EIE209 Basic Electronics

Basic circuit analysis

Fundamental quantities

Voltage — potential difference bet. 2 points

"across" quantity

analogous to 'pressure' between two points

Current — flow of charge through a material

"through" quantity

analogous to fluid flowing along a pipe

$$I = \lim_{\delta t \to 0} \frac{\delta q}{\delta t} = \frac{dq}{dt}$$

Units of measurement

- Voltage: volt (V)
- Current: ampere (A)

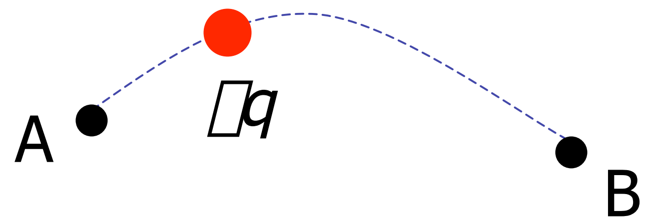
- NOT Volt, Ampere!!

Prefix	Multiplier (abbreviation)
Peta	$\times 10^{15}$ (P)
Tera	$\times 10^{12}$ (T)
Giga	$\times 10^{9}$ (G)
Mega	$\times 10^{6}$ (M)
Kilo	$\times 10^{3}$ (k)
Hecto	$\times 10^{2}$ (h)
Deca	$\times 10$ (da)
deci	$\times 10^{-1}$ (d)
centi	$\times 10^{-2}$ (c)
milli	$\times 10^{-3}$ (m)
micro	$\times 10^{-6}$ (μ)
nano	$\times 10^{-9}$ (n)
pico	$\times 10^{-12}$ (p)
femto	$\times 10^{-15}$ (f)

Power and energy

Work done in moving a charge Δq from A to B having a potential difference of V is

$$W = V \Delta q$$



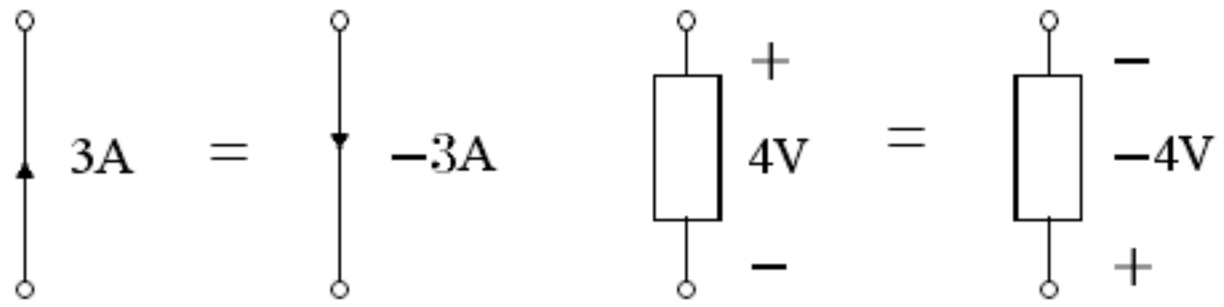
Power is work done per unit time, i.e.,

$$P = \lim_{\delta t \to 0} V \frac{\delta q}{\delta t} = V \frac{dq}{dt} = VI$$

Direction and polarity

- Current direction indicates the direction of flow of positive charge
- Voltage polarity indicates the relative potential between 2 points: **+** assigned to a higher potential point; and **–** assigned to a lower potential point.

- *NOTE: Direction and polarity are arbitrarily assigned on circuit diagrams. Actual direction and polarity will be governed by the sign of the value.*

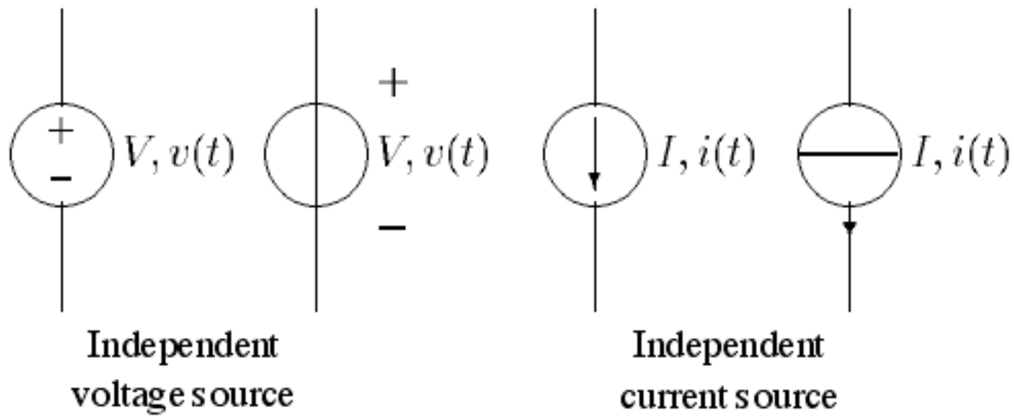


Independent sources

- Voltage sources
- Current sources

Independent — stubborn! never change!

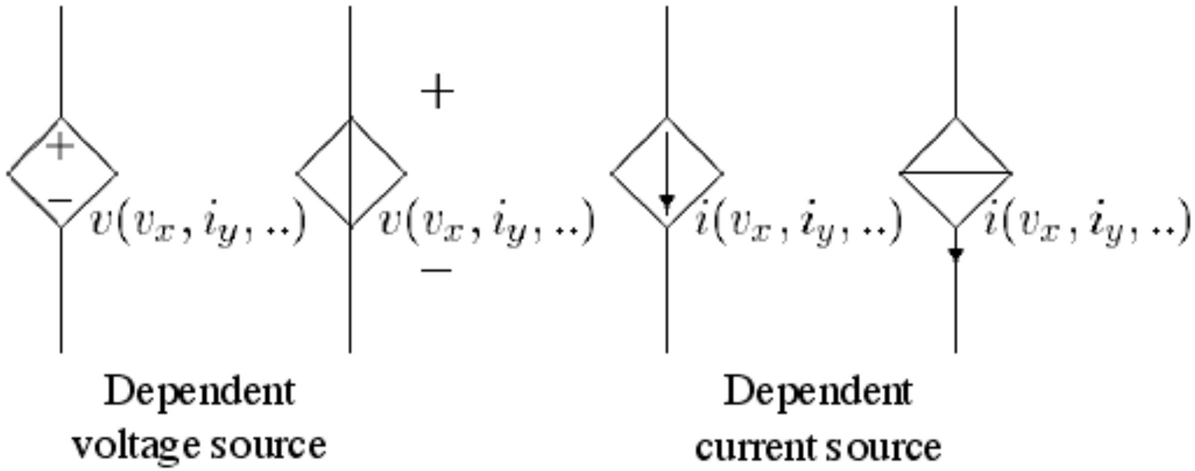
Maintains a voltage/current (fixed or varying) which is not affected by any other quantities.



An independent voltage source can never be shorted.
An independent current source can never be opened.

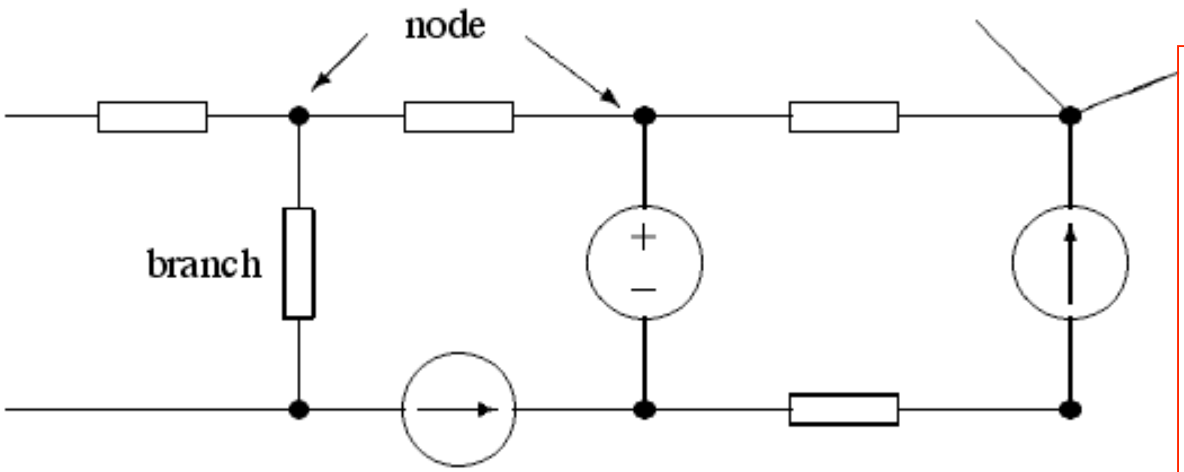
Dependent sources

- Dependent sources — values depend on some other variables



Circuit

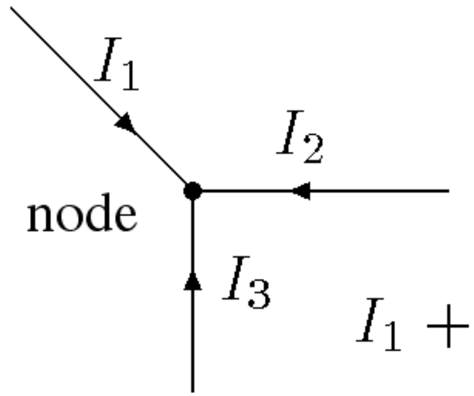
- Collection of devices such as sources and resistors in which terminals are connected together by conducting wires.
  - These wires converge in NODES
  - The devices are called BRANCHES of the circuit



<u>Circuit Analysis Problem:</u>
To find all currents and voltages in the branches of the circuit when the intensities of the sources are known.

