Introduction to Fuzzy Logic Control With Application to Mobile Robotics

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ABSTRACT:
A brief introduction to fuzzy set theory and its application to control systems is provided. Fuzzy sets do not have sharp boundaries and are therefore able to represent linguistic terms which may be considered "gray" or vague. Aspects of fuzzy set theory and fuzzy logic are highlighted in order to illustrate distinct advantages, as contrasted to classical sets and logic, for use in control systems. Using a mobile robot navigation problem as an example, the synthesis of a fuzzy control system is examined.

Keywords: mobile robots, fuzzy logic control, fuzzy sets, rover, autonomy

1. INTRODUCTION

"The world is not black and white but only shades of gray." In 1965, Zadeh [1] wrote a seminal paper in which he introduced fuzzy sets, sets with unsharp boundaries. These sets are considered gray areas rather than black and white in contrast to classical sets which form the basis of binary or Boolean logic. Fuzzy set theory and fuzzy logic are convenient tools for handling uncertain, imprecise, or unmodeled data in intelligent decision-making systems. It has also found many applications in the areas of information sciences and control systems.

In this paper, fundamental concepts of fuzzy sets and logic are briefly presented. Its utility for synthesis of control systems is discussed in the context of an application to mobile robot motion control. In mobile robotics, a fuzzy logic based control system has the advantage that it allows the intuitive nature of collision-free navigation to be easily modeled using linguistic terminology. Due to the relative computational simplicity of fuzzy rule-based systems, intelligent decisions can be made in real-time, thus allowing for uninterrupted robot motion. Moreover, accurate (expensive) sensors and detailed models of the environment are not absolutely necessary for autonomous navigation [2].

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2. FUZZY SET THEORY

In classical set theory a set, C, is comprised of elements, $x \in U$, whose membership in C is described by the characteristic, or membership function

$$\mu_C(x) : U \rightarrow \{0,1\} \tag{1}$$

where U is the universe of discourse, a collection of elements that can be continuous or discrete. The membership function $\mu_C(x)$ implies that the element x either belongs to the set ($\mu_C(x) = 1$) or it does not ($\mu_C(x) = 0$). In fuzzy set theory a fuzzy set, $\tilde{F}$, is described by the membership function

$$\mu_{\tilde{F}}(x) : U \rightarrow [0,1] \tag{2}$$

where elements, $x \in U$, have degrees of membership in $\tilde{F}$ with any value between 0 and 1 inclusive. Note that a fuzzy membership function is a so-called possibility function and not a probability function. A membership value of zero corresponds to the case where the element is definitely not a member of the fuzzy set. A membership value of one corresponds to elements with full membership in the fuzzy set. Membership values in the open interval (0, 1) correspond to partial membership and indicate a measure of uncertainty or imprecision associated with the element.

A comparative example of a crisp set and a fuzzy set can be illustrated by using the linguistic term 'far' in reference to relative distance between objects. The term 'far' can take on different meanings to different individuals, and in different contexts. For illustrative purposes, let 'far' be 2 meters (approximately 2 meters in the fuzzy set case). A graphical representation of a crisp set and a fuzzy set for 'far' is shown in Figure 1.

Membership functions can be defined as functions which take on a variety of possible shapes determined at the discretion of the fuzzy system designer. Commonly used function shapes (fuzzy logic terminology given in parentheses) include triangular (Λ), trapezoidal (Π), delta (singleton), positively sloped ramp (Γ), and negatively sloped ramp (L). These are shown in Figure 2. The ramp functions are sometimes referred to as right shoulders (Γ) and left shoulders (L).

