars; the $j5\,\Omega$ inductor is absorbing $|37.5 + j25|^2/5 = 406.25$ vars. Thus,

$$\sum Q_{\text{abs}} = 2625 + 406.25 = 3031.25\,\text{VAR}$$

$$\sum Q_{\text{dev}} = (12.5)^2(3) + 256.25(10) = 3031.25\,\text{VAR} = \sum Q_{\text{abs}}$$

P 10.44 [a]



$$360\underline{/0^\circ} = 10\mathbf{I}_1 + j40\mathbf{I}_1 + j30(\mathbf{I}_2 - \mathbf{I}_1) - j30\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = j40(\mathbf{I}_2 - \mathbf{I}_1) + j30\mathbf{I}_1 + 70\mathbf{I}_2$$

Solving,

$$\mathbf{I}_2 = 2\underline{/0^\circ}\,\text{A(rms)}; \qquad \therefore \quad \mathbf{V}_o = (2\underline{/0^\circ})(70) = 140\underline{/0^\circ}\,\text{V(rms)}$$

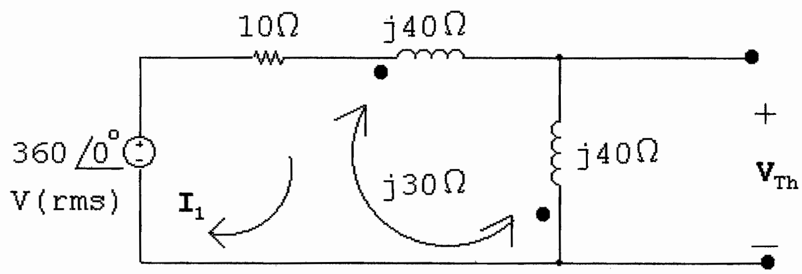
[b] $P = 70|\mathbf{I}_2|^2 = 70(4) = 280\,\text{W}$

[c] $360\underline{/0^\circ} = (10 + j20)\mathbf{I}_1 - j10(2 + j0); \qquad \therefore \quad \mathbf{I}_1 = 8 - j14\,\text{A}$

$$P_g = (360)(8) = 2880\,\text{W}$$

$$\%\ \text{delivered} = \frac{280}{2880}(100) = 9.72\%$$

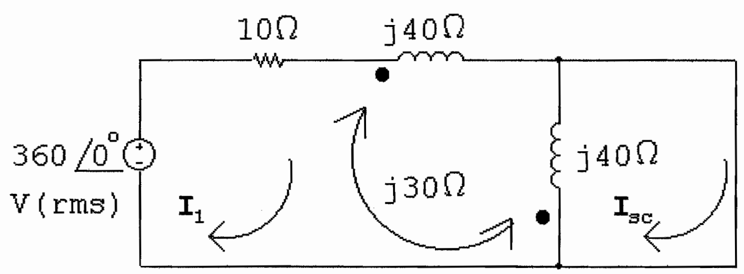
P 10.45 [a]



$$360 = 10\mathbf{I}_1 + j40\mathbf{I}_1 - j30\mathbf{I}_1 + j40\mathbf{I}_1 - j30\mathbf{I}_1$$

$$\therefore \quad \mathbf{I}_1 = 7.2 - j14.4\,\text{A(rms)}$$

$$\mathbf{V}_{\text{Th}} = j40\mathbf{I}_1 - j30\mathbf{I}_1 = j10\mathbf{I}_1 = 144 + j72\,\text{V}$$



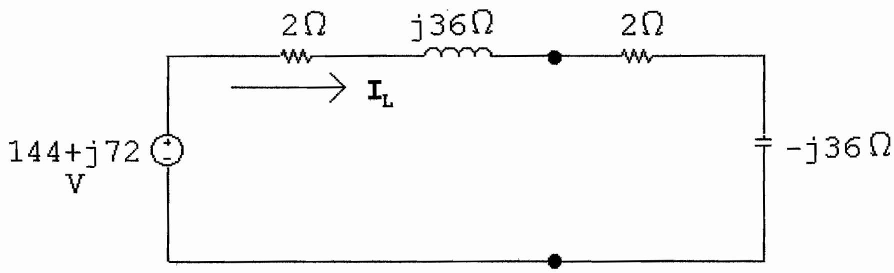
$$360 = (10 + j20)\mathbf{I}_1 - j10\mathbf{I}_{\text{sc}}$$

$$0 = -j10\mathbf{I}_1 + j40\mathbf{I}_{\text{sc}}$$

Solving,

$$\mathbf{I}_{\text{sc}} = 2.215 - j3.877\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{144 + j72}{2.215 - j3.877} = 2 + j36\,\Omega$$



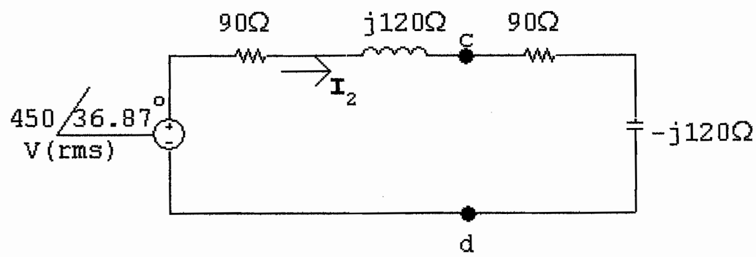
$$\mathbf{I}_L = \frac{144 + j72}{4} = 36 + j18\,\text{A}; \qquad \therefore\quad |\mathbf{I}_L| = 18\sqrt{5}\,\text{A}$$

$$P_L = (18)^2(5)(2) = 3240\,\text{W}$$

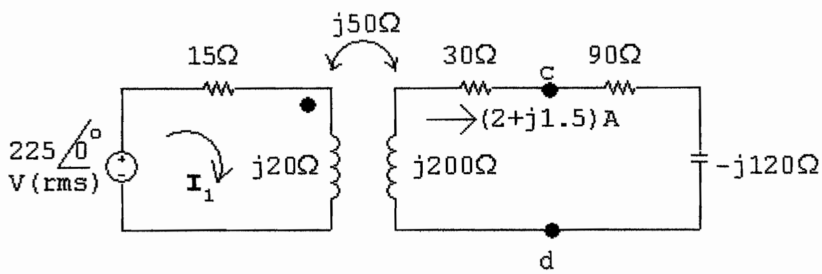
[b] $360 = (10 + j20)\mathbf{I}_1 - j10(36 + j18); \qquad \therefore\quad \mathbf{I}_1 = 18\underline{/0^\circ}\,\text{A}$

$$\therefore\quad P_g = (360)(18) = 6480\,\text{W}$$

P 10.46 [a] From Problem 9.74, $Z_{\text{Th}} = 90 + j120\,\Omega$ and $\mathbf{V}_{\text{Th}} = 450\underline{/36.87^\circ}\,\text{V}$. Thus, for maximum power transfer, $Z_L = Z_{\text{Th}}^* = 90 - j120\,\Omega$:



$$\mathbf{I}_2 = \frac{450\underline{/36.87^\circ}}{180} = 2.5\underline{/36.87^\circ} = 2 + j1.5\,\text{A}$$



$$225\underline{/0^\circ} = (15 + j20)\mathbf{I}_1 - j50(2 + j1.5)$$

$$\therefore\quad \mathbf{I}_1 = \frac{150 + j100}{15 + j20} = 6.8 - j2.4\,\text{A}$$

$$S_g(\text{del}) = 225(6.8 + j2.4) = 1530 + j540\,\text{VA}$$

$$P_g = 1530\,\text{W}$$

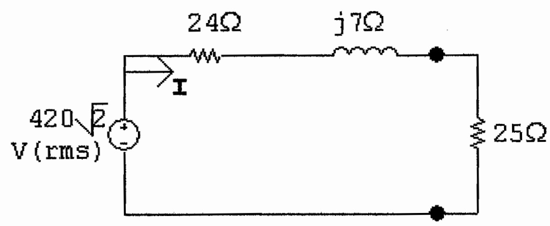
[b] $P_{\text{loss}} = |\mathbf{I}_1|^2(15) + |\mathbf{I}_2|^2(30) = 780 + 187.5 = 967.5\,\text{W}$

$$\%\ \text{loss} = \frac{967.50}{1530}(100) = 63.24\%$$

P 10.47 [a] $Z_{\text{Th}} = 8 + j15 + \dfrac{(-j24)(18+j6)}{18-j18} = 24 + j7 = 25\underline{/16.26^\circ}\,\Omega$

$\therefore\ R = |Z_{\text{Th}}| = 25\,\Omega$

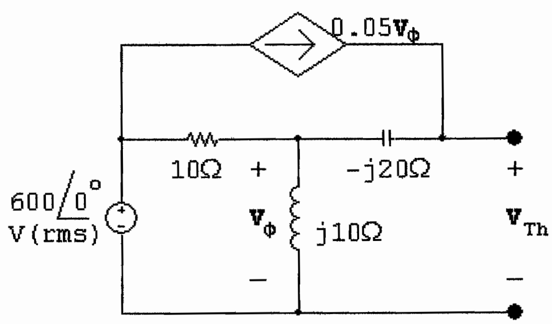
[b] $\mathbf{V}_{\text{Th}} = \dfrac{-j24}{18+j6-j24}(630\underline{/0^\circ}) = 420 - j420 = 420\sqrt{2}\underline{/-45^\circ}\,\text{V(rms)}$



$$\mathbf{I} = \frac{420\sqrt{2}\underline{/0^\circ}}{49+j7}; \qquad |\mathbf{I}| = \frac{60\sqrt{2}}{\sqrt{50}}$$

$$P = \frac{(3600)(2)}{50}(25) = 3600\,\text{W} = 3.6\,\text{kW}$$

P 10.48 [a]

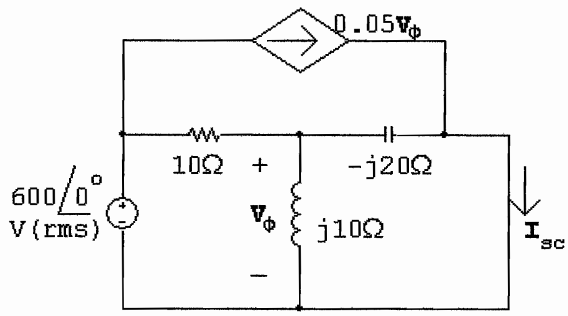


$$\frac{\mathbf{V}_\phi - 600}{10} + \frac{\mathbf{V}_\phi}{j10} - 0.05\mathbf{V}_\phi = 0$$

$\therefore\ \mathbf{V}_\phi = 240 + j480\,\text{V(rms)}$

$\mathbf{V}_{\text{Th}} = \mathbf{V}_\phi + 0.05\mathbf{V}_\phi(-j20) = \mathbf{V}_\phi(1-j1) = 720 + j240\,\text{V(rms)}$

Short circuit current:



$$\mathbf{I}_{sc} = 0.05\mathbf{V}_\phi + \frac{\mathbf{V}_\phi}{-j20} = (0.05 + j0.05)\mathbf{V}_\phi$$

$$\frac{\mathbf{V}_\phi - 600}{10} + \frac{\mathbf{V}_\phi}{j10} + \frac{\mathbf{V}_\phi}{-j20} = 0$$

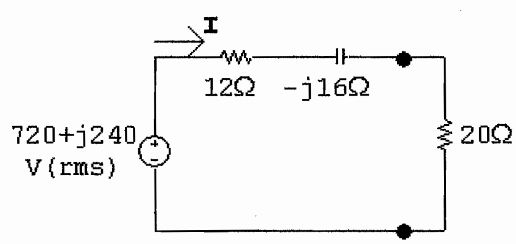
$$\therefore \quad \mathbf{V}_\phi = 480 + j240\,\text{V(rms)}$$

$$\mathbf{I}_{sc} = (0.05 + j0.05)(480 + j240) = 12 + j36\,\text{A(rms)}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{sc}} = \frac{720 + j240}{12 + j36} = 12 - j16 = 20\underline{/-53.13^\circ}\,\Omega$$

$$\therefore \quad R_o = 20\,\Omega$$

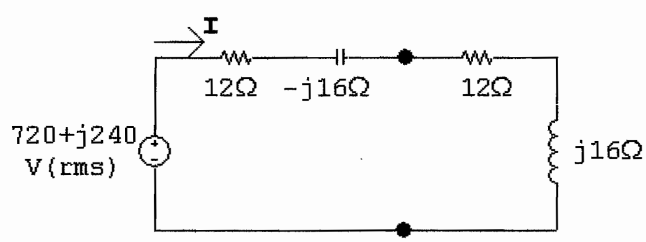
[b]



$$\mathbf{I} = \frac{720 + j240}{32 - j16} = 15 + j15 = 15\sqrt{2}\underline{/45^\circ}\,\text{A(rms)}$$

$$P = (15\sqrt{2})^2(20) = 9000\,\text{W} = 9\,\text{kW}$$

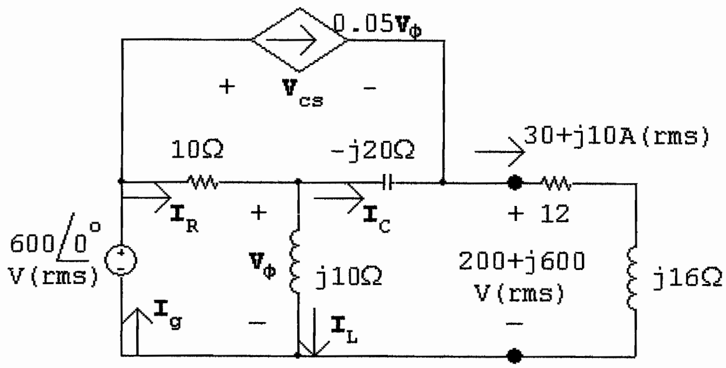
[c]



$$\mathbf{I} = \frac{720 + j240}{24} = 30 + j10\,\text{A(rms)}$$

$$P = (\sqrt{1000})^2(12) = 12\,\text{kW}$$

[d]



$$\frac{\mathbf{V}_\phi - 600}{10} + \frac{\mathbf{V}_\phi}{j10} + \frac{\mathbf{V}_\phi - 200 - j600}{-j20} = 0$$

$$\therefore \quad \mathbf{V}_\phi = 200 + j200\,\text{V}$$

$$0.05\mathbf{V}_\phi = 10 + j10\,\text{A}$$

$$10 + j10 + \mathbf{I}_\text{C} = 30 + j10; \qquad \therefore \quad \mathbf{I}_\text{C} = 20 + j0\,\text{A}$$

$$\mathbf{I}_\text{L} = \frac{\mathbf{V}_\phi}{j10} = 20 - j20\,\text{A}$$

$$\mathbf{I}_\text{R} = \mathbf{I}_\text{C} + \mathbf{I}_\text{L} = 40 - j20\,\text{A}$$

$$\mathbf{I}_g = \mathbf{I}_\text{R} + 0.05\mathbf{V}_\phi = 50 - j10\,\text{A(rms)}$$

$$S_g = -600\mathbf{I}_g^* = -30{,}000 - j6000\,\text{VA}$$

$$600 = \mathbf{V}_\text{cs} + 200 + j600; \qquad \mathbf{V}_\text{cs} = 400 - j600\,\text{V}$$

$$S_\text{cs} = (400 - j600)(10 - j10) = -2000 - j10{,}000\,\text{VA}$$

$$\sum P_\text{dev} = 30{,}000 + 2000 = 32{,}000\,\text{W} = 32\,\text{kW}$$

$$\%\ \text{delivered to}\ Z_o = \frac{12}{32}(100) = 37.50\%$$

Check:

$$\sum P_\text{abs} = 12{,}000 + \mathbf{I}_\text{R}^2(10) = 32\,\text{kW} = \sum P_\text{dev}$$

$$\sum Q_\text{dev} = 6000 + 10{,}000 + |\mathbf{I}_\text{C}|^2(20) = 24\,\text{kVAR}$$

$$\sum Q_\text{abs} = |\mathbf{I}_\text{L}|^2(10) + |\mathbf{I}_o|^2(16) = 24\,\text{kVAR} = \sum Q_\text{dev}$$

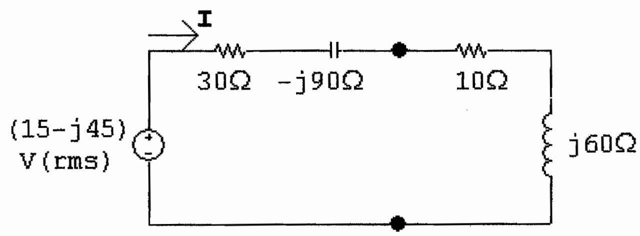
P 10.49 [a] First find the Thévenin equivalent:

$$\frac{1}{j\omega C} = \frac{10^6}{j10^4} = -j100\,\Omega$$

$$Z_{\text{Th}} = \frac{300(-j100)}{300 - j100} = 30 - j90\,\Omega$$

$$\mathbf{V}_{\text{Th}} = \frac{150(-j100)}{300 - j100} = 15 - j45\,\text{V(rms)}$$

$$j\omega L = j10^4(6 \times 10^{-3}) = j60\,\Omega$$



$$\mathbf{I} = \frac{15 - j45}{40 - j30} = \frac{1.5}{25}(13 - j9)\,\text{A(rms)}$$

$$|\mathbf{I}| = \frac{1.5}{25}\sqrt{250}\,\text{A(rms)}$$

$$P = \frac{2.25}{625}(250)(10) = 9\,\text{W}$$

[b] Set $L_o = 8\,\text{mH}$; Set R_o as close as possible to

$$R_o = \sqrt{(30)^2 + (10)^2} = \sqrt{1000} = 31.62\,\Omega$$

$$\therefore \quad R_o = 20\,\Omega$$

[c] $$\mathbf{I} = \frac{15 - j45}{50 - j10} = \frac{3 - j9}{10 - j2}\,\text{A(rms)}$$

$$\therefore \quad |\mathbf{I}| = \frac{\sqrt{90}}{104}$$

$$P = |\mathbf{I}|^2(20) = \frac{(90)(20)}{104} = 17.31\,\text{W}$$

Yes; $17.31\,\text{W} > 9\,\text{W}$

[d] $$\mathbf{I} = \frac{15 - j45}{60} = \frac{1 - j3}{4}\,\text{A(rms)}$$

$$P = \left(\frac{\sqrt{10}}{4}\right)^2 30 = 18.75\,\text{W}$$

[e] $R_o = 30\,\Omega$; $L_o = 9\,\text{mH}$

[f] Yes; $18.75\,\text{W} > 17.31\,\text{W}$

P 10.50 [a] $L_o = 8\,\text{mH}; \qquad R_o = \sqrt{30^2 + 10^2} = 31.62\,\Omega$

$$\mathbf{I} = \frac{15(1 - j3)}{61.62 - j10} = \frac{15\sqrt{10}}{62.43}\underline{/-62.35^\circ}\ \text{A(rms)}$$

$$P = \left(\frac{15\sqrt{10}}{62.43}\right)^2 (31.62) = 18.26\,\text{W}$$

[b] Yes; $18.26\,\text{W} > 17.31\,\text{W}$

[c] Yes; $18.26\,\text{W} < 18.75\,\text{W}$

P 10.51 [a] $\frac{1}{\omega C} = 240\,\Omega; \qquad C = \frac{1}{(240)(120\pi)} = 11.05\,\mu\text{F}$

[b] $\mathbf{I}_{\text{wo}} = \frac{4800}{160} + \frac{4800}{j240} = 30 - j20\,\text{A(rms)}$

$$\mathbf{V}_{\text{swo}} = 4800 + (30 - j20)(1 + j8) = 4990 + j220$$

$$= 4994.85\underline{/2.52^\circ}\ \text{V(rms)}$$

$$\mathbf{I}_{\text{w}} = \frac{4800}{160} + \frac{4800}{j240} + \frac{4800}{-j240} = 30 + j0\,\text{A(rms)}$$

$$\mathbf{V}_{\text{sw}} = 4800 + 30(1 + j8) = 4830 + j240 = 4835.96\underline{/2.84^\circ}\ \text{V(rms)}$$

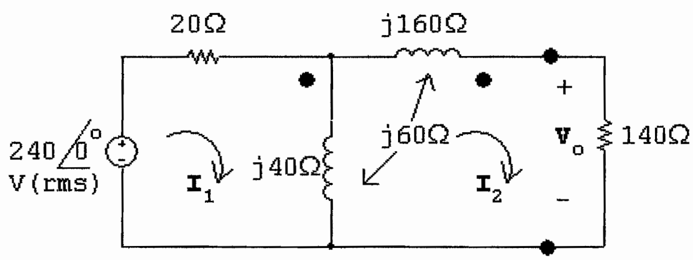
$$\%\ \text{increase} = \left(\frac{4994.85}{4835.96} - 1\right)(100) = 3.29\%$$

[c] $P_{\ell\text{wo}} = |30 - j20|^2 1 = 1300\,\text{W}$

$$P_{\ell\text{w}} = 30^2(1) = 900\,\text{W}$$

$$\%\ \text{increase} = \left(\frac{1300}{900} - 1\right)(100) = 44.44\%$$

P 10.52 [a]



$$240 = 20\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2) - j60\mathbf{I}_2$$

$$0 = j40(\mathbf{I}_2 - \mathbf{I}_1) + j60\mathbf{I}_2 + j160\mathbf{I}_2 + j60(\mathbf{I}_2 - \mathbf{I}_1) + 140\mathbf{I}_2$$

Solving,

$\mathbf{I}_1 = 6.4 - j2.8\,\text{A(rms)}; \qquad \mathbf{I}_2 = 2\underline{/0^\circ}\ \text{A(rms)}$

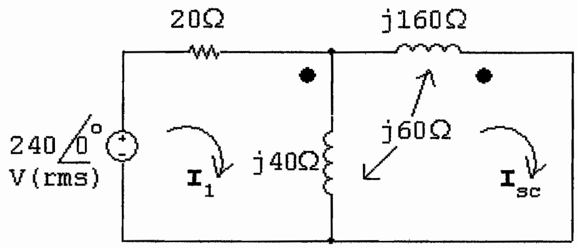
$$\mathbf{V}_o = 140\mathbf{I}_2 = 280\underline{/0^\circ}\ \text{V(rms)}$$

[b] $P = |\mathbf{I}_2|^2(140) = 560\,\text{W}$

[c] $P_g = (240)(6.4) = 1536\,\text{W}$

$$\%\ \text{delivered} = \frac{560}{1536}(100) = 36.46\%$$

P 10.53 [a] $\mathbf{V}_{\text{Th}} = \frac{240/0^\circ}{20+j40}(j40) + \frac{240/0^\circ}{20+j40}(j60) = 480 + j240\,\text{V(rms)}$



From the solution to Problem 10.49 we can write

$240 = (20 + j40)\mathbf{I}_1 - j100\mathbf{I}_{\text{sc}}$

$0 = -j100\mathbf{I}_1 + j320\mathbf{I}_{\text{sc}}$

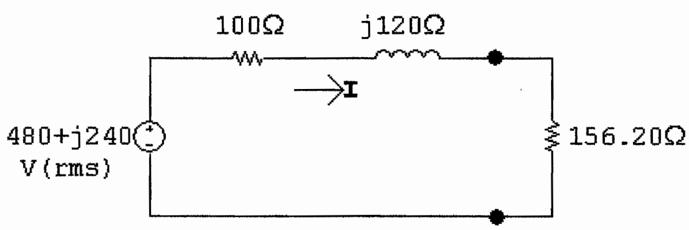
Solving,

$\mathbf{I}_{\text{sc}} = 3.15 - j1.377$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{480 + j240}{3.15 - j1.377} = 100 + j120 = 156.20/50.19^\circ\,\Omega$$

$\therefore\ R_L = 156.20\,\Omega$

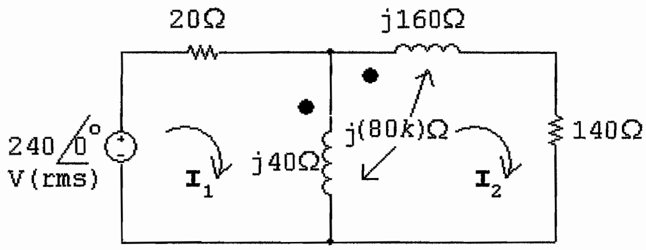
[b]



$$\mathbf{I} = \frac{536.66/26.57^\circ}{282.92/25.10^\circ} = 1.90/1.47^\circ$$

$P = |\mathbf{I}|^2(156.20) = 562.05\,\text{W}$

P 10.54 [a]



$$240 = 20\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2) + j80k\mathbf{I}_2$$

$$0 = j40(\mathbf{I}_2 - \mathbf{I}_1) - j80k\mathbf{I}_2 + j160\mathbf{I}_2 + j80k(\mathbf{I}_1 - \mathbf{I}_2) + 140\mathbf{I}_2$$

or

$$12 = (1 + j2)\mathbf{I}_1 + j(4k - 2)\mathbf{I}_2$$

$$0 = j(4k - 2)\mathbf{I}_1 + [7 + j(10 - 8k)]\mathbf{I}_2$$

$N_2 = -j(4k - 2)(12);$ $\qquad \mathbf{I}_2 = 0$ when $N_2 = 0$

$\mathbf{V}_o = 0$ when $\mathbf{I}_2 = 0$

$\therefore \quad k = 0.5$

[b] When $\mathbf{I}_2 = 0$

$$\mathbf{I}_1 = \frac{12}{1 + j2} = 2.4 - j4.8\,\text{A(rms)}$$

$$P_g = (240)(2.4) = 576\,\text{W}$$

Check:

$$P_{\text{loss}} = |\mathbf{I}_1|^2(20) = 576\,\text{W}$$

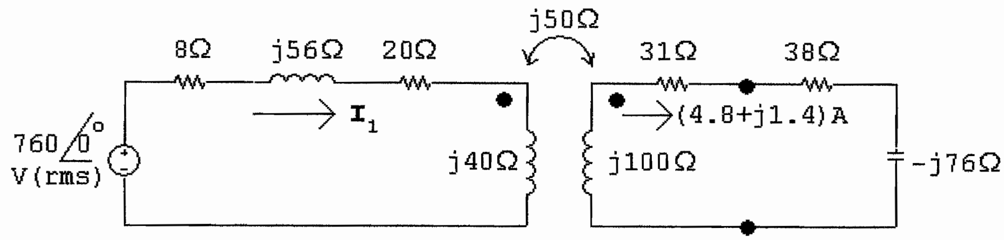
P 10.55 [a] $\mathbf{V}_{\text{Th}} = \dfrac{760/0^\circ}{28 + j96}(j50) = 380/16.26^\circ\,\text{V}$

$$Z_{\text{Th}} = 31 + j100 + \left(\frac{50}{100}\right)^2 (28 - j96) = 38 + j76\,\Omega$$

$\therefore \quad Z_{\text{L}} = 38 - j76\,\Omega$

$$\mathbf{I}_{\text{L}} = \frac{380/16.26^\circ}{76} = 4.8 + j1.4 = 5/16.26^\circ\,\text{A(rms)}$$

$$P_{\text{L}} = |\mathbf{I}_{\text{L}}|^2(38) = 950\,\text{W}$$



$$760\underline{/0^\circ} = \mathbf{I}_1(28 + j96) - j50(4.8 + j1.4)$$

$$\therefore \quad \mathbf{I}_1 = \frac{690 + j240}{100\underline{/73.74^\circ}} = 7.31\underline{/-54.56^\circ} = 4.24 - j5.95\,\text{A}$$

$$S_g(\text{delivered}) = 760(4.24 + j5.95) = 3219.36 + j4523.52\,\text{VA}$$

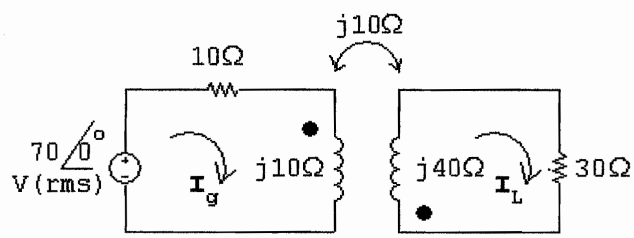
$$P_{\text{loss}} = |\mathbf{I}_1|^2(8) = 426.96\,\text{W}$$

$$P_{\text{in}}(\text{transformer}) = 3219.36 - 426.96 = 2792.40\,\text{W}$$

$$\%\text{ delivered to } Z_{\text{L}} = \frac{950}{2792.4}(100) = 34.02\%$$

P 10.56 [a] $j\omega L_1 = j(5000)(2 \times 10^{-3}) = j10\,\Omega$

$$j\omega L_2 = j(5000)(8 \times 10^{-3}) = j40\,\Omega$$



$$70 = (10 + j10)\mathbf{I}_g + j10\mathbf{I}_{\text{L}}$$

$$0 = j10\mathbf{I}_g + (30 + j40)\mathbf{I}_{\text{L}}$$

Solving,

$$\mathbf{I}_g = 4 - j3\,\text{A}; \qquad \mathbf{I}_{\text{L}} = -1\,\text{A}$$

Thus,

$$i_g = 5\cos(5000t - 36.87^\circ)\,\text{A}$$

$$i_{\text{L}} = 1\cos(5000t - 180^\circ)\,\text{A}$$

[b] $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{2}{\sqrt{16}} = 0.5$

[c] When $t = 100\pi\ \mu$s:

$$5000t = (5000)(100\pi) \times 10^{-6} = 0.5\pi \text{ rad } = 90°$$

$$i_g(100\pi\ \mu\text{s}) = 5\cos(53.15°) = 3\,\text{A}$$

$$i_\text{L}(100\pi\ \mu\text{s}) = 1\cos(-90°) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = \frac{1}{2}(2\times 10^{-3})(9) + 0 + 0 = 9\,\text{mJ}$$

When $t = 200\pi\ \mu$s:

$$5000t = \pi \text{ rad } = 180°$$

$$i_g(200\pi\ \mu\text{s}) = 5\cos(180 - 36.87°) = -4\,\text{A}$$

$$i_\text{L}(200\pi\ \mu\text{s}) = 1\cos(180 - 180°) = 1\,\text{A}$$

$$w = \frac{1}{2}(2\times 10^{-3})(16) + \frac{1}{2}(8\times 10^{-3})(1) + 2\times 10^{-3}(-4)(1) = 12\,\text{mJ}$$

[d] From (a), $I_m = 1$ A,

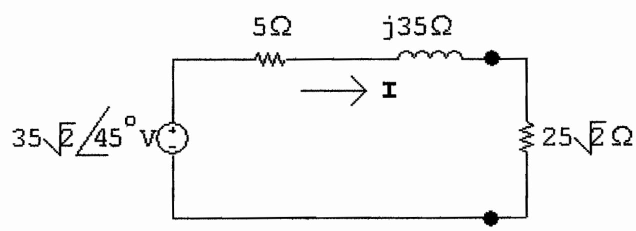
$$\therefore\quad P = \frac{1}{2}(1)^2(30) = 15\,\text{W}$$

[e] $$\mathbf{V}_\text{Th} = \frac{70}{10 + j10}(j10) = 35\sqrt{2}\underline{/45°}\,\text{V}$$

$$Z_\text{Th} = j40 + \left(\frac{10}{10\sqrt{2}}\right)^2 (10 - j10) = 5 + j35 = \sqrt{1250}\underline{/81.78°}\,\Omega$$

$$\therefore\quad R_\text{L} = 25\sqrt{2}\,\Omega$$

[f]



$$\mathbf{I} = \frac{35\sqrt{2}\underline{/45°}}{(5 + 25\sqrt{2}) + j35} = 0.93\underline{/4.07°}\,\text{A}$$

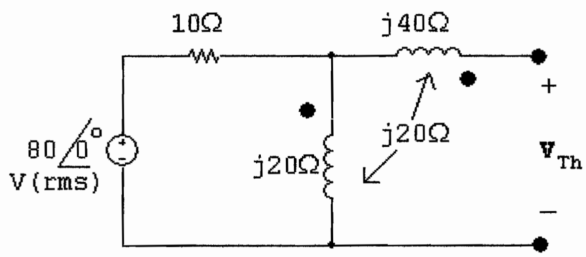
$$P = \frac{1}{2}(0.93)^2(25\sqrt{2}) = 15.18\,\text{W}$$

[g] $Z_\text{L} = Z_\text{Th}^* = 5 - j35\,\Omega$

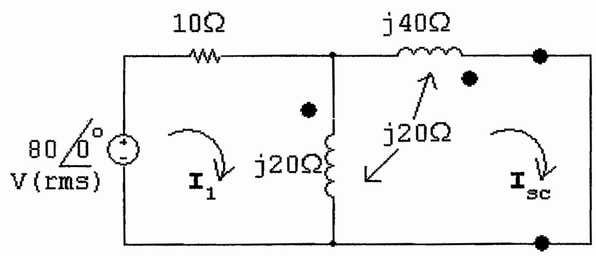
[h] $\mathbf{I} = \frac{35\sqrt{2}\underline{/45^\circ}}{10} = 3.5\sqrt{2}\underline{/45^\circ}$

$$P = \frac{1}{2}(3.5\sqrt{2})^2(5) = 61.25\,\text{W}$$

P 10.57



$$\mathbf{V}_{\text{Th}} = \frac{80}{10+j20}(j20) + \frac{80}{10+j20}(j20) = 128 + j64\,\text{V(rms)}$$



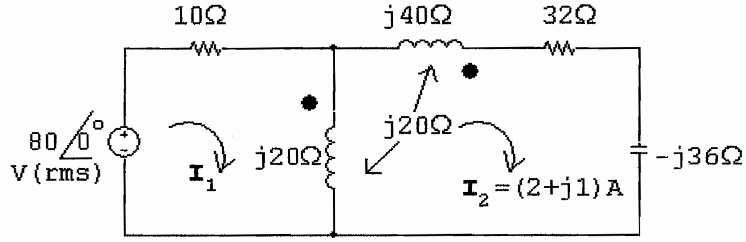
$$80 = 10\mathbf{I}_1 + j20(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) - j20\mathbf{I}_{\text{sc}}$$

$$0 = j20(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) + j20\mathbf{I}_{\text{sc}} + j40\mathbf{I}_{\text{sc}} - j20(\mathbf{I}_1 - \mathbf{I}_{\text{sc}})$$

Solving,

$\mathbf{I}_{\text{sc}} = 2.76 - j1.10\,\text{A};\qquad Z_{\text{Th}} = \frac{128+j64}{2.76-j1.10} = 32 + j36\,\Omega$

$$\therefore\quad \mathbf{I}_{\text{L}} = \frac{128+j64}{64} = 2 + j1\,\text{A}$$



$$80 = 10\mathbf{I}_1 + j20(\mathbf{I}_1 - \mathbf{I}_2) - j20\mathbf{I}_2$$

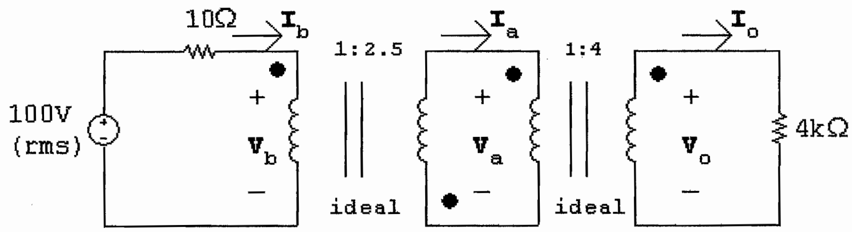
$\mathbf{I}_2 = 2 + j1\,\text{A}$

Solving,

$I_1 = 4\underline{/0^\circ}\,\text{A}$

$Z_g = 80/4 = 20 + j0\,\Omega$

P 10.58

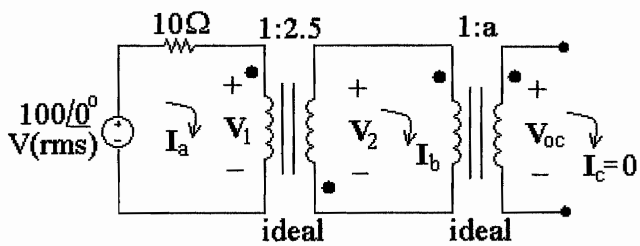


$\mathbf{V}_o = 4\mathbf{V}_a; \qquad 4\mathbf{I}_o = \mathbf{I}_a; \qquad \text{therefore } \frac{\mathbf{V}_a}{\mathbf{I}_a} = 250\,\Omega$

$\frac{\mathbf{V}_b}{1} = \frac{-\mathbf{V}_a}{2.5}; \qquad \mathbf{I}_b = -2.5\mathbf{I}_a; \qquad \text{therefore } \frac{\mathbf{V}_b}{\mathbf{I}_b} = \frac{250}{6.25} = 40\,\Omega$

Therefore $\mathbf{I}_b = [100/(10 + 40)] = 2\,\text{A}$ (rms); since the ideal transformers are lossless, $P_{4k\Omega} = P_{40\Omega}$, and the power delivered to the $4\,\text{k}\Omega$ resistor is $2^2(40)$ or 160 W.

P 10.59 [a]

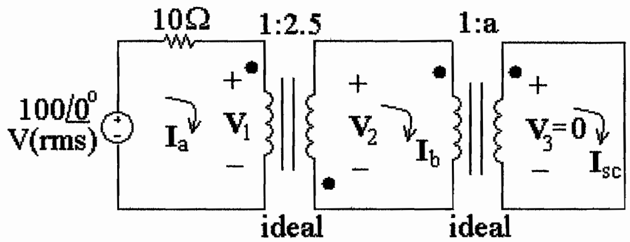


$10\mathbf{I}_a + \mathbf{V}_1 = 100; \qquad \mathbf{I}_a = -2.5\mathbf{I}_b; \qquad \mathbf{V}_1 = -\mathbf{V}_2/2.5$

$\therefore \quad 10(-2.5\mathbf{I}_b) - \mathbf{V}_2/2.5 = 100$

$\mathbf{I}_b = a\mathbf{I}_c = 0; \qquad \mathbf{V}_2 = \mathbf{V}_{oc}/a; \qquad 10[-2.5(0)] - \mathbf{V}_{oc}/2.5a = 100$

$\therefore \quad \mathbf{V}_{oc} = -250a$



$10\mathbf{I}_a + \mathbf{V}_1 = 100; \qquad \mathbf{I}_a = -2.5\mathbf{I}_b; \qquad \mathbf{V}_1 = -\mathbf{V}_2/2.5$

$\therefore \quad 10(-2.5\mathbf{I}_b) - \mathbf{V}_2/2.5 = 100$

$\mathbf{V}_2 = \mathbf{V}_3/a = 0; \qquad \mathbf{I}_b = a\mathbf{I}_{sc}; \qquad 10[-2.5(a\mathbf{I}_{sc})] - 0 = 100$

$\therefore \quad \mathbf{I}_{sc} = 100/(-2.5a) = -4/a$

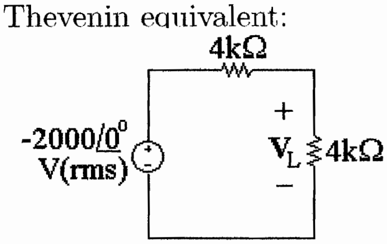
Thus,

$$Z_{Th} = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = \frac{-250a}{-4/a} = 62.5a^2$$

For maximum power to the 4 kΩ load,

$4000 = Z_{Th} = 62.5a^2; \qquad \text{so} \qquad a = 8$

[b] The circuit, with everything to the left of the 4 kΩ load resistor replaced by its Thevenin equivalent:



$$P_L = \frac{\mathbf{V}_L^2}{4000} = \frac{(-1000)^2}{4000} = 250\,\text{W}$$

P 10.60 [a] $Z_{Th} = 32 + j124 + \left(\frac{20}{5}\right)^2 (3 - j4) = 80 + j60 = 100\underline{/36.87^\circ}\,\Omega$

$\therefore \quad Z_{ab} = 100\,\Omega$

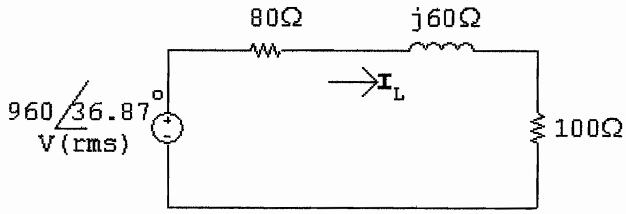
$$Z_{ab} = \frac{Z_L}{(1 + N_1/N_2)^2}$$

$(1 + N_1/N_2)^2 = 3600/100 = 36$

$\therefore \quad N_1/N_2 = 5 \quad \text{or} \quad N_2 = N_1/5$

$\therefore \quad N_2 = 300$ turns

[b] $\mathbf{V}_{\mathrm{Th}} = \dfrac{240/0^\circ}{3 + j4}(j20) = 960\underline{/36.87^\circ}\,\mathrm{V}$

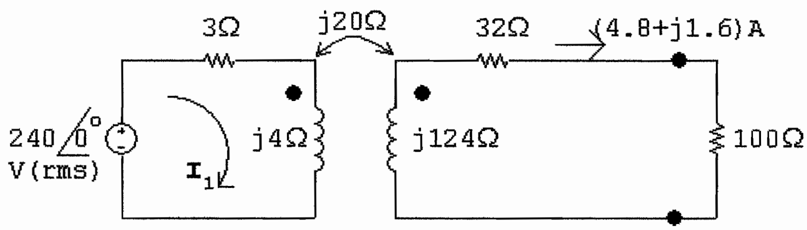


$$\mathbf{I} = \frac{960/36.87^\circ}{180 + j60} = 1.6\sqrt{10}\underline{/18.43^\circ}\,\mathrm{A(rms)}$$

$$|\mathbf{I}| = 1.6\sqrt{10}\,\mathrm{A(rms)}$$

$$P = |\mathbf{I}|^2(100) = 2560\,\mathrm{W}$$

[c]



$$240\underline{/0^\circ} = (3 + j4)\mathbf{I}_1 - j20(4.8 + j1.6)$$

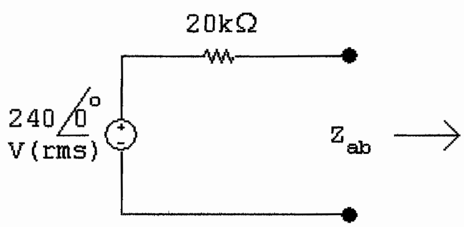
$$\therefore \quad \mathbf{I}_1 = 40.32 - j21.76\,\mathrm{A(rms)}$$

$$P_{\mathrm{gen}} = (240)(40.32) = 9676.80\,\mathrm{W}$$

$$P_{\mathrm{diss}} = 9676.80 - 2560 = 7116.80\,\mathrm{W}$$

$$\%\ \text{dissipated} = \frac{7116.80}{9676.80}(100) = 73.54\%$$

P 10.61 [a]



For maximum power transfer, $Z_{\mathrm{ab}} = 20\,\mathrm{k}\Omega$

$$Z_{\mathrm{ab}} = \left(1 - \frac{N_1}{N_2}\right)^2 Z_{\mathrm{L}}$$

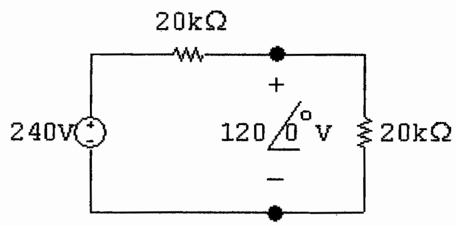
$$\therefore \quad \left(1 - \frac{N_1}{N_2}\right)^2 = \frac{20{,}000}{50} = 400$$

$$1 - \frac{N_1}{N_2} = \pm 20; \qquad \frac{N_1}{N_2} = 1 \mp 20$$

$$\frac{N_1}{N_2} > 0 \qquad \therefore \quad \frac{N_1}{N_2} = 21$$

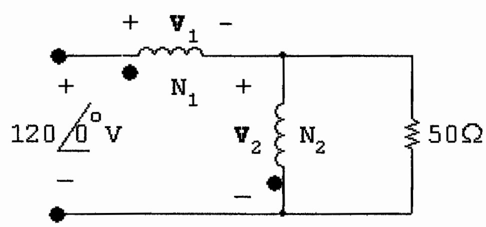
$$N_2 = \frac{N_1}{21} = \frac{2520}{21} = 120 \text{ turns}$$

[b]



$$P_{50\Omega} = P_{20k\Omega} = \frac{(120)^2}{20} \times 10^{-3} = 720\,\text{mW}$$

[c]



$$\mathbf{V}_1 + \mathbf{V}_2 = 120; \qquad \frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2}$$

$$\mathbf{V}_2 = -\frac{N_2}{N_1}\mathbf{V}_1 = -\frac{\mathbf{V}_1}{21}$$

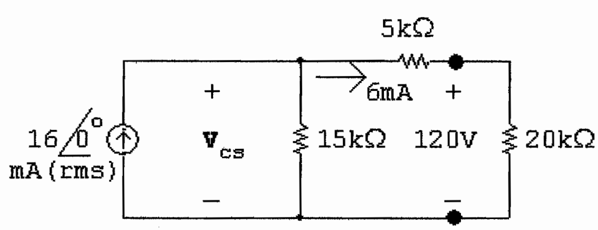
$$\mathbf{V}_1 - \frac{\mathbf{V}_1}{21} = 120; \qquad \therefore \quad \mathbf{V}_1 = 126\,\text{V}$$

$$\therefore \quad \mathbf{V}_2 = -6\,\text{V}$$

Check the power calculation:

$$P_{50\Omega} = \frac{36}{50} = 0.72\,\text{W} = 720\,\text{mW}$$

[d]

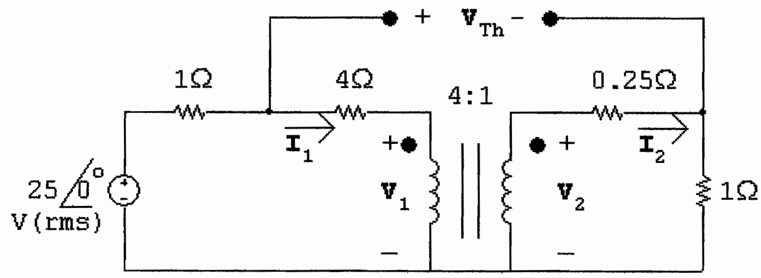


$$\mathbf{V}_{cs} = 120 + (6)(5) = 150\,\text{V}$$

$P_{cs}(\text{del}) = (150)(16) = 2400\,\text{mW}$

$$\%\ \text{delivered} = \frac{720}{2400}(100) = 30\%$$

P 10.62 **[a]**



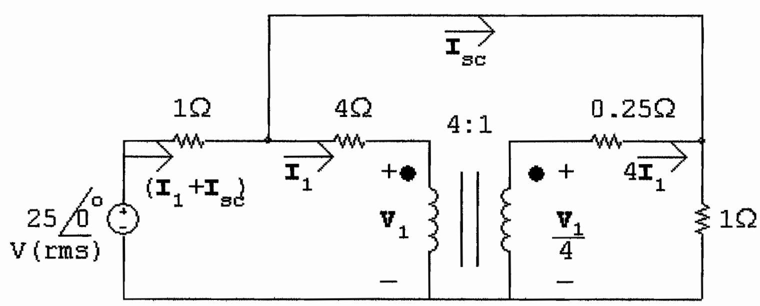
$$\mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_2 = 4\mathbf{I}_1$$

$$25 = 5\mathbf{I}_1 + \mathbf{V}_1$$

$$0 = -\mathbf{V}_2 + 1.25\mathbf{I}_2$$

$$\therefore\ \mathbf{I}_1 = 1\,\text{A}; \qquad \mathbf{I}_2 = 4\,\text{A}$$

$$25 = (1)\mathbf{I}_1 + \mathbf{V}_{\text{Th}} + (1)\mathbf{I}_2; \qquad \therefore\ \mathbf{V}_{\text{Th}} = 20\,\text{V}$$



$$25 = (\mathbf{I}_{sc} + \mathbf{I}_1)(1) + 4\mathbf{I}_1 + \mathbf{V}_1$$

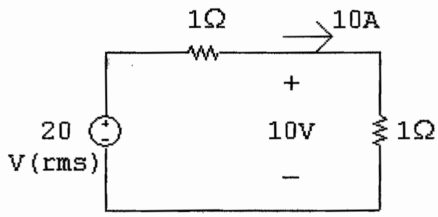
$$25 = (\mathbf{I}_{sc} + \mathbf{I}_1)(1) + (\mathbf{I}_{sc} + 4\mathbf{I}_1)(1)$$

$$\frac{\mathbf{V}_1}{4} = 4\mathbf{I}_1(0.25) + (\mathbf{I}_{sc} + 4\mathbf{I}_1)(1)$$

Solving,

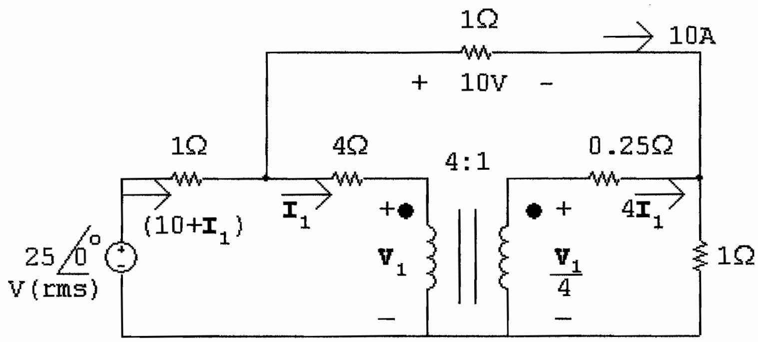
$$\mathbf{I}_{sc} = 20\,\text{A}$$

$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{sc}} = \frac{20}{20} = 1\,\Omega$$



$P = (10)^2(1) = 100\,\text{W}$

[b]



$25 = (10 + \mathbf{I}_1)(1) + 4\mathbf{I}_1 + \mathbf{V}_1$

$\frac{\mathbf{V}_1}{4} = 4\mathbf{I}_1(0.25) + (4\mathbf{I}_1 + 10)(1)$

Solving,

$\mathbf{I}_1 = -1\,\text{A}$

$\therefore \quad P_{\text{source}} = (25)(10 - 1) = 225\,\text{W}$

$\% \text{ delivered} \;=\; \frac{100}{225}(100) = 44.44\%$

[c] $P_{\text{dev}} = 25(10 - 1) = 225\,\text{W}$

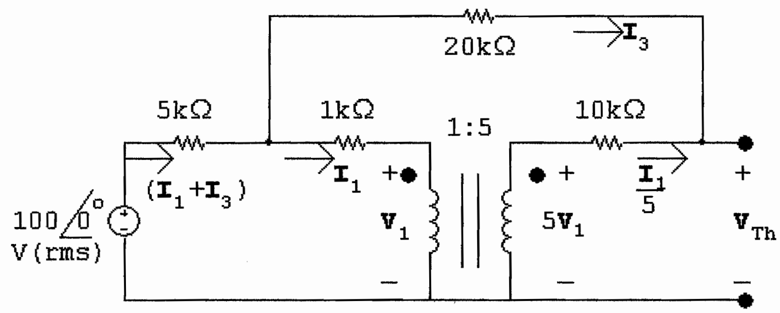
$P_{1\Omega} = (9)^2(1) = 81\,\text{W}; \quad P_{4\Omega} = (-1)^2(4) = 4\,\text{W}$

$P_{1\Omega} = (10)^2(1) = 100\,\text{W}; \quad P_{0.25\Omega} = (-4)^2(0.25) = 4\,\text{W}$

$P_{1\Omega} = (10 - 4)^2(1) = 36\,\text{W}$

$\sum P_{\text{abs}} = 81 + 4 + 100 + 4 + 36 = 225\,\text{W} = \sum P_{\text{dev}}$

P 10.63 [a] Open circuit voltage:



$100\underline{/0^\circ} = 5000(\mathbf{I}_1 + \mathbf{I}_3) + 20{,}000\mathbf{I}_3 + \mathbf{V}_{\rm Th}$

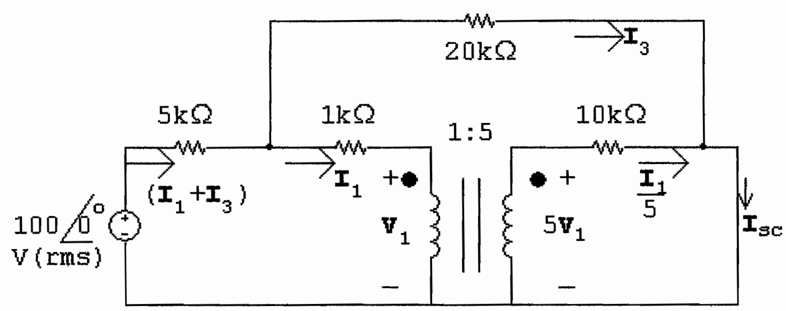
$\mathbf{I}_1 = -5\mathbf{I}_3$

$\therefore \quad 100 = 5000(-5\mathbf{I}_3 + \mathbf{I}_3) + 20{,}000\mathbf{I}_3 + \mathbf{V}_{\rm Th}$

Solving,

$\mathbf{V}_{\rm Th} = 100\underline{/0^\circ}\,\mathrm{V}$

Short circuit current:



$100\underline{/0^\circ} = 5000\mathbf{I}_1 + 5000\mathbf{I}_3 + 1000\mathbf{I}_1 + \mathbf{V}_1$

$5\mathbf{V}_1 = 25{,}000(\mathbf{I}_1/5); \qquad \therefore \quad \mathbf{V}_1 = 1000\mathbf{I}_1$

$\therefore \quad 100\underline{/0^\circ} = 7000\mathbf{I}_1 + 5000\mathbf{I}_3$

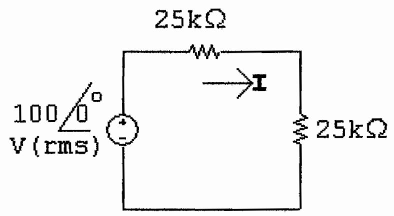
Also,

$100\underline{/0^\circ} = 5000(\mathbf{I}_1 + \mathbf{I}_3) + 20{,}000\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 13.33\,\mathrm{mA}; \qquad \mathbf{I}_3 = 1.33\,\mathrm{mA}; \qquad \mathbf{I}_{\rm sc} = \mathbf{I}_1/5 + \mathbf{I}_3 = 4\,\mathrm{mA}$

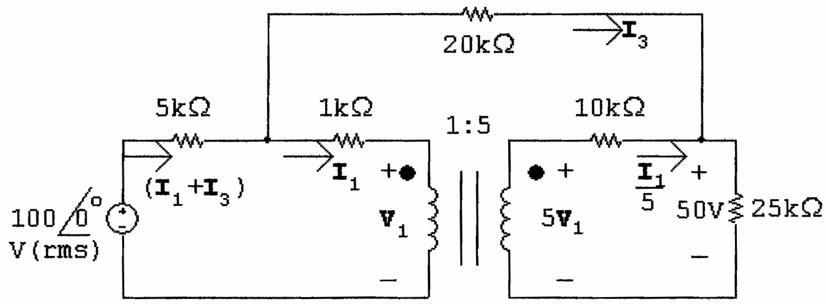
$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{100}{0.004} = 25\,\text{k}\Omega$$



$$\mathbf{I} = \frac{100\underline{/0^\circ}}{50{,}000} = 2\underline{/0^\circ}\,\text{mA(rms)}$$

$$P = (0.002)^2(25{,}000) = 100\,\text{mW}$$

[b]



$$100 = 5000(\mathbf{I}_1 + \mathbf{I}_3) + 20{,}000\mathbf{I}_3 + 50$$

$$5\mathbf{V}_1 = 10{,}000\left(\frac{\mathbf{I}_1}{5}\right) + 50$$

$$100 = 5000(\mathbf{I}_1 + \mathbf{I}_3) + 1000\mathbf{I}_1 + \mathbf{V}_1$$

$$\therefore\quad \mathbf{I}_1 = 14.82\,\text{mA};\qquad \mathbf{I}_3 = -0.963\,\text{mA};\qquad \mathbf{I}_1 + \mathbf{I}_3 = 13.857\underline{/0^\circ}\,\text{mA}$$

$$P_{100V}(\text{developed}) = 100(13.857\,\text{m}) = 1386\,\text{mW}$$

$$\therefore\quad \%\ \text{delivered} = \frac{100}{1386}(100) = 7.22\%$$

[c] $P_{R_L} = 100\,\text{mW};\qquad P_{10\text{k}\Omega} = (2.96\,\text{m})^2(10\,\text{k}) = 87.9\,\text{mW}$

$$P_{20\text{k}\Omega} = (0.963\,\text{m})^2(20\,\text{k}) = 18.6\,\text{mW};\qquad P_{5\text{k}\Omega} = (13.857\,\text{m})^2(5000) = 960.1\,\text{mW}$$

$$P_{1\text{k}\Omega} = (14.82\,\text{m})^2(1000) = 219.6\,\text{mW}$$

$$\sum P_{\text{abs}} = 100 + 87.9 + 18.6 + 960.1 + 219.6 = 1386\,mW = \sum P_{\text{dev}}$$

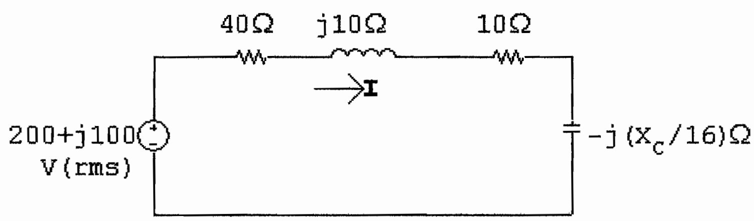
P 10.64 [a] Replace the circuit to the left of the primary winding with a Thévenin equivalent:

$$\mathbf{V}_{\text{Th}} = \frac{250/0^\circ}{25 + j50}(j50) = 200 + j100\,\text{mV}$$

$$Z_{\text{Th}} = 20 + \frac{(25)(j50)}{25 + j50} = 40 + j10\,\Omega$$

Transfer the secondary impedance to the primary side:

$$Z_p = \frac{1}{16}(160 - jX_C) = 10 - j\frac{X_C}{16}\,\Omega$$



Now maximize **I** by setting $(X_C/16) = 10\,\Omega$:

$$\therefore \quad C = \frac{10^{-3}}{(160)(50)} = 125\,\text{nF}$$

[b] $\mathbf{I} = \dfrac{200 + j100}{50} = 4 + j2\,\text{mA}$

$$|\mathbf{I}| = \sqrt{20}\,\text{mA}$$

$$P = (20 \times 10^{-6})(10) = 200\,\mu\text{W}$$

[c] $\dfrac{R_o}{16} = 40\,\Omega; \qquad \therefore \quad R_o = 640\,\Omega$

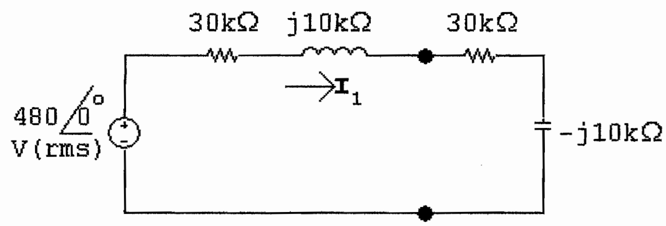
[d] $\mathbf{I} = \dfrac{200 + j100}{80} = 2.5 + j1.25\,\text{mA}$

$$P = |\mathbf{I}|^2(40) = 312.50\,\mu\text{W}$$

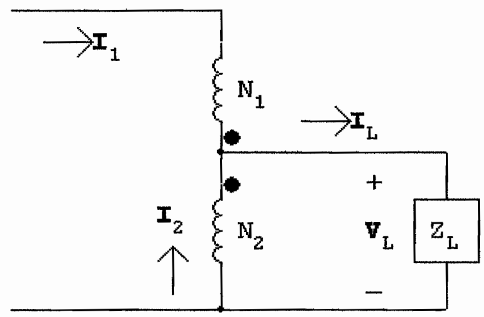
P 10.65 [a] $Z_{\text{ab}} = 30{,}000 - j10{,}000 = \left(1 - \dfrac{N_1}{N_2}\right)^2 Z_{\text{L}}$

$$\therefore \quad Z_{\text{L}} = \frac{1}{4}(30{,}000 - j10{,}000) = 7500 - j2500\,\Omega$$

[b]



$$\mathbf{I}_1 = \frac{480}{60} \times 10^{-3} = 8\underline{/0^\circ}\,\text{mA}$$



$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

$$\mathbf{I}_2 = -3\mathbf{I}_1 = -24\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{I}_\text{L} = \mathbf{I}_1 + \mathbf{I}_2 = -16\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{V}_\text{L} = (7500 - j2500)\mathbf{I}_\text{L} = -120 + j40 = 126.49\underline{/161.57^\circ}\,\text{V(rms)}$$

P 10.66 [a] Begin with the MEDIUM setting, as shown in Fig. 10.31, as it involves only the resistor R_2. Then,

$$P_\text{med} = 500\,\text{W} = \frac{V^2}{R_2} = \frac{120^2}{R_2}$$

Thus,

$$R_2 = \frac{120^2}{500} = 28.8\,\Omega$$

[b] Now move to the LOW setting, as shown in Fig. 10.30, which involves the resistors R_1 and R_2 connected in series:

$$P_\text{low} = \frac{V^2}{R_1 + R_2} = \frac{V^2}{R_1 + 28.8} = 250\,\text{W}$$

Thus,

$$R_1 = \frac{120^2}{250} - 28.8 = 28.8\,\Omega$$

[c] Note that the HIGH setting has R_1 and R_2 in parallel:

$$P_{\text{high}} = \frac{V^2}{R_1 \| R_2} = \frac{120^2}{28.8\|28.8} = 1000\,\text{W}$$

If the HIGH setting has required power other than 1000 W, this problem could not have been solved. In other words, the HIGH power setting was chosen in such a way that it would be satisfied once the two resistor values were calculated to satisfy the LOW and MEDIUM power settings.

P 10.67 [a] $P_L = \dfrac{V^2}{R_1 + R_2}; \qquad R_1 + R_2 = \dfrac{V^2}{P_L}$

$$P_M = \frac{V^2}{R_2}; \qquad R_2 = \frac{V^2}{P_M}$$

$$P_H = \frac{V^2(R_1 + R_2)}{R_1 R_2}$$

$$R_1 + R_2 = \frac{V^2}{P_L}; \qquad R_1 = \frac{V^2}{P_L} - \frac{V^2}{P_M}$$

$$P_H = \frac{V^2 V^2/P_L}{\left(\frac{V^2}{P_L} - \frac{V^2}{P_M}\right)\left(\frac{V^2}{P_M}\right)} = \frac{P_M P_L P_M}{P_L(P_M - P_L)}$$

$$P_H = \frac{P_M^2}{P_M - P_L}$$

[b] $P_H = \dfrac{(750)^2}{(750 - 250)} = 1125\,\text{W}$

P 10.68 First solve the expression derived in P10.67 for P_M as a function of P_L and P_H. Thus

$$P_M - P_L = \frac{P_M^2}{P_H} \quad \text{or} \quad \frac{P_M^2}{P_H} - P_M + P_L = 0$$

$$P_M^2 - P_M P_H + P_L P_H = 0$$

$$\therefore \quad P_M = \frac{P_H}{2} \pm \sqrt{\left(\frac{P_H}{2}\right)^2 - P_L P_H}$$

$$= \frac{P_H}{2} \pm P_H \sqrt{\frac{1}{4} - \left(\frac{P_L}{P_H}\right)}$$

For the specified values of P_L and P_H

$$P_M = 500 \pm 1000\sqrt{0.25 - 0.24} = 500 \pm 100$$

$\therefore \quad P_{M1} = 600\,\text{W}; \qquad P_{M2} = 400\,\text{W}$

Note in this case we design for two medium power ratings
If $P_{M1} = 600\,\text{W}$

$$R_2 = \frac{(120)^2}{600} = 24\,\Omega$$

$$R_1 + R_2 = \frac{(120)^2}{240} = 60\,\Omega$$

$$R_1 = 60 - 24 = 36\,\Omega$$

$$\text{CHECK: } P_{\text{H}} = \frac{(120)^2(60)}{(36)(24)} = 1000\,\text{W}$$

If $P_{M2} = 400\,\text{W}$

$$R_2 = \frac{(120)^2}{400} = 36\,\Omega$$

$$R_1 + R_2 = 60\,\Omega \quad \text{(as before)}$$

$$R_1 = 24\,\Omega$$

$$\text{CHECK: } P_{\text{H}} = 1000\,\text{W}$$

P 10.69 $R_1 + R_2 + R_3 = \dfrac{(120)^2}{600} = 24\,\Omega$

$$R_2 + R_3 = \frac{(120)^2}{900} = 16\,\Omega$$

$$\therefore \quad R_1 = 24 - 16 = 8\,\Omega$$

$$R_3 + R_1 \| R_2 = \frac{(120)^2}{1200} = 12\,\Omega$$

$$\therefore \quad 16 - R_2 + \frac{8R_2}{8 + R_2} = 12$$

$$R_2 - \frac{8R_2}{8 + R_2} = 4$$

$$8R_2 + R_2^2 - 8R_2 = 32 + 4R_2$$

$$R_2^2 - 4R_2 - 32 = 0$$

$$R_2 = 2 \pm \sqrt{4 + 32} = 2 \pm 6$$

$$\therefore \quad R_2 = 8\,\Omega; \qquad \therefore \quad R_3 = 8\,\Omega$$

P 10.70 $R_2 = \dfrac{(220)^2}{500} = 96.8\,\Omega$

$$R_1 + R_2 = \frac{(220)^2}{250} = 193.6\,\Omega$$

$$\therefore \quad R_1 = 96.8\,\Omega$$

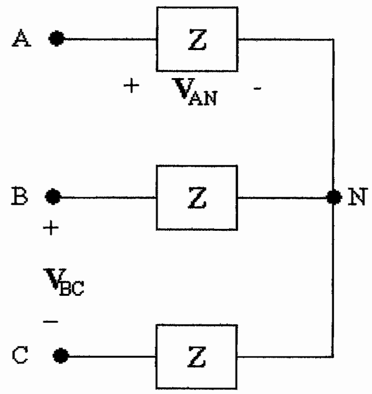
CHECK: $R_1 \| R_2 = 48.4\,\Omega$

$$P_{\mathrm{H}} = \frac{(220)^2}{48.4} = 1000\,\mathrm{W}$$

11
Balanced Three-Phase Circuits

Assessment Problems

AP 11.1 Make a sketch:



We know $\mathbf{V}_{AN}$ and wish to find $\mathbf{V}_{BC}$. To do this, write a KVL equation to find $\mathbf{V}_{AB}$, and use the known phase angle relationship between $\mathbf{V}_{AB}$ and $\mathbf{V}_{BC}$ to find $\mathbf{V}_{BC}$.

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} + \mathbf{V}_{NB} = \mathbf{V}_{AN} - \mathbf{V}_{BN}$$

Since $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$ form a balanced set, and $\mathbf{V}_{AN} = 240\angle{-30^\circ}$V, and the phase sequence is positive,

$$\mathbf{V}_{BN} = |\mathbf{V}_{AN}|\angle{\angle\mathbf{V}_{AN} - 120^\circ} = 240\angle{-30^\circ - 120^\circ} = 240\angle{-150^\circ}\,\text{V}$$

Then,

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = (240\angle{-30^\circ}) - (240\angle{-150^\circ}) = 415.46\angle{0^\circ}\,\text{V}$$

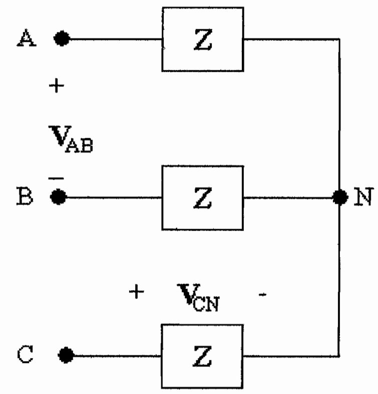
Since $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ form a balanced set with a positive phase sequence, we can find $\mathbf{V}_{BC}$ from $\mathbf{V}_{AB}$:

$$\mathbf{V}_{BC} = |\mathbf{V}_{AB}|\angle{(\angle\mathbf{V}_{AB} - 120^\circ)} = 415.69\angle{0^\circ - 120^\circ} = 415.69\angle{-120^\circ}\,\text{V}$$

Thus,

$$\mathbf{V}_{BC} = 415.69\angle{-120^\circ}\,\text{V}$$

AP 11.2 Make a sketch:



We know $\mathbf{V}_{CN}$ and wish to find $\mathbf{V}_{AB}$. To do this, write a KVL equation to find $\mathbf{V}_{BC}$, and use the known phase angle relationship between $\mathbf{V}_{AB}$ and $\mathbf{V}_{BC}$ to find $\mathbf{V}_{AB}$.

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} + \mathbf{V}_{NC} = \mathbf{V}_{BN} - \mathbf{V}_{CN}$$

Since $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$ form a balanced set, and $\mathbf{V}_{CN} = 450\underline{/-25^\circ}$ V, and the phase sequence is negative,

$$\mathbf{V}_{BN} = |\mathbf{V}_{CN}|\underline{/\angle \mathbf{V}_{CN} - 120^\circ} = 450\underline{/-23^\circ - 120^\circ} = 450\underline{/-145^\circ}\text{ V}$$

Then,

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = (450\underline{/-145^\circ}) - (450\underline{/-25^\circ}) = 779.42\underline{/-175^\circ}\text{ V}$$

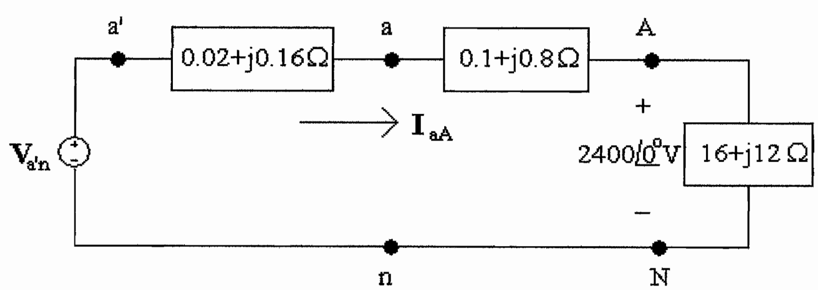
Since $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ form a balanced set with a negative phase sequence, we can find $\mathbf{V}_{AB}$ from $\mathbf{V}_{BC}$:

$$\mathbf{V}_{AB} = |\mathbf{V}_{BC}|\underline{/\angle \mathbf{V}_{BC} - 120^\circ} = 779.42\underline{/-295^\circ}\text{ V}$$

But we normally want phase angle values between +180° and −180°. We add 360° to the phase angle computed above. Thus,

$$\mathbf{V}_{AB} = 779.42\underline{/65^\circ}\text{ V}$$

AP 11.3 Sketch the a-phase circuit:



[a] We can find the line current using Ohm's law, since the a-phase line current is the current in the a-phase load. Then we can use the fact that $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$ form a balanced set to find the remaining line currents. Note that since we were not given any phase angles in the problem statement, we can assume that the phase voltage given, $\mathbf{V}_{AN}$, has a phase angle of 0°.

$$2400\underline{/0^\circ} = \mathbf{I}_{aA}(16 + j12)$$

so

$$\mathbf{I}_{aA} = \frac{2400\underline{/0^\circ}}{16 + j12} = 96 - j72 = 120\underline{/-36.87^\circ}\,\text{A}$$

With an acb phase sequence,

$$\underline{/\mathbf{I}_{bB}} = \underline{/\mathbf{I}_{aA}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{I}_{cC}} = \underline{/\mathbf{I}_{aA}} - 120^\circ$$

so

$$\mathbf{I}_{aA} = 120\underline{/-36.87^\circ}\,\text{A}$$

$$\mathbf{I}_{bB} = 120\underline{/83.13^\circ}\,\text{A}$$

$$\mathbf{I}_{cC} = 120\underline{/-156.87^\circ}\,\text{A}$$

[b] The line voltages at the source are $\mathbf{V}_{ab}$ $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$. They form a balanced set. To find $\mathbf{V}_{ab}$, use the a-phase circuit to find $\mathbf{V}_{AN}$, and use the relationship between phase voltages and line voltages for a y-connection (see Fig. 11.9[b]). From the a-phase circuit, use KVL:

$$\mathbf{V}_{an} = \mathbf{V}_{aA} + \mathbf{V}_{AN} = (0.1 + j0.8)\mathbf{I}_{aA} + 2400\underline{/0^\circ}$$

$$= (0.1 + j0.8)(96 - j72) + 2400\underline{/0^\circ} = 2467.2 + j69.6$$

$$2468.18\underline{/1.62^\circ}\,\text{V}$$

From Fig. 11.9(b),

$$\mathbf{V}_{ab} = \mathbf{V}_{an}(\sqrt{3}\underline{/-30^\circ}) = 4275.02\underline{/-28.38^\circ}\,\text{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{bc}} = \underline{/\mathbf{V}_{ab}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{ca}} = \underline{/\mathbf{V}_{ab}} - 120^\circ$$

so

$$\mathbf{V}_{ab} = 4275.02\underline{/-28.38^\circ}\,\text{V}$$

$$\mathbf{V}_{bc} = 4275.02\underline{/91.62^\circ}\,\text{V}$$

$$\mathbf{V}_{ca} = 4275.02\underline{/-148.38^\circ}\,\text{V}$$

[c] Using KVL on the a-phase circuit

$$\mathbf{V}_{\mathrm{a'n}} = \mathbf{V}_{\mathrm{a'a}} + \mathbf{V}_{\mathrm{an}} = (0.2 + j0.16)\mathbf{I}_{\mathrm{aA}} + \mathbf{V}_{\mathrm{an}}$$

$$= (0.02 + j0.16)(96 - j72) + (2467.2 + j69.9)$$

$$= 2480.64 + j83.52 = 2482.05\underline{/1.93^\circ}\,\mathrm{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{\mathrm{b'n}}} = \underline{/\mathbf{V}_{\mathrm{a'n}}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{\mathrm{c'n}}} = \underline{/\mathbf{V}_{\mathrm{a'n}}} - 120^\circ$$

so

$$\mathbf{V}_{\mathrm{a'n}} = 2482.05\underline{/1.93^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{b'n}} = 2482.05\underline{/121.93^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{c'n}} = 2482.05\underline{/-118.07^\circ}\,\mathrm{V}$$

AP 11.4

$$\mathbf{I}_{\mathrm{cC}} = (\sqrt{3}\underline{/-30^\circ})\mathbf{I}_{\mathrm{CA}} = (\sqrt{3}\underline{/-30^\circ}) \cdot 8\underline{/-15^\circ} = 13.86\underline{/-45^\circ}\,\mathrm{A}$$

AP 11.5

$$\mathbf{I}_{\mathrm{aA}} = 12\underline{/(65^\circ - 120^\circ)} = 12\underline{/-55^\circ}$$

$$\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right]\mathbf{I}_{\mathrm{aA}} = \left(\frac{\underline{/-30^\circ}}{\sqrt{3}}\right) \cdot 12\underline{/-55^\circ}$$

$$= 6.93\underline{/-85^\circ}\,\mathrm{A}$$

AP 11.6 [a] $\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/20^\circ}\,\mathrm{A}$

Therefore $Z_\phi = \dfrac{4160\underline{/0^\circ}}{40\underline{/20^\circ}} = 104\underline{/-20^\circ}\,\Omega$

[b] $\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/-40^\circ}\,\mathrm{A}$

Therefore $Z_\phi = 104\underline{/40^\circ}\,\Omega$

AP 11.7

$$\mathbf{I}_\phi = \frac{110}{3.667} + \frac{110}{j2.75} = 30 - j40 = 50\underline{/-53.13^\circ}\,\mathrm{A}$$

Therefore $|\mathbf{I}_{\mathrm{aA}}| = \sqrt{3}\mathbf{I}_\phi = \sqrt{3}(50) = 86.60\,\mathrm{A}$

AP 11.8 [a] $|S| = \sqrt{3}(208)(73.8) = 26{,}587.67\,\mathrm{VA}$

$$Q = \sqrt{(26{,}587.67)^2 - (22{,}659)^2} = 13{,}909.50\,\mathrm{VAR}$$

[b] $\text{pf} = \dfrac{22{,}659}{26{,}587.67} = 0.8522 \quad \text{lagging}$

AP 11.9 [a] $\mathbf{V}_{\text{AN}} = \left(\dfrac{4160}{\sqrt{3}}\right)\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{\text{AN}}\mathbf{I}^*_{\text{aA}} = S_\phi = 384 + j288\,\text{kVA}$

Therefore

$$\mathbf{I}^*_{\text{aA}} = \frac{(384 + j288)1000}{4160/\sqrt{3}} = (159.88 + j119.91)\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 159.88 - j119.91 = 199.85\underline{/-36.87^\circ}\,\text{A}$$

$$|\mathbf{I}_{\text{aA}}| = 199.85\,\text{A}$$

[b] $P = \dfrac{(4160)^2}{R}; \qquad \text{therefore} \quad R = \dfrac{(4160)^2}{384{,}000} = 45.07\,\Omega$

$$Q = \frac{(4160)^2}{X}; \qquad \text{therefore} \quad X = \frac{(4160)^2}{288{,}000} = 60.09\,\Omega$$

[c] $Z_\phi = \dfrac{\mathbf{V}_{\text{AN}}}{\mathbf{I}_{\text{aA}}} = \dfrac{4160/\sqrt{3}}{199.85\underline{/-36.87^\circ}} = 12.02\underline{/36.87^\circ} = (9.61 + j7.21)\,\Omega$

$$\therefore \quad R = 9.61\,\Omega, \qquad X = 7.21\,\Omega$$

Problems

P 11.1 [a] First, convert the cosine waveforms to phasors:

$\mathbf{V}_a = 120\underline{/54^\circ}$; $\mathbf{V}_b = 120\underline{/-66^\circ}$; $\mathbf{V}_c = 120\underline{/174^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_a} = 54^\circ - 54^\circ = 0^\circ$

$\underline{/\mathbf{V}'_b} = -66^\circ - 54^\circ = -120^\circ$

$\underline{/\mathbf{V}'_c} = 174^\circ - 54^\circ = 120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore abc

[b] First, convert the cosine waveforms to phasors:

$\mathbf{V}_a = 3240\underline{/-26^\circ}$; $\mathbf{V}_b = 3240\underline{/94^\circ}$; $\mathbf{V}_c = 3240\underline{/-146^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_a} = -26^\circ + 26^\circ = 0^\circ$

$\underline{/\mathbf{V}'_b} = 94^\circ + 26^\circ = 120^\circ$

$\underline{/\mathbf{V}'_c} = -146^\circ + 26^\circ = -120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore acb

P 11.2 [a] $\mathbf{V}_a = 339\underline{/0^\circ}$ V

$\mathbf{V}_b = 339\underline{/-120^\circ}$ V

$\mathbf{V}_c = 339\underline{/120^\circ}$ V

Balanced, positive phase sequence

[b] $\mathbf{V}_a = 622\underline{/0^\circ}$ V

$\mathbf{V}_b = 622\underline{/-240^\circ}$ V $= 622\underline{/120^\circ}$ V

$\mathbf{V}_c = 622\underline{/240^\circ}$ V $= 622\underline{/-120^\circ}$ V

Balanced, negative phase sequence

[c] $\mathbf{V}_a = 933\underline{/-90^\circ}$ V

$\mathbf{V}_b = 933\underline{/150^\circ}$ V

$\mathbf{V}_c = 933\underline{/30^\circ}$ V

Balanced, positive phase sequence

[d] $\mathbf{V}_a = 170\underline{/-30^\circ}\,\text{V}$

$\mathbf{V}_b = 170\underline{/90^\circ}\,\text{V}$

$\mathbf{V}_c = 170\underline{/-150^\circ}\,\text{V}$

Balanced, negative phase sequence

[e] Unbalanced, due to unequal amplitudes

[f] Unbalanced, due to unequal phase angle separation

P 11.3 $\mathbf{V}_a = V_m\underline{/0^\circ} = V_m + j0$

$\mathbf{V}_b = V_m\underline{/-120^\circ} = -V_m(0.5 + j0.866)$

$\mathbf{V}_c = V_m\underline{/120^\circ} = V_m(-0.5 + j0.866)$

$$\begin{aligned}\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c &= (V_m)(1 + j0 - 0.5 - j0.866 - 0.5 + j0.866)\\ &= V_m(0) = 0\end{aligned}$$

P 11.4 $\mathbf{I} = \dfrac{\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c}{3(R_W + jX_W)} = 0$

P 11.5 [a] The circuit is unbalanced, because the impedance in each phase of the load is not the same.

[b] $\mathbf{I}_{aA} = \dfrac{240\underline{/0^\circ}}{10 + j30} = 2.4 - j7.2\,\text{A}$

$\mathbf{I}_{bB} = \dfrac{240\underline{/120^\circ}}{20 + j20} = 2.2 + j8.2\,\text{A}$

$\mathbf{I}_{cC} = \dfrac{240\underline{/-120^\circ}}{20 - j40} = 2.96 - j4.48\,\text{A}$

$\mathbf{I}_o = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 7.55 - j3.48 = 8.32\underline{/-24.75^\circ}\,\text{A}$

P 11.6 [a] $\mathbf{I}_{aA} = \dfrac{240\underline{/0^\circ}}{80 + j60} = 2.4\underline{/-36.87^\circ}\,\text{A}$

$\mathbf{I}_{bB} = \dfrac{240\underline{/120^\circ}}{80 + j60} = 2.4\underline{/83.13^\circ}\,\text{A}$

$\mathbf{I}_{cC} = \dfrac{240\underline{/-120^\circ}}{80 + j60} = 2.4\underline{/-156.87^\circ}\,\text{A}$

$\mathbf{I}_o = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 0$

[b] $\mathbf{V}_{AN} = (79 + j55)\mathbf{I}_{aA} = (79 + j55)(2.4\underline{/-36.87^\circ}) = 231.0\underline{/-2.02^\circ}\,\text{V}$

[c] $\mathbf{V}_{\rm BN} = (79 + j52)\mathbf{I}_{\rm bB} = 226.99\underline{/116.48^\circ}\,\text{V}$

$$\therefore \qquad \mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm BN} = 393.6\underline{/-32.5^\circ}\,\text{V}$$

[d] Unbalanced

P 11.7 $Z_{ga} + Z_{la} + Z_{La} = 80 + j60\,\Omega$

$$Z_{gb} + Z_{lb} + Z_{Lb} = 40 + j30\Omega$$

$$Z_{gc} + Z_{lc} + Z_{Lc} = 160 + j120\Omega$$

$$\frac{\mathbf{V}_N - 480}{80 + j60} + \frac{\mathbf{V}_N - 480\underline{/-120^\circ}}{40 + j30} + \frac{\mathbf{V}_N - 480\underline{/120^\circ}}{160 + j120} + \frac{\mathbf{V}_N}{20} = 0$$

Solving for $\mathbf{V}_N$ yields

$$\mathbf{V}_N = 78.61\underline{/-122.69^\circ}\,\text{V}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_N}{20} = 3.93\underline{/-122.69^\circ}\,\text{A}$$

P 11.8 $\mathbf{V}_{\rm AN} = 7967\underline{/0^\circ}\,\text{V}$

$$\mathbf{V}_{\rm BN} = 7967\underline{/+120^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm CN} = 7967\underline{/-120^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm BN} = 13{,}799.25\underline{/-30^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN} = 13{,}799.25\underline{/90^\circ}\,\text{V}$$

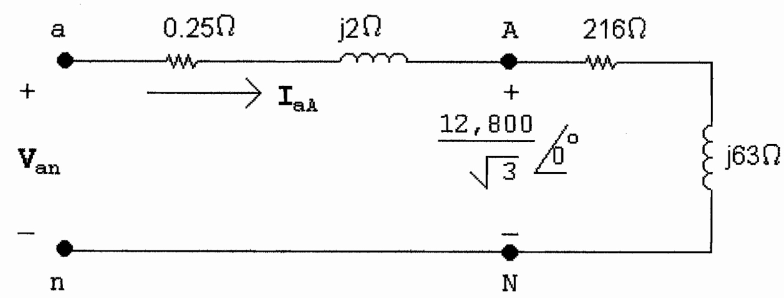
$$\mathbf{V}_{\rm CA} = \mathbf{V}_{\rm CN} - \mathbf{V}_{\rm AN} = 13{,}799.25\underline{/-150^\circ}\,\text{V}$$

$$v_{\rm AB} = 13{,}799.25\cos(\omega t - 30^\circ)\,\text{V}$$

$$v_{\rm BC} = 13{,}799.25\cos(\omega t + 90^\circ)\,\text{V}$$

$$v_{\rm CA} = 13{,}799.25\cos(\omega t - 150^\circ)\,\text{V}$$

P 11.9 [a]



$$\mathbf{I}_{aA} = \frac{12{,}800}{\sqrt{3}(216 + j63)} = 32.84\underline{/-16.26^\circ}\,\mathrm{A(rms)}$$

$$|\mathbf{I}_{aA}| = |\mathbf{I}_L| = 32.84\,\mathrm{A(rms)}$$

[b] $$\mathbf{V}_{an} = \frac{12{,}800}{\sqrt{3}} + (32.84\underline{/-16.26^\circ})(0.25 + j2) = 7416.61\underline{/0.47^\circ}$$

$$|\mathbf{V}_{AB}| = \sqrt{3}(7416.61) = 12{,}845.94\,\mathrm{V(rms)}$$

P 11.10 [a] $$\mathbf{I}_{aA} = \frac{4800\underline{/0^\circ}}{192 + j56} = 24\underline{/-16.26^\circ}\,\mathrm{A}$$

$$\mathbf{I}_{bB} = 24\underline{/120 - 16.26^\circ} = 24\underline{/103.74^\circ}\,\mathrm{A}$$

$$\mathbf{I}_{cC} = 24\underline{/-136.26^\circ}\,\mathrm{A}$$

[b] $$\mathbf{V}_{an} = 4800\underline{/0^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{bn} = 4800\underline{/120^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{cn} = 4800\underline{/-120^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{ab} = \sqrt{3}\underline{/-30^\circ}\mathbf{V}_{an} = 8313.84\underline{/-30^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{bc} = 8313.84\underline{/90^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{ca} = 8313.84\underline{/-150^\circ}\,\mathrm{V}$$

[c] $$\mathbf{V}_{AN} = (24\underline{/-16.26^\circ})(190 + j40) = 4659.96\underline{/-4.37^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{BN} = 4659.96\underline{/115.63^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{CN} = 4659.96\underline{/-124.37^\circ}\,\mathrm{V}$$

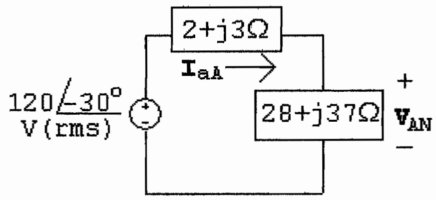
[d] $$\mathbf{V}_{AB} = \sqrt{3}\underline{/-30^\circ}\mathbf{V}_{AN} = 8071.28\underline{/-34.37^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{BC} = 8071.28\underline{/85.63^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{CA} = 8071.28\underline{/-154.37^\circ}\,\mathrm{V}$$

P 11.11 [a] $\mathbf{V}_{an} = 1/\sqrt{3}\underline{/-30^\circ}\mathbf{V}_{ab} = 120\underline{/-30^\circ}\,\text{V(rms)}$

The a-phase circuit is



[b] $\mathbf{I}_{aA} = \dfrac{120\underline{/-30^\circ}}{30 + j40} = 2.4\underline{/-83.13^\circ}\,\text{A(rms)}$

[c] $\mathbf{V}_{AN} = (28 + j37)\mathbf{I}_{aA} = 111.36\underline{/-30.25^\circ}\,\text{V(rms)}$

$\mathbf{V}_{AB} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{AN} = 192.88\underline{/-0.25^\circ}\,\text{A(rms)}$

P 11.12 [a] $\mathbf{I}_{AB} = \dfrac{33{,}000}{360 + j105} = 88\underline{/-16.26^\circ}\,\text{A}$

$\mathbf{I}_{BC} = 88\underline{/-136.26^\circ}\,\text{A}$

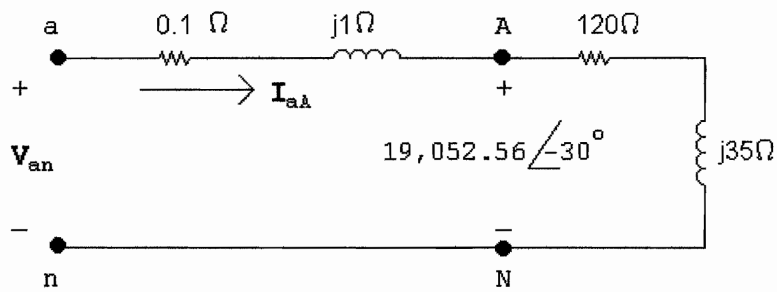
$\mathbf{I}_{CA} = 88\underline{/103.74^\circ}\,\text{A}$

[b] $\mathbf{I}_{aA} = \sqrt{3}\underline{/-30^\circ}\mathbf{I}_{AB} = 152.42\underline{/-46.26^\circ}\,\text{A}$

$\mathbf{I}_{bB} = 152.42\underline{/-166.26^\circ}\,\text{A}$

$\mathbf{I}_{cC} = 152.42\underline{/73.74^\circ}\,\text{A}$

[c]



$\mathbf{V}_{an} = 19{,}052.56\underline{/-30^\circ} + (0.1 + j1.0)(152.42\underline{/-46.26^\circ})$

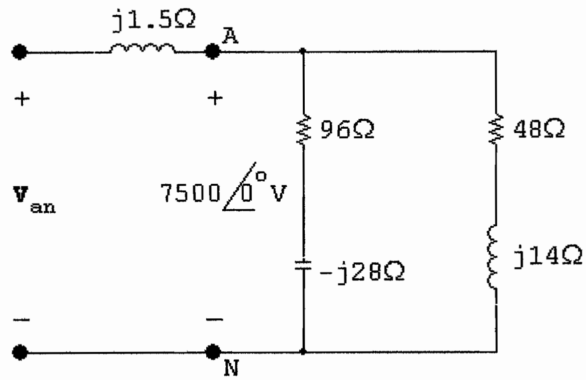
$= 19{,}110.40\underline{/-29.57^\circ}\,\text{V}$

$\mathbf{V}_{ab} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{an} = 33{,}100.18\underline{/0.43^\circ}\,\text{V}$

$\mathbf{V}_{bc} = 33{,}100.18\underline{/-119.57^\circ}\,\text{V}$

$\mathbf{V}_{ca} = 33{,}100.18\underline{/120.43^\circ}\,\text{V}$

P 11.13 [a]



$$\mathbf{I}_{aA} = \frac{7500}{96 - j28} + \frac{7500}{48 + j14} = 217.02\underline{/-5.55^\circ}\text{ A}$$

$$|\mathbf{I}_{aA}| = 217.02\text{ A}$$

[b] $$\mathbf{I}_{AB} = \frac{7500\sqrt{3}\underline{/30^\circ}}{144 + j42} = 86.60\underline{/13.74^\circ}\text{ A}$$

$$|\mathbf{I}_{AB}| = 86.60\text{ A}$$

[c] $$\mathbf{I}_{AN} = \frac{7500\underline{/0^\circ}}{96 - j28} = 75\underline{/16.26^\circ}\text{ A}$$

$$|\mathbf{I}_{AN}| = 75\text{ A}$$

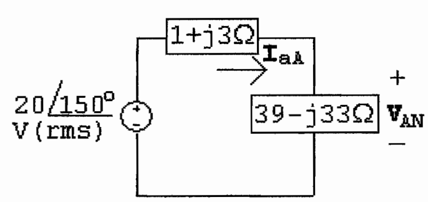
[d] $$\mathbf{V}_{an} = (216 - j21)(j1.5) + 7500\underline{/0^\circ} = 7538.47\underline{/2.46^\circ}\text{ V}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(7538.47) = 13{,}057.01\text{ V}$$

P 11.14 [a] $\mathbf{V}_{an} = \mathbf{V}_{bn} - \underline{/120^\circ} = 20\underline{/-210^\circ} = 20\underline{/150^\circ}\text{ V(rms)}$

$$Z_y = Z_\Delta/3 = 39 - j33\,\Omega$$

The a-phase circuit is



$$\mathbf{I}_{aA} = \frac{20\underline{/150^\circ}}{40 - j30} = 0.4\underline{/-173.13^\circ}\text{ A(rms)}$$

$$\mathbf{V}_{AN} = (39 + j33)\mathbf{I}_{aA} = 20.44\underline{/146.63^\circ}\text{ V(rms)}$$

$$\mathbf{V}_{AB} = \sqrt{3}\underline{/-30^\circ}\mathbf{V}_{AN} = 35.39\underline{/116.63^\circ}\text{ A(rms)}$$

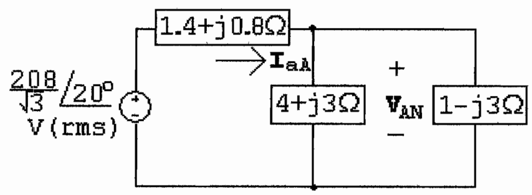
[b] $\mathbf{I}_{AB} = \frac{1}{\sqrt{3}}\underline{/-30^\circ}\mathbf{I}_{aA} = 0.23\underline{/156.87^\circ}\,\text{A(rms)}$

[c] $\mathbf{V}_{AB} = (117 - j99)\mathbf{I}_{AB} = 35.3\underline{/116.63^\circ}\,\text{V(rms)}$

P 11.15 $\mathbf{V}_{an} = 1/\sqrt{3}\underline{/-30^\circ}\mathbf{V}_{ab} = \frac{208}{\sqrt{3}}\underline{/20^\circ}\,\text{V(rms)}$

$Z_y = Z_\Delta/3 = 1 - j3\,\Omega$

The a-phase circuit is



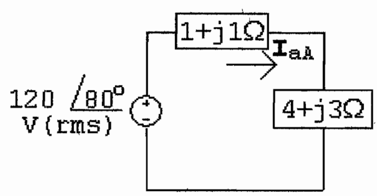
$$Z_{eq} = (4 + j3)\|(1 - j3) = 2.6 - j1.8\,\Omega$$

$$\mathbf{V}_{AN} = \frac{2.6 - j1.8}{(1.4 + j0.8) + (2.6 - j1.8)}\left(\frac{208}{\sqrt{3}}\right)\underline{/20^\circ} = 92.1\underline{/-0.66^\circ}\,\text{V(rms)}$$

$$\mathbf{V}_{AB} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{AN} = 159.5\underline{/29.34^\circ}\,\text{V(rms)}$$

P 11.16 $Z_y = Z_\Delta/3 = 4 + j3\,\Omega$

The a-phase circuit is



$$\mathbf{I}_{aA} = \frac{120\underline{/80^\circ}}{(1 + j1) + (4 + j3)} = 18.74\underline{/41.34^\circ}\,\text{A(rms)}$$

$$\mathbf{I}_{AB} = \frac{1}{\sqrt{3}}\underline{/30^\circ}\mathbf{I}_{aA} = 10.82\underline{/71.34^\circ}\,\text{A(rms)}$$

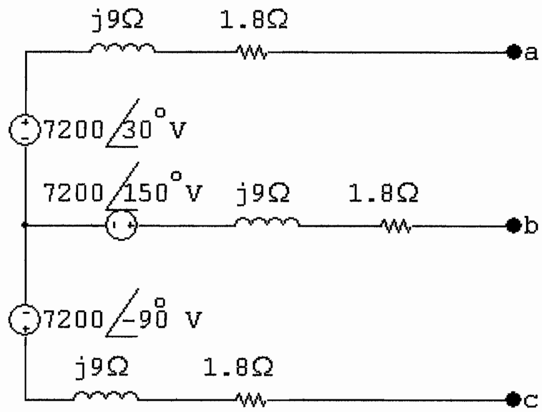
P 11.17 [a] Since the phase sequence is acb (negative) we have:

$$\mathbf{V}_{an} = 7200\underline{/30^\circ}\text{ V}$$

$$\mathbf{V}_{bn} = 7200\underline{/150^\circ}\text{ V}$$

$$\mathbf{V}_{cn} = 7200\underline{/-90^\circ}\text{ V}$$

$$Z_Y = \frac{1}{3}Z_\Delta = 1.8 + j9.0\,\Omega/\phi$$

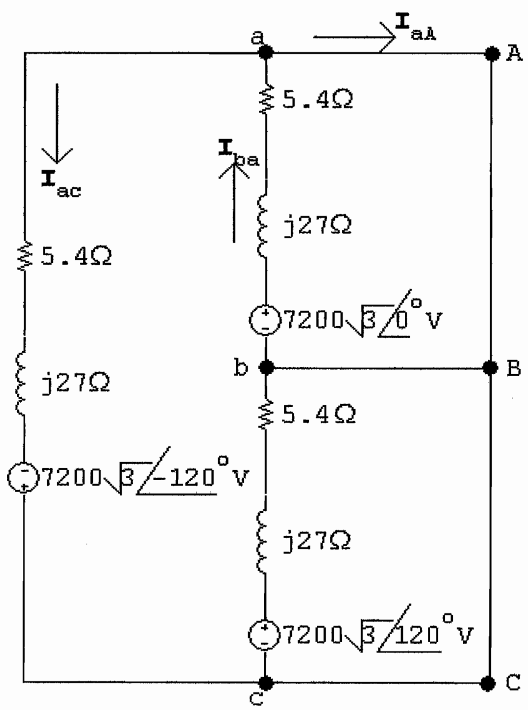


[b] $\mathbf{V}_{ab} = 7200\underline{/30^\circ} - 7200\underline{/150^\circ} = 7200\sqrt{3}\underline{/0^\circ}\text{ V}$

Since the phase sequence is negative, it follows that

$$\mathbf{V}_{bc} = 7200\sqrt{3}\underline{/120^\circ}\text{ V}$$

[c]



$$\mathbf{I}_{ba} = \frac{7200\sqrt{3}}{5.4 + j27} = 452.91\underline{/-78.69^\circ}\text{ A}$$

$$\mathbf{I}_{\rm ac} = \frac{7200\sqrt{3}\underline{/-120^\circ}}{5.4 + j27} = 452.91\underline{/-198.69^\circ}\,\text{A}$$

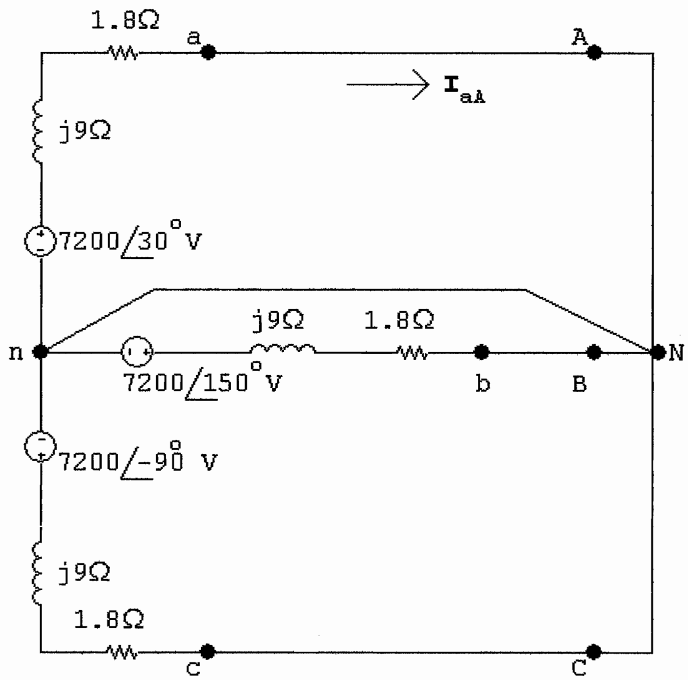
$$\mathbf{I}_{\rm aA} = \mathbf{I}_{\rm ba} - \mathbf{I}_{\rm ac} = 784.46\underline{/-48.69^\circ}\,\text{A}$$

Since we have a balanced three-phase circuit and a negative phase sequence we have:

$$\mathbf{I}_{\rm bB} = 784.46\underline{/71.31^\circ}\,\text{A}$$

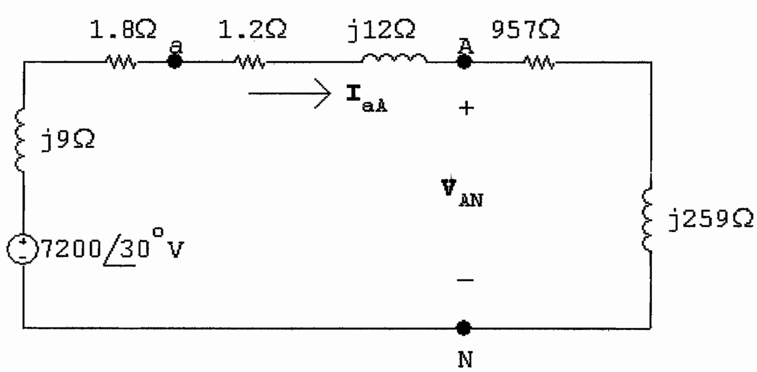
$$\mathbf{I}_{\rm cC} = 784.46\underline{/-168.69^\circ}\,\text{A}$$

[d]



$$\mathbf{I}_{\rm aA} = \frac{7200\underline{/30^\circ}}{1.8 + j9} = 784.46\underline{/-48.69^\circ}\,\text{A}$$

P 11.18 [a]



[b] $\mathbf{I}_{aA} = \dfrac{7200\underline{/30^\circ}}{960 + j280} = 7.2\underline{/13.74^\circ}\text{ A}$

$\mathbf{V}_{AN} = (957 + j259)(7.2\underline{/13.74^\circ}) = 7138.28\underline{/28.88^\circ}\text{ V}$

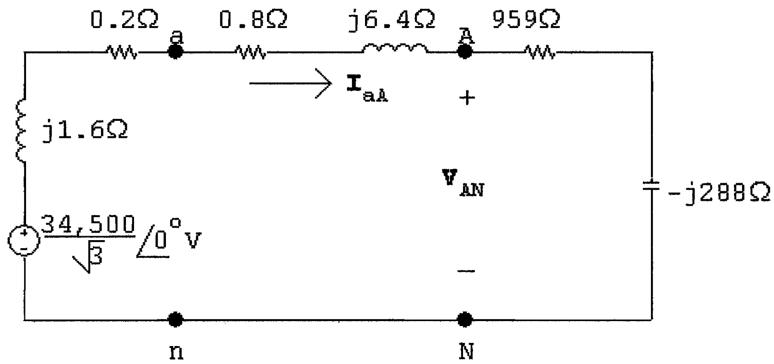
$|\mathbf{V}_{AB}| = \sqrt{3}(7138.28) = 12{,}363.87\text{ V}$

[c] $|\mathbf{I}_{ba}| = \dfrac{7.2}{\sqrt{3}} = 4.16\text{ A}$

[d] $\mathbf{V}_{an} = (958.2 + j271)(7.20\underline{/13.74^\circ}) = 7169.65\underline{/29.54^\circ}\text{ V}$

$|\mathbf{V}_{ab}| = \sqrt{3}(7169.65) = 12{,}418.20\text{ V}$

P 11.19 [a]



[b] $\mathbf{I}_{aA} = \dfrac{34{,}500}{\sqrt{3}(960 - j280)} = 19.92\underline{/16.26^\circ}\text{ A}$

$|\mathbf{I}_{aA}| = 19.92\text{ A}$

[c] $\mathbf{V}_{AN} = (959 - j288)(19.92\underline{/16.26^\circ}) = 19{,}944.71\underline{/-0.46^\circ}\text{ V}$

$|\mathbf{V}_{AB}| = \sqrt{3}|\mathbf{V}_{AN}| = 34{,}545.25\text{ V}$

[d] $\mathbf{V}_{an} = (959.8 - j281.6)(19.92\underline{/16.26^\circ}) = 19{,}923.71\underline{/-0.09^\circ}\text{ V}$

$|\mathbf{V}_{ab}| = \sqrt{3}|\mathbf{V}_{an}| = 34{,}508.88\text{ V}$

[e] $|\mathbf{I}_{AB}| = \dfrac{|\mathbf{I}_{aA}|}{\sqrt{3}} = 11.50\text{ A}$

[f] $|\mathbf{I}_{ba}| = |\mathbf{I}_{AB}| = 11.50\text{ A}$

P 11.20 [a] $\mathbf{I}_{AB} = \dfrac{69{,}000\underline{/0^\circ}}{600 + j450} = 92\underline{/-36.87^\circ}\text{ A}$

$\mathbf{I}_{BC} = 92\underline{/-156.87^\circ}\text{ A}$

$\mathbf{I}_{CA} = 92\underline{/83.13^\circ}\text{ A}$

[b] $\mathbf{I}_{aA} = \sqrt{3}\underline{/-30^\circ}\mathbf{I}_{AB} = 159.35\underline{/-66.87^\circ}\text{ A}$

$\mathbf{I}_{bB} = 159.35\underline{/-186.87^\circ}\text{ A}$

$\mathbf{I}_{cC} = 159.35\underline{/53.13^\circ}\text{ A}$

[c] $\mathbf{I}_{ba} = \mathbf{I}_{AB} = 92\underline{/-36.87^\circ}$ A

$\mathbf{I}_{cb} = \mathbf{I}_{BC} = 92\underline{/-156.87^\circ}$ A

$\mathbf{I}_{ac} = \mathbf{I}_{CA} = 92\underline{/83.13^\circ}$ A

P 11.21 [a] $\mathbf{I}_{AB} = \dfrac{720\underline{/0^\circ}}{4.8 + j1.4} = 144\underline{/-16.26^\circ}$ A

$$\mathbf{I}_{BC} = \frac{720\underline{/-120^\circ}}{16 - j12} = 36\underline{/-83.13^\circ}\text{ A}$$

$$\mathbf{I}_{CA} = \frac{720\underline{/120^\circ}}{25 + j25} = 20.36\underline{/75^\circ}\text{ A}$$

[b] $\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$

$= 138.24 - j40.32 - 5.27 - j19.67$

$= 132.97 - j59.99 = 145.88\underline{/-24.28^\circ}$ A

$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB}$

$= 4.31 - j35.74 - 138.24 + j40.32$

$= -133.93 + j4.58 = 134.01\underline{/178.04^\circ}$ A

$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC}$

$= 5.27 + j19.67 - 4.31 + j35.74$

$= 0.96 + j55.41 = 55.42\underline{/89.01^\circ}$ A

P 11.22 The complex power of the source per phase is $S_s = 30{,}000\underline{/(\cos^{-1} 0.8)} = 30{,}000\underline{/36.87^\circ} = 24{,}000 + j18{,}000$ kVA. This complex power per phase must equal the sum of the per-phase impedances of the two loads:

$$S_s = S_1 + S_2 \quad \text{so} \quad 24{,}000 + j18{,}000 = 20{,}000 + S_2$$

$$\therefore \quad S_2 = 4000 + j18{,}000 \text{ VA}$$

Also, $S_2 = \dfrac{|V_{rms}|^2}{Z_2^*}$

$$|V_{rms}| = \frac{|\mathrm{V}_{load}|}{\sqrt{3}} = \frac{415.69}{\sqrt{3}} = 240 \text{ V(rms)}$$

Thus, $Z_2^* = \dfrac{|V_{rms}|^2}{S_2} = \dfrac{(240)^2}{4000 + j18{,}000} = 0.68 - j3.05\,\Omega$

$$\therefore \quad Z_2 = 0.68 + j3.05\,\Omega$$

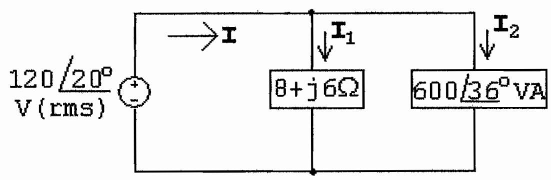
P 11.23 $|I_{\rm line}| = \dfrac{1200}{208/\sqrt{3}} = 10 \text{ A(rms)}$

$$|Z_y| = \frac{|V|}{|I|} = \frac{208/\sqrt{3}}{10} = 12$$

$$Z_y = 12\underline{/25^\circ}\,\Omega$$

$$Z_\Delta = 3Z_y = 36\underline{/25^\circ} = 32.63 + j15.21\,\Omega/\phi$$

P 11.24 The a-phase of the circuit is shown below:



$$\mathbf{I}_1 = \frac{120\underline{/20^\circ}}{8 + j6} = 12\underline{/-16.87^\circ}\text{A(rms)}$$

$$\mathbf{I}_2^* = \frac{600\underline{/36^\circ}}{120\underline{/20^\circ}} = 5\underline{/16^\circ}\text{A(rms)}$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 12\underline{/-16.87^\circ} + 5\underline{/-16^\circ} = 17\underline{/-16.61^\circ}\text{ A(rms)}$$

$$S_a = \mathbf{VI}^* = (120\underline{/20^\circ})(17\underline{/16.61^\circ}) = 2040\underline{/36.61^\circ}\text{ VA}$$

$$S_T = 3S_a = 6120\underline{/36.61^\circ}\text{ VA}$$

P 11.25 [a] $S_{T\Delta} = 14{,}000\underline{/41.41^\circ} - 9000\underline{/53.13^\circ} = 5.5\underline{/22^\circ}$ kVA

$$S_\Delta = S_{T\Delta}/3 = 1833.46\underline{/22^\circ}\text{ VA}$$

[b] $|\mathbf{V}_{\rm an}| = \left|\dfrac{3000\underline{/53.13^\circ}}{10\underline{/-30^\circ}}\right| = 300$ V(rms)

$$|\mathbf{V}_{\rm line}| = |\mathbf{V}_{\rm ab}| = \sqrt{3}|\mathbf{V}_{\rm an}| = 300\sqrt{3} = 519.62\text{ V(rms)}$$

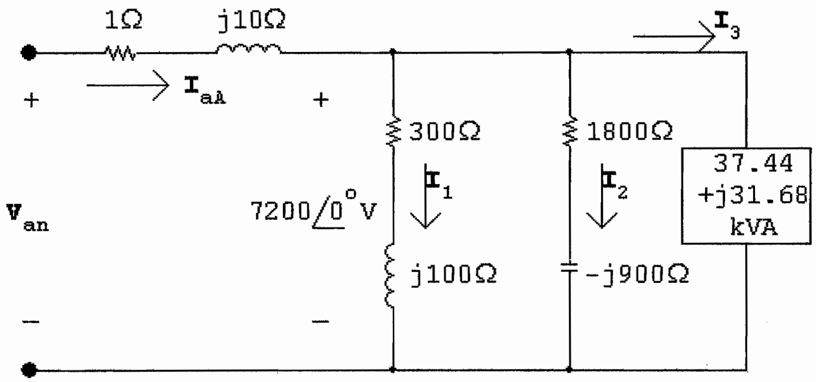
P 11.26 From the solution to Problem 11.21 we have:

$$S_{\rm AB} = (720\underline{/0^\circ})(144\underline{/16.26^\circ}) = 99{,}532.9 + j29{,}030.04\,\text{VA}$$

$$S_{\rm BC} = (720\underline{/-120^\circ})(36\underline{/83.13^\circ}) = 20{,}735.97 - j15{,}552.04\,\text{VA}$$

$$S_{\rm CA} = (720\underline{/120^\circ})(20.36\underline{/-75^\circ}) = 10{,}365.62 + j10{,}365.62\,\text{VA}$$

P 11.27 [a]



$$\mathbf{I}_1 = \frac{7200\underline{/0^\circ}}{300 + j100} = 21.6 - j7.2\,\text{A}$$

$$\mathbf{I}_2 = \frac{7200\underline{/0^\circ}}{1800 - j900} = 3.2 + j1.6\,\text{A}$$

$$\mathbf{I}_3^* = \frac{37{,}440 + j31{,}680}{7200} = 5.2 + j4.4$$

$$\mathbf{I}_3 = 5.2 - j4.4\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 30 - j10\,\text{A} = \sqrt{1000}\underline{/-18.43^\circ}\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 7200 + j0 + (30 - j10)(1 + j10) = 7330 + j290\,\text{V}$$

$$S_\phi = \mathbf{V}_{\text{an}}\mathbf{I}_{\text{aA}}^* = (7330 + j290)(30 + j10) = 217{,}000 + j82{,}000\,\text{VA}$$

$$S_T = 3S_\phi = 651 + j246\,\text{kVA}$$

[b] $S_{1/\phi} = 7200(21.6 + j7.2) = 155.52 + j51.84\,\text{kVA}$

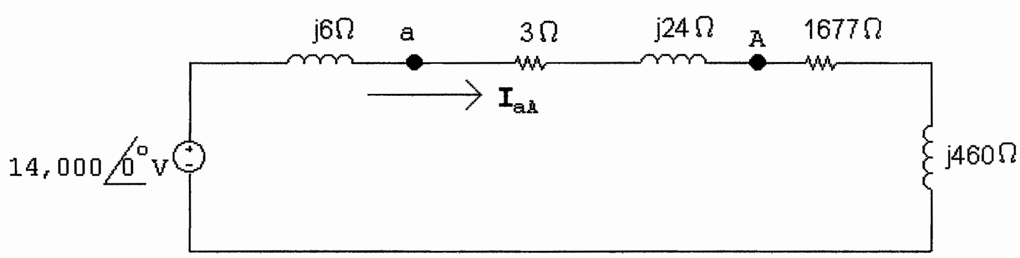
$$S_{2\phi} = 7200(3.2 - j1.6) = 23.04 - j11.52\,\text{kVA}$$

$$S_{3\phi} = 37.44 + j31.68\,\text{kVA}$$

$$S_\phi(\text{load}) = 216 + j72\,\text{kVA}$$

$$\%\text{ delivered } = \left(\frac{216}{217}\right)(100) = 99.54\%$$

P 11.28 [a]



$$\mathbf{I}_{\text{aA}} = \frac{14{,}000\underline{/0^\circ}}{1680 + j490} = 8\underline{/-16.26^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{CA}} = \frac{\mathbf{I}_{\text{aA}}}{\sqrt{3}}\underline{/150^\circ} = 4.62\underline{/133.74^\circ}\,\text{A}$$

[b] $S_{g/\phi} = -14{,}000\mathbf{I}^*_{\text{aA}} = -107{,}520 - j31{,}360\,\text{VA}$

$$\therefore \quad P_{\text{developed/phase}} = 107.52\,\text{kW}$$

$$P_{\text{absorbed/phase}} = |\mathbf{I}_{\text{aA}}|^2 1677 = 107.328\,\text{kW}$$

$$\%\ \text{delivered} = \frac{107.328}{107.52}(100) = 99.82\%$$

P 11.29 Let p_{a}, p_{b}, and p_{c} represent the instantaneous power of phases a, b, and c, respectively. Then assuming a positive phase sequence, we have

$$p_{\text{a}} = v_{\text{an}} i_{\text{aA}} = [V_m \cos\omega t][I_m \cos(\omega t - \theta_\phi)]$$

$$p_{\text{b}} = v_{\text{bn}} i_{\text{bB}} = [V_m \cos(\omega t - 120^\circ)][I_m \cos(\omega t - \theta_\phi - 120^\circ)]$$

$$p_{\text{c}} = v_{\text{cn}} i_{\text{cC}} = [V_m \cos(\omega t + 120^\circ)][I_m \cos(\omega t - \theta_\phi + 120^\circ)]$$

The total instantaneous power is $p_T = p_{\text{a}} + p_{\text{b}} + p_{\text{c}}$, so

$$p_T = V_m I_m [\cos\omega t \cos(\omega t - \theta_\phi) + \cos(\omega t + 120^\circ)\cos(\omega t - \theta_\phi - 120^\circ)$$
$$+ \cos(\omega t - 120^\circ)\cos(\omega t - \theta_\phi + 120^\circ)]$$

Now simplify using trigonometric identities. In simplifying, collect the coefficients of $\cos(\omega t - \theta_\phi)$ and $\sin(\omega t - \theta_\phi)$. We get

$$\begin{aligned} p_T &= V_m I_m [\cos\omega t (1 + 2\cos^2 120^\circ)\cos(\omega t - \theta_\phi) \\ &\quad + 2\sin\omega t \sin^2 120^\circ \sin(\omega t - \theta_\phi)] \\ &= 1.5 V_m I_m [\cos\omega t \cos(\omega t - \theta_\phi) + \sin\omega t \sin(\omega t - \theta_\phi)] \\ &= 1.5 V_m I_m \cos\theta_\phi \end{aligned}$$

P 11.30 [a] $S_1 = 72 - j21\,\text{kVA}$

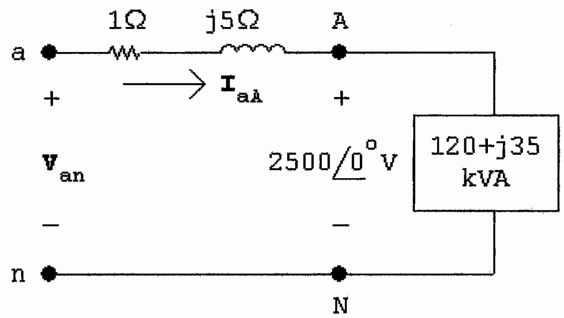
$$S_2 = 120 + j90\,\text{kVA}$$

$$S_3 = 168 + j36\,\text{kVA}$$

$$S_T = S_1 + S_2 + S_3 = 360 + j105\,\text{kVA}$$

$$S_T/\phi = 120 + j35\,\text{kVA}$$

Single phase equivalent circuit



$$\therefore \quad \mathbf{I}_{aA}^* = \frac{120{,}000 + j35{,}000}{2500} = 48 + j14$$

$$\therefore \quad \mathbf{I}_{aA} = 48 - j14\,\text{A} = 50\underline{/-16.26^\circ}\,\text{A}$$

$$\mathbf{V}_{an} = 2500 + (1 + j5)(48 - j14) = 2618 + j226$$

$$= 2627.74\underline{/4.93^\circ}\,\text{V}$$

$$\therefore \quad |\mathbf{V}_{ab}| = \sqrt{3}(2627.74) = 4551.4\,\text{V}$$

[b] $P_L/\phi = 120\,\text{kW}$

$$P_S/\phi = 120{,}000 + |\mathbf{I}_{aA}|^2(1) = 122{,}500\,\text{W} = 122.5\,\text{kW}$$

$$\eta = \left(\frac{120}{122.5}\right)100 = 97.96\%$$

P 11.31 [a] $S_1 = (5.742 + j4.008)\,\text{kVA}$

$$S_2 = 18.566(0.93) + j18.566(0.37) = (17.266 + j6.824)\,\text{kVA}$$

$$\sqrt{3}V_L I_L \sin\theta_3 = 11{,}623; \qquad \sin\theta_3 = \frac{11{,}623}{\sqrt{3}(208)(81.6)} = 0.395$$

Therefore $\cos\theta_3 = 0.919$

Therefore

$$P_3 = \frac{11{,}623}{0.395} \times 0.919 = 27{,}041.67\,\text{W}$$

$$S_3 = 27.042 + j11.623\,\text{kVA}$$

$$S_T = S_1 + S_2 + S_3 = 50.05 + j22.455\,\text{kVA}$$

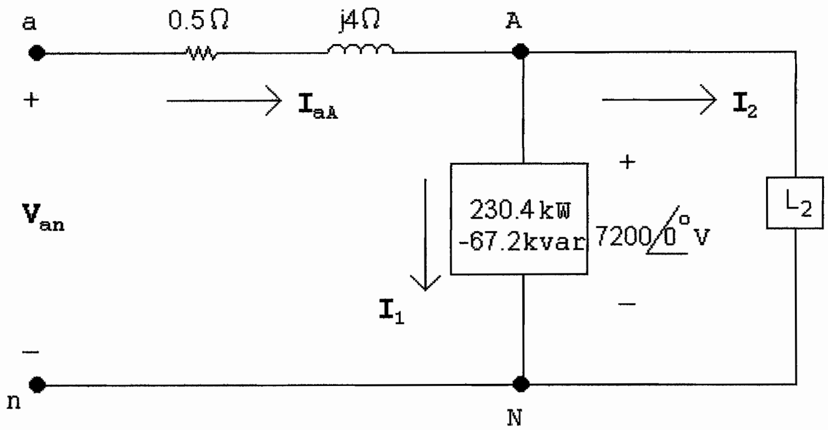
$$S_{T/\phi} = \frac{1}{3}S_T = 16.68 + j7.49\,\text{kVA}$$

$$\frac{208}{\sqrt{3}}\mathbf{I}_{aA}^* = (16.68 + j7.49)10^3; \qquad \mathbf{I}_{aA}^* = 138.92 + j62.33\,\text{A}$$

$$\mathbf{I}_{aA} = 138.92 - j62.33 = 152.26\underline{/-24.16^\circ}\,\text{A} \quad \text{(rms)}$$

[b] pf $= \cos(-24.16^\circ) = 0.912$ leading

P 11.32



$$7200\mathbf{I}_1^* = (230.4 - j67.2)10^3$$

$$\mathbf{I}_1^* = 32 - j9.33\,\text{A}$$

$$\mathbf{I}_1 = 32 + j9.33\,\text{A}$$

$$Z_y = \frac{1}{3}Z_\Delta = 207.36 + j60.48\,\Omega$$

$$\mathbf{I}_2 = \frac{7200\underline{/0^\circ}}{207.36 + j60.48} = 32 - j9.33\,\text{A}$$

$$\therefore\quad \mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 = 64 + j0\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 7200 + j0 + 64(0.5 + j4) = 7236.53\underline{/2.03^\circ}\,\text{V}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 12{,}534.04\,\text{V}$$

P 11.33 [a] $P_{\text{OUT}} = 746 \times 200 = 149{,}200\,\text{W}$

$$P_{\text{IN}} = 149{,}200/(0.96) = 155{,}416.67\,\text{W}$$

$$\sqrt{3}V_L I_L \cos\theta = 155{,}416.67$$

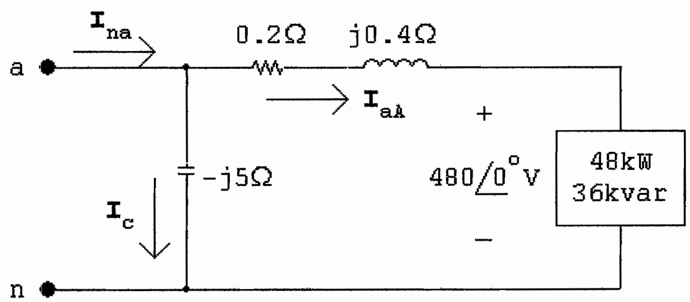
$$I_L = \frac{155{,}416.67}{\sqrt{3}(208)(0.92)} = 468.91\,\text{A}$$

[b] $Q = \sqrt{3}V_L I_L \sin\phi = \sqrt{3}(208)(468.91)(0.39) = 66{,}207.79\,\text{VAR}$

P 11.34 $\mathbf{I}_{aA}^* = \dfrac{(48 + j36)10^3}{480} = 100 + j75$

$$\mathbf{I}_{aA} = 100 - j75\,\text{A}$$

$$\mathbf{V}_{an} = 480 + j0 + (100 - j75)(0.2 + j0.4) = 530 + j25\,\text{V}$$



$$\mathbf{I}_C = \frac{530 + j25}{-j5} = -5 + j106\,\text{A}$$

$$\mathbf{I}_{na} = \mathbf{I}_{aA} + \mathbf{I}_C = 95 + j31 = 99.93\underline{/18.07^\circ}\,\text{A}$$

[b] $S_{g/\phi} = (530 + j25)(95 - j31) = 51{,}125 - j14{,}055\,\text{VA}$

$$S_{gT} = 3S_{g/\phi} = 153{,}375 - j42{,}165\,\text{VA}$$

Therefore, the source is delivering 153,375 W and absorbing 42,165 vars.