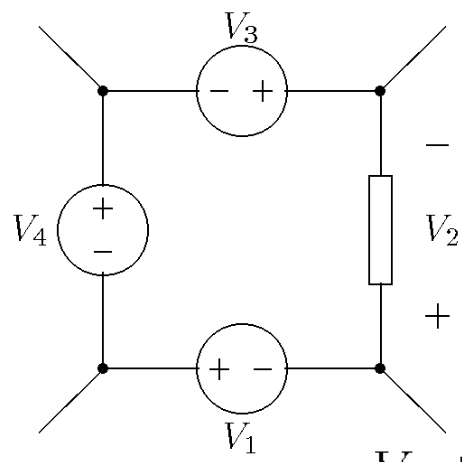
Kirchhoff's laws

- Kirchhoff's current law (KCL)
  - The algebraic sum of the currents in all branches which converge to a common node is equal to zero.



$$I_1 + I_2 + I_3 = 0$$

- Kirchhoff's voltage law (KVL)
  - The algebraic sum of all voltages between successive nodes in a closed path in the circuit is equal to zero.



$$V_1 + V_2 + V_3 + V_4 = 0$$

Overview of analysis

- Ad hoc methods (not general)
  - Series/parallel reduction
  - Ladder circuit
  - Voltage/current division
  - Star-delta conversion
- More general
  - Mesh and nodal methods

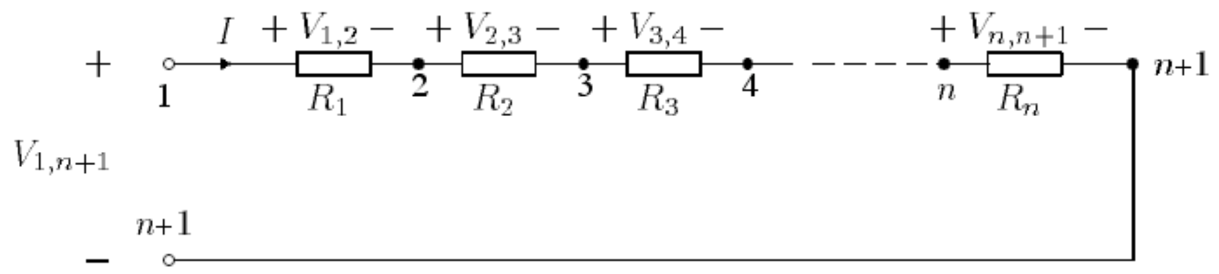
} **Done in Basic Electronics!**

- Completely general
  - Loop and cutset approach (requires graph theory)

NEW

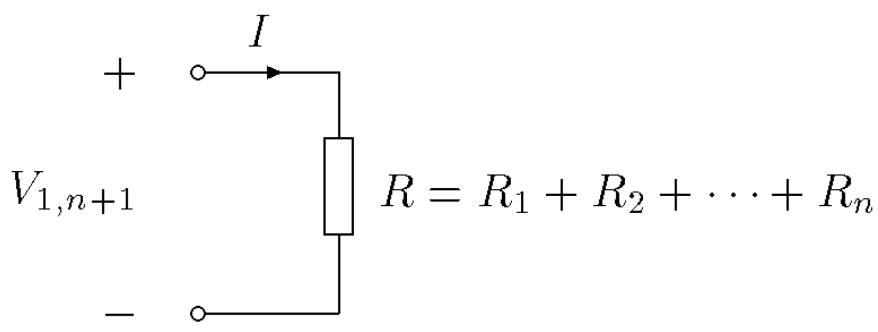
Series/parallel reduction

- Series circuit— each node is incident to just two branches of the circuit



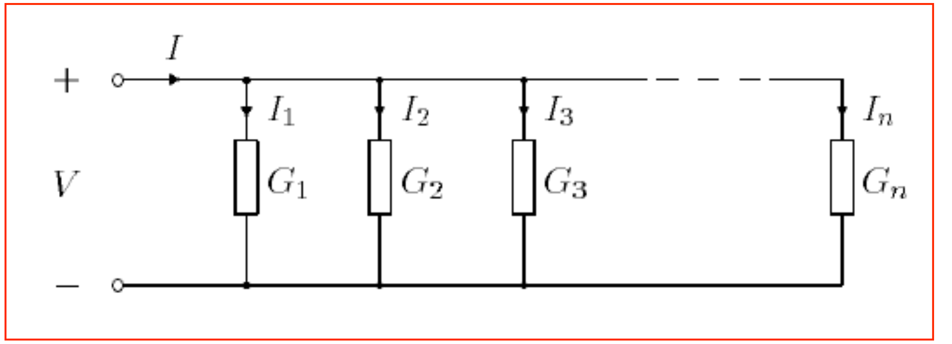
KVL gives
$$V_{1,n+1} = V_{12} + V_{23} + \cdots + V_{n,n+1}$$
$$= (R_1 + R_2 + \cdots + R_n)I$$

Hence, the equivalent resistance is:



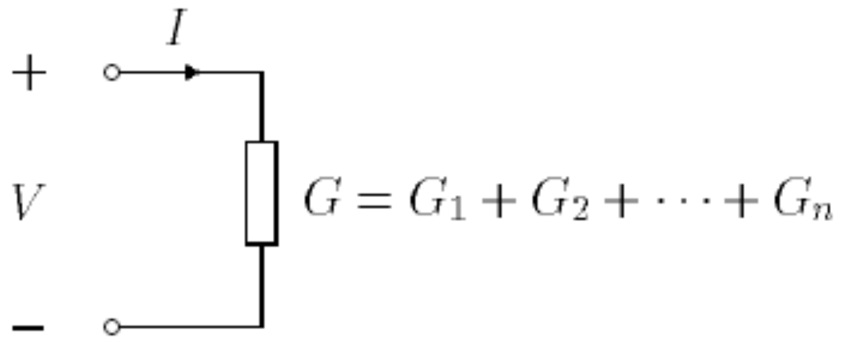
Series/parallel reduction

- Parallel circuit— one terminal of each element is connected to a node of the circuit while other terminals of the elements are connected to another node of the circuit



KCL gives $I = (G_1 + G_2 + \cdots + G_n)V$

Hence, the equivalent resistance is:



Note on algebra

- For algebraic brevity and simplicity:
  - For series circuits, R is preferably used.
  - For parallel circuits, G is preferably used.

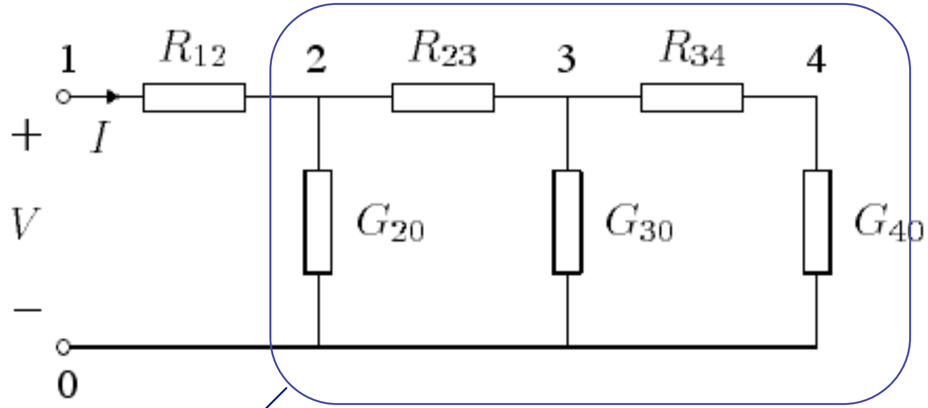
For example, if we use R for the parallel circuit, we get the equivalent resistance as

$$R = \frac{R_1 R_2 R_3 \cdots R_n}{R_2 R_3 \cdots R_n + R_1 R_3 R_4 \cdots R_n + \cdots + R_1 R_2 \cdots R_{n-1}}$$

which is more complex than the formula in terms of G:

$$G = G_1 + G_2 + \ldots + G_n$$

Ladder circuit



- We can find the resistance looking into the terminals 0 and 1, by apply the series/ parallel reduction successively.