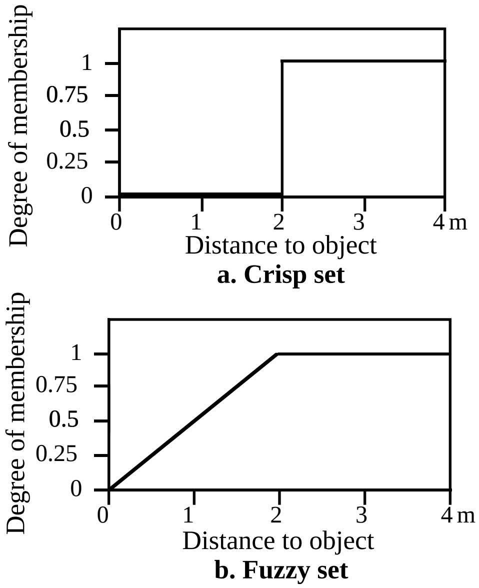

Figure 1 Graphical representations of ‘far’.

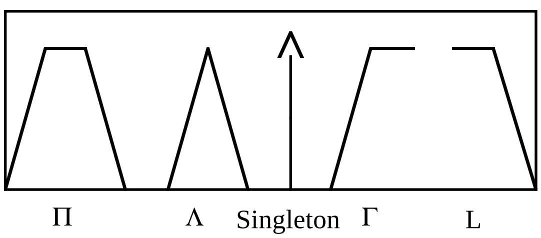

Figure 2 Common fuzzy membership functions.

Fuzzy sets, like classical crisp sets, are subject to set operations such as union, intersection, and complement [1] which are used to express logic statements or propositions. The union of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ with membership functions $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$ is a fuzzy set $\tilde{C} = \tilde{A} \cup \tilde{B}$, whose membership function is related to those of $\tilde{A}$ and $\tilde{B}$ as follows:

$$\mu_{\tilde{C}}(x) = \mu_{\tilde{A} \cup \tilde{B}}(x) = \max[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] \qquad (3)$$

The operator in this equation is referred to as the max-operator and is represented by the logical term OR. The intersection of $\tilde{A}$ and $\tilde{B}$ is a fuzzy set $\tilde{D} = \tilde{A} \cap \tilde{B}$ whose membership function is given by:

$$\mu_{\tilde{D}}(x) = \mu_{\tilde{A} \cap \tilde{B}}(x) = \min[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)] \qquad (4)$$

The operator in this equation is referred to as the min-operator represented by the logical term AND. For details on complements and other fuzzy logical operations see [1] or [3].

Consider the Cartesian product of two universes U and V defined by

$$U \times V = \{(u, v) \mid u \in U; v \in V\}$$

which combines elements of U and V in a set of ordered pairs. A fuzzy relation **R** is a mapping:

$$\mathbf{R} : U \times V \to [0,1]$$

where

$$\mu_{\mathbf{R}}(u, v) = \mu_{\tilde{A} \times \tilde{B}}(u, v) = \min[\mu_{\tilde{A}}(u), \mu_{\tilde{B}}(v)] \qquad (5)$$

The composition of two relations, **R**(u,v) and **S**(v,w), is denoted by $\mathbf{T} = \mathbf{R} \circ \mathbf{S}$. Its membership value can be determined by the following expression

$$\mu_{\mathbf{T}}(u, w) = \max[\mu_{\mathbf{R}}(u, v) \bullet \mu_{\mathbf{S}}(v, w)] \qquad (6)$$

which is called the max-product composition. Another common compositional rule of inference is the max-min composition [3].

Fuzzy relations can be represented linguistically by natural language statements in the form of fuzzy *if-then* rules. A collection of such rules is referred to as a rule-base. Accompanied by suitable membership functions, the rule-base is a core ingredient of any fuzzy rule-based expert system.

3. FUZZY LOGIC CONTROL

Fuzzy logic based controllers are expert control systems that smoothly interpolate between rules. Rules fire to continuous degrees and the multiple resultant actions are combined into an interpolated result. Processing of uncertain information and savings of energy using common-sense rules and natural language statements are the bases for fuzzy logic control. As pointed out by Lee [4], fuzzy logic controllers provide a means of transforming the linguistic control strategy based on expert knowledge into an automatic control strategy.