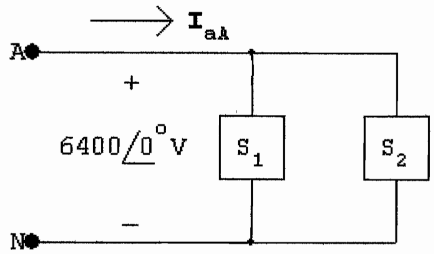
[c] $P_{\text{del}} = 153{,}375\,\text{W}$

$$P_{\text{abs}} = 3(48{,}000) + 3|\mathbf{I}_{aA}|^2(0.2) = 144{,}000 + 9375$$
$$= 153{,}375\,\text{W} = P_{\text{del}}$$

[d] $Q_{\text{del}} = 3|\mathbf{I}_C|^2(5) = 168{,}915\,\text{VAR}$

$$Q_{\text{abs}} = 3(36{,}000) + 42{,}165 + 3|\mathbf{I}_{aA}|^2(0.4)$$
$$= 168{,}915\,\text{VAR} = Q_{\text{del}}$$

P 11.35 [a]



$$S_1 = \frac{1}{3}(1800)(0.96 - j0.28) = 576 - j168\,\text{kVA}$$

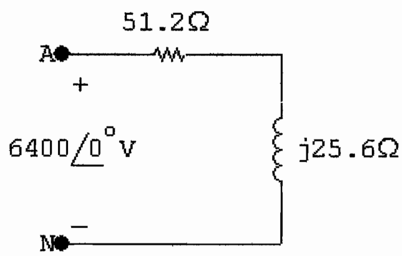
$$S_2 = \frac{1}{3}(192 + j1464) = 64 + j488\,\text{kVA}$$

$$S_1 + S_2 = 640 + j320\,\text{kVA}$$

$$\therefore\quad \mathbf{I}^*_{\text{aA}} = \frac{(640 + j320)10^3}{6400} = 100 + j50$$

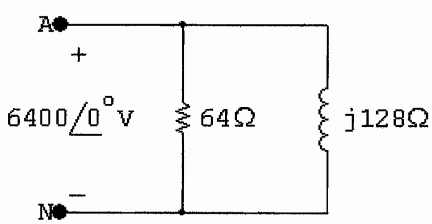
$$\mathbf{I}_{\text{aA}} = 100 - j50\,\text{A}$$

$$Z = \frac{6400}{100 - j50} = 51.2 + j25.6\,\Omega$$



[b] $$R = \frac{(6400)^2}{640 \times 10^3} = 64\,\Omega$$

$$X_{\text{L}} = \frac{(6400)^2}{320 \times 10^3} = 128\,\Omega$$

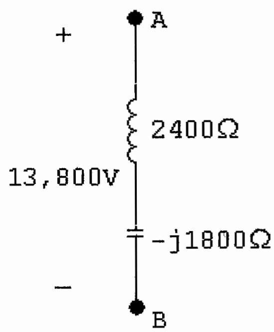


P 11.36 Assume a Δ-connect load (series):

$$S_{\phi} = \frac{1}{3}(190.44 \times 10^3)(0.8 - j0.6) = 50{,}784 - j38{,}088\,\text{VA}$$

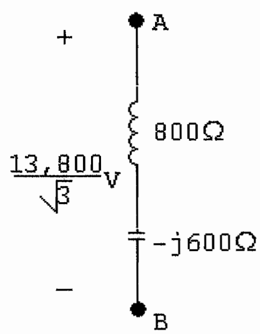
$$Z^*_{\Delta\phi} = \frac{|13{,}800|^2}{50{,}784 - j38{,}088} = 3000\angle 36.87^\circ\,\Omega$$

$$Z_{\Delta\phi} = 3000\angle -36.87^\circ = 2400 - j1800\,\Omega$$



Now assume a Y-connected load (series):

$$Z_{Y\phi} = \frac{1}{3}Z_{\Delta\phi} = 800 - j600\,\Omega$$



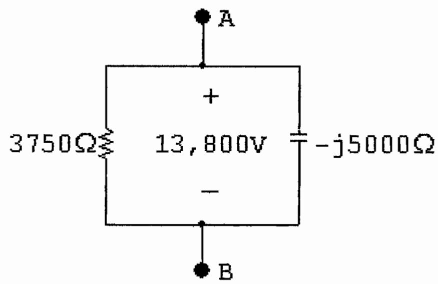
Now assume a Δ-connected load (parallel):

$$P_{\phi} = \frac{|13{,}800|^2}{R_{\Delta}}$$

$$R_{\Delta\phi} = \frac{|13{,}800|^2}{50{,}784} = 3750\,\Omega$$

$$Q_{\phi} = \frac{|13{,}800|^2}{X_{\Delta}}$$

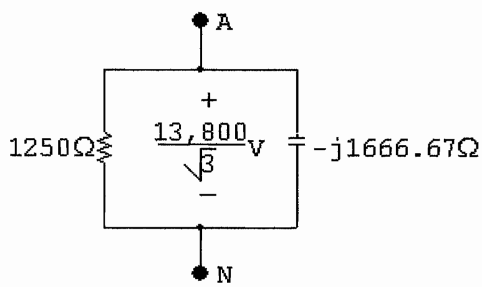
$$X_{\Delta}\phi = \frac{|13{,}800|^2}{-38{,}088} = -5000\,\Omega$$



Now assume a Y-connected load (parallel):

$$R_{Y\phi} = \frac{1}{3}R_{\Delta\phi} = 1250\,\Omega$$

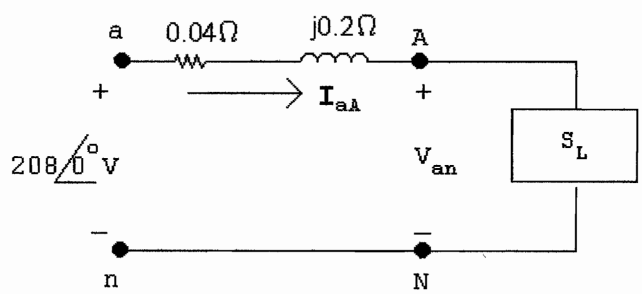
$$X_{Y\phi} = \frac{1}{3}X_{\Delta\phi} = -1666.67\,\Omega$$



P 11.37 $S_{g/\phi} = \frac{1}{3}(78)(0.8 - j0.6) \times 10^3 = 20{,}800 - j15{,}600\,\text{VA}$

$$\mathbf{I}^*_{\text{aA}} = \frac{20{,}800 - j15{,}600}{208} = 100 - j75\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 100 + j75\,\text{A}$$



$$\mathbf{V}_{\text{AN}} = 208 - (100 + j75)(0.04 + j0.20)$$

$$= 219 - j23 = 220.20\underline{/-6^\circ}\,\text{V}$$

$$|\mathbf{V}_{AB}| = \sqrt{3}(220.20) = 381.41\,\text{V}$$

[b] $S_{L/\phi} = (219 - j23)(100 - j75) = 20{,}175 - j18{,}725\,\text{VA}$

$$S_L = 3S_{L/\phi} = 60{,}525 - j56{,}175\,\text{VA}$$

Check:

$$S_g = 3(20{,}800 - j15{,}600) = 62{,}400 - j46{,}800\,\text{VA}$$

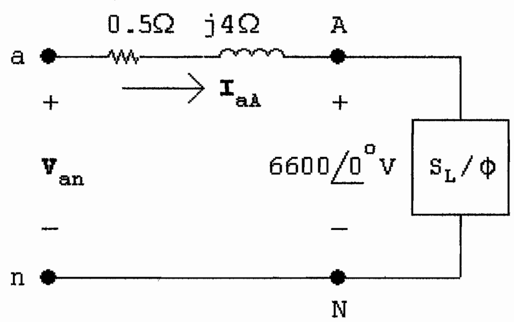
$$P_\ell = 3|\mathbf{I}_{aA}|^2(0.04) = 1875\,\text{W}$$

$$P_g = P_L + P_\ell = 60{,}525 + 1875 = 62{,}400\,\text{W} \quad \text{(checks)}$$

$$Q_\ell = 3|\mathbf{I}_{aA}|^2(0.20) = 9375\,\text{VAR}$$

$$Q_g = Q_L + Q_\ell = -56{,}175 + 9375 = -46{,}800\,\text{VAR} \quad \text{(checks)}$$

P 11.38 [a]



$$S_{L/\phi} = \frac{1}{3}\left[1188 + j\frac{1188}{0.6}(0.8)\right]10^3 = 396{,}000 + j528{,}000\,\text{VA}$$

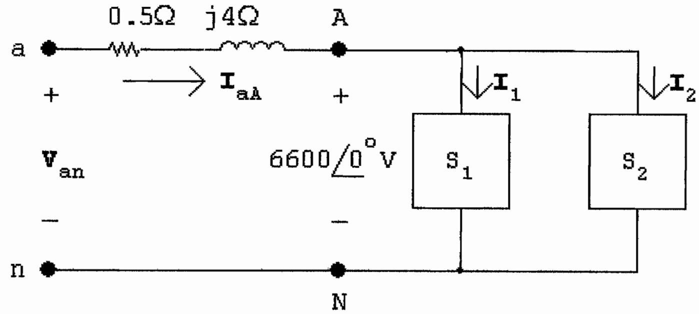
$$\mathbf{I}_{aA}^* = \frac{396{,}000 + j528{,}000}{6600} = 60 + j80\,\text{A}$$

$$\mathbf{I}_{aA} = 60 - j80\,\text{A}$$

$$\mathbf{V}_{an} = 6600 + (60 - j80)(0.5 + j4)$$
$$= 6950 + j200 = 6952.88\underline{/1.65^\circ}\,\text{V}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(6952.88) = 12{,}042.74\,\text{V}$$

[b]



$\mathbf{I}_1 = 60 - j80\,\text{A}$ (from part [a])

$$S_2 = 0 - j\frac{1}{3}(1920) \times 10^3 = -j640{,}000\,\text{VAR}$$

$$\mathbf{I}_2^* = \frac{-j640{,}000}{6600} = -j96.97\,\text{A}$$

$$\therefore\ \mathbf{I}_2 = j96.97\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 60 - j80 + j96.97 = 60 + j16.97\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 6600 + (60 + j16.97)(0.5 + j4)$$

$$= 6562.12 + j248.485 = 6566.82\underline{/2.17^\circ}\,\text{V}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(6566.82) = 11{,}374.07\,\text{V}$$

[c] $|\mathbf{I}_{\text{aA}}| = 100\,\text{A}$

$$P_{\text{loss}/\phi} = (100)^2(0.5) = 5000\,\text{W}$$

$$P_{g/\phi} = 396{,}000 + 5000 = 401\,\text{kW}$$

$$\%\,\eta = \frac{396}{401}(100) = 98.75\%$$

[d] $|\mathbf{I}_{\text{aA}}| = 62.354\,\text{A}$

$$P_{\ell/\phi} = (3887.98)(0.5) = 1943.99\,\text{W}$$

$$\%\,\eta = \frac{396{,}000}{397{,}944}(100) = 99.51\%$$

[e] $Z_{\text{cap/Y}} = -j\dfrac{6600}{96.97} = -j68.062\,\Omega$

$$Z_{\text{cap}/\Delta} = 3Z_{\text{cap/Y}} = -j204.187\,\Omega$$

$$\therefore\ \frac{1}{\omega C} = 204.187; \qquad C = \frac{1}{(204.187)(120\pi)} = 12.99\,\mu\text{F}$$

P 11.39 [a] From Assessment Problem 11.9, $\mathbf{I}_{\text{aA}} = (159.88 - j119.91)\,\text{A}$

Therefore $\mathbf{I}_{\text{cap}} = j119.91\,\text{A}$

Therefore $Z_{CY} = \dfrac{4160/\sqrt{3}}{j119.91} = -j20.03\,\Omega$

Therefore $C_Y = \dfrac{1}{(20.03)(2\pi)(60)} = 132.43\,\mu\text{F}$

$Z_{C\Delta} = (-j20.03)(3) = -j60.09\,\Omega$

Therefore $C_\Delta = \dfrac{132.43}{3} = 44.14\,\mu\text{F}$

[b] $C_Y = 132.43\,\mu\text{F}$

[c] $|\mathbf{I}_{\text{aA}}| = 159.88\,\text{A}$

P 11.40 $Z_\phi = |Z|\underline{/\theta} = \dfrac{\mathbf{V}_{\text{AN}}}{\mathbf{I}_{\text{aA}}}$

$\theta = \underline{/\mathbf{V}_{\text{AN}}} - \underline{/\mathbf{I}_{\text{aA}}}$

$\theta_1 = \underline{/\mathbf{V}_{\text{AB}}} - \underline{/\mathbf{I}_{\text{aA}}}$

For a positive phase sequence,

$\underline{/\mathbf{V}_{\text{AB}}} = \underline{/\mathbf{V}_{\text{AN}}} + 30^\circ$

Thus,

$\theta_1 = \underline{/\mathbf{V}_{\text{AN}}} + 30^\circ - \underline{/\mathbf{I}_{\text{aA}}} = \theta + 30^\circ$

Similarly,

$Z_\phi = |Z|\underline{/\theta} = \dfrac{\mathbf{V}_{\text{CN}}}{\mathbf{I}_{\text{cC}}}$

$\theta = \underline{/\mathbf{V}_{\text{CN}}} - \underline{/\mathbf{I}_{\text{cC}}}$

$\theta_2 = \underline{/\mathbf{V}_{\text{CB}}} - \underline{/\mathbf{I}_{\text{cC}}}$

For a positive phase sequence,

$\underline{/\mathbf{V}_{\text{CB}}} = \underline{/\mathbf{V}_{\text{BA}}} - 120^\circ = \underline{/\mathbf{V}_{\text{AB}}} + 60^\circ$

$\underline{/\mathbf{I}_{\text{cC}}} = \underline{/\mathbf{I}_{\text{aA}}} + 120^\circ$

Thus,

$$\theta_2 = \underline{/\mathbf{V}_{\text{AB}}} + 60^\circ - \underline{/\mathbf{I}_{\text{aA}}} + 120^\circ = \theta_1 - 60^\circ$$
$$= \theta + 30^\circ - 60^\circ = \theta - 30^\circ$$

P 11.41 $W_{m1} = |\mathbf{V}_{\text{AB}}||\mathbf{I}_{\text{aA}}|\cos(\underline{/\mathbf{V}_{\text{AB}}} - \underline{/\mathbf{I}_{\text{aA}}}) = (199.58)(2.4)\cos(65.68^\circ) = 197.26\,W$

$W_{m2} = |\mathbf{V}_{\text{CB}}||\mathbf{I}_{\text{cC}}|\cos(\underline{/\mathbf{V}_{\text{CB}}} - \underline{/\mathbf{I}_{\text{cC}}}) = (199.58)(2.4)\cos(5.68^\circ) = 476.64\,W$

CHECK: $W_1 + W_2 = 673.9 = (2.4)^2(39)(3) = 673.9\,W$

P 11.42 [a]
$$\begin{aligned} W_2 - W_1 &= V_L I_L[\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)] \\ &= V_L I_L[\cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ \\ &\qquad - \cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ] \\ &= 2V_L I_L \sin\theta\sin 30^\circ = V_L I_L \sin\theta, \end{aligned}$$

therefore $\sqrt{3}(W_2 - W_1) = \sqrt{3}V_L I_L \sin\theta = Q_T$

[b] $Z_\phi = (8 + j6)\,\Omega$

$Q_T = \sqrt{3}[2476.25 - 979.75] = 2592\text{ VAR},$

$Q_T = 3(12)^2(6) = 2592\text{ VAR};$

$Z_\phi = (8 - j6)\,\Omega$

$Q_T = \sqrt{3}[979.75 - 2476.25] = -2592\text{ VAR},$

$Q_T = 3(12)^2(-6) = -2592\text{ VAR};$

$Z_\phi = 5(1 + j\sqrt{3})\,\Omega$

$Q_T = \sqrt{3}[2160 - 0] = 3741.23\text{ VAR},$

$Q_T = 3(12)^2(5\sqrt{3}) = 3741.23\text{ VAR};$

$Z_\phi = 10\underline{/75^\circ}\,\Omega$

$Q_T = \sqrt{3}[-645.53 - 1763.63] = -4172.79\text{ VAR},$

$Q_T = 3(12)^2[-10\sin 75^\circ] = -4172.79\text{ VAR}$

P 11.43 $\mathbf{I}_{\text{aA}} = \dfrac{\mathbf{V}_{\text{AN}}}{Z_\phi} = |\mathbf{I}_L|\underline{/-\theta_\phi}\,\text{A},$

$Z_\phi = |Z|\underline{/\theta_\phi}, \qquad \mathbf{V}_{\text{BC}} = |\mathbf{V}_L|\underline{/-90^\circ}\,\text{V},$

$$\begin{aligned} W_m &= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos[-90^\circ - (-\theta_\phi)] \\ &= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos(\theta_\phi - 90^\circ) \\ &= |\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi, \end{aligned}$$

therefore $\sqrt{3}W_m = \sqrt{3}|\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi = Q_{\text{total}}$

P 11.44 [a] $Z = 96 + j72 = 120\underline{/36.87^\circ}\,\Omega$

$$\mathbf{V}_{\rm AN} = 720\underline{/0^\circ}\,\text{V}; \qquad \therefore\ \mathbf{I}_{\rm aA} = 6\underline{/-36.87^\circ}\,\text{A}$$

$$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN} = 720\sqrt{3}\underline{/-90^\circ}\,\text{V}$$

$$W_m = (720\sqrt{3})(6)\cos(-90 + 36.87^\circ) = 4489.48\,\text{W}$$

$$\sqrt{3}W_m = 7776\,\text{VAR}$$

[b] $Q_\phi = (36)(72) = 2592\,\text{VAR}$

$$Q_T = 3Q_\phi = 7776\,\text{VAR} = \sqrt{3}W_m$$

P 11.45 [a] $Z_\phi = 600 + j450 = 750\underline{/36.87^\circ}\,\Omega$

$$S_\phi = \frac{(69\times 10^3)^2}{750\underline{/-36.87^\circ}} = 5{,}078{,}400 + j3{,}808{,}800\,\text{VA}$$

$$S_T = 3S_\phi = 15{,}235{,}200 + j11{,}426{,}400\,\text{VA}$$

[b] $W_{m1} = (69{,}000)\sqrt{3}(92)\cos(0 + 66.87^\circ) = 4{,}318{,}082.44\,\text{W}$

$$W_{m2} = (69{,}000)\sqrt{3}(92)\cos(60 - 53.13^\circ) = 10{,}916{,}117.56\,\text{W}$$

Check: $\qquad P_T = 15{,}235{,}200\,\text{W} = W_{m1} + W_{m2}$.

P 11.46 [a] $\mathbf{I}^*_{\rm aA} = \dfrac{(192 + j56)10^3}{4800} = 41.67\underline{/16.26^\circ}\,\text{A}$

$$\mathbf{I}_{\rm aA} = 41.67\underline{/-16.26^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm bB} = 41.67\underline{/-136.26^\circ}\,\text{A}$$

$$\mathbf{V}_{\rm AB} = 4800\sqrt{3}\underline{/30^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm BC} = 4800\sqrt{3}\underline{/-90^\circ}\,\text{V}$$

$$W_1 = (4800\sqrt{3})(41.67)\cos 46.26^\circ = 239{,}502.58\,\text{W}$$

[b] Current coil in line aA, measure $\mathbf{I}_{\rm aA}$.
Voltage coil across AC, measure $\mathbf{V}_{\rm AC}$.

[c] $I_{\rm aA} = 41.67\underline{/-16.76^\circ}\,\text{A}$

$$\mathbf{V}_{\rm CA} = 4800\sqrt{3}\underline{/150^\circ}\,\text{V}$$

$$\therefore\ \mathbf{V}_{\rm AC} = 4800\sqrt{3}\underline{/-30^\circ}\,\text{V}$$

$$W_2 = (4800\sqrt{3})(41.67)\cos 13.74^\circ = 336{,}497.42\,\text{W}$$

[d] $W_1 + W_2 = 576{,}000 = 576\text{kW}$

$$P_T = 600(0.96) = 576\,\text{kW} = W_1 + W_2$$

P 11.47 [a] $W_1 = |\mathbf{V}_{BA}||\mathbf{I}_{bB}|\cos\theta$

Positive phase sequence, using the equivalent Y-connected load impedances:

$$\mathbf{V}_{BA} = 480\sqrt{3}\underline{/-150^\circ}\,\text{V}$$

$$\mathbf{I}_{aA} = \frac{480\underline{/0^\circ}}{20\underline{/30^\circ}} = 24\underline{/-30^\circ}\,\text{A}$$

$$\mathbf{I}_{bB} = 24\underline{/-150^\circ}\,\text{A}$$

$$W_1 = (24)(480)\sqrt{3}\cos 0^\circ = 19{,}953.23\,\text{W}$$

$$W_2 = |\mathbf{V}_{CA}||\mathbf{I}_{cC}|\cos\theta$$

$$\mathbf{V}_{CA} = 480\sqrt{3}\underline{/150^\circ}\,\text{V}$$

$$\mathbf{I}_{cC} = 24\underline{/90^\circ}\,\text{A}$$

$$W_2 = (24)(480)\sqrt{3}\cos 60^\circ = 9976.61\,\text{W}$$

[b] $P_\phi = (24)^2(20)\cos 30^\circ = 5760\sqrt{3}\,\text{W}$

$$P_T = 3P_\phi = 17{,}280\sqrt{3}\,\text{W}$$

$$W_1 + W_2 = 11{,}520\sqrt{3} + 5760\sqrt{3} = 17{,}280\sqrt{3}\,\text{W}$$

$$\therefore \quad W_1 + W_2 = P_T \qquad \text{(checks)}$$

P 11.48 [a] Negative phase sequence:

$$\mathbf{V}_{AB} = 480\sqrt{3}\underline{/-30^\circ}\,\text{V}$$

$$\mathbf{V}_{BC} = 480\sqrt{3}\underline{/90^\circ}\,\text{V}$$

$$\mathbf{V}_{CA} = 480\sqrt{3}\underline{/-150^\circ}\,\text{V}$$

$$\mathbf{I}_{AB} = \frac{480\sqrt{3}\underline{/-30^\circ}}{60\underline{/-30^\circ}} = 8\sqrt{3}\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_{BC} = \frac{480\sqrt{3}\underline{/90^\circ}}{24\underline{/30^\circ}} = 20\sqrt{3}\underline{/60^\circ}\,\text{A}$$

$$\mathbf{I}_{CA} = \frac{480\sqrt{3}\underline{/-150^\circ}}{80\underline{/0^\circ}} = 6\sqrt{3}\underline{/-150^\circ}\,\text{A}$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} + \mathbf{I}_{AC}$$
$$= 8\sqrt{3}\underline{/0^\circ} + 6\sqrt{3}\underline{/30^\circ} = 23.44\underline{/12.81^\circ}\text{ A}$$
$$\mathbf{I}_{cC} = \mathbf{I}_{CB} + \mathbf{I}_{CA}$$
$$= 20\sqrt{3}\underline{/-120^\circ} + 6\sqrt{3}\underline{/-150^\circ} = 43.95\underline{/-126.79^\circ}\text{ A}$$

$$W_{m1} = 480\sqrt{3}(23.44)\cos(-30 - 12.81^\circ) = 14{,}296.61\text{ W}$$

$$W_{m2} = 480\sqrt{3}(43.95)\cos(-90 + 126.79^\circ) = 29{,}261.53\text{ W}$$

[b] $W_{m1} + W_{m2} = 43{,}558.14\text{ W}$

$$P_A = (8\sqrt{3})^2(60\cos 30^\circ) = 9976.61\text{ W}$$

$$P_B = (20\sqrt{3})^2(24\cos 30^\circ) = 24{,}941.53\text{ W}$$

$$P_C = (6\sqrt{3})^2(80) = 8640\text{ W}$$

$$P_A + P_B + P_C = 43{,}558.14 = W_{m1} + W_{m2}$$

P 11.49 $\tan\phi = \dfrac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} = \dfrac{873{,}290.66}{732{,}777.88} = 1.1918$

$\therefore\ \phi = 50^\circ$

$\therefore\ 7600\sqrt{3}|\mathbf{I}_L|\cos 80^\circ = 114{,}291.64$

$|\mathbf{I}_L| = 50\text{ A}$

$|Z| = \dfrac{7600}{50} = 152\,\Omega \qquad \therefore\ Z = 152\underline{/50^\circ}\,\Omega$

P 11.50 [a] $Z = 276 - j207 = 345\underline{/-36.87^\circ}\,\Omega$

$$\mathbf{I}_{aA} = \frac{6900\underline{/0^\circ}}{345\underline{/-36.87^\circ}} = 20\underline{/36.87^\circ}\text{ A}$$

$$\mathbf{I}_{bB} = 20\underline{/-83.13^\circ}\text{ A}$$

$$\mathbf{V}_{AC} = 6900\sqrt{3}\underline{/-30^\circ}\text{ V}$$

$$\mathbf{V}_{BC} = 6900\sqrt{3}\underline{/-90^\circ}\text{ V}$$

$$W_1 = (6900\sqrt{3})(20)\cos(-30 - 36.87^\circ) = 93{,}893.10\text{ W}$$

$$W_2 = (6900\sqrt{3})(20)\cos(-90 + 83.13^\circ) = 237{,}306.90\text{ W}$$

[b] $W_1 + W_2 = 331{,}200\,\text{W}$

$$P_T = 3(20)^2(276) = 331{,}200\,\text{W}$$

[c] $\sqrt{3}(W_1 - W_2) = -248{,}400\,\text{VAR}$

$$Q_T = 3(20)^2(-207) = -248{,}400\,\text{VAR}$$

P 11.51 From the solution to Prob. 11.21 we have

$$\mathbf{I}_{\text{aA}} = 145.88\underline{/-24.28^\circ}\,\text{A} \qquad \text{and} \qquad \mathbf{I}_{\text{bB}} = 134.01\underline{/178.04^\circ}\,\text{A}$$

[a] $W_1 = |\mathbf{V}_{\text{ac}}|\,|\mathbf{I}_{\text{aA}}|\cos(\theta_{\text{ac}} - \theta_{\text{aA}})$

$$= 720(145.88)\cos(-60^\circ + 24.28^\circ) = 85{,}274.70\,\text{W}$$

[b] $W_2 = |\mathbf{V}_{\text{bc}}|\,|\mathbf{I}_{\text{bB}}|\cos(\theta_{\text{bc}} - \theta_{\text{bB}})$

$$= 720(134.01)\cos(-120^\circ - 178.04^\circ) = 45{,}357.50\,\text{W}$$

[c] $W_1 + W_2 = 130{,}632\,\text{W}$

$$P_{\text{AB}} = (144)^2(4.8) = 99{,}532.8\,\text{W}$$

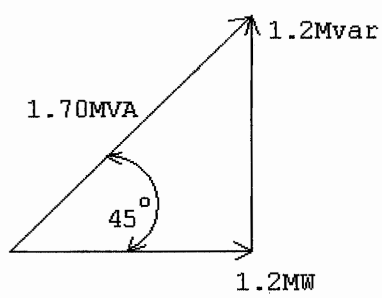
$$P_{\text{BC}} = (36)^2(16) = 20{,}736\,\text{W}$$

$$P_{\text{CA}} = (20.36)^2(25) = 10{,}363.2\,\text{W}$$

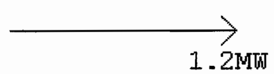
$$P_{\text{AB}} + P_{\text{BC}} + P_{\text{CA}} = 130{,}632$$

therefore $W_1 + W_2 = P_{\text{total}}$

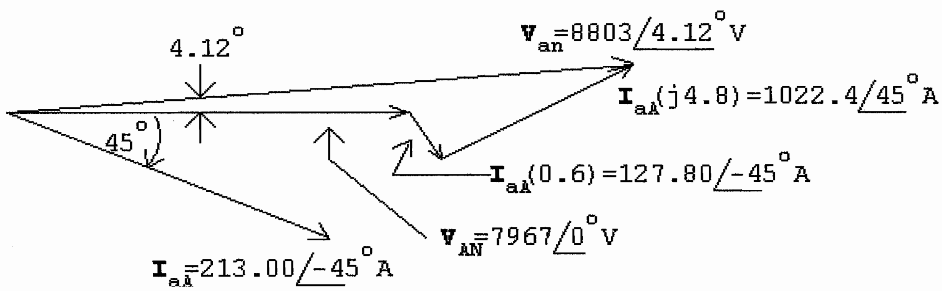
P 11.52 **[a]**



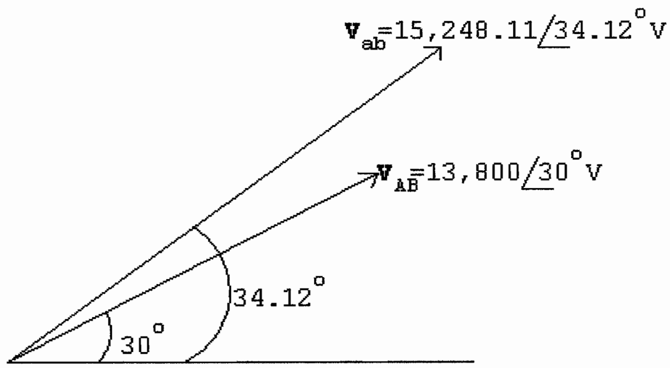
[b]



[c]



[d]



P 11.53 [a] $Q = \frac{|\mathbf{V}|^2}{X_C}$

$$\therefore \quad |X_C| = \frac{(13{,}800)^2}{1.2 \times 10^6} = 158.70\,\Omega$$

$$\therefore \quad \frac{1}{\omega C} = 158.70; \qquad C = \frac{1}{2\pi(60)(158.70)} = 16.71\,\mu\text{F}$$

[b] $|X_C| = \frac{(13{,}800/\sqrt{3})^2}{1.2 \times 10^6} = \frac{1}{3}(158.70)$

$$\therefore \quad C = 3(16.71) = 50.14\,\mu\text{F}$$

P 11.54 If the capacitors remain connected when the substation drops its load, the expression for the line current becomes

$$\frac{13{,}800}{\sqrt{3}}\mathbf{I}_{aA}^* = -j1.2 \times 10^6$$

or $\quad \mathbf{I}_{aA}^* = -j150.61\,\text{A}$

Hence $\quad \mathbf{I}_{aA} = j150.61\,\text{A}$

Now,

$$\mathbf{V}_{an} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(j150.61) = 7244.49 + j90.37 = 7245.05\underline{/0.71^\circ}\,\text{V}$$

The magnitude of the line-to-line voltage at the generating plant is

$$|\mathbf{V}_{ab}| = \sqrt{3}(7245.05) = 12{,}548.80\,\text{V}.$$

This is a problem because the voltage is below the acceptable minimum of 13 kV. Thus when the load at the substation drops off, the capacitors must be switched off.

P 11.55 Before the capacitors are added the total line loss is

$$P_L = 3|150.61 + j150.61|^2(0.6) = 81.66\,\text{kW}$$

After the capacitors are added the total line loss is

$$P_L = 3|150.61|^2(0.6) = 40.83\,\text{kW}$$

Note that adding the capacitors to control the voltage level also reduces the amount of power loss in the lines, which in this example is cut in half.

P 11.56 [a] $\frac{13{,}800}{\sqrt{3}}\mathbf{I}^*_{aA} = 80 \times 10^3 + j200 \times 10^3 - j1200 \times 10^3$

$$\mathbf{I}^*_{aA} = \frac{80\sqrt{3} - j1000\sqrt{3}}{13.8} = 10.04 - j125.51\,\text{A}$$

$$\therefore\quad \mathbf{I}_{aA} = 10.04 + j125.51\,\text{A}$$

$$\mathbf{V}_{an} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(10.04 + j125.51)$$

$$= 7371.01 + j123.50 = 7372.04\underline{/0.96^\circ}\,\text{V}$$

$$\therefore\quad |\mathbf{V}_{ab}| = \sqrt{3}(7372.04) = 12{,}768.75\,\text{V}$$

[b] Yes, the magnitude of the line-to-line voltage at the power plant is less than the allowable minimum of 13 kV.

P 11.57 [a] $\frac{13{,}800}{\sqrt{3}}\mathbf{I}^*_{aA} = (80 + j200) \times 10^3$

$$\mathbf{I}^*_{aA} = \frac{80\sqrt{3} + j200\sqrt{3}}{13.8} = 10.04 + j25.1\,\text{A}$$

$$\therefore\quad \mathbf{I}_{aA} = 10.04 - j25.1\,\text{A}$$

$$\mathbf{V}_{an} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(10.04 - j25.1)$$

$$= 8093.95 + j33.13 = 8094.02\underline{/0.23^\circ}\,\text{V}$$

$$\therefore\quad |\mathbf{V}_{ab}| = \sqrt{3}(8094.02) = 14{,}019.25\,\text{V}$$

[b] Yes: $13\,\text{kV} < 14{,}019.25 < 14.6\,\text{kV}$

[c] $P_{\text{loss}} = 3|10.04 + j125.51|^2(0.6) = 28.54\,\text{kW}$

[d] $P_{\text{loss}} = 3|10.04 + j25.1|^2(0.6) = 1.32\,\text{kW}$

[e] Yes, the voltage at the generating plant is at an acceptable level and the line loss is greatly reduced.

12

Introduction to the Laplace Transform

Assessment Problems

AP 12.1 [a] $\cosh \beta t = \dfrac{e^{\beta t} + e^{-\beta t}}{2}$

Therefore,

$$\mathcal{L}\{\cosh \beta t\} = \frac{1}{2}\int_{0^-}^{\infty} [e^{(s-\beta)t} + e^{-(s-\beta)t}]dt$$

$$= \frac{1}{2}\left[\frac{e^{-(s-\beta)t}}{-(s-\beta)}\Bigg|_{0^-}^{\infty} + \frac{e^{-(s+\beta)t}}{-(s+\beta)}\Bigg|_{0^-}^{\infty}\right]$$

$$= \frac{1}{2}\left(\frac{1}{s-\beta} + \frac{1}{s+\beta}\right) = \frac{s}{s^2-\beta^2}$$

[b] $\sinh \beta t = \dfrac{e^{\beta t} - e^{-\beta t}}{2}$

Therefore,

$$\mathcal{L}\{\sinh \beta t\} = \frac{1}{2}\int_{0^-}^{\infty} \left[e^{-(s-\beta)t} - e^{-(s+\beta)t}\right] dt$$

$$= \frac{1}{2}\left[\frac{e^{-(s-\beta)t}}{-(s-\beta)}\right]_{0^-}^{\infty} - \frac{1}{2}\left[\frac{e^{-(s+\beta)t}}{-(s+\beta)}\right]_{0^-}^{\infty}$$

$$= \frac{1}{2}\left(\frac{1}{s-\beta} - \frac{1}{s+\beta}\right) = \frac{\beta}{(s^2-\beta^2)}$$

AP 12.2 [a] Let $f(t) = te^{-at}$:

$$F(s) = \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

Now, $\quad \mathcal{L}\{tf(t)\} = -\dfrac{dF(s)}{ds}$

So, $$\mathcal{L}\{t \cdot te^{-at}\} = -\frac{d}{ds}\left[\frac{1}{(s+a)^2}\right] = \frac{2}{(s+a)^3}$$

[b] Let $f(t) = e^{-at}\sinh\beta t$, then

$$\mathcal{L}\{f(t)\} = F(s) = \frac{\beta}{(s+a)^2 - \beta^2}$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-) = \frac{s(\beta)}{(s+a)^2 - \beta^2} - 0 = \frac{\beta s}{(s+a)^2 - \beta^2}$$

[c] Let $f(t) = \cos\omega t$. Then

$$F(s) = \frac{s}{(s^2+\omega^2)} \quad \text{and} \quad \frac{dF(s)}{ds} = \frac{-(s^2-\omega^2)}{(s^2+\omega^2)^2}$$

Therefore $$\mathcal{L}\{t\cos\omega t\} = -\frac{dF(s)}{ds} = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$$

AP 12.3

$$F(s) = \frac{6s^2+26s+26}{(s+1)(s+2)(s+3)} = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

$$K_1 = \frac{6-26+26}{(1)(2)} = 3; \qquad K_2 = \frac{24-52+26}{(-1)(1)} = 2$$

$$K_3 = \frac{54-78+26}{(-2)(-1)} = 1$$

Therefore $f(t) = [3e^{-t} + 2e^{-2t} + e^{-3t}]\,u(t)$

AP 12.4

$$F(s) = \frac{7s^2+63s+134}{(s+3)(s+4)(s+5)} = \frac{K_1}{s+3} + \frac{K_2}{s+4} + \frac{K_3}{s+5}$$

$$K_1 = \frac{63-189-134}{1(2)} = 4; \qquad K_2 = \frac{112-252+134}{(-1)(1)} = 6$$

$$K_3 = \frac{175-315+134}{(-2)(-1)} = -3$$

$$f(t) = [4e^{-3t} + 6e^{-4t} - 3e^{-5t}]u(t)$$

AP 12.5

$$F(s) = \frac{10(s^2 + 119)}{(s + 5)(s^2 + 10s + 169)}$$

$$s_{1,2} = -5 \pm \sqrt{25 - 169} = -5 \pm j12$$

$$F(s) = \frac{K_1}{s+5} + \frac{K_2}{s+5-j12} + \frac{K_2^*}{s+5+j12}$$

$$K_1 = \frac{10(25 + 119)}{25 - 50 + 169} = 10$$

$$K_2 = \frac{10[(-5 + j12)^2 + 119]}{(j12)(j24)} = j4.17 = 4.17\underline{/90^\circ}$$

Therefore

$$\begin{aligned} f(t) &= [10e^{-5t} + 8.33e^{-5t}\cos(12t + 90^\circ)]\,u(t) \\ &= [10e^{-5t} - 8.33e^{-5t}\sin 12t]\,u(t) \end{aligned}$$

AP 12.6

$$F(s) = \frac{4s^2 + 7s + 1}{s(s+1)^2} = \frac{K_0}{s} + \frac{K_1}{(s+1)^2} + \frac{K_2}{s+1}$$

$$K_0 = \frac{1}{(1)^2} = 1; \qquad K_1 = \frac{4 - 7 + 1}{-1} = 2$$

$$\begin{aligned} K_2 &= \frac{d}{ds}\left[\frac{4s^2 + 7s + 1}{s}\right]_{s=-1} = \left.\frac{s(8s+7) - (4s^2 + 7s + 1)}{s^2}\right|_{s=-1} \\ &= \frac{1+2}{1} = 3 \end{aligned}$$

Therefore $f(t) = [1 + 2te^{-t} + 3e^{-t}]\,u(t)$

AP 12.7

$$\begin{aligned} F(s) &= \frac{40}{(s^2 + 4s + 5)^2} = \frac{40}{(s + 2 - j1)^2(s + 2 + j1)^2} \\ &= \frac{K_1}{(s + 2 - j1)^2} + \frac{K_2}{(s + 2 - j1)} + \frac{K_1^*}{(s + 2 + j1)^2} \\ &\quad + \frac{K_2^*}{(s + 2 + j1)} \end{aligned}$$

$$K_1 = \frac{40}{(j2)^2} = -10 = 10\underline{/180^\circ} \quad \text{and} \quad K_1^* = -10$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{s^3[4 + (7/s) + (1/s)^2]}{s^3[1 + (1/s)]^2}\right] = 4$$

$$\therefore \quad f(0^+) = 4$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{4s^2 + 7s + 1}{(s + 1)^2}\right] = 1$$

$$\therefore \quad f(\infty) = 1$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{40s}{s^4[1 + (4/s) + (5/s^2)]^2}\right] = 0$$

$$\therefore \quad f(0^+) = 0$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{40s}{(s^2 + 4s + 5)^2}\right] = 0$$

$$\therefore \quad f(\infty) = 0$$

Problems

P 12.1 **[a]** $f(t) = 120 + 30t \qquad -4\,\text{s} \le t \le 0$

$$f(t) = 120 - 30t \qquad 0 \le t \le 8\,\text{s}$$

$$f(t) = -360 + 30t \qquad 8\,\text{s} \le t \le 12\,\text{s}$$

$$f(t) = 0 \qquad \text{elsewhere}$$

$$f(t) = (120 + 30t)[(u(t+4) - u(t)] + (120 - 30t)[u(t) - u(t-8)]$$
$$+(-360 + 30t)[u(t-8) - u(t-12)]$$

[b] $f(t) = 50\sin\frac{\pi}{2}t[u(t) - u(t-4)]$

$$= (50\sin\tfrac{\pi}{2}t)u(t) - (50\sin\tfrac{\pi}{2}t)u(t-4)$$

[c] $f(t) = (30 - 3t)t[u(t) - u(t-10)]$

P 12.2 **[a]** $(50 + 2.5t)[u(t+20) - u(t)] + (50 - 5t)[u(t) - u(t-10)]$

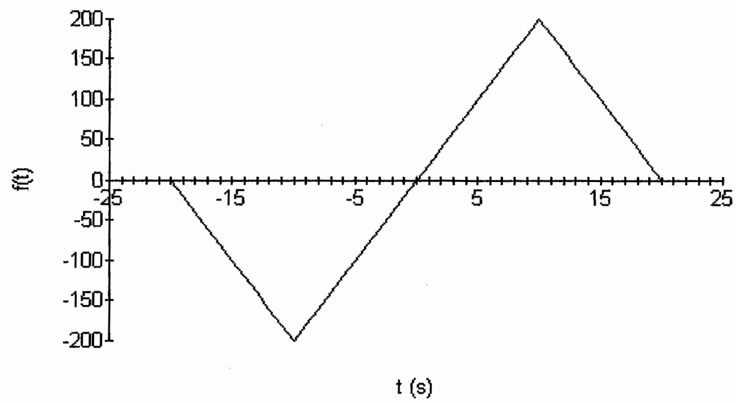
$$= (2.5t + 50)u(t+20) - 2.5tu(t) + (5t - 50)u(t-10)$$

[b] $(5t + 45)[u(t+9) - u(t+6)] + 15[u(t+6) - u(t+3)] - 5t[u(t+3) - u(t-3)]$

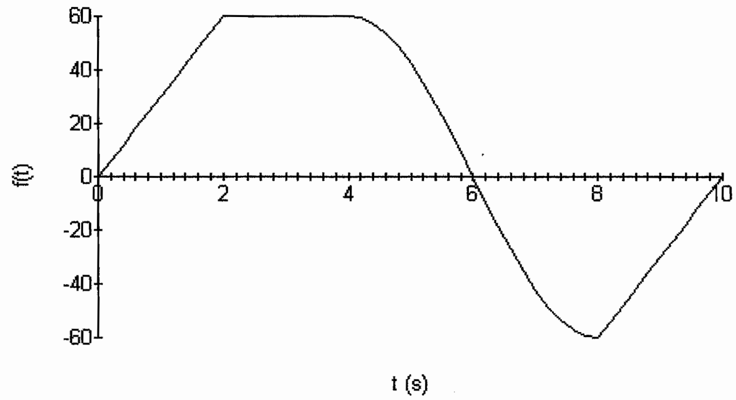
$$-15[u(t-3) - u(t-6)] + (5t - 45)[u(t-6) - u(t-9)]$$

$$= 5(t+9)u(t+9) - 5(t+6)u(t+6) - 5(t+3)u(t+3) + 5(t-3)u(t-3)$$
$$+5(t-6)u(t-6) - 5(t-9)u(t-9)$$

P 12.3



P 12.4 [a]



[b] $f(t) = 30t[u(t) - u(t-2)] + 60[u(t-2) - u(t-4)]$
$+60\cos(\frac{\pi}{4}t - \pi)[u(t-4) - u(t-8)]$
$+(30t - 300)[u(t-8) - u(t-10)]$

P 12.5 [a] $A = \left(\frac{1}{2}\right) bh = \left(\frac{1}{2}\right)(2\varepsilon)\left(\frac{1}{\varepsilon}\right) = 1.0$

[b] 0; [c] ∞

P 12.6 [a] $I = \int_{-2}^{4} (t^3+4)\delta(t)\,dt + \int_{-2}^{4} 4(t^3+4)\delta(t-2)\,dt$

$= 4 + 4(8+4) = 52$

[b] $I = \int_{-3}^{4} t^2\delta(t)\,dt + \int_{-3}^{4} t^2\delta(t+2.5)\,dt + 0$

$= 0^2 + (-2.5)^2 + 0 = 6.25$

P 12.7 $f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{(3+j\omega)}{(4+j\omega)} \cdot \pi\delta(\omega) \cdot e^{jt\omega}\,d\omega = \left(\frac{1}{2\pi}\right)\left(\frac{3+j0}{4+j0}\pi e^{-jt0}\right) = \frac{3}{8}$

P 12.8 As $\varepsilon \to 0$ the amplitude $\to \infty$; the duration $\to 0$; and the area is independent of ε, i.e.,

$$A = \int_{-\infty}^{\infty} \frac{\varepsilon}{\pi}\frac{1}{\varepsilon^2 + t^2}\,dt = 1$$

P 12.9 $F(s) = \int_{-\varepsilon}^{\varepsilon} \frac{1}{2\varepsilon}e^{-st}\,dt = \frac{e^{s\varepsilon} - e^{-s\varepsilon}}{2\varepsilon s}$

$$F(s) = \frac{1}{2s}\lim_{\varepsilon\to 0}\left[\frac{se^{s\varepsilon} + se^{-s\varepsilon}}{1}\right] = \frac{1}{2s}\cdot\frac{2s}{1} = 1$$

P 12.10 [a] Let $dv = \delta'(t-a)\,dt, \qquad v = \delta(t-a)$

$$u = f(t), \qquad du = f'(t)\,dt$$

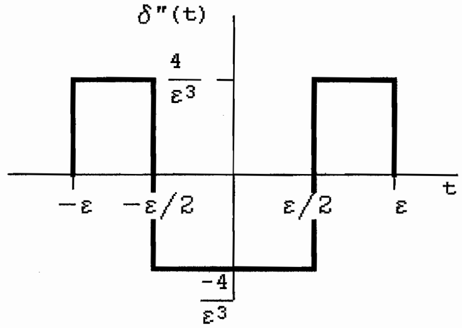
Therefore

$$\int_{-\infty}^{\infty} f(t)\delta'(t-a)\,dt = f(t)\delta(t-a)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \delta(t-a)f'(t)\,dt$$

$$= 0 - f'(a)$$

[b] $\mathcal{L}\{\delta'(t)\} = \int_{0^-}^{\infty} \delta'(t)e^{-st}\,dt = -\left[\frac{d(e^{-st})}{dt}\right]_{t=0} = -\left[-se^{-st}\right]_{t=0} = s$

P 12.11



$$F(s) = \int_{-\varepsilon}^{-\varepsilon/2} \frac{4}{\varepsilon^3}e^{-st}\,dt + \int_{-\varepsilon/2}^{\varepsilon/2} \left(\frac{-4}{\varepsilon^3}\right) e^{-st}\,dt + \int_{\varepsilon/2}^{\varepsilon} \frac{4}{\varepsilon^3}e^{-st}\,dt$$

Therefore $F(s) = \dfrac{4}{s\varepsilon^3}[e^{s\varepsilon} - 2e^{s\varepsilon/2} + 2e^{-s\varepsilon/2} - e^{-s\varepsilon}]$

$$\mathcal{L}\{\delta''(t)\} = \lim_{\varepsilon\to 0} F(s)$$

After applying L'Hopital's rule three times, we have

$$\lim_{\varepsilon\to 0} \frac{2s}{3}\left[se^{s\varepsilon} - \frac{s}{4}e^{s\varepsilon/2} - \frac{s}{4}e^{-s\varepsilon/2} + se^{-s\varepsilon}\right] = \frac{2s}{3}\left(\frac{3s}{2}\right)$$

Therefore $\mathcal{L}\{\delta''(t)\} = s^2$

P 12.12 $\mathcal{L}\left\{\dfrac{d^n f(t)}{dt^n}\right\} = s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \cdots,$

Therefore

$$\mathcal{L}\{\delta^n(t)\} = s^n(1) - s^{n-1}\delta(0^-) - s^{n-2}\delta'(0^-) - s^{n-3}\delta''(0^-) - \cdots = s^n$$

P 12.13 [a] $\mathcal{L}\{t\} = \dfrac{1}{s^2}$; therefore $\mathcal{L}\{te^{-at}\} = \dfrac{1}{(s+a)^2}$

[b] $\sin\omega t = \dfrac{e^{j\omega t} - e^{-j\omega t}}{j2}$

Therefore

$$\mathcal{L}\{\sin\omega t\} = \left(\frac{1}{j2}\right)\left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right) = \left(\frac{1}{j2}\right)\left(\frac{2j\omega}{s^2+\omega^2}\right)$$

$$= \frac{\omega}{s^2+\omega^2}$$

[c] $\sin(\omega t + \theta) = (\sin\omega t\cos\theta + \cos\omega t\sin\theta)$

Therefore

$$\mathcal{L}\{\sin(\omega t+\theta)\} = \cos\theta\mathcal{L}\{\sin\omega t\} + \sin\theta\mathcal{L}\{\cos\omega t\}$$

$$= \frac{\omega\cos\theta + s\sin\theta}{s^2+\omega^2}$$

[d] $\mathcal{L}\{t\} = \displaystyle\int_0^\infty te^{-st}\,dt = \frac{e^{-st}}{s^2}(-st-1)\Big|_0^\infty = 0 - \frac{1}{s^2}(0-1) = \frac{1}{s^2}$

[e] $f(t) = \cosh t\cosh\theta + \sinh t\sinh\theta$

From Assessment Problem 12.1(a)

$$\mathcal{L}\{\cosh t\} = \frac{s}{s^2-1}$$

From Assessment Problem 12.1(b)

$$\mathcal{L}\{\sinh t\} = \frac{1}{s^2-1}$$

$$\therefore\quad \mathcal{L}\{\cosh(t+\theta)\} = \cosh\theta\left[\frac{s}{(s^2-1)}\right] + \sinh\theta\left[\frac{1}{s^2-1}\right]$$

$$= \frac{\sinh\theta + s[\cosh\theta]}{(s^2-1)}$$

P 12.14 [a] $\mathcal{L}\{te^{-at}\} = \displaystyle\int_{0-}^\infty te^{-(s+a)t}\,dt$

$$= \frac{e^{-(s+a)t}}{(s+a)^2}\Big[-(s+a)t-1\Big]_{0-}^\infty$$

$$= 0 + \frac{1}{(s+a)^2}$$

$$\therefore\quad \mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

[b] $\mathcal{L}\left\{\dfrac{d}{dt}(te^{-at})u(t)\right\} = \dfrac{s}{(s+a)^2} - 0$

$$\mathcal{L}\left\{\frac{d}{dt}(te^{-at})u(t)\right\} = \frac{s}{(s+a)^2}$$

[c] $\frac{d}{dt}(te^{-at}) = -ate^{-at} + e^{-at}$

$$\mathcal{L}\{-ate^{-at} + e^{-at}\} = \frac{-a}{(s+a)^2} + \frac{1}{(s+a)} = \frac{-a}{(s+a)^2} + \frac{s+a}{(s+a)^2}$$

$$\therefore \quad \mathcal{L}\left\{\frac{d}{dt}(te^{-at})\right\} = \frac{s}{(s+a)^2} \qquad \text{CHECKS}$$

P 12.15 [a] $\mathcal{L}\{f'(t)\} = \int_{-\varepsilon}^{\varepsilon} \frac{e^{-st}}{2\varepsilon}\,dt + \int_{\varepsilon}^{\infty} -ae^{-a(t-\varepsilon)}e^{-st}\,dt$

$$= \frac{1}{2s\varepsilon}(e^{s\varepsilon} - e^{-s\varepsilon}) - \left(\frac{a}{s+a}\right)e^{-s\varepsilon} = F(s)$$

$$\lim_{\varepsilon\to 0} F(s) = 1 - \frac{a}{s+a} = \frac{s}{s+a}$$

[b] $\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$

Therefore $\mathcal{L}\{f'(t)\} = sF(s) - f(0^-) = \frac{s}{s+a} - 0 = \frac{s}{s+a}$

P 12.16 $\mathcal{L}\{e^{-at}f(t)\} = \int_{0^-}^{\infty}[e^{-at}f(t)]e^{-st}\,dt = \int_{0^-}^{\infty} f(t)e^{-(s+a)t}\,dt = F(s+a)$

P 12.17 [a] $\mathcal{L}\left\{\int_{0^-}^{t} e^{-ax}\,dx\right\} = \frac{F(s)}{s} = \frac{1}{s(s+a)}$

[b] $\mathcal{L}\left\{\int_{0^-}^{t} y\,dy\right\} = \frac{1}{s}\left(\frac{1}{s^2}\right) = \frac{1}{s^3}$

[c] $\int_{0^-}^{t} e^{-ax}\,dx = \frac{1}{a} - \frac{e^{-at}}{a}$

$$\mathcal{L}\left\{\frac{1}{a} - \frac{e^{-at}}{a}\right\} = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{s+a}\right] = \frac{1}{s(s+a)}$$

$$\int_{0^-}^{t} y\,dy = \frac{t^2}{2}; \qquad \mathcal{L}\left\{\frac{t^2}{2}\right\} = \frac{1}{2}\cdot\frac{2}{s^3} = \frac{1}{s^3}$$

P 12.18 [a] $\mathcal{L}\left\{\frac{d\sin\omega t}{dt}u(t)\right\} = \frac{s\omega}{s^2+\omega^2} - \sin(0) = \frac{s\omega}{s^2+\omega^2}$

[b] $\mathcal{L}\left\{\frac{d\cos\omega t}{dt}u(t)\right\} = \frac{s^2}{s^2+\omega^2} - \cos(0) = \frac{s^2}{s^2+\omega^2} - 1 = \frac{-\omega^2}{s^2+\omega^2}$

[c] $\mathcal{L}\left\{\frac{d^3(t^2)}{dt^3}u(t)\right\} = s^3\left(\frac{2}{s^3}\right) - s^2(0) - s(0) - 2(0) = 2$

[d] $\frac{d\sin\omega t}{dt} = (\cos\omega t)\cdot\omega, \qquad \mathcal{L}\{\omega\cos\omega t\} = \frac{\omega s}{s^2+\omega^2}$

$$\frac{d\cos\omega t}{dt} = -\omega\sin\omega t$$

$$\mathcal{L}\{-\omega\sin\omega t\} = -\frac{\omega^2}{s^2+\omega^2}$$

$$\frac{d^3(t^2)}{dt^3} = 2\delta(t); \qquad \mathcal{L}\{2\delta(t)\} = 2$$

P 12.19 [a] $f(t) = 4t[u(t) - u(t-4)]$

$$+(32-4t)[u(t-4) - u(t-12)]$$

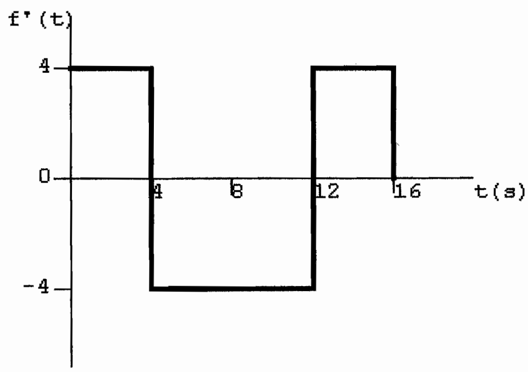
$$+(4t-64)[u(t-12) - u(t-16)]$$

$$= 4tu(t) - 8(t-4)u(t-4)$$

$$+8(t-12)u(t-12) - 4(t-16)u(t-16)$$

$$\therefore \quad F(s) = \frac{4[1 - 2e^{-4s} + 2e^{-12s} - e^{-16s}]}{s^2}$$

[b]



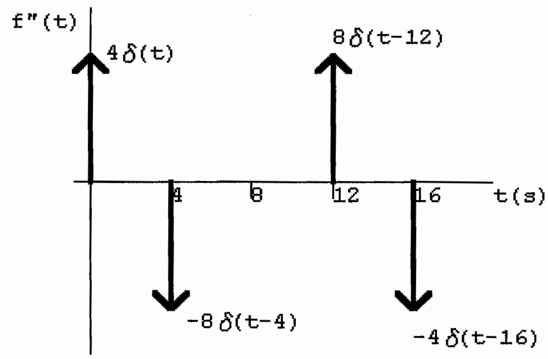
$$f'(t) = 4[u(t) - u(t-4)] - 4[u(t-4) - u(t-12)]$$

$$+4[u(t-12) - u(t-16)]$$

$$= 4u(t) - 8u(t-4) + 8u(t-12) - 4u(t-16)$$

$$\mathcal{L}\{f'(t)\} = \frac{4[1 - 2e^{-4s} + 2e^{-12s} - e^{-16s}]}{s}$$

[c]



$$f''(t) = 4\delta(t) - 8\delta(t-4) + 8\delta(t-12) - 4\delta(t-16)$$

$$\mathcal{L}\{f''(t)\} = 4[1 - 2e^{-4s} + 2e^{-12s} - e^{-16s}]$$

P 12.20 [a] $\int_{0^-}^{t} x\,dx = \frac{t^2}{2}$

$$\mathcal{L}\left\{\frac{t^2}{2}\right\} = \frac{1}{2}\int_{0^-}^{\infty} t^2 e^{-st}\,dt$$

$$= \frac{1}{2}\left[\frac{e^{-st}}{-s^3}(s^2t^2 + 2st + 2)\Big|_{0^-}^{\infty}\right]$$

$$= \frac{1}{2s^3}(2) = \frac{1}{s^3}$$

$$\therefore \quad \mathcal{L}\left\{\int_{0^-}^{t} x\,dx\right\} = \frac{1}{s^3}$$

[b] $\mathcal{L}\left\{\int_{0^-}^{t} x\,dx\right\} = \frac{\mathcal{L}\{t\}}{s} = \frac{1/s^2}{s} = \frac{1}{s^3}$

$$\therefore \quad \mathcal{L}\left\{\int_{0^-}^{t} x\,dx\right\} = \frac{1}{s^3} \qquad \text{CHECKS}$$

P 12.21 [a] $\mathcal{L}\{-20e^{-5(t-2)}u(t-2)\} = \frac{-20e^{-2s}}{(s+5)}$

[b] First rewrite $f(t)$ as

$$f(t) = (8t-8)u(t-1) + (24 - 8t - 8t + 8)u(t-2)$$
$$+(8t - 40 - 24 + 8t)u(t-4) - (8t-40)u(t-5)$$
$$= 8(t-1)u(t-1) - 16(t-2)u(t-2)$$
$$+16(t-4)u(t-4) - 8(t-5)u(t-5)$$

$$\therefore \quad F(s) = \frac{8[e^{-s} - 2e^{-2s} + 2e^{-4s} - e^{-5s}]}{s^2}$$

P 12.22 $\mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(at)e^{-st}\,dt$

Let $u = at$, $du = a\,dt$, $u = 0^-$ when $t = 0^-$

and $u = \infty$ when $t = \infty$

Therefore $\mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(u)e^{-(u/a)s}\dfrac{du}{a} = \dfrac{1}{a}F(s/a)$

P 12.23 [a] $f_1(t) = e^{-at}\sin\omega t$; $F_1(s) = \dfrac{\omega}{(s+a)^2+\omega^2}$

$$F(s) = sF_1(s) - f_1(0^-) = \frac{s\omega}{(s+a)^2+\omega^2} - 0$$

[b] $f_1(t) = e^{-at}\cos\omega t$; $F_1(s) = \dfrac{s+a}{(s+a)^2+\omega^2}$

$$F(s) = \frac{F_1(s)}{s} = \frac{s+a}{s[(s+a)^2+\omega^2]}$$

[c] $\dfrac{d}{dt}[e^{-at}\sin\omega t] = \omega e^{-at}\cos\omega t - ae^{-at}\sin\omega t$

Therefore $F(s) = \dfrac{\omega(s+a)-\omega a}{(s+a)^2+\omega^2} = \dfrac{\omega s}{(s+a)^2+\omega^2}$

$$\int_{0^-}^{t} e^{-ax}\cos\omega x\,dx = \frac{-ae^{-at}\cos\omega t + \omega e^{-at}\sin\omega t + a}{a^2+\omega^2}$$

Therefore

$$F(s) = \frac{1}{a^2+\omega^2}\left[\frac{-a(s+a)}{(s+a)^2+\omega^2} + \frac{\omega^2}{(s+a)^2+\omega^2} + \frac{a}{s}\right]$$

$$= \frac{s+a}{s[(s+a)^2+\omega^2]}$$

P 12.24 [a] $\dfrac{dF(s)}{ds} = \dfrac{d}{ds}\left[\int_{0^-}^{\infty} f(t)e^{-st}\,dt\right] = -\int_{0^-}^{\infty} tf(t)e^{-st}\,dt$

Therefore $\mathcal{L}\{tf(t)\} = -\dfrac{dF(s)}{ds}$

[b] $\dfrac{d^2F(s)}{ds^2} = \int_{0^-}^{\infty} t^2f(t)e^{-st}\,dt$; $\dfrac{d^3F(s)}{ds^3} = \int_{0^-}^{\infty} -t^3f(t)e^{-st}\,dt$

Therefore $\dfrac{d^nF(s)}{ds^n} = (-1)^n\int_{0^-}^{\infty} t^nf(t)e^{-st}\,dt = (-1)^n\mathcal{L}\{t^nf(t)\}$

[c] $\mathcal{L}\{t^5\} = \mathcal{L}\{t^4 t\} = (-1)^4 \frac{d^4}{ds^4}\left(\frac{1}{s^2}\right) = \frac{120}{s^6}$

$$\mathcal{L}\{t \sin \beta t\} = (-1)^1 \frac{d}{ds}\left(\frac{\beta}{s^2+\beta^2}\right) = \frac{2\beta s}{(s^2+\beta^2)^2}$$

$\mathcal{L}\{te^{-t} \cosh t\}$:

From Assessment Problem 12.1(a),

$$F(s) = \mathcal{L}\{\cosh t\} = \frac{s}{s^2-1}$$

$$\frac{dF}{ds} = \frac{(s^2-1)1 - s(2s)}{(s^2-1)^2} = -\frac{s^2+1}{(s^2-1)^2}$$

Therefore $\quad -\frac{dF}{ds} = \frac{s^2+1}{(s^2-1)^2}$

Thus

$$\mathcal{L}\{t \cosh t\} = \frac{s^2+1}{(s^2-1)^2}$$

$$\mathcal{L}\{e^{-t} t \cosh t\} = \frac{(s+1)^2+1}{[(s+1)^2-1]^2} = \frac{s^2+2s+2}{s^2(s+2)^2}$$

P 12.25 [a] $\int_s^\infty F(u)du = \int_s^\infty \left[\int_{0^-}^\infty f(t)e^{-ut}\,dt\right] du = \int_{0^-}^\infty \left[\int_s^\infty f(t)e^{-ut}\,du\right] dt$

$$= \int_{0^-}^\infty f(t) \int_s^\infty e^{-ut}\,du\,dt = \int_{0^-}^\infty f(t) \left[\frac{e^{-tu}}{-t}\Big|_s^\infty\right] dt$$

$$= \int_{0^-}^\infty f(t) \left[\frac{-e^{-st}}{-t}\right] dt = \mathcal{L}\left\{\frac{f(t)}{t}\right\}$$

[b] $\mathcal{L}\{t \sin \beta t\} = \frac{2\beta s}{(s^2+\beta^2)^2}$

therefore $\quad \mathcal{L}\left\{\frac{t \sin \beta t}{t}\right\} = \int_s^\infty \left[\frac{2\beta u}{(u^2+\beta^2)^2}\right] du$

Let $\omega = u^2 + \beta^2$, then $\omega = s^2 + \beta^2$ when $u = s$, and $\omega = \infty$ when $u = \infty$; also $d\omega = 2u\,du$, thus

$$\mathcal{L}\left\{\frac{t \sin \beta t}{t}\right\} = \beta \int_{s^2+\beta^2}^\infty \left[\frac{d\omega}{\omega^2}\right] = \beta\left(\frac{-1}{\omega}\right)\Big|_{s^2+\beta^2}^\infty = \frac{\beta}{s^2+\beta^2}$$

P 12.26 $i_g(t) = 5\cos 10t u(t);$ so $I_g(s) = \dfrac{5s^2}{s^2+100}$

$$\frac{1}{RC} = 40; \qquad \frac{1}{LC} = 64; \qquad \frac{1}{C} = 40$$

Therefore $V = \dfrac{(40)(5)s^2}{(s^2+40s+64)(s^2+100)} = \dfrac{200s^2}{(s^2+40s+64)(s^2+100)}$

P 12.27 [a] $\dfrac{v_o - V_{\rm dc}}{R} + \dfrac{1}{L}\displaystyle\int_0^t v_o\,dx + C\dfrac{dv_o}{dt} = 0$

$$\therefore \quad v_o + \frac{R}{L}\int_0^t v_o\,dx + RC\frac{dv_o}{dt} = V_{\rm dc}$$

[b] $V_o + \dfrac{R}{L}\dfrac{V_o}{s} + RCSV_o = \dfrac{V_{\rm dc}}{s}$

$$\therefore \quad sLV_o + RV_o + RCLs^2V_o = LV_{\rm dc}$$

$$\therefore \quad V_o(s) = \frac{(1/RC)V_{\rm dc}}{s^2 + (1/RC)s + (1/LC)}$$

[c] $i_o = \dfrac{1}{L}\displaystyle\int_0^t v_o\,dx$

$$I_o(s) = \frac{V_o}{sL} = \frac{(1/RCL)V_{\rm dc}}{s[s^2 + (1/RC)s + (1/LC)]}$$

P 12.28 [a] $I_{\rm dc} = \dfrac{1}{L}\displaystyle\int_0^t v_o\,dx + \dfrac{v_o}{R} + C\dfrac{dv_o}{dt}$

[b] $\dfrac{I_{\rm dc}}{s} = \dfrac{V_o(s)}{sL} + \dfrac{V_o(s)}{R} + sCV_o(s)$

$$\therefore \quad V_o(s) = \frac{I_{\rm dc}/C}{s^2 + (1/RC)s + (1/LC)}$$

[c] $i_o = C\dfrac{dv_o}{dt}$

$$\therefore \quad I_o(s) = sCV_o(s) = \frac{sI_{\rm dc}}{s^2 + (1/RC)s + (1/LC)}$$

P 12.29 [a] $\dfrac{1}{L}\displaystyle\int_0^t v_1\,d\tau + \dfrac{v_1 - v_2}{R} = i_g$

and

$$C\frac{dv_2}{dt} + \frac{v_2}{R} - \frac{v_1}{R} = 0$$

[b] $\dfrac{V_1}{sL} + \dfrac{V_1 - V_2}{R} = I_g$

$$\frac{V_2 - V_1}{R} + sCV_2 = 0$$

or

$$(R + sL)V_1(s) - sLV_2(s) = RLsI_g(s)$$

$$-V_1(s) + (RCs + 1)V_2(s) = 0$$

Solving,

$$V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}$$

P 12.30 [a] For $t \geq 0^+$:

$$\frac{v_o}{R} + C\frac{dv_o}{dt} + i_o = 0$$

$$v_o = L\frac{di_o}{dt}; \qquad \frac{dv_o}{dt} = L\frac{d^2i_o}{dt^2}$$

$$\therefore \quad \frac{L}{R}\frac{di_o}{dt} + LC\frac{d^2i_o}{dt^2} + i_o = 0$$

or $\dfrac{d^2i_o}{dt^2} + \dfrac{1}{RC}\dfrac{di_o}{dt} + \dfrac{1}{LC}i_o = 0$

[b] $s^2I_o(s) - sI_{\text{dc}} - 0 + \dfrac{1}{RC}[sI_o(s) - I_{\text{dc}}] + \dfrac{1}{LC}I_o(s) = 0$

$$I_o(s)\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right] = I_{\text{dc}}(s + 1/RC)$$

$$I_o(s) = \frac{I_{\text{dc}}[s + (1/RC)]}{[s^2 + (1/RC)s + (1/LC)]}$$

P 12.31 [a] For $t \geq 0^+$:

$$Ri_o + L\frac{di_o}{dt} + v_o = 0$$

$$i_o = C\frac{dv_o}{dt} \qquad \frac{di_o}{dt} = C\frac{d^2v_o}{dt^2}$$

$$\therefore \quad RC\frac{dv_o}{dt} + LC\frac{d^2v_o}{dt^2} + v_o = 0$$

or

$$\frac{d^2v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC}v_o = 0$$

[b] $s^2V_o(s) - sV_{\rm dc} - 0 + \frac{R}{L}[sV_o(s) - V_{\rm dc}] + \frac{1}{LC}V_o(s) = 0$

$$V_o(s)\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right] = V_{\rm dc}(s + R/L)$$

$$V_o(s) = \frac{V_{\rm dc}[s + (R/L)]}{[s^2 + (R/L)s + (1/LC)]}$$

P 12.32 **[a]** $300 = 60i_1 + 25\frac{di_1}{dt} + 10\frac{d}{dt}(i_2 - i_1) + 5\frac{d}{dt}(i_1 - i_2) - 10\frac{di_1}{dt}$

$$0 = 5\frac{d}{dt}(i_2 - i_1) + 10\frac{di_1}{dt} + 40i_2$$

Simplifying the above equations gives:

$$300 = 60i_1 + 10\frac{di_1}{dt} + 5\frac{di_2}{dt}$$

$$0 = 40i_2 + 5\frac{di_1}{dt} + 5\frac{di_2}{dt}$$

[b] $\frac{300}{s} = (10s + 60)I_1(s) + 5sI_2(s)$

$$0 = 5sI_1(s) + (5s + 40)I_2(s)$$

[c] Solving the equations in (b),

$$I_1(s) = \frac{60(s+8)}{s(s+4)(s+24)}$$

$$I_2(s) = \frac{-60}{(s+4)(s+24)}$$

P 12.33 $V(s) = \frac{200s^2}{(s^2 + 40s + 64)(s^2 + 100)}$

$s^2 + 40s + 64 = (s + 38.33)(s + 1.67);\qquad s^2 + 100 = (s - j10)(s + j10)$

Therefore

$$V(s) = \frac{200s^2}{(s + 38.33)(s + 1.67)(s - j10)(s + j10)}$$

$$= \frac{K_1}{s + 1.67} + \frac{K_2}{s + 38.33} + \frac{K_3}{s - j10} + \frac{K_3^*}{s + j10}$$

$$K_1 = \frac{200s^2}{(s + 38.33)(s^2 + 100)}\Bigg|_{s=-1.67} = 0.15$$

$$K_2 = \frac{200s^2}{(s+1.67)(s^2+100)}\bigg|_{s=-38.33} = -5.11$$

$$K_3 = \frac{200s^2}{(s+1.67)(s+38.33)(s+j10)}\bigg|_{s=j10} = 2.49\underline{/-5.14^\circ}$$

Therefore

$$v(t) = [4.98\cos(10t - 5.14^\circ) + 0.15e^{-1.67t} - 5.11e^{-38.33t}]u(t)\text{ V}$$

P 12.34 [a] $\dfrac{1}{LC} = \dfrac{10^9}{(0.8)(100)} = 1250\times 10^4$

$$\frac{1}{RC} = \frac{10^6}{(10)(100)} = 1000$$

$$V_o(s) = \frac{70{,}000}{(s^2+1000s+1250\times 10^4)}$$

$$s_{1,2} = -500 \pm \sqrt{25\times 10^4 - 1250\times 10^4} = -500 \pm j3500 \text{ rad/s}$$

$$V_o(s) = \frac{70{,}000}{(s+500-j3500)(s+500+j3500)}$$

$$= \frac{K}{s+500-j3500} + \frac{K^*}{s+500+j3500}$$

$$K = \frac{70{,}000}{(j7000)} = 10\underline{/-90^\circ}$$

$$\text{V}_o(s) = \frac{10\underline{/-90^\circ}}{s+500-j3500} + \frac{10\underline{/90^\circ}}{s+500+j3500}$$

$$v_o(t) = [20e^{-500t}\cos(3500t - 90^\circ)]u(t)\text{ V} = [20e^{-500t}\sin 3500t]u(t)\text{ V}$$

[b] $I_o(s) = \dfrac{87{,}500}{s(s+500-j3500)(s+500+j3500)}$

$$= \frac{K_1}{s} + \frac{K_2}{s+500-j3500} + \frac{K_2^*}{s+500+j3500}$$

$$K_1 = \frac{87{,}500}{1250\times 10^4} = 7\text{ mA}$$

$$K_2 = \frac{87{,}500}{(-500+j3500)(j7000)} = 3.5\underline{/171.87^\circ}\text{ mA}$$

$$i_o(t) = [7 + 7e^{-500t}\cos(3500t + 171.87^\circ)]u(t)\text{ mA}$$

P 12.35 [a] $\dfrac{1}{RC} = \dfrac{10^9}{(4\times 10^3)(25)} = 10^4$

$$\frac{1}{LC} = \frac{10^9}{(2.5)(25)} = 16\times 10^6$$

$$V_o(s) = \frac{40\times 10^6 I_{\rm dc}}{s + 10{,}000s + 16\times 10^6}$$

$$= \frac{40\times 10^6 I_{\rm dc}}{(s+2000)(s+8000)}$$

$$= \frac{120{,}000}{(s+2000)(s+8000)}$$

$$= \frac{K_1}{s+2000} + \frac{K_2}{s+8000}$$

$$K_1 = \frac{120{,}000}{6000} = 20; \qquad K_2 = \frac{120{,}000}{-6000} = -20$$

$$V_o(s) = \frac{20}{s+2000} - \frac{20}{s+8000}$$

$$v_o(t) = [20e^{-2000t} - 20e^{-8000t}]u(t)\,\text{V}$$

[b] $I_o(s) = \dfrac{3\times 10^{-3}s}{(s+2000)(s+8000)}$

$$= \frac{K_1}{s+2000} + \frac{K_2}{s+8000}$$

$$K_1 = \frac{-(3\times 10^{-3})(2000)}{6000} = -10^{-3}$$

$$K_2 = \frac{(3\times 10^{-3})(-8000)}{-6000} = 4\times 10^{-3}$$

$$I_o(s) = \frac{-10^{-3}}{s+2000} + \frac{4\times 10^{-3}}{s+8000}$$

$$i_o(t) = (4e^{-8000t} - e^{-2000t})u(t)\,\text{mA}$$

[c] $i_o(0) = 4 - 1 = 3\,\text{mA}$

Yes. The initial inductor current is zero by hypothesis, the initial resistor current is zero because the initial capacitor voltage is zero by hypothesis. Thus at $t = 0$ the source current appears in the capacitor.

P 12.36 $\frac{1}{C} = 2 \times 10^6; \qquad \frac{1}{LC} = 4 \times 10^6; \qquad \frac{R}{L} = 5000; \qquad I_g = \frac{0.015}{s}$

$$V_2(s) = \frac{30{,}000}{s^2 + 5000s + 4 \times 10^6}$$

$$s_1 = -1000; \qquad s_2 = -4000$$

$$V_2(s) = \frac{30{,}000}{(s + 1000)(s + 4000)}$$

$$= \frac{10}{s + 1000} - \frac{10}{s + 4000}$$

$$v_2(t) = [10e^{-1000t} - 10e^{-4000t}]u(t)\text{ V}$$

P 12.37 $\frac{1}{RC} = 10{,}000; \qquad \frac{1}{LC} = 16 \times 10^6$

$$I_o(s) = \frac{0.1(s + 10{,}000)}{s^2 + 10{,}000s + 16 \times 10^6}$$

$$s_1 = -2000; \qquad s_2 = -8000$$

$$I_o(s) = \frac{0.1(s + 10{,}000)}{(s + 2000)(s + 8000)} = \frac{K_1}{s + 2000} + \frac{K_2}{s + 8000}$$

$$K_1 = \frac{0.1(8000)}{6000} = 0.133$$

$$K_2 = \frac{0.1(2000)}{-6000} = -0.033$$

$$I_o(s) = \frac{0.133}{s + 2000} - \frac{0.033}{s + 8000}$$

$$i_o(t) = [133.33e^{-2000t} - 33.33e^{-8000t}]u(t)\text{ mA}$$

P 12.38 $\frac{R}{L} = 5000; \qquad \frac{1}{LC} = 4 \times 10^6$

$$V_o(s) = \frac{15(s + 5000)}{s^2 + 5000s + 4 \times 10^6}$$

$$s_{1,2} = -2500 \pm \sqrt{6.25 \times 10^6 - 4 \times 10^6}$$

$s_1 = -1000$ rad/s; $s_2 = -4000$ rad/s

$$V_o(s) = \frac{15(s + 5000)}{(s + 1000)(s + 4000)} = \frac{K_1}{s + 1000} + \frac{K_2}{s + 4000}$$

$$K_1 = \frac{15(4000)}{3000} = 20\,\text{V}; \qquad K_2 = \frac{15(1000)}{-3000} = -5\,\text{V}$$

$$V_o(s) = \frac{20}{s + 1000} - \frac{5}{s + 4000}$$

$$v_o(t) = [20e^{-1000t} - 5e^{-4000t}]u(t)\,\text{V}$$

P 12.39 [a] $I_1(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s + 4} + \dfrac{K_3}{s + 24}$

$$K_1 = \frac{(60)(8)}{(4)(24)} = 5; \qquad K_2 = \frac{(60)(4)}{(-4)(20)} = -3$$

$$K_3 = \frac{(60)(-16)}{(-24)(-20)} = -2$$

$$I_1(s) = \left(\frac{5}{s} - \frac{3}{s + 4} - \frac{2}{s + 24}\right)$$

$$i_1(t) = (5 - 3e^{-4t} - 2e^{-24t})u(t)\,\text{A}$$

$$I_2(s) = \frac{K_1}{s + 4} + \frac{K_2}{s + 24}$$

$$K_1 = \frac{-60}{20} = -3; \qquad K_2 = \frac{-60}{-20} = 3$$

$$I_2(s) = \left(\frac{-3}{s + 4} + \frac{3}{s + 24}\right)$$

$$i_2(t) = (3e^{-24t} - 3e^{-4t})u(t)\,\text{A}$$

[b] $i_1(\infty) = 5\,\text{A}$; $i_2(\infty) = 0\,\text{A}$

[c] Yes, at $t = \infty$

$$i_1 = \frac{300}{60} = 5\,\text{A}$$

Since i_1 is a dc current at $t = \infty$ there is no voltage induced in the 10 H inductor; hence, $i_2 = 0$. Also note that $i_1(0) = 0$ and $i_2(0) = 0$. Thus our solutions satisfy the condition of no initial energy stored in the circuit.

P 12.40 [a] $F(s) = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$

$$K_1 = \frac{18 - 66 + 54}{(1)(2)} = 3; \qquad K_2 = \frac{72 - 132 + 54}{(-1)(1)} = 6$$

$$K_3 = \frac{162 - 198 + 54}{(-2)(-1)} = 9$$

$$\therefore \quad f(t) = [3e^{-t} + 6e^{-2t} + 9e^{-3t}]u(t)$$

[b] $F(s) = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+3} + \frac{K_4}{s+5}$

$$K_1 = \frac{8s^3 + 89s^2 + 311s + 300}{(s+2)(s+3)(s+5)}\bigg|_{s=0} = 10$$

$$K_2 = \frac{8s^3 + 89s^2 + 311s + 300}{s(s+3)(s+5)}\bigg|_{s=-2} = 5$$

$$K_3 = \frac{8s^3 + 89s^2 + 311s + 300}{s(s+2)(s+5)}\bigg|_{s=-3} = -8$$

$$K_4 = \frac{8s^3 + 89s^2 + 311s + 300}{s(s+2)(s+3)}\bigg|_{s=-5} = 1$$

$$f(t) = [10 + 5e^{-2t} - 8e^{-3t} + e^{-5t}]u(t)$$

[c] $s_{1,2} = -6 \pm \sqrt{36 - 100} = -6 \pm j8$

$$F(s) = \frac{11s^2 + 172s + 700}{(s+2)(s+6-j8)(s+6+j8)}$$

$$= \frac{K_1}{s+2} + \frac{K_2}{s+6-j8} + \frac{K_2^*}{s+6+j8}$$

$$K_1 = \frac{44 - 344 + 700}{4 - 24 + 100} = 5$$

$$K_2 = \frac{11(-6+j8)^2 + 172(-6+j8) + 700}{(-4+j8)j16}$$

$$= 3 - j4 = 5\underline{/-53.13^\circ}$$

$$\therefore \quad f(t) = [5e^{-2t} + 10e^{-6t}\cos(8t - 53.13^\circ)]u(t)$$

[d] $s_{1,2} = -7 \pm \sqrt{49-625} = -7 \pm j24$

$$F(s) = \frac{56s^2+112s+5000}{s(s+7-j24)(s+7+j24)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+7-j24} + \frac{K_2^*}{s+7+j24}$$

$$K_1 = \frac{5000}{625} = 8$$

$$K_2 = \frac{56(-7+j24)^2+112(-7+j24)+5000}{(-7+j24)j48}$$

$$= 24 + j7 = 25\underline{/16.26^\circ}$$

$$\therefore \quad f(t) = [8 + 50e^{-7t}\cos(24t+16.26^\circ)]u(t)$$

P 12.41 [a] $F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+10}$

$$K_1 = \frac{8(s^2-5s+50)}{s+10}\bigg|_{s=0} = \frac{400}{10} = 40$$

$$K_2 = \frac{d}{ds}\left\{\frac{8(s^2-5s+50)}{s+10}\right\}\bigg|_{s=0}$$

$$= \frac{8(s+10)(2s-5) - 8(s^2-5s+50)(1)}{(s+10)^2}\bigg|_{s=0}$$

$$= \frac{10(-40)-8(50)}{100} = -8$$

$$K_3 = \frac{8(s^2-5s+50)}{s^2}\bigg|_{s=-10} = \frac{8(100+50+50)}{100} = 16$$

$$F(s) = \frac{40}{s^2} - \frac{8}{s} + \frac{16}{s+10}$$

$$f(t) = [40t - 8 + 16e^{-10t}]u(t)$$

[b] $F(s) = \frac{K_1}{s} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2}$

$$K_1 = \frac{10(4)}{4} = 10; \qquad K_2 = \frac{10(12-8+4)}{-2} = -40$$

$$K_3 = \frac{d}{ds}\left\{\frac{10(3s^2+4s+4)}{s}\right\}\bigg|_{s=-2}$$

$$= \frac{10[(s)(6s+4)-(3s^2+4s+4)]}{s^2}\bigg|_{s=-2} = 20$$

$$F(s) = \frac{10}{s} - \frac{40}{(s+2)^2} + \frac{20}{s+2}$$

$$f(t) = [10 - 40te^{-2t} + 20e^{-2t}]u(t)$$

[c] $s_{1,2} = -2 \pm \sqrt{4-5} = -2 \pm j1$

$$F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+2-j1} + \frac{K_3^*}{s+2+j1}$$

$$K_1 = \frac{50}{5} = 10$$

$$K_2 = \frac{d}{ds}\left\{\frac{s^3 - 6s^2 + 15s + 50}{s^2 + 4s + 5}\right\}\Bigg|_{s=0}$$

$$= \frac{(s^2+4s+5)(3s^2-12s+15) - (s^3-6s^2+15s+50)(2s+4)}{(s^2+4s+5)^2}\Bigg|_{s=0}$$

$$= \frac{5(15) - 50(4)}{25} = -5$$

$$K_3 = \frac{s^3 - 6s^2 + 15s + 50}{s^2(s+2+j1)}\Bigg|_{s=-2+j1}$$

$$(-2+j1)^3 = -2 + j11; \qquad (-2+j1)^2 = 3 - j4$$

$$K_3 = \frac{-2 + j11 - 6(3-j4) + 15(-2+j1) + 50}{(3-j4)(j2)}$$

$$= 3 + j4 = 5\underline{/53.13^\circ}$$

$$F(s) = \frac{10}{s^2} - \frac{5}{s} + \frac{5\underline{/53.13^\circ}}{s+2-j1} + \frac{5\underline{/-53.13^\circ}}{s+2+j1}$$

$$f(t) = [10t - 5 + 10e^{-2t}\cos(t + 53.13^\circ)]u(t)$$

[d] $F(s) = \dfrac{K_1}{(s+2)^3} + \dfrac{K_2}{(s+2)^2} + \dfrac{K_3}{s+2}$

$$K_1 = s^2 + 6s + 5\Big|_{s=-2} = -3$$

$$K_2 = \frac{d}{ds}\{s^2 + 6s + 5\}\Big|_{s=-2} = 2s + 6\Big|_{s=-2} = 2$$

$$2K_3 = \frac{d}{ds}(2s+6)\Big|_{s=-2} = 2; \qquad K_3 = 1$$

$$F(s) = \frac{-3}{(s+2)^3} + \frac{2}{(s+2)^2} + \frac{1}{s+2}$$

$$f(t) = -\frac{3t^2e^{-2t}}{2} + 2te^{-2t} + e^{-2t} = [(2t - 1.5t^2 + 1)e^{-2t}]u(t)$$

[e] $s_{1,2} = -1 \pm \sqrt{1-5} = -1 \pm j2$

$$F(s) = \frac{K_1}{(s+1-j2)^2} + \frac{K_1^*}{(s+1+j2)^2} + \frac{K_2}{s+1-j2} + \frac{K_2^*}{(s+1+j2)}$$

$$K_1 = \left.\frac{16s^3 + 72s^2 + 216s - 128}{(s+1+j2)^2}\right|_{s=-1+j2}$$

$$(-1+j2)^3 = 11 - j2; \qquad (-1+j2)^2 = -3 - j4$$

$$K_1 = \frac{176 - j32 - 216 - j288 - 216 + j432 - 128}{-16}$$

$$= 24 - j7 = 25\underline{/-16.26^\circ}$$

$$K_2 = \frac{d}{ds}\left\{\left.\frac{16s^3 + 72s^2 + 216s - 128}{(s+1+j2)^2}\right|_{s=-1+j2}\right\}$$

$$= \left.\frac{(s+1+j2)^2(48s^2 + 144s + 216)}{(s+1+j2)^4}\right|_{s=-1+j2}$$

$$- \left.\frac{(16s^3 + 72s^2 + 216s - 128)2(s+1+j2)}{(s+1+j2)^4}\right|_{s=-1+j2}$$

$$= \frac{(j4)^2(-144 - j192 - 144 + j288 + 216) - (-384 + j112)(j8)}{(j4)^4}$$

$$= \frac{2048 + j1536}{256} = 8 + j6 = 10\underline{/36.87^\circ}$$

$$F(s) = \frac{25\underline{/-16.26^\circ}}{(s+1-j2)^2} + \frac{25\underline{/16.26^\circ}}{(s+1+j2)^2} + \frac{10\underline{/36.87^\circ}}{s+1-j2} + \frac{10\underline{/-36.87^\circ}}{s+1+j2}$$

$$f(t) = [50te^{-t}\cos(2t - 16.26^\circ) + 20e^{-t}\cos(2t + 36.87)]u(t)$$

P 12.42 [a]

$$F(s) = \qquad s^2 + 6s + 5 \;\overline{\big)\;10s^2 + 85s + 95}$$

$$\begin{array}{r} 10 \\ s^2+6s+5 \;\big)\; 10s^2 + 85s + 95 \\ \underline{10s^2 + 60s + 50} \\ 25s + 45 \end{array}$$

$$F(s) = 10 + \frac{25s + 45}{s^2 + 6s + 5} = 10 + \frac{K_1}{s+1} + \frac{K_2}{s+5}$$

$$K_1 = \left.\frac{25s + 45}{s+5}\right|_{s=-1} = 5$$

$$K_2 = \left.\frac{25s + 45}{s+1}\right|_{s=-5} = 20$$

$$F(s) = 10 + \frac{5}{s+1} + \frac{20}{s+5}$$

$$f(t) = 10\delta(t) + [5e^{-t} + 20e^{-5t}]u(t)$$

[b]

$$F(s) = \quad s^2 + 4s + 5 \;\overline{\big)\; 5s^2 + 40s + 25} \quad \text{quotient: } 5$$

$$\begin{array}{r} 5s^2 + 40s + 25 \\ 5s^2 + 20s + 25 \\ \hline 20s \end{array}$$

$$F(s) = 5 + \frac{20s}{s^2 + 4s + 5} = 5 + \frac{K_1}{s+2-j} + \frac{K_1^*}{s+2+j}$$

$$K_1 = \left.\frac{20s}{s+2+j}\right|_{s=-2+j} = 10 + j20 = 22.36\underline{/63.43^\circ}$$

$$F(s) = 5 + \frac{22.36\underline{/63.43^\circ}}{s+2-j} + \frac{22.36\underline{/-63.43^\circ}}{s+2+j}$$

$$f(t) = 5\delta(t) + 44.72e^{-2t}\cos(t + 63.43^\circ)u(t)$$

[c]

$$F(s) = \quad s + 20 \;\overline{\big)\; s^2 + 25s + 150} \quad \text{quotient: } s+5$$

$$\begin{array}{r} s^2 + 25s + 150 \\ s^2 + 20s \\ \hline 5s + 150 \\ 5s + 100 \\ \hline 50 \end{array}$$

$$F(s) = s + 5 + \frac{50}{(s+20)} = s + 5 + \frac{50}{s+20}$$

$$f(t) = \delta'(t) + 5\delta(t) + 50e^{-20t}u(t)$$

P 12.43 [a] $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s+1-j2} + \dfrac{K_3^*}{s+1+j2}$

$$K_1 = \left.\frac{100(s+1)}{s^2 + 2s + 5}\right|_{s=0} = 20$$

$$K_2 = \frac{d}{ds}\left[\frac{100(s+1)}{s^2 + 2s + 5}\right] = \left[\frac{100}{s^2 + 2s + 5} - \frac{100(s+1)(2s+2)}{(s^2 + 2s + 5)^2}\right]_{s=0}$$

$$= 20 - 8 = 12$$

$$K_3 = \left.\frac{100(s+1)}{s^2(s+1+j2)}\right|_{s=-1+j2} = -6 + j8 = 10\underline{/126.87^\circ}$$

$$f(t) = [20t + 12 + 20e^{-t}\cos(2t + 126.87^\circ)]u(t)$$

[b] $F(s) = \dfrac{20s^2}{(s+1)^3} = \dfrac{K_1}{(s+1)^3} + \dfrac{K_2}{(s+1)^2} + \dfrac{K_3}{s+1}$

$\therefore \quad 20s^2 = K_1 + K_2(s+1) + K_3(s+1)^2$

$K_1 = 20s^2 \Big|_{s=-1} = 20$

After differentiating each side

$40s = 0 + K_2 + 2K_3(s+1); \qquad \therefore \quad K_2 = 40s\Big|_{s=-1} = -40$

After differentiating again

$40 = 0 + 2K_3; \qquad \therefore \quad K_3 = 20$

$$\therefore \quad \frac{20s^2}{(s+1)^3} = \frac{20}{(s+1)^3} - \frac{40}{(s+1)^2} + \frac{20}{s+1}$$

Test at $s = 0$:

$0 = 20 - 40 + 20 = 0 \qquad$ OK

$$f(t) = \frac{20t^2e^{-t}}{2!} - 40te^{-t} + 20e^{-t} = (10t^2 - 40t + 20)e^{-t}u(t)$$

[c] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+1)^3} + \dfrac{K_3}{(s+1)^2} + \dfrac{K_4}{s+1}$

$$K_1 = \frac{40(s+2)}{(s+1)^3}\Big|_{s=0} = 80$$

$$K_2 = \frac{40(s+2)}{s}\Big|_{s=-1} = -40$$

$$K_3 = \frac{d}{ds}\left[\frac{40(s+2)}{s}\right] = \left[\frac{40}{s} - \frac{40(s+2)}{s^2}\right]_{s=-1} = -40 - 40 = -80$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{40}{s} - \frac{40(s+2)}{s^2}\right]$$

$$= \frac{1}{2}\left[\frac{-40}{s^2} - \frac{40}{s^2} + \frac{80(s+2)}{s^3}\right]_{s=-1} = \frac{1}{2}(-40 - 40 - 80) = -80$$

$$f(t) = [80 - 20t^2e^{-t} - 80te^{-t} - 80e^{-t}]u(t)$$

[d] $F(s) = \dfrac{5(s+2)^2}{s^4(s+1)} = \dfrac{K_1}{s+1} + \dfrac{K_2}{s^4} + \dfrac{K_3}{s^3} + \dfrac{K_4}{s^2} + \dfrac{K_5}{s}$

$$K_1 = \frac{5(s+2)^2}{s^4}\Big|_{s=-1} = 5; \qquad K_2 = \frac{5(s+2)^2}{s+1}\Big|_{s=0} = 20$$

$$\frac{5(s+2)^2}{s+1} = \frac{K_1 s^4}{s+1} + K_2 + K_3 s + K_4 s^2 + K_5 s^3$$

Differentiating each side gives

$$5\left[\frac{(s+1)2(s+2)-(s+2)^2}{(s+1)^2}\right] = \frac{K_1[4s^3(s+1)-s^4]}{(s+1)^2}$$

$$+0 + K_3 + 2K_4 s + 3K_5 s^2$$

$$\frac{5s(s+2)}{(s+1)^2} = \frac{K_1 s^3(3s+4)}{(s+1)^2} + K_3 + 2K_4 s + 3K_5 s^2$$

$$K_3 = \frac{5s(s+2)}{(s+1)^2}\bigg|_{s=0} = 0$$

Note that two more derivatives of the term involving K_1 will drop out at $s = 0$. Hence,

$$2K_4 = 5\frac{d}{ds}\left[\frac{s(s+2)}{(s+1)^2}\right]_{s=0} - 6K_5 s\bigg|_{s=0}$$

$$2K_4 = 5\left\{\frac{(s+1)^2(2s+2)-s(s+2)2(s+1)}{(s+1)^4}\right\}\bigg|_{s=0}$$

$$= 5(s+1)\frac{2(s+1)^2-2s(s+2)}{(s+1)^4}\bigg|_{s=0}$$

$$= (5)\frac{2}{(s+1)^3}\bigg|_{s=0} = 10$$

$$\therefore\quad K_4 = 5$$

Now differentiate once more to get

$$6K_5 = \frac{d}{ds}\left\{\frac{10}{(s+1)^3}\right\}\bigg|_{s=0}$$

$$= \frac{-30(s+1)^2}{(s+1)^6}\bigg|_{s=0}$$

$$= \frac{-30}{(s+1)^4}\bigg|_{s=0} = -30$$

$$\therefore\quad K_5 = -5$$

$$\frac{5(s+2)^2}{s^4(s+1)} = \frac{5}{s+1} + \frac{20}{s^4} + \frac{0}{s^3} + \frac{5}{s^2} - \frac{5}{s}$$

$$= \frac{5}{s+1} + \frac{20}{s^4} + \frac{5}{s^2} - \frac{5}{s}$$

Test at $s = -2$:

$$0 = -5 + \frac{20}{16} + \frac{5}{4} + \frac{5}{2} = 0 \qquad \text{OK}$$

$$\therefore \quad F(s) = \frac{5}{s+1} + \frac{20}{s^4} + \frac{5}{s^2} - \frac{5}{s}$$

$$f(t) = 5e^{-t} + \frac{20t^3}{3!} + 5t - 5$$

$$= (5e^{5t} + \tfrac{10}{3}t^3 + 5t - 5)u(t)$$

P 12.44 $f(t) = \mathcal{L}^{-1}\left\{\frac{K}{s+\alpha-j\beta} + \frac{K^*}{s+\alpha+j\beta}\right\}$

$$= Ke^{-\alpha t}e^{j\beta t} + K^*e^{-\alpha t}e^{-j\beta t}$$

$$= |K|e^{-\alpha t}[e^{j\theta}e^{j\beta t} + e^{-j\theta}e^{-j\beta t}]$$

$$= |K|e^{-\alpha t}[e^{j(\beta t+\theta)} + e^{-j(\beta t+\theta)}]$$

$$= 2|K|e^{-\alpha t}\cos(\beta t + \theta)$$

P 12.45 [a] $\mathcal{L}\{t^n f(t)\} = (-1)^n \left[\frac{d^nF(s)}{ds^n}\right]$

Let $f(t) = 1$, then $F(s) = \frac{1}{s}$, thus $\frac{d^nF(s)}{ds^n} = \frac{(-1)^n n!}{s^{(n+1)}}$

Therefore $\mathcal{L}\{t^n\} = (-1)^n \left[\frac{(-1)^n n!}{s^{(n+1)}}\right] = \frac{n!}{s^{(n+1)}}$

It follows that $\mathcal{L}\{t^{(r-1)}\} = \frac{(r-1)!}{s^r}$

and $\mathcal{L}\{t^{(r-1)}e^{-at}\} = \frac{(r-1)!}{(s+a)^r}$

Therefore $\frac{K}{(r-1)!}\mathcal{L}\{t^{r-1}e^{-at}\} = \frac{K}{(s+a)^r} = \mathcal{L}\left\{\frac{Kt^{r-1}e^{-at}}{(r-1)!}\right\}$

[b] $f(t) = \mathcal{L}^{-1}\left\{\frac{K}{(s+\alpha-j\beta)^r} + \frac{K^*}{(s+\alpha+j\beta)^r}\right\}$

Therefore

$$f(t) = \frac{Kt^{r-1}}{(r-1)!}e^{-(\alpha-j\beta)t} + \frac{K^*t^{r-1}}{(r-1)!}e^{-(\alpha+j\beta)t}$$

$$= \frac{|K|t^{r-1}e^{-\alpha t}}{(r-1)!}\left[e^{j\theta}e^{j\beta t} + e^{-j\theta}e^{-j\beta t}\right]$$

$$= \left[\frac{2|K|t^{r-1}e^{-\alpha t}}{(r-1)!}\right]\cos(\beta t + \theta)$$

P 12.46 [a] $\lim_{s\to\infty} sV(s) = \lim_{s\to\infty}\left[\frac{200s^3}{s^4[1+(40/s)+(64/s^2)][1+(100/s^2)]}\right] = 0$

Therefore $v(0^+) = 0$

[b] Yes, all of the poles of V are in the left-half of the complex plane. Therefore,

$$\lim_{s\to 0} sV(s) = \lim_{s\to 0}\left[\frac{200s^3}{(s^2+40s+64)(s^2+100)}\right] = 0$$

Therefore $v(\infty) = 0$

P 12.47 [a] $sF(s) = \frac{18s^3+66s^2+54s}{(s+1)(s+2)(s+3)}$

$\lim_{s\to 0} sF(s) = 0, \quad \therefore\ f(\infty) = 0$

$\lim_{s\to\infty} sF(s) = 18, \quad \therefore\ f(0^+) = 18$

[b] $sF(s) = \frac{8s^3+89s^2+311s+300}{(s+2)(s^2+8s+15)}$

$\lim_{s\to 0} sF(s) = 10; \quad \therefore\ f(\infty) = 10$

$\lim_{s\to\infty} sF(s) = 8, \quad \therefore\ f(0^+) = 8$

[c] $sF(s) = \frac{11s^3+172s^2+700s}{(s+2)(s^2+12s+100)}$

$\lim_{s\to 0} sF(s) = 0, \quad \therefore\ f(\infty) = 0$

$\lim_{s\to\infty} sF(s) = 11, \quad \therefore\ f(0^+) = 11$

[d] $sF(s) = \frac{56s^2+112s+5000}{(s^2+14s+625)}$

$\lim_{s\to 0} sF(s) = \frac{5000}{625} = 8, \quad \therefore\ f(\infty) = 8$

$\lim_{s\to\infty} sF(s) = 56, \quad \therefore\ f(0^+) = 56$

P 12.48 [a] $sF(s) = \dfrac{8(s^2 - 5s + 50)}{s(s+10)}$

$F(s)$ has a second-order pole at the origin so we cannot use the final value theorem.

$$\lim_{s\to\infty} sF(s) = 8, \qquad \therefore \quad f(0^+) = 8$$

[b] $sF(s) = \dfrac{10(3s^2 + 4s + 4)}{(s+2)^2}$

$$\lim_{s\to 0} sF(s) = \frac{40}{4} = 10, \qquad \therefore \quad f(\infty) = 10$$

$$\lim_{s\to\infty} sF(s) = 30, \qquad \therefore \quad f(0^+) = 30$$

[c] $sF(s) = \dfrac{s^3 - 6s^2 + 15s + 50}{s(s^2 + 4s + 5)}$

$F(s)$ has a second-order pole at the origin so we cannot use the final value theorem.

$$\lim_{s\to\infty} sF(s) = 1, \qquad \therefore \quad f(0^+) = 1$$

[d] $sF(s) = \dfrac{s^3 + 6s^2 + 5s}{(s+2)^3}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to\infty} sF(s) = 1, \qquad \therefore \quad f(0^+) = 1$$

[e] $sF(s) = \dfrac{16s^4 + 72s^3 + 216s^2 - 128s}{(s^2 + 2s + 5)^2}$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to\infty} sF(s) = 16, \qquad \therefore \quad f(0^+) = 16$$

P 12.49 All of the $F(s)$ functions referenced in this problem are improper rational functions, and thus the corresponding $f(t)$ functions contain impulses ($\delta(t)$). Thus, neither the initial value theorem nor the final value theorem may be applied to these $F(s)$ functions!

P 12.50 $sV_o(s) = \dfrac{sV_{\rm dc}/RC}{s^2 + (1/RC)s + (1/LC)}$

$$\lim_{s\to 0} sV_o(s) = 0, \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to\infty} sV_o(s) = 0, \qquad \therefore \quad v_o(0^+) = 0$$

$$sI_o(s) = \frac{V_{dc}/RC)}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s\to 0} sI_o(s) = \frac{V_{dc}/RLC}{1/LC} = \frac{V_{dc}}{R}, \qquad \therefore \quad i_o(\infty) = \frac{V_{dc}}{R}$$

$$\lim_{s\to\infty} sI_o(s) = 0, \qquad \therefore \quad i_o(0^+) = 0$$

P 12.51 $$sV_o(s) = \frac{(I_{dc}/C)s}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s\to 0} sV_o(s) = 0, \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to\infty} sV_o(s) = 0, \qquad \therefore \quad v_o(0^+) = 0$$

$$sI_o(s) = \frac{s^2 I_{dc}}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s\to 0} sI_o(s) = 0, \qquad \therefore \quad i_o(\infty) = 0$$

$$\lim_{s\to\infty} sI_o(s) = I_{dc}, \qquad \therefore \quad v_o(0^+) = I_{dc}$$

P 12.52 $$sI_o(s) = \frac{I_{dc}s[s + (1/RC)]}{s^2 + (1/RC)s + (1/LC)}$$

$$\lim_{s\to 0} sI_o(s) = 0, \qquad \therefore \quad i_o(\infty) = 0$$

$$\lim_{s\to\infty} sI_o(s) = I_{dc}, \qquad \therefore \quad i_o(0^+) = I_{dc}$$

P 12.53 **[a]** $$sF(s) = \frac{100(s+1)}{s(s^2 + 2s + 5)}$$

$F(s)$ has a second-order pole at the origin, so we cannot use the final value theorem here.

$$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

[b] $$sF(s) = \frac{20s^3}{(s+1)^3}$$

$$\lim_{s\to 0} sF(s) = 0, \qquad \therefore \quad f(\infty) = 0$$

$$\lim_{s\to\infty} sF(s) = 20, \qquad \therefore \quad f(0^+) = 20$$

[c] $sF(s) = \dfrac{40(s+2)}{(s+1)^3}$

$$\lim_{s\to 0} sF(s) = 80, \qquad \therefore \quad f(\infty) = 80$$

$$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

[d] $sF(s) = \dfrac{5s(s+2)^2}{s^4(s+1)} = \dfrac{5(s+2)^2}{s^3(s+1)}$

$$\lim_{s\to\infty} sF(s) = 0, \qquad \therefore \quad f(0^+) = 0$$

The final value theorem cannot be applied here, as $F(s)$ violates that requirement that all poles lie in the left-half plane, with the exception of a single pole at the origin. This $F(s)$ has four poles at the origin!

13

The Laplace Transform in Circuit Analysis

Assessment Problems

AP 13.1 [a] $Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{C[s^2 + (1/RC)s + (1/LC)]}{s}$

$$\frac{1}{RC} = \frac{10^6}{(500)(0.025)} = 80{,}000; \qquad \frac{1}{LC} = 25 \times 10^8$$

$$\text{Therefore} \quad Y = \frac{25 \times 10^{-9}(s^2 + 80{,}000s + 25 \times 10^8)}{s}$$

[b] $z_{1,2} = -40{,}000 \pm \sqrt{16 \times 10^8 - 25 \times 10^8} = -40{,}000 \pm j30{,}000$ rad/s

$-z_1 = -40{,}000 - j30{,}000$ rad/s

$-z_2 = -40{,}000 + j30{,}000$ rad/s

$p_1 = 0$ rad/s

AP 13.2 [a] $Z = 2000 + \frac{1}{Y} = 2000 + \frac{4 \times 10^7 s}{s^2 + 80{,}000s + 25 \times 10^8}$

$$= \frac{2000(s^2 + 10^5 s + 25 \times 10^8)}{s^2 + 80{,}000s + 25 \times 10^8} = \frac{2000(s + 50{,}000)^2}{s^2 + 80{,}000s + 25 \times 10^8}$$

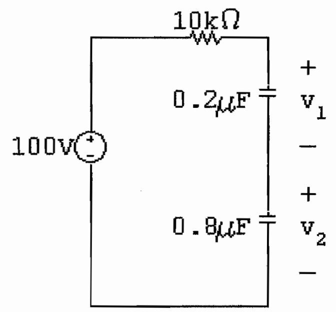
[b] $-z_1 = -z_2 = -50{,}000$ rad/s

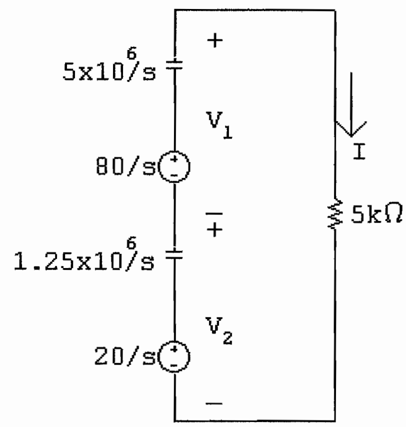
$-p_1 = -40{,}000 - j30{,}000$ rad/s

$-p_2 = -40{,}000 + j30{,}000$ rad/s

AP 13.3 [a] At $t = 0^-$, $0.2v_1 = (0.8)v_2$; $v_1 = 4v_2$; $v_1 + v_2 = 100\,\text{V}$

Therefore $v_1(0^-) = 80V = v_1(0^+)$; $v_2(0^-) = 20V = v_2(0^+)$





$$I = \frac{(80/s) + (20/s)}{5000 + [(5 \times 10^6)/s] + (1.25 \times 10^6/s)} = \frac{20 \times 10^{-3}}{s + 1250}$$

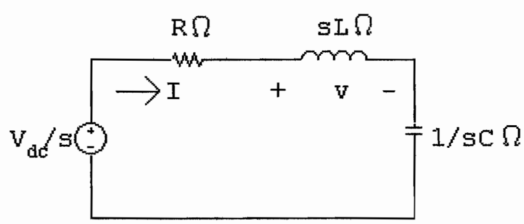
$$V_1 = \frac{80}{s} - \frac{5 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{80}{s + 1250}$$

$$V_2 = \frac{20}{s} - \frac{1.25 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{20}{s + 1250}$$

[b] $i = 20e^{-1250t}u(t)\,\text{mA}$; $v_1 = 80e^{-1250t}u(t)\,\text{V}$

$v_2 = 20e^{-1250t}u(t)\,\text{V}$

AP 13.4 [a]



$$I = \frac{V_{dc}/s}{R + sL + (1/sC)} = \frac{V_{dc}/L}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{V_{dc}}{L} = 40; \qquad \frac{R}{L} = 1.2; \qquad \frac{1}{LC} = 1.0$$

$$I = \frac{40}{(s + 0.6 - j0.8)(s + 0.6 + j0.8)} = \frac{K_1}{s + 0.6 - j0.8} + \frac{K_1^*}{s + 0.6 + j0.8}$$

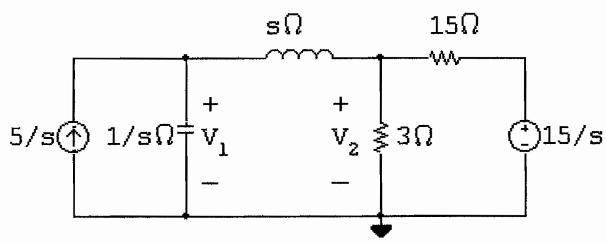
$$K_1 = \frac{40}{j1.6} = -j25 = 25\underline{/-90^\circ}; \qquad K_1^* = 25\underline{/90^\circ}$$

[b] $i = 50e^{-0.6t}\cos(0.8t - 90^\circ) = [50e^{-0.6t}\sin 0.8t]u(t)$ A

[c] $$V = sLI = \frac{160s}{(s+0.6-j0.8)(s+0.6+j0.8)}$$
$$= \frac{K_1}{s+0.6-j0.8} + \frac{K_1^*}{s+0.6+j0.8}$$
$$K_1 = \frac{160(-0.6+j0.8)}{j1.6} = 100\underline{/36.87^\circ}$$

[d] $v(t) = [200e^{-0.6t}\cos(0.8t + 36.87^\circ)]u(t)$ V

AP 13.5 [a]



The two node voltage equations are

$$\frac{V_1 - V_2}{s} + V_1 s = \frac{5}{s} \quad \text{and} \quad \frac{V_2}{3} + \frac{V_2 - V_1}{s} + \frac{V_2 - (15/s)}{15} = 0$$

Solving for V_1 and V_2 yields

$$V_1 = \frac{5(s+3)}{s(s^2+2.5s+1)}, \qquad V_2 = \frac{2.5(s^2+6)}{s(s^2+2.5s+1)}$$

[b] The partial fraction expansions of V_1 and V_2 are

$$V_1 = \frac{15}{s} - \frac{50/3}{s+0.5} + \frac{5/3}{s+2} \quad \text{and} \quad V_2 = \frac{15}{s} - \frac{125/6}{s+0.5} + \frac{25/3}{s+2}$$

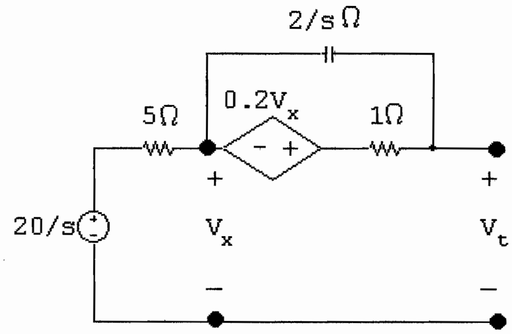
It follows that

$$v_1(t) = \left[15 - \frac{50}{3}e^{-0.5t} + \frac{5}{3}e^{-2t}\right]u(t)\text{ V} \quad \text{and}$$
$$v_2(t) = \left[15 - \frac{125}{6}e^{-0.5t} + \frac{25}{3}e^{-2t}\right]u(t)\text{ V}$$

[c] $$v_1(0^+) = 15 - \frac{50}{3} + \frac{5}{3} = 0$$
$$v_2(0^+) = 15 - \frac{125}{6} + \frac{25}{3} = 2.5\text{ V}$$

[d] $v_1(\infty) = 15$ V; $\qquad v_2(\infty) = 15$ V

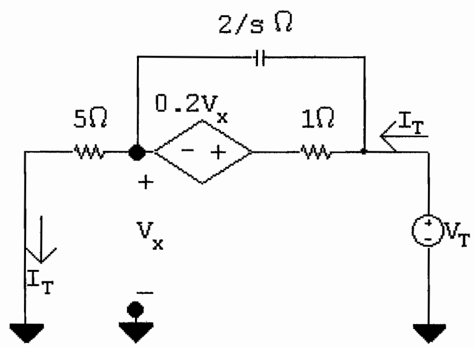
AP 13.6 [a]



With no load across terminals $a-b$ $V_x = 20/s$:

$$\frac{1}{2}\left[\frac{20}{s} - V_{\rm Th}\right]s + \left[1.2\left(\frac{20}{s}\right) - V_{\rm Th}\right] = 0$$

therefore $V_{\rm Th} = \dfrac{20(s+2.4)}{s(s+2)}$



$V_x = 5I_T$ and $Z_{\rm Th} = \dfrac{V_T}{I_T}$

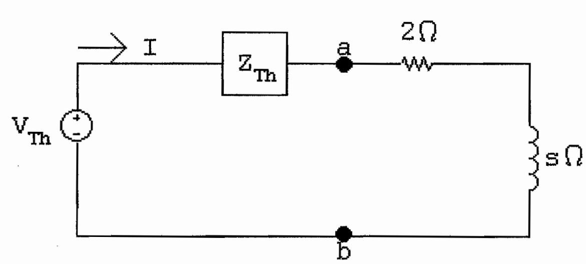
Solving for I_T gives

$$I_T = \frac{(V_T - 5I_T)s}{2} + V_T - 6I_T$$

Therefore

$14I_T = V_Ts + 5sI_T + 2V_T;$ therefore $Z_{\rm Th} = \dfrac{5(s+2.8)}{s+2}$

[b]



$$I = \frac{V_{\rm Th}}{Z_{\rm Th} + 2 + s} = \frac{20(s+2.4)}{s(s+3)(s+6)}$$

AP 13.7 [a] $i_2 = 1.25e^{-t} - 1.25e^{-3t}$; therefore $\frac{di_2}{dt} = -1.25e^{-t} + 3.75e^{-3t}$

Therefore $\frac{di_2}{dt} = 0$ when

$1.25e^{-t} = 3.75e^{-3t}$ or $e^{2t} = 3$, $t = 0.5(\ln 3) = 549.31\,\text{ms}$

$i_2(\text{max}) = 1.25[e^{-0.549} - e^{-3(0.549)}] = 481.13\,\text{mA}$

[b] From Eqs. 13.68 and 13.69, we have

$\Delta = 12(s^2 + 4s + 3) = 12(s + 1)(s + 3)$ and $N_1 = 60(s + 2)$

Therefore $I_1 = \frac{N_1}{\Delta} = \frac{5(s + 2)}{(s + 1)(s + 3)}$

A partial fraction expansion leads to the expression

$I_1 = \frac{2.5}{s + 1} + \frac{2.5}{s + 3}$

Therefore we get

$i_1 = 2.5[e^{-t} + e^{-3t}]u(t)\ \text{A}$

[c] $\frac{di_1}{dt} = -2.5[e^{-t} + 3e^{-3t}]$; $\frac{di_1(0.54931)}{dt} = -2.89\ \text{A/s}$

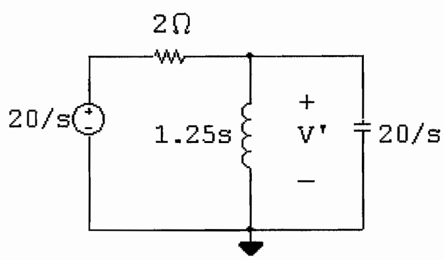
[d] When i_2 is at its peak value,

$\frac{di_2}{dt} = 0$

Therefore $L_2\left(\frac{di_2}{dt}\right) = 0$ and $i_2 = -\left(\frac{M}{12}\right)\left(\frac{di_1}{dt}\right)$

[e] $i_2(\text{max}) = \frac{-2(-2.89)}{12} = 481.13\,\text{mA}$ (checks)

AP 13.8 [a] The s-domain circuit with the voltage source acting alone is

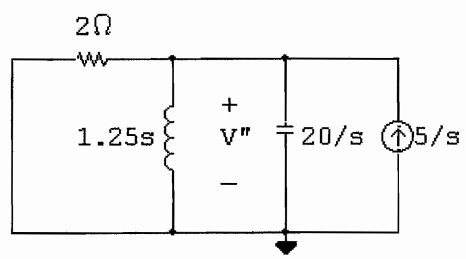


$$\frac{V' - (20/s)}{2} + \frac{V'}{1.25s} + \frac{V's}{20} = 0$$

$$V' = \frac{200}{(s+2)(s+8)} = \frac{100/3}{s+2} - \frac{100/3}{s+8}$$

$$v' = \frac{100}{3}[e^{-2t} - e^{-8t}]u(t)\text{ V}$$

[b] With the current source acting alone,



$$\frac{V''}{2} + \frac{V''}{1.25s} + \frac{V''s}{20} = \frac{5}{s}$$

$$V'' = \frac{100}{(s+2)(s+8)} = \frac{50/3}{s+2} - \frac{50/3}{s+8}$$

$$v'' = \frac{50}{3}[e^{-2t} - e^{-8t}]u(t)\text{ V}$$

[c] $v = v' + v'' = [50e^{-2t} - 50e^{-8t}]u(t)\text{ V}$

AP 13.9 **[a]** $\frac{V_o}{s+2} + \frac{V_o s}{10} = I_g;$ therefore $\frac{V_o}{I_g} = H(s) = \frac{10(s+2)}{s^2+2s+10}$

[b] $-z_1 = -2$ rad/s; $-p_1 = -1 + j3$ rad/s; $-p_2 = -1 - j3$ rad/s

AP 13.10 **[a]**

$$V_o = \frac{10(s+2)}{s^2+2s+10} \cdot \frac{1}{s} = \frac{K_o}{s} + \frac{K_1}{s+1-j3} + \frac{K_1^*}{s+1+j3}$$

$$K_o = 2; \qquad K_1 = 5/3\underline{/-126.87^\circ}; \qquad K_1^* = 5/3\underline{/126.87^\circ}$$

$$v_o = [2 + (10/3)e^{-t}\cos(3t - 126.87^\circ)]u(t)\text{ V}$$

[b] $V_o = \frac{10(s+2)}{s^2+2s+10} \cdot 1 = \frac{K_2}{s+1-j3} + \frac{K_2^*}{s+1+j3}$

$$K_2 = 5.27\underline{/-18.43^\circ}; \qquad K_2^* = 5.27\underline{/18.43^\circ}$$

$$v_o = [10.54e^{-t}\cos(3t - 18.43^\circ)]u(t)\text{ V}$$

AP 13.11 [a]

$$H(s) = \mathcal{L}\{h(t)\} = \mathcal{L}\{v_o(t)\}$$

$$v_o(t) = 10{,}000\cos\theta e^{-70t}\cos 240t - 10{,}000\sin\theta e^{-70t}\sin 240t$$

$$= 9600e^{-70t}\cos 240t - 2800e^{-70t}\sin 240t$$

Therefore $H(s) = \dfrac{9600(s+70)}{(s+70)^2 + (240)^2} - \dfrac{2800(240)}{(s+70)^2+(240)^2}$

$$= \frac{9600s}{s^2 + 140s + 62{,}500}$$

[b] $V_o(s) = H(s)\cdot\dfrac{1}{s} = \dfrac{9600}{s^2+140s+62{,}500}$

$$= \frac{K_1}{s+70-j240} + \frac{K_1^*}{s+70+j240}$$

$$K_1 = \frac{9600}{j480} = -j20 = 20\underline{/-90^\circ}$$

Therefore

$$v_o(t) = [40e^{-70t}\cos(240t - 90^\circ)]u(t)\text{ V} = [40e^{-70t}\sin 240t]u(t)\text{ V}$$

AP 13.12 From Assessment Problem 13.9:

$$H(s) = \frac{10(s+2)}{s^2+2s+10}$$

Therefore $H(j4) = \dfrac{10(2+j4)}{10-16+j8} = 4.47\underline{/-63.43^\circ}$

Thus,

$$v_o = (10)(4.47)\cos(4t - 63.43^\circ) = 44.7\cos(4t - 63.43^\circ)\text{ V}$$

AP 13.13 [a]

Let $R_1 = 10\,\text{k}\Omega,\quad R_2 = 50\,\text{k}\Omega,\quad C = 400\text{ pF},\quad R_2C = 2\times 10^{-5}$

then $V_1 = V_2 = \dfrac{V_gR_2}{R_2 + (1/sC)}$

Also $\dfrac{V_1 - V_g}{R_1} + \dfrac{V_1 - V_o}{R_1} = 0$

therefore $V_o = 2V_1 - V_g$

Now solving for V_o/V_g, we get $H(s) = \dfrac{R_2Cs - 1}{R_2Cs + 1}$

It follows that $H(j50{,}000) = \dfrac{j-1}{j+1} = j1 = 1\underline{/90^\circ}$

Therefore $v_o = 10\cos(50{,}000t + 90^\circ)$ V

[**b**] Replacing R_2 by R_x gives us $H(s) = \dfrac{R_xCs - 1}{R_xCs + 1}$

Therefore

$$H(j50{,}000) = \frac{j20\times 10^{-6}R_x - 1}{j20\times 10^{-6}R_x + 1} = \frac{R_x + j50{,}000}{R_x - j50{,}000}$$

Thus,

$$\frac{50{,}000}{R_x} = \tan 60^\circ = 1.7321, \qquad R_x = 28{,}867.51\,\Omega$$

Problems

P 13.1 $I_{sc_{ab}} = I_N = \frac{-LI_0}{sL} = \frac{-I_0}{s}; \qquad Z_N = sL$

Therefore, the Norton equivalent is the same as the circuit in Fig. 13.4.

P 13.2 $i = \frac{1}{L}\int_{0^-}^{t} v d\tau + I_0; \qquad$ therefore $I = \left(\frac{1}{L}\right)\left(\frac{V}{s}\right) + \frac{I_0}{s} = \frac{V}{sL} + \frac{I_0}{s}$

P 13.3 $V_{\mathrm{Th}} = V_{\mathrm{ab}} = CV_o\left(\frac{1}{sC}\right) = \frac{V_o}{s}; \qquad Z_{\mathrm{Th}} = \frac{1}{sC}$

P 13.4 [a] $Z = R + sL + \frac{1}{sC} = \frac{L[s^2 + (R/L)s + (1/LC)]}{s}$

$$= \frac{5[s^2 + 2000s + 10^7]}{s}$$

[b] $s_{1,2} = -1000 \pm \sqrt{10^6 - 10^7} = -1000 \pm j3000$ rad/s

Zeros at $-1000 + j3000$ rad/s and $-1000 - j3000$ rad/s

Pole at 0.

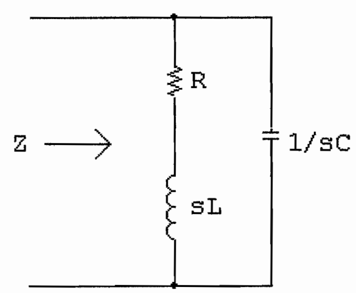
P 13.5 [a] $Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{C[s^2 + (1/RC)s + (1/LC)]}{s}$

$$Z = \frac{1}{Y} = \frac{s/C}{s^2 + (1/RC)s + (1/LC)} = \frac{25 \times 10^6 s}{s^2 + 5000s + 4 \times 10^6}$$

[b] zero at $z_1 = 0$

poles at $-p_1 = -1000$ rad/s and $-p_2 = -4000$ rad/s

P 13.6 [a]



$$Z = \frac{(R + sL)(1/sC)}{R + sL + (1/sC)} = \frac{(1/C)(s + R/L)}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{R}{L} = \frac{1000}{0.5} = 2000; \qquad \frac{1}{LC} = \frac{10^6}{0.2} = 5 \times 10^6$$

$$Z = \frac{2.5 \times 10^6(s + 2000)}{s^2 + 2000s + 5 \times 10^6}$$

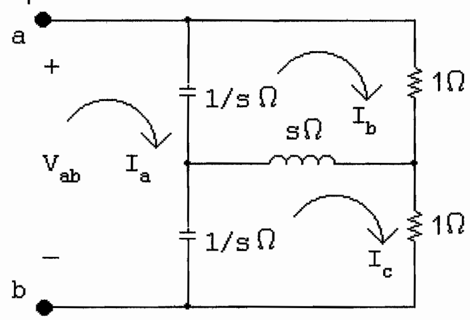
[b] $s_{1,2} = -1000 \pm \sqrt{10^6 - 5 \times 10^6} = -1000 \pm j2000$

$$Z = \frac{2.5 \times 10^6(s + 2000)}{(s + 1000 - j2000)(s + 1000 + j2000)}$$

$$-z_1 = -2000 \text{ rad/s}; \quad -p_1 = -1000 + j2000 \text{ rad/s}$$

$$-p_2 = -1000 - j2000 \text{ rad/s}$$

P 13.7 Transform the Y-connection of the two resistors and the inductor into the equivalent delta-connection:



where

$$Z_a = \frac{(s)(1) + (1)(s) + (1)(1)}{s} = \frac{2s + 1}{s}$$

$$Z_b = Z_c = \frac{(s)(1) + (1)(s) + (1)(1)}{1} = 2s + 1$$

Then

$$Z_{ab} = Z_a \| [(1/s \| Z_c) + (1/s \| Z_b)] = Z_a \| 2(1/s \| Z_b)$$

$$1/s \| Z_b = \frac{\frac{1}{s}(2s + 1)}{\frac{1}{s} + 2s + 1} = \frac{2s + 1}{2s^2 + s + 1}$$

$$Z_{ab} = \left(\frac{2s + 1}{s}\right) \| \frac{2(2s + 1)}{2s^2 + s + 1}$$

$$= \frac{2(2s + 1)^2}{(2s + 1)(2s^2 + s + 1) + 2s(2s + 1)} = \frac{2}{s + 1}$$

No zeros; one pole at -1 rad/s.

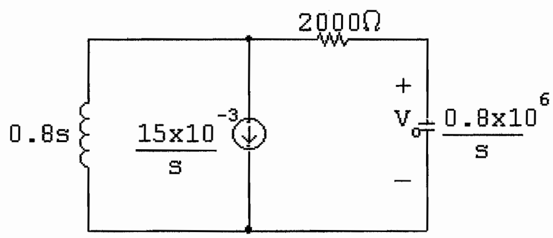
P 13.8 $Z_1 = 0.5s + \dfrac{2(50/s)}{(2 + 50/s)} = \dfrac{s^2 + 25s + 100}{2s + 50}$

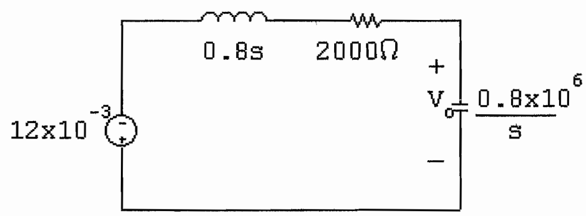
$$Y_{\text{ab}} = \frac{1}{25} + \frac{2s + 50}{s^2 + 25s + 100} = \frac{s^2 + 75s + 1350}{25(s^2 + 25s + 100}$$

$$Z_{\text{ab}} = \frac{25(s^2 + 25s + 100)}{s^2 + 75s + 1350} = \frac{25(s + 5)(s + 20)}{(s + 30)(s + 45)}$$

Zeros at -5 rad/s and -20 rad/s; poles at -30 rad/s and -45 rad/s.

