UNIVERSITY PHYSICS

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PART FIVE

ELECTRICITY AND MAGNETISM

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PREFACE

This book is primarily intended for students taking a First and Second Year Course in Physics at a University. It is designed for preparation for examinations of the standard of Part I of the Natural Sciences Tripos at Cambridge, the B.Sc. General Degree of London and, by the omission of those sections marked with an asterisk, for Intermediate students who have already studied the elements of Physics at school or elsewhere.

It must be remembered that at the stage covered by this book, students will not yet have become specialists in Physics. The writer has had frequent experience of students who, during a period when they are studying two or three additional subjects, feel a great need for **one book** on Physics which contains the basic information which they must acquire. It is not suggested that this book has made others unnecessary or, more particularly, that it has rendered lectures superfluous. It remains as important as ever for students to read widely and to acquire experience of the different methods of treatment of a subject which only a diversity of Lecturers can supply.

Finally, it is becoming more and more recognized, at least as an ideal, that material usually given in formal lectures can be quite as well acquired from good text-books and that lectures will gradually develop into a tuitional tutorial system under which the time and energy of the lecturer can be devoted to the detailed elucidation of difficult points, apt illustrations and demonstrations, the discussion of essays and exercises done by the student, and the exercise of personality to engender an enthusiasm without which a subject remains " dry bones ".

F. C. CHAMPION.

CONTENTS

CHAPTER I

Introductory

Unlike many physical phenomena, electric and magnetic forces are not directly detectable by the senses. Further, according to modern ideas the branches of physics classified under the title of Electricity and Magnetism are on a different footing from those branches already discussed in Vols. I–IV. In problems involving General Physics, Heat, Sound, and to a lesser extent Light, the majority of the experimental results are adequately explained in terms of mechanics and the molecular theory of matter, together with the mathematical aid afforded by thermodynamics. With electric and magnetic phenomena, however, the molecular theory is of little assistance, the simplest experiment requiring interpretation on the modern view in terms of particles smaller than atoms. Of these sub-atomic particles, the electron plays at present the dominant role. It is true that electricity and magnetism achieved tremendous advances during the hundred years prior to the discovery of the electron, yet to attempt to interpret those advances to-day without constant reference to the electron theory would be analogous to an attempt to account for chemical phenomena in terms of phlogiston theory. In fact, just as the student was urged to interpret phenomena in other branches of physics in terms of the molecular theory, so all phenomena in electricity and magnetism should be referred to electron theory. From the point of view of a student approaching the subject for the first time, there is, however, a great difference between the two theories. The molecular theory was familiar as a philosophical speculation for two thousand years before chemical experiment demanded its adoption. The electron theory was a product of several years of patient experiment but had no philosophical history of comparable duration. Consequently, while molecular theory is comparatively easily appreciated, a considerable knowledge of electric and magnetic phenomena is required before the need for the electron theory, as opposed to other possible explanations, is realized. The student will therefore be in a much better position to understand why electric and magnetic phenomena are interpreted in terms of the electron after having arrived at the end rather than the beginning of this book. Provisionally, then, the student is asked to accept the following tenets of the electron theory. Electricity, like matter on the molecular theory, does not exist in any arbitrary amount but only in discrete amounts. These discrete amounts are whole number multiples of a definite amount of electricity, termed the electronic charge. That is, any quantity of electricity consists of one or more electrons, each electron being pictured in the first instance as a small ball of electric charge. Electrons at rest produce only electrical and not magnetic effects.

An electric current consists simply in the movement of electrons. Magnetic effects which accompany electric currents therefore arise only when electrons move and are a direct result of electrons in motion.

Now the electron theory has turned out to be of far greater significance than just a useful idea to explain electric and magnetic phenomena. The atoms and molecules of molecular theory have been shown themselves to consist of electric charges and therefore of electrons. Consequently, electron theory is much more fundamental than atomic theory. In fact, *all* physical phenomena are now explicable in terms of electron theory. However, as indicated in the Introduction to Vol. I, such a fundamental analysis is often unnecessary, the atomic theory giving sufficient explanation of the phenomenon without recourse to electron theory. For example, nothing is to be gained by interpreting sound waves in terms of electron theory. On the other hand, anomalous dispersion (see Vol. III, Light, p. 110) requires the electron theory if it is to be satisfactorily explained.

CHAPTER II

Elementary Electrostatics

1. Introduction.

The fact that amber which had been rubbed with cloth acquired the property of attracting light bodies such as dust and chaff, was known to the ancients. Such a body is said to have acquired an **electric charge.** The properties of electrified bodies with the charge at rest is the domain of **electrostatics.** If the charge is in motion it is said to constitute an **electric current.** An electric current is always accompanied by a **magnetic field** in the neighbourhood of the current. Since a static charge is unaccompanied by a magnetic field, problems in electrostatics are physically simpler than in electromagnetics and we therefore commence our study of electricity with electrostatics.

2. Negative and Positive Electricity.

If an ebonite rod which has been electrified by friction with cat's fur is suspended in a light stirrup supported by a torsionless fibre of unspun silk as shown in fig. 1, it is found that another similarly elec-

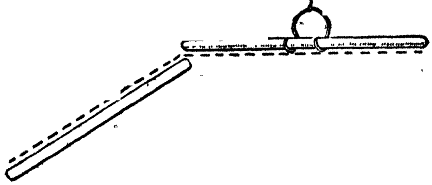

Fig. 1. — Repulsion of Electrified Ebonite Rods

trified ebonite rod exerts a repulsion on the former rod when the two are brought into proximity. Similarly, two glass rods electrified by friction with flannel will also exhibit a mutual repulsion. Should, however, the electrified glass rod be presented to the electrified ebonite rod an attraction takes place. These simple experiments show

(1) There are at least two kinds of electricity;

(2) Bodies charged with the same kind of electricity repel each other, while those charged with different kinds attract each other.

All subsequent experiments have shown that only these two kinds of electricity exist. The electricity acquired by the ebonite rod when rubbed with cat's fur is *arbitrarily* termed **negative electricity,** while that acquired by the glass rod when rubbed with flannel is termed **positive electricity.** The first law of electrostatics is therefore: **Like charges repel, and unlike charges attract each other.**

3. Production of Electricity by Friction.

Consider the simple experiment illustrated in fig. 2. An ebonite rod is fitted with a close-fitting cap of cat's fur and electricity is generated by rotating the cap. On removing the cap and presenting the rod to an electroscope (see section 5) the leaves diverge. If the cap

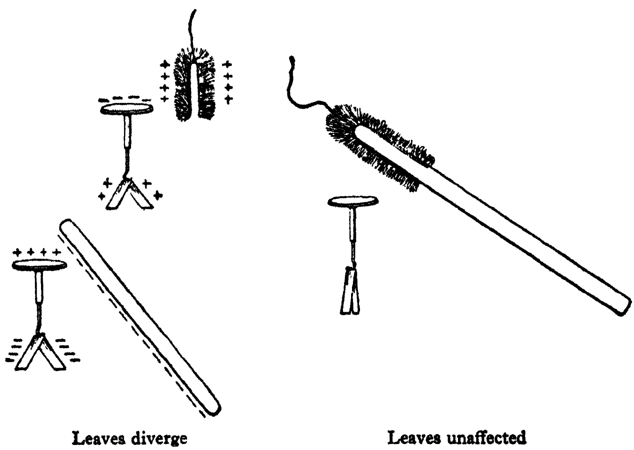

Fig. 2. — Equal and Opposite Charges.

alone is presented the leaves again diverge. If, however, the unseparated but excited cap and rod are simultaneously presented to the electroscope, no divergence results. This clearly indicates that the charges generated are equal and opposite.

These results receive a ready explanation on the electrical theory of matter which is developed more fully in Chap. XX. *Ordinary, uncharged matter consists of equal quantities of positive and negative electricity.* Some of the negative electricity, which consists of electrons, has the property of being fairly easily transferred from one body to another. Simple mechanical friction is sufficient to effect this transfer. Consequently the body which gains the electrons acquires an excess negative charge, while the body which loses them acquires a net positive charge equal in magnitude to the negative charge which it has lost. With ebonite and cat's fur, friction transfers the electrons to the ebonite, while with glass and flannel they are transferred from glass to flannel.

On the modern view, therefore, electricity is not "generated" but merely *separated*. Paradoxically a body is said to " acquire " a positive charge when in fact it has lost the corresponding number of electrons.

4. Conductors and Insulators.

If a metal rod which is held in the hand is subjected to friction, then simple experiments will fail to detect any electrification of the rod, although the cat's fur will show some charge. Should, however, the metal be attached to a piece of ebonite or glass and should it be held by the non-metal, it will become electrified by friction in the usual manner. If the metal is touched with a succession of substances held in the hand it will be found that touching produces little result with some substances, such as glass and ebonite, whereas metals or water produce immediate discharge. Materials like dry wood occupy an intermediate position and cause a slow discharge. Substances are accordingly classified as **conductors, insulators** and **semi-conductors** of electricity. A thin film of moisture will completely ruin the insulating properties of the best insulator. Ordinary glass, being hygroscopic, is therefore very unsuitable for insulation in experiments on static electricity.

With a conductor the charge spreads itself out to the boundary of the conductor, for the individual electrons which constitute the charge, being all of one sign, exert a continual mutual repulsion. Consequently, if a charged conductor is touched by another conductor the charge will continue to spread until it again reaches the boundary of the new conductor. Clearly since the new conductor, say the hand, is normally connected through the rest of the body to the earth, the charge will spread itself to the boundaries of the planet and consequently a negligible fraction will remain on the initially charged conductor.

5. Gold-leaf Electroscope.

The gold-leaf electroscope, a diagram of which (single-leaf type) is shown in fig. 3, is of great value in demonstrating the laws of electrostatics and in measuring quantities such as charge, potential and capacity. It consists essentially of a vertical brass rod to the lower end of which are gummed one or two small strips of very thin gold leaf. The brass rod is insulated from the cubical tin container by an insulating sulphur plug. Charge is communicated to the leaves from a brass disk attached to the top of the brass rod. Owing to the mutual repulsion the leaves diverge at their lower end, and observation of the magnitude of the divergence as registered on a graduated scale allows an estimate of the charge to be made. For accurate work the movement of the image of the leaves over a micrometer scale in the eyepiece

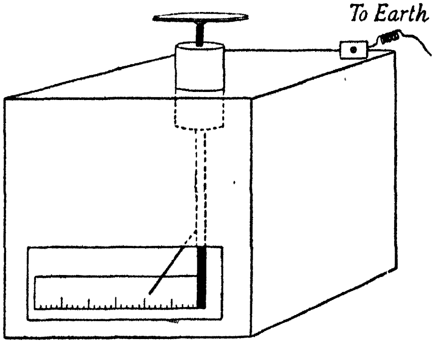

Fig. 3. — Gold-leaf Electroscope

of a microscope is registered. The tin container should be connected to earth throughout the measurements so as to ensure that it is at a definite potential (see section 10).

6. Electrostatic Induction.

If a charged body is brought up to the disk of an electroscope, the leaves of the latter diverge before any charge has been communicated to the leaves by actual contact of the charged body with the disk. This divergence is normally temporary and disappears when the charged rod is removed from the neighbourhood. Such behaviour is due to **electrostatic induction.** When the charged rod, which we shall suppose to be negatively charged, is brought close to the conducting system of disk, support and leaves, some of the electrons in the disk are repelled to the leaves which constitute the boundary farthest removed from the exciting rod. Consequently an excess negative charge is temporarily present on the leaves and the latter diverge. When the exciting rod is removed, the electrons redistribute themselves uniformly, and since the system initially contained equal quantities of positive and negative electricity at all points, it resumes that condition and the leaves collapse.

Suppose, however, that while the leaves are still under the influence of the electrostatic induction, the disk is touched with the finger. The electrons which previously were unable to travel farther from the exciting rod than the leaves, are now able to get to earth. Consequently the temporary excess of electrons travels away from the leaves and the latter collapse. If now the finger is removed and finally the exciting rod is removed, the leaf system will be left with a net positive charge, for the electrons which have escaped will be unable to return to the system. The remaining electrons distribute themselves as evenly as possible among the positive charges, but the net effect is that a *positive* charge has been produced by *electrostatic induction* from the *negatively-charged* exciting rod. The student will notice that it is always the electrons which move and never the positive residues, in which the main mass of the atom resides. Such immobility of the positive part of the atom is confined to solids. In electrical conduction through fluids both positive and negative charges are in motion.

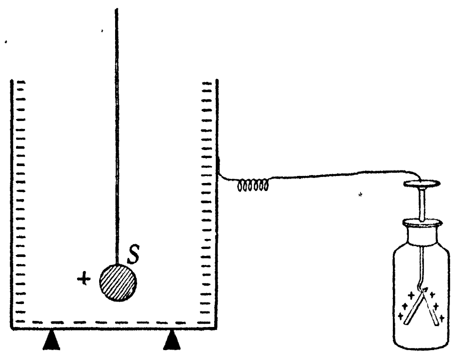

Fig. 4. — Electrostatic Induction

There is complete analogy between the equal and opposite charges produced by friction in that the *maximum* charge which can be communicated by induction is equal and opposite to the inducing charge. To demonstrate this experimentally an apparatus such as that shown in fig. 4 is used. A charged metal sphere S is suspended by an insulating thread of unspun silk in an insulated metal container connected by a metallic wire to an electroscope. Once the sphere is well inside the container, but not in contact with it, the divergence of the electroscope leaves remains constant. The container is then touched momentarily with the finger and the sphere is removed. The divergence now shown by the leaves gives the value of the permanent induced charge. If the sphere is then again introduced and allowed to come into contact with the container the whole charge disappears on both leaves and sphere, showing that the induced charge is equal and opposite to the inducing charge. This neutralization may not appear quite complete in practice, since there is no perfect insulator, and the leak of electricity which is continually taking place from all charged bodies may occur at different rates from the sphere and the other system respectively. By conducting the experiment skilfully and rapidly, however, the proposition may be proved to within a few per cent.

It follows that the attraction between uncharged bodies and charged bodies is due to induction. Under the inducing action of the charged body, like charges in the other body are repelled (more or less effectively) to the far end, and an excess of charges of opposite sign are situated close to the charged body. The attraction between the unlike charges exceeds the repulsion between the like charges owing to the greater proximity of the former; and if the particles are light, the electrostatic forces are sufficient to overcome the force of gravity and to lift the particles.

7. Distribution of Charge.

In fig. 5 are shown a number of conducting bodies of different shapes supported on insulating stands and charged with electricity either by contact or by induction from an ebonite rod. To examine the distribution of the charge over the conductors a **proof plane** is

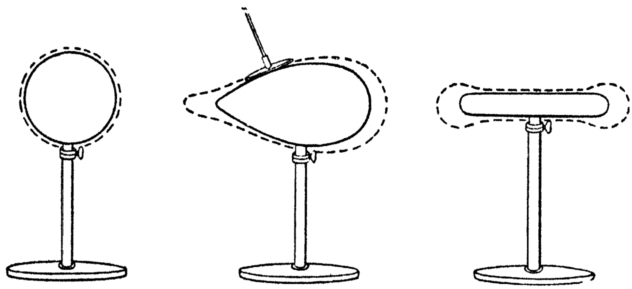

Fig. 5. — Distribution of Charge

used. This consists of a small brass disk attached to an insulating handle; it is placed in contact with the conductor at various points and then presented at a standard distance to an uncharged gold-leaf electroscope. For the charged sphere it is found on sampling that the distribution is uniform over the surface. If the sphere is hollow it may easily be shown that **no charge resides inside the conductor.** This is to be expected, since the various parts of the charge exert a mutual repulsion and consequently are urged to the farthest boundaries of the conductor.

The cylindrical conductor with hemispherical ends will be found to have a greater **surface density** of charge at the ends and least where the curvature is least. This is shown still more strikingly with the pear-shaped conductor where the surface density on the tip is very large. Now the air, although a moderately good insulator for small surface densities, becomes less good for large ones, and consequently

11. Electrostatic Potential.

In dynamics the concept of energy is of great value in solving problems, as an alternative method to working from first principles with force concepts. Similarly, in electrostatic problems it is convenient to deal with the electrical potential energy (or electrical potential as it is briefly called) in the field due to an electric charge. A unit charge, of the same sign as the fixed charge q in fig. 10, has a potential (energy) at a point P, a distance r from q, equal to the kinetic energy which the unit charge would derive in being repelled from r to infinity. This kinetic energy is clearly equal to the work done in the reverse process of forcing the unit charge from an infinite distance up to the point r against the mutual repulsion.

While mathematically the potential falls to zero only at an infinite distance from the charge, in practice, any body which is connected to earth is considered to be at zero potential. In the majority of instances

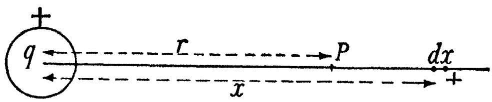

Fig. 10. — Potential due to Point Charge

the earth acts as a perfect sink of charge and consequently reduces all charged bodies placed in communication with it to its own zero potential.

To calculate the potential at r, we note that the element of potential dV gained when the unit charge is moved from x to $(x - dx)$ from q is

$$dV = \text{force} \times \text{distance} = F_x(-dx), \qquad (2.5)$$

$$= \frac{q}{x^2}(-dx). \qquad (2.6)$$

The potential at r (in electrostatic units) is therefore

$$V_r = \int_\infty^r \frac{q}{x^2}(-dx) = \left[\frac{q}{x}\right]_\infty^r$$

$$= \frac{q}{r}. \qquad (2.7)$$

The **potential therefore varies inversely as the distance from a point charge.** The surfaces of equal potential will therefore be spheres, and since the lines of force from an isolated point charge are radial the appearance will be as in fig. 11. This diagram is an example of a general principle that the **lines of force cut the equipotential surfaces orthogonally.** The conducting surface of a charged conductor is therefore an equipotential surface. This deduction also follows from physical reasoning, since if different potentials occurred throughout the same conductor, charge would flow from regions of high to regions of low potential, just as water flows from high to low level (compare equation (2.5)).

Equation (2.5) may be written

$$F_x = -\frac{\partial V}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.8)$$

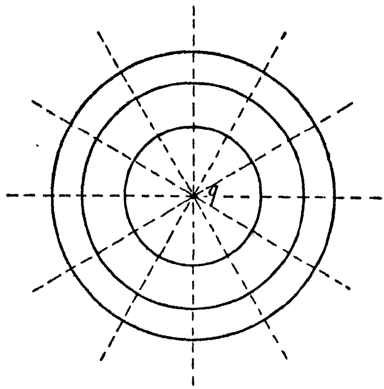

Fig. 11.—Field round Point Charge

When the potential is known, it is often very convenient to derive the forces on systems in any direction by partial differentiation of the potential with respect to that direction.

12. Gauss's Theorem.

In much the same way that potential is a useful tool derived mathematically from the law of force, so Gauss's theorem is a convenient alternative statement of the force law, of great value in mathematical problems in electrostatics. Consider the point charge q in fig. 12 to be enclosed by a surface of arbitrary shape. If the mean electrostatic force component normal to any small area dS is F, the **normal induction** through that surface element is defined as $F \cdot dS$. Gauss's theorem states that the **total normal induction through any surface completely enclosing a charge is ($4\pi \times$ charge enclosed), while through any surface enclosing no charge it is zero.**

To prove this, consider the contribution to the T.N.I. made by the elementary cone shown in fig. 13. This is

$$\Delta \text{T.N.I.} = F_1 \cos\theta_1 \, dS_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.9)$$

Now from solid geometry, the elementary solid angle $d\omega$ is defined by

$$d\omega = \frac{dS_1 \cos\theta_1}{r_1^2}. \qquad (2.10)$$

Hence

$$\Delta \text{T.N.I.} = F_1 \, d\omega r_1^2. \qquad (2.11)$$

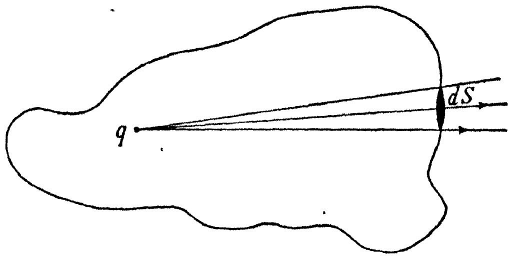

Fig. 12. — Induction through a Surface

But, from the inverse square law,

$$F_1 = \frac{q}{r_1^2}. \qquad (2.12)$$

Hence from (2.11) and (2.12)

$$\Delta \text{T.N.I.} = q\, d\omega.$$

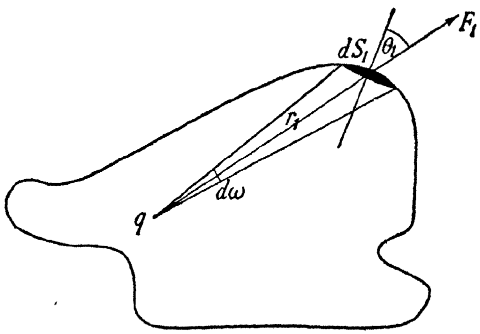

Fig. 13. — Gauss's Theorem

The total normal induction is therefore the product of the whole solid angle subtended by the enclosing surface at the charge, multiplied by the charge itself. Hence

$$\text{T.N.I.} = 4\pi q. \qquad (2.13)$$

If the charge is situated outside the closed surface, the total normal induction is zero, for the angle between the outward normal and the force may be either acute or obtuse, and the contributions to the T.N.I. from any elementary cone, with vertex at q, are alternately positive and negative, but of the same numerical value.

As an example of Gauss's theorem, we shall deduce that for points *outside* a charged sphere the forces are equal to those which would be produced by a point charge equal to the charge on the sphere and situated at its centre. Consider a point P situated a distance x from the centre of a sphere S of radius r and charge q, as shown in fig. 14. Then to determine the force at P, construct an imaginary sphere of radius x passing through P and concentric with S. The total normal induction through the outer sphere is

$$\text{T.N.I.} = F \cdot 4\pi x^2.$$

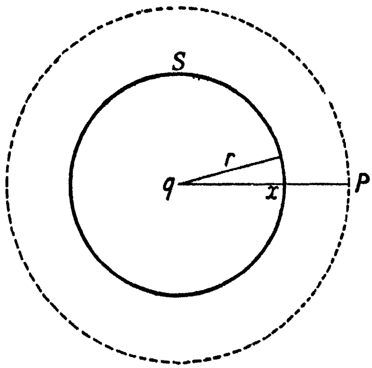

Fig. 14. — Force due to Charged Sphere

Now by Gauss's theorem,

$$\text{T.N.I.} = 4\pi q.$$

Hence
$$F = \frac{q}{x^2},$$

or the charge produces a force at P equal to that which would be produced if the charge q were concentrated at the centre of the sphere.

EXERCISES

1. Describe experiments which show that equal and opposite quantities of electricity may be produced by (*a*) friction, (*b*) induction.

2. Explain what factors govern the distribution of electricity over the surface of conductors. How is the corona discharge caused and how is it reduced?

3. Define (*a*) unit charge, (*b*) strength of an electric field. Quote examples illustrating the use of the conception of lines and tubes of force in understanding electrical phenomena.

4. Upon what evidence is it assumed that the law of force between two electrical charges varies inversely as the square of their distance apart?

5. Define electrical potential at a point and calculate the potential at a point a distance r from a point charge q. Of what value is the potential concept in electrical theory?

6. State and prove Gauss's theorem as applied to electrostatic forces and use the theorem to show that for points external to a charged sphere the forces exerted are the same as would be produced if all the charge were concentrated at the centre of the sphere.

7. Show that if a line of force starts from a charge e_1, in a direction making an angle α with a straight line joining e_1 to a second charge e_2 (of opposite sign), this line of force meets e_2 at an angle β given by

$$e_1 \sin^2 \frac{\alpha}{2} = e_2 \sin^2 \frac{\beta}{2}.$$

[Consider the induction through cones of semi-vertical angles α and β at the two charges.]

CHAPTER III

Electrostatics

1. Capacity.

The potential of a charged sphere of radius r and charge q is q/r, from equation (2.7), and hence the ratio

$$\frac{\text{charge}}{\text{potential}} = r = \text{constant}. \quad . \quad . \quad . \quad . \quad (3.1)$$

This relation, that the potential is directly proportional to the charge for a given conductor, holds quite generally, and the ratio is defined as the **capacity** of the conductor. Since then

$$C = \frac{Q}{V}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.2)$$

by putting $V = 1$ we may define the capacity alternatively as **the charge required to raise the potential of the conductor by unit amount.** The capacity of a sphere (in electrostatic units) is, from (3.1), equal to its radius and is therefore measured in centimetres on the C.G.S. system of units. The capacity of any system of given form is proportional to the first power of its linear dimensions.

2. Condensers.

It is usual to reserve the term electrical condenser for a system composed of two charged surfaces one of which is charged and insulated, while the other, which is earthed, acquires by induction an equal and opposite charge. For example, two parallel plates or two concentric spheres might constitute a condenser. The capacity of a condenser, as opposed to that of an isolated conductor, will depend on the proximity of the insulated and earthed plates of the condenser as well as upon their linear dimensions. We calculate below the capacities of various systems of practical importance.

The method is to calculate the potential V due to a charge Q on the condenser, and then form the ratio Q/V, which is by definition the capacity.

(i) *Two Concentric Spheres, Inner Sphere Insulated, Outer Sphere Earthed.*

The charge $+Q$ on the inner insulated sphere induces an equal and opposite charge $-Q$ on the inside of the outer sphere as shown in fig. 1. The outer sphere, being earth connected, is at zero potential; we wish to calculate the potential of the inner sphere due to the two charges $+Q$ and $-Q$. The potential due to the inner sphere itself is equal to the potential at its centre, i.e.

$$V_1 = \frac{Q}{a}, \quad \ldots \ldots \ldots \quad (3.3)$$

for the potential at all points *inside* a closed conductor is the same and equal to that at the surface of the conductor. This follows from the experimental fact that there is no force anywhere inside a closed conductor (Chap. II, section 10), for by equation (2.8) the force is equal

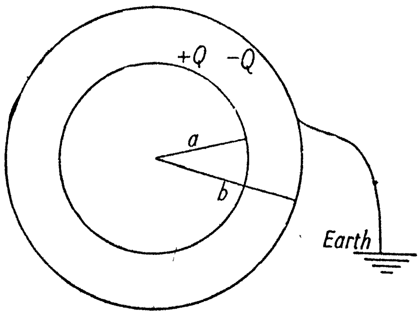

Fig. 1. — Capacity of Spherical Shell (Outer Sphere Earthed)

to the rate of change of potential with distance and since the force is zero there can be no change in potential. By the same argument, the potential of the inner sphere due to the charge $-Q$ on the outer sphere is therefore the same as the potential of the outer sphere due to its own charge, that is

$$V_2 = \frac{-Q}{b}. \quad \ldots \ldots \ldots \quad (3.4)$$

The total potential of the inner sphere is

$$V = V_1 + V_2 = Q\left(\frac{1}{a} - \frac{1}{b}\right), \quad \ldots \ldots \quad (3.5)$$

and the capacity of the system is therefore

$$C = \frac{Q}{V} = \frac{ab}{(b-a)}. \quad \ldots \ldots \ldots \quad (3.6)$$

The capacity is inversely proportional to the distance between the conductors composing the spherical condenser.

(ii) *Two Concentric Spheres, Inner Sphere Earthed, Outer Sphere Insulated.*

In the previous example, all the lines of force which arose from the central charged sphere cut the outer sphere, and hence the induced charge was equal and opposite to the inducing charge. With the outer sphere insulated and the inner sphere earthed, however, only some of the lines of force proceed inwards to the inner sphere, a large number going outwards to the walls as shown in fig. 2. The induced charge $-q$ is therefore only a fraction of the inducing charge $+Q$, and before we can calculate the potential V of the outer sphere

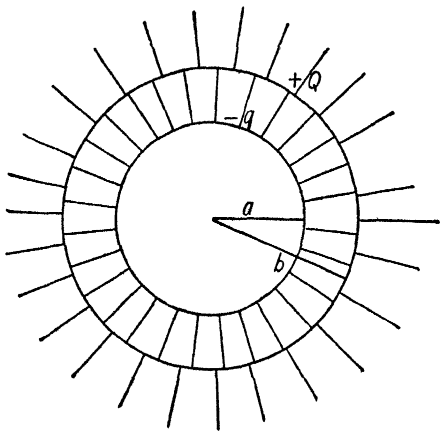

Fig. 2. — Capacity of Spherical Shell (Inner Sphere Earthed)

we require to calculate q in terms of Q. This is accomplished by considering that under the influence of the two charges, the potential of the inner sphere is zero. Hence, from the previous example,

$$0 = -\frac{q}{a} + \frac{Q}{b},$$

or

$$q = Q\frac{a}{b}. \qquad (3.7)$$

The potential of the outer sphere is therefore

$$V = \frac{Q}{b} - \frac{q}{b}$$

$$= Q\frac{(b-a)}{b^2};$$

and

$$C = \frac{Q}{V} = \frac{b^2}{(b-a)}. \qquad (3.8)$$

(iii) *Parallel Plate Condenser.*

If a spherical condenser is imagined to expand indefinitely until its radius becomes very large compared with the separation of the two conductors, it will approach the case of a parallel plate condenser of infinite extent. Hence, putting $b = a$ in the numerator but not in the denominator of (3.6) and (3.8), we have

$$C = \frac{a^2}{t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.9)$$

where $t = (b - a)$.

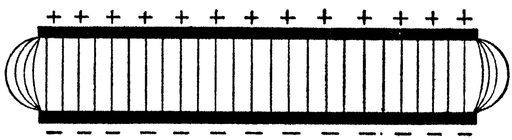

Fig. 3. — Parallel Plate Condenser

Since the area of the surface of the sphere is $4\pi a^2 = S$, equation (3.9) becomes $C = S/4\pi t$. The capacity for a finite area A is therefore

$$C = \frac{A}{4\pi t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.10)$$

which is the required formula for the capacity of a parallel plate condenser, where A is the area of either plate, and t is the distance between them. As shown in fig. 3, the lines of force will be perpendicular to the plates in the central regions but will curve at the edges owing to the attraction of the walls of the room. The formula is in error in so far as it neglects this edge effect, but the correction is small if the ratios of plate length and breadth to plate separation is kept large.

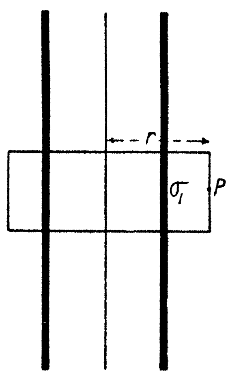

Fig. 4. — Cylindrical Condenser

*(iv) *Cylindrical Condenser.*

The case of two concentric cylinders is of great practical importance since cables and lines generally occur in this form. We first require to calculate the potential at any point a distance r from the common axis of the two cylinders. As in fig. 4, let the charge *per unit length* of the cylinder be σ_1. Then the force at a point P is found by constructing a concentric cylindrical surface through P and applying Gauss's theorem. If the force at P is F, the total normal induction

$$\text{T.N.I.} = F \times (2\pi r \times 1) = 4\pi\sigma_1.$$

Hence
$$F = \frac{2\sigma_1}{r}. \qquad (3.11)$$

The force due to a charged cylinder of infinite length therefore falls off inversely as the distance from the axis of the cylinder. With a cylinder of finite length the lines of force are no longer perpendicular to the axis of the cylinder and an error is involved as with the parallel plate condenser of finite extent. This error is again small if the distance r from the cylinder is small compared with its length.

The difference in potential between any two points r_1 and r_2 from the axis of the cylinder is calculated by the method of Chap. II, section 11. We therefore have

$$(V_{r_1} - V_{r_2}) = \int_{r_2}^{r_1} F_r(-dr) = -\int_{r_2}^{r_1} \frac{2\sigma_1}{r}\,dr$$

$$= 2\sigma_1 \log_e \frac{r_2}{r_1}. \qquad (3.12)$$

Hence, the capacity of a length l of a cylindrical condenser is given by

$$C = \frac{\sigma_1}{V} = \frac{l}{2\log_e(b/a)}. \qquad (3.13)$$

2. Electric Force between Two Charged Plates.

The lines of force are parallel (fig. 3). By applying Gauss's Theorem (p. 15) to a cylinder with ends perpendicular to these lines, we find that the force is everywhere of the same magnitude. If we represent the force by F, the work done in taking a unit charge from one plate to the other is

$$V = Ft, \qquad (3.14)$$

and this work is by definition equal to the potential difference V between the two plates. Now since

$$V = \frac{Q}{C},$$

we have, by (3.10),

$$V = (A\sigma)\frac{4\pi t}{A}, \qquad (3.15)$$

$$= 4\pi\sigma t,$$

where σ is the surface density of charge *per unit area* of the plates. (Compare σ_1 in (3.11).) Hence from (3.14) and (3.15)

$$F = 4\pi\sigma. \qquad (3.16)$$

Each plate contributes symmetrically to the total force and hence **the force close to a single charged plate is**

$$F = 2\pi\sigma. \qquad (3.17)$$

3. Mechanical Force of Attraction between Two Flat Condenser Plates.

The following device enables us to calculate the mechanical force of attraction between two condenser plates. Imagine a unit charge to be situated in a small hole in one condenser plate as shown in fig. 5. Then the force on this charge arises entirely from the other plate, since this alone gives rise to lines of force passing through the hole. Hence the force on the charge is

$$F = 2\pi\sigma \times 1.$$

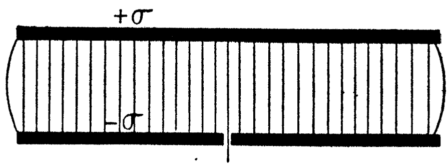

Fig. 5. — Attraction between two Condenser Plates

If the hole is replaced by an element of surface dA with charge density σ, the force becomes

$$F = 2\pi\sigma^2 dA,$$

and hence the total force of attraction between the plates is

$$F = 2\pi\sigma^2 A. \qquad (3.18)$$

4. Energy Stored in a Charged Condenser.

The energy stored in a charged condenser is equal to the work done in the process of forcing the charge on to the plates against the repul-

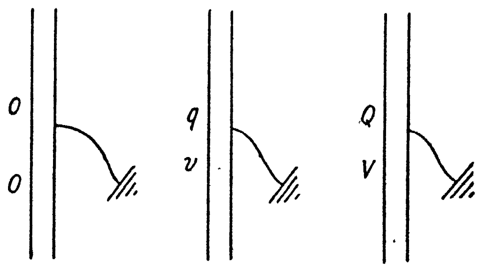

Fig. 6. — Energy stored in Charged Condenser

sion of the charge which has already been communicated. In fig. 6 we illustrate three conditions: (*a*) when the charge and potential are zero; (*b*) some intermediate stage in the process when the charge and potential are q and v respectively; and (*c*) the fully charged state, with final values Q and V. The element of work done in communicating a further charge dq when the existing potential is v is

$$dW = v\,dq;$$

for by definition, the potential v is the work done in bringing unit charge to the system, and hence the work done in bringing an element of charge dq is $v\,dq$. The total work done in raising the charge from 0 to Q is therefore

$$W = \int_0^Q v\,dq, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.19)$$

and this is the energy stored in the condenser. Since v varies with q, equation (3.19) cannot be integrated directly, but the capacity C of the system is constant throughout and

$$C = \frac{q}{v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.20)$$

Substituting for v from (3.20) in (3.19) we therefore have

$$W = \int_0^Q \frac{q}{C}\,dq$$

$$= \tfrac{1}{2}\frac{Q^2}{C}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.21)$$

Since $C = Q/V$, equation (3.21) may alternatively be expressed in the forms:

$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.22)$$

5. Capacities in Parallel and in Series.

The total capacity of two condensers connected in parallel is equal to the sum of their separate capacities. Connexion in parallel implies that the earthed plates have a common connexion and the insulated plates another common connexion as shown in fig. 7. Since the effect is simply to enlarge the areas of the insulated and earthed plates respectively, the proposition is obvious physically. Mathematically we have, with reference to fig. 7,

$$Q = Q_1 + Q_2 = VC, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.23)$$

where C is the final total capacity.

Again,

$$Q_1 = VC_1,$$
$$Q_2 = VC_2,$$

whence, by addition and use of (3.23),

$$C = C_1 + C_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.24)$$

The reasoning clearly applies to any number of condensers.

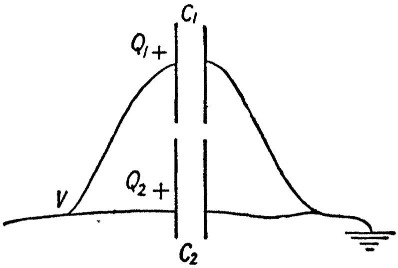

Fig. 7.—Capacities in Parallel

In fig. 8 two condensers are connected in series. In series connexion only one plate is earthed and the potentials of the other plates increase each time we pass across a condenser. The two insulated middle plates are connected, and are therefore at the same potential. Since they are initially uncharged, the total charge on them is zero. The charges facing each other on the interior faces of the plates of the condensers all have the values $\pm Q$ (fig. 8). (There may in addition be a charge

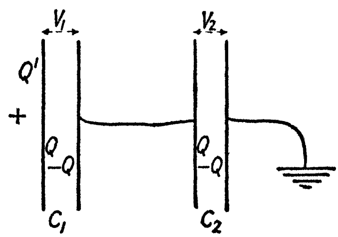

Fig. 8.—Capacities in Series

Q' on the outer surface of the left-hand plate.) Then, if the potential drops are as shown in fig. 8,

$$V_1 = \frac{Q}{C_1},$$

$$V_2 = \frac{Q}{C_2},$$

so that the total fall in potential is

$$V = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right). \quad . \quad . \quad . \quad (3.25)$$

Now the total capacity of the system is defined by

$$C = \frac{Q}{V}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.26)$$

Hence from (3.25) and (3.26)

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.27)$$

6. Effect of Dielectrics.

The preceding calculations have been based on Gauss's thorem, that is upon a mathematical formulation of the inverse square law. We have already stated in Chap. II, section 10, that should the charges be embedded in material of dielectric constant k, the force is reduced, being inversely proportional to k. Since the force is the rate of charge of potential with distance, the potential is likewise reduced and the potential at a distance r from a point charge q in an infinite dielectric of dielectric constant k is

$$V = \frac{q}{kr}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.28)$$

Now the capacity is by definition the ratio of the charge to the potential, and since the former is independent of the dielectric, the capacity must be directly proportional to the dielectric constant. Should therefore the space between the plates of a condenser be completely filled with dielectric, the capacity becomes k times its previous value. In fact the most convenient method of defining the *dielectric constant is that it is the ratio of the capacities of a condenser when the space separating the plates is filled with the dielectric and when it is a vacuum, respectively.*

If the insulated plate is removed with insulating tongs from a charged condenser filled with dielectric, then it will be found that only a fraction of the charge is removed with the plate. If the plate is now replaced, the charge it acquires on contact with the dielectric will be almost equal to the original charge. This experiment shows that the charge is absorbed by the dielectric somewhat as a sponge absorbs water. The penetration is small, as is shown by the fact that it is readily given back to the plate when it is replaced. Dielectrics differ considerably amongst themselves in this property of absorption, which is a function of the molecular structure of the material. The phenomenon is associated with the electrical hysteresis (see Chap. XV) of the specimen.

In terms of the electron theory a good insulator is a material in which the electrons move with extreme difficulty. The negative charges on the condenser plate do, however, move a *short* distance into the dielectric, and similarly the electrons in the dielectric move a *short* distance out of the material towards the positive plate. This general drift extends throughout the entire dielectric, which consequently develops charges of opposite sign on its two surfaces and is said to be in a state of *electrical strain*. The commonly used dielectrics such as mica and ebonite have dielectric constants lying between 1 and 10. These substances are insulators, but conductors also have dielectric properties. Indeed, water has the abnormally high value of 80 for its dielectric constant. The measurement of dielectric constants is described in the next chapter.

EXERCISES

1. Define the term "capacity of a conductor".

Regarding the earth as a sphere of radius $2 \times 10^9/\pi$ cm., show that its electrical capacity is about 700 microfarads. (1 microfarad $= 9 \times 10^5$ electrostatic units.)

2. Distinguish between the electrical force in the region between two flat condenser plates and the mechanical force of attraction between the plates, and show that the ratio of these two magnitudes is $2/\sigma A$, where A is the area of one of the plates and σ is the surface density of charge.

3. Obtain an expression for the electrical energy stored in a charged condenser of any type. Deduce the same result for the special case of a spherical conductor by the principle of virtual work.

4. Show that when two equal condensers are connected in parallel the system has four times the capacity of that obtained when the condensers are connected in series.

5. Explain fully the meaning of the term "dielectric constant" of a material. Upon what factors does the value of the dielectric constant depend?

CHAPTER IV

Electrostatic Measurements

1. Introduction.

For measuring small charges and potentials the gold-leaf electroscope is very suitable, as the charged system may be made extremely small and therefore of small capacity. However, for many purposes a more robust instrument is required, and electrometers and electrostatic voltmeters satisfy this requirement.

*2. Quadrant Electrometer.

The quadrant electrometer, as shown in fig. 1, consists of a flat cylindrical brass pillbox B divided into four separate quadrants. Between the top and bottom of the quadrants is a light paddle C of

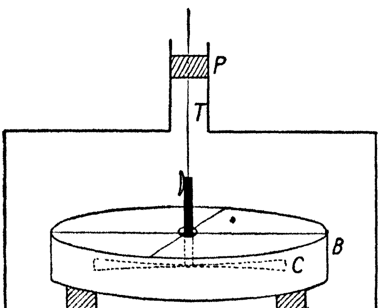

Fig. 1. — Quadrant Electrometer

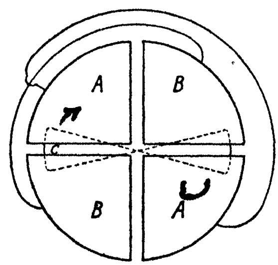

Fig. 2. — Quadrant Electrometer: Connections

aluminium suspended by a delicate torsion fibre T of phosphor bronze. The latter is insulated from the rest of the instrument by an amber plug P, and the brass quadrants likewise are supported on amber blocks B_1. Opposite pairs of quadrants are connected as shown in fig. 2, and the paddle may either be completely insulated from them or may be connected with either pair of quadrants as required.

When the quadrants and paddle acquire electric charges, electrostatic forces are set up which rotate the paddle until an equal and opposite torque is introduced by the suspension. The deflection is magnified by reflection of light from a mirror attached rigidly to the paddle.

Let V_A, V_B and V_C be the potentials of the two pairs of quadrants and paddle respectively, and suppose the paddle twists through an angle θ so that it passes towards the quadrants A and away from B. The capacity of the A-C system is increased and that of the B-C system is decreased by an amount $P\theta$, where P is a constant. Then the change in electrical potential energy of the system is, from (3.22),

$$\begin{aligned} W_E &= \tfrac{1}{2}P\theta(V_1^2 - V_2^2) \\ &= \tfrac{1}{2}P\theta\{(V_C - V_B)^2 - (V_C - V_A)^2\} \\ &= P\theta(V_A - V_B)\left(V_C - \frac{V_A + V_B}{2}\right), \quad . \quad . \quad (4.1) \end{aligned}$$

where $V_1 = (V_C - V_B)$ and $V_2 = (V_C - V_A)$.

The gain in potential energy of the suspension is

$$W_G = \int_0^\theta G\theta \, . \, d\theta = \tfrac{1}{2}G\theta^2, \quad . \quad . \quad . \quad . \quad (4.2)$$

where G is the couple required to produce unit angle of twist. Now the sources of supply to which the quadrants and paddle are connected will have supplied energy at *constant* potentials V_A, V_B and V_C respectively. Consequently in equation (3.19) the integration may be effected directly with v constant and will give

$$W = QV = CV^2 = Q^2/C, \quad . \quad . \quad . \quad . \quad (4.3)$$

which is *twice* the value obtained when the charged conductors are not connected to steady sources of potential. This energy derived from the source will therefore be equal to the sum of (4.1) and (4.2) and will actually be numerically equal to twice (4.1). We therefore have

$$W = W_G + W_E,$$

that is,

$$\begin{aligned} 2P\theta(V_A - V_B)&\left(V_C - \frac{V_A + V_B}{2}\right) \\ &= \tfrac{1}{2}G\theta^2 + P\theta(V_A - V_B)\left(V_C - \frac{V_A + V_B}{2}\right), \end{aligned}$$

or

$$\theta = K(V_A - V_B)\left(V_C - \frac{V_A + V_B}{2}\right), \quad . \quad . \quad . \quad (4.4)$$

where $K = 2P/G =$ constant.

The constant K, which involves the linear dimensions of the apparatus, cannot be accurately calculated and consequently the instrument is *not absolute* but requires calibration with sources of known potential. The two points whose potential difference is required are connected to A and B respectively, and the paddle C is connected either (1) to an independent potential of much greater magnitude than either V_A or V_B, or (2) to A or B. The former or *heterostatic arrangement* is most frequently used for measuring steady potentials. The steady potential difference is connected across A and B while the paddle C is connected to a separate and much higher potential. Since then $V_C \gg V_A$ or V_B, if small potentials are being measured,

$$\left(V_C - \frac{V_A + V_B}{2}\right) \simeq V_C = \text{constant};$$

so that equation (4.4) becomes

$$\theta = K'(V_A - V_B), \qquad \ldots\ldots \quad (4.5)$$

where $K' = KV_C =$ constant. The potential difference is therefore directly proportional to the deflection.

If the potential to be measured across A and B is alternating, the previous arrangement will show no deflection, for the paddle is alternately attracted and repelled by a given pair of quadrants too rapidly for it to respond. A unidirectional effect is produced by connecting the paddle to either A or B so that $V_C = V_A$, say. Then equation (4.4) becomes

$$\theta = \frac{K}{2}(V_A - V_B)^2. \quad . \quad (4.6)$$

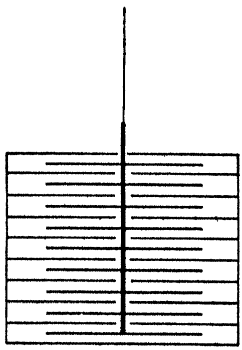

Fig. 3.—Multiple Quadrant Electrometer

The deflection is now proportional to the square of the potential difference and its direction is therefore independent of the sign of the potential. Physically, the quadrant and leaves which are connected together always acquire charge of the same sign simultaneously and therefore always exert a mutual repulsion.

By increasing the number of paddles and quadrant boxes, as shown in fig. 3, the deflection may be proportionately magnified. A more robust suspension then becomes possible without decrease of sensitivity and such instruments constitute **electrostatic voltmeters.**

3. Attracted Disk or Absolute Electrometer.

In fig. 4 is shown a diagram of the attracted disk electrometer. This instrument requires careful manipulation and consequently is used only for standard calibrations. It consists of two flat circular condenser plates P_1 and P_2 set horizontally. One plate is suspended by inclined strings from a balance arm which can be suitably counterpoised. The other plate is insulated and attached to a micrometer screw M so that the distance between the two plates can be varied. When a potential difference is applied between the plates, an attraction takes place between the equal and opposite charges acquired by the plates, the force of attraction being

$$F = 2\pi\sigma^2 A,$$

according to equation (3.18).

This force is directly measured from the value of the counterpoise on the opposite arm of the balance and hence

$$F = mg = 2\pi\sigma^2 A. \quad \ldots \quad (4.7)$$

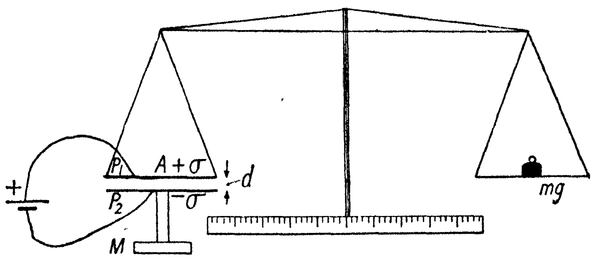

Fig. 4. — Attracted Disk Electrometer

Since m, g and A are known, σ can be calculated **absolutely** from (4.7). Finally since the potential difference V between the two plates is given by

$$V = 4\pi\sigma d,$$

equation (4.7) may be written

$$V = 4\pi d\sqrt{\frac{mg}{2\pi A}}. \quad \ldots \ldots \quad (4.8)$$

It is much easier to achieve a balance by varying d than by varying m, and consequently the final adjustment is made with the micrometer screw.

Since the calculations apply to the attraction between two plates when the lines of force *all* run perpendicular to the plates, a correction is necessary for the curvature of the peripheral lines if the apparatus is as in fig. 4. In practice, the use of a correction is avoided by using annular metal guard rings (not shown) in the plane of the plates. The gap between the plates and the rings is very small and the moving plates behave essentially as the central portion of larger plates. The lines of force are therefore all practically perpendicular to the moving plates.

Measurement of Dielectric Constant.

As an example of the use of the quadrant electrometer we shall describe the determination of the dielectric constant of a slab of material. We require a parallel plate condenser which fits the slab; the capacity C_1 of this condenser is calculated from equation (3.10) when the area of the plates and the plate separation have been measured. It is first essential to determine the capacity C_E of the electrometer itself. The electrometer is initially charged with an unknown charge Q which gives a deflection θ_1. This charge is then shared with the condenser and the deflection falls to θ_2. Introducing a constant of proportionality P, we have

$$Q = C_E \, . \, P\theta_1 = (C_E + C_1)P\theta_2,$$

whence

$$C_E = C_1 \frac{\theta_2}{(\theta_1 - \theta_2)}. \qquad (4.9)$$

The slab of dielectric is then inserted and the deflection falls still farther to θ_3 as the capacity of the condenser increases to C_2, where the dielectric constant k is defined by $k = C_2/C_1$. By analogy with our deduction of (4.9) we have

$$Q = C_E \, . \, P\theta_1 = (C_2 + C_E)P\theta_3,$$

whence

$$C_2 = C_E \frac{(\theta_1 - \theta_3)}{\theta_3}, \qquad (4.10)$$

and from (4.9) and (4.10)

$$C_2 = \frac{C_1\theta_2}{\theta_3} \frac{(\theta_1 - \theta_3)}{(\theta_1 - \theta_2)},$$

or

$$k = \frac{C_2}{C_1} = \frac{\theta_2}{\theta_3} \frac{(\theta_1 - \theta_3)}{(\theta_1 - \theta_2)}. \qquad (4.11)$$

The dielectric constant of gases and conducting liquids is found by filling a container with the fluid and inserting it as a close fit in the condenser. The correction due to the container itself is found by a subsidiary experiment. The dielectric constant of a gas is directly proportional to the pressure over a wide range.

These values of the dielectric constant are for a steady electric field between the plates. If the field is varying (see Chap. XV) the dielectric constant is a function of the frequency of the electrical oscillations. This is because the electrical strain set up in the dielectric takes a finite time to change when the field variation takes place.

5. Electrostatic Units of Measurement.

We have already defined the *electrostatic unit charge* (p. 9); it remains to specify units of potential and capacity.

Unit electrostatic potential difference is said to exist between two points when one erg of work is done in transferring one electrostatic unit of charge from one point to the other.

The *unit of capacity in the electrostatic system* is the capacity of a condenser in which electrostatic unit charge is associated with electrostatic unit difference of potential; it has the dimensions of a length, and therefore may be said to be 1 cm.

When electromagnetic phenomena have been considered we shall find that a second system of units, the **electromagnetic system,** arises quite naturally from the phenomena considered. Owing to the physical connexion between electrostatic and electromagnetic phenomena these two systems of units are interrelated and it is frequently necessary to convert from one system to the other.

In practice, the potentials, charges, currents and other electrical quantities involved are often of a magnitude which is of a different order from the units defined in the electrostatic and electromagnetic systems. There is therefore a third or **practical system of units.** The subject is discussed more fully in Chap. VIII.

EXERCISES

1. Describe the quadrant electrometer and explain how it may be used to measure (*a*) alternating potential, (*b*) direct potential.

2. Compare and contrast the gold-leaf electroscope and the quadrant electrometer as potential measuring instruments.

3. Describe the operation of the attracted disk electrometer, deducing any formulæ required from first principles.

CHAPTER V

Electrostatic Machines

1. The Electrophorus.

The amount of electricity which can be obtained by friction with cat's fur on an ebonite rod is very limited, and consequently machines have been devised to supply larger quantities of electricity at higher potentials. The simplest of these devices is the electrophorus, a diagram of which is shown in fig. 1. It consists essentially of a flat ebonite disk D, the upper surface of which is electrified by friction with cat's fur in the usual fashion. A flat brass covering disk C fitted with an insulating handle rests on the excited ebonite disk and a charge is conveyed to C by induction. This, of course, necessitates earthing C

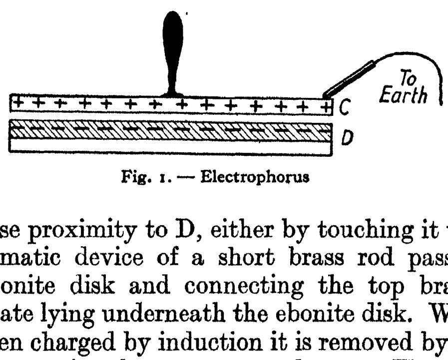

Fig. 1. — Electrophorus

while it is in close proximity to D, either by touching it with the finger or by the automatic device of a short brass rod passing vertically through the ebonite disk and connecting the top brass plate with another brass plate lying underneath the ebonite disk. When the covering plate has been charged by induction it is removed by the insulating handle and gives up its charge to a condenser. The disk C is then returned to the ebonite disk, which throughout the process has retained its original charge, and the whole operation is repeated. In this way, theoretically an infinite charge may be taken away on the brass disk. The electrical energy is derived from the mechanical energy required to separate the opposite induced and inducing charges. In practice, the charge which may be conveyed to a given condenser is limited by the rise in potential of that condenser as the charging process continues. A time arrives when the potential acquired by the charging disk C of the electrophorus is little more than that of the insulated plate of the charged condenser. Very little additional charge is then conveyed in each further operation as the charging disk is put in contact with the condenser.

It must be noted that the production of a charge on C is entirely due to induction from the ebonite disk D. That is, although C is laid on D, the latter is such a poor conductor that C acquires little charge by contact from D. If C and D are exceptionally smooth, C may carry away very little charge indeed, for the charge obtained by good contact with D may exactly neutralize the charge produced by induction. A high degree of planarity of D is therefore deliberately avoided, so that good contact with C is made at only a few unavoidable places when the latter is laid over the former.

2. The Wimshurst Influence Machine.

To avoid the labour of manipulation with the electrophorus and to provide still greater electrical energy, continuously operated machines were devised. These were originally friction machines in which charges were generated by rotating glass cylinders against rubbers which pressed in contact with the surface. Metal foils fixed to the surface of the cylinder served to collect the charges which were removed by wire brushes which were in light contact with the surface of the cylinder. Such friction machines were found to be very erratic and gave way to various influence machines, of which the Wimshurst is a typical example.

As shown in fig. 2, the Wimshurst machine consists of two flat glass disks mounted on a common axis and capable of rotation in opposite directions. The disks are coated with shellac to render them non-hygroscopic, and are rotated either by hand or by electric motor. A number of tin-foil sectors are fixed symmetrically to the disks and the latter revolve between two forks F, F′ provided with sharp points which are situated close to the disks but which do not actually come in contact with them. The forks or *collectors* are connected by brass rods to two metal spheres S, S′ on which the charges collect, and the spheres are usually fitted with condensers C, C′, so that the charge, which is generated continuously, may be continuously stored. Two mutually

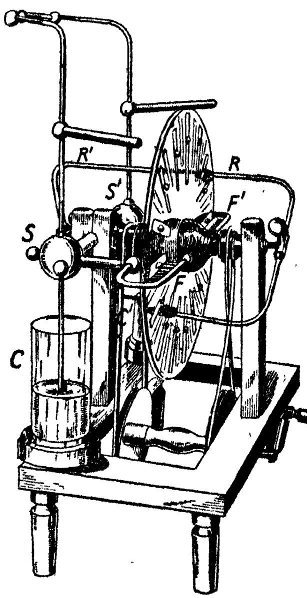

Fig. 2. — Wimshurst Machine

perpendicular brass induction rods R, R′, terminated by fine wire brushes which pass over the surface of the metal sectors, remain stationary as the disks rotate.

The operation of the machine may be explained by reference to fig. 3, in which the two disks are for clarity drawn of unequal size, although in reality they are as identical as possible. The machine requires a small residual charge on the sector A before the generation of subsequent charge will take place. This initial charge is usually resident on the surface of the machine when it has once been used, but in damp weather it may be necessary to convey a charge to the surface by induction from an excited ebonite rod in the usual fashion. The positive charge on A induces a negative charge on the sector B, and since the latter is connected to the brass rod R, a corresponding

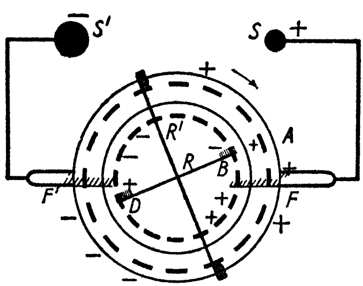

Fig. 3. — Action of Wimshurst Machine

induced positive charge develops on the conductor D which is at the remote end of R. Owing to the rotation of the two disks in opposite directions, the R-B and R-D contacts are broken while they are still under the influence of the charge on A. Consequently a positive and negative charge is left on the sectors D and B respectively. The charge on A has therefore now generated two additional charges on the system. These two charges repeat the inducing effect when they come opposite the ends of the second rod R′. In this way more charges are produced on the back plate containing the original sector A. This mutual induction process continues until large charges and consequently high potentials are produced on the sectors of both plates. It will be observed that the rotation of the two plates in opposite directions is such as to convey charges of one sign to one side of the apparatus and of the opposite sign to the other side. This has an important result on the collection process which thereby becomes unidirectional. The collectors on the fork F acquire a negative charge which is so high that the insulation resistance of the air gap breaks down. The negative electricity streams from the points and partially neutralizes the positively charged sectors. An equal and opposite positive charge therefore develops on the sphere S, so the effect is as if an equal positive charge were conveyed from sector to points. Meanwhile, at the other fork F′, the negative charge streams from the sector to the points under the action of the intense electric field produced between the high negative charge on the sectors and the high positive charge which is induced on the points. The sphere S′ therefore acquires a net negative charge. We emphasize that at both collectors it is the negative electricity which moves, in one case from sector to points, and in the other in the reverse direction from points to sector. Actually, when the insulation resistance breaks down, the air in the spark becomes ionized (see Chap. XVIII), and ions of both signs being present, there is a certain amount of electricity of both signs travelling across the gaps between sectors and points at both F and F′ under the potential difference which exists.

