O N E

Introduction

ANSWERS TO REVIEW QUESTIONS

1. Guided missiles, automatic gain control in radio receivers, satellite tracking antenna

2. Yes - power gain, remote control, parameter conversion; No - Expense, complexity

3. Motor, low pass filter, inertia supported between two bearings

4. Closed-loop systems compensate for disturbances by measuring the response, comparing it to the input response (the desired output), and then correcting the output response.

5. Under the condition that the feedback element is other than unity

6. Actuating signal

7. Multiple subsystems can time share the controller. Any adjustments to the controller can be implemented with simply software changes.

8. Stability, transient response, and steady-state error

9. Steady-state, transient

10. It follows a growing transient response until the steady-state response is no longer visible. The system will either destroy itself, reach an equilibrium state because of saturation in driving amplifiers, or hit limit stops.

11. Transient response

12. True

13. Transfer function, state-space, differential equations

14. Transfer function - the Laplace transform of the differential equation

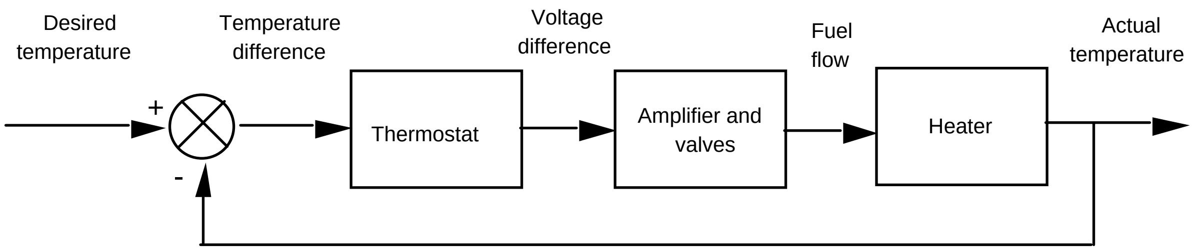
State-space - representation of an nth order differential equation as n simultaneous first-order differential equations

Differential equation - Modeling a system with its differential equation

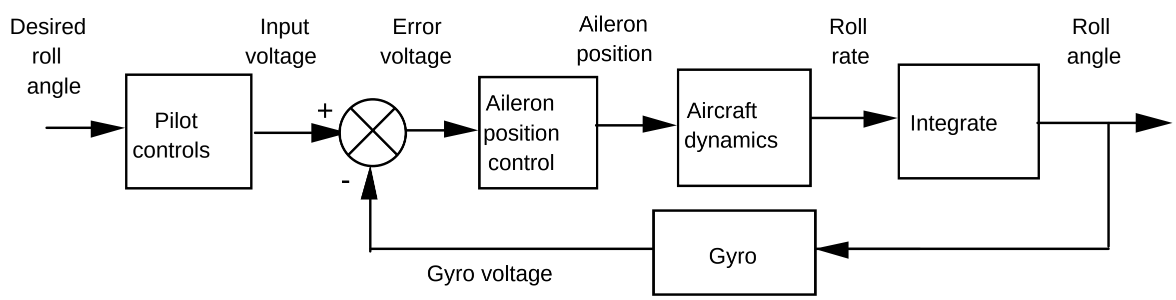
SOLUTIONS TO PROBLEMS

1. Five turns yields 50 v. Therefore $K = \dfrac{50 \text{ volts}}{5 \text{ x } 2\pi \text{ rad}} = 1.59$

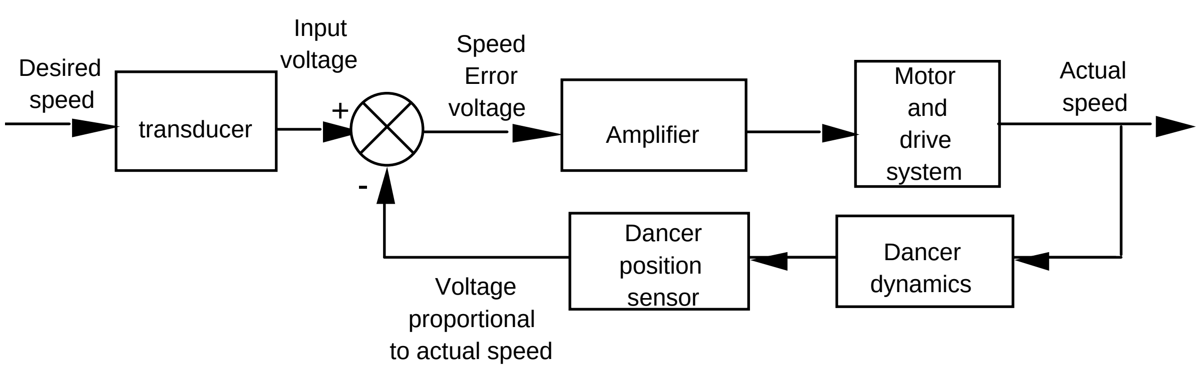
2.