First, lumping everything beyond node 2 as G_2, we have $\frac{V}{I} = R_{12} + \frac{1}{G_2}$

Then, we focus on this G_2, which is just G_{20} in parallel with another subcircuit, i.e.,

$$G_2 = G_{20} + \frac{1}{R_2'}$$

We continue to focus on the remaining subcircuit. Eventually we get

$$\frac{V}{I} = R_{12} + \cfrac{1}{G_{20} + \cfrac{1}{R_{23} + \cfrac{1}{G_{30} + \cfrac{1}{R_{34} + \cfrac{1}{G_{40}}}}}}$$

Voltage/current division

For the series circuit, we can find the voltage across each resistor by the formula:

$$V_{i,i+1} = R_i I = \frac{R_i V}{R_1 + R_2 + \cdots + R_n}$$

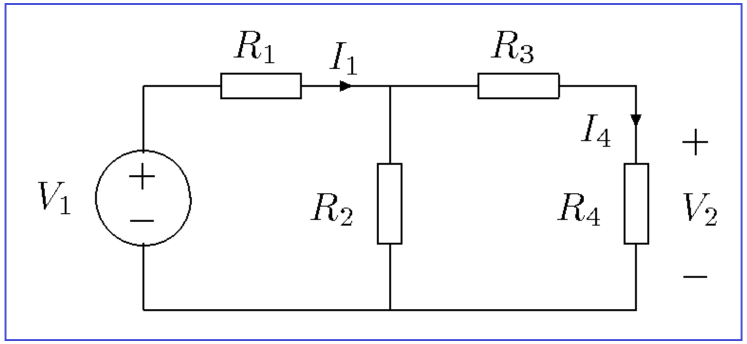
For the parallel circuit, we can find the voltage across each resistor by the formula:

$$I_i = G_i V = \frac{G_i I}{G_1 + G_2 + \cdots + G_n}$$

Note the choice of R and G in the formulae!

Example (something that can be done with series/parallel reduction)

Consider this circuit, which is created deliberately so that you can solve it using series/parallel reduction technique. **Find V_2.**



Solution:

Resistance seen by the voltage source is

$$R = \frac{V_1}{I_1} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3 + R_4}} = R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}$$

Hence, $$I_1 = \frac{(R_2+R_3+R_4)V_1}{(R_3+R_4)(R_1+R_2)+R_1R_2}$$

Current division gives:

$$I_4 = I_1 \times \frac{\left(\frac{1}{R_3+R_4}\right)}{\left(\frac{1}{R_3+R_4}\right) + \frac{1}{R_2}} = \frac{R_2 I_1}{R_2+R_3+R_4}$$

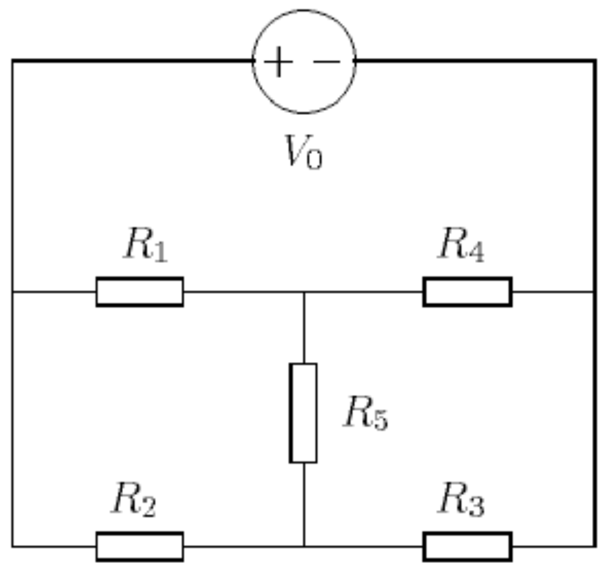
Then, using $V_2 = I_4 R_4$, we get

$$V_2 = \frac{R_2 R_4 V_1}{(R_1+R_2)(R_3+R_4)+R_1R_2}$$

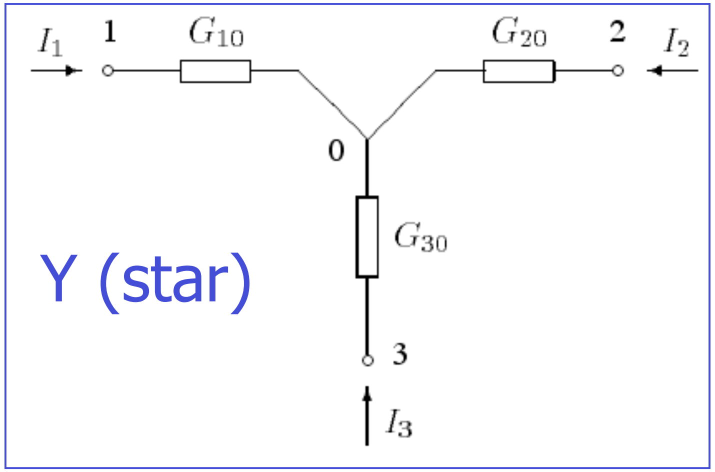
Oops!

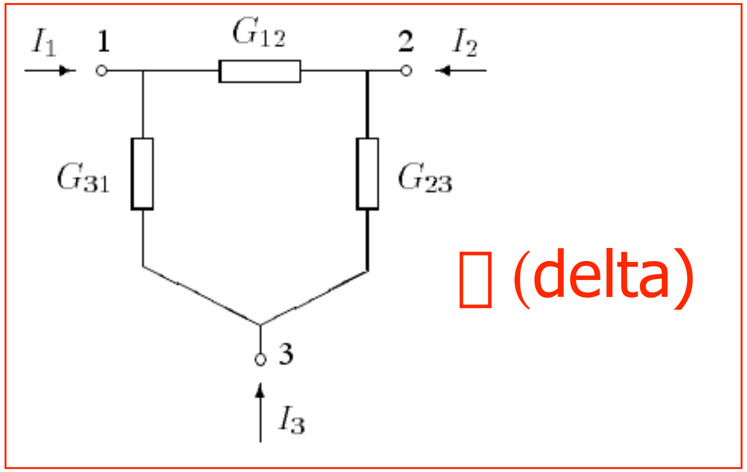
Series/parallel reduction *fails* for this bridge circuit!

Is there some *ad hoc* solution?



Equivalence of star and delta

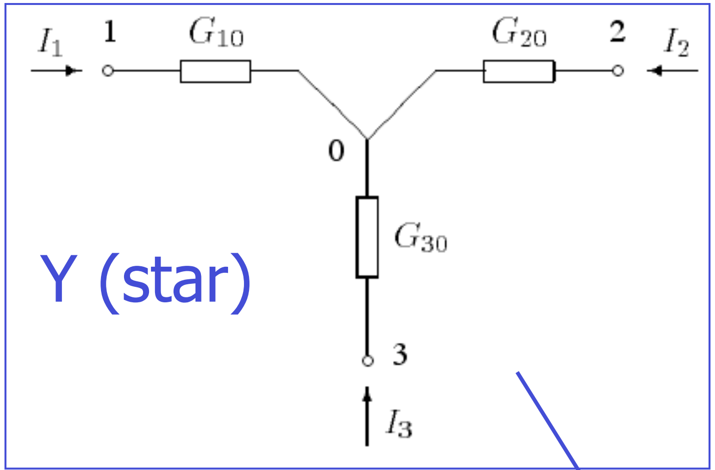


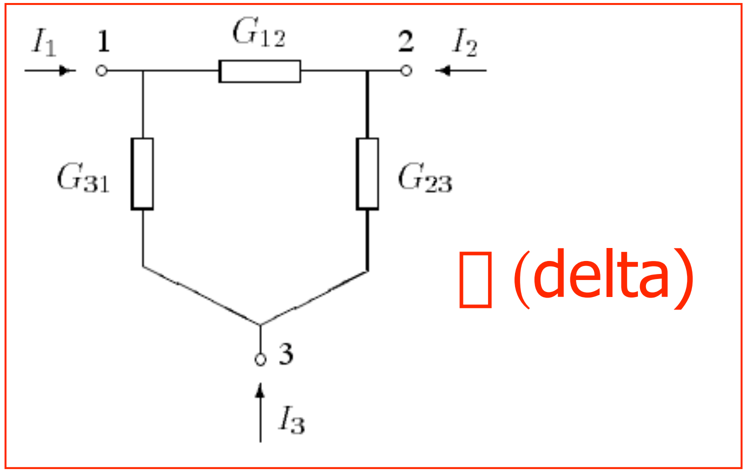


Problems:

1. Given a star circuit, find the delta equivalence. That means, suppose you have all the G's in the star. Find the G's in the delta such that the two circuits are "equivalent" from the external viewpoint.
2. The reverse problem.

