Robust Control of Linear DC Motor using DSP

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Abstract

We have developed a wire-driven system with a linear actuator. After parameter identification as a 2-mass model, three robust control methods are tested: observer-based control, sliding mode control and H_∞ control. The H_∞ controller shows the best performance. Further this paper shows details of the design method of the H_∞ controller.

1 Introduction

In the field of Mechatronics, small-sized actuators with multi-degrees of freedom are desired so that dextrous performance is achieved, especially for small size robot development. One potential solution is to use the linear type actuator instead of rotational one because the linear type actuator can be set at a convenient location and forces can be transmitted using wires. This saves space and it is fairly simple to realize multi-degrees of freedom systems by gathering several linear actuators at the same place. For example, the orientation and effective force of a robot hand are controlled by wire-driven linear actuators. The robot hand should be made small by placing the flat and thin linear actuators aside from it.

However, there are serious problems in applying linear actuators with wire to mechatronics [1]. The characteristics of wire-drives are rather complicated because of its unknown stiffness, backlash and friction. It is very difficult to model the dynamics of the wire-driven system. There must be uncertainties for structure and parameters of the dynamical estimated model.

Recently several robust control methods are available to solve the above problem of uncertain plants such as H_∞, observer-based control, sliding mode, etc. The authors applied these control methods to the control of a wire-driven DC linear actuator and compared the efficiency of each method.

In this paper, firstly the wire-driven system is derived and its parameters are identified by experiments. Secondly three type of robust control methods are summarized briefly. Thirdly the detailed design of the H_∞ controller is described. Finally the experimental results are discussed and compared with each other.

2 Wire-driven System

2.1 Configuration

The configuration of wire-driven system in this study is shown in Figure 1. The following three problems are known to exist in the wire-driven systems:

1. Resonance frequency because of wire flexibility.
2. Backlash because of wire slack.
3. Coulomb friction between pulley and wire.

These problems should be solved by control in this work.

2.2 Plant Modeling

The wire-driven system is modeled as a 2-mass system which has a mass at the driving side and at the output side. The model is shown in Figure 2 and Table 1 shows the parameters. This model is a fourth-order system as

$$G_{y_p u} = \frac{\frac{k_f}{LM}(k_j + d_j s)}{s\left(s^3 + (\frac{d_j}{M} + \frac{d_j}{L} + \frac{k_a}{M})s^2 + (\frac{k_j}{M} + \frac{k_j}{L} + \frac{k_a d_j}{LM})s + \frac{k_a k_j}{LM}\right)} \quad (1)$$

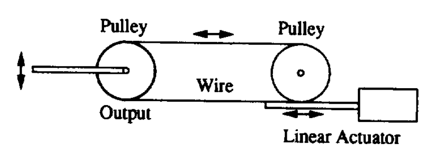

Figure 1: Wire-driven System

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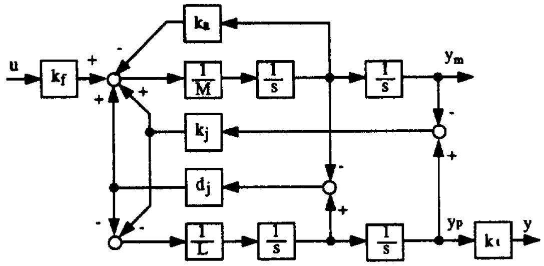


Figure 2: 2-mass Model of Wire-driven System

Table 1: Coefficients of 2-mass Model

k_f	from motor input to force	(N)
k_a	dumping of motor	(N/(m/s))
M	mass of motor	(Kg)
L	mass of load	(Kg)
k_j	stiffness of wire	(N/m)
d_j	dumping of wire	(N/(m/s))
k_t	wire position to rotor	(rad/m)
u	input (PWM signal)	
y	output position	(rad)
y_p	wire position	(m)
y_m	motor position	(m)

The values of parameters are obtained by identification experiments. The transfer function of the system is

$$G_{y_p u} = \frac{5608s + 7852444}{s(s^3 + 364s^2 + 320733s + 43960000)} \tag{2}$$

Figure 3 shows the bode diagram, the resonance frequency is around 500rad/s, Coulomb friction is 6N and the actuator side has 0.3mm backlash.

3 Robust Control Methods

In this section, three robust control methods are described briefly.

