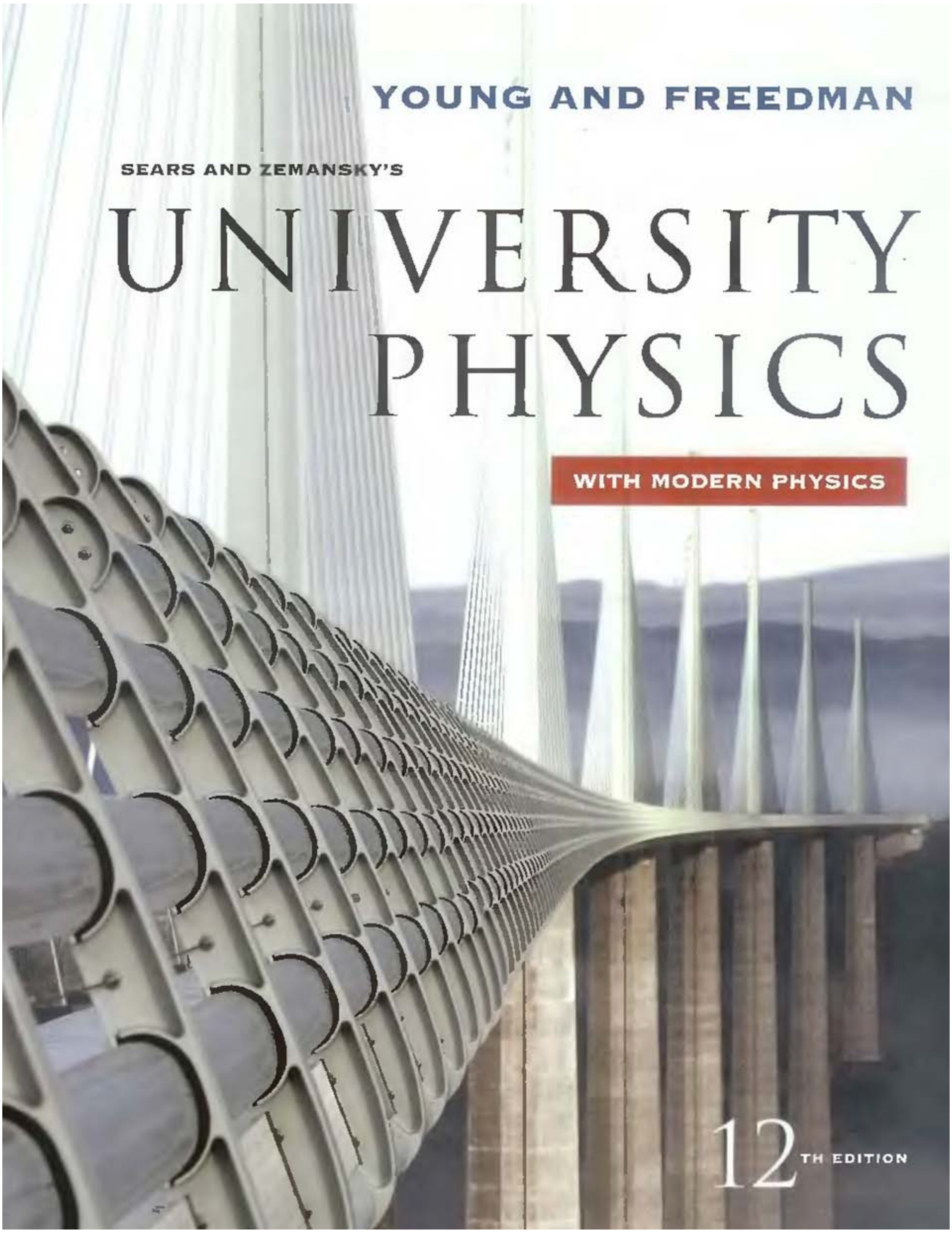
YOUNG AND FREEDMAN
SEARS AND ZEMANSKY'S
UNIVERSITY PHYSICS
WITH MODERN PHYSICS
12TH EDITION

SEARS AND ZEMANSKY'S

UNIVERSITY PHYSICS

12TH EDITION

WITH MODERN PHYSICS

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ABOUT THE AUTHORS

Hugh D. Young is Emeritus Professor of Physics at Carnegie Mellon University in Pittsburgh, PA. He attended Carnegie Mellon for both undergraduate and graduate study and earned his Ph.D. in fundamental particle theory under the direction of the late Richard Cutkosky. He joined the faculty of Carnegie Mellon in 1956 and has also spent two years as a Visiting Professor at the University of California at Berkeley.

Prof. Young's career has centered entirely around undergraduate education. He has written several undergraduate-level textbooks, and in 1973 he became a co-author with Francis Sears and Mark Zemansky for their well-known introductory texts. With their deaths, he assumed full responsibility for new editions of these books until joined by Prof. Freedman for ***University Physics***.

Prof. Young is au enthusiastic skier, climber, and hiker. He also served for several years as Associate Organist at St. Paul's Cathedral in Pittsburgh, and has played numerous organ recitals in the Pittsburgh area. Prof. Young and his wife Alice usually travel extensively in the summer, especially in Europe and in the desert canyon country of southern Utah.

Roger A. Freedman is a Lecturer in Physics at the University of California, Santa Barbara. Dr. Freedman was an undergraduate at the University of California campuses in San Diego and Los Angeles, and did his doctoral research in nuclear theory at Stanford University under the direction of Professor J. Dirk Walecka. He came to UCSB in 1981 after three years teaching and doing research at the University of Washington.

At UCSB, Dr. Freedman has taught in both the Department of Physics and the College of Creative Studies, a branch of the university intended for highly gifted and motivated undergraduates. He has published research in nuclear physics, elementary particle physics, and laser physics. In recent years, he has helped to develop computer-based tools for learning introductory physics and astronomy.

When not in the classroom or slaving over a computer, Dr. Freedman can be found either flying (he holds a commercial pilot's license) or driving with his wife, Caroline, in their 1960 Nash Metropolitan convertible.

A. Lewis Ford is Professor of Physics at Texas A&M University. He received a B.A. from Rice University in 1968 and a Ph.D. in chemical physics from the University of Texas at Austin in 1972. After a one-year postdoc at Harvard University, he joined the Texas A&M physics faculty in 1973 and has been there ever since. Professor Ford's research area is theoretical atomic physics, with a specialization in atomic collisions. At Texas A&M he has taught a variety of undergraduate and graduate courses, but primarily introductory physics.

TO THE STUDENT

HOW TO SUCCEED IN PHYSICS BY REALLY TRYING

Mark Hollabaugh *Normandale Community College*

Physics encompasses the large and the small, the old and the new. From the atom to galaxies, from electrical circuitry to aerodynamics, physics is very much a part of the world around us. You probably are taking this introductory course in calculus-based physics because it is required for subsequent courses you plan to take in preparation for a career in science or engineering. Your professor wants you to learn physics and to enjoy the experience. He or she is very interested in helping you learn this fascinating subject. That is part of the reason your professor chose this textbook for your course. That is also the reason Drs. Young and Freedman asked me to write this introductory section. We want you to succeed!

The purpose of this section of *University Physics* is to give you some ideas that will assist your learning. Specific suggestions on how to use the textbook will follow a brief discussion of general study habits and strategies.

Preparation for This Course

If you had high school physics, you will probably learn concepts faster than those who have not because you will be familiar with the language of physics. If English is a second language for you, keep a glossary of new terms that you encounter and make sure you understand how they are used in physics. Likewise, if you are farther along in your mathematics courses, you will pick up the mathematical aspects of physics faster. Even if your mathematics is adequate, you may find a book such as Arnold D. Pickar's *Preparing for General Physics: Math Skill Drills and Other Useful Help (Calculus Version)* to be useful. Your professor may actually assign sections of this math review to assist your learning.

Learning to Learn

Each of us has a different learning style and a preferred means of learning. Understanding your own learning style will help you to focus on aspects of physics that may give you difficulty and to use those components of your course that will help you overcome the difficulty. Obviously you will want to spend more time on those aspects that give you the most trouble. If you learn by hearing, lectures will be very important. If you learn by explaining, then working with other students will be useful to you. If solving problems is difficult for you, spend more time learning how to solve problems. Also, it is important to understand and develop good study habits. Perhaps the most important thing you can do for yourself is to set aside adequate, regularly scheduled study time in a distraction-free environment.

Answer the following questions for yourself:

- Am I able to use fundamental mathematical concepts from algebra, geometry and trigonometry? (If not, plan a program of review with help from your professor.)
- In similar courses, what activity has given me the most trouble? (Spend more time on this.) What has been the easiest for me? (Do this first; it will help to build your confidence.)

- Do I understand the material better if I read the book before or after the lecture? (You may learn best by skimming the material, going to lecture, and then undertaking an in-depth reading.)
- Do I spend adequate time in studying physics? (A rule of thumb for a class like this is to devote, on the average, 2.5 hours out of class for each hour in class. For a course meeting 5 hours each week, that means you should spend about 10 to 15 hours per week studying physics.)
- Do I study physics every day? (Spread that 10 to 15 hours out over an entire week!) At what time of the day am I at my best for studying physics? (Pick a specific time of the day and stick to it.)
- Do I work in a quiet place where I can maintain my focus? (Distractions will break your routine and cause you to miss important points.)

Working with Others

Scientists or engineers seldom work in isolation from one another but rather work cooperatively. You will learn more physics and have more fun doing it if you work with other students. Some professors may formalize the use of cooperative learning or facilitate the formation of study groups. You may wish to form your own informal study group with members of your class who live in your neighborhood or dorm. If you have access to e-mail, use it to keep in touch with one another. Your study group is an excellent resource when reviewing for exams.

Lectures and Taking Notes

An important component of any college course is the lecture. In physics this is especially important because your professor will frequently do demonstrations of physical principles, run computer simulations, or show video clips. All of these are learning activities that will help you to understand the basic principles of physics. Don't miss lectures, and if for some reason you do, ask a friend or member of your study group to provide you with notes and let you know what happened.

Take your class notes in outline form, and fill in the details later. It can be very difficult to take word for word notes, so just write down key ideas. Your professor may use a diagram from the textbook. Leave a space in your notes and just add the diagram later. After class, edit your notes, filling in any gaps or omissions and noting things you need to study further. Make references to the textbook by page, equation number, or section number.

Make sure you ask questions in class, or see your professor during office hours. Remember the only "dumb" question is the one that is not asked. Your college may also have teaching assistants or peer tutors who are available to help you with difficulties you may have.

Examinations

Taking an examination is stressful. But if you feel adequately prepared and are well-rested, your stress will be lessened. Preparing for an exam is a continual process; it begins the moment the last exam is over. You should immediately go over the exam and understand any mistakes you made. If you worked a problem and made substantial errors, try this: Take a piece of paper and divide it down the middle with a line from top to bottom. In one column, write the proper solution to the problem. In the other column, write what you did and why, if you know, and why your solution was incorrect. If you are uncertain why you made your mistake, or how to avoid making it again, talk with your professor. Physics continually builds on fundamental ideas and it is important to correct any misunderstandings immediately. Warning: While cramming at the last minute may get you through the present exam, you will not adequately retain the concepts for use on the next exam.

TO THE INSTRUCTOR

PREFACE

This book is the product of more than half a century of leadership and innovation in physics education. When the first edition of University Physics by Francis W. Sears and Mark W. Zemansky was published in 1949, it was revolutionary among calculus-based physics textbooks in its emphasis on the fundamental principles of physics and how to apply them. The success of University Physics with generations of (several million) students and educators around the world is a testament to the merits of this approach, and to the many innovations it has introduced subsequently.

In preparing this new Twelfth Edition, we have further enhanced and developed *University Physics* to assimilate the best ideas from education research with enhanced problem-solving instruction, pioneering visual and conceptual pedagogy, the first systematically enhanced problems, and the most pedagogically proven and widely used online homework and tutorial system in the world.

New to This Edition

- **Problem solving.** The acclaimed, research-based **four-step problem-solving framework** (Identify, Set Up, Execute, and Evaluate) is now used throughout every Worked Example, chapter-specific Problem-Solving Strategy, and every Solution in the Instructor and Student Solutions Manuals. Worked Examples now incorporate black-and-white Pencil Sketches to focus students on this critical step—one that research shows students otherwise tend to skip when illustrated with highly rendered figures.

- **Instruction followed by practice.** A streamlined and systematic learning path of instruction followed by practice includes **Learning Goals** at the start of each chapter and **Visual Chapter Summaries** that consolidate each concept in words, math, and figures. Popular **Test Your Understanding** conceptual questions at the end of each section now use **multiple-choice and ranking formats** to allow students to instantly check their knowledge.

- **Instructional power of figures.** The instructional power of figures is enhanced using the research-proven technique of **"annotation"** (chalkboard-style commentary integrated into the figure to guide the student in interpreting the figure) and by **streamlined use of color and detail** (in mechanics, for example, color is used to focus the student on the object of interest while the rest of the image is in grayscale and without distracting detail).

- **Enhanced end-of-chapter problems.** Renowned for providing the most wide-ranging and best-tested problems available, the Twelfth Edition goes still further: It provides **the first library of physics problems systematically enhanced** based on student performance nationally. Using this analysis, more than 800 new problems make up the entire library of 3700.

- **MasteringPhysics™** (www.masteringphysics.com). Launched with the Eleventh Edition, MasteringPhysics is now the most widely adopted, educationally proven, and technically advanced online homework and tutorial system in the world. For the Twelfth Edition, MasteringPhysics provides a wealth of new content and technological enhancements. In addition to a library of more than 1200 tutorials and all the end-of-chapter problems, MasteringPhysics

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now also provides specific tutorials for every Problem-Solving Strategy and key Test Your Understanding questions from each chapter. Answer types include algebraic, numerical, and multiple-choice answers, as well as ranking, sorting, graph drawing, vector drawing, and ray tracing.

Key Features of *University Physics*

A Guide for the Student Many physics students experience difficulty simply because they don't know how to use their textbook. The section entitled "How to Succeed in Physics by Really Trying," which precedes this preface, is a "user's manual" to all the features of this book. This section, written by Professor Mark Hollabaugh (Normandale Community College), also gives a number of helpful study hints. *Every* student should read this section!

Chapter Organization The first section of each chapter is an *Introduction* that gives specific examples of the chapter's content and connects it with what has come before. There are also a *Chapter Opening Question* and a list of *Learning Goals* to make the reader think about the subject matter of the chapter ahead. (To find the answer to the question, look for the ? icon.) Most sections end with a *Test Your Understanding Question,* which can be conceptual or quantitative in nature. At the end of the last section of the chapter is a *Visual Chapter Summary* of the most important principles in the chapter, as well as a list of *Key Terms* with reference to the page number where each term is introduced. The answers to the Chapter Opening Question and Test Your Understanding Questions follow the Key Terms.

Questions and Problems At the end of each chapter is a collection of *Discussion Questions* that probe and extend the student's conceptual understanding. Following these are *Exercises,* which are single-concept problems keyed to specific sections of the text; *Problems,* usually requiring one or two nontrivial steps; and *Challenge Problems,* intended to challenge the strongest students. The problems include applications to such diverse fields as astrophysics, biology, and aerodynamics. Many problems have a conceptual part in which students must discuss and explain their results. The new questions, exercises, and problems for this edition were created and organized by Wayne Anderson (Sacramento City College), Laird Kramer (Florida International University), and Charlie Hibbard.

Problem-Solving Strategies and Worked Examples Throughout the book, *Problem-Solving Strategy* boxes provide students with specific tactics for solving particular types of problems. They address the needs of any students who have ever felt that they "understand the concepts but can't do the problems."

All Problem-Solving Strategy boxes follow the ISEE approach (Identify, Set Up, Execute, and Evaluate) to solving problems. This approach helps students see how to begin with a seemingly complex situation, identify the relevant physical concepts, decide what tools are needed to solve the problem, carry out the solution, and then evaluate whether the result makes sense.

Each Problem-Solving Strategy box is followed by one or more worked-out *Examples* that illustrate the strategy. Many other worked-out Examples are found in each chapter. Like the Problem-Solving Strategy boxes, all of the quantitative Examples use the ISEE approach. Several of the examples are purely qualitative and are labeled as *Conceptual Examples;* see, for instance, Conceptual Examples 6.5 (Comparing kinetic energies, p. 191), 8.1 (Momentum versus kinetic energy, p. 251) and 20.7 (A reversible adiabatic process, p. 693).

"Caution" paragraphs Two decades of physics education research have revealed a number of conceptual pitfalls that commonly plague beginning physics students. These include the ideas that force is required for motion, that electric current is "used up" as it goes around a circuit, and that the product of an object's mass and its acceleration is itself a force. The "Caution" paragraphs alert students to these and other pitfalls, and explain why the wrong way to think about a certain situation (which may have occurred to the student first) is indeed wrong. (See, for example, pp. 118, 159, and 559.)

Notation and units Students often have a hard time keeping track of which quantities are vectors and which are not. We use boldface italic symbols with an arrow on top for vector quantities, such as $\vec{\boldsymbol{v}}$, $\vec{\boldsymbol{a}}$, and $\vec{\boldsymbol{F}}$; unit vectors such as $\hat{\boldsymbol{\imath}}$, have a caret on top. Boldface $\mathbf{+}$, $\mathbf{-}$, $\mathbf{\times}$, and $\mathbf{=}$ signs are used in vector equations to emphasize the distinction between vector and scalar mathematical operations.

SI units are used exclusively (English unit conversions are included where appropriate). The joule is used as the standard unit of energy of all forms, including heat.

Flexibility The book is adaptable to a wide variety of course outlines. There is plenty of material for a three-semester or a five-quarter course. Most instructors will find that there is too much material for a one-year course, but it is easy to tailor the book to a variety of one-year course plans by omitting certain chapters or sections. For example, any or all of the chapters on fluid mechanics, sound and hearing, electromagnetic waves, or relativity can be omitted without loss of continuity. In any case, no instructor should feel constrained to work straight through the entire book.

Instructor Supplements

The **Instructor Solutions Manuals,** prepared by A. Lewis Ford (Texas A&M University), contain complete and detailed solutions to all end-of-chapter problems. All solutions follow consistently the same Identify/Set Up/Execute/Evaluate problem-solving framework used in the textbook. The *Instructor Solutions Manual for Volume 1* (ISBN 0-321-49968-9) covers Chapters 1–20, and the *Instructor Solutions Manual for Volumes 2 and 3* (ISBN 0-321-49210-2) covers Chapters 21–44.

The cross-platform **Media Manager CD-ROM** (ISBN 0-321-49916-6) provides a comprehensive library of more than 220 applets from ActivPhysics OnLine™ as well as all line figures from the textbook in JPEG format. In addition, all the key equations, Problem-Solving Strategies, tables, and chapter summaries are provided in editable Word format. In-class weekly multiple-choice questions for use with various Classroom Response Systems (CRS) are also provided, based on the Test Your Understanding questions in the text. The CD-ROM also provides the Instructor Solutions Manual in convenient editable Word format and as PDFs.

MasteringPhysics™ (www.masteringphysics.com) is the most advanced, educationally effective, and widely used physics homework and tutorial system in the world. It provides instructors with a library of extensively pretested end-of-chapter problems and rich, Socratic tutorials that incorporate a wide variety of answer types, wrong-answer feedback, and adaptive help (comprising hints or simpler sub-problems upon request). MasteringPhysics™ allows instructors to quickly build wide-ranging homework assignments of just the right difficulty and duration and provides them with efficient tools to analyze class trends—or the work of any student—in unprecedented detail and to compare the results either with the national average or with the performance of previous classes.

Five Easy Lessons: Strategies for Successful Physics Teaching (ISBN 0-8053-8702-1) by Randall D. Knight (California Polytechnic State University, San Luis Obispo) is packed with creative ideas on how to enhance any physics course. It is an invaluable companion for both novice and veteran physics instructors.

The **Transparency Acetates** (ISBN 0-321-50034-2) contain more than 200 key figures from *University Physics,* Twelfth Edition, in full color.

The **Printed Test Bank** (ISBN 0-321-50035-0) provides more than 2000 multiple-choice questions.

The **Computerized Test Bank** (ISBN 0-321-50126-8) includes all of the questions from the Printed Test Bank on a cross-platform CD-ROM. More than half the questions have numerical values that can be randomly assigned for each student.

Student Supplements

The **Study Guide,** by James R. Gaines, William F. Palmer, and Laird Kramer, reinforces the text's emphasis on problem-solving strategies and student misconceptions. The *Study Guide for Volume 1* (ISBN 0-321-50033-4) covers Chapters 1–20, and the *Study Guide for Volumes 2 and 3* (ISBN 0-321-50037-7) covers Chapters 21–44.

The **Student Solutions Manual,** by A. Lewis Ford (Texas A&M University), contains detailed, step-by-step solutions to more than half of the odd-numbered end-of-chapter problems from the textbook. All solutions follow consistently the same Identify/Set Up/Execute/Evaluate problem-solving framework used in the textbook. The *Student Solutions Manual for Volume 1* (ISBN 0-321-50063-6) covers Chapters 1–20, and the *Student Solutions Manual for Volumes 2 and 3* (ISBN 0-321-50038-5) covers Chapters 21–44.

MasteringPhysics™ (www.masteringphysics.com) is the most advanced, widely used, and educationally proven physics tutorial system in the world. It is the result of eight years of detailed studies of how real students work physics problems, and of precisely where they need help. Studies show that students who use MasteringPhysics™ significantly improve their scores on final exams and conceptual tests such as the Force Concept Inventory. MasteringPhysics™ achieves this by providing students with instantaneous feedback specific to their wrong answers, simpler sub-problems upon request when they get stuck, and partial credit for their method. This individualized, 24/7 tutor system is recommended by nine out of ten students to their peers as the most effective and time-efficient way to study.

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The **Addison-Wesley Tutor Center** (www.aw.com/tutorcenter) provides one-on-one tutoring via telephone, fax, e-mail, or interactive website. Qualified instructors answer questions and provide instruction with examples, problems, and other content from *University Physics,* Twelfth Edition, as well as help with MasteringPhysics™.

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Please Tell Us What You Think!

We welcome communications from students and professors, especially concerning errors or deficiencies that you find in this edition. We have devoted a lot of time and effort to writing the best book we know how to write, and we hope it will help you to teach and learn physics. In turn, you can help us by letting us know what still needs to be improved! Please feel free to contact us either electronically or by ordinary mail. Your comments will be greatly appreciated.

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DETAILED CONTENTS

MECHANICS

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APPENDICES

UNITS, PHYSICAL QUANTITIES, AND VECTORS

1

? Being able to predict the path of a hurricane is essential for minimizing the damage it does to lives and property. If a hurricane is moving at 20 km/h in a direction 53° north of east, how far north does the hurricane move in one h?

LEARNING GOALS

By studying this chapter, you will learn:

- What the fundamental quantities of mechanics are, and the units physicists use to measure them.
- How to keep track of significant figures in your calculations.
- The difference between scalars and vectors, and how to add and subtract vectors graphically.
- What the components of a vector are, and how to use them in calculations.
- What unit vectors are, and how to use them with components to describe vectors.
- Two ways of multiplying vectors.

The study of physics is important because physics is one of the most fundamental of the sciences. Scientists of all disciplines make use of the ideas of physics, including chemists who study the structure of molecules, paleontologists who try to reconstruct how dinosaurs walked, and climatologists who study how human activities affect the atmosphere and oceans. Physics is also the foundation of all engineering and technology. No engineer could design a flat-screen TV, an interplanetary spacecraft, or even a better mousetrap without first understanding the basic laws of physics.

The study of physics is also an adventure. You will find it challenging, sometimes frustrating, occasionally painful, and often richly rewarding and satisfying. It will appeal to your sense of beauty as well as to your rational intelligence. If you've ever wondered why the sky is blue, how radio waves can travel through empty space, or how a satellite stays in orbit, you can find the answers by using fundamental physics. Above all, you will come to see physics as a towering achievement of the human intellect in its quest to understand our world and ourselves.

In this opening chapter, we'll go over some important preliminaries that we'll need throughout our study. We'll discuss the nature of physical theory and the use of idealized models to represent physical systems. We'll introduce the systems of units used to describe physical quantities and discuss ways to describe the accuracy of a number. We'll look at examples of problems for which we can't (or don't want to) find a precise answer, but for which rough estimates can be useful and interesting. Finally, we'll study several aspects of vectors and vector algebra. Vectors will be needed throughout our study of physics to describe and analyze physical quantities, such as velocity and force, that have direction as well as magnitude.

1.1 The Nature of Physics

Physics is an *experimental* science. Physicists observe the phenomena of nature and try to find patterns and principles that relate these phenomena. These patterns are called physical theories or, when they are very well established and of broad use, physical laws or principles.

CAUTION **The meaning of the word "theory"** Calling an idea a theory does *not* mean that it's just a random thought or an unproven concept. Rather, a theory is an explanation of natural phenomena based on observation and accepted fundamental principles. An example is the well-established theory of biological evolution, which is the result of extensive research and observation by generations of biologists.

The development of physical theory requires creativity at every stage. The physicist has to learn to ask appropriate questions, design experiments to try to answer the questions, and draw appropriate conclusions from the results. Figure 1.1 shows two famous experimental facilities.

1.1 Two research laboratories. **(a)** According to legend, Galileo investigated falling bodies by dropping them from the Leaning Tower in Pisa, Italy, and he studied pendulum motion by observing the swinging of the chandelier in the adjacent cathedral. **(b)** The Hubble Space Telescope is the first major telescope to operate outside the earth's atmosphere. Measurements made with this telescope have helped determine the age and expansion rate of the universe.

(a)

(b)

Legend has it that Galileo Galilei (1564–1642) dropped light and heavy objects from the top of the Leaning Tower of Pisa (Fig. 1.1a) to find out whether their rates of fall were the same or different. Galileo recognized that only experimental investigation could answer this question. From examining the results of his experiments (which were actually much more sophisticated than in the legend), he made the inductive leap to the principle, or theory, that the acceleration of a falling body is independent of its weight.

The development of physical theories such as Galileo's is always a two-way process that starts and ends with observations or experiments. This development often takes an indirect path, with blind alleys, wrong guesses, and the discarding of unsuccessful theories in favor of more promising ones. Physics is not simply a collection of facts and principles; it is also the *process* by which we arrive at general principles that describe how the physical universe behaves.

No theory is ever regarded as the final or ultimate truth. The possibility always exists that new observations will require that a theory be revised or discarded. It is in the nature of physical theory that we can disprove a theory by finding behavior that is inconsistent with it, but we can never prove that a theory is always correct.

Getting back to Galileo, suppose we drop a feather and a cannonball. They certainly do *not* fall at the same rate. This does not mean that Galileo was wrong; it means that his theory was incomplete. If we drop the feather and the cannonball *in a vacuum* to eliminate the effects of the air, then they do fall at the same rate. Galileo's theory has a **range of validity:** It applies only to objects for which the force exerted by the air (due to air resistance and buoyancy) is much less than the weight. Objects like feathers or parachutes are clearly outside this range.

Every physical theory has a range of validity outside of which it is not applicable. Often a new development in physics extends a principle's range of validity. Galileo's analysis of falling bodies was greatly extended half a century later by Newton's laws of motion and law of gravitation.

1.2 Solving Physics Problems

At some point in their studies, almost all physics students find themselves thinking, "I understand the concepts, but I just can't solve the problems." But in physics, truly understanding a concept or principle is the same thing as being able to apply it to a variety of practical problems. Learning how to solve problems is absolutely essential; you don't *know* physics unless you can *do* physics.

How do you learn to solve physics problems? In every chapter of this book you will find *Problem-Solving Strategies* that offer techniques for setting up and solving problems efficiently and accurately. Following each *Problem-Solving Strategy* are one or more worked *Examples* that show these techniques in action.

(The *Problem-Solving Strategies* will also steer you away from some *incorrect* techniques that you may be tempted to use.) You'll also find additional examples that aren't associated with a particular *Problem-Solving Strategy*. Study these strategies and examples carefully, and work through each example for yourself on a piece of paper.

Different techniques are useful for solving different kinds of physics problems, which is why this book offers dozens of *Problem-Solving Strategies*. No matter what kind of problem you're dealing with, however, there are certain key steps that you'll always follow. (These same steps are equally useful for problems in math, engineering, chemistry, and many other fields.) In this book we've organized these steps into four stages of solving a problem.

All of the *Problem-Solving Strategies* and *Examples* in this book will follow these four steps. (In some cases we will combine the first two or three steps.) We encourage you to follow these same steps when you solve problems yourself. You may find it useful to remember the acronym ***I SEE***—short for *Identify, Set up, Execute,* and *Evaluate.*

Problem-Solving Strategy 1.1 Solving Physics Problems

IDENTIFY *the relevant concepts:* First, decide which physics ideas are relevant to the problem. Although this step doesn't involve any calculations, it's sometimes the most challenging part of solving the problem. Don't skip over this step, though; choosing the wrong approach at the beginning can make the problem more difficult than it has to be, or even lead you to an incorrect answer.

At this stage you must also identify the **target variable** of the problem—that is, is the quantity whose value you're trying to find. It could be the speed at which a projectile hits the ground, the intensity of a sound made by a siren, or the size of an image made by a lens. (Sometimes the goal will be to find a mathematical expression rather than a numerical value. Sometimes, too, the problem will have more than one target variable.) The target variable is the goal of the problem-solving process; don't lose sight of this goal as you work through the solution.

SET UP *the problem:* Based on the concepts you selected in the *Identify* step, choose the equations that you'll use to solve the problem and decide how you'll use them. If appropriate, draw a sketch of the situation described in the problem.

EXECUTE *the solution:* In this step, you "do the math." Before you launch into a flurry of calculations, make a list of all known and unknown quantities, and note which are the target variable or variables. Then solve the equations for the unknowns.

EVALUATE *your answer:* The goal of physics problem solving isn't just to get a number or a formula; it's to achieve better understanding. That means you must examine your answer to see what it's telling you. Be sure to ask yourself, "Does this answer make sense?" If your target variable was the radius of the earth and your answer is 6.38 centimeters (or if your answer is a negative number!), something went wrong in your problem-solving process. Go back and check your work, and revise your solution as necessary.

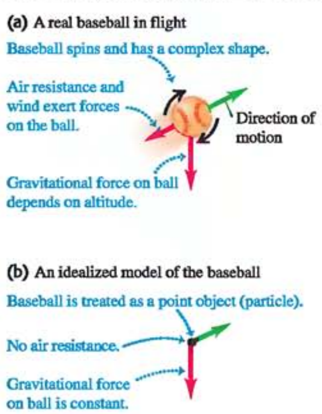
Idealized Models

In everyday conversation we use the word "model" to mean either a small-scale replica, such as a model railroad, or a person who displays articles of clothing (or the absence thereof). In physics a **model** is a simplified version of a physical system that would be too complicated to analyze in full detail.

For example, suppose we want to analyze the motion of a thrown baseball (Fig. 1.2a). How complicated is this problem? The ball is not a perfect sphere (it has raised seams), and it spins as it moves through the air. Wind and air resistance influence its motion, the ball's weight varies a little as its distance from the center of the earth changes, and so on. If we try to include all these things, the analysis gets hopelessly complicated. Instead, we invent a simplified version of the problem. We neglect the size and shape of the ball by representing it as a point object, or **particle.** We neglect air resistance by making the ball move in a vacuum, and we make the weight constant. Now we have a problem that is simple enough to deal with (Fig. 1.2b). We will analyze this model in detail in Chapter 3.

To make an idealized model, we have to overlook quite a few minor effects to concentrate on the most important features of the system. Of course, we have to be careful not to neglect too much. If we ignore the effects of gravity completely,

1.2 To simplify the analysis of (a) a baseball in flight, we use (b) an idealized model.



then our model predicts that when we throw the ball up, it will go in a straight line and disappear into space. We need to use some judgment and creativity to construct a model that simplifies a problem enough to make it manageable, yet keeps its essential features.

When we use a model to predict how a system will behave, the validity of our predictions is limited by the validity of the model. For example, Galileo's prediction about falling bodies (see Section 1.1) corresponds to an idealized model that does not include the effects of air resistance. This model works fairly well for a dropped cannonball, but not so well for a feather.

When we apply physical principles to complex systems in physical science and technology, we always use idealized models, and we have to be aware of the assumptions we are making. In fact, the principles of physics themselves are stated in terms of idealized models; we speak about point masses, rigid bodies, ideal insulators, and so on. Idealized models play a crucial role throughout this book. Watch for them in discussions of physical theories and their applications to specific problems.

1.3 Standards and Units

As we learned in Section 1.1, physics is an experimental science. Experiments require measurements, and we generally use numbers to describe the results of measurements. Any number that is used to describe a physical phenomenon quantitatively is called a **physical quantity.** For example, two physical quantities that describe you are your weight and your height. Some physical quantities are so fundamental that we can define them only by describing how to measure them. Such a definition is called an **operational definition.** Two examples are measuring a distance by using a ruler and measuring a time interval by using a stopwatch. In other cases we define a physical quantity by describing how to calculate it from other quantities that we *can* measure. Thus we might define the average speed of a moving object as the distance traveled (measured with a ruler) divided by the time of travel (measured with a stopwatch).

When we measure a quantity, we always compare it with some reference standard. When we say that a Porsche Carrera GT is 4.61 meters long, we mean that it is 4.61 times as long as a meter stick, which we define to be 1 meter long. Such a standard defines a **unit** of the quantity. The meter is a unit of distance, and the second is a unit of time. When we use a number to describe a physical quantity, we must always specify the unit that we are using; to describe a distance as simply "4.61" wouldn't mean anything.

1.3 In 1791 the distance from the North Pole to the equator was defined to be exactly 10^7 m. With the modern definition of the meter, this distance is about 0.02% more than 10^7 m.

To make accurate, reliable measurements, we need units of measurement that do not change and that can be duplicated by observers in various locations. The system of units used by scientists and engineers around the world is commonly called "the metric system," but since 1960 it has been known officially as the **International System,** or **SI** (the abbreviation for its French name, *Système International*). A list of all SI units is given in Appendix A, as are definitions of the most fundamental units.

The definitions of the basic units of the metric system have evolved over the years. When the metric system was established in 1791 by the French Academy of Sciences, the meter was defined as one ten-millionth of the distance from the North Pole to the equator (Fig. 1.3). The second was defined as the time required for a pendulum one meter long to swing from one side to the other. These definitions were cumbersome and hard to duplicate precisely, and by international agreement they have been replaced with more refined definitions.

Time

From 1889 until 1967, the unit of time was defined as a certain fraction of the mean solar day, the average time between successive arrivals of the sun at its highest point in the sky. The present standard, adopted in 1967, is much more precise. It is based on an atomic clock, which uses the energy difference between the two lowest energy states of the cesium atom. When bombarded by microwaves of precisely the proper frequency, cesium atoms undergo a transition from one of these states to the other. One **second** (abbreviated s) is defined as the time required for 9,192,631,770 cycles of this microwave radiation.

Length

In 1960 an atomic standard for the meter was also established, using the wavelength of the orange-red light emitted by atoms of krypton (^{86}Kr) in a glow discharge tube. Using this length standard, the speed of light in a vacuum was measured to be 299,792,458 m/s. In November 1983, the length standard was changed again so that the speed of light in a vacuum was *defined* to be precisely 299,792,458 m/s. The meter is defined to be consistent with this number and with the above definition of the second. Hence the new definition of the **meter** (abbreviated m) is the distance that light travels in a vacuum in 1/299,792,458 second. This provides a much more precise standard of length than the one based on a wavelength of light.

Mass

The standard of mass, the **kilogram** (abbreviated kg), is defined to be the mass of a particular cylinder of platinum–iridium alloy kept at the International Bureau of Weights and Measures at Sèvres, near Paris (Fig. 1.4). An atomic standard of mass would be more fundamental, but at present we cannot measure masses on an atomic scale with as much accuracy as on a macroscopic scale. The *gram* (which is not a fundamental unit) is 0.001 kilogram.

1.4 The metal object carefully enclosed within these nested glass containers is the international standard kilogram.

Unit Prefixes

Once we have defined the fundamental units, it is easy to introduce larger and smaller units for the same physical quantities. In the metric system these other units are related to the fundamental units (or, in the case of mass, to the gram) by multiples of 10 or $\frac{1}{10}$. Thus one kilometer (1 km) is 1000 meters, and one centimeter (1 cm) is $\frac{1}{100}$ meter. We usually express multiples of 10 or $\frac{1}{10}$ in exponential notation: $1000 = 10^3$, $\frac{1}{1000} = 10^{-3}$, and so on. With this notation, $1 \text{ km} = 10^3 \text{ m}$ and $1 \text{ cm} = 10^{-2} \text{ m}$.

The names of the additional units are derived by adding a **prefix** to the name of the fundamental unit. For example, the prefix "kilo-," abbreviated k, always means a unit larger by a factor of 1000; thus

$$1 \text{ kilometer} = 1 \text{ km} = 10^3 \text{ meters} = 10^3 \text{ m}$$
$$1 \text{ kilogram} = 1 \text{ kg} = 10^3 \text{ grams} = 10^3 \text{ g}$$
$$1 \text{ kilowatt} = 1 \text{ kW} = 10^3 \text{ watts} = 10^3 \text{ W}$$

A table on the inside back cover of this book lists the standard SI prefixes, with their meanings and abbreviations.

Here are several examples of the use of multiples of 10 and their prefixes with the units of length, mass, and time. Figure 1.5 shows how these prefixes help describe both large and small distances.

Length

$1 \text{ nanometer} = 1 \text{ nm} = 10^{-9} \text{ m}$ (a few times the size of the largest atom)

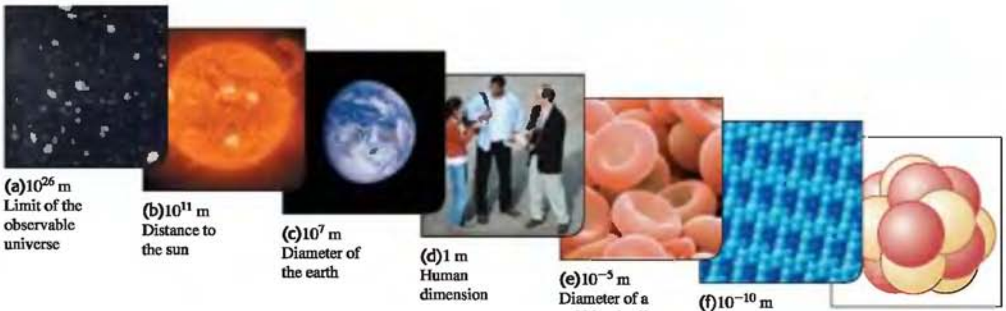
$1 \text{ micrometer} = 1\ \mu\text{m} = 10^{-6} \text{ m}$ (size of some bacteria and living cells)

$1 \text{ millimeter} = 1 \text{ mm} = 10^{-3} \text{ m}$ (diameter of the point of a ballpoint pen)

$1 \text{ centimeter} = 1 \text{ cm} = 10^{-2} \text{ m}$ (diameter of your little finger)

$1 \text{ kilometer} = 1 \text{ km} = 10^3 \text{ m}$ (a 10-minute walk)

1.5 Some typical lengths in the universe. (a) The distance to the most remote galaxies we can see is about 10^{26} m, or 10^{23} km. (b) The sun is 1.50×10^{11} m, or 1.50×10^{8} km, from earth. (c) The diameter of the earth is 1.28×10^{7} m, or 12,800 km. (d) A typical human is about 1.7 m, or 170 cm, tall. (e) Human red blood cells are about 8×10^{-6} m (0.008 mm, or 8 μm) in diameter. (f) These oxygen atoms, shown arrayed on the surface of a crystal, are about 10^{-10} m, or 10^{-4} μm, in radius. (g) Typical atomic nuclei (shown in an artist's impression) have radii of about 10^{-14} m, or 10^{-5} nm.



Mass

1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a very small dust particle)
1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (mass of a grain of salt)
1 gram = 1 g = 10^{-3} kg (mass of a paper clip)

Time

1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 0.3 m)
1 microsecond = 1 μs = 10^{-6} s (time for an orbiting space shuttle to travel 8 mm)
1 millisecond = 1 ms = 10^{-3} s (time for sound to travel 0.35 m)

The British System

Finally, we mention the British system of units. These units are used only in the United States and a few other countries, and in most of these they are being replaced by SI units. British units are now officially defined in terms of SI units, as follows:

Length: 1 inch = 2.54 cm (exactly)
Force: 1 pound = 4.448221615260 newtons (exactly)

The newton, abbreviated N, is the SI unit of force. The British unit of time is the second, defined the same way as in SI. In physics, British units are used only in mechanics and thermodynamics; there is no British system of electrical units.

In this book we use SI units for all examples and problems, but we occasionally give approximate equivalents in British units. As you do problems using SI units, you may also wish to convert to the approximate British equivalents if they are more familiar to you (Fig. 1.6). But you should try to *think* in SI units as much as you can.

1.6 Many everyday items make use of both SI and British units. An example is this speedometer from a U.S.-built automobile, which shows the speed in both kilometers per hour (inner scale) and miles per hour (outer scale).

1.4 Unit Consistency and Conversions

We use equations to express relationships among physical quantities, represented by algebraic symbols. Each algebraic symbol always denotes both a number and a unit. For example, d might represent a distance of 10 m, t a time of 5 s, and v a speed of 2 m/s.

An equation must always be **dimensionally consistent.** You can't add apples and automobiles; two terms may be added or equated only if they have the same units. For example, if a body moving with constant speed v travels a distance d in a time t, these quantities are related by the equation

$$d = vt$$

If d is measured in meters, then the product vt must also be expressed in meters. Using the above numbers as an example, we may write

$$10\,\text{m} = \left(2\frac{\text{m}}{\cancel{\text{s}}}\right)(5\,\cancel{\text{s}})$$

Because the unit $1/\text{s}$ on the right side of the equation cancels the unit s, the product has units of meters, as it must. In calculations, units are treated just like algebraic symbols with respect to multiplication and division.

CAUTION **Always use units in calculations** When a problem requires calculations using numbers with units, *always* write the numbers with the correct units and carry the units through the calculation as in the example above. This provides a very useful check for calculations. If at some stage in a calculation you find that an equation or an expression has inconsistent units, you know you have made an error somewhere. In this book we will *always* carry units through all calculations, and we strongly urge you to follow this practice when you solve problems.

Problem-Solving Strategy 1.2 Unit Conversions

IDENTIFY *the relevant concepts:* Unit conversion is important, but it's also important to recognize when it's needed. In most cases, you're best off using the fundamental SI units (lengths in meters, masses in kilograms, and time in seconds) within a problem. If you need the answer to be in a different set of units (such as kilometers, grams, or hours), wait until the end of the problem to make the conversion. In the following examples, we'll concentrate on unit conversion alone, so we'll skip the *Identify* step.

SET UP *the problem* and **EXECUTE** *the solution:* Units are multiplied and divided just like ordinary algebraic symbols. This gives us an easy way to convert a quantity from one set of units to another. The key idea is to express the same physical quantity in two different units and form an equality.

For example, when we say that $1\,\text{min} = 60\,\text{s}$, we don't mean that the number 1 is equal to the number 60; rather, we mean that 1 min represents the same physical time interval as 60 s. For this reason, the ratio $(1\,\text{min})/(60\,\text{s})$ equals 1, as does its reciprocal $(60\,\text{s})/(1\,\text{min})$. We may multiply a quantity by either of these factors without changing that quantity's physical meaning. For example, to find the number of seconds in 3 min, we write

$$3\,\text{min} = (3\,\cancel{\text{min}})\left(\frac{60\,\text{s}}{1\,\cancel{\text{min}}}\right) = 180\,\text{s}$$

EVALUATE *your answer:* If you do your unit conversions correctly, unwanted units will cancel, as in the example above. If instead you had multiplied 3 min by $(1\,\text{min})/(60\,\text{s})$, your result would have been $\frac{1}{20}\,\text{min}^2/\text{s}$, which is a rather odd way of measuring time. To be sure you convert units properly, you must write down the units at *all* stages of the calculation.

Finally, check whether your answer is reasonable. Is the result $3\,\text{min} = 180\,\text{s}$ reasonable? The answer is yes; the second is a smaller unit than the minute, so there are more seconds than minutes in the same time interval.

Example 1.1 Converting speed units

The official world land speed record is 1228.0 km/h, set on October 15, 1997, by Andy Green in the jet engine car *Thrust SSC.* Express this speed in meters per second.

SOLUTION

IDENTIFY AND SET UP: We want to convert the units of a speed from km/h to m/s.

EXECUTE: The prefix k means 10^3, so the speed $1228.0\,\text{km/h} = 1228.0 \times 10^3\,\text{m/h}$. We also know that there are 3600 s in 1 h. So we must combine the speed of $1228.0 \times 10^3\,\text{m/h}$ and a factor of 3600. But should we multiply or divide by this factor? If we treat the factor as a pure number without units, we're forced to guess how to proceed.

The correct approach is to carry the units with each factor. We then arrange the factor so that the hour unit cancels:

$$1228.0\,\text{km/h} = \left(1228.0 \times 10^3\frac{\text{m}}{\cancel{\text{h}}}\right)\left(\frac{1\,\cancel{\text{h}}}{3600\,\text{s}}\right) = 341.11\,\text{m/s}$$

If you multiplied by $(3600\,\text{s})/(1\,\text{h})$ instead of $(1\,\text{h})/(3600\,\text{s})$, the hour unit wouldn't cancel, and you would be able to easily

Continued

recognize your error. Again, the *only* way to be sure that you correctly convert units is to carry the units throughout the calculation.

EVALUATE: While you probably have a good intuition for speeds in kilometers per hour or miles per hour, speeds in meters per second are likely to be a bit more mysterious. It helps to remember that a typical walking speed is about 1 m/s: the length of an average person's stride is about one meter, and a good walking pace is about one stride per second. By comparison, a speed of 341.11 m/s is rapid indeed!

Example 1.2 Converting volume units

The world's largest cut diamond is the First Star of Africa (mounted in the British Royal Sceptre and kept in the Tower of London). Its volume is 1.84 cubic inches. What is its volume in cubic centimeters? In cubic meters?

SOLUTION

IDENTIFY AND SET UP: Here we are to convert the units of a volume from cubic inches (in.^3) to cubic centimeters (cm^3) and cubic meters (m^3).

EXECUTE: To convert cubic inches to cubic centimeters, we multiply by $[(2.54\ \text{cm})/(1\ \text{in.})]^3$, not just $(2.54\ \text{cm})/(1\ \text{in.})$. We find

$$1.84\ \text{in.}^3 = (1.84\ \text{in.}^3)\left(\frac{2.54\ \text{cm}}{1\ \text{in.}}\right)^3$$

$$= (1.84)(2.54)^3\frac{\text{in.}^3\ \text{cm}^3}{\text{in.}^3} = 30.2\ \text{cm}^3$$

Also, $1\ \text{cm} = 10^{-2}\ \text{m}$, and

$$30.2\ \text{cm}^3 = (30.2\ \text{cm}^3)\left(\frac{10^{-2}\ \text{m}}{1\ \text{cm}}\right)^3$$

$$= (30.2)(10^{-2})^3\frac{\text{cm}^3\ \text{m}^3}{\text{cm}^3} = 30.2\times10^{-6}\ \text{m}^3$$

$$= 3.02\times10^{-5}\ \text{m}^3$$

EVALUATE: While 1 centimeter is 10^{-2} of a meter (that is, $1\ \text{cm} = 10^{-2}\ \text{m}$), our answer shows that a cubic centimeter $(1\ \text{cm}^3)$ is *not* 10^{-2} of a cubic meter. Rather, it is the volume of a cube whose sides are 1 cm long. So $1\ \text{cm}^3 = (1\ \text{cm})^3 = (10^{-2}\ \text{m})^3 = (10^{-2})^3\ \text{m}^3$, or $1\ \text{cm}^3 = 10^{-6}\ \text{m}^3$.

1.5 Uncertainty and Significant Figures

Measurements always have uncertainties. If you measure the thickness of the cover of this book using an ordinary ruler, your measurement is reliable only to the nearest millimeter, and your result will be 3 mm. It would be *wrong* to state this result as 3.00 mm; given the limitations of the measuring device, you can't tell whether the actual thickness is 3.00 mm, 2.85 mm, or 3.11 mm. But if you use a micrometer caliper, a device that measures distances reliably to the nearest 0.01 mm, the result will be 2.91 mm. The distinction between these two measurements is in their **uncertainty.** The measurement using the micrometer caliper has a smaller uncertainty; it's a more accurate measurement. The uncertainty is also called the **error** because it indicates the maximum difference there is likely to be between the measured value and the true value. The uncertainty or error of a measured value depends on the measurement technique used.

We often indicate the **accuracy** of a measured value—that is, how close it is likely to be to the true value—by writing the number, the symbol $\pm$, and a second number indicating the uncertainty of the measurement. If the diameter of a steel rod is given as 56.47 ± 0.02 mm, this means that the true value is unlikely to be less than 56.45 mm or greater than 56.49 mm. In a commonly used shorthand notation, the number 1.6454(21) means 1.6454 ± 0.0021. The numbers in parentheses show the uncertainty in the final digits of the main number.

We can also express accuracy in terms of the maximum likely **fractional error** or **percent error** (also called *fractional uncertainty* and *percent uncertainty*). A resistor labeled "47 ohms $\pm$ 10%" probably has a true resistance that differs from 47 ohms by no more than 10% of 47 ohms—that is, about 5 ohms. The resistance is probably between 42 and 52 ohms. For the diameter of the steel rod given above, the fractional error is $(0.02\ \text{mm})/(56.47\ \text{mm})$, or about 0.0004; the percent error is $(0.0004)(100\%)$, or about 0.04%. Even small percent errors can sometimes be very significant (Fig. 1.7).

1.7 This spectacular mishap was the result of a very small percent error—traveling a few meters too far in a journey of hundreds of thousands of meters.

In many cases the uncertainty of a number is not stated explicitly. Instead, the uncertainty is indicated by the number of meaningful digits, or **significant figures,** in the measured value. We gave the thickness of the cover of this book as 2.91 mm, which has three significant figures. By this we mean that the first two digits are known to be correct, while the third digit is uncertain. The last digit is in the hundredths place, so the uncertainty is about 0.01 mm. Two values with the *same* number of significant figures may have *different* uncertainties; a distance given as 137 km also has three significant figures, but the uncertainty is about 1 km.

When you use numbers having uncertainties to compute other numbers, the computed numbers are also uncertain. When numbers are multiplied or divided, the number of significant figures in the result can be no greater than in the factor with the fewest significant figures. For example, $3.1416 \times 2.34 \times 0.58 = 4.3$. When we add and subtract numbers, it's the location of the decimal point that matters, not the number of significant figures. For example, $123.62 + 8.9 = 132.5$. Although 123.62 has an uncertainty of about 0.01, 8.9 has an uncertainty of about 0.1. So their sum has an uncertainty of about 0.1 and should be written as 132.5, not 132.52. Table 1.1 summarizes these rules for significant figures.

Table 1.1 Using Significant Figures

Mathematical Operation	Significant Figures in Result
Multiplication or division	No more than in the number with the fewest significant figures *Example:* $(0.745 \times 2.2)/3.885 = 0.42$ *Example:* $(1.32578 \times 10^7) \times (4.11 \times 10^{-3}) = 5.45 \times 10^4$
Addition or subtraction	Determined by the number with the largest uncertainty (i.e., the fewest digits to the right of the decimal point) *Example:* $27.153 + 138.2 - 11.74 = 153.6$

Note: In this book we will usually give numerical values with three significant figures.

As an application of these ideas, suppose you want to verify the value of π, the ratio of the circumference of a circle to its diameter. The true value of this ratio to ten digits is 3.141592654. To test this, you draw a large circle and measure its circumference and diameter to the nearest millimeter, obtaining the values 424 mm and 135 mm (Fig. 1.8). You punch these into your calculator and obtain the quotient 3.140740741. This may seem to disagree with the true value of π, but keep in mind that each of your measurements has three significant figures, so your measured value of π, equal to $(424\text{ mm})/(135\text{ mm})$, can have only three significant figures. It should be stated simply as 3.14. Within the limit of three significant figures, your value does agree with the true value.

1.8 Determining the value of π from the circumference and diameter of a circle.

In the examples and problems in this book we usually give numerical values with three significant figures, so your answers should usually have no more than three significant figures. (Many numbers in the real world have even less accuracy. An automobile speedometer, for example, usually gives only two significant figures.) Even if you do the arithmetic with a calculator that displays ten digits, it would be wrong to give a ten-digit answer because it misrepresents the accuracy of the results. Always round your final answer to keep only the correct number of significant figures or, in doubtful cases, one more at most. In Example 1.1 it would have been wrong to state the answer as 341.11111 m/s. Note that when you reduce such an answer to the appropriate number of significant figures, you must *round,* not *truncate.* Your calculator will tell you that the ratio of 525 m to 311 m is 1.688102894; to three significant figures, this is 1.69, not 1.68.

When we calculate with very large or very small numbers, we can show significant figures much more easily by using **scientific notation,** sometimes called **powers-of-10 notation.** The distance from the earth to the moon is about 384,000,000 m, but writing the number in this form doesn't indicate the number of significant figures. Instead, we move the decimal point eight places to the left (corresponding to dividing by 10^8) and multiply by 10^8; that is,

$$384{,}000{,}000\text{ m} = 3.84 \times 10^8\text{ m}$$

In this form, it is clear that we have three significant figures. The number 4.00×10^{-7} also has three significant figures, even though two of them are zeros. Note that in scientific notation the usual practice is to express the quantity as a number between 1 and 10 multiplied by the appropriate power of 10.

When an integer or a fraction occurs in a general equation, we treat that number as having no uncertainty at all. For example, in the equation $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, which is Eq. (2.13) in Chapter 2, the coefficient 2 is *exactly* 2. We can consider this coefficient as having an infinite number of significant figures $(2.000000\ldots)$. The same is true of the exponent 2 in v_x^2 and v_{0x}^2.

Finally, let's note that **precision** is not the same as *accuracy*. A cheap digital watch that gives the time as 10:35:17 A.M. is very *precise* (the time is given to the second), but if the watch runs several minutes slow, then this value isn't very *accurate*. On the other hand, a grandfather clock might be very accurate (that is, display the correct time), but if the clock has no second hand, it isn't very precise. A high-quality measurement, like those used to define standards (see Section 1.3), is both precise *and* accurate.

Example 1.3 Significant figures in multiplication

The rest energy E of an object with rest mass m is given by Einstein's equation

$$E = mc^2$$

where c is the speed of light in a vacuum. Find E for an object with $m = 9.11 \times 10^{-31}$ kg (to three significant figures, the mass of an electron). The SI unit for E is the joule (J); $1\text{ J} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2$.

SOLUTION

IDENTIFY AND SET UP: Our target variable is the energy E. We are given the equation to use and the value of the mass m; from Section 1.3 the exact value of the speed of light is $c = 299{,}792{,}458\text{ m/s} = 2.99792458 \times 10^8\text{ m/s}$.

EXECUTE: Substituting the values of m and c into Einstein's equation, we find

$$\begin{aligned} E &= (9.11 \times 10^{-31}\text{ kg})(2.99792458 \times 10^8\text{ m/s})^2 \\ &= (9.11)(2.99792458)^2(10^{-31})(10^8)^2\text{ kg}\cdot\text{m}^2/\text{s}^2 \\ &= (81.87659678)(10^{[-31+(2\times 8)]})\text{ kg}\cdot\text{m}^2/\text{s}^2 \\ &= 8.187659678 \times 10^{-14}\text{ kg}\cdot\text{m}^2/\text{s}^2 \end{aligned}$$

Since the value of m was given to only three significant figures, we must round this to

$$E = 8.19 \times 10^{-14}\text{ kg}\cdot\text{m}^2/\text{s}^2 = 8.19 \times 10^{-14}\text{ J}$$

Most calculators use scientific notation and add exponents automatically, but you should be able to do such calculations by hand when necessary.

EVALUATE: While the rest energy contained in an electron may seem ridiculously small, on the atomic scale it is tremendous. Compare our answer to 10^{-19} J, the energy gained or lost by a single atom during a typical chemical reaction; the rest energy of an electron is about 1,000,000 times larger! (We will discuss the significance of rest energy in Chapter 37.)

Test Your Understanding of Section 1.5 The density of a material is equal to its mass divided by its volume. What is the density (in kg/m^3) of a rock of mass 1.80 kg and volume $6.0 \times 10^{-4}\text{ m}^3$? (i) $3 \times 10^3\text{ kg/m}^3$; (ii) $3.0 \times 10^3\text{ kg/m}^3$; (iii) $3.00 \times 10^3\text{ kg/m}^3$; (iv) $3.000 \times 10^3\text{ kg/m}^3$; (v) any of these—all of these answers are mathematically equivalent.

1.6 Estimates and Orders of Magnitude

We have stressed the importance of knowing the accuracy of numbers that represent physical quantities. But even a very crude estimate of a quantity often gives us useful information. Sometimes we know how to calculate a certain quantity, but we have to guess at the data we need for the calculation. Or the calculation might be too complicated to carry out exactly, so we make some rough approximations. In either case our result is also a guess, but such a guess can be useful even if it is uncertain by a factor of two, ten, or more. Such calculations are often called **order-of-magnitude estimates.** The great Italian-American nuclear physicist Enrico Fermi (1901–1954) called them "back-of-the-envelope calculations."

Exercises 1.18 through 1.29 at the end of this chapter are of the estimating, or "order-of-magnitude," variety. Some are silly, and most require guesswork for the needed input data. Don't try to look up a lot of data; make the best guesses you can. Even when they are off by a factor of ten, the results can be useful and interesting.

Example 1.4 An order-of-magnitude estimate

You are writing an adventure novel in which the hero escapes across the border with a billion dollars' worth of gold in his suitcase. Is this possible? Would that amount of gold fit in a suitcase? Would it be too heavy to carry?

SOLUTION

IDENTIFY, SET UP, AND EXECUTE: Gold sells for around \$400 an ounce. On a particular day the price might be \$200 or \$600, but never mind. An ounce is about 30 grams. Actually, an ordinary (avoirdupois) ounce is 28.35 g; an ounce of gold is a troy ounce, which is 9.45% more. Again, never mind. Ten dollars' worth of gold has a mass somewhere around one gram, so a billion (10^9) dollars' worth of gold is a hundred million (10^8) grams, or a hundred thousand (10^5) kilograms. This corresponds to a weight in British units of around 200,000 lb, or 100 tons. Whether the precise number is closer to 50 tons or 200 tons doesn't matter. Either way, the hero is not about to carry it across the border in a suitcase.

We can also estimate the *volume* of this gold. If its density were the same as that of water $(1\ \text{g/cm}^3)$, the volume would be $10^8\ \text{cm}^3$, or $100\ \text{m}^3$. But gold is a heavy metal; we might guess its density to be 10 times that of water. Gold is actually 19.3 times as dense as water. But by guessing 10, we find a volume of $10\ \text{m}^3$. Visualize 10 cubical stacks of gold bricks, each 1 meter on a side, and ask yourself whether they would fit in a suitcase!

EVALUATE: Clearly, your novel needs rewriting. Try the calculation again with a suitcase full of five-carat (1-gram) diamonds, each worth \$100,000. Would this work?

Test Your Understanding of Section 1.6 Can you estimate the total number of teeth in all the mouths of everyone (students, staff, and faculty) on your campus? (*Hint:* How many teeth are in your mouth? Count them!)

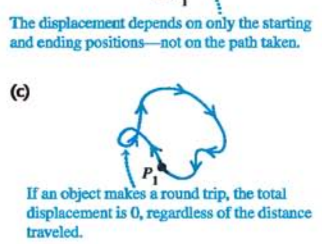
1.7 Vectors and Vector Addition

Some physical quantities, such as time, temperature, mass, and density, can be described completely by a single number with a unit. But many other important quantities in physics have a *direction* associated with them and cannot be described by a single number. A simple example is the motion of an airplane. To describe this motion completely, we must say not only how fast the plane is moving, but also in what direction. To fly from Chicago to New York, a plane has to head east, not south. The speed of the airplane combined with its direction of motion together constitute a quantity called *velocity*. Another example is *force*, which in physics means a push or pull exerted on a body. Giving a complete description of a force means describing both how hard the force pushes or pulls on the body and the direction of the push or pull.

When a physical quantity is described by a single number, we call it a **scalar quantity.** In contrast, a **vector quantity** has both a **magnitude** (the "how much" or "how big" part) and a direction in space. Calculations that combine scalar quantities use the operations of ordinary arithmetic. For example, $6\ \text{kg} + 3\ \text{kg} = 9\ \text{kg}$, or $4 \times 2\ \text{s} = 8\ \text{s}$. However, combining vectors requires a different set of operations.

To understand more about vectors and how they combine, we start with the simplest vector quantity, **displacement.** Displacement is simply a change in position of a point. (The point may represent a particle or a small body.) In Fig. 1.9a we represent the change of position from point P_1 to point P_2 by a line from P_1 to P_2, with an arrowhead at P_2 to represent the direction of motion. Displacement is a vector quantity because we must state not only how far the particle moves, but also in what direction. Walking 3 km north from your front door doesn't get you

1.9 Displacement as a vector quantity. A displacement is always a straight-line segment directed from the starting point to the ending point, even if the path is curved.

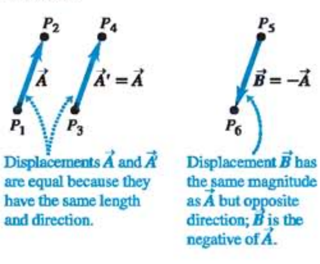


to the same place as walking 3 km southeast; these two displacements have the same magnitude, but different directions.

We usually represent a vector quantity such as displacement by a single letter, such as $\vec{A}$ in Fig. 1.9a. In this book we always print vector symbols in ***boldface italic type with an arrow above them.*** We do this to remind you that vector quantities have different properties from scalar quantities; the arrow is a reminder that vectors have direction. In handwriting, vector symbols are usually underlined or written with an arrow above them (see Fig. 1.9a). When you write a symbol for a vector, *always* write it with an arrow on top. If you don't distinguish between scalar and vector quantities in your notation, you probably won't make the distinction in your thinking either, and hopeless confusion will result.

We always *draw* a vector as a line with an arrowhead at its tip. The length of the line shows the vector's magnitude, and the direction of the line shows the vector's direction. Displacement is always a straight-line segment, directed from the starting point to the ending point, even though the actual path of the particle may be curved. In Fig. 1.9b the particle moves along the curved path shown from P_1 to P_2, but the displacement is still the vector $\vec{A}$. Note that displacement is not related directly to the total *distance* traveled. If the particle were to continue on past P_2 and then return to P_1, the displacement for the entire trip would be *zero* (Fig. 1.9c).

1.10 The meaning of vectors that have the same magnitude and the same or opposite direction.



If two vectors have the same direction, they are **parallel.** If they have the same magnitude *and* the same direction, they are *equal,* no matter where they are located in space. The vector $\vec{A}'$ from point P_3 to point P_4 in Fig. 1.10 has the same length and direction as the vector $\vec{A}$ from P_1 to P_2. These two displacements are equal, even though they start at different points. We write this as $\vec{A}' = \vec{A}$ in Fig. 1.10; the boldface equals sign emphasizes that equality of two vector quantities is not the same relationship as equality of two scalar quantities. Two vector quantities are equal only when they have the same magnitude *and* the same direction.

The vector $\vec{B}$ in Fig. 1.10, however, is not equal to $\vec{A}$ because its direction is *opposite* to that of $\vec{A}$. We define the **negative of a vector** as a vector having the same magnitude as the original vector but the *opposite* direction. The negative of vector quantity $\vec{A}$ is denoted as $-\vec{A}$, and we use a boldface minus sign to emphasize the vector nature of the quantities. If $\vec{A}$ is 87 m south, then $-\vec{A}$ is 87 m north. Thus we can write the relationship between $\vec{A}$ and $\vec{B}$ in Fig. 1.10 as $\vec{A} = -\vec{B}$ or $\vec{B} = -\vec{A}$. When two vectors $\vec{A}$ and $\vec{B}$ have opposite directions, whether their magnitudes are the same or not, we say that they are **antiparallel.**

We usually represent the *magnitude* of a vector quantity (in the case of a displacement vector, its length) by the same letter used for the vector, but in *light italic type* with *no* arrow on top, rather than boldface italic with an arrow (which is reserved for vectors). An alternative notation is the vector symbol with vertical bars on both sides:

$$(\text{Magnitude of } \vec{A}) = A = |\vec{A}| \tag{1.1}$$

By definition the magnitude of a vector quantity is a scalar quantity (a number) and is *always positive.* We also note that a vector can never be equal to a scalar because they are different kinds of quantities. The expression "$\vec{A} = 6$ m" is just as wrong as "2 oranges = 3 apples" or "6 lb = 7 km"!

When drawing diagrams with vectors, we'll generally use a scale similar to those used for maps. For example, a displacement of 5 km might be represented in a diagram by a vector 1 cm long, and a displacement of 10 km by a vector 2 cm long. In a diagram for velocity vectors, we might use a scale in which a vector that is 1 cm long represents a velocity of magnitude 5 meters per second (5 m/s). A velocity of 20 m/s would then be represented by a vector 4 cm long, with the appropriate direction.

Vector Addition

Suppose a particle undergoes a displacement $\vec{A}$ followed by a second displacement $\vec{B}$ (Fig. 1.11a). The final result is the same as if the particle had started at the same initial point and undergone a single displacement $\vec{C}$, as shown. We call displacement $\vec{C}$ the **vector sum, or resultant,** of displacements $\vec{A}$ and $\vec{B}$. We express this relationship symbolically as

$$\vec{C} = \vec{A} + \vec{B} \tag{1.2}$$

The boldface plus sign emphasizes that adding two vector quantities requires a geometrical process and is not the same operation as adding two scalar quantities such as $2 + 3 = 5$. In vector addition we usually place the *tail* of the *second* vector at the *head,* or tip, of the *first* vector (Fig. 1.11a).

If we make the displacements $\vec{A}$ and $\vec{B}$ in reverse order, with $\vec{B}$ first and $\vec{A}$ second, the result is the same (Fig. 1.11b). Thus

$$\vec{C} = \vec{B} + \vec{A} \quad \text{and} \quad \vec{A} + \vec{B} = \vec{B} + \vec{A} \tag{1.3}$$

This shows that the order of terms in a vector sum doesn't matter. In other words, vector addition obeys the commutative law.

Figure 1.11c shows another way to represent the vector sum: If vectors $\vec{A}$ and $\vec{B}$ are both drawn with their tails at the same point, vector $\vec{C}$ is the diagonal of a parallelogram constructed with $\vec{A}$ and $\vec{B}$ as two adjacent sides.

CAUTION **Magnitudes in vector addition** It's a common error to conclude that if $\vec{C} = \vec{A} + \vec{B}$, then the magnitude C should just equal the magnitude A plus the magnitude B. In general, this conclusion is *wrong;* for the vectors shown in Fig. 1.11, you can see that $C < A + B$. The magnitude of $\vec{A} + \vec{B}$ depends on the magnitudes of $\vec{A}$ and $\vec{B}$ *and* on the angle between $\vec{A}$ and $\vec{B}$ (see Problem 1.92). Only in the special case in which $\vec{A}$ and $\vec{B}$ are *parallel* is the magnitude of $\vec{C} = \vec{A} + \vec{B}$ equal to the sum of the magnitudes of $\vec{A}$ and $\vec{B}$ (Fig. 1.12a). By contrast, when the vectors are *antiparallel* (Fig. 1.12b) the magnitude of $\vec{C}$ equals the *difference* of the magnitudes of $\vec{A}$ and $\vec{B}$. If you're careful about distinguishing between scalar and vector quantities, you'll avoid making errors about the magnitude of a vector sum.

When we need to add more than two vectors, we may first find the vector sum of any two, add this vectorially to the third, and so on. Figure 1.13a shows three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$. In Fig. 1.13b, we first add $\vec{A}$ and $\vec{B}$ to give a vector sum $\vec{D}$; we then add vectors $\vec{C}$ and $\vec{D}$ by the same process to obtain the vector sum $\vec{R}$:

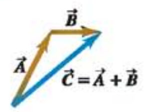
$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C} = \vec{D} + \vec{C}$$

Alternatively, we can first add $\vec{B}$ and $\vec{C}$ to obtain vector $\vec{E}$ (Fig. 1.13c), and then add $\vec{A}$ and $\vec{E}$ to obtain $\vec{R}$:

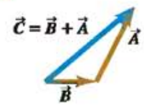
$$\vec{R} = \vec{A} + (\vec{B} + \vec{C}) = \vec{A} + \vec{E}$$

1.11 Three ways to add two vectors. As shown in (b), the order in vector addition doesn't matter; vector addition is commutative.

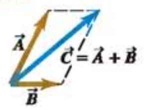
(a) We can add two vectors by placing them head to tail.



(b) Adding them in reverse order gives the same result.



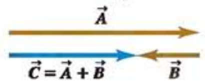
(c) We can also add them by constructing a parallelogram.



1.12 (a) Only when two vectors $\vec{A}$ and $\vec{B}$ are parallel does the magnitude of their sum equal the sum of their magnitudes: $C = A + B$. (b) When $\vec{A}$ and $\vec{B}$ are antiparallel, the magnitude of their sum equals the *difference* of their magnitudes: $C = |A - B|$.

(a) The sum of two parallel vectors

(b) The sum of two antiparallel vectors



1.13 Several constructions for finding the vector sum $\vec{A} + \vec{B} + \vec{C}$.

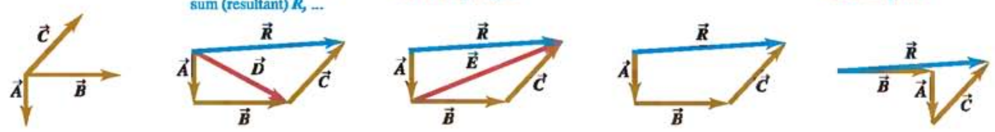
(a) To find the sum of these three vectors ...

(b) we could add $\vec{A}$ and $\vec{B}$ to get $\vec{D}$ and then add $\vec{C}$ to $\vec{D}$ to get the final sum (resultant) $\vec{R}$, ...

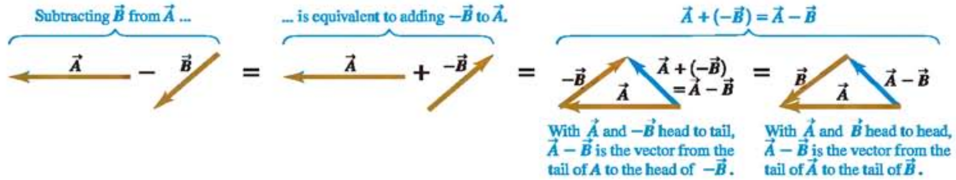
(c) or we could add $\vec{B}$ and $\vec{C}$ to get $\vec{E}$ and then add $\vec{A}$ to $\vec{E}$ to get $\vec{R}$, ...

(d) or we could add $\vec{A}$, $\vec{B}$, and $\vec{C}$ to get $\vec{R}$ directly, ...

(e) or we could add $\vec{A}$, $\vec{B}$, and $\vec{C}$ in any other order and still get $\vec{R}$.



1.14 To construct the vector difference $\vec{A} - \vec{B}$, you can either place the tail of $-\vec{B}$ at the head of $\vec{A}$ or place the two vectors $\vec{A}$ and $\vec{B}$ head to head.



We don't even need to draw vectors $\vec{D}$ and $\vec{E}$; all we need to do is draw $\vec{A}$, $\vec{B}$, and $\vec{C}$ in succession, with the tail of each at the head of the one preceding it. The sum vector $\vec{R}$ extends from the tail of the first vector to the head of the last vector (Fig. 1.13d). The order makes no difference; Fig. 1.13e shows a different order, and we invite you to try others. We see that vector addition obeys the associative law.

We can *subtract* vectors as well as add them. To see how, recall that the vector $-\vec{A}$ has the same magnitude as $\vec{A}$ but the opposite direction. We define the difference $\vec{A} - \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ to be the vector sum of $\vec{A}$ and $-\vec{B}$:

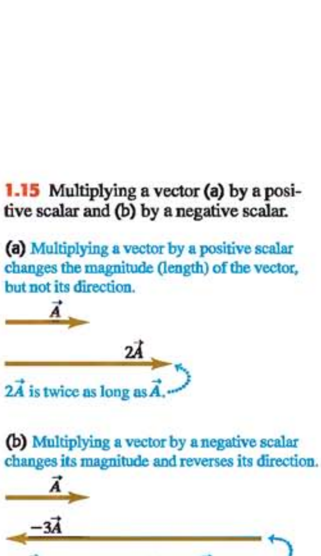
$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \tag{1.4}$$

Figure 1.14 shows an example of vector subtraction.

A vector quantity such as a displacement can be multiplied by a scalar quantity (an ordinary number). The displacement $2\vec{A}$ is a displacement (vector quantity) in the same direction as the vector $\vec{A}$ but twice as long; this is the same as adding $\vec{A}$ to itself (Fig. 1.15a). In general, when a vector $\vec{A}$ is multiplied by a scalar c, the result $c\vec{A}$ has magnitude $|c|A$ (the absolute value of c multiplied by the magnitude of the vector $\vec{A}$). If c is positive, $c\vec{A}$ is in the same direction as $\vec{A}$; if c is negative, $c\vec{A}$ is in the direction opposite to $\vec{A}$. Thus $3\vec{A}$ is parallel to $\vec{A}$, while $-3\vec{A}$ is antiparallel to $\vec{A}$ (Fig. 1.15b).

The scalar quantity used to multiply a vector may also be a physical quantity having units. For example, you may be familiar with the relationship $\vec{F} = m\vec{a}$; the net force $\vec{F}$ (a vector quantity) that acts on a body is equal to the product of the body's mass m (a positive scalar quantity) and its acceleration $\vec{a}$ (a vector quantity). The direction of $\vec{F}$ is the same as that of $\vec{a}$ because m is positive, and the magnitude of $\vec{F}$ is equal to the mass m (which is positive and equals its own absolute value) multiplied by the magnitude of $\vec{a}$. The unit of force is the unit of mass multiplied by the unit of acceleration.

1.15 Multiplying a vector (a) by a positive scalar and (b) by a negative scalar.



Example 1.5 Vector addition

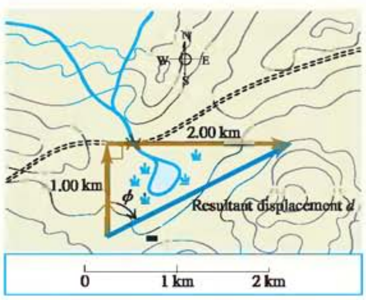
A cross-country skier skis 1.00 km north and then 2.00 km east on a horizontal snow field. How far and in what direction is she from the starting point?

SOLUTION

IDENTIFY: The problem involves combining displacements, so we can solve it using vector addition. The target variables are the skier's total distance and direction from her starting point. The distance is just the magnitude of her resultant displacement vector from the point of origin to where she stops, and the direction we want is the direction of the resultant displacement vector.

SET UP: Figure 1.16 is a scale diagram of the skier's displacements. We describe the direction from the starting point by the angle ϕ (the Greek letter phi). By careful measurement we find that the distance from the starting point to the ending point is about 2.2 km and that

1.16 The vector diagram, drawn to scale, for a cross-country ski trip.



ϕ is about 63°. But we can *calculate* a much more accurate result by adding the 1.00-km and 2.00-km displacement vectors.

EXECUTE: The vectors in the diagram form a right triangle; the distance from the starting point to the ending point is equal to the length of the hypotenuse. We find this length by using the Pythagorean theorem:

$$\sqrt{(1.00\text{ km})^2 + (2.00\text{ km})^2} = 2.24\text{ km}$$

The angle ϕ can be found with a little simple trigonometry. If you need a review, the trigonometric functions and identities are summarized in Appendix B, along with other useful mathematical and geometrical relationships. By the definition of the tangent function,

$$\tan\phi = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{2.00\text{ km}}{1.00\text{ km}}$$

$$\phi = 63.4°$$

We can describe the direction as 63.4° east of north or $90° - 63.4° = 26.6°$ north of east. Take your choice!

EVALUATE: It's good practice to check the results of a vector-addition problem by making measurements on a drawing of the situation. Happily, the answers we found by calculation (2.24 km and $\phi = 63.4°$) are very close to the cruder results we found by measurement (about 2.2 km and about 63°). If they were substantially different, we would have to go back and check for errors.

Test Your Understanding of Section 1.7 Two displacement vectors, $\vec{S}$ and $\vec{T}$, have magnitudes $S = 3$ m and $T = 4$ m. Which of the following could be the magnitude of the difference vector $\vec{S} - \vec{T}$? (There may be more than one correct answer.) (i) 9 m; (ii) 7 m; (iii) 5 m; (iv) 1 m; (v) 0 m; (vi) -1 m.

1.8 Components of Vectors

In Section 1.7 we added vectors by using a scale diagram and by using properties of right triangles. Measuring a diagram offers only very limited accuracy, and calculations with right triangles work only when the two vectors are perpendicular. So we need a simple but general method for adding vectors. This is called the method of *components*.

To define what we mean by the components of a vector $\vec{A}$, we begin with a rectangular (Cartesian) coordinate system of axes (Fig. 1.17a). We then draw the vector with its tail at O, the origin of the coordinate system. We can represent any vector lying in the xy-plane as the sum of a vector parallel to the x-axis and a vector parallel to the y-axis. These two vectors are labeled $\vec{A}_x$ and $\vec{A}_y$ in Fig. 1.17a; they are called the **component vectors** of vector $\vec{A}$, and their vector sum is equal to $\vec{A}$. In symbols,

$$\vec{A} = \vec{A}_x + \vec{A}_y \qquad (1.5)$$

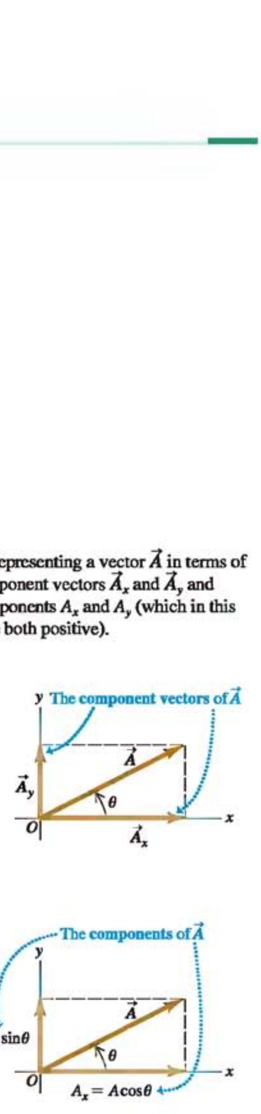
Since each component vector lies along a coordinate-axis direction, we need only a single number to describe each one. When the component vector $\vec{A}_x$ points in the positive x-direction, we define the number A_x to be equal to the magnitude of $\vec{A}_x$. When the component vector $\vec{A}_x$ points in the negative x-direction, we define the number A_x to be equal to the negative of that magnitude (the magnitude of a vector quantity is itself never negative). We define the number A_y in the same way. The two numbers A_x and A_y are called the **components** of $\vec{A}$ (Fig. 1.17b).

CAUTION **Components are not vectors** The components A_x and A_y of a vector $\vec{A}$ are just numbers; they are *not* vectors themselves. This is why we print the symbols for components in light italic type with *no* arrow on top instead of the boldface italic with an arrow, which is reserved for vectors.

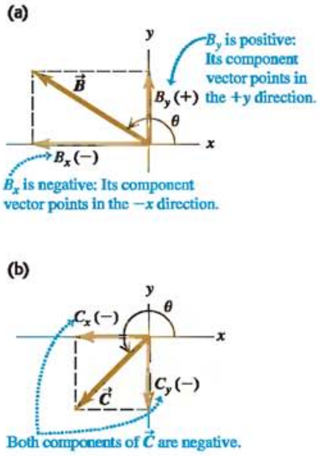
We can calculate the components of the vector $\vec{A}$ if we know its magnitude A and its direction. We'll describe the direction of a vector by its angle relative to some reference direction. In Fig. 1.17b this reference direction is the

1.17 Representing a vector $\vec{A}$ in terms of (a) component vectors $\vec{A}_x$ and $\vec{A}_y$ and (b) components A_x and A_y (which in this case are both positive).

(a)



1.18 The components of a vector may be positive or negative numbers.



positive x-axis, and the angle between vector $\vec{A}$ and the positive x-axis is θ (the Greek letter theta). Imagine that the vector $\vec{A}$ originally lies along the $+x$-axis and that you then rotate it to its correct direction, as indicated by the arrow in Fig. 1.17b on the angle θ. If this rotation is from the $+x$-axis toward the $+y$-axis, as shown in Fig. 1.17b, then θ is *positive;* if the rotation is from the $+x$-axis toward the $-y$-axis, θ is *negative.* Thus the $+y$-axis is at an angle of 90°, the $-x$-axis at 180°, and the $-y$-axis at 270° (or $-90°$). If θ is measured in this way, then from the definition of the trigonometric functions,

$$\frac{A_x}{A} = \cos\theta \quad \text{and} \quad \frac{A_y}{A} = \sin\theta$$
$$A_x = A\cos\theta \quad \text{and} \quad A_y = A\sin\theta \qquad (1.6)$$
$$(\theta \text{ measured from the } +x\text{-axis, rotating toward the } +y\text{-axis})$$

In Fig. 1.17b, A_x is positive because its direction is along the positive x-axis, and A_y is positive because its direction is along the positive y-axis. This is consistent with Eqs. (1.6); θ is in the first quadrant (between 0° and 90°), and both the cosine and the sine of an angle in this quadrant are positive. But in Fig. 1.18a the component B_x is negative; its direction is opposite to that of the positive x-axis. Again, this agrees with Eqs. (1.6); the cosine of an angle in the second quadrant is negative. The component B_y is positive ($\sin\theta$ is positive in the second quadrant). In Fig. 1.18b, both C_x and C_y are negative (both $\cos\theta$ and $\sin\theta$ are negative in the third quadrant).

CAUTION **Relating a vector's magnitude and direction to its components** Equations (1.6) are correct *only* when the angle θ is measured from the positive x-axis as described above. If the angle of the vector is given from a different reference direction or using a different sense of rotation, the relationships are different. Be careful! Example 1.6 illustrates this point.

Example 1.6 Finding components

(a) What are the x- and y-components of vector $\vec{D}$ in Fig. 1.19a? The magnitude of the vector is $D = 3.00$ m and the angle $\alpha = 45°$. (b) What are the x- and y-components of vector $\vec{E}$ in Fig. 1.19b? The magnitude of the vector is $E = 4.50$ m and the angle $\beta = 37.0°$.

SOLUTION

IDENTIFY: In each case we are given the magnitude and direction of a vector, and we are asked to find its components.

1.19 Calculating the x- and y-components of vectors.

SET UP: It would seem that all we need is Eqs. (1.6). However, we need to be careful because the angles in Fig. 1.19 are *not* measured from the $+x$-axis toward the $+y$-axis.

EXECUTE: (a) The angle between $\vec{D}$ and the positive x-axis is α (the Greek letter alpha), but this angle is measured toward the *negative* y-axis. So the angle we must use in Eqs. (1.6) is $\theta = -\alpha = -45°$. We find

$$D_x = D\cos\theta = (3.00\text{ m})(\cos(-45°)) = +2.1\text{ m}$$
$$D_y = D\sin\theta = (3.00\text{ m})(\sin(-45°)) = -2.1\text{ m}$$

The vector has a positive x-component and a negative y-component, as shown in the figure. Had you been careless and substituted $+45°$ for θ in Eqs. (1.6), you would have gotten the wrong sign for D_y.

(b) The x-axis isn't horizontal in Fig. 1.19b, nor is the y-axis vertical. Don't worry, though: *Any* orientation of the x- and y-axes is permissible, just so the axes are mutually perpendicular. (In Chapter 5 we'll use axes like these to study an object sliding on an incline; one axis will lie along the incline and the other will be perpendicular to the incline.)

Here the angle β (the Greek letter beta) is the angle between $\vec{E}$ and the positive y-axis, *not* the positive x-axis, so we *cannot* use this angle in Eqs. (1.6). Instead, note that $\vec{E}$ defines the hypotenuse of a right triangle; the other two sides of the triangle are the magnitudes of E_x and E_y, the x- and y-components of $\vec{E}$. The sine of β is the opposite side (the magnitude of E_x) divided by the hypotenuse (the magnitude E), and the cosine of β is the adjacent side (the magnitude of E_y) divided by the hypotenuse (again, the magnitude E). Both components of $\vec{E}$ are positive, so

$$E_x = E\sin\beta = (4.50\ \text{m})(\sin 37.0°) = +2.71\ \text{m}$$
$$E_y = E\cos\beta = (4.50\ \text{m})(\cos 37.0°) = +3.59\ \text{m}$$

Had you used Eqs. (1.6) directly and written $E_x = E\cos 37.0°$ and $E_y = E\sin 37.0°$, your answers for E_x and E_y would have been reversed!

If you insist on using Eqs. (1.6), you must first find the angle between $\vec{E}$ and the positive x-axis, measured toward the positive y-axis; this is $\theta = 90.0° - \beta = 90.0° - 37.0° = 53.0°$. Then $E_x = E\cos\theta$ and $E_y = E\sin\theta$. You can substitute the values of E and θ into Eqs. (1.6) to show that the results for E_x and E_y are the same as those given above.

EVALUATE: Notice that the answers to part (b) have three significant figures, but the answers to part (a) have only two. Can you see why?

Doing Vector Calculations Using Components

Using components makes it relatively easy to do various calculations involving vectors. Let's look at three important examples.

1. Finding a vector's magnitude and direction from its components. We can describe a vector completely by giving either its magnitude and direction or its x- and y-components. Equations (1.6) show how to find the components if we know the magnitude and direction. We can also reverse the process: We can find the magnitude and direction if we know the components. By applying the Pythagorean theorem to Fig. 1.17b, we find that the magnitude of vector $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2} \qquad (1.7)$$

(We always take the positive root.) Equation (1.7) is valid for any choice of x-axis and y-axis, as long as they are mutually perpendicular. The expression for the vector direction comes from the definition of the tangent of an angle. If θ is measured from the positive x-axis, and a positive angle is measured toward the positive y-axis (as in Fig. 1.17b), then

$$\tan\theta = \frac{A_y}{A_x} \quad \text{and} \quad \theta = \arctan\frac{A_y}{A_x} \qquad (1.8)$$

We will always use the notation arctan for the inverse tangent function. The notation $\tan^{-1}$ is also commonly used, and your calculator may have an INV or 2ND button to be used with the TAN button.

CAUTION **Finding the direction of a vector from its components** There is one slight complication in using Eqs. (1.8) to find θ. Suppose $A_x = 2$ m and $A_y = -2$ m as in Fig. 1.20; then $\tan\theta = -1$. But there are two angles that have tangents of -1—namely, 135° and 315° (or $-45°$). In general, any two angles that differ by 180° have the same tangent. To decide which is correct, we have to look at the individual components. Because A_x is positive and A_y is negative, the angle must be in the fourth quadrant; thus $\theta = 315°$ (or $-45°$) is the correct value. Most pocket calculators give $\arctan(-1) = -45°$. In this case that is correct; but if instead we have $A_x = -2$ m and $A_y = 2$ m, then the correct angle is 135°. Similarly, when A_x and A_y are both negative, the tangent is positive, but the angle is in the third quadrant. You should *always* draw a sketch like Fig. 1.20 to check which of the two possibilities is the correct one.

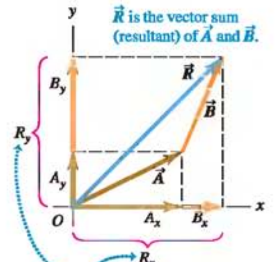
1.20 Drawing a sketch of a vector reveals the signs of its x- and y-components.

2. Multiplying a vector by a scalar. If we multiply a vector $\vec{A}$ by a scalar c, each component of the product $\vec{D} = c\vec{A}$ is just the product of c and the corresponding component of $\vec{A}$:

$$D_x = cA_x \quad D_y = cA_y \quad (\text{components of } \vec{D} = c\vec{A}) \qquad (1.9)$$

For example, Eq. (1.9) says that each component of the vector $2\vec{A}$ is twice as great as the corresponding component of the vector $\vec{A}$, so $2\vec{A}$ is in the same direction as $\vec{A}$ but has twice the magnitude. Each component of the vector $-3\vec{A}$ is three times as great as the corresponding component of the vector $\vec{A}$ but has the opposite sign, so $-3\vec{A}$ is in the opposite direction from $\vec{A}$ and has three times the magnitude. Hence Eqs. (1.9) are consistent with our discussion in Section 1.7 of multiplying a vector by a scalar (see Fig. 1.15).

1.21 Finding the vector sum (resultant) of $\vec{A}$ and $\vec{B}$ using components.



3. Using components to calculate the vector sum (resultant) of two or more vectors. Figure 1.21 shows two vectors $\vec{A}$ and $\vec{B}$ and their vector sum $\vec{R}$, along with the x- and y-components of all three vectors. You can see from the diagram that the x-component R_x of the vector sum is simply the sum $(A_x + B_x)$ of the x-components of the vectors being added. The same is true for the y-components. In symbols,

$$R_x = A_x + B_x \qquad R_y = A_y + B_y \qquad (\text{components of } \vec{R} = \vec{A} + \vec{B}) \tag{1.10}$$

Figure 1.21 shows this result for the case in which the components A_x, A_y, B_x, and B_y are all positive. You should draw additional diagrams to verify for yourself that Eqs. (1.10) are valid for *any* signs of the components of $\vec{A}$ and $\vec{B}$.

If we know the components of any two vectors $\vec{A}$ and $\vec{B}$, perhaps by using Eqs. (1.6), we can compute the components of the vector sum $\vec{R}$. Then if we need the magnitude and direction of $\vec{R}$, we can obtain them from Eqs. (1.7) and (1.8) with the A's replaced by R's.

We can extend this procedure to find the sum of any number of vectors. If $\vec{R}$ is the vector sum of $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}, \ldots$, the components of $\vec{R}$ are

$$\begin{aligned} R_x &= A_x + B_x + C_x + D_x + E_x + \cdots \\ R_y &= A_y + B_y + C_y + D_y + E_y + \cdots \end{aligned} \tag{1.11}$$

We have talked only about vectors that lie in the xy-plane, but the component method works just as well for vectors having any direction in space. We introduce a z-axis perpendicular to the xy-plane; then in general a vector $\vec{A}$ has components A_x, A_y, and A_z in the three coordinate directions. The magnitude A is given by

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \tag{1.12}$$

Again, we always take the positive root. Also, Eqs. (1.11) for the components of the vector sum $\vec{R}$ have an additional member:

$$R_z = A_z + B_z + C_z + D_z + E_z + \cdots$$

Finally, while our discussion of vector addition has centered on combining *displacement* vectors, the method is applicable to all other vector quantities as well. When we study the concept of force in Chapter 4, we'll find that forces are vectors that obey the same rules of vector addition that we've used with displacement. Other vector quantities will make their appearance in later chapters.

Problem-Solving Strategy 1.3 Vector Addition

MP

IDENTIFY *the relevant concepts:* Decide what your target variable is. It may be the magnitude of the vector sum, the direction, or both.

SET UP *the problem:* Draw the individual vectors being summed and the coordinate axes being used. In your drawing, place the tail of the first vector at the origin of coordinates; place the tail of the second vector at the head of the first vector; and so on. Draw the vector sum $\vec{R}$ from the tail of the first vector to the head of the last vector. Use your drawing to make rough estimates of the magnitude and direction of $\vec{R}$; you'll use these estimates later to check your calculations.

EXECUTE *the solution* as follows:

1. Find the x- and y-components of each individual vector and record your results in a table. If a vector is described by its magnitude A and its angle θ, measured from the $+x$-axis toward the $+y$-axis, then the components are given by

$$A_x = A\cos\theta \qquad A_y = A\sin\theta$$

Some components may be positive and some may be negative, depending on how the vector is oriented (that is, what quadrant θ lies in). You can use this sign table as a check:

Quadrant	I	II	III	IV
A_x	+	−	−	+
A_y	+	+	−	−

If the angles of the vectors are given in some other way, perhaps using a different reference direction, convert them to angles measured from the $+x$-axis as described above. Be particularly careful with signs.

2. Add the individual x-components algebraically, including signs, to find R_x, the x-component of the vector sum. Do the same for the y-components to find R_y.

3. Then the magnitude R and direction θ of the vector sum are given by

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \arctan\frac{R_y}{R_x}$$

EVALUATE *your answer:* Check your results for the magnitude and direction of the vector sum by comparing them with the rough estimates you made from your drawing. Remember that the magnitude R is *always* positive and that θ is measured from the positive x-axis. The value of θ that you find with a calculator may be the correct one, or it may be off by 180°. You can decide by examining your drawing.

If your calculations disagree totally with the estimates from your drawing, check whether your calculator is set in "radians" or "degrees" mode. If it's in "radians" mode, entering angles in degrees will give nonsensical answers.

Example 1.7 Adding vectors with components

Three players on a reality TV show are brought to the center of a large, flat field. Each is given a meter stick, a compass, a calculator, a shovel, and (in a different order for each contestant) the following three displacements:

72.4 m, 32.0° east of north
57.3 m, 36.0° south of west
17.8 m straight south

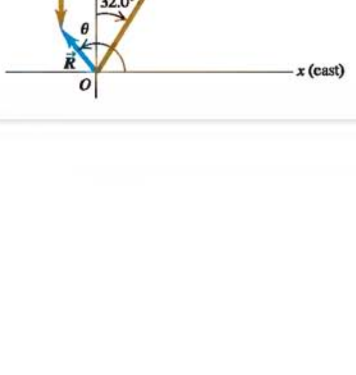
The three displacements lead to the point where the keys to a new Porsche are buried. Two players start measuring immediately, but the winner first *calculates* where to go. What does she calculate?

SOLUTION

IDENTIFY: The goal is to find the sum (resultant) of the three displacements, so this is a problem in vector addition.

SET UP: Figure 1.22 shows the situation. We have chosen the $+x$-axis as east and the $+y$-axis as north, the usual choice for maps. Let $\vec{A}$ be the first displacement, $\vec{B}$ the second, and $\vec{C}$ the third. We can estimate from the diagram that the vector sum $\vec{R}$ is about 10 m, 40° west of north.

1.22 Three successive displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$ and the resultant (vector sum) displacement $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.



EXECUTE: The angles of the vectors, measured from the $+x$-axis toward the $+y$-axis, are $(90.0° - 32.0°) = 58.0°$, $(180.0° + 36.0°) = 216.0°$, and 270.0°. We have to find the components of each. Because of our choice of axes, we may use Eqs. (1.6), and so the components of $\vec{A}$ are

$$A_x = A\cos\theta_A = (72.4\text{ m})(\cos 58.0°) = 38.37\text{ m}$$
$$A_y = A\sin\theta_A = (72.4\text{ m})(\sin 58.0°) = 61.40\text{ m}$$

Note that we have kept one too many significant figures in the components; we will wait until the end to round to the correct number of significant figures. The table shows the components of all the displacements, the addition of the components, and the other calculations. Always arrange your component calculations systematically like this.

Distance	Angle	x-component	y-component
$A = 72.4$ m	58.0°	38.37 m	61.40 m
$B = 57.3$ m	216.0°	−46.36 m	−33.68 m
$C = 17.8$ m	270.0°	0.00 m	−17.80 m
		$R_x = -7.99$ m	$R_y = 9.92$ m

$$R = \sqrt{(-7.99\text{ m})^2 + (9.92\text{ m})^2} = 12.7\text{ m}$$
$$\theta = \arctan\frac{9.92\text{ m}}{-7.99\text{ m}} = 129° = 39° \text{ west of north}$$

The losers try to measure three angles and three distances totaling 147.5 m, one meter at a time. The winner measured only one angle and one much shorter distance.

EVALUATE: Our calculated answers for R and θ are not too different from our estimates of 10 m and 40° west of north; that's good! Notice that $\theta = -51°$, or 51° south of east, also satisfies the equation for θ. But since the winner has made a drawing of the displacement vectors (Fig. 1.22), she knows that $\theta = 129°$ is the only correct solution for the angle.

Example 1.8 A vector in three dimensions

After an airplane takes off, it travels 10.4 km west, 8.7 km north, and 2.1 km up. How far is it from the takeoff point?

SOLUTION

Let the $+x$-axis be east, the $+y$-axis north, and the $+z$-axis up. Then $A_x = -10.4$ km, $A_y = 8.7$ km, and $A_z = 2.1$ km; Eq. (1.12) gives

$$A = \sqrt{(-10.4\text{ km})^2 + (8.7\text{ km})^2 + (2.1\text{ km})^2} = 13.7\text{ km}$$

Test Your Understanding of Section 1.8 Two vectors $\vec{A}$ and $\vec{B}$ both lie in the xy-plane. (a) Is it possible for $\vec{A}$ to have the same magnitude as $\vec{B}$ but different components? (b) Is it possible for $\vec{A}$ to have the same components as $\vec{B}$ but a different magnitude?

1.9 Unit Vectors

A **unit vector** is a vector that has a magnitude of 1, with no units. Its only purpose is to *point*—that is, to describe a direction in space. Unit vectors provide a convenient notation for many expressions involving components of vectors. We will always include a caret or "hat" (^) in the symbol for a unit vector to distinguish it from ordinary vectors whose magnitude may or may not be equal to 1.

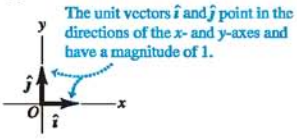
In an x-y coordinate system we can define a unit vector $\hat{\imath}$ that points in the direction of the positive x-axis and a unit vector $\hat{\jmath}$ that points in the direction of the positive y-axis (Fig. 1.23a). Then we can express the relationship between component vectors and components, described at the beginning of Section 1.8, as follows:

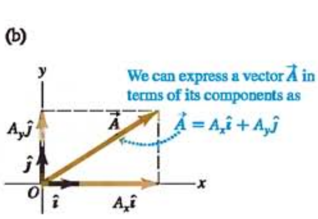
$$\vec{A}_x = A_x\hat{\imath}$$
$$\vec{A}_y = A_y\hat{\jmath} \qquad (1.13)$$

Similarly, we can write a vector $\vec{A}$ in terms of its components as

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} \qquad (1.14)$$

1.23 (a) The unit vectors $\hat{\imath}$ and $\hat{\jmath}$. (b) Expressing a vector $\vec{A}$ in terms of its components.





Equations (1.13) and (1.14) are vector equations; each term, such as $A_x\hat{\imath}$, is a vector quantity (Fig. 1.23b). The boldface equals and plus signs denote vector equality and addition.

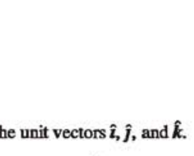
When two vectors $\vec{A}$ and $\vec{B}$ are represented in terms of their components, we can express the vector sum $\vec{R}$ using unit vectors as follows:

$$\begin{aligned}\vec{A} &= A_x\hat{\imath} + A_y\hat{\jmath}\\ \vec{B} &= B_x\hat{\imath} + B_y\hat{\jmath}\\ \vec{R} &= \vec{A} + \vec{B}\\ &= (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath})\\ &= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}\\ &= R_x\hat{\imath} + R_y\hat{\jmath}\end{aligned} \qquad (1.15)$$

Equation (1.15) restates the content of Eqs. (1.10) in the form of a single vector equation rather than two component equations.

If the vectors do not all lie in the xy-plane, then we need a third component. We introduce a third unit vector $\hat{k}$ that points in the direction of the positive z-axis (Fig. 1.24). Then Eqs. (1.14) and (1.15) become

1.24 The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.



$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k} \qquad (1.16)$$

$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$
$$= R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k} \qquad (1.17)$$

Example 1.9 Using unit vectors

Given the two displacements

$$\vec{D} = (6\hat{\imath} + 3\hat{\jmath} - \hat{k})\text{ m} \quad \text{and} \quad \vec{E} = (4\hat{\imath} - 5\hat{\jmath} + 8\hat{k})\text{ m}$$

find the magnitude of the displacement $2\vec{D} - \vec{E}$.

SOLUTION

IDENTIFY: We are to multiply the vector $\vec{D}$ by 2 (a scalar) and then subtract the vector $\vec{E}$ from the result.

SET UP: Equation (1.9) says that to multiply $\vec{D}$ by 2, we simply multiply each of its components by 2. Then Eq. (1.17) tells us that to subtract $\vec{E}$ from $2\vec{D}$, we simply subtract the components of $\vec{E}$ from the components of $2\vec{D}$. (Recall from Section 1.7 that subtracting a vector is the same as adding the negative of that vector.) In each of these mathematical operations, the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ remain unchanged.

EXECUTE: Letting $\vec{F} = 2\vec{D} - \vec{E}$, we have

$$\vec{F} = 2(6\hat{\imath} + 3\hat{\jmath} - \hat{k})\text{ m} - (4\hat{\imath} - 5\hat{\jmath} + 8\hat{k})\text{ m}$$
$$= [(12 - 4)\hat{\imath} + (6 + 5)\hat{\jmath} + (-2 - 8)\hat{k}]\text{ m}$$
$$= (8\hat{\imath} + 11\hat{\jmath} - 10\hat{k})\text{ m}$$

The units of the vectors $\vec{D}$, $\vec{E}$, and $\vec{F}$ are meters, so the components of these vectors are also in meters. From Eq. (1.12),

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$
$$= \sqrt{(8\text{ m})^2 + (11\text{ m})^2 + (-10\text{ m})^2} = 17\text{ m}$$

EVALUATE: Working with unit vectors makes vector addition and subtraction no more complicated than adding and subtracting ordinary numbers. Still, be sure to check for simple arithmetic errors.

Test Your Understanding of Section 1.9 Arrange the following vectors in order of their magnitude, with the vector of largest magnitude first. (i) $\vec{A} = (3\hat{\imath} + 5\hat{\jmath} - 2\hat{k})$ m; (ii) $\vec{B} = (-3\hat{\imath} + 5\hat{\jmath} - 2\hat{k})$ m; (iii) $\vec{C} = (3\hat{\imath} - 5\hat{\jmath} - 2\hat{k})$ m; (iv) $\vec{D} = (3\hat{\imath} + 5\hat{\jmath} + 2\hat{k})$ m.

1.10 Products of Vectors

We have seen how addition of vectors develops naturally from the problem of combining displacements, and we will use vector addition for calculating many other vector quantities later. We can also express many physical relationships concisely by using *products* of vectors. Vectors are not ordinary numbers, so ordinary multiplication is not directly applicable to vectors. We will define two different kinds of products of vectors. The first, called the *scalar product,* yields a result that is a scalar quantity. The second, the *vector product,* yields another vector.

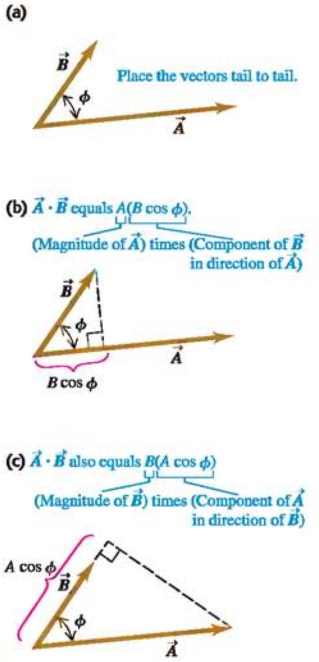
Scalar Product

The **scalar product** of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}\cdot\vec{B}$. Because of this notation, the scalar product is also called the **dot product.** Although $\vec{A}$ and $\vec{B}$ are vectors, the quantity $\vec{A}\cdot\vec{B}$ is a scalar.

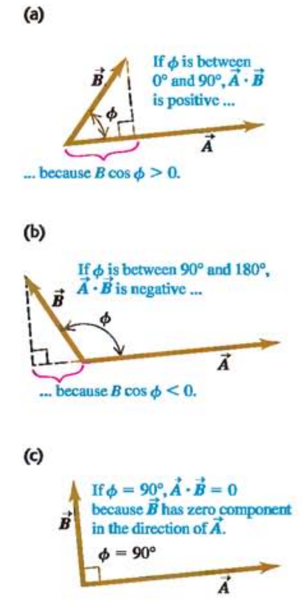
To define the scalar product $\vec{A}\cdot\vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$, we draw the two vectors with their tails at the same point (Fig. 1.25a). The angle ϕ (the Greek letter phi) between their directions ranges from 0° to 180°. Figure 1.25b shows the projection of the vector $\vec{B}$ onto the direction of $\vec{A}$; this projection is the component of $\vec{B}$ in the direction of $\vec{A}$ and is equal to $B\cos\phi$. (We can take components along any direction that's convenient, not just the x- and y-axes.) We define $\vec{A}\cdot\vec{B}$ to be the magnitude of $\vec{A}$ multiplied by the component of $\vec{B}$ in the direction of $\vec{A}$. Expressed as an equation,

$$\vec{A}\cdot\vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi \qquad \text{(definition of the scalar (dot) product)} \qquad (1.18)$$

1.25 Calculating the scalar product of two vectors, $\vec{A}\cdot\vec{B} = AB\cos\phi$.



1.26 The scalar product $\vec{A} \cdot \vec{B} = AB\cos\phi$ can be positive, negative, or zero, depending on the angle between $\vec{A}$ and $\vec{B}$.



Alternatively, we can define $\vec{A} \cdot \vec{B}$ to be the magnitude of $\vec{B}$ multiplied by the component of $\vec{A}$ in the direction of $\vec{B}$, as in Fig. 1.25c. Hence $\vec{A} \cdot \vec{B} = B(A\cos\phi) = AB\cos\phi$, which is the same as Eq. (1.18).

The scalar product is a scalar quantity, not a vector, and it may be positive, negative, or zero. When ϕ is between 0° and 90°, $\cos\phi > 0$ and the scalar product is positive (Fig. 1.26a). When ϕ is between 90° and 180° so that $\cos\phi < 0$, the component of $\vec{B}$ in the direction of $\vec{A}$ is negative, and $\vec{A} \cdot \vec{B}$ is negative (Fig. 1.26b). Finally, when $\phi = 90°$, $\vec{A} \cdot \vec{B} = 0$ (Fig. 1.26c). *The scalar product of two perpendicular vectors is always zero.*

For any two vectors $\vec{A}$ and $\vec{B}$, $AB\cos\phi = BA\cos\phi$. This means that $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$. The scalar product obeys the commutative law of multiplication; the order of the two vectors does not matter.

We will use the scalar product in Chapter 6 to describe work done by a force. When a constant force $\vec{F}$ is applied to a body that undergoes a displacement $\vec{s}$, the work W (a scalar quantity) done by the force is given by

$$W = \vec{F} \cdot \vec{s}$$

The work done by the force is positive if the angle between $\vec{F}$ and $\vec{s}$ is between 0° and 90°, negative if this angle is between 90° and 180°, and zero if $\vec{F}$ and $\vec{s}$ are perpendicular. (This is another example of a term that has a special meaning in physics; in everyday language, "work" isn't something that can be positive or negative.) In later chapters we'll use the scalar product for a variety of purposes, from calculating electric potential to determining the effects that varying magnetic fields have on electric circuits.

Calculating the Scalar Product Using Components

We can calculate the scalar product $\vec{A} \cdot \vec{B}$ directly if we know the x-, y-, and z-components of $\vec{A}$ and $\vec{B}$. To see how this is done, let's first work out the scalar products of the unit vectors. This is easy, since $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ all have magnitude 1 and are perpendicular to each other. Using Eq. (1.18), we find

$$\begin{aligned} \hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} &= (1)(1)\cos 0° = 1 \\ \hat{\imath} \cdot \hat{\jmath} = \hat{\imath} \cdot \hat{k} = \hat{\jmath} \cdot \hat{k} &= (1)(1)\cos 90° = 0 \end{aligned} \tag{1.19}$$

Now we express $\vec{A}$ and $\vec{B}$ in terms of their components, expand the product, and use these products of unit vectors:

$$\begin{aligned} \vec{A} \cdot \vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \cdot (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\ &= A_x\hat{\imath} \cdot B_x\hat{\imath} + A_x\hat{\imath} \cdot B_y\hat{\jmath} + A_x\hat{\imath} \cdot B_z\hat{k} \\ &\quad + A_y\hat{\jmath} \cdot B_x\hat{\imath} + A_y\hat{\jmath} \cdot B_y\hat{\jmath} + A_y\hat{\jmath} \cdot B_z\hat{k} \\ &\quad + A_z\hat{k} \cdot B_x\hat{\imath} + A_z\hat{k} \cdot B_y\hat{\jmath} + A_z\hat{k} \cdot B_z\hat{k} \\ &= A_xB_x\hat{\imath} \cdot \hat{\imath} + A_xB_y\hat{\imath} \cdot \hat{\jmath} + A_xB_z\hat{\imath} \cdot \hat{k} \\ &\quad + A_yB_x\hat{\jmath} \cdot \hat{\imath} + A_yB_y\hat{\jmath} \cdot \hat{\jmath} + A_yB_z\hat{\jmath} \cdot \hat{k} \\ &\quad + A_zB_x\hat{k} \cdot \hat{\imath} + A_zB_y\hat{k} \cdot \hat{\jmath} + A_zB_z\hat{k} \cdot \hat{k} \end{aligned} \tag{1.20}$$

From Eqs. (1.19) we see that six of these nine terms are zero, and the three that survive give simply

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z \quad \text{(scalar (dot) product in terms of components)} \tag{1.21}$$

Thus *the scalar product of two vectors is the sum of the products of their respective components.*

The scalar product gives a straightforward way to find the angle ϕ between any two vectors $\vec{A}$ and $\vec{B}$ whose components are known. In this case, Eq. (1.21) can be used to find the scalar product of $\vec{A}$ and $\vec{B}$. From Eq. (1.18) the scalar product is also equal to $AB\cos\phi$ The vector magnitudes A and B can be found from the vector components with Eq. (1.12), so $\cos\phi$ and hence the angle ϕ can be determined (see Example 1.11).

Example 1.10 Calculating a scalar product

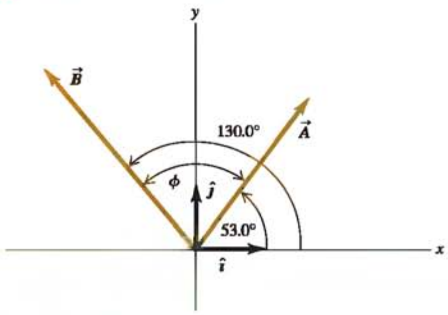
Find the scalar product $\vec{A}\cdot\vec{B}$ of the two vectors in Fig. 1.27. The magnitudes of the vectors are $A = 4.00$ and $B = 5.00$.

SOLUTION

IDENTIFY: We are given the magnitudes and directions of $\vec{A}$ and $\vec{B}$, and we wish to calculate their scalar product.

SET UP: We will calculate the scalar product in two ways: using the magnitudes of the vectors and the angle between them (Eq. 1.18), and using the components of the two vectors (Eq. 1.21).

1.27 Two vectors in two dimensions.



EXECUTE: With the first approach, the angle between the two vectors is $\phi = 130.0° - 53.0° = 77.0°$, so

$$\vec{A}\cdot\vec{B} = AB\cos\phi = (4.00)(5.00)\cos77.0° = 4.50$$

This is positive because the angle between $\vec{A}$ and $\vec{B}$ is between 0° and 90°.

To use the second approach, we first need to find the components of the two vectors. Since the angles of $\vec{A}$ and $\vec{B}$ are given with respect to the $+x$-axis, and these angles are measured in the sense from the $+x$-axis to the $+y$-axis, we can use Eqs. (1.6):

$$A_x = (4.00)\cos53.0° = 2.407$$
$$A_y = (4.00)\sin53.0° = 3.195$$
$$A_z = 0$$
$$B_x = (5.00)\cos130.0° = -3.214$$
$$B_y = (5.00)\sin130.0° = 3.830$$
$$B_z = 0$$

The z-components are zero because both vectors lie in the xy-plane. As in Example 1.7, we are keeping one too many significant figures in the components; we'll round to the correct number at the end. From Eq. (1.21) the scalar product is

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$
$$= (2.407)(-3.214) + (3.195)(3.830) + (0)(0) = 4.50$$

EVALUATE: We get the same result for the scalar product with both methods, as we should.

Example 1.11 Finding angles with the scalar product

Find the angle between the two vectors

$$\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k} \quad \text{and} \quad \vec{B} = -4\hat{\imath} + 2\hat{\jmath} - \hat{k}$$

SOLUTION

IDENTIFY: We are given the x-, y-, and z-components of two vectors. Our target variable is the angle ϕ between them.

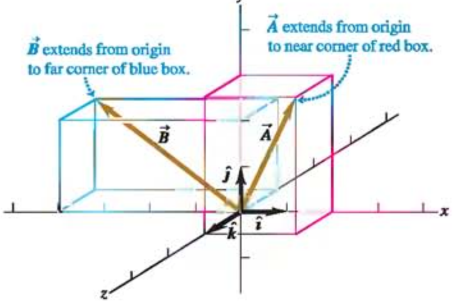
SET UP: Figure 1.28 shows the two vectors. The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is related to the angle ϕ between them and to the magnitudes A and B by Eq. (1.18). The scalar product is also related to the components of the two vectors by Eq. (1.21). If we are given the components of the vectors (as we are in this example), we first determine the scalar product $\vec{A}\cdot\vec{B}$ and the values of A and B, and then determine the target variable ϕ.

EXECUTE: We set our two expressions for the scalar product, Eq. (1.18) and Eq. (1.121), equal to each other. Rearranging, we obtain

$$\cos\phi = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

This formula can be used to find the angle between *any* two vectors $\vec{A}$ and $\vec{B}$. For our example the components of $\vec{A}$ are $A_x = 2$,

1.28 Two vectors in three dimensions.



Continued

$A_y = 3$, and $A_z = 1$, and the components of $\vec{B}$ are $B_x = -4$, $B_y = 2$, and $B_z = -1$. Thus

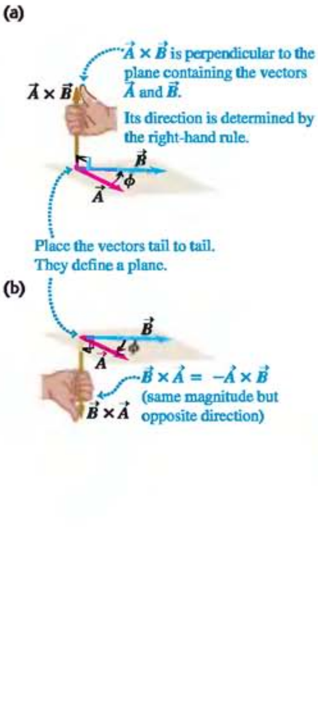
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$
$$= (2)(-4) + (3)(2) + (1)(-1) = -3$$
$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$$
$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(-4^2) + 2^2 + (-1)^2} = \sqrt{21}$$

$$\cos\phi = \frac{A_xB_x + A_yB_y + A_zB_z}{AB} = \frac{-3}{\sqrt{14}\sqrt{21}} = -0.175$$
$$\phi = 100°$$

EVALUATE: As a check on this result, note that the scalar product $\vec{A}\cdot\vec{B}$ is negative. This means that ϕ is between 90° and 180° (see Fig. 1.26), in agreement with our answer.

Vector Product

1.29 (a) The vector product $\vec{A} \times \vec{B}$ determined by the right-hand rule. (b) $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$; the vector product is anticommutative.



The **vector product** of two vectors $\vec{A}$ and $\vec{B}$, also called the **cross product,** is denoted by $\vec{A} \times \vec{B}$. As the name suggests, the vector product is itself a vector. We will use this product in Chapter 10 to describe torqne and angular momentum; in Chapters 27 and 28 we will use it extensively to describe magnetic fields and forces.

To define the vector product $\vec{A} \times \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ we again draw the two vectors with their tails at the same point (Fig. 1.29a). The two vectors then lie in a plane. We define the vector product to be a vector quantity with a direction perpendicular to this plane (that is, perpendicular to both $\vec{A}$ and $\vec{B}$) and a magnitude equal to $AB\sin\phi$. That is, if $\vec{C} = \vec{A} \times \vec{B}$, then

$$C = AB\sin\phi \quad \text{(magnitude of the vector (cross) product of } \vec{A} \text{ and } \vec{B}) \qquad (1.22)$$

We measure the angle ϕ from $\vec{A}$ toward $\vec{B}$ and take it to be the smaller of the two possible angles, so ϕ ranges from 0° to 180°. Then $\sin\phi \geq 0$ and C in Eq. (1.22) is never negative, as must be the case for a vector magnitude. Note also that when $\vec{A}$ and $\vec{B}$ are parallel or antiparallel, $\phi = 0$ or 180° and $C = 0$. That is, *the vector product of two parallel or antiparallel vectors is always zero.* In particular, *the vector product of any vector with itself is zero.*

CAUTION **Vector product vs. scalar product** Be careful not to confuse the expression $AB\sin\phi$ for the magnitude of the vector product $\vec{A} \times \vec{B}$ with the similar expression $AB\cos\phi$ for the scalar product $\vec{A}\cdot\vec{B}$. To see the contrast between these two expressions, imagine that we vary the angle between $\vec{A}$ and $\vec{B}$ while keeping their magnitudes constant. When $\vec{A}$ and $\vec{B}$ are parallel, the magnitude of the vector product will be zero and the scalar product will be maximum. When $\vec{A}$ and $\vec{B}$ are perpendicular, the magnitude of the vector product will be maximum and the scalar product will be zero.

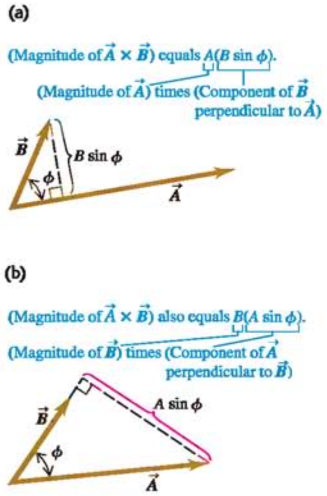
There are always *two* directions perpendicular to a given plane, one on each side of the plane. We choose which of these is the direction of $\vec{A} \times \vec{B}$ as follows. Imagine rotating vector $\vec{A}$ about the perpendicular line until it is aligned with $\vec{B}$, choosing the smaller of the two possible angles between $\vec{A}$ and $\vec{B}$. Curl the fingers of your right hand around the perpendicular line so that the fingertips point in the direction of rotation; your thumb will then point in the direction of $\vec{A} \times \vec{B}$. Figure 1.29a shows this **right-hand rule.**

Similarly, we determine the direction of $\vec{B} \times \vec{A}$ by rotating $\vec{B}$ into $\vec{A}$ as in Fig. 1.29b. The result is a vector that is *opposite* to the vector $\vec{A} \times \vec{B}$. The vector product is *not* commutative! In fact, for any two vectors $\vec{A}$ and $\vec{B}$,

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \qquad (1.23)$$

Just as we did for the scalar product, we can give a geometrical interpretation of the magnitude of the vector product. In Fig. 1.30a, $B\sin\phi$ is the component of vector $\vec{B}$ that is *perpendicular* to the direction of vector $\vec{A}$ From Eq. (1.22) the magnitude of $\vec{A} \times \vec{B}$ equals the magnitude of $\vec{A}$ multiplied by the component of $\vec{B}$ perpendicular to $\vec{A}$. Figure 1.30b shows that the magnitude of $\vec{A} \times \vec{B}$ also equals the magnitude of $\vec{B}$ multiplied by the component of $\vec{A}$ perpendicular to $\vec{B}$. Note that Fig. 1.30 shows the case in which ϕ is between 0° and 90°; you should draw a similar diagram for ϕ between 90° and 180° to show that the same geometrical interpretation of the magnitude of $\vec{A} \times \vec{B}$ still applies.

1.30 Calculating the magnitude $AB\sin\phi$ of the vector product of two vectors, $\vec{A} \times \vec{B}$.



Calculating the Vector Product Using Components

If we know the components of $\vec{A}$ and $\vec{B}$, we can calculate the components of the vector product using a procedure similar to that for the scalar product. First we work out the multiplication table for the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, all three of which are perpendicular to each other (Fig. 1.31a). The vector product of any vector with itself is zero, so

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \mathbf{0}$$

The boldface zero is a reminder that each product is a zero *vector*—that is, one with all components equal to zero and an undefined direction. Using Eqs. (1.22) and (1.23) and the right-hand rule, we find

$$\begin{aligned} \hat{\imath} \times \hat{\jmath} &= -\hat{\jmath} \times \hat{\imath} = \hat{k} \\ \hat{\jmath} \times \hat{k} &= -\hat{k} \times \hat{\jmath} = \hat{\imath} \\ \hat{k} \times \hat{\imath} &= -\hat{\imath} \times \hat{k} = \hat{\jmath} \end{aligned} \qquad (1.24)$$

You can verify these equations by referring to Fig. 1.31a.

Next we express $\vec{A}$ and $\vec{B}$ in terms of their components and the corresponding unit vectors, and we expand the expression for the vector product:

$$\begin{aligned} \vec{A} \times \vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\ &= A_x\hat{\imath} \times B_x\hat{\imath} + A_x\hat{\imath} \times B_y\hat{\jmath} + A_x\hat{\imath} \times B_z\hat{k} \\ &\quad + A_y\hat{\jmath} \times B_x\hat{\imath} + A_y\hat{\jmath} \times B_y\hat{\jmath} + A_y\hat{\jmath} \times B_z\hat{k} \\ &\quad + A_z\hat{k} \times B_x\hat{\imath} + A_z\hat{k} \times B_y\hat{\jmath} + A_z\hat{k} \times B_z\hat{k} \end{aligned} \qquad (1.25)$$

We can also rewrite the individual terms in Eq. (1.25) as $A_x\hat{\imath} \times B_y\hat{\jmath} = (A_xB_y)\hat{\imath} \times \hat{\jmath}$, and so on. Evaluating these by using the multiplication table for the unit vectors in Eqs. (1.24) and then grouping the terms, we find

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\,\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k} \qquad (1.26)$$

Thus the components of $\vec{C} = \vec{A} \times \vec{B}$ are given by

$$C_x = A_yB_z - A_zB_y \qquad C_y = A_zB_x - A_xB_z \qquad C_z = A_xB_y - A_yB_x$$
$$(\text{components of } \vec{C} = \vec{A} \times \vec{B}) \qquad (1.27)$$

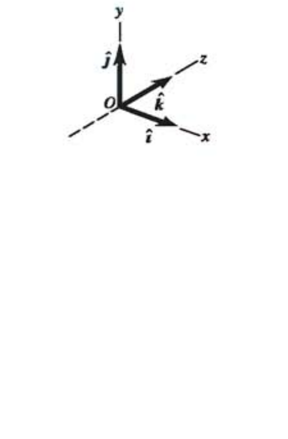
The vector product can also be expressed in determinant form as

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

If you aren't familiar with determinants, don't worry about this form.

With the axis system of Fig. 1.31a, if we reverse the direction of the z-axis, we get the system shown in Fig. 1.31b. Then, as you may verify, the definition of the vector product gives $\hat{\imath} \times \hat{\jmath} = -\hat{k}$ instead of $\hat{\imath} \times \hat{\jmath} = \hat{k}$. In fact, all vector products of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ would have signs opposite to those in Eqs. (1.24). We see that there are two kinds of coordinate systems, differing in the signs of the vector products of unit vectors. An axis system in which $\hat{\imath} \times \hat{\jmath} = \hat{k}$, as in Fig. 1.31a, is called a **right-handed system.** The usual practice is to use *only* right-handed systems, and we will follow that practice throughout this book.

1.31 (a) We will always use a right-handed coordinate system, like this one. (b) We will never use a left handed coordinate system (in which $\hat{\imath} \times \hat{\jmath} = -\hat{k}$, and so on).



Example 1.12 Calculating a vector product

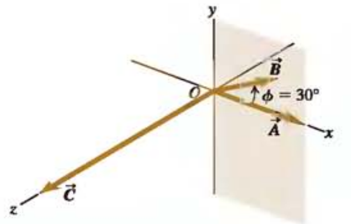
Vector $\vec{A}$ has magnitude 6 units and is in the direction of the $+x$-axis. Vector $\vec{B}$ has magnitude 4 units and lies in the xy-plane, making an angle of 30° with the $+x$-axis (Fig. 1.32). Find the vector product $\vec{A} \times \vec{B}$.

SOLUTION

IDENTIFY: We are given the magnitude and direction for each vector, and we want to find their vector product.

SET UP: We can find the vector product in one of two ways. The first way is to use Eq. (1.22) to determine the magnitude of $\vec{A} \times \vec{B}$ and then use the right-hand rule to find the direction of the vector product. The second way is to use the components of $\vec{A}$ and $\vec{B}$ to find the components of the vector product $\vec{C} = \vec{A} \times \vec{B}$ using Eqs. (1.27).

1.32 Vectors $\vec{A}$ and $\vec{B}$ and their vector product $\vec{C} = \vec{A} \times \vec{B}$. The vector $\vec{B}$ lies in the xy-plane.



EXECUTE: With the first approach, from Eq. (1.22) the magnitude of the vector product is

$$AB\sin\phi = (6)(4)(\sin 30°) = 12$$

From the right-hand rule, the direction of $\vec{A} \times \vec{B}$ is along the $+z$-axis, so we have $\vec{A} \times \vec{B} = 12\hat{k}$.

To use the second approach, we first write the components of $\vec{A}$ and $\vec{B}$:

$$A_x = 6 \qquad A_y = 0 \qquad A_z = 0$$
$$B_x = 4\cos 30° = 2\sqrt{3} \qquad B_y = 4\sin 30° = 2 \qquad B_z = 0$$

Defining $\vec{C} = \vec{A} \times \vec{B}$, we have from Eqs. (1.27) that

$$C_x = (0)(0) - (0)(2) = 0$$
$$C_y = (0)(2\sqrt{3}) - (6)(0) = 0$$
$$C_z = (6)(2) - (0)(2\sqrt{3}) = 12$$

The vector product $\vec{C}$ has only a z-component, and it lies along the $+z$-axis. The magnitude agrees with the result we obtained with the first approach, as it should.

EVALUATE: For this example the first approach was more direct because we knew the magnitudes of each vector and the angle between them, and furthermore, both vectors lay in one of the planes of the coordinate system. But often you will need to find the vector product of two vectors that are not so conveniently oriented or for which only the components are given. In such a case the second approach, using components, is more direct.

Test Your Understanding of Section 1.10 Vector $\vec{A}$ has magnitude 2 and vector $\vec{B}$ has magnitude 3. The angle ϕ between $\vec{A}$ and $\vec{B}$ is known to be either 0°, 90°, or 180°. For each of the following situations, state what the value of ϕ must be. (In each situation there may be more than one correct answer.) (a) $\vec{A} \cdot \vec{B} = 0$; (b) $\vec{A} \times \vec{B} = \mathbf{0}$; (c) $\vec{A} \cdot \vec{B} = 6$; (d) $\vec{A} \cdot \vec{B} = -6$; (e) (magnitude of $\vec{A} \times \vec{B}$) $= 6$.

CHAPTER 1 SUMMARY

Physical quantities and units: The fundamental physical quantities of mechanics are mass, length, and time. The corresponding basic SI units are the kilogram, the meter, and the second. Derived units for other physical quantities are products or quotients of the basic units. Equations must be dimensionally consistent; two terms can be added only when they have the same units. (See Examples 1.1 and 1.2.)

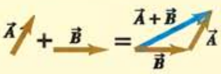
Significant figures: The accuracy of a measurement can be indicated by the number of significant figures or by a stated uncertainty. The result of a calculation usually has no more significant figures than the input data. When only crude estimates are available for input data, we can often make useful order-of-magnitude estimates. (See Examples 1.3 and 1.4.)

Significant figures in magenta

$$\pi = \frac{C}{2r} = \frac{0.424\ \text{m}}{2(0.06750\ \text{m})} = 3.14$$

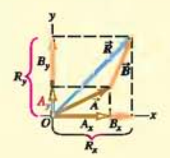
$$123.62 + 8.9 = 132.5$$

Scalars, vectors, and vector addition: Scalar quantities are numbers and combine with the usual rules of arithmetic. Vector quantities have direction as well as magnitude and combine according to the rules of vector addition. The negative of a vector has the same magnitude but points in the opposite direction. (See Example 1.5.)



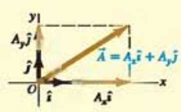
Vector components and vector addition: Vector addition can be carried out using components of vectors. The x-component of $\vec{R} = \vec{A} + \vec{B}$ is the sum of the x-components of $\vec{A}$ and $\vec{B}$, and likewise for the y- and z-components. (See Examples 1.6–1.8.)

$$R_x = A_x + B_x$$
$$R_y = A_y + B_y \qquad (1.10)$$
$$R_z = A_z + B_z$$



Unit vectors: Unit vectors describe directions in space. A unit vector has a magnitude of one, with no units. The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, aligned with the x-, y-, and z-axes of a rectangular coordinate system, are especially useful. (See Example 1.9.)

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \qquad (1.16)$$



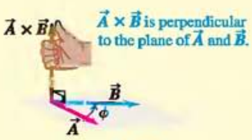
Scalar product: The scalar product $C = \vec{A}\cdot\vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is a scalar quantity. It can be expressed in terms of the magnitudes of $\vec{A}$ and $\vec{B}$ and the angle ϕ between the two vectors, or in terms of the components of $\vec{A}$ and $\vec{B}$. The scalar product is commutative; $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$. The scalar product of two perpendicular vectors is zero. (See Examples 1.10 and 1.11.)

$$\vec{A}\cdot\vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi \qquad (1.18)$$
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \qquad (1.21)$$

Scalar product $\vec{A}\cdot\vec{B} = AB\cos\phi$

Vector product: The vector product $\vec{C} = \vec{A}\times\vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is another vector $\vec{C}$. The magnitude of $\vec{A}\times\vec{B}$ depends on the magnitudes of $\vec{A}$ and $\vec{B}$ and the angle ϕ between the two vectors. The direction of $\vec{A}\times\vec{B}$ is perpendicular to the plane of the two vectors being multiplied, as given by the right-hand rule. The components of $\vec{C} = \vec{A}\times\vec{B}$ can be expressed in terms of the components of $\vec{A}$ and $\vec{B}$. The vector product is not commutative; $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$. The vector product of two parallel or antiparallel vectors is zero. (See Example 1.12.)

$$C = AB\sin\phi \qquad (1.22)$$
$$C_x = A_yB_z - A_zB_y$$
$$C_y = A_zB_x - A_xB_z \qquad (1.27)$$
$$C_z = A_xB_y - A_yB_x$$



(Magnitude of $\vec{A}\times\vec{B}$) = $AB\sin\phi$

Key Terms

range of validity, *2*
target variable, *3*
model, *3*
particle, *3*
physical quantity, *4*
operational definition, *4*
unit, *4*
International System (SI), *4*
second, *5*
meter, *5*
kilogram, *5*
prefix, *5*
dimensionally consistent, *7*
uncertainty (error), *8*
accuracy, *8*
fractional error, *8*
percent error, *8*
significant figures, *9*
scientific (powers-of-10) notation, *9*
precision, *10*
order-of-magnitude estimates, *11*
scalar quantity, *11*
vector quantity, *11*
magnitude of a vector, *11*
displacement, *11*
parallel vectors, *12*
negative of a vector, *12*
antiparallel vectors, *12*
vector sum (resultant), *13*
component vectors, *15*
components, *15*
unit vector, *20*
scalar (dot) product, *21*
vector (cross) product, *24*
right-hand rule, *24*
right-handed system, *25*

Answer to Chapter Opening Question ?

Take the $+x$-axis to point east and the $+y$-axis to point north. Then what we are trying to find is the y-component of the velocity vector, which has magnitude $v = 20\text{ km/h}$ and is at an angle $\theta = 53°$ measured from the $+x$-axis toward the $+y$-axis. From Eqs. (1.6) we have $v_y = v\sin\theta = (20\text{ km/h})\sin 53° = 16\text{ km/h}$. So the hurricane moves 16 km north in 1 h.

Answers to Test Your Understanding Questions

1.5 Answer: (ii) Density $= (1.80\text{ kg})/(6.0\times10^{-4}\text{ m}^3) = 3.0\times10^3\text{ kg/m}^3$. When we multiply or divide, the number with the fewest significant figures controls the number of significant figures in the result.

1.6 The answer depends on how many students are enrolled at your campus.

1.7 Answers: (ii), (iii), and (iv) The vector $-\vec{T}$ has the same magnitude as the vector $\vec{T}$, so $\vec{S}-\vec{T}=\vec{S}+(-\vec{T})$ is the *sum* of one vector of magnitude 3 m and one of magnitude 4 m. This sum has magnitude 7 m if $\vec{S}$ and $-\vec{T}$ are parallel and magnitude 1 m if $\vec{S}$ and $-\vec{T}$ are antiparallel. The magnitude of $\vec{S}-\vec{T}$ is 5 m if $\vec{S}$ and $-\vec{T}$ are perpendicular, so that the vectors $\vec{S}$, $\vec{T}$, and $\vec{S}-\vec{T}$ form a 3-4-5 right triangle. Answer (i) is impossible because the magnitude of the sum of two vectors cannot be greater than the sum of the magnitudes; answer (v) is impossible because the sum of two vectors can be zero only if the two vectors are antiparallel and have the same magnitude; and answer (vi) is impossible because the magnitude of a vector cannot be negative.

1.8 Answers: (a) yes, (b) no Vectors $\vec{A}$ and $\vec{B}$ can have the same magnitude but different components if they point in different directions. If they have the same components, however, they are the same vector $(\vec{A}=\vec{B})$ and so must have the same magnitude.

1.9 Answer: all have the same magnitude The four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ all point in different directions, but all have the same magnitude:

$$A = B = C = D = \sqrt{(\pm3\text{ m})^2+(\pm5\text{ m})^2+(\pm2\text{ m})^2}$$
$$= \sqrt{9\text{ m}^2+25\text{ m}^2+4\text{ m}^2} = \sqrt{38\text{ m}^2} = 6.2\text{ m}$$

1.10 Answers: (a) $\phi = 90°$, (b) $\phi = 0°$ or $\phi = 180°$, (c) $\phi = 0°$, (d) $\phi = 180°$, (e) $\phi = 90°$ (a) The scalar product is zero only if $\vec{A}$ and $\vec{B}$ are perpendicular. (b) The vector product is zero only if $\vec{A}$ and $\vec{B}$ are either parallel or antiparallel. (c) The scalar product is equal to the product of the magnitudes $(\vec{A}\cdot\vec{B}=AB)$ only if $\vec{A}$ and $\vec{B}$ are parallel. (d) The scalar product is equal to the negative of the product of the magnitudes $(\vec{A}\cdot\vec{B}=-AB)$ only if $\vec{A}$ and $\vec{B}$ are antiparallel. (e) The magnitude of the vector product is equal to the product of the magnitudes $[(\text{magnitude of }\vec{A}\times\vec{B})=AB]$ only if $\vec{A}$ and $\vec{B}$ are perpendicular.

PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com MP

Discussion Questions

Q1.1. How many correct experiments do we need to disprove a theory? How many to prove a theory? Explain.

Q1.2. A guidebook describes the rate of climb of a mountain trail as 120 meters per kilometer. How can you express this as a number with no units?

Q1.3. Suppose you are asked to compute the tangent of 5.00 meters. Is this possible? Why or why not?

Q1.4. A highway contractor stated that in building a bridge deck he poured 250 yards of concrete. What do you think he meant?

Q1.5. What is your height in centimeters? What is your weight in newtons?

Q1.6. The U.S. National Institute of Science and Technology (NIST) maintains several accurate copies of the international standard kilogram. Even after careful cleaning, these national standard kilograms are gaining mass at an average rate of about $1\ \mu\text{g/y}$ $(1\text{ y} = 1\text{ year})$ when compared every ten years or so to the standard international kilogram. Does this apparent change have any importance? Explain.

Q1.7. What physical phenomena (other than a pendulum or cesium clock) could you use to define a time standard?

Q1.8. Describe how you could measure the thickness of a sheet of paper with an ordinary ruler.

Q1.9. The quantity $\pi = 3.14159\ldots$ is a number with no dimensions, since it is a ratio of two lengths. Describe two or three other geometrical or physical quantities that are dimensionless.

Q1.10. What are the units of volume? Suppose another student tells you that a cylinder of radius r and height h has volume given by $\pi r^3 h$. Explain why this cannot be right.

Q1.11. Three archers each fire four arrows at a target. Joe's four arrows hit at points 10 cm above, 10 cm below, 10 cm to the left, and 10 cm to the right of the center of the target. All four of Moe's arrows hit within 1 cm of a point 20 cm from the center, and Flo's four arrows all hit within 1 cm of the center. The contest judge says that one of the archers is precise but not accurate, another archer is accurate but not precise, and the third archer is both accurate and precise. Which description goes with which archer? Explain your reasoning.

Q1.12. A circular racetrack has a radius of 500 m. What is the displacement of a bicyclist when she travels around the track from the north side to the south side? When she makes one complete circle around the track? Explain your reasoning.

Q1.13. Can you find two vectors with different lengths that have a vector sum of zero? What length restrictions are required for three vectors to have a vector sum of zero? Explain your reasoning.

Q1.14. One sometimes speaks of the "direction of time," evolving from past to future. Does this mean that time is a vector quantity? Explain your reasoning.

Q1.15. Air traffic controllers give instructions to airline pilots telling them in which direction they are to fly. These instructions are called "vectors." If these are the only instructions given, is the name "vector" used correctly? Why or why not?

Q1.16. Can you find a vector quantity that has a magnitude of zero but components that are different from zero? Explain. Can the magnitude of a vector be less than the magnitude of any of its components? Explain.

Q1.17. (a) Does it make sense to say that a vector is *negative?* Why? (b) Does it make sense to say that one vector is the negative of another? Why? Does your answer here contradict what you said in part (a)?

Q1.18. If $\vec{C}$ is the vector sum of $\vec{A}$ and $\vec{B}$, $\vec{C} = \vec{A} + \vec{B}$, what must be true if $C = A + B$? What must be true if $C = 0$?

Q1.19. If $\vec{A}$ and $\vec{B}$ are nonzero vectors, is it possible for $\vec{A}\cdot\vec{B}$ and $\vec{A}\times\vec{B}$ *both* to be zero? Explain.

Q1.20. What does $\vec{A}\cdot\vec{A}$, the scalar product of a vector with itself, give? What about $\vec{A}\times\vec{A}$, the vector product of a vector with itself?

Q1.21. Let $\vec{A}$ represent any nonzero vector. Why is $\vec{A}/A$ a unit vector and what is its direction? If θ is the angle that $\vec{A}$ makes with the $+x$-axis, explain why $(\vec{A}/A)\cdot\hat{\imath}$ is called the *direction cosine* for that axis.

Q1.22. Which of the following are legitimate mathematical operations: (a) $\vec{A}\cdot(\vec{B}-\vec{C})$; (b) $(\vec{A}-\vec{B})\times\vec{C}$; (c) $\vec{A}\cdot(\vec{B}\times\vec{C})$; (d) $\vec{A}\times(\vec{B}\times\vec{C})$; (e) $\vec{A}\times(\vec{B}\cdot\vec{C})$? In each case, give the reason for your answer.

Q1.23. Consider the two repeated vector products $\vec{A}\times(\vec{B}\times\vec{C})$ and $(\vec{A}\times\vec{B})\times\vec{C}$. Give an example that illustrates the general rule that these two vector products do not have the same magnitude or direction. Can you choose the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ such that these two vector products *are* equal? If so, give an example.

Q1.24. Show that, no matter what $\vec{A}$ and $\vec{B}$ are, $\vec{A}\cdot(\vec{A}\times\vec{B}) = 0$. (*Hint:* Do not look for an elaborate mathematical proof. Rather look at the definition of the direction of the cross product.)

Q1.25. (a) If $\vec{A}\cdot\vec{B} = 0$, does it necessary follow that $A = 0$ or $B = 0$? Explain. (b) If $\vec{A}\times\vec{B} = \mathbf{0}$, does it necessary follow that $A = 0$ or $B = 0$? Explain.

Q1.26. If $\vec{A} = \mathbf{0}$ for a vector in the xy plane, does it follow that $A_x = -A_y$? What *can* you say about A_x and A_y?

Exercises

Section 1.3 Standards and Units
Section 1.4 Unit Consistency and Conversions

1.1. Starting with the definition 1 in. = 2.54 cm, find the number of (a) kilometers in 1.00 mile and (b) feet in 1.00 km.

1.2. According to the label on a bottle of salad dressing, the volume of the contents is 0.473 liter (L). Using only the conversions $1\ \text{L} = 1000\ \text{cm}^3$ and 1 in. = 2.54 cm, express this volume in cubic inches.

1.3. How many nanoseconds does it take light to travel 1.00 ft in vacuum? (This result is a useful quantity to remember.)

1.4. The density of lead is $11.3\ \text{g/cm}^3$. What is this value in kilograms per cubic meter?

1.5. The most powerful engine available for the classic 1963 Chevrolet Corvette Sting Ray developed 360 horsepower and had a displacement of 327 cubic inches. Express this displacement in liters (L) by using only the conversions $1\ \text{L} = 1000\ \text{cm}^3$ and 1 in. = 2.54 cm.

1.6. A square field measuring 100.0 m by 100.0 m has an area of 1.00 hectare. An acre has an area of $43{,}600\ \text{ft}^2$. If a country lot has an area of 12.0 acres, what is the area in hectares?

1.7. How many years older will you be 1.00 billion seconds from now? (Assume a 365-day year.)

1.8. While driving in an exotic foreign land you see a speed limit sign on a highway that reads 180,000 furlongs per fortnight. How many miles per hour is this? (One furlong is $\frac{1}{8}$ mile, and a fortnight is 14 days. A furlong originally referred to the length of a plowed furrow.)

1.9. A certain fuel-efficient hybrid car gets gasoline mileage of 55.0 mpg (miles per gallon). (a) If you are driving this car in Europe and want to compare its mileage with that of other European cars, express this mileage in km/L (L = liter). Use the conversion factors in Appendix E. (b) If this car's gas tank holds 45 L, how many tanks of gas will you use to drive 1500 km?

1.10. The following conversions occur frequently in physics and are very useful. (a) Use 1 mi = 5280 ft and 1 h = 3600 s to convert 60 mph to units of ft/s. (b) The acceleration of a freely falling object is $32\ \text{ft/s}^2$. Use 1 ft = 30.48 cm to express this acceleration in units of m/s^2. (c) The density of water is $1.0\ \text{g/cm}^3$. Convert this density to units of kg/m^3.

1.11. Neptunium. In the fall of 2002, a group of scientists at Los Alamos National Laboratory determined that the critical mass of neptunium-237 is about 60 kg. The critical mass of a fissionable material is the minimum amount that must be brought together to start a chain reaction. This element has a density of $19.5\ \text{g/cm}^3$. What would be the radius of a sphere of this material that has a critical mass?

Section 1.5 Uncertainty and Significant Figures

1.12. A useful and easy-to-remember approximate value for the number of seconds in a year is $\pi \times 10^7$. Determine the percent error in this approximate value. (There are 365.24 days in one year.)

1.13. Figure 1.7 shows the result of unacceptable error in the stopping position of a train. (a) If a train travels 890 km from Berlin to Paris and then overshoots the end of the track by 10 m, what is the percent error in the total distance covered? (b) Is it correct to write the total distance covered by the train as 890,010 m? Explain.

1.14. With a wooden ruler you measure the length of a rectangular piece of sheet metal to be 12 mm. You use micrometer calipers to measure the width of the rectangle and obtain the value 5.98 mm. Give your answers to the following questions to the correct number of significant figures. (a) What is the area of the rectangle? (b) What is the ratio of the rectangle's width to its length? (c) What is the perimeter of the rectangle? (d) What is the difference between the length and width? (e) What is the ratio of the length to the width?

1.15. Estimate the percent error in measuring (a) a distance of about 75 cm with a meter stick; (b) a mass of about 12 g with a chemical balance; (c) a time interval of about 6 min with a stopwatch.

1.16. A rectangular piece of aluminum is 5.10 ± 0.01 cm long and 1.90 ± 0.01 cm wide. (a) Find the area of the rectangle and the uncertainty in the area. (b) Verify that the fractional uncertainty in the area is equal to the sum of the fractional uncertainties in the length and in the width. (This is a general result; see Challenge Problem 1.98.)

1.17. As you eat your way through a bag of chocolate chip cookies, you observe that each cookie is a circular disk with a diameter of 8.50 ± 0.02 cm and a thickness of 0.050 ± 0.005 cm. (a) Find the average volume of a cookie and the uncertainty in the volume. (b) Find the ratio of the diameter to the thickness and the uncertainty in this ratio.

Section 1.6 Estimates and Orders of Magnitude

1.18. How many gallons of gasoline are used in the United States in one day? Assume two cars for every three people, that each car is driven an average of 10,000 mi per year, and that the average car gets 20 miles per gallon.

1.19. A rather ordinary middle-aged man is in the hospital for a routine check-up. The nurse writes the quantity 200 on his medical chart but forgets to include the units. Which of the following quantities could the 200 plausibly represent? (a) his mass in kilograms; (b) his height in meters; (c) his height in centimeters; (d) his height in millimeters; (e) his age in months.

1.20. How many kernels of corn does it take to fill a 2-L soft drink bottle?

1.21. How many words are there in this book?

1.22. Four astronauts are in a spherical space station. (a) If, as is typical, each of them breathes about 500 cm^3 of air with each breath, approximately what volume of air (in cubic meters) do these astronauts breathe in a year? (b) What would the diameter (in meters) of the space station have to be to contain all this air?

1.23. How many times does a typical person blink her eyes in a lifetime?

1.24. How many times does a human heart beat during a lifetime? How many gallons of blood does it pump? (Estimate that the heart pumps 50 cm^3 of blood with each beat.)

1.25. In Wagner's opera *Das Rheingold,* the goddess Freia is ransomed for a pile of gold just tall enough and wide enough to hide her from sight. Estimate the monetary value of this pile. The density of gold is 19.3 g/cm^3, and its value is about \$10 per gram (although this varies).

1.26. You are using water to dilute small amounts of chemicals in the laboratory, drop by drop. How many drops of water are in a 1.0 L bottle? (*Hint:* Start by estimating the diameter of a drop of water.)

1.27. How many pizzas are consumed each academic year by students at your school?

1.28. How many dollar bills would you have to stack to reach the moon? Would that be cheaper than building and launching a spacecraft? (*Hint:* Start by folding a dollar bill to see how many thicknesses make 1.0 mm.)

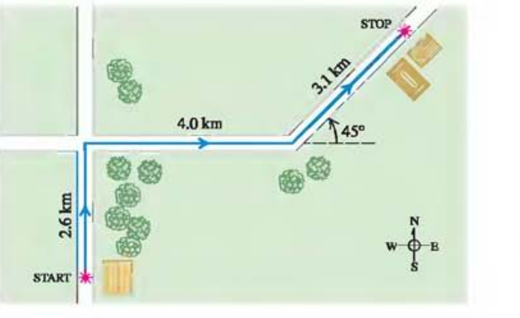
1.29. How much would it cost to paper the entire United States (including Alaska and Hawaii) with dollar bills? What would be the cost to each person in the United States?

Section 1.7 Vectors and Vector Addition

1.30. Hearing rattles from a snake, you make two rapid displacements of magnitude 1.8 m and 2.4 m. In sketches (roughly to scale), show how your two displacements might add up to give a resultant of magnitude (a) 4.2 m; (b) 0.6 m; (c) 3.0 m.

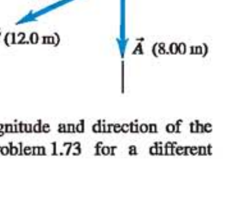
1.31. A postal employee drives a delivery truck along the route shown in Fig. 1.33. Determine the magnitude and direction of the resultant displacement by drawing a scale diagram. (See also Exercise 1.38 for a different approach to this same problem.)

Figure 1.33 Exercises 1.31 and 1.38.



1.32. For the vectors $\vec{A}$ and $\vec{B}$ in Fig. 1.34, use a scale drawing to find the magnitude and direction of (a) the vector sum $\vec{A} + \vec{B}$ and (b) the vector difference $\vec{A} - \vec{B}$. Use your answers to find the magnitude and direction of (c) $-\vec{A} - \vec{B}$ and (d) $\vec{B} - \vec{A}$. (See also Exercise 1.39 for a different approach to this problem.)

Figure 1.34 Exercises 1.32, 1.35, 1.39, 1.47, 1.53, and 1.57, and Problem 1.72.



1.33. A spelunker is surveying a cave. She follows a passage 180 m straight west, then 210 m in a direction 45° east of south, and then 280 m at 30° east of north. After a fourth unmeasured displacement, she finds herself back where she started. Use a scale drawing to determine the magnitude and direction of the fourth displacement. (See also Problem 1.73 for a different approach to this problem.)

Section 1.8 Components of Vectors

1.34. Use a scale drawing to find the x- and y-components of the following vectors. For each vector the numbers given are the magnitude of the vector and the angle, measured in the sense from the $+x$-axis toward the $+y$-axis, that it makes with the $+x$-axis: (a) magnitude 9.30 m, angle 60.0°; (b) magnitude 22.0 km, angle 135°; (c) magnitude 6.35 cm, angle 307°.

1.35. Compute the x- and y-components of the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ in Fig. 1.34.

1.36. Let the angle θ be the angle that the vector $\vec{A}$ makes with the $+x$-axis, measured counterclockwise from that axis. Find the angle θ for a vector that has the following components: (a) $A_x = 2.00$ m, $A_y = -1.00$ m; (b) $A_x = 2.00$ m, $A_y = 1.00$ m; (c) $A_x = -2.00$ m, $A_y = 1.00$ m; (d) $A_x = -2.00$ m, $A_y = -1.00$ m.

1.37. A rocket fires two engines simultaneously. One produces a thrust of 725 N directly forward, while the other gives a 513-N thrust at 32.4° above the forward direction. Find the magnitude and direction (relative to the forward direction) of the resultant force that these engines exert on the rocket.

1.38. A postal employee drives a delivery truck over the route shown in Fig. 1.33. Use the method of components to determine the magnitude and direction of her resultant displacement. In a vector-addition diagram (roughly to scale), show that the resultant displacement found from your diagram is in qualitative agreement with the result you obtained using the method of components.

1.39. For the vectors $\vec{A}$ and $\vec{B}$ in Fig. 1.34, use the method of components to find the magnitude and direction of (a) the vector sum $\vec{A} + \vec{B}$; (b) the vector sum $\vec{B} + \vec{A}$; (c) the vector difference $\vec{A} - \vec{B}$; (d) the vector difference $\vec{B} - \vec{A}$.

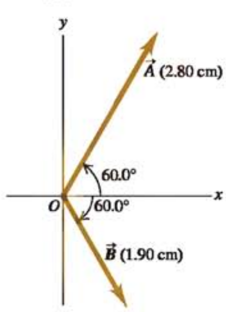
1.40. Find the magnitude and direction of the vector represented by the following pairs of components: (a) $A_x = -8.60$ cm, $A_y = 5.20$ cm; (b) $A_x = -9.70$ m, $A_y = -2.45$ m; (c) $A_x = 7.75$ km, $A_y = -2.70$ km.

1.41. A disoriented physics professor drives 3.25 km north, then 4.75 km west, and then 1.50 km south. Find the magnitude and direction of the resultant displacement, using the method of components. In a vector addition diagram (roughly to scale), show that the resultant displacement found from your diagram is in qualitative agreement with the result you obtained using the method of components.

1.42. Vector $\vec{A}$ has components $A_x = 1.30$ cm, $A_y = 2.25$ cm; vector $\vec{B}$ has components $B_x = 4.10$ cm, $B_y = -3.75$ cm. Find (a) the components of the vector sum $\vec{A} + \vec{B}$; (b) the magnitude and direction of $\vec{A} + \vec{B}$; (c) the components of the vector difference $\vec{B} - \vec{A}$; (d) the magnitude and direction of $\vec{B} - \vec{A}$.

1.43. Vector $\vec{A}$ is 2.80 cm long and is 60.0° above the x-axis in the first quadrant. Vector $\vec{B}$ is 1.90 cm long and is 60.0° below the x-axis in the fourth quadrant (Fig. 1.35). Use components to find the magnitude and direction of (a) $\vec{A} + \vec{B}$; (b) $\vec{A} - \vec{B}$; (c) $\vec{B} - \vec{A}$. In each case, sketch the vector addition or subtraction and show that your numerical answers are in qualitative agreement with your sketch.

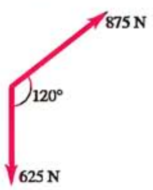
Figure 1.35 Exercises 1.43 and 1.59.



1.44. A river flows from south to north at 5.0 km/h. On this river, a boat is heading east to west perpendicular to the current at 7.0 km/h. As viewed by an eagle hovering at rest over the shore, how fast and in what direction is this boat traveling?

1.45. Use vector components to find the magnitude and direction of the vector needed to balance the two vectors shown in Figure 1.36. Let the 625-N vector be along the $-y$-axis and let the $+x$-axis be perpendicular to it toward the right.

Figure 1.36 Exercise 1.45.



1.46. Two ropes in a vertical plane exert equal magnitude forces on a hanging weight but pull with an angle of 86.0° between them. What pull does each one exert if their resultant pull is 372 N directly upward?

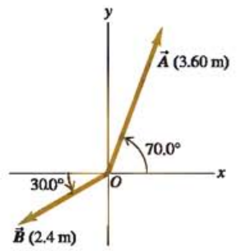
Section 1.9 Unit Vectors

1.47. Write each vector in Fig. 1.34 in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

1.48. In each case, find the x- and y-components of vector $\vec{A}$: (a) $\vec{A} = 5.0\hat{\imath} - 6.3\hat{\jmath}$; (b) $\hat{A} = 11.2\hat{\jmath} - 9.91\hat{\imath}$; (c) $\vec{A} = -15.0\hat{\imath} + 22.4\hat{\jmath}$; (d) $\vec{A} = 5.0\hat{B}$, where $\hat{B} = 4\hat{\imath} - 6\hat{\jmath}$.

1.49. (a) Write each vector in Fig. 1.37 in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$. (b) Use unit vectors to express the vector $\vec{C}$, where $\vec{C} = 3.00\vec{A} - 4.00\vec{B}$. (c) Find the magnitude and direction of $\vec{C}$.

Figure 1.37 Exercise 1.49 and Problem 1.86.



1.50. Given two vectors $\vec{A} = 4.00\hat{\imath} + 3.00\hat{\jmath}$ and $\vec{B} = 5.00\hat{\imath} - 2.00\hat{\jmath}$, (a) find the magnitude of each vector; (b) write an expression for the vector difference $\vec{A} - \vec{B}$ using unit vectors; (c) find the magnitude and direction of the vector difference $\vec{A} - \vec{B}$. (d) In a vector diagram show $\vec{A}$, $\vec{B}$, and $\vec{A} - \vec{B}$, and also show that your diagram agrees qualitatively with your answer in part (c).

1.51. (a) Is the vector $(\hat{\imath} + \hat{\jmath} + \hat{k})$ a unit vector? Justify your answer. (b) Can a unit vector have any components with magnitude greater than unity? Can it have any negative components? In each case justify your answer. (c) If $\vec{A} = a(3.0\hat{\imath} + 4.0\hat{\jmath})$, where a is a constant, determine the value of a that makes $\vec{A}$ a unit vector.

Section 1.10 Products of Vectors

1.52. (a) Use vector components to prove that two vectors commute for both addition and the scalar product. (b) Prove that two vectors *anticommute* for the vector product; that is, prove that $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$.

1.53. For the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Fig. 1.34, find the scalar products (a) $\vec{A} \cdot \vec{B}$; (b) $\vec{B} \cdot \vec{C}$; (c) $\vec{A} \cdot \vec{C}$.

1.54. (a) Find the scalar product of the two vectors $\vec{A}$ and $\vec{B}$ given in Exercise 1.50. (b) Find the angle between these two vectors.

1.55. Find the angle between each of the following pairs of vectors:

(a) $\vec{A} = -2.00\hat{\imath} + 6.00\hat{\jmath}$ and $\vec{B} = 2.00\hat{\imath} - 3.00\hat{\jmath}$
(b) $\vec{A} = 3.00\hat{\imath} + 5.00\hat{\jmath}$ and $\vec{B} = 10.00\hat{\imath} + 6.00\hat{\jmath}$
(c) $\vec{A} = -4.00\hat{\imath} + 2.00\hat{\jmath}$ and $\vec{B} = 7.00\hat{\imath} + 14.00\hat{\jmath}$

1.56. By making simple sketches of the appropriate vector products, show that (a) $\vec{A} \cdot \vec{B}$ can be interpreted as the product of the magnitude of $\vec{A}$ times the component of $\vec{B}$ along $\vec{A}$, or the magnitude of $\vec{B}$ times the component of $\vec{A}$ along $\vec{B}$; (b) $|\vec{A} \times \vec{B}|$ can be interpreted as the product of the magnitude of $\vec{A}$ times the component of $\vec{B}$ perpendicular to $\vec{A}$, or the magnitude of $\vec{B}$ times the component of $\vec{A}$ perpendicular to $\vec{B}$.

1.57. For the vectors $\vec{A}$ and $\hat{D}$ in Fig. 1.34, (a) find the magnitude and direction of the vector product $\vec{A} \times \hat{D}$; (b) find the magnitude and direction of $\hat{D} \times \vec{A}$.

1.58. Find the vector product $\vec{A} \times \vec{B}$ (expressed in unit vectors) of the two vectors given in Exercise 1.50. What is the magnitude of the vector product?

1.58. For the two vectors in Fig. 1.35, (a) find the magnitude and direction of the vector product $\vec{A} \times \vec{B}$; (b) find the magnitude and direction of $\vec{B} \times \vec{A}$.

Problems

1.60. An acre, a unit of land measurement still in wide use, has a length of one furlong $\left(\frac{1}{8}\text{ mi}\right)$ and a width one-tenth of its length. (a) How many acres are in a square mile? (b) How many square feet are in an acre? See Appendix E. (c) An acre-foot is the volume of water that would cover 1 acre of flat land to a depth of 1 foot. How many gallons are in 1 acre-foot?

1.61. An Earthlike Planet. In January 2006, astronomers reported the discovery of a planet comparable in size to the earth orbiting another star and having a mass of about 5.5 times the earth's mass. It is believed to consist of a mixture of rock and ice, similar to Neptune. If this planet has the same density as Neptune (1.76 g/cm^3), what is its radius expressed (a) in kilometers and (b) as a multiple of earth's radius? Consult Appendix F for astronomical data.

1.62. The Hydrogen Maser. You can use the radio waves generated by a hydrogen maser as a standard of frequency. The frequency of these waves is 1,420,405,751.786 hertz. (A hertz is another name for one cycle per second.) A clock controlled by a hydrogen maser is off by only 1 s in 100,000 years. For the following questions, use only three significant figures. (The large number of significant figures given for the frequency simply illustrates the remarkable accuracy to which it has been measured.) (a) What is the time for one cycle of the radio wave? (b) How many cycles occur in 1 h? (c) How many cycles would have occurred during the age of the earth, which is estimated to be 4.6×10^9 years? (d) By how many seconds would a hydrogen maser clock be off after a time interval equal to the age of the earth?

1.63. Estimate the number of atoms in your body. (*Hint:* Based on what you know about biology and chemistry, what are the most common types of atom in your body? What is the mass of each type of atom? Appendix D gives the atomic masses for different elements, measured in atomic mass units; you can find the value of an atomic mass unit, or 1 u, in Appendix F.)

1.64. Biological tissues are typically made up of 98% water. Given that the density of water is $1.0 \times 10^3\text{ kg/m}^3$, estimate the mass of (a) the heart of an adult human; (b) a cell with a diameter of 0.5 μm; (c) a honey bee.

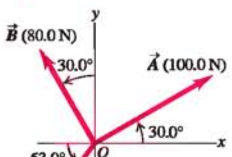
1.65. Iron has a property such that a 1.00-m^3 volume has a mass of 7.86×10^3 kg (density equals $7.86 \times 10^3\text{ kg/m}^3$). You want to manufacture iron into cubes and spheres. Find (a) the length of the side of a cube of iron that has a mass of 200.0 g and (b) the radius of a solid sphere of iron that has a mass of 200.0 g.

1.66. Stars in the Universe Astronomers frequently say that there are more stars in the universe than there are grains of sand on all the beaches on the earth. (a) Given that a typical grain of sand is about 0.2 mm in diameter, estimate the number of grains of sand on all the earth's beaches, and hence the approximate number of stars in the universe. It would be helpful to consult an atlas and do some measuring. (b) Given that a typical galaxy contains about 100 billion stars and there are more than 100 billion galaxies in the known universe, estimate the number of stars in the universe and compare this number with your result from part (a).

1.67. Physicists, mathematicians, and others often deal with large numbers. The number 10^{100} has been given the whimsical name *googol* by mathematicians. Let us compare some large numbers in physics with the googol. (*Note:* This problem requires numerical values that you can find in the appendices of the book, with which you should become familiar.) (a) Approximately how many atoms make up our planet? For simplicity, assume the average atomic mass of the atoms is 14 g/mol. Avogadro's number gives the number of atoms in a mole. (b) Approximately how many neutrons are in a neutron star? Neutron stars are composed almost entirely of neutrons and have approximately twice the mass of the sun. (c) In the leading theory of the origin of the universe, the entire universe that we can now observe occupied, at a very early time, a sphere whose radius was approximately equal to the present distance of the earth to the sun. At that time the universe had a density (mass divided by volume) of 10^{15} g/cm^3. Assuming that one-third of the particles were protons, one-third of the particles were neutrons, and the remaining one-third were electrons, how many particles then made up the universe?

1.60. Three horizontal ropes pull on a large stone stuck in the ground, producing the vector forces $\vec{A}$, $\vec{B}$, and $\vec{C}$ shown in Fig. 1.38. Find the magnitude and direction of a fourth force on the stone that will make the vector sum of the four forces zero.

Figure 1.36 Problem 1.68.



1.69. Two workers pull horizontally on a heavy box, but one pulls twice as hard as the other. The larger pull is directed at 25.0° west of north, and the resultant of these two pulls is 350.0 N directly northward. Use vector components to find the magnitude of each of these pulls and the direction of the smaller pull.

1.70. Emergency Landing. A plane leaves the airport in Galisteo and flies 170 km at 68° east of north and then changes direction to fly 230 km at 48° south of east, after which it makes an immediate emergency landing in a pasture. When the airport sends out a rescue crew, in which direction and how far should this crew fly to go directly to this plane?

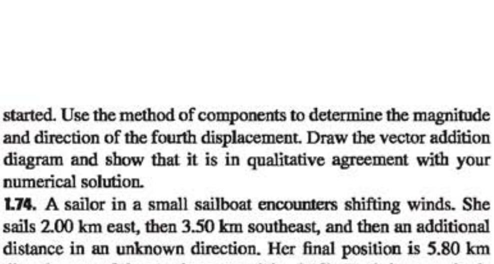
1.71. You are to program a robotic arm on an assembly line to move in the *xy*-plane. Its first displacement is $\vec{A}$; its second displacement is $\vec{B}$, of magnitude 6.40 cm and direction 63.0° measured in the sense from the $+x$-axis toward the $-y$-axis. The resultant $\vec{C} = \vec{A} + \vec{B}$ of the two displacements should also have a magnitude of 6.40 cm, but a direction 22.0° measured in the sense from the $+x$-axis toward the $+y$-axis. (a) Draw the vector addition diagram for these vectors, roughly to scale. (b) Find the components of $\vec{A}$. (c) Find the magnitude and direction of $\vec{A}$.

1.72. (a) Find the magnitude and direction of the vector $\vec{R}$ that is the sum of the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Fig. 1.34. In a diagram, show how $\vec{R}$ is formed from these three vectors. (b) Find the magnitude and direction of the vector $\vec{S} = \vec{C} - \vec{A} - \vec{B}$. In a diagram, show how $\vec{S}$ is formed from these three vectors.

1.73. As noted in Exercise 1.33, a spelunker is surveying a cave. She follows a passage 180 m straight west, then 210 m in a direction 45° east of south, and then 280 m at 30° east of north. After a fourth unmeasured displacement she finds herself back where she started. Use the method of components to determine the magnitude and direction of the fourth displacement. Draw the vector addition diagram and show that it is in qualitative agreement with your numerical solution.

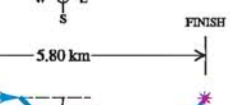
1.74. A sailor in a small sailboat encounters shifting winds. She sails 2.00 km east, then 3.50 km southeast, and then an additional distance in an unknown direction. Her final position is 5.80 km directly east of the starting point (Fig. 1.39). Find the magnitude and direction of the third leg of the journey. Draw the vector addition diagram and show that it is in qualitative agreement with your numerical solution.

Figure 1.39 Problem 1.74.



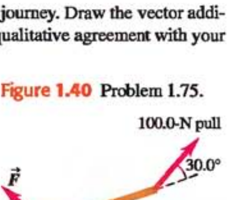
1.75. Equilibrium. We say an object is in *equilibrium* if all the forces on it balance (add up to zero). Figure 1.40 shows a beam weighing 124 N that is supported in equilibrium by a 100.0-N pull and a force $\vec{F}$ at the floor. The third force on the beam is the 124-N weight that acts vertically downward. (a) Use vector components to find the magnitude and direction of $\vec{F}$. (b) Check the reasonableness of your answer in part (a) by doing a graphical solution approximately to scale.

Figure 1.40 Problem 1.75.



1.76. On a training flight, a student pilot flies from Lincoln, Nebraska to Clarinda, Iowa, then to St. Joseph, Missouri, and then to Manhattan, Kansas (Fig. 1.41). The directions are shown relative to north: 0° is north, 90° is east, 180° is south, and 270° is west. Use the method of components to find (a) the distance she has to fly from Manhattan to get back to Lincoln, and (b) the direction (relative to north) she must fly to get there. Illustrate your solutions with a vector diagram.

Figure 1.41 Problem 1.76.



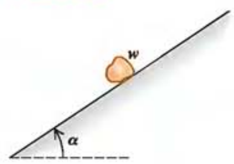
1.77. A graphic artist is creating a new logo for her company's website. In the graphics program she is using, each pixel in an image file has coordinates (x, y), where the origin $(0, 0)$ is at the upper left corner of the image, the $+x$-axis points to the right, and the $+y$-axis points down. Distances are measured in pixels. (a) The artist draws a line from the pixel location $(10, 20)$ to the location $(210, 200)$. She wishes to draw a second line that starts at $(10, 20)$, is 250 pixels long, and is at angle of 30° measured clockwise from the first line. At which pixel location should this second line end? Give your answer to the nearest pixel. (b) The artist now draws an arrow that connects the lower right end of the first line to the lower right end of the second line. Find the length and direction of this arrow. Draw a diagram showing all three lines.

1.78. Getting Back. An explorer in the dense jungles of equatorial Africa leaves his hut. He takes 40 steps northeast, then 80 steps 60° north of west, then 50 steps due south. Assume his steps all have equal length. (a) Sketch, roughly to scale, the three vectors and their resultant. (b) Save the explorer from becoming hopelessly lost in the jungle by giving him the displacement, calculated using the method of components, that will return him to his hut.

1.79. A ship leaves the island of Guam and sails 285 km at 40.0° north of west. In which direction must it now head and how far must it sail so that its resultant displacement will be 115 km directly east of Guam?

1.80. A boulder of weight w rests on a hillside that rises at a constant angle α above the horizontal, as shown in Fig. 1.42. Its weight is a force on the boulder that has direction vertically downward. (a) In terms of α and w, what is the component of the weight of the boulder in the direction parallel to the surface of the hill? (b) What is the component of the weight in the direction perpendicular to the surface of the hill? (c) An air conditioner unit is fastened to a roof that slopes upward at an angle of 35.0°. In order that the unit not slide down the roof, the component of the unit's weight parallel to the roof cannot exceed 550 N. What is the maximum allowed weight of the unit?

Figure 1.42 Problem 1.80.



1.81. Bones and Muscles. A patient in therapy has a forearm that weighs 20.5 N and that lifts a 112.0-N weight. These two forces have direction vertically downward. The only other significant forces on his forearm come from the biceps muscle (which acts perpendicularly to the forearm) and the force at the elbow. If the biceps produces a pull of 232 N when the forearm is raised 43° above the horizontal, find the magnitude and direction of the force that the elbow exerts on the forearm. (The sum of the elbow force and the biceps force must balance the weight of the arm and the weight it is carrying, so their vector sum must be 132.5 N, upward.)

1.82. You are hungry and decide to go to your favorite neighborhood fast-food restaurant. You leave your apartment and take the elevator 10 flights down (each flight is 3.0 m) and then go 15 m south to the apartment exit. You then proceed 0.2 km east, turn north, and go 0.1 km to the entrance of the restaurant. (a) Determine the displacement from your apartment to the restaurant. Use unit vector notation for your answer, being sure to make clear your choice of coordinates. (b) How far did you travel along the path you took from your apartment to the restaurant, and what is the magnitude of the displacement you calculated in part (a)?

1.83. While following a treasure map, you start at an old oak tree. You first walk 825 m directly south, then turn and walk 1.25 km at 30.0° west of north, and finally walk 1.00 km at 40.0° north of east, where you find the treasure: a biography of Isaac Newton! (a) To return to the old oak tree, in what direction should you head and how far will you walk? Use components to solve this problem.

(b) To see whether your calculation in part (a) is reasonable, check it with a graphical solution drawn roughly to scale.

1.64. You are camping with two friends, Joe and Karl. Since all three of you like your privacy, you don't pitch your tents close together. Joe's tent is 21.0 m from yours, in the direction 23.0° south of east. Karl's tent is 32.0 m from yours, in the direction 37.0° north of east. What is the distance between Karl's tent and Joe's tent?

1.85. Vectors $\vec{A}$ and $\vec{B}$ are drawn from a common point. Vector $\vec{A}$ has magnitude A and angle θ_A measured in the sense from the $+x$-axis to the $+y$-axis. The corresponding quantities for vector $\vec{B}$ are B and θ_B. Then $\vec{A} = A\cos\theta_A\hat{\imath} + A\sin\theta_A\hat{\jmath}$, $\vec{B} = B\cos\theta_B\hat{\imath} + B\sin\theta_B\hat{\jmath}$, and $\phi = |\theta_B - \theta_A|$ is the angle between $\vec{A}$ and $\vec{B}$. (a) Derive Eq. (1.18) from Eq. (1.21). (b) Derive Eq. (1.22) from Eqs. (1.27).

1.86. For the two vectors $\vec{A}$ and $\vec{B}$ in Fig. 1.37, (a) find the scalar product $\vec{A}\cdot\vec{B}$, and (b) find the magnitude and direction of the vector product $\vec{A}\times\vec{B}$.

1.67. Figure 1.11c shows a parallelogram based on the two vectors $\vec{A}$ and $\vec{B}$. (a) Show that the magnitude of the cross product of these two vectors is equal to the area of the parallelogram. (*Hint:* Area = base × height.) (b) What is the angle between the cross product and the plane of the parallelogram?

1.88. The vector $\vec{A}$ is 3.50 cm long and is directed into this page. Vector $\vec{B}$ points from the lower right corner of this page to the upper left corner of this page. Define an appropriate right-handed coordinate system and find the three components of the vector product $\vec{A}\times\vec{B}$, measured in cm^2. In a diagram, show your coordinate system and the vectors $\vec{A}$, $\vec{B}$, and $\vec{A}\times\vec{B}$.

1.89. Given two vectors $\vec{A} = -2.00\hat{\imath} + 3.00\hat{\jmath} + 4.00\hat{k}$ and $\vec{B} = 3.00\hat{\imath} + 1.00\hat{\jmath} - 3.00\hat{k}$, do the following. (a) Find the magnitude of each vector. (b) Write an expression for the vector difference $\vec{A} - \vec{B}$, using unit vectors. (c) Find the magnitude of the vector difference $\vec{A} - \vec{B}$. Is this the same as the magnitude of $\vec{B} - \vec{A}$? Explain.

1.98. Bond Angle in Methane. In the methane molecule, CH_4, each hydrogen atom is at a corner of a regular tetrahedron with the carbon atom at the center. In coordinates where one of the C—H bonds is in the direction of $\hat{\imath} + \hat{\jmath} + \hat{k}$, an adjacent C—H bond is in the $\hat{\imath} - \hat{\jmath} - \hat{k}$ direction. Calculate the angle between these two bonds.

1.91. The two vectors $\vec{A}$ and $\vec{B}$ are drawn from a common point, and $\vec{C} = \vec{A} + \vec{B}$. (a) Show that if $C^2 = A^2 + B^2$, the angle between the vectors $\vec{A}$ and $\vec{B}$ is 90°. (b) Show that if $C^2 < A^2 + B^2$, the angle between the vectors $\vec{A}$ and $\vec{B}$ is greater than 90°. (c) Show that if $C^2 > A^2 + B^2$, the angle between the vectors $\vec{A}$ and $\vec{B}$ is between 0° and 90°.

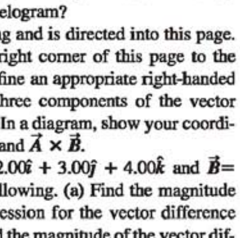
1.92. When two vectors $\vec{A}$ and $\vec{B}$ are drawn from a common point, the angle between them is ϕ. (a) Using vector techniques, show that the magnitude of their vector sum is given by

$$\sqrt{A^2 + B^2 + 2AB\cos\phi}$$

(b) If $\vec{A}$ and $\vec{B}$ have the same magnitude, for which value of ϕ will their vector sum have the same magnitude as $\vec{A}$ or $\vec{B}$?

1.93. A cube is placed so that one corner is at the origin and three edges are along the x-, y-, and z-axes of a coordinate system (Fig. 1.43). Use vectors to compute (a) the angle between the edge along the z-axis (line ab) and the diagonal from the origin to the opposite corner (line ad), and (b) the angle between line ac (the diagonal of a face) and line ad.

Figure 1.43 Problem 1.93.



1.94. Obtain a *unit vector* perpendicular to the two vectors given in Problem 1.89.

1.95. You are given vectors $\vec{A} = 5.0\hat{\imath} - 6.5\hat{\jmath}$ and $\vec{B} = -3.5\hat{\imath} + 7.0\hat{\jmath}$. A third vector $\vec{C}$ lies in the xy-plane. Vector $\vec{C}$ is perpendicular to vector $\vec{A}$, and the scalar product of $\vec{C}$ with $\vec{B}$ is 15.0. From this information, find the components of vector $\vec{C}$.

1.96. Two vectors $\vec{A}$ and $\vec{B}$ have magnitude $A = 3.00$ and $B = 3.00$. Their vector product is $\vec{A}\times\vec{B} = -5.00\hat{k} + 2.00\hat{\imath}$. What is the angle between $\vec{A}$ and $\vec{B}$?

1.97. Later in our study of physics we will encounter quantities represented by $(\vec{A}\times\vec{B})\cdot\vec{C}$. (a) Prove that for any three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$, $\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$. (b) Calculate $(\vec{A}\times\vec{B})\cdot\vec{C}$ for the three vectors $\vec{A}$ with magnitude $A = 5.00$ and angle $\theta_A = 26.0°$ measured in the sense from the $+x$-axis toward the $+y$-axis, $\vec{B}$ with $B = 4.00$ and $\theta_B = 63.0°$, and $\vec{C}$ with magnitude 6.00 and in the $+z$-direction. Vectors $\vec{A}$ and $\vec{B}$ are in the xy-plane.