Star-to-delta conversion





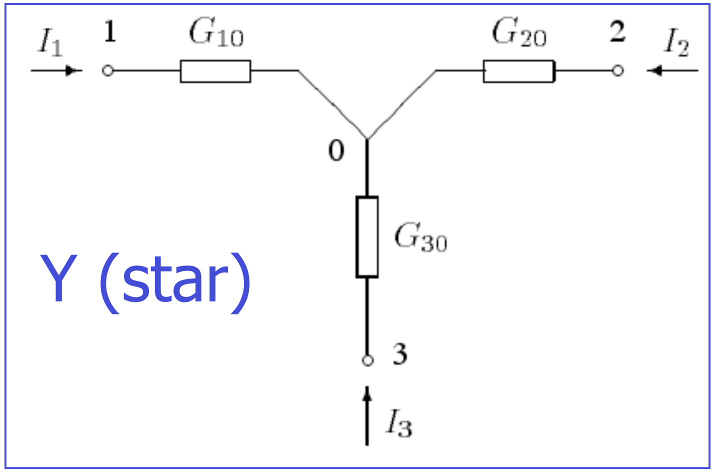
For the Y circuit, we consider summing up all currents into the centre node: $I_1+I_2+I_3=0$, where

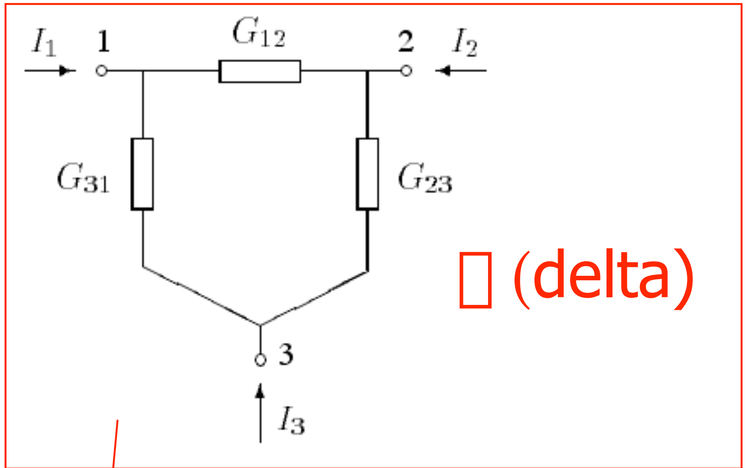
$$
\begin{aligned}
I_1 &= G_{10}(V_1 - V_0)\\
I_2 &= G_{20}(V_2 - V_0)\\
I_3 &= G_{30}(V_3 - V_0)
\end{aligned}
$$

Thus, $V_0 = \dfrac{G_{10}V_1 + G_{20}V_2 + G_{30}V_3}{G_{10}+G_{20}+G_{30}}$, and

$$
\begin{aligned}
I_1 &= \frac{(G_{20}+G_{30})V_1 - G_{20}V_2 - G_{30}V_3}{G_{10}+G_{20}+G_{30}V_3} \times G_{10}\\
I_2 &= \frac{-G_{10}V_1 + (G_{30}+G_{10})V_2 - G_{30}V_3}{G_{10}+G_{20}+G_{30}} \times G_{20}\\
I_3 &= \frac{-G_{10}V_1 - G_{20}V_2 + (G_{10}+G_{20})V_3}{G_{10}+G_{20}+G_{30}} \times G_{30}
\end{aligned}
$$

Star-to-delta conversion





For the Δ circuit, we have

$$
\begin{aligned}
I_1 &= (G_{12}+G_{31})V_1 - G_{12}V_2 - G_{31}V_3 \\
I_2 &= -G_{12}V_1 + (G_{12}+G_{23})V_2 - G_{23}V_3 \\
I_3 &= -G_{31}V_1 - G_{23}V_2 + (G_{31}+G_{23})V_3
\end{aligned}
$$

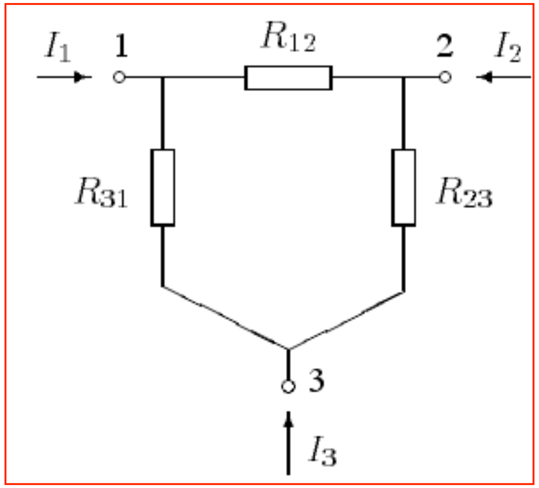
Star-to-delta conversion

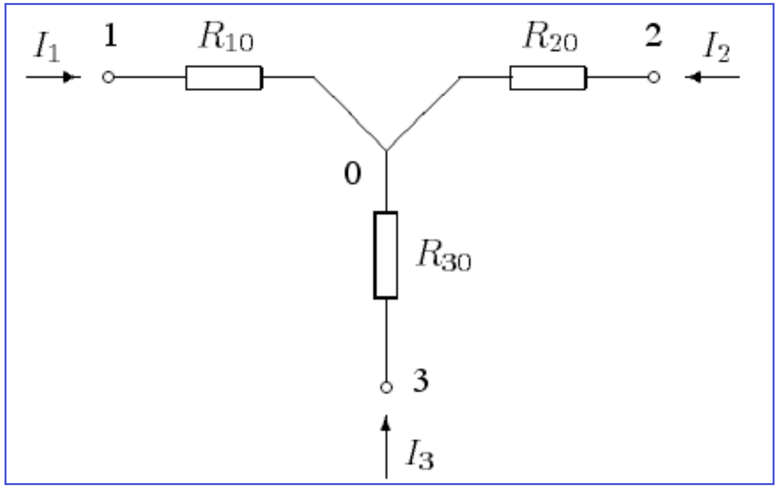
Now, equating the two sets of I_1, I_2 and I_3, we get

$$\begin{aligned} G_{12} &= \frac{G_{10}G_{20}}{G_{10}+G_{20}+G_{30}} \\ G_{23} &= \frac{G_{20}G_{30}}{G_{10}+G_{20}+G_{30}} \\ G_{31} &= \frac{G_{10}G_{30}}{G_{10}+G_{20}+G_{30}} \end{aligned}$$

The first problem is solved.

Delta-to-star conversion





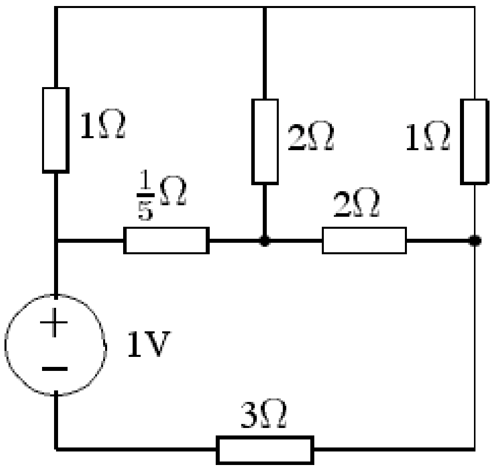
This problem is more conveniently handled in terms of R. The answer is:

$$R_{10} = \frac{R_{12}R_{31}}{R_{23}+R_{31}+R_{12}}$$

$$R_{20} = \frac{R_{23}R_{12}}{R_{23}+R_{31}+R_{12}}$$

$$R_{30} = \frac{R_{31}R_{23}}{R_{23}+R_{31}+R_{12}}$$

Example — the bridge circuit again

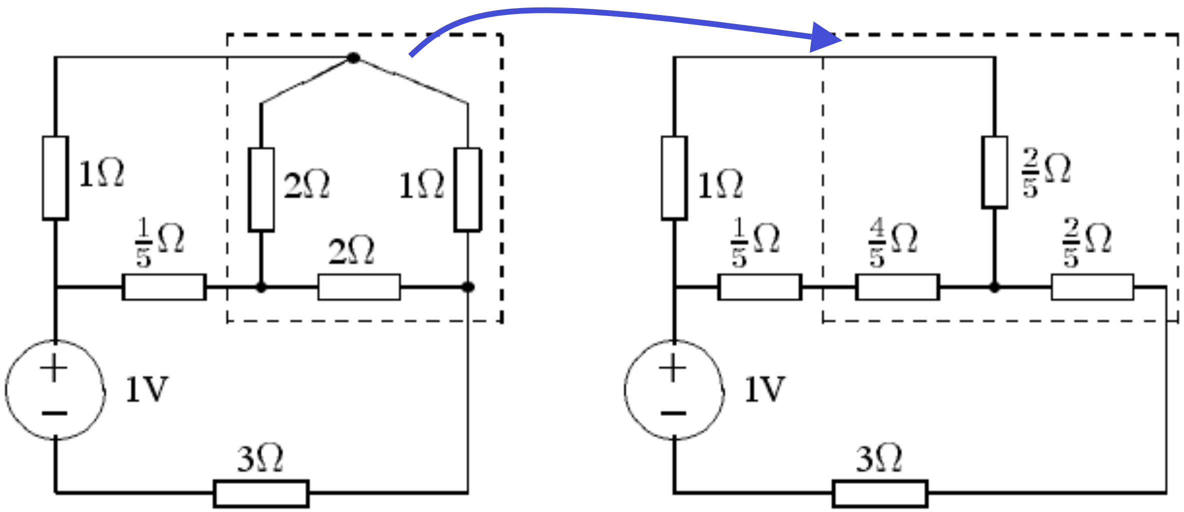


We know that the series/parallel reduction method is not useful for this circuit!

The star-delta transformation may solve this problem.

The question is how to apply the transformation so that the circuit can become solvable using the series/parallel reduction or other ac hoc methods.