Fuzzy controller rule-bases typically take the form of a set of if-then rules whose antecedents (‘if’ parts) and consequents (‘then’ parts) are propositions involving fuzzy membership functions. If X and Y are input and output universes of discourse of a fuzzy controller with a rule-base of size n, the usual if-then rule takes the following form

$$\text{IF } x \text{ is } \tilde{A}_i \text{ THEN } y \text{ is } \tilde{B}_i$$

where x and y represent input and output fuzzy linguistic variables, respectively, and $\tilde{A}_i \in X$ and $\tilde{B}_i \in Y$ $(1 \leq i \leq n)$ are fuzzy sets representing linguistic values of x and y. Typically in robotics applications, the input x refers to sensory data and y to actuator control signals. In general, the rule antecedent consisting of the proposition "x is $\tilde{A}_i$" could be replaced by a conjunction of similar propositions; the same holds for the rule consequent "y is $\tilde{B}_i$". We can formally define a fuzzy system behavior (rule-base) as a function (B), from sensor space (S) to actuator space (A), i.e. $B: S \to A$, where the universes of discourse for S and A are such that $S \subset \Re^n$ and $A \subset \Re^n$. Embodied in this function is a fuzzy relation between fuzzy sets defined over S and fuzzy sets over A. This fuzzy relation is the actual rule-base of the fuzzy control system.

3.1 Mobile robot application

Mobile robots are typically equipped with several sensor modalities which may include range sensors, tactile/contact sensors, encoders, and vision systems. Given such sensor modalities, the usual procedure for fuzzy control synthesis consists of first defining linguistic terminology for the inputs and outputs, partitioning the sensor space and actuator space using appropriate fuzzy sets (membership functions), and formulating fuzzy rules that satisfactorily govern the desired response of the robot in all practical situations.

The subject of discussion in this paper is a mobile robot modeled after LOBOT, a custom-built robot driven by a 2-wheel differential configuration with two supporting casters. It is octagonal in shape, stands about 75 cm tall and measures about 60 cm in width. Range sensing is achieved using a layout of 16 ultrasonic transducers (arranged primarily on the front, sides, and forward-facing obliques); optical encoders on each driven wheel provide position information. Assuming a constant linear speed of 5cm/sec, we synthesize a fuzzy controller that uses four inputs and one output. The inputs are relative obstacle ranges to the front, left and right, and the angle in the direction of a designated goal location. Their respective linguistic terms are: `FS` (front sensor), `LS` (left sensor), `RS` (right sensor), and `DIR`. The individual sensor inputs are derived from pre-processed data from multiple sensors on the corresponding sides of the robot. The output is a direction in which to turn in order to satisfy avoiding obstacles and navigating to the goal. Its linguistic term is `TURN-ANGLE`. The range input space was partitioned based on a relevant maximum sensor range measurement of 4m, i.e. the universe of discourse for range spans the interval [0m, 4m]. The goal direction input covers a universe spanning ±π radians. The actuator control, or steering direction, covers ±π/2 radians. The corresponding fuzzy sets are shown in Figure 3 where the labels of Figure 3b are listed in Table I.