Challenge Problems

1.98. The length of a rectangle is given as $L \pm l$ and its width as $W \pm w$. (a) Show that the uncertainty in its area A is $a = Lw + lW$. Assume that the uncertainties l and w are small, so that the product lw is very small and you can ignore it. (b) Show that the fractional uncertainty in the area is equal to the sum of the fractional uncertainty in length and the fractional uncertainty in width. (c) A rectangular solid has dimensions $L \pm l$, $W \pm w$, and $H \pm h$. Find the fractional uncertainty in the volume, and show that it equals the sum of the fractional uncertainties in the length, width, and height.

1.99. Completed Pass. At Enormous State University (ESU), the football team records its plays using vector displacements, with the origin taken to be the position of the ball before the play starts. In a certain pass play, the receiver starts at $+1.0\hat{\imath} - 5.0\hat{\jmath}$, where the units are yards, $\hat{\imath}$ is to the right, and $\hat{\jmath}$ is downfield. Subsequent displacements of the receiver are $+9.0\hat{\imath}$ (in motion before the snap), $+11.0\hat{\jmath}$ (breaks downfield), $-6.0\hat{\imath} + 4.0\hat{\jmath}$ (zigs), and $+12.0\hat{\imath} + 18.0\hat{\jmath}$ (zags). Meanwhile, the quarterback has dropped straight back to a position $-7.0\hat{\jmath}$. How far and in which direction must the quarterback throw the ball? (Like the coach, you will be well advised to diagram the situation before solving it numerically.)

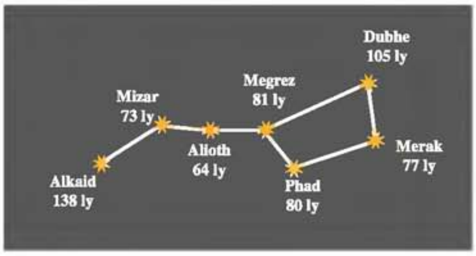
1.100. Navigating in the Solar System. The *Mars Polar Lander* spacecraft was launched on January 3, 1999. On December 3, 1999, the day that *Mars Polar Lander* touched down on the Martian surface, the positions of the earth and Mars were given by these coordinates:

	x	y	z
Earth	0.3182 AU	0.9329 AU	0.0000 AU
Mars	1.3087 AU	−0.4423 AU	−0.0414 AU

In these coordinates, the sun is at the origin and the plane of the earth's orbit is the xy-plane. The earth passes through the $+x$-axis once a year on the autumnal equinox, the first day of autumn in the northern hemisphere (on or about September 22). One AU, or *astronomical unit*, is eqnal to 1.496×10^8 km, the average distance from the earth to the sun. (a) In a diagram, show the positions of the sun, the earth, and Mars on December 3, 1999. (b) Find the following distances in AU on December 3, 1999: (i) from the sun to the earth; (ii) from the sun to Mars; (iii) from the earth to Mars. (c) As seen from the earth, what was the angle between the direction to the sun and the direction to Mars on December 3, 1999? (d) Explain whether Mars was visible from your location at midnight on December 3, 1999. (When it is midnight at your location, the sun is on the opposite side of the earth from you.)

1.101. Navigating in the Big Dipper. All the stars of the Big Dipper (part of the constellation Ursa Major) may appear to be the same distance from the earth, but in fact they are very far from each other. Figure 1.44 shows the distances from the earth to each of these stars. The distances are given in light years (ly), the distance that light travels in one year. One light year equals 9.461×10^{15} m. (a) Alkaid and Merak are 25.6° apart in the earth's sky. In a diagram, show the relative positions of Alkaid, Merak, and our sun. Find the distance in light years from Alkaid to Merak. (b) To an inhabitant of a planet orbiting Merak, how many degrees apart in the sky would Alkaid and our sun be?

Figure 1.44 Challenge Problem 1.101.



1.102. The vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, called the *position vector*, points from the origin $(0, 0, 0)$ to an arbitrary point in space with coordinates (x, y, z). Use what you know about vectors to prove the following: All points (x, y, z) that satisfy the equation $Ax + By + Cz = 0$, where A, B, and C are constants, lie in a plane that passes through the origin and that is perpendicular to the vector $A\hat{\imath} + B\hat{\jmath} + C\hat{k}$. Sketch this vector and the plane.

2 MOTION ALONG A STRAIGHT LINE

LEARNING GOALS

By studying this chapter, you will learn:

- How to describe straight-line motion in terms of average velocity, instantaneous velocity, average acceleration, and instantaneous acceleration.
- How to interpret graphs of position versus time, velocity versus time, and acceleration versus time for straight-line motion.
- How to solve problems involving straight-line motion with constant acceleration, including free-fall problems.
- How to analyze straight-line motion when the acceleration is not constant.

? A typical sprinter speeds up during the first third of a race and slows gradually over the rest of the course. Is it accurate to say that a sprinter is *accelerating* as he slows during the final two-thirds of the race?

What distance must an airliner travel down a runway before reaching takeoff speed? When you throw a baseball straight up in the air, how high does it go? When a glass slips from your hand, how much time do you have to catch it before it hits the floor? These are the kinds of questions you will learn to answer in this chapter. We are beginning our study of physics with *mechanics,* the study of the relationships among force, matter, and motion. In this chapter and the next we will study *kinematics,* the part of mechanics that enables us to describe motion. Later we will study *dynamics,* which relates motion to its causes.

In this chapter we concentrate on the simplest kind of motion: a body moving along a straight line. To describe this motion, we introduce the physical quantities *velocity* and *acceleration.* These quantities have simple definitions in physics; however, those definitions are more precise and slightly different than the ones used in everyday language. An important part of how a physicist defines velocity and acceleration is that these quantities are *vectors.* As you learned in Chapter 1, this means that they have both magnitude and direction. Our concern in this chapter is with motion along a straight line only, so we won't need the full mathematics of vectors just yet. But using vectors will be essential in Chapter 3 when we consider motion in two or three dimensions.

We'll develop simple equations to describe straight-line motion in the important special case when the acceleration is constant. An example is the motion of a freely falling body. We'll also consider situations in which the acceleration varies during the motion; in this case, it's necessary to use integration to describe the motion. (If you haven't studied integration yet, Section 2.6 is optional.)

2.1 Displacement, Time, and Average Velocity

Suppose a drag racer drives her AA-fuel dragster along a straight track (Fig. 2.1). To study the dragster's motion, we need a coordinate system. We choose the x-axis to lie along the dragster's straight-line path, with the origin O at the starting line. We also choose a point on the dragster, such as its front end, and represent the entire dragster by that point. Hence we treat the dragster as a **particle.**

A useful way to describe the motion of the particle—that is, the point that represents the dragster—is in terms of the change in the particle's coordinate x over a time interval. Suppose that 1.0 s after the start the front of the dragster is at point P_1, 19 m from the origin, and 4.0 s after the start it is at point P_2, 277 m from the origin. The *displacement* of the particle is a vector that points from P_1 to P_2 (see Section 1.7). Figure 2.1 shows that this vector points along the x-axis. The x-component of the displacement is just the change in the value of x, $(277\text{ m} - 19\text{ m}) = 258\text{ m}$, that took place during the time interval of $(4.0\text{ s} - 1.0\text{ s}) = 3.0\text{ s}$. We define the dragster's **average velocity** during this time interval as a *vector* quantity whose x-component is the change in x divided by the time interval: $(258\text{ m})/(3.0\text{ s}) = 86\text{ m/s}$.

In general, the average velocity depends on the particular time interval chosen. For a 3.0-s time interval *before* the start of the race, the average velocity would be zero because the dragster would be at rest at the starting line and would have zero displacement.

Let's generalize the concept of average velocity. At time t_1 the dragster is at point P_1, with coordinate x_1, and at time t_2 it is at point P_2, with coordinate x_2. The displacement of the dragster during the time interval from t_1 to t_2 is the vector from P_1 to P_2. The x-component of the displacement, denoted Δx, is just the change in the coordinate x:

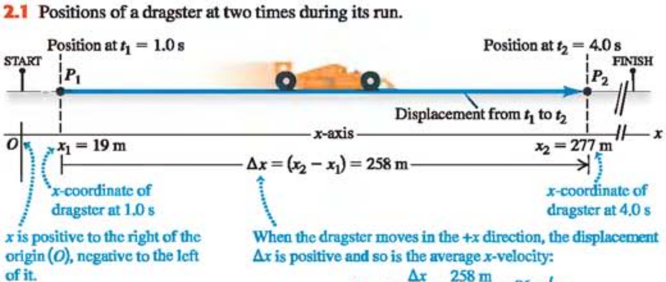
$$\Delta x = x_2 - x_1 \tag{2.1}$$

The dragster moves along the x-axis only, so the y- and z-components of the displacement are equal to zero.

CAUTION **The meaning of Δx** Note that Δx is *not* the product of Δ and x; it is a single symbol that means "the change in the quantity x." We always use the Greek capital letter Δ (delta) to represent a *change* in a quantity, equal to the *final* value of the quantity minus the *initial* value—never the reverse. Likewise, the time interval from t_1 to t_2 is Δt, the change in the quantity t: $\Delta t = t_2 - t_1$ (final time minus initial time).

The x-component of average velocity, or **average x-velocity,** is the x-component of displacement, Δx, divided by the time interval Δt during which the displacement occurs. We use the symbol $v_{av\text{-}x}$ for average x-velocity (the

2.1 Positions of a dragster at two times during its run.



subscript "av" signifies average value and the subscript x indicates that this is the x-component):

$$v_{av\text{-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \quad \text{(average } x\text{-velocity, straight-line motion)} \quad (2.2)$$

As an example, for the dragster $x_1 = 19$ m, $x_2 = 277$ m, $t_1 = 1.0$ s, and $t_2 = 4.0$ s, so Eq. (2.2) gives

$$v_{av\text{-}x} = \frac{277\text{ m} - 19\text{ m}}{4.0\text{ s} - 1.0\text{ s}} = \frac{258\text{ m}}{3.0\text{ s}} = 86\text{ m/s}$$

The average x-velocity of the dragster is positive. This means that during the time interval, the coordinate x increased and the dragster moved in the positive x-direction (to the right in Fig. 2.1).

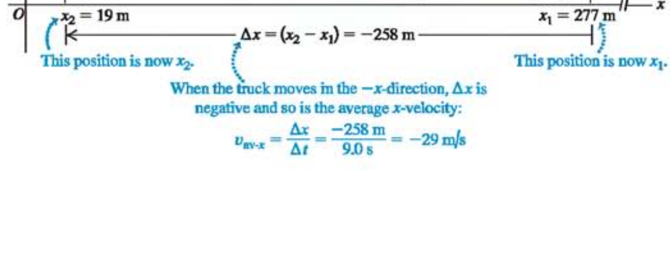
If a particle moves in the *negative* x-direction during a time interval, its average velocity for that time interval is negative. For example, suppose an official's truck moves to the left along the track (Fig. 2.2). The truck is at $x_1 = 277$ m at $t_1 = 16.0$ s and is at $x_2 = 19$ m at $t_2 = 25.0$ s. Then $\Delta x = (19\text{ m} - 277\text{ m}) = -258$ m and $\Delta t = (25.0\text{ s} - 16.0\text{ s}) = 9.0$ s. The x-component of average velocity is $v_{av\text{-}x} = \Delta x/\Delta t = (-258\text{ m})/(9.0\text{ s}) = -29$ m/s.

Here are some simple rules for the average x-velocity. **Whenever x is positive and increasing or is negative and becoming less negative, the particle is moving in the $+x$-direction and $v_{av\text{-}x}$ is positive** (Fig. 2.1). **Whenever x is positive and decreasing or is negative and becoming more negative, the particle is moving in the $-x$-direction and $v_{av\text{-}x}$ is negative** (Fig. 2.2).

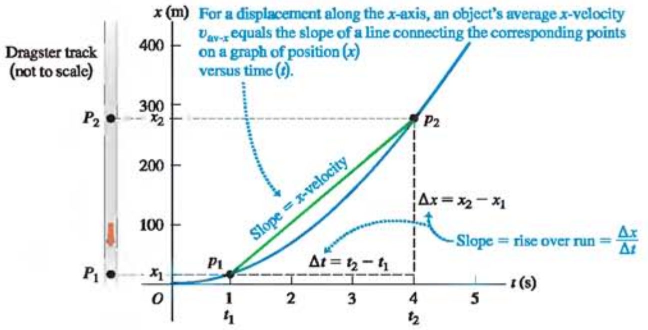
CAUTION Choice of the positive x-direction You might be tempted to conclude that positive average x-velocity must mean motion to the right, as in Fig. 2.1, and that negative average x-velocity must mean motion to the left, as in Fig. 2.2. But that's correct *only* if the positive x-direction is to the right, as we chose it to be in Figs. 2.1 and 2.2. Had we chosen the positive x-direction to be to the left, with the origin at the finish line, the dragster would have negative average x-velocity and the official's truck would have positive average x-velocity. In most problems the direction of the coordinate axis will be yours to choose. Once you've made your choice, you *must* take it into account when interpreting the signs of $v_{av\text{-}x}$ and other quantities that describe motion!

With straight-line motion we sometimes call Δx simply the displacement and $v_{av\text{-}x}$ simply the average velocity. But be sure to remember that these are really the x-components of vector quantities that, in this special case, have *only* x-components. In Chapter 3, displacement, velocity, and acceleration vectors will have two or three nonzero components.

Figure 2.3 is a graph of the dragster's position as a function of time—that is, an ***x-t* graph.** The curve in the figure *does not* represent the dragster's path in space; as Fig. 2.1 shows, the path is a straight line. Rather, the graph is a pictorial way to represent how the dragster's position changes with time. The points p_1



2.2 Positions of an official's truck at two times during its motion. The points P_1 and P_2 now indicate the positions of the truck, and so are the reverse of Fig. 2.1.



2.3 The position of a dragster as a function of time.

and p_2 on the graph correspond to the points P_1 and P_2 along the dragster's path. Line p_1p_2 is the hypotenuse of a right triangle with vertical side $\Delta x = x_2 - x_1$ and horizontal side $\Delta t = t_2 - t_1$. The average x-velocity $v_{av\text{-}x} = \Delta x/\Delta t$ of the dragster equals the *slope* of the line p_1p_2—that is, the ratio of the triangle's vertical side Δx to its horizontal side Δt.

The average x-velocity depends only on the total displacement $\Delta x = x_2 - x_1$ that occurs during the time interval $\Delta t = t_2 - t_1$, not on the details of what happens during the time interval. At time t_1 a motorcycle might have raced past the dragster at point P_1 in Fig. 2.1, then blown its engine and slowed down to pass through point P_2 at the same time t_2 as the dragster. Both vehicles have the same displacement during the same time interval and so have the same average x-velocity.

If distance is given in meters and time in seconds, average velocity is measured in meters per second (m/s). Other common units of velocity are kilometers per hour (km/h), feet per second (ft/s), miles per hour (mi/h), and knots $(1 \text{ knot} = 1 \text{ nautical mile/h} = 6080 \text{ ft/h})$. Table 2.1 lists some typical velocity magnitudes.

Table 2.1 Typical Velocity Magnitudes

A snail's pace	10^{-3} m/s
A brisk walk	2 m/s
Fastest human	11 m/s
Running cheetah	35 m/s
Fastest car	341 m/s
Random motion of air molecules	500 m/s
Fastest airplane	1000 m/s
Orbiting communications satellite	3000 m/s
Electron orbiting in a hydrogen atom	2×10^6 m/s
Light traveling in a vacuum	3×10^8 m/s

Test Your Understanding of Section 2.1 Each of the following automobile trips takes one hour. The positive x-direction is to the east. (i) Automobile *A* travels 50 km due east. (ii) Automobile *B* travels 50 km due west. (iii) Automobile *C* travels 60 km due east, then turns around and travels 10 km due west. (iv) Automobile *D* travels 70 km due east. (v) Automobile *E* travels 20 km due west, then turns around and travels 20 km due east. (a) Rank the five trips in order of average x-velocity from most positive to most negative. (b) Which trips, if any, have the same average x-velocity? (c) For which trip, if any, is the average x-velocity eqnal to zero?

2.2 Instantaneous Velocity

Sometimes the average velocity is all you need to know about a particle's motion. For example, a race along a straight line is really a competition to see whose average velocity, $v_{av\text{-}x}$, has the greatest magnitude. The prize goes to the competitor who can travel the displacement Δx from the start to the finish line in the shortest time interval, Δt (Fig. 2.4).

But the average velocity of a particle during a time interval can't tell us how fast, or in what direction, the particle was moving at any given time during the interval. To do this we need to know the velocity at any specific instant of time or specific point along the path. This is called **instantaneous velocity,** and it needs to be defined carefully.

CAUTION **How long is an instant?** Note that the word "instant" has a somewhat different definition in physics than in everyday language. You might use the phrase "It lasted just an instant" to refer to something that lasted for a very short time interval. But in physics an instant has no duration at all; it refers to a single value of time.

2.4 The winner of a 50-m swimming race is the swimmer whose average velocity has the greatest magnitude—that is, the swimmer who traverses a displacement Δx of 50 m in the shortest elapsed time Δt.

2.5 Even when he's moving forward, this cyclist's instantaneous x-velocity can be negative—if he's traveling in the negative x-direction. In any problem, the choice of which direction is positive and which is negative is entirely up to you.

To find the instantaneous velocity of the dragster in Fig. 2.1 at the point P_1, we move the second point P_2 closer and closer to the first point P_1 and compute the average velocity $v_{av\text{-}x} = \Delta x/\Delta t$ over the ever-shorter displacement and time interval. Both Δx and Δt become very small, but their ratio does not necessarily become small. In the language of calculus, the limit of $\Delta x/\Delta t$ as Δt approaches zero is called the **derivative** of x with respect to t and is written dx/dt. *The instantaneous velocity is the limit of the average velocity as the time interval approaches zero; it equals the instantaneous rate of change of position with time.* We use the symbol v_x, with no "av" subscript, for the instantaneous velocity along the x-axis, or the **instantaneous x-velocity:**

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad \text{(instantaneous } x\text{-velocity, straight-line motion)} \quad (2.3)$$

The time interval Δt is always positive, so v_x has the same algebraic sign as Δx. A positive value of v_x means that x is increasing and the motion is in the positive x-direction; a negative value of v_x means that x is decreasing and the motion is in the negative x-direction. A body can have positive x and negative v_x, or the reverse; x tells us where the body is, while v_x tells us how it's moving (Fig. 2.5).

Instantaneous velocity, like average velocity, is a vector quantity; Eq. (2.3) defines its x-component. In straight-line motion, all other components of instantaneous velocity are zero. In this case we often call v_x simply the instantaneous velocity. (In Chapter 3 we'll deal with the general case in which the instantaneous velocity can have nonzero x-, y-, and z-components.) When we use the term "velocity," we will always mean instantaneous rather than average velocity.

The terms "velocity" and "speed" are used interchangeably in everyday language, but they have distinct definitions in physics. We use the term **speed** to denote distance traveled divided by time, on either an average or an instantaneous basis. We use the symbol v with *no* subscripts to denote instantaneous speed. Instantaneous *speed* measures how fast a particle is moving; instantaneous *velocity* measures how fast *and* in what direction it's moving. For example, a particle with instantaneous velocity $v_x = 25$ m/s and a second particle with $v_x = -25$ m/s are moving in opposite directions at the same instantaneous speed 25 m/s. Instantaneous speed is the magnitude of instantaneous velocity, and so instantaneous speed can never be negative.

CAUTION **Average speed and average velocity** Average speed is *not* the magnitude of average velocity. When Alexander Popov set a world record in 1994 by swimming 100.0 m in 46.74 s, his average speed was $(100.0\text{ m})/(46.74\text{ s}) = 2.139$ m/s. But because he swam two lengths in a 50-m pool, he started and ended at the same point and so had zero total displacement and zero average *velocity!* Both average speed and instantaneous speed are scalars, not vectors, because these quantities contain no information about direction.

Example 2.1 Average and instantaneous velocities

A cheetah is crouched 20 m to the east of an observer's vehicle (Fig. 2.6a). At time $t = 0$ the cheetah charges an antelope and begins to run along a straight line. During the first 2.0 s of the attack, the cheetah's coordinate x varies with time according to the equation $x = 20\text{ m} + (5.0\text{ m/s}^2)t^2$. (a) Find the displacement of the cheetah between $t_1 = 1.0$ s and $t_2 = 2.0$ s. (b) Find the average velocity during the same time interval. (c) Find the instantaneous velocity at time $t_1 = 1.0$ s by taking $\Delta t = 0.1$ s, then $\Delta t = 0.01$ s, then $\Delta t = 0.001$ s. (d) Derive a general expression for the instantaneous velocity as a function of time, and from it find v_x at $t = 1.0$ s and $t = 2.0$ s.

SOLUTION

IDENTIFY: We use the definitions of displacement, average velocity, and instantaneous velocity. Using the first two of these involves algebra; the last one requires using calculus to take a derivative.

SET UP: Figure 2.6b shows our sketch of the cheetah's motion. To analyze this problem we use Eq. (2.1) for displacement, Eq. (2.2) for average velocity, and Eq. (2.3) for instantaneous velocity.

EXECUTE: (a) At time $t_1 = 1.0$ s the cheetah's position x_1 is

$$x_1 = 20\text{ m} + (5.0\text{ m/s}^2)(1.0\text{ s})^2 = 25\text{ m}$$

At time $t_2 = 2.0$ s its position x_2 is

$$x_2 = 20\text{ m} + (5.0\text{ m/s}^2)(2.0\text{ s})^2 = 40\text{ m}$$

The displacement during this interval is

$$\Delta x = x_2 - x_1 = 40\text{ m} - 25\text{ m} = 15\text{ m}$$

(b) The average x-velocity during this time interval is

$$v_{\text{av-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{40\text{ m} - 25\text{ m}}{2.0\text{ s} - 1.0\text{ s}} = \frac{15\text{ m}}{1.0\text{ s}} = 15\text{ m/s}$$

(c) With $\Delta t = 0.1$ s, the time interval is from $t_1 = 1.0$ s to $t_2 = 1.1$ s. At time t_2, the position is

$$x_2 = 20\text{ m} + (5.0\text{ m/s}^2)(1.1\text{ s})^2 = 26.05\text{ m}$$

The average x-velocity during this interval is

$$v_{\text{av-}x} = \frac{26.05\text{ m} - 25\text{ m}}{1.1\text{ s} - 1.0\text{ s}} = 10.5\text{ m/s}$$

You should follow this same pattern to work out the average x-velocities for the 0.01-s and 0.001-s intervals. The results are 10.05 m/s and 10.005 m/s. As Δt gets smaller, the average x-velocity gets closer to 10.0 m/s, so we conclude that the instantaneous x-velocity at time $t = 1.0$ s is 10.0 m/s.

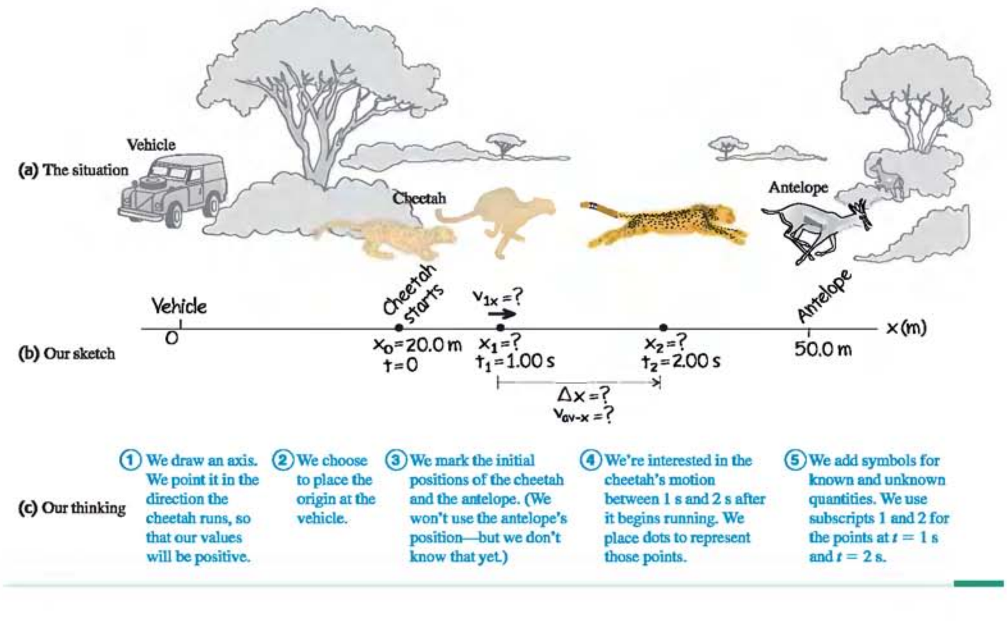
(d) To find the instantaneous x-velocity as a function of time, take the derivative of the expression for x with respect to t. The derivative of a constant is zero, and for any n the derivative of t^n is nt^{n-1}, so the derivative of t^2 is $2t$. Therefore

$$v_x = \frac{dx}{dt} = (5.0\text{ m/s}^2)(2t) = (10\text{ m/s}^2)t$$

At time $t = 1.0$ s, $v_x = 10$ m/s as we found in part (c). At time $t = 2.0$ s, $v_x = 20$ m/s.

EVALUATE: Our results show that the cheetah picked up speed from $t = 0$ (when it was at rest) to $t = 1.0$ s $(v_x = 10\text{ m/s})$ to $t = 2.0$ s $(v_x = 20\text{ m/s})$. This makes sense; the cheetah covered only 5 m during the interval $t = 0$ to $t = 1.0$ s, but covered 15 m during the interval $t = 1.0$ s to $t = 2.0$ s.

2.6 A cheetah attacking an antelope from ambush. The animals are not drawn to the same scale as the axis.



Finding Velocity on an x-t Graph

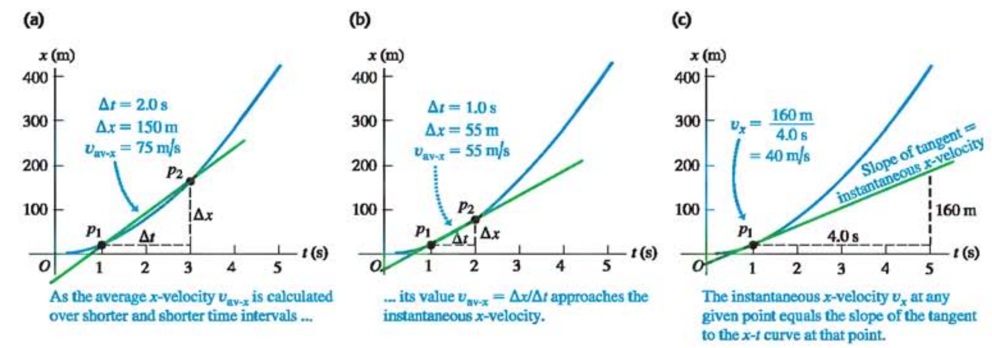
ActivPhysics Online
1.1 Analyzing Motion Using Diagrams

The x-velocity of a particle can also be found from the graph of its position as a function of time. Suppose we want to find the x-velocity of the dragster in Fig. 2.1 at point P_1. As point P_2 in Fig. 2.1 approaches point P_1, point p_2 in the x-t graphs of Figs. 2.7a and 2.7b approaches point p_1 and the average x-velocity is calculated over shorter time intervals Δt. In the limit that $\Delta t \rightarrow 0$, shown in Fig. 2.7c, the slope of the line p_1p_2 equals the slope of the line tangent to the curve at point p_1. Thus, *on a graph of position as a function of time for straight-line motion, the instantaneous x-velocity at any point is equal to the slope of the tangent to the curve at that point.*

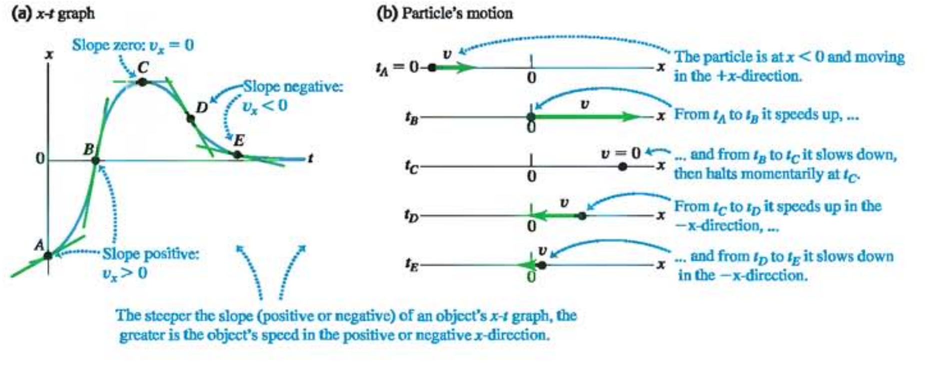
If the tangent to the x-t curve slopes upward to the right, as in Fig. 2.7c, then its slope is positive, the x-velocity is positive, and the motion is in the positive x-direction. If the tangent slopes downward to the right, the slope of the x-t graph and the x-velocity are negative, and the motion is in the negative x-direction. When the tangent is horizontal, the slope and the x-velocity are zero. Figure 2.8 illustrates these three possibilities.

Figure 2.8 actually depicts the motion of a particle in two ways: as (a) an x-t graph and (b) a **motion diagram.** A motion diagram shows the particle's posi-

2.7 Using an x-t graph to go from **(a)**, **(b)** average x-velocity to **(c)** instantaneous x-velocity v_x. In **(c)** we find the slope of the tangent to the x-t curve by dividing any vertical interval (with distance units) along the tangent by the corresponding horizontal interval (with time units).



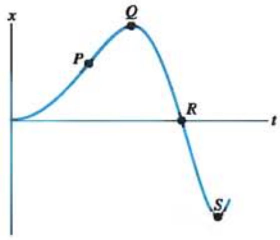
2.8 (a) The x-t graph of the motion of a particular particle. The slope of the tangent at any point equals the velocity at that point. (b) A motion diagram showing the position and velocity of the particle at each of the times labeled on the x-t graph.



tion at various times (like frames from a video of the particle's motion) as well as arrows to represent the particle's velocity at each instant. We will use both *x-t* graphs and motion diagrams in this chapter to help you understand motion. You will find it worth your while to draw *both* an *x-t* graph and a motion diagram as part of solving any problem involving motion.

Test Your Understanding of Section 2.2 Figure 2.9 is an *x-t* graph of the motion of a particle. (a) Rank the values of the particle's *x*-velocity v_x at the points *P*, *Q*, *R*, and *S* from most positive to most negative. (b) At which points is v_x positive? (c) At which points is v_x negative? (d) At which points is v_x zero? (e) Rank the values of the particle's *speed* at the points *P*, *Q*, *R*, and *S* from fastest to slowest.

2.9 An *x-t* graph for a particle.



2.3 Average and Instantaneous Acceleration

Just as velocity describes the rate of change of position with time, *acceleration* describes the rate of change of velocity with time. Like velocity, acceleration is a vector quantity. When the motion is along a straight line, its only nonzero component is along that line. As we'll see, acceleration in straight-line motion can refer to either speeding up or slowing down.

Average Acceleration

Let's consider again a particle moving along the *x*-axis. Suppose that at time t_1 the particle is at point P_1 and has *x*-component of (instantaneous) velocity v_{1x}, and at a later time t_2 it is at point P_2 and has *x*-component of velocity v_{2x}. So the *x*-component of velocity changes by an amount $\Delta v_x = v_{2x} - v_{1x}$ during the time interval $\Delta t = t_2 - t_1$.

We define the **average acceleration** of the particle as it moves from P_1 to P_2 to be a vector quantity whose *x*-component $a_{\text{av-}x}$ (called the **average *x*-acceleration**) equals Δv_x, the change in the *x*-component of velocity, divided by the time interval Δt:

$$a_{\text{av-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t} \quad \text{(average } x\text{-acceleration, straight-line motion)} \qquad (2.4)$$

For straight-line motion along the *x*-axis we will often call $a_{\text{av-}x}$ simply the average acceleration. (We'll encounter the other components of the average acceleration vector in Chapter 3.)

If we express velocity in meters per second and time in seconds, then average acceleration is in meters per second per second, or $(\text{m/s})/\text{s}$. This is usually written as m/s^2 and is read "meters per second squared."

CAUTION **Acceleration vs. velocity** Be very careful not to confuse acceleration with velocity! Velocity describes how a body's position changes with time; it tells us how fast and in what direction the body moves. Acceleration describes how the velocity changes with time; it tells us how the speed and direction of motion are changing. It may help to remember the phrase "acceleration is to velocity as velocity is to position." It can also help to imagine yourself riding along with the moving body. If the body accelerates forward and gains speed, you would feel pushed backward in your seat; if it accelerates backward and loses speed, you would feel pushed forward. If the velocity is constant and there's no acceleration, you would feel neither sensation. (We'll see the reason for these sensations in Chapter 4.)

Example 2.2 Average acceleration

An astronaut has left an orbiting spacecraft to test a new personal maneuvering unit. As she moves along a straight line, her partner on the spacecraft measures her velocity every 2.0 s, starting at time $t = 1.0$ s:

t	v_x	t	v_x
1.0 s	0.8 m/s	9.0 s	−0.4 m/s
3.0 s	1.2 m/s	11.0 s	−1.0 m/s
5.0 s	1.6 m/s	13.0 s	−1.6 m/s
7.0 s	1.2 m/s	15.0 s	−0.8 m/s

Find the average x-acceleration, and describe whether the speed of the astronaut increases or decreases, for each of these time intervals: (a) $t_1 = 1.0$ s to $t_2 = 3.0$ s; (b) $t_1 = 5.0$ s to $t_2 = 7.0$ s; (c) $t_1 = 9.0$ s to $t_2 = 11.0$ s; (d) $t_1 = 13.0$ s to $t_2 = 15.0$ s.

SOLUTION

IDENTIFY: We'll need the definition of average acceleration $a_{av\text{-}x}$. To find the changes in speed, we'll use the idea that speed v is the magnitude of the instantaneous velocity v_x.

SET UP: Figure 2.10 shows our graphs. We use Eq. (2.4) to find the value of $a_{av\text{-}x}$ from the change in *velocity* for each time interval.

EXECUTE: In the upper part of Fig. 2.10, we graph the x-velocity as a function of time. On this v_x-t graph, the slope of the line connecting the points at the beginning and end of each interval equals the average x-acceleration $a_{av\text{-}x} = \Delta v_x/\Delta t$ for that interval. In the lower part of Fig. 2.10, we graph the values of $a_{av\text{-}x}$. We find:

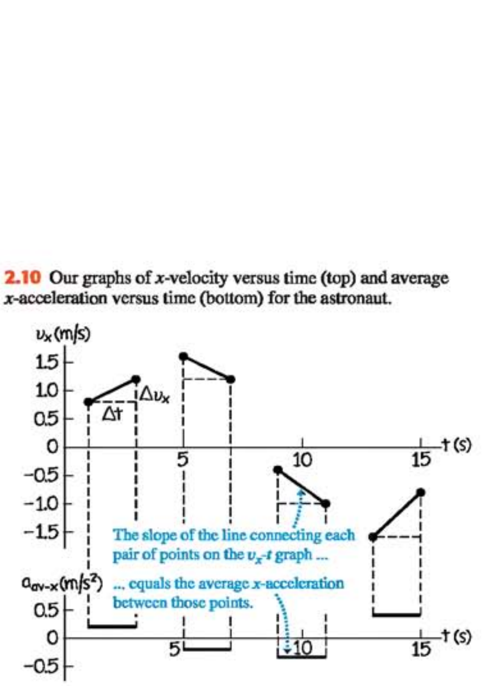
(a) $a_{av\text{-}x} = (1.2\text{ m/s} - 0.8\text{ m/s})/(3.0\text{ s} - 1.0\text{ s}) = 0.2\text{ m/s}^2$. The speed (magnitude of instantaneous x-velocity) increases from 0.8 m/s to 1.2 m/s.

(b) $a_{av\text{-}x} = (1.2\text{ m/s} - 1.6\text{ m/s})/(7.0\text{ s} - 5.0\text{ s}) = -0.2\text{ m/s}^2$. The speed decreases from 1.6 m/s to 1.2 m/s.

(c) $a_{av\text{-}x} = [-1.0\text{ m/s} - (-0.4\text{ m/s})]/(11.0\text{ s} - 9.0\text{ s}) = -0.3\text{ m/s}^2$. The speed increases from 0.4 m/s to 1.0 m/s.

(d) $a_{av\text{-}x} = [-0.8\text{ m/s} - (-1.6\text{ m/s})]/(15.0\text{ s} - 13.0\text{ s}) = 0.4\text{ m/s}^2$. The speed decreases from 1.6 m/s to 0.8 m/s.

2.10 Our graphs of x-velocity versus time (top) and average x-acceleration versus time (bottom) for the astronaut.



EVALUATE: Our results show that when the average x-acceleration has the *same* direction (same algebraic sign) as the initial velocity, as in intervals (a) and (c), the astronaut goes faster; when it has the *opposite* direction (opposite algebraic sign), as in intervals (b) and (d), she slows down. Thus positive x-acceleration means speeding up if the x-velocity is positive [interval (a)] but slowing down if the x-velocity is negative [interval (d)]. Similarly, negative x-acceleration means speeding up if the x-velocity is negative [interval (c)] but slowing down if the x-velocity is positive [interval (b)].

Instantaneous Acceleration

We can now define **instantaneous acceleration** following the same procedure that we used to define instantaneous velocity. As an example, suppose a race car driver is driving along a straightaway as shown in Fig. 2.11. To define the instantaneous acceleration at point P_1, we take the second point P_2 in Fig. 2.11 to be closer and closer to P_1 so that the average acceleration is computed over shorter and shorter time intervals. *The instantaneous acceleration is the limit of the average acceleration as the time interval approaches zero.* In the language of calculus, *instantaneous acceleration equals the instantaneous rate of change of velocity with time.* Thus

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad \text{(instantaneous } x\text{-acceleration, straight-line motion)} \tag{2.5}$$

2.11 A Grand Prix car at two points on the straightaway.

Note that a_x in Eq. (2.5) is really the x-component of the acceleration vector, or the **instantaneous x-acceleration;** in straight-line motion, all other components of this vector are zero. From now on, when we use the term "acceleration," we will always mean instantaneous acceleration, not average acceleration.

Example 2.3 Average and instantaneous accelerations

Suppose the x-velocity v_x of the car in Fig. 2.11 at any time t is given by the equation

$$v_x = 60\ \text{m/s} + (0.50\ \text{m/s}^3)t^2$$

(a) Find the change in x-velocity of the car in the time interval between $t_1 = 1.0$ s and $t_2 = 3.0$ s. (b) Find the average x-acceleration in this time interval. (c) Find the instantaneous x-acceleration at time $t_1 = 1.0$ s by taking Δt to be first 0.1 s, then 0.01 s, then 0.001 s. (d) Derive an expression for the instantaneous x-acceleration at any time, and use it to find the x-acceleration at $t = 1.0$ s and $t = 3.0$ s.

SOLUTION

IDENTIFY: This example is analogous to Example 2.1 in Section 2.2. (Now is a good time to review that example.) There we found the average x-velocity over shorter and shorter time intervals from the change in position, and we determined the instantaneous x-velocity by differentiating the position as a function of time. In this example, we find the *average* x-acceleration from the change in x-velocity over a time interval. Likewise, we find the *instantaneous* x-acceleration by differentiating the x-velocity as a function of time.

SET UP: We'll use Eq. (2.4) for average x-acceleration and Eq. (2.5) for instantaneous x-acceleration.

EXECUTE: (a) We first find the x-velocity at each time by substituting each value of t into the equation. At time $t_1 = 1.0$ s,

$$v_{1x} = 60\ \text{m/s} + (0.50\ \text{m/s}^3)(1.0\ \text{s})^2 = 60.5\ \text{m/s}$$

At time $t_2 = 3.0$ s,

$$v_{2x} = 60\ \text{m/s} + (0.50\ \text{m/s}^3)(3.0\ \text{s})^2 = 64.5\ \text{m/s}$$

The change in x-velocity Δv_x is

$$\Delta v_x = v_{2x} - v_{1x} = 64.5\ \text{m/s} - 60.5\ \text{m/s} = 4.0\ \text{m/s}$$

The time interval is $\Delta t = 3.0\ \text{s} - 1.0\ \text{s} = 2.0$ s.

(b) The average x-acceleration during this time interval is

$$a_{\text{av-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{4.0\ \text{m/s}}{2.0\ \text{s}} = 2.0\ \text{m/s}^2$$

During the time interval from $t_1 = 1.0$ s to $t_2 = 3.0$ s, the x-velocity and average x-acceleration have the same algebraic sign (in this case, positive), and the car speeds up.

(c) When $\Delta t = 0.1$ s, $t_2 = 1.1$ s and we find

$$v_{2x} = 60\ \text{m/s} + (0.50\ \text{m/s}^3)(1.1\ \text{s})^2 = 60.605\ \text{m/s}$$

$$\Delta v_x = 0.105\ \text{m/s}$$

$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{0.105\ \text{m/s}}{0.1\ \text{s}} = 1.05\ \text{m/s}^2$$

You should do these calculations for $\Delta t = 0.01$ s and $\Delta t = 0.001$ s; the results are $a_{\text{av-}x} = 1.005\ \text{m/s}^2$ and $a_{\text{av-}x} = 1.0005\ \text{m/s}^2$, respectively. As Δt gets smaller, the average x-acceleration gets closer to $1.0\ \text{m/s}^2$, so the instantaneous x-acceleration at $t = 1.0$ s is $1.0\ \text{m/s}^2$.

(d) The instantaneous x-acceleration is $a_x = dv_x/dt$. The derivative of a constant is zero and the derivative of t^2 is $2t$, so

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}[60\ \text{m/s} + (0.50\ \text{m/s}^3)t^2]$$

$$= (0.50\ \text{m/s}^3)(2t) = (1.0\ \text{m/s}^3)t$$

When $t = 1.0$ s,

$$a_x = (1.0\ \text{m/s}^3)(1.0\ \text{s}) = 1.0\ \text{m/s}^2$$

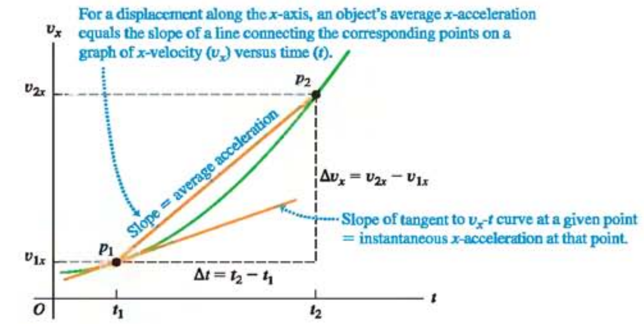
When $t = 3.0$ s,

$$a_x = (1.0\ \text{m/s}^3)(3.0\ \text{s}) = 3.0\ \text{m/s}^2$$

EVALUATE: Note that neither of the values we found in part (d) is equal to the average x-acceleration found in part (b). That's because the car's instantaneous x-acceleration varies with time. The rate of change of acceleration with time is sometimes called the "jerk."

Finding Acceleration on a v_x-t Graph or an x-t Graph

In Section 2.2 we interpreted average and instantaneous x-velocity in terms of the slope of a graph of position versus time. In the same way, we can interpret average and instantaneous x-acceleration by using a graph with instantaneous velocity v_x on the vertical axis and time t on the horizontal axis—that is, a **v_x-t graph** (Fig. 2.12). The points on the graph labeled p_1 and p_2 correspond to points P_1 and P_2 in Fig. 2.11. The average x-acceleration $a_{\text{av-}x} = \Delta v_x/\Delta t$ during this interval is the slope of the line p_1p_2. As point P_2 in Fig. 2.11 approaches point P_1, point p_2 in the v_x-t graph of Fig. 2.12 approaches point p_1, and the slope of the line p_1p_2 approaches the slope of the line tangent to the curve at point p_1. Thus, *on a graph of x-velocity as a function of time, the instantaneous x-acceleration at any point is equal to the slope of the tangent to the curve at that point.* Tangents drawn at



2.12 A v_x-t graph of the motion in Fig. 2.11.

different points along the curve in Fig. 2.12 have different slopes, so the instantaneous x-acceleration varies with time.

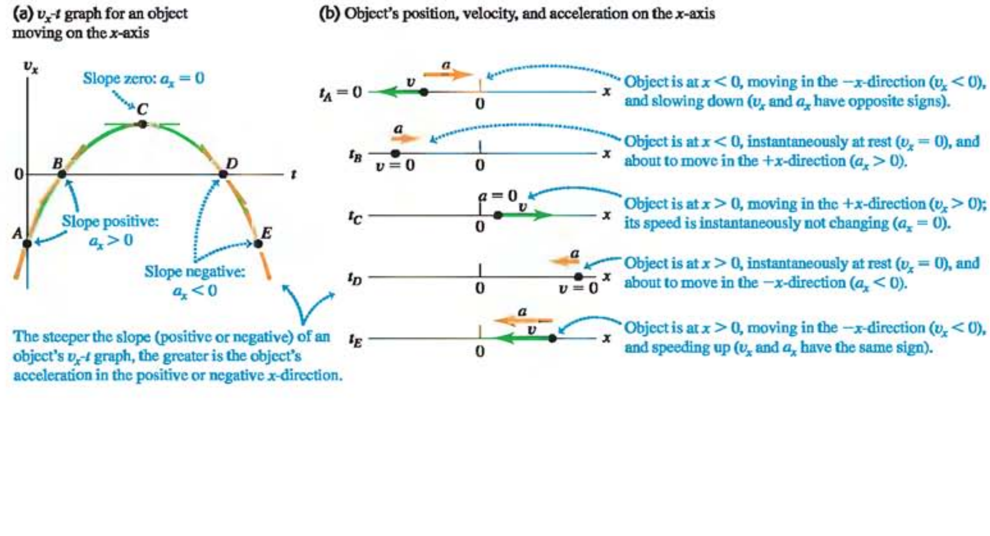
CAUTION **The signs of x-acceleration and x-velocity** By itself, the algebraic sign of the x-acceleration does *not* tell you whether a body is speeding up or slowing down. You must compare the signs of the x-velocity and the x-acceleration. When v_x and a_x have the *same* sign, the body is speeding up. If both are positive, the body is moving in the positive direction with increasing speed. If both are negative, the body is moving in the negative direction with an x-velocity that is becoming more and more negative, and again the speed is increasing. When v_x and a_x have *opposite* signs, the body is slowing down. If v_x is positive and a_x is negative, the body is moving in the positive direction with decreasing speed; if v_x is negative and a_x is positive, the body is moving in the negative direction with an x-velocity that is becoming less negative, and again the body is slowing down. Figure 2.13 illustrates some of these possibilities.

The term "deceleration" is sometimes used for a decrease in speed. Because it may mean positive or negative a_x, depending on the sign of v_x, we avoid this term.

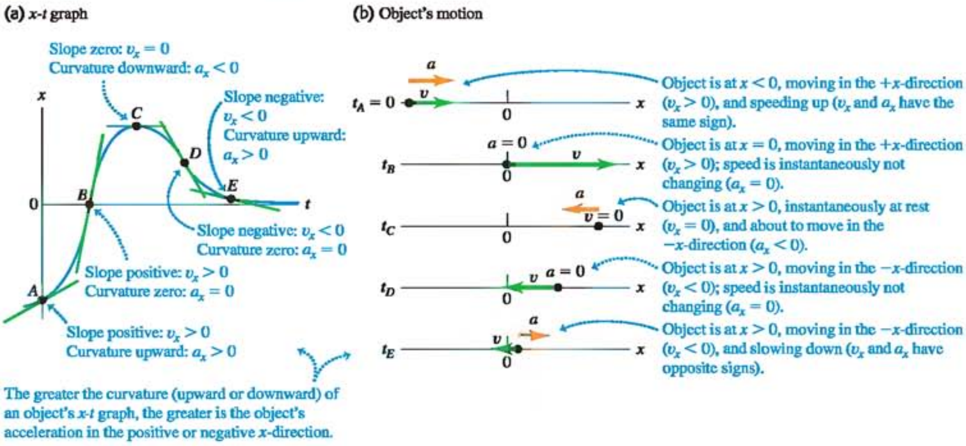
We can also learn about the acceleration of a body from a graph of its *position* versus time. Because $a_x = dv_x/dt$ and $v_x = dx/dt$, we can write

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \tag{2.6}$$

2.13 **(a)** A v_x-t graph of the motion of a different particle than that shown in Fig. 2.8. The slope of the tangent at any point equals the x-acceleration at that point. **(b)** A motion diagram showing the position, velocity, and acceleration of the particle at each of the times labeled on the v_x-t graph. The positions are consistent with the v_x-t graph; for instance, from t_A to t_B the velocity is negative, so at t_B the particle is at a more negative value of x than at t_A.



2.14 (a) The same x-t graph as shown in Fig. 2.8a. The x-velocity is equal to the *slope* of the graph, and the acceleration is given by the *concavity* or *curvature* of the graph. (b) A motion diagram showing the position, velocity, and acceleration of the particle at each of the times labeled on the x-t graph.



That is, a_x is the second derivative of x with respect to t. The second derivative of any function is directly related to the *concavity* or *curvature* of the graph of that function. At a point where the x-t graph is concave up (curved upward), the x-acceleration is positive and v_x is increasing; at a point where the x-t graph is concave down (curved downward), the x-acceleration is negative and v_x is decreasing. At a point where the x-t graph has no curvature, such as an inflection point, the x-acceleration is zero and the velocity is not changing. Figure 2.14 shows all three of these possibilities.

Examining the curvature of an x-t graph is an easy way to decide what the *sign* of acceleration is. This technique is less helpful for determining numerical values of acceleration because the curvature of a graph is hard to measure accurately.

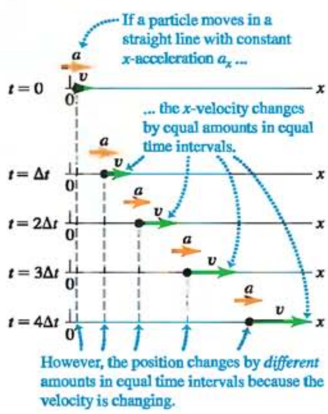
Test Your Understanding of Section 2.3 Look again at the x-t graph in Fig. 2.9 at the end of Section 2.2. (a) At which of the points P, Q, R, and S is the x-acceleration a_x positive? (b) At which points is the x-acceleration negative? (c) At which points does the x-acceleration appear to be zero? (d) At each point state whether the speed is increasing, decreasing, or not changing.

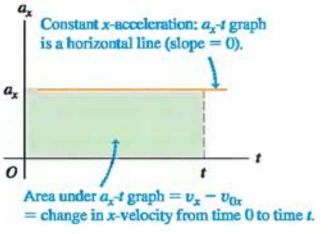
2.4 Motion with Constant Acceleration

The simplest kind of accelerated motion is straight-line motion with *constant* acceleration. In this case the velocity changes at the same rate throughout the motion. This is a very special situation, yet one that occurs often in nature. A falling body has a constant acceleration if the effects of the air are not important. The same is true for a body sliding on an incline or along a rough horizontal surface. Straight-line motion with nearly constant acceleration also occurs in technology, such as an airplane being catapulted from the deck of an aircraft carrier.

Figure 2.15 is a motion diagram showing the position, velocity, and acceleration for a particle moving with constant acceleration. Figures 2.16 and 2.17 depict this same motion in the form of graphs. Since the x-acceleration is constant, the **a_x-t graph** (graph of x-acceleration versus time) in Fig. 2.16 is a horizontal line. The graph of x-velocity versus time, or v_x-t graph, has a constant *slope* because the acceleration is constant, so this graph is a straight line (Fig. 2.17).

2.15 A motion diagram for a particle moving in a straight line in the positive x-direction with constant positive x-acceleration a_x. The position, velocity, and acceleration are shown at five equally spaced times.





2.16 An acceleration-time (a_x-t) graph for straight-line motion with constant positive x-acceleration a_x.

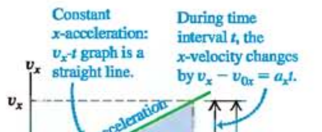
When the x-acceleration a_x is constant, the average x-acceleration $a_{av\text{-}x}$ for any time interval is the same as a_x. This makes it easy to derive equations for the position x and the x-velocity v_x as functions of time. To find an expression for v_x, we first replace $a_{av\text{-}x}$ in Eq. (2.4) by a_x:

$$a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \tag{2.7}$$

Now we let $t_1 = 0$ and let t_2 be any later time t. We use the symbol v_{0x} for the x-velocity at the initial time $t = 0$; the x-velocity at the later time t is v_x. Then Eq. (2.7) becomes

$$a_x = \frac{v_x - v_{0x}}{t - 0} \quad \text{or}$$

$$v_x = v_{0x} + a_x t \quad \text{(constant } x\text{-acceleration only)} \tag{2.8}$$



2.17 A velocity-time (v_x-t) graph for straight-line motion with constant positive x-acceleration a_x. The initial x-velocity v_{0x} is also positive in this case.

We can interpret this equation as follows. The x-acceleration a_x is the constant rate of change of x-velocity—that is, the change in x-velocity per unit time. The term $a_x t$ is the product of the change in x-velocity per unit time, a_x, and the time interval t. Therefore it equals the *total* change in x-velocity from the initial time $t = 0$ to the later time t. The x-velocity v_x at any time t then equals the initial x-velocity v_{0x} (at $t = 0$) plus the change in x-velocity $a_x t$ (see Fig. 2.17).

Another interpretation of Eq. (2.8) is that the change in x-velocity $v_x - v_{0x}$ of the particle between $t = 0$ and any later time t equals the *area* under the a_x-t graph between those two times. In Fig. 2.16, the area under the graph of x-acceleration versus time is a rectangle of vertical side a_x and horizontal side t. The area of this rectangle is $a_x t$, which from Eq. (2.8) is indeed equal to the change in velocity $v_x - v_{0x}$. In Section 2.6 we'll show that even if the x-acceleration is not constant, the change in x-velocity during a time interval is still equal to the area under the a_x-t curve, although in that case Eq. (2.8) does not apply.

Next we'll derive an equation for the position x as a function of time when the x-acceleration is constant. To do this, we use two different expressions for the average x-velocity $v_{av\text{-}x}$ during the interval from $t = 0$ to any later time t. The first expression comes from the definition of $v_{av\text{-}x}$, Eq. (2.2), which is true whether or not the acceleration is constant. We call the position at time $t = 0$ the *initial position*, denoted by x_0. The position at the later time t is simply x. Thus for the time interval $\Delta t = t - 0$ the displacement is $\Delta x = x - x_0$, and Eq. (2.2) gives

$$v_{av\text{-}x} = \frac{x - x_0}{t} \tag{2.9}$$

ActivPhysics ONLINE

1.1 Analyzing Motion Using Diagrams
1.2 Analyzing Motion Using Graphs
1.3 Predicting Motion from Graphs
1.4 Predicting Motion from Equations
1.5 Problem-Solving Strategies for Kinematics
1.6 Skier Races Downhill

We can also get a second expression for $v_{av\text{-}x}$ that is valid only when the x-acceleration is constant, so that the v_x-t graph is a straight line (as in Fig. 2.17) and the x-velocity changes at a constant rate. In this case the average x-velocity during any time interval is simply the arithmetic average of the x-velocities at the beginning and end of the interval. For the time interval 0 to t,

$$v_{av\text{-}x} = \frac{v_{0x} + v_x}{2} \quad \text{(constant } x\text{-acceleration only)} \tag{2.10}$$

(This equation is *not* true if the x-acceleration varies and the v_x-t graph is a curve, as in Fig. 2.13.) We also know that with constant x-acceleration, the x-velocity v_x at any time t is given by Eq. (2.8). Substituting that expression for v_x into Eq. (2.10), we find

$$v_{av\text{-}x} = \frac{1}{2}(v_{0x} + v_{0x} + a_x t)$$
$$= v_{0x} + \frac{1}{2}a_x t \quad \text{(constant } x\text{-acceleration only)} \tag{2.11}$$

Finally, we set Eqs. (2.9) and (2.11) equal to each other and simplify:

$$v_{0x} + \frac{1}{2}a_x t = \frac{x - x_0}{t} \quad \text{or}$$

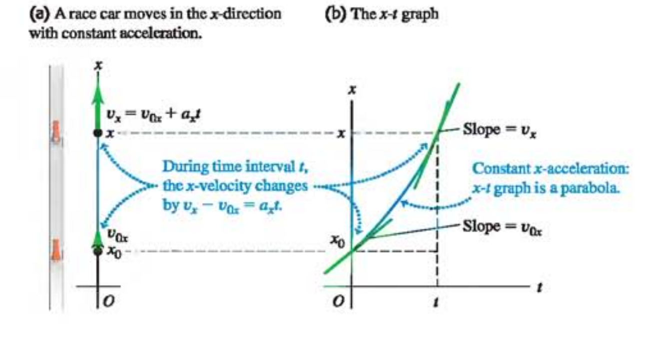
$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \quad \text{(constant } x\text{-acceleration only)} \quad (2.12)$$

Here's what Eq. (2.12) tells us: If at time $t = 0$ a particle is at position x_0 and has x-velocity v_{0x}, its new position x at any later time t is the sum of three terms—its initial position x_0, plus the distance $v_{0x}t$ that it would move if its x-velocity were constant, plus au additional distance $\frac{1}{2}a_x t^2$ caused by the change in x-velocity.

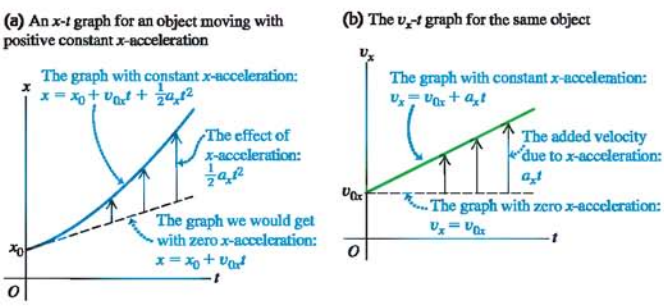
A graph of Eq. (2.12)—that is, an x-t graph for motion with constant x-acceleration (Fig. 2.18a)—is always a *parabola*. Figure 2.18b shows such a graph. The curve intercepts the vertical axis (x-axis) at x_0, the position at $t = 0$. The slope of the tangent at $t = 0$ equals v_{0x}, the initial x-velocity, and the slope of the tangent at any time t equals the x-velocity v_x at that time. The slope and x-velocity are continuously increasing, so the x-acceleration a_x is positive; you can also see this because the graph in Fig. 2.18b is concave up (it curves upward). If a_x is negative, the x-t graph is a parabola that is concave down (has a downward curvature).

If there is zero x-acceleration, the x-t graph is a straight line; if there is a constant x-acceleration, the additional $\frac{1}{2}a_x t^2$ term in Eq. (2.12) for x as a function of t curves the graph into a parabola (Fig. 2.19a). We can analyze the v_x-t graph in the same way. If there is zero x-acceleration this graph is a horizontal line (the x-velocity is constant); adding a constant x-acceleration gives a slope to the v_x-t graph (Fig. 2.19b).

ActivPhysics ONLINE
1.8 Seat Belts Save Lives
1.9 Screeching to a Halt
1.10 Car Starts, Then Stops
1.11 Solving Two-Vehicle Problems
1.12 Car Catches Truck
1.13 Avoiding a Rear-End Collision



2.18 (a) Straight-line motion with constant acceleration. (b) A position-time (x-t) graph for this motion (the same motion as is shown in Figs. 2.15, 2.16, and 2.17). For this motion the initial position x_0, the initial velocity v_{0x}, and the acceleration a_x are all positive.



2.19 (a) How a constant x-acceleration affects a body's (a) x-t graph and (b) v_x-t graph.

Just as the change in x-velocity of the particle equals the area under the a_x-t graph, the displacement—that is, the change in position—equals the area under the v_x-t graph. To be specific, the displacement $x - x_0$ of the particle between $t = 0$ and any later time t equals the area under the v_x-t graph between those two times. In Fig. 2.17 the area under the graph is divided into a dark-colored rectangle of vertical side v_{0x} and horizontal side t and a light-colored right triangle of vertical side $a_x t$ and horizontal side t. The area of the rectangle is $v_{0x}t$ and the area of the triangle is $\frac{1}{2}(a_x t)(t) = \frac{1}{2}a_x t^2$, so the total area under the v_x-t graph is

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

in agreement with Eq. (2.12).

The displacement during a time interval can always be found from the area under the v_x-t curve. This is true even if the acceleration is *not* constant, although in that case Eq. (2.12) does not apply. (We'll show this in Section 2.6.)

We can check whether Eqs. (2.8) and (2.12) are consistent with the assumption of constant acceleration by taking the derivative of Eq. (2.12). We find

$$v_x = \frac{dx}{dt} = v_{0x} + a_x t$$

which is Eq. (2.8). Differentiating again, we find simply

$$\frac{dv_x}{dt} = a_x$$

which agrees with the definition of instantaneous x-acceleration.

It's often useful to have a relationship between position, x-velocity, and (constant) x-acceleration that does not involve the time. To obtain this, we first solve Eq. (2.8) for t, then substitute the resulting expression into Eq. (2.12), and simplify:

$$t = \frac{v_x - v_{0x}}{a_x}$$

$$x = x_0 + v_{0x}\left(\frac{v_x - v_{0x}}{a_x}\right) + \frac{1}{2}a_x\left(\frac{v_x - v_{0x}}{a_x}\right)^2$$

We transfer the term x_0 to the left side and multiply through by $2a_x$:

$$2a_x(x - x_0) = 2v_{0x}v_x - 2v_{0x}^2 + v_x^2 - 2v_{0x}v_x + v_{0x}^2$$

Finally, simplifying gives us

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad \text{(constant } x\text{-acceleration only)} \qquad (2.13)$$

We can get one more useful relationship by equating the two expressions for $v_{\text{av-}x}$, Eqs. (2.9) and (2.10), and multiplying through by t. Doing this, we obtain

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t \qquad \text{(constant } x\text{-acceleration only)} \qquad (2.14)$$

Note that Eq. (2.14) does not contain the x-acceleration a_x. This equation can be handy when a_x is constant but its value is unknown.

Equations (2.8), (2.12), (2.13), and (2.14) are the *equations of motion with constant acceleration.* By using these equations, we can solve *any* problem involving straight-line motion of a particle with constant acceleration.

For the particular case of motion with constant x-acceleration depicted in Fig. 2.15 and graphed in Figs. 2.16, 2.17, and 2.18, the values of x_0, v_{0x}, and a_x are all positive. We invite you to redraw these figures for cases in which one, two, or all three of these quantities are negative.

A special case of motion with constant x-acceleration occurs when the x-acceleration is *zero.* The x-velocity is then constant, and the equations of motion become simply

$$v_x = v_{0x} = \text{constant}$$
$$x = x_0 + v_x t$$

Problem-Solving Strategy 2.1 Motion with Constant Acceleration

MP

IDENTIFY *the relevant concepts:* In most straight-line motion problems, you can use the constant-acceleration equations. Occasionally, however, you will encounter a situation in which the acceleration *isn't* constant. In such a case, you'll need a different approach (see Section 2.6).

SET UP *the problem* using the following steps:

1. First decide where the origin of coordinates is and which axis direction is positive. It is often easiest to place the particle at the origin at time $t = 0$; then $x_0 = 0$. It helps to make a motion diagram showing the coordinates and some later positions of the particle.
2. Remember that your choice of the positive axis direction automatically determines the positive directions for x-velocity and x-acceleration. If x is positive to the right of the origin, then v_x and a_x are also positive toward the right.
3. Restate the problem in words, and then translate it into symbols and equations. *When* does the particle arrive at a certain point (that is, what is the value of t)? *Where* is the particle when its x-velocity has a specified value (that is, what is the value of x when v_x has the specified value)? Example 2.4 asks, "Where is the motorcyclist when his velocity is 25 m/s?" In symbols, this says "What is the value of x when $v_x = 25$ m/s?"
4. Make a list of quantities such as x, x_0, v_x, v_{0x}, a_x, and t. In general, some of them will be known and some will be unknown. Write down the values of the known quantities, and decide which of the unknowns are the target variables. Be on the lookout for implicit information. For example, "A car sits at a stoplight" usually means $v_{0x} = 0$.

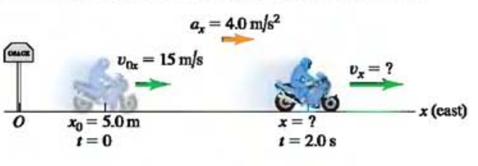
EXECUTE *the solution:* Choose an equation from Eqs. (2.8), (2.12), (2.13), and (2.14) that contains only one of the target variables. Solve this equation for the target variable, using symbols only. Then substitute the known values and compute the value of the target variable. Sometimes you will have to solve two simultaneous equations for two unknown quantities.

EVALUATE *your answer:* Take a hard look at your results to see whether they make sense. Are they within the general range of values you expected?

Example 2.4 Constant-acceleration calculations

A motorcyclist heading east through a small Iowa city accelerates after he passes the signpost marking the city limits (Fig. 2.20). His acceleration is a constant 4.0 m/s². At time $t = 0$ he is 5.0 m east of the signpost, moving east at 15 m/s. (a) Find his position and velocity at time $t = 2.0$ s. (b) Where is the motorcyclist when his velocity is 25 m/s?

2.20 A motorcyclist traveling with constant acceleration.



SOLUTION

IDENTIFY: The problem statement tells us that the acceleration is constant, so we can use the constant-acceleration equations.

SET UP: We take the signpost as the origin of coordinates ($x = 0$), and choose the positive x-axis to point east (see Fig. 2.20, which also serves as a motion diagram). At the initial time $t = 0$, the initial position is $x_0 = 5.0$ m and the initial x-velocity is $v_{0x} = 15$ m/s. The constant x-acceleration is $a_x = 4.0$ m/s². The unknown target variables in part (a) are the values of the position x and the x-velocity v_x at the later time $t = 2.0$ s; the target variable in part (b) is the value of x when $v_x = 25$ m/s.

Continued

EXECUTE: (a) We can find the position x at $t = 2.0$ s by using Eq. (2.12), which gives x as a function of time t:

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$= 5.0\text{ m} + (15\text{ m/s})(2.0\text{ s}) + \frac{1}{2}(4.0\text{ m/s}^2)(2.0\text{ s})^2$$
$$= 43\text{ m}$$

We can find the x-velocity v_x at this same time by using Eq. (2.8), which gives v_x as a function of time t:

$$v_x = v_{0x} + a_x t$$
$$= 15\text{ m/s} + (4.0\text{ m/s}^2)(2.0\text{ s}) = 23\text{ m/s}$$

(b) We want to find the value of x when $v_x = 25$ m/s, but we don't know the time when the motorcycle has this x-velocity. Hence we use Eq. (2.13), which involves x, v_x, and a_x but does not involve t:

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

Solving for x and substituting in the known values, we find

$$x = x_0 + \frac{v_x^2 - v_{0x}^2}{2a_x}$$
$$= 5.0\text{ m} + \frac{(25\text{ m/s})^2 - (15\text{ m/s})^2}{2(4.0\text{ m/s}^2)}$$
$$= 55\text{ m}$$

An alternative but longer route to the same answer is to use Eq. (2.8) to first find the time when $v_x = 25$ m/s:

$$v_x = v_{0x} + a_x t \quad \text{so}$$
$$t = \frac{v_x - v_{0x}}{a_x} = \frac{25\text{ m/s} - 15\text{ m/s}}{4.0\text{ m/s}^2} = 2.5\text{ s}$$

Given the time t, we can find x using Eq. (2.12):

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$= 5.0\text{ m} + (15\text{ m/s})(2.5\text{ s}) + \frac{1}{2}(4.0\text{ m/s}^2)(2.5\text{ s})^2$$
$$= 55\text{ m}$$

EVALUATE: Do these results make sense? According to our results in part (a), the motorcyclist accelerates from 15 m/s (about 34 mi/h, or 54 km/h) to 23 m/s (about 51 mi/h, or 83 km/h) in 2.0 s while traveling a distance of 38 m (about 125 ft). This is pretty brisk acceleration, but well within the capabilities of a high-performance bike.

Comparing our results in part (b) with those in part (a) tells us that the motorcycle attains an x-velocity $v_x = 25$ m/s at a later time and after traveling a greater distance than when the motorcycle had $v_x = 23$ m/s. This makes sense, since the motorcycle has a positive x-acceleration and so its x-velocity is increasing.

Example 2.5 Two bodies with different accelerations

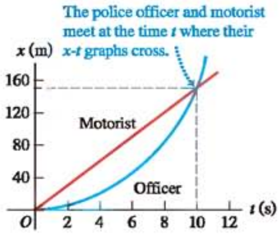
A motorist traveling with a constant speed of 15 m/s (about 34 mi/h) passes a school-crossing corner, where the speed limit is 10 m/s (about 22 mi/h). Just as the motorist passes, a police officer on a motorcycle stopped at the corner starts off in pursuit with constant acceleration of 3.0 m/s^2 (Fig. 2.21a). (a) How much time elapses before the officer catches up with the motorist? (b) What is the officer's speed at that point? (c) What is the total distance each vehicle has traveled at that point?

SOLUTION

IDENTIFY: The police officer and the motorist both move with constant acceleration (equal to zero for the motorist), so we can use the formulas we have developed.

SET UP: We take the origin at the corner, so $x_0 = 0$ for both, and we take the positive direction to the right. Let x_P (for police) be the officer's position and x_M (for motorist) be the motorist's position at any time. The initial x-velocities are $v_{P0x} = 0$ for the officer and $v_{M0x} = 15$ m/s for the motorist; the constant x-accelerations are $a_{Px} = 3.0$ m/s^2 for the officer and $a_{Mx} = 0$ for the motorist. Our target variable in part (a) is the time when the officer catches the motorist—that is, when the two vehicles are at the same position. In part (b) we're looking for the officer's speed v (the magnitude of his velocity) at the time found in part (a). In part (c) we want to find the position of either vehicle at this same time. Hence we use Eq. (2.12) (which relates position and time) in

2.21 (a) Motion with constant acceleration overtaking motion with constant velocity. (b) A graph of x versus t for each vehicle.



parts (a) and (c), and Eq. (2.8) (which relates velocity and time) in part (b).

EXECUTE: (a) To find the value of the time t when the motorist and the police officer are at the same position, we apply Eq. (2.12), $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, to each vehicle:

$$x_M = 0 + v_{M0x}t + \frac{1}{2}(0)t^2 = v_{M0x}t$$

$$x_P = 0 + (0)t + \frac{1}{2}a_{Px}t^2 = \frac{1}{2}a_{Px}t^2$$

Since $x_M = x_P$ at time t, we set these two expressions equal to each other and solve for t:

$$v_{M0x}t = \frac{1}{2}a_{Px}t^2$$

$$t = 0 \quad \text{or} \quad t = \frac{2v_{M0x}}{a_{Px}} = \frac{2(15\ \text{m/s})}{3.0\ \text{m/s}^2} = 10\ \text{s}$$

There are *two* times when both the vehicles have the same x-coordinate. The first, $t = 0$, is the time when the motorist passes the parked motorcycle at the corner. The second, $t = 10$ s, is the time when the officer catches up with the motorist.

(b) We want the magnitude of the officer's x-velocity v_{Px} at the time t found in part (a). Her velocity at any time is given by Eq. (2.8):

$$v_{Px} = v_{P0x} + a_{Px}t = 0 + (3.0\ \text{m/s}^2)t$$

Using $t = 10$ s, we find $v_{Px} = 30$ m/s. When the officer overtakes the motorist, she is traveling twice as fast as the motorist is.

(c) In 10 s the distance the motorist travels is

$$x_M = v_{M0x}t = (15\ \text{m/s})(10\ \text{s}) = 150\ \text{m}$$

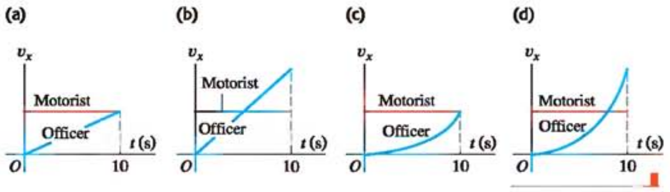
and the distance the officer travels is

$$x_P = \frac{1}{2}a_{Px}t^2 = \frac{1}{2}(3.0\ \text{m/s}^2)(10\ \text{s})^2 = 150\ \text{m}$$

This verifies that at the time the officer catches the motorist, they have gone equal distances.

EVALUATE: Figure 2.21b shows graphs of x versus t for each vehicle. We see again that there are two times when the two positions are the same (where the two graphs cross). At neither of these times do the two vehicles have the same velocity (i.e., where the two graphs cross, their slopes are different). At $t = 0$, the officer is at rest; at $t = 10$ s, the officer has twice the speed of the motorist.

Test Your Understanding of Section 2.4 Four possible v_x-t graphs are shown for the two vehicles in Example 2.5. Which graph is correct?



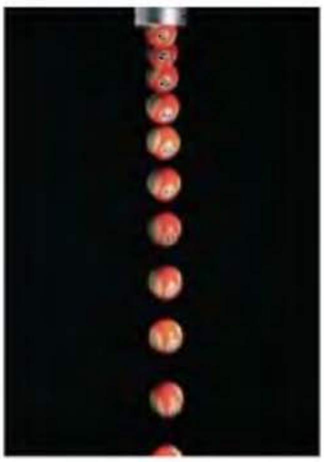
2.5 Freely Falling Bodies

The most familiar example of motion with (nearly) constant acceleration is a body falling under the influence of the earth's gravitational attraction. Such motion has held the attention of philosophers and scientists since ancient times. In the fourth century B.C., Aristotle thought (erroneously) that heavy bodies fall faster than light bodies, in proportion to their weight. Nineteen centuries later, Galileo (see Section 1.1) argued that a body should fall with a downward acceleration that is constant and independent of its weight.

Experiment shows that if the effects of the air can be neglected, Galileo is right; all bodies at a particular location fall with the same downward acceleration, regardless of their size or weight. If in addition the distance of the fall is small compared with the radius of the earth, and if we ignore small effects due to the earth's rotation, the acceleration is constant. The idealized motion that results under all of these assumptions is called **free fall,** although it includes rising as well as falling motion. (In Chapter 3 we will extend the discussion of free fall to include the motion of projectiles, which move both vertically and horizontally.)

Figure 2.22 is a photograph of a falling ball made with a stroboscopic light source that produces a series of short, intense flashes. As each flash occurs, an image of the ball at that instant is recorded on the photograph. There are equal

2.22 Multiflash photo of a freely falling ball.



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1.7 Balloonist Drops Lemonade

1.10 Pole-Vaulter Lands

time intervals between flashes, so the average velocity of the ball between successive flashes is proportional to the distance between corresponding images. The increasing distances between images show that the velocity is continuously changing; the ball is accelerating downward. Careful measurement shows that the velocity change is the same in each time interval, so the acceleration of the freely falling ball is constant.

The constant acceleration of a freely falling body is called the **acceleration due to gravity,** and we denote its magnitude with the letter g. We will frequently use the approximate value of g at or near the earth's surface:

$$g = 9.8\ \text{m/s}^2 = 980\ \text{cm/s}^2$$
$$= 32\ \text{ft/s}^2 \quad \text{(approximate value near the earth's surface)}$$

The exact value varies with location, so we will often give the value of g at the earth's surface to only two significant figures. Because g is the magnitude of a vector quantity, it is always a *positive* number. On the surface of the moon, the acceleration due to gravity is caused by the attractive force of the moon rather than the earth, and $g = 1.6\ \text{m/s}^2$. Near the surface of the sun, $g = 270\ \text{m/s}^2$.

In the following examples we use the constant-acceleration equations developed in Section 2.4. You should review Problem-Solving Strategy 2.1 in that section before you study the next examples.

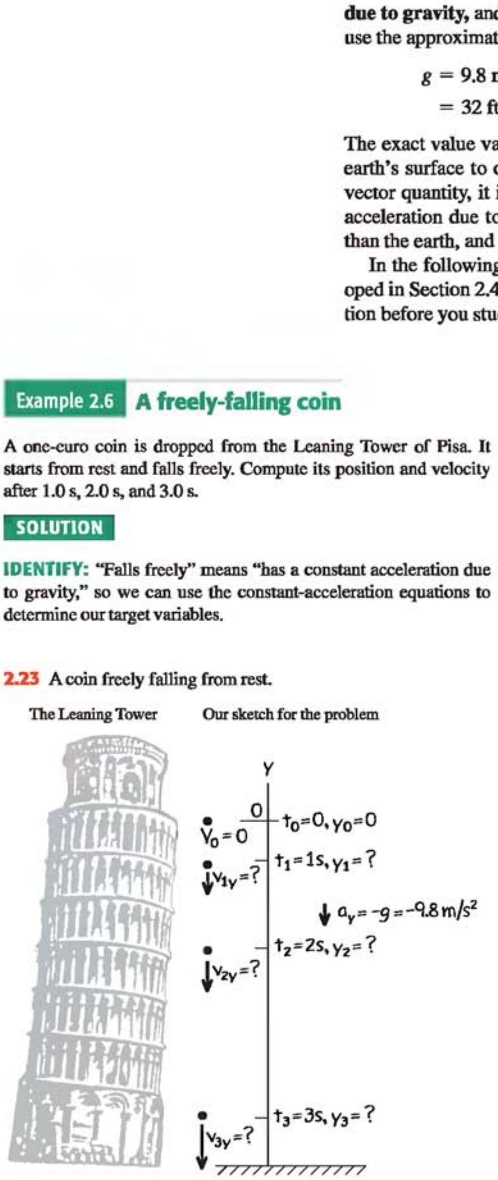
Example 2.6 A freely-falling coin

A one-euro coin is dropped from the Leaning Tower of Pisa. It starts from rest and falls freely. Compute its position and velocity after 1.0 s, 2.0 s, and 3.0 s.

SOLUTION

IDENTIFY: "Falls freely" means "has a constant acceleration due to gravity," so we can use the constant-acceleration equations to determine our target variables.

2.23 A coin freely falling from rest.



SET UP: The right side of Fig. 2.23 shows our motion diagram for the coin. The motion is vertical, so we use a vertical coordinate axis and call the coordinate y instead of x. Then we replace all the x's in the constant-acceleration equations by y's. We take the origin O at the starting point and the upward direction as positive. The initial coordinate y_0 and the initial y-velocity v_{0y} are both zero. The y-acceleration is downward, in the negative y-direction, so $a_y = -g = -9.8\ \text{m/s}^2$. (Remember that, by definition, g itself is *always* positive.) Our target variables are the values of y and v_y at the three given times. To find these, we use Eqs. (2.12) and (2.8) with x replaced by y.

EXECUTE: At a time t after the coin is dropped, its position and y-velocity are

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{1}{2}(-g)t^2 = (-4.9\ \text{m/s}^2)t^2$$

$$v_y = v_{0y} + a_y t = 0 + (-g)t = (-9.8\ \text{m/s}^2)t$$

When $t = 1.0\ \text{s}$, $y = (-4.9\ \text{m/s}^2)(1.0\ \text{s})^2 = -4.9\ \text{m}$ and $v_y = (-9.8\ \text{m/s}^2)(1.0\ \text{s}) = -9.8\ \text{m/s}$; after 1 s, the coin is 4.9 m below the origin (y is negative) and has a downward velocity (v_y is negative) with magnitude 9.8 m/s.

The position and y-velocity at 2.0 s and 3.0 s are found in the same way. Can you show that $y = -19.6\ \text{m}$ and $v_y = -19.6\ \text{m/s}$ at $t = 2.0\ \text{s}$, and that $y = -44.1\ \text{m}$ and $v_y = -29.4\ \text{m/s}$ at $t = 3.0\ \text{s}$?

EVALUATE: All our answers for v_y are negative because we chose the positive y-axis to point upward. But we could just as well have chosen the positive y-axis to point downward. In that case the acceleration would have been $a_y = +g$ and all our answers for v_y would have been positive. Either choice of axis is fine; just make sure that you state your choice explicitly in your solution and confirm that the acceleration has the correct sign.

Example 2.7 Up-and-down motion in free fall

You throw a ball vertically upward from the roof of a tall building. The ball leaves your hand at a point even with the roof railing with an upward speed of 15.0 m/s; the ball is then in free fall. On its way back down, it just misses the railing. At the location of the building, $g = 9.80\ \text{m/s}^2$. Find (a) the position and velocity of the ball 1.00 s and 4.00 s after leaving your hand; (b) the velocity when the ball is 5.00 m above the railing; (c) the maximum height reached and the time at which it is reached; and (d) the acceleration of the ball when it is at its maximum height.

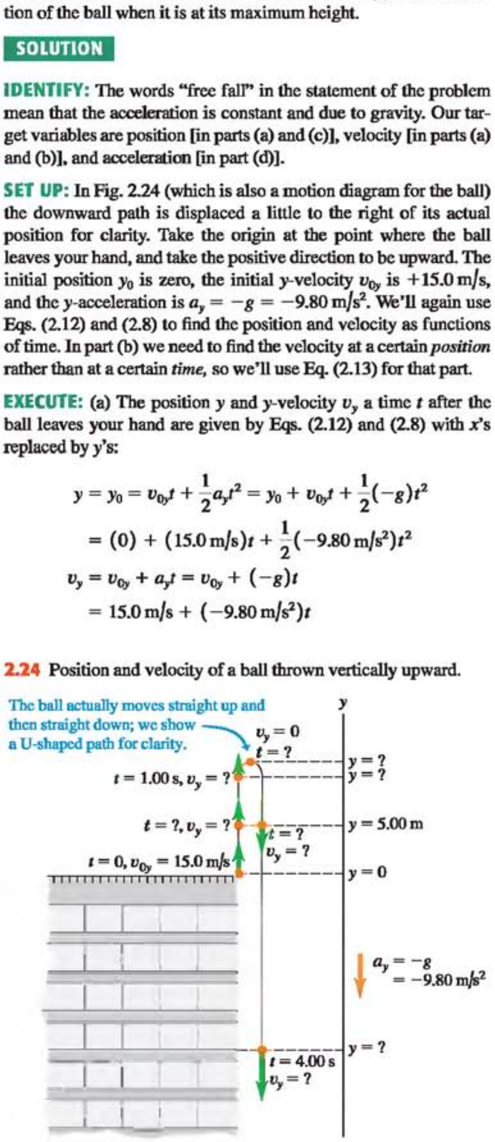
SOLUTION

IDENTIFY: The words "free fall" in the statement of the problem mean that the acceleration is constant and due to gravity. Our target variables are position [in parts (a) and (c)], velocity [in parts (a) and (b)], and acceleration [in part (d)].

SET UP: In Fig. 2.24 (which is also a motion diagram for the ball) the downward path is displaced a little to the right of its actual position for clarity. Take the origin at the point where the ball leaves your hand, and take the positive direction to be upward. The initial position y_0 is zero, the initial y-velocity v_{0y} is $+15.0\ \text{m/s}$, and the y-acceleration is $a_y = -g = -9.80\ \text{m/s}^2$. We'll again use Eqs. (2.12) and (2.8) to find the position and velocity as functions of time. In part (b) we need to find the velocity at a certain *position* rather than at a certain *time*, so we'll use Eq. (2.13) for that part.

EXECUTE: (a) The position y and y-velocity v_y a time t after the ball leaves your hand are given by Eqs. (2.12) and (2.8) with x's replaced by y's:

$$y = y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = y_0 + v_{0y}t + \frac{1}{2}(-g)t^2$$
$$= (0) + (15.0\ \text{m/s})t + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$$
$$v_y = v_{0y} + a_y t = v_{0y} + (-g)t$$
$$= 15.0\ \text{m/s} + (-9.80\ \text{m/s}^2)t$$



2.24 Position and velocity of a ball thrown vertically upward.

When $t = 1.00$ s, these equations give

$$y = +10.1\ \text{m} \qquad v_y = +5.2\ \text{m/s}$$

The ball is 10.1 m above the origin (y is positive) and moving upward (v_y is positive) with a speed of 5.2 m/s. This is less than the initial speed because the ball slows as it ascends.

When $t = 4.00$ s, the equations for y and v_y as functions of time t give

$$y = -18.4\ \text{m} \qquad v_y = -24.2\ \text{m/s}$$

The ball has passed its highest point and is 18.4 m *below* the origin (y is negative). It has a *downward* velocity (v_y is negative) with magnitude 24.2 m/s. The ball loses speed as it ascends, then gains speed as it descends; it is moving at the initial 15.0-m/s speed as it moves downward past the ball's launching point (the origin), and continues to gain speed as it descends below this point.

(b) The y-velocity v_y at any position y is given by Eq. (2.13) with x's replaced by y's:

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) = v_{0y}^2 + 2(-g)(y - 0)$$
$$= (15.0\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)y$$

When the ball is 5.00 m above the origin, $y = +5.00$ m, so

$$v_y^2 = (15.0\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(5.00\ \text{m}) = 127\ \text{m}^2/\text{s}^2$$
$$v_y = \pm 11.3\ \text{m/s}$$

We get *two* values of v_y because the ball passes through the point $y = +5.00$ m twice (see Fig. 2.24), once on the way up so v_y is positive and once on the way down so v_y is negative.

(c) Just at the instant when the ball reaches the highest point, it is momentarily at rest and $v_y = 0$. The maximum height y_1 can then be found in two ways. The first way is to use Eq. (2.13) and substitute $v_y = 0$, $y_0 = 0$, and $a_y = -g$:

$$0 = v_{0y}^2 + 2(-g)(y_1 - 0)$$
$$y_1 = \frac{v_{0y}^2}{2g} = \frac{(15.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = +11.5\ \text{m}$$

The second way is find the time at which $v_y = 0$ using Eq. (2.8), $v_y = v_{0y} + a_y t$, and then substitute this value of t into Eq. (2.12) to find the position at this time. From Eq. (2.8), the time t_1 when the ball reaches the highest point is given by

$$v_y = 0 = v_{0y} + (-g)t_1$$
$$t_1 = \frac{v_{0y}}{g} = \frac{15.0\ \text{m/s}}{9.80\ \text{m/s}^2} = 1.53\ \text{s}$$

Substituting this value of t into Eq. (2.12), we find

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = (0) + (15\ \text{m/s})(1.53\ \text{s})$$
$$+ \frac{1}{2}(-9.8\ \text{m/s}^2)(1.53\ \text{s})^2 = +11.5\ \text{m}$$

Notice that the first way of finding the maximum height is easier, since it's not necessary to find the time first.

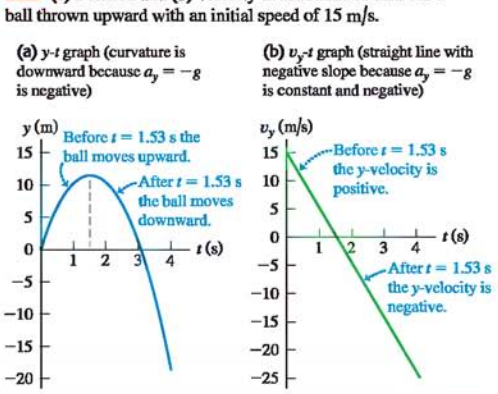
Continued

(d) **CAUTION** **A free-fall misconception** It's a common misconception that at the highest point of free-fall motion the velocity is zero *and* the acceleration is zero. If this were so, once the ball reached the highest point it would hang there suspended in midair! Remember that acceleration is the rate of change of velocity. If the acceleration were zero at the highest point, the ball's velocity would no longer change, and once the ball was instantaneously at rest, it would remain at rest forever.

At the highest point, the acceleration is still $a_y = -g = -9.80\ \mathrm{m/s^2}$, the same value as when the ball is moving up and when it's moving down. That's because the ball's velocity is continuously changing, from positive values through zero to negative values.

EVALUATE: A useful way to check any motion problem is to draw the graphs of position and velocity versus time. Figure 2.25 shows these graphs for this problem. Since the y-acceleration is constant and negative, the y-t graph is a parabola with downward curvature and the v_y-t graph is a straight line with a negative slope.

2.25 (a) Position and (b) velocity as functions of time for a ball thrown upward with an initial speed of 15 m/s.



Example 2.8 Two solutions or one?

Find the time when the ball in Example 2.7 is 5.00 m below the roof railing.

SOLUTION

IDENTIFY: Again this a constant-acceleration problem. The target variable is the time when the ball is at a certain position.

SET UP: We again choose the y-axis as in Fig. 2.24, so y_0, v_{0y}, and $a_y = -g$ have the same values as in Example 2.7. The position y as a function of time t is again given by Eq. (2.12):

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = y_0 + v_{0y}t + \frac{1}{2}(-g)t^2$$

We want to solve this for the value of t when $y = -5.00$ m. Since this equation involves t^2, it is a *quadratic* equation for t.

EXECUTE: We first rearrange the equation into the standard form of a quadratic equation for an unknown x, $Ax^2 + Bx + C = 0$:

$$\left(\frac{1}{2}g\right)t^2 + (-v_{0y})t + (y - y_0) = At^2 + Bt + C = 0$$

so $A = g/2$, $B = -v_{0y}$, and $C = y - y_0$. Using the quadratic formula (see Appendix B), we find that this equation has *two* solutions:

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-(-v_{0y}) \pm \sqrt{(-v_{0y})^2 - 4(g/2)(y - y_0)}}{2(g/2)}$$

$$= \frac{v_{0y} \pm \sqrt{v_{0y}^2 - 2g(y - y_0)}}{g}$$

Substituting the values $y_0 = 0$, $v_{0y} = +15.0\ \mathrm{m/s}$, $g = 9.80\ \mathrm{m/s^2}$, and $y = -5.00$ m, we find

$$t = \frac{(15.0\ \mathrm{m/s}) \pm \sqrt{(15.0\ \mathrm{m/s})^2 - 2(9.80\ \mathrm{m/s^2})(-5.00\ \mathrm{m} - 0)}}{9.80\ \mathrm{m/s^2}}$$

$t = +3.36$ s or $t = -0.30$ s

To decide which of these is the right answer, the key question to ask is, "Are these answers reasonable?" The second answer, $t = -0.30$ s, is simply not reasonable; it refers to a time 0.30 s *before* the ball left your hand! The correct answer is $t = +3.36$ s. The ball is 5.00 m below the railing 3.36 s *after* it leaves your hand.

EVALUATE: Where did the erroneous "solution" $t = -0.30$ s come from? Remember that the equation $y = y_0 + v_{0y}t + \frac{1}{2}(-g)t^2$ is based on the assumption that the acceleration is constant for *all* values of t, whether positive, negative, or zero. Taken at face value, this equation tells us that the ball has been moving upward in free fall ever since the dawn of time; it eventually passes your hand at $y = 0$ at the special instant we chose to call $t = 0$, then continues in free fall. But anything that this equation describes happening before $t = 0$ is pure fiction, since the ball went into free fall only after leaving your hand at $t = 0$; the "solution" $t = -0.30$ s is part of this fiction.

You should repeat these calculations to find the times when the ball is 5.00 m *above* the origin ($y = +5.00$ m). The two answers are $t = +0.38$ s and $t = +2.68$ s. These are both positive values of t, and both refer to the real motion of the ball after leaving your hand. The earlier time is when the ball passes through $y = +5.00$ m moving upward; the later time is when it passes through this point moving downward. [Compare this with part (b) of Example 2.7.]

You should also solve for the times at which $y = +15.0$ m. In this case, both solutions involve the square root of a negative number, so there are *no* real solutions. This makes sense; we found in part (c) of Example 2.7 that the ball's maximum height is only $y = +11.5$ m, so it *never* reaches $y = +15.0$ m. While a quadratic equation such as Eq. (2.12) always has two solutions, in some situations one or both of the solutions will not be physically reasonable.

Test Your Understanding of Section 2.5 If you toss a ball upward with a certain initial speed, it falls freely and reaches a maximum height h a time t after it leaves your hand. (a) If you throw the ball upward with double the initial speed, what new maximum height does the ball reach? (i) $h\sqrt{2}$; (ii) $2h$; (iii) $4h$; (iv) $8h$; (v) $16h$. (b) If you throw the ball upward with double the initial speed, how long does it take to reach its new maximum height? (i) $t/2$; (ii) $t/\sqrt{2}$; (iii) t; (iv) $t\sqrt{2}$; (v) $2t$.

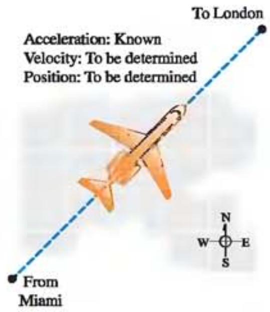
2.6 *Velocity and Position by Integration

This optional section is intended for students who have already learned a little integral calculus. In Section 2.4 we analyzed the special case of straight-line motion with constant acceleration. When a_x is not constant, as is frequently the case, the equations that we derived in that section are no longer valid (Fig. 2.26). But even when a_x varies with time, we can still use the relationship $v_x = dx/dt$ to find the x-velocity v_x as a function of time if the position x is a known function of time. And we can still use $a_x = dv_x/dt$ to find the x-acceleration a_x as a function of time if the x-velocity v_x is a known function of time.

2.26 When you push your car's accelerator pedal to the floorboard, the resulting acceleration is *not* constant: the greater the car's speed, the more slowly it gains additional speed. A typical car takes twice as long to accelerate from 50 km/h to 100 km/h as it does to accelerate from 0 to 50 km/h.

In many situations, however, position and velocity are not known as functions of time, while acceleration is. How can we find the position and velocity from the acceleration function $a_x(t)$? This problem arises in navigating an airliner between North America and Europe (Fig. 2.27). The pilots must know their position precisely at all times, but over the ocean an airliner is usually out of range of both radio navigation beacons on land and air traffic controllers' radar. To determine their position, airliners carry a device called an inertial navigation system (INS), which measures the airliner's acceleration. This is done in much the same way that you can sense changes in the velocity of a car in which you're riding, even when your eyes are closed. (In Chapter 4 we'll discuss how your body detects acceleration.) Given this information, along with the airliner's initial position (say, a particular gate at Miami International Airport) and its initial velocity (zero when parked at the gate), the INS calculates the airliner's current velocity and position at all times during the flight. (Airliners also use the Global Positioning System, or GPS, for navigation, but this supplements INS rather than replacing it.) Our goal in this section is to see how these calculations are done for the simpler case of motion in a straight line with time-varying acceleration.

2.27 The position and velocity of an airliner crossing the Atlantic are found by integrating its acceleration with respect to time.

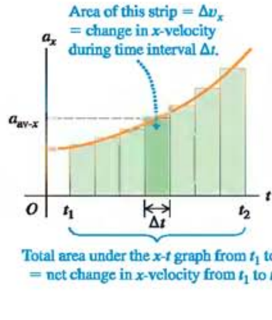


We first consider a graphical approach. Figure 2.28 is a graph of x-acceleration versus time for a body whose acceleration is not constant. We can divide the time interval between times t_1 and t_2 into many smaller intervals, calling a typical one Δt. Let the average x-acceleration during Δt be $a_{av\text{-}x}$. From Eq. (2.4) the change in x-velocity Δv_x during Δt is

$$\Delta v_x = a_{av\text{-}x}\,\Delta t$$

Graphically, Δv_x equals the area of the shaded strip with height $a_{av\text{-}x}$ and width Δt—that is, the area under the curve between the left and right sides of Δt. The total change in x-velocity during any interval (say, t_1 to t_2) is the sum of the x-velocity changes Δv_x in the small subintervals. So the total x-velocity change is represented graphically by the *total* area under the a_x-t curve between the vertical lines t_1 and t_2. (In Section 2.4 we showed this for the special case in which the acceleration is constant.)

2.28 An a_x-t graph for a body whose x-acceleration is not constant.



In the limit that all the Δt's become very small and their number very large, the value of $a_{av\text{-}x}$ for the interval from any time t to $t + \Delta t$ approaches the instantaneous x-acceleration a_x at time t. In this limit, the area under the a_x-t curve is the *integral* of a_x (which is in general a function of t) from t_1 to t_2. If v_{1x} is the x-velocity of the body at time t_1 and v_{2x} is the velocity at time t_2, then

$$v_{2x} - v_{1x} = \int_{v_{1x}}^{v_{2x}} dv_x = \int_{t_1}^{t_2} a_x\,dt \qquad (2.15)$$

The change in the x-velocity v_x is the time integral of the x-acceleration a_x.

We can carry out exactly the same procedure with the curve of x-velocity versus time. If x_1 is a body's position at time t_1 and x_2 is its position at time t_2, from Eq. (2.2) the displacement Δx during a small time interval Δt is equal to $v_{av\text{-}x}\,\Delta t$, where $v_{av\text{-}x}$ is the average x-velocity during Δt. The total displacement $x_2 - x_1$ during the interval $t_2 - t_1$ is given by

$$x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x\,dt \tag{2.16}$$

The change in position x—that is, the displacement—is the time integral of x-velocity v_x. Graphically, the displacement between times t_1 and t_2 is the area under the v_x-t curve between those two times. [This is the same result that we obtained in Section 2.4 for the special case in which v_x is given by Eq. (2.8).]

If $t_1 = 0$ and t_2 is any later time t, and if x_0 and v_{0x} are the position and velocity, respectively, at time $t = 0$, then we can rewrite Eqs. (2.15) and (2.16) as follows:

$$v_x = v_{0x} + \int_0^t a_x\,dt \tag{2.17}$$

$$x = x_0 + \int_0^t v_x\,dt \tag{2.18}$$

Here x and v_x are the position and x-velocity at time t. If we know the x-acceleration a_x as a function of time and we know the initial velocity v_{0x}, we can use Eq. (2.17) to find the x-velocity v_x at any time; in other words, we can find v_x as a function of time. Once we know this function, and given the initial position x_0, we can use Eq. (2.18) to find the position x at any time.

Example 2.9 Motion with changing acceleration

Sally is driving along a straight highway in her classic 1965 Mustang. At time $t = 0$, when Sally is moving at 10 m/s in the positive x-direction, she passes a signpost at $x = 50$ m. Her x-acceleration is a function of time:

$$a_x = 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t$$

(a) Find her x-velocity and position as functions of time. (b) When is her x-velocity greatest? (c) What is the maximum x-velocity? (d) Where is the car when it reaches the maximum x-velocity?

SOLUTION

IDENTIFY: The x-acceleration is a function of time, so we cannot use the constant-acceleration formulas of Section 2.4.

SET UP: We use Eqs. (2.17) and (2.18) to find the x-velocity and position as functions of time. Once we have those functions, we'll be able to answer a variety of questions about the motion.

EXECUTE: (a) At $t = 0$, Sally's position is $x_0 = 50$ m and her x-velocity is $v_{0x} = 10$ m/s. Since we are given the x-acceleration a_x as a function of time, we first use Eq. (2.17) to find the x-velocity v_x as a function of time t. The integral of t^n is $\int t^n\,dt = \frac{1}{n+1}t^{n+1}$ for $n \neq -1$, so

$$v_x = 10\ \text{m/s} + \int_0^t [2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t]\,dt$$
$$= 10\ \text{m/s} + (2.0\ \text{m/s}^2)t - \frac{1}{2}(0.10\ \text{m/s}^3)t^2$$

Then we use Eq. (2.18) to find x as a function of t:

$$x = 50\ \text{m} + \int_0^t \left[10\ \text{m/s} + (2.0\ \text{m/s}^2)t - \frac{1}{2}(0.10\ \text{m/s}^3)t^2\right] dt$$
$$= 50\ \text{m} + (10\ \text{m/s})t + \frac{1}{2}(2.0\ \text{m/s}^2)t^2 - \frac{1}{6}(0.10\ \text{m/s}^3)t^3$$

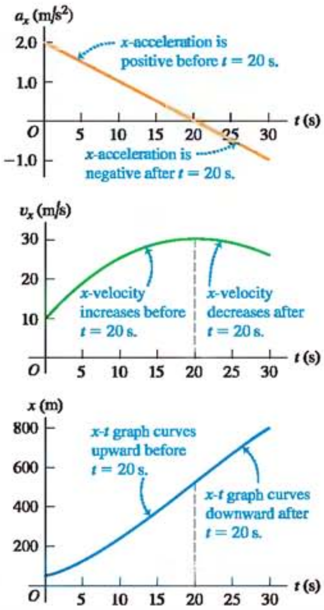
Figure 2.29 shows graphs of a_x, v_x, and x as functions of time. Note that for any time t, the slope of the v_x-t graph equals the value of a_x and the slope of the x-t graph equals the value of v_x.

(b) The maximum value of v_x occurs when the x-velocity stops increasing and begins to decrease. At this instant, $dv_x/dt = a_x = 0$. Setting the expression for a_x equal to zero, we obtain

$$0 = 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t$$
$$t = \frac{2.0\ \text{m/s}^2}{0.10\ \text{m/s}^3} = 20\ \text{s}$$

(c) We find the maximum x-velocity by substituting $t = 20$ s (when x-velocity is maximum) into the equation for v_x from part (a):

$$v_{\text{max-}x} = 10\ \text{m/s} + (2.0\ \text{m/s}^2)(20\ \text{s}) - \frac{1}{2}(0.10\ \text{m/s}^3)(20\ \text{s})^2$$
$$= 30\ \text{m/s}$$



2.29 The position, velocity, and acceleration of the car in Example 2.9 as functions of time. Can you show that if this motion continues, the car will stop at $t = 44.5$ s?

(d) The maximum value of v_x occurs at time $t = 20$ s. To obtain the position of the car at that time, we substitute $t = 20$ s into the expression for x from part (a):

$$x = 50\,\text{m} + (10\,\text{m/s})(20\,\text{s}) + \frac{1}{2}(2.0\,\text{m/s}^2)(20\,\text{s})^2 - \frac{1}{6}(0.10\,\text{m/s}^3)(20\,\text{s})^3 = 517\,\text{m}$$

EVALUATE: Figure 2.29 helps us interpret our results. The top graph in this figure shows that a_x is positive between $t = 0$ and $t = 20$ s and negative after that. It is zero at $t = 20$ s, the time at which v_x is maximum (the high point in the middle graph). The car speeds up until $t = 20$ s (because v_x and a_x have the same sign) and slows down after $t = 20$ s (because v_x and a_x have opposite signs).

Since v_x is maximum at $t = 20$ s, the x-t graph (the bottom graph in Fig. 2.29) has its maximum positive slope at this time. Note that the x-t graph is concave up (curved upward) from $t = 0$ to $t = 20$ s, when a_x is positive. The graph is concave down (curved downward) after $t = 20$ s, when a_x is negative.

Example 2.10 Constant-acceleration formulas via integration

Use Eqs. (2.17) and (2.18) to find v_x and x as functions of time in the case in which the acceleration is constant.

SOLUTION

IDENTIFY: This example serves as a check on the equations we've derived in this section. If they are correct, we should end up with the same constant-acceleration equations we derived in Section 2.4 without using integration.

SET UP: We follow the same steps as in Example 2.9. The only difference is that a_x is a constant.

EXECUTE: From Eq. (2.17) the x-velocity is given by

$$v_x = v_{0x} + \int_0^t a_x\,dt = v_{0x} + a_x\int_0^t dt = v_{0x} + a_x t$$

We were able to take a_x outside the integral because it is constant. Substituting this expression for v_x into Eq. (2.18), we get

$$x = x_0 + \int_0^t v_x\,dt = x_0 + \int_0^t (v_{0x} + a_x t)\,dt$$

Since v_{0x} and a_x are constants, we can take them outside the integral:

$$x = x_0 + v_{0x}\int_0^t dt + a_x\int_0^t t\,dt = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

EVALUATE: Our results are the same as Eqs. (2.8) and (2.12) from Section 2.4, as they should be! Although we developed Eqs. (2.17) and (2.18) to deal with cases in which acceleration depends on time, they can be used just as well when the acceleration is constant.

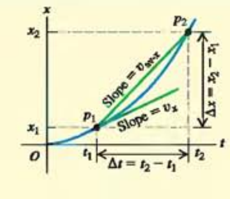
Test Your Understanding of Section 2.6 If the x-acceleration a_x is increasing with time, will the v_x-t graph be (i) a straight line, (ii) concave up (i.e., with an upward curvature), or (iii) concave down (i.e., with a downward curvature)?

CHAPTER 2 SUMMARY

Straight-line motion, average and instantaneous x-velocity: When a particle moves along a straight line, we describe its position with respect to an origin O by means of a coordinate such as x. The particle's average x-velocity $v_{av\text{-}x}$ during a time interval $\Delta t = t_2 - t_1$ is equal to its displacement $\Delta x = x_2 - x_1$ divided by Δt. The instantaneous x-velocity v_x at any time t is equal to the average x-velocity for the time interval from t to $t + \Delta t$ in the limit that Δt goes to zero. Equivalently, v_x is the derivative of the position function with respect to time. (See Example 2.1)

$$v_{av\text{-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad (2.2)$$

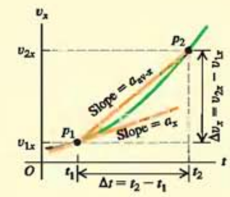
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.3)$$



Average and instantaneous x-acceleration: The average x-acceleration $a_{av\text{-}x}$ during a time interval Δt is equal to the change in velocity $\Delta v_x = v_{2x} - v_{1x}$ during that time interval divided by Δt. The instantaneous x-acceleration a_x is the limit of $a_{av\text{-}x}$ as Δt goes to zero, or the derivative of v_x with respect to t. (See Examples 2.2 and 2.3.)

$$a_{av\text{-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t} \qquad (2.4)$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad (2.5)$$



Straight-line motion with constant acceleration: When the x-acceleration is constant, four equations relate the position x and the x-velocity v_x at any time t to the initial position x_0, the initial x-velocity v_{0x} (both measured at time $t = 0$), and the x-acceleration a_x. (See Examples 2.4 and 2.5.)

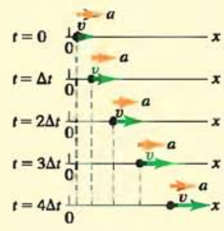
Constant x-acceleration only:

$$v_x = v_{0x} + a_x t \qquad (2.8)$$

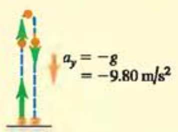
$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \qquad (2.12)$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad (2.13)$$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right) t \qquad (2.14)$$



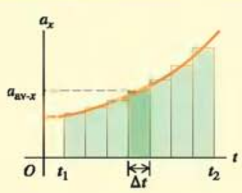
Freely falling bodies: Free fall is a case of motion with constant acceleration. The magnitude of the acceleration due to gravity is a positive quantity, g. The acceleration of a body in free fall is always downward. (See Examples 2.6–2.8.)



Straight-line motion with varying acceleration: When the acceleration is not constant but is a known function of time, we can find the velocity and position as functions of time by integrating the acceleration function. (See Examples 2.9 and 2.10.)

$$v_x = v_{0x} + \int_0^t a_x\, dt \qquad (2.17)$$

$$x = x_0 + \int_0^t v_x\, dt \qquad (2.18)$$



Key Terms

particle, *37*
average velocity, *37*
average *x*-velocity, *37*
x-t graph, *38*
instantaneous velocity, *39*
derivative, *40*
instantaneous *x*-velocity, *40*
speed, *40*
motion diagram, *42*
average acceleration, *43*
average *x*-acceleration, *43*
instantaneous acceleration, *44*
instantaneous *x*-acceleration, *45*
v_x-t graph, *45*
a_x-t graph, *47*
free fall, *53*
acceleration due to gravity, *54*

Answer to Chapter Opening Question ?

Yes. Acceleration refers to *any* change in velocity, including both speeding up and slowing down.

Answers to Test Your Understanding Questions

2.1 Answers to (a): (iv), (i) and (iii) (tie), (v), (ii); answer to (b): (i) and (iii); answer to (c): (v) In (a) the average *x*-velocity is $v_{av\text{-}x} = \Delta x/\Delta t$. For all five trips, $\Delta t = 1$ h. For the individual trips, we have (i) $\Delta x = +50$ km, $v_{av\text{-}x} = +50$ km/h; (ii) $\Delta x = -50$ km, $v_{av\text{-}x} = -50$ km/h; (iii) $\Delta x = 60\text{ km} - 10\text{ km} = +50$ km, $v_{av\text{-}x} = +50$ km/h; (iv) $\Delta x = +70$ km, $v_{av\text{-}x} = +70$ km/h; (v) $\Delta x = \Delta x = -20\text{ km} + 20\text{ km} = 0$, $v_{av\text{-}x} = 0$. In (b) both have $v_{av\text{-}x} = +50$ km/h.

2.2 Answers: (a) *P*, *Q* and *S* (tie), *R* The *x*-velocity is **(b)** positive when the slope of the *x-t* graph is positive (*P*), **(c)** negative when the slope is negative (*R*), and **(d)** zero when the slope is zero (*Q* and *S*). **(e) *R*, *P*, *Q* and *S* (tie)** The speed is greatest when the slope of the *x-t* graph is steepest (either positive or negative) and zero when the slope is zero.

2.3 Answers: (a) *S*, where the *x-t* graph is curved upward (concave up). **(b) *Q*,** where the *x-t* graph is curved downward (concave down). **(c) *P* and *R*,** where the *x-t* graph is not curved either up or down. **(d)** At *P*, $v_x > 0$ and $a_x = 0$ (speed is **not changing**); at *Q*, $v_x > 0$ and $a_x < 0$ (speed is **decreasing**); at *R*, $v_x < 0$ and $a_x = 0$ (speed is **not changing**); and at *S*, $v_x < 0$ and $a_x > 0$ (speed is **decreasing**).

2.4 Answer: (b) The officer's *x*-acceleration is constant, so her v_x-t graph is a straight line, and the officer's motorcycle is moving faster than the motorist's car when the two vehicles meet at $t = 10$ s.

2.5 Answers: (a) (iii) Use Eq. (2.13) with *x* replaced by *y* and $a_y = g$; $v_y^2 = v_{0y}^2 - 2g(y - y_0)$. The starting height is $y_0 = 0$ and the *y*-velocity at the maximum height $y = h$ is $v_y = 0$, so $0 = v_{0y}^2 - 2gh$ and $h = v_{0y}^2/2g$. If the initial *y*-velocity is increased by a factor of 2, the maximum height increases by a factor of $2^2 = 4$ and the ball goes to height 4*h*. **(b) (v)** Use Eq. (2.8) with *x* replaced by *y* and $a_y = g$; $v_y = v_{0y} - gt$. The *y*-velocity at the maximum height is $v_y = 0$, so $0 = v_{0y} - gt$ and $t = v_{0y}/g$. If the initial *y*-velocity is increased by a factor of 2, the time to reach the maximum height increases by a factor of 2 and becomes 2*t*.

2.6 Answer: (ii) The acceleration a_x is equal to the slope of the v_x-t graph. If a_x is increasing, the slope of the v_x-t graph is also increasing and the graph is concave up.

PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com MP

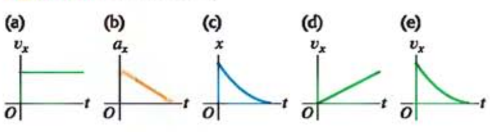
Discussion Questions

Q2.1. Does the speedometer of a car measure speed or velocity? Explain.

Q2.2. Figure 2.30 shows a series of high-speed photographs of an insect flying in a straight line from left to right (in the positive *x*-direction). Which of the graphs in Fig. 2.31 most plausibly depicts this insect's motion?

Figure 2.30 Question Q2.2.

Figure 2.31 Question Q2.2.



Q2.3. Can an object with constant acceleration reverse its direction of travel? Can it reverse its direction *twice*? In each case, explain your reasoning.

Q2.4. Under what conditions is average velocity equal to instantaneous velocity?

Q2.5. Is is possible for an object (a) to be slowing down while its acceleration is increasing in magnitude; (b) to be speeding up while its acceleration is decreasing? In each case, explain your reasoning.

Q2.6. Under what conditions does the magnitude of the average velocity equal the average speed?

Q2.7. When a Dodge Viper is at Elwood's Car Wash, a BMW Z3 is at Elm and Main. Later, when the Dodge reaches Elm and Main, the BMW reaches Elwood's Car Wash. How are the cars' average velocities between these two times related?

Q2.6. A driver in Massachusetts was sent to traffic court for speeding. The evidence against the driver was that a policewoman observed the driver's car alongside a second car at a certain moment, and the policewoman had already clocked the second car as going faster than the speed limit. The driver argued, "The second car was passing me. I was not speeding." The judge ruled against the driver because, in the judge's words, "If two cars were side by side, you were both speeding." If you were a lawyer representing the accused driver, how would you argue this case?

Q2.9. Can you have a zero displacement and a nonzero average velocity? A nonzero velocity? Illustrate your answers on an *x-t* graph.

Q2.10. Can you have zero acceleration and nonzero velocity? Explain using a v_x-t graph.

Q2.11. Can you have zero velocity and nonzero average acceleration? Zero velocity and nonzero acceleration? Explain using a v_x-t graph, and give an example of such motion.

Q2.12. An automobile is traveling west. Can it have a velocity toward the west and at the same time have an acceleration toward the east? Under what circumstances?

Q2.13. The official's truck in Fig. 2.2 is at $x_1 = 277$ m at $t_1 = 16.0$ s and is at $x_2 = 19$ m at $t_2 = 25.0$ s. (a) Sketch *two* different possible *x-t* graphs for the motion of the truck. (b) Does the average velocity $v_{av\text{-}x}$ during the time interval from t_1 to t_2 have the same value for both of your graphs? Why or why not?

Q2.14. Under constant acceleration the average velocity of a particle is half the sum of its initial and final velocities. Is this still true if the acceleration is *not* constant? Explain.

Q2.15. You throw a baseball straight up in the air so that it rises to a maximum height much greater than your height. Is the magnitude of the acceleration greater while it is being thrown or after it leaves your hand? Explain.

Q2.16. Prove these statements: (a) As long as you can neglect the effects of the air, if you throw anything vertically upward, it will have the same speed when it returns to the release point as when it was released. (b) The time of flight will be twice the time it takes to get to its highest point.

Q2.17. A dripping water faucet steadily releases drops 1.0 s apart. As these drops fall, will the distance between them increase, decrease, or remain the same? Prove your answer.

Q2.18. If the initial position and initial velocity of a vehicle are known and a record is kept of the acceleration at each instant, can you compute the vehicle's position after a certain time from these data? If so, explain how this might be done.

Q2.18. From the top of a tall building you throw one ball straight up with speed v_0 and one ball straight down with speed v_0. (a) Which ball has the greater speed when it reaches the ground? (b) Which ball gets to the ground first? (c) Which ball has a greater displacement when it reaches the ground? (d) Which ball has traveled the greater distance when it hits the ground?

Q2.20. A ball is dropped from rest from the top of a building of height h. At the same instant, a second ball is projected vertically upward from ground level, such that it has zero speed when it reaches the top of the building. When the two balls pass each other, which ball has the greater speed, or do they have the same speed? Explain. Where will the two balls be when they are alongside each other: at height $h/2$ above the ground, below this height, or above this height? Explain.

Exercises

Section 2.1 Displacement, Time, and Average Velocity

2.1. A rocket carrying a satellite is accelerating straight up from the earth's surface. At 1.15 s after liftoff, the rocket clears the top of its launch platform, 63 m above the ground. After an additional 4.75 s, it is 1.00 km above the ground. Calculate the magnitude of the average velocity of the rocket for (a) the 4.75-s part of its flight and (b) the first 5.90 s of its flight.

2.2. In an experiment, a shearwater (a seabird) was taken from its nest, flown 5150 km away, and released. The bird found its way back to its nest 13.5 days after release. If we place the origin in the nest and extend the $+x$-axis to the release point, what was the bird's average velocity in m/s (a) for the return flight, and (b) for the whole episode, from leaving the nest to returning?

2.3. Trip Home. You normally drive on the freeway between San Diego and Los Angeles at an average speed of 105 km/h (65 mi/h), and the trip takes 2 h and 20 min. On a Friday afternoon, however, heavy traffic slows you down and you drive the same distance at an average speed of only 70 km/h (43 mi/h). How much longer does the trip take?

2.4. From Pillar to Post. Starting from a pillar, you run 200 m east (the $+x$-direction) at an average speed of 5.0 m/s, and then run 280 m west at an average speed of 4.0 m/s to a post. Calculate (a) your average speed from pillar to post and (b) your average velocity from pillar to post.

2.5. Two runners start simultaneously from the same point on a circular 200-m track and run in *opposite* directions. One runs at a constant speed of 6.20 m/s, and the other runs at a constant speed of 5.50 m/s. When they first meet, (a) for how long a time will they have been running, and (b) how far will each one have run along the track?

2.6. Suppose the two runners in Exercise 2.5 start at the same time from the same place but run in the *same* direction. (a) When will the fast one first overtake ("lap") the slower one, and how far from the starting point will each have run? (b) When will the fast one overtake the slower one for the *second* time, and how far from the starting point will they be at that instant?

2.7. Earthquake Analysis. Earthquakes produce several types of shock waves. The most well known are the P-waves (P for *primary* or *pressure*) and the S-waves (S for *secondary* or *shear*). In the earth's crust, the P-waves travel at around 6.5 km/s, while the S-waves move at about 3.5 km/s. The actual speeds vary depending on the type of material they are going through. The time delay between the arrival of these two waves at a seismic recording station tells geologists how far away the earthquake occurred. If the time delay is 33 s, how far from the seismic station did the earthquake occur?

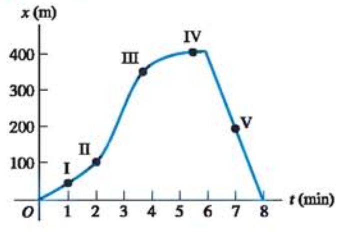
2.8. A Honda Civic travels in a straight line along a road. Its distance x from a stop sign is given as a function of time t by the equation $x(t) = \alpha t^2 - \beta t^3$, where $\alpha = 1.50\ \text{m/s}^2$ and $\beta = 0.0500\ \text{m/s}^3$. Calculate the average velocity of the car for each time interval: (a) $t = 0$ to $t = 2.00$ s; (b) $t = 0$ to $t = 4.00$ s; (c) $t = 2.00$ s to $t = 4.00$ s.

Section 2.2 Instantaneous Velocity

2.9. A car is stopped at a traffic light. It then travels along a straight road so that its distance from the light is given by $x(t) = bt^2 - ct^3$, where $b = 2.40\ \text{m/s}^2$ and $c = 0.120\ \text{m/s}^3$. (a) Calculate the average velocity of the car for the time interval $t = 0$ to $t = 10.0$ s. (b) Calculate the instantaneous velocity of the car at $t = 0$, $t = 5.0$ s, and $t = 10.0$ s. (c) How long after starting from rest is the car again at rest?

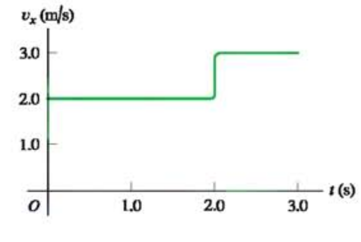
2.10. A physics professor leaves her house and walks along the sidewalk toward campus. After 5 min it starts to rain and she returns home. Her distance from her house as a function of time is shown in Fig. 2.32. At which of the labeled points is her velocity (a) zero? (b) constant and positive? (c) constant and negative? (d) increasing in magnitude? (e) decreasing in magnitude?

Figure 2.32 Exercise 2.10.



2.11. A ball moves in a straight line (the x-axis). The graph in Fig. 2.33 shows this ball's velocity as a function of time. (a) What are the ball's average speed and average velocity during the first 3.0 s? (b) Suppose that the ball moved in such a way that the graph segment after 2.0 s was -3.0 m/s instead of $+3.0$ m/s. Find the ball's average speed and average velocity in this case.

Figure 2.33 Exercise 2.11.



Section 2.3 Average and Instantaneous Acceleration

2.12. A test driver at Incredible Motors, Inc., is testing a new model car with a speedometer calibrated to read m/s rather than mi/h. The following series of speedometer readings was obtained during a test run along a long, straight road:

Time (s)	0	2	4	6	8	10	12	14	16
Speed (m/s)	0	0	2	6	10	16	19	22	22

(a) Compute the average acceleration during each 2-s interval. Is the acceleration constant? Is it constant during any part of the test run? (b) Make a v_x-t graph of the data, using scales of 1 cm = 1 s horizontally and 1 cm = 2 m/s vertically. Draw a smooth curve through the plotted points. By measuring the slope of your curve, find the instantaneous acceleration at $t = 9$ s, 13 s, and 15 s.

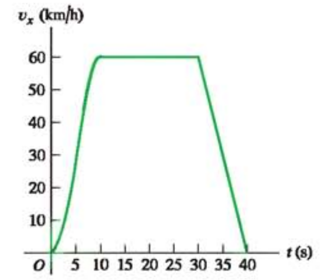
2.13. The Fastest (and Most Expensive) Car! The table shows test data for the Bugatti Veyron, the fastest car made. The car is moving in a straight line (the x-axis).

Time (s)	0	2.1	20.0	53
Speed (mi/h)	0	60	200	253

(a) Make a v_x-t graph of this car's velocity (in mi/h) as a function of time. Is its acceleration constant? (b) Calculate the car's average acceleration (in m/s^2) between (i) 0 and 2.1 s; (ii) 2.1 s and 20.0 s; (iii) 20.0 s and 53 s. Are these results consistent with your graph in part (a)? (Before you decide to buy this car, it might be helpful to know that only 300 will be built, it runs out of gas in 12 minutes at top speed, and it costs \$1.25 million!)

2.14. Figure 2.34 shows the velocity of a solar-powered car as a function of time. The driver accelerates from a stop sign, cruises for 20 s at a constant speed of 60 km/h, and then brakes to come to a stop 40 s after leaving the stop sign. (a) Compute the average acceleration during the following time intervals: (i) $t = 0$ to $t = 10$ s; (ii) $t = 30$ s to $t = 40$ s; (iii) $t = 10$ s to $t = 30$ s; (iv) $t = 0$ to $t = 40$ s. (b) What is the instantaneous acceleration at $t = 20$ s and at $t = 35$ s?

Figure 2.34 Exercise 2.14.



2.15. A turtle crawls along a straight line, which we will call the x-axis with the positive direction to the right. The equation for the turtle's position as a function of time is $x(t) = 50.0\text{ cm} + (2.00\text{ cm/s})t - (0.0625\text{ cm/s}^2)t^2$. (a) Find the turtle's initial velocity, initial position, and initial acceleration. (b) At what time t is the velocity of the turtle zero? (c) How long after starting does it take the turtle to return to its starting point? (d) At what times t is the turtle a distance of 10.0 cm from its starting point? What is the velocity (magnitude and direction) of the turtle at each of these times? (e) Sketch graphs of x versus t, v_x versus t, and a_x versus t, for the time interval $t = 0$ to $t = 40$ s.

2.16. An astronaut has left the International Space Station to test a new space scooter. Her partner measures the following velocity changes, each taking place in a 10-s interval. What are the magnitude, the algebraic sign, and the direction of the average acceleration in each interval? Assume that the positive direction is to the right. (a) At the beginning of the interval the astronaut is moving toward the right along the x-axis at 15.0 m/s, and at the end of the interval she is moving toward the right at 5.0 m/s. (b) At the beginning she is moving toward the left at 5.0 m/s, and at the end she is moving toward the left at 15.0 m/s. (c) At the beginning she is moving toward the right at 15.0 m/s, and at the end she is moving toward the left at 15.0 m/s.

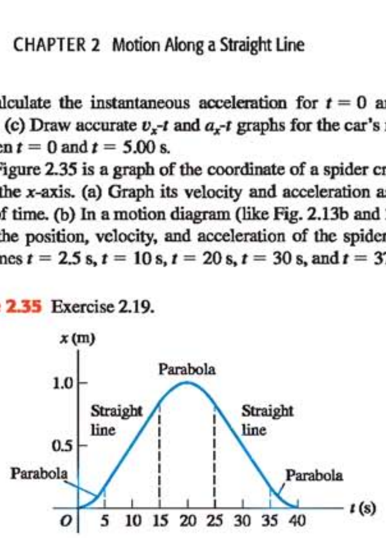
2.17. Auto Acceleration. Based on your experiences of riding in automobiles, estimate the magnitude of a car's average acceleration when it (a) accelerates onto a freeway from rest to 65 mi/h, and (b) brakes from highway speeds to a sudden stop. (c) Explain why the average acceleration in each case could be regarded as either positive or negative.

2.18. A car's velocity as a function of time is given by $v_x(t) = \alpha + \beta t^2$, where $\alpha = 3.00$ m/s and $\beta = 0.100$ m/s^3. (a) Calculate the average acceleration for the time interval $t = 0$ to $t = 5.00$ s.

(b) Calculate the instantaneous acceleration for $t = 0$ and $t = 5.00$ s. (c) Draw accurate v_x-t and a_x-t graphs for the car's motion between $t = 0$ and $t = 5.00$ s.

2.19. Figure 2.35 is a graph of the coordinate of a spider crawling along the x-axis. (a) Graph its velocity and acceleration as functions of time. (b) In a motion diagram (like Fig. 2.13b and 2.14b), show the position, velocity, and acceleration of the spider at the five times $t = 2.5$ s, $t = 10$ s, $t = 20$ s, $t = 30$ s, and $t = 37.5$ s.

Figure 2.35 Exercise 2.19.



2.28. The position of the front bumper of a test car under microprocessor control is given by $x(t) = 2.17\text{ m} + (4.80\text{ m/s}^2)t^2 - (0.100\text{ m/s}^6)t^6$. (a) Find its position and acceleration at the instants when the car has zero velocity. (b) Draw x-t, v_x-t, and a_x-t graphs for the motion of the bumper between $t = 0$ and $t = 2.00$ s.

Section 2.4 Motion with Constant Acceleration

2.21. An antelope moving with constant acceleration covers the distance between two points 70.0 m apart in 7.00 s. Its speed as it passes the second point is 15.0 m/s. (a) What is its speed at the first point? (b) What is its acceleration?

2.22. The catapult of the aircraft carrier USS *Abraham Lincoln* accelerates an F/A-18 Hornet jet fighter from rest to a takeoff speed of 173 mi/h in a distance of 307 ft. Assume constant acceleration. (a) Calculate the acceleration of the fighter in m/s^2. (b) Calculate the time required for the fighter to accelerate to takeoff speed.

2.23. A Fast Pitch. The fastest measured pitched baseball left the pitcher's hand at a speed of 45.0 m/s. If the pitcher was in contact with the ball over a distance of 1.50 m and produced constant acceleration, (a) what acceleration did he give the ball, and (b) how much time did it take him to pitch it?

2.24. A Tennis Serve. In the fastest measured tennis serve, the ball left the racquet at 73.14 m/s. A served tennis ball is typically in contact with the racquet for 30.0 ms and starts from rest. Assume constant acceleration. (a) What was the ball's acceleration during this serve? (b) How far did the ball travel during the serve?

2.25. Automobile Airbags. The human body can survive an acceleration trauma incident (sudden stop) if the magnitude of the acceleration is less than 250 m/s^2. If you are in an automobile accident with an initial speed of 105 km/h (65 mi/h) and you are stopped by an airbag that inflates from the dashboard, over what distance must the airbag stop you for you to survive the crash?

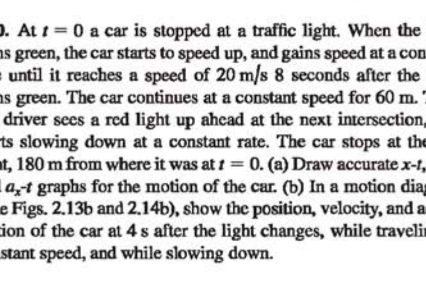
2.26. Entering the Freeway. A car sits in an entrance ramp to a freeway, waiting for a break in the traffic. The driver accelerates with constant acceleration along the ramp and onto the freeway. The car starts from rest, moves in a straight line, and has a speed of 20 m/s (45 mi/h) when it reaches the end of the 120-m-long ramp. (a) What is the acceleration of the car? (b) How much time does it take the car to travel the length of the ramp? (c) The traffic on the freeway is moving at a constant speed of 20 m/s. What distance does the traffic travel while the car is moving the length of the ramp?

2.27. Launch of the Space Shuttle. At launch the space shuttle weighs 4.5 million pounds. When it is launched from rest, it takes 8.00 s to reach 161 km/h, and at the end of the first 1.00 min its speed is 1610 km/h. (a) What is the average acceleration (in m/s^2) of the shuttle (i) during the first 8.00 s, and (ii) between 8.00 s and the end of the first 1.00 min? (b) Assuming the acceleration is constant during each time interval (but not necessarily the same in both intervals), what distance does the shuttle travel (i) during the first 8.00 s, and (ii) during the interval from 8.00 s to 1.00 min?

2.28. According to recent test data, an automobile travels 0.250 mi in 19.9 s, starting from rest. The same car, when braking from 60.0 mi/h on dry pavement, stops in 146 ft. Assume constant acceleration in each part of the motion, but not necessarily the same acceleration when slowing down as when speeding up. (a) Find the acceleration of this car when it is speeding up and when it is braking. (b) If its acceleration is constant, how fast (in mi/h) should this car be traveling after 0.250 mi of acceleration? The actual measured speed is 70.0 mi/h; what does this tell you about the motion? (c) How long does it take this car to stop while braking from 60.0 mi/h?

2.29. A cat walks in a straight line, which we shall call the x-axis with the positive direction to the right. As an observant physicist, you make measurements of this cat's motion and construct a graph of the feline's velocity as a function of time (Fig. 2.36). (a) Find the cat's velocity at $t = 4.0$ s and at $t = 7.0$ s. (b) What is the cat's acceleration at $t = 3.0$ s? At $t = 6.0$ s? At $t = 7.0$ s? (c) What distance does the cat move during the first 4.5 s? From $t = 0$ to $t = 7.5$ s? (d) Sketch clear graphs of the cat's acceleration and position as functions of time, assuming that the cat started at the origin.

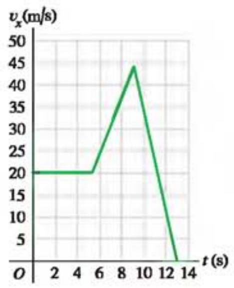
Figure 2.36 Exercise 2.29.



2.30. At $t = 0$ a car is stopped at a traffic light. When the light turns green, the car starts to speed up, and gains speed at a constant rate until it reaches a speed of 20 m/s 8 seconds after the light turns green. The car continues at a constant speed for 60 m. Then the driver sees a red light up ahead at the next intersection, and starts slowing down at a constant rate. The car stops at the red light, 180 m from where it was at $t = 0$. (a) Draw accurate x-t, v_x-t, and a_x-t graphs for the motion of the car. (b) In a motion diagram (like Figs. 2.13b and 2.14b), show the position, velocity, and acceleration of the car at 4 s after the light changes, while traveling at constant speed, and while slowing down.

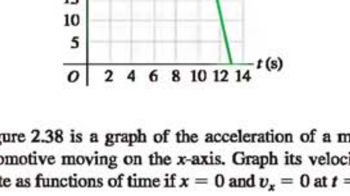
2.31. The graph in Fig. 2.37 shows the velocity of a motorcycle police officer plotted as a function of time. (a) Find the instantaneous acceleration at $t = 3$ s, at $t = 7$ s, and at $t = 11$ s. (b) How far does the officer go in the first 5 s? The first 9 s? The first 13 s?

Figure 2.37 Exercise 2.31.



2.32. Figure 2.38 is a graph of the acceleration of a model railroad locomotive moving on the x-axis. Graph its velocity and x-coordinate as functions of time if $x = 0$ and $v_x = 0$ at $t = 0$.

Figure 2.38 Exercise 2.32.

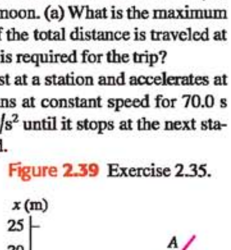


2.33. A spaceship ferrying workers to Moon Base I takes a straight-line path from the earth to the moon, a distance of 384,000 km. Suppose the spaceship starts from rest and accelerates at $20.0\ \text{m/s}^2$ for the first 15.0 min of the trip, and then travels at constant speed until the last 15.0 min, when it slows down at a rate of $20.0\ \text{m/s}^2$, just coming to rest as it reaches the moon. (a) What is the maximum speed attained? (b) What fraction of the total distance is traveled at constant speed? (c) What total time is required for the trip?

2.34. A subway train starts from rest at a station and accelerates at a rate of $1.60\ \text{m/s}^2$ for 14.0 s. It runs at constant speed for 70.0 s and slows down at a rate of $3.50\ \text{m/s}^2$ until it stops at the next station. Find the *total* distance covered.

2.35. Two cars, A and B, move along the x-axis. Figure 2.39 is a graph of the positions of A and B versus time. (a) In motion diagrams (like Figs. 2.13b and 2.14b), show the position, velocity, and acceleration of each of the two cars at $t = 0$, $t = 1$ s, and $t = 3$ s. (b) At what time(s), if any, do A and B have the same position? (c) Graph velocity versus time for both A and B. (d) At what time(s), if any, do A and B have the same velocity? (e) At what time(s), if any, does car A pass car B? (f) At what time(s), if any, does car B pass car A?

Figure 2.39 Exercise 2.35.



2.36. At the instant the traffic light turns green, a car that has been waiting at an intersection starts ahead with a constant acceleration of $3.20\ \text{m/s}^2$. At the same instant a truck, traveling with a constant speed of 20.0 m/s, overtakes and passes the car. (a) How far beyond its starting point does the car overtake the truck? (b) How fast is the car traveling when it overtakes the truck? (c) Sketch an x-t graph of the motion of both vehicles. Take $x = 0$ at the intersection. (d) Sketch a v_x-t graph of the motion of both vehicles.

2.37. Mars Landing. In January 2004, NASA landed exploration vehicles on Mars. Part of the descent consisted of the following stages:

Stage A: Friction with the atmosphere reduced the speed from 19,300 km/h to 1600 km/h in 4.0 min.

Stage B: A parachute then opened to slow it down to 321 km/h in 94 s.

Stage C: Retro rockets then fired to reduce its speed to zero over a distance of 75 m.

Assume that each stage followed immediately after the preceding one and that the acceleration during each stage was constant. (a) Find the rocket's acceleration (in m/s^2) during each stage. (b) What total distance (in km) did the rocket travel during stages A, B, and C?

Section 2.5 Freely Falling Bodies

2.38. Raindrops. If the effects of the air acting on falling raindrops are ignored, then we can treat raindrops as freely falling objects. (a) Rain clouds are typically a few hundred meters above the ground. Estimate the speed with which raindrops would strike the ground if they were freely falling objects. Give your estimate in m/s, km/h, and mi/h. (b) Estimate (from your own personal observations of rain) the speed with which raindrops actually strike the ground. (c) Based on your answers to parts (a) and (b), is it a good approximation to neglect the effects of the air on falling raindrops? Explain.

2.39. (a) If a flea can jump straight up to a height of 0.440 m, what is its initial speed as it leaves the ground? (b) How long is it in the air?

2.40. Touchdown on the Moon. A lunar lander is making its descent to Moon Base I (Fig. 2.40). The lander descends slowly under the retro-thrust of its descent engine. The engine is cut off when the lander is 5.0 m above the surface and has a downward speed of 0.8 m/s. With the engine off, the lander is in free fall. What is the speed of the lander just before it touches the surface? The acceleration due to gravity on the moon is $1.6\ \text{m/s}^2$.

Figure 2.40 Exercise 2.40.

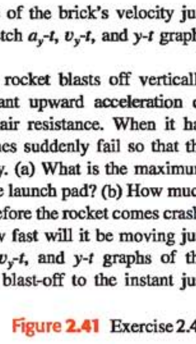
2.41. A Simple Reaction-Time Test. A meter stick is held vertically above your hand, with the lower end between your thumb and first finger. On seeing the meter stick released, you grab it with these two fingers. You can calculate your reaction time from the distance the meter stick falls, read directly from the point where your fingers grabbed it. (a) Derive a relationship for your reaction time in terms of this measured distance, d. (b) If the measured distance is 17.6 cm, what is the reaction time?

2.42. A brick is dropped (zero initial speed) from the roof of a building. The brick strikes the ground in 2.50 s. You may ignore air resistance, so the brick is in free fall. (a) How tall, in meters, is the building? (b) What is the magnitude of the brick's velocity just before it reaches the ground? (c) Sketch a_y-t, v_y-t, and y-t graphs for the motion of the brick.

2.43. Launch Failure. A 7500-kg rocket blasts off vertically from the launch pad with a constant upward acceleration of $2.25\ m/s^2$ and feels no appreciable air resistance. When it has reached a height of 525 m, its engines suddenly fail so that the only force acting on it is now gravity. (a) What is the maximum height this rocket will reach above the launch pad? (b) How much time after engine failure will elapse before the rocket comes crashing down to the launch pad, and how fast will it be moving just before it crashes? (c) Sketch a_y-t, v_y-t, and y-t graphs of the rocket's motion from the instant of blast-off to the instant just before it strikes the launch pad.

2.44. A hot-air balloonist, rising vertically with a constant velocity of magnitude 5.00 m/s, releases a sandbag at an instant when the balloon is 40.0 m above the ground (Fig. 2.41). After it is released, the sandbag is in free fall. (a) Compute the position and velocity of the sandbag at 0.250 s and 1.00 s after its release. (b) How many seconds after its release will the bag strike the ground? (c) With what magnitude of velocity does it strike the ground? (d) What is the greatest height above the ground that the sandbag reaches? (e) Sketch a_y-t, v_y-t, and y-t graphs for the motion.

Figure 2.41 Exercise 2.44.



2.45. A student throws a water balloon vertically downward from the top of a building. The balloon leaves the thrower's hand with a speed of 6.00 m/s. Air resistance may be ignored, so the water balloon is in free fall after it leaves the thrower's hand. (a) What is its speed after falling for 2.00 s? (b) How far does it fall in 2.00 s? (c) What is the magnitude of its velocity after falling 10.0 m? (d) Sketch a_y-t, v_y-t, and y-t graphs for the motion.

2.46. An egg is thrown nearly vertically upward from a point near the cornice of a tall building. It just misses the cornice on the way down and passes a point 50.0 m below its starting point 5.00 s after it leaves the thrower's hand. Air resistance may be ignored. (a) What is the initial speed of the egg? (b) How high does it rise above its starting point? (c) What is the magnitude of its velocity at the highest point? (d) What are the magnitude and direction of its acceleration at the highest point? (e) Sketch a_y-t, v_y-t, and y-t graphs for the motion of the egg.

2.47. The rocket-driven sled *Sonic Wind No. 2*, used for investigating the physiological effects of large accelerations, runs on a straight, level track 1070 m (3500 ft) long. Starting from rest, it can reach a speed of 224 m/s (500 mi/h) in 0.900 s. (a) Compute the acceleration in m/s^2, assuming that it is constant. (b) What is the ratio of this acceleration to that of a freely falling body (g)? (c) What distance is covered in 0.900 s? (d) A magazine article states that at the end of a certain run, the speed of the sled decreased from 283 m/s (632 mi/h) to zero in 1.40 s and that during this time the magnitude of the acceleration was greater than $40g$. Are these figures consistent?

2.48. A large boulder is ejected vertically upward from a volcano with an initial speed of 40.0 m/s. Air resistance may be ignored. (a) At what time after being ejected is the boulder moving at 20.0 m/s upward? (b) At what time is it moving at 20.0 m/s downward? (c) When is the displacement of the boulder from its initial position zero? (d) When is the velocity of the boulder zero? (e) What are the magnitude and direction of the acceleration while the boulder is (i) moving upward? (ii) Moving downward? (iii) At the highest point? (f) Sketch a_y-t, v_y-t, and y-t graphs for the motion.

2.49. A 15-kg rock is dropped from rest on the earth and reaches the ground in 1.75 s. When it is dropped from the same height on Saturn's satellite Enceladus, it reaches the ground in 18.6 s. What is the acceleration due to gravity on Enceladus?

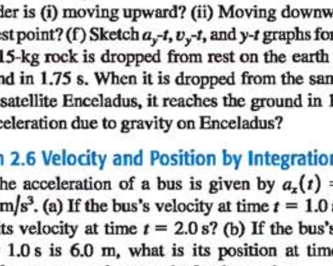
*Section 2.6 Velocity and Position by Integration

***2.50.** The acceleration of a bus is given by $a_x(t) = \alpha t$, where $\alpha = 1.2\ m/s^3$. (a) If the bus's velocity at time $t = 1.0$ s is 5.0 m/s, what is its velocity at time $t = 2.0$ s? (b) If the bus's position at time $t = 1.0$ s is 6.0 m, what is its position at time $t = 2.0$ s? (c) Sketch a_x-t, v_x-t, and x-t graphs for the motion.

***2.51.** The acceleration of a motorcycle is given by $a_x(t) = At - Bt^2$, where $A = 1.50\ m/s^3$ and $B = 0.120\ m/s^4$. The motorcycle is at rest at the origin at time $t = 0$. (a) Find its position and velocity as functions of time. (b) Calculate the maximum velocity it attains.

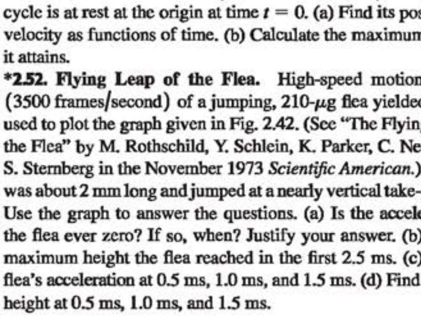
***2.52. Flying Leap of the Flea.** High-speed motion pictures (3500 frames/second) of a jumping, 210-μg flea yielded the data used to plot the graph given in Fig. 2.42. (See "The Flying Leap of the Flea" by M. Rothschild, Y. Schlein, K. Parker, C. Neville, and S. Sternberg in the November 1973 *Scientific American*.) This flea was about 2 mm long and jumped at a nearly vertical take-off angle. Use the graph to answer the questions. (a) Is the acceleration of the flea ever zero? If so, when? Justify your answer. (b) Find the maximum height the flea reached in the first 2.5 ms. (c) Find the flea's acceleration at 0.5 ms, 1.0 ms, and 1.5 ms. (d) Find the flea's height at 0.5 ms, 1.0 ms, and 1.5 ms.

Figure 2.42 Exercise 2.52.



***2.53.** The graph in Fig. 2.43 describes the acceleration as a function of time for a stone rolling down a hill starting from rest. (a) Find

Figure 2.43 Exercise 2.53



the change in the stone's velocity between $t = 2.5$ s and $t = 7.5$ s. (b) Sketch a graph of the stone's velocity as a function of time.

Problems

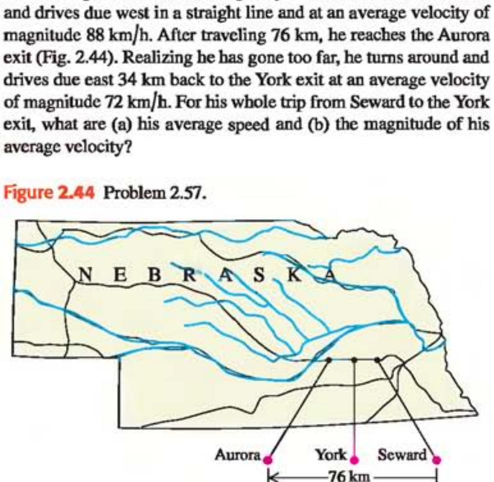
2.54. On a 20-mile bike ride, you ride the first 10 miles at an average speed of 8 mi/h. What must your average speed over the next 10 miles be to have your average speed for the total 20 miles be (a) 4 mi/h? (b) 12 mi/h? (c) Given this average speed for the first 10 miles, can you possibly attain an average speed of 16 mi/h for the total 20-mile ride? Explain.

2.55. The position of a particle between $t = 0$ and $t = 2.00$ s is given by $x(t) = (3.00\ \text{m/s}^3)t^3 - (10.0\ \text{m/s}^2)t^2 + (9.00\ \text{m/s})t$. (a) Draw the x-t, v_x-t, and a_x-t graphs of this particle. (b) At what time(s) between $t = 0$ and $t = 2.00$ s is the particle instantaneously at rest? Does your numerical result agree with the v_x-t graph in part (a)? (c) At each time calculated in part (b) is the acceleration of the particle positive or negative? Show that in each case the same answer is deduced from $a_x(t)$ and from the v_x-t graph. (d) At what time(s) between $t = 0$ and $t = 2.00$ s is the velocity of the particle instantaneously not changing? Locate this point on the v_x-t and a_x-t graphs of part (a). (e) What is the particle's greatest distance from the origin $(x = 0)$ between $t = 0$ and $t = 2.00$ s? (f) At what time(s) between $t = 0$ and $t = 2.00$ s is the particle *speeding up* at the greatest rate? At what time(s) between $t = 0$ and $t = 2.00$ s is the particle *slowing down* at the greatest rate? Locate these points on the v_x-t and a_x-t graphs of part (a).

2.56. Relay Race. In a relay race, each contestant runs 25.0 m while carrying an egg balanced on a spoon, turns around, and comes back to the starting point. Edith runs the first 25.0 m in 20.0 s. On the return trip she is more confident and takes only 15.0 s. What is the magnitude of her average velocity for (a) the first 25.0 m? (b) The return trip? (c) What is her average velocity for the entire round trip? (d) What is her average speed for the round trip?

2.57. Dan gets on Interstate Highway I–80 at Seward, Nebraska, and drives due west in a straight line and at an average velocity of magnitude 88 km/h. After traveling 76 km, he reaches the Aurora exit (Fig. 2.44). Realizing he has gone too far, he turns around and drives due east 34 km back to the York exit at an average velocity of magnitude 72 km/h. For his whole trip from Seward to the York exit, what are (a) his average speed and (b) the magnitude of his average velocity?

Figure 2.44 Problem 2.57.



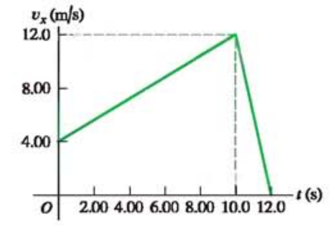
2.56. Freeway Traffic. According to a *Scientific American* article (May 1990), current freeways can sustain about 2400 vehicles per lane per hour in smooth traffic flow at 96 km/h (60 mi/h). With more vehicles the traffic flow becomes "turbulent" (stop-and-go). (a) If a vehicle is 4.6 m (15 ft) long on the average, what is the average spacing between vehicles at the above traffic density? (b) Collision-avoidance automated control systems, which operate by bouncing radar or sonar signals off surrounding vehicles and then accelerate or brake the car when necessary, could greatly reduce the required spacing between vehicles. If the average spacing is 9.2 m (two car lengths), how many vehicles per hour can a lane of traffic carry at 96 km/h?

2.59. A world-class sprinter accelerates to his maximum speed in 4.0 s. He then maintains this speed for the remainder of a 100-m race, finishing with a total time of 9.1 s. (a) What is the runner's average acceleration during the first 4.0 s? (b) What is his average acceleration during the last 5.1 s? (c) What is his average acceleration for the entire race? (d) Explain why your answer to part (c) is not the average of the answers to parts (a) and (b).

2.60. A sled starts from rest at the top of a hill and slides down with a constant acceleration. At some later time it is 14.4 m from the top; 2.00 s after that it is 25.6 m from the top, 2.00 s later 40.0 m from the top, and 2.00 s later it is 57.6 m from the top. (a) What is the magnitude of the average velocity of the sled during each of the 2.00-s intervals after passing the 14.4-m point? (b) What is the acceleration of the sled? (c) What is the speed of the sled when it passes the 14.4-m point? (d) How much time did it take to go from the top to the 14.4-m point? (e) How far did the sled go during the first second after passing the 14.4-m point?

2.61. A gazelle is running in a straight line (the x-axis). The graph in Fig. 2.45 shows this animal's velocity as a function of time. During the first 12.0 s, find (a) the total distance moved and (b) the displacement of the gazelle. (c) Sketch an a_x-t graph showing this gazelle's acceleration as a function of time for the first 12.0 s.

Figure 2.45 Problem 2.61.

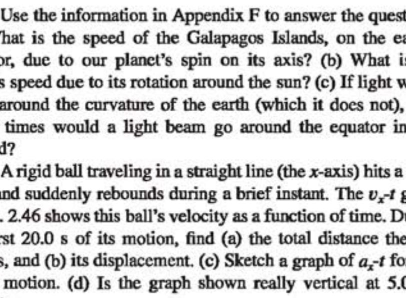


2.62. In air or vacuum light travels at a constant speed of 3.0×10^8 m/s. To answer some of these questions you may need to look up astronomical data in Appendix F. (a) One light year is defined as the distance light travels in 1 year. Use this information to determine how many meters there are in 1 light-year. (b) How far in meters does light travel in 1 nanosecond? (c) When a solar flare occurs on our sun, how soon after its occurrence can we first observe it? (d) By bouncing laser beams off a reflector placed on our moon by the Apollo astronauts, astronomers can make very accurate measurements of the earth–moon distance. How long after it is sent does it take such a laser beam (which is just a light beam) to return to earth? (e) The *Voyager* probe, which passed by Neptune in August 1989, was about 3.0 billion miles from earth at that time. Photographs and other information were sent to earth by radio waves, which travel at the speed of light. How long did it take these waves to reach earth from *Voyager*?

2.63. Use the information in Appendix F to answer the questions. (a) What is the speed of the Galapagos Islands, on the earth's equator, due to our planet's spin on its axis? (b) What is the earth's speed due to its rotation around the sun? (c) If light would bend around the curvature of the earth (which it does not), how many times would a light beam go around the equator in one second?

2.64. A rigid ball traveling in a straight line (the x-axis) hits a solid wall and suddenly rebounds during a brief instant. The v_x-t graph in Fig. 2.46 shows this ball's velocity as a function of time. During the first 20.0 s of its motion, find (a) the total distance the ball moves, and (b) its displacement. (c) Sketch a graph of a_x-t for this ball's motion. (d) Is the graph shown really vertical at 5.00 s? Explain.

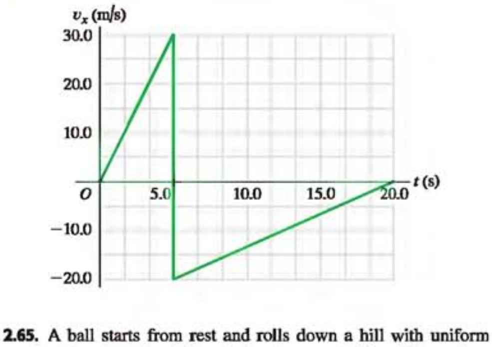
Figure 2.46 Problem 2.64.



2.65. A ball starts from rest and rolls down a hill with uniform acceleration, traveling 150 m during the second 5.0 s of its motion. How far did it roll during the first 5.0 s of motion?

2.66. Collision. The engineer of a passenger train traveling at 25.0 m/s sights a freight train whose caboose is 200 m ahead on the same track (Fig. 2.47). The freight train is traveling at 15.0 m/s in the same direction as the passenger train. The engineer of the passenger train immediately applies the brakes, causing a constant acceleration of $-0.100\ \text{m/s}^2$, while the freight train continues with constant speed. Take $x = 0$ at the location of the front of the passenger train when the engineer applies the brakes. (a) Will the cows nearby witness a collision? (b) If so, where will it take place? (c) On a single graph, sketch the positions of the front of the passenger train and the back of the freight train.

Figure 2.47 Problem 2.66.



2.67. Large cockroaches can run as fast as 1.50 m/s in short bursts. Suppose you turn on the light in a cheap motel and see one scurrying directly away from you at a constant 1.50 m/s. If you start 0.90 m behind the cockroach with an initial speed of 0.80 m/s toward it, what minimum constant acceleration would you need to catch up with it when it has traveled 1.20 m, just short of safety under a counter?

2.68. Two cars start 200 m apart and drive toward each other at a steady 10 m/s. On the front of one of them, an energetic grasshopper jumps back and forth between the cars (he has strong legs!) with a constant horizontal velocity of 15 m/s relative to the ground. The insect jumps the instant he lands, so he spends no time resting on either car. What total distance does the grasshopper travel before the cars hit?

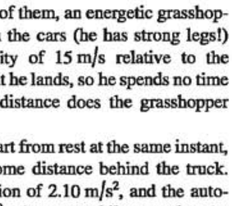
2.69. An automobile and a truck start from rest at the same instant, with the automobile initially at some distance behind the truck. The truck has a constant acceleration of $2.10\ \text{m/s}^2$, and the automobile an acceleration of $3.40\ \text{m/s}^2$. The automobile overtakes the truck after the truck has moved 40.0 m. (a) How much time does it take the automobile to overtake the truck? (b) How far was the automobile behind the truck initially? (c) What is the speed of each when they are abreast? (d) On a single graph, sketch the position of each vehicle as a function of time. Take $x = 0$ at the initial location of the truck.

2.70. Two stunt drivers drive directly toward each other. At time $t = 0$ the two cars are a distance D apart, car 1 is at rest, and car 2 is moving to the left with speed v_0. Car 1 begins to move at $t = 0$, speeding up with a constant acceleration a_x. Car 2 continues to move with a constant velocity. (a) At what time do the two cars collide? (b) Find the speed of car 1 just before it collides with car 2. (c) Sketch x-t and v_x-t graphs for car 1 and car 2. For each of the two graphs, draw the curves for both cars on the same set of axes.

2.71. A marble is released from one rim of a hemispherical bowl of diameter 50.0 cm and rolls down and up to the opposite rim in 10.0 s. Find (a) the average speed and (b) the average velocity of the marble.

2.72. You may have noticed while driving that your car's velocity does not continue to increase, even though you keep your foot on the gas pedal. This behavior is due to air resistance and friction between the moving parts of the car. Figure 2.48 shows a qualitative v_x-t graph for a typical car if it starts from rest at the origin and travels in a straight line (the x-axis). Sketch qualitative a_x-t and x-t graphs for this car.

Figure 2.48 Problem 2.72.



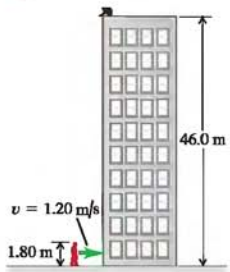
2.73. Passing. The driver of a car wishes to pass a truck that is traveling at a constant speed of 20.0 m/s (about 45 mi/h). Initially, the car is also traveling at 20.0 m/s and its front bumper is 24.0 m behind the truck's rear bumper. The car accelerates at a constant $0.600\ \text{m/s}^2$, then pulls back into the truck's lane when the rear of the car is 26.0 m ahead of the front of the truck. The car is 4.5 m long and the truck is 21.0 m long. (a) How much time is required for the car to pass the truck? (b) What distance does the car travel during this time? (c) What is the final speed of the car?

***2.74.** An object's velocity is measured to be $v_x(t) = \alpha - \beta t^2$, where $\alpha = 4.00$ m/s and $\beta = 2.00\ \text{m/s}^3$. At $t = 0$ the object is at $x = 0$. (a) Calculate the object's position and acceleration as functions of time. (b) What is the object's maximum *positive* displacement from the origin?

***2.75.** The acceleration of a particle is given by $a_x(t) = -2.00\ \mathrm{m/s^2} + (3.00\ \mathrm{m/s^3})t$. (a) Find the initial velocity v_{0x} such that the particle will have the same x-coordinate at $t = 4.00$ s as it had at $t = 0$. (b) What will be the velocity at $t = 4.00$ s?

2.76. Egg Drop. You are on the roof of the physics building, 46.0 m above the ground (Fig. 2.49). Your physics professor, who is 1.80 m tall, is walking alongside the building at a constant speed of 1.20 m/s. If you wish to drop an egg on your professor's head, where should the professor be when you release the egg? Assume that the egg is in free fall.

Figure 2.49 Problem 2.76.



2.77. A certain volcano on earth can eject rocks vertically to a maximum height H. (a) How high (in terms of H) would these rocks go if a volcano on Mars ejected them with the same initial velocity? The acceleration due to gravity on Mars is 3.71 m/s², and you can neglect air resistance on both planets. (b) If the rocks are in the air for a time T on earth, for how long (in terms of T) will they be in the air on Mars?

2.78. An entertainer juggles balls while doing other activities. In one act, she throws a ball vertically upward, and while it is in the air, she runs to and from a table 5.50 m away at a constant speed of 2.50 m/s, returning just in time to catch the falling ball. (a) With what minimum initial speed must she throw the ball upward to accomplish this feat? (b) How high above its initial position is the ball just as she reaches the table?

2.79. Visitors at an amusement park watch divers step off a platform 21.3 m (70 ft) above a pool of water. According to the announcer, the divers enter the water at a speed of 56 mi/h (25 m/s). Air resistance may be ignored. (a) Is the announcer correct in this claim? (b) Is it possible for a diver to leap directly upward off the board so that, missing the board on the way down, she enters the water at 25.0 m/s? If so, what initial upward speed is required? Is the required initial speed physically attainable?

2.60. A flowerpot falls off a windowsill and falls past the window below. You may ignore air resistance. It takes the pot 0.420 s to pass this window, which is 1.90 m high. How far is the top of the window below the windowsill from which the flowerpot fell?

2.81. Certain rifles can fire a bullet with a speed of 965 m/s just as it leaves the muzzle (this speed is called the *muzzle velocity*). If the muzzle is 70.0 cm long and if the bullet is accelerated uniformly from rest within it, (a) what is the acceleration (in g's) of the bullet in the muzzle, and (b) for how long (in ms) is it in the muzzle? (c) If, when this rifle is fired vertically, the bullet reaches a maximum height H, what would be the maximum height (in terms of H) for a new rifle that produced half the muzzle velocity of this one?

2.62. A Multi-stage Rocket. In the first stage of a two-stage rocket, the rocket is fired from the launch pad starting from rest but with a constant acceleration of 3.50 m/s² upward. At 25.0 s after launch, the rocket fires the second stage, which suddenly boosts its speed to 132.5 m/s upward. This firing uses up all the fuel, however, so then the only force acting on the rocket is gravity. Air resistance is negligible. (a) Find the maximum height that the stage-two rocket reaches above the launch pad. (b) How much time after the stage-two firing will it take for the rocket to fall back to the launch pad? (c) How fast will the stage-two rocket be moving just as it reaches the launch pad?

2.83. Look Out Below. Sam heaves a 16-lb shot straight upward, giving it a constant upward acceleration from rest of 45.0 m/s² for 64.0 cm. He releases it 2.20 m above the ground. You may ignore air resistance. (a) What is the speed of the shot when Sam releases it? (b) How high above the ground does it go? (c) How much time does he have to get out of its way before it returns to the height of the top of his head, 1.83 m above the ground?

2.64. A physics teacher performing an outdoor demonstration suddenly falls from rest off a high cliff and simultaneously shouts "Help." When she has fallen for 3.0 s, she hears the echo of her shout from the valley floor below. The speed of sound is 340 m/s. (a) How tall is the cliff? (b) If air resistance is neglected, how fast will she be moving just before she hits the ground? (Her actual speed will be less than this, due to air resistance.)

2.65. Juggling Act. A juggler performs in a room whose ceiling is 3.0 m above the level of his hands. He throws a ball upward so that it just reaches the ceiling. (a) What is the initial velocity of the ball? (b) What is the time required for the ball to reach the ceiling? At the instant when the first ball is at the ceiling, the juggler throws a second ball upward with two-thirds the initial velocity of the first. (c) How long after the second ball is thrown did the two balls pass each other? (d) At what distance above the juggler's hand do they pass each other?

2.66. A helicopter carrying Dr. Evil takes off with a constant upward acceleration of 5.0 m/s². Secret agent Austin Powers jumps on just as the helicopter lifts off the ground. After the two men struggle for 10.0 s, Powers shuts off the engine and steps out of the helicopter. Assume that the helicopter is in free fall after its engine is shut off, and ignore the effects of air resistance. (a) What is the maximum height above ground reached by the helicopter? (b) Powers deploys a jet pack strapped on his back 7.0 s after leaving the helicopter, and then he has a constant downward acceleration with magnitude 2.0 m/s². How far is Powers above the ground when the helicopter crashes into the ground?

2.87. Building Height. Spider-Man steps from the top of a tall building. He falls freely from rest to the ground a distance of h. He falls a distance of $h/4$ in the last 1.0 s of his fall. What is the height h of the building?

2.88. Cliff Height. You are climbing in the High Sierra where you suddenly find yourself at the edge of a fog-shrouded cliff. To find the height of this cliff, you drop a rock from the top and 10.0 s later hear the sound of it hitting the ground at the foot of the cliff. (a) Ignoring air resistance, how high is the cliff if the speed of sound is 330 m/s? (b) Suppose you had ignored the time it takes the sound to reach you. In that case, would you have overestimated or underestimated the height of the cliff? Explain your reasoning.

2.69. Falling Can. A painter is standing on scaffolding that is raised at constant speed. As he travels upward, he accidentally nudges a paint can off the scaffolding and it falls 15.0 m to the ground. You are watching, and measure with your stopwatch that it takes 3.25 s for the can to reach the ground. Ignore air resistance. (a) What is the speed of the can just before it hits the ground? (b) Another painter is standing on a ledge, with his hands 4.00 m above the can when it falls off. He has lightning-fast reflexes and if the can passes in front of him, he can catch it. Does he get the chance?

2.90. Determined to test the law of gravity for himself, a student walks off a skyscraper 180 m high, stopwatch in hand, and starts his free fall (zero initial velocity). Five seconds later, Superman arrives at the scene and dives off the roof to save the student. Superman leaves the roof with an initial speed v_0 that he produces by pushing himself downward from the edge of the roof with his legs of steel. He then falls with the same acceleration as any freely falling body. (a) What must the value of v_0 be so that Superman catches the student just before they reach the ground? (b) On the same graph, sketch the positions of the student and of Superman as functions of time. Take Superman's initial speed to have the value calculated in part (a). (c) If the height of the skyscraper is less than some minimum value, even Superman can't reach the student before he hits the ground. What is this minimum height?

2.91. During launches, rockets often discard unneeded parts. A certain rocket starts from rest on the launch pad and accelerates upward at a steady 3.30 m/s^2. When it is 235 m above the launch pad, it discards a used fuel canister by simply disconnecting it. Once it is disconnected, the only force acting on the canister is gravity (air resistance can be ignored). (a) How high is the rocket when the canister hits the launch pad, assuming that the rocket does not change its acceleration? (b) What total distance did the canister travel between its release and its crash onto the launch pad?

2.92. A ball is thrown straight up from the ground with speed v_0. At the same instant, a second ball is dropped from rest from a height H, directly above the point where the first ball was thrown upward. There is no air resistance. (a) Find the time at which the two balls collide. (b) Find the value of H in terms of v_0 and g so that at the instant when the balls collide, the first ball is at the highest point of its motion.

2.93. Two cars, A and B, travel in a straight line. The distance of A from the starting point is given as a function of time by $x_A(t) = \alpha t + \beta t^2$, with $\alpha = 2.60$ m/s and $\beta = 1.20\ m/s^2$. The distance of B from the starting point is $x_B(t) = \gamma t^2 - \delta t^3$, with $\gamma = 2.80\ m/s^2$ and $\delta = 0.20\ m/s^3$. (a) Which car is ahead just after they leave the starting point? (b) At what time(s) are the cars at the same point? (c) At what time(s) is the distance from A to B neither increasing nor decreasing? (d) At what time(s) do A and B have the same acceleration?

2.94. An apple drops from the tree and falls freely. The apple is originally at rest a height H above the top of the grass of a thick lawn, which is made of blades of grass of height h. When the apple enters the grass, it slows down at a constant rate so that its speed is 0 when it reaches ground level. (a) Find the speed of the apple just before it enters the grass. (b) Find the acceleration of the apple while it is in the grass. (c) Sketch the y-t, v_y-t, and a_y-t graphs for the apple's motion.

Challenge Problems

2.95. Catching the Bus. A student is running at her top speed of 5.0 m/s to catch a bus, which is stopped at the bus stop. When the student is still 40.0 m from the bus, it starts to pull away, moving with a constant acceleration of 0.170 m/s^2. (a) For how much time and what distance does the student have to run at 5.0 m/s before she overtakes the bus? (b) When she reaches the bus, how fast is the bus traveling? (c) Sketch an x-t graph for both the student and the bus. Take $x = 0$ at the initial position of the student. (d) The equations you used in part (a) to find the time have a second solution, corresponding to a later time for which the student and bus are again at the same place if they continue their specified motions. Explain the significance of this second solution. How fast is the bus traveling at this point? (e) If the student's top speed is 3.5 m/s, will she catch the bus? (f) What is the *minimum* speed the student must have to just catch up with the bus? For what time and what distance does she have to run in that case?

2.90. In the vertical jump, an athlete starts from a crouch and jumps upward to reach as high as possible. Even the best athletes spend little more than 1.00 s in the air (their "hang time"). Treat the athlete as a particle and let y_{max} be his maximum height above the floor. To explain why he seems to hang in the air, calculate the ratio of the time he is above $y_{max}/2$ to the time it takes him to go from the floor to that height. You may ignore air resistance.

2.97. A ball is thrown straight up from the edge of the roof of a building. A second ball is dropped from the roof 1.00 s later. You may ignore air resistance. (a) If the height of the building is 20.0 m, what must the initial speed of the first ball be if both are to hit the ground at the same time? On the same graph, sketch the position of each ball as a function of time, measured from when the first ball is thrown. Consider the same situation, but now let the initial speed v_0 of the first ball be given and treat the height h of the building as an unknown. (b) What must the height of the building be for both balls to reach the ground at the same time (i) if v_0 is 6.0 m/s and (ii) if v_0 is 9.5 m/s? (c) If v_0 is greater than some value v_{max}, a value of h does not exist that allows both balls to hit the ground at the same time. Solve for v_{max}. The value v_{max} has a simple physical interpretation. What is it? (d) If v_0 is less than some value v_{min}, a value of h does not exist that allows both balls to hit the ground at the same time. Solve for v_{min}. The value v_{min} also has a simple physical interpretation. What is it?

2.90. An alert hiker sees a boulder fall from the top of a distant cliff and notes that it takes 1.30 s for the boulder to fall the last third of the way to the ground. You may ignore air resistance. (a) What is the height of the cliff in meters? (b) If in part (a) you get two solutions of a quadratic equation and you use one for your answer, what does the other solution represent?

MOTION IN TWO OR THREE DIMENSIONS

3

? If a car is going around a curve at constant speed, is it accelerating? If so, in what direction is it accelerating?

LEARNING GOALS

By studying this chapter, you will learn:

- How to represent the position of a body in two or three dimensions using vectors.
- How to determine the vector velocity of a body from a knowledge of its path.
- How to find the vector acceleration of a body, and why a body can have an acceleration even if its speed is constant.
- How to interpret the components of a body's acceleration parallel to and perpendicular to its path.
- How to describe the curved path followed by a projectile.
- The key ideas behind motion in a circular path, with either constant speed or varying speed.
- How to relate the velocity of a moving body as seen from two different frames of reference.

What determines where a batted baseball lands? How do you describe the motion of a roller coaster car along a curved track or the flight of a circling hawk? If you throw a water balloon horizontally from your window, will it take the same amount of time to hit the ground as a balloon that you simply drop?

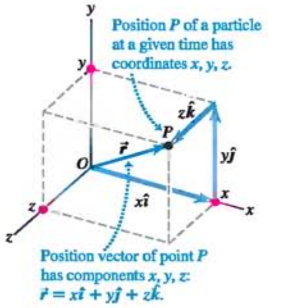
We can't answer these kinds of questions using the techniques of Chapter 2, in which particles moved only along a straight line. Instead, we need to extend our descriptions of motion to two- and three-dimensional situations. We'll still use the vector quantities displacement, velocity, and acceleration, but now these quantities will no longer lie along a single line. We'll find that several important kinds of motion take place in two dimensions only—that is, in a *plane*. These motions can be described with two components of position, velocity, and acceleration.

We also need to consider how the motion of a particle is described by different observers who are moving relative to each other. The concept of *relative velocity* will play an important role later in the book when we study collisions, when we explore electromagnetic phenomena, and when we introduce Einstein's special theory of relativity.

This chapter merges the vector mathematics of Chapter 1 with the kinematic language of Chapter 2. As before, we are concerned with describing motion, not with analyzing its causes. But the language you learn here will be an essential tool in later chapters when we study the relationship between force and motion.

3.1 Position and Velocity Vectors

3.1 The position vector $\vec{r}$ from the origin to point P has components x, y, and z. The path that the particle follows through space is in general a curve (Fig. 3.2).



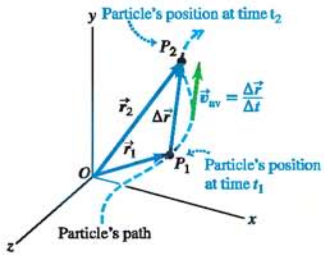
To describe the *motion* of a particle in space, we must first be able to describe the particle's *position*. Consider a particle that is at a point P at a certain instant. The **position vector $\vec{r}$** of the particle at this instant is a vector that goes from the origin of the coordinate system to the point P (Fig. 3.1). The Cartesian coordinates x, y, and z of point P are the x-, y-, and z-components of vector $\vec{r}$. Using the unit vectors we introduced in Section 1.9, we can write

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad \text{(position vector)} \tag{3.1}$$

During a time interval Δt the particle moves from P_1, where its position vector is $\vec{r}_1$, to P_2, where its position vector is $\vec{r}_2$. The change in position (the displacement) during this interval is $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$. We define the **average velocity $\vec{v}_{av}$** during this interval in the same way we did in Chapter 2 for straight-line motion, as the displacement divided by the time interval:

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t} \qquad \text{(average velocity vector)} \tag{3.2}$$

3.2 The average velocity $\vec{v}_{av}$ between points P_1 and P_2 has the same direction as the displacement $\Delta\vec{r}$.



Dividing a vector by a scalar is really a special case of *multiplying* a vector by a scalar, described in Section 1.7; the average velocity $\vec{v}_{av}$ is equal to the displacement vector $\Delta\vec{r}$ multiplied by $1/\Delta t$, the reciprocal of the time interval. Note that the x-component of Eq. (3.2) is $v_{av\text{-}x} = (x_2 - x_1)/(t_2 - t_1) = \Delta x/\Delta t$. This is just Eq. (2.2), the expression for average x-velocity that we found in Section 2.1 for one-dimensional motion.

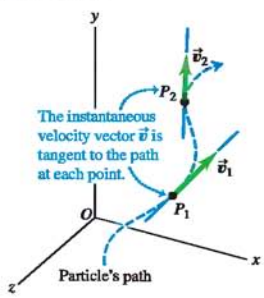
We now define **instantaneous velocity** just as we did in Chapter 2: It is the limit of the average velocity as the time interval approaches zero, and it equals the instantaneous rate of change of position with time. The key difference is that position $\vec{r}$ and instantaneous velocity $\vec{v}$ are now both vectors:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \qquad \text{(instantaneous velocity vector)} \tag{3.3}$$

The *magnitude* of the vector $\vec{v}$ at any instant is the *speed* v of the particle at that instant. The *direction* of $\vec{v}$ at any instant is the same as the direction in which the particle is moving at that instant.

Note that as $\Delta t \to 0$, points P_1 and P_2 in Fig. 3.2 move closer and closer together. In this limit, the vector $\Delta\vec{r}$ becomes tangent to the path. The direction of $\Delta\vec{r}$ in the limit is also the direction of the instantaneous velocity $\vec{v}$. This leads to an important conclusion: *At every point along the path, the instantaneous velocity vector is tangent to the path at that point* (Fig. 3.3).

3.3 The vectors $\vec{v}_1$ and $\vec{v}_2$ are the instantaneous velocities at the points P_1 and P_2 shown in Fig. 3.2.



It's often easiest to calculate the instantaneous velocity vector using components. During any displacement $\Delta\vec{r}$, the changes Δx, Δy, and Δz in the three coordinates of the particle are the *components* of $\Delta\vec{r}$. It follows that the components v_x, v_y, and v_z of the instantaneous velocity $\vec{v}$ are simply the time derivatives of the coordinates x, y, and z. That is,

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt} \qquad \text{(components of instantaneous velocity)} \tag{3.4}$$

The x-component of $\vec{v}$ is $v_x = dx/dt$, which is the same as Eq. (2.3)—the expression for instantaneous velocity for straight-line motion that we obtained in Section 2.2. Hence Eq. (3.4) is a direct extension of the idea of instantaneous velocity to motion in three dimensions.

We can also get this result by taking the derivative of Eq. (3.1). The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are constant in magnitude and direction, so their derivatives are zero, and we find

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} \tag{3.5}$$

This shows again that the components of $\vec{v}$ are dx/dt, dy/dt, and dz/dt.

The magnitude of the instantaneous velocity vector $\vec{v}$—that is, the speed—is given in terms of the components v_x, v_y, and v_z by the Pythagorean relation

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2} \tag{3.6}$$

Figure 3.4 shows the situation when the particle moves in the xy-plane. In this case, z and v_z are zero. Then the speed (the magnitude of $\vec{v}$) is

$$v = \sqrt{v_x^2 + v_y^2}$$

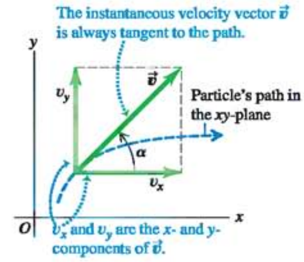
and the direction of the instantaneous velocity $\vec{v}$ is given by the angle α in the figure. We see that

$$\tan\alpha = \frac{v_y}{v_x} \tag{3.7}$$

(We always use Greek letters for angles. We use α for the direction of the instantaneous velocity vector to avoid confusion with the direction θ of the *position* vector of the particle.)

The instantaneous velocity vector is usually more interesting and useful than the average velocity vector. From now on, when we use the word "velocity," we will always mean the instantaneous velocity vector $\vec{v}$ (rather than the average velocity vector). Usually, we won't even bother to call $\vec{v}$ a vector; it's up to you to remember that velocity is a vector quantity with both magnitude and direction.

3.4 The two velocity components for motion in the xy-plane.



Example 3.1 Calculating average and instantaneous velocity

A robotic vehicle, or rover, is exploring the surface of Mars. The landing craft is the origin of coordinates, and the surrounding Martian surface lies in the xy-plane. The rover, which we represent as a point, has x- and y-coordinates that vary with time:

$$x = 2.0\text{ m} - (0.25\text{ m/s}^2)t^2$$
$$y = (1.0\text{ m/s})t + (0.025\text{ m/s}^3)t^3$$

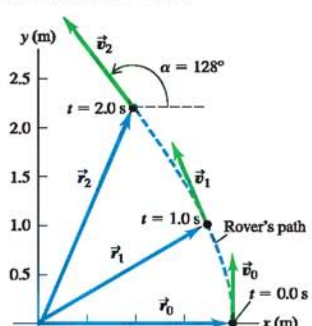
(a) Find the rover's coordinates and its distance from the lander at $t = 2.0$ s. (b) Find the rover's displacement and average velocity vectors during the interval from $t = 0.0$ s to $t = 2.0$ s. (c) Derive a general expression for the rover's instantaneous velocity vector. Express the instantaneous velocity at $t = 2.0$ s in component form and also in terms of magnitude and direction.