Example — the bridge circuit again

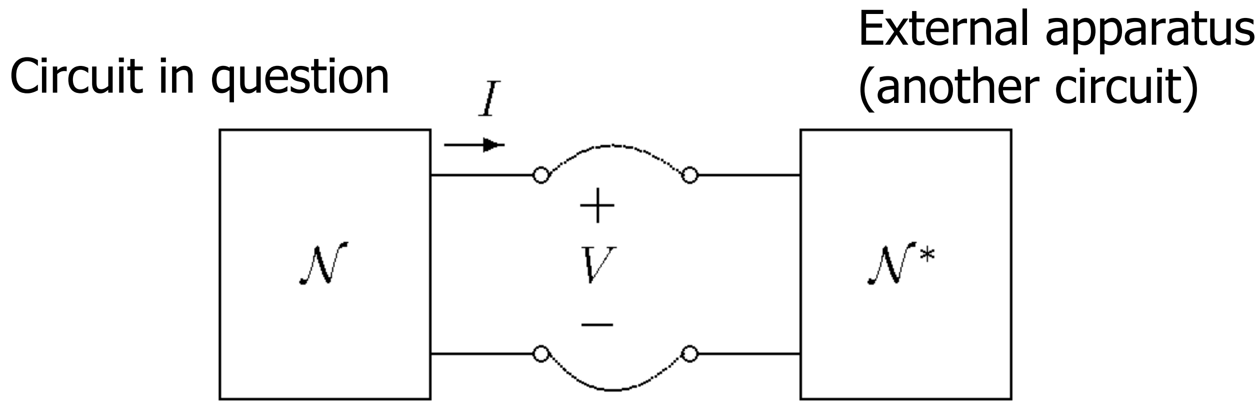


After we do the conversion from Y to D, we can easily solve the circuit with parallel/series reduction.

Useful/important theorems

- Thévenin Theorem
- Norton Theorem
- Maximum Power Transfer Theorem

Thévenin and Norton theorems



Problem:
Find the simplest equivalent circuit model for *N*, such that the external circuit *N** would not feel any difference if *N* is replaced by that equivalent model.

The solution is contained in two theorems due to Thévenin and Norton.

Thévenin and Norton theorems

Let's look at the logic behind these theorems (quite simple really).

If we write down KVL, KCL, and Ohm's law equations correctly, we will have a number of equations with the same number of unknowns. Then, we can try to solve them to get what we want.

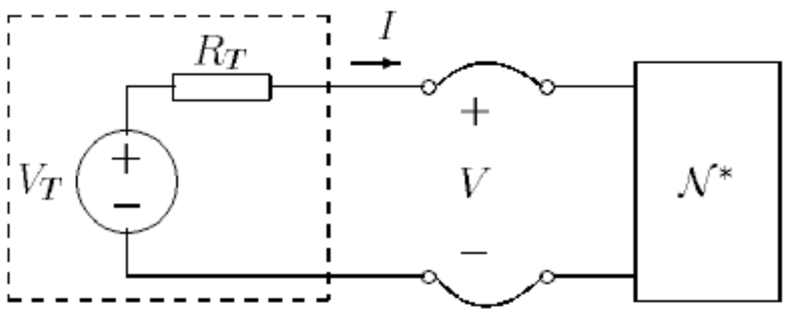
Now suppose everything is linear. We are sure that we can get the following equation after elimination/substitution (some high school algebra):

$$aV + bI - c = 0$$

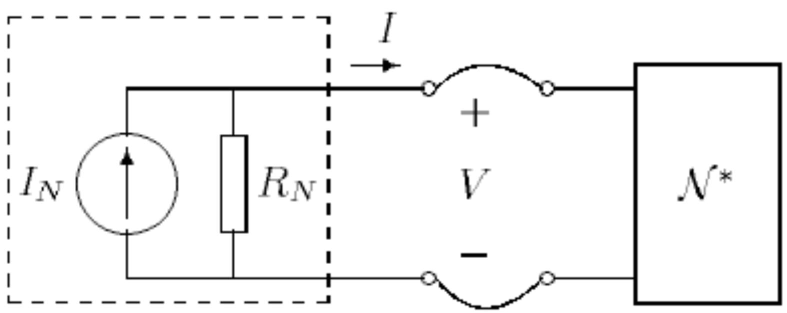
Case 1: $a \neq 0$ $\quad V = \frac{-b}{a} I + \frac{c}{a} = -R_T I + V_T$ → Thévenin

Case 2: $b \neq 0$ $\quad I = \frac{-a}{b} V + \frac{c}{b} = -\frac{V}{R_N} + I_N$ → Norton

Equivalent models

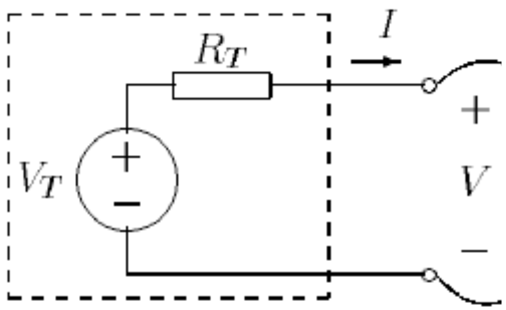


Thévenin equiv. ckt
Voltage source in series with a resistor
i.e., $V + IR_T = V_T$
which is consistent with case 1 equation



Norton equiv. ckt
Current source in parallel with a resistor
i.e., $I = I_N + V/R_N$
which is consistent with case 2 equation

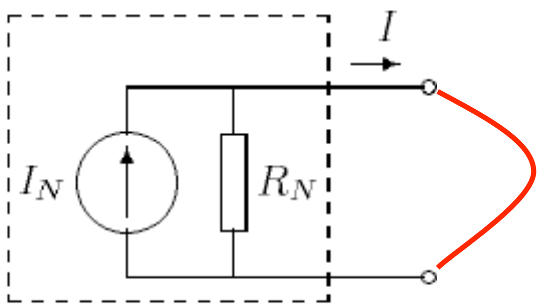
How to find V_T and I_N



$V = V_T$

<u>Thévenin equiv. ckt</u>
Open-circuit the terminals (I=0), we get V_T as the observed value of V.

Easy! V_T is just the open-circuit voltage!

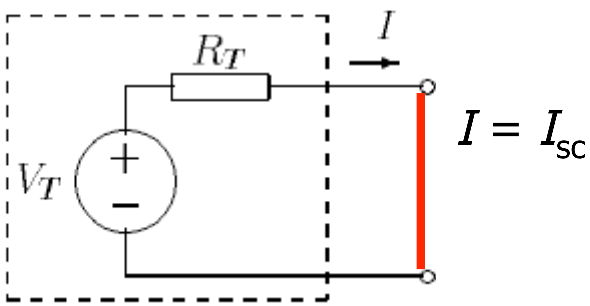


$I = I_N$

<u>Norton equiv. ckt</u>
Short-circuit the terminals (V=0), we get I_N as the observed current I.

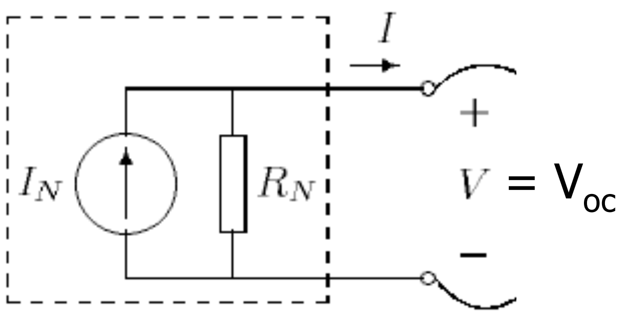
Easy! I_N is just the short-circuit current!

How to find R_T and R_N (they are equal)



Thévenin equiv. ckt

Short-circuit the terminals (V=0), find I which is equal to V_T/R_T. Thus, $R_T = V_T / I_{sc}$



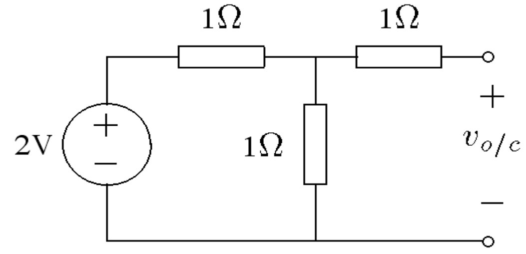
Norton equiv. ckt

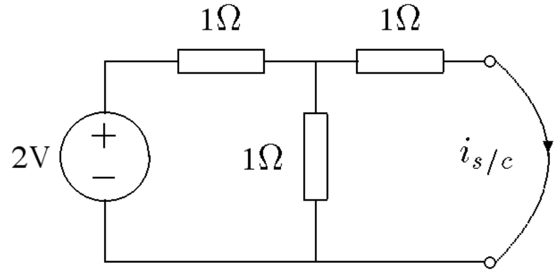
Open-circuit the terminals (I=0), find V which is equal to $I_N R_N$. Thus, $R_N = V_{oc} / I_N$.