Desired temperature
Temperature difference
Voltage difference
Fuel flow
Actual temperature
+
-
Thermostat
Amplifier and valves
Heater

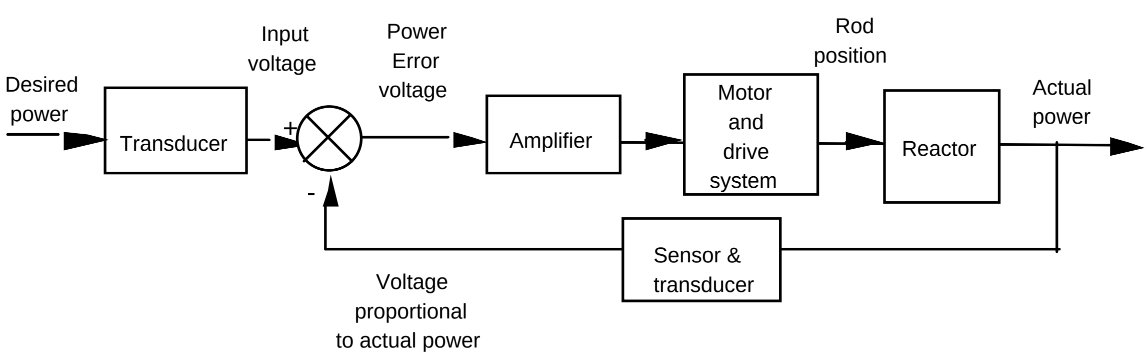
3.


Desired roll angle
Input voltage
Error voltage
Aileron position
Roll rate
Roll angle
Pilot controls
+
-
Aileron position control
Aircraft dynamics
Integrate
Gyro
Gyro voltage

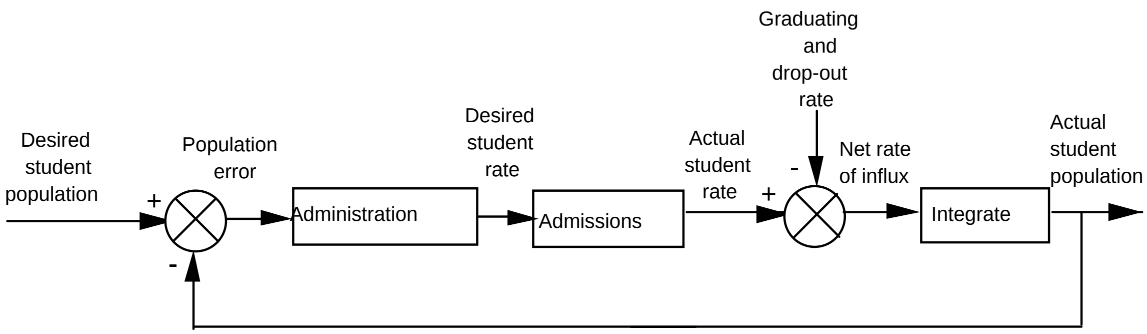
4.