SOLUTION

IDENTIFY: This problem involves motion in two dimensions—that is, in a plane. Hence we must use the expressions for the displacement, average velocity, and instantaneous velocity *vectors* obtained in this section. (The simpler expressions in Sections 2.1 and 2.2 don't involve vectors; they apply only to motion along a straight line.)

SET UP: Figure 3.5 shows the rover's path. We'll use Eq. (3.1) for position $\vec{r}$, the expression $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ for displacement, Eq. (3.2) for average velocity, and Eqs. (3.5) and (3.6) for instantaneous velocity and its direction. The target variables are stated in the problem.

3.5 At $t = 0$ the rover has position vector $\vec{r}_0$ and instantaneous velocity vector $\vec{v}_0$. Likewise, $\vec{r}_1$ and $\vec{v}_1$ are the vectors at $t = 1.0$ s; $\vec{r}_2$ and $\vec{v}_2$ are the vectors at $t = 2.0$ s.



Continued

EXECUTE: (a) At time $t = 2.0$ s the rover's coordinates are

$$x = 2.0\text{ m} - (0.25\text{ m/s}^2)(2.0\text{ s})^2 = 1.0\text{ m}$$
$$y = (1.0\text{ m/s})(2.0\text{ s}) + (0.025\text{ m/s}^3)(2.0\text{ s})^3 = 2.2\text{ m}$$

The rover's distance from the origin at this time is

$$r = \sqrt{x^2 + y^2} = \sqrt{(1.0\text{ m})^2 + (2.2\text{ m})^2} = 2.4\text{ m}$$

(b) To find the displacement and average velocity, we express the position vector $\vec{r}$ as a function of time t. From Eq. (3.1), this is

$$\begin{aligned}\vec{r} &= x\hat{\imath} + y\hat{\jmath}\\ &= [2.0\text{ m} - (0.25\text{ m/s}^2)t^2]\hat{\imath}\\ &\quad + [(1.0\text{ m/s})t + (0.025\text{ m/s}^3)t^3]\hat{\jmath}\end{aligned}$$

At time $t = 0.0$ s the position vector $\vec{r}_0$ is

$$\vec{r}_0 = (2.0\text{ m})\hat{\imath} + (0.0\text{ m})\hat{\jmath}$$

From part (a) the position vector $\vec{r}_2$ at time $t = 2.0$ s is

$$\vec{r}_2 = (1.0\text{ m})\hat{\imath} + (2.2\text{ m})\hat{\jmath}$$

Therefore the displacement from $t = 0.0$ s to $t = 2.0$ s is

$$\begin{aligned}\Delta\vec{r} = \vec{r}_2 - \vec{r}_0 &= (1.0\text{ m})\hat{\imath} + (2.2\text{ m})\hat{\jmath} - (2.0\text{ m})\hat{\imath}\\ &= (-1.0\text{ m})\hat{\imath} + (2.2\text{ m})\hat{\jmath}\end{aligned}$$

During the time interval from $t = 0.0$ s to $t = 2.0$ s, the rover moves 1.0 m in the negative x-direction and 2.2 m in the positive y-direction. From Eq. (3.2), the average velocity during this interval is the displacement divided by the elapsed time:

$$\begin{aligned}\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} &= \frac{(-1.0\text{ m})\hat{\imath} + (2.2\text{ m})\hat{\jmath}}{2.0\text{ s} - 0.0\text{ s}}\\ &= (-0.50\text{ m/s})\hat{\imath} + (1.1\text{ m/s})\hat{\jmath}\end{aligned}$$

The components of this average velocity are

$$v_{av\text{-}x} = -0.50\text{ m/s} \qquad v_{av\text{-}y} = 1.1\text{ m/s}$$

(c) From Eq. (3.4), the components of instantaneous velocity are the time derivatives of the coordinates:

$$v_x = \frac{dx}{dt} = (-0.25\text{ m/s}^2)(2t)$$
$$v_y = \frac{dy}{dt} = 1.0\text{ m/s} + (0.025\text{ m/s}^3)(3t^2)$$

Then we can write the instantaneous velocity vector $\vec{v}$ as

$$\begin{aligned}\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} &= (-0.50\text{ m/s}^2)t\hat{\imath}\\ &\quad + [1.0\text{ m/s} + (0.075\text{ m/s}^3)t^2]\hat{\jmath}\end{aligned}$$

At time $t = 2.0$ s, the components of instantaneous velocity are

$$v_x = (-0.50\text{ m/s}^2)(2.0\text{ s}) = -1.0\text{ m/s}$$
$$v_y = 1.0\text{ m/s} + (0.075\text{ m/s}^3)(2.0\text{ s})^2 = 1.3\text{ m/s}$$

The magnitude of the instantaneous velocity (that is, the speed) at $t = 2.0$ s is

$$\begin{aligned}v &= \sqrt{v_x^2 + v_y^2} = \sqrt{(-1.0\text{ m/s})^2 + (1.3\text{ m/s})^2}\\ &= 1.6\text{ m/s}\end{aligned}$$

The direction of $\vec{v}$ with respect to the positive x-axis is given by the angle α, where, from Eq. (3.7),

$$\tan\alpha = \frac{v_y}{v_x} = \frac{1.3\text{ m/s}}{-1.0\text{ m/s}} = -1.3 \qquad \text{so} \qquad \alpha = 128°$$

Your calculator will tell you that the inverse tangent of -1.3 is $-52°$. But as we learned in Section 1.8, you have to examine a sketch of a vector to decide on its direction. Figure 3.5 shows that the correct answer for α is $-52° + 180° = 128°$.

EVALUATE: Take a moment to compare the components of *average* velocity that we found in part (b) for the interval from $t = 0.0$ s to $t = 2.0$ s $(v_{av\text{-}x} = -0.50\text{ m/s},\ v_{av\text{-}y} = 1.1\text{ m/s})$ with the components of *instantaneous* velocity at $t = 2.0$ s that we found in part (c) $(v_x = -1.0\text{ m/s},\ v_y = 1.3\text{ m/s})$. The comparison shows that, just as in one dimension, the average velocity vector $\vec{v}_{av}$ over an interval is in general *not* equal to the instantaneous velocity $\vec{v}$ at the end of the interval (see Example 2.1).

You should calculate the position vector, instantaneous velocity vector, speed, and direction of motion at $t = 0.0$ s and $t = 1.0$ s. Figure 3.5 shows the position vectors $\vec{r}$ and instantaneous velocity vectors $\vec{v}$ at $t = 0.0$ s, 1.0 s, and 2.0 s. Notice that at every point, $\vec{v}$ is tangent to the path. The magnitude of $\vec{v}$ increases as the rover moves, which shows that its speed is increasing.

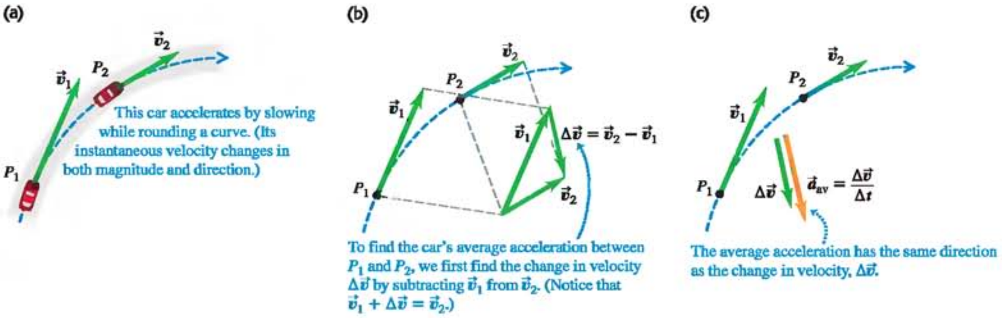
Test Your Understanding of Section 3.1 In which of these situations *would* the average velocity vector $\vec{v}_{av}$ over an interval be equal to the instantaneous velocity $\vec{v}$ at the end of the interval? (i) a body moving along a curved path at constant speed; (ii) a body moving along a curved path and speeding up; (iii) a body moving along a straight line at constant speed; (iv) a body moving along a straight line and speeding up.

3.2 The Acceleration Vector

Now let's consider the *acceleration* of a particle moving in space. Just as for motion in a straight line, acceleration describes how the velocity of the particle changes. But since we now treat velocity as a vector, acceleration will describe changes in the velocity magnitude (that is, the speed) *and* changes in the direction of velocity (that is, the direction in which the particle is moving).

In Fig. 3.6a, a car (treated as a particle) is moving along a curved road. The vectors $\vec{v}_1$ and $\vec{v}_2$ represent the car's instantaneous velocities at time t_1, when the

3.6 (a) A car moving along a curved road from P_1 to P_2. (b) Obtaining $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ by vector subtraction. (c) The vector $\vec{a}_{av} = \Delta\vec{v}/\Delta t$ represents the average acceleration between P_1 and P_2.



car is at point P_1, and at time t_2, when the car is at point P_2. The two velocities may differ in both magnitude and direction. During the time interval from t_1 to t_2, the *vector change in velocity* is $\vec{v}_2 - \vec{v}_1 = \Delta\vec{v}$ (Fig. 3.6b). We define the **average acceleration** $\vec{a}_{av}$ of the car during this time interval as the velocity change divided by the time interval $t_2 - t_1 = \Delta t$:

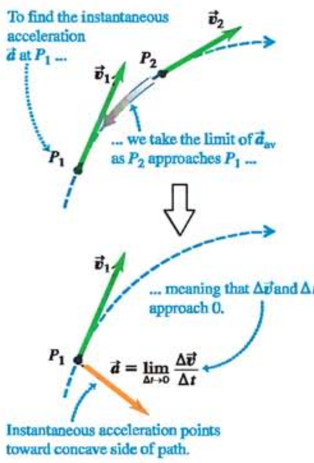
$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \quad \text{(average acceleration vector)} \qquad (3.8)$$

Average acceleration is a *vector* quantity in the same direction as the vector $\Delta\vec{v}$ (Fig. 3.6c). Note that $\vec{v}_2$ is the vector sum of the original velocity $\vec{v}_1$ and the change $\Delta\vec{v}$ (Fig. 3.6b). The x-component of Eq. (3.8) is $a_{av\text{-}x} = (v_{2x} - v_{1x})/(t_2 - t_1) = \Delta v_x/\Delta t$, which is just Eq. (2.4) for the average acceleration in straight-line motion.

As in Chapter 2, we define the **instantaneous acceleration** $\vec{a}$ at point P_1 as the limit of the average acceleration when point P_2 approaches point P_1 and $\Delta\vec{v}$ and Δt both approach zero. The instantaneous acceleration is also equal to the instantaneous rate of change of velocity with time. Because we are not restricted to straight-line motion, instantaneous acceleration is now a vector (Fig. 3.7):

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \quad \text{(instantaneous acceleration vector)} \qquad (3.9)$$

3.7 Instantaneous acceleration $\vec{a}$ at point P_1 in Fig. 3.6.

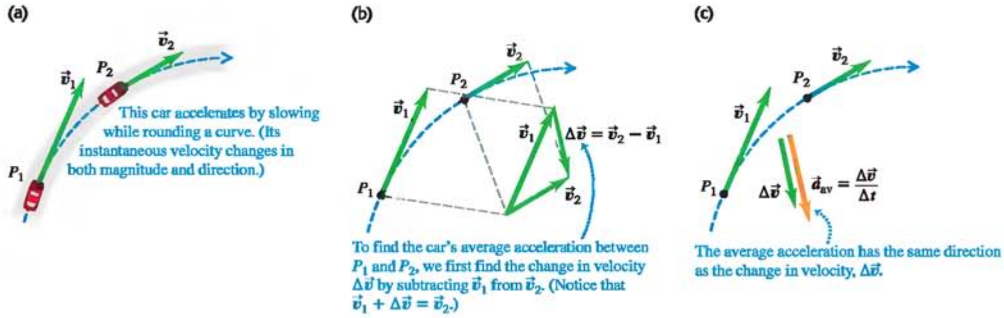


The velocity vector $\vec{v}$, as we have seen, is tangent to the path of the particle. But Figs. 3.6c and 3.7 show that if the path is curved, the instantaneous acceleration vector $\vec{a}$ always points toward the concave side of the path—that is, toward the inside of any turn that the particle is making.

CAUTION **Any particle following a curved path is accelerating** When a particle is moving in a curved path, it always has nonzero acceleration, even when it moves with constant speed. This conclusion may seem contrary to your intuition, but it's really just contrary to the everyday use of the word "acceleration" to mean that speed is increasing. The more precise definition given in Eq. (3.9) shows that there is a nonzero acceleration whenever the velocity vector changes in any way, whether there is a change of speed, direction, or both.

To convince yourself that a particle has a nonzero acceleration when moving on a curved path with constant speed, think of your sensations when you ride in a car. When the car accelerates, you tend to move inside the car in a

3.6 (a) A car moving along a curved road from P_1 to P_2. (b) Obtaining $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ by vector subtraction. (c) The vector $\vec{a}_{av} = \Delta\vec{v}/\Delta t$ represents the average acceleration between P_1 and P_2.



car is at point P_1, and at time t_2, when the car is at point P_2. The two velocities may differ in both magnitude and direction. During the time interval from t_1 to t_2, the *vector change in velocity* is $\vec{v}_2 - \vec{v}_1 = \Delta\vec{v}$ (Fig. 3.6b). We define the **average acceleration** $\vec{a}_{av}$ of the car during this time interval as the velocity change divided by the time interval $t_2 - t_1 = \Delta t$:

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \quad \text{(average acceleration vector)} \quad (3.8)$$

Average acceleration is a *vector* quantity in the same direction as the vector $\Delta\vec{v}$ (Fig. 3.6c). Note that $\vec{v}_2$ is the vector sum of the original velocity $\vec{v}_1$ and the change $\Delta\vec{v}$ (Fig. 3.6b). The x-component of Eq. (3.8) is $a_{av\text{-}x} = (v_{2x} - v_{1x})/(t_2 - t_1) = \Delta v_x/\Delta t$, which is just Eq. (2.4) for the average acceleration in straight-line motion.

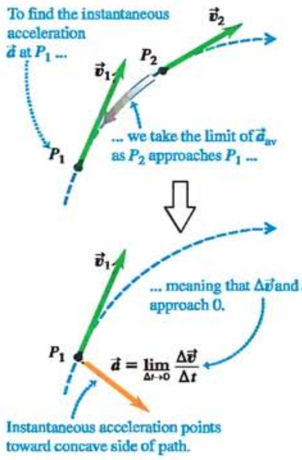
As in Chapter 2, we define the **instantaneous acceleration** $\vec{a}$ at point P_1 as the limit of the average acceleration when point P_2 approaches point P_1 and $\Delta\vec{v}$ and Δt both approach zero. The instantaneous acceleration is also equal to the instantaneous rate of change of velocity with time. Because we are not restricted to straight-line motion, instantaneous acceleration is now a vector (Fig. 3.7):

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \quad \text{(instantaneous acceleration vector)} \quad (3.9)$$

The velocity vector $\vec{v}$, as we have seen, is tangent to the path of the particle. But Figs. 3.6c and 3.7 show that if the path is curved, the instantaneous acceleration vector $\vec{a}$ always points toward the concave side of the path—that is, toward the inside of any turn that the particle is making.

CAUTION **Any particle following a curved path is accelerating** When a particle is moving in a curved path, it always has nonzero acceleration, even when it moves with constant speed. This conclusion may seem contrary to your intuition, but it's really just contrary to the everyday use of the word "acceleration" to mean that speed is increasing. The more precise definition given in Eq. (3.9) shows that there is a nonzero acceleration whenever the velocity vector changes in any way, whether there is a change of speed, direction, or both.

3.7 Instantaneous acceleration $\vec{a}$ at point P_1 in Fig. 3.6.



To convince yourself that a particle has a nonzero acceleration when moving on a curved path with constant speed, think of your sensations when you ride in a car. When the car accelerates, you tend to move inside the car in a

We can write the instantaneous acceleration vector $\vec{a}$ as

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = (-0.50\ \text{m/s}^2)\hat{\imath} + (0.15\ \text{m/s}^3)t\hat{\jmath}$$

At time $t = 2.0$ s, the components of instantaneous acceleration are

$$a_x = -0.50\ \text{m/s}^2 \qquad a_y = (0.15\ \text{m/s}^3)(2.0\ \text{s}) = 0.30\ \text{m/s}^2$$

The acceleration vector at this time is

$$\vec{a} = (-0.50\ \text{m/s}^2)\hat{\imath} + (0.30\ \text{m/s}^2)\hat{\jmath}$$

The magnitude of acceleration at this time is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.50\ \text{m/s}^2)^2 + (0.30\ \text{m/s}^2)^2} = 0.58\ \text{m/s}^2$$

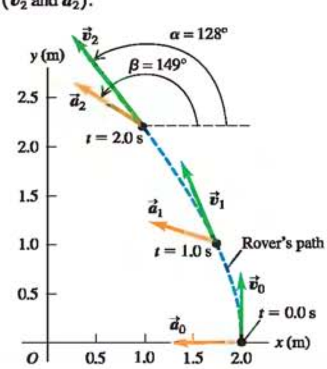
The direction of $\vec{a}$ with respect to the positive x-axis is given by the angle β, where

$$\tan\beta = \frac{a_y}{a_x} = \frac{0.30\ \text{m/s}^2}{-0.50\ \text{m/s}^2} = -0.60$$

$$\beta = 180° - 31° = 149°$$

EVALUATE: You should use the results of part (b) to calculate the instantaneous acceleration at $t = 0.0$ s and $t = 1.0$ s. Figure 3.9 shows the rover's path and the velocity and acceleration vectors at $t = 0.0$ s, 1.0 s, and 2.0 s. Note that $\vec{v}$ and $\vec{a}$ are *not* in the same direction at any of these times. The velocity vector $\vec{v}$ is tangent to the path at each point, and the acceleration vector $\vec{a}$ points toward the concave side of the path.

3.9 The path of the robotic rover, showing the velocity and acceleration at $t = 0.0$ s ($\vec{v}_0$ and $\vec{a}_0$), $t = 1.0$ s ($\vec{v}_1$ and $\vec{a}_1$), and $t = 2.0$ s ($\vec{v}_2$ and $\vec{a}_2$).



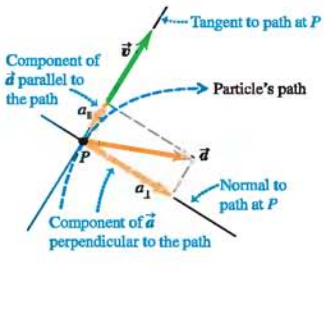
Parallel and Perpendicular Components of Acceleration

The acceleration vector $\vec{a}$ for a particle can describe changes in the particle's speed, its direction of motion, or both. It's useful to note that the component of acceleration *parallel* to a particle's path—that is, parallel to the velocity—tells us about changes in the particle's *speed*, while the acceleration component *perpendicular* to the path—and hence perpendicular to the velocity—tells us about changes in the particle's *direction of motion*. Figure 3.10 shows these components, which we label $a_\parallel$ and $a_\perp$. To see why the parallel and perpendicular components of $\vec{a}$ have these properties, let's consider two special cases.

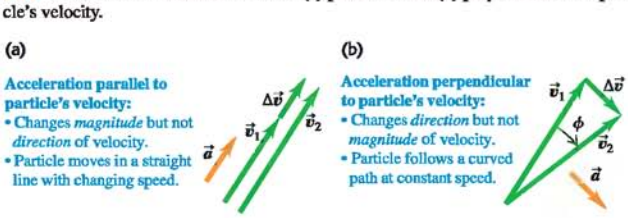
In Fig. 3.11a the acceleration vector is in the same direction as the velocity $\vec{v}_1$, so $\vec{a}$ has only a parallel component $a_\parallel$ (that is, $a_\perp = 0$). The velocity change $\Delta\vec{v}$ during a small time interval Δt is in the same direction as $\vec{a}$ and hence in the same direction as $\vec{v}_1$. The velocity $\vec{v}_2$ at the end of Δt, given by $\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$, is in the same direction as $\vec{v}_1$ but has greater magnitude. Hence during the time interval Δt the particle in Fig. 3.11a moved in a straight line with increasing speed.

In Fig. 3.11b the acceleration is *perpendicular* to the velocity, so $\vec{a}$ has only a perpendicular component $a_\perp$ (that is, $a_\parallel = 0$). In a small time interval Δt, the velocity change $\Delta\vec{v}$ is very nearly perpendicular to $\vec{v}_1$. Again $\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$, but in this case $\vec{v}_1$ and $\vec{v}_2$ have different directions. As the time interval Δt

3.10 The acceleration can be resolved into a component $a_\parallel$ parallel to the path (that is, along the tangent to the path) and a component $a_\perp$ perpendicular to the path (that is, along the normal to the path).



3.11 The effect of acceleration directed (a) parallel to and (b) perpendicular to a particle's velocity.

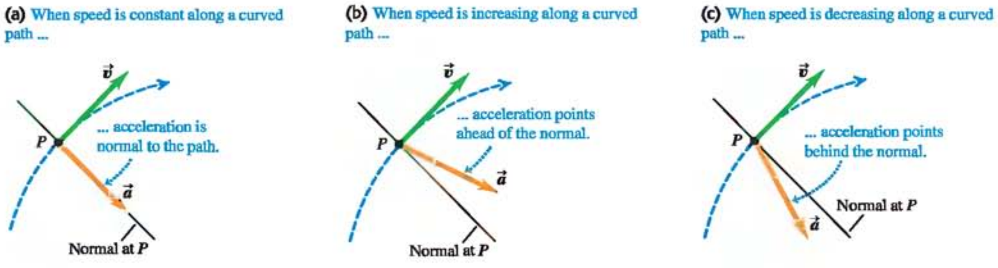


approaches zero, the angle ϕ in the figure also approaches zero, $\Delta\vec{v}$ becomes perpendicular to *both* $\vec{v}_1$ and $\vec{v}_2$, and $\vec{v}_1$ and $\vec{v}_2$ have the same magnitude. In other words, the speed of the particle stays the same, but the direction of motion changes and the path of the particle curves.

In the most general case, the acceleration $\vec{a}$ has components *both* parallel and perpendicular to the velocity $\vec{v}$, as in Fig. 3.10. Then the particle's speed will change (described by the parallel component $a_{\parallel}$) *and* its direction of motion will change (described by the perpendicular component $a_{\perp}$) so that it follows a curved path.

Figure 3.12 shows a particle moving along a curved path for three different situations: constant speed, increasing speed, and decreasing speed. If the speed is constant, $\vec{a}$ is perpendicular, or *normal*, to the path and to $\vec{v}$ and points toward the concave side of the path (Fig. 3.12a). If the speed is increasing, there is still a perpendicular component of $\vec{a}$, but there is also a parallel component having the same direction as $\vec{v}$ (Fig. 3.12b). Then $\vec{a}$ points ahead of the normal to the path. (This was the case in Example 3.2.) If the speed is decreasing, the parallel component has the direction opposite to $\vec{v}$, and $\vec{a}$ points behind the normal to the path (Fig. 3.12c). We will use these ideas again in Section 3.4 when we study the special case of motion in a circle.

3.12 Velocity and acceleration vectors for a particle moving through a point P on a curved path with (a) constant speed, (b) increasing speed, and (c) decreasing speed.



Example 3.3 Calculating parallel and perpendicular components of acceleration

For the rover of Examples 3.1 and 3.2, find the parallel and perpendicular components of the acceleration at $t = 2.0$ s.

SOLUTION

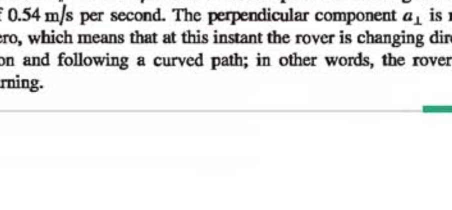
IDENTIFY: We want to find the components of the acceleration vector $\vec{a}$ that are parallel and perpendicular to the velocity vector $\vec{v}$.

SET UP: We found the directions of $\vec{a}$ and $\vec{v}$ in Examples 3.2 and 3.1, respectively. This will allow us to find the angle between the two vectors and hence the components of $\vec{a}$.

EXECUTE: In Example 3.2 we found that at $t = 2.0$ s the particle has an acceleration of magnitude $0.58\ \text{m/s}^2$ at an angle of 149° with respect to the positive x-axis. From Example 3.1, at this same time the velocity vector is at an angle of 128° with respect to the positive x-axis. So Fig. 3.9 shows that the angle between $\vec{a}$ and $\vec{v}$ is $149° - 128° = 21°$ (Fig. 3.13). The parallel and perpendicular components of acceleration are then

$$a_{\parallel} = a\cos 21° = (0.58\ \text{m/s}^2)\cos 21° = 0.54\ \text{m/s}^2$$
$$a_{\perp} = a\sin 21° = (0.58\ \text{m/s}^2)\sin 21° = 0.21\ \text{m/s}^2$$

3.13 The parallel and perpendicular components of the acceleration of the rover at $t = 2.0$ s.



EVALUATE: The parallel component $a_{\parallel}$ is in the same direction as $\vec{v}$, which means that the speed is increasing at this instant; the value of $a_{\parallel} = 0.54\ \text{m/s}^2$ means that the speed is increasing at a rate of 0.54 m/s per second. The perpendicular component $a_{\perp}$ is not zero, which means that at this instant the rover is changing direction and following a curved path; in other words, the rover is turning.

Conceptual Example 3.4 Acceleration of a skier

A skier moves along a ski-jump ramp as shown in Fig. 3.14a. The ramp is straight from point *A* to point *C* and curved from point *C* onward. The skier picks up speed as she moves downhill from point *A* to point *E*, where her speed is maximum. She slows down after passing point *E*. Draw the direction of the acceleration vector at points *B*, *D*, *E*, and *F*.

SOLUTION

Figure 3.14b shows our solution. At point *B* the skier is moving in a straight line with increasing speed, so her acceleration points downhill, in the same direction as her velocity.

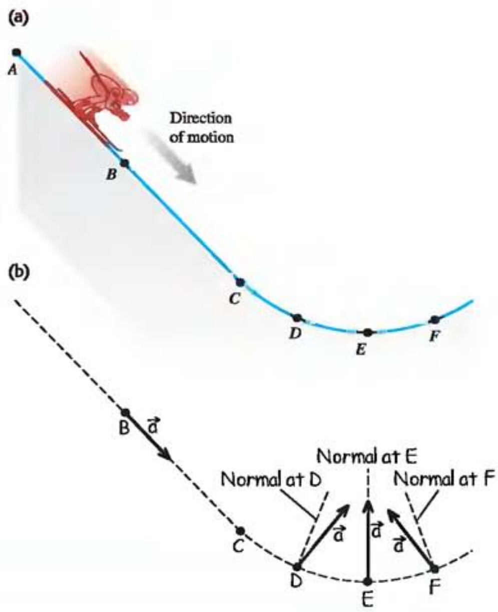
At point *D* the skier is moving along a curved path, so her acceleration has a component perpendicular to the path. There is also a component in the direction of her motion because she is still speeding up at this point. So the acceleration vector points *ahead* of the normal to her path at point *D*.

The skier's speed is instantaneously not changing at point *E*; the speed is maximum at this point, so its derivative is zero. There is no parallel component of $\vec{a}$, and the acceleration is perpendicular to her motion.

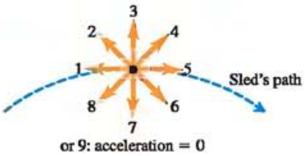
Finally, at point *F* the acceleration has a perpendicular component (because her path is curved at this point) and a parallel component *opposite* to the direction of her motion (because she's slowing down). So at this point, the acceleration vector points *behind* the normal to her path.

In the next section we'll examine the skier's acceleration after she flies off the ramp.

3.14 (a) The skier's path. (b) Our solution.



Test Your Understanding of Section 3.2 A sled travels over the crest of a snow-covered hill. The sled slows down as it climbs up one side of the hill and gains speed as it descends on the other side. Which of the vectors (1 through 9) in the figure correctly shows the direction of the sled's acceleration at the crest? (Choice 9 is that the acceleration is zero.)



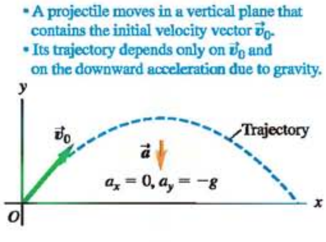
3.3 Projectile Motion

A **projectile** is any body that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance. A batted baseball, a thrown football, a package dropped from an airplane, and a bullet shot from a rifle are all projectiles. The path followed by a projectile is called its **trajectory.**

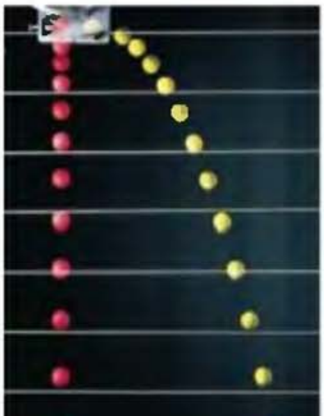
To analyze this common type of motion, we'll start with an idealized model, representing the projectile as a single particle with an acceleration (due to gravity) that is constant in both magnitude and direction. We'll neglect the effects of air resistance and the curvature and rotation of the earth. Like all models, this one has limitations. Curvature of the earth has to be considered in the flight of long-range missiles, and air resistance is of crucial importance to a sky diver. Nevertheless, we can learn a lot from analysis of this simple model. For the remainder of this chapter the phrase "projectile motion" will imply that we're ignoring air resistance. In Chapter 5 we will see what happens when air resistance cannot be ignored.

Projectile motion is always confined to a vertical plane determined by the direction of the initial velocity (Fig. 3.15). This is because the acceleration due to

3.15 The trajectory of a projectile.



3.16 The red ball is dropped from rest, and the yellow ball is simultaneously projected horizontally; successive images in this stroboscopic photograph are separated by equal time intervals. At any given time, both balls have the same y-position, y-velocity, and y-acceleration, despite having different x-positions and x-velocities.



gravity is purely vertical; gravity can't move the projectile sideways. Thus projectile motion is *two-dimensional.* We will call the plane of motion the xy-coordinate plane, with the x-axis horizontal and the y-axis vertically upward.

The key to analyzing projectile motion is that we can treat the x- and y-coordinates separately. The x-component of acceleration is zero, and the y-component is constant and equal to $-g$. (By definition, g is always positive; with our choice of coordinate directions, a_y is negative.) So *we can analyze projectile motion as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.* Figure 3.16 shows two projectiles with different x-motion but identical y-motion; one is dropped from rest and the other is projected horizontally, but both projectiles fall the same distance in the same time.

We can then express all the vector relationships for the projectile's position, velocity, and acceleration by separate equations for the horizontal and vertical components. The components of $\vec{a}$ are

$$a_x = 0 \qquad a_y = -g \qquad \text{(projectile motion, no air resistance)} \tag{3.14}$$

Since the x-acceleration and y-acceleration are both constant, we can use Eqs. (2.8), (2.12), (2.13), and (2.14) directly. For example, suppose that at time $t = 0$ our particle is at the point (x_0, y_0) and that at this time its velocity components have the initial values v_{0x} and v_{0y}. The components of acceleration are $a_x = 0$, $a_y = -g$. Considering the x-motion first, we substitute 0 for a_x in Eqs. (2.8) and (2.12). We find

$$v_x = v_{0x} \tag{3.15}$$

$$x = x_0 + v_{0x}t \tag{3.16}$$

For the y-motion we substitute y for x, v_y for v_x, v_{0y} for v_{0x}, and $a_y = -g$ for a_x:

$$v_y = v_{0y} - gt \tag{3.17}$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \tag{3.18}$$

It's usually simplest to take the initial position (at $t = 0$) as the origin; then $x_0 = y_0 = 0$. This might be the position of a ball at the instant it leaves the thrower's hand or the position of a bullet at the instant it leaves the gun barrel.

Figure 3.17 shows the path of a projectile that starts at (or passes through) the origin at time $t = 0$. The position, velocity, and velocity components are shown

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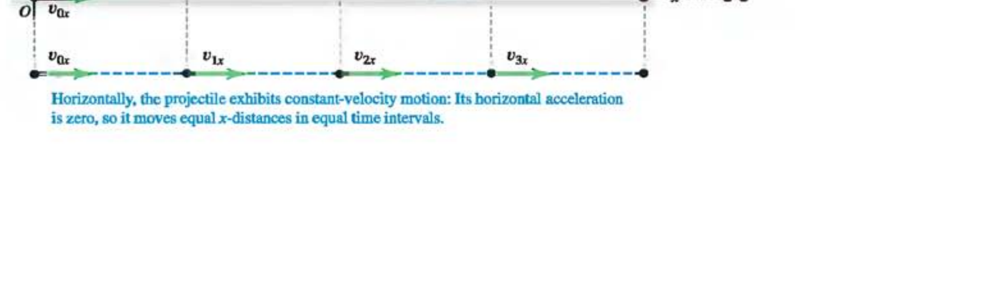
3.1 Solving Projectile Motion Problems
3.2 Two Balls Falling
3.3 Changing the x-velocity
3.4 Projectile x-y-Accelerations

3.17 If air resistance is negligible, the trajectory of a projectile is a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.



at equal time intervals. The x-component of acceleration is zero, so v_x is constant. The y-component of acceleration is constant and not zero, so v_y changes by equal amounts in equal times, just the same as if the projectile were launched vertically with the same initial y-velocity. At the highest point in the trajectory, $v_y = 0$.

We can also represent the initial velocity $\vec{v}_0$ by its magnitude v_0 (the initial speed) and its angle α_0 with the positive x-axis (Fig. 3.18). In terms of these quantities, the components v_{0x} and v_{0y} of the initial velocity are

$$v_{0x} = v_0 \cos\alpha_0 \qquad v_{0y} = v_0 \sin\alpha_0 \tag{3.19}$$

Using these relationships in Eqs. (3.15) through (3.18) and setting $x_0 = y_0 = 0$, we find

$$x = (v_0 \cos\alpha_0)t \quad \text{(projectile motion)} \tag{3.20}$$

$$y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2 \quad \text{(projectile motion)} \tag{3.21}$$

$$v_x = v_0 \cos\alpha_0 \quad \text{(projectile motion)} \tag{3.22}$$

$$v_y = v_0 \sin\alpha_0 - gt \quad \text{(projectile motion)} \tag{3.23}$$

These equations describe the position and velocity of the projectile in Fig. 3.17 at any time t.

We can get a lot of information from these equations. For example, at any time the distance r of the projectile from the origin (the magnitude of the position vector $\vec{r}$) is given by

$$r = \sqrt{x^2 + y^2} \tag{3.24}$$

The projectile's speed (the magnitude of its velocity) at any time is

$$v = \sqrt{v_x^2 + v_y^2} \tag{3.25}$$

The *direction* of the velocity, in terms of the angle α it makes with the positive x-direction (see Fig. 3.17), is given by

$$\tan\alpha = \frac{v_y}{v_x} \tag{3.26}$$

The velocity vector $\vec{v}$ is tangent to the trajectory at each point.

We can derive an equation for the trajectory's shape in terms of x and y by eliminating t. From Eqs. (3.20) and (3.21), which assume $x_0 = y_0 = 0$, we find $t = x/(v_0 \cos\alpha_0)$ and

$$y = (\tan\alpha_0)x - \frac{g}{2v_0^2\cos^2\alpha_0}x^2 \tag{3.27}$$

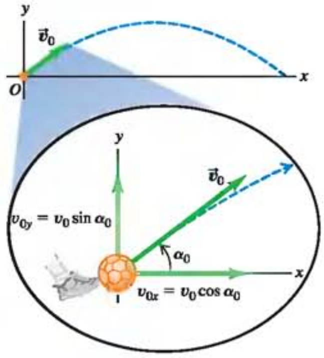
Don't worry about the details of this equation; the important point is its general form. The quantities v_0, $\tan\alpha_0$, $\cos\alpha_0$, and g are constants, so the equation has the form

$$y = bx - cx^2$$

where b and c are constants. This is the equation of a *parabola*. In projectile motion, with our simple model, the trajectory is always a parabola (Fig. 3.19).

When air resistance *isn't* always negligible and has to be included, calculating the trajectory becomes a lot more complicated; the effects of air resistance

3.18 The initial velocity components v_{0x} and v_{0y} of a projectile (such as a kicked soccer ball) are related to the initial speed v_0 and initial angle α_0.



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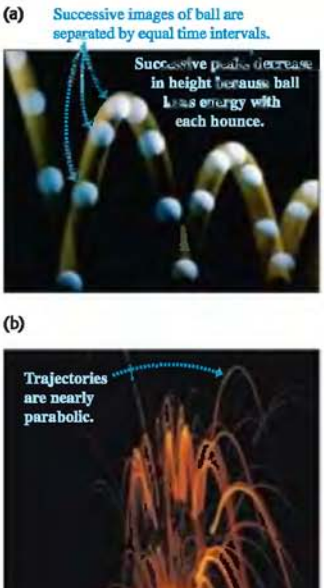
3.5 Initial Velocity Components
3.6 Target Practice I
3.7 Target Practice II

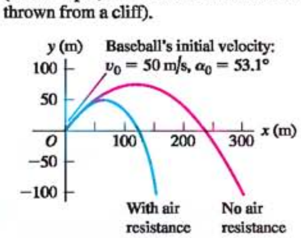
3.19 The nearly parabolic trajectories of (a) a bouncing ball and (b) blobs of molten rock ejected from a volcano.

(a)



(b)

3.20 Air resistance has a large cumulative effect on the motion of a baseball. In this simulation we allow the baseball to fall below the height from which it was thrown (for example, the baseball could have been thrown from a cliff).



depend on velocity, so the acceleration is no longer constant. Figure 3.20 shows a computer simulation of the trajectory of a baseball both without air resistance and with air resistance proportional to the square of the baseball's speed. We see that air resistance has a very large effect; the maximum height and range both decrease, and the trajectory is no longer a parabola. (If you look closely at Fig. 3.19b, you'll see that the trajectories of the volcanic blobs deviate in a similar way from a parabolic shape.)

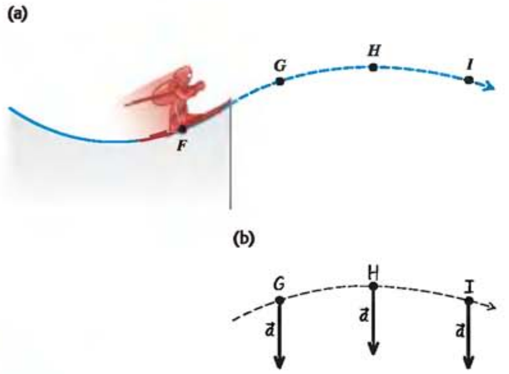
Conceptual Example 3.5 Acceleration of a skier, continued

Let's consider again the skier in Conceptual Example 3.4. What is her acceleration at points G, H, and I in Fig. 3.21a *after* she flies off the ramp? Neglect air resistance.

SOLUTION

Figure 3.21b shows our answer. The skier's acceleration changed from point to point while she was on the ramp. But as soon as she leaves the ramp, she becomes a projectile. So at points G, H, and I, and indeed at *all* points after she leaves the ramp, the skier's acceleration points vertically downward and has magnitude g. No matter how complicated the acceleration of a particle before it becomes a projectile, its acceleration as a projectile is given by $a_x = 0$, $a_y = -g$.

3.21 (a) The skier's path during the jump. (b) Our solution.



Problem Solving Strategy 3.1 Projectile Motion

NOTE: The strategies we used in Sections 2.4 and 2.5 for straight-line, constant-acceleration problems are also useful here.

IDENTIFY *the relevant concepts:* The key concept to remember is that throughout projectile motion, the acceleration is downward and has a constant magnitude g. Note that the projectile-motion equations don't apply to throwing a ball, because during the throw the ball is acted on by both the thrower's hand and gravity. These equations come into play only after the ball leaves the thrower's hand.

SET UP *the problem* using the following steps:

1. Define your coordinate system and make a sketch showing your axes. Usually it's easiest to take the x-axis as being horizontal and the y-axis as being upward and to place the origin at the initial $(t = 0)$ position where the body first becomes a projectile (such as where a ball leaves the thrower's hand). Then the components of the (constant) acceleration are $a_x = 0$, $a_y = -g$, and the initial position is $x_0 = 0$, $y_0 = 0$.

2. List the unknown and known quantities, and decide which unknowns are your target variables. For example, you might be given the initial velocity (either the components or the magnitude and direction) and asked to find the coordinates and velocity components at some later time. In any case, you'll be using Eqs. (3.20) through (3.23). (Certain other equations given in Section 3.3 may be useful as well.) Make sure that you have as many equations as there are target variables to be found.
3. State the problem in words and then translate those words into symbols. For example, *when* does the particle arrive at a certain point? (That is, at what value of t?) *Where* is the particle when its velocity has a certain value? (That is, what are the values of x and y when v_x or v_y has the specified value?) Since $v_y = 0$ at the highest point in a trajectory, the question "When does the projectile reach its highest point?" translates into "What is the value of t when $v_y = 0$?" Similarly, "When does the projectile return to its initial elevation?" translates into "What is the value of t when $y = y_0$?"

EXECUTE *the solution:* Use Eqs. (3.20) through (3.23) to find the target variables. Resist the temptation to break the trajectory into segments and analyze each segment separately. You don't have to start all over when the projectile reaches its highest point! It's almost always easier to use the same axes and time scale throughout the problem. Use the value $g = 9.8\ \text{m/s}^2$.

EVALUATE *your answer:* As always, look at your results to see whether they make sense and whether the numerical values seem reasonable.

Example 3.6 A body projected horizontally

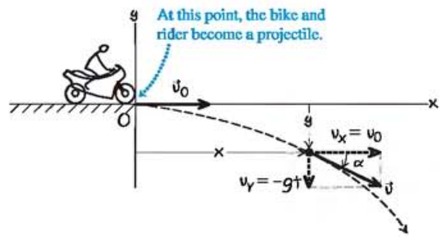
A motorcycle stunt rider rides off the edge of a cliff. Just at the edge his velocity is horizontal, with magnitude 9.0 m/s. Find the motorcycle's position, distance from the edge of the cliff, and velocity after 0.50 s.

SOLUTION

IDENTIFY: Once the rider leaves the cliff, he is in projectile motion. His velocity at the edge of the cliff is therefore his initial velocity.

SET UP: Figure 3.22 shows our sketch. We place the origin of our coordinate system at the edge of the cliff, where the motorcycle first becomes a projectile, so $x_0 = 0$ and $y_0 = 0$. The initial velocity is purely horizontal (that is, $\alpha_0 = 0$), so the initial velocity components are $v_{0x} = v_0\cos\alpha_0 = 9.0\ \text{m/s}$ and $v_{0y} = v_0\sin\alpha_0 = 0$. To find the motorcycle's position at time $t = 0.50$ s, we use Eqs. (3.20) and (3.21), which give x and y as functions of time. We then find the distance from the origin using Eq. (3.24). Finally, we use Eqs. (3.22) and (3.23) to find the velocity components v_x and v_y at $t = 0.50$ s.

3.22 Our sketch for this problem.



EXECUTE: Where is the motorcycle at $t = 0.50$ s? From Eqs. (3.20) and (3.21), the x- and y-coordinates are

$$x = v_{0x}t = (9.0\ \text{m/s})(0.50\ \text{s}) = 4.5\ \text{m}$$

$$y = -\frac{1}{2}gt^2 = -\frac{1}{2}(9.8\ \text{m/s}^2)(0.50\ \text{s})^2 = -1.2\ \text{m}$$

The negative value of y shows that at this time the motorcycle is below its starting point.

What is the motorcycle's distance from the origin at this time? From Eq. (3.24),

$$r = \sqrt{x^2 + y^2} = \sqrt{(4.5\ \text{m})^2 + (-1.2\ \text{m})^2} = 4.7\ \text{m}$$

What is the velocity at time $t = 0.50$ s? From Eqs. (3.22) and (3.23), the components of velocity at this time are

$$v_x = v_{0x} = 9.0\ \text{m/s}$$

$$v_y = -gt = (-9.8\ \text{m/s}^2)(0.50\ \text{s}) = -4.9\ \text{m/s}$$

The motorcycle has the same horizontal velocity v_x as when it left the cliff at $t = 0$, but in addition there is a downward (negative) vertical velocity v_y. If we use unit vectors, the velocity at $t = 0.50$ s is

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (9.0\ \text{m/s})\hat{\imath} + (-4.9\ \text{m/s})\hat{\jmath}$$

We can also express the velocity in terms of magnitude and direction. From Eq. (3.25), the speed (magnitude of the velocity) at this time is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9.0\ \text{m/s})^2 + (-4.9\ \text{m/s})^2} = 10.2\ \text{m/s}$$

From Eq. (3.26), the angle α of the velocity vector is

$$\alpha = \arctan\frac{v_y}{v_x} = \arctan\left(\frac{-4.9\ \text{m/s}}{9.0\ \text{m/s}}\right) = -29°$$

At this time the velocity is 29° below the horizontal.

EVALUATE: Just as shown in Fig. 3.17, the horizontal aspect of the motion is unchanged by gravity; the motorcycle continues to move horizontally at 9.0 m/s, covering 4.5 m in 0.50 s. The motorcycle initially has zero vertical velocity, so it falls vertically just like a body released from rest and descends a distance $\frac{1}{2}gt^2 = 1.2$ m in 0.50 s.

Example 3.7 Height and range of a projectile I: A batted baseball

A batter hits a baseball so that it leaves the bat at speed $v_0 = 37.0\ \text{m/s}$ at an angle $\alpha_0 = 53.1°$, at a location where $g = 9.80\ \text{m/s}^2$. (a) Find the position of the ball, and the magnitude and direction of its velocity, at $t = 2.00$ s. (b) Find the time when the ball reaches the highest point of its flight and find its height h at this point. (c) Find the *horizontal range R*—that is, the horizontal distance from the starting point to where the ball hits the ground.

SOLUTION

IDENTIFY: As Fig. 3.20 shows, the effects of air resistance on the motion of a baseball aren't really negligible. For the sake of simplicity, however, we'll ignore air resistance for this example and use the projectile-motion equations to describe the motion.

SET UP: Figure 3.23 shows our sketch. We use the same coordinate system as in Fig. 3.17 or 3.18 so we can use Eqs. (3.20) through (3.23) without any modifications. Our target variables are (1) the position and velocity of the ball 2.00 s after it leaves the bat, (2) the elapsed time after leaving the bat when the ball is at its maximum height—that is, when $v_y = 0$—and the y-coordinate at this time, and (3) the x-coordinate at the time when the y-coordinate is equal to the initial value y_0.

The ball leaves the bat a meter or so above ground level, but we neglect this distance and assume that it starts at ground level $(y_0 = 0)$. The initial velocity of the ball has components

$$v_{0x} = v_0\cos\alpha_0 = (37.0\ \text{m/s})\cos 53.1° = 22.2\ \text{m/s}$$
$$v_{0y} = v_0\sin\alpha_0 = (37.0\ \text{m/s})\sin 53.1° = 29.6\ \text{m/s}$$

EXECUTE: (a) We want to find x, y, v_x, and v_y at time $t = 2.00$ s. From Eqs. (3.20) through (3.23),

$$x = v_{0x}t = (22.2\ \text{m/s})(2.00\ \text{s}) = 44.4\ \text{m}$$
$$y = v_{0y}t - \frac{1}{2}gt^2$$
$$= (29.6\ \text{m/s})(2.00\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(2.00\ \text{s})^2$$
$$= 39.6\ \text{m}$$
$$v_x = v_{0x} = 22.2\ \text{m/s}$$
$$v_y = v_{0y} - gt = 29.6\ \text{m/s} - (9.80\ \text{m/s}^2)(2.00\ \text{s})$$
$$= 10.0\ \text{m/s}$$

The y-component of velocity is positive, which means that the ball is still moving upward at this time (Fig. 3.23). The magnitude and direction of the velocity are found from Eqs. (3.25) and (3.26):

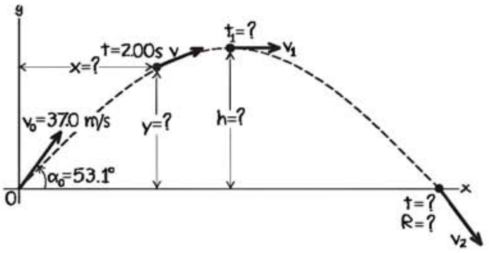
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(22.2\ \text{m/s})^2 + (10.0\ \text{m/s})^2}$$
$$= 24.3\ \text{m/s}$$
$$\alpha = \arctan\left(\frac{10.0\ \text{m/s}}{22.2\ \text{m/s}}\right) = \arctan 0.450 = 24.2°$$

The direction of the velocity (that is, the direction of motion) is 24.2° above the horizontal.

(b) At the highest point, the vertical velocity v_y is zero. When does this happen? Call the time t_1; then

$$v_y = v_{0y} - gt_1 = 0$$
$$t_1 = \frac{v_{0y}}{g} = \frac{29.6\ \text{m/s}}{9.80\ \text{m/s}^2} = 3.02\ \text{s}$$

3.23 Our sketch for this problem.



The height h at this time is the value of y when $t = t_1 = 3.02$ s:

$$h = v_{0y}t_1 - \frac{1}{2}gt_1^2$$
$$= (29.6\ \text{m/s})(3.02\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(3.02\ \text{s})^2$$
$$= 44.7\ \text{m}$$

(c) We'll find the horizontal range in two steps. First, *when* does the ball hit the ground? This occurs when $y = 0$. Call this time t_2; then

$$y = 0 = v_{0y}t_2 - \frac{1}{2}gt_2^2 = t_2\left(v_{0y} - \frac{1}{2}gt_2\right)$$

This is a quadratic equation for t_2. It has two roots:

$$t_2 = 0 \quad \text{and} \quad t_2 = \frac{2v_{0y}}{g} = \frac{2(29.6\ \text{m/s})}{9.80\ \text{m/s}^2} = 6.04\ \text{s}$$

There are two times at which $y = 0$; $t_2 = 0$ is the time the ball *leaves* the ground, and $t_2 = 2v_{0y}/g = 6.04$ s is the time of its return. This is exactly twice the time to reach the highest point that we found in part (b), $t_1 = v_{0y}/g = 3.02$ s, so the time of descent equals the time of ascent. This is *always* true if the starting and end points are at the same elevation and air resistance can be neglected.

The horizontal range R is the value of x when the ball returns to the ground—that is, at $t = 6.04$ s:

$$R = v_{0x}t_2 = (22.2\ \text{m/s})(6.04\ \text{s}) = 134\ \text{m}$$

The vertical component of velocity when the ball hits the ground is

$$v_y = v_{0y} - gt_2 = 29.6\ \text{m/s} - (9.80\ \text{m/s}^2)(6.04\ \text{s})$$
$$= -29.6\ \text{m/s}$$

That is, v_y has the same magnitude as the initial vertical velocity v_{0y} but the opposite direction (down). Since v_x is constant, the angle $\alpha = -53.1°$ (below the horizontal) at this point is the negative of the initial angle $\alpha_0 = 53.1°$.

EVALUATE: It's often useful to check results by getting them in a different way. For example, we can check our answer for the maximum height in part (b) by applying the constant-acceleration formula Eq. (2.13) to the y-motion:

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) = v_{0y}^2 - 2g(y - y_0)$$

At the highest point, $v_y = 0$ and $y = h$. Substituting these, along with $y_0 = 0$, we find

$$0 = v_{0y}^2 - 2gh$$

$$h = \frac{v_{0y}^2}{2g} = \frac{(29.6\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 44.7\ \text{m}$$

which is the same height we obtained in part (b).

It's interesting to note that $h = 44.7$ m in part (b) is comparable to the 52.4-m height above the playing field of the roof of the Hubert H. Humphrey Metrodome in Minneapolis, and the horizontal range $R = 134$ m in part (c) is greater than the 99.7-m distance from home plate to the right-field fence at Safeco Field in Seattle. (The ball's height when it crosses the fence is more than enough to clear it, so this ball is a home run.)

In real life, a batted ball with the initial speed and angle we've used here won't go as high or as far as we've calculated. (If it did, home runs would be far more common and baseball would be a far less interesting game.) The reason is that air resistance, which we neglected in this example, is actually an important factor at the typical speeds of pitched and batted balls (see Fig. 3.20).

Example 3.8 Height and range of a projectile II: Maximum height, maximum range

For a projectile launched with speed v_0 at initial angle α_0 (between 0° and 90°), derive general expressions for the maximum height h and horizontal range R (Fig. 3.23). For a given v_0, what value of α_0 gives maximum height? What value gives maximum horizontal range?

SOLUTION

IDENTIFY: This is really the same exercise as parts (b) and (c) of Example 3.7. The difference is that we are looking for general expressions for h and R. We'll also be looking for the values of α_0 that give the maximum values of h and R.

SET UP: In part (b) of Example 3.7 we found that the projectile reaches the high point of its trajectory (so that $v_y = 0$) at time $t_1 = v_{0y}/g$, and in part (c) of Example 3.7 we found that the projectile returns to its starting height (so that $y = y_0$) at time $t_2 = 2v_{0y}/g$. (As we saw in Example 3.7, $t_2 = 2t_1$.) To determine the height h at the high point of the trajectory, we use Eq. (3.21) to find the y-coordinate at t_1. To determine R, we substitute t_2 into Eq. (3.20) to determine the x-coordinate at t_2. We'll express our answers in terms of the launch speed v_0 and launch angle α_0 using Eq. (3.19).

EXECUTE: From Eq. (3.19), $v_{0x} = v_0 \cos\alpha_0$ and $v_{0y} = v_0 \sin\alpha_0$. Hence we can write the time t_1 when $v_y = 0$ as

$$t_1 = \frac{v_{0y}}{g} = \frac{v_0 \sin\alpha_0}{g}$$

Then, from Eq. (3.21), the height at this time is

$$h = (v_0 \sin\alpha_0)\left(\frac{v_0 \sin\alpha_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\alpha_0}{g}\right)^2$$

$$= \frac{v_0^2 \sin^2\alpha_0}{2g}$$

For a given launch speed v_0, the maximum value of h occurs when $\sin\alpha_0 = 1$ and $\alpha_0 = 90°$—that is, when the projectile is launched straight up. That's what we should expect. If it is launched horizontally, as in Example 3.6, $\alpha_0 = 0$ and the maximum height is zero!

The time t_2 when the projectile returns to the ground is

$$t_2 = \frac{2v_{0y}}{g} = \frac{2v_0 \sin\alpha_0}{g}$$

The horizontal range R is the value of x at this time. From Eq. (3.20),

$$R = (v_0 \cos\alpha_0)t_2 = (v_0 \cos\alpha_0)\frac{2v_0 \sin\alpha_0}{g}$$

We can now use the trigonometric identity $2\sin\alpha_0 \cos\alpha_0 = \sin 2\alpha_0$ to rewrite this as

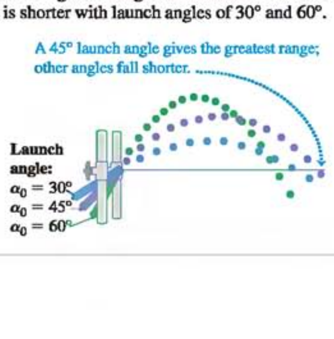
$$R = \frac{v_0^2 \sin 2\alpha_0}{g}$$

The maximum value of $\sin 2\alpha_0$ is 1; this occurs when $2\alpha_0 = 90°$, or $\alpha_0 = 45°$. This angle gives the maximum range for a given initial speed.

EVALUATE: Figure 3.24 is based on a composite photograph of three trajectories of a ball projected from a spring gun at angles of 30°, 45°, and 60°. The initial speed v_0 is approximately the same in all three cases. The horizontal ranges are nearly the same for the 30° and 60° angles, and the range for 45° is greater than either. Can you prove that for a given value of v_0 the range is the same for both an initial angle α_0 and an initial angle $90° - \alpha_0$?

CAUTION **Height and range of a projectile** We don't recommend memorizing the above expressions for h and R. They are applicable only in the special circumstances we have described. In particular, the expression for the range R can be used *only* when launch and landing heights are equal. There are many end-of-chapter problems to which these equations do *not* apply.

3.24 A launch angle of 45° gives the maximum horizontal range. The range is shorter with launch angles of 30° and 60°.



Example 3.9 Different initial and final heights

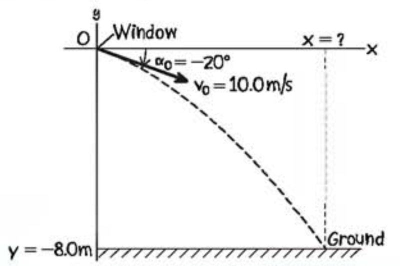
You toss a ball from your window 8.0 m above the ground. When the ball leaves your hand, it is moving at 10.0 m/s at an angle of 20° below the horizontal. How far horizontally from your window will the ball hit the ground? Ignore air resistance.

SOLUTION

IDENTIFY: As in our calculation of the horizontal range in Examples 3.7 and 3.8, we are trying to find the horizontal coordinate of a projectile when it is at a given value of y. The difference here is that this value of y is *not* equal to the initial y-coordinate.

SET UP: Once again we choose the x-axis to be horizontal and the y-axis to be upward, and we place the origin of coordinates at the point where the ball leaves your hand (Fig. 3.25). We have $v_0 = 10.0\ \text{m/s}$ and $\alpha_0 = -20°$; the angle is negative because the initial velocity is below the horizontal. Our target variable is the value of x at the point where the ball reaches the ground—that is, when $y = -8.0\ \text{m}$. Because the initial and final heights of the ball are different, we can't simply use the expression for the horizontal range found in Example 3.8. Instead, we first use Eq. (3.21) to find the time t when the ball reaches $y = -8.0\ \text{m}$ and then calculate the value of x at this time using Eq. (3.20).

3.25 Our sketch for this problem.



EXECUTE: To determine t, we rewrite Eq. (3.21) in the standard form for a quadratic equation for t:

$$\frac{1}{2}gt^2 - (v_0 \sin\alpha_0)t + y = 0$$

The roots of this equation are

$$t = \frac{v_0\sin\alpha_0 \pm \sqrt{(-v_0\sin\alpha_0)^2 - 4\left(\frac{1}{2}g\right)y}}{2\left(\frac{1}{2}g\right)}$$

$$= \frac{v_0\sin\alpha_0 \pm \sqrt{v_0^2\sin^2\alpha_0 - 2gy}}{g}$$

$$= \frac{\begin{bmatrix}(10.0\ \text{m/s})\sin(-20°) \\ \pm\sqrt{(10.0\ \text{m/s})^2\sin^2(-20°) - 2(9.80\ \text{m/s}^2)(-8.0\ \text{m})}\end{bmatrix}}{9.80\ \text{m/s}^2}$$

$$= -1.7\ \text{s} \quad \text{or} \quad 0.98\ \text{s}$$

We can discard the negative root, since it refers to a time before the ball left your hand. The positive root tells us that the ball takes 0.98 s to reach the ground. From Eq. (3.20), the ball's x-coordinate at that time is

$$x = (v_0\cos\alpha_0)t = (10.0\ \text{m/s})[\cos(-20°)](0.98\ \text{s})$$
$$= 9.2\ \text{m}$$

The ball hits the ground a horizontal distance of 9.2 m from your window.

EVALUATE: The root $t = -1.7\ \text{s}$ is an example of a "fictional" solution to a quadratic equation. We discussed these in Example 2.8 in Section 2.5; you should review that discussion.

With our choice of origin we had initial and final heights $y_0 = 0$ and $y = -8.0\ \text{m}$. Can you use Eqs. (3.16) and (3.18) to show that you get the same answers for t and x if you choose the origin to be at the point on the ground directly below where the ball leaves your hand?

Example 3.10 The zookeeper and the monkey

A monkey escapes from the zoo and climbs a tree. After failing to entice the monkey down, the zookeeper fires a tranquilizer dart directly at the monkey (Fig. 3.26). The clever monkey lets go at the same instant the dart leaves the gun barrel, intending to land on the ground and escape. Show that the dart *always* hits the monkey, regardless of the dart's muzzle velocity (provided that it gets to the monkey before he hits the ground).

SOLUTION

IDENTIFY: In this example we have *two* bodies in projectile motion: the tranquilizer dart and the monkey. The dart and the monkey have different initial positions and initial velocities, but they go into projectile motion at the same time. To show that the dart hits the monkey, we have to prove that at some time the monkey and the dart have the same x-coordinate and the same y-coordinate.

SET UP: We make the usual choice for the x- and y-directions, and place the origin of coordinates at the end of the barrel of the tranquilizer gun (Fig. 3.26). We'll first use Eq. (3.20) to find the time t when the x-coordinates x_{monkey} and x_{dart} are the same. Then we'll use Eq. (3.21) to check whether y_{monkey} and y_{dart} are also equal at this time; if they are, the dart hits the monkey.

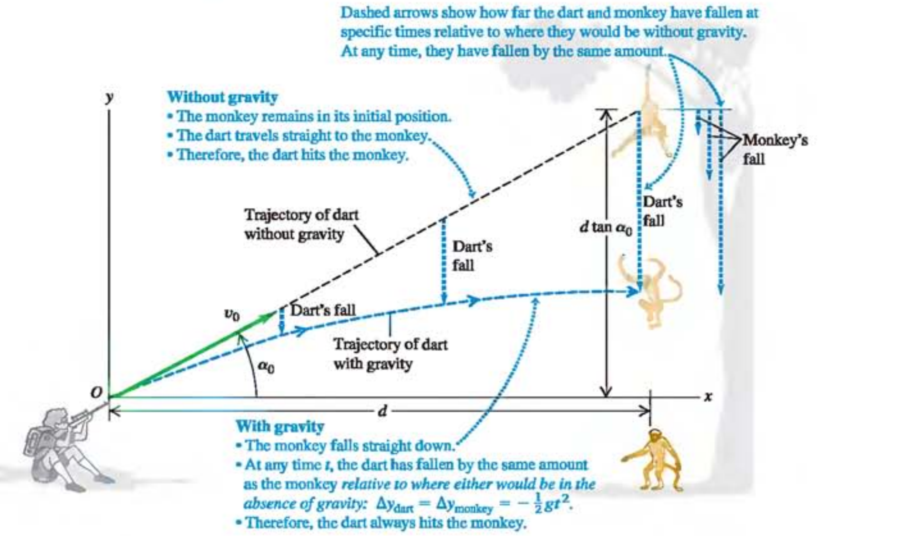
EXECUTE: The monkey drops straight down, so $x_{\text{monkey}} = d$ at *all* times. For the dart, Eq. (3.20) tells us that $x_{\text{dart}} = (v_0\cos\alpha_0)t$. When these x-coordinates are equal, $d = (v_0\cos\alpha_0)t$, or

$$t = \frac{d}{v_0\cos\alpha_0}$$

To have the dart hit the monkey, it must be true that $y_{\text{monkey}} = y_{\text{dart}}$ at this same time. The monkey is in one-dimensional free fall; his position at any time is given by Eq. (2.12), with appropriate symbol changes. Figure 3.26 shows that the monkey's initial height is $d\tan\alpha_0$ (the opposite side of a right triangle with angle α_0 and adjacent side d), and we find

$$y_{\text{monkey}} = d\tan\alpha_0 - \frac{1}{2}gt^2$$

3.26 The tranquilizer dart hits the falling monkey.



For the dart we use Eq. (3.21):

$$y_{dart} = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$$

So we see that if $d\tan\alpha_0 = (v_0 \sin\alpha_0)t$ at the time when the two x-coordinates are equal, then $y_{monkey} = y_{dart}$, and we have a hit. To prove that this happens, we replace t with $d/(v_0 \cos\alpha_0)$, the time when $x_{monkey} = x_{dart}$. Sure enough, we find that

$$(v_0 \sin\alpha_0)t = (v_0 \sin\alpha_0)\frac{d}{v_0 \cos\alpha_0} = d\tan\alpha_0$$

EVALUATE: We have proved that at the time the x-coordinates are equal, the y-coordinates are also equal; a dart aimed at the initial position of the monkey *always* hits it, no matter what v_0 is. This result is also independent of the value of g, the acceleration due to gravity. With no gravity $(g = 0)$, the monkey would remain motionless, and the dart would travel in a straight line to hit him. With gravity, both "fall" the same distance $(\frac{1}{2}gt^2)$ below their $g = 0$ positions, and the dart still hits the monkey (Fig. 3.26).

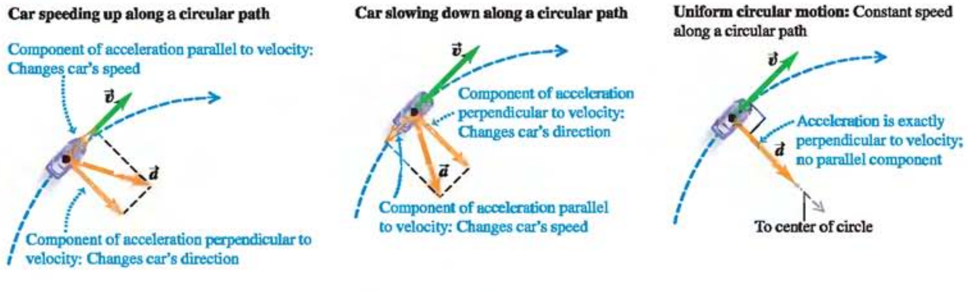
Test Your Understanding of Section 3.3 In Example 3.10, suppose the tranquilizer dart has a relatively low muzzle velocity so that the dart reaches a maximum height at a point P before striking the monkey, as shown in the figure. When the dart is at point P, will the monkey be (i) at point A (higher than P), (ii) at point B (at the same height as P), or (iii) at point C (lower than P)? Ignore air resistance.

3.4 Motion in a Circle

When a particle moves along a curved path, the direction of its velocity changes. As we saw in Section 3.2, this means that the particle *must* have a component of acceleration perpendicular to the path, even if its speed is constant (see Fig. 3.11b). In this section we'll calculate the acceleration for the important special case of motion in a circle.

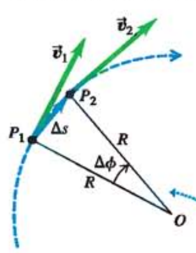
ActivPhysics ONLINE
4.1 Magnitude of Centripetal Acceleration

3.27 A car in uniform circular motion. The speed is constant and the acceleration is directed toward the center of the circular path.

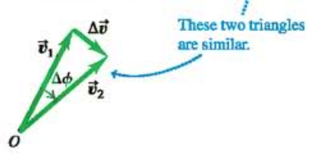


3.28 Finding the velocity change $\Delta\vec{v}$, average acceleration $\vec{a}_{av}$, and instantaneous acceleration $\vec{a}_{rad}$ for a particle moving in a circle with constant speed.

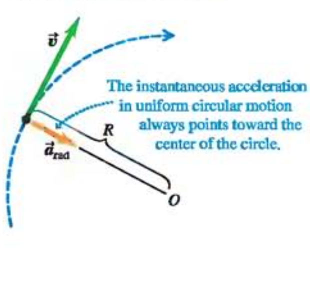
(a) A point moves a distance Δs at constant speed along a circular path.



(b) The corresponding change in velocity and average acceleration



(c) The instantaneous acceleration



Uniform Circular Motion

When a particle moves in a circle with *constant speed,* the motion is called **uniform circular motion.** A car rounding a curve with constant radius at constant speed, a satellite moving in a circular orbit, and an ice skater skating in a circle with constant speed are all examples of uniform circular motion (Fig. 3.27; compare Fig. 3.12). There is no component of acceleration parallel (tangent) to the path; otherwise, the speed would change. The acceleration vector is perpendicular (normal) to the path and hence directed inward (never outward!) toward the center of the circular path. This causes the direction of the velocity to change without changing the speed. Our next project is to show that the magnitude of the acceleration in uniform circular motion is related in a simple way to the speed of the particle and the radius of the circle.

Figure 3.28a shows a particle moving with constant speed in a circular path of radius R with center at O. The particle moves from P_1 to P_2 in a time Δt. The vector change in velocity $\Delta\vec{v}$ during this time is shown in Fig. 3.28b.

The angles labeled $\Delta\phi$ in Figs. 3.28a and 3.28b are the same because $\vec{v}_1$ is perpendicular to the line OP_1 and $\vec{v}_2$ is perpendicular to the line OP_2. Hence the triangles in Figs. 3.28a and 3.28b are *similar.* The ratios of corresponding sides of similar triangles are equal, so

$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R} \quad \text{or} \quad |\Delta\vec{v}| = \frac{v_1}{R}\Delta s$$

The magnitude a_{av} of the average acceleration during Δt is therefore

$$a_{av} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}$$

The magnitude a of the *instantaneous* acceleration $\vec{a}$ at point P_1 is the limit of this expression as we take point P_2 closer and closer to point P_1:

$$a = \lim_{\Delta t \to 0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}$$

But the limit of $\Delta s/\Delta t$ is the speed v_1 at point P_1. Also, P_1 can be any point on the path, so we can drop the subscript and let v represent the speed at any point. Then

$$a_{rad} = \frac{v^2}{R} \quad \text{(uniform circular motion)} \qquad (3.28)$$

We have added the subscript "rad" as a reminder that the direction of the instantaneous acceleration at each point is always along a radius of the circle, toward its center. Because the speed is constant, the acceleration is always perpendicular to the instantaneous velocity. This is shown in Fig. 3.28c; compare with the right-hand illustration in Fig. 3.27.

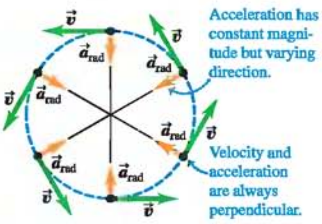
We have found that *in uniform circular motion, the magnitude a of the instantaneous acceleration is equal to the square of the speed v divided by the radius R of the circle. Its direction is perpendicular to $\vec{v}$ and inward along the radius.*

Because the acceleration is always directed toward the center of the circle, it is sometimes called **centripetal acceleration.** The word "centripetal" is derived from two Greek words meaning "seeking the center." Figure 3.29a shows the directions of the velocity and acceleration vectors at several points for a particle moving with uniform circular motion.

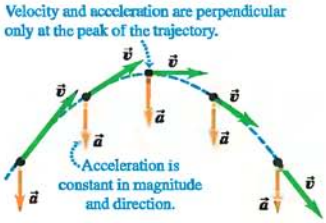
CAUTION Uniform circular motion vs. projectile motion The acceleration in uniform circular motion has some similarities to the acceleration in projectile motion without air resistance, but there are also some important differences. In both uniform circular motion (Fig. 3.29a) and projectile motion (Fig. 3.29b) the *magnitude* of acceleration is the same at all times. However, in uniform circular motion the *direction* of $\vec{a}$ changes continuously so that it always points toward the center of the circle. (At the top of the circle the acceleration points down; at the bottom of the circle the acceleration points up.) In projectile motion, by contrast, the direction of $\vec{a}$ remains the same at all times.

3.29 Acceleration and velocity (a) for a particle in uniform circular motion and (b) for a projectile with no air resistance.

(a) Uniform circular motion



(b) Projectile motion



We can also express the magnitude of the acceleration in uniform circular motion in terms of the **period** T of the motion, the time for one revolution (one complete trip around the circle). In a time T the particle travels a distance equal to the circumference $2\pi R$ of the circle, so its speed is

$$v = \frac{2\pi R}{T} \tag{3.29}$$

When we substitute this into Eq. (3.28), we obtain the alternative expression

$$a_{rad} = \frac{4\pi^2 R}{T^2} \quad \text{(uniform circular motion)} \tag{3.30}$$

Example 3.11 Centripetal acceleration on a curved road

An Aston Martin V8 Vantage sports car has a "lateral acceleration" of $0.96g$, which is $(0.96)(9.8\ \text{m/s}^2) = 9.4\ \text{m/s}^2$. This represents the maximum centripetal acceleration that the car can attain without skidding out of the circular path. If the car is traveling at a constant 40 m/s (about 89 mi/h, or 144 km/h), what is the minimum radius of curve it can negotiate? (Assume that the curve is unbanked.)

SOLUTION

IDENTIFY: Because the car is moving at a constant speed along a curve that is a segment of a circle, we can apply the ideas of uniform circular motion.

SET UP: We use Eq. (3.28) to find the target variable R (the radius of the curve) in terms of the given centripetal acceleration a_{rad} and speed v.

EXECUTE: We are given a_{rad} and v, so we solve Eq. (3.28) for R:

$$R = \frac{v^2}{a_{rad}} = \frac{(40\ \text{m/s})^2}{9.4\ \text{m/s}^2} = 170\ \text{m (about 560 ft)}$$

EVALUATE: Our result shows that the required turning radius R is proportional to the *square* of the speed. Hence even a small reduction in speed can make R substantially smaller. For example, reducing v by 20% (from 40 m/s to 32 m/s) would decrease R by 36% (from 170 m to 109 m).

Another way to make the required turning radius smaller is to *bank* the curve. We will investigate this option in Chapter 5.

Example 3.12 Centripetal acceleration on a carnival ride

In a carnival ride, the passengers travel at constant speed in a circle of radius 5.0 m. They make one complete circle in 4.0 s. What is their acceleration?

SOLUTION

IDENTIFY: The speed is constant, so this is a problem involving uniform circular motion.

SET UP: We are given the radius $R = 5.0\text{ m}$ and the period $T = 4.0\text{ s}$, so we can use Eq. (3.30) to calculate the acceleration. Alternatively, we can first calculate the speed v using Eq. (3.29) and then find the acceleration using Eq. (3.28).

EXECUTE: From Eq. (3.30),

$$a_{rad} = \frac{4\pi^2(5.0\text{ m})}{(4.0\text{ s})^2} = 12\text{ m/s}^2$$

We'll check this answer by using Eq. (3.28) after first determining the speed v. From Eq. (3.29), the speed is the circumference of the circle divided by the period T:

$$v = \frac{2\pi R}{T} = \frac{2\pi(5.0\text{ m})}{4.0\text{ s}} = 7.9\text{ m/s}$$

The centripetal acceleration is then

$$a_{rad} = \frac{v^2}{R} = \frac{(7.9\text{ m/s})^2}{5.0\text{ m}} = 12\text{ m/s}^2$$

Happily, we get the same answer for a_{rad} with both approaches.

EVALUATE: As in Example 3.11, the direction of $\vec{a}$ is always toward the center of the circle. The magnitude of $\vec{a}$ is greater than g, the acceleration due to gravity, so this is not a ride for the faint-hearted. (Some roller coasters subject their passengers to accelerations as great as $4g$.)

Nonuniform Circular Motion

We have assumed throughout this section that the particle's speed is constant. If the speed varies, we call the motion **nonuniform circular motion.** An example is a roller coaster car that slows down and speeds up as it moves around a vertical loop. In nonuniform circular motion, Eq. (3.28) still gives the *radial* component of acceleration $a_{rad} = v^2/R$, which is always *perpendicular* to the instantaneous velocity and directed toward the center of the circle. But since the speed v has different values at different points in the motion, the value of a_{rad} is not constant. The radial (centripetal) acceleration is greatest at the point in the circle where the speed is greatest.

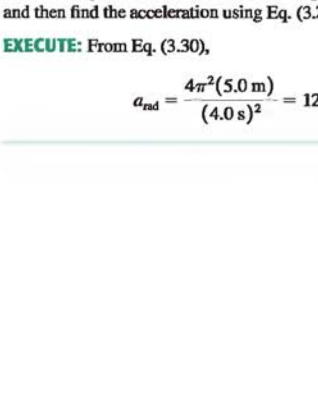
In nonuniform circular motion there is also a component of acceleration that is *parallel* to the instantaneous velocity. This is the component $a_\parallel$ that we discussed in Section 3.2; here we call this component a_{tan} to emphasize that it is *tangent* to the circle. From the discussion at the end of Section 3.2 we see that the tangential component of acceleration a_{tan} is equal to the rate of change of *speed*. Thus

$$a_{rad} = \frac{v^2}{R} \quad \text{and} \quad a_{tan} = \frac{d|\vec{v}|}{dt} \quad \text{(nonuniform circular motion)} \tag{3.31}$$

The vector acceleration of a particle moving in a circle with varying speed is the vector sum of the radial and tangential components of accelerations. The tangential component is in the same direction as the velocity if the particle is speeding up, and in the opposite direction if the particle is slowing down (Fig. 3.30).

In *uniform* circular motion there is no tangential component of acceleration, but the radial component is the magnitude of $d\vec{v}/dt$.

3.30 A particle moving in a vertical loop with a varying speed, like a roller coaster car.



CAUTION **Uniform vs. nonuniform circular motion** Note that the two quantities

$$\frac{d|\vec{v}|}{dt} \quad \text{and} \quad \left|\frac{d\vec{v}}{dt}\right|$$

are *not* the same. The first, equal to the tangential acceleration, is the rate of change of speed; it is zero whenever a particle moves with constant speed, even when its direction of motion changes (such as in *uniform* circular motion). The second is the magnitude of the vector acceleration; it is zero only when the particle's acceleration *vector* is zero—that is, when the particle moves in a straight line with constant speed. In *uniform* circular motion $|d\vec{v}/dt| = a_{rad} = v^2/r$; in *nonuniform* circular motion there is also a tangential component of acceleration, so $|d\vec{v}/dt| = \sqrt{a_{rad}^2 + a_{tan}^2}$.

Test Your Understanding of Section 3.4 Suppose that the particle in Fig. 3.30 experiences four times the acceleration at the bottom of the loop as it does at the top of the loop. Compared to its speed at the top of the loop, is its speed at the bottom of the loop (i) $\sqrt{2}$ times as great; (ii) 2 times as great; (iii) $2\sqrt{2}$ times as great; (iv) 4 times as great; or (v) 16 times as great.

3.5 Relative Velocity

You've no doubt observed how a car that is moving slowly forward appears to be moving backward when you pass it. In general, when two observers measure the velocity of a moving body, they get different results if one observer is moving relative to the other. The velocity seen by a particular observer is called the velocity *relative* to that observer, or simply **relative velocity.** Figure 3.31 shows a situation in which understanding relative velocity is extremely important.

We'll first consider relative velocity along a straight line, then generalize to relative velocity in a plane.

3.31 Airshow pilots face a complicated problem involving relative velocities. They must keep track of their motion relative to the air (to maintain enough airflow over the wings to sustain lift), relative to each other (to keep a tight formation without colliding), and relative to their audience (to remain in sight of the spectators).

Relative Velocity in One Dimension

A passenger walks with a velocity of 1.0 m/s along the aisle of a train that is moving with a velocity of 3.0 m/s (Fig. 3.32a). What is the passenger's velocity? It's a simple enough question, but it has no single answer. As seen by a second passenger sitting in the train, she is moving at 1.0 m/s. A person on a bicycle standing beside the train sees the walking passenger moving at 1.0 m/s + 3.0 m/s = 4.0 m/s. An observer in another train going in the opposite direction would give still another answer. We have to specify which observer we mean, and we speak of the velocity *relative* to a particular observer. The walking passenger's velocity relative to the train is 1.0 m/s, her velocity relative to the cyclist is 4.0 m/s, and so on. Each observer, equipped in principle with a meter stick and a stopwatch, forms what we call a **frame of reference.** Thus a frame of reference is a coordinate system plus a time scale.

Let's use the symbol A for the cyclist's frame of reference (at rest with respect to the ground) and the symbol B for the frame of reference of the moving train. In straight-line motion the position of a point P relative to frame A is given by $x_{P/A}$ (the position of P with respect to A), and the position of P relative to frame B is given by $x_{P/B}$ (see Fig. 3.32b). The position of the origin of A with respect to the origin of B is $x_{B/A}$. Figure 3.32b shows that

$$x_{P/A} = x_{P/B} + x_{B/A} \tag{3.32}$$

In words, the total distance from the origin of A to point P equals the distance from the origin of B to point P plus the distance from the origin of A to the origin of B.

The x-velocity of P relative to frame A, denoted by $v_{P/A\text{-}x}$, is the derivative of $x_{P/A}$ with respect to time. The other velocities are similarly obtained. So the time derivative of Eq. (3.32) gives us a relationship among the various velocities:

$$\frac{dx_{P/A}}{dt} = \frac{dx_{P/B}}{dt} + \frac{dx_{B/A}}{dt} \quad \text{or}$$

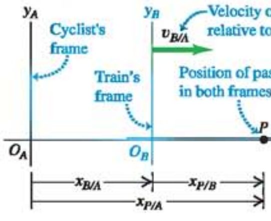
$$v_{P/A\text{-}x} = v_{P/B\text{-}x} + v_{B/A\text{-}x} \quad \text{(relative velocity along a line)} \tag{3.33}$$

Getting back to the passenger on the train in Fig. 3.32, we see that A is the cyclist's frame of reference, B is the frame of reference of the train, and point P represents the passenger. Using the above notation, we have

$$v_{P/B\text{-}x} = +1.0 \text{ m/s} \qquad v_{B/A\text{-}x} = +3.0 \text{ m/s}$$

3.32 (a) A passenger walking in a train. (b) The position of the passenger relative to the cyclist's frame of reference and the train's frame of reference.

(a)



(b)

From Eq. (3.33) the passenger's velocity $v_{P/A}$ relative to the cyclist is

$$v_{P/A\text{-}x} = +1.0\ \text{m/s} + 3.0\ \text{m/s} = +4.0\ \text{m/s}$$

as we already knew.

In this example, both velocities are toward the right, and we have taken this as the positive x-direction. If the passenger walks toward the *left* relative to the train, then $v_{P/B\text{-}x} = -1.0\ \text{m/s}$, and her x-velocity relative to the cyclist is $v_{P/A\text{-}x} = -1.0\ \text{m/s} + 3.0\ \text{m/s} = +2.0\ \text{m/s}$. The sum in Eq. (3.33) is always an algebraic sum, and any or all of the x-velocities may be negative.

When the passenger looks out the window, the stationary cyclist on the ground appears to her to be moving backward; we can call the cyclist's velocity relative to her $v_{A/P\text{-}x}$. Clearly, this is just the negative of $v_{P/A\text{-}x}$. In general, if A and B are any two points or frames of reference,

$$v_{A/B\text{-}x} = -v_{B/A\text{-}x} \tag{3.34}$$

Problem Solving Strategy 3.2 Relative Velocity

MP

IDENTIFY *the relevant concepts:* Whenever you see the phrase "velocity relative to" or "velocity with respect to," it's likely that the concepts of relative velocity will be helpful.

SET UP *the problem:* Label each frame of reference in the problem. Each moving body has its own frame of reference; in addition, you'll almost always have to include the frame of reference of the earth's surface. (Statements such as "The car is traveling north at 90 km/h" implicitly refer to the car's velocity relative to the surface of the earth.) Use the labels to help identify the target variable. For example, if you want to find the x-velocity of a car (C) with respect to a bus (B), your target variable is $v_{C/B\text{-}x}$.

EXECUTE *the solution:* Solve for the target variable using Eq. (3.33). (If the velocities are not along the same direction, you'll need to use the vector form of this equation, derived later in this section.) It's important to note the order of the double subscripts in Eq. (3.33): $v_{A/B\text{-}x}$ always means "x-velocity of A relative to B." These subscripts obey an interesting kind of algebra, as Eq. (3.33) shows. If we regard each one as a fraction, then the fraction on the left side is the *product* of the fractions on the right sides: $P/A = (P/B)(B/A)$. This is a handy rule you can use when applying Eq. (3.33) to any number of frames of reference. For example, if there are three different frames of reference A, B, and C, we can write immediately

$$v_{P/A\text{-}x} = v_{P/C\text{-}x} + v_{C/B\text{-}x} + v_{B/A\text{-}x}$$

EVALUATE *your answer:* Be on the lookout for stray minus signs in your answer. If the target variable is the x-velocity of a car relative to a bus $(v_{C/B\text{-}x})$, make sure that you haven't accidentally calculated the x-velocity of the *bus* relative to the *car* $(v_{B/C\text{-}x})$. If you have made this mistake, you can recover using Eq. (3.34).

Example 3.13 Relative velocity on a straight road

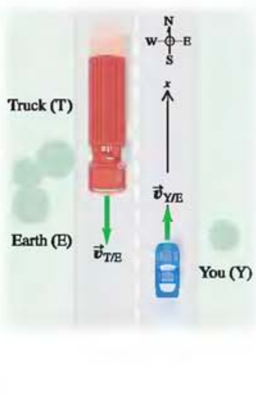
You are driving north on a straight two-lane road at a constant 88 km/h. A truck traveling at a constant 104 km/h approaches you (in the other lane, fortunately). (a) What is the truck's velocity relative to you? (b) What is your velocity with respect to the truck? (c) How do the relative velocities change after you and the truck have passed each other?

SOLUTION

IDENTIFY: This example is about relative velocities along a line.

SET UP: Let you be Y, the truck be T, and the earth's surface be E, and let the positive x-direction be north (Fig. 3.33). Then your x-velocity relative to the earth is $v_{Y/E\text{-}x} = +88\ \text{km/h}$. As the truck is initially approaching you, it must be moving south and its x-velocity with respect to the earth is $v_{T/E\text{-}x} = -104\ \text{km/h}$. The target variable in part (a) is $v_{T/Y\text{-}x}$; the target variable in part (b) is $v_{Y/T\text{-}x}$. We'll find both target variables by using Eq. (3.33) for relative velocity.

3.33 Reference frames for you and the truck.



EXECUTE: (a) To find $v_{T/Y\text{-}x}$, we first write Eq. (3.33) for the three frames Y, T, and E, and then rearrange:

$$v_{T/E\text{-}x} = v_{T/Y\text{-}x} + v_{Y/E\text{-}x}$$
$$v_{T/Y\text{-}x} = v_{T/E\text{-}x} - v_{Y/E\text{-}x}$$
$$= -104\text{ km/h} - 88\text{ km/h} = -192\text{ km/h}$$

The truck is moving at 192 km/h in the negative x-direction (south) relative to you.

(b) From Eq. (3.34),

$$v_{Y/T\text{-}x} = -v_{T/Y\text{-}x} = -(-192\text{ km/h}) = +192\text{ km/h}$$

You are moving at 192 km/h in the positive x-direction (north) relative to the truck.

(c) The relative velocities do *not* change at all after you and the truck pass each other. The relative positions of the bodies don't matter. The truck is still moving at 192 km/h toward the south relative to you, but it is now moving away from you instead of toward you.

EVALUATE: To check your answer in part (b), try using Eq. (3.33) directly in the form $v_{Y/T\text{-}x} = v_{Y/E\text{-}x} + v_{E/T\text{-}x}$. (Remember that the x-velocity of the earth with respect to the truck is the opposite of the x-velocity of the truck with respect to the earth: $v_{E/T\text{-}x} = -v_{T/E\text{-}x}$.) Do you get the same result?

Relative Velocity in Two or Three Dimensions

We can extend the concept of relative velocity to include motion in a plane or in space by using vector addition to combine velocities. Suppose that the passenger in Fig. 3.32a is walking not down the aisle of the railroad car but from one side of the car to the other, with a speed of 1.0 m/s (Fig. 3.34a). We can again describe the passenger's position P in two different frames of reference: A for the stationary ground observer and B for the moving train. But instead of coordinates x, we use position vectors [&*lcacc*{~bfit~r~normal~}{lArarrl}&] because the problem is now two-dimensional. Then, as Fig. 3.34b shows,

$$\vec{r}_{P/A} = \vec{r}_{P/B} + \vec{r}_{B/A} \tag{3.35}$$

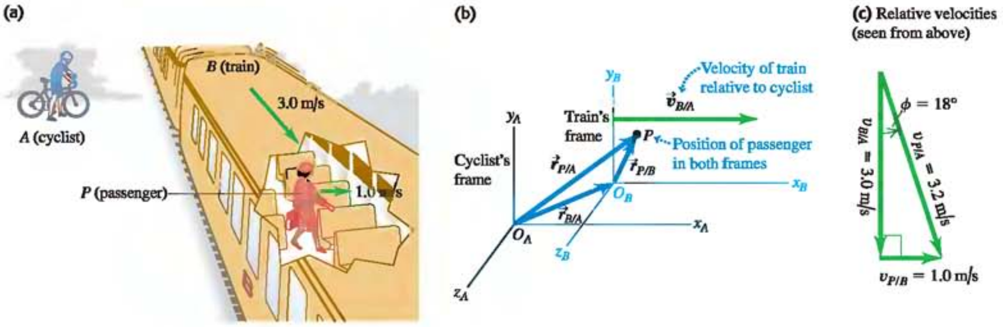
Just as we did before, we take the time derivative of this equation to get a relationship among the various velocities; the velocity of P relative to A is $\vec{v}_{P/A} = d\vec{r}_{P/A}/dt$ and so on for the other velocities. We get

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A} \quad \text{(relative velocity in space)} \tag{3.36}$$

Equation (3.36) is known as the *Galilean velocity transformation*. It relates the velocity of a body P with respect to frame A and its velocity with respect to frame B ($\vec{v}_{P/A}$ and $\vec{v}_{P/B}$, respectively) to the velocity of frame B with respect to frame A $(\vec{v}_{B/A})$. If all three of these velocities lie along the same line, then Eq. (3.36) reduces to Eq. (3.33) for the components of the velocities along that line.

If the train is moving at $v_{B/A} = 3.0$ m/s relative to the ground and the passenger is moving at $v_{P/B} = 1.0$ m/s relative to the train, then the passenger's velocity

3.34 (a) A passenger walking across a railroad car. (b) Position of the passenger relative to the cyclist's frame and the train's frame. (c) Vector diagram for the velocity of the passenger relative to the ground (the cyclist's frame), $\vec{v}_{P/A}$.



vector $\vec{v}_{P/A}$ relative to the ground is as shown in Fig. 3.34c. The Pythagorean theorem then gives us

$$v_{P/A} = \sqrt{(3.0\text{ m/s})^2 + (1.0\text{ m/s})^2} = \sqrt{10\text{ m}^2/\text{s}^2} = 3.2\text{ m/s}$$

Figure 3.34c also shows that the *direction* of the passenger's velocity vector relative to the ground makes an angle ϕ with the train's velocity vector $\vec{v}_{B/A}$, where

$$\tan\phi = \frac{v_{P/B}}{v_{B/A}} = \frac{1.0\text{ m/s}}{3.0\text{ m/s}} \quad \text{and} \quad \phi = 18°$$

As in the case of motion along a straight line, we have the general rule that if A and B are *any* two points or frames of reference,

$$\vec{v}_{A/B} = -\vec{v}_{B/A} \tag{3.37}$$

The velocity of the passenger relative to the train is the negative of the velocity of the train relative to the passenger, and so on.

In the early 20th century Albert Einstein showed in his special theory of relativity that the velocity-addition relationship given in Eq. (3.36) has to be modified when speeds approach the speed of light, denoted by c. It turns out that if the passenger in Fig. 3.32a could walk down the aisle at $0.30c$ and the train could move at $0.90c$, then her speed relative to the ground would be not $1.20c$ but $0.94c$; nothing can travel faster than light! We'll return to the special theory of relativity in Chapter 37.

Example 3.14 Flying in a crosswind

The compass of an airplane indicates that it is headed due north, and its airspeed indicator shows that it is moving through the air at 240 km/h. If there is a wind of 100 km/h from west to east, what is the velocity of the airplane relative to the earth?

SOLUTION

IDENTIFY: This problem involves velocities in two dimensions (northward and eastward), so it is a relative velocity problem using vectors.

SET UP: We are given the magnitude and direction of the velocity of the plane (P) relative to the air (A). We are also given the magnitude and direction of the wind velocity, which is the velocity of the air (A) with respect to the earth (E):

$$\vec{v}_{P/A} = 240\text{ km/h} \quad \text{due north}$$
$$\vec{v}_{A/E} = 100\text{ km/h} \quad \text{due east}$$

Our target variables are the magnitude and direction of the velocity of the plane (P) relative to the earth (E), $\vec{v}_{P/E}$. We'll find these using Eq. (3.36).

EXECUTE: Using Eq. (3.36), we have

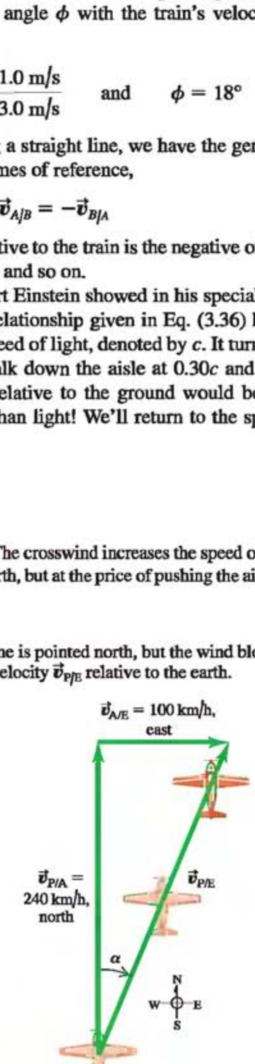
$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$$

Figure 3.35 shows the three relative velocities and their relationship; the unknowns are the speed $v_{P/E}$ and the angle α. From this diagram we find

$$v_{P/E} = \sqrt{(240\text{ km/h})^2 + (100\text{ km/h})^2} = 260\text{ km/h}$$
$$\alpha = \arctan\left(\frac{100\text{ km/h}}{240\text{ km/h}}\right) = 23°\text{ E of N}$$

EVALUATE: The crosswind increases the speed of the airplane relative to the earth, but at the price of pushing the airplane off course.

3.35 The plane is pointed north, but the wind blows east, giving the resultant velocity $\vec{v}_{P/E}$ relative to the earth.



Example 3.15 Correcting for a crosswind

In Example 3.14, in what direction should the pilot head to travel due north? What will be her velocity relative to the earth? (Assume that her airspeed and the velocity of the wind are the same as in Example 3.14.)

SOLUTION

IDENTIFY: Like Example 3.14, this is a relative velocity problem with vectors.

SET UP: Figure 3.36 illustrates the situation. The vectors are arranged in accordance with the vector relative-velocity equation, Eq. (3.36):

$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$$

As Fig. 3.36 shows, the pilot points the nose of the airplane at an angle β into the wind to compensate for the crosswind. This angle, which tells us the direction of the vector $\vec{v}_{P/A}$ (the velocity of the airplane relative to the air), is one of our target variables. The other target variable is the speed of the airplane over the ground, which is the magnitude of the vector $\vec{v}_{P/E}$ (the velocity of the airplane relative to the earth). Here are the known and unknown quantities:

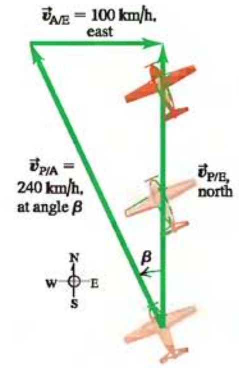
$\vec{v}_{P/E}$ = magnitude unknown	due north
$\vec{v}_{P/A}$ = 240 km/h	direction unknown
$\vec{v}_{A/E}$ = 100 km/h	due east

We can solve for the unknown target variables using Fig. 3.36 and trigonometry.

EXECUTE: From the diagram, the speed $v_{P/E}$ and the angle β are given by

$$v_{P/E} = \sqrt{(240 \text{ km/h})^2 - (100 \text{ km/h})^2} = 218 \text{ km/h}^2$$

$$\beta = \arcsin\left(\frac{100 \text{ km/h}}{240 \text{ km/h}}\right) = 25°$$



3.36 The pilot must point the plane in the direction of the vector $\vec{v}_{P/A}$ to travel due north relative to the earth.

The pilot should point the airplane 25° west of north, and her ground speed is then 218 km/h.

EVALUATE: Note that there were two target variables—the magnitude of a vector and the direction of a vector—in both this example and Example 3.14. The difference is that in Example 3.14, the magnitude and direction referred to the *same* vector $(\vec{v}_{P/E})$, whereas in this example they referred to *different* vectors $(\vec{v}_{P/E}$ and $\vec{v}_{P/A})$.

It's no surprise that a headwind reduces an airplane's speed relative to the ground. This example shows that a *crosswind* also slows an airplane down—an unfortunate fact of aeronautical life.

Test Your Understanding of Section 3.5 Suppose the nose of an airplane is pointed due east and the airplane has an airspeed of 150 km/h. Due to the wind, the airplane is moving due *north* relative to the ground and its speed relative to the ground is 150 km/h. What is the velocity of the air relative to the earth? (i) 150 km/h from east to west; (ii) 150 km/h from south to north; (iii) 150 km/h from southeast to northwest; (iv) 212 km/h from east to west; (v) 212 km/h from south to north; (vi) 212 km/h from southeast to northwest; (vii) there is no possible wind velocity that could cause this.

CHAPTER 3 SUMMARY

Position, velocity, and acceleration vectors: The position vector $\vec{r}$ of a point P in space is the vector from the origin to P. Its components are the coordinates x, y, and z.

The average velocity vector $\vec{v}_{av}$ during the time interval Δt is the displacement $\Delta\vec{r}$ (the change in the position vector $\vec{r}$) divided by Δt. The instantaneous velocity vector $\vec{v}$ is the time derivative of $\vec{r}$, and its components are the time derivatives of x, y, and z. The instantaneous speed is the magnitude of $\vec{v}$. The velocity $\vec{v}$ of a particle is always tangent to the particle's path. (See Example 3.1.)

The average acceleration vector $\vec{a}_{av}$ during the time interval Δt equals $\Delta\vec{v}$ (the change in the velocity vector $\vec{v}$) divided by Δt. The instantaneous acceleration vector $\vec{a}$ is the time derivative of $\vec{v}$, and its components are the time derivatives of v_x, v_y, and v_z. (See Example 3.2.)

The component of acceleration parallel to the direction of the instantaneous velocity affects the speed, while the component of $\vec{a}$ perpendicular to $\vec{v}$ affects the direction of motion. (See Examples 3.3 and 3.4.)

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad (3.1)$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t} \qquad (3.2)$$

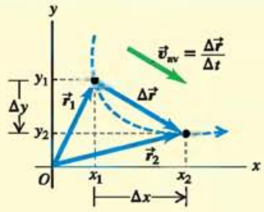
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \qquad (3.3)$$

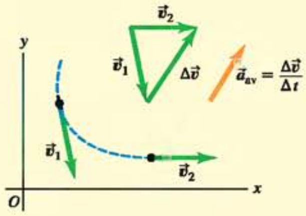
$$v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \quad v_z = \frac{dz}{dt} \qquad (3.4)$$

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \qquad (3.8)$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad (3.9)$$

$$a_x = \frac{dv_x}{dt} \quad a_y = \frac{dv_y}{dt} \quad a_z = \frac{dv_z}{dt} \qquad (3.10)$$





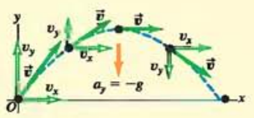
Projectile motion: In projectile motion with no air resistance, $a_x = 0$ and $a_y = -g$. The coordinates and velocity components are simple functions of time, and the shape of the path is always a parabola. We usually choose the origin to be at the initial position of the projectile. (See Examples 3.5–3.10.)

$$x = (v_0\cos\alpha_0)t \qquad (3.20)$$

$$y = (v_0\sin\alpha_0)t - \frac{1}{2}gt^2 \qquad (3.21)$$

$$v_x = v_0\cos\alpha_0 \qquad (3.22)$$

$$v_y = v_0\sin\alpha_0 - gt \qquad (3.23)$$



Uniform and nonuniform circular motion: When a particle moves in a circular path of radius R with constant speed v (uniform circular motion), its acceleration $\vec{a}$ is directed toward the center of the circle and perpendicular to $\vec{v}$. The magnitude a_{rad} of the acceleration can be expressed in terms of v and R or in terms of R and the period T (the time for one revolution), where $v = 2\pi R/T$. (See Examples 3.11 and 3.12.)

If the speed is not constant in circular motion (nonuniform circular motion), there is still a radial component of $\vec{a}$ given by Eq. (3.28) or (3.30), but there is also a component of $\vec{a}$ parallel (tangential) to the path. This tangential component is equal to the rate of change of speed, dv/dt.

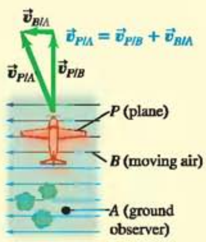
$$a_{rad} = \frac{v^2}{R} \qquad (3.28)$$

$$a_{rad} = \frac{4\pi^2 R}{T^2} \qquad (3.30)$$

Relative velocity: When a body P moves relative to a body (or reference frame) B, and B moves relative to A, we denote the velocity of P relative to B by $\vec{v}_{P/B}$, the velocity of P relative to A by $\vec{v}_{P/A}$, and the velocity of B relative to A by $\vec{v}_{B/A}$. If these velocities are all along the same line, their components along that line are related by Eq. (3.33). More generally, these velocities are related by Eq. (3.36). (See Examples 3.13–3.15)

$$v_{P/A\text{-}x} = v_{P/B\text{-}x} + v_{B/A\text{-}x} \qquad (3.33)$$
(relative velocity along a line)

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A} \qquad (3.36)$$
(relative velocity in space)



Key Terms

position vector, *72*
average velocity, *72*
instantaneous velocity, *72*
average acceleration, *75*
instantaneous acceleration, *75*
projectile, *79*
trajectory, *79*
uniform circular motion, *88*
centripetal acceleration, *89*
period, *89*
nonuniform circular motion, *90*
relative velocity, *91*
frame of reference, *91*

Answer to Chapter Opening Question ?

A car going around a curve at constant speed has an acceleration directed toward the inside of the curve (see Section 3.2, especially Fig. 3.12a).

Answers to Test Your Understanding Questions

3.1 Answer: (iii) If the instantaneous velocity $\vec{v}$ is constant over an interval, its value at any point (including the end of the interval) is the same as the average velocity $\vec{v}_{av}$ over the interval. In (i) and (ii) the direction of $\vec{v}$ at the end of the interval is tangent to the path at that point, while the direction of $\vec{v}_{av}$ points from the beginning of the path to its end (in the direction of the net displacement). In (iv) $\vec{v}$ and $\vec{v}_{av}$ are both directed along the straight line, but $\vec{v}$ has a greater magnitude because the speed has been increasing.

3.2 Answer: vector 7 At the high point of the sled's path, the speed is minimum. At that point the speed is neither increasing nor decreasing, and the parallel component of the acceleration (that is, the horizontal component) is zero. The acceleration has only a perpendicular component toward the inside of the sled's curved path. In other words, the acceleration is downward.

3.3 Answer: (i) If there were no gravity $(g = 0)$, the monkey would not fall and the dart would follow a straight-line path (shown as a dashed line). The effect of gravity is to make the monkey and the dart both fall the same distance $\frac{1}{2}gt^2$ below their $g = 0$ positions. Point A is the same distance below the monkey's initial position as point P is below the dashed straight line, so point A is where we would find the monkey at the time in question.

3.4 Answer: (ii) At both the top and bottom of the loop, the acceleration is purely radial and is given by Eq. (3.28). The radius R is the same at both points, so the difference in acceleration is due purely to differences in speed. Since a_{rad} is proportional to the square of v, the speed must be twice as great at the bottom of the loop as at the top.

3.5 Answer: (vi) The effect of the wind is to cancel the airplane's eastward motion and give it a northward motion. So the velocity of the air relative to the ground (the wind velocity) must have one 150-km/h component to the west and one 150-km/h component to the north. The combination of these is a vector of magnitude $\sqrt{(150\text{ km/h})^2 + (150\text{ km/h})^2} = 212\text{ km/h}$ that points to the northwest.

PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com

Discussion Questions

Q3.1. A simple pendulum (a mass swinging at the end of a string) swings back and forth in a circular arc. What is the direction of the acceleration off the mass at the ends of the swing? At the midpoint? In each case, explain how you obtain your answer.

Q3.2. Redraw Fig. 3.11a if $\vec{a}$ is antiparallel to $\vec{v}_1$. Does the particle move in a straight line? What happens to its speed?

Q3.3. A projectile moves in a parabolic path without air resistance. Is there any point at which $\vec{a}$ is parallel to $\vec{v}$? Perpendicular to $\vec{v}$? Explain.

Q3.4. When a rifle is fired at a distant target, the barrel is not lined up exactly on the target. Why not? Does the angle of correction depend on the distance of the target?

Q3.5. At the same instant that you fire a bullet horizontally from a gun, you drop a bullet from the height of the barrel. If there is no air resistance, which bullet hits the ground first? Explain.

Q3.6. A package falls out of an airplane that is flying in a straight line at a constant altitude and speed. If you could ignore air resistance, what would be the path of the package as observed by the pilot? As observed by a person on the ground?

Q3.7. Sketch the six graphs of the x- and y-components of position, velocity, and acceleration versus time for projectile motion with $x_0 = y_0 = 0$ and $0 < \alpha_0 < 90°$.

Q3.8. An object is thrown straight up into the air and feels no air resistance. How is it possible for it to have an acceleration when it has stopped moving at its highest point?

Q3.9. If a jumping frog can give itself the same initial speed regardless of the direction in which it jumps (forward or straight up), how is the maximum vertical height to which it can jump related to its maximum horizontal range $R_{max} = v_0^2/g$?

Q3.10. A projectile is fired upward at an angle θ above the horizontal with an initial speed v_0. At its maximum height, what are its velocity vector, its speed, and its acceleration vector?

Q3.11. In uniform circular motion, what are the *average* velocity and *average* acceleration for one revolution? Explain.

Q3.12. In uniform circular motion, how does the acceleration change when the speed is increased by a factor of 3? When the radius is decreased by a factor of 2?

Q3.13. In uniform circular motion, the acceleration is perpendicular to the velocity at every instant. Is this still true when the motion is not uniform—that is, when the speed is not constant?

Q3.14. Raindrops hitting the side windows of a car in motion often leave diagonal streaks even if there is no wind. Why? Is the explanation the same or different for diagonal streaks on the windshield?

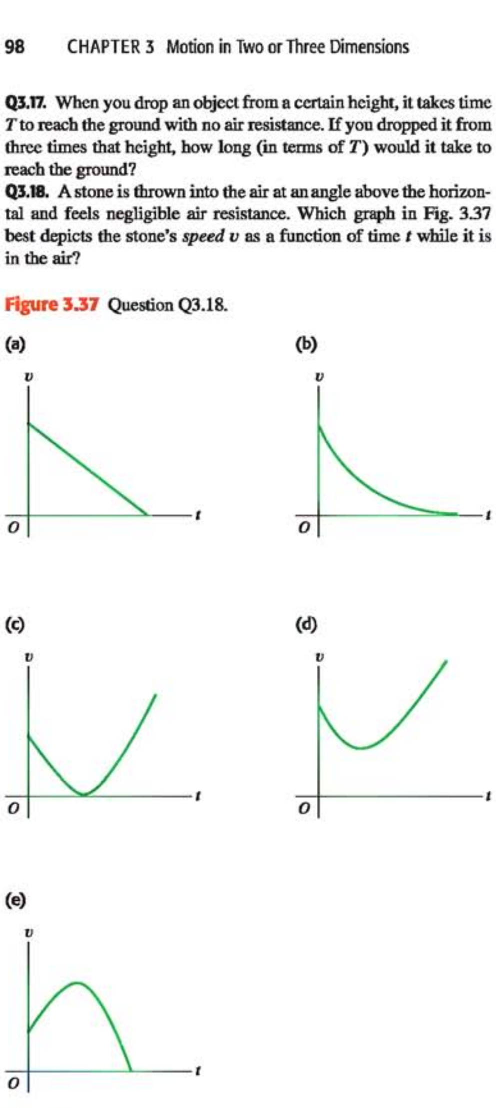
Q3.15. In a rainstorm with a strong wind, what determines the best position in which to hold an umbrella?

Q3.16. You are on the west bank of a river that is flowing north with a speed of 1.2 m/s. Your swimming speed relative to the water is 1.5 m/s, and the river is 60 m wide. What is your path relative to earth that allows you to cross the river in the shortest time? Explain your reasoning.

Q3.17. When you drop an object from a certain height, it takes time T to reach the ground with no air resistance. If you dropped it from three times that height, how long (in terms of T) would it take to reach the ground?

Q3.18. A stone is thrown into the air at an angle above the horizontal and feels negligible air resistance. Which graph in Fig. 3.37 best depicts the stone's *speed* v as a function of time t while it is in the air?

Figure 3.37 Question Q3.18.



Exercises

Section 3.1 Position and Velocity Vectors

3.1. A squirrel has x- and y-coordinates $(1.1\text{ m}, 3.4\text{ m})$ at time $t_1 = 0$ and coordinates $(5.3\text{ m}, -0.5\text{ m})$ at time $t_2 = 3.0\text{ s}$. For this time interval, find (a) the components of the average velocity, and (b) the magnitude and direction of the average velocity.

3.2. A rhinoceros is at the origin of coordinates at time $t_1 = 0$. For the time interval from $t_1 = 0$ to $t_2 = 12.0\text{ s}$, the rhino's average velocity has x-component -3.8 m/s and y-component 4.9 m/s. At time $t_2 = 12.0\text{ s}$, (a) what are the x- and y-coordinates of the rhino? (b) How far is the rhino from the origin?

3.3. A web page designer creates an animation in which a dot on a computer screen has a position of $\vec{r} = [4.0\text{ cm} + (2.5\text{ cm/s}^2)t^2]\hat{i} + (5.0\text{ cm/s})t\hat{j}$. (a) Find the magnitude and direction of the dot's average velocity between $t = 0$ and $t = 2.0\text{ s}$. (b) Find the magnitude and direction of the instantaneous velocity at $t = 0$, $t = 1.0\text{ s}$, and $t = 2.0\text{ s}$. (c) Sketch the dot's trajectory from $t = 0$ to $t = 2.0\text{ s}$, and show the velocities calculated in part (b).

3.4. If $\vec{r} = bt^2\hat{i} + ct^3\hat{j}$, where b and c are positive constants, when does the velocity vector make an angle of 45.0° with the x- and y-axes?

Section 3.2 The Acceleration Vector

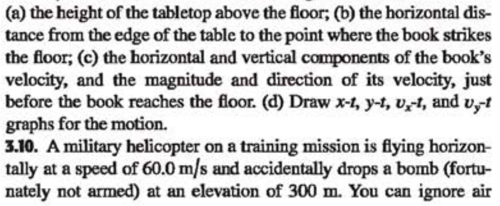
3.5. A jet plane is flying at a constant altitude. At time $t_1 = 0$ it has components of velocity $v_x = 90\text{ m/s}$, $v_y = 110\text{ m/s}$. At time $t_2 = 30.0\text{ s}$ the components are $v_x = -170\text{ m/s}$, $v_y = 40\text{ m/s}$. (a) Sketch the velocity vectors at t_1 and t_2. How do these two vectors differ? For this time interval calculate (b) the components of the average acceleration, and (c) the magnitude and direction of the average acceleration.

3.6. A dog running in an open field has components of velocity $v_x = 2.6\text{ m/s}$ and $v_y = -1.8\text{ m/s}$ at $t_1 = 10.0\text{ s}$. For the time interval from $t_1 = 10.0\text{ s}$ to $t_2 = 20.0\text{ s}$, the average acceleration of the dog has magnitude 0.45 m/s^2 and direction 31.0° measured from the $+x$-axis toward the $+y$-axis. At $t_2 = 20.0\text{ s}$, (a) what are the x- and y-components of the dog's velocity? (b) What are the magnitude and direction of the dog's velocity? (c) Sketch the velocity vectors at t_1 and t_2. How do these two vectors differ?

3.7. The coordinates of a bird flying in the xy-plane are given by $x(t) = \alpha t$ and $y(t) = 3.0\text{ m} - \beta t^2$, where $\alpha = 2.4\text{ m/s}$ and $\beta = 1.2\text{ m/s}^2$. (a) Sketch the path of the bird between $t = 0$ and $t = 2.0\text{ s}$. (b) Calculate the velocity and acceleration vectors of the bird as functions of time. (c) Calculate the magnitude and direction of the bird's velocity and acceleration at $t = 2.0\text{ s}$. (d) Sketch the velocity and acceleration vectors at $t = 2.0\text{ s}$. At this instant, is the bird speeding up, is it slowing down, or is its speed instantaneously not changing? Is the bird turning? If so, in what direction?

3.8. A particle moves along a path as shown in Fig. 3.38. Between points B and D, the path is a straight line. Sketch the acceleration vectors at A, C, and E in the cases in which (a) the particle moves with a constant speed; (b) the particle moves with a steadily increasing speed; (c) the particle moves with a steadily decreasing speed.

Figure 3.38 Exercise 3.8.



Section 3.3 Projectile Motion

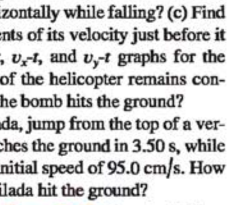
3.9. A physics book slides off a horizontal tabletop with a speed of 1.10 m/s. It strikes the floor in 0.350 s. Ignore air resistance. Find (a) the height of the tabletop above the floor; (b) the horizontal distance from the edge of the table to the point where the book strikes the floor; (c) the horizontal and vertical components of the book's velocity, and the magnitude and direction of its velocity, just before the book reaches the floor. (d) Draw x-t, y-t, v_x-t, and v_y-t graphs for the motion.

3.10. A military helicopter on a training mission is flying horizontally at a speed of 60.0 m/s and accidentally drops a bomb (fortunately not armed) at an elevation of 300 m. You can ignore air resistance. (a) How much time is required for the bomb to reach the earth? (b) How far does it travel horizontally while falling? (c) Find the horizontal and vertical components of its velocity just before it strikes the earth. (d) Draw x-t, y-t, v_x-t, and v_y-t graphs for the bomb's motion. (e) If the velocity of the helicopter remains constant, where is the helicopter when the bomb hits the ground?

3.11. Two crickets, Chirpy and Milada, jump from the top of a vertical cliff. Chirpy just drops and reaches the ground in 3.50 s, while Milada jumps horizontally with an initial speed of 95.0 cm/s. How far from the base of the cliff will Milada hit the ground?

3.12. A daring 510-N swimmer dives off a cliff with a running horizontal leap, as shown in Fig. 3.39. What must her minimum speed be just as she leaves the top of the cliff so that she will miss the ledge at the bottom, which is 1.75 m wide and 9.00 m below the top of the cliff?

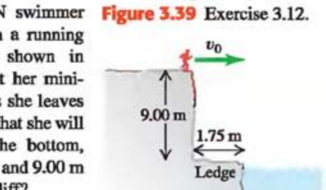
Figure 3.39 Exercise 3.12.



3.13. Leaping the River I. A car comes to a bridge during a storm and finds the bridge washed out. The driver must get to the other side, so he decides to try leaping it with his car. The side of the road the car is on is 21.3 m above the river, while the opposite side is a mere 1.8 m above the river. The river itself is a raging torrent 61.0 m wide. (a) How fast should the car be traveling at the time it leaves the road in order just to clear the river and land safely on the opposite side? (b) What is the speed of the car just before it lands on the other side?

3.14. A small marble rolls horizontally with speed v_0 off the top of a platform 2.75 m tall and feels no appreciable air resistance. On the level ground, 2.00 m from the base of the platform, there is a gaping hole in the ground (Fig. 3.40.) For what range of marble speeds v_0 will the marble land in the hole?

Figure 3.40 Exercise 3.14.



3.15. Inside a starship at rest on the earth, a ball rolls off the top of a horizontal table and lands a distance D from the foot of the table. This starship now lands on the unexplored Planet X. The commander, Captain Curious, rolls the same ball off the same table with the same initial speed as on earth and finds that it lands a distance $2.76D$ from the foot of the table. What is the acceleration due to gravity on Planet X?

3.16. A rookie quarterback throws a football with an initial upward velocity component of 16.0 m/s and a horizontal velocity component of 20.0 m/s. Ignore air resistance. (a) How much time is required for the football to reach the highest point of the trajectory? (b) How high is this point? (c) How much time (after it is thrown) is required for the football to return to its original level? How does this compare with the time calculated in part (a)? (d) How far has the football traveled horizontally during this time? (e) Draw x-t, y-t, v_x-t, and v_y-t graphs for the motion.

3.17. On level ground a shell is fired with an initial velocity of 80.0 m/s at 60.0° above the horizontal and feels no appreciable air resistance. (a) Find the horizontal and vertical components of the shell's initial velocity. (b) How long does it take the shell to reach its highest point? (c) Find its maximum height above the ground. (d) How far from its firing point does the shell land? (e) At its highest point, find the horizontal and vertical components of its acceleration and velocity.

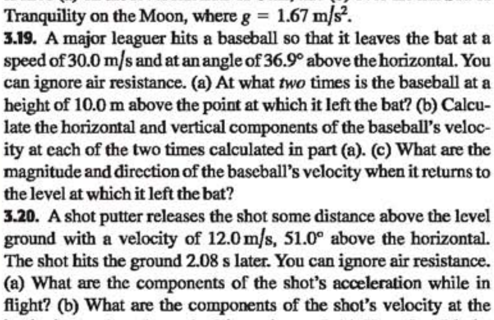
3.18. A pistol that fires a signal flare gives it an initial velocity (muzzle velocity) of 125 m/s at an angle of 55.0° above the horizontal. You can ignore air resistance. Find the flare's maximum height and the distance from its firing point to its landing point if it is fired (a) on the level salt flats of Utah, and (b) over the flat Sea of Tranquility on the Moon, where $g = 1.67\ \text{m/s}^2$.

3.19. A major leaguer hits a baseball so that it leaves the bat at a speed of 30.0 m/s and at an angle of 36.9° above the horizontal. You can ignore air resistance. (a) At what *two* times is the baseball at a height of 10.0 m above the point at which it left the bat? (b) Calculate the horizontal and vertical components of the baseball's velocity at each of the two times calculated in part (a). (c) What are the magnitude and direction of the baseball's velocity when it returns to the level at which it left the bat?

3.20. A shot putter releases the shot some distance above the level ground with a velocity of 12.0 m/s, 51.0° above the horizontal. The shot hits the ground 2.08 s later. You can ignore air resistance. (a) What are the components of the shot's acceleration while in flight? (b) What are the components of the shot's velocity at the beginning and at the end of its trajectory? (c) How far did she throw the shot horizontally? (d) Why does the expression for R in Example 3.8 *not* give the correct answer for part (c)? (e) How high was the shot above the ground when she released it? (f) Draw x-t, y-t, v_x-t, and v_y-t graphs for the motion.

3.21. Win the Prize. In a carnival booth, you win a stuffed giraffe if you toss a quarter into a small dish. The dish is on a shelf above the point where the quarter leaves your hand and is a horizontal distance of 2.1 m from this point (Fig. 3.41). If you toss the coin with a velocity of 6.4 m/s at an angle of 60° above the horizontal, the coin lands in the dish. You can ignore air resistance. (a) What is the height of the shelf above the point where the quarter leaves your hand? (b) What is the vertical component of the velocity of the quarter just before it lands in the dish?

Figure 3.41 Exercise 3.21.



3.22. Suppose the departure angle α_0 in Fig. 3.26 is 42.0° and the distance d is 3.00 m. Where will the dart and monkey meet if the initial speed of the dart is (a) 12.0 m/s? (b) 8.0 m/s? (c) What will happen if the initial speed of the dart is 4.0 m/s? Sketch the trajectory in each case.

3.23. A man stands on the roof of a 15.0-m-tall building and throws a rock with a velocity of magnitude 30.0 m/s at an angle of 33.0° above the horizontal. You can ignore air resistance. Calculate

(a) the maximum height above the roof reached by the rock; (b) the magnitude of the velocity of the rock just before it strikes the ground; and (c) the horizontal range from the base of the building to the point where the rock strikes the ground. (d) Draw x-t, y-t, v_x-t, and v_y-t graphs for the motion.

3.24. Firemen are shooting a stream of water at a burning building using a high-pressure hose that shoots out the water with a speed of 25.0 m/s as it leaves the end of the hose. Once it leaves the hose, the water moves in projectile motion. The firemen adjust the angle of elevation α of the hose until the water takes 3.00 s to reach a building 45.0 m away. You can ignore air resistance; assume that the end of the hose is at ground level. (a) Find the angle of elevation α. (b) Find the speed and acceleration of the water at the highest point in its trajectory. (c) How high above the ground does the water strike the building, and how fast is it moving just before it hits the building?

3.25. A 124-kg balloon carrying a 22-kg basket is descending with a constant downward velocity of 20.0 m/s. A 1.0-kg stone is thrown from the basket with an initial velocity of 15.0 m/s perpendicular to the path of the descending balloon, as measured relative to a person at rest in the basket. The person in the basket sees the stone hit the ground 6.00 s after being thrown. Assume that the balloon continues its downward descent with the same constant speed of 20.0 m/s. (a) How high was the balloon when the rock was thrown out? (b) How high is the balloon when the rock hits the ground? (c) At the instant the rock hits the ground, how far is it from the basket? (d) Just before the rock hits the ground, find its horizontal and vertical velocity components as measured by an observer (i) at rest in the basket and (ii) at rest on the ground.

3.26. A cannon, located 60.0 m from the base of a vertical 25.0-m-tall cliff, shoots a 15-kg shell at 43.0° above the horizontal toward the cliff. (a) What must the minimum muzzle velocity be for the shell to clear the top of the cliff? (b) The ground at the top of the cliff is level, with a constant elevation of 25.0 m above the cannon. Under the conditions of part (a), how far does the shell land past the edge of the cliff?

3.27. An airplane is flying with a velocity of 90.0 m/s at an angle of 23.0° above the horizontal. When the plane is 114 m directly above a dog that is standing on level ground, a suitcase drops out of the luggage compartment. How far from the dog will the suitcase land? You can ignore air resistance.

Section 3.4 Motion in a Circle

3.20. On your first day at work for an appliance manufacturer, you are told to figure out what to do to the period of rotation during a washer spin cycle to triple the centripetal acceleration. You impress your boss by answering immediately. What do you tell her?

3.29. The earth has a radius of 6380 km and turns around once on its axis in 24 h. (a) What is the radial acceleration of an object at the earth's equator? Give your answer in m/s^2 and as a fraction of g. (b) If a_{rad} at the equator is greater than g, objects would fly off the earth's surface and into space. (We will see the reason for this in Chapter 5.) What would the period of the earth's rotation have to be for this to occur?

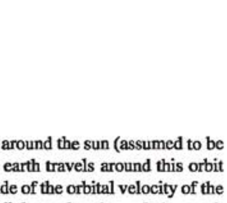
3.30. A model of a helicopter rotor has four blades, each 3.40 m long from the central shaft to the blade tip. The model is rotated in a wind tunnel at 550 rev/min. (a) What is the linear speed of the blade tip, in m/s? (b) What is the radial acceleration of the blade tip expressed as a multiple of the acceleration of gravity, g?

3.31. In a test of a "g-suit," a volunteer is rotated in a horizontal circle of radius 7.0 m. What must the period of rotation be so that the centripetal acceleration has a magnitude of (a) $3.0g$? (b) $10g$?

3.32. The radius of the earth's orbit around the sun (assumed to be circular) is 1.50×10^8 km, and the earth travels around this orbit in 365 days. (a) What is the magnitude of the orbital velocity of the earth, in m/s? (b) What is the radial acceleration of the earth toward the sun, in m/s^2? (c) Repeat parts (a) and (b) for the motion of the planet Mercury (orbit radius $= 5.79 \times 10^7$ km, orbital period $= 88.0$ days).

3.33. A Ferris wheel with radius 14.0 m is turning about a horizontal axis through its center (Fig. 3.42). The linear speed of a passenger on the rim is constant and equal to 7.00 m/s. What are the magnitude and direction of the passenger's acceleration as she passes through (a) the lowest point in her circular motion? (b) The highest point in her circular motion? (c) How much time does it take the Ferris wheel to make one revolution?

Figure 3.42 Exercises 3.33 and 3.34.



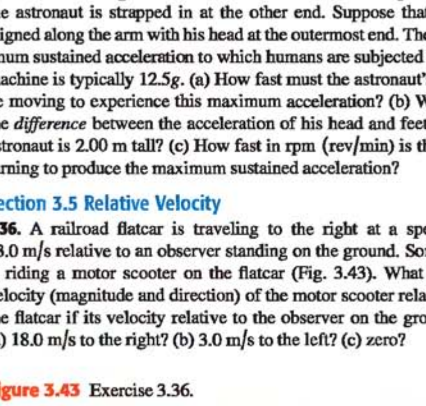
3.34. The Ferris wheel in Fig. 3.42, which rotates counterclockwise, is just starting up. At a given instant, a passenger on the rim of the wheel and passing through the lowest point of his circular motion is moving at 3.00 m/s and is gaining speed at a rate of 0.500 m/s^2. (a) Find the magnitude and the direction of the passenger's acceleration at this instant. (b) Sketch the Ferris wheel and the passenger, showing his velocity and acceleration vectors.

3.35. Hypergravity. At its Ames Research Center, NASA uses its large "20-G" centrifuge to test the effects of very large accelerations ("hypergravity") on test pilots and astronauts. In this device, an arm 8.84 m long rotates about one end in a horizontal plane, and the astronaut is strapped in at the other end. Suppose that he is aligned along the arm with his head at the outermost end. The maximum sustained acceleration to which humans are subjected in this machine is typically $12.5g$. (a) How fast must the astronaut's head be moving to experience this maximum acceleration? (b) What is the *difference* between the acceleration of his head and feet if the astronaut is 2.00 m tall? (c) How fast in rpm (rev/min) is the arm turning to produce the maximum sustained acceleration?

Section 3.5 Relative Velocity

3.36. A railroad flatcar is traveling to the right at a speed of 13.0 m/s relative to an observer standing on the ground. Someone is riding a motor scooter on the flatcar (Fig. 3.43). What is the velocity (magnitude and direction) of the motor scooter relative to the flatcar if its velocity relative to the observer on the ground is (a) 18.0 m/s to the right? (b) 3.0 m/s to the left? (c) zero?

Figure 3.43 Exercise 3.36.



3.37. A "moving sidewalk" in an airport terminal building moves at 1.0 m/s and is 35.0 m long. If a woman steps on at one end and walks at 1.5 m/s relative to the moving sidewalk, how much time does she require to reach the opposite end if she walks (a) in the same direction the sidewalk is moving? (b) In the opposite direction?

3.38. Two piers, A and B, are located on a river: B is 1500 m downstream from A (Fig. 3.44). Two friends must make round trips from pier A to pier B and return. One rows a boat at a constant speed of 4.00 km/h relative to the water; the other walks on the shore at a constant speed of 4.00 km/h. The velocity of the river is 2.80 km/h in the direction from A to B. How much time does it take each person to make the round trip?

Figure 3.44 Exercise 3.38.

3.39. A canoe has a velocity of 0.40 m/s southeast relative to the earth. The canoe is on a river that is flowing 0.50 m/s east relative to the earth. Find the velocity (magnitude and direction) of the canoe relative to the river.

3.40. An airplane pilot wishes to fly due west. A wind of 80.0 km/h (about 50 mi/h) is blowing toward the south. (a) If the airspeed of the plane (its speed in still air) is 320.0 km/h (about 200 mi/h), in which direction should the pilot head? (b) What is the speed of the plane over the ground? Illustrate with a vector diagram.

3.41. Crossing the River I. A river flows due south with a speed of 2.0 m/s. A man steers a motorboat across the river; his velocity relative to the water is 4.2 m/s due east. The river is 800 m wide. (a) What is his velocity (magnitude and direction) relative to the earth? (b) How much time is required to cross the river? (c) How far south of his starting point will he reach the opposite bank?

3.42. Crossing the River II. (a) In which direction should the motorboat in Exercise 3.41 head in order to reach a point on the opposite bank directly east from the starting point? (The boat's speed relative to the water remains 4.2 m/s.) (b) What is the velocity of the boat relative to the earth? (c) How much time is required to cross the river?

3.43. The nose of an ultralight plane is pointed south, and its airspeed indicator shows 35 m/s. The plane is in a 10-m/s wind blowing toward the southwest relative to the earth. (a) In a vector-addition diagram, show the relationship of $\vec{v}_{P/E}$ (the velocity of the plane relative to the earth) to the two given vectors. (b) Letting x be east and y be north, find the components of $\vec{v}_{P/E}$. (c) Find the magnitude and direction of $\vec{v}_{P/E}$.

Problems

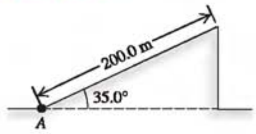
3.44. A faulty model rocket moves in the xy-plane (the positive y-direction is vertically upward). The rocket's acceleration has components $a_x(t) = \alpha t^2$ and $a_y(t) = \beta - \gamma t$, where $\alpha = 2.50\ \text{m/s}^4$, $\beta = 9.00\ \text{m/s}^2$, and $\gamma = 1.40\ \text{m/s}^3$. At $t = 0$ the rocket is at the origin and has velocity $\vec{v}_0 = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath}$ with $v_{0x} = 1.00$ m/s and $v_{0y} = 7.00$ m/s. (a) Calculate the velocity and position vectors as functions of time. (b) What is the maximum height reached by the rocket? (c) Sketch the path of the rocket. (d) What is the horizontal displacement of the rocket when it returns to $y = 0$?

3.45. A rocket is fired at an angle from the top of a tower of height $h_0 = 50.0$ m. Because of the design of the engines, its position coordinates are of the form $x(t) = A + Bt^2$ and $y(t) = C + Dt^3$, where A, B, C, and D are constants. Furthermore, the acceleration of the rocket 1.00 s after firing is $\vec{a} = (4.00\hat{\imath} + 3.00\hat{\jmath})\ \text{m/s}^2$. Take the origin of coordinates to be at the base of the tower. (a) Find the constants A, B, C, and D, including their SI units. (b) At the instant after the rocket is fired, what are its acceleration vector and its velocity? (c) What are the x- and y-components of the rocket's velocity 10.0 s after it is fired, and how fast is it moving? (d) What is the position vector of the rocket 10.0 s after it is fired?

3.46. A bird flies in the xy-plane with a velocity vector given by $\vec{v} = (\alpha - \beta t^2)\hat{\imath} + \gamma t\hat{\jmath}$, with $\alpha = 2.4$ m/s, $\beta = 1.6\ \text{m/s}^3$, and $\gamma = 4.0\ \text{m/s}^2$. The positive y-direction is vertically upward. At $t = 0$ the bird is at the origin. (a) Calculate the position and acceleration vectors of the bird as functions of time. (b) What is the bird's altitude (y-coordinate) as it flies over $x = 0$ for the first time after $t = 0$?

3.47. A test rocket is launched by accelerating it along a 200.0-m incline at 1.25 m/s² starting from rest at point A (Figure 3.45.) The incline rises at 35.0° above the horizontal, and at the instant the rocket leaves it, its engines turn off and it is subject only to gravity (air resistance can be ignored). Find (a) the maximum height above the ground that the rocket reaches, and (b) the greatest horizontal range of the rocket beyond point A.

Figure 3.45 Problem 3.47.



3.48. Martian Athletics. In the long jump, an athlete launches herself at an angle above the ground and lands at the same height, trying to travel the greatest horizontal distance. Suppose that on earth she is in the air for time T, reaches a maximum height h, and achieves a horizontal distance D. If she jumped in *exactly* the same way during a competition on Mars, where g_{Mars} is 0.379 of its earth value, find her time in the air, maximum height, and horizontal distance. Express each of these three quantities in terms of its earth value. Air resistance can be neglected on both planets.

3.49. Dynamite! A demolition crew uses dynamite to blow an old building apart. Debris from the explosion flies off in all directions and is later found at distances as far as 50 m from the explosion. Estimate the maximum speed at which debris was blown outward by the explosion. Describe any assumptions that you make.

3.50. Spiraling Up. It is common to see birds of prey rising upward on thermals. The paths they take may be spiral-like. You can model the spiral motion as uniform circular motion combined with a constant upward velocity. Assume a bird completes a circle of radius 8.00 m every 5.00 s and rises vertically at a rate of 3.00 m/s. Determine: (a) the speed of the bird relative to the ground; (b) the bird's acceleration (magnitude and direction); and (c) the angle between the bird's velocity vector and the horizontal.

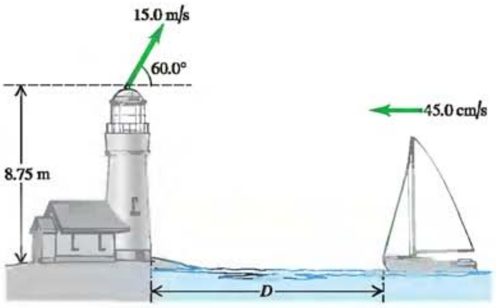
3.51. A jungle veterinarian with a blow-gun loaded with a tranquilizer dart and a sly 1.5-kg monkey are each 25 m above the ground in trees 90 m apart. Just as the hunter shoots horizontally at the monkey, the monkey drops from the tree in a vain attempt to escape being hit. What must the minimum muzzle velocity of the dart have been for the hunter to hit the monkey before it reached the ground?

3.52. A movie stuntwoman drops from a helicopter that is 30.0 m above the ground and moving with a constant velocity whose components are 10.0 m/s upward and 15.0 m/s horizontal and toward the south. You can ignore air resistance. (a) Where on the ground (relative to the position of the helicopter when she drops) should the stuntwoman have placed the foam mats that break her fall? (b) Draw x-t, y-t, v_x-t, and v_y-t graphs of her motion.

3.53. In fighting forest fires, airplanes work in support of ground crews by dropping water on the fires. A pilot is practicing by dropping a canister of red dye, hoping to hit a target on the ground below. If the plane is flying in a horizontal path 90.0 m above the ground and with a speed of 64.0 m/s (143 mi/h), at what horizontal distance from the target should the pilot release the canister? Ignore air resistance.

3.54. As a ship is approaching the dock at 45.0 cm/s, an important piece of landing equipment needs to be thrown to it before it can dock. This equipment is thrown at 15.0 m/s at 60.0° above the horizontal from the top of a tower at the edge of the water, 8.75 m above the ship's deck (Fig. 3.46.) For this equipment to land at the front of the ship, at what distance D from the dock should the ship be when the equipment is thrown? Air resistance can be neglected.

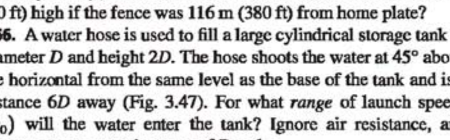
Figure 3.46 Problem 3.54.



3.55. The Longest Home Run. According to the *Guinness Book of World Records*, the longest home run ever measured was hit by Roy "Dizzy" Carlyle in a minor league game. The ball traveled 188 m (618 ft) before landing on the ground outside the ballpark. (a) Assuming the ball's initial velocity was 45° above the horizontal and ignoring air resistance, what did the initial speed of the ball need to be to produce such a home run if the ball was hit at a point 0.9 m (3.0 ft) above ground level? Assume that the ground was perfectly flat. (b) How far would the ball be above a fence 3.0 m (10 ft) high if the fence was 116 m (380 ft) from home plate?

3.56. A water hose is used to fill a large cylindrical storage tank of diameter D and height $2D$. The hose shoots the water at 45° above the horizontal from the same level as the base of the tank and is a distance $6D$ away (Fig. 3.47). For what *range* of launch speeds (v_0) will the water enter the tank? Ignore air resistance, and express your answer in terms of D and g.

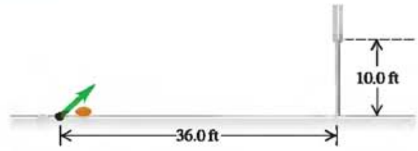
Figure 3.47 Problem 3.56.



3.57. A projectile is being launched from ground level with no air resistance. You want to avoid having it enter a temperature inversion layer in the atmosphere a height h above the ground. (a) What is the maximum launch speed you could give this projectile if you shot it straight up? Express your answer in terms of h and g. (b) Suppose the launcher available shoots projectiles at twice the maximum launch speed you found in part (a). At what maximum angle above the horizontal should you launch the projectile? (c) How far (in terms of h) from the launcher does the projectile in part (b) land?

3.50. Kicking a Field Goal. In U.S. football, after a touchdown the team has the opportunity to earn one more point by kicking the ball over the bar between the goal posts. The bar is 10.0 ft above the ground, and the ball is kicked from ground level, 36.0 ft horizontally from the bar (Fig. 3.48). Football regulations are stated in English units, but convert to SI units for this problem. (a) There is a minimum angle above the ground such that if the ball is launched below this angle, it can never clear the bar, no matter how fast it is kicked. What is this angle? (b) If the ball is kicked at 45.0° above the horizontal, what must its initial speed be if it to just clear the bar? Express your answer in m/s and km/h.

Figure 3.48 Problem 3.58.



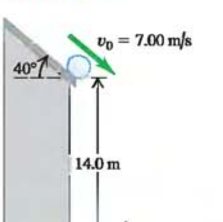
3.59. A projectile is launched with speed v_0 at an angle α_0 above the horizontal. The launch point is a height h above the ground. (a) Show that if air resistance is ignored, the horizontal distance that the projectile travels before striking the ground is

$$x = \frac{v_0 \cos\alpha_0}{g}\left(v_0 \sin\alpha_0 + \sqrt{v_0^2 \sin^2\alpha_0 + 2gh}\right)$$

Verify that if the launch point is at ground level so that $h = 0$, this is equal to the horizontal range R found in Example 3.8. (b) For the case where $v_0 = 10$ m/s and $h = 5.0$ m, graph x as a function of launch angle α_0 for values of α_0 from 0° to 90°. Your graph should show that x is zero if $\alpha_0 = 90°$, but x is nonzero if $\alpha_0 = 0$; explain why this is so. (c) We saw in Example 3.8 that for a projectile that lands at the same height from which it is launched, the horizontal range is maximum for $\alpha_0 = 45°$. For the case graphed in part (b), is the angle for maximum horizontal distance equal to, less than, or greater than 45°? (This is a general result for the situation where a projectile is launched from a point higher than where it lands.)

3.60. Look Out! A snowball rolls off a barn roof that slopes downward at an angle of 40° (Fig. 3.49). The edge of the roof is 14.0 m above the ground, and the snowball has a speed of 7.00 m/s as it rolls off the roof. Ignore air resistance. (a) How far from the edge of the barn does the snowball strike the ground if it doesn't strike anything else while falling? (b) Draw x-t, y-t, v_x-t, and v_y-t graphs for the motion in part (a). (c) A man 1.9 m tall is standing 4.0 m from the edge of the barn. Will he be hit by the snowball?

Figure 3.49 Problem 3.60.



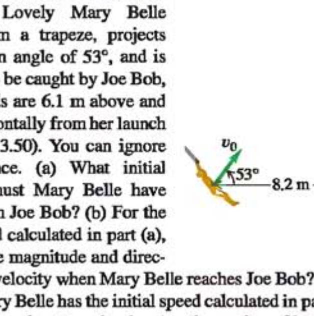
3.61. (a) Prove that a projectile launched at angle α_0 has the same horizontal range as one launched with the same speed at angle $(90° - \alpha_0)$. (b) A frog jumps at a speed of 2.2 m/s and lands 25 cm from its starting point. At which angles above the horizontal could it have jumped?

3.62. On the Flying Trapeze. A new circus act is called the Texas Tumblers. Lovely Mary Belle swings from a trapeze, projects herself at an angle of 53°, and is supposed to be caught by Joe Bob, whose hands are 6.1 m above and 8.2 m horizontally from her launch point (Fig. 3.50). You can ignore air resistance. (a) What initial speed v_0 must Mary Belle have just to reach Joe Bob? (b) For the initial speed calculated in part (a), what are the magnitude and direction of her velocity when Mary Belle reaches Joe Bob? (c) Assuming that Mary Belle has the initial speed calculated in part (a), draw x-t, y-t, v_x-t, and v_y-t graphs showing the motion of both tumblers. Your graphs should show the motion up until the point where Mary Belle reaches Joe Bob. (d) The night of their debut performance, Joe Bob misses her completely as she flies past. How far horizontally does Mary Belle travel, from her initial launch point, before landing in the safety net 8.6 m below her starting point?

Figure 3.50 Problem 3.62.

3.63. Leaping the River II. A physics professor did daredevil stunts in his spare time. His last stunt was an attempt to jump across a river on a motorcycle (Fig. 3.51). The takeoff ramp was inclined at 53.0°, the river was 40.0 m wide, and the far bank was 15.0 m lower than the top of the ramp. The river itself was 100 m below the ramp. You can ignore air resistance. (a) What should his speed have been at the top of the ramp to have just made it to the edge of the far bank? (b) If his speed was only half the value found in (a), where did he land?

Figure 3.51 Problem 3.63.



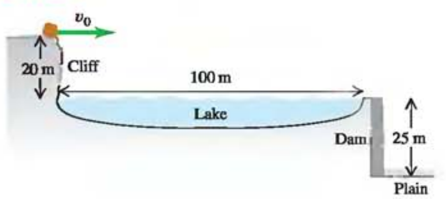
3.64. A rock is thrown from the roof of a building with a velocity v_0 at an angle of α_0 from the horizontal. The building has height h. You can ignore air resistance. Calculate the magnitude of the velocity of the rock just before it strikes the ground, and show that this speed is independent of α_0.

3.65. A 5500-kg cart carrying a vertical rocket launcher moves to the right at a constant speed of 30.0 m/s along a horizontal track. It launches a 45.0-kg rocket vertically upward with an initial speed of 40.0 m/s relative to the cart. (a) How high will the rocket go? (b) Where, relative to the cart, will the rocket land? (c) How far does the cart move while the rocket is in the air? (d) At what angle, relative to the horizontal, is the rocket traveling just as it leaves the cart, as measured by an observer at rest on the ground? (e) Sketch the rocket's trajectory as seen by an observer (i) stationary on the cart and (ii) stationary on the ground.

3.66. A 2.7-kg ball is thrown upward with an initial speed of 20.0 m/s from the edge of a 45.0-m-high cliff. At the instant the ball is thrown, a woman starts running away from the base of the cliff with a constant speed of 6.00 m/s. The woman runs in a straight line on level ground, and air resistance acting on the ball can be ignored. (a) At what angle above the horizontal should the ball be thrown so that the runner will catch it just before it hits the ground, and how far does the woman run before she catches the ball? (b) Carefully sketch the ball's trajectory as viewed by (i) a person at rest on the ground and (ii) the runner.

3.67. A 76.0-kg boulder is rolling horizontally at the top of a vertical cliff that is 20 m above the surface of a lake, as shown in Fig. 3.52. The top of the vertical face of a dam is located 100 m from the foot of the cliff, with the top of the dam level with the surface of the water in the lake. A level plain is 25 m below the top of the dam. (a) What must be the minimum speed of the rock just as it leaves the cliff so it will travel to the plain without striking the dam? (b) How far from the foot of the dam does the rock hit the plain?

Figure 3.52 Problem 3.67.



3.60. Tossing Your Lunch. Henrietta is going off to her physics class, jogging down the sidewalk at 3.05 m/s. Her husband Bruce suddenly realizes that she left in such a hurry that she forgot her lunch of bagels, so he runs to the window of their apartment, which is 43.9 m above the street level and directly above the sidewalk, to throw them to her. Bruce throws them horizontally 9.00 s after Henrietta has passed below the window, and she catches them on the run. You can ignore air resistance. (a) With what initial speed must Bruce throw the bagels so Henrietta can catch them just before they hit the ground? (b) Where is Henrietta when she catches the bagels?

3.69. Two tanks are engaged in a training exercise on level ground. The first tank fires a paint-filled training round with a muzzle speed of 250 m/s at 10.0° above the horizontal while advancing toward the second tank with a speed of 15.0 m/s relative to the ground. The second tank is retreating at 35.0 m/s relative to the ground, but is hit by the shell. You can ignore air resistance and assume the shell hits at the same height above ground from which it was fired. Find the distance between the tanks (a) when the round was first fired and (b) at the time of impact.

3.70. Bang! A student sits atop a platform a distance h above the ground. He throws a large firecracker horizontally with a speed v. However, a wind blowing parallel to the ground gives the firecracker a constant horizontal acceleration with magnitude a. This results in the firecracker reaching the ground directly under the student. Determine the height h in terms of v, a, and g. You can ignore the effect of air resistance on the vertical motion.

3.71. A rocket is launched vertically from rest with a constant upward acceleration of $1.75\ \text{m/s}^2$. Suddenly 22.0 s after launch, an unneeded fuel tank is jettisoned by shooting it away from the rocket. A crew member riding in the rocket measures that the initial speed of the tank is 25.0 m/s and that it moves perpendicular to the rocket's path. The fuel tank feels no appreciable air resistance and feels only the force of gravity once it leaves the rocket. (a) How fast is the rocket moving at the instant the fuel tank is jettisoned? (b) What are the horizontal and vertical components of the fuel tank's velocity just as it is jettisoned as measured by (i) a crew member in the rocket and (ii) a technician standing on the ground? (c) At what angle with respect to the horizontal does the jettisoned fuel tank initially move, as measured by (i) a crew member in the rocket and (ii) a technician standing on the ground? (d) What maximum height above the launch pad does the jettisoned tank reach?

3.72. When it is 145 m above the ground, a rocket traveling vertically upward at a constant 8.50 m/s relative to the ground launches a secondary rocket at a speed of 12.0 m/s at an angle of 53.0° above the horizontal, both quantities being measured by an astronaut sitting in the rocket. Air resistance is too small to worry about. (a) Just as the secondary rocket is launched, what are the horizontal and vertical components of its velocity relative to (i) the astronaut sitting in the rocket and (ii) Mission Control on the ground? (b) Find the initial speed and launch angle of the secondary rocket as measured by Mission Control. (c) What maximum height above the ground does the secondary rocket reach?

3.73. In a Fourth of July celebration, a firecracker is launched from ground level with an initial velocity of 25.0 m/s at 30.0° from the *vertical*. At its maximum height it explodes in a starburst into many fragments, two of which travel forward initially at 20.0 m/s at ±53.0° with respect to the horizontal, both quantities measured *relative to the original firecracker just before it exploded*. With what angles with respect to the horizontal do the two fragments initially move right after the explosion, as measured by a spectator standing on the ground?

3.74. In an action-adventure film, the hero is supposed to throw a grenade from his car, which is going 90.0 km/h, to his enemy's car, which is going 110 km/h. The enemy's car is 15.8 m in front of the hero's when he lets go of the grenade. If the hero throws the grenade so its initial velocity relative to him is at an angle of 45° above the horizontal, what should the magnitude of the initial velocity be? The cars are both traveling in the same direction on a level road. You can ignore air resistance. Find the magnitude of the velocity both relative to the hero and relative to the earth.

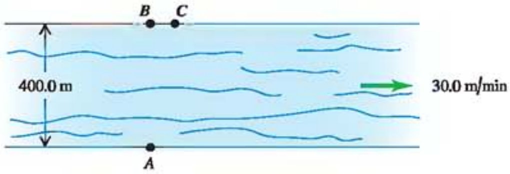
3.75. A rock tied to a rope moves in the xy-plane. Its coordinates are given as functions of time by

$$x(t) = R\cos\omega t \qquad y(t) = R\sin\omega t$$

where R and ω are constants. (a) Show that the rock's distance from the origin is constant and equal to R—that is, that its path is a circle of radius R. (b) Show that at every point the rock's velocity is perpendicular to its position vector. (c) Show that the rock's acceleration is always opposite in direction to its position vector and has magnitude $\omega^2 R$. (d) Show that the magnitude of the rock's velocity is constant and equal to ωR. (e) Combine the results of parts (c) and (d) to show that the rock's acceleration has constant magnitude v^2/R.

3.76. A 400.0-m-wide river flows from west to east at 30.0 m/min. Your boat moves at 100.0 m/min relative to the water no matter which direction you point it. To cross this river, you start from a dock at point A on the south bank. There is a boat landing directly opposite at point B on the north bank, and also one at point C, 75.0 m downstream from B (Fig. 3.53). (a) Where on the north shore will you land if you point your boat perpendicular to the water current, and what distance will you have traveled? (b) If you initially aim your boat directly toward point C and do not change that bearing relative to the shore, where on the north shore will you land? (c) To reach point C: (i) at what bearing must you aim your boat, (ii) how long will it take to cross the river, (iii) what distance do you travel, and (iv) and what is the speed of your boat as measured by an observer standing on the river bank?

Figure 3.53 Problem 3.76.



3.77. Cycloid. A particle moves in the xy-plane. Its coordinates are given as functions of time by

$$x(t) = R(\omega t - \sin\omega t) \qquad y(t) = R(1 - \cos\omega t)$$

where R and ω are constants. (a) Sketch the trajectory of the particle. (This is the trajectory of a point on the rim of a wheel that is rolling at a constant speed on a horizontal surface. The curve traced out by such a point as it moves through space is called a *cycloid*.) (b) Determine the velocity components and the acceleration components of the particle at any time t. (c) At which times is the particle momentarily at rest? What are the coordinates of the particle at these times? What are the magnitude and direction of the acceleration at these times? (d) Does the magnitude of the acceleration depend on time? Compare to uniform circular motion.

3.78. A projectile is fired from point A at an angle above the horizontal. At its highest point, after having traveled a horizontal distance D from its launch point, it suddenly explodes into two identical fragments that travel horizontally with equal but opposite velocities as measured *relative to the projectile just before it exploded*. If one fragment lands back at point A, how far from A (in terms of D) does the other fragment land?

3.79. Centrifuge on Mercury. A laboratory centrifuge on earth makes n rpm (rev/min) and produces an acceleration of $5.00g$ at its outer end. (a) What is the acceleration (in g's) at a point halfway out to the end? (b) This centrifuge is now used in a space capsule on the planet Mercury, where g_{Mercury} is 0.378 what it is on earth. How many rpm (in terms of n) should it make to produce $5g_{\text{Mercury}}$ at its outer end?

3.80. Raindrops. When a train's velocity is 12.0 m/s eastward, raindrops that are falling vertically with respect to the earth make traces that are inclined 30.0° to the vertical on the windows of the train. (a) What is the horizontal component of a drop's velocity with respect to the earth? With respect to the train? (b) What is the magnitude of the velocity of the raindrop with respect to the earth? With respect to the train?

3.81. An airplane pilot sets a compass course due west and maintains an airspeed of 220 km/h. After flying for 0.500 h, she finds herself over a town 120 km west and 20 km south of her starting point. (a) Find the wind velocity (magnitude and direction). (b) If the wind velocity is 40 km/h due south, in what direction should the pilot set her course to travel due west? Use the same airspeed of 220 km/h.

3.82. An elevator is moving upward at a constant speed of 2.50 m/s. A bolt in the elevator ceiling 3.00 m above the elevator floor works loose and falls. (a) How long does it take for the bolt to fall to the elevator floor? What is the speed of the bolt just as it hits the elevator floor (b) according to an observer in the elevator? (c) According to an observer standing on one of the floor landings of the building? (d) According to the observer in part (c), what distance did the bolt travel between the ceiling and the floor of the elevator?

3.83. Suppose the elevator in Problem 3.82 starts from rest and maintains a constant upward acceleration of 4.00 m/s², and the bolt falls out the instant the elevator begins to move. (a) How long does it take for the bolt to reach the floor of the elevator? (b) Just as it reaches the floor, how fast is the bolt moving according to an observer (i) in the elevator? (ii) Standing on the floor landings of the building? (c) According to each observer in part (b), how far has the bolt traveled between the ceiling and floor of the elevator?

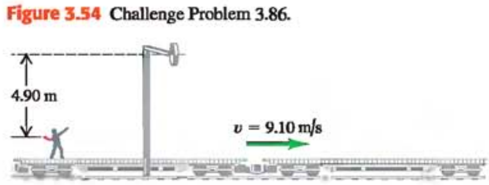
3.84. City *A* lies directly west of city *B*. When there is no wind, an airliner makes the 5550-km round-trip flight between them in 6.60 h of flying time while traveling at the same speed in both directions. When a strong, steady 225-km/h wind is blowing from west to east and the airliner has the same airspeed as before, how long will the trip take?

3.85. In a World Cup soccer match, Juan is running due north toward the goal with a speed of 8.00 m/s relative to the ground. A teammate passes the ball to him. The ball has a speed of 12.0 m/s and is moving in a direction of 37.0° east of north, relative to the ground. What are the magnitude and direction of the ball's velocity relative to Juan?

Challenge Problems

3.86. A man is riding on a flatcar traveling at a constant speed of 9.10 m/s (Fig. 3.54). He wishes to throw a ball through a stationary hoop 4.90 m above the height of his hands in such a manner that the ball will move horizontally as it passes through the hoop. He throws the ball with a speed of 10.8 m/s with respect to himself. (a) What must the vertical component of the initial velocity of the ball be? (b) How many seconds after he releases the ball will it pass through the hoop? (c) At what horizontal distance in front of the hoop must he release the ball? (d) When the ball leaves the man's hands, what is the direction of its velocity relative to the frame of reference of the flatcar? Relative to the frame of reference of an observer standing on the ground?

Figure 3.54 Challenge Problem 3.86.

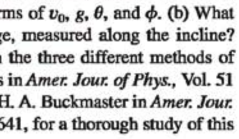


3.87. A shotgun fires a large number of pellets upward, with some pellets traveling very nearly vertically and others as much as 1.0° from the vertical. Assume that the initial speed of the pellets is uniformly 150 m/s, and ignore air resistance. (a) Within what radius from the point of firing will the pellets land? (b) If there are 1000 pellets, and they fall in a uniform distribution over a circle with the radius calculated in part (a), what is the probability that at least one pellet will fall on the head of the person who fires the shotgun? Assume that his head has a radius of 10 cm. (c) Air resistance, in fact, has several effects. It slows down the rising pellets, decreases their horizontal component of velocity, and limits the speed with which they fall. Which of these effects will tend to make the radius larger than calculated in part (a), and which will tend to make it smaller? What do you think the overall effect of air resistance will be? (The effect of air resistance on a velocity component increases as the magnitude of the component increases.)

3.88. A projectile is thrown from a point *P*. It moves in such a way that its distance from *P* is always increasing. Find the maximum angle above the horizontal with which the projectile could have been thrown. You can ignore air resistance.

3.89. Projectile Motion on an Incline I. A baseball is given an initial velocity with magnitude v_0 at an angle ϕ above the surface of an incline, which is in turn inclined at an angle θ above the horizontal (Fig. 3.55) (a) Calculate the distance, measured along the incline, from the launch point to where the baseball strikes the incline. Your answer will be in terms of v_0, g, θ, and ϕ. (b) What angle ϕ gives the maximum range, measured along the incline? (*Note:* You might be interested in the three different methods of solution presented by I. R. Lapidus in *Amer. Jour. of Phys.*, Vol. 51 (1983), pp. 806 and 847. See also H. A. Buckmaster in *Amer. Jour. of Phys.*, Vol. 53 (1985), pp. 638–641, for a thorough study of this and some similar problems.)

Figure 3.55 Challenge Problem 3.89.



3.90. Projectile Motion on an Incline II. Refer to Challenge Problem 3.89. (a) An archer on ground that has a constant upward slope of 30.0° aims at a target 60.0 m farther up the incline. The arrow in the bow and the bull's-eye at the center of the target are each 1.50 m above the ground. The initial velocity of the arrow just after it leaves the bow has magnitude 32.0 m/s. At what angle above the *horizontal* should the archer aim to hit the bull's-eye? If there are two such angles, calculate the smaller of the two. You might have to solve the equation for the angle by iteration—that is, by trial and error. How does the angle compare to that required when the ground is level, with 0 slope? (b) Repeat the above for ground that has a constant *downward* slope of 30.0°.

3.91. For no apparent reason, a poodle is running at a constant speed of $v = 5.00$ m/s in a circle with radius $R = 2.50$ m. Let $\vec{v}_1$ be the velocity vector at time t_1, and let $\vec{v}_2$ be the velocity vector at time t_2. Consider $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ and $\Delta t = t_2 - t_1$. Recall that $\vec{a}_{av} = \Delta\vec{v}/\Delta t$. For $\Delta t = 0.5$ s, 0.1 s, and 0.05 s, calculate the magnitude (to four significant figures) and direction (relative to $\vec{v}_1$) of the average acceleration $\vec{a}_{av}$. Compare your results to the general expression for the instantaneous acceleration $\vec{a}$ for uniform circular motion that is derived in the text.

3.92. A rocket designed to place small payloads into orbit is carried to an altitude of 12.0 km above sea level by a converted airliner. When the airliner is flying in a straight line at a constant speed of 850 km/h, the rocket is dropped. After the drop, the airliner maintains the same altitude and speed and continues to fly in a straight line. The rocket falls for a brief time, after which its rocket motor turns on. Once its rocket motor is on, the combined effects of thrust and gravity give the rocket a constant acceleration of magnitude 3.00*g* directed at an angle of 30.0° above the horizontal. For reasons of safety, the rocket should be at least 1.00 km in front of the airliner when it climbs through the airliner's altitude. Your job is to determine the minimum time that the rocket must fall before its engine starts. You can ignore air resistance.

Your answer should include (i) a diagram showing the flight paths of both the rocket and the airliner, labeled at several points with vectors for their velocities and accelerations; (ii) an x-t graph showing the motions of both the rocket and the airliner; and (iii) a y-t graph showing the motions of both the rocket and the airliner. In the diagram and the graphs, indicate when the rocket is dropped, when the rocket motor turns on, and when the rocket climbs through the altitude of the airliner.

3.93. Two students are canoeing on a river. While heading upstream, they accidentally drop an empty bottle overboard. They then continue paddling for 60 minutes, reaching a point 2.0 km farther upstream. At this point they realize that the bottle is missing and, driven by ecological awareness, they turn around and head downstream. They catch up with and retrieve the bottle (which has been moving along with the current) 5.0 km downstream from the turn-around point. (a) Assuming a constant paddling effort throughout, how fast is the river flowing? (b) What would the canoe speed in a still lake be for the same paddling effort?

NEWTON'S LAWS OF MOTION

4

? The standing child is pushing the child seated on the swing. Is the seated child pushing back? If so, is he pushing with the same amount of force or a different amount?

LEARNING GOALS

By studying this chapter, you will learn:

- What the concept of force means in physics, and why forces are vectors.
- The significance of the net force on an object, and what happens when the net force is zero.
- The relationship among the net force on an object, the object's mass, and its acceleration.
- How the forces that two bodies exert on each other are related.

We've seen in the last two chapters how to describe motion in one, two, or three dimensions. But what are the underlying *causes* of motion? For example, how can a tugboat push a cruise ship that's much heavier than the tug? Why is it harder to control a car on wet ice than on dry concrete? The answers to these and similar questions take us into the subject of **dynamics,** the relationship of motion to the forces that cause it. In the two preceding chapters we studied *kinematics,* the language for *describing* motion. Now we are ready to think about what makes bodies move the way they do.

In this chapter we will use two new concepts, *force* and *mass,* to analyze the principles of dynamics. These principles can be wrapped up in just three statements that were clearly stated for the first time by Sir Isaac Newton (1642–1727), who published them in 1687 in his *Philosophiae Naturalis Principia Mathematica* ("Mathematical Principles of Natural Philosophy"). These three statements are called **Newton's laws of motion.** The first law states that when the net force on a body is zero, its motion doesn't change. The second law relates force to acceleration when the net force is *not* zero. The third law is a relationship between the forces that two interacting bodies exert on each other.

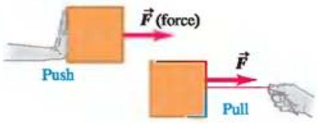
Newton's laws are not the product of mathematical derivations, but rather a synthesis of what physicists have learned from a multitude of *experiments* about how objects move. (Newton used the ideas and observations of many scientists before him, including Copernicus, Brahe, Kepler, and especially Galileo Galilei, who died the same year Newton was born.) These laws are truly fundamental, for they cannot be deduced or proved from other principles. Newton's laws are the foundation of **classical mechanics** (also called **Newtonian mechanics**); using them we can understand most familiar kinds of motion. Newton's laws need modification only for situations involving extremely high speeds (near the speed of light) or very small sizes (such as within the atom).

Newton's laws are very simple to state, yet many students find these laws difficult to grasp and to work with. The reason is that before studying physics, you've spent years walking, throwing balls, pushing boxes, and doing dozens of things that involve motion. Along the way, you've developed a set of "common sense" ideas about motion and its causes. But many of these "common sense" ideas don't stand up to logical analysis. A big part of the job of this chapter—and of the rest of our study of physics—is helping you to recognize how "common sense" ideas can sometimes lead you astray, and how to adjust your understanding of the physical world to make it consistent with what experiments tell us.

4.1 Force and Interactions

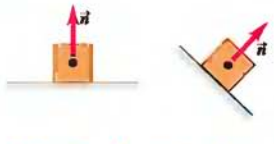
4.1 Some properties of forces.

- A force is a push or a pull.
- A force is an interaction between two objects or between an object and its environment.
- A force is a vector quantity, with magnitude and direction.

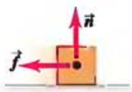


4.2 Four common types of forces.

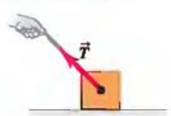
(a) Normal force $\vec{n}$: When an object rests or pushes on a surface, the surface exerts a push on it that is directed perpendicular to the surface.



(b) Friction force $\vec{f}$: In addition to the normal force, a surface may exert a frictional force on an object, directed parallel to the surface.



(c) Tension force $\vec{T}$: A pulling force exerted on an object by a rope, cord, etc.



(d) Weight $\vec{w}$: The pull of gravity on an object is a long-range force (a force that acts over a distance).

In everyday language, a **force** is a push or a pull. A better definition is that a force is an *interaction* between two bodies or between a body and its environment (Fig. 4.1). That's why we always refer to the force that one body *exerts* on a second body. When you push on a car that is stuck in the suow, you exert a force on the car; a steel cable exerts a force on the beam it is hoisting at a construction site; and so on. As Fig. 4.1 shows, force is a *vector* quantity; you can push or pull a body in different directions.

When a force involves direct contact between two bodies, such as a push or pull that you exert on an object with your hand, we call it a **contact force.** Figures 4.2a, 4.2b, and 4.2c show three common types of contact forces. The **normal force** (Fig. 4.2a) is exerted on an object by any surface with which it is contact. The adjective *normal* means that the force always acts perpendicular to the surface of contact, no matter what the angle of that surface. By contrast, the **friction force** (Fig. 4.2b) exerted on an object by a surface acts *parallel* to the surface, in the direction that opposes sliding. The pulling force exerted by a stretched rope or cord on an object to which it's attached is called a **tension force** (Fig. 4.2c). When you tug on your dog's leash, the force that pulls on her collar is a tension force.

In addition to contact forces, there are **long-range forces** that act even when the bodies are separated by empty space. The force between two magnets is an example of a long-range force, as is the force of gravity (Fig. 4.2d); the earth pulls a dropped object toward it even though there is no direct contact between the object and the earth. The gravitational force that the earth exerts on your body is called your **weight.**

To describe a force vector $\vec{F}$, we need to describe the *direction* in which it acts as well as its *magnitude*, the quantity that describes "how much" or "how hard" the force pushes or pulls. The SI unit of the magnitude of force is the *newton*, abbreviated N. (We'll give a precise definition of the newton in Section 4.3.) Table 4.1 lists some typical force magnitudes.

Table 4.1 Typical Force Magnitudes

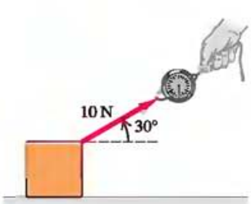
Sun's gravitational force on the earth	3.5×10^{22} N
Thrust of a space shuttle during launch	3.1×10^{7} N
Weight of a large blue whale	1.9×10^{6} N
Maximum pulling force of a locomotive	8.9×10^{5} N
Weight of a 250-lb linebacker	1.1×10^{3} N
Weight of a medium apple	1 N
Weight of smallest insect eggs	2×10^{-6} N
Electric attraction between the proton and the electron in a hydrogen atom	8.2×10^{-8} N
Weight of a very small bacterium	1×10^{-18} N
Weight of a hydrogen atom	1.6×10^{-26} N
Weight of an electron	8.9×10^{-30} N
Gravitational attraction between the proton and the electron in a hydrogen atom	3.6×10^{-47} N

A common instrument for measuring force magnitudes is the *spring balance*. It consists of a coil spring enclosed in a case with a pointer attached to one end. When forces are applied to the ends of the spring, it stretches by an amount that depends on the force. We can make a scale for the pointer by using a number of identical bodies with weights of exactly 1 N each. When one, two, or more of these are suspended simultaneously from the balance, the total force stretching the spring is 1 N, 2 N, and so on, and we can label the corresponding positions of the pointer 1 N, 2 N, and so on. Then we cau use this instrument to measure the magnitude of an unknown force. We can also make a similar instrument that measures pushes instead of pulls.

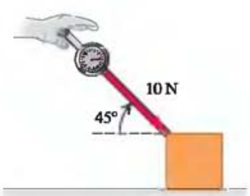
Figure 4.3 shows a spring balance being used to measure a pull or push that we apply to a box. In each case we draw a vector to represent the applied force. The labels indicate the magnitude and direction of the force. The length of the vector also shows the magnitude; the longer the vector, the greater the force magnitude.

4.3 Using a vector arrow to denote the force that we exert when **(a)** pulling a block with a string or **(b)** pushing a block with a stick.

(a) A 10-N pull directed 30° above the horizontal



(b) A 10-N push directed 45° below the horizontal



Superposition of Forces

When you throw a ball, there are at least two forces acting on it: the push of your hand and the downward pull of gravity. Experiment shows that when two forces $\vec{F}_1$ and $\vec{F}_2$ act at the same time at a point A of a body (Fig. 4.4), the effect on the body's motion is the same as if a single force $\vec{R}$ were acting equal to the *vector sum* of the original forces: $\vec{R} = \vec{F}_1 + \vec{F}_2$. More generally, *any number of forces applied at a point on a body have the same effect as a single force equal to the vector sum of the forces.* This important principle is called **superposition of forces.**

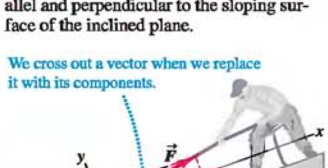
The experimental discovery that forces combine according to vector addition is of the utmost importance, and we will use this fact throughout our study of physics. It allows us to replace a force by its component vectors, as we did with displacements in Section 1.8. For example, in Fig. 4.5a, force $\vec{F}$ acts on a body at point O. The component vectors of $\vec{F}$ in the directions Ox and Oy are $\vec{F}_x$ and $\vec{F}_y$. When $\vec{F}_x$ and $\vec{F}_y$ are applied simultaneously, as in Fig. 4.5b, the effect is exactly the same as the effect of the original force $\vec{F}$. Hence *any force can be replaced by its component vectors, acting at the same point.*

It's frequently more convenient to describe a force $\vec{F}$ in terms of its x- and y-components F_x and F_y rather than by its component vectors (recall from Section 1.8 that *component vectors* are vectors, but *components* are just numbers). For the case shown in Fig. 4.5, both F_x and F_y are positive; for other orientations of the force $\vec{F}$, either F_x or F_y can be negative or zero.

There is no law that says our coordinate axes have to be vertical and horizontal. Figure 4.6 shows a crate being pulled up a ramp by a force $\vec{F}$, represented by its components F_x and F_y parallel and perpendicular to the sloping surface of the ramp.

4.4 Superposition of forces.

Two forces $\vec{F}_1$ and $\vec{F}_2$ acting on a point A have the same effect as a single force $\vec{R}$ equal to their vector sum.



4.5 The force $\vec{F}$, which acts at an angle θ from the x-axis, may be replaced by its rectangular component vectors $\vec{F}_x$ and $\vec{F}_y$.

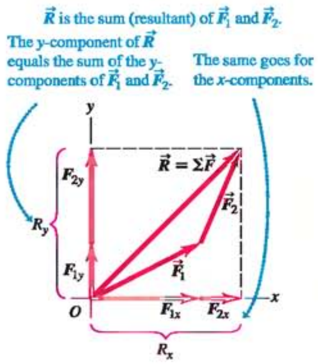
(a) Component vectors: $\vec{F}_x$ and $\vec{F}_y$
Components: $F_x = F\cos\theta$ and $F_y = F\sin\theta$

(b) Component vectors $\vec{F}_x$ and $\vec{F}_y$ together have the same effect as original force $\vec{F}$.

4.6 F_x and F_y are the components of $\vec{F}$ parallel and perpendicular to the sloping surface of the inclined plane.

We cross out a vector when we replace it with its components.

4.7 Finding the components of the vector sum (resultant) $\vec{R}$ of two forces $\vec{F}_1$ and $\vec{F}_2$.



CAUTION **Using a wiggly line in force diagrams** In Fig. 4.6 we draw a wiggly line through the force vector $\vec{F}$ to show that we have replaced it by its x- and y-components. Otherwise, the diagram would include the same force twice. We will draw such a wiggly line in any force diagram where a force is replaced by its components. Look for this wiggly line in other figures in this and subsequent chapters.

We will often need to find the vector sum (resultant) of *all* the forces acting on a body. We call this the **net force** acting on the body. We will use the Greek letter Σ (capital sigma, equivalent to the Roman *S*) as a shorthand notation for a sum. If the forces are labeled $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, and so on, we abbreviate the sum as

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum\vec{F} \tag{4.1}$$

We read $\sum\vec{F}$ as "the vector sum of the forces" or "the net force." The component version of Eq. (4.1) is the pair of component equations

$$R_x = \sum F_x \qquad R_y = \sum F_y \tag{4.2}$$

Here $\sum F_x$ is the sum of the x-components and $\sum F_y$ is the sum of the y-components (Fig. 4.7). Each component may be positive or negative, so be careful with signs when you evaluate the sums in Eq. (4.2).

Once we have R_x and R_y, we can find the magnitude and direction of the net force $\vec{R} = \sum\vec{F}$ acting on the body. The magnitude is

$$R = \sqrt{R_x^2 + R_y^2}$$

and the angle θ between $\vec{R}$ and the $+x$-axis can be found from the relation $\tan\theta = R_y/R_x$. The components R_x and R_y may be positive, negative, or zero, and the angle θ may be in any of the four quadrants.

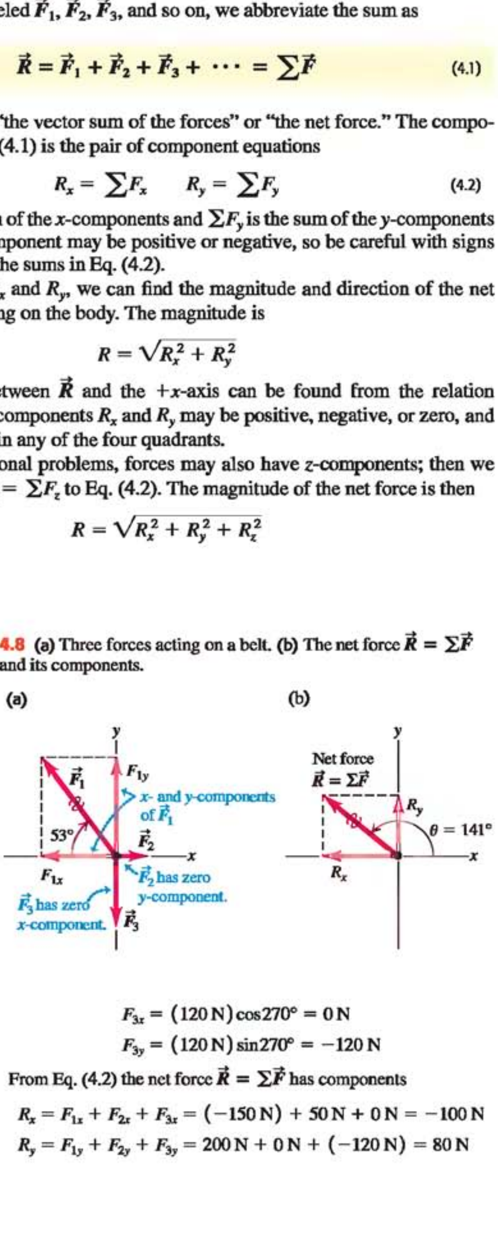
In three-dimensional problems, forces may also have *z*-components; then we add the equation $R_z = \sum F_z$ to Eq. (4.2). The magnitude of the net force is then

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

Example 4.1 Superposition of forces

Three professional wrestlers are fighting over the same champion's belt. As viewed from above, they apply the three horizontal forces to the belt that are shown in Fig. 4.8a. The magnitudes of the three forces are $F_1 = 250$ N, $F_2 = 50$ N, and $F_3 = 120$ N. Find the x- and y-components of the net force on the belt, and find the magnitude and direction of the net force.

4.8 (a) Three forces acting on a belt. (b) The net force $\vec{R} = \sum\vec{F}$ and its components.



SOLUTION

IDENTIFY: This example is just a problem in vector addition. The only new feature is that the vectors represent forces.

SET UP: We need to find the x- and y-components of the net force $\vec{R}$, so we'll use the component method of vector addition expressed by Eq. (4.2). Once we have the components of $\vec{R}$, we can find its magnitude and direction.

EXECUTE: From Fig. 4.8a, the angles between the three forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ and the $+x$-axis are $\theta_1 = 180° - 53° = 127°$, $\theta_2 = 0°$, and $\theta_3 = 270°$. The x- and y-components of the three forces are

$$F_{1x} = (250\text{ N})\cos 127° = -150\text{ N}$$
$$F_{1y} = (250\text{ N})\sin 127° = 200\text{ N}$$
$$F_{2x} = (50\text{ N})\cos 0° = 50\text{ N}$$
$$F_{2y} = (50\text{ N})\sin 0° = 0\text{ N}$$
$$F_{3x} = (120\text{ N})\cos 270° = 0\text{ N}$$
$$F_{3y} = (120\text{ N})\sin 270° = -120\text{ N}$$

From Eq. (4.2) the net force $\vec{R} = \sum\vec{F}$ has components

$$R_x = F_{1x} + F_{2x} + F_{3x} = (-150\text{ N}) + 50\text{ N} + 0\text{ N} = -100\text{ N}$$
$$R_y = F_{1y} + F_{2y} + F_{3y} = 200\text{ N} + 0\text{ N} + (-120\text{ N}) = 80\text{ N}$$

The net force has a negative x-component and a positive y-component, so it points to the left and toward the top of the page in Fig. 4.8b (that is, in the second quadrant).

The magnitude of the net force $\vec{R} = \Sigma\vec{F}$ is

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-100\ \text{N})^2 + (80\ \text{N})^2} = 128\ \text{N}$$

To find the angle between the net force and the $+x$-axis, we use the relation $\tan\theta = R_y/R_x$, or

$$\theta = \arctan\frac{R_y}{R_x} = \arctan\left(\frac{80\ \text{N}}{-100\ \text{N}}\right) = \arctan(-0.80)$$

The two possible solutions are $\theta = -39°$ and $\theta = -39° + 180° = 141°$. Since the net force lies in the second quadrant, as mentioned earlier, the correct answer is 141° (see Fig. 4.8b).

EVALUATE: In this situation the net force is *not* zero, and you can see intuitively that wrestler 1 (who exerts the largest force, $\vec{F}_1$, on the belt) is likely to walk away with the belt at the end of the struggle. In Section 4.2 we will explore in detail what happens in situations in which the net force *is* zero.

Test Your Understanding of Section 4.1 Figure 4.6 shows a force $\vec{F}$ acting on a crate. With the x- and y-axes shown in the figure, which statement about the components of the *gravitational* force that the earth exerts on the crate (the crate's weight) is *correct?* (i) The x- and y-components are both positive. (ii) The x-component is zero and the y-component is positive. (iii) The x-component is negative and the y-component is positive. (iv) The x- and y-components are both negative. (v) The x-component is zero and the y-component is negative. (vi) The x-component is positive and the y-component is negative.