For both cases,

$R_T = R_N = V_{oc} / I_{sc}$

Simple example





Step 1: open-circuit
The o/c terminal voltage is

$$v_{o/c} = 2 \times \frac{1}{1+1} = 1\text{V}$$

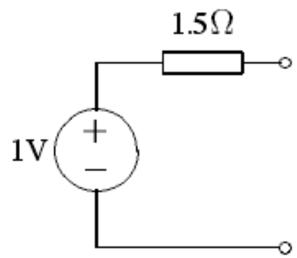
Step 2: short-circuit
The s/c current is

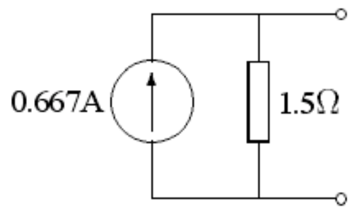
$$i_{s/c} = \frac{2}{1+0.5} \times \frac{1}{2} = \frac{2}{3}\text{A}$$

Step 3: Thévenin or Norton resistance

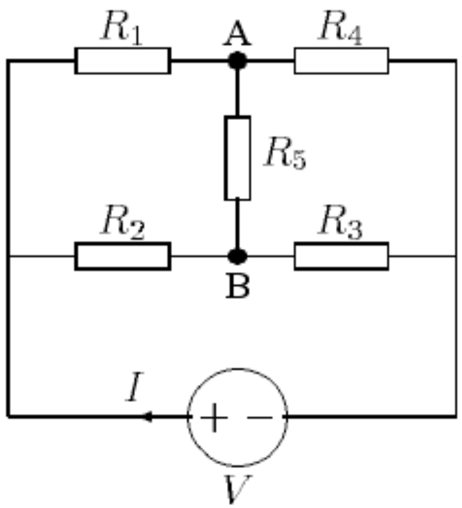
$$R_T = R_N = \frac{v_{o/c}}{i_{s/c}} = \frac{1}{2/3} = \frac{3}{2}\Omega$$

Hence, the equiv. ckts are:





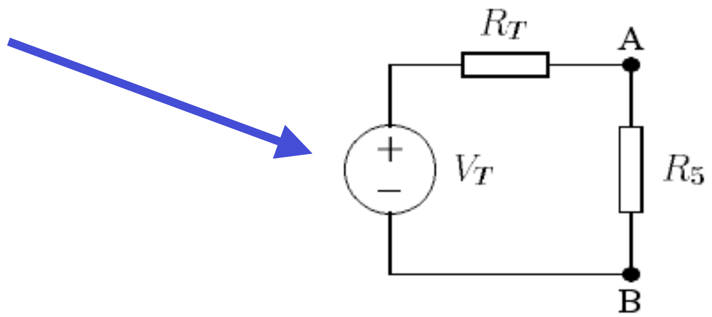
Example — the bridge again



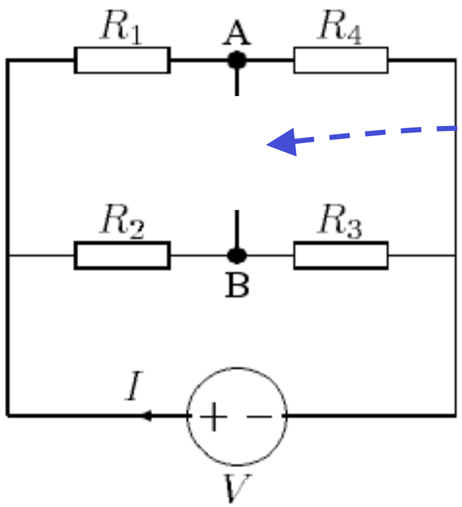
Problem: Find the current flowing in R5.

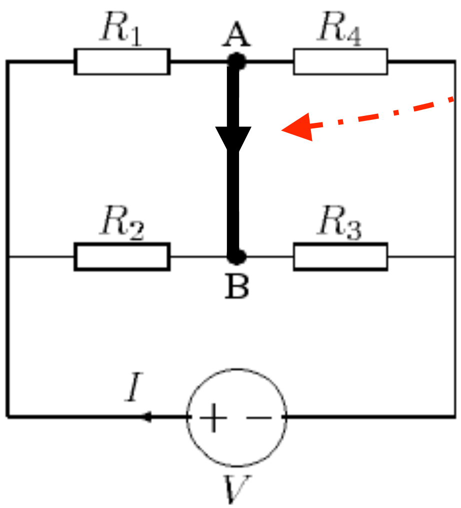
One solution is by delta-star conversion (as done before).

Another simpler method is to find the Thévenin equivalent circuit seen from R5.



Example — the bridge again





Step 1: open circuit

The o/c voltage across A and B is

$$v_{o/c} = V \times \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right) = V_T$$

Step 2: short circuit

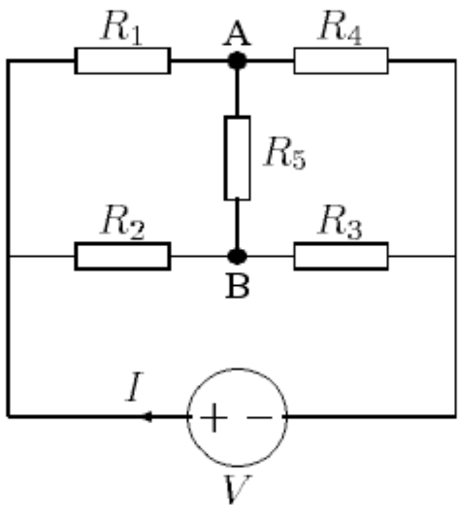
The s/c current is

$$\begin{aligned} i_{s/c} &= (\text{current in } R_1) - (\text{current in } R_4) \\ &= I \times \left(\frac{G_1}{G_1 + G_2} - \frac{G_4}{G_3 + G_4} \right) \end{aligned}$$

Step 3: R_T

$$\begin{aligned} R_T &= \frac{v_{o/c}}{i_{s/c}} \\ &= \frac{G_1 + G_2 + G_3 + G_4}{(G_1 + G_4)(G_2 + G_3)} \\ &= (R_1 \| R_4) + (R_2 \| R_3) \end{aligned}$$

Example — the bridge again



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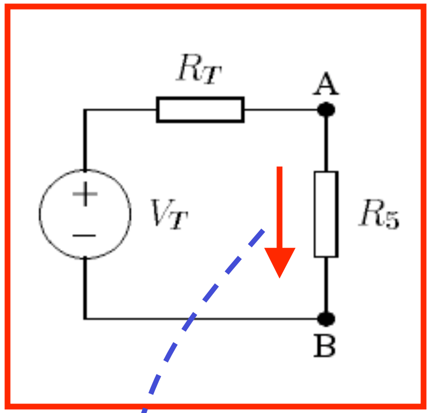
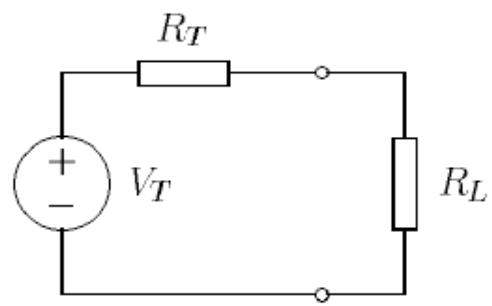


$$\text{Current in R5} = \frac{V_T}{R_5 + R_T}$$

Maximum power transfer theorem

We consider the power dissipated by R_L.
The current in R_L is

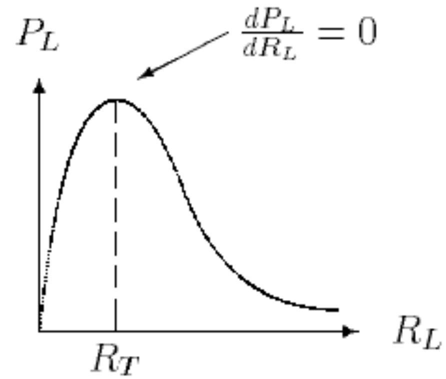
$$I = \frac{V_T}{R_T + R_L}$$



Thus, the power is

$$P_L = I^2 R_L = \frac{V_T^2 R_L}{(R_T + R_L)^2}$$

This power has a maximum, when plotted against R_L.



$$\frac{dP_L}{dR_L} = V_T^2 \frac{R_T - R_L}{(R_T + R_L)^3} = 0$$ gives $R_L = R_T$.

A misleading interpretation

It seems counter-intuitive that the MPT theorem suggests a maximum power at $R_L = R_T$.

Shouldn't maximum power occur when we have all power go to the load? That is, when $R_T = 0$!

Is the MPT theorem wrong?

Discussion: what is the condition required by the theorem?

Systematic analysis techniques

So far, **we have solved circuits on an *ad hoc* manner.** We are able to treat circuits with parallel/series reduction, star-delta conversion, with the help of some theorems.

How about very *general arbitrary circuit styles*?

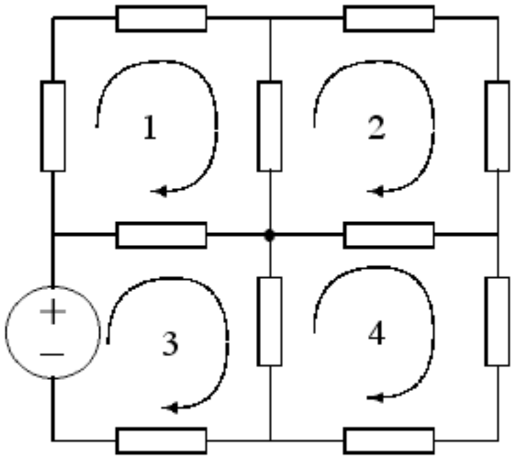
In *Basic Electronics,* you have learnt the use of MESH and NODAL methods.