Theoretically there is no limit to the electrical energy which may be obtained. In practice, frictional losses are inevitable in the moving system, but the real limit to the electrical energy available lies in electrical losses. The air in the whole neighbourhood of the machine becomes slightly conducting owing to the ionization at the collectors and unavoidable sparking at the brushes. Consequently losses occur through the air from various parts of the machine, the losses being proportionately greater where the surface density is largest, that is at angles and points. The losses there are usually so intense that a blue glow is observed in the neighbourhood; the process is commonly termed "brushing" and is said to be due to *corona discharge.* Again, the surface insulation between the sectors breaks down as the potential difference between different sectors becomes very large, and brushing to earth across various parts of the insulators is always taking place to a certain extent. In spite of these difficulties, several Wimshurst machines connected in parallel and suitably motor-driven will supply a current of several milliamperes at a hundred thousand volts.

3. The Van der Graaf Generator.

If potentials of millions of volts are required, a machine like the Van der Graaf generator must be used. This consists of a vertical tube of insulating material which may be several metres high and up to a metre in diameter. As shown in fig. 4, the tube is surmounted by a large metal sphere which is the system ultimately to be charged to the high potential. Inside the vertical tube runs a silk band which is connected electrically at the top to the inside of the metal sphere so that any charge on the band may be communicated to the inside, and hence automatically to the outside, of the sphere. The silk band is caused to rotate, and charge is sprayed on the band either by a battery of mechanically-driven Wimshurst machines situated at the base of the tube or from a high-voltage transformer (see Chap. XIII). In this way the charge on the sphere may be continuously increased until it is several million volts above earth potential. If one such system is

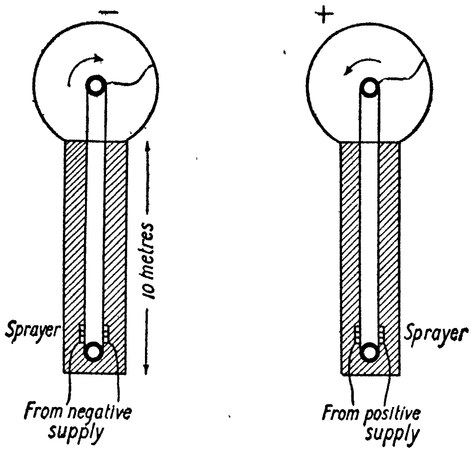

Fig. 4. — Van der Graaf Generator

charged with positive, and another with negative electricity a potential difference of ten million volts may be set up comparatively easily between the two spheres.

EXERCISES

1. Explain in detail the operation of an electrostatic induction machine.

2. Compare the advantages and disadvantages of a battery of Wimshurst machines and a transformer as sources of high potential.

CHAPTER VI

Elementary Magnetism

1. Introduction.

The fact that certain iron ore found in Magnesia in Asia Minor possessed the property of attracting small pieces of iron and steel was known to the ancients. This ore, which is termed **magnetite,** is said to exhibit magnetic properties or to possess **magnetism** just in the same way as rubbed amber is said to possess electrification.

2. Magnetic Substances.

All materials, if they are sufficiently light, show electrical attraction; but unless the apparatus is extremely sensitive, only a few substances exhibit any attraction under magnetic forces. These substances form the *ferro-magnetic group* and consist essentially of iron, nickel and cobalt and certain of their alloys. Again, while a closed conducting surface acts as a perfect *electrostatic screen* in that enormous charges outside or on the surface fail to produce any electrical forces within the conductor, magnetic forces readily penetrate conductors. For example, iron filings inside a brass vase respond as readily to magnetite placed outside the vase as when the separation is due to air only.

In the case of electric charge, mere contact between the electrified body and an initially uncharged conductor is sufficient to convey charge to the latter, especially if the electrified body is itself a conductor. On the other hand, mere contact between a magnet and a magnetic body is quite insufficient to produce any **permanent magnetism** of the latter. Magnetic substances readily become magnets, however, if stroked systematically by natural magnetite. This process may be repeated indefinitely without any appreciable diminution of the magnetic power of the magnets used. This again contrasts with the transference of electric charge, the latter being continually reduced in amount by sharing.

If a bar of iron which has been magnetized—that is a **bar magnet**—is plunged into a bed of iron filings the latter are found to adhere most strongly at the ends of the bar. These regions of intense magnetic force are termed the **poles** of the magnet. An imaginary line joining the two poles is termed the **magnetic axis,** and an imaginary line running round the centre of the magnet is termed the **magnetic equator.**

3. Directional Properties.

If a bar magnet is suspended by a torsionless fibre it is found that it sets with its axis pointing approximately geographically north and south, and if disturbed always returns to this position. This property—the knowledge of which, as of magnetic attraction, is of great antiquity,—constitutes the basis of the *magnetic compass*. The pole which points to the north, or the *north-seeking pole*, is termed the **north pole** and the other is termed the **south pole** of the magnet. There is therefore a formal correspondence with the N-pole and S-pole and positive and negative electricity. Experiment shows that this correspondence is much deeper, for just as like charges repel and unlike charges attract each other, **like poles repel and unlike poles attract each other.** Thus, if one bar magnet is freely suspended, and another is brought up to it, repulsion takes place if the two N-poles or two S-poles are presented to each other and attraction if a N-pole and a S-pole are so presented.

4. Magnetic Field and Unit Pole.

The region around a magnetic pole is termed its **magnetic field** and, just as in the electrical case, magnetic forces are imagined as due to myriads of **magnetic lines of force.** A magnetic line of force is therefore an *imaginary line showing the direction of the magnetic force at any point in the magnetic field.* Similarly, attractions and repulsions are

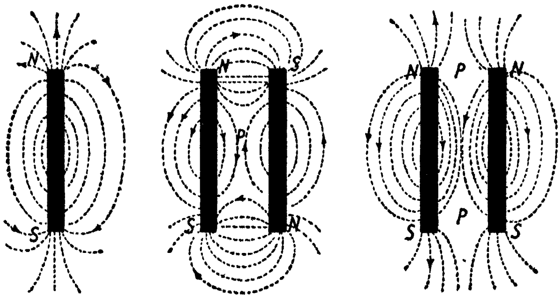

Fig. 1. — Magnetic Lines of Force

pictured mechanically as arising from the tension and repulsion of *magnetic tubes of force.*

In the same way that electrostatic lines of force are mapped out with crystals of oxalic acid, so magnetic lines of force may be mapped out with iron fillings. In fig. 1 is shown the distribution of the lines of force for various arrangements of one or more magnets. The centres P of the diamond-shaped regions which are free from magnetic force are termed *neutral points.*

Again, the definition of unit magnetic pole is absolutely analogous to that of unit charge. *A unit magnetic pole is that pole which exerts a force of* 1 *dyne on an equal pole at a distance of* 1 *cm., both poles being situated in a vacuum.*

Similarly, the *intensity of a magnetic field at any point is measured by the force which would be exerted on a unit pole placed at that point.* The unit of magnetic field strength is termed the *oersted* (originally *gauss*).

Experiment shows, however, one fundamental difference between magnetic poles and electric charges in that while it is easy and usual to obtain separate positive and negative charges, N-poles and S-poles always occur together, as in a bar magnet. If the bar magnet is broken diametrically in an effort to separate the N-pole from the S-pole, another N-pole and S-pole will be found to have been generated at the point of fracture. This process is repeated until the fragments are of molecular size. The concept of a unit pole is therefore a purely theoretical one, the experimental unit being a short bar magnet or *magnetic dipole.*

5. Molecular Theory of Magnetism.

The behaviour of magnetic substances is readily explained on the molecular theory of magnetism. This states that magnetic substances consist permanently of large numbers of molecular magnets. In an unmagnetized bar these molecular magnets are arranged completely

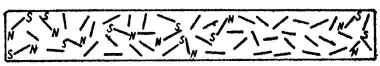

Fig. 2. — Unmagnetized Bar

at random as shown in fig. 2. The bar as a whole therefore exhibits no magnetism, neither attractive nor directive. The process of magnetization consists in the gradual adjustment of these molecular magnets as shown in fig. 3. Under the influence of the magnetic field of the stroking magnet, the small molecular magnets turn slightly into the

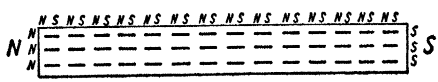

Fig. 3. — Process of Magnetisation

direction of the external magnetic field. The motion of the molecular magnets is opposed partly by friction and partly by their mutual magnetic interactions. Consequently the stroking process has to be repeated several times before appreciable magnetization is produced. When all the molecular magnets have become orientated in one direction, further magnetization is clearly impossible. This prediction of a limiting state by the molecular theory accords well with experimental observation. It is termed **magnetic saturation.**

Again, since mechanical fracture results in the separation of groups of molecules and not the fracture of any one molecule, the impossibility of separating a N-pole from a S-pole is easily explained. The development of fresh N-poles and S-poles on fracture is merely the separation of two adjacent layers of N-poles and S-poles already present. The large magnetic intensity at the poles of a magnet is clearly to be attributed to the large number of unbalanced N-poles and S-poles at the two ends respectively. Farther towards the centre of the magnet there is negligible external field, as the N-poles and S-poles are extremely close to each other and all the lines of force which arise from them run straight from one to the other.

It is to be expected that any process which tends to derange the molecules of a magnet would be likely to demagnetize it. This is found experimentally, percussion or heating being well-known methods of demagnetization.

6. Magnetic Induction.

If an unmagnetized iron bar is presented to a compass needle it attracts both ends. On the other hand, one end of a magnetized bar will repel one end of the compass needle when like poles are presented to each other. Consider now the experiment shown in fig. 4, where an

Fig. 4. — Magnetic Induction

unmagnetized bar is placed close to a compass needle and a bar magnet is brought up to the far end of the unmagnetized bar. It is found that the hitherto unmagnetized bar will repel one end of the compass needle, showing that under the influence of the neighbouring magnet it becomes temporarily magnetized. This process is termed **magnetic induction.** If the unmagnetized bar is of soft iron the effect will be large while the magnet is in the neighbourhood, but will completely disappear when the magnet is removed. Soft iron is therefore said to possess high magnetic **susceptibility** but poor **retentivity.** On the other hand, if the bar is of steel, the effect is small but persists slightly after the exciting magnet is removed. Steel therefore possesses low susceptibility but high retentivity.

The analogy with electrostatic induction is complete in that the attraction of unmagnetized bodies is due to induction. Under the inducing field, a pole of opposite sign is temporarily formed at the end closest to the magnet and a pole of the same sign is formed at the far end. Owing to the closer proximity of the unlike poles, a net attraction occurs between the magnetic substance and the magnet.

7. Terrestrial Magnetism.

The directional properties shown by a freely suspended bar magnet are due to the presence of a magnetic field associated with the earth. The precise cause of the earth's magnetic field is still uncertain, but its properties are conveniently summarized by noting that except for local irregularities it is the field which would be expected if a short bar magnet were situated at the earth's centre, and lay so that its S-pole pointed not quite N along the earth's geographical axis. This

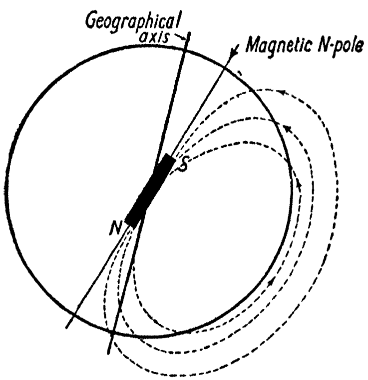

Fig. 5. — Approximation to Earth's Magnetic Field

implies a distribution of lines of force as shown in fig. 5. The points where the magnetic axis cuts the earth's surface are termed the magnetic poles.

It will be observed that the lines of force do not in general lie parallel to the earth's surface. If a magnet were freely suspended it would therefore be expected to set at an angle to the horizontal. This angle is termed the **angle of dip**, and is characteristic of the magnetic field at any place. As we travel from the magnetic equator to the magnetic pole, the angle of dip varies from 0° to 90°. In practice the angle of dip is measured with a dip circle, as described in the next chapter. The **isoclinal lines** are imaginary lines drawn over the earth's surface connecting points of equal dip. They lie parallel to the lines of latitude except for local variations. It is clear that the horizontal force exerted on a bar magnet as ordinarily suspended, or on a compass needle, is only the *horizontal component H* of a *total magnetic intensity I* which is directed along the angle of dip. It is often convenient to regard I as possessing both the horizontal component H and a vertical component V and to make use of the relations

$$I^2 = H^2 + V^2, \qquad (6.1)$$

and

$$\tan D = \frac{V}{H}, \qquad (6.2)$$

where D is the angle of dip.

The deviation of a compass needle from the geographical meridian at any point is termed the **angle of declination** at that point. The **isogonal lines** or lines of equal declination run roughly parallel to the geographical lines of longitude but large irregularities are common. For example, there is an **agonal line** of zero declination which forms a closed curve termed the *Siberian oval.* Such irregularities are usually associated with massive ore deposits of magnetic material. Lines connecting points of equal H are termed **isodynamic lines.**

Apart from the variations in terrestrial magnetic properties across the earth's surface, temporal variations occur of both a regular and an irregular nature. For example, records show that there is a cyclic variation in the magnetic declination with a complete period of about 1000 years. Its present value in London is about 14° W., its maximum of $24\frac{1}{2}$° W. was reached in 1820, and it is expected to reach zero in 2139. Superposed on this long-period variation are annual and daily variations. The annual variation, which is oppositely directed in the northern and southern hemispheres, is about $2\frac{1}{4}'$: the maximum westerly deviation occurring in the spring and the maximum easterly deviation in the autumn. Daily variations are more irregular, but are generally to be distinguished from *magnetic storms.* These give rise to large and irregular fluctuations in magnetic intensity and are usually quite unpredictable.

As sources of the earth's magnetism, the existence of permanent magnets in the interior of the earth, of magnetism arising from electric currents in the earth's interior, of effects due to the circulation of large electric currents in the upper atmosphere, and of magnetic influences from the sun and other celestial bodies have all been suggested. Little agreement has been reached among authorities except as to the inadequacy of the proposed suggestions.

EXERCISES

1. Compare and contrast electrification and magnetization. What is meant by the statement that the natural unit of magnetism is a dipole?

2. Define *unit magnetic pole* and *strength of a magnetic field.* Give a brief account of the molecular theory of magnetism.

3. Write a short essay on terrestrial magnetism.

CHAPTER VII

Magnetic Measurements

1. Magnetic Moment.

We have seen that experimentally it is impossible to separate completely a N-pole and a S-pole and that the natural magnetic element is the dipole rather than the unit pole. Similarly, on a larger scale the behaviour of a magnet is characterized by the distance between the poles as well as the pole strengths themselves. We shall now show that the characteristic quantity is the **magnetic moment** M, which is defined as the *product of the pole strength* m *and the distance between the poles* $2l$, that is

$$M = 2ml. \qquad (7.1)$$

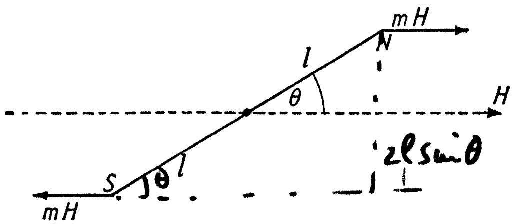

Fig. 1. — Moment of a Magnet

If a magnet is freely suspended so as to turn in a horizontal plane, it can be seen from fig. 1 that when the displacement from the magnetic meridian is θ, there is a *restoring couple or moment* G given by

$$G = 2Hml \sin\theta. \qquad (7.2)$$

If the field H is of unit strength and the **magnetic axis** is at right angles to the field so that $\theta = 90°$, equation (7.2) becomes

$$G = 2ml. \qquad (7.3)$$

The magnetic moment, or moment of the magnet, $2ml$, is thus connected with a mechanical turning couple or moment.

2. Magnetic Law of Force.

The law of force between two magnetic poles, just as for two electric charges, is that of the inverse square. We have seen how a very indirect proof is necessary in the electrostatic case, owing to the inevitable leakage of charge if a direct proof is attempted. In the magnetic case, since separate N- and S-poles cannot be obtained, a proof based upon the forces between two dipoles must be devised. It is, however, convenient to note here that since the same law is obeyed as in electrostatics, Gauss's theorem and all the other theorems there derived may be taken over directly and applied to the magnetic case.

For example, *magnetic potential at a point in a magnetic field is defined as the work done in bringing a unit pole from infinity up to that point.* Further, its value at a distance r from a point magnetic pole of strength m will be

$$V = \frac{m}{r}, \quad \ldots \ldots \quad (7.3)$$

by analogy with (2.7). The lines of force will be radial and the equipotential surfaces concentric spheres.

3. Force due to a Magnetic Dipole.

Since the dipole is the natural magnetic unit, we require an expression for the force due to a magnetic dipole of pole-strength m, separation $2l$, at a distance r from the centre of the dipole, the line joining the point to the centre of the dipole being inclined at θ to the magnetic axis. We shall assume that $r \gg 2l$, to simplify the calculation. The total magnetic potential at P in fig. 2 is then approximately, from equation (7.3),

$$\begin{aligned} V &= \frac{m}{(r - l\cos\theta)} - \frac{m}{(r + l\cos\theta)} \\ &= \frac{2ml\cos\theta}{(r^2 - l^2\cos^2\theta)} \\ &= \frac{M\cos\theta}{r^2}, \quad \ldots \ldots \quad (7.4) \end{aligned}$$

since $r \gg l$.

Now by equation (2.8), the force in any direction is equal to the derivative of the potential in that direction, with sign changed. Hence the force at P directed along r is

$$F_r = -\frac{\partial V}{\partial r} = \frac{2M\cos\theta}{r^3}. \quad \ldots \ldots \quad (7.5)$$

In a direction perpendicular to r the force is

$$F_\theta = -\frac{1}{r}\cdot\frac{\partial V}{\partial \theta} = \frac{M \sin\theta}{r^3}, \quad . \quad . \quad . \quad . \quad (7.6)$$

and hence the total force F_{T} at P may be obtained if required from

$$F_{\mathrm{T}}^2 = F_r^2 + F_\theta^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.7)$$

At a point on the magnetic axis produced, $\theta = 0$, hence $F_\theta = 0$, the force is directed along the axis, and

$$F = \frac{2M}{r^3}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.8)$$

while for a point on the perpendicular bisector of the axis $\theta = 90°$, hence $F_{\mathrm{R}} = 0$, the force is directed parallel to the axis and

$$F = \frac{M}{r^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.9)$$

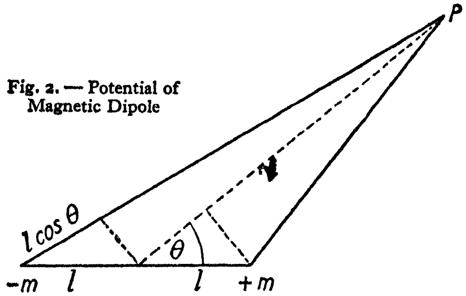

Fig. 2. — Potential of Magnetic Dipole

For an accurate test of the inverse square law we cannot neglect $l \cos\theta$ compared with r in the equation preceding (7.4). We shall, however, be concerned with only the two symmetrical positions, namely on the axis and on the perpendicular bisector of the axis. For these positions, which are known as the Gauss A and Gauss B positions respectively, we have $\theta = 0$ and $\theta = 90°$, so for the first position

$$F_1 = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{M}{r^2 - l^2}\right) = \frac{2Mr}{(r^2 - l^2)^2}. \quad . \quad (7.10)$$

For the second position, which lies equidistant from the N- and S-poles, the potential is zero. Applying the inverse-square law, however, the force is easily calculated directly: thus

$$F_2 = 2 \cdot \frac{m}{(r^2 + l^2)} \cdot \frac{l}{(r^2 + l^2)^{1/2}}$$

$$= \frac{M}{(r^2 + l^2)^{3/2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.11)$$

4. The Deflection Magnetometer.

To test equations (7.8)–(7.11) the deflection magnetometer is used. As shown in fig. 3, this consists of a short bar magnet to which is attached a light aluminium pointer moving over a horizontal circular scale. In the most accurate instruments the pointer is replaced by a mirror and the system is usually suspended rather than pivoted from below. The case enclosing the needle is mounted accurately at the centre of a metre scale of some non-magnetic material such as wood, and a short bar magnet is placed on the arm of the magnetometer at a suitable distance from the suspended needle. Initially the bar magnet is removed

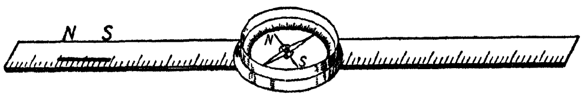

Fig. 3.—Deflection Magnetometer

and the needle comes to rest under the influence of the earth's horizontal component alone. If the Gauss A or *end-on* position is to be examined, the arms of the magnetometer are rotated until the suspended magnet lies at right angles to them. On putting the magnet in the end-on position, the needle swings through an angle θ which is then read. The conditions are as shown in fig. 4, that is,

$$F_1 = H \tan \theta_1 = \frac{2Mr}{(r^2 - l^2)^2} = \frac{2M}{r^3}, \text{ approx.} \quad (7.12)$$

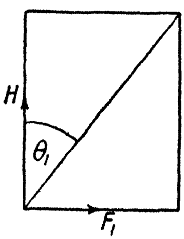

Fig. 4

Both ends of the pointer are read, the readings are repeated with the magnet reversed, and another set is taken with the magnet on the other side of the magnetometer. The final value of θ_1 is the average of the eight readings.

The arms of the magnetometer are then swung through 90° and the magnet placed in the *broadside* or Gauss B position and the new angle θ_2 observed. Hence from equation (7.9)

$$F_2 = H \tan \theta_2 = \frac{M}{r^3}, \quad . \quad . \quad . \quad . \quad . \quad (7.13)$$

whence from (7.12) and (7.13)

$$\tan \theta_1 = 2 \tan \theta_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.14)$$

This affords only an approximate test of the inverse square law as we have neglected l compared with r. More accurately, we may concentrate on a series of readings for different values of r in either the Gauss A or Gauss B positions. For the latter,

$$F_2 = H \tan \theta = \frac{M}{(r^2 + l^2)^{3/2}}, \quad . \quad . \quad . \quad . \quad (7.15)$$

so that, since M and H are constant, a straight line should be obtained if $\cot \theta$ is plotted against $(r^2 + l^2)^{3/2}$. Now actually the poles of a magnet are not situated exactly at its geometrical ends; in fact $2l$ represents the *equivalent length* of an ideal magnet equal in strength to the actual magnet but with ideal point poles. As the quantity l is not known, the test of (7.15) is obtained by writing it in the form

$$(r^2 + l^2) = \left(\frac{M}{H} \cot \theta\right)^{2/3}. \quad . \quad . \quad . \quad . \quad (7.16)$$

Plotting $(\cot \theta)^{2/3}$ against r^2 is found to give a straight line in accordance with the inverse square law on which the deduction of the formula is based. Further, the intercept on the r^2 axis when (7.16) is plotted gives the square of the equivalent length of the magnet.

5. Measurement of M and H.

Consideration of equations (7.12) and (7.13) shows that they are of such a form that even by taking several positions it is impossible to determine either M or H unless the other is known. Some other experiment is therefore required. This is obtained by suspending the short bar magnet, used on the arm of the deflection magnetometer, from a thread of unspun silk and allowing it to undergo torsional oscillations under the action of the earth's horizontal field. From fig. 1, the restoring couple for an angle of twist θ is

$$G = MH \sin \theta = MH\theta, \quad . \quad . \quad . \quad . \quad (7.17)$$

if θ is small. Hence from the treatment in Part I, Chap. IV, section 5, the period of oscillation is

$$t = 2\pi\sqrt{\frac{I}{MH}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.18)$$

where I is the moment of inertia of the magnet about the axis of suspension. In practice the magnet is suspended in a draught-free enclosure and oscillations are timed with a stop-clock. The moment of inertia is obtained from the mass and linear dimensions of the magnet. Application of (7.18) then gives the product MH and the ratio M/H is obtained from experiments in either the Gauss A or Gauss B positions. Hence the values of M and H are obtained absolutely.

6. The Oscillation or Vibration Magnetometer.

An oscillation or vibration magnetometer is a useful device for estimating the strengths of the horizontal components of magnetic fields. It consists simply of a very short magnet carrying a light aluminium pointer symmetrically at right angles and suspended inside a small cylindrical glass enclosure. For all positions of the magnetometer M and I remain constant, so that from (7.18)

$$H \propto \frac{1}{t^2} \propto n^2,$$

where n is the number of oscillations executed in a given time. If H varies rapidly from place to place, that is if the *field gradient* is steep, the magnet must be correspondingly small or the field will be different at the two ends of the magnet.

7. The Dip Circle.

The angle of dip which we have discussed in the previous chapter is measured with a dip circle, a diagram of which is shown in fig. 5. It consists of a compass needle which is initially unmagnetized and balanced about a horizontal axis. The needle is then magnetized and it is found that it dips at some angle to the horizontal. If the supporting axis is in any arbitrary position the dip observed will not in general

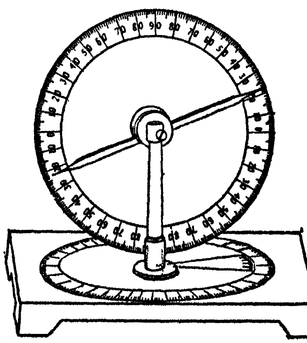

Fig. 5.—Dip Circle

be the true angle of dip. This is clear from fig. 6, for if the supporting axis is at an angle α to the magnetic meridian, the effective horizontal component acting on the *dip needle* is only $H \cos \alpha$. As the axis is rotated, therefore, the effective horizontal component varies from H, when the axis lies in the magnetic meridian, to zero, when the axis is perpendicular to this direction. In this last position the dip needle is under the influence of the earth's vertical component V only, and the dip needle therefore stands vertically. The horizontal axis is itself attached to a vertical axis, and the process of setting the dip needle is to locate the position when the dip needle stands vertical and then to rotate through 90° about the vertical axis. The horizontal axis then lies perpendicular to the magnetic meridian and the true angle of dip is registered.

In using the instrument, it is first levelled with a spirit-level and

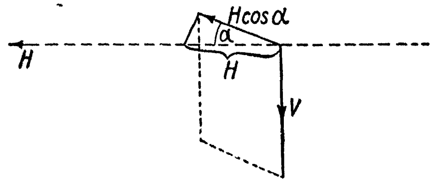

Fig. 6.— Forces on Dip Needle

then rotated so that the dip needle stands vertical on the circular scale which lies in a vertical plane. Rotation through 90° then brings the needle into the magnetic meridian and the readings of the two ends of the needle are taken. Further operations are:

(1) The horizontal axis is turned through 180° and a further pair of readings is taken. This is to eliminate error due to the zero line of the vertical scale not being horizontal (fig. 7*a*).

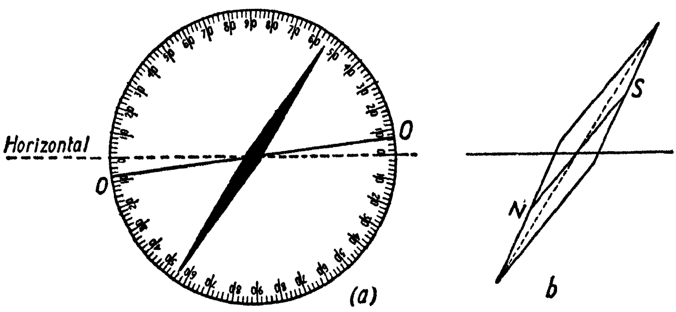

Fig. 7.— Dip Needle

(2) The needle is then turned over sideways and four further readings are taken. This allows for the magnetic axis of the needle not coinciding with the geometrical axis, as shown in fig. 7*b*.

(3) The needle is removed and remagnetized in the reverse direction and the preceding eight readings are repeated. This is to allow for the needle not being accurately supported through its C.G.

The average of the sixteen readings is taken as the angle of dip.

We consider magnetism further in later chapters after we have discussed the magnetic effect of an electric current.

EXERCISES

1. Define (*a*) the moment of a magnet, (*b*) the equivalent length of a magnet.

How may the equivalent length of a bar magnet be determined?

2. Upon what evidence may it be assumed that the force between two magnetic poles varies inversely as the square of their distance apart?

3. Define the term *magnetic potential* and calculate an expression for the magnetic potential at a point a distance r from the centre of a short magnet of moment M if the point lies on the magnetic axis produced. [M/r^2.]

4. Describe *two* experiments by which the moments of two magnets may be compared. Which method do you consider to be the better?

5. Describe carefully how the total intensity of the earth's magnetic field may be measured absolutely and accurately at any place.

6. Describe the use of the dip circle. If a dip needle which has been correctly set is then slightly displaced and allowed to oscillate it makes 3 vibrations per second at one place and 4 vibrations per second at another place. Compare the total intensity of the earth's magnetic field at the two places. [9 : 16.]

CHAPTER VIII

Elementary Properties of the Electric Current

1. Production of Electric Current.

An electric current consists of movement of electric charge. In solids generally, and in conductors in particular, motion is confined to the negative charges or electrons. We have seen in Chap. II that a charge moves, if it is free to do so, when a potential difference exists. For example, the charged disk of an electrophorus continues to convey charge to a conductor until the potential of disk and conductor is the same; that is, an electric current flows into the conductor until the potential is everywhere the same. Strictly speaking, the charge will only flow from the disk to the conductor if the former is negatively charged. If it is positively charged, electrons flow from the conductor to the disk until, as before, the potentials of the two are the same. Of course, the effect is the same as if positive charge had flowed from the disk to the conductor, and it is customary to use this description, as is quite permissible if it is borne in mind that in reality the flow is that of electrons in the reverse direction.

In electrostatics the flow of current is clearly very temporary, the potential being equalized in a very short space of time unless the bodies are very poor conductors indeed. The basic problem of current electricity is the continuous production of a potential difference so that a continuous current may be produced. This can be effected by some device such as the Wimshurst machine in which fresh charge is being continually generated. It suffers from the disadvantage, however, of producing too small a current for the majority of purposes, besides being cumbersome and requiring a source of mechanical power to drive the machine. Fortunately nature provides a great many ways in which a potential difference may be established. We tabulate some of these below. Only a few of them are of practical importance, but all are of interest in explaining electrical phenomena.

(1) *Percussion.*—This is closely allied to the production of electricity by friction between two bodies.

(2) *Vibration.*—This is simply internal friction in a body as opposed to external friction.

(3) *Disruption and Cleavage.*—These are further examples of violent friction.

(4) *Crystallization and Solidification.*

(5) *Combustion.*

(6) *Evaporation.*

(7) *Atmospheric Electricity.*—The analogy between electric sparks between two electrostatic charges at greatly differing potentials and lightning flashes is obvious. Franklin performed classical experiments to show that the two phenomena were identical. A kite flown in the clouds yielded abundant sparks between the string and the ground in thundery weather, especially if the string was rendered conducting by moisture. The function of pointed metal lightning conductors is to discharge clouds before they acquire sufficient potential to strike with

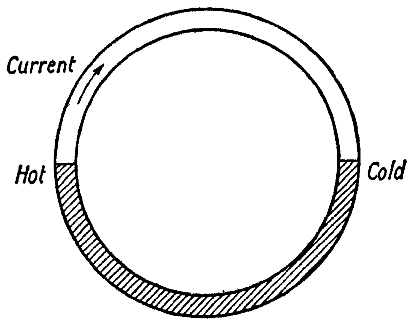

Fig. 1. — Thermoelectric Circuit

dangerous violence. As a source of electricity, atmospheric electricity is much too uncontrollable to be of practical importance.

(8) *Pressure.*—This is termed **piezo-electricity** and has important applications in the quartz oscillator which has been discussed in Part IV, Chap. IV.

(9) *Pyro-electricity.*—This is the development of differences of potential between different parts of certain crystals when they are heated. It is too small to be of use as a source of potential.

(10) *Thermo-electricity.*—If two dissimilar metals are joined together and one junction is at a higher temperature than the other, a steady potential difference is set up and hence a steady current flows around the wires which, when connected as in fig. 1, are said to constitute an **electric circuit.** The potential difference is small, but has important practical applications, and is discussed fully in Chap. XVII.

(11) *Contact of Dissimilar Metals.*—The ease with which electrons leave a given metal varies from metal to metal. Consequently, when two dissimilar metals are joined there is a temporary flow of electrons

from one metal to the other. This is said to be due to contact potential. It is not suitable as a source of electric current, but it must frequently be taken into account in interpreting electrical phenomena.

(12) *Chemical Action.*—If a zinc plate and a copper plate are placed side by side but not in contact in a dilute solution of sulphuric acid and the two plates are joined by a wire outside the solution, a steady current flows from one plate to the other. This source of potential difference is termed a **simple cell.** The process is accompanied by the formation of zinc sulphate and hydrogen, thus showing that chemical action is accompanying the electrical phenomena. This process is of great practical importance and forms the basis of **electric batteries.**

(13) *Magneto-electricity.*—Whenever a conductor moves so as to cut across the lines of force of a magnetic field, a potential difference is caused between the ends of the conductor. If the conductor forms a closed circuit a current will flow round the circuit. This process is the most important of all in the practical generation of electric current.

2. Passage of a Current through Solid Conductors.

The potential difference which exists between the zinc and copper plates, or **electrodes,** of the simple cell discussed above is not large enough to create an appreciable spark when the electrodes are joined by a wire. The presence of the current is shown most easily by two other distinct effects which it produces. These are the **heating effect** and the **magnetic effect.**

If the wire is short and thin it may actually be raised in temperature sufficiently to glow; otherwise the effect may be shown by winding the wire round a thermometer.

That a steady magnetic field is established in the neighbourhood of a current may be shown by the deflection of a compass needle placed in the vicinity of the wire.

A third distinct effect of an electric current is that it will produce chemical action when it passes through solutions, though not through solids. We discuss this further in Chap. XII.

The fact that an electric current produces a magnetic field was discovered by Oersted. It suggested the connexion between the hitherto unconnected phenomena of electricity and magnetism. The complete reciprocity of this connexion was established by Faraday's discovery that motion of a conductor in a magnetic field produces electricity. It should be noted that the magnetic field due to electric charge exists only while the charge is in motion; and conversely, electric current is generated only while the conductor and the field are in relative motion.

If the electrodes or **poles** of a simple cell are connected to one of the electrometers described in Chap. IV, the existence of a steady potential difference between the poles may be established. In particular, the copper pole is at a higher potential than the zinc pole. The former is therefore said to be positive and the latter negative. This distinction is only relative to the pair of metals concerned. For example, other cells may be constructed in which the zinc is positive with respect to the second electrode. Before the electronic nature of solid conduction was realized, it was always considered that the electric current flowed from regions of high potential to regions of low potential. Consequently in all literature up to 1900 the current is depicted as flowing from the copper to the zinc electrode, whereas in reality the electrons which constitute the current are proceeding in the reverse direction. Now unfortunately many rules had been established governing, for example, the direction of the magnetic field consequent upon a current flowing in a particular direction, and all these rules would need to be put the other way round if the direction of the current were reversed. It is therefore conventionally accepted that in circumstances where the electronic nature of the circuit need not be considered, the current shall be considered to flow from regions of high potential to regions of low potential. Such a distinction is only an academic one since, for example, in the highly practical case of radio valves, electronic considerations are paramount.

3. Unit of Current.

For the electrostatic (or E.S.) system, since a current consists of a flow of electrical charge, no further unit is necessary in fixing the unit of current. Unit current on the electrostatic system is that current which is present when unit quantity of electricity as defined on the E.S. system passes any point in the electric circuit per second. We wish, however, in practice to make use of the heating or the magnetic effects to define the unit of current. The heating effect is unsatisfactory since it increases with time and is not easily measured with accuracy. It also involves the heat capacity of the wire composing the circuit and introduces uncertain corrections due to heat lost by conduction along the wire itself. The magnetic effect, however, is stationary with time and is independent of the nature of the wire provided it is not of magnetic material.

Now we have to make use of a compass needle, or ideally a unit magnetic pole, in measuring the intensity of the magnetic field produced by the current. We have, however, already defined the unit magnetic pole from purely magnetic considerations. The unit of current which we shall now define is therefore dependent on the properties of the unit magnetic pole, and consequently no simple connexion might be expected to exist between the unit of current on the electromagnetic or E.M. system, and the unit of current on the E.S. system. Actually a very fundamental and simple relation does exist between the units on the two systems, but this was not realized at the time of the formulation of the two systems. It constituted a separate physical deduction of profound significance and led to the discovery of electromagnetic waves and to the electromagnetic theory of light as discussed in Chap. XVI.

The effect on a compass needle was found by Ampère to depend on (1) the shape of the conductor carrying the current, (2) the distance of the magnet or unit pole from the conductor. Eventually a symmetrical circuit was chosen, and the unit of current of the E.M. system is defined as follows: *Unit* E.M. *current is said to flow in a circular conductor of unit radius* (1 *cm.*) *if a force of* 2π *dynes acts on a unit pole placed at the centre of the circle.*

Whereas on the E.S. system the unit of charge is defined first and the E.S. unit of current derived from it by considering the charge in motion, the E.M. unit of charge is derived from the E.M. current as first defined above. *The unit of charge on the* E.M. *system is that charge which has flowed past a point in a conductor when unit* E.M. *current has been flowing for* 1 *sec.*

4. Electromagnetic Unit of Potential Difference.

Just as the E.S. unit of P.D. was defined as that P.D. which exists when one erg of work is performed in moving unit E.S. charge through that P.D., so unit P.D. will exist on the E.M. system when one erg of work is done in moving unit E.M. charge through that P.D. Actually it is more usual to speak of unit P.D. " maintaining unit E.M. current for 1 sec. between the two points ", but this is clearly the same as conveying unit E.M. charge.

5. Electrical Resistance.

Measurement with an electrometer shows that the potential difference between the copper and zinc plates of a simple cell is quite independent of the area of the plates, or their separation. If, however, the wire connecting the poles is long and thin, the current as estimated from its magnetic effect is much smaller than if the wire is short and thick. Similarly, a platinum wire will allow much less current to pass than a copper wire of the same dimensions. Materials are therefore said to offer a resistance to the passage of an electric current. The behaviour of this resistance is closely analogous to the resistance of a pipe to the flow of liquid through it. In fact the comparison between flow of liquid along a pipe and flow of current along a wire may be extended considerably if pressure difference in the former is identified with potential difference in the latter.

For example, we have seen in Part I, Chap. XII, that, provided the flow is streamline, the quantity of liquid flowing through a tube per second is directly proportional to the pressure difference. In exact analogy, for a given wire, the current is directly proportional to the potential difference across the wire. This important relation is termed **Ohm's law,** and is written mathematically

$$\frac{E}{I} = \text{constant} = R, \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.1)$$

where E and I are the potential difference and the current respectively, and the constant R is termed the **resistance** of the wire. Since the units of potential difference and current have already been fixed on both the E.S. and E.M. systems, the unit of resistance on both systems is fixed automatically from equation (8.1).

If one material only is considered, and wires of different cross-sections and lengths are inserted into a circuit operating at fixed potential, it will be found that the current is directly proportional to the area of cross-section and inversely proportional to the length of the wire. Since from (8.1) the resistance is inversely proportional to the current for a fixed potential,

$$R = \frac{sl}{A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.2)$$

where l is the length, A is the area of cross-section assumed uniform, and s is a constant which depends on the nature of the material of the wire. The quantity s is defined by (8.2) and is termed the **specific resistance** of the material. If the conductor is in the form of a unit cube, that is, if $l = 1$, $A = 1$, then $s = R$, or the specific resistance may be defined as the *resistance of a unit cube of the material.* In practice it is found that the resistance varies considerably with the temperature of the wire, and indeed this property, when reversed, forms the basis of the *platinum resistance thermometer* which is mentioned in Part II, Chap. I, and the use of which is described more fully in Chap. XI.

We shall see in Chaps. XII and XVIII that liquids also obey Ohm's law, but that gases show deviations from the law. The existence of these deviations is responsible for many of the most important applications of gaseous conduction.

6. Practical Electrical Units.

The electrical units based on the E.S. and E.M. systems are either too large or too small for practical purposes. Since measurement of current is more frequently required than measurement of charge, the practical system is linked directly with the E.M. rather than with the E.S. system. The E.M. or **absolute** unit of current is too large for practical purposes, and the practical unit or **ampere** is defined as **one-tenth the E.M. unit of current.** The ampere is also defined from the amount of electrochemical action which occurs when the current traverses certain solutions, as discussed in Chap. XII.

On the other hand, the E.M. unit of potential difference is much too small for practical purposes and the practical unit or **volt** is defined as **10^8 E.M. units of P.D.** Now by Ohm's law, if the units of current and P.D. are fixed, the unit of resistance is automatically fixed. The practical unit of resistance is termed the **ohm,** and from equation (8.1)

$$\mathbf{1\ ohm} = \frac{10^8}{10^{-1}} = \mathbf{10^9\ E.M.\ units\ of\ resistance.}$$

It is also, of course, equal to 1 volt/1 amp. Since the resistance of a given conductor at a given temperature is fixed if the length and area of cross-section of the conductor are fixed, *standard ohms* are available consisting of conductors of given material and dimensions. The resistance of a solid depends to some extent on the drawing and annealing of the material as well as upon its nature. It is also very dependent on the presence of small quantities of impurities. The choice of material for standard ohms is therefore very restricted. Copper and silver might be used but are very good conductors and an inconvenient length would be required to construct a standard ohm. Platinum is more suitable but is again dependent on the previous treatment of the specimen. In practice, the liquid metal mercury has been chosen, although copies made of platinum and other solids are convenient to use if they can be repeatedly calibrated against a mercury standard. *The standard mercury ohm, sometimes termed the* **international ohm,** *is the resistance of a column of mercury of length* 106·300 *cm. and weight* 14·4521 *gm. at* 0° *C.* This corresponds to an area of cross-section of about 1 mm.², but owing to the difficulty of drawing a glass tube container of uniform bore, the standard is preferably specified by the weight of mercury present.

7. Magnetic Field due to a Straight Wire.

If a long straight wire is arranged to pass vertically through a board supporting a piece of drawing paper, the nature of the magnetic field due to the current in the wire may be explored with iron filings in the usual manner, or with a compass needle, as shown in fig. 2. If the current is high and the needle is kept close to the wire the effect of the earth's magnetic field may be neglected, and on tracing the lines of force they will be found to consist of **concentric circles** round the wire. The magnetic field F therefore acts in a direction perpendicular to the wire and perpendicular to a line joining the point under consideration to the wire itself. The direction in which the needle sets when it is at a considerable distance from the wire, and free to turn in a horizontal plane, is therefore the direction of the force R resulting from F and H as shown in fig. 3, H being the horizontal component of the earth's field.

The direction of the magnetic field due to the current alone, that is, the direction in which a free N-pole would be urged, is given by certain useful rules of which we select the **corkscrew rule.** This states that if the point of an ordinary right-handed corkscrew is directed along the current, a free N-pole would be urged in the direction of rotation of the handle of the corkscrew in its forward travel. Reversal of the direction of the current reverses the direction of its magnetic field but otherwise leaves it unchanged. If the direction of the current is upward, at some point due W of the wire a neutral point will occur when the fields due to the earth and the current are exactly equal and

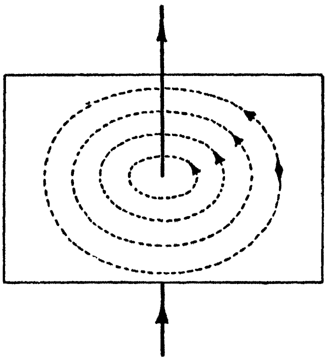

Fig. 2. — Magnetic Field due to Current in Straight Wire

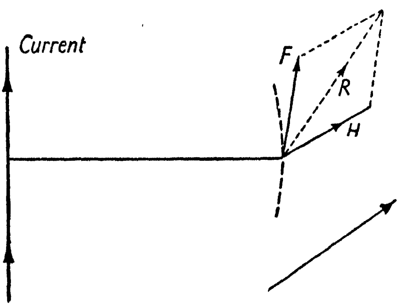

Fig. 3. — Resultant of H and Field due to Current

opposite. Again, at points on a N-S line, the inclination θ to this line at which the needle sets will be given by

$$\tan\theta = \frac{F}{H}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.3)$$

where F is the magnetic field due to the current in the wire, and H is the earth's horizontal component. Since the latter remains constant, $F \propto \tan\theta$, and hence the variation of F with distance from the wire is easily examined. It is found that the *force varies inversely as the distance from the wire.* As we are defining the strength of the current from the magnetic effect, the strength of the magnetic field is assumed to be directly proportional to the electric current. For a long straight wire, the complete relation is shown in Chap. X to be

$$F = \frac{2i}{r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.4)$$

where i is the current strength in E.M. units.

8. Magnetic Field due to a Circular Coil.

If the previous experiment is repeated using a circular coil in place of the long straight wire, the lines of force are found to be as in fig. 4. At the centre of the coil the lines of force run perpendicular to the plane of the coil. If, therefore, the latter is placed in the magnetic meridian, the force due to the current and the earth's horizontal component are again at right angles, and the tangent of the angle of deflection is again a measure of the force. If the number of turns in the coil is increased, the force is found to increase proportionally: *the force at the centre of the coil is directly proportional to the length of the*

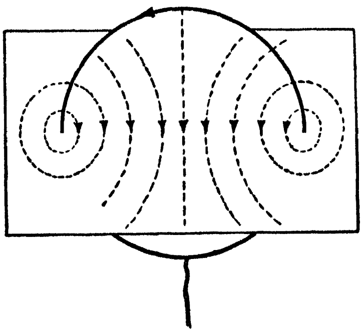

Fig. 4.—Magnetic Field due to Circular Coil

circuit present. Again, if coils of different radii are used, the *force varies inversely as the radius.* For a circular coil we therefore have

$$F \propto \frac{ni}{r} = \frac{kni}{r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.5)$$

where k is a constant. Now for $n = r = i = 1$, $F = 2\pi$ from the definition of unit current. Hence, for a circular coil,

$$F = \frac{2\pi ni}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.6)$$

For circuits of any shape we require a general theorem in an integral form expressing the force at any point distant r from a circuit element of length ds. This theorem may be conveniently derived from (8.6); for if the force varies inversely as r^x we have for a circular coil

$$\int_0^s \frac{i\,ds}{r^x} = \frac{2\pi ni}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.7)$$

Now $\int_0^s ds = 2\pi rn$ for the circular coil: hence from (8.7)

$$x = 2.$$

Finally, remembering that the force always acts perpendicular to the wire and in a direction at right angles to the perpendicular on to the wire, if the line joining the circuit element ds to the point is inclined at an angle θ to ds, the complete general expression is

$$F = \int \frac{i\,ds}{r^2} \sin\theta. \qquad (8.8)$$

9. Action of a Magnetic Field on a Conductor carrying a Current.

The fact so far observed that the magnetic needle moves while the conductor remains stationary is solely due to the light suspension of the magnet and the heavy or fixed nature of the conductor. The action and reaction between the two must be equal and opposite, and

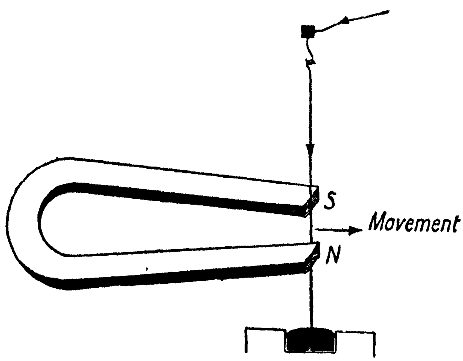

Fig. 5.—Force on Conductor carrying a Current in a Magnetic Field

if the conductor is light and free to move it will do so. In fig. 5 is shown a copper wire hanging between the poles of a powerful horseshoe magnet. The wire is lightly suspended at the top and dips into a pool of mercury at the bottom so as to complete the electrical circuit. On switching on the current the wire will move either way according to the direction of the current. If the permanent magnetic field and the current are situated perpendicular to each other, the wire will move in the remaining direction at right angles to the other two. Experiment shows that the force is given directly by

$$F = Hil,$$

where H is the strength of the permanent magnetic field, i is the current strength, and l is the length of the conductor in the uniform field. If the conductor lies at an angle θ to the field, the force is given generally by the equation

$$F = Hil \sin\theta, \qquad (8.9)$$

and in accordance with this relation the force is zero if the magnetic field and the conductor are parallel to each other. The direction of motion of the conductor is given by the *left-hand rule*, which states that if the *F*orefinger of the left hand points in the direction of the *F*ield, the m*I*ddle finger in the direction of the current I, then the thu*M*b gives the direction of *M*otion.

10. Electrical Energy Absorbed by Resistance.

In the passage of a viscous fluid down a tube under a given pressure difference, work is done by the viscous friction and is converted into heat. The work done is equal to the product of the pressure difference and the volume of liquid which has passed. In exact analogy, the electrical work done when a wire conveys a current is equal to the product of the potential difference (p. 57) and the electrical charge which has passed, and this electrical energy is converted into heat energy. Thus, if the potential difference and current respectively are e, i in E.M. units, and E, I in practical units, then

$$W = e(it) \text{ ergs} = E(It)10^7 \text{ ergs},$$

where W is the electrical work done in t sec.; or, if the heat H is expressed in calories, since (Part II, Chap. VI) $W = JH$, where $J = 4{\cdot}2 \times 10^7$ ergs/cal.,

$$H = \frac{EIt}{4{\cdot}2}. \qquad \qquad (8.10)$$

This heating is referred to as the **Joule heating.** Again, since from Ohm's law $E = IR$, equation (8.10) may be written

$$H = \frac{E^2t}{4{\cdot}2R} = \frac{I^2Rt}{4{\cdot}2}. \qquad \qquad (8.11)$$

The heating effect is seen to be proportional to the square of the current and heat is therefore always *evolved,* independent of the direction of the current.

Additional Practical Units.—It has been found convenient to introduce further units on the practical system.

The practical unit of quantity of electricity is termed the **coulomb,** and is the quantity of electricity which flows past when 1 amp flows for 1 sec.

The practical unit of *electrical energy* is termed the **joule,** and

$$\begin{aligned} 1 \text{ joule} &= 1 \text{ volt} \times 1 \text{ coulomb} = 1 \text{ volt} \times 1 \text{ amp} \times 1 \text{ sec.} \\ &= 10^7 \text{ ergs.} \end{aligned}$$

Electrical power or *rate of working* is the electrical energy expended per second; the power P is therefore given by

$$P = EI.$$

The unit of electrical power, 1 joule per second, is termed the **watt:**

$$1 \text{ watt} = 1 \text{ volt} \times 1 \text{ amp.};$$

$$1 \text{ horse-power} = 746 \text{ watts} = 0{\cdot}746 \text{ kilowatt}.$$

EXERCISES

1. Discuss the methods available for producing an electric current, indicating the practical value or otherwise of the methods described.

2. State three distinct physical effects accompanying the passage of an electric current. Which of these effects is considered to be best suited to the definition of unit current, and why?

3. Define unit current on the electromagnetic system. Hence define unit potential difference and unit resistance on the same system. In what way are these units related to (*a*) the practical system, (*b*) the electrostatic system, of units?

4. Upon what factors does the resistance of a solid conductor depend? Define the specific resistance of a material and indicate how far Ohm's law is obeyed by solids, liquids and gases.

5. Sketch the lines of force arising from a current flowing in (*a*) a straight wire, (*b*) a circular coil, (*c*) a solenoid. What arrangement will produce a uniform magnetic field over a volume of a few cubic centimetres?

6. State *one* rule governing the direction of the magnetic field arising from a current flowing in a conductor. State also one rule governing the direction of motion of a conductor carrying a current and situated in a magnetic field. What determines the magnitude of the force on a conductor carrying a current and situated in a magnetic field?

7. Define the ampere, volt, ohm, coulomb, joule and watt. State Ohm's law and compare the heat generated in two wires connected in parallel if the first has twice the resistance of the second. [1 : 2.]

CHAPTER IX

Galvanometers, Ammeters and Voltmeters

1. The Tangent Galvanometer.

To measure the strength of a current from its magnetic effect an instrument is required in which a magnetic needle is suspended at a definite distance from a circuit of symmetrical shape. Such instruments are termed galvanometers, and one of the simplest is the tangent galvanometer shown in fig. 1. It consists essentially of a small magnet carrying a light pointer at right angles (this part of the apparatus is identical with the deflection magnetometer described in Chap. VII), situated at the centre of a circular coil of wire which is supported rigidly in a vertical plane. The radius of the coil must be large compared with the dimensions of the suspended magnet, so that the latter may be considered to be subject to the magnetic field at the centre of the coil. Then from equations (8.6) and (8.3)

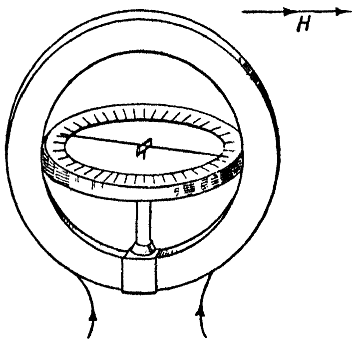

Fig. 1. — Tangent Galvanometer

$$F = \frac{2\pi ni}{r} = H\tan\theta,$$

or expressing the current in amperes,

$$I = \left(\frac{10Hr}{2\pi n}\right)\tan\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.1)$$

The expression in brackets in (9.1), which is constant for a given instrument, is termed the *galvanometer constant* or *reduction factor*. The disadvantages of the instrument are:

(1) It requires setting with the plane of the coil in the magnetic meridian;

(2) It uses a tangent relation and therefore becomes inaccurate at angles greater than 70°;

(3) It is bulky, and the supported needle system is not robust.

2. The Astatic Galvanometer.

The first of these disadvantages is avoided in the astatic galvanometer, a diagram of which is shown in fig. 2. It consists of two short, light magnetic needles joined rigidly together by a short light vertical rod attached to their centres, the whole being suspended by a torsion fibre of phosphor-bronze. The two needles are magnetically oppositely directed and each is supplied with a small surrounding coil, the two coils being connected in series. Since the needles are oppositely directed and are otherwise equal in all respects, the system will set in any position, and does not respond directively to the earth's field. There is no simple relation connecting the deflection with the current strength, and the instrument needs calibration against some absolute instrument such as the tangent galvanometer. The current is proportional to the angle of deflection for small deflections.

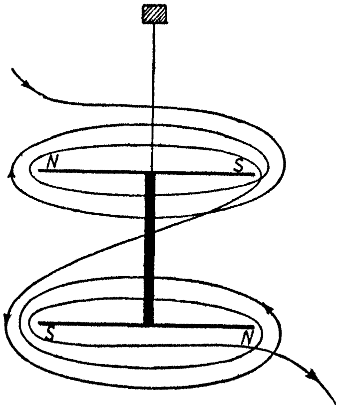

Fig. 2. — Astatic Galvanometer

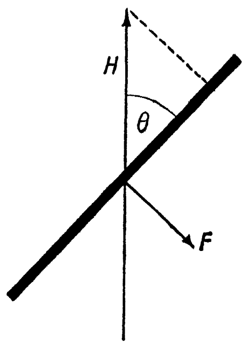

Fig. 3. — Sine Galvanometer

3. Sine Galvanometer.

If the large coil of a tangent galvanometer is supported by a vertical axle so that it may be rotated in a vertical plane, it may be used as a sine galvanometer. The principle is to rotate the coil until it catches up the deflected needle and both needle and coil lie in one common plane. The forces acting are shown in fig. 3, whence $F = H \sin\theta$, and hence

$$I = \left(\frac{10Hr}{2\pi n}\right) \sin\theta. \qquad (9.2)$$

The instrument has the advantage that deflections up to 90° may be used, but the numerical value of the maximum current which may be read is clearly equal to the reduction factor of the galvanometer.

4. Moving-coil Galvanometer.

There is a rather low limit to the pole-strength which the suspended needle may have if it is not to suffer from self-demagnetization due to the proximity of its own poles. Consequently, since the force between the magnetic field of the current and the suspended magnet depends on the strength of the latter as well as the former, the sensitivity is too small when very small currents are to be measured, even if the pointer is replaced by a mirror, lamp and scale device. In sensitive current measuring instruments, therefore, the roles of magnet and conductor are reversed. The magnetic needle is replaced by a powerful permanent horse-shoe magnet as shown in fig. 4, and the conductor

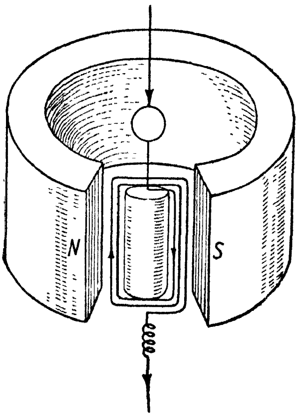

Fig. 4. — Moving-coil Galvanometer

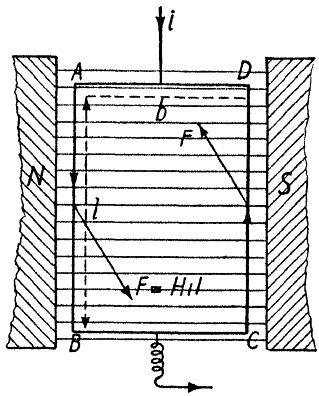

Fig. 5. — Moving-coil Galvanometer

carrying the current is in the shape of a circular or rectangular coil suspended between the poles of the permanent magnet.

For a **rectangular coil,** from the considerations of section 9 of the preceding chapter, we have by reference to fig. 5 a force $F = Hil$ on each vertical side of the rectangle, and zero force on the top and bottom sides since these are parallel to the field. The current traverses the sides AB and CD in opposite directions, so that the forces on these sides are oppositely directed, and a net couple

$$G = 2F\frac{b}{2} = Hilb = HAi \quad . \quad . \quad . \quad . \quad (9.3)$$

is produced, where A is the area of the rectangle, causing it to rotate until an equal and opposite couple $c\theta$ is brought into play owing to the twist in the suspension.