P 13.9 [a] For $t > 0$:





[b] $V_o = \dfrac{-12 \times 10^{-3}(0.8/s) \times 10^6}{0.8s + 2000 + (0.8 \times 10^6)/s}$

$$= \frac{-9600}{0.8s^2 + 2000s + 0.8 \times 10^6}$$

$$= \frac{-12{,}000}{s^2 + 2500s + 10^6}$$

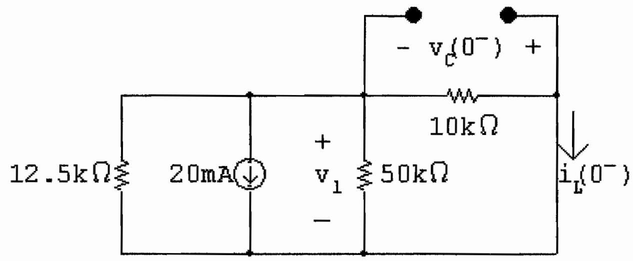
[c] $V_o = \dfrac{-12{,}000}{(s + 500)(s + 2000)} = \dfrac{K_1}{s + 500} + \dfrac{K_2}{s + 2000}$

$K_1 = -8; \qquad K_2 = 8$

$$V_o = \frac{-8}{s + 500} + \frac{8}{s + 2000}$$

$$v_o(t) = (-8e^{-500t} + 8e^{-2000t})u(t)\ \text{V}$$

P 13.10 [a] For $t < 0$:



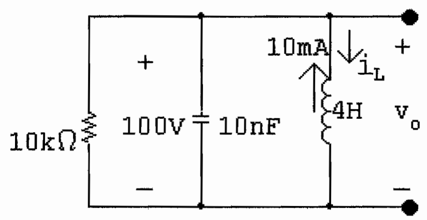
$$\frac{1}{R_e} = \frac{1}{12.5} + \frac{1}{50} + \frac{1}{10} = \frac{1}{5}; \qquad R_e = 5\,\text{k}\Omega$$

$$v_1 = -20(5) = -100\,\text{V}$$

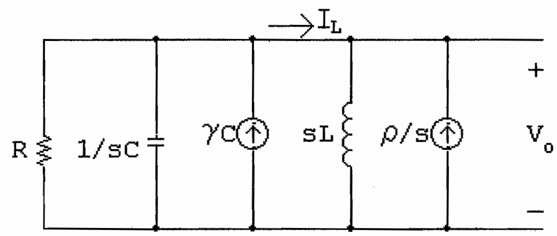
$$i_L(0^-) = \frac{-100}{10} \times 10^{-3} = -10\,\text{mA}$$

$$v_C(0^-) = -v_1 = 100\,\text{V}$$

For $t = 0^+$:



s-domain circuit:



where

$R = 10\,\text{k}\Omega$; $\quad C = 10\,\text{nF}$; $\quad \gamma = 100\,\text{V}$;

$L = 4\,\text{H}$; $\quad$ and $\quad \rho = 10\,\text{mA}$

[b] $$\frac{V_o}{R} + V_o sC - \gamma C + \frac{V_o}{sL} - \frac{\rho}{s} = 0$$

$$\therefore \quad V_o = \frac{\gamma[s + (\rho/\gamma C)]}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{\rho}{\gamma C} = \frac{10 \times 10^{-3}}{(100)(10)10^{-9}} = 10^4$$

$$\frac{1}{RC} = \frac{10^9}{10^5} = 10^4$$

$$\frac{1}{LC} = \frac{10^9}{40} = 25 \times 10^6$$

$$V_o = \frac{100(s + 10^4)}{s^2 + 10^4 s + 25 \times 10^6}$$

[c] $I_{\rm L} = \dfrac{V_o}{sL} - \dfrac{\rho}{s} = \dfrac{V_o}{4s} - \dfrac{10 \times 10^{-3}}{s}$

$$I_{\rm L} = \frac{25(s + 10^4)}{s(s^2 + 10^4 s + 25 \times 10^6)} - \frac{10^{-2}}{s} = \frac{-0.01(s + 7500)}{(s + 5000)^2}$$

[d] $V_o = \dfrac{100(s + 10^4)}{s^2 + 10^4 s + 25 \times 10^6}$

$$= \frac{100(s + 10^4)}{(s + 5000)^2} = \frac{K_1}{(s + 5000)^2} + \frac{K_2}{s + 5000}$$

$$K_1 = 100(5000) = 5 \times 10^5$$

$$K_2 = \frac{d}{ds}\left[100(s + 10{,}000)\right]_{s=-5000} = 100$$

$$V_o = \frac{5 \times 10^5}{(s + 5000)^2} + \frac{100}{s + 5000}$$

$$v_o = [5 \times 10^5 t e^{-5000t} + 100 e^{-5000t}]u(t)\ \text{V}$$

[e] $I_{\rm L} = \dfrac{-0.01(s + 7500)}{(s + 5000)^2}$

$$= \frac{K_1}{(s + 5000)^2} + \frac{K_2}{(s + 5000)}$$

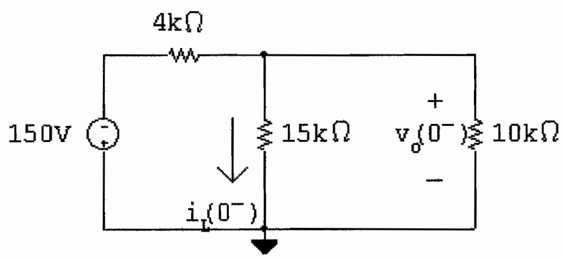
$$K_1 = -0.01(2500) = -25$$

$$K_2 = \frac{d}{ds}[-0.01(s + 7500)]_{s=-5000} = -0.01$$

$$I_{\rm L} = \left[\frac{-25{,}000}{(s + 5000)^2} - \frac{10}{s + 5000}\right] \times 10^{-3}$$

$$i_{\rm L} = -[25{,}000t + 10]e^{-5000t}u(t)\ \text{mA}$$

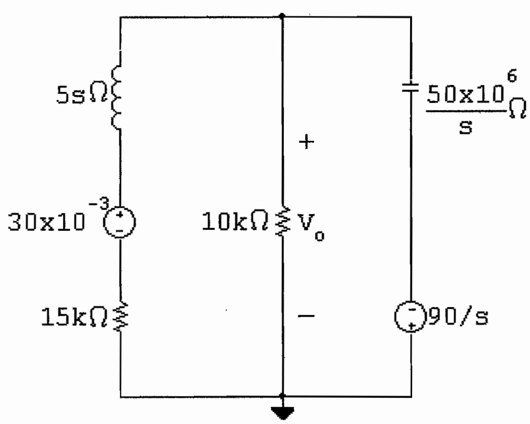
P 13.11 For $t < 0$:



$$\frac{v_o(0^-) + 150}{4000} + \frac{v_o(0^-)}{15{,}000} + \frac{v_o(0^-)}{10{,}000} = 0$$

$$\therefore \quad v_o(0^-) = -90\,\text{V}; \qquad \therefore \quad i_L(0^-) = -6\,\text{mA}$$

For $t > 0$:



$$\frac{V_o - 30 \times 10^{-3}}{5s + 15{,}000} + \frac{V_o}{10^4} + \frac{(V_o + 90/s)s}{50 \times 10^6} = 0$$

$$V_o = \frac{30(1000 - 3s)}{s^2 + 8000s + 25 \times 10^6}$$

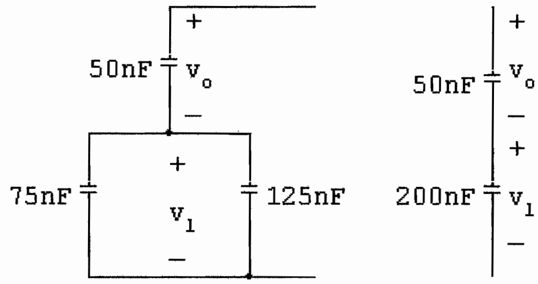
$$= \frac{30(1000 - 3s)}{(s + 4000 - j3000)(s + 4000 + j3000)}$$

$$K_1 = \frac{30(1000 + 12{,}000 - j9000)}{j6000} = 79.06\underline{/-124.70^\circ}\,\text{V}$$

$$v_o(t) = 158.11e^{-4000t}\cos(3000t - 124.70^\circ)u(t)\,V$$

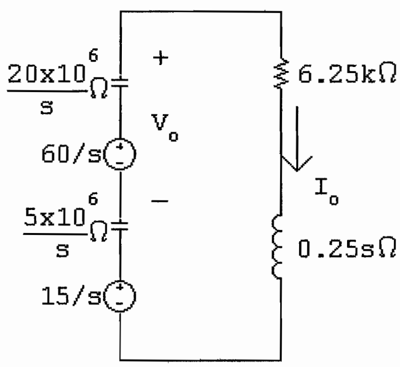
Check: $v_o(0) = 158.11\cos(-124.70^\circ) = -90\,\text{V}$

P 13.12 [a] For $t > 0$:



$v_1 = 75 - v_o; \qquad 50v_o = 200(75 - v_0);$

$\therefore \quad v_o = 60\,\text{V}; \qquad v_1 = 15\,\text{V}$



[b] $$I_o = \frac{75/s}{(25 \times 10^6/s) + 6250 + 0.25s} = \frac{300}{s^2 + 25{,}000s + 10^8}$$

$$= \frac{300}{(s+5000)(s+20{,}000)} = \frac{20 \times 10^{-3}}{s+5000} - \frac{20 \times 10^{-3}}{s+20{,}000}$$

$$i_o(t) = (20e^{-5000t} - 20e^{-20{,}000t})u(t)\,\text{mA}$$

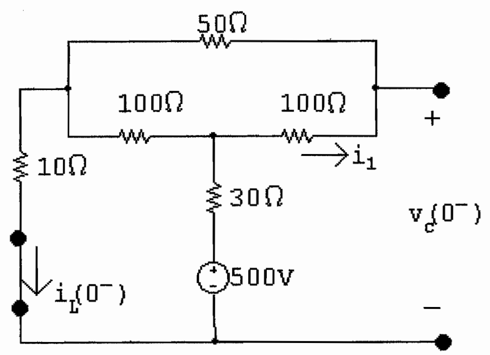
[c] $$V_o = \frac{60}{s} - \frac{20 \times 10^6}{s} \cdot \frac{300}{(s+5000)(s+20{,}000)}$$

$$= \frac{60}{s} - \left[\frac{60}{s} - \frac{80}{s+5000} + \frac{20}{s+20{,}000}\right]$$

$$= \frac{80}{s+5000} + \frac{-20}{s+20{,}000}$$

$$v_o(t) = (80e^{-5000t} - 20e^{-20{,}000t})u(t)\,\text{V}$$

P 13.13 [a] For $t < 0$:

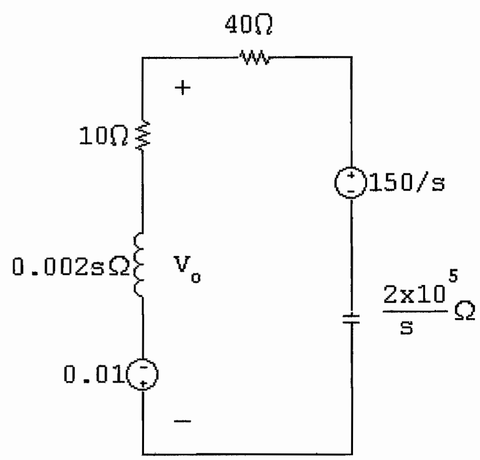


$$i_L(0^-) = \frac{500}{30+60+10} = 5\,\text{A}$$

$$i_1 = \frac{5(100)}{250} = 2\,\text{A}$$

$$v_C(0^-) = 500 - 5(30) - 2(100) = 500 - 350 = 150\,\text{V}$$

For $t > 0$:



[b] $$\frac{V_o + 0.01}{10 + 0.002s} + \frac{V_o - 150/s}{40 + 2\times 10^5/s} = 0$$

$$V_o\left[\frac{1}{10+0.002s} + \frac{s}{40s + 2\times 10^5}\right] = \frac{150}{40s+2\times 10^5} - \frac{0.01}{10+0.002s}$$

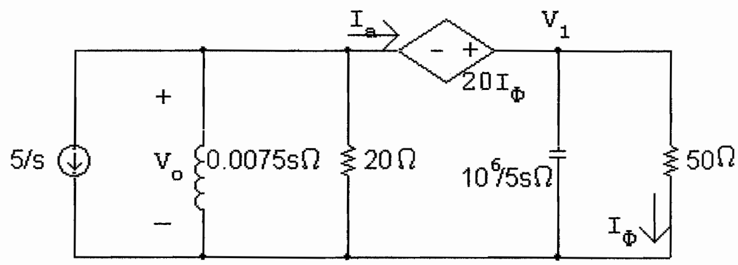
$$V_o = \frac{-50(s+5000)}{s^2 + 25{,}000s + 10^8} = \frac{-50(s+5000)}{(s+5000)(s+20{,}000)}$$

$$V_o = \frac{-50}{s+20{,}000}$$

[c] $v_o(t) = -50e^{-20{,}000t}u(t)\,\text{V}$

P 13.14 [a] $i_L(0^-) = i_L(0^+) = 5\,\text{A}$, down

$$v_C(0^-) = v_C(0^+) = 0$$



$$\frac{V_o}{20} + \frac{V_o}{0.0075s} + I_a = \frac{-5}{s}$$

$$I_a = \frac{V_1(5s)}{10^6} + \frac{V_1}{50} = \left(\frac{250s + 10^6}{50 \times 10^6}\right) V_1$$

$$V_o + 20I_\phi = V_1; \qquad V_o + 20\frac{V_1}{50} = V_1; \qquad \therefore\ 0.6V_1 = V_o$$

$$\therefore\ \frac{V_o}{20} + \frac{V_o}{0.0075s} + \frac{250s + 10^6}{30 \times 10^6} V_o = \frac{-5}{s}$$

$$\therefore\ (s^2 + 10{,}000s + 16 \times 10^6)V_o = -6 \times 10^5$$

$$V_o = \frac{-6 \times 10^5}{s^2 + 10{,}000s + 16 \times 10^6}$$

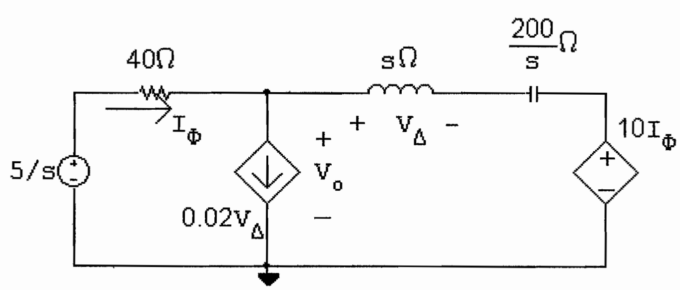
[b] $$V_o = \frac{-6 \times 10^5}{(s + 2000)(s + 8000)} = \frac{K_1}{(s + 8000)} + \frac{K_2}{(s + 2000)}$$

$$K_1 = \frac{-6 \times 10^5}{-6000} = 100$$

$$K_2 = \frac{-6 \times 10^5}{6000} = -100$$

$$v_o(t) = [100e^{-8000t} - 100e^{-2000t}]u(t)\ \text{V}$$

P 13.15 [a]



$$\frac{V_o - 5/s}{40} + 0.02V_\Delta + \frac{V_o - 10I_\phi}{s + (200/s)} = 0$$

$$V_\Delta = \left[\frac{V_o - 10I_\phi}{s + (200/s)}\right] s; \qquad I_\phi = \frac{(5/s) - V_o}{40}$$

Solving for V_o yields:

$$V_o = \frac{3s^2 + 25s + 500}{s(s^2 + 25s + 100)} = \frac{3s^2 + 25s + 500}{s(s+5)(s+20)}$$

$$V_o = \frac{K_1}{s} + \frac{K_2}{s+5} + \frac{K_3}{s+20}$$

$$K_1 = \left.\frac{3s^2 + 25s + 500}{(s+5)(s+20}\right|_{s=0} = 5$$

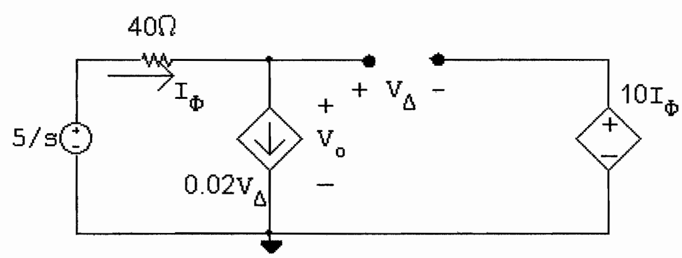
$$K_2 = \left.\frac{3s^2 + 25s + 500}{s(s+20}\right|_{s=-5} = -6$$

$$K_3 = \left.\frac{3s^2 + 25s + 500}{s(s+5}\right|_{s=-20} = 4$$

$$\therefore \quad V_o = \frac{5}{s} + \frac{-6}{s+5} + \frac{4}{s+20}$$

$$\therefore \quad v_o(t) = [5 - 6e^{-5t} + 4e^{-20t}]u(t)\text{ V}$$

[b] At $t = 0^+$ $\qquad v_o = 5 - 6 + 4 = 3\text{ V}$



$$v_o = v_\Delta + 10i_\phi$$

$$i_\phi = \frac{5 - v_o}{40}$$

$$\therefore \quad v_o = v_\Delta + 10\frac{(5 - v_o)}{40} = v_\Delta + 1.25 - 0.25v_o$$

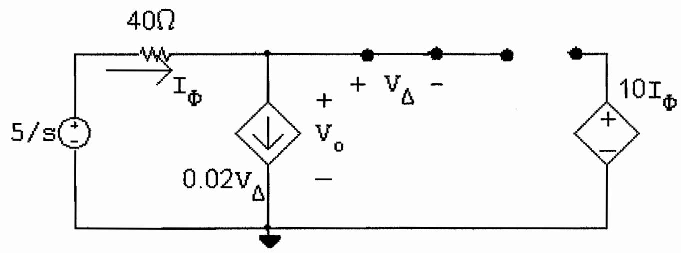
$$\therefore \quad 1.25v_o - 1.25 = v_\Delta$$

$$\frac{v_o - 5}{40} + 0.02v_\Delta = 0$$

$$v_o = 5 + 0.8v_\Delta = 0$$

$$v_o - 5 + v_o - 1 = 0 \quad \text{so} \quad v_o = 3\text{ V(checks)}$$

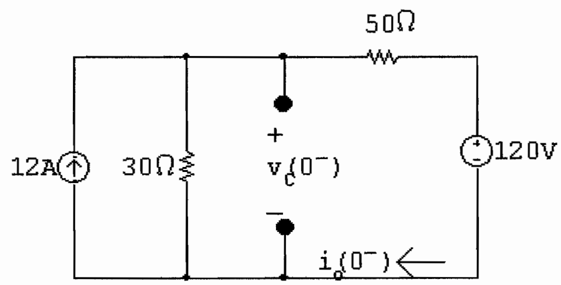
At $t = \infty$, the circuit is



From the equation for $v_o(t), v_o(\infty) = 5$ V. From the circuit,

$v_\Delta = 0, \quad i_\phi = 0 \qquad \therefore \quad v_o = 5\,\text{V(checks)}$

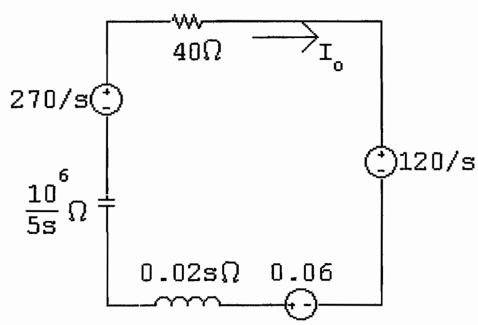
P 13.16 [a] For $t < 0$:



$$-12 + \frac{v_C(0^-)}{30} + \frac{v_C(0^-) - 120}{50} = 0$$

$$8v_C(0^-) = 2160; \qquad \therefore \quad v_C(0^-) = 270\,\text{V}$$

$$i_o(0^-) = \frac{270 - 120}{50} = 3\,\text{A}$$



[b] $$I_o = \frac{(270/s) + 0.06 - (120/s)}{40 + 0.02s + (10^6/5s)}$$

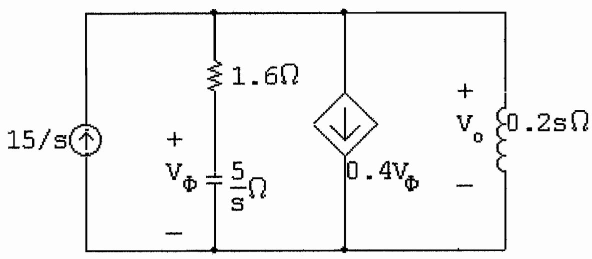
$$= \frac{3(s + 2500)}{s^2 + 2000s + 10^7}$$

$$= \frac{3(s + 2500)}{(s + 1000 - j3000)(s + 1000 + j3000)}$$

$$K_1 = \frac{3(1500 + j3000)}{j6000} = 0.75\sqrt{5}\underline{/-26.57^\circ}$$

[c] $i_o(t) = 3.35e^{-1000t}\cos(3000t - 26.57^\circ)u(t)$ A

P 13.17



$$\frac{15}{s} = \frac{V_o}{1.6 + 5/s} + 0.4V_\phi + \frac{V_o}{0.2s}$$

$$V_\phi = \frac{5/s}{1.6 + 5/s}V_o = \frac{5V_o}{1.6s + 5}$$

$$\therefore \quad \frac{15}{s} = \frac{V_o s}{1.6s + 5} + \frac{2V_o}{1.6s + 5} + \frac{5V_o}{s}$$

$$= V_o\left[\frac{s(s + 2) + 5(1.6s + 5)}{s(1.6s + 5)}\right]$$

$$15(1.6s + 5) = V_o(s^2 + 10s + 25)$$

$$\therefore \quad V_o = \frac{15(1.6s + 5)}{(s + 5)^2} = \frac{K_1}{(s + 5)^2} + \frac{K_2}{s + 5}$$

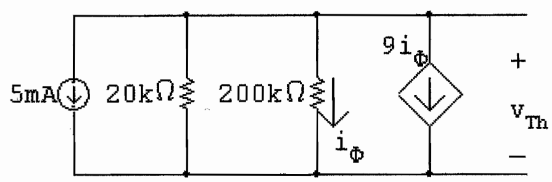
$$K_1 = 15(-8 + 5) = -45; \qquad K_2 = 24$$

$$V_o = \frac{-45}{(s + 5)^2} + \frac{24}{s + 5}$$

$$v_o(t) = [-45te^{-5t} + 24e^{-5t}]u(t)\ \text{V}$$

P 13.18 $v_C(0^-) = v_C(0^+) = 0$

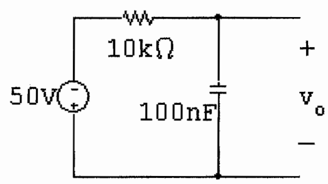
Find the Thévenin equivalent with respect to the capacitor:



$$\frac{v_{Th}}{20{,}000} + \frac{v_{Th}}{200{,}000} + \frac{9v_{Th}}{200{,}000} = -0.005$$

$$\therefore \quad v_{Th} = -50\,\text{V}$$

$$i_{sc} = -5\,\text{mA}; \qquad \therefore \quad R_{Th} = 10\,\text{k}\Omega$$

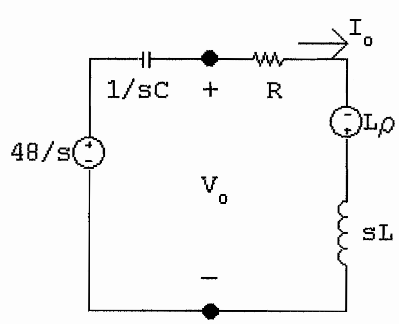


$$V_o = \frac{-50/s}{10{,}000 + (10^7/s)} \cdot \frac{10^7}{s}$$

$$= \frac{-50 \times 10^3}{s(s+1000)} = \frac{-50}{s} + \frac{50}{s+1000}$$

$$v_o(t) = [-50 + 50e^{-1000t}]u(t)\,\text{V}$$

P 13.19 [a] $i_o(0^-) = \dfrac{48}{4} \times 10^{-3} = 12\,\text{mA} = \rho$



$$\frac{V_o - 48/s}{(1/sC)} + \frac{V_o + \rho L}{R + sL} = 0$$

$$\therefore\ V_o = \frac{48(s + R/L) - \rho/C}{s^2 + (R/L)s + (1/LC)}$$

When the numerical values are substituted we get

$$V_o = \frac{48(s + 4875)}{(s + 4000 - j3000)(s + 4000 + j3000)}$$

$$K_1 = \frac{48(875 + j3000)}{j6000} = 25\underline{/-16.26^\circ}$$

$$v_o(t) = 50e^{-4000t}\cos(3000t - 16.26^\circ)u(t)\ \text{V}$$

Check: $v_o(0^+) = 50\cos(-16.26^\circ) = 48$ V, which agrees with the fact that the initial capacitor voltage is zero.

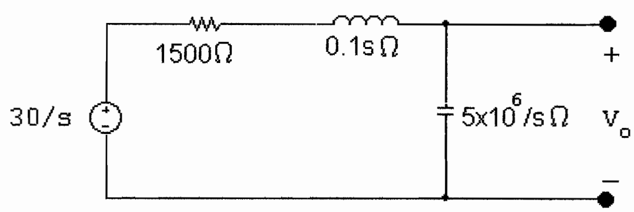
[b] $$I_o = \frac{48/s + \rho L}{R + sL + (1/sC)} = \frac{\rho[s + (48/\rho L)]}{s^2 + (R/L)s + (1/LC)}$$

$$I_o = \frac{12 \times 10^{-3}(s + 8000)}{(s + 4000 - j3000)(s + 4000 + j3000)}$$

$$K_1 = \frac{12 \times 10^{-3}(4000 + j3000)}{j6000} = 10 \times 10^{-3}\underline{/-53.13^\circ}$$

$$i_o(t) = 20e^{-4000t}\cos(3000t - 53.13^\circ)u(t)\ \text{mA}$$

P 13.20



$$V_o = \frac{(30/s)(5 \times 10^6/s)}{1500 + 0.1s + (5 \times 10^6/s)}$$

$$= \frac{1500 \times 10^6}{s(s^2 + 15{,}000s + 50 \times 10^6)}$$

$$= \frac{1500 \times 10^6}{s(s + 5000)(s + 10{,}000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 5000} + \frac{K_3}{s + 10{,}000}$$

$$K_1 = \frac{1500 \times 10^6}{(5000)(10{,}000)} = 30$$

$$K_2 = \frac{1500 \times 10^6}{(-5000)(5000)} = -60$$

$$K_3 = \frac{1500 \times 10^6}{(-5000)(-10{,}000)} = 30$$

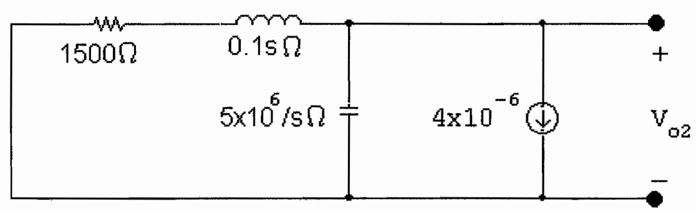
$$V_o = \frac{30}{s} - \frac{60}{s + 5000} + \frac{30}{s + 10{,}000}$$

$$v_o(t) = [30 - 60e^{-5000t} + 30e^{-10{,}000t}]u(t)\text{ V}$$

P 13.21 Since we already have the solution for $v_o(t)$ when the initial voltage is zero, we will use superposition to determine the contribution of the initial voltage of -20 V.

$$V_{o1} = \text{ output when } \gamma = 0$$

$$V_{o2} = \text{ output when } \gamma = -20\text{ V}$$



$$4 \times 10^{-6} + \frac{V_{o2}s}{5 \times 10^6} + \frac{V_{o2}}{1500 + 0.1s} = 0$$

$$\therefore \quad V_{o2} = \frac{-20(s + 15{,}000)}{s^2 + 15{,}000s + 50 \times 10^6}$$

$$= \frac{K_1}{s + 5000} + \frac{K_2}{s + 10{,}000}$$

$$K_1 = \frac{-20(10{,}000)}{5000} = -40$$

$$K_2 = \frac{-20(5000)}{-5000} = 20$$

$$V_{o2} = \frac{-40}{s + 5000} + \frac{20}{s + 10{,}000}$$

From the solution to Problem 13.20we have

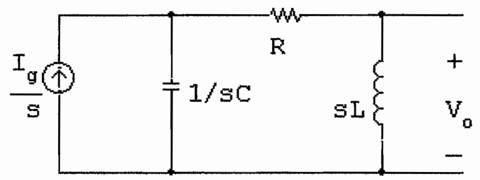
$$V_{o1} = \frac{30}{s} - \frac{60}{s + 5000} + \frac{30}{s + 10{,}000}$$

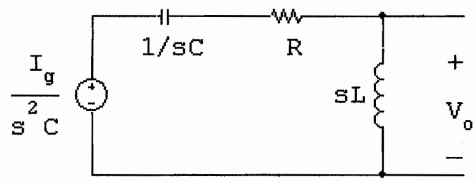
$$V_o = V_{o1} + V_{o2}$$

$$\therefore \quad V_o = \frac{30}{s} - \frac{100}{s + 5000} + \frac{50}{s + 10{,}000}$$

$$v_o(t) = [30 - 100e^{-5000t} + 50e^{-10{,}000t}]u(t)\ \mathrm{V}$$

P 13.22 [a]





$$V_o = \frac{(I_g/s^2C)(sL)}{R + sL + (1/sC)} = \frac{I_g/C}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{I_g}{C} = \frac{9.6 \times 10^{-3}}{3.2 \times 10^{-9}} = 3000 \times 10^3 = 3 \times 10^6$$

$$\frac{R}{L} = \frac{7000}{0.5} = 14{,}000; \qquad \frac{1}{LC} = \frac{2}{3.2} \times 10^9 = 625 \times 10^6$$

$$V_o = \frac{3 \times 10^6}{s^2 + 14{,}000s + 625 \times 10^6}$$

[b] $$sV_o = \frac{3 \times 10^6 s}{s^2 + 14{,}000s + 625 \times 10^6}$$

$$\lim_{s\to 0} sV_o = 0; \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to \infty} sV_o = 0; \qquad \therefore \quad v_o(0^+) = 0$$

[c] $s_{1,2} = -7000 \pm \sqrt{49 \times 10^6 - 625 \times 10^6} = -7000 \pm j24{,}000$ rad/s

$$V_o = \frac{3{,}000{,}000}{(s + 7000 - j24{,}000)(s + 7000 + j24{,}000)}$$

$$K_1 = \frac{3 \times 10^6}{j48{,}000} = -j62.5 = 62.5\underline{/-90^\circ}$$

$$v_o = 125e^{-7000t}\cos(24{,}000t - 90^\circ) = [125e^{-7000t}\sin 24{,}000t]u(t)\text{ V}$$

P 13.23 $I_C = \dfrac{I_g}{s} - \dfrac{V_o}{sL}$

$$= \frac{9.6 \times 10^{-3}}{s} - \frac{2}{s}\left[\frac{3 \times 10^6}{(s + 7000 - j24{,}000)(s + 7000 + j24{,}000)}\right]$$

$$= \frac{9.6 \times 10^{-3}}{s} - \frac{6 \times 10^6}{s(s + 7000 - j24{,}000)(s + 7000 + j24{,}000)}$$

$$= \frac{9.6 \times 10^{-3}}{s} - \frac{K_1}{s} - \frac{K_2}{s + 7000 - j24{,}000} - \frac{K_2^*}{s + 7000 + j24{,}000}$$

$$K_1 = \frac{6 \times 10^6}{625 \times 10^6} = 9.6 \times 10^{-3}$$

$$K_2 = \frac{6 \times 10^6}{(-7000 + j24{,}000)(j48{,}000)}$$

$$= \frac{6}{(-7 + j24)(j48)} = 5 \times 10^{-3}\underline{/163.74^\circ}$$

$$\therefore\ I_C = \frac{9.6 \times 10^{-3}}{s} - \frac{9.6 \times 10^{-3}}{s} - [\text{congugate terms}]$$

$$= \left[\frac{-5\underline{/163.74^\circ}}{s + 7000 - j24{,}000} + \text{ congugate}\right] \times 10^{-3}$$

$$= \left[\frac{5\underline{/-16.26^\circ}}{s + 7000 - j24{,}000} + \text{ congugate}\right] \times 10^{-3}$$

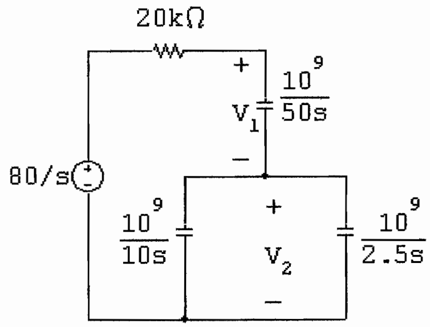
$$i_C = 10e^{-7000t}\cos(24{,}000t - 16.26^\circ)u(t)\text{ mA}$$

Check:

$i_C(0^+) = 10\cos(-16.26^\circ) = 9.6\,\text{mA}$ (ok)

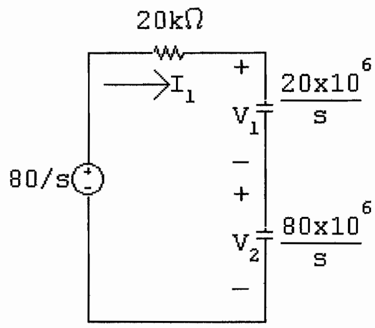
$i_C(\infty) = 0$ (ok)

P 13.24 [a]



$$Y_e = \frac{10s}{10^9} + \frac{2.5s}{10^9} = \frac{12.5s}{10^9}$$

$$Z_e = \frac{10^9}{12.5s} = \frac{80 \times 10^6}{s}$$



[b] $I_1 = \dfrac{80/s}{20{,}000 + (100 \times 10^6/s)} = \dfrac{4 \times 10^{-3}}{s + 5000}$

$$V_1 = \frac{4 \times 10^{-3}}{s+5000} \cdot \frac{20 \times 10^6}{s} = \frac{80{,}000}{s(s+5000)}$$

$$V_2 = \frac{4 \times 10^{-3}}{s+5000} \cdot \frac{80 \times 10^6}{s} = \frac{320{,}000}{s(s+5000)}$$

[c] $i_1(t) = 4e^{-5000t}u(t)\,\text{mA}$

$$V_1 = \frac{16}{s} - \frac{16}{s+5000}; \qquad v_1(t) = (16 - 16e^{-5000t})u(t)\,\text{V}$$

$$V_2 = \frac{64}{s} - \frac{64}{s+5000}; \qquad v_2(t) = (64 - 64e^{-5000t})u(t)\,\text{V}$$

[d] $i_1(0^+) = 4\,\text{mA}$

$$i_1(0^+) = \frac{80}{20} \times 10^{-3} = 4\,\text{mA(checks)}$$

$$v_1(0^+) = 0; \qquad v_2(0^+) = 0\text{(checks)}$$

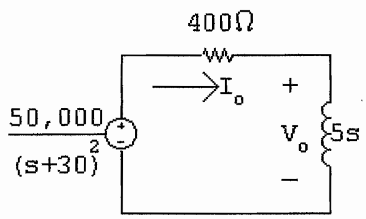
$$v_1(\infty) = 16\,\text{V}; \qquad v_2(\infty) = 64\,\text{V(checks)}$$

$$v_1(\infty) + v_2(\infty) = 80\,\text{V(checks)}$$

$$(50 \times 10^{-9})v_1(\infty) = 800\,\text{nC}$$

$$(12.5 \times 10^{-9})v_2(\infty) = 800\,\text{nC(checks)}$$

P 13.25 [a] $V_g = \dfrac{50{,}000}{(s+30)^2}$



$$I_o = \frac{50{,}000}{(s+30)^2(5s+400)} = \frac{10{,}000}{(s+30)^2(s+80)}$$

$$V_o = 5sI_o = \frac{50{,}000s}{(s+30)^2(s+80)}$$

[b] $I_o = \dfrac{K_1}{(s+30)^2} + \dfrac{K_2}{s+30} + \dfrac{K_3}{s+80}$

$$K_1 = \frac{10{,}000}{50} = 200$$

$$K_2 = \frac{d}{ds}\left[\frac{10{,}000}{s+80}\right]_{s=-30} = -4$$

$$K_3 = \frac{10{,}000}{(-50)^2} = 4$$

$$I_o = \frac{200}{(s+30)^2} - \frac{4}{s+30} + \frac{4}{s+80}$$

$$i_o(t) = [200te^{-30t} - 4e^{-30t} + 4e^{-80t}]u(t)\,\text{A}$$

$$V_o = \frac{K_1}{(s+30)^2} + \frac{K_2}{s+30} + \frac{K_3}{s+80}$$

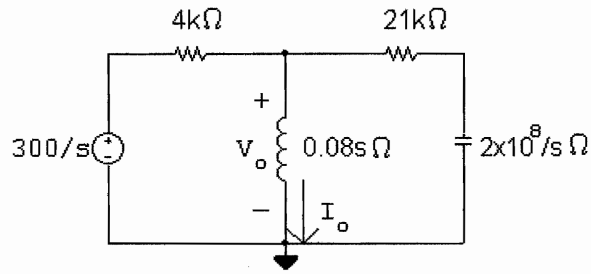
$$K_1 = \frac{50{,}000(-30)}{50} = -30{,}000$$

$$K_2 = \frac{d}{ds}\left[\frac{50{,}000s}{s+80}\right]_{s=-30} = 1600$$

$$K_3 = \frac{50{,}000(-80)}{(-50)^2} = -1600$$

$$v_o(t) = [-30{,}000te^{-30t} + 1600e^{-30t} - 1600e^{-80t}]u(t)\,\text{V}$$

P 13.26 [a]



$$\frac{V_o - 300/s}{4000} + \frac{12.5V_o}{s} + \frac{V_o s}{21{,}000s + 2\times 10^8} = 0$$

$$\therefore \quad V_o = \frac{12(21s + 20\times 10^4)}{(s+10{,}000)(s+40{,}000)} = \frac{K_1}{s+10{,}000} + \frac{K_2}{s+40{,}000}$$

$K_1 = -4; \qquad K_2 = 256$

$$V_o = \frac{-4}{s+10{,}000} + \frac{256}{s+40{,}000}$$

$$v_o(t) = (256e^{-40{,}000t} - 4e^{-10{,}000t})u(t)\ \text{V}$$

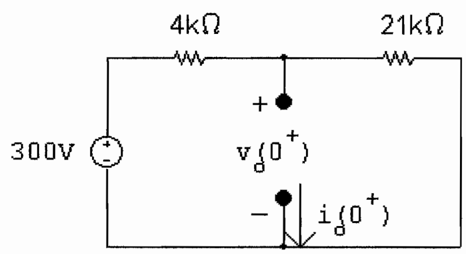
[b] $I_o = \dfrac{V_o}{0.08s} = \dfrac{12.5V_o}{s}$

$$I_o = \frac{150(21s + 20\times 10^4)}{s(s+10{,}000)(s+40{,}000)} = \frac{K_1}{s} + \frac{K_2}{s+10{,}000} + \frac{K_3}{s+40{,}000}$$

$K_1 = 75\times 10^{-3}; \qquad K_2 = 5\times 10^{-3}; \qquad K_3 = -80\times 10^{-3}$

$$i_o(t) = (75 + 5e^{-10{,}000t} - 80e^{-40{,}000t})u(t)\ \text{mA}$$

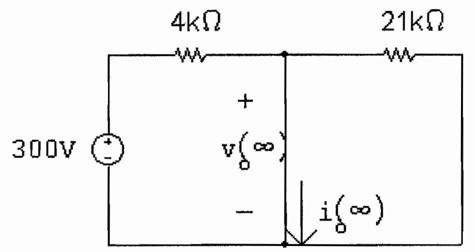
[c] At $t = 0^+$ the circuit is



$$\therefore \quad v_o(0^+) = \frac{300}{25}(21) = 252\ \text{V}; \qquad i_o(0^+) = 0$$

Both values agree with our solutions for v_o and i_o.

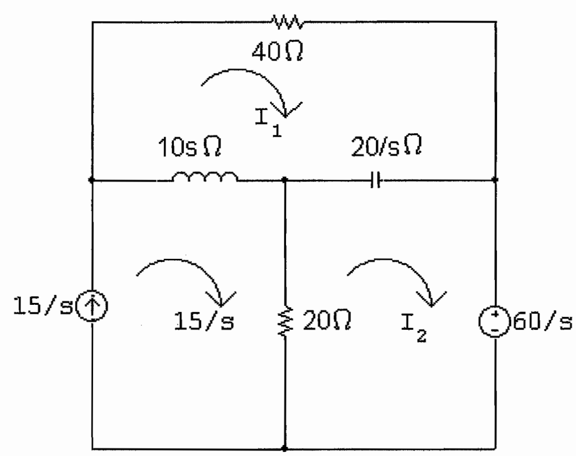
At $t = \infty$ the circuit is



$\therefore \quad v_o(\infty) = 0; \qquad i_o(\infty) = 75\,\text{mA}$

Both values agree with our solutions for v_o and i_o.

P 13.27 [a]



$$40I_1 + \frac{20}{s}(I_1 - I_2) + 10s(I_1 - 15/s) = 0$$

$$20(I_2 - 15/s) + \frac{20}{s}(I_2 - I_1) + \frac{60}{s} = 0$$

or

$$(s^2 + 4s + 2)I_1 - 2I_2 = 15s$$

$$-I_1 + (s + 1)I_2 = 12$$

$$\Delta = \begin{vmatrix} (s^2 + 4s + 2) & -2 \\ -1 & (s + 1) \end{vmatrix} = s(s + 2)(s + 3)$$

$$N_1 = \begin{vmatrix} 15s & -2 \\ 12 & (s + 1) \end{vmatrix} = 15s^2 + 15s + 24$$

$$I_1 = \frac{N_1}{\Delta} = \frac{15s^2 + 15s + 24}{s(s+2)(s+3)}$$

$$N_2 = \begin{vmatrix} (s^2 + 4s + 2) & 15s \\ -1 & 12 \end{vmatrix} = 12s^2 + 63s + 24$$

$$I_2 = \frac{N_2}{\Delta} = \frac{12s^2 + 63s + 24}{s(s+2)(s+3)}$$

[b] $$sI_1 = \frac{15s^2 + 15s + 24}{(s+2)(s+3)}$$

$$\lim_{s\to\infty} sI_1 = 15 \qquad \therefore \; i_1(0^+) = 15\,\text{A}$$

$$\lim_{s\to 0} sI_1 = 4 \qquad \therefore \; i_1(\infty) = 4\,\text{A}$$

$$sI_2 = \frac{12s^2 + 63s + 24}{(s+2)(s+3)}$$

$$\lim_{s\to\infty} sI_2 = 12 \qquad \therefore \; i_2(0^+) = 12\,\text{A}$$

$$\lim_{s\to 0} sI_2 = 4 \qquad \therefore \; i_2(\infty) = 4\,\text{A}$$

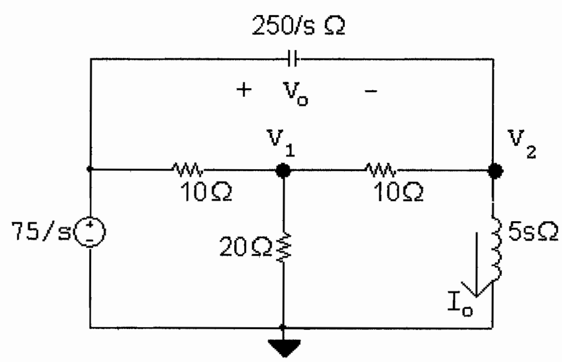
[c] $$I_1 = \frac{4}{s} - \frac{27}{s+2} + \frac{38}{s+3}$$

$$i_1(t) = (4 - 27e^{-2t} + 38e^{-3t})u(t)\,\text{A}$$

$$I_2 = \frac{4}{s} + \frac{27}{s+2} - \frac{19}{s+3}$$

$$i_2(t) = (4 + 27e^{-2t} - 19e^{-3t})u(t)\,\text{A}$$

P 13.28 [a]



$$\frac{V_1 - 75/s}{10} + \frac{V_1}{20} + \frac{V_1 - V_2}{10} = 0$$

$$\frac{V_2}{5s} + \frac{V_2 - V_1}{10} + \frac{(V_2 - 75/s)s}{250} = 0$$

Thus,

$$5V_1 - 2V_2 = \frac{150}{s}$$

$$-25sV_1 + (s^2 + 25s + 50)V_2 = 75s$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -25s & s^2 + 25s + 50 \end{vmatrix} = 5(s+5)(s+10)$$

$$N_2 = \begin{vmatrix} 5 & 150/s \\ -25s & 75s \end{vmatrix} = 375(s+10)$$

$$V_2 = \frac{N_2}{\Delta} = \frac{375(s+10)}{5(s+5)(s+10)} = \frac{75}{s+5}$$

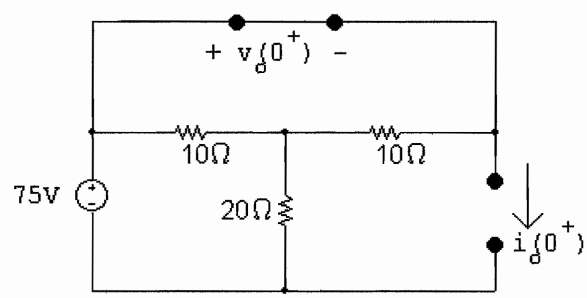
$$V_o = \frac{75}{s} - \frac{75}{s+5} = \frac{375}{s(s+5)}$$

$$I_o = \frac{V_2}{5s} = \frac{15}{s(s+5)} = \frac{3}{s} - \frac{3}{s+5}$$

[b] $v_o(t) = (75 - 75e^{-5t})u(t)\text{ V}$

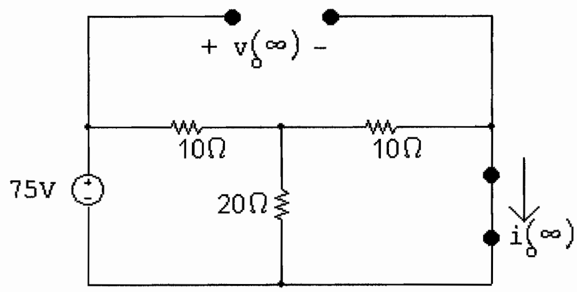
$i_o(t) = (3 - 3e^{-5t})u(t)\text{ A}$

[c] At $t = 0^+$ the circuit is



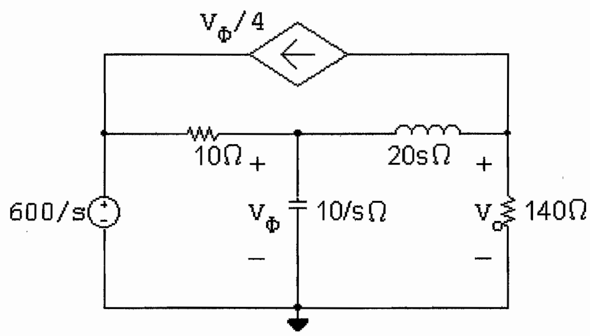
$v_o(0^+) = 0; \qquad i_o(0^+) = 0 \qquad$ Checks

At $t = \infty$ the circuit is



$$v_o(\infty) = 75\,\text{V}; \qquad i_o(\infty) = \frac{75}{10 + (200/30)} \cdot \frac{20}{30} = 3\,\text{A} \quad \text{Checks}$$

P 13.29 **[a]**



$$\frac{V_\phi}{10/s} + \frac{V_\phi - (600/s)}{10} + \frac{V_\phi - V_o}{20s} = 0$$

$$\frac{V_o}{140} + \frac{V_o - V_\phi}{20s} + \frac{V_\phi}{4} = 0$$

Simplfying,

$$(2s^2 + 2s + 1)V_\phi - V_o = 1200$$

$$(35s - 7)V_\phi + (s + 7)V_o = 0$$

$$\Delta = \begin{vmatrix} 2s^2 + 2s + 1 & -1 \\ 35s - 7 & s + 7 \end{vmatrix} = 2s(s^2 + 8s + 25)$$

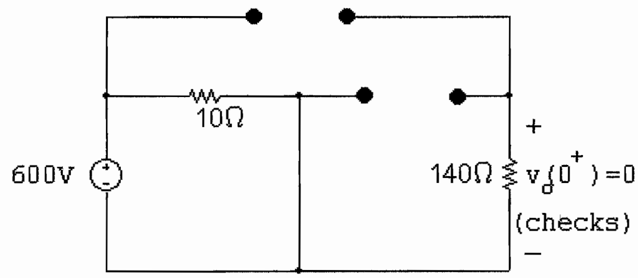
$$N_2 = \begin{vmatrix} 2s^2 + 2s + 1 & 1200 \\ 35s - 7 & 0 \end{vmatrix} = -42{,}000s + 8400$$

$$V_o = \frac{N_2}{\Delta} = \frac{-21{,}000s + 4200}{s(s^2 + 8s + 25)} = \frac{-4200(5s - 1)}{s(s^2 + 8s + 25)}$$

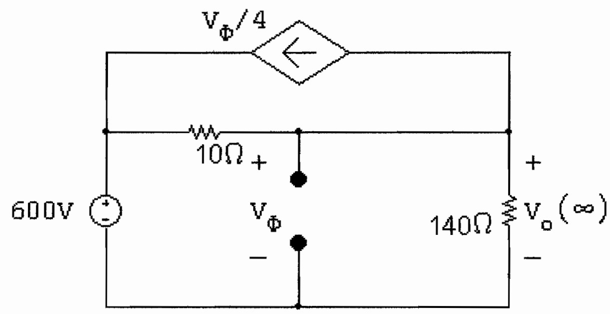
[b] $v_o(0^+) = \lim_{s \to \infty} sV_o = 0$

$$v_o(\infty) = \lim_{s \to 0} sV_o = \frac{4200}{25} = 168$$

[c] At $t = 0^+$ the circuit is



At $t = \infty$ the circuit is



$$\frac{V_\phi - 600}{10} + \frac{V_\phi}{140} + \frac{V_\phi}{4} = 0$$

$$\therefore \quad V_\phi = 168\,\text{V} = V_o(\infty) \qquad \text{(checks)}$$

[d] $V_o = \dfrac{N_2}{\Delta} = \dfrac{-21{,}000s + 4200}{s(s^2 + 8s + 25)} = \dfrac{K_1}{s} + \dfrac{K_2}{s + 4 - j3} + \dfrac{K_2^*}{s + 4 + j3}$

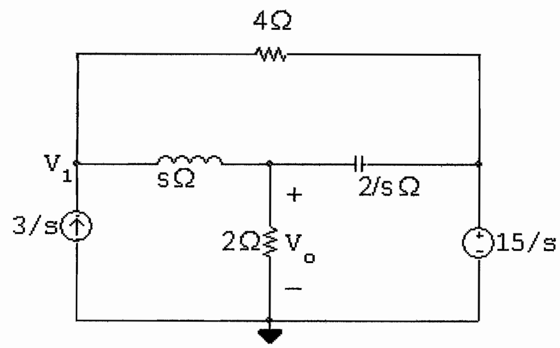
$$K_1 = \frac{4200}{25} = 168$$

$$K_2 = \frac{-21{,}000(-4 + j3) + 4200}{(-4 + j3)(j6)} = -84 + j3612 = 3612.98\underline{/91.33^\circ}$$

$$v_o(t) = [168 + 7225.95e^{-4t}\cos(3t + 91.33^\circ)]u(t)\,\text{V}$$

Check: $\quad v_o(0^+) = 0\,\text{V}; \qquad v_o(\infty) = 168\,\text{V}$

P 13.30 [a]



$$\frac{-3}{s} + \frac{V_1 - V_o}{s} + \frac{V_1 - (15/s)}{4} = 0$$

$$\frac{V_o}{2} + \frac{V_o - V_1}{s} + \frac{V_o - (15/s)}{2/s} = 0$$

Simplfying,

$$(s+4)V_1 - 4V_o = 27$$

$$(s^2 + s + 2)V_o - 2V_1 = 15s$$

$$\Delta = \begin{vmatrix} s+4 & -4 \\ -2 & s^2+s+2 \end{vmatrix} = s(s+2)(s+3)$$

$$N_2 = \begin{vmatrix} s+4 & 27 \\ -2 & 15s \end{vmatrix} = 15s^2 + 60s + 54$$

$$V_o = \frac{N_2}{\Delta} = \frac{15s^2 + 60s + 54}{s(s+2)(s+3)} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

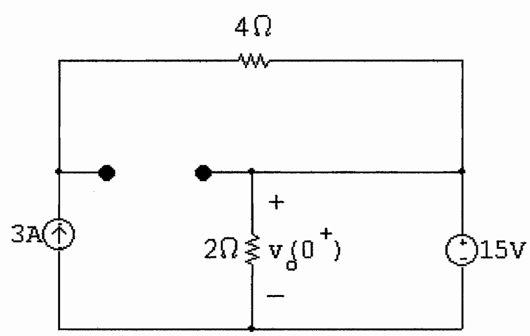
$$K_1 = \frac{54}{(2)(3)} = 9; \quad K_2 = \frac{60 - 120 + 54}{(-2)(1)} = 3$$

$$K_3 = \frac{135 - 180 + 54}{(-3)(-1)} = 3$$

$$\therefore \quad V_o = \frac{9}{s} + \frac{3}{s+2} + \frac{3}{s+3}$$

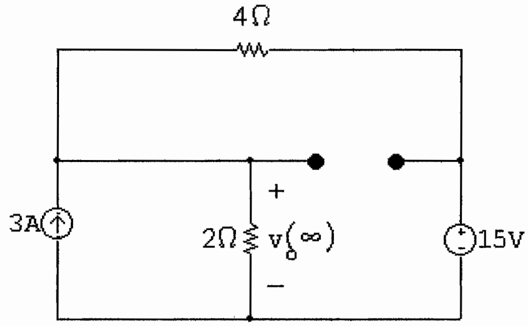
[b] $v_o(t) = (9 + 3e^{-2t} + 3e^{-3t})u(t)$ V

[c] At $t = 0^+$:



$v_o(0^+) = 15$ V(checks)

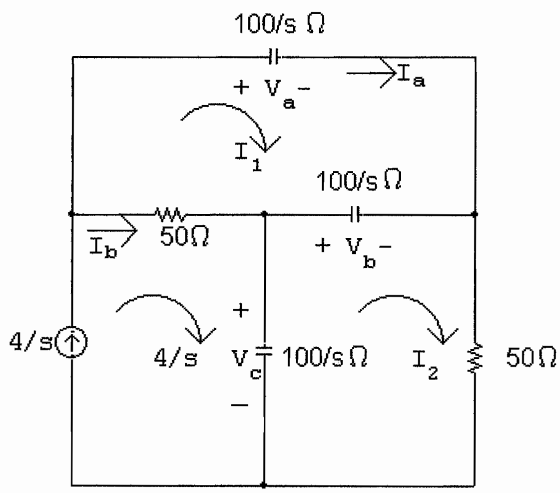
At $t = \infty$:



$$\frac{v_o(\infty)}{2} - 3 + \frac{v_o(\infty) - 15}{4} = 0$$

$\therefore\quad 3v_o(\infty) = 27; \qquad \therefore\quad v_o(\infty) = 9\,\text{V(checks)}$

P 13.31 [a]



$$\frac{100}{s}I_1 + \frac{100}{s}(I_1 - I_2) + 50(I_1 - 4/s) = 0$$

$$\frac{100}{s}(I_2 - 4/s) + \frac{100}{s}(I_2 - I_1) + 50I_2 = 0$$

Simplifying,

$$(s+4)I_1 - 2I_2 = 4$$

$$-2I_1 + (s+4)I_2 = \frac{8}{s}$$

$$\Delta = \begin{vmatrix} (s+4) & -2 \\ -2 & (s+4) \end{vmatrix} = s^2 + 8s + 12 = (s+2)(s+6)$$

$$N_1 = \begin{vmatrix} 4 & -2 \\ 8/s & (s+4) \end{vmatrix} = \frac{4s^2+16s+16}{s} = \frac{4(s+2)^2}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{4(s+2)^2}{s(s+2)(s+6)} = \frac{4(s+2)}{s(s+6)} = \frac{4/3}{s} + \frac{8/3}{s+6}$$

$$N_2 = \begin{vmatrix} (s+4) & 4 \\ -2 & 8/s \end{vmatrix} = \frac{16s+32}{s} = \frac{16(s+2)}{s}$$

$$I_2 = \frac{N_2}{\Delta} = \frac{16(s+2)}{s(s+2)(s+6)} = \frac{16}{s(s+6)} = \frac{8/3}{s} - \frac{8/3}{s+6}$$

$$I_a = I_1 = \frac{4/3}{s} + \frac{8/3}{s+6}$$

$$I_b = \frac{4}{s} - I_1 = \frac{8/3}{s} - \frac{8/3}{s+6}$$

[b] $i_a(t) = (4/3)(1+2e^{-6t})u(t)$ A

$i_b(t) = (8/3)(1-e^{-6t})u(t)$ A

[c] $$V_a = \frac{100}{s}I_a = \frac{100}{s}\left(\frac{4/3}{s} + \frac{8/3}{s+6}\right)$$

$$= \frac{400/3}{s^2} + \frac{800/3}{s(s+6)} = \frac{400/3}{s^2} + \frac{400/9}{s} - \frac{400/9}{s+6}$$

$$V_b = \frac{100}{s}(I_2 - I_1) = \frac{100}{s}\left(\frac{4/3}{s} - \frac{16/3}{s+6}\right)$$

$$= \frac{400/3}{s^2} - \frac{1600/3}{s(s+6)} = \frac{400/3}{s^2} - \frac{800/9}{s} + \frac{800/9}{s+6}$$

$$V_c = \frac{100}{s}(4/s - I_2) = \frac{100}{s}\left(\frac{4/3}{s} + \frac{8/3}{s+6}\right)$$

$$= \frac{400/3}{s^2} + \frac{800/3}{s(s+6)} = \frac{400/3}{s^2} + \frac{400/9}{s} - \frac{400/9}{s+6}$$

[d] $v_a(t) = (400/9)(3t+1-e^{-6t})u(t)$ V

$v_b(t) = (400/9)(3t-2+2e^{-6t})u(t)$ V

$v_c(t) = (400/9)(3t+1-e^{-6t})u(t)$ V

[e] Calculating the time when a capacitor's voltage drop first reaches 1000 V:

For $v_a(t)$ or $v_c(t)$:

$$1000\left(\frac{9}{400}\right) = 3t + 1 - e^{-6t} = 22.5$$

$$3t - e^{-6t} = 21.5$$

$$t = 7.17\,\text{s}$$

For $v_b(t)$:

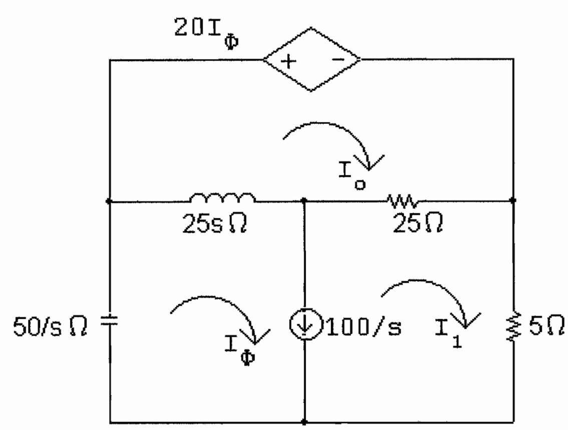
$$3t - 2 + 2e^{-6t} = 22.5$$

$$3t + 2e^{-6t} = 24.5$$

$$t = 8.17\,\text{s}$$

Thus, the capacitors whose voltage drops are designated v_a and v_c will break down first, at a time of 7.17 s.

P 13.32 [a]



$$20I_\phi + 25s(I_o - I_\phi) + 25(I_o - I_1) = 0$$

$$25s(I_\phi - I_o) + \frac{50}{s}I_\phi + 5I_1 + 25(I_1 - I_o) = 0$$

$$I_\phi - I_1 = \frac{100}{s}; \qquad \therefore \quad I_1 = I_\phi - \frac{100}{s}$$

Simplifying,

$$(-5s - 1)I_\phi + (5s + 5)I_o = -500/s$$

$$(5s^2 + 6s + 10)I_\phi + (-5s^2 - 5s)I_o = 600$$

$$\Delta = \begin{vmatrix} -5s - 1 & 5s + 5 \\ 5s^2 + 6s + 10 & -5s^2 - 5s \end{vmatrix} = -25(s^2 + 3s + 2)$$

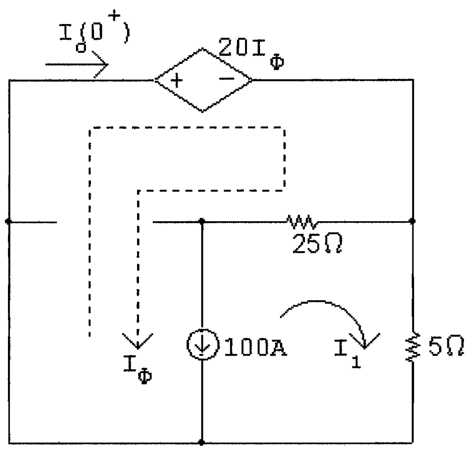
$$N_2 = \begin{vmatrix} -5s-1 & -500/s \\ 5s^2+6s+10 & 600 \end{vmatrix} = -\frac{500}{s}(s^2 - 4.8s - 10)$$

$$I_o = \frac{N_2}{\Delta} = \frac{20s^2 - 96s - 200}{s(s+1)(s+2)}$$

[b] $i_o(0^+) = \lim_{s\to\infty} sI_o = 20\,\text{A}$

$$i_o(\infty) = \lim_{s\to 0} sI_o = \frac{-200}{2} = -100\,\text{A}$$

[c] At $t = 0^+$ the circuit is

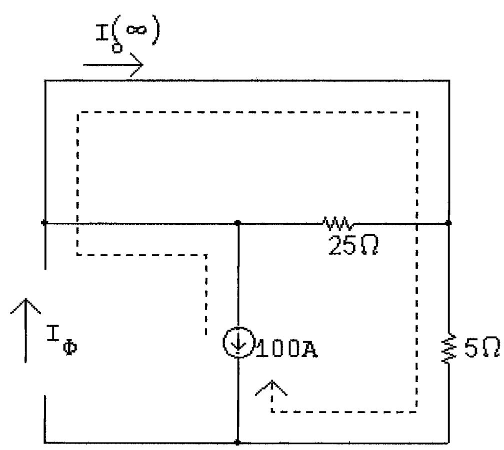


$$20I_\phi + 5I_1 = 0; \quad I_\phi - I_1 = 100$$

$$\therefore\ 20I_\phi + 5(I_\phi - 100) = 0; \qquad 25I_\phi = 500$$

$$\therefore\ I_\phi = I_o(0^+) = 20\,\text{A(checks)}$$

At $t + \infty$ the circuit is



$$I_o(\infty) = -100\,\text{A(checks)}$$

[d] $I_o = \frac{20s^2 - 96s - 200}{s(s+1)(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$

$$K_1 = \frac{-200}{(1)(2)} = -100; \qquad K_2 = \frac{20 + 96 - 200}{(-1)(1)} = 84$$

$$K_3 = \frac{80 + 192 - 200}{(-2)(-1)} = 36$$

$$I_o = \frac{-100}{s} + \frac{84}{s+1} + \frac{36}{s+2}$$

$$i_o(t) = (-100 + 84e^{-t} + 36e^{-2t})u(t)\,\text{A}$$

$$i_o(\infty) = -100\,\text{A(checks)}$$

$$i_o(0^+) = -100 + 84 + 36 = 20\,\text{A(checks)}$$

P 13.33 $v_C = 12 \times 10^5 te^{-5000t}\,\text{V}, \quad C = 5\,\mu\text{F}; \qquad$ therefore

$$i_C = C\left(\frac{dv_C}{dt}\right) = 6e^{-5000t}(1 - 5000t)\,\text{A}$$

$i_C > 0$ when $1 > 5000t$ or $i_C \geq 0$ when $0 \leq t \leq 200\,\mu\text{s}$

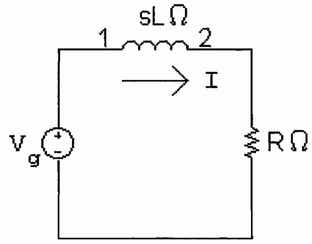
and $i_C < 0$ when $t > 200\,\mu\text{s}$

$i_C = 0$ when $1 - 5000t = 0,$ or $t = 200\,\mu\text{s}$

$$\frac{dv_C}{dt} = 12 \times 10^5 e^{-5000t}[1 - 5000t]$$

$$\therefore \quad i_C = 0 \quad \text{when} \quad \frac{dv_C}{dt} = 0$$

P 13.34 [a] The s-domain equivalent circuit is



$$I = \frac{V_g}{R + sL} = \frac{V_g/L}{s + (R/L)}, \qquad V_g = \frac{V_m(\omega\cos\phi + s\sin\phi)}{s^2 + \omega^2}$$

$$I = \frac{K_0}{s+R/L} + \frac{K_1}{s-j\omega} + \frac{K_1^*}{s+j\omega}$$

$$K_0 = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2+\omega^2L^2}, \qquad K_1 = \frac{V_m/\underline{\phi - 90 - \theta(\omega)}}{2\sqrt{R^2+\omega^2L^2}}$$

where $\tan\theta(\omega) = \omega L/R$. Therefore, we have

$$i(t) = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2+\omega^2L^2}e^{-(R/L)t} + \frac{V_m\sin[\omega t+\phi-\theta(\omega)]}{\sqrt{R^2+\omega^2L^2}}$$

[b] $i_{ss}(t) = \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}\sin[\omega t+\phi-\theta(\omega)]$

[c] $i_{\text{tr}} = \dfrac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2+\omega^2L^2}e^{-(R/L)t}$

[d] $\mathbf{I} = \dfrac{\mathbf{V}_g}{R+j\omega L}, \qquad \mathbf{V}_g = V_m/\underline{\phi}$

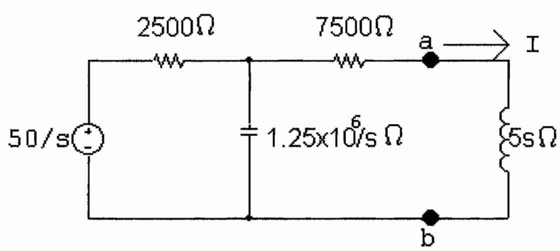
Therefore $\mathbf{I} = \dfrac{V_m/\underline{\phi}}{\sqrt{R^2+\omega^2L^2}/\underline{\theta(\omega)}} = \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}/\underline{\phi - \theta(\omega)}$

Therefore $i_{ss} = \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}\sin[\omega t+\phi-\theta(\omega)]$

[e] The transient component vanishes when

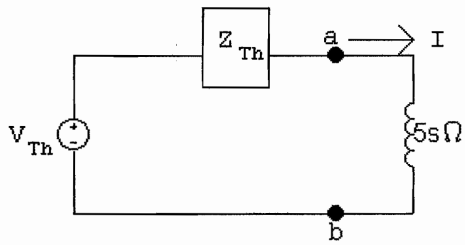
$$\omega L\cos\phi = R\sin\phi \quad \text{or} \quad \tan\phi = \frac{\omega L}{R} \quad \text{or} \quad \phi = \theta(\omega)$$

P 13.35



$$V_{\text{Th}} = \frac{50/s}{2500 + (1.25\times10^6/s)}\cdot\frac{1.25\times10^6}{s} = \frac{25{,}000}{s(s+500)}$$

$$Z_{\text{Th}} = 7500 + \frac{2500(1.25\times10^6/s)}{2500+(1.25\times10^6/s)} = \frac{7500s + 5\times10^6}{s+500}$$



$$I = \frac{25{,}000/s(s+500)}{5s + \dfrac{7500s + 5\times 10^6}{s+500}}$$

$$= \frac{5000}{s(s^2 + 2000s + 10^6)} = \frac{5000}{s(s+1000)^2}$$

$$= \frac{K_1}{s} + \frac{K_2}{(s+1000)^2} + \frac{K_3}{s+1000}$$

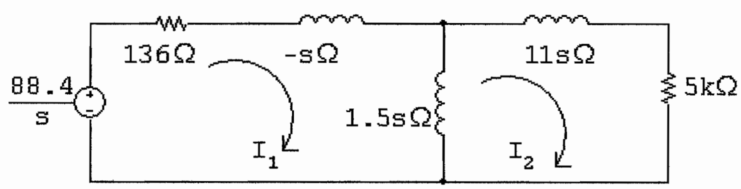
$$K_1 = \frac{5000}{10^6} = 5\times 10^{-3}$$

$$K_2 = \frac{5000}{-1000} = -5000\times 10^{-3}$$

$$K_3 = \frac{d}{ds}\left(\frac{5000}{s}\right)_{s=-1000} = -5\times 10^{-3}$$

$$i(t) = [5 - 5000te^{-1000t} - 5e^{-1000t}]u(t)\,\text{mA}$$

P 13.36 [a]



$$\frac{88.4}{s} = 136I_1 - sI_1 + 1.5s(I_1 - I_2)$$

$$0 = 1.5s(I_2 - I_1) + 11sI_2 + 5000I_2$$

Simplifying,

$$\frac{88.4}{s} = (0.5s + 136)I_1 - 1.5sI_2$$

$$0 = -1.5sI_1 + (12.5s + 5000)I_2$$

$$\Delta = \begin{vmatrix} 0.5s + 136 & -1.5s \\ -1.5s & 12.5s + 5000 \end{vmatrix} = 4(s+200)(s+850)$$

$$N_1 = \begin{vmatrix} 88.4/s & -1.5s \\ 0 & 12.5s + 5000 \end{vmatrix} = \frac{1105(s+400)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{276.25(s+400)}{s(s+200)(s+850)}$$

[b] $sI_1 = \dfrac{276.25(s+400)}{(s+200)(s+850)}$

$$\lim_{s\to 0} sI_1 = i_1(\infty) = 650\,\text{mA}$$

$$\lim_{s\to\infty} sI_1 = i_1(0) = 0$$

[c] $I_1 = \dfrac{K_1}{s} + \dfrac{K_2}{s+200} + \dfrac{K_3}{s+850}$

$K_1 = 650 \times 10^{-3}$; $K_2 = -425 \times 10^{-3}$; $K_3 = -225 \times 10^{-3}$

$$i_1(t) = (650 - 425e^{-200t} - 225e^{-850t})u(t)\,\text{mA}$$

P 13.37 [a] From the solution to Problem 13.36 we have

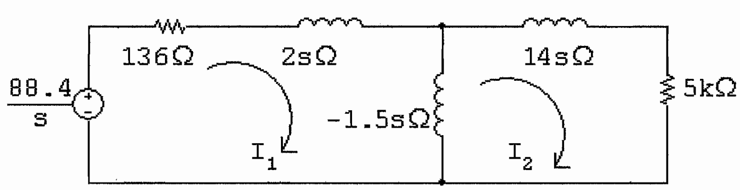
$$N_2 = \begin{vmatrix} 0.5s + 136 & 88.4/s \\ -1.5s & 0 \end{vmatrix} = 132.6$$

$$\therefore\quad I_2 = \frac{132.6}{4(s+200)(s+850)} = \frac{33.15}{(s+200)(s+850)}$$

$$= \frac{51 \times 10^{-3}}{s+200} - \frac{51 \times 10^{-3}}{s+850}$$

$$i_2(t) = (51e^{-200t} - 51e^{-850t})u(t)\,\text{mA}$$

[b] Reversing the dot on the 12.5 H coil will reverse the sign of M, thus the circuit becomes



The two simulanteous equations are

$$\frac{88.4}{s} = (136 + 0.5s)I_1 + 1.5sI_2$$

$$0 = 1.5sI_1 + (12.5s + 5000)I_2$$

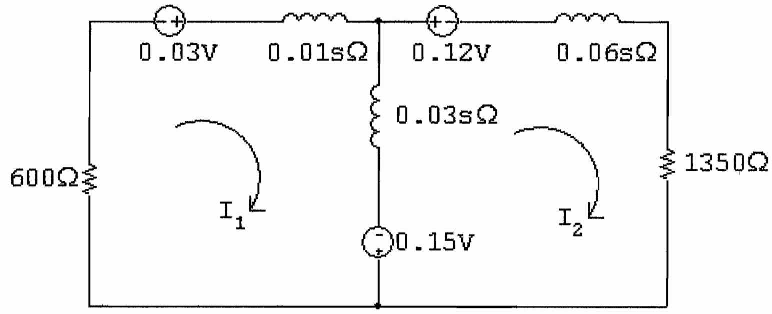
When these equations are compared to those derived in Problem 13.39 we see the only difference is the algebraic sign of the 1.5s term. Thus reversing the dot will have no effect on I_1 and will reverse the sign of I_2. Hence,

$$i_2(t) = (-51e^{-200t} + 51e^{-850t})u(t)\,\text{mA}$$

P 13.38 [a] $w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + Mi_1 i_2$

$$w = \left[\frac{1}{2}(40)(9) + \frac{1}{2}(90)(4) + 30(6)\right] \times 10^{-3} = 540\,\text{mJ}$$

[b] The s-domain circuit:



$$(600 + 0.04s)I_1 - 0.03sI_2 = 0.18$$

$$-0.03sI_1 + (0.09s + 1350)I_2 = -0.27$$

$$\Delta = \begin{vmatrix} 0.04(s + 15{,}000) & -0.03s \\ -0.03s & 0.09(s + 15{,}000) \end{vmatrix}$$

$$= 27 \times 10^{-4}(s + 10{,}000)(s + 30{,}000)$$

$$N_1 = \begin{vmatrix} 0.18 & -0.03s \\ -0.27 & 0.09(s + 15{,}000) \end{vmatrix} = 81 \times 10^{-4}(s + 30{,}000)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{3}{s + 10{,}000}$$

$$N_2 = \begin{vmatrix} 0.04(s + 15{,}000) & 0.18 \\ -0.03s & -0.27 \end{vmatrix} = -54 \times 10^{-4}(s + 30{,}000)$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-2}{s + 10{,}000}$$

[c] $i_1(t) = 3e^{-10{,}000t}u(t)$ A; $\quad i_2(t) = -2e^{-10{,}000t}u(t)$ A

[d] $p_{600\Omega} = (600)(9e^{-20,000t}) = 5400e^{-20,000t}$ W

$p_{1350\Omega} = (1350)(4e^{-20,000t}) = 5400e^{-20,000t}$ W

$$w_{600} = \frac{5400}{20} \times 10^{-3} = 270\,\text{mJ}$$

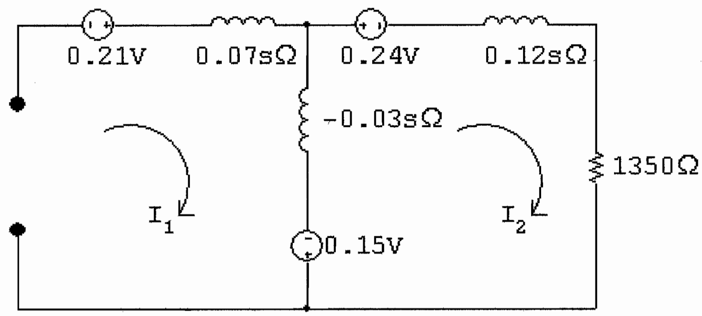
$$w_{1350} = \frac{5400}{20} \times 10^{-3} = 270\,\text{mJ}$$

$w_T = 540\,\text{mJ}$

[e] With the dot reversed,

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2 = 180 + 180 - 180 = 180\,\text{mJ}$$

The s-domain equivalent circuit is



Solving for I_1 and I_2 yields

$$I_1 = \frac{3}{s + 30,000}; \qquad I_2 = \frac{-2}{s + 30,000}$$

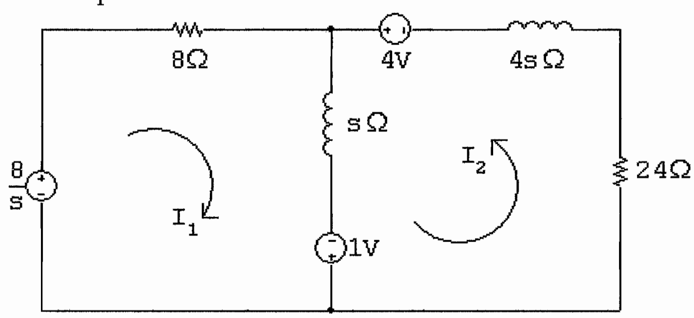
$\therefore\ i_1(t) = 3e^{-30,000t}u(t)$ A; $\qquad i_2(t) = -2e^{-30,000t}u(t)$ A

$$w_{600} = 5400 \int_0^\infty e^{-60,000t}\,dt = 90\,\text{mJ}$$

$$w_{1350} = 5400 \int_0^\infty e^{-60,000t}\,dt = 90\,\text{mJ}$$

$w_T = 180\,\text{mJ}$

P 13.39 [a] s-domain equivalent circuit is



[b] $\frac{8}{s} = 8I_1 + s(I_1 + I_2) - 1$

$$0 = -1 + s(I_2 + I_1) + 4sI_2 - 4 + 24I_2$$

or

$$\frac{8}{s} + 1 = (s+8)I_1 + sI_2$$

$$5 = sI_1 + (5s + 24)I_2$$

$$\Delta = \begin{vmatrix} s+8 & s \\ s & 5s+24 \end{vmatrix} = 4(s+4)(s+12)$$

$$I_2 = \frac{N_2}{\Delta}$$

$$N_2 = \begin{vmatrix} s+8 & (8/s)+1 \\ s & 5 \end{vmatrix} = 4(s+8)$$

$$\therefore \quad I_2 = \frac{s+8}{(s+4)(s+12)}$$

[c] $sI_2 = \frac{s(s+8)}{(s+4)(s+12)}$

$$\lim_{s\to\infty} sI_2 = i_2(0^+) = 1\text{ A}$$

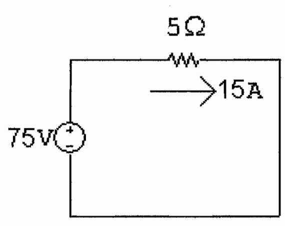
$$\lim_{s\to 0} sI_2 = i_2(\infty) = 0$$

[d] $I_2 = \frac{K_1}{s+4} + \frac{K_2}{s+12}$

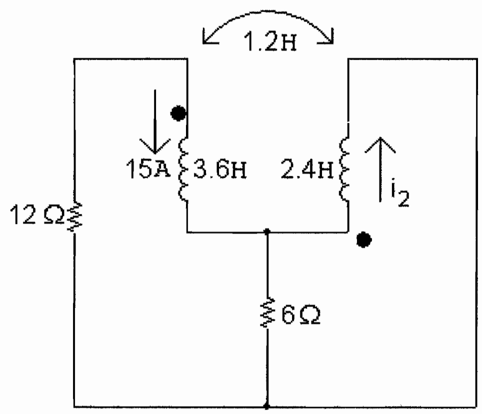
$$K_1 = K_2 = 1/2; \qquad \therefore \quad I_2 = \frac{1/2}{s+4} + \frac{1/2}{s+12}$$

$$i_2(t) = \frac{1}{2}[e^{-4t} + e^{-12t}]u(t)\text{ A}$$

P 13.40 For $t < 0$:

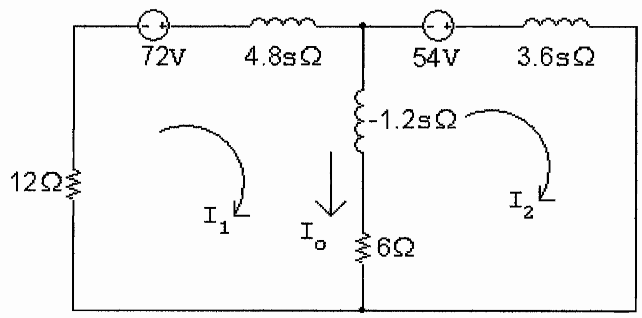


For $t > 0^+$:



$$L_1 + M = 3.6 + 1.2 = 4.8\,\text{H}; \qquad M - L_2 = 1.2 - 2.4 = -1.2\,\text{H}$$

$$15 \times 4.8 = 72; \qquad 15 \times 3.6 = 54$$



$$12I_o + 4.8sI_o - 72 + (I_o - I_2)(6 - 1.2s) = 0$$

$$(6 - 1.2s)(I_2 - I_o) + 3.6sI_2 - 54 = 0$$

$$\therefore \quad \Delta = \begin{vmatrix} 3(s+5) & -(5-s) \\ -(5-s) & 2(s+2.5) \end{vmatrix} = 5(s+1)(s+10)$$

$$N_o = \begin{vmatrix} 60 & -(5-s) \\ 45 & 2(s+2.5) \end{vmatrix} = 75(s+7)$$

$$I_o = \frac{N_o}{\Delta} \frac{75(s+7)}{5(s+1)(s+10)}$$

$$= \frac{K_1}{(s+1)} + \frac{K_2}{(s+10)}$$

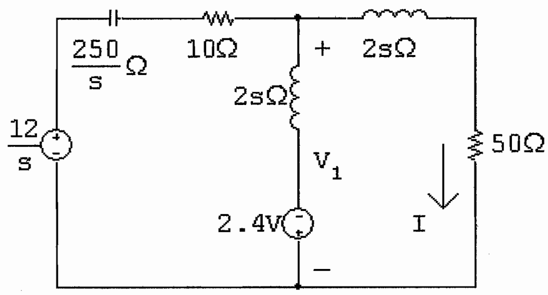
$$K_1 = \frac{(15)(6)}{9} = 10$$

$$K_2 = \frac{(15)(-3)}{-9} = 5$$

$$\therefore \quad I_o = \frac{10}{s+1} + \frac{5}{s+10}$$

$$\therefore \quad i_o(t) = [10e^{-t} + 5e^{-10t}]u(t) \text{ A}$$

P 13.41 The s-domain equivalent circuit is



$$\frac{V_1 - 12/s}{10 + (250/s)} + \frac{V_1 + 2.4}{2s} + \frac{V_1}{2s + 50} = 0$$

$$V_1 = \frac{-300(s+25)}{(s+25)(s^2+10s+125)} = \frac{-300}{s^2+10s+125}$$

$$I_o = \frac{-300}{(2s+50)(s^2+10s+125)}$$

$$= \frac{-150}{(s+25)(s+5-j10)(s+5+j10)}$$

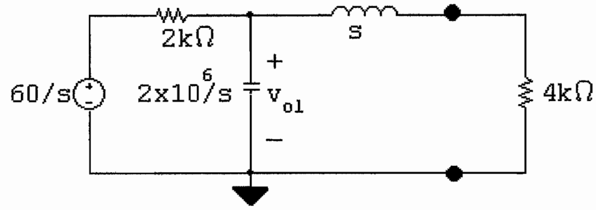
$$= \frac{K_1}{s+25} + \frac{K_2}{s+5-j10} + \frac{K_2^*}{s+5+j10}$$

$$K_1 = \frac{-150}{625 - 250 + 125} = -300 \times 10^{-3}$$

$$K_2 = \frac{-150}{(-5+j10+25)(j20)} = 150\sqrt{5} \times 10^{-3} \underline{/63.43^\circ}$$

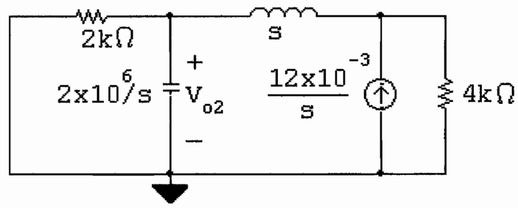
$$i_o(t) = [-300e^{-25t} + 300\sqrt{5}e^{-5t}\cos(10t + 63.43^\circ)]u(t) \text{ mA}$$

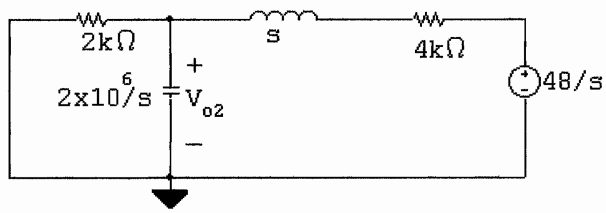
P 13.42 **[a]** Voltage source acting alone:



$$\frac{V_{o1} - 60/s}{2000} + \frac{V_{01}s}{2 \times 10^6} + \frac{V_{01}}{s + 4000} = 0$$

$$\therefore \quad V_{01} = \frac{60{,}000(s + 4000)}{s(s + 2000)(s + 3000)}$$





$$\frac{V_{o2}}{2000} + \frac{V_{02}s}{2 \times 10^6} + \frac{V_{02} - 48/s}{4000 + s} = 0$$

$$\therefore \quad V_{02} = \frac{96 \times 10^6}{s(s + 2000)(s + 3000)}$$

$$V_o = V_{o1} + V_{o2} = \frac{6 \times 10^4(s + 4000) + 96 \times 10^6}{s(s + 2000)(s + 3000)}$$

[b] $V_o = \dfrac{K_1}{s} + \dfrac{K_2}{s + 2000} + \dfrac{K_3}{s + 3000}$

$$= \frac{56}{s} - \frac{108}{s + 2000} + \frac{52}{s + 3000}$$

$$v_o(t) = (56 - 108e^{-2000t} + 52e^{-3000t})u(t)\text{ V}$$

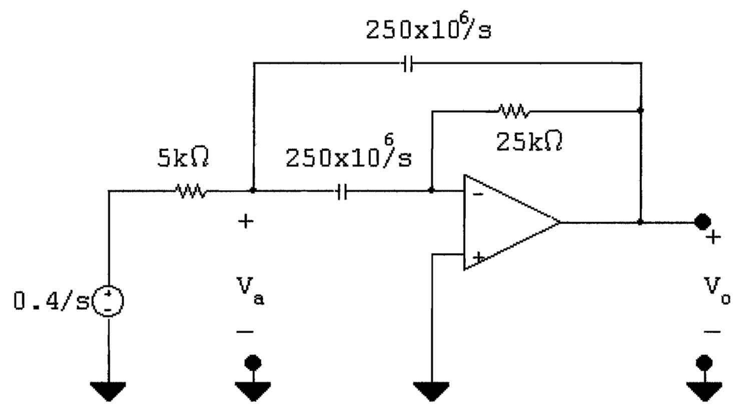
P 13.43 $\Delta = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{vmatrix} = Y_{11}Y_{22} - Y_{12}^2$

$$N_2 = \begin{vmatrix} Y_{11} & [(V_g/R_1) + \gamma C - (\rho/s)] \\ Y_{12} & (I_g - \gamma C) \end{vmatrix}$$

$$V_2 = \frac{N_2}{\Delta}$$

Substitution and simplification lead directly to Eq. 13.90.

P 13.44



$$\frac{V_a - 0.4/s}{5000} + \frac{V_a s}{250 \times 10^6} + \frac{(V_a - V_o)s}{250 \times 10^6} = 0$$

$$\frac{(0 - V_a)s}{250 \times 10^6} + \frac{(0 - V_o)}{25{,}000} = 0$$

$$V_a = \frac{-10^4 V_o}{s}$$

$$\therefore \quad V_o(s^2 + 20{,}000s + 500 \times 10^6) = -20{,}000$$

$$V_o = \frac{-20{,}000}{(s + 10{,}000 - j20{,}000)(s + 10{,}000 + j20{,}000)}$$

$$K_1 = \frac{-20{,}000}{j40{,}000} = j0.5 = 0.5\underline{/90^\circ}$$

$$v_o(t) = e^{-10{,}000t}\cos(20{,}000t + 90^\circ) = -e^{-10{,}000t}\sin(20{,}000t)u(t) \text{ V}$$

P 13.45 [a] $V_o = -\frac{Z_f}{Z_i}V_g$

$$Z_f = \frac{10^8}{s + \left[\frac{10^9}{(10)(2)\times 10^4}\right]} = \frac{10^8}{s + 5000}$$

$$Z_i = \frac{8000}{s}\left(s + \frac{10^9}{(50)(8000)}\right) = \frac{8000}{s}(s + 2500)$$

$$V_g = \frac{20{,}000}{s^2}$$

$$\therefore \quad V_o = \frac{-250 \times 10^6}{s(s+2500)(s+5000)}$$

[b] $V_o = \dfrac{K_1}{s} + \dfrac{K_2}{s+2500} + \dfrac{K_3}{s+5000}$

$$K_1 = \frac{-250 \times 10^6}{(5000)(2500)} = -20$$

$$K_2 = \frac{-250 \times 10^6}{(-2500)(2500)} = 40$$

$$K_3 = \frac{-250 \times 10^6}{(-5000)(-2500)} = -20$$

$$\therefore \quad v_o(t) = (-20 + 40e^{-2500t} - 20e^{-5000t})u(t)\,\text{V}$$

[c] $-20 + 40e^{-2500t_s} - 20e^{-5000t_s} = -5$

$$\therefore \quad 40e^{-2500t_s} - 20e^{-5000t_s} = 15$$

Let $x = e^{-2500t_s}$. Then

$40x - 20x^2 = 15;\qquad$ or $x^2 - 2x + 0.75 = 0$

Solving,

$x = 1 \pm 0.5 \quad$ so $\quad x = 0.5$

$$\therefore \quad e^{-2500t_s} = 0.5; \qquad \therefore \quad t_s = \frac{\ln 2}{0.0025} \times 10^{-6} = 277.26\,\mu\text{s}$$

[d] $v_g = \text{m}\,tu(t); \qquad V_g = \dfrac{\text{m}}{s^2}$

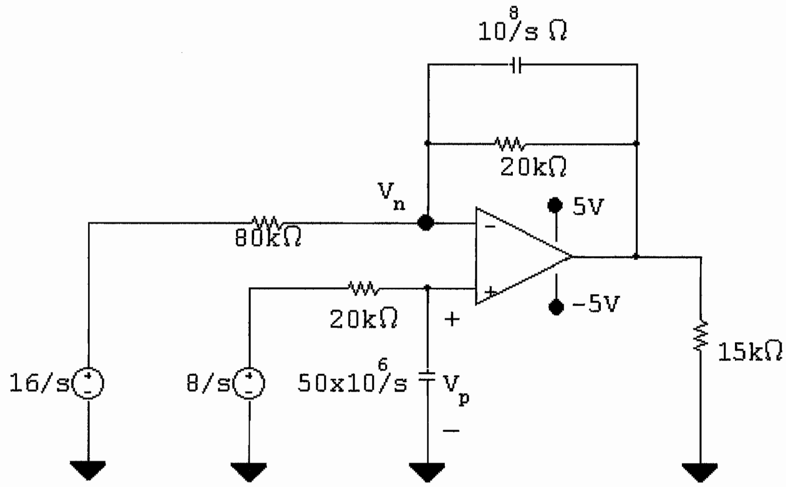
$$V_o = \frac{-10^8 s}{8000(s+2500)(s+5000)} \cdot \frac{\text{m}}{s^2}$$

$$= \frac{-12{,}500\text{m}}{s(s+2500)(s+5000)}$$

$$K_1 = \frac{-12{,}500\text{m}}{(2500)(5000)} = -\text{m} \times 10^{-3}$$

$$\therefore \quad -5 = -\text{m} \times 10^{-3} \qquad \therefore \quad \text{m} = 5000\,\text{V/s}$$

P 13.46 [a]



$$\frac{V_p s}{50 \times 10^6} + \frac{V_p - V_{g2}}{20{,}000} = 0; \qquad V_p = \frac{2500}{s+2500} V_{g2}$$

$$\frac{V_p - V_{g1}}{80{,}000} + \frac{V_p - V_o}{20{,}000} + \frac{(V_p - V_o)s}{10^8} = 0$$

$$(s + 6250)V_p - (s + 5000)V_o = 1250 V_{g1}$$

$$\therefore \quad (s + 5000)V_o = \frac{(s + 6250)(2500)}{(s + 2500)} V_{g2} - 1250 V_{g1}$$

$$V_{g1} = \frac{16}{s}; \qquad V_{g2} = \frac{8}{s}$$

$$\therefore \quad V_o = \frac{7500 \times 10^4}{s(s + 2500)(s + 5000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 2500} + \frac{K_3}{s + 5000}$$

$$K_1 = \frac{7500 \times 10^4}{(2500)(5000)} = \frac{750}{125} = 6$$

$$K_2 = \frac{7500 \times 10^4}{(-2500)(2500)} = -12$$

$$K_3 = \frac{7500 \times 10^4}{(-5000)(-2500)} = 6$$

$$v_o = [6 - 12e^{-2500t} + 6e^{-5000t}]u(t)\,\text{V}$$

[b] $6 - 12e^{-2500t_s} + 6e^{-5000t_s} = 5;$ let $x = e^{-2500t_s}$

$$6 - 12x + 6x^2 = 5$$

$$x^2 - 2x + \frac{1}{6} = 0$$

$$x = 1 - \sqrt{5/6} = 0.0871$$

$$\therefore \quad e^{-2500t} = 0.0871; \qquad t = 976.15\,\mu\text{s}$$

P 13.47 $$Z_{i1} = 400{,}000 + \frac{(4\times10^5/s)(2\times10^5)}{2\times10^5 + (4\times10^5/s)} = \frac{4\times10^5(s+3)}{s+2}$$

$$Z_{f1} = 8\times10^5$$

$$V_{o1} = -\frac{Z_{f1}}{Z_{i1}}V_g = \frac{-8\times10^5(s+2)}{4\times10^5(s+3)}\frac{(0.18)}{s} = \frac{-0.36(s+2)}{s(s+3)}$$

The final value of v_{01} is

$$v_{o1}(\infty) = \lim_{s\to0}\left(\frac{-0.36(s+2)}{s+3}\right) = -0.24\,\text{V}$$

Thus, the first stage will not saturate.

$$V_o = -\frac{Z_{f2}}{Z_{i2}}V_{o1}$$

$$Z_{f2} = \frac{10^9}{250s} = \frac{4\times10^6}{s}; \qquad Z_{i2} = 50\times10^3$$

$$V_o = \frac{-0.36(s+2)}{s(s+3)}\left(\frac{-80}{s}\right) = \frac{28.8(s+2)}{s^2(s+3)}$$

$$= \frac{19.2}{s^2} + \frac{3.2}{s} - \frac{3.2}{s+3}$$

$$v_o(t) = (19.2t + 3.2 - 3.2e^{-3t})u(t)\,\text{V}$$

The second stage saturates when v_o reaches 6.4 V. Thus

$$19.2t_s + 3.2 - 3.2e^{-3t_s} = 6.4; \qquad \therefore \quad 6t_s - 1 = e^{-3t_s}$$

t_s must be greater than $\frac{1}{6}$ or 166.68 ms. Using trial and error we find

$$t_s = 246.28\,\text{ms}$$

P 13.48 [a] Let V_a be the voltage across the 0.2 μF capacitor, positive at the upper terminal and let V_b be the voltage across the 200 kΩ resistor, positive at the upper terminal. Then

$$\frac{V_a s}{5\times 10^6} + \frac{V_a - V_g}{400{,}000} + \frac{V_a}{400{,}000} = 0; \qquad \therefore \quad V_a = \frac{12.5}{s+25}V_g$$

$$\frac{-V_a}{400{,}000} - \frac{sV_b}{10^7} = 0; \qquad \therefore \quad V_b = \frac{-25}{s}V_a = \frac{-312.5}{s(s+25)}V_g$$

$$\frac{V_b}{200{,}000} + \frac{sV_b}{10^7} + \frac{(V_b - V_o)s}{10^7} = 0$$

$$\therefore \quad V_o = \frac{2(s+25)}{s}V_b = \left[\frac{2(s+25)}{s}\right]\left[\frac{-312.5}{s(s+25)}\right]\left(\frac{8}{s}\right) = \frac{-5000}{s^3}$$

[b] $v_o(t) = -2500t^2u(t)$ V

[c] The op amp will saturate when $v_o = -12.5$ V.

$$-12.5 = -2500t^2; \qquad t^2 = 0.005; \qquad \therefore \quad t = 0.071 = 71\,\text{ms}$$

P 13.49 [a] $$\frac{V_o}{V_i} = \frac{1/sC}{R + 1/sC} = \frac{1}{RCs+1}$$

$$H(s) = \frac{(1/RC)}{s + (1/RC)} = \frac{50}{s+50}; \qquad -p_1 = -50\,\text{rad/s}$$

[b] $$\frac{V_o}{V_i} = \frac{R}{R+1/sC} = \frac{RCs}{RCs+1} = \frac{s}{s+(1/RC)}$$

$$= \frac{s}{s+50}; \qquad z_1 = 0, \quad -p_1 = -50\,\text{rad/s}$$

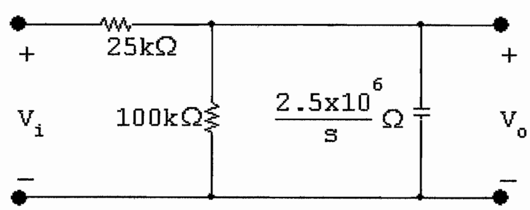
[c] $$\frac{V_o}{V_i} = \frac{sL}{R+sL} = \frac{s}{s+R/L} = \frac{s}{s+3\times 10^6}$$

$$z_1 = 0; \qquad -p_1 = -3\times 10^6\,\text{rad/s}$$

[d] $$\frac{V_o}{V_i} = \frac{R}{R+sL} = \frac{R/L}{s+(R/L)} = \frac{3\times 10^6}{s+3\times 10^6}$$

$$-p_1 = -3\times 10^6\,\text{rad/s}$$

[e]



$$\frac{V_o s}{2.5\times 10^6} + \frac{V_o}{10^5} + \frac{V_o - V_i}{25\times 10^3} = 0$$

$$sV_o + 25V_o + 100V_o = 100V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{100}{s+125}$$

$$-p_1 = -125\,\text{rad/s}$$

P 13.50 [a] Let $R_1 = 40\,\text{k}\Omega$; $R_2 = 10\,\text{k}\Omega$; $C_2 = 500\,\text{nF}$; and $C_f = 250\,\text{nF}$. Then

$$Z_f = \frac{(R_2 + 1/sC_2)1/sC_f}{\left(R_2 + \frac{1}{sC_2} + \frac{1}{sC_f}\right)} = \frac{(s + 1/R_2C_2)}{C_f s\left(s + \frac{C_2+C_f}{C_2C_fR_2}\right)}$$

$$\frac{1}{C_f} = 4 \times 10^6$$

$$\frac{1}{R_2C_2} = 200\,\text{rad/s}$$

$$\frac{C_2 + C_f}{C_2C_fR_2} = \frac{750 \times 10^{-9}}{1.25 \times 10^{-9}} = 600\,\text{rad/s}$$

$$\therefore\ Z_f = \frac{4 \times 10^6(s+200)}{s(s+600)}\Omega$$

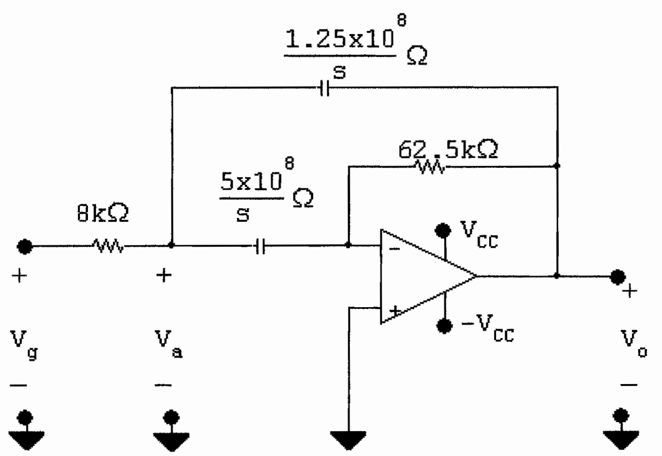
$$Z_i = R_1 = 40 \times 10^3\,\Omega$$

$$H(s) = \frac{V_o}{V_g} = \frac{-Z_f}{Z_i} = \frac{-100(s+200)}{s(s+600)}$$

[b] $-z_1 = -200\,\text{rad/s}$

$-p_1 = 0;\qquad -p_2 = -600\,\text{rad/s}$

P 13.51 [a]



$$\frac{V_a - V_g}{8000} + \frac{V_a s}{5 \times 10^8} + \frac{(V_a - V_o)s}{1.25 \times 10^8} = 0$$

$$\frac{-V_a s}{5\times 10^8} - \frac{V_o}{62{,}500} = 0; \qquad V_a = \frac{-8000V_o}{s}$$

$$\therefore \quad \frac{-8000V_o}{s}(5s + 62{,}500) - 4sV_o = 62{,}500V_g$$

$$\therefore \quad H(s) = \frac{V_o}{V_g} = \frac{-15{,}625s}{s^2 + 10{,}000s + 125\times 10^6}$$

$$s_{1,2} = -5000 \pm \sqrt{25\times 10^6 - 125\times 10^6} = -5000 \pm j10{,}000$$

$$H(s) = \frac{-15{,}625s}{(s + 5000 - j10{,}000)(s + 5000 + j10{,}000)}$$

[b] $-p_1 = -5000 + j10{,}000$ rad/s

$-p_2 = -5000 - j10{,}000$ rad/s

$z = 0$

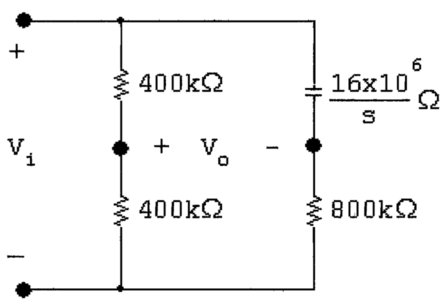
P 13.52 [a] $Z_i = 10{,}000 + \frac{10^9}{20s} = \frac{10^4(s + 5000)}{s}$

$$Z_f = \frac{25{,}000}{(25{,}000)(4\times 10^{-9})s + 1} = \frac{250\times 10^6}{s + 10{,}000}$$

$$H(s) = -\frac{Z_f}{Z_i} = \frac{-25{,}000s}{(s + 5000)(s + 10{,}000)}$$

[b] Zero at $s = 0$
Poles at $-p_1 = -5000$ rad/s and $-p_2 = -10{,}000$ rad/s.

P 13.53 [a]



$$\frac{4}{8}V_i = V_o + \frac{800{,}000V_i}{800{,}000 + (16\times 10^6/s)}$$

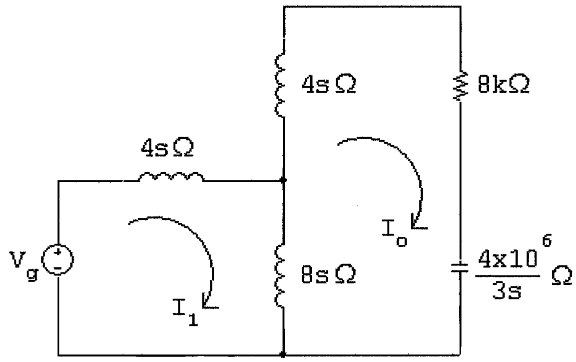
$$0.5V_i - \frac{sV_i}{s + 20} = V_o$$

$$\therefore \quad \frac{V_o}{V_i} = H(s) = \frac{-0.5(s - 20)}{(s + 20)}$$

[b] $-z_1 = 20\,\text{rad/s}$

$-p_1 = -20\,\text{rad/s}$

P 13.54



$$V_g = 12sI_1 - 8sI_o$$

$$0 = -8sI_1 + (12s + 8000 + 4\times10^6/3s)I_o$$

$$\Delta = \begin{vmatrix} 12s & -8s \\ -8s & 12s + 8000 + 4\times10^6/3s \end{vmatrix} = 80(s+200)(s+1000)$$

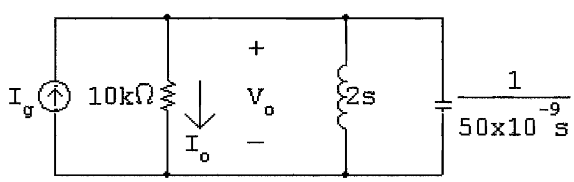
$$N_o = \begin{vmatrix} 12s & V_g \\ -8s & 0 \end{vmatrix} = 8sV_g$$

$$I_o = \frac{N_o}{\Delta} = \frac{8sV_g}{80(s+200)(s+1000)}$$

$$H(s) = \frac{I_o}{V_g} = \frac{0.1s}{(s+200)(s+1000)}$$

$z_1 = 0;\qquad -p_1 = -200$ rad/s; $\qquad -p_2 = -1000$ rad/s

P 13.55 **[a]**



$$\frac{V_o}{10{,}000} + \frac{V_o}{2s} + V_o(50\times10^{-9})s = I_g$$

$$\therefore \quad V_o = \frac{20 \times 10^6 s}{s^2 + 2000s + 10 \times 10^6} \cdot I_g$$

$$I_g = \frac{60 \times 10^{-3} s}{s^2 + 16 \times 10^6}; \qquad I_o = \frac{V_o}{10^4}$$

$$\therefore \quad H(s) = \frac{2000s}{s^2 + 2000s + 10^7}$$

[b] $$I_o = \frac{(2000s)(60 \times 10^{-3}s)}{(s + 1000 - j3000)(s + 1000 + j3000)(s^2 + 16 \times 10^6)}$$

$$I_o = \frac{120s^2}{(s + 1000 - j3000)(s + 1000 + j3000)(s + j4000)(s - j4000)}$$

[c] Damped sinusoid of the form

$$Me^{-1000t} \cos(3000t + \theta_1)$$

[d] Steady-state sinusoid of the form

$$N \cos(4000t + \theta_2)$$

[e] $$I_o = \frac{K_1}{s + 1000 - j3000} + \frac{K_1^*}{s + 1000 + j3000} + \frac{K_2}{s - j4000} + \frac{K_2^*}{s + j4000}$$

$$K_1 = \frac{120(-1000 + j3000)^2}{(j6000)(-1000 - j1000)(-j1000 + j7000)} = 20 \times 10^{-3}\underline{/163.74^\circ}$$

$$K_2 = \frac{120(-16 \times 10^6)}{(j8000)(1000 + j1000)(j1000 + j7000)} = 24 \times 10^{-3}\underline{/-36.87^\circ}$$

$$i_o(t) = [40e^{-1000t} \cos(3000t + 163.74^\circ) + 48 \cos(4000t - 36.87^\circ)] \text{ mA}$$

Test:

$$i_o(0) = 40 \cos(163.74^\circ) + 48 \cos(-36.87^\circ) = -384 + 384 = 0$$

$$Z = \frac{1}{Y}; \qquad Y = \frac{1}{10{,}000} + \frac{1}{j8000} + \frac{1}{-j5000} = \frac{1 + j0.75}{10{,}000}$$

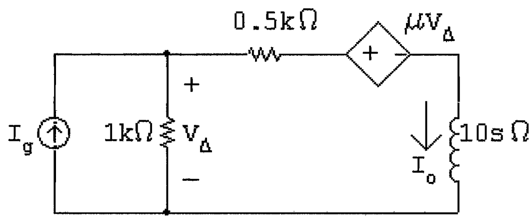
$$\therefore \quad Z = \frac{10{,}000}{1 + j0.75} = 8000\underline{/-36.87^\circ}\,\Omega$$

$$\mathbf{V}_o = \mathbf{I}_g Z = (60 \times 10^{-3}\underline{/0^\circ})(8000\underline{/-36.87^\circ}) = 480\underline{/-36.87^\circ}\text{ V}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{10^4} = 48\underline{/-36.87^\circ}\text{ mA}$$

$$i_{oss} = 48 \cos(4000t - 36.87^\circ) \text{ mA(checks)}$$

P 13.56 [a]



$$1000(I_o - I_g) + 500I_o + \mu(I_g - I_o)(1000) + 10sI_o = 0$$

$$\therefore\quad I_o = \frac{100(1-\mu)}{s + 100(1.5-\mu)}I_g$$

$$\therefore\quad H(s) = \frac{100(1-\mu)}{s + 100(1.5-\mu)}$$

[b] $\mu < 1.5$

[c]

μ	$H(s)$	I_o
-0.5	$150/(s+200)$	$1500/s(s+200)$
0	$100/(s+150)$	$1000/s(s+150)$
1.0	0	0
1.5	$-50/s$	$-500/s^2$
2.0	$-100/(s-50)$	$-1000/s(s-50)$

$\mu = -0.5$:

$$I_o = \frac{7.5}{s} - \frac{7.5}{(s+200)};\qquad i_o = [7.5 - 7.5e^{-200t}]u(t),\text{A}$$

$\mu = 0$:

$$I_o = \frac{20/3}{s} - \frac{20/3}{s+150};\qquad i_o = \frac{20}{3}[1 - e^{-150t}]u(t),\text{A}$$

$\mu = 1:\qquad i_o = 0\,\text{A}$

$\mu = 1.5$:

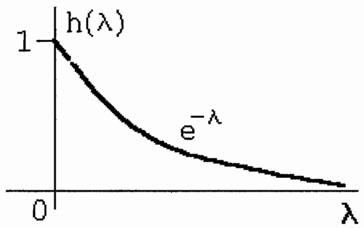
$$I_o = \frac{-500}{s^2};\qquad i_o = -500t\,u(t)\,\text{A}$$

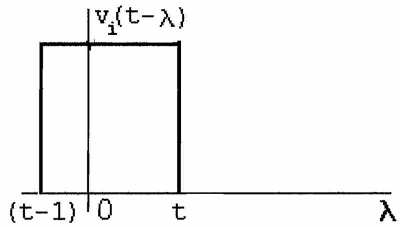
$\mu = 2$:

$$I_o = \frac{20}{s} - \frac{20}{s-50};\qquad i_o = 20[1 - e^{50t}]u(t),\text{A}$$

P 13.57 $H(s) = \dfrac{V_o}{V_i} = \dfrac{1}{s+1}; \qquad h(t) = e^{-t}$

For $0 \leq t \leq 1$:





$$v_o = \int_0^t e^{-\lambda}\,d\lambda = (1 - e^{-t})\,\mathrm{V}$$

For $1 \leq t \leq \infty$:

$$v_o = \int_{t-1}^t e^{-\lambda}\,d\lambda = (e-1)e^{-t}\,\mathrm{V}$$

P 13.58 $H(s) = \dfrac{V_o}{V_i} = \dfrac{s}{s+1} = 1 - \dfrac{1}{s+1}; \qquad h(t) = \delta(t) - e^{-t}$

$$h(\lambda) = \delta(\lambda) - e^{-\lambda}$$

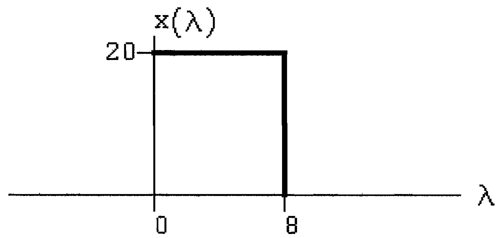
For $0 \leq t \leq 1$:

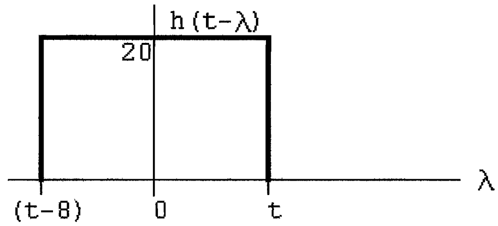
$$v_o = \int_0^t [\delta(\lambda) - e^{-\lambda}]\,d\lambda = [1 + e^{-\lambda}]\,\Big|_0^t = e^{-t}V$$

For $1 \leq t \leq \infty$:

$$v_o = \int_{t-1}^t (-e^{-\lambda})\,d\lambda = e^{-\lambda}\Big|_{t-1}^t = (1-e)e^{-t}\,\mathrm{V}$$

P 13.59 [a]



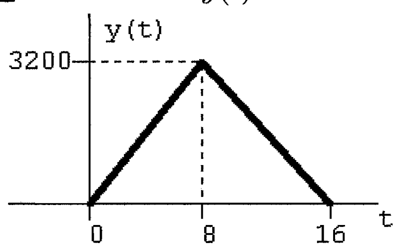


$y(t) = 0 \qquad t < 0$

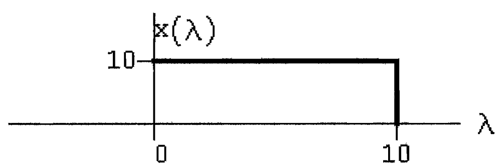
$0 \le t \le 8: \qquad y(t) = \int_0^t 400\, d\lambda = 400t$

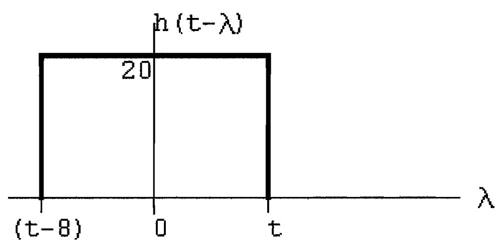
$8 \le t \le 16: \qquad y(t) = \int_{t-8}^{8} 400\, d\lambda = 400(8 - t + 8) = 400(16 - t)$

$16 \le t < \infty: \qquad y(t) = 0$



[b]





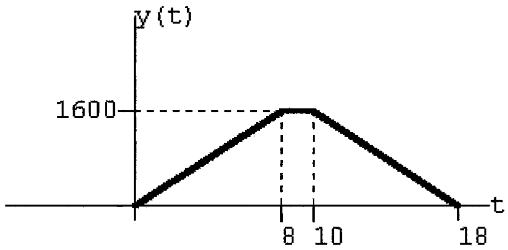
$y(t) = 0 \qquad t < 0$

$0 \le t \le 8: \qquad y(t) = \int_0^t 200\, d\lambda = 200t$

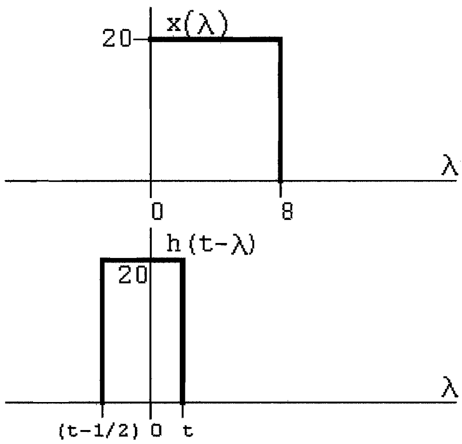
$$8 \le t \le 10: \qquad y(t) = \int_{t-8}^{t} 200\,d\lambda = 200(t - t + 8) = 1600$$

$$10 \le t \le 18: \qquad y(t) = \int_{t-8}^{10} 200\,d\lambda = 200(18 - t)$$

$$18 \le t < \infty: \qquad y(t) = 0$$



[c]



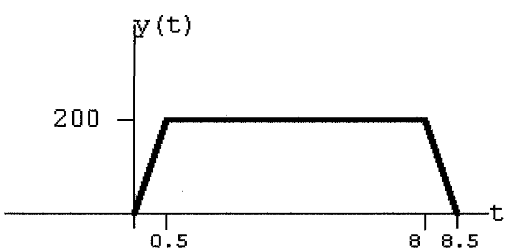
$$y(t) = 0 \qquad t < 0$$

$$0 \le t \le 0.5: \qquad y(t) = \int_{0}^{t} 400\,d\lambda = 400t$$

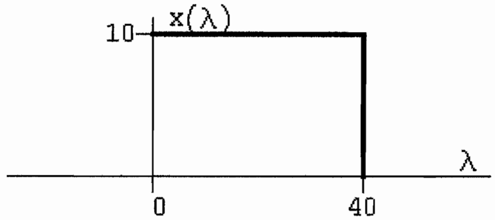
$$0.5 \le t \le 8: \qquad y(t) = \int_{t-0.5}^{t} 400\,d\lambda = 400(t - t + 0.5) = 200$$

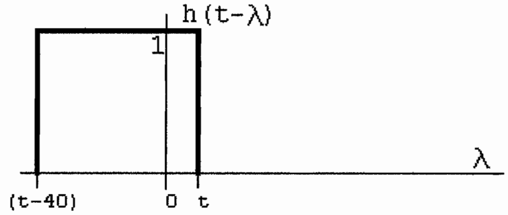
$$8 \le t \le 8.5: \qquad y(t) = \int_{t-0.5}^{8} 400\,d\lambda = 400(8.5 - t)$$

$$8.5 \le t < \infty: \qquad y(t) = 0$$



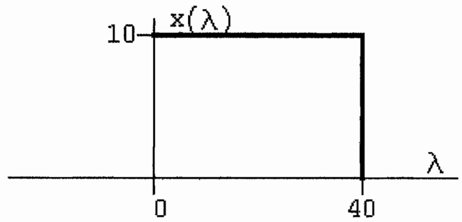
P 13.60 [a] $0 \leq t \leq 40$:

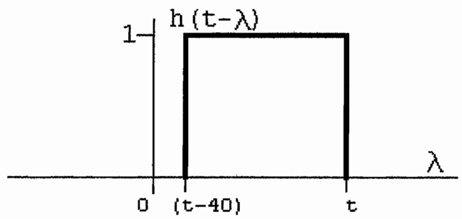




$$y(t) = \int_0^t (10)(1)(d\lambda) = 10\lambda \Big|_0^t = 10t$$

$40 \leq t \leq 80$:

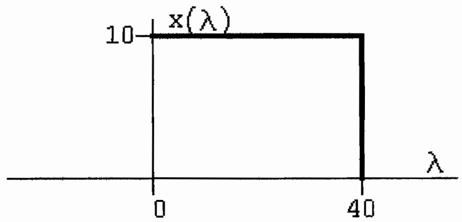


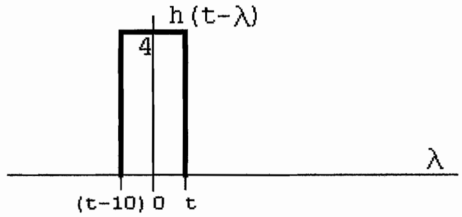


$$y(t) = \int_{t-40}^{40} (10)(1)(d\lambda) = 10\lambda \Big|_{t-40}^{40} = 10(80 - t)$$

$t \geq 80: \qquad y(t) = 0$

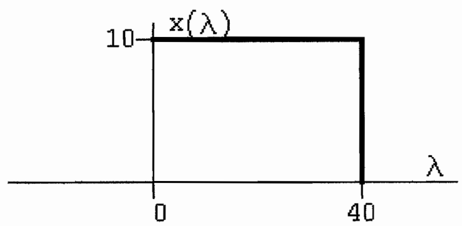
[b] $0 \le t \le 10$:

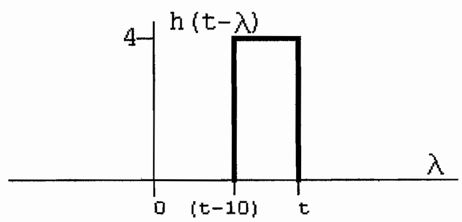




$$y(t) = \int_0^t 40\,d\lambda = 40\lambda \Big|_0^t = 40t$$

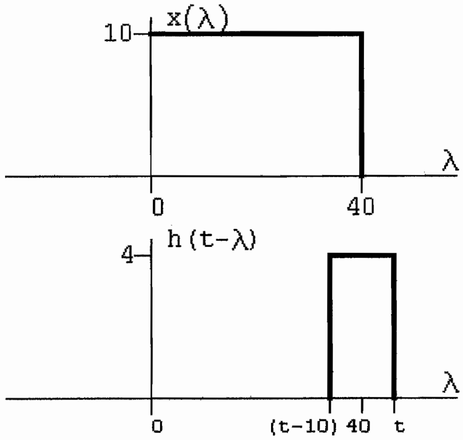
$10 \le t \le 40$:





$$y(t) = \int_{t-10}^t 40\,d\lambda = 40\lambda \Big|_{t-10}^t = 400$$

$40 \leq t \leq 50$:



$$y(t) = \int_{t-10}^{40} 40\,d\lambda = 40\lambda \Big|_{t-10}^{40} = 40(50 - t)$$

$t \geq 50: \qquad y(t) = 0$

[c] The expressions are

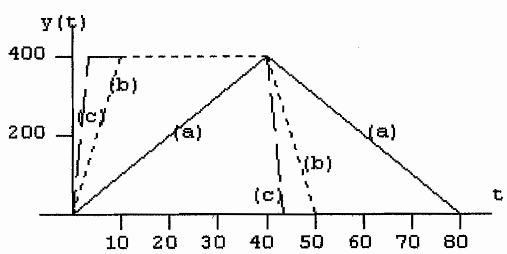
$$0 \leq t \leq 1.0: \qquad y(t) = \int_0^t 400\,d\lambda = 400\lambda \Big|_0^t = 400t$$

$$1.0 \leq t \leq 40: \qquad y(t) = \int_{t-1}^t 400\,d\lambda = 400\lambda \Big|_{t-1}^t = 400$$

$$40 \leq t \leq 41: \qquad y(t) = \int_{t-1}^{40} 400\,d\lambda = 400\lambda \Big|_{t-1}^{40} = 400(41 - t)$$

$$41 \leq t < \infty: \qquad y(t) = 0$$

[d]



[e] Yes, note that $h(t)$ is approaching $40\delta(t)$, therefore $y(t)$ must approach $40x(t)$, i.e.

$$y(t) = \int_0^t h(t-\lambda)x(\lambda)\,d\lambda \rightarrow \int_0^t 40\delta(t-\lambda)x(\lambda)\,d\lambda$$

$$\rightarrow 40x(t)$$

This can be seen in the plot, e.g., in part (c), $y(t) \cong 40x(t)$.

P 13.61 [a] $-1 \leq t \leq 4$:

$$v_o = 20\int_0^{t+1} 3\lambda\, d\lambda = 30\lambda^2 \Big|_0^{t+1} = 30t^2 + 60t + 30$$

$4 \leq t \leq 7$:

$$v_o = 20\int_0^5 3\lambda\, d\lambda + 20\int_5^{t+1} (20-\lambda)\, d\lambda$$

$$= 30\lambda^2 \Big|_0^5 + 400\lambda \Big|_5^{t+1} - 10\lambda^2 \Big|_5^{t+1}$$

$$= -10t^2 + 380t - 610$$

$7 \leq t \leq 12$:

$$v_o = 20\int_{t-7}^5 3\lambda\, d\lambda + 20\int_5^{t+1} (20-\lambda)\, d\lambda$$

$$= 30\lambda^2 \Big|_{t-7}^5 + 400\lambda \Big|_5^{t+1} - 10\lambda^2 \Big|_5^{t+1}$$

$$= -40t^2 + 800t - 2080$$

$12 \leq t \leq 19$:

$$v_o = 20\int_{t-7}^{t+1} (20-\lambda)\, d\lambda = 400\lambda \Big|_{t-7}^{t+1} - 10\lambda^2 \Big|_{t-7}^{t+1}$$

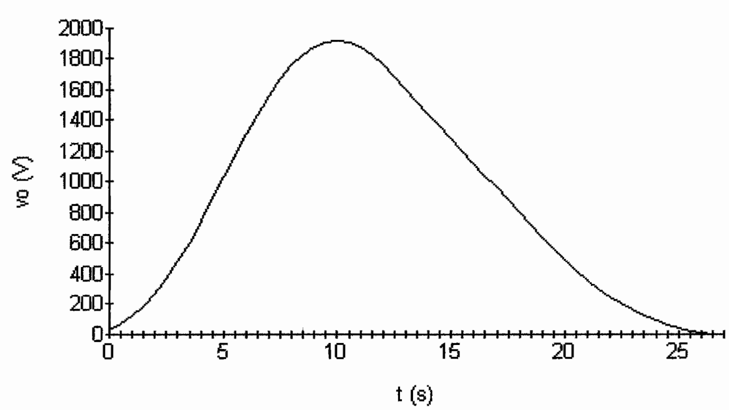
$$= -160t + 3680$$

$19 \leq t \leq 27$:

$$v_o = 20\int_{t-7}^{20} (20-\lambda)\, d\lambda = 400\lambda \Big|_{t-7}^{20} - 10\lambda^2 \Big|_{t-7}^{20}$$

$$= 10t^2 - 540t + 7290$$

[b]



P 13.62 [a] $h(\lambda) = \frac{5}{10}\lambda \qquad 0 \leq \lambda \leq 10\,\mathrm{s}$

$$h(\lambda) = 10 - \frac{5}{10}\lambda \qquad 10 \leq \lambda \leq 20\,\mathrm{s}$$

$$h(\lambda) = 0 \qquad 20 \leq \lambda \leq \infty$$

$0 \leq t \leq 10\,\mathrm{s}$:

$$v_o = \int_0^t (0.5\lambda)(4)\,d\lambda = 2\frac{\lambda^2}{2}\Big|_0^t = t^2$$

$10 \leq t \leq 20\,\mathrm{s}$:

$$v_o = \int_0^{10} 2\lambda\,d\lambda + \int_{10}^t 4(10 - 0.5\lambda)\,d\lambda$$

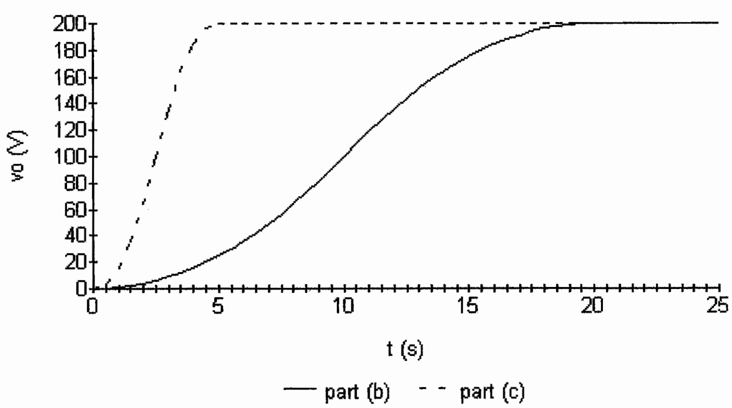
$$v_o = 100 + 40t - 400 - t^2 + 100 = 40t - 200 - t^2\,\mathrm{V}$$

$20 \leq t \leq \infty$:

$$v_o = \int_0^{10} 2\lambda\,d\lambda + \int_{10}^{20} 4(10 - 0.5\lambda)\,d\lambda$$

$$v_o = 100 + 400 - (400 - 100) = 200\,\mathrm{V}$$

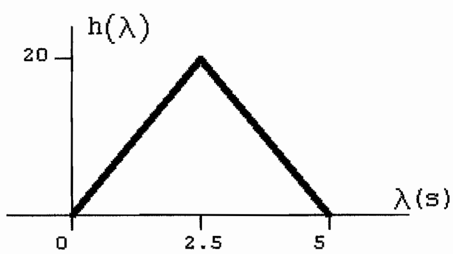
[b]



[c] $h(\lambda) = 8\lambda \qquad 0 \leq \lambda \leq 2.5\,\mathrm{s}$

$$h(\lambda) = 40 - 8\lambda \qquad 2.5 \leq \lambda \leq 5\,\mathrm{s}$$

$$h(\lambda) = 0 \qquad 5 \leq \lambda \leq \infty$$



$0 \le t \le 2.5$ s:

$$v_o = \int_0^t 32\lambda\, d\lambda = 16t^2 \text{ V}$$

$2.5 \le t \le 5$ s:

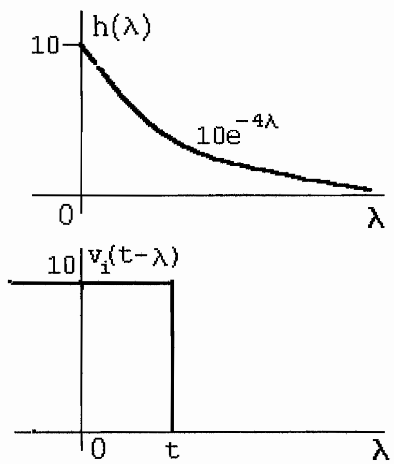
$$v_o = \int_0^{2.5} 32\lambda\, d\lambda + \int_{2.5}^{t} 4(40 - 8\lambda)\, d\lambda = 160t - 200 - 16t^2 \text{ V}$$

$5 \le t \le \infty$:

$$v_o = \int_0^{2.5} 32\lambda\, d\lambda + \int_{2.5}^{5} 4(40 - 8\lambda)\, d\lambda = 200 \text{ V}$$

[d] The waveform in part (c) is closer to replicating the input waveform because in part (c) $h(\lambda)$ is closer to being an ideal impulse response. That is, the area was preserved as the base was shortened.

P 13.63 [a]

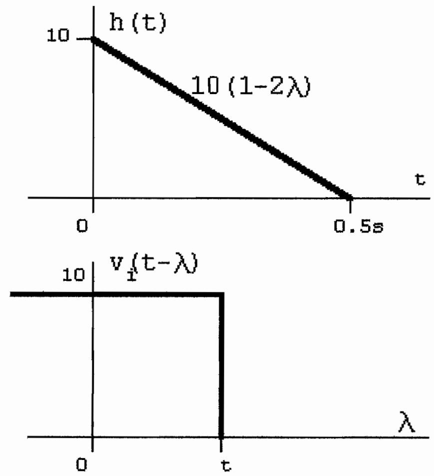


$$v_o = \int_0^t 10(10e^{-4\lambda})\, d\lambda$$

$$= 100\frac{e^{-4\lambda}}{-4}\Bigg|_0^t = -25[e^{-4t} - 1]$$

$$= 25(1 - e^{-4t}) \text{ V}, \qquad 0 \le t \le \infty$$

[b]

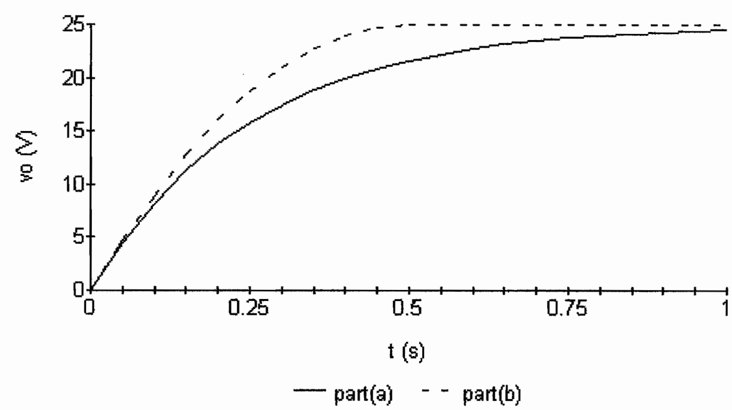


$0 \le t \le 0.5$:

$$v_o = \int_0^t 100(1-2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^t = 100t(1-t)$$

$0.5 \le t \le \infty$:

$$v_o = \int_0^{0.5} 100(1-2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^{0.5} = 25$$



P 13.64 [a] From Problem 13.49(d)

$$H(s) = \frac{3000}{s+3000}$$

$$h(\lambda) = 3000e^{-3000\lambda}$$

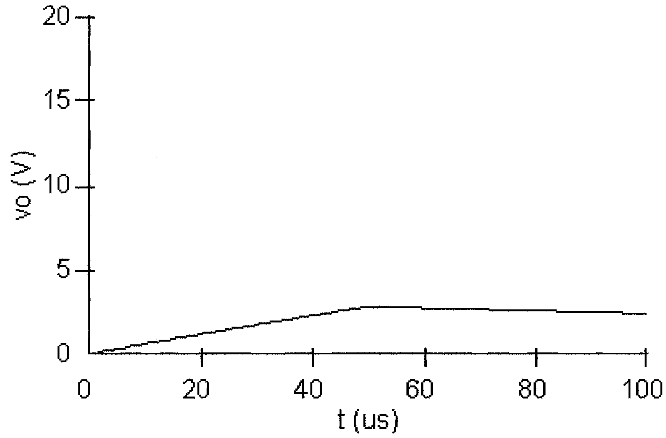
$0 \le t \le 50\,\mu$s:

$$v_o = \int_0^t 20(3000)e^{-3000\lambda}\,d\lambda = 20(1 - e^{-3000t})\text{ V}$$

$50\,\mu\text{s} \le t \le \infty$:

$$v_o = \int_{t-50\times10^{-6}}^{t} 20(3000)e^{-3000\lambda}\,d\lambda = 20(e^{0.15} - 1)e^{-3000t}\text{ V}$$

[b]



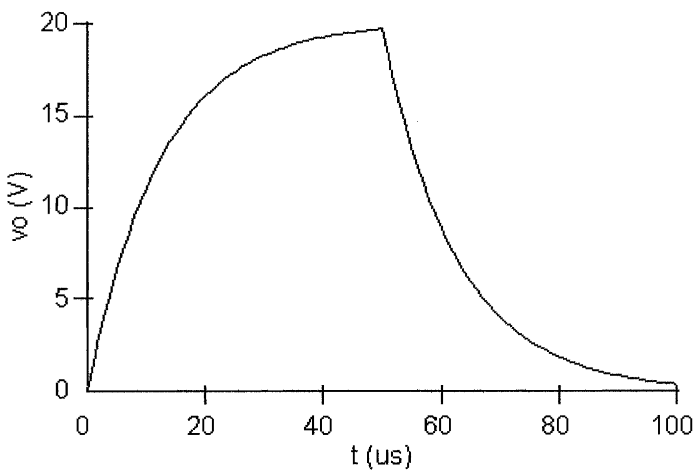
P 13.65 [a] $H(s) = \dfrac{80{,}000}{s + 80{,}000}$ $\quad \therefore \quad h(\lambda) = 80{,}000e^{-80{,}000\lambda}$

$0 \le t \le 50\,\mu$s:

$$v_o = \int_0^t 20(80 \times 10^3)e^{-80{,}000\lambda}\,d\lambda = 20(1 - e^{-80{,}000t})\text{ V}$$

$50\,\mu\text{s} \le t \le \infty$:

$$v_o = \int_{t-50\times 10^{-6}}^t 20(80 \times 10^3)e^{-80{,}000\lambda}\,d\lambda = 20(e^4 - 1)e^{-80{,}000t}\text{ V}$$



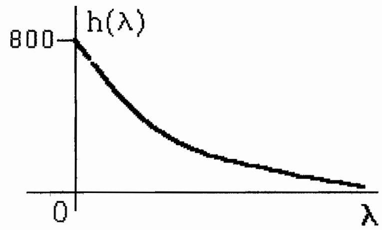
[b] decrease

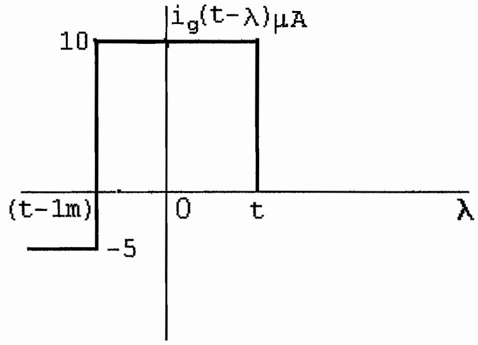
[c] The circuit with $R = 400\,\Omega$.