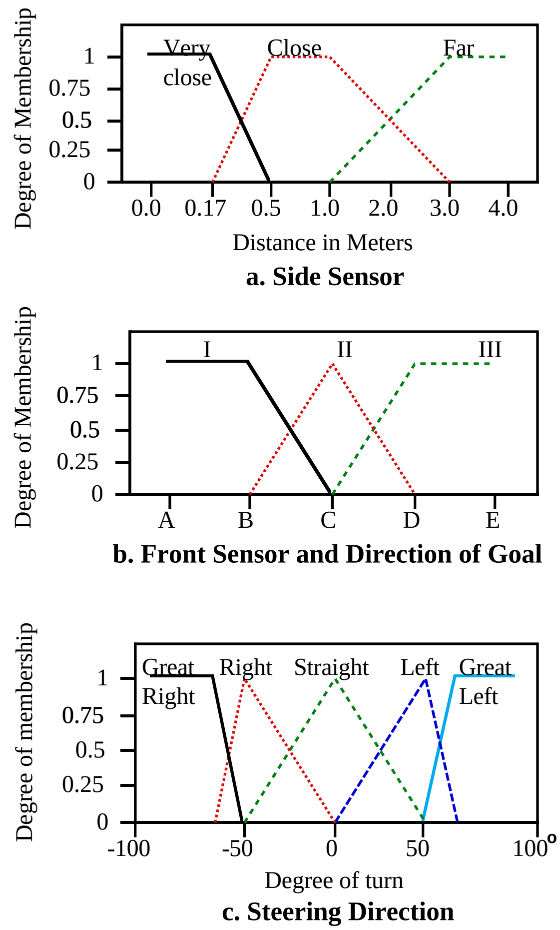

Figure 3. Membership functions

Table I.

Input	Front sensor	Goal Direction
I	Very close	Right
II	Close	Zero
III	Far	Left
A	0.0	-π
B	0.2	-π/2
C	0.3	0
D	0.75	π/2
E	1.0	π
Units	meters	radians

Based on the membership functions selected, a rule-base was designed to effect motion behavior suitable for collision-free navigation to designated goal locations. A total of 36 rules were formulated. An example of one of these is:

IF: `LS` is *Close* and `RS` is *Close* and `FS` is *Far*
and `DIR` is *Zero*
THEN: `TURN-ANGLE` is *Straight*.

4. SIMULATION

A simulation is described here to demonstrate the behavior of the fuzzy controller described above. It is a two-dimensional mock simulation of a Mars rover navigation task. The scenario is as follows. A planetary rover is deployed at a scientifically interesting landing site near Ares Tiu on Mars. Human operators on Earth command the rover to navigate to a designated location where experiments are to be performed. The rover's immediate task is to autonomously navigate to the goal under sensor-based control, i.e. no internal map of the environment is used.

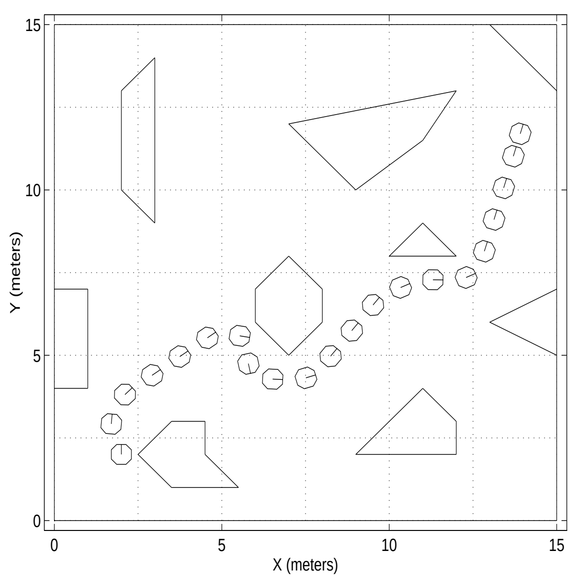

Figure 4. Fuzzy controlled sensor-based navigation.

The simulation result is illustrated in Figure 4 which shows a 225m^2 region cluttered with an arbitrary distribution of obstacles. The simulated mobile robot (rover) is displayed as an octagon with a radial line segment indicating the robot's heading. It's initial location is (x y θ) = (2.0m 2.0m π rad.) and the goal is located at (X, Y) = (14.0m, 12.0m).

Using a fuzzy controller as developed above, the rover was able to successfully negotiate a smooth path to the goal location. In the figure, the robot icon is displayed every 10 seconds as it traverses the path.

5. SUMMARY

A brief introduction to fuzzy sets and logic was given with emphasis on its application to intelligent control of mobile robots. Insights into the synthesis procedure of such fuzzy control systems is provided via an exercise in developing a fuzzy controller for autonomous navigation. The performance of the resulting control system was demonstrated using a simulated navigation task described in the context of a simplified two-dimensional Mars rover mission scenario.

At the ACE Center, research is ongoing in the area of intelligent control of autonomous mobile robots. Focal areas include hierarchical fuzzy control, genetic programming applications to intelligent controller design, and embedded fuzzy control at the microprocessor and integrated circuit level.

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