We must note, however, that as soon as the coil rotates from its rest position, in which it lies with its area parallel to the field, the couple is reduced to $HAi \cos\theta$, so that

$$c\theta = HAi \cos\theta, \quad \ldots\ldots\ldots \quad (9.4)$$

where θ is the inclination of the plane of the coil to the field. The top and bottom of the rectangle are also now subject to forces since they are inclined to the direction of the field. These forces are, however, oppositely directed in a vertical plane and therefore produce no effect on the suspended system. If there are n turns of wire in the rectangle, the complete expression for the current becomes

$$i = \frac{c}{nAH} \cdot \frac{\theta}{\cos\theta}. \quad \ldots\ldots\ldots \quad (9.5)$$

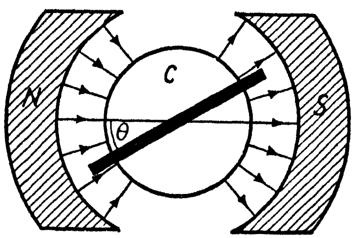

FIG. 6. — Radial Field

A scale which varied as $\theta/\cos\theta$ would be very inconvenient, and consequently a device is introduced which reduces the denominator to unity for all positions of the coil. This device consists of a soft-iron cylinder C, which together with the circular-shaped pole-pieces produces a **radial field.** The coil now lies along the lines of force for all values of θ and consequently, as shown in fig. 6, the couple maintains a uniform value of $nAiH$ for all orientations. Equation (9.5) then reduces to

$$i = \left(\frac{c}{nAH}\right)\theta, \quad \ldots\ldots\ldots \quad (9.6)$$

which gives a linear scale.

The ordinary **direct-current ammeter,** shown in fig. 7, is simply a moving-coil galvanometer of robust design. The phosphor-bronze suspension is replaced by a pivot and the coil is returned to zero by a hair-spring. Since an ammeter is introduced into a circuit in series in order to measure the current in that circuit, the resistance of the coil of the ammeter must be exceedingly small if its introduction is not to change the total resistance, and hence the current, in the circuit. This low resistance may be effected by using a few turns of good conducting wire for the coil. Alternatively, a low resistance shunt may be used.

5. The Hot-wire Ammeter.

In fig. 8 is shown a diagram of the hot-wire ammeter, the action of which depends on the thermal expansion of the wire W when it is heated by the passage of the electrical current. The slack which results from the expansion of the wire W is taken up by the spring S, the movement of which rotates the pointer over the scale of the ammeter. The instrument requires calibration with one of the absolute current measuring instruments. An important advantage of the hot-wire ammeter is that since the heat evolved is independent of the direction of the current, the instrument will measure alternating current as well as direct current. However, the scale of the instrument is not linear, since the heating effect is proportional to the square of the current.

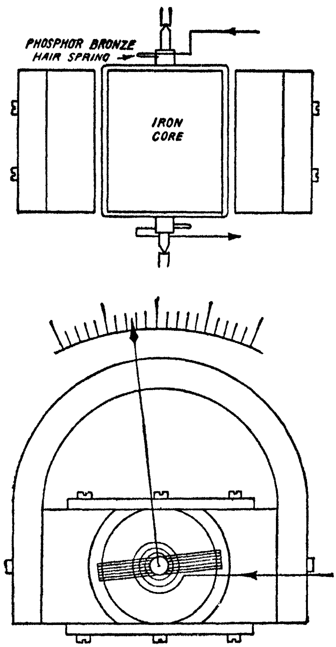

Fig. 7. — Ammeter

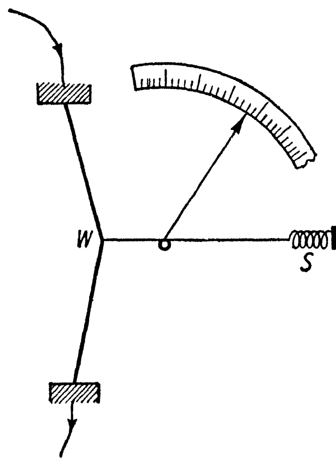

Fig. 8. — Hot-wire Ammeter

6. Soft Iron Ammeter.

This ammeter also is suitable for measuring A.C. or D.C. As shown in fig. 9, it consists of two soft iron rods lying inside and parallel to the axis of a long coil or **solenoid.** The current to be measured flows through the solenoid, and under the influence of the magnetic field produced, the soft iron rods become magnetized. One rod is fixed but the other is pivoted, and since the rods acquire the same polarity at neighbouring ends, the pivoted rod is repelled by the fixed rod. A suitable counterpoise or control spring limits the movement of the movable rod, and since the latter is attached to a pointer the current is read off directly. On reversal of the current in the solenoid the polarity of both rods reverses simultaneously, the repulsion is maintained, and the instrument is therefore suitable for A.C. measurement. As with the hot-wire ammeter, the deflection is proportional to the square of the current.

Another modification of the soft iron ammeter is shown in fig. 10, where an eccentrically mounted soft iron core is attracted by the magnetic field of the solenoid. The core rotates into the solenoid until prevented from doing so by the couple in the hair-spring.

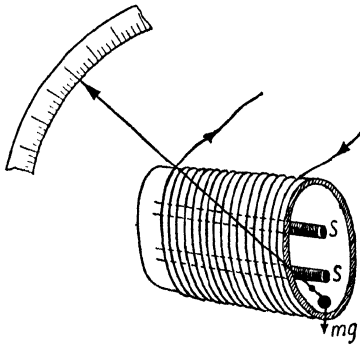

Fig. 9. — Soft Iron Ammeter

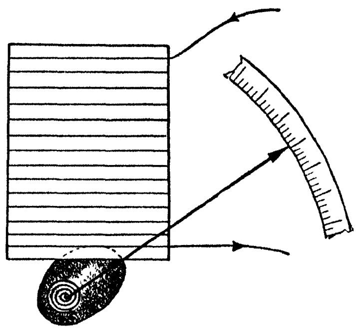
Fig. 10. — Soft iron Ammeter

6. Einthoven String Galvanometer.

To measure currents of short duration such as occur in condenser discharges, the ballistic galvanometer described in Chap. XIII is often employed. A simpler instrument is the Einthoven string galvanometer, which has the advantage of great rapidity of response so that it is especially suited to the measurement of discharges which occur with only short time intervals. As shown in fig. 11, it consists of a straight wire held in a state of tension and situated perpendicular to the pole-pieces of a powerful permanent magnet. When a discharge passes down the wire, the latter is urged mechanically at right angles to the field of the permanent magnet, but as soon as the discharge has passed the wire returns to its original position. The pole pieces of the magnet are bored so that a beam of light travels from one through the other and throws a shadow of the wire on a screen. When the discharge passes, the shadow flicks sideways temporarily, and if the shadow falls on sensitive photographic paper which is travelling parallel to the wire or " string ", a succession of discharges gives a succession of " kicks " on a straight base line as shown in fig. 12. The length of the duration

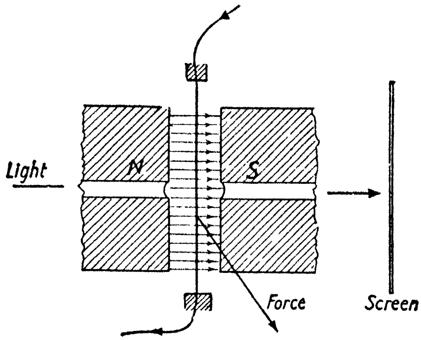

Fig. 11. — Einthoven String Galvanometer

of the kicks can be estimated if the speed of movement of the photographic paper is known. The relative magnitude of the discharges is shown by the relative height of the kicks.

Fig. 12. — Einthoven Galvanometer Record

7. Moving-coil Voltmeter.

The electrometer described in Chap. IV may be used to measure potential difference, but unless the voltage is greater than about 500 it is difficult to make an electrostatic voltmeter which is both sensitive and robust. It is therefore usual to reserve such instruments for high potentials, modified ammeters being used for ordinary voltages. The

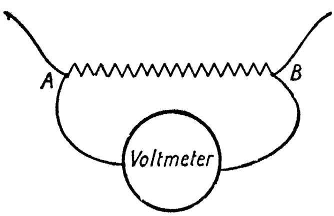

Fig. 13. — Moving-coil Voltmeter

ordinary moving-coil voltmeter simply consists of a moving-coil ammeter with a high resistance fitted in series with the low resistance moving-coil. If the potential difference across AB in fig. 13 were required, the voltmeter would be connected across AB, that is, in parallel with the conductor. Now it clearly would not do to connect a low resistance ammeter across AB in parallel with the conductor for the current would take the path of least resistance and pass through the ammeter rather than through the conductor. Hence a resistance is included in series with the ammeter and this resistance is as high as is consistent with a suitable deflection on the voltmeter scale. The use of a high resistance introduces the minimum disturbance into the circuit when the voltmeter is introduced, just as the use of a low resistance ammeter in series introduces the minimum disturbance when a current measurement is required.

An ammeter may therefore always be used as a voltmeter by the inclusion of a separate resistance in series with the moving coil. Some voltmeters are made directly with moving coils of large resistance, consisting of a large number of turns of fine wire to ensure a high sensitivity. Such instruments cannot be used as ammeters since their high resistance is a permanent feature of the voltmeter.

The hot-wire and soft iron ammeters are directly convertible to voltmeters by the insertion of resistance in series with the hot wire and the solenoid respectively. These instruments are then suitable for measuring either alternating or direct voltages.

The **sensitivity** of galvanometers generally is defined as the deflection in scale divisions per microamp. Alternatively, the *figure of merit* of a galvanometer is defined as the current required to produce a deflection of one-scale division.

EXERCISES

1. Describe and give the theory of the tangent galvanometer. In what way does it differ from the sine galvanometer?

2. Compare the moving magnet and moving-coil galvanometers. How are the latter modified to form ammeters?

3. Give the theory of the moving-coil galvanometer with a linear scale. What are the essentials of a ballistic-coil galvanometer?

4. Describe some form of ammeter suitable for measuring alternating current. Has the instrument a linear scale?

5. What is the essential difference between an ammeter and a voltmeter? Describe some instrument suitable for measuring a series of electrical discharges produced at short time intervals.

CHAPTER X

Magnetic Interactions of the Electric Current

* 1. Magnetic Shell.

We have already considered in Chaps. VIII and IX some elementary properties and applications of the magnetic field which accompanies an electric current. More complicated instances, such as the magnetic interactions of two or more coils carrying currents, require a more general approach to the problem such as is provided by the concept of the **magnetic shell.** By a series of experiments Ampère showed that the magnetic effect produced by a conductor carrying a current was the same as that produced by regarding the area enclosed by the conductor as a magnetic polar sheet or magnetic shell. The magnetic moment per unit area or **strength** of such a shell is commonly represented by σ, and the total magnetic moment of the shell in fig. 1

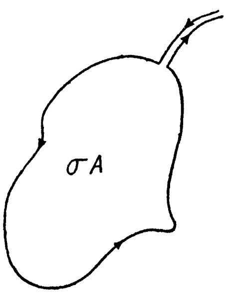

Fig. 1. — Magnetic Shell

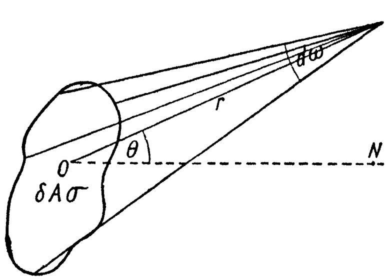

Fig. 2. — Potential due to Magnetic Shell

is σA, where A is the area bounded by the conductor. By equation (7.4) the magnetic potential at the point P in fig. 2 due to a small element of area δA of the shell is

$$dV = \sigma \frac{\delta A \cos \theta}{r^2}, \qquad \text{. . . . . . (10.1)}$$

where θ is the angle between OP ($= r$) and the normal ON to the shell at O.

Now if $d\omega$ is the solid angle subtended at P by the area δA, by definition $d\omega = \delta A \cos\theta/r^2$. Hence equation (10.1) becomes

$$\int dV = \int \sigma\, d\omega,$$

and the total magnetic potential at P becomes

$$V = \sigma\Omega, \qquad \qquad (10.2)$$

where Ω is the whole solid angle subtended by the conductor at P.

2. Helmholtz Coils.

We have derived from elementary considerations in Chap. VIII an expression for the force at the centre of a circular conductor. We now require a more general expression for the force at any point on the axis perpendicular to the plane of the circular coil. The solid angle subtended at the point P, at distance x from the centre of the coil, is A/r^2, where r is the radius of a sphere with centre P as shown in fig. 3, and A is the area of the cap DCB. Since the surface area of the cap is equal to that of a circumscribing cylinder of height h equal to that of the cap,

$$A = 2\pi rh = 2\pi r(r - x),$$

and

$$\Omega = 2\pi r(r - x)/r^2 = 2\pi\left(1 - \frac{x}{r}\right).$$

The magnetic potential at P is therefore, from (10.2),

$$V = 2\pi\sigma\left(1 - \frac{x}{r}\right). \qquad \qquad (10.3)$$

Now $r^2 = x^2 + a^2$: hence

$$V = 2\pi\sigma\left\{1 - \frac{x}{(x^2 + a^2)^{1/2}}\right\}. \qquad \qquad (10.4)$$

The magnetic force F at P is from symmetry directed along the axis and hence

$$F = -\frac{\partial V}{\partial x} = \frac{2\pi a^2\sigma}{(x^2 + a^2)^{3/2}}. \qquad \qquad (10.5)$$

Equation (10.5) must reduce to (8.6) when $x = 0$, that is at the centre of the coil

$$F = \frac{2\pi\sigma}{a} = \frac{2\pi i}{a}, \qquad \qquad (10.6)$$

and hence $\sigma = i$, or *the strength of the shell is equal to the current in the conductor.*

Helmholtz's Arrangement.—Let two equal coils be mounted coaxially as in fig. 4, the currents being in such directions that the magnetic fields assist each other. The object of the arrangement is to obtain as uniform a field as possible near the point midway between the coils; this is achieved by proper choice of the distance $2b$ between them.

Now, by (10.5), the force F at distance x from the mid-point is

$$F = 2\pi a^2 i\,[\{a^2 + (b + x)^2\}^{-3/2} + \{a^2 + (b - x)^2\}^{-3/2}].$$

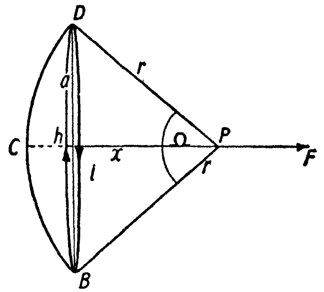

Fig. 3.—Magnetic Force on Axis of Circular Coil

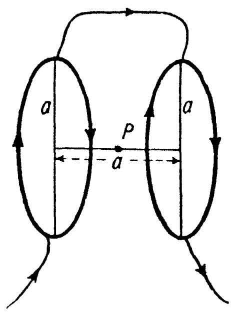

Fig. 4.—Helmholtz Coils

The field gradient is $\partial F/\partial x$, and this changes very slowly if $\partial^2 F/\partial x^2 = 0$, when $x = 0$.

By carrying out the differentiations, it is easily found that $2b = a$, or the separation of the coils is equal to their radius, as shown in fig. 4.

3. Field of a Solenoid.

The magnetic field at any point on the axis of a solenoid may be calculated by considering the solenoid to consist of a series of plane circular coils and integrating the total effect of all the coils. Referring

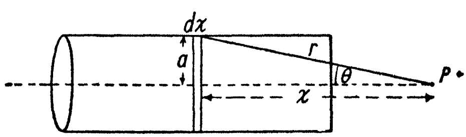

Fig. 5.—Field of a Solenoid

to fig. 5, the strength of the field at P due to the coil of elementary length dx, if i is the current per unit length, is

$$dF = \frac{2\pi a^2 i}{(x^2 + a^2)^{3/2}}\,dx. \quad . \quad . \quad . \quad . \quad . \quad (10.8)$$

Now $r\,d\theta = dx \,.\, \sin\theta$, as shown in fig. 6. Hence

$$dF = \frac{2\pi a^2 i}{(x^2 + a^2)^{3/2}} \frac{r d\theta}{\sin\theta}. \quad . \quad . \quad . \quad . \quad (10.9)$$

Since $(x^2 + a^2) = r^2$ and $a = r \sin\theta$, substituting in equation (10.9) we have

$$F = 2\pi i \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta \quad . \quad . \quad . \quad . \quad . \quad (10.10)$$

$$= 2\pi i \Big[- \cos\theta \Big]_{\theta_1}^{\theta_2}. \quad . \quad . \quad . \quad . \quad (10.11)$$

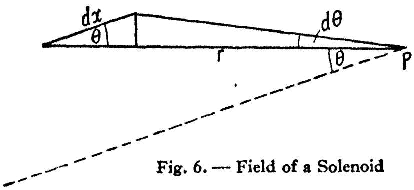

Fig. 6. — Field of a Solenoid

If the solenoid contains n turns per unit length,

$$F = 2\pi n i(\cos\theta_1 - \cos\theta_2). \quad . \quad . \quad . \quad . \quad (10.12)$$

For an infinitely long solenoid $\theta_1 = 0$ and $\theta_2 = \pi$, and hence from (10.12)

$$F = 4\pi n i. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10.13)$$

* 4. Work done in taking a Unit Pole round a Current.

If we consider a point P_1 close to the surface of a magnetic shell, from equation (10.2) the magnetic potential at P_1 is given by $i\Omega$, where i is the current in the conductor to which the shell is equivalent, and Ω is the solid angle subtended by the conductor at P_1. Suppose now that the unit pole is taken once round any closed path encircling the current and brought back from the other side to a point P_2 infinitely close to P_1. Then the solid angle subtended by the equivalent magnetic shell at P_2 is $(\Omega - 4\pi)$ and the magnetic potential of this point is therefore $\sigma(\Omega - 4\pi) = i(\Omega - 4\pi)$. If we make P_2 coincide with P_1 in the limit, the change in magnetic potential which occurs in carrying a unit pole round a current is clearly

$$V = i\Omega - i(\Omega - 4\pi) = 4\pi i,$$

or the work done in taking a unit pole once round a current by any closed path is 4π times the current encircled. If the strength of the magnetic field at any point is H and the unit pole is taken an infinitesimally small distance ds, the work done in this short element of path is $H\,ds$. Hence the total work done, or **line integral of the magnetic field,** or the **magnetomotive force** involved in once encircling a current i, is

$$\text{M.M.F.} = \int H\,ds = V = 4\pi i. \quad . \quad . \quad . \quad (10.14)$$

5. Applications of the Work Theorem.

(i) *Infinite Straight Wire.*

If the field is H at a point a distance r from the wire, the work done in once encircling the wire and returning to the initial position is

$$\begin{aligned} W &= \text{force} \times \text{distance} \\ &= H \, . \, 2\pi r = 4\pi i \end{aligned}$$

by equation (10.14).

Hence

$$H = \frac{2i}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10.15)$$

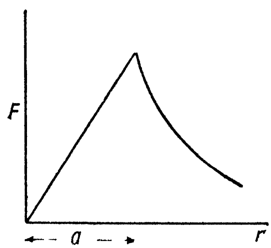

Fig. 7. — Field for Wire of Finite Thickness

We note that if the wire is replaced by a tube there is no field inside the tube, since no current is encircled by a closed curve entirely within the tube. Similarly if the conductor is a wire of finite thickness, the only contribution to the magnetic field at a point inside the wire is due to current flowing nearer to the axis than the point under consideration. If ρ represents the current density per unit area of the conductor of radius a, and the point is situated a distance r from the axis, the field is

$$F = \frac{2}{r}(\pi r^2 \rho) = 2\pi r \rho = \frac{2ri}{a^2}, \quad . \quad . \quad . \quad (10.16)$$

since the whole current $i = \pi a^2 \rho$. From equation (10.16), therefore, the field increases proportionally to the distance from the axis reaching a maximum value of $2i/a$ at the surface of the conductor and then falling inversely as the distance for points external to the wire, as shown in fig. 7.

(ii) *Endless Solenoid or Anchor Ring.*

If the magnetic field is F in the endless solenoid or anchor ring shown in fig. 8, the work done in one complete revolution of a unit pole is

$$W = 2\pi r \,.\, F = 4\pi(2\pi rni)$$

by the work theorem, if n is the number of turns per unit length of the solenoid and r is the mean radius of the ring. Hence

$$F = 4\pi ni, \qquad \ldots\ldots \qquad (10.17)$$

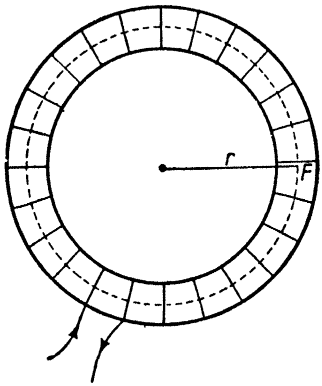

Fig. 8. — Field of an Endless Solenoid

which shows that the field is independent both of the radius and the thickness of the ring.

6. Force Between Currents.

(i) *Two Infinite Parallel Straight Conductors.*

The force on a unit pole (i.e. the magnetic field) at a distance r from an infinite straight wire carrying a current i is from equation (10.15) $F = 2i_1/r$, and this force is at right angles to the plane containing the wire and the pole. Hence, by Chap. VIII, section 9, the mechanical force on unit length of a second wire parallel to the first, at distance r from it, and carrying a current i_2, is

$$\frac{2i_1i_2}{r}. \qquad \ldots\ldots \qquad (10.18)$$

The force is perpendicular to both wires, and is an attraction if the currents are in the same direction.

(ii) *Two Coaxial Coils.*

By applying the principles which we have already considered, the force between two coaxial coils may be obtained. In general, the expressions obtained are complicated and only approximate. For the special case where $r_1 \ll r_2$, the force may be shown to be

$$F = \frac{6\pi^2 r_1^2 r_2^2 \,.\, i_1 i_2}{(r_2^2 + x^2)^{5/2}}\, x, \quad . \quad . \quad . \quad . \quad . \quad . \quad (10.19)$$

where i_1 and i_2 are the currents through the two coils of radii r_1 and r_2 respectively and x is the distance between them. The expression has a maximum when $x = r_2/2$.

7. Kelvin Current Balance.

In fig. 9 is shown the essential part of the Kelvin current balance, which is an instrument designed to operate from the magnetic force of interaction of coaxial coils. It consists of four fixed coils arranged at

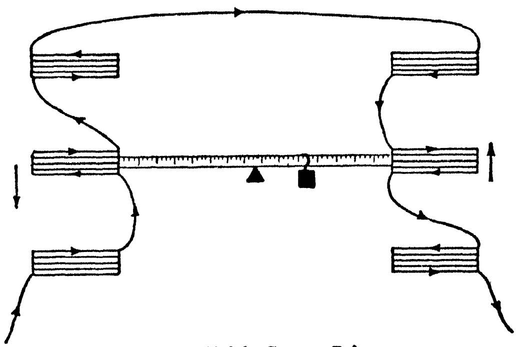

Fig. 9. — Kelvin Current Balance

the corners of a rectangle, with two similar coils attached to the arms of a centrally pivoted horizontal lever. The coils are arranged in series and the current circulates in such a direction that one coil is urged upwards and the other downwards. To restore the balance of the lever, a small rider is slid along the graduated lever arm. The expression for the forces between the coils cannot be calculated with sufficient accuracy, so the instrument has to be calibrated with some instrument such as the silver voltameter. If the rider has to be displaced a distance d to obtain equilibrium when the current changes by i, the couple is proportional to

$$kd = i^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10.20)$$

since the force between the coils is proportional to the current in both the fixed- and moving-coil systems. The lever is therefore graduated with a current scale obeying a square root law.

8. Kelvin Watt Balance.

The coils of the Kelvin current balance must all be of low resistance, since they are connected in series and in series with the circuit in which the current is to be measured. In the Kelvin watt balance, the outward appearance of the instrument is the same. The two moving coils, however, are now of high resistance, and only the four fixed coils have a low resistance. The current i_1 in the main circuit flows through the fixed low resistance coils as shown in fig. 10, but the current in the high resistance coils is obtained by connecting them in parallel with the system whose power absorption is required. The current i_2 in the high resistance coils is proportional to the potential difference E across

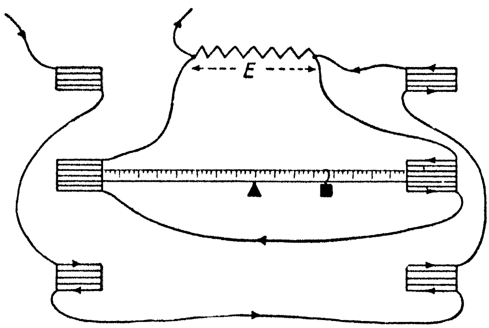

FIG. 10.—Kelvin Watt Balance

them, and hence the force F on the moving coils is a measure of the power W absorbed, for we have

$$W = Ei_1 = (i_2R)i_1 = ki_1i_2 = k'F = k''d,$$

where R is the resistance of the moving coils, and k, k' and k'' are constants. A linear scale is thus obtained between the displacement d of the rider and the power absorbed.

9. Siemens Electrodynamometer.

As shown in fig. 11, this instrument consists of two coils mounted in a vertical plane, one coil being fixed and the other free to move about a vertical axis which lies along a common diameter of the coils. In the zero position the coils are perpendicular to each other, but under the action of a common current i the movable coil tends to parallelism with the other until returned to the perpendicular position by twist applied to the torsion head. Then we have

$$\text{Couple} \propto \theta \propto i^2,$$

or

$$i = k\theta^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10.21)$$

so that the instrument acts as a current balance with a square-root torsion scale. The dynamometer may easily be modified for use as a wattmeter by inserting a high resistance in series with one of the coils and connecting this high resistance arrangement in parallel with the system whose power absorption is required. The other low resistance coil is, of course, placed in series with the system and takes its common current i. If i_1 is the current through the high resistance circuit, this

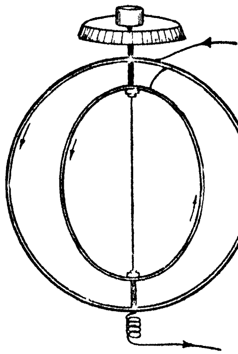

Fig. 11. — Siemens Electrodynamometer

is proportional to the potential difference E across it. Hence if W is the power absorbed,

$$W = Ei = ki_1i = k'\theta,$$

where k and k' are constants. The torsion head therefore gives a linear scale with respect to power absorbed (wattage).

EXERCISES

1. Explain the concept of the magnetic shell and use it to find the strength of the magnetic field at a point on the axis of a solenoid carrying a current.

2. Prove that the work done in conveying a unit pole once round a current is 4π times the current encircled. Apply this theorem to find the strength of the magnetic field inside an endless solenoid or anchor ring.

3. Describe the Kelvin current balance. In what essential features does it differ from the watt balance?

CHAPTER XI

Measurement of Resistance and Potential

1. Resistances in Series and in Parallel.

If resistances are connected in series, the total resistance is equal to the sum of the individual resistances. With reference to fig. 1 we have, if E represents the total fall in potential between A and D,

$$E = E_1 + E_2 + E_3 = IR, \quad . \quad . \quad . \quad (11.1)$$

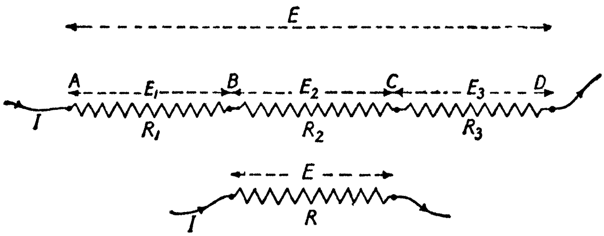

Fig. 1. — Resistances in Series

where R is the equivalent resistance. For the separate resistances, however, we have, by application of Ohm's law,

$$\begin{aligned} E_1 &= IR_1, \\ E_2 &= IR_2, \\ E_3 &= IR_3. \end{aligned} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11.2)$$

Hence, adding,

$$E_1 + E_2 + E_3 = I(R_1 + R_2 + R_3), \quad . \quad . \quad (11.3)$$

and, equating (11.1) and (11.3),

$$R = R_1 + R_2 + R_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11.4)$$

If the resistances are connected in parallel, the total resistance is such that its reciprocal is equal to the sum of the reciprocals of the individual resistances. From fig. 2 we have, on application of Ohm's law,

$$E = IR = I_1R_1 = I_2R_2 = I_3R_3. \quad . \quad . \quad . \quad (11.5)$$

Now $$I = I_1 + I_2 + I_3,$$

or, from (11.5),

$$\frac{E}{R} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3},$$

or $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \qquad (11.6)$$

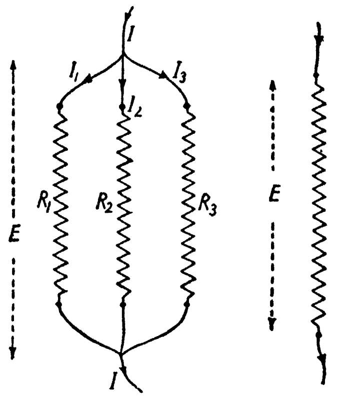

Fig. 2. — Resistances in Parallel

2. Currents in Divided Circuits.

We shall now calculate the currents in a circuit divided into two branches, as shown in fig. 3. Let the main current I divide into I_1 through R_1 and I_2 through R_2. Then, since the total current is unchanged,

$$I = I_1 + I_2. \qquad (11.7)$$

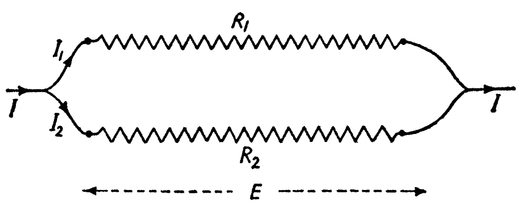

Fig. 3. — Currents in divided Circuit

Application of Ohm's law to first one resistance and then the other gives

$$E = I_1R_1 = I_2R_2. \qquad (11.8)$$

Hence, from (11.7) and (11.8),

$$I_1 = I\frac{R_2}{(R_1 + R_2)},$$

and

$$I_2 = I\frac{R_1}{(R_1 + R_2)}. \qquad . \; . \; . \; . \; . \; . \quad (11.9)$$

In such a circuit, therefore, the current divides inversely as the sum of the resistances and directly as the other resistance.

3. Currents in any Network: Kirchhoff's Laws.

The magnitude of the current in any portion of a network, and the potential difference acting between any two parts of the network, are readily derived by the application of **Kirchhoff's laws.** The first law states that the *sum of the currents arriving at any point equals the sum of the currents leaving that point*; or, in algebraic form,

$$\Sigma I = 0. \qquad . \; . \; . \; . \; . \; . \; . \quad (11.10)$$

This law is a statement of the experimental fact that there is no accumulation of charge at any point in a complete electrical circuit. It corresponds exactly to the *equation of continuity* of a perfectly incompressible fluid, mentioned in Part I.

Kirchhoff's second law is simply Ohm's law applied to closed networks or closed portions of networks. It states that the *algebraic sum of the potential differences acting around separate portions of a closed network is equal to zero*, that is

$$\Sigma(E - IR) = 0, \qquad . \; . \; . \; . \; . \; . \quad (11.11)$$

where E represents the potential difference arising from batteries present and IR that from the operation of Ohm's law. The combination of (11.10) and (11.11), which we have already used in a simple manner in deriving expressions in sections 1 and 2 of this chapter, is sufficient to solve completely any problem concerning the distribution of steady currents in networks.

4. Approximate Resistance Measurements.

Perhaps the simplest way of measuring the resistance of a wire is to connect it in series with an ammeter and a cell and place a voltmeter across the resistance as shown in fig. 4. Then application of Ohm's law gives $R = E/I$, where E is the reading on the voltmeter and I that on the ammeter. The result can only be approximately correct, because when the voltmeter is placed in parallel with the resistance alone as in fig. 4, the current I registered by the ammeter is only partially going through the resistance, part going through the voltmeter itself. Strictly Ohm's law would require $R = E/I_R$, where I_R is the current in the resistance. If, however, I_R is determined by placing the voltmeter across the ammeter and resistance together as in fig. 5, the

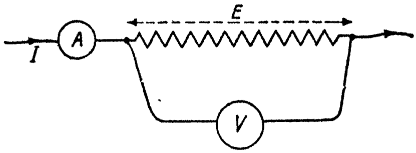

Fig. 4. — Approximate Measurement of Resistance

voltage E is now no longer that across the resistance alone but across ammeter and resistance together. However, the lower the resistance of the ammeter and the higher the resistance of the voltmeter, the more accurate the result, and if an electrostatic voltmeter is used, an accurate

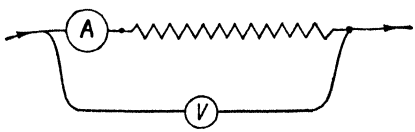

Fig. 5. — Approximate Measurement of Resistance

value for the resistance may be obtained, since the electrostatic voltmeter takes no current.

If a **resistance box** containing known resistances is available, the *method of substitution* affords a simple means of determining an unknown resistance. A tangent galvanometer, a cell and the resistance box are

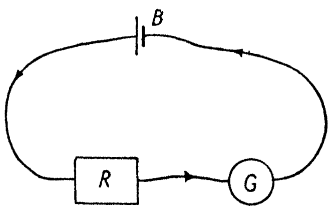

Fig. 6. — Use of Resistance Box

connected in series as in fig. 6, and a graph is obtained of R against $\cot\theta$. From the equation $I = k\tan\theta$ and by Ohm's law we have, if E is the voltage of the cell and r its **internal resistance** (see p. 94),

$$E = I(R + G + r), \quad . \quad . \quad . \quad . \quad (11.12)$$

where G is the resistance of the galvanometer coil. Hence

$$R + G + r = \frac{E}{k\tan\theta} = k'\cot\theta, \quad . \quad . \quad (11.13)$$

where $k' = E/k =$ constant. Since G and r remain constant a straight-line graph will be obtained, the intercept of which on the resistance axis gives the combined resistances of galvanometer and battery. To find the value of an unknown resistance, the latter is simply inserted in place of the resistance box and, when the deflection has been noted, reference to the already obtained $R \cot \theta$ graph gives the value of the unknown resistance.

5. Wheatstone Bridge.

Very accurate values of resistance may be obtained with a special arrangement of resistances termed the *Wheatstone bridge* or *net.* As shown in fig. 7, this consists of three known resistances P, Q and R and the unknown resistance X, connected to form the four sides of a quadrilateral. A battery is connected across the junction of P and R

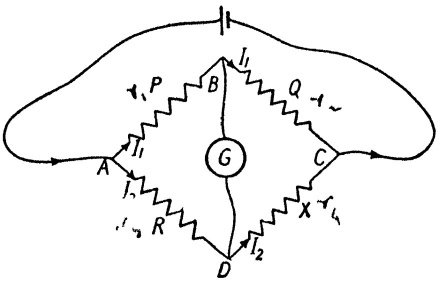

FIG. 7. — Wheatstone Bridge

and the junction of Q and X, while a galvanometer connected across the junctions (P-Q) and (R-X) completes the symmetry of the circuit. In general P and Q, which are said to constitute the *ratio arms*, may be varied within the simple limits $P = Q$, $P = 10Q$ or $P = 100Q$ respectively. The relation is reciprocal, that is Q may be made equal to $10P$ or $100P$; the basic value when $P = Q$ is usually 10 ohms. The resistance R is variable over a wide range usually from 1 ohm to several thousand ohms. Any sensitive galvanometer is suitable.

The action of the instrument depends on the adjustment of the value of R until there is no deflection in the galvanometer. This *null condition* is realized when no current flows in the galvanometer, that is, when the potential of B equals the potential of D. From fig. 7 we have

$$V_{AB} = I_1 P, \qquad \text{and} \qquad V_{AD} = I_2 R. \qquad \qquad (11.14)$$

Now the potential of A is common to both P and R, and if no current flows in the galvanometer, the potentials of B and D are equal. Hence

$$V_{AB} = V_{AD},$$

and therefore, from (11.14),

$$I_1 P = I_2 R. \quad \ldots \ldots \quad (11.15)$$

Similarly for the resistances Q and X,

$$V_{BC} = V_{DC},$$

and hence

$$I_1 Q = I_2 X. \quad \ldots \ldots \quad (11.16)$$

Hence, finally from (11.15) and (11.16) we have

$$\frac{Q}{P} = \frac{X}{R}, \quad \ldots \ldots \quad (11.17)$$

from which X is easily determined if P, Q and R are known.

In using the bridge, the resistance R is changed until ideally the galvanometer shows no deflection. In practice, for values of R slightly greater than a certain amount the galvanometer will generally show a small deflection in one direction, while for values of R slightly less than the same amount, the galvanometer will be deflected in the reverse direction. In general, R can be varied by steps of one ohm or larger so that the value of X can be found only within a certain accuracy. If the ratio arms are equal, so that $P = Q$, from (11.17) $R = X$ for a balance to be obtained, so X will usually lie somewhere between two integral values of R differing by 1 ohm.

When an approximate value for X has been obtained as above, the ratio arms are changed so that $P = 10Q$. The value of R, which now gives a balance, is from (11.17) equal to $10X$. Hence the accuracy of determination of X is carried to one-tenth of an ohm. By further increasing the ratio to $P = 100Q$, the value of $R = 100X$, and X is determined to one-hundredth of an ohm. Of course, the value of X must be such that the resistances available for R in the given instrument are sufficient to cover the range $R = 100X$, or alternatively $R = X/100$ if the ratio arms are reversed. Theoretically by having greater ratio arms the accuracy could be extended indefinitely, but in practice the introduction of very large resistances reduces the current in the circuit to such an extent that the galvanometer sensitivity becomes insufficient to respond to small charges in R, and a natural limit is thereby set to the accuracy obtainable.

6. Post Office Box and Metre Bridge.

The Wheatstone bridge is available in many different forms, two of which are the *Post Office box* and the *metre bridge*. The former, which is shown diagrammatically in fig. 8, is fitted with two tapping keys which are situated in the battery and galvanometer circuits respectively. Incidentally, the bridge is symmetrical in that the battery and galvanometer may be interchanged without affecting the balance point. Owing to the existence of electromagnetic induction (see Chap. XIII), the battery circuit should be closed before the galvanometer circuit, so that steady current conditions are already established in the circuit. The resistances in the box are usually *non-inductively wound* so as to reduce induction effects to a minimum. In any case, it is the final steady deflection of the galvanometer which is observed and not any temporary kick which may occur when the keys are depressed. Since resistance varies with temperature, only small currents are permis-

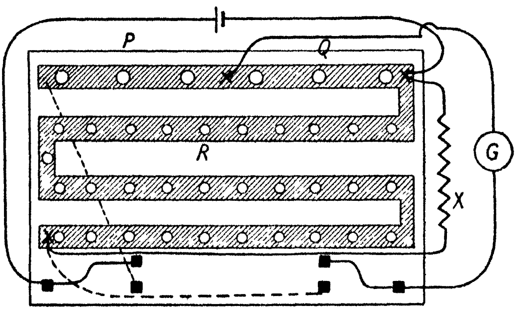

Fig. 8.—Post Office Box

sible and boxes are usually *rated* at a definite maximum current which can be carried with safety. This should on no account be exceeded or the resistances may be permanently damaged and acquire values differing from their nominal amounts.

The **metre bridge or slide-wire bridge** was devised to allow a continuous change in the value of the resistances instead of only integral values such as the Post Office box affords. As shown in fig. 9(*a*), it consists of a rectangular board to which is attached a metre rule, stretched beside which is a wire about a metre long. The wire is made of some resistance material, such as Eureka alloy, so that the whole wire has a resistance of about 1 ohm. The ends of the wire are soldered to terminals which are in turn connected to stout copper strips of negligible resistance. These strips are provided with two gaps in which are inserted a known resistance R and the unknown resistance X respectively. A third copper strip joins R and X, and a third terminal is attached to the centre of this strip. This terminal is joined by a wire to a galvanometer, the other lead of which goes to a metal slider or *jockey* which slides along the bare wire of the metre bridge. The battery is joined to the extremities of the bridge wire, and the whole constitutes the usual Wheatstone quadrilateral. A balance is obtained by sliding the jockey until no deflection is shown in the galvanometer. If the two resistances into which the bridge wire is then divided are R_1 and R_2, we have by (11.17)

$$\frac{R_1}{R_2} = \frac{R}{X}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11.18)$$

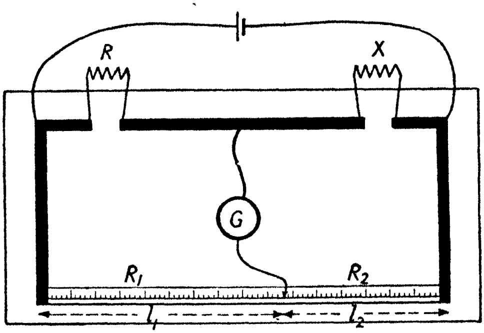

Fig. 9 (*a*). — Metre Bridge

The wire is assumed to be of uniform cross-section; hence the resistance of any portion of it is proportional to the length. Therefore $R_1 \propto l_1$ and $R_2 \propto l_2$, whence from (11.18)

$$X = R\frac{l_2}{l_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11.19)$$

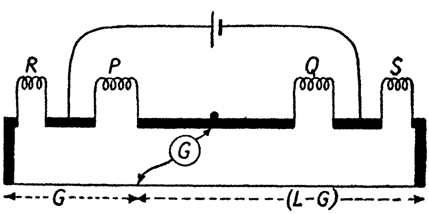

Fig. 9 (*b*).—Carey Foster Bridge

Since readings of l_1 and l_2 may be taken to a fraction of a millimetre, good accuracy is obtainable in the value of X if R is chosen to be nearly equal to X. If R is very different from X, l_2 is very different from l_1 and the ratio l_2/l_1 becomes very susceptible to error.

The *Carey Foster bridge*, a diagram of which is shown in fig. 9 (*b*), is a modification of the metre bridge to allow the accurate comparison of two nearly equal resistances R and S. From the diagram, if a balance occurs when the jockey is G from one end of the bridge of total length L and resistance per unit length ρ, we have

$$\frac{P}{Q}=\frac{R+\rho G}{S+\rho(L-G)}.$$

The resistances R and S are now interchanged, a new balance point is found at l_2, and as before we have

$$\frac{P}{Q}=\frac{S+\rho l_2}{R+\rho(L-l_2)}.$$

Hence, equating the two preceding expressions, adding unity to both sides of the equation and cancelling the numerators which then become identical, we have

$$R-S=\rho(l_2-G).$$

To find ρ, we put $S=0$, whereupon ρ is obtained in terms of R.

7. Approximate Measurement of Potential.

The electrostatic voltmeter is the ideal instrument, since it takes no current from the circuit and therefore leaves undisturbed the circuit to which it is applied. It is, however, insensitive for potentials less than about 500 volts. The ordinary moving-coil voltmeter is fairly satisfactory if its resistance is high, but clearly it will cause great disturbance if it is introduced across a circuit in which resistances of several thousand ohms are present. The *potentiometer* is a device which allows the measurement of potential without taking any current from the system.

8. The Potentiometer.

In construction, the potentiometer is identical with the metre bridge, which is often used as a potentiometer, except that the resistances R and X are omitted, these spaces being left unoccupied. The instrument therefore consists, in its simplest form, of a wire stretched beside a metre scale and connected to a *steady* supply battery such as an accumulator (see Chap. XII) as shown in fig. 10. The potentiometer will only *compare* voltages (potential differences, electromotive forces), that is, a standard voltage is first necessary to calibrate the potentiometer wire. Such a standard voltage is provided by the E.M.F. of a standard cell (see p. 108); we denote this by E_S. The positive pole of the standard cell is connected to that end of the potentiometer wire to which is attached the positive pole of the accumulator. The negative pole of the standard cell is connected through a sensitive galvanometer to a jockey sliding on the potentiometer wire.

Now at certain positions of the jockey it will be found that the galvanometer shows no deflection. This means that the drop in potential between the common point A and the point of contact C of the jockey, due to the current supplied from the accumulator to the bridge wire, is exactly equal to the potential E_s between the poles of the standard cell; the E.M.F. of a cell being by definition the potential between its poles when no current is flowing through the cell. Since the wire is uniform, the drop E in potential across any length l is known by simple ratio,

$$\frac{E}{E_s} = \frac{l}{l_s}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11.20)$$

where l_s is the distance AC.

To determine any other potential, therefore, the source to be measured simply replaces the standard cell and the new balance point

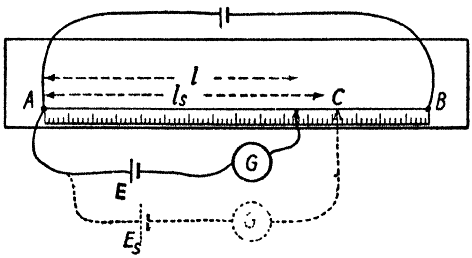

Fig. 10. — Potentiometer

is obtained. To avoid errors due to fluctuations in the voltage of the accumulator, it is usual to have a two-way switch included in the circuit so that readings of the balance points for E and E_s may be taken alternately at regular intervals and an average result obtained. The voltage to be measured must not exceed that of the accumulator; otherwise no balance point will be found anywhere along the wire. To overcome this difficulty it is not usually sufficient to increase the number of accumulators since a prohibitively large current may be produced in the slide wire. Conversely, very small potentials could not be measured accurately if the standard voltage drop had its ordinary value of 2 volts per 100 cm. of potentiometer wire. These difficulties are overcome by the use of *auxiliary resistances* in series with the slide wire of the potentiometer, and commercial potentiometers are available for measuring potentials over a very wide range.

If the potential to be measured is, say, between 99 and 100 volts, an auxiliary resistance B of some value such as 98 ohms is included in series with the wire as shown in fig. 11. The resistance of the bridge wire itself must have been accurately determined simply by inserting it in one arm of a Wheatstone bridge in the usual fashion. If we assume for simplicity that its resistance is 2 ohms, a steady applied voltage

E, which must be somewhat greater than 100 volts, would produce a current of $E/100$ amps and consequently a drop in potential of $E/50$ volts down the wire. The value of $E/50$ is easily determined by connecting a standard cell to the bridge wire but not across the auxiliary resistance; for

$$\frac{E_s}{E/50} = \frac{l_s}{100},$$

or

$$\frac{E}{50} = 100\frac{E_s}{l_s}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11.21)$$

The standard cell is now replaced by the unknown potential difference, which, however, is connected across the auxiliary resistance as well as

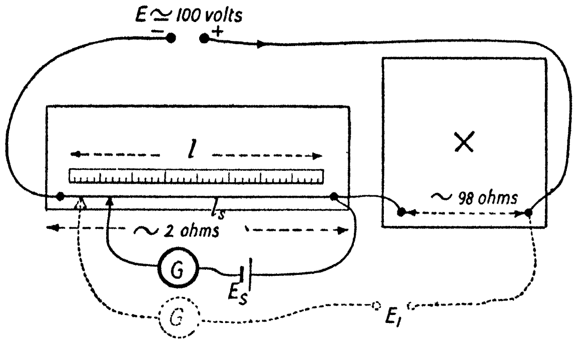

Fig. 11. — Potentiometer with Auxiliary Resistance

across the bridge wire. If the balance point occurs at some point l, we have, by reference to fig. 11,

$$\begin{aligned} E_1 &= IR \\ &= \frac{E}{100}\left(98 + \frac{l}{50}\right); \quad . \quad . \quad . \quad . \quad (11.22) \end{aligned}$$

whence, from (11.21) and (11.22),

$$E_1 = 50\frac{E_s}{l_s}\left(98 + \frac{l}{50}\right). \quad . \quad . \quad . \quad . \quad (11.23)$$

If the potential to be measured is of the order of millivolts or microvolts, as when thermoelectric potentials (see Chap. XVIII) are under consideration, the auxiliary resistance is raised to some suitable value, such that when a 2-volt accumulator is connected across auxiliary resistance and bridge wire the potential drop across the latter is of the order of several millivolts or microvolts. The actual method of procedure in this case is described in Chap. XVII.

9. Measurement of Current and Resistance with a Potentiometer.

A potentiometer will only compare potentials, but since, by Ohm's law, a current may be estimated by the potential which is required to drive it through a known resistance, the indirect measurement of current may be carried out with great accuracy with the potentiometer. Referring to fig. 12, a *small* known resistance R is inserted in the circuit carrying the current I whose value is required, and the ends of R are then connected to the potentiometer, in place of the normal standard cell. If the potentiometer wire has been calibrated with a standard cell we have

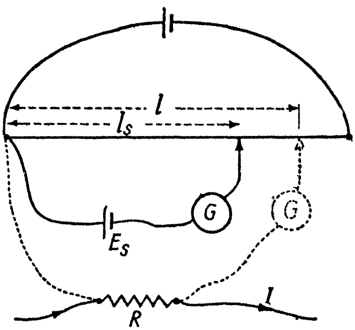

Fig. 12. — Measurement of Current

$$\frac{E_s}{l_s} = \frac{IR}{l},$$

whence
$$I = \frac{E_s}{R} \frac{l}{l_s}. \quad \ldots\ldots\ldots \quad (11.24)$$

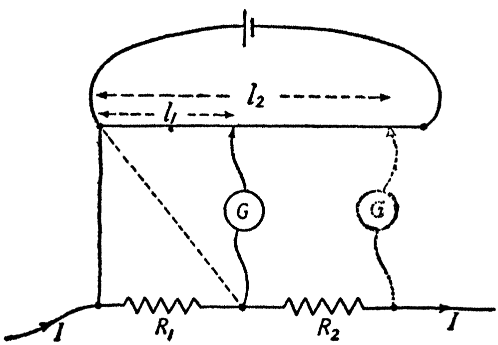

Fig. 13. — Comparison of Resistances

Two resistances R_1 and R_2 may be accurately compared by comparing the potentials E_1 and E_2 developed across them when they are carrying a common current, as in fig. 13. If balances are obtained for lengths l_1 and l_2 when the ends of first R_1 and then R_2 are connected to the potentiometer, then

$$\frac{E_1}{E_2} = \frac{IR_1}{IR_2} = \frac{l_1}{l_2}. \quad \ldots\ldots \quad (11.25)$$

10. Measurement of the Internal Resistance of a Battery.

If the poles of a battery are joined by a short thick wire of negligible resistance, the current which results, although large, does not rise to infinity as it should according to Ohm's law, where $I = E/R$ and R has been made equal to zero. This is because the electric current which flows from one pole of the battery to the other does not stop when it reaches the negative pole. On the contrary, the passage of an electric current is more correctly described as the *circulation* of electric current. The current circulates right round the circuit back through the liquid in the cell from the negative to the positive plate. Now this liquid conductor also has a resistance, which is called the **internal resistance** r of the battery. If the external resistance of the circuit is R, the current round the circuit is, by Ohm's law, given by $E = I(R + r)$. Hence if $R = 0$, $I = E/r$, i.e. the current has a finite limiting value depending on the internal resistance of the battery. Further, the potential difference available for the external circuit varies with the external resistance or load. Thus, for an external resistance R, this potential difference V is by Ohm's law

$$V = IR = E\frac{R}{(R + r)}. \quad . \quad . \quad . \quad . \quad (11.26)$$

If, therefore, the external resistance and internal resistance were equal, that is $R = r$, from (11.26)

$$V = E/2,$$

or the available external potential difference is only half the maximum potential difference which the cell can deliver, the other half being used to drive the current through the cell itself.

The total potential difference of which a cell is capable is termed the **electromotive force** (E.M.F.) of the cell, on the analogy that a force is necessary to drive a current through a resistance. The term is in common use, but in some respects is unfortunate since it is actually simply the maximum potential difference available. It is therefore measured in energy or work done per unit charge conveyed, and not in force units as the name would imply.

11. Platinum Resistance Thermometer.

As an example of the practical application of the Wheatstone bridge circuit we shall now describe its use in conjunction with the platinum resistance thermometer for temperature measurement. This thermometer when used for the measurement of high temperatures consists of a spiral of platinum wire wound on a mica former and inserted in a silica tube, as shown in fig. 14. The leads from the thermometer are connected to one arm of the Wheatstone bridge which incorporates a slide wire MO. The ratio arms are represented by P and Q, and these are generally arranged to be equal. The balance arm contains a suitable resistance R approximately equal to the resistance of the thermometer. Included in the balance arm are a pair of dummy leads identical in size, position and resistance with the leads which go to the platinum resistance thermometer. Let the resistance of the thermometer be T and let the resistance of MN be X, where

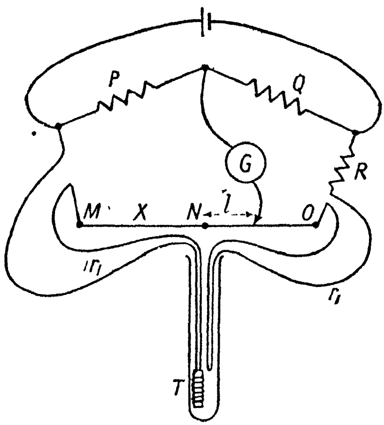

Fig. 14.— Platinum Resistance Thermometer

N is the mid-point of MO. Then if ρ is the resistance per centimetre of MN and the resistance of the leads is r_1, since $P = Q$ we have

$$r_1 + R + (X - l\rho) = r_1 + T + (X + l\rho)$$

where a balance occurs l cm. from the centre of MO. Hence

$$T = R - 2l\rho.$$

Owing to the passage of current through T even in the balance position, a certain rise in temperature occurs in the thermometer quite apart from that due to the heat supply in which it is inserted. To obtain the resistance for zero current an ammeter is inserted in the battery circuit, and a curve is plotted of T against I. This curve may be extrapolated to zero as shown in fig. 15 and hence the value of T for zero current may be estimated.

To deduce the relation between the temperature as measured on the resistance scale and that which would be found on the gas scale, we may note that Callendar found on placing a platinum resistance thermometer inside a gas thermometer that the resistance-temperature variation over a range of a few hundred degrees from 0° C. was given by

$$R_t = R_0(1 + at + \beta t^2), \quad . \quad . \quad . \quad . \quad (11.27)$$

where t is measured on the gas thermometer and R_t and R_0 are the resistances at t and 0° C. respectively. The quantities a and β are constants depending on the nature of the metal.

Now from the definition of temperature on the Centigrade scale (see Part II), the temperature on the platinum resistance scale is

$$t_{Pt} = \frac{R_t - R_0}{1/100\,(R_{100} - R_0)}. \quad . \quad . \quad . \quad . \quad (11.28)$$

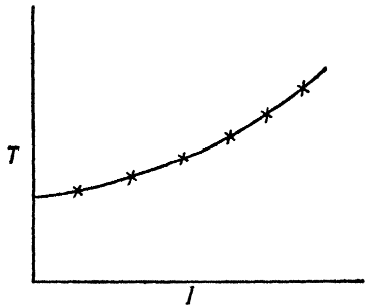

Fig. 15.—Extrapolation for Zero Current

Substituting for R_t from (11.27) in (11.28) we have

$$t_{Pt} = \frac{t(a + \beta t)}{(a + 100\beta)}. \quad . \quad . \quad . \quad . \quad . \quad (11.29)$$

We express (11.29) in the form of a difference correction thus:

$$(t - t_{Pt}) = \frac{100\beta}{(a + 100\beta)} t\left(1 - \frac{t}{100}\right), \quad . \quad . \quad (11.30)$$

whence $$(t - t_{Pt}) = \frac{\Delta t}{100}\left(\frac{t}{100} - 1\right), \quad . \quad . \quad . \quad (11.30)$$

where $\Delta = -10^4\beta/(a + 100\beta) =$ constant.

We note that if the variation of resistance with gas-scale temperature had been a linear one, that is $\beta = 0$, then $\Delta = 0$, and from (11.30) the platinum resistance scale and the gas scale would have been identical. Actually, for platinum $\Delta \simeq 1{\cdot}57$. Its value is usually found by calculating t_{Pt} from (11.28) for some fixed point such as the melting-point of sulphur, which is 444° C. on the gas scale, and substituting these values in equation (11.30). A slightly different value of Δ is required at higher temperatures, since α and β are not quite constant, the most suitable values for Δ being found by choosing suitable higher fixed points such as the melting-point of gold and other metals.

EXERCISES

1. Two wires of equal resistance are connected first in series and then in parallel. Show that the equivalent resistance of the former arrangement is four times that of the latter.

2. State and explain Kirchhoff's laws of the electric circuit. What modification if any is necessary to their formulation for a circuit containing varying currents?

3. Describe two distinct methods for finding the value of an unknown resistance suspected to be about 100 ohms.

4. Give the theory of the Wheatstone bridge circuit for steady currents. Give examples of the application of the Wheatstone bridge to physical problems.

5. Describe the use of the metre bridge in the comparison of two nearly equal resistances. How would you compare two resistances whose ratio is expected to be about 100 : 1?

6. How may a potentiometer be used (*a*) to compare E.M.F.s, (*b*) to measure a current and (*c*) to compare two resistances?

7. Owing to the resistance between wire and binding-post a potentiometer has a certain zero error. Calculate this error in the form of a length to be added to the observed balance length, given that one cell is balanced by a length of 50 cm., a second cell by a length of 40 cm., whereas when the two cells are used in series, the balance length is 93 cm. [3 cm.]

8. Describe an accurate method for finding the internal resistance of a battery. How does the internal resistance of a cell vary with temperature.

9. Explain fully how the temperature of a furnace is measured with the aid of a platinum resistance thermometer.

CHAPTER XII

Electrolysis

1. Qualitative Laws.

When an electric current is passed through a solution, in general chemical action occurs at the electrodes. For example, when an electric current is passed through a solution of copper sulphate in water it is found that metallic copper is deposited at the negative electrode or cathode. What occurs at the anode depends on the chemical nature of the electrodes. If these are of copper, the anode gradually dissolves, forming copper sulphate solution. Moreover, it dissolves at the same rate as copper is deposited at the cathode. The concentration of the solution therefore remains constant as a whole, although the distribution of the concentration, as shown by the intensity of the blue colour of the solution, is such that a decrease occurs in the neighbourhood of the cathode and a corresponding equal increase in the concentration near the anode. Although it is sometimes stated that the electrolysis of copper sulphate between copper electrodes consists in the transfer of copper from anode to cathode, it is clear from the above experimental considerations that the copper which dissolves from the anode does not proceed immediately to the cathode. The explanation of electrolysis lies in the *ionic hypothesis*, which we discuss in section 4.

If the copper electrodes are replaced by platinum electrodes, copper is still deposited at the cathode but no solution of the platinum occurs at the anode. Instead, oxygen gas is evolved. Briefly it may be stated that the products obtained by electrolysis depend on the

(i) Nature of the solute;
(ii) Nature of the solvent;
(iii) Nature of the electrodes;
(iv) Current density, that is the current divided by the area of the electrodes immersed.

These qualitative laws are illustrated by various examples which we shall consider throughout this chapter.

2. Quantitative Laws.

Faraday's laws of electrolysis are:

(1) *The amount of chemical action is the same at all points.*

This law is illustrated in fig. 1, showing several different solutions subjected simultaneously to a common current. Electrolysis takes place in all the solutions just as if the other solutions were not present.