P 13.66 [a] $I_o = \dfrac{20I_g}{25 + 0.025s} = \dfrac{800I_g}{s + 1000}$

$$\frac{I_o}{I_g} = H(s) = \frac{800}{s + 1000}$$

$$h(\lambda) = 800e^{-1000\lambda}u(\lambda)$$



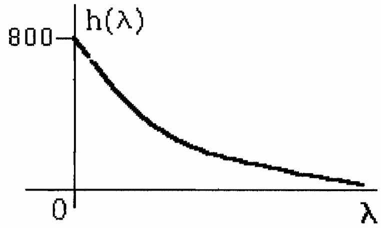


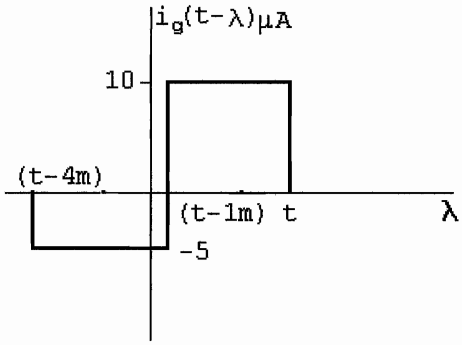
$0 \le t \le 1\,\text{ms}$:

$$i_o = \int_0^t (10 \times 10^{-6})(800)e^{-1000\lambda}\,d\lambda = 0.008\frac{e^{-1000\lambda}}{-1000}\bigg|_0^t$$

$$= 8(1 - e^{-1000t})\,\mu\text{A}$$

$1\,\text{ms} \le t \le 4\,\text{ms}$:





$$i_o = \int_0^{t-1\times10^{-3}} (-5\times10^{-6})(800e^{-1000\lambda}\,d\lambda)$$

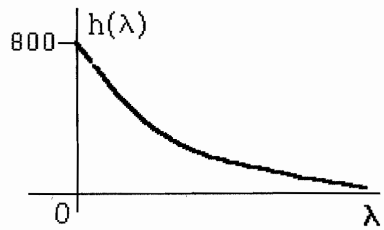
$$+\int_{t-1\times10^{-3}}^{t} (10\times10^{-6})(800e^{-1000\lambda}\,d\lambda)$$

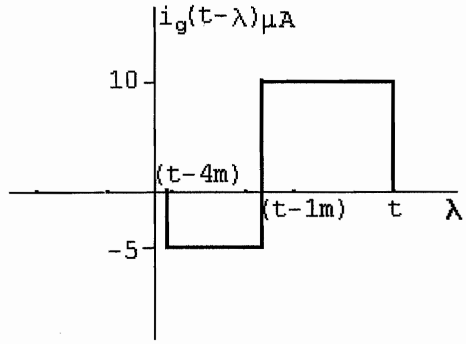
$$= -0.004\frac{e^{-1000\lambda}}{-1000}\Big|_0^{t-1\times10^{-3}} + 0.008\frac{e^{-1000\lambda}}{-1000}\Big|_{t-1\times10^{-3}}^{t}$$

$$= 4\left[e^{-1000(t-0.001)} - 1\right] - 8\left[e^{-1000t} - e^{-1000(t-0.001)}\right]$$

$$i_o = [12e^{-1000(t-0.001)} - 8e^{-1000t} - 4]\,\mu\text{A}$$

$4\,\text{ms} < t < \infty$:





$$i_o = \int_{t-0.004}^{t-0.001} -0.004e^{-1000\lambda}\,d\lambda + \int_{t-0.001}^{t} 0.008e^{-1000\lambda}\,d\lambda$$

$$= \left[4e^{-1000\lambda}\Big|_{t-0.004}^{t-0.001} - 8e^{-1000\lambda}\Big|_{t-0.001}^{t}\right] \times 10^{-6}$$

$$i_o = [12e^{-1000(t-0.001)} - 4e^{-1000(t-0.004)} - 8e^{-1000t}]\ \mu\text{A}$$

[b] $V_o = 0.025sI_o = \dfrac{20sI_g}{s+1000}$

$$\frac{V_o}{I_g} = H(s) = \frac{20s}{s+1000} = 20 - \frac{20{,}000}{s+1000}$$

$$h(\lambda) = 20\delta(\lambda) - 20{,}000e^{-1000\lambda}$$

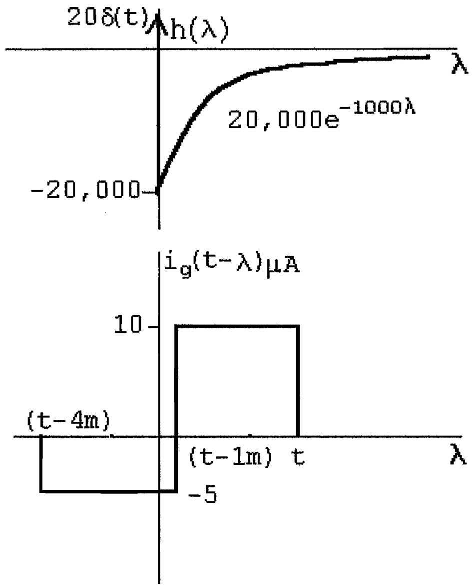
$0 < t < 0.001\,\text{s}$:

$$v_o = \int_0^t (10\times 10^{-6})[20\delta(\lambda) - 20{,}000e^{-1000\lambda}]\,d\lambda$$

$$= 200\times 10^{-6} - 0.2\frac{e^{-1000\lambda}}{-1000}\Big|_0^t$$

$$= 200\times 10^{-6} + 200\times 10^{-6}[e^{-1000t} - 1] = 200e^{-1000t}\ \mu\text{V}$$

$0.001\,\mathrm{s} < t < 0.004\,\mathrm{s}$:



$$v_o = \int_0^{t-0.001} (-5\times10^{-6})[20\delta(\lambda) - 20{,}000e^{-1000\lambda}]\,d\lambda$$

$$+\int_{t-0.001}^{t} (10\times10^{-6})(-20{,}000e^{-1000\lambda})\,d\lambda$$

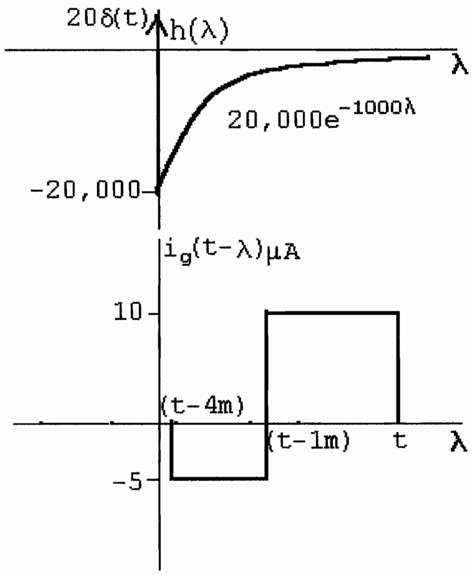
$$= -100\times10^{-6} + 0.1\frac{e^{-1000\lambda}}{-1000}\bigg|_0^{t-0.001} - 0.2\frac{e^{-1000\lambda}}{-1000}\bigg|_{t-0.001}^{t}$$

$$= -100\times10^{-6} - 0.1\times10^{-3}e^{-1000(t-0.001)} + 0.1\times10^{-3}$$

$$+0.2\times10^{-3}e^{-1000t} - 0.2\times10^{-3}e^{-1000(t-0.001)}$$

$$= 200e^{-1000t} - 300e^{-1000(t-0.001)}\,\mu\mathrm{V}$$

$0.004\,\mathrm{s} < t < \infty$:



$$v_o = \int_{t-0.004}^{t-0.001} (-5 \times 10^{-6})(-20{,}000e^{-1000\lambda})\,d\lambda$$

$$+\int_{t-0.001}^{t} (10 \times 10^{-6})(-20{,}000e^{-1000\lambda})\,d\lambda$$

$$= 200e^{-1000t} - 300e^{-1000(t-0.001)} + 100e^{-1000(t-0.004)}\,\mu\mathrm{V}$$

[c] At $t = 0.001^-$:

$i_o = 8(1 - e^{-1}) = 5.06\,\mu\mathrm{A}$; $\qquad i_{20\Omega} = (10 - 5.06) = 4.94\,\mu\mathrm{A}$

$\therefore\ v_o = 20(4.94 \times 10^{-6}) - 5(5.06 \times 10^{-6}) = 73.58\,\mu\mathrm{V}$

From the solution for v_o we have

$v_o(0.001^-) = 200e^{-1} = 73.58\,\mu\mathrm{V}$ (checks)

At $t = 0.001^+$:

$i_o(0.001^+) = i_o(0.001^-) = 5.06\,\mu\mathrm{A}$

$i_{20\Omega} = (-5 - 5.06)\,\mu\mathrm{A} = -10.06\,\mu\mathrm{A}$

$\therefore\ v_o(0.001^+) = 20(-10.06 \times 10^{-6}) + 5(5.06 \times 10^{-6}) = -226.42\,\mu\mathrm{V}$

From the solution for v_o we have

$v_o(0.001^+) = 200e^{-1} - 300 = -226.42\,\mu\mathrm{V}$ (checks)

At $t = 0.004^-$:

$i_o = 12e^{-3} - 8e^{-4} - 4 = -3.55\,\mu\mathrm{A}$

$i_{20\Omega} = (-5 + 3.55) = -1.45\,\mu\text{A}$

$v_o = 20(-1.45 \times 10^{-6}) - 5(-3.55 \times 10^{-6}) = -11.27\,\mu\text{V}$

From the solution for v_o,

$v_o((0.004^-) = 200e^{-4} - 300e^{-3} = -11.27\,\mu\text{V}$ (checks)

At $t = 0.004^+$:

$i_o(0.004^+) = i_o(0.004^-) = -3.55\,\mu\text{A}; \qquad i_{20\Omega} = 3.55\,\mu\text{A}$

$i_o = 20(3.55 \times 10^{-6}) + 5(3.55 \times 10^{-6}) = 88.73\,\mu\text{V}$

From the solution for v_o,

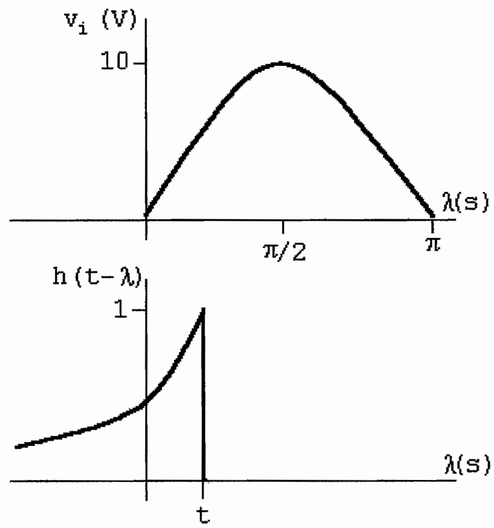
$v_o(0.004^+) = 200e^{-4} - 300e^{-3} + 100 = 88.73\,\mu\text{V}$(checks)

P 13.67 $v_i = 10\,\sin\lambda\,[u(\lambda) - u(\lambda - \pi)]$

$$H(s) = \frac{1}{s+1}$$

$$h(\lambda) = e^{-\lambda}$$

$$h(t-\lambda) = e^{-(t-\lambda)} = e^{-t}e^{\lambda}$$



$$v_o = 10e^{-t}\int_0^t e^{\lambda}\sin\lambda\,d\lambda$$

$$= 10e^{-t}\left[\frac{e^{\lambda}}{2}(\sin\lambda - \cos\lambda\,\Big|_0^t\right]$$

$$= 5e^{-t}[e^t(\sin t - \cos t) + 1]$$

$$= 5(\sin t - \cos t + e^{-t})$$

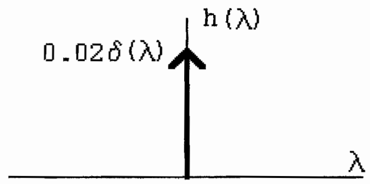
$v_o(2.2) = 7.539\,\text{V}$

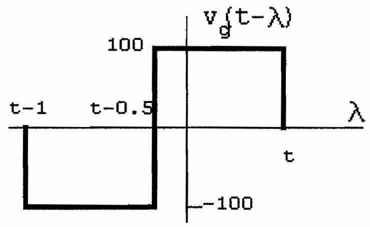
P 13.68 [a] $I_o = \dfrac{60}{100} I_g; \qquad I_g = \dfrac{V_g}{30}$

$$\therefore \quad I_o = \frac{V_g}{50}; \qquad H(s) = \frac{I_o}{V_g} = \frac{1}{50}$$

$$h(\lambda) = 0.02\delta(\lambda)$$

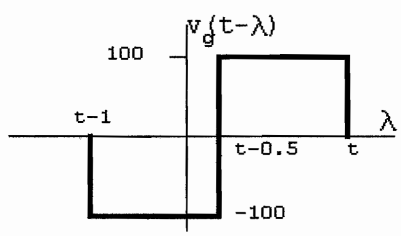
[b]





$0 < t < 0.5\,\text{s}: \qquad i_o = \displaystyle\int_0^t 100[0.02\delta(\lambda)]\,d\lambda = 2\,\text{A}$

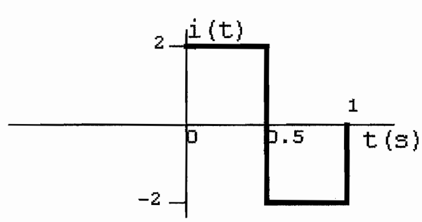
$0.5\,\text{s} \leq t \leq 1.0\,\text{s}:$



$$i_o = \int_0^{t-0.5} -100[0.02\delta(\lambda)]\,d\lambda = -2\,\text{A}$$

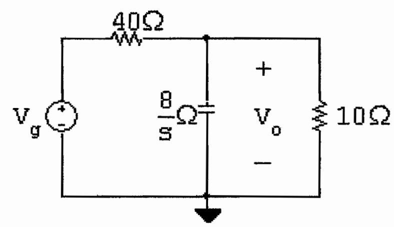
$1\,\text{s} < t < \infty: \qquad v_o = 0$

[c]



Yes, because the circuit has no memory.

P 13.69 [a]

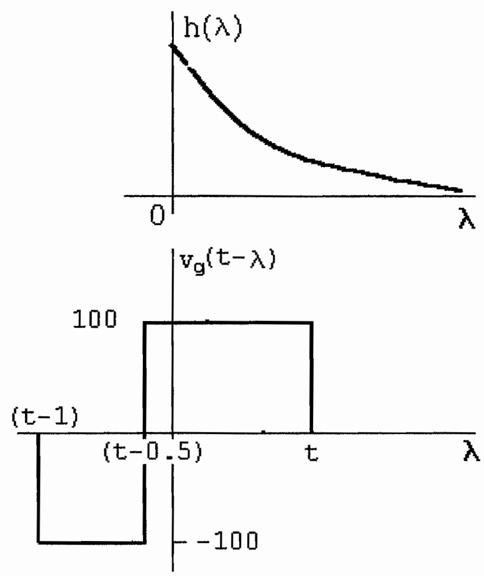


$$\frac{V_o - V_g}{40} + \frac{V_o s}{8} + \frac{V_o}{10} = 0$$

$$(5s + 5)V_o = V_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{0.2}{s+1}; \qquad h(\lambda) = 0.2e^{-\lambda}u(\lambda)$$

[b]

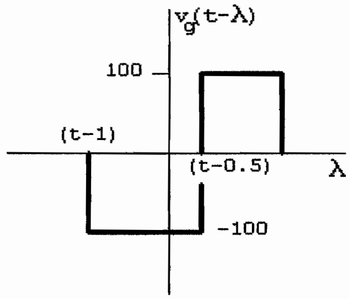


$0 \leq t \leq 0.5$ s;

$$v_o = \int_0^t 100(0.2e^{-\lambda})\,d\lambda = 20\frac{e^{-\lambda}}{-1}\Big|_0^t$$

$$v_o = 20 - 20e^{-t}\,\text{V}, \qquad 0 \leq t \leq 0.5\,\text{s}$$

$0.5\,\text{s} \leq t \leq 1\,\text{s}$:



$$v_o = \int_0^{t-0.5} (-100)(0.2e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 100(0.2e^{-\lambda})\,d\lambda$$

$$= -20\frac{e^{-\lambda}}{-1}\Big|_0^{t-0.5} + 20\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

$$= 40e^{-(t-0.5)} - 20e^{-t} - 20\,\text{V}, \qquad 0.5\,\text{s} \leq t \leq 1\,\text{s}$$

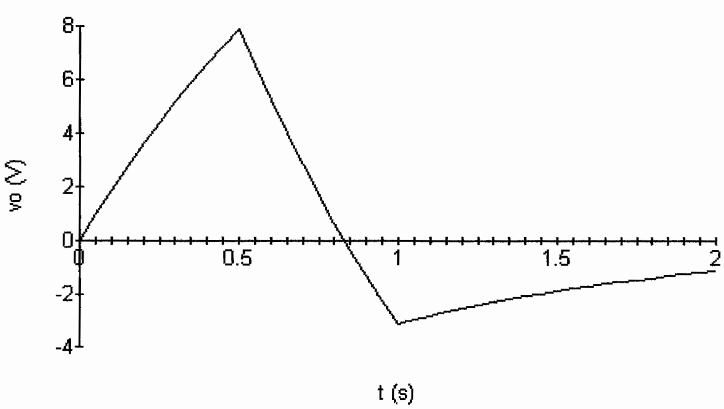
$1\,\text{s} \leq t \leq \infty$;

$$v_o = \int_{t-1}^{t-0.5} (-100)(0.2e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 100(0.2e^{-\lambda})\,d\lambda$$

$$= -20\frac{e^{-\lambda}}{-1}\Big|_{t-1}^{t-0.5} + 20\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

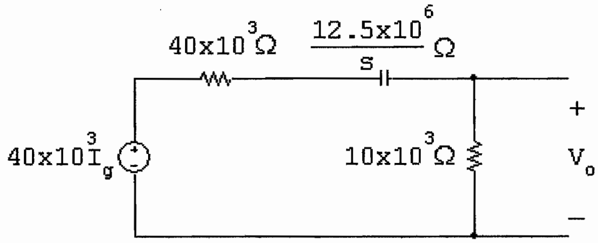
$$= 40e^{-(t-0.5)} - 20e^{-(t-1)} - 20e^{-t}\,\text{V}, \qquad 1\,\text{s} \leq t \leq \infty$$

[c]



[d] No, the circuit has memory because of the capacitive storage element.

P 13.70



$$V_o = \frac{40 \times 10^3 I_g}{50 \times 10^3 + 12.5 \times 10^6/s}(10 \times 10^3)$$

$$\frac{V_o}{I_g} = H(s) = \frac{8000s}{s + 250}$$

$$H(s) = 8000\left[1 - \frac{250}{s + 250}\right] = 8000 - \frac{2 \times 10^6}{s + 250}$$

$$h(t) = 8000\delta(t) - 2 \times 10^6 e^{-250t}$$

$$v_o = \int_0^{5\times10^{-3}} (-10 \times 10^{-3})[8000\delta(\lambda) - 2 \times 10^6 e^{-250\lambda}]\, d\lambda$$

$$+\int_{5\times10^{-3}}^{7\times10^{-3}} (5 \times 10^{-3})[-2 \times 10^6 e^{-250\lambda}]\, d\lambda$$

$$= -80 + 20{,}000\int_0^{5\times10^{-3}} e^{-250\lambda}\, d\lambda - 10{,}000\int_{5\times10^{-3}}^{7\times10^{-3}} e^{-250\lambda}\, d\lambda$$

$$= -80 - 80(e^{-1.25} - 1) + 40(e^{-1.75} - e^{-1.25})$$

$$= -120e^{-1.25} + 40e^{-1.75} = -27.43\text{ V}$$

Alternate:

$$I_g = \int_0^{2\times10^{-3}} (5 \times 10^{-3})e^{-st}\, dt + \int_{2\times10^{-3}}^{8\times10^{-3}} (-10 \times 10^{-3})e^{-st}\, dt$$

$$= \left[\frac{5}{s} - \frac{15}{s}e^{-2\times10^{-3}s} + \frac{10}{s}e^{-8\times10^{-3}s}\right] \times 10^{-3}$$

$$V_o = I_g H(s) = \frac{8}{s + 250}[5 - 15e^{-2\times10^{-3}s} + 10e^{-8\times10^{-3}s}]$$

$$= \frac{40}{s + 250} - \frac{120e^{-2\times10^{-3}s}}{s + 250} + \frac{80e^{-8\times10^{-3}s}}{s + 250}]$$

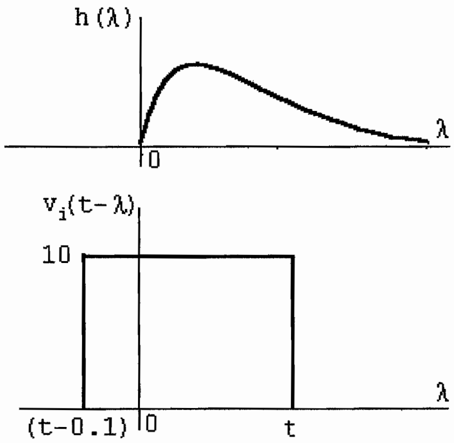
$$v_o(t) = 40e^{-250t} - 120e^{-250(t-2\times10^{-3})}u(t-2\times10^{-3})$$
$$+80e^{-250(t-8\times10^{-3})}u(t-8\times10^{-3})$$

$$v_o(7\times10^{-3}) = 40e^{-1.75} - 120e^{-1.25} + 0 = -27.43\,\text{V} \quad \text{(checks)}$$

P 13.71 [a] $H(s) = \frac{V_o}{V_i} = \frac{1/LC}{s^2 + (R/L)s + (1/LC)}$

$$= \frac{25}{s^2+10s+25} = \frac{25}{(s+5)^2}$$

$$h(\lambda) = 25\lambda e^{-5\lambda}u(\lambda)$$



$0 \le t \le 0.10s$:

$$v_o = 250\int_0^t \lambda e^{-5\lambda}\,d\lambda$$

$$= 250\left\{\frac{e^{-5\lambda}}{25}(-5\lambda-1)\Big|_0^t\right\}$$

$$= 10[1 - e^{-5t}(5t+1)]$$

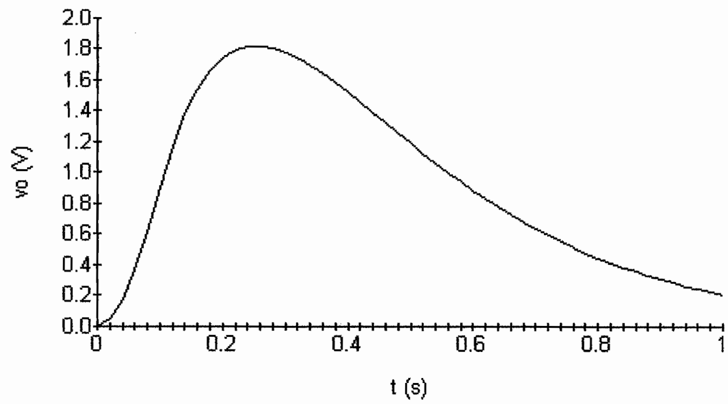
$0.1 \le t \le \infty$:

$$v_o = 250\int_{t-0.1}^t \lambda e^{-5\lambda}\,d\lambda$$

$$= 250\left\{\frac{e^{-5\lambda}}{25}(-5\lambda-1)\Big|_{t-0.1}^t\right\}$$

$$= -10e^{-5t}[(5t+1) - e^{0.5}(5t+0.5)]$$

[b]



P 13.72 $H(s) = \frac{V_o}{V_i} = \frac{8s}{50+10s} = \frac{0.8s}{s+5}$

$$= 0.8\left[1 - \frac{5}{s+5}\right] = 0.8 - \frac{4}{s+5}$$

$$h(t) = 0.8\delta(t) - 4e^{-5t}$$

$$v_o = \int_0^t 75[0.8\delta(\lambda) - 4e^{-5\lambda}]\,d\lambda$$

$$= \int_0^t 60\delta(\lambda)\,d\lambda - 300\int_0^t e^{-5\lambda}\,d\lambda$$

$$= 60 - 300\frac{e^{-5\lambda}}{-5}\Big|_0^t$$

$$= 60 + 60[e^{-5t} - 1] = 60e^{-5t}\,\text{V} \qquad 0 \leq t \leq \infty$$

P 13.73 [a] $Y(s) = \int_0^\infty y(t)e^{-st}\,dt$

$$Y(s) = \int_0^\infty e^{-st}\left[\int_0^\infty h(\lambda)x(t-\lambda)\,d\lambda\right]dt$$

$$= \int_0^\infty \int_0^\infty e^{-st}h(\lambda)x(t-\lambda)\,d\lambda\,dt$$

$$= \int_0^\infty h(\lambda) \textstyle\int_0^\infty e^{-st}x(t-\lambda)\,dt\,d\lambda$$

But $x(t-\lambda) = 0$ when $t < \lambda$

Therefore $Y(s) = \int_0^\infty h(\lambda)\int_\lambda^\infty e^{-st}x(t-\lambda)\,dt\,d\lambda$

Let $u = t - \lambda;\quad du = dt;\quad u = 0,\quad t = \lambda;\quad u = \infty,\quad t = \infty$

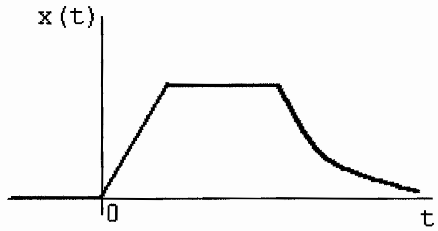
$$Y(s) = \int_0^\infty h(\lambda)\int_0^\infty e^{-s(u+\lambda)}x(u)\,du\,d\lambda$$

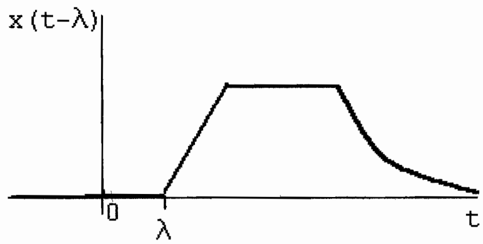
$$= \int_0^\infty h(\lambda)e^{-s\lambda}\int_0^\infty e^{-su}x(u)\,du\,d\lambda$$

$$= \int_0^\infty h(\lambda)e^{-s\lambda}X(s)\,d\lambda = H(s)\,X(s)$$

Note on $x(t-\lambda) = 0, \qquad t < \lambda$

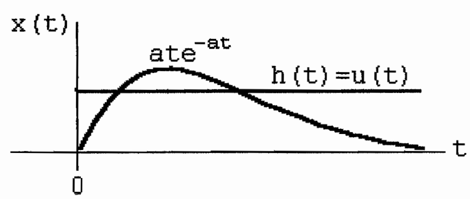
We are using one-sided Laplace transforms; therefore $h(t)$ and $X(t)$ are assumed zero for $t < 0$.





[b] $F(s) = \dfrac{a}{s(s+a)^2} = \dfrac{1}{s}\cdot\dfrac{a}{(s+a)^2} = H(s)X(s)$

$\therefore\ h(t) = u(t$



$$\therefore\ f(t) = \int_0^t (1)a\lambda e^{-a\lambda}\,d\lambda = a\left[\frac{e^{-a\lambda}}{a^2}(-a\lambda - 1)\right|_0^t$$

$$= \frac{1}{a}[e^{-at}(-at-1) - 1(-1)] = \frac{1}{a}[1 - e^{-at} - ate^{-at}]$$

$$= \left[\frac{1}{a} - \frac{1}{a}e^{-at} - te^{-at}\right]u(t)$$

Check:

$$F(s) = \frac{a}{s(s+a)^2} = \frac{K_0}{s} + \frac{K_1}{(s+a)^2} + \frac{K_2}{s+a}$$

$$K_0 = \frac{1}{a}; \qquad K_1 = -1; \qquad K_2 = \frac{d}{ds}\left(\frac{a}{s}\right)_{s=-a} = -\frac{1}{a}$$

$$f(t) = \left[\frac{1}{a} - te^{-at} - \frac{1}{a}e^{-at}\right]u(t)$$

P 13.74 $$H(j8000) = \frac{10^4(6000 + j8000)}{-64 \times 10^6 + j7 \times 10^6 + 88 \times 10^6}$$

$$= \frac{10^7(6+j8)}{10^6(24+j7)} = 4\underline{/36.87^\circ}$$

$$\therefore \quad v_o(t) = 50\cos(8000t + 36.87^\circ)\text{ V}$$

P 13.75 **[a]** $$H(s) = \frac{-Z_f}{Z_i}$$

$$Z_f = \frac{(1/C_f)}{s + (1/R_fC_f)} = \frac{4 \times 10^9}{s + 16{,}000}$$

$$Z_i = \frac{R_i[s + (1/R_iC_i)]}{s} = \frac{25{,}000(s + 8000)}{s}$$

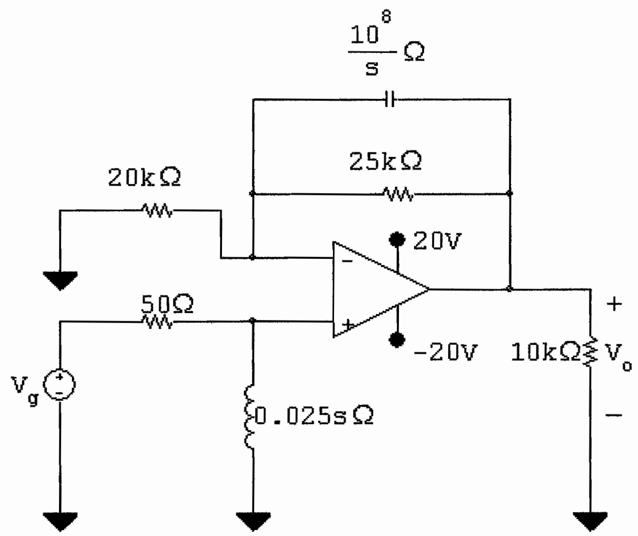
$$H(s) = \frac{-16 \times 10^4 s}{(s+8000)(s+16{,}000)}$$

[b] $$H(j8000) = \frac{-16 \times 10^4(j8000)}{(8000 + j8000)(16{,}000 + j8000)} = \sqrt{40}\underline{/-161.57^\circ}$$

$$v_o(t) = (200\sqrt{10}) \times 10^{-3}(\sqrt{40})\cos(8000t - 161.57^\circ)$$

$$= 4\cos(8000t - 161.57^\circ)\text{ V}$$

P 13.76 [a]



$$V_p = \frac{0.025s}{50 + 0.025s}V_g = \frac{s}{s + 2000}V_g$$

$$V_n = V_p$$

$$\frac{V_p}{20{,}000} + \frac{V_p - V_o}{25{,}000} + \frac{(V_p - V_o)s}{10^8} = 0$$

$$\therefore \quad V_p = \frac{(s + 4000)}{(s + 9000)}V_o$$

$$\frac{sV_g}{s + 2000} = \frac{s + 4000}{s + 9000}V_o$$

$$\therefore \quad H(s) = \frac{V_o}{V_g} = \frac{s(s + 9000)}{(s + 2000)(s + 4000)}$$

[b] $v_g = 10u(t); \qquad V_g = \frac{10}{s}$

$$V_o = \frac{10(s + 9000)}{(s + 2000)(s + 4000)} = \frac{K_1}{s + 2000} + \frac{K_2}{s + 4000}$$

$$K_1 = \frac{70{,}000}{2000} = 35; \qquad K_2 = \frac{50{,}000}{-2000} = -25$$

$$\therefore \quad v_o(t) = (35e^{-2000t} - 25e^{-4000t})u(t)\text{ V}$$

[c] $\omega = 2000$ rad/s

$$H(j\omega) = \frac{j2000(9000 + j2000)}{(2000 + j2000)(4000 + j2000)}$$

$$= 1.25 + j0.75 = 1.46\underline{/30.96^\circ}$$

$$\therefore\quad V_{oss} = (8)(1.46)\cos(2000t + 30.96^\circ)$$
$$= 11.68\cos(2000t + 30.96^\circ)\text{ V}$$

P 13.77 $V_o = \dfrac{75}{s} - \dfrac{100}{s+800} + \dfrac{25}{s+3200} = \dfrac{192\times 10^6}{s(s+800)(s+3200)}$

$$V_o = H(s)V_g = H(s)\left(\frac{240}{s}\right)$$

$$\therefore\quad H(s) = \frac{800{,}000}{(s+800)(s+3200)}$$

$$H(j1600) = \frac{8\times 10^5}{(800+j1600)(3200+j1600)} = 0.125\underline{/-90^\circ}$$

$$\therefore\quad v_o(t) = (40)(0.125)\cos(1600t - 90^\circ)\text{ V} = 5\sin 1600t\text{ V}$$

P 13.78 Original charge on C_1; $q_1 = V_0C_1$

The charge transferred to C_2; $q_2 = V_0C_e = \dfrac{V_0C_1C_2}{C_1+C_2}$

The charge remaining on C_1; $q_1' = q_1 - q_2 = \dfrac{V_0C_1^2}{C_1+C_2}$

Therefore $V_2 = \dfrac{q_2}{C_2} = \dfrac{V_0C_1}{C_1+C_2}$ and $V_1 = \dfrac{q_1'}{C_1} = \dfrac{V_0C_1}{C_1+C_2}$

P 13.79 [a] $Z_1 = \dfrac{1/C_1}{s+1/R_1C_1} = \dfrac{20\times 10^{10}}{s+20\times 10^4}\Omega$

$$Z_2 = \frac{1/C_2}{s+1/R_2C_2} = \frac{5\times 10^{10}}{s+12{,}500}\Omega$$

$$\frac{V_0}{Z_2} + \frac{V_0 - 10/s}{Z_1} = 0$$

$$\frac{V_0(s+12{,}500)}{5\times 10^{10}} + \frac{V_0(s+20\times 10^4)}{20\times 10^{10}} = \frac{10}{s}\,\frac{(s+20\times 10^4)}{20\times 10^{10}}$$

$$V_0 = \frac{2(s+200{,}000)}{s(s+50{,}000)} = \frac{K_1}{s} + \frac{K_2}{s+50{,}000}$$

$$K_1 = \frac{2(200{,}000)}{50{,}000} = 8$$

$$K_2 = \frac{2(150{,}000)}{-50{,}000} = -6$$

$$\therefore\quad v_o = [8 - 6e^{-50{,}000t}]u(t)\text{ V}$$

[b] $I_0 = \frac{V_0}{Z_2} = \frac{2(s+200{,}000)(s+12{,}500)}{s(s+50{,}000)5\times 10^{10}}$

$$= 40\times 10^{-12}\left[1+\frac{162{,}500s+25\times 10^8}{s(s+50{,}000)}\right]$$

$$= 40\times 10^{-12}\left[1+\frac{K_1}{s}+\frac{K_2}{s+50{,}000}\right]$$

$K_1 = 50{,}000; \qquad K_2 = 112{,}500$

$i_o = 40\delta(t) + [2\times 10^6 + 4.5\times 10^6 e^{-50{,}000t}]u(t)\, pA$

[c] When $C_1 = 80\,\text{pF}$

$$Z_1 = \frac{125\times 10^8}{s+12{,}500}\Omega$$

$$\frac{V_0(s+12{,}500)}{500\times 10^8}+\frac{V_0(s+12{,}500)}{125\times 10^8} = \frac{10}{s}\frac{(s+12{,}500)}{125\times 10^8}$$

$$\therefore\quad V_0 + 4V_0 = \frac{40}{s}$$

$$V_0 = \frac{8}{s}$$

$v_o = 8u(t)\,\text{V}$

$$I_0 = \frac{V_0}{Z_2} = \frac{8}{s}\frac{(s+12{,}500)}{5\times 10^{10}} = 160\times 10^{-12}\left[1+\frac{12{,}500}{s}\right]$$

$i_o(t) = 160\delta(t) + 2\times 10^{-6}u(t)\,\text{pA}$

P 13.80 Let $a = \frac{1}{R_1C_1} = \frac{1}{R_2C_2}$

Then $Z_1 = \frac{1}{C_1(s+a)}$ and $Z_2 = \frac{1}{C_2(s+a)}$

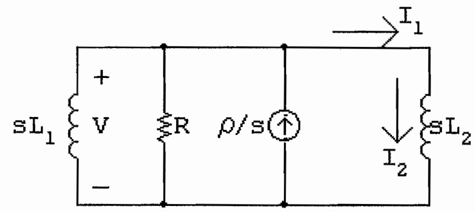
$$\frac{V_o}{Z_2}+\frac{V_o}{Z_1} = \frac{10/s}{Z_1}$$

$$V_oC_2(s+a) + V_0C_1(s+a) = (10/s)C_1(s+a)$$

$$V_o = \frac{10}{s}\left(\frac{C_1}{C_1+C_2}\right)$$

Thus, v_o is the input scaled by the factor $\frac{C_1}{C_1+C_2}$

P 13.81 [a] The s-domain circuit is



The node-voltage equation is $\frac{V}{sL_1} + \frac{V}{R} + \frac{V}{sL_2} = \frac{\rho}{s}$

Therefore $V = \frac{\rho R}{s + (R/L_e)}$ where $L_e = \frac{L_1 L_2}{L_1 + L_2}$

Therefore $v = \rho R e^{-(R/L_e)t} u(t)$ V

[b] $I_1 = \frac{V}{R} + \frac{V}{sL_2} = \frac{\rho[s + (R/L_2)]}{s[s + (R/L_e)]} = \frac{K_0}{s} + \frac{K_1}{s + (R/L_e)}$

$K_0 = \frac{\rho L_1}{L_1 + L_2}$; $K_1 = \frac{\rho L_2}{L_1 + L_2}$

Thus we have $i_1 = \frac{\rho}{L_1 + L_2}[L_1 + L_2 e^{-(R/L_e)t}]u(t)$ A

[c] $I_2 = \frac{V}{sL_2} = \frac{(\rho R/L_2)}{s[s + (R/L_e)]} = \frac{K_2}{s} + \frac{K_3}{s + (R/L_e)}$

$K_2 = \frac{\rho L_1}{L_1 + L_2}$; $K_3 = \frac{-\rho L_1}{L_1 + L_2}$

Therefore $i_2 = \frac{\rho L_1}{L_1 + L_2}[1 - e^{-(R/L_e)t}]u(t)$

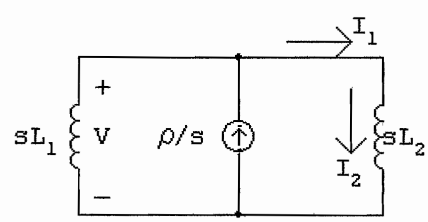
[d] $\lambda(t) = L_1 i_1 + L_2 i_2 = \rho L_1$

P 13.82 [a] As $R \to \infty$, $v(t) \to \rho L_e \delta(t)$ since the area under the impulse generating function is ρL_e.

$i_1(t) \to \frac{\rho L_1}{L_1 + L_2}$ as $R \to \infty$

$i_2(t) \to \frac{\rho L_1}{L_1 + L_2}$ as $R \to \infty$

[b] The s-domain circuit is



$\frac{V}{sL_1} + \frac{V}{sL_2} = \frac{\rho}{s}$; therefore $V = \frac{\rho L_1 L_2}{L_1 + L_2} = \rho L_e$

Therefore $v(t) = \rho L_e \delta(t)$

$$I_1 = I_2 = \frac{V}{sL_2} = \left(\frac{\rho L_1}{L_1 + L_2}\right)\left(\frac{1}{s}\right)$$

Therefore $i_1 = i_2 = \dfrac{\rho L_1}{L_1 + L_2} u(t)$ A

P 13.83 [a] For $t < 0$, $0.5v_1 = 2v_2$; therefore $v_1 = 4v_2$

$v_1 + v_2 = 100$; therefore $v_1(0^-) = 80$ V

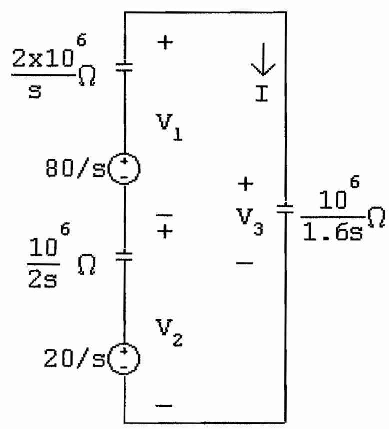
[b] $v_2(0^-) = 20$ V

[c] $v_3(0^-) = 0$ V

[d] For $t > 0$:

$$I = \frac{100/s}{3.125/s} \times 10^{-6} = 32 \times 10^{-6}$$

$i(t) = 32\delta(t)\,\mu$A



[e] $v_1(0^+) = -\dfrac{10^6}{0.5}\displaystyle\int_{0^-}^{0^+} 32 \times 10^{-6}\delta(t)\,dt + 80 = -64 + 80 = 16$ V

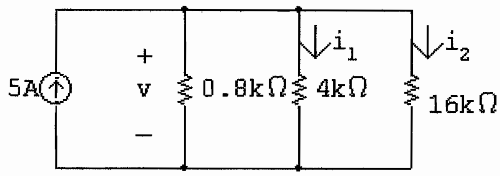
[f] $v_2(0^+) = -\dfrac{10^6}{2}\displaystyle\int_{0^-}^{0^+} 32 \times 10^{-6}\delta(t)\,dt + 20 = -16 + 20 = 4$ V

[g] $V_3 = \dfrac{0.625 \times 10^6}{s} \cdot 32 \times 10^{-6} = \dfrac{20}{s}$

$v_3(t) = 20u(t)$ V; $v_3(0^+) = 20$ V

Check: $v_1(0^+) + v_2(0^+) = v_3(0^+)$

P 13.84 [a] For $t < 0$:



$$R_{eq} = 0.8\,\text{k}\Omega \| 4\,\text{k}\Omega \| 16\,\text{k}\Omega = 0.64\,\text{k}\Omega; \qquad v = 5(640) = 3200\,\text{V}$$

$$i_1(0^-) = \frac{3200}{4000} = 0.8\,\text{A}; \qquad i_2(0^-) = \frac{3200}{1600} = 0.2\,\text{A}$$

[b] For $t > 0$:

$$i_1 + i_2 = 0$$

$$8(\Delta i_1) = 2(\Delta i_2)$$

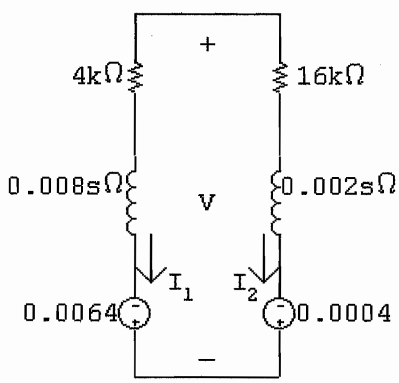
$$i_1(0^-) + \Delta i_1 + i_2(0^-) + \Delta i_2 = 0; \qquad \text{therefore} \quad \Delta i_1 = -0.2\,\text{A}$$

$$\Delta i_2 = -0.8\,\text{A}; \qquad i_1(0^+) = 0.8 - 0.2 = 0.6\,\text{A}$$

[c] $i_2(0^-) = 0.2\,\text{A}$

[d] $i_2(0^+) = 0.2 - 0.8 = -0.6\,\text{A}$

[e] The s-domain equivalent circuit for $t > 0$ is



$$I_1 = \frac{0.006}{0.01s + 20{,}000} = \frac{0.6}{s + 2 \times 10^6}$$

$$i_1(t) = 0.6e^{-2\times 10^6 t}u(t)\,\text{A}$$

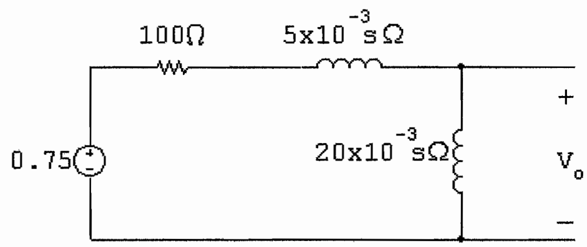
[f] $i_2(t) = -i_1(t) = -0.6e^{-2\times 10^6 t}u(t)\,\text{A}$

[g] $V = -0.0064 + (0.008s + 4000)I_1 = \dfrac{-0.0016(s + 6.5 \times 10^6)}{s + 2 \times 10^6}$

$$= -1.6 \times 10^{-3} - \frac{7200}{s + 2 \times 10^6}$$

$$v(t) = [-1.6 \times 10^{-3}\delta(t)] - [7200e^{-2\times 10^6 t}u(t)]\,\text{V}$$

P 13.85 [a]



$$V_o = \frac{0.75}{100 + 25 \times 10^{-3}s} \cdot 20 \times 10^{-3}s$$

$$= \frac{0.6s}{s + 4000} = 0.6 - \frac{2400}{s + 4000}$$

$$v_o(t) = 0.6\delta(t) - 2400e^{-4000t}u(t)\,\text{V}$$

[b] At $t = 0$ the voltage impulse establishes a current in the inductors; thus

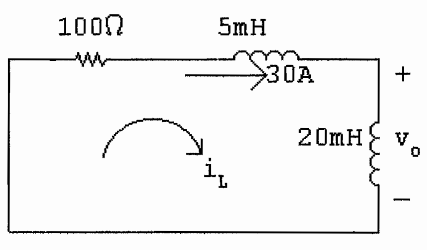
$$i_L(0) = \frac{10^3}{25}\int_{0^-}^{0^+} 750 \times 10^{-3}\delta(t)\,dt = 30\,\text{A}$$

It follows that since $i_L(0^-) = 0$ that

$$\frac{di_L}{dt}(0) = 30\delta(t)$$

$$\therefore \quad v_o(0) = (20 \times 10^{-3})(30\delta(t)) = 0.6\delta(t)$$

This agrees with our solution.
At $t = 0^+$ our circuit is



$$\therefore \quad i_L(t) = 30e^{-t/\tau}\,\text{A}, \qquad t \geq 0^+$$

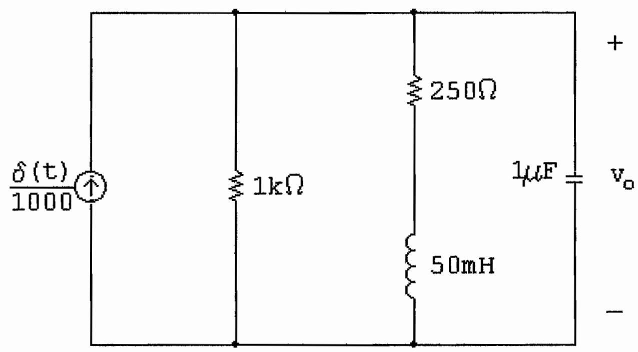
$$\tau = L/R = 0.25\,\text{ms}$$

$$\therefore \quad i_L(t) = 30e^{-4000t}\,\text{A}, \qquad t \geq 0^+$$

$$v_o(t) = 20 \times 10^{-3}\frac{di_L}{dt} = -2400e^{-4000t}\,\text{V}, \qquad t \geq 0^+$$

which agrees with our solution.

P 13.86 [a] After making a source transformation, the circuit is as shown. The impulse current will pass through the capacitive branch since it appears as a short circuit to the impulsive current,



$$\text{Therefore} \quad v_o(0^+) = 10^6\int_{0^-}^{0^+}\left[\frac{\delta(t)}{1000}\right]dt = 1000\,\text{V}$$

$$\text{Therefore} \quad w_C = (0.5)Cv^2 = 0.5\,\text{J}$$

[b] $i_L(0^+) = 0;$ therefore $w_L = 0\,\text{J}$

[c] $V_o(10^{-6})s + \dfrac{V_o}{250 + 0.05s} + \dfrac{V_o}{1000} = 10^{-3}$

Therefore

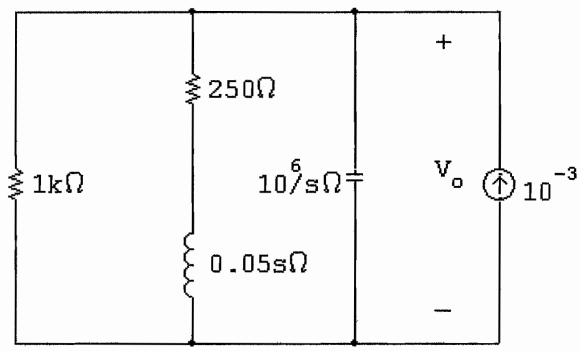
$$V_o = \frac{1000(s + 5000)}{s^2 + 6000s + 25 \times 10^6}$$

$$= \frac{K_1}{s + 3000 - j4000} + \frac{K_1^*}{s + 3000 + j4000}$$

$$K_1 = 559.02\underline{/-26.57^\circ}; \qquad K_1^* = 559.02\underline{/26.57^\circ}$$

$$v_o = [1118.03e^{-3000t}\cos(4000t - 26.57^\circ)]u(t)\,\text{V}$$

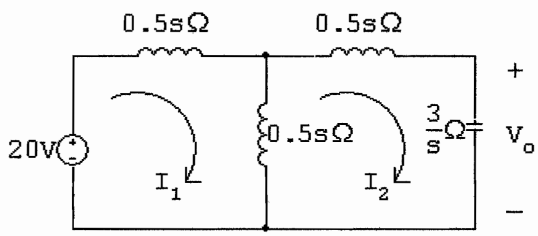
[d] The s-domain circuit is



$$\frac{V_o s}{10^6} + \frac{V_o}{250 + 0.05s} + \frac{V_o}{1000} = 10^{-3}$$

Note that this equation is identical to that derived in part [c], therefore the solution for V_o will be the same.

P 13.87 [a]



$$20 = sI_1 - 0.5sI_2$$

$$0 = -0.5sI_1 + \left(s + \frac{3}{s}\right) I_2$$

$$\Delta = \begin{vmatrix} s & -0.5s \\ -0.5s & (s + 3/s) \end{vmatrix} = s^2 + 3 - 0.25s^2 = 0.75(s^2 + 4)$$

$$N_1 = \begin{vmatrix} 20 & -0.5s \\ 0 & (s + 3/s) \end{vmatrix} = 20s + \frac{60}{s} = \frac{20s^2 + 60}{s} = \frac{20(s^2 + 3)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{20(s^2 + 3)}{s(0.75)(s^2 + 4)} = \frac{80}{3} \cdot \frac{s^2 + 3}{s(s^2 + 4)}$$

$$= \frac{K_0}{s} + \frac{K_1}{s - j2} + \frac{K_1^*}{s + j2}$$

$$K_0 = \frac{80}{3}\left(\frac{3}{4}\right) = 20; \qquad K_1 = \frac{80}{3}\left[\frac{-4 + 3}{(j2)(j4)}\right] = \frac{10}{3}\underline{/0^\circ}$$

$$\therefore \quad i_1 = \left[20 + \frac{20}{3}\cos 2t\right] u(t)\ \text{A}$$

[b] $$N_2 = \begin{vmatrix} s & 20 \\ -0.5s & 0 \end{vmatrix} = 10s$$

$$I_2 = \frac{N_2}{\Delta} = \frac{10s}{0.75(s^2+4)} = \frac{40}{3}\left(\frac{s}{s^2+4}\right) = \frac{K_1}{s-j2} + \frac{K_1^*}{s+j2}$$

$$K_1 = \frac{40}{3}\left(\frac{j2}{j4}\right) = \frac{20}{3}\underline{/0^\circ}$$

$$i_2 = \frac{40}{3}\cos 2tu(t)\ \text{A}$$

[c] $$V_0 = \frac{3}{s}I_2 = \left(\frac{3}{s}\right)\frac{40}{3}\left(\frac{s}{s^2+4}\right) = \frac{40}{s^2+4} = \frac{K_1}{s-j2} = \frac{K_1^*}{s+j2}$$

$$K_1 = \frac{40}{j4} = -j10 = 10\underline{/90^\circ}$$

$$v_o = 20\cos(2t - 90^\circ) = 20\sin 2t$$

$$v_o = [20\sin 2t]u(t)\ \text{V}$$

[d] Let us begin by noting i_1 jumps from 0 to (80/3) A between 0^- and 0^+ and in this same interval i_2 jumps from 0 to (40/3) A. Therefore in the derivatives of i_1 and i_2 there will be impulses of $(80/3)\delta(t)$ and $(40/3)\delta(t)$, respectively. Thus

$$\frac{di_1}{dt} = \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t\ \text{A/s}$$

$$\frac{di_2}{dt} = \frac{40}{3}\delta(t) - \frac{80}{3}\sin 2t\ \text{A/s}$$

From the circuit diagram we have

$$20\delta(t) = 1\frac{di_1}{dt} - 0.5\frac{di_2}{dt}$$

$$= \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t - \frac{20\delta(t)}{3} + \frac{40}{3}\sin 2t$$

$$= 20\delta(t)$$

Thus our solutions for i_1 and i_2 are in agreement with known circuit behavior.

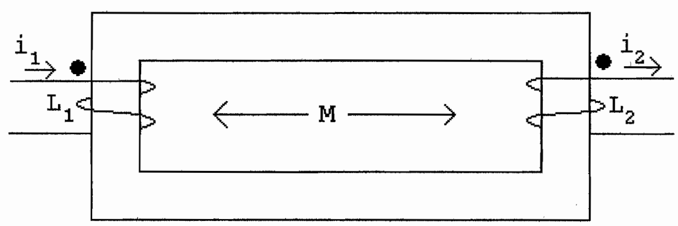
Let us also note the impulsive voltage will impart energy into the circuit. Since there is no resistance in the circuit, the energy will not dissipate.

Thus the fact that i_1, i_2, and v_o exist for all time is consistent with known circuit behavior.
Also note that although i_1 has a dc component, i_2 does not. This follows from known transformer behavior.
Finally we note the flux linkage prior to the appearance of the impulsive voltage is zero. Now since $v = d\lambda/dt$, the impulsive voltage source must be matched to an instantaneous change in flux linkage at $t = 0^+$ of 20.
For the given polarity dots and reference directions of i_1 and i_2 we have

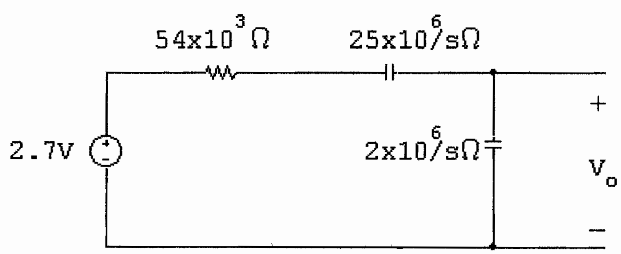
$$\lambda(0^+) = L_1 i_1(0^+) + M i_1(0^+) - L_2 i_2(0^+) - M i_2(0^+)$$

$$\lambda(0^+) = 1\left(\frac{80}{3}\right) + 0.5\left(\frac{80}{3}\right) - 1\left(\frac{40}{3}\right) - 0.5\left(\frac{40}{3}\right)$$

$$= \frac{120}{3} - \frac{60}{3} = 20 \quad \text{(checks)}$$



P 13.88 [a]



$$V_o = \frac{2.7}{54\times10^3 + 25\times10^6/s + 2\times10^6/s}\cdot\frac{2\times10^6}{s}$$

$$= \frac{5.4\times10^6}{54\times10^3 s + 27\times10^6} = \frac{100}{s+500}$$

$$v_o(t) = 100e^{-500t}u(t)\,\text{V}$$

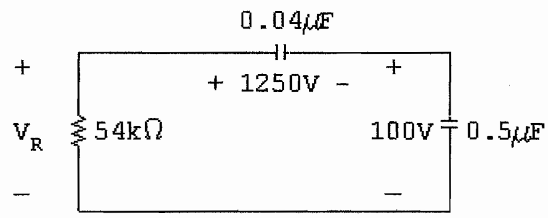
At $t = 0$ the impulsive current passes through the two capacitors. The voltage on the $0.04\,\mu$F capacitor at $t = 0^+$ is

$$v_{0.04} = 25\times10^6\int_{0^-}^{0^+} 50\times10^{-6}\delta(t)\,dt = 1250\,\text{V}$$

The voltage on the $0.5\,\mu$F capacitor at $t = 0^+$ is

$$v_{0.5} = 2\times10^6\int_{0^-}^{0^+} 50\times10^{-6}\delta(t)\,dt = 100\,\text{V}$$

Note this agrees with our solution.
At $t = 0^+$ the circuit is



The equivalent capacitance is

$$C_e = \frac{(0.04)(0.5) \times 10^{-12}}{0.54 \times 10^{-6}} = \frac{1}{27}\,\mu\text{F}$$

Thus, the time constant is

$$\tau = 54 \times 10^3 C_e = 2\,\text{ms}$$

Therefore, $1/\tau = 500$, which agrees with our solution.
It follows that

$$v_R(t) = 1350e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

Therefore

$$v_o(t) = \frac{0.04}{0.54}v_R = 100e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

which also agrees with our solution.

P 13.89 [a] The circuit parameters are

$$R_\text{a} = \frac{120^2}{1200} = 12\,\Omega \qquad R_\text{b} = \frac{120^2}{1800} = 8\,\Omega \qquad X_\text{a} = \frac{120^2}{350} = \frac{1440}{35}\,\Omega$$

The branch currents are

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{12} = 10\underline{/0^\circ}\text{ A(rms)} \qquad \mathbf{I}_2 = \frac{120\underline{/0^\circ}}{j1440/35} = -j\frac{35}{12} = \frac{35}{12}\underline{/-90^\circ}\text{ A(rms)}$$

$$\mathbf{I}_3 = \frac{120\underline{/0^\circ}}{8} = 15\underline{/0^\circ}\text{ A(rms)}$$

$$\therefore\ \mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 25 - j\frac{35}{12} = 25.17\underline{/-6.65^\circ}\text{ A(rms)}$$

Therefore,

$$i_2 = \left(\frac{35}{12}\right)\sqrt{2}\cos(\omega t - 90^\circ)\text{ A} \qquad \text{and} \qquad i_L = 25.17\sqrt{2}\cos(\omega t - 6.65^\circ)\text{ A}$$

Thus,

$$i_2(0^-) = i_2(0^+) = 0\,\text{A} \qquad \text{and} \qquad i_L(0^-) = i_L(0^+) = 25\sqrt{2}\,\text{A}$$

[b] Begin by using the s-domain circuit in Fig. 13.60 to solve for V_0 symbolically Write a single node voltage equation:

$$\frac{V_0 - (V_g + L_\ell I_o)}{sL_\ell} + \frac{V_0}{R_a} + \frac{V_0}{sL_a} = 0$$

$$\therefore \quad V_0 = \frac{(R_a/L_\ell)V_g + I_oR_a}{s + [R_a(L_a + L_\ell)]/L_aL_\ell}$$

where $L_\ell = 1/120\pi$ H, $L_a = 12/35\pi$ H, $R_a = 12\,\Omega$, and $I_0R_a = 300\sqrt{2}$ V. Thus,

$$V_0 = \frac{1440\pi(122.92\sqrt{2}s - 3000\pi\sqrt{2})}{(s + 1475\pi)(s^2 + 14{,}400\pi^2)} + \frac{300\sqrt{2}}{s + 1475\pi}$$

$$= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi} + \frac{300\sqrt{2}}{s + 1475\pi}$$

The coefficients are

$K_1 = -121.18\sqrt{2}\,\text{V} \qquad K_2 = 61.03\sqrt{2}\underline{/6.85^\circ}\,\text{V} \qquad K_2^* = 61.03\sqrt{2}\underline{/-6.85^\circ}$

Note that $K_1 + 300\sqrt{2} = 178.82\sqrt{2}$ V. Thus, the inverse transform of V_0 is

$v_0 = 178.82\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)\,\text{V}$

Initially,

$v_0(0^+) = 178.82\sqrt{2} + 122.06\sqrt{2}\cos 6.85^\circ = 300\sqrt{2}\,\text{V}$

Note that at $t = 0^+$ the initial value of i_L, which is $25\sqrt{2}$ A, exists in the $12\,\Omega$ resistor R_a. Thus, the initial value of V_0 is $(25\sqrt{2})(12) = 300\sqrt{2}$ V.

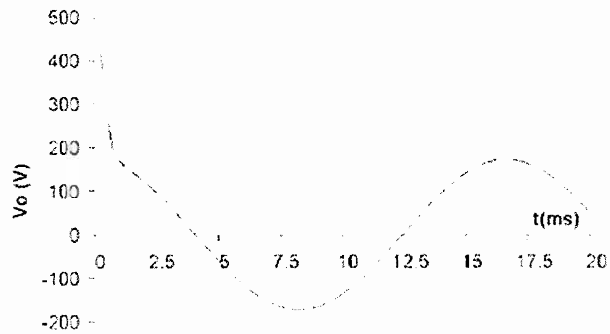
[c] The phasor domain equivalent circuit has a $j1\,\Omega$ inductive impedance in series with the parallel combination of a $12\,\Omega$ resistive impedance and a $j1440/35\,\Omega$ inductive impedance (remember that $\omega = 120\pi$ rad/s). Note that $\mathbf{V}_g = 120\underline{/0^\circ} + (25.17\underline{/-6.65^\circ})(j1) = 125.43\underline{/11.50^\circ}$ V(rms). The node voltage equation in the phasor domain circuit is

$$\frac{\mathbf{V}_0 - 125.43\underline{/11.50^\circ}}{j1} + \frac{\mathbf{V}_0}{12} + \frac{35\mathbf{V}_0}{1440} = 0$$

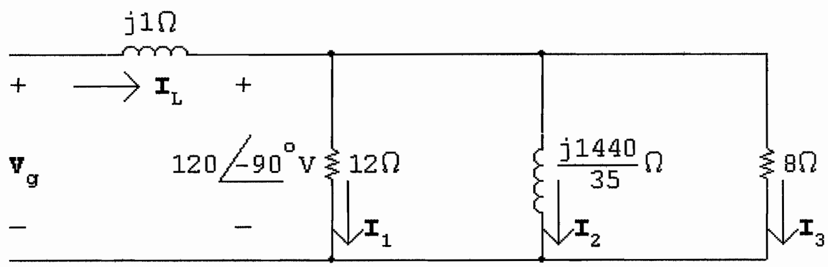
$$\therefore \quad \mathbf{V}_0 = 122.06\underline{/6.85^\circ}\ \text{V(rms)}$$

Therefore, $v_0 = 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)$ V, agreeing with the steady-state component of the result in part (b).

[d] A plot of v_0, generated in Excel, is shown below.



P 13.90 [a] At $t = 0^-$ the phasor domain equivalent circuit is



$$\mathbf{I}_1 = \frac{-j120}{12} = -j10 = 10\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{-j120(35)}{j1440} = -\frac{35}{12} = \frac{35}{12}\underline{/180^\circ}\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{-j120}{8} = -j15 = 15\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = -\frac{35}{12} - j25 = 25.17\underline{/-96.65^\circ}\text{A (rms)}$$

$$i_L = 25.17\sqrt{2}\cos(120\pi t - 96.65^\circ)\text{A}$$

$$i_L(0^-) = i_L(0^+) = -2.92\sqrt{2}\text{A}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t + 180^\circ)\text{A}$$

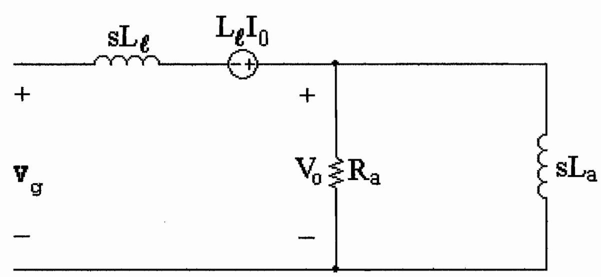
$$i_2(0^-) = i_2(0^+) = -\frac{35}{12}\sqrt{2} = -2.92\sqrt{2}\text{A}$$

$$\mathbf{V}_g = \mathbf{V}_o + j1\mathbf{I}_L$$

$$\mathbf{V}_g = -j120 + 25 - j\frac{35}{12}$$
$$= 25 - j122.92 = 125.43\underline{/-78.50^\circ}\text{V (rms)}$$
$$v_g = 125.43\sqrt{2}\cos(120\pi t - 78.50^\circ)\text{V}$$
$$= 125.43\sqrt{2}[\cos 120\pi t \cos 78.50^\circ + \sin 120\pi t \sin 78.50^\circ]$$
$$= 25\sqrt{2}\cos 120\pi t + 122.92\sqrt{2}\sin 120\pi t$$

$$\therefore \quad V_g = \frac{25\sqrt{2}s + 122.92\sqrt{2}(120\pi)}{s^2 + (120\pi)^2}$$

s-domain circuit:



where

$$L_l = \frac{1}{120\pi}\text{ H}; \qquad L_a = \frac{12}{35\pi}\text{ H}; \qquad R_a = 12\,\Omega$$

$$i_L(0) = -2.92\sqrt{2}\text{A}; \qquad i_2(0) = -2.92\sqrt{2}\text{A}$$

The node voltage equation is

$$0 = \frac{V_o - (V_g + i_L(0)L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + i_2(0)L_a}{sL_a}$$

Solving for V_o yields

$$V_o = \frac{V_gR_a/L_l}{[s + R_a(L_l + L_a)/L_aL_l]} + \frac{R_a[i_L(0) - i_2(0)]}{[s + R_a(L_l + L_a)/L_lL_a]}$$

$$\frac{R_a}{L_l} = 1440\pi$$

$$\frac{R_a(L_l + L_a)}{L_lL_a} = \frac{12(\frac{1}{120\pi} + \frac{12}{35\pi})}{(\frac{12}{35\pi})(\frac{1}{120\pi})} = 1475\pi$$

$$i_L(0) - i_2(0) = -2.92\sqrt{2} + 2.92\sqrt{2} = 0$$

$$\therefore \quad V_o = \frac{1440\pi[25\sqrt{2}s + 122.92\sqrt{2}(120\pi)]}{(s + 1475\pi)[s^2 + (120\pi)^2]}$$
$$= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}$$

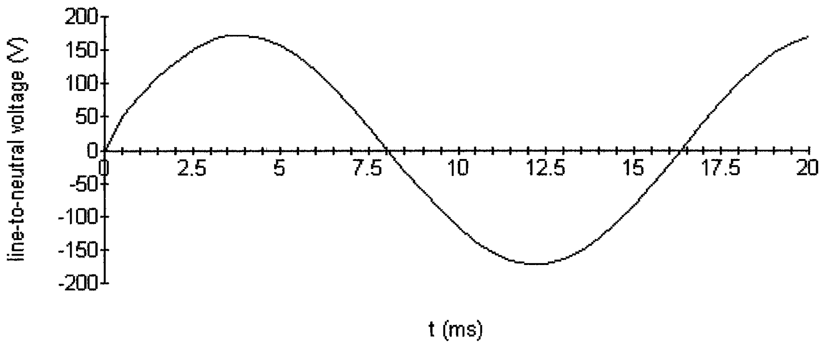
$K_1 = -14.55\sqrt{2}$ $\qquad$ $K_2 = 61.03\sqrt{2}/\underline{-83.15^\circ}$

$\therefore \quad v_o(t) = -14.55\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t - 83.15^\circ)\text{V}$

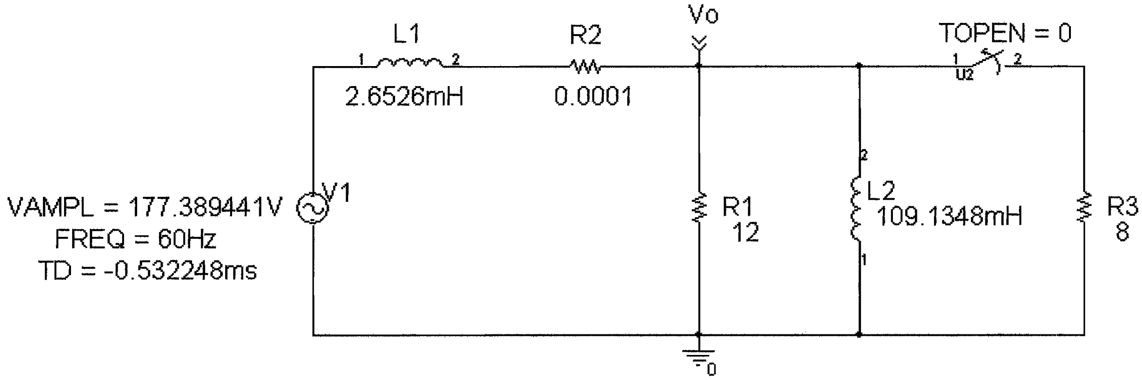
Check:

$v_o(0) = (-14.55 + 14.55)\sqrt{2} = 0$

[b]



PSpice schematic



PSpice output file

```
**** 07/15/01 07:40:45 ************** PSpice Lite (Mar 2000) *****************

 ** Profile: "SCHEMATIC1-tran"  [ C:\shortcircuits\solutions\p9_76-SCHEMATIC1-tran.sim ]


 ****     CIRCUIT DESCRIPTION


******************************************************************************




** Creating circuit file "p9_76-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

*Libraries:
* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
.TRAN  0 20ms 0
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_76-SCHEMATIC1.net"



**** INCLUDING p9_76-SCHEMATIC1.net ****
* source P9_76
V_V1          N00637 0
+SIN 0 177.389441V 60Hz -0.532248ms 0 0
L_L1          N00637 N01311  2.6526mH IC=0
L_L2          0 VO  109.1348mH IC=0
R_R1          0 VO  12
R_R2          VO N01311  0.0001
R_R3          0 N01959  8
X_U2          VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg

**** RESUMING p9_76-SCHEMATIC1-tran.sim.cir ****
.END
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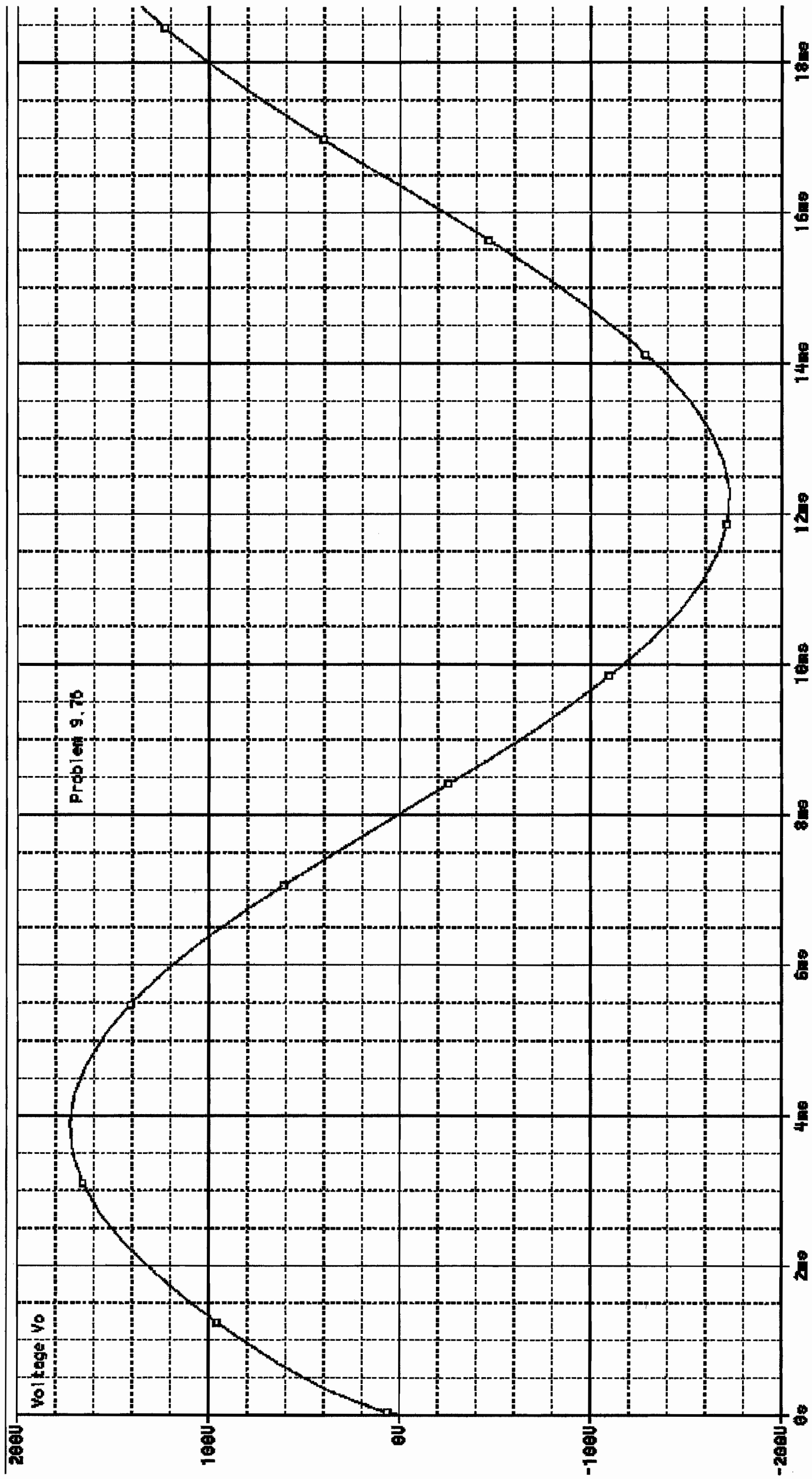
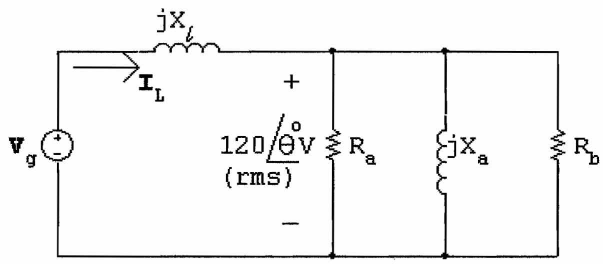

Voltage Vo
Problem 9.76
200V
100V
0V
-100V
-200V
0s
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
□ V(VO)

[c] In the Practical Perspective the line-to-neutral voltage spikes at $300\sqrt{2}$ V. In Prob. 13.89(c) the line-to-neutral voltage has no spike. Thus the amount of voltage disturbance depends on what part of the cycle the sinusoidal steady-state voltage is switched.

P 13.91 [a] First find V_g before R_b is disconnected. The phasor domain circuit is



$$\mathbf{I}_L = \frac{120\underline{/\theta^\circ}}{R_a} + \frac{120\underline{/\theta^\circ}}{R_b} + \frac{120\underline{/\theta^\circ}}{jX_a}$$
$$= \frac{120\underline{/\theta^\circ}}{R_a R_b X_a}[(R_a + R_b)X_a = jR_a R_b]$$

Since $X_l = 1\,\Omega$ we have

$$\mathbf{V}_g = 120\underline{/\theta^\circ} + \frac{120\underline{/\theta^\circ}}{R_a R_b X_a}[R_a R_b + j(R_a + R_b)X_a]$$

$$R_a = 12\,\Omega; \qquad R_b = 8\,\Omega; \qquad X_a = \frac{1440}{35}\,\Omega$$

$$\mathbf{V}_g = \frac{120\underline{/\theta^\circ}}{1400}(1475 + j300)$$
$$= \frac{25}{12}\underline{/\theta^\circ}(59 + j12) = 125.43\underline{/(\theta + 11.50)^\circ}$$

$$v_g = 125.43\sqrt{2}\cos(120\pi t + \theta + 11.50^\circ)\text{V}$$

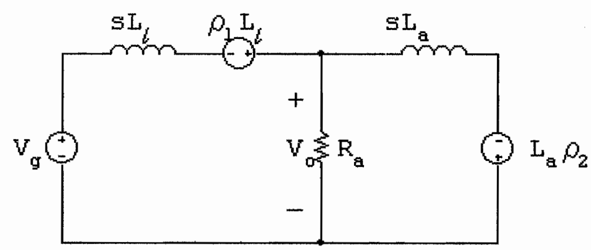
Let $\beta = \theta + 11.50^\circ$. Then

$$v_g = 125.43\sqrt{2}(\cos 120\pi t \cos\beta - \sin 120\pi t \sin\beta)\text{V}$$

Therefore

$$V_g = \frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + (120\pi)^2}$$

The s-domain circuit becomes



where $\rho_1 = i_L(0^+)$ and $\rho_2 = i_2(0^+)$.
The s-domain node voltage equation is

$$\frac{V_o - (V_g + \rho_1 L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + \rho_2 L_a}{sL_a} = 0$$

Solving for V_o yields

$$V_o = \frac{V_g R_a/L_l + (\rho_1 - \rho_2)R_a}{[s + \frac{(L_a+L_l)R_a}{L_a L_l}]}$$

Substituting the numerical values

$$L_l = \frac{1}{120\pi}\text{ H}; \qquad L_a = \frac{12}{35\pi}\text{ H}; \qquad R_a = 12\,\Omega; \qquad R_b = 8\,\Omega;$$

gives

$$V_o = \frac{1440\pi V_g + 12(\rho_1 - \rho_2)}{(s + 1475\pi)}$$

Now determine the values of ρ_1 and ρ_2.

$$\rho_1 = i_L(0^+) \quad \text{and} \quad \rho_2 = i_2(0^+)$$

$$\mathbf{I}_L = \frac{120\underline{/\theta^\circ}}{R_a R_b X_a}[(R_a + R_b)X_a - jR_aR_b]$$

$$= \frac{120\underline{/\theta^\circ}}{96(1440/35)}\left[\frac{(20)(1440)}{35} - j96\right]$$

$$= 25.17\underline{/(\theta - 6.65)^\circ}\text{A(rms)}$$

$$\therefore \quad i_L = 25.17\sqrt{2}\cos(120\pi t + \theta - 6.65^\circ)\text{A}$$

$$i_L(0^+) = \rho_1 = 25.17\sqrt{2}\cos(\theta - 6.65^\circ)\text{A}$$

$$\therefore \quad \rho_1 = 25\sqrt{2}\cos\theta + 2.92\sqrt{2}\sin\theta\text{A}$$

$$\mathbf{I}_2 = \frac{120\underline{/\theta^\circ}}{j(1440/35)} = \frac{35}{12}\underline{/(\theta - 90)^\circ}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t + \theta - 90^\circ)\text{A}$$

$$\rho_2 = i_2(0^+) = \frac{35}{12}\sqrt{2}\sin\theta = 2.92\sqrt{2}\sin\theta\text{A}$$

$$\therefore \quad \rho_1 = \rho_2 = 25\sqrt{2}\cos\theta$$

$$(\rho_1 - \rho_2)R_a = 300\sqrt{2}\cos\theta$$

$$\therefore \quad \text{V}_o = \frac{1440\pi}{s + 1475\pi}\cdot V_g + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{1440\pi}{s + 1475\pi}\left[\frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + 14{,}400\pi^2}\right] + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{K_1 + 300\sqrt{2}\cos\theta}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}$$

Now

$$K_1 = \frac{(1440\pi)(125.43\sqrt{2})[-1475\pi\cos\beta - 120\pi\sin\beta]}{1475^2\pi^2 + 14{,}400\pi^2}$$

$$= \frac{-1440(125.43\sqrt{2})[1475\cos\beta + 120\sin\beta]}{1475^2 + 14{,}000}$$

Since $\beta = \theta + 11.50^\circ$, K_1 reduces to

$$K_1 = -121.18\sqrt{2}\cos\theta + 14.55\sqrt{2}\sin\theta$$

From the partial fraction expansion for V_o we see $v_o(t)$ will go directly into steady state when $K_1 = -300\sqrt{2}\cos\theta$. It follow that

$$14.55\sqrt{2}\sin\theta = -178.82\sqrt{2}\cos\theta$$

or $\quad \tan\theta = -12.29$

Therefore, $\quad \theta = -85.35^\circ$

[b] When $\theta = -85.35^\circ$, $\beta = -73.85^\circ$

$$\therefore \quad K_2 = \frac{1440\pi(125.43\sqrt{2})[-120\pi\sin(-73.85^\circ) + j120\pi\cos(-73.85^\circ)}{(1475\pi + j120\pi)(j240\pi)}$$

$$= \frac{720\sqrt{2}(120.48 + j34.88)}{-120 + j1475}$$

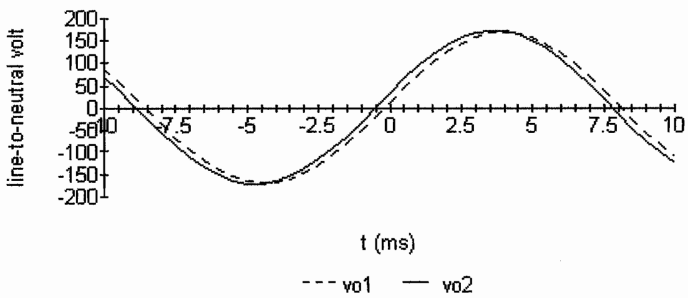
$$= 61.03\sqrt{2}\underline{/-78.50^\circ}$$

$$\therefore \quad v_o = 122.06\sqrt{2}\cos(120\pi t - 78.50^\circ)\text{V} \quad t > 0$$

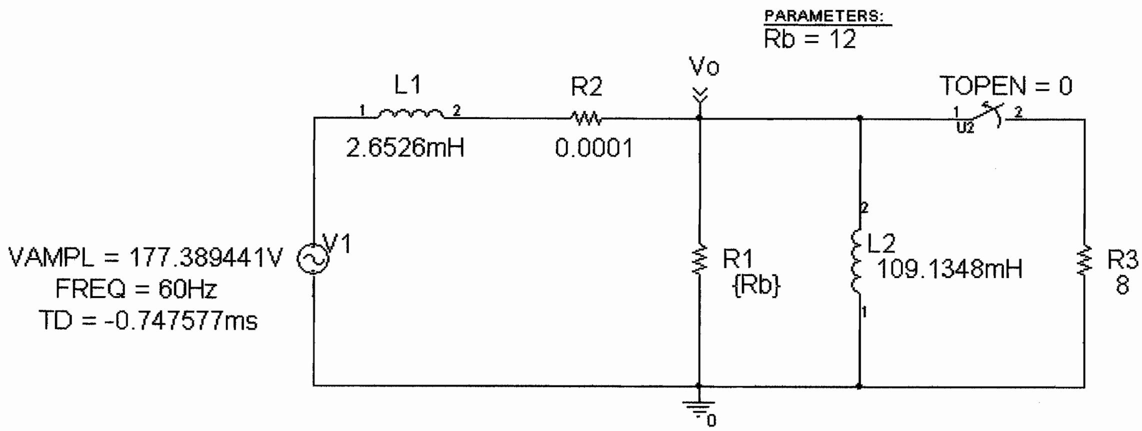
$$= 172.61\cos(120\pi t - 78.50^\circ)\text{V} \quad t > 0$$

[c] $v_{o1} = 169.71\cos(120\pi t - 85.35^\circ)\text{V} \qquad t < 0$

$v_{o2} = 172.61\cos(120\pi t - 78.50^\circ)\text{V} \qquad t > 0$



PSpice schematic



PSpice output file

```
** Creating circuit file "p9_77-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

*Libraries:
* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
.TRAN  0 20ms 0
.STEP PARAM Rb LIST 4.8  12
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_77-SCHEMATIC1.net"



**** INCLUDING p9_77-SCHEMATIC1.net ****
* source P9_77
V_V1         N00637 0
+SIN 0 177.389441V 60Hz -0.747577ms 0 0
L_L1         N00637 N01311  2.6526mH IC=0
L_L2         0 VO  109.1348mH IC=0
R_R1         0 VO  {Rb}
R_R2         VO N01311  0.0001
R_R3         0 N01959  8
X_U2         VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg
.PARAM  Rb=12

**** RESUMING p9_77-SCHEMATIC1-tran.sim.cir ****
.END
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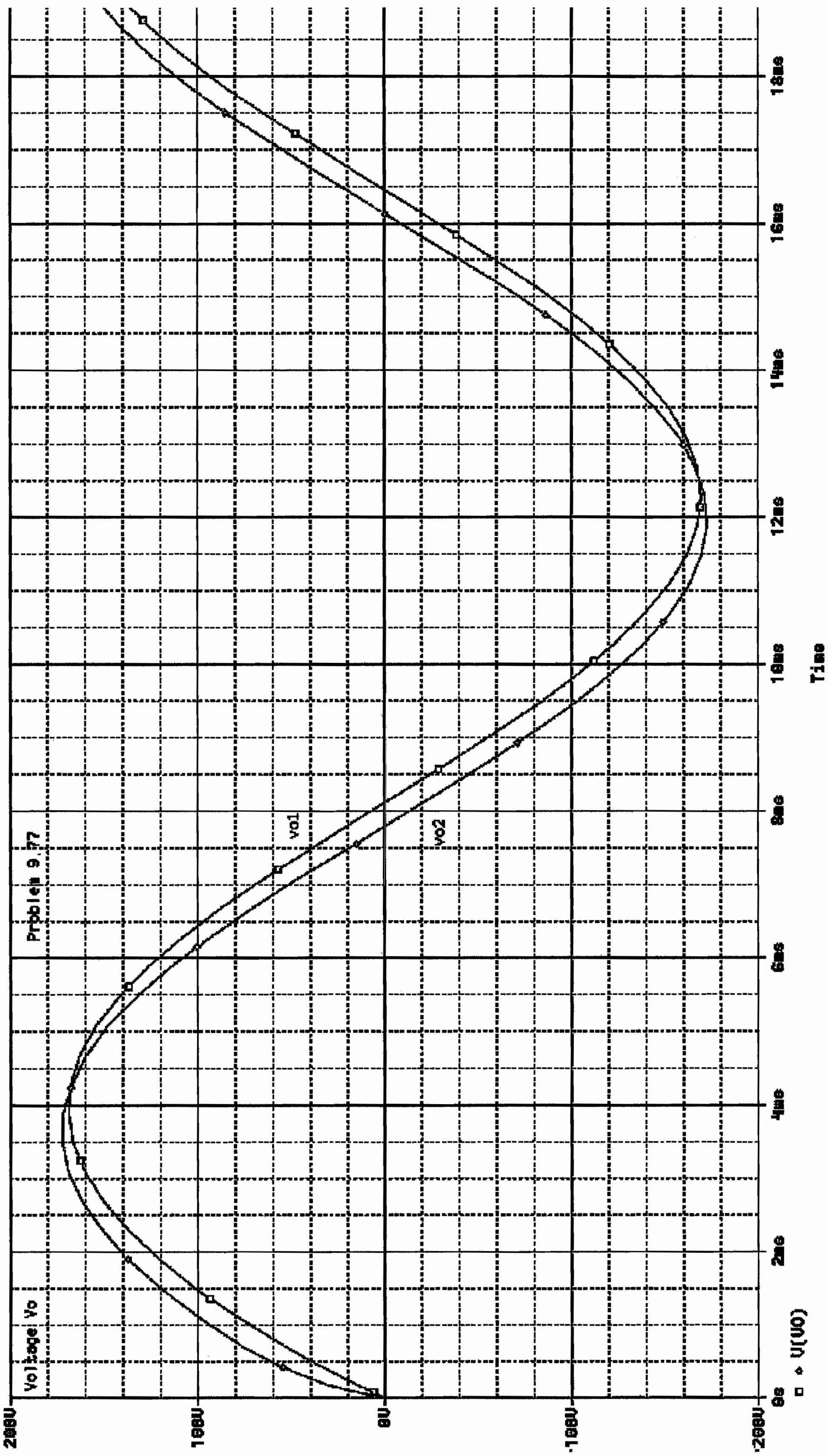

Voltage Vo
Problem 9.77
vo1
vo2
200V
100V
0V
-100V
-200V
0s
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
Time
□ ◇ V(VO)

14

Introduction to Frequency-Selective Circuits

Assessment Problems

AP 14.1

$$f_c = 8\,\text{kHz}, \quad \omega_c = 2\pi f_c = 16\pi\,\text{krad/s}$$

$$\omega_c = \frac{1}{RC}; \qquad R = 10\,\text{k}\Omega;$$

$$\therefore \quad C = \frac{1}{\omega_c R} = \frac{1}{(16\pi \times 10^3)(10^4)} = 1.99\,\text{nF}$$

AP 14.2 [a] $\omega_c = 2\pi f_c = 2\pi(2000) = 4\pi\,\text{krad/s}$

$$L = \frac{R}{\omega_c} = \frac{5000}{4000\pi} = 0.40\,\text{H}$$

[b] $H(j\omega) = \dfrac{\omega_c}{\omega_c + j\omega} = \dfrac{4000\pi}{4000\pi + j\omega}$

When $\omega = 2\pi f = 2\pi(50{,}000) = 100{,}000\pi\,\text{rad/s}$

$$H(j100{,}000\pi) = \frac{4000\pi}{4000\pi + j100{,}000\pi} = \frac{1}{1 + j25} = 0.04\underline{/87.71^\circ}$$

$$\therefore \quad |H(j100{,}000\pi)| = 0.04$$

[c] $\therefore \quad \theta(100{,}000\pi) = -87.71^\circ$

AP 14.3

$$\omega_c = \frac{R}{L} = \frac{5000}{3.5 \times 10^{-3}} = 1.43\,\text{Mrad/s}$$

AP 14.4 [a] $\omega_c = \dfrac{1}{RC} = \dfrac{10^6}{R} = \dfrac{10^6}{100} = 10\,\text{krad/s}$

[b] $\omega_c = \dfrac{10^6}{5000} = 200\,\text{rad/s}$

[c] $\omega_c = \dfrac{10^6}{3 \times 10^4} = 33.33\,\text{rad/s}$

AP 14.5 Let Z represent the parallel combination of $(1/SC)$ and R_L. Then

$$Z = \frac{R_L}{(R_L C s + 1)}$$

Thus $$H(s) = \frac{Z}{R+Z} = \frac{R_L}{R(R_L C s + 1) + R_L}$$

$$= \frac{(1/RC)}{s + \frac{R+R_L}{R_L}\left(\frac{1}{RC}\right)} = \frac{(1/RC)}{s + \frac{1}{K}\left(\frac{1}{RC}\right)}$$

where $$K = \frac{R_L}{R + R_L}$$

AP 14.6

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(24\pi \times 10^3)^2(0.1 \times 10^{-6})} = 1.76\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{R/L} \quad \text{so} \quad R = \frac{\omega_o L}{Q} = \frac{(24\pi \times 10^3)(1.76 \times 10^{-3})}{6} = 22.10\,\Omega$$

AP 14.7

$$\omega_o = 2\pi(2000) = 4000\pi\,\text{rad/s};$$

$$\beta = 2\pi(500) = 1000\pi\,\text{rad/s}; \qquad R = 250\,\Omega$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad C = \frac{1}{\beta R} = \frac{1}{(1000\pi)(250)} = 1.27\,\mu\text{F}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(4000\pi)^2(1.27)} = 4.97\,\text{mH}$$

AP 14.8

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(10^4\pi)^2(0.2 \times 10^{-6})} = 5.07\,\text{mH}$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad R = \frac{1}{\beta C} = \frac{1}{400\pi(0.2 \times 10^{-6})} = 3.98\,\text{k}\Omega$$

AP 14.9

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(400\pi)^2(0.2 \times 10^{-6})} = 31.66\,\text{mH}$$

$$Q = \frac{f_o}{\beta} = \frac{5 \times 10^3}{200} = 25 = \omega_o RC$$

$$\therefore \quad R = \frac{Q}{\omega_o C} = \frac{25}{(400\pi)(0.2 \times 10^{-6})} = 9.95\,\text{k}\Omega$$

AP 14.10

$$\omega_o = 8000\pi\,\text{rad/s}$$

$$C = 500\,\text{nF}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = 3.17\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$\therefore \quad R = \frac{1}{\omega_o CQ} = \frac{1}{(8000\pi)(500)(5 \times 10^{-9})} = 15.92\,\Omega$$

AP 14.11

$$\omega_o = 2\pi f_o = 2\pi(20{,}000) = 40\pi\,\text{krad/s}; \qquad R = 100\,\Omega; \qquad Q = 5$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{(R/L)} \quad \text{so} \quad L = \frac{QR}{\omega_o} = \frac{5(100)}{(40\pi \times 10^3)} = 3.98\,\text{mH}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad C = \frac{1}{\omega_o^2 L} = \frac{1}{(40\pi \times 10^3)^2(3.98 \times 10^{-3})} = 15.92\,\text{nF}$$

Problems

P 14.1 [a] $\omega_c = \frac{R}{L} = \frac{1.5 \times 10^3}{0.25} = 6000 \text{ rad/s}$

$$\therefore \quad f_c = \frac{6000}{2\pi} = 954.93 \text{ Hz}$$

[b] $H(s) = \frac{R/L}{s + R/L} = \frac{6000}{s + 6000}$

$$H(j\omega) = \frac{6000}{6000 + j\omega}$$

$$H(j\omega_c) = \frac{6000}{6000 + j6000} = 0.7071\underline{/-45^\circ}$$

$$H(j0.3\omega_c) = \frac{6000}{6000 + j1800} = 0.9578\underline{/-16.70^\circ}$$

$$H(j3\omega_c) = \frac{6000}{6000 + j18{,}000} = 0.3162\underline{/-71.57^\circ}$$

[c] $v_o(\omega_c) = 35.36\cos(6000t - 45^\circ) \text{ V}$

$$v_o(0.3\omega_c) = 47.89\cos(1800t - 16.70^\circ) \text{ V}$$

$$v_o(3\omega_c) = 15.81\cos(18{,}000t - 71.57^\circ) \text{ V}$$

P 14.2 [a] $\frac{R}{L} = 5000\pi \text{ rad/s}$

$$R = (0.025)(5000)(\pi) = 392.70\,\Omega$$

[b] $R_e = 392.70 \| 750 = 257.74\,\Omega$

$$\omega_{\text{loaded}} = \frac{R_e}{L} = 10{,}309.78 \text{ rad/s}$$

$$\therefore \quad f_{\text{loaded}} = 1640.85 \text{ Hz}$$

P 14.3 [a] $H(s) = \frac{V_o}{V_i} = \frac{R}{sL + R + R_l} = \frac{(R/L)}{s + (R + R_l)/L}$

[b] $H(j\omega) = \frac{(R/L)}{\left(\frac{R+R_l}{L}\right) + j\omega}$

$$|H(j\omega)| = \frac{(R/L)}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega^2}}$$

$|H(j\omega)|_{\text{max}}$ occurs when $\omega = 0$

[c] $|H(j\omega)|_{\text{max}} = \dfrac{R}{R + R_l}$

[d] $|H(j\omega_c)| = \dfrac{R}{\sqrt{2}(R + R_l)} = \dfrac{R/L}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega_c^2}}$

$$\therefore\ \omega_c^2 = \left(\frac{R + R_l}{L}\right)^2; \qquad \therefore\ \omega_c = (R + R_l)/L$$

[e] $\omega_c = \dfrac{1575}{0.25} = 6300\ \text{rad/s}$

$$H(j\omega) = \frac{6000}{6300 + j\omega}$$

$$H(j0) = 0.9524$$

$$H(j6300) = \frac{0.9524}{\sqrt{2}}\underline{/-45^\circ} = 0.6734\underline{/-45^\circ}$$

$$H(j1890) = \frac{6000}{6300 + j1890} = 0.9122\underline{/-16.70^\circ}$$

$$H(j18{,}900) = \frac{6000}{6300 + j18{,}900} = 0.3012\underline{/-71.57^\circ}$$

P 14.4 [a] $\omega_c = \dfrac{10^9}{80 \times 10^3} = 12{,}500\ \text{rad/s}$

$$f_c = 1989.44\,\text{Hz}$$

[b] $H(j\omega) = \dfrac{12{,}500}{12{,}500 + j\omega}$

$$\therefore\ H(j\omega_c) = 0.7071\underline{/-45^\circ}$$

$$H(j0.2\omega_c) = \frac{12{,}500}{12{,}500 + j2500} = 0.9806\underline{/-11.31^\circ}$$

$$H(j8\omega_c) = \frac{12{,}500}{12{,}500 + j100{,}000} = 0.1240\underline{/-82.87^\circ}$$

[c] $v_o(\omega_c) = 339.41\cos(12{,}500t - 45^\circ)\,\text{mV}$

$$v_o(0.2\omega_c) = 470.68\cos(2500t - 11.31^\circ)\,\text{mV}$$

$$v_o(8\omega_c) = 59.54\cos(100{,}000t - 82.87^\circ)\,\text{mV}$$

P 14.5 [a] Let $Z = \dfrac{R_L(1/SC)}{R_L + 1/SC} = \dfrac{R_L}{R_L Cs + 1}$

Then $H(s) = \dfrac{Z}{Z+R}$

$$= \frac{R_L}{RR_L Cs + R + R_L}$$

$$= \frac{(1/RC)}{s + \left(\dfrac{R+R_L}{RR_L C}\right)}$$

[b] $|H(j\omega)| = \dfrac{(1/RC)}{\sqrt{\omega^2 + [(R+R_L)/RR_L C]^2}}$

$|H(j\omega)|$ is maximum at $\omega = 0$

[c] $|H(j\omega)|_{\max} = \dfrac{R_L}{R+R_L}$

[d] $|H(j\omega_c)| = \dfrac{R_L}{\sqrt{2}(R+R_L)} = \dfrac{(1/RC)}{\sqrt{\omega_C^2 + [(R+R_L)/RR_L C]^2}}$

$$\therefore \quad \omega_c = \frac{R+R_L}{RR_L C} = \frac{1}{RC}(1 + (R/R_L))$$

[e] $\omega_c = 12{,}500\left(1 + \dfrac{20}{300}\right) = 13{,}333.33$ rad/s

$$H(j0) = \frac{300}{320} = 0.9375$$

$$H(j\omega_c) = \frac{12{,}500}{13{,}333.33 + j13{,}333.33} = 0.6629\underline{/-45^\circ}$$

$$H(j0.2\omega_c) = \frac{12{,}500}{13{,}333.33 + j2666.67} = 0.9193\underline{/-11.31^\circ}$$

$$H(j8\omega_c) = \frac{12{,}500}{13{,}333.33 + j106{,}666.67} = 0.1163\underline{/-82.87^\circ}$$

P 14.6 [a] $f_c = \dfrac{160}{2\pi} \times 10^3 = 25.46\,\text{kHz}$

[b] $\dfrac{1}{RC} = 160 \times 10^3$

$$R = \frac{1}{(160 \times 10^3)(25 \times 10^{-9})} = 250\,\Omega$$

[c] $\omega_c = \dfrac{1}{RC}\left(1 + \dfrac{R}{R_L}\right)$

$\therefore \quad \dfrac{R}{R_L} = 0.08 \qquad \therefore \quad R_L = 12.5R = 3125\,\Omega$

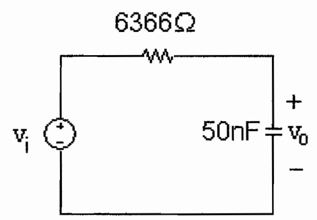
[d] $H(j0) = \dfrac{R_L}{R + R_L} = \dfrac{3125}{3375} = 0.9259$

$H(j0) = 0.9259$

P 14.7 [a] $\omega_c = 2\pi(500) = 3141.59$ rad/s

[b] $\omega_c = \dfrac{1}{RC}$ so $R = \dfrac{1}{\omega_c C} = \dfrac{1}{(3141.59)(50 \times 10^{-9})} = 6366\,\Omega$

[c]



[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{1/sC}{R + 1/sC} = \dfrac{1/RC}{s + 1/RC} = \dfrac{3141.59}{s + 3141.59}$

[e] $H(s) = \dfrac{V_o}{V_i} = \dfrac{(1/sC)\|R_L}{R + (1/sC)\|R_L} = \dfrac{1/RC}{s + \left(\dfrac{R + R_L}{R_L}\right) 1/RC} = \dfrac{3141.59}{s + 2(3141.59)}$

[f] $\omega_c = 2(3141.59) = 6283.19$ rad/s

[g] $H(0) = 1/2$

P 14.8 [a] $Z_L = j\omega L = j0L = 0$ so it is a short circuit

At $\omega = 0$, $V_o = V_i$

[b] $Z_L = j\omega L = j\infty L = \infty$ so it is an open circuit

At $\omega = \infty$, $V_o = 0$

[c] This is a low pass filter, with a gain of 1 at low frequencies and a gain of 0 at high frequencies.

[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + sL} = \dfrac{R/L}{s + R/L}$

[e] $\omega_c = \dfrac{R}{L} = \dfrac{1000}{0.02} = 50$ krad/s

P 14.9 [a] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R\|R_L}{R\|R_L + sL} = \dfrac{\dfrac{R}{L}\left(\dfrac{R_L}{R + R_L}\right)}{s + \dfrac{R}{L}\left(\dfrac{R_L}{R + R_L}\right)}$

[b] $\omega_{c(UL)} = \dfrac{R}{L}; \qquad \omega_{c(L)} = \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right)$ so the cutoff frequencies are different

$H(0)_{(UL)} = 1; \qquad H(0)_{(L)} = 1$ so the passband gains are the same

[c] $\omega_{c(UL)} = 50{,}000$ rad/s

$\omega_{c(L)} = 50{,}000 - 0.1(50{,}000) = 45{,}000$ rad/s

$$45{,}000 = \frac{1000}{0.02}\left(\frac{R_L}{1000+R_L}\right) \quad \text{so} \quad \frac{R_L}{1000+R_L} = 0.9$$

$\therefore \quad 0.1R_L = 900 \quad$ so $\quad R_L \geq 9\,\text{k}\Omega$

P 14.10 [a] $\dfrac{1}{RC} = \dfrac{10^9}{(40\times 10^3)(2.5)} = 10\,\text{krad/s}$

$$f_c = \frac{5000}{\pi} = 1591.55\,\text{Hz}$$

[b] $H(j\omega) = \dfrac{j\omega}{10{,}000 + j\omega}$

$$H(j\omega_c) = \frac{j10{,}000}{10{,}000 + j10{,}000} = 0.7071\underline{/45^\circ}$$

$$H(j0.1\omega_c) = \frac{j1000}{10{,}000 + j1000} = 0.0995\underline{/84.29^\circ}$$

$$H(j10\omega_c) = \frac{j100{,}000}{10{,}000 + j100{,}000} = 0.9950\underline{/5.71^\circ}$$

[c] $v_o(\omega_c) = 565.69\cos(10{,}000t + 45^\circ)\,\text{mV}$

$v_o(0.1\omega_c) = 79.60\cos(1000t + 84.29^\circ)\,\text{mV}$

$v_o(10\omega_c) = 796.03\cos(100{,}000t + 5.71^\circ)\,\text{mV}$

P 14.11 [a] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + R_c + (1/sC)}$

$$= \frac{R}{R+R_c}\cdot\frac{s}{[s + (1/(R+R_c)C)]}$$

[b] $H(j\omega) = \dfrac{R}{R+R_c}\cdot\dfrac{j\omega}{j\omega + (1/(R+R_c)C)}$

$$|H(j\omega)| = \frac{R}{R+R_c}\cdot\frac{\omega}{\sqrt{\omega^2 + \frac{1}{(R+R_c)^2C^2}}}$$

The magnitude will be maximum when $\omega = \infty$

[c] $|H(j\omega)|_{\max} = \dfrac{R}{R+R_c}$

[d] $|H(j\omega_c)| = \dfrac{R\omega_c}{(R+R_c)\sqrt{\omega_c^2 + [1/(R+R_c)C]^2}}$

$$\therefore \quad |H(j\omega)| = \frac{R}{\sqrt{2}(R+R_c)} \qquad \text{when}$$

$$\therefore \quad \omega_c^2 = \frac{1}{(R+R_c)^2C^2}$$

$$\text{or } \omega_c = \frac{1}{(R+R_c)C}$$

[e] $\omega_c = \dfrac{1}{(R+R_C)C} = \dfrac{10^9}{(50\times 10^3)(2.5)} = 8000 \text{ rad/s}$

$$H(j\omega_c) = \left(\frac{40}{50}\right)\frac{j8000}{8000+j8000} = 0.5657\underline{/45^\circ}$$

$$H(j0.1\omega_c) = \frac{(0.8)j800}{8000+j800} = 0.0796\underline{/84.29^\circ}$$

$$H(j10\omega_c) = \frac{(0.8)j80{,}000}{8000+j80{,}000} = 0.7960\underline{/5.71^\circ}$$

P 14.12 [a] $\dfrac{1}{RC} = 2\pi(800) = 1600\pi \text{ rad/s}$

$$\therefore \quad R = \frac{10^9}{(1600\pi)(20)} = 9.95\,\text{k}\Omega$$

[b] $R_e = 9.95\|68 = 8.68\,\text{k}\Omega$

$$\omega_c = \frac{10^9}{(8.68)(10^3)(20)} = 5761.84 \text{ rad/s}$$

$$f_c = \frac{5761.84}{2\pi} = 917.03\,\text{Hz}$$

P 14.13 [a] $R = \omega_c L = (160\times 10^3)(25\times 10^{-3}) = 4000\,\Omega = 4\,\text{k}\Omega$

[b] $\dfrac{R}{L}\cdot\dfrac{R_L}{R+R_L} = 150{,}000$

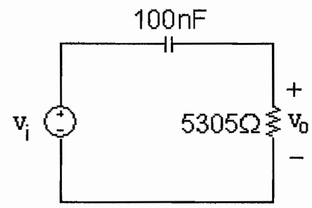
$$\therefore \quad \frac{R_L}{R+R_L} = \frac{150{,}000}{160{,}000} = 0.9375$$

$$\therefore \quad 0.0625R_L = (0.9375)(4000); \qquad \therefore \quad R_L = 60\,\text{k}\Omega$$

P 14.14 [a] $\omega_c = 2\pi(300) = 1884.96$ rad/s

[b] $\omega_c = \dfrac{1}{RC}$ so $R = \dfrac{1}{\omega_c C} = \dfrac{1}{(1884.96)(100 \times 10^{-9})} = 5305\,\Omega$

[c]



[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + 1/sC} = \dfrac{s}{s + 1/RC} = \dfrac{s}{s + 1884.96}$

[e] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R\|R_L}{R\|R_L + (1/sC)} = \dfrac{s}{s + \left(\dfrac{R + R_L}{R_L}\right) 1/RC} = \dfrac{s}{s + 2(1884.96)}$

[f] $\omega_c = 2(1884.96) = 3769.91$ rad/s

[g] $H(\infty) = 1$

P 14.15 [a] For $\omega = 0$, the inductor behaves as a short circuit, so $V_o = 0$.
For $\omega = \infty$, the inductor behaves as an open circuit, so $V_o = V_i$.
Thus, the circuit is a high pass filter.

[b] $H(s) = \dfrac{sL}{R + sL} = \dfrac{s}{s + R/L} = \dfrac{s}{s + 20{,}000}$

[c] $\omega_c = \dfrac{R}{L} = 20{,}000$ rad/s

[d] $|H(jR/L)| = \left|\dfrac{jR/L}{jR/L + R/L}\right| = \left|\dfrac{j}{j+1}\right| = \dfrac{1}{\sqrt{2}}$

P 14.16 [a] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R_L\|sL}{R + R_L\|sL} = \dfrac{s\left(\dfrac{R_L}{R + R_L}\right)}{s + \dfrac{R}{L}\left(\dfrac{R_L}{R + R_L}\right)}$

$$= \frac{\frac{1}{2}s}{s + \frac{1}{2}(20{,}000)}$$

[b] $\omega_c = \dfrac{R}{L}\left(\dfrac{R_L}{R + R_L}\right) = \dfrac{1}{2}(20{,}000) = 10{,}000$ rad/s

[c] $\omega_{c(L)} = \dfrac{1}{2}\omega_{c(UL)}$

[d] The gain in the passband is also reduced by a factor of 1/2 for the loaded filter.

P 14.17 By definition $Q = \omega_o/\beta$ therefore $\beta = \omega_o/Q$. Substituting this expression into Eqs. 14.34 and 14.35 yields

$$\omega_{c1} = -\frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

$$\omega_{c2} = \frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

Now factor ω_o out to get

$$\omega_{c1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{c2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

P 14.18 $\omega_o = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{(180)(200)} = 189.74\,\text{krad/s}$

$$f_o = \frac{\omega_o}{2\pi} = 30.20\,\text{kHz}$$

$$\beta = 200 - 180 = 20\,\text{krad/s} = 3.18\,\text{kHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{189.74}{20} = 9.49 = \frac{30.20}{3.18}$$

P 14.19 $\beta = \frac{\omega_o}{Q} = \frac{80}{8} = 10\,\text{krad/s} = \frac{5}{\pi} = 1.59\,\text{kHz}$

$$\omega_{c2} = 80\left[\frac{1}{16} + \sqrt{1 + \left(\frac{1}{16}\right)^2}\right] = 85.16\,\text{krad/s}$$

$$f_{c2} = \frac{85.16}{2\pi} = 13.55\,\text{kHz}$$

$$\omega_{c1} = 80\left[-\frac{1}{16} + \sqrt{1 + \left(\frac{1}{16}\right)^2}\right] = 75.16\,\text{krad/s}$$

$$f_{c1} = \frac{75.16}{2\pi} = 11.96\,\text{kHz}$$

P 14.20 [a] $L = \dfrac{1}{[2\pi(20{,}000)]^2(20 \times 10^{-9})} = 3.17\,\text{mH}$

$$R = \frac{\omega_o L}{Q} = \frac{40\pi \times 10^3(3.17 \times 10^{-3})}{5} = 79.58\,\Omega$$

[b] $f_{c1} = 20\left[-\dfrac{1}{10} + \sqrt{1 + \dfrac{1}{100}}\right] = 18.10\,\text{kHz}$

[c] $f_{c2} = 20\left[\dfrac{1}{10} + \sqrt{1 + \dfrac{1}{100}}\right] = 22.10\,\text{kHz}$

[d] $\beta = f_{c2} - f_{c1} = 4\,\text{kHz}$

or

$$\beta = \frac{f_o}{Q} = \frac{20}{5} = 4\,\text{kHz}$$

P 14.21 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{(10^6)(10^9)}{(40)(25)} = 10^{12}$

$$\omega_o = 10^6\ \text{rad/s} = 1\,\text{Mrad/s}$$

[b] $f_o = \dfrac{500}{\pi}\,\text{kHz} = 159.15\,\text{kHz}$

[c] $Q = \omega_o RC = (10^6)(300)(25 \times 10^{-9}) = 7.5$

[d] $\omega_{c1} = 10^6\left[-\dfrac{1}{15} + \sqrt{1 + \dfrac{1}{225}}\right] = 935.55\,\text{krad/s}$

[e] $\therefore\ f_{c1} = 148.90\,\text{kHz}$

[f] $\omega_{c2} = 10^6\left[\dfrac{1}{15} + \sqrt{1 + \dfrac{1}{225}}\right] = 1068.89\,\text{krad/s}$

[g] $\therefore\ f_{c2} = 170.12\,\text{kHz}$

[h] $\beta = \dfrac{\omega_o}{Q} = 133.33\,\text{krad/s or } 21.22\,\text{kHz}$

P 14.22 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{L(25)} = 25 \times 10^8$

$$\therefore\ L = \frac{10^9}{625 \times 10^8} = 16\,\text{mH}; \qquad R = \frac{10 \times 10^9}{(50 \times 10^3)(25)} = 8\,\text{k}\Omega$$

[b] $\omega_{c2} = 50\left[\dfrac{1}{20} + \sqrt{1 + \dfrac{1}{400}}\right] = 52.56\,\text{krad/s}$

$$\therefore\ f_{c2} = 8.37\,\text{kHz}$$

$$\omega_{c1} = 50\left[-\frac{1}{20} + \sqrt{1 + \frac{1}{400}}\right] = 47.56\,\text{krad/s}$$

$$\therefore \quad f_{c1} = 7.57\,\text{kHz}$$

[c] $\beta = \dfrac{\omega_o}{Q} = 5000 \text{ rad/s} = 795.77\,\text{Hz}$

Check: $\beta = f_{c2} - f_{c1} = 795.77\,\text{Hz}$

P 14.23 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{(10^3)(10^{12})}{(312.5)(1.25)} = 2.56 \times 10^{12}$

$$\omega_o = 1.6 \times 10^6 \text{ rad/s}$$

$$f_o = \frac{800}{\pi} = 254.65\,\text{kHz}$$

[b] $Q = \dfrac{\omega_o L}{R + R_i} = \dfrac{(1.6 \times 10^6)(312.5 \times 10^{-3})}{(50 + 12.5)10^3} = 8$

[c] $f_{c1} = \dfrac{800}{\pi}\left[-\dfrac{1}{16} + \sqrt{1 + \dfrac{1}{256}}\right] = 239.23\,\text{kHz}$

[d] $f_{c2} = \dfrac{800}{\pi}\left[\dfrac{1}{16} + \sqrt{1 + \dfrac{1}{256}}\right] = 271.06\,\text{kHz}$

[e] $\beta = f_{c2} - f_{c1} = 31.83\,\text{kHz}$

or

$$\beta = \frac{\omega_o}{Q} = 200\,\text{krad/s} = \frac{100}{\pi}\,\text{kHz} = 31.83\,\text{kHz}$$

P 14.24 [a] $H(s) = \dfrac{(R/L)s}{s^2 + \frac{(R+R_i)}{L}s + \frac{1}{LC}}$

For the numerical values in Problem 14.23 we have

$$H(s) = \frac{16 \times 10^4 s}{s^2 + 2 \times 10^5 s + 2.56 \times 10^{12}}$$

$$\therefore \quad H(j\omega) = \frac{j16 \times 10^4 \omega}{(2.56 \times 10^{12} - \omega^2) + j2 \times 10^5 \omega}$$

$$H(j\omega_o) = \frac{j16 \times 10^4 (1.6 \times 10^6)}{j2 \times 10^5 (1.6 \times 10^6)} = 0.8\underline{/0^\circ}$$

$$\therefore \quad v_o(t) = 640 \cos \omega t\,\text{mV}$$

[b] $\omega_{c1} = 1.6 \times 10^6 \left[-\frac{1}{16} + \sqrt{1 + \frac{1}{256}} \right] = 1.5 \times 10^6$ rad/s

$$H(j\omega_{c1}) = \frac{j16 \times 10^4 (1.5 \times 10^6)}{2.56 \times 10^{12} - 1.5^2 \times 10^{12} + j2 \times 10^5 (1.5 \times 10^6)}$$

$$= 0.57\underline{/45^\circ}$$

$\therefore \quad v_o(t) = 452.55 \cos(1.5 \times 10^6 t + 45^\circ)$ mV

[c] $\omega_{c2} = 1.6 \times 10^6 \left[\frac{1}{16} + \sqrt{1 + \frac{1}{256}} \right] = 1.7 \times 10^6$ rad/s

$$H(j\omega_{c2}) = \frac{j16 \times 10^4 (1.7 \times 10^6)}{2.56 \times 10^{12} - 1.7^2 \times 10^{12} + j2 \times 10^5 (1.7 \times 10^6)}$$

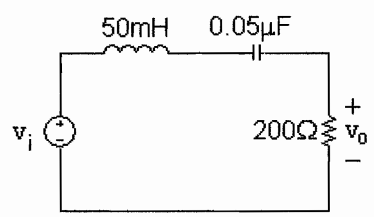
$$= 0.57\underline{/45^\circ}$$

$\therefore \quad v_o(t) = 452.55 \cos(1.7 \times 10^6 t - 45^\circ)$ mV

P 14.25 [a] $\omega_o = \sqrt{1/LC}$ so $L = \frac{1}{\omega_o^2 C} = \frac{(20{,}000)^2}{(50 \times 10^{-9})} = 50$ mH

$Q = \frac{\omega_o}{\beta}$ so $\beta = \frac{\omega o}{Q} = \frac{20{,}000}{5} = 4000$ rad/s

$\beta = \frac{R}{L}$ so $R = L\beta = (50 \times 10^{-3})(4000) = 200\,\Omega$



[b] $\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm\frac{4000}{2} + \sqrt{\left(\frac{4000}{2}\right)^2 + 20{,}000^2} = \pm 2000 + 20{,}099.75$

$\omega_{c1} = 18{,}099.75$ rad/s $\qquad \omega_{c2} = 22{,}099.75$ rad/s

P 14.26 $H(j\omega) = \frac{j\omega(4000)}{20{,}000^2 - \omega^2 + j\omega(4000)}$

[a] $H(j20{,}000) = \frac{j20{,}000(4000)}{20{,}000^2 - 20{,}000^2 + j(20{,}000)(4000)} = 1$

$V_o = (1)V_i \quad \therefore \quad v_o(t) = 200 \cos 20{,}000t$ mV

[b] $H(j18{,}099.75) = \dfrac{j18{,}099.75(4000)}{20{,}000^2 - 18{,}099.75^2 + j(18{,}099.75)(4000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ} V_i \quad \therefore \quad v_o(t) = 141.42\cos(18{,}099.75t + 45^\circ)\,\text{mV}$

[c] $H(j22{,}099.75) = \dfrac{j22{,}099.75(4000)}{20{,}000^2 - 22{,}099.75^2 + j(22{,}099.75)(4000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ} V_i \quad \therefore \quad v_o(t) = 141.42\cos(22{,}099.75t - 45^\circ)\,\text{mV}$

[d] $H(j2000) = \dfrac{j2000(4000)}{20{,}000^2 - 2000^2 + j(2000)(4000)} = 0.02\underline{/88.8^\circ}$

$V_o = 0.02\underline{/88.8^\circ} V_i \quad \therefore \quad v_o(t) = 4\cos(2000t + 88.8^\circ)\,\text{mV}$

[e] $H(j200{,}000) = \dfrac{j200{,}000(4000)}{20{,}000^2 - 200{,}000^2 + j(200{,}000)(4000)} = 0.02\underline{/-88.8^\circ}$

$V_o = 0.02\underline{/-88.8^\circ} V_i \quad \therefore \quad v_o(t) = 4\cos(200{,}000t - 88.8^\circ)\,\text{mV}$

P 14.27 $H(s) = 1 - \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \dfrac{s^2 + (1/LC)}{s^2 + (R/L)s + (1/LC)}$

$H(j\omega) = \dfrac{20{,}000^2 - \omega^2}{20{,}000^2 - \omega^2 + j\omega(4000)}$

[a] $H(j20{,}000) = \dfrac{20{,}000^2 - 20{,}000^2}{20{,}000^2 - 20{,}000^2 + j(20{,}000)(4000)} = 0$

$V_o = (0)V_i \quad \therefore \quad v_o(t) = 0\,\text{mV}$

[b] $H(j18{,}099.75) = \dfrac{20{,}000^2 - 18{,}099.75^2}{20{,}000^2 - 18{,}099.75^2 + j(18{,}099.75)(4000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ} V_i \quad \therefore \quad v_o(t) = 141.42\cos(18{,}099.75t - 45^\circ)\,\text{mV}$

[c] $H(j22{,}099.75) = \dfrac{20{,}000^2 - 22{,}099.75^2}{20{,}000^2 - 22{,}099.75^2 + j(22{,}099.75)(4000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ} V_i \quad \therefore \quad v_o(t) = 141.42\cos(22{,}099.75t + 45^\circ)\,\text{mV}$

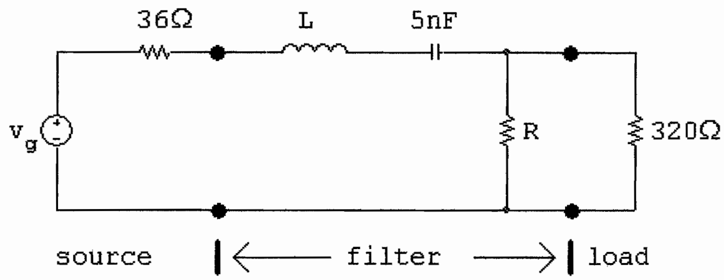
[d] $H(j2000) = \dfrac{20{,}000^2 - 2000^2}{20{,}000^2 - 2000^2 + j(2000)(4000)} = 0.9998\underline{/-1.16^\circ}$

$V_o = 0.9998\underline{/-1.16^\circ} V_i \quad \therefore \quad v_o(t) = 199.96\cos(2000t - 1.16^\circ)\,\text{mV}$

[e] $H(j200{,}000) = \dfrac{20{,}000^2 - 200{,}000^2}{20{,}000^2 - 200{,}000^2 + j(200{,}000)(4000)} = 0.9998\underline{/1.16^\circ}$

$V_o = 0.9998\underline{/1.16^\circ}V_i \quad \therefore \quad v_o(t) = 199.96\cos(200{,}000t + 1.16^\circ)\,\text{mV}$

P 14.28 [a]



[b] $L = \dfrac{1}{\omega_o^2 C} = \dfrac{10^9}{(625 \times 10^8)5} = 3.2 \times 10^{-3} = 3.2\,\text{mH}$

$R = \dfrac{\omega_o L}{Q} = \dfrac{800}{10} = 80\,\Omega$

[c] $R_e = 80\|320 = 64\,\Omega$

$R_e + R_i = 64 + 36 = 100\,\Omega$

$Q_{\text{system}} = \dfrac{\omega_o L}{R_e + R_i} = \dfrac{800}{100} = 8$

[d] $\beta_{\text{system}} = \dfrac{\omega_o}{Q_{\text{system}}} = \dfrac{250 \times 10^3}{8} = 31.25\,\text{krad/s}$

$\beta_{\text{system}}(\text{kHz}) = \dfrac{31.25}{2\pi} = 4.97\,\text{kHz} = 4973.59\,\text{Hz}$

P 14.29 [a] $\dfrac{V_o}{V_i} = \dfrac{Z}{Z+R}$ where $Z = \dfrac{1}{Y}$

and $Y = sC + \dfrac{1}{sL} + \dfrac{1}{R_L} = \dfrac{LCR_L s^2 + sL + R_L}{R_L L s}$

$$H(s) = \frac{R_L L s}{R_L R L C s^2 + (R + R_L)Ls + RR_L}$$

$$= \frac{(1/RC)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{\left(\frac{R_L}{R+R_L}\right)\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{K\beta s}{s^2 + \beta s + \omega_o^2}, \qquad K = \frac{R_L}{R + R_L}$$

[b] $\beta = \left(\frac{R+R_L}{R_L}\right)\frac{1}{RC}$

[c] $\beta_u = \frac{1}{RC}$

$$\therefore \quad \beta_L = \left(\frac{R+R_L}{R_L}\right)\beta_u = \left(1+\frac{R}{R_L}\right)\beta_u$$

[d] $Q = \frac{\omega_o}{\beta} = \frac{\omega_o RC}{\left(\frac{R+R_L}{R_L}\right)}$

[e] $Q_u = \omega_o RC$

$$\therefore \quad Q_L = \left(\frac{R_L}{R+R_L}\right)Q_u = \frac{1}{[1+(R/R_L)]}Q_u$$

[f] $H(j\omega) = \frac{Kj\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta}$

$$H(j\omega_o) = K$$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}} = \frac{K\omega_c\beta}{\sqrt{(\omega_o^2-\omega_c^2)^2+\omega_c^2\beta^2}}$$

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{\omega_c\beta}{\sqrt{(\omega_o^2-\omega_c^2)^2+\omega_c^2\beta^2}}$$

Squaring both sides leads to

$$(\omega_o^2-\omega_c^2)^2 = \omega_c^2\beta^2 \text{ or } (\omega_o^2-\omega_c^2) = \pm\omega_c\beta$$

$$\therefore \quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4}+\omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4}+\omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4}+\omega_o^2}$$

where

$$\beta = \left(1+\frac{R}{R_L}\right)\frac{1}{RC} \text{ and } \omega_o^2 = \frac{1}{LC}$$

P 14.30 [a] $\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^{12})}{(10)(4)} = 25\times10^{12}$

$$\omega_o = 5\,\text{Mrad/s}$$

[b] $\beta = \frac{R+R_L}{R_L} \cdot \frac{1}{RC} = \left(\frac{6.25}{5.0}\right)\left(\frac{10^{12}}{5\times 10^6}\right) = 250\,\text{krad/s}$

[c] $Q = \frac{\omega_o}{\beta} = \frac{5}{0.25} = 20$

[d] $H(j\omega_o) = \frac{R_L}{R+R_L} = 0.8\underline{/0^\circ}$

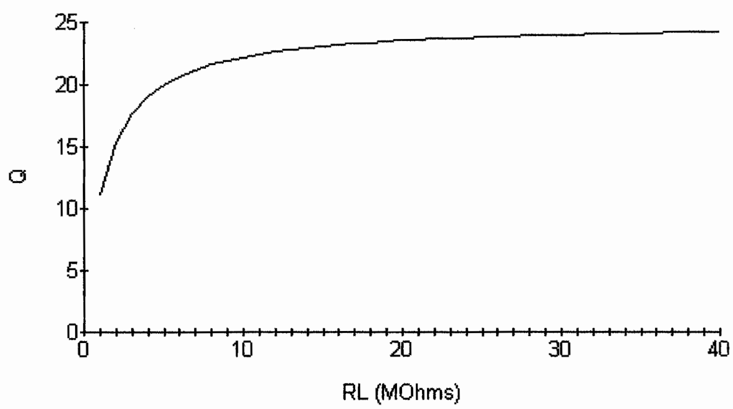
$\therefore\ v_o(t) = 600\cos(5\times 10^6 t)\,\text{mV}$

[e] $\beta = \left(1+\frac{R}{R_L}\right)\frac{1}{RC} = \left(1+\frac{1.25}{R_L}\right)(200\times 10^3)\ \text{rad/s}$

$\omega_o = 5\times 10^6\ \text{rad/s}$

$Q = \frac{\omega_o}{\beta} = \frac{25}{1+(1.25/R_L)}$ where R_L is in megohms

[f]



P 14.31 $\omega_o^2 = \frac{1}{LC} = \frac{(10^6)(10^{12})}{(400)(4)} = 625\times 10^{12}$

$\omega_o = 25\,\text{Mrad/s}$

$Q_u = \omega_o RC = (25\times 10^6)(100\times 10^3)(4\times 10^{-12}) = 10$

$\therefore\ \left(\frac{R_L}{R+R_L}\right)10 = 9; \qquad \therefore\ R_L = 9R = 900\,\text{k}\Omega$

P 14.32 [a] In analyzing the circuit qualitatively we visualize v_i is a sinusoidal voltage and we seek the steady-state nature of the output voltage v_o.
At zero frequency the inductor provides a direct connection between the input and the output, hence $v_o = v_i$ when $\omega = 0$.
At infinite frequency the capacitor provides the direct connection, hence $v_o = v_i$ when $\omega = \infty$.

At the resonant frequency of the parallel combination of L and C the impedance of the combination is infinite and hence the output voltage will be zero when $\omega = \omega_o$.
At frequencies on either side of ω_o the amplitude of the output voltage will be nonzero but less than the amplitude of the input voltage.
Thus the circuit behaves like a band-reject filter.