Desired speed
Input voltage
Speed Error voltage
Actual speed
transducer
+
-
Amplifier
Motor and drive system
Dancer position sensor
Dancer dynamics
Voltage proportional to actual speed

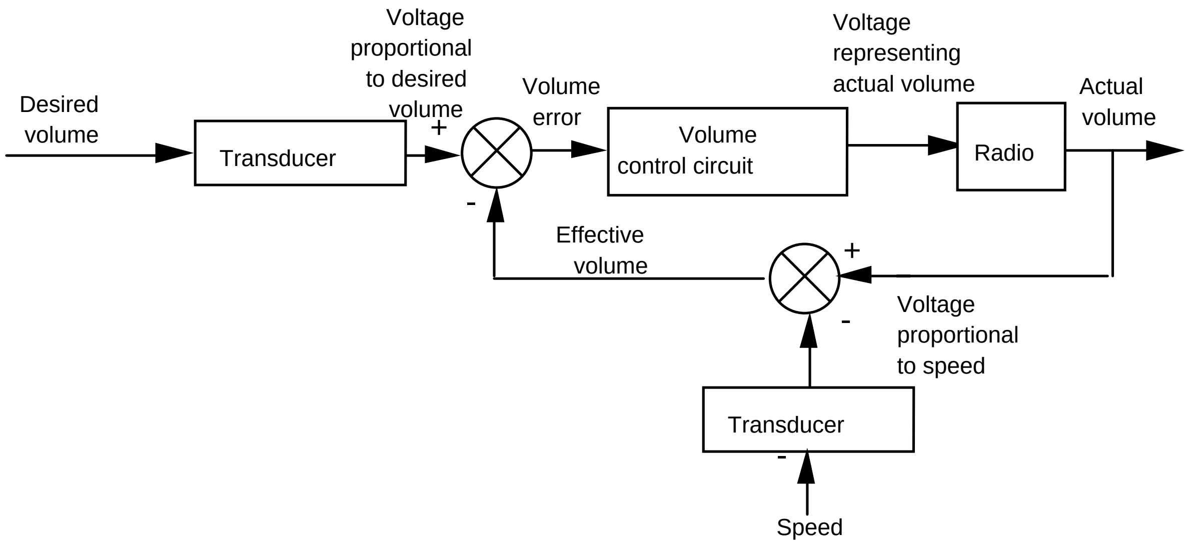
5.


Desired power
Input voltage
Power Error voltage
Rod position
Actual power
Transducer
+
-
Amplifier
Motor and drive system
Reactor
Sensor & transducer
Voltage proportional to actual power

6.

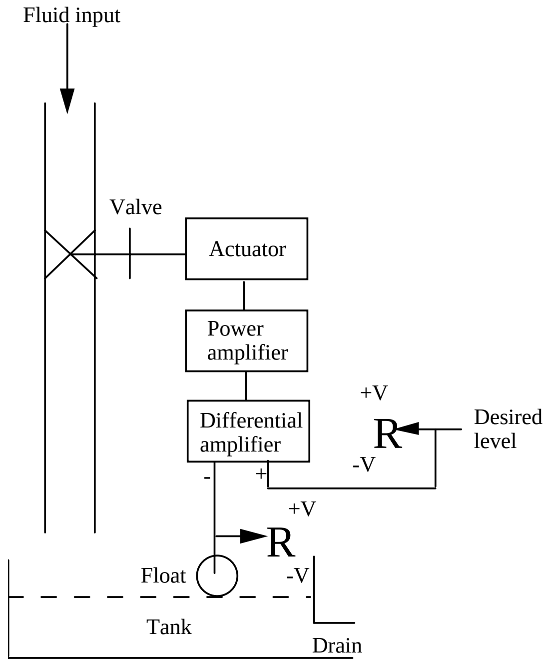

Graduating
and
drop-out
rate
Desired
student
population
Population
error
Desired
student
rate
Actual
student
rate
Net rate
of influx
Actual
student
population
+
-
Administration
Admissions
+
-
Integrate

7.