3.1 Observer-Based Robust Control

The observer is designed to include pure disturbance and the mismatched dynamics between actual system and observer model [2]. It is equivalent to assume the system dynamics as the observer model (nominal model). Figure 4 shows the system. P is actual system dynamics and P_M is nominal system dynamics (observer model). The transfer function from u to y is

$$T_{yu} = kP_M \tag{3}$$

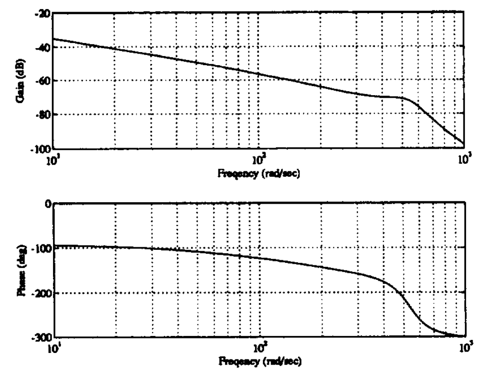


Figure 3: Frequency Response of the Wire-driven System

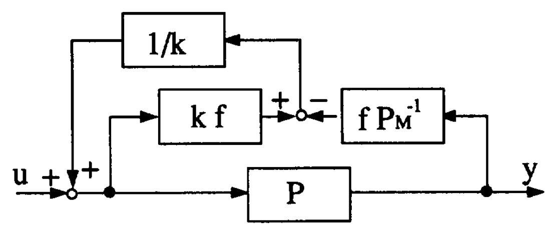


Figure 4: Observer-Based Controller

For the application of motion control the nominal system dynamics P_M is usually set as $1/s^2$. In this case the reverse of the nominal dynamics is not proper and not realizable. Therefore the low pass filter is inserted in series to make the system proper. The observer can suppress the differences between the actual dynamics and nominal dynamics. Of course it works well in the range of limited frequency. In this case the low frequency domain dependent on the low pass filter is valid to this observer.

The difficult point of this design is how to choose the low pass filter. And this system is sensitive to measurement noises because the loop gain is very high to assure good convergence of the observer.

3.2 Sliding Mode Control

The sliding mode based on Variable Structure Systems is characterized by its change of control structure. The system in sliding mode is constrained to the prescribed switching hyper plane. The control structure is altered depending on the side of the hyper plane to let the system approach to it and then the system during sliding mode is identical to the hyper plane.

In this study the design of switching hyper plane is based on the LQG design method [3].

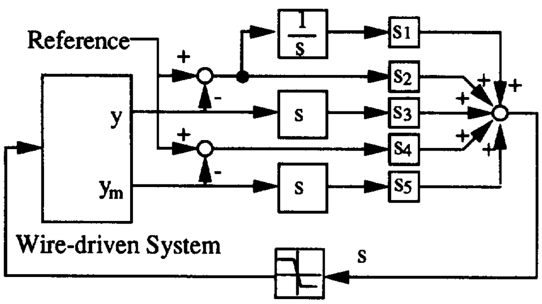


Figure 5: Sliding Mode Controller

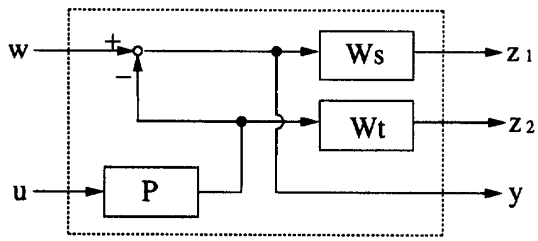


Figure 6: Generalized Plant

3.3 H_∞ Control

H_∞ is a kind of loop shaping method of the closed loop system so that H_∞ norm is minimized under the unknown disturbances and plant uncertainties. In this study, mixed sensitivity method is used to get robustness. The sensitivity function $S = 1/(1 + CP)$ and the complementary sensitivity function $T = CP/(1 + CP)$ have the relation S+T=1. With this constraint the introducing the weighting function W_S and W_T so that the H_∞ norm is

$$\left\| \begin{array}{c} SW_S \\ TW_T \end{array} \right\|_\infty < 1 \tag{4}$$

The H_∞ control method gives the stable controller to satisfy (4) automatically by solving the riccati equation. For the 2-mass system the weighting function W_T work not to excite the resonant frequency and the W_S guarantees the high gain in the low frequency to get the good robustness to the parameter mismatch and disturbances.