[b] Let Z represent the impedance of the parallel branches L and C, thus

$$Z = \frac{sL(1/sC)}{sL + 1/sC} = \frac{sL}{s^2LC + 1}$$

Then

$$H(s) = \frac{V_o}{V_i} = \frac{R}{Z + R} = \frac{R(s^2LC + 1)}{sL + R(s^2LC + 1)}$$

$$= \frac{[s^2 + (1/LC)]}{s^2 + \left(\frac{1}{RC}\right)s + \left(\frac{1}{LC}\right)}$$

$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}$$

[c] From part (b) we have

$$H(j\omega) = \frac{\omega_o^2 - \omega^2}{w_o^2 - \omega^2 + j\omega\beta}$$

It follows that $H(j\omega) = 0$ when $\omega = \omega_o$

$$\therefore \quad \omega_o = \frac{1}{\sqrt{LC}}$$

[d] $$|H(j\omega)| = \frac{\omega_o^2 - \omega^2}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{2}} \text{ when } \omega^2\beta^2 = (\omega_o^2 - \omega^2)^2$$

or $\pm\,\omega\beta = \omega_o^2 - \omega^2$, thus

$$\omega^2 \pm \beta\omega - \omega_o^2 = 0$$

The two positive roots of this quadratic are

$$\omega_{c_1} = \frac{-\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

$$\omega_{c_2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

Also note that since $\beta = \omega_o/Q$

$$\omega_{c_1} = \omega_o\left[\frac{-1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{c_2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

[e] It follows from the equations derived in part (d) that

$$\beta = \omega_{c_2} - \omega_{c_1} = 1/RC$$

[f] By definition $Q = \omega_o/\beta = \omega_o RC$

P 14.33 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{(10^6)(10^{12})}{(625)(25)} = 64 \times 10^{12}$

$$\therefore \quad \omega_o = 8\,\text{Mrad/s}$$

[b] $f_o = \dfrac{\omega_o}{2\pi} = 1.27\,\text{MHz}$

[c] $Q = \omega_o RC = (8 \times 10^6)(80 \times 10^3)(25 \times 10^{-12}) = 16$

[d] $\omega_{c1} = 8 \times 10^6\left[-\dfrac{1}{32} + \sqrt{1 + \dfrac{1}{1024}}\right] = 7.75\,\text{Mrad/s}$

[e] $f_{c1} = \dfrac{\omega_{c1}}{2\pi} = 1.234\,\text{MHz}$

[f] $\omega_{c2} = 8 \times 10^6\left[\dfrac{1}{32} + \sqrt{1 + \dfrac{1}{1024}}\right] = 8.25\,\text{Mrad/s}$

[g] $f_{c2} = \dfrac{\omega_{c1}}{2\pi} = 1.31\,\text{MHz}$

[h] $\beta = f_{c2} - f_{c1} = 79.58\,\text{kHz}$

or

$$\beta = \frac{\omega_o}{2\pi Q} = \frac{500 \times 10^3}{2\pi} = 79.58\,\text{kHz}$$

P 14.34 [a] $\omega_o = 2\pi f_o = 100\pi\,\text{krad/s}$

$$L = \frac{1}{\omega_o^2 C} = \frac{10^6}{10^4\pi \times 10^6(0.1)} = 101.32\,\mu\text{H}$$

$$R = \frac{Q}{\omega_o C} = \frac{8 \times 10^6}{(100\pi)(0.1 \times 10^3)} = 254.65\,\Omega$$

[b] $f_{c2} = 50\text{k}\left[\frac{1}{16} + \sqrt{1 + \frac{1}{256}}\right] = 53.22\,\text{kHz}$

$$f_{c1} = 50\text{k}\left[-\frac{1}{16} + \sqrt{1 + \frac{1}{256}}\right] = 46.97\,\text{kHz}$$

[c] $\beta = f_{c2} - f_{c1} = 6.25\,\text{kHz}$

or

$$\beta = \frac{f_o}{Q} = \frac{50\text{k}}{8} = 6.25\,\text{kHz}$$

P 14.35 [a] $R_e = 254.65 \| 932 = 200\,\Omega$

$$Q = \omega_o R_e C = 100\pi \times 10^3(200)(0.1)10^{-6} = 2\pi = 6.28$$

[b] $\beta = \frac{f_o}{Q} = \frac{50}{2\pi} = 7.96\,\text{kHz}$

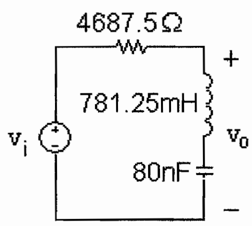
[c] $f_{c2} = 50\left[\frac{1}{4\pi} + \sqrt{1 + \frac{1}{16\pi^2}}\right] = 54.14\,\text{kHz}$

[d] $f_{c1} = 50\left[-\frac{1}{4\pi} + \sqrt{1 + \frac{1}{16\pi^2}}\right] = 46.18\,\text{kHz}$

P 14.36 [a] $\omega_o = \sqrt{1/LC}$ so $L = \frac{1}{\omega_o^2 C} = \frac{1}{(4000)^2(80 \times 10^{-9})} = 781.25\,\text{mH}$

$$Q = \frac{\omega_o}{\beta} \quad \text{so} \quad \beta = \frac{\omega o}{Q} = \frac{4000}{2/3} = 6000 \text{ rad/s}$$

$$\beta = \frac{R}{L} \quad \text{so} \quad R = L\beta = (781.25 \times 10^{-3})(6000) = 4687.5\,\Omega$$



[b] $\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm\frac{6000}{2} + \sqrt{\left(\frac{6000}{2}\right)^2 + 4000^2} = \pm 3000 + 5000$

$$\omega_{c1} = 2000 \text{ rad/s} \qquad \omega_{c2} = 8000 \text{ rad/s}$$

P 14.37 $H(j\omega) = \frac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2 + j\omega\beta} = \frac{4000^2 - \omega^2}{4000^2 - \omega^2 + j\omega(6000)}$

[a] $H(j4000) = \dfrac{4000^2 - 4000^2}{4000^2 - 4000^2 + j(4000)(6000)} = 0$

$V_o = (0)V_i \quad \therefore \quad v_o(t) = 0\,\text{mV}$

[b] $H(j2000) = \dfrac{4000^2 - 2000^2}{4000^2 - 2000^2 + j(2000)(6000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 88.39\cos(2000t - 45^\circ)\,\text{mV}$

[c] $H(j8000) = \dfrac{4000^2 - 8000^2}{4000^2 - 8000^2 + j(8000)(6000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 88.39\cos(8000t + 45^\circ)\,\text{mV}$

[d] $H(j400) = \dfrac{4000^2 - 400^2}{4000^2 - 400^2 + j(400)(6000)} = 0.989\underline{/-8.62^\circ}$

$V_o = 0.989\underline{/-8.62^\circ}V_i \quad \therefore \quad v_o(t) = 123.6\cos(400t - 8.62^\circ)\,\text{mV}$

[e] $H(j40{,}000) = \dfrac{4000^2 - 40{,}000^2}{4000^2 - 40{,}000^2 + j(40{,}000)(6000)} = 0.989\underline{/8.62^\circ}$

$V_o = 0.989\underline{/8.62^\circ}V_i \quad \therefore \quad v_o(t) = 123.6\cos(40{,}000t + 8.62^\circ)\,\text{mV}$

P 14.38 $H(j\omega) = \dfrac{j\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta} = \dfrac{j\omega(6000)}{4000^2 - \omega^2 + j\omega(6000)}$

[a] $H(j4000) = \dfrac{j(4000)(6000)}{4000^2 - 4000^2 + j(4000)(6000)} = 1$

$V_o = (1)V_i \quad \therefore \quad v_o(t) = 125\cos 4000t\,\text{mV}$

[b] $H(j2000) = \dfrac{j(2000)(6000)}{4000^2 - 2000^2 + j(2000)(6000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 88.39\cos(2000t + 45^\circ)\,\text{mV}$

[c] $H(j8000) = \dfrac{j(8000)(6000)}{4000^2 - 8000^2 + j(8000)(6000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 88.39\cos(8000t - 45^\circ)\,\text{mV}$

[d] $H(j400) = \dfrac{j(400)(6000)}{4000^2 - 400^2 + j(400)(6000)} = 0.15\underline{/81.4^\circ}$

$V_o = 0.15\underline{/81.4^\circ}V_i \quad \therefore \quad v_o(t) = 18.73\cos(400t + 81.4^\circ)\,\text{mV}$

[e] $H(j40{,}000) = \dfrac{j(40{,}000)(6000)}{4000^2 - 40{,}000^2 + j(40{,}000)(6000)} = 0.15\underline{/-81.4^\circ}$

$V_o = 0.15\underline{/-81.4^\circ}V_i \quad \therefore \quad v_o(t) = 18.73\cos(40{,}000t - 81.4^\circ)\,\text{mV}$

P 14.39 [a] Let $Z = \dfrac{R_L(sL + (1/sC))}{R_L + sL + (1/sC)}$

$$Z = \frac{R_L(s^2LC + 1)}{s^2LC + R_LCs + 1}$$

Then $H(s) = \dfrac{V_o}{V_i} = \dfrac{s^2R_LCL + R_L}{(R + R_L)LCs^2 + RR_LCs + R + R_L}$

Therefore

$$H(s) = \left(\frac{R_L}{R + R_L}\right) \cdot \frac{[s^2 + (1/LC)]}{\left[s^2 + \left(\frac{RR_L}{R+R_L}\right)\frac{s}{L} + \frac{1}{LC}\right]}$$

$$= \frac{K(s^2 + \omega_o^2)}{s^2 + \beta s + \omega_o^2}$$

where $K = \dfrac{R_L}{R + R_L}$; $\omega_o^2 = \dfrac{1}{LC}$; $\beta = \left(\dfrac{RR_L}{R + R_L}\right)\dfrac{1}{L}$

[b] $\omega_o = \dfrac{1}{\sqrt{LC}}$

[c] $\beta = \left(\dfrac{RR_L}{R + R_L}\right)\dfrac{1}{L}$

[d] $Q = \dfrac{\omega_o}{\beta} = \dfrac{\omega_o L}{[RR_L/(R + R_L)]}$

[e] $H(j\omega) = \dfrac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$H(j\omega_o) = 0$

[f] $H(j0) = \dfrac{K\omega_o^2}{\omega_o^2} = K$

[g] $H(j\omega) = \dfrac{K\left[(\omega_o/\omega)^2 - 1\right]}{\left\{\left[(\omega_o/\omega)^2 - 1\right] + j\beta/\omega\right\}}$

$H(j\infty) = \dfrac{-K}{-1} = K$

[h] $H(j\omega) = \dfrac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$H(j0) = H(j\infty) = K$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}}$$

$$\therefore \quad \frac{K}{\sqrt{2}} = \frac{K(\omega_o^2 - \omega_c^2)}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

Squaring both sides leads to

$$(\omega_o^2 - \omega_c^2)^2 = \omega_c^2\beta^2 \text{ or } (\omega_o^2 - \omega_c^2) = \pm\omega_c\beta$$

$$\therefore \quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

where

$$\beta = \frac{RR_L}{R + R_L} \cdot \frac{1}{L} \text{ and } \omega_o^2 = \frac{1}{LC}$$

P 14.40 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(400 \times 10^{-3})(250 \times 10^{-12})} = 10^{10}$

$$\omega_o = 10^5 = 100\,\text{krad/s} = 15.9\,\text{kHz}$$

$$\beta = \frac{RR_L}{R + R_L} \cdot \frac{1}{L} = \frac{(5000)(20{,}000)}{25{,}000} \cdot \frac{1}{0.4} = 10^4\,\text{rad/s} = 1.59\,\text{kHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{10^5}{10^4} = 10$$

[b] $H(j0) = \dfrac{R_L}{R + R_L} = \dfrac{20{,}000}{25{,}000} = 0.8$

$$H(j\infty) = \frac{R_L}{R + R_L} = 0.8$$

[c] $f_{c2} = \dfrac{10^5}{2\pi}\left[\dfrac{1}{20} + \sqrt{1 + \dfrac{1}{400}}\right] = 16.73\,\text{kHz}$

$$f_{c1} = \frac{10^5}{2\pi}\left[-\frac{1}{20} + \sqrt{1 + \frac{1}{400}}\right] = 15.14\,\text{kHz}$$

Check: $\beta = f_{c2} - f_{c1} = 1.59\,\text{kHz}.$

[d] $Q = \frac{\omega_o}{\beta} = \frac{10^5}{\frac{RR_L}{R+R_L} \cdot \frac{1}{L}}$

$= \frac{40(R + R_L)}{RR_L} = 8\left(1 + \frac{5}{R_L}\right)$

where R_L is in kilohms.

[e]

P 14.41 [a] $\omega_o^2 = \frac{1}{LC} = 10^{12}$

$\therefore \quad L = \frac{1}{(10^{12})(400 \times 10^{-12})} = 2.5\,\text{mH}$

$\frac{R_L}{R + R_L} = 0.96; \qquad \therefore \quad 0.04R_L = 0.96R$

$\therefore \qquad R_L = 24R \quad \therefore \quad R = \frac{36{,}000}{24} = 1.5\,\text{k}\Omega$

[b] $\beta = \left(\frac{R_L}{R + R_L}\right) R \cdot \frac{1}{L} = 576 \times 10^3$

$Q = \frac{\omega_o}{\beta} = \frac{10^6}{576 \times 10^3} = 1.74$

P 14.42 [a] $|H(j\omega)| = \frac{4 \times 10^6}{\sqrt{(4 \times 10^6 - \omega^2)^2 + (500\omega)^2}} = 1$

$\therefore \qquad 16 \times 10^{12} = (4 \times 10^6 - \omega^2)^2 + (500\omega)^2$

$= -8 \times 10^6\omega^2 + \omega^4 + 25 \times 10^4\omega^2$

$\therefore \qquad \omega^2 = 8 \times 10^6 - 25 \times 10^4 \quad \text{so} \quad \omega = 2783.88\,\text{rad/s}$

[b] From the equation for $|H(j\omega)|$ in part (a), the frequency for which the magnitude is maximum is the frequency for which the denominator is minimum. This is the frequency at which

$(4 \times 10^6 - \omega^2)^2 = 0 \quad \text{so} \quad \omega = \sqrt{4 \times 10^6} = 2000\,\text{rad/s}$

[c] $$|H(j2000)| = \frac{4 \times 10^6}{\sqrt{(4 \times 10^6 - 2000^2)^2 + [500(2000)]^2}} = 4$$

P 14.43 [a] Use the cutoff frequencies to calculate the bandwidth:

$$\omega_{c1} = 2\pi(697) = 4379.38 \text{ rad/s} \qquad \omega_{c2} = 2\pi(941) = 5912.48 \text{ rad/s}$$

Thus $\beta = \omega_{c2} - \omega_{c1} = 1533.10$ rad/s

Calculate inductance using Eq. (14.32) and capacitance using Eq. (14.31):

$$L = \frac{R}{\beta} = \frac{600}{1533.10} = 0.39\,\text{H}$$

$$C = \frac{1}{L\omega_{c1}\omega_{c2}} = \frac{1}{(0.39)(4379.38)(5912.48)} = 0.10\,\mu\text{F}$$

[b] At the outermost two frequencies in the low-frequency group (687 Hz and 941 Hz) the amplitudes are

$$|V_{697Hz}| = |V_{941Hz}| = \frac{|V_{\text{peak}}|}{\sqrt{2}} = 0.707|V_{\text{peak}}|$$

because these are cutoff frequencies. We calculate the amplitudes at the other two low frequencies using Eq. (14.32):

$$|V| = (|V_{\text{peak}}|)(|H(j\omega)|) = |V_{\text{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + (\omega\beta)^2}}$$

Therefore

$$|V_{770Hz}| = |V_{\text{peak}}| = \frac{(4838.05)(1533.10)}{\sqrt{(5088.52^2 - 4838.05^2)^2 + [(4838.05)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

and

$$|V_{852Hz}| = |V_{\text{peak}}| = \frac{(5353.27)(1533.10)}{\sqrt{(5088.52^2 - 5353.27^2)^2 + [(5353.27)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

It is not a coincidence that these two magnitudes are the same. The frequencies in both bands of the DTMF system were carefully chosen to produce this type of predictable behavior with linear filters. In other words, the frequencies were chosen to be equally far apart with respect to the response produced by a linear filter. Most musical scales consist of tones designed with this dame property – note intervals are selected to place the notes equally far apart. That is why the DTMF tones remind us of musical notes! Unlike musical scales, DTMF frequencies were selected to be harmonically unrelated, to lower the risk of misidentifying a tone's frequency if the circuit elements are not perfectly linear.

[c] The high-band frequency closest to the low-frequency band is 1209 Hz. The amplitude of a tone with this frequency is

$$|V_{1209Hz}| = |V_{\text{peak}}| = \frac{(7596.37)(1533.10)}{\sqrt{(5088.52^2 - 7596.37^2)^2 + [(7596.37)(1533.10)]^2}}$$

$$= 0.344|V_{\text{peak}}|$$

This is less than one half the amplitude of the signals with the low-band cutoff frequencies, ensuring adequate separation of the bands.

P 14.44 The cutoff frequencies and bandwidth are

$$\omega_{c_1} = 2\pi(1209) = 7596\,\text{rad/s}$$

$$\omega_{c_2} = 2\pi(1633) = 10.26\,\text{krad/s}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = 2664\,\text{rad/s}$$

Telephone circuits always have $R = 600\,\Omega$. Therefore, the filters inductance and capacitance values are

$$L = \frac{R}{\beta} = \frac{600}{2664} = 0.225\,\text{H}$$

$$C = \frac{1}{\omega_{c_1}\omega_{c_2}L} = 0.057\,\mu\text{F}$$

At the highest of the low-band frequencies, 941 Hz, the amplitude is

$$|V_\omega| = |V_{\text{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

where $\quad \omega_o = \sqrt{\omega_{c_1}\omega_{c_2}}$. Thus,

$$|V_\omega| = \frac{|V_{\text{peak}}|(5912)(2664)}{\sqrt{[(8828)^2 - (5912)^2]^2 + [(5912)(2664)]^2}}$$

$$= 0.344\ |V_{\text{peak}}|$$

Again it is not coincidental that this result is the same as the response of the low-band filter to the lowest of the high-band frequencies.

P 14.45 From Problem 14.43 the response to the largest of the DTMF low-band tones is $0.948|V_{\text{peak}}|$. The response to the 20 Hz tone is

$$|V_{20\text{Hz}}| = \frac{|V_{\text{peak}}|(125.6)(1533)}{[(5089^2 - 125.6^2)^2 + [(125.6)(1533)]^2]^{1/2}}$$

$$= 0.00744|V_{\text{peak}}|$$

$$\therefore \quad \frac{|V_{20\text{Hz}}|}{|V_{770\text{Hz}}|} = \frac{|V_{20\text{Hz}}|}{|V_{852\text{Hz}}|} = \frac{0.00744|V_{\text{peak}}|}{0.948|V_{\text{peak}}|} = 0.5$$

$$\therefore \quad |V_{20\text{Hz}}| = 63.7|V_{770\text{Hz}}|$$

Thus, the 20Hz signal can be 63.7 times as large as the DTMF tones.

15

Active Filter Circuits

Assessment Problems

AP 15.1

$$H(s) = \frac{-(R_2/R_1)s}{s + (1/R_1C)}$$

$$\frac{1}{R_1C} = 1\,\text{rad/s}; \qquad R_1 = 1\,\Omega, \quad \therefore \quad C = 1\,\text{F}$$

$$\frac{R_2}{R_1} = 1, \quad \therefore \quad R_2 = R_1 = 1\,\Omega$$

$$\therefore \qquad H_{\text{prototype}}(s) = \frac{-s}{s+1}$$

AP 15.2

$$H(s) = \frac{-(1/R_1C)}{s + (1/R_2C)} = \frac{-20{,}000}{s + 5000}$$

$$\frac{1}{R_1C} = 20{,}000; \quad C = 5\,\mu\text{F}$$

$$\therefore \quad R_1 = \frac{1}{(20{,}000)(5\times10^{-6})} = 10\,\Omega$$

$$\frac{1}{R_2C} = 5000$$

$$\therefore \quad R_2 = \frac{1}{(5000)(5\times10^{-6})} = 40\,\Omega$$

AP 15.3

$$\omega_c = 2\pi f_c = 2\pi \times 10^4 = 20{,}000\pi\,\text{rad/s}$$

$$\therefore \quad k_f = 20{,}000\pi = 62{,}831.85$$

$$C' = \frac{C}{k_f k_m} \quad \therefore \quad 0.5 \times 10^{-6} = \frac{1}{k_f k_m}$$

$$\therefore \quad k_m = \frac{1}{(0.5 \times 10^{-6})(62{,}831.85)} = 31.83$$

AP 15.4 For a 2nd order Butterworth high pass filter

$$H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

For the circuit in Fig. 15.25

$$H(s) = \frac{s^2}{s^2 + \left(\frac{2}{R_2C}\right)s + \left(\frac{1}{R_1R_2C^2}\right)}$$

Equate the transfer functions. For $C = 1\text{F}$,

$$\frac{2}{R_2C} = \sqrt{2}, \quad \therefore \quad R_2 = \sqrt{2} = 0.707\,\Omega$$

$$\frac{1}{R_1R_2C^2} = 1, \quad \therefore \quad R_1 = \frac{1}{\sqrt{2}} = 1.414\,\Omega$$

AP 15.5

$Q = 8, K = 5, \omega_o = 1000\,\text{rad/s}, C = 1\,\mu\text{F}$

For the circuit in Fig 15.26

$$H(s) = \frac{-\left(\frac{1}{R_1C}\right)s}{s^2 + \left(\frac{2}{R_3C}\right)s + \left(\frac{R_1 + R_2}{R_1R_2R_3C^2}\right)} = \frac{K\beta s}{s^2 + \beta s + \omega_o^2}$$

$$\beta = \frac{2}{R_3C}, \quad \therefore \quad R_3 = \frac{2}{\beta C}$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{8} = 125\,\text{rad/s}$$

$$\therefore \quad R_3 = \frac{2 \times 10^6}{(125)(1)} = 16\,\text{k}\Omega$$

$$K\beta = \frac{1}{R_1 C}$$

$$\therefore \quad R_1 = \frac{1}{K\beta C} = \frac{1}{5(125)(1 \times 10^{-6})} = 1.6\,\text{k}\Omega$$

$$\omega_o^2 = \frac{R_1 + R_2}{R_1 R_2 R_3 C^2}$$

$$10^6 = \frac{(1600 + R_2)}{(1600)(R_2)(16{,}000)(10^{-6})^2}$$

Solving for R_2,

$$R_2 = \frac{(1600 + R_2)10^6}{256 \times 10^5}, \qquad 246R_2 = 16{,}000, \qquad R_2 = 65.04\,\Omega$$

AP 15.6

$$\omega_o = 1000\,\text{rad/s}; \qquad Q = 4;$$

$$C = 2\,\mu\text{F}$$

$$H(s) = \frac{s^2 + (1/R^2C^2)}{s^2 + \left[\dfrac{4(1-\sigma)}{RC}\right] s + \left(\dfrac{1}{R^2C^2}\right)}$$
$$= \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}; \qquad \omega_o = \frac{1}{RC}; \qquad \beta = \frac{4(1-\sigma)}{RC}$$

$$R = \frac{1}{\omega_o C} = \frac{1}{(1000)(2 \times 10^{-6})} = 500\,\Omega$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{4} = 250$$

$$\therefore \qquad \frac{4(1-\sigma)}{RC} = 250$$

$$4(1-\sigma) = 250RC = 250(500)(2 \times 10^{-6}) = 0.25$$

$$1 - \sigma = \frac{0.25}{4} = 0.0625; \quad \therefore \qquad \sigma = 0.9375$$

Problems

P 15.1 Summing the currents at the inverting input node yields

$$\frac{0 - V_i}{Z_i} + \frac{0 - V_o}{Z_f} = 0$$

$$\therefore \quad \frac{V_o}{Z_f} = -\frac{V_i}{Z_i}$$

$$\therefore \quad H(s) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i}$$

P 15.2 [a] $Z_f = \dfrac{R_2(1/sC_2)}{[R_2 + (1/sC_2)]} = \dfrac{R_2}{R_2C_2s + 1}$

$$= \frac{(1/C_2)}{s + (1/R_2C_2)}$$

Likewise

$$Z_i = \frac{(1/C_1)}{s + (1/R_1C_1)}$$

$$\therefore \quad H(s) = \frac{-(1/C_2)[s + (1/R_1C_1)]}{[s + (1/R_2C_2)](1/C_1)}$$

$$= -\frac{C_1\,[s + (1/R_1C_1)]}{C_2\,[s + (1/R_2C_2)]}$$

[b] $H(j\omega) = \dfrac{-C_1}{C_2}\left[\dfrac{j\omega + (1/R_1C_1)}{j\omega + (1/R_2C_2)}\right]$

$$H(j0) = \frac{-C_1}{C_2}\left(\frac{R_2C_2}{R_1C_1}\right) = \frac{-R_2}{R_1}$$

[c] $H(j\infty) = -\dfrac{C_1}{C_2}\left(\dfrac{j}{j}\right) = \dfrac{-C_1}{C_2}$

[d] As $\omega \to 0$ the two capacitor branches become open and the circuit reduces to a resistive inverting amplifier having a gain of $-R_2/R_1$.

As $\omega \to \infty$ the two capacitor branches approach a short circuit and in this case we encounter an indeterminate situation; namely $v_n \to v_i$ but $v_n = 0$ because of the ideal op amp. At the same time the gain of the ideal op amp is infinite so we have the indeterminate form $0 \cdot \infty$. Although $\omega = \infty$ is indeterminate we can reason that for finite large values of ω $H(j\omega)$ will approach $-C_1/C_2$ in value. In other words, the circuit approaches a purely capacitive inverting amplifier with a gain of $(-1/j\omega C_2)/(1/j\omega C_1)$ or $-C_1/C_2$.

P 15.3 [a] $Z_f = \dfrac{(1/C_2)}{s + (1/R_2C_2)}$

$$Z_i = R_1 + \frac{1}{sC_1} = \frac{R_1}{s}[s + (1/R_1C_1)]$$

$$H(s) = -\frac{(1/C_2)}{[s + (1/R_2C_2)]} \cdot \frac{s}{R_1[s + (1/R_1C_1)]}$$

$$= -\frac{1}{R_1C_2}\frac{s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

[b] $H(j\omega) = -\dfrac{1}{R_1C_2}\dfrac{j\omega}{\left(j\omega + \frac{1}{R_1C_1}\right)\left(j\omega + \frac{1}{R_2C_2}\right)}$

$$H(j0) = 0$$

[c] $H(j\infty) = 0$

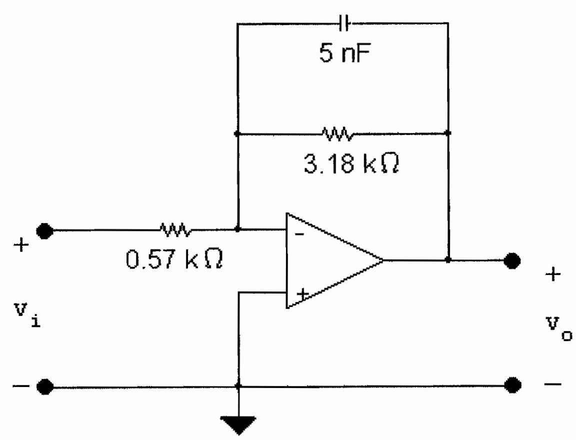
[d] As $\omega \to 0$ the capacitor C_1 disconnects v_i from the circuit. Therefore $v_o = v_n = 0$.
As $\omega \to \infty$ the capacitor short circuits the feedback network, thus $Z_F = 0$ and therefore $v_o = 0$.

P 15.4 [a] $K = 10^{0.75} = 5.62 = \dfrac{R_2}{R_1}$

$$R_2 = \frac{1}{\omega_c C} = \frac{10^9}{(2\pi)(10^4)(5)} = 3.18\,\text{k}\Omega$$

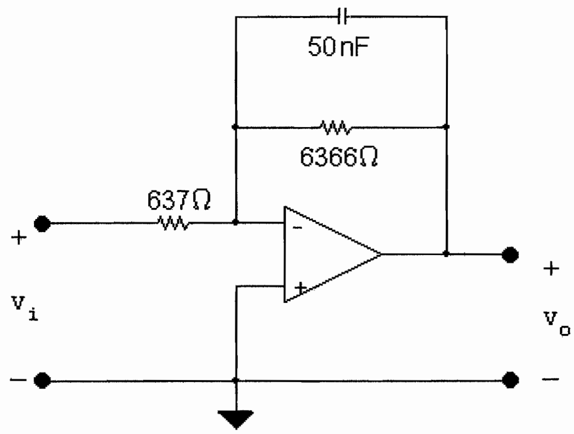
$$R_1 = \frac{R_2}{K} = \frac{3.18}{5.62} = 0.57\,\text{k}\Omega$$

[b]



P 15.5 [a] $\omega_c = \dfrac{1}{R_2C}$ so $R_2 = \dfrac{1}{\omega_c C} = \dfrac{1}{2\pi(500)(50 \times 10^{-9})} = 6366\,\Omega$

$$K = \frac{R_2}{R_1} \quad \text{so} \quad R_1 = \frac{R_2}{K} = \frac{6366}{10} = 637\,\Omega$$



[b] Both the cutoff frequency and the passband gain are changed.

P 15.6 [a] $10(0.2) = 2\,\text{V} \quad \text{so} \quad V_{cc} \geq 2\,\text{V}$

[b] $H(j\omega) = \dfrac{-10(2\pi)(500)}{j\omega + 2\pi(500)}$

$$H(j1000\pi) = \frac{-10(1000\pi)}{1000\pi + j1000\pi} = -5 + j5 = \frac{10}{\sqrt{2}}\underline{/135^\circ}$$

$$V_o = \frac{10}{\sqrt{2}}\underline{/135^\circ}V_i \quad \text{so} \quad v_o(t) = 1.414\cos(1000\pi t + 135^\circ)\,\text{V}$$

[c] $H(j100\pi) = \dfrac{-10(1000\pi)}{1000\pi + j100\pi} = 9.95\underline{/174.3^\circ}$

$$V_o = 9.95\underline{/174.3^\circ}V_i \quad \text{so} \quad v_o(t) = 1.99\cos(100\pi t + 174.3^\circ)\,\text{V}$$

[d] $H(j10{,}000\pi) = \dfrac{-10(1000\pi)}{1000\pi + j10{,}000\pi} = 0.995\underline{/95.7^\circ}$

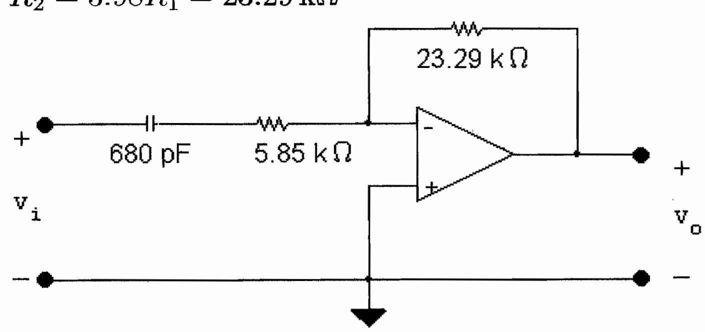
$$V_o = 0.995\underline{/95.7^\circ}V_i \quad \text{so} \quad v_o(t) = 199\cos(10{,}000\pi t + 95.7^\circ)\,\text{mV}$$

P 15.7 [a] $R_1 = \dfrac{1}{\omega_c C} = \dfrac{10^{12}}{(2\pi)(40)(10^3)(680)} = 5.85\,\text{k}\Omega$

$$K = 10^{0.6} = 3.98 = \frac{R_2}{R_1}$$

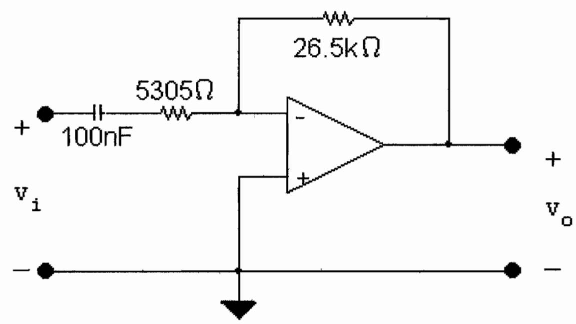
$$\therefore \quad R_2 = 3.98R_1 = 23.29\,\text{k}\Omega$$

[b]



P 15.8 [a] $\omega_c = \dfrac{1}{R_1 C}$ so $R_1 = \dfrac{1}{\omega_c C} = \dfrac{1}{2\pi(300)(100\times 10^{-9})} = 5305\,\Omega$

$$K = \frac{R_2}{R_1} \quad \text{so} \quad R_2 = KR_1 = (5)(5305) = 26.5\,\text{k}\Omega$$



[b] The passband gain changes but the cutoff frequency is unchanged.

P 15.9 [a] $5(0.15) = 0.75\,\text{V}$ so $V_{cc} \geq 0.75\,\text{V}$

[b] $H(j\omega) = \dfrac{-5j\omega}{j\omega + 600\pi}$

$$H(j600\pi) = \frac{-5(j600\pi)}{600\pi + j600\pi} = \frac{5}{\sqrt{2}}\underline{/-135^\circ}$$

$$V_o = \frac{5}{\sqrt{2}}\underline{/-135^\circ}V_i \quad \text{so} \quad v_o(t) = 530.33\cos(600\pi t - 135^\circ)\,\text{mV}$$

[c] $H(j60\pi) = \dfrac{-5(j60\pi)}{600\pi + j60\pi} = 0.5\underline{/-95.7^\circ}$

$$V_o = 0.5\underline{/-95.7^\circ}V_i \quad \text{so} \quad v_o(t) = 74.63\cos(60\pi t - 95.7^\circ)\,\text{mV}$$

[d] $H(j6000\pi) = \dfrac{-5(j6000\pi)}{600\pi + j6000\pi} = 4.98\underline{/-174.3^\circ}$

$$V_o = 4.98\underline{/-174.3^\circ}V_i \quad \text{so} \quad v_o(t) = 746.3\cos(6000\pi t - 174.3^\circ)\,\text{mV}$$

P 15.10 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{(1/RC)}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = k_m R\frac{C}{k_m k_f} = \frac{1}{k_f}RC = \frac{1}{k_f}$$

$$\frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{(1/R'C')}{s + (1/R'C')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

For the RL circuit

$$R' = k_m R; \qquad L' = \frac{k_m}{k_f} L$$

$$\frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f} L} = k_f \left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{(R'/L')}{s + (R'/L')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

P 15.11 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{s}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = \frac{RC}{k_f} = \frac{1}{k_f}; \qquad \frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{s}{s + (1/R'C')} = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

For the RL circuit

$$H(s) = \frac{s}{s + (R/L)}$$

$$R' = k_m R; \qquad L' = \frac{k_m L}{k_f}$$

$$\frac{R'}{L'} = k_f \left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

P 15.12 $H(s) = \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \dfrac{\beta s}{s^2 + \beta s + \omega_o^2}$

For the prototype circuit $\omega_o = 1$ and $\beta = \omega_o/Q = 1/Q$.
For the scaled circuit

$$H'(s) = \frac{(R'/L')s}{s^2 + (R'/L')s + (1/L'C')}$$

where $R' = k_m R$; $L' = \dfrac{k_m}{k_f}L$; and $C' = \dfrac{C}{k_f k_m}$

$$\therefore \quad \frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f}L} = k_f\left(\frac{R}{L}\right) = k_f\beta$$

$$\frac{1}{L'C'} = \frac{k_f k_m}{\frac{k_m}{k_f}LC} = \frac{k_f^2}{LC} = k_f^2$$

$$Q' = \frac{w_o'}{\beta'} = \frac{k_f w_o}{k_f \beta} = Q$$

therefore the Q of the scaled circuit is the same as the Q of the unscaled circuit. Also note $\beta' = k_f\beta$.

$$\therefore \quad H'(s) = \frac{\left(\frac{k_f}{Q}\right)s}{s^2 + \left(\frac{k_f}{Q}\right)s + k_f^2}$$

$$H'(s) = \frac{\left(\frac{1}{Q}\right)\left(\frac{s}{k_f}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \frac{1}{Q}\left(\frac{s}{k_f}\right) + 1\right]}$$

P 15.13 [a] $L = 1\,\text{H}$; $\quad C = 1\,\text{F}$

$$R = \frac{1}{Q} = \frac{1}{25} = 0.04\,\Omega$$

[b] $k_f = 100{,}000$; $\quad k_m = \dfrac{3600}{0.04} = 90{,}000$

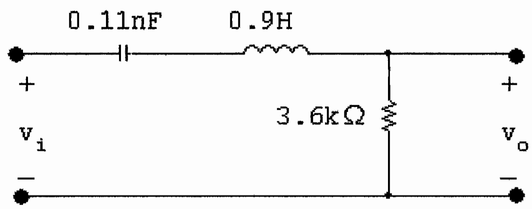
Thus,

$$R' = (0.04)(90{,}000) = 3.6\,\text{k}\Omega$$

$$L' = \frac{90{,}000}{100{,}000}(1) = 0.9\,\text{H}$$

$$C' = \frac{1}{(10^5)(9\times 10^4)} = \frac{1}{9}\,\text{nF} = 0.11\,\text{nF}$$

[c]



P 15.14 [a] By hypothesis, $LC = 1$; Thus,

$$C = \frac{1}{L} = \frac{1}{Q}F$$

[b] $H(s) = \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$

$$H(s) = \frac{(1/Q)s}{s^2 + (1/Q)s + 1}$$

[c] In the prototype circuit

$$R = 1\,\Omega; \qquad L = 20\,\text{H}; \qquad C = 0.05\,\text{F}$$

$$\therefore \quad k_m = 5000; \qquad k_f = 50{,}000$$

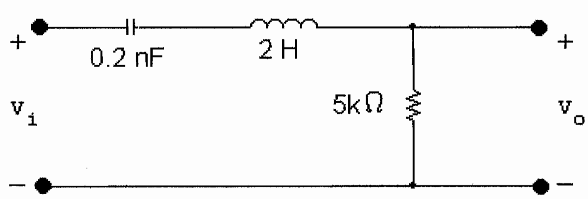
Thus

$$R' = 5\,\text{k}\Omega$$

$$L' = \frac{5000}{50{,}000}(20) = 2\,\text{H}$$

$$C' = \frac{0.05}{(5000)(50{,}000)} = 0.2 \times 10^{-9} = 0.2\,\text{nF}$$

[d]



[e] $H'(s) = \dfrac{\frac{1}{20}\left(\frac{s}{50{,}000}\right)}{\left(\frac{s}{50{,}000}\right)^2 + \frac{1}{20}\left(\frac{s}{50{,}000}\right) + 1}$

$$H'(s) = \frac{2500s}{s^2 + 2500s + 25 \times 10^8}$$

P 15.15 [a] Using the first prototype

$$\omega_o = 1 \text{ rad/s}; \qquad C = 1\,\text{F}; \qquad L = 1\,\text{H}; \qquad R = 16\,\Omega$$

$$k_m = \frac{80{,}000}{16} = 5000; \qquad k_f = 80{,}000$$

Thus,

$$R' = 80\,\text{k}\Omega; \qquad L' = \frac{5}{80}(1) = 62.5\,\text{mH};$$

$$C' = \frac{1}{400 \times 10^6} = 2.5\,\text{nF}$$

Using the second prototype

$$\omega_o = 1 \text{ rad/s}; \qquad C = 16\,\text{F}$$

$$L = \frac{1}{16} = 6.25\,\text{mH}; \qquad R = 1\,\Omega$$

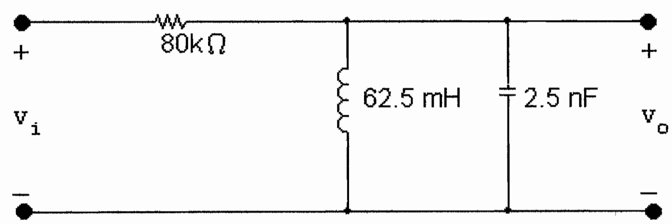
$$k_m = 80{,}000; \qquad k_f = 80{,}000$$

Thus,

$$R' = 80\,\text{k}\Omega; \qquad L' = \frac{80}{80}(6.25) = 6.25\,\text{mH};$$

$$C' = \frac{16}{64 \times 10^8} = 2.5\,\text{nF}$$

[b]



P 15.16 For the scaled circuit

$$H'(s) = \frac{s^2 + \left(\frac{1}{L'C'}\right)}{s^2 + \left(\frac{R'}{L'}\right)s + \left(\frac{1}{L'C'}\right)}$$

$$L' = \frac{k_m}{k_f}L; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}; \qquad R' = \text{k}_m R$$

$$\therefore \quad \frac{R'}{L'} = k_f\left(\frac{R}{L}\right)$$

It follows then that

$$H'(s) = \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{R}{L}\right) k_f s + \frac{k_f^2}{LC}}$$

$$= \frac{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{LC}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \left(\frac{R}{L}\right)\left(\frac{s}{k_f}\right) + \left(\frac{1}{LC}\right)\right]}$$

$$= H(s)|_{s=s/k_f}$$

P 15.17 For the circuit in Fig. 15.31

$$H(s) = \frac{s^2 + \left(\frac{1}{LC}\right)}{s^2 + \frac{s}{RC} + \left(\frac{1}{LC}\right)}$$

It follows that

$$H'(s) = \frac{s^2 + \frac{1}{L'C'}}{s^2 + \frac{s}{R'C'} + \frac{1}{L'C'}}$$

where $R' = k_m R; \qquad L' = \frac{k_m}{k_f} L;$

$$C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}$$

$$\frac{1}{R'C'} = \frac{k_f}{RC}$$

$$H'(s) = \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{k_f}{RC}\right) s + \frac{k_f^2}{LC}}$$

$$= \frac{\left(\frac{s}{k_f}\right)^2 + \frac{1}{LC}}{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{RC}\right)\left(\frac{s}{k_f}\right) + \frac{1}{LC}}$$

$$= H(s)|_{s=s/k_f}$$

P 15.18 [a] For the circuit in Fig. P15.18(a)

$$H(s) = \frac{V_o}{V_i} = \frac{s + \frac{1}{s}}{\frac{1}{Q} + s + \frac{1}{s}} = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For the circuit in Fig. P15.18(b)

$$H(s) = \frac{V_o}{V_i} = \frac{Qs + \frac{Q}{s}}{1 + Qs + \frac{Q}{s}}$$

$$= \frac{Q(s^2 + 1)}{Qs^2 + s + Q}$$

$$H(s) = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

[b] $$H'(s) = \frac{\left(\frac{s}{50{,}000}\right)^2 + 1}{\left(\frac{s}{50{,}000}\right)^2 + \frac{1}{5}\left(\frac{s}{50{,}000}\right) + 1}$$

$$= \frac{s^2 + 25 \times 10^8}{s^2 + 10{,}000s + 25 \times 10^8}$$

P 15.19 For prototype circuit (a):

$$H(s) = \frac{V_o}{V_i} = \frac{Q}{Q + \frac{1}{s + \frac{1}{s}}} = \frac{Q}{Q + \frac{s}{s^2+1}}$$

$$H(s) = \frac{Q(s^2 + 1)}{Q(s^2 + 1) + s} = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For prototype circuit (b):

$$H(s) = \frac{V_o}{V_i} = \frac{1}{1 + \frac{(s/Q)}{(s^2+1)}}$$

$$= \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

P 15.20 From the solution to Problem 14.21, $\omega_o = 10^6$ rad/s and $\beta = 133.33$ krad/s. Compute the two scale factors:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(250 \times 10^3)}{10^6} = \pi/2$$

$$k_m = \frac{1}{k_f}\frac{C}{C'} = \frac{2}{\pi}\frac{25 \times 10^{-9}}{10 \times 10^{-9}} = \frac{5}{\pi}$$

Thus,

$$R' = k_m R = \frac{5}{\pi}(300) = 477.46\,\Omega \qquad L' = \frac{k_m}{k_f}L = \frac{5/\pi}{\pi/2}(40 \times 10^{-6}) = 40.53\,\mu\text{H}$$

Calculate the cutoff frequencies:

$$\omega'_{c1} = k_f \omega_{c1} = (\pi/2)(935.56 \times 10^3) = 1469.57 \text{ krad/s}$$

$$\omega'_{c2} = k_f \omega_{c2} = (\pi/2)(1068.89 \times 10^3) = 1679.01 \text{ krad/s}$$

To check, calculate the bandwidth:

$$\beta' = \omega'_{c2} - \omega'_{c1} = 209.44 \text{ krad/s} = (\pi/2)\beta \text{ (checks!)}$$

P 15.21 From the solution to Problem 14.33, $\omega_o = 8 \times 10^6$ rad/s and $\beta = 500$ krad/s. Calculate the scale factors:

$$k_f = \frac{\omega'_o}{\omega_o} = \frac{500 \times 10^3}{8 \times 10^6} = 0.0625$$

$$k_m = \frac{k_f L'}{L} = \frac{0.0625(50 \times 10^{-6})}{625 \times 10^{-6}} = 0.005$$

Thus,

$$R' = k_m R = (0.005)(80{,}000) = 400\,\Omega \qquad C' = \frac{C}{k_m k_f} = \frac{25 \times 10^{-12}}{(0.005)(0.0625)} = 800\,\text{nF}$$

Calculate the bandwidth:

$$\beta' = k_f \beta = (0.0625)(500 \times 10^3) = 31{,}250 \text{ rad/s}$$

To check, calculate the quality factor:

$$Q = \frac{\omega_o}{\beta} = \frac{8 \times 10^6}{500 \times 10^3} = 16$$

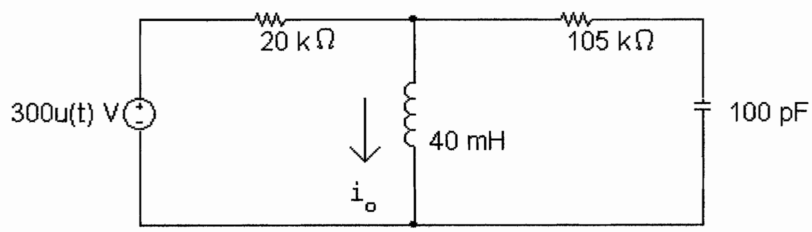
$$Q' = \frac{\omega'_o}{\beta'} = \frac{500 \times 10^3}{31{,}250} = 16 \text{ (checks)}$$

P 15.22 [a] $k_m = \dfrac{20}{4} = 5$

$$\therefore \quad 100 \times 10^{-12} = \frac{5 \times 10^{-9}}{5k_f}; \qquad \therefore \quad k_f = 10$$

$$L_{\text{scaled}} = \frac{5}{10}(80) = 40\,\text{mH}$$

$$R_{2\text{scaled}} = (21)(5 \times 10^3) = 105\,\text{k}\Omega$$



[b] From the soltion to Problem 13.26(b) we have

$$i_o = [75 + 5e^{-10{,}000t} - 80e^{-40{,}000t}]u(t)\,\text{mA}$$

Since $k_m = 5$ the amplitude of i_o in the scaled circuit will be one-fifth the original amplitude.
Since $k_f = 10$ the coefficients of t in the exponents will increase by a factor of 10. Thus,

$$i_o = [15 + e^{-100{,}000t} - 16e^{-400{,}000t}]u(t)\,\text{mA}$$

P 15.23 $k_m = \dfrac{1000}{10} = 100; \qquad k_f = 1000$

$$C = \frac{100 \times 10^{-3}}{10^5} = 1\,\mu\text{F}; \qquad 10\,\Omega \rightarrow 1\,\text{k}\Omega;$$

$$140\,\Omega \rightarrow 14\,\text{k}\Omega; \qquad L = \frac{100}{1000}(20) = 2\,\text{H}$$

$$0.25 \rightarrow \frac{0.25}{k_m} = 25 \times 10^{-4}$$

$$v_o = [16.8 + 722.4e^{-4000t}\cos(3000t + 91.33^\circ)]u(t)\,\text{V}$$

P 15.24 [a] From Eq 15.1 we have

$$H(s) = \frac{-K\omega_c}{s + \omega_c}$$

$$\text{where } K = \frac{R_2}{R_1}, \qquad \omega_c = \frac{1}{R_2 C}$$

$$\therefore \quad H'(s) = \frac{-K'\omega_c'}{s + \omega_c'}$$

where $K' = \dfrac{R_2'}{R_1'} \qquad \omega_c' = \dfrac{1}{R_2'C'}$

By hypothesis $R_1' = k_m R_1; \qquad R_2' = k_m R_2,$

and $C' = \dfrac{C}{k_f k_m}$. It follows that

$K' = K$ and $\omega_c' = k_f\omega_c$, therefore

$$H'(s) = \frac{-Kk_f\omega_c}{s + k_f\omega_c} = \frac{-K\omega_c}{\left(\frac{s}{k_f}\right) + \omega_c}$$

[b] $H(s) = \dfrac{-K}{(s+1)}$

[c] $H'(s) = \dfrac{-K}{\left(\frac{s}{k_f}\right) + 1} = \dfrac{-Kk_f}{s + k_f}$

P 15.25 [a] From Eq. 15.4

$H(s) = \dfrac{-Ks}{s + \omega_c}$ where $K = \dfrac{R_2}{R_1}$ and

$$\omega_c = \frac{1}{R_1 C}$$

$\therefore \quad H'(s) = \dfrac{-K's}{s + \omega_c'}$ where $K' = \dfrac{R_2'}{R_1'}$

and $\omega_c' = \dfrac{1}{R_1'C'}$

By hypothesis

$$R_1' = k_m R_1; \qquad R_2' = k_m R_2; \qquad C' = \frac{C}{k_m k_f}$$

It follows that

$K' = K$ and $\omega_c' = k_f\omega_c$

$$\therefore \quad H'(s) = \frac{-Ks}{s + k_f\omega_c} = \frac{-K(s/k_f)}{\left(\frac{s}{k_f}\right) + \omega_c}$$

[b] $H(s) = \dfrac{-Ks}{(s+1)}$

[c] $H'(s) = \dfrac{-K(s/k_f)}{\left(\frac{s}{k_f}+1\right)} = \dfrac{-Ks}{(s+k_f)}$

P 15.26 [a] $H_{\rm hp} = \dfrac{s}{s+1}; \qquad k_f = 4000\pi$

$$\therefore \quad H'_{\rm hp} = \frac{s}{s+4000\pi}$$

$$\frac{1}{R_H C_H} = 4000\pi; \qquad \therefore \quad R_H = \frac{10^6}{(4000\pi)(0.02)} = 3.98\,\text{k}\Omega$$

$$H_{\rm lp} = \frac{1}{s+1}; \qquad k_f = 16{,}000\pi$$

$$\therefore \quad H'_{\rm lp} = \frac{16{,}000\pi}{s+16{,}000\pi}$$

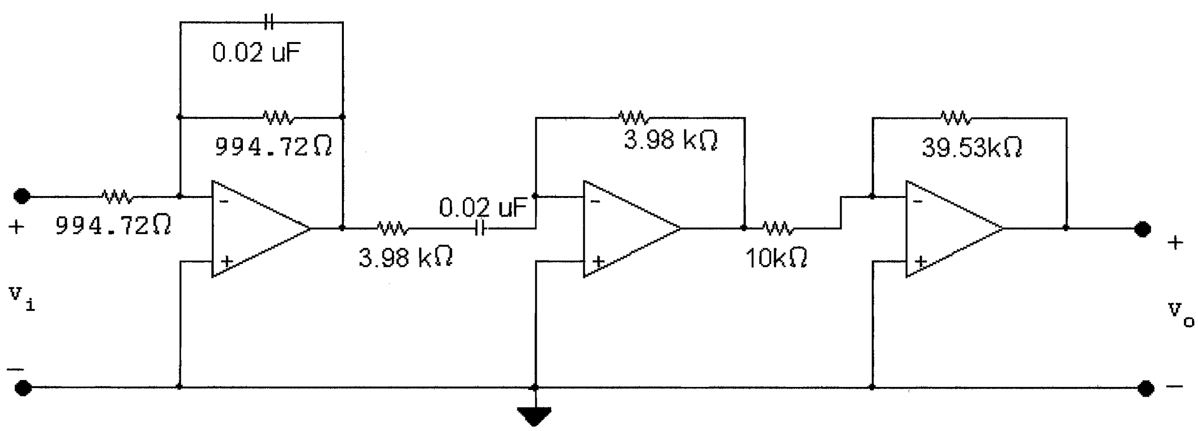
$$\frac{1}{R_L C_L} = 16{,}000\pi; \qquad \therefore \quad R_H = \frac{10^6}{(16{,}000\pi)(0.02)} = 994.72\,\Omega$$

$$H(j\omega_o) = \frac{K\omega_{c2}}{\omega_{c1}+\omega_{c2}} = 0.8K$$

$$20\log_{10}(0.8K) = 10; \qquad \therefore \quad K = 1.25\sqrt{10}$$

$$\therefore \quad \frac{R_f}{R_i} = 1.25\sqrt{10}$$

$$R_i = 10\,\text{k}\Omega; \qquad R_f = 12.5\sqrt{10} = 39.53\,\text{k}\Omega$$



[b] $H'(s) = \dfrac{s}{s+4000\pi} \cdot \dfrac{16{,}000\pi}{s+16{,}000\pi} \cdot \dfrac{39.53}{10}$

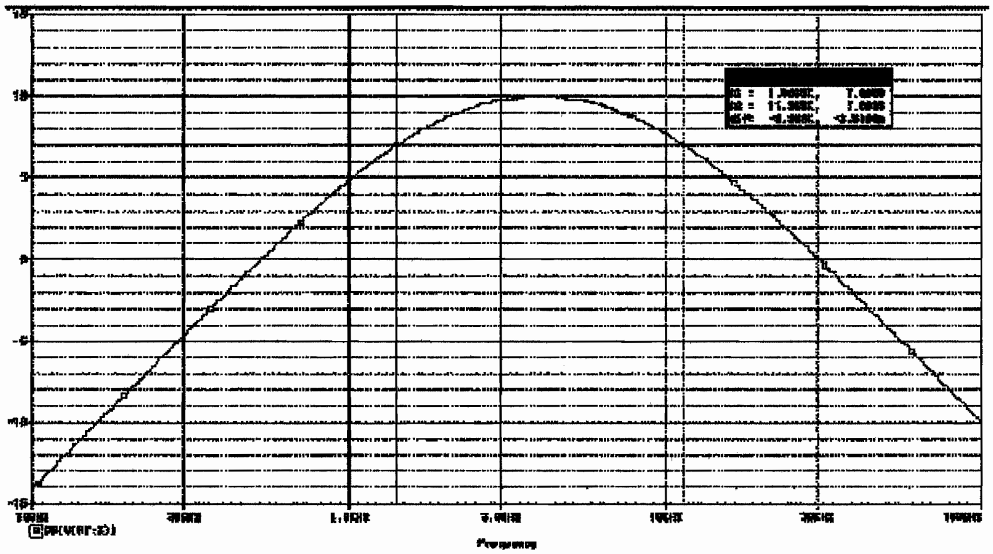
[c] $\omega_o = \sqrt{\omega_{c1}\omega_{c1}} = 8000\pi$ rad/s

$$H'(j\omega_o) = \frac{(16{,}000\pi)(j8000\pi)}{(4000\pi + j8000\pi)(16{,}000\pi + j8000\pi)} \cdot \frac{39.53}{10}$$

$$= (0.8)(3.953) = 3.16 = \sqrt{10}$$

[d] $20\log_{10}|H'(j\omega_o)| = 20\log_{10}\sqrt{10} = 10$ dB

[e]



P 15.27 [a] $\omega_{c1} = \dfrac{1}{R_LC_L} = 2000\pi$ rad/s

$$R_L = \frac{10^9}{(2000\pi)(5)} = 31.83\,\text{k}\Omega$$

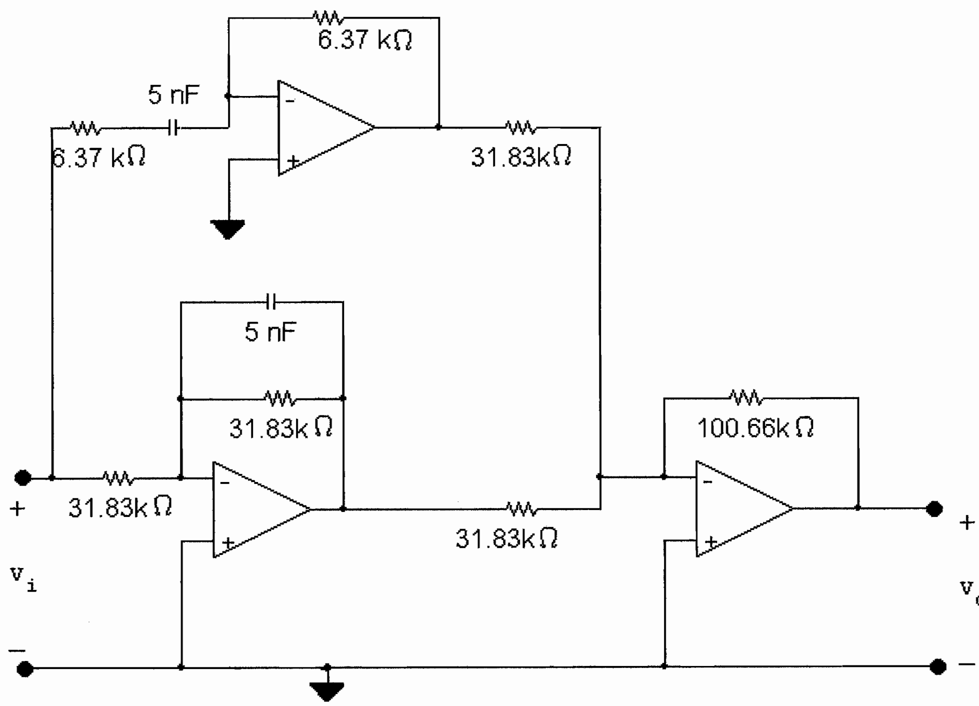
$$\omega_{c2} = \frac{1}{R_HC_H} = 10{,}000\pi \text{ rad/s}$$

$$R_H = \frac{10^9}{(10{,}000\pi)(5)} = 6.37\,\text{k}\Omega$$

$$20\log_{10}\left(\frac{R_f}{R_i}\right) = 10; \qquad \therefore \quad R_f = \sqrt{10}R_i$$

Choose $R_i = 31.83\,\text{k}\Omega$; then $R_f = 100.66\,\text{k}\Omega$

[b]



[c] $H(s)_{\rm LP} = \dfrac{-1}{s/k_f + 1} = \dfrac{-2000\pi}{s + 2000\pi}$

$$H(s)_{\rm HP} = \frac{-s/k_f}{s/k_f + 1} = \frac{-s}{s + 10{,}000\pi}$$

$$-\frac{R_f}{R_i} = -\sqrt{10}$$

$$H(s) = \sqrt{10}\left[\frac{2000\pi}{s + 2000\pi} + \frac{s}{s + 10{,}000\pi}\right]$$

$$= \sqrt{10}\left[\frac{s^2 + 4000\pi s + 20 \times 10^6\pi^2}{(s + 2000\pi)(s + 10{,}000\pi)}\right]$$

[d] $\omega_o = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{20 \times 10^6\pi^2}$

$$= 1000\pi\sqrt{20} = 2000\pi\sqrt{5} \text{ rad/s}$$

$$H(j\omega_o) = \sqrt{10}\left[\frac{j4000\pi(2000\pi\sqrt{5})}{(2000\pi + j2000\pi\sqrt{5})(10{,}000\pi + j2000\pi\sqrt{5})}\right]$$

$$= \frac{j2\sqrt{5}\sqrt{10}}{(1+j\sqrt{5})(5+j\sqrt{5})} = \frac{j2\sqrt{5}\sqrt{10}}{j6\sqrt{5}}$$

$$= \frac{\sqrt{10}}{3} = 1.05$$

[e] $20\log_{10}|H(j\omega_o)| = 20\log_{10}1.05 = 0.46$ dB

[f] $$H(j\omega) = \frac{\left[1 - \left(\frac{\omega}{1000\sqrt{20}\pi}\right)^2\right] + j\frac{4}{\sqrt{20}}\cdot\frac{w}{100\sqrt{20}\pi}}{\left(1 + j\frac{\omega}{2000\pi}\right)\left(1 + j\frac{\omega}{10{,}000\pi}\right)}$$

$$2\zeta = \frac{4}{\sqrt{20}}; \qquad \zeta = \frac{2}{\sqrt{20}}; \qquad \zeta^2 = 0.20$$

$$\omega_o = 2000\pi\sqrt{5}; \qquad f_o = 1000\sqrt{5} = 2236.07\,\text{Hz}$$

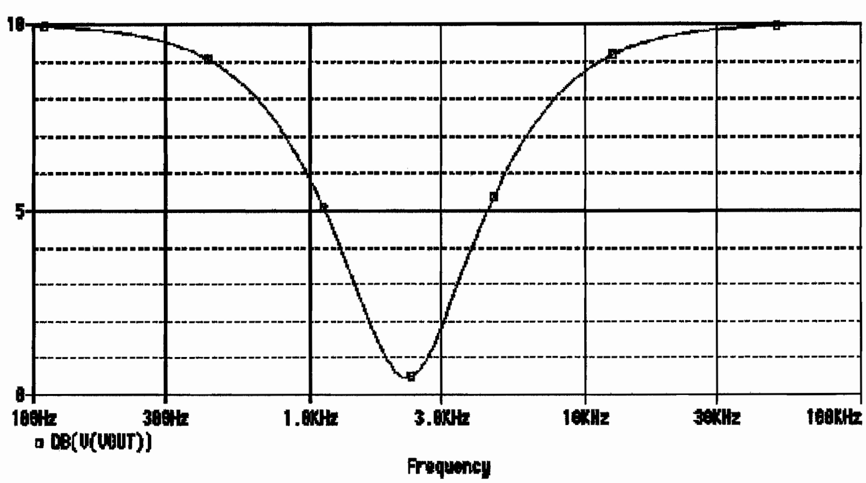
$$f_p = f_o\sqrt{1 - 2\zeta^2} = f_o\sqrt{0.6} = 1732.05\,\text{Hz}$$

$$A_{\text{dB}}(f_p) = 10\log_{10}[4\zeta^2(1-\zeta^2)] = 10\log_{10}0.64 = -1.94 \text{ dB}$$

$$A_{\text{dB}}(f_o/2) = 10\log_{10}0.7625 = -1.18 \text{ dB}$$

$$A_{\text{dB}}(f_o) = 20\log_{10}2\zeta = -0.97 \text{ dB}$$

For the quadratic term, $A_{\text{dB}} = 0$ when $f = \sqrt{2}f_p = 2449.48\,\text{Hz}$.



P 15.28 $H(s) = \frac{V_o}{V_i} = \frac{-Z_f}{Z_i}$

$$Z_f = \frac{1}{sC_2}\|R_2 = \frac{(1/C_2)}{s + (1/R_2C_2)}; \qquad Z_i = R_1 + \frac{1}{sC_1} = \frac{sR_1C_1 + 1}{sC_1}$$

$$\therefore \quad H(s) = \frac{\dfrac{-1/C_2}{s + (1/R_2C_2)}}{\dfrac{s + (1/R_1C_1)}{s/R_1}} = \frac{-(1/R_1C_2)s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

$$= \frac{-K\beta s}{s^2 + \beta s + \omega_o^2}$$

[a] $H(s) = \frac{-250s}{(s + 50)(s + 20)} = \frac{-250s}{s^2 + 70s + 1000} = \frac{-3.57(70s)}{s^2 + 70s + (\sqrt{1000})^2}$

$\omega_o = \sqrt{1000} = 31.6$ rad/s

$\beta = 70$ rad/s

$K = -3.57$

[b] $Q = \frac{\omega_o}{\beta} = 0.45$

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2} = \pm 35 + \sqrt{35^2 + 1000} = \pm 35 + 47.17$$

$\omega_{c1} = 12.17$ rad/s $\qquad \omega_{c2} = 82.17$ rad/s

P 15.29 [a] $H(s) = \frac{(1/sC)}{R + (1/sC)} = \frac{(1/RC)}{s + (1/RC)}$

$$H(j\omega) = \frac{(1/RC)}{j\omega + (1/RC)}$$

$$|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + (1/RC)^2}}$$

$$|H(j\omega)|^2 = \frac{(1/RC)}{\omega^2 + (1/RC)^2}$$

[b] Let V_a be the voltage across the capacitor, positive at the upper terminal. Then

$$\frac{V_a - V_{in}}{R_1} + sCV_a + \frac{V_a}{R_2 + sL} = 0$$

Solving for V_a yields

$$V_a = \frac{(R_2 + sL)V_{in}}{R_1LCs^2 + (R_1R_2C + L)s + (R_1 + R_2)}$$

But

$$v_o = \frac{sLV_a}{R_2 + sL}$$

Therefore

$$V_o = \frac{sLV_{in}}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(s) = \frac{sL}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(j\omega) = \frac{j\omega L}{[(R_1 + R_2) - R_1LC\omega^2] + j\omega(L + R_1R_2C)}$$

$$|H(j\omega)| = \frac{\omega L}{\sqrt{[R_1 + R_2 - R_1LC\omega^2]^2 + \omega^2(L + R_1R_2C)^2}}$$

$$|H(j\omega)|^2 = \frac{\omega^2L^2}{(R_1 + R_2 - R_1LC\omega^2)^2 + \omega^2(L + R_1R_2C)^2}$$

$$= \frac{\omega^2L^2}{R_1^2L^2C^2\omega^4 + (L^2 + R_1^2R_2^2C^2 - 2R_1^2LC)\omega^2 + (R_1 + R_2)^2}$$

[c] Let V_a be the voltage across R_2 positive at the upper terminal. Then

$$\frac{V_a - V_{in}}{R_1} + \frac{V_a}{R_2} + V_asC + V_asC = 0$$

$$(0 - V_a)sC + (0 - V_a)sC + \frac{0 - V_o}{R_3} = 0$$

$$\therefore\quad V_a = \frac{R_2V_{in}}{2R_1R_2Cs + R_1 + R_2}$$

and $V_a = -\dfrac{V_o}{2R_3Cs}$

It follows directly that

$$H(s) = \frac{V_o}{V_{in}} = \frac{-2R_2R_3Cs}{2R_1R_2Cs + (R_1 + R_2)}$$

$$H(j\omega) = \frac{-2R_2R_3C(j\omega)}{(R_1 + R_2) + j\omega(2R_1R_2C)}$$

$$|H(j\omega)| = \frac{2R_2R_3C\omega}{\sqrt{(R_1 + R_2)^2 + \omega^2 4R_1^2R_2^2C^2}}$$

$$|H(j\omega)|^2 = \frac{4R_2^2R_3^2C^2\omega^2}{(R_1 + R_2)^2 + 4R_1^2R_2^2C^2\omega^2}$$

P 15.30 $\omega_o = 50{,}000\,\text{rad/s}$

$\beta = 300{,}000\,\text{rad/s}$

$\therefore \quad \omega_{c_2} - \omega_{c_1} = 300{,}000$

$\sqrt{\omega_{c_1}\omega_{c_2}} = \omega_o = 50{,}000$

Solve for the cutoff frequencies:

$\omega_{c_1}\omega_{c_2} = 25 \times 10^8$

$$\omega_{c_2} = \frac{25 \times 10^8}{\omega_{c_1}}$$

$$\therefore \quad \frac{25 \times 10^8}{\omega_{c_1}} - \omega_{c_1} = 300{,}000$$

or $\omega_{c_1}^2 + 300{,}000\omega_{c_1} - 25 \times 10^8 = 0$

$\omega_{c_1} = 8113.88\,\text{rad/s}$

$\therefore \quad \omega_{c_2} = 300{,}000 + 8113.88 = 308{,}113.88\,\text{rad/s}$

Thus, $f_{c1} = 1291.4$ Hz and $f_{c2} = 49{,}037.85$ Hz

$$\omega_{c2} = \frac{1}{R_LC_L} = 308{,}113.88$$

$$R_L = \frac{1}{(308{,}113.88)(150 \times 10^{-9})} = 21.64\,\Omega$$

$$\omega_{c1} = \frac{1}{R_HC_H} = 8113.88$$

$$R_H = \frac{1}{(8113.88)(150 \times 10^{-9})} = 821.64\,\Omega$$

P 15.31 $\omega_o = 2\pi(5000)\,\text{rad/s};\qquad \text{GAIN} = 4$

$$\beta = 2\pi(30{,}000)\ \text{rad/s};\qquad C = 250\,\text{nF}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = 60{,}000\pi$$

$$\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}} = 10{,}000\pi$$

Solve for the cutoff frequencies:

$$\therefore\quad \omega_{c_1}^2 + 60{,}000\pi\omega_{c_1} - (10{,}000\pi)^2 = 0$$

$$\omega_{c_1} = 5098.1\,\text{rad/s}$$

$$\omega_{c_2} = 60{,}000\pi + \omega_{c_1} = 193{,}593.7\,\text{rad/s}$$

$$\omega_{c_1} = \frac{1}{R_L C_L}$$

$$\therefore\quad R_L = \frac{1}{(250\times10^{-9})(5098.1)} = 784.6\,\Omega$$

$$\frac{1}{R_H C_H} = \omega_{c2}$$

$$R_H = \frac{1}{(250\times10^{-9})(193{,}593.7)} = 20.7\,\Omega$$

$$\frac{R_f}{R_i} = 4$$

If $R_i = 1\,\text{k}\Omega \qquad R_f = 4R_i = 4\,\text{k}\Omega$

P 15.32 [a] $y = 20\log_{10}\dfrac{1}{\sqrt{1+\omega^{2n}}} = -10\log_{10}(1+\omega^{2n})$

From the laws of logarithms we have

$$y = \left(\frac{-10}{\ln 10}\right)\ln(1+\omega^{2n})$$

Thus

$$\frac{dy}{d\omega} = \left(\frac{-10}{\ln 10}\right)\frac{2n\omega^{2n-1}}{(1+\omega^{2n})}$$

$$x = \log_{10}\omega = \frac{\ln\omega}{\ln 10}$$

$$\therefore \quad \ln\omega = x\ln 10$$

$$\frac{1}{\omega}\frac{d\omega}{dx} = \ln 10, \quad \frac{d\omega}{dx} = \omega\ln 10$$

$$\frac{dy}{dx} = \left(\frac{dy}{d\omega}\right)\left(\frac{d\omega}{dx}\right) = \frac{-20n\omega^{2n}}{1+\omega^{2n}}\ \text{dB/decade}$$

at $\omega = \omega_c = 1\,\text{rad/s}$

$$\frac{dy}{dx} = -10\text{n dB/decade.}$$

[b] $$y = 20\log_{10}\frac{1}{[\sqrt{1+\omega^2}]^n} = -10\text{n}\log_{10}(1+\omega^2)$$

$$= \frac{-10n}{\ln 10}\ln(1+\omega^2)$$

$$\frac{dy}{d\omega} = \frac{-10}{\ln 10}\left(\frac{1}{1+\omega^2}\right)2\omega = \frac{-20\text{n}\omega}{(\ln 10)(1+\omega^2)}$$

As before

$$\frac{d\omega}{dx} = \omega(\ln 10); \qquad \therefore \quad \frac{dy}{dx} = \frac{-20\text{n}\omega^2}{(1+\omega^2)}$$

At the corner $\omega_c = \sqrt{2^{1/n}-1}$ $\quad \therefore \quad \omega_c^2 = 2^{1/n}-1$

$$\frac{dy}{dx} = \frac{-20n[2^{1/n}-1]}{2^{1/n}}\ \text{dB/decade.}$$

[c] For the Butterworth Filter

n	dy/dx (dB/decade)
1	−10
2	−20
3	−30
4	−40
∞	−∞

For the cascade of identical sections

n	dy/dx (dB/decade)
1	−10
2	−11.72
3	−12.38
4	−12.73
∞	−12.36

[d] It is apparent from the calculations in part (c) that as n increases the amplitude characteristic at the cut off frequency decreases at a much faster rate for the Butterworth filter.
Hence the transition region of the Butterworth filter will be much narrower than that of the cascaded sections.

P 15.33 [a] $n \cong \dfrac{(-0.05)(-40)}{\log_{10}(4000/1000)} \cong 3.32$

$\therefore \quad n = 4$

[b] Gain $= 20\log_{10}\dfrac{1}{\sqrt{1+(4)^8}} = -10\log_{10}(1+4^8) = -48.16$ dB

P 15.34 [a] For the scaled circuit

$$H'(s) = \frac{1/(R')^2C_1'C_2'}{s^2 + \frac{2}{R'C_1'}s + \frac{1}{(R')^2C_1'C_2'}}$$

where

$R' = k_mR; \qquad C_1' = C_1/k_fk_m; \qquad C_2' = C_2/k_fk_m$

It follows that

$$\frac{1}{(R')^2C_1'C_2'} = \frac{k_f^2}{R^2C_1C_2}$$

$$\frac{2}{R'C_1'} = \frac{2k_f}{RC_1}$$

$$\therefore \quad H'(s) = \frac{k_f^2/RC_1C_2}{s^2 + \frac{2k_f}{RC_1}s + \frac{k_f^2}{R^2C_1C_2}}$$

$$= \frac{1/RC_1C_2}{\left(\frac{s}{k_f}\right)^2 + \frac{2}{RC_1}\left(\frac{s}{k_f}\right) + \frac{1}{R^2C_1C_2}}$$

P 15.35 [a] $H(s) = \dfrac{1}{(s^2+0.765s+1)(s^2+1.848s+1)}$

[b] $f_c = 1000\,\text{Hz}; \qquad \omega_c = 2000\pi$ rad/s; $\qquad k_f = 2000\pi$

$$H'(s) = \frac{1}{\left[\left(\frac{s}{2000\pi}\right)^2 + \frac{0.765s}{2000\pi} + 1\right]\left[\left(\frac{s}{2000\pi}\right)^2 + \frac{1.848s}{2000\pi} + 1\right]}$$

$$= \frac{(4\times10^6\pi^2)^2}{(s^2 + 1530\pi s + 4\times10^6\pi^2)(s^2 + 3696\pi s + 4\times10^6\pi^2)}$$

[c] $H'(j8000\pi) = \dfrac{16}{(-60+j12.24)(-60+j29.568)}$

$$|H'(j8000\pi)| = \frac{16}{(61.24)(66.89)} = 3.91\times10^{-3}$$

Gain $= 20\log_{10}|H(j8000\pi)| = -48.16$ dB

P 15.36 [a] $k_m = 2000; \qquad k_f = 2000\pi$

First stage:

$$\frac{2}{C_1} = 0.765; \qquad \therefore \quad C_1 = \frac{2}{0.765}$$

$$C_1' = \frac{2}{(0.765)(2000)(2000\pi)} = 208.05\,\text{nF}$$

$$C_2 = \frac{1}{C_1} = \frac{0.765}{2}$$

$$C_2' = \frac{0.765}{2(2000)(2000\pi)} = 30.44\,\text{nF}$$

Second stage:

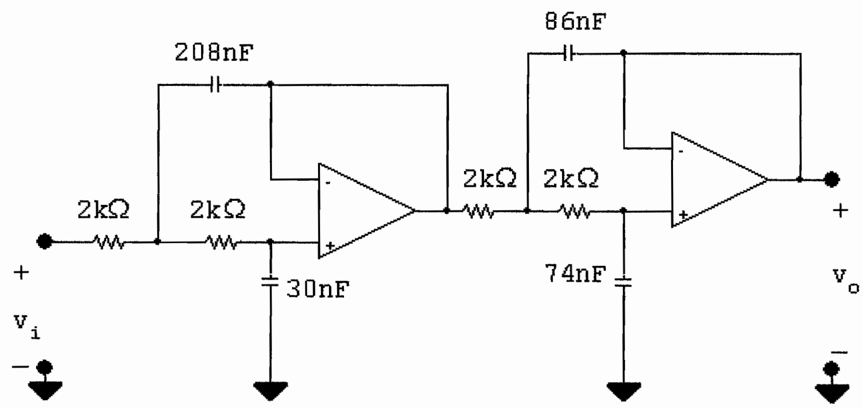
$$\frac{2}{C_1} = 1.848; \qquad \therefore \quad C_1 = \frac{2}{1.848}$$

$$C_1' = \frac{2}{(1.848)(2000)(2000\pi)} = 86.12\,\text{nF}$$

$$C_2 = \frac{1}{C_1} = \frac{1.848}{2}$$

$$C_2' = \frac{1.848}{2(2000)(2000\pi)} = 73.53\,\text{nF}$$

[b]



P 15.37 [a] $n \cong \dfrac{(-0.05)(-25)}{\log_{10}(5/1)} = 1.79; \qquad \therefore \quad n = 2$

$$\therefore \quad H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

$$\frac{2}{R_2} = \sqrt{2}; \qquad R_2 = \sqrt{2}\,\Omega; \qquad R_1 = \frac{1}{R_2} = \frac{1}{\sqrt{2}}\,\Omega$$

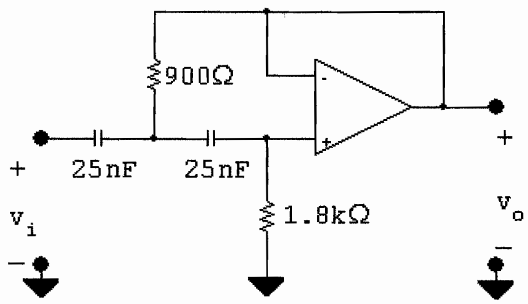
$$k_f = 10{,}000\pi$$

$$\therefore \quad k_m = \frac{10^9}{(10{,}000\pi)(25)} = \frac{4000}{\pi}$$

$$R_1 = \frac{1}{\sqrt{2}} \cdot \frac{4000}{\pi} = 900.32\,\Omega$$

$$R_2 = \sqrt{2}\left(\frac{4000}{\pi}\right) = 1800.63\,\Omega$$

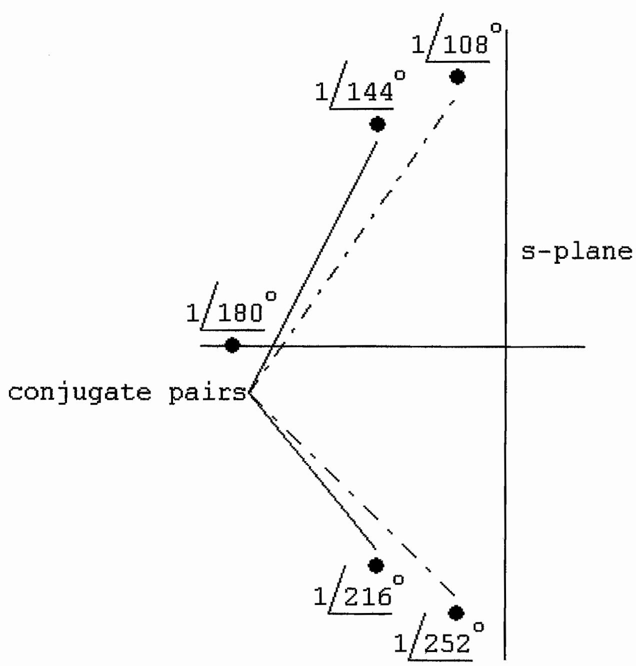
[**b**]



P 15.38 $n = 5$: $1 + (-1)^5 s^{10} = 0$; $\quad s^{10} = 1$

$s^{10} = 1\underline{/(0 + 36k)^\circ}$

k	s_{k+1}
0	$1\underline{/0^\circ}$
1	$1\underline{/36^\circ}$
2	$1\underline{/72^\circ}$
3	$1\underline{/108^\circ}$
4	$1\underline{/144^\circ}$
5	$1\underline{/180^\circ}$
6	$1\underline{/216^\circ}$
7	$1\underline{/252^\circ}$
8	$1\underline{/288^\circ}$
9	$1\underline{/324^\circ}$



Group by conjugate pairs to form denominator polynomial.

$$(s+1)[s - (\cos 108^\circ + j\sin 108^\circ)][(s - (\cos 252^\circ + j\sin 252^\circ)]$$

$$\cdot [(s - (\cos 144^\circ + j \sin 144^\circ)][(s - (\cos 216^\circ + j \sin 216^\circ)]$$

$$(s + 1)(s + 0.309 - j0.951)(s + 0.309 + j0.951)\cdot$$

$$(s + 0.809 - j0.588)(s + 0.809 + j0.588)$$

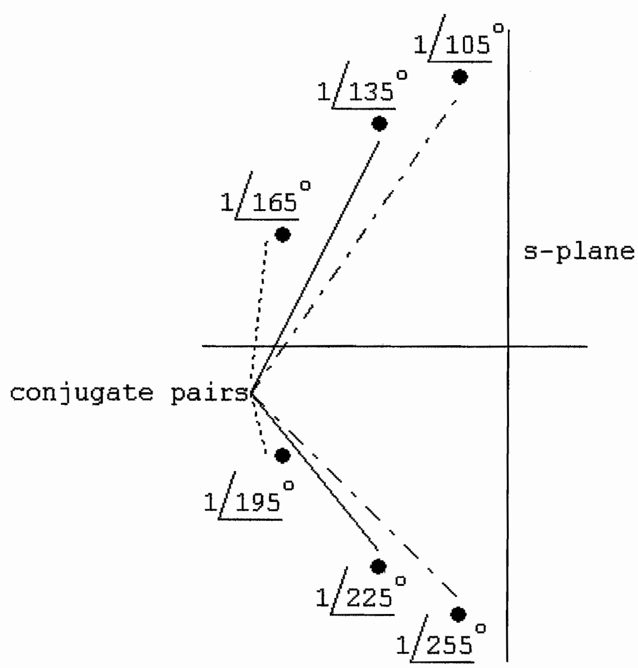
which reduces to

$$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$$

$n = 6$: $1 + (-1)^6 s^{12} = 0 \qquad s^{12} = -1$

$s^{12} = 1\underline{/180^\circ + 360k}$

k	s_{k+1}
0	$1\underline{/15^\circ}$
1	$1\underline{/45^\circ}$
2	$1\underline{/75^\circ}$
3	$1\underline{/105^\circ}$
4	$1\underline{/135^\circ}$
5	$1\underline{/165^\circ}$
6	$1\underline{/195^\circ}$
7	$1\underline{/225^\circ}$
8	$1\underline{/255^\circ}$
9	$1\underline{/285^\circ}$
10	$1\underline{/315^\circ}$
11	$1\underline{/345^\circ}$



Grouping by conjugate pairs yields

$$(s + 0.2588 - j0.9659)(s + 0.2588 + j0.9659)\times$$

$$(s + 0.7071 - j0.7071)(s + 0.7071 + j0.7071)\times$$

$$(s + 0.9659 - j0.2588)(s + 0.9659 + j0.2588)$$

or $(s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.9319s + 1)$

P 15.39 $$H'(s) = \frac{s^2}{s^2 + \frac{2}{k_m R_2 (C/k_m k_f)} s + \frac{1}{k_m R_1 k_m R_2 (C^2/k_m^2 k_f^2)}}$$

$$H'(s) = \frac{s^2}{s^2 + \frac{2k_f}{R_2 C} s + \frac{k f^2}{R_1 R_2 C^2}}$$

$$= \frac{(s/k_f)^2}{(s/k_f)^2 + \frac{2}{R_2 C}\left(\frac{s}{k_f}\right) + \frac{1}{R_1 R_2 C^2}}$$

P 15.40 [a] $n \cong \frac{(-0.05)(-25)}{\log_{10}(100/20)} = 1.79; \therefore \quad n = 2$

$$\therefore \quad H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

$$\frac{2}{C_1} = \sqrt{2} : \qquad C_1 = \sqrt{2}\,\text{F}$$

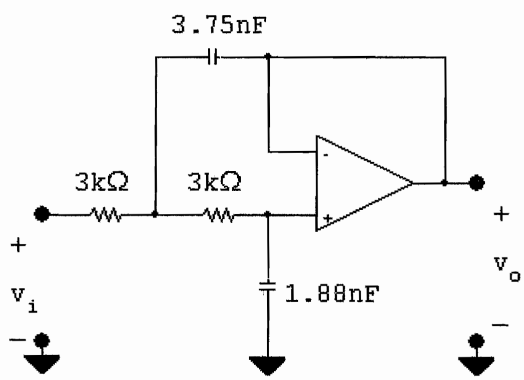
$$C_2 = \frac{1}{C_1} = \frac{1}{\sqrt{2}} = 0.5\sqrt{2}\,\text{F}$$

$$k_m = 3000; \qquad k_f = 40{,}000\pi$$

$$C_1' = \frac{\sqrt{2}}{(3000)(40{,}000\pi)} = 3.75\,\text{nF}$$

$$C_2' = \frac{1}{2}C_1' = 1.88\,\text{nF}; \qquad R_1 = R_2 = 3\,\text{k}\Omega$$

[b]



P 15.41 [a] A bandpass filter.

[b] $f_{c1} = 5000\,\text{Hz}; \qquad f_{c2} = 20{,}000\,\text{Hz}$

$$f_o = \sqrt{f_{c1} f_{c2}} = 10{,}000\,\text{Hz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{f_o}{f_{c2} - f_{c1}} = \frac{10{,}000}{15{,}000} = 0.67$$

[c] $H(s)_{\rm hp} = \dfrac{s^2}{s^2+\sqrt{2}s+1}$

$$H'(s)_{\rm hp} = \frac{(s/10^4\pi)^2}{(s/10^4\pi)^2+\sqrt{2}(s/10^4\pi)+1}$$

$$= \frac{s^2}{s^2+\pi\sqrt{2}\times 10^4 s+10^8\pi^2}$$

$$H(s)_{\rm lp} = \frac{1}{s^2+\sqrt{2}s+1}$$

$$H'(s)_{\rm lp} = \frac{1}{(s/10^4\pi)^2+\sqrt{2}(s/10^4\pi)+1}$$

$$= \frac{16\times 10^8\pi^2}{s^2+4\pi\sqrt{2}\times 10^4 s+16\times 10^8\pi^2}$$

$$H(s) = H'(s)_{\rm hp}\cdot H'(s)_{\rm lp}$$

$$= \frac{16\times 10^8\pi^2 s^2}{(s^2+\pi\sqrt{2}10^4 s+10^8\pi^2)(s^2+4\pi\sqrt{2}\times 10^4 s+16\times 10^8\pi^2)}$$

[d] $\omega_o = 20{,}000\pi$ rad/s $= 2\times 10^4$ krad/s

$$H(s) = \frac{16\times 10^8\pi^2(-4\times 10^8\pi^2)}{(-3\times 10^8\pi^2+j\pi\sqrt{2}10^4(2\times 10^4\pi))}$$

$$\times\frac{1}{(12\times 10^8\pi^2+j4\sqrt{2}\pi 10^4(2\times 10^4\pi))}$$

$$= \frac{-64}{(-3+j2\sqrt{2})(12+j8\sqrt{2})} = \frac{-64}{-68} = 0.9412$$

P 15.42 [a] $20\log_{10}K = 40; \qquad \therefore\ K = 10^2 = 100$

$$R_1 = \frac{Q}{K} = 0.20\,\Omega$$

$$R_2 = \frac{20}{800-100} = \frac{20}{700} = \frac{1}{35}\,\Omega$$

$$R_3 = 2Q = 40\,\Omega$$

$$k_f = 16{,}000\pi$$

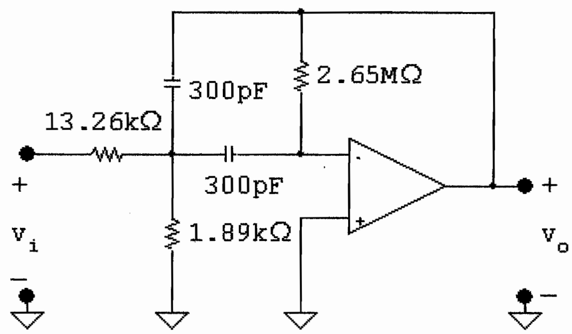
$$\therefore\ k_m = \frac{10^{12}}{(16{,}000\pi)(300)} = 66{,}314.56$$

$$R_1 = 0.2k_m = 13.26\,{\rm k}\Omega$$

$$R_2 = \frac{1}{35}k_m = 1.89\,{\rm k}\Omega$$

$$R_3 = 40k_m = 2.65\,{\rm M}\Omega$$

[b]



P 15.43 From Eq 15.58 we can write

$$H(s) = \frac{-\left(\frac{2}{R_3C}\right)\left(\frac{R_3C}{2}\right)\left(\frac{1}{R_1C}\right)s}{s^2 + \frac{2}{R_3C}s + \frac{R_1R_2}{R_1R_2R_3C^2}}$$

or

$$H(s) = \frac{-\left(\frac{R_3}{2R_1}\right)\left(\frac{2}{R_3C}s\right)}{s^2 + \frac{2}{R_3C}s + \frac{R_1R_2}{R_1R_2R_3C^2}}$$

Therefore

$$\frac{2}{R_3C} = \beta = \frac{\omega_o}{Q}; \qquad \frac{R_1 + R_2}{R_1R_2R_3C^2} = \omega_o^2;$$

and $K = \dfrac{R_3}{2R_1}$

By hypothesis $C = 1\,\text{F}$ and $\omega_o = 1\,\text{rad/s}$

$$\therefore \quad \frac{2}{R_3} = \frac{1}{Q} \text{ or } R_3 = 2Q$$

$$R_1 = \frac{R_3}{2K} = \frac{Q}{K}$$

$$\frac{R_1 + R_2}{R_1R_2R_3} = 1$$

$$\frac{Q}{K} + R_2 = \left(\frac{Q}{K}\right)(2Q)R_2$$

$$\therefore \quad R_2 = \frac{Q}{2Q^2 - K}$$

P 15.44 [a] First we will design a unity gain filter and then provide the passband gain with an inverting amplifier. For the high pass section the cut-off frequency is 1000 Hz. The order of the Butterworth is

$$n = \frac{(-0.05)(-20)}{\log_{10}(1000/400)} = 2.51$$

$$\therefore \quad n = 3$$

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\text{F}$$

For the prototype second-order section

$$R_1 = 0.5\,\Omega, \quad R_2 = 2\,\Omega, \quad C = 1\,\text{F}$$

The scaling factors are

$$k_f = 2\pi(1000) = 2000\pi$$

$$k_m = \frac{10^9}{50(2000\pi)} = \frac{10^4}{\pi}$$

In the scaled first-order section

$$R_1 = R_2 = \frac{10^4}{\pi}(1) = 3.183\,\text{k}\Omega$$

$$C = 50\text{nF}$$

In the scaled second-order section

$$R_1 = 0.5k_m = 1591.55\,\Omega$$

$$R_2 = 2k_m = 6.366\,\text{k}\Omega$$

$$C = 50\,\text{nF}$$

For the low-pass section the cut-off frequency is 8000 Hz. The order of the Butterworth filter is

$$n = \frac{(-0.05)(-20)}{\log_{10}(20{,}000/8000)} = 2.51; \qquad \therefore \quad n = 3$$

$$H_{\text{lp}}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\text{F}$$

For the prototype second-order section

$R_1 = R_2 = 1\,\Omega; \qquad C_1 = 2\,\text{F}; \qquad C_2 = 0.5\,\text{F}$

The low-pass scaling factors are

$k_m = 5 \times 10^3; \qquad k_f = (8000)(2\pi) = 16{,}000\pi$

For the scaled first-order section

$$R_1 = R_2 = 5\,\text{k}\Omega; \qquad C = \frac{1}{(16{,}000\pi)(5 \times 10^3)} = 3.98\,\text{nF}$$

For the scaled second-order section

$R_1 = R_2 = 5\,\text{k}\Omega$

$$C_1 = \frac{2}{8\pi \times 10^7} = 7.96\,\text{nF}$$

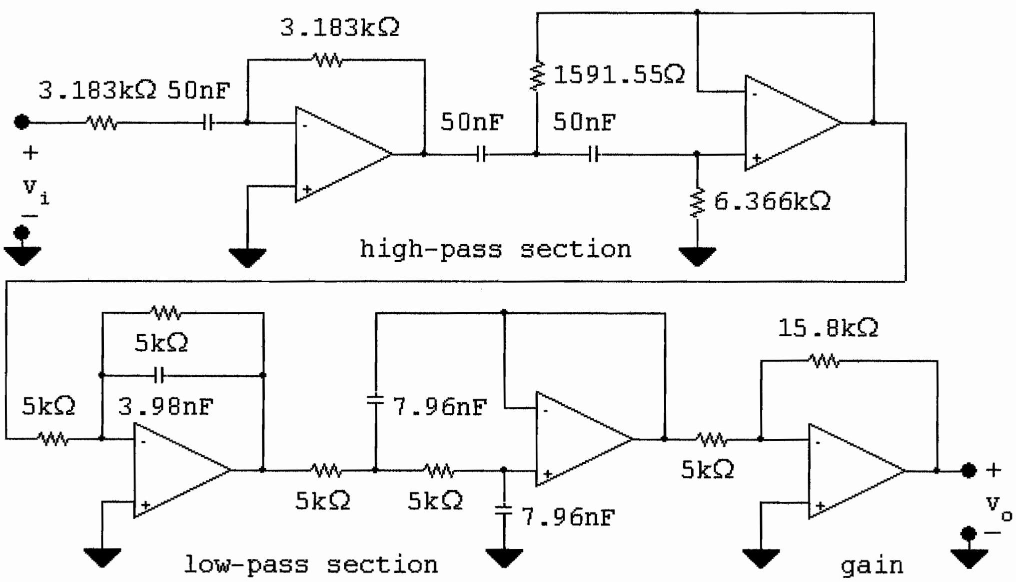
$$C_2 = \frac{0.5}{8\pi \times 10^7} = 1.99\,\text{nF}$$

GAIN AMPLIFIER

$20\log_{10} K = 10$ dB, $\quad \therefore \quad K = 3.16$

Since we are using 5 kΩ resistors in the low-pass stage, we will use $R_f = 15.8\,\text{k}\Omega$ and $R_i = 5\,\text{k}\Omega$ in the inverting amplifier stage.

[b]



P 15.45 [a] Unscaled high-pass stage

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

Frequency scaling factor $k_f = 2000\pi$. Therefore the scaled transfer function is

$$H'_{hp}(s) = \frac{(s/2000\pi)^3}{\left(\frac{s}{2000\pi}+1\right)\left[\left(\frac{s}{2000\pi}\right)^3 + \frac{s}{2000\pi}+1\right]}$$

$$= \frac{s^3}{(s+2000\pi)[s^2+2000\pi s + 4\times 10^6\pi^2]}$$

Unscaled low-pass stage

$$H_{lp}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

Frequency scaling factor $k_f = 16{,}000\pi$. Therefore the scaled transfer function is

$$H'_{lp}(s) = \frac{1}{\left(\frac{s}{16{,}000\pi}+1\right)\left[\left(\frac{s}{16{,}000\pi}\right)^2 + \left(\frac{s}{16{,}000\pi}\right)+1\right]}$$

$$= \frac{(16{,}000\pi)^3}{(s+16{,}000\pi)(s^2+16{,}000\pi s + 256\times 10^6\pi^2)}$$

Thus the transfer function for the filter is

$$H'(s) = 10H'_{hp}(s)H'_{lp}(s) = \frac{4096\times 10^{10}\pi^3 s^3}{D_1D_2D_3D_4}$$

where

$$D_1 = s + 2000\pi$$

$$D_2 = s + 16{,}000\pi$$

$$D_3 = s^2 + 2000\pi s + 4\times 10^6\pi^2$$

$$D_4 = s^2 + 16{,}000\pi s + 256\times 10^6\pi^2$$

[b] At 400 Hz $\quad \omega = 800\pi$ rad/s

$$D_1(j800\pi) = 800\pi(2.5 + j1)$$

$$D_2(j800\pi) = 800\pi(20 + j1)$$

$$D_3(j800\pi) = 16\times 10^5\pi^2(2.1 + j1.0)$$

$$D_4(j800\pi) = 128\times 10^5\pi^2(19.95 + j1)$$

Therefore

$$D_1D_2D_3D_4(j800\pi) = 131{,}072\pi^6 10^{14}(2505.11\underline{/53^\circ})$$

$$H'(j800\pi) = \frac{(4096\pi^3\times 10^{10})(512\times 10^6\pi^3)}{131{,}072\times 10^{14}\pi^6(2505.11\underline{/53^\circ})}$$

$$= 0.639\underline{/-53^\circ}$$

$$\therefore \quad 20\log_{10}|H'(j800\pi)| = 20\log_{10}(0.639) = -3.89 \text{ dB}$$

At $f = 5000\,\text{Hz}, \qquad \omega = 10{,}000\pi\,\text{rad/s}$

Then

$$D_1(j10{,}000\pi) = 2000\pi(1 + j5)$$

$$D_2(j10{,}000\pi) = 10{,}000\pi(1.6 + j1)$$

$$D_3(j10{,}000\pi) = 10^7\pi^2(-9.6 + j2)$$

$$D_4(j10{,}000\pi) = 10^7\pi^2(15.6 + j16)$$

$$H'(j10{,}000\pi) = \frac{(4096 \times \pi^3 \times 10^{10})(10^{12}\pi^3)}{2 \times 10^{21}\pi^6(2108.22\underline{/-35.35^\circ})}$$

$$= 9.71\underline{/35.35^\circ}$$

$$\therefore \quad 20\log_{10}|H'(j10{,}000\pi)| = 19.74 \text{ dB}$$

[c] From the transfer function the gain is down $19.74 + 3.89$ or 23.63 dB at 400 Hz. Because the upper cut-off frequency is eight times the lower cut-off frequency we would expect the high-pass stage of the filter to predict the loss in gain at 400 Hz. For a 3nd order Butterworth

$$\text{GAIN} = 20\log_{10}\frac{1}{\sqrt{1 + (1000/400)^6}} = -23.89 \text{ dB}.$$

5000 Hz is in the passband for this bandpass filter. Hence we expect the gain at 5000 Hz to nearly equal 20 dB as specified in Problem 15.37. Thus our scaled transfer function confirms that the filter meets the specifications.

P 15.46 [a] From Table 15.1

$$H_{lp}(s) = \frac{1}{(s+1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)}$$

$$H_{hp}(s) = \frac{1}{[(1/s) + 1][(1/s)^2 + 0.618(1/s) + 1][(1/s)^2 + 1.618(1/s) + 1]}$$

$$H_{hp}(s) = \frac{s^5}{(s+1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)}$$

P 15.47 [a] $k_f = 10{,}000$

$$H'_{\text{hp}}(s) = \frac{(s/10{,}000)^5}{[(s/10{,}000) + 1]}$$

$$\cdot \frac{1}{[(s/10{,}000)^2 + 0.618s/10{,}000 + 1][(s/10{,}000)^2 + 1.618s/10{,}000 + 1]}$$

$$= \frac{s^5}{(s + 10{,}000)(s^2 + 6180s + 10^8)(s^2 + 16{,}180s + 10^8)}$$

[b] $$H'(j10{,}000) = \frac{j(10{,}000)^5}{[10{,}000(j+1)][6180(j10{,}000)][16{,}180(j10{,}000)]}$$

$$= \frac{j(10{,}000)^2}{(1+j)(6180)(16{,}180)j^2}$$

$$= 0.7072\underline{/-45^\circ}$$

$$20\log_{10}|H'(j10{,}000)| = -3.01 \text{ dB}$$

P 15.48 [a] At very low frequencies the two capacitor branches are open and because the op amp is ideal the current in R_3 is zero. Therefore at low frequencies the circuit behaves as an inverting amplifier with a gain of R_2/R_1. At very high frequencies the capacitor branches are short circuits and hence the output voltage is zero.

[b] Let the node where R_1, R_2, R_3, and C_2 join be denoted as a, then

$$(V_a - V_i)G_1 + V_a sC_2 + (V_a - V_o)G_2 + V_a G_3 = 0$$

$$-V_a G_3 - V_o sC_1 = 0$$

or

$$(G_1 + G_2 + G_3 + sC_2)V_a - G_2 V_o = G_1 V_i$$

$$V_a = \frac{-sC_1}{G_3} V_o$$

Solving for V_o/V_i yields

$$H(s) = \frac{-G_1G_3}{(G_1 + G_2 + G_3 + sC_2)sC_1 + G_2G_3}$$

$$= \frac{-G_1G_3}{s^2C_1C_2 + (G_1 + G_2 + G_3)C_1 s + G_2G_3}$$

$$= \frac{-G_1G_3/C_1C_2}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right]s + \frac{G_2G_3}{C_1C_2}}$$

$$= \frac{-\frac{G_1G_2G_3}{G_2C_1C_2}}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right]s + \frac{G_2G_3}{C_1C_2}}$$

$$= \frac{-Kb_o}{s^2 + b_1 s + b_o}$$

where $K = \dfrac{G_1}{G_2}$; $\qquad b_o = \dfrac{G_2G_3}{C_1C_2}$

and $b_1 = \dfrac{G_1 + G_2 + G_3}{C_2}$

[c] Equating coefficients we see that

$$G_1 = KG_2$$

$$G_3 = \frac{b_oC_1C_2}{G_2} = \frac{b_oC_1}{G_2}$$

since by hypothesis $C_2 = 1\,\text{F}$

$$b_1 = \frac{G_1 + G_2 + G_3}{C_2} = G_1 + G_2 + G_3$$

$$\therefore\ b_1 = KG_2 + G_2 + \frac{b_oC_1}{G_2}$$

$$b_1 = G_2(1+K) + \frac{b_oC_1}{G_2}$$

Solving this quadratic equation for G_2 we get

$$G_2 = \frac{b_1}{2(1+K)} \pm \sqrt{\frac{b_1^2 - b_oC_14(1+K)}{4(1+K)^2}}$$

$$= \frac{b_1 \pm \sqrt{b_1^2 - 4b_o(1+K)C_1}}{2(1+K)}$$

For G_2 to be realizable

$$C_1 < \frac{b_1^2}{4b_o(1+K)}$$

[d] 1. Select $C_2 = 1\,\text{F}$

2. Select C_1 such that $C_1 < \dfrac{b_1^2}{4b_o(1+K)}$

3. Calculate $G_2(R_2)$

4. Calculate $G_1(R_1)$; $G_1 = KG_2$

5. Calculate $G_3(R_3)$; $G_3 = b_oC_1/G_2$

P 15.49 $b_1 = b_o = 1$

[a] $C_1 = \dfrac{1}{4(1+K)} = \dfrac{1}{36}\,\text{F}$

[b] $G_2 = \dfrac{1}{2(1+K)} = \dfrac{1}{18}\,\text{S}; \qquad \therefore\quad R_2 = 18\,\Omega$

$G_1 = 8G_2 = \dfrac{8}{18}\,\text{S}; \qquad \therefore\quad R_1 = \dfrac{18}{8} = 2.25\,\Omega$

$G_3 = \dfrac{1}{G_2}C_1 = (18)\left(\dfrac{1}{36}\right) = \dfrac{1}{2}\,\text{S}; \qquad \therefore\quad R_3 = 2\,\Omega$

[c] $f_c = 50\,\text{kHz}; \qquad \omega_c = 100\pi\,\text{krad/s}$

$k_f = 10^5\pi; \qquad 250\times10^{-12} = \dfrac{1}{10^5\pi k_m}; \qquad \therefore\quad k_m = \dfrac{40}{\pi}\times10^3$

$R_1 = 2.25(40/\pi)10^3 = 28.65\,\text{k}\Omega$

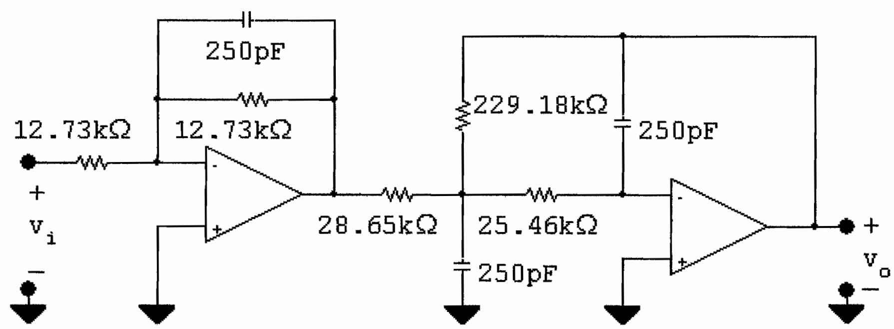
$R_2 = 18(40/\pi)10^3 = 229.18\,\text{k}\Omega$

$R_3 = 2(40/\pi)10^3 = 25.46\,\text{k}\Omega$

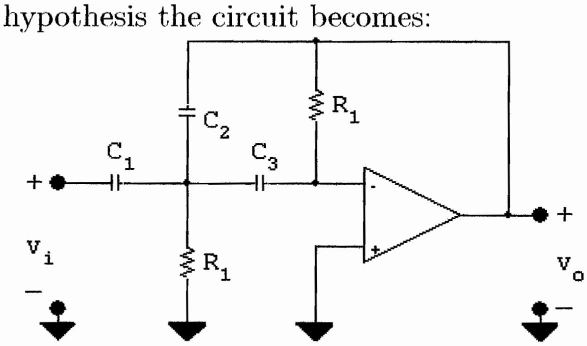
[d] $R_1 = R_2 = k_m = \dfrac{40}{\pi}\times10^3 = 12.73\,\text{k}\Omega$

$C = \dfrac{1}{k_f k_m} = 250\,\text{pF}$

[e]



P 15.50 [a] By hypothesis the circuit becomes:



For very small frequencies the capacitors behave as open circuits and therefore v_o is zero. As the frequency increases, the capacitive branch impedances become small compared to the resistive branches. When this happens the circuit becomes an inverting amplifier with the capacitor C_2 dominating the feedback path. Hence the gain of the amplifier approaches $(1/j\omega C_2)/(1/j\omega C_1)$ or C_1/C_2. Therefore the circuit behaves like a high-pass filter with a passband gain of C_1/C_2.

[b] Summing the currents away from the upper terminal of R_2 yields

$$V_aG_2 + (V_a - V_i)sC_1 + (V_a - V_o)sC_2 + V_asC_3 = 0$$

or

$$V_a[G_2 + s(C_1 + C_2 + C_3)] - V_osC_2 = sC_1V_i$$

Summing the currents away from the inverting input terminal gives

$$(0 - V_a)sC_3 + (0 - V_o)G_1 = 0$$

or

$$sC_3V_a = -G_1V_o; \qquad V_a = \frac{-G_1V_o}{sC_3}$$

Therefore we can write

$$\frac{-G_1V_o}{sC_3}[G_2 + s(C_1 + C_2 + C_3)] - sC_2V_o = sC_1V_i$$

Solving for V_o/V_i gives

$$H(s) = \frac{V_o}{V_i} = \frac{-C_1C_3s^2}{C_2C_3s^2 + G_1(C_1 + C_2 + C_3)s + G_1G_2]}$$

$$= \frac{\frac{-C_1}{C_2}s^2}{\left[s^2 + \frac{G_1}{C_2C_3}(C_1 + C_2 + C_3)s + \frac{G_1G_2}{C_2C_3}\right]}$$

$$= \frac{-Ks^2}{s^2 + b_1s + b_o}$$

Therefore the circuit implements a second-order high-pass filter with a passband gain of C_1/C_2.

[c] $C_1 = K$:

$$b_1 = \frac{G_1}{(1)(1)}(K + 2) = G_1(K + 2)$$

$$\therefore \quad G_1 = \frac{b_1}{K + 2}; \qquad R_1 = \left(\frac{K + 2}{b_1}\right)$$

$$b_o = \frac{G_1G_2}{(1)(1)} = G_1G_2$$

$$\therefore \quad G_2 = \frac{b_o}{G_1} = \frac{b_o}{b_1}(K + 2)$$

$$\therefore \quad R_2 = \frac{b_1}{b_o(K + 2)}$$

[d] From Table 15.1 the transfer function of the second-order section of a third-order high-pass Butterworth filter is

$$H(s) = \frac{Ks^2}{s^2 + s + 1}$$

Therefore $b_1 = b_o = 1$

Thus

$$C_1 = K = 8\,\text{F}$$

$$R_1 = \frac{8+2}{1} = 10\,\Omega$$

$$R_2 = \frac{1}{1(8+2)} = 0.10\,\Omega$$

P 15.51 [a] Low-pass filter with a gain of 0 dB (handle 20 dB passband gain in a separate gain section):

$$n = \frac{(-0.05)(-20)}{\log_{10}(1500/800)} = 3.66; \qquad \therefore \quad n = 4$$

In the first prototype second-order section: $b_1 = 0.765$, $b_o = 1$, $C_2 = 1\,\text{F}$

$$C_1 \leq \frac{b_1^2}{4b_o(1+K)} \leq \frac{(0.765)^2}{(4)(2)} \leq 0.073$$

choose $C_1 = 0.05\,\text{F}$

$$G_2 = \frac{0.765 \pm \sqrt{(0.765)^2 - 4(2)(0.05)}}{2(1+1)} = \frac{0.765 \pm 0.430}{4}$$

Arbitrarily select the larger value for G_2, then

$$G_2 = 0.3\ \text{S}; \therefore \quad R_2 = 3.33\,\Omega$$

$$G_1 = KG_2 = 0.3\ \text{S}; \qquad R_1 = 3.33\,\Omega$$

$$G_3 = \frac{b_oC_1}{G_2} = \frac{(1)(0.05)}{0.3} = 0.167$$

$$R_3 = 1/G_3 = 6\,\Omega$$

Therefore in the first second-order prototype circuit

$$R_1 = 3.33\,\Omega; \qquad R_2 = 3.33\,\Omega; \qquad R_3 = 6\,\Omega$$

$$C_1 = 0.05\,\text{F}; \qquad C_2 = 1\,\text{F}$$

In the second second-order prototype circuit:
$b_1 = 1.848$, $b_0 = 1$, $C_2 = 1\,\text{F}$

$$\therefore \quad C_1 \leq \frac{(1.848)^2}{8} \leq 0.427$$

choose $C_1 = 0.3\,\mathrm{F}$

$$G_2 = \frac{1.848 \pm \sqrt{(1.848)^2 - 8(0.3)}}{4}$$

$$= \frac{1.848 \pm 1.008}{4}$$

Arbitrarily select the larger value, then

$G_2 = 0.71\ \mathrm{S}; \therefore\ R_2 = 1.4\,\Omega$

$G_1 = KG_2 = 0.71\ \mathrm{S}; \qquad R_1 = 1.4\,\Omega$

$$G_3 = \frac{b_oC_1}{G_2} = \frac{(1)(0.3)}{0.71} = 0.42\ \mathrm{S}$$

$R_3 = 1/G_3 = 2.4\,\Omega$

In the low-pass section of the filter

$k_f = 2\pi(800) = 1600\pi$

$$k_m = \frac{C_2}{C_2' k_f} = \frac{1}{50 \times 10^{-9} k_f} = \frac{12{,}500}{\pi}$$

Therefore in the first scaled second-order section

$R_1 = 3.33k_m = 13.25\,\mathrm{k}\Omega$

$R_2 = 3.33k_m = 13.25\,\mathrm{k}\Omega$

$R_3 = 6k_m = 23.87\,\mathrm{k}\Omega$

$$C_1 = \frac{0.05}{(1600\pi)(12{,}500/\pi)} = 2.5\,\mathrm{nF}$$

$C_2 = 50\,\mathrm{nF}$

In the second scaled second-order section

$R_1 = 1.4k_m = 5.57\,\mathrm{k}\Omega$

$R_2 = 1.4k_m = 5.57\,\mathrm{k}\Omega$

$R_3 = 2.4k_m = 9.55\,\mathrm{k}\Omega$

$$C_1 = \frac{0.3}{(1600\pi)(12{,}500/\pi)} = 15\,\mathrm{nF}$$

$C_2 = 50\,\mathrm{nF}$

High-pass filter section with a gain of 0 dB (handle 20 dB passband gain in a separate gain section):

$$n = \frac{(-0.05)(-20)}{\log_{10}(13{,}500/7200)} = 3.66; \qquad n = 4.$$

In the first prototype second-order section:
$b_1 = 0.765$; $b_o = 1$; $C_2 = C_3 = 1\,\text{F}$

$$C_1 = K = 1\,\text{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{0.765} = 3.92\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{0.765}{3} = 0.255\,\Omega$$

In the second prototype second-order section: $b_1 = 1.848$; $b_o = 1$; $C_2 = C_3 = 1\,\text{F}$

$$C_1 = K = 1\,\text{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{1.848} = 1.62\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{1.848}{3} = 0.616\,\Omega$$

In the high-pass section of the filter

$$k_f = 2\pi(7200) = 14{,}400\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{50 \times 10^{-9}k_f} = \frac{1389}{\pi}$$

In the first scaled second-order section

$$R_1 = 3.92k_m = 1.73\,\text{k}\Omega$$

$$R_2 = 0.255k_m = 113\,\Omega$$

$$C_1 = C_2 = C_3 = 50\,\text{nF}$$

In the second scaled second-order section

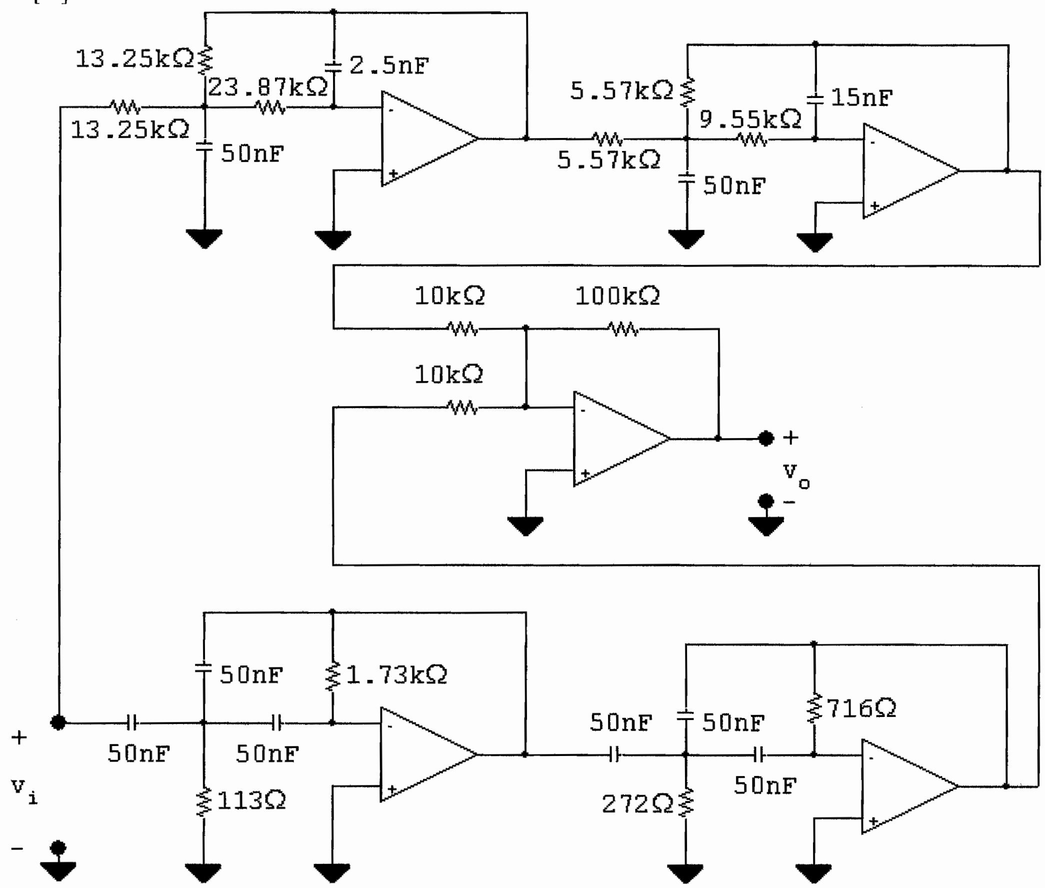
$$R_1 = 1.62k_m = 716\,\Omega$$

$$R_2 = 0.616k_m = 272\,\Omega$$

$$C_1 = C_2 = C_3 = 50\,\text{nF}$$

In the gain section, let $R_i = 10\,\text{k}\Omega$ and $R_f = 100\,\text{k}\Omega$.

[b]



P 15.52 [a] The prototype low-pass transfer function is

$$H_{lp}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The low-pass frequency scaling factor is

$$k_{f_{lp}} = 2\pi(800) = 1600\pi$$

The scaled transfer function for the low-pass filter is

$$H'_{lp}(s) = \frac{1}{\left[\left(\frac{s}{1600\pi}\right)^2 + \frac{0.765s}{1600\pi} + 1\right]\left[\left(\frac{s}{1600\pi}\right)^2 + \frac{1.848s}{1600\pi} + 1\right]}$$

$$= \frac{65{,}536 \times 10^8\pi^4}{[s^2 + 1224\pi s + (1600\pi)^2]\,[s^2 + 2956.8\pi s + (1600\pi)^2]}$$

The prototype high-pass transfer function is

$$H_{hp}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The high-pass frequency scaling factor is

$$k_{f_{hp}} = 2\pi(7200) = 14{,}400\pi$$

The scaled transfer function for the high-pass filter is

$$H'_{hp}(s) = \frac{(s/14{,}400\pi)^4}{\left[\left(\frac{s}{14{,}400\pi}\right)^2 + \frac{0.765s}{14{,}400\pi} + 1\right]\left[\left(\frac{s}{14{,}400\pi}\right)^2 + \frac{1.848s}{14{,}400\pi} + 1\right]}$$

$$= \frac{s^4}{[s^2 + 11{,}016\pi s + (14{,}400\pi)^2][s^2 + 26{,}611.2\pi s + (14{,}400\pi)^2]}$$

The transfer function for the filter is

$$H'(s) = \left[H'_{lp}(s) + H'_{hp}(s)\right](-10)$$

[b] $f_o = \sqrt{f_{c1}f_{c2}} = \sqrt{800)(7200)} = 2400\,\text{Hz}$

$$\omega_o = 4800\pi\,\text{rad/s}$$

$$(j\omega_o)^2 = -2304 \times 10^4\pi^2$$

$$(j\omega_o)^4 = 5{,}308{,}416 \times 10^8\pi^4$$

$$H'_{lp}(j\omega_o) = \frac{65{,}536 \times 10^8\pi^4}{[-2048 \times 10^4\pi^2 + j1224(4800\pi^2)]} \times \frac{1}{[-2048 \times 10^4\pi^2 + j2956.8(4800\pi^2)]}$$

$$= 0.0123\underline{/50.73^\circ}$$

$$H'_{hp}(j\omega_o) = \frac{5{,}308{,}416 \times 10^8\pi^4}{[18{,}432 \times 10^4\pi^2 + j11{,}016(4800\pi^2)]} \frac{1}{[18{,}432 \times 10^4\pi^2 + j26{,}611.2(4800\pi^2)]}$$

$$= 0.0123\underline{/-50.73^\circ}$$

$$\therefore \quad H'(j\omega_o) = 0.0123(1\underline{/50.73^\circ} + 1\underline{/-50.73^\circ})(-10) = -0.1557\underline{/0^\circ}$$

$$G = 20\log_{10}|H'(j\omega_o)| = 20\log_{10}(0.1557) = -16.15\text{ dB}$$

P 15.53 [a] At low frequencies the capacitor branches are open; $v_o = v_i$. At high frequencies the capacitor branches are short circuits and the output voltage is zero. Hence the circuit behaves like a unity-gain low-pass filter.

[b] Let v_a represent the voltage-to-ground at the right-hand terminal of R_1. Observe this will also be the voltage at the left-hand terminal of R_2. The s-domain equations are

$$(V_a - V_i)G_1 + (V_a - V_o)sC_1 = 0$$

$$(V_o - V_a)G_2 + sC_2V_o = 0$$

or

$$(G_1 + sC_1)V_a - sC_1V_o = G_1V_i$$

$$-G_2V_a + (G_2 + sC_2)V_o = 0$$

$$\therefore \quad V_a = \frac{G_2 + sC_2V_o}{G_2}$$

$$\therefore \quad \left[(G_1 + sC_1)\frac{(G_2 + sC_2)}{G_2} - sC_1\right] V_o = G_1V_i$$

$$\therefore \quad \frac{V_o}{V_i} = \frac{G_1G_2}{(G_1 + sC_1)(G_2 + sC_2) - C_1G_2s}$$

which reduces to

$$\frac{V_o}{V_i} = \frac{G_1G_2/C_1C_2}{s^2 + \frac{G_1}{C_1}s + \frac{G_1G_2}{C_1C_2}} = \frac{b_o}{s^2 + b_1s + b_o}$$

[c] There are four circuit components and two restraints imposed by $H(s)$; therefore there are two free choices.

[d] $b_1 = \dfrac{G_1}{C_1} \quad \therefore \quad G_1 = b_1C_1$

$$b_o = \frac{G_1G_2}{C_1C_2} \quad \therefore \quad G_2 = \frac{b_o}{b_1}C_2$$

[e] No, all physically realizeable capacitors will yield physically realizeable resistors.

[f] From Table 15.1 we know the transfer function of the prototype 4th order Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

In the first section $b_o = 1, \quad b_1 = 0.765$

$$\therefore \quad G_1 = (0.765)(1) = 0.765 \text{ S}$$

$$R_1 = 1/G_1 = 1.307\,\Omega$$

$$G_2 = \frac{1}{0.765}(1) = 1.307 \text{ S}$$

$$R_2 = 1/G_2 = 0.765\,\Omega$$

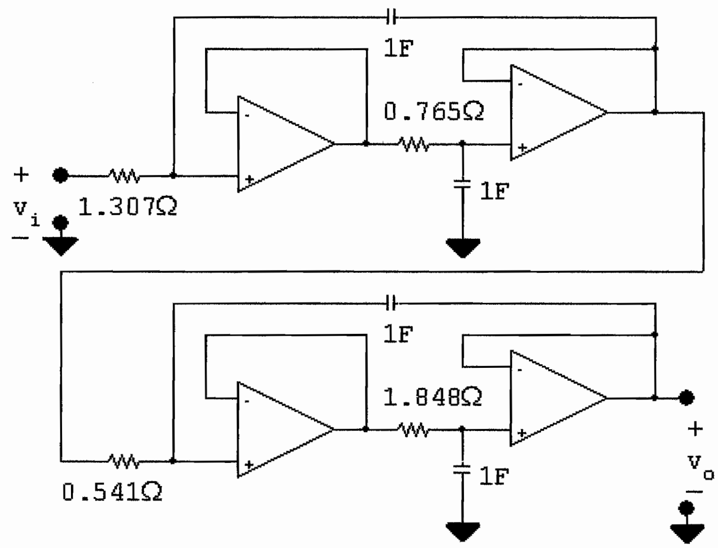
In the second section $b_o = 1, \quad b_1 = 1.848$

$$\therefore \quad G_1 = 1.848\,\text{S}$$

$$R_1 = 1/G_1 = 0.541\,\Omega$$

$$G_2 = \left(\frac{1}{1.848}\right)(1) = 0.541\,\text{S}$$

$$R_2 = 1/G_2 = 1.848\,\Omega$$



P 15.54 [a] $k_f = 2\pi(25) \times 10^3 = 50\pi \times 10^3$

$$k_m = \frac{10^{12}}{50\pi \times 10^3(750)} = \frac{80}{3\pi} \times 10^3$$

In the first section

$$R_1 = \frac{1}{0.765} \cdot \frac{80}{3\pi}(10^3) = 11.10\,\text{k}\Omega$$

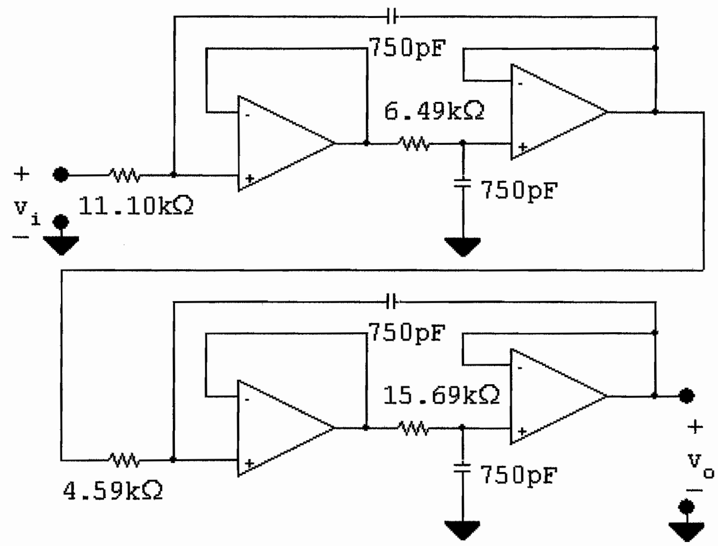
$$R_2 = (0.765)\frac{80}{3\pi}(10^3) = 6.49\,\text{k}\Omega$$

In the second section

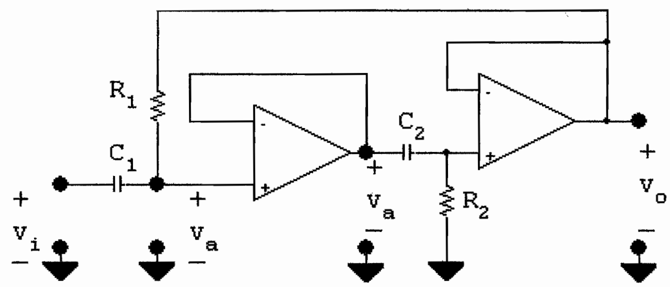
$$R_1 = \frac{1}{1.848} \cdot \frac{80}{3\pi}(10^3) = 4.59\,\text{k}\Omega$$

$$R_2 = (1.848)\frac{80}{3\pi}(10^3) = 15.69\,\text{k}\Omega$$

[b]



P 15.55 [a] Interchanging the Rs and Cs yields the following circuit.



At low frequencies the capacitors appear as open circuits and hence the output voltage is zero. As the frequency increases the capacitor branches approach short circuits and $v_a = v_i = v_o$. Thus the circuit is a unity-gain, high-pass filter.

[b] The s-domain equations are

$$(V_a - V_i)sC_1 + (V_a - V_o)G_1 = 0$$

$$(V_o - V_a)sC_2 + V_oG_2 = 0$$

It follows that

$$V_a(G_1 + sC_1) - G_1V_o = sC_1V_i$$

and $V_a = \dfrac{(G_2 + sC_2)V_o}{sC_2}$

Thus

$$\left\{\left[\frac{(G_2 + sC_2)}{sC_2}\right](G_1 + sC_1) - G_1\right\}V_o = sC_1V_i$$

$$V_o\{s^2C_1C_2 + sC_1G_2 + G_1G_2\} = s^2C_1C_2V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{s^2}{\left(s^2 + \frac{G_2}{C_2}s + \frac{G_1G_2}{C_1C_2}\right)}$$

$$= \frac{V_o}{V_i} = \frac{s^2}{s^2 + b_1 s + b_o}$$

[c] There are 4 circuit components: R_1, R_2, C_1 and C_2.
There are two transfer function constraints: b_1 and b_o.
Therefore there are two free choices.

[d] $b_o = \frac{G_1G_2}{C_1C_2}; \qquad b_1 = \frac{G_2}{C_2}$

$$\therefore \quad G_2 = b_1C_2; \qquad R_2 = \frac{1}{b_1C_2}$$

$$G_1 = \frac{b_o}{b_1}C_1 \therefore \quad R_1 = \frac{b_1}{b_oC_1}$$

[e] No, all realizeable capacitors will produce realizeable resistors.

[f] The second-order section in a 3rd-order Butterworth high-pass filter is $s^2/(s^2 + s + 1)$. Therefore $b_o = b_1 = 1$ and

$$R_1 = \frac{1}{(1)(1)} = 1\,\Omega.$$

$$R_2 = \frac{1}{(1)(1)} = 1\,\Omega.$$

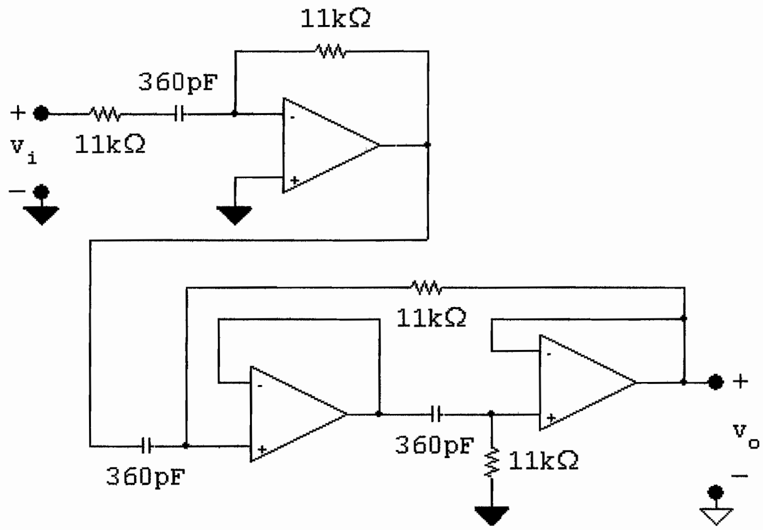
P 15.56 [a] $f_c = 40\,\text{kHz}; \qquad \omega_c = 80\pi\,\text{krad/s}; \qquad \therefore \quad k_f = 8\pi \times 10^4$

$$k_m = \frac{10^{12}}{8\pi \times 10^4(360)} = 11.05 \times 10^3$$

$$\therefore \quad R_1 = R_2 = k_m = 11\,\text{k}\Omega$$

[b] $C = 360\,\text{pF}$

[c]



[d] $H'(s) = \dfrac{(s/8\pi \times 10^4)^3}{\left[\left(\frac{s}{8\pi\times10^4}\right)+1\right]\left[\left(\frac{s}{8\pi\times10^4}\right)^2+\frac{s}{8\pi\times10^4}+1\right]}$

$$= \frac{s^3}{(s+8\pi\times10^4)(s^2+8\pi\times10^4 s+64\pi^2\times10^8)}$$

[e] $H'(j8\pi\times10^4) = \dfrac{(j8\pi\times10^4)^3}{(8\pi\times10^4+j8\pi\times10^4)(j(8\pi\times10^4)(8\pi\times10^4)}$

$$= \frac{-j}{j(1+j1)} = \frac{1}{\sqrt{2}}\underline{/135^\circ}$$

$$\text{GAIN} = 20\log_{10}\frac{1}{\sqrt{2}} = -3.01 \text{ dB}$$

P 15.57 [a] It follows directly from Eq 15.65 that

$$H(s) = \frac{s^2+1}{s^2+4(1-\sigma)s+1}$$

Now note from Eq 15.69 that $(1-\sigma)$ equals $1/4Q$, hence

$$H(s) = \frac{s^2+1}{s^2+\frac{1}{Q}s+1}$$

[b] For Example 15.13 $\omega_o = 5000$ rad/s and $Q = 5$. Therefore $k_f = 5000$ and

$$H'(s) = \frac{(s/5000)^2+1}{(s/5000)^2+\frac{1}{5}\left(\frac{s}{5000}\right)+1}$$

$$= \frac{s^2+25\times10^6}{s^2+1000s+25\times10^6}$$

P 15.58 [a] $\omega_o = 8000\pi$ rad/s

$$\therefore \quad k_f = \frac{\omega_o'}{\omega_o} = 8000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(150 \times 10^{-9})(8000\pi)} = \frac{833.33}{\pi}$$

$$R' = k_m R = \frac{833.33}{\pi}(1) = 265\,\Omega$$

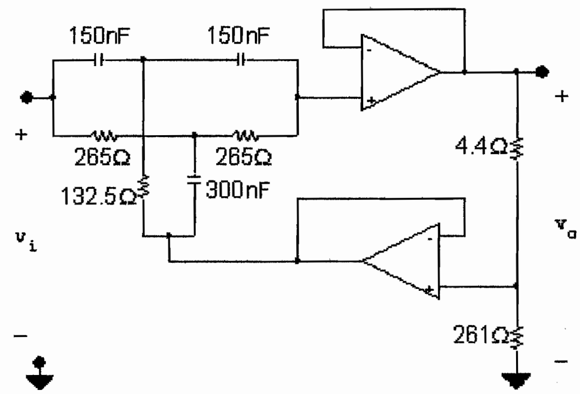
$$\sigma = 1 - \frac{1}{4Q} = 1 - \frac{1}{4(15)} = 0.9833$$

$$\sigma R' = 261\,\Omega; \qquad (1-\sigma)R' = 4.4\,\Omega$$

$$C' = 150\,\text{nF}$$

$$2C' = 300\,\text{nF}$$

[b]



[c] $k_f = 8000\pi$

$$H(s) = \frac{(s/8000\pi)^2 + 1}{(s/8000\pi)^2 + \frac{1}{15}(s/8000\pi) + 1}$$

$$= \frac{s^2 + 64 \times 10^6\pi^2}{s^2 + 533.33\pi s + 64 \times 10^6\pi^2}$$

P 15.59 To satisfy the gain specification of 20 dB at $\omega = 0$ and $\alpha = 1$ requires

$$\frac{R_1 + R_2}{R_1} = 10 \qquad \text{or} \qquad R_2 = 9R_1$$

Choose a standard resistor of 11.1 kΩ for R_1 and a 100 kΩ potentiometer for R_2. Since $(R_1 + R_2)/R_1 \gg 1$ the value of C_1 is

$$C_1 = \frac{1}{2\pi(40)(10^5)} = 39.79 \text{ nF}$$

Choose a standard capacitor value of 39 nF. Using the selected values of R_1 and R_2 the maximum gain for $\alpha = 1$ is

$$20\log_{10}\left(\frac{111.1}{11.1}\right)_{\alpha=1} = 20.01 \text{ dB}$$

When $C_1 = 39$ nF the frequency $1/R_2C_1$ is

$$\frac{1}{R_2C_1} = \frac{10^9}{10^5(39)} = 256.41 \text{ rad/s} = 40.81 \text{ Hz}$$

The magnitude of the transfer function at 256.41 rad/s is

$$|H(j256.41)|_{\alpha=1} = \frac{|111.1 \times 10^3 + j256.41(11.1)(100)(39)10^{-3}|}{11.1 \times 10^3 + j256.41(11.1)(100)(39)10^{-3}|} = 7.11$$

Therefore the gain at 40.81 Hz is

$$20\log_{10}(7.11)_{\alpha=1} = 17.04 \text{ dB}$$

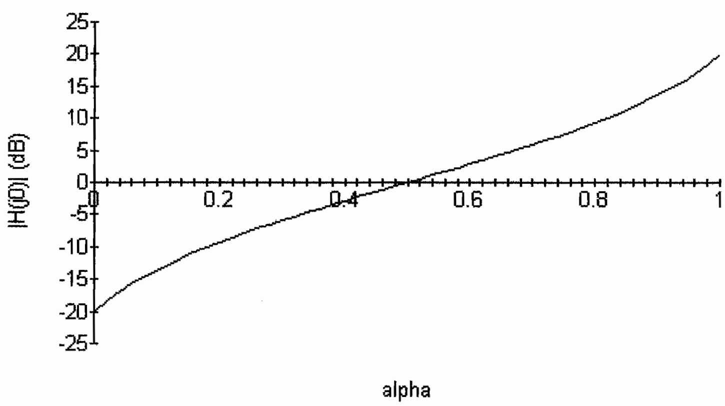
P 15.60 $20\log_{10}\left(\frac{R_1+R_2}{R_1}\right) = 13.98$

$$\therefore \quad \frac{R_1+R_2}{R_1} = 5; \qquad \therefore \quad R_2 = 4R_1$$

Choose $R_1 = 100\,\text{k}\Omega$. Then $R_2 = 400\,\text{k}\Omega$

$$\frac{1}{R_2C_1} = 100\pi \text{ rad/s}; \qquad \therefore \quad C_1 = \frac{1}{(100\pi)(400\times 10^3)} = 7.96\,\text{nF}$$

P 15.61 [a] $|H(j0)| = \dfrac{R_1 + \alpha R_2}{R_1 + (1-\alpha)R_2} = \dfrac{11.1 + \alpha(100)}{11.1 + (1-\alpha)100}$

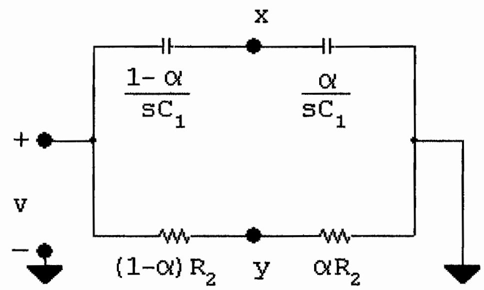


P 15.62 [a] Combine the impedances of the capacitors in series in Fig. P15.62(b) to get

$$C_{eq} = \frac{1-\alpha}{sC_1} + \frac{\alpha}{sC_1} = \frac{1}{sC_1}$$

which is identical to the impedance of the capacitor in Fig. P15.58(a).