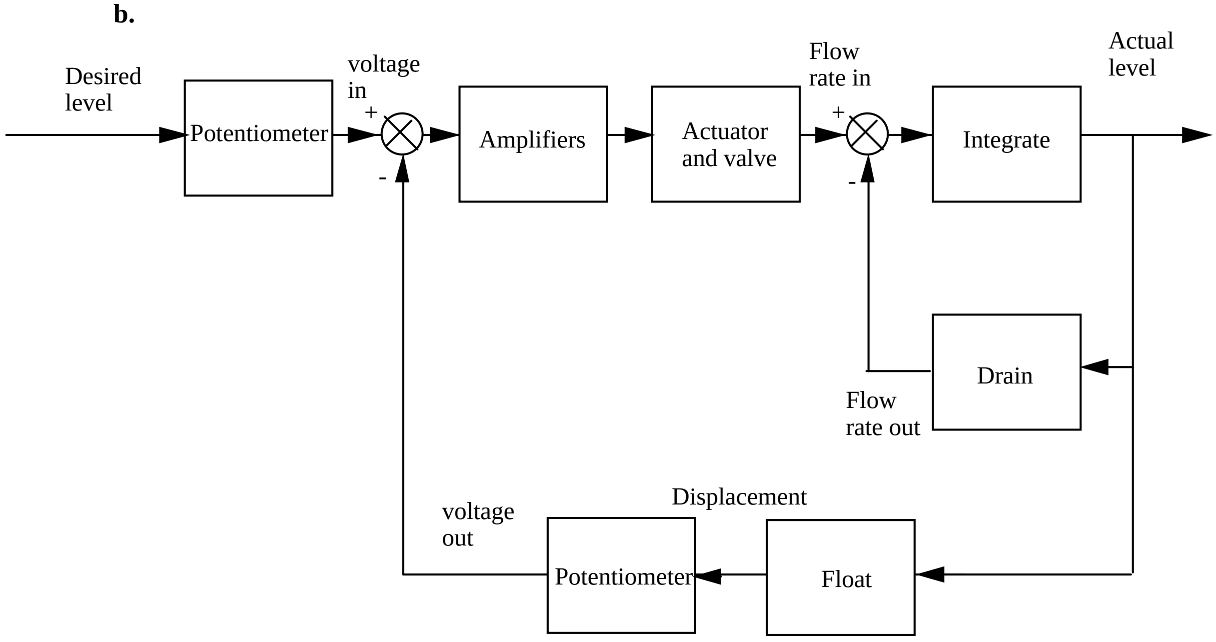

Voltage
proportional
to desired
volume
Voltage
representing
actual volume
Desired
volume
Volume
error
Actual
volume
Transducer
+
Volume
control circuit
Radio
-
Effective
volume
+
-
Voltage
proportional
to speed
Transducer
-
Speed

8.

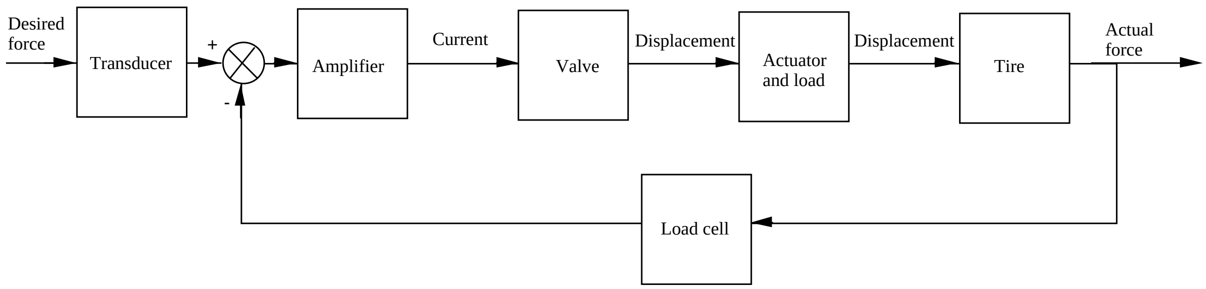
a.


Fluid input
Valve
Actuator
Power amplifier
Differential amplifier
+V
R
-V
Desired level
-
+
+V
R
Float
-V
Tank
Drain

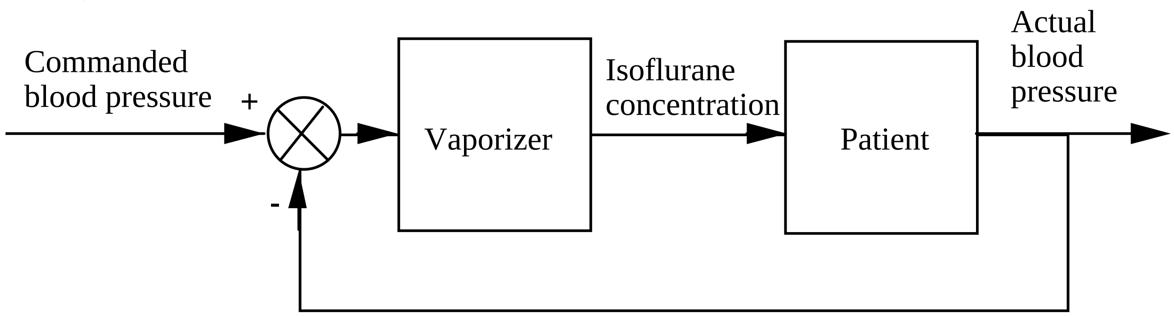
b.