MESH — planar circuits only; solution in terms of mesh currents.
NODAL — any circuit; solution in terms of nodal voltages.

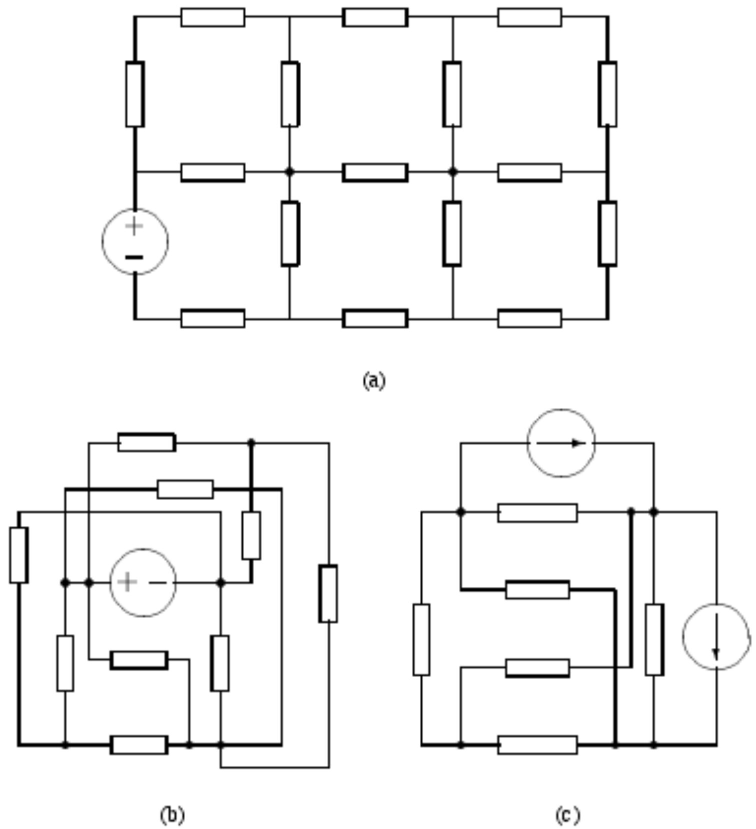
BUT THEY ARE NOT EFFICIENT!

Mesh analysis (for planar circuits only)

Meshes — windows

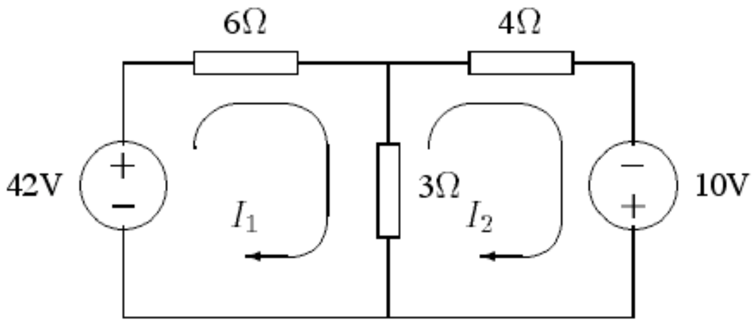


Planar or not?



Mesh analysis

Step 1: Define meshes and unknowns
Each window is a mesh. Here, we have two meshes. For each one, we "imagine" a current circulating around it. So, we have two such currents, I_1 and I_2 — unknowns to be found.



Step 2: Set up KVL equations

Mesh 1: $-42 + 6I_1 + 3(I_1 - I_2) = 0$

Mesh 2: $3(I_2 - I_1) + 4I_2 - 10 = 0$

Step 3: Simplify and solve

$$\begin{aligned} 9I_1 - 3I_2 &= 42 \\ -3I_1 + 7I_2 &= 10 \end{aligned}$$

which gives I_1 = 6 A and I_2 = 4 A.

Once we know the mesh currents, we can find anything in the circuit!

e.g., current flowing down the 3Ω resistor in the middle is equal to $I_1 - I_2$;
current flowing up the 42V source is I_1 ;
current flowing down the 10V source is I_2 ;
and voltages can be found via Ohm's law.

Mesh analysis

In general, we formulate the solution in terms of unknown mesh currents:

[R] [I] = [V] — mesh equation

where [R] is the resistance matrix
[I] is the unknown mesh current vector
[V] is the source vector

For a short cut in setting up the above matrix equation, see *Sec. 3.2.1.2 of the textbook*. This may be picked up in the tutorial.

Mesh analysis — observing *superposition*

Consider the previous example. The mesh equation is given by:

$$\begin{aligned} 9I_1 - 3I_2 &= 42 \\ -3I_1 + 7I_2 &= 10 \end{aligned} \quad \text{or} \quad \begin{pmatrix} 9 & -3 \\ -3 & 7 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 42 \\ 10 \end{pmatrix}$$

Thus, the solution can be written as

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 9 & -3 \\ -3 & 7 \end{pmatrix}^{-1} \begin{pmatrix} 42 \\ 10 \end{pmatrix}$$

Remember what 42 and 10 are? They are the sources! The above solution can also be written as

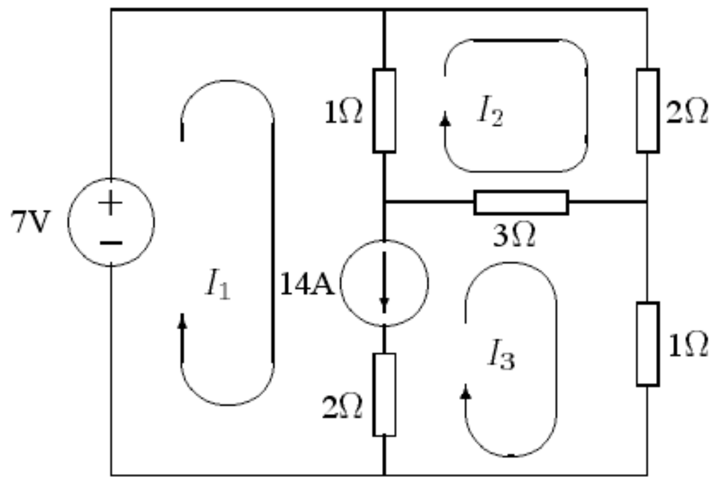
$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 9 & -3 \\ -3 & 7 \end{pmatrix}^{-1} \begin{pmatrix} 42 \\ 0 \end{pmatrix} + \begin{pmatrix} 9 & -3 \\ -3 & 7 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 10 \end{pmatrix}$$

or

$$\begin{aligned} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} &= \mathcal{R}^{-1} \begin{pmatrix} V_1 \\ 0 \end{pmatrix} + \mathcal{R}^{-1} \begin{pmatrix} 0 \\ V_2 \end{pmatrix} \\ &= \mathcal{R}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} V_1 + \mathcal{R}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} V_2 \\ &= AV_1 + BV_2 \end{aligned}$$

SUPERPOSITION of two sources

Problem with current sources

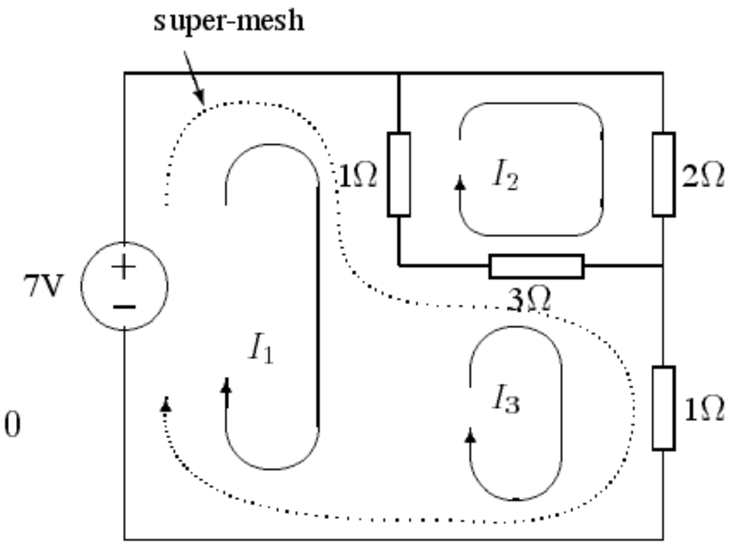


The mesh method may run into trouble if the circuit has current source(s).

Suppose we define the unknowns in the same way, i.e., I_1, I_2 and I_3 .

The trouble is that we don't know what voltage is dropped across the 14A source! How can we set up the KVL equation for meshes 1 and 3?

One solution is to ignore meshes 1 and 3. Instead we look at the **supermesh** containing 1 and 3.



So, we set up KVL equations for mesh 2 and the supermesh:

Mesh 2: $(I_2 - I_1) \times 1 + I_2 \times 2 + (I_2 - I_3) \times 3 = 0$

Supermesh: $-7 + (I_1 - I_2) \times 1 + (I_3 - I_2) \times 3 + I_3 \times 1 = 0$

One more equation: $I_1 - I_3 = 14$

Finally, solve the equations.