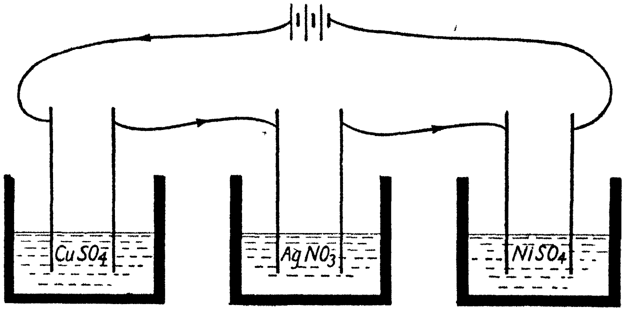

Fig. 1. — Simultaneous Electrolysis

(2) *The mass deposited or evolved is proportional to the current, the chemical equivalent of the solute and the time.*

If an ammeter is included in the circuit in which electrolysis of copper sulphate is taking place, then it is easily shown that the mass deposited is directly proportional to the time during which the current passes, and to the magnitude of the current.

Suppose now that a second electrolytic cell or **voltameter,** containing silver nitrate, is included in the circuit containing the copper voltameter. Then it is found on comparing the weights of copper and silver deposited that they are in the ratio of their chemical equivalents. The same law is obeyed if the substances are evolved as gases, as may be shown if the latter are collected, as in the *hydrogen and oxygen voltameter* shown in fig. 2, in which the liquid is dilute H_2SO_4.

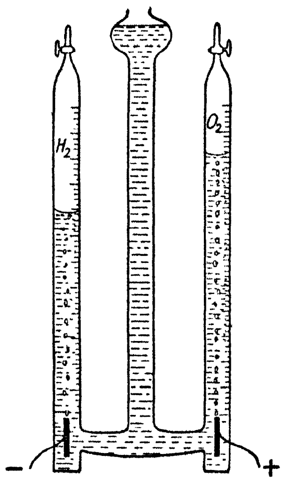

Fig. 2. — Hydrogen and Oxygen Voltameter

Faraday's laws may be expressed mathematically thus:

$$m = Izt, \qquad (12.1)$$

where m = mass deposited,
t = time of passage of current,
I = current,
z = electrochemical equivalent.

The **electrochemical equivalent** is simply the chemical equivalent multiplied by the necessary number (constant) to make equation (12.1) valid numerically. The product (It) is, of course, the **quantity** of electricity which has passed, measured in coulombs. Since in (12.1) $z = m$ if $It = 1$, the electrochemical equivalent is the mass deposited by 1 ampere in 1 sec. It is found that it requires 98,470 coulombs to deposit the chemical equivalent in grammes of any element. The numerical value of the electrochemical equivalent is therefore slightly larger than 10^{-5} of its chemical equivalent.

3. Electrochemical or International Ampere.

Since the mass deposited is directly proportional to the current, the ampere may be defined on an electrochemical basis. In practice, the amount deposited depends to a small extent on the concentration of the solution, the temperature, the nature of the electrodes, the current density and the nature of the other ions (see section 4) present. Consequently, precise details must be stipulated in the definition of the unit of current. Actually, silver nitrate solution is preferred, and the electrochemical or international ampere is defined as that current which will deposit 0·001118 gm. of silver per second from a specified solution under specified conditions.

4. Nature of Electrolysis.

The nature of electrolysis is intimately connected with the processes which occur when a solute forms a solution. On the molecular hypothesis it was originally thought that the process of solution consisted merely in the suspension of molecules of the solute between molecules of the solvent. Owing to their heat energy, the molecules of both solute and solvent were in continual random motion, as evidenced by Brownian movement, but the molecules still retained their molecular structure. Now on this hypothesis the production of copper during the electrolysis of copper sulphate would consist in the disruption of the copper sulphate molecule, followed by the transfer of the copper atom to the cathode. It is clearly difficult to understand why the copper atom should proceed to the cathode except under electrical forces, which implies that the copper atom has somehow acquired a positive charge. This might conceivably take place on disruption of the molecule by the electric force acting between anode and cathode, by analogy with electrification known to be produced by cleavage. In view of the now well established electrical theory of matter it would be a waste of time to pursue the above arguments further, but we may note one experimental fact that was strongly against the disruption hypothesis. If measurement is made of the potential E across, and the current I passing through, a solution undergoing electrolysis, the electrical energy consumed is given by EIt, where t is the time of passage of the current. Now, if the water equivalent of the electrolytic cell is known and a thermometer is inserted, it will be found that heat is evolved just as in the passage of the current through a solid conductor. Moreover, the heat generated is found to be exactly equal to the electrical energy consumed (but compare section 11). There is therefore *no energy available to disrupt the molecules of the solute.*

From this fact, and other considerations, Arrhenius argued that the process of solution consisted in far more than the distribution of molecules of solute between molecules of solvent. He suggested that **dissociation** of the solute molecule into **ions** takes place immediately the solute dissolves. A copper sulphate solution therefore consists of copper and sulphion (SO_4) ions in solution. Electrolysis then consists in the attraction of the copper ions, which are positively charged, to the cathode and of sulphions to the anode. The hypothesis was very revolutionary and met with the bitterest opposition, but that it represents the facts there is not now any question. Anomalies regarding the magnitude of osmotic pressure, variation of electrical conductivity with concentration, change in the boiling- and freezing-points of solutions, and many other phenomena, receive entirely satisfactory and natural explanation on the **ionic hypothesis.**

In a dilute solution, dissociation is complete. With concentrated solutions, dissociation is only partial and, of course, there is a balance between the number of ions which dissociate in any given time and the number which recombine. Incidentally, the analogy with gaseous conduction, which is also ionic, is complete (see Chapter XVIII). Further, the charge carried by a monovalent ion is exactly equal to the electronic charge, so the ionic hypothesis is in complete agreement with modern electron theory. Indeed, the latter received much initial support and encouragement from the ionic theory. Silver, being monovalent, carries one positive electronic charge, while the nitrate radical (NO_3) carries the corresponding negative charge. With copper sulphate, in which the binding is divalent, the copper ion carries a double positive electronic charge and the sulphion radical a double negative charge. The fact that the mass deposited is directly proportional to the chemical equivalent is therefore a natural result of the number of charges carried being numerically equal to the valency. That the charge carried is independent of the chemical nature of the substance suggests strongly what has been proved to be the case, namely that the same electronic charge is associated with all matter.

5. Further Examples.

We shall now discuss further examples of electrolysis in the light of the ionic hypothesis. Under the action of the electric field between anode and cathode, two streams of ions commence to move in opposite directions. The accelerations under the field are opposed by the viscous friction of the solute molecules, and a steady velocity is ultimately reached in agreement with Stokes's law (see Part I, p. 137). These ionic velocities will clearly be different since, while the charges of the two ions may be equal and opposite, their masses will in general be different. The velocity of an ion for unit field gradient is defined as the **mobility** of the ion; methods of measuring mobilities are described in section 6.

In the electrolysis of copper sulphate between platinum electrodes, oxygen is evolved from the anode because platinum is not susceptible to attack by the neutralized SO_4 ion. Instead, the SO_4 ions attack the solvent, water, liberating oxygen and forming sulphuric acid, which, of course, remains dissociated in solution. We express these results thus:

$$\begin{aligned} &CuSO_4 \rightleftharpoons Cu^{++} + SO_4^{--}, \\ &SO_4 + H_2O \rightarrow H_2SO_4 + O, \qquad \qquad (12.2) \\ &H_2SO_4 \rightleftharpoons H^+ + H^+ + SO_4^{--}. \end{aligned}$$

This example illustrates how the products of electrolysis depend on the nature of the solvent and the electrodes.

The behaviour of water is instructive. Pure distilled water conducts very little. The explanation of this is that it consists almost entirely of undissociated molecules (actually they are often associated into higher groups of nH_2O, where n is integral). Consequently there are few ions to carry the current. If now a small quantity of sulphuric acid is added to the water, hydrogen and oxygen are evolved when the solution is electrolysed between platinum electrodes. The equations representing the reaction are

$$\begin{aligned} &H_2SO_4 \rightleftharpoons H^+ + H^+ + SO_4^{--}, \\ &SO_4 + H_2O \rightarrow H_2SO_4 + O. \end{aligned} \qquad \qquad (12.3)$$

Since the sulphuric acid is reformed in the process while water is removed, the solution grows continually more acid. In fact, the sulphuric acid is acting like a chemical catalyst, for it remains unchanged in amount throughout the process.

With sodium chloride in water, if platinum electrodes are used, hydrogen is evolved at the cathode and platinic chloride forms in the solution. Soon after the process has commenced, therefore, platinum may be deposited together with hydrogen at the cathode. The hydrogen is formed indirectly, owing to the chemical reaction between the sodium and the water. The equations are:

$$\begin{aligned} &NaCl \rightleftharpoons Na^+ + Cl^-, \\ &Na + H_2O \rightarrow NaOH + H, \\ &4Cl + Pt \rightarrow PtCl_4, \\ &PtCl_4 \rightleftharpoons Pt^{++++} + Cl^- + Cl^- + Cl^- + Cl^-. \end{aligned} \qquad \qquad (12.4)$$

The sodium hydroxide itself also undergoes electrolysis with the liberation of oxygen at the anode. If carbon electrodes are used, these are moderately resistant chemically to chlorine, and consequently the latter is evolved from the carbon anode instead of attacking it. For rapid evolution of chlorine, however, the current density should be high, for chlorine reacts slowly with water according to the equation

$$2Cl + H_2O \rightarrow 2HCl + O, \quad . \quad . \quad . \quad . \quad (12.5)$$

so that some oxygen is also evolved at the anode.

Finally, if the current density is high, acidulated water gives some ozone as well as oxygen at platinum electrodes.

***6. Measurement of Ionic Velocities.**

Simple but approximate methods of measuring ionic velocities are based upon chemical action between the travelling ions and the material through which they travel. In Lodge's method, two vessels

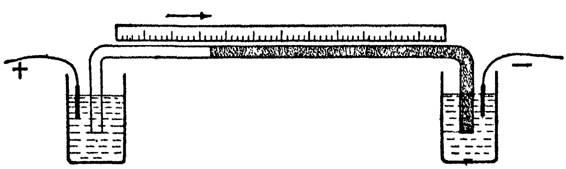

Fig. 3. — Measurement of Ionic Velocities

containing dilute sulphuric acid are joined by a horizontal tube containing sodium chloride suspended in gelatine, together with a trace of phenolphthalein as indicator, as shown in fig. 3. As electrolysis proceeds, the H^+ ions form HCl and decolorize the phenolphthalein. The velocity of the H^+ ions may therefore be determined directly from the rate of recession of the coloured edge along the tube. Alternative methods may be used based upon the change of refractive index along a tube containing the electrolyte.

More accurate methods depend on a determination of the change in concentration in the region of the anode and cathode, either by direct chemical sampling or by refractive index measurements. If u and v are the velocities of the positive and negative ions respectively, the total current and total deposition in unit time are proportional to $(u + v)$. If the current flows for such a time that $(u + v)$ gram-molecules of solute are removed from the solution, in the region near the cathode $(u + v)$ gram-atoms of positive ions have been removed, but u gram-atoms have been gained by migration, leaving a total loss of v gram-atoms of positive ions near this electrode. An equal loss of v gram-atoms of negative ions is lost by migration from the same neighbourhood. By a similar process u gram-atoms of solute are lost near the anode and hence we have

$$\frac{\text{Loss of concentration near the cathode}}{\text{Loss of concentration near the anode}} = \frac{v}{u}. \quad . \quad (12.6)$$

If the concentration of completely dissociated solute is w gram-equivalents per c.c., since 1 gram equivalent requires 1 Faraday, that is 96,470 coulombs for its deposition, the current density accompanying the electrolysis is given by

$$\rho = w(u + v)\ 96,470. \quad . \quad . \quad . \quad . \quad . \quad (12.7)$$

Since w may be measured, equations (12.6) and (12.7) are sufficient to determine v and u completely. It is usual to determine the conductivity σ of the electrolyte by some method such as that described in Chap. XIII, section 16 (iv). Then if the potential acting across the solution is V and the distance between the electrodes is l, the *field gradient* E is defined by $E = V/l$. Applying Ohm's law we have

$$\rho = \sigma E,$$

where ρ is measured in amperes per sq. cm., σ in ohm^{-1}/cm.$^{-1}$, and E in volts per cm., and hence equation (12.7) acquires its usual form

$$(u + v) = \frac{\sigma}{w}\frac{E}{96,470}. \quad . \quad . \quad . \quad . \quad . \quad (12.8)$$

7. Applications of Electrolysis.

The great variety of chemical products which may be obtained by suitable adjustment of the physical parameters makes electrolysis one of the most valuable processes in industry. We select the following typical applications of electrolysis.

(*a*) *Production of Metals.*

Aluminium is now rarely extracted by chemical smelting. The general process is to dissolve alumina, the oxide Al_2O_3, in fused cryolite, Na_3AlF_6, as solvent. Pure aluminium forms at the cathode and the process goes on as though it were simple electrolysis of the alumina, fresh supplies of which are added from time to time.

(*b*) *Refining of Copper.*

Crude copper obtained by smelting is used as the anode in a solution of copper sulphate, a pure copper rod being used as cathode. Arsenic and other impurities form at the anode and drop to the bottom of the vessel, where they form *anode sludge.*

(c) *Electroplating.*

Valuable metals such as gold and silver, or metals which resist atmospheric corrosion such as chromium, may be deposited on baser metals. In gold and silver plating, the articles are suspended to form the cathode in a solution of the double cyanides $KAu(CN)_2$ or $KAg(CN)_2$. To obtain a strongly adhesive coating the articles must be scrupulously clean and the current density must be very low. The latter consideration requires the plating process to occupy several weeks.

(d) *Electrotyping.*

If moulds of plaster or wax are coated with graphite to render them conducting, metal type may be built up by using suitable solutions and using the cast as cathode.

(e) *Anodizing.*

Aluminium and its alloys do not readily take up paint and dyes. By using the aluminium as the anode in an acidulated solution, a thin film of oxide is formed on the aluminium, after which it will readily take up the dye.

(f) *Alloys.*

By using certain concentrations of mixed solutions of metals together with suitable current densities, alloys may be formed directly by electrolysis. Examples are alloys of Cu-Zn, Cu-Sn and Zn-Cd.

Many of the processes are trade secrets, which depend on small changes in the conditions. For example, addition of certain colloids adds a lustrous finish to some electroplated articles, thus obviating subsequent polishing. Other applications are to the formation of seamless metal tubing, to the surfacing of machine tools with more durable materials, and to the building up of worn machine parts.

8. The Voltaic Cell.

The student will now realize that since electric current flows not only from the positive pole to the negative pole outside a cell but circulates right through the cell, the latter is undergoing continual electrolysis. The solution in a voltaic cell is dilute sulphuric acid, and the poles are of Zn and Cu. Reference to fig. 4 shows that the positive pole for the external circuit acts as the negative pole for the internal circuit. Consequently the copper electrode acts as the cathode and the hydrogen is given off at that electrode. At the zinc anode, the SO_4 attacks the electrode to form zinc sulphate. In practice such a cell is found to give a steady current only if the latter is very small or if the cell is in action only intermittently. The explanation of this effect, which is termed **polarization** (not to be confused with optical polarization, with which there is no connexion), is simply that some of the hydrogen bubbles adhere to the copper plate. This results in a twofold action. First, the gas is an excellent insulator and introduces an internal layer of very high resistance. Secondly, owing to the electric field present, a double layer of positive and negative ions forms on the surface of the hydrogen and the cell actually tries to send a current in the reverse direction or a **back E.M.F.** develops. Clearly the two opposing E.M.F.s eventually balance, and the current falls to zero.

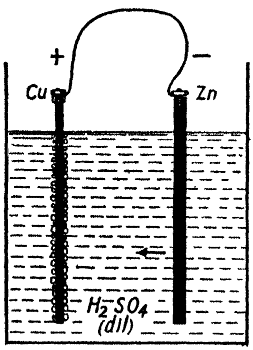

Fig. 4. — Voltaic Cell

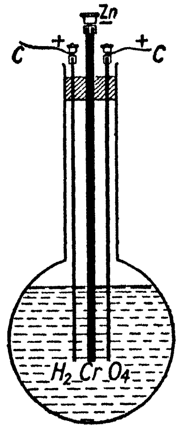

Fig. 5. — Bichromate Cell

9. Remedies for Polarization.

The remedies for polarization are of three distinct types:

(*a*) *Mechanical.*

If the copper plate is brushed the hydrogen may be removed. Such a process is clearly impracticable and suggested the construction of plates with pointed surfaces to burst the bubbles, but such methods have a very limited success.

(*b*) *Chemical.*

If a strong oxidizing agent is present, the hydrogen may be oxidized to water as fast as it is formed. Cells based on this process are fairly successful, but usually use highly corrosive liquids which require careful handling.

The **bichromate cell,** shown in fig. 5, consists of two carbon plates dipping in chromic acid, which is formed by dissolving potassium bichromate in concentrated sulphuric acid and then diluting somewhat with water. The negative pole is still zinc, but must be removed from the cell except when it is in operation or the zinc will dissolve.

(*c*) *Electrochemical.*

These cells are the most satisfactory, and the best of them is the Daniell cell. The general principle is to separate the two electrodes by a porous pot and to surround each electrode with a different solution. The porous pot prevents these solutions from mixing except by very slow diffusion but it allows free passage of ions under the influence of the electric field.

The **Daniell cell** consists of a copper vessel which forms the positive electrode. This vessel contains copper sulphate solution and a porous pot, as shown in fig. 6. The zinc electrode stands in a bath of dilute

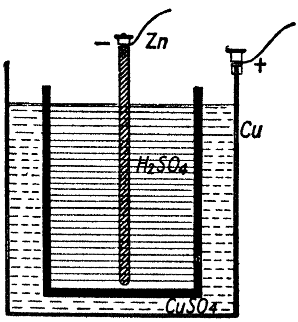

Fig. 6. — Daniell Cell

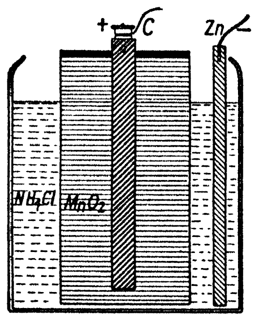

Fig. 7. — Leclanché Cell

sulphuric acid contained in the porous pot. Hydrogen ions which travel towards the copper electrode displace the copper ions in solution, and it is the latter which are deposited on the copper electrode. The chemical action therefore consists in the solution of zinc and the deposition of copper, and polarization is prevented.

Other cells are the **Bunsen cell,** using Zn and H_2SO_4 outside the porous pot, nitric acid as depolarizer and carbon as the positive electrode. **Grove's cell** uses the same materials as the Bunsen cell except that platinum is used in place of carbon.

A cell of great practical importance is the **Leclanché cell.** As shown in fig. 7, this consists of a zinc rod dipping in ammonium chloride solution outside the porous pot, inside which a carbon rod serves as anode, surrounded by solid powdered manganese dioxide as depolarizer. The equations expressing the reactions are:

$$\begin{aligned} 2NH_4Cl + Zn &\rightarrow ZnCl_2 + 2NH_3 + H_2, \\ H_2 + 2MnO_2 &\rightarrow Mn_2O_3 + H_2O. \end{aligned} \qquad (12.9)$$

The E.M.F. is about 1·4 volts. Owing to the fairly slow action of the solid depolarizer, the cell is only suitable for small or intermittent currents.

The common dry battery is a set of Leclanché cells constructed in portable form. The liquid ammonium chloride solution is in the form of a paste and the china porous pot is replaced by a linen bag.

10. Standard Cells.

The E.M.F. of the cells described so far varies somewhat with the conditions under which they are used. They are therefore quite unsuitable as standards. Two cells which suffer little variation of E.M.F. with temperature and which have been chosen as standards are the

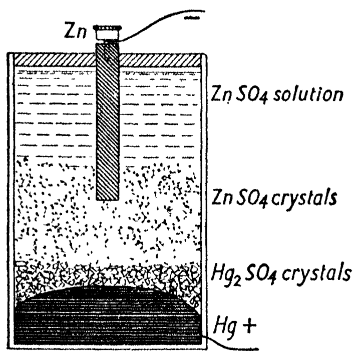

FIG. 8. — Latimer Clark Cell

Latimer Clark cell and the **Weston cadmium cell.** The former is shown diagrammatically in fig. 8. The electrodes are zinc-mercury amalgam and mercury, and zinc sulphate crystals are in contact with the zinc electrode, while zinc sulphate solution fills the main bulk of the cell. A layer of mercury sulphate crystals covers the positive mercury electrode. The E.M.F. of this cell is given by the equation

$$E = 1{\cdot}4328 - 0{\cdot}00119(t - 15^\circ \text{ C.}) - 0{\cdot}000007(t - 15)^2 \text{ volts.} \quad (12.10)$$

In the Weston cell, the zinc-mercury amalgam is replaced by a cadmium-mercury amalgam, and the zinc sulphate by cadmium sulphate. The E.M.F.-temperature equation is

$$E = 1{\cdot}0184 - 0{\cdot}0000406(t - 20) - 0{\cdot}000000(t - 20)^2 + 0{\cdot}00000001(t - 20)^3 \text{ volts.} \quad (12.11)$$

The Weston cell therefore has a much lower temperature coefficient than the Latimer Clark cell. A high resistance is always included in the circuit of a standard cell, for the E.M.F. is constant only for the passage of a very small current. The cells are therefore usually used in potentiometer and other null circuits.

11. Energetics of a Cell.

We shall now consider the source of energy in electric cells, selecting for the purpose the Daniell cell. When an equivalent weight of zinc dissolves to form zinc sulphate, a definite amount of chemical energy disappears and a corresponding amount of heat energy H_1 makes its appearance. A definite amount of heat energy H_2 likewise appears when an equivalent weight of copper dissolves to form copper sulphate. Now in the Daniell cell for every equivalent weight of zinc which passes into solution an equivalent weight of copper is deposited. Hence a net amount of energy $(H_1 - H_2)$ is available and this represents the source of electrical energy. Now the passage of 96,470 coulombs is required to deposit a gram-equivalent of any element. Hence, from the conservation of energy, we should have

$$E\,\frac{96{,}470}{4{\cdot}2} = (H_1 - H_2), \quad . \quad . \quad . \quad . \quad (12.12)$$

where E is the E.M.F. of the Daniell cell. Equation (12.12) therefore allows E to be calculated and the value obtained agrees well with the experimentally observed E.M.F. of the cell. It should be noted, however, that this agreement is due largely to the fact that the Daniell cell has a very small E.M.F.-temperature coefficient. Referring to the thermodynamical argument in Part II, Chap. XIII, p. 120, we have here $dE/d\theta = 0$, and therefore $E = H$. The constancy of the standard cells is to be attributed to the fact that they are strictly reversible and therefore have a constant E.M.F. if the temperature is kept constant, and a constant temperature coefficient if the temperature varies.

12. Secondary Cells.

The cells described so far have been **primary cells,** that is, cells in which the negative electrode is dissolved away irreversibly as time goes on. Such cells therefore require replacement of the negative electrode, of the acid solution and of the depolarizer. Secondary cells are, however, available, in which the electrodes may be reformed by electrolysis, so that effectively the cell gives current in one direction when in use, that is when *discharging*, and is then subjected to electrolysis by a current from an external supply passing in the opposite direction, until the electrodes and the original acid solution have been completely reformed. The best-known secondary cell is the **lead accumulator,** which consists of electrodes of lead and lead peroxide respectively, dipping in dilute sulphuric acid. Such a cell acts as a primary cell, the peroxide electrode being at a steady potential of about 2 volts above the lead electrode. The chemical process which occurs on discharge is shown by the following equations:

$$PbO_2 + H_2 \rightarrow PbO + H_2O,$$
$$PbO + H_2SO_4 \rightarrow PbSO_4 + H_2O, \quad . \quad . \quad (12.13)$$

and
$$Pb + SO_4 \rightarrow PbSO_4.$$

The discharging process therefore results in the formation of two electrodes each covered with lead sulphate, and therefore showing no difference in potential when the process is complete or the cell is " discharged ".

In the charging process, current is passed through the cell in such a direction that the original lead electrode is reconverted into lead according to the equation:

$$PbSO_4 + H_2 \rightarrow H_2SO_4 + Pb, \quad . \quad . \quad . \quad (12.14)$$

while the lead peroxide is reformed according to the equation

$$PbSO_4 + SO_4 + 2H_2O \rightarrow PbO_2 + 2H_2SO_4. \quad . \quad (12.15)$$

It is clear from equations (12.13) that in the discharging process water is formed, so that the specific gravity of the acid solution drops steadily. Conversely, in the charging process, equations (12.14) and (12.15) show that the acid concentration increases. Indeed, the state of charge of an accumulator is estimated from the density of the electrolyte, which varies from about 1.15 when completely discharged to 1.21 when fully charged. Throughout all these processes the E.M.F. remains approximately constant at 2.1 volts and is therefore useless as a sign of the degree of charge in the battery.

The lead peroxide plate is formed in two distinct ways. It may be formed over a long period by electrolysis, using a lead plate as an anode and allowing it to undergo oxidation. This process is lengthy and costly in the electrical energy assumed. Alternatively, red lead Pb_3O_4, which is easily obtained, is compressed into lead grids. On immersion in sulphuric acid peroxide is formed immediately according to the chemical reaction

$$Pb_3O_4 + 2H_2SO_4 \rightarrow 2PbSO_4 + PbO_2 + 2H_2O, \quad (12.16)$$

and the lead sulphate is readily converted to lead peroxide by charging over a moderate time.

13. Properties of a Good Cell.

The following properties should be possessed by the ideal cell:

(*a*) The E.M.F. should be high and constant;

(*b*) The internal resistance should be small;

(*c*) A constant current should be delivered and the cell should have a long life;

(*d*) It should be quite inactive chemically when on open circuit;

(*e*) The electrodes and electrolyte should consist of cheap and durable materials;

(*f*) It should be manageable and should not emit corrosive fumes.

All these properties are possessed by the lead accumulator, which suffers only from the disadvantage of its heavy weight. Other types of secondary cell have been attempted such as the *Drum battery*, which consists of nickel and zinc electrodes with a sodium hydroxide electrolyte. A lighter cell results, but the E.M.F. is little more than half that of the lead accumulator and the durability much less. How far the various primary cells obey the criteria (*a*) to (*f*) may be considered by the student. The low internal resistance of the lead accumulator is in part due to the method of construction in which plates of large area are situated a fraction of an inch apart. Great care must be taken not to short-circuit the accumulator, for owing to its low internal resistance the current will rise to a prohibitively high value. The internal heat developed may then be sufficient to buckle the plates, and even if this should not occur, the chemical action is so rapid that partial disintegration of the plates occurs and their life is considerably shortened. Likewise, in charging the accumulator, the current density must not be too high or fragmentation of the plates will occur.

14. Grouping of Cells.

The choice of connexion of cells in series or in parallel depends on the use to which they are to be put. For the best *economy* in electrical energy, as little as possible should be wasted as heat internally. This requires the internal resistance of the combination to be as small as possible compared with the external resistance, and the cells are therefore grouped in parallel. For the *quickest action*, such as may be required in a circuit of *high induction* (see Chap. XIII), for example an electromagnet, the total resistance of the circuit should be as high as possible. The cells are therefore connected in series.

To obtain the *maximum current* in a given external circuit the grouping of the cells varies according to the particular value of the external resistance. We shall now show that the cells must be grouped partly in series and partly in parallel, so that the resistance of the combination is as nearly as possible equal to the external resistance.

Suppose there are n files of m cells in series as shown in fig. 9, and that the internal resistance and E.M.F. of each cell are r and e respectively, the external resistance being R. The total number N of cells is then given by

$$N = mn, \quad . \quad . \quad . \quad . \quad . \quad . \quad (12.17),$$

while the total E.M.F. E is

$$E = me. \qquad (12.18)$$

Applying Ohm's law, the current I is given by

$$I = \frac{me}{mr/n + R}, \qquad (12.19)$$

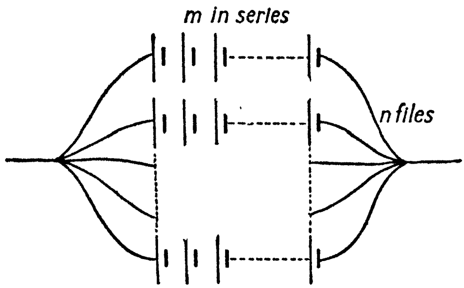

Fig. 9. — Grouping of Cells for a maximum Current

since the total internal resistance of the cells as grouped is mr/n. Substituting for m in terms of n from (12.17) in (12.19) we have

$$I = \frac{Nne}{Nr + Rn^2}. \qquad (12.20)$$

For a maximum value of I, forming the expression $\frac{dI}{dn}$ and equating to zero, we find $Nr = Rn^2$, whence

$$\frac{mr}{n} = R, \qquad (12.21)$$

or the external resistance is equal to the combined internal resistance of the group of cells.

EXERCISES

1. State the laws of electrolysis and discuss the factors which govern the ultimate products obtained by electrolytic processes.

2. Outline the application of the ionic theory to the explanation of electrolysis. What other phenomena are readily accounted for on the ionic theory?

3. Describe one method for the determination of the velocity of ions during electrolysis.

If a given solution shows a loss in concentration near the cathode and anode respectively in the ratio 2 : 1, calculate the ionic velocities, given that the concentration of completely dissociated solute is 10^{-3} gm.-equivalents/c.c., the field gradient is 3 volts/cm., and the conductivity of the solution 0·289 ohm^{-1} cm.$^{-1}$. [3×10^{-3} cm./sec.; 6×10^{-3} cm./sec.]

4. Write a short essay on the applications of electrolysis to industry.

5. Give a brief account of the processes by which polarization in cells is overcome. What are the essential properties of a good cell?

6. From what source is the energy of a primary cell derived and in what respect are the energetics of a Daniell cell unique?

7. Describe in detail the construction and mode of action of some form of secondary cell.

CHAPTER XIII

Electromagnetism

1. Faraday's Experiments.

Faraday made the most important discovery that any change in the magnetic flux threading (or " linked with ") a conductor resulted in an E.M.F. acting along that conductor. Experiments to illustrate this fact are shown in figs. 1*a* and 1*b*. In fig. 1*a* a solenoid is connected in series with a galvanometer and a bar magnet is inserted into the solenoid. A deflection of the galvanometer takes place at the instant that the magnet is introduced, but falls to zero again as soon as movement ceases, even though magnetic flux is still threading the solenoid.

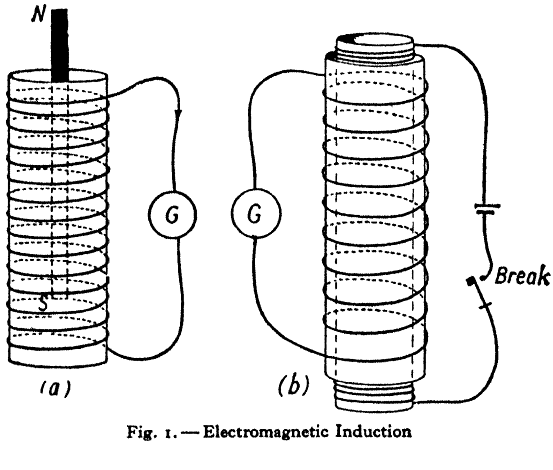

Fig. 1.—Electromagnetic Induction

If relative movement of magnetic field and conductor again occurs owing to removal of the magnet, a temporary deflection of the galvanometer in the reverse direction is observed. The permanent magnet may be replaced by a second solenoid carrying a current and thereby producing a magnetic field, with the same result as shown in fig. 1*b*. If the sign of the magnetic pole which is introduced is changed, the deflection is in the reverse direction. The magnitude of the deflection is found to depend on the speed of movement of the magnet. Faraday summed up these results in the following **Laws of Electromagnetic Induction:**

(1) *If the magnetic flux through a conductor is changed, an* E.M.F. *acts round the conductor.*

(2) *The magnitude of the* E.M.F. *is equal to the rate of change of the flux N, that is*

$$E = -\frac{dN}{dt}. \qquad \qquad (13.1)$$

If N is expressed in oersteds and t in seconds, E is obtained in *absolute electromagnetic units.* Hence to obtain the potential difference in volts (13.1) must be multiplied by 10^{-8}.

The negative sign in (13.1) indicates that the E.M.F. which is established produces a current which in turn produces a magnetic

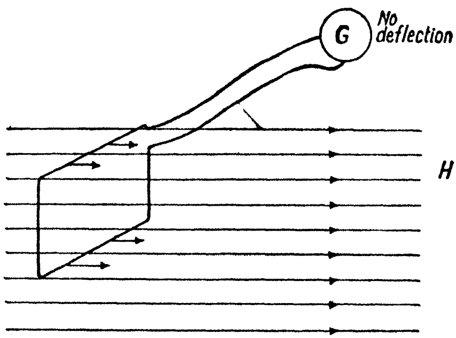

FIG. 2

field of such a sign as to prevent relative motion of the primary magnet and the conductor. This third law of electromagnetic induction, which is often termed **Lenz's law,** and which has important practical applications, may be expressed as follows:

The magnetic field produced by the excited current is always in such a direction as to oppose the change of flux being produced by the primary magnetic field.

Since a change in flux is all that is required to produce an **induced E.M.F.**, it is sufficient to leave the inner and outer solenoids(in fig. 1*b*) completely at rest and simply to vary the primary magnetic field by varying the current in the centre or *primary coil.* This may be achieved by starting and stopping the current in the primary, since this results in a changing primary magnetic flux. Many important practical applications, such as the induction coil, described in section 9, depend on this principle.

It will be noted that if a conductor moves *along* the lines of force of a magnetic field as shown in fig. 2, there is no change in the particular lines of force which cut the conductor. There is therefore no induced E.M.F. in this case.

2. Mutual Induction.

The magnitude of the magnetic flux threading the *secondary circuit* in fig. 1(*b*) depends on the number of turns of wire in the primary and secondary coils and the geometry of their disposition. Clearly, the galvanometer could be connected to either coil and the other coil used as the primary coil. This mutual effect of two or more circuits in close proximity is termed *mutual induction.* Its quantitative measure may be defined in the two following ways:

(i) If N is the total flux linked with the secondary circuit, and I is the current in the primary responsible for the flux, the **mutual inductance** M is defined by the equation

$$N = MI. \qquad (13.2)$$

The mutual inductance may therefore be defined as the magnetic flux linked with the secondary when unit current flows in the primary.

(ii) Alternatively, combining (13.1) and (13.2) we have

$$E = -M\frac{dI}{dt}, \qquad (13.3)$$

and hence the *mutual inductance is equal to the E.M.F. induced in the secondary when* $dI/dt = 1$, *that is, when the current is changing at unit rate in the primary.*

The practical unit of inductance is termed the **henry.** This is defined so that equation (13.3) still holds in practical units. Hence the inductance in henries is equal to the E.M.F. in volts induced in the secondary when the current in the primary charges at the rate of one ampere per second.

3. Self-induction.

If an ammeter is placed in a circuit containing a battery and a large electromagnet it will be found on completing the circuit that the ammeter creeps rather sluggishly to its final steady reading, which is that consistent with Ohm's law for the circuit. This effect is due to *self-induction,* which we may explain by reference to fig. 3. The current as it enters one turn after another gradually builds up a larger and larger field to thread the remaining turns. The remaining turns are therefore subject to a changing magnetic flux and consequently an E.M.F. is induced in them. By Lenz's law this E.M.F. acts in a back direction to oppose the increasing primary current, so that a finite time is required for the latter to become established.

By analogy with the definition of mutual inductance, the self-inductance L is defined by

$$N = LI, \qquad \ldots\ldots\ldots \quad (13.4)$$

where N is the magnetic flux linked with the circuit due to the current I. Alternatively, combining (13.1) and (13.4),

$$E = -L\frac{dI}{dt}, \qquad \ldots\ldots\ldots \quad (13.5)$$

which shows that the *self-inductance in henries is equal to the back* E.M.F. *in volts set up in the circuit when the current is growing at the rate of* 1 *amp./sec.*

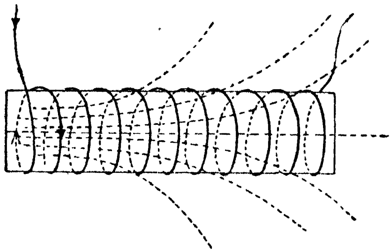

Fig. 3 — Self Induction

4. Calculation of Mutual and Self-inductance.

The value of self-inductances and mutual inductances is calculated with the aid of equations (13.5) and (13.3). For example, the self-inductance of a long solenoid is derived as follows. From equation (10.13), the field due to a steady current I in the solenoid is

$$F = 4\pi nI.$$

Hence if the area of cross-section of the solenoid is A, the magnetic flux through the area is

$$N = 4\pi nAI.$$

As I varies, the resultant change in flux threads the solenoid nl times, where l is the length of the solenoid, and nl therefore the total number of turns. Hence

$$E = -\frac{dN}{dt} = -nl\frac{d}{dt}(4\pi nAI) = -4\pi n^2lA\frac{dI}{dt}. \quad (13.6)$$

Since by (13.5) L is defined by

$$E = -L\frac{dI}{dt},$$

from this equation and (13.6) we have

$$L = 4\pi n^2lA. \qquad \ldots\ldots\ldots \quad (13.7)$$

By a similar argument if a mutual inductance consists of a primary in the form of a long solenoid with n_1 turns per unit length, and n_2 turns of wire are wound round the centre of the primary to constitute the secondary, we have

$$E = -n_2 \frac{d}{dt}(4\pi n_1 A I) = -4\pi n_1 n_2 A \frac{dI}{dt}$$

and, since $E = -M\frac{dI}{dt}$,

$$M = 4\pi n_1 n_2 A. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.8)$$

*** 5. Growth and Decay of Currents.**

The student will now realize that the steady currents governed by Ohm's law represent a particular case. In general in any circuit such as that shown in fig. 4, there is present *resistance, inductance* and *capacity.* Although steady conditions governed by Ohm's law may be

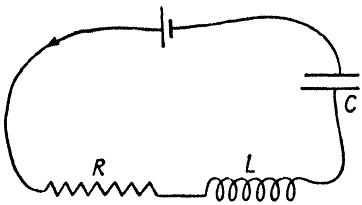

Fig. 4.— Circuit with Resistance, Inductance and Capacity

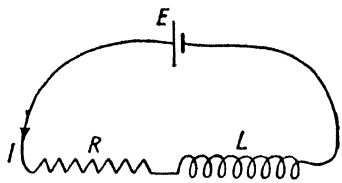

Fig. 5.— Circuit with Resistance and Inductance

eventually set up, more general considerations are required during the starting and cessation of the current. Actually a most important condition arises for certain particular values of resistance, inductance and capacity, where no steady value of current is ever attained but *electrical oscillations* are produced, the current swinging between two equal extreme values, first in one direction and then in the other.

We concentrate first on the growth of current in a circuit containing only *resistance* and *inductance*, as shown in fig. 5. Initially, before the battery is connected, the current in the circuit is zero. Eventually, the current attains a steady value I_0 given by $I_0 = E/R$, where E is the E.M.F. of the battery and R is the resistance in the circuit. We require an expression for the current I at any time t after the circuit has been completed. If E' representes the back E.M.F. present at this instant, application of Ohm's law gives

$$IR = E - E'; \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.9)$$

and, substituting for E' from (13.5) and remembering that we have already allowed for the sign of E', we obtain

$$IR = E - L\frac{dI}{dt}. \quad . \quad . \quad . \quad . \quad . \quad (13.10)$$

We therefore obtain a differential equation of the first order. Rearranging (13.10), we have

$$\frac{L\,dI}{(E - IR)} = dt; \qquad \ldots\ldots \quad (13.11)$$

whence, by integration, we obtain

$$-\frac{L}{R}\log_e(E - IR) = t + A, \qquad \ldots \quad (13.12)$$

where A is a constant.

Now when $t = 0$, that is before the circuit is completed, $I = 0$, so substitution in (13.12) gives $A = -(L\log_e E)/R$. Hence

$$\log_e\frac{(E - IR)}{E} = -\frac{R}{L}t, \qquad \ldots\ldots \quad (13.13)$$

or

$$E - IR = E\exp.\left(-\frac{R}{L}t\right), \qquad \ldots \quad (13.14)$$

or

$$I = \frac{E}{R}\left(1 - \left(\exp. - \frac{R}{L}t\right)\right). \qquad . \quad (13.15)$$

Now the final steady current $I_0 = E/R$, so that (13.15) becomes

$$I = I_0\left(1 - \exp. - \frac{R}{L}t\right). \qquad \ldots \quad (13.16)$$

In fig. 6 the full curve is the graph of I against t. It shows that the final steady value I_0 is theoretically reached only when $t = \infty$. The important fact emerges that the speed with which the current rises to its final steady value is governed not by the magnitude of L alone but rather by the ratio L/R. This ratio is termed the **time constant** of the circuit.

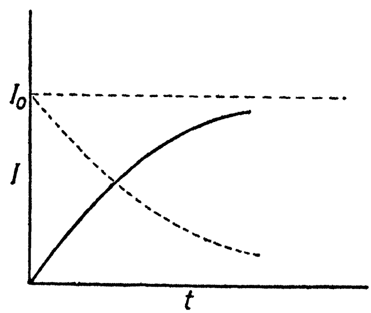

Fig. 6.—Growth and Decay of Current

By analogy with the preceding considerations, if the battery is cut out and the circuit immediately closed again, the current I at some subsequent time t will be given by

$$IR = E', \quad . \quad (13.17)$$

where E' is the back E.M.F. at time t. This E.M.F. opposes the decaying E.M.F. and tends to prolong the current. Its value is given by $E' = -L\,dI/dt$ as usual, and hence we have

$$\frac{dI}{I} = -\frac{R}{L}dt. \qquad \ldots\ldots \quad (13.18)$$

By integration of (13.18) and substitution of the initial condition $I = I_0$ when $t = 0$, we obtain

$$I = I_0 \exp.\left(-\frac{R}{L}t\right). \quad . \quad . \quad . \quad . \quad (13.19)$$

This expression is shown graphically by the dotted curve in fig. 6. The growth and decay curves have the same time constant and are in every way complementary.

We discuss further cases in Chap. XV.

6. Coil rotating in a Magnetic Field.

We have seen that for a continual E.M.F. to be generated, relative movement must take place between the lines of force and the conductor. This is most easily achieved by rotating a coil in a uniform magnetic field, as shown in fig. 7, where the rectangular coil rotates about an axis (shown dotted) in the plane of the coil and perpendicular to the field. To calculate the E.M.F. acting round the coil at any instant when the inclination of the plane of the coil to the uniform magnetic field H is θ, we note that the flux N changes from HA to zero as the coil rotates from $\theta = \pi/2$ to $\theta = 0$, and that the effective flux at any intermediate value is

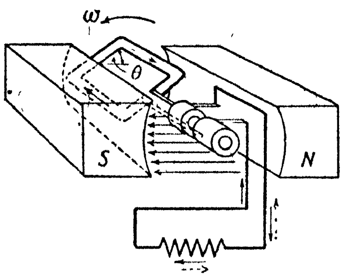

Fig. 7.—Coil rotating in Magnetic Field

$$N = nHA \sin\theta, \quad . \quad . \quad . \quad . \quad . \quad (13.20)$$

where n is the number of turns of wire in the coil. Hence from (13.20) and (13.1)

$$E = - nHA \cos\theta \frac{d\theta}{dt}. \quad . \quad . \quad . \quad . \quad (13.21)$$

Now the steady angular velocity ω of rotation of the coil equals $d\theta/dt$. Hence (13.21) becomes

$$E = - nHA\omega \cos\theta. \quad . \quad . \quad . \quad . \quad (13.22)$$

The change in flux and in E.M.F. during the rotation are shown in fig. 8. The flux is a maximum when $\theta = \pi/2$, that is when the coil is perpendicular to the field, but the E.M.F. is then zero. Conversely, when the plane of the coil and the field direction coincide and the flux is zero, the E.M.F. attains its maximum value $nAH\omega$. The explanation lies in the fact that the *rate of change* of flux is a maximum in the latter position.

If the coil circuit is completed by the **slip rings** (see fig. 7) being joined to some external circuit, the current in the latter will clearly be in one direction for one half-cycle of revolution and in the reverse

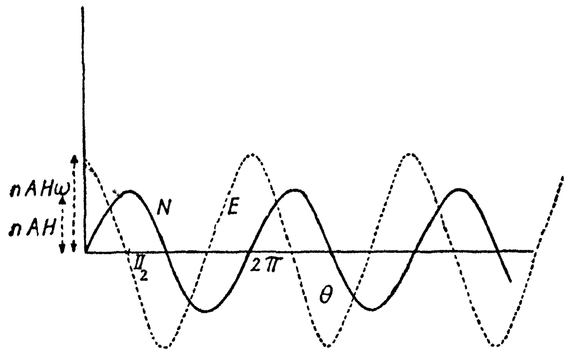

Fig. 8.—Flux and E.M.F. for Rotating Coil

direction for the second half-cycle. Such an arrangement therefore acts as a source of **alternating current.**

By a mechanical split-ring device, known as a **commutator** and shown in fig. 9, the connexions to the external circuit are reversed each half-cycle of revolution. In this way the current in the external circuit

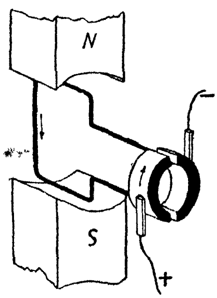

Fig. 9.—Fixed Split Ring Commutator

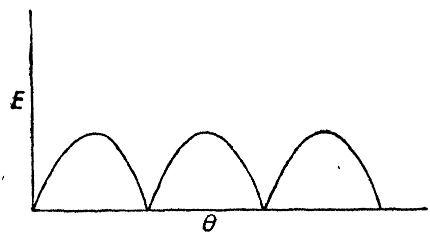

Fig. 10.—Half Sine Curve

becomes **direct current,** though, of course, it varies in magnitude according to the *half sine curve* shown in fig. 10. The alternating current is said to have undergone mechanical **rectification** by the commutator.

The **earth-inductor** shown in fig. 11 simply consists of a coil which may be rotated in the earth's magnetic field. It is mounted in gimbals so that it may be rotated in any orientation and is fitted with a commutator. Clearly, if it is rotated with its axis horizontal and in the magnetic meridian, the coil effectively cuts the earth's vertical magnetic component V. Again, if it is rotated with its axis in a vertical plane, it is cutting the earth's horizontal component H. If the deflections on a tangent galvanometer connected to the inductor are θ_1 and θ_2 respectively, and the speed of rotation is the same in both experiments,

$$\frac{V}{H} = \frac{\tan\theta_1}{\tan\theta_2} = \tan D, \quad (13.23)$$

where D is the angle of dip.

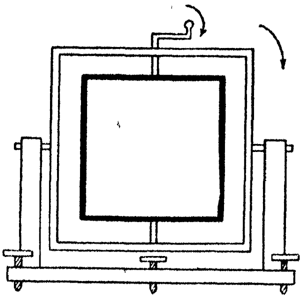

Fig. 11.—Earth-Inductor

Finally, if the coil is rotated with its axle along the total intensity of the earth's field, no deflection will be produced in the galvanometer, as there is no effective change of flux. The angle of dip of the axle is then the magnetic angle of dip. Such an instrument is instructive but does not give such an accurate value of the angle of dip as the method described in Chap. VII.

7. The Dynamo.

The rotating coil described in section 6 is a simple form of *dynamo*. In practice, each dynamo is designed for a specific purpose, but certain general rules are applicable. First, a more powerful magnetic field than that of the earth is required, and this is generally provided by an electromagnet or **field coils** excited by the current from the coil or **armature** itself. These field coils may either be connected *in series* or *in parallel* with the armature windings and, more commonly, part of the windings of the field coil may be in series and part in parallel with the armature. Such an instrument is said to be **compound wound,** and we shall now consider the advantage of a compound winding.

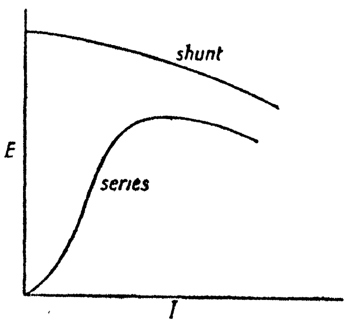

Fig. 12.—Characteristic Curves for Shunt and Series Windings

Consider first the curve obtained by plotting the E.M.F. available in an external circuit of the **series-wound** dynamo, against the current in the external circuit. As shown in the lower curve in fig. 12, this curve starts at the origin, rises rapidly to a fairly flat maximum and then falls slowly. The explanation of this *characteristic curve* is as follows. If the resistance of the external circuit is high, the current which is produced by revolution of the armature is necessarily low, since armature, field coils and external circuit are all in series. The-electromagnetism which can be excited in the field coils is therefore small, and consequently the field cut by the armature is small. Hence only a small E.M.F. develops, and a small current accompanies the small E.M.F. As the external resistance is reduced, the current increases correspondingly, more excitation of the field coils takes place and a larger E.M.F. develops in the armature. Eventually, should the resistance of the external circuit fall below that of the armature and the field coils, the E.M.F. available for the external circuit becomes progressively less, and the E.M.F. at the terminals of the dynamo becomes correspondingly smaller.

In the **shunt-wound** dynamo, where the field coils and the external circuit are in parallel, a very different characteristic curve is obtained, as shown in the top curve in fig. 12. When the external resistance is high, almost all the current generated in the armature goes through the field coils. The latter are therefore highly excited, and the E.M.F. produced by the armature starts with its maximum value. As the resistance of the external circuit is gradually reduced, it allows more current to pass through it and less current is available for the field coils. The excitation of the latter therefore drops, and there is a corresponding fall in the E.M.F. generated by the armature. The characteristic curve of the shunt-wound dynamo therefore falls steadily as shown in fig. 12.

Now it is clearly undesirable that the voltage available at a main supply should vary with the external resistance, that is with the **load** taken by the external circuit. Since the characteristic curves of the series-wound and shunt-wound dynamos are oppositely directed over a considerable range, a compound winding may be constructed to give a reasonably constant voltage characteristic over a moderate range of load.

Quite apart from load variations, a single coil and commutator, as shown by the graph in fig. 10, will provide only a sinusoidal and not a steady E.M.F. The difficulty is overcome by using a number of conductors arranged around a central cylindrical axis or *drum* to constitute the *drum armature.* These conductors are connected partly in series and partly in parallel according to the precise design of the particular dynamo, but their essential function is that each contributes a certain amount to the total E.M.F. When one conductor moves to a fresh position its former place is occupied by a neighbouring conductor, so that the total E.M.F. remains constant except for a slight *ripple,* the elimination of which, if required, is obtained by some *smoothing device* such as the *choke coils and condensers* described on p. 158.

8. The Transformer.

The simple arrangement of the two solenoids described on p. 114 constitutes a form of *transformer.* If the secondary coil has a large number of turns compared with the primary coil it may be found that on completing the primary circuit with a battery of a few volts, the temporary E.M.F. set up in the secondary is of the order of several thousand volts. The low voltage in the primary has therefore been transformed into a much greater voltage by a step-up transformer. The process is entirely reciprocal and a large voltage applied to the secondary from some external source would result in a correspondingly small voltage being temporarily produced in the primary, the latter now acting as a low-voltage secondary. Such arrangements are known as **step-up** and **step-down transformers** respectively. To increase the magnetic flux between the primary and secondary circuits, the solenoids are usually wound concentrically on formers of magnetic materials. For such instruments no simple relation exists between the voltage and the number of turns in the coil, but for the less efficient case of an *air-core* transformer it is approximately true that

$$\frac{E_s}{E_p} = \frac{n_s}{n_p}, \qquad \qquad (13.24)$$

where E_s and E_p are the secondary and primary E.M.F.s and n_s and n_p the numbers of turns in the secondary and primary coils respectively.

In order to provide a continual difference of potential in the secondary, arrangements must be made to maintain a continual change in the magnetic flux. This may be achieved (1) by making and breaking the circuit repeatedly, (2) by using an alternating current in the primary.

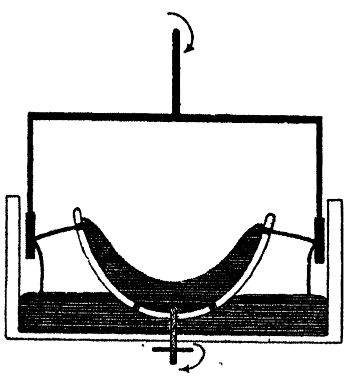

Fig. 13.—Rotary Mercury Make and Break

The latter method is generally the more convenient. With the former, either an electromagnetic trembler or a rotary mercury make and break may be used. The operation of the trembler is described as part of the induction coil in section 9. The rotary mercury make and break as shown in fig. 13 commonly consists of an electric motor attached to a centrifugal device which shoots a jet of mercury against revolving metal blades. The instrument runs in an atmosphere of coal gas to prevent oxidation at the spark.

The power output available from the secondary of a transformer cannot be greater than the power input in the primary. Consequently the high E.M.F. in the secondary of a step-up transformer is accompanied by a corresponding reduction in the secondary current, whereas for a step-down transformer a corresponding increase in current accompanies the reduction in E.M.F. With no losses, the ideal relation would be

$$E_p I_p = E_s I_s, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.25)$$

where E_p, I_p are the E.M.F. and current at the primary and E_s, I_s the E.M.F. and current at the secondary.

While the introduction of ferromagnetic material increases the magnetic linkage between primary and secondary, it introduces other losses in the form of hysteresis and eddy currents.

9. The Induction Coil.

The induction coil is a particular form of transformer usually designed to give a potential of several thousand volts when a few volts and amps are applied to the primary. It is of great practical

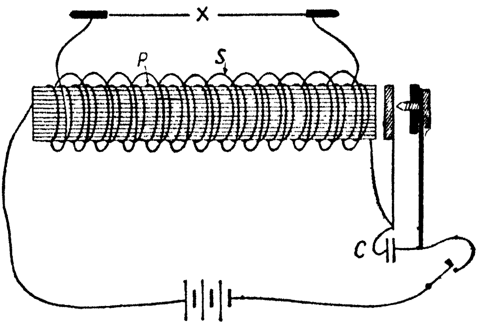

Fig. 14.—Induction Coil

importance in providing the spark for automobile ignition, and in providing a suitable source of power to operate X-ray and other discharge tubes. As shown in fig. 14, it consists of concentric solenoidal primary and secondary coils P and S wound on a bundle of iron wires which act as a core. Intermittent current is provided by the electromagnetic make and break across which a large condenser C is usually inserted. The practical function of the latter is to reduce the spark at the break by providing an alternative circuit for the battery which charges the condenser when it is no longer able to send a current through the primary. The condenser, of course, discharges through the primary at the make.

10. The Electric Motor.

As shown in fig. 15, the electric motor is in principle very similar in construction to the dynamo, and in fact operates by the reverse process. It consists of an armature and field coils and may be series wound, shunt wound or compound wound. In the series-wound motor, for example, rotation is produced by the interaction of the magnetic fields excited in the field coils and the armature when current is sent through the motor from an external supply. To produce continuous rotation a commutator is necessary to reverse the current so that portions of the armature which are attracted at any instant to the field coils are repelled at the appropriate instant later. The angular momentum of the armature is sufficient to carry it beyond the "dead" position and a continuous rotation results.

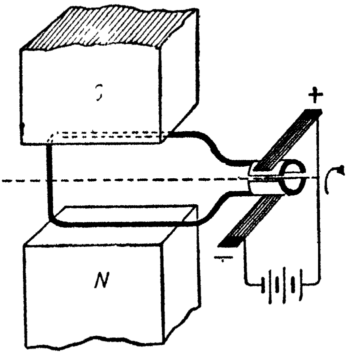

Fig. 15.—Simple Electric Motor

Now a motor when running constitutes a dynamo, and therefore generates an E.M.F. which by Lenz's law opposes the externally applied E.M.F. This back E.M.F. substantially reduces the current through the motor, when it is running steadily, below the value it has when the motor is first connected and no back E.M.F. has been generated. Motors are therefore fitted with variable resistances to avoid overheating of the machine by initial excess currents. These resistances are gradually cut out as the steady operating conditions are reached.

The **efficiency** of electric motors is usually considered from three aspects. The *electrical efficiency* is defined as

$$\frac{\text{Power obtained on rotation}}{\text{Electrical power supplied}}.$$

The *mechanical efficiency* is defined as

$$\frac{\text{Brake output (see Part I, p. 43)}}{\text{Electrical power spent in producing rotation}}.$$

The *commercial efficiency* is defined as

$$\frac{\text{Brake output}}{\text{Electrical power supplied}}.$$

We are particularly interested in the electrical efficiency F. If the back E.M.F. developed is E', and the current in the armature is I, the power obtained on rotation is $E'I$, while that supplied to the motor is EI, where E is the E.M.F. applied to the terminals. Hence

$$F = \frac{E'I}{EI} = \frac{E'}{E}. \qquad (13.26)$$

For maximum possible efficiency $F = 1$, and hence from (13.26) $E' = E$. Clearly such a state of affairs could never be reached, for no current would then flow through the motor. In practice when such a condition is almost reached, the motor rotates very rapidly but very little power is available for external work, as the following considerations show. If the power obtained is P,

$$P = E'I = I(E - IR), \quad . \quad . \quad . \quad . \quad (13.27)$$

since $E' = E - IR$, where R is the resistance of the motor. The condition for maximum power is obtained by differentiating (13.27) thus:

$$\frac{dP}{dI} = 0 = E - 2IR$$

or

$$E' = \tfrac{1}{2}E. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.28)$$

Analogous considerations hold for the efficiencies of the dynamo, but, as with design, each machine requires special treatment and general considerations are of little value.

* 11. Ballistic Galvanometers.

The galvanometers described in Chap. IX were intended for use in measuring "steady" currents either in the form of direct current or of alternating current of constant amplitude. Ballistic galvanometers are designed to measure currents of short duration, such as result from the charging or discharging of a condenser, or that arise from the presence of self or mutual inductance in the circuit. Both suspended magnet and suspended coil instruments are available, but the formulæ deduced differ from steady current relations in that they involve the time of oscillation of the suspended system.