4.2 Newton's First Law

We have discussed some of the properties of forces, but so far have said nothing about how forces affect motion. To begin, let's consider what happens when the net force on a body is *zero*. You would almost certainly agree that if a body is at rest, and if no net force acts on it (that is, no net push or pull), that body will remain at rest. But what if there is zero net force acting on a body in *motion?*

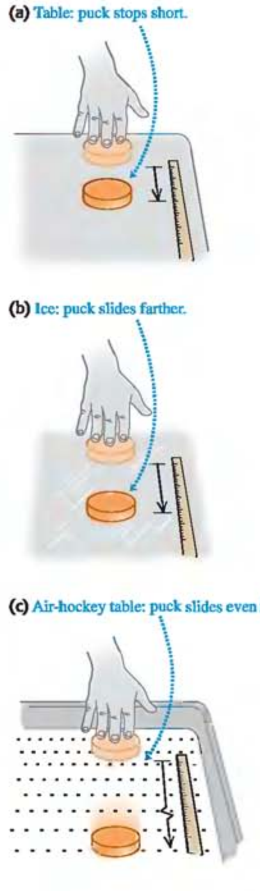
To see what happens in this case, suppose you slide a hockey puck along a horizontal tabletop, applying a horizontal force to it with your hand (Fig. 4.9a). After you stop pushing, the puck *does not* continue to move indefinitely; it slows down and stops. To keep it moving, you have to keep pushing (that is, applying a force). You might come to the "common sense" conclusions that bodies in motion naturally come to rest and that a force is required to sustain motion.

But now imagine pushing the puck across a smooth surface of ice (Fig. 4.9b). After you quit pushing, the puck will slide a lot farther before it stops. Put it on an air-hockey table, where it floats on a thin cushion of air, and it moves still farther (Fig. 4.9c). In each case, what slows the puck down is *friction,* an interaction between the lower surface of the puck and the surface on which it slides. Each surface exerts a frictional force on the puck that resists the puck's motion; the difference in the three cases is the magnitude of the frictional force. The ice exerts less friction than the tabletop, so the puck travels farther. The gas molecules of the air-hockey table exert the least friction of all. If we could eliminate friction completely, the puck would never slow down, and we would need no force at all to keep the puck moving once it had been started. Thus the "common sense" idea that a force is required to sustain motion is *incorrect.*

Experiments like the ones we've just described show that when no net force acts on a body, the body either remains at rest or moves with constant velocity in a straight line. Once a body has been set in motion, no net force is needed to keep it moving. We now call this observation *Newton's first law of motion:*

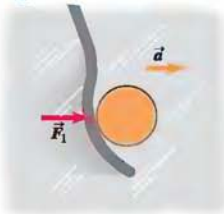
Newton's first law of motion: A body acted on by no net force moves with constant velocity (which may be zero) and zero acceleration.

4.9 The slicker the surface, the farther a puck slides after being given an initial velocity. On an air-hockey table (c) the friction force is practically zero, so the puck continues with almost constant velocity.

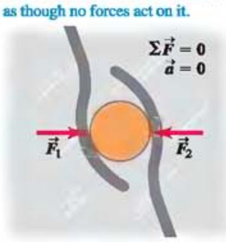


4.10 (a) A hockey puck accelerates in the direction of a net applied force $\vec{F}_1$. (b) When the net force is zero, the acceleration is zero, and the puck is in equilibrium.

(a) A puck on a frictionless surface accelerates when acted on by a single horizontal force.



(b) An object acted on by forces whose vector sum is zero behaves as though no forces act on it.



The tendency of a body to keep moving once it is set in motion results from a property called **inertia.** You use inertia when you try to get ketchup out of a bottle by shaking it. First you start the bottle (and the ketchup inside) moving forward; when you jerk the bottle back, the ketchup tends to keep moving forward and, you hope, ends up on your burger. The tendency of a body at rest to remain at rest is also due to inertia. You may have seen a tablecloth yanked out from under the china without breaking anything. The force on the china isn't great enough to make it move appreciably during the short time it takes to pull the tablecloth away.

It's important to note that the *net* force is what matters in Newton's first law. For example, a physics book at rest on a horizontal tabletop has two forces acting on it: an upward supporting force, or normal force, exerted by the tabletop (see Fig. 4.2a) and the downward force of the earth's gravitational attraction (a long-range force that acts even if the tabletop is elevated above the ground; see Fig. 4.2d). The upward push of the surface is just as great as the downward pull of gravity, so the *net* force acting on the book (that is, the vector sum of the two forces) is zero. In agreement with Newton's first law, if the book is at rest on the tabletop, it remains at rest. The same principle applies to a hockey puck sliding on a horizontal, frictionless surface: The vector sum of the upward push of the surface and the downward pull of gravity is zero. Once the puck is in motion, it continues to move with constant velocity because the *net* force acting on it is zero.

Here's another example. Suppose a hockey puck rests on a horizontal surface with negligible friction, such as an air-hockey table or a slab of wet ice. If the puck is initially at rest and a single horizontal force $\vec{F}_1$ acts on it (Fig. 4.10a), the puck starts to move. If the puck is in motion to begin with, the force changes its speed, its direction, or both, depending on the direction of the force. In this case the net force is equal to $\vec{F}_1$, which is *not* zero. (There are also two vertical forces: the earth's gravitational attraction and the upward normal force exerted by the surface. But as we mentioned earlier, these two forces cancel.)

Now suppose we apply a second force $\vec{F}_2$ (Fig. 4.10b), equal in magnitude to $\vec{F}_1$ but opposite in direction. The two forces are negatives of each other, $\vec{F}_2 = -\vec{F}_1$, and their vector sum is zero:

$$\sum\vec{F} = \vec{F}_1 + \vec{F}_2 = \vec{F}_1 + (-\vec{F}_1) = \mathbf{0}$$

Again, we find that if the body is at rest at the start, it remains at rest; if it is initially moving, it continues to move in the same direction with constant speed. These results show that in Newton's first law, *zero net force is equivalent to no force at all.* This is just the principle of superposition of forces that we saw in Section 4.1.

When a body is either at rest or moving with constant velocity (in a straight line with constant speed), we say that the body is in **equilibrium.** For a body to be in equilibrium, it must be acted on by no forces, or by several forces such that their vector sum—that is, the net force—is zero:

$$\sum\vec{F} = \mathbf{0} \quad \text{(body in equilibrium)} \tag{4.3}$$

For this to be true, each component of the net force must be zero, so

$$\sum F_x = 0 \qquad \sum F_y = 0 \quad \text{(body in equilibrium)} \tag{4.4}$$

We are assuming that the body can be represented adequately as a point particle. When the body has finite size, we also have to consider *where* on the body the forces are applied. We will return to this point in Chapter 11.

Conceptual Example 4.2 Zero net force means constant velocity

In the classic 1950 science fiction film *Rocketship X-M*, a spaceship is moving in the vacuum of outer space, far from any planet, when its engine dies. As a result, the spaceship slows down and stops. What does Newton's first law say about this event?

SOLUTION

In this situation there are no forces acting on the spaceship, so according to Newton's first law, it will *not* stop. It continues to move in a straight line with constant speed. Some science fiction movies have made use of very accurate science; this was not one of them.

Conceptual Example 4.3 Constant velocity means zero net force

You are driving a Porsche Carrera GT on a straight testing track at a constant speed of 150 km/h. You pass a 1971 Volkswagen Beetle doing a constant 75 km/h. For which car is the net force greater?

SOLUTION

The key word in this question is "net." Both cars are in equilibrium because their velocities are both constant; therefore the *net* force on each car is *zero*.

This conclusion seems to contradict the "common sense" idea that the faster car must have a greater force pushing it. It's true that the forward force on your Porsche is much greater than that on the Volkswagen (thanks to your Porsche's high-power engine). But there is also a *backward* force acting on each car due to road friction and air resistance. The only reason these cars need engines is to counteract this backward force so that the vector sum of the forward and backward forces will be zero and the car will travel with constant velocity. The backward force on your Porsche is greater because of its greater speed, so its engine has to be more powerful than the Volkswagen's.

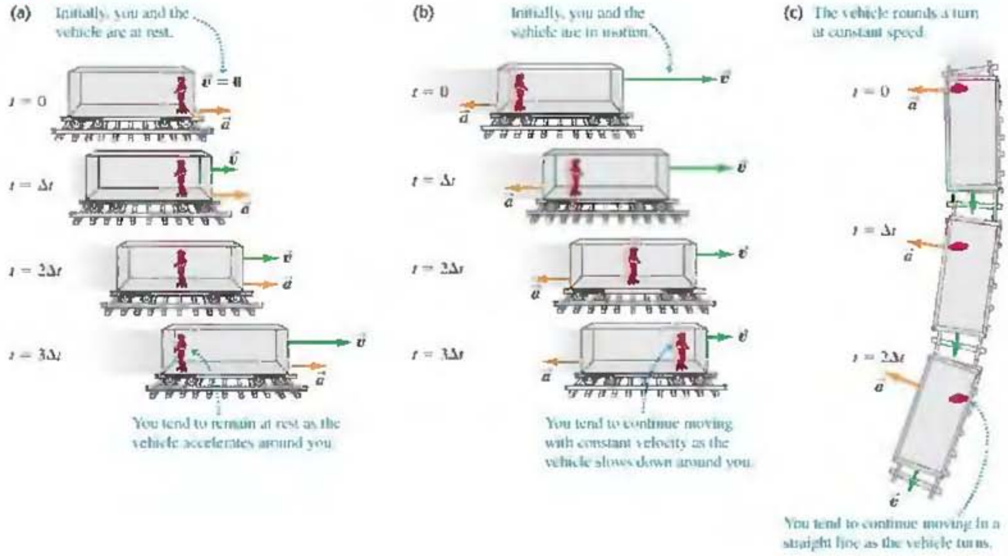
Inertial Frames of Reference

In discussing relative velocity in Section 3.5, we introduced the concept of *frame of reference*. This concept is central to Newton's laws of motion. Suppose you are in a bus that is traveling on a straight road and speeding up. If you could stand in the aisle on roller skates, you would start moving *backward* relative to the bus as the bus gains speed. If instead the bus was slowing to a stop, you would start moving forward down the aisle. In either case, it looks as though Newton's first law is not obeyed; there is no net force acting on you, yet your velocity changes. What's wrong?

The point is that the bus is accelerating with respect to the earth and is *not* a suitable frame of reference for Newton's first law. This law is valid in some frames of reference and not valid in others. A frame of reference in which Newton's first law *is* valid is called an **inertial frame of reference.** The earth is at least approximately an inertial frame of reference, but the bus is not. (The earth is not a completely inertial frame, owing to the acceleration associated with its rotation and its motion around the sun. These effects are quite small, however; see Exercises 3.29 and 3.32.) Because Newton's first law is used to define what we mean by an inertial frame of reference, it is sometimes called the *law of inertia.*

Figure 4.11 helps us understand what you experience when riding in a vehicle that's accelerating. In Fig. 4.11a, a vehicle is initially at rest and then begins to accelerate to the right. A passenger on roller skates (which nearly eliminate the effects of friction) has virtually no net force acting on her, so she tends to remain at rest relative to the inertial frame of the earth. As the vehicle accelerates around her, she moves backward relative to the vehicle. In the same way, a passenger in a vehicle that is slowing down tends to continue moving with constant velocity relative to the earth, and so moves forward relative to the vehicle (Fig. 4.11b). A vehicle is also accelerating if it moves at a constant speed but is turning (Fig. 4.11c). In this case a passenger tends to continue moving relative to the earth at constant speed in a straight line; relative to the vehicle, the passenger moves to the side of the vehicle on the outside of the turn.

4.11 Riding in an accelerating vehicle.



In each case shown in Fig. 4.11, an observer in the vehicle's frame of reference might be tempted to conclude that there *is* a net force acting on the passenger, since the passenger's velocity *relative to the vehicle* changes in each case. This conclusion is simply wrong; the net force on the passenger is indeed zero. The vehicle observer's mistake is in trying to apply Newton's first law in the vehicle's frame of reference, which is *not* an inertial frame and in which Newton's first law isn't valid (Fig. 4.12). In this book we will use *only* inertial frames of reference.

We've mentioned only one (approximately) inertial frame of reference: the earth's surface. But there are many inertial frames. If we have an inertial frame of reference A, in which Newton's first law is obeyed, then *any* second frame of reference B will also be inertial if it moves relative to A with constant velocity $\vec{v}_{B/A}$. We can prove this using the relative velocity relation Eq. (3.36) from Section 3.5:

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

Suppose that P is a body that moves with constant velocity $\vec{v}_{P/A}$ with respect to an inertial frame A. By Newton's first law the net force on this body is zero. The velocity of P relative to another frame B has a different value, $\vec{v}_{P/B} = \vec{v}_{P/A} - \vec{v}_{B/A}$. But if the relative velocity $\vec{v}_{B/A}$ of the two frames is constant, then $\vec{v}_{P/B}$ is constant as well. Thus B is also an inertial frame; the velocity of P in this frame is constant, and the net force on P is zero, so Newton's first law is obeyed in B. Observers in frames A and B will disagree about the velocity of P, but they will agree that P has a constant velocity (zero acceleration) and has zero net force acting on it.

There is no single inertial frame of reference that is preferred over all others for formulating Newton's laws. If one frame is inertial, then every other frame moving relative to it with constant velocity is also inertial. Viewed in this light, the state of rest and the state of motion with constant velocity are not very different; both occur when the vector sum of forces acting on the body is zero.

4.12 From the frame of reference of the car, it seems as though a force is pushing the crash test dummies forward as the car comes to a sudden stop. But there is really no such force: As the car stops, the dummies keep moving forward as a consequence of Newton's first law.

Test Your Understanding of Section 4.2 In which of the following situations is there zero net force on the body? (i) an airplane flying due north at a steady 120 m/s and at a constant altitude; (ii) a car driving straight up a hill with a 3° slope at a constant 90 km/h; (iii) a hawk circling at a constant 20 km/h at a constant height of 15 m above an open field; (iv) a box with slick, frictionless surfaces in the back of a truck as the truck accelerates forward on a level road at 5 m/s^2.

4.3 Newton's Second Law

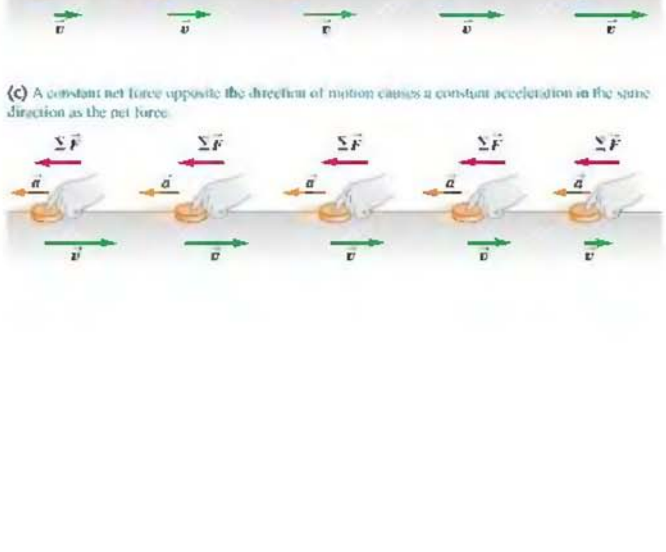
Newton's first law tells us that when a body is acted on by zero net force, it moves with constant velocity and zero acceleration. In Fig. 4.13a, a hockey puck is sliding to the right on wet ice. There is negligible friction, so there are no horizontal forces acting on the puck; the downward force of gravity and the upward normal force exerted by the ice surface sum to zero. So the net force $\sum\vec{F}$ acting on the puck is zero, the puck has zero acceleration, and its velocity is constant.

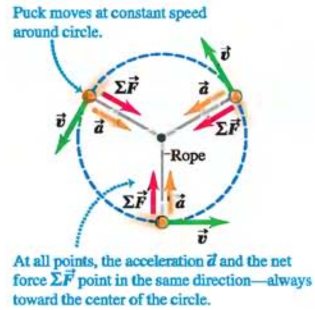
But what happens when the net force is *not* zero? In Fig. 4.13b we apply a constant horizontal force to a sliding puck in the same direction that the puck is moving. Then $\sum\vec{F}$ is constant and in the same horizontal direction as $\vec{v}$. We find that during the time the force is acting, the velocity of the puck changes at a constant rate; that is, the puck moves with constant acceleration. The speed of the puck increases, so the acceleration $\vec{a}$ is in the same direction as $\vec{v}$ and $\sum\vec{F}$.

In Fig. 4.13c we reverse the direction of the force on the puck so that $\sum\vec{F}$ acts opposite to $\vec{v}$. In this case as well, the puck has an acceleration; the puck moves more and more slowly to the right. The acceleration $\vec{a}$ in this case is to the left, in the same direction as $\sum\vec{F}$. As in the previous case, experiment shows that the acceleration is constant if $\sum\vec{F}$ is constant.

We conclude that ***a net force acting on a body causes the body to accelerate in the same direction as the net force.*** If the magnitude of the net force is constant, as in Figs. 4.13b and 4.13c, then so is the magnitude of the acceleration.

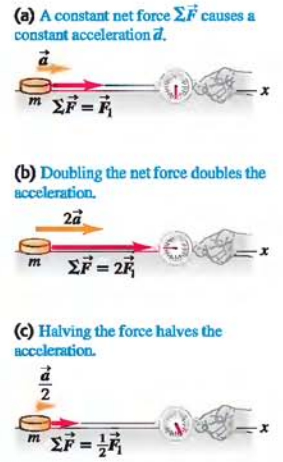
4.13 Exploring the relationship between the acceleration of a body and the net force acting on the body (in this case, a hockey puck on a frictionless surface).





4.14 A top view of a hockey puck in uniform circular motion on a frictionless horizontal surface.

4.15 For a body of a given mass m, the magnitude of the body's acceleration is directly proportional to the magnitude of the net force acting on the body.



These conclusions about net force and acceleration also apply to a body moving along a curved path. For example, Fig. 4.14 shows a hockey puck moving in a horizontal circle on an ice surface of negligible friction. A rope attaching the puck to the ice exerts a tension force of constant magnitude toward the center of the circle. The result is a net force and an acceleration that are constant in magnitude and directed toward the center of the circle. The speed of the puck is constant, so this is uniform circular motion, as discussed in Section 3.4.

Figure 4.15a shows another experiment to explore the relationship between acceleration and net force. We apply a constant horizontal force to a puck on a frictionless horizontal surface, using the spring balance described in Section 4.1 with the spring stretched a constant amount. As in Figs. 4.13b and 4.13c, this horizontal force equals the net force on the puck. If we change the magnitude of the net force, the acceleration changes in the same proportion. Doubling the net force doubles the acceleration (Fig. 4.15b), halving the net force halves the acceleration (Fig. 4.15c), and so on. Many such experiments show that *for any given body, the magnitude of the acceleration is directly proportional to the magnitude of the net force acting on the body.*

Mass and Force

Our results mean that for a given body, the *ratio* of the magnitude $|\Sigma\vec{F}|$ of the net force to the magnitude $a = |\vec{a}|$ of the acceleration is constant, regardless of the magnitude of the net force. We call this ratio the *inertial mass,* or simply the **mass,** of the body and denote it by m. That is,

$$m = \frac{|\Sigma\vec{F}|}{a} \quad \text{or} \quad |\Sigma\vec{F}| = ma \quad \text{or} \quad a = \frac{|\Sigma\vec{F}|}{m} \qquad (4.5)$$

Mass is a quantitative measure of inertia, which we discussed in Section 4.2. The last of the equations in Eq. (4.5) says that the greater its mass, the more a body "resists" being accelerated. When you hold a piece of fruit in your hand at the supermarket and move it slightly up and down to estimate its heft, you're applying a force and seeing how much the fruit accelerates up and down in response. If a force causes a large acceleration, the fruit has a small mass; if the same force causes only a small acceleration, the fruit has a large mass. In the same way, if you hit a table-tennis ball and then a basketball with the same force, the basketball has much smaller acceleration because it has much greater mass.

The SI unit of mass is the **kilogram.** We mentioned in Section 1.3 that the kilogram is officially defined to be the mass of a cylinder of platinum–iridium alloy kept in a vault near Paris. We can use this standard kilogram, along with Eq. (4.5), to define the **newton:**

One newton is the amount of net force that gives an acceleration of 1 meter per second squared to a body with a mass of 1 kilogram.

This definition allows us to calibrate the spring balances and other instruments used to measure forces. Because of the way we have defined the newton, it is related to the units of mass, length, and time. For Eq. (4.5) to be dimensionally consistent, it must be true that

$$1 \text{ newton} = (1 \text{ kilogram})(1 \text{ meter per second squared})$$

or

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

We will use this relationship many times in the next few chapters, so keep it in mind.

We can also use Eq. (4.5) to compare a mass with the standard mass and thus to *measure* masses. Suppose we apply a constant net force $\Sigma\vec{F}$ to a body having a known mass m_1 and we find an acceleration of magnitude a_1 (Fig. 4.16a). We then apply the same force to another body having an unknown mass m_2, and we find an acceleration of magnitude a_2 (Fig. 4.16b). Then, according to Eq. (4.5),

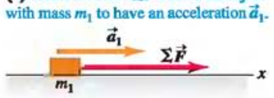
$$m_1 a_1 = m_2 a_2$$
$$\frac{m_2}{m_1} = \frac{a_1}{a_2} \quad \text{(same net force)} \tag{4.6}$$

For the same net force, the ratio of the masses of two bodies is the inverse of the ratio of their accelerations. In principle we could use Eq. (4.6) to measure an unknown mass m_2, but it is usually easier to determine mass indirectly by measuring the body's *weight.* We'll return to this point in Section 4.4.

When two bodies with masses m_1 and m_2 are fastened together, we find that the mass of the composite body is always $m_1 + m_2$ (Fig. 4.16c). This additive property of mass may seem obvious, but it has to be verified experimentally. Ultimately, the mass of a body is related to the number of protons, electrons, and neutrons it contains. This wouldn't be a good way to *define* mass because there is no practical way to count these particles. But the concept of mass is the most fundamental way to characterize the quantity of matter in a body.

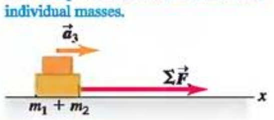
4.16 For a given net force $\sum\vec{F}$ acting on a body, the acceleration is inversely proportional to the mass of the body. Masses add like ordinary scalars.

(a) A known force $\sum\vec{F}$ causes an object with mass m_1 to have an acceleration $\vec{a}_1$.



(b) Applying the same force $\sum\vec{F}$ to a second object and noting the acceleration allow us to measure the mass.

(c) When the two objects are fastened together, the same method shows that their composite mass is the sum of their individual masses.



Stating Newton's Second Law

We've been careful to state that the *net* force on a body is what causes that body to accelerate. Experiment shows that if a combination of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3$, and so on is applied to a body, the body will have the same acceleration (magnitude and direction) as when only a single force is applied, if that single force is equal to the vector sum $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$. In other words, the principle of superposition of forces (see Fig. 4.4) also holds true when the net force is not zero and the body is accelerating.

Equation (4.5) relates the magnitude of the net force on a body to the magnitude of the acceleration that it produces. We have also seen that the direction of the net force is the same as the direction of the acceleration, whether the body's path is straight or curved. Newton wrapped up all these relationships and experimental results in a single concise statement that we now call *Newton's second law of motion:*

Newton's second law of motion: If a net external force acts on a body, the body accelerates. The direction of acceleration is the same as the direction of the net force. The mass of the body times the acceleration of the body equals the net force vector.

In symbols,

$$\sum\vec{F} = m\vec{a} \quad \text{(Newton's second law of motion)} \tag{4.7}$$

An alternative statement is that the acceleration of a body is in the same direction as the net force acting on the body, and is equal to the net force divided by the body's mass:

$$\vec{a} = \frac{\sum\vec{F}}{m}$$

Newton's second law is a fundamental law of nature, the basic relationship between force and motion. Most of the remainder of this chapter and all of the next are devoted to learning how to apply this principle in various situations.

Equation (4.7) has many practical applications (Fig. 4.17). You've actually been using it all your life to measure your body's acceleration. In your inner ear, microscopic hair cells sense the magnitude and direction of the force that they must exert to canse small membranes to accelerate along with the rest of your body. By Newton's second law, the acceleration of the membranes—and hence

4.17 The design of high-performance motorcycles depends fundamentally on Newton's second law. To maximize the forward acceleration, the designer makes the motorcycle as light as possible (that is, minimizes the mass) and uses the most powerful engine possible (thus maximizing the forward force).

that of your body as a whole—is proportional to this force and has the same direction. In this way, you can sense the magnitude and direction of your acceleration even with your eyes closed!

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2.1.3 Tension Change
2.1.4 Sliding on an Incline

Using Newton's Second Law

There are at least four aspects of Newton's second law that deserve special attention. First, Eq. (4.7) is a *vector* equation. Usually we will use it in component form, with a separate equation for each component of force and the corresponding acceleration:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \qquad \text{(Newton's second law of motion)} \tag{4.8}$$

This set of component equations is equivalent to the single vector equation (4.7). Each component of the net force equals the mass times the corresponding component of acceleration.

Second, the statement of Newton's second law refers to *external* forces. By this we mean forces exerted on the body by other bodies in its environment. It's impossible for a body to affect its own motion by exerting a force on itself; if it were possible, you could lift yourself to the ceiling by pulling up on your belt! That's why only external forces are included in the sum $\sum\vec{F}$ in Eqs. (4.7) and (4.8).

Third, Eqs. (4.7) and (4.8) are valid only when the mass m is *constant*. It's easy to think of systems whose masses change, such as a leaking tank truck, a rocket ship, or a moving railroad car being loaded with coal. But such systems are better handled by using the concept of momentum; we'll get to that in Chapter 8.

Finally, Newton's second law is valid only in inertial frames of reference, just like the first law. Thus it is not valid in the reference frame of any of the accelerating vehicles in Fig. 4.11; relative to any of these frames, the passenger accelerates even though the net force on the passenger is zero. We will usually assume that the earth is an adequate approximation to an inertial frame, although because of its rotation and orbital motion it is not precisely inertial.

CAUTION **$m\vec{a}$ is not a force** You must keep in mind that even though the vector $m\vec{a}$ is equal to the vector sum $\sum\vec{F}$ of all the forces acting on the body, the vector $m\vec{a}$ is *not* a force. Acceleration is a *result* of a nonzero net force; it is not a force itself. It's "common sense" to think that there is a "force of acceleration" that pushes you back into your seat when your car accelerates forward from rest. But *there is no such force;* instead, your inertia causes you to tend to stay at rest relative to the earth, and the car accelerates around you (see Fig. 4.11a). The "common sense" confusion arises from trying to apply Newton's second law where it isn't valid, in the noninertial reference frame of an accelerating car. We will always examine motion relative to *inertial* frames of reference only.

In learning how to use Newton's second law, we will begin in this chapter with examples of straight-line motion. Then in Chapter 5 we will consider more general cases and develop more detailed problem-solving strategies.

Example 4.4 Determining acceleration from force

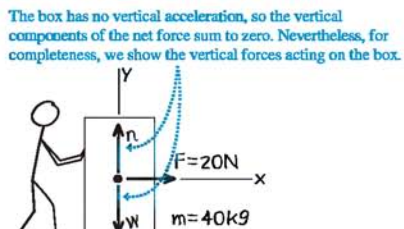
A worker applies a constant horizontal force with magnitude 20 N to a box with mass 40 kg resting on a level floor with negligible friction. What is the acceleration of the box?

SOLUTION

IDENTIFY: This problem involves force and acceleration. Whenever you encounter a problem of this kind, you should approach it using Newton's second law.

SET UP: In *any* problem involving forces, the first steps are to choose a coordinate system and then identify all of the forces acting on the body in question.

It's usually convenient to take one axis either along or opposite the direction of the body's acceleration, which in this case is horizontal. Hence we take the $+x$-axis to be in the direction of the applied horizontal force (that is, the direction in which the box accelerates) and the $+y$-axis to be upward (Fig. 4.18). In most



4.18 Our sketch for this problem. The tiles under the box are freshly waxed, so we assume that friction is negligible.

force problems that you'll encounter (including this one), the force vectors all lie in a plane, so the z-axis isn't used.

The forces acting on the box are (i) the horizontal force $\vec{F}$ exerted by the worker, of magnitude 20 N; (ii) the weight $\vec{w}$ of the box—that is, the downward gravitational force exerted by the earth; and (iii) the upward supporting force $\vec{n}$ exerted by the floor. As in Section 4.2, we call $\vec{n}$ a *normal* force because it is normal (perpendicular) to the surface of contact. (We use an italic letter n to avoid confusion with the abbreviation N for newton.) We are told that friction is negligible, so no friction force is present.

Since the box doesn't move vertically at all, the y-acceleration is zero: $a_y = 0$. Our target variable is the x-component of acceleration, a_x. We'll find it using Newton's second law in component form as given by Eq. (4.8).

EXECUTE: From Fig. 4.18, only the 20-N force has a nonzero x-component. Hence the first relation in Eqs. (4.8) tells us that

$$\sum F_x = F = 20\,\text{N} = ma_x$$

Hence the x-component of acceleration is

$$a_x = \frac{\sum F_x}{m} = \frac{20\,\text{N}}{40\,\text{kg}} = \frac{20\,\text{kg}\cdot\text{m/s}^2}{40\,\text{kg}} = 0.50\,\text{m/s}^2$$

EVALUATE: The acceleration is in the +x-direction, the same direction as the net force. The net force is constant, so the acceleration is also constant. If we are given the initial position and velocity of the box, we can find the position and velocity at any later time from the equations of motion with constant acceleration we derived in Chapter 2.

Notice that to determine a_x, we didn't have to use the y-component of Newton's second law from Eq. (4.8), $\sum F_y = ma_y$. By using this equation, can you show that the magnitude n of the normal force in this situation is equal to the weight of the box?

Example 4.5 Determining force from acceleration

A waitress shoves a ketchup bottle with mass 0.45 kg to the right along a smooth, level lunch counter. The bottle leaves her hand moving at 2.8 m/s, then slows down as it slides because of the constant horizontal friction force exerted on it by the counter top. It slides a distance of 1.0 m before coming to rest. What are the magnitude and direction of the friction force acting on the bottle?

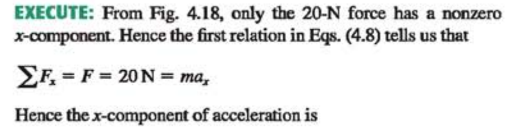
SOLUTION

IDENTIFY: Like Example 4.4, this problem involves forces and acceleration (the slowing of the ketchup bottle), so we'll use Newton's second law to solve it.

SET UP: As in Example 4.4, we first choose a coordinate system and then identify the forces acting on the body (in this case, the ketchup bottle). As Fig. 4.19 shows, we choose the +x-axis to be in the direction that the bottle slides, and we take the origin to be where the bottle leaves the waitress's hand moving at 2.8 m/s. Figure 4.19 also shows the forces acting on the bottle. The friction force $\vec{f}$ acts to slow the bottle down, so its direction must be opposite the direction of velocity (see Fig. 4.13c).

Our target variable is the magnitude f of the friction force. We'll find it using the x-component of Newton's second law from Eq. (4.8). To do so, we'll first need to know the x-component of the bottle's acceleration, a_x. We aren't told the value of a_x in the problem, but we are told that the friction force is constant. Hence the acceleration is constant as well, and we can calculate a_x by using one of the constant-acceleration formulas from Section 2.4. Since we know the bottle's initial x-coordinate and x-velocity ($x_0 = 0$, $v_{0x} = 2.8\,\text{m/s}$) as well as its final x-coordinate and x-velocity ($x = 1.0\,\text{m}, v_x = 0$), the easiest equation to use to determine a_x is Eq. (2.13), $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$.

4.19 Our sketch for this problem.



EXECUTE: From Eq. (2.13),

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(0\,\text{m/s})^2 - (2.8\,\text{m/s})^2}{2(1.0\,\text{m} - 0\,\text{m})} = -3.9\,\text{m/s}^2$$

The negative sign means that the acceleration is toward the *left;* the velocity is in the opposite direction to acceleration, as it must be, since the bottle is slowing down. The net force in the x-direction is the x-component $-f$ of the friction force, so

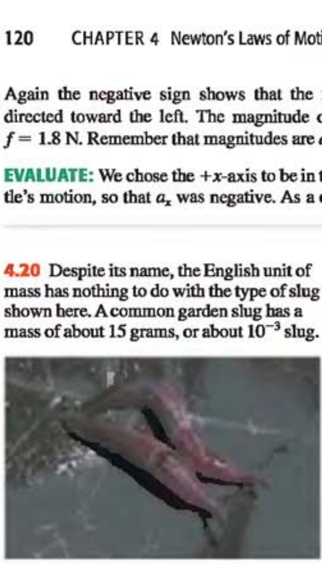
$$\sum F_x = -f = ma_x = (0.45\,\text{kg})(-3.9\,\text{m/s}^2)$$
$$= -1.8\,\text{kg}\cdot\text{m/s}^2 = -1.8\,\text{N}$$

Continued

Again the negative sign shows that the force on the bottle is directed toward the left. The magnitude of the friction force is $f = 1.8$ N. Remember that magnitudes are *always* positive!

EVALUATE: We chose the $+x$-axis to be in the direction of the bottle's motion, so that a_x was negative. As a check on the result, try repeating the calculation with the $+x$-axis directed *opposite* to the motion (to the left in Fig. 4.19) so that a_x is positive. In this case you should find that $\sum F_x$ is equal to $+f$ (because the friction force is now in the $+x$-direction), which in turn is equal to $+1.8$ N. Your answers for the *magnitudes* of forces (which are always positive numbers) should never depend on your choice of coordinate axes!

4.20 Despite its name, the English unit of mass has nothing to do with the type of slug shown here. A common garden slug has a mass of about 15 grams, or about 10^{-3} slug.



Some Notes on Units

A few words about units are in order. In the cgs metric system (not used in this book), the unit of mass is the gram, equal to 10^{-3} kg, and the unit of distance is the centimeter, equal to 10^{-2} m. The cgs unit of force is called the *dyne:*

$$1 \text{ dyne} = 1 \text{ g} \cdot \text{cm/s}^2 = 10^{-5}\text{ N}$$

In the British system, the unit of force is the *pound* (or pound-force) and the unit of mass is the *slug* (Fig. 4.20). The unit of acceleration is 1 foot per second squared, so

$$1 \text{ pound} = 1 \text{ slug} \cdot \text{ft/s}^2$$

The official definition of the pound is

$$1 \text{ pound} = 4.448221615260 \text{ newtons}$$

It is handy to remember that a pound is about 4.4 N and a newton is about 0.22 pound. Next time you want to order a "quarter-pounder," try asking for a "one-newtoner" and see what happens. Another useful fact: A body with a mass of 1 kg has a weight of about 2.2 lb at the earth's surface.

Table 4.2 summarizes the units of force, mass, and acceleration in the three systems.

Table 4.2 Units of Force, Mass, and Acceleration

System of Units	Force	Mass	Acceleration
SI	newton (N)	kilogram (kg)	m/s^2
cgs	dyne (dyn)	gram (g)	cm/s^2
British	pound (lb)	slug	ft/s^2

Test Your Understanding of Section 4.3 Rank the following situations in order of the magnitude of the object's acceleration, from lowest to highest. Are there any cases that have the same magnitude of acceleration? (i) a 2.0-kg object acted on by a 2.0-N net force; (ii) a 2.0-kg object acted on by an 8.0-N net force; (iii) an 8.0-kg object acted on by a 2.0-N net force; (iv) an 8.0-kg object acted on by a 8.0-N net force.

4.4 Mass and Weight

One of the most familiar forces is the *weight* of a body, which is the gravitational force that the earth exerts on the body. (If you are on another planet, your weight is the gravitational force that planet exerts on you.) Unfortunately, the terms *mass* and *weight* are often misused and interchanged in everyday conversation. It is absolutely essential for you to understand clearly the distinctions between these two physical quantities.

Mass characterizes the *inertial* properties of a body. Mass is what keeps the china on the table when you yank the tablecloth out from under it. The greater the mass, the greater the force needed to cause a given acceleration; this is reflected in Newton's second law, $\sum \vec{F} = m\vec{a}$.

Weight, on the other hand, is a *force* exerted on a body by the pull of the earth. Mass and weight are related: Bodies having large mass also have large weight. A large stone is hard to throw because of its large *mass,* and hard to lift off the ground because of its large *weight.*

To understand the relationship between mass and weight, note that a freely falling body has an acceleration of magnitude g. Newton's second law tells us that a force must act to produce this acceleration. If a 1-kg body falls with an acceleration of 9.8 m/s^2, the required force has magnitude

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2.9 Pole-Vaulter Vaults

$$F = ma = (1 \text{ kg})(9.8 \text{ m/s}^2) = 9.8 \text{ kg} \cdot \text{m/s}^2 = 9.8 \text{ N}$$

The force that makes the body accelerate downward is its weight. Any body near the surface of the earth that has a mass of 1 kg *must* have a weight of 9.8 N to give it the acceleration we observe when it is in free fall. More generally, a body with mass m must have weight with magnitude w given by

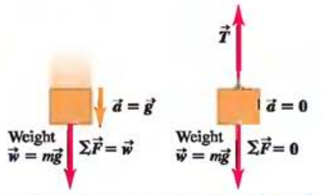
$$w = mg \quad \text{(magnitude of the weight of a body of mass } m\text{)} \tag{4.9}$$

Hence the magnitude w of a body's weight is directly proportional to its mass m. The weight of a body is a force, a vector quantity, and we can write Eq. (4.9) as a vector equation (Fig. 4.21):

$$\vec{w} = m\vec{g} \tag{4.10}$$

Remember that g is the *magnitude* of $\vec{g}$, the acceleration due to gravity, so g is always a positive number, by definition. Thus w, given by Eq. (4.9), is the *magnitude* of the weight and is also always positive.

4.21 The relationship of mass and weight.



CAUTION A body's weight acts at all times It is important to understand that the weight of a body acts on the body *all the time,* whether it is in free fall or not. If we suspend an object from a chain, it is in equilibrium, and its acceleration is zero. But its weight, given by Eq. (4.10), is still pulling down on it (Fig. 4.21). In this case the chain pulls up on the object, applying an upward force. The *vector sum* of the forces is zero, but the weight still acts.

Conceptual Example 4.6 Net force and acceleration in free fall

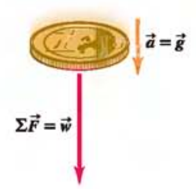
In Example 2.6 (Section 2.5) a one-euro coin was dropped from rest from the Leaning Tower of Pisa. If the coin falls freely, so that the effects of the air are negligible, how does the net force on the coin vary as it falls?

SOLUTION

In free fall, the acceleration $\vec{a}$ of the coin is constant and equal to $\vec{g}$. Hence by Newton's second law the net force $\Sigma\vec{F} = m\vec{a}$ is also constant and equal to $m\vec{g}$, which is the coin's weight $\vec{w}$ (Fig. 4.22). The coin's velocity changes as it falls, but the net force acting on it remains constant. If this surprises you, perhaps you still believe in the erroneous "common sense" idea that greater speed implies greater force. If so, you should reread Conceptual Example 4.3.

The net force on a freely falling coin is constant even if you initially toss it upward. The force that your hand exerts on the coin to toss it is a contact force, and it disappears the instant that the coin loses contact with your hand. From then on, the only force acting on the coin is its weight $\vec{w}$.

4.22 The acceleration of a freely falling object is constant, and so is the net force acting on the object.

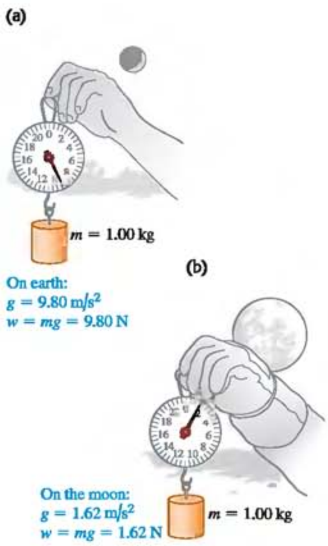


Variation of g with Location

We will use $g = 9.80\ \text{m/s}^2$ for problems on the earth (or, if the other data in the problem are given to only two significant figures, $g = 9.8\ \text{m/s}^2$). In fact, the value of g varies somewhat from point to point on the earth's surface, from about 9.78 to $9.82\ \text{m/s}^2$, because the earth is not perfectly spherical and because of effects due to its rotation and orbital motion. At a point where $g = 9.80\ \text{m/s}^2$, the weight of a standard kilogram is $w = 9.80\ \text{N}$. At a different point, where $g = 9.78\ \text{m/s}^2$, the weight is $w = 9.78\ \text{N}$ but the mass is still 1 kg. The weight of a body varies from one location to another; the mass does not.

If we take a standard kilogram to the surface of the moon, where the acceleration of free fall (equal to the value of g at the moon's surface) is $1.62\ \text{m/s}^2$, its

4.23 The weight of a 1-kilogram mass (a) on earth and (b) on the moon.



weight is 1.62 N, but its mass is still 1 kg (Fig. 4.23). An 80.0-kg astronaut has a weight on earth of $(80.0\,\text{kg})(9.80\,\text{m/s}^2) = 784\,\text{N}$, but on the moon the astronaut's weight would be only $(80.0\,\text{kg})(1.62\,\text{m/s}^2) = 130\,\text{N}$. In Chapter 12 we'll see how to calculate the value of g at the surface of the moon or on other worlds.

Measuring Mass and Weight

In Section 4.3 we described a way to compare masses by comparing their accelerations when subjected to the same net force. Usually, however, the easiest way to measure the mass of a body is to measure its weight, often by comparing with a standard. Equation (4.9) says that two bodies that have the same weight at a particular location also have the same mass. We can compare weights very precisely; the familiar equal-arm balance (Fig. 4.24) can determine with great precision (up to 1 part in 10^6) when the weights of two bodies are equal and hence when their masses are equal. (This method doesn't work in the apparent "zero-gravity" environment of outer space. Instead, we apply a known force to the body, measure its acceleration, and compute the mass as the ratio of force to acceleration. This method, or a variation of it, is used to measure the masses of astronauts in orbiting space stations as well as the masses of atomic aud subatomic particles.)

The concept of mass plays two rather different roles in mechanics. The weight of a body (the gravitational force acting on it) is proportional to its mass; we call the property related to gravitational interactions *gravitational mass.* On the other hand, we call the inertial property that appears in Newton's second law the *inertial mass.* If these two quantities were different, the acceleration due to gravity might well be different for different bodies. However, extraordinarily precise experiments have established that in fact the two *are* the same to a precision of better than one part in 10^{12}.

CAUTION **Don't confuse mass and weight** The SI units for mass and weight are often misused in everyday life. Incorrect expressions such as "This box weighs 6 kg" are nearly universal. What is meant is that the *mass* of the box, probably determined indirectly by *weighing,* is 6 kg. Be careful to avoid this sloppy usage in your own work! In SI units, weight (a force) is measured in newtons, while mass is measured in kilograms.

Example 4.7 Mass and weight

A $2.49 \times 10^4\,\text{N}$ Rolls-Royce Phantom traveling in the $+x$-direction makes a fast stop; the x-component of the net force acting on it is $-1.83 \times 10^4\,\text{N}$. What is its acceleration?

SOLUTION

IDENTIFY: Again we will use Newton's second law to relate force and acceleration. To use this relationship, we need to know the car's mass. However, because the newton is a unit for force, we know that $2.49 \times 10^4\,\text{N}$ is the car's weight, not its mass. So we'll also have to use the relationship between a body's mass and its weight.

SET UP: Our target variable is the x-component of acceleration of the car, a_x. (The motion is purely in the x-direction.) We use Eq. (4.9) to determine the car's mass from its weight and then use the x-component of Newton's second law from Eq. (4.8) to determine a_x.

EXECUTE: The mass m of the car is

$$m = \frac{w}{g} = \frac{2.49 \times 10^4\,\text{N}}{9.80\,\text{m/s}^2} = \frac{2.49 \times 10^4\,\text{kg}\cdot\text{m/s}^2}{9.80\,\text{m/s}^2}$$
$$= 2540\,\text{kg}$$

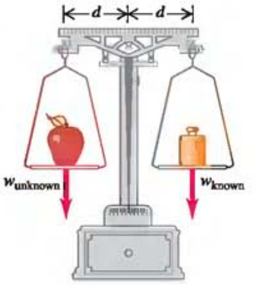
Then $\sum F_x = ma_x$ gives

$$a_x = \frac{\sum F_x}{m} = \frac{-1.83 \times 10^4\,\text{N}}{2540\,\text{kg}} = \frac{-1.83 \times 10^4\,\text{kg}\cdot\text{m/s}^2}{2540\,\text{kg}}$$
$$= -7.20\,\text{m/s}^2$$

EVALUATE: The negative sign means that the acceleration vector points in the negative x-direction. This makes sense: The car is moving in the positive x-direction and is slowing down.

Note that the acceleration can alternatively be written as $-0.735g$. It's of interest that -0.735 is also the ratio of $-1.83 \times 10^4\,\text{N}$ (the x-component of the net force) to $2.49 \times 10^4\,\text{N}$ (the weight). Indeed, the acceleration of a body expressed as a multiple of g is always equal to the ratio of the net force on the body to its weight. Can you see why?

Test Your Understanding of Section 4.4 Suppose an astronaut landed on a planet where $g = 19.6\ \text{m/s}^2$. Compared to earth, would it be easier, harder, or just as easy for her to walk around? Would it be easier, harder, or just as easy for her to catch a ball that is moving horizontally at 12 m/s? (Assume that the astronaut's spacesuit is a lightweight model that doesn't impede her movements in any way.)



4.24 An equal-arm balance determines the mass of a body (such as an apple) by comparing its weight to a known weight.

4.5 Newton's Third Law

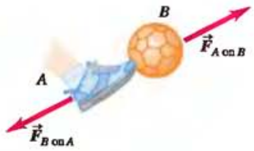
A force acting on a body is always the result of its interaction with another body, so forces always come in pairs. You can't pull on a doorknob without the doorknob pulling back on you. When you kick a football, the forward force that your foot exerts on the ball launches it into its trajectory, but you also feel the force the ball exerts back on your foot. If you kick a boulder, the pain you feel is due to the force that the boulder exerts on your foot.

In each of these cases, the force that you exert on the other body is in the opposite direction to the force that body exerts on you. Experiments show that whenever two bodies interact, the two forces that they exert on each other are always *equal in magnitude* and *opposite in direction.* This fact is called *Newton's third law of motion:*

Newton's third law of motion: If body *A* exerts a force on body *B* (an "action"), then body *B* exerts a force on body *A* (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on *different* bodies.

For example, in Fig. 4.25 $\vec{F}_{A\text{ on }B}$ is the force applied *by* body *A* (first subscript) *on* body *B* (second subscript), and $\vec{F}_{B\text{ on }A}$ is the force applied *by* body *B* (first subscript) *on* body *A* (second subscript). The mathematical statement of Newton's third law is

$$\vec{F}_{A\text{ on }B} = -\vec{F}_{B\text{ on }A} \quad \text{(Newton's third law of motion)} \quad (4.11)$$



4.25 If body *A* exerts a force $\vec{F}_{A\text{ on }B}$ on body *B*, then body *B* exerts a force $\vec{F}_{B\text{ on }A}$ on body *A* that is equal in magnitude and opposite in direction: $\vec{F}_{A\text{ on }B} = -\vec{F}_{B\text{ on }A}$.

It doesn't matter whether one body is inanimate (like the soccer ball in Fig. 4.25) and the other is not (like the kicker): They necessarily exert forces on each other that obey Eq. (4.11). ?

In the statement of Newton's third law, "action" and "reaction" are the two opposite forces (in Fig. 4.25, $\vec{F}_{A\text{ on }B}$ and $\vec{F}_{B\text{ on }A}$); we sometimes refer to them as an **action–reaction pair.** This is *not* meant to imply any cause-and-effect relationship; we can consider either force as the "action" and the other as the "reaction." We often say simply that the forces are "equal and opposite," meaning that they have equal magnitudes and opposite directions.

CAUTION **The two forces in an action–reaction pair act on different bodies** We stress that the two forces described in Newton's third law act on *different* bodies. This is important in problems involving Newton's first or second law, which involve the forces that act on a body. For instance, the net force acting on the soccer ball in Fig. 4.25 is the vector sum of the weight of the ball and the force $\vec{F}_{A\text{ on }B}$ exerted by the kicker. You would not include the force $\vec{F}_{B\text{ on }A}$ because this force acts on the kicker, not on the ball.

In Fig. 4.25 the action and reaction forces are *contact* forces that are present only when the two bodies are touching. But Newton's third law also applies to *long-range* forces that do not require physical contact, such as the force of gravitational attraction. A table-tennis ball exerts an upward gravitational force on the earth that's equal in magnitude to the downward gravitational force the earth exerts on the ball. When you drop the ball, both the ball and the earth accelerate toward each other. The net force on each body has the same magnitude, but the earth's acceleration is microscopically small because its mass is so great. Nevertheless, it does move!

Conceptual Example 4.8 Which force is greater?

After your sports car breaks down, you start to push it to the nearest repair shop. While the car is starting to move, how does the force you exert on the car compare to the force the car exerts on you? How do these forces compare when you are pushing the car along at a constant speed?

SOLUTION

In *both* cases, the force you exert on the car is equal in magnitude and opposite in direction to the force the car exerts on you. It's true that you have to push harder to get the car going than to keep it going. But no matter how hard you push on the car, the car pushes just as hard back on you. Newton's third law gives the same result whether the two bodies are at rest, moving with constant velocity, or accelerating.

You may wonder how the car "knows" to push back on you with the same magnitude of force that you exert on it. It may help to remember that the forces you and the car exert on each other are really interactions between the atoms at the surface of your hand and the atoms at the surface of the car. These interactions are analogous to miniature springs between adjacent atoms, and a compressed spring exerts equally strong forces on both of its ends.

Fundamentally, though, the reason we know that objects of different masses exert equally strong forces on each other is that *experiment tells us so.* Never forget that physics isn't merely a collection of rules and equations; rather, it's a systematic description of the natural world based on experiment and observation.

Conceptual Example 4.9 Applying Newton's third law: Objects at rest

An apple sits on a table in equilibrium. What forces act on it? What is the reaction force to each of the forces acting on the apple? What are the action–reaction pairs?

SOLUTION

Figure 4.26a shows the forces acting on the apple. In the diagram, $\vec{F}_{\text{earth on apple}}$ is the weight of the apple—that is, the downward gravitational force exerted *by* the earth (first subscript) *on* the apple (second subscript). Similarly, $\vec{F}_{\text{table on apple}}$ is the upward force exerted *by* the table (first subscript) *on* the apple (second subscript).

As the earth pulls down on the apple, the apple exerts an equally strong upward pull $\vec{F}_{\text{apple on earth}}$ on the earth, as shown in Fig. 4.26b. $\vec{F}_{\text{apple on earth}}$ and $\vec{F}_{\text{earth on apple}}$ are an action–reaction pair, representing the mutual interaction of the apple and the earth, so

$$\vec{F}_{\text{apple on earth}} = -\vec{F}_{\text{earth on apple}}$$

Also, as the table pushes up on the apple with force $\vec{F}_{\text{table on apple}}$, the corresponding reaction is the downward force $\vec{F}_{\text{apple on table}}$ exerted by the apple on the table (Fig. 4.26c). So we have

$$\vec{F}_{\text{apple on table}} = -\vec{F}_{\text{table on apple}}$$

The two forces acting on the apple are $\vec{F}_{\text{table on apple}}$ and $\vec{F}_{\text{earth on apple}}$. Are they an action–reaction pair? No, they aren't, despite being equal and opposite. They do not represent the mutual interaction of two bodies; they are two different forces acting on the *same* body. ***The two forces in an action–reaction pair never act on the same body.*** Here's another way to look at it. Suppose we suddenly yank the table out from under the apple (Fig. 4.26d). The two forces $\vec{F}_{\text{apple on table}}$ and $\vec{F}_{\text{table on apple}}$ then become zero, but $\vec{F}_{\text{apple on earth}}$ and $\vec{F}_{\text{earth on apple}}$ are still there (the gravitational interaction is still present). Since $\vec{F}_{\text{table on apple}}$ is now zero, it can't be the negative of $\vec{F}_{\text{earth on apple}}$, and these two forces can't be an action–reaction pair.

4.26 The two forces in an action–reaction pair always act on different bodies.

Conceptual Example 4.10 Applying Newton's third law: Objects in motion

A stonemason drags a marble block across a floor by pulling on a rope attached to the block (Fig. 4.27a). The block may or may not be in equilibrium. How are the various forces related? What are the action–reaction pairs?

SOLUTION

We'll use subscripts on all the forces to help explain things: B for the block, R for the rope, and M for the mason. Vector $\vec{F}_{\text{M on R}}$ represents the force exerted by the *mason* on the *rope*. Its reaction is the equal and opposite force $\vec{F}_{\text{R on M}}$ exerted by the *rope* on the *mason*. Vector $\vec{F}_{\text{R on B}}$ represents the force exerted by the *rope* on the *block*. The reaction to it is the equal and opposite force $\vec{F}_{\text{B on R}}$ exerted by the *block* on the *rope*. For these two action-reaction pairs (Fig. 4.27b), we have

$$\vec{F}_{\text{R on M}} = -\vec{F}_{\text{M on R}} \quad \text{and} \quad \vec{F}_{\text{B on R}} = -\vec{F}_{\text{R on B}}$$

Be sure you understand that the forces $\vec{F}_{\text{M on R}}$ and $\vec{F}_{\text{B on R}}$ are *not* an action–reaction pair (Fig. 4.27c) because both of these forces act on the *same* body (the rope); an action and its reaction *must* always act on *different* bodies. Furthermore, the forces $\vec{F}_{\text{M on R}}$ and $\vec{F}_{\text{B on R}}$ are not necessarily equal in magnitude. Applying Newton's second law to the rope, we get

$$\sum\vec{F} = \vec{F}_{\text{M on R}} + \vec{F}_{\text{B on R}} = m_{\text{rope}}\vec{a}_{\text{rope}}$$

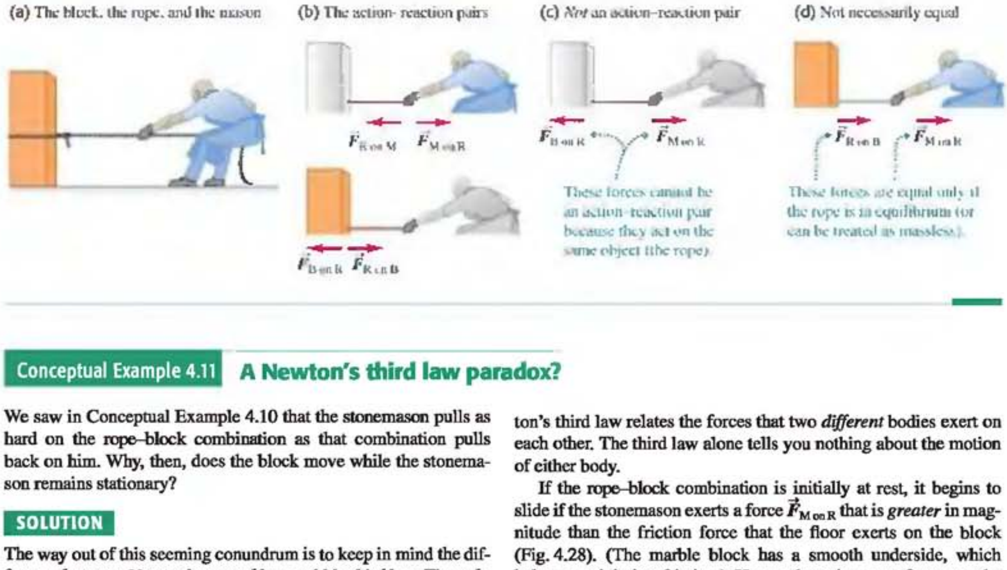
If the block and rope are accelerating (that is, speeding up or slowing down), the rope is not in equilibrium, and $\vec{F}_{\text{M on R}}$ must have a different magnitude than $\vec{F}_{\text{B on R}}$. By contrast, the action–reaction forces $\vec{F}_{\text{M on R}}$ and $\vec{F}_{\text{R on M}}$ are always equal in magnitude, as are $\vec{F}_{\text{R on B}}$ and $\vec{F}_{\text{B on R}}$. Newton's third law holds whether or not the bodies are accelerating.

In the special case in which the rope is in equilibrium, the forces $\vec{F}_{\text{M on R}}$ and $\vec{F}_{\text{B on R}}$ are equal in magnitude. But this is an example of Newton's *first* law, not his *third*. Another way to look at this is that in equilibrium, $\vec{a}_{\text{rope}} = \mathbf{0}$ in the preceding equation. Then $\vec{F}_{\text{B on R}} = -\vec{F}_{\text{M on R}}$ because of Newton's first or second law.

This is also true if the rope is accelerating but has negligibly small mass compared to the block or the mason. In this case, $m_{\text{rope}} = 0$ in the above equation, so again $\vec{F}_{\text{B on R}} = -\vec{F}_{\text{M on R}}$. Since $\vec{F}_{\text{B on R}}$ *always* equals $-\vec{F}_{\text{R on B}}$ by Newton's third law (they are an action–reaction pair), in these same special cases, $\vec{F}_{\text{R on B}}$ also equals $\vec{F}_{\text{M on R}}$ (Fig. 4.27d). In other words, in these cases the force of the rope on the block equals the force of the mason on the rope, and we can then think of the rope as "transmitting" to the block, without change, the force the person exerts on the rope. This is a useful point of view, but you have to remember that it is valid *only* when the rope has negligibly small mass or is in equilibrium.

If you feel as though you're drowning in subscripts at this point, take heart. Go over this discussion again, comparing the symbols with the vector diagrams, until you're sure you see what's going on.

4.27 Identifying the forces that act when a mason pulls on a rope attached to a block.



Conceptual Example 4.11 A Newton's third law paradox?

We saw in Conceptual Example 4.10 that the stonemason pulls as hard on the rope–block combination as that combination pulls back on him. Why, then, does the block move while the stonemason remains stationary?

SOLUTION

The way out of this seeming conundrum is to keep in mind the difference between Newton's *second* law and his *third* law. The only forces involved in Newton's second law are those that act *on* that body. The vector sum of these forces determines how the body accelerates (and whether it accelerates at all). By contrast, Newton's third law relates the forces that two *different* bodies exert on each other. The third law alone tells you nothing about the motion of either body.

If the rope–block combination is initially at rest, it begins to slide if the stonemason exerts a force $\vec{F}_{\text{M on R}}$ that is *greater* in magnitude than the friction force that the floor exerts on the block (Fig. 4.28). (The marble block has a smooth underside, which helps to minimize friction.) Hence there is a net force on the rope–block combination to the right, and so it accelerates to the right. By contrast, the stonemason *doesn't* move because the net force acting on him is *zero*. His shoes have nonskid soles that don't

Continued

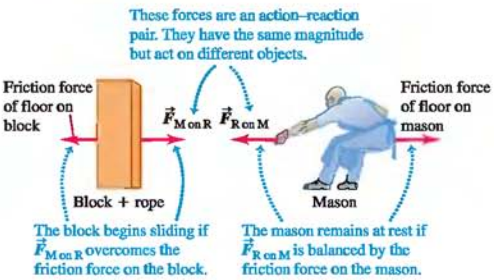
slip on the floor, so the friction force that the floor exerts on him is strong enough to exactly balance the pull of the rope, $\vec{F}_{R\,on\,M}$. (Both the block and the stonemason also experience a downward force of gravity and an upward normal force exerted by the floor. These balance each other and cancel out, so we haven't included them in Fig. 4.28.)

Once the block is moving, the stonemason doesn't need to pull quite so hard; he need exert only enough force to exactly balance the friction force on the block. Then the net force on the moving block is zero, and the block continues to move toward the mason at a constant velocity in accordance with Newton's first law.

We conclude that the block moves while the stonemason doesn't because different amounts of friction act on them. If the floor were freshly waxed, so that there was little friction between the floor and the stonemason's shoes, pulling on the rope would start the block sliding to the right *and* start him sliding to the left.

The moral of this example is that when analyzing the motion of a body, remember that only forces acting *on* the body determine its motion. From this perspective, Newton's third law is merely a tool that can help you determine what those forces are.

4.28 The horizontal forces acting on the block–rope combination (left) and the mason (right). (The vertical forces are not shown.)



A body, such as the rope in Fig. 4.27, that has pulling forces applied at its ends is said to be in *tension*. The **tension** at any point is the magnitude of force acting at that point (see Fig. 4.2c). In Fig. 4.27b the tension at the right end of the rope is the magnitude of $\vec{F}_{M\,on\,R}$ (or of $\vec{F}_{R\,on\,M}$), and the tension at the left end equals the magnitude of $\vec{F}_{B\,on\,R}$ (or of $\vec{F}_{R\,on\,B}$). If the rope is in equilibrium and if no forces act except at its ends, the tension is the *same* at both ends and throughout the rope. Thus, if the magnitudes of $\vec{F}_{B\,on\,R}$ and $\vec{F}_{M\,on\,R}$ are 50 N each, the tension in the rope is 50 N (*not* 100 N). The *total* force vector $\vec{F}_{B\,on\,R} + \vec{F}_{M\,on\,R}$ acting on the rope in this case is zero!

We emphasize once more a fundamental truth: The two forces in an action–reaction pair *never* act on the same body. Remembering this simple fact can often help you avoid confusion about action–reaction pairs and Newton's third law.

Test Your Understanding of Section 4.5 You are driving your car on a country road when a mosquito splatters itself on the windshield. Which has the greater magnitude, the force that the car exerted on the mosquito or the force that the mosquito exerted on the car? Or are the magnitudes the same? If they are different, how can you reconcile this fact with Newton's third law? If they are equal, why is the mosquito splattered while the car is undamaged?

4.6 Free-Body Diagrams

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2.1.1 Force Magnitudes

Newton's three laws of motion contain all the basic principles we need to solve a wide variety of problems in mechanics. These laws are very simple in form, but the process of applying them to specific situations can pose real challenges. In this brief section we'll point out three key ideas and techniques to use in any problems involving Newton's laws. You'll learn others in Chapter 5, which also extends the use of Newton's laws to cover more complex situations.

1. *Newton's first and second laws apply to a specific body.* Whenever you use Newton's first law, $\sum\vec{F} = \mathbf{0}$, for an equilibrium situation or Newton's second law, $\sum\vec{F} = m\vec{a}$, for a nonequilibrium situation, you must decide at the

beginning to which body you are referring. This decision may sound trivial, but it isn't.

2. *Only forces acting on the body matter.* The sum $\sum\vec{F}$ includes all the forces that act *on* the body in question. Hence, once you've chosen the body to analyze, you have to identify all the forces acting on it. Don't get confused between the forces acting on a body and the forces exerted by that body on some other body. For example, to analyze a person walking, you would include in $\sum\vec{F}$ the force that the ground exerts on the person as he walks, but *not* the force that the person exerts on the ground (Fig. 4.29). These forces form an action–reaction pair and are related by Newton's third law, but only the member of the pair that acts on the body you're working with goes into $\sum\vec{F}$.
3. *Free-body diagrams are essential to help identify the relevant forces.* A **free-body diagram** is a diagram showing the chosen body by itself, "free" of its surroundings, with vectors drawn to show the magnitudes and directions of all the forces applied to the body by the various other bodies that interact with it. We have already shown some free-body diagrams in Figs. 4.18, 4.19, 4.21, and 4.26a. Be careful to include all the forces acting *on* the body, but be equally careful *not* to include any forces that the body exerts on any other body. In particular, the two forces in an action–reaction pair must *never* appear in the same free-body diagram because they never act on the same body. Furthermore, forces that a body exerts on itself are never included, since these can't affect the body's motion.

4.29 The simple act of walking depends crucially on Newton's third law. To start moving forward, you push backward on the ground with your foot. As a reaction, the ground pushes forward on your foot (and hence on your body as a whole) with a force of the same magnitude. This *external* force provided by the ground is what accelerates your body forward.

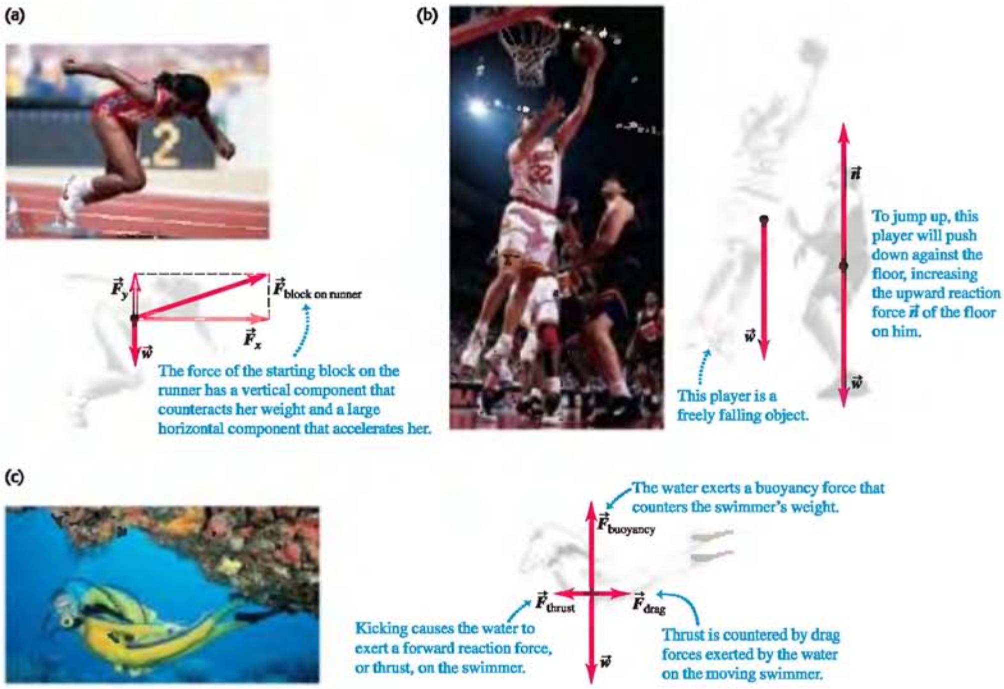
CAUTION **Forces in free-body diagrams** When you have a complete free-body diagram, you *must* be able to answer for each force the question: What other body is applying this force? If you can't answer that question, you may be dealing with a nonexistent force. Be especially on your guard to avoid nonexistent forces such as "the force of acceleration" or "the $m\vec{a}$ force," discussed in Section 4.3.

When a problem involves more than one body, you have to take the problem apart and draw a separate free-body diagram for each body. For example, Fig. 4.27c shows a separate free-body diagram for the rope in the case in which the rope is considered massless (so that no gravitational force acts on it). Figure 4.28 also shows diagrams for the block and the mason, but these are *not* complete free-body diagrams because they don't show all the forces acting on each body. (We left out the vertical forces—the weight force exerted by the earth and the upward normal force exerted by the floor.)

Figure 4.30 on page 128 presents some real-life situations and the corresponding complete free-body diagrams. Note that in each situation a person exerts a force on something in his or her surroundings, but the force that shows up in the person's free-body diagram is the surroundings pushing back *on* the person.

Test Your Understanding of Section 4.6 The buoyancy force shown in Fig. 4.30c is one half of an action–reaction pair. What force is the other half of this pair? (i) the weight of the swimmer; (ii) the forward thrust force; (iii) the backward drag force; (iv) the downward force that the swimmer exerts on the water; (v) the backward force that the swimmer exerts on the water by kicking.

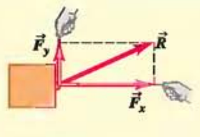
4.30 Examples of free-body diagrams. In each case, the free-body diagram shows all the external forces that act on the object in question.



CHAPTER 4 SUMMARY

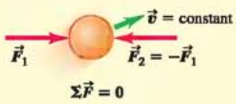
Force as a vector: Force is a quantitative measure of the interaction between two bodies. It is a vector quantity. When several forces act on a body, the effect on its motion is the same as when a single force, equal to the vector sum (resultant) of the forces, acts on the body. (See Example 4.1.)

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F} \quad (4.1)$$



The net force on a body and Newton's first law: Newton's first law states that when the vector sum of all forces acting on a body (the *net force*) is zero, the body is in equilibrium and has zero acceleration. If the body is initially at rest, it remains at rest; if it is initially in motion, it continues to move with constant velocity. This law is valid only in inertial frames of reference. (See Examples 4.2 and 4.3.)

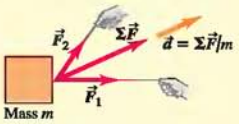
$$\sum \vec{F} = 0 \quad (4.3)$$



Mass, acceleration, and Newton's second law: The inertial properties of a body are characterized by its *mass*. The acceleration of a body under the action of a given set of forces is directly proportional to the vector sum of the forces (the *net force*) and inversely proportional to the mass of the body. This relationship is Newton's second law. Like Newton's first law, this law is valid only in inertial frames of reference. The unit of force is defined in terms of the units of mass and acceleration. In SI units, the unit of force is the newton (N), equal to 1 kg · m/s². (See Examples 4.4 and 4.5.)

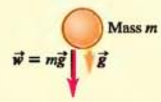
$$\sum \vec{F} = m\vec{a} \quad (4.7)$$

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y \quad (4.8)$$
$$\sum F_z = ma_z$$



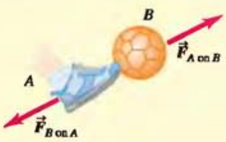
Weight: The weight $\vec{w}$ of a body is the gravitational force exerted on it by the earth. Weight is a vector quantity. The magnitude of the weight of a body at any specific location is equal to the product of its mass m and the magnitude of the acceleration due to gravity g at that location. While the weight of a body depends on its location, the mass is independent of location. (See Examples 4.6 and 4.7.)

$$w = mg \quad (4.9)$$



Newton's third law and action–reaction pairs: Newton's third law states that when two bodies interact, they exert forces on each other that at each instant are equal in magnitude and opposite in direction. These forces are called action and reaction forces. Each of these two forces acts on only one of the two bodies; they never act on the same body. (See Examples 4.8–4.11.)

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A} \quad (4.11)$$



Key Terms

dynamics, *107*
Newton's laws of motion, *107*
classical (Newtonian) mechanics, *107*
force, *108*
contact force, *108*
normal force, *108*
friction force, *108*
tension force, *108*
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tension, *126*
free-body diagram, *127*

Answer to Chapter Opening Question ?

Newton's third law tells us that the seated child (who we'll call Ryder) pushes on the standing child (who we'll call Stan) just as hard as Stan pushes on Ryder, but in the opposite direction. This is true whether Ryder pushes on Stan "actively" (for instance, if Ryder pushed his hand against Stan's) or "passively" (if Ryder's back does the pushing, as in the photograph that opens the chapter). The force magnitudes would be greater in the "active" case than in the "passive" case, but either way Ryder's push on Stan is just as strong as Stan's push on Ryder.

Answers to Test Your Understanding Questions

4.1 Answer: (iv) The gravitational force on the crate points straight downward. In Fig. 4.6 the x-axis points up and to the right, and the y-axis points up and to the left. Hence the gravitational force has both an x-component and a y-component, and both are negative.

4.2 Answer: (i), (ii), and (iv) In (i), (ii), and (iv) the body is not accelerating, so the net force on the body is zero. [In (iv), the box remains stationary as seen in the inertial reference frame of the ground as the truck accelerates forward, like the skater in Fig. 4.11a.] In (iii), the hawk is moving in a circle; hence it is accelerating and is *not* in equilibrium.

4.3 Answer: (iii), (i) and (iv) (tie), (ii) The acceleration is equal to the net force divided by the mass. Hence the magnitude of the acceleration in each situation is
(i) $a = (2.0\text{ N})/(2.0\text{ kg}) = 1.0\text{ m/s}^2$;
(ii) $a = (8.0\text{ N})/(2.0\text{ N}) = 4.0\text{ m/s}^2$;
(iii) $a = (2.0\text{ N})/(8.0\text{ kg}) = 0.25\text{ m/s}^2$;
(iv) $a = (8.0\text{ N})/(8.0\text{ kg}) = 1.0\text{ m/s}^2$.

4.4 It would take twice the effort for the astronaut to walk around because her weight on the planet would be twice as much as on the earth. But it would be just as easy to catch a ball moving horizontally. The ball's *mass* is the same as on earth, so the horizontal force the astronaut would have to exert to bring it to a stop (i.e., to give it the same acceleration) would also be the same as on earth.

4.5 By Newton's third law, the two forces have equal magnitudes. Because the car has much greater mass than the mosquito, it undergoes only a tiny, imperceptible acceleration in response to the force of the impact. By contrast, the mosquito, with its minuscule mass, undergoes a catastrophically large acceleration.

4.6 Answer: (iv) The buoyancy force is an *upward* force that the *water* exerts on the *swimmer*. By Newton's third law, the other half of the action–reaction pair is a *downward* force that the *swimmer* exerts on the *water* and has the same magnitude as the buoyancy force. It's true that the weight of the swimmer is also downward and has the same magnitude as the buoyancy force; however, the weight acts on the same body (the swimmer) as the buoyancy force, and so these forces aren't an action–reaction pair.

PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com

Discussion Questions

Q4.1. Can a body be in equilibrium when only one force acts on it? Explain.

Q4.2. A ball thrown straight up has zero velocity at its highest point. Is the ball in equilibrium at this point? Why or why not?

Q4.3. A helium balloon hovers in midair, neither ascending nor descending. Is it in equilibrium? What forces act on it?

Q4.4. When you fly in an airplane at night in smooth air, there is no sensation of motion, even though the plane may be moving at 800 km/h (500 mi/h). Why is this?

Q4.5. If the two ends of a rope in equilibrium are pulled with forces of equal magnitude and opposite direction, why is the total tension in the rope not zero?

Q4.6. You tie a brick to the end of a rope and whirl the brick around you in a horizontal circle. Describe the path of the brick after you suddenly let go of the rope.

Q4.7. When a car stops suddenly, the passengers tend to move forward relative to their seats. Why? When a car makes a sharp turn, the passengers tend to slide to one side of the car. Why?

Q4.8. Some people say that the "force of inertia" (or "force of momentum") throws the passengers forward when a car brakes sharply. What is wrong with this explanation?

Q4.9. A passenger in a moving bus with no windows notices that a ball that has been at rest in the aisle suddenly starts to move toward the rear of the bus. Think of two different possible explanations, and devise a way to decide which is correct.

Q4.10. Suppose you chose the fundamental SI units to be force, length, and time instead of mass, length, and time. What would be the units of mass in terms of those fundamental units?

Q4.11. Some of the ancient Greeks thought that the "natural state" of an object was to be at rest, so objects would seek their natural state by coming to rest if left alone. Explain why this view can actually seem quite plausible in the everyday world.

Q4.12. Why is the earth only approximately an inertial reference frame?

Q4.13. Does Newton's second law hold true for an observer in a van as it speeds up, slows down, or rounds a corner? Explain.

Q4.14. Some students refer to the quantity $m\vec{a}$ as "the force of acceleration." Is it correct to refer to this quantity as a force? If so, what exerts this force? If not, what is a better description of this quantity?

Q4.15. The acceleration of a falling body is measured in an elevator traveling upward at a constant speed of 9.8 m/s. What result is obtained?

Q4.16. You can play catch with a softball in a bus moving with constant speed on a straight road, just as though the bus were at rest. Is this still possible when the bus is making a turn at constant speed on a level road? Why or why not?

Q4.17. Students sometimes say that the force of gravity on an object is $9.8\ m/s^2$. What is wrong with this view?

Q4.18. The head of a hammer begins to come loose from its wooden handle. How should you strike the handle on a concrete sidewalk to reset the head? Why does this work?

Q4.19. Why can it hurt your foot more to kick a big rock than a small pebble? *Must* the big rock hurt more? Explain.

Q4.20. "It's not the fall that hurts you; it's the sudden stop at the bottom." Translate this saying into the language of Newton's laws of motion.

Q4.21. A person can dive into water from a height of 10 m without injury, but a person who jumps off the roof of a 10-m-tall building and lands on a concrete street is likely to be seriously injured. Why is there a difference?

Q4.22. Why are cars designed to crumple up in front and back for safety? Why not for side collisions and rollovers?

Q4.23. When a bullet is fired from a gun, what is the origin of the force that accelerates the bullet?

Q4.24. When a string barely strong enough lifts a heavy weight, it can lift the weight by a steady pull; but if you jerk the string, it will break. Explain in terms of Newton's laws of motion.

Q4.25. A large crate is suspended from the end of a vertical rope. Is the tension in the rope greater when the crate is at rest or when it is moving upward at constant speed? If the crate is traveling upward, is the tension in the rope greater when the crate is speeding up or when it is slowing down? In each case explain in terms of Newton's laws of motion.

Q4.26. Which feels a greater pull due to the earth's gravity, a 10-kg stone or a 20-kg stone? If you drop them, why does the 20-kg stone not fall with twice the acceleration of the 10-kg stone? Explain your reasoning.

Q4.27. Why is it incorrect to say that 1.0 kg *equals* 2.2 lb?

Q4.28. A horse is hitched to a wagon. Since the wagon pulls back on the horse just as hard as the horse pulls on the wagon, why does the wagon not remain in equilibrium, no matter how hard the horse pulls?

Q4.28. True or false? You exert a push P on an object and it pushes back on you with a force F. If the object is moving at constant velocity, then F is equal to P, but if the object is being accelerated, then P must be greater than F.

Q4.30. A large truck and a small compact car have a head-on collision. During the collision, the truck exerts a force $\vec{F}_{T\text{ on }C}$ on the car, and the car exerts a force $\vec{F}_{C\text{ on }T}$ on the truck. Which force has the larger magnitude, or are they the same? Does your answer depend on how fast each vehicle was moving before the collision? Why or why not?

Q4.31. When a car comes to a stop on a level highway, what force causes it to slow down? When the car increases its speed on the same highway, what force causes it to speed up? Explain.

Q4.32. A small compact car is pushing a large van that has broken down, and they travel along the road with equal velocities and accelerations. While the car is speeding up, is the force it exerts on the van larger than, smaller than, or the same magnitude as the force the van exerts on it? Which object, the car or the van, has the larger net force on it, or are the net forces the same? Explain.

Q4.33. Consider a tug-of-war between two people who pull in opposite directions on the ends of a rope. By Newton's third law, the force that A exerts on B is just as great as the force that B exerts on A. So what determines who wins? (*Hint:* Draw a free-body diagram showing all the forces that act on each person.)

Q4.34. On the moon, $g = 1.62\ m/s^2$. If a 2-kg brick drops on your foot from a height of 2 m, will this hurt more, or less, or the same if it happens on the moon instead of on the earth? Explain. If a 2-kg brick is thrown and hits you when it is moving horizontally at 6 m/s, will this hurt more, less, or the same if it happens on the moon instead of on the earth? Explain. (On the moon, assume that you are inside a pressurized structure, so you are not wearing a spacesuit.)

Q4.35. A manual for student pilots contains the following passage: "When an airplane flies at a steady altitude, neither climbing nor descending, the upward lift force from the wings equals the airplane's weight. When the airplane is climbing at a steady rate, the upward lift is greater than the weight; when the airplane is descending at a steady rate, the upward lift is less than the weight." Are these statements correct? Explain.

Q4.36. If your hands are wet and no towel is handy, you can remove some of the excess water by shaking them. Why does this get rid of the water?

Q4.37. If you are squatting down (such as when you are examining the books on the bottom shelf in a library or bookstore) and suddenly get up, you can temporarily feel light-headed. What do Newton's laws of motion have to say about why this happens?

Q4.38. When a car is hit from behind, the passengers can receive a whiplash. Use Newton's laws of motion to explain what causes this to occur.

Q4.38. In a head-on auto collision, passengers not wearing seat belts can be thrown through the windshield. Use Newton's laws of motion to explain why this happens.

Q4.40. In a head-on collision between a compact 1000-kg car and a large 2500-kg car, which one experiences the greater force? Explain. Which one experiences the greater acceleration? Explain why. Now explain why passengers in the smaller car are more likely to be injured than those in the large car, even if the bodies of both cars are equally strong.

Q4.41. Suppose you are in a rocket with no windows, traveling in deep space far from any other objects. Without looking outside the rocket or making any contact with the outside world, explain how you could determine if the rocket is (a) moving forward at a constant 80% of the speed of light and (b) accelerating in the forward direction.

Exercises

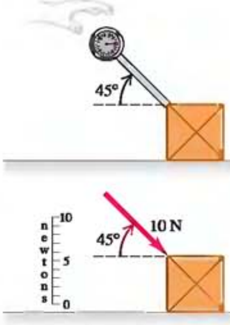
Section 4.1 Force and Interactions

4.1. Two forces have the same magnitude F. What is the angle between the two vectors if their sum has a magnitude of (a) $2F$? (b) $\sqrt{2}F$? (c) zero? Sketch the three vectors in each case.

4.2. Instead of using the x- and y-axes of Fig. 4.8 to analyze the situation of Example 4.1, use axes rotated 37.0° counterclockwise, so the y-axis is parallel to the 250-N force. (a) For these axes find the x- and y-components of the net force on the belt. (b) From the components computed in part (a) find the magnitude and direction of the net force. Compare your results to Example 4.1.

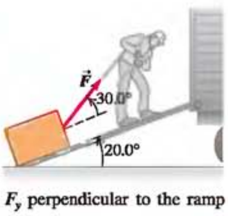
4.3. A warehouse worker pushes a crate along the floor, as shown in Fig. 4.31, with a force of 10 N that points downward at an angle of 45° below the horizontal. Find the horizontal and vertical components of the force.

Figure 4.31 Exercise 4.3.



4.4. A man is dragging a trunk up the loading ramp of a mover's truck. The ramp has a slope angle of 20.0°, and the man pulls upward with a force $\vec{F}$ whose direction makes an angle of 30.0° with the ramp (Fig. 4.32). (a) How large a force $\vec{F}$ is necessary for the component F_x parallel to the ramp to be 60.0 N? (b) How large will the component F_y perpendicular to the ramp then be?

Figure 4.32 Exercise 4.4.



4.5. Two dogs pull horizontally on ropes attached to a post; the angle between the ropes is 60.0°. If dog *A* exerts a force of 270 N and dog *B* exerts a force of 300 N, find the magnitude of the resultant force and the angle it makes with dog *A*'s rope.

4.6. Two forces, $\vec{F}_1$ and $\vec{F}_2$, act at a point. The magnitude of $\vec{F}_1$ is 9.00 N, and its direction is 60.0° above the *x*-axis in the second quadrant. The magnitude of $\vec{F}_2$ is 6.00 N, and its direction is 53.1° below the *x*-axis in the third quadrant. (a) What are the *x*- and *y*-components of the resultant force? (b) What is the magnitude of the resultant force?

Section 4.3 Newton's Second Law

4.7. If a net horizontal force of 132 N is applied to a person with mass 60 kg who is resting on the edge of a swimming pool, what horizontal acceleration is produced?

4.8. What magnitude of net force is required to give a 135-kg refrigerator an acceleration of magnitude 1.40 m/s^2?

4.9. A box rests on a frozen pond, which serves as a frictionless horizontal surface. If a fisherman applies a horizontal force with magnitude 48.0 N to the box and produces an acceleration of magnitude 3.00 m/s^2, what is the mass of the box?

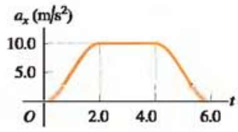
4.10. A dockworker applies a constant horizontal force of 80.0 N to a block of ice on a smooth horizontal floor. The frictional force is negligible. The block starts from rest and moves 11.0 m in 5.00 s. (a) What is the mass of the block of ice? (b) If the worker stops pushing at the end of 5.00 s, how far does the block move in the next 5.00 s?

4.11. A hockey puck with mass 0.160 kg is at rest at the origin $(x = 0)$ on the horizontal, frictionless surface of the rink. At time $t = 0$ a player applies a force of 0.250 N to the puck, parallel to the *x*-axis; he continues to apply this force until $t = 2.00$ s. (a) What are the position and speed of the puck at $t = 2.00$ s? (b) If the same force is again applied at $t = 5.00$ s, what are the position and speed of the puck at $t = 7.00$ s?

4.12. A crate with mass 32.5 kg initially at rest on a warehouse floor is acted on by a net horizontal force of 140 N. (a) What acceleration is produced? (b) How far does the crate travel in 10.0 s? (c) What is its speed at the end of 10.0 s?

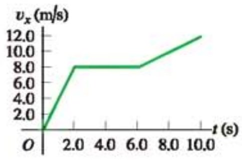
4.13. A 4.50-kg toy cart undergoes an acceleration in a straight line (the *x*-axis). The graph in Fig. 4.33 shows this acceleration as a function of time. (a) Find the maximum net force on this cart. When does this maximum force occur? (b) During what times is the net force on the cart a constant? (c) When is the net force equal to zero?

Figure 4.33 Exercise 4.13.



4.14. A 2.75-kg cat moves in a straight line (the *x*-axis). Figure 4.34 shows a graph of the *x*-component of this cat's velocity as a function of time. (a) Find the maximum net force on this cat. When does this force occur? (b) When is the net force on the cat equal to zero? (c) What is the net force at time 8.5 s?

Figure 4.34 Exercise 4.14.



4.15. A small 8.00-kg rocket burns fuel that exerts a time-varying upward force on the rocket. This force obeys the equation $F = A + Bt^2$. Measurements show that at $t = 0$, the force is 100.0 N, and at the end of the first 2.00 s, it is 150.0 N. (a) Find the constants *A* and *B*, including their SI units. (b) Find the *net* force on this rocket and its acceleration (i) the instant after the fuel ignites and (ii) 3.00 s after fuel ignition. (c) Suppose you were using this rocket in outer space, far from all gravity. What would its acceleration be 3.00 s after fuel ignition?

4.16. An electron $(\text{mass} = 9.11 \times 10^{-31}\ \text{kg})$ leaves one end of a TV picture tube with zero initial speed and travels in a straight line to the accelerating grid, which is 1.80 cm away. It reaches the grid with a speed of 3.00×10^6 m/s. If the accelerating force is constant, compute (a) the acceleration; (b) the time to reach the grid; (c) the net force, in newtons. (You can ignore the gravitational force on the electron.)

Section 4.4 Mass and Weight

4.17. Superman throws a 2400-N boulder at an adversary. What horizontal force must Superman apply to the boulder to give it a horizontal acceleration of 12.0 m/s^2?

4.18. A bowling ball weighs 71.2 N (16.0 lb). The bowler applies a horizontal force of 160 N (36.0 lb) to the ball. What is the magnitude of the horizontal acceleration of the ball?

4.19. At the surface of Jupiter's moon Io, the acceleration due to gravity is $g = 1.81 \text{ m/s}^2$. A watermelon weighs 44.0 N at the surface of the earth. (a) What is the watermelon's mass on the earth's surface? (b) What are its mass and weight on the surface of Io?

4.20. An astronaut's pack weighs 17.5 N when she is on earth but only 3.24 N when she is at the surface of an asteroid. (a) What is the acceleration due to gravity on this asteroid? (b) What is the mass of the pack on the asteroid?

Section 4.5 Newton's Third Law

4.21. World-class sprinters can accelerate out of the starting blocks with an acceleration that is nearly horizontal and has magnitude 15 m/s^2. How much horizontal force must a 55-kg sprinter exert on the starting blocks during a start to produce this acceleration? Which body exerts the force that propels the sprinter: the blocks or the sprinter herself?

4.22. Imagine that you are holding a book weighing 4 N at rest on the palm of your hand. Complete the following sentences: (a) A downward force of magnitude 4 N is exerted on the book by __________. (b) An upward force of magnitude __________ is exerted on __________ by your hand. (c) Is the upward force in part (b) the reaction to the downward force in part (a)? (d) The reaction to the force in part (a) is a force of magnitude __________, exerted on __________ by __________. Its direction is __________. (e) The reaction to the force in part (b) is a force of magnitude __________, exerted on __________ by __________. Its direction is __________. (f) The forces in parts (a) and (b) are equal and opposite because of Newton's __________ law. (g) The forces in parts (b) and (e) are equal and opposite because of Newton's __________ law. Now suppose that you exert an upward force of magnitude 5 N on the book. (h) Does the book remain in equilibrium? (i) Is the force exerted on the book by your hand equal and opposite to the force exerted on the book by the earth? (j) Is the force exerted on the book by the earth equal and opposite to the force exerted on the earth by the book? (k) Is the force exerted on the book by your hand equal and opposite to the force exerted on your hand by the book? Finally, suppose you snatch your hand away while the book is moving upward. (l) How many forces then act on the book? (m) Is the book in equilibrium?

4.23. A bottle is given a push along a tabletop and slides off the edge of the table. Do *not* ignore air resistance. (a) What forces are exerted on the bottle while it is falling from the table to the floor? (b) What is the reaction to each force; that is, on which body and by which body is the reaction exerted?

4.24. The upward normal force exerted by the floor is 620 N on an elevator passenger who weighs 650 N. What are the reaction forces to these two forces? Is the passenger accelerating? If so, what are the magnitude and direction of the acceleration?

4.25. A student with mass 45 kg jumps off a high diving board. Using 6.0×10^{24} kg for the mass of the earth, what is the acceleration of the earth toward her as she accelerates toward the earth with an acceleration of 9.8 m/s^2? Assume that the net force on the earth is the force of gravity she exerts on it.

Section 4.6 Free-Body Diagrams

4.26. An athlete throws a ball of mass m directly upward, and it feels no appreciable air resistance. Draw a free-body diagram of this ball while it is free of the athlete's hand and (a) moving upward; (b) at its highest point; (c) moving downward. (d) Repeat parts (a), (b), and (c) if the athlete throws the ball at a 60° angle above the horizontal instead of directly upward.

4.27. Two crates, A and B, sit at rest side by side on a frictionless horizontal surface. The crates have masses m_A and m_B. A horizontal force $\vec{F}$ is applied to crate A and the two crates move off to the right. (a) Draw clearly labeled free-body diagrams for crate A and for crate B. Indicate which pairs of forces, if any, are third-law action–reaction pairs. (b) If the magnitude of force $\vec{F}$ is less than the total weight of the two crates, will it cause the crates to move? Explain.

4.20. A person pulls horizontally on block B in Fig. 4.35, causing both blocks to move together as a unit. While this system is moving, make a carefully labeled free-body diagram of block A if (a) the table is frictionless and (b) there is friction between block B and the table and the pull is equal to the friction force on block B due to the table.

Figure 4.35 Exercise 4.28.

4.29. A ball is hanging from a long string that is tied to the ceiling of a train car traveling eastward on horizontal tracks. An observer inside the train car sees the ball hang motionless. Draw a clearly labeled free-body diagram for the ball if (a) the train has a uniform velocity, and (b) the train is speeding up uniformly. Is the net force on the ball zero in either case? Explain.

4.30. A large box containing your new computer sits on the bed of your pickup truck. You are stopped at a red light. The light turns green and you stomp on the gas and the truck accelerates. To your horror, the box starts to slide toward the back of the truck. Draw clearly labeled free-body diagrams for the truck and for the box. Indicate pairs of forces, if any, that are third-law action–reaction pairs. (The bed of the truck is *not* frictionless.)

4.31. A chair of mass 12.0 kg is sitting on the horizontal floor; the floor is not frictionless. You push on the chair with a force $F = 40.0$ N that is directed at an angle of 37.0° below the horizontal and the chair slides along the floor. (a) Draw a clearly labeled free-body diagram for the chair. (b) Use your diagram and Newton's laws to calculate the normal force that the floor exerts on the chair.

4.32. A skier of mass 65.0 kg is pulled up a snow-covered slope at constant speed by a tow rope that is parallel to the ground. The ground slopes upward at a constant angle of 26.0° above the horizontal, and you can ignore friction. (a) Draw a clearly labeled free-body diagram for the skier. (b) Calculate the tension in the tow rope.

4.33. A truck is pulling a car on a horizontal highway using a horizontal rope. The car is in neutral gear, so we can assume that there is no appreciable friction between its tires and the highway. As the truck is accelerating to highway speeds, draw a free-body diagram of (a) the car and (b) the truck. (c) What force accelerates this system forward? Explain how this force originates.

Problems

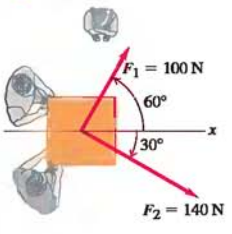
4.34. A .22 rifle bullet, traveling at 350 m/s, strikes a large tree, which it penetrates to a depth of 0.130 m. The mass of the bullet is 1.80 g. Assume a constant retarding force. (a) How much time is required for the bullet to stop? (b) What force, in newtons, does the tree exert on the bullet?

4.35. Two horses pull horizontally on ropes attached to a stump. The two forces $\vec{F}_1$ and $\vec{F}_2$ that they apply to the stump are such that the net (resultant) force $\vec{R}$ has a magnitude equal to that of $\vec{F}_1$ and makes an angle of 90° with $\vec{F}_1$. Let $F_1 = 1300$ N and $R = 1300$ N also. Find the magnitude of $\vec{F}_2$ and its direction (relative to $\vec{F}_1$).

4.36. You have just landed on Planet X. You take out a 100-g ball, release it from rest from a height of 10.0 m, and measure that it takes 2.2 s to reach the ground. You can ignore any force on the ball from the atmosphere of the planet. How much does the 100-g ball weigh on the surface of Planet X?

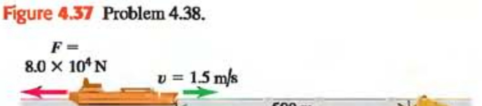
4.37. Two adults and a child want to push a wheeled cart in the direction marked x in Fig. 4.36. The two adults push with horizontal forces $\vec{F}_1$ and $\vec{F}_2$ as shown in the figure. (a) Find the magnitude and direction of the *smallest* force that the child should exert. You can ignore the effects of friction. (b) If the child exerts the minimum force found in part (a), the cart accelerates at $2.0\ \text{m/s}^2$ in the $+x$-direction. What is the weight of the cart?

Figure 4.36 Problem 4.37.



4.38. An oil tanker's engines have broken down, and the wind is blowing the tanker straight toward a reef at a constant speed of 1.5 m/s (Fig. 4.37). When the tanker is 500 m from the reef, the wind dies down just as the engineer gets the engines going again. The rudder is stuck, so the only choice is to try to accelerate straight backward away from the reef. The mass of the tanker and cargo is 3.6×10^7 kg, and the engines produce a net horizontal force of 8.0×10^4 N on the tanker. Will the ship hit the reef? If it does, will the oil be safe? The hull can withstand an impact at a speed of 0.2 m/s or less. You can ignore the retarding force of the water on the tanker's hull.

Figure 4.37 Problem 4.38.



4.39. A Standing Vertical Jump. Basketball player Darrell Griffith is on record as attaining a standing vertical jump of 1.2 m (4 ft). (This means that he moved upward by 1.2 m after his feet left the floor.) Griffith weighed 890 N (200 lb). (a) What is his speed as he leaves the floor? (b) If the time of the part of the jump before his feet left the floor was 0.300 s, what was his average acceleration (magnitude and direction) while he was pushing against the floor? (c) Draw his free-body diagram (see Section 4.6). In terms of the forces on the diagram, what is the net force on him? Use Newton's laws and the results of part (b) to calculate the average force he applied to the ground.

4.40. An advertisement claims that a particular automobile can "stop on a dime." What net force would actually be necessary to stop a 850-kg automobile traveling initially at 45.0 km/h in a distance equal to the diameter of a dime, which is 1.8 cm?

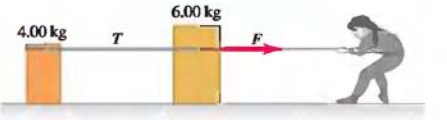
4.41. A 4.80-kg bucket of water is accelerated upward by a cord of negligible mass whose breaking strength is 75.0 N. (a) Draw the free-body force diagram for the bucket. In terms of the forces on your diagram, what is the net force on the bucket? (b) Apply Newton's second law to the bucket and find the maximum upward acceleration that can be given to the bucket without breaking the cord.

4.42. A parachutist relies on air resistance (mainly on her parachute) to decrease her downward velocity. She and her parachute have a mass of 55.0 kg, and air resistance exerts a total upward force of 620 N on her and her parachute. (a) What is the weight of the parachutist? (b) Draw a free-body diagram for the parachutist (see Section 4.6). Use that diagram to calculate the net force on the parachutist. Is the net force upward or downward? (c) What is the acceleration (magnitude and direction) of the parachutist?

4.43. Two crates, one with mass 4.00 kg and the other with mass 6.00 kg, sit on the frictionless surface of a frozen pond, connected by a light rope (Fig. 4.38). A woman wearing golf shoes (so she can get traction on the ice) pulls horizontally on the 6.00-kg crate with a force F that gives the crate an acceleration of $2.50\ \text{m/s}^2$. (a) What is the acceleration of the 4.00-kg crate? (b) Draw a free-body diagram for the 4.00-kg crate. Use that diagram and Newton's second law to find the tension T in the rope that connects the two crates. (c) Draw a free-body diagram for the 6.00-kg crate. What is the direction of the net force on the 6.00-kg crate? Which is larger in magnitude, force T or force F? (d) Use part (c) and Newton's second law to calculate the magnitude of the force F.

4.44. An astronaut is tethered by a strong cable to a spacecraft. The astronaut and her spacesuit have a total mass of 105 kg, while the mass of the cable is negligible. The mass of the spacecraft is 9.05×10^4 kg. The spacecraft is far from any large astronomical bodies, so we can ignore the gravitational forces on it and the astronaut. We also assume that both the spacecraft and the astronaut are initially at rest in an inertial reference frame. The astronaut then pulls on the cable with a force of 80.0 N. (a) What force does the cable exert on the astronaut? (b) Since $\Sigma\vec{F} = m\vec{a}$, how can a "massless" $(m = 0)$ cable exert a force? (c) What is the astronaut's acceleration? (d) What force does the cable exert on the spacecraft? (e) What is the acceleration of the spacecraft?

Figure 4.38 Problem 4.43.



4.45. To study damage to aircraft that collide with large birds, you design a test gun that will accelerate chicken-sized objects so that their displacement along the gun barrel is given by $x = (9.0 \times 10^3\ \text{m/s}^2)t^2 - (8.0 \times 10^4\ \text{m/s}^3)t^3$. The object leaves the end of the barrel at $t = 0.025$ s. (a) How long must the gun barrel be? (b) What will be the speed of the objects as they leave the end of the barrel? (c) What net force must be exerted on a 1.50-kg object at (i) $t = 0$ and (ii) $t = 0.025$ s?

4.46. A spacecraft descends vertically near the surface of Planet X. An upward thrust of 25.0 kN from its engines slows it down at a rate of $1.20\ \text{m/s}^2$, but it speeds up at a rate of $0.80\ \text{m/s}^2$ with an upward thrust of 10.0 kN. (a) In each case, what is the direction of the acceleration of the spacecraft? (b) Draw a free-body diagram for the spacecraft. In each case, speeding up or slowing down, what is the direction of the net force on the spacecraft? (c) Apply Newton's second law to each case, slowing down or speeding up, and use this to find the spacecraft's weight near the surface of Planet X.

4.47. A 6.50-kg instrument is hanging by a vertical wire inside a space ship that is blasting off at the surface of the earth. This ship starts from rest and reaches an altitude of 276 m in 15.0 s with constant acceleration. (a) Draw a free-body diagram for the instrument during this time. Indicate which force is greater. (b) Find the force that the wire exerts on the instrument.

4.40. Suppose the rocket in Problem 4.47 is coming in for a vertical landing instead of blasting off. The captain adjusts the engine thrust so that the magnitude of the rocket's acceleration is the same as it was during blast-off. Repeat parts (a) and (b).

4.49. A gymnast of mass m climbs a vertical rope attached to the ceiling. You can ignore the weight of the rope. Draw a free-body diagram for the gymnast. Calculate the tension in the rope if the gymnast (a) climbs at a constant rate; (b) hangs motionless on the rope; (c) accelerates up the rope with an acceleration of magnitude $|\vec{a}|$; (d) slides down the rope with a downward acceleration of magnitude $|\vec{a}|$.

4.50. A loaded elevator with very worn cables has a total mass of 2200 kg, and the cables can withstand a maximum tension of 28,000 N. (a) Draw the free-body force diagram for the elevator. In terms of the forces on your diagram, what is the net force on the elevator? Apply Newton's second law to the elevator and find the maximum upward acceleration for the elevator if the cables are not to break. (b) What would be the answer to part (a) if the elevator were on the moon, where $g = 1.62\ \text{m/s}^2$?

4.51. Jumping to the Ground. A 75.0-kg man steps off a platform 3.10 m above the ground. He keeps his legs straight as he falls, but at the moment his feet touch the ground his knees begin to bend, and, treated as a particle, he moves an additional 0.60 m before coming to rest. (a) What is his speed at the instant his feet touch the ground? (b) Treating him as a particle, what is his acceleration (magnitude and direction) as he slows down, if the acceleration is assumed to be constant? (c) Draw his free-body diagram (see Section 4.6). In terms of the forces on the diagram, what is the net force on him? Use Newton's laws and the results of part (b) to calculate the average force his feet exert on the ground while he slows down. Express this force in newtons and also as a multiple of his weight.

4.52. A 4.9-N hammer head is stopped from an initial downward velocity of 3.2 m/s in a distance of 0.45 cm by a nail in a pine board. In addition to its weight, there is a 15-N downward force on the hammer head applied by the person using the hammer. Assume that the acceleration of the hammer head is constant while it is in contact with the nail and moving downward. (a) Draw a free-body diagram for the hammer head. Identify the reaction force to each action force in the diagram. (b) Calculate the downward force $\vec{F}$ exerted by the hammer head on the nail while the hammer head is in contact with the nail and moving downward. (c) Suppose the nail is in hardwood and the distance the hammer head travels in coming to rest is only 0.12 cm. The downward forces on the hammer head are the same as on part (b). What then is the force $\vec{F}$ exerted by the hammer head on the nail while the hammer head is in contact with the nail and moving downward?

4.53. A uniform cable of weight w hangs vertically downward, supported by an upward force of magnitude w at its top end. What is the tension in the cable (a) at its top end; (b) at its bottom end; (c) at its middle? Your answer to each part must include a free-body diagram. (*Hint:* For each question choose the body to analyze to be a section of the cable or a point along the cable.) (d) Graph the tension in the rope versus the distance from its top end.

4.54. The two blocks in Fig. 4.39 are connected by a heavy uniform rope with a mass of 4.00 kg. An upward force of 200 N is applied as shown. (a) Draw three free-body diagrams, one for the 6.00-kg block, one for the 4.00-kg rope, and another one for the 5.00-kg block. For each force, indicate what body exerts that force. (b) What is the acceleration of the system? (c) What is the tension at the top of the heavy rope? (d) What is the tension at the midpoint of the rope?

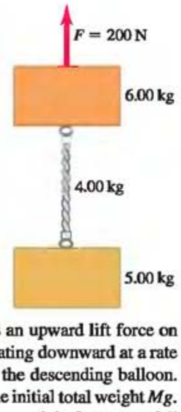


Figure 4.39 Problem 4.54.

4.55. An athlete whose mass is 90.0 kg is performing weight-lifting exercises. Starting from the rest position, he lifts, with constant acceleration, a barbell that weighs 490 N. He lifts the barbell a distance of 0.60 m in 1.6 s. (a) Draw a clearly labeled free-body force diagram for the barbell and for the athlete. (b) Use the diagrams in part (a) and Newton's laws to find the total force that his feet exert on the ground as he lifts the barbell.

4.56. A hot-air balloon consists of a basket, one passenger, and some cargo. Let the total mass be M. Even though there is an upward lift force on the balloon, the balloon is initially accelerating downward at a rate of $g/3$. (a) Draw a free-body diagram for the descending balloon. (b) Find the upward lift force in terms of the initial total weight Mg. (c) The passenger notices that he is heading straight for a waterfall and decides he needs to go up. What fraction of the total weight must he drop overboard so that the balloon accelerates *upward* at a rate of $g/2$? Assume that the upward lift force remains the same.

4.57. A student tries to raise a chain consisting of three identical links. Each link has a mass of 300 g. The three-piece chain is connected to a string and then suspended vertically, with the student holding the upper end of the string and pulling upward. Because of the student's pull, an upward force of 12 N is applied to the chain by the string. (a) Draw a free-body diagram for *each* of the links in the chain and also for the entire chain considered as a single body. (b) Use the results of part (a) and Newton's laws to find (i) the acceleration of the chain and (ii) the force exerted by the top link on the middle link.

4.56. The position of a 2.75×10^5 N training helicopter under test is given by $\vec{r} = (0.020\ \text{m/s}^3)t^3\hat{\imath} + (2.2\ \text{m/s})t\hat{\jmath} - (0.060\ \text{m/s}^2)t^2\hat{k}$. Find the net force on the helicopter at $t = 5.0$ s.

4.59. An object with mass m moves along the x-axis. Its position as a function of time is given by $x(t) = At - Bt^3$, where A and B are constants. Calculate the net force on the object as a function of time.

4.60. An object with mass m initially at rest is acted on by a force $\vec{F} = k_1\hat{\imath} + k_2t^3\hat{\jmath}$, where k_1 and k_2 are constants. Calculate the velocity $\vec{v}(t)$ of the object as a function of time.

Challenge Problems

4.61. If we know $F(t)$, the force as a function of time, for straight-line motion, Newton's second law gives us $a(t)$, the acceleration as a function of time. We can then integrate $a(t)$ to find $v(t)$ and $x(t)$. However, suppose we know $F(v)$ instead. (a) The net force on a body moving along the x-axis equals $-Cv^2$. Use Newton's second law written as $\sum F = m\,dv/dt$ and two integrations to show that $x - x_0 = (m/C)\ln(v_0/v)$. (b) Show that Newton's second law can be written as $\sum F = mv\,dv/dx$. Derive the same expression as in part (a) using this form of the second law and one integration.

4.62. An object of mass m is at rest in equilibrium at the origin. At $t = 0$ a new force $\vec{F}(t)$ is applied that has components

$$F_x(t) = k_1 + k_2y \qquad F_y(t) = k_3t$$

where k_1, k_2, and k_3 are constants. Calculate the position $\vec{r}(t)$ and velocity $\vec{v}(t)$ vectors as functions of time.

5 APPLYING NEWTON'S LAWS

LEARNING GOALS

By studying this chapter, you will learn:

- How to use Newton's first law to solve problems involving the forces that act on a body in equilibrium.
- How to use Newton's second law to solve problems involving the forces that act on an accelerating body.
- The nature of the different types of friction forces—static friction, kinetic friction, rolling friction, and fluid resistance—and how to solve problems that involve these forces.
- How to solve problems involving the forces that act on a body moving along a circular path.
- The key properties of the four fundamental forces of nature.

? Suppose a gliding bird is caught in an updraft so that it ascends at a steady rate. In this situation, which has a greater magnitude: the force of gravity or the upward force of the air on the bird?

We saw in Chapter 4 that Newton's three laws of motion, the foundation of classical mechanics, can be stated very simply. But *applying* these laws to situations such as an iceboat skating across a frozen lake, a toboggan sliding down a hill, or an airplane making a steep turn requires analytical skills and problem-solving technique. In this chapter we'll help you extend the problem-solving skills you began to develop in Chapter 4.

We begin with equilibrium problems, in which a body is at rest or moving with constant velocity. Then we generalize our problem-solving techniques to include bodies that are not in equilibrium, for which we need to deal precisely with the relationships between forces and motion. We will learn how to describe and analyze the contact force acting on a body when it rests or slides on a surface. Finally, we study the important case of uniform circular motion, in which a body moves in a circle with constant speed.

All these situations involve the concept of force, a concept we'll use throughout our study of physics. We close the chapter with a brief look at the fundamental nature of force and the classes of forces found in our physical universe.

5.1 Using Newton's First Law: Particles in Equilibrium

We learned in Chapter 4 that a body is in *equilibrium* when it is at rest or moving with constant velocity in an inertial frame of reference. A hanging lamp, a suspension bridge, an airplane flying straight and level at a constant speed—all are examples of equilibrium situations. In this section we consider only equilibrium of a body that can be modeled as a particle. (In Chapter 11, we'll consider the additional principles needed when the body can't be represented adequately as a particle.) The essential physical principle is Newton's first law: When a particle is at rest or is moving with constant velocity in an inertial frame of reference, the net force acting on it—that is, the vector sum of all the forces acting on it—must be zero:

$$\sum \vec{F} = \mathbf{0} \quad \text{(particle in equilibrium, vector form)} \tag{5.1}$$

We most often use this equation in component form:

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \text{(particle in equilibrium, component form)} \tag{5.2}$$

This section is about using Newton's first law to solve problems dealing with bodies in equilibrium. Some of these problems may seem complicated, but the important thing to remember is that *all* problems involving particles in equilibrium are done in the same way. Problem-Solving Strategy 5.1 details the steps you need to follow for any and all such problems. Study this strategy carefully, look at how it's applied in the worked-out examples, and try to apply it yourself when you solve assigned problems.

Problem-Solving Strategy 5.1 Newton's First Law: Equilibrium of a Particle

IDENTIFY *the relevant concepts:* You must use Newton's first law for any problem that involves forces acting on a body in equilibrium—that is, either at rest or moving with constant velocity. For example, a car is in equilibrium when it's parked, but also when it's traveling down a straight road at a steady speed.

If the problem involves more than one body and the bodies interact with each other, you'll also need to use Newton's *third* law. This law allows you to relate the force that one body exerts on a second body to the force that the second body exerts on the first one.

Be certain that you identify the target variable(s). Common target variables in equilibrium problems include the magnitude of one of the forces, the components of a force, or the direction (angle) of a force.

SET UP *the problem* using the following steps:

1. Draw a very simple sketch of the physical situation, showing dimensions and angles. You don't have to be an artist!
2. Draw a free-body diagram for each body that is in equilibrium. For the present, we consider the body as a particle, so you can represent it as a large dot. In your free-body diagram, *do not* include the other bodies that interact with it, such as a surface it may be resting on, or a rope pulling on it.
3. Ask yourself what is interacting with the body by touching it or in any other way. On your free-body diagram, draw a force vector for each interaction and label each force with a symbol representing the *magnitude* of the force. If you know the angle at which a force is directed, draw the angle accurately and label it. Include the body's weight, except in cases where the body has negligible mass (and hence negligible weight). If the mass is given, use $w = mg$ to find the weight. A surface in contact with the body exerts a normal force perpendicular to the surface and possibly a friction force parallel to the surface. A rope or chain exerts a pull (never a push) in a direction along its length.
4. *Do not* show in the free-body diagram any forces exerted *by* the body on any other body. The sums in Eqs. (5.1) and (5.2) include only forces that act *on* the body. For each force on the body, ask yourself "What other body causes that force?" If you can't answer that question, you may be imagining a force that isn't there.
5. Choose a set of coordinate axes and include them in your free-body diagram. (If there is more than one body in the problem, choose axes for each body separately.) Label the positive direction for each axis. If a body rests or slides on a plane surface, it usually simplifies the solution to take the axes in the directions parallel and perpendicular to this surface, even when the plane is tilted.

EXECUTE *the solution* as follows:

1. Find the components of each force along each of the body's coordinate axes. Draw a wiggly line through each force vector that has been replaced by its components, so you don't count it twice. Remember that while the *magnitude* of a force is always positive, the *component* of a force along a particular direction may be positive or negative.
2. Set the algebraic sum of all x-components of force equal to zero. In a separate equation, set the algebraic sum of all y-components equal to zero. (*Never* add x- and y-components in a single equation.)
3. If there are two or more bodies, repeat all of the above steps for each body. If the bodies interact with each other, use Newton's third law to relate the forces they exert on each other.
4. Make sure that you have as many independent equations as the number of unknown quantities. Then solve these equations to obtain the target variables.

EVALUATE *your answer:* Look at your results and ask whether they make sense. When the result is a symbolic expression or formula, try to think of special cases (particular values or extreme cases for the various quantities) for which you can guess what the results ought to be. Check to see that your formula works in these particular cases.

Example 5.1 One-dimensional equilibrium: Tension in a massless rope

A gymnast with mass $m_G = 50.0\text{ kg}$ suspends herself from the lower end of a hanging rope. The upper end of the rope is attached to the gymnasium ceiling. What is the gymnast's weight? What force (magnitude and direction) does the rope exert on her? What is the tension at the top of the rope? Assume that the mass of the rope itself is negligible.

SOLUTION

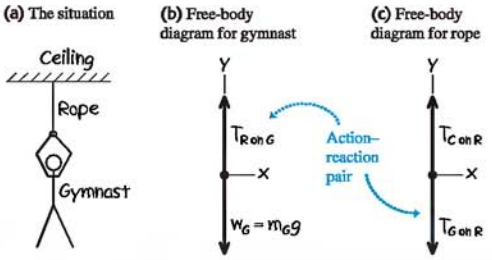
IDENTIFY: The gymnast and the rope are in equilibrium, so we can apply Newton's first law to both bodies. We'll also use Newton's third law to relate the forces that the gymnast and the rope exert on each other. the target variables are the weight of the gymnast, w_G; the force that the rope exerts on the gymnast (call it $T_{R\text{ on }G}$); and the tension that the ceiling exerts on the top of the rope (call it $T_{C\text{ on }R}$).

SET UP: We sketch the situation (Fig. 5.1a) and draw separate free-body diagrams for the gymnast (Fig. 5.1b) and the rope (Fig. 5.1c). We take the positive y-axis to be upward, as shown. Each force acts in the vertical direction and so has only a y-component.

The two forces $T_{R\text{ on }G}$ and $T_{G\text{ on }R}$ are the upward force of the rope on the gymnast (in Fig. 5.1b) and the downward force of the gymnast on the rope (in Fig. 5.1c). These forces form an action–reaction pair, so they must have the same magnitude.

Note also that the gymnast's weight w_G is the attractive (downward) force exerted on the *gymnast* by the *earth*. Its reaction force is the equal to and opposite the attractive (upward) force exerted on the *earth* by the *gymnast*. This force acts on the earth, not on the gymnast, so it doesn't appear in her free-body diagram (Fig. 5.1b). Compare the discussion of the apple in Conceptual Example 4.9 (Section 4.5). Similarly, the force that the rope exerts on the ceiling doesn't appear in Fig. 5.1c.

5.1 Our sketches for this problem.



EXECUTE: The magnitude of the gymnast's weight is the product of her mass and the acceleration due to gravity, g:

$$w_G = m_G g = (50.0\text{ kg})(9.80\text{ m/s}^2) = 490\text{ N}$$

This force points in the negative y-direction, so its y-component is $-w_G$. The upward force exerted by the rope has unknown magnitude $T_{R\text{ on }G}$ and positive y-component $+T_{R\text{ on }G}$. Because the gymnast is in equilibrium, the sum of the y-components of force acting on her must be zero:

$$\text{Gymnast:}\quad \sum F_y = T_{R\text{ on }G} + (-w_G) = 0 \quad\text{so}$$
$$T_{R\text{ on }G} = w_G = 490\text{ N}$$

The rope pulls *up* on the gymnast with a force $T_{R\text{ on }G}$ of magnitude 490 N. By Newton's third law, the gymnast pulls *down* on the rope with a force of the same magnitude, $T_{G\text{ on }R} = 490\text{ N}$.

The rope is also in equilibrium. We have assumed that it is weightless, so the upward force of magnitude $T_{C\text{ on }R}$ that the ceiling exerts on its top end must make the *net* vertical force on the rope equal to zero. Expressed as an equation, this says

$$\text{Rope:}\quad \sum F_y = T_{C\text{ on }R} + (-T_{G\text{ on }R}) = 0 \quad\text{so}$$
$$T_{C\text{ on }R} = T_{G\text{ on }R} = 490\text{ N}$$

EVALUATE: The *tension* at any point in the rope is the force that acts at that point. For this weightless rope, the tension $T_{G\text{ on }R}$ at the lower end has the same value as the tension $T_{C\text{ on }R}$ at the upper end. Indeed, for an ideal weightless rope, the tension has the same value at any point along the rope's length. (Compare the discussion of Conceptual Example 4.10 in Section 4.5.)

Note that we have defined tension to be the *magnitude* of a force, so it is always positive. But the y-component of force acting on the rope at its lower end is $-T_{G\text{ on }R} = -490\text{ N}$.

Example 5.2 One-dimensional equilibrium: Tension in a rope with mass

Suppose that in Example 5.1, the weight of the rope is not negligible but is 120 N. Find the tension at each end of the rope.

SOLUTION

IDENTIFY: As in Example 5.1, the target variables are the magnitudes $T_{G\text{ on }R}$ and $T_{C\text{ on }R}$ of the forces that act at the bottom and top of the rope, respectively. Once again, we'll apply Newton's first law to the gymnast and to the rope, and use Newton's third law to relate the forces that the gymnast and rope exert on each other.

SET UP: Again we draw separate free-body diagrams for the gymnast (Fig. 5.2a) and the rope (Fig. 5.2b). The only difference from Example 5.1 is that there are now *three* forces acting on the rope: the downward force exerted by the gymnast $(T_{G\text{ on }R})$, the upward force exerted by the ceiling $(T_{C\text{ on }R})$, and the weight of the rope, of magnitude $w_R = 120\text{ N}$.

EXECUTE: The gymnast's free-body diagram is the same as in Example 5.1, so her equilibrium condition is also the same. From Newton's third law, $T_{R\text{ on }G} = T_{G\text{ on }R}$, and we have

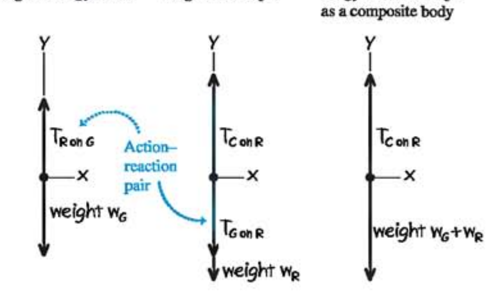
$$\text{Gymnast:}\quad \sum F_y = T_{R\text{ on }G} + (-w_G) = 0 \quad\text{so}$$
$$T_{R\text{ on }G} = T_{G\text{ on }R} = w_G = 490\text{ N}$$

The equilibrium condition $\sum F_y = 0$ for the rope is

$$\text{Rope:}\quad \sum F_y = T_{C\text{ on }R} + (-T_{G\text{ on }R}) + (-w_R) = 0$$

Note that the y-component of $T_{C\text{ on }R}$ is positive because it points in the $+y$-direction, but the y-components of both $T_{G\text{ on }R}$ and w_R are

5.2 Our sketches for this problem, including the weight of the rope.



negative. When we solve for $T_{\text{C on R}}$ and substitute the values $T_{\text{G on R}} = T_{\text{R on G}} = 490\text{ N}$ and $w_R = 120\text{ N}$, we find

$$T_{\text{C on R}} = T_{\text{G on R}} + w_R = 490\text{ N} + 120\text{ N} = 610\text{ N}$$

EVALUATE: When we include the weight of the rope, the tension is *different* at the rope's two ends. The force $T_{\text{C on R}}$ exerted by the ceiling has to hold up both the 490-N weight of the gymnast and the 120-N weight of the rope, so $T_{\text{C on R}} = 610\text{ N}$.

To see this more explicitly, draw a free-body diagram for a composite body consisting of the gymnast and rope considered as a unit (Fig. 5.2c). Only two external forces act on this composite body: the force $T_{\text{C on R}}$ exerted by the ceiling and the total weight $w_G + w_R = 490\text{ N} + 120\text{ N} = 610\text{ N}$. (The forces $T_{\text{G on R}}$ and $T_{\text{R on G}}$ are *internal* to the composite body. Since Newton's first law involves only *external* forces, the internal forces play no role.) Hence Newton's first law applied to this composite body is

$$\text{Composite body:} \quad \sum F_y = T_{\text{C on R}} + [-(w_G + w_R)] = 0$$

and so $T_{\text{C on R}} = w_G + w_R = 610\text{ N}$.

This method of treating the gymnast and rope as a composite body seems a lot simpler, and you may be wondering why we didn't use it first. The answer is that we can't find the tension $T_{\text{G on R}}$ at the bottom of the rope by this method. *Moral: Whenever you have more than one body in a problem involving Newton's laws, the safest approach is to treat each body separately.*

Example 5.3 Two-dimensional equilibrium

In Fig. 5.3a, a car engine with weight w hangs from a chain that is linked at ring O to two other chains, one fastened to the ceiling and the other to the wall. Find the tension in each of the three chains in terms of w. The weights of the ring and chains are negligible.

SOLUTION

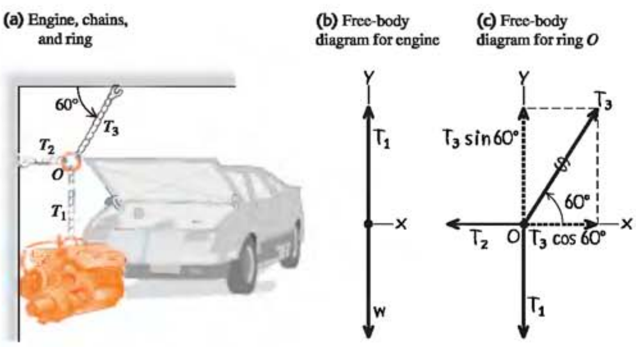
IDENTIFY: The target variables are the tensions T_1, T_2, and T_3 in the three chains (Fig. 5.3a). It may seem strange that we neglect the weight of the chains and ring in this example, while in Example 5.2 we did *not* neglect the weight of a mere rope. The reason is that the weight of the chains or ring is very small compared to the weight of the massive engine. By contrast, in Example 5.2 the rope weighed a reasonable fraction of the gymnast's weight (120 N compared to 490 N).

All the bodies in the example are in equilibrium, so we'll use Newton's first law to determine T_1, T_2, and T_3. We need three simultaneous equations, one for each target variable. However, applying Newton's first law to just one body gives us just *two* equations, as in Eq. (5.2). So to solve the problem, we'll have to consider more than one body in equilibrium. We'll look at the engine (which is acted on by T_1) and the ring (which is connected to all three chains and so is acted on by all three tensions).

SET UP: Figures 5.3b and 5.3c show our free-body diagrams, including an x-y coordinate system, for the engine and the ring, respectively.

The two forces acting on the engine are its weight w and the upward force T_1 exerted by the vertical chain; the three forces acting on the ring are the tensions from the vertical chain (T_1), the horizontal chain (T_2), and the slanted chain (T_3). Because the vertical chain has negligible weight, it exerts forces of the same magnitude T_1 at both of its ends: upward on the engine in Fig. 5.3b and

5.3 (a) The situation. (b), (c) Our free-body diagrams.



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downward on the ring in Fig. 5.3c (see Example 5.1). If the weight were not negligible, these two forces would have different magnitudes, as was the case for the rope in Example 5.2. We're also neglecting the weight of the ring, which is why it isn't included in the forces in Fig. 5.3c.

EXECUTE: The forces acting on the engine are along the y-axis only, so Newton's first law says

$$\text{Engine:} \quad \sum F_y = T_1 + (-w) = 0 \quad \text{and} \quad T_1 = w$$

The horizontal and slanted chains do not exert forces on the engine itself because they are not attached to it. These forces appear when we apply Newton's first law to the ring, however.

In the free-body diagram for the ring (Fig. 5.3c), remember that T_1, T_2, and T_3 are the *magnitudes* of the forces. We first resolve the force with magnitude T_3 into its x- and y-components. The ring is in equilibrium, so we then write separate equations stating that the x- and y-components of the net force on the ring are zero. (Remember from Problem-Solving Strategy 5.1 that we *never* add x- and y-components together in a single equation.) We find

$$\text{Ring:} \quad \sum F_x = T_3 \cos 60° + (-T_2) = 0$$
$$\text{Ring:} \quad \sum F_y = T_3 \sin 60° + (-T_1) = 0$$

Because $T_1 = w$ (from the engine equation), we can rewrite the second ring equation as

$$T_3 = \frac{T_1}{\sin 60°} = \frac{w}{\sin 60°} = 1.155w$$

We can now use this result in the first ring equation:

$$T_2 = T_3 \cos 60° = w\frac{\cos 60°}{\sin 60°} = 0.577w$$

So we can express all three tensions as multiples of the weight w of the engine, which we assume is known. To summarize,

$$T_1 = w$$
$$T_2 = 0.577w$$
$$T_3 = 1.155w$$

EVALUATE: Our results show that the chain attached to the ceiling exerts a force on the ring of magnitude T_3, which is *greater* than the weight of the engine. If this seems strange, note that the vertical component of this force is equal to T_1, which in turn is equal to w. But since this force also has a horizontal component, its magnitude T_3 must be somewhat larger than w. Hence the chain attached to the ceiling is under the greatest tension and is the one most susceptible to breaking.

You may have thought at first that the most important body in this problem was the engine. But to get enough equations to solve the problem, we also had to consider the forces acting on a second body (the ring connecting the chains). Situations like this are fairly common in equilibrium problems, so keep this technique in mind.

Example 5.4 An inclined plane

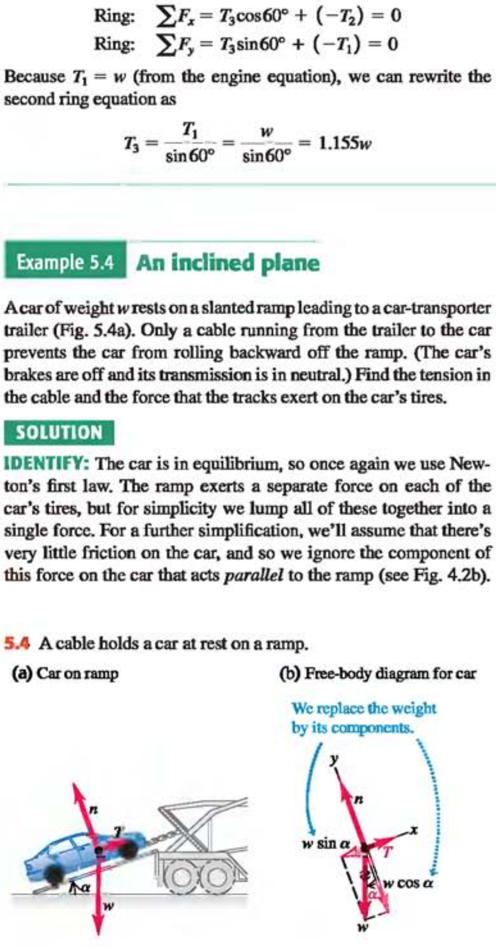
A car of weight w rests on a slanted ramp leading to a car-transporter trailer (Fig. 5.4a). Only a cable running from the trailer to the car prevents the car from rolling backward off the ramp. (The car's brakes are off and its transmission is in neutral.) Find the tension in the cable and the force that the tracks exert on the car's tires.

SOLUTION

IDENTIFY: The car is in equilibrium, so once again we use Newton's first law. The ramp exerts a separate force on each of the car's tires, but for simplicity we lump all of these together into a single force. For a further simplification, we'll assume that there's very little friction on the car, and so we ignore the component of this force on the car that acts *parallel* to the ramp (see Fig. 4.2b).

5.4 A cable holds a car at rest on a ramp.

(a) Car on ramp (b) Free-body diagram for car



(We'll return to the friction force in Section 5.3.) Hence we can say that the ramp only exerts a force on the car that is *perpendicular* to the tracks. This force appears because the atoms on the surface of the track resist having the atoms of the tires squeezed into them. As in Section 4.1, we call this force the *normal* force (see Fig. 4.2a). The two target variables are the magnitude n of the normal force and the magnitude T of the tension in the cable.

SET UP: Figure 5.4b shows a free-body diagram for the car. The three forces acting on the car are its weight (magnitude w), the tension in the cable (magnitude T), and the normal force (magnitude n). Note that the normal force acts up and to the left because it's preventing the car from penetrating into the solid tracks.

We choose the x- and y-axes to be parallel and perpendicular to the ramp as shown. This choice makes the problem easier to analyze because only the weight force has both an x- and y-component. If we chose axes that were horizontal and vertical, our job would be harder because we'd need to find x- and y-components for *two* forces (the normal force and the tension).

Note that the angle α between the ramp and the horizontal is equal to the angle α between the weight vector $\vec{w}$ and the normal to the plane of the ramp.

EXECUTE: To write down the x- and y-components of Newton's first law, we need to find the components of the weight. One complication is that the angle α in Fig. 5.4b is *not* measured from the $+x$-axis toward the $+y$-axis. Hence we *cannot* use Eqs. (1.6) directly to find the components. (You may want to review Section 1.8 to make sure that you understand this important point.)

One approach to finding the components of $\vec{w}$ is to consider the right triangles in Fig. 5.4b. The sine of α is the magnitude of the x-component of $\vec{w}$ (that is, the side of the triangle opposite α) divided by the magnitude w (the hypotenuse of the triangle). Similarly, the cosine of α is the magnitude of the y-component (the side of the triangle adjacent to α) divided by w. Both components are negative, so $w_x = -w\sin\alpha$ and $w_y = -w\cos\alpha$.

Another approach is to recognize that one component of $\vec{w}$ must involve $\sin\alpha$ while the other component involves $\cos\alpha$. To decide which is which, draw the free-body diagram so that the angle α is noticeably smaller or larger than 45°. (You'll have to fight the natural tendency to draw such angles as being close to 45°.) We've drawn Fig. 5.4b so that α is smaller than 45°, so $\sin\alpha$ is less than $\cos\alpha$. The figure shows that the x-component of $\vec{w}$ is smaller than the y-component, so the x-component must involve $\sin\alpha$ and the y-component must involve $\cos\alpha$. We again find $w_x = -w\sin\alpha$ and $w_y = -w\cos\alpha$.

In Fig. 5.4b we draw a wiggly line through the original vector representing the weight to remind us not to count it twice. The equilibrium conditions then give us

$$\sum F_x = T + (-w\sin\alpha) = 0$$
$$\sum F_y = n + (-w\cos\alpha) = 0$$

Be sure you understand how these signs are related to our choice of coordinates. Remember that, by definition, T, w, and n are all *magnitudes* of vectors and are therefore all positive.

Solving these equations for T and n, we find

$$T = w\sin\alpha$$
$$n = w\cos\alpha$$

EVALUATE: Our answers for T and n depend on the value of α; we can check this dependence by looking at some special cases. If the angle α is zero, then $\sin\alpha = 0$ and $\cos\alpha = 1$. In this case, the ramp is horizontal; our answers tell us that no cable tension T is needed to hold the car, and the normal force n is equal in magnitude to the weight. If the angle is 90°, then $\sin\alpha = 1$ and $\cos\alpha = 0$. Then the cable tension T equals the weight w, and the normal force n is zero. Are these the results you would expect for these particular cases?

CAUTION **Normal force and weight may not be equal** It's a common error to automatically assume that the magnitude n of the normal force is equal to the weight w. But our result shows that this is *not* true in general. It's always best to treat n as a variable and solve for its value, as we have done here. ❙

How would the answers for T and n be affected if the car were not stationary but were being pulled up the ramp at a constant speed? This, too, is an equilibrium situation, since the car's velocity is constant. So the calculation is exactly the same, and T and n have the same values as when the car is at rest. (It's true that T must be greater than $w\sin\alpha$ to *start* the car moving up the ramp, but that's not what we asked.) **?**

Example 5.5 Tension over a frictionless pulley

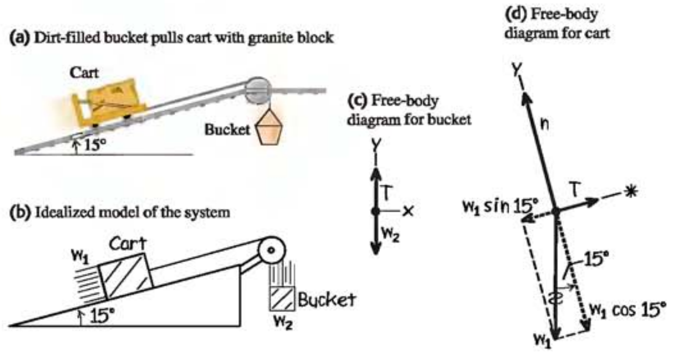
Blocks of granite are to be hauled up a 15° slope out of a quarry, and dirt is to be dumped into the quarry to fill up old holes. To simplify the process, you design a system in which a granite block on a cart with steel wheels (weight w_1, including both block and cart) is pulled uphill on steel rails by a dirt-filled bucket (weight w_2, including both dirt and bucket) dropping vertically into the quarry (Fig. 5.5a). How must the weights w_1 and w_2 be related in order for the system to move with constant speed? Ignore friction in the pulley and wheels and the weight of the cable.

SOLUTION

IDENTIFY: The cart and bucket each move with a constant velocity (that is, in a straight line at constant speed). Hence each body is in equilibrium, and we can apply Newton's first law to each.

Our two target variables are the weights w_1 and w_2. The forces that act on the bucket are its weight w_2 and an upward tension exerted by the cable. The cart has *three* forces acting on it: its weight w_1, a normal force of magnitude n exerted by the rails, and

5.5 (a) The situation. (b) Our idealized model. (c), (d) Our free-body diagrams.



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a tension force from the cable. (We're ignoring friction, so we're assuming that the rails exert no force parallel to the incline.) This is exactly like the situation for the car on the ramp in Example 5.4. As in that example, the forces on the cart are not all along the same direction, so we'll need to use both components of Newton's first law in Eq. (5.2).

We're assuming that the cable has negligible weight, so the tension forces that the rope exerts on the cart and on the bucket have the same magnitude T.

SET UP: Figure 5.5b shows our idealized model for the system, and Figs. 5.5c and 5.5d show the free-body diagrams we draw. Note that we're free to orient the axes differently for each body; the choices shown are the most convenient ones. As we did for the car in Example 5.4, we represent the weight of the granite block in terms of its x- and y-components.

EXECUTE: Applying $\sum F_y = 0$ to the dirt-filled bucket in Fig. 5.5c, we find

$$\sum F_y = T + (-w_2) = 0 \quad \text{so} \quad T = w_2$$

Applying $\sum F_x = 0$ to the block and cart in Fig. 5.5d, we get

$$\sum F_x = T + (-w_1 \sin 15°) = 0 \quad \text{so} \quad T = w_1 \sin 15°$$

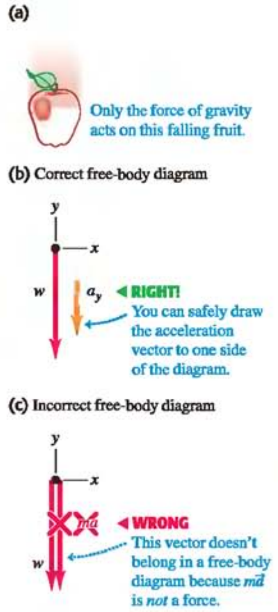
Equating the two expressions for T, we find

$$w_2 = w_1 \sin 15° = 0.26 w_1$$

EVALUATE: Our analysis doesn't depend on the direction of motion, only on the velocity being constant. Hence the system can move with constant speed in *either* direction if the weight of dirt and bucket totals 26% of the weight of the granite block and cart. What would happen if w_2 were greater than $0.26w_1$? If it were less than $0.26w_1$?

Notice that we didn't need to apply the equation $\sum F_y = 0$ to the cart and block; this would be useful only if we wanted to find the value of n. Can you show that $n = w_1 \cos 15°$?

Test Your Understanding of Section 5.1 A traffic light of weight w hangs from two lightweight cables, one on each side of the light. Each cable hangs at a 45° angle from the horizontal. What is the tension in each cable? (i) $w/2$; (ii) $w/\sqrt{2}$; (iii) w; (iv) $w\sqrt{2}$; (v) $2w$.



5.6 Correct and incorrect free-body diagrams for a falling body.

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2.1.5 Car Race
2.2 Lifting a Crate
2.3 Lowering a Crate
2.4 Rocket Blasts Off
2.5 Modified Atwood Machine

5.2 Using Newton's Second Law: Dynamics of Particles

We are now ready to discuss *dynamics* problems. In these problems, we apply Newton's second law to bodies on which the net force is *not* zero, so the bodies are *not* in equilibrium and hence are accelerating. The net force is equal to the mass of the body times its acceleration:

$$\sum \vec{F} = m\vec{a} \quad \text{(Newton's second law, vector form)} \tag{5.3}$$

We most often use this relationship in component form:

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \text{(Newton's second law, component form)} \tag{5.4}$$

The following problem-solving strategy is very similar to Problem-Solving Strategy 5.1 for equilibrium problems in Section 5.1. We urge you to study it carefully, watch how we apply it in our examples, and use it when you tackle the end-of-chapter problems. Remember that you can solve *any* dynamics problem using this strategy.

CAUTION **$m\vec{a}$ doesn't belong in free-body diagrams** Remember that the quantity $m\vec{a}$ is the *result* of forces acting on a body, *not* a force itself; it's not a push or a pull exerted by anything in the body's environment. When you draw the free-body diagram for an accelerating body (like the fruit in Fig. 5.6a), make sure you *never* include the "$m\vec{a}$ force" because *there is no such force* (Fig. 5.6c). You should review Section 4.3 if you're not clear on this point. Sometimes we draw the acceleration vector $\vec{a}$ *alongside* a free-body diagram, as in Fig. 5.6b. But we *never* draw the acceleration vector with its tail touching the body (a position reserved exclusively for the forces that act on the body).

Problem-Solving Strategy 5.2 Newton's Second Law: Dynamics of Particles

IDENTIFY *the relevant concepts:* You have to use Newton's second law for *any* problem that involves forces acting on an accelerating body.

Identify the target variable—usually an acceleration or a force. If the target variable is something else, you'll need to select another concept to use. For example, suppose you want to find how fast a sled is moving when it reaches the bottom of a hill. This means your target variable is the sled's final velocity. Newton's second law will let you find the sled's acceleration; you'll then use the constant-acceleration relationships from Section 2.4 to find velocity from acceleration.

SET UP *the problem* using the following steps:

1. Draw a simple sketch of the situation. Identify one or more moving bodies to which you'll apply Newton's second law.
2. For each body you identified, draw a free-body diagram that shows all the forces acting *on* the body. Remember, the acceleration of a body is determined by the forces that act on it, *not* by the forces that it exerts on anything else. Make sure you can answer the question "What other body is applying this force?" for each force in your diagram. Never include the quantity $m\vec{a}$ in your free-body diagram; it's not a force!
3. Label each force with an algebraic symbol for the force's *magnitude*. (Remember that magnitudes are always positive. Minus signs show up later when you take components of the forces.) Usually, one of the forces will be the body's weight; it's usually best to label this as $w = mg$. If a numerical value of mass is given, you can compute the corresponding weight.
4. Choose your x- and y-coordinate axes for each body, and show them in its free-body diagram. Be sure to indicate the positive direction for each axis. If you know the direction of the acceleration, it usually simplifies things to take one positive axis along that direction. If your problem involves more than one object and the objects accelerate in different directions, you can use a different set of axes for each object.
5. In addition to Newton's second law, $\sum\vec{F} = m\vec{a}$, identify any other equations you might need. (You need as many equations as there are target variables.) For example, you might need one or more of the equations for motion with constant acceleration. If more than one body is involved, there may be relationships among their motions; for example, they may be connected by a rope. Express any such relationships as equations relating the accelerations of the various bodies.

EXECUTE *the solution* as follows:

1. For each object, determine the components of the forces along each of the object's coordinate axes. When you represent a force in terms of its components, draw a wiggly line through the original force vector to remind you not to include it twice.
2. For each object, write a separate equation for each component of Newton's second law, as in Eq. (5.4).
3. Make a list of all the known and unknown quantities. In your list, identify the target variable or variables.
4. Check that you have as many equations as there are unknowns. If you have too few equations, go back to step 5 of "Set up the problem." If you have too many equations, perhaps there is an unknown quantity that you haven't identified as such.
5. Do the easy part—the math! Solve the equations to find the target variable(s).

EVALUATE *your answer:* Does your answer have the correct units? (When appropriate, use the conversion $1\text{ N} = 1\text{ kg}\cdot\text{m/s}^2$.) Does it have the correct algebraic sign? (If the problem is about a sled sliding downhill, you probably took the positive x-axis to point down the hill. If you then find that the sled has a negative acceleration—that is, the acceleration is uphill—something went wrong in your calculations.) When possible, consider particular values or extreme cases of quantities and compare the results with your intuitive expectations. Ask, "Does this result make sense?"

Example 5.6 Straight-line motion with a constant force

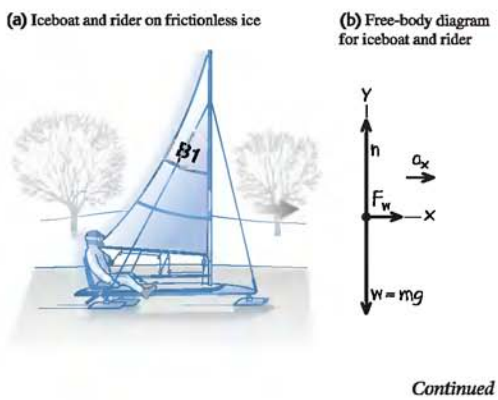
An iceboat is at rest on a perfectly frictionless horizontal surface (Fig. 5.7a). A wind is blowing (along the direction of the runners) so that 4.0 s after the iceboat is released, it attains a velocity of 6.0 m/s (about 22 km/h, or 13 mi/h). What constant horizontal force F_W does the wind exert on the iceboat? The mass of iceboat and rider is 200 kg.

SOLUTION

IDENTIFY: Our target variable is one of the forces (F_W) acting on the iceboat, so we'll need to use Newton's second law. This law involves forces and acceleration, but the acceleration isn't given; we'll need to find it. Since the wind is assumed to exert a constant force, the resulting acceleration is constant and we'll be able to use one of the constant-acceleration formulas from Section 2.4.

SET UP: Figure 5.7b shows our free-body diagram for the iceboat and rider considered as a unit. The forces acting on this body are the weight w, the normal force n exerted by the surface, and the

5.7 (a) The situation. (b) Our free-body diagram.



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horizontal force F_W (our target variable). The net force and hence the acceleration are to the right, so we chose the positive x-axis in this direction.

To find the x-acceleration, note what we are told about the iceboat's motion: It starts at rest so that its initial x-velocity is $v_{0x} = 0$, and it attains an x-velocity $v_x = 6.0\ \text{m/s}$ after an elapsed time $t = 4.0\ \text{s}$. An equation we can use to relate the x-acceleration a_x to these quantities is Eq. (2.8), $v_x = v_{0x} + a_x t$.

EXECUTE: The *known* quantities are the mass $m = 200\ \text{kg}$, the initial and final x-velocities $v_{0x} = 0$ and $v_x = 6.0\ \text{m/s}$, and the elapsed time $t = 4.0\ \text{s}$. The three *unknown* quantities are the acceleration a_x, the normal force n, and the horizontal force F_W (the target variable). Hence we need three equations.

The first two equations are the x- and y-equations for Newton's second law. The force F_W is in the positive x-direction, while the forces n and mg are in the positive and negative y-directions, respectively. Hence we have

$$\sum F_x = F_W = ma_x$$
$$\sum F_y = n + (-mg) = 0$$

The third equation we need is the constant-acceleration relationship

$$v_x = v_{0x} + a_x t$$

To find F_W, we first solve the constant-acceleration equation for a_x and then substitute this into the $\sum F_x$ equation:

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{6.0\ \text{m/s} - 0\ \text{m/s}}{4.0\ \text{s}} = 1.5\ \text{m/s}^2$$
$$F_W = ma_x = (200\ \text{kg})(1.5\ \text{m/s}^2) = 300\ \text{kg}\cdot\text{m/s}^2$$

One $\text{kg}\cdot\text{m/s}^2$ is the same as 1 newton (N), so the final answer is

$$F_W = 300\ \text{N} \qquad (\text{about } 67\ \text{lb})$$

Note that we did not need the $\sum F_y$ equation at all to find F_W. We would need this equation if we wanted to find the normal force n:

$$n - mg = 0$$
$$n = mg = (200\ \text{kg})(9.8\ \text{m/s}^2)$$
$$= 2.0 \times 10^3\ \text{N} \qquad (\text{about } 450\ \text{lb})$$

EVALUATE: Our answers for F_W and n have the correct units for a force, as they should. The magnitude n of the normal force is equal to mg, the combined weight of the iceboat and rider, because the surface is horizontal and these are the only vertical forces that act. Does it seem reasonable that the force F_W is substantially *less* than mg?

Example 5.7 Straight-line motion with friction

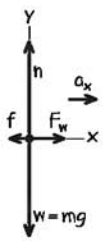
Suppose a constant horizontal friction force with magnitude 100 N opposes the motion of the iceboat in Example 5.6. In this case, what constant force F_W must the wind exert on the iceboat to cause the same constant x-acceleration $a_x = 1.5\ \text{m/s}^2$?

SOLUTION

IDENTIFY: Once again the target variable is F_W. We are given the x-acceleration, so to find F_W all we need is Newton's second law.

SET UP: Figure 5.8 shows our new free-body diagram. The only difference from Fig. 5.7b is the addition of the friction force $\vec{f}$, which points opposite to the motion. (Note that its *magnitude*, $f = 100\ \text{N}$, is a positive quantity but that its *component* in the x-direction is negative, equal to $-f$ or $-100\ \text{N}$.)

5.8 Our free-body diagram for the iceboat and rider with a friction force $\vec{f}$ opposing the motion.



EXECUTE: Two forces now have x-components: the force of the wind and the friction force. The x-component of Newton's second law gives

$$\sum F_x = F_W + (-f) = ma_x$$
$$F_W = ma_x + f = (200\ \text{kg})(1.5\ \text{m/s}^2) + (100\ \text{N}) = 400\ \text{N}$$

EVALUATE: Because there is friction, a greater force F_W is needed than in Example 5.6. We need 100 N to overcome friction and 300 N more to give the iceboat the necessary acceleration.

Example 5.8 Tension in an elevator cable

An elevator and its load have a total mass of 800 kg (Fig. 5.9a). The elevator is originally moving downward at $10.0\ \text{m/s}$; it slows to a stop with constant acceleration in a distance of 25.0 m. Find the tension T in the supporting cable while the elevator is being brought to rest.

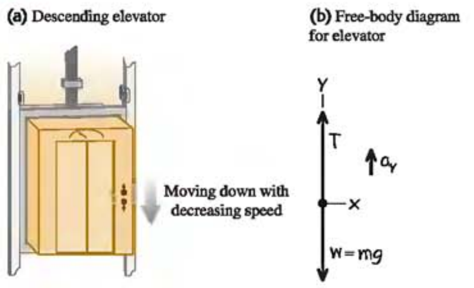
SOLUTION

IDENTIFY: The target variable is the tension T, which we will find using Newton's second law. As in Example 5.6, we'll have to determine the acceleration using the constant-acceleration formulas.

SET UP: Our free-body diagram in Fig. 5.9b shows the two forces acting on the elevator: its weight w and the tension force T of the cable. The elevator is moving downward with decreasing speed, so its acceleration is upward; we chose the positive y-axis to be in this direction.

The elevator is moving in the negative y-direction, so its initial y-velocity v_{0y} and its y-displacement $y - y_0$ are both negative: $v_{0y} = -10.0\ \text{m/s}$ and $y - y_0 = -25.0\ \text{m}$. The final y-velocity is $v_y = 0$. To find the y-acceleration a_y from this information, we'll use Eq. (2.13) in the form $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$. Once we

5.9 (a) The situation. (b) Our free-body diagram.



have a_y, we'll substitute it into the y-component of Newton's second law from Eq. (5.4).

EXECUTE: First let's write out Newton's second law. The tension force acts upward and the weight acts downward, so

$$\sum F_y = T + (-w) = ma_y$$

We solve for the target variable T:

$$T = w + ma_y = mg + ma_y = m(g + a_y)$$

To determine a_y, we rewrite the constant-acceleration equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$:

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{(0)^2 - (-10.0\ \text{m/s})^2}{2(-25.0\ \text{m})} = +2.00\ \text{m/s}^2$$

The acceleration is upward (positive), just as it should be for downward motion with decreasing speed.

Now we can substitute the acceleration into the equation for the tension:

$$T = m(g + a_y) = (800\ \text{kg})(9.80\ \text{m/s}^2 + 2.00\ \text{m/s}^2) = 9440\ \text{N}$$

EVALUATE: The tension is 1600 N *greater* than the weight. This makes sense: The net force must be upward to provide the upward acceleration that brings the elevator to a halt. Can you see that we would get the same answers for a_y and T if the elevator were moving *upward* and *gaining* speed at a rate of $2.00\ \text{m/s}^2$?

Example 5.9 Apparent weight in an accelerating elevator

A 50.0-kg woman stands on a bathroom scale while riding in the elevator in Example 5.8 (Fig. 5.10a). What is the reading on the scale?

SOLUTION

IDENTIFY: The scale reads the magnitude of the downward force exerted *by* the woman *on* the scale. By Newton's third law, this equals the magnitude of the upward normal force exerted *by* the scale *on* the woman. Hence our target variable is the magnitude n of the normal force.

We'll find n by applying Newton's second law to the woman. We already know her acceleration; it's the same as the acceleration of the elevator, which we calculated in Example 5.8.

SET UP: Figure 5.10b shows our free-body diagram for the woman. The forces acting on her are the normal force n exerted by the scale and her weight $w = mg = (50.0\ \text{kg})(9.80\ \text{m/s}^2) =$ 490 N. (The tension force, which played a major role in Example 5.8, doesn't appear here because it doesn't act directly on the woman. What pushes upward on her feet is the scale, *not* the elevator cable.) From Example 5.8, the y-acceleration of the elevator and of the woman is $a_y = +2.00\ \text{m/s}^2$.

5.10 (a) The situation. (b) Our free-body diagram.

EXECUTE: Newton's second law gives

$$\sum F_y = n + (-mg) = ma_y$$
$$n = mg + ma_y = m(g + a_y)$$
$$= (50.0\ \text{kg})(9.80\ \text{m/s}^2 + 2.00\ \text{m/s}^2) = 590\ \text{N}$$

EVALUATE: Our answer for n means that while the elevator is stopping, the scale pushes up on the woman with a force of 590 N. By Newton's third law, she pushes down on the scale with the same force; so the scale reads 590 N, which is 100 N more than her actual weight. The scale reading is called the passenger's **apparent weight.** The woman *feels* the floor pushing up harder on her feet than when the elevator is stationary or moving with constant velocity.

What would the woman feel if the elevator were accelerating *downward* so that $a_y = -2.00\ \text{m/s}^2$? This would be the case if the elevator were moving upward with decreasing speed or moving downward with increasing speed. To find the answer for this situation, we just insert the new value of a_y in our equation for n:

$$n = m(g + a_y) = (50.0\ \text{kg})[9.80\ \text{m/s}^2 + (-2.00\ \text{m/s}^2)] = 390\ \text{N}$$

Now the woman feels as though she weighs only 390 N, or 100 N *less* than her actual weight.

You can feel these effects yourself; try taking a few steps in an elevator that is coming to a stop after descending (when your apparent weight is greater than your true weight w) or coming to a stop after ascending (when your apparent weight is less than w).

5.11 Astronauts in orbit feel "weightless" because they have the same acceleration as their spacecraft—*not* because they are "outside the pull of the earth's gravity." (If no gravity acted on them, the astronauts and their spacecraft wouldn't remain in orbit, but would fly off into deep space.)

Apparent Weight and Apparent Weightlessness

Let's generalize the result of Example 5.9. When a passenger with mass m rides in an elevator with y-acceleration a_y, a scale shows the passenger's apparent weight to be

$$n = m(g + a_y)$$

When the elevator is accelerating upward, a_y is positive and n is greater than the passenger's weight $w = mg$. When the elevator is accelerating downward, a_y is negative and n is less than the weight. If the passenger doesn't know the elevator is accelerating, she may feel as though her weight is changing; indeed, this is just what the scale shows.

The extreme case occurs when the elevator has a downward acceleration $a_y = -g$, that is, when it is in free fall. In that case, $n = 0$ and the passenger *seems* to be weightless. Similarly, an astronaut orbiting the earth in a spacecraft experiences *apparent weightlessness* (Fig. 5.11). In each case, the person is not truly weightless because there is still a gravitational force acting. But the person's sensations in this free-fall condition are exactly the same as though the person were in outer space with no gravitational force at all. In both cases the person and the vehicle (elevator or spacecraft) are falling together with the same acceleration g, so nothing pushes the person against the floor or walls of the vehicle.

Example 5.10 Acceleration down a hill

A toboggan loaded with vacationing students (total weight w) slides down a long, snow-covered slope. The hill slopes at a constant angle α, and the toboggan is so well waxed that there is virtually no friction. What is its acceleration?

SOLUTION

IDENTIFY: Our target variable is the acceleration, which we'll find using Newton's second law. There is no friction, so only two forces act on the toboggan: its weight w and the normal force n exerted by the hill. As in Example 5.4 (Section 5.1), the surface is inclined so the normal force is not vertical and is not opposite to the weight. Hence we must use both components of $\sum\vec{F} = m\vec{a}$ in Eq. (5.4).

SET UP: Figure 5.12 shows our sketch and free-body diagram. We take axes parallel and perpendicular to the surface of the hill, so that the acceleration (which is parallel to the hill) is along the positive x-direction.

5.12 Our sketches for this problem.

(a) The situation (b) Free-body diagram for toboggan

EXECUTE: The normal force has only a y-component, but the weight has both x- and y-components: $w_x = w\sin\alpha$ and $w_y = -w\cos\alpha$. (Compare to Example 5.4, in which the x-component of weight was $-w\sin\alpha$. The difference is that the positive x-axis was uphill in Example 5.4, while in Fig. 5.12b it is downhill.) The wiggly line in Fig. 5.12b reminds us that we have resolved the weight into its components.

The acceleration is purely in the $+x$-direction, so $a_y = 0$. Newton's second law in component form then tells us that

$$\sum F_x = w\sin\alpha = ma_x$$
$$\sum F_y = n - w\cos\alpha = ma_y = 0$$

Since $w = mg$, the x-component equation tells us that $mg\sin\alpha = ma_x$, or

$$a_x = g\sin\alpha$$

Note that we didn't need the y-component equation to find the acceleration. That's the beauty of choosing the x-axis to lie along the acceleration direction! What the y-components tell us is the magnitude of the normal force that the hill exerts on the toboggan:

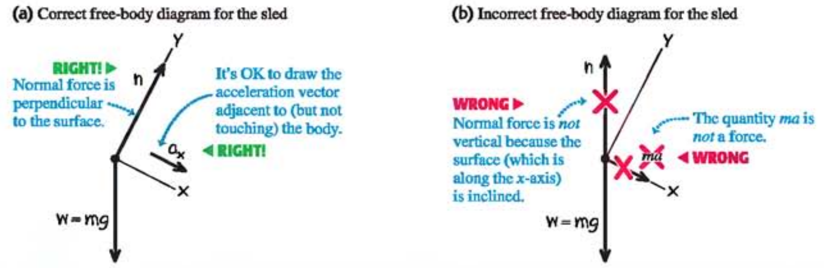
$$n = w\cos\alpha = mg\cos\alpha$$

EVALUATE: Notice that the mass does not appear in our answer for the acceleration. This means that *any* toboggan, regardless of its mass or number of passengers, slides down a frictionless hill with an acceleration of $g\sin\alpha$. In particular, if the plane is horizontal, $\alpha = 0$ and $a_x = 0$ (the toboggan does not accelerate); if the plane is vertical, $\alpha = 90°$ and $a_x = g$ (the toboggan is in free fall).

Notice also that the normal force n is not equal to the toboggan's weight (compare Example 5.4 in Section 5.1). We don't need this result here, but it will be useful in a later example.

CAUTION Common free-body diagram errors Figure 5.13 shows both the correct way (Fig. 5.13a) and a common *incorrect* way (Fig. 5.13b) to draw the free-body diagram for the toboggan. The diagram in Fig. 5.13b is wrong for two reasons: the normal force must be drawn perpendicular to the surface, and there's no such thing as the "$m\vec{a}$ force." If you remember that "normal" means "perpendicular" and that $m\vec{a}$ is not itself a force, you'll be well on your way to always drawing correct free-body diagrams.

5.13 Correct and incorrect diagrams for a toboggan on a frictionless hill.



Example 5.11 Two bodies with the same acceleration

You push a 1.00-kg food tray through the cafeteria line with a constant 9.0-N force. As the tray moves, it pushes on a 0.50-kg carton of milk (Fig. 5.14a). The tray and carton slide on a horizontal surface that is so greasy that friction can be neglected. Find the acceleration of the tray and carton and the horizontal force that the tray exerts on the carton.

SOLUTION

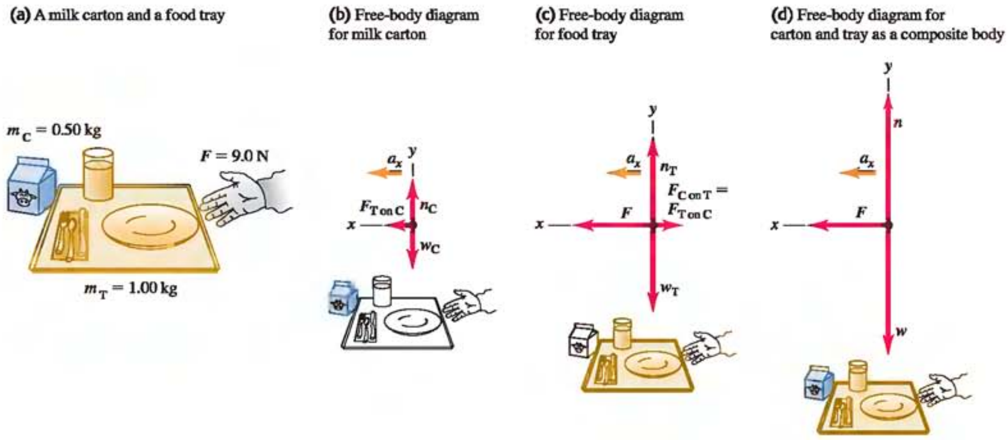
IDENTIFY: Our *two* target variables are the acceleration of the system of tray and carton and the force of the tray on the carton. Again we will use Newton's second law, but we'll have to apply it to two different bodies to get two equations (one for each target variable).

SET UP: We can set up the problem in two ways.

Method 1: We can treat the milk carton (mass m_C) and tray (mass m_T) as separate bodies, each with its own free-body diagram (Figs. 5.14b and 5.14c). Note that the force F that you exert on the tray doesn't appear in the free-body diagram for the milk carton. Instead, what makes the carton accelerate is the force of magnitude $F_{\text{T on C}}$ exerted on it by the tray. By Newton's third law, the carton exerts a force of equal magnitude on the tray: $F_{\text{C on T}} = F_{\text{T on C}}$. We take the acceleration to be in the positive x-direction; both the tray and milk carton move with the same x-acceleration a_x.

Method 2: We can treat the tray and milk carton as a composite body of mass $m = m_T + m_C = 1.50$ kg (Fig. 5.14d). The only

5.14 Pushing a food tray and milk carton in the cafeteria line.



Continued

horizontal force acting on this composite body is the force F that you exert. The forces $F_{\mathrm{T\,on\,C}}$ and $F_{\mathrm{C\,on\,T}}$ don't come into play because they're *internal* to this composite body, and Newton's second law tells us that only *external* forces affect a body's acceleration (see Section 4.3). Hence we'll need an additional equation to find the magnitude $F_{\mathrm{T\,on\,C}}$ using this method; we'll get this equation by also applying Newton's second law to the milk carton, as in Method 1.

EXECUTE: *Method 1:* The x-component equations of Newton's second law for the tray and for the carton are

$$\text{Tray:}\quad \sum F_x = F - F_{\mathrm{C\,on\,T}} = F - F_{\mathrm{T\,on\,C}} = m_{\mathrm{T}}a_x$$
$$\text{Carton:}\quad \sum F_x = F_{\mathrm{T\,on\,C}} = m_{\mathrm{C}}a_x$$

These are two simultaneous equations for the two target variables a_x and $F_{\mathrm{T\,on\,C}}$. (Two equations are all we need, which means that the y-components don't play a role in this example.) An easy way to solve the two equations for a_x is to add them; this eliminates $F_{\mathrm{T\,on\,C}}$, giving

$$F = m_{\mathrm{T}}a_x + m_{\mathrm{C}}a_x = (m_{\mathrm{T}} + m_{\mathrm{C}})a_x$$

and

$$a_x = \frac{F}{m_{\mathrm{T}} + m_{\mathrm{C}}} = \frac{9.0\ \mathrm{N}}{1.00\ \mathrm{kg} + 0.50\ \mathrm{kg}} = 6.0\ \mathrm{m/s^2}$$

Substituting this back into the equation for the carton gives

$$F_{\mathrm{T\,on\,C}} = m_{\mathrm{C}}a_x = (0.50\ \mathrm{kg})(6.0\ \mathrm{m/s^2}) = 3.0\ \mathrm{N}$$

Method 2: The x-component of Newton's second law for the composite body of mass m is

$$\sum F_x = F = ma_x$$

and the acceleration of this composite body is

$$a_x = \frac{F}{m} = \frac{9.0\ \mathrm{N}}{1.50\ \mathrm{kg}} = 6.0\ \mathrm{m/s^2}$$

Then, looking at the milk carton by itself, we see that to give it an acceleration of 6.0 m/s² requires that the tray exert a force:

$$F_{\mathrm{T\,on\,C}} = m_{\mathrm{C}}a_x = (0.50\ \mathrm{kg})(6.0\ \mathrm{m/s^2}) = 3.0\ \mathrm{N}$$

EVALUATE: The answers are the same with either method, as they should be. To check the answers, note that there are different forces on the two sides of the tray: $F = 9.0\ \mathrm{N}$ on the right and $F_{\mathrm{C\,on\,T}} = 3.0\ \mathrm{N}$ on the left. Hence the net horizontal force on the tray is $F - F_{\mathrm{C\,on\,T}} = 6.0\ \mathrm{N}$, exactly enough to accelerate a 1.00-kg tray at 6.0 m/s².

The method of treating the two bodies as a single composite body works *only* if the two bodies have the same magnitude *and* direction of acceleration. If the accelerations are different, we must treat the two bodies separately, as in the next example.

Example 5.12 Two bodies with the same magnitude of acceleration

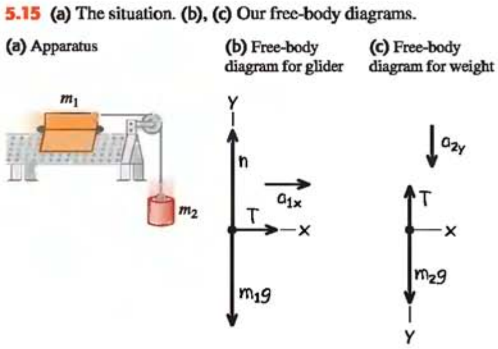
Figure 5.15a shows an air-track glider with mass m_1 moving on a level, frictionless air track in the physics lab. The glider is connected to a lab weight with mass m_2 by a light, flexible, nonstretching string that passes over a small frictionless pulley. Find the acceleration of each body and the tension in the string.

SOLUTION

IDENTIFY: The glider and weight are accelerating, so again we must use Newton's second law. Our three target variables are the tension T in the string and the accelerations of the two bodies.

SET UP: The two bodies move in different directions—one horizontal, one vertical—so we can't consider them together as we did the bodies in Example 5.11. Figures 5.15b and 5.15c show our free-body diagrams and coordinate systems. It's convenient to have both bodies accelerate in the positive axis directions, so we chose the positive y-direction for the lab weight to be downward. (It's perfectly all right to use different coordinate axes for the two bodies.)

5.15 (a) The situation. (b), (c) Our free-body diagrams.



There is no friction in the pulley and we consider the string to be massless, so the tension T in the string is the same throughout; it applies a force of the same magnitude T to each body. (You may want to review Conceptual Example 4.10 in Section 4.5, where we discussed the tension force exerted by a massless string.) The weights are m_1g and m_2g.

While the *directions* of the two accelerations are different, their *magnitudes* are the same. That's because the string doesn't stretch. Hence the two bodies must move equal distances in equal times, and so their speeds at any instant must be equal. When the speeds change, they change by equal amounts in a given time, so the accelerations of the two bodies must have the same magnitude a. We can express this relationship as

$$a_{1x} = a_{2y} = a$$

Thanks to this relationship, we actually have only *two* target variables: a and the tension T.

EXECUTE: For the glider on the track, Newton's second law gives

$$\text{Glider:}\quad \sum F_x = T = m_1a_{1x} = m_1a$$
$$\text{Glider:}\quad \sum F_y = n + (-m_1g) = m_1a_{1y} = 0$$

For the lab weight, the only forces are in the y-direction, and

$$\text{Lab weight:}\quad \sum F_y = m_2g + (-T) = m_2a_{2y} = m_2a$$

In these equations we've used the relationships $a_{1y} = 0$ (the glider doesn't accelerate vertically) and $a_{1x} = a_{2y} = a$ (the two objects have the same magnitude of acceleration).

The x-equation for the glider and the equation for the lab weight give us two simultaneous equations for the target variables T and a:

$$\text{Glider:} \quad T = m_1 a$$

$$\text{Lab weight:} \quad m_2 g - T = m_2 a$$

We add the two equations to eliminate T, giving

$$m_2 g = m_1 a + m_2 a = (m_1 + m_2)a$$

and so the magnitude of each body's acceleration is

$$a = \frac{m_2}{m_1 + m_2} g$$

Substituting this back into the first equation (for the glider), we get

$$T = \frac{m_1 m_2}{m_1 + m_2} g$$

EVALUATE: The acceleration is less than g, as you might expect; the lab weight accelerates more slowly because the string tension pulls it back.

The tension T is *not* equal to the weight $m_2 g$ of the lab weight, but is *less* by a factor of $m_1/(m_1 + m_2)$. If T *were* equal to $m_2 g$, then the lab weight would be in equilibrium, and it isn't.

CAUTION **Tension and weight may not be equal** It's a common mistake to assume that if an object is attached to a vertical string, the string tension must be equal to the object's weight. That was the case in Example 5.5, where the acceleration was zero, but it would certainly be wrong in this example! The only safe approach is to *always* treat the tension as a variable, as we have done here.

Finally, let's check some special cases. If $m_1 = 0$, then the lab weight would fall freely and there would be no tension in the string. The equations do give $T = 0$ and $a = g$ when $m_1 = 0$. Also, if $m_2 = 0$, we expect no tension and no acceleration; for this case the equations do indeed tell us that $T = 0$ and $a = 0$.

Test Your Understanding of Section 5.2 Suppose you hold the glider in Example 5.12 so that it and the weight are initially at rest. You give the glider a push to the left in Fig. 5.15a and then release it. The string remains taut as the glider moves to the left, comes instantaneously to rest, then moves to the right. At the instant the glider has zero velocity, what is the tension in the string? (i) greater than in Example 5.12; (ii) the same as in Example 5.12; (iii) less than in Example 5.12, but greater than zero; (iv) zero.

5.3 Frictional Forces

We have seen several problems where a body rests or slides on a surface that exerts forces on the body. Whenever two bodies interact by direct contact (touching) of their surfaces, we describe the interaction in terms of *contact forces*. The normal force is one example of a contact force; in this section we'll look in detail at another contact force, the force of friction.

Friction is important in many aspects of everyday life. The oil in a car engine minimizes friction between moving parts, but without friction between the tires and the road we couldn't drive or turn the car. Air drag—the frictional force exerted by the air on a body moving through it—decreases automotive fuel economy but makes parachutes work. Without friction, nails would pull out, light bulbs would unscrew effortlessly, and ice hockey would be hopeless (Fig. 5.16).

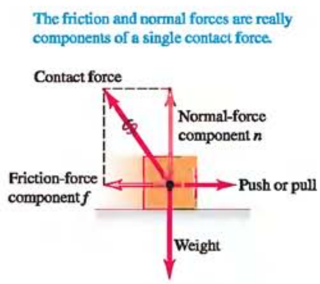
5.16 The sport of ice hockey depends on having the right amount of friction between a player's skates and the ice. If there were too much friction, the players would move too slowly; if there were too little friction, they would fall over.

Kinetic and Static Friction

When you try to slide a heavy box of books across the floor, the box doesn't move at all unless you push with a certain minimum force. Then the box starts moving, and you can usually keep it moving with less force than you needed to get it started. If you take some of the books out, you need less force than before to get it started or keep it moving. What general statements can we make about this behavior?

First, when a body rests or slides on a surface, we can think of the surface as exerting a single contact force on the body, with force components perpendicular and parallel to the surface (Fig. 5.17). The perpendicular component vector is the normal force, denoted by $\vec{n}$. The component vector parallel to the surface (and perpendicular to $\vec{n}$) is the **friction force,** denoted by $\vec{f}$. If the surface is frictionless, then $\vec{f}$ is zero but there is still a normal force. (Frictionless surfaces are an unattainable idealization, like a massless rope. But we can approximate a surface

5.17 When a block is pushed or pulled over a surface, the surface exerts a contact force on it.



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2.5 Truck Pulls Crate
2.6 Pushing a Crate Up a Wall
2.7 Skier Goes Down a Slope
2.8 Skier and Rope Tow
2.10 Truck Pulls Two Crates

as frictionless if the effects of friction are negligibly small.) The direction of the friction force is always such as to oppose relative motion of the two surfaces.

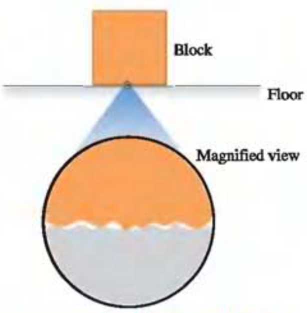
The kind of friction that acts when a body slides over a surface is called a **kinetic friction force** $\vec{f}_k$. The adjective "kinetic" and the subscript "k" remind us that the two surfaces are moving relative to each other. The *magnitude* of the kinetic friction force usually increases when the normal force increases. This is why it takes more force to slide a box full of books across the floor than to slide the same box when it is empty. This principle is also used in automotive braking systems: The harder the brake pads are squeezed against the rotating brake disks, the greater the braking effect. In many cases the magnitude of the kinetic friction force f_k is found experimentally to be approximately *proportional* to the magnitude n of the normal force. In such cases we represent the relationship by the equation

$$f_k = \mu_k n \quad \text{(magnitude of kinetic friction force)} \qquad (5.5)$$

where μ_k (pronounced "mu-sub-k") is a constant called the **coefficient of kinetic friction.** The more slippery the surface, the smaller the coefficient of friction. Because it is a quotient of two force magnitudes, μ_k is a pure number, without units.

CAUTION Friction and normal forces are always perpendicular Remember that Eq. (5.5) is *not* a vector equation because $\vec{f}_k$ and $\vec{n}$ are always perpendicular. Rather, it is a scalar relationship between the magnitudes of the two forces.

5.18 The normal and friction forces arise from interactions between molecules at high points on the surfaces of the block and the floor.



On a microscopic level, even smooth surfaces are rough; they tend to catch and cling.

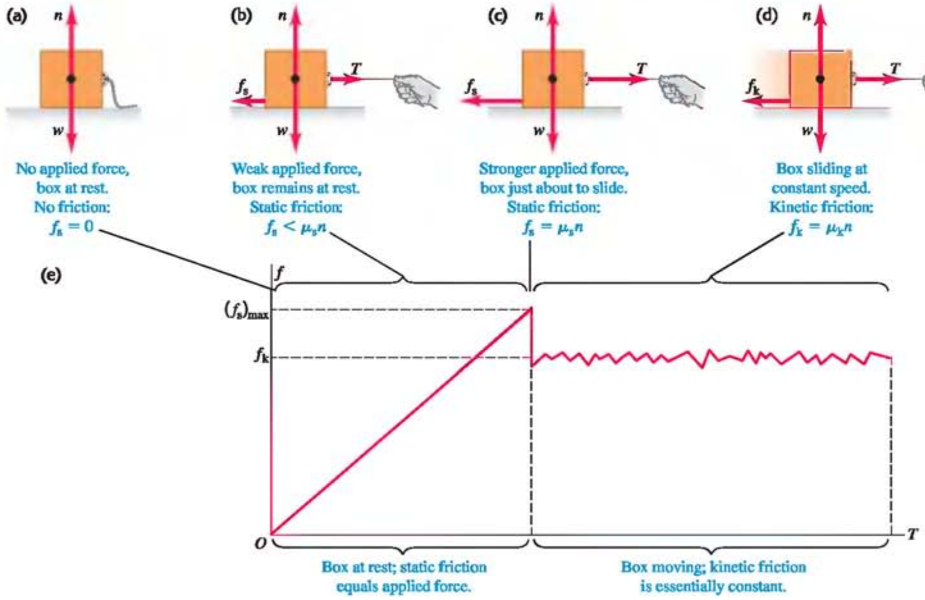
Equation (5.5) is only an approximate representation of a complex phenomenon. On a microscopic level, friction and normal forces result from the intermolecular forces (fundamentally electrical in nature) between two rough surfaces at points where they come into contact (Fig. 5.18). As a box slides over the floor, bonds between the two surfaces form and break, and the total number of such bonds varies; hence the kinetic friction force is not perfectly constant. Smoothing the surfaces can actually increase friction, since more molecules are able to interact and bond; bringing two smooth surfaces of the same metal together can cause a "cold weld." Lubricating oils work because an oil film between two surfaces (such as the pistons and cylinder walls in a car engine) prevents them from coming into actual contact.

Table 5.1 lists some representative values of μ_k. Although these values are given with two significant figures, they are only approximate, since friction forces

Table 5.1 Approximate Coefficients of Friction

Materials	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Brass on steel	0.51	0.44
Zinc on cast iron	0.85	0.21
Copper on cast iron	1.05	0.29
Glass on glass	0.94	0.40
Copper on glass	0.68	0.53
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

5.19 (a), (b), (c) When there is no relative motion, the magnitude of the static friction force f_s is less than or equal to $\mu_s n$. (d) When there is relative motion, the magnitude of the kinetic friction force f_k equals $\mu_k n$. (e) A graph of the friction force magnitude f as a function of the magnitude T of the applied force T. The kinetic friction force varies somewhat as intermolecular bonds form and break.



can also depend on the speed of the body relative to the surface. For now we'll ignore this effect and assume that μ_k and f_k are independent of speed, in order to concentrate on the simplest cases. Table 5.1 also lists coefficients of static friction; we'll define these shortly.

Friction forces may also act when there is *no* relative motion. If you try to slide a box across the floor, the box may not move at all because the floor exerts an equal and opposite friction force on the box. This is called a **static friction force** $\vec{f}_s$. In Fig. 5.19a, the box is at rest, in equilibrium, under the action of its weight $\vec{w}$ and the upward normal force $\vec{n}$. The normal force is equal in magnitude to the weight $(n = w)$ and is exerted on the box by the floor. Now we tie a rope to the box (Fig. 5.19b) and gradually increase the tension T in the rope. At first the box remains at rest because, as T increases, the force of static friction f_s also increases (staying equal in magnitude to T).