Complexity of mesh method

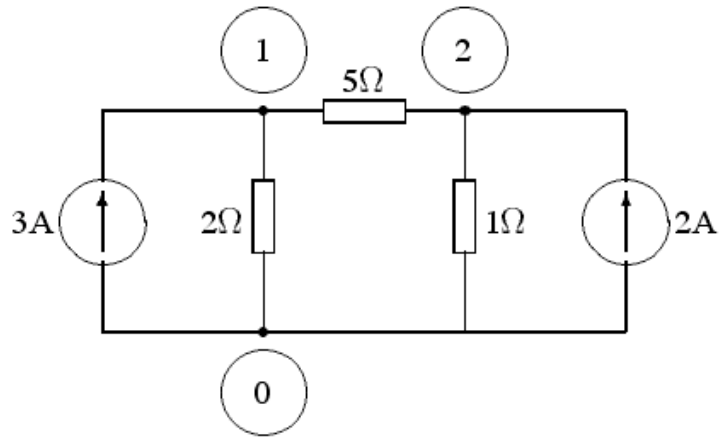
In all cases, we see that the mesh method ends up with N equations and N unknowns, where N is the **number of meshes (windows) of the circuit.**

One important point:
The mesh method is over-complex when applied to circuits with current source(s). WHY?

We don't need N equations for circuits with current source(s) because the currents are partly known!

In the previous example, it seems unnecessary to solve for both I_1 and I_3 because their difference is known to be 14! This is a waste of efforts! Can we improve it?

Nodal analysis



Step 1: Define unknowns
Each node is assigned a number. Choose a reference node which has zero potential. Then, each node has a voltage w.r.t. the reference node. Here, we have V_1 and V_2 — unknowns to be found.

Step 2: Set up KCL equation for each node

Node 1: $$-3+\frac{V_1}{2}+\frac{V_1-V_2}{5}=0$$

Node 2: $$\frac{V_2-V_1}{5}+\frac{V_2}{1}-2=0$$

Step 3: Simplify and solve

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5}+1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

which gives V_1 = 5 V and V_2 = 2.5 V.

Once we know the nodal voltages, we can find anything in the circuit!

e.g., voltage across the 5Ω resistor in the middle is equal to $V_1 - V_2$;
voltage across the 3A source is V_1 ;
voltage across the 2A source is V_2 ;
and currents can be found via Ohm's law.

Nodal analysis

In general, we formulate the solution in terms of unknown nodal voltages:

[G] [V] = [I]   — nodal equation

where [G] is the conductance matrix
[V] is the unknown node voltage vector
[I] is the source vector

For a short cut in setting up the above matrix equation, see *Sec. 3.3.1.2 of the textbook*. This may be picked up in the tutorial.

Nodal analysis — observing *superposition*

Consider the previous example. The nodal equation is given by:

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5}+1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Thus, the solution can be written as $\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} \frac{7}{10} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{6}{5} \end{pmatrix}^{-1} \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Remember what 3 and 2 are? They are the sources! The above solution can also be written as

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} \frac{7}{10} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{6}{5} \end{pmatrix}^{-1} \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{7}{10} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{6}{5} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

or

$$\begin{aligned} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} &= \mathcal{G}^{-1} \begin{pmatrix} I_1 \\ 0 \end{pmatrix} + \mathcal{G}^{-1} \begin{pmatrix} 0 \\ I_2 \end{pmatrix} \\ &= \mathcal{G}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} I_1 + \mathcal{G}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} I_2 \\ &= AI_1 + BI_2 \end{aligned}$$

SUPERPOSITION of two sources

Problem with voltage sources

The nodal method may run into trouble if the circuit has voltage source(s).

Suppose we define the unknowns in the same way, i.e., V_1, V_2 and V_3 .

The trouble is that we don't know what current is flowing through the 2V source! How can we set up the KCL equation for nodes 2 and 3?

One solution is to ignore nodes 1 and 3. Instead we look at the **supernode** merging 2 and 3.

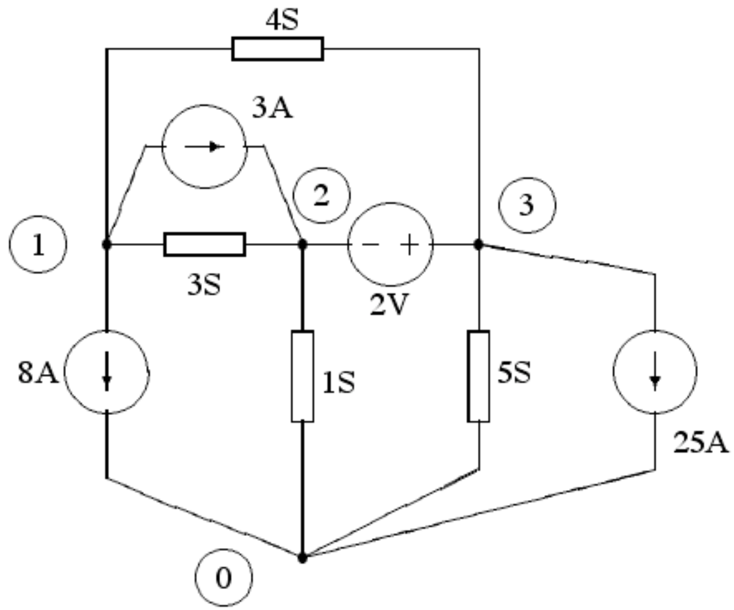
So, we set up KCL equations for node 1 and the supernode:

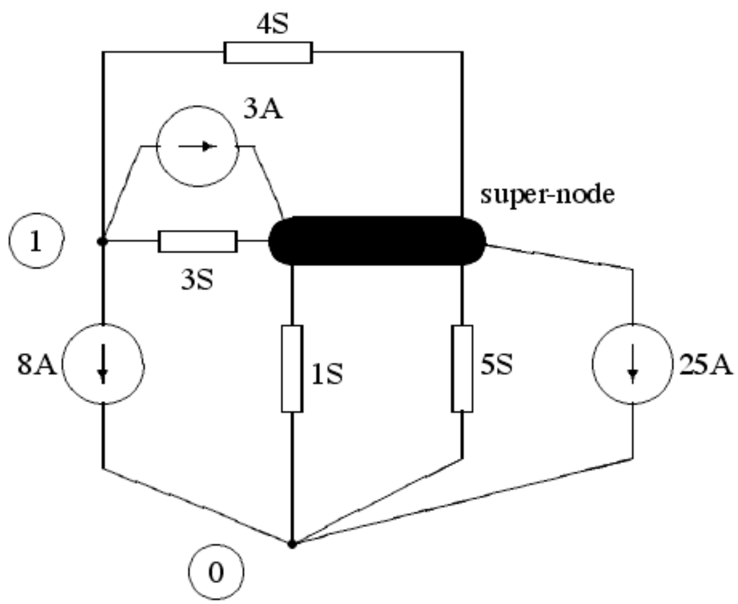
$$8 + (V_1 - V_2) \times 3 + 3 + (V_1 - V_3) \times 4 = 0$$

$$(V_2 - V_1) \times 3 - 3 + V_2 \times 1 + (V_3 - V_1) \times 4 + V_3 \times 5 + 25 = 0$$

One more equation: $V_3 - V_2 = 2$

Finally, solve the equations.





Complexity of nodal method

In all cases, we see that the mesh method ends up with N equations and N unknowns, where N is the **number of nodes of the circuit minus 1.**

One important point:
The nodal method is over-complex when applied to circuits with voltage source(s). WHY?

We don't need N equations for circuits with voltage source(s) because the node voltages are partly known!

In the previous example, it seems unnecessary to solve for both V_2 and V_3 because their difference is known to be 2! This is a waste of efforts! Can we improve it?

Final note on superposition

Superposition is a consequence of linearity.

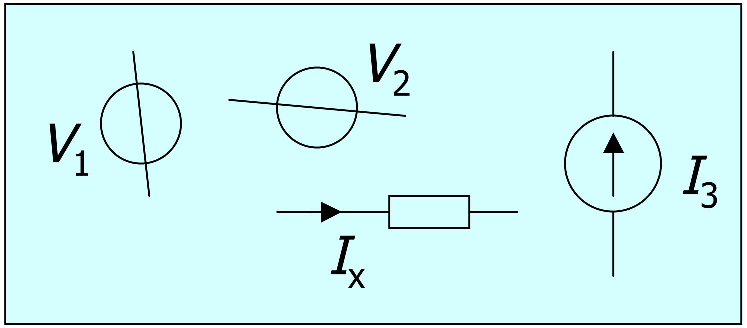
We may conclude that for any linear circuit, any voltage or current can be written as linear combination of the sources.

Suppose we have a circuit which contains two voltage sources V_1, V_2 and I_3. And, suppose we wish to find I_x.

Without doing anything, we know for sure that the following is correct:

$I_x = a\, V_1 + b\, V_2 + c\, I_3$

where a, b and c are some constants.



Is this property useful?
Can we use this property for analysis?

We may pick this up in the tutorial.