Desired level
Potentiometer
voltage in
+
-
Amplifiers
Actuator and valve
Flow rate in
+
-
Integrate
Actual level
Drain
Flow rate out
Displacement
voltage out
Potentiometer
Float

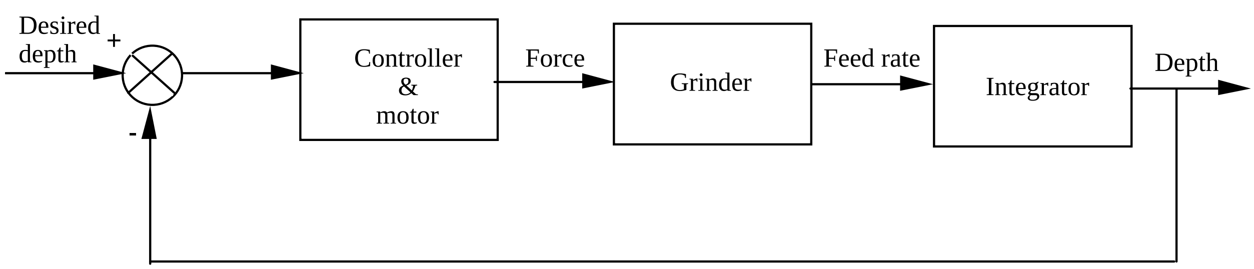
9.



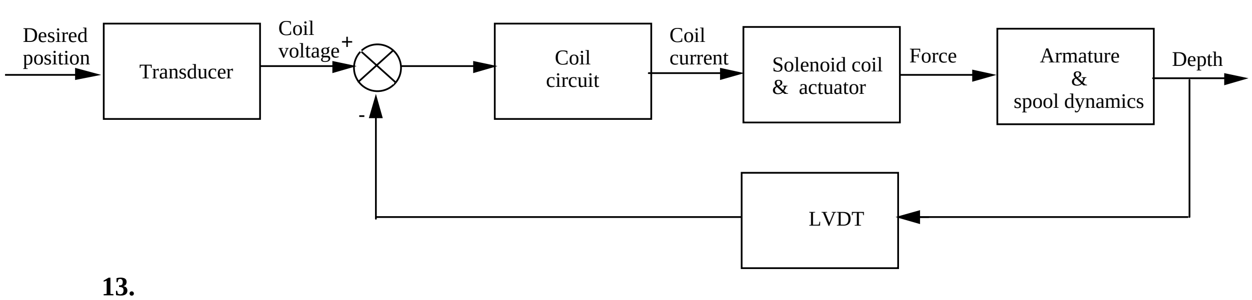
10.



11.



12.



13.