4 H_∞ Controller Design

4.1 Generalized Plant and its H_∞ Solution

The mixed sensitivity problem is to obtain a controller which satisfies (4). To solve this problem, the generalized plant G, which is shown in Figure 6, is constructed. In Figure 6, w is reference signal, u is input from controller, y is observing output, and W_S, W_T are weight functions. In this case, controller is designed as the H_∞ norm of the transfer function from w to z (T_{ZW}) is less than 1.

To make the state space expression of the generalized plant G stable, a controller C is obtained by the DGKF method[4]. But this method requires some conditions. One of them, the transfer function G_{21} from w to y has no zeros on the imaginary axis and all poles of G on the imaginary axis are included in G_{21}. This plant has integral element, i.e. there are some poles at zero point, it can't be solved by the DGKF method. A modification of G is required.

4.2 Modified Generalized Plant

In this section, the method to obtain H_∞ controller when plant has poles on the imaginary axis is described[5]. The plant is assumed to be a SISO system, and can be expressed as

$$P = P_1 P_2 \qquad\qquad P_2 = \frac{s+a}{s} \tag{5}$$

In (5), P_1 does not have zeros or poles on the imaginary axis and P_S satisfies the following conditions:

1. All poles on the imaginary axis are included in P_2.
2. The order of numerator and denominator are equal.

If (5) is obtained, the modified generalized plant can be constructed. The process is shown below.

1. Select Ws, as $W_S = \widehat{W}_S P_2$, $\widehat{W}_S \in RH_\infty$
2. Design the generalized plant as follows.

$$\begin{aligned} \begin{bmatrix} z_1 \\ z_2 \\ y \end{bmatrix} &= \begin{bmatrix} W_S & -W_S P_1 \\ 0 & W_T P_1 \\ P_2 & -P \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \\ &= \begin{bmatrix} \widehat{W}_S P_2 & -\widehat{W}_S P_2 P_1 \\ 0 & W_T P_1 \\ P_2 & -P_2 P_1 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \end{aligned} \tag{6}$$

The transfer function Φ from w to z is

$$\Phi = \begin{bmatrix} W_S S \\ W_T T \end{bmatrix}. \tag{7}$$

It is the same as the normal mixed sensitivity problem. This modified generalized plant is shown in Figure 7. This G satisfies the condition of poles on the imaginary axis. So this problem can be solved by the DGKF

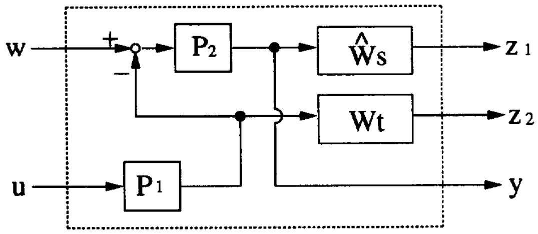


Figure 7: Modefied Generalized Plant

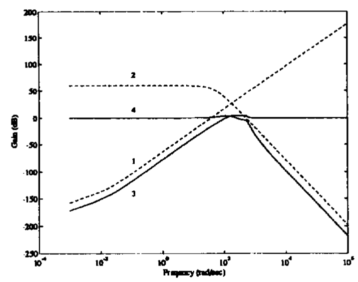


Figure 8: Mixed Sensitivity Problem
1:$1/W_S$ 2:$1/W_T$ 3:S 4:T

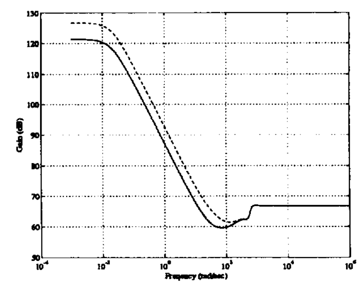


Figure 9: H_∞ Controller
Solid : 4th, Broken : 8th Order Controller

method. Each transfer function can be expressed by the state space form.

$$\widehat{W}_S = C_{\widehat{W}_S}(sI - A_{\widehat{W}_S})B_{\widehat{W}_S} + D_{\widehat{W}_S} \tag{8}$$
$$W_T = C_{W_T}(sI - A_{W_T})B_{W_T} + D_{W_T} \tag{9}$$
$$P_1 = C_{P_1}(sI - A_{P_1})B_{P_1} + D_{P_1} \tag{10}$$
$$P_2 = C_{P_2}(sI - A_{P_2})B_{P_2} + D_{P_2} \tag{11}$$

The state space expression of the modified generalized plant is shown as follows.