Considering first the *moving magnet type of ~~tangent~~ galvanometer* design, it is essential that the inertia of the moving system should be large enough to ensure that the discharge has passed before appreciable rotation has occurred. Suppose the current in the coil is i at any time t, that the magnetic field at the centre due to unit current in the galvanometer is G, and that the pole-strength of the suspended magnet is m. Then we have that the total impulse

$$P = \int_0^T Gim\,dt = Gm\int_0^T i\,dt = Gmq, \quad . \quad . \quad (13.29)$$

where q is the total quantity of electricity discharged. If $2l$ is the equivalent length of the magnet, the moment of impulse about the axis is

$$2l \,.\, Gmq = GMq, \quad . \quad . \quad . \quad . \quad . \quad (13.30)$$

where M is the moment of the suspended magnet. This equals the product of the angular velocity of rotation ω of the magnet and its moment of inertia K. Hence

$$K\omega = GMq. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.31)$$

If θ is the maximum angle of swing, the total kinetic energy imparted to the magnet eventually becomes potential energy in the earth's field. We therefore have

$$\tfrac{1}{2}K\omega^2 = MH(1 - \cos\theta), \quad . \quad . \quad . \quad . \quad (13.32)$$

from fig. 16. Eliminating ω from (13.21) and (13.22), we obtain

$$q^2 = \frac{K}{MH} \cdot \frac{4H^2}{G^2} \cdot \sin^2\frac{\theta}{2}. \quad . \quad . \quad . \quad . \quad (13.33)$$

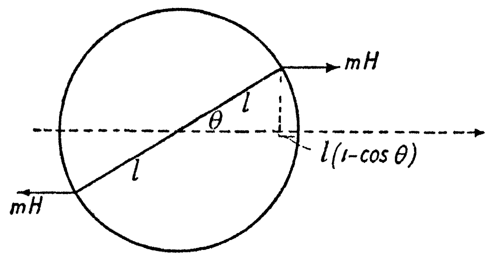

Fig. 16.—Potential Energy of Magnet

Now the time of oscillation T of the suspended magnet under the earth's field alone is given by

$$T = 2\pi\sqrt{\frac{K}{MH}}, \quad . \quad . \quad . \quad . \quad . \quad (13.34)$$

so that, eliminating K and M from (13.33) and (13.34), we find

$$q = \frac{HT}{\pi G}\sin\frac{\theta}{2}. \quad . \quad . \quad . \quad . \quad . \quad (13.35)$$

The quantity of electricity discharged is therefore proportional to the sine of half the angle of throw.

With the *suspended coil galvanometer*, if we consider initially for simplicity a rectangular coil of vertical side l and breadth b, the total impulse on one side is

$$P = \int_0^T Fil\,dt = Flq, \quad . \quad . \quad . \quad . \quad (13.36)$$

where F is the force of the magnetic field in which the coil is situated. The moment of the impulse about the axis is therefore

$$Flqb = FqA = K\omega \quad . \quad . \quad . \quad . \quad . \quad (13.37)$$

(the angular momentum), A being the area of cross-section of the coil. If the final angle of swing is θ, then the kinetic energy is ultimately all converted to potential energy in the suspension. Hence we have

$$\tfrac{1}{2}K\omega^2 = \tfrac{1}{2}c\theta^2, \quad \ldots \ldots \quad (13.38)$$

where c is the constant of the suspension. Eliminating ω from (13.37) and (13.38), and remembering that the time of free oscillation of the suspended coil under the torsion in the fibre is given by

$$T = 2\pi\sqrt{\frac{K}{c}}, \quad \ldots \ldots \quad (13.39)$$

we find

$$q = \frac{cT}{2\pi FA}\theta. \quad \ldots \ldots \quad (13.40)$$

The quantity of electricity discharged through the suspended coil galvanometer is therefore directly proportional to the maximum throw θ.

*** 12. Damping.**

The maximum throw which is observed is not as large as the throw to be expected theoretically, owing to two damping factors. The first of these is the air resistance to the motion of the coil. The second factor is the electromagnetic damping. As the coil swings in the permanent magnetic field F, by Lenz's law induced currents are set up in the coil which produce magnetic fields which interact with the primary field F so as to oppose the continual motion of the coil. If successive values of the maximum throws of the galvanometer as the energy dies away are given by θ_1, θ_2, θ_3, θ_4, &c., then it is found that

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4}, \text{ \&c.} = d = \text{constant.} \quad \ldots \quad (13.41)$$

The constant ratio of successive throws is termed the **decrement,** and we introduce, for reasons which will be clear in a moment, the logarithmic decrement $\log_e d = \lambda$ or $d = \exp. \lambda$. From (13.41) we have for two successive throws on the same side

$$\frac{\theta_1}{\theta_3} = \exp.(2\lambda). \quad \ldots \ldots \quad (13.42)$$

Since for a whole swing the decrement is exp. (2λ) while for half a swing it is exp. λ, we infer that for a quarter swing such as constitutes the ordinary throw from zero to its maximum position,

$$\frac{\theta}{\theta_1} = \exp.\frac{\lambda}{2}, \quad \ldots \ldots \quad (13.43)$$

where θ is the theoretical value of the swing and θ_1 is the observed swing. Expanding the exponential and taking the first two terms as a sufficiently good approximation,

$$\theta = \theta_1 \exp. \frac{\lambda}{2} = \theta_1 \left(1 + \frac{\lambda}{2}\right). \quad . \quad . \quad . \quad (13.44)$$

It is important to note that T in (13.40) is the time of oscillation on open circuit.

Ballistic galvanometers are calibrated by the use of a small direct current. If the potential is E across the galvanometer, and the resistance of the circuit is R, then assuming it is a tangent instrument and the deflection is θ_1,

$$I = \frac{E}{R} = \frac{H}{G} \tan \theta_1 \quad . \quad . \quad . \quad . \quad . \quad (13.45)$$

Hence, substituting for H/G in (13.35) from (13.45), for small angles of θ and θ_1 we have

$$q = \frac{ET}{2\pi R} \frac{\theta}{\theta_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.46)$$

For the suspended coil galvanometer, the steady deflection θ_1 due to a current $I = E/R$ is from (9.6) given by

$$\frac{E}{R} = \frac{c}{AF} \theta_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.47)$$

Substituting $E/R\theta_1$ for c/AF in (13.40) we obtain

$$q = \frac{ET}{2\pi R} \frac{\theta}{\theta_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.48)$$

thus showing from the equivalence of (13.46) and (13.48) that the two galvanometers obey identical formulæ for small deflections.

* 13. Applications of Ballistic Galvanometers.

(i) *Comparison of Capacities.*

Much more convenient than the electrostatic methods described in Chap. IV, section 4, is the comparison of capacities with the ballistic galvanometer. A typical circuit for this purpose is shown in fig. 17, where a depression of the tapping key charges the condenser with a charge $q = EC$, where E is the potential supplied by the battery and C is the capacity of the condenser. On releasing the key, the condenser is discharged through the ballistic galvanometer, and a deflection proportional to the charge is obtained. If the applied potential is kept constant, the deflections given by the discharge of different condensers are directly proportional to their capacities.

The practical unit of capacity is the **farad** which is defined as one coulomb divided by one volt. Small capacities are expressed in microfarads.

(ii) *Measurement of Mutual Inductance.*

Suppose the current in the primary of a mutual inductance at any instant is I_1 and in the secondary I_2, and that the mutual inductance is M, while the self-inductance of the secondary is L_2. Then applying Ohm's law to the secondary after the fashion of section 4 of this chapter we have

$$L_2\frac{dI_2}{dt}+M\frac{dI_1}{dt}+I_2R_2=0, \quad . \quad . \quad . \quad (13.49)$$

where R_2 is the resistance of the secondary circuit. Integrating (13.49) with respect to time from $t=0$ to $t=\infty$ we get

$$\frac{L_2}{R_2}\int_0^\infty\frac{dI_2}{dt}dt+\frac{M}{R_2}\int_0^\infty\frac{dI_1}{dt}dt=-\int_0^\infty I_2\,dt. \quad (13.50)$$

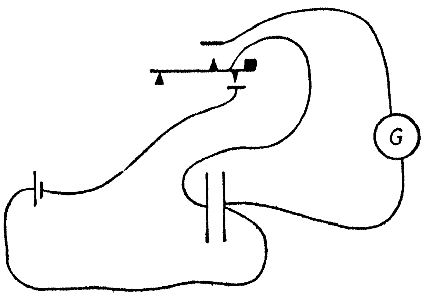

Fig. 17.—Ballistic Galvanometer and Capacities

The first term in (13.50) is zero at both limits, so we obtain

$$\frac{M}{R_2}\int_0^\infty dI_1=-\int_0^\infty I_2\,dt. \quad . \quad . \quad . \quad . \quad (13.51)$$

Now the term on the R.H.S. represents (ignoring sign) the quantity of electricity q which has circulated round the secondary circuit, and

$$\int_0^\infty dI=\Big|I_1\Big|_0^\infty=I_0.$$

Hence
$$q=\frac{MI_0}{R_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.52)$$

If the current in the primary had been reversed from $+I_0$ to $-I_0$, the quantity circulating in the secondary would be double that given by (13.52). Since the quantity q can be easily measured with the ballistic galvanometer, application of (13.52) allows the mutual inductance to be calculated.

(iii) *Measurement of Magnetic Field Strength.*

The strength of a magnetic field which is uniform over a large volume may be measured with the earth-inductor described in section 6. For the powerful magnetic field between the poles of an electromagnet where the homogeneity of the field is restricted to a very small volume, the best method is to use the fluxmeter described in the next section. The ballistic galvanometer, however, provides a method both convenient and accurate, as follows. A small flat circular coil termed a **search coil** is connected in series with the ballistic galvanometer and is initially situated between the poles of the electromagnet. It is then rapidly flicked away and the throw of the ballistic galvanometer is a direct measure of the magnetic field strength. Provided the induced charge has circulated through the ballistic galvanometer before the suspended coil of the latter has started to move, the magnitude of the throw is independent of speed of removal of the coil. If R is the resistance of the galvanometer and search coil, and the magnetic flux is N, then the current circulating through the coil at any time t during the process of removal is, by equation (13.1),

$$I = -\frac{1}{R}\frac{dN}{dt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.53)$$

and since the quantity of electricity dq which flows through the circuit in time dt is $I\,dt$, (13.53) may be written

$$q = -\int_0^N \frac{dN}{R}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.54)$$

and hence the total charge q which circulates through the ballistic galvanometer is

$$q = -\frac{N}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.55)$$

Now the flux $N = FAn$, where n is the number of turns in the search coil of area A and F is the strength of the magnetic field. Hence we have

$$F = -\frac{qR}{nA}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.56)$$

(iv) *Hysteresis Curve.*

It frequently happens that it is inconvenient to use a specimen in the form of a short rod when the hysteresis characteristics (see next Chapter, section 6) are required. A dynamical method is available in which the specimen is in the form of an anchor ring, thus forming a closed magnetic circuit. A primary circuit P consists of an endless solenoid of n_1 turns per unit length wound on the ring: the secondary circuit S, which is connected to the ballistic galvanometer G, consists of a few turns n_2 of wire wound close together round the primary at one spot. Then, when the current is established in the primary, as shown in fig. 18, if the flux linked with the secondary circuit is N, by equation (13.55) the quantity of electricity which circulates through the secondary circuit is $q = N/R$, where the sign has been omitted as it only signifies the direction in which the charge circulates. Now

$$N = BAn_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.57)$$

where B is the magnetic induction in the ring and A is its area of cross-section, so we obtain

$$B = \frac{qR}{n_2A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.58)$$

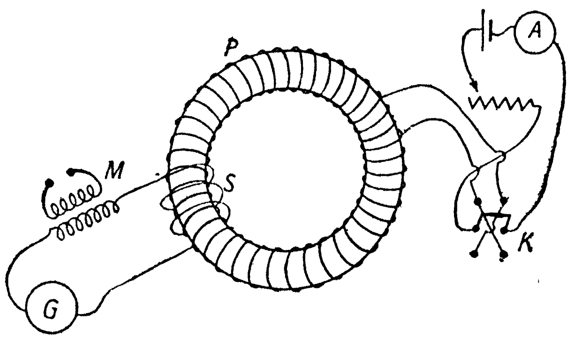

Fig. 18. — Determination of Hysteresis Curve

The magnetic field producing this flux is that due to an endless solenoid as given by equation (10.17). Hence, since $q \propto \theta$ where θ is the throw of the ballistic galvanometer, $B \propto \theta$; and H the magnetizing field is proportional to the primary current I, so that the curve obtained by plotting θ against I will give the B-H curve. This gives, of course, one half of the hysteresis loop if different values of I are used. The other half of the hysteresis loop is plotted from symmetry with the first half. In practice, the maximum saturation current is first used, and it is usual to reverse the current so as to obtain double the throw in the ballistic galvanometer. Indeed a reversing key K must always be included in the primary circuit to reduce the iron to a *cyclic state* by subjecting it to several reversals of current between each reading. To obtain B absolutely, the ballistic galvanometer must be calibrated by reversing a current through a known mutual inductance M, which must remain in the circuit throughout the hysteresis experiment, so that the resistance R of the secondary circuit remains constant throughout the experiment.

The standard mutual inductance is usually in the form of a long straight primary solenoid, round the centre of which are wound a few turns of wire to form a secondary circuit. The flux through the secondary consequent on reversing the current in the primary is, from equation (13.8),

$$N = 2 \times 4\pi n_1 n_2 AI, \quad . \quad . \quad . \quad . \quad . \quad (13.59)$$

where n_1 is the number of turns per unit length of the primary, n_2 is the total number of turns of the secondary, A is the area of cross-section of the solenoid, and I is the maximum steady current in the primary in electromagnetic units. Such a standard magnetic flux is useful in calibrating the ballistic galvanometer when using it to measure the strength of a magnetic field as described in section (iii).

* 14. Grassot Fluxmeter.

The Grassot fluxmeter consists essentially of a moving-coil galvanometer in which the restoring torque in the suspension is negligibly small. The pointer attached to the coil therefore has no definite zero but remains normally at rest in any position on the scale. It is connected to a standard search coil and is initially calibrated by establishing a known magnetic flux through the coil. The point of particular interest about the instrument is that the deflection obtained on removing the coil from a given field whose value is required depends only on the initial and final value of the flux through the search coil and is quite independent of the speed of removal. To prove this, suppose that the current through the coil is I at any instant when the E.M.F. is E, and that the resistance of the fluxmeter circuit is R. Then $I = E/R$, and from the theory of the moving-coil galvanometer in Chap. IX, the couple on the moving coil is

$$G = nIAH = \frac{nEAH}{R}, \quad . \quad . \quad . \quad . \quad (13.60)$$

where n is the number of turns in the galvanometer coil, H is the magnetic field in which it moves and A is its area of cross-section. This couple results in an angular acceleration such that

$$G = K\frac{d\omega}{dt} = \frac{nEAH}{R}, \quad . \quad . \quad . \quad . \quad (13.61)$$

where K is the moment of inertia of the moving coil about its axis of suspension, and ω is its angular velocity of rotation. Now the E.M.F. is the difference between the E.M.F. dN/dt due to the search coil cutting the flux of the magnetic field, and the back E.M.F. due to the rotation of the galvanometer coil in its own magnetic field H. Hence

$$E = \left(\frac{dN}{dt} - AH\omega\right), \quad . \quad . \quad . \quad . \quad (13.62)$$

since from equation (13.22) the back E.M.F. produced by the rotating coil is $AH\omega$ if θ is small. Substituting from (13.61) in (13.62) and integrating with respect to t, we obtain

$$\frac{nAH}{R}\int_0^t \left(\frac{dN}{dt} - AH\frac{d\theta}{dt}\right) dt = K\int_0^t \frac{d\omega}{dt}\, dt, \quad . \quad (13.63)$$

where $\omega = d\theta/dt$. The expression on the R.H.S. is zero at both limits since the galvanometer coil goes from rest to rest, that is $\omega = 0$. Hence (13.63) becomes

$$\Big[N\Big]_0^N = AH\Big[\theta\Big]_0^{\theta_0},$$

or

$$N = AH\theta_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.64)$$

where θ_0 is the change in reading of the pointer of the fluxmeter and N is the total change in the magnetic flux linked with the search coil.

* 15. Discharge through a Divided Circuit.

We have seen in Chap. XI that with steady currents the current divides in a two-branched circuit in inverse proportion to the resistances of the branches. We shall now obtain a corresponding result

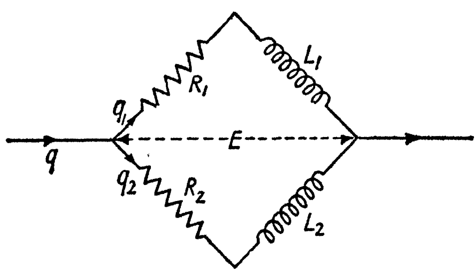

Fig. 19.—Discharge through Divided Circuit

when discharge of a system takes place through a two-branched circuit, such as that shown in fig. 19, where the inductance and resistance of one arm are L_1 and R_1 and of the other, L_2 and R_2. If the E.M.F. across each partial circuit is E at any time t, we have

$$L_1\frac{dI_1}{dt} + R_1 I_1 = E$$

for the first circuit, and

$$L_2 \frac{dI_2}{dt} + R_2 I_2 = E$$

for the second circuit. Hence

$$L_1 \int_0^t \frac{dI_1}{dt}\, dt + R_1 \int_0^t I_1\, dt = L_2 \int_0^t \frac{dI_2}{dt}\, dt + R_2 \int_0^t I_2\, dt,$$

where we have integrated with respect to t. Now I_1 and I_2 are both zero at the beginning and end of the discharge. Hence the first terms on both the R.H.S. and L.H.S. of the preceding equation are zero. But $\int_0^t I_1\, dt = q_1$, and $\int_0^t I_2\, dt = q_2$, where q_1 and q_2 are the charges which pass through the two arms respectively. Hence

$$\frac{q_1}{q_2} = \frac{R_2}{R_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.65)$$

so that the distribution of charge depends only on the resistance and not upon the inductance of the circuit. A ballistic galvanometer may therefore be used with a shunt which has been fitted for steady current conditions.

* 16. Application of Bridge Methods to Varying Currents.

We have already seen in Chap. XI how the Wheatstone bridge affords a very valuable null method for the accurate measurement of resistance to steady currents. Exactly analogous methods have been devised to compare inductances, capacities and resistances, or even an inductance with a capacity, and so on. The number of such bridge circuits, especially when used in conjunction with oscillatory circuits described in the next chapter, is legion, and we shall select just a few simple examples for our purpose.

(i) *Comparison of Self-Inductances.*

The two self-inductances L_1 and L_2 are connected in the arms of the bridge, as shown in fig. 20, together with resistances P, Q, R and S, of which either P and Q, or R and S, must *both* be variable. A balance is first obtained for a steady current. We then have from the standard Wheatstone relation:

$$\frac{P}{R} = \frac{Q}{S}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.65a)$$

The additional condition which is now required is that no momentary discharge is registered by the ballistic galvanometer when the battery circuit is made or broken. We proceed to show that on this condition

$$\frac{L_1}{L_2} = \frac{P}{R} = \frac{Q}{S}. \quad . \quad . \quad . \quad . \quad . \quad (13.66)$$

In general, the particular ratio of P/R chosen for a balance on steady currents will not satisfy equation (13.66) simultaneously, and the ratio P/R must be changed until this condition is satisfied. The operation of the bridge depends on rate of growth of potential at A and C remaining the same as the currents rise to their steady values in the two arms on completing the battery circuit. From our considerations of section 5 this equality of growth requires that the time constants shall be the same for the two arms. Hence

$$\frac{L_1}{P+Q} = \frac{L_2}{R+S},$$

and on combining this equation with (13.65*a*) we obtain equation (13.66).

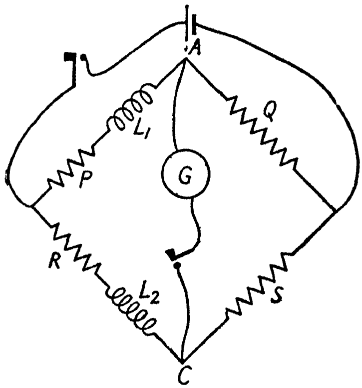

Fig. 20.—Bridge with Self-Inductances

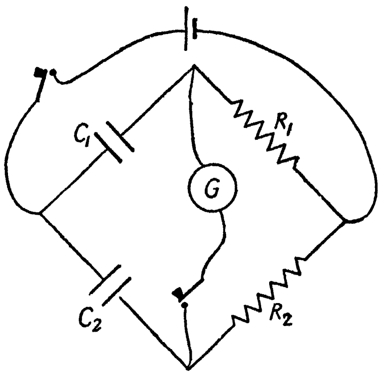

Fig. 21.—Bridge with Capacities

(ii) *Comparison of Capacities.*

This bridge, due to de Sauty, is shown in fig. 21. In operation it is completely analogous to the inductance bridge described in the preceding section. There is, of course, no question of balancing the circuit for steady currents in this case. The condition for no throw in the ballistic galvanometer on completing the battery circuit is that the time constants of the two arms of the bridge shall be equal. From considerations analogous to those of section 5, the equality of the time constants of circuits containing capacity and resistance requires

$$C_1R_1 = C_2R_2;$$

and hence $$\frac{C_1}{C_2} = \frac{R_2}{R_1}. \qquad \qquad (13.67)$$

(iii) *Comparison of Capacity and Mutual Inductance by Campbell's Bridge.*

As an example of the use of an alternating current bridge to compare capacity and mutual inductance, we use the circuit shown in fig. 22. With A.C. circuits the galvanometer could be either of the soft-iron or hot-wire type, but the sensitivity of such instruments is generally too low for accurate work. In practice the most common device is to replace the galvanometer by a telephone T, and to determine by the ear when no current is flowing through the telephone and the circuit is therefore balanced. The A.C. supply which replaces the battery used for steady current work must, of course, be of audio-frequency. With the current disposition shown in fig. 22, for no current through the telephone we have, considering circuit DEA,

$$L\frac{dI_1}{dt} - M\frac{dI}{dt} + R_2 I_1 = 0,$$

or, since $I = I_1 + I_2$,

$$L\frac{dI_1}{dt} - M\frac{dI_1}{dt} + R_2 J_1 = M\frac{dI_2}{dt}. \qquad (13.68)$$

For the circuit DBA, since A and D are always at the same potential,

$$RI_2 - R_1 I_1 = \frac{q}{C},$$

where q is the charge on the condenser and equals $\int I_1\, dt$. Hence

$$RI_2 - R_1 I_1 = \frac{1}{C}\int I_1\, dt,$$

or $$R_1\frac{dI_1}{dt} + \frac{I_1}{C} = R\frac{dI_2}{dt}. \qquad (13.69)$$

Eliminating dI_2/dt from equations (13.68) and (13.69) we obtain

$$\left(L - M - \frac{MR_1}{R}\right)\frac{dI_1}{dt} + \left(R_2 - \frac{M}{RC}\right) I_1 = 0. \qquad (13.70)$$

Now the first term in equation (13.70) is a function of the rate of change of the current while the second is not. Hence each term must vanish separately, so that

$$\left(L - M - M\frac{R_1}{R}\right) = 0 = \left(R_2 - \frac{M}{RC}\right)$$

or $$\frac{L}{M} = \frac{R + R_1}{R} \text{ and } \frac{M}{C} = RR_2. \qquad (13.71)$$

The resistances are therefore adjusted until no sound is heard in the telephone, when application of equation (13.71) gives the comparison of the inductances and capacities.

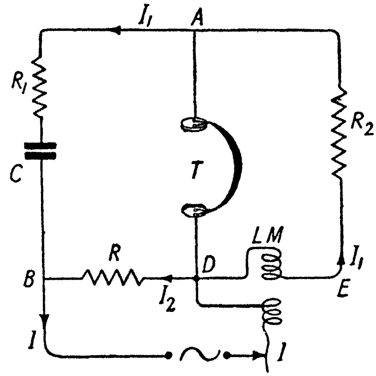

Fig. 22.—Campbell's Bridge

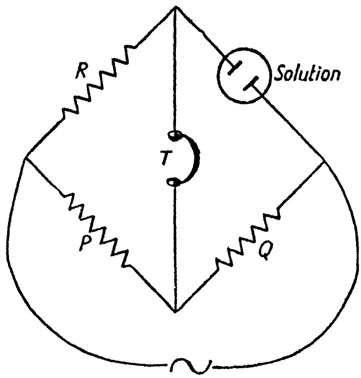

Fig. 23.—Conductivity of an Electrolyte

(iv) *Conductivity of an Electrolyte.*

The resistance or the conductivity of an electrolyte is required for various purposes, as for example when an estimate of the mobility of the ions is required as described in Chap. XII, section 6. Owing to the presence of polarization and the chemical action at the electrodes attendant upon electrolysis, the use of a direct current is inadmissible. By the use of an alternating current, the ordinary Wheatstone bridge methods may be applied directly to the solution, as shown in fig. 23. The resistances P and Q represent the ratio arms in the usual fashion, and R is adjusted until minimum sound is heard in the telephone T, which replaces the galvanometer normally used in D.C. work.

* 17. Absolute Determination of Resistance.

We have already described how the absolute unit of current is defined and measured by its magnetic or chemical effects. Of the remaining quantities, the unit of resistance or absolute ohm is determined independently, while the unit of E.M.F. is derived from the other two units by application of Ohm's law. The absolute ohm has been determined by methods based on: (i) the measurement of the deflection of a compass needle from the magnetic meridian due to the magnetic field of a rotating earth-inductor. The resultant deflection depends only on the area of the coil, its resistance and number of turns, and its angular velocity of rotation; (ii) the method of Lorenz which we describe below; (iii) the use of a suspended magnet ballistic galvanometer and a standard mutual inductance.

Combining equations (13.52) and (13.35) we have

$$\frac{MI_0}{R_2} = \frac{HT}{\pi G} \sin\frac{\theta}{2}. \quad . \quad . \quad . \quad . \quad . \quad (13.72)$$

If θ_1 is the steady deflection produced in the galvanometer when the steady potential across a resistance R in the primary (carrying a steady current I_0) is applied to the secondary circuit, including the galvanometer, we have

$$\frac{I_0 R}{R_2} = \frac{H}{G} \tan \theta_1. \quad . \quad . \quad . \quad . \quad . \quad (13.73)$$

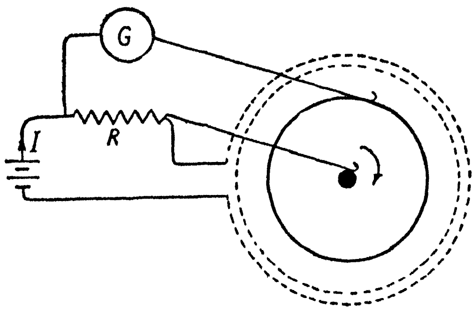

Fig. 24.—Resistance by Lorenz's Method

Hence from (13.72) and (13.73) we have

$$R = M \frac{\pi}{T} \frac{\tan \theta_1}{\sin \theta_2}, \quad . \quad . \quad . \quad . \quad . \quad (13.74)$$

so that on measuring T and calculating M from the dimensions of the mutual inductance, R is obtained absolutely.

In Lorenz's method, a metal disk is rotated in the magnetic field produced by a current passing through a mutual inductance consisting of two coaxial circular coils. As shown in fig. 24, the E.M.F. generated between the centre and the circumference of the disk is balanced against the difference of potential between the ends of the resistance R, through which is passing the current which energises the coils of the mutual inductance. If the disk makes n revolutions per second we have

$$IR = nMI;$$

hence

$$R = nM,$$

and by calculating M from the dimensions of the mutual inductance, R is obtained absolutely. Special devices must be introduced to obtain an accurately known low resistance for R, and thermoelectric effects at the junctions of different parts of the circuit must be eliminated.

EXERCISES

1. State the laws of electromagnetic induction and give examples of their application.

2. Define the terms "mutual induction" and "self-induction" and calculate the self-induction of a solenoid of length l, cross-sectional area A, and having a total of N turns. [$4\pi N^2A/l$.]

3. Show that for a circuit containing inductance and resistance, the growth and decay curves for the current are exponential and complementary.

4. Show that for a coil rotating uniformly in a magnetic field, the E.M.F. produced is sinusoidal and proportional to the angular velocity of rotation. How is the current rectified and smoothed?

5. Describe the use of the earth-inductor for finding the angle of magnetic dip.

6. Compare and contrast the operation of a dynamo and an electric motor, both of simple form.

7. What are the essentials of a good transformer, and how are certain phenomena, avoided in transformer construction, put to use in other instruments?

8. Describe the construction and mode of action of an induction coil and give examples of its application.

9. Obtain an expression for the throw of a ballistic coil galvanometer duly corrected for damping. How is such a galvanometer usually calibrated?

10. Describe how the strength of a magnetic field is determined with a search coil and a ballistic galvanometer.

11. How may a hysteresis curve be obtained by a dynamical method and what advantages does this method possess over the magnetometer method?

12. Give the theory of the Grassot fluxmeter, explaining clearly why the observed deflection is independent of the speed of removal of the search coil.

13. Obtain an expression for the distribution of discharge through a two-divided circuit, and hence show that a shunt fitted to a ballistic galvanometer under steady current conditions is still suitable when used for sudden discharges.

14. Describe two methods by which two capacities may be accurately compared.

15. Explain clearly how the conductivity of an electrolyte is determined.

16. Describe one method by which resistance may be measured absolutely.

CHAPTER XIV

Magnetic Properties of Materials

1. Introduction.

All substances show reaction to a magnetic field if the latter is sufficiently intense. This is consistent with the electron theory of matter (see Chap. XX), since the incessant motion of the electrons, on the dynamic model of the atom, is equivalent to electric currents, which are always associated with magnetic fields. Certain groupings of electrons, however, are far more effective in producing a resultant magnetic field than others, and the most effective of these constitutes the ferromagnetic group. We shall commence with a series of definitions based on the observed behaviour of materials in this strongly magnetic group.

2. Intensity of Magnetization.

As we have seen in Chap. VI, the effect produced by a magnet depends on the distance apart of the poles as well as on the pole strength, and consequently the magnetic moment is a better criterion of the "strength" of a magnet than the pole strength alone. Since a magnet is three-dimensional, a concept still more relevant to a real magnet is the *intensity of magnetization*, I, of a specimen, which is defined as the magnetic moment per unit volume, that is

$$I = \frac{M}{V}, \quad \ldots \ldots \quad (14.1)$$

where V is the volume of the specimen, and M its magnetic moment. If the magnet is rectangular as shown in fig. 1, and the area of cross-

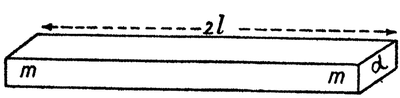

Fig. 1.— Intensity of Magnetization

section is a, the pole strength m and the length $2l$, since $M = 2ml$ and $V = 2la$, (14.1) may be written

$$I = \frac{2ml}{2la} = \frac{m}{a}, \quad \ldots \ldots \quad (14.2)$$

or the intensity of magnetization is equal to the *pole strength per unit area* or *polar density*. This last definition is useful but must be accepted with reserve, since (1) the magnetic pole is not in fact spread over the area of the end of the magnet in the same way as electric charge in the corresponding case of a charged conductor, (2) the length $2l$ used in defining the magnetic moment is the equivalent length of the magnet whereas in computing the volume it is the geometrical length.

3. Magnetic Susceptibility.

If a specimen acquires an intensity of magnetization I as a result of being in a magnetic field H, the *magnetic susceptibility* k is defined by

$$k = \frac{I}{H}. \quad \text{. . . . .} \quad (14.3)$$

The quantitative definition is consistent with the qualitative concept of susceptibility as a measure of the ease of magnetization of a specimen.

4. Magnetic Intensity near Flat Poles.

By analogy with the electric intensity between two flat condenser plates, the magnetic intensity near two flat magnetic poles is obtained directly. The laws are exactly the same since they are based in both instances on the inverse square law. Referring to fig. 2, and assuming that the polar density is I, we have by analogy with (3.16), if F is the strength of the magnetic field between two (infinite) flat poles,

$$F = 4\pi I, \quad \text{. .} \quad (14.4)$$

while the effect due to a single pole is, from (3.17),

$$F = 2\pi I. \quad \text{. .} \quad (14.5)$$

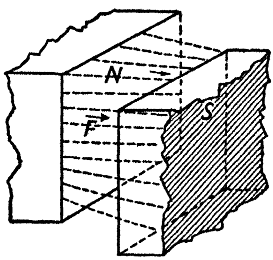

Fig. 2.—Field between two Flat Poles

Finally, by analogy with (3.18), the mechanical force of attraction P between two flat poles of area A situated close together is

$$P = 2\pi I^2 A. \quad \text{. . . . . . .} \quad (14.6)$$

Gauss's theorem is directly applicable to problems in magnetism just as in electrostatics.

5. Magnetic Induction and Permeability.

If a bar of ferro-magnetic material is placed with its longest axis parallel to a uniform magnetic field H as shown in fig. 3, the number of lines of force crossing unit area in the bar perpendicular to its axis is increased above H due to the original field, owing to the partial magnetization of the specimen. If the intensity of magnetization is I, then from the preceding section, the intensity of the field at P in fig. 3 due to the molecular orientation is $4\pi I$. Hence, the total magnetic intensity or the number of lines per unit area is given by

$$B = H + 4\pi I, \quad . \quad . \quad . \quad . \quad . \quad . \quad (14.7)$$

where B is termed the **magnetic induction** in the specimen.

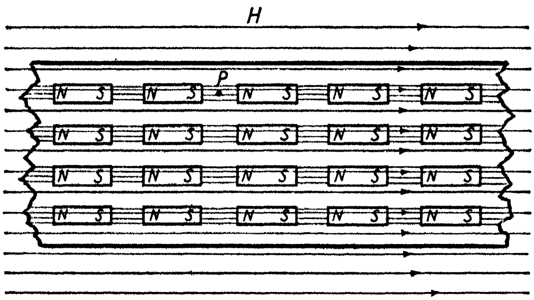

Fig. 3.—Magnetic Induction

The ratio of the magnetic induction to the inducing field is defined as the **permeability** μ of the specimen, that is,

$$\mu = \frac{B}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14.8)$$

Substituting from (14.3) and (14.8) in (14.7) we obtain

$$\mu = 1 + 4\pi k. \quad . \quad . \quad . \quad . \quad . \quad . \quad (14.9)$$

Hence, if one of the magnetic constants μ or k is known, the other is easily calculated from (14.9). If the space between and around two theoretical unit poles is filled with material of permeability μ, there is a reduction in the force of attraction between the poles proportional to the permeability of the medium, and the law of force becomes

$$F = \frac{m_1 m_2}{\mu r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (14.10)$$

6. Magnetic Constants and Magnetic Hysteresis.

To determine the magnetic susceptibility of a specimen of ferromagnetic material the apparatus shown in fig. 4 may be used. It consists essentially of a deflection magnetometer similar to that described in Chap. VII, section 4. On one of the arms of the magnetometer is a solenoid S to enclose the specimen, which is in the form of a thin rod. On the other arm is a compensating coil C, the purpose of which is to neutralize the effect of the magnetic field of the solenoid as distinct from the magnetic field produced by the magnetized specimen. Compensation is first obtained by removing the specimen, using a fairly high magnetizing current and adjusting the position of solenoid and compensating coil until no deflection is shown in the magnetometer. The current is then cut off, the unmagnetized specimen in-

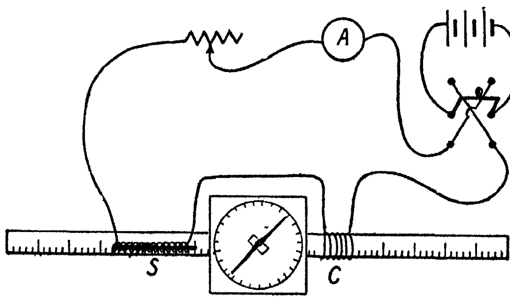

Fig. 4. — Determination of Magnetic Susceptibility

serted and a small current sent through the solenoid. The magnetic moment of the specimen is calculated from equation (7.12) for the standard end-on or Gauss A position. The intensity of magnetization is obtained by dividing the measured magnetic moment by the measured volume of the rod. Finally, the intensity H of the magnetizing field of the solenoid is assumed to be that given in equation (10.13) for an infinite solenoid. Hence the magnetizing field is proportional to the current I, and the intensity of magnetization to $\tan\theta$, where θ is the magnetometer deflection. A graph of $\tan\theta$ against I is shown in fig. 5. The susceptibility k, defined from (14.3), is given by the slope of the tangent to the curve at any point. The susceptibility of ferromagnetic substances therefore varies with the inducing field as shown in fig. 6. As the specimen acquires magnetic saturation the susceptibility approaches zero. The permeability is calculated from (14.9) when the susceptibility has been measured. Similarly the B-H curve is constructed from (14.7) after the I-H curve has been obtained.

It is very important in plotting fig. 5 that the current should be increased continually by small amounts, and that no temporary reduction of current is allowed to occur during the experiment. After saturation has been reached, if the current is reduced steadily by increments equal to those by which it was increased it is found that

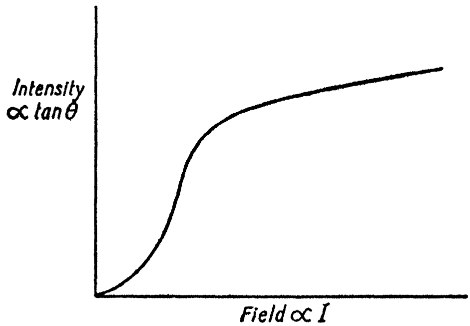

Fig. 5.— Intensity of Magnetization and Field

the specimen does not retrace its original path. The intensity of magnetization lags behind the reduced inducing field giving the curve BC shown in fig. 7. The ordinate OC is termed the **residual magnetism** of the specimen. To reduce the magnetization to zero, the current through the solenoid must be reversed in direction. If this is done and

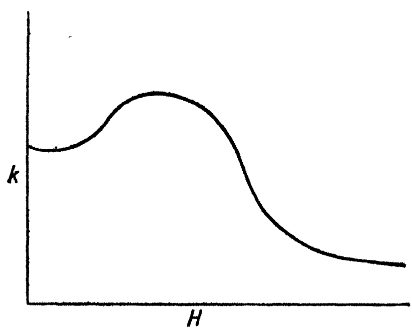

Fig. 6.— Susceptibility and Field

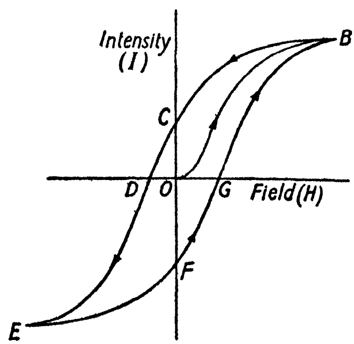

Fig. 7.— Hysteresis Cycle or Loop

the strength of the current continually increased, the magnetization is eventually destroyed. The abscissa DO is termed the **coercive force** of the specimen, as it represents the strength of field required to demagnetize the specimen. Further increase of the current in the reverse direction results in magnetic saturation in the reverse direction. Finally, if the current is reduced to zero and again increased to its maximum in the original direction, the path EFGB is traversed.

The path BCDEFGB is said to constitute a **hysteresis cycle or loop** and has many important properties. For example, it is shown in section 7 that the area enclosed by the loop represents electromagnetic energy which is converted, usually wastefully, into heat. Such a transformation occurs in the iron cores of transformers running on alternating current in the primary. Consideration of the hysteresis curves of various ferromagnetic materials makes it possible to choose a thin loop with minimum hysteresis loss. Soft iron has a thin loop while hard steel embraces a considerable area.

* 7. Energy Loss due to Hysteresis.

We shall show that the area of the I-H hysteresis loop represents the loss of energy when the material is taken round the loop. From the molecular theory of magnetism, let M be the magnetic moment of any one of the molecular magnets, and suppose that at some stage the axis of this molecular magnet is inclined at θ to the field direction. Then the component of magnetic moment parallel to the field is $M\cos\theta$, and perpendicular to the field it is $M\sin\theta$. Hence for all the molecules in unit volume, remembering that the magnetic moment per unit volume is by definition the intensity of magnetization I,

$$\begin{aligned} \Sigma M\cos\theta &= I, \\ \Sigma M\sin\theta &= 0, \end{aligned} \quad . \quad . \quad . \quad . \quad . \quad (14.11)$$

since there is no resultant magnetism perpendicular to the magnetic axis.

Now the couple acting on any one of the molecular magnets when it is inclined at θ to the field is, from (7.2),

$$G = MH\sin\theta, \quad . \quad . \quad . \quad . \quad . \quad (14.12)$$

and the work done in rotating the molecular magnets through further angles $(-d\theta)$, resulting in an increase in the intensity of magnetization dI of the specimen is

$$\begin{aligned} dW &= \Sigma MH\sin\theta(-d\theta) \\ &= -H\,.\,\Sigma M\sin\theta\,.\,d\theta. \quad . \quad . \quad (14.13) \end{aligned}$$

Differentiating the first of equations (14.11),

$$dI = -\Sigma M\sin\theta\,.\,d\theta, \quad . \quad . \quad . \quad . \quad (14.14)$$

and substituting from (14.14) in (14.13) we have

$$dW = H\,.\,dI,$$

and hence the work done in a complete hysteresis cycle is

$$W = \int_{-I_{sat}}^{+I_{sat}} H\,dI = \text{area BCDEFGB}. \quad . \quad (14.15)$$

Now if B is the magnetic induction,

$$B = H + 4\pi I,$$

or, by differentiating,

$$dB = dH + 4\pi dI. \quad \ldots \quad (14.16)$$

Substituting from (14.16) in (14.15) we obtain

$$W = \int_{-B_{sat}}^{+B_{sat}} \frac{H\,dB}{4\pi} - \int_{-H_{sat}}^{+H_{sat}} \frac{H\,dH}{4\pi}. \quad \ldots \quad (14.17)$$

The second term on the R.H.S. in (14.17) is zero, since $H\,dH = d(\tfrac{1}{2}H^2)$, and hence

$$W = \frac{1}{4\pi}\int_{-B_{sat}}^{+B_{sat}} H\,dB. \quad \ldots \quad (14.18)$$

The definite integral in (14.18) represents the area of the loop of the B-H curve, which can be drawn by means of (14.7), and is of similar shape to the I-H curve (fig. 7).

The following approximate expression was given by Steinmetz for the energy loss due to hysteresis:

$$\text{Work done per cycle} = \eta B_{max}^{1.68},$$

where η is a constant for a particular substance, usually lying between 10^{-2} and 10^{-3}, and B_{max} is the saturation value of the magnetic induction.

8. Paramagnetism, Diamagnetism and Ferromagnetism.

Careful examination of the behaviour of substances in a magnetic field shows that they may be classified in two distinct groups according to the behaviour of a rod or elongated portion of the material. If the substance moves so that the axis of the rod or the main bulk of the material moves from a weaker to the strongest portion of the field when the latter is established (say by switching on an electromagnet as shown in fig. 8) the substance is said to be **paramagnetic.** If a rod of the material sets transversely to the field, or if its main bulk moves from stronger to weaker portions of the field when the latter is established, the substance is said to be **diamagnetic.** The **ferromagnetic** group behaves essentially as a very powerful paramagnetic group. One standard apparatus for examining the magnetic behaviour of substances is the *Curie balance.* As shown in fig. 8, this consists of a powerful electromagnet with its axes inclined at an angle so as to form a magnetic field with a pronounced field gradient along the x-axis. It may be shown that the force along the x-axis at any point is

$$F_x = kH_x\frac{dH_x}{dx} \quad \ldots \quad (14.19)$$

per unit volume, where k is the susceptibility of the material. By suspending the material from the arm of a delicate torsion balance, the susceptibility of very feebly magnetic substances may be determined. The magnetic characteristics of gases are obtained by using

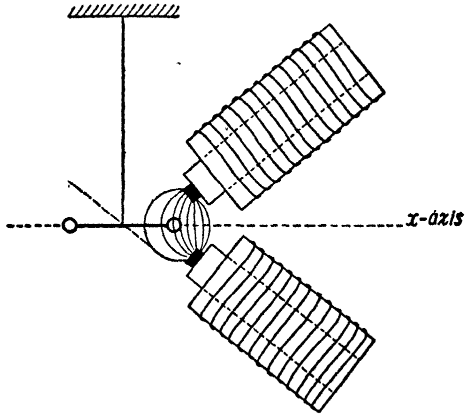

Fig. 8.—Curie Balance

a hollow suspended quartz sphere filled with the gas. The effect of the quartz is determined by a preliminary experiment with the sphere exhausted. A comparison of the properties of diamagnetic, paramagnetic and ferromagnetic substances is shown in the accompanying Table.

Property	Diamagnetic	Paramagnetic	Ferromagnetic
1. Nature.	Shown by solids, liquids and gases.		A few solids only.
2. k.	Negative.	Positive.	Positive.
3. μ.	< 1.	> 1.	$\gg 1$.
4. Variation of k with temperature.	Independent	Inversely as absolute temperature.	Irregular, but decreases.
5. Variation of k with H.	Constant.	Constant.	Irregular, curve tending towards zero k at saturation.
6. Behaviour of compounds.	k additive	Irregular.	Irregular, e.g. Heusler alloys.

The fact that k is positive for paramagnetic and ferromagnetic substances implies that the polarity induced in these substances is opposite in sign to that of the inducing pole. On the other hand, with diamagnetic substances, induction results in poles of the same sign being formed in closest proximity, and hence the movement of diamagnetic materials to the weakest part of the field is explained.

While diamagnetic substances show independence of susceptibility with temperature, **Curie's law** states that the susceptibility of paramagnetic substances varies inversely as the absolute temperature. There is a final residual negative susceptibility shown both by paramagnetic and ferromagnetic substances when the temperature is raised sufficiently, that is, all substances becomes diamagnetic at some temperature. Another peculiarity in magnetic properties is the dependence on crystalline structure. For example, grey tin is diamagnetic and crystalline tin is paramagnetic. In compounds, the total diamagnetic susceptibility is equal to the sum of the susceptibilities of the components. Paramagnetic and ferromagnetic compounds are quite irregular in their behaviour. For example, the gas iron carbonyl is very feebly magnetic, while the Heusler alloys, the individual component metals of which are very feebly magnetic, have susceptibilities comparable with those of ferromagnetic substances. Since k is independent of H for diamagnetic and paramagnetic substances, the I-H curve is a straight line. These substances therefore do not show magnetic hysteresis or residual magnetism like the ferromagnetic group. We discuss the explanation of magnetism on the electron theory in Chap. XX.

9. Magnetic Circuit.

If we consider the magnetic condition of a closed iron circuit of irregular cross-section as shown in fig. 9, we note that the number of lines of induction or the magnetic flux N is constant round the circuit. The magnetic induction B, therefore, defined as the number of lines of induction per unit area of cross-section varies from place to place according to the relation

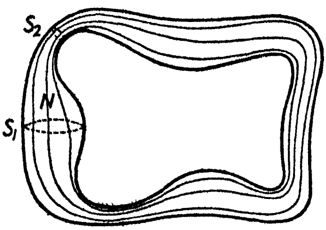

Fig. 9.—Magnetic Circuit

$$N = B_1 S_1 = B_2 S_2 = \&c., \quad (14.20)$$

where B_1, B_2, &c., are the magnetic inductions at points where the areas of cross-section are S_1, S_2, &c. Now from section 4, Chap. X, the work done in taking a unit pole round a complete circuit of induction is the magnetomotive force or M.M.F., that is

$$\text{M.M.F.} = \int H\,dl, \quad . \quad . \quad . \quad . \quad . \quad (14.21)$$

where the integral extends over the length of the circuit, and H is the field strength at any point. Now from equation (14.8), $B = \mu H$ and hence, from (14.20) and (14.21), we have

$$\text{M.M.F.} = N\int\frac{dl}{\mu S},$$

or

$$N = \frac{\text{M.M.F.}}{\int dl/\mu S}. \quad . \quad . \quad . \quad . \quad . \quad (14.22)$$

Equation (14.22) is formally analogous to Ohm's law of the electric circuit, if we compare M.M.F. with E.M.F., and the magnetic flux N with the electric current I. The quantity $\int dl/\mu S$, which depends only on the length, area of cross-section and nature of the material comprising the circuit, is then clearly analogous to the electrical resistance and is termed the **magnetic resistance** or **reluctance.**

Equation (14.22) provides a useful general method of solving electromagnetic flux problems. As examples we shall consider first an anchor ring magnet with a small air gap, and then an electromagnet of the conventional design shown in fig. 10. The M.M.F. is given by the work theorem, that is, it is $4\pi \times$ current linked. Hence as at equation (10.17) we have for an *anchor ring*

$$\text{M.M.F.} = 4\pi(2\pi rni), \quad . \quad . \quad . \quad . \quad (14.23)$$

where r is the radius of the ring and n is the number of turns per unit length.

The total magnetic resistance is the sum of two resistances in series, that of the ring and that of the air gap. If the width of the gap is d, we have

$$\int\frac{dl}{\mu S} = \frac{d}{\mu_1 S} + \frac{(2\pi r - d)}{\mu S}, \quad . \quad . \quad . \quad . \quad (14.24)$$

where $\mu_1 = 1$ for the air gap, and the cross-section S is uniform.

Hence, substituting from (14.23) and (14.24) in (14.22) we have

$$N = 4\pi(2\pi rni)\,\frac{\mu S}{2\pi r + (\mu - 1)d}. \quad . \quad . \quad (14.25)$$

From (14.20)

$$B = \frac{N}{S} = \frac{4\pi(2\pi rni)\mu}{2\pi r + (\mu - 1)d}, \quad . \quad . \quad . \quad (14.26)$$

and the strength H of the field within the gap is obtained from

$$H = B/\mu_1 = B,$$

since $\mu_1 = 1$, and within the iron by

$$H = B/\mu = \frac{4\pi(2\pi rni)}{2\pi r + (\mu - 1)d}. \quad . \quad . \quad . \quad (14.27)$$

The field due to the *electromagnet* in fig. 10 is calculated in a precisely similar manner except that the total magnetic resistance becomes

$$\int\frac{dl}{\mu S} = \frac{l}{S_1} + \frac{2l_1}{\mu_1 S_1} + \frac{2l_2}{\mu_2 S_2} + \frac{l_3}{\mu_3 S_3},$$

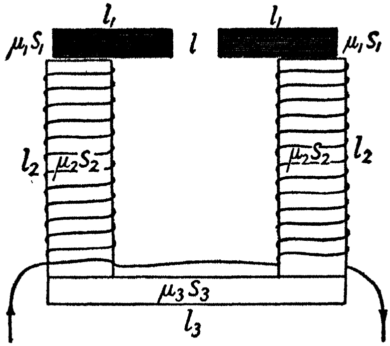

Fig. 10.—Reluctance of Electromagnet

where S_1, S_2 and S_3 are the areas of cross-section of the pole-pieces, the arms and the yoke respectively, and μ_1, μ_2, and μ_3 are the permeabilities of the materials of which the three portions are made.

EXERCISES

1. Define the terms *intensity of magnetization*, *magnetic susceptibility*, *magnetic permeability* and *magnetic induction*. How are these various quantities inter-related?

2. Show how and why the general expressions for the field strength between infinite plane magnetic poles and infinite plane condenser plates are formally similar. Are the formulæ derived for the magnetic or the electrical case likely to be in better agreement with practical observations?

3. Obtain an expression for the electromagnetic energy lost in a hysteresis cycle. Do paramagnetic bodies show hysteresis?

4. Write a short essay on diamagnetism, paramagnetism and ferromagnetism.

5. Explain the significance of the term *magnetic circuit* and use this concept to calculate an expression for the strength of the magnetic field inside a gap in the circumference of an electromagnet of anchor ring type.

CHAPTER XV

*Electrical Oscillations

1. Introduction.

The varying currents which we have so far encountered when considering the presence of inductance and resistance in a circuit (as in Chap. XIII), or the discharge which passes through a ballistic galvanometer when connected to a search coil which is being used to explore a magnetic field, have all been pulses of short duration. By suitable choice of circuit, however, it is possible to produce continuous electrical oscillations in which the current surges first in one direction and then in the other. These oscillations may be damped away gradually as in the condenser discharge considered in section 2, or they may be maintained indefinitely either from an alternating supply or with an oscillation generator such as a radio valve circuit.

2. Circuit containing Capacity and Inductance.

Consider the circuit shown in fig. 1, in which a condenser C is discharging through an inductance L. The resistance in the circuit is assumed to be negligible, to simplify the calculation, but we shall consider later the effect of introducing resistance into the circuit. Suppose at any time t the quantities of electricity on the plates of the condenser are $+Q$, $-Q$ respectively, and let I be the current through the inductance from $+Q$ to $-Q$, so that $I = -dQ/dt$.

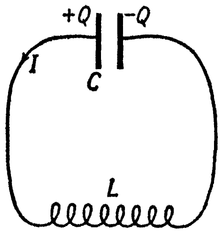

Fig. 1.—Oscillatory Circuit without Damping

The E.M.F. acting round the wire is the difference of potential on the plates, i.e. Q/C. Hence (p. 117)

$$L\frac{dI}{dt} = \frac{Q}{C}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15.1)$$

or

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (15.2)$$

This is the well-known Simple Harmonic Equation discussed in detail in Part I, Chap. III, section 9. Initially let the plates be insulated from each other, and carry charges $\pm Q_0$. At time $t = 0$, let them be short-circuited by the wire of inductance L. Then the solution of (15.2) is

$$Q = Q_0 \cos \frac{t}{\sqrt{LC}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (15.3)$$

for this gives $Q = Q_0$ and $I = 0$, for $t = 0$. The discharge is therefore oscillatory, the period being given by

$$T = 2\pi\sqrt{LC}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (15.4)$$

The energy W at any time t consists of two parts, associated respectively with the capacity and the inductance. When the charge is Q, the

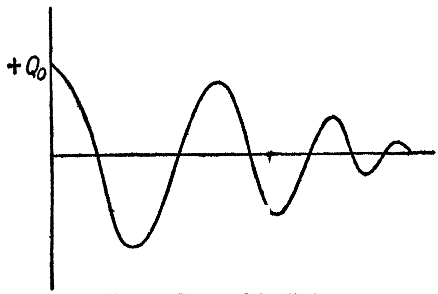

Fig. 2.—Decay of Oscillations

former part is $\frac{1}{2}Q^2/C$ (p. 24); the rate of increase of the latter part is $I \times \text{E.M.F.} = I \,.\, L dI/dt = \frac{d}{dt}\left(\frac{1}{2}LI^2\right)$.

Hence
$$W = \tfrac{1}{2}Q^2/C + \tfrac{1}{2}LI^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (15.5)$$

no constant being required, since initially

$$W = \tfrac{1}{2}Q_0{}^2/C, \text{ and } I = 0.$$

Note that (15.1) may be obtained from (15.5) by using the fact that $dW/dt = 0$.

Since $I = -dQ/dt$, (15.3) gives

$$I = \frac{Q_0}{\sqrt{LC}} \sin \frac{t}{\sqrt{LC}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (15.6)$$

the maximum value of which is

$$I_0 = Q_0/\sqrt{LC}.$$

In practice the oscillations gradually die away as shown in fig. 2. This is due to the presence of resistance which transforms some of the electrical energy into heat energy at each oscillation. It may be shown that, in the presence of resistance R, equation (15.2) transforms to

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0, \quad . \quad . \quad . \quad . \quad (15.7)$$

and (15.4) becomes

$$T = \frac{2\pi}{\sqrt{1/LC - R^2/4L^2}}, \quad . \quad . \quad . \quad . \quad (15.8)$$

so the time of oscillation is increased in the presence of resistance. It will be observed that the frequency of the oscillation depends on the *product* LC and the *ratio* R/L and not on the individual values of these three quantities.

3. Forced Oscillations.

A circuit containing inductance and capacity alone has a **natural frequency** of oscillation given by $n = 1/(2\pi\sqrt{LC})$, according to the considerations of the preceding paragraph. Consequently it corresponds exactly to any other vibrating system such as an organ pipe or a stretched string. We have seen in Part IV, Chap. IV, how any such system may be forced to oscillate if excited by some external oscillating supply, but that the magnitude of the excited oscillations is generally small except at resonance. Precisely similar relations are found if an alternating E.M.F., such as is provided by a dynamo without a commutator, is applied to a circuit containing inductance and capacity. At any time t, let the applied E.M.F. be $E = E_0 \sin pt$, where E_0 is the maximum amplitude of the E.M.F. and its period is $2\pi/p$. The total E.M.F. is now $Q/C + E_0 \sin pt$, and equation (15.2) becomes

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = -E_0 \sin pt. \quad . \quad . \quad . \quad . \quad (15.9)$$

The forced oscillations which are set up will clearly be of the same frequency as the forcing supply and of simple harmonic form. The solution of (15.9) corresponding to the forced oscillations is therefore of the form

$$Q = A \sin pt + B \cos pt, \quad . \quad . \quad . \quad . \quad (15.10)$$

where A and B are constants.

Just as in our treatment of the S.H. equation in Part I, p. 23, to determine A and B we differentiate (15.10) and substitute in (15.9). Hence we obtain

$$\frac{dQ}{dt} = pA \cos pt - pB \sin pt$$

and

$$\frac{d^2Q}{dt^2} = -p^2(A \sin pt + B \cos pt). \quad . \quad . \quad (15.11)$$

Substituting from (15.10) and (15.11) in (15.9) we obtain

$$-p^2L(A \sin pt + B \cos pt) + \frac{1}{C}(A \sin pt + B \cos pt) = -E_0 \sin pt. \quad (15.12)$$

When $t = 0$, $\sin pt = 0$ and $\cos pt = 1$; hence, substituting this condition in (15.12), $(-p^2L + 1/C)B = 0$, and therefore $B = 0$ (if we assume that $-p^2L + 1/C$ is not zero). When $pt = \pi/2$, $\sin pt = 1$ and $\cos pt = 0$; substituting this condition in (15.12)

$$-p^2LA + \frac{A}{C} = -E_0,$$

or

$$A = -\frac{E_0}{(1/C - p^2L)} \quad . \quad . \quad . \quad . \quad . \quad (15.13)$$

and equation (15.10) becomes

$$Q = -\frac{E_0}{(1/C - p^2L)} \sin pt. \quad . \quad . \quad . \quad . \quad (15.14)$$

The forced oscillation of Q is therefore in phase with the forcing oscillation, if p^2L is greater than $1/C$. If $Lp = 1/Cp$ or $p = 1/\sqrt{LC}$, that is, if the forced oscillations are of the same frequency as the natural period of the circuit, according to (15.14) the charge Q becomes infinite. With a circuit of zero resistance, if radiated energy is neglected (see Chap. XVI), this situation would occur. In practice, however, the resistance which is actually present acts as a damping factor and keeps the charge (and current) finite although large.