[b]



$$V_x = \frac{\alpha/sC_1}{(1-\alpha)/sC_1 + \alpha/sC_1}V = \alpha$$

$$V_y = \frac{\alpha R_2}{(1-\alpha)R_2 + \alpha R_2} = \alpha = V_x$$

[c] Since x and y are both at the same potential, they can be shorted together, and the circuit in Fig. 15.34 can thus be drawn as shown in Fig. 15.53(c).

[d] The feedback path between V_o and V_s containing the resistance $R_4 + 2R_3$ has no effect on the ratio V_o/V_s, as this feedback path is not involved in the nodal equation that defines the voltage ratio. In addition, the resistor attached to the inverting terminal has no effect on the voltage ratio, since for an ideal op amp no current flows through this resistor. Thus, the circuit in Fig. 15.62(c) can be simplified into the form of Fig. 15.2, where the input impedance is the equivalent impedance of R_1 in series with the parallel combination of $(1-\alpha)/sC_1$ and $(1-\alpha)R_2$, and the feedback impedance is the equivalent impedance of R_1 in series with the parallel combination of α/sC_1 and αR_2:

$$Z_i = R_1 + \frac{\frac{(1-\alpha)}{sC_1}\cdot(1-\alpha)R_2}{(1-\alpha)R_2 + \frac{(1-\alpha)}{sC_1}}$$

$$= \frac{R_1 + (1-\alpha)R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

$$Z_f = R_1 + \frac{\frac{\alpha}{sC_1}\cdot\alpha R_2}{\alpha R_2 + \frac{\alpha}{sC_1}}$$

$$= \frac{R_1 + \alpha R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

P 15.63 As $\omega \rightarrow 0$

$$|H(j\omega)| \rightarrow \frac{2R_3 + R_4}{2R_3 + R_4} = 1$$

Therefore the circuit would have no effect on low frequency signals. As $\omega \to \infty$

$$|H(j\omega)| \to \frac{[(1-\beta)R_4 + R_o](\beta R_4 + R_3)}{[(1-\beta)R_4 + R_3](\beta R_4 + R_o)}$$

When $\beta = 1$

$$|H(j\infty)|_{\beta=1} = \frac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=1} \cong \frac{R_o}{R_3} > 1$$

Thus, when $\beta = 1$ we have amplification or "boost". When $\beta = 0$

$$|H(j\infty)|_{\beta=0} = \frac{R_3(R_4 + R_3)}{R_o(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=0} \cong \frac{R_3}{R_0} < 1$$

Thus, when $\beta = 0$ we have attenuation or "cut".
Also note that when $\beta = 0.5$

$$|H(j\omega)|_{\beta=0.5} = \frac{(0.5R_4 + R_o)(0.5R_4 + R_3)}{(0.5R_4 + R_3)(0.5R_4 + R_o)} = 1$$

Thus, the transition from amplification to attenuation occurs at $\beta = 0.5$. If $\beta > 0.5$ we have amplification, and if $\beta < 0.5$ we have attenuation.
Also note the amplification an attenuation are symmetric about $\beta = 0.5$. i.e.

$$|H(j\omega)|_{\beta=0.6} = \frac{1}{|H(j\omega)|_{\beta=0.4}}$$

Yes, the circuit can be used as a treble volume control because

- The circuit has no effect on low frequency signals
- Depending on β the circuit can either amplify ($\beta > 0.5$) or attenuate ($\beta < 0.5$) signals in the treble range
- The amplification (boost) and attenuation (cut) are symmetric around $\beta = 0.5$. When $\beta = 0.5$ the circuit has no effect on signals in the treble frequency range.

P 15.64 [a] $$|H(j\infty)|_{\beta=1} = \frac{R_o(R_4+R_3)}{R_3(R_4+R_o)} = \frac{(65.9)(505.9)}{(5.9)(565.9)} = 9.99$$

$\therefore$ maximum boost $= 20\log_{10} 9.99 = 19.99$ dB

[b] $$|H(j\infty)|_{\beta=0} = \frac{R_3(R_4+R_3)}{R_o(R_4+R_o)}$$

$\therefore$ maximum cut $= -19.99$ dB

[c] $R_4 = 500\,\text{k}\Omega;\qquad R_o = R_1 + R_3 + 2R_2 = 65.9\,\text{k}\Omega$

$\therefore\ R_4 = 7.59R_o$

Yes, R_4 is significantly greater than R_o.

[d] $$|H(j/R_3C_2)|_{\beta=1} = \left|\frac{(2R_3+R_4) + j\frac{R_o}{R_3}(R_4+R_3)}{(2R_3+R_4)+j(R_4+R_o)}\right|$$

$$= \left|\frac{511.8 + j\frac{65.9}{5.9}(505.9)}{511.8 + j565.9}\right|$$

$$= 7.44$$

$20\log_{10}|H(j/R_3C_2)|_{\beta=1} = 20\log_{10} 7.44 = 17.43$ dB

[e] When $\beta = 0$

$$|H(j/R_3C_2)|_{\beta=0} = \frac{(2R_3+R_4)+j(R_4+R_o)}{(2R_3+R_4)+j\dfrac{R_o}{R_3}(R_4+R_3)}$$

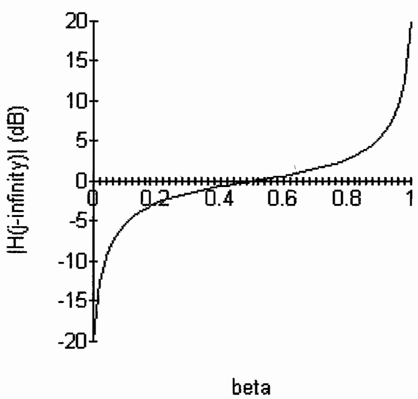
Note this is the reciprocal of $|H(j/R_3C_2)|_{\beta=1}$.

$\therefore\ 20\log_{10}|H(j/R_3C_2)|_{\beta+0} = -17.43$ dB

[f] The frequency $1/R_3C_2$ is very nearly where the gain is 3 dB off from its maximum boost or cut. Therefore for frequencies higher than $1/R_3C_2$ the circuit designer knows that gain or cut will be within 3 dB of the maximum.

P 15.65 $$|H(j\infty)| = \frac{[(1-\beta)R_4 + R_o][\beta R_4 + R_3]}{[(1-\beta R_4 + R_3][\beta R_4 + R_3]}$$

$$= \frac{[(1-\beta)500 + 65.9][\beta 500 + 5.9]}{[(1-\beta)500 + 5.9][\beta 500 + 65.9]}$$



16

Fourier Series

Assessment Problems

AP 16.1

$$a_v = \frac{1}{T}\int_0^{2T/3} V_m\,dt + \frac{1}{T}\int_{2T/3}^{T}\left(\frac{V_m}{3}\right)dt = \frac{7}{9}V_m = 7\pi\,\text{V}$$

$$a_k = \frac{2}{T}\left[\int_0^{2T/3} V_m \cos\,k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\cos\,k\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\sin\left(\frac{4k\pi}{3}\right) = \left(\frac{6}{k}\right)\sin\left(\frac{4k\pi}{3}\right)$$

$$b_k = \frac{2}{T}\left[\int_0^{2T/3} V_m \sin k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\sin k\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\left[1 - \cos\left(\frac{4k\pi}{3}\right)\right] = \left(\frac{6}{k}\right)\left[1 - \cos\left(\frac{4k\pi}{3}\right)\right]$$

AP 16.2 [a] $a_v = 7\pi = 21.99\,\text{V}$

[b] $a_1 = -5.196 \quad a_2 = 2.598 \quad a_3 = 0 \quad a_4 = -1.299 \quad a_5 = 1.039$

$b_1 = 9 \quad b_2 = 4.5 \quad b_3 = 0 \quad b_4 = 2.25 \quad b_5 = 1.8$

[c] $w_0 = \left(\frac{2\pi}{T}\right) = 50\,\text{rad/s}$

[d] $f_3 = 3f_0 = 23.87\,\text{Hz}$

[e] $v(t) = 21.99 - 5.2\cos 50t + 9\sin 50t + 2.6\sin 100t + 4.5\cos 100t$

$-1.3\sin 200t + 2.25\cos 200t + 1.04\sin 250t + 1.8\cos 250t + \cdots\,\text{V}$

AP 16.3 Odd function with both half- and quarter-wave symmetry.

$$v_g(t) = \left(\frac{6V_m}{T}\right)t, \qquad 0 \le t \le T/6; \qquad a_v = 0, \qquad a_k = 0 \quad \text{for all } k$$

$b_k = 0 \quad$ for k even

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt, \qquad k \text{ odd}$$

$$= \frac{8}{T}\int_0^{T/6}\left(\frac{6V_m}{T}\right)t\sin k\omega_0 t\,dt + \frac{8}{T}\int_{T/6}^{T/4} V_m \sin k\omega_0 t\,dt$$

$$= \left(\frac{12V_m}{k^2\pi^2}\right)\sin\left(\frac{k\pi}{3}\right)$$

$$v_g(t) = \frac{12V_m}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{3}\sin n\omega_0 t \text{ V}$$

AP 16.4 [a] $A_1 = -5.2 - j9 = 10.4\underline{/-120^\circ}; \qquad A_2 = 2.6 - j4.5 = 5.2\underline{/-60^\circ}$

$A_3 = 0; \qquad A_4 = -1.3 - j2.25 = 2.6\underline{/-120^\circ}$

$A_5 = 1.04 - j1.8 = 2.1\underline{/-60^\circ}$

$\theta_1 = -120^\circ; \qquad \theta_2 = -60^\circ; \qquad \theta_3$ not defined;

$\theta_4 = -120^\circ; \qquad \theta_5 = -60^\circ$

[b] $v(t) = 21.99 + 10.4\cos(50t - 120^\circ) + 5.2\cos(100t - 60^\circ)$

$+2.6\cos(200t - 120^\circ) + 2.1\cos(250t - 60^\circ) + \cdots \text{ V}$

AP 16.5 The Fourier series for the input voltage is

$$v_i = \frac{8A}{\pi^2}\sum_{n=1,3,5}^{\infty}\left(\frac{1}{n^2}\sin\frac{n\pi}{2}\right)\sin n\omega_0(t + T/4)$$

$$= \frac{8A}{\pi^2}\sum_{n=1,3,5}^{\infty}\left(\frac{1}{n^2}\sin^2\frac{n\pi}{2}\right)\cos n\omega_0 t$$

$$= \frac{8A}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}\cos n\omega_0 t$$

$$\frac{8A}{\pi^2} = \frac{8(281.25\pi^2)}{\pi^2} = 2250\,\text{mV}$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{200\pi}\times 10^3 = 10$$

$$\therefore \quad v_i = 2250 \sum_{n=1,3,5}^{\infty} \frac{1}{n^2} \cos 10nt \,\text{mV}$$

From the circuit we have

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + (1/j\omega C)} \cdot \frac{1}{j\omega C} = \frac{\mathbf{V}_i}{1 + j\omega RC}$$

$$\mathbf{V}_o = \frac{1/RC}{1/RC + j\omega}\mathbf{V}_i = \frac{100}{100 + j\omega}\mathbf{V}_i$$

$$\mathbf{V}_{i1} = 2250\underline{/0^\circ}\,\text{mV}; \qquad \omega_0 = 10 \text{ rad/s}$$

$$\mathbf{V}_{i3} = \frac{2250}{9}\underline{/0^\circ} = 250\underline{/0^\circ}\,\text{mV}; \qquad 3\omega_0 = 30 \text{ rad/s}$$

$$\mathbf{V}_{i5} = \frac{2250}{25}\underline{/0^\circ} = 90\underline{/0^\circ}\,\text{mV}; \qquad 5\omega_0 = 50 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{100}{100 + j10}(2250\underline{/0^\circ}) = 2238.83\underline{/-5.71^\circ}\,\text{mV}$$

$$\mathbf{V}_{o3} = \frac{100}{100 + j30}(250\underline{/0^\circ}) = 239.46\underline{/-16.70^\circ}\,\text{mV}$$

$$\mathbf{V}_{o5} = \frac{100}{100 + j50}(90\underline{/0^\circ}) = 80.50\underline{/-26.57^\circ}\,\text{mV}$$

$$\therefore \quad v_o = 2238.33\cos(10t - 5.71^\circ) + 239.46\cos(30t - 16.70^\circ)$$

$$+80.50\cos(50t - 26.57^\circ) + \ldots \,\text{mV}$$

AP 16.6 [a] $\omega_o = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.2\pi}(10^3) = 10^4 \text{ rad/s}$

$$v_g(t) = 840 \sum_{n=1,3,5}^{\infty} \frac{1}{n} \sin\frac{n\pi}{2} \cos n10{,}000t \,\text{V}$$

$$= 840\cos 10{,}000t - 280\cos 30{,}000t + 168\cos 50{,}000t$$

$$- 120\cos 70{,}000t + \cdots \,\text{V}$$

$$\mathbf{V}_{g1} = 840\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{g3} = 280\underline{/180^\circ}\,\text{V}$$

$$\mathbf{V}_{g5} = 168\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{g7} = 120\underline{/180^\circ}\,\text{V}$$

$$H(s) = \frac{V_o}{V_g} = \frac{\beta s}{s^2 + \beta s + \omega_c^2}$$

$$\beta = \frac{1}{RC} = \frac{10^9}{10^4(20)} = 5000 \text{ rad/s}$$

$$\omega_c^2 = \frac{1}{LC} = \frac{(10^9)(10^3)}{400} = 25 \times 10^8$$

$$H(s) = \frac{5000s}{s^2 + 5000s + 25 \times 10^8}$$

$$H(j\omega) = \frac{j5000\omega}{25 \times 10^8 - \omega^2 + j5000\omega}$$

$$H_1 = \frac{j5 \times 10^7}{24 \times 10^8 + j5 \times 10^7} = 0.02\underline{/88.81^\circ}$$

$$H_3 = \frac{j15 \times 10^7}{16 \times 10^8 + j15 \times 10^7} = 0.09\underline{/84.64^\circ}$$

$$H_5 = \frac{j25 \times 10^7}{25 \times 10^7} = 1\underline{/0^\circ}$$

$$H_7 = \frac{j35 \times 10^7}{-24 \times 10^8 + j35 \times 10^7} = 0.14\underline{/-81.70^\circ}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{g1}H_1 = 17.50\underline{/88.81^\circ} \text{ V}$$

$$\mathbf{V}_{o3} = \mathbf{V}_{g3}H_3 = 26.14\underline{/-95.36^\circ} \text{ V}$$

$$\mathbf{V}_{o5} = \mathbf{V}_{g5}H_5 = 168\underline{/0^\circ} \text{ V}$$

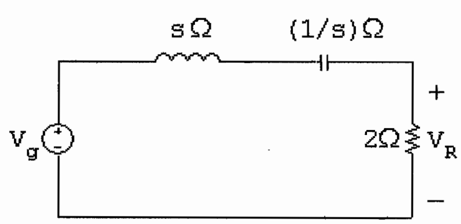
$$\mathbf{V}_{o7} = \mathbf{V}_{g7}H_7 = 17.32\underline{/98.30^\circ} \text{ V}$$

$$v_o = 17.50\cos(10{,}000t + 88.81^\circ) + 26.14\cos(30{,}000t - 95.36^\circ)$$
$$+ 168\cos(50{,}000t) + 17.32\cos(70{,}000t + 98.30^\circ) + \cdots \text{ V}$$

[b] The 5th harmonic because the circuit is a passive bandpass filter with a Q of 10 and a center frequency of 50 krad/s.

AP 16.7

$$w_0 = \frac{2\pi \times 10^3}{2094.4} = 3 \text{ rad/s}$$



$$j\omega_0 k = j3k$$

$$V_R = \frac{2}{2+s+1/s}(V_g) = \frac{2sV_g}{s^2+2s+1}$$

$$H(s) = \left(\frac{V_R}{V_g}\right) = \frac{2s}{s^2+2s+1}$$

$$H(j\omega_0 k) = H(j3k) = \frac{j6k}{(1-9k^2)+j6k}$$

$$v_{g1} = 25.98\sin\omega_0 t\,\text{V}; \qquad V_{g1} = 25.98\underline{/0^\circ}\,\text{V}$$

$$H(j3) = \frac{j6}{-8+j6} = 0.6\underline{/-53.13^\circ}; \qquad V_{R1} = 15.588\underline{/-53.13^\circ}\,\text{V}$$

$$P_1 = \frac{(15.588/\sqrt{2})^2}{2} = 60.75\,\text{W}$$

$$v_{g3} = 0, \quad \text{therefore} \quad P_3 = 0\,\text{W}$$

$$v_{g5} = -1.04\sin 5\omega_0 t\,\text{V}; \qquad V_{g5} = 1.04\underline{/180^\circ}$$

$$H(j15) = \frac{j30}{-224+j30} = 0.1327\underline{/-82.37^\circ}$$

$$V_{R5} = (1.04\underline{/180^\circ})(0.1327\underline{/-82.37^\circ}) = 138\underline{/97.63^\circ}\,\text{mV}$$

$$P_5 = \frac{(0.1396/\sqrt{2})^2}{2} = 4.76\,\text{mW}; \qquad \text{therefore} \quad P \cong P_1 \cong 60.75\,\text{W}$$

AP 16.8 Odd function with half- and quarter-wave symmetry, therefore $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for k even; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/8} 2\sin \text{k}\omega_0 t\,dt + \frac{8}{T}\int_{T/8}^{T/4} 8\sin k\omega_0 t\,dt$$
$$= \left(\frac{8}{\pi k}\right)\left[1+3\cos\left(\frac{k\pi}{4}\right)\right], \quad k \text{ odd}$$

$$\text{Therefore} \quad C_n = \left(\frac{-j4}{n\pi}\right)\left[1+3\cos\left(\frac{n\pi}{4}\right)\right], \quad n \text{ odd}$$

AP 16.9 [a] $I_{\text{rms}} = \sqrt{\frac{2}{T}\left[(2)^2\left(\frac{T}{8}\right)(2) + (8)^2\left(\frac{3T}{8}-\frac{T}{8}\right)\right]} = \sqrt{34} = 5.7683\,\text{A}$

[b] $C_1 = \dfrac{-j12.5}{\pi}; \quad C_3 = \dfrac{j1.5}{\pi}; \quad C_5 = \dfrac{j0.9}{\pi};$

$$C_7 = \frac{-j1.8}{\pi}; \quad C_9 = \frac{-j1.4}{\pi}; \quad C_{11} = \frac{j0.4}{\pi}$$

$$I_{\rm rms} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}(12.5^2 + 1.5^2 + 1.8^2 + 1.4^2 + 0.4^2)}$$

$$\cong 5.777\,\text{A}$$

[c] $\% \text{ Error} = \dfrac{5.777 - 5.831}{5.831} \times 100 = -1.08\%$

[d] Using just the terms $C_1 - C_9$,

$$I_{\rm rms} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}(12.5^2 + 1.5^2 + 1.8^2 + 1.4^2)}$$

$$\cong 5.774\,\text{A}$$

$$\% \text{ Error} = \frac{5.774 - 5.831}{5.831} \times 100 = -0.98\%$$

Thus, the % error is still less than 1%.

AP 16.10 $T = 32\,\text{ms}$, therefore 8 ms requires shifting the function $T/4$ to the right.

$$i = \sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} -j\frac{4}{n\pi}\left(1 + 3\cos\frac{n\pi}{4}\right) e^{jn\omega_0(t-T/4)}$$

$$= \frac{4}{\pi}\sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} \frac{1}{n}\left(1 + 3\cos\frac{n\pi}{4}\right) e^{-j(n+1)(\pi/2)} e^{jn\omega_0 t}$$

Problems

P 16.1 [a] $\omega_{\rm oa} = \frac{2\pi}{90}(10^6) = 69{,}813.17 \text{ rad/s}$

$$\omega_{\rm ob} = \frac{2\pi}{T} = \frac{2\pi}{8}(10^6) = 785{,}398.16 \text{ rad/s}$$

[b] $f_{\rm oa} = \frac{1}{T} = \frac{10^6}{90} = 11{,}111.11\,\text{Hz}; \qquad f_{\rm ob} = \frac{1}{T} = \frac{10^6}{8} = 125{,}000\,\text{Hz}$

[c] $a_{va} = 0; \qquad a_{vb} = \frac{2(50 \times 1 + 25 \times 1)}{8} = 18.75\,\text{V}$

[d] The periodic function in Fig. P16.1(a) is odd with half-wave and quarter-wave symmetry. Therefore,

$$a_v = 0; \qquad a_{ka} = 0 \qquad \text{for all } k; \qquad b_{ka} = 0 \qquad \text{for } k \text{ even}$$

For k odd,

$$b_{ka} = \frac{8}{T}\int_0^{T/6} 100 \sin\frac{2\pi kt}{T}\,dt + \frac{8}{T}\int_{T/6}^{T/4} 50 \sin\frac{2\pi kt}{T}\,dt$$

$$= \frac{400}{T}\left\{\frac{2T}{2\pi k}\left(-\cos\frac{2\pi kt}{T}\Big|_0^{T/6}\right) + \frac{T}{2\pi k}\left(-cos\frac{2\pi kt}{T}\Big|_{T/6}^{T/4}\right)\right\}$$

$$= \frac{-200}{\pi k}\left\{2\left(\cos\frac{\pi k}{3} - 1\right) + \cos\frac{\pi}{2}k - \cos\frac{\pi k}{3}\right\}$$

$$= \frac{200}{\pi k}\left\{2 - \cos\frac{\pi k}{3} - \cos\frac{\pi}{2}k\right\}\ \text{V}$$

Since k is odd, $\cos \pi k/2 = 0$.

$$\therefore\quad b_{ka} = \frac{200}{\pi k}\left[2 - \cos\frac{\pi k}{3}\right]\ \text{V}, \qquad k \text{ odd}$$

The periodic function in Fig. P16.1(b) is even; therefore $b_{kb} = 0$ for all k.

$a_{vb} = 18.75\,\text{V}$

$$a_{kb} = \frac{4}{T}\left\{\int_0^{T/8} 50\cos k\omega_o t\,dt + \int_{T/8}^{T/4} 25\cos k\omega_o t\,dt + \int_{T/4}^{T/2} 0\cos k\omega_o t\,dt\right\}$$

$$= \frac{4}{T}\left\{\frac{50}{k\omega_o}\sin k\omega_o t\,\Big|_0^{T/8} + \frac{25}{k\omega_o}\sin k\omega_o t\,\Big|_{T/8}^{T/4}\right\}$$

$$= \frac{50}{k\pi}\left\{2\sin\frac{k\pi}{4} + \sin\frac{k\pi}{2} - \sin\frac{k\pi}{4}\right\}$$

$$= \frac{50}{k\pi}\left\{\sin\frac{k\pi}{4} + \sin\frac{k\pi}{2}\right\}\text{ V}$$

[e] For the periodic function in 16.1(a):

$$v(t) = \frac{200}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\left(2 - \cos\frac{n\pi}{3}\right)\sin n\omega_o t\text{ V}$$

For the periodic function in 16.1(b):

$$v(t) = 18.75 + \frac{50}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\sin\frac{n\pi}{4} + \sin\frac{n\pi}{2}\right)\cos n\omega_o t\text{ V}$$

P 16.2 [a] Odd function with half- and quarter-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for even k; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/4} V_m \sin k\omega_0 t\,dt = \frac{4V_m}{k\pi}, \qquad k\text{ odd}$$

and $$v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin n\omega_0 t\text{ V}$$

[b] Even function: $b_k = 0$ for k

$$a_v = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{\pi}{T}t\,dt = \frac{2V_m}{\pi}$$

$$a_k = \frac{4}{T}\int_0^{T/2} V_m \sin\frac{\pi}{T}t\cos k\omega_0 t\,dt = \frac{2V_m}{\pi}\left(\frac{1}{1-2k} + \frac{1}{1+2k}\right)$$

$$= \frac{4V_m/\pi}{1-4k^2}$$

and $$v(t) = \frac{2V_m}{\pi}\left[1 + 2\sum_{n=1}^{\infty}\frac{1}{1-4n^2}\cos n\omega_0 t\right]\text{ V}$$

[c] $$a_v = \frac{1}{T}\int_0^{T/2} V_m \sin\left(\frac{2\pi}{T}\right)t\,dt = \frac{V_m}{\pi}$$

$$a_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t\cos k\omega_0 t\,dt = \frac{V_m}{\pi}\left(\frac{1+\cos k\pi}{1-k^2}\right)$$

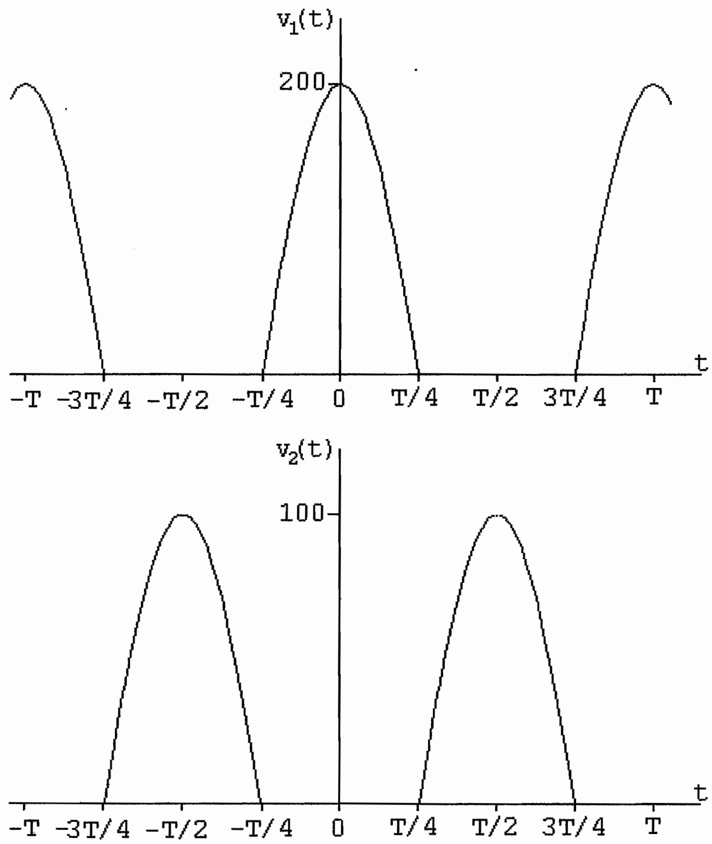
Note: $a_k = 0$ for k-odd, $a_k = \dfrac{2V_m}{\pi(1-k^2)}$ for k even,

$$b_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t \sin k\omega_0 t\,dt = 0 \quad \text{for} \quad k = 2, 3, 4, \ldots$$

For $k = 1$, we have $b_1 = \dfrac{V_m}{2}$; therefore

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin\omega_0 t + \frac{2V_m}{\pi}\sum_{n=2,4,6}^{\infty}\frac{1}{1-n^2}\cos n\omega_0 t \text{ V}$$

P 16.3 In studying the periodic function in Fig. P16.3 note that it can be visualized as the combination of two half-wave rectified sine waves, as shown in the figure below. Hence we can use the Fourier series for a half-wave rectified sine wave which is given as the answer to Problem 16.2(c).



In using the previously derived Fourier series for the half-wave rectified sine wave we note $v_1(t)$ has been shifted $T/4$ units to the left and $v_2(t)$ has been shifted $T/4$ units to the right. Thus,

$$v_1(t) = \frac{200}{\pi} + 100\sin\omega_o(t+T/4) - \frac{400}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos n\omega_o(t+T/4)}{(n^2-1)}\text{ V}$$

Now observe the following:

$$\sin\omega_o(t+T/4) = sin(\omega_o t + \pi/2) = \cos\omega_o t$$

$$\cos n\omega_o(t+T/4) = cos(n\omega_o t + n\pi/2) = \cos\frac{n\pi}{2}\cos n\omega_o t$$

because n is even.

$$\therefore \quad v_1(t) = \frac{200}{\pi} + 100\cos\omega_o t - \frac{400}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos(n\pi/2)\cos(n\omega_o t)}{(n^2-1)}\text{ V}$$

$$\therefore \quad v_2(t) = \frac{100}{\pi} + 50\sin\omega_o(t-T/4) - \frac{200}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos n\omega_o(t-T/4)}{(n^2-1)}\text{ V}$$

Again, observe the following:

$$\sin(\omega_o t - \pi/2) = -\cos\omega_o t$$

$$\cos(n\omega_o t - n\pi/2) = \cos(n\pi/2)\cos n\omega_o t$$

because n is even.

$$\therefore \quad v_2(t) = \frac{100}{\pi} - 50\cos\omega_o t - \frac{200}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos(n\pi/2)\cos(n\omega_o t)}{(n^2-1)}\text{ V}$$

Thus: $\quad v = v_1 + v_2$

$$\therefore \quad v(t) = \frac{300}{\pi} + 50\cos\omega_o t - \frac{600}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos(n\pi/2)\cos(n\omega_o t)}{(n^2-1)}\text{ V}$$

P 16.4 $f(t)\sin k\omega_0 t = a_v\sin k\omega_0 t + \sum_{n=1}^{\infty} a_n\cos n\omega_0 t\sin k\omega_0 t + \sum_{n=1}^{\infty} b_n\sin n\omega_0 t\sin k\omega_0 t$

Now integrate both sides from t_o to t_o+T. All the integrals on the right-hand side reduce to zero except in the last summation when $n=k$, therefore we have

$$\int_{t_o}^{t_o+T} f(t)\sin k\omega_0 t\,dt = 0 + 0 + b_k\left(\frac{T}{2}\right) \quad \text{or} \quad b_k = \frac{2}{T}\int_{t_o}^{t_o+T} f(t)\sin k\omega_0 t\,dt$$

P 16.5 [a] $I_6 = \int_{t_o}^{t_o+T} \sin m\omega_0 t\,dt = -\frac{1}{m\omega_0}\cos m\omega_0 t\Big|_{t_o}^{t_o+T}$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0(t_o+T) - \cos m\omega_0 t_o]$$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0 t_o \cos m\omega_0 T - \sin m\omega_0 t_o \sin m\omega_0 T - \cos m\omega_0 t_o]$$

$$= (-1m\omega_0)[\cos m\omega_0 t_o - 0 - \cos m\omega_0 t_o] = 0 \quad \text{for all } m,$$

$$I_7 = \int_{t_o}^{t_o+T} \cos m\omega_0 t_o\,dt = \frac{1}{m\omega_0}[\sin m\omega_0 t]\Big|_{t_o}^{t_o+T}$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0(t_o+T) - \sin m\omega_0 t_o]$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0 t_o - \sin m\omega_0 t_o] = 0 \quad \text{for all } m$$

[b] $I_8 = \int_{t_o}^{t_o+T} \cos m\omega_0 t \sin n\omega_0 t\,dt = \frac{1}{2}\int_{t_o}^{t_o+T}[\sin(m+n)\omega_0 t - \sin(m-n)\omega_0 t]\,dt$

But $(m+n)$ and $(m-n)$ are integers, therefore from I_6 above, $I_8 = 0$ for all m, n.

[c] $I_9 = \int_{t_o}^{t_o+T} \sin m\omega_0 t \sin n\omega_0 t\,dt = \frac{1}{2}\int_{t_o}^{t_o+T}[\cos(m-n)\omega_0 t - \cos(m+n)\omega_0 t]\,dt$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we get

$$I_9 = \frac{1}{2}\int_{t_o}^{t_o+T} dt - \frac{1}{2}\int_{t_o}^{t_o+T} \cos 2m\omega_0 t\,dt = \frac{T}{2} - 0 = \frac{T}{2}$$

[d] $I_{10} = \int_{t_o}^{t_o+T} \cos m\omega_0 t \cos n\omega_0 t\,dt$

$$= \frac{1}{2}\int_{t_o}^{t_o+T}[\cos(m-n)\omega_0 t + \cos(m+n)\omega_0 t]\,dt$$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we have

$$I_{10} = \frac{1}{2}\int_{t_o}^{t_o+T} dt + \frac{1}{2}\int_{t_o}^{t_o+T} \cos 2m\omega_0 t\,dt = \frac{T}{2} + 0 = \frac{T}{2}$$

P 16.6 $a_v = \frac{1}{T}\int_{t_o}^{t_o+T} f(t)\,dt = \frac{1}{T}\left\{\int_{-T/2}^{0} f(t)\,dt + \int_{0}^{T/2} f(t)\,dt\right\}$

Let $t = -x, \quad dt = -dx, \quad x = \frac{T}{2} \quad \text{when} \quad t = \frac{-T}{2}$

and $x = 0$ when $t = 0$

Therefore $\dfrac{1}{T}\displaystyle\int_{-T/2}^{0} f(t)\,dt = \dfrac{1}{T}\int_{T/2}^{0} f(-x)(-dx) = -\dfrac{1}{T}\int_{0}^{T/2} f(x)\,dx$

Therefore $a_v = -\dfrac{1}{T}\displaystyle\int_{0}^{T/2} f(t)\,dt + \dfrac{1}{T}\int_{0}^{T/2} f(t)\,dt = 0$

$$a_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t\,dt + \frac{2}{T}\int_{0}^{T/2} f(t)\cos k\omega_0 t\,dt$$

Again, let $t = -x$ in the first integral and we get

$$\frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t\,dt = -\frac{2}{T}\int_{0}^{T/2} f(x)\cos k\omega_0 x\,dx$$

Therefore $a_k = 0$ for all k.

$$b_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt + \frac{2}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$$

Using the substitution $t = -x$, the first integral becomes

$$\frac{2}{T}\int_{0}^{T/2} f(x)\sin k\omega_0 x\,dx$$

Therefore we have $b_k = \dfrac{4}{T}\displaystyle\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$

P 16.7 $b_k = \dfrac{2}{T}\displaystyle\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt + \dfrac{2}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$

Now let $t = x - T/2$ in the first integral, then $dt = dx$, $x = 0$ when $t = -T/2$ and $x = T/2$ when $t = 0$, also
$\sin k\omega_0(x - T/2) = \sin(k\omega_0 x - k\pi) = \sin k\omega_0 x \cos k\pi$. Therefore

$\dfrac{2}{T}\displaystyle\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt = -\dfrac{2}{T}\int_{0}^{T/2} f(x)\sin k\omega_0 x\cos k\pi\,dx$ and

$$b_k = \frac{2}{T}(1 - \cos k\pi)\int_{0}^{T/2} f(x)\sin k\omega_0 t\,dt$$

Now note that $1 - \cos k\pi = 0$ when k is even, and $1 - \cos k\pi = 2$ when k is odd. Therefore $b_k = 0$ when k is even, and

$b_k = \dfrac{4}{T}\displaystyle\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$ when k is odd

P 16.8 Because the function is even and has half-wave symmetry, we have $a_v = 0$, $a_k = 0$ for k even, $b_k = 0$ for all k and

$$a_k = \frac{4}{T}\int_0^{T/2} f(t)\cos k\omega_0 t\,dt, \qquad k \text{ odd}$$

The function also has quarter-wave symmetry;
therefore $f(t) = -f(T/2 - t)$ in the interval $T/4 \le t \le T/2$;
thus we write

$$a_k = \frac{4}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt + \frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\,dt$$

Now let $t = (T/2 - x)$ in the second integral, then $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$. Therefore we get

$$\frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\,dt = -\frac{4}{T}\int_0^{T/4} f(x)\cos k\pi\cos k\omega_0 x\,dx$$

Therefore we have

$$a_k = \frac{4}{T}(1 - \cos k\pi)\int_0^{T/4} f(t)\cos k\omega_0 t\,dt$$

But k is odd, hence

$$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt, \qquad k \text{ odd}$$

P 16.9 Because the function is odd and has half-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, and $b_k = 0$ for k even. For k odd we have

$$b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin k\omega_0 t\,dt$$

The function also has quarter-wave symmetry, therefore $f(t) = f(T/2 - t)$ in the interval $T/4 \le t \le T/2$. Thus we have

$$b_k = \frac{4}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt + \frac{4}{T}\int_{T/4}^{T/2} f(t)\sin k\omega_0 t\,dt$$

Now let $t = (T/2 - x)$ in the second integral and note that $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$, thus

$$\frac{4}{T}\int_{T/4}^{T/2} f(t)\sin k\omega_0 t\,dt = -\frac{4}{T}\cos k\pi\int_0^{T/4} f(x)(\sin k\omega_0 x)\,dx$$

But k is odd, therefore the expression becomes

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt$$

P 16.10 [a] $f = \frac{1}{T} = \frac{10^3}{10} = 100\,\text{Hz}$

[b] no

[c] yes

[d] yes

[e] yes

[f] $a_v = 0,\qquad$ function is odd

$a_k = 0,\qquad$ for all k; the function is odd

$b_k = 0,\qquad$ for k even, the function has half-wave symmetry

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t, \qquad k \text{ odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8} 4000t\sin k\omega_o t\,dt + \int_{T/8}^{T/4} 5\sin k\omega_o t\,dt\right\}$$

$$= \frac{8}{T}\{\text{Int1} + \text{Int2}\}$$

$$\text{Int1} = 4000\int_0^{T/8} t\sin k\omega_o t\,dt$$

$$= 4000\left[\frac{1}{k^2\omega_o^2}\sin k\omega_o t - \frac{t}{k\omega_o}\cos k\omega_o t\,\Big|_0^{T/8}\right]$$

$$= \frac{4000}{k^2\omega_o^2}\sin\frac{k\pi}{4} - \frac{500T}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int2} = 5\int_{T/8}^{T/4}\sin k\omega_o t\,dt = \frac{-5}{k\omega_o}\cos k\omega_o t\,\Big|_{T/8}^{T/4} = \frac{5}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int1} + \text{Int2} = \frac{4000}{k^2\omega_o^2}\sin\frac{k\pi}{4} + \left(\frac{5}{k\omega_o} - \frac{500T}{k\omega_o}\right)\cos\frac{k\pi}{4}$$

$$500T = (500)(10\times 10^{-3}) = 5$$

$$\therefore\quad \text{Int1} + \text{Int2} = \frac{4000}{k^2\omega_o^2}\sin\frac{k\pi}{4}$$

$$b_k = \left[\frac{8}{T}\cdot\frac{4000}{4\pi^2k^2}\cdot T^2\right]\sin\frac{k\pi}{4} = \frac{80}{\pi^2k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

$$i(t) = \frac{80}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{\sin(n\pi/4)}{n^2}\sin n\omega_o t\,\text{A}$$

P 16.11 [a] $\omega_o = \frac{2\pi}{T} = \frac{\pi}{6}$ rad/s

[b] no

[c] yes

[d] no

P 16.12 [a] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for all k, $a_k = 0$ for k-even; for odd k we have

$$a_k = \frac{8}{T}\int_0^{T/4} V_m \cos k\omega_0 t\, dt = \frac{4V_m}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

$$v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5}^{\infty}\left[\frac{1}{n}\sin\frac{n\pi}{2}\right]\cos n\omega_0 t \text{ V}$$

[b] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for k-even, $a_k = 0$ for all k; for k-odd we have

$$a_k = \frac{8}{T}\int_0^{T/4}\left(\frac{4V_p}{T}t - V_p\right)\cos k\omega_0 t\, dt = \frac{-8V_p}{\pi^2 k^2}$$

$$\text{Therefore}\quad v(t) = \frac{-8V_p}{\pi^2}\sum_{n=1,3,5}^{\infty}\left[\frac{1}{n^2}\cos\frac{n\pi}{2}\right]\cos n\omega_0 t \text{ V}$$

P 16.13 [a] $i(t)$ is even, therefore $b_k = 0$ for all k.

$$a_v = \frac{1}{2}\cdot\frac{T}{4}\cdot I_m \cdot 2\cdot\frac{1}{T} = \frac{I_m}{4}\text{ A}$$

$$a_k = \frac{4}{T}\int_0^{T/4}\left(I_m - \frac{4I_m}{T}t\right)\cos k\omega_o t\, dt$$

$$= \frac{4I_m}{T}\int_0^{T/4}\cos k\omega_o t\, dt - \frac{16I_m}{T^2}\int_0^{T/4} t\cos k\omega_o t\, dt$$

$$= \text{Int}_1 - \text{Int}_2$$

$$\text{Int}_1 = \frac{4I_m}{T}\int_0^{T/4}\cos k\omega_o t\, dt = \frac{2I_m}{\pi k}\sin\frac{k\pi}{2}$$

$$\text{Int}_2 = \frac{16I_m}{T^2}\int_0^{T/4} t\cos k\omega_o t\, dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{1}{k^2\omega_o^2}\cos k\omega_o t + \frac{t}{k\omega_o}\sin k\omega_o t \Big|_0^{T/4}\right\}$$

$$= \frac{4I_m}{\pi^2 k^2}\left(\cos\frac{k\pi}{2} - 1\right) + \frac{2I_m}{k\pi}\sin\frac{k\pi}{2}$$

$$\therefore\quad a_k = \frac{4I_m}{\pi^2 k^2}\left(1 - \cos\frac{k\pi}{2}\right)\text{ A}$$

$$\therefore \quad i(t) = \frac{I_m}{4} + \frac{4I_m}{\pi^2}\sum_{n=1}^{\infty}\frac{1-\cos(n\pi/2)}{n^2}\cos n\omega_o t \text{ A}$$

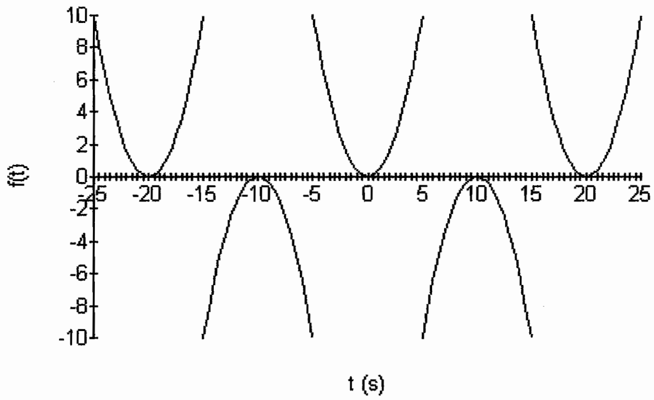
[b] Shifting the reference axis to the left is equivalent to shifting the periodic function to the right:

$$\cos n\omega_o(t - T/2) = \cos n\pi \cos n\omega_o t$$

Thus

$$i(t) = \frac{I_m}{4} + \frac{4I_m}{\pi^2}\sum_{n=1}^{\infty}\frac{(1-\cos(n\pi/2))\cos n\pi}{n^2}\cos n\omega_o t \text{ A}$$

P 16.14 [a]



[b] even

[c] yes

[d] $a_v = 0; \qquad b_k = 0 \qquad$ for all k; the function is even

$a_k = 0, \qquad k$ even, half-wave symmetry

$$a_k = \frac{8}{T}\int_0^{T/4} 0.4t^2\cos k\omega_o t\,dt$$

$$= \frac{3.2}{T}\int_0^{T/4} t^2\cos k\omega_o t\,dt$$

$$= \frac{3.2}{T}\left\{\frac{2t}{k^2\omega_o^2}\cos k\omega_o t + \frac{k^2\omega_o^2 t^2 - 2}{k^3\omega_o^3}\sin k\omega_o t\,\Big|_0^{T/4}\right\}$$

First term is 0 at both $T/4$ and 0; second term is 0 at 0, hence

$$a_k = \frac{3.2}{k^3\omega_o^3 T}\left\{\frac{k^2\omega_o^2T^2 - 32}{16}\right\}\sin\frac{k\pi}{2}$$

$$= \frac{T^2}{5k^3(8\pi^3)}(k^2 4\pi^2 - 32)\sin\frac{k\pi}{2}$$

$$T^2 = 400$$

$$\therefore \quad a_k = \frac{40}{\pi^3 k^3}(k^2\pi^2 - 8)\sin\frac{k\pi}{2}$$

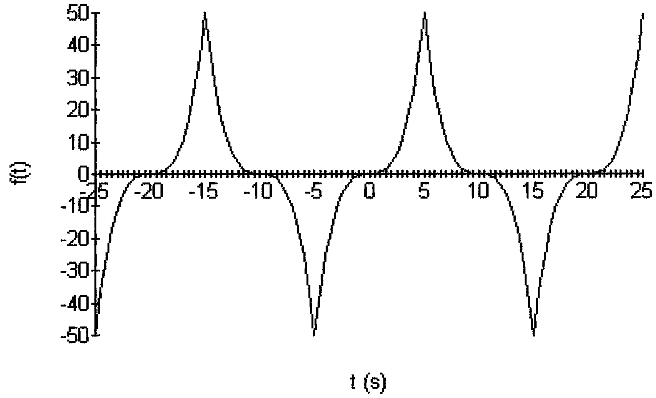
$$f(t) = \frac{40}{\pi^3}\sum_{n=1,3,5}^{\infty}\left(\frac{n^2\pi^2 - 8}{n^3}\right)\sin\frac{n\pi}{2}\cos n\omega_o t$$

[e] $\cos n\omega_o(t - T/4) = \cos(n\omega_o t - n\pi/2)$

$$= \sin(n\pi/2)\sin n\omega_O t \qquad \text{since } n \text{ is odd}$$

$$\therefore \quad f(t) = \frac{40}{\pi^3}\sum_{n=1,3,5}^{\infty}\left(\frac{n^2\pi^2 - 8}{n^3}\right)\sin n\omega_o t$$

P 16.15 [a]



[b] odd

[c] yes

[d] $a_v = 0; \qquad a_k = 0 \quad$ for all k since the function is odd

$b_k = 0 \quad$ for k even, since the function has half-wave symmetry

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t\,dt, \qquad k \text{ odd}$$

$$= \frac{3.2}{T}\int_0^{T/4} t^3 \sin k\omega_o t\,dt$$

$$= \frac{3.2}{T}\left[\frac{3k^2\omega_o^2 t^2 - 6}{k^4\omega_o^4}\sin k\omega_o t\Big|_0^{T/4} + \frac{t(6 - k^2\omega_o^2 t^2)}{k^3\omega_o^3}\cos k\omega_o t\Big|_0^{T/4}\right]$$

Note that the first term is zero at the lower limit and the second term is zero at both limits because

$\cos k\omega_o T/4 = \cos k\pi/2, \quad k$ odd

Thus

$$b_k = \left\{\frac{(3k^2\omega_o^2 T^2/16) - 6}{k^4\omega_o^4}\sin\frac{k\pi}{2}\right\}\frac{3.2}{T}$$

$$\omega_o T = 2\pi$$

$$b_k = \frac{3.2}{T}\left\{\frac{12(k^2\pi^2 - 8)T^4}{256k^4\pi^4}\right\}\sin\frac{k\pi}{2}$$

$$= \frac{3(k^2\pi^2 - 8)T^3}{20k^4\pi^4}\sin\frac{k\pi}{2}$$

$$T = 20\,\text{s}$$

$$b_k = \frac{1200(k^2\pi^2 - 8)}{k^4\pi^4}\sin\frac{k\pi}{2}$$

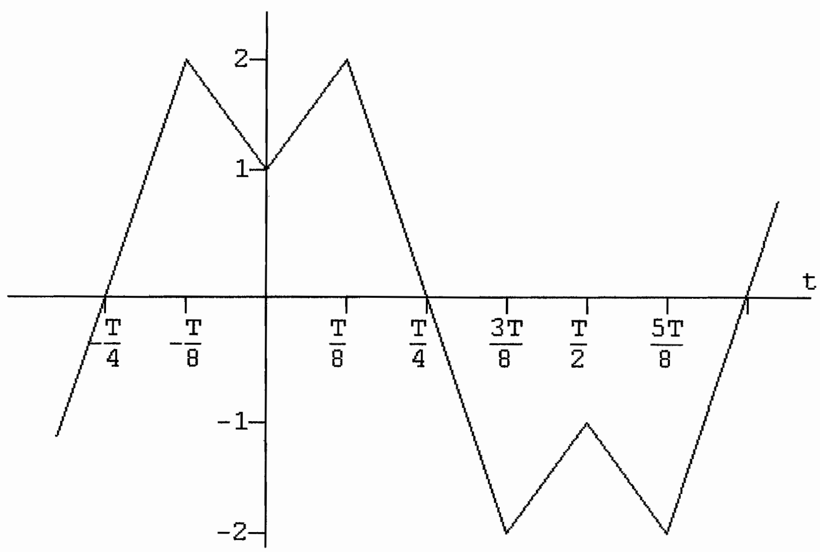
$$f(t) = \frac{1200}{\pi^4}\sum_{n=1,3,5}^{\infty}\left(\frac{n^2\pi^2 - 8}{n^4}\right)\sin\frac{n\pi}{2}\sin n\omega_o t$$

[e] $\sin n\omega_o(t - T/4) = \sin(n\omega_o t - n\pi/2)$

$$= -\cos n\omega_o t \sin n\pi/2 \qquad (n \text{ is odd})$$

$$f(t) = -\frac{1200}{\pi^4}\sum_{n=1,3,5}^{\infty}\left(\frac{n^2\pi^2 - 8}{n^4}\right)\cos n\omega_o t$$

P 16.16 **[a]**



[b] $a_v = 0$; $\quad a_k = 0$ for all k even; $\quad b_k = 0$ for all k

For k odd, $\quad a_k = \dfrac{8}{T}\displaystyle\int_0^{T/4} f(t)\cos k\omega_o t\,dt$

$$a_k = \frac{8}{T}\int_0^{T/8}\left(1 + \frac{8t}{T}\right)\cos k\omega_o t\,dt + \frac{8}{T}\int_{T/8}^{T/4}\left(4 - \frac{16t}{T}\right)\cos k\omega_o t\,dt$$

$$= \text{Int1} + \text{Int2}$$

$$\text{Int1} = \frac{8}{T}\int_0^{T/8} \cos k\omega_o t\,dt + \frac{64}{T^2}\int_0^{T/8} t\cos k\omega_o t\,dt$$

$$= \frac{8}{T}\frac{\sin k\omega_o t}{k\omega_o}\Big|_0^{T/8} + \frac{64}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t}{k\omega_o}\sin k\omega_o t\right]_0^{T/8}$$

$$k\omega_o T = 2k\pi; \qquad (k\omega_o T)^2 = 4k^2\pi^2$$

$$\text{Int1} = \frac{8}{k\pi}\sin\frac{k\pi}{4} + \frac{16}{k^2\pi^2}\left[\cos\left(\frac{k\pi}{4}\right) - 1\right] \qquad k \text{ odd}$$

$$\text{Int2} = \frac{32}{T}\int_{T/8}^{T/4} \cos k\omega_o t\,dt - \frac{128}{T^2}\int_{T/8}^{T/4} t\cos k\omega_o t\,dt$$

$$= \frac{32}{T}\frac{\sin k\omega_o t}{k\omega_o}\Big|_{T/8}^{T/4} - \frac{128}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t}{k\omega_o}\sin k\omega_o t\right]_{T/8}^{T/4}$$

$$\text{Int2} = \frac{-8}{k\pi}\sin\frac{k\pi}{4} + \frac{32}{k^2\pi^2}\cos\frac{k\pi}{4} \qquad k \text{ odd}$$

$$a_k = \text{Int1} + \text{Int2}$$

$$= \frac{16}{k^2\pi^2}\left[3\cos\frac{k\pi}{4} - 1\right]$$

[c] $a_1 = \frac{48}{\pi^2}\cos\frac{\pi}{4} - \frac{16}{\pi^2} = 1.8178$

$$a_3 = \frac{48}{9\pi^2}\cos\frac{3\pi}{4} - \frac{16}{9\pi^2} = -0.5622$$

$$a_5 = \frac{48}{25\pi^2}\cos\frac{5\pi}{4} - \frac{16}{25\pi^2} = -0.2024$$

$$f(t) = 1.8178\cos\omega_o t - 0.5622\cos 3\omega_o t - 0.2024\cos 5\omega_o t - \cdots$$

[d] $f(T/8) = 1.8178\cos(\pi/4) - 0.5622\cos(3\pi/4) - 0.2024\cos(5\pi/4) = 1.8261$

P 16.17 Let $f(t) = v_2(t - T/6)$.

$a_v = -(2V_m/3)(T/3)(1/T) = -(2V_m/9)$ and $b_k = 0$ since $f(t)$ is even

$$a_k = \frac{4}{T}\int_0^{T/6}\left(-\frac{2V_m}{3}\right)\cos k\omega_o t dt = -\frac{4}{T}\frac{2V_m}{3}\frac{1}{k\omega_o}\sin k\omega_o t\Big|_0^{T/6}$$

$$= -\frac{8V_m}{3k2\pi}\sin\left(k\frac{\pi}{3}\right) = -\frac{4V_m}{3k\pi}\sin\left(k\frac{\pi}{3}\right)$$

Therefore, $v_2(t - T/6) = -\frac{2V_m}{9} - \frac{4V_m}{3\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{3}\right)\cos n\omega_o t$

and $$v_2(t) = -\frac{2V_m}{9} - \frac{4V_m}{3\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{3}\right)\cos n\omega_o(t+T/6)$$

Then, $v(t) = v_1(t) + v_2(t)$. Simplifying,

$$v(t) = \frac{7V_m}{9} - \frac{4V_m}{3\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\sin\left(\frac{n\pi}{3}\right)\cos\left(\frac{n\pi}{3}\right)\right]\cos n\omega_o t$$

$$+\frac{4V_m}{3\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\sin^2\left(\frac{n\pi}{3}\right)\right]\sin n\omega_o t\,\text{V}$$

If $V_m = 9\pi$ then $a_v = 7\pi = 21.99$ (Checks)

$$a_k = -\left(\frac{12}{n}\right)\sin\left(\frac{n\pi}{3}\right)\cos\left(\frac{n\pi}{3}\right) = -\left(\frac{12}{n}\right)\left(\frac{1}{2}\right)\sin\left(\frac{2n\pi}{3}\right) = \left(\frac{6}{n}\right)\sin\left(\frac{4n\pi}{3}\right)$$

$$b_k = \left(\frac{12}{n}\right)\sin^2\left(\frac{n\pi}{3}\right) = \left(\frac{12}{n}\right)\left(\frac{1}{2}\right)\left[1-\cos\left(\frac{2n\pi}{3}\right)\right] = \left(\frac{6}{n}\right)\left[1-\cos\left(\frac{4n\pi}{3}\right)\right]$$

$a_1 = 6\sin(4\pi/3) = -5.2;\quad b_1 = 6[1-\cos(4\pi/3)] = 9$

$a_2 = 3\sin(8\pi/3) = 2.6;\quad b_2 = 3[1-\cos(8\pi/3)] = 4.5$

$a_3 = 2\sin(12\pi/3) = 0;\quad b_3 = 2[1-\cos(12\pi/3)] = 0$

$a_4 = 1.5\sin(16\pi/3) = -1.3;\quad b_4 = 1.5[1-\cos(16\pi/3)] = 2.25$

$a_5 = 1.2\sin(20\pi/3) = 1.04;\quad b_5 = 1.2[1-\cos(20\pi/3)] = 1.8$

All coefficients check!

P 16.18 [a] The voltage has half-wave symmetry. Therefore,

$$a_v = 0;\qquad a_k = b_k = 0,\quad k \text{ even}$$

$$a_k = \frac{4}{T}\int_0^{T/2}\left(V_m - \frac{2V_m}{T}t\right)\cos k\omega_o t\,dt,\quad k \text{ odd}$$

$$b_k = \frac{4}{T}\int_0^{T/2}\left(V_m - \frac{2V_m}{T}t\right)\sin k\omega_o t\,dt,\quad k \text{ odd}$$

$$a_k = \frac{4V_m}{T}\int_0^{T/2}\cos k\omega_o t\,dt - \frac{8V_m}{T^2}\int_0^{T/2}t\cos k\omega_o t\,dt$$

$$= \text{Int1} - \text{Int2}$$

$$\text{Int1} = \frac{4V_m}{T}\int_0^{T/2}\cos k\omega_o t\,dt = \frac{4V_m}{T}\cdot\frac{1}{k\omega_o}\sin k\omega_o t\Big|_0^{T/2} = 0$$

$$\text{Int2} = \frac{8V_m}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t\sin k\omega_o t}{k\omega_o}\right|_0^{T/2}$$

$$= \frac{8V_m}{T^2}\left[\frac{1}{k^2\omega_o^2}(\cos k\pi - 1)\right]$$

$$= \frac{-16V_m}{k^2(4\pi^2)} = \frac{-4V_m}{\pi^2k^2}, \quad k \text{ odd}$$

$$\therefore\quad a_k = \frac{4V_m}{\pi^2k^2}, \quad k \text{ odd}$$

$$b_k = \frac{4V_m}{T}\int_0^{T/2}\sin k\omega_o t\,dt - \frac{8V_m}{T^2}\int_0^{T/2} t\sin k\omega_o t\,dt$$

$$= \text{Int1} - \text{Int2}$$

$$\text{Int1} = \frac{4V_m}{T}\int_0^{T/2}\sin k\omega_o t\,dt = \frac{4V_m}{T}\cdot\frac{-1}{k\omega_o}\cos k\omega_o t\Big|_0^{T/2}$$

$$= \frac{-4V_m}{Tk\omega_o}[\cos k\pi - 1] = \frac{8V_m}{k\omega_o T} = \frac{4V_m}{\pi k}$$

$$\text{Int2} = \frac{8V_m}{T^2}\int_0^{T/2} t\sin k\omega_o t\,dt$$

$$= \frac{8V_m}{T^2}\left[\frac{\sin k\omega_o t}{k^2\omega_o^2} - \frac{t\cos k\omega_o t}{k\omega_o}\right|_0^{T/2}$$

$$= \frac{8V_m}{T^2}\left[0 - \frac{T}{2k\omega_o}\cos k\pi - 0 - 0\right] = \frac{2V_m}{k\pi}$$

$$\therefore\quad b_k = \frac{4V_m}{\pi k} - \frac{2V_m}{\pi k} = \frac{2V_m}{\pi k}$$

$$\therefore\quad A_k\underline{/\varGamma\,\theta_k} = a_k - jb_k = \frac{2V_m}{\pi k}\left(\frac{2}{\pi k} - j1\right)$$

$$V_m = 378\pi\,\text{mV}$$

$$A_k\underline{/-\theta_k} = \frac{756}{k}\left(\frac{2}{\pi k} - j1\right)\text{ mV}$$

$$v(t) = \sum_{n=1,3,5}^{\infty} A_n\cos(n\omega_o t - \theta_n)$$

$$A_1\underline{/-\theta_1} = 896.20\underline{/-57.52^\circ}\text{ mV}$$

$$A_3\underline{/-\theta_3} = 257.61\underline{/-78.02^\circ}\,\text{mV}$$

$$A_5\underline{/-\theta_5} = 152.42\underline{/-82.74^\circ}\,\text{mV}$$

$$A_7\underline{/-\theta_7} = 108.45\underline{/-84.80^\circ}\,\text{mV}$$

$$A_9\underline{/-\theta_9} = 84.21\underline{/-85.95^\circ}\,\text{mV}$$

$$\begin{aligned} v(t) &= 896.20\cos(\omega_o t - 57.52^\circ) + 257.61\cos(3\omega_o t - 78.02^\circ) \\ &\quad +152.42\cos(5\omega_o t - 82.74^\circ) + 108.45\cos(7\omega_o t - 84.80^\circ) \\ &= +84.21\cos(9\omega_o t - 85.95^\circ) + \ldots \end{aligned}$$

[b] $$\begin{aligned} v(T/8) &= 896.20\cos(45 - 57.52^\circ) + 257.61\cos(135 - 78.02^\circ) \\ &\quad +152.42\cos(225 - 82.74^\circ) + 108.45\cos(315 - 84.80^\circ) \\ &= +84.21\cos(405 - 85.95^\circ) = 888.92\,\text{mV} \end{aligned}$$

$$v(T/8) = 378\pi - \frac{2(378\pi)}{T}\left(\frac{T}{8}\right) = 378\pi(1 - \frac{1}{4}) = 890.64\,\text{mV}$$

The % difference based on the exact value is

$$\left(\frac{888.92 - 890.64}{890.64}\right)(100) = -0.19\%$$

P 16.19 The periodic function in Fig. P16.1(a) is odd, so $a_v = 0$ and $a_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = 0 - jb_n = b_n\underline{/-90^\circ}$$

From Problem 16.1(a),

$$b_n = \frac{200}{\pi n}\left[2 - \cos\frac{\pi n}{3}\right]\,\text{V}, \qquad n \text{ odd}$$

Therefore,

$$A_n = \frac{200}{\pi n}\left[2 - \cos\frac{\pi n}{3}\right]\,\text{V}, \qquad n \text{ odd}$$

and

$$-\theta_n = -90^\circ, \qquad n \text{ odd}$$

Thus, $v(t) = \dfrac{200}{\pi}\displaystyle\sum_{n=1,3,5}^{\infty}\frac{1}{n}\left(2 - \cos\frac{n\pi}{3}\right)\cos(n\omega_o t - 90^\circ)\,\text{V}$

The periodic function in Fig. P16.1(b) is even, so $b_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = a_n = a_n\underline{/0^\circ}$$

From Problem 16.1(b),

$$a_v = 18.75\,\text{V} = A_0$$

$$a_n = \frac{50}{n\pi}\left\{\sin\frac{n\pi}{4} + \sin\frac{n\pi}{2}\right\}\,\text{V}$$

Therefore,

$$A_n = \frac{50}{n\pi}\left\{\sin\frac{n\pi}{4} + \sin\frac{n\pi}{2}\right\}\,\text{V}$$

and

$$-\theta_n = 0^\circ$$

Thus, $v(t) = 18.75 + \dfrac{50}{\pi}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\left(\sin\frac{n\pi}{4} + \sin\frac{n\pi}{2}\right)\cos n\omega_o t\,\text{V}$

P 16.20 The periodic function in Problem 16.10 is odd, so $a_v = 0$ and $a_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = 0 - jb_n = b_n\underline{/-90^\circ}$$

From Problem 16.10,

$$b_k = \frac{80}{\pi^2 k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

Therefore,

$$A_n = \frac{80}{\pi^2 k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

and

$$-\theta_n = -90^\circ, \qquad n \text{ odd}$$

Thus, $i(t) = \dfrac{80}{\pi^2}\displaystyle\sum_{n=1,3,5}^{\infty}\frac{\sin(n\pi/4)}{n^2}\cos(n\omega_o t - 90^\circ)\,\text{A}$

P 16.21 The periodic function in Problem 16.14 is even, so $b_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = a_n = a_n\underline{/0^\circ}$$

From Problem 16.14,

$$a_v = 0 = A_0$$

$$a_n = \frac{40}{\pi^3 n^3}(n^2\pi^2 - 8)\sin\frac{n\pi}{2}$$

Therefore,

$$A_n = \frac{40}{\pi^3 n^3}(n^2\pi^2 - 8)\sin\frac{n\pi}{2}$$

and

$$-\theta_n = 0^\circ$$

Thus, $f(t) = \dfrac{40}{\pi^3}\displaystyle\sum_{n=1,3,5}^{\infty}\left(\frac{n^2\pi^2 - 8}{n^3}\right)\sin\frac{n\pi}{2}\cos n\omega_o t$

P 16.22 The function has half-wave symmetry, thus $a_k = b_k = 0$ for k-even, $a_v = 0$; for k-odd

$$a_k = \frac{4}{T}\int_0^{T/2} V_m\cos k\omega_0 t\,dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC}\cos k\omega_0 t\,dt$$

where $\rho = \left[1 + e^{-T/2RC}\right]$.

Upon integrating we get

$$a_k = \frac{4V_m}{T}\frac{\sin k\omega_0 t}{k\omega_0}\Big|_0^{T/2}$$

$$-\frac{8V_m}{\rho T}\cdot\left\{\frac{e^{-t/RC}}{(1/RC)^2 + (k\omega_0)^2}\cdot\left[\frac{-\cos k\omega_0 t}{RC} + k\omega_0\sin k\omega_0 t\right]\Big|_0^{T/2}\right\}$$

$$= \frac{-8V_m RC}{T[1 + (k\omega_0 RC)^2]}$$

$$b_k = \frac{4}{T}\int_0^{T/2} V_m\sin k\omega_0 t\,dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC}\sin k\omega_0 t\,dt$$

$$= -\frac{4V_m}{T}\frac{\cos k\omega_0 t}{k\omega_0}\Big|_0^{T/2}$$

$$-\frac{8V_m}{\rho T}\cdot\left\{\frac{-e^{-t/RC}}{(1/RC)^2 + (k\omega_0)^2}\cdot\left[\frac{\sin k\omega_0 t}{RC} + k\omega_0\cos k\omega_0 t\right]\Big|_0^{T/2}\right\}$$

$$= \frac{4V_m}{\pi k} - \frac{8k\omega_0 V_m R^2C^2}{T[1 + (k\omega_0 RC)^2]}$$

P 16.23 [a] $a_k^2 + b_k^2 = a_k^2 + \left(\frac{4V_m}{\pi k} + k\omega_0 RC a_k\right)^2$

$$= a_k^2\left[1 + (k\omega_0 RC)^2\right] + \tfrac{8V_m}{\pi k}\left[\tfrac{2V_m}{\pi k} + k\omega_0 RC a_k\right]$$

But $\quad a_k = \left\{\dfrac{-8V_m RC}{T\left[1 + (k\omega_0 RC)^2\right]}\right\}$

Therefore $\quad a_k^2 = \left\{\dfrac{64V_m^2 R^2 C^2}{T^2[1 + (k\omega_0 RC)^2]^2}\right\}, \quad$ thus we have

$$a_k^2 + b_k^2 = \frac{64V_m^2 R^2 C^2}{T^2[1 + (k\omega_0 RC)^2]} + \frac{16V_m^2}{\pi^2 k^2} - \frac{64V_m^2 k\omega_0 R^2 C^2}{\pi k T[1 + (k\omega_0 RC)^2]}$$

Now let $\alpha = k\omega_0 RC$ and note that $T = 2\pi/\omega_0$, thus the expression for $a_k^2 + b_k^2$ reduces to $a_k^2 + b_k^2 = 16V_M^2/\pi^2 k^2(1 + \alpha^2)$. It follows that

$$\sqrt{a_k^2 + b_k^2} = \frac{4V_m}{\pi k\sqrt{1 + (k\omega_0 RC)^2}}$$

[b] $b_k = k\omega_0 RC a_k + \dfrac{4V_m}{\pi k}$

Thus $\quad \dfrac{b_k}{a_k} = k\omega_0 RC + \dfrac{4V_m}{\pi k a_k} = \alpha - \dfrac{1 + \alpha^2}{\alpha} = -\dfrac{1}{\alpha}$

Therefore $\quad \dfrac{a_k}{b_k} = -\alpha = -k\omega_0 RC$

P 16.24 Since $a_v = 0$ (half-wave symmetry), Eq. 16.38 gives us

$$v_o(t) = \sum_{1,3,5}^{\infty} \frac{4V_m}{n\pi}\frac{1}{\sqrt{1 + (n\omega_0 RC)^2}}\cos(n\omega_0 t - \theta_n) \quad \text{where} \quad \tan\theta_n = \frac{b_n}{a_n}$$

But from Eq. 16.58, we have $\tan\beta_k = k\omega_0 RC$. It follows from Eq. 16.72 that $\tan\beta_k = -a_k/b_k$ or $\tan\theta_n = -\cot\beta_n$. Therefore $\theta_n = 90 + \beta_n$ and $\cos(n\omega_0 t - \theta_n) = \cos(n\omega_0 t - \beta_n - 90^\circ) = \sin(n\omega_0 t - \beta_n)$, thus our expression for v_o becomes

$$v_o = \frac{4V_m}{\pi}\sum_{n=1,3,5}^{\infty}\frac{\sin(n\omega_0 t - \beta_n)}{n\sqrt{1 + (n\omega_0 RC)^2}}$$

P 16.25 [a] $e^{-x} \cong 1 - x \quad$ for small x; $\qquad$ therefore

$$e^{-t/RC} \cong \left(1 - \frac{t}{RC}\right) \quad \text{and} \quad e^{-T/2RC} \cong \left(1 - \frac{T}{2RC}\right)$$

$$v_o = V_m - \frac{2V_m[1 - (t/RC)]}{2 - (T/2RC)} = \left(\frac{V_m}{RC}\right)\left[\frac{2t - (T/2)}{2 - (T/2RC)}\right]$$

$$= \left(\frac{V_m}{RC}\right)\left(t - \frac{T}{4}\right) = \left(\frac{V_m}{RC}\right)t - \frac{V_m T}{4RC} \qquad \text{for} \quad 0 \le t \le \frac{T}{2}$$

[b] $a_k = \left(\frac{-8}{\pi^2 k^2}\right) V_p = \left(\frac{-8}{\pi^2 k^2}\right)\left(\frac{V_m T}{4RC}\right) = \frac{-4V_m}{\pi\omega_0 RCk^2}$

P 16.26 [a] Express v_g as a constant plus a symmetrical square wave. The constant is $V_m/2$ and the square wave has an amplitude of $V_m/2$, is odd, and has half- and quarter-wave symmetry. Therefore the Fourier series for v_g is

$$v_g = \frac{V_m}{2} + \frac{2V_m}{\pi}\sum_{n=1,3,5}^{\infty} \frac{1}{n}\sin n\omega_0 t$$

The dc component of the current is $V_m/2R$ and the k th harmonic phase current is

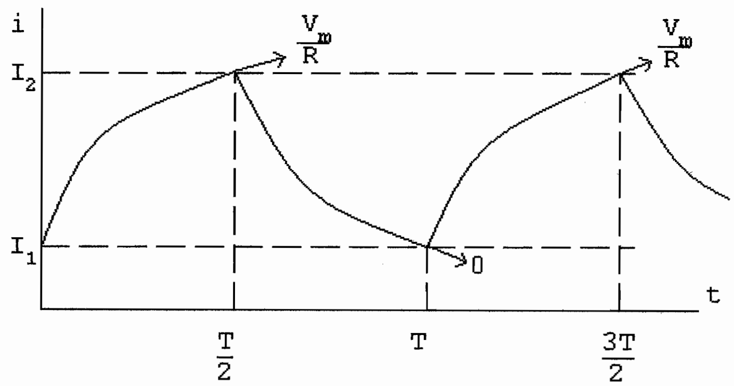
$$\mathbf{I}_k = \frac{2V_m/k\pi}{R + jk\omega_0 L} = \frac{2V_m}{k\pi\sqrt{R^2 + (k\omega_0 L)^2}}\underline{/-\theta_k}$$

$$\text{where}\quad \theta_k = \tan^{-1}\left(\frac{k\omega_0 L}{R}\right)$$

Thus the Fourier series for the steady-state current is

$$i = \frac{V_m}{2R} + \frac{2V_m}{\pi}\sum_{n=1,3,5}^{\infty} \frac{\sin(n\omega_0 t - \theta_n)}{n\sqrt{R^2 + (n\omega_0 L)^2}}\ \text{A}$$

[b]



The steady-state current will alternate between I_1 and I_2 in exponential traces as shown. Assuming $t = 0$ at the instant i increases toward (V_m/R), we have

$$i = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-t/\tau} \quad \text{for} \quad 0 \le t \le \frac{T}{2}$$

and $i = I_2 e^{-[t-(T/2)]/\tau}$ for $T/2 \le t \le T$, where $\tau = L/R$. Now we solve for I_1 and I_2 by noting that

$$I_1 = I_2 e^{-T/2\tau} \quad \text{and} \quad I_2 = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-T/2\tau}$$

These two equations are now solved for I_1. Letting $x = T/2\tau$, we get

$$I_1 = \frac{(V_m/R)e^{-x}}{1+e^{-x}}$$

Therefore the equations for i become

$$i = \frac{V_m}{R} - \left[\frac{V_m}{R(1+e^{-x})}\right]e^{-t/\tau} \qquad \text{for} \quad 0 \le t \le \frac{T}{2} \quad \text{and}$$

$$i = \left[\frac{V_m}{R(1+e^{-x})}\right]e^{-[t-(T/2)]/\tau} \qquad \text{for} \quad \frac{T}{2} \le t \le T$$

A check on the validity of these expressions shows they yield an average value of $(V_m/2R)$:

$$I_{\text{avg}} = \frac{1}{T}\left\{\int_0^{T/2}\left[\frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-t/\tau}\right]dt + \int_{T/2}^{T} I_2 e^{-[t-(T/2)]/\tau}\,dt\right\}$$

$$= \frac{1}{T}\left\{\frac{V_m T}{2R} + \tau(1-e^{-x})\left(I_1 - \frac{V_m}{R} + I_2\right)\right\}$$

$$= \frac{V_m}{2R} \quad \text{since} \quad I_1 + I_2 = \frac{V_m}{R}$$

P 16.27 $$v_i(t) = \frac{4A}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin n\omega_o(t+T/4)$$

$$= 240\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin\frac{n\pi}{2}\cos n\omega_o t$$

$$\omega_o = \frac{2\pi}{T} = 2000 \text{ rad/s}$$

$$v_{i1} = 240\cos 2000t \text{ V}; \qquad \mathbf{V}_{i1} = 240\underline{/0^\circ}\text{ V}$$

$$v_{i3} = -80\cos 6000t \text{ V}; \qquad \mathbf{V}_{i3} = 80\underline{/180^\circ}\text{ V}$$

$$v_{i5} = 48\cos 10{,}000t \text{ V}; \qquad \mathbf{V}_{i5} = 48\underline{/0^\circ}\text{ V}$$

$$H(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{(R/L)}{s+(R/L)}$$

$$\frac{R}{L} = \frac{100}{25}\times 10^3 = 4000 \text{ rad/s}$$

$$H(j\omega) = \frac{4000}{4000+j\omega}$$

$$H_1 = \frac{4000}{4000 + j2000} = 0.89\underline{/-26.57^\circ}$$

$$H_3 = \frac{4000}{4000 + j6000} = 0.55\underline{/-56.31^\circ}$$

$$H_5 = \frac{4000}{4000 + j10{,}000} = 0.37\underline{/-68.20^\circ}$$

$$\mathbf{V}_{o1} = (240\underline{/0^\circ})(0.89\underline{/-26.57^\circ}) = 214.66\underline{/-26.57^\circ}$$

$$\mathbf{V}_{o3} = (80\underline{/180^\circ})(0.55\underline{/-56.31^\circ}) = 44.38\underline{/123.69^\circ}$$

$$\mathbf{V}_{o5} = (48\underline{/0^\circ})(0.37\underline{/-68.20^\circ}) = 17.83\underline{/-68.20^\circ}$$

$$v_o = 214.66\cos(2000t - 26.57^\circ) + 44.38\cos(6000t + 123.69^\circ) + 17.83\cos(10{,}000t - 68.20^\circ) + \ldots$$

P 16.28 [a] For the circuit in Fig. P16.28

$$H(s) = \frac{s^2 + (1/LC)}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{LC} = 25 \times 10^8; \qquad \frac{1}{RC} = 5000$$

$$H(s) = \frac{s^2 + 25 \times 10^8}{s^2 + 5000s + 25 \times 10^8}$$

$$H(j\omega) = \frac{25 \times 10^8 - \omega^2}{(25 \times 10^8 - \omega^2) + j5000\omega}$$

$$H_1 = \frac{24 \times 10^8}{24 \times 10^8 + j5 \times 10^7} = 0.99978\underline{/-1.19^\circ}$$

$$H_3 = \frac{16 \times 10^8}{16 \times 10^8 + j15 \times 10^7} = 0.99563\underline{/-5.36^\circ}$$

$$H_5 = \frac{0}{j25 \times 10^7} = 0$$

$$H_7 = \frac{-24 \times 10^8}{-24 \times 10^8 + j35 \times 10^7} = 0.98953\underline{/8.30^\circ}$$

From Assessment Problem 16.6

$$\mathbf{V}_{g1} = 840\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{g3} = 280\underline{/180^\circ}\,\text{V}$$

$\mathbf{V}_{g5} = 168\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{g7} = 120\underline{/180^\circ}\,\text{V}$

Thus,

$\mathbf{V}_{o1} = 840\underline{/0^\circ}H_1 = 839.82\underline{/-1.19^\circ}\,\text{V}$

$\mathbf{V}_{o3} = 280\underline{/180^\circ}H_3 = 278.78\underline{/174.64^\circ}\,\text{V}$

$\mathbf{V}_{o5} = 168\underline{/0^\circ}H_5 = 0\,\text{V}$

$\mathbf{V}_{o7} = 120\underline{/180^\circ}H_7 = 118.74\underline{/-171.70^\circ}\,\text{V}$

$$v_o = 839.82\cos(10{,}000t - 1.19^\circ) + 278.78\cos(30{,}000t + 174.64^\circ)$$
$$= +0 + 118.74\cos(70{,}000t - 171.70^\circ) + \cdots\,\text{V}$$

[b] The 5th harmonic, that is, the voltage having a frequency of 50 krad/s. The circuit is a passive bandreject filter with a center frequency of 50 krad/s.

P 16.29 [a] $\omega_o = \frac{2\pi}{T} = 240\pi$ rad/s

$$f(t) = \frac{2(54\pi)}{\pi} - \frac{4(54\pi)}{\pi}\sum_{n=1}^{\infty}\frac{\cos n(240\pi)t}{4n^2-1}$$

$$= 108 - 216\sum_{n=1}^{\infty}\frac{\cos n(240\pi)t}{4n^2-1}$$

$$v_{g1} = \frac{-216}{3}\cos 240\pi t = -72\cos 240\pi t$$

$$v_{g2} = \frac{-216}{15}\cos 480\pi t = -1814.4\cos 480\pi t$$

$$v_{g3} = \frac{-216}{35}\cos 720\pi t$$

$\mathbf{V}_{g1} = 72\underline{/0^\circ}\,\text{V}$

$\mathbf{V}_{g2} = 14.4\underline{/180^\circ}\,\text{V}$

$\mathbf{V}_{g3} = (216/25)\underline{/180^\circ}\,\text{V}$

$$H(s) = \frac{V_o}{V_g} = \frac{(1/LC)}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{LC} = \frac{10^6}{25} = 4\times10^4; \qquad \frac{1}{RC} = \frac{10^6}{(5000)(2.5)} = 80$$

$$H(s) = \frac{4\times10^4}{s^2 + 80s + 4\times10^4}$$

$$H(j\omega) = \frac{4\times 10^4}{4\times 10^4 - \omega^2 + j80\omega}$$

$$H(j0) = 1\underline{/0^\circ}$$

$$H_1(j240\pi) = \frac{4\times 10^4}{4\times 10^4 - 5.76\pi^2\times 10^4 + j1.92\times 10^4\pi}$$

$$= 0.0752\underline{/-173.49^\circ}$$

$$H_2(j480\pi) = \frac{4\times 10^4}{4\times 10^4 - 23.04\pi^2\times 10^4 + j3.84\times 10^4\pi}$$

$$= 0.0179\underline{/-176.91^\circ}$$

$$H_3(j720\pi) = \frac{4\times 10^4}{4\times 10^4 - 51.84\pi^2\times 10^4 + j5.76\times 10^4\pi}$$

$$= 0.0079\underline{/-177.96^\circ}$$

$$\mathbf{V}_{o1} = 72\underline{/180^\circ}H_1 = 5.41\underline{/6.51^\circ}\text{ V}$$

$$\mathbf{V}_{o2} = 14.4\underline{/180^\circ}H_2 = 0.2575\underline{/3.09^\circ}\text{ V}$$

$$\mathbf{V}_{o3} = (216/25)\underline{/180^\circ}H_3 = 0.0486\underline{/2.04^\circ}\text{ V}$$

$$V_{odc} = (108)(1) = 108\text{ V}$$

$$v_o = 108 + 5.41\cos(240\pi t + 6.51^\circ) + 0.2575\cos(480\pi t + 3.09^\circ)$$
$$- 0.0486\cos(720\pi t + 2.04^\circ) + \cdots\text{ V}$$

[b] The circuit is a low pass filter. Hence, the harmonic terms are greatly reduced in the output voltage.

P 16.30 $$H(s) = \frac{I_o}{I_g} = \frac{(1/LC)}{s^2 + \left(\frac{1}{R_1C} + \frac{R_2}{L}\right)s + \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{1}{LC}\right)}$$

where $R_1 = 800\,\Omega$ and $R_2 = 200\,\Omega$. Thus

$$H(s) = \frac{20\times 10^8}{s^2 + 60{,}000s + 25\times 10^8}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{0.2\pi}(10^3) = 10\text{ krad/s}; \qquad 5\omega_o = 50\text{ krad/s}$$

$$H(j50{,}000) = \frac{20\times 10^8}{j(60{,}000)(50{,}000)} = -j\frac{2}{3}$$

$$i_g(t) = \frac{8A}{\pi^2} \sum_{n=1,3,5,}^{\infty} \frac{1}{n^2} \sin(n\pi/2) \sin n\omega_o t$$

$$\therefore \quad i_{g5}(t) = \frac{8(30\pi^2)}{\pi^2} \cdot \frac{1}{25}(1) \sin 50{,}000t$$

$$= 9.6 \sin 50{,}000t \text{ A} = 9.6 \cos(50{,}000t - 90^\circ) \text{ A}$$

$$\mathbf{I}_{g5} = 9.6\underline{/-90^\circ}; \qquad H(j50{,}000) = \frac{2}{3}\underline{/-90^\circ}$$

$$\mathbf{I}_{o5} = (9.6)(2/3)\underline{/-180^\circ} = 6.4\underline{/-180^\circ} \text{ A}$$

$$\therefore \quad i_{o5} = 6.4 \cos(50{,}000t - 180^\circ) = -6.4 \cos(50{,}000t) \text{ A}$$

P 16.31 $\omega_o = \frac{2\pi}{0.1\pi} \times 10^3 = 20$ krad/s

$$\therefore \quad n = \frac{300}{20} = \text{ 15th harmonic}$$

$$\mathbf{V}_{g15} = 45\frac{(\pi^2(15)^2 - 8)}{15^3} \sin 15\left(\frac{\pi}{2}\right)$$

$$= -29.5 \text{ V} = 29.5\underline{/180^\circ} \text{ V}$$

$$H(s) = \frac{(1/RC)s}{s^2 + (1/RC)s + (1/LC)}$$

$$= \frac{10^4 s}{s^2 + 10^4 s + 9 \times 10^{10}}$$

$$H(j300{,}000) = 1\underline{/0^\circ}$$

$$\mathbf{V}_{o15} = (29.5\underline{/180^\circ})(1\underline{/0^\circ}) = 29.5\underline{/180^\circ} \text{ V}$$

$$v_{o25} = 29.5 \cos(300{,}000t + 180^\circ) \text{ V}$$

P 16.32 [a] From Example 16.1

$$a_v = \frac{1}{2}(270\pi) = 135\pi \text{ V}$$

$$a_k = 0, \quad \text{all } k$$

$$b_k = \frac{-270\pi}{\pi k} = \frac{-270}{k} \quad \text{all } k$$

$$\therefore \quad v(t) = 135\pi - 270\sin\omega_o t - 135\sin 2\omega_o t - 90\sin 3\omega_o t - \cdots$$

$$V_{\text{rms}} = \sqrt{(135\pi)^2 + \left(\frac{270}{\sqrt{2}}\right)^2 + \left(\frac{135}{\sqrt{2}}\right)^2 + \left(\frac{90}{\sqrt{2}}\right)^2} = 479.05$$

$$P = \frac{(479.05)^2}{81\pi^2} = 287.06 \text{ W}$$

[b] $V_{\text{rms}} = \dfrac{270\pi}{\sqrt{3}} = 489.73 \text{ V}$

$$\therefore \quad P = \frac{(489.72)^2}{81\pi^2} = 300 \text{ W}$$

[c] $\% \text{ error } = \left(\dfrac{287.06}{300} - 1\right)(100) = -4.31\%$

P 16.33 $v_g(t) = 25 + \dfrac{200}{\pi^2}\displaystyle\sum_{n=1,3,5,}^{\infty} \frac{1}{n^2}\sin(n\pi/2)\sin n\omega_o t \text{ V}$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{2\pi} \times 10^6 = 1 \text{ Mrad/s}$$

$$v_g(t) = 25 + \frac{200}{\pi^2}\sin\omega_o t - \frac{200}{9\pi^2}\sin 3\omega_o t + \frac{200}{25\pi^2}\sin 5\omega_o t - \cdots \text{ V}$$

$$H(s) = \frac{(1/LC)}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{LC} = \frac{(10^3)(10^{12})}{(20)(50)} = 10^{12}; \qquad \frac{1}{RC} = \frac{10^{12}}{(20 \times 10^3)(50)} = 10^6$$

$$H(s) = \frac{10^{12}}{s^2 + 10^6 s + 10^{12}}$$

$$H(j\omega) = \frac{10^{12}}{10^{12} - \omega^2 + j10^6\omega}$$

$$H(j0) = 1$$

$$H(j\omega_o) = -j1$$

$$H(j3\omega_o) = \frac{1}{-8 + j3} = 0.1170\underline{/-159.44^\circ}$$

$$H(j5\omega_o) = \frac{1}{-24 + j5} = 0.0408\underline{/-168.23^\circ}$$

$$\therefore\quad v_o = 25 + 20.26\sin(\omega_o t - 90^\circ) - 0.2635\sin(3\omega_o t - 159.44^\circ)$$
$$+ 0.0331\sin(5\omega_o t - 168.23^\circ) - \cdots \text{ V}$$

Now note that the harmonic terms will have a negligible effect on the rms value of v_o, hence a good estimate of the power delivered to the 20 kΩ resistor can be obtained by assuming $v_o \approx 25 + 20.26\sin(\omega_o t - 90^\circ)$ V.

$$\therefore\quad V_{orms} \approx \sqrt{25^2 + \left(\frac{20.26}{\sqrt{2}}\right)^2} = 28.82\text{ V}$$

$$\therefore\quad P \approx \frac{(28.82)^2}{20 \times 10^3} = 41.52\text{ mW}$$

P 16.34 [a] $a_v = \frac{1}{T}\left[\frac{1}{2}\left(\frac{T}{2}\right)I_m + \frac{T}{2}I_m\right] = \frac{3V_m}{4}$

$$i(t) = \frac{2I_m}{T}t, \qquad 0 \le t \le T/2$$

$$i(t) = I_m, \qquad T/2 \le t \le T$$

$$a_k = \frac{2}{T}\int_0^{T/2} \frac{2I_m}{T} t\cos k\omega_o t\,dt + \frac{2}{T}\int_{T/2}^{T} I_m \cos k\omega_o t\,dt$$

$$= \frac{I_m}{\pi^2 k^2}(\cos k\pi - 1)$$

$$b_k = \frac{2}{T}\int_0^{T/2} \frac{2I_m}{T} t\sin k\omega_o t\,dt + \frac{2}{T}\int_{T/2}^{T} I_m \sin k\omega_o t\,dt$$

$$= \frac{I_m}{\pi k}$$

$$a_1 = \frac{-2I_m}{\pi^2}, \qquad a_2 = 0, \qquad a_v = \frac{3I_m}{4}$$

$$a_3 = \frac{-2I_m}{9\pi^2}$$

$$b_1 = \frac{I_m}{\pi}, \quad b_2 = \frac{I_m}{2\pi}$$

$$\therefore \quad I_{\text{rms}} = I_m\sqrt{\frac{9}{16} + \frac{2}{\pi^4} + \frac{1}{2\pi^2} + \frac{1}{8\pi^2}} = 0.8040 I_m$$

$$I_{\text{rms}} = 192.95\,\text{mA}$$

$$P = (0.19295)^2(1000) = 37.23\,\text{W}$$

[b] Area under i^2:

$$A = \int_0^{T/2} \frac{4I_m^2}{T^2}t^2\,dt + I_m^2\frac{T}{2}$$

$$= \frac{4I_m^2}{T^2}\frac{t^3}{3}\bigg|_0^{T/2} + I_m^2\frac{T}{2}$$

$$= I_m^2 T\left[\frac{1}{6} + \frac{3}{6}\right] = \frac{2}{3}TI_m^2$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\cdot\frac{2}{3}TI_m^2} = \sqrt{\frac{2}{3}}I_m = 195.96\,\text{mA}$$

$$P = (0.19596)^2(1000) = 38.4\,\text{W}$$

[c] Error $= \left(\frac{37.23}{38.40} - 1\right)100 = -3.05\%$

P 16.35 [a] $v = 80 + 200\cos(500t + 45^\circ) + 60\cos(1500t - 90^\circ)\,\text{V}$

$$i = 10 + 6\cos(500t - 15^\circ) + 3\cos(1500t + 30^\circ)\,\text{A}$$

$$P = (80)(10) + \frac{1}{2}(200)(6)\cos(60^\circ) + \frac{1}{2}(60)(3)\cos(-120^\circ) = 1055\,\text{W}$$

[b] $V_{\text{rms}} = \sqrt{(80)^2 + \left(\frac{200}{\sqrt{2}}\right)^2 + \left(\frac{60}{\sqrt{2}}\right)^2} = 167.93\,\text{V}$

[c] $I_{\text{rms}} = \sqrt{(10)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = 11.07\,\text{A}$

P 16.36 [a] Area under $v^2 = A = 4\int_0^{T/6} \frac{36V_m^2}{T^2}t^2\,dt + 2V_m^2\left(\frac{T}{3} - \frac{T}{6}\right)$

$$= \frac{2V_m^2T}{9} + \frac{V_m^2T}{3}$$

Therefore $V_{\text{rms}} = \sqrt{\frac{1}{T}\left(\frac{2V_m^2T}{9} + \frac{V_m^2T}{3}\right)} = V_m\sqrt{\frac{2}{9} + \frac{1}{3}} = 74.5356\,\text{V}$

[b] From Asssessment Problem 16.3,

$$v_g = 105.30\sin\omega_0 t - 4.21\sin 5\omega_0 t + 2.15\sin 7\omega_0 t + \cdots \text{ V}$$

$$\text{Therefore} \quad V_{\text{rms}} \cong \sqrt{\frac{(105.30)^2 + (4.21)^2 + (2.15)^2}{2}} = 74.5306\text{ V}$$

P 16.37 **[a]** $v(t) \approx \frac{320}{\pi}\left[\sin 200\pi t + \frac{1}{3}\sin 600\pi t + \frac{1}{5}\sin 1000\pi t + \frac{1}{7}\sin 1400\pi t\right]$

$$v_{\text{rms}} \approx \frac{320}{\pi}\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{5\sqrt{2}}\right)^2 + \left(\frac{1}{7\sqrt{2}}\right)^2}$$

$$\approx \frac{320}{\pi}\sqrt{\frac{1}{2} + \frac{1}{18} + \frac{1}{50} + \frac{1}{98}} \approx 77.9578\text{ V}$$

[b] $V_{\text{rms}} = 80\text{ V}$

$$\%\text{ Error} = \left(\frac{77.9578}{80} - 1\right)100 = -2.55\%$$

[c] $v(t) \approx \frac{640}{\pi^2}\left[\sin 200\pi t - \frac{1}{9}\sin 600\pi t + \frac{1}{25}\sin 1000\pi t - \frac{1}{49}\sin 1400\pi t\right]$

$$v_{\text{rms}} \approx \frac{640}{\pi^2}\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{9\sqrt{2}}\right)^2 + \left(\frac{1}{25\sqrt{2}}\right)^2 + \left(\frac{1}{49\sqrt{2}}\right)^2}$$

$$\approx \frac{640}{\pi}\sqrt{\frac{1}{2} + \frac{1}{162} + \frac{1}{1250} + \frac{1}{4802}} \approx 46.1808\text{ V}$$

$$V_{\text{rms}} = \frac{80}{\sqrt{3}} = 46.1880\text{ V}$$

$$\%\text{ Error} = \left(\frac{46.1808}{46.1880} - 1\right)100 = -0.0156\%$$

P 16.38 **[a]** $v(t) \approx \frac{340}{\pi} - \frac{680}{\pi}\left\{\frac{1}{3}\cos\omega_o t + \frac{1}{15}\cos 2\omega_o t + \cdots\right\}$

$$V_{\text{rms}} \approx \sqrt{\left(\frac{340}{\pi}\right)^2 + \left(\frac{680}{\pi}\right)^2\left[\left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{15\sqrt{2}}\right)^2\right]}$$

$$= \frac{340}{\pi}\sqrt{1 + 4\left(\frac{1}{18} + \frac{1}{450}\right)} = 120.0819\text{ V}$$

[b] $V_{\text{rms}} = \frac{170}{\sqrt{2}} = 120.2082$

$$\%\text{ error } = \left(\frac{120.0819}{120.2082} - 1\right)(100) = -0.11\%$$

[c] $v(t) \approx \frac{170}{\pi} + 85\sin\omega_o t - \frac{340}{3\pi}\cos 2\omega_o t$

$$V_{\rm rms} \approx \sqrt{\left(\frac{170}{\pi}\right)^2 + \left(\frac{85}{\sqrt{2}}\right)^2 + \left(\frac{340}{3\sqrt{2}\pi}\right)^2} \approx 84.8021\,\text{V}$$

$$V_{\rm rms} = \frac{170}{2} = 85\,\text{V}$$

$$\%\ \text{error} = -0.23\%$$

P 16.39 [a] Half-wave symmetry $a_v = 0$, $a_k = b_k = 0$, even k

$$a_k = \frac{4}{T}\int_0^{T/4}\frac{4I_m}{T}t\cos k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4} t\cos k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\cos k\omega_0 t}{k^2\omega_0^2} + \frac{t}{k\omega_0}\sin k\omega_0 t\,\Big|_0^{T/4}\right\}$$

$$= \frac{16I_m}{T^2}\left\{0 + \frac{T}{4k\omega_0}\sin\frac{k\pi}{2} - \frac{1}{k^2\omega_0^2}\right\}$$

$$a_k = \frac{2I_m}{\pi k}\left[\sin\left(\frac{k\pi}{2}\right) - \frac{2}{\pi k}\right], \qquad k\text{—odd}$$

$$b_k = \frac{4}{T}\int_0^{T/4}\frac{4I_m}{T}t\sin k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4} t\sin k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t}{k\omega_0}\cos k\omega_0 t\,\Big|_0^{T/4}\right\} = \frac{4I_m}{\pi^2 k^2}\sin\left(\frac{k\pi}{2}\right)$$

[b] $a_k - jb_k = \frac{2I_m}{\pi k}\left\{\left[\sin\left(\frac{k\pi}{2}\right) - \frac{2}{\pi k}\right] - \left[j\frac{2}{\pi k}\sin\left(\frac{k\pi}{2}\right)\right]\right\}$

$$a_1 - jb_1 = \frac{2I_m}{\pi}\left\{\left(1 - \frac{2}{\pi}\right) - j\frac{2}{\pi}\right\} = 0.47I_m\underline{/-60.28^\circ}$$

$$a_3 - jb_3 = \frac{2I_m}{3\pi}\left\{\left(-1 - \frac{2}{3\pi}\right) + j\left(\frac{2}{3\pi}\right)\right\} = 0.26I_m\underline{/170.07^\circ}$$

$$a_5 - jb_5 = \frac{2I_m}{5\pi}\left\{\left(1 - \frac{2}{5\pi}\right) - j\left(\frac{2}{5\pi}\right)\right\} = 0.11I_m\underline{/-8.30^\circ}$$

$$a_7 - jb_7 = \frac{2I_m}{7\pi}\left\{\left(-1 - \frac{2}{7\pi}\right) + j\left(\frac{2}{7\pi}\right)\right\} = 0.10I_m\underline{/175.23^\circ}$$

$$i_g = 0.47I_m\cos(\omega_0 t - 60.28^\circ) + 0.26I_m\cos(3\omega_0 t + 170.07^\circ)$$
$$+0.11I_m\cos(5\omega_0 t - 8.30^\circ) + 0.10I_m\cos(7\omega_0 t + 175.23^\circ) + \cdots$$

[c] $I_g = \sqrt{\sum_{n=1,3,5}^{\infty} \left(\frac{A_n^2}{2}\right)}$

$$\cong I_m\sqrt{\frac{(0.47)^2 + (0.26)^2 + (0.11)^2 + (0.10)^2}{2}} = 0.39I_m$$

[d] $\text{Area} = 2\int_0^{T/4}\left(\frac{4I_m}{T}t\right)^2 dt = \left(\frac{32I_m^2}{T^2}\right)\left(\frac{t^3}{3}\right)\Big|_0^{T/4} = \frac{I_m^2 T}{6}$

$$I_g = \sqrt{\frac{1}{T}\left(\frac{I_m^2 T}{6}\right)} = \frac{I_m}{\sqrt{6}} = 0.41I_m$$

[e] $\%\ \text{error} = \left(\frac{\text{estimated}}{\text{exact}} - 1\right)100 = \left(\frac{0.3927I_m}{(I_m/\sqrt{6})} - 1\right)100 = -3.8\%$

P 16.40 [a] v_g has hws, qws, and is odd

$\therefore\ a_v = 0,\ a_k = 0$ all k, $b_k = 0$ k-even

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t\,dt, \qquad k\text{-odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8} V_m \sin k\omega_o t\,dt + \int_{T/8}^{T/4}\frac{V_m}{2}\sin k\omega_o t\,dt\right\}$$

$$= \frac{8V_m}{T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\Big|_0^{T/8} + \frac{8V_m}{2T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\Big|_{T/8}^{T/4}\right.\right.$$

$$= \frac{8V_m}{k\omega_o T}\left[1 - \cos\frac{k\pi}{4}\right] + \frac{8V_m}{2Tk\omega_o}\left[\cos\frac{k\pi}{4} - 0\right]$$

$$= \frac{8V_m}{k\omega_o T}\left\{1 - \cos\frac{k\pi}{4} + \frac{1}{2}\cos\frac{k\pi}{4}\right\}$$

$$= \frac{4V_m}{\pi k}\left\{1 - 0.5\cos\frac{k\pi}{4}\right\}$$

$$b_1 = \frac{4V_m}{\pi}\left(1 - 0.5\cos\frac{\pi}{4}\right) = 0.8231V_m$$

$$b_3 = \frac{4V_m}{3\pi}\left(1 - 0.5\cos\frac{3\pi}{4}\right) = 0.5745V_m$$

$$b_5 = \frac{4V_m}{5\pi}\left(1 - 0.5\cos\frac{5\pi}{4}\right) = 0.3447V_m$$

$$b_7 = \frac{4V_m}{7\pi}\left(1 - 0.5\cos\frac{7\pi}{4}\right) = 0.1176V_m$$

$$V_{grms} \approx \mathbf{V}_m\sqrt{\frac{(0.8231)^2 + (0.5745)^2 + (0.3447)^2 + (0.1176)^2}{2}}$$

$$V_{grms} \approx 0.7550V_m$$

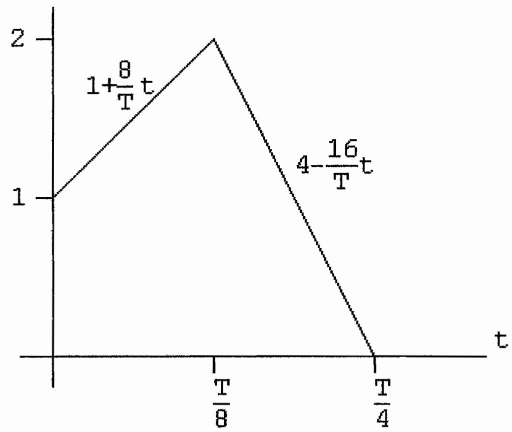
[b] $$\text{Area} = 2\left[2V_m^2\left(\frac{T}{8}\right) + \frac{V_m^2}{4}\left(\frac{T}{4}\right)\right] = \frac{5}{8}V_m^2T$$

$$V_{grms} = \sqrt{\frac{1}{T}\,\frac{5V_m^2}{8}T} = V_m\sqrt{\frac{5}{8}} = 0.7906\,V_m$$

[c] $$\%\ \text{Error} = \left[\frac{0.7550V_m}{0.7906V_m} - 1\right]100$$

$$\text{Error} = -4.5\%$$

P 16.41 [a]



Area under i^2:

$$A = 4\left[\int_0^{T/8}\left(1 + \frac{8}{T}t\right)^2 dt + \int_{T/8}^{T/4}\left(4 - \frac{16}{T}t\right)^2 dt\right]$$

$$= 4\left[\frac{T}{8} + \frac{T}{8} + \frac{T}{24} + 2T - 4T + T + \frac{4T}{3} - \frac{T}{6}\right]$$

$$= \frac{44T}{24}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\left(\frac{44T}{24}\right)} = \sqrt{\frac{44}{24}} = 1.35$$

[b] $P = I_{\text{rms}}^2(54) = 99\,\text{W}$

[c] From Problem 16.16:

$$a_1 = 1.8178\text{A}$$

$$i_g \approx 1.8178\cos\omega_o t\,\text{A}$$

$$P = \left(\frac{1.8178}{\sqrt{2}}\right)^2 (54) = 89.22\,\text{W}$$

[d] $\%\ \text{error} = \left(\frac{89.22}{99} - 1\right) = -9.88\%$

P 16.42 Figure P16.42(b): $t_a = 0.2s;\quad t_b = 0.6s$

$$v = 50t \quad 0 \le t \le 0.2$$

$$v = -50t + 20 \quad 0.2 \le t \le 0.6$$

$$v = 25t - 25 \quad 0.6 \le t \le 1.0$$

$$\text{Area } 1 = A_1 = \int_0^{0.2} 2500t^2\,dt = \frac{20}{3}$$

$$\text{Area } 2 = A_2 = \int_{0.2}^{0.6} 100(4 - 20t + 25t^2)\,dt = \frac{40}{3}$$

$$\text{Area } 3 = A_3 = \int_{0.6}^{1.0} 625(t^2 - 2t + 1)\,dt = \frac{40}{3}$$

$$A_1 + A_2 + A_3 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

Figure P16.42(c): $t_a = t_b = 0.4s$

$$v(t) = 25t \quad 0 \le t \le 0.4$$

$$v(t) = \frac{50}{3}(t-1) \quad 0.4 \le t \le 1$$

$$A_1 = \int_0^{0.4} 625t^2\,dt = \frac{40}{3}$$

$$A_2 = \int_{0.4}^{1.0} \frac{2500}{9}(t^2 - 2t + 1)\,dt = \frac{60}{3}$$

$$A_1 + A_2 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}(A_1 + A_2)} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

Figure P16.42 (d): $t_a = t_b = 1$

$$v = 10t \quad 0 \le t \le 1$$

$$A_1 = \int_0^1 100t^2\,dt = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

P 16.43 $$C_n = \frac{1}{T}\int_{-T/4}^{0} -V_m e^{-jn\omega_o t}\,dt + \frac{1}{T}\int_0^{T/4} V_m e^{-jn\omega_o t}\,dt$$

$$= \frac{-V_m}{T}\left[\frac{e^{-jn\omega_o t}}{-jn\omega_o}\Big|_{T/4}^{0}\right] + \frac{V_m}{T}\left[\frac{e^{-jn\omega_o t}}{-jn\omega_o}\Big|_0^{T/4}\right]$$

$$= -j\frac{V_m}{\pi n}\left(1 - \cos\frac{n\pi}{2}\right)$$

$$v(t) = \sum_{n=-\infty}^{\infty} -j\frac{V_m}{\pi n}\left(1 - \cos\frac{n\pi}{2}\right)e^{jn\omega_o t}$$

P 16.44 $$c_0 = a_v = \left(\frac{1}{2}(\frac{T}{4})I_m(2)\right)\frac{1}{T} = \frac{I_m}{4}$$

$$c_n = \frac{1}{T}\int_{-T/4}^{0} -\frac{4I_m}{T}te^{-jn\omega_o t}\,dt + \frac{1}{T}\int_0^{T/4}\frac{4I_m}{T}te^{-jn\omega_o t}\,dt$$

$$= \text{Int1} + \text{Int2}$$

$$\text{Int1} = \frac{-4I_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2}(-jn\omega_o t - 1)\Big|_{-T/4}^{0}\right]$$

$$= \frac{-I_m}{(n\pi)^2}\left[1 - e^{jn\pi/2}(-jn\pi/2 + 1)\right]$$

$$\text{Int2} = \frac{4I_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2}(-jn\omega_o t - 1)\Big|_0^{T/4}\right]$$

$$= \frac{I_m}{(n\pi)^2}\left[e^{-jn\pi/2}(jn\pi/2 + 1) - 1\right]$$

$$\therefore \quad c_n = \frac{I_m}{n^2\pi^2}[e^{-jn\pi/2}(1 + jn\pi/2) - 1 + e^{jn\pi/2}(1 - jn\pi/2) - 1]$$

$$= \frac{I_m}{n^2\pi^2}[2\cos(n\pi/2) + n\pi\sin(n\pi/2) - 2]$$

P 16.45 [a] $I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{2}{T}\int_0^{T/4}\left(\frac{16I_m^2}{T^2}\right)t^2\,dt}$

$$= \sqrt{\frac{32I_m^2}{T^3}\cdot\frac{t^3}{3}\Big|_0^{T/4}} = \frac{I_m}{\sqrt{6}} = \frac{20}{\sqrt{6}} = 8.16\,\text{A}$$

$$P = 60I_m^2 = 60\left(\frac{400}{6}\right) = 4000\,\text{W}$$

[b] From the solution to Problem 16.44

$$c_0 = \frac{20}{4} = 5\,\text{A}$$

$$c_1 = \frac{20}{\pi^2}[\pi\sin(\pi/2) - 2] = 2.31$$

$$c_2 = \frac{20}{4\pi^2}[-2 - 2] = -2.03$$

$$c_3 = \frac{20}{9\pi^2}[3\pi\sin(3\pi/2) - 2] = -2.57$$

$$c_4 = \frac{20}{16\pi^2}[2 - 2] = 0$$

$$c_5 = \frac{20}{25\pi^2}[5\pi - 2] = 1.11$$

$$I_{\text{rms}} = \sqrt{c_o^2 + 2\sum_{n=1}^{\infty}|c_n|^2}$$

$$= \sqrt{25 + 2(2.31^2 + 2.03^2 + 2.57^2 + 1.11^2)}$$

$$= \sqrt{25 + 34.62} = 7.72\,\text{A}$$

[c] $P = (7.72)^2(60) = 3577.17\,\text{W}$

$$\%\ \text{error} = \left(\frac{3577.17}{4000} - 1\right)(100) = -10.57\%$$

P 16.46 [a] $c_n = \frac{1}{T}\int_{-T/2}^{T/2} \frac{2V_m}{T} te^{-jn\omega_o t}\,dt$

$$= \frac{2V_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2}(-jn\omega_o t - 1)\Big|_{-T/2}^{T/2}\right]$$

$$= \frac{2V_m}{4\pi^2 n^2}[e^{-jn\pi}(jn\pi + 1) - e^{jn\pi}(-jn\pi + 1)]$$

$$= \frac{-jV_m}{\pi^2 n^2}[\sin n\pi - n\pi\cos n\pi]$$

$\sin n\pi = 0$ for all n

$$c_n = \frac{jV_m}{\pi^2 n^2} n\pi\cos n\pi = j\frac{V_m}{n\pi}\cos n\pi$$

[b] $c_{-1} = j72$; $c_1 = -j72$

$c_{-2} = -j36$; $c_2 = j36$

$c_{-3} = j24$; $c_3 = -j24$

$c_{-4} = -j18$; $c_4 = j18$

[c] $\frac{V_o}{R_2} + V_o sC + \frac{V_o}{sL} + \frac{V_o - V_g}{R_1} = 0$

$$\therefore\ H(s) = \frac{V_o}{V_G} = \frac{(1/R_1C)s}{s^2 + \left(\frac{R_1+R_2}{R_1R_2C}\right)s + (1/LC)}$$

$$= \frac{3200s}{s^2 + 4000s + 16\times 10^8}$$

$$H(jn\omega_o) = \frac{j3200n\omega_o}{16\times 10^8 - n^2\omega_o^2 + j4000n\omega_o}$$

$$\omega_o = \frac{2\pi}{50\pi}\times 10^6 = 40{,}000 \text{ rad/s}$$

$$\therefore\ H(jn\omega_o) = \frac{j1.28n}{16(1-n^2) + j1.6n}$$

$H_{-1} = 0.8\underline{/0^\circ}$; $H_1 = 0.8\underline{/0^\circ}$

$H_{-2} = 0.0532\underline{/86.19^\circ}$; $H_2 = 0.0532\underline{/-86.19^\circ}$

$H_{-3} = 0.0300\underline{/87.85^\circ}$; $H_3 = 0.0300\underline{/-87.85^\circ}$

$H_{-4} = 0.0213\underline{/88.47^\circ}$; $H_4 = 0.0213\underline{/-88.47^\circ}$

$c_o = 0$

$c_{-1} = (72\underline{/90^\circ})(0.8\underline{/0^\circ}) = 57.60\underline{/90^\circ}$

$c_1 = 57.60\underline{/-90^\circ}$

$c_{-2} = (36\underline{/-90^\circ})(0.0532\underline{/86.18^\circ}) = 1.92\underline{/-3.81^\circ}$

$c_2 = 1.92\underline{/3.81^\circ}$

$c_{-3} = (24\underline{/90^\circ})(0.0300\underline{/87.85^\circ}) = 0.72\underline{/177.85^\circ}$

$c_3 = 0.72\underline{/-177.85^\circ}$

$c_{-4} = (18\underline{/-90^\circ})(0.0213\underline{/88.47^\circ}) = 0.38\underline{/-1.53^\circ}$

$c_4 = 0.38\underline{/1.53^\circ}$

[d] $V_{orms} \approx \sqrt{2\sum_{n=1}^{4}|c_n|^2}$

$$= \sqrt{2(57.6^2 + 1.92^2 + 0.72^2 + 0.38^2)} = 81.51\,\text{V}$$

$$P = \frac{(81.51)^2}{200} \times 10^{-3} = 33.22\,\text{mW}$$

P 16.47 [a] $V_{rms} = \sqrt{\frac{2}{T}\int_0^{T/2}\frac{4v_m^2}{T^2}t^2\,dt} = \frac{V_m}{\sqrt{3}} = \frac{72\pi}{\sqrt{3}} = 130.59\,\text{V}$

[b] $V_{rms} \approx \sqrt{2\sum_{n=1}^{4}|c_n|^2} = \sqrt{2(72^2 + 36^2 + 24^2 + 18^2)} = 121.49\,\text{V}$

[c] % error $= \left(\frac{121.49}{130.59} - 1\right)(100) = -6.97\%$

P 16.48 [a] From Example 16.3 we have:

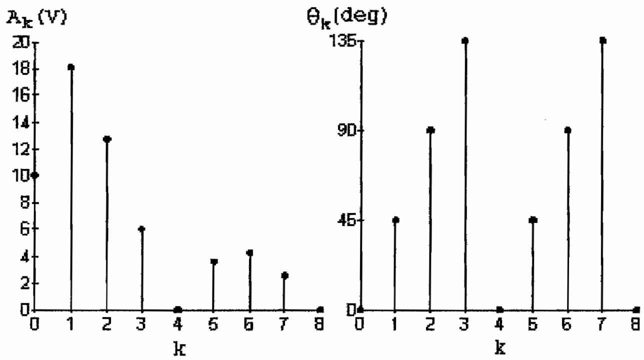
$$a_v = \frac{40}{4} = 10\,\text{V}, \qquad a_k = \frac{40}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

$$b_k = \frac{40}{\pi k}\left[1 - \cos\left(\frac{k\pi}{2}\right)\right], \qquad A_k\underline{/-\theta_k^\circ} = a_k - jb_k$$

$A_1 = 18.01\,\text{V} \quad \theta_1 = 45^\circ, \quad A_2 = 12.73\,\text{V}, \quad \theta_2 = 90^\circ$

$A_3 = 6\,\text{V}, \quad \theta_3 = 135^\circ, \quad A_4 = 0, \quad A_5 = 3.6\,\text{V}, \quad \theta_5 = 45^\circ$

$A_6 = 4.24\,\text{V}, \quad \theta_6 = 90^\circ, \quad A_7 = 2.57\,\text{V}, \quad \theta_7 = 135^\circ$



[b] $C_n = \dfrac{a_n - jb_n}{2}, \qquad C_{-n} = \dfrac{a_n + jb_n}{2} = C_n^*$

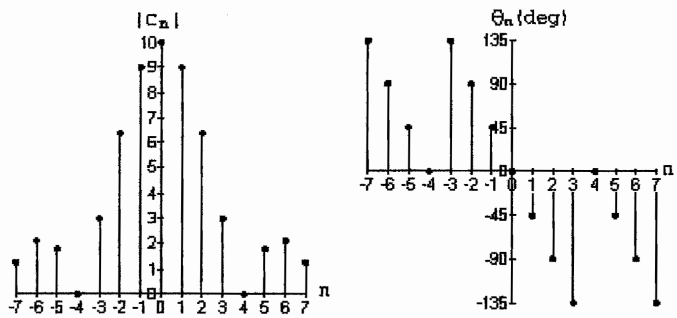
$$C_0 = a_v = 10\,\text{V} \qquad C_3 = 3\underline{/135^\circ}\,\text{V} \qquad C_6 = 2.12\underline{/90^\circ}\,\text{V}$$

$$C_1 = 9\underline{/45^\circ}\,\text{V} \qquad C_{-3} = 3\underline{/-135^\circ}\,\text{V} \qquad C_{-6} = 2.12\underline{/-90^\circ}\,\text{V}$$

$$C_{-1} = 9\underline{/45^\circ}\,\text{V} \qquad C_4 = C_{-4} = 0 \qquad C_7 = 1.29\underline{/135^\circ}\,\text{V}$$

$$C_2 = 6.37\underline{/90^\circ}\,\text{V} \qquad C_5 = 1.8\underline{/45^\circ}\,\text{V} \qquad C_{-7} = 1.29\underline{/-135^\circ}\,\text{V}$$

$$C_{-2} = 6.37\underline{/-90^\circ}\,\text{V} \qquad C_{-5} = 1.8\underline{/-45^\circ}\,\text{V}$$



P 16.49 **[a]** From the solution to Problem 16.36 we have

$$a_v = 135\pi\,\text{V}; \qquad a_k = 0, \quad \text{all } k$$

$$b_k = \frac{-270}{k} \quad \text{all } k$$

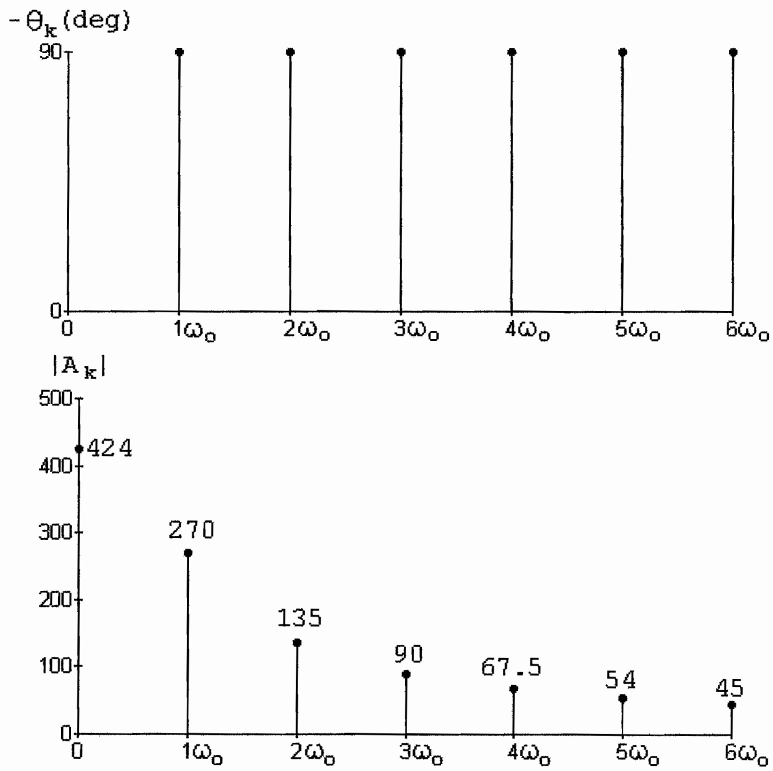
$$A_k\underline{/-\theta_k} = a_k - jb_k = j\frac{270}{k} = \frac{270}{k}\underline{/90^\circ}$$

$$\therefore \quad \theta_k = -90, \quad \text{all } k$$

$$A_1\underline{/-\theta_1} = 270\underline{/90^\circ}; \qquad A_2\underline{/-\theta_2} = 135\underline{/90^\circ}$$

$A_3\underline{/-\theta_3} = 90\underline{/90^\circ}$; $A_4\underline{/-\theta_4} = 67.5\underline{/90^\circ}$

$A_5\underline{/-\theta_5} = 54\underline{/90^\circ}$; $A_6\underline{/-\theta_6} = 45\underline{/90^\circ}$



[b] $c_n = \frac{1}{2}(a_n - jb_n) = j\frac{135}{n} = c_n\underline{/\theta_n}$ (see Eq.[16.87])

$$c_{-n} = \frac{1}{2}(a_n + jb_n) = -j\frac{135}{n}$$

$c_1 = 135\underline{/90^\circ}$; $c_{-1} = 135\underline{/-90^\circ}$

$c_2 = 67.5\underline{/90^\circ}$; $c_{-2} = 67.5\underline{/-90^\circ}$

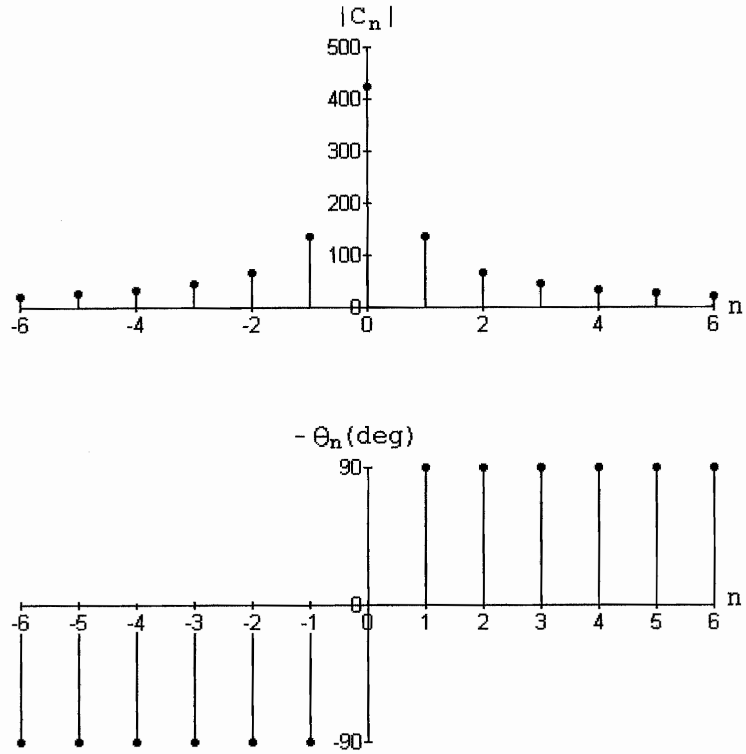
$c_3 = 45\underline{/90^\circ}$; $c_{-3} = 45\underline{/-90^\circ}$

$c_4 = 33.75\underline{/90^\circ}$; $c_{-4} = 33.75\underline{/-90^\circ}$

$c_5 = 27\underline{/90^\circ}$; $c_{-5} = 27\underline{/-90^\circ}$

$c_6 = 22.5\underline{/90^\circ}$; $c_{-6} = 22.5\underline{/-90^\circ}$

$c_o = a_v = 424.12$



P 16.50 [a] $v = A_1 \cos(\omega_o t - 90^\circ) + A_3 \cos(3\omega_o t + 90^\circ)$

$$+A_5 \cos(5\omega_o t - 90^\circ) + A_7 \cos(7\omega_o t + 90^\circ)$$

$$v = A_1 \sin \omega_o t - A_3 \sin 3\omega_o t + A_5 \sin 5\omega_o t - A_7 \sin 7\omega_o t$$

[b] $v(-t) = -A_1 \sin \omega_o t + A_3 \sin 3\omega_o t - A_5 \sin 5\omega_o t + A_7 \sin 7\omega_o t$

$\therefore\ v(-t) = -v(t);$ odd function

[c] $v(t - T/2) = A_1 \sin(\omega_o t - \pi) - A_3 \sin(3\omega_o t - 3\pi)$

$$+A_5 \sin(5\omega_o t - 5\pi) - A_7 \sin(7\omega_o t - 7\pi)$$

$$= -A_1 \sin \omega_o t + A_3 \sin 3\omega_o t - A_5 \sin 5\omega_o t + A_7 \sin 7\omega_o t$$

$\therefore\ v(t - T/2) = -v(t)$, yes, the function has half-wave symmetry

[d] Since the function is odd, with hws, we test to see if

$f(T/2 - t) = f(t)$

$$f(T/2 - t) = A_1 \sin(\pi - \omega_o t) - A_3 \sin(3\pi - 3\omega_o t)$$

$$+A_5 \sin(5\pi - 5\omega_o t) - A_7 \sin(7\pi - 7\omega_o t)$$

$$= A_1 \sin \omega_o t - A_3 \sin 3\omega_o t + A_5 \sin 5\omega_o t - A_7 \sin 7\omega_o t$$

$\therefore \quad f(T/2-t)=f(t)$ and the voltage has quarter-wave symmetry

P 16.51 [a] $i = 441\cos(1000t-90^\circ) + 49\cos(3000t+90^\circ) + 17.64\cos(5000t-90^\circ) + 9\cos(7000t+90^\circ)$ mA

$= 441\sin 1000t - 49\sin 3000t + 17.64\sin 5000t - 9\sin 7000t$ mA

[b] $i(t) = -i(-t)$ odd

[c] Yes $A_o = 0,\quad A_n = 0$ for n even

[d] $I_{\rm rms} = \sqrt{\dfrac{441^2+49^2+17.64^2+9^2}{2}} = 314.07\,\text{mA}$

[e] $c_{-1} = 220.50\underline{/90^\circ}; \qquad c_1 = 220.50\underline{/-90^\circ}$

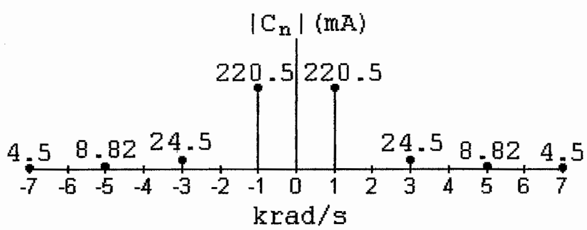
$c_{-3} = 24.50\underline{/-90^\circ}; \qquad c_3 = 24.50\underline{/90^\circ}$

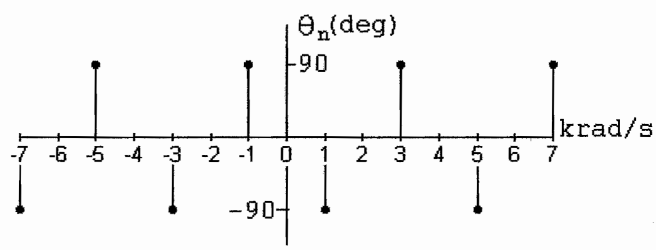
$c_{-5} = 8.82\underline{/90^\circ}; \qquad c_5 = 8.82\underline{/-90^\circ}$

$c_{-7} = 4.50\underline{/-90^\circ}; \qquad c_7 = 4.50\underline{/90^\circ}$

$$i = j4.5e^{-j7000t} + j8.82e^{-j5000t} + j24.5e^{-j3000t} - j220.5e^{-j1000t} + j220.50e^{j1000t} - 24.5e^{j3000t} + j8.82e^{j5000t} - j4.5e^{j7000t}\ \text{mA}$$

[f]





P 16.52 $v_g = \dfrac{8(\pi^2/8)}{\pi^2}\left[\sum_{n=1,3,5,}^{\infty} \dfrac{1}{n^2}\cos n\omega_o t\right]$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{4\pi} = 0.5\ \text{rad/s}$$

$$v_g = \cos 0.5t + \frac{1}{9}\cos 1.5t + \frac{1}{25}\cos 2.5t + \cdots \text{ V}$$

$$H(j0.5k) = \frac{1}{(1-0.5k^2) + jk(1-0.125k^2)}$$

$$H_1 = \frac{1}{(1-0.5) + j(1-0.125)} = 0.9923\underline{/-60.26^\circ}$$

$$H_3 = \frac{1}{[1-0.5(9)] + j3[1-0.125(9)]} = 0.2841\underline{/173.88^\circ}$$

$$H_5 = \frac{1}{[1-0.5(25)] + j5[1-0.125(25)]} = 0.0639\underline{/137.26^\circ}$$

$$v_o = 0.9923\cos(0.5t - 60.26^\circ) + 0.0316\cos(1.5t + 173.88^\circ)$$

$$+ 0.0026\cos(2.5t + 137.26^\circ) + \cdots \text{ V}$$

P 16.53 $$v_g = \frac{2(2.5\pi)}{\pi} - \frac{4(2.5\pi)}{\pi}\frac{\cos 5000t}{4-1} = 5 - (10/3)\cos 5000t - \cdots \text{ V}$$

$$H(j0) = 1$$

$$H(j5000) = \frac{10^6}{(10^6 - 25\times 10^6) + j5\sqrt{2}\times 10^6} = 0.04\underline{/-163.58^\circ}$$

$$\therefore \quad v_o(t) = 5 - 0.1332\cos(5000t - 163.58^\circ) - \cdots \text{ V}$$

P 16.54 [a] Let V_a represent the node voltage across R_2, then the node-voltage equations are

$$\frac{V_a - V_g}{R_1} + \frac{V_a}{R_2} + V_a sC_2 + (V_a - V_o)sC_1 = 0$$

$$(0 - V_a)sC_2 + \frac{0 - V_o}{R_3} = 0$$

Solving for V_o in terms of V_g yields

$$\frac{V_o}{V_g} = H(s) = \frac{\frac{-1}{R_1C_1}s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

It follows that

$$\omega_o^2 = \frac{R_1 + R_2}{R_1R_2R_3C_1C_2}$$

$$\beta = \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$K_o = \frac{R_3}{R_1}\left(\frac{C_2}{C_1 + C_2}\right)$$

Note that

$$H(s) = \frac{-\frac{R_3}{R_1}\left(\frac{C_2}{C_1+C_2}\right)\frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \left(\frac{R_1+R_2}{R_1R_2R_3C_1C_2}\right)}$$

[b] For the given values of $R_1, R_2, R_3, C_1,$ and C_2 we have

$$H(s) = \frac{-400s}{s^2 + 400s + 10^8}$$

$$v_g = \frac{(8)(2.25\pi^2)}{\pi^2}\sum_{n=1,3,5,}^{\infty}\frac{1}{n^2}\cos n\omega_o t$$

$$= 18\left[\cos\omega_o t + \frac{1}{9}\cos 3\omega_o t + \frac{1}{25}\cos 5\omega_o t + \cdots\right] \text{ mV}$$

$$= [18\cos\omega_o t + 2\cos 3\omega_o t + 0.72\cos 5\omega_o t + \cdots] \text{ mV}$$

$$\omega_o = \frac{2\pi}{0.2\pi}\times 10^3 = 10^4 \text{ rad/s}$$

$$H(jk10^4) = \frac{-400jk10^4}{10^8 - k^2 10^8 + j400k10^4} = \frac{-jk}{25(1-k^2) + jk}$$

$$H_1 = -1 = 1\underline{/180^\circ}$$

$$H_3 = \frac{-j3}{-200 + j3} = 0.015\underline{/90.86^\circ}$$

$$H_5 = \frac{-j5}{-600 + j5} = 0.0083\underline{/90.48^\circ}$$

$$v_o = 18\cos(\omega_o t + 180^\circ) + 0.03\cos(3\omega_o t + 90.86^\circ)$$

$$+ 0.006\cos(5\omega_o t + 90.48^\circ) + \cdots \text{ mV}$$

Note – $\omega_o = 10^4$ rad/s and $\beta = 400$ rad/s. Therefore, $Q = 10{,}000/400 = 25$. We expect the output voltage to be dominated by the fundamental frequency component since the bandpass filter is tuned to this frequency!