$$G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \tag{12}$$

$$A = \begin{bmatrix} A_{P_1} & 0 & 0 & 0 \\ -B_{P_2}C_{P_1} & A_{P_2} & 0 & 0 \\ -B_{\widehat{W}_S}D_{P_2}C_{P_1} & B_{\widehat{W}_S}C_{P_2} & A_{\widehat{W}_S} & 0 \\ 0 & 0 & 0 & A_{W_T} \end{bmatrix} \tag{13}$$

$$B = \begin{bmatrix} 0 & B_{P_1} \\ B_{P_2} & -B_{P_2}D_{P_1} \\ B_{\widehat{W}_S}D_{P_2} & -B_{\widehat{W}_S}D_{P_2}D_{P_1} \\ 0 & B_{W_T}D_{P_1} \end{bmatrix} \tag{14}$$

$$C = \begin{bmatrix} -D_{\widehat{W}_S}D_{P2}C_{P_1} & D_{\widehat{W}_S}C_{P_2} & C_{\widehat{W}_S} & 0 \\ 0 & 0 & 0 & C_{W_T} \\ -D_{P_2}C_{P_1} & C_{P_2} & 0 & 0 \end{bmatrix} \tag{15}$$

$$D = \begin{bmatrix} D_{\widehat{W}_S} & -D_{\widehat{W}_S}D_{P_2}D_{P_1} \\ 0 & D_{W_T} \\ D_{P_2} & -D_{P_2}D_{P_1} \end{bmatrix} \tag{16}$$

4.3 Calculation of Controller

The controller for the wire-driven system is designed as previously described. Each transfer function is set as follows.

$$P_1 = \frac{5608s + 785244}{(s^3 + 364s^2 + 320733s + 43960000)(s + 50)} \tag{17}$$

$$P_2 = \frac{s + 50}{s} \tag{18}$$

$$W_T = \frac{(s + 50)^2}{1000 \times 50^3} \tag{19}$$

$$\widehat{W}_s = \frac{\rho}{(s + 0.02)(s + 50)} \tag{20}$$

Consequently the transfer function W_S is $\rho/s(s+2)$ so real weight function for S is $\rho/(s+0.02)$ except for $1/s$. The practical controller is designed using MATLAB together with the Robust Control Toolbox ($\rho = 1500$). An 8th order controller is obtained, but it can be reduced to 4th order very easily. The gain becomes a little smaller, but the shape does not change very much. This operation is executed manually. The bode-diagram of the controller is shown in Figure 8. The simulation result of the closed loop system is shown in Figure 10. It has a large overshoot, but is stable. The discrete controller is obtained by bilinear transformation.

$$C = \frac{\begin{array}{c} 2183.5528s^4 + 852531.35s^3 + 722098600s^2 \\ +114823090000s + 2719365800000 \end{array}}{\begin{array}{c} s^4 + 597.88482s^3 + 433674.55s^2 \\ +116623830s + 2332305.0 \end{array}} \tag{21}$$

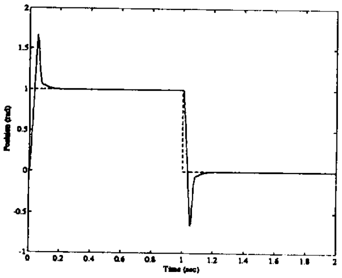


Figure 10: Simulation Result

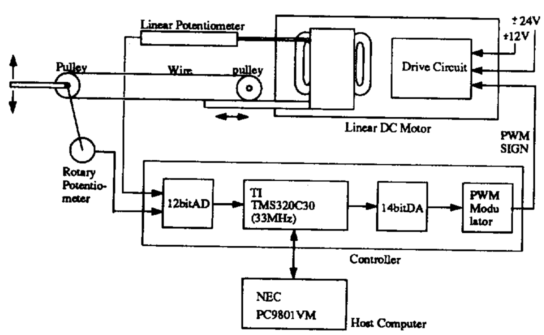


Figure 11: Experimental System

5 Experiments and Results

5.1 Experimental Setup

The experimental system configuration is shown in Figure 11. A DSP (TMS320C30) is used for the controller. AD, DA converters are 14bit and 12bit respectively and their conversion time is $5\mu s$. The control program is coded in C language and a 20th order controller at $200\mu s$ sampling time can be implemented.