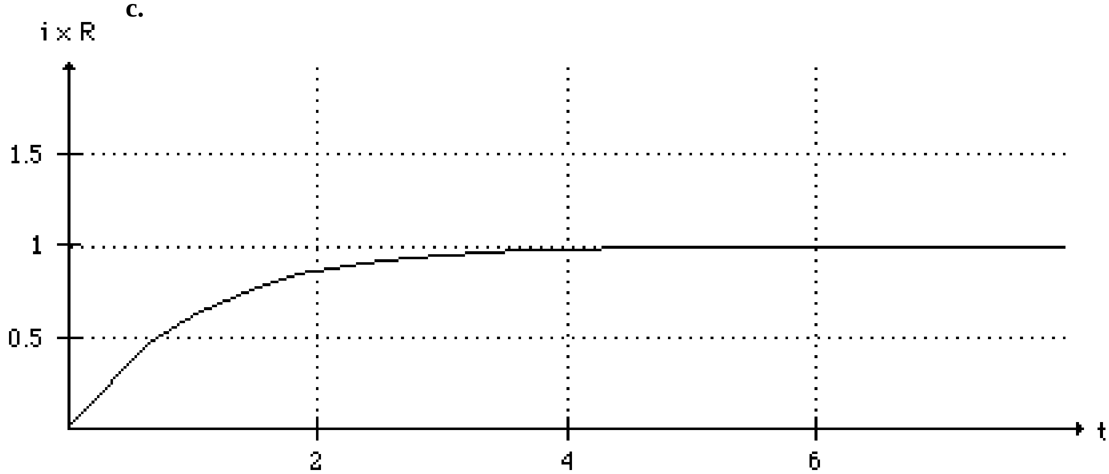
a. $L\dfrac{di}{dt} + Ri = u(t)$

b. Assume a steady-state solution i_{SS} = B. Substituting this into the differential equation yields RB =

1,

from which B = $\frac{1}{R}$. The characteristic equation is LM + R = 0, from which M = -$\frac{R}{L}$. Thus, the total solution is $i(t) = Ae^{-(R/L)t} + \frac{1}{R}$. Solving for the arbitrary constants, i(0) = A + $\frac{1}{R}$ = 0. Thus, A = -$\frac{1}{R}$. The final solution is i(t) = $\frac{1}{R}$ -- $\frac{1}{R}$ $e^{-(R/L)t}$ = $\frac{1}{R}(1 - e^{-(R/L)t})$.

c.



14.

a. Writing the loop equation, $Ri + L\frac{di}{dt} + \frac{1}{C}\int idt + v_C(0) = v(t)$

b. Differentiating and substituting values, $\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 30i = 0$

Writing the characteristic equation and factoring,

$$M^2 + 2M + 30 = \left(M + 1 + \sqrt{29}\ i\right)\left(M + 1 - \sqrt{29}\ i\right).$$

The general form of the solution and its derivative is

$$i = e^{-t}\cos\left(\sqrt{29}\ t\right) A + \left(B \sin\left(\sqrt{29}\ t\right)\right) e^{-t}$$

$$\frac{di}{dt} = \left(-A + \sqrt{29}\ B\right) e^{-t}\cos\left(\sqrt{29}\ t\right) - \left(\sqrt{29}\ A + B\right) e^{-t}\sin\left(\sqrt{29}\ t\right)$$

Using $i(0) = 0; \frac{di}{dt}(0) = \frac{v_L(0)}{L} = \frac{1}{L} = 2$

$i\ (0) = A = 0$

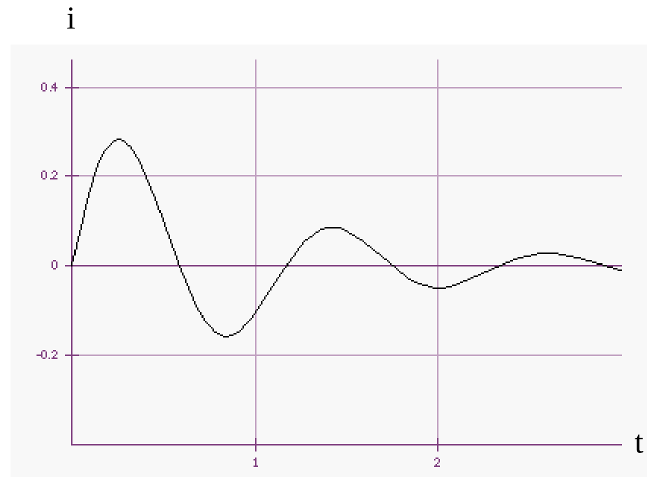
$\frac{di}{dt}(0) = -A + \sqrt{29}B = 2$

Thus, $A = 0$ and $B = \frac{2}{\sqrt{29}}$.

The solution is

$$i = \frac{2}{29}\sqrt{29}\, e^{-t} \sin\left(\sqrt{29}\, t\right)$$

c.



15.

a. Assume a particular solution of

Substitute into the differential equation and obtain

$$(7C + 2D)\cos(2t) + (-2C + 7D)\sin(2t) = 5\cos(2t)$$

Equating like coefficients,

$$7C + 2D = 5$$

$$-2C + 7D = 0$$

From which, $C = \frac{35}{53}$ and $D = \frac{10}{53}$.