17

The Fourier Transform

Assessment Problems

AP 17.1 [a] $F(\omega) = \int_{-\tau/2}^{0} (-Ae^{-j\omega t})\,dt + \int_{0}^{\tau/2} Ae^{-j\omega t}\,dt$

$$= \frac{A}{j\omega}[2 - e^{j\omega\tau/2} - e^{-j\omega\tau/2}]$$
$$= \frac{2A}{j\omega}\left[1 - \frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{2}\right]$$
$$= \frac{-j2A}{\omega}[1 - \frac{\cos\omega\tau}{2}]$$

[b] $F(\omega) = \int_{0}^{\infty} te^{-at}e^{-j\omega t}\,dt = \int_{0}^{\infty} te^{-(a+j\omega)t}\,dt = \frac{1}{(a+j\omega)^2}$

AP 17.2

$$f(t) = \frac{1}{2\pi}\left\{\int_{-3}^{-2} 4e^{jt\omega}\,d\omega + \int_{-2}^{2} e^{jt\omega}\,d\omega + \int_{2}^{3} 4e^{jt\omega}\,d\omega\right\}$$

$$= \frac{1}{j2\pi t}\{4e^{-j2t} - 4e^{-j3t} + e^{j2t} - e^{-j2t} + 4e^{j3t} - 4e^{j2t}\}$$

$$= \frac{1}{\pi t}\left[\frac{3e^{-j2t} - 3e^{j2t}}{j2} + \frac{4e^{j3t} - 4e^{-j3t}}{j2}\right]$$

$$= \frac{1}{\pi t}(4\sin 3t - 3\sin 2t)$$

AP 17.3 [a] $F(\omega) = F(s)\,|_{s=j\omega} = \mathcal{L}\{e^{-at} sin\omega_0 t\}_{s=j\omega}$

$$= \left.\frac{\omega_0}{(s+a)^2 + \omega_0^2}\right|_{s=j\omega} = \frac{\omega_0}{(a+j\omega)^2 + \omega_0^2}$$

[b] $F(\omega) = \mathcal{L}\{f^-(t)\}_{s=-j\omega} = \left[\frac{1}{(s+a)^2}\right]_{s=-j\omega} = \frac{1}{(a-j\omega)^2}$

[c] $f^+(t) = te^{-at}, \qquad f^-(t) = -te^{-at}$

$$\mathcal{L}\{f^+(t)\} = \frac{1}{(s+a)^2}, \quad \mathcal{L}\{f^-(t)\} = \frac{-1}{(s+a)^2}$$

Therefore $F(\omega) = \dfrac{1}{(a+j\omega)^2} - \dfrac{1}{(a-j\omega)^2} = \dfrac{-j4a\omega}{(a^2+\omega^2)^2}$

AP 17.4 [a] $f'(t) = \dfrac{2A}{\tau}, \quad \dfrac{-\tau}{2} < t < 0; \qquad f'(t) = \dfrac{-2A}{\tau}, \quad 0 < t < \dfrac{\tau}{2}$

$$\therefore \quad f'(t) = \frac{2A}{\tau}[u(t+\tau/2) - u(t)] - \frac{2A}{\tau}[u(t) - u(t-\tau/2)]$$

$$= \frac{2A}{\tau}u(t+\tau/2) - \frac{4A}{\tau}u(t) + \frac{2A}{\tau}u(t-\tau/2)$$

$$\therefore \quad f''(t) = \frac{2A}{\tau}\delta\left(t+\frac{\tau}{2}\right) - \frac{4A}{\tau} + \frac{2A}{\tau}\delta\left(t-\frac{\tau}{2}\right)$$

[b] $\mathcal{F}\{f''(t)\} = \left[\dfrac{2A}{\tau}e^{j\omega\tau/2} - \dfrac{4A}{\tau} + \dfrac{2A}{\tau}e^{-j\omega\tau/2}\right]$

$$= \frac{4A}{\tau}\left[\frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2} - 1\right] = \frac{4A}{\tau}\left[\cos\left(\frac{\omega\tau}{2}\right) - 1\right]$$

[c] $\mathcal{F}\{f''(t)\} = (j\omega)^2F(\omega) = -\omega^2F(\omega); \qquad$ therefore $F(\omega) = -\dfrac{1}{\omega^2}\mathcal{F}\{f''(t)\}$

Thus we have $F(\omega) = -\dfrac{1}{\omega^2}\left\{\dfrac{4A}{\tau}\left[\cos\left(\dfrac{\omega\tau}{2}\right) - 1\right]\right\}$

AP 17.5

$$v(t) = V_m\left[u\left(t+\frac{\tau}{2}\right) - u\left(t-\frac{\tau}{2}\right)\right]$$

$$\mathcal{F}\left\{u\left(t+\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{j\omega\tau/2}$$

$$\mathcal{F}\left\{u\left(t-\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{-j\omega\tau/2}$$

Therefore $V(\omega) = V_m\left[\pi\delta(\omega) + \dfrac{1}{j\omega}\right]\left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$

$$= j2V_m\pi\delta(\omega)\sin\left(\frac{\omega\tau}{2}\right) + \frac{2V_m}{\omega}\sin\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{(V_m\tau)\sin(\omega\tau/2)}{\omega\tau/2}$$

AP 17.6 [a] $I_g(\omega) = \mathcal{F}\{10\text{sgn}\, t\} = \dfrac{20}{j\omega}$

[b] $H(s) = \dfrac{V_o}{I_g}$

Using current division and Ohm's law,

$$V_o = -I_2 s = -\left[\frac{4}{4+1+s}\right](-I_g)s = \frac{4s}{5+s}I_g$$

$$H(s) = \frac{4s}{s+5}, \qquad H(j\omega) = \frac{j4\omega}{5+j\omega}$$

[c] $V_o(\omega) = H(j\omega)\cdot I_g(\omega) = \left(\dfrac{j4\omega}{5+j\omega}\right)\left(\dfrac{20}{j\omega}\right) = \dfrac{80}{5+j\omega}$

[d] $v_o(t) = 80e^{-5t}u(t)$ V

[e] Using current division,

$$i_1(0^-) = \frac{1}{5}i_g = \frac{1}{5}(-10) = -2\,\text{A}$$

[f] $i_1(0^+) = i_g + i_2(0^+) = 10 + i_2(0^-) = 10 + 8 = 18\,\text{A}$

[g] Using current division,

$$i_2(0^-) = \frac{4}{5}(10) = 8\,\text{A}$$

[h] Since the current in an inductor must be continuous,

$$i_2(0^+) = i_2(0^-) = 8\,\text{A}$$

[i] Since the inductor behaves as a short circuit for $t < 0$,

$$v_o(0^-) = 0\,\text{V}$$

[j] $v_o(0^+) = 1i_2(0^+) + 4i_1(0^+) = 80\,\text{V}$

AP 17.7 [a] $V_g(\omega) = \dfrac{1}{1-j\omega} + \pi\delta(\omega) + \dfrac{1}{j\omega}$

$$H(s) = \frac{V_a}{V_g} = \frac{0.5\|(1/s)}{1+0.5\|(1/s)} = \frac{1}{s+3}, \qquad H(j\omega) = \frac{1}{3+j\omega}$$

$$\begin{aligned} V_a(\omega) &= H(j\omega)V_g(j\omega) \\ &= \frac{1}{(1-j\omega)(3+j\omega)} + \frac{1}{j\omega(3+j\omega)} + \frac{\pi\delta(\omega)}{3+j\omega} \\ &= \frac{1/4}{1-j\omega} + \frac{1/4}{3+j\omega} + \frac{1/3}{j\omega} - \frac{1/3}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \\ &= \frac{1/4}{1-j\omega} + \frac{1/3}{j\omega} - \frac{1/12}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \end{aligned}$$

Therefore $v_a(t) = \left[\dfrac{1}{4}e^{t}u(-t) + \dfrac{1}{6}\text{sgn}\, t - \dfrac{1}{12}e^{-3t}u(t) + \dfrac{1}{6}\right]$ V

[b] $v_a(0^-) = \frac{1}{4} - \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{4}\,\text{V}$

$$v_a(0^+) = 0 + \frac{1}{6} - \frac{1}{12} + \frac{1}{6} = \frac{1}{4}\,\text{V}$$

$$v_a(\infty) = 0 + \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{3}\,\text{V}$$

AP 17.8

$$v(t) = 4te^{-t}u(t); \qquad V(\omega) = \frac{4}{(1+j\omega)^2}$$

Therefore $|V(\omega)| = \dfrac{4}{1+\omega^2}$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{\sqrt{3}} \left[\frac{4}{(1+\omega^2)}\right]^2 d\omega$$

$$= \frac{16}{\pi}\left\{\frac{1}{2}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\sqrt{3}}\right\}$$

$$= 16\left[\frac{\sqrt{3}}{8\pi} + \frac{1}{6}\right] = 3.769\,\text{J}$$

$$W_{1\Omega}(\text{total}) = \frac{8}{\pi}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\infty} = \frac{8}{\pi}\left[0 + \frac{\pi}{2}\right] = 4\,\text{J}$$

Therefore $\% = \dfrac{3.769}{4}(100) = 94.23\%$

AP 17.9

$$|V(\omega)| = 6 - \left(\frac{6}{2000\pi}\right)\omega, \qquad 0 \le \omega \le 2000\pi$$

$$|V(\omega)|^2 = 36 - \left(\frac{72}{2000\pi}\right)\omega + \left(\frac{36}{4\pi^2 \times 10^6}\right)\omega^2$$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{2000\pi}\left[36 - \frac{72\omega}{2000\pi} + \frac{36 \times 10^{-6}}{4\pi^2}\omega^2\right] d\omega$$

$$= \frac{1}{\pi}\left[36\omega - \frac{72\omega^2}{4000\pi} + \frac{36 \times 10^{-6}\omega^3}{12\pi^2}\right]_0^{2000\pi}$$

$$= \frac{1}{\pi}\left[36(2000\pi) - \frac{72}{4000\pi}(2000\pi)^2 + \frac{36 \times 10^{-6}(2000\pi)^3}{12\pi^2}\right]$$

$$= 36(2000) - \frac{72(2000)^2}{4000} + \frac{36 \times 10^{-6}(2000)^3}{12}$$

$$= 24\,\mathrm{kJ}$$

$$W_{6\mathrm{k}\Omega} = \frac{24 \times 10^3}{6 \times 10^3} = 4\,\mathrm{J}$$

Problems

P 17.1 [a] $F(\omega) = \int_{-2}^{2}\left[A\sin\left(\frac{\pi}{2}\right)t\right]e^{-j\omega t}\,dt = \frac{-j4\pi A}{\pi^2 - 4\omega^2}\sin 2\omega$

[b] $F(\omega) = \int_{-\tau/2}^{0}\left(\frac{2A}{\tau}t + A\right)e^{-j\omega t}\,dt + \int_{0}^{\tau/2}\left(\frac{-2A}{\tau}t + A\right)e^{-j\omega t}\,dt$

$$= \frac{4A}{\omega^2\tau}\left[1 - \cos\left(\frac{\omega\tau}{2}\right)\right]$$

P 17.2 [a] $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$

$$= \int_{-\tau/2}^{0}\frac{-2A}{\tau}te^{-j\omega t}\,dt + \int_{0}^{\tau/2}\frac{2A}{\tau}te^{-j\omega t}\,dt$$

$$= \text{Int1} + \text{Int2}$$

$$\text{Int1} = \frac{-2A}{\tau}\int_{-\tau/2}^{0} te^{-j\omega t}\,dt$$

$$= \frac{-2A}{\tau}\left\{\frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_{-\tau/2}^{0}\right\}$$

$$= \frac{-2A}{\omega^2\tau}\left\{1 - [e^{j\omega\tau/2}(-j\omega\tau/2 + 1)]\right\}$$

$$= \frac{2A}{\omega^2\tau}\left\{e^{j\omega\tau/2}(1 - j\omega\tau/2) - 1\right\}$$

$$\text{Int2} = \frac{2A}{\tau}\int_{0}^{\tau/2} te^{-j\omega t}\,dt$$

$$= \frac{2A}{\tau}\left\{\frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_{0}^{\tau/2}\right\}$$

$$= \frac{2A}{\omega^2\tau}\left\{e^{j\omega\tau/2}(j\omega\tau/2 + 1) - 1)]\right\}$$

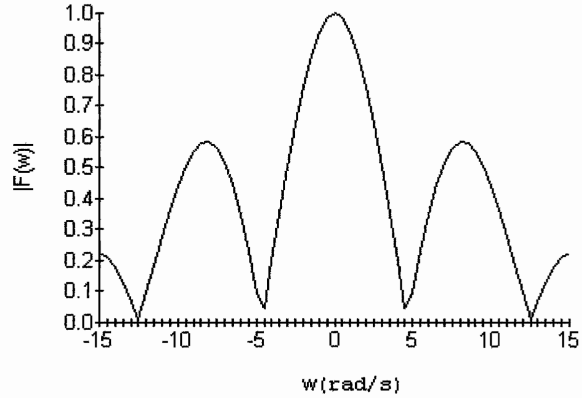
$$F(\omega) = \text{Int1} + \text{Int2}$$

$$= \frac{2A}{\omega^2\tau}\left\{2\cos\frac{\omega\tau}{2} + \omega\tau\sin\frac{\omega\tau}{2} - 2\right\}$$

[b] After using L'Hopital's rule we have

$$F(0) = \lim_{\omega\to 0}\frac{2A\tau\cos(\omega\tau/2)}{4} = \frac{A\tau}{2}$$

[c]



P 17.3 [a] $F(\omega) = j\frac{2A}{\omega_o}\omega \qquad -\frac{\omega_o}{2} \leq \omega \leq \frac{\omega_o}{2}$

$$f(t) = \frac{1}{2\pi}\int_{-\omega_o/2}^{\omega_o/2} \frac{j2A}{\omega_o}\omega e^{j\omega t}\,d\omega$$

$$= \frac{jA}{\pi\omega_o}\left[\frac{e^{jt\omega}}{-t^2}(jt\omega - 1)\Big|_{-\omega_o/2}^{\omega_o/2}\right]$$

$$= \frac{A}{\pi\omega_o t^2}[\omega_o t\cos(\omega_o t/2) - 2\sin(\omega_o t/2)]$$

[b] $f(t) = \frac{A}{\pi\omega_o}\left[\frac{\omega_o t\cos(\omega_o t/2) - 2\sin(\omega_o t/2)}{t^2}\right]$

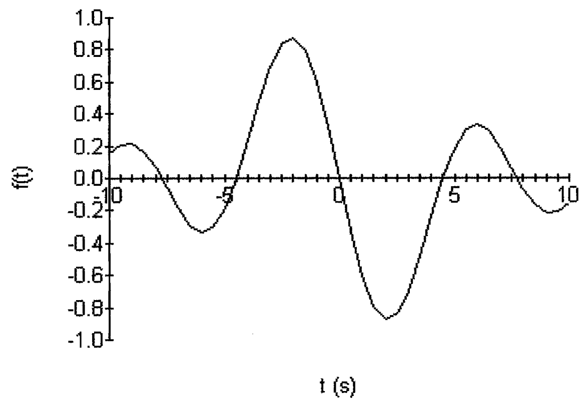
$$f(0) = \lim_{t\to 0}\left\{\frac{A}{\pi\omega_o}\left[\frac{\omega_o t(-\frac{\omega_o}{2}\sin\frac{\omega_o t}{2}) + \omega_o\cos\frac{\omega_o t}{2} - \omega_o\cos\frac{\omega_o t}{2}}{2t}\right]\right\}$$

$$= \lim_{t\to 0}\left\{\frac{A}{\pi\omega_o}\left[\frac{-\omega_o^2}{4}\sin\left(\frac{\omega_o t}{2}\right)\right]\right\} = 0$$

[c] When $A = 2\pi$ and $\omega_o = 2$ rad/s

$$f(t) = \frac{1}{t^2}[2t\cos t - 2\sin t]$$

$f(-t) = -f(t) \qquad$ odd function



P 17.4 [a] $F(s) = \mathcal{L}\{te^{-at}\} = \dfrac{1}{(s+a)^2}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \left[\frac{1}{(a+j\omega)^2}\right] + \left[\frac{1}{(a-j\omega)^2}\right]$$

$$= \frac{2(a^2-\omega^2)}{(a^2-\omega^2)^2+4a^2\omega^2} = \frac{2(a^2-\omega^2)}{(a^2+\omega^2)^2}$$

[b] $F(s) = \mathcal{L}\{t^3e^{-at}\} = \dfrac{-6}{(s+a)^4}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{-6}{(a+j\omega)^4} + \frac{-6}{(a-j\omega)^4} = -j48a\omega\frac{a^2-\omega^2}{(a^2+\omega^2)^4}$$

[c] $F(s) = \mathcal{L}\{e^{-at}\cos\omega_0 t\} = \dfrac{s+a}{(s+a)^2+\omega_0^2} = \dfrac{0.5}{(s+a)-j\omega_0} + \dfrac{0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{0.5}{(a+j\omega)-j\omega_0} + \frac{0.5}{(a+j\omega)+j\omega_0}$$

$$+ \frac{0.5}{(a-j\omega)-j\omega_0} + \frac{0.5}{(a-j\omega)+j\omega_0}$$

$$= \frac{a}{a^2+(\omega-\omega_0)^2} + \frac{a}{a^2+(\omega+\omega_0)^2}$$

[d] $F(s) = \mathcal{L}\{e^{-at}\sin\omega_0 t\} = \dfrac{\omega_0}{(s+a)^2+\omega_0^2} = \dfrac{-j0.5}{(s+a)-j\omega_0} + \dfrac{j0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{-(\omega-\omega_0)}{a^2+(\omega-\omega_0)^2} + \frac{(\omega+\omega_0)}{a^2+(\omega+\omega_0)^2}$$

[e] $F(\omega) = \int_{-\infty}^{\infty} \delta(t-t_o)e^{-j\omega t}\,dt = e^{-j\omega t_o}$

(Use the sifting property of the Dirac delta function.)

P 17.5 $\mathcal{F}\{\sin\omega_0 t\} = \mathcal{F}\left\{\dfrac{e^{j\omega_0 t}}{2j}\right\} - \mathcal{F}\left\{\dfrac{e^{-j\omega_0 t}}{2j}\right\}$

$$= \frac{1}{2j}[2\pi\delta(\omega-\omega_0) - 2\pi\delta(\omega+\omega_0)]$$

$$= j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$$

P 17.6 $f(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}[A(\omega)+jB(\omega)][\cos t\omega + j\sin t\omega]\,d\omega$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega - B(\omega)\sin t\omega]\,d\omega$$

$$+ \frac{j}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\sin t\omega + B(\omega)\cos t\omega]\,d\omega$$

But $f(t)$ is real, therefore the second integral in the sum is zero.

P 17.7 By hypothesis, $f(t) = -f(-t)$. From Problem 17.6, we have

$$f(-t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega + B(\omega)\sin t\omega]\,d\omega$$

For $f(t) = -f(-t)$, the integral $\int_{-\infty}^{\infty} A(\omega)\cos t\omega\,d\omega$ must be zero. Therefore, if $f(t)$ is real and odd, we have

$$f(t) = \frac{-1}{2\pi}\int_{-\infty}^{\infty} B(\omega)\sin t\omega\,d\omega$$

P 17.8 $F(\omega) = \dfrac{-j2}{\omega}$; therefore $B(\omega) = \dfrac{-2}{\omega}$; thus we have

$$f(t) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{-2}{\omega}\right)\sin t\omega\,d\omega = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin t\omega}{\omega}\,d\omega$$

But $\dfrac{\sin t\omega}{\omega}$ is even; therefore $f(t) = \dfrac{2}{\pi}\displaystyle\int_0^{\infty}\frac{\sin t\omega}{\omega}\,d\omega$

Therefore,

$$\left.\begin{aligned} f(t) &= \frac{2}{\pi}\cdot\frac{\pi}{2} = 1 \qquad t>0 \\ f(t) &= \frac{2}{\pi}\cdot\left(\frac{-\pi}{2}\right) = -1\; t<0 \end{aligned}\right\}\text{from a table of definite integrals}$$

Therefore $\quad f(t) = \operatorname{sgn} t$

P 17.9 From Problem 17.4[c] we have

$$F(\omega) = \frac{\epsilon}{\epsilon^2 + (\omega-\omega_0)^2} + \frac{\epsilon}{\epsilon^2 + (\omega+\omega_0)^2}$$

Note that as $\epsilon \to 0$, $F(\omega) \to 0$ everywhere except at $\omega = \pm\omega_0$. At $\omega = \pm\omega_0$, $F(\omega) = 1/\epsilon$, therefore $F(\omega) \to \infty$ at $\omega = \pm\omega_0$ as $\epsilon \to 0$. The area under each bell-shaped curve is independent of ϵ, that is

$$\int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega-\omega_0)^2} = \int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega+\omega_0)^2} = \pi$$

Therefore as $\epsilon \to 0$, $\quad F(\omega) \to \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$

P 17.10 $A(\omega) = \int_{-\infty}^{\infty} f(t)\cos\omega t\, dt$

$$= \int_{-\infty}^{0} f(t)\cos\omega t\, dt + \int_{0}^{\infty} f(t)\cos\omega t\, dt$$

$$= 2\int_{0}^{\infty} f(t)\cos\omega t\, dt, \quad \text{since } f(t)\cos\omega t \text{ is also even.}$$

$B(\omega) = 0$, since $f(t)\sin\omega t$ is an odd function and

$$\int_{-\infty}^{0} f(t)\sin\omega t\, dt = -\int_{0}^{\infty} f(t)\sin\omega t\, dt$$

P 17.11 $A(\omega) = \int_{-\infty}^{0} f(t)\cos\omega t\, dt + \int_{0}^{\infty} f(t)\cos\omega t\, dt = 0$

since $f(t)\cos\omega t$ is an odd function.

$$B(\omega) = -2\int_{0}^{\infty} f(t)\sin\omega t\, dt, \quad \text{since } f(t)\sin\omega t \text{ is an even function.}$$

P 17.12 [a] $\mathcal{F}\left\{\frac{df(t)}{dt}\right\} = \int_{-\infty}^{\infty} \frac{df(t)}{dt} e^{-j\omega t}\, dt$

Let $u = e^{-j\omega t}$, then $du = -j\omega e^{-j\omega t}\,dt$; let $dv = [df(t)/dt]\,dt$, then $v = f(t)$.

$$\text{Therefore}\quad \mathcal{F}\left\{\frac{df(t)}{dt}\right\} = f(t)e^{-j\omega t}\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(t)[-j\omega e^{-j\omega t}\,dt]$$

$$= 0 + j\omega F(\omega)$$

[b] Fourier transform of $f(t)$ exists, i.e., $f(\infty) = f(-\infty) = 0$.

[c] To find $\mathcal{F}\left\{\frac{d^2f(t)}{dt^2}\right\}$, let $g(t) = \frac{df(t)}{dt}$

$$\text{Then}\quad \mathcal{F}\left\{\frac{d^2f(t)}{dt^2}\right\} = \mathcal{F}\left\{\frac{dg(t)}{dt}\right\} = j\omega G(\omega)$$

$$\text{But}\quad G(\omega) = \mathcal{F}\left\{\frac{df(t)}{dt}\right\} = j\omega F(\omega)$$

$$\text{Therefore we have}\quad \mathcal{F}\left\{\frac{d^2f(t)}{dt^2}\right\} = (j\omega)^2F(\omega)$$

Repeated application of this thought process gives

$$\mathcal{F}\left\{\frac{d^nf(t)}{dt^n}\right\} = (j\omega)^nF(\omega)$$

P 17.13 [a] $\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{t} f(x)\,dx\right]e^{-j\omega t}\,dt$

$$\text{Now let}\quad u = \int_{-\infty}^{t} f(x)\,dx, \qquad \text{then}\quad du = f(t)dt$$

$$\text{Let}\quad dv = e^{-j\omega t}\,dt, \qquad \text{then}\quad v = \frac{e^{-j\omega t}}{-j\omega}$$

Therefore,

$$\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \frac{e^{-j\omega t}}{-j\omega}\int_{-\infty}^{t} f(x)\,dx\,\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\left[\frac{e^{-j\omega t}}{-j\omega}\right]f(t)\,dt$$

$$= 0 + \frac{F(\omega)}{j\omega}$$

[b] We require $\int_{-\infty}^{\infty} f(x)\,dx = 0$

[c] No, because $\int_{-\infty}^{\infty} e^{-ax}u(x)\,dx = \frac{1}{a} \neq 0$

P 17.14 [a] $\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(at)e^{-j\omega t}\,dt$

Let $u = at$, $du = a\,dt$, $u = \pm\infty$ when $t = \pm\infty$

Therefore,

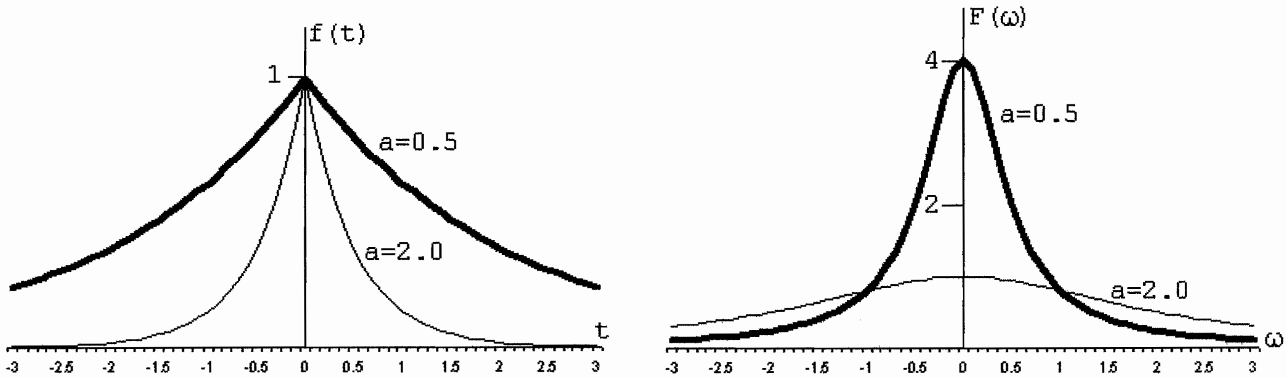
$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega u/a}\left(\frac{du}{a}\right) = \frac{1}{a}F\left(\frac{1}{a}\right), \qquad a > 0$$

[b] $\mathcal{F}\{e^{-|t|}\} = \frac{1}{1+j\omega} + \frac{1}{1-j\omega} = \frac{2}{1+\omega^2}$

Therefore $\mathcal{F}\{e^{-a|t|}\} = \frac{(1/a)2}{(\omega/a)^2+1}$

Therefore $\mathcal{F}\{e^{-0.5|t|}\} = \frac{4}{4\omega^2+1}$, $\qquad \mathcal{F}\{e^{-|t|}\} = \frac{2}{\omega^2+1}$

$\mathcal{F}\{e^{-2|t|}\} = 1/[0.25\omega^2+1]$, yes as "$a$" increases, the sketches show that $f(t)$ approaches zero faster and $F(\omega)$ flattens out over the frequency spectrum.



P 17.15 [a] $\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(t-a)e^{-j\omega t}\,dt$

Let $u = t - a$, then $du = dt$, $t = u + a$, and $u = \pm\infty$ when $t = \pm\infty$.
Therefore,

$$\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega(u+a)}\,du$$

$$= e^{-j\omega a}\int_{-\infty}^{\infty} f(u)e^{-j\omega u}\,du = e^{-j\omega a}F(\omega)$$

[b] $\mathcal{F}\{e^{j\omega_0 t}f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_0)t}\,dt = F(\omega-\omega_0)$

[c] $\mathcal{F}\{f(t)\cos\omega_0 t\} = \mathcal{F}\left\{f(t)\left[\frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right]\right\}$

$$= \frac{1}{2}F(\omega-\omega_0) + \frac{1}{2}F(\omega+\omega_0)$$

P 17.16 $Y(\omega) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\lambda) h(t-\lambda)\, d\lambda \right] e^{-j\omega t}\, dt$

$$= \int_{-\infty}^{\infty} x(\lambda) \left[\int_{-\infty}^{\infty} h(t-\lambda) e^{-j\omega t}\, dt \right] d\lambda$$

Let $u = t - \lambda$, $du = dt$, and $u = \pm\infty$, when $t = \pm\infty$.

Therefore $Y(\omega) = \int_{-\infty}^{\infty} x(\lambda) \left[\int_{-\infty}^{\infty} h(u) e^{-j\omega(u+\lambda)}\, du \right] d\lambda$

$$= \int_{-\infty}^{\infty} x(\lambda) \left[e^{-j\omega\lambda} \int_{-\infty}^{\infty} h(u) e^{-j\omega u}\, du \right] d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda) e^{-j\omega\lambda} H(\omega)\, d\lambda = H(\omega)X(\omega)$$

P 17.17 $\mathcal{F}\{f_1(t)f_2(t)\} = \int_{-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(u) e^{jtu} du \right] f_2(t) e^{-j\omega t}\, dt$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} F_1(u) f_2(t) e^{-j\omega t} e^{jtu}\, du \right] dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[F_1(u) \int_{-\infty}^{\infty} f_2(t) e^{-j(\omega-u)t}\, dt \right] du$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(u) F_2(\omega - u)\, du$$

P 17.18 [a] $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt$

$$\frac{dF}{d\omega} = \int_{-\infty}^{\infty} \frac{d}{d\omega} \left[f(t) e^{-j\omega t}\, dt \right] = -j \int_{-\infty}^{\infty} t f(t) e^{-j\omega t}\, dt = -j\mathcal{F}\{tf(t)\}$$

Therefore $j\dfrac{dF(\omega)}{d\omega} = \mathcal{F}\{tf(t)\}$

$$\frac{d^2F(\omega)}{d\omega^2} = \int_{-\infty}^{\infty} (-jt)(-jt) f(t) e^{-j\omega t}\, dt = (-j)^2 \mathcal{F}\{t^2 f(t)\}$$

Note that $(-j)^n = \dfrac{1}{j^n}$

Thus we have $j^n \left[\dfrac{d^nF(\omega)}{d\omega^n} \right] = \mathcal{F}\{t^n f(t)\}$

[b] (i) $\mathcal{F}\{e^{-at}u(t)\} = \dfrac{1}{a + j\omega} = F(\omega);$ $\qquad \dfrac{dF(\omega)}{d\omega} = \dfrac{-j}{(a+j\omega)^2}$

Therefore $j \left[\dfrac{dF(\omega)}{d\omega} \right] = \dfrac{1}{(a+j\omega)^2}$

$$\text{Therefore}\quad \mathcal{F}\{te^{-at}u(t)\} = \frac{1}{(a+j\omega)^2}$$

(ii) $\mathcal{F}\{|t|e^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} - \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} - j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} + \frac{1}{(a-j\omega)^2}$$

(iii) $\mathcal{F}\{te^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} + \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} + j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} - \frac{1}{(a-j\omega)^2}$$

P 17.19 [a] $f_1(t) = \cos\omega_0 t, \qquad F_1(u) = \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]$

$f_2(t) = 1, \quad -\tau/2 < t < \tau/2, \qquad$ and $f_2(t) = 0$ elsewhere

$$\text{Thus}\quad F_2(u) = \frac{\tau\sin(u\tau/2)}{u\tau/2}$$

Using convolution,

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega-u)\,du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]\tau\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\int_{-\infty}^{\infty} \delta(u+\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$+ \frac{\tau}{2}\int_{-\infty}^{\infty} \delta(u-\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\cdot\frac{\sin[(\omega+\omega_0)\tau/2]}{(\omega+\omega_0)(\tau/2)} + \frac{\tau}{2}\cdot\frac{\sin[(\omega-\omega_0)\tau/2]}{(\omega-\omega_0)\tau/2}$$

[b] As τ increases, the amplitude of $F(\omega)$ increases at $\omega = \pm\omega_0$ and at the same time the duration of $F(\omega)$ approaches zero as ω deviates from $\pm\omega_0$. The area under the $[\sin x]/x$ function is independent of τ, that is

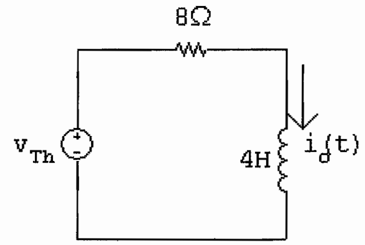
$$\frac{\tau}{2}\int_{-\infty}^{\infty}\frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}\,d\omega = \int_{-\infty}^{\infty}\frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}[(\tau/2)\,d\omega] = \pi$$

Therefore as $t \to \infty$,

$$f_1(t)f_2(t) \to \cos\omega_0 t \quad \text{and} \quad F(\omega) \to \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$$

P 17.20 [a] Find the Thévenin equivalent with respect to the terminals of the inductor. Thus,

$$v_{\text{Th}} = \frac{40}{50}v_g = 0.8v_g; \qquad R_{\text{Th}} = 10\|40 = 8\,\Omega$$



$$I_o = \frac{0.8V_g}{8+4s} = \frac{0.2V_g}{s+2}$$

$$H(s) = \frac{I_o}{V_g} = \frac{0.2}{s+2}$$

$$H(j\omega) = \frac{0.2}{j\omega+2}$$

$$V_g(\omega) = 125\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$

$$I_o(\omega) = V_g(\omega)H(j\omega)$$

$$= \frac{25}{j\omega+2}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$

$$= \frac{25\pi\delta(\omega)}{j\omega+2} + \frac{25}{j\omega(2+j\omega)}$$

$$= I_1(\omega) + I_2(\omega)$$

$$i_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{25\pi\delta(\omega)e^{j\omega t}}{2+j\omega}\,dt = 6.25\,\text{A}$$

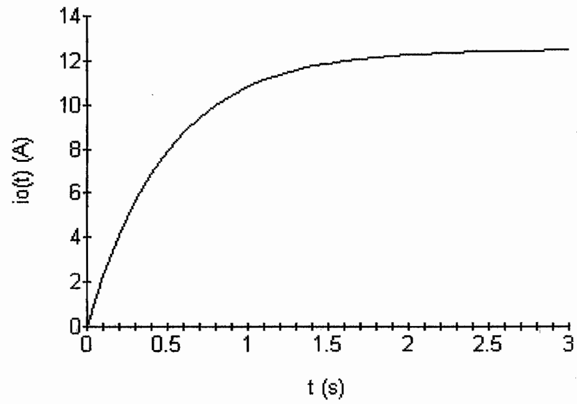
$$I_2(\omega) = \frac{12.5}{j\omega} - \frac{12.5}{j\omega+2}$$

$$i_2(t) = 6.25\text{sgn}(t) - 12.5e^{-2t}u(t)\,\text{A}$$

$$i_o = i_1 + i_2 = 6.25 + 6.25\text{sgn}(t) - 12.5e^{-2t}u(t)\,\text{A}$$

$$i_o(t) = 12.5u(t) - 12.5e^{-2t}u(t)\,\text{A}$$

[b]



P 17.21 [a] From the solution to Problem 17.20 we have

$$H(s) = \frac{I_o}{V_g} = \frac{0.2}{s+2}$$

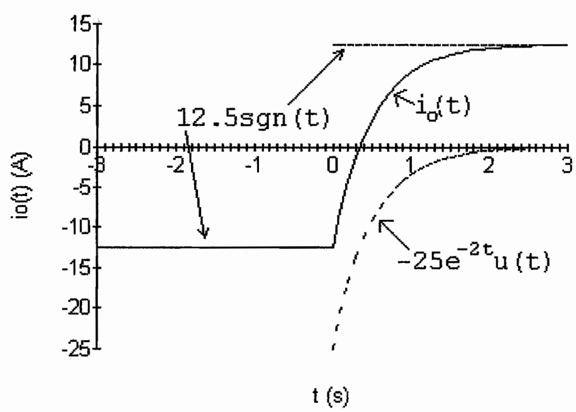
$$H(j\omega) = \frac{0.2}{j\omega + 2}$$

$$v_g = 125\text{sgn}(t)\text{ V}; \qquad V_g(\omega) = \frac{250}{j\omega}$$

$$I_o = V_g H(j\omega) = \frac{50}{j\omega(j\omega+2)} = \frac{25}{j\omega} - \frac{25}{j\omega+2}$$

$$\therefore \quad i_o(t) = 12.5\text{sgn}(t) - 25e^{-2t}u(t)\text{ A}$$

[b]



P 17.22 [a] $H(s) = \dfrac{1/sC}{R + 1/sC} = \dfrac{1/RC}{s + 1/RC} = \dfrac{50}{s+50}$

$$H(\omega) = \frac{50}{j\omega + 50}$$

$$V_g(\omega) = \frac{40}{j\omega}$$

$$V_o(\omega) = \left(\frac{40}{j\omega}\right)\left(\frac{50}{j\omega+50}\right) = \frac{2000}{j\omega(j\omega+50)}$$

$$= \frac{40}{j\omega} - \frac{40}{j\omega+50}$$

$$v_o(t) = 20\text{sgn}(t) - 40e^{-50t}u(t)\text{ V}$$

[b] $v_o()^-) = -20$ V. This makes sense because the capacitor will be charged to -20 V when $t < 0$.
$v_o(0^+) = 20 - 40 = -20$ V. This makes sense because there cannot be an instantaneous change in the voltage drop across the capacitor.
$v(\infty) = 20$ v. This makes sense because the capacitor will charge to 20 V after the signal voltage reverses polarity.
The circuit is a first-order circuit with a time constant of RC or 0.02 s. Therefore, $1/\tau = 50$. We would expect the transition from -20 V to $+20$ V to be exponential with a time constant of 0.02 s.

P 17.23 [a] $H(s) = \dfrac{I_o}{V_g} = \dfrac{1}{R + 1/sC} = \dfrac{(1/R)s}{s + 1/RC}$

$$H(s) = \frac{25\times10^{-6}s}{s+50}; \qquad H(\omega) = \frac{25\times10^{-6}j\omega}{j\omega+50}$$

$$\therefore\quad I_o(\omega) = \frac{25\times10^{-6}j\omega}{j\omega+50}\frac{40}{j\omega} = \frac{10^{-3}}{j\omega+50}$$

$$i_o(t) = 10^{-3}e^{-50t}u(t) = e^{-50t}u(t)\text{ mA}$$

[b] $i_o(0^-) = 0$

This makes sense because v_g and v_o equal; -20 V at $t = 0$.

$i_o(0^+) = 1\,\text{mA}$

This makes sense because $v_o = -20$ V and $v_g = +20$ V at $t = 0^+$. Thus,

$i_o(0^+) = [20 - (-20)]/(40\times10^3) = 1\,\text{mA}$

$i_o(\infty) = 0$

This makes sense because at $t = \infty$, $v_g = v_o = 20$ V.
We have a first-order circuit with a time constant of 0.02 s and therefore we expect $i_o(t)$ to decay exponentially with an exponent of $-t/\tau$ or $-50t$.

P 17.24 [a] $H(s) = \dfrac{1/RC}{s + 1/RC} = \dfrac{100}{s+100}$

$$H(\omega) = \frac{100}{j\omega+100}; \qquad V_g(\omega) = \frac{30}{j\omega}$$

$$V_o(\omega) = \left(\frac{30}{j\omega}\right)\left(\frac{100}{j\omega+100}\right) = \frac{3000}{j\omega(j\omega+100)}$$

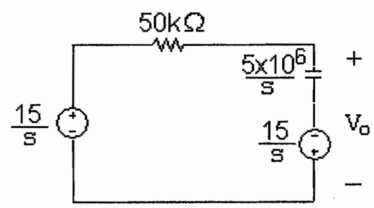
$$= \frac{30}{j\omega} - \frac{30}{j\omega+100}$$

$$\therefore \quad v_o(t) = 15\text{sgn}(t) - 30e^{-100t}u(t)\,\text{V}$$

[b] $v_o(0^-) = -15\,\text{V}$

[c] $v_o(0^+) = 15 - 30 = -15\,\text{V}$

[d]



$$\frac{V_o - 15/s}{50{,}000} + \frac{(V_o + 15/s)s}{5\times10^6} = 0$$

$$100V_o - \frac{1500}{s} + V_os + 15 = 0$$

$$\therefore \quad V_o = \frac{15(100-s)}{s(s+100)} = \frac{K_1}{s} + \frac{K_2}{s+100}$$

$$K_1 = \frac{15(100)}{100} = 15; \qquad K_2 = \frac{15(200)}{-100} = -30$$

$$v_o(t) = (15 - 30e^{-100t})u(t)\,\text{V}$$

[e] Yes, they agree. The solution from part (a) for $t > 0$ is

$$v_o(t) = (15 - 30e^{-100t})u(t)\,\text{V}$$

P 17.25 [a] $H(s) = \dfrac{I_o}{V_g} = \dfrac{(1/R)s}{s + 1/RC}$

$$H(s) = \frac{20\times10^{-6}s}{s+100}; \qquad H(\omega) = \frac{20\times10^{-6}(j\omega)}{j\omega+100}$$

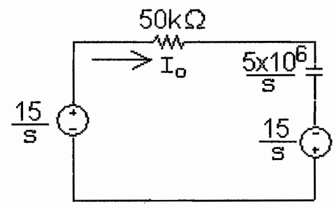
$$I_o(\omega) = \frac{20\times10^{-6}(j\omega)}{j\omega+100}\cdot\frac{30}{j\omega} = \frac{600\times10^{-6}}{j\omega+100}$$

$$i_o(t) = 600e^{-100t}u(t)\,\mu\text{A}$$

[b] $i_o(0^-) = 0$

[c] $i_o(0^+) = 600\,\mu\text{A}$

[d]

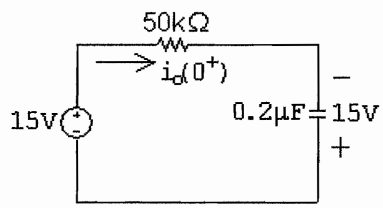


$$I_o = \frac{30/s}{50{,}000 + (5 \times 10^6/s)} = \frac{30}{50{,}000s + 5 \times 10^6}$$

$$= \frac{600 \times 10^{-6}}{s + 100)}$$

$$i_o(t) = 600e^{-100t}u(t)\ \mu\text{A}$$

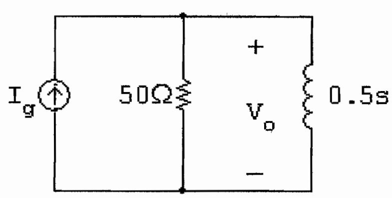
[e] Yes they agree. Also note that at $t = 0^+$ the circuit is



$$i_o(0^+) = \frac{30}{50{,}000} = 600\ \mu\text{A}$$

which agrees with our solution.

P 17.26 [a]



$$\frac{V_o}{50} + \frac{2V_o}{s} = I_g$$

$$V_o\left[\frac{1}{50} + \frac{2}{s}\right] = I_g$$

$$\frac{V_o}{I_g} = H(s) = \frac{50s}{s + 100}$$

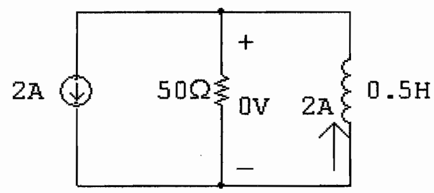
$$H(j\omega) = \frac{j\omega 50}{j\omega + 100}$$

$$I_g(\omega) = \frac{4}{j\omega}$$

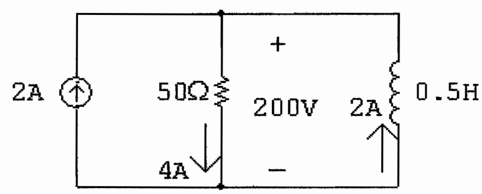
$$V_o(\omega) = \frac{4}{j\omega} \cdot \frac{50(j\omega)}{j + 100} = \frac{200}{j\omega + 100}$$

$$\therefore \quad v_o(t) = 200e^{-100t}u(t)\,\text{V}$$

[b] At $t = 0^-$ the circuit is



At $t = 0^+$ the circuit is



From the circuit

$$v_o(0^+) = (4)(50) = 200\,\text{V}$$

which agrees with our solution.
At $t = \infty$

$$v_o(\infty) = 0$$

since the inductor short-circuits the dc current source. This is also in agreement with our solution.

$$\tau = L/R = 0.5/50 = 1/100; \qquad \therefore \quad 1/\tau = 100$$

which agrees with our solution.

P 17.27 [a] $I_o = \dfrac{V_o}{0.5s} = \dfrac{2}{s}\left(\dfrac{50sI_g}{s + 100}\right)$

$$\frac{I_o}{I_g} = H(s) = \frac{100}{s + 100}$$

$$H(j\omega) = \frac{100}{j\omega + 100}$$

$$I_g(\omega) = \frac{4}{j\omega}$$

$$I_o(\omega) = \frac{400}{j\omega(j\omega + 100)} = \frac{4}{j\omega} - \frac{4}{j\omega + 100}$$

$$\therefore \quad i_o(t) = 2\text{sgn}(t) - 4e^{-100t}u(t)\text{ A}$$

[b]
- From the solution to Problem 17.21(b) we note $i_o(0^-) = -2$ A and $i_o(0^+) = -2$ A. Our solution agrees with these results.
- From the circuit, $i_o(\infty) = 2$ A. Our solution agrees with this value.
- From the circuit, $\tau = 0.01$ s which agrees with our solution.

P 17.28 [a] $V_o = \dfrac{V_g(1/sC)}{R + (1/sC)} = \dfrac{V_g}{RCs + 1}$

$$\frac{V_o}{V_g} = H(s) = \frac{1/RC}{s + (1/RC)} = \frac{1}{s+1}$$

$$H(j\omega) = \frac{1}{j\omega + 1}$$

$$V_g(\omega) = \frac{30}{-j\omega + 5} + \frac{30}{j\omega + 5}$$

$$V_o(\omega) = \frac{30}{(-j\omega + 5)(j\omega + 1)} + \frac{30}{(j\omega + 5)(j\omega + 1)}$$

$$= \frac{K_1}{-j\omega + 5} + \frac{K_2}{j\omega + 1} + \frac{K_3}{j\omega + 5} + \frac{K_4}{j\omega + 1}$$

$$K_1 = \frac{30}{6} = 5; \qquad K_2 = \frac{30}{6} = 5; \qquad K_3 = \frac{30}{-4} = -7.5; \qquad K_4 = \frac{30}{4} = 7.5$$

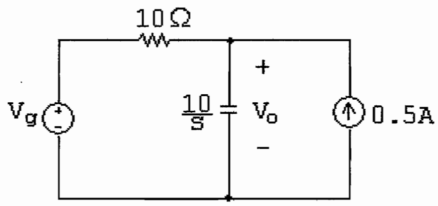
$$V_o(\omega) = \frac{5}{-j\omega + 5} + \frac{12.5}{j\omega + 1} - \frac{7.5}{j\omega + 5}$$

$$v_o(t) = 5e^{5t}u(-t) + (12.5e^{-t} - 7.5e^{-5t})u(t)\text{ V}$$

[b] $v_o(0^-) = 5$ V

[c] $v_o(0^+) = 12.5 - 7.5 = 5$ V

[d]



$$\frac{V_o - V_g}{10} + \frac{V_o s}{10} - 0.5 = 0$$

$$V_o - V_g + V_o s - 5 = 0$$

$$V_o(s+1) = 5 + V_g$$

$$V_g = \frac{30}{s+5}$$

$$\therefore \quad V_o = \frac{5}{s+1} + \frac{30}{(s+1)(s+5)} = \frac{5}{s+1} + \frac{7.5}{s+1} - \frac{7.5}{s+5} = \frac{12.5}{s+1} - \frac{7.5}{s+5}$$

$$v_o(t) = (12.5e^{-t} - 7.5e^{-5t})u(t)\text{ V}$$

[e] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$v_o(t) = (12.5e^{-t} - 7.5e^{-5t})u(t)\text{ V}$$

P 17.29 [a] $I_o = \dfrac{V_g}{10 + 10/s} = \dfrac{V_g s}{10s + 10}$

$$H(s) = \frac{I_o}{V_g} = \frac{0.1}{s+1}$$

$$H(j\omega) = \frac{0.1}{j\omega + 1}$$

$$V_g(\omega) = \frac{30}{-j\omega + 5} + \frac{30}{j\omega + 5}$$

$$I_o(\omega) = H(j\omega)V_g(j\omega) = \frac{0.1j\omega}{j\omega + 1}\left[\frac{30}{-j\omega + 5} + \frac{30}{j\omega + 5}\right]$$

$$= \frac{3j\omega}{(j\omega + 1)(-j\omega + 5)} + \frac{3j\omega}{(j\omega + 1)(j\omega + 5)}$$

$$= \frac{K_1}{j\omega + 1} + \frac{K_2}{-j\omega + 5} + \frac{K_3}{j\omega + 1} + \frac{K_4}{j\omega + 5}$$

$$K_1 = \frac{3(-1)}{6} = -0.5; \qquad K_2 = \frac{3(5)}{6} = 2.5$$

$$K_3 = \frac{3(-1)}{4} = -0.75; \qquad K_4 = \frac{3(-5)}{-4} = 3.75$$

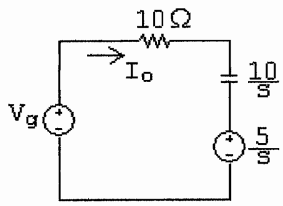
$$\therefore \quad I_o(\omega) = \frac{-1.25}{j\omega + 1} + \frac{2.5}{-j\omega + 5} + \frac{3.75}{j\omega + 5}$$

$$i_o(t) = 2.5e^{5t}u(-t) + [-1.25e^{-t} + 3.75e^{-5t}]u(t)\text{ A}$$

[b] $i_o(0^-) = 2.5\text{ V}$

[c] $i_o(0^+) = 2.5\text{ V}$

[d] Note – since $i_o(0^+) = 2.5$ A, $v_o(0^+) = 30 - 25 = 5$ V.



$$I_o = \frac{V_g - (5/s)}{10 + (10/s)} = \frac{sV_g - 5}{10s + 10}; \qquad V_g = \frac{30}{s+5}$$

$$\therefore \quad I_o = \frac{25s - 25}{10(s+1)(s+5)} = \frac{2.5(s-1)}{(s+1)(s+5)} = \frac{-1.25}{s+1} + \frac{3.75}{s+5}$$

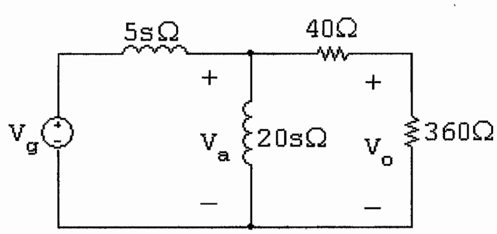
$$i_o(t) = (-1.25e^{-t} + 3.75e^{-5t})u(t)\text{ A}$$

[e] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$i_o(t) = (-1.25e^{-t} + 3.75e^{-5t})u(t)\text{ A}$$

P 17.30 [a] $v_g = 125\cos 75t$

$$V_g(\omega) = 125\pi[\delta(\omega + 75) + \delta(\omega - 75)]$$



$$\frac{V_a}{20s} + \frac{V_a - V_g}{5s} + \frac{V_a}{400} = 0$$

$$V_a\left[\frac{1}{20s} + \frac{1}{5s} + \frac{1}{400}\right] = \frac{V_g}{5s}$$

$$V_a[20 + 80 + s] = 80V_g$$

$$V_a = \frac{80V_g}{s+100}; \qquad V_o = \frac{V_a}{400}(360) = 0.9V_a$$

$$H(s) = \frac{V_o}{V_g} = \frac{72}{s+100}$$

$$H(\omega) = \frac{72}{j\omega + 100}$$

$$V_o(\omega) = V_g(\omega)H(\omega) = \frac{9000\pi[\delta(\omega+75) + \delta(\omega-75)]}{j\omega+100}$$

$$v_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V_o(\omega)e^{jt\omega}\,d\omega$$

$$= 4500\left[\frac{e^{j75t}}{100+j75} + \frac{e^{-j75t}}{100-j75}\right]$$

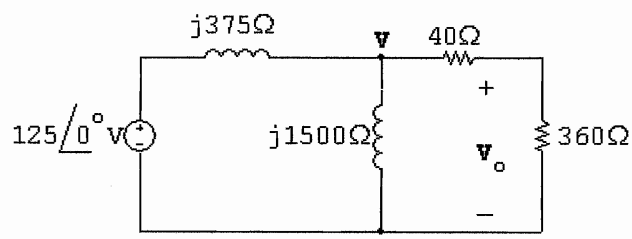
$$= 180\left[\frac{e^{j75t}}{4+j3} + \frac{e^{-j75t}}{4-j3}\right]$$

$$= 36[e^{j75t}e^{-j36.87^\circ} + e^{-j75t}e^{j36.87^\circ}]$$

$$= 36[e^{j(75t-j36.87^\circ)} + e^{-j(75t+36.87^\circ)}]$$

$$v_o(t) = 72\cos(75t - 36.87^\circ)\text{ V}$$

[**b**] In the phasor domain:



$$\frac{\mathbf{V}-125}{j375} + \frac{\mathbf{V}}{j1500} + \frac{\mathbf{V}}{400} = 0$$

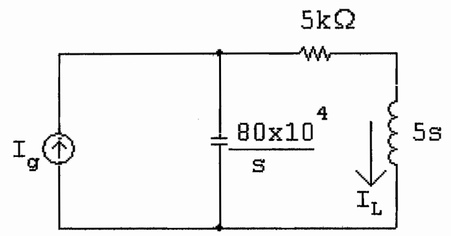
$$\mathbf{V}\left[\frac{1}{j375} + \frac{1}{j1500} + \frac{1}{400}\right] = \frac{125}{j375}$$

$$\mathbf{V} = \frac{(144+j192)(125)}{j375} = 64 - j48 = 80\underline{/-36.87^\circ}\text{ V}$$

$$\mathbf{V}_o = \frac{360}{400}(\mathbf{V}) = 72\underline{/-36.87^\circ}\text{ V}$$

$$v_o(t) = 72\cos(75t - 36.87^\circ)\text{ V}$$

P 17.31 [a]



$$I_L = \frac{80\times 10^4/s}{5000+5s+80\times 10^4/s}I_g = \frac{80\times 10^4}{5s^2+5000s+80\times 10^4}I_g$$

$$\frac{I_L}{I_g} = H(s) = \frac{16\times 10^4}{s^2+1000s+16\times 10^4} = \frac{16\times 10^4}{(s+200)(s+800)}$$

$$H(j\omega) = \frac{16\times 10^4}{(j\omega+200)(j\omega+800)}$$

$$I_g(\omega) = \frac{-45}{(-j\omega+400)} + \frac{45}{(j\omega+400)}$$

$$I_L(\omega) = \frac{-45(16\times 10^4)}{(j\omega+200)(j\omega+800)(-j\omega+400)} + \frac{45(16\times 10^4)}{(j\omega+200)(j\omega+800)(j\omega+400)}$$

$$= I_{L1} + I_{L2}$$

I_{L1}:

$$K_1 = \frac{-45(16\times 10^4)}{(600)(600)} = -20$$

$$K_2 = \frac{-45(16\times 10^4)}{(-600)(1200)} = 10$$

$$K_3 = \frac{-45(16\times 10^4)}{(600)(1200)} = -10$$

$$I_{L1} = \frac{-20}{j\omega+200} + \frac{10}{j\omega+800} - \frac{10}{-j\omega+400}$$

I_{L2}:

$$K_1 = \frac{45(16\times 10^4)}{(200)(600)} = 60$$

$$K_2 = \frac{45(16\times 10^4)}{(-600)(-400)} = 30$$

$$K_3 = \frac{45(16\times 10^4)}{(-200)(400)} = -90$$

$$I_{L2} = \frac{60}{j\omega + 200} + \frac{30}{j\omega + 800} - \frac{90}{j\omega + 400}$$

$$\therefore \quad I_L = \frac{40}{j\omega + 200} + \frac{40}{j\omega + 800} - \frac{10}{-j\omega + 400} - \frac{90}{j\omega + 400}$$

$$i_L(t) = (40e^{-200t} + 40e^{-800t} - 90e^{-400t})u(t) - 10e^{400t}u(-t)\text{ A}$$

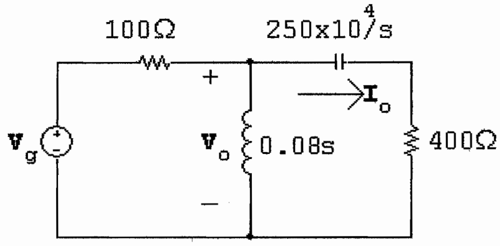
[b] $i_L(0^-) = -10e^{400(0^-)}u(0^-) = -10\text{ A}$

[c] $i_L(0^+) = (40e^{-200(0^+)} + 40e^{-800(0^+)} - 90e^{-400(0^+)} = -10\text{ A}$

[d] Yes, there cannot be an instantaneous change in the inductor current,

$$\therefore \quad i_L(0^-) = i_L(0^+)$$

P 17.32



$$\frac{V_o - V_g}{100} + \frac{V_o}{0.08s} + \frac{V_o s}{400s + 250\times 10^4} = 0$$

$$\therefore \quad V_o = \frac{32s(s + 6250)V_g}{40(s^2 + 6000s + 625\times 10^4)}$$

$$I_o = \frac{sV_o}{400(s + 6250)}$$

$$H(s) = \frac{I_o}{V_g} = \frac{2\times 10^{-3}s^2}{s^2 + 6000s + 625\times 10^4}$$

$$H(j\omega) = \frac{-2\times 10^{-3}\omega^2}{(625\times 10^4 - \omega^2) + j6000\omega}$$

$$V_g(\omega) = 200\pi[\delta(\omega + 2500) + \delta(\omega - 2500)]$$

$$I_o(\omega) = H(j\omega)V_g(\omega) = \frac{-0.4\pi\omega^2[\delta(\omega + 2500) + \delta(\omega - 2500)]}{(625\times 10^4 - \omega^2) + j6000\omega}$$

$$i_o(t) = \frac{-0.4\pi}{2\pi}\int_{-\infty}^{\infty} \frac{\omega^2[\delta(\omega+2500)+\delta(\omega-2500)]}{(625\times 10^4-\omega^2)+j6000\omega}e^{jt\omega}\,d\omega$$

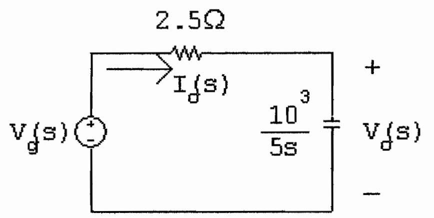
$$= -0.2\left\{\frac{625\times 10^4 e^{-j2500t}}{-j(6000)(2500)} + \frac{625\times 10^4 e^{j2500t}}{j(6000)(2500)}\right\}$$

$$= \frac{1}{12}\left\{\frac{e^{-j2500t}}{-j} + \frac{e^{j2500t}}{j}\right\}$$

$$= 0.0833[e^{-j(2500t+90^\circ)} + e^{j(2500t+90^\circ)}]$$

$$i_o(t) = 166.67\cos(2500t+90^\circ)\,\text{mA}$$

P 17.33 [a]



$$v_g(t) = 18e^{4t}u(-t) - 12u(t); \qquad \therefore\quad V_g(\omega) = \frac{18}{4-j\omega} - 12\pi\delta(\omega) - \frac{12}{j\omega}$$

Using voltage division,

$$V_o(s) = \frac{(10^3/5s)}{(10^3/5s)+2.5}V_g(s) = \frac{80}{s+80}V_g$$

$$\therefore\quad H(s) = \frac{V_o(s)}{V_g(s)} = \frac{80}{s+80}$$

$$\therefore\quad H(j\omega) = \frac{80}{j\omega+80}$$

$$V_o(j\omega) = H(j\omega)\cdot V_g(\omega)$$

$$= \frac{(80)(18)}{(j\omega+80)(4-j\omega)} - \frac{(80)12\pi\delta(\omega)}{j\omega+80} - \frac{(12)(80)}{j\omega(j\omega+80)}$$

$$= \frac{(120/7)}{j\omega+80} + \frac{(120/7)}{4-j\omega} - \frac{960\pi\delta(\omega)}{j\omega+80} - \frac{12}{j\omega} + \frac{12}{j\omega+80}$$

$$v_o(t) = \frac{120}{7}e^{-80t}u(t) + \frac{120}{7}e^{4t}u(-t) - 6 - 6\text{sgn}(t) + 12e^{-80t}u(t)\text{V}$$

$$\therefore\quad v_o(0^-) = \frac{120}{7} - 6 + 6 = \frac{120}{7}\text{V}; \qquad v_o(0^+) = \frac{120}{7} - 6 - 6 + 12 = \frac{120}{7}\text{V}$$

The voltages at 0^- and 0^+ must be the same since the voltage cannot change instantaneously across a capacitor.

[b] $I_o(s) = \dfrac{V_g(s)}{(10^3/5s) + 2.5} = \dfrac{0.4s}{s+80}V_g(s)$

$$H(s)\ \frac{I_o(s)}{V_g(s)} = \frac{0.4s}{s+80}; \qquad \therefore\ H(j\omega) = \frac{0.4j\omega}{j\omega+80}$$

$$I_o(j\omega) = H(j\omega)\cdot V_g(\omega)$$

$$= \frac{7.2j\omega}{(4-j\omega)(j\omega+80)} - \frac{4.8\pi\delta(\omega)j\omega}{j\omega+80} - \frac{4.8j\omega}{j\omega(j\omega+80)}$$

$$= \frac{(24/70)}{4-j\omega} - \frac{(48/7)}{j\omega+80} - \frac{4.8}{j\omega+80}$$

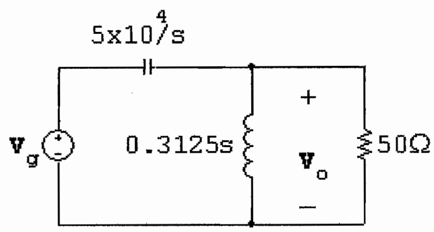
$$= \frac{(24/70)}{4-j\omega} - \frac{(816/70)}{j\omega+80}$$

$$i_o(t) = \frac{24}{70}e^{4t}u(-t) - \frac{816}{70}e^{-80t}u(t)\text{A}$$

$$\therefore\ i_o(0^-) = 24/70\text{A}; \qquad i_o(0^+) = -816/70\text{A}$$

[c] $v_o(t) = \dfrac{120}{7}e^{-80t}u(t) + \dfrac{120}{7}e^{4t}u(-t) - 6 - 6\text{sgn}(t) + 12e^{-80t}u(t)\text{V}$

P 17.34 [a]



$$\frac{(V_o - V_g)s}{5\times 10^4} + \frac{V_o}{0.3125s} + \frac{V_o}{50} = 0$$

$$\therefore\ V_o = \frac{s^2V_g}{s^2 + 1000s + 16\times 10^4}$$

$$\frac{V_o}{V_g} = H(s) = \frac{s^2}{(s+200)(s+800)}$$

$$H(j\omega) = \frac{(j\omega)^2}{(j\omega+200)(j\omega+800)}$$

$$v_g = 90e^{-400|t|}; \qquad V_g(\omega) = \frac{72{,}000}{(j\omega+400)(-j\omega+400)}$$

$$\therefore\ V_o(\omega) = H(j\omega)V_g(\omega) = \frac{72{,}000(j\omega)^2}{(j\omega+200)(j\omega+400)(j\omega+800)(-j\omega+400)}$$

$$= \frac{K_1}{j\omega+200} + \frac{K_2}{j\omega+400} + \frac{K_3}{j\omega+800} + \frac{K_4}{-j\omega+400}$$

$$K_1 = \frac{72{,}000(-200)^2}{(200)(600)(600)} = 40$$

$$K_2 = \frac{72{,}000(-400)^2}{(-200)(400)(800)} = -180$$

$$K_3 = \frac{72{,}000(-800)^2}{(-600)(-400)(1200)} = 160$$

$$K_4 = \frac{72{,}000(400)^2}{(600)(800)(1200)} = 20$$

$$\therefore\quad v_o(t) = [40e^{-200t} - 180e^{-400t} + 160e^{-800t}]u(t) + 20e^{400t}u(-t)\text{ V}$$

[b] $v_o(0^-) = 20\text{ V};\qquad V_o(0^+) = 40 - 180 + 160 = 20\text{ V}$

$$v_o(\infty) = 0\text{ V}$$

[c] $I_L = \dfrac{V_o}{0.3125s} = \dfrac{3.2sV_g}{(s+200)(s+800)}$

$$H(s) = \frac{I_L}{V_o} = \frac{3.2s}{(s+200)(s+800)}$$

$$H(j\omega) = \frac{3.2(j\omega)}{(j\omega+200)(j\omega+800)}$$

$$I_L(\omega) = \frac{3.2(j\omega)(72{,}000)}{(j\omega+200)(j\omega+400)(j\omega+800)(-j\omega+400)}$$

$$= \frac{K_1}{j\omega+200} + \frac{K_2}{j\omega+400} + \frac{K_3}{j\omega+800} + \frac{K_4}{-j\omega+400}$$

$$K_4 = \frac{(3.2)(400)(72{,}000)}{(600)(800)(1200)} = 160\text{ mA}$$

$$i_L(t) = 160e^{400t}u(-t);\qquad \therefore\quad i_L(0^-) = 160\text{ mA}$$

$$K_1 = \frac{(3.2)(-200)(72{,}000)}{(200)(600)(600)} = -640\text{ mA}$$

$$K_2 = \frac{(3.2)(-400)(72{,}000)}{(-200)(400)(800)} = 1440\text{ mA}$$

$$K_3 = \frac{(3.2)(-800)(72{,}000)}{(-600)(-400)(1200)} = -640\text{ mA}$$

$$\therefore\quad i_L(0^+) = K_1 + K_2 + K_3 = -640 + 1440 - 640 = 160\text{ mA}$$

Checks, i.e., $\quad i_L(0^+) = i_L(0^-) = 160\text{ mA}$

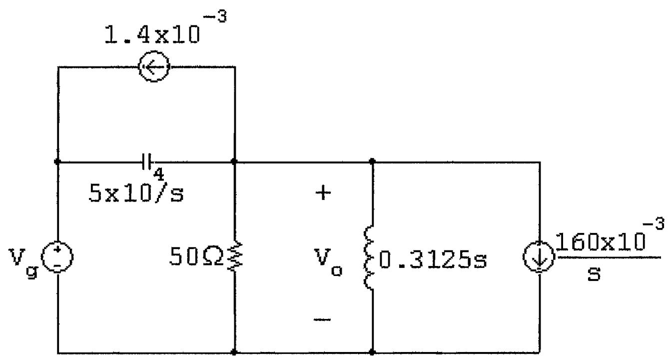
At $t = 0^-$:

$v_C(0^-) = 90 - 20 = 70\,\text{V}$

At $t = 0^+$:

$v_C(0^+) = 90 - 20 = 70\,\text{V}$

[d] We can check the correctness of out solution for $t \geq 0^+$ by using the Laplace transform. Our circuit becomes



$$\frac{V_o}{50} + \frac{V_o}{0.3125s} + \frac{(V_o - V_g)s}{5 \times 10^4} + 1.4 \times 10^{-3} + \frac{160 \times 10^{-3}}{s} = 0$$

$$\therefore \quad (s^2 + 1000s + 16 \times 10^4)V_o = s^2 V_g - (70s + 8000)$$

$$v_g(t) = 90e^{-400t}u(t)\,\text{V}; \qquad V_g = \frac{90}{s + 400}$$

$$\therefore \quad (s + 200)(s + 800)V_o = \frac{90s^2 - (70s + 8000)(s + 400)}{(s + 400)}$$

$$\therefore \quad V_o = \frac{20s^2 - 36{,}000s - 320 \times 10^4}{(s + 200)(s + 400)(s + 800)}$$

$$= \frac{40}{s + 200} - \frac{180}{s + 400} + \frac{160}{s + 800}$$

$$\therefore \quad v_o(t) = [40e^{-200t} - 180e^{-400t} + 160e^{-800t}]u(t)\,\text{V}$$

This agrees with our solution for $v_o(t)$ for $t \geq 0^+$.

P 17.35 [a] $V_g(\omega) = \dfrac{60}{-j\omega + 5} + \dfrac{900}{(j\omega + 5)^2}$

$$\frac{V_o \,\Gamma\, V_g}{12} + \frac{V_o}{4s + 20} + \frac{sV_o}{300} = 0$$

$$\therefore \quad H(s) = \frac{V_o}{V_g} = \frac{25(s + 5)}{(s + 10)(s + 20)}$$

$$H(\omega) = \frac{25(j\omega + 5)}{(j\omega + 10)(j\omega + 20)}$$

$$V_o(\omega) = V_g(\omega)H(\omega)$$

$$= \frac{1500(j\omega + 5)}{(j\omega + 10)(j\omega + 20)(-j\omega + 5)} + \frac{22{,}500}{(j\omega + 10)(j\omega + 20)(j\omega + 5)^2}$$

$$= V_1(\omega) + V_2(\omega)$$

$$V_1(\omega) = \frac{K_1}{j\omega + 10} + \frac{K_2}{j\omega + 20} + \frac{K_3}{-j\omega + 5}$$

$$K_1 = \frac{1500(-5)}{(10)(15)} = -50$$

$$K_2 = \frac{1500(-15)}{(-10)(25)} = 90$$

$$K_3 = \frac{1500(10)}{(15)(25)} = 40$$

$$V_2(\omega) = \frac{K_4}{j\omega + 10} + \frac{K_5}{j\omega + 20} + \frac{K_6}{(j\omega + 5)^2} + \frac{K_7}{(j\omega + 5)}$$

$$K_4 = \frac{22{,}500}{(10)(-5)^2} = 90$$

$$K_5 = \frac{22{,}500}{(-10)(-15)^2} = -10$$

$$K_6 = \frac{22{,}500}{(5)(15)} = 300$$

$$K_7 = \frac{-22{,}500}{(5)^2(15)} + \frac{-22{,}500}{(5)(15)^2} = -80$$

$$V_o(\omega) = \frac{-50}{j\omega + 10} + \frac{90}{j\omega + 20} + \frac{40}{-j\omega + 5} + \frac{90}{j\omega + 10} - \frac{10}{j\omega + 20} + \frac{300}{(j\omega + 5)^2} - \frac{80}{j\omega + 5}$$

$$= \frac{40}{j\omega + 10} + \frac{80}{j\omega + 20} + \frac{40}{-j\omega + 5} + \frac{300}{(j\omega + 5)^2} - \frac{80}{j\omega + 5}$$

$$\therefore \quad v_o(t) = [40e^{-10t} + 80e^{-20t} - 80e^{-5t} + 300te^{-5t}]u(t) + 40e^{5t}u(-t)\text{ V}$$

[b] $v_o(0^-) = 40\text{ V}$

[c] $v_o(0^+) = 40 + 80 - 80 = 40\text{ V}$

P 17.36 $V_o = \frac{60}{s} - \frac{40}{s+5} + \frac{20}{s+20} = \frac{40(s^2+20s+150)}{s(s+5)(s+20)}$

$$V_i = \frac{8}{s}$$

$$H(s) = \frac{5(s^2+20s+150)}{(s+5)(s+20)}$$

$$H(j\omega) = \frac{5[(j\omega)^2+20(j\omega)+150]}{(j\omega+5)(j\omega+20)}$$

$$V_i(\omega) = \frac{16}{(j\omega)}$$

$$V_o(\omega) = \frac{80[(j\omega)^2+20(j\omega)+150]}{j\omega(j\omega+5)(j\omega+20)}$$

$$= \frac{K_1}{j\omega} + \frac{K_2}{j\omega+5} + \frac{K_3}{j\omega+20}$$

$$K_1 = \frac{(80)(150)}{100} = 120$$

$$K_2 = \frac{(80)(25-100+150)}{(-5)(15)} = -80$$

$$K_3 = \frac{(80)(150)}{300} = 40$$

$$\therefore\ v_o(t) = 60\text{sgn}(t) - 80e^{-5t}u(t) + 40e^{-20t}u(t)\text{ V}$$

P 17.37 [a] $f(t) = \frac{1}{2\pi}\left\{\int_{-\infty}^{0} e^{\omega}e^{jt\omega}\,d\omega + \int_0^{\infty} e^{-\omega}e^{jt\omega}\,d\omega\right\} = \frac{1/\pi}{1+t^2}$

[b] $W = 2\int_0^{\infty}\frac{(1/\pi)^2}{(1+t^2)^2}\,dt = \frac{2}{\pi^2}\int_0^{\infty}\frac{dt}{(1+t^2)^2} = \frac{1}{2\pi}$ J

[c] $W = \frac{1}{\pi}\int_0^{\infty} e^{-2\omega}\,d\omega = \frac{1}{\pi}\frac{e^{-2\omega}}{-2}\Big|_0^{\infty} = \frac{1}{2\pi}$ J

[d] $\frac{1}{\pi}\int_0^{\omega_1} e^{-2\omega}\,d\omega = \frac{0.9}{2\pi}, \qquad 1 - e^{-2\omega_1} = 0.9, \qquad e^{2\omega_1} = 10$

$$\omega_1 = (1/2)\ln 10 \cong 1.15\text{ rad/s}$$

P 17.38 $I_o = \frac{I_g(10)}{10 + 0.2s} = \frac{50I_g}{s + 50}$

$$H(s) = \frac{I_o}{I_g} = \frac{50}{s + 50}$$

$$H(j\omega) = \frac{50}{j\omega + 50}$$

$$I_g(\omega) = \frac{3}{j\omega + 25}$$

$$I_o(\omega) = \frac{150}{(j\omega + 25)(j\omega + 50)}$$

$$|I_o(\omega)| = \frac{150}{\sqrt{(\omega^2 + 625)(\omega^2 + 2500)}}$$

$$|I_o(\omega)|^2 = \frac{22{,}500}{(\omega^2 + 625)(\omega^2 + 2500)} = \frac{12}{\omega^2 + 625} - \frac{12}{\omega^2 + 2500}$$

$$W_o = \frac{1}{\pi}\int_0^\infty \frac{12d\omega}{\omega^2 + 625} - \frac{1}{\pi}\int_0^\infty \frac{12d\omega}{\omega^2 + 2500}$$

$$= \frac{12}{\pi} \cdot \frac{1}{25} \cdot \tan^{-1}\left(\frac{\omega}{25}\right)\Big|_0^\infty - \frac{12}{\pi} \cdot \frac{1}{50} \cdot \tan^{-1}\left(\frac{\omega}{50}\right)\Big|_0^\infty$$

$$= \frac{12}{25\pi} \cdot \frac{\pi}{2} - \frac{12}{50\pi} \cdot \frac{\pi}{2} = \frac{12}{50} - \frac{6}{50} = \frac{6}{50} = 120\,\text{mJ}$$

$\therefore \quad W_o(\text{total}) = 120\,\text{mJ}$

$$W_o(10\text{rad/s}) = \frac{12}{\pi}\left(\frac{1}{25}\tan^{-1}(0.4)\right) - \frac{12}{\pi}\left(\frac{1}{50}\tan^{-1}(0.2)\right) = 43.06\,\text{mJ}$$

$$\% = \frac{43.06}{120}(100) = 35.88\%$$

P 17.39 [a] $V_g(\omega) = \frac{600}{(j\omega + 5)(-j\omega + 5)}$

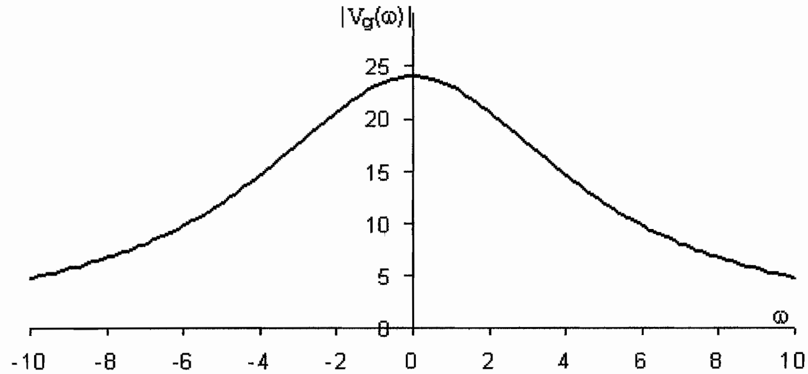
$$H(s) = \frac{V_o}{V_g} = \frac{5}{s + 25}; \qquad H(\omega) = \frac{5}{(j\omega + 25)}$$

$$V_o(\omega) = \frac{3000}{(j\omega + 5)(j\omega + 25)(-j\omega + 5)}$$

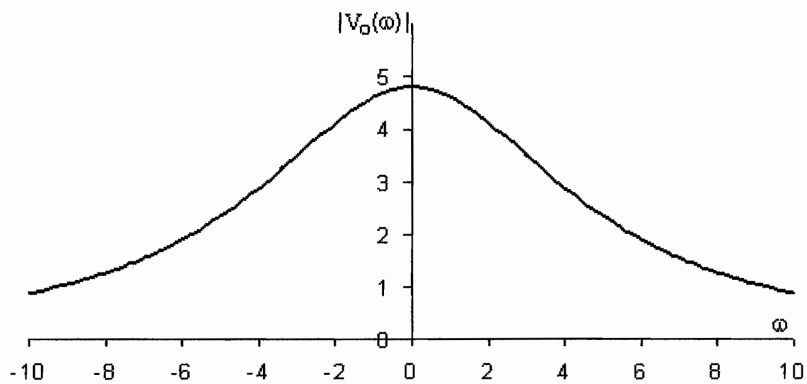
$$V_o(\omega) = \frac{15}{j\omega + 5} - \frac{5}{j\omega + 25} + \frac{10}{-j\omega + 5}$$

$$v_o(t) = [15e^{-5t} - 5e^{-25t}]u(t) + 10e^{5t}u(-t)\,\text{V}$$

[b] $|V_g(\omega)| = \dfrac{600}{(\omega^2 + 25)}$



[c] $|V_o(\omega)| = \dfrac{3000}{(\omega^2 + 25)\sqrt{\omega^2 + 625}}$



[d] $W_i = 2\int_0^\infty 3600e^{-10t}\,dt = 7200\,\dfrac{e^{-10t}}{-10}\Big|_0^\infty = 720\,\text{J}$

[e] $W_o = \int_{-\infty}^0 100e^{10t}\,dt + \int_0^\infty (15e^{-5t} - 5e^{-25t})^2\,dt$

$$= 10 + \int_0^\infty [225e^{-10t} - 150e^{-30t} + 25e^{-50t}]\,dt$$

$$= 10 + 22.5 - 5 + 0.5 = 28\,\text{J}$$

[f] $|V_g(\omega)| = \dfrac{600}{\omega^2+25}, \quad |V_g^2(\omega)| = \dfrac{36\times 10^4}{(\omega^2+25)^2}$

$$W_g = \frac{36\times 10^4}{\pi}\int_0^{10}\frac{d\omega}{(\omega^2+25)^2}$$

$$= \frac{36\times 10^4}{\pi}\left\{\frac{1}{2(25)}\left(\frac{\omega}{\omega^2+25}+\frac{1}{5}\tan^{-1}\frac{\omega}{5}\right)\Big|_0^{10}\right\}$$

$$= \frac{7200}{\pi}\left(\frac{10}{125}+\frac{1}{5}\tan^{-1}2\right) = 690.8\,\mathrm{J}$$

$$\therefore\quad \% = \left(\frac{690.8}{720}\right)\times 100 = 95.95\%$$

[g] $|V_o(\omega)|^2 = \dfrac{9\times 10^6}{(\omega^2+25)^2(\omega^2+625)}$

$$= \frac{15{,}000}{(\omega^2+25)^2} - \frac{25}{\omega^2+25} + \frac{25}{(\omega^2+625)}$$

$$W_o = \frac{1}{\pi}\left\{15{,}000\left(\frac{1}{2(25)}\right)\left(\frac{\omega}{\omega^2+25}+\frac{1}{5}\tan^{-1}\frac{\omega}{5}\right)\Big|_0^{10} -25\left(\frac{1}{5}\right)\tan^{-1}\frac{\omega}{5}\Big|_0^{10}\right.$$

$$\left. +25\left(\frac{1}{25}\right)\tan^{-1}\frac{\omega}{25}\Big|_0^{10}\right\}$$

$$= \frac{300}{\pi}\left(\frac{10}{125}+\frac{1}{5}\tan^{-1}2\right) - \frac{5}{\pi}\tan^{-1}2 + \frac{1}{\pi}\tan^{-1}0.4$$

$$= 27.14\,\mathrm{J}$$

$$\% = \frac{27.14}{28}\times 100 = 96.93\%$$

P 17.40 $I_g(\omega) = \dfrac{30\times 10^{-6}}{j\omega+2}$

$$H(s) = \frac{V_o}{I_g} = \frac{1/C}{s+1/RC} = \frac{800{,}000}{s+8}$$

$$H(\omega) = \frac{8\times 10^5}{j\omega+8}; \qquad V_o(\omega) = I_g(\omega)H(\omega)$$

$$\therefore\quad V_o(\omega) = \frac{24}{(j\omega+2)(j\omega+8)}$$

$$|V_o(\omega)| = \frac{24}{\sqrt{\omega^2+4}\cdot\sqrt{\omega^2+64}}$$

$$|V_o(\omega)|^2 = \frac{576}{(\omega^2+4)(\omega^2+64)} = \frac{9.6}{\omega^2+4} - \frac{9.6}{\omega^2+64}$$

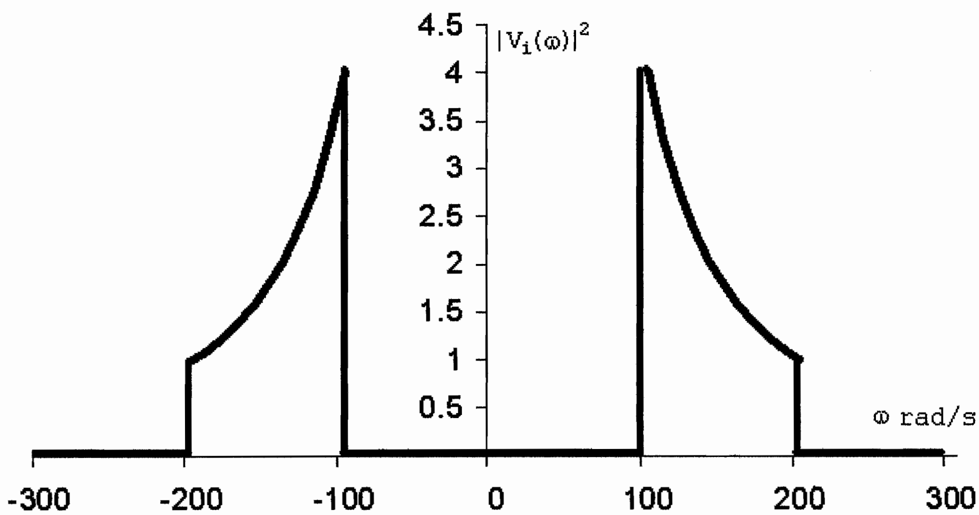
$$W_o = \frac{1}{\pi}\int_0^\infty \frac{9.6d\omega}{\omega^2+4} - \frac{1}{\pi}\int_0^\infty \frac{9.6d\omega}{\omega^2+64}$$

$$= \frac{9.6}{\pi}\cdot\frac{1}{2}\cdot\frac{\pi}{2} - \frac{9.6}{\pi}\cdot\frac{1}{8}\cdot\frac{\pi}{2} = 1.8\,\text{J TOTAL}$$

$$W_{\text{to 4 rad/s}} = \frac{4.8}{\pi}\tan^{-1}2 - \frac{1.2}{\pi}\tan^{-1}0.5 = 1.5145\,\text{J}$$

$$\% = \left(\frac{1.5145}{1.8}\right)100 = 84.14\%$$

P 17.41 **[a]** $|V_i(\omega)|^2 = \frac{4\times10^4}{\omega^2}$; $\quad |V_i(100)|^2 = \frac{4\times10^4}{100^2} = 4$; $\quad |V_i(200)|^2 = \frac{4\times10^4}{200^2} = 1$



[b] $V_o = \frac{V_iR}{R+(1/sC)} = \frac{sRCV_i}{RCs+1}$

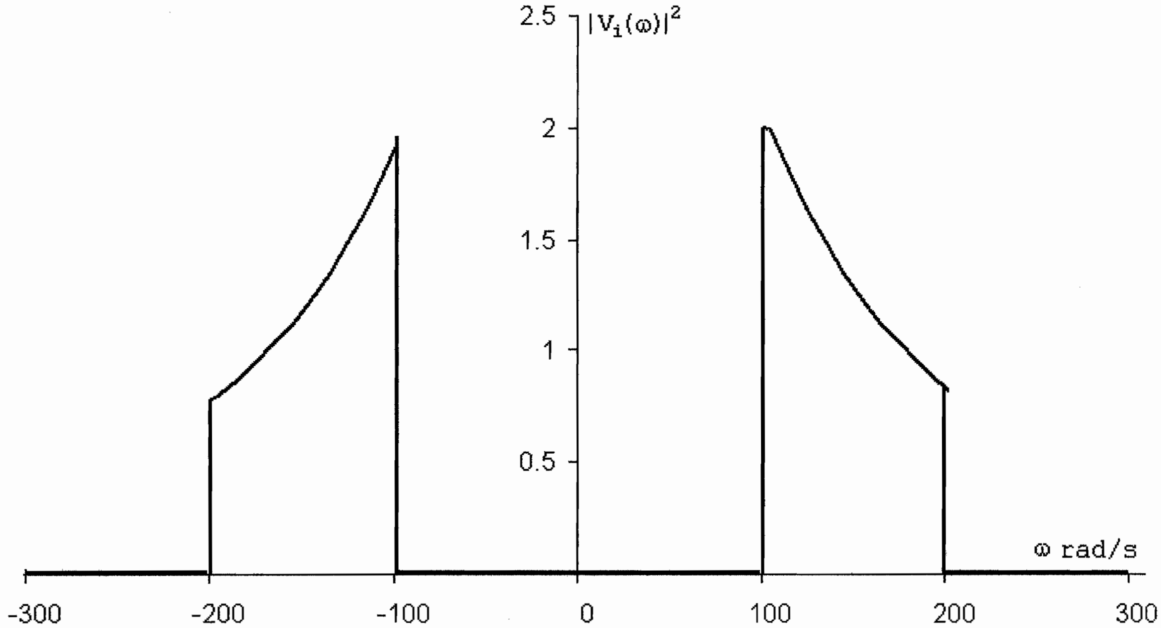
$$H(s) = \frac{V_o}{V_i} = \frac{s}{s+(1/RC)}; \qquad \frac{1}{RC} = \frac{10^6 10^{-3}}{(0.5)(20)} = \frac{1000}{10} = 100$$

$$H(j\omega) = \frac{j\omega}{j\omega+100}$$

$$|V_o(\omega)| = \frac{200}{|\omega|}\cdot\frac{|\omega|}{\sqrt{\omega^2+10^4}} = \frac{200}{\sqrt{\omega^2+10^4}}$$

$$|V_o(\omega)|^2 = \frac{4\times10^4}{\omega^2+10^4}, \quad 100\le\omega\le200\ \text{rad/s}; \quad |V_o(\omega)|^2 = 0, \quad \text{elsewhere}$$

$$|V_o(100)|^2 = \frac{4\times 10^4}{10^4+10^4} = 2; \qquad |V_o(200)|^2 = \frac{4\times 10^4}{5\times 10^4} = 0.8$$



[c] $W_{1\Omega} = \frac{1}{\pi}\int_{100}^{200} \frac{4\times 10^4}{\omega^2}\,d\omega = \frac{4\times 10^4}{\pi}\left[-\frac{1}{\omega}\right]_{100}^{200}$

$$= \frac{4\times 10^4}{\pi}\left[\frac{1}{100} - \frac{1}{200}\right] = \frac{200}{\pi} \cong 63.66\,\text{J}$$

[d] $W_{1\Omega} = \frac{1}{\pi}\int_{100}^{200} \frac{4\times 10^4}{\omega^2+10^4}\,d\omega = \frac{4\times 10^4}{\pi}\cdot \tan^{-1}\frac{\omega}{100}\Big|_{100}^{200}$

$$= \frac{400}{\pi}[\tan^{-1}2 - \tan^{-1}1] =\cong 40.97\,\text{J}$$

P 17.42 [a] $V_i(\omega) = \frac{A}{a+j\omega}; \qquad |V_i(\omega)| = \frac{A}{\sqrt{a^2+\omega^2}}$

$$H(s) = \frac{s}{s+\alpha}; \qquad H(j\omega) = \frac{j\omega}{\alpha+j\omega}; \qquad |H(\omega)| = \frac{\omega}{\sqrt{\alpha^2+\omega^2}}$$

Therefore $|V_o(\omega)| = \frac{\omega A}{\sqrt{(a^2+\omega^2)(\alpha^2+\omega^2)}}$

Therefore $|V_o(\omega)|^2 = \frac{\omega^2 A^2}{(a^2+\omega^2)(\alpha^2+\omega^2)}$

$W_{\text{IN}} = \int_0^\infty A^2 e^{-2at}\,dt = \frac{A^2}{2a}$; when $\alpha = a$ we have

$$W_{\rm OUT} = \frac{A^2}{\pi}\int_0^a \frac{\omega^2\,d\omega}{(\omega^2+a^2)^2} = \frac{A^2}{\pi}\left\{\int_0^a \frac{d\omega}{a^2+\omega^2} - \int_0^a \frac{a^2\,d\omega}{(a^2+\omega^2)^2}\right\}$$

$$= \frac{A^2}{4a\pi}\left(\frac{\pi}{2}-1\right)$$

$$W_{\rm OUT}({\rm total}) = \frac{A^2}{\pi}\int_0^\infty \left[\frac{\omega^2}{(a^2+\omega^2)^2}\right] d\omega = \frac{A^2}{4a}$$

Therefore $\dfrac{W_{\rm OUT}(a)}{W_{\rm OUT}({\rm total})} = 0.5 - \dfrac{1}{\pi} = 0.1817$ or 18.17%

[b] When $\alpha \neq a$ we have

$$W_{\rm OUT}(\alpha) = \frac{1}{\pi}\int_0^\alpha \frac{\omega^2 A^2 d\omega}{(a^2+\omega^2)(\alpha^2+\omega^2)}$$

$$= \frac{A^2}{\pi}\left\{\int_0^\alpha \left[\frac{K_1}{a^2+\omega^2} + \frac{K_2}{\alpha^2+\omega^2}\right] d\omega\right\}$$

where $K_1 = \dfrac{a^2}{a^2-\alpha^2}$ and $K_2 = \dfrac{-\alpha^2}{a^2-\alpha^2}$

Therefore

$$W_{\rm OUT}(\alpha) = \frac{A^2}{\pi(a^2-\alpha^2)}\left[a\tan^{-1}\left(\frac{\alpha}{a}\right) - \frac{\alpha\pi}{4}\right]$$

$$W_{\rm OUT}({\rm total}) = \frac{A^2}{\pi(a^2-\alpha^2)}\left[a\frac{\pi}{2} - \alpha\frac{\pi}{2}\right] = \frac{A^2}{2(a+\alpha)}$$

Therefore $\dfrac{W_{\rm OUT}(\alpha)}{W_{\rm OUT}({\rm total})} = \dfrac{2}{\pi(a-\alpha)}\cdot\left[a\tan^{-1}\left(\dfrac{\alpha}{a}\right) - \dfrac{\alpha\pi}{4}\right]$

For $\alpha = a\sqrt{3}$, this ratio is 0.2723, or 27.23% of the output energy lies in the frequency band between 0 and $a\sqrt{3}$.

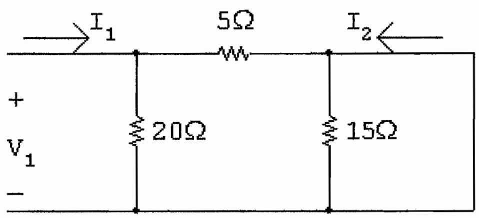
[c] For $\alpha = a/\sqrt{3}$, the ratio is 0.1057, or 10.57% of the output energy lies in the frequency band between 0 and $a/\sqrt{3}$.

18

Two-Port Circuits

Assessment Problems

AP 18.1 With port 2 short-circuited, we have



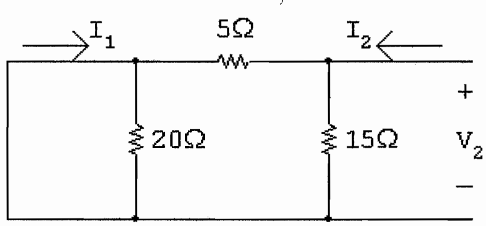
$$I_1 = \frac{V_1}{20} + \frac{V_1}{5}; \qquad \frac{I_1}{V_1} = y_{11} = 0.25\,\text{S}; \qquad I_2 = \left(\frac{-20}{25}\right) I_1 = -0.8 I_1$$

When $V_2 = 0$, we have $I_1 = y_{11}V_1$ and $I_2 = y_{21}V_1$

Therefore $I_2 = -0.8(y_{11}V_1) = -0.8y_{11}V_1$

Thus $y_{21} = -0.8y_{11} = -0.2\,\text{S}$

With port 1 short-circuited, we have



$$I_2 = \frac{V_2}{15} + \frac{V_2}{5}; \qquad \frac{I_2}{V_2} = y_{22} = \left(\frac{4}{15}\right)\,\text{S}$$

$$I_1 = \left(\frac{-15}{20}\right) I_2 = -0.75 I_2 = -0.75 y_{22} V_2$$

$$\text{Therefore} \quad y_{12} = (-0.75)\frac{4}{15} = -0.2\,\text{S}$$

AP 18.2

$$h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\text{S}$$

$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\text{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

AP 18.3

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \frac{5\times 10^{-6}}{50\times 10^{-3}} = 0.1\,\text{mS}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{200\times 10^{-3}}{50\times 10^{-3}} = 4$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = \frac{2\times 10^{-6}}{0.5\times 10^{-6}} = 4$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{10\times 10^{-3}}{0.5\times 10^{-6}} = 20\,\text{k}\Omega$$

AP 18.4 First calculate the *b*-parameters:

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} = \frac{15}{10} = 1.5\,\Omega; \qquad b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} = \frac{30}{10} = 3\,\text{S}$$

$$b_{12} = \frac{-V_2}{I_1}\bigg|_{V_1=0} = \frac{-10}{-5} = 2\,\Omega; \qquad b_{22} = \frac{-I_2}{I_1}\bigg|_{V_1=0} = \frac{-4}{-5} = 0.8$$

Now the *z*-parameters are calculated:

$$z_{11} = \frac{b_{22}}{b_{21}} = \frac{0.8}{3} = \frac{4}{15}\,\Omega; \qquad z_{12} = \frac{1}{b_{21}} = \frac{1}{3}\,\Omega$$

$$z_{21} = \frac{\Delta b}{b_{21}} = \frac{(1.5)(0.8) - 6}{3} = -1.6\,\Omega; \qquad z_{22} = \frac{b_{11}}{b_{21}} = \frac{1.5}{3} = \frac{1}{2}\,\Omega$$

AP 18.5

$$z_{11} = z_{22}, \quad z_{12} = z_{21}, \quad 95 = z_{11}(5) + z_{12}(0)$$

Therefore, $z_{11} = z_{22} = 95/5 = 19\,\Omega$

$$11.52 = 19I_1 - z_{12}(2.72)$$

$$0 = z_{12}I_1 - 19(2.72)$$

Solving these simultaneous equations for z_{12} yields the quadratic equation

$$z_{12}^2 + \left(\frac{72}{17}\right) z_{12} - \frac{6137}{17} = 0$$

For a purely resistive network, it follows that $z_{12} = z_{21} = 17\,\Omega$.

AP 18.6 [a] $I_2 = \dfrac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_gZ_L + a_{22}Z_g}$

$$= \frac{-50 \times 10^{-3}}{(5 \times 10^{-4})(5 \times 10^3) + 10 + (10^{-6})(100)(5 \times 10^3) + (-3 \times 10^{-2})(100)}$$

$$= \frac{-50 \times 10^{-3}}{10} = -5\,\text{mA}$$

$$P_L = \frac{1}{2}(5 \times 10^{-3})^2(5 \times 10^3) = 62.5\,\text{mW}$$

[b] $Z_{\text{Th}} = \dfrac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \dfrac{10 + (-3 \times 10^{-2})(100)}{5 \times 10^{-4} + (10^{-6})(100)}$

$$= \frac{7}{6 \times 10^{-4}} = \frac{70}{6}\,\text{k}\Omega$$

[c] $V_{\text{Th}} = \dfrac{V_g}{a_{11} + a_{21}Z_g} = \dfrac{50 \times 10^{-3}}{6 \times 10^{-4}} = \dfrac{500}{6}\,\text{V}$

Therefore $V_2 = \dfrac{250}{6}\,\text{V}; \qquad P_{\text{max}} = \dfrac{(1/2)(250/6)^2}{(70/6) \times 10^3} = 74.4\,\text{mW}$

AP 18.7 [a] For the given bridged-tee circuit, we have

$a'_{11} = a'_{22} = 1.25, \qquad a'_{21} = \dfrac{1}{20}\,\text{S}, \qquad a'_{12} = 11.25\,\Omega$

The a-parameters of the cascaded networks are

$a_{11} = (1.25)^2 + (11.25)(0.05) = 2.125$

$a_{12} = (1.25)(11.25) + (11.25)(1.25) = 28.125\,\Omega$

$a_{21} = (0.05)(1.25) + (1.25)(0.05) = 0.125\,\text{S}$

$a_{22} = a_{11} = 2.125, \qquad R_{\text{Th}} = (45.125/3.125) = 14.44\,\Omega$

[b] $V_t = \dfrac{100}{3.125} = 32\,\text{V}; \qquad$ therefore $V_2 = 16\,\text{V}$

[c] $P = \dfrac{16^2}{14.44} = 17.73\,\text{W}$

Problems

P 18.1 $h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\mathrm{S}$$

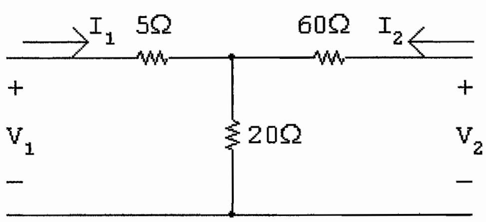
$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\mathrm{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

P 18.2



$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = 5 + 20 = 25\,\Omega$$

$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = 20\,\Omega$$

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = 20\,\Omega$$

$$z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} = 60 + 20 = 80\,\Omega$$

P 18.3 $\Delta z = (25)(80) - (20)(20) = 1600$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{80}{1600} = \frac{1}{20}\,\text{S}$$

$$y_{12} = \frac{-z_{12}}{\Delta z} = \frac{-20}{1600} = \frac{-1}{80}\,\text{S}$$

$$y_{21} = \frac{-z_{21}}{\Delta z} = \frac{-20}{1600} = \frac{-1}{80}\,\text{S}$$

$$y_{22} = \frac{-z_{11}}{\Delta z} = \frac{25}{1600} = \frac{1}{64}\,\text{S}$$

P 18.4 $V_1 = z_{11}I_1 + z_{12}I_2$

$V_1 = z_{21}I_1 + z_{22}I_2$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 5\|20 + 16 = 20\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 16 + (10)(5/25) = 18\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 16 + (10/25)(5) = 18\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 10\|15 + 6 = 22\,\Omega$$

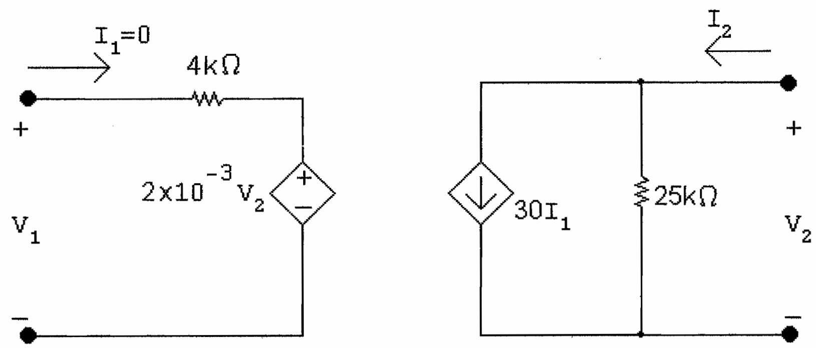
Summary:

$z_{11} = 20\,\Omega \quad z_{12} = 18\,\Omega \quad z_{21} = 18\,\Omega \quad z_{22} = 22\,\Omega$

P 18.5 $V_2 = b_{11}V_1 - b_{12}I_1$

$I_2 = b_{21}V_1 - b_{22}I_1$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$



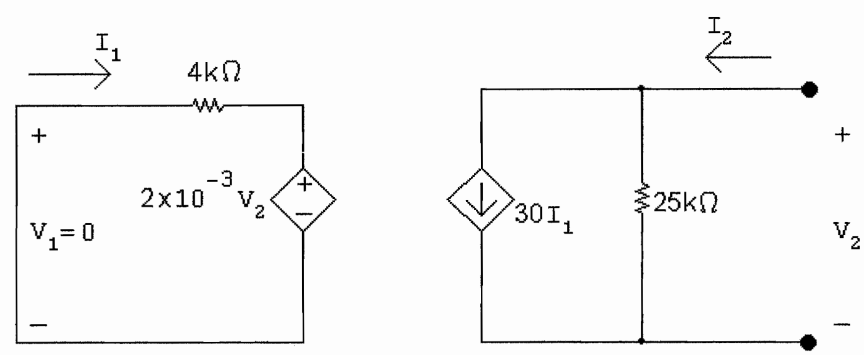
$V_1 = 2 \times 10^{-3} V_2$

$$\therefore \quad b_{11} = \frac{1}{2 \times 10^{-3}} = 500$$

$V_2 = 25{,}000 I_2; \qquad \text{so} \qquad V_1 = (2 \times 10^{-3})(25{,}000) I_2 = 50 I_2$

$$\therefore \quad b_{21} = \frac{1}{50} = 20 \text{ mS}$$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0}; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$



$$I_1 = -\frac{2 \times 10^{-3} V_2}{4000}; \qquad \therefore \quad b_{12} = \frac{4000}{2 \times 10^{-3}} = 2\,\text{M}\Omega$$

$$I_2 = 30 I_1 + \frac{V_2}{25{,}000} = 30 I_1 - \frac{4000}{(2 \times 10^{-3})(25{,}000)} I_1 = -50 I_1; \qquad \therefore \quad b_{22} = 50$$

Summary

$b_{11} = 500; \quad b_{12} = 2\,\text{M}\Omega; \quad b_{21} = 20\,\text{mS}; \quad b_{22} = 50$

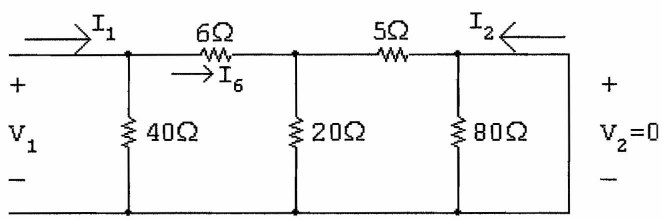
P 18.6 $g_{11} = \frac{b_{21}}{b_{22}} = \frac{20 \times 10^{-3}}{50} = 0.4\,\text{mS}$

$$g_{12} = \frac{-1}{b_{22}} = \frac{-1}{50} = -0.02$$

$$g_{21} = \frac{\Delta b}{b_{22}} = \frac{(500)(50) - (2 \times 10^6)(20 \times 10^{-3})}{50} = -300$$

$$g_{22} = \frac{b_{12}}{b_{22}} = \frac{2 \times 10^6}{50} = 40\,\text{k}\Omega$$

P 18.7 $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$

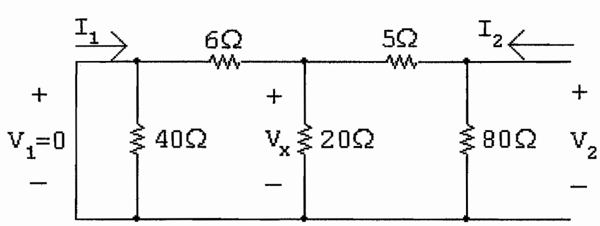


$$\frac{V_1}{I_1} = 40\|[6 + 20\|5] = 40\|10 = 8\,\Omega \qquad \therefore \quad h_{11} = 8\,\Omega$$

$$I_6 = \frac{40}{40+10}I_1 = 0.8I_1$$

$$I_2 = \frac{-20}{20+5}I_6 = -0.8I_6 = -0.8(0.8)I_1 = -0.64I_1 \qquad \therefore \quad h_{21} = -0.64$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$\frac{V_2}{I_2} = 80\|[5 + 20\|(40+6)] = 15.314\,\Omega \qquad \therefore \quad h_{22} = \frac{1}{15.314} = 65.3\text{ mS}$$

$$V_x = \frac{20\|46}{5 + 20\|46} V_2$$

$$V_1 = \frac{40}{40+6} V_x = \frac{40(20\|46)}{46(5 + 20\|46)} V_2 = \frac{557.5758}{871.2121} V_2$$

$$\therefore \qquad h_{12} = 0.64$$

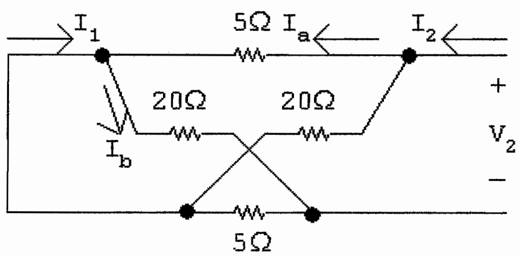
Summary:

$$h_{11} = 8\,\Omega; \quad h_{12} = 0.64; \quad h_{21} = -0.64; \quad h_{22} = 65.3 \text{ mS}$$

P 18.8 $V_2 = b_{11}V_1 - b_{12}I_1$

$$I_2 = b_{21}V_1 - b_{22}I_1$$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0}; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$



$$5\|20 = 4\,\Omega$$

$$I_2 = \frac{V_2}{4+4} = \frac{V_2}{8}; \qquad I_1 = I_b - I_a$$

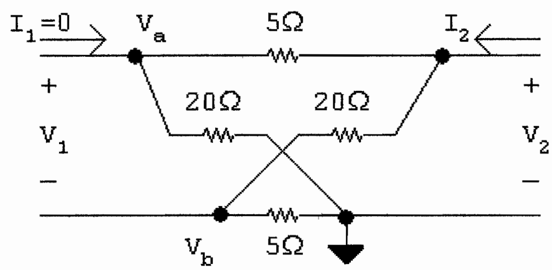
$$I_a = \frac{20}{25} I_2; \qquad I_b = \frac{5}{25} I_2$$

$$I_1 = \left(\frac{5}{25} - \frac{20}{25}\right) I_2 = \frac{-15}{25} I_2 = \frac{-3}{5} I_2$$

$$b_{22} = \frac{-I_2}{I_1} = \frac{5}{3}$$

$$b_{12} = \frac{-V_2}{I_1} = \frac{-V_2}{I_2}\left(\frac{I_2}{I_1}\right) = 8\left(\frac{5}{3}\right) = \frac{40}{3}\,\Omega$$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$



$$V_1 = V_a - V_b; \quad V_a = \frac{20}{25}V_2; \quad V_b = \frac{5}{25}V_2$$

$$V_1 = \frac{20}{25}V_2 - \frac{5}{25}V_2 = \frac{15}{25}V_2 = \frac{3}{5}V_2$$

$$b_{11} = \frac{V_2}{V_1} = \frac{5}{3}$$

$$V_2 = (20+5)\|(20+5)I_2 = 12.5I_2$$

$$b_{21} = \frac{I_2}{V_1} = \left(\frac{I_2}{V_2}\right)\left(\frac{V_2}{V_1}\right) = \left(\frac{1}{12.5}\right)\left(\frac{5}{3}\right) = \frac{2}{15}\,\text{S}$$

P 18.9 $$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}; \qquad V_2 = \frac{V_1}{R_1+R_3}R_3$$

$$\therefore \quad a_{11} = \frac{R_1+R_3}{R_3} = 1 + \frac{R_1}{R_3} = 1.2 \qquad \therefore \quad \frac{R_1}{R_3} = 0.2$$

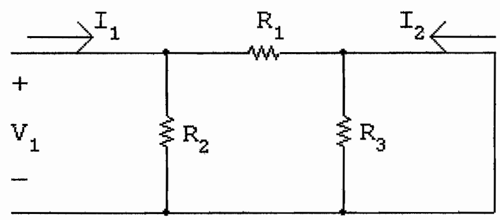
$$\therefore \quad R_1 = 0.2R_3 \qquad \text{(Eq 1)}$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0}; \qquad V_2 = I_3R_3 = \frac{R_2}{R_1+R_2+R_3}I_1R_3$$

$$\therefore \quad a_{21} = \frac{R_1+R_2+R_3}{R_2R_3} = 20\times10^{-3} \qquad \text{(Eq 2)}$$

Substitute Eq 1 into Eq 2:

$$\frac{0.2R_3+R_2+R_3}{R_2R_3} = \frac{R_2+1.2R_3}{R_2R_3} = 20\times10^{-3} \qquad \text{(Eq 3)}$$



$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0}; \qquad I_2 = \frac{-R_2}{R_1+R_2}I_1; \qquad \therefore\ a_{22} = \frac{R_1+R_2}{R_2} = 1.4$$

$$\frac{R_1}{R_2} = 0.4; \qquad \therefore\ R_2 = \frac{R_1}{0.4} = \frac{0.2R_3}{0.4} = 0.5R_3 \qquad \text{(Eq 4)}$$

Substitute Eq 4 into Eq 3:

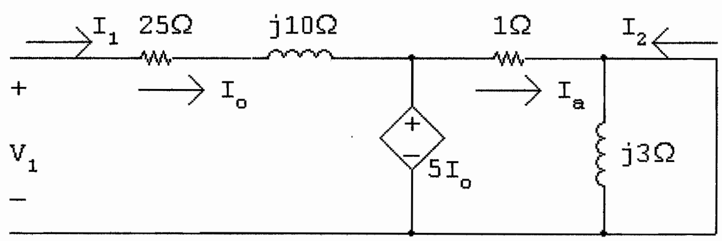
$$\frac{0.5R_3 + 1.2R_3}{(0.5R_3)R_3} = \frac{3.4}{R_3} = 20\times10^{-3} \qquad \therefore\ R_3 = 170\,\Omega$$

Therefore,

$$R_1 = 0.2R_3 = 0.2(170) = 34\,\Omega; \qquad R_2 = 0.5R_3 = 0.5(170) = 85\,\Omega$$

Summary: $R_1 = 34\,\Omega$; $R_2 = 85\,\Omega$; $R_3 = 170\,\Omega$

P 18.10 $h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$; $\qquad h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$

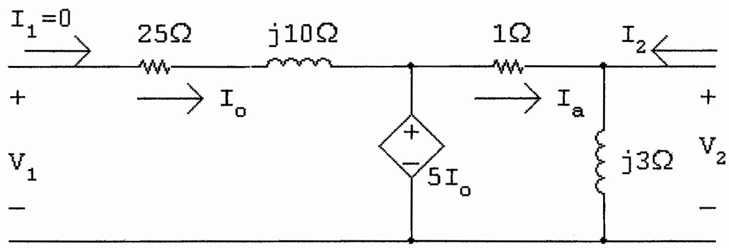


$$I_a = \frac{5I_o}{1} = 5I_1 = -I_2; \qquad \therefore\ h_{21} = -5$$

$$V_1 = (25+j10)I_1 + 5I_1 = (30+j10)I_1 = (30+j10)I_1$$

$$\therefore\ h_{11} = 30 + j10\,\Omega$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}; \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}$$



$I_o = 0 \quad$ thus $\quad 5I_o = 0 \quad$ cs is a short circuit

$V_1 = 5I_o = 0; \qquad \therefore \quad h_{12} = 0$

$$h_{22} = \frac{I_2}{V_2} = \frac{1 + j3}{j3} = (1 - j/3)\,\text{S}$$

Summary:

$h_{11} = 30 + j10\,\Omega; \quad h_{12} = 0; \quad h_{21} = -5; \quad h_{22} = 1 - j/3\,\text{S}$

P 18.11 $V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

$I_1 = 0$:

$1 \times 10^{-3} = h_{12}(10); \qquad \therefore \quad h_{12} = 1 \times 10^{-4}$

$200 \times 10^{-6} = h_{22}(10); \qquad \therefore \quad h_{22} = 20 \times 10^{-6}\,\text{S}$

$V_1 = 0$:

$80 \times 10^{-6} = h_{21}(-0.5 \times 10^{-6}) + (20 \times 10^{-6})(5); \qquad \therefore \quad h_{21} = 40$

$0 = h_{11}(-0.5 \times 10^{-6}) + (1 \times 10^{-4})(5); \qquad \therefore \quad h_{11} = 1000\,\Omega$

P 18.12 [a] $V_1 = a_{11}V_2 - a_{12}I_2$

$I_1 = a_{21}V_2 - a_{22}I_2$

From $I_1 = 0$: $\qquad 1 \times 10^{-3} = a_{11}(10) - a_{12}(200 \times 10^{-6})$

From $V_1 = 0$: $\qquad 0 = a_{11}(5) - a_{12}(80 \times 10^{-6})$

Solving simultaneously yields

$a_{11} = -4 \times 10^{-4}; \qquad a_{12} = -25\,\Omega$

From $I_1 = 0$: $\qquad 0 = a_{21}(10) - a_{22}(200 \times 10^{-6})$

From $V_1 = 0$: $\qquad -0.5 \times 10^{-6} = a_{21}(5) - a_{22}(80 \times 10^{-6})$

Solving simultaneously yields

$a_{21} = -5 \times 10^{-7}\,\text{S}; \qquad a_{22} = -0.025$

[b] $$a_{11} = -\frac{\Delta h}{h_{21}} = \frac{-[(1000)(20 \times 10^{-6}) - (1 \times 10^{-4})(40)]}{40} = -4 \times 10^{-4}$$

$$a_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{40} = -25\,\Omega$$

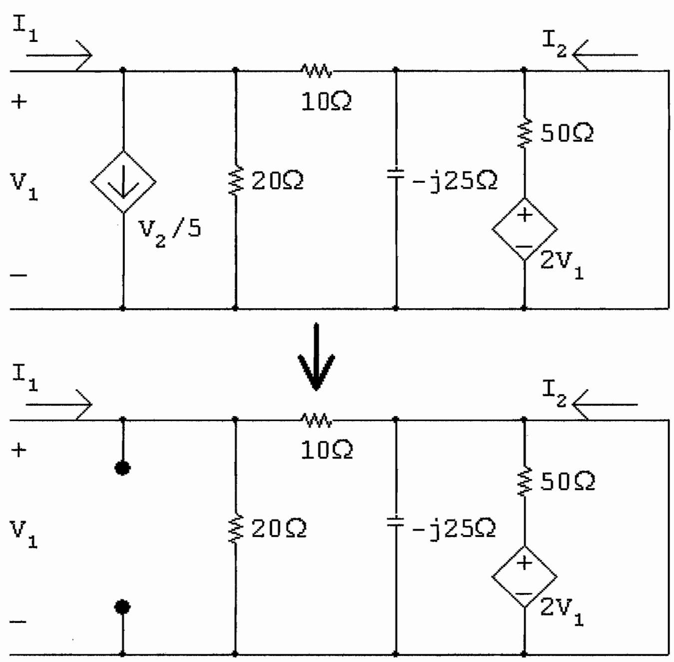
$$a_{21} = \frac{-h_{22}}{h_{21}} = \frac{-20 \times 10^{-6}}{40} = -5 \times 10^{-7}\,\text{S}$$

$$a_{22} = \frac{-1}{h_{21}} = \frac{-1}{40} = -0.025$$

Summary:

$a_{11} = -4 \times 10^{-4}; \quad a_{12} = -25\,\Omega; \quad a_{21} = -5 \times 10^{-7}\,\text{S}; \quad a_{22} = -0.025$

P 18.13 $$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}; \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

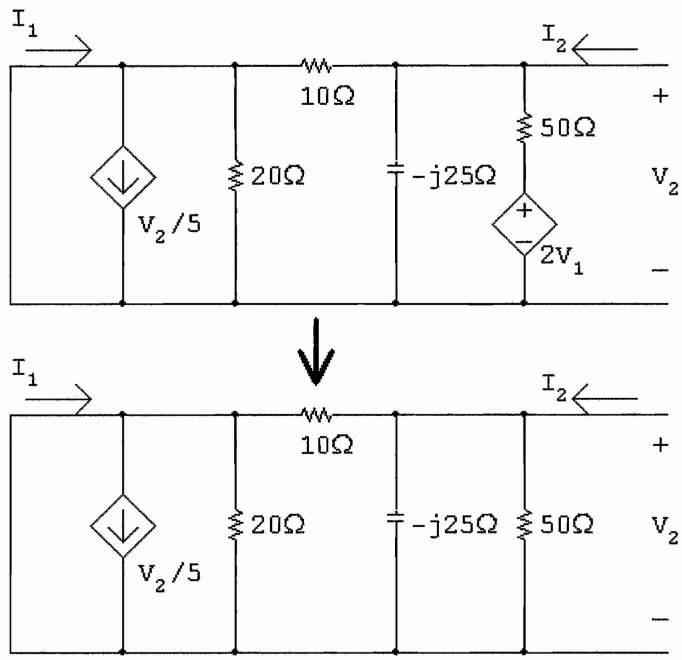


$$I_1 = \frac{V_1}{20} + \frac{V_1}{10} = \frac{3V_1}{20}; \qquad \therefore \quad y_{11} = \frac{I_1}{V_1} = \frac{3}{20} = 0.15\,\text{S}$$

$$I_2 = -\frac{2V_1}{50} - (I_1 - V_1/20) = -\frac{V_1}{25} - \frac{3V_1}{20} + \frac{V_1}{20} = -\frac{7V_1}{50}$$

$$\therefore \quad y_{21} = \frac{I_2}{V_1} = -\frac{7}{50} = -0.14\,\text{S}$$

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0}; \qquad y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$



$$I_1 = \frac{V_2}{5} - \frac{V_2}{10} = 0.1V_2; \qquad \therefore \quad y_{12} = \frac{I_1}{V_2} = 0.1\,\text{S}$$

$$I_2 = \frac{V_2}{50} + \frac{V_2}{-j25} + \frac{V_2}{10} = \frac{6 + j2}{50}V_2$$

$$\therefore \quad y_{22} = \frac{I_2}{V_2} = \frac{6 + j2}{50} = 0.12 + j0.04\,\text{S}$$

Summary:

$$y_{11} = 0.15\,\text{S}; \quad y_{12} = 0.1\,\text{S}; \quad y_{21} = -0.14\,\text{S}; \quad y_{22} = 0.12 + j0.04\,\text{S}$$

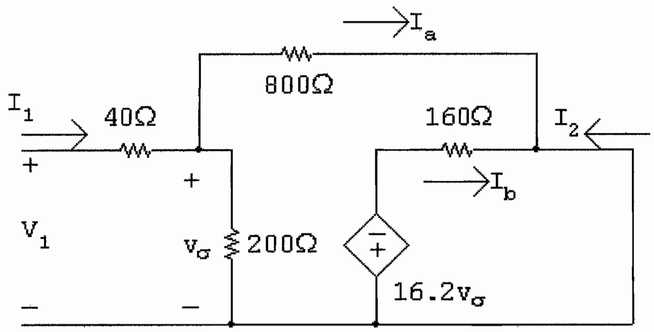
P 18.14 $b_{11} = -\frac{y_{11}}{y_{12}} = \frac{-0.15}{0.1} = -1.5$

$$b_{12} = -\frac{1}{y_{12}} = \frac{-1}{0.1} = -10\,\Omega$$

$$b_{21} = -\frac{\Delta y}{y_{12}} = \frac{-[(0.15)(0.12+j0.04)+(0.1)(0.14)]}{0.1} = -0.32 - j0.06\,\text{S}$$

$$b_{22} = \frac{y_{22}}{y_{12}} = \frac{0.12+j0.04}{0.1} = 1.2 + j0.4$$

P 18.15 $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0};\qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$



$$\frac{V_1}{I_1} = 40 + \frac{(800)(200)}{1000} = 40 + 160 = 200\,\Omega$$

$\therefore\quad h_{11} = 200\,\Omega$

$$I_a = I_1\left(\frac{200}{1000}\right) = 0.2I_1$$

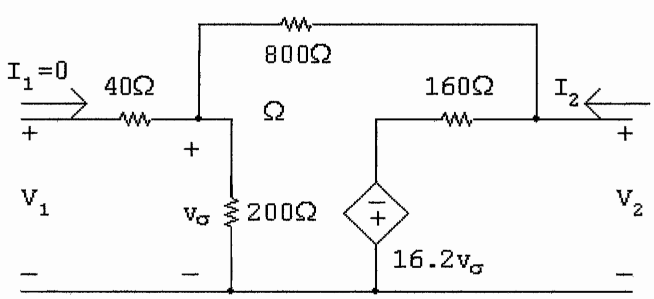
$16.2v_\sigma + 160I_b = 0;\qquad v_\sigma = 160I_1$

$\therefore\quad 160I_b = -2592I_1;\qquad I_b = -16.2I_1$

$\therefore\quad I_a + I_b + I_2 = 0;\quad 0.2I_1 - 16.2I_1 + I_2 = 0;\quad I_2 = 16I_1$

$\therefore\quad h_{21} = 16$

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0};\qquad h_{21} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$



$I_1 = 0;\qquad v_\sigma = V_1$

$$\frac{V_1}{200} + \frac{V_1 - V_2}{800} = 0; \quad 4V_1 + V_1 - V_2 = 0; \quad 5V_1 = V_2$$

$$\therefore \quad h_{12} = \frac{1}{5} = 0.2$$

$$I_2 = \frac{V_2 + 16.2V_1}{160} + \frac{V_2 - V_1}{800}; \qquad 800I_2 = 6V_2 + 80V_1$$

$$800I_2 = 6V_2 + 80(0.2V_2) = 22V_2$$

$$\therefore \quad h_{22} = \frac{I_2}{V_2} = \frac{22}{800} = 27.5\,\text{mS}$$

Summary:

$$h_{11} = 200\,\Omega; \quad h_{12} = 0.20; \quad h_{21} = 16; \quad h_{22} = 27.5\,\text{mS}$$

P 18.16 $V_1 = a_{11}V_2 - a_{12}I_2; \qquad I_1 = a_{21}V_2 - a_{22}I_2$

$$V_1 = h_{11}I_1 + h_{12}V_2; \qquad I_2 = h_{21}I_1 + h_{22}V_2$$

$$V_1 = -a_{12}I_2 + a_{11}V_2; \qquad I_2 = \frac{a_{21}V_2 - I_1}{a_{22}}$$

$$\therefore \quad V_1 = -a_{12}\left(\frac{a_{21} - I_1}{a_{22}}\right) + a_{11}V_2$$

$$V_1 = \frac{a_{12}}{a_{22}}I_1 + \left(\frac{a_{11}a_{22} - a_{12}a_{21}}{a_{22}}\right)V_2$$

$$\therefore \quad h_{11} = \frac{a_{12}}{a_{22}}; \qquad h_{12} = \frac{\Delta a}{a_{22}}$$

$$I_2 = -\frac{1}{a_{22}}I_1 + \frac{a_{21}}{a_{22}}V_2$$

$$\therefore \quad h_{21} = -\frac{1}{a_{22}}; \qquad h_{22} = \frac{a_{21}}{a_{22}}$$

P 18.17 $I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$

$$V_2 = b_{11}V_1 - b_{12}I_1; \qquad I_2 = b_{21}V_1 - b_{22}I_1$$

$$I_1 = \frac{b_{11}}{b_{12}}V_1 - \frac{1}{b_{12}}V_2$$

$$\therefore \quad y_{11} = \frac{b_{11}}{b_{12}}; \qquad y_{12} = -\frac{1}{b_{12}}$$

$$I_2 = b_{21}V_1 - b_{22}\left[\frac{b_{11}}{b_{12}}V_1 - \frac{1}{b_{12}}V_2\right]$$

$$I_2 = \frac{b_{21}b_{12} - b_{11}b_{22}}{b_{12}}V_1 + \frac{b_{22}}{b_{12}}V_2$$

$$\therefore \quad y_{21} = -\frac{\Delta b}{b_{12}}; \qquad y_{22} = \frac{b_{22}}{b_{12}}$$

P 18.18 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_2 = g_{21}V_1 + g_{22}I_2$

$$V_1 = z_{11}I_1 + z_{12}I_2; \qquad V_2 = z_{21}I_1 + z_{22}I_2$$

$$I_1 = \frac{V_1}{z_{11}} - \frac{z_{12}}{z_{11}}I_2$$

$$\therefore \quad g_{11} = \frac{1}{z_{11}}; \qquad g_{12} = \frac{-z_{12}}{z_{11}}$$

$$V_2 = z_{21}\left(\frac{V_1}{z_{11}} - \frac{z_{12}}{z_{11}}I_2\right) + z_{22}I_2 = \frac{z_{21}}{z_{11}}V_1 + \left(\frac{z_{11}z_{22} - z_{12}z_{21}}{z_{11}}\right)I_2$$

$$\therefore \quad g_{21} = \frac{z_{21}}{z_{11}}; \qquad g_{22} = \frac{\Delta z}{z_{11}}$$

P 18.19 $g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}; \qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$

$$V_1 = 200I_1 + 800I_1 = 1000I_1; \qquad \therefore \quad g_{11} = 10^{-3}\,\text{S}$$

$$V_- = \frac{1000}{1500}V_2 = V_+; \qquad V_+ = \frac{800}{1000}V_1$$

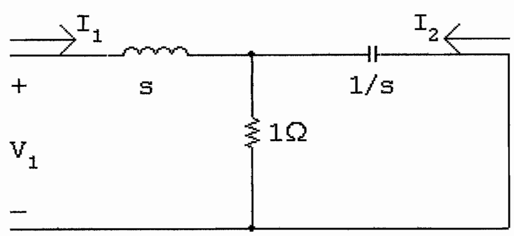
$$\therefore\quad \frac{1000}{1500}V_2 = \frac{800}{1000}V_1; \qquad \therefore\quad g_{21} = 1.2$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}; \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

$$I_1 = 0; \qquad \therefore\quad g_{12} = 0$$

$$\text{Also, } V_o = 0; \qquad \therefore\quad g_{22} = \frac{V_2}{I_2} = 40\,\Omega$$

P 18.20 $V_2 = 0$:



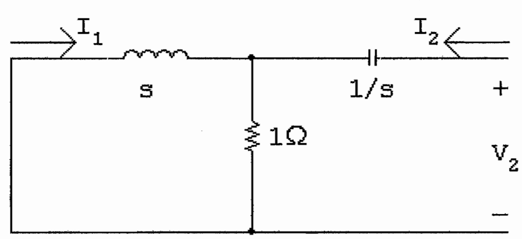
$$\frac{V_1}{I_1} = s + [1\|(1/s)] = \frac{s^2+s+1}{s+1}$$

$$\therefore\quad y_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{s+1}{s^2+s+1}$$

$$I_2 = \frac{-1}{1+(1/s)}I_1 = \frac{-s}{s+1}I_1 = \frac{-s}{s+1}\left(\frac{s+1}{s^2+s+1}\right)V_1$$

$$\therefore\quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-s}{s^2+s+1}$$

$V_1 = 0$:



$$\frac{V_2}{I_2} = (1/s) + 1\|s = \frac{1}{s} + \frac{s}{s+1} = \frac{s^2+s+1}{s(s+1)}$$

$$\therefore\quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{s(s+1)}{s^2+s+1}$$

$$I_1 = \frac{-1}{s+1}I_2 = \frac{-1}{s+1}\left[\frac{s(s+1)}{s^2+s+1}\right]V_2$$

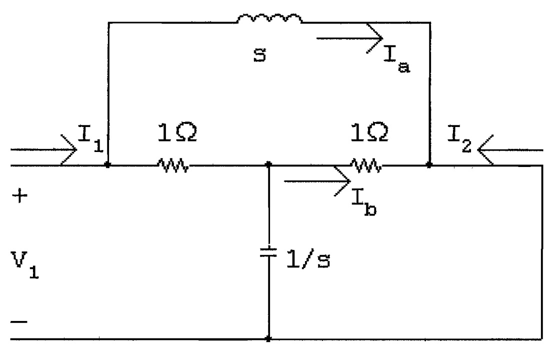
$$\therefore \quad y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = \frac{-s}{s^2+s+1}$$

P 18.21 First, find the y parameters:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

Since the two-port is symmetric and reciprocal we only need to calculate two parameters since $y_{11} = y_{22}$ and $y_{12} = y_{21}$.



$$I_1 = \frac{V_1}{s} + \frac{V_1}{1+\left(\frac{1}{s+1}\right)} = \left[\frac{1}{s} + \frac{1}{1+\frac{1}{s+1}}\right]V_1$$

$$\frac{I_1}{V_1} = \frac{s^2+2s+2}{s(s+2)}$$

$$y_{11} = y_{22} = \frac{s^2+2s+2}{s(s+2)}$$

$$I_a = \frac{V_1}{s}$$

$$I_b = \frac{V_1}{1+\frac{1}{s+1}} \cdot \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{V_1}{s+2}$$

$$I_2 = -(I_a + I_b) = -\left[\frac{V_1}{s} + \frac{V_1}{s+2}\right]$$

$$\frac{I_2}{V_1} = -\frac{2s+2}{s(s+2)}$$

$$y_{12} = y_{21} = -\frac{2(s+1)}{s(s+2)}$$

Now, transform to the a parameters:

$$a_{11} = \frac{-y_{22}}{y_{21}} = \frac{s^2+2s+2}{2(s+1)}$$

$$a_{12} = \frac{-1}{y_{21}} = \frac{s(s+2)}{2(s+1)}$$

$$a_{21} = \frac{-\Delta y}{y_{21}} = \frac{-1}{y_{21}} = \frac{s(s+2)}{2(s+1)}$$

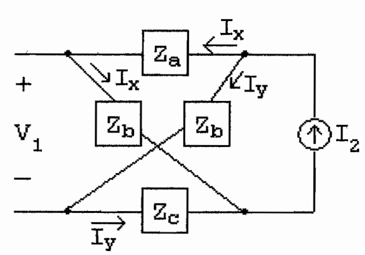
$$a_{22} = \frac{-y_{11}}{y_{21}} = \frac{s^2+2s+2}{2(s+1)}$$

P 18.22 First we note that

$$z_{11} = \frac{(Z_b+Z_c)(Z_a+Z_b)}{Z_a+2Z_b+Z_c} \quad \text{and} \quad z_{22} = \frac{(Z_a+Z_b)(Z_b+Z_c)}{Z_a+2Z_b+Z_c}$$

Therefore $z_{11} = z_{22}$.