We next consider the case of a circuit containing inductance and resistance. The electromotive force equation becomes

$$L\frac{dI}{dt} + RI = E_0 \sin pt, \quad . \quad . \quad . \quad . \quad (15.15)$$

and we solve this by precisely the same method as we have already used in this section. Assuming $I = A \sin pt + B \cos pt$, we find

$$A = \frac{RE_0}{L^2p^2 + R^2},$$

and

$$B = \frac{-LpE_0}{L^2p^2 + R^2}. \quad . \quad . \quad . \quad . \quad . \quad (15.16)$$

Hence

$$I = \frac{E_0}{L^2p^2 + R^2}(R \sin pt - Lp \cos pt). \quad . \quad (15.17)$$

If we construct a triangle as in fig. 3 with $\cos \theta = R/\sqrt{(L^2p^2 + R^2)}$,

$\sin\theta = Lp/\sqrt{(L^2p^2 + R^2)}$, and $\tan\theta = Lp/R$, we may write (15.17) in the form

$$I = \frac{E_0}{\sqrt{(L^2p^2 + R^2)}} \sin(pt - \theta). \quad . \quad . \quad (15.18)$$

The current oscillations are therefore of amplitude $E_0/\sqrt{(L^2p^2 + R^2)}$, and they are no longer in phase with the applied E.M.F., but lag behind it by an amount θ. Of course, if there is no inductance present, that is if $L = 0$, then $\theta = 0$, and the current is given by $E_0 \sin pt/R$. The current is then in phase with the E.M.F. and is given by the ordinary ohmic relation. These considerations show that while capacity and resistance respond instantaneously to the applied E.M.F., *inductance behaves as inertia.*

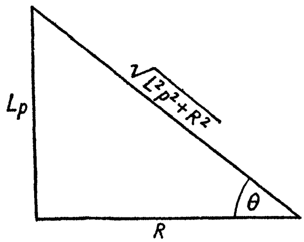

Fig. 3.—Phase of Current

When inductance, resistance and capacity are present simultaneously, the treatment is similar to that already given. As it is rather lengthy we simply quote the results here. The equation differs from (15.7) by having $-E_0 \sin pt$ on the right. The charge Q on the condenser at any time t is given by

$$Q = -\frac{E_0(1/Cp - Lp)}{p(1/Cp - Lp)^2 + pR^2} \sin pt + \frac{E_0 R}{p(1/Cp - Lp)^2 + pR^2} \cos pt, \quad . \quad . \quad . \quad (15.19)$$

and constructing a triangle as before with

$$\tan\theta = \frac{1/Cp - Lp}{R}, \quad . \quad . \quad . \quad . \quad (15.20)$$

we obtain (15.19) in the simpler form

$$I = \frac{E_0}{\sqrt{(1/Cp - Lp)^2 + R^2}} \sin(pt + \theta). \quad . \quad (15.21)$$

The phase angle θ is positive or negative according as $1/Cp$ is greater or less than Lp.

Equation (15.21) bears a formal resemblance to Ohm's law for steady currents if we consider the simple resistance R in the denominator replaced by the quantity $\sqrt{(1/Cp - Lp)^2 + R^2}$. This quantity is termed the **impedance** of the oscillatory circuit. The difference of Lp and $1/Cp$ is called the **reactance.** If the circuit contains no capacity, the impedance reduces to $\sqrt{R^2 + L^2p^2}$. At very high frequencies $R^2 \ll L^2p^2$

and under these conditions the impedance consists of reactance only and has the value Lp.)

4. Choke Coil.

If we require to reduce the current in a circuit conveying steady current, the procedure is to introduce additional resistance. This process is wasteful, as electrical energy is converted wastefully into heat in the added resistance. With varying currents a much more economical process is available. Suppose a coil of negligible resistance but appreciable inductance L is included in the circuit. Consideration of equation (15.18) then shows that $\theta = 90°$ or the current is 90° out of phase with the E.M.F. applied across the coil. The average energy absorbed per cycle by the coil is therefore

$$W = \frac{\int_0^{2\pi} E_0 \sin a \, I \, da}{\int_0^{2\pi} da}, \quad . \quad . \quad . \quad . \quad (15.22)$$

where a is written for pt for convenience. Substituting for I from (15.21) in (15.22) and integrating, we obtain

$$W = \frac{E_0^2}{Lp} \frac{\int_0^{2\pi} \sin a \cos a \, da}{\int_0^{2\pi} da} = 0. \quad . \quad . \quad (15.23)$$

Hence no energy is consumed by the coil although the current is reduced by the insertion of the choke coil by a factor $1/Lp$.

5. Measurement of Alternating Current and Voltage.

We have already stated that the hot-wire and soft-iron ammeters give a steady deflection when subjected to alternating current, but we have not yet considered what value of the alternating current amplitude this steady reading represents. It clearly depends upon the mean value of the *square* of the current, since the action of the ammeters considered depends on effects which are proportional to the square of the current. Representing the mean square current by $\overline{I^2}$, we have by its definition

$$\overline{I^2} = \frac{\int_0^{2\pi} I_0^2 \sin^2 a \, da}{\int_0^{2\pi} da} = \frac{I_0^2}{2}. \quad . \quad . \quad . \quad (15.24)$$

The steady continuous current which would give the same reading on the ammeter as the alternating current is therefore $I = I_0/\sqrt{2}$; It is termed the *virtual current*. In the same way, the steady voltage which would give the same reading as the applied A.C. voltage is $E = E_0/\sqrt{2}$.

If we have a circuit containing inductance and resistance we can use an A.C. ammeter and voltmeter to determine its impedance just as we can use a D.C. ammeter and voltmeter to determine the resistance of a circuit to steady current. We may then apply the relation

$$\frac{I_0}{\sqrt{2}} = \frac{E_0}{\sqrt{2}} \frac{1}{\sqrt{L^2p^2 + R^2}}; \quad . \quad . \quad . \quad . \quad (15.25)$$

here $I_0/\sqrt{2}$ and $E_0/\sqrt{2}$ are measured, R is measured by application of a steady current and observation of ammeter and voltmeter readings, and p is calculated from the relation $p = 2\pi n$, where n is the frequency of the applied A.C.; thus L can be calculated from (15.25).

The nature of the electrical oscillations occurring in any circuit are best examined with the cathode ray oscillograph described on p. 194.

6. Foucault or Eddy Currents.

When an oscillatory circuit is in the neighbourhood of a mass of metal, since the latter constitutes a conductor in a varying magnetic field, currents are set up in the metal. The electrical energy of these Foucault or eddy currents is dissipated as heat in the metal, and the phenomenon constitutes a serious problem in the construction of the iron cores of such instruments as transformers. Transformer cores are therefore susceptible to two forms of electrical energy loss, hysteresis and eddy currents, the energy being transformed into heat in both instances. Hysteresis loss is reduced to a minimum by using material with a narrow hysteresis loop (see Chap. XIV). Eddy current loss is reduced to a minimum by making the core of as high an electrical resistance as possible. This is achieved by constructing it of thin laminations covered with oxide and bolted together. The energy loss due to an eddy current circulating round any part of the core is E^2/R, where E is the induced E.M.F. and R the resistance of the circuit. Hence by increasing R, the energy loss is reduced to a minimum. Eddy current action is put to a useful purpose in the *eddy current heater*. This consists essentially of a device for producing high-frequency oscillations of great intensity. When such an instrument is held close to any mass of metal the eddy currents are of considerable magnitude and are often sufficient to raise the metal to red or white heat. One of its most important applications is to the degassing of the metal parts of radio valves (see Chap. XIX), which could not be heated in situ conveniently in any other fashion.

7. Skin Effect.

With a steady current supply, the current is of uniform density across the section of a uniform wire. At high-frequency the current tends to be confined more to the outer surface of the wire. This is termed the *skin effect*; it results in an undesirable decrease of the effective cross-section, and therefore increase of the effective resistance of the conductor. The difficulty is in part overcome by using a stranded conductor of insulated wires, since these present a larger surface area for any cross-section than the corresponding single solid conductor. If a single conductor is used this is often made hollow to save material in the centre which would not be traversed by current. The reason for the skin effect may be explained with reference to fig. 4, where A and B represent two cross-sections of equal area, the former constituting the core of the conductor and the latter the outer cylindrical area. It may be shown that the self-inductance of a straight cylindrical conductor decreases if its radius increases. Hence the self-inductance of the core A is greater than that of the annulus B. The impedance of A is therefore greater than that of B, and a higher proportion of current will therefore flow through B.

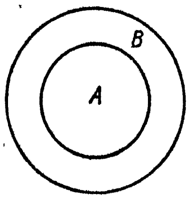

Fig. 4.—Skin Effect

EXERCISES

1. Obtain an expression for the time of oscillation of a circuit containing inductance and capacity.

Show that a circuit containing an inductance of 10^{-4} microhenries and a capacity of 1·9 microfarads will oscillate with a frequency corresponding to a wave-length in free space of about 26 m.

2. Distinguish between free and forced oscillations, and show that the forced oscillations set up in a circuit containing inductance and resistance lag behind the applied E.M.F. by an amount dependent on the value of the inductance, the resistance and the applied frequency.

3. Explain what is meant by the impedance of a circuit, and show how the magnitude of an alternating current may be reduced without loss of energy as heat.

4. Define the terms virtual current and virtual voltage as applied to A.C. circuits and describe how these quantities may be measured.

5. Explain the existence of eddy currents and the skin effect. To what practical purposes have eddy currents been applied?

CHAPTER XVI

Electromagnetic Waves

1. Hertz's Experiments.

We have seen in the preceding chapter how a circuit containing inductance, capacity and resistance may be the seat of electrical oscillations of a period depending upon the magnitudes of the three quantities involved. Hertz showed that these electrical oscillations were, in certain circumstances, not simply confined to the oscillating circuit, but that part of the electromagnetic energy could be radiated away completely from the oscillating system. The oscillatory system used for radiation and termed the **transmitter** is shown in fig. 1. It

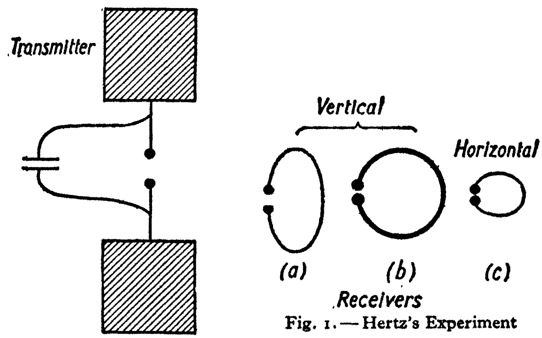

Fig. 1.—Hertz's Experiment

consisted of two square sheets of metal on each of which was fixed a short rod ending in a polished ball. The system is charged by connexion to an induction coil and then discharges across the ball-gap at regular intervals. If now a simple **receiver** is constructed from a circle of wire ending in two spheres with a short gap, a remarkable phenomenon may be observed. The receiver is placed so far from the transmitter that the direct induction effect of surges in the transmitter is negligible, and it is found that when the spheres are in the position (*a*) and (*b*) a spark jumps across, whereas when they are in the position (*c*) no effect is observed. The interpretation is that electromagnetic energy travels across the intervening space between transmitter and receiver, and the dependence on orientation of the receiver suggests that the radiation is in the form of a plane polarized electromagnetic wave (see Part III).

2. Tubes of Force and Electromagnetic Waves.

The fact that electrical oscillations might lead to the production of electromagnetic waves in free space had been deduced some years before Hertz's experiments by Maxwell, from consideration of electromagnetic theory (see Section 5). These considerations are too mathematical for this book, but we can arrive at a physical explanation of the production of electromagnetic waves using Faraday's concept of tubes of force, already mentioned in Chap. II. Referring to fig. 2(*a*), before the discharge across the gap commences, the distribution of (one half of) the tubes of force will be as shown, where we have considered a negative charge on the pole A and an equal positive charge

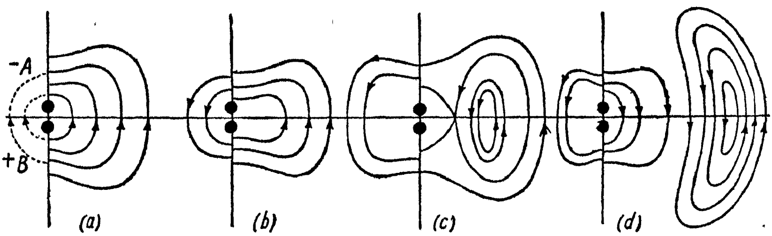

Fig. 2.—Electromagnetic Waves

on the pole B. When the insulation resistance of the gap breaks down, the ends of the tubes in contact with the conductors coalesce and the tubes collapse, causing the electric current which flows across the gap. Now owing to the existence of lateral pressure between the tubes, the outer tubes move in to the left to occupy the position originally held by the collapsed inner tubes and hence the tubes acquire a velocity perpendicular to the gap. If we postulate that the tubes possess inertia, the approaching ends of the tube will cross over at the gap while the body of the tube will pass through the gap and expand again on the other side as shown in fig. 2(*b*). In this way the polarity of A and B will have changed sign after the first oscillation, a situation which we know occurs in an oscillatory circuit from our considerations in Chap. XV, section 2. The process will clearly be repeated again as the tubes surge in the reverse direction, and would continue indefinitely were it not for the gradual loss of energy as heat in the charged rods and as heat, light and sound in the spark. Consider now the case shown in fig. 2(*c*). Here the ends of the tube have crossed at the gap before the main body of the tube has passed through the gap. A very short interval of time later the crossed portions coalesce and a completely closed tube of electromagnetic energy is left situated in free space and repelled away by the lateral pressure of neighbouring tubes. In this way electromagnetic energy is radiated away into space, and the result when several tubes are present simultaneously is shown in fig. 2(*d*). These tubes exert a mutual repulsion, so that the radiation spreads out until it becomes a plane surface with the electrical force directed parallel to the spark gap. Maxwell's theoretical calculations showed that the radiated energy took the form of electromagnetic waves travelling with a velocity equal to the square root of the ratio of the units of capacity on the electromagnetic and electrostatic systems respectively. On the C.G.S. system this quantity is found experimentally to be equal to the velocity of light, and consequently strong evidence was provided that light itself consisted of electromagnetic radiation. We have discussed the subject further from this point of view in Chap. IX, Part III. We therefore proceed here to give evidence that electromagnetic waves as generated by the **Hertzian oscillator** obey the laws of light.

3. Experiments with Electromagnetic Waves.

By using a parabolic metal reflector with the Hertzian oscillator at the focus, a parallel beam of electromagnetic radiation is produced. This may be received on a Hertzian receiver set at the focus of a receiving parabolic reflector. The existence of polarization, requiring definite orientation of the receiver to the transmitter, has already been mentioned. Using a large prism of paraffin wax, the laws of refraction are found to be obeyed for electromagnetic waves as for light.

To determine the wave-length of the radiation, the standard method of producing stationary waves by interference between incident waves and those reflected by a plane metal sheet is applicable, as in the corresponding experiment with sound waves described in Part IV, Chap. III. The receiver shows maximum response when situated at regular distances from the reflector, these distances clearly corresponding to half a wave-length. Further, all the general relations for wave motion derived in Part IV now become applicable. In particular, the well-known relation $V = \nu\lambda$, between velocity, frequency and wave-length may be examined, and if any two of these are known, the third may be calculated. For example, the frequency of the radiation will equal that of the oscillating source, which can be calculated according to the principles of section 2, Chap. XV. The wave-length may be measured by a stationary wave method, and the product of frequency and wave-length then formed to show that the resulting velocity in free space is equal to that of light.

The source of oscillations now almost invariably used is the valve oscillator, the action of which is described briefly in Chap. XIX. Such an instrument allows of extremely sensitive control of frequency and radiated power.

4. Waves along Conductors.

If an oscillator such as the Hertzian oscillator is connected to two parallel conductors, then waves are set up in the conductors similar to the waves radiated into free space. The waves which pass down the wires are reflected either at the open end of the wires or at the end closed by a wire bridge, so that eventually stationary waves are formed just as with an open or closed organ pipe subjected to vibrations of a tuning fork held at the mouth. The presence of stationary nodes and antinodes spaced at intervals of half a wave-length along the wires is easily demonstrated with some device such as the neon lamp. This consists of two electrodes sealed in a glass envelope containing neon, and it has the property of glowing if the potential difference across the two electrodes exceeds a certain amount, although the current needed to produce the glow is very small. Consequently if such a lamp is

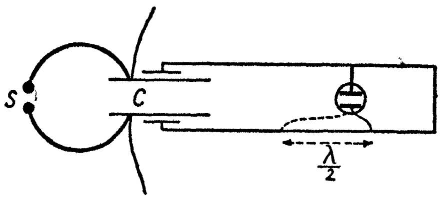

Fig. 3.—Dielectric Constant and Frequency

moved along the pair of **Lecher wires,** the distance between two points of maximum glow is the distance between two potential antinodes and is equal to half a wave-length. For an open-ended conductor, the free end is a potential maximum, whereas for a closed end the maximum occurs at $\lambda/4$ from the closed end, in complete analogy with the resonance tube. It may be shown that the current antinodes and nodes occur half-way between, and are completely out of phase with, the potential nodes and antinodes.

One important application of Lecher wires is the examination of the change of dielectric constant with frequency. In the circuit shown in fig. 3, the condenser C is first filled with dielectric and the wave-length of the oscillations is found on the associated Lecher wire system. The condenser is then emptied of dielectric, whereupon the wave-length will be found to have changed. Now since the velocity of the waves remains constant, the ratio of the two wave-lengths is inversely proportional to the ratios of the frequencies of the oscillations in the two circumstances. But the frequencies are given by $n = 1/2\pi\sqrt{LC}$, where n is the frequency, L the inductance and C the capacity of the oscillating circuit. Since L remains constant, $n/n' = \sqrt{C'/C} = \sqrt{k}$, where k is the dielectric constant by definition (Chap. III).

The dielectric constant is found by these experiments not to be constant at all but to vary with the frequency of the electrical oscillations to which it is submitted. The value normally taken for a dielectric constant is its electrostatic value, that is its asymptotic value for infinite wave-length and zero frequency. These experiments also show the existence of *electrical hysteresis* quite analogous to magnetic hysteresis. The response of the dielectric medium to the alternating electric field lags behind the change in the field, and electrical energy loss reappears as heat in the material.

5. Dimensional Relations between E.S. and E.M. Units.

Dimensional considerations similar to those developed in Part I, Chap. V, may be applied to electrical and magnetic quantities. As an illustration we shall consider the dimensional expressions for charge, potential and capacity in the two systems.

(i) *Electrostatic Units.*

The dimensions of electric charge on the E.S. system are obtained from the law of force between two equal charges q; thus

$$F = \frac{q^2}{k d^2}, \qquad (16.1)$$

where k is the dielectric constant of the surrounding medium and d is the distance between the charges. Hence, dimensionally,

$$\mathrm{q} = \mathrm{k}^{\frac{1}{2}}\mathrm{L}\mathrm{F}^{\frac{1}{2}}, \qquad (16.2)$$

since the dimensions of d are those of length. We may note that we have introduced k as a primary quantity along with mass, length and time. Now F has dimensions

$$\mathrm{F} = \text{mass} \times \text{acceleration} = \mathrm{MLT}^{-2}.$$

Hence (16.2) becomes

$$\mathrm{q} = \mathrm{k}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}\mathrm{M}^{\frac{1}{2}}\mathrm{T}^{-1}. \qquad (16.3)$$

Next, the potential e is defined by

$$\text{Work done} = eq. \qquad (16.4)$$

Hence from (16.3) and (16.4)

$$\mathrm{e} = \frac{\text{force} \times \text{distance}}{\mathrm{q}}$$
$$= \mathrm{ML^2T^{-2}} \,.\, \mathrm{k}^{-\frac{1}{2}}\mathrm{L}^{-\frac{3}{2}}\mathrm{M}^{-\frac{1}{2}}\mathrm{T}$$
$$= \mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}\mathrm{T}^{-1}\mathrm{k}^{-\frac{1}{2}}. \qquad (16.5)$$

Finally, capacity is given by the quotient of (16.3) and (16.5), or

$$c = \frac{q}{e} = kL. \qquad (16.6)$$

(ii) *Electromagnetic Units.*

On the E.M. system charge is defined by

$$\text{charge } q = \text{current} \times \text{time},$$

so the dimensions of current must first be obtained. From the E.M. definition of unit current, if H is the magnetic field produced at the centre of a circular coil of radius a,

$$H = \frac{2\pi i}{a},$$

whence dimensionally

$$i = HL, \qquad (16.7)$$

since a has dimensions L and 2π is dimensionless. Now the strength of the magnetic field is the force exerted on unit magnetic pole m, hence

$$F = Hm,$$

or dimensionally,

$$H = \frac{F}{m} = ML\,T^{-2}\,m^{-1}. \qquad (16.8)$$

We can proceed further, expressing m dimensionally from the inverse square law of force, thus

$$F' = \frac{m^2}{\mu d^2},$$

where μ is the permeability of the medium in which the poles are situated a distance d apart. We therefore have dimensionally,

$$m = \mu^{\frac{1}{2}} L F'^{\frac{1}{2}}$$
$$= \mu^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}. \qquad (16.9)$$

Substituting from (16.9) in (16.8) we obtain

$$H = \mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}. \qquad (16.10)$$

Hence from (16.10) and (16.7) we have

$$i = \mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}. \qquad (16.11)$$

Finally

$$q = iT = \mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}}. \qquad (16.12)$$

Again, the potential difference on the E.M. system will be given by

$$\text{rate of working} = ei,$$

and hence from (16.11)

$$\begin{aligned} e &= \frac{\text{force} \times \text{distance}}{\text{time}} \frac{1}{i} \\ &= ML^2T^{-3}\mu^{\frac{1}{2}}M^{-\frac{1}{2}}L^{-\frac{1}{2}}T \\ &= \mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}. \qquad \ldots \ldots \quad (16.13) \end{aligned}$$

Finally, capacity is given by

$$c = \frac{q}{e} = \mu^{-1}L^{-1}T^2. \qquad \ldots \ldots \quad (16.14)$$

From (16.6) and (16.14) therefore we have

$$\frac{[c]\text{E.S.}}{[c]\text{E.M.}} = \frac{kL}{\mu^{-1}L^{-1}T^2} = \mu k \left(\frac{L}{T}\right)^2. \qquad \ldots \quad (16.15)$$

This expression indicates that if the capacity of a condenser is derived from its linear measurements according to the formulæ obtained in Chap. III, and then determined in the E.M. system, say by the use of a ballistic galvanometer and equation (13.40), the ratio of the two numerical values should represent a $(\text{velocity})^2$. The actual value of the ratio obtained is $(3 \times 10^{10}\ \text{cm./sec.})^2$, that is it is exactly equal to the square of the velocity of light in vacuo. While this result alone does not "prove" that light is an electromagnetic wave, these dimensional considerations indicate strongly that light and electromagnetism are intimately connected.

EXERCISES

1. Describe experiments illustrating the production of electromagnetic waves. How may the wave-length of such waves be determined?

2. Use the concept of tubes of force to explain the radiation of energy from an oscillatory circuit. What is the connexion between electromagnetic waves and light waves?

3. Write a short essay on "Electromagnetic wave propagation along conductors and its applications".

4. Show by the method of dimensions that the ratio of the units of quantity of electricity in the electrostatic and electromagnetic systems is a function of a velocity. What is the significance of this result?

CHAPTER XVII

Thermoelectricity

1. Seebeck Effect.

It was discovered by Seebeck in 1821 that if the junctions of two dissimilar metals are at different temperatures, a current flows round the circuit. A list of metals was constructed such that the order of the metals indicated the direction in which the thermoelectric E.M.F.s acted when any pair of metals was chosen. Near opposite ends of the list occur antimony and bismuth, and a couple constructed of these metals gives one of the most sensitive couples available. However, as the melting-points of these metals are low they are unsuited for use over a large temperature range.

Experiment has shown that there are two laws of the thermoelectric effect which may be stated as follows:

(1) The total E.M.F. between two metals A and C is equal to the sum of the E.M.F.s of the couples AB + BC over the same temperature range.

(2) The total E.M.F. $E_{\theta_3\theta_1}$ when the junctions are at temperatures θ_3 and θ_1, is equal to the sum of the E.M.F.s $E_{\theta_3\theta_2} + E_{\theta_2\theta_1}$, where these are the E.M.F.s when the junctions are at (θ_3, θ_2) and (θ_2, θ_1) respectively.

These laws are sometimes known as the *Law of Intermediate Metals* and the *Law of Intermediate Temperatures*, respectively.

2. Peltier Effect.

In 1834, Peltier showed that the thermoelectric effect was reversible, that is heat is evolved at one junction and absorbed at the other if a current from some external source is sent round a circuit of two dissimilar metals as in fig. 1. The heating effect, which is confined entirely to the junctions, is quite distinct from the Joule heating, which takes place at all points of a conductor and arises from its resistance. Further, if the current is reversed in direction, the Peltier effect is reversed so that a junction which previously evolved heat now absorbs it and vice versa. This dependence on the direction or sign of the current implies that the Peltier heating depends on some odd power of the current, and experiment shows that it is directly proportional to it. The Peltier coefficient Π for two metals is defined by

$$H = \frac{\Pi I}{J}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17.1)$$

where H is the heat evolved in calories/sec., I is the current in amperes and J is Joule's equivalent, equal to 4.2 joules/cal. Hence Π has the dimensions of an E.M.F. The Peltier coefficient is a function of the temperature, and the Peltier effect is connected directly with the Seebeck E.M.F. in that the latter is (approximately) equal to the difference in the two Peltier E.M.F.s which are present when the two junctions of a thermocouple are at different temperatures.

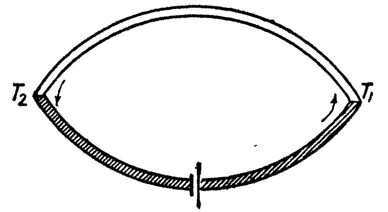

Fig. 1. —Peltier Effect

3. Thomson or Kelvin Effect.

From the complementary character of the Seebeck and Peltier effects it is clear that the thermoelectric effect behaves like a reversible heat engine. Application of thermodynamics according to the principles of Part II, Chap. XIII, then shows that there should be a straight line relation between Seebeck E.M.F. and absolute temperature. In practice the curves are parabolic as shown in fig. 2. Kelvin therefore suggested that a third thermoelectric effect exists such that an E.M.F. acts along a single metal if it is not at uniform temperature. Since both the metals in a couple are subject to a temperature gradient, two Kelvin E.M.F.s will be present, and the Seebeck E.M.F. is therefore the result of the Peltier E.M.F.s and the Kelvin E.M.F.s acting together. The Kelvin E.M.F.s or coefficients are usually denoted by σ, where σ represents the E.M.F. acting under unit uniform temperature gradient. The complete expression connecting the Seebeck, Peltier and Kelvin E.M.F.s is therefore

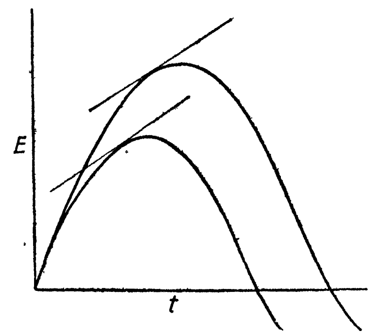

Fig. 2.—Seebeck E.M.F. and Absolute Temperature

$$E_{AB}(\theta_1\theta_2) = \Pi_{AB}(\theta_1) - \Pi_{AB}(\theta_2) - \int_{\theta_1}^{\theta_2}(\sigma_A - \sigma_B)\,dT. \quad (17.2)$$

Regarding the thermoelectric couple as a reversible heat engine, we have shown in Part II, p. 123, that it follows that the Peltier coefficient is given by

$$\Pi = T\frac{dE}{dT}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17.3)$$

and the difference in the Kelvin coefficients by

$$(\sigma_A - \sigma_B) = -T\frac{d^2E}{dT^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (17.4)$$

As the Peltier and Kelvin coefficients are difficult to measure directly, these relations are very valuable since they allow the coefficients to be calculated from the gradient of the Seebeck curve and the slope of the thermoelectric power diagram (see next section) respectively.

4. Thermoelectric Power Diagram.

It is very inconvenient mathematically to deal with sets of parabolic curves in estimating thermoelectric behaviour, and the usual procedure is as follows. The thermoelectric Seebeck curves are first plotted for a series of metals, the second metal of the junction being lead. The reason for this is that the Kelvin effect for lead is zero, so the Kelvin effect of the other metal can be obtained directly from the data. Tangents are then drawn to the Seebeck curves at a number of points, and a second graph as shown in fig. 3 is constructed of the slope of the tangents dE/dT against T. This diagram is termed the **thermo-electric power diagram** (the term is unfortunate since it is not measured in units of power but in volts per degree) and is a straight line. This follows from the parabolic nature of the Seebeck curve, for, representing this algebraically by

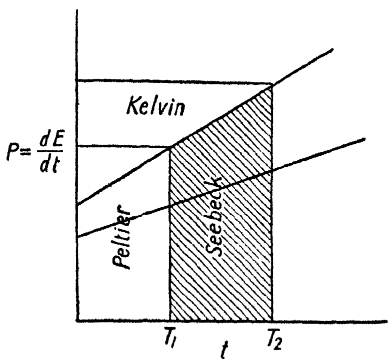

Fig. 3.—Thermoelectric Power Diagram

$$E = aT + bT^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17.5)$$

where a and b are constants, we have by differentiation with respect to T

$$\frac{dE}{dT} = P = a + 2bT, \quad . \quad . \quad . \quad . \quad . \quad . \quad (17.6)$$

which is the equation of the thermoelectric power line. We note that the thermoelectric constants are given by the slope of the line and its intercept on the P axis respectively. From this diagram, the Seebeck, Peltier and Kelvin E.M.F.s are all calculable as areas. Thus the Seebeck E.M.F. is

$$\int_{T_1}^{T_2} P\,dT = \int_{T_1}^{T_2} \frac{dE}{dT}\,dT = \text{area of trapezium enclosed between ordinates at } T_1 \text{ and } T_2, \text{ the } T \text{ axis and the thermoelectric power line.}$$

The Peltier E.M.F. is, from (17.3),

$$T\frac{dE}{dT} = TP = \text{area of rectangle enclosed between the co-ordinate axes and perpendiculars dropped to them from the point on the thermoelectric power line.}$$

Finally, from equation (17.2), the Kelvin E.M.F. is given by

$$\int_{T_1}^{T_2} (\sigma_A - \sigma_B)\,dT = \Pi_{AB}(T_1) - \Pi_{AB}(T_2) - E_{AB}(T_1, T_2);$$

that is, it is represented by the difference between the areas of the two Peltier rectangles less the trapezoidal area which represents the Seebeck E.M.F. This area is clearly that of the trapezium enclosed between the abscissæ at T_1 and T_2, the P-axis and the thermoelectric power line.

5. Experimental Demonstration of the Peltier and Kelvin Effects.

To demonstrate the existence of the Peltier effect directly, an apparatus may be used as in fig. 4. Two bars of bismuth are connected

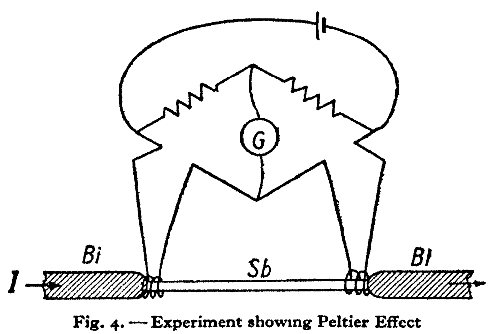

Fig. 4.—Experiment showing Peltier Effect

to a bar of antimony, and the junctions are wound with coils of insulated wire which are connected to the opposite arms of a Wheatstone bridge. The bridge is balanced and a current is then sent through the bismuth-antimony junctions. Owing to the heat evolved at one junction and absorbed at the other, the resistances of the two coils are changed, one being increased and the other decreased. Consequently the balance of the bridge is disturbed, and if the heat capacity of the junctions is known together with the current in the couple, application of (17.1) allows a rough estimate of the Peltier coefficient to be made. If the current is reversed, the Wheatstone bridge is thrown out of balance in the reverse direction, thus demonstrating that the Peltier effect depends on current direction.

To demonstrate the Kelvin effect, which is much smaller than the Seebeck or Peltier effects, an apparatus as shown in fig. 5 may be used. An iron rod several feet long is bent into the shape of a large hairpin and heated to redness at the loop, the ends of the bar being cooled in

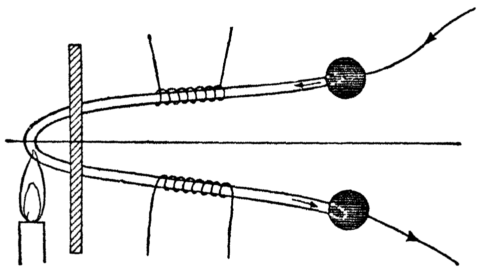

Fig. 5.—Experiment showing Kelvin Effect

pools of mercury. In this way a steep temperature gradient is created between the loop and the ends of the bar. A current of several amperes is then sent through the bar, and this current passes up the temperature gradient in one arm and down the temperature gradient in the other. Owing to the existence of the Kelvin effect the current is therefore travelling against the Kelvin E.M.F. in one arm and with the Kelvin E.M.F. in the other. Superimposed upon the Joule heating, therefore, additional heat will be evolved in one arm and absorbed in the other. The effect is shown as with the Peltier experiment, by winding a coil of wire round each arm, connecting the coils to the opposite arms of a Wheatstone bridge and establishing a balance when no current flows through the bar. The balance is disturbed in opposite directions according to the direction of the current, thus showing that, like the Peltier effect, the Kelvin effect is directly proportional to the current.

6. Characteristics of the Thermoelectric Curves.

The parabolic nature of the thermoelectric curves implies that a maximum E.M.F. occurs for a certain temperature difference between the junctions, after which the E.M.F. decreases again to zero and then increases again in the reverse direction. The temperature at which the maximum occurs is termed the **neutral temperature,** and the temperature at which a reversal in direction of the E.M.F. takes place is termed the **inversion temperature.**

Since the thermoelectric power lines are all plotted with respect to lead, some metals will be positive with respect to lead at the hot junction while others will be negative. The corresponding thermoelectric power lines will have slopes in the reverse direction. To estimate the Seebeck E.M.F. between any pair of metals, application of the principles outlined in the preceding section shows that the Seebeck E.M.F. is equal to the trapezoidal area enclosed between the two thermoelectric power lines and the two ordinates corresponding to the temperatures of the junctions.

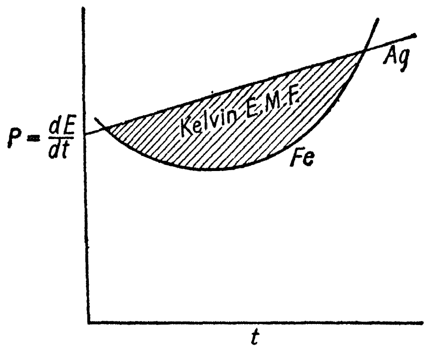

Fig. 6.—Thermoelectric Diagram for Iron

Certain cases are of particular interest. Thus the thermoelectric power lines for platinum and a certain platinum-iridium alloy have the same constant *b* and therefore the same slope. For a couple composed of these metals, therefore, the thermoelectric E.M.F. is proportional to the temperature difference, and thus a thermoelectric temperature measuring instrument may be constructed which operates on a linear scale. Again, the behaviour of iron is irregular, no definite parabolic law being obeyed. Consequently the thermoelectric power diagram for iron is non-linear and consists in fact of a loop as shown in fig. 6. This loop crosses the thermoelectric power line for silver at temperatures of 310° C. and 620° C. Now the Peltier coefficients for both metals must be the same for both metals at these points of intersection, yet an E.M.F. acts round the couple equal in magnitude to the area enclosed between the loop and the line. Hence we have an example of a couple operating on the Kelvin coefficients only of the two metals.

7. Measurement of Thermoelectric E.M.F.s.

The main practical use of the thermoelectric effect is in the measurement of temperature, and the apparatus used to measure the thermoelectric E.M.F. depends upon whether the temperature is steady or variable. If the temperature is varying, rapid reading is essential and the couple is connected directly to a microvoltmeter, which is simply a sensitive moving-coil milliammeter with a suitable resistance in series. Such an instrument will measure to about 1/50° C.

For steady temperatures a more accurate method is to use a potentiometer circuit as in fig. 7. The potentiometer wire AB is connected in series with two variable high resistances X and Y. One end of the thermocouple is connected to A, while the other end goes to a sensitive galvanometer which is in turn connected to the tapping point P of

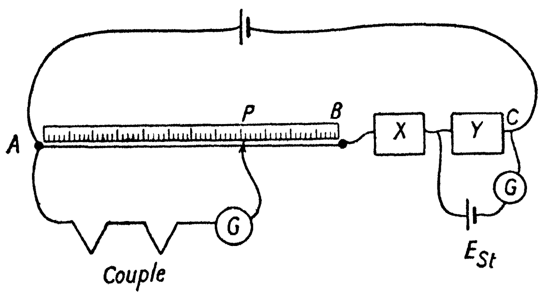

Fig. 7.—Measurement of Thermoelectric E.M.F.

the potentiometer wire. A standard cell is connected across Y as shown, with a second sensitive galvanometer in the standard cell circuit. The drop in potential across AC is about 2 volts, and since X and Y are each several thousand ohms while the resistance of AB is about 1 ohm, the drop in potential across AB is about a thousand microvolts. Consequently the balance point P occurs at a convenient distance along AB. Initially the procedure consists in varying the resistance Y but making corresponding alterations to the resistance X so as to keep the total resistance $(X + Y)$ constant. The resistance Y is varied until the standard cell is balanced. Then if I is the (unknown) current in the potentiometer, we have from Ohm's law

$$E_{\mathrm{Th}} = l\rho I, \quad \ldots\ldots \quad (17.7)$$

where E_{Th} is the E.M.F. of the thermocouple, l is the distance along AB corresponding to a balance, and ρ is the resistance per unit length of the potentiometer wire AB. (This must be determined by a separate experiment using AB as the unknown resistance in a post office box circuit.) Again,

$$E_{St} = IY, \quad \ldots \ldots \quad (17.8)$$

where E_{St} is the E.M.F. of the standard cell. Hence from (17.7) and (17.8), eliminating I we have

$$E_{Th} = \frac{l\rho}{Y} E_{St}. \quad \ldots \ldots \quad (17.9)$$

8. Applications of the Thermoelectric Effect.

The thermoelectric effect is far too small to serve as a practical source of electrical power, but it is invaluable as a means of measuring temperatures accurately and quickly and of measuring small quantities of heat. Its advantage over other heat-measuring appliances lies in

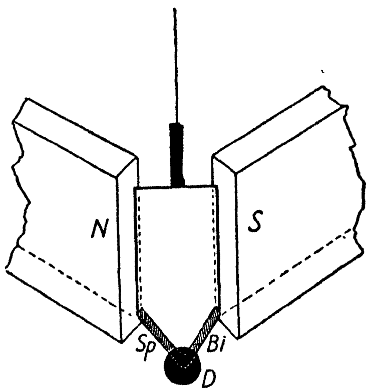

Fig. 8.—Boys' Radiomicrometer

its small bulk and in its small heat capacity, which results in the minimum mechanical and thermal disturbance of the system into which it is inserted. The application to thermometry is mentioned in Chap. II, Part II. As a typical instrument we shall describe first *Boys' radiomicrometer* for detecting very small intensity of heat radiation. As shown in fig. 8, it consists of a sensitive suspended coil galvanometer the coil of which has the circuit completed by an antimony-bismuth couple. The junction of the couple is covered with a small blackened disk D on to which the radiation is focused. This instrument is so sensitive that it will detect radiant heat equivalent to that from a candle at several miles. It is used to examine the heat radiation from stars.

The radiobalance shown in fig. 9 is an instrument used to detect the heat produced by elements undergoing radioactive decay (see

Chap. XX). It consists of two thermocouple units P and Q constructed of an array of iron and constantan wires joined together by a cup containing the radioactive element. The two cups are also at the junction of another iron-constantan couple which is connected to a galvanometer. The couple Q is a dummy arrangement to preserve symmetry: a variable current is passed through the couple Q, the magnitude of the current being registered on the ammeter A. Owing to the operation of the Peltier cooling, the heat emitted by the radioactive source is prevented from causing any rise in temperature and consequently the couple in the galvanometer circuit gives rise to no E.M.F. and no deflection in the galvanometer. Since some Joule heating inevitably

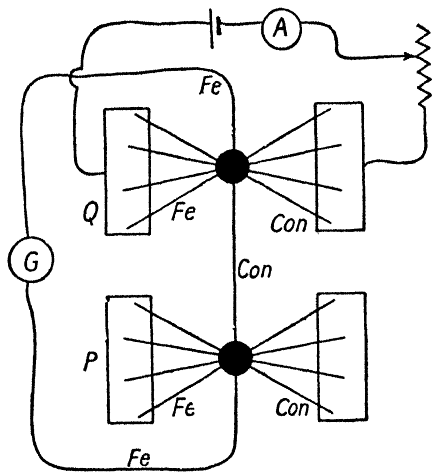

Fig. 9.—Radiobalance

arises due to the electrical resistance at the cup, a subsidiary experiment is made to allow for this. If I_0 is the current which produces no temperature rise in the junction when the radioactive element is absent, equating the Joule heating to the Peltier cooling we have

$$I_0^2 R = \Pi I_0. \quad \quad (17.10)$$

This current must be reduced to I when the radioactive substance is inserted if the junction is still to be free from rise in temperature. Hence if the radioactive heat generated per second is H, we have

$$H + I^2 R = \Pi I, \quad \quad (17.11)$$

or, substituting from (17.10) in (17.11),

$$H = \frac{\Pi I(I_0 - I)}{I_0}. \quad \quad (17.12)$$

EXERCISES

1. What are the three thermoelectric effects and how are they related?

2. How may the Seebeck E.M.F. be measured accurately? For what practical purposes has the existence of the Seebeck E.M.F. been used?

3. How may the Peltier and Kelvin effects be (*a*) demonstrated, (*b*) measured?

4. Define thermoelectric power and explain fully the significance of the areas on a thermoelectric power diagram.

5. What are the laws of intermediate temperatures and intermediate metals as applied to thermoelectric circuits? Is there any practical example of a current being produced entirely by the Kelvin effect?

CHAPTER XVIII

Electrical Discharge through Gases

1. Introduction.

We have seen in Chap. XII how liquids will conduct electricity only if there are carriers or ions present. The high insulation properties of gases in their normal state arises from the almost complete absence of ions. Now certain agents such as X-rays and the rays from radioactive substances are capable of ionizing gases, and a current will then flow if two electrodes at different potential are inserted in the

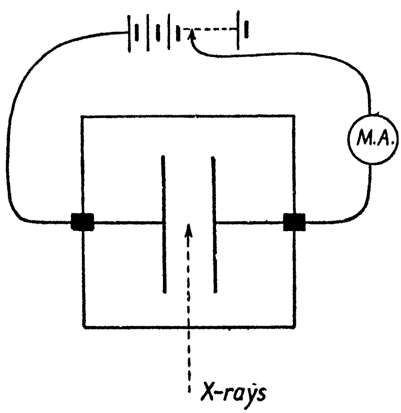

Fig. 1.—Ionization Chamber

gas. A suitable experimental arrangement, sometimes termed an *ionization chamber*, is shown in fig. 1, and the curve showing the variation of current with the applied potential difference is shown in fig. 2. Initially the curve approximates to a straight line passing through the origin, showing that Ohm's law is obeyed just as for conduction in solids and liquids. Soon, however, the curve starts to bend over parallel to the potential axis and ultimately *saturation conditions* are produced. This clearly occurs when the ions are removed as rapidly as they are generated by the ionizing radiation. If the potential is raised sufficiently, the current suddenly rises to an enormous value corresponding to a spark discharge between the electrodes.

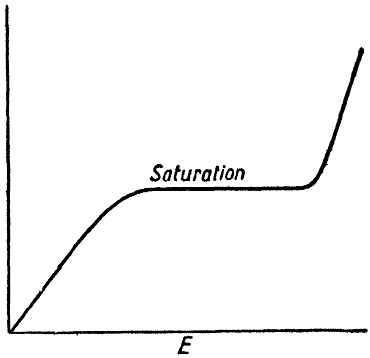

FIG. 2. — Current and Potential Difference

2. Effect of Gas Pressure on Conduction.

If a steady source of potential of several thousand volts is available, the conductivity of gases may be examined without recourse to external ionizing agents. If the vessel containing the gas is connected to a pump it will be found that the potential required to produce a spark, that is to effect conduction, is considerably reduced as the gas pressure

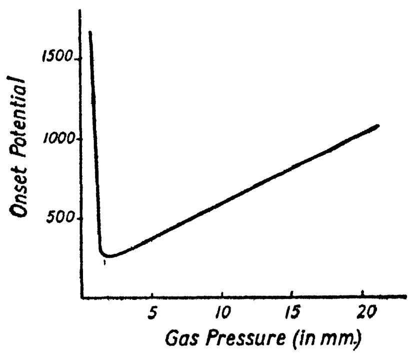

FIG. 3. — Onset Potential and Gas Pressure

is lowered. The onset potential-pressure curve is shown in fig. 3, which indicates that a minimum effective potential occurs at about 1 mm. pressure for air. Above this pressure the necessary voltage rises steadily and slowly while below this pressure it rises very steeply indeed. Consequently the potential necessary to create a discharge across a hard vacuum is very great.

In the region of minimum potential the discharge occurs even if the electrodes are situated a distance apart much larger than the sparking potential. This silent steady discharge is accompanied by a visible glow in the discharge tube, the general appearance being shown in fig. 4. The colour of the discharge depends on the nature of the gas, but

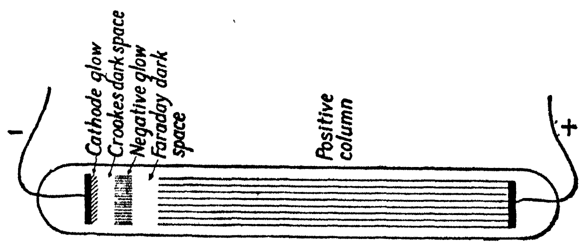

Fig. 4.—Silent Steady Discharge

certain general characteristics are in evidence. At moderately low pressures, proceeding from the cathode we note the *cathode glow*, after which comes the *Crookes dark space*. This is followed by another glowing region termed the *negative glow*, and this in turn by the *Faraday dark space*. Finally, we arrive at the glowing *positive column*, which stretches to the anode and may occupy most of the tube. As the pressure is reduced the positive column gradually splits up into a

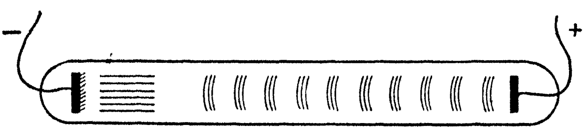

Fig. 5.—Striæ

number of *striæ*, as shown in fig. 5. Further reduction in pressure results in continual shrinkage of the positive column accompanied by a corresponding increase in the space occupied by the negative glow and the Crookes dark space. A degree of evacuation is eventually reached when the Crookes dark space occupies the entire tube and the glow is now confined to the walls of the containing vessel, being particularly intense near the anode and therefore opposite to the cathode.

3. Cathode Rays.

A number of simple experiments show that, when the discharge tube reaches the operating condition such that the Crookes dark space occupies the entire tube, the latter is filled with *cathode rays* proceeding from the cathode. These cathode rays are subject to rectilinear propagation, as is shown in fig 6, a sharp shadow of the obstacle being thrown on the glow at the end of the tube.

Another most important property of cathode rays is that they are

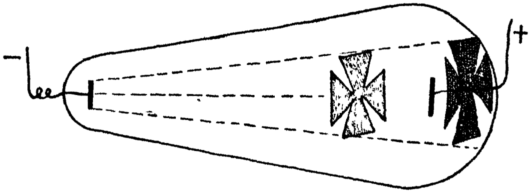

Fig. 6.—Cathode Rays

deflected by electric or magnetic fields. For example, if a bar magnet is presented to the rays as in fig. 7, the glow at the end of the tube is deflected up or down according to the sign of the pole presented. That

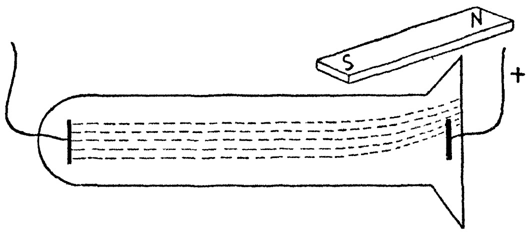

Fig. 7.—Deflection in Magnetic Field

magnetic deflection occurs implies that the cathode radiation consists of an *electric current*, the deflection process being analogous to the motion of a conductor carrying a current in a magnetic field and obeying the same rules.

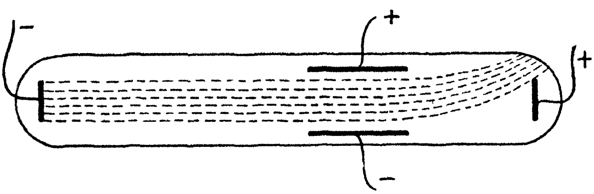

Fig. 8.—Deflection in Electric Field

Electrical deflection is less easily shown, but if the pressure is adjusted correctly, application of a difference of potential of a few thousand volts between the two flat condenser plates shown in fig. 8 results in deflection of the cathode ray beam in the electric field.

By using an apparatus as in fig. 9, the cathode radiation may be collected in a small brass cylinder (termed a *Faraday cylinder*) con-

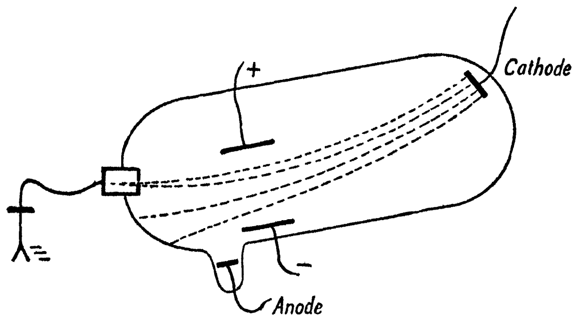

Fig. 9.— Collection in Faraday Cylinder

nected to an electroscope or electrometer. In this way the cathode rays may be shown directly to consist of negative electricity.

4. Positive Rays.

If a discharge tube is fitted with a perforated cathode, a glow will develop on the glass container immediately behind the cathode. This glow is due to the impact of *positive rays* proceeding along the discharge tube from anode to cathode and passing through the perforated cathode. The properties of the positive rays may be examined by experiments similar to those with cathode rays, and in this way it has been shown that they are positively charged and are deflected by electric and magnetic fields.

5. Electrons.

We have seen how electrolysis requires the assumption that each ion possesses a definite electric charge whose magnitude depends only on the valency and is quite independent of the nature of the solute. This fact implies that in conduction through liquids electricity is atomistic in nature, that is it occurs in definite quantities only, being either one ionic charge, two, or so on according to the valency. The possibility therefore arose that *all* electricity was atomistic and consisted of integral numbers of a small unit of charge termed the **electronic charge.** Thomson carried out experiments to show that a value for e/m, the ratio of the charge to the mass of an electron, could be obtained from the deflection of cathode rays in electric and magnetic fields, if the assumption was made that the cathode radiation consisted of a stream of negatively charged corpuscles or electrons. By means of the apparatus shown in fig. 10, electric and magnetic fields were simultaneously applied in directions mutually perpendicular to each other. Since the charged particles move in the direction of the applied electric field between the condenser plates C, the action of the electric field alone would cause the electron stream to move upwards, as shown by the deflection of a glowing spot at the end of the tube where the cathode rays impinge on a sensitive screen of zinc sulphide or willemite. The application of a magnetic field at right angles to the electric field may cause the spot to return to its original position, since the electron stream behaves as a conductor carrying a current and therefore obeys the left-hand rule.

If e is the charge of each of the electrons, all of which are assumed to be identical, and if the velocity with which they are moving is v,

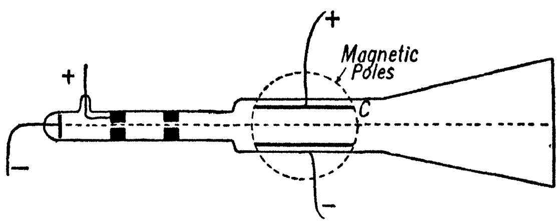

Fig. 10.—Simultaneous Electric and Magnetic Fields

the condition that no deflection shall occur is that the electric and magnetic forces shall be equal and opposite, that is:

$$Xe = Hev, \qquad (18.1)$$

or

$$v = \frac{X}{H}, \qquad (18.2)$$

where X and H are the strengths of the electric and magnetic fields respectively. If the electric field is now cut off, the spot shows a deflection of the cathode rays in the magnetic field alone. Since the electron stream and the magnetic field are mutually perpendicular, the deflection of the electrons is always in the direction at right angles to both, and it is therefore subject to a constant force perpendicular to its direction of motion. The path traversed by the electron stream is therefore circular according to Chap. III, Part I. If the mass of the electron is m, and the radius of curvature of its path is ρ, equating the magnetic force to the centrifugal force in the circular orbit we have

$$Hev = \frac{mv^2}{\rho}, \qquad (18.3)$$

or

$$\frac{e}{m} = \frac{v}{\rho H} = \frac{X}{\rho H^2}, \qquad (18.4)$$

from (18.2). Now the radius of curvature ρ can be calculated from the geometry of the apparatus if the deflection of the cathode spot is measured together with the distance from the magnetic field. Consequently the ratio of the charge to the mass of the electron may be derived. In this way Thomson showed that the ratio was independent of the nature of the gas in the discharge tube, thus indicating that electrons were a constituent of all gases. Subsequent experiment showed that electrons were constituents of all matter, and in this way arose the concept of the **electrical structure of matter.**

6. Positive Ray Analysis.

By deflection of the positive rays in electric and magnetic fields, the ratio of their charge to their mass, E/M, was likewise obtained, but no such generality as obtained for electrons was forthcoming. The values obtained depended greatly on the nature of the gas in the discharge tube, a decrease in E/M being observed for positive rays of the same velocity but increasing atomic weight.

For positive rays, Thomson introduced the use of *parallel* electric and magnetic fields. This results in the electric and magnetic forces acting at right angles to each other, and leads to the formation of a parabolic streak on a screen placed at right angles to the general direction of the positive ray beam. The equations governing the positive ray deflection are

$$x \propto \frac{1}{\rho} \propto \frac{EH}{MV}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18.5)$$

according to (18.4), where x is the deflection due to the magnetic field, E and M are the charge and mass of the positive corpuscle, V is its velocity, and H is the magnetic field strength. Similarly, the deflection y in the electrostatic field is given by

$$y = \tfrac{1}{2}ft^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18.6)$$

where f is the acceleration under the electric force maintained for a time t. If the length of path in the electric field is l, $t = l/V$, and since $f = XE/M$, equation (18.6) becomes

$$y \propto X\frac{E}{M}\frac{l^2}{V^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18.7)$$

Hence from (18.5) and (18.7), eliminating the velocity,

$$\frac{x^2}{y} \propto \frac{E}{M}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18.8)$$

Equation (18.8) is the equation of a parabola and hence any stream of positive corpuscles, independently of their velocity, will lie on one parabola only if E/M is constant. By analysing the parabolas obtained by photographic recording, Thomson found that E/M, when hydrogen gas occupied the discharge tube, was about 1/1800 of e/m for electrons. Now the total charge Q required to deposit one chemical equivalent of an element is easily determined by electrolysis. Further, a rough estimate of Avogadro's number N, that is, the number of ions present in a chemical equivalent, had been afforded by measurements on Brownian movement (see Chap. XI, Part II). Hence the charge carried by one ion is

$$e = \frac{Q}{N},$$

and the value of this charge is exactly equal to that of the electron. Although these results had not been proved accurately at the time, Thomson suggested that very probably the charge E carried by the hydrogen ion in the discharge tube was equal to e, the electronic charge. Hence $m = M/1800$, and consequently the existence of a universal particle with a mass much less than that of the lightest atom had been demonstrated. Since electrons were found in the discharge tube quite independently of the nature of the gas present, very strong evidence was provided for assuming that electrons were a fundamental constituent of all matter.

EXERCISES

1. Describe the effect of pressure on gaseous conduction and explain the action of an internal ionizing agent in rendering a gas conducting.

2. Explain how a beam of cathode rays may be produced and how their nature has been elucidated.

3. How has the ratio e/m of the charge to the mass of an electron been determined? Why does the value obtained vary with the speed of the electrons?

4. Write a short essay on positive ray analysis.

CHAPTER XIX

Electronics

1. Electronic Charge and Mass.

Since the whole of electrical theory and modern ideas on the structure of matter depend largely on the properties and behaviour of electrons, we shall now devote considerable attention to what is, in fact, the essence of our subject. We shall see that not only is theory dependent on the electronic concept but that vitally important practical applications such as radio valves, X-ray tubes and photo-electric cells could never otherwise have been invented or understood.