The characteristic polynomial is

$$M + 7 = 0$$

Thus, the total solution is

$$x(t) = A e^{-7t} + \left(\frac{35}{53}\cos[2t] + \frac{10}{53}\sin[2t]\right)$$

Solving for the arbitrary constants, $x(0) = A + \frac{35}{53} = 0$. Therefore, $A = -\frac{35}{53}$. The final solution is

$$x(t) = \left(-\frac{35}{53}\right)e^{-7t} + \left(\frac{35}{53}\cos[2t] + \frac{10}{53}\sin[2t]\right)$$

b. Assume a particular solution of

$$x_p = A\sin 3t + B\cos 3t$$

Substitute into the differential equation and obtain

$$(18A - B)\cos(3t) - (A + 18B)\sin(3t) = 5\sin(3t)$$

Therefore, 18A – B = 0 and –(A + 18B) = 5. Solving for A and B we obtain

$$x_p = (-1/65)\sin 3t + (-18/65)\cos 3t$$

The characteristic polynomial is

$$M^2 + 6M + 8 = (M + 4)(M + 2)$$

Thus, the total solution is

$$x = C\,e^{-4t} + D\,e^{-2t} + \left(-\frac{18}{65}\cos(3t) - \frac{1}{65}\sin(3t)\right)$$

Solving for the arbitrary constants, $x(0) = C + D - \frac{18}{65} = 0$.

Also, the derivative of the solution is

$$\frac{dx}{dt} = -\frac{3}{65}\cos(3t) + \frac{54}{65}\sin(3t) - 4C\,e^{-4t} - 2D\,e^{-2t}$$

Solving for the arbitrary constants, $\dot{x}(0) \; -\frac{3}{65} - 4C - 2D = 0$, or $C = -\frac{3}{10}$ and $D = \frac{15}{26}$.

The final solution is

$$x = -\frac{18}{65}\cos(3t) - \frac{1}{65}\sin(3t) - \frac{3}{10}e^{-4t} + \frac{15}{26}e^{-2t}$$

c. Assume a particular solution of

$$x_p = A$$

Substitute into the differential equation and obtain 25A = 10, or A = 2/5.

The characteristic polynomial is

$$M^2 + 8M + 25 = (M + 4 + 3i)(M + 4 - 3i)$$

Thus, the total solution is

$$x = \frac{2}{5} + e^{-4t}\left(B\sin(3t) + C\cos(3t)\right)$$

Solving for the arbitrary constants, x(0) = C + 2/5 = 0. Therefore, C = -2/5. Also, the derivative of the solution is

$$\frac{dx}{dt} = \left((3B - 4C)\cos(3t) - (4B + 3C)\sin(3t)\right)e^{-4t}$$

Solving for the arbitrary constants, $\dot{x}(0) = 3B - 4C = 0$. Therefore, B = -8/15. The final solution is

$$x(t) = \frac{2}{5} - e^{-4t}\left(\frac{8}{15}\sin(3t) + \frac{2}{5}\cos(3t)\right)$$

16.

a. Assume a particular solution of

$$x_p(t) = C\cos(2t) + D\sin(2t)$$

Substitute into the differential equation and obtain

$$-2(C - 2D)\cos(2t) - 4\left(C + \frac{1}{2}D\right)\sin(2t) = \sin(2t)$$

Equating like coefficients,

$$-2(C - 2D) = 0$$
$$-4\left(C + \frac{1}{2}D\right) = 1$$

From which, C = $-\frac{1}{5}$ and D = $-\frac{1}{10}$.

The characteristic polynomial is

$$M^2 + 2M + 2 = (M + 1 + i)(M + 1 - i)$$

Thus, the total solution is

$$x = -\frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t) + e^{-t}(A\cos[t] + B\sin[t])$$

Solving for the arbitrary constants, x(0) = A $-\frac{1}{5}$ = 2. Therefore, A = $\frac{11}{5}$. Also, the derivative of the solution is

$$\frac{dx}{dt} = -\frac{1}{5}\cos(2t) + \frac{2}{5}\sin(2t) + (-A + B)e^{-t}\cos(t) - (A + B)e^{-t}\sin(t)$$

Solving for the arbitrary constants, $\dot{x}(0) = -A + B - 0.2 = -3$. Therefore, B = $-\frac{3}{5}$. The final solution is

$$x(t) = -\frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t) + e^{-t}\left(\frac{11}{5}\cos(t) - \frac{3}{5}\sin(t)\right)$$

b. Assume a particular solution of

$$x_p = Ce^{-2t} + Dt + E$$

Substitute into the differential equation and obtain

$$Ce^{-2t} + Dt + 2D + E = 5e^{-2t} + t$$

Equating like coefficients, C = 5, D = 1, and 2D + E = 0.