At some point T becomes greater than the maximum static friction force f_s the surface can exert. Then the box "breaks loose" (the tension T is able to break the bonds between molecules in the surfaces of the box and floor) and starts to slide. Figure 5.19c shows the forces when T is at this critical value. If T exceeds this value, the box is no longer in equilibrium. For a given pair of surfaces the maximum value of f_s depends on the normal force. Experiment shows that in many cases this maximum value, called $(f_s)_{max}$, is approximately *proportional* to n; we call the proportionality factor μ_s the **coefficient of static friction.** Table 5.1 lists some representative values of μ_s. In a particular situation, the actual force of static friction can have any magnitude between zero (when there is no other force parallel to the surface) and a maximum value given by $\mu_s n$. In symbols,

$$f_s \leq \mu_s n \quad \text{(magnitude of static friction force)} \qquad (5.6)$$

Like Eq. (5.5), this is a relationship between magnitudes, *not* a vector relationship. The equality sign holds only when the applied force T has reached the critical value at which motion is about to start (Fig. 5.19c). When T is less than this value (Fig. 5.19b), the inequality sign holds. In that case we have to use the equilibrium conditions $(\sum\vec{F} = \mathbf{0})$ to find f_s. If there is no applied force $(T = 0)$ as in Fig. 5.19a, then there is no static friction force either $(f_s = 0)$.

As soon as the box starts to slide (Fig. 5.19d), the friction force usually *decreases;* it's easier to keep the box moving than to start it moving. Hence the coefficient of kinetic friction is usually *less* than the coefficient of static friction for any given pair of surfaces, as Table 5.1 shows. If we start with no applied force $(T = 0)$ and gradually increase the force, the friction force varies somewhat, as shown in Fig. 5.19e.

In some situations the surfaces will alternately stick (static friction) and slip (kinetic friction). This is what causes the horrible sound made by chalk held at the wrong angle while writing on the blackboard. Other stick-slip phenomena are the squeak of windshield wipers on dry glass and the shriek of tires sliding on asphalt pavement. A more positive example is the motion of a violin bow against the string.

When a body slides on a layer of gas, friction can be made very small. In the linear air track used in physics laboratories, the gliders are supported on a layer of air. The frictional force is velocity dependent, but at typical speeds the effective coefficient of friction is of the order of 0.001.

Example 5.13 Friction in horizontal motion

You are trying to move a 500-N crate across a level floor. To start the crate moving, you have to pull with a 230-N horizontal force. Once the crate "breaks loose" and starts to move, you can keep it moving at constant velocity with only 200 N. What are the coefficients of static and kinetic friction?

SOLUTION

IDENTIFY: The crate is in equilibrium whether it is at rest or moving with constant velocity, so we use Newton's first law as expressed by Eq. (5.2). We'll also need the relationships in Eqs. (5.5) and (5.6) to find the target variables μ_s and μ_k.

SET UP: In either situation there are four forces acting on the crate: the downward weight force (magnitude $w = 500$ N), the upward normal force (magnitude n) exerted by the ground, a tension force (magnitude T) to the right exerted by the rope, and a friction force to the left exerted by the ground. Figures 5.20a and 5.20b show our sketch and free-body diagram for the instant just before the crate starts to move, when the static friction force has its maximum possible value $(f_s)_{max} = \mu_s n$. Once the crate is moving to the right at constant velocity, the friction force changes to its kinetic form (Fig. 5.20c). Because the rope in Fig. 5.20a is in equilibrium, the tension is the same at both ends. Hence the tension force that the rope exerts on the crate has the same magnitude as the force you exert on the rope.

5.20 Our sketches for this problem.

EXECUTE: Just before the crate starts to move (Fig. 5.20b), we have

$$\sum F_x = T + (-(f_s)_{max}) = 0 \quad \text{so} \quad (f_s)_{max} = T = 230\text{ N}$$
$$\sum F_y = n + (-w) = 0 \quad \text{so} \quad n = w = 500\text{ N}$$

Then we use Eq. (5.6), $(f_s)_{max} = \mu_s n$, to find the value of μ_s:

$$\mu_s = \frac{(f_s)_{max}}{n} = \frac{230\text{ N}}{500\text{ N}} = 0.46$$

After the crate starts to move, the forces are as shown in Fig. 5.20c, and we have

$$\sum F_x = T + (-f_k) = 0 \quad \text{so} \quad f_k = T = 200\text{ N}$$
$$\sum F_y = n + (-w) = 0 \quad \text{so} \quad n = w = 500\text{ N}$$

Using $f_k = \mu_k n$ from Eq. (5.5), we find

$$\mu_k = \frac{f_k}{n} = \frac{200\text{ N}}{500\text{ N}} = 0.40$$

EVALUATE: It's easier to keep the crate moving than to start it moving, and so the coefficient of kinetic friction is less than the coefficient of static friction.

Example 5.14 Static friction can be less than the maximum

In Example 5.13, what is the friction force if the crate is at rest on the surface and a horizontal force of 50 N is applied to it?

SOLUTION

IDENTIFY: The applied force is less than the maximum force of static friction, $(f_s)_{max} = 230\ \text{N}$. Hence the crate remains at rest and the net force acting on it is zero. The target variable is the magnitude f_s of the friction force.

SET UP: The free-body diagram is the same as in Fig. 5.20b, but with $(f_s)_{max}$ replaced by f_s and $T = 230\ \text{N}$ replaced by $T = 50\ \text{N}$.

EXECUTE: From the equilibrium conditions, Eq. (5.2), we have

$$\sum F_x = T + (-f_s) = 0 \qquad \text{so} \qquad f_s = T = 50\ \text{N}$$

EVALUATE: In this case, f_s is less than the maximum value $(f_s)_{max} = \mu_s n$. The frictional force can prevent motion for any horizontal applied force up to 230 N.

Example 5.15 Minimizing kinetic friction

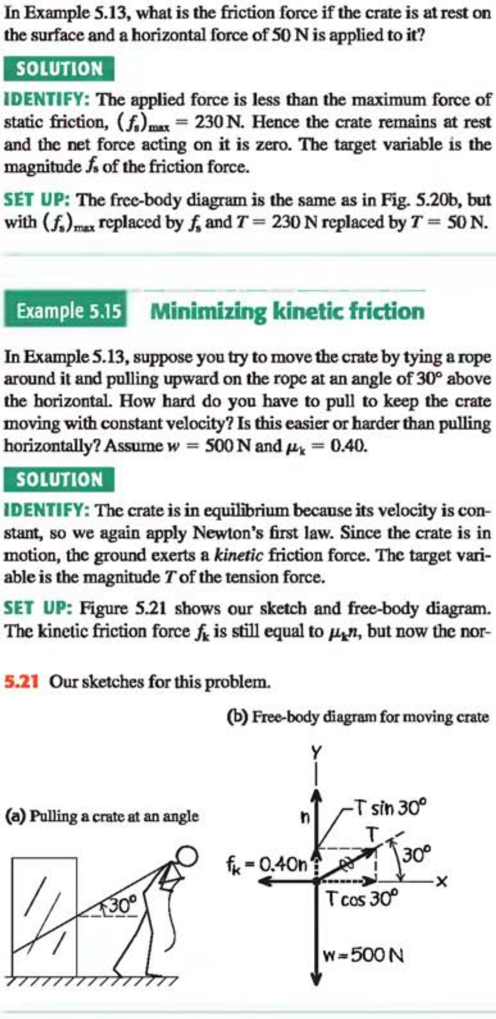
In Example 5.13, suppose you try to move the crate by tying a rope around it and pulling upward on the rope at an angle of 30° above the horizontal. How hard do you have to pull to keep the crate moving with constant velocity? Is this easier or harder than pulling horizontally? Assume $w = 500\ \text{N}$ and $\mu_k = 0.40$.

SOLUTION

IDENTIFY: The crate is in equilibrium because its velocity is constant, so we again apply Newton's first law. Since the crate is in motion, the ground exerts a *kinetic* friction force. The target variable is the magnitude T of the tension force.

SET UP: Figure 5.21 shows our sketch and free-body diagram. The kinetic friction force f_k is still equal to $\mu_k n$, but now the normal force n is *not* equal in magnitude to the weight of the crate. The force exerted by the rope has an additional vertical component that tends to lift the crate off the floor.

5.21 Our sketches for this problem.



EXECUTE: From the equilibrium conditions and the equation $f_k = \mu_k n$, we have

$$\sum F_x = T\cos 30° + (-f_k) = 0 \qquad \text{so} \qquad T\cos 30° = \mu_k n$$

$$\sum F_y = T\sin 30° + n + (-w) = 0 \qquad \text{so} \qquad n = w - T\sin 30°$$

These are two equations for the two unknown quantities T and n. To solve them, we can eliminate one unknown and solve for the other. There are many ways to do this; one way is to substitute the expression for n in the second equation back into the first equation:

$$T\cos 30° = \mu_k(w - T\sin 30°)$$

Then we solve this equation for T, with the result

$$T = \frac{\mu_k w}{\cos 30° + \mu_k \sin 30°} = 188\ \text{N}$$

We can substitute this result back into either of the original equations to obtain n. If we use the second equation to do this, we get

$$n = w - T\sin 30° = (500\ \text{N}) - (188\ \text{N})\sin 30° = 406\ \text{N}$$

EVALUATE: The normal force is *less* than the weight of the box $(w = 500\ \text{N})$ because the vertical component of tension pulls upward on the crate. Despite this, the tension required is a little less than the 200-N force needed when you pulled horizontally in Example 5.13. Try pulling at 22°; you'll find you need even less force (see Challenge Problem 5.123).

Example 5.16 Toboggan ride with friction I

Let's go back to the toboggan we studied in Example 5.10 (Section 5.2). The wax has worn off and there is now a nonzero coefficient of kinetic friction μ_k. The slope has just the right angle to make the toboggan slide with constant speed. Derive an expression for the slope angle in terms of w and μ_k.

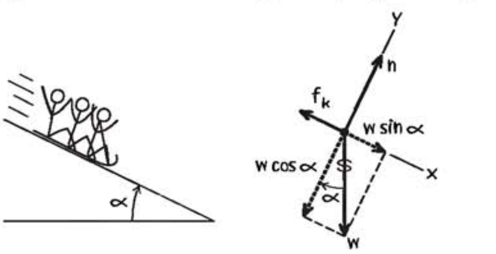
SOLUTION

IDENTIFY: Our target variable is the slope angle α. The toboggan is in equilibrium because its velocity is constant, so we use Newton's first law. There are three forces acting on the toboggan: its weight, the normal force, and the kinetic friction force. Since the motion is downhill, the kinetic friction force (which opposes the motion of the toboggan over the hill) is directed uphill.

SET UP: Figure 5.22 shows our sketch and free-body diagram. We take axes perpendicular and parallel to the surface and represent the weight in terms of its components in these two directions, as shown. (Compare Fig. 5.12b in Example 5.10.) The magnitude of the friction force is given by Eq. (5.5), $f_k = \mu_k n$.

Continued

5.22 Our sketches for this problem.



EXECUTE: The equilibrium conditions are

$$\sum F_x = w\sin\alpha + (-f_k) = w\sin\alpha - \mu_k n = 0$$
$$\sum F_y = n + (-w\cos\alpha) = 0$$

(We used the relationship $f_k = \mu_k n$ in the equation for the x-components.) Rearranging, we get

$$\mu_k n = w\sin\alpha \quad \text{and} \quad n = w\cos\alpha$$

Just as in Example 5.10, the normal force n is *not* equal to the weight w. When we divide the first of these equations by the second, we find

$$\mu_k = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha \quad \text{so} \quad \alpha = \arctan\mu_k$$

EVALUATE: The weight w doesn't appear in this expression. *Any* toboggan, regardless of its weight, slides down an incline with constant speed if the coefficient of kinetic friction equals the tangent of the slope angle of the incline. The greater the coefficient of friction, the steeper the slope has to be for the toboggan to slide with constant velocity.

Example 5.17 Toboggan ride with friction II

The same toboggan with the same coefficient of friction as in Example 5.16 *accelerates* down a steeper hill. Derive an expression for the acceleration in terms of g, α, μ_k, and w.

SOLUTION

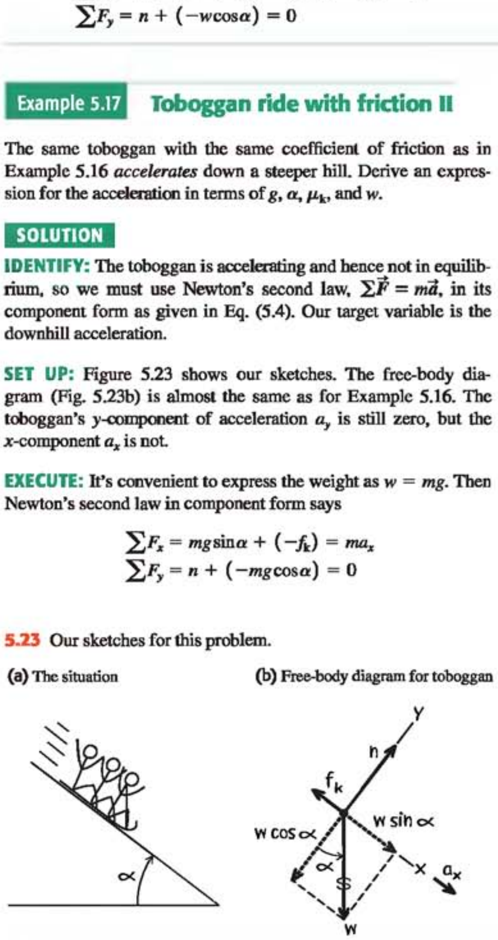
IDENTIFY: The toboggan is accelerating and hence not in equilibrium, so we must use Newton's second law, $\sum \vec{F} = m\vec{a}$, in its component form as given in Eq. (5.4). Our target variable is the downhill acceleration.

SET UP: Figure 5.23 shows our sketches. The free-body diagram (Fig. 5.23b) is almost the same as for Example 5.16. The toboggan's y-component of acceleration a_y is still zero, but the x-component a_x is not.

EXECUTE: It's convenient to express the weight as $w = mg$. Then Newton's second law in component form says

$$\sum F_x = mg\sin\alpha + (-f_k) = ma_x$$
$$\sum F_y = n + (-mg\cos\alpha) = 0$$

5.23 Our sketches for this problem.



From the second equation and Eq. (5.5) we get an expression for f_k:

$$n = mg\cos\alpha$$
$$f_k = \mu_k n = \mu_k mg\cos\alpha$$

We substitute this back into the x-component equation:

$$mg\sin\alpha + (-\mu_k mg\cos\alpha) = ma_x$$
$$a_x = g(\sin\alpha - \mu_k\cos\alpha)$$

EVALUATE: Does this result make sense? Let's check some special cases. First, if the hill is vertical, $\alpha = 90°$; then $\sin\alpha = 1$, $\cos\alpha = 0$, and $a_x = g$. This is free fall, just what we would expect. Second, on a hill at angle α with no friction, $\mu_k = 0$. Then $a_x = g\sin\alpha$. The situation is the same as in Example 5.10; happily, we get the same result. Next, suppose that there is just enough friction to make the toboggan move with constant velocity. In that case $a_x = 0$, so it must be that

$$\sin\alpha = \mu_k\cos\alpha \quad \text{and} \quad \mu_k = \tan\alpha$$

This agrees with our result from Example 5.16. Finally, note that there may be so much friction that $\mu_k\cos\alpha$ is actually greater than $\sin\alpha$. In that case, a_x is negative; if we give the toboggan an initial downhill push to start it moving, it will slow down and eventually stop.

We have pretty much beaten the toboggan problem to death, but look what we've done: Starting with a simple problem, we extended it to more and more general situations. The general result we found in this example includes *all* the previous ones as special cases. Don't memorize this general result; it is useful only for this one set of problems. But make sure you understand how we obtained it and what it means.

One final variation that you may want to try out is the case in which we give the toboggan an initial push up the hill. The direction of the kinetic friction force is now reversed, so the acceleration is different from the downhill value. It turns out that the expression for a_x is the same as for downhill motion except that the minus sign becomes plus. Can you prove this?

Rolling Friction

It's a lot easier to move a loaded filing cabinet across a horizontal floor using a cart with wheels than to slide it. How much easier? We can define a **coefficient of rolling friction** μ_r, which is the horizontal force needed for constant speed on a flat surface divided by the upward normal force exerted by the surface. Transportation engineers call μ_r the *tractive resistance*. Typical values of μ_r are 0.002 to 0.003 for steel wheels on steel rails and 0.01 to 0.02 for rubber tires on concrete. These values show one reason railroad trains are generally much more fuel efficient than highway trucks.

Example 5.18 Motion with rolling friction

A typical car weighs about 12,000 N (about 2700 lb). If the coefficient of rolling friction is $\mu_r = 0.015$, what horizontal force is needed to make the car move with constant speed on a level road? Neglect air resistance.

SOLUTION

IDENTIFY: The car is moving with constant velocity, so this is an equilibrium problem that uses Newton's first law. The four forces on the car are the weight, the upward normal force, the backward force of rolling friction, and the unknown forward force F (our target variable).

SET UP: The free-body diagram is much like the one in Fig. 5.20c of Example 5.13, but with the kinetic friction force replaced by the rolling friction force f_r and with the tension force replaced by the unknown force F.

EXECUTE: As in Example 5.13, Newton's first law for the *vertical* components tells us that the normal force is equal in magnitude to the car's weight. Hence, from the definition of μ_r, the rolling friction force f_r is

$$f_r = \mu_r n = (0.015)(12{,}000\ \text{N}) = 180\ \text{N} \qquad (\text{about } 40\ \text{lb})$$

Newton's first law for the *horizontal* components tells us that a forward force with this magnitude is needed to keep the car moving with constant speed.

EVALUATE: The required force is rather small, which is why it's possible to push a car by hand. (As in the case of sliding, it's easier to keep a car rolling than it is to start it rolling.) We've ignored the effects of air resistance, which is a pretty good approximation if the car is moving slowly. But at typical highway speeds, air resistance is a larger effect than rolling friction.

Try applying this analysis to the crate in Example 5.13. If the crate is on a rubber-wheeled dolly with $\mu_r = 0.02$, only a 10-N force is needed to keep it moving at constant velocity. Can you verify this?

Fluid Resistance and Terminal Speed

ActivPhysics ONLINE 2.1.2 Skydiver

Sticking your hand out the window of a fast-moving car will convince you of the existence of **fluid resistance**, the force that a fluid (a gas or liquid) exerts on a body moving through it. The moving body exerts a force on the fluid to push it out of the way. By Newton's third law, the fluid pushes back on the body with an equal and opposite force.

The *direction* of the fluid resistance force acting on a body is always opposite the direction of the body's velocity relative to the fluid. The *magnitude* of the fluid resistance force usually increases with the speed of the body through the fluid. This is very different from the kinetic friction force between two surfaces in contact, which we can usually regard as independent of speed. For very low speeds, the magnitude f of the fluid resistance force is approximately proportional to the body's speed v:

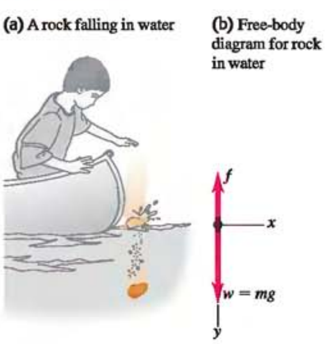
$$f = kv \qquad \text{(fluid resistance at low speed)} \qquad (5.7)$$

where k is a proportionality constant that depends on the shape and size of the body and the properties of the fluid. In motion through air at the speed of a tossed tennis ball or faster, the resisting force is approximately proportional to v^2 rather than to v. It is then called **air drag** or simply *drag*. Airplanes, falling raindrops, and bicyclists all experience air drag. In this case we replace Eq. (5.7) by

$$f = Dv^2 \qquad \text{(fluid resistance at high speed)} \qquad (5.8)$$

Because of the v^2 dependence, air drag increases rapidly with increasing speed. The air drag on a typical car is negligible at low speeds but comparable to or greater than rolling resistance at highway speeds. The value of D depends on the shape and size of the body and on the density of the air. You should verify that the units of the constant k in Eq. (5.7) are $N \cdot s/m$ or kg/s, and that the units of the constant D in Eq. (5.8) are $N \cdot s^2/m^2$ or kg/m.

5.24 A rock falling through a fluid (water).



Because of the effects of fluid resistance, an object falling in a fluid does *not* have a constant acceleration. To describe its motion, we can't use the constant-acceleration relationships from Chapter 2; instead, we have to start over using Newton's second law. As an example, suppose you drop a rock at the surface of a pond and let it fall to the bottom (Fig. 5.24a). The fluid resistance force in this situation is given by Eq. (5.7). What are the acceleration, velocity, and position of the rock as functions of time?

Figure 5.24b shows the free-body diagram. We take the positive y-direction to be downward and neglect any force associated with buoyancy in the water. Since the rock is moving downward, its speed v is equal to its y-velocity v_y and the fluid resistance force is in the $-y$-direction. There are no x-components, so Newton's second law gives

$$\sum F_y = mg + (-kv_y) = ma_y$$

When the rock first starts to move, $v_y = 0$, the resisting force is zero, and the initial acceleration is $a_y = g$. As the speed increases, the resisting force also increases, until finally it is equal in magnitude to the weight. At this time $mg - kv_y = 0$, the acceleration becomes zero, and there is no further increase in speed. The final speed v_t, called the **terminal speed**, is given by $mg - kv_t = 0$, or

$$v_t = \frac{mg}{k} \quad \text{(terminal speed, fluid resistance } f = kv\text{)} \tag{5.9}$$

Figure 5.25 shows how the acceleration, velocity, and position vary with time. As time goes by, the acceleration approaches zero and the velocity approaches v_t (remember that we chose the positive y-direction to be down). The slope of the graph of y versus t becomes constant as the velocity becomes constant.

To see how the graphs in Fig. 5.25 are derived, we must find the relationship between speed and time during the interval before the terminal speed is reached. We go back to Newton's second law, which we rewrite using $a_y = dv_y/dt$:

$$m\frac{dv_y}{dt} = mg - kv_y$$

After rearranging terms and replacing mg/k by v_t, we integrate both sides, noting that $v_y = 0$ when $t = 0$:

$$\int_0^v \frac{dv_y}{v_y - v_t} = -\frac{k}{m}\int_0^t dt$$

5.25 Graphs of the motion of a body falling without fluid resistance and with fluid resistance proportional to the speed.

which integrates to

$$\ln\frac{v_t - v_y}{v_t} = -\frac{k}{m}t \quad \text{or} \quad 1 - \frac{v_y}{v_t} = e^{-(k/m)t}$$

and finally

$$v_y = v_t[1 - e^{-(k/m)t}] \tag{5.10}$$

Note that v_y becomes equal to the terminal speed v_t only in the limit that $t \to \infty$; the rock cannot attain terminal speed in any finite length of time.

The derivative of v_y gives a_y as a function of time, and the integral of v_y gives y as a function of time. We leave the derivations for you to complete (see Exercise 5.46); the results are

$$a_y = ge^{-(k/m)t} \tag{5.11}$$

$$y = v_t\left[t - \frac{m}{k}\left(1 - e^{-(k/m)t}\right)\right] \tag{5.12}$$

Now look again at Fig. 5.25, which shows graphs of these three relationships.

In deriving the terminal speed in Eq. (5.9), we assumed that the fluid resistance force is proportional to the speed. For an object falling through the air at high speeds, so that the fluid resistance is equal to Dv^2 as in Eq. (5.8), the terminal speed is reached when Dv^2 equals the weight mg (Fig. 5.26a). You can show that the terminal speed v_t is given by

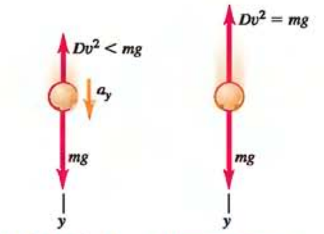
$$v_t = \sqrt{\frac{mg}{D}} \quad \text{(terminal speed, fluid resistance } f = Dv^2\text{)} \tag{5.13}$$

This expression for terminal speed explains why heavy objects in air tend to fall faster than light objects. Two objects with the same physical size but different mass (say, a table-tennis ball and a lead ball with the same radius) have the same value of D but different values of m. The more massive object has a higher terminal speed and falls faster. The same idea explains why a sheet of paper falls faster if you first crumple it into a ball; the mass m is the same, but the smaller size makes D smaller (less air drag for a given speed) and v_t larger. Skydivers use the same principle to control their descent (Fig. 5.26b).

Figure 5.27 shows the trajectories of a baseball with and without air drag, assuming a coefficient $D = 1.3 \times 10^{-3}$ kg/m (appropriate for a batted ball at sea level). You can see that both the range of the baseball and the maximum height reached are substantially less than the zero-drag calculation would lead you to believe. Hence the baseball trajectory we calculated in Example 3.8 (Section 3.3) by ignoring air drag is quite unrealistic. Air drag is an important part of the game of baseball!

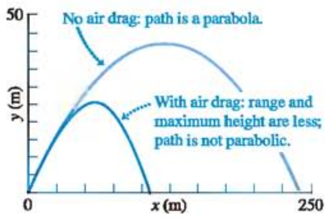
5.26 (a) Air drag and terminal speed. (b) By changing the positions of their arms and legs while falling, skydivers can change the value of the constant D in Eq. (5.8) and hence adjust the terminal speed of their fall [Eq. (5.13)].

(a) Free-body diagrams for falling with air drag



(b) A skydiver falling at terminal speed

5.27 Computer-generated trajectories of a baseball launched at 50 m/s at 35° above the horizontal. Note that the scales are different on the horizontal and vertical axes.



Example 5.19 Terminal speed of a skydiver

For a human body falling through air in a spread-eagle position (Fig. 5.26b), the numerical value of the constant D in Eq. (5.8) is about 0.25 kg/m. Find the terminal speed for a lightweight 50-kg skydiver.

SOLUTION

IDENTIFY: This example uses the relationship among terminal speed, mass, and drag coefficient.

SET UP: We use Eq. (5.13) to find the target variable v_t.

EXECUTE: We find for $m = 50$ kg:

$$v_t = \sqrt{\frac{mg}{D}} = \sqrt{\frac{(50\text{ kg})(9.8\text{ m/s}^2)}{0.25\text{ kg/m}}}$$
$$= 44\text{ m/s} \quad \text{(about 160 km/h, or 99 mi/h)}$$

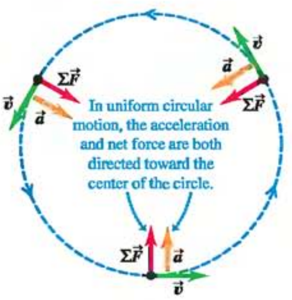
Continued

EVALUATE: The terminal speed is proportional to the square root of the skydiver's mass, so a more robust skydiver with the same drag coefficient D but twice the mass would have a terminal speed $\sqrt{2} = 1.41$ times greater, or 63 m/s. (A skydiver with more mass would also have more frontal area and hence a larger drag coefficient, so his terminal speed would be a bit less than 63 m/s.) Even the lightweight skydiver's terminal speed is quite high, so skydives don't last very long. A drop from 2800 m (9200 ft) to the surface at the terminal speed takes only $(2800\text{ m})/(44\text{ m/s}) = 64\text{ s}$.

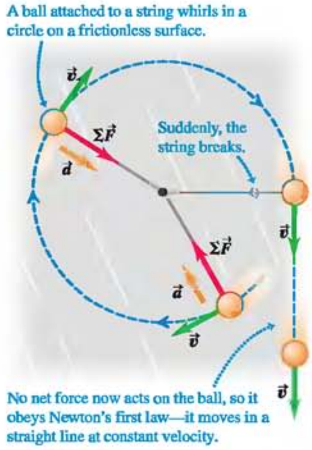
When the sky diver deploys the parachute, the value of D increases greatly. Hence the terminal speed of the skydiver and parachute decreases dramatically to a much slower value.

Test Your Understanding of Section 5.3 Consider a box that is placed on different surfaces. (a) In which situation(s) is there *no* friction force acting on the box? (b) In which situation(s) is there a *static* friction force acting on the box? (c) In which situation(s) is there a *kinetic* friction force on the box? (i) The box is at rest on a rough horizontal surface. (ii) The box is at rest on a rough tilted surface. (iii) The box is on the rough-surfaced flat bed of a truck—the truck is moving at a constant velocity on a straight, level road, and the box remains in the same place in the middle of the truck bed. (iv) The box is on the rough-surfaced flat bed of a truck—the truck is speeding up on a straight, level road, and the box remains in the same place in the middle of the truck bed. (v) The box is on the rough-surfaced flat bed of a truck—the truck is climbing a hill, and the box is sliding toward the back of the truck.

5.28 In uniform circular motion, both the acceleration and the net force are directed toward the center of the circle.



5.29 What happens if the inward radial force suddenly ceases to act on a body in circular motion?



5.4 Dynamics of Circular Motion

We talked about uniform circular motion in Section 3.4. We showed that when a particle moves in a circular path with constant speed, the particle's acceleration is always directed toward the center of the circle (perpendicular to the instantaneous velocity). The magnitude a_{rad} of the acceleration is constant and is given in terms of the speed v and the radius R of the circle by

$$a_{rad} = \frac{v^2}{R} \quad \text{(uniform circular motion)} \tag{5.14}$$

The subscript "rad" is a reminder that at each point the acceleration is radially inward toward the center of the circle, perpendicular to the instantaneous velocity. We explained in Section 3.4 why this acceleration is often called *centripetal acceleration.*

We can also express the centripetal acceleration a_{rad} in terms of the *period* T, the time for one revolution:

$$T = \frac{2\pi R}{v} \tag{5.15}$$

In terms of the period, a_{rad} is

$$a_{rad} = \frac{4\pi^2 R}{T^2} \quad \text{(uniform circular motion)} \tag{5.16}$$

Uniform circular motion, like all other motion of a particle, is governed by Newton's second law. To make the particle accelerate toward the center of the circle, the net force $\sum\vec{F}$ on the particle must always be directed toward the center (Fig. 5.28). The magnitude of the acceleration is constant, so the magnitude F_{net} of the net force must also be constant. If the inward net force stops acting, the particle flies off in a straight line tangent to the circle (Fig. 5.29).

The magnitude of the radial acceleration is given by $a_{rad} = v^2/R$, so the magnitude F_{net} of the net force on a particle with mass m in uniform circular motion must be

$$F_{net} = ma_{rad} = m\frac{v^2}{R} \quad \text{(uniform circular motion)} \tag{5.17}$$

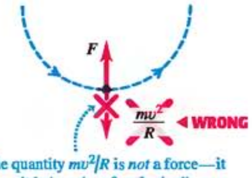
Uniform circular motion can result from *any* combination of forces, just so the net force $\sum\vec{F}$ is always directed toward the center of the circle and has a constant magnitude. Note that the body need not move around a complete circle: Equation (5.17) is valid for *any* path that can be regarded as part of a circular arc.

CAUTION **Avoid using "centrifugal force"** Figure 5.30 shows both a correct free-body diagram for uniform circular motion (Fig. 5.30a) and a common *incorrect* diagram (Fig. 5.30b). Figure 5.30b is incorrect because it includes an extra outward force of magnitude $m(v^2/R)$ to "keep the body out there" or to "keep it in equilibrium." There are three reasons not to include such an outward force, usually called *centrifugal force* ("centrifugal" means "fleeing from the center") First, the body does *not* "stay out there": It is in constant motion around its circular path. Because its velocity is constantly changing in direction, the body accelerates and is *not* in equilibrium. Second, if there *were* an additional outward force that balanced the inward force, the net force would be zero and the body would move in a straight line, not a circle (Fig. 5.29). And third, the quantity $m(v^2/R)$ is *not* a force; it corresponds to the $m\vec{a}$ side of $\sum\vec{F} = m\vec{a}$ and does not appear in $\sum\vec{F}$ (Fig. 5.30a). It's true that when you ride in a car that goes around a circular path, you tend to slide to the outside of the turn as though there was a "centrifugal force." But we saw in Section 4.2 that what really happens is that you tend to keep moving in a straight line, and the outer side of the car "runs into" you as the car turns (Fig. 4.11c). *In an inertial frame of reference there is no such thing as "centrifugal force."* We won't mention this term again, and we strongly advise you to avoid using it as well.

5.30 (a) Correct and (b) incorrect free-body diagrams for a body in uniform circular motion.

(a) Correct free-body diagram

(b) Incorrect free-body diagram



Example 5.20 Force in uniform circular motion

A sled with a mass of 25.0 kg rests on a horizontal sheet of essentially frictionless ice. It is attached by a 5.00-m rope to a post set in the ice. Once given a push, the sled revolves uniformly in a circle around the post (Fig. 5.31a). If the sled makes five complete revolutions every minute, find the force F exerted on it by the rope.

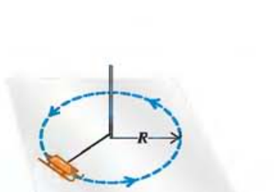
SOLUTION

IDENTIFY: The sled is moving in uniform circular motion, so it has a radial acceleration. We will apply Newton's second law to the sled to find the magnitude F of the force exerted by the rope (our target variable).

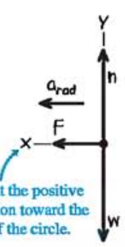
SET UP: Figure 5.31b shows our free-body diagram for the sled. The acceleration has only an x-component; this is toward the center of the circle, so we denote it as a_{rad}. The acceleration isn't given, so we'll need to determine its value using either Eq. (5.14) or Eq. (5.16).

5.31 (a) The situation. (b) Our free-body diagram.

(a) A sled in uniform circular motion



(b) Free-body diagram for the sled



EXECUTE: The acceleration in the y-direction is zero, so the net force in that direction is zero and the normal force and weight have the same magnitude. For the x-direction, Newton's second law gives

$$\sum F_x = F = ma_{rad}$$

We can find the centripetal acceleration a_{rad} using Eq. (5.16). The sled moves in a circle of radius $R = 5.00\text{ m}$ with a period $T = (60.0\text{ s})/(5\text{ rev}) = 12.0\text{ s}$, so

$$a_{rad} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(5.00\text{ m})}{(12.0\text{ s})^2} = 1.37\text{ m/s}^2$$

Alternatively, we can first use Eq. (5.15) to find the speed v:

$$v = \frac{2\pi R}{T} = \frac{2\pi(5.00\text{ m})}{12.0\text{ s}} = 2.62\text{ m/s}$$

Then, using Eq. (5.14),

$$a_{rad} = \frac{v^2}{R} = \frac{(2.62\text{ m/s})^2}{5.00\text{ m}} = 1.37\text{ m/s}^2$$

Hence the magnitude F of the force exerted by the rope is

$$F = ma_{rad} = (25.0\text{ kg})(1.37\text{ m/s}^2) = 34.3\text{ kg}\cdot\text{m/s}^2 = 34.3\text{ N}$$

EVALUATE: A greater force would be needed if the sled moved around the circle at a higher speed v. In fact, if v were doubled while R remained the same, F would be four times greater. Can you show this? How would F change if v remained the same but the radius R were doubled?

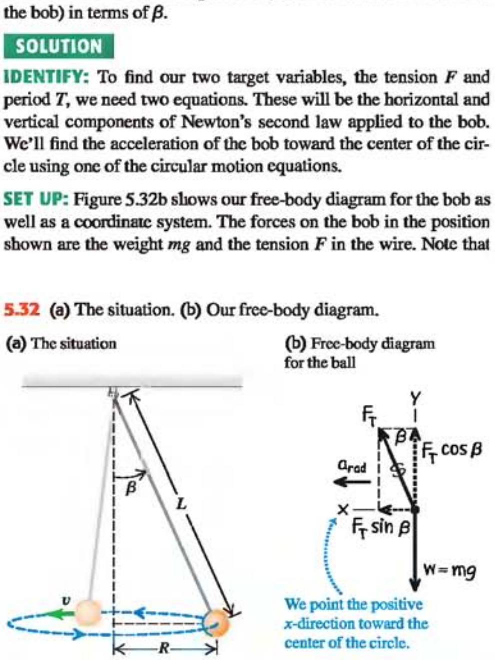
Example 5.21 The conical pendulum

An inventor proposes to make a pendulum clock using a pendulum bob with mass m at the end of a thin wire of length L. Instead of swinging back and forth, the bob moves in a horizontal circle with constant speed v, with the wire making a constant angle β with the vertical direction (Fig. 5.32a). This system is called a *conical pendulum* because the suspending wire traces out a cone. Find the tension F in the wire and the period T (the time for one revolution of the bob) in terms of β.

SOLUTION

IDENTIFY: To find our two target variables, the tension F and period T, we need two equations. These will be the horizontal and vertical components of Newton's second law applied to the bob. We'll find the acceleration of the bob toward the center of the circle using one of the circular motion equations.

SET UP: Figure 5.32b shows our free-body diagram for the bob as well as a coordinate system. The forces on the bob in the position shown are the weight mg and the tension F in the wire. Note that the center of the circular path is in the same horizontal plane as the bob, *not* at the top end of the wire. The horizontal component of tension is the force that produces the horizontal acceleration a_{rad} toward the center of the circle.



5.32 (a) The situation. (b) Our free-body diagram.

EXECUTE: The bob has zero vertical acceleration; the horizontal acceleration is toward the center of the circle, which is why we use the symbol a_{rad}. The $\sum \vec{F} = m\vec{a}$ equations are

$$\sum F_x = F\sin\beta = ma_{rad}$$
$$\sum F_y = F\cos\beta + (-mg) = 0$$

These are two equations for the two unknowns F and β. The equation for $\sum F_y$ gives $F = mg/\cos\beta$; substituting this result into the equation for $\sum F_x$ and using $\sin\beta/\cos\beta = \tan\beta$, we find

$$\tan\beta = \frac{a_{rad}}{g}$$

To relate β to the period T, we use Eq. (5.16) for a_{rad}. The radius of the circle is $R = L\sin\beta$, so

$$a_{rad} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 L\sin\beta}{T^2}$$

Substituting this into $\tan\beta = a_{rad}/g$, we obtain

$$\tan\beta = \frac{4\pi^2 L\sin\beta}{gT^2}$$

which we can rewrite as

$$T = 2\pi\sqrt{\frac{L\cos\beta}{g}}$$

EVALUATE: For a given length L, as the angle β increases, $\cos\beta$ decreases, the period T becomes smaller, and the tension $F = mg/\cos\beta$ increases. The angle can never be 90°, however; this would require that $T = 0$, $F = \infty$, and $v = \infty$. A conical pendulum would not make a very good clock because the period depends on the angle β in such a direct way.

Example 5.22 Rounding a flat curve

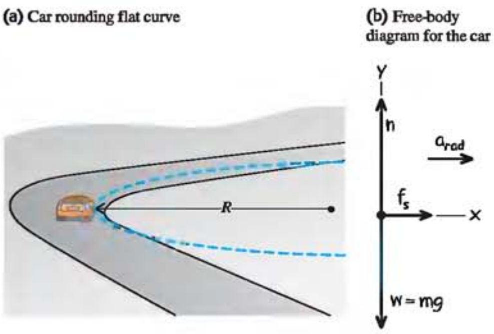
The sports car in Example 3.11 (Section 3.4) is rounding a flat, unbanked curve with radius R (Fig. 5.33a). If the coefficient of static friction between tires and road is μ_s, what is the maximum speed v_{max} at which the driver can take the curve without sliding?

SOLUTION

IDENTIFY: The car's acceleration as it rounds the curve has magnitude $a_{rad} = v^2/R$. Hence the maximum speed v_{max} (our target variable) corresponds to the maximum acceleration a_{rad} and to the maximum horizontal force on the car toward the center of its circular path. The only horizontal force acting on the car is the friction force exerted by the road. So we'll need Newton's second law and our knowledge of the friction force from Section 5.3.

SET UP: The free-body diagram in Fig. 5.33b includes the car's weight $w = mg$ and the two forces exerted by the road, the normal force n and the horizontal friction force f. The friction force must

5.33 (a) The situation. (b) Our free-body diagram.



point toward the center of the circular path in order to cause the radial acceleration. Since the car doesn't move in the radial direction (it doesn't slide toward or away from the center of the circle), the friction force is *static* friction with a maximum magnitude $f_{max} = \mu_s n$ [see Eq. (5.6)].

EXECUTE: The acceleration toward the center of the circular path is $a_{rad} = v^2/R$ and there is no vertical acceleration. Thus we have

$$\sum F_x = f = ma_{rad} = m\frac{v^2}{R}$$

$$\sum F_y = n + (-mg) = 0$$

The second equation shows that $n = mg$. The first equation shows that the friction force *needed* to keep the car moving in its circular path increases with the car's speed. But the maximum friction force *available* is $f_{max} = \mu_s n = \mu_s mg$, and this determines the car's maximum speed. Substituting f_{max} for f and v_{max} for v in the $\sum F_x$ equation, we find

$$\mu_s mg = m\frac{v_{max}{}^2}{R}$$

so the maximum speed is

$$v_{max} = \sqrt{\mu_s gR}$$

As an example, if $\mu_s = 0.96$ and $R = 230$ m, then

$$v_{max} = \sqrt{(0.96)(9.8\ \text{m/s}^2)(230\ \text{m})} = 47\ \text{m/s}$$

or about $170\ \text{km/h}\ (100\ \text{mi/h})$. This is the maximum speed for this radius.

EVALUATE: If the car's speed is slower than $\sqrt{\mu_s gR}$, the required friction force is less than the maximum possible value $f_{max} = \mu_s mg$ and the car can easily make the curve. If we try to take the curve going *faster* than the maximum speed, the car can still go in a circle without skidding, but the radius will be larger and the car will run off the road.

Note that the maximum centripetal acceleration (called the "lateral acceleration" in Example 3.11) is equal to $\mu_s g$. If the coefficient of friction is reduced, the maximum centripetal acceleration and v_{max} are also reduced. That's why it's best to take curves at a lower speed if the road is wet or icy (either of which can reduce the value of μ_s).

Example 5.23 Rounding a banked curve

For a car traveling at a certain speed, it is possible to bank a curve at just the right angle so that no friction at all is needed to maintain the car's turning radius. Then a car can safely round the curve even on wet ice. (Bobsled racing depends on this same idea.) Your engineering firm plans to rebuild the curve in Example 5.22 so that a car moving at speed v can safely make the turn even with no friction (Fig. 5.34a). At what angle β should the curve be banked?

SOLUTION

IDENTIFY: With no friction, the only two forces acting on the car are its weight and the normal force. Because the road is banked, the normal force (which acts perpendicular to the road surface) has a horizontal component. This component causes the car's horizontal acceleration toward the center of the car's circular path. Since forces and acceleration are involved, we'll use Newton's second law to find the target variable β.

SET UP: Our free-body diagram (Fig. 5.34b) is very similar to the diagram for the conical pendulum in Example 5.21 (Fig. 5.32b). The normal force acting on the car plays the role of the tension acting on the pendulum bob.

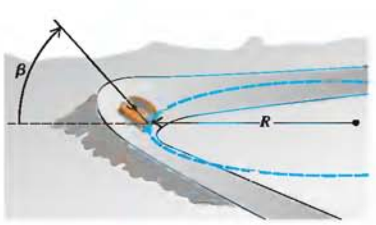
EXECUTE: The normal force $\vec{n}$ is perpendicular to the roadway at an angle β with the vertical. Thus it has a vertical component $n\cos\beta$ and a horizontal component $n\sin\beta$, as Fig. 5.34b shows. The acceleration in the x-direction is the centripetal acceleration, $a_{rad} = v^2/R$; there is no acceleration in the y-direction. Thus the equations of Newton's second law are

$$\sum F_x = n\sin\beta = ma_{rad}$$

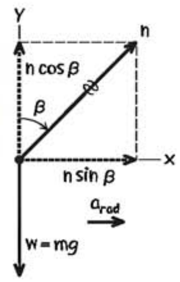
$$\sum F_y = n\cos\beta + (-mg) = 0$$

5.34 (a) The situation. (b) Our free-body diagram.

(a) Car rounding banked curve



(b) Free-body diagram for the car



Continued

From the $\sum F_y$ equation, $n = mg/\cos\beta$. Substituting this into the $\sum F_x$ equation gives an expression for the bank angle:

$$\tan\beta = \frac{a_{rad}}{g}$$

This is the same expression we found in Example 5.21. Finally, substituting the expression $a_{rad} = v^2/R$, we have

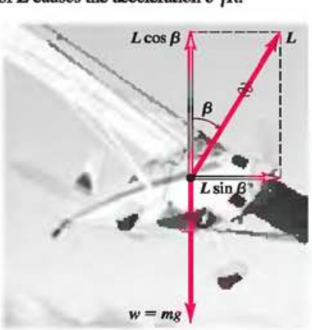
$$\tan\beta = \frac{v^2}{gR}$$

EVALUATE: The bank angle depends on the speed and the radius. For a given radius, no one angle is correct for all speeds. In the design of highways and railroads, curves are often banked for the average speed of the traffic over them. If $R = 230$ m and $v = 25$ m/s (equal to a highway speed of 88 km/h, or 55 mi/h), then

$$\beta = \arctan\frac{(25\ \text{m/s})^2}{(9.8\ \text{m/s}^2)(230\ \text{m})} = 15°$$

This is within the range of banking angles actually used in highways. With the same radius and $v = 47$ m/s, as in Example 5.22, $\beta = 44°$; such steeply banked curves are found at automobile raceways.

5.35 An airplane banks to one side in order to turn in that direction. The vertical component of the lift force $\vec{L}$ balances the force of gravity; the horizontal component of $\vec{L}$ causes the acceleration v^2/R.



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4.2 Circular Motion Problem Solving
4.3 Cart Goes Over Circular Path
4.4 Ball Swings on a String
4.5 Car Circles a Track

Banked Curves and the Flight of Airplanes

The results of Example 5.23 also apply to an airplane when it makes a turn in level flight (Fig. 5.35). When an airplane is flying in a straight line at a constant speed and at a steady altitude, the airplane's weight is exactly balanced by the lift force $\vec{L}$ exerted by the air. (The upward lift force that the air exerts on the wings is a reaction to the downward push the wings exert on the air as they move through it.) To make the airplane turn, the pilot banks the airplane to one side so that the lift force has a horizontal component as Fig. 5.35 shows. (The pilot also changes the angle at which the wings "bite" into the air so that the vertical component of lift continues to balance the weight.) The bank angle is related to the airplane's speed v and the radius R of the turn by the same expression as in Example 5.23: $\tan\beta = v^2/gR$. For an airplane to make a tight turn (small R) at high speed (large v), $\tan\beta$ must be large and the required bank angle β must approach 90°.

We can also apply the results of Example 5.23 to the *pilot* of an airplane. The free-body diagram for the pilot of the airplane is exactly as shown in Fig. 5.34b; the normal force $n = mg/\cos\beta$ is exerted on the pilot by the seat. As in Example 5.9, n is equal to the apparent weight of the pilot, which is greater than the pilot's true weight mg. In a tight turn with a large bank angle β, the pilot's apparent weight can be tremendous: $n = 5.8mg$ at $\beta = 80°$ and $n = 9.6mg$ at $\beta = 84°$. Pilots black out in such tight turns because the apparent weight of their blood increases by the same factor, and the human heart isn't strong enough to pump such apparently "heavy" blood to the brain.

Motion in a Vertical Circle

In Examples 5.20, 5.21, 5.22, and 5.23 the body moved in a horizontal circle. Motion in a *vertical* circle is no different in principle, but the weight of the body has to be treated carefully. The following example shows what we mean.

Example 5.24 Uniform circular motion in a vertical circle

A passenger on a carnival Ferris wheel moves in a vertical circle of radius R with constant speed v. The seat remains upright during the motion. Find expressions for the force the seat exerts on the passenger at the top of the circle and at the bottom.

SOLUTION

IDENTIFY: At both the top and bottom of the circle, the target variable is the magnitude n of the normal force that the seat exerts on the passenger. We'll find this force at each position using Newton's second law and the equations of uniform circular motion.

SET UP: Figure 5.36a shows the passenger's velocity and acceleration at the two positions. Note that the acceleration points *downward* at the *top* of the circle but *upward* at the *bottom* of the circle. At each position the only forces acting are vertical: the upward normal force and the downward force of gravity. Hence we need only the vertical component of Newton's second law.

EXECUTE: Figures 5.36b and 5.36c show free-body diagrams for the two positions. We take the positive y-direction as upward in both cases. Let n_T be the upward normal force the seat applies to the passenger at the top of the circle, and let n_B be the normal force at the bottom. At the top the acceleration has magnitude v^2/R, but its vertical component is negative because its direction is downward. Hence $a_y = -v^2/R$, and Newton's second law tells us that

$$\text{Top:}\quad \sum F_y = n_T + (-mg) = -m\frac{v^2}{R} \quad \text{or}$$

$$n_T = m\left(g - \frac{v^2}{R}\right)$$

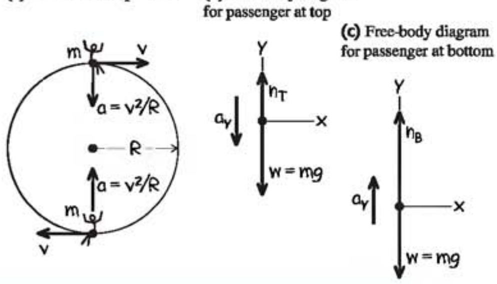
At the bottom the acceleration is upward, so $a_y = +v^2/R$ and Newton's second law is

$$\text{Bottom:}\quad \sum F_y = n_B + (-mg) = +m\frac{v^2}{R} \quad \text{or}$$

$$n_B = m\left(g + \frac{v^2}{R}\right)$$

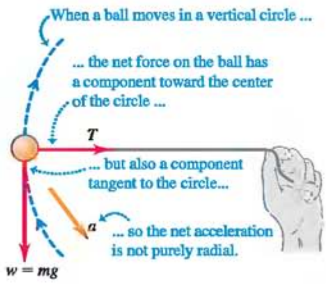
EVALUATE: Our result for n_T tells us that at the top of the Ferris wheel, the upward force the seat applies to the passenger is *smaller* in magnitude than the passenger's weight $w = mg$. If the ride goes fast enough that $g - v^2/R$ becomes zero, the seat applies *no* force, and the passenger is about to become airborne. If v becomes still larger, n_T becomes negative; this means that a *downward* force (such as from a seat belt) is needed to keep the passenger in the seat. By contrast, the normal force n_B at the bottom is always *greater than* the passenger's weight. You feel the seat pushing up on you more firmly than when you are at rest. You can see that n_T and n_B are the values of the passenger's *apparent weight* at the top and bottom of the circle (see Section 5.2).

5.36 Our sketches for this problem.



When we tie a string to an object and whirl it in a vertical circle, the analysis in Example 5.24 isn't directly applicable. The reason is that v is *not* constant in this case; except at the top and bottom of the circle, the net force (and hence the acceleration) does *not* point toward the center of the circle (Fig. 5.37). So both $\sum\vec{F}$ and $\vec{a}$ have a component tangent to the circle, which means that the speed changes. Hence this is a case of *nonuniform* circular motion (see Section 3.4). Even worse, we can't use the constant-acceleration formulas to relate the speeds at various points because *neither* the magnitude nor the direction of the acceleration is constant. The speed relationships we need are best obtained by using the concept of energy. We'll consider such problems in Chapter 7.

5.37 A ball moving in a vertical circle.



Test Your Understanding of Section 5.4 Satellites are held in orbit by the force of our planet's gravitational attraction. A satellite in a small-radius orbit moves at a higher speed than a satellite in an orbit of large radius. Based on this information, what you can conclude about the earth's gravitational attraction for the satellite? (i) It increases with increasing distance from the earth. (ii) It is the same at all distances from the earth. (iii) It decreases with increasing distance from the earth. (iv) This information by itself isn't enough to answer the question.

*5.5 The Fundamental Forces of Nature

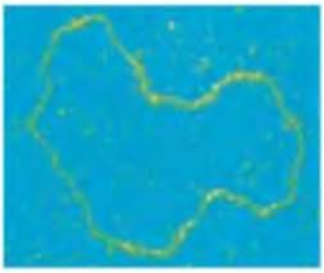
We have discussed several kinds of forces—including weight, tension, friction, fluid resistance, and the normal force—and we will encounter others as we continue our study of physics. But just how many kinds of forces are there? Our current understanding is that all forces are expressions of just four distinct classes of *fundamental* forces, or interactions between particles (Fig. 5.38). Two are familiar in everyday experience. The other two involve interactions between subatomic particles that we cannot observe with the unaided senses.

Gravitational interactions include the familiar force of your *weight*, which results from the earth's gravitational attraction acting on you. The mutual gravitational attraction of various parts of the earth for each other holds our planet

5.38 Examples of the fundamental interactions in nature. **(a)** The moon and the earth are held together and held in orbit by gravitational forces. **(b)** This molecule of bacterial plasmid DNA is held together by electromagnetic forces between its atoms. **(c)** The sun shines because in its core, strong forces between nuclear particles cause the release of energy. **(d)** When a massive star explodes into a supernova, a flood of energy is released by weak interactions between the star's nuclear particles.

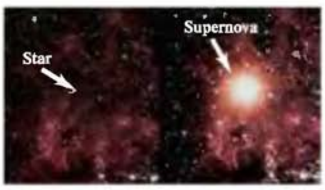
(a) Gravitational forces hold planets together.

(b) Electromagnetic forces hold molecules together.



(c) Strong forces release energy to power the sun.

(d) Weak forces play a role in exploding stars.



together (Fig. 5.38a). Newton recognized that the sun's gravitational attraction for the earth keeps the earth in its nearly circular orbit around the sun. In Chapter 12 we will study gravitational interactions in greater detail, and we will analyze their vital role in the motions of planets and satellites.

The second familiar class of forces, **electromagnetic interactions,** includes electric and magnetic forces. If you run a comb through your hair, the comb ends up with an electric charge; you can use the electric force exerted by this charge to pick up bits of paper. All atoms contain positive and negative electric charge, so atoms and molecules can exert electric forces on each other (Fig. 5.38b). Contact forces, including the normal force, friction, and fluid resistance, are the combination of all such forces exerted on the atoms of a body by atoms in its surroundings. *Magnetic* forces, such as those between magnets or between a magnet and a piece of iron, are actually the result of electric charges in motion. For example, an electromagnet causes magnetic interactions because electric charges move through its wires. We will study electromagnetic interactions in detail in the second half of this book.

On the atomic or molecular scale, gravitational forces play no role because electric forces are enormously stronger: The electrical repulsion between two protons is stronger than their gravitational attraction by a factor of about 10^{35}. But in bodies of astronomical size, positive and negative charges are usually present in nearly equal amounts, and the resulting electrical interactions nearly cancel out. Gravitational interactions are thus the dominant influence in the motion of planets and in the internal structure of stars.

The other two classes of interactions are less familiar. One, the **strong interaction,** is responsible for holding the nucleus of an atom together. Nuclei contain electrically neutral neutrons and positively charged protons. The electric force between charged protons tries to push them apart; the strong attractive force between nuclear particles counteracts this repulsion and makes the nucleus stable. In this context the strong interaction is also called the *strong nuclear force.* It has much shorter range than electrical interactions, but within its range it is much stronger. The strong interaction plays a key role in thermonuclear reactions that take place at the sun's core and generate the sun's heat and light (Fig. 5.38c).

Finally, there is the **weak interaction.** Its range is so short that it plays a role only on the scale of the nucleus or smaller. The weak interaction is responsible for a common form of radioactivity called beta decay, in which a neutron in a radioactive nucleus is transformed into a proton while ejecting an electron and a nearly massless particle called an antineutrino. The weak interaction between the antineutrino and ordinary matter is so feeble that an antineutrino could easily penetrate a wall of lead a million kilometers thick! Yet when a giant star undergoes a cataclysmic explosion called a supernova, most of the energy is released by way of the weak interaction (Fig. 5.38d).

In the 1960s physicists developed a theory that described the electromagnetic and weak interactions as aspects of a single *electroweak* interaction. This theory has passed every experimental test to which it has been put. Encouraged by this success, physicists have made similar attempts to describe the strong, electromagnetic, and weak interactions in terms of a single *grand unified theory* (GUT), and have taken steps toward a possible unification of all interactions into a *theory of everything* (TOE). Such theories are still speculative, and there are many unanswered questions in this very active field of current research.

CHAPTER 5 SUMMARY

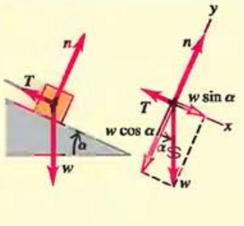
Using Newton's first law: When a body is in equilibrium in an inertial frame of reference—that is, either at rest or moving with constant velocity—the vector sum of forces acting on it must be zero (Newton's first law). Free-body diagrams are essential in identifying the forces that act on the body being considered.

Newton's third law (action and reaction) is also frequently needed in equilibrium problems. The two forces in an action–reaction pair *never* act on the same body. (See Examples 5.1–5.5.)

The normal force exerted on a body by a surface is *not* always equal to the body's weight. (See Example 5.3.)

$$\sum \vec{F} = 0 \quad \text{(vector form)} \qquad (5.1)$$

$$\sum F_x = 0, \quad \sum F_y = 0 \quad \text{(component form)} \qquad (5.2)$$



Using Newton's second law: If the vector sum of forces on a body is *not* zero, the body accelerates. The acceleration is related to the net force by Newton's second law.

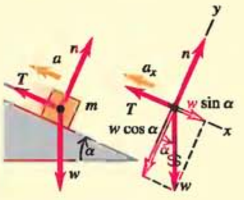
Just as for equilibrium problems, free-body diagrams are essential for solving problems involving Newton's second law, and the normal force exerted on a body is not always equal to its weight. (See Examples 5.6–5.12.)

Vector form:

$$\sum \vec{F} = m\vec{a} \qquad (5.3)$$

Component form:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad (5.4)$$



Friction and fluid resistance: The contact force between two bodies can always be represented in terms of a normal force $\vec{n}$ perpendicular to the surface of contact and a friction force $\vec{f}$ parallel to the surface.

When a body is sliding over the surface, the friction force is called *kinetic* friction. Its magnitude f_k is approximately equal to the normal force magnitude n multiplied by the coefficient of kinetic friction μ_k. When a body is *not* moving relative to a surface, the friction force is called *static* friction. The *maximum* possible static friction force is approximately equal to the magnitude n of the normal force multiplied by the coefficient of static friction μ_s. The *actual* static friction force may be anything from zero to this maximum value, depending on the situation. Usually μ_s is greater than μ_k for a given pair of surfaces in contact. (See Examples 5.13–5.17.)

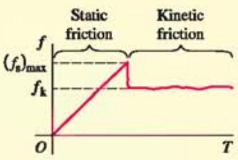
Rolling friction is similar to kinetic friction, but the force of fluid resistance depends on the speed of an object through a fluid. (See Examples 5.18 and 5.19.)

Magnitude of kinetic friction force:

$$f_k = \mu_k n \qquad (5.5)$$

Magnitude of static friction force:

$$f_s \le \mu_s n \qquad (5.6)$$



Forces in circular motion: In uniform circular motion, the acceleration vector is directed toward the center of the circle. The motion is governed by Newton's second law, $\sum \vec{F} = m\vec{a}$. (See Examples 5.20–5.24.)

Acceleration in uniform circular motion:

$$a_{rad} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2} \qquad (5.14), (5.16)$$

Key Terms

apparent weight, *145*
friction force, *149*
kinetic friction force, *150*
coefficient of kinetic friction, *150*
static friction force, *151*
coefficient of static friction, *151*
coefficient of rolling friction, *155*
fluid resistance, *155*
air drag, *155*
terminal speed, *156*
gravitational interaction, *163*
electromagnetic interaction, *164*
strong interaction, *164*
weak interaction, *164*

Answer to Chapter Opening Question ?

Neither; the upward force of the air has the *same* magnitude as the force of gravity. Although the bird is ascending, its vertical velocity is constant and so its vertical acceleration is zero. Hence the net vertical force on the bird must also be zero, and the individual vertical forces must balance.

Answers to Test Your Understanding Questions

5.1 Answer: (ii) The two cables are arranged symmetrically, so the tension in either cable has the same magnitude T. The vertical component of the tension from each cable is $T\sin 45°$ (or, equivalently, $T\cos 45°$), so Newton's first law applied to the vertical forces tells us that $2T\sin 45° - w = 0$. Hence $T = w/(2\sin 45°) = w/\sqrt{2} = 0.71w$. Each cable supports half of the weight of the traffic light, but the tension is greater than $w/2$ because only the vertical component of the tension counteracts the weight.

5.2 Answer: (ii) No matter what the instantaneous velocity of the glider, its acceleration is constant and has the value found in Example 5.12. In the same way, the acceleration of a body in free fall is the same whether it is ascending, descending, or at the high point of its motion (see Section 2.5).

5.3 Answers to (a): (i), (iii); answers to (b): (ii), (iv); answer to (c): (v) In situations (i) and (iii) the box is not accelerating (so the net force on it must be zero) and there is no other force acting parallel to the horizontal surface; hence no friction force is needed to prevent sliding. In situations (ii) and (iv) the box would start to slide over the surface if no friction were present, so a static friction force must act to prevent this. In situation (v) the box is sliding over a rough surface, so a kinetic friction force acts on it.

5.4 Answer: (iii) A satellite of mass m orbiting the earth at speed v in an orbit of radius r has an acceleration of magnitude v^2/r, so the net force acting on it from the earth's gravity has magnitude $F = mv^2/r$. The farther the satellite is from earth, the greater the value of r, the smaller the value of v, and hence the smaller the values of v^2/r and of F. In other words, the earth's gravitational force decreases with increasing distance.

PROBLEMS

For instructor-assigned homework, go to **www.masteringphysics.com** MP

Discussion Questions

Q5.1. A man sits in a seat that is suspended from a rope. The rope passes over a pulley suspended from the ceiling, and the man holds the other end of the rope in his hands. What is the tension in the rope, and what force does the seat exert on the man? Draw a free-body force diagram for the man.

Q5.2. "In general, the normal force is not equal to the weight." Give an example where the two forces are equal in magnitude, and at least two examples where they are not.

Q5.3. A clothesline hangs between two poles. No matter how tightly the line is stretched, it always sags a little at the center. Explain why.

Q5.4. A car is driven up a steep hill at constant speed. Discuss all the forces acting on the car. What pushes it up the hill?

Q5.5. For medical reasons it is important for astronauts in outer space to determine their body mass at regular intervals. Devise a scheme for measuring body mass in an apparently weightless environment.

Q5.6. To push a box up a ramp, is the force required smaller if you push horizontally or if you push parallel to the ramp? Why?

Q5.7. A woman in an elevator lets go of her briefcase but it does not fall to the floor. How is the elevator moving?

Q5.8. You can classify scales for weighing objects as those that use springs and those that use standard masses to balance unknown masses. Which group would be more accurate when you use it in an accelerating spaceship? When you use it on the moon?

Q5.9. When you tighten a nut on a bolt, how are you increasing the frictional force? How does a lock washer work?

Q5.10. A block rests on an inclined plane with enough friction to prevent it from sliding down. To start the block moving, is it easier to push it up the plane or down the plane? Why?

Q5.11. A crate of books rests on a level floor. To move it along the floor at a constant velocity, why do you exert a smaller force if you pull it at an angle θ above the horizontal than if you push it at the same angle below the horizontal?

Q5.12. In a world without friction, which of the following activities could you do (or not do)? Explain your reasoning. (a) drive around an unbanked highway curve; (b) jump into the air; (c) start walking on a horizontal sidewalk; (d) climb a vertical ladder; (e) change lanes on the freeway.

Q5.13. Walking on horizontal slippery ice can be much more tiring than walking on ordinary pavement. Why?

Q5.14. When you stand with bare feet in a wet bathtub, the grip feels fairly secure, and yet a catastrophic slip is quite possible. Explain this in terms of the two coefficients of friction.

Q5.15. You are pushing a large crate from the back of a freight elevator to the front as the elevator is moving to the next floor. In which situation is the force you must apply to move the crate the smallest and in which is it the largest: when the elevator is accelerating upward, when it is accelerating downward, or when it is traveling at constant speed? Explain.

Q5.16. The moon is accelerating toward the earth. Why isn't it getting closer to us?

Q5.17. An automotive magazine calls decreasing-radius curves "the bane of the Sunday driver." Explain.

Q5.18. You often hear people say that "friction always opposes motion." Give at least one example where (a) static friction *causes* motion, and (b) kinetic friction *causes* motion.

Q5.19. If there is a net force on a particle in uniform circular motion, why doesn't the particle's speed change?

Q5.20. A curve in a road has the banking angle calculated and posted for 80 km/h. However, the road is covered with ice so you cautiously plan to drive slower than this limit. What may happen to your car? Why?

Q5.21. You swing a ball on the end of a lightweight string in a horizontal circle at constant speed. Can the string ever be truly horizontal? If not, would it slope above the horizontal or below the horizontal? Why?

Q5.22. The centrifugal force is not included in the free-body diagrams of Figs. 5.34b and 5.35. Explain why not.

Q5.23. A professor swings a rubber stopper in a horizontal circle on the end of a string in front of his class. He tells Caroline, in the first row, that he is going to let the string go when the stopper is directly in front of her face. Should Caroline worry?

Q5.24. To keep the forces on the riders within allowable limits, loop-the-loop roller coaster rides are often designed so that the loop, rather than being a perfect circle, has a larger radius of curvature at the bottom than at the top. Explain.

Q5.25. A tennis ball drops from rest at the top of a tall glass cylinder, first with the air pumped out of the cylinder so there is no air resistance, and then a second time after the air has been readmitted to the cylinder. You examine multiflash photographs of the two drops. From these photos how can you tell which one is which, or can you?

Q5.26. If you throw a baseball straight upward with speed v_0, how does its speed, when it returns to the point from where you threw it, compare to v_0 (a) in the absence of air resistance and (b) in the presence of air resistance? Explain.

Q5.27. You throw a baseball straight upward. If air resistance is *not* ignored, how does the time required for the ball to go from the height at which it was thrown up to its maximum height compare to the time required for it to fall from its maximum height back down to the height from which it was thrown? Explain your answer.

Q5.28. You take two identical tennis balls and fill one with water. You release both balls simultaneously from the top of a tall building. If air resistance is negligible, which ball strikes the ground first? Explain. What is the answer if air resistance is *not* negligible?

Q5.29. A ball is dropped from rest and feels air resistance as it falls. Which of the graphs in Fig. 5.39 best represents its acceleration as a function of time?

Q5.30. A ball is dropped from rest and feels air resistance as it falls. Which of the graphs in Fig. 5.40 best represents its vertical velocity component as a function of time?

Q5.31. When does a baseball in flight have an acceleration with a positive upward component? Explain in terms of the forces on the ball and also in terms of the velocity components compared to the terminal speed. Do *not* ignore air resistance.

Q5.32. When a batted baseball moves with air drag, does it travel a greater horizontal distance while climbing to its maximum height or while descending from its maximum height back to the ground? Or is the horizontal distance traveled the same for both? Explain in terms of the forces acting on the ball.

Figure 5.39 Question Q5.29.

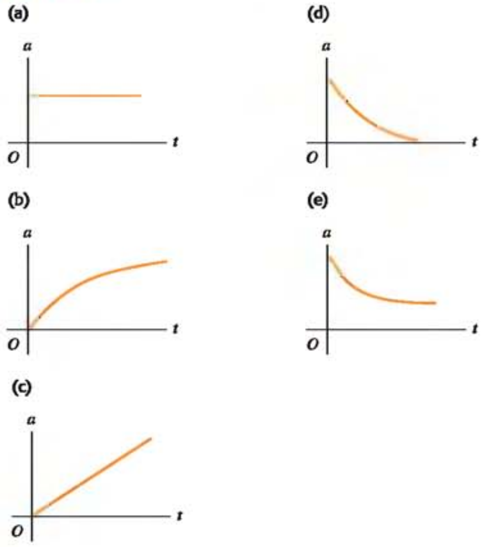
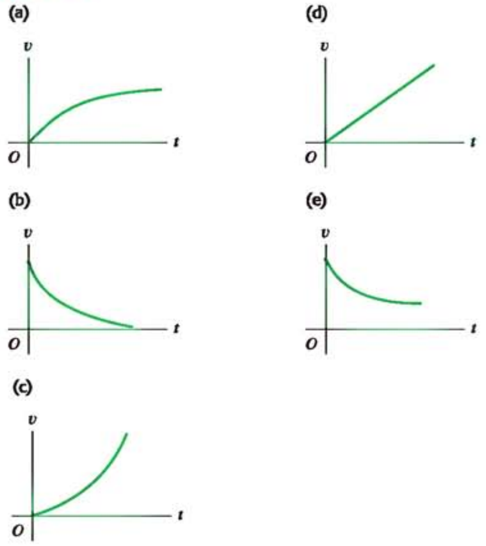


Figure 5.40 Question Q5.30.



Q5.33. "A ball is thrown from the edge of a high cliff. No matter what the angle at which it is thrown, due to air resistance, the ball will eventually end up moving vertically downward." Justify this statement.

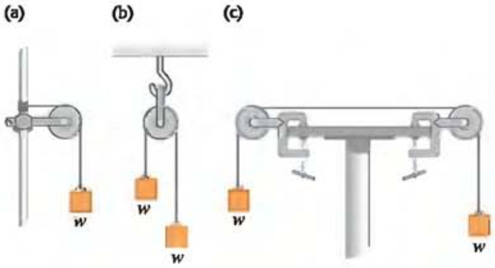
Exercises

Section 5.1 Using Newton's First Law: Particles in Equilibrium

5.1. Two 25.0-N weights are suspended at opposite ends of a rope that passes over a light, frictionless pulley. The pulley is attached to a chain that goes to the ceiling. (a) What is the tension in the rope? (b) What is the tension in the chain?

5.2. In Fig. 5.41 each of the suspended blocks has weight w. The pulleys are frictionless and the ropes have negligible weight. Calculate, in each case, the tension T in the rope in terms of the weight w. In each case, include the free-body diagram or diagrams you used to determine the answer.

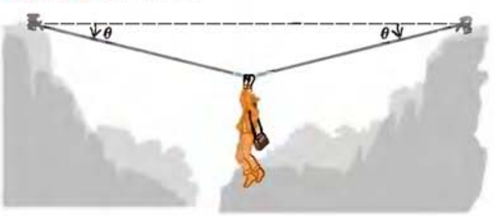
Figure 5.41 Exercise 5.2.



5.3. A 75.0-kg wrecking ball hangs from a uniform heavy-duty chain having a mass of 26.0 kg. (a) Find the maximum and minimum tension in the chain. (b) What is the tension at a point three-fourths of the way up from the bottom of the chain?

5.4. An adventurous archaeologist crosses between two rock cliffs by slowly going hand over hand along a rope stretched between the cliffs. He stops to rest at the middle of the rope (Fig. 5.42). The rope will break if the tension in it exceeds 2.50×10^4 N, and our hero's mass is 90.0 kg. (a) If the angle θ is 10.0°, find the tension in the rope. (b) What is the smallest value the angle θ can have if the rope is not to break?

Figure 5.42 Exercise 5.4.



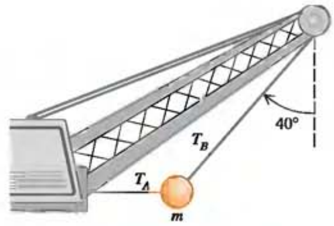
5.5. A picture frame hung against a wall is suspended by two wires attached to its upper corners. If the two wires make the same angle with the vertical, what must this angle be if the tension in each wire is equal to 0.75 of the weight of the frame? (Ignore any friction between the wall and the picture frame.)

5.6. Solve the problem in Example 5.5 using coordinate axes where the y-axis is vertical and the x-axis is horizontal. Do you get the same answers using this different set of axes?

5.7. Certain streets in San Francisco make an angle of 17.5° with the horizontal. What force parallel to the street surface is required to keep a loaded 1967 Corvette of mass 1390 kg from rolling down such a street?

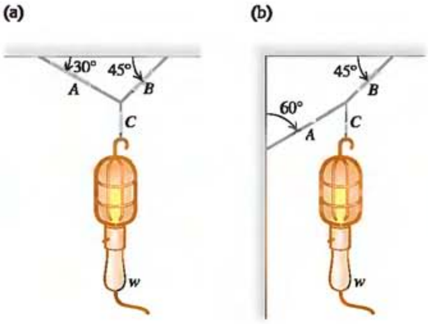
5.8. A large wrecking ball is held in place by two light steel cables (Fig. 5.43). If the mass m of the wrecking ball is 4090 kg, what are (a) the tension T_B in the cable that makes an angle of 40° with the vertical and (b) the tension T_A in the horizontal cable?

Figure 5.43 Exercise 5.8.



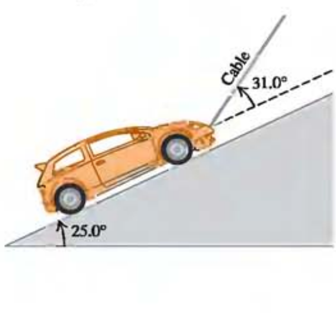
5.9. Find the tension in each cord in Fig. 5.44 if the weight of the suspended object is w.

Figure 5.44 Exercise 5.9.



5.10. A 1130-kg car is held in place by a light cable on a very smooth (frictionless) ramp, as shown in Fig. 5.45. The cable makes an angle of 31.0° above the surface of the ramp, and the ramp itself rises at 25.0° above the horizontal. (a) Draw a free-body diagram for the car. (b) Find the tension in the cable. (c) How hard does the surface of the ramp push on the car?

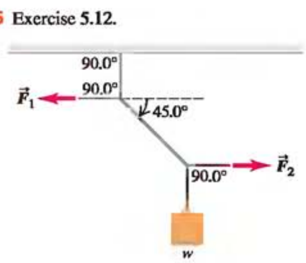
Figure 5.45 Exercise 5.10.



5.11. A man pushes on a piano with mass 180 kg so that it slides at constant velocity down a ramp that is inclined at 11.0° above the horizontal floor. Neglect any friction acting on the piano. Calculate the magnitude of the force applied by the man if he pushes (a) parallel to the incline and (b) parallel to the floor.

5.12. In Fig. 5.46 the weight w is 60.0 N. (a) What is the tension in the diagonal string? (b) Find the magnitudes of the horizontal forces $\vec{F}_1$ and $\vec{F}_2$ that must be applied to hold the system in the position shown.

Figure 5.46 Exercise 5.12.

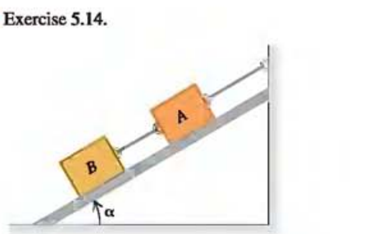


5.13. A solid uniform 45.0-kg ball of diameter 32.0 cm is supported against a vertical frictionless wall using a thin 30.0-cm wire of negligible mass, as shown in Fig. 5.47. (a) Make a free-body diagram for the ball and use it to find the tension in the wire. (b) How hard does the ball push against the wall?

Figure 5.47 Exercise 5.13.

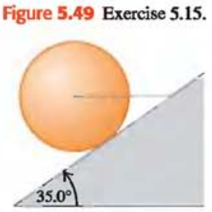
5.14. Two blocks, each with weight w, are held in place on a frictionless incline (Fig. 5.48). In terms of w and the angle α of the incline, calculate the tension in (a) the rope connecting the blocks and (b) the rope that connects block A to the wall. (c) Calculate the magnitude of the force that the incline exerts on each block. (d) Interpret your answers for the cases $\alpha = 0$ and $\alpha = 90°$.

Figure 5.48 Exercise 5.14.



5.15. A horizontal wire holds a solid uniform ball of mass m in place on a tilted ramp that rises 35.0° above the horizontal. The surface of this ramp is perfectly smooth, and the wire is directed away from the center of the ball (Fig. 5.49). (a) Draw a free-body diagram for the ball. (b) How hard does the surface of the ramp push on the ball? (c) What is the tension in the wire?

Figure 5.49 Exercise 5.15.



Section 5.2 Using Newton's Second Law: Dynamics of Particles

5.16. A 125-kg (including all the contents) rocket has an engine that produces a constant vertical force (the *thrust*) of 1720 N. Inside this rocket, a 15.5-N electrical power supply rests on the floor. (a) Find the acceleration of the rocket. (b) When it has reached an altitude of 120 m, how hard does the floor push on the power supply? (*Hint:* Start with a free-body diagram for the power supply.)

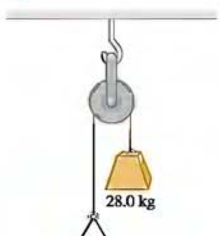
5.17. ***Genesis* Crash.** On September 8, 2004, the *Genesis* spacecraft crashed in the Utah desert because its parachute did not open. The 210-kg capsule hit the ground at 311 km/h and penetrated the soil to a depth of 81.0 cm. (a) Assuming it to be constant, what was its acceleration (in m/s^2 and in g's) during the crash? (b) What force did the ground exert on the capsule during the crash? Express the force in newtons and as a multiple of the capsule's weight. (c) For how long did this force last?

5.18. Three sleds are being pulled horizontally on frictionless horizontal ice using horizontal ropes (Fig. 5.50). The pull is horizontal and of magnitude 125 N. Find (a) the acceleration of the system and (b) the tension in ropes A and B.

Figure 5.50 Exercise 5.18.

5.19. Atwood's Machine. A 15.0-kg load of bricks hangs from one end of a rope that passes over a small, frictionless pulley. A 28.0-kg counterweight is suspended from the other end of the rope, as shown in Fig. 5.51. The system is released from rest. (a) Draw two free-body diagrams, one for the load of bricks and one for the counterweight. (b) What is the magnitude of the upward acceleration of the load of bricks? (c) What is the tension in the rope while the load is moving? How does the tension compare to the weight of the load of bricks? To the weight of the counterweight?

Figure 5.51 Exercise 5.19.



5.20. A 8.00-kg block of ice, released from rest at the top of a 1.50-m-long frictionless ramp, slides downhill, reaching a speed of 2.50 m/s at the bottom. (a) What is the angle between the ramp and the horizontal? (b) What would be the speed of the ice at the bottom if the motion were opposed by a constant friction force of 10.0 N parallel to the surface of the ramp?

5.21. A light rope is attached to a block with mass 4.00 kg that rests on a frictionless, horizontal surface. The horizontal rope passes over a frictionless, massless pulley, and a block with mass m is suspended from the other end. When the blocks are released, the tension in the rope is 10.0 N. (a) Draw two free-body diagrams, one for the 4.00-kg block and one for the block with mass m. (b) What is the acceleration of either block? (c) Find the mass m of the hanging block. (d) How does the tension compare to the weight of the hanging block?

5.22. Runway Design. A transport plane takes off from a level landing field with two gliders in tow, one behind the other. The mass of each glider is 700 kg, and the total resistance (air drag plus friction with the runway) on each may be assumed constant and equal to 2500 N. The tension in the towrope between the transport plane and the first glider is not to exceed 12,000 N. (a) If a speed of 40 m/s is required for takeoff, what minimum length of runway is needed? (b) What is the tension in the towrope between the two gliders while they are accelerating for the takeoff?

5.23. A 750.0-kg boulder is raised from a quarry 125 m deep by a long uniform chain having a mass of 575 kg. This chain is of uniform strength, but at any point it can support a maximum tension no greater than 2.50 times its weight without breaking. (a) What is the maximum acceleration the boulder can have and still get out of the quarry, and (b) how long does it take to be lifted out at maximum acceleration if it started from rest?

5.24. Apparent Weight. A 550-N physics student stands on a bathroom scale in an 850-kg (including the student) elevator that is supported by a cable. As the elevator starts moving, the scale reads 450 N. (a) Find the acceleration of the elevator (magnitude and direction). (b) What is the acceleration if the scale reads 670 N? (c) If the scale reads zero, should the student worry? Explain. (d) What is the tension in the cable in parts (a) and (c)?

5.25. A physics student playing with an air hockey table (a frictionless surface) finds that if she gives the puck a velocity of 3.80 m/s along the length (1.75 m) of the table at one end, by the time it has reached the other end the puck has drifted 2.50 cm to the right but still has a velocity component along the length of 3.80 m/s. She correctly concludes that the table is not level and correctly calculates its inclination from the given information. What is the angle of inclination?

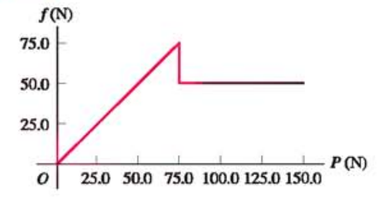
5.26. A 2540-kg test rocket is launched vertically from the launch pad. Its fuel (of negligible mass) provides a thrust force so that its vertical velocity as a function of time is given by $v(t) = At + Bt^2$, where A and B are constants and time is measured from the instant the fuel is ignited. At the instant of ignition, the rocket has an upward acceleration of $1.50\ m/s^2$ and 1.00 s later an upward velocity of 2.00 m/s. (a) Determine A and B, including their SI units. (b) At 4.00 s after fuel ignition, what is the acceleration of the rocket, and (c) what thrust force does the burning fuel exert on it, assume no air resistance? Express the thrust in newtons and as a multiple of the rocket's weight. (d) What was the initial thrust due to the fuel?

Section 5.3 Frictional Forces

5.27. Free-Body Diagrams. The first two steps in the solution of Newton's second-law problems are to select an object for analysis and then to draw free-body diagrams for that object. Draw free-body diagrams for the following situations: (a) a mass M sliding down a frictionless inclined plane of angle α, and (b) a mass M sliding up a frictionless inclined plane of angle α; (c) a mass M sliding up an inclined plane of angle α with kinetic friction present.

5.20. In a laboratory experiment on friction, a 135-N block resting on a rough horizontal table is pulled by a horizontal wire. The pull gradually increases until the block begins to move and continues to increase thereafter. Figure 5.52 shows a graph of the friction force on this block as a function of the pull. (a) Identify the regions of the graph where static and kinetic friction occur. (b) Find the coefficients of static and kinetic friction between the block and the table. (c) Why does the graph slant upward in the first part but then level out? (d) What would the graph look like if a 135-N brick were placed on the box, and what would be the coefficients of friction be in that case?

Figure 5.52 Exercise 5.28.



5.29. A stockroom worker pushes a box with mass 11.2 kg on a horizontal surface with a constant speed of 3.50 m/s. The coefficient of kinetic friction between the box and the surface is 0.20. (a) What horizontal force must the worker apply to maintain the motion? (b) If the force calculated in part (a) is removed, how far does the box slide before coming to rest?

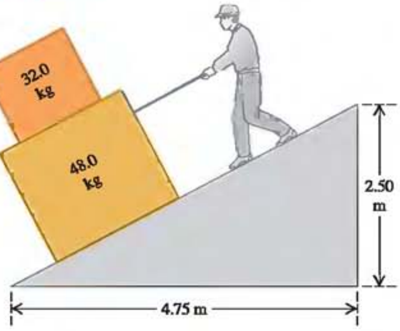
5.30. A box of bananas weighing 40.0 N rests on a horizontal surface. The coefficient of static friction between the box and the surface is 0.40, and the coefficient of kinetic friction is 0.20. (a) If no horizontal force is applied to the box and the box is at rest, how large is the friction force exerted on the box? (b) What is the magnitude of the friction force if a monkey applies a horizontal force of 6.0 N to the box and the box is initially at rest? (c) What minimum horizontal force must the monkey apply to start the box in motion? (d) What minimum horizontal force must the monkey apply to keep the box moving at constant velocity once it has been started? (e) If the monkey applies a horizontal force of 18.0 N, what is the magnitude of the friction force and what is the box's acceleration?

5.31. A crate of 45.0-kg tools rests on a horizontal floor. You exert a gradually increasing horizontal push on it and observe that the crate just begins to move when your force exceeds 313 N. After that you must reduce your push to 208 N to keep it moving at a steady 25.0 cm/s. (a) What are the coefficients of static and kinetic friction between the crate and the floor? (b) What push must you exert to give it an acceleration of $1.10\ m/s^2$? (c) Suppose you were performing the same experiment on this crate but were doing it on the moon instead, where the acceleration due to gravity is $1.62\ m/s^2$. (i) What magnitude push would cause it to move? (ii) What would its acceleration be if you maintained the push in part (b)?

5.32. An 85-N box of oranges is being pushed across a horizontal floor. As it moves, it is slowing at a constant rate of 0.90 m/s each second. The push force has a horizontal component of 20 N and a vertical component of 25 N downward. Calculate the coefficient of kinetic friction between the box and floor.

5.33. You are lowering two boxes, one on top of the other, down the ramp shown in Figure 5.53 by pulling on a rope parallel to the surface of the ramp. Both boxes move together at a constant speed of 15.0 cm/s. The coefficient of kinetic friction between the ramp and the lower box is 0.444, and the coefficient of static friction between the two boxes is 0.800. (a) What force do you need to

Figure 5.53 Exercise 5.33.



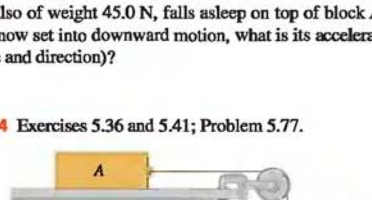
exert to accomplish this? (b) What are the magnitude and direction of the friction force on the upper box?

5.34. Stopping Distance. (a) If the coefficient of kinetic friction between tires and dry pavement is 0.80, what is the shortest distance in which you can stop an automobile by locking the brakes when traveling at 28.7 m/s (about 65 mi/h)? (b) On wet pavement the coefficient of kinetic friction may be only 0.25. How fast should you drive on wet pavement in order to be able to stop in the same distance as in part (a)? (*Note:* Locking the brakes is *not* the safest way to stop.)

5.35. Coefficient of Friction. A clean brass washer slides along a horizontal clean steel surface until it stops. Using the values from Table 5.1, how many times farther would it slide with the same initial speed if the washer were Teflon-coated?

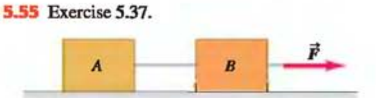
5.36. Consider the system shown in Fig. 5.54. Block A weighs 45.0 N and block B weighs 25.0 N. Once block B is set into downward motion, it descends at a constant speed. (a) Calculate the coefficient of kinetic friction between block A and the tabletop. (b) A cat, also of weight 45.0 N, falls asleep on top of block A. If block B is now set into downward motion, what is its acceleration (magnitude and direction)?

Figure 5.54 Exercises 5.36 and 5.41; Problem 5.77.



5.37. Two crates connected by a rope lie on a horizontal surface (Fig. 5.55). Crate A has mass m_A and crate B has mass m_B. The coefficient of kinetic friction between each crate and the surface is μ_k. The crates are pulled to the right at constant velocity by a horizontal force $\vec{F}$. In terms of m_A, m_B, and μ_k, calculate (a) the magnitude of the force $\vec{F}$ and (b) the tension in the rope connecting the blocks. Include the free-body diagram or diagrams you used to determine each answer.

Figure 5.55 Exercise 5.37.



5.38. Rolling Friction. Two bicycle tires are set rolling with the same initial speed of 3.50 m/s on a long, straight road, and the distance each travels before its speed is reduced by half is measured. One tire is inflated to a pressure of 40 psi and goes 18.1 m; the other is at 105 psi and goes 92.9 m. What is the coefficient of rolling friction μ_r for each? Assume that the net horizontal force is due to rolling friction only.

5.39. Wheels. You find that it takes a horizontal force of 160 N to slide a box along the surface of a level floor at constant speed. The coefficient of static friction is 0.52, and the coefficient of kinetic friction is 0.47. If you place the box on a dolly of mass 5.3 kg and with coefficient of rolling friction 0.018, what horizontal acceleration would that 160-N force provide?

5.40. You find it takes 200 N of horizontal force to move an empty pickup truck along a level road at a speed of 2.4 m/s. You then load the pickup and pump up its tires so that its total weight increases by 42% while the coefficient of rolling friction decreases by 19%. Now what horizontal force will you need to move the pickup along the same road at the same speed? The speed is low enough that you can ignore air resistance.

5.41. As shown in Fig. 5.54, block A (mass 2.25 kg) rests on a tabletop. It is connected by a horizontal cord passing over a light, frictionless pulley to a hanging block B (mass 1.30 kg). The coefficient of kinetic friction between block A and the tabletop is 0.450. After the blocks are released from rest, find (a) the speed of each block after moving 3.00 cm and (b) the tension in the cord. Include the free-body diagram or diagrams you used to determine the answers.

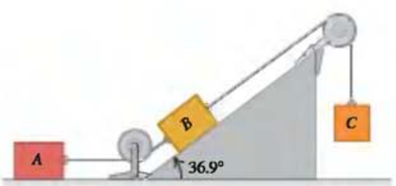
5.42. A 25.0-kg box of textbooks rests on a loading ramp that makes an angle α with the horizontal. The coefficient of kinetic friction is 0.25, and the coefficient of static friction is 0.35. (a) As the angle α is increased, find the minimum angle at which the box starts to slip. (b) At this angle, find the acceleration once the box has begun to move. (c) At this angle, how fast will the box be moving after it has slid 5.0 m along the loading ramp?

5.43. A large crate with mass m rests on a horizontal floor. The coefficients of friction between the crate and the floor are μ_s and μ_k. A woman pushes downward at an angle θ below the horizontal on the crate with a force $\vec{F}$. (a) What magnitude of force $\vec{F}$ is required to keep the crate moving at constant velocity? (b) If μ_s is greater than some critical value, the woman cannot start the crate moving no matter how hard she pushes. Calculate this critical value of μ_s.

5.44. A box with mass m is dragged across a level floor having a coefficient of kinetic friction μ_k by a rope that is pulled upward at an angle θ above the horizontal with a force of magnitude F. (a) In terms of m, μ_k, θ, and g, obtain an expression for the magnitude of force required to move the box with constant speed. (b) Knowing that you are studying physics, a CPR instructor asks you how much force it would take to slide a 90-kg patient across a floor at constant speed by pulling on him at an angle of 25° above the horizontal. By dragging some weights wrapped in an old pair of pants down the hall with a spring balance, you find that $\mu_k = 0.35$. Use the result of part (a) to answer the instructor's question.

5.45. Blocks *A*, *B*, and *C* are placed as in Fig. 5.56 and connected by ropes of negligible mass. Both *A* and *B* weigh 25.0 N each, and the coefficient of kinetic friction between each block and the surface is 0.35. Block *C* descends with constant velocity. (a) Draw two separate free-body diagrams showing the forces acting on *A* and on *B*. (b) Find the tension in the rope connecting blocks *A* and *B*. (c) What is the weight of block *C*? (d) If the rope connecting *A* and *B* were cut, what would be the acceleration of *C*?

Figure 5.56 Exercise 5.45.



5.46. Starting from Eq. (5.10), derive Eqs. (5.11) and (5.12).

5.47. (a) In Example 5.19 (Section 5.3), what value of D is required to make $v_t = 42\ \text{m/s}$ for the skydiver? (b) If the skydiver's daughter, whose mass is 45 kg, is falling through the air and has the same D $(0.25\ \text{kg/m})$ as her father, what is the daughter's terminal speed?

5.48. You throw a baseball straight up. The drag force is proportional to v^2. In terms of g, what is the y-component of the ball's acceleration when its speed is half its terminal speed and (a) it is moving up? (b) It is moving back down?

Section 5.4 Dynamics of Circular Motion

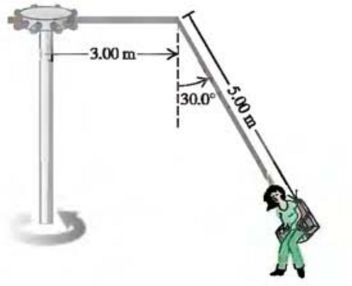
5.48. A machine part consists of a thin 40.0-cm-long bar with small 1.15-kg masses fastened by screws to its ends. The screws can support a maximum force of 75.0 N without pulling out. This bar rotates about an axis perpendicular to it at its center. (a) As the bar is turning at a constant rate on a horizontal frictionless surface, what is the maximum speed the masses can have without pulling out the screws? (b) Suppose the machine is redesigned so that the bar turns at a constant rate in a vertical circle. Will one of the screws be more likely to pull out when the mass is at the top of the circle or at the bottom? Use a free-body diagram to see why. (c) Using the result of part (b), what is the greatest speed the masses can have without pulling a screw?

5.50. A flat (unbanked) curve on a highway has a radius of 220.0 m. A car rounds the curve at a speed of 25.0 m/s. (a) What is the minimum coefficient of friction that will prevent sliding? (b) Suppose the highway is icy and the coefficient of friction between the tires and pavement is only one-third what you found in part (a). What should be the maximum speed of the car so it can round the curve safely?

5.51. A 1125-kg car and a 2250-kg pickup truck aproach a curve on the expressway that has a radius of 225 m. (a) At what angle should the highway engineer bank this curve so that vehicles traveling at 65.0 mi/h can safely round it regardless of the condition of their tires? Should the heavy truck go slower than the lighter car? (b) As the car and truck round the curve at 65.0 mi/h, find the normal force on each one due to the highway surface.

5.52. The "Giant Swing" at a county fair consists of a vertical central shaft with a number of horizontal arms attached at its upper end (Fig. 5.57). Each arm supports a seat suspended from a cable 5.00 m long, the upper end of the cable being fastened to the arm at a point 3.00 m from the central shaft. (a) Find the time of one revolution of the swing if the cable supporting a seat makes an angle of 30.0° with the vertical. (b) Does the angle depend on the weight of the passenger for a given rate of revolution?

Figure 5.57 Exercise 5.52.



5.53. In another version of the "Giant Swing" (see Exercise 5.52), the seat is connected to two cables as shown in Fig. 5.58, one of which is horizontal. The seat swings in a horizontal circle at a rate of 32.0 rpm (rev/min). If the seat weighs 255 N and a 825-N person is sitting in it, find the tension in each cable.

Figure 5.56 Exercise 5.53.

5.54. A small button placed on a horizontal rotating platform with diameter 0.320 m will revolve with the platform when it is brought up to a speed of 40.0 rev/min, provided the button is no more than 0.150 m from the axis. (a) What is the coefficient of static friction between the button and the platform? (b) How far from the axis can the button be placed, without slipping, if the platform rotates at 60.0 rev/min?

5.55. Rotating Space Stations. One problem for humans living in outer space is that they are apparently weightless. One way around this problem is to design a space station that spins about its center at a constant rate. This creates "artificial gravity" at the outside rim of the station. (a) If the diameter of the space station is 800 m, how many revolutions per minute are needed for the "artificial gravity" acceleration to be $9.80\ \text{m/s}^2$? (b) If the space station is a waiting area for travelers going to Mars, it might be desirable to simulate the acceleration due to gravity on the Martian surface $(3.70\ \text{m/s}^2)$. How many revolutions per minute are needed in this case?

5.56. The Cosmoclock 21 Ferris wheel in Yokohama City, Japan, has a diameter of 100 m. Its name comes from its 60 arms, each of which can function as a second hand (so that it makes one revolution every 60.0 s). (a) Find the speed of the passengers when the Ferris wheel is rotating at this rate. (b) A passenger weighs 882 N at the weight-guessing booth on the ground. What is his apparent weight at the highest and at the lowest point on the Ferris wheel? (c) What would be the time for one revolution if the passenger's apparent weight at the highest point were zero? (d) What then would be the passenger's apparent weight at the lowest point?

5.57. An airplane flies in a loop (a circular path in a vertical plane) of radius 150 m. The pilot's head always points toward the center of the loop. The speed of the airplane is not constant; the airplane goes slowest at the top of the loop and fastest at the bottom. (a) At the top of the loop, the pilot feels weightless. What is the speed of the airplane at this point? (b) At the bottom of the loop, the speed of the airplane is 280 km/h. What is the apparent weight of the pilot at this point? His true weight is 700 N.

5.58. A 50.0-kg stunt pilot who has been diving her airplane vertically pulls out of the dive by changing her course to a circle in a vertical plane. (a) If the plane's speed at the lowest point of the circle is 95.0 m/s, what is the minimum radius of the circle for the acceleration at this point not to exceed 4.00g? (b) What is the apparent weight of the pilot at the lowest point of the pullout?

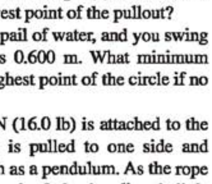
5.59. Stay Dry! You tie a cord to a pail of water, and you swing the pail in a vertical circle of radius 0.600 m. What minimum speed must you give the pail at the highest point of the circle if no water is to spill from it?

5.60. A bowling ball weighing 71.2 N (16.0 lb) is attached to the ceiling by a 3.80-m rope. The ball is pulled to one side and released; it then swings back and forth as a pendulum. As the rope swings through the vertical, the speed of the bowling ball is 4.20 m/s. (a) What is the acceleration of the bowling ball, in magnitude and direction, at this instant? (b) What is the tension in the rope at this instant?

Problems

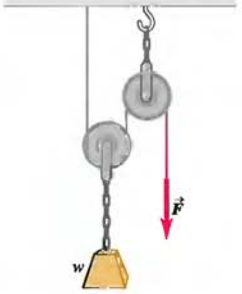
5.61. Two ropes are connected to a steel cable that supports a hanging weight as shown in Fig. 5.59. (a) Draw a free-body diagram showing all of the forces acting at the knot that connects the two ropes to the steel cable. Based on your force diagram, which of the two ropes will have the greater tension? (b) If the maximum tension either rope can sustain without breaking is 5000 N, determine the maximum value of the hanging weight that these ropes can safely support. You can ignore the weight of the ropes and the steel cable.

Figure 5.59 Problem 5.61.



5.62. In Fig. 5.60 a worker lifts a weight w by pulling down on a rope with a force $\vec{F}$. The upper pulley is attached to the ceiling by a chain, and the lower pulley is attached to the weight by another chain. In terms of w, find the tension in each chain and the magnitude of the force $\vec{F}$ if the weight is lifted at constant speed. Include the free-body diagram or diagrams you used to determine your answers. Assume that the rope, pulleys, and chains all have negligible weights.

Figure 5.60 Problem 5.62.



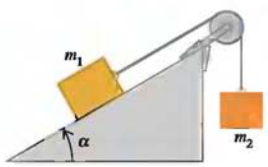
5.63. A Rope with Mass. In most problems in this book, the ropes, cords, or cables have so little mass compared to other objects in the problem that you can safely ignore their mass. But if the rope is the *only* object in the problem, then clearly you cannot ignore its mass. For example, suppose we have a clothesline attached to two poles (Fig. 5.61). The clothesline has a mass M, and each end makes an angle θ with the horizontal. What are (a) the tension at the ends of the clothesline and (b) the tension at the lowest point? (c) Why can't we have $\theta = 0$? (See Discussion Question Q5.3.) (d) Discuss your results for parts (a) and (b) in the limit that $\theta \to 90°$. The curve of the clothesline, or of any flexible cable hanging under its own weight, is called a *catenary*. [For a more advanced treatment of this curve, see K. R. Symon, *Mechanics*, 3rd ed. (Reading, MA: Addison-Wesley, 1971), pp. 237–241.]

Figure 5.61 Problem 5.63.

5.64. Another Rope with Mass. A block with mass M is attached to the lower end of a vertical, uniform rope with mass m and length L. A constant upward force $\vec{F}$ is applied to the top of the rope, causing the rope and block to accelerate upward. Find the tension in the rope at a distance x from the top end of the rope, where x can have any value from 0 to L.

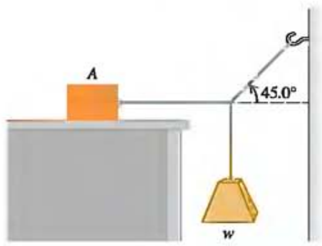
5.65. A block with mass m_1 is placed on an inclined plane with slope angle α and is connected to a second hanging block with mass m_2 by a cord passing over a small, frictionless pulley (Fig. 5.62). The coefficient of static friction is μ_s and the coefficient of kinetic friction is μ_k. (a) Find the mass m_2 for which block m_1 moves up the plane at constant speed once it is set in motion. (b) Find the mass m_2 for which block m_1 moves down the plane at constant speed once it is set in motion. (c) For what range of values of m_2 will the blocks remain at rest if they are released from rest?

Figure 5.62 Problem 5.65.



5.66. (a) Block A in Fig. 5.63 weighs 60.0 N. The coefficient of static friction between the block and the surface on which it rests is

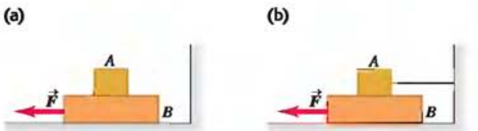
Figure 5.63 Problem 5.66.



0.25. The weight w is 12.0 N and the system is in equilibrium. Find the friction force exerted on block A. (b) Find the maximum weight w for which the system will remain in equilibrium.

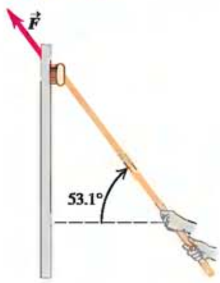
5.67. Block A in Fig. 5.64 weighs 1.20 N and block B weighs 3.60 N. The coefficient of kinetic friction between all surfaces is 0.300. Find the magnitude of the horizontal force $\vec{F}$ necessary to drag block B to the left at constant speed (a) if A rests on B and moves with it (Fig. 5.64a) and (b) if A is held at rest (Fig. 5.64b).

Figure 5.64 Problem 5.67.



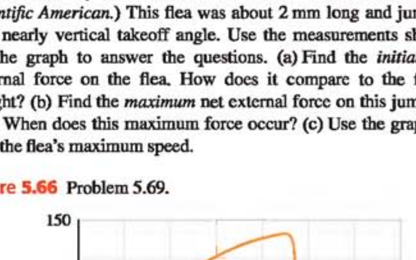
5.68. A window washer pushes his scrub brush up a vertical window at constant speed by applying a force $\vec{F}$ as shown in Fig. 5.65. The brush weighs 12.0 N and the coefficient of kinetic friction is $\mu_k = 0.150$. Calculate (a) the magnitude of the force $\vec{F}$ and (b) the normal force exerted by the window on the brush.

Figure 5.65 Problem 5.68.



5.69. The Flying Leap of a Flea. High-speed motion pictures (3500 frames/second) of a jumping 210-μg flea yielded the data to plot the flea's acceleration as a function of time as shown in Fig. 5.66. (See "The Flying Leap of the Flea," by M. Rothschild et al. in the November 1973 *Scientific American*.) This flea was about 2 mm long and jumped at a nearly vertical takeoff angle. Use the measurements shown on the graph to answer the questions. (a) Find the *initial* net external force on the flea. How does it compare to the flea's weight? (b) Find the *maximum* net external force on this jumping flea. When does this maximum force occur? (c) Use the graph to find the flea's maximum speed.

Figure 5.66 Problem 5.69.



5.70. A 25,000-kg rocket blasts off vertically from the earth's surface with a constant acceleration. During the motion considered in the problem, assume that g remains constant (see Chapter 12). Inside the rocket, a 15.0-N instrument hangs from a wire that can support a maximum tension of 35.0 N. (a) Find the minimum time for this rocket to reach the sound barrier (330 m/s) without breaking the inside wire and the maximum vertical thrust of the rocket engines under these conditions. (b) How far is the rocket above the earth's surface when it breaks the sound barrier?

5.71. You are standing on a bathroom scale in an elevator in a tall building. Your mass is 72 kg. The elevator starts from rest and travels upward with a speed that varies with time according to $v(t) = (3.0\ \text{m/s}^2)t + (0.20\ \text{m/s}^3)t^2$. When $t = 4.0$ s, what is the reading of the bathroom scale?

5.72. Elevator Design. You are designing an elevator for a hospital. The force exerted on a passenger by the floor of the elevator is not to exceed 1.60 times the passenger's weight. The elevator accelerates upward with constant acceleration for a distance of 3.0 m and then starts to slow down. What is the maximum speed of the elevator?

5.73. You are working for a shipping company. Your job is to stand at the bottom of a 8.0-m-long ramp that is inclined at 37° above the horizontal. You grab packages off a conveyor belt and propel them up the ramp. The coefficient of kinetic friction between the packages and the ramp is $\mu_k = 0.30$. (a) What speed do you need to give a package at the bottom of the ramp so that it has zero speed at the top of the ramp? (b) Your coworker is supposed to grab the packages as they arrive at the top of the ramp, but she misses one and it slides back down. What is its speed when it returns to you?

5.74. A hammer is hanging by a light rope from the ceiling of a bus. The ceiling of the bus is parallel to the roadway. The bus is traveling in a straight line on a horizontal street. You observe that the hammer hangs at rest with respect to the bus when the angle between the rope and the ceiling of the bus is 74°. What is the acceleration of the bus?

5.75. A steel washer is suspended inside an empty shipping crate from a light string attached to the top of the crate. The crate slides down a long ramp that is inclined at an angle of 37° above the horizontal. The crate has mass 180 kg. You are sitting inside the crate (with a flashlight); your mass is 55 kg. As the crate is sliding down the ramp, you find the washer is at rest with respect to the crate when the string makes an angle of 68° with the top of the crate. What is the coefficient of kinetic friction between the ramp and the crate?

5.76. Lunch Time! You are riding your motorcycle one day down a wet street that slopes downward at an angle of 20° below the horizontal. As you start to ride down the hill, you notice a construction crew has dug a deep hole in the street at the bottom of the hill. A Siberian tiger, escaped from the City Zoo, has taken up residence in the hole. You apply the brakes and lock your wheels at the top of the hill, where you are moving with a speed of 20 m/s. The inclined street in front of you is 40 m long. (a) Will you plunge into the hole and become the tiger's lunch, or do you skid to a stop before you reach the hole? (The coefficients of friction between your motorcycle tires and the wet pavement are $\mu_s = 0.90$ and $\mu_k = 0.70$.) (b) What must your initial speed be if you are to stop just before reaching the hole?

5.77. In the system shown in Fig. 5.54, block A has mass m_A, block B has mass m_B, and the rope connecting them has a *nonzero* mass m_{rope}. The rope has a total length L, and the pulley has a very small radius. You can ignore any sag in the horizontal part of the rope. (a) If there is no friction between block A and the tabletop, find the acceleration of the blocks at an instant when a length d of rope hangs vertically between the pulley and block B. As block B falls, will the magnitude of the acceleration of the system increase, decrease, or remain constant? Explain. (b) Let $m_A = 2.00$ kg, $m_B = 0.400$ kg, $m_{rope} = 0.160$ kg, and $L = 1.00$ m. If there is friction between block A and the tabletop, with $\mu_k = 0.200$ and $\mu_s = 0.250$, find the minimum value of the distance d such that the blocks will start to move if they are initially at rest. (c) Repeat part (b) for the case $m_{rope} = 0.040$ kg. Will the blocks move in this case?

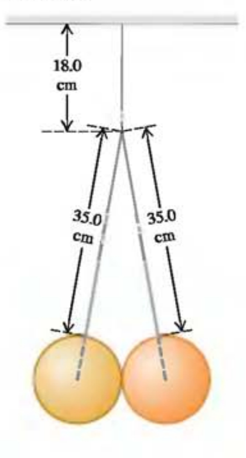
5.76. If the coefficient of static friction between a table and a uniform massive rope is μ_s, what fraction of the rope can hang over the edge of the table without the rope sliding?

5.79. A 30.0-kg packing case is initially at rest on the floor of a 1500-kg pickup truck. The coefficient of static friction between the case and the truck floor is 0.30, and the coefficient of kinetic friction is 0.20. Before each acceleration given below, the truck is traveling due north at constant speed. Find the magnitude and direction of the friction force acting on the case (a) when the truck accelerates at 2.20 m/s^2 northward and (b) when it accelerates at 3.40 m/s^2 southward.

5.68. Traffic Court. You are called as an expert witness in the trial of a traffic violation. The facts are these: A driver slammed on his brakes and came to a stop with constant acceleration. Measurements of his tires and the skid marks on the pavement indicate that he locked his car's wheels, the car traveled 192 ft before stopping, and the coefficient of kinetic friction between the road and his tires was 0.750. The charge is that he was speeding in a 45-mi/h zone. He pleads innocent. What is your conclusion, guilty or innocent? How fast was he going when he hit his brakes?

5.81. Two identical 15.0-kg balls, each 25.0 cm in diameter, are suspended by two 35.0-cm wires as shown in Fig. 5.67. The entire apparatus is supported by a single 18.0-cm wire, and the surfaces of the balls are perfectly smooth. (a) Find the tension in each of the three wires. (b) How hard does each ball push on the other one?

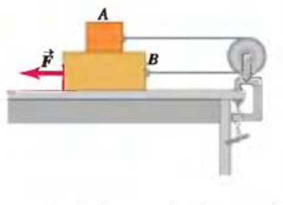
Figure 5.67 Problem 5.81.



5.82. Losing Cargo. A 12.0-kg box rests on the flat floor of a truck. The coefficients of friction between the box and floor are $\mu_s = 0.19$ and $\mu_k = 0.15$. The truck stops at a stop sign and then starts to move with an acceleration of 2.20 m/s^2. If the box is 1.80 m from the rear of the truck when the truck starts, how much time elapses before the box falls off the truck? How far does the truck travel in this time?

5.83. Block A in Fig. 5.68 weighs 1.40 N, and block B weighs 4.20 N. The coefficient of kinetic friction between all surfaces is 0.30. Find the magnitude of the horizontal force $\vec{F}$ necessary to drag block B to the left at constant speed if A and B are connected by a light, flexible cord passing around a fixed, frictionless pulley.

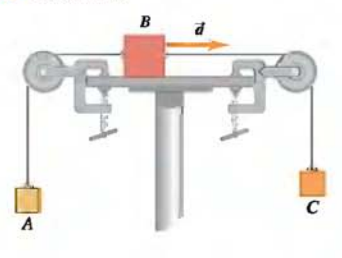
Figure 5.68 Problem 5.83.



5.64. You are part of a design team for future exploration of the planet Mars, where $g = 3.7\ m/s^2$. An explorer is to step out of a survey vehicle traveling horizontally at 33 m/s when it is 1200 m above the surface and then fall freely for 20 s. At that time, a portable advanced propulsion system (PAPS) is to exert a constant force that will decrease the explorer's speed to zero at the instant she touches the surface. The total mass (explorer, suit, equipment, and PAPS) is 150 kg. Assume the change in mass of the PAPS to be negligible. Find the horizontal and vertical components of the force the PAPS must exert, and for what interval of time the PAPS must exert it. You can ignore air resistance.

5.85. Block A in Fig. 5.69 has a mass of 4.00 kg, and block B has mass 12.0 kg. The coefficient of kinetic friction between block B and the horizontal surface is 0.25. (a) What is the mass of block C if block B is moving to the right and speeding up with an acceleration 2.00 m/s^2? (b) What is the tension in each cord when block B has this acceleration?

Figure 5.69 Problem 5.85.

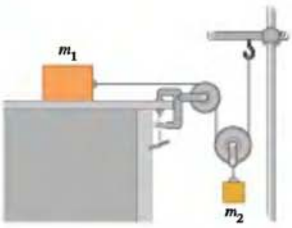


5.68. Two blocks connected by a cord passing over a small, frictionless pulley rest on frictionless planes (Fig. 5.70). (a) Which way will the system move when the blocks are released from rest? (b) What is the acceleration of the blocks? (c) What is the tension in the cord?

Figure 5.70 Problem 5.86.

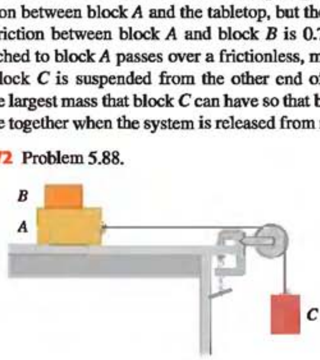
5.67. In terms of m_1, m_2, and g, find the accelerations of each block in Fig. 5.71. There is no friction anywhere in the system.

Figure 5.71 Problem 5.87.



5.68. Block B, with mass 5.00 kg, rests on block A, with mass 8.00 kg, which in turn is on a horizontal tabletop (Fig. 5.72). There is no friction between block A and the tabletop, but the coefficient of static friction between block A and block B is 0.750. A light string attached to block A passes over a frictionless, massless pulley, and block C is suspended from the other end of the string. What is the largest mass that block C can have so that blocks A and B still slide together when the system is released from rest?

Figure 5.72 Problem 5.88.



5.68. Two objects with masses 5.00 kg and 2.00 kg hang 0.600 m above the floor from the ends of a cord 6.00 m long passing over a frictionless pulley. Both objects start from rest. Find the maximum height reached by the 2.00-kg object.

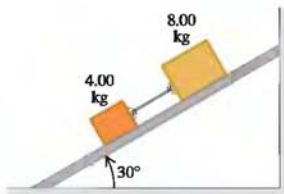
5.90. Friction in an Elevator. You are riding in an elevator on the way to the 18th floor of your dormitory. The elevator is accelerating upward with $a = 1.90\ \text{m/s}^2$. Beside you is the box containing your new computer; the box and its contents have a total mass of 28.0 kg. While the elevator is accelerating upward, you push horizontally on the box to slide it at constant speed toward the elevator door. If the coefficient of kinetic friction between the box and the elevator floor is $\mu_k = 0.32$, what magnitude of force must you apply?

5.91. A block is placed against the vertical front of a cart as shown in Fig. 5.73. What acceleration must the cart have so that block A does not fall? The coefficient of static friction between the block and the cart is μ_s. How would an observer on the cart describe the behavior of the block?

Figure 5.73 Problem 5.91.

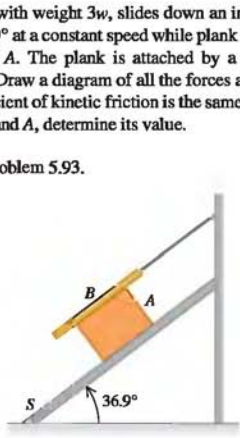
5.92. Two blocks with masses 4.00 kg and 8.00 kg are connected by a string and slide down a 30.0° inclined plane (Fig. 5.74). The coefficient of kinetic friction between the 4.00-kg block and the plane is 0.25; that between the 8.00-kg block and the plane is 0.35. (a) Calculate the acceleration of each block. (b) Calculate the tension in the string. (c) What happens if the positions of the blocks are reversed, so the 4.00-kg block is above the 8.00-kg block?

Figure 5.74 Problem 5.92.



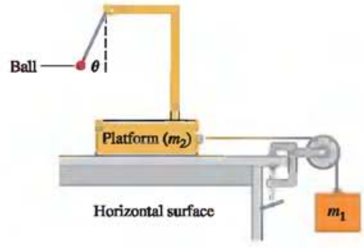
5.93. Block A, with weight $3w$, slides down an inclined plane S of slope angle 36.9° at a constant speed while plank B, with weight w, rests on top of A. The plank is attached by a cord to the wall (Fig. 5.75). (a) Draw a diagram of all the forces acting on block A. (b) If the coefficient of kinetic friction is the same between A and B and between S and A, determine its value.

Figure 5.75 Problem 5.93.



5.94. Accelerometer. The system shown in Fig. 5.76 can be used to measure the acceleration of the system. An observer riding on the platform measures the angle θ that the thread supporting the light ball makes with the vertical. There is no friction anywhere. (a) How is θ related to the acceleration of the system? (b) If $m_1 = 250$ kg and $m_2 = 1250$ kg, what is θ? (c) If you can vary m_1 and m_2, what is the largest angle θ you could achieve? Explain how you need to adjust m_1 and m_2 to do this.

Figure 5.76 Problem 5.94.



5.95. Banked Curve I. A curve with a 120-m radius on a level road is banked at the correct angle for a speed of 20 m/s. If an automobile rounds this curve at 30 m/s, what is the minimum coefficient of static friction needed between tires and road to prevent skidding?

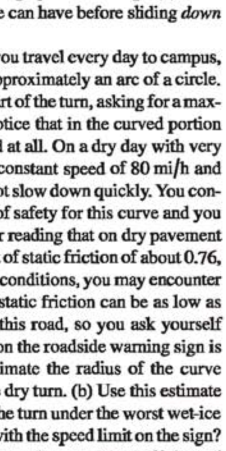
5.96. Banked Curve II. Consider a wet roadway banked as in Example 5.23 (Section 5.4), where there is a coefficient of static friction of 0.30 and a coefficient of kinetic friction of 0.25 between the tires and the roadway. The radius of the curve is $R = 50$ m. (a) If the banking angle is $\beta = 25°$, what is the *maximum* speed the automobile can have before sliding *up* the banking? (b) What is the *minimum* speed the automobile can have before sliding *down* the banking?

5.97. Maximum Safe Speed. As you travel every day to campus, the road makes a large turn that is approximately an arc of a circle. You notice the warning sign at the start of the turn, asking for a maximum speed of 55 mi/h. You also notice that in the curved portion the road is level—that is, not banked at all. On a dry day with very little traffic, you enter the turn at a constant speed of 80 mi/h and feel that the car may skid if you do not slow down quickly. You conclude that your speed is at the limit of safety for this curve and you slow down. However, you remember reading that on dry pavement new tires have an average coefficient of static friction of about 0.76, while under the worst winter driving conditions, you may encounter wet ice for which the coefficient of static friction can be as low as 0.20. Wet ice is not unheard of on this road, so you ask yourself whether the speed limit for the turn on the roadside warning sign is for the worst-case scenario. (a) Estimate the radius of the curve from your 80-mi/h experience in the dry turn. (b) Use this estimate to find the maximum speed limit in the turn under the worst wet-ice conditions. How does this compare with the speed limit on the sign? Is the sign misleading drivers? (c) On a rainy day, the coefficient of static friction would be about 0.37. What is the maximum safe speed for the turn when the road is wet? Does your answer help you understand the maximum-speed sign?

5.98. You are riding in a school bus. As the bus rounds a flat curve at constant speed, a lunch box with mass 0.500 kg, suspended from the ceiling of the bus by a string 1.80 m long, is found to hang at rest relative to the bus when the string makes an angle of 30.0° with the vertical. In this position the lunch box is 50.0 m from the center of curvature of the curve. What is the speed v of the bus?

5.99. The Monkey and Bananas Problem. A 20-kg monkey has a firm hold on a light rope that passes over a frictionless pulley and is attached to a 20-kg bunch of bananas (Fig. 5.77). The monkey looks up, sees the bananas, and starts to climb the rope to get them. (a) As the monkey climbs, do the bananas move up, down, or remain at rest? (b) As the monkey climbs, does the distance between the monkey and the bananas decrease, increase, or remain constant? (c) The monkey releases her hold on the rope. What happens to the distance between the monkey and the bananas while she is falling? (d) Before reaching the ground, the monkey grabs the rope to stop her fall. What do the bananas do?

Figure 5.77 Problem 5.99.



5.100. You throw a rock downward into water with a speed of $3mg/k$, where k is the coefficient in Eq. (5.7). Assume that the relationship between fluid resistance and speed is as given in Eq. (5.7), and calculate the speed of the rock as a function of time.

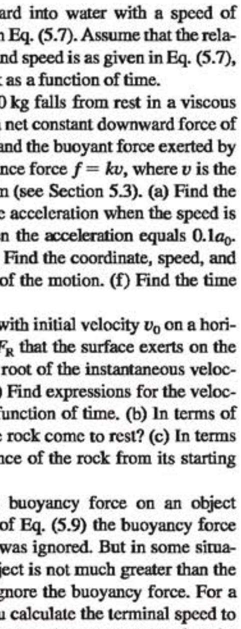
5.101. A rock with mass $m = 3.00$ kg falls from rest in a viscous medium. The rock is acted on by a net constant downward force of 18.0 N (a combination of gravity and the buoyant force exerted by the medium) and by a fluid resistance force $f = kv$, where v is the speed in m/s and $k = 2.20\ \text{N} \cdot \text{s/m}$ (see Section 5.3). (a) Find the initial acceleration a_0. (b) Find the acceleration when the speed is 3.00 m/s. (c) Find the speed when the acceleration equals $0.1a_0$. (d) Find the terminal speed v_t. (e) Find the coordinate, speed, and acceleration 2.00 s after the start of the motion. (f) Find the time required to reach a speed $0.9v_t$.

5.102. A rock with mass m slides with initial velocity v_0 on a horizontal surface. A retarding force F_R that the surface exerts on the rock is proportional to the square root of the instantaneous velocity of the rock $(F_R = -kv^{1/2})$. (a) Find expressions for the velocity and position of the rock as a function of time. (b) In terms of m, k, and v_0, at what time will the rock come to rest? (c) In terms of m, k, and v_0, what is the distance of the rock from its starting point when it comes to rest?

5.103. A fluid exerts an upward buoyancy force on an object immersed in it. In the derivation of Eq. (5.9) the buoyancy force exerted on an object by the fluid was ignored. But in some situations, where the density of the object is not much greater than the density of the fluid, you cannot ignore the buoyancy force. For a plastic sphere falling in water, you calculate the terminal speed to be 0.36 m/s when you ignore buoyancy, but you measure it to be 0.24 m/s. The buoyancy force is what fraction of the weight?

5.104. The 4.00-kg block in Fig. 5.78 is attached to a vertical rod by means of two strings. When the system rotates about the axis of the rod, the strings are extended as shown in the diagram and the tension in the upper string is 80.0 N. (a) What is the tension in the lower cord? (b) How many revolutions per minute does the system make? (c) Find the number of revolutions per minute at which the lower cord just goes slack. (d) Explain what happens if the number of revolutions per minute is less than in part (c).

Figure 5.78 Problem 5.104.



5.105. Equation (5.10) applies to the case where the initial velocity is zero. (a) Derive the corresponding equation for $v_y(t)$ when the falling object has an initial downward velocity with magnitude v_0. (b) For the case where $v_0 < v_t$, sketch a graph of v_y as a function of t and label v_t on your graph. (c) Repeat part (b) for the case where $v_0 > v_t$. (d) Discuss what your result says about $v_y(t)$ when $v_0 = v_t$.

5.106. A small rock moves in water, and the force exerted on it by the water is given by Eq. (5.7). The terminal speed of the rock is measured and found to be 2.0 m/s. The rock is projected *upward* at an initial speed of 6.0 m/s. You can ignore the buoyancy force on the rock. (a) In the absence of fluid resistance, how high will the rock rise and how long will it take to reach this maximum height?

(b) When the effects of fluid resistance are included, what are the answers to the questions in part (a)?

5.107. You observe a 1350-kg sports car rolling along flat pavement in a straight line. The only horizontal forces acting on it are a constant rolling friction and air resistance (proportional to the square of its speed). You take the following data during a time interval of 25 s: When its speed is 32 m/s, the car slows down at a rate of $-0.42\ \text{m/s}^2$, and when its speed is decreased to 24 m/s, it slows down at $-0.30\ \text{m/s}^2$. (a) Find the coefficient of rolling friction and the air drag constant D. (b) At what constant speed will this car move down an incline that makes a 2.2° angle with the horizontal? (c) How is the constant speed for an incline of angle β related to the terminal speed of this sports car if the car drops off a high cliff? Assume that in both cases the air resistance force is proportional to the square of the speed, and the air drag constant is the same.

5.108. A 70-kg person rides in a 30-kg cart moving at 12 m/s at the top of a hill that is in the shape of an arc of a circle with a radius of 40 m. (a) What is the apparent weight of the person as the cart passes over the top of the hill? (b) Determine the maximum speed that the cart may travel at the top of the hill without losing contact with the surface. Does your answer depend on the mass of the cart or the mass of the person? Explain.

5.109. Merry-Go-Round. One December identical twins Jena and Jackie are playing on a large merry-go-round (a disk mounted parallel to the ground, on a vertical axle through its center) in their school playground in northern Minnesota. Each twin has mass 30.0 kg. The icy coating on the merry-go-round surface makes it frictionless. The merry-go-round revolves at a constant rate as the twins ride on it. Jena, sitting 1.80 m from the center of the merry-go-round, must hold on to one of the metal posts attached to the merry-go-round with a horizontal force of 60.0 N to keep from sliding off. Jackie is sitting at the edge, 3.60 m from the center. (a) With what horizontal force must Jackie hold on to keep from falling off? (b) If Jackie falls off, what will be her horizontal velocity when she becomes airborne?

5.110. A passenger with mass 85 kg rides in a Ferris wheel like that in Example 5.24 (Section 5.4). The seats travel in a circle of radius 35 m. The Ferris wheel rotates at constant speed and makes one complete revolution every 25 s. Calculate the magnitude and direction of the net force exerted on the passenger by the seat when she is (a) one-quarter revolution past her lowest point and (b) one-quarter revolution past her highest point.

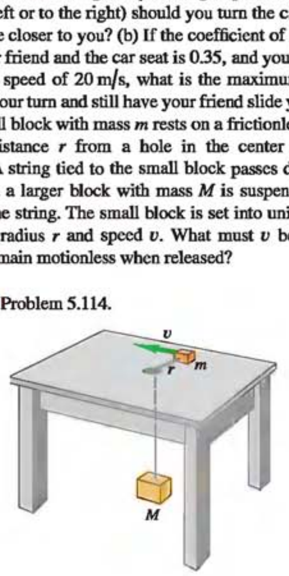
5.111. On the ride "Spindletop" at the amusement park Six Flags Over Texas, people stood against the inner wall of a hollow vertical cylinder with radius 2.5 m. The cylinder started to rotate, and when it reached a constant rotation rate of 0.60 rev/s, the floor on which people were standing dropped about 0.5 m. The people remained pinned against the wall. (a) Draw a force diagram for a person on this ride, after the floor has dropped. (b) What minimum coefficient of static friction is required if the person on the ride is not to slide downward to the new position of the floor? (c) Does your answer in part (b) depend on the mass of the passenger? (*Note:* When the ride is over, the cylinder is slowly brought to rest. As it slows down, people slide down the walls to the floor.)

5.112. A physics major is working to pay his college tuition by performing in a traveling carnival. He rides a motorcycle inside a hollow transparent plastic sphere. After gaining sufficient speed, he travels in a vertical circle with a radius of 13.0 m. The physics major has mass 70.0 kg, and his motorcycle has mass 40.0 kg. (a) What minimum speed must he have at the top of the circle if the tires of the motorcycle are not to lose contact with the sphere? (b) At the bottom of the circle, his speed is twice the value calculated in part (a). What is the magnitude of the normal force exerted on the motorcycle by the sphere at this point?

5.113. Ulterior Motives. You are driving a classic 1954 Nash Ambassador with a friend who is sitting to your right on the passenger side of the front seat. The Ambassador has flat bench seats. You would like to be closer to your friend and decide to use physics to achieve your romantic goal by making a quick turn. (a) Which way (to the left or to the right) should you turn the car to get your friend to slide closer to you? (b) If the coefficient of static friction between your friend and the car seat is 0.35, and you keep driving at a constant speed of 20 m/s, what is the maximum radius you could make your turn and still have your friend slide your way?

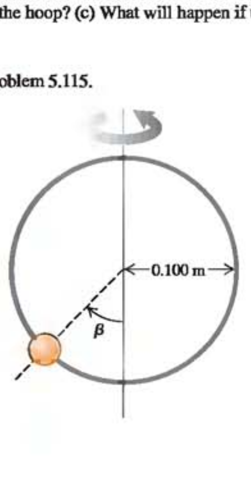
5.114. A small block with mass m rests on a frictionless horizontal tabletop a distance r from a hole in the center of the table (Fig. 5.79). A string tied to the small block passes down through the hole, and a larger block with mass M is suspended from the free end of the string. The small block is set into uniform circular motion with radius r and speed v. What must v be if the large block is to remain motionless when released?

Figure 5.79 Problem 5.114.



5.115. A small bead can slide without friction on a circular hoop that is in a vertical plane and has a radius of 0.100 m. The hoop rotates at a constant rate of 4.00 rev/s about a vertical diameter (Fig. 5.80). (a) Find the angle β at which the bead is in vertical equilibrium. (Of course, it has a radial acceleration toward the axis.) (b) Is it possible for the bead to "ride" at the same elevation as the center of the hoop? (c) What will happen if the hoop rotates at 1.00 rev/s?

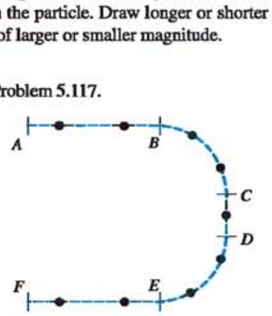
Figure 5.80 Problem 5.115.



5.116. A model airplane with mass 2.20 kg moves in the *xy*-plane such that its *x*- and *y*-coordinates vary in time according to $x(t) = \alpha - \beta t^3$ and $y(t) = \gamma t - \delta t^2$, where $\alpha = 1.50$ m, $\beta = 0.120\ \text{m/s}^3$, $\gamma = 3.00$ m/s, and $\delta = 1.00\ \text{m/s}^2$. (a) Calculate the *x*- and *y*-components of the net force on the plane as functions of time. (b) Sketch the trajectory of the airplane between $t = 0$ and $t = 3.00$ s, and draw on your sketch vectors showing the net force on the airplane at $t = 0$, $t = 1.00$ s, $t = 2.00$ s, and $t = 3.00$ s. For each of these times, relate the direction of the net force to the direction that the airplane is turning, and to whether the airplane is speeding up or slowing down (or neither). (c) What are the magnitude and direction of the net force at $t = 3.00$ s?

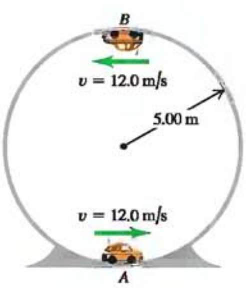
5.117. A particle moves on a frictionless surface along a path as shown in Fig. 5.81. (The figure gives a view looking down on the surface.) The particle is initially at rest at point *A* and then begins to move toward *B* as it gains speed at a constant rate. From *B* to *C*, the particle moves along a circular path at a constant speed. The speed remains constant along the straight-line path from *C* to *D*. From *D* to *E*, the particle moves along a circular path, but now its speed is decreasing at a constant rate. The speed continues to decrease at a constant rate as the particle moves from *E* to *F*; the particle comes to a halt at *F*. (The time intervals between the marked points are not equal.) At each point marked with a dot, draw arrows to represent the velocity, the acceleration, and the net force acting on the particle. Draw longer or shorter arrows to represent vectors of larger or smaller magnitude.

Figure 5.81 Problem 5.117.



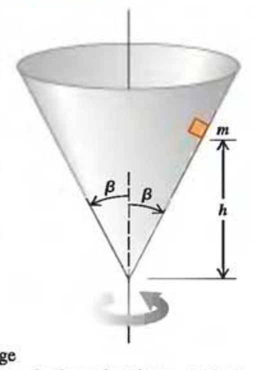
5.119. A small remote-control car with mass 1.60 kg moves at a constant speed of $v = 12.0$ m/s in a vertical circle inside a hollow metal cylinder that has a radius of 5.00 m (Fig 5.82). What is the magnitude of the normal force exerted on the car by the walls of the cylinder at (a) point *A* (at the bottom of the vertical circle) and (b) point *B* (at the top of the vertical circle)?

Figure 5.82 Problem 5.118.



5.119. A small block with mass *m* is placed inside an inverted cone that is rotating about a vertical axis such that the time for one revolution of the cone is *T* (Fig. 5.83). The walls of the cone make an angle β with the vertical. The coefficient of static friction between the block and the cone is μ_s. If the block is to remain at a constant height *h* above the apex of the cone, what are the maximum and minimum values of *T*?

Figure 5.83 Problem 5.119.



Challenge Problems

5.120. Moving Wedge. A wedge with mass *M* rests on a frictionless, horizontal tabletop. A block with mass *m* is placed on the wedge (Fig. 5.84a). There is no friction between the block and the wedge. The system is released from rest. (a) Calculate the acceleration of the wedge and the horizontal and vertical components of the acceleration of the block. (b) Do your answers to part (a) reduce to the correct results when *M* is very large? (c) As seen by a stationary observer, what is the shape of the trajectory of the block?

Figure 5.84 Challenge Problems 5.120 and 5.121.

(a) (b)

5.121. A wedge with mass *M* rests on a frictionless horizontal tabletop. A block with mass *m* is placed on the wedge and a horizontal force $\vec{F}$ is applied to the wedge (Fig. 5.84b). What must the magnitude of $\vec{F}$ be if the block is to remain at a constant height above the tabletop?

5.122. A box of weight *w* is accelerated up a ramp by a rope that exerts a tension *T*. The ramp makes an angle α with the horizontal, and the rope makes an angle θ above the ramp. The coefficient of kinetic friction between the box and the ramp is μ_k. Show that no matter what the value of α, the acceleration is maximum if $\theta = \arctan \mu_k$ (as long as the box remains in contact with the ramp).

5.123. Angle for Minimum Force. A box with weight *w* is pulled at constant speed along a level floor by a force $\vec{F}$ that is at an angle θ above the horizontal. The coefficient of kinetic friction between the floor and box is μ_k. (a) In terms of θ, μ_k, and *w*, calculate *F*. (b) For $w = 400$ N and $\mu_k = 0.25$, calculate *F* for θ ranging from 0° to 90° in increments of 10°. Graph *F* versus θ. (c) From the general expression in part (a), calculate the value of θ for which the value of *F*, required to maintain constant speed, is a minimum. (*Hint:* At a point where a function is minimum, what are the first and second derivatives of the function? Here *F* is a function of θ.) For the special case of $w = 400$ N and $\mu_k = 0.25$, evaluate this optimal θ and compare your result to the graph you constructed in part (b).

5.124. Falling Baseball. You drop a baseball from the roof of a tall building. As the ball falls, the air exerts a drag force proportional to the square of the ball's speed ($f = Dv^2$). (a) In a diagram, show the direction of motion and indicate, with the aid of vectors, all the forces acting on the ball. (b) Apply Newton's second law and infer from the resulting equation the general properties of the motion. (c) Show that the ball acquires a terminal speed that is as given in Eq. (5.13). (d) Derive the equation for the speed at any time. (*Note:*

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a}\text{arctanh}\left(\frac{x}{a}\right)$$

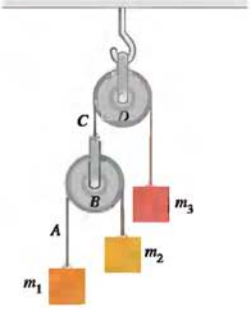
where

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

defines the hyperbolic tangent.)

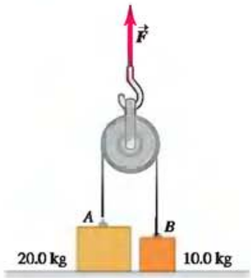
5.125. Double Atwood's Machine. In Fig. 5.85 masses m_1 and m_2 are connected by a light string A over a light, frictionless pulley B. The axle of pulley B is connected by a second light string C over a second light, frictionless pulley D to a mass m_3. Pulley D is suspended from the ceiling by an attachment to its axle. The system is released from rest. In terms of m_1, m_2, m_3, and g, what are (a) the acceleration of block m_3; (b) the acceleration of pulley B; (c) the acceleration of block m_1; (d) the acceleration of block m_2; (e) the tension in string A; (f) the tension in string C? (g) What do your expressions give for the special case of $m_1 = m_2$ and $m_3 = m_1 + m_2$? Is this sensible?

Figure 5.85 Challenge Problem 5.125.



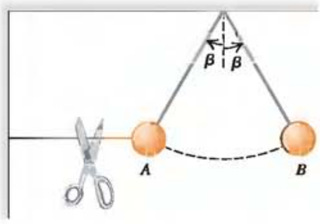
5.126. The masses of blocks A and B in Fig. 5.86 are 20.0 kg and 10.0 kg, respectively. The blocks are initially at rest on the floor and are connected by a massless string passing over a massless and frictionless pulley. An upward force $\vec{F}$ is applied to the pulley. Find the accelerations $\vec{a}_A$ of block A and $\vec{a}_B$ of block B when F is (a) 124 N; (b) 294 N; (c) 424 N.

Figure 5.86 Challenge Problem 5.126.



5.127. A ball is held at rest at position A in Fig. 5.87 by two light strings. The horizontal string is cut and the ball starts swinging as a pendulum. Point B is the farthest to the right the ball goes as it swings back and forth. What is the ratio of the tension in the supporting string in position B to its value at A before the horizontal string was cut?

Figure 5.87 Challenge Problem 5.127.



WORK AND KINETIC ENERGY

6

?When a shotgun fires, the expanding gases in the barrel push the shell out. According to Newton's third law, the shell exerts as much force on the gases as the gases exert on the shell. Would it be correct to say that the *shell* does work on the *gases?*

LEARNING GOALS

By studying this chapter, you will learn:

- What it means for a force to do work on a body, and how to calculate the amount of work done.
- The definition of the kinetic energy (energy of motion) of a body, and what it means physically.
- How the total work done on a body changes the body's kinetic energy, and how to use this principle to solve problems in mechanics.
- How to use the relationship between total work and change in kinetic energy when the forces are not constant, the body follows a curved path, or both.
- How to solve problems involving power (the rate of doing work).

Suppose you try to find the speed of an arrow that has been shot from a bow. You apply Newton's laws and all the problem-solving techniques that we've learned, but you run across a major stumbling block: After the archer releases the arrow, the bow string exerts a *varying* force that depends on the arrow's position. As a result, the simple methods that we've learned aren't enough to calculate the speed. Never fear; we aren't by any means finished with mechanics, and there are other methods for dealing with such problems.

The new method that we're about to introduce uses the ideas of *work* and *energy*. The importance of the energy idea stems from the ***principle of conservation of energy***: Energy is a quantity that can be converted from one form to another but cannot be created or destroyed. In an automobile engine, chemical energy stored in the fuel is converted partially to the energy of the automobile's motion and partially to thermal energy. In a microwave oven, electromagnetic energy obtained from your power company is converted to thermal energy of the food being cooked. In these and all other processes, the *total* energy—the sum of all energy present in all different forms—remains the same. No exception has ever been found.

We'll use the energy idea throughout the rest of this book to study a tremendous range of physical phenomena. This idea will help you understand why a sweater keeps you warm, how a camera's flash unit can produce a short burst of light, and the meaning of Einstein's famous equation $E = mc^2$.

In this chapter, though, our concentration will be on mechanics. We'll learn about one important form of energy called *kinetic energy*, or energy of motion, and how it relates to the concept of *work*. We'll also consider *power*, which is the time rate of doing work. In Chapter 7 we'll expand the ideas of work and kinetic energy into a deeper understanding of the concepts of energy and the conservation of energy.

6.1 Work

6.1 These people are doing work as they push on the stalled car because they exert a force on the car as it moves.

You'd probably agree that it's hard work to pull a heavy sofa across the room, to lift a stack of encyclopedias from the floor to a high shelf, or to push a stalled car off the road. Indeed, all of these examples agree with the everyday meaning of *work*—any activity that requires muscular or mental effort.

In physics, work has a much more precise definition. By making use of this definition we'll find that in any motion, no matter how complicated, the total work done on a particle by all forces that act on it equals the change in its *kinetic energy*—a quantity that's related to the particle's speed. This relationship holds even when the forces acting on the particle aren't constant, a situation that can be difficult or impossible to handle with the techniques you learned in Chapters 4 and 5. The ideas of work and kinetic energy enable us to solve problems in mechanics that we could not have attempted before.

In this section we'll see how work is defined and how to calculate work in a variety of situations involving *constant* forces. Even though we already know how to solve problems in which the forces are constant, the idea of work is still useful in such problems. Later in this chapter we'll relate work and kinetic energy, and then apply these ideas to problems in which the forces are *not* constant.

The three examples of work described above—pulling a sofa, lifting encyclopedias, and pushing a car—have something in common. In each case you do work by exerting a *force* on a body while that body *moves* from one place to another—that is, undergoes a *displacement* (Fig. 6.1). You do more work if the force is greater (you push harder on the car) or if the displacement is greater (you push the car farther down the road).

6.2 The work done by a constant force acting in the same direction as the displacement.

The physicist's definition of work is based on these observations. Consider a body that undergoes a displacement of magnitude s along a straight line. (For now, we'll assume that any body we discuss can be treated as a particle so that we can ignore any rotation or changes in shape of the body.) While the body moves, a constant force $\vec{F}$ acts on it in the same direction as the displacement $\vec{s}$ (Fig. 6.2). We define the **work** W done by this constant force under these circumstances as the product of the force magnitude F and the displacement magnitude s:

$$W = Fs \qquad \text{(constant force in direction of straight-line displacement)} \qquad (6.1)$$

The work done on the body is greater if either the force F or the displacement s is greater, in agreement with our observations above.

CAUTION **Work $= W$, weight $= w$** Don't confuse W (work) with w (weight). Though the symbols are similar, work and weight are different quantities.

The SI unit of work is the **joule** (abbreviated J, pronounced "jewel," and named in honor of the 19th-century English physicist James Prescott Joule). From Eq. (6.1) we see that in any system of units, the unit of work is the unit of force multiplied by the unit of distance. In SI units the unit of force is the newton and the unit of distance is the meter, so 1 joule is equivalent to 1 *newton-meter* $(\text{N} \cdot \text{m})$:

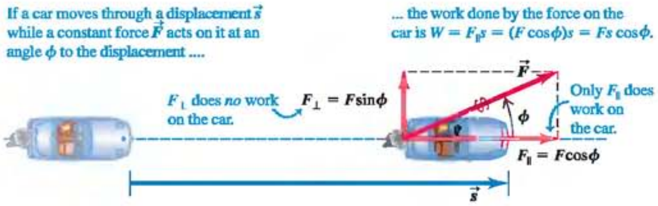
$$1 \text{ joule} = (1 \text{ newton})(1 \text{ meter}) \quad \text{or} \quad 1 \text{ J} = 1 \text{ N} \cdot \text{m}$$

In the British system the unit of force is the pound (lb), the unit of distance is the foot (ft), and the unit of work is the *foot-pound* $(\text{ft} \cdot \text{lb})$. The following conversions are useful:

$$1 \text{ J} = 0.7376 \text{ ft} \cdot \text{lb} \qquad 1 \text{ ft} \cdot \text{lb} = 1.356 \text{ J}$$

As an illustration of Eq. (6.1), think of a person pushing a stalled car. If he pushes the car through a displacement $\vec{s}$ with a constant force $\vec{F}$ in the direction of motion, the amount of work he does on the car is given by Eq. (6.1): $W = Fs$. But what if the person pushes at an angle ϕ with the car's displacement (Fig. 6.3)? Then $\vec{F}$ has a component $F_{\parallel} = F\cos\phi$ in the direction of the displacement and a component $F_{\perp} = F\sin\phi$ that acts perpendicular to the displacement. (Other

6.3 The work done by a constant force acting at an angle to the displacement.



forces must act on the car so that it moves along $\vec{s}$, not in the direction of $\vec{F}$. We're interested only in the work that the person does, however, so we'll consider only the force he exerts.) In this case only the parallel component $F_{\parallel}$ is effective in moving the car, so we define the work as the product of this force component and the magnitude of the displacement. Hence $W = F_{\parallel}s = (F\cos\phi)s$, or

ActivPhysics ONLINE 5.1 Work Calculations

$$W = Fs\cos\phi \quad \text{(constant force, straight-line displacement)} \qquad (6.2)$$

We are assuming that F and ϕ are constant during the displacement. If $\phi = 0$, so that $\vec{F}$ and $\vec{s}$ are in the same direction, then $\cos\phi = 1$ and we are back to Eq. (6.1).

Equation (6.2) has the form of the *scalar product* of two vectors, which we introduced in Section 1.10: $\vec{A} \cdot \vec{B} = AB\cos\phi$. You may want to review that definition. Hence we can write Eq. (6.2) more compactly as

$$W = \vec{F} \cdot \vec{s} \quad \text{(constant force, straight-line displacement)} \qquad (6.3)$$

CAUTION **Work is a scalar** Here's an essential point: Work is a *scalar* quantity, even though it's calculated by using two vector quantities (force and displacement). A 5-N force toward the east acting on a body that moves 6 m to the east does exactly the same amount of work as a 5-N force toward the north acting on a body that moves 6 m to the north.

Example 6.1 Work done by a constant force

(a) Steve exerts a steady force of magnitude 210 N (about 47 lb) on the stalled car in Fig. 6.3 as he pushes it a distance of 18 m. The car also has a flat tire, so to make the car track straight Steve must push at an angle of 30° to the direction of motion. How much work does Steve do? (b) In a helpful mood, Steve pushes a second stalled car with a steady force $\vec{F} = (160\,\text{N})\hat{\imath} - (40\,\text{N})\hat{\jmath}$. The displacement of the car is $\vec{s} = (14\,\text{m})\hat{\imath} + (11\,\text{m})\hat{\jmath}$. How much work does Steve do in this case?

SOLUTION

IDENTIFY: In both parts (a) and (b), the target variable is the work W done by Steve. In each case the force is constant and the displacement is along a straight line, so we can use Eq. (6.2) or (6.3).

SET UP: The angle between $\vec{F}$ and $\vec{s}$ is given explicitly in part (a), so we can apply Eq. (6.2) directly. In part (b) the angle isn't given, so we're better off calculating the scalar product in Eq. (6.3) from the components of $\vec{F}$ and $\vec{s}$, as in Eq. (1.21): $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$.

EXECUTE: (a) From Eq. (6.2),

$$W = Fs\cos\phi = (210\,\text{N})(18\,\text{m})\cos 30° = 3.3 \times 10^3\,\text{J}$$

(b) The components of $\vec{F}$ are $F_x = 160$ N and $F_y = -40$ N, and the components of $\vec{s}$ are $x = 14$ m and $y = 11$ m. (There are no z-components for either vector.) Hence, using Eqs. (1.21) and (6.3),

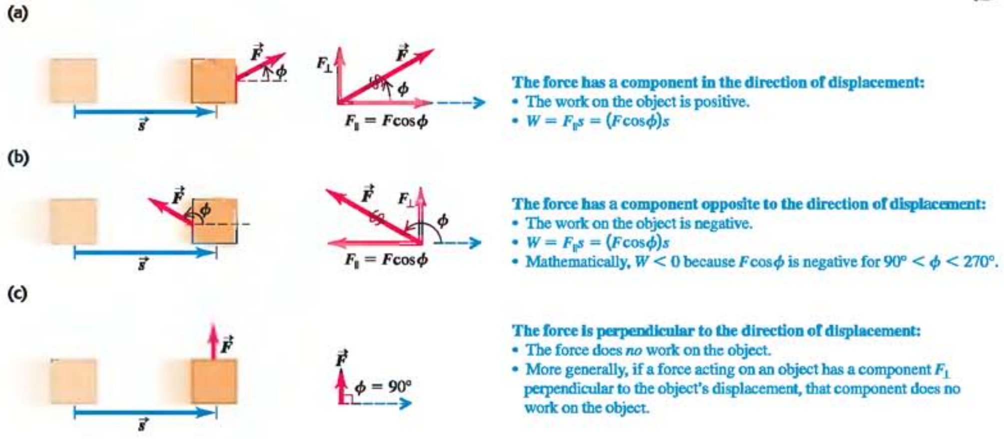
$$\begin{aligned} W = \vec{F} \cdot \vec{s} &= F_x x + F_y y \\ &= (160\,\text{N})(14\,\text{m}) + (-40\,\text{N})(11\,\text{m}) \\ &= 1.8 \times 10^3\,\text{J} \end{aligned}$$

EVALUATE: In each case the work that Steve does is more than 1000 J. This shows that 1 joule is a rather small amount of work.

Work: Positive, Negative, or Zero

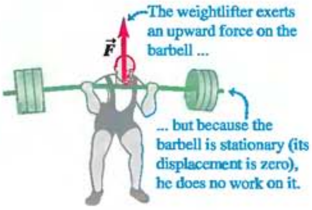
In Example 6.1 the work done in pushing the cars was positive. But it's important to understand that work can also be negative or zero. This is the essential way in which work as defined in physics differs from the "everyday" definition of work. When the force has a component in the *same direction* as the displacement

6.4 A constant force $\vec{F}$ can do positive, negative, or zero work depending on the angle between $\vec{F}$ and the displacement $\vec{s}$.



(ϕ between zero and 90°), $\cos\phi$ in Eq. (6.2) is positive and the work W is *positive* (Fig. 6.4a). When the force has a component *opposite* to the displacement (ϕ between 90° and 180°), $\cos\phi$ is negative and the work is *negative* (Fig. 6.4b). When the force is *perpendicular* to the displacement, $\phi = 90°$ and the work done by the force is *zero* (Fig. 6.4c). The cases of zero work and negative work bear closer examination, so let's look at some examples.

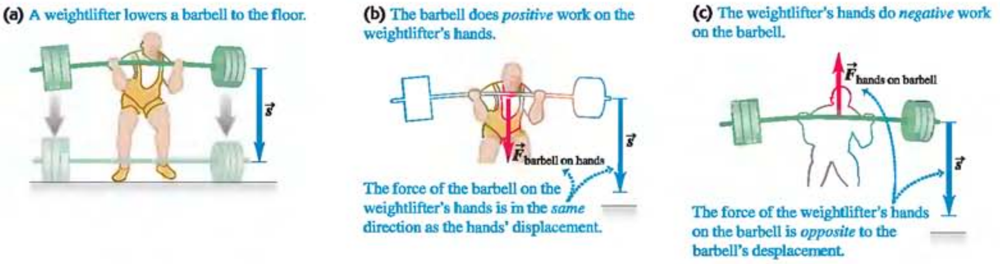
6.5 A weightlifter does no work on a barbell as long as he holds it stationary.



There are many situations in which forces act but do zero work. You might think it's "hard work" to hold a barbell motionless in the air for 5 minutes (Fig. 6.5). But in fact, you aren't doing any work at all on the barbell because there is no displacement. You get tired because the components of muscle fibers in your arm do work as they continually contract and relax. This is work done by one part of the arm exerting force on another part, however, *not* on the barbell. (We'll say more in Section 6.2 about work done by one part of a body on another part.) Even when you walk with constant velocity on a level floor while carrying a book, you still do no work on it. The book has a displacement, but the (vertical) supporting force that you exert on the book has no component in the direction of the (horizontal) motion. Then $\phi = 90°$ in Eq. (6.2), and $\cos\phi = 0$. When a body slides along a surface, the work done on the body by the normal force is zero; and when a ball on a string moves in uniform circular motion, the work done on the ball by the tension in the string is also zero. In both cases the work is zero because the force has no component in the direction of motion.

What does it really mean to do *negative* work? The answer comes from Newton's third law of motion. When a weightlifter lowers a barbell as in Fig. 6.6a, his hands and the barbell move together with the same displacement $\vec{s}$. The barbell exerts a force $\vec{F}_{\text{barbell on hands}}$ on his hands in the same direction as the hands' displacement, so the work done by the *barbell* on his *hands* is positive. (Fig. 6.6b). But by Newton's third law the weightlifter's hands exert an equal and opposite force $\vec{F}_{\text{hands on barbell}} = -\vec{F}_{\text{barbell on hands}}$ on the barbell (Fig. 6.6c). This force, which keeps the barbell from crashing to the floor, acts opposite to the barbell's displacement. Thus the work done by his *hands* on the *barbell* is negative. Because the weightlifter's hands and the barbell have the same displacement, the work that his hands do on the barbell is just the negative of the work that the barbell does on his hands. In general, when one body does negative work on a second body, the second body does an equal amount of *positive* work on the first body.

CAUTION **Keep track of who's doing the work** We always speak of work done *on* a particular body *by* a specific force. Always be sure to specify exactly what force is doing the



6.6 This weightlifter's hands do negative work on a barbell as the barbell does positive work on his hands.

work you are talking about. When you lift a book, you exert an upward force on the book and the book's displacement is upward, so the work done by the lifting force on the book is positive. But the work done by the *gravitational* force (weight) on a book being lifted is *negative* because the downward gravitational force is opposite to the upward displacement.

Total Work

How do we calculate work when *several* forces act on a body? One way is to use Eq. (6.2) or (6.3) to compute the work done by each separate force. Then, because work is a scalar quantity, the *total* work W_{tot} done on the body by all the forces is the algebraic sum of the quantities of work done by the individual forces. An alternative way to find the total work W_{tot} is to compute the vector sum of the forces (that is, the net force) and then use this vector sum as $\vec{F}$ in Eq. (6.2) or (6.3). The following example illustrates both of these techniques.

Example 6.2 Work done by several forces

A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground (Fig. 6.7a). The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000-N force at an angle of 36.9° above the horizontal, as shown in Fig. 6.7b. There is a 3500-N friction force opposing the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces.

SOLUTION

IDENTIFY: Each force is constant and the displacement is along a straight line, so we can calculate the work using the ideas of this section. We'll find the total work in two ways: (1) by adding together the work done on the sled by each force and (2) by finding the amount of work done by the net force on the sled.

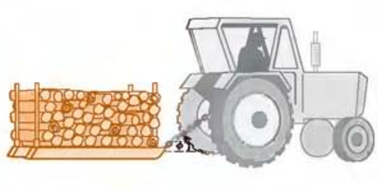
SET UP: Since we're working with forces, we first draw a free-body diagram showing all of the forces acting on the sled and we choose a coordinate system (Fig. 6.7b). For each force—weight, normal force, force of the tractor, and friction force—we know the angle between the displacement (in the positive x-direction) and the force. Hence we can calculate the work each force does using Eq. (6.2).

As we did in Chapter 5, we'll find the net force by adding the components of the four forces. Newton's second law tells us that because the sled's motion is purely horizontal, the net force has only a horizontal component.

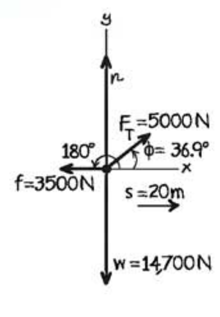
EXECUTE: The work W_w done by the weight is zero because its direction is perpendicular to the displacement (compare Fig. 6.4c). For the same reason, the work W_n done by the normal force is

6.7 Calculating the work done on a sled of firewood being pulled by a tractor.

(a)



(b) Free-body diagram for sled



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also zero. So $W_w = W_n = 0$. (Incidentally, can you see that the magnitude of the normal force is less than the weight? Compare Example 5.15 in Section 5.3, which has a very similar free-body diagram.)

That leaves the force F_T exerted by the tractor and the friction force f. From Eq. (6.2) the work W_T done by the tractor is

$$W_T = F_T s\cos\phi = (5000\text{ N})(20\text{ m})(0.800) = 80{,}000\text{ N}\cdot\text{m}$$
$$= 80\text{ kJ}$$

The friction force $\vec{f}$ is opposite to the displacement, so for this force $\phi = 180°$ and $\cos\phi = -1$. The work W_f done by the friction force is

$$W_f = fs\cos 180° = (3500\text{ N})(20\text{ m})(-1) = -70{,}000\text{ N}\cdot\text{m}$$
$$= -70\text{ kJ}$$

The total work W_{tot} done on the sled by all forces is the *algebraic* sum of the work done by the individual forces:

$$W_{tot} = W_w + W_n + W_T + W_f = 0 + 0 + 80\text{ kJ} + (-70\text{ kJ})$$
$$= 10\text{ kJ}$$

In the alternative approach, we first find the *vector* sum of all the forces (the net force) and then use it to compute the total work. The vector sum is best found by using components. From Fig. 6.7b,

$$\sum F_x = F_T\cos\phi + (-f) = (5000\text{ N})\cos 36.9° - 3500\text{ N}$$
$$= 500\text{ N}$$
$$\sum F_y = F_T\sin\phi + n + (-w)$$
$$= (5000\text{ N})\sin 36.9° + n - 14{,}700\text{ N}$$

We don't really need the second equation; we know that the y-component of force is perpendicular to the displacement, so it does no work. Besides, there is no y-component of acceleration, so $\sum F_y$ has to be zero anyway. The total work is therefore the work done by the total x-component:

$$W_{tot} = \left(\sum \vec{F}\right)\cdot\vec{s} = \left(\sum F_x\right)s = (500\text{ N})(20\text{ m}) = 10{,}000\text{ J}$$
$$= 10\text{ kJ}$$

EVALUATE: We get the same result for W_{tot} with either method, as we should.

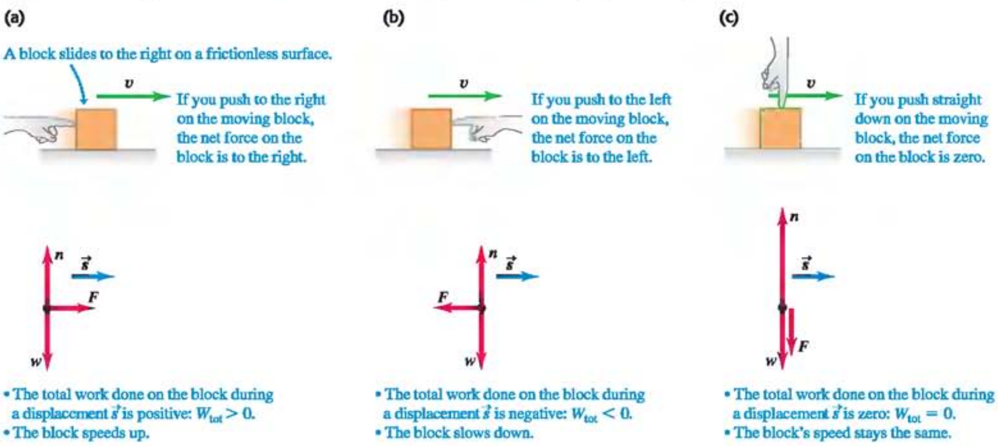
Note that the net force in the x-direction is *not* zero, and so the sled must accelerate as it moves. In Section 6.2 we'll return to this example and see how to use the concept of work to explore the sled's motion.

Test Your Understanding of Section 6.1 An electron moves in a straight line toward the east with a constant speed of 8×10^7 m/s. It has electric, magnetic, and gravitational forces acting on it. During a 1-m displacement, the total work done on the electron is (i) positive; (ii) negative; (iii) zero; (iv) not enough information given to decide.

6.2 Kinetic Energy and the Work–Energy Theorem

The total work done on a body by external forces is related to the body's displacement—that is, to changes in its position. But the total work is also related to changes in the *speed* of the body. To see this, consider Fig. 6.8, which shows

6.8 The relationship between the total work done on a body and how the body's speed changes.

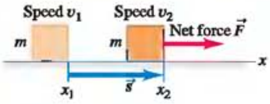


three examples of a block sliding on a frictionless table. The forces acting on the block are its weight $\vec{w}$, the normal force $\vec{n}$, and the force $\vec{F}$ exerted on it by the hand.

In Fig. 6.8a the net force on the block is in the direction of its motion. From Newton's second law, this means that the block speeds up; from Eq. (6.1), this also means that the total work W_{tot} done on the block is positive. The total work is *negative* in Fig. 6.8b because the net force opposes the displacement; in this case the block slows down. The net force is zero in Fig. 6.8c, so the speed of the block stays the same and the total work done on the block is zero. We can conclude that *when a particle undergoes a displacement, it speeds up if* $W_{tot} > 0$, *slows down if* $W_{tot} < 0$, *and maintains the same speed if* $W_{tot} = 0$.

Let's make these observations more quantitative. Consider a particle with mass m moving along the x-axis under the action of a constant net force with magnitude F directed along the positive x-axis (Fig. 6.9). The particle's acceleration is constant and given by Newton's second law, $F = ma_x$. Suppose the speed changes from v_1 to v_2 while the particle undergoes a displacement $s = x_2 - x_1$ from point x_1 to x_2. Using a constant-acceleration equation, Eq. (2.13), and replacing v_{0x} by v_1, v_x by v_2, and $(x - x_0)$ by s, we have

6.9 A constant net force $\vec{F}$ does work on a moving body.



$$v_2^2 = v_1^2 + 2a_x s$$
$$a_x = \frac{v_2^2 - v_1^2}{2s}$$

When we multiply this equation by m and equate ma_x to the net force F, we find

$$F = ma_x = m\frac{v_2^2 - v_1^2}{2s} \quad \text{and}$$
$$Fs = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \qquad (6.4)$$

The product Fs is the work done by the net force F and thus is equal to the total work W_{tot} done by all the forces acting on the particle. The quantity $\frac{1}{2}mv^2$ is called the **kinetic energy** K of the particle:

$$K = \frac{1}{2}mv^2 \quad \text{(definition of kinetic energy)} \qquad (6.5)$$

Like work, the kinetic energy of a particle is a scalar quantity; it depends on only the particle's mass and speed, not its direction of motion (Fig. 6.10). A car (viewed as a particle) has the same kinetic energy when going north at 10 m/s as when going east at 10 m/s. Kinetic energy can never be negative, and it is zero only when the particle is at rest.

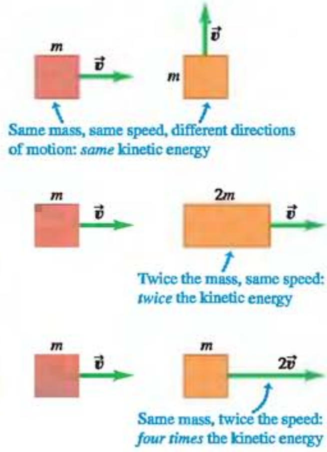
We can now interpret Eq. (6.4) in terms of work and kinetic energy. The first term on the right side of Eq. (6.4) is $K_2 = \frac{1}{2}mv_2^2$, the final kinetic energy of the particle (that is, after the displacement). The second term is the initial kinetic energy, $K_1 = \frac{1}{2}mv_1^2$, and the difference between these terms is the *change* in kinetic energy. So Eq. (6.4) says:

The work done by the net force on a particle equals the change in the particle's kinetic energy:

$$W_{tot} = K_2 - K_1 = \Delta K \quad \text{(work–energy theorem)} \qquad (6.6)$$

This result is the **work–energy theorem.**

6.10 Comparing the kinetic energy $K = \frac{1}{2}mv^2$ of different bodies.



The work–energy theorem agrees with our observations about the block in Fig. 6.8. When W_{tot} is *positive*, the kinetic energy *increases* (the final kinetic energy K_2 is greater than the initial kinetic energy K_1) and the particle is going faster at the end of the displacement than at the beginning. When W_{tot} is *negative*, the kinetic energy *decreases* (K_2 is less than K_1) and the speed is less after the displacement. When $W_{tot} = 0$, the kinetic energy stays the same ($K_1 = K_2$) and the speed is unchanged. Note that the work–energy theorem by itself tells us only about changes in *speed*, not velocity, since the kinetic energy doesn't depend on the direction of motion.

From Eq. (6.4) or (6.6), kinetic energy and work must have the same units. Hence the joule is the SI unit of both work and kinetic energy (and, as we will see later, of all kinds of energy). To verify this, note that in SI units the quantity $K = \frac{1}{2}mv^2$ has units $\text{kg}\cdot(\text{m/s})^2$ or $\text{kg}\cdot\text{m}^2/\text{s}^2$; we recall that $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$, so

$$1\ \text{J} = 1\ \text{N}\cdot\text{m} = 1\ (\text{kg}\cdot\text{m/s}^2)\cdot\text{m} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$$

In the British system the unit of kinetic energy and of work is

$$1\ \text{ft}\cdot\text{lb} = 1\ \text{ft}\cdot\text{slug}\cdot\text{ft/s}^2 = 1\ \text{slug}\cdot\text{ft}^2/\text{s}^2$$

Because we used Newton's laws in deriving the work–energy theorem, we can use this theorem only in an inertial frame of reference. Note also that the work–energy theorem is valid in *any* inertial frame, but the values of W_{tot} and $K_2 - K_1$ may differ from one inertial frame to another (because the displacement and speed of a body may be different in different frames).

We have derived the work–energy theorem for the special case of straight-line motion with constant forces, and in the following examples we'll apply it to this special case only. We'll find in the next section that the theorem is valid in general, even when the forces are not constant and the particle's trajectory is curved.

Problem-Solving Strategy 6.1 Work and Kinetic Energy

MP

IDENTIFY *the relevant concepts:* The work–energy theorem, $W_{tot} = K_2 - K_1$, is extremely useful when you want to relate a body's speed v_1 at one point in its motion to its speed v_2 at a different point. (It's less useful for problems that involve the *time* it takes a body to go from point 1 to point 2, because the work–energy theorem doesn't involve time at all. For such problems it's usually best to use the relationships among time, position, velocity, and acceleration described in Chapters 2 and 3.)

SET UP *the problem* using the following steps:

1. Choose the initial and final positions of the body, and draw a free-body diagram showing all the forces that act on the body.
2. Choose a coordinate system. (If the motion is along a straight line, it's usually easiest to have both the initial and final positions lie along the x-axis.)
3. List the unknown and known quantities, and decide which unknowns are your target variables. The target variable may be the body's initial or final speed, the magnitude of one of the forces acting on the body, or the body's displacement.

EXECUTE *the solution:* Calculate the work W done by each force. If the force is constant and the displacement is a straight line, you can use Eq. (6.2) or (6.3). (Later in this chapter we'll see how to handle varying forces and curved trajectories.) Be sure to check signs; W must be positive if the force has a component in the direction of the displacement, negative if the force has a component opposite to the displacement, and zero if the force and displacement are perpendicular.

Add the amounts of work done by each force to find the total work W_{tot}. Sometimes it's easier to calculate the vector sum of the forces (the net force) and then find the work done by the net force; this value is also equal to W_{tot}.

Write expressions for the initial and final kinetic energies, K_1 and K_2. Note that kinetic energy involves *mass*, not *weight*; if you are given the body's weight, you'll need to use the relationship $w = mg$ to find the mass.

Finally, use $W_{tot} = K_2 - K_1$ to solve for the target variable. Remember that the right-hand side of this equation is the *final* kinetic energy minus the *initial* kinetic energy, never the other way around.

EVALUATE *your answer:* Check whether your answer makes physical sense. A key point to remember is that kinetic energy $K = \frac{1}{2}mv^2$ can never be negative. If you come up with a negative value of K, perhaps you interchanged the initial and final kinetic energies in $W_{tot} = K_2 - K_1$ or made a sign error in one of the work calculations.

Example 6.3 Using work and energy to calculate speed

Let's look again at the sled in Fig. 6.7 and the numbers at the end of Example 6.2. Suppose the initial speed v_1 is 2.0 m/s. What is the speed of the sled after it moves 20 m?

SOLUTION

IDENTIFY: We'll use the work–energy theorem, Eq. (6.6) ($W_{tot} = K_2 - K_1$), since we are given the initial speed $v_1 = 2.0\ \text{m/s}$ and want to find the final speed.

SET UP: Figure 6.11 shows our sketch of the situation. The motion is in the positive *x*-direction.

EXECUTE: In Example 6.2 we calculated the total work done by all the forces: $W_{tot} = 10\ \text{kJ}$. Hence the kinetic energy of the sled and its load must increase by 10 kJ.

To write expressions for the initial and final kinetic energies, we need the mass of the sled and load. We are given that the *weight* is 14,700 N, so the mass is

$$m = \frac{w}{g} = \frac{14{,}700\ \text{N}}{9.8\ \text{m/s}^2} = 1500\ \text{kg}$$

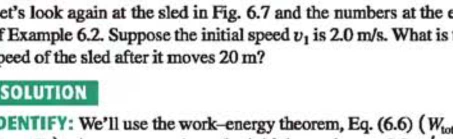
Then the initial kinetic energy K_1 is

$$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(1500\ \text{kg})(2.0\ \text{m/s})^2 = 3000\ \text{kg}\cdot\text{m}^2/\text{s}^2$$
$$= 3000\ \text{J}$$

The final kinetic energy K_2 is

$$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(1500\ \text{kg})v_2^2$$

6.11 Our sketch for this problem.



where v_2 is the unknown speed we want to find. Equation (6.6) gives

$$K_2 = K_1 + W_{tot} = 3000\ \text{J} + 10{,}000\ \text{J} = 13{,}000\ \text{J}$$

Setting these two expressions for K_2 equal, substituting $1\ \text{J} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$, and solving for v_2, we find

$$v_2 = 4.2\ \text{m/s}$$

EVALUATE: The total work is positive, so the kinetic energy increases ($K_2 > K_1$) and the speed increases ($v_2 > v_1$).

This problem can also be done without the work–energy theorem. We can find the acceleration from $\sum\vec{F} = m\vec{a}$ and then use the equations of motion for constant acceleration to find v_2. Since the acceleration is along the *x*-axis,

$$a = a_x = \frac{\sum F_x}{m} = \frac{(5000\ \text{N})\cos 36.9° - 3500\ \text{N}}{1500\ \text{kg}}$$
$$= 0.333\ \text{m/s}^2$$

Then, using Eq. (2.13),

$$v_2^2 = v_1^2 + 2as = (2.0\ \text{m/s})^2 + 2(0.333\ \text{m/s}^2)(20\ \text{m})$$
$$= 17.3\ \text{m}^2/\text{s}^2$$
$$v_2 = 4.2\ \text{m/s}$$

This is the same result we obtained with the work–energy approach, but there we avoided the intermediate step of finding the acceleration. You will find several other examples in this chapter and the next that *can* be done without using energy considerations but that are easier when energy methods are used. When a problem can be done by two different methods, doing it by both methods (as we did in this example) is a very good way to check your work.

Example 6.4 Forces on a hammerhead

In a pile driver, a steel hammerhead with mass 200 kg is lifted 3.00 m above the top of a vertical I-beam being driven into the ground (Fig. 6.12a). The hammer is then dropped, driving the I-beam 7.4 cm farther into the ground. The vertical rails that guide the hammerhead exert a constant 60-N friction force on the hammerhead. Use the work–energy theorem to find (a) the speed of the hammerhead just as it hits the I-beam and (b) the average force the hammerhead exerts on the I-beam. Ignore the effects of the air.

SOLUTION

IDENTIFY: We'll use the work–energy theorem to relate the hammerhead's speed at different locations and the forces acting on it. There are *three* locations of interest: point 1, where the hammerhead starts from rest; point 2, where it first contacts the I-beam; and point 3, where the hammerhead comes to a halt (see Fig. 6.12a). The two unknowns are the hammerhead's speed at point 2 and the force the hammerhead exerts between points 2 and 3. Hence we'll apply the work–energy theorem twice: once for the motion from 1 to 2, and once for the motion from 2 to 3.

SET UP: Figure 6.12b shows the vertical forces on the hammerhead as it falls from point 1 to point 2. (We can ignore any horizontal forces that may be present because they do no work as the hammerhead moves vertically.) For this part of the motion, our target variable is the hammerhead's speed v_2.

Figure 6.12c shows the vertical forces on the hammerhead during the motion from point 2 to point 3. In addition to the forces shown in Fig. 6.12b, the I-beam exerts an upward normal force of magnitude *n* on the hammerhead. This force actually varies as the hammerhead comes to a halt, but for simplicity we'll treat *n* as a

Continued

constant. Hence n represents the *average* value of this upward force during the motion. Our target variable for this part of the motion is the force that the *hammerhead* exerts on the I-beam; it is the reaction force to the normal force exerted by the I-beam, so by Newton's third law its magnitude is also n.

EXECUTE: (a) From point 1 to point 2, the vertical forces are the downward weight $w = mg = (200\text{ kg})(9.8\text{ m/s}^2) = 1960\text{ N}$ and the upward friction force $f = 60\text{ N}$. Thus the net downward force is $w - f = 1900\text{ N}$. The displacement of the hammerhead from point 1 to point 2 is downward and equal to $s_{12} = 3.00\text{ m}$. The total work done on the hammerhead as it moves from point 1 to point 2 is then

$$W_{tot} = (w - f)s_{12} = (1900\text{ N})(3.00\text{ m}) = 5700\text{ J}$$

At point 1 the hammerhead is at rest, so its initial kinetic energy K_1 is zero. Hence the kinetic energy K_2 at point 2 equals the total work done on the hammerhead between points 1 and 2:

$$W_{tot} = K_2 - K_1 = K_2 - 0 = \frac{1}{2}mv_2^2 - 0$$

$$v_2 = \sqrt{\frac{2W_{tot}}{m}} = \sqrt{\frac{2(5700\text{ J})}{200\text{ kg}}} = 7.55\text{ m/s}$$

This is the hammerhead's speed at point 2, just as it hits the I-beam.

(b) As the hammerhead moves downward between points 2 and 3, the net downward force acting on it is $w - f - n$ (see Fig. 6.12c). The total work done on the hammerhead during this displacement is

$$W_{tot} = (w - f - n)s_{23}$$

The initial kinetic energy for this part of the motion is K_2, which from part (a) equals 5700 J. The final kinetic energy is $K_3 = 0$, since the hammerhead ends at rest. Then, from the work–energy theorem,

$$W_{tot} = (w - f - n)s_{23} = K_3 - K_2$$

$$n = w - f - \frac{K_3 - K_2}{s_{23}}$$

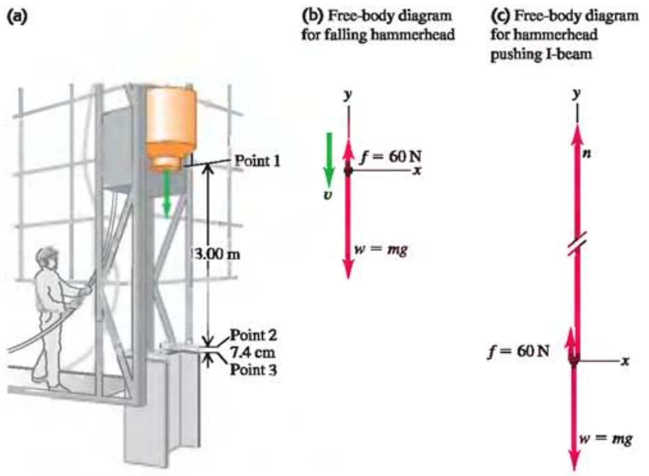
$$= 1960\text{ N} - 60\text{ N} - \frac{0\text{ J} - 5700\text{ J}}{0.074\text{ m}}$$

$$= 79{,}000\text{ N}$$

The downward force that the hammerhead exerts on the I-beam has this same magnitude, 79,000 N (about 9 tons)—more than 40 times the weight of the hammerhead.

EVALUATE: The net change in the hammerhead's kinetic energy from point 1 to point 3 is zero; a relatively small net force does positive work over a large distance, and then a much larger net force does negative work over a much smaller distance. The same thing happens if you speed up your car gradually and then drive it into a brick wall. The very large force needed to reduce the kinetic energy to zero over a short distance is what does the damage to your car—and possibly to you.

6.12 (a) A pile driver pounds an I-beam into the ground. (b), (c) Free-body diagrams. Vector lengths are not to scale.



The Meaning of Kinetic Energy

Example 6.4 gives insight into the physical meaning of kinetic energy. The hammerhead is dropped from rest, and its kinetic energy when it hits the I-beam equals the total work done on it up to that point by the net force. This result is true in general: To accelerate a particle of mass m from rest (zero kinetic energy) up to a speed v, the total work done on it must equal the change in kinetic energy from zero to $K = \frac{1}{2}mv^2$:

$$W_{tot} = K - 0 = K$$

So *the kinetic energy of a particle is equal to the total work that was done to accelerate it from rest to its present speed* (Fig. 6.13). The definition $K = \frac{1}{2}mv^2$, Eq. (6.5), wasn't chosen at random; it's the *only* definition that agrees with this interpretation of kinetic energy.

In the second part of Example 6.4 the kinetic energy of the hammerhead did work on the I-beam and drove it into the ground. This gives us another interpretation of kinetic energy: *The kinetic energy of a particle is equal to the total work that particle can do in the process of being brought to rest.* This is why you pull your hand and arm backward when you catch a ball. As the ball comes to rest, it does an amount of work (force times distance) on your hand equal to the ball's initial kinetic energy. By pulling your hand back, you maximize the distance over which the force acts and so minimize the force on your hand.