We saw in the last chapter how a value of e/m could be obtained from the deflection of cathode rays in crossed electric and magnetic fields. We now have to consider the experimental determination of the individual values of the charge e and mass m respectively. There are very many methods available depending on various phenomena such as the photo-electric effect, X-radiation, spectral theory and so on, and even a brief comparison of the methods is beyond the scope of this book. We shall therefore describe only **Millikan's method,** which afforded the first accurate measurement of these fundamental quantities. The principle of the method was to charge a microscopic oil drop with a few electrons, and then to hold the drop balanced between the gravitational force downwards and the electric force upwards due to the drop being situated between two condenser plates. As shown in fig. 1, two circular horizontal condenser plates a few centimetres in diameter were separated by an ebonite ring about 1 cm. high fitted with three glass windows. Two of the windows were at opposite ends of a diameter of the ring, but the third was situated so that a radius to this window made an angle of about 20° with the diameter joining the other two windows. The purpose of the windows was the illumination and observation of the interior of the condenser. The condenser was placed at the bottom of a metal cylinder and drops of oil were squirted from an atomizer into the region above the condenser. Owing to the presence of a small hole in the top condenser plate, an occasional oil drop fell through the condenser. It was observed with a microscope by scattered light through one of the windows, illumination being provided by a powerful light beam via one of the other windows. In general, on applying an electric field to the region between the condenser plates the drop either moved more rapidly, more slowly or even moved upwards owing to the electric field being greater than the gravitational field. For the very special case where a drop could be held completely stationary we have, by equating the electrical and gravitational forces,

$$X \, . \, ne = mg, \qquad \qquad (19.1)$$

where X is the strength of the electric field, e is the electronic charge, n is the number of electrons on the drop, m is the mass of the drop, and g the acceleration due to gravity. In this equation all the quantities except ne and m are known. The latter is determined by allowing

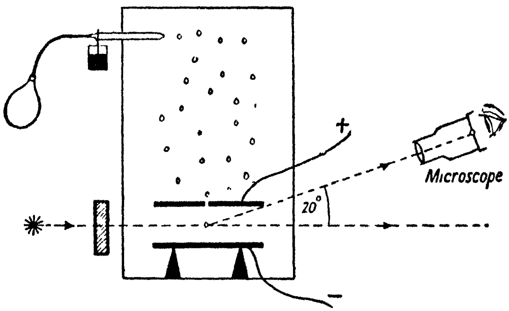

Fig. 1.—Millikan's Method for e

the drop to fall freely under gravity, and then, applying Stokes's law of Part I, Chap. XII, we have

$$mg = 6\pi\eta av = \tfrac{4}{3}\,\pi a^3\,(\rho - \sigma)g, \qquad (19.2)$$

where a is the radius of the drop assumed spherical, η is the coefficient of viscosity of the gas surrounding the drop, v is the terminal velocity of the drop, and ρ and σ are the densities of the oil and the gas respectively. By using a special oil Millikan overcame difficulties associated with evaporation and oxidation, and thus obtained a series of values of ne. These were found all to be multiples of a certain definite value, which clearly corresponded to the electronic charge e when $n = 1$. In practice the method could be applied even when the electrical and gravitational fields did not exactly balance, from measurement of the velocities of the drop with and without an electric field present. The present accepted value for the electronic charge in terms of the unit electrostatic charge defined on p. 9 is $e = 4{\cdot}800 \times 10^{-10}$ e.s.u. By using the value of e/m obtained by deflection experiments or otherwise, the mass m of the electron is then found to be about 9×10^{-28} gm.

There are many other methods available for determining the electronic charge, but we shall discuss these when considering the various phenomena involved.

2. Conduction of Electricity through Solids.

Since the electric current must be essentially the same phenomenon whether it takes place through solids, liquids or gases, it follows that conduction through solids must consist in the passage of electrons through the conductor from the *negative* to the *positive* pole. Experiments on conduction through liquids and gases have shown that positive electricity is always accompanied by matter, that is by the positive ion. Since no transference of matter takes place in conduction through solids, the current is a one-way phenomenon, involving the passage of electrons only. Good conductors are substances through which the electrons move freely, while insulators are substances in which the electrons are tightly bound to their positive counterpart. In metals, therefore, we conceive the structure to consist of positively charged centres which are fixed except for thermal vibration and which contain the main mass of the atom. Moving freely amongst these positive residues are the electrons, which have a kinetic energy which is in equilibrium with the vibrational heat energy of the positive residues. Since the metal is uncharged in its normal state, the number of free electrons present must be equal to the number of positive residues. If now a difference in potential is applied between two parts of the metal, the electrons will be attracted to the positive and repelled by the negative pole. At first it would appear that a very large saturation current might be produced however small the applied potential, all the electrons present moving rapidly from the negative to the positive pole. Such behaviour does not occur and would, of course, be in disagreement with Ohm's law, which states that the number of electrons passing any point per second is proportional to the applied potential difference. It was first shown by Drude that a satisfactory qualitative and quantitative explanation of electron conduction in metals could be evolved if it were remembered that the electrons will collide with the atomic centres and that the effect will be that a resistance is provided to the free motion of the electrons. Further, as the temperature of the metal is raised, the amplitude of swing of the atomic centres is increased and consequently the area over which they become effective in stopping the passage of the electrons is also increased. Theory then agrees with experiment in indicating a rise of resistance with temperature. Drude's theory also allows a deduction of the **Wiedemann-Franz law,** which embodies the experimental observation that the ratio of the electrical to the thermal conductivities of good conductors is constant. Owing to the existence of the recently discovered wave-like properties of the electron (see section 8), Drude's theory has been much modified, but it still remains a useful concept in any initial approach to the explanation of conduction through solids.

Since the number of electrons per unit volume or **electron density** will vary from metal to metal, the existence of contact potentials between two dissimilar metals is easily explained. The two dissimilar metals will in general have a different electron pressure, and when they are placed in contact electrons will flow temporarily from one to the other until some equilibrium is attained. In the same way thermoelectric effects receive a ready explanation on electron theory. If the temperature of one junction differs from that of the other, the electrons at the hot junction will have greater energy than those at the cold junction. This fact, combined with the differing electron pressures of dissimilar metals, is sufficient to account for the Seebeck and Peltier effects. The Kelvin effect is similarly explained as due to a difference in electron pressure between different parts of the same metal if it is not at uniform temperature.

3. Thermionics.

If the temperature of a metal is raised sufficiently, the kinetic energy of the electrons may become so great that they leave the metal completely. The analogy with evaporation is exceedingly close, and in

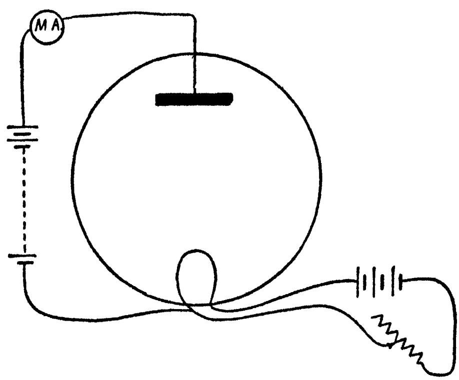

Fig. 2.—Thermionic Effect

fact thermodynamics may be applied directly to the phenomenon with excellent results. To demonstrate this **thermionic effect** experimentally a fine tungsten *filament* is sealed into a glass vessel together with another electrode as shown in fig. 2, and the vessel is exhausted as highly as possible. Such an arrangement is said to constitute a *diode*. When the filament or *cathode* is heated (usually electrically), if a positive potential is applied to the other electrode or *anode*, a milliammeter included in the circuit will show the presence of a current flowing from anode to cathode, corresponding to a stream of electrons from cathode to anode. By experiments similar to those described in Chap. XIX, it may be proved that these thermionic electrons are identical with the electrons in a cathode ray beam. Two *characteristic curves* may be obtained for a *diode*, one showing the variation of current with potential for a fixed temperature of the filament as shown in fig. 3, and the other showing the variation of the saturation current with temperature of the filament. The first curve shows that at first the current increases rapidly with the applied potential, but that ultimately a saturation current is reached. The explanation is that at first a reservoir of electrons is provided owing to the number evolved from the filament per second being greater than the number removed per second

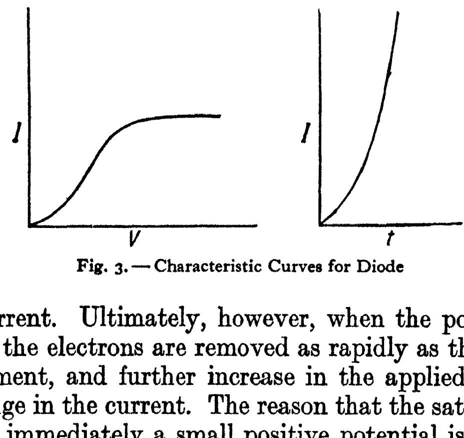

Fig. 3.—Characteristic Curves for Diode

as electric current. Ultimately, however, when the potential is sufficiently great, the electrons are removed as rapidly as they are evolved from the filament, and further increase in the applied potential produces no change in the current. The reason that the saturation current does not flow immediately a small positive potential is applied to the anode is due to the **space charge effect.** Thus, the region surrounding the filament consists of an atmosphere of electrons termed the space charge. Owing to the mutual repulsion which exists between charges of the same sign a fair proportion of the electrons which are emitted are repelled back to the cathode. When a small potential is applied the general behaviour of the electrons is still governed largely by their mutual repulsions, and consequently the space charge behaves as a reservoir of electrons. This behaviour holds until the potential is large enough to overcome the space charge control and then saturation rapidly sets in. The experimental law, which has a theoretical support, for the relation between anode current and anode potential for a fixed filament temperature, over the space charge controlled region of the curve, is

$$I = kV^{3/2}, \qquad (19.3)$$

where k is a constant.

The second characteristic curve of a diode is that between saturation current and temperature. Experimentally, the equation deduced theoretically by Richardson connecting these two quantities is $I = AT^2 \exp. \frac{-b}{T}$, where A and b are constants depending on the nature of the filament.

Now since electrons travel from cathode to anode when the latter is positive but not when it is negative (for then the electrons are repelled back to the cathode), the diode allows current to pass in *one direction only*. The general term **valve** is therefore applied to such contrivances, in analogy with a mechanical valve which has the same essentially one-way function. The practical applications of such an

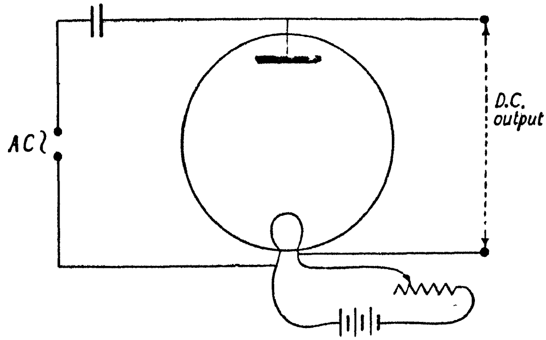

Fig. 4.—Valve Action of Diode

instrument are legion. For example, the diode will act as a *current limiter* in any circuit in which it is in series, for no matter how the potential may rise, no current larger than the saturation current can flow and this current is uniquely determined by the filament temperature. Again, consider the circuit shown in fig. 4 where an alternating potential is applied between cathode and anode. The current in the associated *anode circuit* is unidirectional, for, owing to the valve action, current flows in this circuit only during the positive half-cycle of the alternating potential. The diode is then said to function as a **rectifier.**

4. The Triode.

In the triode valve, a third electrode is inserted in the form of a wire grid, the three electrodes usually having cylindrical symmetry represented by a central line filament surrounded by a concentric grid and finally a solid cylindrical anode. The presence of the third electrode or **grid** acts as an extremely sensitive control of the current flowing between the cathode and the anode. Owing to the open nature of the grid very few electrons travel to this electrode, but its potential profoundly affects the magnitude of the anode current. For example, if the grid is made strongly negative with respect to the filament, although a high positive potential may be applied to the anode, the anode current is almost entirely suppressed. As the negative grid potential is gradually raised to zero the anode current increases, while if the grid becomes positive, the anode current increases still further. When the grid potential is strongly positive, a small grid current is created consisting of a small proportion of electrons attracted from the anode current. If the anode is subjected to very vigorous electron bombardment, the anode itself may emit electrons. This phenomenon is termed *secondary emission*. The applications of the triode are very

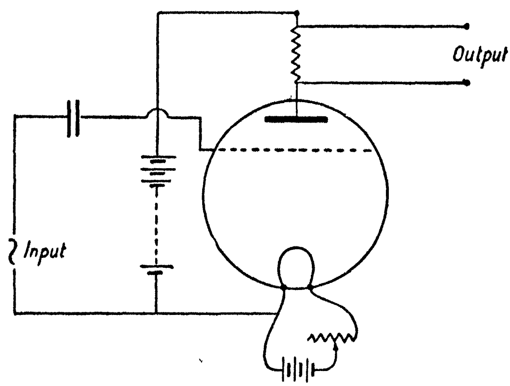

Fig. 5.—Triode as Amplifier

numerous, and form a considerable section in the science of radio-engineering. As an illustration of its properties we shall consider its use as an **amplifier.** Since small changes in the potential of the grid produce large changes in the anode current, if a large *anode resistance* is included in the anode circuit as shown in fig. 5, large fluctuations in potential are produced across the ends of this resistance. Consequently small fluctuations in potential in the grid circuit may be amplified and examined as large fluctuations in potential in the anode circuit. Moreover, if the large oscillations in potential in the anode circuit are fed into the primary of a mutual inductance, the secondary of which is in the grid circuit, the initial oscillations or fluctuations in potential in the grid circuit may be so augmented as to overcome entirely the natural decay of the oscillations due to ohmic losses. Consequently the valve itself becomes a permanent seat of oscillations and we obtain a **valve oscillator.**

Just as with the diode, characteristic curves are plotted for any given triode. Owing to the presence of a third electrode two characteristic curves are obtained. The first shows the variation of anode current with grid potential for a fixed anode voltage, and is shown in fig. 6, where a family of curves is drawn each corresponding to a cer-

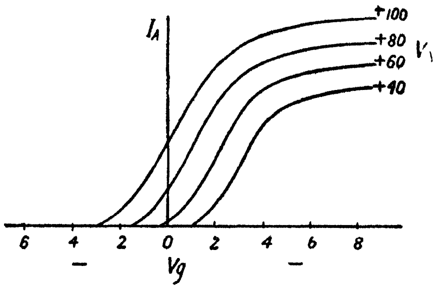

Fig. 6.—Anode Current and Grid Potential

tain anode voltage. The second type of characteristic shown in fig. 7 is obtained by plotting the anode current against anode voltage for a fixed grid potential. Again a family of curves is obtained, this time for different grid potentials.

Besides the triode, multiple-electrode valves containing up to ten

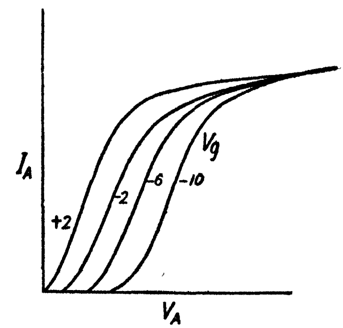

Fig. 7.—Anode Current and Anode Potential

electrodes are now available for a great variety of purposes. Additional electrodes up to the pentode are usually additional control grids. When more electrodes than five are present the valve usually functions as two or more valves in one envelope. Again, in modern valves the cathode is usually heated indirectly by a small heating coil close to it. In this way the whole of the cathode may be maintained at a uniform potential, whereas if it carries a heating current there is a potential drop along the filament.

5. Cathode Ray Oscillograph.

One of the most important applications of the cathode ray stream is to the cathode ray oscillograph, a diagram of which is shown in fig. 8. Many commercial variations are available, but it consists essentially of a hot cathode K emitting a canalized beam of electrons which

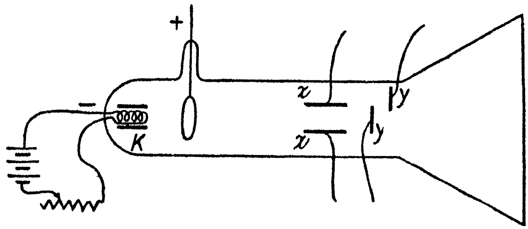

Fig. 8.—Cathode Ray Oscillograph

produce a glowing spot on a sensitive screen at the end of the tube. Two sets of plates (x, x) and (y, y) are placed on each side of the beam and to these are applied electric fields which deflect the spot. The (x, x) pair of plates is connected to a condenser which is charged at regular intervals to a known potential and then allowed to discharge through a high resistance. By a calculation similar to that of section 5, Chap. XIII, in which the inductance is replaced by the capacity of the condenser, the discharge may be shown to be exponential. Consequently the spot is first drawn to one side as the condenser receives its maximum potential and then returns to its original position as the condenser discharges. The time taken for the spot to traverse its path is easily calculated from the *time constant* of the condenser and resistance, and thus a *time base* is provided for the oscillograph. In practice the condenser charges and discharges many times per second, and owing to the persistence of vision the spot appears drawn out into a horizontal line. If now an alternating potential is applied to the (y, y) plates the spot will be subjected to two forces at right angles, and owing to the oscillating nature of the (y, y) potential will show the wave-form directly as in fig. 9. Since the horizontal time base is known for the instrument, the frequency and shape of the oscillating potential is obtained from direct measurement on the observed curve. Photographic fixing allows the trace to be

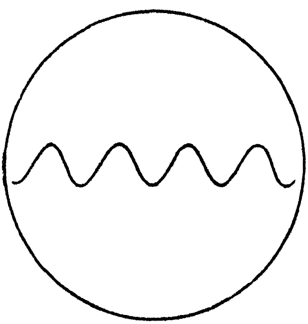

Fig. 9.—Oscillating Potential

studied at leisure, and in this way the instrument has many applications to the examination of electrical oscillations, or of sound oscillations (see Part IV) which have been converted to electrical oscillations.

6. Photoelectric Effect.

We have mentioned in Part III, Chap. IX, the experiments of Lenard which led to the theory of the photoelectric effect. On the electron theory of metals a free electron, if given sufficient energy, will leave the metal as in thermionic evaporation. This energy may be supplied by light or other radiation and then constitutes the photo-

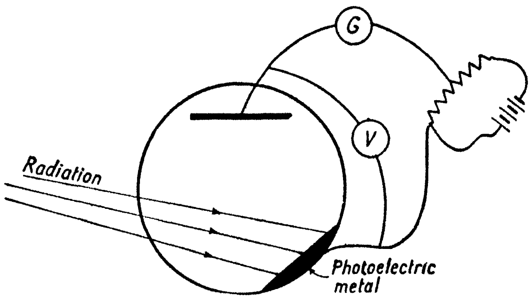

Fig. 10.—Determination of Planck's Constant

electric effect. Lenard's experiments are summed up in *Einstein's equation of the photoelectric effect* which states that

$$E = \tfrac{1}{2}mv^2 = h\nu - P, \quad . \quad . \quad . \quad . \quad . \quad (19.4)$$

where E is the energy with which the electron escapes, h is Planck's constant of action, ν is the frequency of the radiation, and P represents the energy required just to liberate the electron from the metal with no appreciable velocity. In practice, examination is made of the photoelectric effect with an apparatus as in fig. 10. The energy of the electrons is estimated indirectly from the positive *retarding potential* V, which must be applied to the illuminated plate so that electrons are just unable to escape from the metal. Equation (19.4) then becomes

$$Ve = h\nu - P, \quad . \quad . \quad . \quad . \quad . \quad . \quad (19.5)$$

where e is the charge on the electron. Hence if for a series of different frequencies V is plotted against ν, the slope of the line determines h/e, and since e is known from Millikan's experiment, h may be calculated. This method is one of the most accurate available for determining Planck's constant. The intercept of the straight line on the ν-axis gives the quantity P, which is sometimes termed the *work* *function* of the metal. Clearly for frequencies less than P/h, no electrons are emitted. There is therefore a **threshold frequency,** for which $\nu_0 = P/h$. The threshold frequency is in the red end of the visible spectrum for some metals like calcium and rubidium, and hence these are more suitable for use in photoelectric cells than other metals like copper, whose photoelectric threshold lies at higher frequencies. By the use of ultra-violet radiation, the photoelectric effect may be shown to exist in gases as well as in metals, and therefore to be characteristic of all matter subject to radiation.

7. X-rays.

If a photographic plate wrapped in black paper is in the neighbourhood of a cathode ray tube running at high potential, the photographic plate is found to be affected. Likewise a photoelectric cell which is screened from visible and ultra-violet radiation is shown to

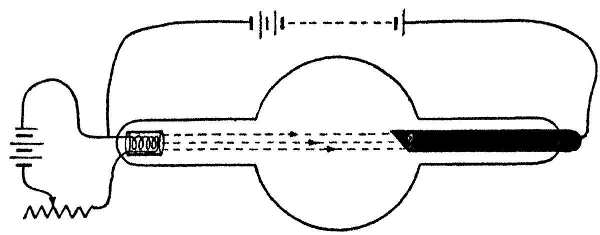

Fig. 11.—X-Ray Tube

emit electrons when close to such a discharge tube. These and other experiments have shown that the impact of the cathode rays on the anode of the discharge tube produces an extremely penetrating radiation, termed X-radiation, which has all the properties of ultra-violet light except that it has greater penetrating power, penetrating layers of material normally regarded as opaque. A typical X-ray tube is shown in fig. 11, where the cathode rays are thermionic electrons: the anode is usually of tungsten, since the latter has a high melting-point and the anode becomes intensely hot if the cathode ray bombardment is continued for any length of time. By measurements on the photoelectric effect or otherwise, since P and h remain constant for a given metal, ν may be calculated for the X-radiation. In this way a definite frequency and wave-length may be ascribed to the X-rays. Since X-radiation as produced above contains a wide range of wave-lengths, some form of spectrometer is required if individual wave-lengths are to be separated. In light radiation the diffraction grating provides the standard instrument, but since X-radiation has wave-lengths several hundred times shorter than visible radiation, application of the diffraction grating equation on p. 77, Part III, shows that the diffraction angle is so small as to render it impossible to distinguish the diffracted from the incident beam. Fortunately Nature has provided a three-dimensional diffraction grating in the form of crystals, which consists of layers of atoms arranged with a regular separation of the order of the wave-length of X-radiation. By

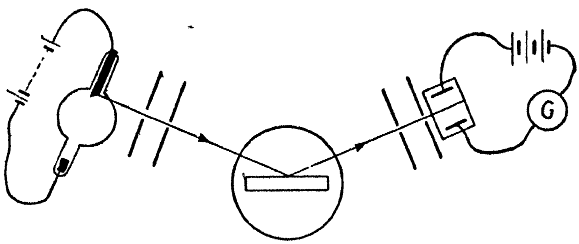

Fig. 12.—X-Ray Spectrometer

a treatment similar to that used in Part III, p. 64, where the effect of reflected light from the top and bottom layers of a thin film is considered, it may be shown that if a wave-length λ is present in an X-ray beam, regular reflection occurs from a crystal surface at a **glancing angle** θ given by

$$2d \sin \theta = n\lambda, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19.6)$$

where n is integral and d is the **grating constant,** that is the separation of the atomic layers constituting the crystal. Now d may be calculated

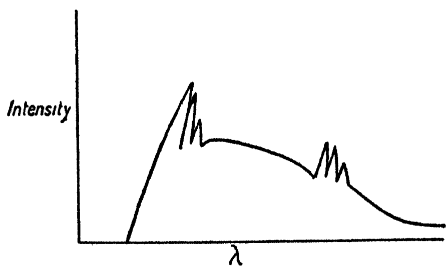

Fig. 13.—Intensity and Wave-length of X-radiation

from the density of the crystal together with a knowledge of Avogadro's number, and θ may be measured by mounting the crystal on an **X-ray spectrometer.** This consists of a slit to define the X-ray beam and an ionization chamber or photographic plate to detect the reflected beam as shown in fig. 12, the crystal being mounted on a rotating table as in the ordinary optical spectrometer, and the slit and detector replacing the collimator and telescope of the optical spectrometer.

In fig. 13 a curve is plotted of the intensity of the X-radiation (measured from the degree of blackness of the photographic plate or the intensity of current in the ionization chamber) obtained from an X-ray tube, against the wave-length of the radiation. This shows that there is a definite lower limit of wave-length, below which no radiation occurs. A general or "white" background of radiation is present, superposed on which are sharp peaks at certain positions. These latter are termed the characteristic X-rays and change their position as the nature of the anode is changed. If V is the potential across the X-ray tube and ν_{max} is the limiting (highest) frequency of the radiation present, it is found that

$$Ve = h\nu_{max}, \quad \ldots \ldots \quad (19.7)$$

where e is the electronic charge and h is Planck's constant. The interpretation of the production of "white" X-rays is therefore that it is the reverse of the photoelectric effect. The kinetic energy acquired by the electrons in falling through the potential difference V in the X-ray tube is equal to the loss in potential energy Ve. This therefore represents the maximum energy which can be converted into radiation, and since the conversion is governed by Planck's radiation law, equation (19.7) represents the X-ray production process. In general, the efficiency of the tube is low, by far the greater part of the electronic energy being converted into heat at the anode. The other frequencies are therefore to be attributed to partial conversion only of the electron energy into radiant energy. We return to the consideration of the characteristic X-rays in the next chapter.

8. Electron Interference and Diffraction.

If a beam of electrons is reflected from the surface of a crystal, just as in the X-ray spectrometer it is found that for a particular glancing angle specular reflection is particularly pronounced. The analogy with the behaviour of X-radiation is complete, so that a definite wave-length λ may be said to be associated with the electrons, given as in equation (19.6) by

$$2d \sin \theta = n\lambda,$$

where d is the grating constant and θ is the glancing angle. Further, if the velocity of the electrons is v, the value of λ derived from the above equation is found to be

$$\lambda = \frac{h}{mv}, \quad \ldots \ldots \quad (19.8)$$

where h is Planck's constant. The electrons used in these experiments are of low velocity and hence the mass m may be taken as equal to the rest mass.

In a similar experiment, if electrons are sent normally through a thin foil, a photographic plate placed behind the foil shows in general a series of concentric diffraction rings very similar to those produced by the passage of a light wave through a circular aperture (see Part III, Chap. VII). Again, just as the wave-length of light may be estimated from measurements on its diffraction pattern, so the wave-length to be associated with the electron may be determined in the same way.

Although the nature of the wave motion associated with the electron is not well understood, and the wave is certainly not an ether wave, electron diffraction has already expanded into a separate science of *electron optics* with important applications in the *electron microscope*. We have seen in Part III that if structures to be examined are much finer than the wave-length of the light with which they are illuminated, the images of the structures will be very blurred and the structures will be said not to be resolved. Now substitution in (19.8) shows that for electrons which have fallen through a potential of a few hundred volts, λ is of the order of 10^{-8} cm. Hence a ray treatment is applicable to much finer structures in electron optics than with ordinary light.

EXERCISES

1. Describe Millikan's method of measuring the electronic charge. Of what significance is the fact that the electronic mass is much less than that of the hydrogen atom?

2. Outline Drude's theory of electronic conduction in solids. What other evidence is available that lightly bound electrons exist in good conductors?

3. Explain the phenomenon of the thermionic effect and describe how a diode may be used (*a*) as a rectifier of A,C. potential, (*b*) as a current limiter.

4. Describe the construction and mode of action of a simple triode and explain the shape of the characteristic curves obtained when anode current is plotted against grid potential at different anode potentials.

5. Give an account of the cathode ray oscillograph with special reference to its use in elucidating the wave form of electromagnetic oscillations.

6. Enumerate the main points associated with the photoelectric effect. In what sense may the production of X-rays be regarded as the inverse of the photoelectric effect?

7. Describe the construction of a typical X-ray tube and explain how the wave-length of X-radiation may be measured.

8. Describe experiments which indicate that cathode rays behave simultaneously as particles and as waves.

CHAPTER XX

Atomic and Nuclear Physics

1. Radioactivity.

It was discovered by Becquerel that uranium compounds, when placed close to a photographic plate covered with black paper, possessed the property of blackening the photographic plate. Similarly an electroscope could be discharged when in the neighbourhood of the uranium, and in fact all the general properties of X-rays were observed. In consequence it was established that uranium, and still more powerfully radium, emitted ionizing radiations *spontaneously* without any previous excitation by electrical discharge or any other means. The radiations from these **radioactive** substances were then examined by Rutherford and others with the following general results:

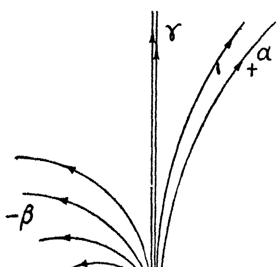

Fig. 1.—Separation of Rays by Magnetic Field

(i) The ionizing power of the radiations was much reduced by covering the radioactive substance with a thin layer of material, while the photographic action was little affected. Hence the radiations emitted are of at least two types. The easily absorbed but strongly ionizing component was termed **α-radiation.**

(ii) By subjecting the more penetrating radiations to magnetic and electric fields it was established that these were of two further types. One, termed **β-radiation,** was deflected by the fields and shown to consist simply of electrons moving with very great velocity, even approaching that of light. The other component, termed **γ-radiation**, which was more penetrating still, was unaffected by the fields and subsequently shown to be similar to X-rays but of shorter wave-length than this. The α-rays were deflected by the field in the reverse direction but less strongly than the β-rays, so that the application of a magnetic field effects a separation as shown in fig. 1.

(iii) In the process of emitting the radiations, the parent element is transformed into another element lower in the periodic table. The radioactive substance therefore decays, and the decay follows the simple law that the rate of decay is proportional to the mass of the radioactive element present. Expressing this mathematically, we have

$$\frac{dN}{dt} = -\lambda N, \qquad \ldots \ldots \quad (20.1)$$

where N is the number of atoms of the radioactive material present at any time t, and λ is termed the **decay constant.** Integrating equation (20.1), we obtain

$$\frac{dN}{N} = -\int \lambda\, dt,$$

or

$$\log_e N = -\lambda t + A, \qquad \ldots \ldots \quad (20.2)$$

where A is a constant. If $N = N_0$ at time $t = 0$, $A = \log_e N_0$, so (20.2) becomes

$$N = N_0 \exp.^{-\lambda t}, \qquad \ldots \ldots \quad (20.3)$$

and an exponential decay law is obeyed.

In general, the new element which is formed is itself radioactive, and hence a radioactive chain is formed. This disintegration process continues down the periodic table until the element *lead* is reached, after which radioactive action ceases. Neither change in temperature nor electrical excitation nor any change in physical conditions affects the rate of decay of radioactive substances.

2. α-rays.

By deflection of α-rays in electric and magnetic fields according to the principles described in the preceding chapter, the ratio E/M was found to be half the value $(E/M)_H$ for the hydrogen ion in electrolysis. That α-rays consist of positively charged helium atoms may be shown directly by allowing them to accumulate in an exhausted tube and applying an electrical potential difference. Discharge gradually sets in, showing that a gas is forming, and examination of the glow with a spectrometer shows the characteristic lines of the spectrum of helium. We are therefore led to conclude that the charge on the α-particle is twice the electronic charge. The velocity of the α-particles emitted is calculated from the E/M experiment. It is the same for all α-rays from the same radioactive element, and is usually about 10^9 cm./sec., but varies from one element to another. The positive charge and velocity of the particles explain the easy absorption of the rays and their strong ionizing power.

In the disintegration of radium, α-rays are emitted in the first disintegration process. As may be seen from the periodic table on p. 202, the new element falls in the rare gas group and, in fact, constitutes the rare gas **radon.**

PERIODIC TABLE OF THE ELEMENTS

	I	II	III	IV	V	VI	VII	VIII
1	1 H 1·0078							2 He 4·002
2	3 Li 6·940	4 Be 9·02	5 B 10·82	6 C 12·00	7 N 14·008	8 O 16·0000	9 F 19·000	10 Ne 20·183
3	11 Na 22·997	12 Mg 24·32	13 Al 26·97	14 Si 28·06	15 P 31·02	16 S 32·06	17 Cl 35·457	18 A ← 39·944
→ 4	19 K 39·096 29 Cu 63·57	20 Ca 40·08 30 Zn 65·38	21 Sc 45·10 31 Ga 69·72	22 Ti 47·90 32 Ge 72·60	23 V 50·95 33 As 74·91	24 Cr 52·01 34 Se 78·96	25 Mn 54·93 35 Br 79·916	26 Fe 55·84, 27 Co 58·94 ⟷ 28 Ni 58·69 36 Kr 83·7
5	37 Rb 85·44 47 Ag 107·880	38 Sr 87·63 48 Cd 112·41	39 Y 88·92 49 In 114·76	40 Zr 91·22 50 Sn 118·70	41 Cb 93·3 51 Sb 121·76	42 Mo 96·0 52 Te 127·61	43 Ma ⟷ 53 I 126·92	44 Ru 101·7, 45 Rh 102·91, 46 Pd 106·7 54 Xe 131·3
6	55 Cs 132·91 79 Au 197·2	56 Ba 137·36 80 Hg 200·61	57 La 138·92 81 Tl 204·39	72 Hf 178·6 82 Pb 207·22	73 Ta 181·4 83 Bi 209·00	74 W 184·0 84 Po (210·0)	75 Re 186·31 85 —	76 Os 191·5, 77 Ir 193·1, 78 Pt 195·23 86 Rn 222
7	87 —	88 Ra 225·97	89 Ac (227)	90 Th ⟷ 232·12	91 Pa (231)	92 U 238·14		

THE RARE EARTHS (to be inserted between 57 La and 72 Hf)

58 Ce 140·13	59 Pr 140·92	60 Nd 144·27	61 —	62 Sm 150·43	63 Eu 152·0	64 Gd 157·3
65 Tb 159·2	66 Dy 162·46	67 Ho 163·5	68 Er 167·64	69 Tm 169·4	70 Yb 173·04	71 Lu 175·0

The numbers in front of the symbols of the elements denote the atomic numbers; the numbers underneath are the atomic weights. The latter are taken, with a few modifications, from the Report of the International Commission on Atomic Weights for 1932. The double arrow ⟷ indicates the places where the order of atomic weights and that of atomic numbers do not agree.

3. β-rays.

By carrying out experiments similar to those devised to establish the nature of electrons, β-rays may be shown to be electrons moving with velocities which vary over an enormous range both with different elements and for any one element. While a value e/m exactly equal to that of electrons is obtained when the velocity of the β-rays is small, for large velocities the measured value becomes progressively less. This is due to the fact that these higher velocities approach that of light, and from our considerations in Part III, Chap. XII, we saw that ordinary mechanics has to be replaced by relativistic mechanics under such conditions. In particular, while the charge e is unaffected by the velocity, the mass m increases with the speed according to the simple relation

$$m = \frac{m_0}{\sqrt{1 - (v^2/c^2)}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (20.4)$$

where m_0 is the rest mass, and m the mass when the velocity is v; c is the velocity of light. The deviation of the observed value of e/m for fast-moving electrons from its measured value for slow electrons affords one of the most direct and accurate tests of the relativistic mass equation (20.4).

4. γ-rays.

That γ-rays are similar to X-rays but of shorter wave-length is shown by a variety of experiments similar to those used for identifying the nature of X-rays. The less penetrating γ-rays may have their wave-length determined by the method of crystal reflection. By application of the photoelectric effect and using Einstein's equation, the frequency may be measured directly from the energy of the electrons ejected. Owing to the high retarding potentials which would be required to prevent photoelectric emission, it is impracticable to use exactly the same methods as for ultra-violet light. The energy of the electrons has therefore to be determined by magnetic deflection, and we may note that the relativistic expression

$$E_{\text{kin}} = m_0c^2\left[\frac{1}{\sqrt{1 - (v^2/c^2)}} - 1\right], \quad . \quad . \quad . \quad (20.5)$$

for kinetic energy must replace the Newtonian expression $E_{\text{kin}} = \frac{1}{2}mv^2$ at higher velocities.

5. Scattering of α-Particles by Matter.

Rutherford suggested that the energetic particles emitted by radioactive substances could be used as powerful probes to examine the structure of matter subject to the radiation. Experiments were carried out in which thin metal foils were bombarded by a stream of α-particles, and the number of α-particles scattered at various angles to the incident beam was observed. Now, Avogadro's number being known, the number of atoms present in a given layer of the scatterer could be calculated, and hence the chance of an α-particle being scattered through any given angle could also be calculated if the atoms were regarded as hard spheres of the diameter calculated from the kinetic theory of matter (see Part II, Chap. XI). The result showed that the observed scattering was in complete disagreement with that calculated on classical theory, but behaved instead as though the mass of the atom were concentrated in a region of about 10^{-13} cm. diameter instead of 10^{-8} cm. diameter as expected on classical theory. Rutherford was therefore led to suggest that the main mass of the atom was concentrated in a central **nucleus.** The scattering arose from the mutual repulsive electrostatic action between the positively charged nucleus and the positive α-particle. The electrons in the atom were assumed to be situated outside the nucleus and to play no part (owing to their small mass relative to that of the α-particle) in the scattering process.

*6. Bohr Theory of the Hydrogen Atom.

Guided by the α-particle scattering experiment and other considerations, Rutherford and Bohr suggested that an atom of matter consisted of a positively charged nucleus around which revolved one or more extra-nuclear electrons, the number of such electrons, multiplied by the electronic charge, being numerically equal to the positive charge on the nucleus, so as to render the matter electrically neutral in its normal state. The stability of such a system arises from the fact that the electrical force of attraction between the nucleus and the extra-nuclear electrons is just balanced by the centrifugal force of the latter, which are regarded as being in orbital motion about the central nucleus, in complete analogy with the planetary system revolving round the sun.

Bohr developed this theory mathematically to account for the properties of the hydrogen atom, which was chosen since it represented the simplest system, consisting of one electron revolving around a singly charged positive nucleus termed the **proton.** In particular, Bohr deduced the properties of the line spectrum of hydrogen (see Part III). From experimental measurement of the position of the hydrogen lines, Balmer had shown many years earlier that the frequency of any line was given by the simple empirical formula

$$\nu = R\left(\frac{1}{2^2} - \frac{1}{m^2}\right), \quad \ldots \ldots \quad (20.6)$$

where R is a constant termed the Rydberg constant, and m is integral and equal to 3, 4, 5, etc. We shall now deduce this relation from Bohr's theory. Equating the centrifugal force to the electrical force for the special case of an electron revolving in a circular orbit as shown in fig. 2, we have

$$\frac{mv^2}{a} = \frac{(Ze)e}{a^2}, \quad \ldots \ldots \ldots \quad (20.7)$$

where a is the radius of the orbit, v is the velocity of the electron of mass m and charge e, and (Ze) is the nuclear charge where $Z = 1$ for hydrogen. Bohr then made the assumption that the angular momentum of the electron about the nucleus could not have any arbitrary

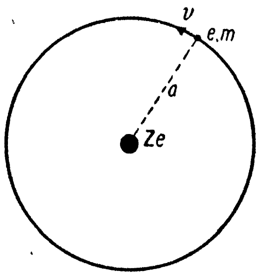

Fig. 2.— Electron in Circular Orbit

value (as in the gravitational case), but was "quantized" according to the relation

$$mva = \frac{nh}{2\pi}, \quad \ldots \ldots \ldots \quad (20.8)$$

where h is Planck's constant and n is integral. The reason for this suggestion, which at first seems quite arbitrary, will be explained later. For the present, eliminating a from (20.7) and (20.8), we have

$$v = \frac{2\pi Ze^2}{nh}, \quad \ldots \ldots \ldots \quad (20.9)$$

while eliminating v, we have

$$a = \frac{n^2h^2}{4\pi^2Zme^2}. \quad \ldots \ldots \ldots \quad (20.10)$$

The *total* energy of an electron in its orbit is partly kinetic and partly potential. The kinetic energy

$$E_{\text{kin}} = \tfrac{1}{2}mv^2, \quad \ldots \ldots \ldots \quad (20.11)$$

while the potential energy is

$$E_{\text{pot}} = -\frac{(Ze)e}{a} = -\frac{4\pi^2 Z^2 me^4}{n^2 h^2} = -mv^2, \quad . \quad (20.12)$$

from equations (20.10) and (20.9).

Hence the total energy is

$$E = -\tfrac{1}{2}mv^2 = -\frac{2\pi^2 Z^2 me^4}{n^2 h^2}. \quad . \quad . \quad . \quad (20.13)$$

Now from equation (20.10) it is clear that the radius of any orbit is uniquely determined by the value of the integer n. Hence Bohr's assumptions implied that the electron could not rotate in any circular orbit but only in certain allowed orbits or stationary states governed by (20.8). Bohr's final assumption was that radiation of the spectral line was due to the change in energy which resulted when an electron jumped from one orbit to another, the general quantum relation

$$E_2 - E_1 = h\nu \quad . \quad . \quad . \quad . \quad . \quad (20.14)$$

being obeyed, where ν is the frequency of the radiation emitted, and E_2 and E_1 are the energies of the initial and final orbit respectively. Hence, substituting in (20.14) from (20.13), we have

$$h\nu = \frac{2\pi^2 Z^2 me^4}{h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right),$$

$$\nu = \frac{2\pi^2 Z^2 me^4}{h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right). \quad . \quad . \quad . \quad (20.15)$$

Comparison of this result with Balmer's formula (20.6) shows that the two equations are identical if $n_1 = 2$ and $n_2 = m = 3, 4, 5$, &c. This general similarity is not surprising, since all Bohr's assumptions were made to lead to this result. The real test came from the fact that R, which is called the Rydberg constant, could be measured experimentally from spectroscopic data and (20.6), while (20.15) requires

$$R = \frac{2\pi^2 Z^2 me^4}{h^3}. \quad . \quad . \quad . \quad . \quad . \quad (20.16)$$

Putting $Z = 1$, the Rydberg constant for hydrogen could therefore be calculated from the values of e, m and h derived from quite different experiments. The agreement between the observed and calculated values of R was complete, thus showing that Bohr's conception and assumptions about the nature of the hydrogen atom represented reality to a marked degree.

Owing to the fact that the electron behaves as a wave as well as a particle it has been found necessary to refine the original Bohr model of the atom. Unfortunately this refinement leads to great mathematical complication in which the simple dynamical picture of the atom is largely lost. It seems likely, however, that Bohr's original model, like that of the billiard ball atom in kinetic theory, will always serve as a useful tool in understanding at least a limited group of phenomena.

7. X-ray Spectra.

We have already described the type of spectrum obtained when the intensity of X-radiation from a discharge tube is plotted against the wave-length, and we have mentioned that certain peaks occur on the curve which are said to be the characteristic X-rays of the element composing the anode. These characteristic X-rays may be studied free from the "white" background by making use of the following phenomenon. If X-radiation of greater frequency than the characteristic X-rays is allowed to fall on an element, **secondary X-rays** are emitted from the element. On examination of these radiations with an X-ray spectrometer they are found to consist of the characteristic X-rays of the element. Moseley took a series of elements and excited their characteristic X-rays with high-frequency X-radiation. He demonstrated the remarkable facts: (1) that the **X-ray spectrum** of all elements consisted of a few lines only, in great contrast to the complicated visible spectrum; (2) that the appearance of the lines was similar for all elements, and that they occurred in groups termed the K-lines, L-lines, &c.; and (3) that the frequency of corresponding lines increased in a perfectly regular manner in the order of the elements in the periodic system.

From these facts it was deduced that some quantity associated with the atoms of the elements increased uniformly throughout the periodic system. In the light of this and further evidence it became clear that the quantity could only be the *atomic number* Z of the element, that is the number of the element in the periodic table. These results, together with Rutherford's experiments on the scattering of α-particles, led to the basic assumption of the modern electrical theory of matter, namely that the charge on the nucleus is simply equal to the electronic charge multiplied by the atomic number. To preserve neutrality of the element in its normal state it was inferred that the number of extra-nuclear electrons was equal to the atomic number. Now although $Z = 2$ for helium, since it is the second element in the periodic table, its mass is approximately 4 times that of hydrogen. It was therefore suggested that the helium nucleus consisted of four protons, but that since this would give $Z = 4$ it must also contain two electrons, thus giving a *net* positive charge $Z = 2$ to the nucleus. Incidentally the presence of the two negative electrons would explain the stability of the nucleus, for if it consisted of purely positive charges it would be expected to explode spontaneously owing to the mutual electrostatic repulsion of charges of one sign only. Very recent experiments have shown that the simple interpretation of a nucleus as consisting of protons and electrons is not tenable, and we discuss this further in section 14. For the moment, however, such a concept of nuclear constitution is satisfactory as giving an initial insight into nuclear structure.

Following his success with the theory of the hydrogen atom, Bohr suggested that for elements of higher atomic number all the electrons were arranged in orbits governed by quantum rules. To determine the arrangement of the electrons recourse was made to the periodic system. It was suggested that the regular behaviour of certain groups such as the rare gases, the alkali metals and so on, was due to the

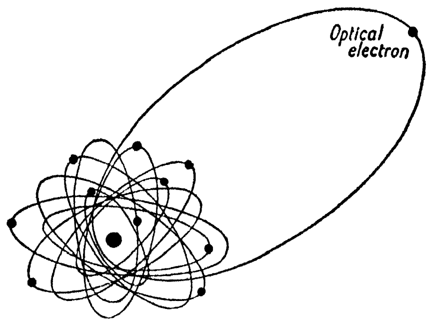

Fig. 3.—Electric Orbits in Sodium Atoms

similarity of the arrangement of the electron orbits for all elements in any one group. It was eventually shown that the first rare gas, helium, consisted of a doubly charged nucleus with $Z = 2$, surrounded by two electrons in equal orbits. The next element, lithium, has quite different chemical properties from helium, and it is therefore assumed that the third electron starts a new orbit outside the inner or **K-group.** As we proceed up the periodic system, additional electrons continue to occupy this second or **L-group** until eight more electrons have been added. We then arrive at the next rare gas neon with $Z = 10$, and since the properties of neon correspond to those of helium it is assumed that this second group is now complete. The next element, sodium, will then have the 11th electron commencing a fresh or **M-group,** and so on. Since this 11th electron is circulating in an orbit outside all the other electrons and the nucleus (as shown in fig. 3), the K- and L-groups and the nucleus constitute a sort of inner electric field in which the 11th electron moves. Now by Bohr's theory of the origin of spectra, in the case of hydrogen the spectrum originates when an electron switches from one possible allowed orbit to another which is vacant. It is clear that with sodium the only electron which can "jump" freely from one orbit to another is the outermost electron, for the K- and L-orbits are all occupied. Hence we should expect the optical spectrum of sodium and all alkali metals to be similar to the hydrogen spectrum, a prediction which agrees well with experiment.

We are now in a position to interpret the characteristic X-rays of the elements. Ordinary photoelectric action is due simply to the removal of the outermost electron by the light quantum, for the electrostatic force of attraction between the nucleus and the electrons in the inner levels is too great for them to be affected even by ultra-violet light. The frequency of X-rays, however, is much greater, and hence the energy $h\nu$ of the X-ray quantum is often sufficient to remove electrons from any of the inner groups. Consider now what will occur if a K-electron is removed by such high energy photoelectric action. The vacant K-ring will now be filled by one of the outer electrons jumping down into the vacated orbit. But by Bohr's theory this will result in the emission of radiation of energy equal to the energy difference in the two orbits. The frequency of this radiation is very high, since the switch occurs to an orbit much closer to the nucleus than those of the outer or "optical electron". In fact, the energy of the emitted radiation is that of the K-radiation of the characteristic X-rays. In general, the gap in the K-ring will be filled by an electron dropping from the L-ring, but it will sometimes happen that electrons from still higher orbits will take part in the transition. Hence the K-radiation consists of a group of lines of approximately the same frequency, the difference in frequency being equal to

$$\nu = (E_L - E_K)/h \quad \text{or} \quad (E_M - E_K)/h, \text{ \&c.},$$

according to the transition which occurs.

The gap created in the L-ring due to the switch to the vacant K-ring is in turn filled up by electron switches from still higher levels. These transitions give rise to the characteristic L-radiation, and so on. The great similarity in X-ray spectra of different elements shows conclusively that all elements are built up on the same structure, that is first a K-group, then an L-group, and so on.

Many physical properties and all the chemical properties of the elements are explicable in terms of the arrangement of the electrons in the extra-nuclear structure. For example, the inert chemical nature of the rare gases is due to the existence of the completed shell in these gases. Again, the alkali metals are monovalent because they have one less tightly bound electron operating outside the inner closed shell systems. The affinity of hydrogen for chlorine or sodium for chlorine lies in the tendency to form closed shells. Thus chlorine lacks one electron from its outermost shell to complete the number required for a rare gas grouping and therefore chemical inertness. This one electron is conspicuously present in hydrogen or the alkali metals, and hence chemical combination occurs with the additional electron entering the vacant space in the chlorine structure, giving the latter a structure like argon. Correspondingly the alkali metal by loss of an electron acquires an extra-nuclear structure like the rare gas which precedes it in the periodic table. Both elements are thereby chemically "satisfied".

8. Magnetism on the Electron Theory.

On Bohr's model of the atom, each circulating extra-nuclear electron will produce a magnetic field just as if it consisted of an electric current traversing an elliptical conducting circuit. The total magnetic field possessed by the atom will be the vector sum of the magnetic fields of each electronic orbit. Now in the rare gas atoms, the electronic orbits are highly symmetrical and no resultant magnetic field exists external to the atom. If we consider the effect of establishing a magnetic field in the neighbourhood of such a gas, we know by Lenz's law that in a conducting circuit induced currents would be set up so as to produce an oppositely directed field to the inducing field. The electronic orbits are precisely similar to closed electrical circuits, and hence the effect of establishing the magnetic field is to cause the electrons to accelerate or decelerate (according to the sense of rotation of the electron) so as to produce an oppositely directed field to the inducing field. A repulsion is therefore set up between the opposing fields and the material moves away from the applied magnet, that is it behaves as a *diamagnetic* substance. In so far as the atoms of all elements contain these electronic orbits, so all matter is diamagnetic in its behaviour. If the material behaves like a paramagnetic or ferromagnetic body, this is due to some other mechanism which more than counterbalances the diamagnetic repulsion which is always present. The change in velocity of the electrons when a magnetic field is present results in a change in the frequency of the spectral lines which are normally emitted by the element. This behaviour is known as the **Zeeman effect**; a similar phenomenon in the presence of a strong electric field is termed the **Stark effect.**

If the magnetic moments of the individual orbits present in an atom show a resultant, the atom will behave as a magnetic dipole and will respond positively to a magnetic field. The diamagnetic opposition due to the operation of Lenz's law on each orbit will still be present, but will be feebler than the reaction of the resultant magnetic dipole. Materials which behave in this way constitute the *paramagnetic* group.

Finally, the most strongly magnetic or *ferromagnetic* group are generally assumed to consist of larger groupings of paramagnetic atoms, each group constituting the elementary "molecular" magnet of section 5, Chap. VI.

9. Isotopes.

At each α-ray change in a radioactive series a mass 4 (in terms of hydrogen as unity) is lost, whereas each β-ray change results in no appreciable alteration in mass. Hence between the beginning and the end of a radioactive series, the difference in atomic weight between the initial and final elements is $4n$, where n is the number of α-ray changes. Now radium and thorium, of atomic weights 226 and 232, undergo five and six α-ray changes respectively, both elements eventually being transformed into lead. But the atomic weight of the lead in the former case is $226 - 5 \times 4 = 206$, while in the latter it is $232 - 6 \times 4 = 208$. We are therefore led to conclude that the same element, that is, one with the same position in the periodic table, can exist with two different atomic weights. Ordinary lead and all elements which have fractional atomic weights are in fact mixtures of whole number components or **isotopes.** Thus chlorine with a chemical atomic weight about 35·5 may be shown to consist of a mixture of isotopes of masses 35 and 37.

The existence of isotopes of non-radioactive elements is shown by **positive ray analysis,** that is by examination of the positive ray parabolæ described in Chap. XVIII. If the discharge takes place in chlorine, two parabolæ are formed, one corresponding to each isotope of mass 35 and 37.

10. Origin of α-, β- and γ-rays.

Since the whole of the mass of the atom resides in the nucleus, α-rays must originate from the nucleus of the atom. At first it appears possible for the β-particles to originate either in the extra-nuclear structure or to consist of nuclear electrons. Now the radioactive element which results from a β-ray transition in radioactive decay is found to occupy a position in the periodic table with Z one unit higher than the parent element. Hence the β-ray must have originated from the nucleus so as to increase the effective positive charge of the latter by one unit.

Finally, γ-radiation has an energy greater than that of any X-ray due to a transition of extra-nuclear electrons to the lowermost or K-ring, and hence these radiations also are of nuclear origin. The inability of change in physical condition to affect radioactive decay is therefore easily explained as due to the fact that external physical changes are of insignificant energy compared with that involved in the strong electric fields within the nucleus.

11. Artificial Disintegration.

By using the α-particles which have an energy comparable with that of the nuclear electric fields, the nuclei of stable elements like nitrogen may be disintegrated. If energetic α-particles are sent through nitrogen gas, hydrogen (which is made up of protons) is produced and, since the nitrogen has disintegrated, a new element is formed. The equation governing the reaction is

$$ {}_{14}^{7}\mathrm{N} + {}_{4}^{2}\mathrm{He} \rightarrow {}_{1}^{1}\mathrm{H} + {}_{17}^{8}\mathrm{O}, \quad . \quad . \quad . \quad . \quad (20.17) $$

where the top suffix gives the atomic number and the lower suffix the atomic weight of the element involved. We note that the α-particle amalgamates with the disintegrated nitrogen nucleus to give an element with $Z = 8$, which is therefore oxygen. The mass of this oxygen is, however, 17 instead of the customary 16 as found by a

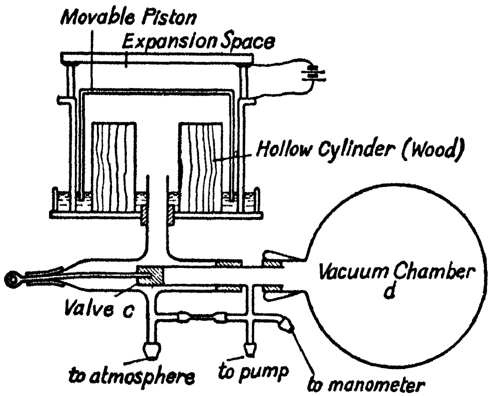

Fig. 4.—Wilson Cloud Chamber Diagram
The movable piston is suddenly lowered by opening the valve *c* and so connecting the vacuum chamber *d* with the part of the apparatus beneath the piston.

chemical determination of atomic weights. This ${}_{17}^{8}\mathrm{O}$ is a rare isotope of oxygen which has subsequently been shown to exist in a very small proportion in ordinary oxygen.

Such nuclear reactions are shown in a pictorial manner by the Wilson cloud chamber which is illustrated in fig. 4. This instrument consists essentially of a glass vessel filled with saturated vapour. Now electrical charges, like dust particles (see Part II, p. 80), have the property of causing condensation of saturated vapour. Consequently if an ionizing particle such as an α-particle traverses the vapour-laden atmosphere, it leaves in its wake a trail of ionization, that is a trail of positive and negative charges on which the vapour condenses as a line of drops. If this line of drops is strongly illuminated and photographed, a permanent record of **α-ray tracks** is obtained. Now should the vapour be present in nitrogen, about one track in a million shows the curious appearance as shown in fig 5. This shows the disintegration of the nitrogen nucleus. An α-particle has entered from below and pierced a nitrogen nucleus. The latter has then disintegrated into the long proton (hydrogen) track, the short recoil track being the relatively heavy nucleus of ${}^{17}_{8}O$. The whole process is subject to the same laws as the collision of billiard balls, momentum and energy being conserved in the process. Of course, in a disinte-

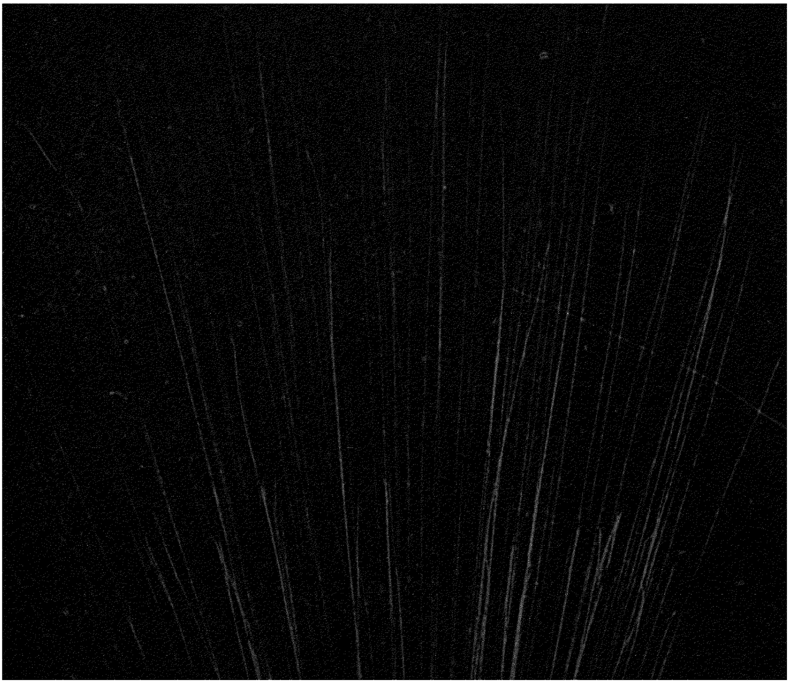

Fig 5 — Cloud-track photograph showing the transmutation of a nitrogen nucleus by the capture of an α-particle and emission of a proton (after Blackett)

gration *kinetic energy* is not conserved, as the potential energy of the various nuclei involved has to be considered.

Nuclear transmutations may be achieved by any atomic particles of sufficiently high energy, and besides α-particles modern technique uses positive ions as the bombarding particles. Various ingenious devices such as the *cyclotron* have been made to produce bombarding particles of the requisite energy. In this way all elements in the periodic table have been disintegrated, and a vast chemistry of nuclear transformations governed by equations of the type of (20.17) is now in existence. Apart from the instantaneous disintegrations such as occur when α-particles disintegrate nitrogen, "delayed action" disintegrations occur. In fact, a whole range of new radioactive elements such as radioactive chlorine, radioactive arsenic, &c., may be manufactured.

12. The Neutron.

If lithium is bombarded by high energy α-particles, radiation is emitted with a penetrating power millions of times that of the most penetrating γ-radiation known. By ingenious experiments these rays have been shown to owe their high penetrating power *not* to their enormous energy as was first thought, but to the fact that they interact little with matter and therefore do not readily lose their energy to anything else. It has been shown that these radiations are *particles* of mass equal to that of the proton but possessing no electric charge at all. They are termed **neutrons** and have been shown by disintegration experiments to be constituents of all nuclei.

13. Positive Electrons.

If the radiations emitted by *artificially prepared radioactive substances* are examined it is found that some elements emit β-particles with a positive charge instead of a negative charge. These positive electrons have values of e and m identical with those of ordinary electrons. They are, however, very short-lived, as they easily undergo annihilation by combination with the negative extra-nuclear electrons present in all matter. The mass and energy of the two electrons are not destroyed, for the process of annihilation results in the creation of γ-radiation of such energy that energy is conserved.

Positive electrons were first discovered by examination of **cosmic radiation.** The latter consists of a very penetrating ionizing radiation which originates in outer space and is continually bombarding this planet. By the application of the technique used for identifying and examining radioactive radiations, these cosmic radiations have been shown to consist very largely of extremely high energy positive and negative electrons present in about equal quantity.

14. Nuclear Structure.

Since matter can be made to emit both neutrons and positive electrons as a result of bombardment by high-speed particles, the model of the atom as composed entirely of protons and electrons is clearly inadequate. The model of the atom at present accepted is as follows. The extra-nuclear structure consists entirely of negative electrons as on the original Rutherford-Bohr model. All chemical and many physical changes are due to interactions of the extra-nuclear electrons, and the nucleus is not involved except in so far as it provides a central electric field in which the extra-nuclear electrons carry out their transitions.