From which, C = 5, D = 1, and E = - 2.

The characteristic polynomial is

$$M^2 + 2M + 1 = (M+1)^2$$

Thus, the total solution is

$$x(t) = Ae^{-t} + Be^{-t}t + 5e^{-2t} + t - 2$$

Solving for the arbitrary constants, x(0) = A + 5 - 2 = 2 Therefore, A = -1. Also, the derivative of the solution is

$$\frac{dx}{dt} = (-A+B)e^{-t} - Bte^{-t} - 10e^{-2t} + 1$$

Solving for the arbitrary constants, $\dot{x}(0)$ = B - 8 = 1. Therefore, B = 9. The final solution is

$$x(t) = -e^{-t} + 9te^{-t} + 5e^{-2t} + t - 2$$

c. Assume a particular solution of

$$x_p = Ct^2 + Dt + E$$

Substitute into the differential equation and obtain

$$4Ct^2 + 4Dt + 2C + 4E = t^2$$

Equating like coefficients, C = $\frac{1}{4}$, D = 0, and 2C + 4E = 0.

From which, C = $\frac{1}{4}$, D = 0, and E = - $\frac{1}{8}$.

The characteristic polynomial is

$$M^2 + 4 = (M+2i)(M-2i)$$

Thus, the total solution is

$$x(t) = A\cos(2t) + B\sin(2t) + \frac{1}{4}t^2 - \frac{1}{8}$$

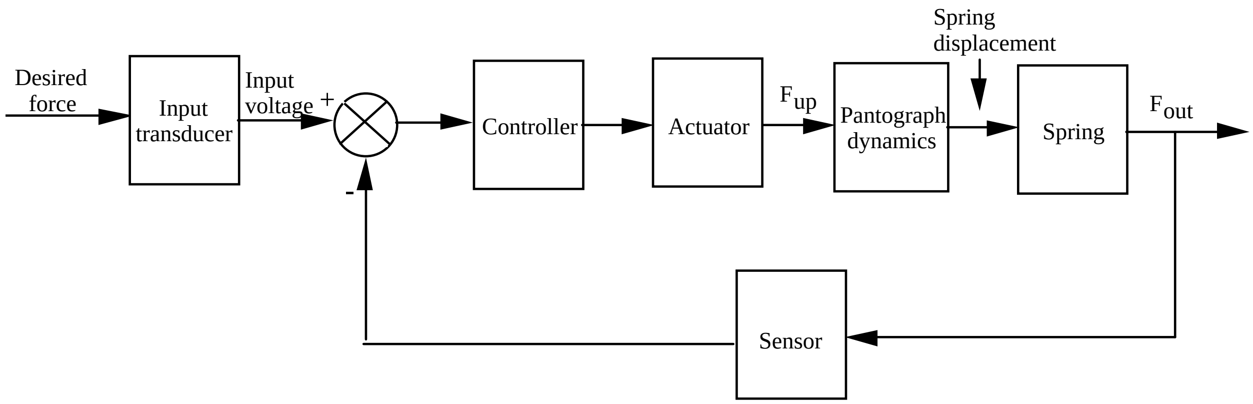
Solving for the arbitrary constants, x(0) = A - $\frac{1}{8}$ = 1 Therefore, A = $\frac{9}{8}$. Also, the derivative of the solution is

$$\frac{dx}{dt} = 2B\cos(2t) - 2A\sin(2t) + \frac{1}{2}t$$

Solving for the arbitrary constants, $\dot{x}(0)$ = 2B = 2. Therefore, B = 1. The final solution is

$$x(t) = \frac{9}{8}\cos(2t) + \sin(2t) + \frac{1}{4}t^2 - \frac{1}{8}$$

17.



Desired force
Input transducer
Input voltage
+
-
Controller
Actuator
F_{up}
Pantograph dynamics
Spring displacement
Spring
F_{out}
Sensor