6.13 When a billiards player hits a cue ball at rest, the ball's kinetic energy after being hit is equal to the work that was done on it by the cue. The greater the force exerted by the cue and the greater the distance the ball moves while in contact with it, the greater the ball's kinetic energy.

Conceptual Example 6.5 Comparing kinetic energies

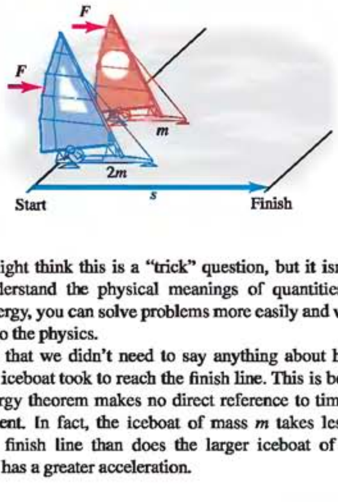
Two iceboats like the one in Example 5.6 (Section 5.2) hold a race on a frictionless horizontal lake (Fig. 6.14). The two iceboats have masses m and $2m$. Each iceboat has an identical sail, so the wind exerts the same constant force $\vec{F}$ on each iceboat. The two iceboats start from rest and cross the finish line a distance s away. Which iceboat crosses the finish line with greater kinetic energy?

SOLUTION

If you use the mathematical definition of kinetic energy, $K = \frac{1}{2}mv^2$, Eq. (6.5), the answer to this problem isn't immediately obvious. The iceboat of mass $2m$ has greater mass, so you might guess that the larger iceboat attains a greater kinetic energy at the finish line. But the smaller iceboat, of mass m, crosses the finish line with a greater speed, and you might guess that *this* iceboat has the greater kinetic energy. How can we decide?

The correct way to approach this problem is to remember that *the kinetic energy of a particle is equal to the total work done to accelerate it from rest.* Both iceboats travel the same distance s, and only the horizontal force F in the direction of motion does work on either iceboat. Hence the total work done between the starting line and the finish line is the *same* for each iceboat, $W_{tot} = Fs$. At the finish line, each iceboat has a kinetic energy equal to the work W_{tot} done on it, because each iceboat started from rest. So both iceboats have the *same* kinetic energy at the finish line!

6.14 A race between iceboats.

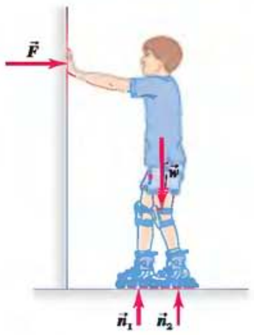


You might think this is a "trick" question, but it isn't. If you really understand the physical meanings of quantities such as kinetic energy, you can solve problems more easily and with better insight into the physics.

Notice that we didn't need to say anything about how much time each iceboat took to reach the finish line. This is because the work–energy theorem makes no direct reference to time, only to displacement. In fact, the iceboat of mass m takes less time to reach the finish line than does the larger iceboat of mass $2m$ because it has a greater acceleration.

Work and Kinetic Energy in Composite Systems

In this section we've been careful to apply the work–energy theorem only to bodies that we can represent as *particles*—that is, as moving point masses. New subtleties appear for more complex systems that have to be represented as many particles with different motions. We can't go into these subtleties in detail in this chapter, but here's an example.



6.15 The external forces acting on a skater pushing off a wall. The work done by these forces is zero, but the skater's kinetic energy changes nonetheless.

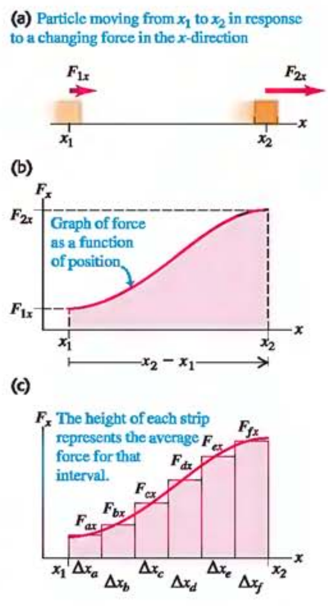
Suppose a boy stands on frictionless roller skates on a level surface, facing a rigid wall (Fig. 6.15). He pushes against the wall, which makes him move to the right. The forces acting on him are his weight $\vec{w}$, the upward normal forces $\vec{n}_1$ and $\vec{n}_2$ exerted by the ground on his skates, and the horizontal force $\vec{F}$ exerted on him by the wall. There is no vertical displacement, so $\vec{w}$, $\vec{n}_1$, and $\vec{n}_2$ do no work. Force $\vec{F}$ accelerates him to the right, but the parts of his body where that force is applied (the man's hands) do not move while the force acts. Thus the force $\vec{F}$ also does no work. Where, then, does the boy's kinetic energy come from?

The explanation is that it's not adequate to represent the boy as a single point mass. Different parts of the boy's body have different motions; his hands remain stationary against the wall while his torso is moving away from the wall. The various parts of his body interact with each other, and one part can exert forces and do work on another part. Therefore the *total* kinetic energy of this *composite* system of body parts can change, even though no work is done by forces applied by bodies (such as the wall) that are outside the system. In Chapter 8 we'll consider further the motion of a collection of particles that interact with each other. We'll discover that just as for the boy in this example, the total kinetic energy of such a system can change even when no work is done on any part of the system by anything outside it.

Test Your Understanding of Section 6.2 Rank the following bodies in order of their kinetic energy, from least to greatest. (i) a 2.0-kg body moving at 5.0 m/s; (ii) a 1.0 kg body that initially was at rest and then had 30 J of work done on it; (iii) a 1.0-kg body that initially was moving at 4.0 m/s and then had 20 J of work done on it; (iv) a 2.0 kg body that initially was moving at 10 m/s and then did 80 J of work on another body. ∎

6.16 Calculating the work done by a varying force F_x in the x-direction as a particle moves from x_1 to x_2.

(a) Particle moving from x_1 to x_2 in response to a changing force in the x-direction



6.3 Work and Energy with Varying Forces

So far in this chapter we've considered work done by *constant forces* only. But what happens when you stretch a spring? The more you stretch it, the harder you have to pull, so the force you exert is *not* constant as the spring is stretched. We've also restricted our discussion to *straight-line* motion. There are many situations in which a body moves along a curved path and is acted on by a force that varies in magnitude, direction, or both. We need to be able to compute the work done by the force in these more general cases. Fortunately, we'll find that the work–energy theorem holds true even when varying forces are considered and when the body's path is not straight.

Work Done by a Varying Force, Straight-Line Motion

To add only one complication at a time, let's consider straight-line motion along the x-axis with a force whose x-component F_x may change as the body moves. (A real-life example is driving a car along a straight road with stop signs, so the driver has to alternately step on the gas and apply the brakes.) Suppose a particle moves along the x-axis from point x_1 to x_2 (Fig. 6.16a). Figure 6.16b is a graph of the x-component of force as a function of the particle's coordinate x. To find the work done by this force, we divide the total displacement into small segments Δx_a, Δx_b, and so on (Fig. 6.16c). We approximate the work done by the force during segment Δx_a as the average x-component of force F_{ax} in that segment multiplied by the x-displacement Δx_a. We do this for each segment and then add the results for all the segments. The work done by the force in the total displacement from x_1 to x_2 is approximately

$$W = F_{ax}\Delta x_a + F_{bx}\Delta x_b + \cdots$$

In the limit that the number of segments becomes very large and the width of each becomes very small, this sum becomes the *integral* of F_x from x_1 to x_2:

$$W = \int_{x_1}^{x_2} F_x\,dx \qquad \text{(varying } x\text{-component of force, straight-line displacement)} \tag{6.7}$$

Note that $F_{ax}\Delta x_a$ represents the *area* of the first vertical strip in Fig. 6.16c and that the integral in Eq. (6.7) represents the area under the curve of Fig. 6.16b between x_1 and x_2. *On a graph of force as a function of position, the total work done by the force is represented by the area under the curve between the initial and final positions.* An alternative interpretation of Eq. (6.7) is that the work W equals the average force that acts over the entire displacement, multiplied by the displacement.

In the special case that F_x, the x-component of the force, is constant, it may be taken outside the integral in Eq. (6.7):

$$W = \int_{x_1}^{x_2} F_x\,dx = F_x\int_{x_1}^{x_2} dx = F_x(x_2 - x_1) \qquad \text{(constant force)}$$

But $x_2 - x_1 = s$, the total displacement of the particle. So in the case of a constant force F, Eq. (6.7) says that $W = Fs$, in agreement with Eq. (6.1). The interpretation of work as the area under the curve of F_x as a function of x also holds for a constant force; $W = Fs$ is the area of a rectangle of height F and width s (Fig. 6.17).

Now let's apply these ideas to the stretched spring. To keep a spring stretched beyond its unstretched length by an amount x, we have to apply a force of equal magnitude at each end (Fig. 6.18). If the elongation x is not too great, the force we apply to the right-hand end has an x-component directly proportional to x:

$$F_x = kx \qquad \text{(force required to stretch a spring)} \tag{6.8}$$

where k is a constant called the **force constant** (or spring constant) of the spring. The units of k are force divided by distance: N/m in SI units and lb/ft in British units. A floppy toy spring such as a Slinky™ has a force constant of about 1 N/m; for the much stiffer springs in an automobile's suspension, k is about 10^5 N/m. The observation that force is directly proportional to elongation for elongations that are not too great was made by Robert Hooke in 1678 and is known as **Hooke's law.** It really shouldn't be called a "law," since it's a statement about a specific device and not a fundamental law of nature. Real springs don't always obey Eq. (6.8) precisely, but it's still a useful idealized model. We'll discuss Hooke's law more fully in Chapter 11.

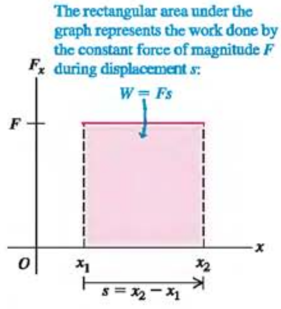
To stretch a spring, we must do work. We apply equal and opposite forces to the ends of the spring and gradually increase the forces. We hold the left end stationary, so the force we apply at this end does no work. The force at the moving end *does* do work. Figure 6.19 is a graph of F_x as a function of x, the elongation of the spring. The work done by this force when the elongation goes from zero to a maximum value X is

$$W = \int_0^X F_x\,dx = \int_0^X kx\,dx = \frac{1}{2}kX^2 \tag{6.9}$$

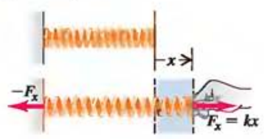
We can also obtain this result graphically. The area of the shaded triangle in Fig. 6.19, representing the total work done by the force, is equal to half the product of the base and altitude, or

$$W = \frac{1}{2}(X)(kX) = \frac{1}{2}kX^2$$

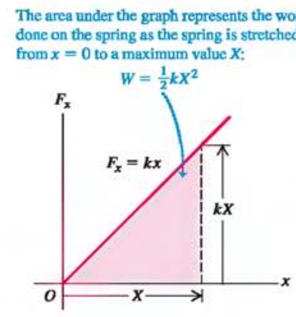
6.17 The work done by a constant force F in the x-direction as a particle moves from x_1 to x_2.



6.18 The force needed to stretch an ideal spring is proportional to the spring's elongation: $F_x = kx$.



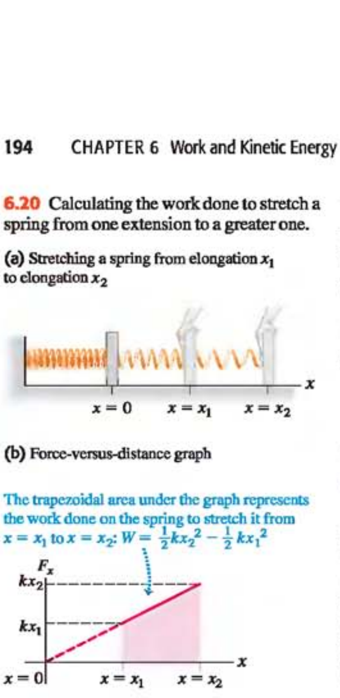
6.19 Calculating the work done to stretch a spring by a length X.



6.20 Calculating the work done to stretch a spring from one extension to a greater one.

(a) Stretching a spring from elongation x_1 to elongation x_2

(b) Force-versus-distance graph



This equation also says that the work is the *average* force $kX/2$ multiplied by the total displacement X. We see that the total work is proportional to the *square* of the final elongation X. To stretch an ideal spring by 2 cm, you must do four times as much work as is needed to stretch it by 1 cm.

Equation (6.9) assumes that the spring was originally unstretched. If initially the spring is already stretched a distance x_1, the work we must do to stretch it to a greater elongation x_2 (Fig. 6.20a) is

$$W = \int_{x_1}^{x_2} F_x\,dx = \int_{x_1}^{x_2} kx\,dx = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 \qquad (6.10)$$

You should use your knowledge of geometry to convince yourself that the trapezoidal area under the graph in Fig. 6.20b is given by the expression in Eq. (6.10).

If the spring has spaces between the coils when it is unstretched, then it can also be compressed, and Hooke's law holds for compression as well as stretching. In this case the force and displacement are in the opposite directions from those shown in Fig. 6.18, and so F_x and x in Eq. (6.8) are both negative. Since both F_x and x are reversed, the force again is in the same direction as the displacement, and the work done by F_x is again positive. So the total work is still given by Eq. (6.9) or (6.10), even when X is negative or either or both of x_1 and x_2 are negative.

CAUTION **Work done *on* a spring vs. work done *by* a spring** Note that Eq. (6.10) gives the work that *you* must do *on* a spring to change its length. For example, if you stretch a spring that's originally relaxed, then $x_1 = 0$, $x_2 > 0$, and $W > 0$: The force you apply to one end of the spring is in the same direction as the displacement, and the work you do is positive. By contrast, the work that the *spring* does on whatever it's attached to is given by the *negative* of Eq. (6.10). Thus, as you pull on the spring, the spring does negative work on you. Paying careful attention to the sign of work will eliminate confusion later on!

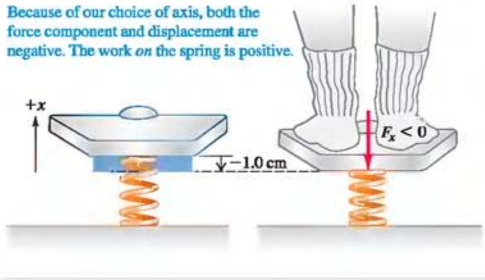
Example 6.6 Work done on a spring scale

A woman weighing 600 N steps on a bathroom scale containing a stiff spring (Fig. 6.21). In equilibrium the spring is compressed 1.0 cm under her weight. Find the force constant of the spring and the total work done on it during the compression.

SOLUTION

IDENTIFY: In equilibrium the upward force exerted by the spring balances the downward force of the woman's weight. We'll use this principle and Eq. (6.8) to determine the force constant k, and we'll use Eq. (6.10) to calculate the work W that the woman does on the spring to compress it.

6.21 Compressing a spring in a bathroom scale.



SET UP: We take positive values of x to correspond to elongation (upward in Fig. 6.21), so that the displacement of the spring (x) and the x-component of the force that the woman exerts on it (F_x) are both negative.

EXECUTE: The top of the spring is displaced by $x = -1.0\text{ cm} = -0.010\text{ m}$, and the woman exerts a force $F_x = -600\text{ N}$ on the spring. From Eq. (6.8) the force constant is

$$k = \frac{F_x}{x} = \frac{-600\text{ N}}{-0.010\text{ m}} = 6.0 \times 10^4\text{ N/m}$$

Then, using $x_1 = 0$ and $x_2 = -0.010\text{ m}$ in Eq. (6.10),

$$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$
$$= \frac{1}{2}(6.0 \times 10^4\text{ N/m})(-0.010\text{ m})^2 - 0 = 3.0\text{ J}$$

EVALUATE: The applied force and the displacement of the end of the spring were in the same direction, so the work done must have been positive—just as we found. Our arbitrary choice of the positive direction has no effect on the answer for W. (You can test this by taking the positive x-direction to be downward, corresponding to compression. You'll get the same values for k and W.)

Work–Energy Theorem for Straight-Line Motion, Varying Forces

In Section 6.2 we derived the work–energy theorem, $W_{tot} = K_2 - K_1$, for the special case of straight-line motion with a constant net force. We can now prove that this theorem is true even when the force varies with position. As in Section 6.2, let's consider a particle that undergoes a displacement x while being acted on by a net force with x-component F_x, which we now allow to vary. Just as in Fig. 6.16, we divide the total displacement x into a large number of small segments Δx. We can apply the work–energy theorem, Eq. (6.6), to each segment because the value of F_x in each small segment is approximately constant. The change in kinetic energy in segment Δx_a is equal to the work $F_{ax}\Delta x_a$, and so on. The total change of kinetic energy is the sum of the changes in the individual segments, and thus is equal to the total work done on the particle during the entire displacement. So $W_{tot} = \Delta K$ holds for varying forces as well as for constant ones.

Here's an alternative derivation of the work–energy theorem for a force that may vary with position. It involves making a change of variable from x to v_x in the work integral. As a preliminary, we note that the acceleration a of the particle can be expressed in various ways, using $a_x = dv_x/dt$, $v_x = dx/dt$, and the chain rule for derivatives:

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = v_x\frac{dv_x}{dx} \tag{6.11}$$

From this result, Eq. (6.7) tells us that the total work done by the *net* force F_x is

$$W_{tot} = \int_{x_1}^{x_2} F_x\,dx = \int_{x_1}^{x_2} ma_x\,dx = \int_{x_1}^{x_2} mv_x\frac{dv_x}{dx}\,dx \tag{6.12}$$

Now $(dv_x/dx)\,dx$ is the change in velocity dv_x during the displacement dx, so in Eq. (6.12) we can substitute dv_x for $(dv_x/dx)\,dx$. This changes the integration variable from x to v_x, so we change the limits from x_1 and x_2 to the corresponding x-velocities v_1 and v_2 at these points. This gives us

$$W_{tot} = \int_{v_1}^{v_2} mv_x\,dv_x$$

The integral of $v_x\,dv_x$ is just $v_x^2/2$. Substituting the upper and lower limits, we finally find

$$W_{tot} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \tag{6.13}$$

This is the same as Eq. (6.6), so the work–energy theorem is valid even without the assumption that the net force is constant.

Example 6.7 Motion with a varying force

An air-track glider of mass 0.100 kg is attached to the end of a horizontal air track by a spring with force constant 20.0 N/m (Fig. 6.22a). Initially the spring is unstretched and the glider is moving at 1.50 m/s to the right. Find the maximum distance d that the glider moves to the right (a) if the air track is turned on so that there is no friction, and (b) if the air is turned off so that there is kinetic friction with coefficient $\mu_k = 0.47$.

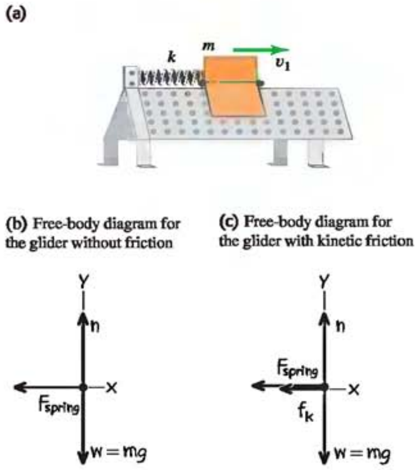
SOLUTION

IDENTIFY: The force exerted by the spring is not constant, so we *cannot* use the constant-acceleration formulas of Chapter 2 to solve this problem. Instead, we'll use the work–energy theorem, which involves the distance moved (our target variable) through the formula for work.

SET UP: In Figs. 6.22b and 6.22c we chose the positive x-direction to be to the right (in the direction of the glider's motion). We take $x = 0$ at the glider's initial position (where the spring is unstretched) and $x = d$ (the target variable) at the position where the glider stops. The motion is purely horizontal, so only the horizontal forces do work. Note that Eq. (6.10) gives the work done *on* the spring as it stretches, but to use the work–energy theorem we

Continued

6.22 (a) A glider attached to an air track by a spring. (b), (c) Our free-body diagrams.



need the work done *by* the spring *on* the glider—which is the negative of Eq. (6.10).

EXECUTE: (a) As the glider moves from $x_1 = 0$ to $x_2 = d$, it does an amount of work on the spring given by Eq. (6.10): $W = \frac{1}{2}kd^2 - \frac{1}{2}k(0)^2 = \frac{1}{2}kd^2$. The amount of work that the spring does on the glider is the negative of this value, or $-\frac{1}{2}kd^2$. The spring stretches until the glider comes instantaneously to rest, so the final kinetic energy K_2 is zero. The initial kinetic energy is $\frac{1}{2}mv_1^2$, where $v_1 = 1.50\ \text{m/s}$ is the glider's initial speed. Using the work–energy theorem, we find

$$-\frac{1}{2}kd^2 = 0 - \frac{1}{2}mv_1^2$$

We solve for the distance d the glider moves:

$$d = v_1\sqrt{\frac{m}{k}} = (1.50\ \text{m/s})\sqrt{\frac{0.100\ \text{kg}}{20.0\ \text{N/m}}}$$
$$= 0.106\ \text{m} = 10.6\ \text{cm}$$

The stretched spring subsequently pulls the glider back to the left, so the glider is at rest only instantaneously.

(b) If the air is turned off, we must also include the work done by the constant force of kinetic friction. The normal force n is equal in magnitude to the weight of the glider, since the track is horizontal and there are no other vertical forces. Hence the magnitude of the kinetic friction force is $f_k = \mu_k n = \mu_k mg$. The friction force is directed opposite to the displacement, so the work done by friction is

$$W_{fric} = f_k d\cos 180° = -f_k d = -\mu_k mgd$$

The total work is the sum of W_{fric} and the work done by the spring, $-\frac{1}{2}kd^2$. The work–energy theorem then says that

$$-\mu_k mgd - \frac{1}{2}kd^2 = 0 - \frac{1}{2}mv_1^2$$

$$-(0.47)(0.100\ \text{kg})(9.8\ \text{m/s}^2)d - \frac{1}{2}(20.0\ \text{N/m})d^2$$
$$= -\frac{1}{2}(0.100\ \text{kg})(1.50\ \text{m/s})^2$$
$$(10.0\ \text{N/m})d^2 + (0.461\ \text{N})d - (0.113\ \text{N}\cdot\text{m}) = 0$$

This is a quadratic equation for d. The solutions are

$$d = \frac{-(0.461\ \text{N}) \pm \sqrt{(0.461\ \text{N})^2 - 4(10.0\ \text{N/m})(-0.113\ \text{N}\cdot\text{m})}}{2(10.0\ \text{N/m})}$$
$$= 0.086\ \text{m} \quad \text{or} \quad -0.132\ \text{m}$$

We have used d as the symbol for a positive displacement, so only the positive value of d makes sense. Thus with friction the glider moves a distance

$$d = 0.086\ \text{m} = 8.6\ \text{cm}$$

EVALUATE: With friction present, the glider goes a shorter distance and the spring stretches less, as you might expect. Again the glider stops instantaneously, and again the spring force pulls the glider to the left; whether it moves or not depends on how great the *static* friction force is. How large would the coefficient of static friction μ_s have to be to keep the glider from springing back to the left?

Work–Energy Theorem for Motion Along a Curve

We can generalize our definition of work further to include a force that varies in direction as well as magnitude, and a displacement that lies along a curved path. Suppose a particle moves from point P_1 to P_2 along a curve, as shown in Fig. 6.23a. We divide the portion of the curve between these points into many infinitesimal vector displacements, and we call a typical one of these $d\vec{l}$. Each $d\vec{l}$ is tangent to the path at its position. Let $\vec{F}$ be the force at a typical point along the path, and let ϕ be the angle between $\vec{F}$ and $d\vec{l}$ at this point. Then the small element of work dW done on the particle during the displacement $d\vec{l}$ may be written as

$$dW = F\cos\phi\, dl = F_{\parallel}\, dl = \vec{F}\cdot d\vec{l}$$

where $F_{\parallel} = F\cos\phi$ is the component of $\vec{F}$ in the direction parallel to $d\vec{l}$ (Fig. 6.23b). The total work done by $\vec{F}$ on the particle as it moves from P_1 to P_2 is then

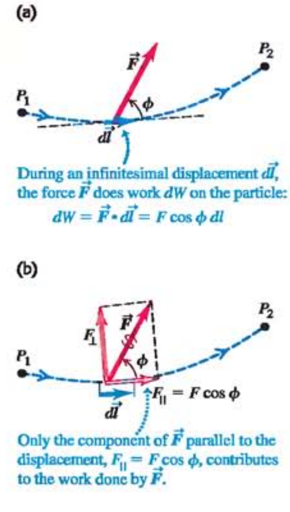
$$W = \int_{P_1}^{P_2} F\cos\phi\, dl = \int_{P_1}^{P_2} F_{\parallel}\, dl = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l} \quad \text{(work done on a curved path)} \qquad (6.14)$$

We can now show that the work–energy theorem, Eq. (6.6), holds true even with varying forces and a displacement along a curved path. The force $\vec{F}$ is essentially constant over any given infinitesimal segment $d\vec{l}$ of the path, so we can apply the work–energy theorem for straight-line motion to that segment. Thus the change in the particle's kinetic energy K over that segment equals the work $dW = F_{\parallel}\, dl = \vec{F}\cdot d\vec{l}$ done on the particle. Adding up these infinitesimal quantities of work from all the segments along the whole path gives the total work done, Eq. (6.14), which equals the total change in kinetic energy over the whole path. So $W_{tot} = \Delta K = K_2 - K_1$ is true *in general,* no matter what the path and no matter what the character of the forces. This can be proved more rigorously by using steps like those in Eqs. (6.11) through (6.13) (see Challenge Problem 6.104).

Note that only the component of the net force parallel to the path, $F_{\parallel}$, does work on the particle, so only this component can change the speed and kinetic energy of the particle. The component perpendicular to the path, $F_{\perp} = F\sin\phi$, has no effect on the particle's speed; it acts only to change the particle's direction.

The integral in Eq. (6.14) is called a *line integral.* To evaluate this integral in a specific problem, we need some sort of detailed description of the path and of the way in which $\vec{F}$ varies along the path. We usually express the line integral in terms of some scalar variable, as in the following example.

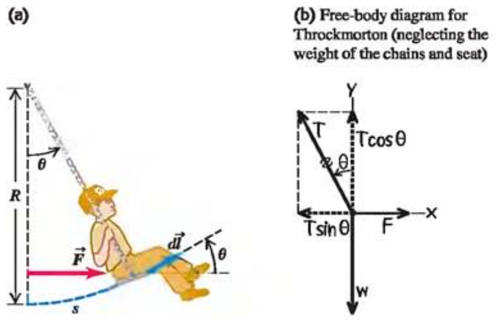
6.23 A particle moves along a curved path from point P_1 to P_2, acted on by a force $\vec{F}$ that varies in magnitude and direction.



Example 6.8 Motion on a curved path I

At a family picnic you are appointed to push your obnoxious cousin Throckmorton in a swing (Fig. 6.24a). His weight is w, the length of the chains is R, and you push Throcky until the chains make an angle θ_0 with the vertical. To do this, you exert a varying horizontal force $\vec{F}$ that starts at zero and gradually increases just enough so that Throcky and the swing move very slowly and remain very nearly in equilibrium. What is the total work done on Throcky by all forces? What is the work done by the tension T in the chains? What is the work you do by exerting the force $\vec{F}$? (Neglect the weight of the chains and seat.)

6.24 (a) Pushing cousin Throckmorton in a swing. (b) Our free-body diagram.



SOLUTION

IDENTIFY: The motion is along a curve, so we will use Eq. (6.14) to calculate the work done by the net force, by the tension force, and by the force $\vec{F}$.

SET UP: Figure 6.24b shows our free-body diagram and coordinate system. We have replaced the tensions in the two chains with a single tension T.

EXECUTE: There are two ways to find the total work done during the motion: (1) by calculating the work done by each force and then adding the quantities of work together, and (2) by calculating the work done by the net force. The second approach is far easier in this situation because Throcky is in equilibrium at every point. Hence the net force on him is zero, the integral of the net force in Eq. (6.14) is zero, and the total work done on him by all forces is zero.

It's also easy to find the work done by the chain tension on Throcky because this force is perpendicular to the direction of motion at all points along the path. Hence at all points the angle between the chain tension and the displacement vector $d\vec{l}$ is 90° and the scalar product in Eq. (6.14) is zero. Thus the chain tension does zero work.

Continued

To compute the work done by $\vec{F}$, we need to know how this force varies with the angle θ. The net force on Throcky is zero, so $\sum F_x = 0$ and $\sum F_y = 0$. From Fig. 6.24b, we get

$$\sum F_x = F + (-T\sin\theta) = 0$$
$$\sum F_y = T\cos\theta + (-w) = 0$$

By eliminating T from these two equations, we obtain

$$F = w\tan\theta$$

The point where $\vec{F}$ is applied swings through the arc s. The arc length s equals the radius R of the circular path multiplied by the length θ (in radians), so $s = R\theta$. Therefore the displacement $d\vec{l}$ corresponding to a small change of angle $d\theta$ has a magnitude $dl = ds = R\,d\theta$. The work done by $\vec{F}$ is

$$W = \int \vec{F}\cdot d\vec{l} = \int F\cos\theta\, ds$$

Now we express everything in terms of the angle θ, whose value increases from 0 to θ_0:

$$W = \int_0^{\theta_0} (w\tan\theta)\cos\theta\,(R\,d\theta) = wR\int_0^{\theta_0}\sin\theta\,d\theta$$
$$= wR(1 - \cos\theta_0)$$

EVALUATE: If $\theta_0 = 0$, there is no displacement; then $\cos\theta_0 = 1$ and $W = 0$, as we should expect. If $\theta_0 = 90°$, then $\cos\theta_0 = 0$ and $W = wR$. In that case the work you do is the same as if you had lifted Throcky straight up a distance R with a force equal to his weight w. In fact, the quantity $R(1 - \cos\theta_0)$ is the increase in his height above the ground during the displacement, so for any value of θ_0 the work done by the force $\vec{F}$ is the change in height multiplied by the weight. This is an example of a more general result that we'll prove in Section 7.1.

Example 6.9 Motion on a curved path II

In Example 6.8 the infinitesimal displacement $d\vec{l}$ (Fig. 6.24a) has a magnitude of ds, an x-component of $ds\cos\theta$, and a y-component of $ds\sin\theta$. Hence $d\vec{l} = \hat{\imath}\,ds\cos\theta + \hat{\jmath}\,ds\sin\theta$. Use this expression and Eq. (6.14) to calculate the work done during the motion by the chain tension, by the force of gravity, and by the force $\vec{F}$.

SOLUTION

IDENTIFY: We again use Eq. (6.14), but now we'll use Eq. (1.21) to find the scalar product in terms of components.

SET UP: We use the same free-body diagram, Fig. 6.24b, as in Example 6.8.

EXECUTE: From Fig. 6.24b, we can write the three forces in terms of unit vectors:

$$\vec{T} = \hat{\imath}(-T\sin\theta) + \hat{\jmath}T\cos\theta$$
$$\vec{w} = \hat{\jmath}(-w)$$
$$\vec{F} = \hat{\imath}F$$

To use Eq. (6.14), we must calculate the scalar product of each of these forces with $d\vec{l}$. Using Eq. (1.21),

$$\vec{T}\cdot d\vec{l} = (-T\sin\theta)(ds\cos\theta) + (T\cos\theta)(ds\sin\theta) = 0$$
$$\vec{w}\cdot d\vec{l} = (-w)(ds\sin\theta) = -w\sin\theta\, ds$$
$$\vec{F}\cdot d\vec{l} = F(ds\cos\theta) = F\cos\theta\, ds$$

Since $\vec{T}\cdot d\vec{l} = 0$, the integral of this quantity is zero and the work done by the chain tension is zero (just as we found in Example 6.8). Using $ds = R\,d\theta$ as in Example 6.8, we find the work done by the force of gravity is

$$\int \vec{w}\cdot d\vec{l} = \int (-w\sin\theta)R\,d\theta = -wR\int_0^{\theta_0}\sin\theta\,d\theta$$
$$= -wR(1 - \cos\theta_0)$$

The work done by gravity is negative because gravity pulls down while Throcky moves upward. Finally, the work done by the force $\vec{F}$ is the integral $\int\vec{F}\cdot d\vec{l} = \int F\cos\theta\,ds$, which we calculated in Example 6.8; the answer is $+wR(1 - \cos\theta_0)$.

EVALUATE: As a check on our answers, note that the sum of all three quantities of work is zero. This is just what we concluded in Example 6.8 using the work–energy theorem.

The method of components is often the most convenient way to calculate scalar products. Use it when it makes your life easier!

Test Your Understanding of Section 6.3 In Example 5.21 (Section 5.4) we examined a conical pendulum. The speed of the pendulum bob remains constant as it travels around the circle shown in Fig. 5.32a. (a) Over one complete circle, how much work does the tension force F do on the bob? (i) a positive amount; (ii) a negative amount; (iii) zero. (b) Over one complete circle, how much work does the weight do on the bob? (i) a positive amount; (ii) a negative amount; (iii) zero.

6.4 Power

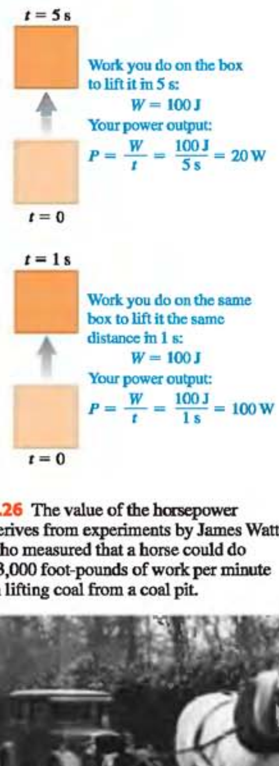
The definition of work makes no reference to the passage of time. If you lift a barbell weighing 100 N through a vertical distance of 1.0 m at constant velocity, you do $(100\text{ N})(1.0\text{ m}) = 100\text{ J}$ of work whether it takes you 1 second, 1 hour, or 1 year to do it. But often we need to know how quickly work is done. We describe this in terms of *power*. In ordinary conversation the word "power" is often synonymous with "energy" or "force." In physics we use a much more precise definition: **Power** is the time *rate* at which work is done. Like work and energy, power is a scalar quantity.

When a quantity of work ΔW is done during a time interval Δt, the average work done per unit time or **average power** P_{av} is defined to be

$$P_{av} = \frac{\Delta W}{\Delta t} \quad \text{(average power)} \tag{6.15}$$

The rate at which work is done might not be constant. We can define **instantaneous power** P as the quotient in Eq. (6.15) as Δt approaches zero:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \quad \text{(instantaneous power)} \tag{6.16}$$



6.25 The same amount of work is done in both of these situations, but the power (the rate at which work is done) is different.

The SI unit of power is the **watt** (W), named for the English inventor James Watt. One watt equals 1 joule per second: $1\text{ W} = 1\text{ J/s}$ (Fig. 6.25). The kilowatt $(1\text{ kW} = 10^3\text{ W})$ and the megawatt $(1\text{ MW} = 10^6\text{ W})$ are also commonly used. In the British system, work is expressed in foot-pounds, and the unit of power is the foot-pound per second. A larger unit called the *horsepower* (hp) is also used (Fig. 6.26):

$$1\text{ hp} = 550\text{ ft}\cdot\text{lb/s} = 33{,}000\text{ ft}\cdot\text{lb/min}$$

That is, a 1-hp motor running at full load does 33,000 ft · lb of work every minute. A useful conversion factor is

$$1\text{ hp} = 746\text{ W} = 0.746\text{ kW}$$

The watt is a familiar unit of *electrical* power; a 100-W light bulb converts 100 J of electrical energy into light and heat each second. But there's nothing inherently electrical about a watt. A light bulb could be rated in horsepower, and an engine can be rated in kilowatts.

The *kilowatt-hour* $(\text{kW}\cdot\text{h})$ is the usual commercial unit of electrical energy. One kilowatt-hour is the total work done in 1 hour (3600 s) when the power is 1 kilowatt (10^3 J/s), so

$$1\text{ kW}\cdot\text{h} = (10^3\text{ J/s})(3600\text{ s}) = 3.6 \times 10^6\text{ J} = 3.6\text{ MJ}$$

The kilowatt-hour is a unit of *work* or *energy*, not power.

6.26 The value of the horsepower derives from experiments by James Watt, who measured that a horse could do 33,000 foot-pounds of work per minute in lifting coal from a coal pit.

In mechanics we can also express power in terms of force and velocity. Suppose that a force $\vec{F}$ acts on a body while it undergoes a vector displacement $\Delta\vec{s}$. If $F_\parallel$ is the component of $\vec{F}$ tangent to the path (parallel to $\Delta\vec{s}$), then the work done by the force is $\Delta W = F_\parallel \Delta s$. The average power is

$$P_{av} = \frac{F_\parallel \Delta s}{\Delta t} = F_\parallel \frac{\Delta s}{\Delta t} = F_\parallel v_{av} \tag{6.17}$$

Instantaneous power P is the limit of this expression as $\Delta t \to 0$:

$$P = F_\parallel v \tag{6.18}$$

where v is the magnitude of the instantaneous velocity. We can also express Eq. (6.18) in terms of the scalar product:

$$P = \vec{F} \cdot \vec{v} \quad \text{(instantaneous rate at which force } \vec{F} \text{ does work on a particle)} \tag{6.19}$$

Example 6.10 Force and power

Each of the two jet engines in a Boeing 767 airliner develops a thrust (a forward force on the airplane) of 197,000 N (44,300 lb). When the airplane is flying at 250 m/s (900 km/h, or roughly 560 mi/h), what horsepower does each engine develop?

SOLUTION

IDENTIFY: Our target variable is the instantaneous power P, which is the rate at which the thrust does work.

SET UP: We use Eq. (6.18). The thrust is in the direction of motion, so $F_\parallel$ is just equal to the thrust.

EXECUTE: At $v = 250\ \text{m/s}$, the power developed by each engine is

$$P = F_\parallel v = (1.97 \times 10^5\ \text{N})(250\ \text{m/s}) = 4.93 \times 10^7\ \text{W}$$
$$= (4.93 \times 10^7\ \text{W})\frac{1\ \text{hp}}{746\ \text{W}} = 66{,}000\ \text{hp}$$

EVALUATE: The speed of modern airliners is directly related to the power of their engines (Fig. 6.27). The largest propeller-driven airliners of the 1950s had engines that developed about 3400 hp $(2.5 \times 10^6\ \text{W})$, giving them maximum speeds of about 600 km/h (370 mi/h). Each engine in a Boeing 767 develops nearly 20 times more power, enabling it to fly at about 900 km/h (560 mi/h) and to carry a much heavier load.

If the engines are at maximum thrust while the airliner is at rest on the ground so that $v = 0$, the engines develop *zero* power. Force and power are not the same thing!

6.27 (a) Propeller-driven and (b) jet airliners.

(a)

(b)

Example 6.11 A "power climb"

A 50.0-kg marathon runner runs up the stairs to the top of Chicago's 443-m-tall Sears Tower, the tallest building in the United States (Fig. 6.28). To lift herself to the top in 15.0 minutes, what must be her average power output in watts? In kilowatts? In horsepower?

SOLUTION

IDENTIFY: We'll treat the runner as a particle of mass m. Her average power output P_{av} must be enough to lift her at constant speed against gravity.

SET UP: We can find P_{av} in two ways: (1) by first determining how much work she must do and then dividing it by the elapsed time, as in Eq. (6.15), or (2) by calculating the average upward force she must exert (in the direction of the climb) and then multiplying it by her upward velocity, as in Eq. (6.17).

EXECUTE: As in Example 6.8, lifting a mass m against gravity requires an amount of work equal to the weight mg multiplied by the height h it is lifted. Hence the work she must do is

$$W = mgh = (50.0\ \text{kg})(9.80\ \text{m/s}^2)(443\ \text{m})$$
$$= 2.17 \times 10^5\ \text{J}$$

6.28 How much power is required to run up the stairs of Chicago's Sears Tower in 15 minutes?

The time is 15.0 min = 900 s, so from Eq. (6.15) the average power is

$$P_{av} = \frac{2.17 \times 10^5\,\text{J}}{900\,\text{s}} = 241\ \text{W} = 0.241\ \text{kW} = 0.323\ \text{hp}$$

Let's try the calculation again using Eq. (6.17). The force exerted is vertical, and the average vertical component of velocity is $(443\ \text{m})/(900\ \text{s}) = 0.492\ \text{m/s}$, so the average power is

$$P_{av} = F_{\parallel}v_{av} = (mg)v_{av}$$
$$= (50.0\ \text{kg})(9.80\ \text{m/s}^2)(0.492\ \text{m/s}) = 241\ \text{W}$$

which is the same result as before.

EVALUATE: The runner's *total* power output will be several times greater than 241 W. The reason is that the runner isn't really a particle but a collection of parts that exert forces on each other and do work, such as the work done to inhale and exhale and to make her arms and legs swing. What we've calculated is only the part of her power output that lifts her to the top of the building.

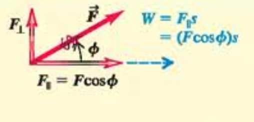
Test Your Understanding of Section 6.4 The air surrounding an airplane in flight exerts a drag force that acts opposite to the airplane's motion. When the Boeing 767 in Example 6.10 is flying in a straight line at a constant altitude at a constant 250 m/s, what is the rate at which the drag force does work on it? (i) 132,000 hp; (ii) 66,000 hp; (iii) 0; (iv) −66,000 hp; (v) −132,000 hp.

CHAPTER 6 SUMMARY

Work done by a force: When a constant force $\vec{F}$ acts on a particle that undergoes a straight-line displacement $\vec{s}$, the work done by the force on the particle is defined to be the scalar product of $\vec{F}$ and $\vec{s}$. The unit of work in SI units is 1 joule = 1 newton-meter $(1\ \text{J} = 1\ \text{N}\cdot\text{m})$. Work is a scalar quantity; it can be positive or negative, but it has no direction in space. (See Examples 6.1 and 6.2.)

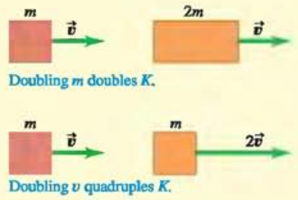
$$W = \vec{F}\cdot\vec{s} = Fs\cos\phi \qquad (6.2), (6.3)$$

ϕ = angle between $\vec{F}$ and $\vec{s}$



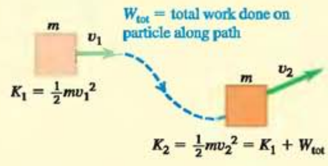
Kinetic energy: The kinetic energy K of a particle equals the amount of work required to accelerate the particle from rest to speed v. It is also equal to the amount of work the particle can do in the process of being brought to rest. Kinetic energy is a scalar that has no direction in space; it is always positive or zero. Its units are the same as the units of work: $1\ \text{J} = 1\ \text{N}\cdot\text{m} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$.

$$K = \frac{1}{2}mv^2 \qquad (6.5)$$



The work–energy theorem: When forces act on a particle while it undergoes a displacement, the particle's kinetic energy changes by an amount equal to the total work done on the particle by all the forces. This relationship, called the work–energy theorem, is valid whether the forces are constant or varying and whether the particle moves along a straight or curved path. It is applicable only to bodies that can be treated as a particle. (See Examples 6.3–6.5)

$$W_{tot} = K_2 - K_1 = \Delta K. \qquad (6.6)$$



Work done by a varying force or on a curved path: When a force varies during a straight-line displacement, the work done by the force is given by an integral, Eq. (6.7). (See Examples 6.6 and 6.7.) When a particle follows a curved path, the work done on it by a force $\vec{F}$ is given by an integral that involves the angle ϕ between the force and the displacement. This expression is valid even if the force magnitude and the angle ϕ vary during the displacement. (See Examples 6.8 and 6.9.)

$$W = \int_{x_1}^{x_2} F_x\,dx \qquad (6.7)$$

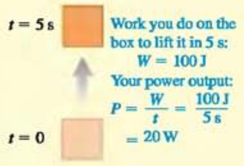
$$W = \int_{P_1}^{P_2} F\cos\phi\,dl = \int_{P_1}^{P_2} F_\parallel\,dl = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l} \qquad (6.14)$$

Power: Power is the time rate of doing work. The average power P_{av} is the amount of work ΔW done in time Δt divided by that time. The instantaneous power is the limit of the average power as Δt goes to zero. When a force $\vec{F}$ acts on a particle moving with velocity $\vec{v}$, the instantaneous power (the rate at which the force does work) is the scalar product of $\vec{F}$ and $\vec{v}$. Like work and kinetic energy, power is a scalar quantity. The SI unit of power is 1 watt = 1 joule/second $(1\ \text{W} = 1\ \text{J/s})$. (See Examples 6.10 and 6.11.)

$$P_{av} = \frac{\Delta W}{\Delta t} \qquad (6.15)$$

$$P = \lim_{\Delta t \to 0}\frac{\Delta W}{\Delta t} = \frac{dW}{dt} \qquad (6.16)$$

$$P = \vec{F}\cdot\vec{v} \qquad (6.19)$$



Key Terms

work, *182*
joule, *182*
kinetic energy, *187*
work–energy theorem, *187*
force constant, *193*
Hooke's law, *193*
power, *199*
average power, *199*
instantaneous power, *199*
watt, *199*

Answer to Chapter Opening Question

It is indeed true that the shell does work on the gases. However, because the shell exerts a backward force on the gases as the gases and shell move forward through the barrel, the work done by the shell is *negative* (see Section 6.1).

Answers to Test Your Understanding Questions

6.1 Answer: (iii) The electron has constant velocity, so its acceleration is zero and (by Newton's second law) the net force on the electron is also zero. Therefore the total work done by all the forces (equal to the work done by the net force) must be zero as well. The individual forces may do nonzero work, but that's not what the question asks.

6.2 Answer: (iv), (i), (iii), (ii) Body (i) has kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2}(2.0\,\text{kg})(5.0\,\text{m/s})^2 = 25\,\text{J}$. Body (ii) had zero kinetic energy initially and then had 30 J of work done it, so its final kinetic energy is $K_2 = K_1 + W = 0 + 30\,\text{J} = 30\,\text{J}$. Body (iii) had initial kinetic energy $K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(1.0\,\text{kg})(4.0\,\text{m/s})^2 = 8.0\,\text{J}$ and then had 20 J of work done on it, so its final kinetic energy is $K_2 = K_1 + W = 8.0\,\text{J} + 20\,\text{J} = 28\,\text{J}$. Body (iv) had initial kinetic energy $K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(2.0\,\text{kg})(10\,\text{m/s})^2 = 100\,\text{J}$; when it did 80 J of work on another body, the other body did -80 J of work on body (iv), so the final kinetic energy of body (iv) is $K_2 = K_1 + W = 100\,\text{J} + (-80\,\text{J}) = 20\,\text{J}$.

6.3 Answers:(a) (iii), (b) (iii) At any point during the pendulum bob's motion, the tension force and the weight both act perpendicular to the motion—that is, perpendicular to an infinitesimal displacement $d\vec{l}$ of the bob. (In Fig. 5.32b, the displacement $d\vec{l}$ would be directed outward from the plane of the free-body diagram.) Hence for either force the scalar product inside the integral in Eq. (6.14) is $\vec{F} \cdot d\vec{l} = 0$, and the work done along any part of the circular path (including a complete circle) is $W = \int \vec{F} \cdot d\vec{l} = 0$.

6.4 Answer: (v) The airliner has a constant horizontal velocity, so the net horizontal force on it must be zero. Hence the backward drag force must have the same magnitude as the forward force due to the combined thrust of the two engines. This means that the drag force must do *negative* work on the airplane at the same rate that the combined thrust force does *positive* work. The combined thrust does work at a rate of $2(66{,}000\,\text{hp}) = 132{,}000\,\text{hp}$, so the drag force must do work at a rate of $-132{,}000$ hp.

PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com

Discussion Questions

Q6.1. The sign of many physical quantities depends on the choice of coordinates. For example, g can be negative or positive, depending on whether we choose upward or downward as positive. Is the same thing true of work? In other words, can we make positive work negative by a different choice of coordinates? Explain.

Q6.2. An elevator is hoisted by its cables at constant speed. Is the total work done on the elevator positive, negative, or zero? Explain.

Q6.3. A rope tied to a body is pulled, causing the body to accelerate. But according to Newton's third law, the body pulls back on the rope with an equal and opposite force. Is the total work done then zero? If so, how can the body's kinetic energy change? Explain.

Q6.4. If it takes total work W to give an object a speed v and kinetic energy K, starting from rest, what will be the object's speed (in terms of v) and kinetic energy (in terms of K) if we do twice as much work on it, again starting from rest?

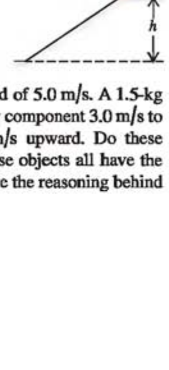
Q6.5. If there is a net nonzero force on a moving object, is it possible for the total work done on the object to be zero? Explain, with an example that illustrates your answer.

Q6.6. In Example 5.5 (Section 5.1), how does the work done on the bucket by the tension in the cable compare to the work done on the cart by the tension in the cable?

Q6.7. In the conical pendulum in Example 5.21 (Section 5.4), which of the forces do work on the bob while it is swinging?

Q6.8. For the cases shown in Fig. 6.29, the object is released from rest at the top and feels no friction or air resistance. In which (if any) cases will the mass have (i) the greatest speed at the bottom and (ii) the most work done on it by the time it reaches the bottom?

Figure 6.29 Question Q6.8.



Q6.9. A force $\vec{F}$ is in the x-direction and has a magnitude that depends on x. Sketch a possible graph of F versus x such that the force does zero work on an object that moves from x_1 to x_2, even though the force magnitude is not zero at all x in this range.

Q6.10. Does the kinetic energy of a car change more when it speeds up from 10 to 15 m/s or from 15 to 20 m/s? Explain.

Q6.11. A falling brick has a mass of 1.5 kg and is moving straight downward with a speed of 5.0 m/s. A 1.5-kg physics book is sliding across the floor with a speed of 5.0 m/s. A 1.5-kg melon is traveling with a horizontal velocity component 3.0 m/s to the right and a vertical component 4.0 m/s upward. Do these objects all have the same velocity? Do these objects all have the same kinetic energy? For each question, give the reasoning behind your answer.

Q6.12. Can the *total* work done on an object during a displacement be negative? Explain. If the total work is negative, can its magnitude be larger than the initial kinetic energy of the object? Explain.

Q6.13. A net force acts on an object and accelerates it from rest to a speed v_1. In doing so, the force does an amount of work W_1. By what factor must the work done on the object be increased to produce three times the final speed, with the object again starting from rest?

Q6.14. A truck speeding down the highway has a lot of kinetic energy relative to a stopped state trooper, but no kinetic energy relative to the truck driver. In these two frames of reference, is the same amount of work required to stop the truck? Explain.

Q6.15. You are holding a briefcase by the handle, with your arm straight down by your side. Does the force your hand exerts do work on the briefcase when (a) you walk at a constant speed down a horizontal hallway and (b) you ride an escalator from the first to second floor of a building? In each case justify your answer.

Q6.16. When a book slides along a tabletop, the force of friction does negative work on it. Can friction ever do *positive* work? Explain. (*Hint:* Think of a box in the back of an accelerating truck.)

Q6.17. Time yourself while running up a flight of steps, and compute the average rate at which you do work against the force of gravity. Express your answer in watts and in horsepower.

Q6.10. Fractured Physics. Many terms from physics are badly misused in everyday language. In each case, explain the errors involved. (a) A *strong* person is called *powerful.* What is wrong with this use of *power*? (b) When a worker carries a bag of concrete along a level construction site, people say he did a lot of *work.* Did he?

Q6.19. An advertisement for a portable electrical generating unit claims that the unit's diesel engine produces 28,000 hp to drive an electrical generator that produces 30 MW of electrical power. Is this possible? Explain.

Q6.20. A car speeds up while the engine delivers constant power. Is the acceleration greater at the beginning of this process or at the end? Explain.

Q6.21. Consider a graph of instantaneous power versus time, with the vertical P axis starting at $P = 0$. What is the physical significance of the area under the P versus t curve between vertical lines at t_1 and t_2? How could you find the average power from the graph? Draw a P versus t curve that consists of two straight-line sections and for which the peak power is equal to twice the average power.

Q6.22. A nonzero net force acts on an object. Is it possible for any of the following quantities to be constant: (a) the particle's speed; (b) the particle's velocity; (c) the particle's kinetic energy.

Q6.23. When a certain force is applied to an ideal spring, the spring stretches a distance x from its unstretched length and does work W. If instead twice the force is applied, what distance (in terms of x) does the spring stretch from its unstretched length, and how much work (in terms of W) is required to stretch it this distance?

Q6.24. If work W is required to stretch a spring a distance x from its unstretched length, what work (in terms of W) is required to stretch the spring an *additional* distance x?

Exercises

Section 6.1 Work

6.1. An old oaken bucket of mass 6.75 kg hangs in a well at the end of a rope. The rope passes over a frictionless pulley at the top of the well, and you pull horizontally on the end of the rope to raise the bucket slowly a distance of 4.00 m. (a) How much work do you do on the bucket in pulling it up? (b) How much work does gravity do on the bucket? (c) What is the total work done on the bucket?

6.2. A tow truck pulls a car 5.00 km along a horizontal roadway using a cable having a tension of 850 N. (a) How much work does the cable do on the car if it pulls horizontally? If it pulls at 35.0° above the horizontal? (b) How much work does the cable do on the tow truck in both cases of part (a)? (c) How much work does gravity do on the car in part (a)?

6.3. A factory worker pushes a 30.0-kg crate a distance of 4.5 m along a level floor at constant velocity by pushing horizontally on it. The coefficient of kinetic friction between the crate and the floor is 0.25. (a) What magnitude of force must the worker apply? (b) How much work is done on the crate by this force? (c) How much work is done on the crate by friction? (d) How much work is done on the crate by the normal force? By gravity? (e) What is the total work done on the crate?

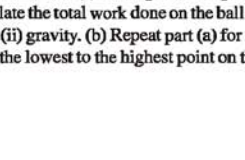
6.4. Suppose the worker in Exercise 6.3 pushes downward at an angle of 30° below the horizontal. (a) What magnitude of force must the worker apply to move the crate at constant velocity? (b) How much work is done on the crate by this force when the crate is pushed a distance of 4.5 m? (c) How much work is done on the crate by friction during this displacement? (d) How much work is done on the crate by the normal force? By gravity? (e) What is the total work done on the crate?

6.5. A 75.0-kg painter climbs a ladder that is 2.75 m long leaning against a vertical wall. The ladder makes an 30.0° angle with the wall. (a) How much work does gravity do on the painter? (b) Does the answer to part (a) depend on whether the painter climbs at constant speed or accelerates up the ladder?

6.6. Two tugboats pull a disabled supertanker. Each tug exerts a constant force of 1.80×10^6 N, one 14° west of north and the other 14° east of north, as they pull the tanker 0.75 km toward the north. What is the total work they do on the supertanker?

6.7. Two blocks are connected by a very light string passing over a massless and frictionless pulley (Figure 6.30). Traveling at constant speed, the 20.0-N block moves 75.0 cm to the right and the 12.0-N block moves 75.0 cm downward. During this process, how much work is done (a) on the 12.0-N block by (i) gravity and (ii) the tension in the string? (b) On the 20.0-N block by (i) gravity, (ii) the tension in the string, (iii) friction, and (iv) the normal force? (c) Find the total work done on each block.

Figure 6.30 Exercise 6.7.



6.6. A loaded grocery cart is rolling across a parking lot in a strong wind. You apply a constant force $\vec{F} = (30\,\text{N})\hat{\imath} - (40\,\text{N})\hat{\jmath}$ to the cart as it undergoes a displacement $\vec{s} = (-9.0\,\text{m})\hat{\imath} - (3.0\,\text{m})\hat{\jmath}$. How much work does the force you apply do on the grocery cart?

6.9. A 0.800-kg ball is tied to the end of a string 1.60 m long and swung in a vertical circle. (a) During one complete circle, starting anywhere, calculate the total work done on the ball by (i) the tension in the string and (ii) gravity. (b) Repeat part (a) for motion along the semicircle from the lowest to the highest point on the path.

Section 6.2 Kinetic Energy and the Work–Energy Theorem

6.10. (a) How many joules of kinetic energy does a 750-kg automobile traveling at a typical highway speed of 65 mi/h have? (b) By what factor would its kinetic energy decrease if the car traveled half as fast? (c) How fast (in mi/h) would the car have to travel to have half as much kinetic energy as in part (a)?

6.11. Meteor Crater. About 50,000 years ago, a meteor crashed into the earth near present-day Flagstaff, Arizona. Recent (2005) measurements estimate that this meteor had a mass of about 1.4×10^8 kg (around 150,000 tons) and hit the ground at 12 km/s. (a) How much kinetic energy did this meteor deliver to the ground? (b) How does this energy compare to the energy released by a 1.0-megaton nuclear bomb? (A megaton bomb releases the same energy as a million tons of TNT, and 1.0 ton of TNT releases 4.184×10^9 J of energy.)

6.12. Some Typical Kinetic Energies. (a) How many joules of kinetic energy does a 75-kg person have when walking and when running? (b) In the Bohr model of the atom, the ground-state electron in hydrogen has an orbital speed of 2190 km/s. What is its kinetic energy? (Consult Appendix F.) (c) If you drop a 1.0-kg weight (about 2 lb) from shoulder height, how many joules of kinetic energy will it have when it reaches the ground? (d) Is it reasonable that a 30-kg child could run fast enough to have 100 J of kinetic energy?

6.13. The mass of a proton is 1836 times the mass of an electron. (a) A proton is traveling at speed V. At what speed (in terms of V) would an electron have the same kinetic energy as the proton? (b) An electron has kinetic energy K. If a proton has the same speed as the electron, what is its kinetic energy (in terms of K)?

6.14. A 4.80-kg watermelon is dropped from rest from the roof of a 25.0-m-tall building and feels no appreciable air resistance. (a) Calculate the work done by gravity on the watermelon during its displacement from the roof to the ground. (b) Just before it strikes the ground, what is the watermelon's (i) kinetic energy and (ii) speed? (c) Which of the answers in parts (a) and (b) would be *different* if there were appreciable air resistance?

6.15. Use the work–energy theorem to solve each of these problems. You can use Newton's laws to check your answers. Neglect air resistance in all cases. (a) A branch falls from the top of a 95.0-m-tall redwood tree, starting from rest. How fast is it moving when it reaches the ground? (b) A volcano ejects a boulder directly upward 525 m into the air. How fast was the boulder moving just as it left the volcano? (c) A skier moving at 5.00 m/s encounters a long, rough horizontal patch of snow having coefficient of kinetic friction 0.220 with her skis. How far does she travel on this patch before stopping? (d) Suppose the rough patch in part (c) was only 2.90 m long? How fast would the skier be moving when she reached the end of the patch? (e) At the base of a frictionless icy hill that rises at 25.0° above the horizontal, a toboggan has a speed of 12.0 m/s toward the hill. How high vertically above the base will it go before stopping?

6.16. You throw a 20-N rock vertically into the air from ground level. You observe that when it is 15.0 m above the ground, it is traveling at 25.0 m/s upward. Use the work–energy theorem to find (a) the rock's speed just as it left the ground and (b) its maximum height.

6.17. You are a member of an Alpine Rescue Team. You must project a box of supplies up an incline of constant slope angle α so that it reaches a stranded skier who is a vertical distance h above the bottom of the incline. The incline is slippery, but there is some friction present, with kinetic friction coefficient μ_k. Use the work–energy theorem to calculate the minimum speed you must give the box at the bottom of the incline so that it will reach the skier. Express your answer in terms of g, h, μ_k, and α.

6.18. A mass m slides down a smooth inclined plane from an initial vertical height h, making an angle α with the horizontal. (a) The work done by a force is the sum of the work done by the components of the force. Consider the components of gravity parallel and perpendicular to the surface of the plane. Calculate the work done on the mass by each of the components, and use these results to show that the work done by gravity is exactly the same as if the mass had fallen straight down through the air from a height h. (b) Use the work–energy theorem to prove that the speed of the mass at the bottom of the incline is the same as if it had been dropped from height h, independent of the angle α of the incline. Explain how this speed can be independent of the slope angle. (c) Use the results of part (b) to find the speed of a rock that slides down an icy frictionless hill, starting from rest 15.0 m above the bottom.

6.19. A car is stopped in a distance D by a constant friction force that is independent of the car's speed. What is the stopping distance (in terms of D) (a) if the car's initial speed is tripled, and (b) if the speed is the same as it originally was but the friction force is tripled? (Solve using the work–energy theorem.)

6.20. A moving electron has kinetic energy K_1. After a net amount of work W has been done on it, the electron is moving one-quarter as fast in the opposite direction. (a) Find W in terms of K_1. (b) Does your answer depend on the final direction of the electron's motion?

6.21. A sled with mass 8.00 kg moves in a straight line on a frictionless horizontal surface. At one point in its path, its speed is 4.00 m/s; after it has traveled 2.50 m beyond this point, its speed is 6.00 m/s. Use the work–energy theorem to find the force acting on the sled, assuming that this force is constant and that it acts in the direction of the sled's motion.

6.22. A soccer ball with mass 0.420 kg is initially moving with speed 2.00 m/s. A soccer player kicks the ball, exerting a constant force of magnitude 40.0 N in the same direction as the ball's motion. Over what distance must the player's foot be in contact with the ball to increase the ball's speed to 6.00 m/s?

6.23. A 12-pack of Omni-Cola (mass 4.30 kg) is initially at rest on a horizontal floor. It is then pushed in a straight line for 1.20 m by a trained dog that exerts a horizontal force with magnitude 36.0 N. Use the work–energy theorem to find the final speed of the 12-pack if (a) there is no friction between the 12-pack and the floor, and (b) the coefficient of kinetic friction between the 12-pack and the floor is 0.30.

6.24. A batter hits a baseball with mass 0.145 kg straight upward with an initial speed of 25.0 m/s. (a) How much work has gravity done on the baseball when it reaches a height of 20.0 m above the bat? (b) Use the work–energy theorem to calculate the speed of the baseball at a height of 20.0 m above the bat. You can ignore air resistance. (c) Does the answer to part (b) depend on whether the baseball is moving upward or downward at a height of 20.0 m? Explain.

6.25. A little red wagon with mass 7.00 kg moves in a straight line on a frictionless horizontal surface. It has an initial speed of 4.00 m/s and then is pushed 3.0 m in the direction of the initial velocity by a force with a magnitude of 10.0 N. (a) Use the work–energy theorem to calculate the wagon's final speed. (b) Calculate the acceleration produced by the force. Use this acceleration in the kinematic relationships of Chapter 2 to calculate the wagon's final speed. Compare this result to that calculated in part (a).

6.26. A block of ice with mass 2.00 kg slides 0.750 m down an inclined plane that slopes downward at an angle of 36.9° below the horizontal. If the block of ice starts from rest, what is its final speed? You can ignore friction.

6.27. Stopping Distance. A car is traveling on a level road with speed v_0 at the instant when the brakes lock, so that the tires slide rather than roll. (a) Use the work–energy theorem to calculate the minimum stopping distance of the car in terms of v_0, g, and the coefficient of kinetic friction μ_k between the tires and the road. (b) By what factor would the minimum stopping distance change if (i) the coefficient of kinetic friction were doubled, or (ii) the initial speed were doubled, or (iii) both the coefficient of kinetic friction and the initial speed were doubled?

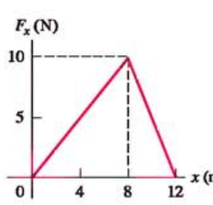
Section 6.3 Work and Energy with Varying Forces

6.26. To stretch a spring 3.00 cm from its unstretched length, 12.0 J of work must be done. (a) What is the force constant of this spring? (b) What magnitude force is needed to stretch the spring 3.00 cm from its unstretched length? (c) How much work must be done to compress this spring 4.00 cm from its unstretched length, and what force is needed to stretch it this distance?

6.29. A force of 160 N stretches a spring 0.050 m beyond its unstretched length. (a) What magnitude of force is required to stretch the spring 0.015 m beyond its unstretched length? To compress the spring 0.020 m? (b) How much work must be done to stretch the spring 0.015 m beyond its unstretched length? To compress the spring 0.020 m from its unstretched length?

6.30. A child applies a force $\vec{F}$ parallel to the x-axis to a 10.0-kg sled moving on the frozen surface of a small pond. As the child controls the speed of the sled, the x-component of the force she applies varies with the x-coordinate of the sled as shown in Fig. 6.31. Calculate the work done by the force $\vec{F}$ when the sled moves (a) from $x = 0$ to $x = 8.0$ m; (b) from $x = 8.0$ m to $x = 12.0$ m; (c) from $x = 0$ to 12.0 m.

Figure 6.31 Exercises 6.30 and 6.31.



6.31. Suppose the sled in Exercise 6.30 is initially at rest at $x = 0$. Use the work–energy theorem to find the speed of the sled at (a) $x = 8.0$ m and (b) $x = 12.0$ m. You can ignore friction between the sled and the surface of the pond.

6.32. A balky cow is leaving the barn as you try harder and harder to push her back in. In coordinates with the origin at the barn door, the cow walks from $x = 0$ to $x = 6.9$ m as you apply a force with x-component $F_x = -[20.0\,\text{N} + (3.0\,\text{N/m})x]$. How much work does the force you apply do on the cow during this displacement?

6.33. A 6.0-kg box moving at 3.0 m/s on a horizontal, frictionless surface runs into a light spring of force constant 75 N/cm. Use the work–energy theorem to find the maximum compression of the spring.

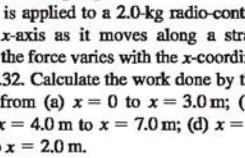
6.34. Leg Presses. As part of your daily workout, you lie on your back and push with your feet against a platform attached to two stiff springs arranged side by side so that they are parallel to each other. When you push the platform, you compress the springs. You do 80.0 J of work when you compress the springs 0.200 m from their uncompressed length. (a) What magnitude of force must you apply to hold the platform in this position? (b) How much *additional* work must you do to move the platform 0.200 m *farther*, and what maximum force must you apply?

6.35. (a) In Example 6.7 (Section 6.3) it was calculated that with the air track turned off, the glider travels 8.6 cm before it stops instantaneously. How large would the coefficient of static friction μ_s have to be to keep the glider from springing back to the left? (b) If the coefficient of static friction between the glider and the track is $\mu_s = 0.60$, what is the maximum initial speed v_1 that the glider can be given and still remain at rest after it stops instantaneously? With the air track turned off, the coefficient of kinetic friction is $\mu_k = 0.47$.

6.36. A 4.00-kg block of ice is placed against a horizontal spring that has force constant $k = 200$ N/m and is compressed 0.025 m. The spring is released and accelerates the block along a horizontal surface. You can ignore friction and the mass of the spring. (a) Calculate the work done on the block by the spring during the motion of the block from its initial position to where the spring has returned to its uncompressed length. (b) What is the speed of the block after it leaves the spring?

6.37. A force $\vec{F}$ is applied to a 2.0-kg radio-controlled model car parallel to the x-axis as it moves along a straight track. The x-component of the force varies with the x-coordinate of the car as shown in Fig. 6.32. Calculate the work done by the force $\vec{F}$ when the car moves from (a) $x = 0$ to $x = 3.0$ m; (b) $x = 3.0$ m to $x = 4.0$ m; (c) $x = 4.0$ m to $x = 7.0$ m; (d) $x = 0$ to $x = 7.0$ m; (e) $x = 7.0$ m to $x = 2.0$ m.

Figure 6.32 Exercises 6.37 and 6.38.



6.30. Suppose the 2.0-kg model car in Exercise 6.37 is initially at rest at $x = 0$ and $\vec{F}$ is the net force acting on it. Use the work–energy theorem to find the speed of the car at (a) $x = 3.0$ m; (b) $x = 4.0$ m; (c) $x = 7.0$ m.

6.39. At a waterpark, sleds with riders are sent along a slippery, horizontal surface by the release of a large compressed spring. The spring with force constant $k = 40.0$ N/cm and negligible mass rests on the frictionless horizontal surface. One end is in contact with a stationary wall. A sled and rider with total mass 70.0 kg are pushed against the other end, compressing the spring 0.375 m. The sled is then released with zero initial velocity. What is the sled's speed when the spring (a) returns to its uncompressed length and (b) is still compressed 0.200 m?

6.40. Half of a Spring. (a) Suppose you cut a massless ideal spring in half. If the full spring had a force constant k, what is the force constant of each half, in terms of k? (*Hint:* Think of the original spring as two equal halves, each producing the same force as the entire spring. Do you see why the forces must be equal?) (b) If you cut the spring into three equal segments instead, what is the force constant of each one, in terms of k?

6.41. A small glider is placed against a compressed spring at the bottom of an air track that slopes upward at an angle of 40.0° above the horizontal. The glider has mass 0.0900 kg. The spring has $k = 640$ N/m and negligible mass. When the spring is released, the glider travels a maximum distance of 1.80 m along the air track before sliding back down. Before reaching this maximum distance, the glider loses contact with the spring. (a) What distance was the spring originally compressed? (b) When the glider has traveled along the air track 0.80 m from its initial position against the compressed spring, is it still in contact with the spring? What is the kinetic energy of the glider at this point?

6.42. An ingenious bricklayer builds a device for shooting bricks up to the top of the wall where he is working. He places a brick on a vertical compressed spring with force constant $k = 450\ \text{N/m}$ and negligible mass. When the spring is released, the brick is propelled upward. If the brick has mass 1.80 kg and is to reach a maximum height of 3.6 m above its initial position on the compressed spring, what distance must the bricklayer compress the spring initially? (The brick loses contact with the spring when the spring returns to its uncompressed length. Why?)

Section 6.4 Power

6.43. How many joules of energy does a 100-watt light bulb use per hour? How fast would a 70-kg person have to run to have that amount of kinetic energy?

6.44. The total consumption of electrical energy in the United States is about 1.0×10^{19} J per year. (a) What is the average rate of electrical energy consumption in watts? (b) The population of the United States is about 300 million people. What is the average rate of electrical energy consumption per person? (c) The sun transfers energy to the earth by radiation at a rate of approximately 1.0 kW per square meter of surface. If this energy could be collected and converted to electrical energy with 40% efficiency, how great an area (in square kilometers) would be required to collect the electrical energy used in the United States?

6.45. Magnetar. On December 27, 2004, astronomers observed the greatest flash of light ever recorded from outside the solar system. It came from the highly magnetic neutron star SGR 1806-20 (a *magnetar*). During 0.20 s, this star released as much energy as our sun does in 250,000 years. If P is the average power output of our sun, what was the average power output (in terms of P) of this magnetar?

6.46. A 20.0-kg rock is sliding on a rough, horizontal surface at 8.00 m/s and eventually stops due to friction. The coefficient of kinetic friction between the rock and the surface is 0.200. What average power is produced by friction as the rock stops?

6.47. A tandem (two-person) bicycle team must overcome a force of 165 N to maintain a speed of 9.00 m/s. Find the power required per rider, assuming that each contributes equally. Express your answer in watts and in horsepower.

6.46. When its 75-kW (100-hp) engine is generating full power, a small single-engine airplane with mass 700 kg gains altitude at a rate of 2.5 m/s (150 m/min, or 500 ft/min). What fraction of the engine power is being used to make the airplane climb? (The remainder is used to overcome the effects of air resistance and of inefficiencies in the propeller and engine.)

6.49. Working Like a Horse. Your job is to lift 30-kg crates a vertical distance of 0.90 m from the ground onto the bed of a truck. (a) How many crates would you have to load onto the truck in 1 minute for the average power output you use to lift the crates to equal 0.50 hp? (b) How many crates for an average power output of 100 W?

6.58. An elevator has mass 600 kg, not including passengers. The elevator is designed to ascend, at constant speed, a vertical distance of 20.0 m (five floors) in 16.0 s, and it is driven by a motor that can provide up to 40 hp to the elevator. What is the maximum number of passengers that can ride in the elevator? Assume that an average passenger has mass 65.0 kg.

6.51. Automotive Power. It is not unusual for a 1000-kg car to get 30 mi/gal when traveling at 60 mi/h on a level road. If this car makes a 200-km trip, (a) how many joules of energy does it consume, and (b) what is the average rate of energy consumption during the trip? Note that 1.0 gal of gasoline yields 1.3×10^{9} J (although this can vary). Consult Appendix E.

6.52. The aircraft carrier *John F. Kennedy* has mass 7.4×10^{7} kg. When its engines are developing their full power of 280,000 hp, the *John F. Kennedy* travels at its top speed of 35 knots (65 km/h). If 70% of the power output of the engines is applied to pushing the ship through the water, what is the magnitude of the force of water resistance that opposes the carrier's motion at this speed?

6.53. A ski tow operates on a 15.0° slope of length 300 m. The rope moves at 12.0 km/h and provides power for 50 riders at one time, with an average mass per rider of 70.0 kg. Estimate the power required to operate the tow.

6.54. A typical flying insect applies an average force equal to twice its weight during each downward stroke while hovering. Take the mass of the insect to be 10 g, and assume the wings move an average downward distance of 1.0 cm during each stroke. Assuming 100 downward strokes per second, estimate the average power output of the insect.

Problems

6.55. Rotating Bar. A thin, uniform 12.0-kg bar that is 2.00 m long rotates uniformly about a pivot at one end, making 5.00 complete revolutions every 3.00 seconds. What is the kinetic energy of this bar? (*Hint:* Different points in the bar have different speeds. Break the bar up into infinitesimal segments of mass dm and integrate to add up the kinetic energy of all these segments.)

6.56. A Near-Earth Asteroid. On April 13, 2029 (Friday the 13th!), the asteroid 99942 Apophis will pass within 18,600 mi of the earth—about 1/13 the distance to the moon! It has a density of $2600\ \text{kg/m}^3$, can be modeled as a sphere 320 m in diameter, and will be traveling at 12.6 km/s. (a) If, due to a small disturbance in its orbit, the asteroid were to hit the earth, how much kinetic energy would it deliver? (b) The largest nuclear bomb ever tested by the United States was the "Castle/Bravo" bomb, having a yield of 15 megatons of TNT. (A megaton of TNT releases 4.184×10^{15} J of energy.) How many Castle/Bravo bombs would be equivalent to the energy of Apophis?

6.57. A luggage handler pulls a 20.0-kg suitcase up a ramp inclined at 25.0° above the horizontal by a force $\vec{F}$ of magnitude 140 N that acts parallel to the ramp. The coefficient of kinetic friction between the ramp and the incline is $\mu_k = 0.300$. If the suitcase travels 3.80 m along the ramp, calculate (a) the work done on the suitcase by the force $\vec{F}$; (b) the work done on the suitcase by the gravitational force; (c) the work done on the suitcase by the normal force; (d) the work done on the suitcase by the friction force; (e) the total work done on the suitcase. (f) If the speed of the suitcase is zero at the bottom of the ramp, what is its speed after it has traveled 3.80 m along the ramp?

6.58. Chin-Ups. While doing a chin-up, a man lifts his body 0.40 m. (a) How much work must the man do per kilogram of body mass? (b) The muscles involved in doing a chin-up can generate about 70 J of work per kilogram of muscle mass. If the man can just barely do a 0.40-m chin-up, what percentage of his body's mass do these muscles constitute? (For comparison, the *total* percentage of muscle in a typical 70-kg man with 14% body fat is about 43%.) (c) Repeat part (b) for the man's young son, who has arms half as long as his father's but whose muscles can also generate 70 J of work per kilogram of muscle mass. (d) Adults and children have about the same percentage of muscle in their bodies. Explain why children can commonly do chin-ups more easily than their fathers.

6.59. Simple Machines. Ramps for the disabled are used because a large weight w can be raised by a relatively small force equal to $w \sin \alpha$ plus the small friction force. Such inclined planes are an example of a class of devices called *simple machines*. An input force F_{in} is applied to the system and results in an output force F_{out} applied to the object that is moved. For a simple machine the ratio of these forces, F_{out}/F_{in}, is called the actual mechanical advantage (AMA). The inverse ratio of the distances that the points of application of these forces move through during the motion of the object, s_{in}/s_{out}, is called the ideal mechanical advantage (IMA). (a) Find the IMA for an inclined plane. (b) What can we say about the relationship between the work supplied to the machine, W_{in}, and the work output of the machine, W_{out}, if AMA = IMA? (c) Sketch a single pulley arranged to give IMA = 2. (d) We define the efficiency e of a simple machine to equal the ratio of the output work to the input work, $e = W_{out}/W_{in}$. Show that $e = \text{AMA}/\text{IMA}$.

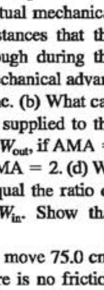
6.60. Consider the blocks in Exercise 6.7 as they move 75.0 cm. Find the total work done on each one (a) if there is no friction between the table and the 20.0-N block, and (b) if $\mu_s = 0.500$ and $\mu_k = 0.325$ between the table and the 20.0-N block.

6.61. The space shuttle *Endeavour*, with mass 86,400 kg, is in a circular orbit of radius 6.66×10^6 m around the earth. It takes 90.1 min for the shuttle to complete each orbit. On a repair mission, the shuttle is cautiously moving 1.00 m closer to a disabled satellite every 3.00 s. Calculate the shuttle's kinetic energy (a) relative to the earth and (b) relative to the satellite.

6.62. A 5.00-kg package slides 1.50 m down a long ramp that is inclined at 12.0° below the horizontal. The coefficient of kinetic friction between the package and the ramp is $\mu_k = 0.310$. Calculate (a) the work done on the package by friction; (b) the work done on the package by gravity; (c) the work done on the package by the normal force; (d) the total work done on the package. (e) If the package has a speed of 2.20 m/s at the top of the ramp, what is its speed after sliding 1.50 m down the ramp?

6.63. Springs in Parallel. Two springs are in *parallel* if they are parallel to each other and are connected at their ends (Figure 6.33). We can think of this combination as being equivalent to a single spring. The force constant of the equivalent single spring is called the *effective* force constant, k_{eff}, of the combination. (a) Show that the effective force constant of this combination is $k_{eff} = k_1 + k_2$. (b) Generalize this result for N springs in parallel.

Figure 6.33 Problem 6.63.



6.64. Springs in Series. Two massless springs are connected in series when they are attached one after the other, head to tail. (a) Show that the effective force constant (see Problem 6.63) of a series combination is given by $\dfrac{1}{k_{eff}} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$. (*Hint:* For a given force, the total distance stretched by the equivalent single spring is the sum of the distances stretched by the springs in combination. Also, each spring must exert the same force. Do you see why?) (b) Generalize this result for N springs in series.

6.65. An object is attracted toward the origin with a force given by $F_x = -k/x^2$. (Gravitational and electrical forces have this distance dependence.) (a) Calculate the work done by the force F_x when the object moves in the x-direction from x_1 to x_2. If $x_2 > x_1$, is the work done by F_x positive or negative? (b) The only other force acting on the object is a force that you exert with your hand to move the object slowly from x_1 to x_2. How much work do you do? If $x_2 > x_1$, is the work you do positive or negative? (c) Explain the similarities and differences between your answers to parts (a) and (b).

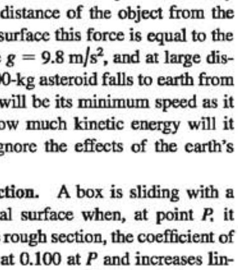
6.66. The gravitational pull of the earth on an object is inversely proportional to the square of the distance of the object from the center of the earth. At the earth's surface this force is equal to the object's normal weight mg, where $g = 9.8\ \text{m/s}^2$, and at large distances, the force is zero. If a 20,000-kg asteroid falls to earth from a very great distance away, what will be its minimum speed as it strikes the earth's surface, and how much kinetic energy will it impart to our planet? You can ignore the effects of the earth's atmosphere.

6.67. Varying Coefficient of Friction. A box is sliding with a speed of 4.50 m/s on a horizontal surface when, at point P, it encounters a rough section. On the rough section, the coefficient of friction is not constant, but starts at 0.100 at P and increases linearly with distance past P, reaching a value of 0.600 at 12.5 m past point P. (a) Use the work–energy theorem to find how far this box slides before stopping. (b) What is the coefficient of friction at the stopping point? (c) How far would the box have slid if the friction coefficient didn't increase but instead had the constant value of 0.100?

6.68. Consider a spring that does not obey Hooke's law very faithfully. One end of the spring is fixed. To keep the spring stretched or compressed an amount x, a force along the x-axis with x-component $F_x = kx - bx^2 + cx^3$ must be applied to the free end. Here $k = 100\ \text{N/m}$, $b = 700\ \text{N/m}^2$, and $c = 12{,}000\ \text{N/m}^3$. Note that $x > 0$ when the spring is stretched and $x < 0$ when it is compressed. (a) How much work must be done to stretch this spring by 0.050 m from its unstretched length? (b) How much work must be done to *compress* this spring by 0.050 m from its unstretched length? (c) Is it easier to stretch or compress this spring? Explain why in terms of the dependence of F_x on x. (Many real springs behave qualitatively in the same way.)

6.68. A small block with a mass of 0.120 kg is attached to a cord passing through a hole in a frictionless, horizontal surface (Fig. 6.34). The block is originally revolving at a distance of 0.40 m from the hole with a speed of 0.70 m/s. The cord is then pulled from below, shortening the radius of the circle in which the block revolves to 0.10 m. At this new distance, the speed of the block is observed to be 2.80 m/s. (a) What is the tension in the cord in the original situation when the block has speed $v = 0.70$ m/s? (b) What is the tension in the cord in the final situation when the block has speed $v = 2.80$ m/s? (c) How much work was done by the person who pulled on the cord?

Figure 6.34 Problem 6.69.



6.70. Proton Bombardment. A proton with mass 1.67×10^{-27} kg is propelled at an initial speed of 3.00×10^5 m/s directly toward a uranium nucleus 5.00 m away. The proton is repelled by the uranium nucleus with a force of magnitude $F = \alpha/x^2$, where x is the separation between the two objects and $\alpha = 2.12 \times 10^{-26}\ \text{N} \cdot \text{m}^2$. Assume that the uranium nucleus remains at rest.

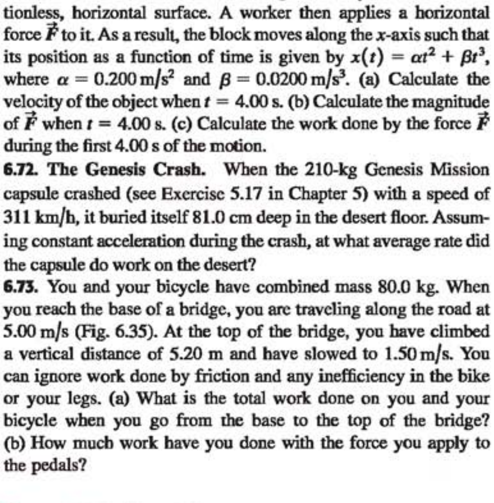
(a) What is the speed of the proton when it is 8.00×10^{-10} m from the uranium nucleus? (b) As the proton approaches the uranium nucleus, the repulsive force slows down the proton until it comes momentarily to rest, after which the proton moves away from the uranium nucleus. How close to the uranium nucleus does the proton get? (c) What is the speed of the proton when it is again 5.00 m away from the uranium nucleus?

6.71. A block of ice with mass 6.00 kg is initially at rest on a frictionless, horizontal surface. A worker then applies a horizontal force $\vec{F}$ to it. As a result, the block moves along the x-axis such that its position as a function of time is given by $x(t) = \alpha t^2 + \beta t^3$, where $\alpha = 0.200\ \text{m/s}^2$ and $\beta = 0.0200\ \text{m/s}^3$. (a) Calculate the velocity of the object when $t = 4.00$ s. (b) Calculate the magnitude of $\vec{F}$ when $t = 4.00$ s. (c) Calculate the work done by the force $\vec{F}$ during the first 4.00 s of the motion.

6.72. The Genesis Crash. When the 210-kg Genesis Mission capsule crashed (see Exercise 5.17 in Chapter 5) with a speed of 311 km/h, it buried itself 81.0 cm deep in the desert floor. Assuming constant acceleration during the crash, at what average rate did the capsule do work on the desert?

6.73. You and your bicycle have combined mass 80.0 kg. When you reach the base of a bridge, you are traveling along the road at 5.00 m/s (Fig. 6.35). At the top of the bridge, you have climbed a vertical distance of 5.20 m and have slowed to 1.50 m/s. You can ignore work done by friction and any inefficiency in the bike or your legs. (a) What is the total work done on you and your bicycle when you go from the base to the top of the bridge? (b) How much work have you done with the force you apply to the pedals?

Figure 6.35 Problem 6.73.



6.74. A force in the $+x$-direction has magnitude $F = b/x^n$, where b and n are constants. (a) For $n > 1$, calculate the work done on a particle by this force when the particle moves along the x-axis from $x = x_0$ to infinity. (b) Show that for $0 < n < 1$, even though F becomes zero as x becomes very large, an infinite amount of work is done by F when the particle moves from $x = x_0$ to infinity.

6.75. You are asked to design spring bumpers for the walls of a parking garage. A freely rolling 1200-kg car moving at 0.65 m/s is to compress the spring no more than 0.070 m before stopping. What should be the force constant of the spring? Assume that the spring has negligible mass.

6.76. The spring of a spring gun has force constant $k = 400$ N/m and negligible mass. The spring is compressed 6.00 cm, and a ball with mass 0.0300 kg is placed in the horizontal barrel against the compressed spring. The spring is then released, and the ball is propelled out the barrel of the gun. The barrel is 6.00 cm long, so the ball leaves the barrel at the same point that it loses contact with the spring. The gun is held so the barrel is horizontal. (a) Calculate the speed with which the ball leaves the barrel if you can ignore friction. (b) Calculate the speed of the ball as it leaves the barrel if a constant resisting force of 6.00 N acts on the ball as it moves along the barrel. (c) For the situation in part (b), at what position along the barrel does the ball have the greatest speed, and what is that speed? (In this case, the maximum speed does not occur at the end of the barrel.)

6.77. A 2.50-kg textbook is forced against a horizontal spring of negligible mass and force constant 250 N/m, compressing the spring a distance of 0.250 m. When released, the textbook slides on a horizontal tabletop with coefficient of kinetic friction $\mu_k = 0.30$. Use the work–energy theorem to find how far the textbook moves from its initial position before coming to rest.

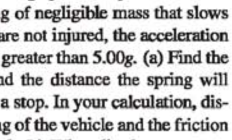
6.78. Pushing a Cat. Your cat "Ms." (mass 7.00 kg) is trying to make it to the top of a frictionless ramp 2.00 m long and inclined upward at 30.0° above the horizontal. Since the poor cat can't get any traction on the ramp, you push her up the entire length of the ramp by exerting a constant 100-N force parallel to the ramp. If Ms. takes a running start so that she is moving at 2.40 m/s at the bottom of the ramp, what is her speed when she reaches the top of the incline? Use the work–energy theorem.

6.79. Crash Barrier. A student proposes a design for an automobile crash barrier in which a 1700-kg sport utility vehicle moving at 20.0 m/s crashes into a spring of negligible mass that slows it to a stop. So that the passengers are not injured, the acceleration of the vehicle as it slows can be no greater than $5.00g$. (a) Find the required spring constant k, and find the distance the spring will compress in slowing the vehicle to a stop. In your calculation, disregard any deformation or crumpling of the vehicle and the friction between the vehicle and the ground. (b) What disadvantages are there to this design?

6.80. A physics professor is pushed up a ramp inclined upward at 30.0° above the horizontal as he sits in his desk chair that slides on frictionless rollers. The combined mass of the professor and chair is 85.0 kg. He is pushed 2.50 m along the incline by a group of students who together exert a constant horizontal force of 600 N. The professor's speed at the bottom of the ramp is 2.00 m/s. Use the work–energy theorem to find his speed at the top of the ramp.

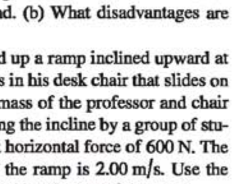
6.81. A 5.00-kg block is moving at $v_0 = 6.00$ m/s along a frictionless, horizontal surface toward a spring with force constant $k = 500$ N/m that is attached to a wall (Fig. 6.36). The spring has negligible mass. (a) Find the maximum distance the spring will be compressed. (b) If the spring is to compress by no more than 0.150 m, what should be the maximum value of v_0?

Figure 6.36 Problem 6.81.



6.82. Consider the system shown in Fig. 6.37. The rope and pulley have negligible mass, and the pulley is frictionless. The coefficient of kinetic friction between the 8.00-kg block and the tabletop is $\mu_k = 0.250$. The blocks are released from rest. Use energy methods to calculate the speed of the 6.00-kg block after it has descended 1.50 m.

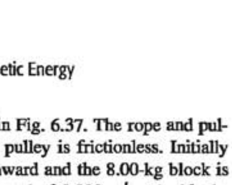
Figure 6.37 Problems 6.82 and 6.83.



6.83. Consider the system shown in Fig. 6.37. The rope and pulley have negligible mass, and the pulley is frictionless. Initially the 6.00-kg block is moving downward and the 8.00-kg block is moving to the right, both with a speed of 0.900 m/s. The blocks come to rest after moving 2.00 m. Use the work–energy theorem to calculate the coefficient of kinetic friction between the 8.00-kg block and the tabletop.

6.84. Bow and Arrow. Figure 6.38 shows how the force exerted by the string of a compound bow on an arrow varies as a function of how far back the arrow is pulled (the draw length). Assume that the same force is exerted on the arrow as it moves forward after being released. Full draw for this bow is at a draw length of 75.0 cm. If the bow shoots a 0.0250-kg arrow from full draw, what is the speed of the arrow as it leaves the bow?

Figure 6.38 Problem 6.84.



6.65. On an essentially frictionless, horizontal ice rink, a skater moving at 3.0 m/s encounters a rough patch that reduces her speed by 45% due to a friction force that is 25% of her weight. Use the work–energy theorem to find the length of this rough patch.

6.66. Rescue. Your friend (mass 65.0 kg) is standing on the ice in the middle of a frozen pond. There is very little friction between her feet and the ice, so she is unable to walk. Fortunately, a light rope is tied around her waist and you stand on the bank holding the other end. You pull on the rope for 3.00 s and accelerate your friend from rest to a speed of 6.00 m/s while you remain at rest. What is the average power supplied by the force you applied?

6.67. A pump is required to lift 800 kg of water (about 210 gallons) per minute from a well 14.0 m deep and eject it with a speed of 18.0 m/s. (a) How much work is done per minute in lifting the water? (b) How much work is done in giving the water the kinetic energy it has when ejected? (c) What must be the power output of the pump?

6.80. Find the power output of the worker in Problem 6.71 as a function of time. What is the numerical value of the power (in watts) at $t = 4.00$ s?

6.80. A physics student spends part of her day walking between classes or for recreation, during which time she expends energy at an average rate of 280 W. The remainder of the day she is sitting in class, studying, or resting; during these activities, she expends energy at an average rate of 100 W. If she expends a total of 1.1×10^7 J of energy in a 24-hour day, how much of the day did she spend walking?

6.90. All birds, independent of their size, must maintain a power output of 10–25 watts per kilogram of body mass in order to fly by flapping their wings. (a) The Andean giant hummingbird (*Patagona gigas*) has mass 70 g and flaps its wings 10 times per second while hovering. Estimate the amount of work done by such a hummingbird in each wingbeat. (b) A 70-kg athlete can maintain a power output of 1.4 kW for no more than a few seconds; the *steady* power output of a typical athlete is only 500 W or so. Is it possible for a human-powered aircraft to fly for extended periods by flapping its wings? Explain.

6.91. The Grand Coulee Dam is 1270 m long and 170 m high. The electrical power output from generators at its base is approximately 2000 MW. How many cubic meters of water must flow from the top of the dam per second to produce this amount of power if 92% of the work done on the water by gravity is converted to electrical energy? (Each cubic meter of water has a mass of 1000 kg.)

6.92. The engine of a car with mass m supplies a constant power P to the wheels to accelerate the car. You can ignore rolling friction and air resistance. The car is initially at rest. (a) Show that the speed of the car is given as a function of time by $v = (2Pt/m)^{1/2}$. (b) Show that the acceleration of the car is not constant but is given as a function of time by $a = (P/2mt)^{1/2}$. (c) Show that the displacement as a function of time is given by $x - x_0 = (8P/9m)^{1/2} t^{3/2}$.

6.93. Power of the Human Heart. The human heart is a powerful and extremely reliable pump. Each day it takes in and discharges about 7500 L of blood. Assume that the work done by the heart is equal to the work required to lift this amount of blood a height equal to that of the average American woman (1.63 m). The density (mass per unit volume) of blood is $1.05 \times 10^5\ kg/m^3$. (a) How much work does the heart do in a day? (b) What is the heart's power output in watts?

6.94. Six diesel units in series can provide 13.4 MW of power to the lead car of a freight train. The diesel units have total mass 1.10×10^6 kg. The average car in the train has mass 8.2×10^4 kg and requires a horizontal pull of 2.8 kN to move at a constant 27 m/s on level tracks. (a) How many cars can be in the train under these conditions? (b) This would leave no power for accelerating or climbing hills. Show that the extra force needed to accelerate the train is about the same for a 0.10-m/s^2 acceleration or a 1.0% slope (slope angle $\alpha = \arctan 0.010$). (c) With the 1.0% slope, show that an extra 2.9 MW of power is needed to maintain the 27-m/s speed of the diesel units. (d) With 2.9 MW less power available, how many cars can the six diesel units pull up a 1.0% slope at a constant 27-m/s?

6.95. It takes a force of 53 kN on the lead car of a 16-car passenger train with mass 9.1×10^5 kg to pull it at a constant 45 m/s (101 mi/h) on level tracks. (a) What power must the locomotive provide to the lead car? (b) How much more power to the lead car than calculated in part (a) would be needed to give the train an acceleration of 1.5 m/s^2, at the instant that the train has a speed of 45 m/s on level tracks? (c) How much more power to the lead car than that calculated in part (a) would be needed to move the train up a 1.5% grade (slope angle $\alpha = \arctan 0.015$) at a constant 45 m/s?

6.96. An object has several forces acting on it. One of these forces is $\vec{F} = \alpha x y \hat{\imath}$, a force in the x-direction whose magnitude depends on the position of the object, with $\alpha = 2.50\ N/m^2$. Calculate the work done on the object by this force for the following displacements of the object: (a) The object starts at the point $x = 0$, $y = 3.00$ m and moves parallel to the x-axis to the point $x = 2.00$ m, $y = 3.00$ m. (b) The object starts at the point $x = 2.00$ m, $y = 0$ and moves in the y-direction to the point $x = 2.00$ m, $y = 3.00$ m. (c) The object starts at the origin and moves on the line $y = 1.5x$ to the point $x = 2.00$ m, $y = 3.00$ m.

6.97. Cycling. For a touring bicyclist the drag coefficient $C(f_{air} = \frac{1}{2}CA\rho v^2)$ is 1.00, the frontal area A is 0.463 m^2, and the coefficient of rolling friction is 0.0045. The rider has mass 50.0 kg, and her bike has mass 12.0 kg. (a) To maintain a speed of 12.0 m/s (about 27 mi/h) on a level road, what must the rider's power output to the rear wheel be? (b) For racing, the same rider uses a different bike with coefficient of rolling friction 0.0030 and mass 9.00 kg. She also crouches down, reducing her drag coefficient to 0.88 and reducing her frontal area to $0.366\ m^2$. What must her power output to the rear wheel be then to maintain a speed of 12.0 m/s? (c) For the situation in part (b), what power output is required to maintain a speed of 6.0 m/s? Note the great drop in power requirement when the speed is only halved. (For more on aerodynamic speed limitations for a wide variety of human-powered vehicles, see "The Aerodynamics of Human-Powered Land Vehicles," *Scientific American*, December 1983.)

6.98. Automotive Power I. A truck engine transmits 28.0 kW (37.5 hp) to the driving wheels when the truck is traveling at a constant velocity of magnitude 60.0 km/h (37.3 mi/h) on a level road. (a) What is the resisting force acting on the truck? (b) Assume that 65% of the resisting force is due to rolling friction and the remainder is due to air resistance. If the force of rolling friction is independent of speed, and the force of air resistance is proportional to the square of the speed, what power will drive the truck at 30.0 km/h? At 120.0 km/h? Give your answers in kilowatts and in horsepower.

6.99. Automotive Power II. (a) If 8.00 hp are required to drive a 1800-kg automobile at 60.0 km/h on a level road, what is the total retarding force due to friction, air resistance, and so on? (b) What power is necessary to drive the car at 60.0 km/h up a 10.0% grade (a hill rising 10.0 m vertically in 100.0 m horizontally)? (c) What power is necessary to drive the car at 60.0 km/h *down* a 1.00% grade? (d) Down what percent grade would the car coast at 60.0 km/h?

Challenge Problems

6.100. On a winter's day in Maine, a warehouse worker is shoving boxes up a rough plank inclined at an angle α above the horizontal. The plank is partially covered with ice, with more ice near the bottom of the plank than near the top, so that the coefficient of friction increases with the distance x along the plank: $\mu = Ax$, where A is a positive constant and the bottom of the plank is at $x = 0$. (For this plank the coefficients of kinetic and static friction are equal: $\mu_k = \mu_s = \mu$.) The worker shoves a box up the plank so that it leaves the bottom of the plank moving at speed v_0. Show that when the box first comes to rest, it will remain at rest if

$$v_0^2 \geq \frac{3g\sin^2\alpha}{A\cos\alpha}$$

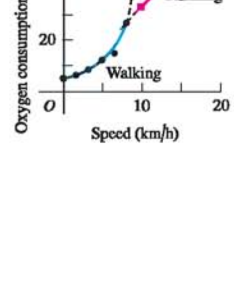
6.101. A Spring with Mass. We usually ignore the kinetic energy of the moving coils of a spring, but let's try to get a reasonable approximation to this. Consider a spring of mass M, equilibrium length L_0, and spring constant k. The work done to stretch or compress the spring by a distance L is $\frac{1}{2}kX^2$, where $X = L - L_0$. (a) Consider a spring, as described above, that has one end fixed and the other end moving with speed v. Assume that the speed of points along the length of the spring varies linearly with distance l from the fixed end. Assume also that the mass M of the spring is distributed uniformly along the length of the spring. Calculate the kinetic energy of the spring in terms of M and v. (*Hint:* Divide the spring into pieces of length dl; find the speed of each piece in terms of l, v, and L; find the mass of each piece in terms of dl, M, and L; and integrate from 0 to L. The result is *not* $\frac{1}{2}Mv^2$, since not all of the spring moves with the same speed.) In a spring gun, a spring of mass 0.243 kg and force constant 3200 N/m is compressed 2.50 cm from its unstretched length. When the trigger is pulled, the spring pushes horizontally on a 0.053-kg ball. The work done by friction is negligible. Calculate the ball's speed when the spring reaches its uncompressed length (b) ignoring the mass of the spring and (c) including, using the results of part (a), the mass of the spring. (d) In part (c), what is the final kinetic energy of the ball and of the spring?

6.102. An airplane in flight is subject to an air resistance force proportional to the square of its speed v. But there is an additional resistive force because the airplane has wings. Air flowing over the wings is pushed down and slightly forward, so from Newton's third law the air exerts a force on the wings and airplane that is up and slightly backward (Fig. 6.39). The upward force is the lift force that keeps the airplane aloft, and the backward force is called *induced drag*. At flying speeds, induced drag is inversely proportional to v^2, so that the total air resistance force can be expressed by $F_{air} = \alpha v^2 + \beta/v^2$, where α and β are positive constants that depend on the shape and size of the airplane and the density of the air. For a Cessna 150, a small single-engine airplane, $\alpha = 0.30\ N \cdot s^2/m^2$ and $\beta = 3.5 \times 10^5\ N \cdot m^2/s^2$. In steady flight, the engine must provide a forward force that exactly balances the air resistance force. (a) Calculate the speed (in km/h) at which this airplane will have the maximum *range* (that is, travel the greatest distance) for a given quantity of fuel. (b) Calculate the speed (in km/h) for which the airplane will have the maximum *endurance* (that is, remain in the air the longest time).

Figure 6.39 Challenge Problem 6.102.

6.103. Figure 6.40 shows the oxygen consumption rate of men walking and running at different speeds. The vertical axis shows the volume of oxygen (in cm^3) that a man consumes per kilogram

Figure 6.40 Challenge Problem 6.103.



of body mass per minute. Note the transition from walking to running that occurs naturally at about 9 km/h. The metabolism of 1 cm^3 of oxygen releases about 20 J of energy. Using the data in the graph, calculate the energy required for a 70-kg man to travel 1 km on foot at (a) 5 km/h (walking); (b) 10 km/h (running); (c) 15 km/h (running). (d) Which speed is the most efficient—that is, requires the least energy to travel 1 km?

6.104. General Proof of the Work–Energy Theorem. Consider a particle that moves along a curved path in space from (x_1, y_1, z_1) to (x_2, y_2, z_2). At the initial point, the particle has velocity $\vec{v} = v_{1x}\hat{\imath} + v_{1y}\hat{\jmath} + v_{1z}\hat{k}$. The path that the particle follows may be divided into infinitesimal segments $d\vec{l} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$. As the particle moves, it is acted on by a net force $\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$. The force components F_x, F_y, and F_z are in general functions of position. By the same sequence of steps used in Eqs. (6.11) through (6.13), prove the work–energy theorem for this general case. That is, prove that

$$W_{tot} = K_2 - K_1$$

where

$$W_{tot} = \int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} \vec{F} \cdot d\vec{l} = \int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} (F_x\,dx + F_y\,dy + F_z\,dz)$$

POTENTIAL ENERGY AND ENERGY CONSERVATION

7

? As this diver enters the water, is the force of gravity doing positive or negative work on him? Is the water doing positive or negative work on him?

LEARNING GOALS

By studying this chapter, you will learn:

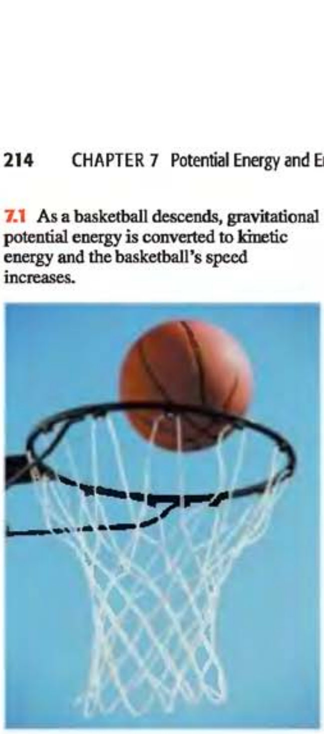
- How to use the concept of gravitational potential energy in problems that involve vertical motion.
- How to use the concept of elastic potential energy in problems that involve a moving body attached to a stretched or compressed spring.
- The distinction between conservative and nonconservative forces, and how to solve problems in which both kinds of forces act on a moving body.
- How to calculate the properties of a conservative force if you know the corresponding potential-energy function.
- How to use energy diagrams to understand the motion of an object moving in a straight line under the influence of a conservative force.

When a diver jumps off a high board into a swimming pool, he hits the water moving pretty fast, with a lot of kinetic energy. Where does that energy come from? The answer we learned in Chapter 6 was that the gravitational force (his weight) does work on the diver as he falls. The diver's kinetic energy—energy associated with his *motion*—increases by an amount equal to the work done.

However, there is a very useful alternative way to think about work and kinetic energy. This new approach is based on the concept of *potential energy,* which is energy associated with the *position* of a system rather than its motion. In this approach, there is *gravitational potential energy* even while the diver is standing on the high board. Energy is not added to the earth–diver system as the diver falls, but rather a storehouse of energy is *transformed* from one form (potential energy) to another (kinetic energy) as he falls. In this chapter we'll see how the work–energy theorem explains this transformation.

If the diver bounces on the end of the board before he jumps, the bent board stores a second kind of potential energy called *elastic potential energy.* We'll discuss elastic potential energy of simple systems such as a stretched or compressed spring. (An important third kind of potential energy is associated with the positions of electrically charged particles relative to each other. We'll encounter this potential energy in Chapter 23.)

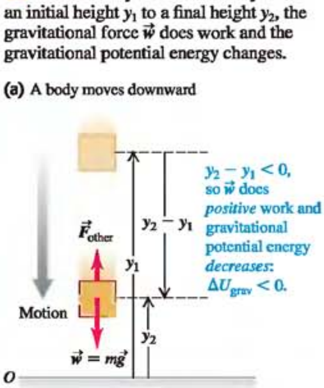
We will prove that in some cases the sum of a system's kinetic and potential energy, called the *total mechanical energy* of the system, is constant during the motion of the system. This will lead us to the general statement of the *law of conservation of energy,* one of the most fundamental and far-reaching principles in all of science.



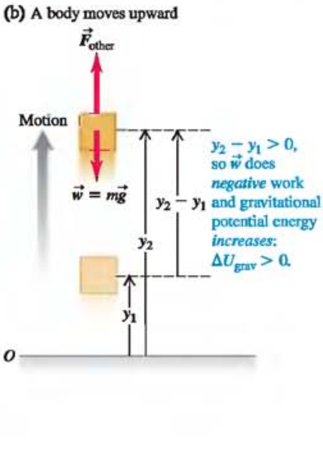
7.1 As a basketball descends, gravitational potential energy is converted to kinetic energy and the basketball's speed increases.

7.2 When a body moves vertically from an initial height y_1 to a final height y_2, the gravitational force $\vec{w}$ does work and the gravitational potential energy changes.

(a) A body moves downward



(b) A body moves upward



7.1 Gravitational Potential Energy

We learned in Chapter 6 that a particle gains or loses kinetic energy because it interacts with other objects that exert forces on it. During any interaction, the change in a particle's kinetic energy is equal to the total work done on the particle by the forces that act on it.

In many situations it seems as though energy has been stored in a system, to be recovered later. For example, you must do work to lift a heavy stone over your head. It seems reasonable that in hoisting the stone into the air you are storing energy in the system, energy that is later converted into kinetic energy when you let the stone fall.

This example points to the idea of an energy associated with the *position* of bodies in a system. This kind of energy is a measure of the *potential* or *possibility* for work to be done; when a stone is raised into the air, there is a potential for work to be done on it by the gravitational force, but only if the stone is allowed to fall to the ground. For this reason, energy associated with position is called **potential energy.** Our discussion suggests that there is potential energy associated with a body's weight and its height above the ground. We call this *gravitational potential energy* (Fig. 7.1).

We now have *two* ways to describe what happens when a body falls without air resistance. One way is to say that gravitational potential energy decreases and the falling body's kinetic energy increases. The other way, which we learned in Chapter 6, is that a falling body's kinetic energy increases because the force of the earth's gravity (the body's weight) does work on the body. Later in this section we'll use the work–energy theorem to show that these two descriptions are equivalent.

To begin with, however, let's derive the expression for gravitational potential energy. Suppose a body with mass m moves along the (vertical) y-axis, as in Fig. 7.2. The forces acting on it are its weight, with magnitude $w = mg$, and possibly some other forces; we call the vector sum (resultant) of all the other forces $\vec{F}_{other}$. We'll assume that the body stays close enough to the earth's surface that the weight is constant. (We'll find in Chapter 12 that weight decreases with altitude.) We want to find the work done by the weight when the body moves downward from a height y_1 above the origin to a lower height y_2 (Fig. 7.2a). The weight and displacement are in the same direction, so the work W_{grav} done on the body by its weight is positive;

$$W_{grav} = Fs = w(y_1 - y_2) = mgy_1 - mgy_2 \tag{7.1}$$

This expression also gives the correct work when the body moves *upward* and y_2 is greater than y_1 (Fig. 7.2b). In that case the quantity $(y_1 - y_2)$ is negative, and W_{grav} is negative because the weight and displacement are opposite in direction.

Equation (7.1) shows that we can express W_{grav} in terms of the values of the quantity mgy at the beginning and end of the displacement. This quantity, the product of the weight mg and the height y above the origin of coordinates, is called the **gravitational potential energy,** U_{grav}:

$$U_{grav} = mgy \quad \text{(gravitational potential energy)} \tag{7.2}$$

Its initial value is $U_{grav,1} = mgy_1$ and its final value is $U_{grav,2} = mgy_2$. The change in U_{grav} is the final value minus the initial value, or $\Delta U_{grav} = U_{grav,2} - U_{grav,1}$. We can express the work W_{grav} done by the gravitational force during the displacement from y_1 to y_2 as

$$W_{grav} = U_{grav,1} - U_{grav,2} = -(U_{grav,2} - U_{grav,1}) = -\Delta U_{grav} \tag{7.3}$$

The negative sign in front of ΔU_{grav} is *essential*. When the body moves up, y increases, the work done by the gravitational force is negative, and the gravitational potential energy increases $(\Delta U_{grav} > 0)$. When the body moves down, y decreases, the gravitational force does positive work, and the gravitational potential energy decreases $(\Delta U_{grav} < 0)$. It's like drawing money out of the bank (decreasing U_{grav}) and spending it (doing positive work). As Eq. (7.3) shows, the unit of potential energy is the joule (J), the same unit as is used for work.

CAUTION **To what body does gravitational potential energy "belong"?** It is *not* correct to call $U_{grav} = mgy$ the "gravitational potential energy of the body." The reason is that gravitational potential energy U_{grav} is a *shared* property of the body and the earth. The value of U_{grav} increases if the earth stays fixed and the body moves upward, away from the earth; it also increases if the body stays fixed and the earth is moved away from it. Notice that the formula $U_{grav} = mgy$ involves characteristics of both the body (its mass m) and the earth (the value of g).

Conservation of Mechanical Energy (Gravitational Forces Only)

To see what gravitational potential energy is good for, suppose the body's weight is the *only* force acting on it, so $\vec{F}_{other} = \mathbf{0}$. The body is then falling freely with no air resistance, and can be moving either up or down. Let its speed at point y_1 be v_1 and let its speed at y_2 be v_2. The work–energy theorem, Eq. (6.6), says that the total work done on the body equals the change in the body's kinetic energy: $W_{tot} = \Delta K = K_2 - K_1$. If gravity is the only force that acts, then from Eq. (7.3), $W_{tot} = W_{grav} = -\Delta U_{grav} = U_{grav,1} - U_{grav,2}$. Putting these together, we get

$$\Delta K = -\Delta U_{grav} \qquad \text{or} \qquad K_2 - K_1 = U_{grav,1} - U_{grav,2}$$

which we can rewrite as

$$K_1 + U_{grav,1} = K_2 + U_{grav,2} \qquad \text{(if only gravity does work)} \qquad (7.4)$$

or

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \qquad \text{(if only gravity does work)} \qquad (7.5)$$

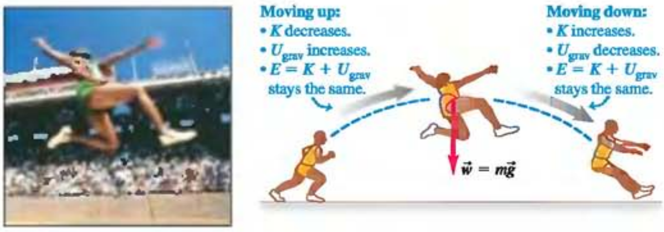
The sum $K + U_{grav}$ of kinetic and potential energy is called E, the **total mechanical energy of the system.** By "system" we mean the body of mass m and the earth considered together, because gravitational potential energy U is a shared property of both bodies. Then $E_1 = K_1 + U_{grav,1}$ is the total mechanical energy at y_1 and $E_2 = K_2 + U_{grav,2}$ is the total mechanical energy at y_2. Equation (7.4) says that when the body's weight is the only force doing work on it, $E_1 = E_2$. That is, E is constant; it has the same value at y_1 and y_2. But since the positions y_1 and y_2 are arbitrary points in the motion of the body, the total mechanical energy E has the same value at *all* points during the motion:

$$E = K + U_{grav} = \text{constant} \qquad \text{(if only gravity does work)}$$

A quantity that always has the same value is called a *conserved* quantity. *When only the force of gravity does work, the total mechanical energy is constant—that is, is conserved* (Fig. 7.3). This is our first example of the **conservation of mechanical energy.**

When we throw a ball into the air, its speed decreases on the way up as kinetic energy is converted to potential energy; $\Delta K < 0$ and $\Delta U_{grav} > 0$. On the way back down, potential energy is converted back to kinetic energy and the ball's speed increases; $\Delta K > 0$ and $\Delta U_{grav} < 0$. But the *total* mechanical energy (kinetic plus potential) is the same at every point in the motion, provided that no force other than gravity does work on the ball (that is, air resistance must be

7.3 While this athlete is in midair, only gravity does work on him (if we neglect the minor effects of air resistance). Mechanical energy E—the sum of kinetic and gravitational potential energy—is conserved.



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5.2 Upward-Moving Elevator Stops
5.3 Stopping a Downward-Moving Elevator
5.6 Skier Speed

negligible). It's still true that the gravitational force does work on the body as it moves up or down, but we no longer have to calculate work directly; keeping track of changes in the value of U_{grav} takes care of this completely.

CAUTION **Choose "zero height" to be wherever you like** When working with gravitational potential energy, we may choose any height to be $y = 0$. If we shift the origin for y, the values of y_1 and y_2 change, as do the values of $U_{grav,1}$ and $U_{grav,2}$. But this shift has no effect on the *difference* in height $y_2 - y_1$ or on the *difference* in gravitational potential energy $U_{grav,2} - U_{grav,1} = mg(y_2 - y_1)$. As the following example shows, the physically significant quantity is not the value of U_{grav} at a particular point, but only the *difference* in U_{grav} between two points. So we can define U_{grav} to be zero at whatever point we choose without affecting the physics.

Example 7.1 Height of a baseball from energy conservation

You throw a 0.145-kg baseball straight up in the air, giving it an initial upward velocity of magnitude 20.0 m/s. Find how high it goes, ignoring air resistance.

SOLUTION

IDENTIFY: After the ball leaves your hand, the only force doing work on the ball is gravity. Hence we can use conservation of mechanical energy.

SET UP: We'll use Eqs. (7.4) and (7.5), taking point 1 to be where the ball leaves your hand and point 2 to be where it reaches its maximum height. As in Fig. 7.2, we take the positive y-direction to be upward. The ball's speed at point 1 is $v_1 = 20.0$ m/s; at its maximum height the ball is instantaneously at rest, so $v_2 = 0$.

We want to know how far the ball moves vertically between the two points, so our target variable is the displacement $y_2 - y_1$. If we take the origin to be where the ball leaves your hand (point 1), then $y_1 = 0$ (Fig. 7.4) and the target variable is just y_2.

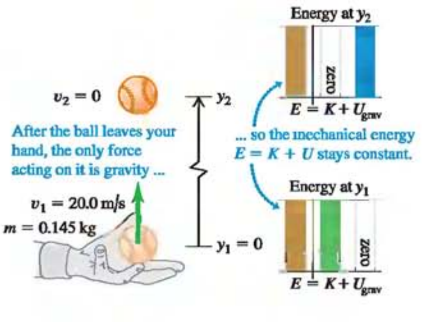
EXECUTE: Since $y_1 = 0$, the potential energy at point 1 is $U_{grav,1} = mgy_1 = 0$. Furthermore, since the ball is at rest at point 2, the kinetic energy at that point is $K_2 = \frac{1}{2}mv_2^2 = 0$. Hence Eq. (7.4), which says that $K_1 + U_{grav,1} = K_2 + U_{grav,2}$, becomes

$$K_1 = U_{grav,2}$$

As the energy bar graphs in Fig. 7.4 show, the kinetic energy of the ball at point 1 is completely converted to gravitational potential energy at point 2. At point 1 the kinetic energy is

$$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.145\ \text{kg})(20.0\ \text{m/s})^2 = 29.0\ \text{J}$$

7.4 After a baseball leaves your hand, mechanical energy $E = K + U$ is conserved.



This equals the gravitational potential energy $U_{grav,2} = mgy_2$ at point 2, so

$$y_2 = \frac{U_{grav,2}}{mg} = \frac{29.0\ \text{J}}{(0.145\ \text{kg})(9.80\ \text{m/s}^2)} = 20.4\ \text{m}$$

We can also solve the equation $K_1 = U_{grav,2}$ algebraically for y_2:

$$\frac{1}{2}mv_1^2 = mgy_2$$

$$y_2 = \frac{v_1^2}{2g} = \frac{(20.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 20.4\ \text{m}$$

EVALUATE: The mass divides out, as we should expect; we learned in Chapter 2 that the motion of a body in free fall doesn't depend on its mass. Indeed, we could have derived the result $y_2 = v_1^2/2g$ using Eq. (2.13).

In our calculation we chose the origin to be at point 1, so $y_1 = 0$ and $U_{grav,1} = 0$. What happens if we make a different choice? As an example, suppose we choose the origin to be 5.0 m below point 1, so $y_1 = 5.0$ m. Then the total mechanical energy at point 1 is part kinetic and part potential, while at point 2 it's purely potential energy. If you work through the calculation again with this choice of origin, you'll find $y_2 = 25.4$ m; this is 20.4 m above point 1, just as with the first choice of origin. In problems like this, the choice of height at which $U_{grav} = 0$ is up to you; don't agonize over the choice, though, because the physics of the answer doesn't depend on your choice.

When Forces Other Than Gravity Do Work

If other forces act on the body in addition to its weight, then $\vec{F}_{other}$ in Fig. 7.2 is *not* zero. For the pile driver described in Example 6.4 (Section 6.2), the force applied by the hoisting cable and the friction with the vertical guide rails are examples of forces that might be included in $\vec{F}_{other}$. The gravitational work W_{grav} is still given by Eq. (7.3), but the total work W_{tot} is then the sum of W_{grav} and the work done by $\vec{F}_{other}$. We will call this additional work W_{other}, so the total work done by all forces is $W_{tot} = W_{grav} + W_{other}$. Equating this to the change in kinetic energy, we have

$$W_{other} + W_{grav} = K_2 - K_1 \tag{7.6}$$

Also, from Eq. (7.3), $W_{grav} = U_{grav,1} - U_{grav,2}$, so

$$W_{other} + U_{grav,1} - U_{grav,2} = K_2 - K_1$$

which we can rearrange in the form

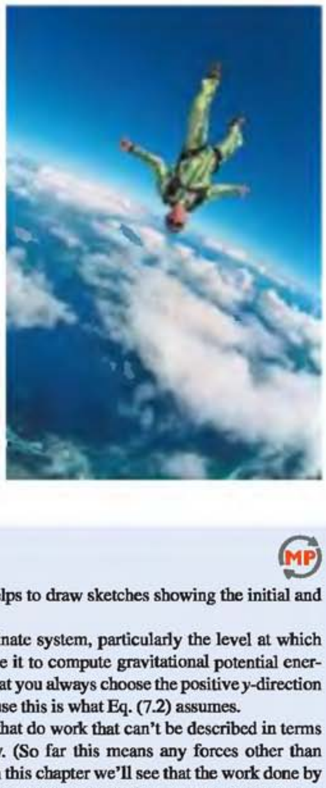
$$K_1 + U_{grav,1} + W_{other} = K_2 + U_{grav,2} \quad \text{(if forces other than gravity do work)} \tag{7.7}$$

Finally, using the appropriate expressions for the various energy terms, we obtain

$$\frac{1}{2}mv_1^2 + mgy_1 + W_{other} = \frac{1}{2}mv_2^2 + mgy_2 \quad \text{(if forces other than gravity do work)} \tag{7.8}$$

The meaning of Eqs. (7.7) and (7.8) is this: *The work done by all forces* other than the gravitational force *equals the change in the total mechanical energy* $E = K + U_{grav}$ *of the system, where* U_{grav} *is the gravitational potential energy.* When W_{other} is positive, E increases, and $K_2 + U_{grav,2}$ is greater than $K_1 + U_{grav,1}$. When W_{other} is negative, E decreases (Fig. 7.5). In the special case in which no forces other than the body's weight do work, $W_{other} = 0$. The total mechanical energy is then constant, and we are back to Eq. (7.4) or (7.5).

7.5 As this skydiver moves downward, the upward force of air resistance does negative work W_{other} on him. Hence the total mechanical energy $E = K + U$ decreases: The skydiver's speed and kinetic energy K stay the same, while the gravitational potential energy U goes down.



Problem-Solving Strategy 7.1 Problems Using Mechanical Energy I

IDENFITY *the relevant concepts:* Decide whether the problem should be solved by energy methods, by using $\sum\vec{F} = m\vec{a}$ directly, or by a combination of these. The energy approach is best when the problem involves varying forces, motion along a curved path (discussed later in this section), or both. If the problem involves elapsed time, the energy approach is usually *not* the best choice, because it doesn't involve time directly.

SET UP *the problem* using the following steps:

1. When using the energy approach, first decide what the initial and final states (the positions and velocities) of the system are. Use the subscript 1 for the initial state and the subscript 2 for the final state. It helps to draw sketches showing the initial and final states.
2. Define your coordinate system, particularly the level at which $y = 0$. You will use it to compute gravitational potential energies. We suggest that you always choose the positive y-direction to be upward because this is what Eq. (7.2) assumes.
3. Identify all forces that do work that can't be described in terms of potential energy. (So far this means any forces other than gravity. But later in this chapter we'll see that the work done by an ideal spring can also be expressed as a change in potential energy.) A free-body diagram is always helpful.

Continued

4. List the unknown and known quantities, including the coordinates and velocities at each point. Decide which unknowns are your target variables.

EXECUTE *the solution:* Write expressions for the initial and final kinetic and potential energies—that is, K_1, K_2, $U_{grav,1}$, and $U_{grav,2}$. Then relate the kinetic and potential energies and the work done by other forces, W_{other}, using Eq. (7.7). (You will have to calculate W_{other} in terms of these forces.) If no other forces do work, this expression becomes Eq. (7.4). It's helpful to draw bar graphs showing the initial and final values of K, U_{grav}, and $E = K + U_{grav}$. Then solve to find whatever unknown quantity is required.

EVALUATE *your answer:* Check whether your answer makes physical sense. Keep in mind, here and in later sections, that the work done by each force must be represented either in $U_{grav,1} - U_{grav,2} = -\Delta U_{grav}$ or as W_{other}, but *never* in both places. The gravitational work is included in ΔU_{grav}, so make sure you did not include it again in W_{other}.

Example 7.2 Work and energy in throwing a baseball

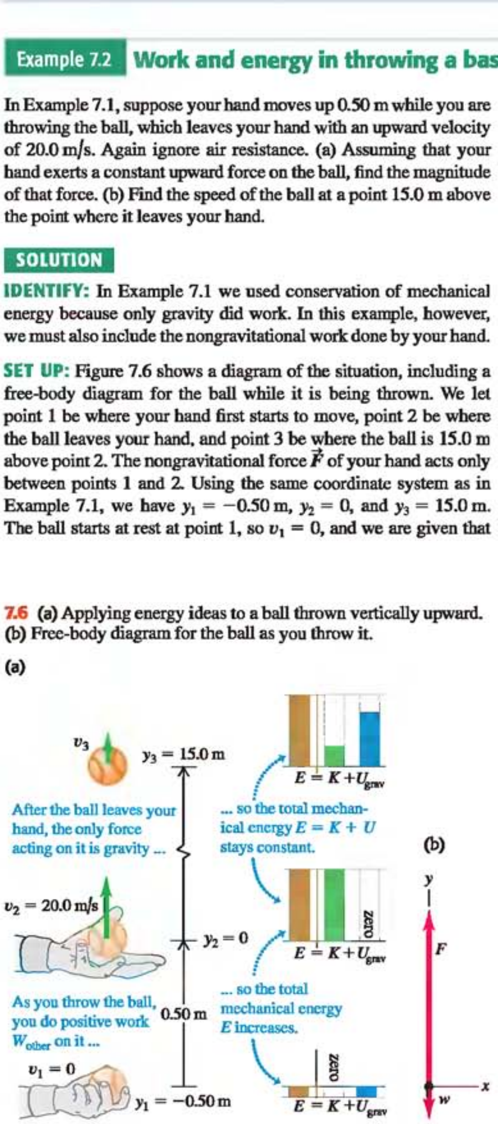
In Example 7.1, suppose your hand moves up 0.50 m while you are throwing the ball, which leaves your hand with an upward velocity of 20.0 m/s. Again ignore air resistance. (a) Assuming that your hand exerts a constant upward force on the ball, find the magnitude of that force. (b) Find the speed of the ball at a point 15.0 m above the point where it leaves your hand.

SOLUTION

IDENTIFY: In Example 7.1 we used conservation of mechanical energy because only gravity did work. In this example, however, we must also include the nongravitational work done by your hand.

SET UP: Figure 7.6 shows a diagram of the situation, including a free-body diagram for the ball while it is being thrown. We let point 1 be where your hand first starts to move, point 2 be where the ball leaves your hand, and point 3 be where the ball is 15.0 m above point 2. The nongravitational force $\vec{F}$ of your hand acts only between points 1 and 2. Using the same coordinate system as in Example 7.1, we have $y_1 = -0.50$ m, $y_2 = 0$, and $y_3 = 15.0$ m. The ball starts at rest at point 1, so $v_1 = 0$, and we are given that the ball's speed as it leaves your hand is $v_2 = 20.0$ m/s. Our target variables are (a) the magnitude F of the force of your hand and (b) the speed v_3 at point 3.

7.6 (a) Applying energy ideas to a ball thrown vertically upward. (b) Free-body diagram for the ball as you throw it.



EXECUTE: (a) To determine the magnitude of $\vec{F}$, we'll first use Eq. (7.7) to calculate the work W_{other} done by this force. We have

$$K_1 = 0$$
$$U_{grav,1} = mgy_1 = (0.145\text{ kg})(9.80\text{ m/s}^2)(-0.50\text{ m}) = -0.71\text{ J}$$
$$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.145\text{ kg})(20.0\text{ m/s})^2 = 29.0\text{ J}$$
$$U_{grav,2} = mgy_2 = (0.145\text{ kg})(9.80\text{ m/s}^2)(0) = 0$$

The initial potential energy $U_{grav,1}$ is *negative* because the ball was initially below the origin. (Don't worry about having a potential energy that's less than zero. Remember, all that matters is the *difference* in potential energy from one point to another.) According to Eq. (7.7), $K_1 + U_{grav,1} + W_{other} = K_2 + U_{grav,2}$, so

$$W_{other} = (K_2 - K_1) + (U_{grav,2} - U_{grav,1})$$
$$= (29.0\text{ J} - 0) + (0 - (-0.71\text{ J})) = 29.7\text{ J}$$

The kinetic energy of the ball increases by $K_2 - K_1 = 29.0$ J, and the potential energy increases by $U_{grav,2} - U_{grav,1} = 0.71$ J; the sum is $E_2 - E_1$, the change in total mechanical energy, which is equal to W_{other}.

Assuming the upward force $\vec{F}$ that your hand applies is constant, the work W_{other} done by this force is equal to the magnitude F of the force multiplied by the upward displacement $y_2 - y_1$ over which it acts:

$$W_{other} = F(y_2 - y_1)$$
$$F = \frac{W_{other}}{y_2 - y_1} = \frac{29.7\text{ J}}{0.50\text{ m}} = 59\text{ N}$$

This is about 40 times greater than the weight of the ball.

(b) To find the speed at point 3, note that between points 2 and 3, total mechanical energy is conserved; the force of your hand no longer acts, so $W_{other} = 0$. We can then find the kinetic energy at point 3 using Eq. (7.4):

$$K_2 + U_{grav,2} = K_3 + U_{grav,3}$$
$$U_{grav,3} = mgy_3 = (0.145\text{ kg})(9.80\text{ m/s}^2)(15.0\text{ m}) = 21.3\text{ J}$$
$$K_3 = (K_2 + U_{grav,2}) - U_{grav,3}$$

$$= (29.0\,\text{J} + 0\,\text{J}) - 21.3\,\text{J} = 7.7\,\text{J}$$

Since $K_3 = \frac{1}{2}mv_{3y}^2$, where v_{3y} is the y-component of the ball's velocity at point 3, we have

$$v_{3y} = \pm\sqrt{\frac{2K_3}{m}} = \pm\sqrt{\frac{2(7.7\,\text{J})}{0.145\,\text{kg}}} = \pm 10\,\text{m/s}$$

The significance of the plus-or-minus sign is that the ball passes point 3 *twice*, once on the way up and again on the way down. The total mechanical energy E is constant and equal to 29.0 J while the ball is in free fall, and the potential energy at point 3 is $U_{grav,3} = 21.3$ J whether the ball is moving up or down. So at point 3, the ball's kinetic energy K_3 and *speed* don't depend on the direction the ball is moving. The velocity v_{3y} is positive $(+10\,\text{m/s})$ when the ball is moving up and negative $(-10\,\text{m/s})$ when it is moving down; the speed v_3 is 10 m/s in either case.

EVALUATE: As a check on our result, recall from Example 7.1 that the ball reaches a maximum height $y = 20.4$ m. At that point all of the kinetic energy that the ball had when it left your hand at $y = 0$ has been converted to gravitational potential energy. At $y = 15.0$ m, the ball is about three-fourths of the way to its maximum height, so about three-fourths of its mechanical energy should be in the form of potential energy. (This is shown in the energy bar graphs in Fig. 7.6a.) Can you show that this is true from our results for K_3 and $U_{grav,3}$?

Gravitational Potential Energy for Motion Along a Curved Path

In our first two examples the body moved along a straight vertical line. What happens when the path is slanted or curved (Fig. 7.7a)? The body is acted on by the gravitational force $\vec{w} = m\vec{g}$ and possibly by other forces whose resultant we call $\vec{F}_{other}$. To find the work done by the gravitational force during this displacement, we divide the path into small segments $\Delta\vec{s}$; Fig. 7.7b shows a typical segment. The work done by the gravitational force over this segment is the scalar product of the force and the displacement. In terms of unit vectors, the force is $\vec{w} = m\vec{g} = -mg\hat{j}$ and the displacement is $\Delta\vec{s} = \Delta x\hat{\imath} + \Delta y\hat{j}$, so the work done by the gravitational force is

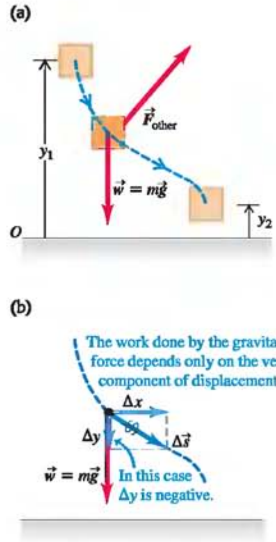
$$\vec{w}\cdot\Delta\vec{s} = -mg\hat{j}\cdot(\Delta x\hat{\imath} + \Delta y\hat{j}) = -mg\Delta y$$

The work done by gravity is the same as though the body had been displaced vertically a distance Δy, with no horizontal displacement. This is true for every segment, so the *total* work done by the gravitational force is $-mg$ multiplied by the *total* vertical displacement $(y_2 - y_1)$:

$$W_{grav} = -mg(y_2 - y_1) = mgy_1 - mgy_2 = U_{grav,1} - U_{grav,2}$$

This is the same as Eq. (7.1) or (7.3), in which we assumed a purely vertical path. So even if the path a body follows between two points is curved, the total work done by the gravitational force depends only on the difference in height between the two points of the path. This work is unaffected by any horizontal motion that may occur. So *we can use the same expression for gravitational potential energy whether the body's path is curved or straight.*

7.7 Calculating the change in gravitational potential energy for a displacement along a curved path.



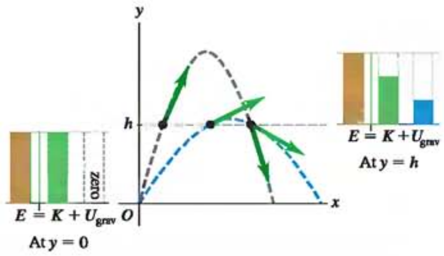
Conceptual Example 7.3 Energy in projectile motion

A batter hits two identical baseballs with the same initial speed and height but different initial angles. Prove that at a given height h, both balls have the same speed if air resistance can be neglected.

SOLUTION

If there is no air resistance, the only force acting on each ball after it is hit is its weight. Hence the total mechanical energy for each ball is constant. Figure 7.8 shows the trajectories of two balls batted at the same height with the same initial speed, and thus the same total mechanical energy, but with different initial angles. At all points at the same height the potential energy is the same. Thus the kinetic energy at this height must be the same for both balls, and the speeds are the same.

7.8 For the same initial speed and initial height, the speed of a projectile at a given elevation h is always the same, neglecting air resistance.



Example 7.4 Calculating speed along a vertical circle

Your cousin Throckmorton skateboards down a curved playground ramp. If we treat Throcky and his skateboard as a particle, he moves through a quarter-circle with radius $R = 3.00$ m (Fig. 7.9). The total mass of Throcky and his skateboard is 25.0 kg. He starts from rest and there is no friction. (a) Find his speed at the bottom of the ramp. (b) Find the normal force that acts on him at the bottom of the curve.

SOLUTION

IDENTIFY: We can't use the constant-acceleration equations because Throcky's acceleration isn't constant; the slope decreases as he descends. Instead, we'll use the energy approach. Since Throcky moves along a circular arc, we'll also use what we learned about circular motion in Section 5.4.

SET UP: Since there is no friction, the only force other than Throcky's weight is the normal force $\vec{n}$ exerted by the ramp (Fig. 7.9b). Although this force acts all along the path, it does *zero* work because $\vec{n}$ is perpendicular to Throcky's displacement at every point. Hence $W_{other} = 0$ and mechanical energy is conserved.

We take point 1 at the starting point and point 2 at the bottom of the curved ramp, and we let $y = 0$ be at the bottom of the ramp (Fig. 7.9a). Then $y_1 = R$ and $y_2 = 0$. (We are treating Throcky as if his entire mass were concentrated at his center.) Throcky starts at rest at the top, so $v_1 = 0$. Our target variable in part (a) is his speed at the bottom, v_2. In part (b) we want to find the magnitude n of the normal force at point 2. Because this force does no work, it doesn't appear in the energy equation, so we'll use Newton's second law instead.

EXECUTE: (a) The various energy quantities are

$$K_1 = 0 \qquad U_{grav,1} = mgR$$
$$K_2 = \frac{1}{2}mv_2^2 \qquad U_{grav,2} = 0$$

From conservation of mechanical energy,

$$K_1 + U_{grav,1} = K_2 + U_{grav,2}$$
$$0 + mgR = \frac{1}{2}mv_2^2 + 0$$
$$v_2 = \sqrt{2gR}$$
$$= \sqrt{2(9.80\ \text{m/s}^2)(3.00\ \text{m})} = 7.67\ \text{m/s}$$

Notice that this answer doesn't depend on the ramp being circular; no matter what the shape of the ramp, Throcky will have the same speed $v_2 = \sqrt{2gR}$ at the bottom. This would be true even if the wheels of his skateboard lost contact with the ramp during the ride, because only the gravitational force would still do work. In fact, the speed is the same as if Throcky had fallen vertically through a height R. The answer is also independent of his mass.

(b) To find n at point 2 using Newton's second law, we need the free-body diagram at that point (Fig. 7.9b). At point 2, Throcky is moving at speed $v_2 = \sqrt{2gR}$ in a circle of radius R; his acceleration is toward the center of the circle and has magnitude

$$a_{rad} = \frac{v_2^2}{R} = \frac{2gR}{R} = 2g$$

If we take the positive y-direction to be upward, the y-component of Newton's second law is

$$\sum F_y = n + (-w) = ma_{rad} = 2mg$$
$$n = w + 2mg = 3mg$$
$$= 3(25.0\ \text{kg})(9.80\ \text{m/s}^2) = 735\ \text{N}$$

At point 2 the normal force is three times Throcky's weight. This result is independent of the radius of the circular ramp. We learned in Example 5.9 (Section 5.2) and Example 5.24 (Section 5.4) that the magnitude of n is the *apparent weight*, so Throcky feels as though he weighs three times his true weight mg. But as soon as he reaches the horizontal part of the ramp to the right of point 2, the normal force decreases to $w = mg$ and Throcky feels normal again. Can you see why?

EVALUATE: This example shows a general rule about the role of forces in problems in which we use energy techniques: What matters is not simply whether a force *acts*, but whether that force *does work*. If the force does no work, like the normal force $\vec{n}$ in this example, then it does not appear at all in Eq. (7.7), $K_1 + U_{grav,1} + W_{other} = K_2 + U_{grav,2}$.

Notice we had to use *both* the energy approach and Newton's second law to solve this problem; energy conservation gave us the speed and $\sum\vec{F} = m\vec{a}$ gave us the normal force. For each part of the problem we used the technique that most easily led us to the answer.

7.9 (a) Throcky skateboarding down a frictionless circular ramp. The total mechanical energy is constant. (b) Free-body diagrams for Throcky and his skateboard at various points on the ramp.

Example 7.5 A vertical circle with friction

In Example 7.4, suppose that the ramp is not frictionless and that Throcky's speed at the bottom is only 6.00 m/s. What work was done by the friction force acting on him?

SOLUTION

IDENTIFY: Figure 7.10 shows that again the normal force does no work, but now there is a friction force $\vec{f}$ that *does* do work. Hence the nongravitational work done on Throcky between points 1 and 2, W_{other}, is not zero.

SET UP: We use the same coordinate system and the same initial and final points as in Example 7.4 (see Fig. 7.10). Our target variable is the work done by friction, W_f; since friction is the only force other than gravity that does work, this is just equal to W_{other}. We'll find W_f using Eq. (7.7).

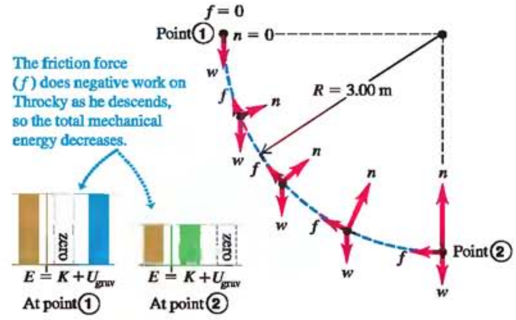
EXECUTE: The energy quantities are

$$K_1 = 0$$
$$U_{grav,1} = mgR = (25.0\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m}) = 735\text{ J}$$
$$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(25.0\text{ kg})(6.00\text{ m/s})^2 = 450\text{ J}$$
$$U_{grav,2} = 0$$

From Eq. (7.7),

$$W_f = K_2 + U_{grav,2} - K_1 - U_{grav,1}$$
$$= 450\text{ J} + 0 - 0 - 735\text{ J} = -285\text{ J}$$

The work done by the friction force is -285 J, and the total mechanical energy *decreases* by 285 J. Do you see why W_f has to be negative?



7.10 Free-body diagram and energy bar graphs for Throcky skateboarding down a ramp with friction.

EVALUATE: Throcky's motion is determined by Newton's second law, $\Sigma\vec{F} = m\vec{a}$. But it would be very difficult to apply the second law directly to this problem because the normal and friction forces and the acceleration are continuously changing in both magnitude and direction as Throcky moves. The energy approach, by contrast, relates the motions at the top and bottom of the ramp without involving the details of what happens in between. Many problems are easy if energy considerations are used but very complex if we try to use Newton's laws directly.

Example 7.6 An inclined plane with friction

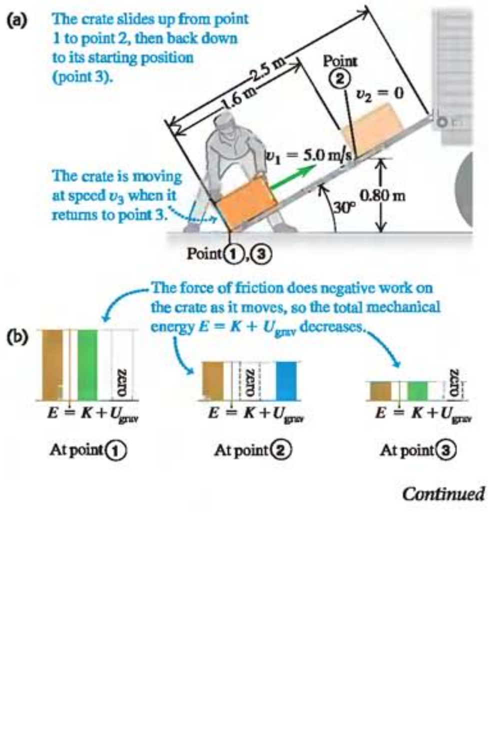
We want to load a 12-kg crate into a truck by sliding it up a ramp 2.5 m long, inclined at 30°. A worker, giving no thought to friction, calculates that he can get the crate up the ramp by giving it an initial speed of 5.0 m/s at the bottom and letting it go. But friction is *not* negligible; the crate slides 1.6 m up the ramp, stops, and slides back down (Fig. 7.11). (a) Assuming that the friction force acting on the crate is constant, find its magnitude. (b) How fast is the crate moving when it reaches the bottom of the ramp?

SOLUTION

IDENTIFY: The friction force does work on the crate as it slides. As in Example 7.2, we'll use the energy approach in part (a) to find the magnitude of the nongravitational force that does work (in this case, friction). In part (b) we'll calculate how much nongravitational work this force does as the crate slides back down and then use the energy approach to find the crate's speed at the bottom of the ramp.

SET UP: The first part of the motion is from point 1, at the bottom of the ramp, to point 2, where the crate stops instantaneously. In the second part of the motion, the crate returns to the bottom of the ramp, which we'll also call point 3 (Fig. 7.11a). We take $y = 0$ (and hence $U_{grav} = 0$) to be at ground level, so $y_1 = 0$,

7.11 (a) A crate slides partway up the ramp, stops, and slides back down. (b) Energy bar graphs for points 1, 2, and 3.



Continued

$y_2 = (1.6\text{ m})\sin 30° = 0.80\text{ m}$, and $y_3 = 0$. We are given that $v_1 = 5.0\text{ m/s}$ and $v_2 = 0$ (the crate is instantaneously at rest at point 2). Our target variable in part (a) is f, the magnitude of the friction force. In part (b) our target variable is v_3, the speed at the bottom of the ramp.

EXECUTE: (a) The energy quantities are

$$K_1 = \frac{1}{2}(12\text{ kg})(5.0\text{ m/s})^2 = 150\text{ J}$$
$$U_{\text{grav},1} = 0$$
$$K_2 = 0$$
$$U_{\text{grav},2} = (12\text{ kg})(9.8\text{ m/s}^2)(0.80\text{ m}) = 94\text{ J}$$
$$W_{\text{other}} = -fs$$

Here f is the unknown magnitude of the friction force and $s = 1.6\text{ m}$. Using Eq. (7.7), we find

$$K_1 + U_{\text{grav},1} + W_{\text{other}} = K_2 + U_{\text{grav},2}$$
$$W_{\text{other}} = -fs = (K_2 + U_{\text{grav},2}) - (K_1 + U_{\text{grav},1})$$
$$f = -\frac{(K_2 + U_{\text{grav},2}) - (K_1 + U_{\text{grav},1})}{s}$$
$$= -\frac{(0 + 94\text{ J}) - (150\text{ J} + 0)}{1.6\text{ m}} = 35\text{ N}$$

The friction force of 35 N, acting over 1.6 m, causes the mechanical energy of the crate to decrease from 150 J to 94 J (Fig. 7.11b).

(b) On the way down from point 2 to point 3 at the bottom of the ramp, the friction force and the displacement both reverse direction but have the same magnitudes, so the frictional work has the same negative value as from point 1 to point 2. The total work done by friction between points 1 and 3 is

$$W_{\text{other}} = W_{\text{fric}} = -2fs = -2(35\text{ N})(1.6\text{ m}) = -112\text{ J}$$

From part (a), $K_1 = 150\text{ J}$ and $U_{\text{grav},1} = 0$. Equation (7.7) then gives

$$K_1 + U_{\text{grav},1} + W_{\text{other}} = K_3 + U_{\text{grav},3}$$
$$K_3 = K_1 + U_{\text{grav},1} - U_{\text{grav},3} + W_{\text{other}}$$
$$= 150\text{ J} + 0 - 0 + (-112\text{ J}) = 38\text{ J}$$

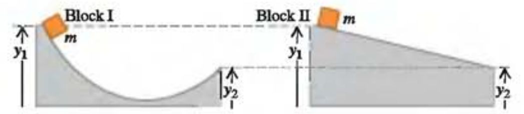
The crate returns to the bottom of the ramp with only 38 J of the original 150 J of mechanical energy (Fig. 7.11b). Using $K_3 = \frac{1}{2}mv_3^2$, we get

$$v_3 = \sqrt{\frac{2K_3}{m}} = \sqrt{\frac{2(38\text{ J})}{12\text{ kg}}} = 2.5\text{ m/s}$$

EVALUATE: The crate's speed when it returns to the bottom of the ramp, $v_3 = 2.5\text{ m/s}$, is less than the speed $v_1 = 5.0\text{ m/s}$ at which it left that point. That's good—energy was lost due to friction.

In part (b) we applied Eq. (7.7) to points 1 and 3, considering the entire round trip as a whole. Alternatively, we could have considered the second part of the motion by itself and applied Eq. (7.7) to points 2 and 3. Try it and see whether you get the same result for v_3.

Test Your Understanding of Section 7.1 The figure shows two different frictionless ramps. The heights y_1 and y_2 are the same for both ramps. If a block of mass m is released from rest at the left-hand end of each ramp, which block arrives at the right-hand end with the greater speed? (i) block I; (ii) block II; (iii) the speed is the same for both blocks.



7.12 The Achilles tendon, which runs along the back of the ankle to the heel bone, acts like a natural spring. When it stretches and then relaxes, this tendon stores and then releases elastic potential energy. This spring action reduces the amount of work your leg muscles must do as you run.

7.2 Elastic Potential Energy

There are many situations in which we encounter potential energy that is not gravitational in nature. One example is a rubber-band slingshot. Work is done on the rubber band by the force that stretches it, and that work is stored in the rubber band until you let it go. Then the rubber band gives kinetic energy to the projectile.

This is the same pattern we saw with the pile driver in Section 7.1: Do work on the system to store energy, which can later be converted to kinetic energy. We'll describe the process of storing energy in a deformable body such as a spring or rubber band in terms of *elastic potential energy* (Fig. 7.12). A body is called *elastic* if it returns to its original shape and size after being deformed.

To be specific, we'll consider storing energy in an ideal spring, like the ones we discussed in Section 6.3. To keep such an ideal spring stretched by a distance x, we must exert a force $F = kx$, where k is the force constant of the spring. The ideal spring is a useful idealization because many elastic bodies show this same direct proportionality between force $\vec{F}$ and displacement x, provided that x is sufficiently small.

We proceed just as we did for gravitational potential energy. We begin with the work done by the elastic (spring) force and then combine this with the work–energy theorem. The difference is that gravitational potential energy is a shared property of a body and the earth, but elastic potential energy is stored just in the spring (or other deformable body).

Figure 7.13 shows the ideal spring from Fig. 6.18, with its left end held stationary and its right end attached to a block with mass m that can move along the x-axis. In Fig. 7.13a the body is at $x = 0$ when the spring is neither stretched nor compressed. We move the block to one side, thereby stretching or compressing the spring, and then let it go. As the block moves from one position x_1 to another position x_2, how much work does the elastic (spring) force do on the block?

We found in Section 6.3 that the work we must do *on* the spring to move one end from an elongation x_1 to a different elongation x_2 is

$$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 \qquad \text{(work done } on \text{ a spring)}$$

where k is the force constant of the spring. If we stretch the spring farther, we do positive work on the spring; if we let the spring relax while holding one end, we do negative work on it. We also saw that this expression for work is still correct if the spring is compressed, not stretched, so that x_1 or x_2 or both are negative. Now we need to find the work done *by* the spring. From Newton's third law the two quantities of work are just negatives of each other. Changing the signs in this equation, we find that in a displacement from x_1 to x_2 the spring does an amount of work W_{el} given by

$$W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 \qquad \text{(work done } by \text{ a spring)}$$

The subscript "el" stands for *elastic*. When x_1 and x_2 are both positive and $x_2 > x_1$ (Fig. 7.13b), the spring does negative work on the block, which moves in the $+x$-direction while the spring pulls on it in the $-x$-direction. The spring stretches farther, and the block slows down. When x_1 and x_2 are both positive and $x_2 < x_1$ (Fig. 7.13c), the spring does positive work as it relaxes and the block speeds up. If the spring can be compressed as well as stretched, x_1 or x_2 or both may be negative, but the expression for W_{el} is still valid. In Fig. 7.13d, both x_1 and x_2 are negative, but x_2 is less negative than x_1; the compressed spring does positive work as it relaxes, speeding the block up.

Just as for gravitational work, we can express the work done by the spring in terms of a given quantity at the beginning and end of the displacement. This quantity is $\frac{1}{2}kx^2$, and we define it to be the **elastic potential energy:**

$$U_{el} = \frac{1}{2}kx^2 \qquad \text{(elastic potential energy)} \qquad (7.9)$$

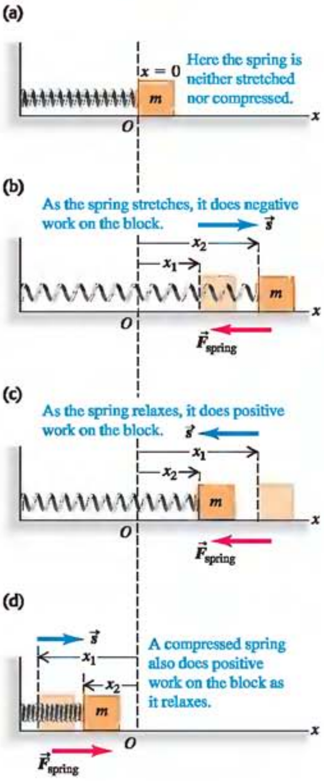
Figure 7.14 is a graph of Eq. (7.9). The unit of U_{el} is the joule (J), the unit used for *all* energy and work quantities; to see this from Eq. (7.9), recall that the units of k are N/m and that $1\ \text{N} \cdot \text{m} = 1\ \text{J}$.

We can use Eq. (7.9) to express the work W_{el} done on the block by the elastic force in terms of the change in elastic potential energy:

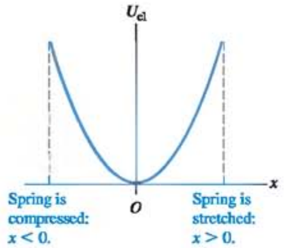
$$W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = U_{el,1} - U_{el,2} = -\Delta U_{el} \qquad (7.10)$$

When a stretched spring is stretched farther, as in Fig. 7.13b, W_{el} is negative and U_{el} *increases;* a greater amount of elastic potential energy is stored in the spring. When a stretched spring relaxes, as in Fig. 7.13c, x decreases, W_{el} is positive, and U_{el} *decreases;* the spring loses elastic potential energy. Negative values of x refer

7.13 Calculating the work done by a spring attached to a block on a horizontal surface. The quantity x is the extension or compression of the spring.



7.14 The graph of elastic potential energy for an ideal spring is a parabola: $U_{el} = \frac{1}{2}kx^2$, where x is the extension or compression of the spring. Elastic potential energy U_{el} is never negative.



to a compressed spring. But, as Fig. 7.14 shows, U_{el} is positive for both positive and negative x, and Eqs. (7.9) and (7.10) are valid for both cases. The more a spring is compressed *or* stretched, the greater its elastic potential energy.

CAUTION **Gravitational potential energy vs. elastic potential energy** An important difference between gravitational potential energy $U_{grav} = mgy$ and elastic potential energy $U_{el} = \frac{1}{2}kx^2$ is that we do *not* have the freedom to choose $x = 0$ to be wherever we wish. To be consistent with Eq. (7.9), $x = 0$ *must* be the position at which the spring is neither stretched nor compressed. At that position, its elastic potential energy and the force that it exerts are both zero.

The work–energy theorem says that $W_{tot} = K_2 - K_1$, no matter what kind of forces are acting on a body. If the elastic force is the *only* force that does work on the body, then

$$W_{tot} = W_{el} = U_{el,1} - U_{el,2}$$

The work–energy theorem $W_{tot} = K_2 - K_1$ then gives us

$$K_1 + U_{el,1} = K_2 + U_{el,2} \quad \text{(if only the elastic force does work)} \quad (7.11)$$

Here U_{el} is given by Eq. (7.9), so

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2 \quad \text{(if only the elastic force does work)} \quad (7.12)$$

In this case the total mechanical energy $E = K + U_{el}$—the sum of kinetic and *elastic* potential energy—is *conserved*. An example of this is the motion of the block in Fig. 7.13, provided the horizontal surface is frictionless so that no force does work other than that exerted by the spring.

For Eq. (7.12) to be strictly correct, the ideal spring that we've been discussing must also be *massless*. If the spring has a mass, it also has kinetic energy as the coils of the spring move back and forth. We can neglect the kinetic energy of the spring if its mass is much less than the mass m of the body attached to the spring. For instance, a typical automobile has a mass of 1200 kg or more. The springs in its suspension have masses of only a few kilograms, so their mass can be neglected if we want to study how a car bounces on its suspension.

Situations with Both Gravitational and Elastic Potential Energy

ActivPhysics ONLINE
5.4 Inverse Bungee Jumper
5.5 Spring-Launched Bowler

Equations (7.11) and (7.12) are valid when the only potential energy in the system is elastic potential energy. What happens when we have *both* gravitational and elastic forces, such as a block attached to the lower end of a vertically hanging spring? And what if work is also done by other forces that *cannot* be described in terms of potential energy, such as the force of air resistance on a moving block? Then the total work is the sum of the work done by the gravitational force (W_{grav}), the work done by the elastic force (W_{el}), and the work done by other forces (W_{other}): $W_{tot} = W_{grav} + W_{el} + W_{other}$. Then the work–energy theorem gives

$$W_{grav} + W_{el} + W_{other} = K_2 - K_1$$

The work done by the gravitational force is $W_{grav} = U_{grav,1} - U_{grav,2}$ and the work done by the spring is $W_{el} = U_{el,1} - U_{el,2}$. Hence we can rewrite the work–energy theorem for this most general case as

$$K_1 + U_{grav,1} + U_{el,1} + W_{other} = K_2 + U_{grav,2} + U_{el,2} \quad \text{(valid in general)} \quad (7.13)$$

or, equivalently,

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2 \qquad \text{(valid in general)} \qquad (7.14)$$

where $U = U_{\text{grav}} + U_{\text{el}} = mgy + \frac{1}{2}kx^2$ is the *sum* of gravitational potential energy and elastic potential energy. For short, we call U simply "the potential energy."

Equation (7.14) is *the most general statement* of the relationship among kinetic energy, potential energy, and work done by other forces. It says:

The work done by all forces other than the gravitational force or elastic force equals the change in the total mechanical energy $E = K + U$ of the system, where $U = U_{\text{grav}} + U_{\text{el}}$ is the sum of the gravitational potential energy and the elastic potential energy.

The "system" is made up of the body of mass m, the earth with which it interacts through the gravitational force, and the spring of force constant k.

If W_{other} is positive, $E = K + U$ increases; if W_{other} is negative, E decreases. If the gravitational and elastic forces are the *only* forces that do work on the body, then $W_{\text{other}} = 0$ and the total mechanical energy (including both gravitational and elastic potential energy) is conserved. (You should compare Eq. (7.14) to Eqs. (7.7) and (7.8), which describe situations in which there is gravitational potential energy but no elastic potential energy.)

Bungee jumping (Fig. 7.15) is an example of transformations among kinetic energy, elastic potential energy, and gravitational potential energy. As the jumper falls, gravitational potential energy decreases and is converted into the kinetic energy of the jumper and the elastic potential energy of the bungee cord. Beyond a certain point in the fall, the jumper's speed decreases so that both gravitational potential energy and kinetic energy are converted into elastic potential energy.

7.15 The fall of a bungee jumper involves an interplay among kinetic energy, gravitational potential energy, and elastic potential energy. Due to air resistance and frictional forces within the bungee cord, mechanical energy is not conserved. (If mechanical energy *were* conserved, the bungee jumper would keep bouncing up and down forever!)

Problem-Solving Strategy 7.2 Problems Using Mechanical Energy II

Problem-Solving Strategy 7.1 (Section 7.1) is equally useful in solving problems that involve elastic forces as well as gravitational forces. The only new wrinkle is that the potential energy U now includes the elastic potential energy $U_{\text{el}} = \frac{1}{2}kx^2$, where x is the displacement of the spring *from its unstretched length*. The work done by the gravitational and elastic forces is accounted for by their potential energies; the work of the other forces, W_{other}, has to be included separately.

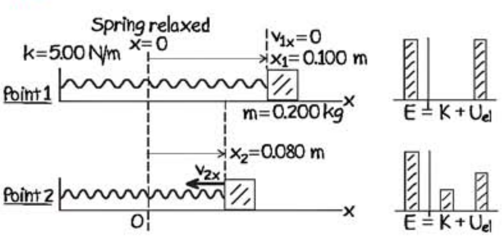
Example 7.7 Motion with elastic potential energy

A glider with mass $m = 0.200$ kg sits on a frictionless horizontal air track, connected to a spring with force constant $k = 5.00$ N/m. You pull on the glider, stretching the spring 0.100 m, and then release it with no initial velocity. The glider begins to move back toward its equilibrium position ($x = 0$). What is its x-velocity when $x = 0.080$ m?

SOLUTION

IDENTIFY: Because the spring force varies with position, this problem can't be solved with the equations for motion with constant acceleration. Instead, we'll use the idea that as the glider starts to move, elastic potential energy is converted into kinetic energy. (The glider remains at the same height throughout the motion, so gravitational potential energy is not a factor. Hence $U = U_{\text{el}} = \frac{1}{2}kx^2$.)

7.16 Our sketches and energy bar graphs for this problem.



SET UP: Figure 7.16 shows our sketches. The spring force is the only force doing work on the glider, so $W_{\text{other}} = 0$ and we may use

Continued

Eq. (7.11). We designate the point where the glider is released as point 1 and $x = 0.080$ m as point 2. We know the velocity at point 1 ($v_{1x} = 0$); our target variable is the x-velocity at point 2, v_{2x}.

EXECUTE: The energy quantities are

$$K_1 = \frac{1}{2}mv_{1x}^2 = \frac{1}{2}(0.200\text{ kg})(0)^2 = 0$$

$$U_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(5.00\text{ n/m})(0.100\text{ m})^2 = 0.0250\text{ J}$$

$$K_2 = \frac{1}{2}mv_{2x}^2$$

$$U_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(5.00\text{ N/m})(0.080\text{ m})^2 = 0.0160\text{ J}$$

Then from Eq. (7.11),

$$K_2 = K_1 + U_1 - U_2 = 0 + 0.0250\text{ J} - 0.0160\text{ J} = 0.0090\text{ J}$$

$$v_{2x} = \pm\sqrt{\frac{2K_2}{m}} = \pm\sqrt{\frac{2(0.0090\text{ J})}{0.200\text{ kg}}} = \pm 0.30\text{ m/s}$$

We choose the negative root because the glider is moving in the $-x$-direction; the answer we want is $v_{2x} = -0.30$ m/s.

EVALUATE: What is the meaning of the second solution, $v_{2x} = +0.30$ m/s? Eventually the spring will compress and push the glider back to the right in the positive x-direction (see Fig. 7.13d). The second solution tells us that when the glider passes through $x = 0.080$ m while moving to the right, its speed will be 0.30 m/s—the same speed as when it passed through this point while moving to the left.

When the glider passes through the point $x = 0$, the spring is relaxed and all of the mechanical energy is in the form of kinetic energy. Can you show that the speed of the glider at this point is 0.50 m/s?

Example 7.8 Motion with elastic potential energy and work done by other forces

For the system of Example 7.7, suppose the glider is initially at rest at $x = 0$, with the spring unstretched. You then apply a constant force $\vec{F}$ in the $+x$-direction with magnitude 0.610 N to the glider. What is the glider's velocity when it has moved to $x = 0.100$ m?

SOLUTION

IDENTIFY: Although the force $\vec{F}$ you apply is constant, the spring force isn't, so the acceleration of the glider won't be constant. Total mechanical energy is not conserved because of the work done by the force $\vec{F}$, so we must use the generalized energy relationship given by Eq. (7.13). (As in Example 7.7, we ignore gravitational potential energy because the glider's height doesn't change. Hence we have only elastic potential energy, and so $U = U_{el} = \frac{1}{2}kx^2$.)

SET UP: Let point 1 be at $x = 0$, where the velocity is $v_{1x} = 0$, and let point 2 be at $x = 0.100$ m. (These points are different from the ones labeled in Fig. 7.16.) Our target variable is v_{2x}, the velocity at point 2.

EXECUTE: The energy quantities are

$$K_1 = 0$$

$$U_1 = \frac{1}{2}kx_1^2 = 0$$

$$K_2 = \frac{1}{2}mv_{2x}^2$$

$$U_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(5.00\text{ N/m})(0.100\text{ m})^2 = 0.0250\text{ J}$$

$$W_{other} = (0.610\text{ N})(0.100\text{ m}) = 0.0610\text{ J}$$

(To calculate W_{other} we multiplied the magnitude of the force by the displacement, since both are in the $+x$-direction.) Initially, the total mechanical energy is zero; the work done by the force $\vec{F}$ increases the total mechanical energy to 0.0610 J, of which 0.0250 J is elastic potential energy. The remainder is kinetic energy. From Eq. (7.13),

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

$$K_2 = K_1 + U_1 + W_{other} - U_2$$

$$= 0 + 0 + 0.0610\text{ J} - 0.0250\text{ J} = 0.0360\text{ J}$$

$$v_{2x} = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.0360\text{ J})}{0.200\text{ kg}}} = 0.60\text{ m/s}$$

We choose the positive square root because the glider is moving in the $+x$-direction.

EVALUATE: To test our answer, think what would be different if we disconnected the glider from the spring. Then $\vec{F}$ would be the only force doing work, there would be zero potential energy at all times, and Eq. (7.13) would give us

$$K_2 = K_1 + W_{other} = 0 + 0.0610\text{ J}$$

$$v_{2x} = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.0610\text{ J})}{0.200\text{ kg}}} = 0.78\text{ m/s}$$

We found a lower velocity than this value because the spring does negative work on the glider as it stretches (see Fig. 7.13b).

If you stop pushing on the glider when it reaches the point $x = 0.100$ m, beyond that point the only force that does work on the glider is the spring force. Hence for $x > 0.100$ m, the total mechanical energy $E = K + U$ is conserved and maintains the same value of 0.0610 J. The glider will slow down as the spring continues to stretch, so the kinetic energy K will decrease as the potential energy increases. The glider will come to rest at a point $x = x_3$; at this point the kinetic energy is zero and the potential energy $U = U_{el} = \frac{1}{2}kx_3^2$ is equal to the total mechanical energy 0.0610 J. You should be able to show that the glider comes to rest at $x_3 = 0.156$ m, which means that it moves an additional 0.056 m after the force $\vec{F}$ is removed at $x_2 = 0.100$ m. (Since there's no friction, the glider will not remain at rest but will start moving back toward $x = 0$ due to the force of the stretched spring.)

Example 7.9 Motion with gravitational, elastic, and friction forces

In a "worst-case" design scenario, a 2000-kg elevator with broken cables is falling at 4.00 m/s when it first contacts a cushioning spring at the bottom of the shaft. The spring is supposed to stop the elevator, compressing 2.00 m as it does so (Fig. 7.17). During the motion a safety clamp applies a constant 17,000-N frictional force to the elevator. As a design consultant, you are asked to determine what the force constant of the spring should be.

SOLUTION

IDENTIFY: We'll use the energy approach to determine the force constant, which appears in the expression for elastic potential energy. Note that this problem involves *both* gravitational and elastic potential energy. Furthermore, total mechanical energy is not conserved because the friction force does negative work W_{other} on the elevator.

SET UP: Since mechanical energy isn't conserved and more than one kind of potential energy is involved, we'll use the most general form of the energy relationship, Eq. (7.13). We take point 1 as the position of the bottom of the elevator when it initially contacts the spring, and take point 2 as its position when it is at rest. We choose the origin to be at point 1, so $y_1 = 0$ and $y_2 = -2.00$ m. With this choice the coordinate of the upper end of the spring is the same as the coordinate of the elevator, so the elastic potential energy at any point between point 1 and point 2 is $U_{el} = \frac{1}{2}ky^2$. (The gravitational potential energy is $U_{grav} = mgy$ as usual.) We know the initial and final speeds of the elevator and the magnitude of the friction force, so the only unknown is the force constant k (our target variable).

EXECUTE: The elevator's initial speed is $v_1 = 4.00$ m/s, so the initial kinetic energy is

$$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(2000\text{ kg})(4.00\text{ m/s})^2 = 16{,}000\text{ J}$$

The elevator stops at point 2, so $K_2 = 0$. The potential energy at point 1, U_1, is zero; U_{grav} is zero because $y_1 = 0$, and $U_{el} = 0$ because the spring is not yet compressed. At point 2 there is both gravitational and elastic potential energy, so

$$U_2 = mgy_2 + \frac{1}{2}ky_2^2$$

The gravitational potential energy at point 2 is

$$mgy_2 = (2000\text{ kg})(9.80\text{ m/s}^2)(-2.00\text{ m}) = -39{,}200\text{ J}$$

The other force is the 17,000-N friction force, acting opposite to the 2.00-m displacement, so

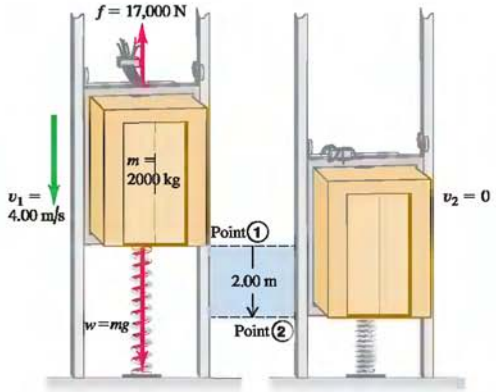
$$W_{other} = -(17{,}000\text{ N})(2.00\text{ m}) = -34{,}000\text{ J}$$

Putting these terms into $K_1 + U_1 + W_{other} = K_2 + U_2$, we have

$$K_1 + 0 + W_{other} = 0 + \left(mgy_2 + \frac{1}{2}ky_2^2\right)$$

so the force constant of the spring is

$$\begin{aligned} k &= \frac{2(K_1 + W_{other} - mgy_2)}{y_2^2} \\ &= \frac{2[16{,}000\text{ J} + (-34{,}000\text{ J}) - (-39{,}200\text{ J})]}{(-2.00\text{ m})^2} \\ &= 1.06 \times 10^4\text{ N/m} \end{aligned}$$



7.17 The fall of an elevator is stopped by a spring and by a constant friction force.

This is about one-tenth the force constant of a spring in an automobile suspension.

EVALUATE: Let's note what might seem to be a paradox in this problem. The elastic potential energy in the spring at point 2 is

$$\frac{1}{2}ky_2^2 = \frac{1}{2}(1.06 \times 10^4\text{ N/m})(-2.00\text{ m})^2 = 21{,}200\text{ J}$$

This is *more* than the total mechanical energy at point 1:

$$E_1 = K_1 + U_1 = 16{,}000\text{ J} + 0 = 16{,}000\text{ J}$$

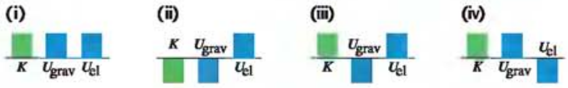
But the friction force caused the mechanical energy of the system to *decrease* by 34,000 J between point 1 and point 2. Does this mean that energy appeared from nowhere? Don't panic; there is no paradox. At point 2 there is also *negative* gravitational potential energy, $mgy_2 = -39{,}200$ J, because point 2 is below the origin. The total mechanical energy at point 2 is

$$\begin{aligned} E_2 &= K_2 + U_2 = 0 + \frac{1}{2}ky_2^2 + mgy_2 \\ &= 0 + 21{,}200\text{ J} + (-39{,}200\text{ J}) = -18{,}000\text{ J} \end{aligned}$$

This is just the initial mechanical energy of 16,000 J, minus 34,000 J lost to friction.

Will the elevator stay at the bottom of the shaft? At point 2 the compressed spring exerts an upward force of magnitude $F_{spring} = (1.06 \times 10^4\text{ N/m})(2.00\text{ m}) = 21{,}200$ N, while the downward force of gravity on the elevator is only $w = mg = (2000\text{ kg})(9.80\text{ m/s}^2) = 19{,}600$ N. So if there were no friction, there would be a net upward force of 21,200 N − 19,600 N = 1600 N and the elevator would bounce back upward. However, there *is* friction in the safety clamp, which can exert a force of as much as 17,000 N; hence the clamp can keep the elevator from rebounding.

Test Your Understanding of Section 7.2 Consider the situation in Example 7.9 at the instant when the elevator is still moving downward and the spring is compressed by 1.00 m. Which of the energy bar graphs in the figure most accurately shows the kinetic energy K, gravitational potential energy U_{grav}, and elastic potential energy U_{el} at this instant?



7.3 Conservative and Nonconservative Forces

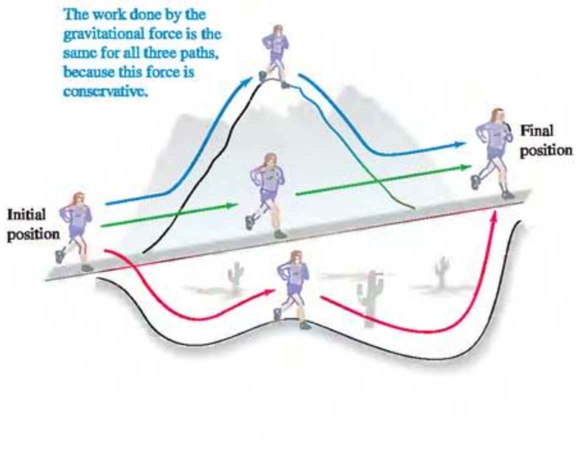
In our discussions of potential energy we have talked about "storing" kinetic energy by converting it to potential energy. We always have in mind that later we may retrieve it again as kinetic energy. For example, when you throw a ball up in the air, it slows down as kinetic energy is converted into potential energy. But on the way down, the conversion is reversed, and the ball speeds up as potential energy is converted back to kinetic energy. If there is no air resistance, the ball is moving just as fast when you catch it as when you threw it.

Another example is a glider moving on a frictionless horizontal air track that runs into a spring bumper at the end of the track. The glider stops as it compresses the spring and then bounces back. If there is no friction, the glider ends up with the same speed and kinetic energy it had before the collision. Again, there is a two-way conversion from kinetic to potential energy and back. In both cases we can define a potential-energy function so that the total mechanical energy, kinetic plus potential, is constant or *conserved* during the motion.

Conservative Forces

A force that offers this opportunity of two-way conversion between kinetic and potential energies is called a **conservative force.** We have seen two examples of conservative forces: the gravitational force and the spring force. (Later in this book we will study another conservative force, the electric force between charged objects.) An essential feature of conservative forces is that their work is always *reversible.* Anything that we deposit in the energy "bank" can later be withdrawn without loss. Another important aspect of conservative forces is that a body may move from point 1 to point 2 by various paths, but the work done by a conservative force is the same for all of these paths (Fig. 7.18). Thus, if a body

7.18 The work done by a conservative force such as gravity depends only on the end points of a path, not on the specific path taken between those points.



stays close to the surface of the earth, the gravitational force $m\vec{g}$ is independent of height, and the work done by this force depends only on the change in height. If the body moves around a closed path, ending at the same point where it started, the *total* work done by the gravitational force is always zero.

The work done by a conservative force *always* has four properties:

1. It can be expressed as the difference between the initial and final values of a *potential-energy* function.
2. It is reversible.
3. It is independent of the path of the body and depends only on the starting and ending points.
4. When the starting and ending points are the same, the total work is zero.

When the *only* forces that do work are conservative forces, the total mechanical energy $E = K + U$ is constant.

Nonconservative Forces

Not all forces are conservative. Consider the friction force acting on the crate sliding on a ramp in Example 7.6 (Section 7.1). When the body slides up and then back down to the starting point, the total work done on it by the friction force is *not* zero. When the direction of motion reverses, so does the friction force, and friction does *negative* work in *both* directions. When a car with its brakes locked skids across the pavement with decreasing speed (and decreasing kinetic energy), the lost kinetic energy cannot be recovered by reversing the motion or in any other way, and mechanical energy is *not* conserved. There is *no* potential-energy function for the friction force.

In the same way, the force of fluid resistance (see Section 5.3) is not conservative. If you throw a ball up in the air, air resistance does negative work on the ball while it's rising *and* while it's descending. The ball returns to your hand with less speed and less kinetic energy than when it left, and there is no way to get back the lost mechanical energy.

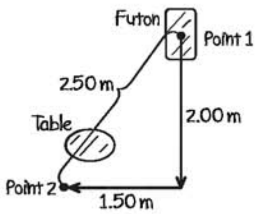
A force that is not conservative is called a **nonconservative force.** The work done by a nonconservative force *cannot* be represented by a potential-energy function. Some nonconservative forces, like kinetic friction or fluid resistance, cause mechanical energy to be lost or dissipated; a force of this kind is called a **dissipative force.** There are also nonconservative forces that *increase* mechanical energy. The fragments of an exploding firecracker fly off with very large kinetic energy, thanks to a chemical reaction of gunpowder with oxygen. The forces unleashed by this reaction are nonconservative because the process is not reversible. (The fragments never spontaneously reassemble themselves into a complete firecracker!)

Example 7.10 Frictional work depends on the path

You are rearranging your furniture and wish to move a 40.0-kg futon 2.50 m across the room. However, the straight-line path is blocked by a heavy coffee table that you don't want to move. Instead, you slide the futon in a dogleg path over the floor; the doglegs are 2.00 m and 1.50 m long. Compared to the straight-line path, how much more work must you do to push the futon in the dogleg path? The coefficient of kinetic friction is 0.200.

SOLUTION

IDENTIFY: Here work is done both by you and by the force of friction, so we must use the energy relationship that includes forces other than elastic or gravitational forces. We'll use this relationship to find a connection between the work that *you* do and the work done by *friction*.



7.19 Our sketch for this problem.

SET UP: Figure 7.19 shows our sketch. The futon is at rest at both point 1 and point 2, so $K_1 = K_2 = 0$. There is no elastic potential

Continued

energy (there are no springs), and the gravitational potential energy does not change because the futon moves only horizontally, so $U_1 = U_2$. From Eq. (7.14) it follows that $W_{other} = 0$. The other work done on the futon is the sum of the positive work you do, W_{you}, and the negative work W_{fric} done by the kinetic friction force. Since the sum of these is zero, we have

$$W_{you} = -W_{fric}$$

Thus to determine W_{you}, we'll calculate the work done by friction.