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0};$$ Use the circuit below:

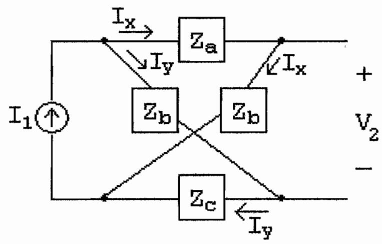


$$V_1 = Z_bI_x - Z_cI_y = Z_bI_x - Z_c(I_2 - I_x) = (Z_b+Z_c)I_x - Z_cI_2$$

$$I_x = \frac{Z_b+Z_c}{Z_a+2Z_b+Z_c}I_2 \quad \text{so} \quad V_1 = \frac{(Z_b+Z_c)^2}{Z_a+2Z_b+Z_c}I_2 - Z_cI_2$$

$$\therefore \quad Z_{12} = \frac{V_1}{I_2} = \frac{(Z_b+Z_c)^2}{Z_a+2Z_b+Z_c} - Z_c = \frac{Z_b^2 - Z_aZ_c}{Z_a+2Z_b+Z_c}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}; \qquad \text{Use the circuit below:}$$



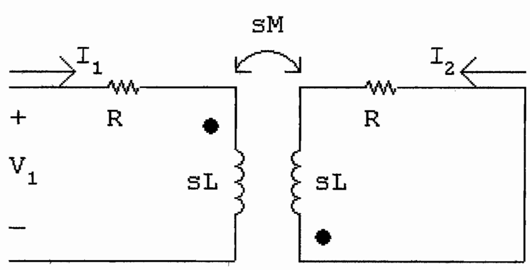
$$V_2 = Z_bI_x - Z_cI_y = Z_bI_x - Z_c(I_1 - I_x) = (Z_b + Z_c)I_x - Z_cI_1$$

$$I_x = \frac{Z_b + Z_c}{Z_a + 2Z_b + Z_c}I_1 \qquad \text{so} \qquad V_2 = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c}I_1 - Z_cI_1$$

$$\therefore \quad z_{21} = \frac{V_2}{I_1} = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} - Z_c = \frac{Z_b^2 - Z_aZ_c}{Z_a + 2Z_b + Z_c} = z_{12}$$

Thus the network is symmetrical and reciprocal.

P 18.23 [a] $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$



$$V_1 = (R + sL)I_1 - sMI_2$$

$$0 = -sMI_1 + (R + sL)I_2$$

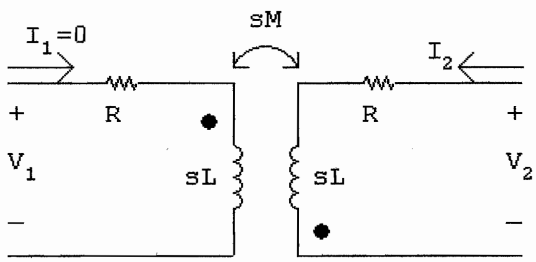
$$\Delta = \begin{vmatrix} (R + sL) & -sM \\ -sM & (R + sL) \end{vmatrix} = (R + sL)^2 - s^2M^2$$

$$N_1 = \begin{vmatrix} V_1 & -sM \\ 0 & (R + sL) \end{vmatrix} = (R + sL)V_1$$

$$I_1 = \frac{N_1}{\Delta} = \frac{(R + sL)V_1}{(R + sL)^2 - s^2M^2}; \qquad h_{11} = \frac{V_1}{I_1} = \frac{(R + sL)^2 - s^2M^2}{R + sL}$$

$$0 = -sMI_1 + (R + sL)I_2; \qquad \therefore \; h_{21} = \frac{I_2}{I_1} = \frac{sM}{R + sL}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$V_1 = -sMI_2; \qquad I_2 = \frac{V_2}{R + sL}$$

$$V_1 = \frac{-sMV_2}{R + sL}; \qquad h_{12} = \frac{V_1}{V_2} = \frac{-sM}{R + sL}$$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{R + sL}$$

[b] $h_{12} = -h_{21}$ (reciprocal)

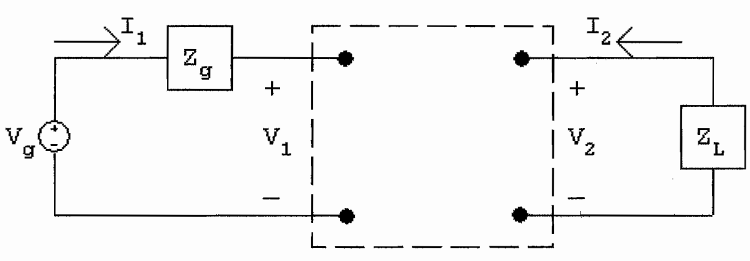
$h_{11}h_{22} - h_{12}h_{21} = 1$ (symmetrical, reciprocal)

$$h_{12} = \frac{-sM}{R + sL}; \qquad h_{21} = \frac{sM}{R + sL} \quad \text{(checks)}$$

$$h_{11}h_{22} - h_{12}h_{21} = \frac{(R + sL)^2 - s^2M^2}{R + sL} \cdot \frac{1}{R + sL} - \frac{(sM)(-sM)}{(R + sL)^2}$$

$$= \frac{(R + sL)^2 - s^2M^2 + s^2M^2}{(R + sL)^2} = 1 \quad \text{(checks)}$$

P 18.24



$$V_2 = b_{11}V_1 - b_{12}I_1; \qquad V_1 = V_g - I_1Z_g$$

$$I_2 = b_{21}V_1 - b_{22}I_1; \qquad V_2 = -Z_LI_2$$

$$I_2 = -\frac{V_2}{Z_L} = \frac{-b_{11}V_1 + b_{12}I_1}{Z_L}$$

$$\frac{-b_{11}V_1 + b_{12}I_1}{Z_L} = b_{21}V_1 - b_{22}I_1$$

$$\therefore\quad V_1\left(\frac{b_{11}}{Z_L} + b_{21}\right) = \left(b_{22} + \frac{b_{12}}{Z_L}\right) I_1$$

$$\frac{V_1}{I_1} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}} = Z_{\text{in}}$$

P 18.25 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_1 = V_g - Z_gI_1$

$$V_2 = g_{21}V_1 + g_{22}I_2; \qquad V_2 = -Z_LI_2$$

$$-Z_LI_2 = g_{21}V_1 + g_{22}I_2; \qquad V_1 = \frac{I_1 - g_{12}I_2}{g_{11}}$$

$$\therefore\quad -Z_LI_2 = \frac{g_{21}}{g_{11}}(I_1 - g_{12}I_2) + g_{22}I_2$$

$$\therefore\quad -Z_LI_2 + \frac{g_{12}g_{21}}{g_{11}}I_2 - g_{22}I_2 = \frac{g_{21}}{g_{11}}I_1$$

$$\therefore\quad (Z_Lg_{11} + \Delta g)I_2 = -g_{21}I_1; \qquad \therefore\quad \frac{I_2}{I_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$$

P 18.26 $I_1 = y_{11}V_1 + y_{12}V_2; \qquad V_1 = V_g - Z_gI_1$

$$I_2 = y_{21}V_1 + y_{22}V_2; \qquad V_2 = -Z_LI_2$$

$$\frac{-V_2}{Z_L} = y_{21}V_1 + y_{22}V_2$$

$$\therefore\quad -y_{21}V_1 = \left(\frac{1}{Z_L} + y_{22}\right) V_2; \qquad -y_{21}Z_LV_1 = (1 + y_{22}Z_L)V_2$$

$$\therefore\quad \frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$$

P 18.27 $V_1 = h_{11}I_1 + h_{12}V_2; \qquad V_1 = V_g - Z_gI_1$

$I_2 = h_{21}I_1 + h_{22}V_2; \qquad V_2 = -Z_LI_2$

$\therefore \quad V_g - Z_gI_1 = h_{11}I_1 + h_{12}V_2; \qquad V_g = (h_{11} + Z_g)I_1 + h_{12}V_2$

$$\therefore \quad I_1 = \frac{V_g - h_{12}V_2}{h_{11} + Z_g}$$

$$\therefore \quad -\frac{V_2}{Z_L} = h_{21}\left[\frac{V_g - h_{12}V_2}{h_{11} + Z_g}\right] + h_{22}V_2$$

$$\frac{-V_2(h_{11} + Z_g)}{Z_L} = h_{21}V_g - h_{12}h_{21}V_2 + h_{22}(h_{11} + Z_g)V_2$$

$$-V_2(h_{11} + Z_g) = h_{21}Z_LV_g - h_{12}h_{21}Z_LV_2 + h_{22}Z_L(h_{11} + Z_g)V_2$$

$$-h_{21}Z_LV_g = (h_{11} + Z_g)\,[V_2 + h_{22}Z_LV_2] - h_{12}h_{21}Z_LV_2$$

$$\therefore \quad \frac{V_2}{V_g} = \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$$

P 18.28 $V_1 = z_{11}I_1 + z_{12}I_2; \qquad V_1 = V_g - Z_gI_1$

$V_2 = z_{21}I_1 + z_{22}I_2; \qquad V_2 = -Z_LI_2$

$$V_{\text{Th}} = V_2\Big|_{I_2=0}; \qquad V_2 = z_{21}I_1; \qquad I_1 = \frac{V_1}{z_{11}} = \frac{V_g - I_1Z_g}{z_{11}}$$

$$\therefore \quad I_1 = \frac{V_g}{z_{11} + Z_g}; \qquad \therefore \quad V_2 = \frac{z_{21}V_g}{z_{11} + Z_g} = V_t$$

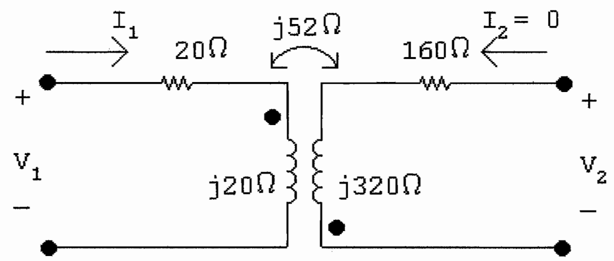
$$Z_{\text{Th}} = \frac{V_2}{I_2}\Big|_{V_g=0}; \qquad V_2 = z_{21}I_1 + z_{22}I_2$$

$$-I_1Z_g = z_{11}I_1 + z_{12}I_2; \qquad I_1 = \frac{-z_{12}I_2}{z_{11} + Z_g}$$

$$\therefore \quad V_2 = z_{21}\left[\frac{-z_{12}I_2}{z_{11} + Z_g}\right] + z_{22}I_2$$

$$\therefore \quad \frac{V_2}{I_2} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g} = Z_{\text{Th}}$$

P 18.29 [a] $a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}$; $\quad a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0}$

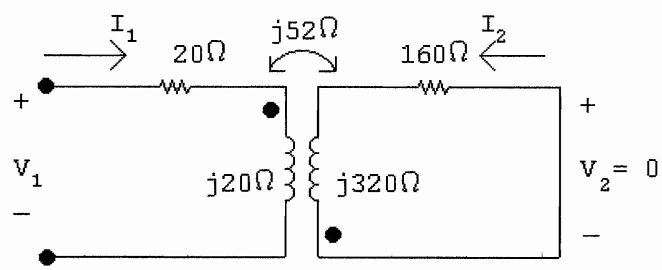


$$V_2 = -j52I_1 = -j52\frac{V_1}{20+j20}$$

$$a_{11} = \frac{V_1}{V_2} = \frac{20+j20}{-j52} = \frac{5}{13}(-1+j)$$

$$a_{21} = \frac{I_1}{V_2} = \frac{1}{-j52} = \frac{j}{52}\text{ S}$$

$$a_{12} = -\left.\frac{V_1}{I_2}\right|_{V_2=0}; \qquad a_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$



$$V_1 = (20+j20)I_1 - j52I_2$$

$$0 = -j52I_1 + (160+j320)I_2$$

$$\Delta = \begin{vmatrix} 20+j20 & -j52 \\ -j52 & 160+j320 \end{vmatrix} = -496 + j9600$$

$$N_2 = \begin{vmatrix} 20+j20 & V_1 \\ -j52 & 0 \end{vmatrix} = j52V_1$$

$$I_2 = \frac{j52V_1}{-496+j9600} \quad \text{so} \quad \frac{V_1}{I_2} = \frac{-496+j9600}{j52} = \frac{1}{52}(9600+j496)$$

$$\therefore \quad a_{12} = -\frac{V_1}{I_2} = \frac{1}{13}(-2400 - j124)$$

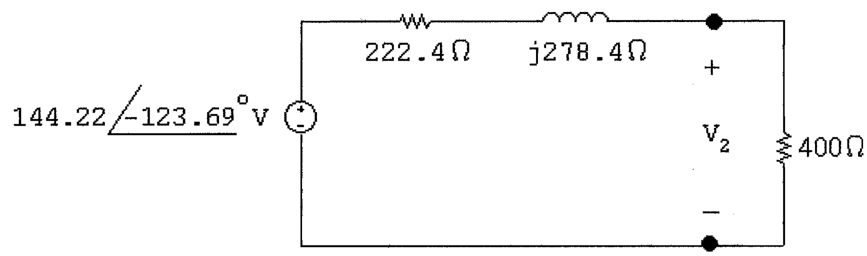
$$j52I_1 = (160 + j320)I_2; \qquad \therefore \quad a_{22} = -\frac{I_1}{I_2} = \frac{-320 + j160}{52}$$

[b] $$V_{\rm Th} = \frac{V_g}{a_{11} + a_{21}Z_g} = \frac{100\underline{/0^\circ}}{(5/13)(-1+j) + (j/52)(10)}$$

$$= -80 - j120 = 144.22\underline{/-123.69^\circ}\,\text{V}$$

$$Z_{\rm Th} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{\frac{1}{13}(-2400 - j124) + \frac{-320+j160}{52}(10)}{(5/13)(-1+j) + (j/52)(10)}$$

$$= 222.4 + j278.4 = 356.33\underline{/51.38^\circ}\,\Omega$$



[c] $$V_2 = \frac{144.22\underline{/-123.69^\circ}}{622.4 + j278.4}(400) = 84.607\underline{/-147.789^\circ}$$

$$v_2(t) = 84.607\cos(2000t - 147.789^\circ)\,\text{V}$$

P 18.30 $$\mathbf{I}_2 = \frac{y_{21}\mathbf{V}_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$$

$$= \frac{-0.25(1)}{1 + (-0.04)(100) + (0.025)(10) + (-0.00125)(10)(100)}$$

$$= 0.0625 \text{ A(rms)}$$

$$P_o = (I_2)^2 Z_L = (0.0625)^2(100) = 390.625\,\text{mW}$$

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L} = \frac{-0.25}{0.025 + (-0.00125)(100)} = 2.5$$

$$\therefore \quad I_1 = \frac{I_2}{2.5} = \frac{0.0625}{2.5} = 25 \text{ mA(rms)}$$

$$P_g = (1)(0.025) = 25\,\text{mW}$$

$$\frac{P_o}{P_g} = \frac{390.625}{25} = 15.625$$

P 18.31 [a] $Z_{\text{Th}} = g_{22} - \dfrac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$

$$g_{12}g_{21} = \left(-\frac{1}{2} + j\frac{1}{2}\right)\left(\frac{1}{2} - j\frac{1}{2}\right) = j\frac{1}{2}$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$\therefore \quad Z_{\text{Th}} = 1.5 + j2.5 - \frac{j3}{2 - j1} = 2.1 + j1.3\,\Omega$$

$$\therefore \quad Z_L = 2.1 - j1.3\,\Omega$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$g_{21}Z_L = \left(\frac{1}{2} - j\frac{1}{2}\right)(2.1 - j1.3) = 0.4 - j1.7$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$g_{22} + Z_L = 1.5 + j2.5 + 2.1 - j1.3 = 3.6 + j1.2$$

$$g_{12}g_{21}Z_g = j3$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{0.4 - j1.7}{(2 - j1)(3.6 + j1.2) - j3} = \frac{0.4 - j1.7}{8.4 - j4.2}$$

$$\mathbf{V}_2 = \frac{0.4 - j1.7}{8.4 - j4.2}(42\underline{/0^\circ}) = 5 - j6\,\text{V(rms)} = 7.81\underline{/-50.19^\circ}\,\text{V(rms)}$$

The rms value of $\mathbf{V}_2$ is 7.81 V.

[b] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{Z_L} = \dfrac{-5 + j6}{2.1 - j1.3} = -3 + j1\,\text{A(rms)}$

$$P = |\mathbf{I}_2|^2(2.1) = 21\,\text{W}$$

[c] $\dfrac{\mathbf{I}_2}{\mathbf{I}_1} = \dfrac{-g_{21}}{g_{11}Z_L + \Delta g}$

$$\Delta g = \left(\frac{1}{6} - j\frac{1}{6}\right)\left(\frac{3}{2} + j\frac{5}{2}\right) - \left(\frac{1}{2} - j\frac{1}{2}\right)\left(-\frac{1}{2} + j\frac{1}{2}\right)$$

$$= \frac{3}{12} + j\frac{5}{12} - j\frac{3}{12} + \frac{5}{12} - j\frac{1}{2} = \frac{2}{3} - j\frac{1}{3}$$

$$g_{11}Z_L = \left(\frac{1}{6} - j\frac{1}{6}\right)(2.1 - j1.3) = \frac{0.8}{6} - j\frac{3.4}{6}$$

$$\therefore \quad g_{11}Z_L + \Delta g = \frac{0.8}{6} - j\frac{3.4}{6} + \frac{4}{6} - j\frac{2}{6} = 0.8 - j0.9$$

P 18.33 [a] $Z_{\text{Th}} = \dfrac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$

From the solution to Problem 18.32

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$y_{22} + \Delta y Z_g = -50 \times 10^{-6} + 10^{-7}(2500) = 200 \times 10^{-6}$$

$$Z_{\text{Th}} = \frac{6}{200} \times 10^6 = 30{,}000\,\Omega$$

$$Z_L = Z_{\text{Th}}^* = 30{,}000\,\Omega$$

[b] $y_{21}Z_L = (100 \times 10^{-3})(30{,}000) = 3000$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(30{,}000) = -15$$

$$1 + y_{11}Z_g = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(30 \times 10^3) = -0.5$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{3000}{-15 - 6(-0.5)} = \frac{3000}{-12} = -250$$

$$\mathbf{V}_2 = -250(80 \times 10^{-3}) = -20 = 20\underline{/180^\circ}\,\text{V(rms)}$$

$$P = \frac{|\mathbf{V}_2|^2}{30{,}000} = \frac{400}{30} \times 10^{-3} = 13.33\,\text{mW}$$

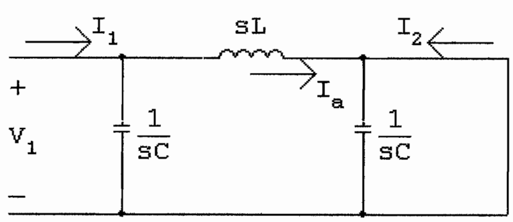
[c] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{30{,}000} = \dfrac{20\underline{/0^\circ}}{30{,}000} = \dfrac{2}{3}\,\text{mA}$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{2 \times 10^{-3} + 10^{-7}(30{,}000)} = \frac{100 \times 10^{-3}}{5 \times 10^{-3}} = 20$$

$$\mathbf{I}_1 = \frac{\mathbf{I}_2}{20} = \frac{2 \times 10^{-3}}{3(20)} = \frac{1}{30}\,\text{mA(rms)}$$

$$P_g(\text{developed}) = (80 \times 10^{-3})\left(\frac{1}{30} \times 10^{-3}\right) = \frac{8}{3}\,\mu\text{W}$$

P 18.34 [a] $h_{11} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\dfrac{I_2}{I_1}\right|_{V_2=0}$



$$h_{11} = \frac{(1/sC)(sL)}{(1/sC) + sL} = \frac{(1/C)s}{s^2 + (1/LC)}$$

$$I_2 = -I_a; \qquad I_a = \frac{I_1(1/sC)}{sL + (1/sC)}$$

$$I_2 = \frac{-I_1}{s^2LC + 1}$$

$$h_{21} = \frac{I_2}{I_1} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}; \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}$$

$$V_1 = \frac{V_2(1/sC)}{sL + (1/sC)} = \frac{V_2}{s^2LC + 1}$$

$$\frac{V_1}{V_2} = h_{12} = \frac{1/LC}{s^2 + (1/LC)}$$

$$\frac{V_2}{I_2} = \frac{(1/sC)[sL + (1/sC)]}{sL + (2/sC)} = \frac{s^2 + (1/LC)}{sC[s^2 + (2/LC)]}$$

$$\frac{I_2}{V_2} = h_{22} = \frac{Cs[s^2 + (2/LC)]}{s^2 + (1/LC)}$$

[b] $$\frac{1}{LC} = \frac{10^9}{(0.1)(400)} = 25 \times 10^6$$

$$h_{11} = \frac{10^7 s}{s^2 + 25 \times 10^6}$$

$$h_{12} = \frac{25 \times 10^6}{s^2 + 25 \times 10^6}$$

$$h_{21} = \frac{-25 \times 10^6}{s^2 + 25 \times 10^6}$$

$$h_{22} = \frac{10^{-7}s(s^2 + 50 \times 10^6)}{(s^2 + 25 \times 10^6)}$$

$$\frac{V_2}{V_1} = \frac{-h_{21}Z_L}{h_{11} + \Delta h Z_L} = \frac{-h_{21}Z_L}{h_{11} + Z_L} = \frac{\left(\frac{25\times10^6}{s^2+25\times10^6}\right)800}{\frac{10^7 s}{(s^2+25\times10^6)} + 800}$$

$$\frac{V_2}{V_1} = \frac{25 \times 10^6}{s^2 + 12{,}500s + 25 \times 10^6} = \frac{25 \times 10^6}{(s + 2500)(s + 10{,}000)}$$

$$V_1 = \frac{45}{s}$$

$$V_2 = \frac{1125 \times 10^6}{s(s + 2500)(s + 10{,}000)} = \frac{45}{s} - \frac{60}{s + 2500} + \frac{15}{s + 10{,}000}$$

$$v_2 = [45 - 60e^{-2500t} + 15e^{-10{,}000t}]u(t) \quad \text{V}$$

P 18.35 [a] $V_1 = z_{11}I_1 + z_{12}I_2$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = s + \frac{1}{s} = \frac{s^2+1}{s}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{1}{s}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{1}{s}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = s + \frac{1}{s} = \frac{s^2+1}{s}$$

[b] $$\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11}+Z_g)(z_{22}+Z_L) - z_{12}z_{21}}$$

$$= \frac{z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}}$$

$$= \frac{1/s}{\left(\frac{s^2+1}{s}+1\right)\left(\frac{s^2+1}{s}+1\right) - \frac{1}{s^2}}$$

$$= \frac{s}{(s^2+s+1)^2 - 1}$$

$$= \frac{s}{s^4+2s^3+3s^2+2s+1-1}$$

$$= \frac{1}{s^3+2s^2+3s+2}$$

$$= \frac{1}{(s+1)(s^2+s+2)}$$

$$\therefore \quad V_2 = \frac{50}{s(s+1)(s^2+s+2)}$$

$$s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

$$V_2 = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{K_3^*}{s+\frac{1}{2}+j\frac{\sqrt{7}}{2}}$$

$$K_1 = 25; \qquad K_2 = -25; \qquad K_3 = 9.45\underline{/90^\circ}$$

$$\therefore \quad v_2(t) = [25 - 25e^{-t} + 18.90e^{-0.5t}\cos(1.32t + 90^\circ)]u(t)\ \text{V}$$

CHECK

$$v_2(0) = 25 - 25 + 18.90\cos 90^\circ = 0$$

$$v_2(\infty) = 25 + 0 + 0 = 25\,\text{V}$$

P 18.36 $$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{100}{1.125} = \frac{800}{9}\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{104}{1.125} = \frac{832}{9}\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{20}{0.25} = 80\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{24}{0.25} = 96\,\Omega$$

$$Z_{\text{Th}} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g} = 96 - \frac{(80)(832/9)}{(800/9) + 0} = 12.8\,\Omega$$

$$\therefore \quad Z_L = 12.8\,\Omega$$

$$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$$

$$\Delta z = \left(\frac{800}{9}\right)96 - 80\left(\frac{832}{9}\right) = \frac{10{,}240}{9}$$

$$\frac{V_2}{V_1} = \frac{(832/9)(12.8)}{(800/9)(12.8) + (10{,}240/9)} = \frac{10{,}649.60}{20{,}480} = 0.52$$

$$V_2 = (0.52)(160) = 83.20\,\text{V}$$

$$P = \frac{(83.2)^2}{12.8} = 540.80\,\text{W}$$

P 18.37 $$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 25\,\Omega; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -0.5$$

From the second set of measurements we have

$$41 = 25(1) + h_{12}(20); \qquad \therefore \quad h_{12} = \frac{41 - 25}{20} = 0.80$$

$$0 = -0.5(1) + h_{22}(20); \qquad \therefore \quad h_{22} = \frac{0.5}{20} = 0.025\,\mho$$

$$R_{\text{Th}} = \frac{23 + 25}{23(0.025) + \Delta h}; \qquad \Delta h = 25(0.025) - (-0.5)(0.8) = 1.025$$

$$\therefore \quad R_{\text{Th}} = \frac{48}{1.6} = 30\,\Omega; \qquad \therefore \quad R_o = 30\,\Omega$$

$$\frac{V_2}{V_g} = \frac{-(-0.5)(30)}{(48)(1.75) + (0.4)(30)} = \frac{15}{96}$$

$$V_2 = 15\,\text{V}; \qquad P = \frac{(15)^2}{30} = 7.5\,\text{W}$$

P 18.38 $a'_{11} = -\dfrac{\Delta h}{h_{21}} = \dfrac{-0.01}{-0.1} = 0.1$

$$a'_{12} = -\frac{h_{11}}{h_{21}} = \frac{-150}{-0.1} = 1500$$

$$a'_{21} = -\frac{h_{22}}{h_{21}} = \frac{-10^{-4}}{-0.1} = 10^{-3}$$

$$a'_{22} = -\frac{1}{h_{21}} = \frac{-1}{-0.1} = 10$$

$$a''_{11} = \frac{1}{g_{21}} = \frac{1}{20} = 0.05$$

$$a''_{12} = \frac{g_{22}}{g_{21}} = \frac{24 \times 10^3}{20} = 1200$$

$$a''_{21} = \frac{g_{11}}{g_{21}} = \frac{0.01}{20} = 5 \times 10^{-4}$$

$$a''_{22} = \frac{\Delta g}{g_{21}} = \frac{320}{20} = 16$$

$$a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = (0.1)(0.05) + (1500)(5 \times 10^{-4}) = 0.755$$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (0.1)(1200) + (1500)(16) = 24{,}120$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (10^{-3})(0.05) + (10)(5 \times 10^{-4}) = 5.05 \times 10^{-3}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (10^{-3})(1200) + (10)(16) = 161.2$$

$$V_2 = \frac{Z_L V_g}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$= \frac{(1000)(109.5)}{[0.755 + (5.05 \times 10^{-3})(20)](1000) + 24{,}120 + (161.2)(20)} = 3.88\,\text{V}$$

P 18.39 The a parameters of the first two port are

$$a'_{11} = \frac{z_{11}}{z_{21}} = \frac{200}{-1.6 \times 10^6} = -125 \times 10^{-6}$$

$$a'_{12} = \frac{\Delta z}{z_{21}} = \frac{40 \times 10^6}{-1.6 \times 10^6} = -25\,\Omega$$

$$a'_{21} = \frac{1}{z_{21}} = \frac{1}{-1.6 \times 10^6} = -625 \times 10^{-9}\,\text{S}$$

$$a'_{22} = \frac{z_{22}}{z_{21}} = \frac{40{,}000}{-1.6 \times 10^6} = -25 \times 10^{-3}$$

The a parameters of the second two port are

$$a''_{11} = \frac{5}{4}; \quad a''_{12} = \frac{3R}{4}; \quad a''_{21} = \frac{3}{4R}; \quad a''_{22} = \frac{5}{4}$$

or $a''_{11} = 1.25; \quad a''_{12} = 6\,\text{k}\Omega; \quad a''_{21} = 93.75\,\mu\text{S}; \quad a''_{22} = 1.25$

The a parameters of the cascade connection are

$$a_{11} = -125 \times 10^{-6}(1.25) + (-25)(93.75 \times 10^{-6}) = -2.5 \times 10^{-3}$$

$$a_{12} = -125 \times 10^{-6}(6000) + (-25)(1.25) = -32\,\Omega$$

$$a_{21} = -625 \times 10^{-9}(1.25) + (-25 \times 10^{-3})(93.75 \times 10^{-6}) = -3.125 \times 10^{-6}\,\text{S}$$

$$a_{22} = -625 \times 10^{-9}(6000) + (-25 \times 10^{-3})(1.25) = -35 \times 10^{-3}$$

$$\frac{V_o}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$a_{21}Z_g = (-3.125 \times 10^{-6})(500) = -1.5625 \times 10^{-3}$$

$$a_{11} + a_{21}Z_g = -2.5 \times 10^{-3} - 1.5625 \times 10^{-3} = -4.0625 \times 10^{-3}$$

$$(a_{11} + a_{21}Z_g)Z_L = (-4.0625 \times 10^{-3})(8000) = -32.5$$

$$a_{22}Z_g = (-35 \times 10^{-3})(500) = -17.5$$

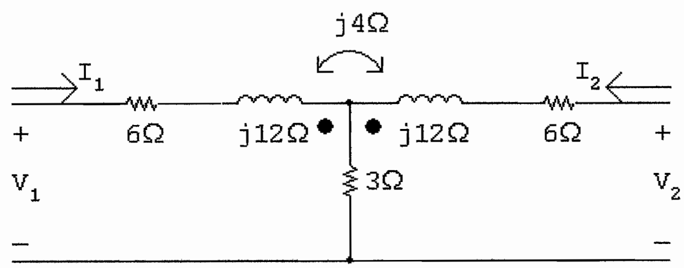
$$\frac{V_o}{V_g} = \frac{8000}{-32.5 - 32.25 - 17.5} = -97.26$$

$$v_o = V_o = -97.26V_g = -1.46\,\text{V}$$

P 18.40 [**a**] From reciprocity and symmetry

$$a'_{11} = a'_{22}, \quad \Delta a' = 1; \qquad \therefore\ 16 - 5a'_{21} = 1, \quad a'_{21} = 3\,\text{S}$$

For network B



$$a''_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{I_2=0}$$

$$\mathbf{V}_1 = (6 + j12 + 3)\mathbf{I}_1 = (9 + j12)\mathbf{I}_1$$

$$\mathbf{V}_2 = 3\mathbf{I}_1 + j4\mathbf{I}_1 = (3 + j4)\mathbf{I}_1$$

$$a''_{11} = \frac{9 + j12}{3 + j4} = 3$$

$$a''_{21} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{I_2=0} = \frac{1}{3 + j4} = 0.12 - j0.16\,\text{S}$$

$$a''_{22} = a''_{11} = 3$$

$$\Delta a'' = 1 = (3)(3) - (0.12 - j0.16)a''_{12}$$

$$\therefore\ a''_{12} = \frac{8}{0.12 - j0.16} = 24 + j32\,\Omega$$

[b] $a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = 12 + 5(0.12 - j0.16) = 12.6 - j0.8$

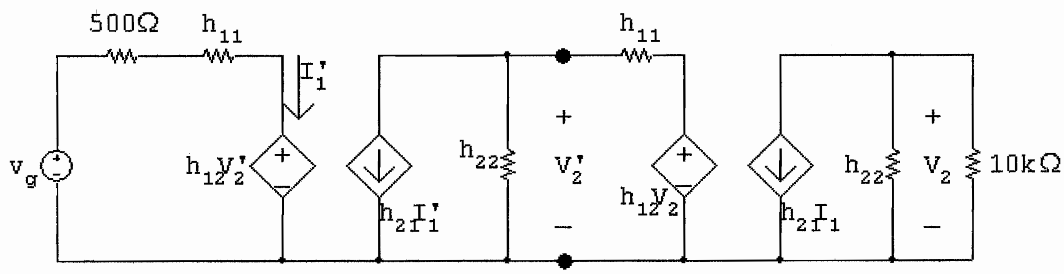
$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (4)(24 + j32) + (5)(3) = 111 + j128\,\Omega$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (3)(3) + (4)(0.12 - j0.16) = 9.48 - j0.64\,\text{S}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (3)(24 + j32) + (4)(3) = 84 + j96$$

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{1}{a_{11}} = \frac{1}{12.6 - j0.8} = 0.079 + j0.005$$

P 18.41 [a] At the input port: $V_1 = h_{11}I_1 + h_{12}V_2$;
At the output port: $I_2 = h_{21}I_1 + h_{22}V_2$



[b] $\dfrac{V_2}{10^4} + (100 \times 10^{-6}V_2) + 100I_1 = 0$

therefore $I_1 = -2 \times 10^{-6}V_2$

$$V'_2 = 1000I_1 + 15 \times 10^{-4}V_2 = -5 \times 10^{-4}V_2$$

$$100I'_1 + 10^{-4}V'_2 + (-2 \times 10^{-6})V_2 = 0$$

therefore $I'_1 = 205 \times 10^{-10}V_2$

$$V_g = 1500I'_1 + 15 \times 10^{-4}V'_2 = 3000 \times 10^{-8}V_2$$

$$\frac{V_2}{V_g} = \frac{10^5}{3} = 33{,}333$$

P 18.42 [a] $V_1 = I_2(z_{12} - z_{21}) + I_1(z_{11} - z_{21}) + z_{21}(I_1 + I_2)$

$$= I_2z_{12} - I_2z_{21} + I_1z_{11} - I_1z_{21} + z_{21}I_1 + z_{21}I_2 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = I_2(z_{22} - z_{21}) + z_{21}(I_1 + I_2) = z_{21}I_1 + z_{22}I_2$$

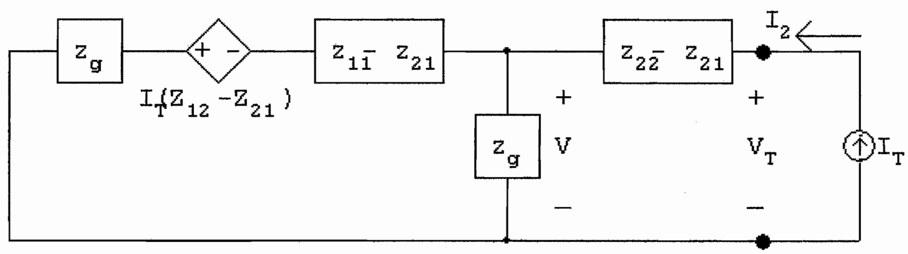
[b] Short circuit V_g and apply a test current source to port 2 as shown. Note that $I_T = I_2$. We have

$$\frac{V}{z_{21}} - I_T + \frac{V + I_T(z_{12} - z_{21})}{Z_g + z_{11} - z_{21}} = 0$$

Therefore

$$V = \left[\frac{z_{21}(Z_g + z_{11} - z_{12})}{Z_g + z_{11}}\right] I_T \quad \text{and} \quad V_T = V + I_T(z_{22} - z_{21})$$

Thus $\dfrac{V_T}{I_T} = Z_{\text{Th}} = z_{22} - \left(\dfrac{z_{12}z_{21}}{Z_g + z_{11}}\right)\,\Omega$



For V_{Th} note that $V_{\text{oc}} = \dfrac{z_{21}}{z_g + z_{11}} V_g$ since $I_2 = 0$.

P 18.43 [a] $V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2) = z_{11}I_1 + z_{12}I_2$

$V_2 = (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_2 + I_1) = z_{21}I_1 + z_{22}I_2$

[b] With port 2 terminated in an impedance Z_L, the two mesh equations are

$V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2)$

$0 = Z_L I_2 + (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_1 + I_2)$

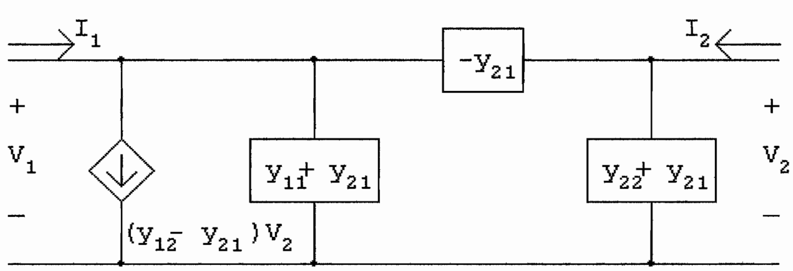
Solving for I_1:

$$I_1 = \frac{V_1(z_{22} + Z_L)}{z_{11}(Z_L + z_{22}) - z_{12}z_{21}}$$

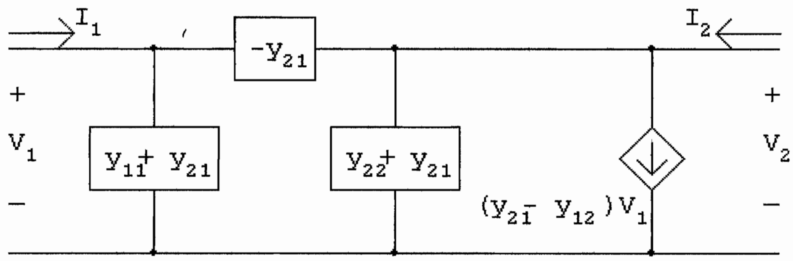
Therefore

$$Z_{\text{in}} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

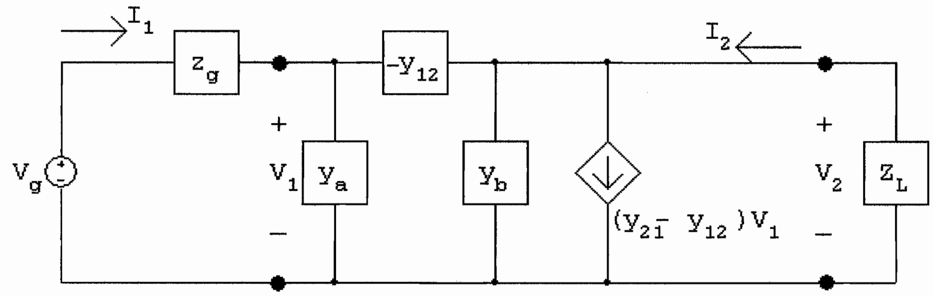
P 18.44 [a] $I_1 = y_{11}V_1 + y_{21}V_2 + (y_{12} - y_{21})V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$



$I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{12}V_1 + y_{22}V_2 + (y_{21} - y_{12})V_1$



[b] Using the second circuit derived in part [a], we have



where $y_a = (y_{11} + y_{12})$ and $y_b = (y_{22} + y_{12})$

At the input port we have

$$I_1 = y_a V_1 - y_{12}(V_1 - V_2) = y_{11}V_1 + y_{12}V_2$$

At the output port we have

$$\frac{V_2}{Z_L} + (y_{21} - y_{12})V_1 + y_b V_2 - y_{12}(V_2 - V_1) = 0$$

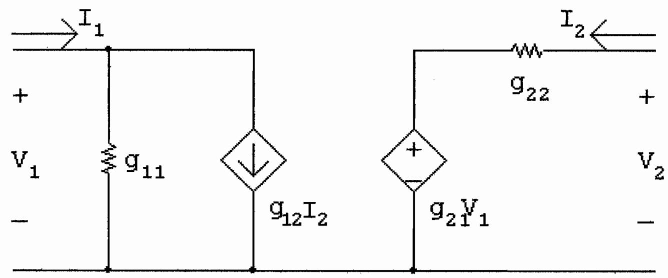
Solving for V_1 gives

$$V_1 = \left(\frac{1 + y_{22}Z_L}{-y_{21}Z_L}\right) V_2$$

Substituting Eq. (18.2) into (18.1) and at the same time using $V_2 = -Z_L I_2$, we get

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

P 18.45 [a] The g-parameter equations are $I_1 = g_{11}V_1 + g_{12}I_2$ and $V_2 = g_{21}V_1 + g_{22}I_2$. These equations are satisfied by the following circuit:



[b] The g parameters for the first two port in Fig P 18.39(a) are

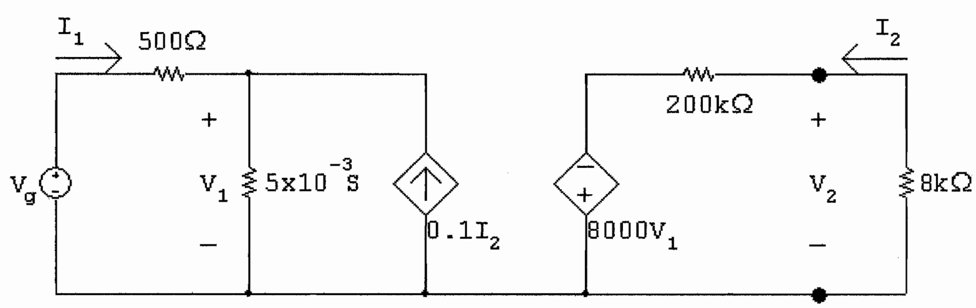
$$g_{11} = \frac{1}{z_{11}} = \frac{1}{200} = 5 \times 10^{-3}\,\text{S}$$

$$g_{12} = \frac{-z_{12}}{z_{11}} = \frac{-20}{200} = -0.10$$

$$g_{21} = \frac{z_{21}}{z_{11}} = \frac{-1.6 \times 10^6}{200} = -8000$$

$$g_{22} = \frac{\Delta z}{z_{11}} = \frac{40 \times 10^6}{200} = 200\,\text{k}\Omega$$

From Problem 3.64, since the load resistor and all resistors in the attenuator pad of the second two-port are equal to 8 kΩ, $R_{cd} = 8\,\text{k}\Omega$, hence our circuit reduces to



$$V_2 = \frac{8000}{8000 + 200{,}000}(-8000V_1)$$

$$I_2 = \frac{-V_2}{8000} = \frac{8000}{208{,}000}V_1 = \frac{8}{208}V_1$$

$$v_g = 15\,\text{mV}$$

$$\frac{V_1 - 15 \times 10^{-3}}{500} + V_1(5 \times 10^{-3}) - 0.1\frac{8V_1}{208} = 0$$

$$V_1\left(\frac{1}{500}+5\times 10^{-3}-\frac{0.8}{208}\right)=\frac{15\times 10^{-3}}{500}$$

$$\therefore \quad V_1 = 9.512\times 10^{-3}$$

$$V_2=\frac{-(8000)^2}{208{,}000}(9.512\times 10^{-3}) = -2.927\,\text{V}$$

Again, from the results of analyzing the attenuator pad in Problem 3.64

$$\frac{V_o}{V_2}=0.5; \qquad \therefore \quad V_o=(0.5)(-2.927)=-1.46\,\text{V}$$

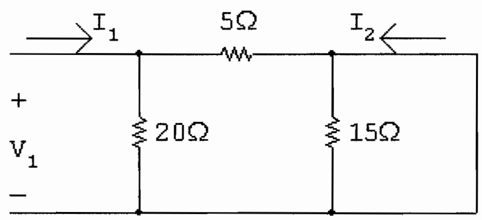
This result matches the solution to Problem 18.38.

18

Two-Port Circuits

Assessment Problems

AP 18.1 With port 2 short-circuited, we have



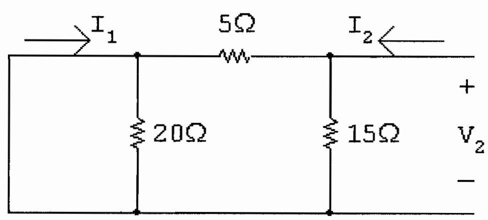
$$I_1 = \frac{V_1}{20} + \frac{V_1}{5}; \qquad \frac{I_1}{V_1} = y_{11} = 0.25\,\text{S}; \qquad I_2 = \left(\frac{-20}{25}\right) I_1 = -0.8 I_1$$

When $V_2 = 0$, we have $I_1 = y_{11}V_1$ and $I_2 = y_{21}V_1$

Therefore $I_2 = -0.8(y_{11}V_1) = -0.8y_{11}V_1$

Thus $y_{21} = -0.8y_{11} = -0.2\,\text{S}$

With port 1 short-circuited, we have



$$I_2 = \frac{V_2}{15} + \frac{V_2}{5}; \qquad \frac{I_2}{V_2} = y_{22} = \left(\frac{4}{15}\right)\,\text{S}$$

$$I_1 = \left(\frac{-15}{20}\right) I_2 = -0.75 I_2 = -0.75 y_{22} V_2$$

$$\text{Therefore} \quad y_{12} = (-0.75)\frac{4}{15} = -0.2\,\text{S}$$

AP 18.2

$$h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\text{S}$$

$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\text{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

AP 18.3

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \frac{5\times10^{-6}}{50\times10^{-3}} = 0.1\,\text{mS}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{200\times10^{-3}}{50\times10^{-3}} = 4$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = \frac{2\times10^{-6}}{0.5\times10^{-6}} = 4$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{10\times10^{-3}}{0.5\times10^{-6}} = 20\,\text{k}\Omega$$

AP 18.4 First calculate the *b*-parameters:

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} = \frac{15}{10} = 1.5\,\Omega; \qquad b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} = \frac{30}{10} = 3\,\text{S}$$

$$b_{12} = \frac{-V_2}{I_1}\bigg|_{V_1=0} = \frac{-10}{-5} = 2\,\Omega; \qquad b_{22} = \frac{-I_2}{I_1}\bigg|_{V_1=0} = \frac{-4}{-5} = 0.8$$

Now the *z*-parameters are calculated:

$$z_{11} = \frac{b_{22}}{b_{21}} = \frac{0.8}{3} = \frac{4}{15}\,\Omega; \qquad z_{12} = \frac{1}{b_{21}} = \frac{1}{3}\,\Omega$$

$$z_{21} = \frac{\Delta b}{b_{21}} = \frac{(1.5)(0.8) - 6}{3} = -1.6\,\Omega; \qquad z_{22} = \frac{b_{11}}{b_{21}} = \frac{1.5}{3} = \frac{1}{2}\,\Omega$$

AP 18.5

$z_{11} = z_{22}, \quad z_{12} = z_{21}, \quad 95 = z_{11}(5) + z_{12}(0)$

Therefore, $\quad z_{11} = z_{22} = 95/5 = 19\,\Omega$

$$11.52 = 19I_1 - z_{12}(2.72)$$

$$0 = z_{12}I_1 - 19(2.72)$$

Solving these simultaneous equations for z_{12} yields the quadratic equation

$$z_{12}^2 + \left(\frac{72}{17}\right) z_{12} - \frac{6137}{17} = 0$$

For a purely resistive network, it follows that $z_{12} = z_{21} = 17\,\Omega$.

AP 18.6 [a]
$$I_2 = \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_gZ_L + a_{22}Z_g}$$

$$= \frac{-50 \times 10^{-3}}{(5 \times 10^{-4})(5 \times 10^3) + 10 + (10^{-6})(100)(5 \times 10^3) + (-3 \times 10^{-2})(100)}$$

$$= \frac{-50 \times 10^{-3}}{10} = -5\,\text{mA}$$

$$P_L = \frac{1}{2}(5 \times 10^{-3})^2(5 \times 10^3) = 62.5\,\text{mW}$$

[b]
$$Z_{\text{Th}} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{10 + (-3 \times 10^{-2})(100)}{5 \times 10^{-4} + (10^{-6})(100)}$$

$$= \frac{7}{6 \times 10^{-4}} = \frac{70}{6}\,\text{k}\Omega$$

[c] $V_{\text{Th}} = \dfrac{V_g}{a_{11} + a_{21}Z_g} = \dfrac{50 \times 10^{-3}}{6 \times 10^{-4}} = \dfrac{500}{6}\,\text{V}$

Therefore $V_2 = \dfrac{250}{6}\,\text{V};\qquad P_{\text{max}} = \dfrac{(1/2)(250/6)^2}{(70/6) \times 10^3} = 74.4\,\text{mW}$

AP 18.7 [a] For the given bridged-tee circuit, we have

$$a'_{11} = a'_{22} = 1.25, \qquad a'_{21} = \frac{1}{20}\,\text{S}, \qquad a'_{12} = 11.25\,\Omega$$

The a-parameters of the cascaded networks are

$$a_{11} = (1.25)^2 + (11.25)(0.05) = 2.125$$

$$a_{12} = (1.25)(11.25) + (11.25)(1.25) = 28.125\,\Omega$$

$$a_{21} = (0.05)(1.25) + (1.25)(0.05) = 0.125\,\text{S}$$

$$a_{22} = a_{11} = 2.125, \qquad R_{\text{Th}} = (45.125/3.125) = 14.44\,\Omega$$

[b] $V_t = \dfrac{100}{3.125} = 32\,\text{V};\qquad$ therefore $V_2 = 16\,\text{V}$

[c] $P = \dfrac{16^2}{14.44} = 17.73\,\text{W}$

Problems

P 18.1 $$h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\mathrm{S}$$

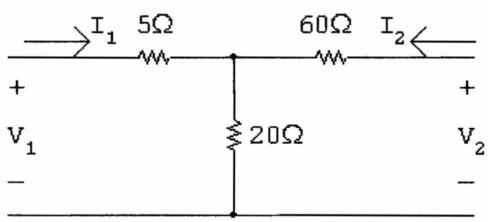
$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\mathrm{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

P 18.2



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 5 + 20 = 25\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 20\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 20\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 60 + 20 = 80\,\Omega$$

P 18.3 $\Delta z = (25)(80) - (20)(20) = 1600$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{80}{1600} = \frac{1}{20}\,\text{S}$$

$$y_{12} = \frac{-z_{12}}{\Delta z} = \frac{-20}{1600} = \frac{-1}{80}\,\text{S}$$

$$y_{21} = \frac{-z_{21}}{\Delta z} = \frac{-20}{1600} = \frac{-1}{80}\,\text{S}$$

$$y_{22} = \frac{-z_{11}}{\Delta z} = \frac{25}{1600} = \frac{1}{64}\,\text{S}$$

P 18.4 $V_1 = z_{11}I_1 + z_{12}I_2$

$V_1 = z_{21}I_1 + z_{22}I_2$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 5\|20 + 16 = 20\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 16 + (10)(5/25) = 18\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 16 + (10/25)(5) = 18\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 10\|15 + 6 = 22\,\Omega$$

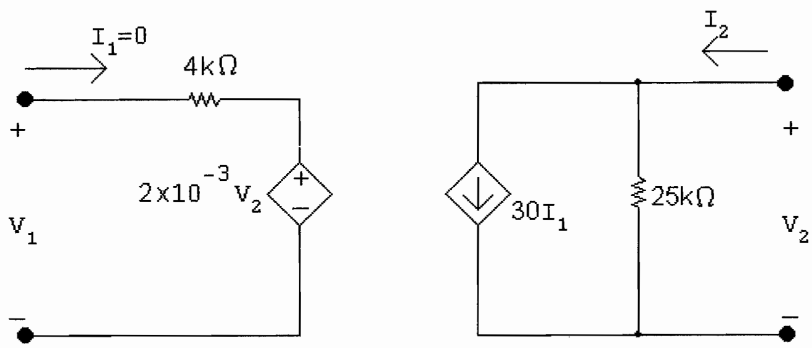
Summary:

$z_{11} = 20\,\Omega \quad z_{12} = 18\,\Omega \quad z_{21} = 18\,\Omega \quad z_{22} = 22\,\Omega$

P 18.5 $V_2 = b_{11}V_1 - b_{12}I_1$

$I_2 = b_{21}V_1 - b_{22}I_1$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$



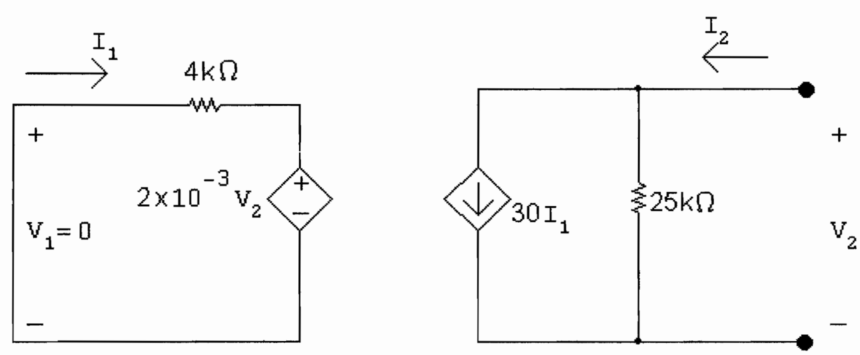
$$V_1 = 2 \times 10^{-3} V_2$$

$$\therefore \quad b_{11} = \frac{1}{2 \times 10^{-3}} = 500$$

$$V_2 = 25{,}000 I_2; \qquad \text{so} \qquad V_1 = (2 \times 10^{-3})(25{,}000) I_2 = 50 I_2$$

$$\therefore \quad b_{21} = \frac{1}{50} = 20 \text{ mS}$$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0}; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$



$$I_1 = -\frac{2 \times 10^{-3} V_2}{4000}; \qquad \therefore \quad b_{12} = \frac{4000}{2 \times 10^{-3}} = 2\,\text{M}\Omega$$

$$I_2 = 30 I_1 + \frac{V_2}{25{,}000} = 30 I_1 - \frac{4000}{(2 \times 10^{-3})(25{,}000)} I_1 = -50 I_1; \qquad \therefore \quad b_{22} = 50$$

Summary

$b_{11} = 500$; $b_{12} = 2\,\text{M}\Omega$; $b_{21} = 20\,\text{mS}$; $b_{22} = 50$

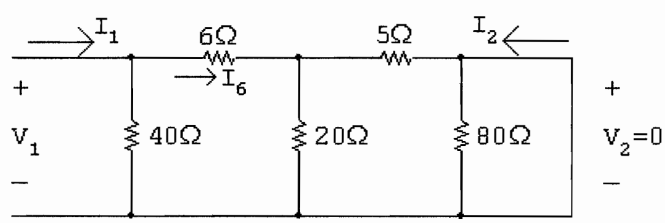
P 18.6 $g_{11} = \frac{b_{21}}{b_{22}} = \frac{20 \times 10^{-3}}{50} = 0.4\,\text{mS}$

$$g_{12} = \frac{-1}{b_{22}} = \frac{-1}{50} = -0.02$$

$$g_{21} = \frac{\Delta b}{b_{22}} = \frac{(500)(50) - (2 \times 10^6)(20 \times 10^{-3})}{50} = -300$$

$$g_{22} = \frac{b_{12}}{b_{22}} = \frac{2 \times 10^6}{50} = 40\,\text{k}\Omega$$

P 18.7 $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$

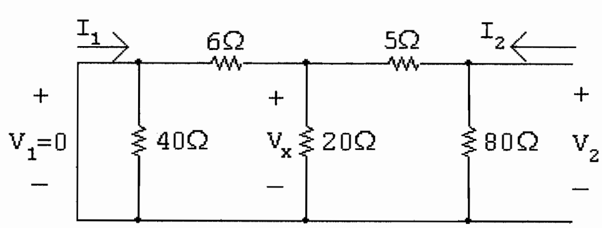


$$\frac{V_1}{I_1} = 40\|[6 + 20\|5] = 40\|10 = 8\,\Omega \qquad \therefore\; h_{11} = 8\,\Omega$$

$$I_6 = \frac{40}{40+10}I_1 = 0.8I_1$$

$$I_2 = \frac{-20}{20+5}I_6 = -0.8I_6 = -0.8(0.8)I_1 = -0.64I_1 \qquad \therefore\; h_{21} = -0.64$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$\frac{V_2}{I_2} = 80\|[5 + 20\|(40+6)] = 15.314\,\Omega \qquad \therefore\; h_{22} = \frac{1}{15.314} = 65.3\text{ mS}$$

$$V_x = \frac{20\|46}{5 + 20\|46} V_2$$

$$V_1 = \frac{40}{40+6} V_x = \frac{40(20\|46)}{46(5 + 20\|46)} V_2 = \frac{557.5758}{871.2121} V_2$$

$\therefore \qquad h_{12} = 0.64$

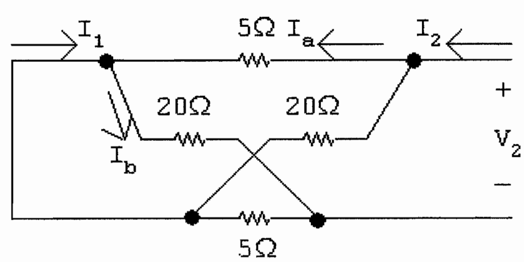
Summary:

$h_{11} = 8\,\Omega; \quad h_{12} = 0.64; \quad h_{21} = -0.64; \quad h_{22} = 65.3$ mS

P 18.8 $V_2 = b_{11}V_1 - b_{12}I_1$

$I_2 = b_{21}V_1 - b_{22}I_1$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0}; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$



$5\|20 = 4\,\Omega$

$$I_2 = \frac{V_2}{4+4} = \frac{V_2}{8}; \qquad I_1 = I_b - I_a$$

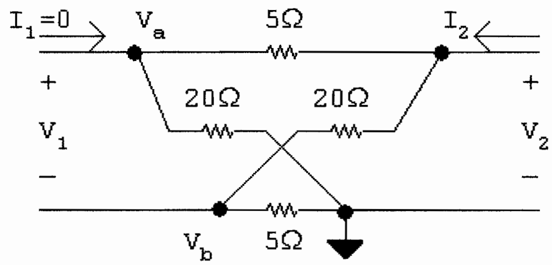
$$I_a = \frac{20}{25} I_2; \qquad I_b = \frac{5}{25} I_2$$

$$I_1 = \left(\frac{5}{25} - \frac{20}{25}\right) I_2 = \frac{-15}{25} I_2 = \frac{-3}{5} I_2$$

$$b_{22} = \frac{-I_2}{I_1} = \frac{5}{3}$$

$$b_{12} = \frac{-V_2}{I_1} = \frac{-V_2}{I_2}\left(\frac{I_2}{I_1}\right) = 8\left(\frac{5}{3}\right) = \frac{40}{3}\,\Omega$$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$



$$V_1 = V_a - V_b; \quad V_a = \frac{20}{25}V_2; \quad V_b = \frac{5}{25}V_2$$

$$V_1 = \frac{20}{25}V_2 - \frac{5}{25}V_2 = \frac{15}{25}V_2 = \frac{3}{5}V_2$$

$$b_{11} = \frac{V_2}{V_1} = \frac{5}{3}$$

$$V_2 = (20+5)\|(20+5)I_2 = 12.5I_2$$

$$b_{21} = \frac{I_2}{V_1} = \left(\frac{I_2}{V_2}\right)\left(\frac{V_2}{V_1}\right) = \left(\frac{1}{12.5}\right)\left(\frac{5}{3}\right) = \frac{2}{15}\,\text{S}$$

P 18.9 $$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}; \qquad V_2 = \frac{V_1}{R_1+R_3}R_3$$

$$\therefore \quad a_{11} = \frac{R_1+R_3}{R_3} = 1 + \frac{R_1}{R_3} = 1.2 \qquad \therefore \quad \frac{R_1}{R_3} = 0.2$$

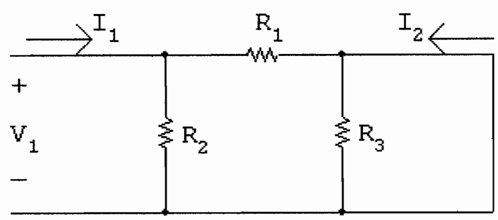
$$\therefore \quad R_1 = 0.2R_3 \qquad \text{(Eq 1)}$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0}; \qquad V_2 = I_3R_3 = \frac{R_2}{R_1+R_2+R_3}I_1R_3$$

$$\therefore \quad a_{21} = \frac{R_1+R_2+R_3}{R_2R_3} = 20\times10^{-3} \qquad \text{(Eq 2)}$$

Substitute Eq 1 into Eq 2:

$$\frac{0.2R_3+R_2+R_3}{R_2R_3} = \frac{R_2+1.2R_3}{R_2R_3} = 20\times10^{-3} \qquad \text{(Eq 3)}$$



$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0}; \qquad I_2 = \frac{-R_2}{R_1+R_2}I_1; \qquad \therefore\ a_{22} = \frac{R_1+R_2}{R_2} = 1.4$$

$$\frac{R_1}{R_2} = 0.4; \qquad \therefore\ R_2 = \frac{R_1}{0.4} = \frac{0.2R_3}{0.4} = 0.5R_3 \qquad \text{(Eq 4)}$$

Substitute Eq 4 into Eq 3:

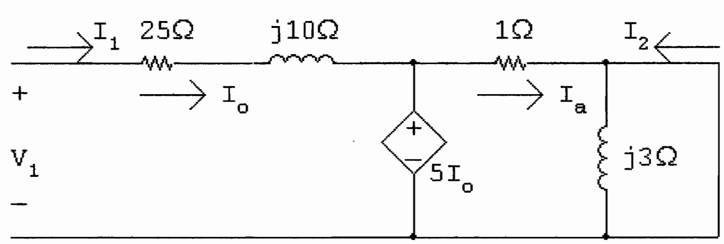
$$\frac{0.5R_3 + 1.2R_3}{(0.5R_3)R_3} = \frac{3.4}{R_3} = 20\times10^{-3} \qquad \therefore\ R_3 = 170\,\Omega$$

Therefore,

$$R_1 = 0.2R_3 = 0.2(170) = 34\,\Omega; \qquad R_2 = 0.5R_3 = 0.5(170) = 85\,\Omega$$

Summary: $R_1 = 34\,\Omega$; $R_2 = 85\,\Omega$; $R_3 = 170\,\Omega$

P 18.10 $h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0}; \qquad h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0}$

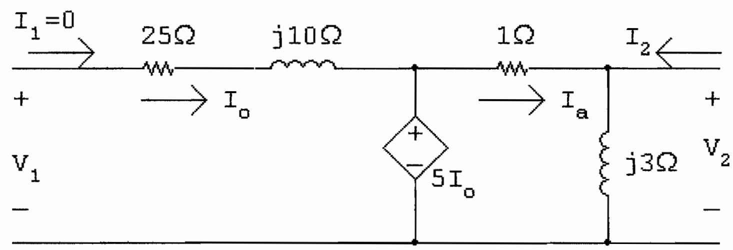


$$I_a = \frac{5I_o}{1} = 5I_1 = -I_2; \qquad \therefore\ h_{21} = -5$$

$$V_1 = (25 + j10)I_1 + 5I_1 = (30 + j10)I_1 = (30 + j10)I_1$$

$$\therefore\ h_{11} = 30 + j10\,\Omega$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}; \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}$$



$I_o = 0$ thus $5I_o = 0$ cs is a short circuit

$V_1 = 5I_o = 0; \quad \therefore \quad h_{12} = 0$

$$h_{22} = \frac{I_2}{V_2} = \frac{1 + j3}{j3} = (1 - j/3)\,\text{S}$$

Summary:

$h_{11} = 30 + j10\,\Omega; \quad h_{12} = 0; \quad h_{21} = -5; \quad h_{22} = 1 - j/3\,\text{S}$

P 18.11 $V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

$I_1 = 0$:

$1 \times 10^{-3} = h_{12}(10); \quad \therefore \quad h_{12} = 1 \times 10^{-4}$

$200 \times 10^{-6} = h_{22}(10); \quad \therefore \quad h_{22} = 20 \times 10^{-6}\,\text{S}$

$V_1 = 0$:

$80 \times 10^{-6} = h_{21}(-0.5 \times 10^{-6}) + (20 \times 10^{-6})(5); \quad \therefore \quad h_{21} = 40$

$0 = h_{11}(-0.5 \times 10^{-6}) + (1 \times 10^{-4})(5); \quad \therefore \quad h_{11} = 1000\,\Omega$

P 18.12 [a] $V_1 = a_{11}V_2 - a_{12}I_2$

$I_1 = a_{21}V_2 - a_{22}I_2$

From $I_1 = 0$: $\quad 1 \times 10^{-3} = a_{11}(10) - a_{12}(200 \times 10^{-6})$

From $V_1 = 0$: $\quad 0 = a_{11}(5) - a_{12}(80 \times 10^{-6})$

Solving simultaneously yields

$a_{11} = -4 \times 10^{-4}; \qquad a_{12} = -25\,\Omega$

From $I_1 = 0$: $\qquad 0 = a_{21}(10) - a_{22}(200 \times 10^{-6})$

From $V_1 = 0$: $\qquad -0.5 \times 10^{-6} = a_{21}(5) - a_{22}(80 \times 10^{-6})$

Solving simultaneously yields

$a_{21} = -5 \times 10^{-7}\,\text{S}; \qquad a_{22} = -0.025$

[b] $a_{11} = -\dfrac{\Delta h}{h_{21}} = \dfrac{-[(1000)(20 \times 10^{-6}) - (1 \times 10^{-4})(40)]}{40} = -4 \times 10^{-4}$

$$a_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{40} = -25\,\Omega$$

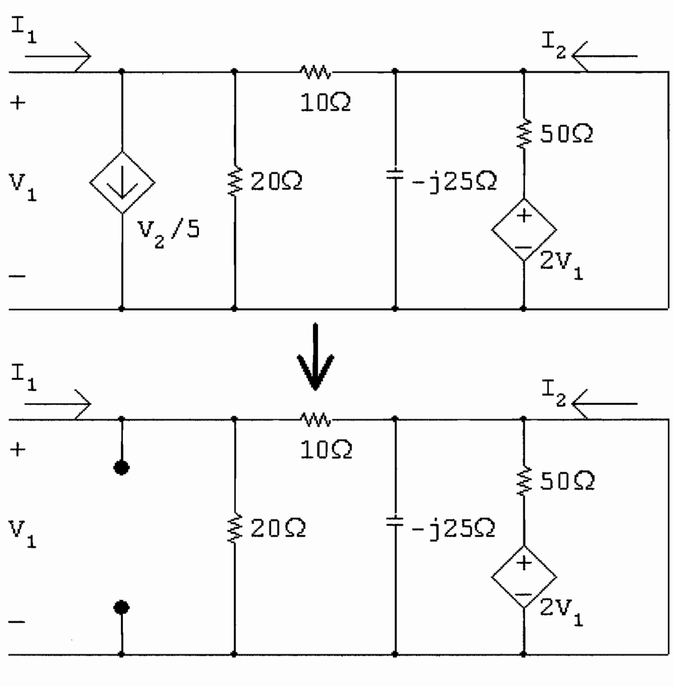
$$a_{21} = \frac{-h_{22}}{h_{21}} = \frac{-20 \times 10^{-6}}{40} = -5 \times 10^{-7}\,\text{S}$$

$$a_{22} = \frac{-1}{h_{21}} = \frac{-1}{40} = -0.025$$

Summary:

$a_{11} = -4 \times 10^{-4}; \quad a_{12} = -25\,\Omega; \quad a_{21} = -5 \times 10^{-7}\,\text{S}; \quad a_{22} = -0.025$

P 18.13 $y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0}; \qquad y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2=0}$

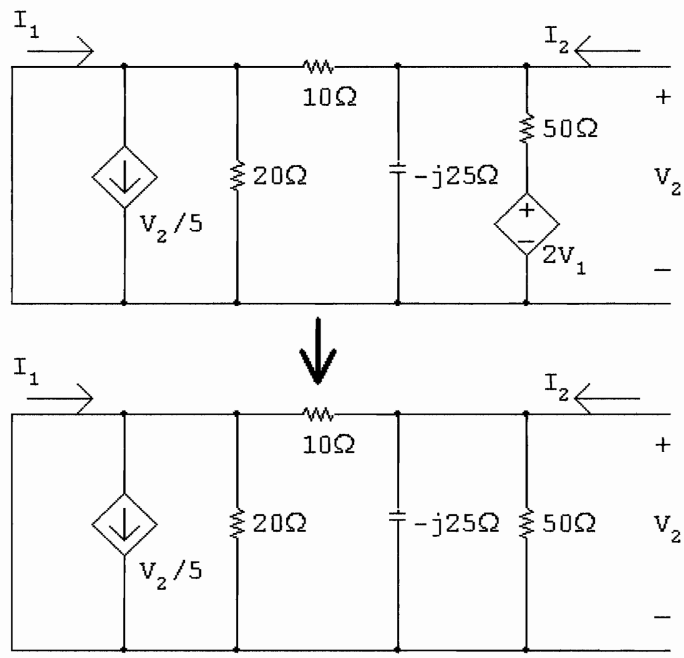


$$I_1 = \frac{V_1}{20} + \frac{V_1}{10} = \frac{3V_1}{20}; \qquad \therefore\ y_{11} = \frac{I_1}{V_1} = \frac{3}{20} = 0.15\,\text{S}$$

$$I_2 = -\frac{2V_1}{50} - (I_1 - V_1/20) = -\frac{V_1}{25} - \frac{3V_1}{20} + \frac{V_1}{20} = -\frac{7V_1}{50}$$

$$\therefore \quad y_{21} = \frac{I_2}{V_1} = -\frac{7}{50} = -0.14\,\text{S}$$

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0}; \qquad y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$



$$I_1 = \frac{V_2}{5} - \frac{V_2}{10} = 0.1V_2; \qquad \therefore \quad y_{12} = \frac{I_1}{V_2} = 0.1\,\text{S}$$

$$I_2 = \frac{V_2}{50} + \frac{V_2}{-j25} + \frac{V_2}{10} = \frac{6+j2}{50}V_2$$

$$\therefore \quad y_{22} = \frac{I_2}{V_2} = \frac{6+j2}{50} = 0.12 + j0.04\,\text{S}$$

Summary:

$y_{11} = 0.15\,\text{S}; \quad y_{12} = 0.1\,\text{S}; \quad y_{21} = -0.14\,\text{S}; \quad y_{22} = 0.12 + j0.04\,\text{S}$

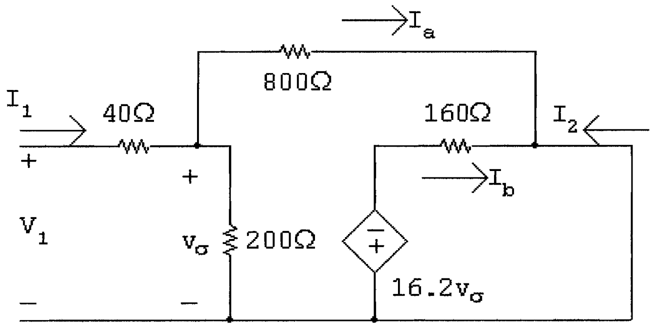
P 18.14 $b_{11} = -\dfrac{y_{11}}{y_{12}} = \dfrac{-0.15}{0.1} = -1.5$

$$b_{12} = -\frac{1}{y_{12}} = \frac{-1}{0.1} = -10\,\Omega$$

$$b_{21} = -\frac{\Delta y}{y_{12}} = \frac{-[(0.15)(0.12 + j0.04) + (0.1)(0.14)]}{0.1} = -0.32 - j0.06\,\text{S}$$

$$b_{22} = \frac{y_{22}}{y_{12}} = \frac{0.12 + j0.04}{0.1} = 1.2 + j0.4$$

P 18.15 $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$



$$\frac{V_1}{I_1} = 40 + \frac{(800)(200)}{1000} = 40 + 160 = 200\,\Omega$$

$$\therefore \quad h_{11} = 200\,\Omega$$

$$I_a = I_1\left(\frac{200}{1000}\right) = 0.2I_1$$

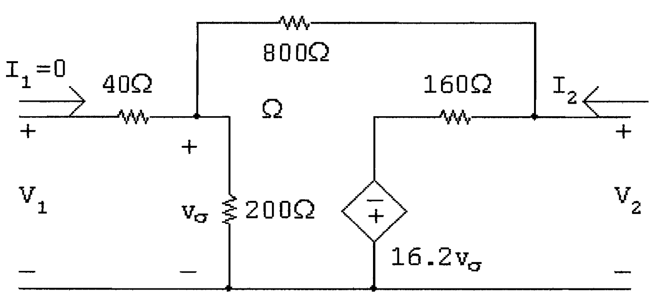
$$16.2v_\sigma + 160I_b = 0; \qquad v_\sigma = 160I_1$$

$$\therefore \quad 160I_b = -2592I_1; \qquad I_b = -16.2I_1$$

$$\therefore \quad I_a + I_b + I_2 = 0; \quad 0.2I_1 - 16.2I_1 + I_2 = 0; \quad I_2 = 16I_1$$

$$\therefore \quad h_{21} = 16$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{21} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$I_1 = 0; \qquad v_\sigma = V_1$$

$$\frac{V_1}{200} + \frac{V_1 - V_2}{800} = 0; \quad 4V_1 + V_1 - V_2 = 0; \quad 5V_1 = V_2$$

$$\therefore \quad h_{12} = \frac{1}{5} = 0.2$$

$$I_2 = \frac{V_2 + 16.2V_1}{160} + \frac{V_2 - V_1}{800}; \qquad 800I_2 = 6V_2 + 80V_1$$

$$800I_2 = 6V_2 + 80(0.2V_2) = 22V_2$$

$$\therefore \quad h_{22} = \frac{I_2}{V_2} = \frac{22}{800} = 27.5\,\text{mS}$$

Summary:

$$h_{11} = 200\,\Omega; \quad h_{12} = 0.20; \quad h_{21} = 16; \quad h_{22} = 27.5\,\text{mS}$$

P 18.16 $V_1 = a_{11}V_2 - a_{12}I_2; \qquad I_1 = a_{21}V_2 - a_{22}I_2$

$$V_1 = h_{11}I_1 + h_{12}V_2; \qquad I_2 = h_{21}I_1 + h_{22}V_2$$

$$V_1 = -a_{12}I_2 + a_{11}V_2; \qquad I_2 = \frac{a_{21}V_2 - I_1}{a_{22}}$$

$$\therefore \quad V_1 = -a_{12}\left(\frac{a_{21} - I_1}{a_{22}}\right) + a_{11}V_2$$

$$V_1 = \frac{a_{12}}{a_{22}}I_1 + \left(\frac{a_{11}a_{22} - a_{12}a_{21}}{a_{22}}\right)V_2$$

$$\therefore \quad h_{11} = \frac{a_{12}}{a_{22}}; \qquad h_{12} = \frac{\Delta a}{a_{22}}$$

$$I_2 = -\frac{1}{a_{22}}I_1 + \frac{a_{21}}{a_{22}}V_2$$

$$\therefore \quad h_{21} = -\frac{1}{a_{22}}; \qquad h_{22} = \frac{a_{21}}{a_{22}}$$

P 18.17 $I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$

$$V_2 = b_{11}V_1 - b_{12}I_1; \qquad I_2 = b_{21}V_1 - b_{22}I_1$$

$$I_1 = \frac{b_{11}}{b_{12}}V_1 - \frac{1}{b_{12}}V_2$$

$$\therefore \quad y_{11} = \frac{b_{11}}{b_{12}}; \qquad y_{12} = -\frac{1}{b_{12}}$$

$$I_2 = b_{21}V_1 - b_{22}\left[\frac{b_{11}}{b_{12}}V_1 - \frac{1}{b_{12}}V_2\right]$$

$$I_2 = \frac{b_{21}b_{12} - b_{11}b_{22}}{b_{12}}V_1 + \frac{b_{22}}{b_{12}}V_2$$

$$\therefore \quad y_{21} = -\frac{\Delta b}{b_{12}}; \qquad y_{22} = \frac{b_{22}}{b_{12}}$$

P 18.18 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_2 = g_{21}V_1 + g_{22}I_2$

$$V_1 = z_{11}I_1 + z_{12}I_2; \qquad V_2 = z_{21}I_1 + z_{22}I_2$$

$$I_1 = \frac{V_1}{z_{11}} - \frac{z_{12}}{z_{11}}I_2$$

$$\therefore \quad g_{11} = \frac{1}{z_{11}}; \qquad g_{12} = \frac{-z_{12}}{z_{11}}$$

$$V_2 = z_{21}\left(\frac{V_1}{z_{11}} - \frac{z_{12}}{z_{11}}I_2\right) + z_{22}I_2 = \frac{z_{21}}{z_{11}}V_1 + \left(\frac{z_{11}z_{22} - z_{12}z_{21}}{z_{11}}\right)I_2$$

$$\therefore \quad g_{21} = \frac{z_{21}}{z_{11}}; \qquad g_{22} = \frac{\Delta z}{z_{11}}$$

P 18.19 $g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}; \qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$

$$V_1 = 200I_1 + 800I_1 = 1000I_1; \qquad \therefore \quad g_{11} = 10^{-3}\,\text{S}$$

$$V_- = \frac{1000}{1500}V_2 = V_+; \qquad V_+ = \frac{800}{1000}V_1$$

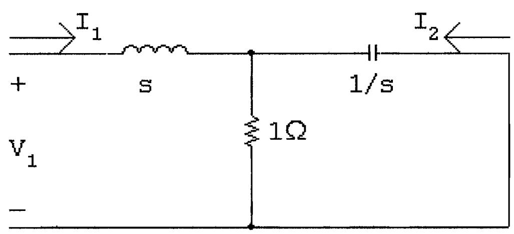
$$\therefore \quad \frac{1000}{1500}V_2 = \frac{800}{1000}V_1; \qquad \therefore \quad g_{21} = 1.2$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}; \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

$$I_1 = 0; \qquad \therefore \quad g_{12} = 0$$

$$\text{Also, } V_o = 0; \qquad \therefore \quad g_{22} = \frac{V_2}{I_2} = 40\,\Omega$$

P 18.20 $V_2 = 0$:



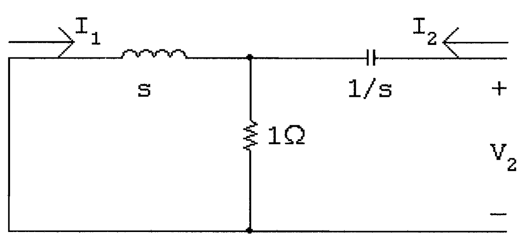
$$\frac{V_1}{I_1} = s + [1\|(1/s)] = \frac{s^2+s+1}{s+1}$$

$$\therefore \quad y_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{s+1}{s^2+s+1}$$

$$I_2 = \frac{-1}{1+(1/s)}I_1 = \frac{-s}{s+1}I_1 = \frac{-s}{s+1}\left(\frac{s+1}{s^2+s+1}\right)V_1$$

$$\therefore \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-s}{s^2+s+1}$$

$V_1 = 0$:



$$\frac{V_2}{I_2} = (1/s) + 1\|s = \frac{1}{s} + \frac{s}{s+1} = \frac{s^2+s+1}{s(s+1)}$$

$$\therefore \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{s(s+1)}{s^2+s+1}$$

$$I_1 = \frac{-1}{s+1} I_2 = \frac{-1}{s+1}\left[\frac{s(s+1)}{s^2+s+1}\right] V_2$$

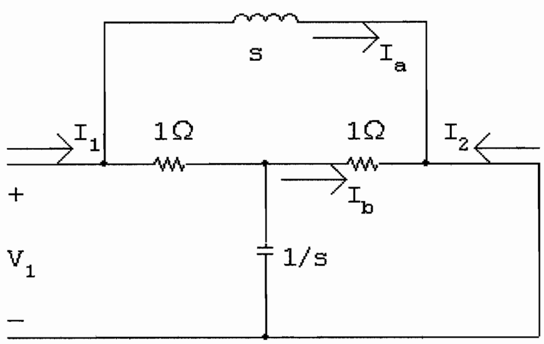
$$\therefore \quad y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = \frac{-s}{s^2+s+1}$$

P 18.21 First, find the y parameters:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

Since the two-port is symmetric and reciprocal we only need to calculate two parameters since $y_{11} = y_{22}$ and $y_{12} = y_{21}$.



$$I_1 = \frac{V_1}{s} + \frac{V_1}{1+\left(\frac{1}{s+1}\right)} = \left[\frac{1}{s} + \frac{1}{1+\frac{1}{s+1}}\right] V_1$$

$$\frac{I_1}{V_1} = \frac{s^2+2s+2}{s(s+2)}$$

$$y_{11} = y_{22} = \frac{s^2+2s+2}{s(s+2)}$$

$$I_a = \frac{V_1}{s}$$

$$I_b = \frac{V_1}{1+\frac{1}{s+1}} \cdot \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{V_1}{s+2}$$

$$I_2 = -(I_a + I_b) = -\left[\frac{V_1}{s} + \frac{V_1}{s+2}\right]$$

$$\frac{I_2}{V_1} = -\frac{2s+2}{s(s+2)}$$

$$y_{12} = y_{21} = -\frac{2(s+1)}{s(s+2)}$$

Now, transform to the a parameters:

$$a_{11} = \frac{-y_{22}}{y_{21}} = \frac{s^2+2s+2}{2(s+1)}$$

$$a_{12} = \frac{-1}{y_{21}} = \frac{s(s+2)}{2(s+1)}$$

$$a_{21} = \frac{-\Delta y}{y_{21}} = \frac{-1}{y_{21}} = \frac{s(s+2)}{2(s+1)}$$

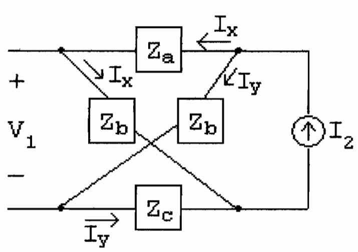
$$a_{22} = \frac{-y_{11}}{y_{21}} = \frac{s^2+2s+2}{2(s+1)}$$

P 18.22 First we note that

$$z_{11} = \frac{(Z_\mathrm{b}+Z_\mathrm{c})(Z_\mathrm{a}+Z_\mathrm{b})}{Z_\mathrm{a}+2Z_\mathrm{b}+Z_\mathrm{c}} \quad \text{and} \quad z_{22} = \frac{(Z_\mathrm{a}+Z_\mathrm{b})(Z_\mathrm{b}+Z_\mathrm{c})}{Z_\mathrm{a}+2Z_\mathrm{b}+Z_\mathrm{c}}$$

Therefore $z_{11} = z_{22}$.

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}; \qquad \text{Use the circuit below:}$$

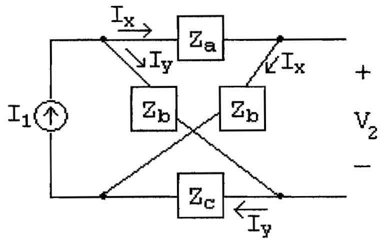


$$V_1 = Z_bI_x - Z_cI_y = Z_bI_x - Z_c(I_2 - I_x) = (Z_b + Z_c)I_x - Z_cI_2$$

$$I_x = \frac{Z_b+Z_c}{Z_a+2Z_b+Z_c}I_2 \qquad \text{so} \qquad V_1 = \frac{(Z_b+Z_c)^2}{Z_a+2Z_b+Z_c}I_2 - Z_cI_2$$

$$\therefore \quad Z_{12} = \frac{V_1}{I_2} = \frac{(Z_b+Z_c)^2}{Z_a+2Z_b+Z_c} - Z_c = \frac{Z_b^2 - Z_aZ_c}{Z_a+2Z_b+Z_c}$$

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0};$$ Use the circuit below:



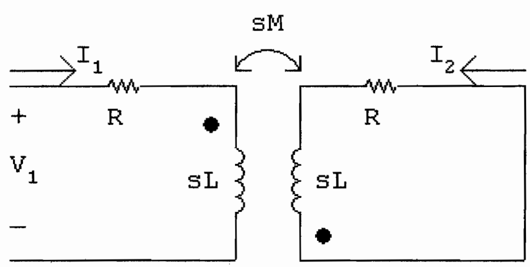
$$V_2 = Z_bI_x - Z_cI_y = Z_bI_x - Z_c(I_1 - I_x) = (Z_b + Z_c)I_x - Z_cI_1$$

$$I_x = \frac{Z_b + Z_c}{Z_a + 2Z_b + Z_c}I_1 \qquad \text{so} \qquad V_2 = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c}I_1 - Z_cI_1$$

$$\therefore \quad z_{21} = \frac{V_2}{I_1} = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} - Z_c = \frac{Z_b^2 - Z_aZ_c}{Z_a + 2Z_b + Z_c} = z_{12}$$

Thus the network is symmetrical and reciprocal.

P 18.23 [a] $h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0}; \qquad h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0}$



$$V_1 = (R + sL)I_1 - sMI_2$$

$$0 = -sMI_1 + (R + sL)I_2$$

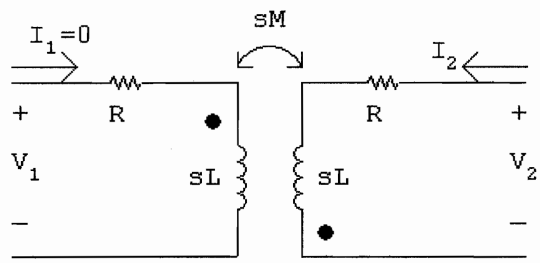
$$\Delta = \begin{vmatrix} (R + sL) & -sM \\ -sM & (R + sL) \end{vmatrix} = (R + sL)^2 - s^2M^2$$

$$N_1 = \begin{vmatrix} V_1 & -sM \\ 0 & (R + sL) \end{vmatrix} = (R + sL)V_1$$

$$I_1 = \frac{N_1}{\Delta} = \frac{(R + sL)V_1}{(R + sL)^2 - s^2M^2}; \qquad h_{11} = \frac{V_1}{I_1} = \frac{(R + sL)^2 - s^2M^2}{R + sL}$$

$$0 = -sMI_1 + (R + sL)I_2; \qquad \therefore\ h_{21} = \frac{I_2}{I_1} = \frac{sM}{R + sL}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$V_1 = -sMI_2; \qquad I_2 = \frac{V_2}{R + sL}$$

$$V_1 = \frac{-sMV_2}{R + sL}; \qquad h_{12} = \frac{V_1}{V_2} = \frac{-sM}{R + sL}$$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{R + sL}$$

[b] $h_{12} = -h_{21}$ (reciprocal)

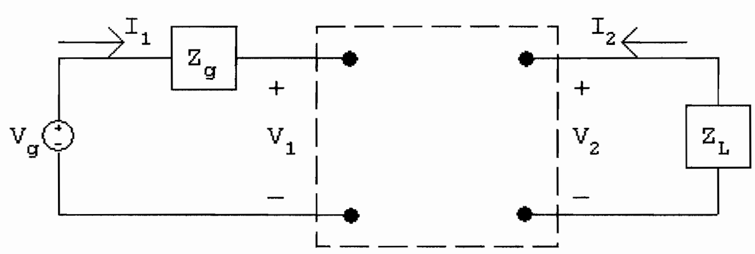
$h_{11}h_{22} - h_{12}h_{21} = 1$ (symmetrical, reciprocal)

$$h_{12} = \frac{-sM}{R + sL}; \qquad h_{21} = \frac{sM}{R + sL} \quad \text{(checks)}$$

$$h_{11}h_{22} - h_{12}h_{21} = \frac{(R + sL)^2 - s^2M^2}{R + sL} \cdot \frac{1}{R + sL} - \frac{(sM)(-sM)}{(R + sL)^2}$$

$$= \frac{(R + sL)^2 - s^2M^2 + s^2M^2}{(R + sL)^2} = 1 \quad \text{(checks)}$$

P 18.24



$$V_2 = b_{11}V_1 - b_{12}I_1; \qquad V_1 = V_g - I_1Z_g$$

$$I_2 = b_{21}V_1 - b_{22}I_1; \qquad V_2 = -Z_LI_2$$

$$I_2 = -\frac{V_2}{Z_L} = \frac{-b_{11}V_1 + b_{12}I_1}{Z_L}$$

$$\frac{-b_{11}V_1 + b_{12}I_1}{Z_L} = b_{21}V_1 - b_{22}I_1$$

$$\therefore \quad V_1\left(\frac{b_{11}}{Z_L} + b_{21}\right) = \left(b_{22} + \frac{b_{12}}{Z_L}\right) I_1$$

$$\frac{V_1}{I_1} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}} = Z_{\text{in}}$$

P 18.25 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_1 = V_g - Z_gI_1$

$$V_2 = g_{21}V_1 + g_{22}I_2; \qquad V_2 = -Z_LI_2$$

$$-Z_LI_2 = g_{21}V_1 + g_{22}I_2; \qquad V_1 = \frac{I_1 - g_{12}I_2}{g_{11}}$$

$$\therefore \quad -Z_LI_2 = \frac{g_{21}}{g_{11}}(I_1 - g_{12}I_2) + g_{22}I_2$$

$$\therefore \quad -Z_LI_2 + \frac{g_{12}g_{21}}{g_{11}}I_2 - g_{22}I_2 = \frac{g_{21}}{g_{11}}I_1$$

$$\therefore \quad (Z_Lg_{11} + \Delta g)I_2 = -g_{21}I_1; \qquad \therefore \quad \frac{I_2}{I_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$$

P 18.26 $I_1 = y_{11}V_1 + y_{12}V_2; \qquad V_1 = V_g - Z_gI_1$

$$I_2 = y_{21}V_1 + y_{22}V_2; \qquad V_2 = -Z_LI_2$$

$$\frac{-V_2}{Z_L} = y_{21}V_1 + y_{22}V_2$$

$$\therefore \quad -y_{21}V_1 = \left(\frac{1}{Z_L} + y_{22}\right)V_2; \qquad -y_{21}Z_LV_1 = (1 + y_{22}Z_L)V_2$$

$$\therefore \quad \frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$$

P 18.27 $V_1 = h_{11}I_1 + h_{12}V_2; \qquad V_1 = V_g - Z_gI_1$

$I_2 = h_{21}I_1 + h_{22}V_2; \qquad V_2 = -Z_LI_2$

$\therefore \quad V_g - Z_gI_1 = h_{11}I_1 + h_{12}V_2; \qquad V_g = (h_{11} + Z_g)I_1 + h_{12}V_2$

$$\therefore \quad I_1 = \frac{V_g - h_{12}V_2}{h_{11} + Z_g}$$

$$\therefore \quad -\frac{V_2}{Z_L} = h_{21}\left[\frac{V_g - h_{12}V_2}{h_{11} + Z_g}\right] + h_{22}V_2$$

$$\frac{-V_2(h_{11} + Z_g)}{Z_L} = h_{21}V_g - h_{12}h_{21}V_2 + h_{22}(h_{11} + Z_g)V_2$$

$$-V_2(h_{11} + Z_g) = h_{21}Z_LV_g - h_{12}h_{21}Z_LV_2 + h_{22}Z_L(h_{11} + Z_g)V_2$$

$$-h_{21}Z_LV_g = (h_{11} + Z_g)\left[V_2 + h_{22}Z_LV_2\right] - h_{12}h_{21}Z_LV_2$$

$$\therefore \quad \frac{V_2}{V_g} = \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$$

P 18.28 $V_1 = z_{11}I_1 + z_{12}I_2; \qquad V_1 = V_g - Z_gI_1$

$V_2 = z_{21}I_1 + z_{22}I_2; \qquad V_2 = -Z_LI_2$

$$V_{\text{Th}} = V_2\Big|_{I_2=0}; \qquad V_2 = z_{21}I_1; \qquad I_1 = \frac{V_1}{z_{11}} = \frac{V_g - I_1Z_g}{z_{11}}$$

$$\therefore \quad I_1 = \frac{V_g}{z_{11} + Z_g}; \qquad \therefore \quad V_2 = \frac{z_{21}V_g}{z_{11} + Z_g} = V_t$$

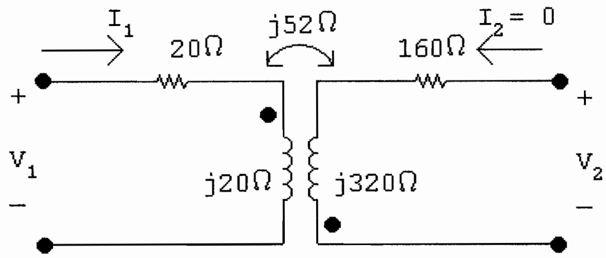
$$Z_{\text{Th}} = \frac{V_2}{I_2}\Big|_{V_g=0}; \qquad V_2 = z_{21}I_1 + z_{22}I_2$$

$$-I_1Z_g = z_{11}I_1 + z_{12}I_2; \qquad I_1 = \frac{-z_{12}I_2}{z_{11} + Z_g}$$

$$\therefore \quad V_2 = z_{21}\left[\frac{-z_{12}I_2}{z_{11} + Z_g}\right] + z_{22}I_2$$

$$\therefore \quad \frac{V_2}{I_2} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g} = Z_{\text{Th}}$$

P 18.29 [a] $a_{11} = \frac{V_1}{V_2}\Big|_{I_2=0}; \qquad a_{21} = \frac{I_1}{V_2}\Big|_{I_2=0}$

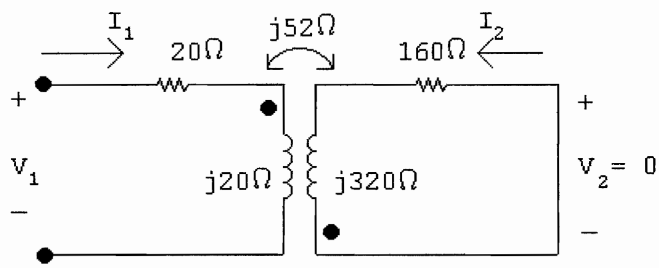


$$V_2 = -j52I_1 = -j52\frac{V_1}{20+j20}$$

$$a_{11} = \frac{V_1}{V_2} = \frac{20+j20}{-j52} = \frac{5}{13}(-1+j)$$

$$a_{21} = \frac{I_1}{V_2} = \frac{1}{-j52} = \frac{j}{52}\,\text{S}$$

$$a_{12} = -\frac{V_1}{I_2}\Big|_{V_2=0}; \qquad a_{22} = -\frac{I_1}{I_2}\Big|_{V_2=0}$$



$$V_1 = (20+j20)I_1 - j52I_2$$

$$0 = -j52I_1 + (160+j320)I_2$$

$$\Delta = \begin{vmatrix} 20+j20 & -j52 \\ -j52 & 160+j320 \end{vmatrix} = -496 + j9600$$

$$N_2 = \begin{vmatrix} 20+j20 & V_1 \\ -j52 & 0 \end{vmatrix} = j52V_1$$

$$I_2 = \frac{j52V_1}{-496+j9600} \quad \text{so} \quad \frac{V_1}{I_2} = \frac{-496+j9600}{j52} = \frac{1}{52}(9600+j496)$$

$$\therefore \quad a_{12} = -\frac{V_1}{I_2} = \frac{1}{13}(-2400 - j124)$$

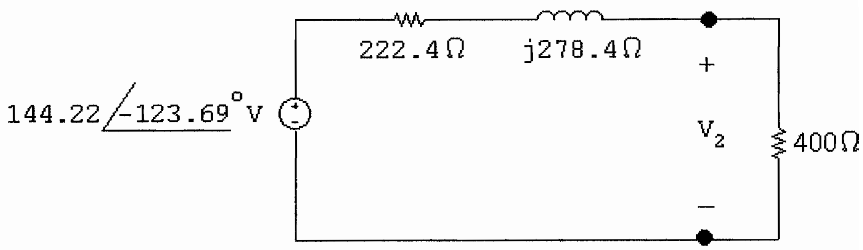
$$j52I_1 = (160 + j320)I_2; \qquad \therefore \quad a_{22} = -\frac{I_1}{I_2} = \frac{-320 + j160}{52}$$

[b] $$V_{\rm Th} = \frac{V_g}{a_{11} + a_{21}Z_g} = \frac{100\underline{/0^\circ}}{(5/13)(-1+j) + (j/52)(10)}$$

$$= -80 - j120 = 144.22\underline{/-123.69^\circ}\,\text{V}$$

$$Z_{\rm Th} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{\frac{1}{13}(-2400 - j124) + \frac{-320+j160}{52}(10)}{(5/13)(-1+j) + (j/52)(10)}$$

$$= 222.4 + j278.4 = 356.33\underline{/51.38^\circ}\,\Omega$$



[c] $$V_2 = \frac{144.22\underline{/-123.69^\circ}}{622.4 + j278.4}(400) = 84.607\underline{/-147.789^\circ}$$

$$v_2(t) = 84.607\cos(2000t - 147.789^\circ)\,\text{V}$$

P 18.30 $$\mathbf{I}_2 = \frac{y_{21}\mathbf{V}_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$$

$$= \frac{-0.25(1)}{1 + (-0.04)(100) + (0.025)(10) + (-0.00125)(10)(100)}$$

$$= 0.0625\ \text{A(rms)}$$

$$P_o = (I_2)^2 Z_L = (0.0625)^2(100) = 390.625\,\text{mW}$$

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L} = \frac{-0.25}{0.025 + (-0.00125)(100)} = 2.5$$

$$\therefore \quad I_1 = \frac{I_2}{2.5} = \frac{0.0625}{2.5} = 25\ \text{mA(rms)}$$

$$P_g = (1)(0.025) = 25\,\text{mW}$$

$$\frac{P_o}{P_g} = \frac{390.625}{25} = 15.625$$

P 18.31 [a] $Z_{\rm Th} = g_{22} - \dfrac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$

$$g_{12}g_{21} = \left(-\frac{1}{2} + j\frac{1}{2}\right)\left(\frac{1}{2} - j\frac{1}{2}\right) = j\frac{1}{2}$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$\therefore \quad Z_{\rm Th} = 1.5 + j2.5 - \frac{j3}{2 - j1} = 2.1 + j1.3\,\Omega$$

$$\therefore \quad Z_L = 2.1 - j1.3\,\Omega$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$g_{21}Z_L = \left(\frac{1}{2} - j\frac{1}{2}\right)(2.1 - j1.3) = 0.4 - j1.7$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$g_{22} + Z_L = 1.5 + j2.5 + 2.1 - j1.3 = 3.6 + j1.2$$

$$g_{12}g_{21}Z_g = j3$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{0.4 - j1.7}{(2 - j1)(3.6 + j1.2) - j3} = \frac{0.4 - j1.7}{8.4 - j4.2}$$

$$\mathbf{V}_2 = \frac{0.4 - j1.7}{8.4 - j4.2}(42\underline{/0^\circ}) = 5 - j6\,\text{V(rms)} = 7.81\underline{/-50.19^\circ}\,\text{V(rms)}$$

The rms value of $\mathbf{V}_2$ is 7.81 V.

[b] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{Z_L} = \dfrac{-5 + j6}{2.1 - j1.3} = -3 + j1\,\text{A(rms)}$

$$P = |\mathbf{I}_2|^2(2.1) = 21\,\text{W}$$

[c] $\dfrac{\mathbf{I}_2}{\mathbf{I}_1} = \dfrac{-g_{21}}{g_{11}Z_L + \Delta g}$

$$\Delta g = \left(\frac{1}{6} - j\frac{1}{6}\right)\left(\frac{3}{2} + j\frac{5}{2}\right) - \left(\frac{1}{2} - j\frac{1}{2}\right)\left(-\frac{1}{2} + j\frac{1}{2}\right)$$

$$= \frac{3}{12} + j\frac{5}{12} - j\frac{3}{12} + \frac{5}{12} - j\frac{1}{2} = \frac{2}{3} - j\frac{1}{3}$$

$$g_{11}Z_L = \left(\frac{1}{6} - j\frac{1}{6}\right)(2.1 - j1.3) = \frac{0.8}{6} - j\frac{3.4}{6}$$

$$\therefore \quad g_{11}Z_L + \Delta g = \frac{0.8}{6} - j\frac{3.4}{6} + \frac{4}{6} - j\frac{2}{6} = 0.8 - j0.9$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-[(1/2) - j(1/2)]}{0.8 - j0.9}$$

$$\therefore \quad \mathbf{I}_1 = \frac{(0.8 - j0.9)\mathbf{I}_2}{-0.5 + j0.5} = \left(\frac{1.6 - j1.8}{-1 + j1}\right)\mathbf{I}_2$$

$$= (-1.7 + j0.1)(-3 + j1) = 5 - j2\,\text{A(rms)}$$

$$\therefore \quad P_g(\text{developed}) = (42)(5) = 210\,\text{W}$$

$$\%\ \text{delivered} = \frac{21}{210}(100) = 10\%$$

P 18.32 [a] $$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(70{,}000) = -35$$

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(70 \times 10^3) = -2.5$$

$$y_{21}Z_L = (100 \times 10^{-3})(70 \times 10^3) = 7000$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{7000}{-35 - (6)(-2.5)} = \frac{7000}{-20} = -350$$

$$\mathbf{V}_2 = -350\mathbf{V}_g = -350(80) \times 10^{-3} = -28\,\text{V(rms)}$$

$$\mathbf{V}_2 = 28\underline{/180^\circ}\,\text{V(rms)}$$

[b] $$P = \frac{|\mathbf{V}_2|^2}{70{,}000} = 11.2 \times 10^{-3} = 11.20\,\text{mW}$$

[c] $$\mathbf{I}_2 = \frac{-28\underline{/180^\circ}}{70{,}000} = -0.4 \times 10^{-3}\underline{/180^\circ} = 400\underline{/0^\circ}\,\mu\text{A}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

$$\Delta y = (2 \times 10^{-3})(-50 \times 10^{-6}) - (-2 \times 10^{-6})(100 \times 10^{-3})$$

$$= 100 \times 10^{-9}$$

$$\Delta y Z_L = (100)(70) \times 10^3 \times 10^{-9} = 7 \times 10^{-3}$$

$$y_{11} + \Delta y Z_L = 2 \times 10^{-3} + 7 \times 10^{-3} = 9 \times 10^{-3}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{9 \times 10^{-3}} = \frac{100}{9}$$

$$\therefore \quad 100\mathbf{I}_1 = 9\mathbf{I}_2; \qquad \mathbf{I}_1 = \frac{9(400 \times 10^{-6})}{100} = 36\,\mu\text{A(rms)}$$

$$P_g = (80)10^{-3}(36) \times 10^{-6} = 2.88\,\mu\text{W}$$

P 18.33 [a] $Z_{\text{Th}} = \frac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$

From the solution to Problem 18.32

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$y_{22} + \Delta y Z_g = -50 \times 10^{-6} + 10^{-7}(2500) = 200 \times 10^{-6}$$

$$Z_{\text{Th}} = \frac{6}{200} \times 10^6 = 30{,}000\,\Omega$$

$$Z_L = Z_{\text{Th}}^* = 30{,}000\,\Omega$$

[b] $y_{21}Z_L = (100 \times 10^{-3})(30{,}000) = 3000$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(30{,}000) = -15$$

$$1 + y_{11}Z_g = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(30 \times 10^3) = -0.5$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{3000}{-15 - 6(-0.5)} = \frac{3000}{-12} = -250$$

$$\mathbf{V}_2 = -250(80 \times 10^{-3}) = -20 = 20\underline{/180^\circ}\,\text{V(rms)}$$

$$P = \frac{|\mathbf{V}_2|^2}{30{,}000} = \frac{400}{30} \times 10^{-3} = 13.33\,\text{mW}$$

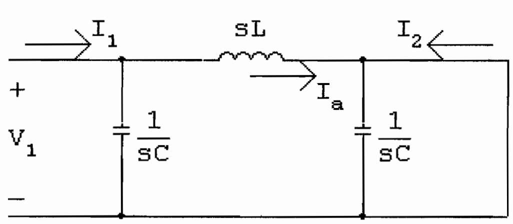
[c] $\mathbf{I}_2 = \frac{-\mathbf{V}_2}{30{,}000} = \frac{20\underline{/0^\circ}}{30{,}000} = \frac{2}{3}\,\text{mA}$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{2 \times 10^{-3} + 10^{-7}(30{,}000)} = \frac{100 \times 10^{-3}}{5 \times 10^{-3}} = 20$$

$$\mathbf{I}_1 = \frac{\mathbf{I}_2}{20} = \frac{2 \times 10^{-3}}{3(20)} = \frac{1}{30}\,\text{mA(rms)}$$

$$P_g(\text{developed}) = (80 \times 10^{-3})\left(\frac{1}{30} \times 10^{-3}\right) = \frac{8}{3}\,\mu\text{W}$$

P 18.34 [a] $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$



$$h_{11} = \frac{(1/sC)(sL)}{(1/sC) + sL} = \frac{(1/C)s}{s^2 + (1/LC)}$$

$$I_2 = -I_a; \qquad I_a = \frac{I_1(1/sC)}{sL + (1/sC)}$$

$$I_2 = \frac{-I_1}{s^2LC + 1}$$

$$h_{21} = \frac{I_2}{I_1} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$V_1 = \frac{V_2(1/sC)}{sL + (1/sC)} = \frac{V_2}{s^2LC + 1}$$

$$\frac{V_1}{V_2} = h_{12} = \frac{1/LC}{s^2 + (1/LC)}$$

$$\frac{V_2}{I_2} = \frac{(1/sC)[sL + (1/sC)]}{sL + (2/sC)} = \frac{s^2 + (1/LC)}{sC[s^2 + (2/LC)]}$$

$$\frac{I_2}{V_2} = h_{22} = \frac{Cs[s^2 + (2/LC)]}{s^2 + (1/LC)}$$

[b] $$\frac{1}{LC} = \frac{10^9}{(0.1)(400)} = 25 \times 10^6$$

$$h_{11} = \frac{10^7 s}{s^2 + 25 \times 10^6}$$

$$h_{12} = \frac{25 \times 10^6}{s^2 + 25 \times 10^6}$$

$$h_{21} = \frac{-25 \times 10^6}{s^2 + 25 \times 10^6}$$

$$h_{22} = \frac{10^{-7}s(s^2 + 50 \times 10^6)}{(s^2 + 25 \times 10^6)}$$

$$\frac{V_2}{V_1} = \frac{-h_{21}Z_L}{h_{11} + \Delta h Z_L} = \frac{-h_{21}Z_L}{h_{11} + Z_L} = \frac{\left(\frac{25\times10^6}{s^2+25\times10^6}\right)800}{\frac{10^7 s}{(s^2+25\times10^6)} + 800}$$

$$\frac{V_2}{V_1} = \frac{25 \times 10^6}{s^2 + 12{,}500s + 25 \times 10^6} = \frac{25 \times 10^6}{(s + 2500)(s + 10{,}000)}$$

$$V_1 = \frac{45}{s}$$

$$V_2 = \frac{1125 \times 10^6}{s(s + 2500)(s + 10{,}000)} = \frac{45}{s} - \frac{60}{s + 2500} + \frac{15}{s + 10{,}000}$$

$$v_2 = [45 - 60e^{-2500t} + 15e^{-10{,}000t}]u(t) \quad \text{V}$$

P 18.35 [a] $V_1 = z_{11}I_1 + z_{12}I_2$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = s + \frac{1}{s} = \frac{s^2+1}{s}$$

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = \frac{1}{s}$$

$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = \frac{1}{s}$$

$$z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} = s + \frac{1}{s} = \frac{s^2+1}{s}$$

[b] $$\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11}+Z_g)(z_{22}+Z_L) - z_{12}z_{21}}$$

$$= \frac{z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}}$$

$$= \frac{1/s}{\left(\frac{s^2+1}{s}+1\right)\left(\frac{s^2+1}{s}+1\right) - \frac{1}{s^2}}$$

$$= \frac{s}{(s^2+s+1)^2 - 1}$$

$$= \frac{s}{s^4+2s^3+3s^2+2s+1-1}$$

$$= \frac{1}{s^3+2s^2+3s+2}$$

$$= \frac{1}{(s+1)(s^2+s+2)}$$

$$\therefore \quad V_2 = \frac{50}{s(s+1)(s^2+s+2)}$$

$$s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

$$V_2 = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{K_3^*}{s+\frac{1}{2}+j\frac{\sqrt{7}}{2}}$$

$$K_1 = 25; \qquad K_2 = -25; \qquad K_3 = 9.45\underline{/90^\circ}$$

$$\therefore \quad v_2(t) = [25 - 25e^{-t} + 18.90e^{-0.5t}\cos(1.32t + 90^\circ)]u(t)\,\text{V}$$

CHECK

$$v_2(0) = 25 - 25 + 18.90\cos 90^\circ = 0$$

$$v_2(\infty) = 25 + 0 + 0 = 25\,\text{V}$$

P 18.36 $z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{100}{1.125} = \frac{800}{9}\,\Omega$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{104}{1.125} = \frac{832}{9}\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{20}{0.25} = 80\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{24}{0.25} = 96\,\Omega$$

$$Z_{\text{Th}} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g} = 96 - \frac{(80)(832/9)}{(800/9) + 0} = 12.8\,\Omega$$

$$\therefore \quad Z_L = 12.8\,\Omega$$

$$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$$

$$\Delta z = \left(\frac{800}{9}\right)96 - 80\left(\frac{832}{9}\right) = \frac{10{,}240}{9}$$

$$\frac{V_2}{V_1} = \frac{(832/9)(12.8)}{(800/9)(12.8) + (10{,}240/9)} = \frac{10{,}649.60}{20{,}480} = 0.52$$

$$V_2 = (0.52)(160) = 83.20\,\text{V}$$

$$P = \frac{(83.2)^2}{12.8} = 540.80\,\text{W}$$

P 18.37 $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 25\,\Omega; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -0.5$

From the second set of measurements we have

$$41 = 25(1) + h_{12}(20); \qquad \therefore \quad h_{12} = \frac{41 - 25}{20} = 0.80$$

$$0 = -0.5(1) + h_{22}(20); \qquad \therefore \quad h_{22} = \frac{0.5}{20} = 0.025\,\mho$$

$$R_{\text{Th}} = \frac{23 + 25}{23(0.025) + \Delta h}; \qquad \Delta h = 25(0.025) - (-0.5)(0.8) = 1.025$$

$$\therefore \quad R_{\text{Th}} = \frac{48}{1.6} = 30\,\Omega; \qquad \therefore \quad R_o = 30\,\Omega$$

$$\frac{V_2}{V_g} = \frac{-(-0.5)(30)}{(48)(1.75) + (0.4)(30)} = \frac{15}{96}$$

$$V_2 = 15\,\text{V}; \qquad P = \frac{(15)^2}{30} = 7.5\,\text{W}$$

P 18.38 $$a'_{11} = -\frac{\Delta h}{h_{21}} = \frac{-0.01}{-0.1} = 0.1$$

$$a'_{12} = -\frac{h_{11}}{h_{21}} = \frac{-150}{-0.1} = 1500$$

$$a'_{21} = -\frac{h_{22}}{h_{21}} = \frac{-10^{-4}}{-0.1} = 10^{-3}$$

$$a'_{22} = -\frac{1}{h_{21}} = \frac{-1}{-0.1} = 10$$

$$a''_{11} = \frac{1}{g_{21}} = \frac{1}{20} = 0.05$$

$$a''_{12} = \frac{g_{22}}{g_{21}} = \frac{24 \times 10^3}{20} = 1200$$

$$a''_{21} = \frac{g_{11}}{g_{21}} = \frac{0.01}{20} = 5 \times 10^{-4}$$

$$a''_{22} = \frac{\Delta g}{g_{21}} = \frac{320}{20} = 16$$

$$a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = (0.1)(0.05) + (1500)(5 \times 10^{-4}) = 0.755$$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (0.1)(1200) + (1500)(16) = 24{,}120$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (10^{-3})(0.05) + (10)(5 \times 10^{-4}) = 5.05 \times 10^{-3}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (10^{-3})(1200) + (10)(16) = 161.2$$

$$V_2 = \frac{Z_L V_g}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$= \frac{(1000)(109.5)}{[0.755 + (5.05 \times 10^{-3})(20)](1000) + 24{,}120 + (161.2)(20)} = 3.88\,\text{V}$$

P 18.39 The a parameters of the first two port are

$$a'_{11} = \frac{z_{11}}{z_{21}} = \frac{200}{-1.6 \times 10^6} = -125 \times 10^{-6}$$

$$a'_{12} = \frac{\Delta z}{z_{21}} = \frac{40 \times 10^6}{-1.6 \times 10^6} = -25\,\Omega$$

$$a'_{21} = \frac{1}{z_{21}} = \frac{1}{-1.6 \times 10^6} = -625 \times 10^{-9}\,\text{S}$$

$$a'_{22} = \frac{z_{22}}{z_{21}} = \frac{40{,}000}{-1.6 \times 10^6} = -25 \times 10^{-3}$$

The a parameters of the second two port are

$$a''_{11} = \frac{5}{4}; \quad a''_{12} = \frac{3R}{4}; \quad a''_{21} = \frac{3}{4R}; \quad a''_{22} = \frac{5}{4}$$

or $a''_{11} = 1.25; \quad a''_{12} = 6\,\text{k}\Omega; \quad a''_{21} = 93.75\,\mu\text{S}; \quad a''_{22} = 1.25$

The a parameters of the cascade connection are

$$a_{11} = -125 \times 10^{-6}(1.25) + (-25)(93.75 \times 10^{-6}) = -2.5 \times 10^{-3}$$

$$a_{12} = -125 \times 10^{-6}(6000) + (-25)(1.25) = -32\,\Omega$$

$$a_{21} = -625 \times 10^{-9}(1.25) + (-25 \times 10^{-3})(93.75 \times 10^{-6}) = -3.125 \times 10^{-6}\,\text{S}$$

$$a_{22} = -625 \times 10^{-9}(6000) + (-25 \times 10^{-3})(1.25) = -35 \times 10^{-3}$$

$$\frac{V_o}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$a_{21}Z_g = (-3.125 \times 10^{-6})(500) = -1.5625 \times 10^{-3}$$

$$a_{11} + a_{21}Z_g = -2.5 \times 10^{-3} - 1.5625 \times 10^{-3} = -4.0625 \times 10^{-3}$$

$$(a_{11} + a_{21}Z_g)Z_L = (-4.0625 \times 10^{-3})(8000) = -32.5$$

$$a_{22}Z_g = (-35 \times 10^{-3})(500) = -17.5$$

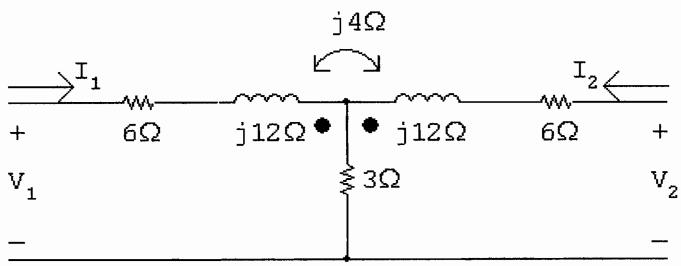
$$\frac{V_o}{V_g} = \frac{8000}{-32.5 - 32.25 - 17.5} = -97.26$$

$$v_o = V_o = -97.26V_g = -1.46\,\text{V}$$

P 18.40 [a] From reciprocity and symmetry

$$a'_{11} = a'_{22}, \quad \Delta a' = 1; \qquad \therefore\ 16 - 5a'_{21} = 1, \quad a'_{21} = 3\,\text{S}$$

For network B



$$a''_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{I_2=0}$$

$$\mathbf{V}_1 = (6 + j12 + 3)\mathbf{I}_1 = (9 + j12)\mathbf{I}_1$$

$$\mathbf{V}_2 = 3\mathbf{I}_1 + j4\mathbf{I}_1 = (3 + j4)\mathbf{I}_1$$

$$a''_{11} = \frac{9 + j12}{3 + j4} = 3$$

$$a''_{21} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{I_2=0} = \frac{1}{3 + j4} = 0.12 - j0.16\,\text{S}$$

$$a''_{22} = a''_{11} = 3$$

$$\Delta a'' = 1 = (3)(3) - (0.12 - j0.16)a''_{12}$$

$$\therefore\ a''_{12} = \frac{8}{0.12 - j0.16} = 24 + j32\,\Omega$$

[b] $a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = 12 + 5(0.12 - j0.16) = 12.6 - j0.8$

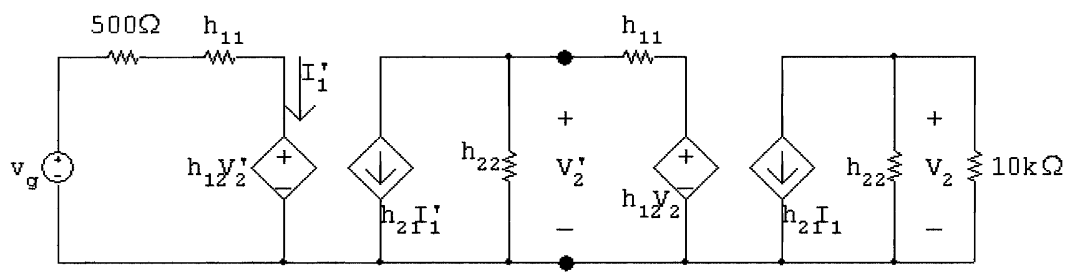
$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (4)(24 + j32) + (5)(3) = 111 + j128\,\Omega$

$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (3)(3) + (4)(0.12 - j0.16) = 9.48 - j0.64\,\text{S}$

$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (3)(24 + j32) + (4)(3) = 84 + j96$

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{1}{a_{11}} = \frac{1}{12.6 - j0.8} = 0.079 + j0.005$$

P 18.41 **[a]** At the input port: $V_1 = h_{11}I_1 + h_{12}V_2$;
At the output port: $I_2 = h_{21}I_1 + h_{22}V_2$



[b] $\frac{V_2}{10^4} + (100 \times 10^{-6}V_2) + 100I_1 = 0$

therefore $I_1 = -2 \times 10^{-6}V_2$

$V'_2 = 1000I_1 + 15 \times 10^{-4}V_2 = -5 \times 10^{-4}V_2$

$100I'_1 + 10^{-4}V'_2 + (-2 \times 10^{-6})V_2 = 0$

therefore $I'_1 = 205 \times 10^{-10}V_2$

$V_g = 1500I'_1 + 15 \times 10^{-4}V'_2 = 3000 \times 10^{-8}V_2$

$$\frac{V_2}{V_g} = \frac{10^5}{3} = 33{,}333$$

P 18.42 **[a]** $V_1 = I_2(z_{12} - z_{21}) + I_1(z_{11} - z_{21}) + z_{21}(I_1 + I_2)$

$= I_2z_{12} - I_2z_{21} + I_1z_{11} - I_1z_{21} + z_{21}I_1 + z_{21}I_2 = z_{11}I_1 + z_{12}I_2$

$V_2 = I_2(z_{22} - z_{21}) + z_{21}(I_1 + I_2) = z_{21}I_1 + z_{22}I_2$

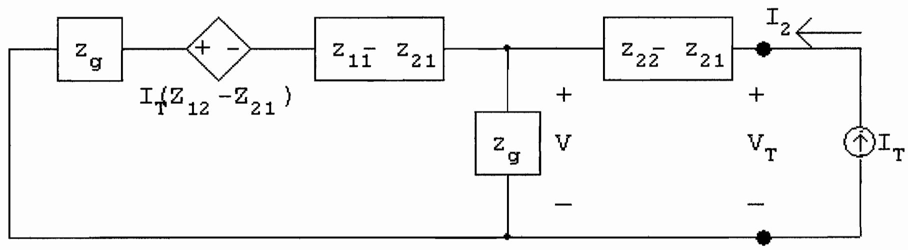
[b] Short circuit V_g and apply a test current source to port 2 as shown. Note that $I_T = I_2$. We have

$$\frac{V}{z_{21}} - I_T + \frac{V + I_T(z_{12} - z_{21})}{Z_g + z_{11} - z_{21}} = 0$$

Therefore

$$V = \left[\frac{z_{21}(Z_g + z_{11} - z_{12})}{Z_g + z_{11}}\right] I_T \quad \text{and} \quad V_T = V + I_T(z_{22} - z_{21})$$

$$\text{Thus} \quad \frac{V_T}{I_T} = Z_{\text{Th}} = z_{22} - \left(\frac{z_{12}z_{21}}{Z_g + z_{11}}\right)\,\Omega$$



For V_{Th} note that $V_{\text{oc}} = \dfrac{z_{21}}{z_g + z_{11}} V_g$ since $I_2 = 0$.

P 18.43 [a] $V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2) = z_{11}I_1 + z_{12}I_2$

$V_2 = (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_2 + I_1) = z_{21}I_1 + z_{22}I_2$

[b] With port 2 terminated in an impedance Z_L, the two mesh equations are

$V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2)$

$0 = Z_L I_2 + (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_1 + I_2)$

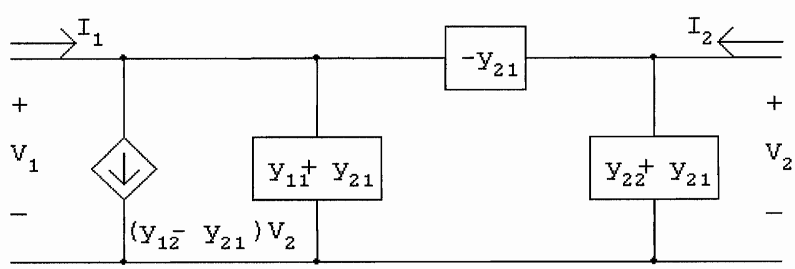
Solving for I_1:

$$I_1 = \frac{V_1(z_{22} + Z_L)}{z_{11}(Z_L + z_{22}) - z_{12}z_{21}}$$

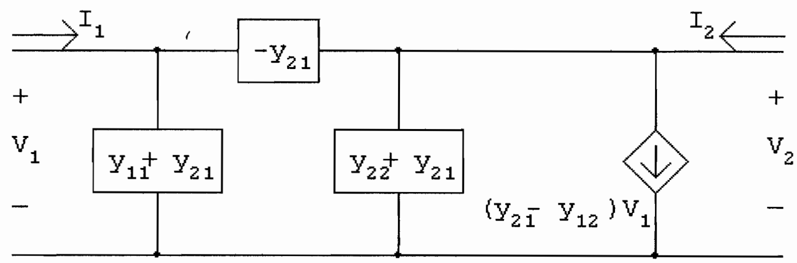
Therefore

$$Z_{\text{in}} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

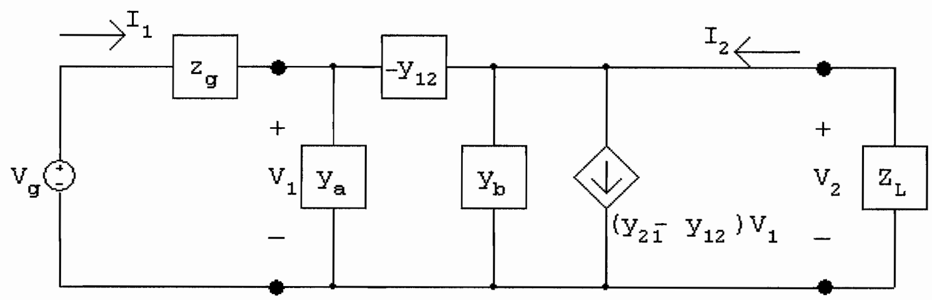
P 18.44 [a] $I_1 = y_{11}V_1 + y_{21}V_2 + (y_{12} - y_{21})V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$



$I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{12}V_1 + y_{22}V_2 + (y_{21} - y_{12})V_1$



[b] Using the second circuit derived in part [a], we have



where $y_a = (y_{11} + y_{12})$ and $y_b = (y_{22} + y_{12})$

At the input port we have

$$I_1 = y_a V_1 - y_{12}(V_1 - V_2) = y_{11}V_1 + y_{12}V_2$$

At the output port we have

$$\frac{V_2}{Z_L} + (y_{21} - y_{12})V_1 + y_b V_2 - y_{12}(V_2 - V_1) = 0$$

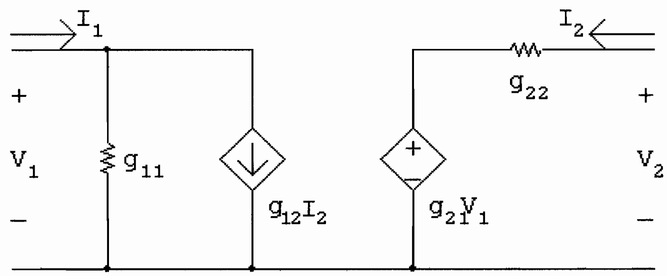
Solving for V_1 gives

$$V_1 = \left(\frac{1 + y_{22}Z_L}{-y_{21}Z_L}\right) V_2$$

Substituting Eq. (18.2) into (18.1) and at the same time using $V_2 = -Z_L I_2$, we get

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

P 18.45 **[a]** The g-parameter equations are $I_1 = g_{11}V_1 + g_{12}I_2$ and $V_2 = g_{21}V_1 + g_{22}I_2$. These equations are satisfied by the following circuit:



[b] The g parameters for the first two port in Fig P 18.39(a) are

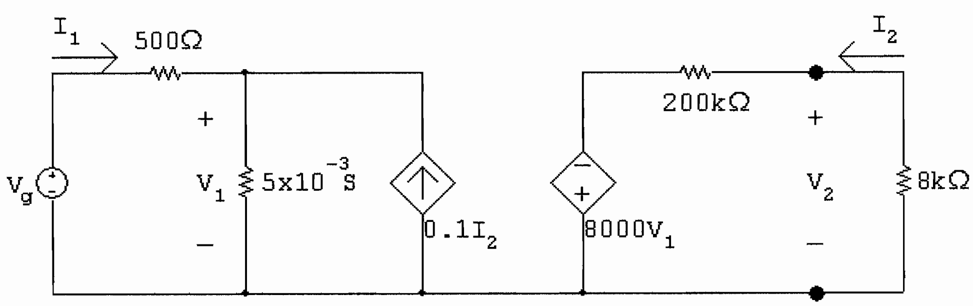
$$g_{11} = \frac{1}{z_{11}} = \frac{1}{200} = 5 \times 10^{-3}\,\text{S}$$

$$g_{12} = \frac{-z_{12}}{z_{11}} = \frac{-20}{200} = -0.10$$

$$g_{21} = \frac{z_{21}}{z_{11}} = \frac{-1.6 \times 10^6}{200} = -8000$$

$$g_{22} = \frac{\Delta z}{z_{11}} = \frac{40 \times 10^6}{200} = 200\,\text{k}\Omega$$

From Problem 3.64, since the load resistor and all resistors in the attenuator pad of the second two-port are equal to 8 kΩ, $R_{cd} = 8\,\text{k}\Omega$, hence our circuit reduces to



$$V_2 = \frac{8000}{8000 + 200{,}000}(-8000V_1)$$

$$I_2 = \frac{-V_2}{8000} = \frac{8000}{208{,}000}V_1 = \frac{8}{208}V_1$$

$$v_g = 15\,\text{mV}$$

$$\frac{V_1 - 15 \times 10^{-3}}{500} + V_1(5 \times 10^{-3}) - 0.1\frac{8V_1}{208} = 0$$

$$V_1\left(\frac{1}{500}+5\times10^{-3}-\frac{0.8}{208}\right)=\frac{15\times10^{-3}}{500}$$

$$\therefore\quad V_1=9.512\times10^{-3}$$

$$V_2=\frac{-(8000)^2}{208{,}000}(9.512\times10^{-3})=-2.927\,\text{V}$$

Again, from the results of analyzing the attenuator pad in Problem 3.64

$$\frac{V_o}{V_2}=0.5;\qquad \therefore\quad V_o=(0.5)(-2.927)=-1.46\,\text{V}$$

This result matches the solution to Problem 18.38.