The linear DC motor is small and very thin (100x250x8mm) type. It has build in driver circuit, so can be operated by supplying only ±24V DC and ±12V DC and PWM, SIGN control signal. The maximum force of this motor is about 17N.

The wire-driven system is configured as follows. Pulleys diameter are 16mm (motor side) and 10mm (load side), each pulley has a bear-ring. The wire is of 0.5mm diameter made of stainless steel. The distance of two pulleys is about 0.4m. Position sensors are a linear potentiometer to detect motor position and a rotary potentiometer for output position. The resolutions are 0.5mm and 0.0015rad, respectively.

As the experiment, the position response of output is detected against 0.5Hz square and sin wave reference signel. The width of wave is 1 rad.

5.2 Results and Discussion

Experimental results are shown bellow. Three types of controller were tested.

The observer-based controller is tuned by trial and error. Parameters to be tuned were filter shape, gain k and PD controller parameters. This PD controller forms a position control loop. Every value is set at the point where the highest gain is obtained and the stability is kept. Sampling time is $200\mu s$. The response is rather smooth(Figure 12(a)(b)). There is no steady state error for step response. But small vibration and noisy sound is generated. It is caused by observer noise and derivative characteristics of this controller.

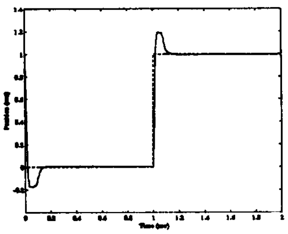
The sliding mode controller is designed by the LQG method. Sampling time was 1ms, because the resolution of motor position is low, so fast sampling time does not show good performance. A responses are shown in Figure 12(c)(d). It is not smooth and there is a steady state error. The noise was large. It seems that such a system which has high order delay and backlash is not a good application for sliding mode control.

H_∞ controller test was executed with $200\mu s$ sampling time. The responses are shown in Figure 12(e)(f). There is no steady state error, because this controller has higher gain at low frequency. This controller also does not generate a noisy sound.

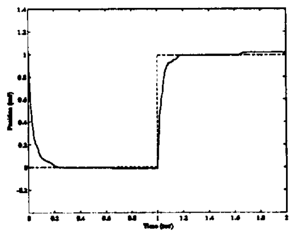
Observer-based controller shows better response, but generate small vibration and noisy sound. The design strategy is not clear, so trial and error was required for all controller parameters. Sliding mode controller is designed easily when identification model is obtained. The weight of LQG method has a room for improvement. The H_∞ controller shows best performance. It avoids the resonance frequency and at the same time implements a high control gain for low frequency. Noisy sound was not heard at all. These characteristics are designed in the frequency domain, so a desirable controller is easily found. H_∞ control theory is effective for wire-driven systems.

6 Conclusion

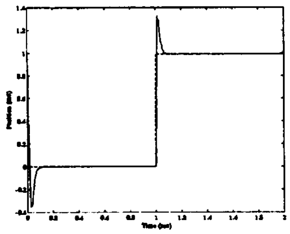
For the combination of wire-driven system and linear DC motor, three robust control methods, observer-based controller, sliding mode controller and H_∞ controller were implemented. Experimental results have shown that the H_∞ controller is the best for this plant, which has a resonance frequency, high friction and backlash. It was shown that wire-driven systems can be effectively controlled by an H_∞ controller achieving good performance.



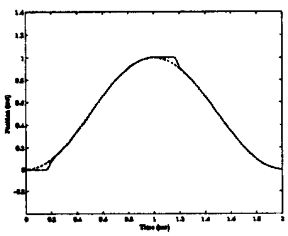
(a) Step Reference
(Observer-Based Control)



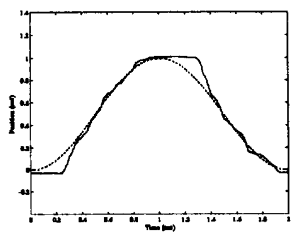
(c) Step Reference
(Sliding Mode Control)



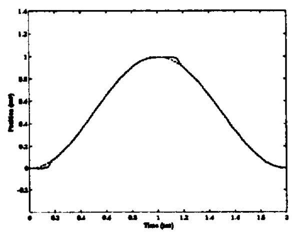
(e) Step Response
(H_∞ control)



(b) Sinusoidal Reference
(Observer-Based Control)



(d) Sinusoidal Reference
(Sliding Mode Control)



(f) Sinusoidal Reference
(H_∞ control)

Figure 12: Time Responses

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