EXECUTE: Because the floor is horizontal, the normal force on the futon equals its weight mg, and the magnitude of the friction force is $f_k = \mu_k n = \mu_k mg$. The work you must do over each path is then

$$\begin{aligned} W_{you} &= -W_{fric} = -(-f_k s) = +\mu_k mgs \\ &= (0.200)(40.0\,\text{kg})(9.80\,\text{m/s}^2)(2.50\,\text{m}) \\ &= 196\,\text{J} \quad \text{(straight-line path)} \end{aligned}$$

$$\begin{aligned} W_{you} &= -W_{fric} \\ &= (0.200)(40.0\,\text{kg})(9.80\,\text{m/s}^2)(2.00\,\text{m} + 1.50\,\text{m}) \\ &= 274\,\text{J} \quad \text{(dogleg path)} \end{aligned}$$

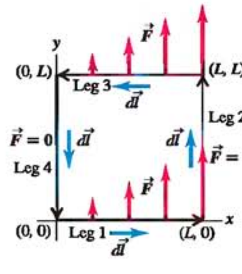
The extra work you must do is $274\,\text{J} - 196\,\text{J} = 78\,\text{J}$.

EVALUATE: The work done by friction is $W_{fric} = -W_{you} = -196\,\text{J}$ on the straight-line path and $-274\,\text{J}$ on the dogleg path. The work done by friction depends on the path taken, which illustrates that friction is a *nonconservative* force.

Example 7.11 Conservative or nonconservative?

In a certain region of space the force on an electron is $\vec{F} = Cx\hat{j}$, where C is a positive constant. The electron moves in a counterclockwise direction around a square loop in the xy-plane (Fig. 7.20). The corners of the square are at $(x, y) = (0, 0)$, $(L, 0)$, (L, L), and $(0, L)$. Calculate the work done on the electron by the force $\vec{F}$ during one complete trip around the square. Is this force conservative or nonconservative?

7.20 An electron moving around a square loop while being acted on by the force $\vec{F} = Cx\hat{j}$.



SOLUTION

IDENTIFY: In Example 7.10 the force of friction was constant in magnitude and always opposite to the displacement, so it was easy to calculate the work done. Here, however, the force $\vec{F}$ is not constant and in general is not in the same direction as the displacement.

SET UP: To calculate the work done by the force $\vec{F}$, we'll use the more general expression for work, Eq. (6.14):

$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l}$$

where $d\vec{l}$ is an infinitesimal displacement. Let's calculate the work done on each leg of the square and then add the results to find the work done on the round trip.

EXECUTE: On the first leg, from $(0, 0)$ to $(L, 0)$, the force varies but is everywhere perpendicular to the displacement. So $\vec{F} \cdot d\vec{l} = 0$, and the work done on the first leg is $W_1 = 0$. The force has the same value $\vec{F} = CL\hat{j}$ everywhere on the second leg from $(L, 0)$ to (L, L). The displacement on this leg is in the $+y$-direction, so $d\vec{l} = dy\hat{j}$ and

$$\vec{F} \cdot d\vec{l} = CL\hat{j} \cdot dy\hat{j} = CL\,dy$$

The work done on the second leg is then

$$W_2 = \int_{(L,0)}^{(L,L)} \vec{F} \cdot d\vec{l} = \int_{y=0}^{y=L} CL\,dy = CL\int_0^L dy = CL^2$$

On the third leg, from (L, L) to $(0, L)$, $\vec{F}$ is again perpendicular to the displacement so $W_3 = 0$. The force is zero on the final leg, from $(0, L)$ to $(0, 0)$, so no work is done and $W_4 = 0$. The work done by the force $\vec{F}$ on the round trip is

$$W = W_1 + W_2 + W_3 + W_4 = 0 + CL^2 + 0 + 0 = CL^2$$

The starting and ending points are the same, but the total work done by $\vec{F}$ is not zero. This is a nonconservative force; it *cannot* be represented by a potential-energy function.

EVALUATE: Because W is positive, the mechanical energy *increases* as the electron goes around the loop. This is not a mathematical curiosity; it's a description of what happens in an electrical generating plant. A loop of wire is moved through a magnetic field, which gives rise to a nonconservative force similar to the one in this example. Electrons in the wire gain energy as they move around the loop, and this energy is carried via transmission lines to the consumer. (We'll discuss how this works in detail in Chapter 29.)

If the electron went around the loop clockwise instead of counterclockwise, the force $\vec{F}$ would be unaffected but the direction of each infinitesimal displacement $d\vec{l}$ would reverse. Thus the sign of work would also reverse, and the work for a clockwise round trip would be $W = -CL^2$. This is a different behavior than the nonconservative friction force. When a body slides over a stationary surface with friction, the work done by friction is always negative, no matter what the direction of motion (see Example 7.6 in Section 7.1).

The Law of Conservation of Energy

Nonconservative forces cannot be represented in terms of potential energy. But we can describe the effects of these forces in terms of kinds of energy other than kinetic and potential energy. When a car with locked brakes skids to a stop, the tires and the road surface both become hotter. The energy associated with this change in the state of the materials is called **internal energy.** Raising the temperature of a body increases its internal energy; lowering the body's temperature decreases its internal energy.

ActivPhysics ONLINE
5.7 Modified Atwood Machine

To see the significance of internal energy, let's consider a block sliding on a rough surface. Friction does *negative* work on the block as it slides, and the change in internal energy of the block and surface (both of which get hotter) is *positive.* Careful experiments show that the increase in the internal energy is *exactly* equal to the absolute value of the work done by friction. In other words,

$$\Delta U_{\text{int}} = -W_{\text{other}}$$

where ΔU_{int} is the change in internal energy. If we substitute this into Eq. (7.7) or (7.14), we find

$$K_1 + U_1 - \Delta U_{\text{int}} = K_2 + U_2$$

Writing $\Delta K = K_2 - K_1$ and $\Delta U = U_2 - U_1$, we can finally express this as

$$\Delta K + \Delta U + \Delta U_{\text{int}} = 0 \quad \text{(law of conservation of energy)} \quad (7.15)$$

This remarkable statement is the general form of the **law of conservation of energy.** In a given process, the kinetic energy, potential energy, and internal energy of a system may all change. But the *sum* of those changes is always zero. If there is a decrease in one form of energy, it is made up for by an increase in the other forms (Fig. 7.21). When we expand our definition of energy to include internal energy, Eq. (7.15) says: *Energy is never created or destroyed; it only changes form.* No exception to this rule has ever been found.

7.21 When 1 liter of gasoline is burned in an automotive engine, it releases 3.3×10^7 J of internal energy. Hence $\Delta U_{\text{int}} = -3.3 \times 10^7$ J, where the minus sign means that the amount of energy stored in the gasoline has decreased. This energy can be converted into kinetic energy (making the car go faster) or into potential energy (enabling the car to climb uphill).

? The concept of work has been banished from Eq. (7.15); instead, it suggests that we think purely in terms of the conversion of energy from one form to another. For example, when you throw a baseball straight up, you convert a portion of the internal energy of your molecules into kinetic energy of the baseball. This is converted into gravitational potential energy as the ball climbs and back to kinetic energy as the ball falls. If there is air resistance, part of the energy is used to heat up the air and the ball and increase their internal energy. Energy is converted back into the kinetic form as the ball falls. If you catch the ball in your hand, whatever energy was not lost to the air once again becomes internal energy; the ball and your hand are now warmer than they were at the beginning.

In Chapters 19 and 20, we will study the relationship of internal energy to temperature changes, heat, and work. This is the heart of the area of physics called *thermodynamics.*

Example 7.12 Work done by friction

Let's look again at Example 7.5 (Section 7.1), in which your cousin Throcky skateboards down a curved ramp. He starts with zero kinetic energy and 735 J of potential energy, and at the bottom he has 450 J of kinetic energy and zero potential energy. So $\Delta K = +450$ J and $\Delta U = -735$ J. The work $W_{\text{other}} = W_{\text{fric}}$ done by the nonconservative friction forces is -285 J, so the change in internal energy is $\Delta U_{\text{int}} = -W_{\text{other}} = +285$ J. The wheels, the bearings, and the ramp all get a little warmer as Throcky rolls down. In accordance with Eq. (7.15), the sum of the energy changes equals zero:

$$\Delta K + \Delta U + \Delta U_{\text{int}} = +450\text{ J} + (-735\text{ J}) + 285\text{ J} = 0$$

The total energy of the system (including nonmechanical forms of energy) is conserved.

Test Your Understanding of Section 7.3 In a hydroelectric generating station, falling water is used to drive turbines ("water wheels"), which in turn run electric generators. Compared to the amount of gravitational potential energy released by the falling water, how much electrical energy is produced? (i) the same; (ii) more; (iii) less.

7.4 Force and Potential Energy

For the two kinds of conservative forces (gravitational and elastic) we have studied, we started with a description of the behavior of the *force* and derived from that an expression for the *potential energy.* For example, for a body with mass m in a uniform gravitational field, the gravitational force is $F_y = -mg$. We found that the corresponding potential energy is $U(y) = mgy$. To stretch an ideal spring by a distance x, we exert a force equal to $+kx$. By Newton's third law the force that an ideal spring exerts on a body is opposite this, or $F_x = -kx$. The corresponding potential energy function is $U(x) = \frac{1}{2}kx^2$.

In studying physics, however, you'll encounter situations in which you are given an expression for the *potential energy* as a function of position and have to find the corresponding *force.* We'll see several examples of this kind when we study electric forces later in this book: it's often far easier to calculate the electric potential energy first and then determine the corresponding electric force afterward.

Here's how we find the force that corresponds to a given potential-energy expression. First let's consider motion along a straight line, with coordinate x. We denote the x-component of force, a function of x, by $F_x(x)$, and the potential energy as $U(x)$. This notation reminds us that both F_x and U are *functions* of x. Now we recall that in any displacement, the work W done by a conservative force equals the negative of the change ΔU in potential energy:

$$W = -\Delta U$$

Let's apply this to a small displacement Δx. The work done by the force $F_x(x)$ during this displacement is approximately equal to $F_x(x)\,\Delta x$. We have to say "approximately" because $F_x(x)$ may vary a little over the interval Δx. But it is at least approximately true that

$$F_x(x)\,\Delta x = -\Delta U \quad \text{and} \quad F_x(x) = -\frac{\Delta U}{\Delta x}$$

You can probably see what's coming. We take the limit as $\Delta x \rightarrow 0$; in this limit, the variation of F_x becomes negligible, and we have the exact relationship

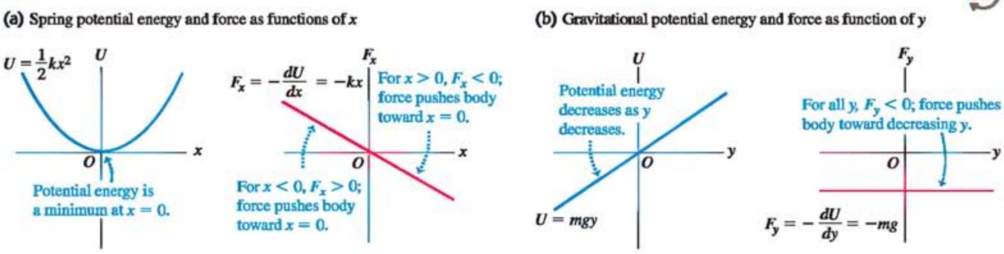
$$F_x(x) = -\frac{dU(x)}{dx} \quad \text{(force from potential energy, one dimension)} \quad (7.16)$$

This result makes sense; in regions where $U(x)$ changes most rapidly with x (that is, where $dU(x)/dx$ is large), the greatest amount of work is done during a given displacement, and this corresponds to a large force magnitude. Also, when $F_x(x)$ is in the positive x-direction, $U(x)$ *decreases* with increasing x. So $F_x(x)$ and $dU(x)/dx$ should indeed have opposite signs. The physical meaning of Eq. (7.16) is that *a conservative force always acts to push the system toward lower potential energy.*

As a check, let's consider the function for elastic potential energy, $U(x) = \frac{1}{2}kx^2$. Substituting this into Eq. (7.16) yields

$$F_x(x) = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

7.22 A conservative force is the negative derivative of the corresponding potential energy.



which is the correct expression for the force exerted by an ideal spring (Fig. 7.22a). Similarly, for gravitational potential energy we have $U(y) = mgy$; taking care to change x to y for the choice of axis, we get $F_y = -dU/dy = -d(mgy)/dy = -mg$, which is the correct expression for gravitational force (Fig. 7.22b).

Example 7.13 An electric force and its potential energy

An electrically charged particle is held at rest at the point $x = 0$, while a second particle with equal charge is free to move along the positive x-axis. The potential energy of the system is

$$U(x) = \frac{C}{x}$$

where C is a positive constant that depends on the magnitude of the charges. Derive an expression for the x-component of force acting on the movable charged particle, as a function of its position.

SOLUTION

IDENTIFY: We are given the potential-energy function $U(x)$, and we want to find the force function $F_x(x)$.

SET UP: We'll use Eq. (7.16), $F_x(x) = -dU(x)/dx$.

EXECUTE: The derivative with respect to x of the function $1/x$ is $-1/x^2$. So the force on the movable charged particle for $x > 0$ is

$$F_x(x) = -\frac{dU(x)}{dx} = -C\left(-\frac{1}{x^2}\right) = \frac{C}{x^2}$$

EVALUATE: The x-component of force is positive, corresponding to a repulsion between like electric charges. The potential energy is very large when the particles are close together (small x) and approaches zero as the particles move farther apart (large x); the force pushes the movable particle toward large positive values of x, for which the potential energy is less. The force $F_x(x) = C/x^2$ gets weaker as the particles move farther apart (x increases). We'll study electric forces in greater detail in Chapter 21.

Force and Potential Energy in Three Dimensions

We can extend this analysis to three dimensions, where the particle may move in the x-, y-, or z-direction, or all at once, under the action of a conservative force that has components F_x, F_y, and F_z. Each component of force may be a function of the coordinates x, y, and z. The potential-energy function U is also a function of all three space coordinates. We can now use Eq. (7.16) to find each component of force. The potential-energy change ΔU when the particle moves a small distance Δx in the x-direction is again given by $-F_x \Delta x$; it doesn't depend on F_y and F_z, which represent force components that are perpendicular to the displacement and do no work. So we again have the approximate relationship

$$F_x = -\frac{\Delta U}{\Delta x}$$

The y- and z-components of force are determined in exactly the same way:

$$F_y = -\frac{\Delta U}{\Delta y} \qquad F_z = -\frac{\Delta U}{\Delta z}$$

To make these relationships exact, we take the limits $\Delta x \rightarrow 0$, $\Delta y \rightarrow 0$, and $\Delta z \rightarrow 0$ so that these ratios become derivatives. Because U may be a function of all three coordinates, we need to remember that when we calculate each of these derivatives, only one coordinate changes at a time. We compute the derivative of U with respect to x by assuming that y and z are constant and only x varies, and so on. Such a derivative is called a *partial derivative*. The usual notation for a partial derivative is $\partial U/\partial x$ and so on; the symbol ∂ is a modified d. So we write

$$F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \quad F_z = -\frac{\partial U}{\partial z} \quad \text{(force from potential energy)} \tag{7.17}$$

We can use unit vectors to write a single compact vector expression for the force $\vec{F}$:

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right) \quad \text{(force from potential energy)} \tag{7.18}$$

The expression inside the parentheses represents a particular operation on the function U, in which we take the partial derivative of U with respect to each coordinate, multiply by the corresponding unit vector, and then take the vector sum. This operation is called the **gradient** of U and is often abbreviated as $\vec{\nabla}U$. Thus the force is the negative of the gradient of the potential-energy function:

$$\vec{F} = -\vec{\nabla}U \tag{7.19}$$

As a check, let's substitute into Eq. (7.19) the function $U = mgy$ for gravitational potential energy:

$$\vec{F} = -\vec{\nabla}(mgy) = -\left(\frac{\partial(mgy)}{\partial x}\hat{\imath} + \frac{\partial(mgy)}{\partial y}\hat{\jmath} + \frac{\partial(mgy)}{\partial z}\hat{k}\right) = (-mg)\hat{\jmath}$$

This is just the familiar expression for the gravitational force.

Example 7.14 Force and potential energy in two dimensions

A puck slides on a level, frictionless air-hockey table. The coordinates of the puck are x and y. It is acted on by a conservative force described by the potential-energy function

$$U(x, y) = \frac{1}{2}k(x^2 + y^2)$$

Derive an expression for the force acting on the puck, and find an expression for the magnitude of the force as a function of position.

SOLUTION

IDENTIFY: Starting with the function $U(x, y)$, we need to find the vector components and magnitude of the corresponding conservative force $\vec{F}$.

SET UP: We'll find the components of the force from $U(x, y)$ using Eq. (7.18). This function doesn't depend on z, so the partial derivative of U with respect to z is $\partial U/\partial z = 0$ and the force has no z-component. We'll then determine the magnitude of the force using the formula for the magnitude of a vector: $F = \sqrt{F_x^2 + F_y^2}$.

EXECUTE: The x- and y-components of the force are

$$F_x = -\frac{\partial U}{\partial x} = -kx \qquad F_y = -\frac{\partial U}{\partial y} = -ky$$

From Eq. (7.18) this corresponds to the vector expression

$$\vec{F} = -k(x\hat{\imath} + y\hat{\jmath})$$

Now $x\hat{\imath} + y\hat{\jmath}$ is just the position vector $\vec{r}$ of the particle, so we can rewrite this expression as $\vec{F} = -k\vec{r}$. This represents a force that at each point is opposite in direction to the position vector of the point—that is, a force that at each point is directed toward the origin. The potential energy is minimum at the origin, so again the force pushes in the direction of decreasing potential energy.

The *magnitude* of the force at any point is

$$F = \sqrt{(-kx)^2 + (-ky)^2} = k\sqrt{x^2 + y^2} = kr$$

where r is the particle's distance from the origin. This is the force that would be exerted on the puck if it were attached to one end of a spring that obeys Hooke's law and has a negligibly small length (compared to the other distances in the problem) when it is not stretched. (The other end is attached to the air-hockey table at the origin.)

EVALUATE: To check our result, note that the potential-energy function can also be expressed as $U = \frac{1}{2}kr^2$. Written this way, U is a function of a single coordinate r, so we can find the force using Eq. (7.16) with x replaced by r:

$$F_r = -\frac{dU}{dr} = -\frac{d}{dr}\left(\frac{1}{2}kr^2\right) = -kr$$

Just as we calculated above, the force has magnitude kr; the minus sign indicates that the force is radially inward (toward the origin).

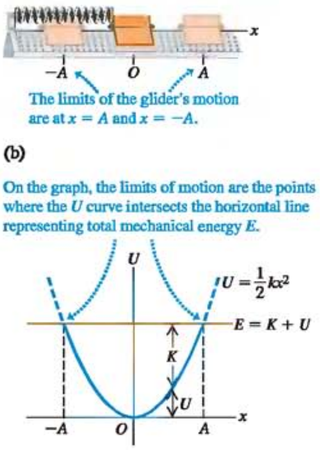
Test Your Understanding of Section 7.4 A particle moving along the x-axis is acted on by a conservative force F_x. At a certain point, the force is zero. (a) Which of the following statements about the value of the potential-energy function $U(x)$ at that point is correct? (i) $U(x) = 0$; (ii) $U(x) > 0$; (iii) $U(x) < 0$; (iv) not enough information is given to decide. (b) Which of the following statements about the value of the derivative of $U(x)$ at that point is correct? (i) $dU(x)/dx = 0$; (ii) $dU(x)/dx > 0$; (iii) $dU(x)/dx < 0$; (iv) not enough information is given to decide.

7.5 Energy Diagrams

When a particle moves along a straight line under the action of a conservative force, we can get a lot of insight into its possible motions by looking at the graph of the potential-energy function $U(x)$. Figure 7.23a shows a glider with mass m that moves along the x-axis on an air track. The spring exerts on the glider a force with x-component $F_x = -kx$. Figure 7.23b is a graph of the corresponding potential-energy function $U(x) = \frac{1}{2}kx^2$. If the elastic force of the spring is the *only* horizontal force acting on the glider, the total mechanical energy $E = K + U$ is constant, independent of x. A graph of E as a function of x is thus a straight horizontal line. We use the term **energy diagram** for a graph like this, which shows both the potential-energy function $U(x)$ and the energy of the particle subjected to the force that corresponds to $U(x)$.

7.23 (a) A glider on an air track. The spring exerts a force $F_x = -kx$. (b) The potential-energy function.

(a)



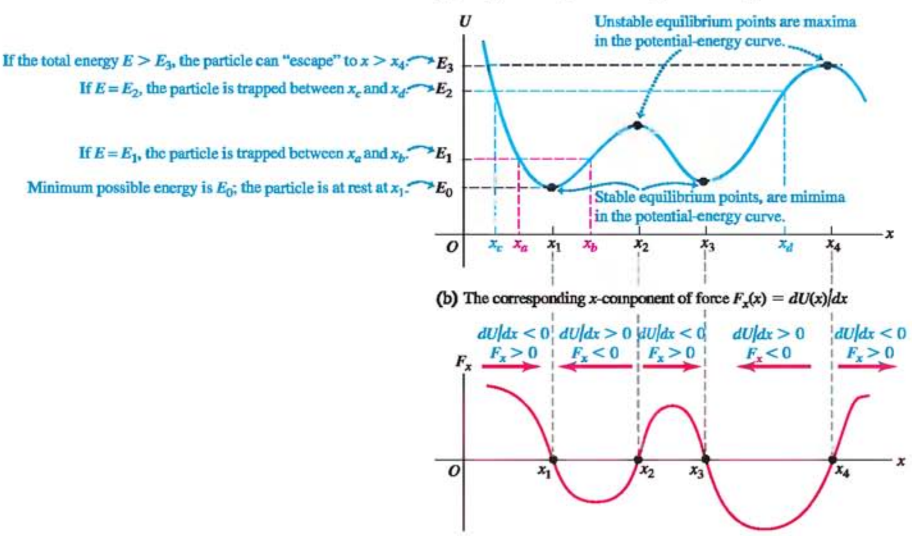
The vertical distance between the U and E graphs at each point represents the difference $E - U$, equal to the kinetic energy K at that point. We see that K is greatest at $x = 0$. It is zero at the values of x where the two graphs cross, labeled A and $-A$ in the diagram. Thus the speed v is greatest at $x = 0$, and it is zero at $x = \pm A$, the points of *maximum* possible displacement from $x = 0$ for a given value of the total energy E. The potential energy U can never be greater than the total energy E; if it were, K would be negative, and that's impossible. The motion is a back-and-forth oscillation between the points $x = A$ and $x = -A$.

At each point, the force F_x on the glider is equal to the negative of the slope of the $U(x)$ curve: $F_x = -dU/dx$ (see Fig. 7.22a). When the particle is at $x = 0$, the slope and the force are zero, so this is an *equilibrium* position. When x is positive, the slope of the $U(x)$ curve is positive and the force F_x is negative, directed toward the origin. When x is negative, the slope is negative and F_x is positive, again toward the origin. Such a force is called a *restoring force;* when the glider is displaced to either side of $x = 0$, the force tends to "restore" it back to $x = 0$. An analogous situation is a marble rolling around in a round-bottomed bowl. We say that $x = 0$ is a point of **stable equilibrium.** More generally, *any minimum in a potential-energy curve is a stable equilibrium position.*

Figure 7.24a shows a hypothetical but more general potential-energy function $U(x)$. Figure 7.24b shows the corresponding force $F_x = -dU/dx$. Points x_1 and x_3 are stable equilibrium points. At each of these points, F_x is zero because the slope of the $U(x)$ curve is zero. When the particle is displaced to either side, the force pushes back toward the equilibrium point. The slope of the $U(x)$ curve is also zero at points x_2 and x_4, and these are also equilibrium points. But when the particle is displaced a little to the right of either point, the slope of the $U(x)$ curve becomes negative, corresponding to a positive F_x that tends to push the particle still farther from the point. When the particle is displaced a little to the left, F_x is negative, again pushing away from equilibrium. This is analogous to a marble rolling on the top of a bowling ball. Points x_2 and x_4 are called **unstable equilibrium** points; *any maximum in a potential-energy curve is an unstable equilibrium position.*

CAUTION **Potential energy and the direction of a conservative force** The direction of the force on a body is *not* determined by the sign of the potential energy U. Rather, it's the sign of $F_x = -dU/dx$ that matters. As we discussed in Section 7.1, the physically significant quantity is the *difference* in the value of U between two points, which is just

7.24 The maxima and minima of a potential-energy function $U(x)$ correspond to points where $F_x = 0$.



what the derivative $F_x = -dU/dx$ measures. This means that you can always add a constant to the potential-energy function without changing the physics of the situation.

If the total energy is E_1 and the particle is initially near x_1, it can move only in the region between x_a and x_b determined by the intersection of the E_1 and U graphs (Fig. 7.24a). Again, U cannot be greater than E_1 because K can't be negative. We speak of the particle as moving in a *potential well,* and x_a and x_b are the *turning points* of the particle's motion (since at these points, the particle stops and reverses direction). If we increase the total energy to the level E_2, the particle can move over a wider range, from x_c to x_d. If the total energy is greater than E_3, the particle can "escape" and move to indefinitely large values of x. At the other extreme, E_0 represents the least possible total energy the system can have.

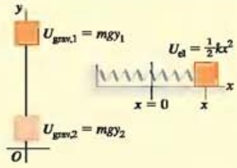
Test Your Understanding of Section 7.5 The curve in Fig. 7.24b has a maximum at a point between x_2 and x_3. Which statement correctly describes what happens to the particle when it is at this point? (i) The particle's acceleration is zero. (ii) The particle accelerates in the positive x-direction; the magnitude of the acceleration is less than at any other point between x_2 and x_3. (iii) The particle accelerates in the positive x-direction; the magnitude of the acceleration is greater than at any other point between x_2 and x_3. (iv) The particle accelerates in the negative x-direction; the magnitude of the acceleration is less than at any other point between x_2 and x_3. (v) The particle accelerates in the negative x-direction; the magnitude of the acceleration is greater than at any other point between x_2 and x_3.

CHAPTER 7 SUMMARY

Gravitational potential energy and elastic potential energy: The work done on a particle by a constant gravitational force can be represented as a change in the gravitational potential energy $U_{grav} = mgy$. This energy is a shared property of the particle and the earth. A potential energy is also associated with the elastic force $F_x = -kx$ exerted by an ideal spring, where x is the amount of stretch or compression. The work done by this force can be represented as a change in the elastic potential energy of the spring, $U_{el} = \frac{1}{2}kx^2$.

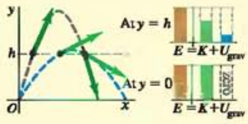
$$W_{grav} = mgy_1 - mgy_2 = U_{grav,1} - U_{grav,2} = -\Delta U_{grav} \qquad (7.1), (7.3)$$

$$W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = U_{el,1} - U_{el,2} = -\Delta U_{el} \qquad (7.10)$$



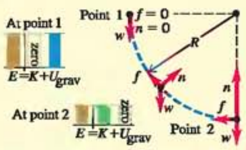
When total mechanical energy is conserved: The total potential energy U is the sum of the gravitational and elastic potential energy: $U = U_{grav} + U_{el}$. If no forces other than the gravitational and elastic forces do work on a particle, the sum of kinetic and potential energy is conserved. This sum $E = K + U$ is called the total mechanical energy. (See Examples 7.1, 7.3, 7.4, and 7.7.)

$$K_1 + U_1 = K_2 + U_2 \qquad (7.4), (7.11)$$



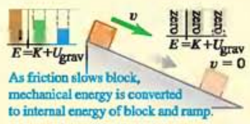
When total mechanical energy is not conserved: When forces other than the gravitational and elastic forces do work on a particle, the work W_{other} done by these other forces equals the change in total mechanical energy (kinetic energy plus total potential energy). (See Examples 7.2, 7.5, 7.6, 7.8, and 7.9.)

$$K_1 + U_1 + W_{other} = K_2 + U_2 \qquad (7.14)$$



Conservative forces, nonconservative forces, and the law of conservation of energy: All forces are either conservative or nonconservative. A conservative force is one for which the work–kinetic energy relationship is completely reversible. The work of a conservative force can always be represented by a potential-energy function, but the work of a nonconservative force cannot. The work done by nonconservative forces manifests itself as changes in the internal energy of bodies. The sum of kinetic, potential, and internal energy is always conserved. (See Examples 7.10–7.12.)

$$\Delta K + \Delta U + \Delta U_{int} = 0 \qquad (7.15)$$



Determining force from potential energy: For motion along a straight line, a conservative force $F_x(x)$ is the negative derivative of its associated potential-energy function U. In three dimensions, the components of a conservative force are negative partial derivatives of U. (See Examples 7.13 and 7.14.)

$$F_x(x) = -\frac{dU(x)}{dx} \qquad (7.16)$$

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial y} \qquad (7.17)$$

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right) \qquad (7.18)$$

Key Terms

Answer to Chapter Opening Question ?

Gravity is doing positive work on the diver, since this force is in the same downward direction as his displacement. This corresponds to a decrease in gravitational potential energy. The water is doing negative work on the diver; it exerts an upward force of fluid resistance as he moves downward. This corresponds to an increase in internal energy of the diver and the water (see Section 7.3).

Answers to Test Your Understanding Questions

7.1 Answer: (iii) The initial kinetic energy $K_1 = 0$, the initial potential energy $U_1 = mgy_1$, and the final potential energy $U_2 = mgy_2$ are the same for both blocks. Mechanical energy is conserved in both cases, so the final kinetic energy $K = \frac{1}{2}mv_2^2$ is also the same for both blocks. Hence the speed at the right-hand end is the *same* in both cases!

7.2 Answer: (iii) The elevator is still moving downward, so the kinetic energy K is positive (remember that K can never be negative); the elevator is below point 1, so $y < 0$ and $U_{grav} < 0$; and the spring is compressed, so $U_{el} > 0$.

7.3 Answer: (iii) Because of friction in the turbines and between the water and turbines, some of the potential energy goes into raising the temperatures of the water and the mechanism.

7.4 Answers: (a) (iv), (b) (i) If $F_x = 0$ at a point, then the derivative of $U(x)$ must be zero at that point because $F_x = -dU(x)/dx$. However, this tells us absolutely nothing about the *value* of $U(x)$ at that point.

7.5 Answers: (iii) Figure 7.24b shows the *x*-component of force, F_x. Where this is maximum (most positive), the *x*-component of force and the *x*-acceleration have more positive values than at adjacent values of *x*.

PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com (MP)

Discussion Questions

Q7.1. A baseball is thrown straight up with initial speed v_0. If air resistance cannot be ignored, when the ball returns to its initial height its speed is less than v_0. Explain why, using energy concepts.

Q7.2. A projectile has the same initial kinetic energy no matter what the angle of projection. Why doesn't it rise to the same maximum height in each case?

Q7.3. Does an object's speed at the bottom of a frictionless ramp depend on the shape of the ramp or just on its height? Explain. What if the ramp is *not* frictionless?

Q7.4. An egg is released from rest from the roof of a building and falls to the ground. Its fall is observed by a student on the roof of the building, who uses coordinates with origin at the roof, and by a student on the ground, who uses coordinates with origin at the ground. Do the two students assign the same or different values to the initial gravitational potential energy, the final gravitational potential energy, the change in gravitational potential energy, and the kinetic energy of the egg just before it strikes the ground? Explain.

Q7.5. A physics teacher had a bowling ball suspended from a very long rope attached to the high ceiling of a large lecture hall. To illustrate his faith in conservation of energy, he would back up to one side of the stage, pull the ball far to one side until the taut rope brought it just to the end of his nose, and then release it. The massive ball would swing in a mighty arc across the stage and then return to stop momentarily just in front of the nose of the stationary, unflinching teacher. However, one day after the demonstration he looked up just in time to see a student at the other side of the stage *push* the ball away from his nose as he tried to duplicate the demonstration. Tell the rest of the story and explain the reason for the potentially tragic outcome.

Q7.6. Lost Energy? The principle of the conservation of energy tells us that energy is never lost, but only changes from one form to another. Yet in many ordinary situations, energy may appear to be lost. In each case, explain what happens to the "lost" energy. (a) A box sliding on the floor comes to a halt due to friction. How did friction take away its kinetic energy, and what happened to that energy? (b) A car stops when you apply the brakes. What happened to its kinetic energy? (c) Air resistance uses up some of the original gravitational potential energy of a falling object. What type of energy did the "lost" potential energy become? (d) When a returning space shuttle touches down on the runway, it has lost almost all its kinetic energy and gravitational potential energy. Where did all that energy go?

Q7.7. Is it possible for a frictional force to *increase* the mechanical energy of a system? If so, give examples.

Q7.8. A woman bounces on a trampoline, going a little higher with each bounce. Explain how she increases the total mechanical energy.

Q7.9. Fractured Physics. People often call their electric bill a *power* bill, yet the quantity on which the bill is based is expressed in *kilowatt-hours*. What are people really being billed for?

Q7.10. A rock of mass m and a rock of mass $2m$ are both released from rest at the same height and feel no air resistance as they fall. Which statements about these rocks are true? (There may be more than one correct choice.) (a) Both have the same initial gravitational potential energy. (b) Both have the same kinetic energy when they reach the ground. (c) Both reach the ground with the same speed. (d) When it reaches the ground, the heavier rock has twice the kinetic energy of the lighter one. (e) When it reaches the ground, the heavier rock has four times the kinetic energy of the lighter one.

Q7.11. On a friction-free ice pond, a hockey puck is pressed against (but not attached to) a fixed ideal spring, compressing the spring by a distance x_0. The maximum energy stored in the spring is U_0, the maximum speed the puck gains after being released is v_0, and its maximum kinetic energy is K_0. Now the puck is pressed so it compresses the spring twice as far as before. In this case, (a) what is the maximum potential energy stored in the spring (in terms of U_0), and (b) what are the puck's maximum kinetic energy and speed (in terms of K_0 and x_0)?

Q7.12. When people are cold, they often rub their hands together to warm them up. How does doing this produce heat? Where did the heat come from?

Q7.13. You often hear it said that most of our energy ultimately comes from the sun. Trace each of the following energies back to the sun. (a) the kinetic energy of a jet plane; (b) the potential energy gained by a mountain climber; (c) the electrical energy used to run a computer; (d) the electrical energy from a hydroelectric plant.

Q7.14. A box slides down a ramp and work is done on the box by the forces of gravity and friction. Can the work of each of these forces be expressed in terms of the change in a potential-energy function? For each force explain why or why not.

Q7.15. In physical terms, explain why friction is a nonconservative force. Does it store energy for future use?

Q7.16. A compressed spring is clamped in its compressed position and then is dissolved in acid. What becomes of its potential energy?

Q7.17. Since only changes in potential energy are important in any problem, a student decides to let the elastic potential energy of a spring be zero when the spring is stretched a distance x_1. The student decides, therefore, to let $U = \frac{1}{2}k(x - x_1)^2$. Is this correct? Explain.

Q7.18. Figure 7.22a shows the potential-energy function for the force $F_x = -kx$. Sketch the potential-energy function for the force $F_x = +kx$. For this force, is $x = 0$ a point of equilibrium? Is this equilibrium stable or unstable? Explain.

Q7.19. Figure 7.22b shows the potential-energy function associated with the gravitational force between an object and the earth. Use this graph to explain why objects always fall toward the earth when they are released.

Q7.20. For a system of two particles we often let the potential energy for the force between the particles approach zero as the separation of the particles approaches infinity. If this choice is made, explain why the potential energy at noninfinite separation is positive if the particles repel one another and negative if they attract.

Q7.21. Explain why the points $x = A$ and $x = -A$ in Fig. 7.23b are called *turning points*. How are the values of E and U related at a turning point?

Q7.22. A particle is in *neutral equilibrium* if the net force on it is zero and remains zero if the particle is displaced slightly in any direction. Sketch the potential-energy function near a point of neutral equilibrium, for the case of one-dimensional motion. Give an example of an object in neutral equilibrium.

Q7.23. The net force on a particle of mass m has the potential-energy function graphed in Fig. 7.24a. If the total energy is E_1, graph the speed v of the particle versus its position x. At what value of x is the speed greatest? Sketch v versus x if the total energy is E_2.

Q7.24. The potential-energy function for a force $\vec{F}$ is $U = \alpha x^3$, where α is a positive constant. What is the direction of $\vec{F}$?

Exercises

Section 7.1 Gravitational Potential Energy

7.1. In one day, a 75-kg mountain climber ascends from the 1500-m level on a vertical cliff to the top at 2400 m. The next day, she descends from the top to the base of the cliff, which is at an elevation of 1350 m. What is her change in gravitational potential energy (a) on the first day and (b) on the second day?

7.2. A 5.00-kg sack of flour is lifted vertically at a constant speed of 3.50 m/s through a height of 15.0 m. (a) How great a force is required? (b) How much work is done on the sack by the lifting force? What becomes of this work?

7.3. A 120-kg mail bag hangs by a vertical rope 3.5 m long. A postal worker then displaces the bag to a position 2.0 m sideways from its original position, always keeping the rope taut. (a) What horizontal force is necessary to hold the bag in the new position? (b) As the bag is moved to this position, how much work is done (i) by the rope and (ii) by the worker?

7.4. A 72.0-kg swimmer jumps into the old swimming hole from a diving board 3.25 m above the water. Use energy conservation to find his speed just he hits the water (a) if he just holds his nose and drops in, (b) if he bravely jumps straight up (but just beyond the board!) at 2.50 m/s, and (c) if he manages to jump downward at 2.50 m/s.

7.5. A baseball is thrown from the roof of a 22.0-m-tall building with an initial velocity of magnitude 12.0 m/s and directed at an angle of 53.1° above the horizontal. (a) What is the speed of the ball just before it strikes the ground? Use energy methods and ignore air resistance. (b) What is the answer for part (a) if the initial velocity is at an angle of 53.1° *below* the horizontal? (c) If the effects of air resistance are included, will part (a) or (b) give the higher speed?

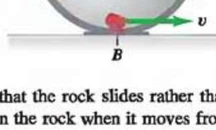
7.6. A crate of mass M starts from rest at the top of a frictionless ramp inclined at an angle α above the horizontal. Find its speed at the bottom of the ramp, a distance d from where it started. Do this in two ways: (a) Take the level at which the potential energy is zero to be at the bottom of the ramp with y positive upward. (b) Take the zero level for potential energy to be at the top of the ramp with y positive upward. (c) Why did the normal force not enter into your solution?

7.7. Answer part (b) of Example 7.6 (Section 7.1) by applying Eq. (7.7) to points 2 and 3, rather than to points 1 and 3 as was done in the example.

7.8. An empty crate is given an initial push down a ramp, starting it with a speed v_0, and reaches the bottom with speed v and kinetic energy K. Some books are now placed in the crate, so that the total mass is quadrupled. The coefficient of kinetic friction is constant and air resistance is negligible. Starting again with v_0 at the top of the ramp, what are the speed and kinetic energy at the bottom? Explain the reasoning behind your answers.

7.9. A small rock with mass 0.20 kg is released from rest at point A, which is at the top edge of a large, hemispherical bowl with radius $R = 0.50$ m (Fig. 7.25). Assume that the size of the rock is small compared to R, so that the rock can be treated as a particle, and assume that the rock slides rather than rolls. The work done by friction on the rock when it moves from point A to point B at the bottom of the bowl has magnitude 0.22 J. (a) Between points A and B, how much work is done on the rock by (i) the normal force and (ii) gravity? (b) What is the speed of the rock as it reaches point B? (c) Of the three forces acting on the rock as it slides down the bowl, which (if any) are constant and which are not? Explain. (d) Just as the rock reaches point B, what is the normal force on it due to the bottom of the bowl?

Figure 7.25 Exercise 7.9.



7.10. A stone of mass m is thrown upward at an angle θ above the horizontal and feels no appreciable air resistance. Use conservation of energy to show that at its highest point, it is a distance $v_0^2(\sin^2\theta)/2g$ above the point where it was launched. (*Hint:* $v_0^2 = v_{0x}^2 + v_{0y}^2$.)

7.11. You are testing a new amusement park roller coaster with an empty car with mass 120 kg. One part of the track is a vertical loop with radius 12.0 m. At the bottom of the loop (point *A*) the car has speed 25.0 m/s, and at the top of the loop (point *B*) it has speed 8.0 m/s. As the car rolls from point *A* to point *B*, how much work is done by friction?

7.12. Tarzan and Jane. Tarzan, in one tree, sights Jane in another tree. He grabs the end of a vine with length 20 m that makes an angle of 45° with the vertical, steps off his tree limb, and swings down and then up to Jane's open arms. When he arrives, his vine makes an angle of 30° with the vertical. Determine whether he gives her a tender embrace or knocks her off her limb by calculating Tarzan's speed just before he reaches Jane. You can ignore air resistance and the mass of the vine.

7.13. A 10.0-kg microwave oven is pushed 8.00 m up the sloping surface of a loading ramp inclined at an angle of 36.9° above the horizontal, by a constant force $\vec{F}$ with a magnitude 110 N and acting parallel to the ramp. The coefficient of kinetic friction between the oven and the ramp is 0.250. (a) What is the work done on the oven by the force $\vec{F}$? (b) What is the work done on the oven by the friction force? (c) Compute the increase in potential energy for the oven. (d) Use your answers to parts (a), (b), and (c) to calculate the increase in the oven's kinetic energy. (e) Use $\sum\vec{F} = m\vec{a}$ to calculate the acceleration of the oven. Assuming that the oven is initially at rest, use the acceleration to calculate the oven's speed after traveling 8.00 m. From this, compute the increase in the oven's kinetic energy, and compare it to the answer you got in part (d).

7.14. Pendulum. A small rock with mass 0.12 kg is fastened to a massless string with length 0.80 m to form a pendulum. The pendulum is swinging so as to make a maximum angle of 45° with the vertical. Air resistance is negligible. (a) What is the speed of the rock when the string passes through the vertical position? (b) What is the tension in the string when it makes an angle of 45° with the vertical? (c) What is the tension in the string as it passes through the vertical?

Section 7.2 Elastic Potential Energy

7.15. A force of 800 N stretches a certain spring a distance of 0.200 m. (a) What is the potential energy of the spring when it is stretched 0.200 m? (b) What is its potential energy when it is compressed 5.00 cm?

7.16. An ideal spring of negligible mass is 12.00 cm long when nothing is attached to it. When you hang a 3.15-kg weight from it, you measure its length to be 13.40 cm. If you wanted to store 10.0 J of potential energy in this spring, what would be its *total* length? Assume that it continues to obey Hooke's law.

7.17. A spring stores potential energy U_0 when it is compressed a distance x_0 from its uncompressed length. (a) In terms of U_0, how much energy does it store when it is compressed (i) twice as much and (ii) half as much? (b) In terms of x_0, how much must it be compressed from its uncompressed length to store (i) twice as much energy and (ii) half as much energy?

7.18. A slingshot will shoot a 10-g pebble 22.0 m straight up. (a) How much potential energy is stored in the slingshot's rubber band? (b) With the same potential energy stored in the rubber band, how high can the slingshot shoot a 25-g pebble? (c) What physical effects did you ignore in solving this problem?

7.19. A spring of negligible mass has force constant $k =$ 1600 N/m. (a) How far must the spring be compressed for 3.20 J of potential energy to be stored in it? (b) You place the spring vertically with one end on the floor. You then drop a 1.20-kg book onto it from a height of 0.80 m above the top of the spring. Find the maximum distance the spring will be compressed.

7.20. A 1.20-kg piece of cheese is placed on a vertical spring of negligible mass and force constant $k = 1800$ N/m that is compressed 15.0 cm. When the spring is released, how high does the cheese rise from this initial position? (The cheese and the spring are *not* attached.)

7.21. Consider the glider of Example 7.7 (Section 7.2) and Fig. 7.16. As in the example, the glider is released from rest with the spring stretched 0.100 m. What is the displacement x of the glider from its equilibrium position when its speed is 0.20 m/s? (You should get more than one answer. Explain why.)

7.22. Consider the glider of Example 7.7 (Section 7.2) and Fig. 7.16. (a) As in the example, the glider is released from rest with the spring stretched 0.100 m. What is the speed of the glider when it returns to $x = 0$? (b) What must the initial displacement of the glider be if its maximum speed in the subsequent motion is to be 2.50 m/s?

7.23. A 2.50-kg mass is pushed against a horizontal spring of force constant 25.0 N/cm on a frictionless air table. The spring is attached to the tabletop, and the mass is not attached to the spring in any way. When the spring has been compressed enough to store 11.5 J of potential energy in it, the mass is suddenly released from rest. (a) Find the greatest speed the mass reaches. When does this occur? (b) What is the greatest acceleration of the mass, and when does it occur?

7.24. (a) For the elevator of Example 7.9 (Section 7.2), what is the speed of the elevator after it has moved downward 1.00 m from point 1 in Fig. 7.17? (b) When the elevator is 1.00 m below point 1 in Fig. 7.17, what is its acceleration?

7.25. You are asked to design a spring that will give a 1160-kg satellite a speed of 2.50 m/s relative to an orbiting space shuttle. Your spring is to give the satellite a maximum acceleration of $5.00g$. The spring's mass, the recoil kinetic energy of the shuttle, and changes in gravitational potential energy will all be negligible. (a) What must the force constant of the spring be? (b) What distance must the spring be compressed?

Section 7.3 Conservative and Nonconservative Forces

7.26. A 75-kg roofer climbs a vertical 7.0-m ladder to the flat roof of a house. He then walks 12 m on the roof, climbs down another vertical 7.0-m ladder, and finally walks on the ground back to his starting point. How much work is done on him by gravity (a) as he climbs up; (b) as he climbs down; (c) as he walks on the roof and on the ground? (d) What is the total work done on him by gravity during this round trip? (e) On the basis of your answer to part (d), would you say that gravity is a conservative or nonconservative force? Explain.

7.27. A 10.0-kg box is pulled by a horizontal wire in a circle on a rough horizontal surface for which the coefficient of kinetic friction is 0.250. Calculate the work done by friction during one complete circular trip if the radius is (a) 2.00 m and (b) 4.00 m. (c) On the basis of the results you just obtained, would you say that friction is a conservative or nonconservative force? Explain.

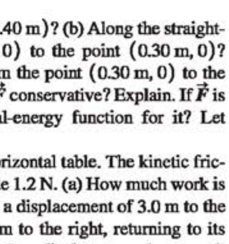
7.20. In an experiment, one of the forces exerted on a proton is $\vec{F} = -\alpha x^2\hat{\imath}$, where $\alpha = 12$ N/m^2. (a) How much work does $\vec{F}$ do when the proton moves along the straight-line path from the point

$(0.10\text{ m}, 0)$ to the point $(0.10\text{ m}, 0.40\text{ m})$? (b) Along the straight-line path from the point $(0.10\text{ m}, 0)$ to the point $(0.30\text{ m}, 0)$? (c) Along the straight-line path from the point $(0.30\text{ m}, 0)$ to the point $(0.10\text{ m}, 0)$? (d) Is the force $\vec{F}$ conservative? Explain. If $\vec{F}$ is conservative, what is the potential-energy function for it? Let $U = 0$ when $x = 0$.

7.29. A 0.60-kg book slides on a horizontal table. The kinetic friction force on the book has magnitude 1.2 N. (a) How much work is done on the book by friction during a displacement of 3.0 m to the left? (b) The book now slides 3.0 m to the right, returning to its starting point. During this second 3.0-m displacement, how much work is done on the book by friction? (c) What is the total work done on the book by friction during the complete round trip? (d) On the basis of your answer to part (c), would you say that the friction force is conservative or nonconservative? Explain.

7.30. You and three friends stand at the corners of a square whose sides are 8.0 m long in the middle of the gym floor, as shown in Fig. 7.26. You take your physics book and push it from one person to the other. The book has a mass of 1.5 kg, and the coefficient of kinetic friction between the book and the floor is $\mu_k = 0.25$. (a) The book slides from you to Beth and then from Beth to Carlos, along the lines connecting these people. What is the work done by friction during this displacement? (b) You slide the book from you to Carlos along the diagonal of the square. What is the work done by friction during this displacement? (c) You slide the book to Kim who then slides it back to you. What is the total work done by friction during this motion of the book? (d) Is the friction force on the book conservative or nonconservative? Explain.

Figure 7.26 Exercise 7.30.



7.31. A block with mass m is attached to an ideal spring that has force constant k. (a) The block moves from x_1 to x_2, where $x_2 > x_1$. How much work does the spring force do during this displacement? (b) The block moves from x_1 to x_2 and then from x_2 to x_1. How much work does the spring force do during the displacement from x_2 to x_1? What is the total work done by the spring during the entire $x_1 \to x_2 \to x_1$ displacement? Explain why you got the answer you did. (c) The block moves from x_1 to x_3, where $x_3 > x_2$. How much work does the spring force do during this displacement? The block then moves from x_3 to x_2. How much work does the spring force do during this displacement? What is the total work done by the spring force during the $x_1 \to x_3 \to x_2$ displacement? Compare your answer to the answer in part (a), where the starting and ending points are the same but the path is different.

Section 7.4 Force and Potential Energy

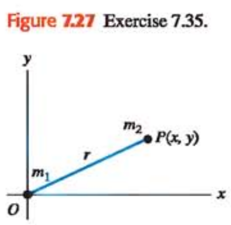
7.32. The potential energy of a pair of hydrogen atoms separated by a large distance x is given by $U(x) = -C_6/x^6$, where C_6 is a positive constant. What is the force that one atom exerts on the other? Is this force attractive or repulsive?

7.33. A force parallel to the x-axis acts on a particle moving along the x-axis. This force produces potential energy $U(x)$ given by $U(x) = \alpha x^4$, where $\alpha = 1.20\text{ J/m}^4$. What is the force (magnitude and direction) when the particle is at $x = -0.800$ m?

7.34. Gravity in One Dimension. Two point masses, m_1 and m_2, lie on the x-axis, with m_1 held in place at the origin and m_2 at position x and free to move. The gravitational potential energy of these masses is found to be $U(x) = -Gm_1m_2/x$, where G is a constant (called the *gravitational constant*). You'll learn more about gravitation in Chapter 12. Find the x-component of the force acting on m_2 due to m_1. Is this force attractive or repulsive? How do you know?

7.35. Gravity in Two Dimensions. Two point masses, m_1 and m_2, lie in the xy-plane, with m_1 held in place at the origin and m_2 free to move a distance r away at a point P having coordinates x and y (Fig. 7.27). The gravitational potential energy of these masses is found to be $U(r) = -Gm_1m_2/r$, where G is the gravitational constant. (a) Show that the components of the force on m_2 due to m_1 are

Figure 7.27 Exercise 7.35.



$$F_x = -\frac{Gm_1m_2x}{(x^2 + y^2)^{3/2}} \quad \text{and} \quad F_y = -\frac{Gm_1m_2y}{(x^2 + y^2)^{3/2}}$$

(*Hint:* First write r in terms of x and y.) (b) Show that the magnitude of the force on m_2 is $F = Gm_1m_2/r^2$. (c) Does m_1 attract or repel m_2? How do you know?

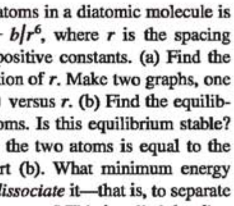
7.36. An object moving in the xy-plane is acted on by a conservative force described by the potential-energy function $U(x, y) = \alpha(1/x^2 + 1/y^2)$, where α is a positive constant. Derive an expression for the force expressed in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

Section 7.5 Energy Diagrams

7.37. The potential energy of two atoms in a diatomic molecule is approximated by $U(r) = a/r^{12} - b/r^6$, where r is the spacing between atoms and a and b are positive constants. (a) Find the force $F(r)$ on one atom as a function of r. Make two graphs, one of $U(r)$ versus r and one of $F(r)$ versus r. (b) Find the equilibrium distance between the two atoms. Is this equilibrium stable? (c) Suppose the distance between the two atoms is equal to the equilibrium distance found in part (b). What minimum energy must be added to the molecule to *dissociate* it—that is, to separate the two atoms to an infinite distance apart? This is called the *dissociation energy* of the molecule. (d) For the molecule CO, the equilibrium distance between the carbon and oxygen atoms is 1.13×10^{-10} m and the dissociation energy is 1.54×10^{-18} J per molecule. Find the values of the constants a and b.

7.38. A marble moves along the x-axis. The potential-energy function is shown in Fig. 7.28. (a) At which of the labeled x-coordinates is the force on the marble zero? (b) Which of the labeled x-coordinates is a position of stable equilibrium? (c) Which of the labeled x-coordinates is a position of unstable equilibrium?

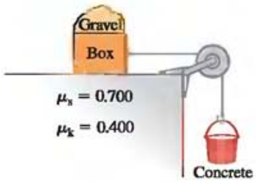
Figure 7.28 Exercise 7.38.



Problems

7.38. At a construction site, a 65.0-kg bucket of concrete hangs from a light (but strong) cable that passes over a light friction-free pulley and is connected to an 80.0-kg box on a horizontal roof (Fig. 7.29). The cable pulls horizontally on the box, and a 50.0-kg bag of gravel rests on top of the box. The coefficients of friction between the box and roof are shown. (a) Find the friction force on the bag of gravel and on the box. (b) Suddenly a worker picks up the bag of gravel. Use energy conservation to find the speed of the bucket after it has descended 2.00 m, from rest. (You can check your answer by solving this problem using Newton's laws.)

Figure 7.29 Problem 7.39.

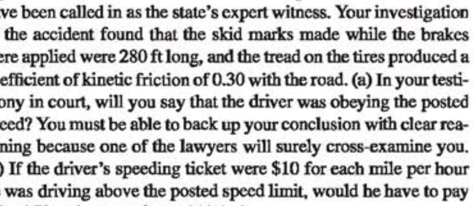


7.40. Two blocks with different mass are attached to either end of a light rope that passes over a light, frictionless pulley that is suspended from the ceiling. The masses are released from rest, and the more massive one starts to descend. After this block has descended 1.20 m, its speed is 3.00 m/s. If the total mass of the two blocks is 15.0 kg, what is the mass of each block?

7.41. Legal Physics. In an auto accident, a car hit a pedestrian and the driver then slammed on the brakes to stop the car. During the subsequent trial, the driver's lawyer claimed that he was obeying the posted 35 mi/h speed limit, but that the legal speed was too high to allow him to see and react to the pedestrian in time. You have been called in as the state's expert witness. Your investigation of the accident found that the skid marks made while the brakes were applied were 280 ft long, and the tread on the tires produced a coefficient of kinetic friction of 0.30 with the road. (a) In your testimony in court, will you say that the driver was obeying the posted speed? You must be able to back up your conclusion with clear reasoning because one of the lawyers will surely cross-examine you. (b) If the driver's speeding ticket were \$10 for each mile per hour he was driving above the posted speed limit, would he have to pay a fine? If so, how much would it be?

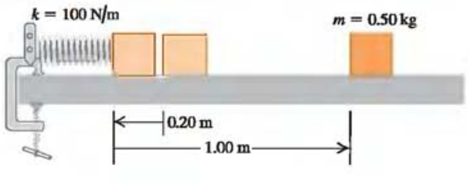
7.42. A 2.00-kg block is pushed against a spring with negligible mass and force constant $k = 400$ N/m, compressing it 0.220 m. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope 37.0° (Fig. 7.30). (a) What is the speed of the block as it slides along the horizontal surface after having left the spring? (b) How far does the block travel up the incline before starting to slide back down?

Figure 7.30 Problem 7.42.



7.43. A block with mass 0.50 kg is forced against a horizontal spring of negligible mass, compressing the spring a distance of 0.20 m (Fig. 7.31). When released, the block moves on a horizontal tabletop for 1.00 m before coming to rest. The spring constant k is 100 N/m. What is the coefficient of kinetic friction μ_k between the block and the tabletop?

Figure 7.31 Problem 7.43.

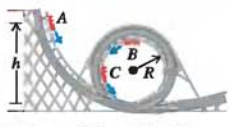


7.44. On a horizontal surface, a crate with mass 50.0 kg is placed against a spring that stores 360 J of energy. The spring is released, and the crate slides 5.60 m before coming to rest. What is the speed of the crate when it is 2.00 m from its initial position?

7.45. Bouncing Ball. A 650-gram rubber ball is dropped from an initial height of 2.50 m, and on each bounce it returns to 75% of its previous height. (a) What is the initial mechanical energy of the ball, just after it is released from its initial height? (b) How much mechanical energy does the ball lose during its first bounce? What happens to this energy? (c) How much mechanical energy is lost during the second bounce?

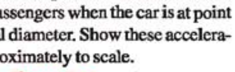
7.46. Riding a Loop-the-Loop. A car in an amusement park ride rolls without friction around the track shown in Fig. 7.32. It starts from rest at point A at a height h above the bottom of the loop. Treat the car as a particle. (a) What is the minimum value of h (in terms of R) such that the car moves around the loop without falling off at the top (point B)? (b) If $h = 3.50R$ and $R = 20.0$ m, compute the speed, radial acceleration, and tangential acceleration of the passengers when the car is at point C, which is at the end of a horizontal diameter. Show these acceleration components in a diagram, approximately to scale.

Figure 7.32 Problem 7.46.



7.47. A 2.0-kg piece of wood slides on the surface shown in Fig. 7.33. The curved sides are perfectly smooth, but the rough horizontal bottom is 30 m long and has a kinetic friction coefficient of 0.20 with the wood. The piece of wood starts from rest 4.0 m above the rough bottom. (a) Where will this wood eventually come to rest? (b) For the motion from the initial release until the piece of wood comes to rest, what is the total amount of work done by friction?

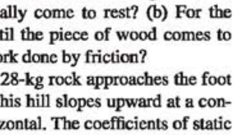
Figure 7.33 Problem 7.47.



7.48. Up and Down the Hill. A 28-kg rock approaches the foot of a hill with a speed of 15 m/s. This hill slopes upward at a constant angle of 40.0° above the horizontal. The coefficients of static and kinetic friction between the hill and the rock are 0.75 and 0.20, respectively. (a) Use energy conservation to find the maximum height above the foot of the hill reached by the rock. (b) Will the rock remain at rest at its highest point, or will it slide back down the hill? (c) If the rock does slide back down, find its speed when it returns to the bottom of the hill.

7.49. A 15.0-kg stone slides down a snow-covered hill (Fig. 7.34), leaving point A with a speed of 10.0 m/s. There is no friction on the hill between points A and B, but there is friction on the level ground at the bottom of the hill, between B and the wall. After

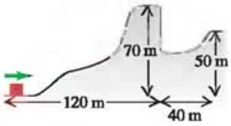
Figure 7.34 Problem 7.49.



entering the rough horizontal region, the stone travels 100 m and then runs into a very long, light spring with force constant 2.00 N/m. The coefficients of kinetic and static friction between the stone and the horizontal ground are 0.20 and 0.80, respectively. (a) What is the speed of the stone when it reaches point B? (b) How far will the stone compress the spring? (c) Will the stone move again after it has been stopped by the spring?

7.50. A 2.8-kg block slides over the smooth, icy hill shown in Fig. 7.35. The top of the hill is horizontal and 70 m higher than its base. What minimum speed must the block have at the base of the hill so that it will not fall into the pit on the far side of the hill?

Figure 7.35 Problem 7.50.



7.51. Bungee Jump. A bungee cord is 30.0 m long and, when stretched a distance x, it exerts a restoring force of magnitude kx. Your father-in-law (mass 95.0 kg) stands on a platform 45.0 m above the ground, and one end of the cord is tied securely to his ankle and the other end to the platform. You have promised him that when he steps off the platform he will fall a maximum distance of only 41.0 m before the cord stops him. You had several bungee cords to select from, and you tested them by stretching them out, tying one end to a tree, and pulling on the other end with a force of 380.0 N. When you do this, what distance will the bungee cord that you should select have stretched?

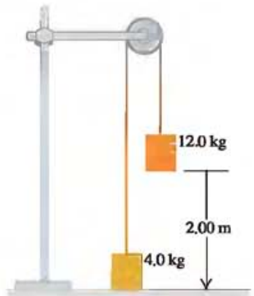
7.52. Ski Jump Ramp. You are designing a ski jump ramp for the next Winter Olympics. You need to calculate the vertical height h from the starting gate to the bottom of the ramp. The skiers push off hard with their ski poles at the start, just above the starting gate, so they typically have a speed of 2.0 m/s as they reach the gate. For safety, the skiers should have a speed of no more than 30.0 m/s when they reach the bottom of the ramp. You determine that for a 85.0-kg skier with good form, friction and air resistance will do total work of magnitude 4000 J on him during his run down the slope. What is the maximum height h for which the maximum safe speed will not be exceeded?

7.53. The Great Sandini is a 60-kg circus performer who is shot from a cannon (actually a spring gun). You don't find many men of his caliber, so you help him design a new gun. This new gun has a very large spring with a very small mass and a force constant of 1100 N/m that he will compress with a force of 4400 N. The inside of the gun barrel is coated with Teflon, so the average friction force will be only 40 N during the 4.0 m he moves in the barrel. At what speed will he emerge from the end of the barrel, 2.5 m above his initial rest position?

7.54. You are designing a delivery ramp for crates containing exercise equipment. The 1470-N crates will move at 1.8 m/s at the top of a ramp that slopes downward at 22.0°. The ramp exerts a 550-N kinetic friction force on each crate, and the maximum static friction force also has this value. Each crate will compress a spring at the bottom of the ramp and will come to rest after traveling a total distance of 8.0 m along the ramp. Once stopped, a crate must not rebound back up the ramp. Calculate the force constant of the spring that will be needed in order to meet the design criteria.

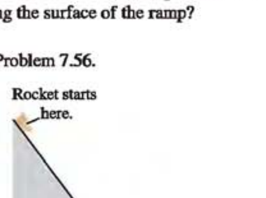
7.55. A system of two paint buckets connected by a lightweight rope is released from rest with the 12.0-kg bucket 2.00 m above the floor (Fig. 7.36). Use the principle of conservation of energy to find the speed with which this bucket strikes the floor. You can ignore friction and the mass of the pulley.

Figure 7.36 Problem 7.55.



7.56. A 1500-kg rocket is to be launched with an initial upward speed of 50.0 m/s. In order to assist its engines, the engineers will start it from rest on a ramp that rises 53° above the horizontal (Fig. 7.37). At the bottom, the ramp turns upward and launches the rocket vertically. The engines provide a constant forward thrust of 2000 N, and friction with the ramp surface is a constant 500 N. How far from the base of the ramp should the rocket start, as measured along the surface of the ramp?

Figure 7.37 Problem 7.56.



7.57. A machine part of mass m is attached to a horizontal ideal spring of force constant k that is attached to the edge of a friction-free horizontal surface. The part is pushed against the spring, compressing it a distance x_0, and then released from rest. Find the maximum (a) speed and (b) acceleration of the machine part. (c) Where in the motion do the maxima in parts (a) and (b) occur? (d) What will be the maximum extension of the spring? (e) Describe the subsequent motion of this machine part. Will it ever stop permanently?

7.58. A wooden rod of negligible mass and length 80.0 cm is pivoted about a horizontal axis through its center. A white rat with mass 0.500 kg clings to one end of the stick, and a mouse with mass 0.200 kg clings to the other end. The system is released from rest with the rod horizontal. If the animals can manage to hold on, what are their speeds as the rod swings through a vertical position?

7.59. A 0.100-kg potato is tied to a string with length 2.50 m, and the other end of the string is tied to a rigid support. The potato is held straight out horizontally from the point of support, with the string pulled taut, and is then released. (a) What is the speed of the potato at the lowest point of its motion? (b) What is the tension in the string at this point?

7.60. These data are from a computer simulation for a batted baseball with mass 0.145 kg, including air resistance:

t	x	y	v_x	v_y
0	0	0	30.0 m/s	40.0 m/s
3.05 s	70.2 m	53.6 m	18.6 m/s	0
6.59 s	124.4 m	0	11.9 m/s	−28.7 m/s

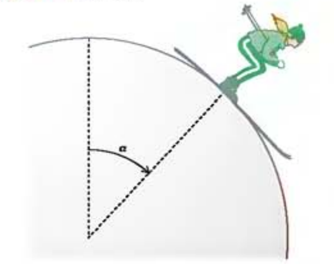
(a) How much work was done by the air on the baseball as it moved from its initial position to its maximum height? (b) How much work was done by the air on the baseball as it moved from its maximum height back to the starting elevation? (c) Explain why the magnitude of the answer in part (b) is smaller than the magnitude of the answer in part (a).

7.61. Down the Pole. A fireman of mass m slides a distance d down a pole. He starts from rest. He moves as fast at the bottom as if he had stepped off a platform a distance $h \le d$ above the ground and descended with negligible air resistance. (a) What average friction force did the fireman exert on the pole? Does your answer make sense in the special cases of $h = d$ and $h = 0$? (b) Find a numerical value for the average friction force a 75-kg fireman exerts, for $d = 2.5$ m and $h = 1.0$ m. (c) In terms of g, h, and d, what is the speed of the fireman when he is a distance y above the bottom of the pole?

7.62. A 60.0-kg skier starts from rest at the top of a ski slope 65.0 m high. (a) If frictional forces do −10.5 kJ of work on her as she descends, how fast is she going at the bottom of the slope? (b) Now moving horizontally, the skier crosses a patch of soft snow, where $\mu_k = 0.20$. If the patch is 82.0 m wide and the average force of air resistance on the skier is 160 N, how fast is she going after crossing the patch? (c) The skier hits a snowdrift and penetrates 2.5 m into it before coming to a stop. What is the average force exerted on her by the snowdrift as it stops her?

7.63. A skier starts at the top of a very large, frictionless snowball, with a very small initial speed, and skis straight down the side (Fig. 7.38). At what point does she lose contact with the snowball and fly off at a tangent? That is, at the instant she loses contact with the snowball, what angle α does a radial line from the center of the snowball to the skier make with the vertical?

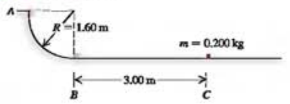
Figure 7.38 Problem 7.63.



7.64. A rock is tied to a cord and the other end of the cord is held fixed. The rock is given an initial tangential velocity that causes it to rotate in a vertical circle. Prove that the tension in the cord at the lowest point exceeds the tension at the highest point by six times the weight of the rock.

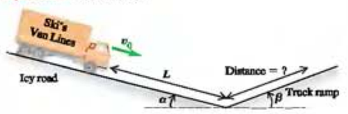
7.65. In a truck-loading station at a post office, a small 0.200-kg package is released from rest at point A on a track that is one-quarter of a circle with radius 1.60 m (Fig. 7.39). The size of the package is much less than 1.60 m, so the package can be treated as a particle. It slides down the track and reaches point B with a speed of 4.80 m/s. From point B, it slides on a level surface a distance of 3.00 m to point C, where it comes to rest. (a) What is the coefficient of kinetic friction on the horizontal surface? (b) How much work is done on the package by friction as it slides down the circular arc from A to B?

Figure 7.39 Problem 7.65.



7.66. A truck with mass m has a brake failure while going down an icy mountain road of constant downward slope angle α (Fig. 7.40). Initially the truck is moving downhill at speed v_0. After careening downhill a distance L with negligible friction, the truck driver steers the runaway vehicle onto a runaway truck ramp of constant upward slope angle β. The truck ramp has a soft sand surface for which the coefficient of rolling friction is μ_r. What is the distance that the truck moves up the ramp before coming to a halt? Solve using energy methods.

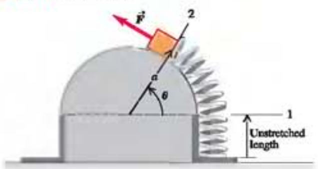
Figure 7.40 Problem 7.66.



7.67. A certain spring is found *not* to obey Hooke's law; it exerts a restoring force $F_x(x) = -\alpha x - \beta x^2$ if it is stretched or compressed, where $\alpha = 60.0$ N/m and $\beta = 18.0$ N/m^2. The mass of the spring is negligible. (a) Calculate the potential-energy function $U(x)$ for this spring. Let $U = 0$ when $x = 0$. (b) An object with mass 0.900 kg on a frictionless, horizontal surface is attached to this spring, pulled a distance 1.00 m to the right (the $+x$-direction) to stretch the spring, and released. What is the speed of the object when it is 0.50 m to the right of the $x = 0$ equilibrium position?

7.68. A variable force $\vec{F}$ is maintained tangent to a frictionless, semicircular surface (Fig. 7.41). By slow variations in the force, a

Figure 7.41 Problem 7.68.

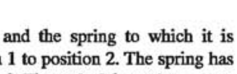


block with weight w is moved, and the spring to which it is attached is stretched from position 1 to position 2. The spring has negligible mass and force constant k. The end of the spring moves in an arc of radius a. Calculate the work done by the force $\vec{F}$.

7.69. A 0.150-kg block of ice is placed against a horizontal, compressed spring mounted on a horizontal tabletop that is 1.20 m above the floor. The spring has force constant 1900 N/m and is initially compressed 0.045 m. The mass of the spring is negligible. The spring is released, and the block slides along the table, goes off the edge, and travels to the floor. If there is negligible friction between the block of ice and the tabletop, what is the speed of the block of ice when it reaches the floor?

7.70. A 3.00-kg block is connected to two ideal horizontal springs having force constants $k_1 = 25.0$ N/cm and $k_2 = 20.0$ N/cm (Fig. 7.42). The system is initially in equilibrium on a horizontal, frictionless surface. The block is now pushed 15.0 cm to the right and released from rest. (a) What is the maximum speed of the block? Where in the motion does the maximum speed occur? (b) What is the maximum compression of spring 1?

Figure 7.42 Problem 7.70.



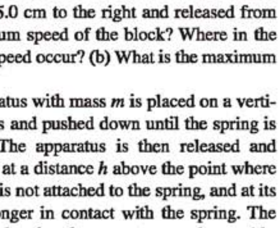
7.71. An experimental apparatus with mass m is placed on a vertical spring of negligible mass and pushed down until the spring is compressed a distance x. The apparatus is then released and reaches its maximum height at a distance h above the point where it is released. The apparatus is not attached to the spring, and at its maximum height it is no longer in contact with the spring. The maximum magnitude of acceleration the apparatus can have without being damaged is a, where $a > g$. (a) What should the force constant of the spring be? (b) What distance x must the spring be compressed initially?

7.72. If a fish is attached to a vertical spring and slowly lowered to its equilibrium position, it is found to stretch the spring by an amount d. If the same fish is attached to the end of the unstretched spring and then allowed to fall from rest, through what maximum distance does it stretch the spring? (*Hint:* Calculate the force constant of the spring in terms of the distance d and the mass m of the fish.)

7.73. A wooden block with mass 1.50 kg is placed against a compressed spring at the bottom of an incline of slope 30.0° (point A). When the spring is released, it projects the block up the incline. At point B, a distance of 6.00 m up the incline from A, the block is moving up the incline at 7.00 m/s and is no longer in contact with the spring. The coefficient of kinetic friction between the block and the incline is $\mu_k = 0.50$. The mass of the spring is negligible. Calculate the amount of potential energy that was initially stored in the spring.

7.74. A 2.00-kg package is released on a 53.1° incline, 4.00 m from a long spring with force constant 120 N/m that is attached at the bottom of the incline (Fig. 7.43). The coefficients of friction between the package and the incline are $\mu_s = 0.40$ and $\mu_k = 0.20$. The mass of the spring is negligible. (a) What is the speed of the package just before it reaches the spring? (b) What is the maximum compression of the spring? (c) The package rebounds back up the incline. How close does it get to its initial position?

Figure 7.43 Problem 7.74.



7.75. A 0.500-kg block, attached to a spring with length 0.60 m and force constant 40.0 N/m, is at rest with the back of the block at point A on a frictionless, horizontal air table (Fig. 7.44). The mass of the spring is negligible. You move the block to the right along the surface by pulling with a constant 20.0-N horizontal force. (a) What is the block's speed when the back of the block reaches point B, which is 0.25 m to the right of point A? (b) When the back of the block reaches point B, you let go of the block. In the subsequent motion, how close does the block get to the wall where the left end of the spring is attached?

Figure 7.44 Problem 7.75.

7.76. Fraternity Physics. The brothers of Iota Eta Pi fraternity build a platform, supported at all four corners by vertical springs, in the basement of their frat house. A brave fraternity brother wearing a football helmet stands in the middle of the platform; his weight compresses the springs by 0.18 m. Then four of his fraternity brothers, pushing down at the corners of the platform, compress the springs another 0.53 m until the top of the brave brother's helmet is 0.90 m below the basement ceiling. They then simultaneously release the platform. You can ignore the masses of the springs and platform. (a) When the dust clears, the fraternity asks you to calculate their fraternity brother's speed just before his helmet hit the flimsy ceiling. (b) Without the ceiling, how high would he have gone? (c) In discussing their probation, the dean of students suggests that the next time they try this, they do it outdoors on another planet. Would the answer to part (b) be the same if this stunt were performed on a planet with a different value of g? Assume that the fraternity brothers push the platform down 0.53 m as before. Explain your reasoning.

7.77. A particle with mass m is acted on by a conservative force and moves along a path given by $x = x_0 \cos\omega_0 t$ and $y = y_0 \sin\omega_0 t$, where x_0, y_0, and ω_0 are constants. (a) Find the components of the force that acts on the particle. (b) Find the potential energy of the particle as a function of x and y. Take $U = 0$ when $x = 0$ and $y = 0$. (c) Find the total energy of the particle when (i) $x = x_0$, $y = 0$ and (ii) $x = 0$, $y = y_0$.

7.76. When it is burned, 1 gallon of gasoline produces 1.3×10^8 J of energy. A 1500-kg car accelerates from rest to 37 m/s in 10 s. The engine of this car is only 15% efficient (which is typical), meaning that only 15% of the energy from the combustion of the gasoline is used to accelerate the car. The rest goes into things like the internal kinetic energy of the engine parts as well as heating of the exhaust air and engine. (a) How many gallons of gasoline does this car use during the acceleration? (b) How many such accelerations will it take to burn up 1 gallon of gas?

7.79. A hydroelectric dam holds back a lake of surface area 3.0×10^6 m^2 that has vertical sides below the water level. The water level in the lake is 150 m above the base of the dam. When the water passes through turbines at the base of the dam, its mechanical energy is converted into electrical energy with 90% efficiency. (a) If gravitational potential energy is taken to be zero at the base of the dam, how much energy is stored in the top meter of the water in the lake? The density of water is 1000 kg/m^3. (b) What volume of water must pass through the dam to produce 1000 kilowatt-hours of electrical energy? What distance does the level of water in the lake fall when this much water passes through the dam?

7.80. How much total energy is stored in the lake in Problem 7.79? As in that problem, take the gravitational potential energy to be zero at the base of the dam. Express your answer in joules and in kilowatt-hours. (*Hint:* Break the lake up into infinitesimal horizontal layers of thickness dy, and integrate to find the total potential energy.)

7.81. Gravity in Three Dimensions. A point mass m_1 is held in place at the origin, and another point mass m_2 is free to move a distance r away at a point P having coordinates x, y, and z. The gravitational potential energy of these masses is found to be $U(r) = -Gm_1m_2/r$, where G is the gravitational constant (see Exercises 7.34 and 7.35). (a) Show that the components of the force on m_2 due to m_1 are

$$F_x = -\frac{Gm_1m_2x}{(x^2+y^2+z^2)^{3/2}} \qquad F_y = -\frac{Gm_1m_2y}{(x^2+y^2+z^2)^{3/2}}$$

$$F_z = -\frac{Gm_1m_2z}{(x^2+y^2+z^2)^{3/2}}$$

(*Hint:* First write r in terms of x, y, and z.) (b) Show that the magnitude of the force on m_2 is $F = Gm_1m_2/r^2$. (c) Does m_1 attract or repel m_2? How do you know?

7.82. (a) Is the force $\vec{F} = Cy^2\hat{j}$, where C is a negative constant with units of N/m^2, conservative or nonconservative? Justify your answer. (b) Is the force $\vec{F} = Cy^2\hat{\imath}$, where C is a negative constant with units of N/m^2, conservative or nonconservative? Justify your answer.

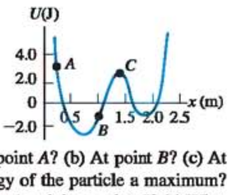
7.83. A cutting tool under microprocessor control has several forces acting on it. One force is $\vec{F} = -\alpha xy^2\hat{j}$, a force in the negative y-direction whose magnitude depends on the position of the tool. The constant is $\alpha = 2.50$ N/m^3. Consider the displacement of the tool from the origin to the point $x = 3.00$ m, $y = 3.00$ m. (a) Calculate the work done on the tool by $\vec{F}$ if this displacement is along the straight line $y = x$ that connects these two points. (b) Calculate the work done on the tool by $\vec{F}$ if the tool is first moved out along the x-axis to the point $x = 3.00$ m, $y = 0$ and then moved parallel to the y-axis to the point $x = 3.00$ m, $y = 3.00$ m. (c) Compare the work done by $\vec{F}$ along these two paths. Is $\vec{F}$ conservative or nonconservative? Explain.

7.84. An object has several forces acting on it. One force is $\vec{F} = \alpha xy\hat{\imath}$, a force in the x-direction whose magnitude depends on the position of the object. (See Problem 6.96.) The constant is $\alpha = 2.00$ N/m^2. The object moves along the following path: (1) It starts at the origin and moves along the y-axis to the point $x = 0$, $y = 1.50$ m; (2) it moves parallel to the x-axis to the point $x = 1.50$ m, $y = 1.50$ m; (3) it moves parallel to the y-axis to the point $x = 1.50$ m, $y = 0$; (4) it moves parallel to the x-axis back to the origin. (a) Sketch this path in the xy-plane. (b) Calculate the work done on the object by $\vec{F}$ for each leg of the path and for the complete round trip. (c) Is $\vec{F}$ conservative or nonconservative? Explain.

7.85. A Hooke's law force $-kx$ and a constant conservative force F in the $+x$-direction act on an atomic ion. (a) Show that a possible potential-energy function for this combination of forces is $U(x) = \frac{1}{2}kx^2 - Fx - F^2/2k$. Is this the *only* possible function? Explain. (b) Find the stable equilibrium position. (c) Graph $U(x)$ (in units of F^2/k) versus x (in units of F/k) for values of x between $-5F/k$ and $5F/k$. (d) Are there any unstable equilibrium positions? (e) If the total energy is $E = F^2/k$, what are the maximum and minimum values of x that the ion reaches in its motion? If the ion has mass m, find its maximum speed if the total energy is $E = F^2/k$. For what value of x is the speed maximum?

7.86. A particle moves along the x-axis while acted on by a single conservative force parallel to the x-axis. The force corresponds to the potential-energy function graphed in Fig. 7.45. The particle is released from rest at point A. (a) What is the direction of the force on the particle when it is at point A? (b) At point B? (c) At what value of x is the kinetic energy of the particle a maximum? (d) What is the force on the particle when it is at point C? (e) What is the largest value of x reached by the particle during its motion? (f) What value or values of x correspond to points of stable equilibrium? (g) Of unstable equilibrium?

Figure 7.45 Problem 7.86.



Challenge Problem

7.87. A proton with mass m moves in one dimension. The potential-energy function is $U(x) = \alpha/x^2 - \beta/x$, where α and β are positive constants. The proton is released from rest at $x_0 = \alpha/\beta$. (a) Show that $U(x)$ can be written as

$$U(x) = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right]$$

Graph $U(x)$. Calculate $U(x_0)$ and thereby locate the point x_0 on the graph. (b) Calculate $v(x)$, the speed of the proton as a function of position. Graph $v(x)$ and give a qualitative description of the motion. (c) For what value of x is the speed of the proton a maximum? What is the value of that maximum speed? (d) What is the force on the proton at the point in part (c)? (e) Let the proton be released instead at $x_1 = 3\alpha/\beta$. Locate the point x_1 on the graph of $U(x)$. Calculate $v(x)$ and give a qualitative description of the motion. (f) For each release point ($x = x_0$ and $x = x_1$), what are the maximum and minimum values of x reached during the motion?

MOMENTUM, IMPULSE, AND COLLISIONS

8

? Which could potentially cause you the greater injury: being tackled by a lightweight, fast-moving football player, or being tackled by a player with double the mass but moving at half the speed?

Learning Goals

By studying this chapter, you will learn:

- the meaning of the momentum of a particle, and how the impulse of the net force acting on a particle causes its momentum to change.
- the conditions under which the total momentum of a system of particles is constant (conserved).
- how to solve problems in which two bodies collide with each other.
- the important distinction among elastic, inelastic, and completely inelastic collisions.
- the definition of the center of mass of a system, and what determines how the center of mass moves.
- how to analyze situations such as rocket propulsion in which the mass of a body changes as it moves.

There are many questions involving forces that cannot be answered by directly applying Newton's second law, $\sum\vec{F} = m\vec{a}$. For example, when an 18-wheeler collides head-on with a compact car, what determines which way the wreckage moves after the collision? In playing pool, how do you decide how to aim the cue ball in order to knock the eight ball into the pocket? And when a meteorite collides with the earth, how much of the meteorite's kinetic energy is released in the impact?

A common theme of all these questions is that they involve forces about which we know very little: the forces between the car and the 18-wheeler, between the two pool balls, or between the meteorite and the earth. Remarkably, we will find in this chapter that we don't have to know *anything* about these forces to answer questions of this kind!

Our approach uses two new concepts, ***momentum*** and ***impulse***, and a new conservation law, ***conservation of momentum***. This conservation law is every bit as important as that of conservation of energy. The law of conservation of momentum is valid even in situations in which Newton's laws are inadequate, such as bodies moving at very high speeds (near the speed of light) or objects on a very small scale (such as the constituents of atoms). Within the domain of Newtonian mechanics, conservation of momentum enables us to analyze many situations that would be very difficult if we tried to use Newton's laws directly. Among these are *collision* problems, in which two bodies collide and can exert very large forces on each other for a short time.

8.1 Momentum and Impulse

In Chapter 6 we re-expressed Newton's second law for a particle, $\sum\vec{F} = m\vec{a}$, in terms of the work–energy theorem. This theorem helped us tackle a great number of physics problems and led us to the law of conservation of energy. Let's now return to $\sum\vec{F} = m\vec{a}$ and see yet another useful way to restate this fundamental law.

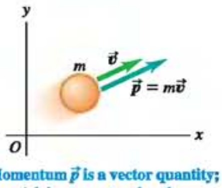
Newton's Second Law in Terms of Momentum

Consider a particle of constant mass m. (Later in this chapter we'll see how to deal with situations in which the mass of a body changes.) Because $\vec{a} = d\vec{v}/dt$, we can write Newton's second law for this particle as

$$\sum\vec{F} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) \tag{8.1}$$

We can take the mass m inside the derivative because it is constant. Thus Newton's second law says that the net force $\sum\vec{F}$ acting on a particle equals the time rate of change of the combination $m\vec{v}$, the product of the particle's mass and velocity. We'll call this combination the **momentum,** or **linear momentum,** of the particle. Using the symbol $\vec{p}$ for momentum, we have

$$\vec{p} = m\vec{v} \quad \text{(definition of momentum)} \tag{8.2}$$



8.1 The velocity and momentum vectors of a particle.

The greater the mass m and speed v of a particle, the greater is its magnitude of momentum mv. Keep in mind, however, that momentum is a *vector* quantity with the same direction as the particle's velocity (Fig. 8.1). Hence a car driving north at 20 m/s and an identical car driving east at 20 m/s have the same *magnitude* of momentum (mv) but different momentum *vectors* ($m\vec{v}$) because their directions are different.

We often express the momentum of a particle in terms of its components. If the particle has velocity components v_x, v_y, and v_z, then its momentum components p_x, p_y, and p_z (which we also call the *x-momentum, y-momentum,* and *z-momentum*) are given by

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z \tag{8.3}$$

These three component equations are equivalent to Eq. (8.2).

The units of the magnitude of momentum are units of mass times speed; the SI units of momentum are kg · m/s. The plural of momentum is "momenta."

If we now substitute the definition of momentum, Eq. (8.2), into Eq. (8.1), we get

$$\sum\vec{F} = \frac{d\vec{p}}{dt} \quad \text{(Newton's second law in terms of momentum)} \tag{8.4}$$

The net force (vector sum of all forces) acting on a particle equals the time rate of change of momentum of the particle. This, not $\sum\vec{F} = m\vec{a}$, is the form in which Newton originally stated his second law (although he called momentum the "quantity of motion"). This law is valid only in inertial frames of reference.

According to Eq. (8.4), a rapid change in momentum requires a large net force, while a gradual change in momentum requires less net force. This principle is used in the design of automobile safety devices such as air bags (Fig. 8.2).

8.2 If a fast-moving automobile stops suddenly in a collision, the driver's momentum (mass times velocity) changes from a large value to zero in a short time. An air bag causes the driver to lose momentum more gradually than would an abrupt collision with the steering wheel, reducing the force exerted on the driver as well as the possibility of injury.

The Impulse–Momentum Theorem

A particle's momentum $\vec{p} = m\vec{v}$ and its kinetic energy $K = \frac{1}{2}mv^2$ both depend on the mass and velocity of the particle. What is the fundamental difference between these two quantities? A purely mathematical answer is that momentum is a vector whose magnitude is proportional to speed, while kinetic energy is a scalar proportional to the speed squared. But to see the *physical* difference between momentum and kinetic energy, we must first define a quantity closely related to momentum called *impulse.*

Let's first consider a particle acted on by a *constant* net force $\sum\vec{F}$ during a time interval Δt from t_1 to t_2. (We'll look at the case of varying forces shortly.)

The **impulse** of the net force, denoted by $\vec{J}$, is defined to be the product of the net force and the time interval:

$$\vec{J} = \sum\vec{F}(t_2 - t_1) = \sum\vec{F}\,\Delta t \qquad \text{(assuming constant net force)} \qquad (8.5)$$

Impulse is a vector quantity; its direction is the same as the net force $\sum\vec{F}$. Its magnitude is the product of the magnitude of the net force and the length of time that the net force acts. The SI unit of impulse is the newton-second $(\text{N}\cdot\text{s})$. Because $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$, an alternative set of units for impulse is $\text{kg}\cdot\text{m/s}$, the same as the units of momentum.

To see what impulse is good for, let's go back to Newton's second law as restated in terms of momentum, Eq. (8.4). If the net force $\sum\vec{F}$ is constant, then $d\vec{p}/dt$ is also constant. In that case, $d\vec{p}/dt$ is equal to the *total* change in momentum $\vec{p}_2 - \vec{p}_1$ during the time interval $t_2 - t_1$, divided by the interval:

$$\sum\vec{F} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1}$$

Multiplying this equation by $(t_2 - t_1)$, we have

$$\sum\vec{F}(t_2 - t_1) = \vec{p}_2 - \vec{p}_1$$

Comparing with Eq. (8.5), we end up with a result called the **impulse–momentum theorem:**

$$\vec{J} = \vec{p}_2 - \vec{p}_1 \qquad \text{(impulse–momentum theorem)} \qquad (8.6)$$

The change in momentum of a particle during a time interval equals the impulse of the net force that acts on the particle during that interval.

The impulse–momentum theorem also holds when forces are not constant. To see this, we integrate both sides of Newton's second law $\sum\vec{F} = d\vec{p}/dt$ over time between the limits t_1 and t_2:

$$\int_{t_1}^{t_2} \sum\vec{F}\,dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt}\,dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$

The integral on the left is defined to be the impulse $\vec{J}$ of the net force $\sum\vec{F}$ during this interval:

$$\vec{J} = \int_{t_1}^{t_2} \sum\vec{F}\,dt \qquad \text{(general definition of impulse)} \qquad (8.7)$$

With this definition, the impulse–momentum theorem $\vec{J} = \vec{p}_2 - \vec{p}_1$, Eq. (8.6), is valid even when the net force $\sum\vec{F}$ varies with time.

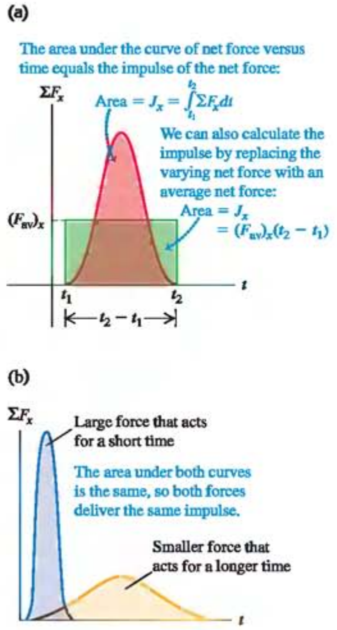
We can define an *average* net force $\vec{F}_{av}$ such that even when $\sum\vec{F}$ is not constant, the impulse $\vec{J}$ is given by

$$\vec{J} = \vec{F}_{av}(t_2 - t_1) \qquad (8.8)$$

When $\sum\vec{F}$ is constant, $\sum\vec{F} = \vec{F}_{av}$ and Eq. (8.8) reduces to Eq. (8.5).

Figure 8.3a shows the x-component of net force $\sum F_x$ as a function of time during a collision. This might represent the force on a soccer ball that is in contact with a player's foot from time t_1 to t_2. The x-component of impulse during this interval is represented by the red area under the curve between t_1 and t_2. This area is equal to the green rectangular area bounded by t_1, t_2, and $(F_{av})_x$, so $(F_{av})_x(t_2 - t_1)$ is

8.3 The meaning of the area under a graph of $\sum F_x$ versus t.



equal to the impulse of the actual time-varying force during the same interval. Note that a large force acting for a short time can have the same impulse as a smaller force acting for a longer time if the areas under the force–time curves are the same (Fig. 8.3b). In this language, an automobile airbag (Fig. 8.2) provides the same impulse to the driver as would the steering wheel or the dashboard by applying a weaker and less injurious force for a longer time.

Impulse and momentum are both vector quantities, and Eqs. (8.5)–(8.8) are all vector equations. In specific problems, it is often easiest to use them in component form:

$$
\begin{aligned}
J_x &= \int_{t_1}^{t_2} \Sigma F_x \, dt = (F_{av})_x (t_2 - t_1) = p_{2x} - p_{1x} = mv_{2x} - mv_{1x} \\
J_y &= \int_{t_1}^{t_2} \Sigma F_y \, dt = (F_{av})_y (t_2 - t_1) = p_{2y} - p_{1y} = mv_{2y} - mv_{1y}
\end{aligned}
\tag{8.9}
$$

and similarly for the z-component.

Momentum and Kinetic Energy Compared

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6.1 Momentum and Energy Change

We can now see the fundamental difference between momentum and kinetic energy. The impulse–momentum theorem $\vec{J} = \vec{p}_2 - \vec{p}_1$ says that changes in a particle's momentum are due to impulse, which depends on the *time* over which the net force acts. By contrast, the work–energy theorem $W_{tot} = K_2 - K_1$ tells us that kinetic energy changes when work is done on a particle; the total work depends on the *distance* over which the net force acts. Consider a particle that starts from rest at t_1 so that $\vec{v}_1 = \mathbf{0}$. Its initial momentum is $\vec{p}_1 = m\vec{v}_1 = \mathbf{0}$, and its initial kinetic energy is $K_1 = \frac{1}{2}mv_1^2 = 0$. Now let a constant net force equal to $\vec{F}$ act on that particle from time t_1 until time t_2. During this interval, the particle moves a distance s in the direction of the force. From Eq. (8.6), the particle's momentum at time t_2 is

$$\vec{p}_2 = \vec{p}_1 + \vec{J} = \vec{J}$$

where $\vec{J} = \vec{F}(t_2 - t_1)$ is the impulse that acts on the particle. So *the momentum of a particle equals the impulse that accelerated it from rest to its present speed;* impulse is the product of the net force that accelerated the particle and the *time* required for the acceleration. By comparison, the kinetic energy of the particle at t_2 is $K_2 = W_{tot} = Fs$, the total *work* done on the particle to accelerate it from rest. The total work is the product of the net force and the *distance* required to accelerate the particle (Fig. 8.4).

8.4 The *kinetic energy* of a pitched baseball is equal to the work the pitcher does on it (force multiplied by the distance the ball moves during the throw). The *momentum* of the ball is equal to the impulse the pitcher imparts to it (force multiplied by the time it took to bring the ball up to speed).

Here's an application of the distinction between momentum and kinetic energy. Suppose you have a choice between catching a 0.50-kg ball moving at 4.0 m/s or a 0.10-kg ball moving at 20 m/s. Which will be easier to catch? Both balls have the same magnitude of momentum, $p = mv = (0.50\text{ kg})(4.0\text{ m/s}) = (0.10\text{ kg})(20\text{ m/s}) = 2.0\text{ kg}\cdot\text{m/s}$. However, the two balls have different values of kinetic energy $K = \frac{1}{2}mv^2$; the large, slow-moving ball has $K = 4.0$ J, while the small, fast-moving ball has $K = 20$ J. Since the momentum is the same for both balls, both require the same *impulse* to be brought to rest. But stopping the 0.10-kg ball with your hand requires five times more *work* than stopping the 0.50-kg ball because the smaller ball has five times more kinetic energy. For a given force that you exert with your hand, it takes the same amount of time (the duration of the catch) to stop either ball, but your hand and arm will be pushed back five times farther if you choose to catch the small, fast-moving ball. To minimize arm strain, you should choose to catch the 0.50-kg ball with its lower kinetic energy.

Both the impulse–momentum and work–energy theorems are relationships between force and motion, and both rest on the foundation of Newton's laws. They are *integral* principles, relating the motion at two different times separated by a finite interval. By contrast, Newton's second law itself (in either of the forms $\Sigma\vec{F} = m\vec{a}$ or $\Sigma\vec{F} = d\vec{p}/dt$) is a *differential* principle, relating the forces to the rate of change of velocity or momentum at each instant.

Conceptual Example 8.1 Momentum versus kinetic energy

Consider again the race described in Conceptual Example 6.5 (Section 6.2) between two iceboats on a frictionless frozen lake. The iceboats have masses m and $2m$, and the wind exerts the same constant horizontal force $\vec{F}$ on each iceboat (see Fig. 6.14). The two iceboats start from rest and cross the finish line a distance s away. Which iceboat crosses the finish line with greater momentum?

SOLUTION

In Conceptual Example 6.5 we asked how the *kinetic energies* of the iceboats compare when they cross the finish line. The way to answer this was not to use the formula $K = \frac{1}{2}mv^2$, but to remember that a body's kinetic energy equals the total work done to accelerate it from rest. Both iceboats started from rest, and the total work done between the starting and finish lines was the same for both iceboats (because the net force and displacement were the same for both). Hence both iceboats cross the finish line with the same kinetic energy.

Similarly, the best way to compare the *momenta* of the iceboats is *not* to use the formula $\vec{p} = m\vec{v}$. By itself this formula isn't enough to determine which iceboat has greater momentum at the finish line. The iceboat of mass $2m$ has greater mass, which suggests greater momentum, but this iceboat crosses the finish line going slower than the other one, which suggests less momentum.

Instead, we use the idea that the momentum of each iceboat equals the impulse that accelerated it from rest. For each iceboat the downward force of gravity and the upward normal force add to zero, so the net force equals the constant horizontal wind force $\vec{F}$. Let Δt be the time an iceboat takes to reach the finish line, so that the impulse on the iceboat during that time is $\vec{J} = \vec{F}\,\Delta t$. Since the iceboat starts from rest, this equals the iceboat's momentum $\vec{p}$ at the finish line:

$$\vec{p} = \vec{F}\,\Delta t$$

Both iceboats are subjected to the same force $\vec{F}$, but they take different amounts of time Δt to reach the finish line. The iceboat of mass $2m$ accelerates more slowly and takes a longer time to travel the distance s; thus there is a greater impulse on this iceboat between the starting and finish lines. So the iceboat of mass $2m$ crosses the finish line with a greater magnitude of momentum than the iceboat of mass m (but with the same kinetic energy). Can you show that the iceboat of mass $2m$ has $\sqrt{2}$ times as much momentum at the finish line as the iceboat of mass m?

Example 8.2 A ball hits a wall

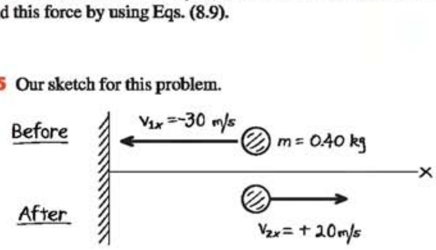
Suppose you throw a ball with a mass of 0.40 kg against a brick wall. It hits the wall moving horizontally to the left at 30 m/s and rebounds horizontally to the right at 20 m/s. (a) Find the impulse of the net force on the ball during its collision with the wall. (b) If the ball is in contact with the wall for 0.010 s, find the average horizontal force that the wall exerts on the ball during the impact.

SOLUTION

IDENTIFY: We're given enough information to determine the initial and final values of the ball's momentum, so we can use the impulse–momentum theorem to find the impulse. We'll then use the definition of impulse to determine the average force.

SET UP: Figure 8.5 shows our sketch. The motion is purely horizontal, so we need only a single axis. We'll take the x-axis to be horizontal and the positive direction to be to the right. Our target variable in part (a) is the x-component of impulse, J_x, which we'll find from the x-components of momentum before and after the impact, using Eqs. (8.9). In part (b), our target variable is the average x-component of force $(F_{av})_x$; once we know J_x, we can also find this force by using Eqs. (8.9).

8.5 Our sketch for this problem.



EXECUTE: (a) With our choice of x-axis, the initial and final x-components of momentum of the ball are

$$p_{1x} = mv_{1x} = (0.40\,\text{kg})(-30\,\text{m/s}) = -12\,\text{kg}\cdot\text{m/s}$$
$$p_{2x} = mv_{2x} = (0.40\,\text{kg})(+20\,\text{m/s}) = +8.0\,\text{kg}\cdot\text{m/s}$$

From the x-equation in Eqs. (8.9), the x-component of impulse equals the *change* in the x-momentum:

$$J_x = p_{2x} - p_{1x}$$
$$= 8.0\,\text{kg}\cdot\text{m/s} - (-12\,\text{kg}\cdot\text{m/s}) = 20\,\text{kg}\cdot\text{m/s} = 20\,\text{N}\cdot\text{s}$$

(b) The collision time is $t_2 - t_1 = \Delta t = 0.010\,\text{s}$. From the x-equation in Eqs. (8.9), $J_x = (F_{av})_x(t_2 - t_1) = (F_{av})_x\,\Delta t$, so

$$(F_{av})_x = \frac{J_x}{\Delta t} = \frac{20\,\text{N}\cdot\text{s}}{0.010\,\text{s}} = 2000\,\text{N}$$

EVALUATE: The x-component of impulse is positive—that is, to the right in Fig. 8.5. This is as it should be: The impulse represents the "kick" that the wall imparts to the ball, and this "kick" is certainly to the right.

CAUTION **Momentum is a vector** Because momentum is a vector, we had to include the negative sign in p_{1x}. Had we carelessly omitted it, we would have calculated the impulse to be $8.0\,\text{kg}\cdot\text{m/s} - (12\,\text{kg}\cdot\text{m/s}) = -4\,\text{kg}\cdot\text{m/s}$. This incorrect answer would say that the wall had somehow given the ball a kick to the *left!* Make sure that you account for the *direction* of momentum in your calculations.

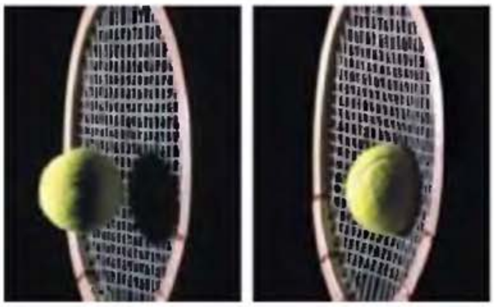
The force that the wall exerts on the ball has to have a large magnitude of 2000 N (equal to 450 lb, or the weight of a 200-kg

Continued

object) to change the ball's momentum in such a short time interval. Other forces that act on the ball during the collision are very weak by comparison; for instance, the gravitational force is only 3.9 N. Thus, during the brief time that the collision lasts, we can ignore all other forces on the ball to a very good approximation. Figure 8.6 is a photograph showing the impact of a tennis ball and racket.

Note that the 2000-N value we calculated is just the *average* horizontal force that the wall exerts on the ball during the impact. It corresponds to the horizontal line $(F_{av})_x$ in Fig. 8.3a. The horizontal force is zero before impact, rises to a maximum, and then decreases to zero when the ball loses contact with the wall. If the ball is relatively rigid, like a baseball or golf ball, the collision lasts a short time and the maximum force is large, as in the blue curve in Fig. 8.3b. If the ball is softer, like a tennis ball, the collision time is longer and the maximum force is less, as in the orange curve in Fig. 8.3b.

8.6 Typically, a tennis ball is in contact with the racket for approximately 0.01 s. The ball flattens noticeably due to the tremendous force exerted by the racket.



Example 8.3 Kicking a soccer ball

A soccer ball has a mass of 0.40 kg. Initially, it is moving to the left at 20 m/s, but then it is kicked and given a velocity at 45° upward and to the right, with a magnitude of 30 m/s (Fig. 8.7a). Find the impulse of the net force and the average net force, assuming a collision time $\Delta t = 0.010$ s.

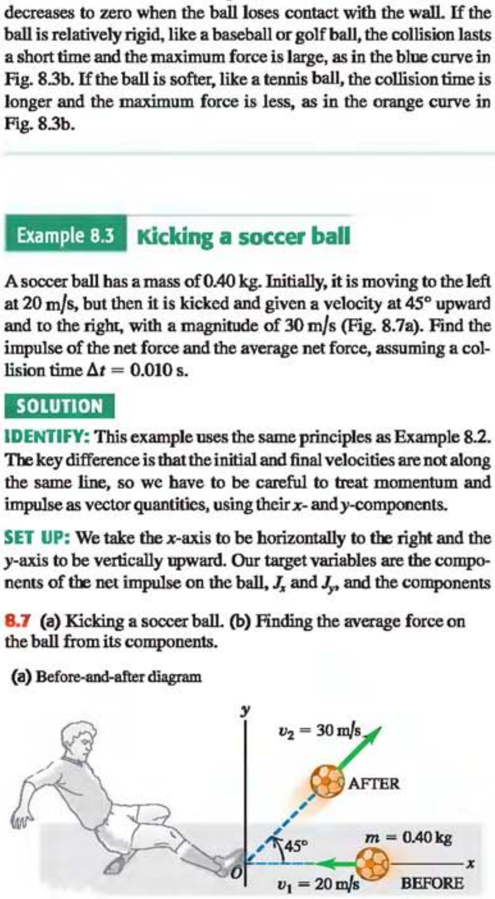
SOLUTION

IDENTIFY: This example uses the same principles as Example 8.2. The key difference is that the initial and final velocities are not along the same line, so we have to be careful to treat momentum and impulse as vector quantities, using their x- and y-components.

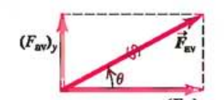
SET UP: We take the x-axis to be horizontally to the right and the y-axis to be vertically upward. Our target variables are the components of the net impulse on the ball, J_x and J_y, and the components of the average net force on the ball, $(F_{av})_x$ and $(F_{av})_y$. We'll find them using the x- and y-components of Eqs. (8.9).

8.7 (a) Kicking a soccer ball. (b) Finding the average force on the ball from its components.

(a) Before-and-after diagram



(b) Average force on the ball



EXECUTE: With our choice of axes, we find the ball's velocity components before (subscript 1) and after (subscript 2) it is kicked:

$$v_{1x} = -20 \text{ m/s} \qquad v_{1y} = 0$$
$$v_{2x} = v_{2y} = (30 \text{ m/s})(0.707) = 21.2 \text{ m/s}$$
$$(\text{since } \cos 45° = \sin 45° = 0.707)$$

The x-component of impulse is equal to the x-component of momentum change, and the same is true for the y-components:

$$J_x = p_{2x} - p_{1x} = m(v_{2x} - v_{1x})$$
$$= (0.40 \text{ kg})[21.2 \text{ m/s} - (-20 \text{ m/s})] = 16.5 \text{ kg}\cdot\text{m/s}$$
$$J_y = p_{2y} - p_{1y} = m(v_{2y} - v_{1y})$$
$$= (0.40 \text{ kg})(21.2 \text{ m/s} - 0) = 8.5 \text{ kg}\cdot\text{m/s}$$

The components of the average net force on the ball are

$$(F_{av})_x = \frac{J_x}{\Delta t} = 1650 \text{ N} \qquad (F_{av})_y = \frac{J_y}{\Delta t} = 850 \text{ N}$$

The magnitude and direction of the average force are

$$F_{av} = \sqrt{(1650 \text{ N})^2 + (850 \text{ N})^2} = 1.9 \times 10^3 \text{ N}$$
$$\theta = \arctan\frac{850 \text{ N}}{1650 \text{ N}} = 27°$$

where θ is measured upward from the $+x$-axis (Fig. 8.7b). Note that because the ball was not initially at rest, the ball's final velocity does *not* have the same direction as the average force that acted on it.

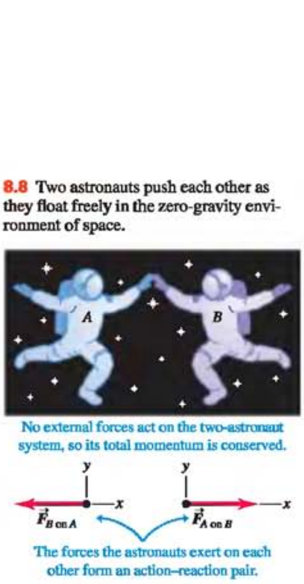
EVALUATE: The average net force $\vec{F}_{av}$ includes the effects of the force of gravity, although these are small; the weight of the ball is only 3.9 N. As in Example 8.2, the average force acting during the collision is exerted almost entirely by the object that the ball hit (in this case, the soccer player's foot).

Test Your Understanding of Section 8.1 Rank the following situations according to the magnitude of the impulse of the net force, from largest value to smallest value. In each situation a 1000-kg automobile is moving along a straight east–west road. (i) The automobile is initially moving east at 25 m/s and comes to a stop in 10 s. (ii) The automobile is initially moving east at 25 m/s and comes to a stop in 5 s. (iii) The automobile is initially at rest, and a 2000-N net force toward the east is applied to it for 10 s. (iv) The automobile is initially moving east at 25 m/s, and a 2000-N net force toward the west is applied to it for 10 s. (v) The automobile is initially moving east at 25 m/s. Over a 30-s period, the automobile reverses direction and ends up moving west at 25 m/s.

8.2 Conservation of Momentum

The concept of momentum is particularly important in situations in which we have two or more *interacting* bodies. To see why, let's consider first an idealized system consisting of two bodies that interact with each other but not with anything else—for example, two astronauts who touch each other as they float freely in the zero-gravity environment of outer space (Fig. 8.8). Think of the astronauts as particles. Each particle exerts a force on the other; according to Newton's third law, the two forces are always equal in magnitude and opposite in direction. Hence, the *impulses* that act on the two particles are equal and opposite, and the changes in momentum of the two particles are equal and opposite.

8.8 Two astronauts push each other as they float freely in the zero-gravity environment of space.



Let's go over that again with some new terminology. For any system, the forces that the particles of the system exert on each other are called **internal forces.** Forces exerted on any part of the system by some object outside it are called **external forces.** For the system shown in Fig. 8.8, the internal forces are $\vec{F}_{B \text{ on } A}$, exerted by particle B on particle A, and $\vec{F}_{A \text{ on } B}$, exerted by particle A on particle B. There are *no* external forces; when this is the case, we have an **isolated system.**

The net force on particle A is $\vec{F}_{B \text{ on } A}$ and the net force on particle B is $\vec{F}_{A \text{ on } B}$, so from Eq. (8.4) the rates of change of the momenta of the two particles are

$$\vec{F}_{B \text{ on } A} = \frac{d\vec{p}_A}{dt} \qquad \vec{F}_{A \text{ on } B} = \frac{d\vec{p}_B}{dt} \tag{8.10}$$

The momentum of each particle changes, but these changes are related to each other by Newton's third law: The two forces $\vec{F}_{B \text{ on } A}$ and $\vec{F}_{A \text{ on } B}$ are always equal in magnitude and opposite in direction. That is, $\vec{F}_{B \text{ on } A} = -\vec{F}_{A \text{ on } B}$, so $\vec{F}_{B \text{ on } A} + \vec{F}_{A \text{ on } B} = \mathbf{0}$. Adding together the two equations in Eq. (8.10), we have

$$\vec{F}_{B \text{ on } A} + \vec{F}_{A \text{ on } B} = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \frac{d(\vec{p}_A + \vec{p}_B)}{dt} = \mathbf{0} \tag{8.11}$$

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6.3 Momentum Conservation and Collisions
6.7 Explosion Problems
6.10 Pendulum Person-Projectile Bowling

The rates of change of the two momenta are equal and opposite, so the rate of change of the vector sum $\vec{p}_A + \vec{p}_B$ is zero. We now define the **total momentum** $\vec{P}$ of the system of two particles as the vector sum of the momenta of the individual particles; that is,

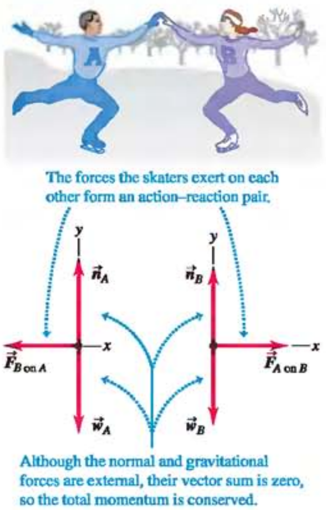
$$\vec{P} = \vec{p}_A + \vec{p}_B \tag{8.12}$$

Then Eq. (8.11) becomes, finally,

$$\vec{F}_{B \text{ on } A} + \vec{F}_{A \text{ on } B} = \frac{d\vec{P}}{dt} = \mathbf{0} \tag{8.13}$$

The time rate of change of the *total* momentum $\vec{P}$ is zero. Hence the total momentum of the system is constant, even though the individual momenta of the particles that make up the system can change.

8.9 Two ice skaters push each other as they skate on a frictionless, horizontal surface. (Compare to Fig. 8.8.)



If external forces are also present, they must be included on the left side of Eq. (8.13) along with the internal forces. Then the total momentum is, in general, not constant. But if the vector sum of the external forces is zero, as in Fig. 8.9, these forces don't contribute to the sum, and $d\vec{P}/dt$ is again zero. Thus we have the following general result:

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.

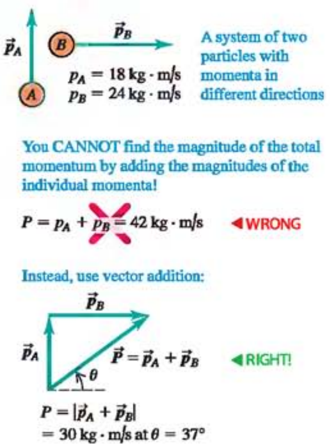
This is the simplest form of the **principle of conservation of momentum.** This principle is a direct consequence of Newton's third law. What makes this principle useful is that it doesn't depend on the detailed nature of the internal forces that act between members of the system. This means that we can apply conservation of momentum even if (as is often the case) we know very little about the internal forces. We have used Newton's second law to derive this principle, so we have to be careful to use it only in inertial frames of reference.

We can generalize this principle for a system that contains any number of particles $A, B, C, \ldots$ interacting only with each other. The total momentum of such a system is

$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots = m_A\vec{v}_A + m_B\vec{v}_B + \cdots \quad \text{(total momentum of a system of particles)} \tag{8.14}$$

We make the same argument as before: The total rate of change of momentum of the system due to each action–reaction pair of internal forces is zero. Thus the total rate of change of momentum of the entire system is zero whenever the vector sum of the external forces acting on it is zero. The internal forces can change the momenta of individual particles in the system but not the *total* momentum of the system.

8.10 When applying conservation of momentum, remember that momentum is a vector quantity!



CAUTION Conservation of momentum means conservation of its components When you apply the conservation of momentum to a system, remember that momentum is a *vector* quantity. Hence you must use vector addition to compute the total momentum of a system (Fig. 8.10). Using components is usually the simplest method. If p_{Ax}, p_{Ay}, and p_{Az} are the components of momentum of particle A, and similarly for the other particles, then Eq. (8.14) is equivalent to the component equations

$$\begin{aligned} P_x &= p_{Ax} + p_{Bx} + \cdots \\ P_y &= p_{Ay} + p_{By} + \cdots \\ P_z &= p_{Az} + p_{Bz} + \cdots \end{aligned} \tag{8.15}$$

If the vector sum of the external forces on the system is zero, then P_x, P_y, and P_z are all constant.

In some ways the principle of conservation of momentum is more general than the principle of conservation of mechanical energy. For example, mechanical energy is conserved only when the internal forces are *conservative*—that is, when the forces allow two-way conversion between kinetic and potential energy—but conservation of momentum is valid even when the internal forces are *not* conservative. In this chapter we will analyze situations in which both momentum and mechanical energy are conserved, and others in which only momentum is conserved. These two principles play a fundamental role in all areas of physics, and we will encounter them throughout our study of physics.

Problem-Solving Strategy 8.1 Conservation of Momentum

IDENTIFY *the relevant concepts:* Before applying conservation of momentum to a problem, you must decide whether momentum *is* conserved! This will be true *only* if the vector sum of the external forces acting on the system of particles is zero. If this is not the case, you can't use conservation of momentum.

SET UP *the problem using the following steps:*

1. Define a coordinate system and show it in a sketch, including the positive direction for each axis. Often it is easiest to choose the x-axis in the direction of one of the initial velocities. Make sure you are using an inertial frame of reference. Most of the problems in this chapter deal with two-dimensional situations, in which the vectors have only x- and y-components, but this strategy can be generalized to include z-components when necessary.
2. Treat each body as a particle. Draw "before" and "after" sketches, and include vectors on each to represent all known velocities. Label the vectors with magnitudes, angles, components, or whatever information is given, and give each unknown magnitude, angle, or component an algebraic symbol. It's helpful to use the subscripts 1 and 2 for velocities before and after the interaction, respectively, and use letters (not numbers) to label each particle.
3. As always, identify the target variable(s) from among the unknowns.

EXECUTE *the solution as follows:*

1. Write an equation in symbols equating the total *initial* x-component of momentum (that is, before the interaction) to the total *final* x-component of momentum (that is, after the interaction), using $p_x = mv_x$ for each particle. Write another equation for the y-components, using $p_y = mv_y$ for each particle. (*Never* add the x- and y-components of velocity or momentum together in the same equation!) Even when all motions are along a line (such as the x-axis), the components of velocity along this line can be positive or negative; be careful with signs!
2. Solve these equations to determine whatever results are required. In some problems you will have to convert from the x- and y-components of a velocity to its magnitude and direction, or the reverse.
3. In some problems, energy considerations give additional relationships among the various velocities, as we will see later in this chapter.

EVALUATE *your answer:* Does your answer make physical sense? If your target variable is a certain body's momentum, check that the direction of the momentum is reasonable.

Example 8.4 Recoil of a rifle

A marksman holds a rifle of mass $m_R = 3.00$ kg loosely in his hands, so as to let it recoil freely when fired. He fires a bullet of mass $m_B = 5.00$ g horizontally with a velocity relative to the ground of $v_{Bx} = 300$ m/s. What is the recoil velocity v_{Rx} of the rifle? What are the final momentum and kinetic energy of the bullet? Of the rifle?

SOLUTION

IDENTIFY: We consider an idealized model in which the horizontal forces the marksman exerts on the rifle are negligible. Then there is no net horizontal force on the system (the bullet and rifle) during the firing of the rifle, and so the total horizontal momentum of the system is the same before and after the rifle is fired (i.e., it is conserved).

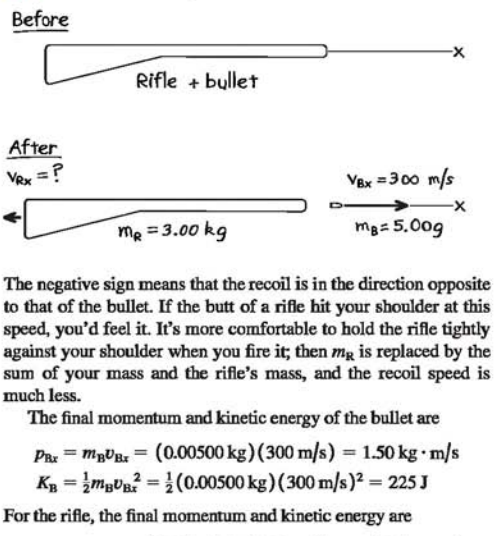
SET UP: Figure 8.11 shows our sketch. We take the positive x-axis to be the direction the rifle is aimed. Initially, both the rifle and the bullet are at rest, so the initial x-component of total momentum is zero. After the shot is fired, the bullet's x-momentum is $p_{Bx} = m_B v_{Bx}$ and the rifle's x-momentum is $p_{Rx} = m_R v_{Rx}$. Our target variables are v_{Rx}, p_{Bx}, p_{Rx}, and $K_B = \frac{1}{2}m_B v_{Bx}^2$ and $K_R = \frac{1}{2}m_R v_{Rx}^2$ (the final kinetic energies of the bullet and rifle, respectively).

EXECUTE: Conservation of the x-component of total momentum gives

$$P_x = 0 = m_B v_{Bx} + m_R v_{Rx}$$

$$v_{Rx} = -\frac{m_B}{m_R}v_{Bx} = -\left(\frac{0.00500\ \text{kg}}{3.00\ \text{kg}}\right)(300\ \text{m/s}) = -0.500\ \text{m/s}$$

8.11 Our sketch for this problem.



The negative sign means that the recoil is in the direction opposite to that of the bullet. If the butt of a rifle hit your shoulder at this speed, you'd feel it. It's more comfortable to hold the rifle tightly against your shoulder when you fire it; then m_R is replaced by the sum of your mass and the rifle's mass, and the recoil speed is much less.

The final momentum and kinetic energy of the bullet are

$$p_{Bx} = m_B v_{Bx} = (0.00500\ \text{kg})(300\ \text{m/s}) = 1.50\ \text{kg}\cdot\text{m/s}$$

$$K_B = \tfrac{1}{2}m_B v_{Bx}^2 = \tfrac{1}{2}(0.00500\ \text{kg})(300\ \text{m/s})^2 = 225\ \text{J}$$

For the rifle, the final momentum and kinetic energy are

$$p_{Rx} = m_R v_{Rx} = (3.00\ \text{kg})(-0.500\ \text{m/s}) = -1.50\ \text{kg}\cdot\text{m/s}$$

$$K_R = \tfrac{1}{2}m_R v_{Rx}^2 = \tfrac{1}{2}(3.00\ \text{kg})(-0.500\ \text{m/s})^2 = 0.375\ \text{J}$$

Continued

EVALUATE: The bullet and the rifle have equal and opposite *momenta* after the interaction. That's because they were subjected to equal and opposite interaction forces for the same amount of *time* (i.e., equal and opposite impulses). But the bullet acquires much greater *kinetic energy* than the rifle because the bullet travels a much greater *distance* than the rifle during the interaction. Thus the force on the bullet does more work than the force on the rifle. The ratio of the two kinetic energies, 600 : 1, is equal to the inverse ratio of the masses; in fact, it can be shown that this always happens in recoil situations. We leave the proof as a problem (see Exercise 8.22).

Our calculation doesn't depend on the details of how the rifle works. In a real rifle, the bullet is propelled forward by an explosive charge; if instead the rifle used a very stiff spring, the answers would have been exactly the same.

Example 8.5 Collision along a straight line

Two gliders move toward each other on a frictionless linear air track (Fig. 8.12a). After they collide (Fig. 8.12b), glider *B* moves away with a final velocity of $+2.0\ \text{m/s}$ (Fig. 8.12c). What is the final velocity of glider *A*? How do the changes in momentum and in velocity compare for the two gliders?

SOLUTION

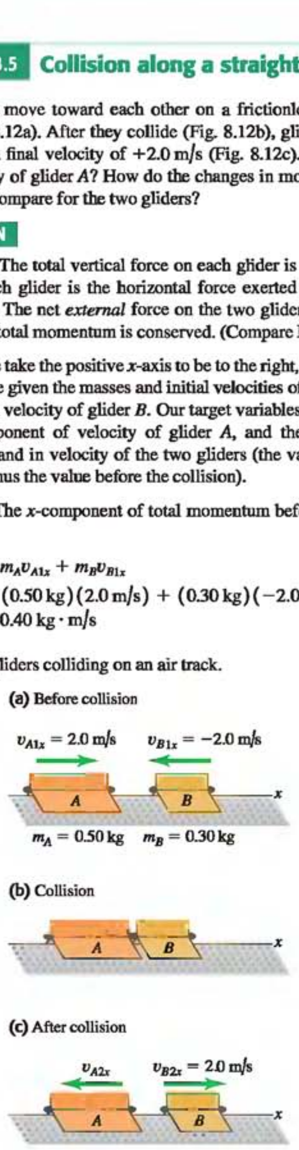
IDENTIFY: The total vertical force on each glider is zero; the net force on each glider is the horizontal force exerted on it by the other glider. The net *external* force on the two gliders together is zero, so the total momentum is conserved. (Compare Fig. 8.9.)

SET UP: We take the positive *x*-axis to be to the right, along the air track. We are given the masses and initial velocities of both gliders and the final velocity of glider *B*. Our target variables are v_{A2x}, the final *x*-component of velocity of glider *A*, and the changes in momentum and in velocity of the two gliders (the value after the collision minus the value before the collision).

EXECUTE: The *x*-component of total momentum before the collision is

$$\begin{aligned} P_x &= m_A v_{A1x} + m_B v_{B1x} \\ &= (0.50\ \text{kg})(2.0\ \text{m/s}) + (0.30\ \text{kg})(-2.0\ \text{m/s}) \\ &= 0.40\ \text{kg} \cdot \text{m/s} \end{aligned}$$

8.12 Two gliders colliding on an air track.



This is positive (to the right in Fig. 8.12) because glider *A* has a greater magnitude of momentum before the collision than does glider *B*. The *x*-component of total momentum has the same value after the collision, so

$$P_x = m_A v_{A2x} + m_B v_{B2x}$$

Solving this equation for v_{A2x}, the final *x*-velocity of *A*, we find

$$\begin{aligned} v_{A2x} &= \frac{P_x - m_B v_{B2x}}{m_A} = \frac{0.40\ \text{kg} \cdot \text{m/s} - (0.30\ \text{kg})(2.0\ \text{m/s})}{0.50\ \text{kg}} \\ &= -0.40\ \text{m/s} \end{aligned}$$

The change in *x*-momentum of glider *A* is

$$\begin{aligned} m_A v_{A2x} - m_A v_{A1x} &= (0.50\ \text{kg})(-0.40\ \text{m/s}) \\ &\quad - (0.50\ \text{kg})(2.0\ \text{m/s}) = -1.2\ \text{kg} \cdot \text{m/s} \end{aligned}$$

and the change in *x*-momentum of glider *B* is

$$\begin{aligned} m_B v_{B2x} - m_B v_{B1x} &= (0.30\ \text{kg})(2.0\ \text{m/s}) \\ &\quad - (0.30\ \text{kg})(-2.0\ \text{m/s}) = +1.2\ \text{kg} \cdot \text{m/s} \end{aligned}$$

The two interacting gliders undergo changes in momentum that are equal in magnitude and opposite in direction. The same is *not* true of their changes in velocity, however. For *A*, $v_{A2x} - v_{A1x} = (-0.40\ \text{m/s}) - 2.0\ \text{m/s} = -2.4\ \text{m/s}$; for *B*, $v_{B2x} - v_{B1x} = 2.0\ \text{m/s} - (-2.0\ \text{m/s}) = +4.0\ \text{m/s}$.

EVALUATE: Why do the momentum changes have the same magnitude for the two gliders, but the velocity changes do not? By Newton's third law, both gliders were acted on for equal amounts of time by an interaction force of the same magnitude. Hence both gliders experienced impulses of the same magnitude, and therefore equal-magnitude changes in momentum. But by Newton's second law, the less massive glider (B) had a greater magnitude of acceleration and hence a greater velocity change.

Here's an application of these ideas. When a large truck collides with a car of normal size, both vehicles undergo equal changes in momentum. The occupants of the car, however, are subjected to greater acceleration (and greater chance of injury) than the occupants of the truck. An even more extreme example is what happens when a truck collides with an insect: The truck driver won't notice the resulting acceleration at all, but the insect surely will!

Example 8.6 Collision in a horizontal plane

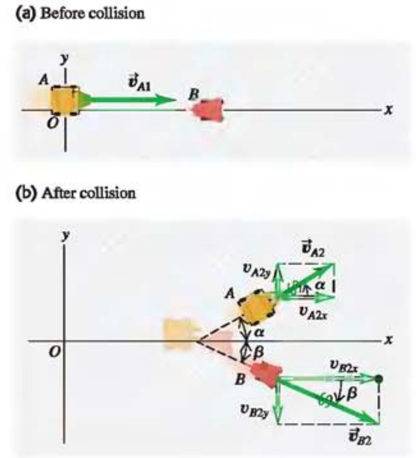
Figure 8.13a shows two battling robots sliding on a frictionless surface. Robot A, with mass 20 kg, initially moves at 2.0 m/s parallel to the x-axis. It collides with robot B, which has mass 12 kg and is initially at rest. After the collision, robot A is moving at 1.0 m/s in a direction that makes an angle $\alpha = 30°$ with its initial direction (Fig. 8.13b). What is the final velocity of robot B?

SOLUTION

IDENTIFY: There are no horizontal (x or y) external forces, so the x-component and the y-component of the total momentum of the system are both conserved in the collision.

SET UP: Figure 8.13 shows the coordinate axes. The velocities are not all along a single line, so we have to treat momentum as a vector quantity. Momentum conservation requires that the sum of the x-components of momentum *before* the collision (subscript 1) must equal the sum *after* the collision (subscript 2), and similarly for the sums of the y-components. We write a separate momentum conservation equation for each component. Our target variable is $\vec{v}_{B2}$, the final velocity of robot B.

8.13 Views from above of the velocities (a) before and (b) after the collision.



EXECUTE: Conservation of the x-component of total momentum says that

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$v_{B2x} = \frac{m_A v_{A1x} + m_B v_{B1x} - m_A v_{A2x}}{m_B}$$

$$= \frac{\begin{bmatrix}(20\,\text{kg})(2.0\,\text{m/s}) + (12\,\text{kg})(0) \\ -(20\,\text{kg})(1.0\,\text{m/s})(\cos 30°)\end{bmatrix}}{12\,\text{kg}}$$

$$= 1.89\,\text{m/s}$$

Similarly, for the y-component of total momentum we have

$$m_A v_{A1y} + m_B v_{B1y} = m_A v_{A2y} + m_B v_{B2y}$$

$$v_{B2y} = \frac{m_A v_{A1y} + m_B v_{B1y} - m_A v_{A2y}}{m_B}$$

$$= \frac{\begin{bmatrix}(20\,\text{kg})(0) + (12\,\text{kg})(0) \\ -(20\,\text{kg})(1.0\,\text{m/s})(\sin 30°)\end{bmatrix}}{12\,\text{kg}}$$

$$= -0.83\,\text{m/s}$$

After the collision, robot B moves in the positive x-direction and the negative y-direction (Fig. 8.13b). The magnitude of $\vec{v}_{B2}$ is

$$v_{B2} = \sqrt{(1.89\,\text{m/s})^2 + (-0.83\,\text{m/s})^2} = 2.1\,\text{m/s}$$

and the angle of its direction from the positive x-axis is

$$\beta = \arctan\frac{-0.83\,\text{m/s}}{1.89\,\text{m/s}} = -24°$$

EVALUATE: We can check our answer by looking at the values of momentum before and after the collision. Initially all of the momentum is in robot A, which has x-momentum $m_A v_{A1x} = (20\,\text{kg})(2.0\,\text{m/s}) = 40\,\text{kg}\cdot\text{m/s}$ and zero y-momentum. After the collision, robot A has x-momentum $m_A v_{A2x} = (20\,\text{kg})(1.0\,\text{m/s})(\cos 30°) = 17\,\text{kg}\cdot\text{m/s}$, while robot B has x-momentum $m_B v_{B2x} = (12\,\text{kg})(1.89\,\text{m/s}) = 23\,\text{kg}\cdot\text{m/s}$; the total x-momentum is $40\,\text{kg}\cdot\text{m/s}$, the same as before the collision (as it should be). In the y-direction, robot A acquires y-momentum $m_A v_{A2y} = (20\,\text{kg})(1.0\,\text{m/s})(\sin 30°) = 10\,\text{kg}\cdot\text{m/s}$, while robot B acquires y-momentum of the same magnitude but opposite direction: $m_B v_{B2y} = (12\,\text{kg})(-0.83\,\text{m/s}) = -10\,\text{kg}\cdot\text{m/s}$. Hence the *total* y-component of momentum after the collision has the same value (zero) as before the collision.

Test Your Understanding of Section 8.2 A spring-loaded toy sits at rest on a horizontal frictionless surface. When the spring releases, the toy breaks into three equal-mass pieces, A, B, and C, which slide along the surface. Piece A moves off in the negative x-direction, while piece B moves off in the negative y-direction. (a) What are the signs of the velocity components of piece C? (b) Which of the three pieces is moving the fastest?

8.3 Momentum Conservation and Collisions

To most people the term *collision* is likely to mean some sort of automotive disaster. We'll use it in that sense, but we'll also broaden the meaning to include any strong interaction between bodies that lasts a relatively short time. So we include

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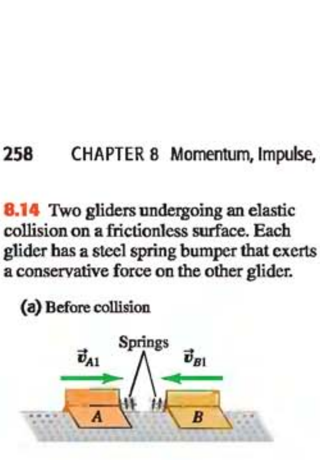
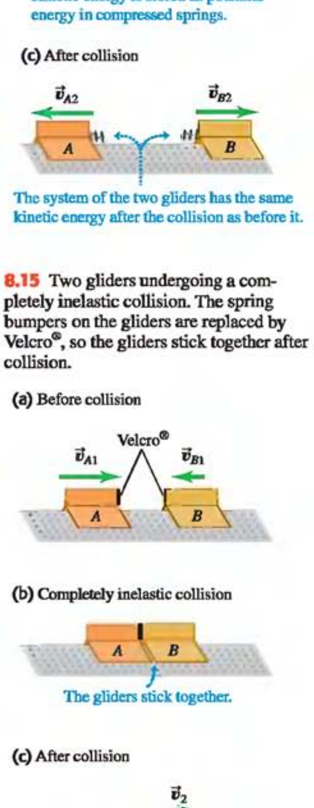
6.4 Collision Problems
6.8 Skier and Cart

8.14 Two gliders undergoing an elastic collision on a frictionless surface. Each glider has a steel spring bumper that exerts a conservative force on the other glider.



8.15 Two gliders undergoing a completely inelastic collision. The spring bumpers on the gliders are replaced by Velcro®, so the gliders stick together after collision.



not only car accidents but also balls colliding on a billiard table, neutrons hitting atomic nuclei in a nuclear reactor, the impact of a meteor on the Arizona desert, and a close encounter of a spacecraft with the planet Saturn.

If the forces between the bodies are much larger than any external forces, as is the case in most collisions, we can neglect the external forces entirely and treat the bodies as an *isolated* system. Then momentum is conserved and the total momentum of the system has the same value before and after the collision. Two cars colliding at an icy intersection provide a good example. Even two cars colliding on dry pavement can be treated as an isolated system during the collision if the forces between the cars are much larger than the friction forces of pavement against tires.

Elastic and Inelastic Collisions

If the forces between the bodies are also *conservative*, so that no mechanical energy is lost or gained in the collision, the total *kinetic* energy of the system is the same after the collision as before. Such a collision is called an **elastic collision.** A collision between two marbles or two billiard balls is almost completely elastic. Figure 8.14 shows a model for an elastic collision. When the gliders collide, their springs are momentarily compressed and some of the original kinetic energy is momentarily converted to elastic potential energy. Then the gliders bounce apart, the springs expand, and this potential energy is converted back to kinetic energy.

A collision in which the total kinetic energy after the collision is *less* than before the collision is called an **inelastic collision.** A meatball landing on a plate of spaghetti and a bullet embedding itself in a block of wood are examples of inelastic collisions. An inelastic collision in which the colliding bodies stick together and move as one body after the collision is often called a **completely inelastic collision.** Figure 8.15 shows an example; we have replaced the spring bumpers in Fig. 8.14 with Velcro®, which sticks the two bodies together.

CAUTION **An inelastic collision doesn't have to be *completely* inelastic** It's a common misconception that the *only* inelastic collisions are those in which the colliding bodies stick together. In fact, inelastic collisions include many situations in which the bodies do *not* stick. If two cars bounce off each other in a "fender bender," the work done to deform the fenders cannot be recovered as kinetic energy of the cars, so the collision is inelastic (Fig. 8.16).

Remember this rule: **In any collision in which external forces can be neglected, momentum is conserved and the total momentum before equals the total momentum after; in elastic collisions *only*, the total kinetic energy before equals the total kinetic energy after.**

Completely Inelastic Collisions

Let's look at what happens to momentum and kinetic energy in a *completely* inelastic collision of two bodies (A and B), as in Fig. 8.15. Because the two bodies stick together after the collision, they have the same final velocity $\vec{v}_2$:

$$\vec{v}_{A2} = \vec{v}_{B2} = \vec{v}_2$$

Conservation of momentum gives the relationship

$$m_A\vec{v}_{A1} + m_B\vec{v}_{B1} = (m_A + m_B)\vec{v}_2 \quad \text{(completely inelastic collision)} \quad (8.16)$$

If we know the masses and initial velocities, we can compute the common final velocity $\vec{v}_2$.

Suppose, for example, that a body with mass m_A and initial x-component of velocity v_{A1x} collides inelastically with a body with mass m_B that is initially at rest $(v_{B1x} = 0)$. From Eq. (8.16) the common x-component of velocity v_{2x} of both bodies after the collision is

$$v_{2x} = \frac{m_A}{m_A + m_B} v_{A1x} \quad \text{(completely inelastic collision, } B \text{ initially at rest)} \tag{8.17}$$

Let's verify that the total kinetic energy after this completely inelastic collision is less than before the collision. The motion is purely along the x-axis, so the kinetic energies K_1 and K_2 before and after the collision, respectively, are

$$K_1 = \tfrac{1}{2} m_A v_{A1x}^2$$

$$K_2 = \tfrac{1}{2}(m_A + m_B) v_{2x}^2 = \tfrac{1}{2}(m_A + m_B)\left(\frac{m_A}{m_A + m_B}\right)^2 v_{A1x}^2$$

The ratio of final to initial kinetic energy is

$$\frac{K_2}{K_1} = \frac{m_A}{m_A + m_B} \quad \text{(completely inelastic collision, } B \text{ initially at rest)} \tag{8.18}$$

The right side is always less than unity because the denominator is always greater than the numerator. Even when the initial velocity of m_B is not zero, it is not hard to verify that the kinetic energy after a completely inelastic collision is always less than before.

Please note: We don't recommend memorizing Eqs. (8.17) or (8.18). We derived them only to prove that kinetic energy is always lost in a completely inelastic collision.

8.16 Automobile collisions are intended to be inelastic, so that the structure of the car absorbs as much of the energy of the collision as possible. This absorbed energy cannot be recovered, since it goes into a permanent deformation of the car.

Example 8.7 A completely inelastic collision

Suppose we repeat the collision described in Example 8.5 (Section 8.2), but this time equip the gliders so that they stick together instead of bouncing apart after they collide. Their masses and initial velocities are the same as in Example 8.5. Find the common final x-velocity v_{2x}, and compare the initial and final kinetic energies.

SOLUTION

IDENTIFY: There are no external forces in the x-direction, so the x-component of momentum is conserved.

SET UP: Figure 8.17 shows our sketch. As in Example 8.5, we take the positive x-axis to point to the right. Our target variables are the final x-velocity v_{2x} and the initial and final kinetic energies of the system.

EXECUTE: From conservation of the x-component of momentum,

$$m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B) v_{2x}$$

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(0.50\,\text{kg})(2.0\,\text{m/s}) + (0.30\,\text{kg})(-2.0\,\text{m/s})}{0.50\,\text{kg} + 0.30\,\text{kg}} = 0.50\,\text{m/s}$$

Because v_{2x} is positive, the gliders move together to the right (the $+x$-direction) after the collision. Before the collision, the kinetic energies of gliders A and B are

$$K_A = \tfrac{1}{2} m_A v_{A1x}^2 = \tfrac{1}{2}(0.50\,\text{kg})(2.0\,\text{m/s})^2 = 1.0\,\text{J}$$

$$K_B = \tfrac{1}{2} m_B v_{B1x}^2 = \tfrac{1}{2}(0.30\,\text{kg})(-2.0\,\text{m/s})^2 = 0.60\,\text{J}$$

(Note that the kinetic energy of glider B is positive, even though the x-components of its velocity v_{B1x} and momentum $m_B v_{B1x}$ are both negative.) The *total* kinetic energy before the collision is 1.6 J. The kinetic energy after the collision is

$$\tfrac{1}{2}(m_A + m_B) v_{2x}^2 = \tfrac{1}{2}(0.50\,\text{kg} + 0.30\,\text{kg})(0.50\,\text{m/s})^2 = 0.10\,\text{J}$$

EVALUATE: The final kinetic energy is only $\frac{1}{16}$ of the original; $\frac{15}{16}$ is converted from mechanical energy to various other forms. If there is a ball of chewing gum between the gliders, it squashes and becomes warmer. If there is a spring between the gliders that is compressed as they lock together, then the energy is stored as potential energy of the spring. In both of these cases the *total* energy of the system is conserved, although *kinetic* energy is not. However, in an isolated system, momentum is *always* conserved, whether the collision is elastic or not.

8.17 Our sketch for this problem.

Example 8.8 The ballistic pendulum

Figure 8.18 shows a ballistic pendulum, a system for measuring the speed of a bullet. The bullet, with mass m_B, is fired into a block of wood with mass m_W, suspended like a pendulum, and makes a completely inelastic collision with it. After the impact of the bullet, the block swings up to a maximum height y. Given the values of y, m_B, and m_W, what is the initial speed v_1 of the bullet?

SOLUTION

IDENTIFY: We'll analyze this event in two stages: (1) the embedding of the bullet in the block and (2) the subsequent swinging of the block on its strings.

During the first stage, the bullet embeds itself in the block so quickly that the block has no time to move appreciably. The supporting strings remain nearly vertical, so negligible external horizontal force acts on the system of bullet plus block, and the *horizontal component of momentum* is conserved. Mechanical energy is *not* conserved in this stage because a nonconservative force does work (the force of friction between bullet and block).

In the second stage, after the collision, the block and bullet move as a unit. The only forces acting on this unit are gravity (a conservative force) and the string tensions (which do no work). Thus, as the block swings upward and to the right, *mechanical energy* is conserved. Momentum is *not* conserved during this stage because there is a net external force (the forces of gravity and string tension don't cancel when the strings are inclined).

SET UP: We take the positive x-axis to be to the right and the positive y-axis to be upward in Fig. 8.18. Our target variable is v_1. Another unknown quantity is the speed v_2 of the block and bullet as a unit just after the collision (that is, just at the end of the first stage). We'll use momentum conservation in the first stage to relate v_1 to v_2, and we'll use energy conservation in the second stage to relate v_2 to the (given) maximum height y.

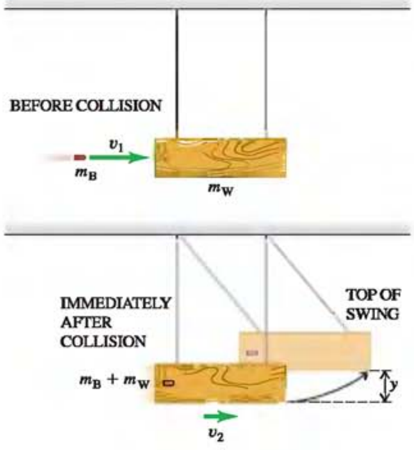
EXECUTE: In the first stage, the velocities are all in the positive x-direction. Momentum conservation gives

$$m_B v_1 = (m_B + m_W)v_2 \qquad v_1 = \frac{m_B + m_W}{m_B}v_2$$

At the beginning of the second stage, the block–bullet unit has kinetic energy $K = \frac{1}{2}(m_B + m_W)v_2^2$. [As in Eq. (8.18), this is less than the kinetic energy before the collision; the collision is inelastic!] The block–bullet unit swings up and comes to rest for an instant at a height y, where its kinetic energy is zero and the potential energy is $(m_B + m_W)gy$; it then swings back down. Energy conservation gives

$$\tfrac{1}{2}(m_B + m_W)v_2^2 = (m_B + m_W)gy \qquad v_2 = \sqrt{2gy}$$

8.18 A ballistic pendulum.



Now we substitute this expression into the momentum equation to find an expression for our target variable v_1:

$$v_1 = \frac{m_B + m_W}{m_B}\sqrt{2gy}$$

Hence measuring m_B, m_W, and y tells us the initial speed of the bullet.

EVALUATE: Let's check our answers by plugging in some realistic numbers. If $m_B = 5.00\text{ g} = 0.00500\text{ kg}$, $m_W = 2.00\text{ kg}$, and $y = 3.00\text{ cm} = 0.0300\text{ m}$, the initial speed of the bullet is

$$v_1 = \frac{0.00500\text{ kg} + 2.00\text{ kg}}{0.00500\text{ kg}}\sqrt{2(9.80\text{ m/s}^2)(0.0300\text{ m})}$$
$$= 307\text{ m/s}$$

The speed v_2 of the block just after impact is

$$v_2 = \sqrt{2gy} = \sqrt{2(9.80\text{ m/s}^2)(0.0300\text{ m})}$$
$$= 0.767\text{ m/s}$$

The kinetic energy of the bullet just before impact is $\frac{1}{2}(0.00500\text{ kg})(307\text{ m/s})^2 = 236\text{ J}$. Just after impact the kinetic energy of the bullet and block is $\frac{1}{2}(2.005\text{ kg})(0.767\text{ m/s})^2 = 0.589\text{ J}$. Nearly all the kinetic energy disappears as the wood splinters and the bullet and block become hotter.

Example 8.9 An automobile collision

A 1000-kg compact car is traveling north at 15 m/s when it collides with a 2000-kg truck traveling east at 10 m/s. All occupants are wearing seat belts and there are no injuries, but the two vehicles are thoroughly tangled and move away from the impact point as one mass. The insurance adjustor has asked you to find the velocity of the wreckage just after impact. What do you tell her?

SOLUTION

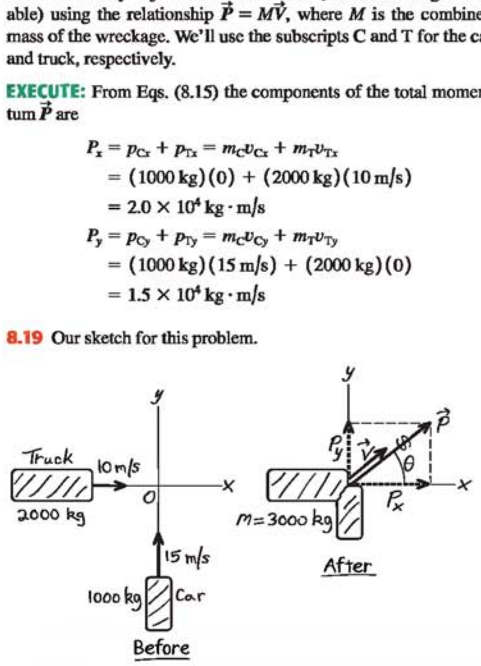
IDENTIFY: We'll assume that we can treat the cars as an isolated system during the collision. We can do so because the horizontal forces that the cars exert on each other during the collision have very large magnitudes, great enough to crumple the cars' metal skins. Compared with these forces, we can neglect any external forces such as friction. (We'll justify this assumption later.) Hence the momentum of the system of two cars has the same value just before and just after the collision.

SET UP: Figure 8.19 shows our sketch. We can find the total momentum before the collision, $\vec{P}$, using Eqs. (8.15) and the coordinate axes shown in Fig. 8.19. The momentum has the same value just after the collision; hence, once we've found $\vec{P}$, we'll be able to find the velocity $\vec{V}$ just after the collision (our second target variable) using the relationship $\vec{P} = M\vec{V}$, where M is the combined mass of the wreckage. We'll use the subscripts C and T for the car and truck, respectively.

EXECUTE: From Eqs. (8.15) the components of the total momentum $\vec{P}$ are

$$\begin{aligned} P_x &= p_{Cx} + p_{Tx} = m_C v_{Cx} + m_T v_{Tx} \\ &= (1000\,\text{kg})(0) + (2000\,\text{kg})(10\,\text{m/s}) \\ &= 2.0 \times 10^4\,\text{kg}\cdot\text{m/s} \\ P_y &= p_{Cy} + p_{Ty} = m_C v_{Cy} + m_T v_{Ty} \\ &= (1000\,\text{kg})(15\,\text{m/s}) + (2000\,\text{kg})(0) \\ &= 1.5 \times 10^4\,\text{kg}\cdot\text{m/s} \end{aligned}$$

8.19 Our sketch for this problem.



The magnitude of $\vec{P}$ is

$$\begin{aligned} P &= \sqrt{(2.0 \times 10^4\,\text{kg}\cdot\text{m/s})^2 + (1.5 \times 10^4\,\text{kg}\cdot\text{m/s})^2} \\ &= 2.5 \times 10^4\,\text{kg}\cdot\text{m/s} \end{aligned}$$

and its direction is given by the angle θ shown in Fig. 8.19, where

$$\tan\theta = \frac{P_y}{P_x} = \frac{1.5 \times 10^4\,\text{kg}\cdot\text{m/s}}{2.0 \times 10^4\,\text{kg}\cdot\text{m/s}} = 0.75 \qquad \theta = 37^\circ$$

The total momentum just after the collision is the same as just before. Assuming that no parts fall off, the total mass of wreckage is $M = m_C + m_T = 3000$ kg. From $\vec{P} = M\vec{V}$, the direction of the velocity $\vec{V}$ just after the collision is the same as that of the momentum, and its magnitude is

$$V = \frac{P}{M} = \frac{2.5 \times 10^4\,\text{kg}\cdot\text{m/s}}{3000\,\text{kg}} = 8.3\,\text{m/s}$$

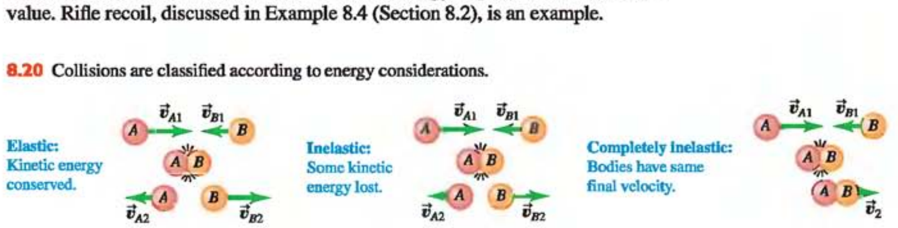
EVALUATE: This is an inelastic collision, so we expect the total kinetic energy to be less after the collision than before. Carry out the calculations yourself; you will find that the initial kinetic energy is 2.1×10^5 J and the final value is 1.0×10^5 J. More than half of the initial kinetic energy is converted to other forms.

We still need to justify our assumption that we can neglect the external forces on the vehicles during the collision. To do so, note that the mass of the truck is 2000 kg, its weight is about 20,000 N, and, if the coefficient of friction is about 0.5, the friction force when it slides across the pavement is about 10,000 N. The truck's kinetic energy just before the impact is $\frac{1}{2}(2000\,\text{kg})(10\,\text{m/s})^2 = 1.0 \times 10^5$ J. The car may crumple 0.2 m or so; to do -1.0×10^5 J of work on the car (required to stop it) in a distance of 0.2 m would require a force of 5.0×10^5 N, which is 50 times greater than the friction force. So it's reasonable to treat the external force of friction as negligible compared with the internal forces that the vehicles exert on each other.

Classifying Collisions

It's important to remember that we can classify collisions according to energy considerations (Fig. 8.20). A collision in which kinetic energy is conserved is called *elastic*. (We'll explore these in more depth in the next section.) A collision in which the total kinetic energy decreases is called *inelastic*. When the two bodies have a common final velocity, we say that the collision is *completely inelastic*. There are also cases in which the final kinetic energy is *greater* than the initial value. Rifle recoil, discussed in Example 8.4 (Section 8.2), is an example.

8.20 Collisions are classified according to energy considerations.



Finally, we emphasize again that we can sometimes use momentum conservation even when there are external forces acting on the system, if the net external force acting on the colliding bodies is small in comparison with the internal forces during the collision (as in Example 8.9)

Test Your Understanding of Section 8.3 For each situation, state whether the collision is elastic or inelastic. If it is inelastic, state whether it is completely inelastic. (a) You drop a ball from your hand. It collides with the floor and bounces back up so that it just reaches your hand. (b) You drop a different ball from your hand and let it collide with the ground. This ball bounces back up to half the height from which it was dropped. (c) You drop a ball of clay from your hand. When it collides with the ground, it stops.

8.4 Elastic Collisions

We saw in Section 8.3 that an *elastic collision* in an isolated system is one in which kinetic energy (as well as momentum) is conserved. Elastic collisions occur when the forces between the colliding bodies are *conservative*. When two billiard balls collide, they squash a little near the surface of contact, but then they spring back. Some of the kinetic energy is stored temporarily as elastic potential energy, but at the end it is reconverted to kinetic energy (Fig. 8.21).

ActivPhysics ONLINE
6.2 Collisions and Elasticity
6.7 Car Collisions: Two Dimensions
6.9 Pendulum Bashes Box

Let's look at an elastic collision between two bodies A and B. We start with a one-dimensional collision, in which all the velocities lie along the same line; we choose this line to be the x-axis. Each momentum and velocity then has only an x-component. We call the x-velocities before the collision v_{A1x} and v_{B1x}, and those after the collision v_{A2x} and v_{B2x}. From conservation of kinetic energy we have

$$\tfrac{1}{2}m_A v_{A1x}{}^2 + \tfrac{1}{2}m_B v_{B1x}{}^2 = \tfrac{1}{2}m_A v_{A2x}{}^2 + \tfrac{1}{2}m_B v_{B2x}{}^2$$

and conservation of momentum gives

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

If the masses m_A and m_B and the initial velocities v_{A1x} and v_{B1x} are known, we can solve these two equations to find the two final velocities v_{A2x} and v_{B2x}.

8.21 Billiard balls deform very little when they collide, and they quickly spring back from any deformation they do undergo. Hence the force of interaction between the balls is almost perfectly conservative, and the collision is almost perfectly elastic.

Elastic Collisions, One Body Initially at Rest

The general solution to the above equations is a little complicated, so we will concentrate on the particular case in which body B is at rest before the collision (so $v_{B1x} = 0$). Think of body B as a target for body A to hit. Then the kinetic energy and momentum conservation equations are, respectively,

$$\tfrac{1}{2}m_A v_{A1x}{}^2 = \tfrac{1}{2}m_A v_{A2x}{}^2 + \tfrac{1}{2}m_B v_{B2x}{}^2 \qquad (8.19)$$

$$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x} \qquad (8.20)$$

We can solve for v_{A2x} and v_{B2x} in terms of the masses and the initial velocity v_{A1x}. This involves some fairly strenuous algebra, but it's worth it. No pain, no gain! The simplest approach is somewhat indirect, but along the way it uncovers an additional interesting feature of elastic collisions.

First we rearrange Eqs. (8.19) and (8.20) as follows:

$$m_B v_{B2x}{}^2 = m_A(v_{A1x}{}^2 - v_{A2x}{}^2) = m_A(v_{A1x} - v_{A2x})(v_{A1x} + v_{A2x}) \qquad (8.21)$$

$$m_B v_{B2x} = m_A(v_{A1x} - v_{A2x}) \qquad (8.22)$$

Now we divide Eq. (8.21) by Eq. (8.22) to obtain

$$v_{B2x} = v_{A1x} + v_{A2x} \qquad (8.23)$$

We substitute this expression back into Eq. (8.22) to eliminate v_{B2x} and then solve for v_{A2x}:

$$m_B(v_{A1x} + v_{A2x}) = m_A(v_{A1x} - v_{A2x})$$

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x} \tag{8.24}$$

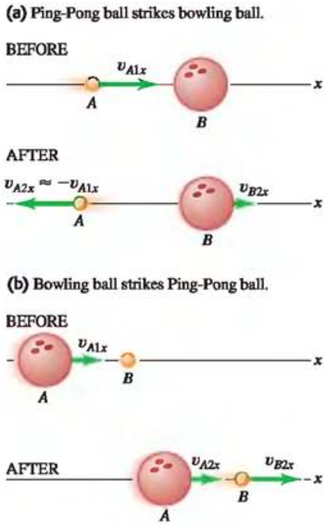
Finally, we substitute this result back into Eq. (8.23) to obtain

$$v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x} \tag{8.25}$$

Now we can interpret the results. Suppose body A is a Ping-Pong ball and body B is a bowling ball. Then we expect A to bounce off after the collision with a velocity nearly equal to its original value but in the opposite direction (Fig. 8.22a), and we expect B's velocity to be much less. That's just what the equations predict. When m_A is much smaller than m_B, the fraction in Eq. (8.24) is approximately equal to (-1), so v_{A2x} is approximately equal to $-v_{A1x}$. The fraction in Eq. (8.25) is much smaller than unity, so v_{B2x} is much less than v_{A1x}. Figure 8.22b shows the opposite case, in which A is the bowling ball and B the Ping-Pong ball and m_A is much larger than m_B. What do you expect to happen then? Check your predictions against Eqs. (8.24) and (8.25).

Another interesting case occurs when the masses are equal (Fig. 8.23). If $m_A = m_B$, then Eqs. (8.24) and (8.25) give $v_{A2x} = 0$ and $v_{B2x} = v_{A1x}$. That is, the body that was moving stops dead; it gives all its momentum and kinetic energy to the body that was at rest. This behavior is familiar to all pool players.

8.22 Collisions between (a) a moving Ping-Pong ball and an initially stationary bowling ball, and (b) a moving bowling ball and an initially stationary Ping-Pong ball.



Elastic Collisions and Relative Velocity

Let's return to the more general case in which A and B have different masses. Equation (8.23) can be rewritten as

$$v_{A1x} = v_{B2x} - v_{A2x} \tag{8.26}$$

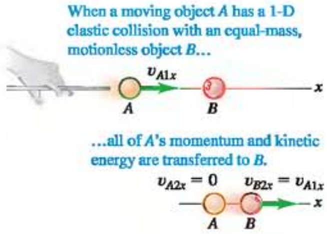
Here $v_{B2x} - v_{A2x}$ is the velocity of B relative to A *after* the collision; from Eq. (8.26), this equals v_{A1x}, which is the *negative* of the velocity of B relative to A *before* the collision. (We discussed relative velocity in Section 3.5.) The relative velocity has the same magnitude, but opposite sign, before and after the collision. The sign changes because A and B are approaching each other before the collision but moving apart after the collision. If we view this collision from a second coordinate system moving with constant velocity relative to the first, the velocities of the bodies are different but the *relative* velocities are the same. Hence our statement about relative velocities holds for *any* straight-line elastic collision, even when neither body is at rest initially. ***In a straight-line elastic collision of two bodies, the relative velocities before and after the collision have the same magnitude but opposite sign.*** This means that if B is moving before the collision, Eq. (8.26) becomes

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x}) \tag{8.27}$$

It turns out that a *vector* relationship similar to Eq. (8.27) is a general property of *all* elastic collisions, even when both bodies are moving initially and the velocities do not all lie along the same line. This result provides an alternative and equivalent definition of an elastic collision: ***In an elastic collision, the relative velocity of the two bodies has the same magnitude before and after the collision.*** Whenever this condition is satisfied, the total kinetic energy is also conserved.

When an elastic two-body collision isn't head-on, the velocities don't all lie along a single line. If they all lie in a plane, then each final velocity has two unknown components, and there are four unknowns in all. Conservation of energy and conservation of the x- and y-components of momentum give only three equations. To determine the final velocities uniquely, we need additional information, such as the direction or magnitude of one of the final velocities.

8.23 A one-dimensional elastic collision between bodies of equal mass.



Example 8.10 An elastic straight-line collision

We repeat the air-track experiment from Example 8.5 (Section 8.2), but now we add ideal spring bumpers to the gliders so that the collision is elastic. What are the velocities of A and B after the collision?

SOLUTION

IDENTIFY: As in Example 8.5, the net external force on the system of two gliders is zero, and the momentum of the system is conserved.

SET UP: Figure 8.24 shows our sketch. We again choose the positive x-axis to point to the right. We'll find our target variables, v_{A2x} and v_{B2x}, using Eq. (8.27) and the equation of momentum conservation.

EXECUTE: From conservation of momentum,

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

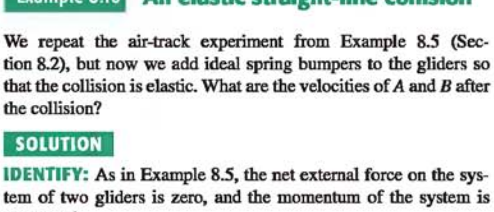
$$\begin{aligned}(0.50\,\text{kg})(2.0\,\text{m/s}) + (0.30\,\text{kg})(-2.0\,\text{m/s}) \\ = (0.50\,\text{kg})v_{A2x} + (0.30\,\text{kg})v_{B2x}\end{aligned}$$

$$0.50v_{A2x} + 0.30v_{B2x} = 0.40\,\text{m/s}$$

(In the last equation we divided through by the unit "kg.") From Eq. (8.27), the relative velocity relationship for an elastic collision, we have

$$\begin{aligned}v_{B2x} - v_{A2x} &= -(v_{B1x} - v_{A1x}) \\ &= -(-2.0\,\text{m/s} - 2.0\,\text{m/s}) = 4.0\,\text{m/s}\end{aligned}$$

8.24 Our sketch for this problem.



Before the collision, the velocity of B relative to A is to the left at 4.0 m/s; after the collision, the velocity of B relative to A is to the right at 4.0 m/s. Solving these equations simultaneously, we find

$$v_{A2x} = -1.0\,\text{m/s} \qquad v_{B2x} = 3.0\,\text{m/s}$$

EVALUATE: Both bodies reverse their directions of motion; A moves to the left at 1.0 m/s and B moves to the right at 3.0 m/s. This is different from the result of Example 8.5 because that collision was *not* elastic.

Note that unlike the situations shown in Fig. 8.22, the two gliders are *both* moving toward each other before the collision. Our results show that A (the more massive glider) moves slower after the collision than before the collision, and so loses kinetic energy. In contrast, B (the less massive glider) gains kinetic energy: It moves faster after the collision than before. The *total* kinetic energy after the elastic collision is

$$\tfrac{1}{2}(0.50\,\text{kg})(-1.0\,\text{m/s})^2 + \tfrac{1}{2}(0.30\,\text{kg})(3.0\,\text{m/s})^2 = 1.6\,\text{J}$$

As expected, this equals the total kinetic energy *before* the collision (which we calculated in Example 8.7 in Section 8.3). Thus kinetic energy is transferred from A to B in the collision, with none of it lost in the process. Much the same happens when a baseball player swings a bat and hits an oncoming baseball. The collision is nearly elastic, and the more massive bat transfers kinetic energy to the less massive baseball. The baseball leaves the bat with a much greater speed—perhaps enough to make a home run.

CAUTION Be careful with the elastic collision equations You might have been tempted to solve this problem using Eqs. (8.24) and (8.25). These equations apply *only* to situations in which body B is initially at rest, which isn't the case here. When in doubt, always solve the problem at hand using equations that are applicable to a broad variety of cases.

Example 8.11 Moderator in a nuclear reactor

The fission of uranium nuclei in a nuclear reactor produces high-speed neutrons. Before a neutron can trigger additional fissions, it has to be slowed down by collisions with nuclei in the *moderator* of the reactor. The first nuclear reactor (built in 1942 at the University of Chicago) and the reactor involved in the 1986 Chernobyl accident both used carbon (graphite) as the moderator material. Suppose a neutron (mass 1.0 u) traveling at 2.6×10^7 m/s undergoes a head-on elastic collision with a carbon nucleus (mass 12 u) initially at rest. The external forces during the collision are negligible. What are the velocities after the collision? (1 u is the *atomic mass unit*, equal to 1.66×10^{-27} kg.)

SOLUTION

IDENTIFY: We are given that the external forces can be neglected (so momentum is conserved in the collision) and that the collision is elastic (so kinetic energy is also conserved).

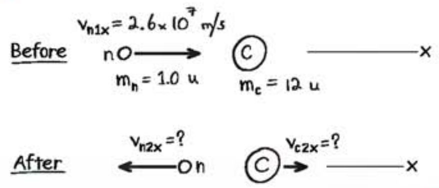
SET UP: Figure 8.25 shows our sketch. We take the x-axis to be in the direction in which the neutron is moving initially. Because the collision is head-on, both the neutron and the carbon nucleus move along this same axis after the collision. Furthermore, because one body is initially at rest, we can use Eqs. (8.24) and (8.25) with A replaced by n (for the neutron) and B replaced by C (for the carbon nucleus). We have $m_n = 1.0\,\text{u}$, $m_C = 12\,\text{u}$, and $v_{n1x} = 2.6 \times 10^7$ m/s, and we need to solve for the target variables v_{n2x} and v_{C2x} (the final velocities of the neutron and the carbon nucleus, respectively).

EXECUTE: We'll let you do the arithmetic; the results are

$$v_{n2x} = -2.2 \times 10^7\,\text{m/s} \qquad v_{C2x} = 0.4 \times 10^7\,\text{m/s}$$

EVALUATE: The neutron ends up with $\frac{11}{13}$ of its initial speed, and the speed of the recoiling carbon nucleus is $\frac{2}{13}$ of the neutron's initial speed. [These ratios are the factors $(m_n - m_C)/(m_n + m_C)$ and $2m_n/(m_n + m_C)$ that appear in Eqs. (8.24) and (8.25), with the subscripts revised for this problem.] Kinetic energy is proportional to speed squared, so the neutron's final kinetic energy is $(\frac{11}{13})^2$, or about 0.72 of its original value. If the neutron makes a second such collision, its kinetic energy is $(0.72)^2$, or about half its original value, and so on. After several collisions, the neutron will be moving quite slowly and will be able to trigger a fission reaction in a uranium nucleus.

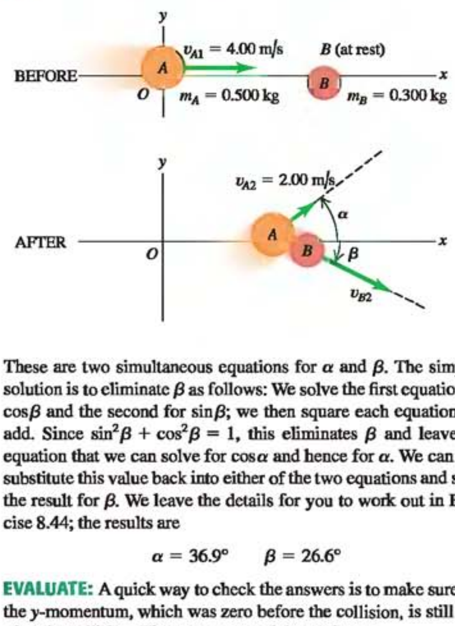
8.25 Our sketch for this problem.



Example 8.12 A two-dimensional elastic collision

Figure 8.26 shows an elastic collision of two pucks on a frictionless air-hockey table. Puck A has mass $m_A = 0.500$ kg and puck B has mass $m_B = 0.300$ kg. Puck A has an initial velocity of 4.00 m/s in the positive x-direction and a final velocity of 2.00 m/s in an unknown direction. Puck B is initially at rest. Find the final speed v_{B2} of puck B and the angles α and β in the figure.

8.26 An elastic collision that isn't head-on.



SOLUTION

IDENTIFY: Although the collision is elastic, it is *not* one-dimensional, so we can't use any of the one-dimensional formulas derived in this section. Instead, we'll use the equations for conservation of energy, conservation of x-momentum, and conservation of y-momentum.

SET UP: The target variables are given in the statement of the problem. We have three equations, which should be enough to solve for our three target variables.

EXECUTE: Because the collision is elastic, the initial and final kinetic energies are equal:

$$\tfrac{1}{2}m_A v_{A1}{}^2 = \tfrac{1}{2}m_A v_{A2}{}^2 + \tfrac{1}{2}m_B v_{B2}{}^2$$

$$v_{B2}{}^2 = \frac{m_A v_{A1}{}^2 - m_A v_{A2}{}^2}{m_B}$$

$$= \frac{(0.500\text{ kg})(4.00\text{ m/s})^2 - (0.500\text{ kg})(2.00\text{ m/s})^2}{0.300\text{ kg}}$$

$$v_{B2} = 4.47\text{ m/s}$$

Conservation of the x-component of total momentum gives

$$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$(0.500\text{ kg})(4.00\text{ m/s}) = (0.500\text{ kg})(2.00\text{ m/s})(\cos\alpha) + (0.300\text{ kg})(4.47\text{ m/s})(\cos\beta)$$

and conservation of the y-component gives

$$0 = m_A v_{A2y} + m_B v_{B2y}$$

$$0 = (0.500\text{ kg})(2.00\text{ m/s})(\sin\alpha) - (0.300\text{ kg})(4.47\text{ m/s})(\sin\beta)$$

These are two simultaneous equations for α and β. The simplest solution is to eliminate β as follows: We solve the first equation for $\cos\beta$ and the second for $\sin\beta$; we then square each equation and add. Since $\sin^2\beta + \cos^2\beta = 1$, this eliminates β and leaves an equation that we can solve for $\cos\alpha$ and hence for α. We can then substitute this value back into either of the two equations and solve the result for β. We leave the details for you to work out in Exercise 8.44; the results are

$$\alpha = 36.9^\circ \qquad \beta = 26.6^\circ$$

EVALUATE: A quick way to check the answers is to make sure that the y-momentum, which was zero before the collision, is still zero after the collision. The y-momenta of the pucks are

$$p_{A2y} = (0.500\text{ kg})(2.00\text{ m/s})(\sin 36.9^\circ) = +0.600\text{ kg}\cdot\text{m/s}$$

$$p_{B2y} = -(0.300\text{ kg})(4.47\text{ m/s})(\sin 26.6^\circ) = -0.600\text{ kg}\cdot\text{m/s}$$

The sum of these values is zero, as it should be.

Test Your Understanding of Section 8.4 Most present-day nuclear reactors use water as a moderator (see Example 8.11). Are water molecules (mass $m_w = 18.0$ u) a better or worse moderator than carbon atoms? (One advantage of water is that it also acts as a coolant for the reactor's radioactive core.)

8.5 Center of Mass

We can restate the principle of conservation of momentum in a useful way by using the concept of **center of mass.** Suppose we have several particles with masses m_1, m_2, and so on. Let the coordinates of m_1 be (x_1, y_1), those of m_2 be (x_2, y_2), and so on. We define the center of mass of the system as the point that has coordinates (x_{cm}, y_{cm}) given by

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_ix_i}{\sum_i m_i}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_ix_i}{\sum_i m_i} \qquad \text{(center of mass) (8.28)}$$

The position vector $\vec{r}_{cm}$ of the center of mass can be expressed in terms of the position vectors $\vec{r}_1, \vec{r}_2, \ldots$ of the particles as

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i} \qquad \text{(center of mass) (8.29)}$$

In statistical language, the center of mass is a *mass-weighted average* position of the particles.

Example 8.13 Center of mass of a water molecule

Figure 8.27 shows a simple model of the structure of a water molecule. The separation between atoms is $d = 9.57 \times 10^{-11}$ m. Each hydrogen atom has mass 1.0 u, and the oxygen atom has mass 16.0 u. Find the position of the center of mass.

SOLUTION

IDENTIFY: Nearly all the mass of each atom is concentrated in its nucleus, which is only about 10^{-5} times the overall radius of the atom. Hence we can safely represent each atom as a point particle.

SET UP: The coordinate system is shown in Fig. 8.27. We'll use Eqs. (8.28) to determine the coordinates x_{cm} and y_{cm}.

EXECUTE: The x-coordinate of each hydrogen atom is $d\cos(105°/2)$; the y-coordinates of the upper and lower hydrogen atoms are $+d\sin(105°/2)$ and $-d\sin(105°/2)$, respectively. The coordinates of the oxygen atom are $x = 0, y = 0$. From Eqs. (8.28) the x-coordinate of the center of mass is

$$x_{cm} = \frac{\begin{bmatrix}(1.0\text{ u})(d\cos 52.5°) + (1.0\text{ u}) \\ \times (d\cos 52.5°) + (16.0\text{ u})(0)\end{bmatrix}}{1.0\text{ u} + 1.0\text{ u} + 16.0\text{ u}} = 0.068d$$

and the y-coordinate is

$$y_{cm} = \frac{\begin{bmatrix}(1.0\text{ u})(d\sin 52.5°) + (1.0\text{ u}) \\ \times (-d\sin 52.5°) + (16.0\text{ u})(0)\end{bmatrix}}{1.0\text{ u} + 1.0\text{ u} + 16.0\text{ u}} = 0$$

Substituting the value $d = 9.57 \times 10^{-11}$ m, we find

$$x_{cm} = (0.068)(9.57 \times 10^{-11}\text{ m}) = 6.5 \times 10^{-12}\text{ m}$$

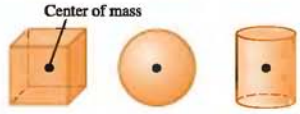
EVALUATE: The center of mass is much closer to the oxygen atom than to either hydrogen atom because the oxygen atom is much more massive. Notice that the center of mass lies along the x-axis, which is the *axis of symmetry* of this molecule. If the molecule is rotated by 180° around this axis, it looks exactly the same as before. The position of the center of mass can't be affected by this rotation, so it must lie on the axis of symmetry.

8.27 Where is the center of mass of a water molecule?

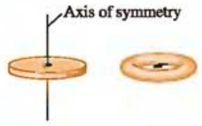
For solid bodies, in which we have (at least on a macroscopic level) a continuous distribution of matter, the sums in Eqs. (8.28) have to be replaced by integrals. The calculations can get quite involved, but we can say three general things about such problems (Fig. 8.28). First, whenever a homogeneous body has a geometric center, such as a billiard ball, a sugar cube, or a can of frozen orange juice, the center of mass is at the geometric center. Second, whenever a body has an axis of symmetry, such as a wheel or a pulley, the center of mass always lies on that axis. Third, there is no law that says the center of mass has to be within the body. For example, the center of mass of a donut is right in the middle of the hole.

We'll talk a little more about locating the center of mass in Chapter 11 in connection with the related concept of *center of gravity.*

8.28 Locating the center of mass of a symmetrical object.



If a homogeneous object has a geometric center, that is where the center of mass is located.



If an object has an axis of symmetry, the center of mass lies along it. As in the case of the donut, the center of mass may not be within the object.

Motion of the Center of Mass

To see the significance of the center of mass of a collection of particles, we must ask what happens to the center of mass when the particles move. The x- and y-components of velocity of the center of mass, $v_{cm\text{-}x}$ and $v_{cm\text{-}y}$, are the time derivatives of x_{cm} and y_{cm}. Also, dx_1/dt is the x-component of velocity of particle 1, and so on, so $dx_1/dt = v_{1x}$, and so on. Taking time derivatives of Eqs. (8.28), we get

$$v_{cm\text{-}x} = \frac{m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} + \cdots}{m_1 + m_2 + m_3 + \cdots}$$
$$v_{cm\text{-}y} = \frac{m_1 v_{1y} + m_2 v_{2y} + m_3 v_{3y} + \cdots}{m_1 + m_2 + m_3 + \cdots} \quad (8.30)$$

These equations are equivalent to the single vector equation obtained by taking the time derivative of Eq. (8.29):

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \quad (8.31)$$

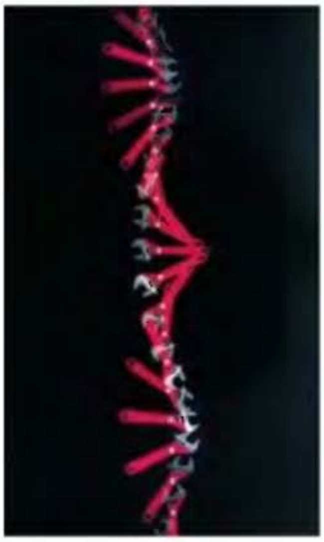
We denote the *total* mass $m_1 + m_2 + \cdots$ by M. We can then rewrite Eq. (8.31) as

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots = \vec{P} \quad (8.32)$$

The right side is simply the total momentum $\vec{P}$ of the system. Thus we have proved that *the total momentum is equal to the total mass times the velocity of the center of mass.* When you catch a baseball, you are really catching a collection of a very large number of molecules of masses $m_1, m_2, m_3, \ldots$. The impulse you feel is due to the total momentum of this entire collection. But this impulse is the same as if you were catching a single particle of mass $M = m_1 + m_2 + m_3 + \cdots$ moving with velocity $\vec{v}_{cm}$, the velocity of the collection's center of mass. So Eq. (8.32) helps to justify representing an extended body as a particle.

For a system of particles on which the net external force is zero, so that the total momentum $\vec{P}$ is constant, the velocity of the center of mass $\vec{v}_{cm} = \vec{P}/M$ is also constant. Suppose we mark the center of mass of a wrench and then slide the wrench with a spinning motion across a smooth, horizontal tabletop (Fig. 8.29). The overall motion appears complicated, but the center of mass follows a straight line, as though all the mass were concentrated at that point.

8.29 The center of mass of this wrench is marked with a white dot. The net external force acting on the wrench is almost zero. As the wrench spins on a smooth horizontal surface, the center of mass moves in a straight line with nearly constant velocity.



Example 8.14 A tug-of-war on the ice

James and Ramon are standing 20.0 m apart on the slippery surface of a frozen pond. Ramon has mass 60.0 kg and James has mass 90.0 kg. Midway between the two men a mug of their favorite beverage sits on the ice. They pull on the ends of a light rope that is stretched between them. When James has moved 6.0 m toward the mug, how far and in what direction has Ramon moved?

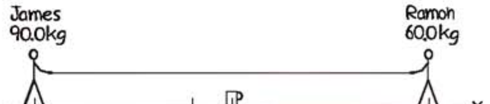
Continued

SOLUTION

IDENTIFY: The frozen surface is horizontal and essentially frictionless, so the net external force on the system of James, Ramon, and the rope is zero. Hence their total momentum is conserved. Initially there is no motion, so the total momentum is zero; thus the velocity of the center of mass is zero, and the center of mass remains at rest. We can use this to relate the positions of James and Ramon.

SET UP: Let's take the origin at the position of the mug, and let the $+x$-axis extend from the mug toward Ramon. Figure 8.30 shows our sketch. Since the rope is light, we can neglect its mass in calculating the position of the center of mass with Eq. (8.28).

8.30 Our sketch for this problem.



EXECUTE: The initial x-coordinates of James and Ramon are -10.0 m and $+10.0$ m, respectively, so the x-coordinate of the center of mass is

$$x_{cm} = \frac{(90.0\text{ kg})(-10.0\text{ m}) + (60.0\text{ kg})(10.0\text{ m})}{90.0\text{ kg} + 60.0\text{ kg}} = -2.0\text{ m}$$

When James moves 6.0 m toward the mug, his new x-coordinate is -4.0 m; we'll call Ramon's new x-coordinate x_2. The center of mass doesn't move, so

$$x_{cm} = \frac{(90.0\text{ kg})(-4.0\text{ m}) + (60.0\text{ kg})x_2}{90.0\text{ kg} + 60.0\text{ kg}} = -2.0\text{ m}$$

$$x_2 = 1.0\text{ m}$$

James has moved 6.0 m in the positive x-direction and is still 4.0 m from the mug, but Ramon has moved 9.0 m in the negative x-direction and is only 1.0 m from it.