The nucleus is now considered to consist of Z protons together with $(A - Z)$ neutrons, where Z is the atomic number and A is the atomic weight. No electrons are considered to be present in the nucleus, but the positive or negative β-particles emitted in the corresponding radioactive changes are considered to be created by the spontaneous transition of a proton into a neutron or a neutron into a proton, thus:

$$p \rightarrow n + e^+,$$
$$n \rightarrow p + e^-.$$

We are finally left with the problem as to why the nucleus holds together if no electron "cement" is present to counterbalance the mutual repulsion of the positively charged protons. It has become essential to assume that the neutrons and protons attract each other, and since the former are neutral the introduction of non-electric forces in the nucleus would seem inevitable. In this way the electrical theory of matter has by its very successes taken us so far that we now require a new theory of matter, in which non-electric as well as electric forces seem destined to play a part.

EXERCISES

1. By what simple experiment may it be shown that the radiations emitted by radioactive substances are of three distinct types? Give a fuller account of *one* of these radiations.

2. What do you understand by the transformation theory of radioactive decay? Given that the rate of decay is proportional to the quantity of radioactive material present, show that the decay follows an exponential law.

3. State briefly how the nature of α-radiation was determined, and give some account of what you consider to be the most important experiment ever performed with these rays as the main agent.

4. What are the properties of β- and γ-rays? Give reasons for believing that these radiations arise from the nucleus and not from the extra-nuclear structure.

5. Give Bohr's theory of the origin of the spectrum of atomic hydrogen. What effect is produced by the fact that the nucleus is not absolutely fixed but that, actually, nucleus and extra-nuclear electron both revolve about the common centre of gravity of the system?

6. Distinguish between continuous X-radiation and characteristic X-rays. Explain the origin of the latter on the Rutherford-Bohr atomic model.

7. How is magnetism explained on the electron theory, and by what means may diamagnetic susceptibilities be measured experimentally?

8. Write a short essay on isotopes.

9. How may one element be transmuted into another? What elements are formed when nitrogen is bombarded with high energy α-particles?

10. Write short notes on the following: neutrons, positive electrons, cosmic radiation. What is your conception of the atomic nucleus, and how do you account for its stability?

EXAMPLES

1. A bar magnet of equivalent length 10 cm. is lying in the magnetic meridian with its N pole pointing N. If the horizontal component of the earth's magnetic field is 0·18 gauss and is just balanced at a point on the perpendicular bisector of the axis of the magnet, find the magnetic moment of the magnet, given that the distance of the neutral point from the centre of the magnet is 12 cm.

2. Two magnets are lying in equilibrium in the same straight line, the distance between their centres being 100 cm. Find the force between them if they have equivalent lengths 20 cm. and 10 cm. and pole strengths 50 units and 25 units respectively.

3. A ball-ended magnet of mass 100 gm. is suspended vertically about a horizontal axis passing through its N pole. A second long ball-ended magnet is then slowly brought up horizontally with its S pole pointing towards the lower end of the suspended magnet until the S pole of the second magnet is vertically below the axis of suspension of the first magnet and 20 cm. from it. If the S poles are in the same horizontal line and the pole strength of each magnet is 200 units, find the maximum angle at which the suspended magnet is inclined to the vertical when in equilibrium.

4. A flat strip of wood capable of being rotated in a horizontal plane about a vertical axis carries a bar magnet in the broadside position. A compass needle is placed immediately over the vertical axis. Show that the needle points along the wooden strip when the latter is inclined to the magnetic meridian at an angle θ given by $\sin\theta = M/HL^3$, where M is the magnetic moment of the bar magnet, L the distance of either pole of the magnet from the vertical axis, and H the strength of the horizontal component of the earth's magnetic field.

5. In Ques. 4, the bar magnet is placed in the end-on position with its centre 100 cm. from the vertical axis. It is then found that there are two positions of the wooden strip in which the needle is perpendicular to the axis of the magnet, the angle between these two positions being 60°. Find the moment of the magnet, given that the earth's horizontal magnetic component is 0·18 gauss.

6. A vibration magnetometer when placed at a point on the

axis of a bar magnet lying in the magnetic meridian has a time of oscillation t_1, and when placed an equal distance along the axis on the opposite side of the magnet has a time of oscillation t_2. Prove that the time of oscillation t_3 in the earth's field alone is given by $t_3^2(t_1^2 - t_2^2) = 2t_1^2t_2^2$.

7. A dip circle lying initially in the magnetic meridian is rotated through an angle α in a horizontal plane. Show that the tangent of the angle of dip is increased in the ratio $\sec\alpha : 1$.

8. A bar magnet is brought up in the end-on position to a point due E of a vibrating magnetometer which is thereby deflected through an angle θ. Show that the time of oscillation of the magnetometer is changed in the ratio $(\cos\theta)^{\frac{1}{2}} : 1$.

9. Regarding the earth's magnetic field as equivalent to that produced by a small magnet of moment μ situated at the centre of the earth, show that if λ is the magnetic latitude at any point, the angle of dip D is given by $\tan D = 2\tan\lambda$ and that the horizontal component H is given by $H = \cos\lambda/R^3$, where R is the radius of the earth.

10. Two small magnets of moments M and M' are so placed that their axes make angles θ and θ' with the line joining their centres. Show that the potential energy of one magnet in the field of the other is

$$W = MM' (\sin\theta \sin\theta' - 2\cos\theta\cos\theta')/d^3,$$

where d is the distance between the centres of the magnets.

11. In Ques. 10, show that the translational force between the magnets along the line of centres is

$$F = 3MM' (\sin\theta \sin\theta' - 2\cos\theta\cos\theta')/d^4.$$

12. In Ques. 10, show that the transverse force acting on either magnet in a direction perpendicular to the line of centres is

$$T = -3MM' \sin(\theta + \theta')/d^4.$$

13. In Ques. 10, show that the rotational couple acting on the second magnet is given by

$$G = MM'(2\cos\theta\cos\theta' - \sin\theta\sin\theta')/d^3.$$

14. A rectangular bar magnet of weight 200 gm., equivalent length 20 cm., and breadth 4 cm., gives a deflection of 45° in the end-on position when the centre of the magnet is 50 cm. from the needle of a deflection magnetometer. When the magnet is freely suspended and allowed to oscillate in the earth's field, it makes 10 oscillations in 100 sec. Find the strength of the earth's horizontal field and the moment of the magnet, neglecting the difference between actual and equivalent length of the magnet.

15. Two spherical pith-balls, each of diameter 0·5 cm. and weight 0·1 gm., are suspended from the same point by strings 13 cm. long. When charged with equal charges of electricity, the balls repel each other to a distance of 10 cm. Find the charge and potential of each ball.

16. Find the position of the neutral points in the neighbourhood of two equal small spheres carrying charges +10 units and −5 units and situated a distance 50 cm. apart, before and after the spheres have been connected by a wire.

17. Point charges $+q$ are placed at the corners of an equilateral triangle of side 3 cm. Find what charge will hold the three charges in equilibrium and where it must be placed.

18. If the three charges of Ques. 17 are held rigidly at the corners of the triangle, find the work done in taking the fourth charge from the centroid of the triangle to the mid-point of one of the sides.

19. Find the ratio of the electrostatic unit of charge on the lb. ft. sec. and C.G.S. systems respectively.

20. Two spherical condensers A and B carry charges of +10 and +15 units respectively. If the internal radii of A and B are 25 cm. and 35 cm. and their external radii are 30 cm. and 40 cm. respectively, find the loss in energy of the system when the condensers are joined by a wire. Find also the final charges on A and B.

21. The capacities of the systems formed by connecting a condenser (with a capacity equivalent to that of an insulated sphere of radius a) and an air-filled parallel plate condenser of plate separation d, in parallel and series respectively, are in the ratio $(a + d)^2 : ad$. Show that there are two possible values for the area of the parallel plate condenser and that these are in the ratio d^2/a^2.

22. Find the maximum value of the surface density of electrification of a conductor in air if the latter breaks down under a tension of more than 700 dynes/sq. cm.

23. Referring to Ques. 22, determine approximately the rise in level of the water of a lake immediately below a thunder-cloud just before the lightning strikes.

24. A piece of metal foil 1 sq. cm. in area and of weight 10^{-4} gm. lies on a flat metal plate of area 10^4 sq. cm. Find approximately the charge which must be communicated to the metal plate so as to lift the piece of foil.

25. Three thin flat parallel metal plates A, B and C form a double condenser, the inner plate B having a surface density of charge σ and the two outer plates A and C being earthed. Find the potential of B if A and C are at distances a and b respectively from B.

26. Two parallel metal plates A and B are situated a distance d apart; A is insulated and B is earthed. Show that the capacity per unit area of the system changes by an amount $t/4\pi d(d-t)$ if an insulated metal plate of thickness t is placed in between and parallel to A and B.

27. A hollow metal sphere of mass 2 gm. and radius 5 cm. rests on an insulating stand and is cut in half in a horizontal plane. It is then given an electric charge of 1000 e.s.u. Determine whether the top hemisphere will be lifted from the lower hemisphere.

28. In Ques. 27, show, in a general case, that if the two hemispheres (of radius a) are earthed, and a charge q is placed at the centre, forces $q^2/8a^2$ are required to separate them.

29. Three concentric spherical conductors of radii a, b and c having $c > b > a$ are given charges q_1, q_2 and q_3. If the innermost sphere is earthed, show that the loss in potential energy of the system is $a(q_1/a + q_2/b + q_3/c)^2/2$.

30. A conductor is charged by repeated contact with the brass disk of an electrophorus which supplies a constant amount of electricity Q to the brass disk. If the conductor acquires a charge q after the first contact, show that the limiting amount of charge which may be communicated to the conductor in this way is $Qq/(Q-q)$.

31. A gold-leaf electroscope is charged with electricity and reads 20 scale divisions. It is then connected to a parallel plate condenser of area 30 sq. cm. and plate separation 0·5 cm., whereupon the electroscope reading falls to 12 scale divisions. Find the capacity of the electroscope.

32. A small particle has an electric charge of $4{\cdot}800 \times 10^{-10}$ e.s.u. Find the mass of the particle if it is suspended in equilibrium between two flat condenser plates 2 cm. apart, charged to a potential difference of 3000 volts.

33. Find the additional weight required to balance an absolute electrometer if the area of the moving plate is 30 sq. cm. and its distance from the lower plate is 0·2 cm., the difference of potential between the two plates being 1000 volts.

34. If the electrometer of Ques. 33 is immersed in oil of dielectric constant 3·5, find the change in balance weight required.

35. The lower half of a spherical condenser is filled with a non-conductor of dielectric constant k. Show that the new capacity of the condenser is the same as if the whole space were filled with material of dielectric constant $(1+k)/2$.

36. A charged parallel plate condenser containing a slab of material of thickness a and dielectric constant k is connected to an electro-

scope whose capacity may be neglected. On removing the slab it is found necessary to move the plates of the condenser closer together by an amount b if the deflection of the electroscope is to remain unaltered. Show that $k = a/(a - b)$.

37. Find the capacity per unit length of a submarine cable from the following data: radius of core = 0·5 cm., outer radius = 1·0 cm., specific inductive capacity of insulating layer = 3·2.

38. A quadrant electrometer reads a deflection of 20 scale divisions when the paddle has been raised to a potential of 1000 volts, the pairs of quadrants having potentials of 10 and zero volts respectively. Find the mean potential of an A.C. supply which registers a deflection of 100 scale divisions when connected idiostatically to the electrometer.

39. What is the least horse-power required to drive a Wimshurst machine which is to deliver one milliampere at a pressure of 5000 volts?

40. Two flat condenser plates are at a potential difference of 3000 volts and are situated in a vacuum. If a small particle (an electron) of mass 9×10^{-28} gm. and carrying a charge of $4{\cdot}800 \times 10^{-10}$ e.s.u. starts out with negligible velocity from the negative plate, find the velocity with which it will strike the other plate.

41. The insulated plate of a parallel plate condenser of area A has a surface density of charge σ. Calculate the work done if the plates move together a distance d owing to the mutual force of attraction.

42. In Ques. 41, if the insulated plate is connected to a battery which supplies a constant potential V, given that the initial plate separation is t, find the energy supplied by the battery.

43. In Ques. 42, find the external work done during the movement.

44. Show that if any two condensers of capacities C_1 and C_2 and carrying charges q_1 and q_2 are connected in parallel by a wire, the system experiences a loss in energy given by

$$(C_2q_1 - C_1q_2)^2/2C_1C_2(C_1 + C_2).$$

45. Find the horse-power required to run a 100 candle-power lamp of 2 c.p. per watt, and the current in the circuit when on a 220-volt circuit.

46. For a given resistance box the heating effect must not exceed 10^{-3} watts per ohm. Find the maximum voltages that may be applied across the box when resistances of 10^4 ohms and 1 ohm are being used respectively.

47. A circuit is to be supplied with a direct current of 200 amp. at a potential difference of 200 volts. Calculate the E.M.F. at the terminals of the dynamo if the leads to the circuit are half a mile long, 1 sq. in. in cross-section, and of copper of specific resistance

$1{\cdot}6 \times 10^{-6}$ ohm. cm. Find also the percentage of power wasted in the leads.

48. Current is supplied to 100 filament lamps arranged in parallel, the resistance of each lamp being 250 ohms. When 50 more lamps are switched into the circuit, the voltage across the lamp terminals drops from 224 to 218. Neglecting the resistance of the leads, find the internal resistance of the supply batteries.

49. In finding the position of a fault in a transmission line AB 100 miles long, when A is maintained at a potential of 200 volts and B is insulated, the latter is found to be at a potential of 50 volts. Conversely, when A is insulated, A is found to be at a potential of 50 volts when B is maintained at a potential of 230 volts. Determine the position of the fault.

50. Show that the maximum current is obtained in an external circuit when its resistance is equal to that of the total effective internal resistance of the supply batteries.

51. Two batteries of E.M.F.s 2 volts and 1·5 volt and of internal resistance 2 ohms and 1 ohm respectively are found to give the same current when connected by a certain wire. Find the resistance of the wire and the current flowing.

52. The potential difference between the poles of a given battery is 1·2 volt when the poles are joined by a wire of resistance 2 ohms, and 1·0 volt when the wire has resistance 1 ohm. Find the internal resistance of the battery and its E.M.F. on open circuit.

53. ABCD is a quadrilateral of which the arms have resistance AB = 1 ohm, BC = 2 ohms, CD = 3 ohms, and DA = 4 ohms. A galvanometer of resistance 5 ohms is placed across BD. If the current of 1 amp. is passed in at A and leaves at C, calculate the current in the galvanometer.

54. The positive poles of two cells are joined by a uniform wire of resistance 6 ohms, and the negative poles by a uniform wire of resistance 10 ohms. The middle of the 10-ohm wire is earthed: the potential of the centre of the 6-ohm wire is then found to be 16/9 volts. If the E.M.F. of one cell is 3/2 volts and its internal resistance is 2 ohms, find the internal resistance of the other cell, given that its E.M.F. is 2 volts.

55. An equilateral triangle ABC has a 2-volt cell joined to the points A and B. The wire of which the triangle is constructed is of different cross-sections, so that these are in the ratio 1 : 2 : 3 for the sides, AB, BC and CA. The internal resistance of the cell which is equal to that of the side of least resistance is required. The current in AB is given to be 5/17 amp.

56. The insulation resistance of a cable between two points P and R is 20,000 ohms. If the insulation resistance between R and an intermediate point Q is 30,000 ohms, find the insulation resistance between P and Q.

57. A uniform circular wire is joined across a diameter by another wire of the same material and cross-section. Show that the equivalent resistance of the combination for a current which enters the circuit at one of the junctions of diameter and circumference and leaves at a point on the circumference midway between the ends of the diameter is given by $\pi R(6 + \pi)/4(4 + \pi)$, where R is the resistance of a length of the wire equal to a radius of the circle.

58. A regular hexagon is formed by six wires each of resistance r, and the corners are joined to the centre by wires of the same resistance. If the current enters at one corner and leaves at the opposite corner, find the equivalent resistance of the combination.

59. In Ques. 58, find the equivalent resistance if the current leaves at a corner adjacent to that at which it enters.

60. Find the equivalent resistance of a cube composed of 12 equal wires of resistance r for a current which enters at one corner and leaves at the opposite corner.

61. In Ques. 60, find the equivalent resistance if the current enters and leaves at the ends of one edge.

62. A circuit is composed of two uniform wires of resistances πr and $2r$, joined together in the form of a semicircle and its diameter. A similar third wire of resistance $2r$ is joined to one point of contact of diameter and semicircle, while the other end may make a sliding contact at any point between the far end of the diameter and halfway round the semicircle. If a current enters at the common junction of the three wires and leaves at the point of contact with the semicircle, show that the maximum resistance of the network is $2r(\pi + 2)/(\pi + 10)$.

63. In Ques. 62, find the minimum resistance.

64. Determine the value of the resistance of a shunt which must be placed in parallel with an ammeter whose coil has a resistance of 80 ohms, if the ammeter, which is to read up to 5 amp., shows a full-scale deflection under a potential difference of 100 millivolts.

65. When a given cell has a resistance of 10 ohms placed across its terminals, it is found that its E.M.F. is balanced by 120 cm. of a given potentiometer wire, whereas on open circuit the balance length is 150 cm. Find the internal resistance of the cell.

66. A vibration magnetometer makes 10 vibrations in 25 sec. when placed at a point 10 cm. due E of a long vertical wire carrying

a current, and 20 vibrations in 60 sec. when placed the same distance due W of the wire. Find the position of a neutral point in the magnetic field.

67. A small magnet is freely suspended at the centre of a metal ring of diameter 20 cm., the axis of the ring being in the magnetic meridian. If the magnet makes 10 vibrations in 30 sec. when under the influence of the earth's horizontal magnetic field alone, find the current in the ring when 20 vibrations are executed in 30 sec., the magnet still pointing in the same direction. $H = 0{\cdot}18$ gauss.

68. Given the same data for the times of oscillation as in Ques. 67, find the current in the ring if the plane of the ring is vertical but inclined at 30° to the magnetic meridian.

69. In Ques. 67, find the current in the ring for the same data but with the magnet situated at a point on the axis of the ring 10 cm. from the centre.

70. Two equal circular coils of radius a are mounted coaxially a distance x apart. Given that for any separation of the coils the magnetic field gradient due to a common similarly directed current in the two coils is least at a point on the axis midway between the two coils, find the separation x for which the magnetic field gradient is least.

71. If two equal circular coils of 10 turns and radius 20 cm. are arranged as in Ques. 70, find the strength of the common current necessary to neutralize the horizontal component of the earth's magnetic field of 0·18 gauss at a point on the axis midway between the two coils.

72. A cell of internal resistance 1 ohm, when used in conjunction with a given tangent galvanometer, may be connected either to a single turn of resistance 1 ohm or to a coil of the same radius but having 50 turns and of resistance 99 ohms. With which coil will the greater deflection be obtained?

73. The deflections produced when (*a*) one cell, (*b*) two such cells in parallel, (*c*) two such cells in series are connected to a given tangent galvanometer are θ_1, θ_2 and θ_3. Show that $2\tan\theta_1(\tan\theta_3 + \tan\theta_2) = 3\tan\theta_2\tan\theta_3$.

74. Find the maximum current which may be measured with a sine galvanometer which has a coil of 50 turns of radius 10 cm. at a place where the earth's magnetic field is 0·18 gauss.

75. Determine the strength of the current which must flow through a narrow solenoid of 1000 turns and of length 1 m., lying in the magnetic meridian, if the earth's magnetic field of 0·18 gauss is to be neutralized at the centre.

76. Find the strength of the current flowing in a solenoid of 10

turns per cm., length 10 cm. and radius 1 cm., if a compass needle is deflected through 45° when the solenoid is placed in the end-on position due E of the needle and with its centre 6 cm. from the needle. The earth's horizontal component may be taken as 0·18 gauss.

77. Calculate the reading which would be shown by a moving-coil galvanometer having a permanent radial magnetic field of 200 gauss, when a current of 1 milliamp. flows in the square suspended coil of 25 turns and of side 2 cm. The instrument has a sensitivity of one division per dyne-cm. of couple.

78. Find the deflection shown by a tangent galvanometer of 50 turns and radius 10 cm., at a point where the earth's horizontal magnetic component is 0·18 gauss, if the galvanometer is placed in a circuit containing also a copper voltameter in which the current deposits 1 dgm. of copper in 30 min. The atomic weight of silver is 107·9, that of copper is 63·6, and the electrochemical equivalent of silver is 0·001118 gm./coulomb.

79. With a given hydrogen voltameter, 214 c.c. of hydrogen are collected in 1 hour at a temperature of 15° C. and a total pressure of 783 mm. of mercury. Find the current flowing, given that the electrochemical equivalent of hydrogen is $1{\cdot}04 \times 10^{-5}$ gm./coulomb, and that its density at N.T.P. is 0·09 gm./litre.

80. If 3800 calories are liberated when 1 gm. of water is formed by the combustion of hydrogen and oxygen, and $9{\cdot}4 \times 10^{-5}$ gm. of water is decomposed during the passage of 1 coulomb through acidulated water, show that the minimum E.M.F. required for the electrolysis of water is about 1·5 volt.

81. Assuming that the increase of resistance with temperature is linear and of amount 5×10^{-3} ohms per °C. for tungsten, find the temperature of the incandescent filament of a tungsten lamp from the following data: resistance of lamp at 0° C. as found by a Wheatstone's bridge is 40 ohms; potential difference between terminals of lamp when incandescent is 200 volts, the current flowing being 0·5 amp.

82. A vertical silica tube of internal area of cross-section 1 sq. cm. contains a 25 per cent solution of sodium chloride in water, the length of the column of fluid at 15° C. being 100·00 cm. One electrode is let into the bottom of the tube and the other electrode floats on the surface of the liquid. Find the resistance of the liquid column at 65° C., given that the coefficient of thermal expansion of the liquid is $4{\cdot}36 \times 10^{-4}$ per °C., the conductivity of the solution at 15° C. is 0·1642 $\text{ohm}^{-1}.\text{cm.}^{-1}$, and the temperature coefficient of conductivity is $3{\cdot}5 \times 10^{-3}$ per °C.

83. Two straight cables carrying current to and from a circuit lie one on top of the other. If the radius of the cables is 5 cm. and their

weight is 10 Kgm. per metre, find the strength of the current for which one cable will just be lifted from the other.

84. If a couple of 10^5 dyne-cm. is required to hold a circular coil of 20 turns and diameter 20 cm. with its plane parallel to that of a magnetic field of strength 50 gauss, find the current in the coil.

85. Find the strength of the magnetic field in a small gap in an iron ring magnet whose intensity of magnetization is 80 C.G.S. units.

86. A short soft iron bar of susceptibility 80 lies horizontally in the direction of the earth's horizontal magnetic component of strength 0·18 gauss. Find the volume of the bar, given that a vibration magnetometer placed at a point on the perpendicular bisector of the axis of the bar and 30 cm. due E of it executes 10 vibrations a minute when the bar is present and 20 vibrations a minute when it is absent.

87. The soft iron bar of Ques. 86 is cut into two equal parts in a direction perpendicular to the axis of the bar. Find the stress now required to separate the two halves of the bar.

88. A ring electromagnet has a mean radius of 25 cm. and is to be excited by a current of 10 amp. to produce a magnetic field of 2500 gauss in a gap in the ring 0·5 cm. wide. If the permeability of the material of the ring is 800, what must be the number of turns of wire on the ring?

89. An electromagnet consists of a semicircular ring of iron of permeability 1000, wound with 600 turns of wire. The radius of the ring is 20 cm. On the ends of the semicircular ring slide two horizontal bars of the same area of cross-section as the ring but of magnetic permeability 800. If the two poles are slid towards each other until they are 0·5 cm. apart, determine the strength of the magnetic field in the gap when the exciting current is 10 amp.

90. A straight wire of length 80 cm. is lying horizontally E and W. Taking $H = 0{\cdot}18$ gauss, find the E.M.F. acting down the wire if it is dropped (*a*) after falling 50 m., (*b*) after it has been falling for 10 sec.

91. Calculate the potential difference between the hub and the rim of the wheel of a locomotive which is running in a direction due magnetic E. The diameter of the wheel is 4 ft., $H = 0{\cdot}18$ gauss, and the speed of the train is 60 m.p.h.

92. In Ques. 91, if the angle of dip is 65°, what is the potential difference acting between the ends of the axle of the wheels? Length of axle is 4 ft. 6 in.

93. A disk of radius 8 cm. is rotated inside a long solenoid of 50 turns per cm., the axis of rotation of the disk coinciding with the axis of the solenoid. If the disk makes 600 rev./min. and the current in the

solenoid is 1 amp., find the potential difference between the centre and the circumference of the disk.

94. An armature conductor of length 1 m. is mounted 1 m. from the axis of a shaft rotating at a speed of 1000 rev./min. Find the E.M.F. in the armature when it passes through the maximum field of 5000 gauss.

95. A solenoid of area of cross-section 2 sq. cm. has 50 windings per cm. and carries a current of 2 amp. Find the E.M.F. induced in 10 turns of wire wound round the centre of the solenoid if the current in the latter is cut off and falls to zero in 10^{-3} sec.

96. The windings of an electromagnet have a resistance of 5 ohms, and the self-inductance associated with the magnet is 4 henries. After how long will the current in the circuit rise to one-half its final steady value when the circuit is completed?

97. A long thin ball-ended magnet of very low susceptibility and of pole strength 60 units is balanced on a knife-edge. One pole is inserted into a small gap between the flat pole pieces of an electromagnet having an intensity of magnetization of 900 units. What weight must be added to the other pole of the balanced magnet if it is to remain horizontal after the magnet has been switched on?

98. Prove that for a series-wound electric motor the speed for maximum output of energy is about one-half that at which the back E.M.F. would equal the applied E.M.F.

99. A flat circular coil fitted with a commutator is rotated at a speed of 10 rev. per sec. in the horizontal component of the earth's magnetic field, and then at a speed of 5 rev. per sec. in the vertical component of the field. Determine the angle of dip if the steady deflections registered by a tangent galvanometer connected to the coil are 30° and 38° respectively.

100. Assuming that the E.M.F. of a thermo-couple is given by the relation $E(\text{micro-volts}) = a + bt + ct^2$, where t is the temperature difference in °C. between the junctions and a, b and c are constants, show from the data given below that a platinum-rhodium platinum-iridium couple has a nearly linear change of E.M.F. with temperature, and calculate the value of this E.M.F. when the temperature difference between the junctions is 50° C.

For a Pt, Pt-Rh couple $a = -307$, $b = 8{\cdot}1$, $c = 0{\cdot}0017$.

For a Pt, Pt-Ir couple $a = -550$, $b = 14{\cdot}8$, $c = 0{\cdot}0016$.

ANSWERS AND HINTS FOR SOLUTION

1. Resultant force due to magnet is $M/(d^2 + l^2)^{3/2}$, where M is moment of magnet, l is half equivalent length, and d is distance of neutral point from centre of magnet. Hence $M = 0{\cdot}18(144 + 25)^{3/2} = 395{\cdot}5$ gauss . cm.^{3}.

2. $1{\cdot}571 \times 10^{-2}$ dynes.

3. Taking moments about the axis $F \times 20 = 100 \times g \times d/2$: also $F = (200/d)^2$; if $g = 981$ cm./sec.^{2}, $d = 2{\cdot}54$ cm.; 7° 14′.

4. Equating forces perpendicular to the strip $F = H \sin\theta$: also for the broadside position $F = M/D^3$.

5. If θ is the angle between the magnetic meridian and the axis of the magnet in one of the positions, since there is to be no force parallel to the axis, $F = H\cos\theta = H\cos(60° - \theta)$. Hence $\theta = 30°$; also $F = \dfrac{2M}{100^3}$; $4{\cdot}5\sqrt{3} \times 10^4$ gauss cm.^{3}.

6. Apply $t = 2\pi\left(\dfrac{I}{MH}\right)^{\frac{1}{2}}$, making H equal to $H \pm F$ in the first two cases.

7. Angle of dip D is given by $\tan D = V/H$, where V and H are vertical and horizontal components of earth's field. If dip-circle is at α to magnetic meridian, effective H is $H\cos\alpha$.

8. Field due to magnet is $H\tan\theta$; total field is $H\sec\theta$; apply

$$t = 2\pi\left(\frac{I}{MH}\right)^{\frac{1}{2}}.$$

9. Defining magnetic potential as the work done in bringing a unit magnetic pole from an infinite distance up to the required point, the magnetic potential at a point a distance r from the centre of a magnet, if θ is the angle between the line r and the axis of the magnet (fig. 1), is the integral of force × distance,

$$= W = \int_\infty^r \left\{\frac{m\cos\alpha}{(r - l\cos\theta)^2} - \frac{m\cos\beta}{(r + l\cos\theta)^2}\right\}(-dr)$$

$$= -\int_\infty^r \frac{2\mu\cos\theta}{r^3}\,dr = \frac{\mu\cos\theta}{r^2},$$

where $\cos\alpha = \cos\beta = 1$ approx., m is the pole strength of the magnet, and l is very small compared with r. Referring to the earth's field,

$\theta = (90 - \lambda)$, the vertical component V is given by $V = -\frac{\delta W}{\delta r} = \frac{2\mu \cos\theta}{r^3}$, the horizontal component H by $H = -\frac{1}{r}\frac{\delta W}{\delta \theta} = \frac{\mu \sin\theta}{r^3}$. Hence $\tan D = V/H = 2\tan\lambda$ and $H = \frac{\mu \cos\lambda}{R^3}$.

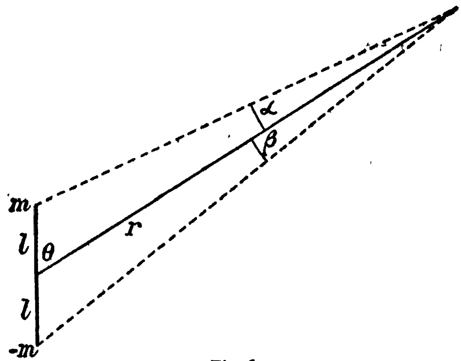

Fig. 1

10. Referring to fig. 2 and using the result of Ques. 9, $W = \frac{\mu \cos\theta}{r^2}$, the potential of the upper pole (strength m') of magnet 2 is $\frac{Mm'}{a^2} \cdot \cos(\theta - \alpha)$, and that of the lower pole is $\frac{Mm'}{b^2}\cos(\theta + \alpha')$. Hence the total magnetic potential of 2 in the field of 1 is

$$\frac{Mm'\cos(\theta - \alpha)}{(d + l'\cos\theta')^2} - \frac{Mm'\cos(\theta + \alpha')}{(d - l'\cos\theta')^2}$$

$$= \frac{Mm'}{d^4}\{(\cos\theta + \alpha\sin\theta)(d - l'\cos\theta')^2 - (\cos\theta - \alpha'\sin\theta)(d + l'\cos\theta')^2\},$$

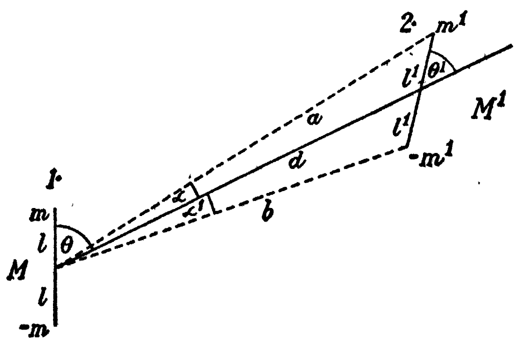

Fig. 2

where $l' \cos\theta' \ll d$. Put $\alpha = \alpha' = \dfrac{l' \sin\theta'}{d}$ approx., then

$$W = \frac{2Mm'l'}{d^3}\{\sin\theta\sin\theta' - 2\cos\theta\cos\theta'\}$$
$$= \frac{MM'}{d^3}(\sin\theta\sin\theta' - 2\cos\theta\cos\theta').$$

11. As in Ques. 9 for the radial force, $F = \dfrac{\delta W}{\delta r}$, hence

$$F = \frac{3MM'}{d^4}(\sin\theta\sin\theta' - 2\cos\theta\cos\theta').$$

12. The transverse force is

$$T = -\frac{1}{r}\left(\frac{\delta W}{\delta\theta} + \frac{\delta W}{\delta\theta'}\right) = -\frac{1}{d}\frac{MM'}{d^3}(\cos\theta\sin\theta' + 2\sin\theta\cos\theta'$$
$$+ \sin\theta\cos\theta' + 2\cos\theta\sin\theta') = \frac{-3MM'}{d^4}\sin(\theta + \theta').$$

13. The couple G on the second magnet is given by $G = -\dfrac{\delta W}{\delta\theta'}$. Hence $G = \dfrac{MM'}{d^3}(2\cos\theta\sin\theta' - \sin\theta\cos\theta')$.

14. Apply $t = 2\pi\sqrt{\dfrac{I}{MH}}$ and $\dfrac{2Md}{(d^2 - l^2)^2} = H\tan\theta$; 0·218 gauss; 12,560 gauss cm.^{3}.

15. Considering the forces in equilibrium for one ball and taking moments about the point of suspension, $12F = 0{\cdot}1 \times 981 \times 5$ dynes; also $F = q^2/10^2$; $V = q/0{\cdot}25$; 64 e.s.u., 256 e.s.u. of potential.

16. $\dfrac{10}{(50 + x)^2} = \dfrac{5}{x^2}$; 120·7 cm.; at mid-point between spheres.

17. $-q/\sqrt{3}$ at the centroid.

18. Work done on unit charge is difference in potential. Potential difference is $\dfrac{q}{1{\cdot}5} + \dfrac{q}{1{\cdot}5} + \dfrac{q}{3}\dfrac{2}{\sqrt{3}} - \dfrac{3q}{\sqrt{3}} = \dfrac{q}{9}(12 - 7\sqrt{3})$; $\dfrac{q^2}{9}(4\sqrt{3} - 7)$.

19. Unit of charge defined from $q^2/d^2 = F$; hence the unit on the lb.-ft. sec. system is greater than that on the C.G.S. system in the ratio

$$Q/q = d\sqrt{F} = 12 \times 2{\cdot}54\sqrt{\frac{1000}{2{\cdot}2} \times 12 \times 2{\cdot}54} = 3588 : 1.$$

20. Capacity of spherical condenser $\dfrac{ab}{b - a}$; hence capacity of $A = 150$ cm. and of $B = 280$ cm. Total capacity on joining in parallel is 430 cm. Energy of charged condenser is $\frac{1}{2}Q^2/C$; hence total initial energy of system is $\dfrac{1}{2}\cdot\dfrac{100}{150} + \dfrac{1}{2}\cdot\dfrac{225}{280}$. Total final energy is $\dfrac{1}{2}\cdot\dfrac{625}{430}$, so loss in energy is $\dfrac{121}{14448}$ ergs. Final potential is $V = Q/C = \dfrac{25}{430}$, hence charge on A is $\dfrac{25}{430} \times 150 = \dfrac{375}{43}$, and on B is $\dfrac{25}{430} \times 280 = \dfrac{700}{43}$.

21. Let C_1 and C_2 be the capacities of the two condensers, where $C_1 = a$ and $C_2 = A/4\pi d$. Capacity in parallel is $C_1 + C_2$, in series is $C_1C_2/(C_1 + C_2)$; hence ratio is $(C_1 + C_2)^2/C_1C_2 = (a + d)^2/ad$. Solving, $C_2 = d$ or a^2/d.

22. Consider unit area of conductor with surface density of charge σ; air is charged by contact. Since air is very close to surface, latter behaves as an infinite plane, and force of repulsion of air will be $F = 2\pi\sigma \,.\, \sigma$ per unit area : $(350/\pi)^{\frac{1}{2}}$ e.s.u.

23. Let water rise d cm. above the general level; then electrostatic force of attraction equals hydrostatic pull downwards; $2\pi \,.\, 350/\pi = d \,.\, 981$; 700/981 cm.

24. Force of repulsion is $2\pi\sigma^2 = mg$; 1249 e.s.u.

25. Capacity of whole system per unit area is $1/4\pi a + 1/4\pi b$; if surface density of charge is σ, potential is therefore $4\pi\sigma ab/(a + b)$.

26. Initial capacity per unit area is $1/4\pi d$; final capacity is $1/4\pi(d - t)$.

27. Consider any sphere of radius r, carrying a charge q. To find the force on any part of the sphere due to the charge on the remainder, describe a concentric sphere of radius R around the first sphere. By Gauss's theorem, total electrical force on second sphere is $4\pi q$. Let R approach r; when R nearly equals r, the force on any point of the second sphere may be considered as arising partly from the charged area immediately below the point and partly from the remainder of the charged sphere. For points sufficiently close to the surface, the area immediately below behaves as an infinite plane which exerts an electric force $2\pi\sigma$, hence the remainder of the sphere exerts a force $2\pi\sigma$, where $\sigma = q/4\pi r^2$. Letting the second sphere coincide with the first, the mechanical force on unit area due to the charge on the remainder is $2\pi\sigma \,.\, \sigma = 2\pi\sigma^2$. For every unit area of the sphere in question there is therefore an outward radial force per unit area of $2\pi\sigma^2$; hence total force on top hemisphere, resolving along a direction perpendicular to the horizontal bisecting plane, is

$$2\pi\sigma^2 \,.\, \pi r^2 = 2\pi^2\sigma^2 r^2 = q^2/8r^2 = 10^6/8 \times 25 = 5000 \text{ dynes},$$

which is greater than 981 dynes due to the weight of the hemisphere.

28. Let induced surface density be σ; then $\sigma = q/4\pi a^2$; radial force of attraction per unit area of the hemisphere is $q\sigma/a^2$; hence total effective attractive force in direction perpendicular to horizontal dividing plane is $(q\sigma/a^2)\pi a^2 = q^2/4a^2$. Repulsion due to charge on sphere itself is, from Ques. 27, $q^2/8a^2$; hence net attractive force on each hemisphere is $q^2/8a^2$.

29. Let V_1, V_2 and V_3 be initial potentials of A, B and C. Then $V_1 = (q_1/a + q_2/b + q_3/c)$, $V_2 = (q_1/b + q_2/b + q_3/c)$, and $V_3 = (q_1/c + q_2/c + q_3/c)$. After inner sphere is earthed, it acquires an induced charge Q given by $Q/a + q_2/b + q_3/c = 0$, or $Q = -a(q_2/b + q_3/c)$. Hence final potentials of B and C are $V_2' = q_2/b + q_3/c - a(q_2/b + q_3/c)/b$ and $V_3' = q_2/c + q_3/c - a(q_2/b + q_3/c)/c$. Hence changes in potentials of the spheres are $\Delta V_1 = q_1/a + q_2/b + q_3/c$, $\Delta V_2 = q_1/b + aq_2/b^2 + aq_3/bc$, and $\Delta V_3 = q_1/c + aq_2/bc + aq_3/c^2$. Hence total change in energy is $\frac{1}{2}(q_1 \,.\, \Delta V_1 + q_2 \,.\, \Delta V_2 + q_3 \,.\, \Delta V_3)$.

30. Ratio of capacities of conductor and whole system is q/Q; after second contact, total charge on conductor and disk is $Q + q$, and charge now acquired by conductor is $q(Q + q)/Q = q(1 + q/Q)$; similarly after third charging, charge acquired by conductor is $q(1 + q/Q + q^2/Q^2)$. This is a geometrical progression with constant ratio q/Q, and the sum to infinity is $qQ/(Q - q)$.

31. Capacity of parallel plate condenser is $A/4\pi d = 4{\cdot}77$ cm. If C is capacity of electroscope and q = charge on electroscope, since scale readings are proportional to potential,

$$q/C : q/(C + 4{\cdot}77) = 20 : 12;\ 7{\cdot}16 \text{ cm.}$$

32. Electric field strength is $3000/2 \times 300 = 5$ e.s.u.; hence

$$5 \times 4{\cdot}8 \times 10^{-10} = m \times 981;\ 2{\cdot}45 \times 10^{-12} \text{ gm.}$$

33. If surface density is σ, $4\pi\sigma \times 0{\cdot}2 = \dfrac{1000}{300}$. Force required is $m \times 981 = 2\pi\sigma^2 A$; 0·34 gm.

34. Capacity of system is increased, so if potential difference is maintained at 1000 volts, surface density is increased in the ratio 3·5 : 1; now force is decreased to $2\pi\sigma^2/3{\cdot}5$; hence total result is to increase force in ratio 3·5 : 1; 1·19 gm.

35. If initial capacity of condenser is C, capacity of lower half becomes $kC/2$, that of top half remaining $C/2$; hence total capacity becomes $C(1 + k)/2$.

36. Let initial separation of plates be d; the capacity of condenser with slab present is equal to $A/4\pi\{(d - a) + a/k\}$. On removing slab and moving plates closer, capacity regains its original value when plate separation is reduced to $(d - b)$; hence capacity is also given by $A/4\pi(d - b)$. Comparing the two expressions, $k = a/(a - b)$.

37. Capacity of two concentric cylinders per unit length $= k/(2 \log_e b/a)$, where b is external radius and a is radius of central core; 2·31 cm.

38. Deflection θ of quadrant electrometer is given by

$$\theta = k(V_A - V_B)\left(V_C - \frac{V_A + V_B}{2}\right),$$

where k is a constant and V_A, V_B and V_C are the potentials of the pairs of quadrants and the paddle respectively. First finding k, we have

$$20 = k(10)(1000 - 5) \quad \text{or} \quad k = 2/995.$$

On connecting idiostatically to the A.C. supply, $\theta = k(V_A - V_B)^2/2$ since $V_C = V_A$ or $V_A - V_B = 315{\cdot}4$ volts.

39. Wattage developed is $5 \times 10^3 \times 10^{-3} = 5$ watts; assuming no frictional losses, since 1 h.p. = 746 watts, h.p. required is 5/746.

40. Gain in kinetic energy = loss in potential energy. If velocity acquired is v, $\frac{1}{2}mv^2 = Ve$; $\frac{1}{2} . 9 \times 10^{-28} \times v^2 = 3000 \times 4{\cdot}8 \times 10^{-10}/300$; $3{\cdot}27 \times 10^9$ cm./sec.

41. Force of attraction between plates is $2\pi\sigma^2 A$; hence work done in moving distance d is $2\pi\sigma^2 Ad$.

42. Initial charge on condenser is $AV/4\pi t$; final charge on condenser is $AV/4\pi(t - d)$; hence energy supplied by battery is $\frac{AV}{4\pi}\left\{\frac{1}{(t-d)} - \frac{1}{t}\right\} . V$.

43. Initial capacity is $A/4\pi t$ and initial energy is $\frac{1}{2}CV^2 = \frac{1}{2}AV^2/4\pi t$. Final capacity is $A/4\pi(t - d)$, hence final energy is $\frac{1}{2}AV^2/4\pi(t - d)$. Hence increase in energy of condenser is $AV^2d/8\pi t(t - d)$. Hence from Ques. 42, external work done = energy supplied by battery less energy gained by condenser $= AV^2d/8\pi t(t - d)$.

44. Total capacity becomes $C_1 + C_2$; total charge becomes $q_1 + q_2$; final energy is therefore $\frac{1}{2}(q_1 + q_2)^2/(C_1 + C_2)$. Initial energy was $\frac{1}{2}(q_1^2/C_1 + q_2^2/C_2)$.

45. Wattage required = 50. Hence h.p. required is 50/746; 5/22 amp.

46. Wattage $= EI = E^2/R$; $100 \times 10^{\frac{1}{2}}$; $1/10 \times 10^{\frac{1}{2}}$ volts.

47. Resistance of leads is $1{\cdot}6 \times 10^{-6} \times 5280 \times 12 \times 2{\cdot}54/2{\cdot}54^2 = 0{\cdot}04$ ohm nearly; 208 volts; percentage wasted, $8 \times 200/208 \times 200 = 3{\cdot}9$ per cent.

48. Initial effective resistance of lamps = 5/2 ohms; final resistance is 5/3 ohms. Let E.M.F. and internal resistance of supply battery be E and r. Then $\frac{E\,.\,(5/2)}{r + 5/2} = 224$ and $\frac{E\,.\,(5/3)}{r + 5/3} = 218$; 15/103 ohms.

49. Since B is insulated, there is no fall of potential between B and fault; hence potential at fault is 50 volts. Similarly in the second case, potential of fault is potential of A = 50 volts. Hence same current must flow through fault and therefore down cable in both cases. Hence $(200 - 50)/l = (230 - 50)/(100 - l)$, where l is distance of fault from A; $45\frac{5}{11}$ miles.

50. Let there be N supply cells which are grouped so that there are n files of m cells in series, that is, $N = mn$. Let R be external resistance and let r be internal resistance of one cell; then effective resistance of battery is mr/n. If E.M.F. of one cell is E, current flowing is

$$I = mE/(R + mr/n) \quad \text{or} \quad I = NnE/(n^2R + Nr),$$

and condition for maximum current is $dI/dn = 0$, hence $R = mr/n$ as required.

51. If resistance of wire is R and current is I, $2 = I(R + 2)$ and $1{\cdot}5 = I(R + 1)$; 2 ohms, $\frac{1}{2}$ amp.

52. $1{\cdot}2 = E\,.\,2/(2 + r)$, $1{\cdot}0 = E\,.\,1/(1 + r)$; $\frac{1}{2}$ ohm, 1·5 volt.

53. Let current along AB $= I_1$ and down BD $= I_2$; current in AD $= 1 - I_1$, in BC $= I_1 - I_2$, and in DC $= 1 - I_1 + I_2$. In circuit ABD, considering points A and D, the drop in potential between these points is the same whether we proceed along AD or via AB, BD. Applying Ohm's law, $4(1 - I_1) = I_1 + 5I_2$ or $5I_1 + 5I_2 = 4$. In circuit BCD, $2(I_1 - I_2) = 5I_2 + 3(1 - I_1 + I_2)$ or $5I_1 - 10I_2 = 3$; 1/15 amp.

54. Current in circuit is

$$(2 - 3/2)/(18 + r) = 1/2(18 + r),$$

where r is internal resistance of second cell. Apply Ohm's law to find potential of centre point of 6-ohm wire, commencing at the centre point of 10-ohm wire and proceeding via 3/2 volt cell. We have $16/9 = (5 + 2 + 3)/2(18 + r) + 3/2$; zero.

55. Let resistance of side AB $= r$; that of BC is then $r/2$, and of AC is $r/3$. Equivalent resistance of AC + CB $= 5r/6$, and with AB in parallel

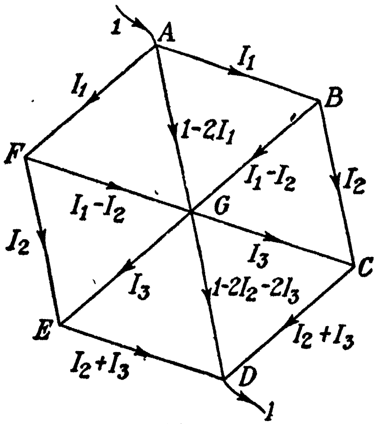

Fig. 3

with this, total external resistance is $5r/11$. Hence total current is $2/(5r/11 + r/3) = 33/13r$; therefore $r = 51/13$; 17/13 ohms.

56. Insulation resistances between P and Q and between Q and R are effectively in parallel; hence if insulation resistance of PQ is r, $1/20{,}000 - 1/r = 1/30{,}000$; $60{,}000\,\omega$.

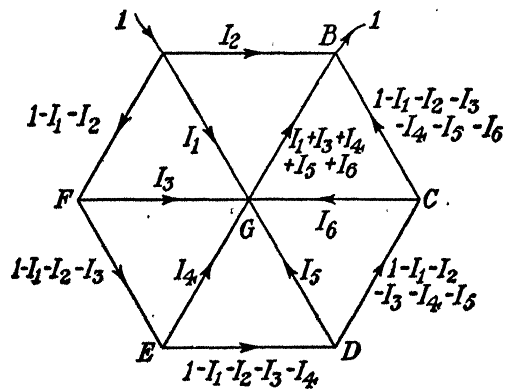

Fig. 4

57. Equivalent resistance of diam. and semicircle is $2\pi R/(\pi + 2)$; adding $\pi R/2$, we finally require equivalent resistance of $\pi R/2$ and $\pi R(6 + \pi)/2(2 + \pi)$ in parallel.

58. The circuit is symmetrical; taking the current entering as unity and putting in currents I_1, I_2 and I_3 as in fig. 3, and considering circuits

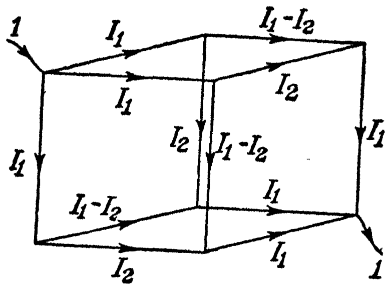

Fig. 5

ABG, BCG and CDG, $1 = 4I_1 - I_2$, $2I_2 = I_1 + I_3$, and $1 = 3I_2 + 4I_3$; $I_1 = 3/10$, $I_2 = 1/5$, $I_3 = 1/10$. Equivalent resistance is $(I_1 + 2I_2 + I_3)r = 4r/5$.

59. Taking the circuits AFG, GFE, GED, GCD, BCG and ABG (fig. 4), six equations are obtained. Find the value of I_2; $11r/20$.

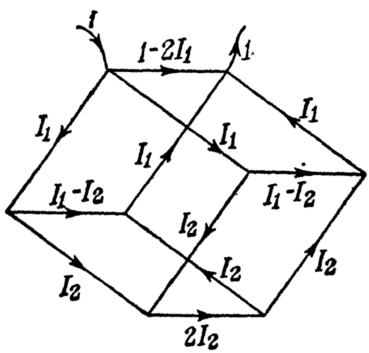

Fig. 6

60. The circuit is symmetrical (fig. 5); $5r/6$.

61. From partial symmetry currents will divide as in fig. 6; $7r/12$.

62. Let max. resistance occur when point of contact is at point on circumference distance x from far end of diam. The equivalent resistance of network is given by $1/R = 1/2r + 1/(2r + x) + 1/(\pi r - x)$. Differentiating and equating to zero, $x = r(\pi - 2)/2$; hence

$$R = 2r(\pi + 2)/(\pi + 10).$$

63. For wires connected in parallel, combined resistance is always less than the least single resistance present. This will occur when $x = r\pi/2$, that is, point of contact is as close to the point of entrance of the current as the conditions of the question will allow; $2\pi r(4 + \pi)/(\pi^2 + 12\pi + 16)$.

64. Max. current to be allowed in coil is 1/800 amp.; 80/3999 ohm.

65. $(E - e)/e = 120/(150 - 120) = IR/Ir = 10/r$; 2·5 ohms.

66. $t_1 = 2\pi\{I/(F + H)\}^{\frac{1}{2}}$, $t_2 = 2\pi\{I/(F - H)\}^{\frac{1}{2}}$, $F = 2I/10^2$. At neutral point $H = F' = 2I/10R$; 55·6 cm. due W.

67. $27/\pi$ amp.

68. Resultant field is given by $H'^2 = H^2 + F^2 + 2FH \cdot \cos 60°$; also $t_1 = 2\pi(I/MH)^{\frac{1}{2}}$ and $t_2 = 2\pi(I/MH')^{\frac{1}{2}}$; $30{\cdot}6/\pi$ amp.

69. As in Ques. 67, $F = 0{\cdot}54$ gauss; also since for point on axis at distance x from centre of coil $F = 2\pi nIr^2/10(r^2 + x^2)^{3/2}$, $I = 54\sqrt{2}/\pi$ amp.

70. Force due to both coils at point $x/2$ will be

$$F = 2 \cdot 2\pi nIa^2/10(a^2 + x^2/4)^{3/2}.$$

For field gradient to be a minimum $d^2F/dx^2 = 0$; $x = a$.

71. $1{\cdot}26/\pi$ amp.

72. $\tan\theta_1 \propto n_1I_1 \propto n_1E/R_1 \propto 1 \times E/2 \propto E/2$; similarly for $\tan\theta_2$; same deflection.

73. Apply $I = 10(Hr/2\pi n)\tan\theta$.

74. $F = H\sin\theta$; hence max. value of $F = H$; $0{\cdot}18/\pi$ amp.

75. $H = 4\pi nI/10$; $0{\cdot}18/4\pi$ amp.

76. Field at point on axis of solenoid is $F = 2\pi ni(\cos\theta_1 - \cos\theta_2)$, where θ_1 and θ_2 are semi-angles subtended at point by extremities of solenoid, and n is number of turns per unit length. Hence $F = H\tan\theta = H$ and $F = 2\pi \times 10 \times i(11/\sqrt{122} - 1/\sqrt{2})$; 1/10 amp. nearly.

77. Couple $G = nAIH/10 = 2$ dynes-cm.; 2 divisions.

78. Mass $= I \times$ electrochemical equivalent $\times$ time; 71° approx.

79. 0·5 amp.

80. Equivalence in calories when 1 coulomb passed is $9{\cdot}4 \times 10^{-5} \times 3800 = 0{\cdot}357$ cal. Voltage required is therefore $4{\cdot}18 \times 0{\cdot}357 = 1{\cdot}5$ approx.

81. $R_t = R_0(1 + at)$; $400 = 40(1 + 5 \times 10^{-3}t)$; 1800° C.

82. $l_t = 102{\cdot}18$ cm., $\sigma_t = 0{\cdot}1642 \times 1{\cdot}175$ ohm^{-1} cm.$^{-1}$; 529·7 ohms.

83. Force of repulsion is $2I^2/100 \times 10$ dynes per unit length; 7000 amp. approx.

84. $G = nAIH/10$; $10/\pi$ amp.

85. Intensity of magnetization = pole density per unit area; $F = 4\pi\sigma$; 320π gauss.

86. Intensity of magnetization $kH = 14 \cdot 4$; hence $M = 14 \cdot 4\ V$: $t_1 = 2\pi(I/H)^{\frac{1}{2}}$, $t_2 = 2\pi\{I/(H - F)\}^{\frac{1}{2}}$; hence $F = 0 \cdot 135$ gauss, also $F = M/d^3$; 253 c.c.

87. $F = 2\pi I^2$ per unit area; $414 \cdot 8\pi$ dynes per sq. cm.

88. Magneto-motive force is M.M.F. $= 4\pi nI/10 = 4\pi n$; also M.M.F. $=$ flux$(N) \times$ magnetic resistance $\left(\int dl/\mu S\right)$ or $4\pi n = \frac{N}{S}\left\{\frac{(2\pi r - d)}{\mu} + d\right\}$. Now $N =$ induction$(B) \times$ area(S); hence $B = N/S$, and in gap $B = H$ since $\mu = 1$; 138 turns.

89. Magnetic resistance is $\pi r/\mu_1 S + (2r - d)/\mu_2 S + d/S$; M.M.F. $= 2400\pi$; hence flux $N = 2400\pi S/\{\pi r/\mu_1 + (2r - d)/\mu_2 + d\}$; now $B = N/S$, and in gap $\mu = 1$ and therefore $B = H$; 12,310 gauss.

90. $v^2 = 2fs$, $v = 100\sqrt{981}$ cm./sec. $E = dN/dt = 100\sqrt{981} \times 80 \times 0 \cdot 18/10^8$; $4 \cdot 51 \times 10^{-4}$ volts; $v = ft = 9810$ cm./sec.; $1 \cdot 41 \times 10^{-3}$ volts.

91. If wheel makes n rev./sec., $2\pi rn = v$; hence area swept per sec. due to rotation is $\pi r^2 n = rv/2$. Hence E.M.F. between hub and rim will be $rvH/2 = 1 \cdot 47 \times 10^{-4}$ volts. Superposed on this is an E.M.F. acting vertically upwards due to the translational movement of the wheel, its maximum value being $rvH = 2 \cdot 94 \times 10^{-4}$ volts. Hence E.M.F. between hub and circumference will depend on point of circumference considered, being $4 \cdot 41 \times 10^{-4}$ volts for a point vertically above the hub acting from hub to rim, and $1 \cdot 47 \times 10^{-4}$ volts for a point vertically below the hub acting from rim to hub.

92. $V/H = \tan D$, hence $V = 0 \cdot 18 \times 2 \cdot 1445$; $E = lvV/10^8 = 1 \cdot 42 \times 10^{-3}$ volts.

93. $E = \pi \times 64 \times 600 \times 4\pi \times 50 \times 1/10 \times 60 \times 10^8 = 1 \cdot 26 \times 10^{-3}$ volts.

94. Linear velocity of conductor is $2\pi \times 100 \times (1000/60)$ cm./sec.; hence $E = 100 \times 2\pi \times 100 \times 1000 \times 5000/(60 \times 10^{-8}) = 50\pi/3$ volts.

95. Total flux $N = 4\pi \times 50 \times 2 \times 2/10 = 80\pi$; hence

$$E = -dN/dt \,.\, n/t = 2 \cdot 51 \times 10^{-2} \text{ volts.}$$

96. Current I at any time t after circuit is completed is given by $I = I_0(1 - exp^{-rt/l})$, where I_0 is final steady current, r is resistance, and l is inductance of circuit. $t = 4 \log_e 2/5 = 0 \cdot 55$ sec.

97. $4\pi\sigma m' = wg$; 692 gm.

98. Let applied E.M.F. $= E$ and back E.M.F. $= E'$; also let total resistance be R. Then current flowing is $I = (E - E')/R$ and energy wasted as heat is $(E - E')I$; hence since energy supplied is EI, electrical energy transformed into rotation is $E'I$. Efficiency of machine is therefore $E'I/EI = E'/E$, and this has its maximum value of unity when $E' = E$ or the back E.M.F. is equal to the applied E.M.F. In practice the most *efficient* running must occur at a speed just less than this, since if $E' = E$, no current would flow through the motor. Now rotational energy is given

by $P = E'I$ or $P = E'(E - E')/R$; hence for P to be a max. $dP/dE' = 0$, hence $E' = E/2$.

99. $E_1 \propto n_1 H$, $E_2 \propto n_2 V$; $V/H = \tan D$; 70° nearly.

100. Since E.M.F.s are additive with metals,

$$E = (a_1 - a_2) + (b_1 - b_2)t + (c_1 - c_2)t^2;$$

hence $E = -243 + 6{\cdot}7t - 0{\cdot}0001t^2$, in which the coefficient of t^2 is very small and E is approximately linear with temperature; 92 microvolts.

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