EVALUATE: The ratio of how far each man moved, $(6.0\text{ m})/(9.0\text{ m}) = \frac{2}{3}$, equals the inverse ratio of their masses. Can you see why? If the two men keep moving (and if the surface is frictionless, they will!), Ramon will reach the mug first. This result is completely independent of how hard either person pulls; pulling harder just helps Ramon quench his thirst sooner.

External Forces and Center-of-Mass Motion

If the net external force on a system of particles is not zero, then total momentum is not conserved and the velocity of the center of mass changes. Let's look at the relationship between the motion of the center of mass and the forces acting on the system.

Equations (8.31) and (8.32) give the *velocity* of the center of mass in terms of the velocities of the individual particles. We take the time derivatives of these equations to show that the *accelerations* are related in the same way. Let $\vec{a}_{cm} = d\vec{v}_{cm}/dt$ be the acceleration of the center of mass; then we find

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots \qquad (8.33)$$

Now $m_1\vec{a}_1$ is equal to the vector sum of forces on the first particle, and so on, so the right side of Eq. (8.33) is equal to the vector sum $\sum\vec{F}$ of *all* the forces on *all* the particles. Just as we did in Section 8.2, we can classify each force as *external* or *internal*. The sum of all forces on all the particles is then

$$\sum\vec{F} = \sum\vec{F}_{ext} + \sum\vec{F}_{int} = M\vec{a}_{cm}$$

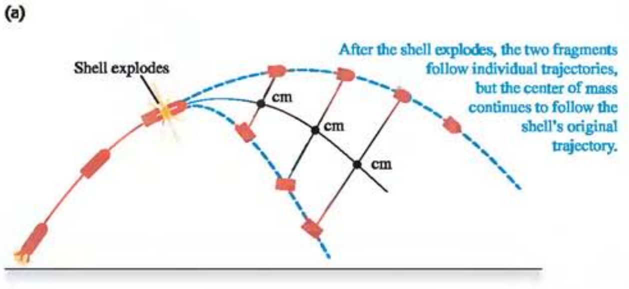
Because of Newton's third law, the internal forces all cancel in pairs, and $\sum\vec{F}_{int} = \mathbf{0}$. What survives on the left side is the sum of only the *external* forces:

$$\sum\vec{F}_{ext} = M\vec{a}_{cm} \qquad \text{(body or collection of particles)} \qquad (8.34)$$

When a body or a collection of particles is acted on by external forces, the center of mass moves just as though all the mass were concentrated at that point and it were acted on by a net force equal to the sum of the external forces on the system.

This result may not sound very impressive, but in fact it is central to the whole subject of mechanics. In fact, we've been using this result all along; without it, we would not be able to represent an extended body as a point particle when we apply Newton's laws. It explains why only *external* forces can affect the motion of an extended body. If you pull upward on your belt, your belt exerts an equal downward force on your hands; these are *internal* forces that cancel and have no effect on the overall motion of your body.

8.31 **(a)** A shell explodes into two fragments in flight. If air resistance is ignored, the center of mass continues on the same trajectory as the shell's path before exploding. **(b)** The same effect occurs with exploding fireworks.



Suppose a cannon shell traveling in a parabolic trajectory (neglecting air resistance) explodes in flight, splitting into two fragments with equal mass (Fig. 8.31a). The fragments follow new parabolic paths, but the center of mass continues on the original parabolic trajectory, just as though all the mass were still concentrated at that point. A skyrocket exploding in air (Fig. 8.31b) is a spectacular example of this effect.

This property of the center of mass is important when we analyze the motion of rigid bodies. We describe the motion of an extended body as a combination of translational motion of the center of mass and rotational motion about an axis through the center of mass. We will return to this topic in Chapter 10. This property also plays an important role in the motion of astronomical objects. It's not correct to say that the moon orbits the earth; rather, the earth and moon both move in orbits around their center of mass.

There's one more useful way to describe the motion of a system of particles. Using $\vec{a}_{cm} = d\vec{v}_{cm}/dt$, we can rewrite Eq. (8.33) as

$$M\vec{a}_{cm} = M\frac{d\vec{v}_{cm}}{dt} = \frac{d(M\vec{v}_{cm})}{dt} = \frac{d\vec{P}}{dt} \tag{8.35}$$

The total system mass M is constant, so we're allowed to take it inside the derivative. Substituting Eq. (8.35) into Eq. (8.34), we find

$$\Sigma\vec{F}_{ext} = \frac{d\vec{P}}{dt} \quad \text{(extended body or system of particles)} \tag{8.36}$$

This equation looks like Eq. (8.4). The difference is that Eq. (8.36) describes a *system* of particles, such as an extended body, while Eq. (8.4) describes a single particle. The interactions between the particles that make up the system can change the individual momenta of the particles, but the *total* momentum $\vec{P}$ of the system can be changed only by external forces acting from outside the system.

Finally, we note that if the net external force is zero, Eq. (8.34) shows that the acceleration $\vec{a}_{cm}$ of the center of mass is zero. So the center-of-mass velocity $\vec{v}_{cm}$ is constant, as for the wrench in Fig. 8.29. From Eq. (8.36) the total momentum $\vec{P}$ is also constant. This reaffirms our statement in Section 8.3 of the principle of conservation of momentum.

Test Your Understanding of Section 8.5 Will the center of mass in Fig. 8.31a continue on the same parabolic trajectory even after one of the fragments hits the ground? Why or why not?

*8.6 Rocket Propulsion

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6.6 Saving an Astronaut

Momentum considerations are particularly useful for analyzing a system in which the masses of parts of the system change with time. In such cases we can't use Newton's second law $\sum\vec{F} = m\vec{a}$ directly because m changes. Rocket propulsion offers a typical and interesting example of this kind of analysis. A rocket is propelled forward by rearward ejection of burned fuel that initially was in the rocket (which is why rocket fuel is also called *propellant*). The forward force on the rocket is the reaction to the backward force on the ejected material. The total mass of the system is constant, but the mass of the rocket itself decreases as material is ejected.

As a simple example, consider a rocket fired in outer space, where there is no gravitational force and no air resistance. Let m denote the mass of the rocket, which will change as it expends fuel. We choose our x-axis to be along the rocket's direction of motion. Figure 8.32a shows the rocket at a time t, when its mass is m and its x-velocity relative to our coordinate system is v. (For simplicity, we will drop the subscript x in this discussion.) The x-component of total momentum at this instant is $P_1 = mv$. In a short time interval dt, the mass of the rocket changes by an amount dm. This is an inherently negative quantity because the rocket's mass m ***decreases*** with time. During dt, a ***positive*** mass $-dm$ of burned fuel is ejected from the rocket. Let v_{ex} be the exhaust *speed* of this material ***relative to the rocket;*** the burned fuel is ejected opposite the direction of motion, so its x-component of ***velocity*** relative to the rocket is $-v_{ex}$. The x-velocity v_{fuel} of the burned fuel relative to our coordinate system is then

$$v_{fuel} = v + (-v_{ex}) = v - v_{ex}$$

and the x-component of momentum of the ejected mass $(-dm)$ is

$$(-dm)v_{fuel} = (-dm)(v - v_{ex})$$

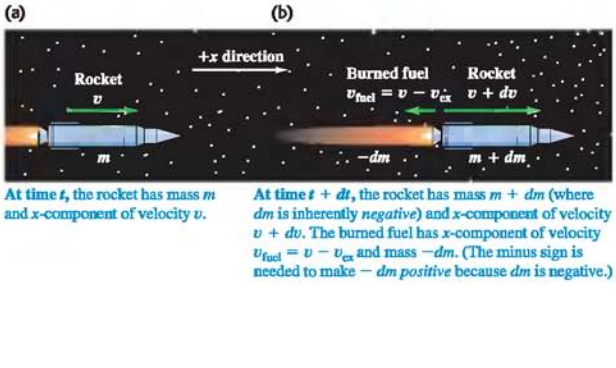
Figure 8.32b shows that at the end of the time interval dt, the x-velocity of the rocket and unburned fuel has increased to $v + dv$, and its mass has decreased to $m + dm$ (remember that dm is negative). The rocket's momentum at this time is

$$(m + dm)(v + dv)$$

Thus the *total* x-component of momentum P_2 of the rocket plus ejected fuel at time $t + dt$ is

$$P_2 = (m + dm)(v + dv) + (-dm)(v - v_{ex})$$

8.32 A rocket moving in gravity-free outer space at (a) time t and (b) time $t + dt$.



According to our initial assumption, the rocket and fuel are an isolated system. Thus momentum is conserved, and the total x-component of momentum of the system must be the same at time t and at time $t + dt$: $P_1 = P_2$. Hence

$$mv = (m + dm)(v + dv) + (-dm)(v - v_{ex})$$

This can be simplified to

$$m\,dv = -dm\,v_{ex} - dm\,dv$$

We can neglect the term $(-dm\,dv)$ because it is a product of two small quantities and thus is much smaller than the other terms. Dropping this term, dividing by dt, and rearranging, we find

$$m\frac{dv}{dt} = -v_{ex}\frac{dm}{dt} \tag{8.37}$$

Now dv/dt is the acceleration of the rocket, so the left side of this equation (mass times acceleration) equals the net force F, or *thrust*, on the rocket,

$$F = -v_{ex}\frac{dm}{dt} \tag{8.38}$$

The thrust is proportional both to the relative speed v_{ex} of the ejected fuel and to the mass of fuel ejected per unit time, $-dm/dt$. (Remember that dm/dt is negative because it is the rate of change of the rocket's mass, so F is positive.)

The x-component of acceleration of the rocket is

$$a = \frac{dv}{dt} = -\frac{v_{ex}}{m}\frac{dm}{dt} \tag{8.39}$$

This is positive because v_{ex} is positive (remember, it's the exhaust *speed*) and dm/dt is negative. The rocket's mass m decreases continuously while the fuel is being consumed. If v_{ex} and dm/dt are constant, the acceleration increases until all the fuel is gone.

Equation (8.38) tells us that an effective rocket burns fuel at a rapid rate (large $-dm/dt$) and ejects the burned fuel at a high relative speed (large v_{ex}), as in Fig. 8.33. In the early days of rocket propulsion, people who didn't understand conservation of momentum thought that a rocket couldn't function in outer space because "it doesn't have anything to push against." On the contrary, rockets work *best* in outer space, where there is no air resistance! The launch vehicle in Fig. 8.33 is *not* "pushing against the ground" to get into the air.

If the exhaust speed v_{ex} is constant, we can integrate Eq. (8.39) to find a relationship between the velocity v at any time and the remaining mass m. At time $t = 0$, let the mass be m_0 and the velocity v_0. Then we rewrite Eq. (8.39) as

$$dv = -v_{ex}\frac{dm}{m}$$

We change the integration variables to v' and m', so we can use v and m as the upper limits (the final speed and mass). Then we integrate both sides, using limits v_0 to v and m_0 to m, and take the constant v_{ex} outside the integral:

$$\int_{v_0}^{v} dv' = -\int_{m_0}^{m} v_{ex}\frac{dm'}{m'} = -v_{ex}\int_{m_0}^{m}\frac{dm'}{m'}$$

$$v - v_0 = -v_{ex}\ln\frac{m}{m_0} = v_{ex}\ln\frac{m_0}{m} \tag{8.40}$$

The ratio m_0/m is the original mass divided by the mass after the fuel has been exhausted. In practical spacecraft this ratio is made as large as possible to maximize the speed gain, which means that the initial mass of the rocket is almost all fuel. The final velocity of the rocket will be greater in magnitude (and is often

8.33 To provide enough thrust to lift its payload into space, this Atlas V launch vehicle exhausts more than 1000 kg of burned fuel per second at speeds of nearly 4000 m/s.

much greater) than the relative speed v_{ex} if $\ln(m_0/m) > 1$—that is, if $m_0/m > e = 2.71828\ldots$

We've assumed throughout this analysis that the rocket is in gravity-free outer space. However, gravity must be taken into account when a rocket is launched from the surface of a planet, as in Fig. 8.33 (see Problem 8.110).

Example 8.15 Acceleration of a rocket

A rocket is in outer space, far from any planet, when the rocket engine is turned on. In the first second of firing, the rocket ejects $\frac{1}{120}$ of its mass with a relative speed of 2400 m/s. What is the rocket's initial acceleration?

SOLUTION

IDENTIFY: We are given the rocket's exhaust speed v_{ex}, but not its mass m or the rate of change of its mass dm/dt. However, we are told what fraction of the initial mass is lost during a given time interval, which should be enough.

SET UP: We'll use Eq. (8.39) to find the acceleration of the rocket.

EXECUTE: The initial rate of change of mass is

$$\frac{dm}{dt} = -\frac{m_0/120}{1\text{ s}} = -\frac{m_0}{120\text{ s}}$$

where m_0 is the initial ($t = 0$) mass of the rocket. From Eq. (8.39) the initial acceleration is

$$a = -\frac{v_{ex}}{m_0}\frac{dm}{dt} = -\frac{2400\text{ m/s}}{m_0}\left(-\frac{m_0}{120\text{ s}}\right) = 20\text{ m/s}^2$$

EVALUATE: Note that the answer didn't depend on the value of m_0. If v_{ex} is the same, the initial acceleration is the same for a 120,000-kg spacecraft that ejects 1000 kg/s as for a 60-kg astronaut equipped with a small rocket that ejects 0.5 kg/s.

Example 8.16 Speed of a rocket

Suppose that $\frac{3}{4}$ of the initial mass m_0 of the rocket in Example 8.15 is fuel, so the final mass is $m = m_0/4$, and that the fuel is completely consumed at a constant rate in a total time $t = 90$ s. If the rocket starts from rest in our coordinate system, find its speed at the end of this time.

SOLUTION

IDENTIFY: We are given the initial velocity v_0 (equal to zero), the exhaust speed v_{ex}, and the final mass m in terms of the initial mass m_0.

SET UP: We'll use Eq. (8.40) directly to find the final speed v.

EXECUTE: We have $m_0/m = 4$, so from Eq. (8.40),

$$v = v_0 + v_{ex}\ln\frac{m_0}{m} = 0 + (2400\text{ m/s})(\ln 4) = 3327\text{ m/s}$$

EVALUATE: Let's examine what happens as the rocket gains speed. At the start of the flight, when the velocity of the rocket is zero, the ejected fuel is moving to the left, relative to our coordinate system, at 2400 m/s. At the end of the first second ($t = 1$ s), the rocket is moving at 20 m/s, and the fuel's speed relative to our system is 2380 m/s. During the next second the acceleration, given by Eq. (8.39), is a little greater. At $t = 2$ s, the rocket is moving a little faster than 40 m/s, and the fuel's speed is a little less than 2360 m/s. Detailed calculation shows that at about $t = 75.6$ s, the rocket's velocity v in our coordinate system equals 2400 m/s. The burned fuel ejected after this time moves *forward*, not backward, in our system. Since the final velocity of the rocket is 3327 m/s and the relative velocity is 2400 m/s, the last portion of the ejected fuel has a forward velocity (relative to our frame of reference) of $(3327 - 2400)$ m/s $= 927$ m/s. (To illustrate our point, we are using more figures than are significant.)

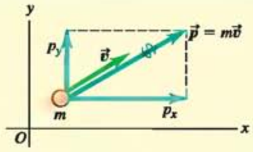
Test Your Understanding of Section 8.6 (a) If a rocket in gravity-free outer space has the same thrust at all times, is its acceleration constant, increasing, or decreasing? (b) If the rocket has the same acceleration at all times, is the thrust constant, increasing, or decreasing?

CHAPTER 8 SUMMARY

Momentum of a particle: The momentum $\vec{p}$ of a particle is a vector quantity equal to the product of the particle's mass m and velocity $\vec{v}$. Newton's second law says that the net force on a particle is equal to the rate of change of the particle's momentum.

$$\vec{p} = m\vec{v} \quad (8.2)$$

$$\sum\vec{F} = \frac{d\vec{p}}{dt} \quad (8.4)$$

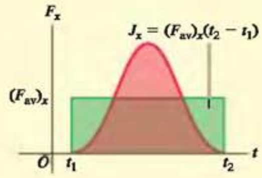


Impulse and momentum: If a constant net force $\sum\vec{F}$ acts on a particle for a time interval Δt from t_1 to t_2, the impulse $\vec{J}$ of the net force is the product of the net force and the time interval. If $\sum\vec{F}$ varies with time, $\vec{J}$ is the integral of the net force over the time interval. In any case, the change in a particle's momentum during a time interval equals the impulse of the net force that acted on the particle during that interval. The momentum of a particle equals the impulse that accelerated it from rest to its present speed. (See Examples 8.1–8.3.)

$$\vec{J} = \sum\vec{F}(t_2 - t_1) = \sum\vec{F}\,\Delta t \quad (8.5)$$

$$\vec{J} = \int_{t_1}^{t_2} \sum\vec{F}\,dt \quad (8.7)$$

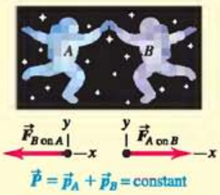
$$\vec{J} = \vec{p}_2 - \vec{p}_1 \quad (8.6)$$



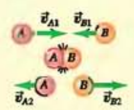
Conservation of momentum: An internal force is a force exerted by one part of a system on another. An external force is a force exerted on any part of a system by something outside the system. If the net external force on a system is zero, the total momentum of the system $\vec{P}$ (the vector sum of the momenta of the individual particles that make up the system) is constant, or conserved. Each component of total momentum is separately conserved. (See Examples 8.4–8.6)

$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots$$
$$= m_A\vec{v}_A + m_B\vec{v}_B + \cdots \quad (8.14)$$

If $\sum\vec{F} = \mathbf{0}$, then $\vec{P}$ = constant.



Collisions: In collisions of all kinds, the initial and final total momenta are equal. In an elastic collision between two bodies, the initial and final total kinetic energies are also equal, and the initial and final relative velocities have the same magnitude. In an inelastic two-body collision, the total kinetic energy is less after the collision than before. If the two bodies have the same final velocity, the collision is completely inelastic. (See Examples 8.7–8.12.)

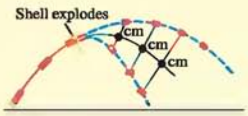


Center of mass: The position vector of the center of mass of a system of particles, $\vec{r}_{cm}$, is a weighted average of the positions $\vec{r}_1, \vec{r}_2, \ldots$ of the individual particles. The total momentum $\vec{P}$ of a system equals its total mass M multiplied by the velocity of its center of mass, $\vec{v}_{cm}$. The center of mass moves as though all the mass M were concentrated at that point. If the net external force on the system is zero, the center-of-mass velocity $\vec{v}_{cm}$ is constant. If the net external force is not zero, the center of mass accelerates as though it were a particle of mass M being acted on by the same net external force. (See Examples 8.13 and 8.14.)

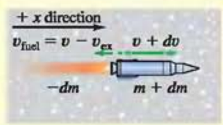
$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$
$$= \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i} \quad (8.29)$$

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots$$
$$= M\vec{v}_{cm} \quad (8.32)$$

$$\sum\vec{F}_{ext} = M\vec{a}_{cm} \quad (8.34)$$



Rocket propulsion: In rocket propulsion, the mass of a rocket changes as the fuel is used up and ejected from the rocket. Analysis of the motion of the rocket must include the momentum carried away by the spent fuel as well as the momentum of the rocket itself. (See Examples 8.15 and 8.16.)



Key Terms

momentum (linear momentum), *2*
impulse, *3*
impulse–momentum theorem, *3*
internal force, *7*
external force, *7*
isolated system, *7*
total momentum, *7*
principle of conservation of momentum, *8*
elastic collision, *12*
inelastic collision, *12*
completely inelastic collision, *12*
center of mass, *20*

Answer to Chapter Opening Question ?

The two players have the same magnitude of momentum $p = mv$ (the product of mass and speed), but the faster, lightweight player has twice as much kinetic energy $K = \frac{1}{2}mv^2$. Hence, the lightweight player can do twice as much work on you (and twice as much damage) in the process of coming to a halt (see Section 8.1).

Answers to Test Your Understanding Questions

8.1 Answer: (v), (i) and (ii) (tied for second place), (iii) and (iv) (tied for third place) We use two interpretations of the impulse of the net force: (1) the net force multiplied by the time that the net force acts, and (2) the change in momentum of the particle on which the net force acts. Which interpretation we use depends on what information we are given. We take the positive x-direction to be to the east. (i) The force is not given, so we use interpretation 2: $J_x = mv_{2x} - mv_{1x} = (1000\text{ kg})(0) - (1000\text{ kg})(25\text{ m/s}) = -25{,}000\text{ kg}\cdot\text{m/s}$, so the magnitude of the impulse is $25{,}000\text{ kg}\cdot\text{m/s} = 25{,}000\text{ N}\cdot\text{s}$. (ii) For the same reason as in (i), we use interpretation 2: $J_x = mv_{2x} - mv_{1x} = (1000\text{ kg})(0) - (1000\text{ kg})(25\text{ m/s}) = -25{,}000\text{ kg}\cdot\text{m/s}$, and the magnitude of the impulse is again $25{,}000\text{ kg}\cdot\text{m/s} = 25{,}000\text{ N}\cdot\text{s}$. (iii) The final velocity is not given, so we use interpretation 1: $J_x = (\Sigma F_x)_{\text{av}}(t_2 - t_1) = (2000\text{ N})(10\text{ s}) = 20{,}000\text{ N}\cdot\text{s}$, so the magnitude of the impulse is $20{,}000\text{ N}\cdot\text{s}$. (iv) For the same reason as in (iii), we use interpretation 1: $J_x = (\Sigma F_x)_{\text{av}}(t_2 - t_1) = (-2000\text{ N})(10\text{ s}) = -20{,}000\text{ N}\cdot\text{s}$, so the magnitude of the impulse is $20{,}000\text{ N}\cdot\text{s}$. (v) The force is not given, so we use interpretation 2: $J_x = mv_{2x} - mv_{1x} = (1000\text{ kg})(-25\text{ m/s}) - (1000\text{ kg})(25\text{ m/s}) = -50{,}000\text{ kg}\cdot\text{m/s}$, so the magnitude of the impulse is $50{,}000\text{ kg}\cdot\text{m/s} = 50{,}000\text{ N}\cdot\text{s}$.

8.2 Answers: (a) $v_{C2x} > 0$, $v_{C2y} > 0$, (b) piece C There are no external horizontal forces, so the x- and y-components of the total momentum of the system are both conserved. Both components of the total momentum are zero before the spring releases, so they must be zero after the spring releases. Hence

$$P_x = 0 = m_A v_{A2x} + m_B v_{B2x} + m_C v_{C2x}$$
$$P_y = 0 = m_A v_{A2y} + m_B v_{B2y} + m_C v_{C2y}$$

We are given that $m_A = m_B = m_C$, $v_{A2x} < 0$, $v_{A2y} = 0$, $v_{B2x} = 0$, and $v_{B2y} < 0$. You can solve the above equations to show that $v_{C2x} = -v_{A2x} > 0$ and $v_{C2y} = -v_{B2y} > 0$, so the velocity components of piece C are both positive. Piece C has speed $\sqrt{v_{C2x}^2 + v_{C2y}^2} = \sqrt{v_{A2x}^2 + v_{B2y}^2}$, which is greater than the speed of either piece A or piece B.

8.3 Answers: (a) inelastic, (b) elastic, (c) completely inelastic In each case gravitational potential energy is converted to kinetic energy as the ball falls, and the collision is between the ball and the ground. In (a) all of the initial energy is converted back to gravitational potential energy, so no kinetic energy is lost in the bounce and the collision is elastic. In (b) there is less gravitational potential energy at the end than at the beginning, so some kinetic energy was lost in the bounce. Hence the collision is inelastic. In (c) the ball loses all the kinetic energy it has to give, the ball and the ground stick together, and the collision is completely inelastic.

8.4 Answer: worse After a collision with a water molecule initially at rest, the speed of the neutron is $|(m_n - m_w)/(m_n + m_w)| = |(1.0\text{ u} - 18\text{ u})/(1.0\text{ u} + 18\text{ u})| = \frac{17}{19}$ of its initial speed, and its kinetic energy is $(\frac{17}{19})^2 = 0.80$ of the initial value. Hence a water molecule is a worse moderator than a carbon atom, for which the corresponding numbers are $\frac{11}{13}$ and $(\frac{11}{13})^2 = 0.72$.

8.5 Answer: no If gravity is the only force acting on the system of two fragments, the center of mass will follow the parabolic trajectory of a freely falling object. Once a fragment lands, however, the ground exerts a normal force on that fragment. Hence the net force on the system has changed, and the trajectory of the center of mass changes in response.

8.6 Answers: (a) increasing, (b) decreasing From Eqs. (8.37) and (8.38), the thrust F is equal to $m(dv/dt)$, where m is the rocket's mass and dv/dt is its acceleration. Because m decreases with time, if the thrust F is constant, then the acceleration must increase with time (the same force acts on a smaller mass); if the acceleration dv/dt is constant, then the thrust must decrease with time (a smaller force is all that's needed to accelerate a smaller mass).

PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com

Discussion Questions

Q8.1. In splitting logs with a hammer and wedge, is a heavy hammer more effective than a lighter hammer? Why?

Q8.2. Suppose you catch a baseball and then someone invites you to catch a bowling ball with either the same momentum or the same kinetic energy as the baseball. Which would you choose? Explain.

Q8.3. When rain falls from the sky, what happens to its momentum as it hits the ground? Is your answer also valid for Newton's famous apple?

Q8.4. A car has the same kinetic energy when it is traveling south at 30 m/s as when it is traveling northwest at 30 m/s. Is the momentum of the car the same in both cases? Explain.

Q8.5. A truck is accelerating as it speeds down the highway. One inertial frame of reference is attached to the ground with its origin at a fence post. A second frame of reference is attached to a police car that is traveling down the highway at constant velocity. Is the momentum of the truck the same in these two reference frames? Explain. Is the rate of change of the truck's momentum the same in these two frames? Explain.

Q8.6. When a large, heavy truck collides with a passenger car, the occupants of the car are more likely to be hurt than the truck driver. Why?

Q8.7. A woman holding a large rock stands on a frictionless, horizontal sheet of ice. She throws the rock with speed v_0 at an angle α above the horizontal. Consider the system consisting of the woman plus the rock. Is the momentum of the system conserved? Why or why not? Is any component of the momentum of the system conserved? Again, why or why not?

Q8.8. In Example 8.7 (Section 8.3), where the two gliders in Fig. 8.15 a stick together after the collision, the collision is inelastic because $K_2 < K_1$. In Example 8.5 (Section 8.2), is the collision inelastic? Explain.

Q8.9. In a completely inelastic collision between two objects, where the objects stick together after the collision, is it possible for the final kinetic energy of the system to be zero? If so, give an example in which this would occur. If the final kinetic energy is zero, what must the initial momentum of the system be? Is the initial kinetic energy of the system zero? Explain.

Q8.10. Since for a particle the kinetic energy is given by $K = \frac{1}{2}mv^2$ and the momentum by $\vec{p} = m\vec{v}$, it is easy to show that $K = p^2/2m$. How, then, is it possible to have an event during which the total momentum of the system is constant but the total kinetic energy changes?

Q8.11. In each of Examples 8.10, 8.11, and 8.12 (Section 8.4), verify that the relative velocity vector of the two bodies has the same magnitude before and after the collision. In each case what happens to the *direction* of the relative velocity vector?

Q8.12. A glass dropped on the floor is more likely to break if the floor is concrete than if it is wood. Why? (Refer to Fig. 8.3b.)

Q8.13. In Fig. 8.22b, the kinetic energy of the Ping-Pong ball is larger after its interaction with the bowling ball than before. From where does the extra energy come? Describe the event in terms of conservation of energy.

Q8.14. A machine gun is fired at a steel plate. Is the average force on the plate from the bullet impact greater if the bullets bounce off or if they are squashed and stick to the plate? Explain.

Q8.15. A net force of 4 N acts on an object initially at rest for 0.25 s and gives it a final speed of 5 m/s. How could a net force of 2 N produce the same final speed?

Q8.16. A net force with x-component $\sum F_x$ acts on an object from time t_1 to time t_2. The x-component of the momentum of the object is the same at t_1 as it is at t_2, but $\sum F_x$ is not zero at all times between t_1 and t_2. What can you say about the graph of $\sum F_x$ versus t?

Q8.17. A tennis player hits a tennis ball with a racket. Consider the system made up of the ball and the racket. Is the total momentum of the system the same just before and just after the hit? Is the total momentum just after the hit the same as 2 s later, when the ball is in midair at the high point of its trajectory? Explain any differences between the two cases.

Q8.18. In Example 8.4 (Section 8.2), consider the system consisting of the rifle plus the bullet. What is the speed of the system's center of mass after the rifle is fired? Explain.

Q8.19. An egg is released from rest from the roof of a building and falls to the ground. As the egg falls, what happens to the momentum of the system of the egg plus the earth?

Q8.20. A woman stands in the middle of a perfectly smooth, frictionless, frozen lake. She can set herself in motion by throwing things, but suppose she has nothing to throw. Can she propel herself to shore *without* throwing anything?

Q8.21. In a zero-gravity environment, can a rocket-propelled spaceship ever attain a speed greater than the relative speed with which the burnt fuel is exhausted?

Q8.22. When an object breaks into two pieces (explosion, radioactive decay, recoil, etc.), the lighter fragment gets more kinetic energy than the heavier one. This is a consequence of momentum conservation, but can you also explain it using Newton's laws of motion?

Q8.23. An apple falls from a tree and feels no air resistance. As it is falling, which of these statements about it are true? (a) Only its momentum is conserved; (b) only its mechanical energy is conserved, (c) both its momentum and its mechanical energy are conserved, (d) its kinetic energy is conserved.

Q8.24. Two pieces of clay collide and stick together. During the collision, which of these statements are true? (a) Only the momentum of the clay is conserved, (b) only the mechanical energy of the clay is conserved, (c) both the momentum and the mechanical energy of the clay are conserved, (d) the kinetic energy of the clay is conserved.

Q8.25. Two marbles are pressed together with a light ideal spring between them, but they are not attached to the spring in any way. They are then released on a frictionless horizontal table and soon move free of the spring. As the marbles are moving away from each other, which of these statements about them are true? (a) Only the momentum of the marbles is conserved, (b) only the mechanical energy of the marbles is conserved, (c) both the momentum and the mechanical energy of the marbles are conserved, (d) the kinetic energy of the marbles is conserved.

Q8.26. A very heavy SUV collides head-on with a very light compact car. Which of these statements about the collision are correct? (a) The amount of kinetic energy lost by the SUV is equal to the amount of kinetic energy gained by the compact, (b) the amount of momentum lost by the SUV is equal to the amount of momentum gained by the compact, (c) The compact feels a considerably greater force during the collision than the SUV does, (d) both cars lose the same amount of kinetic energy.

Exercises

Section 8.1 Momentum and Impulse

8.1. (a) What is the magnitude of the momentum of a 10,000-kg truck whose speed is 12.0 m/s? (b) What speed would a 2,000-kg SUV have to attain in order to have (i) the same momentum? (ii) the same kinetic energy?

8.2. In Conceptual Example 8.1 (Section 8.1), show that the iceboat with mass $2m$ has $\sqrt{2}$ times as much momentum at the finish line as does the iceboat with mass m.

8.3. (a) Show that the kinetic energy K and the momentum magnitude p of a particle with mass m are related by $K = p^2/2m$. (b) A 0.040-kg cardinal (*Richmondena cardinalis*) and a 0.145-kg baseball have the same kinetic energy. Which has the greater magnitude of momentum? What is the ratio of the cardinal's magnitude of momentum to the baseball's? (c) A 700-N man and a 450-N woman have the same momentum. Who has the greater kinetic energy? What is the ratio of the man's kinetic energy to that of the woman?

8.4. In a certain men's track and field event, the shotput has a mass of 7.30 kg and is released with a speed of 15.0 m/s at 40.0° above the horizontal over a man's straight left leg. What are the initial horizontal and vertical components of the momentum of this shotput?

8.5. One 110-kg football lineman is running to the right at 2.75 m/s while another 125-kg lineman is running directly toward him at

2.60 m/s. What are (a) the magnitude and direction of the net momentum of these two athletes, and (b) their total kinetic energy?

8.6. Two vehicles are approaching an intersection. One is a 2500-kg pickup traveling at 14.0 m/s from east to west (the $-x$-direction), and the other is a 1500-kg sedan going from south to north (the $+y$-direction at 23.0 m/s). (a) Find the x- and y-components of the net momentum of this system. (b) What are the magnitude and direction of the net momentum?

8.7. Force of a Golf Swing. A 0.0450-kg golf ball initially at rest is given a speed of 25.0 m/s when a club strikes. If the club and ball are in contact for 2.00 ms, what average force acts on the ball? Is the effect of the ball's weight during the time of contact significant? Why or why not?

8.8. Force of a Baseball Swing. A baseball has mass 0.145 kg. (a) If the velocity of a pitched ball has a magnitude of 45.0 m/s and the batted ball's velocity is 55.0 m/s in the opposite direction, find the magnitude of the change in momentum of the ball and of the impulse applied to it by the bat. (b) If the ball remains in contact with the bat for 2.00 ms, find the magnitude of the average force applied by the bat.

8.9. A 0.160-kg hockey puck is moving on an icy, frictionless, horizontal surface. At $t = 0$, the puck is moving to the right at 3.00 m/s. (a) Calculate the velocity of the puck (magnitude and direction) after a force of 25.0 N directed to the right has been applied for 0.050 s. (b) If, instead, a force of 12.0 N directed to the left is applied from $t = 0$ to $t = 0.050$ s, what is the final velocity of the puck?

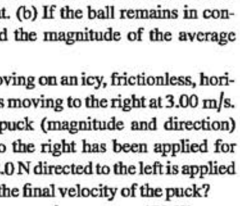
8.10. An engine of the orbital maneuvering system (OMS) on a space shuttle exerts a force of $(26{,}700\ \text{N})\hat{j}$ for 3.90 s, exhausting a negligible mass of fuel relative to the 95,000-kg mass of the shuttle. (a) What is the impulse of the force for this 3.90 s? (b) What is the shuttle's change in momentum from this impulse? (c) What is the shuttle's change in velocity from this impulse? (d) Why can't we find the resulting change in the kinetic energy of the shuttle?

8.11. At time $t = 0$, a 2150-kg rocket in outer space fires an engine that exerts an increasing force on it in the $+x$-direction. This force obeys the equation $F_x = At^2$, where t is time, and has a magnitude of 781.25 N when $t = 1.25$ s. (a) Find the SI value of the constant A, including its units. (b) What impulse does the engine exert on the rocket during the 1.50-s interval starting 2.00 s after the engine is fired? (c) By how much does the rocket's velocity change during this interval?

8.12. A bat strikes a 0.145-kg baseball. Just before impact, the ball is traveling horizontally to the right at 50.0 m/s, and it leaves the bat traveling to the left at an angle of 30° above horizontal with a speed of 65.0 m/s. If the ball and bat are in contact for 1.75 ms, find the horizontal and vertical components of the average force on the ball.

8.13. A 2.00-kg stone is sliding to the right on a frictionless horizontal surface at 5.00 m/s when it is suddenly struck by an object that exerts a large horizontal force on it for a short period of time. The graph in Fig. 8.34 shows the magnitude of this force as a function of time. (a) What impulse does this force exert on the stone? (b) Just after the force stops acting, find the magnitude and direction of the stone's velocity if the force acts (i) to the right or (ii) to the left.

Figure 8.34 Exercise 8.13.



Section 8.2 Conservation of Momentum

8.14. A 68.5-kg astronaut is doing a repair in space on the orbiting space station. She throws a 2.25-kg tool away from her at 3.20 m/s relative to the space station. With what speed and in what direction will she begin to move?

8.15. Animal Propulsion. Squids and octopuses propel themselves by expelling water. They do this by keeping water in a cavity and then suddenly contracting the cavity to force out the water through an opening. A 6.50-kg squid (including the water in the cavity) at rest suddenly sees a dangerous predator. (a) If the squid has 1.75 kg of water in its cavity, at what speed must it expel this water to suddenly achieve a speed of 2.50 m/s to escape the predator? Neglect any drag effects of the surrounding water. (b) How much kinetic energy does the squid create by this maneuver?

8.16. You are standing on a sheet of ice that covers the football stadium parking lot in Buffalo; there is negligible friction between your feet and the ice. A friend throws you a 0.400-kg ball that is traveling horizontally at 10.0 m/s. Your mass is 70.0 kg. (a) If you catch the ball, with what speed do you and the ball move afterward? (b) If the ball hits you and bounces off your chest, so afterward it is moving horizontally at 8.0 m/s in the opposite direction, what is your speed after the collision?

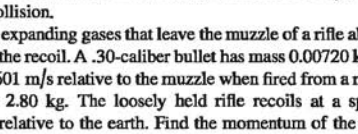
8.17. On a frictionless, horizontal air table, puck A (with mass 0.250 kg) is moving toward puck B (with mass 0.350 kg), which is initially at rest. After the collision, puck A has a velocity of 0.120 m/s to the left, and puck B has a velocity of 0.650 m/s to the right. (a) What was the speed of puck A before the collision? (b) Calculate the change in the total kinetic energy of the system that occurs during the collision.

8.18. When cars are equipped with flexible bumpers, they will bounce off each other during low-speed collisions, thus causing less damage. In one such accident, a 1750-kg car traveling to the right at 1.50 m/s collides with a 1450-kg car going to the left at 1.10 m/s. Measurements show that the heavier car's speed just after the collision was 0.250 m/s in its original direction. You can ignore any road friction during the collision. (a) What was the speed of the lighter car just after the collision? (b) Calculate the change in the combined kinetic energy of the two-car system during this collision.

8.19. The expanding gases that leave the muzzle of a rifle also contribute to the recoil. A .30-caliber bullet has mass 0.00720 kg and a speed of 601 m/s relative to the muzzle when fired from a rifle that has mass 2.80 kg. The loosely held rifle recoils at a speed of 1.85 m/s relative to the earth. Find the momentum of the propellant gases in a coordinate system attached to the earth as they leave the muzzle of the rifle.

8.20. Block A in Fig. 8.35 has mass 1.00 kg, and block B has mass 3.00 kg. The blocks are forced together, compressing a spring S between them; then the system is released from rest on a level, frictionless surface. The spring, which has negligible mass, is not fastened to either block and drops to the surface after it has expanded. Block B acquires a speed of 1.20 m/s. (a) What is the final speed of block A? (b) How much potential energy was stored in the compressed spring?

Figure 8.35 Exercise 8.20.



8.21. A hunter on a frozen, essentially frictionless pond uses a rifle that shoots 4.20-g bullets at 965 m/s. The mass of the hunter (including his gun) is 72.5 kg, and the hunter holds tight to the gun after firing it. Find the recoil velocity of the hunter if he fires the rifle (a) horizontally and (b) at 56.0° above the horizontal.

8.22. An atomic nucleus suddenly bursts apart (fissions) into two pieces. Piece A, of mass m_A, travels off to the left with speed v_A. Piece B, of mass m_B, travels off to the right with speed v_B. (a) Use conservation of momentum to solve for v_B in terms of m_A, m_B, and v_A. (b) Use the results of part (a) to show that $K_A/K_B = m_B/m_A$, where K_A and K_B are the kinetic energies of the two pieces.

8.23. The nucleus of ^{214}Po decays radioactively by emitting an alpha particle (mass 6.65×10^{-27} kg) with kinetic energy 1.23×10^{-12} J, as measured in the laboratory reference frame. Assuming that the Po was initially at rest in this frame, find the recoil velocity of the nucleus that remains after the decay.

8.24. You are standing on a large sheet of frictionless ice and holding a large rock. In order to get off the ice, you throw the rock so it has velocity 12.0 m/s relative to the earth at an angle of 35.0° above the horizontal. If your mass is 70.0 kg and the rock's mass is 15.0 kg, what is your speed after you throw the rock (see Discussion Question Q8.7)?

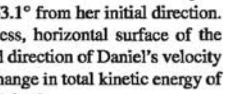
8.25. Two ice skaters, Daniel (mass 65.0 kg) and Rebecca (mass 45.0 kg), are practicing. Daniel stops to tie his shoelace and, while at rest, is struck by Rebecca, who is moving at 13.0 m/s before she collides with him. After the collision, Rebecca has a velocity of magnitude 8.00 m/s at an angle of 53.1° from her initial direction. Both skaters move on the frictionless, horizontal surface of the rink. (a) What are the magnitude and direction of Daniel's velocity after the collision? (b) What is the change in total kinetic energy of the two skaters as a result of the collision?

8.26. An astronaut in space cannot use a scale or balance to weigh objects because there is no gravity. But she does have devices to measure distance and time accurately. She knows her own mass is 78.4 kg, but she is unsure of the mass of a large gas canister in the airless rocket. When this canister is approaching her at 3.50 m/s, she pushes against it, which slows it down to 1.20 m/s (but does not reverse it) and gives her a speed of 2.40 m/s. What is the mass of this canister?

8.27. Changing Mass. An open-topped freight car with mass 24,000 kg is coasting without friction along a level track. It is raining very hard, and the rain is falling vertically downward. Originally, the car is empty and moving with a speed of 4.00 m/s. What is the speed of the car after it has collected 3000 kg of rainwater?

8.28. Asteroid Collision. Two asteroids of equal mass in the asteroid belt between Mars and Jupiter collide with a glancing blow. Asteroid A, which was initially traveling at 40.0 m/s, is deflected 30.0° from its original direction, while asteroid B travels at 45.0° to the original direction of A (Fig. 8.36). (a) Find the speed of each asteroid after the collision. (b) What fraction of the original kinetic energy of asteroid A dissipates during this collision?

Figure 8.36 Exercise 8.28.



Section 8.3 Momentum Conservation and Collisions

8.29. A 15.0-kg fish swimming at 1.10 m/s suddenly gobbles up a 4.50-kg fish that is initially stationary. Neglect any drag effects of the water. (a) Find the speed of the large fish just after it eats the small one. (b) How much mechanical energy was dissipated during this meal?

8.30. Two fun-loving otters are sliding toward each other on a muddy (and hence frictionless) horizontal surface. One of them, of mass 7.50 kg, is sliding to the left at 5.00 m/s, while the other, of mass 5.75 kg, is slipping to the right at 6.00 m/s. They hold fast to each other after they collide. (a) Find the magnitude and direction of the velocity of these free-spirited otters right after they collide. (b) How much mechanical energy dissipates during this play?

8.31. Deep Impact Mission. In July 2005, NASA's "Deep Impact" mission crashed a 372-kg probe directly onto the surface of the comet Tempel 1, hitting the surface at 37,000 km/h. The original speed of the comet at that time was about 40,000 km/h, and its mass was estimated to be in the range $(0.10\text{–}2.5) \times 10^{14}$ kg. Use the smallest value of the estimated mass. (a) What change in the comet's velocity did this collision produce? Would this change be noticeable? (b) Suppose this comet were to hit the earth and fuse with it. By how much would it change our planet's velocity? Would this change be noticeable? (The mass of the earth is 5.97×10^{24} kg.)

8.32. A 1050-kg sports car is moving westbound at 15.0 m/s on a level road when it collides with a 6320-kg truck driving east on the same road at 10.0 m/s. The two vehicles remain locked together after the collision. (a) What is the velocity (magnitude and direction) of the two vehicles just after the collision? (b) At what speed should the truck have been moving so that it and car are both stopped in the collision? (c) Find the change in kinetic energy of the system of two vehicles for the situations of part (a) and part (b). For which situation is the change in kinetic energy greater in magnitude?

8.33. On a very muddy football field, a 110-kg linebacker tackles an 85-kg halfback. Immediately before the collision, the linebacker is slipping with a velocity of 8.8 m/s north and the halfback is sliding with a velocity of 7.2 m/s east. What is the velocity (magnitude and direction) at which the two players move together immediately after the collision?

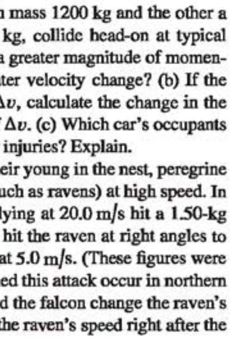
8.34. Two skaters collide and grab on to each other on frictionless ice. One of them, of mass 70.0 kg, is moving to the right at 2.00 m/s, while the other, of mass 65.0 kg, is moving to the left at 2.50 m/s. What are the magnitude and direction of the velocity of these skaters just after they collide?

8.35. Two cars, one a compact with mass 1200 kg and the other a large gas-guzzler with mass 3000 kg, collide head-on at typical freeway speeds. (a) Which car has a greater magnitude of momentum change? Which car has a greater velocity change? (b) If the larger car changes its velocity by Δv, calculate the change in the velocity of the small car in terms of Δv. (c) Which car's occupants would you expect to sustain greater injuries? Explain.

8.36. Bird Defense. To protect their young in the nest, peregrine falcons will fly into birds of prey (such as ravens) at high speed. In one such episode, a 600-g falcon flying at 20.0 m/s hit a 1.50-kg raven flying at 9.0 m/s. The falcon hit the raven at right angles to its original path and bounced back at 5.0 m/s. (These figures were estimated by the author as he watched this attack occur in northern New Mexico.) (a) By what angle did the falcon change the raven's direction of motion? (b) What was the raven's speed right after the collision?

8.37. At the intersection of Texas Avenue and University Drive, a yellow subcompact car with mass 950 kg traveling east on University collides with a red pickup truck with mass 1900 kg that is traveling north on Texas and ran a red light (Fig. 8.37). The two vehicles stick together as a result of the collision, and the wreckage slides at 16.0 m/s in the direction 24.0° east of north. Calculate the

Figure 8.37 Exercise 8.37.



speed of each vehicle before the collision. The collision occurs during a heavy rainstorm; you can ignore friction forces between the vehicles and the wet road.

8.38. A 5.00-g bullet is fired horizontally into a 1.20-kg wooden block resting on a horizontal surface. The coefficient of kinetic friction between block and surface is 0.20. The bullet remains embedded in the block, which is observed to slide 0.230 m along the surface before stopping. What was the initial speed of the bullet?

8.39. A Ballistic Pendulum. A 12.0-g rifle bullet is fired with a speed of 380 m/s into a ballistic pendulum with mass 6.00 kg, suspended from a cord 70.0 cm long (see Example 8.8 in Section 8.3). Compute (a) the vertical height through which the pendulum rises, (b) the initial kinetic energy of the bullet, and (c) the kinetic energy of the bullet and pendulum immediately after the bullet becomes embedded in the pendulum.

8.40. You and your friends are doing physics experiments on a frozen pond that serves as a frictionless, horizontal surface. Sam, with mass 80.0 kg, is given a push and slides eastward. Abigail, with mass 50.0 kg, is sent sliding northward. They collide, and after the collision Sam is moving at 37.0° north of east with a speed of 6.00 m/s and Abigail is moving at 23.0° south of east with a speed of 9.00 m/s. (a) What was the speed of each person before the collision? (b) By how much did the total kinetic energy of the two people decrease during the collision?

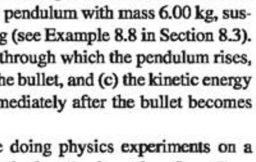
Section 8.4 Elastic Collisions

8.41. Blocks *A* (mass 2.00 kg) and *B* (mass 10.00 kg) move on a frictionless, horizontal surface. Initially, block *B* is at rest and block *A* is moving toward it at 2.00 m/s. The blocks are equipped with ideal spring bumpers, as in Example 8.10. The collision is head-on, so all motion before and after the collision is along a straight line. (a) Find the maximum energy stored in the spring bumpers and the velocity of each block at that time. (b) Find the velocity of each block after they have moved apart.

8.42. A 0.150-kg glider is moving to the right on a frictionless, horizontal air track with a speed of 0.80 m/s. It has a head-on collision with a 0.300-kg glider that is moving to the left with a speed of 2.20 m/s. Find the final velocity (magnitude and direction) of each glider if the collision is elastic.

8.43. A 10.0-g marble slides to the left with a velocity of magnitude 0.400 m/s on the frictionless, horizontal surface of an icy New York sidewalk and has a head-on, elastic collision with a larger 30.0-g marble sliding to the right with a velocity of magnitude 0.200 m/s (Fig. 8.38). (a) Find the velocity of each marble (magnitude and direction) after the collision. (Since the collision is head-on, all the motion is along a line.) (b) Calculate the *change in momentum* (that is, the momentum after the collision minus the momentum before the collision) for each marble. Compare the values you get for each marble. (c) Calculate the *change in kinetic energy* (that is, the kinetic energy after the collision minus the kinetic energy before the collision) for each marble. Compare the values you get for each marble.

Figure 8.38 Exercise 8.43.



8.44. Supply the details of the calculation of α and β in Example 8.12 (Section 8.4).

8.45. Moderators. Canadian nuclear reactors use *heavy water* moderators in which elastic collisions occur between the neutrons and deuterons of mass 2.0 u (see Example 8.11 in Section 8.4). (a) What is the speed of a neutron, expressed as a fraction of its original speed, after a head-on, elastic collision with a deuteron that is initially at rest? (b) What is its kinetic energy, expressed as a fraction of its original kinetic energy? (c) How many such successive collisions will reduce the speed of a neutron to 1/59,000 of its original value?

8.46. You are at the controls of a particle accelerator, sending a beam of 1.50×10^7 m/s protons (mass *m*) at a gas target of an unknown element. Your detector tells you that some protons bounce straight back after a collision with one of the nuclei of the unknown element. All such protons rebound with a speed of 1.20×10^7 m/s. Assume that the initial speed of the target nucleus is negligible and the collision is elastic. (a) Find the mass of one nucleus of the unknown element. Express your answer in terms of the proton mass *m*. (b) What is the speed of the unknown nucleus immediately after such a collision?

Section 8.5 Center of Mass

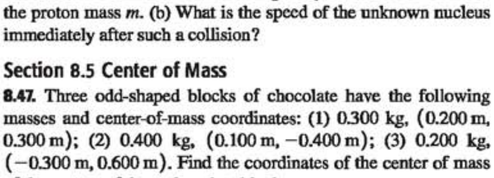
8.47. Three odd-shaped blocks of chocolate have the following masses and center-of-mass coordinates: (1) 0.300 kg, (0.200 m, 0.300 m); (2) 0.400 kg, (0.100 m, −0.400 m); (3) 0.200 kg, (−0.300 m, 0.600 m). Find the coordinates of the center of mass of the system of three chocolate blocks.

8.48. Find the position of the center of mass of the system of the sun and Jupiter. (Since Jupiter is more massive than the rest of the planets combined, this is essentially the position of the center of mass of the solar system.) Does the center of mass lie inside or outside the sun? Use the data in Appendix F.

8.49. Pluto and Charon. Pluto's diameter is approximately 2370 km, and the diameter of its satellite Charon is 1250 km. Although the distance varies, they are often about 19,700 km apart, center-to-center. Assuming that both Pluto and Charon have the same composition and hence the same average density, find the location of the center of mass of this system relative to the center of Pluto.

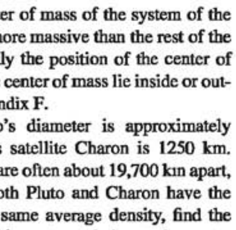
8.50. A 1200-kg station wagon is moving along a straight highway at 12.0 m/s. Another car, with mass 1800 kg and speed 20.0 m/s, has its center of mass 40.0 m ahead of the center of mass of the station wagon (Fig. 8.39). (a) Find the position of the center of mass of the system consisting of the two automobiles. (b) Find the magnitude of the total momentum of the system from the given data. (c) Find the speed of the center of mass of the system. (d) Find the total momentum of the system, using the speed of the center of mass. Compare your result with that of part (b).

Figure 8.39 Exercise 8.50.



8.51. A machine part consists of a thin, uniform 4.00-kg bar that is 1.50 m long, hinged perpendicular to a similar vertical bar of mass 3.00 kg and length 1.80 m. The longer bar has a small but dense 2.00-kg ball at one end (Fig. 8.40). By what distance will the center of mass of this part move horizontally and vertically if the vertical bar is pivoted counterclockwise through 90° to make the entire part horizontal?

Figure 8.40 Exercise 8.51.



8.52. At one instant, the center of mass of a system of two particles is located on the x-axis at $x = 2.0$ m and has a velocity of $(5.0\ \text{m/s})\hat{\imath}$. One of the particles is at the origin. The other particle has a mass of 0.10 kg and is at rest on the x-axis at $x = 8.0$ m. (a) What is the mass of the particle at the origin? (b) Calculate the total momentum of this system. (c) What is the velocity of the particle at the origin?

8.53. In Example 8.14 (Section 8.5), Ramon pulls on the rope to give himself a speed of 0.70 m/s. What is James's speed?

8.54. A system consists of two particles. At $t = 0$ one particle is at the origin; the other, which has a mass of 0.50 kg, is on the y-axis at $y = 6.0$ m. At $t = 0$ the center of mass of the system is on the y-axis at $y = 2.4$ m. The velocity of the center of mass is given by $(0.75\ \text{m/s}^3)t^2\hat{\imath}$. (a) Find the total mass of the system. (b) Find the acceleration of the center of mass at any time t. (c) Find the net external force acting on the system at $t = 3.0$ s.

8.55. A radio-controlled model airplane has a momentum given by $[(-0.75\ \text{kg}\cdot\text{m/s}^3)t^2 + (3.0\ \text{kg}\cdot\text{m/s})]\hat{\imath} + (0.25\ \text{kg}\cdot\text{m/s}^2)t\hat{\jmath}$. What are the x-, y-, and z-components of the net force on the airplane?

*Section 8.6 Rocket Propulsion

***8.56.** A small rocket burns 0.0500 kg of fuel per second, ejecting it as a gas with a velocity relative to the rocket of magnitude 1600 m/s. (a) What is the thrust of the rocket? (b) Would the rocket operate in outer space where there is no atmosphere? If so, how would you steer it? Could you brake it?

***8.57.** A 70-kg astronaut floating in space in a 110-kg MMU (manned maneuvering unit) experiences an acceleration of $0.029\ \text{m/s}^2$ when he fires one of the MMU's thrusters. (a) If the speed of the escaping N_2 gas relative to the astronaut is 490 m/s, how much gas is used by the thruster in 5.0 s? (b) What is the thrust of the thruster?

***8.58.** A rocket is fired in deep space, where gravity is negligible. If the rocket has an initial mass of 6000 kg and ejects gas at a relative velocity of magnitude 2000 m/s, how much gas must it eject in the first second to have an initial acceleration of $25.0\ \text{m/s}^2$?

***8.58.** A rocket is fired in deep space, where gravity is negligible. In the first second it ejects $\frac{1}{160}$ of its mass as exhaust gas and has an acceleration of $15.0\ \text{m/s}^2$. What is the speed of the exhaust gas relative to the rocket?

***8.60.** A C6-5 model rocket engine has an impulse of 10.0 N · s for 1.70 s, while burning 0.0125 kg of propellant. It has a maximum thrust of 13.3 N. The initial mass of the engine plus propellant is 0.0258 kg. (a) What fraction of the maximum thrust is the average thrust? (b) Calculate the relative speed of the exhaust gases, assuming it is constant. (c) Assuming that the relative speed of the exhaust gases is constant, find the final speed of the engine if it was attached to a very light frame and fired from rest in gravity-free outer space.

***8.61.** A single-stage rocket is fired from rest from a deep-space platform, where gravity is negligible. If the rocket burns its fuel in 50.0 s and the relative speed of the exhaust gas is $v_{ex} = 2100$ m/s, what must the mass ratio m_0/m be for a final speed v of 8.00 km/s (about equal to the orbital speed of an earth satellite)?

***8.62.** Obviously, we can make rockets to go very fast, but what is a reasonable top speed? Assume that a rocket is fired from rest at a space station in deep space, where gravity is negligible. (a) If the rocket ejects gas at a relative speed of 2000 m/s and you want the rocket's speed eventually to be $1.00 \times 10^{-3}c$, where c is the speed of light, what fraction of the initial mass of the rocket and fuel is *not* fuel? (b) What is this fraction if the final speed is to be 3000 m/s?

Problems

8.63. A steel ball with mass 40.0 g is dropped from a height of 2.00 m onto a horizontal steel slab. The ball rebounds to a height of 1.60 m. (a) Calculate the impulse delivered to the ball during impact. (b) If the ball is in contact with the slab for 2.00 ms, find the average force on the ball during impact.

8.64. In a volcanic eruption, a 2400-kg boulder is thrown vertically upward into the air. At its highest point, it suddenly explodes (due to trapped gases) into two fragments, one being three times the mass of the other. The lighter fragment starts out with only horizontal velocity and lands 274 m directly north of the point of the explosion. Where will the other fragment land? Neglect any air resistance.

8.65. Just before it is struck by a racket, a tennis ball weighing 0.560 N has a velocity of $(20.0\ \text{m/s})\hat{\imath} - (4.0\ \text{m/s})\hat{\jmath}$. During the 3.00 ms that the racket and ball are in contact, the net force on the ball is constant and equal to $-(380\ \text{N})\hat{\imath} + (110\ \text{N})\hat{\jmath}$. (a) What are the x- and y-components of the impulse of the net force applied to the ball? (b) What are the x- and y-components of the final velocity of the ball?

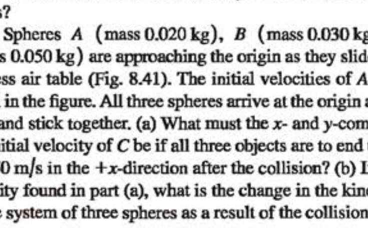
8.66. Three coupled railroad cars roll along and couple with a fourth car, which is initially at rest. These four cars roll along and couple with a fifth car initially at rest. This process continues until the speed of the final collection of railroad cars is one-fifth the speed of the initial three railroad cars. All the cars are identical. Ignoring friction, how many cars are in the final collection?

8.67. A 1500-kg blue convertible is traveling south, and a 2000-kg red SUV is traveling west. If the total momentum of the system consisting of the two cars is 8000 kg · m/s directed at 60.0° west of south, what is the speed of each vehicle?

8.68. Three identical pucks on a horizontal air table have repelling magnets. They are held together and then released simultaneously. Each has the same speed at any instant. One puck moves due west. What is the direction of the velocity of each of the other two pucks?

8.69. Spheres A (mass 0.020 kg), B (mass 0.030 kg), and C (mass 0.050 kg) are approaching the origin as they slide on a frictionless air table (Fig. 8.41). The initial velocities of A and B are given in the figure. All three spheres arrive at the origin at the same time and stick together. (a) What must the x- and y-components of the initial velocity of C be if all three objects are to end up moving at 0.50 m/s in the $+x$-direction after the collision? (b) If C has the velocity found in part (a), what is the change in the kinetic energy of the system of three spheres as a result of the collision?

Figure 8.41 Problem 8.69.



8.70. A railroad handcar is moving along straight, frictionless tracks with negligible air resistance. In the following cases, the car initially has a total mass (car and contents) of 200 kg and is traveling east with a velocity of magnitude 5.00 m/s. Find the *final* *velocity* of the car in each case, assuming that the handcar does not leave the tracks. (a) A 25.0-kg mass is thrown sideways out of the car with a velocity of magnitude 2.00 m/s relative to the car's initial velocity. (b) A 25.0-kg mass is thrown backward out of the car with a velocity of 5.00 m/s relative to the initial motion of the car. (c) A 25.0-kg mass is thrown into the car with a velocity of 6.00 m/s relative to the ground and opposite in direction to the initial velocity of the car.

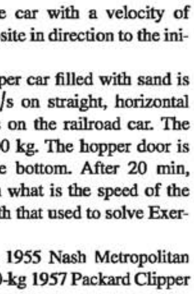
8.71. Changing Mass. A railroad hopper car filled with sand is rolling with an initial speed of 15.0 m/s on straight, horizontal tracks. You can ignore frictional forces on the railroad car. The total mass of the car plus sand is 85,000 kg. The hopper door is not fully closed so sand leaks out the bottom. After 20 min, 13,000 kg of sand has leaked out. Then what is the speed of the railroad car? (Compare your analysis with that used to solve Exercise 8.27.)

8.72. At a classic auto show, a 840-kg 1955 Nash Metropolitan motors by at 9.0 m/s, followed by a 1620-kg 1957 Packard Clipper purring past at 5.0 m/s. (a) Which car has the greater kinetic energy? What is the ratio of the kinetic energy of the Nash to that of the Packard? (b) Which car has the greater magnitude of momentum? What is the ratio of the magnitude of momentum of the Nash to that of the Packard? (c) Let F_N be the net force required to stop the Nash in time t, and let F_P be the net force required to stop the Packard in the same time. Which is larger: F_N or F_P? What is the ratio F_N/F_P of these two forces? (d) Now let F_N be the net force required to stop the Nash in a distance d, and let F_P be the net force required to stop the Packard in the same distance. Which is larger: F_N or F_P? What is the ratio F_N/F_P?

8.73. A soldier on a firing range fires an eight-shot burst from an assault weapon at a full automatic rate of 1000 rounds per minute. Each bullet has a mass of 7.45 g and a speed of 293 m/s relative to the ground as it leaves the barrel of the weapon. Calculate the average recoil force exerted on the weapon during that burst.

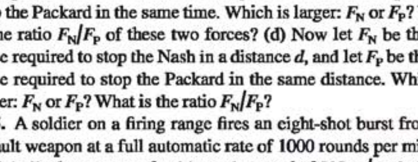
8.74. A 0.150-kg frame, when suspended from a coil spring, stretches the spring 0.050 m. A 0.200-kg lump of putty is dropped from rest onto the frame from a height of 30.0 cm (Fig. 8.42). Find the maximum distance the frame moves downward from its initial position.

Figure 8.42 Problem 8.74.



8.75. A rifle bullet with mass 8.00 g strikes and embeds itself in a block with mass 0.992 kg that rests on a frictionless, horizontal surface and is attached to a coil spring (Fig. 8.43). The impact compresses the spring 15.0 cm. Calibration of the spring shows that a force of 0.750 N is required to compress the spring 0.250 cm. (a) Find the magnitude of the block's velocity just after impact. (b) What was the initial speed of the bullet?

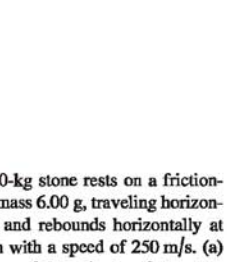
Figure 8.43 Problem 8.75.



8.76. A Ricocheting Bullet. 0.100-kg stone rests on a frictionless, horizontal surface. A bullet of mass 6.00 g, traveling horizontally at 350 m/s, strikes the stone and rebounds horizontally at right angles to its original direction with a speed of 250 m/s. (a) Compute the magnitude and direction of the velocity of the stone after it is struck. (b) Is the collision perfectly elastic?

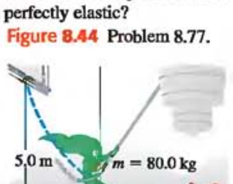
8.77. A movie stuntman (mass 80.0 kg) stands on a window ledge 5.0 m above the floor (Fig. 8.44). Grabbing a rope attached to a chandelier, he swings down to grapple with the movie's villain (mass 70.0 kg), who is standing directly under the chandelier. (Assume that the stuntman's center of mass moves downward 5.0 m. He releases the rope just as he reaches the villain.) (a) With what speed do the entwined foes start to slide across the floor? (b) If the coefficient of kinetic friction of their bodies with the floor is $\mu_k = 0.250$, how far do they slide?

Figure 8.44 Problem 8.77.



8.78. Two identical masses are released from rest in a smooth hemispherical bowl of radius R, from the positions shown in Fig. 8.45. You can ignore friction between the masses and the surface of the bowl. If they stick together when they collide, how high above the bottom of the bowl will the masses go after colliding?

Figure 8.45 Problem 8.78.



8.79. A ball with mass M, moving horizontally at 5.00 m/s, collides elastically with a block with mass $3M$ that is initially hanging at rest from the ceiling on the end of a 50.0-cm wire. Find the maximum angle through which the block swings after it is hit.

8.80. A 20.00-kg lead sphere is hanging from a hook by a thin wire 3.50 m long, and is free to swing in a complete circle. Suddenly it is struck horizontally by a 5.00-kg steel dart that embeds itself in the lead sphere. What must be the minimum initial speed of the dart so that the combination makes a complete circular loop after the collision?

8.81. An 8.00-kg ball, hanging from the ceiling by a light wire 135 cm long, is struck in an elastic collision by a 2.00-kg ball moving horizontally at 5.00 m/s just before the collision. Find the tension in the wire just after the collision.

8.82. A rubber ball of mass m is released from rest at height h above the floor. After its first bounce, it rises to 90% of its original height. What impulse (magnitude and direction) does the floor exert on this ball during its first bounce? Express your answer in terms of the variables m and h.

8.83. A 4.00-g bullet, traveling horizontally with a velocity of magnitude 400 m/s, is fired into a wooden block with mass 0.800 kg, initially at rest on a level surface. The bullet passes through the block and emerges with its speed reduced to 120 m/s. The block slides a distance of 45.0 cm along the surface from its initial position. (a) What is the coefficient of kinetic friction between block and surface? (b) What is the decrease in kinetic energy of the bullet? (c) What is the kinetic energy of the block at the instant after the bullet passes through it?

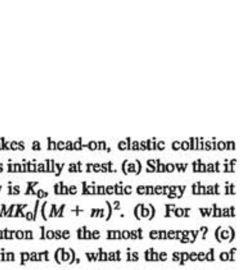
8.84. A 5.00-g bullet is shot *through* a 1.00-kg wood block suspended on a string 2.00 m long. The center of mass of the block rises a distance of 0.45 cm. Find the speed of the bullet as it emerges from the block if its initial speed is 450 m/s.

8.85. A neutron with mass m makes a head-on, elastic collision with a nucleus of mass M, which is initially at rest. (a) Show that if the neutron's initial kinetic energy is K_0, the kinetic energy that it loses during the collision is $4mMK_0/(M + m)^2$. (b) For what value of M does the incident neutron lose the most energy? (c) When M has the value calculated in part (b), what is the speed of the neutron after the collision?

8.86. Energy Sharing in Elastic Collisions. A stationary object with mass m_B is struck head-on by an object with mass m_A that is moving initially at speed v_0. (a) If the collision is elastic, what percentage of the original energy does each object have after the collision? (b) What does your answer in part (a) give for the special cases (i) $m_A = m_B$ and (ii) $m_A = 5m_B$? (c) For what values, if any, of the mass ratio m_A/m_B is the original kinetic energy shared equally by the two objects after the collision?

8.87. In a shipping company distribution center, an open cart of mass 50.0 kg is rolling to the left at a speed of 5.00 m/s (Fig. 8.46). You can ignore friction between the cart and the floor. A 15.0-kg package slides down a chute that is inclined at 37° from the horizontal and leaves the end of the chute with a speed of 3.00 m/s. The package lands in the cart and they roll off together. If the lower end of the chute is a vertical distance of 4.00 m above the bottom of the cart, what are (a) the speed of the package just before it lands in the cart and (b) the final speed of the cart?

Figure 8.46 Problem 8.87.



8.88. A blue puck with mass 0.0400 kg, sliding with a velocity of magnitude 0.200 m/s on a frictionless, horizontal air table, makes a perfectly elastic, head-on collision with a red puck with mass m, initially at rest. After the collision, the velocity of the blue puck is 0.050 m/s in the same direction as its initial velocity. Find (a) the velocity (magnitude and direction) of the red puck after the collision; and (b) the mass m of the red puck.

8.89. Two asteroids with masses m_A and m_B are moving with velocities $\vec{v}_A$ and $\vec{v}_B$ with respect to an astronomer in a space vehicle. (a) Show that the total kinetic energy as measured by the astronomer is

$$K = \tfrac{1}{2}Mv_{cm}^2 + \tfrac{1}{2}(m_A v_A'^2 + m_B v_B'^2)$$

with $\vec{v}_{cm}$ and M defined as in Section 8.5, $\vec{v}_A' = \vec{v}_A - \vec{v}_{cm}$, and $\vec{v}_B' = \vec{v}_B - \vec{v}_{cm}$. In this expression the total kinetic energy of the two asteroids is the energy associated with their center of mass plus the energy associated with the internal motion relative to the center of mass. (b) If the asteroids collide, what is the *minimum* possible kinetic energy they can have after the collision, as measured by the astronomer? Explain.

8.90. Suppose you hold a small ball in contact with, and directly over, the center of a large ball. If you then drop the small ball a short time after dropping the large ball, the small ball rebounds with surprising speed. To show the extreme case, ignore air resistance and suppose the large ball makes an elastic collision with the floor and then rebounds to make an elastic collision with the still-descending small ball. Just before the collision between the two balls, the large ball is moving upward with velocity $\vec{v}$ and the small ball has velocity $-\vec{v}$. (Do you see why?) Assume the large ball has a much greater mass than the small ball. (a) What is the velocity of the small ball immediately after its collision with the large ball? (b) From the answer to part (a), what is the ratio of the small ball's rebound distance to the distance it fell before the collision?

8.91. Jack and Jill are standing on a crate at rest on the frictionless, horizontal surface of a frozen pond. Jack has mass 75.0 kg, Jill has mass 45.0 kg, and the crate has mass 15.0 kg. They remember that they must fetch a pail of water, so each jumps horizontally from the top of the crate. Just after each jumps, that person is moving away from the crate with a speed of 4.00 m/s relative to the crate. (a) What is the final speed of the crate if both Jack and Jill jump simultaneously and in the same direction? (*Hint:* Use an inertial coordinate system attached to the ground.) (b) What is the final speed of the crate if Jack jumps first and then a few seconds later Jill jumps in the same direction? (c) What is the final speed of the crate if Jill jumps first and then Jack, again in the same direction?

8.92. Energy Sharing. An object with mass m, initially at rest, explodes into two fragments, one with mass m_A and the other with mass m_B, where $m_A + m_B = m$. (a) If energy Q is released in the explosion, how much kinetic energy does each fragment have immediately after the collision? (b) What percentage of the total energy released does each fragment get when one fragment has four times the mass of the other?

8.93. Neutron Decay. A neutron at rest decays (breaks up) to a proton and an electron. Energy is released in the decay and appears as kinetic energy of the proton and electron. The mass of a proton is 1836 times the mass of an electron. What fraction of the total energy released goes into the kinetic energy of the proton?

8.94. A ^{232}Th (thorium) nucleus at rest decays to a ^{228}Ra (radium) nucleus with the emission of an alpha particle. The total kinetic energy of the decay fragments is 6.54×10^{-13} J. An alpha particle has 1.76% of the mass of a ^{228}Ra nucleus. Calculate the kinetic energy of (a) the recoiling ^{228}Ra nucleus and (b) the alpha particle.

8.95. Antineutrino. In beta decay, a nucleus emits an electron. A ^{210}Bi (bismuth) nucleus at rest undergoes beta decay to ^{210}Po (polonium). Suppose the emitted electron moves to the right with a momentum of 5.60×10^{-22} kg · m/s. The ^{210}Po nucleus, with mass 3.50×10^{-25} kg, recoils to the left at a speed of 1.14×10^{-3} m/s. Momentum conservation requires that a second particle, called an antineutrino, must also be emitted. Calculate the magnitude and direction of the momentum of the antineutrino that is emitted in this decay.

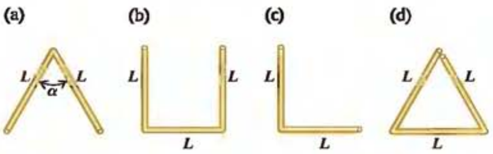
8.96. A proton moving with speed v_{A1} in the $+x$-direction makes an elastic, off-center collision with an identical proton originally at rest. After impact, the first proton moves with speed v_{A2} in the first quadrant at an angle α with the x-axis, and the second moves with speed v_{B2} in the fourth quadrant at an angle β with the x-axis (Fig. 8.13). (a) Write the equations expressing conservation of linear momentum in the x- and y-directions. (b) Square the equations from part (a) and add them. (c) Now introduce the fact that the collision is elastic. (d) Prove that $\alpha + \beta = \pi/2$. (You have shown that this equation is obeyed in any elastic, off-center collision between objects of equal mass when one object is initially at rest.)

8.97. Hockey puck B rests on a smooth ice surface and is struck by a second puck A, which has the same mass. Puck A is initially traveling at 15.0 m/s and is deflected 25.0° from its initial direction. Assume that the collision is perfectly elastic. Find the final speed of each puck and the direction of B's velocity after the collision. [*Hint:* Use the relationship derived in part (d) of Problem 8.96.]

8.90. Jonathan and Jane are sitting in a sleigh that is at rest on frictionless ice. Jonathan's weight is 800 N, Jane's weight is 600 N, and that of the sleigh is 1000 N. They see a poisonous spider on the floor of the sleigh and immediately jump off. Jonathan jumps to the left with a velocity of 5.00 m/s at 30.0° above the horizontal (relative to the ice), and Jane jumps to the right at 7.00 m/s at 36.9° above the horizontal (relative to the ice). Calculate the sleigh's horizontal velocity (magnitude and direction) after they jump out.

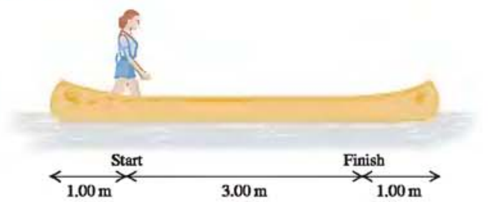
8.99. The objects in Fig. 8.47 are constructed of uniform wire bent into the shapes shown. Find the position of the center of mass of each.

Figure 8.47 Problem 8.99.



8.100. A 45.0-kg woman stands up in a 60.0-kg canoe 5.00 m long. She walks from a point 1.00 m from one end to a point 1.00 m from the other end (Fig. 8.48). If you ignore resistance to motion of the canoe in the water, how far does the canoe move during this process?

Figure 8.48 Problem 8.100.



8.101. You are standing on a concrete slab that in turn is resting on a frozen lake. Assume there is no friction between the slab and the ice. The slab has a weight five times your weight. If you begin walking forward at 2.00 m/s relative to the ice, with what speed, relative to the ice, does the slab move?

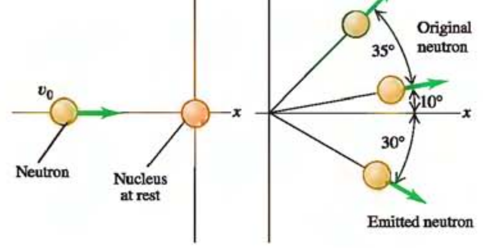
8.102. A 20.0-kg projectile is fired at an angle of 60.0° above the horizontal with a speed of 80.0 m/s. At the highest point of its trajectory, the projectile explodes into two fragments with equal mass, one of which falls vertically with zero initial speed. You can ignore air resistance. (a) How far from the point of firing does the other fragment strike if the terrain is level? (b) How much energy is released during the explosion?

8.103. A fireworks rocket is fired vertically upward. At its maximum height of 80.0 m, it explodes and breaks into two pieces, one with mass 1.40 kg and the other with mass 0.28 kg. In the explosion, 860 J of chemical energy is converted to kinetic energy of the two fragments. (a) What is the speed of each fragment just after the explosion? (b) It is observed that the two fragments hit the ground at the same time. What is the distance between the points on the ground where they land? Assume that the ground is level and air resistance can be ignored.

8.104. A 12.0-kg shell is launched at an angle of 55.0° above the horizontal with an initial speed of 150 m/s. When it is at its highest point, the shell exploded into two fragments, one three times heavier than the other. The two fragments reach the ground at the same time. Assume that air resistance can be ignored. If the heavier fragment lands back at the same point from which the shell was launched, where will the lighter fragment land and how much energy was released in the explosion?

8.105. A Nuclear Reaction. Fission, the process that supplies energy in nuclear power plants, occurs when a heavy nucleus is split into two medium-sized nuclei. One such reaction occurs when a neutron colliding with a ^{235}U (uranium) nucleus splits that nucleus into a ^{141}Ba (barium) nucleus and a ^{92}Kr (krypton) nucleus. In this reaction, two neutrons also are split off from the original ^{235}U. Before the collision, the arrangement is as shown in Fig. 8.49a. After the collision, the ^{141}Ba nucleus is moving in the $+z$-direction and the ^{92}Kr nucleus in the $-z$-direction. The three neutrons are moving in the xy-plane, as shown in Fig. 8.49b. If the incoming neutron has an initial velocity of magnitude 3.0×10^3 m/s and a final velocity of magnitude 2.0×10^3 m/s in the directions shown, what are the speeds of the other two neutrons, and what can you say about the speeds of the ^{141}Ba and ^{92}Kr nuclei? (The mass of the ^{141}Ba nucleus is approximately 2.3×10^{-25} kg, and the mass of ^{92}Kr is about 1.5×10^{-25} kg.)

Figure 8.49 Problem 8.105.



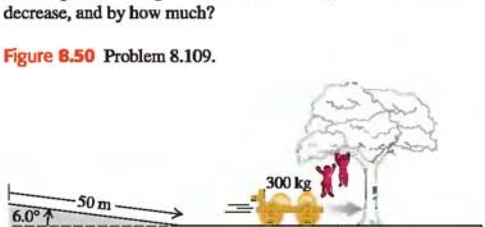
8.106. Center-of-Mass Coordinate System. Puck A (mass m_A) is moving on a frictionless, horizontal air table in the $+x$-direction with velocity $\vec{v}_{A1}$ and makes an elastic, head-on collision with puck B (mass m_B), which is initially at rest. After the collision, both pucks are moving along the x-axis. (a) Calculate the velocity of the center of mass of the two-puck system before the collision. (b) Consider a coordinate system whose origin is at the center of mass and moves with it. Is this an inertial reference frame? (c) What are the initial velocities $\vec{u}_{A1}$ and $\vec{u}_{B1}$ of the two pucks in this center-of-mass reference frame? What is the total momentum in this frame? (d) Use conservation of momentum and energy, applied in the center-of-mass reference frame, to relate the final momentum of each puck to its initial momentum and thus the final velocity of each puck to its initial velocity. Your results should show that a one-dimensional, elastic collision has a very simple description in center-of-mass coordinates. (e) Let $m_A = 0.400$ kg, $m_B = 0.200$ kg, and $v_{A1} = 6.00$ m/s. Find the center-of-mass velocities $\vec{u}_{A1}$ and $\vec{u}_{B1}$, apply the simple result found in part (d), and transform back to velocities in a stationary frame to find the final velocities of the pucks. Does your result agree with Eqs. (8.24) and (8.25)?

8.107. The *coefficient of restitution* ϵ for a head-on collision is defined as the ratio of the relative speed after the collision to the relative speed before. (a) What is ϵ for a completely inelastic collision? (b) What is ϵ for an elastic collision? (c) A ball is dropped from a height h onto a stationary surface and rebounds back to a height H_1. Show that $\epsilon = \sqrt{H_1/h}$. (d) A properly inflated basketball should have a coefficient of restitution of 0.85. When dropped from a height of 1.2 m above a solid wood floor, to what height should a properly inflated basketball bounce? (e) The height of the first bounce is H_1. If ϵ is constant, show that the height of the nth bounce is $H_n = \epsilon^{2n}h$. (f) If ϵ is constant, what is the height of the eighth bounce of a properly inflated basketball dropped from 1.2 m?

8.108. Binding Energy of the Hydrogen Molecule. When two hydrogen atoms of mass m combine to form a diatomic hydrogen molecule (H_2), the potential energy of the system after they combine is $-\Delta$, where Δ is a positive quantity called the *binding energy* of the molecule. (a) Show that in a collision that involves only two hydrogen atoms, it is *impossible* to form an H_2 molecule because momentum and energy cannot simultaneously be conserved. (*Hint:* If you can show this to be true in one frame of reference, then it is true in all frames of reference. Can you see why?) (b) An H_2 molecule can be formed in a collision that involves *three* hydrogen atoms. Suppose that before such a collision, each of the three atoms has speed 1.00×10^3 m/s, and they are approaching at 120° angles so that at any instant, the atoms lie at the corners of an equilateral triangle. Find the speeds of the H_2 molecule and of the single hydrogen atom that remains after the collision. The binding energy of H_2 is $\Delta = 7.23 \times 10^{-19}$ J, and the mass of the hydrogen atom is 1.67×10^{-27} kg.

8.109. A wagon with two boxes of gold, having total mass 300 kg, is cut loose from the horses by an outlaw when the wagon is at rest 50 m up a 6.0° slope (Fig. 8.50). The outlaw plans to have the wagon roll down the slope and across the level ground, and then fall into a canyon where his confederates wait. But in a tree 40 m from the canyon edge wait the Lone Ranger (mass 75.0 kg) and Tonto (mass 60.0 kg). They drop vertically into the wagon as it passes beneath them. (a) If they require 5.0 s to grab the gold and jump out, will they make it before the wagon goes over the edge? The wagon rolls with negligible friction. (b) When the two heroes drop into the wagon, is the kinetic energy of the system of the heroes plus the wagon conserved? If not, does it increase or decrease, and by how much?

Figure 8.50 Problem 8.109.



***8.110.** In Section 8.6, we considered a rocket fired in outer space where there is no air resistance and where gravity is negligible. Suppose instead that the rocket is accelerating vertically upward from rest on the earth's surface. Continue to ignore air resistance and consider only that part of the motion where the altitude of the rocket is small so that g may be assumed to be constant. (a) How is Eq. (8.37) modified by the presence of the gravity force? (b) Derive an expression for the acceleration a of the rocket, analogous to Eq. (8.39). (c) What is the acceleration of the rocket in Example 8.15 (Section 8.6) if it is near the earth's surface rather than in outer space? You can ignore air resistance. (d) Find the speed of the rocket in Example 8.16 (Section 8.6) after 90 s if the rocket is fired from the earth's surface rather than in outer space. You can ignore air resistance. How does your answer compare with the rocket speed calculated in Example 8.16?

***8.111. A Multistage Rocket.** Suppose the first stage of a two-stage rocket has total mass 12,000 kg, of which 9000 kg is fuel. The total mass of the second stage is 1000 kg, of which 700 kg is fuel. Assume that the relative speed v_{ex} of ejected material is constant, and ignore any effect of gravity. (The effect of gravity is small during the firing period if the rate of fuel consumption is large.) (a) Suppose the entire fuel supply carried by the two-stage rocket is utilized in a single-stage rocket with the same total mass of 13,000 kg. In terms of v_{ex}, what is the speed of the rocket, starting from rest, when its fuel is exhausted? (b) For the two-stage rocket, what is the speed when the fuel of the first stage is exhausted if the first stage carries the second stage with it to this point? This speed then becomes the initial speed of the second stage. At this point, the second stage separates from the first stage. (c) What is the final speed of the second stage? (d) What value of v_{ex} is required to give the second stage of the rocket a speed of 7.00 km/s?

***8.112.** For the rocket described in Examples 8.15 and 8.16 (Section 8.6), the mass of the rocket as a function of time is

$$m(t) = \begin{cases} m_0 & \text{for } t < 0 \\ m_0\left(1 - \dfrac{t}{120\text{ s}}\right) & \text{for } 0 \le t \le 90\text{ s} \\ m_0/4 & \text{for } t \ge 90\text{ s} \end{cases}$$

(a) Calculate and graph the velocity of the rocket as a function of time from $t = 0$ to $t = 100$ s. (b) Calculate and graph the acceleration of the rocket as a function of time from $t = 0$ to $t = 100$ s. (c) A 75-kg astronaut lies on a reclined chair during the firing of the rocket. What is the maximum net force exerted by the chair on the astronaut during the firing? How does your answer compare with her weight on earth?

Challenge Problems

8.113. In Section 8.5 we calculated the center of mass by considering objects composed of a *finite* number of point masses or objects that, by symmetry, could be represented by a finite number of point masses. For a solid object whose mass distribution does not allow for a simple determination of the center of mass by symmetry, the sums of Eqs. (8.28) must be generalized to integrals

$$x_{cm} = \frac{1}{M}\int x\,dm \qquad y_{cm} = \frac{1}{M}\int y\,dm$$

where x and y are the coordinates of the small piece of the object that has mass dm. The integration is over the whole of the object. Consider a thin rod of length L, mass M, and cross-sectional area A. Let the origin of the coordinates be at the left end of the rod and the positive x-axis lie along the rod. (a) If the density $\rho = M/V$ of the object is uniform, perform the integration described above to show that the x-coordinate of the center of mass of the rod is at its geometrical center. (b) If the density of the object varies linearly with x—that is, $\rho = \alpha x$, where α is a positive constant—calculate the x-coordinate of the rod's center of mass.

8.114. Use the methods of Challenge Problem 8.113 to calculate the x- and y-coordinates of the center of mass of a semicircular metal plate with uniform density ρ and thickness t. Let the radius of the plate be a. The mass of the plate is thus $M = \frac{1}{2}\rho\pi a^2 t$. Use the coordinate system indicated in Fig. 8.51.

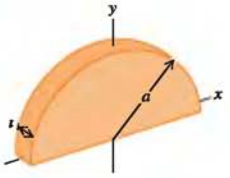


Figure 8.51 Challenge Problem 8.114.

8.115. One-fourth of a rope of length l is hanging down over the edge of a frictionless table. The rope has a uniform, linear density (mass per unit length) λ (Greek lambda), and the end already on the table is held by a person. How much work does the person do when she pulls on the rope to raise the rest of the rope slowly onto the table? Do the problem in two ways as follows. (a) Find the force that the person must exert to raise the rope and from this the work done. Note that this force is variable because at different times, different amounts of rope are hanging over the edge. (b) Suppose the segment of the rope initially hanging over the edge of the table has all of its mass concentrated at its center of mass. Find the work necessary to raise this to table height. You will probably find this approach simpler than that of part (a). How do the answers compare, and why is this so?

***8.116 A Variable-Mass Raindrop.** In a rocket-propulsion problem the mass is variable. Another such problem is a raindrop falling through a cloud of small water droplets. Some of these small droplets adhere to the raindrop, thereby *increasing* its mass as it falls. The force on the raindrop is

$$F_{\text{ext}} = \frac{dp}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

Suppose the mass of the raindrop depends on the distance x that it has fallen. Then $m = kx$, where k is a constant, and $dm/dt = kv$. This gives, since $F_{\text{ext}} = mg$,

$$mg = m\frac{dv}{dt} + v(kv)$$

Or, dividing by k,

$$xg = x\frac{dv}{dt} + v^2$$

This is a differential equation that has a solution of the form $v = at$, where a is the acceleration and is constant. Take the initial velocity of the raindrop to be zero. (a) Using the proposed solution for v, find the acceleration a. (b) Find the distance the raindrop has fallen in $t = 3.00$ s. (c) Given that $k = 2.00$ g/m, find the mass of the raindrop at $t = 3.00$ s. For many more intriguing aspects of this problem, see K. S. Krane, *Amer. Jour. Phys.*, Vol. 49 (1981), pp. 113–117.

ROTATION OF RIGID BODIES

9

? All segments of a rotating helicopter blade have the same angular velocity and angular acceleration. Compared to a given blade segment, how many times greater is the linear speed of a second segment twice as far from the axis of rotation? How many times greater is the linear acceleration?

LEARNING GOALS

By studying this chapter, you will learn:

- How to describe the rotation of a rigid body in terms of angular coordinate, angular velocity, and angular acceleration.
- How to analyze rigid-body rotation when the angular acceleration is constant.
- How to relate the rotation of a rigid body to the linear velocity and linear acceleration of a point on the body.
- The meaning of a body's moment of inertia about a rotation axis, and how it relates to rotational kinetic energy.
- How to calculate the moment of inertia of various bodies.

What do the motions of a compact disc, a Ferris wheel, a circular saw blade, and a ceiling fan have in common? None of these can be represented adequately as a moving *point;* each involves a body that *rotates* about an axis that is stationary in some inertial frame of reference.

Rotation occurs at all scales, from the motion of electrons in atoms to the motions of entire galaxies. We need to develop some general methods for analyzing the motion of a rotating body. In this chapter and the next we consider bodies that have definite size and definite shape, and that in general can have rotational as well as translational motion.

Real-world bodies can be very complicated; the forces that act on them can deform them—stretching, twisting, and squeezing them. We'll neglect these deformations for now and assume that the body has a perfectly definite and unchanging shape and size. We call this idealized model a **rigid body.** This chapter and the next are mostly about rotational motion of a rigid body.

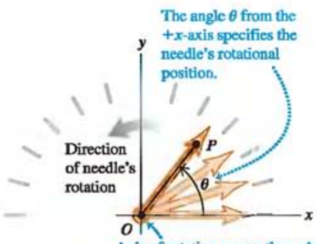
We begin with kinematic language for *describing* rotational motion. Next we look at the kinetic energy of rotation, the key to using energy methods for rotational motion. Then in Chapter 10 we'll develop dynamic principles that relate the forces on a body to its rotational motion.

9.1 Angular Velocity and Acceleration

In analyzing rotational motion, let's think first about a rigid body that rotates about a *fixed axis*—an axis that is at rest in some inertial frame of reference and does not change direction relative to that frame. The rotating rigid body might be a motor shaft, a chunk of beef on a barbecue skewer, or a merry-go-round.

Figure 9.1 shows a rigid body (in this case, the indicator needle of a speedometer) rotating about a fixed axis. The axis passes through point O and is

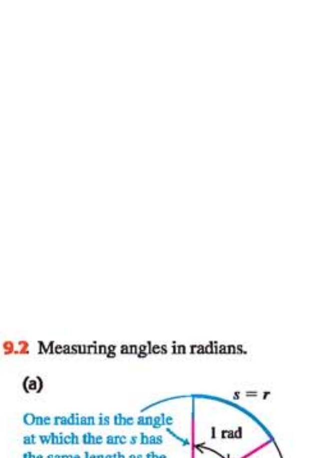
9.1 A speedometer needle (an example of a rigid body) rotating counterclockwise about a fixed axis.



perpendicular to the plane of the diagram, which we choose to call the xy-plane. One way to describe the rotation of this body would be to choose a particular point P on the body and to keep track of the x- and y-coordinates of this point. This isn't a terribly convenient method, since it takes two numbers (the two coordinates x and y) to specify the rotational position of the body. Instead, we notice that the line OP is fixed in the body and rotates with it. The angle θ that this line makes with the $+x$-axis describes the rotational position of the body; we will use this single quantity θ as a *coordinate* for rotation.

The angular coordinate θ of a rigid body rotating around a fixed axis can be positive or negative. If we choose positive angles to be measured counterclockwise from the positive x-axis, then the angle θ in Fig. 9.1 is positive. If we instead choose the positive rotation direction to be clockwise, then θ in Fig. 9.1 is negative. When we considered the motion of a particle along a straight line, it was essential to specify the direction of positive displacement along that line; when we discuss rotation around a fixed axis, it's just as essential to specify the direction of positive rotation.

9.2 Measuring angles in radians.



To describe rotational motion, the most natural way to measure the angle θ is not in degrees, but in **radians.** As shown in Fig. 9.2a, one radian (1 rad) is the angle subtended at the center of a circle by an arc with a length equal to the radius of the circle. In Fig. 9.2b an angle θ is subtended by an arc of length s on a circle of radius r. The value of θ (in radians) is equal to s divided by r:

$$\theta = \frac{s}{r} \quad \text{or} \quad s = r\theta \tag{9.1}$$

An angle in radians is the ratio of two lengths, so it is a pure number, without dimensions. If $s = 3.0\text{ m}$ and $r = 2.0\text{ m}$, then $\theta = 1.5$, but we will often write this as 1.5 rad to distinguish it from an angle measured in degrees or revolutions.

The circumference of a circle (that is, the arc length all the way around the circle) is 2π times the radius, so there are 2π (about 6.283) radians in one complete revolution $(360°)$. Therefore

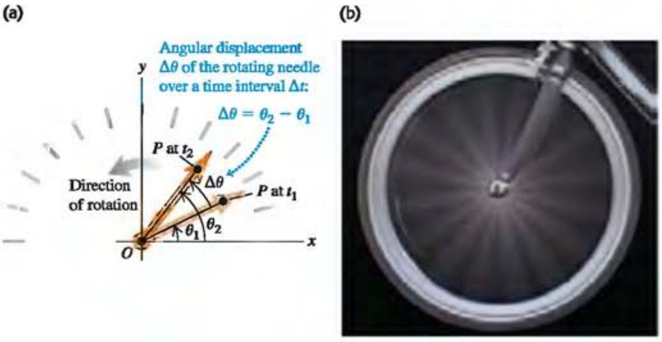
$$1\text{ rad} = \frac{360°}{2\pi} = 57.3°$$

Similarly, $180° = \pi$ rad, $90° = \pi/2$ rad, and so on. If we had insisted on measuring the angle θ in degrees, we would have needed to include an extra factor of $(2\pi/360)$ on the right-hand side of $s = r\theta$ in Eq. (9.1). By measuring angles in radians, we keep the relationship between angle and distance along an arc as simple as possible.

Angular Velocity

The coordinate θ shown in Fig. 9.1 specifies the rotational position of a rigid body at a given instant. We can describe the rotational *motion* of such a rigid body in terms of the rate of change of θ. We'll do this in an analogous way to our description of straight-line motion in Chapter 2. In Fig. 9.3a, a reference line OP in a rotating body makes an angle θ_1 with the $+x$-axis at time t_1. At a later time t_2 the angle has changed to θ_2. We define the **average angular velocity** $\omega_{\text{av-}z}$ (the Greek letter omega) of the body in the time interval $\Delta t = t_2 - t_1$ as the ratio of the **angular displacement** $\Delta\theta = \theta_2 - \theta_1$ to Δt:

$$\omega_{\text{av-}z} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \tag{9.2}$$



9.3 (a) Angular displacement $\Delta\theta$ of a rotating body. (b) Every part of a rotating rigid body has the same angular velocity $\Delta\theta/\Delta t$.

The subscript z indicates that the body in Fig. 9.3a is rotating about the z-axis, which is perpendicular to the plane of the diagram. The **instantaneous angular velocity** ω_z is the limit of $\omega_{\text{av-}z}$ as Δt approaches zero—that is, the derivative of θ with respect to t:

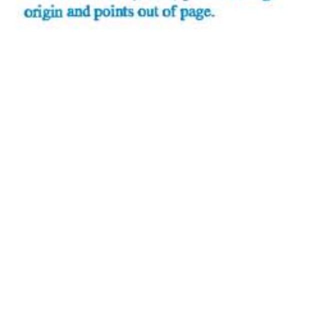
$$\omega_z = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad \text{(definition of angular velocity)} \qquad (9.3)$$

When we refer simply to "angular velocity," we mean the instantaneous angular velocity, not the average angular velocity.

The angular velocity ω_z can be positive or negative, depending on the direction in which the rigid body is rotating (Fig. 9.4). The angular *speed* ω, which we will use extensively in Sections 9.3 and 9.4, is the magnitude of angular velocity. Like ordinary (linear) speed v, the angular speed is never negative.

CAUTION **Angular velocity vs. linear velocity** Keep in mind the distinction between angular velocity ω_z and ordinary velocity, or *linear velocity,* v_x (see Section 2.2). If an object has a velocity v_x, the object as a whole is *moving* along the x-axis. By contrast, if an object has an angular velocity ω_z, then it is *rotating* around the z-axis. We do *not* mean that the object is moving along the z-axis.

9.4 A rigid body's average angular velocity (shown here) and instantaneous angular velocity can be positive or negative.



Different points on a rotating rigid body move different distances in a given time interval, depending on how far the point lies from the rotation axis. But because the body is rigid, *all* points rotate through the same angle in the same time (Fig. 9.3b). Hence *at any instant, every part of a rotating rigid body has the same angular velocity.* The angular velocity is positive if the body is rotating in the direction of increasing θ and negative if it is rotating in the direction of decreasing θ.

If the angle θ is in radians, the unit of angular velocity is the radian per second (rad/s). Other units, such as the revolution per minute (rev/min or rpm), are often used. Since 1 rev $= 2\pi$ rad, two useful conversions are

$$1 \text{ rev/s} = 2\pi \text{ rad/s} \qquad \text{and} \qquad 1 \text{ rev/min} = 1 \text{ rpm} = \frac{2\pi}{60} \text{ rad/s}$$

That is, 1 rad/s is about 10 rpm.

Example 9.1 Calculating angular velocity

The flywheel of a prototype car engine is under test. The angular position θ of the flywheel is given by

$$\theta = (2.0\ \text{rad/s}^3)t^3$$

The diameter of the flywheel is 0.36 m. (a) Find the angle θ, in radians and in degrees, at times $t_1 = 2.0$ s and $t_2 = 5.0$ s. (b) Find the distance that a particle on the rim moves during that time interval. (c) Find the average angular velocity, in rad/s and in rev/min (rpm), between $t_1 = 2.0$ s and $t_2 = 5.0$ s. (d) Find the instantaneous angular velocity at time $t = t_2 = 5.0$ s.

SOLUTION

IDENTIFY: We need to find the values θ_1 and θ_2 of the angular position at times t_1 and t_2, the angular displacement $\Delta\theta$ between t_1 and t_2, the distance traveled and the average angular velocity between t_1 and t_2, and the instantaneous angular velocity at t_2.

SET UP: We're given the angular position θ as a function of time, so we can easily find our first two target variables θ_1 and θ_2; the angular displacement $\Delta\theta$ is the difference between θ_1 and θ_2. Given $\Delta\theta$ we'll find the distance and the average angular velocity using Eqs. (9.1) and (9.2), respectively. To find the instantaneous angular velocity, we'll take the derivative of θ with respect to time, as in Eq. (9.3).

EXECUTE: (a) We substitute the values of t into the given equation:

$$\theta_1 = (2.0\ \text{rad/s}^3)(2.0\ \text{s})^3 = 16\ \text{rad}$$
$$= (16\ \text{rad})\frac{360°}{2\pi\ \text{rad}} = 920°$$
$$\theta_2 = (2.0\ \text{rad/s}^3)(5.0\ \text{s})^3 = 250\ \text{rad}$$
$$= (250\ \text{rad})\frac{360°}{2\pi\ \text{rad}} = 14{,}000°$$

(b) The flywheel turns through an angular displacement of $\Delta\theta = \theta_2 - \theta_1 = 250\ \text{rad} - 16\ \text{rad} = 234\ \text{rad}$. The radius r is half the diameter, or 0.18 m. Equation (9.1) gives

$$s = r\theta = (0.18\ \text{m})(234\ \text{rad}) = 42\ \text{m}$$

To use Eq. (9.1), the angle *must* be expressed in radians. We drop "radians" from the unit for s because θ is really a dimensionless pure number; s is a distance and is measured in meters, the same unit as r.

(c) In Eq. (9.2) we have

$$\omega_{\text{av-}z} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{250\ \text{rad} - 16\ \text{rad}}{5.0\ \text{s} - 2.0\ \text{s}} = 78\ \text{rad/s}$$
$$= \left(78\frac{\text{rad}}{\text{s}}\right)\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 740\ \text{rev/min}$$

(d) We use Eq. (9.3):

$$\omega_z = \frac{d\theta}{dt} = \frac{d}{dt}[(2.0\ \text{rad/s}^3)t^3] = (2.0\ \text{rad/s}^3)(3t^2)$$
$$= (6.0\ \text{rad/s}^3)t^2$$

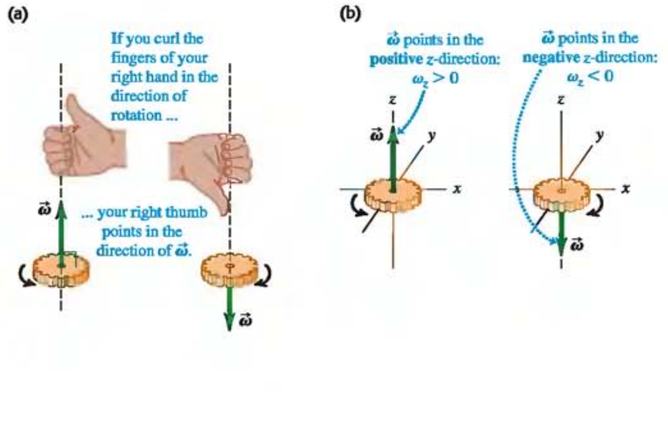
At time $t = 5.0$ s,

$$\omega_z = (6.0\ \text{rad/s}^3)(5.0\ \text{s})^2 = 150\ \text{rad/s}$$

EVALUATE: Our result in part (d) shows that ω_z is proportional to t^2 and hence increases with time. Our numerical results are consistent with this result: The 150-rad/s instantaneous angular velocity at $t = 5.0$ s is greater than the 78-rad/s average angular velocity for the 3.0-s interval leading up to that time (from $t_1 = 2.0$ s to $t_2 = 5.0$ s).

Angular Velocity As a Vector

As we have seen, our notation for the angular velocity ω_z about the z-axis is reminiscent of the notation v_x for the ordinary velocity along the x-axis (see Section 2.2). Just as v_x is the x-component of the velocity vector $\vec{v}$, ω_z is the z-component of an angular velocity *vector* $\vec{\omega}$ directed along the axis of rotation. As Fig. 9.5a



9.5 (a) The right-hand rule for the direction of the angular velocity vector $\vec{\omega}$. Reversing the direction of rotation reverses the direction of $\vec{\omega}$. (b) The sign of ω_z for rotation along the z-axis.

shows, the direction of $\vec{\omega}$ is given by the right-hand rule that we used to define the vector product in Section 1.10. If the rotation is about the z-axis, then $\vec{\omega}$ has only a z-component; this component is positive if $\vec{\omega}$ is along the positive z-axis and negative if $\vec{\omega}$ is along the negative z-axis (Fig. 9.5b).

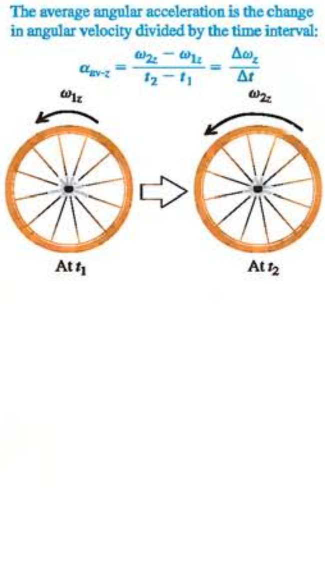
The vector formulation is especially useful in situations in which the direction of the rotation axis *changes.* We'll examine such situations briefly at the end of Chapter 10. In this chapter, however, we'll consider only situations in which the rotation axis is fixed. Hence throughout this chapter we'll use "angular velocity" to refer to ω_z, the component of the angular velocity vector $\vec{\omega}$ along the axis.

Angular Acceleration

When the angular velocity of a rigid body changes, it has an *angular acceleration.* When you pedal your bicycle harder to make the wheels turn faster or apply the brakes to bring the wheels to a stop, you're giving the wheels an angular acceleration. You also impart an angular acceleration whenever you change the rotation speed of a piece of spinning machinery such as an automobile engine's crankshaft.

If ω_{1z} and ω_{2z} are the instantaneous angular velocities at times t_1 and t_2, we define the **average angular acceleration** $\alpha_{\text{av-}z}$ over the interval $\Delta t = t_2 - t_1$ as the change in angular velocity divided by Δt (Fig. 9.6):

$$\alpha_{\text{av-}z} = \frac{\omega_{2z} - \omega_{1z}}{t_2 - t_1} = \frac{\Delta\omega_z}{\Delta t} \tag{9.4}$$



9.6 Calculating the average angular acceleration of a rotating rigid body.

The **instantaneous angular acceleration** α_z is the limit of $\alpha_{\text{av-}z}$ as $\Delta t \to 0$:

$$\alpha_z = \lim_{\Delta t \to 0} \frac{\Delta\omega_z}{\Delta t} = \frac{d\omega_z}{dt} \quad \text{(definition of angular acceleration)} \tag{9.5}$$

The usual unit of angular acceleration is the radian per second per second, or rad/s^2. From now on we will use the term "angular acceleration" to mean the instantaneous angular acceleration rather than the average angular acceleration.

Because $\omega_z = d\theta/dt$, we can also express angular acceleration as the second derivative of the angular coordinate:

$$\alpha_z = \frac{d}{dt}\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} \tag{9.6}$$

You have probably noticed that we are using Greek letters for angular kinematic quantities: θ for angular position, ω_z for angular velocity, and α_z for angular acceleration. These are analogous to x for position, v_x for velocity, and a_x for acceleration, respectively, in straight-line motion. In each case, velocity is the rate of change of position with respect to time and acceleration is the rate of change of velocity with respect to time. We will sometimes use the terms "*linear* velocity" and "*linear* acceleration" for the familiar quantities we defined in Chapters 2 and 3 to distinguish clearly between these and the *angular* quantities introduced in this chapter.

In rotational motion, if the angular acceleration α_z is positive, then the angular velocity ω_z is increasing; if α_z is negative, then ω_z is decreasing. The rotation is speeding up if α_z and ω_z have the same sign and slowing down if α_z and ω_z have opposite signs. (These are exactly the same relationships as those between *linear* acceleration a_x and *linear* velocity v_x for straight-line motion; see Section 2.3.)

Example 9.2 Calculating angular acceleration

In Example 9.1 we found that the instantaneous angular velocity ω_z of the flywheel at any time t is given by

$$\omega_z = (6.0\ \text{rad/s}^3)t^2$$

(a) Find the average angular acceleration between $t_1 = 2.0$ s and $t_2 = 5.0$ s. (b) Find the instantaneous angular acceleration at time $t_2 = 5.0$ s.

SOLUTION

IDENTIFY: This example uses the definitions of average angular acceleration $\alpha_{\text{av-}z}$ and instantaneous angular acceleration α_z.

SET UP: We'll use Eqs. (9.4) and (9.5) to find the value of $\alpha_{\text{av-}z}$ between t_1 and t_2 and the value of α_z at $t = t_2$.

EXECUTE: (a) The values of ω_z at the two times are

$$\omega_{1z} = (6.0\ \text{rad/s}^3)(2.0\ \text{s})^2 = 24\ \text{rad/s}$$
$$\omega_{2z} = (6.0\ \text{rad/s}^3)(5.0\ \text{s})^2 = 150\ \text{rad/s}$$

From Eq. (9.4) the average angular acceleration is

$$\alpha_{\text{av-}z} = \frac{150\ \text{rad/s} - 24\ \text{rad/s}}{5.0\ \text{s} - 2.0\ \text{s}} = 42\ \text{rad/s}^2$$

(b) From Eq. (9.5) the instantaneous angular acceleration at any time t is

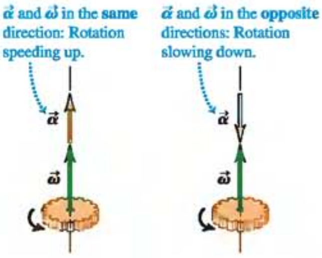
$$\alpha_z = \frac{d\omega_z}{dt} = \frac{d}{dt}[(6.0\ \text{rad/s}^3)(t^2)] = (6.0\ \text{rad/s}^3)(2t)$$
$$= (12\ \text{rad/s}^3)t$$

At time $t = 5.0$ s,

$$\alpha_z = (12\ \text{rad/s}^3)(5.0\ \text{s}) = 60\ \text{rad/s}^2$$

EVALUATE: Note that the angular acceleration is *not* constant in this situation. The angular velocity ω_z is always increasing because α_z is always positive. Furthermore, the rate at which the angular velocity increases is itself increasing, since α_z increases with time.

9.7 When the rotation axis is fixed, the angular acceleration and angular velocity vectors both lie along that axis.

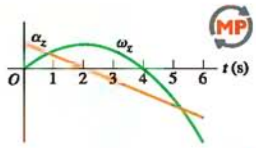


Angular Acceleration As a Vector

Just as we did for angular velocity, it's useful to define an angular acceleration *vector* $\vec{\alpha}$. Mathematically, $\vec{\alpha}$ is the time derivative of the angular velocity vector $\vec{\omega}$. If the object rotates around the fixed z-axis, then $\vec{\alpha}$ has only a z-component; the quantity α_z is just that component. In this case, $\vec{\alpha}$ is in the same direction as $\vec{\omega}$ if the rotation is speeding up and opposite to $\vec{\omega}$ if the rotation is slowing down (Fig. 9.7).

The angular acceleration vector will be particularly useful in Chapter 10 when we discuss what happens when the rotation axis can change direction. In this chapter, however, the rotation axis will always be fixed and we need use only the z-component α_z.

Test Your Understanding of Section 9.1
The figure shows a graph of ω_z and α_z versus time for a particular rotating body. (a) During which time intervals is the rotation speeding up? (i) $0 < t < 2$ s; (ii) $2\ \text{s} < t < 4$ s; (iii) $4\ \text{s} < t < 6$ s. (b) During which time intervals is the rotation slowing down? (i) $0 < t < 2$ s; (ii) $2\ \text{s} < t < 4$ s; (iii) $4\ \text{s} < 5 < 6$ s.



9.2 Rotation with Constant Angular Acceleration

In Chapter 2 we found that straight-line motion is particularly simple when the acceleration is constant. This is also true of rotational motion about a fixed axis. When the angular acceleration is constant, we can derive equations for angular velocity and angular position using exactly the same procedure that we used for straight-line motion in Section 2.4. In fact, the equations we are about to derive are identical to Eqs. (2.8), (2.12), (2.13), and (2.14) if we replace x with θ, v_x with ω_z, and a_x with α_z. We suggest that you review Section 2.4 before continuing.

Let ω_{0z} be the angular velocity of a rigid body at time $t = 0$, and let ω_z be its angular velocity at any later time t. The angular acceleration α_z is constant and equal to the average value for any interval. Using Eq. (9.4) with the interval from 0 to t, we find

$$\alpha_z = \frac{\omega_z - \omega_{0z}}{t - 0} \quad \text{or}$$

$$\omega_z = \omega_{0z} + \alpha_z t \quad \text{(constant angular acceleration only)} \quad (9.7)$$

The product $\alpha_z t$ is the total change in ω_z between $t = 0$ and the later time t; the angular velocity ω_z at time t is the sum of the initial value ω_{0z} and this total change.

With constant angular acceleration, the angular velocity changes at a uniform rate, so its average value between 0 and t is the average of the initial and final values:

$$\omega_{\text{av-}z} = \frac{\omega_{0z} + \omega_z}{2} \quad (9.8)$$

We also know that $\omega_{\text{av-}z}$ is the total angular displacement $(\theta - \theta_0)$ divided by the time interval $(t - 0)$:

$$\omega_{\text{av-}z} = \frac{\theta - \theta_0}{t - 0} \quad (9.9)$$

When we equate Eqs. (9.8) and (9.9) and multiply the result by t, we get

$$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t \quad \text{(constant angular acceleration only)} \quad (9.10)$$

To obtain a relationship between θ and t that doesn't contain ω_z, we substitute Eq. (9.7) into Eq. (9.10):

$$\theta - \theta_0 = \frac{1}{2}[\omega_{0z} + (\omega_{0z} + \alpha_z t)]t \quad \text{or}$$

$$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2 \quad \text{(constant angular acceleration only)} \quad (9.11)$$

That is, if at the initial time $t = 0$ the body is at angular position θ_0 and has angular velocity ω_{0z}, then its angular position θ at any later time t is the sum of three terms: its initial angular position θ_0, plus the rotation $\omega_{0z}t$ it would have if the angular velocity were constant, plus an additional rotation $\frac{1}{2}\alpha_z t^2$ caused by the changing angular velocity.

Following the same procedure as for straight-line motion in Section 2.4, we can combine Eqs. (9.7) and (9.11) to obtain a relationship between θ and ω_z that does not contain t. We invite you to work out the details, following the same procedure we used to get Eq. (2.13). (See Exercise 9.12.) In fact, because of the perfect analogy between straight-line and rotational quantities, we can simply take Eq. (2.13) and replace each straight-line quantity by its rotational analog. We get

$$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0) \quad \text{(constant angular acceleration only)} \quad (9.12)$$

CAUTION **Constant angular acceleration** Keep in mind that all of these results are valid *only* when the angular acceleration α_z is *constant;* be careful not to try to apply them to problems in which α_z is *not* constant. Table 9.1 shows the analogy between Eqs. (9.7), (9.10), (9.11), and (9.12) for fixed-axis rotation with constant angular acceleration and the corresponding equations for straight-line motion with constant linear acceleration.

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7.7 Rotational Kinematics

Table 9.1 Comparison of Linear and Angular Motion with Constant Acceleration

Straight-Line Motion with Constant Linear Acceleration	Fixed-Axis Rotation with Constant Angular Acceleration
$a_x = \text{constant}$	$\alpha_z = \text{constant}$
$v_x = v_{0x} + a_x t$	$\omega_z = \omega_{0z} + \alpha_z t$
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_x + v_{0x})t$	$\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$

Example 9.3 Rotation with constant angular acceleration

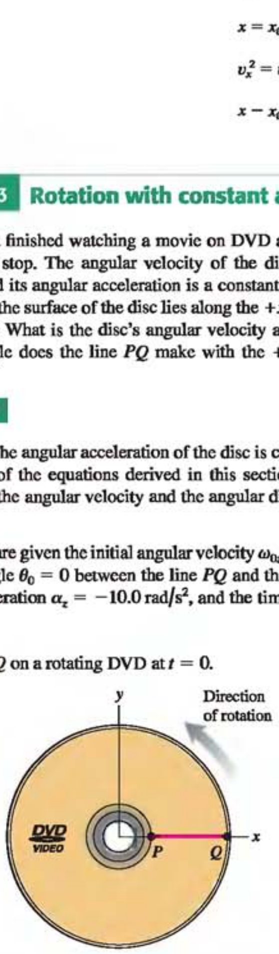
You have just finished watching a movie on DVD and the disc is slowing to a stop. The angular velocity of the disc at $t = 0$ is 27.5 rad/s and its angular acceleration is a constant $-10.0\ \text{rad/s}^2$. A line PQ on the surface of the disc lies along the $+x$-axis at $t = 0$ (Fig. 9.8). (a) What is the disc's angular velocity at $t = 0.300$ s? (b) What angle does the line PQ make with the $+x$-axis at this time?

SOLUTION

IDENTIFY: The angular acceleration of the disc is constant, so we can use any of the equations derived in this section. Our target variables are the angular velocity and the angular displacement at $t = 0.300$ s.

SET UP: We are given the initial angular velocity $\omega_{0z} = 27.5$ rad/s, the initial angle $\theta_0 = 0$ between the line PQ and the $+x$-axis, the angular acceleration $\alpha_z = -10.0\ \text{rad/s}^2$, and the time $t = 0.300$ s.

9.8 A line PQ on a rotating DVD at $t = 0$.



With this information it's easiest to use Eqs. (9.7) and (9.11) to find the target variables ω_z and θ, respectively.

EXECUTE: (a) From Eq. (9.7), at $t = 0.300$ s we have

$$\omega_z = \omega_{0z} + \alpha_z t = 27.5\ \text{rad/s} + (-10.0\ \text{rad/s}^2)(0.300\ \text{s}) = 24.5\ \text{rad/s}$$

(b) From Eq. (9.11),

$$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$$

$$= 0 + (27.5\ \text{rad/s})(0.300\ \text{s}) + \frac{1}{2}(-10.0\ \text{rad/s}^2)(0.300\ \text{s})^2$$

$$= 7.80\ \text{rad} = 7.80\ \text{rad}\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 1.24\ \text{rev}$$

The DVD has turned through one complete revolution plus an additional 0.24 revolution—that is, through an additional angle of $(0.24\ \text{rev})(360°/\text{rev}) = 87°$. Hence the line PQ is at an angle of 87° with the $+x$-axis.

EVALUATE: Our answer to part (a) tells us that the angular velocity has decreased. This is as it should be, since α_z is negative. We can also use our answer for ω_z in part (a) to check our result for θ in part (b). To do so, we solve Eq. (9.12), $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, for the angle θ:

$$\theta = \theta_0 + \left(\frac{\omega_z^2 - \omega_{0z}^2}{2\alpha_z}\right)$$

$$= 0 + \frac{(24.5\ \text{rad/s})^2 - (27.5\ \text{rad/s})^2}{2(-10.0\ \text{rad/s}^2)} = 7.80\ \text{rad}$$

which agrees with the result we found earlier.

Test Your Understanding of Section 9.2 Suppose the DVD in Example 9.3 was initially spinning at twice the rate (55.0 rad/s rather than 27.5 rad/s) and slowed down at twice the rate ($-20.0\ \text{rad/s}^2$ rather than $-10.0\ \text{rad/s}^2$). (a) Compared to the situation in Example 9.3, how long would it take the DVD to come to a stop? (i) the same amount of time; (ii) twice as much time; (iii) 4 times as much time; (iv) $\frac{1}{2}$ as much time; (v) $\frac{1}{4}$ as much time. (b) Compared to the situation in Example 9.3, through how many revolutions would the DVD rotate before coming to a stop? (i) the same number of revolutions; (ii) twice as many revolutions; (iii) 4 times as many revolutions; (iv) $\frac{1}{2}$ as many revolutions; (v) $\frac{1}{4}$ as many revolutions.

9.3 Relating Linear and Angular Kinematics

How do we find the linear speed and acceleration of a particular point in a rotating rigid body? We need to answer this question to proceed with our study of rotation. For example, to find the kinetic energy of a rotating body, we have to start from $K = \frac{1}{2}mv^2$ for a particle, and this requires knowing the speed v for each particle in the body. So it's worthwhile to develop general relationships between the *angular* speed and acceleration of a rigid body rotating about a fixed axis and the *linear* speed and acceleration of a specific point or particle in the body.

Linear Speed in Rigid-Body Rotation

When a rigid body rotates about a fixed axis, every particle in the body moves in a circular path. The circle lies in a plane perpendicular to the axis and is centered on the axis. The speed of a particle is directly proportional to the body's angular velocity; the faster the body rotates, the greater the speed of each particle. In Fig. 9.9, point P is a constant distance r from the axis of rotation, so it moves in a circle of radius r. At any time, the angle θ (in radians) and the arc length s are related by

$$s = r\theta$$

We take the time derivative of this, noting that r is constant for any specific particle, and take the absolute value of both sides:

$$\left|\frac{ds}{dt}\right| = r\left|\frac{d\theta}{dt}\right|$$

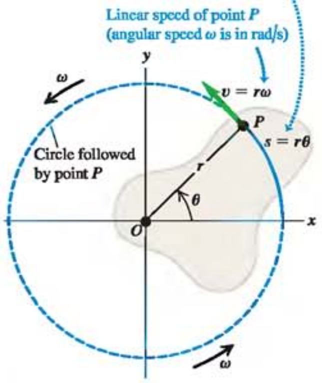
Now $|ds/dt|$ is the absolute value of the rate of change of arc length, which is equal to the instantaneous *linear* speed v of the particle. Analogously, $|d\theta/dt|$, the absolute value of the rate of change of the angle, is the instantaneous **angular speed** ω—that is, the magnitude of the instantaneous angular velocity in rad/s. Thus

$$v = r\omega \quad \text{(relationship between linear and angular speeds)} \quad (9.13)$$

The farther a point is from the axis, the greater its linear speed. The *direction* of the linear velocity *vector* is tangent to its circular path at each point (Fig. 9.9).

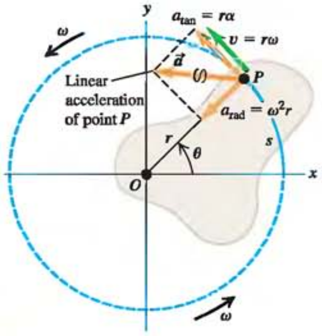
CAUTION **Speed vs. velocity** Keep in mind the distinction between the linear and angular *speeds* v and ω, which appear in Eq. (9.13), and the linear and angular *velocities* v_x and ω_z. The quantities without subscripts, v and ω, are never negative; they are the magnitudes of the vectors $\vec{v}$ and $\vec{\omega}$, respectively, and their values tell you only how fast a particle is moving (v) or how fast a body is rotating (ω). The corresponding quantities with subscripts, v_x and ω_z, can be either positive or negative; their signs tell you the direction of the motion.

9.9 A rigid body rotating about a fixed axis through point O.



9.10 A rigid body whose rotation is speeding up. The acceleration of point P has a component a_{rad} toward the rotation axis (perpendicular to $\vec{v}$) and a component a_{tan} along the circle that point P follows (parallel to $\vec{v}$).

Radial and tangential acceleration components:
- $a_{rad} = \omega^2 r$ is point P's centripetal acceleration.
- $a_{tan} = r\alpha$ means that P's rotation is speeding up (the body has angular acceleration).



Linear Acceleration in Rigid-Body Rotation

We can represent the acceleration of a particle moving in a circle in terms of its centripetal and tangential components, a_{rad} and a_{tan} (Fig. 9.10), as we did in Section 3.4. It would be a good idea to review that section now. We found that the **tangential component of acceleration** a_{tan}, the component parallel to the instantaneous velocity, acts to change the *magnitude* of the particle's velocity (i.e., the speed) and is equal to the rate of change of speed. Taking the derivative of Eq. (9.13), we find

$$a_{tan} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha \quad \begin{array}{l}\text{(tangential acceleration of}\\ \text{a point on a rotating body)}\end{array} \quad (9.14)$$

This component of a particle's acceleration is always tangent to the circular path of the particle.

The quantity $\alpha = d\omega/dt$ in Eq. (9.14) is the rate of change of the angular *speed*. It is not quite the same as $\alpha_z = d\omega_z/dt$, which is the rate of change of the angular *velocity*. For example, consider a body rotating so that its angular velocity vector points in the $-z$-direction (Fig. 9.5b). If the body is gaining angular speed at a rate of 10 rad/s per second, then $\alpha = 10\ \text{rad/s}^2$. But ω_z is negative and becoming more negative as the rotation gains speed, so $\alpha_z = -10\ \text{rad/s}^2$. The rule for rotation about a fixed axis is that α is equal to α_z if ω_z is positive but equal to $-\alpha_z$ if ω_z is negative.

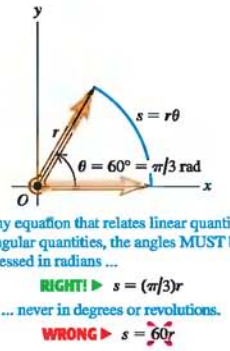
The component of the particle's acceleration directed toward the rotation axis, the **centripetal component of acceleration** a_{rad}, is associated with the change of *direction* of the particle's velocity. In Section 3.4 we worked out the relationship $a_{rad} = v^2/r$. We can express this in terms of ω by using Eq. (9.13):

$$a_{rad} = \frac{v^2}{r} = \omega^2 r \quad \text{(centripetal acceleration of a point on a rotating body)} \qquad (9.15)$$

This is true at each instant, *even when ω and v are not constant*. The centripetal component always points toward the axis of rotation.

The vector sum of the centripetal and tangential components of acceleration of a particle in a rotating body is the linear acceleration $\vec{a}$ (Fig. 9.10).

9.11 Always use radians when relating linear and angular quantities.



CAUTION **Use angles in radians in all equations** It's important to remember that Eq. (9.1), $s = r\theta$, is valid *only* when θ is measured in radians. The same is true of any equation derived from this, including Eqs. (9.13), (9.14), and (9.15). When you use these equations, you *must* express the angular quantities in radians, not revolutions or degrees (Fig. 9.11).

Equations (9.1), (9.13), and (9.14) also apply to any particle that has the same tangential velocity as a point in a rotating rigid body. For example, when a rope wound around a circular cylinder unwraps without stretching or slipping, its speed and acceleration at any instant are equal to the speed and tangential acceleration of the point at which it is tangent to the cylinder. The same principle holds for situations such as bicycle chains and sprockets, belts and pulleys that turn without slipping, and so on. We will have several opportunities to use these relationships later in this chapter and in Chapter 10. Note that Eq. (9.15) for the centripetal component a_{rad} is applicable to the rope or chain *only* at points that are in contact with the cylinder or sprocket. Other points do not have the same acceleration toward the center of the circle that points on the cylinder or sprocket have.

Example 9.4 Throwing a discus

A discus thrower moves the discus in a circle of radius 80.0 cm. At a certain instant, the thrower is spinning at an angular speed of 10.0 rad/s and the angular speed is increasing at 50.0 rad/s². At this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

SOLUTION

IDENTIFY: We model the discus as a particle traveling on a circular path (Fig. 9.12a), so we can use the ideas developed in this section.

SET UP: We are given the radius $r = 0.800$ m, the angular speed $\omega = 10.0$ rad/s, and the rate of change of angular speed $\alpha = 50.0\ \text{rad/s}^2$. (Fig. 9.12b).The first two target variables are the acceleration components a_{tan} and a_{rad}, which we'll find with Eqs. (9.14) and (9.15), respectively. Given these components of the acceleration vector, we'll find its magnitude a (the third target variable) using the Pythagorean theorem.

EXECUTE: From Eqs. (9.14) and (9.15),

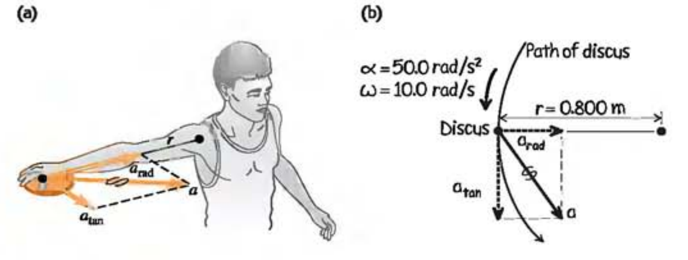
$$a_{tan} = r\alpha = (0.800\ \text{m})(50.0\ \text{rad/s}^2) = 40.0\ \text{m/s}^2$$

$$a_{rad} = \omega^2 r = (10.0\ \text{rad/s})^2(0.800\ \text{m}) = 80.0\ \text{m/s}^2$$

The magnitude of the acceleration vector is

$$a = \sqrt{a_{tan}^2 + a_{rad}^2} = 89.4\ \text{m/s}^2$$

9.12 (a) Whirling a discus in a circle. (b) Our sketch showing the acceleration components for the discus.



EVALUATE: Note that we dropped the unit "radian" from our results for a_{tan}, a_{rad}, and a. We can do this because "radian" is a dimensionless quantity.

The magnitude a is about nine times g, the acceleration due to gravity. Can you show that if the angular speed doubles to 20.0 rad/s while α remains the same, the acceleration magnitude a increases to 322 m/s^2, or almost 33g?

Example 9.5 Designing a propeller

You are asked to design an airplane propeller to turn at 2400 rpm. The forward airspeed of the plane is to be 75.0 m/s (270 km/h, or about 168 mi/h), and the speed of the tips of the propeller blades through the air must not exceed 270 m/s (Fig. 9.13a). (This is about 0.80 times the speed of sound in air. If the propeller tips were to move too close to the speed of sound, they would produce a tremendous amount of noise.) (a) What is the maximum radius the propeller can have? (b) With this radius, what is the acceleration of the propeller tip?

SOLUTION

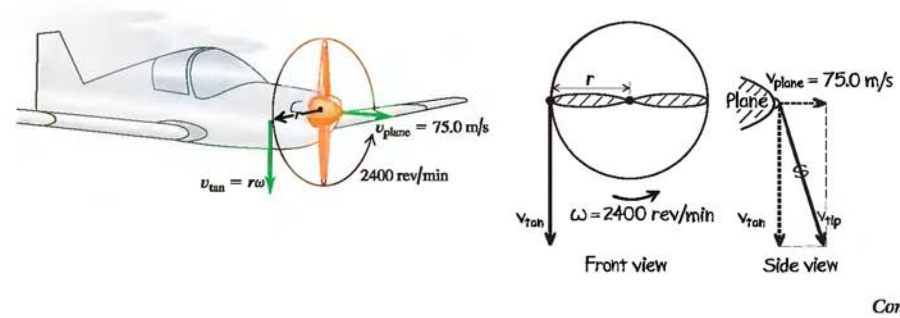
IDENTIFY: The object of interest in this example is a particle at the tip of the propeller; our target variables are the particle's distance from the axis and its acceleration. Note that the speed of this particle through the air (which cannot exceed 270 m/s) is due to both the propeller's rotation *and* the forward motion of the airplane.

SET UP: As Fig. 9.13b shows, the velocity $\vec{v}_{tip}$ of a particle at the propeller tip is the vector sum of its tangential velocity due to the propeller's rotation (magnitude v_{tan}, given by Eq. (9.13)) and the forward velocity of the airplane (magnitude $v_{plane} = 75.0$ m/s). The rotation plane of the propeller is perpendicular to the direction of flight, so these two vectors are perpendicular and we can use the Pythagorean theorem to relate v_{tan} and v_{plane} to v_{tip}. We will then set $v_{tip} = 270$ m/s and solve for the radius r. Note that the angular speed of the propeller is constant, so the acceleration of the propeller tip has only a radial component; we'll find it using Eq. (9.15).

EXECUTE: We first convert ω to rad/s (see Fig. 9.11):

$$\omega = 2400 \text{ rpm} = \left(2400 \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$
$$= 251 \text{ rad/s}$$

9.13 (a) A propeller-driven airplane in flight. (b) Our sketch showing the velocity components for the propeller tip.



Continued

(a) From Fig. 9.13b and Eq. (9.13), the velocity magnitude v_{total} is given by

$$v_{tip}^2 = v_{plane}^2 + v_{tan}^2 = v_{plane}^2 + r^2\omega^2 \quad \text{so}$$

$$r^2 = \frac{v_{tip}^2 - v_{plane}^2}{\omega^2} \quad \text{and} \quad r = \frac{\sqrt{v_{tip}^2 - v_{plane}^2}}{\omega}$$

If $v_{tip} = 270$ m/s, the propeller radius is

$$r = \frac{\sqrt{(270\text{ m/s})^2 - (75.0\text{ m/s})^2}}{251\text{ rad/s}} = 1.03\text{ m}$$

(b) The centripetal acceleration is

$$a_{rad} = \omega^2 r$$
$$= (251\text{ rad/s})^2(1.03\text{ m}) = 6.5 \times 10^4\text{ m/s}^2$$

The *tangential* acceleration is zero because the angular speed is constant.

EVALUATE: From $\sum\vec{F} = m\vec{a}$, the propeller must exert a force of 6.5×10^4 N on each kilogram of material at its tip! This is why propellers are made out of tough material, usually aluminum alloy.

Conceptual Example 9.6 Bicycle gears

How are the angular speeds of the two bicycle sprockets in Fig. 9.14 related to the number of teeth on each sprocket?

SOLUTION

The chain does not slip or stretch, so it moves at the same tangential speed v on both sprockets. From Eq. (9.13),

$$v = r_{front}\omega_{front} = r_{rear}\omega_{rear} \quad \text{so} \quad \frac{\omega_{rear}}{\omega_{front}} = \frac{r_{front}}{r_{rear}}$$

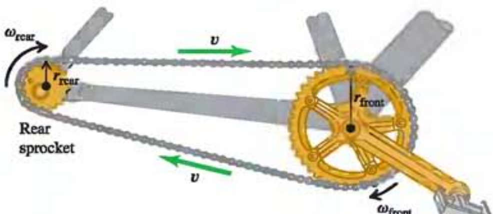
The angular speed is inversely proportional to the radius. This relationship also holds for pulleys connected by a belt, provided the belt doesn't slip. For chain sprockets the teeth must be equally spaced on the circumferences of both sprockets for the chain to mesh properly with both. Let N_{front} and N_{rear} be the numbers of teeth; the condition that the tooth spacing is the same on both sprockets is

$$\frac{2\pi r_{front}}{N_{front}} = \frac{2\pi r_{rear}}{N_{rear}} \quad \text{or} \quad \frac{r_{front}}{r_{rear}} = \frac{N_{front}}{N_{rear}}$$

Combining this with the other equation, we get

$$\frac{\omega_{rear}}{\omega_{front}} = \frac{N_{front}}{N_{rear}}$$

9.14 The sprockets and chain of a bicycle.



The angular speed of each sprocket is inversely proportional to the number of teeth. On a multispeed bike, you get the highest angular speed ω_{rear} of the rear wheel for a given pedaling rate ω_{front} when the ratio N_{front}/N_{rear} is maximum; this means using the largest-radius front sprocket (largest N_{front}) and the smallest-radius rear sprocket (smallest N_{rear}).

Test Your Understanding of Section 9.3 Information is stored on a CD or DVD (see Fig. 9.8) in a coded pattern of tiny pits. The pits are arranged in a track that spirals outward toward the rim of the disc. As the disc spins inside a player, the track is scanned at a constant *linear* speed. How must the rotation speed of the disc change as the player's scanning head moves over the track? (i) The rotation speed must increase. (ii) The rotation speed must decrease. (iii) The rotation speed must stay the same.

9.4 Energy in Rotational Motion

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7.7 Rotational Inertia

A rotating rigid body consists of mass in motion, so it has kinetic energy. As we will see, we can express this kinetic energy in terms of the body's angular speed and a new quantity, called *moment of inertia,* that depends on the body's mass and how the mass is distributed.

To begin, we think of a body as being made up of a large number of particles, with masses $m_1, m_2, \ldots$ at distances $r_1, r_2, \ldots$ from the axis of rotation. We label the particles with the index i: The mass of the ith particle is m_i and its distance from the axis of rotation is r_i. The particles don't necessarily all lie in the same plane, so we specify that r_i is the *perpendicular* distance from the axis to the ith particle.

When a rigid body rotates about a fixed axis, the speed v_i of the ith particle is given by Eq. (9.13), $v_i = r_i\omega$, where ω is the body's angular speed. Different particles have different values of r, but ω is the same for all (otherwise, the body wouldn't be rigid). The kinetic energy of the ith particle can be expressed as

$$\frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i r_i^2\omega^2$$

The *total* kinetic energy of the body is the sum of the kinetic energies of all its particles:

$$K = \frac{1}{2}m_1 r_1^2\omega^2 + \frac{1}{2}m_2 r_2^2\omega^2 + \cdots = \sum_i \frac{1}{2}m_i r_i^2\omega^2$$

Taking the common factor $\omega^2/2$ out of this expression, we get

$$K = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + \cdots)\omega^2 = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

The quantity in parentheses, obtained by multiplying the mass of each particle by the square of its distance from the axis of rotation and adding these products, is denoted by I and is called the **moment of inertia** of the body for this rotation axis:

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots = \sum_i m_i r_i^2 \quad \text{(definition of moment of inertia)} \quad (9.16)$$

The word "moment" means that I depends on how the body's mass is distributed in space; it has nothing to do with a "moment" of time. For a body with a given rotation axis and a given total mass, the greater the distance from the axis to the particles that make up the body, the greater the moment of inertia. In a rigid body, the distances r_i are all constant and I is independent of how the body rotates around the given axis. The SI unit of moment of inertia is the kilogram-meter^2 $(\text{kg}\cdot\text{m}^2)$.

In terms of moment of inertia I, the **rotational kinetic energy** K of a rigid body is

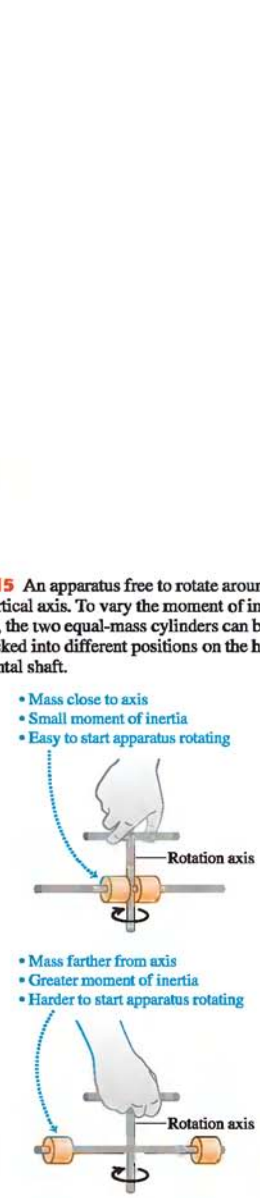
$$K = \frac{1}{2}I\omega^2 \quad \text{(rotational kinetic energy of a rigid body)} \quad (9.17)$$

The kinetic energy given by Eq. (9.17) is *not* a new form of energy; it's simply the sum of the kinetic energies of the individual particles that make up the rotating rigid body. To use Eq. (9.17), ω *must* be measured in radians per second, not revolutions or degrees per second, to give K in joules. That's because we used $v_i = r_i\omega$ in our derivation.

Equation (9.17) gives a simple physical interpretation of moment of inertia: *The greater the moment of inertia, the greater the kinetic energy of a rigid body rotating with a given angular speed* ω. We learned in Chapter 6 that the kinetic energy of a body equals the amount of work done to accelerate that body from rest. So the greater a body's moment of inertia, the harder it is to start the body rotating if it's at rest and the harder it is to stop its rotation if it's already rotating (Fig. 9.15). For this reason, I is also called the *rotational inertia*.

The next example shows how *changing* the rotation axis can affect the value of I.

9.15 An apparatus free to rotate around a vertical axis. To vary the moment of inertia, the two equal-mass cylinders can be locked into different positions on the horizontal shaft.



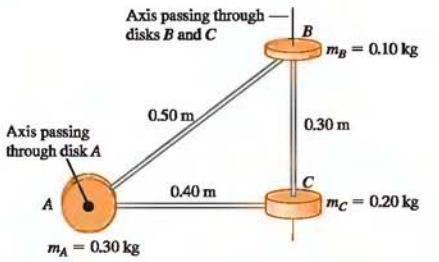
Example 9.7 Moments of inertia for different rotation axes

An engineer is designing a machine part consisting of three heavy disks linked by lightweight struts (Fig. 9.16). (a) What is the moment of inertia of this body about an axis through the center of disk A, perpendicular to the plane of the diagram? (b) What is the moment of inertia about an axis through the centers of disks B and C? (c) If the body rotates about an axis through A perpendicular to the plane of the diagram, with angular speed $\omega = 4.0\ \text{rad/s}$, what is its kinetic energy?

SOLUTION

IDENTIFY: We'll consider the disks as massive particles and the lightweight struts as massless rods. Then we can use the ideas of this section to calculate the moment of inertia of this collection of three particles.

9.16 An oddly shaped machine part.



SET UP: In parts (a) and (b), we'll use Eq. (9.16) to find the moments of inertia for each of the two axes. Given the moment of inertia for axis A, we'll use Eq. (9.17) in part (c) to find the rotational kinetic energy.

EXECUTE: (a) The particle at point A lies *on* the axis. Its distance r from the axis is zero, so it contributes nothing to the moment of inertia. Equation (9.16) gives

$$I = \sum m_i r_i^2 = (0.10\ \text{kg})(0.50\ \text{m})^2 + (0.20\ \text{kg})(0.40\ \text{m})^2$$
$$= 0.057\ \text{kg}\cdot\text{m}^2$$

(b) The particles at B and C both lie *on* the axis, so for them $r = 0$ and neither contributes to the moment of inertia. Only A contributes, and we have

$$I = \sum m_i r_i^2 = (0.30\ \text{kg})(0.40\ \text{m})^2 = 0.048\ \text{kg}\cdot\text{m}^2$$

(c) From Eq. (9.17),

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.057\ \text{kg}\cdot\text{m}^2)(4.0\ \text{rad/s})^2 = 0.46\ \text{J}$$

EVALUATE: Our results show that the moment of inertia for the axis through A is greater than that for the axis through B and C. Hence, of the two axes, it's easier to make the machine part rotate about the axis through B and C.

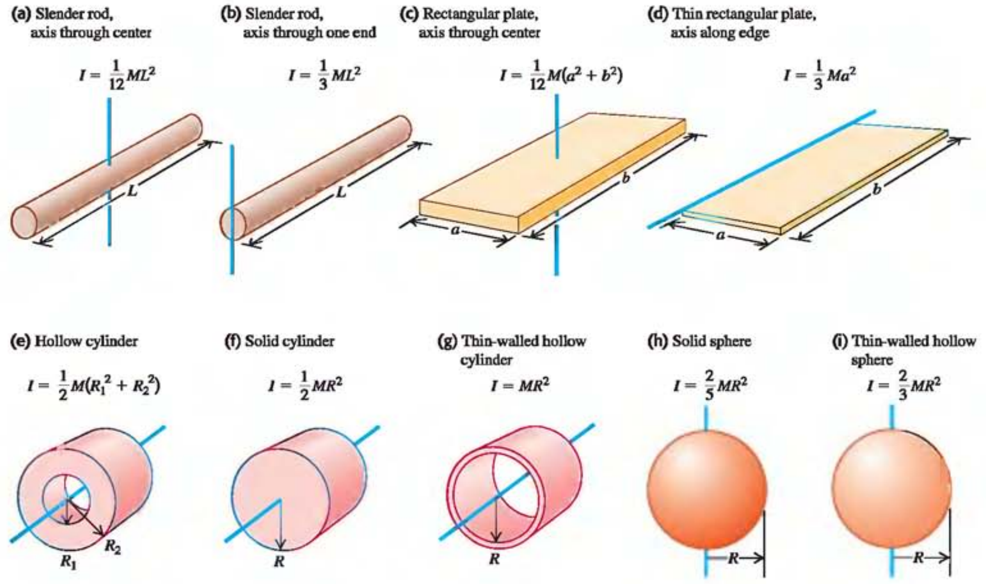
CAUTION **Moment of inertia depends on the choice of axis** The results of parts (a) and (b) of Example 9.7 show that the moment of inertia of a body depends on the location and orientation of the axis. It's not enough to just say, "The moment of inertia of this body is $0.048\ \text{kg}\cdot\text{m}^2$." We have to be specific and say, "The moment of inertia of this body *about the axis through B and C* is $0.048\ \text{kg}\cdot\text{m}^2$."

In Example 9.7 we represented the body as several point masses, and we evaluated the sum in Eq. (9.16) directly. When the body is a *continuous* distribution of matter, such as a solid cylinder or plate, the sum becomes an integral, and we need to use calculus to calculate the moment of inertia. We will give several examples of such calculations in Section 9.6; meanwhile, Table 9.2 gives moments of inertia for several familiar shapes in terms of their masses and dimensions. Each body shown in Table 9.2 is *uniform;* that is, the density has the same value at all points within the solid parts of the body.

CAUTION **Computing the moment of inertia** You may be tempted to try to compute the moment of inertia of a body by assuming that all the mass is concentrated at the center of mass and multiplying the total mass by the square of the distance from the center of mass to the axis. Resist that temptation; it doesn't work! For example, when a uniform thin rod of length L and mass M is pivoted about an axis through one end, perpendicular to the rod, the moment of inertia is $I = ML^2/3$ [case (b) in Table 9.2]. If we took the mass as concentrated at the center, a distance $L/2$ from the axis, we would obtain the *incorrect* result $I = M(L/2)^2 = ML^2/4$.

Now that we know how to calculate the kinetic energy of a rotating rigid body, we can apply the energy principles of Chapter 7 to rotational motion. Here are some points of strategy and some examples.

Table 9.2 Moments of Inertia of Various Bodies



Problem-Solving Strategy 9.1 Rotational Energy

IDENTIFY *the relevant concepts:* You can use work–energy relationships and conservation of energy to find relationships involving position and motion of a rigid body rotating around a fixed axis. As we saw in Chapter 7, the energy method is usually not helpful for problems that involve elapsed time. In Chapter 10 we'll see how to approach rotational problems of this kind.

SET UP *the problem* using the same steps as in Problem-Solving Strategy 7.1 (Section 7.1), with the following addition:

5. Many problems involve a rope or cable wrapped around a rotating rigid body, which functions as a pulley. In these situations, remember that the point on the pulley that contacts the rope has the same linear speed as the rope, provided the rope doesn't slip on the pulley. You can then take advantage of Eqs. (9.13) and (9.14), which relate the linear speed and tangential acceleration of a point on a rigid body to the angular velocity and angular acceleration of the body. Examples 9.8 and 9.9 illustrate this point.

EXECUTE *the solution:* As in Chapter 7, write expressions for the initial and final kinetic and potential energies (K_1, K_2, U_1, and U_2) and the nonconservative work W_{other} (if any). The new feature is rotational kinetic energy, which is expressed in terms of the body's moment of inertia I for the given axis and its angular speed ω $\left(K = \frac{1}{2}I\omega^2\right)$ instead of its mass m and speed v. Substitute these expressions into $K_1 + U_1 + W_{other} = K_2 + U_2$ (if nonconservative work is done) or $K_1 + U_1 = K_2 + U_2$ (if only conservative work is done) and solve for the target variable(s). As in Chapter 7, it's helpful to draw bar graphs showing the initial and final values of K, U, and $E = K + U$.

EVALUATE *your answer:* As always, check whether your answer makes physical sense.

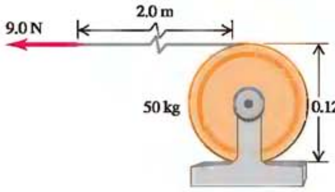
Example 9.8 An unwinding cable I

A light, flexible, nonstretching cable is wrapped several times around a winch drum, a solid cylinder of mass 50 kg and diameter 0.120 m, which rotates about a stationary horizontal axis held by frictionless bearings (Fig. 9.17). The free end of the cable is pulled with a constant 9.0-N force for a distance of 2.0 m. It unwinds without slipping and turns the cylinder. If the cylinder is initially at rest, find its final angular speed and the final speed of the cable.

SOLUTION

IDENTIFY: We will solve this problem using energy methods. Point 1 is when the cylinder first begins to move, and point 2 is when the cable has moved 2.0 m. We'll assume that the light cable is massless, so that only the cylinder has kinetic energy. The cylinder doesn't move vertically, so there are no changes in gravitational potential energy. There is friction between the cable and the cylinder, which is what makes the cylinder rotate when the cable is pulled. But because the cable doesn't slip, there is no sliding of the cable relative to the cylinder and no mechanical energy is lost in friction. Because the cable is massless, the force that the cable exerts on the cylinder rim is equal to the applied force F.

9.17 A cable unwinds from a cylinder (side view).



SET UP: The cylinder starts at rest, so the initial kinetic energy is $K_1 = 0$. Between points 1 and 2 the force F does work on the cylinder over a distance $s = 2.0$ m. As a result, the kinetic energy at point 2 is $K_2 = \frac{1}{2}I\omega^2$. One of our target variables is ω; the other is the speed of the cable at point 2, which is equal to the tangential speed v of the cylinder at that point. We'll find v from ω by using Eq. (9.13).

EXECUTE: The work done on the cylinder is $W_{other} = Fs = (9.0\ \text{N})(2.0\ \text{m}) = 18\ \text{J}$. From Table 9.2 the moment of inertia is

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(50\ \text{kg})(0.060\ \text{m})^2 = 0.090\ \text{kg}\cdot\text{m}^2$$

(The radius R is half the diameter of the cylinder.) The relationship $K_1 + U_1 + W_{other} = K_2 + U_2$ then gives

$$0 + 0 + W_{other} = \frac{1}{2}I\omega^2 + 0$$

$$\omega = \sqrt{\frac{2W_{other}}{I}} = \sqrt{\frac{2(18\ \text{J})}{0.090\ \text{kg}\cdot\text{m}^2}} = 20\ \text{rad/s}$$

The final tangential speed of the cylinder, and hence the final speed of the cable, is

$$v = R\omega = (0.060\ \text{m})(20\ \text{rad/s}) = 1.2\ \text{m/s}$$

EVALUATE: If the mass of the cable can't be neglected, then some of the work done would go into the kinetic energy of the cable. Hence the cylinder would end up with less kinetic energy and a smaller angular speed than we calculated here.

Example 9.9 An unwinding cable II

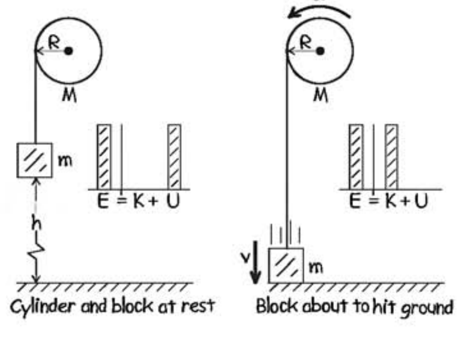
We wrap a light, flexible cable around a solid cylinder with mass M and radius R. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass m and release the object with no initial velocity at a distance h above the floor. As the block falls, the cable unwinds without stretching or slipping, turning the cylinder. Find the speed of the falling block and the angular speed of the cylinder just as the block strikes the floor.

SOLUTION

IDENTIFY: As in Example 9.8, the cable doesn't slip and friction does no work. The cable does no *net* work; at its upper end the force and displacement are in the same direction, and at its lower end they are in opposite directions. Thus the total work done by the two ends of the cable is zero. Hence only gravity does work, and so mechanical energy is conserved.

SET UP: Figure 9.18a shows the situation just before the block begins to fall. At this point the system has no kinetic energy, so

9.18 Our sketches for this problem.



$K_1 = 0$. We take the potential energy to be zero when the block is at floor level; then $U_1 = mgh$ and $U_2 = 0$. (We can ignore the gravitational potential energy for the rotating cylinder, since its height doesn't change.) Just before the block hits the floor (Fig. 9.18b), both the block and the cylinder have kinetic energy. The total kinetic energy K_2 at that instant is

$$K_2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

From Table 9.2 the moment of inertia of the cylinder is $I = \frac{1}{2}MR^2$. Also, v and ω are related by $v = R\omega$, since the speed of the falling block must be equal to the tangential speed at the outer surface of the cylinder. We'll use these relationships to solve for the target variables v and ω shown in Fig. 9.18b.

EXECUTE: We use our expressions for K_1, U_1, K_2, and U_2 and the relationship $\omega = v/R$ in the energy-conservation equation $K_1 + U_1 = K_2 + U_2$. We then solve for v:

$$0 + mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 + 0 = \frac{1}{2}\left(m + \frac{1}{2}M\right)v^2$$

$$v = \sqrt{\frac{2gh}{1 + M/2m}}$$

The final angular speed of the cylinder is $\omega = v/R$.

EVALUATE: Let's check some particular cases. When M is much larger than m, v is very small, as we would expect. When M is much smaller than m, v is nearly equal to $\sqrt{2gh}$, which is the speed of a body that falls freely from an initial height h. Does it surprise you that v doesn't depend on the radius of the cylinder?

Gravitational Potential Energy for an Extended Body

In Example 9.9 the cable was of negligible mass, so we could ignore its kinetic energy as well as the gravitational potential energy associated with it. If the mass is *not* negligible, we need to know how to calculate the *gravitational potential energy* associated with such an extended body. If the acceleration of gravity g is the same at all points on the body, the gravitational potential energy is the same as though all the mass were concentrated at the center of mass of the body. Suppose we take the y-axis vertically upward. Then for a body with total mass M, the gravitational potential energy U is simply

$$U = Mgy_{cm} \quad \text{(gravitational potential energy for an extended body)} \quad (9.18)$$

where y_{cm} is the y-coordinate of the center of mass. This expression applies to any extended body, whether it is rigid or not (Fig. 9.19).

To prove Eq. (9.18), we again represent the body as a collection of mass elements m_i. The potential energy for element m_i is $m_i g y_i$, so the total potential energy is

$$U = m_1 g y_1 + m_2 g y_2 + \cdots = (m_1 y_1 + m_2 y_2 + \cdots)g$$

But from Eq. (8.28), which defines the coordinates of the center of mass,

$$m_1 y_1 + m_2 y_2 + \cdots = (m_1 + m_2 + \cdots)y_{cm} = My_{cm}$$

where $M = m_1 + m_2 + \cdots$ is the total mass. Combining this with the above expression for U, we find $U = Mgy_{cm}$ in agreement with Eq. (9.18).

We leave the application of Eq. (9.18) to the problems. We'll make use of this relationship in Chapter 10 in the analysis of rigid-body problems in which the axis of rotation moves.

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7.12 Woman and Flywheel Elevator—Energy Approach

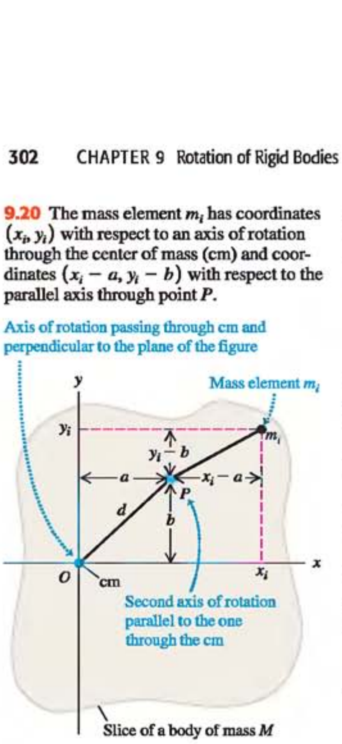
7.13 Rotoride—Energy Approach

9.19 In a technique called the "Fosbury flop" after its innovator, this athlete arches his body as he passes over the bar in the high jump. As a result, his center of mass actually passes *under* the bar. This technique requires a smaller increase in gravitational potential energy [Eq. (9.18)] than the older method of straddling the bar.

Test Your Understanding of Section 9.4 Suppose the cylinder and block in Example 9.9 have the same mass, so $m = M$. Just before the block strikes the floor, which statement is correct about the relationship between the kinetic energy of the falling block and the rotational kinetic energy of the cylinder? (i) The block has more kinetic energy than the cylinder. (ii) The block has less kinetic energy than the cylinder. (iii) The block and the cylinder have equal amounts of kinetic energy.

9.5 Parallel-Axis Theorem

We pointed out in Section 9.4 that a body doesn't have just one moment of inertia. In fact, it has infinitely many, because there are infinitely many axes about which it might rotate. But there is a simple relationship between the moment of inertia I_{cm} of a body of mass M about an axis through its center of



9.20 The mass element m_i has coordinates (x_i, y_i) with respect to an axis of rotation through the center of mass (cm) and coordinates $(x_i - a, y_i - b)$ with respect to the parallel axis through point P.

mass and the moment of inertia I_P about any other axis parallel to the original one but displaced from it by a distance d. This relationship, called the **parallel-axis theorem,** states that

$$I_P = I_{cm} + Md^2 \quad \text{(parallel-axis theorem)} \tag{9.19}$$

To prove this theorem, we consider two axes, both parallel to the z-axis, one through the center of mass and the other through a point P (Fig. 9.20). First we take a very thin slice of the body, parallel to the xy-plane and perpendicular to the z-axis. We take the origin of our coordinate system to be at the center of mass of the body; the coordinates of the center of mass are then $x_{cm} = y_{cm} = z_{cm} = 0$. The axis through the center of mass passes through this thin slice at point O, and the parallel axis passes through point P, whose x- and y-coordinates are (a, b). The distance of this axis from the axis through the center of mass is d, where $d^2 = a^2 + b^2$.

We can write an expression for the moment of inertia I_P about the axis through point P. Let m_i be a mass element in our slice, with coordinates (x_i, y_i, z_i). Then the moment of inertia I_{cm} of the slice about the axis through the center of mass (at O) is

$$I_{cm} = \sum_i m_i(x_i^2 + y_i^2)$$

The moment of inertia of the slice about the axis through P is

$$I_P = \sum_i m_i[(x_i - a)^2 + (y_i - b)^2]$$

These expressions don't involve the coordinates z_i measured perpendicular to the slices, so we can extend the sums to include *all* particles in *all* slices. Then I_P becomes the moment of inertia of the *entire* body for an axis through P. We then expand the squared terms and regroup, and obtain

$$I_P = \sum_i m_i(x_i^2 + y_i^2) - 2a\sum_i m_i x_i - 2b\sum_i m_i y_i + (a^2 + b^2)\sum_i m_i$$

The first sum is I_{cm}. From Eq. (8.28), the definition of the center of mass, the second and third sums are proportional to x_{cm} and y_{cm}; these are zero because we have taken our origin to be the center of mass. The final term is d^2 multiplied by the total mass, or Md^2. This completes our proof that $I_P = I_{cm} + Md^2$.

As Eq. (9.19) shows, a rigid body has a lower moment of inertia about an axis through its center of mass than about any other parallel axis. Thus it's easier to start a body rotating if the rotation axis passes through the center of mass. This suggests that it's somehow most natural for a rotating body to rotate about an axis through its center of mass; we'll make this idea more quantitative in Chapter 10.

Example 9.10 Using the parallel-axis theorem

A part of a mechanical linkage (Fig. 9.21) has a mass of 3.6 kg. We measure its moment of inertia about an axis 0.15 m from its center of mass to be $I_P = 0.132\ \text{kg}\cdot\text{m}^2$. What is the moment of inertia I_{cm} about a parallel axis through the center of mass?

9.21 Calculating I_{cm} from a measurement of I_P.

SOLUTION

IDENTIFY: The parallel-axis theorem allows us to relate the moments of inertia I_{cm} and I_P through the two parallel axes.

SET UP: We'll use Eq. (9.19) to determine the target variable I_{cm}.

EXECUTE: Rearranging the equation and substituting the values,

$$\begin{aligned} I_{cm} &= I_P - Md^2 = 0.132\ \text{kg}\cdot\text{m}^2 - (3.6\ \text{kg})(0.15\ \text{m})^2 \\ &= 0.051\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

EVALUATE: Our result shows that I_{cm} is less than I_P. This is as it should be: As we saw earlier, the moment of inertia for an axis through the center of mass is lower than for any other parallel axis.

Test Your Understanding of Section 9.5 A pool cue is a wooden rod with a uniform composition and tapered with a larger diameter at one end than at the other end. Use the parallel-axis theorem to decide whether a pool cue has a larger moment of inertia (i) for an axis through the thicker end of the rod and perpendicular to the length of the rod, or (ii) for an axis through the thinner end of the rod and perpendicular to the length of the rod.

*9.6 Moment-of-Inertia Calculations

***NOTE:** This optional section is for students who are familiar with integral calculus.*

If a rigid body is a continuous distribution of mass—like a solid cylinder or a solid sphere—it cannot be represented by a few point masses. In this case the sum of masses and distances that defines the moment of inertia [Eq. (9.16)] becomes an integral. Imagine dividing the body into elements of mass dm that are very small, so that all points in a particular element are at essentially the same perpendicular distance from the axis of rotation. We call this distance r, as before. Then the moment of inertia is

$$I = \int r^2\, dm \tag{9.20}$$

To evaluate the integral, we have to represent r and dm in terms of the same integration variable. When the object is effectively one-dimensional, such as the slender rods (a) and (b) in Table 9.2, we can use a coordinate x along the length and relate dm to an increment dx. For a three-dimensional object it is usually easiest to express dm in terms of an element of volume dV and the *density* ρ of the body. Density is mass per unit volume, $\rho = dm/dV$, so we may also write Eq. (9.20) as

$$I = \int r^2 \rho\, dV$$

This expression tells us that a body's moment of inertia depends on how its density varies within its volume (Fig. 9.22). If the body is uniform in density, then we may take ρ outside the integral:

$$I = \rho \int r^2\, dV \tag{9.21}$$

To use this equation, we have to express the volume element dV in terms of the differentials of the integration variables, such as $dV = dx\, dy\, dz$. The element dV must always be chosen so that all points within it are at very nearly the same distance from the axis of rotation. The limits on the integral are determined by the shape and dimensions of the body. For regularly shaped bodies, this integration is often easy to do.

9.22 By measuring small variations in the orbits of satellites, geophysicists can measure the earth's moment of inertia. This tells us how our planet's mass is distributed within its interior. The data show that the earth is far denser at the core than in its outer layers.

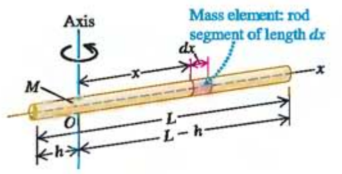
Example 9.11 Uniform thin rod, axis perpendicular to length

Figure 9.23 shows a slender uniform rod with mass M and length L. It might be a baton held by a twirler in a marching band (less the rubber end caps). Compute its moment of inertia about an axis through O, at an arbitrary distance h from one end.

SOLUTION

IDENTIFY: The rod is a continuous distribution of mass, so we must use integration to find the moment of inertia. We choose as an element of mass a short section of rod with length dx at a distance x from point O.

9.23 Finding the moment of inertia of a thin rod about an axis through O.



SET UP: The ratio of the mass dm of an element to the total mass M is equal to the ratio of its length dx to the total length L:

$$\frac{dm}{M} = \frac{dx}{L} \qquad \text{so} \qquad dm = \frac{M}{L}dx$$

We'll determine I from Eq. (9.20) with r replaced by x (see Fig. 9.23).

EXECUTE: Figure 9.23 shows that the integration limits on x are from $-h$ to $(L - h)$. Hence we obtain

$$I = \int x^2\,dm = \frac{M}{L}\int_{-h}^{L-h} x^2\,dx$$

$$= \left[\frac{M}{L}\left(\frac{x^3}{3}\right)\right]_{-h}^{L-h} = \frac{1}{3}M(L^2 - 3Lh + 3h^2)$$

EVALUATE: From this general expression we can find the moment of inertia about an axis through any point on the rod. For example, if the axis is at the left end, $h = 0$ and

$$I = \frac{1}{3}ML^2$$

If the axis is at the right end, we should get the same result. Putting $h = L$, we again get

$$I = \frac{1}{3}ML^2$$

If the axis passes through the center, the usual place for a twirled baton, then $h = L/2$ and

$$I = \frac{1}{12}ML^2$$

These results agree with the expressions in Table 9.2.

Example 9.12 Hollow or solid cylinder, rotating about axis of symmetry

Figure 9.24 shows a hollow, uniform cylinder with length L, inner radius R_1, and outer radius R_2. It might be a steel cylinder in a printing press or a sheet-steel rolling mill. Find the moment of inertia about the axis of symmetry of the cylinder.

SOLUTION

IDENTIFY: Again we must use integration to find the moment of inertia, but now we choose as a volume element a thin cylindrical shell of radius r, thickness dr, and length L. All parts of this element are at very nearly the same distance from the axis.

SET UP: The volume of the element is very nearly equal to that of a flat sheet with thickness dr, length L, and width $2\pi r$ (the circumference of the shell). Then

$$dm = \rho\,dV = \rho(2\pi rL\,dr)$$

We will use this expression in Eq. (9.20) and integrate from $r = R_1$ to $r = R_2$.

EXECUTE: The moment of inertia is given by

$$I = \int r^2\,dm = \int_{R_1}^{R_2} r^2\rho(2\pi rL\,dr)$$

$$= 2\pi\rho L\int_{R_1}^{R_2} r^3\,dr$$

$$= \frac{2\pi\rho L}{4}(R_2^4 - R_1^4)$$

$$= \frac{\pi\rho L}{2}(R_2^2 - R_1^2)(R_2^2 + R_1^2)$$

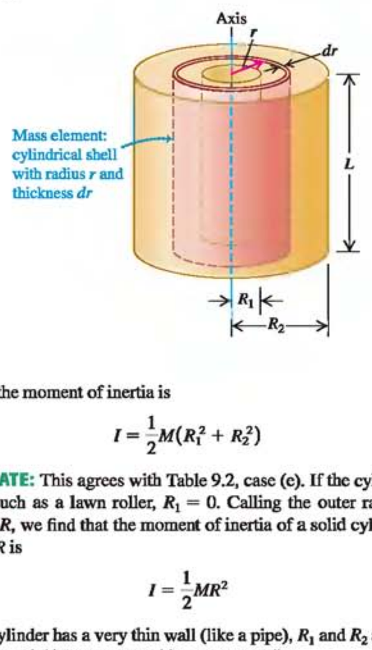
It is usually more convenient to express the moment of inertia in terms of the total mass M of the body, which is its density ρ multiplied by the total volume V. The volume is

$$V = \pi L(R_2^2 - R_1^2)$$

so the total mass M is

$$M = \rho V = \pi L\rho(R_2^2 - R_1^2)$$

9.24 Finding the moment of inertia of a hollow cylinder about its symmetry axis.



Hence the moment of inertia is

$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$

EVALUATE: This agrees with Table 9.2, case (e). If the cylinder is solid, such as a lawn roller, $R_1 = 0$. Calling the outer radius R_2 simply R, we find that the moment of inertia of a solid cylinder of radius R is

$$I = \frac{1}{2}MR^2$$

If the cylinder has a very thin wall (like a pipe), R_1 and R_2 are very nearly equal; if R represents this common radius,

$$I = MR^2$$

We could have predicted this last result; in a thin-walled cylinder, all the mass is the same distance $r = R$ from the axis, so $I = \int r^2\,dm = R^2\int dm = MR^2$.

Example 9.13 Uniform sphere with radius R, axis through center

Find the moment of inertia of a solid, uniform sphere (like a billiard ball or ball bearing) about an axis through its center.

SOLUTION

IDENTIFY: To calculate the moment of inertia we divide the sphere into thin disks of thickness dx (Fig. 9.25), whose moment of inertia we know from Example 9.12. We'll integrate over these to find the total moment of inertia. The only tricky point is that the radius and mass of a disk depend on its distance x from the center of the sphere.

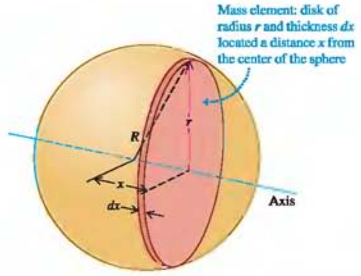
SET UP: The radius r of the disk shown in Fig. 9.25 is

$$r = \sqrt{R^2 - x^2}$$

Its volume is

$$dV = \pi r^2\,dx = \pi(R^2 - x^2)\,dx$$

9.25 Finding the moment of inertia of a sphere about an axis through its center.



and its mass is

$$dm = \rho\,dV = \pi\rho(R^2 - x^2)\,dx$$

EXECUTE: From Example 9.12, the moment of inertia of a disk of radius r and mass dm is

$$dI = \frac{1}{2}r^2\,dm = \frac{1}{2}\left(\sqrt{R^2 - x^2}\right)^2[\pi\rho(R^2 - x^2)\,dx]$$
$$= \frac{\pi\rho}{2}(R^2 - x^2)^2\,dx$$

Integrating this expression from $x = 0$ to $x = R$ gives the moment of inertia of the right hemisphere. The total I for the entire sphere, including both hemispheres, is just twice this:

$$I = (2)\frac{\pi\rho}{2}\int_0^R (R^2 - x^2)^2\,dx$$

Carrying out the integration, we obtain

$$I = \frac{8\pi\rho}{15}R^5$$

The mass M of the sphere of volume $V = 4\pi R^3/3$ is

$$M = \rho V = \frac{4\pi\rho R^3}{3}$$

By comparing the expressions for I and M, we find

$$I = \frac{2}{5}MR^2$$

EVALUATE: This result agrees with the expression in Table 9.2, case (h). Note that the moment of inertia of a solid sphere of mass M and radius R is less than the moment of inertia of a solid *cylinder* of the same mass and radius, $I = \frac{1}{2}MR^2$. The reason is that more of the sphere's mass is located close to the axis.

Test Your Understanding of Section 9.6 Two hollow cylinders have the same inner and outer radii and the same mass, but they have different lengths. One is made of low-density wood and the other of high-density lead. Which cylinder has the greater moment of inertia around its axis of symmetry? (i) the wood cylinder; (ii) the lead cylinder; (iii) the two moments of inertia are equal.

CHAPTER 9 SUMMARY

Rotational kinematics: When a rigid body rotates about a stationary axis (usually called the z-axis), its position is described by an angular coordinate θ. The angular velocity ω_z is the time derivative of θ, and the angular acceleration α_z is the time derivative of ω_z or the second derivative of θ. (See Examples 9.1 and 9.2.) If the angular acceleration is constant, then θ, ω_z, and α_z are related by simple kinematic equations analogous to those for straight-line motion with constant linear acceleration. (See Example 9.3.)

$$\omega_z = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt} \qquad (9.3)$$

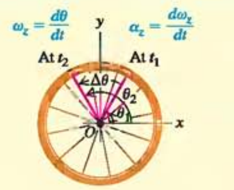
$$\alpha_z = \lim_{\Delta t \to 0} \frac{\Delta \omega_z}{\Delta t} = \frac{d\omega_z}{dt} = \frac{d^2\theta}{dt^2} \qquad (9.5)$$

$$\theta = \theta_0 + \omega_{0z} t + \frac{1}{2}\alpha_z t^2 \qquad (9.11)$$
(constant α_z only)

$$\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t \qquad (9.10)$$
(constant α_z only)

$$\omega_z = \omega_{0z} + \alpha_z t \qquad (9.7)$$
(constant α_z only)

$$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0) \qquad (9.12)$$
(constant α_z only)

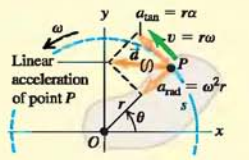


Relating linear and angular kinematics: The angular speed ω of a rigid body is the magnitude of its angular velocity. The rate of change of ω is $\alpha = d\omega/dt$. For a particle in the body a distance r from the rotation axis, the speed v and the components of the acceleration $\vec{a}$ are related to ω and α. (See Examples 9.4–9.6.)

$$v = r\omega \qquad (9.13)$$

$$a_{tan} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha \qquad (9.14)$$

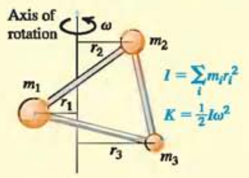
$$a_{rad} = \frac{v^2}{r} = \omega^2 r \qquad (9.15)$$



Moment of inertia and rotational kinetic energy: The moment of inertia I of a body about a given axis is a measure of its rotational inertia: The greater the value of I, the more difficult it is to change the state of the body's rotation. The moment of inertia can be expressed as a sum over the particles m_i that make up the body, each of which is at its own perpendicular distance r_i from the axis. The rotational kinetic energy of a rigid body rotating about a fixed axis depends on the angular speed ω and the moment of inertia I for that rotation axis. (See Examples 9.7–9.9.)

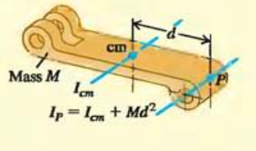
$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots = \sum_i m_i r_i^2 \qquad (9.16)$$

$$K = \frac{1}{2} I\omega^2 \qquad (9.17)$$



Calculating the moment of inertia: The parallel-axis theorem relates the moments of inertia of a rigid body of mass M about two parallel axes: an axis through the center of mass (moment of inertia I_{cm}) and a parallel axis a distance d from the first axis (moment of inertia I_P). (See Example 9.10.) If the body has a continuous mass distribution, the moment of inertia can be calculated by integration. (See Examples 9.11–9.13.)

$$I_P = I_{cm} + Md^2 \qquad (9.19)$$



Key Terms

rigid body, *285*
radian, *286*
average angular velocity, *286*
angular displacement, *286*
instantaneous angular velocity, *287*
average angular acceleration, *289*
instantaneous angular acceleration, *289*
angular speed, *293*
tangential component of acceleration, *293*
centripetal component of acceleration, *294*
moment of inertia, *297*
rotational kinetic energy, *297*
parallel-axis theorem, *302*

Answer to Chapter Opening Question ?

Both segments of the rigid blade have the same angular speed ω. From Eqs. (9.13) and (9.15), doubling the distance r for the same ω doubles the linear speed $v = r\omega$ and doubles the radial acceleration $a_{rad} = \omega^2 r$.

Answers to Test Your Understanding Questions

9.1 Answers: (a) (i) and (iii), (b) (ii) The rotation is speeding up when the angular velocity and angular acceleration have the same sign, and slowing down when they have opposite signs. Hence it is speeding up for $0 < t < 2$ s (ω_z and α_z are both positive) and for $4 \text{ s} < t < 6$ s (ω_z and α_z are both negative), but is slowing down for $2 \text{ s} < t < 4$ s (ω_z is positive and α_z is negative). Note that the body is rotating in one direction for $t < 4$ s (ω_z is positive) and in the opposite direction for $t > 4$ s (ω_z is negative).

9.2 Answers: (a) (i), (b) (ii) When the DVD comes to rest, $\omega_z = 0$. From Eq. (9.7), the *time* when this occurs is $t = (\omega_z - \omega_{0z})/\alpha_z = -\omega_{0z}/\alpha_z$ (this is a positive time because α_z is negative). If we double the initial angular velocity ω_{0z} and also double the angular acceleration α_z, their ratio is unchanged and the rotation stops in the same amount of time. The *angle* through which the DVD rotates is given by Eq. (9.10): $\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t = \frac{1}{2}\omega_{0z}t$ (since the final angular velocity is $\omega_z = 0$). The initial angular velocity ω_{0z} has been doubled but the time t is the same, so the angular displacement $\theta - \theta_0$ (and hence the number of revolutions) has doubled. You can also come to the same conclusion using Eq. (9.12).

9.3 Answer: (ii) From Eq. (9.13), $v = r\omega$. To maintain a constant linear speed v, the angular speed ω must decrease as the scanning head moves outward (greater r).

9.4 Answer: (i) The kinetic energy in the falling block is $\frac{1}{2}mv^2$, and the kinetic energy in the rotating cylinder is $\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{4}mv^2$. Hence the total kinetic energy of the system is $\frac{3}{4}mv^2$, of which two-thirds is in the block and one-third is in the cylinder.

9.5 Answer: (ii) More of the mass of the pool cue is concentrated at the thicker end, so the center of mass is closer to that end. The moment of inertia through a point P at either end is $I_P = I_{cm} + Md^2$; the thinner end is farther from the center of mass, so the distance d and the moment of inertia I_P are greater for the thinner end.

9.6 Answer: (iii) Our result from Example 9.12 does *not* depend on the cylinder length L. The moment of inertia depends only on the *radial* distribution of mass, not on its distribution along the axis.

PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com

Discussion Questions

Q9.1. Which of the following formulas is valid if the angular acceleration of an object is *not* constant? Explain your reasoning in each case. (a) $v = r\omega$; (b) $a_{tan} = r\alpha$; (c) $\omega = \omega_0 + \alpha t$; (d) $a_{tan} = r\omega^2$; (e) $K = \frac{1}{2}I\omega^2$.

Q9.2. A diatomic molecule can be modeled as two point masses, m_1 and m_2, slightly separated (Fig. 9.26). If the molecule is oriented along the y-axis, it has kinetic energy K when it spins about the x-axis. What will its kinetic energy (in terms of K) be if it spins at the same angular speed about (a) the z-axis and (b) the y-axis?

Figure 9.26 Question Q9.2.

Q9.3. What is the difference between tangential and radial acceleration for a point on a rotating body?

Q9.4. In Fig. 9.14, all points on the chain have the same linear speed. Is the magnitude of the linear acceleration also the same for all points on the chain? How are the angular accelerations of the two sprockets related? Explain.

Q9.5. In Fig. 9.14, how are the radial accelerations of points at the teeth of the two sprockets related? Explain the reasoning behind your answer.

Q9.6. A flywheel rotates with constant angular velocity. Does a point on its rim have a tangential acceleration? A radial acceleration? Are these accelerations constant in magnitude? In direction? In each case give the reasoning behind your answer.

Q9.7. What is the purpose of the spin cycle of a washing machine? Explain in terms of acceleration components.

Q9.6. Although angular velocity and angular acceleration can be treated as vectors, the angular displacement θ, despite having a magnitude and a direction, cannot. This is because θ does not follow the commutative law of vector addition (Eq. 1.3). Prove this to yourself in the following way: Lay your physics textbook flat on