S E V E N

Steady-State Errors

SOLUTIONS TO CASE STUDIES CHALLENGES

Antenna Control: Steady-State Error Design via Gain

a. $G(s) = \dfrac{76.39K}{s(s+150)(s+1.32)}$. System is Type 1. Step input: $e(\infty) = 0$; Ramp input:

$e(\infty) = \dfrac{1}{K_v} = \dfrac{1}{\dfrac{76.39K}{150 \text{ x } 1.32}} = \dfrac{2.59}{K}$; Parabolic input: $e(\infty) = \infty$.

b. $\dfrac{1}{K_v} = \dfrac{2.59}{K} = 0.2$. Therefore, K = 12.95. Now test the closed-loop transfer function,

$T(s) = \dfrac{989.25}{s^3+151.32s^2+198s+989.25}$, for stability. Using Routh-Hurwitz, the system is stable.

s^3	1	198
s^2	151.32	989.25
s^1	191.46253	0
s^0	989.25	0

Video Laser Disc Recorder: Steady-State Error Design via Gain

a. The input, $15t^2$, transforms into $30/s^3$. $e(\infty) = 30/K_a = 0.005$.

$K_a = \dfrac{0.2*600}{20000} * K_1K_2K_3 = 6x10^{-3} K_1K_2K_3$. Therefore: $e(\infty) = 30/K_a = \dfrac{30}{6x10^{-3}K_1K_2K_3}$

$= 5x10^{-3}$. Therefore $K_1K_2K_3 = 10^6$.

b. Using $K_1K_2K_3 = 10^6$, $G(s) = \dfrac{2x10^5(s+600)}{s^2(s+2x10^4)}$. Therefore, T(s) =

$\dfrac{2x10^5(s+600)}{s^3+2x10^4s^2+2x10^5s+1.2x10^8}$.

Making a Routh table,

s^3	1	$2x10^5$
s^2	$2x10^4$	$1.2x10^8$
s^1	194000	0
s^0	120000000	0

we see that the system is stable.

c.

Program:

```
numg=200000*[1 600];
deng=poly([0 0 -20000]);
G=tf(numg,deng);
'T(s)'
T=feedback(G,1)
poles=pole(T)
```

Computer response:

```
ans =

T(s)

Transfer function:
         200000 s + 1.2e008
------------------------------------
s^3 + 20000 s^2 + 200000 s + 1.2e008

poles =

  1.0e+004 *

  -1.9990
  -0.0005 + 0.0077i
  -0.0005 - 0.0077I
```

ANSWERS TO REVIEW QUESTIONS

1. Nonlinear, system configuration

2. Infinite

3. Step(position), ramp(velocity), parabola(acceleration)

4. Step(position)-1, ramp(velocity)-2, parabola(acceleration)-3

5. Decreases the steady-state error

6. Static error coefficient is much greater than unity.

7. They are exact reciprocals.

8. A test input of a step is used; the system has no integrations in the forward path; the error for a step input is 1/10001.

9. The number of pure integrations in the forward path

10. Type 0 since there are no poles at the origin

11. Minimizes their effect

12. If each transfer function has no pure integrations, then the disturbance is minimized by decreasing the plant gain and increasing the controller gain. If any function has an integration then there is no control over its effect through gain adjustment.

13. No

14. A unity feedback is created by subtracting one from H(s). G(s) with H(s)-1 as feedback form an equivalent forward path transfer function with unity feedback.

15. The fractional change in a function caused by a fractional change in a parameter

16. Final value theorem and input substitution methods

SOLUTIONS TO PROBLEMS

1.

$$e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}$$

where

$$G(s) = \frac{450(s+12)(s+8)(s+15)}{s(s+38)(s^2+2s+28)}.$$

For step, $e(\infty) = 0$. For 37tu(t) , $R(s) = \frac{37}{s^2}$. Thus, $e(\infty) = 6.075\text{x}10^{-2}$. For parabolic input, $e(\infty) = \infty$

2.

$$e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}$$

$$= \lim_{s \to 0} \frac{s(60/s^3)}{1 + \frac{20(s+3)(s+4)(s+8)}{s^2(s+2)(s+15)}} = 0.9375$$

3.

Reduce the system to an equivalent unity feedback system by first moving 1/s to the left past the summing junction. This move creates a forward path consisting of a parallel pair, $\left(\frac{1}{s}+1\right)$ in cascade with a feedback loop consisting of $G(s) = \frac{2}{s+3}$ and $H(s) = 7$. Thus,

$$G_e(s) = \left(\frac{(s+1)}{s}\right)\left(\frac{2/(s+3)}{1+14/(s+3)}\right) = \frac{2(s+1)}{s(s+17)}$$

Hence, the system is Type 1 and the steady-state errors are as follows:

Steady-state error for $15u(t) = 0$.

Steady-state error for $15tu(t) = \frac{15}{K_v} = \frac{15}{2/17} = 127.5$.

Steady-state error for $15t^2u(t) = \infty$

4.

System is type 0. $K_p = \frac{5}{2}$.

For $40u(t)$, $e(\infty) = \frac{40}{1+K_p} = \frac{80}{7} = 11.43$

For 70tu(t), $e(\infty) = \infty$

For $80t^2u(t)$, $e(\infty) = \infty$

5.

$$E(s) = \frac{R(s)}{1+G(s)} = \frac{72/S^4}{1+\frac{200(S+2)(S+5)(S+7)(S+9)}{S^3(S+3)(S+10)(S+15)}}$$

Thus,

$$e(\infty) = \lim_{s\to 0} sE(s) = \frac{72}{\frac{(200)(2)(5)(7)(9)}{(3)(10)(15)}} = 0.2571$$

6.

$$\mathcal{L}\left[\frac{de}{dt}\right] = sE(s)$$

Therefore, $\dot{e}(\infty) = \lim_{s\to 0} s^2E(s) = \lim_{s\to 0} s^2\frac{R(s)}{1+G(s)} = \lim_{s\to 0} \frac{s^2\frac{6}{s^4}}{1+\frac{100(s+1)(s+2)}{s^2(s+10)(s+3)}} = \frac{9}{10}$.

7.

$e(\infty) = \frac{15}{1+K_p}$; $K_p = \frac{1000(12)(25)(32)}{(61)(73)(87)} = 24.78$. Therefore, $e(\infty) = 0.582$.

8.

For 8u(t), $e_{ss} = \frac{8}{1+K_p} = 2$; For 8tu(t), $e_{ss} = \infty$, since the system is Type 0.

9.

a. The closed-loop transfer function is,

$$T(s) = \frac{5000}{s^2+75s+5000}$$

from which, $\omega_n = \sqrt{5000}$ and $2\zeta\omega_n = 75$. Thus, $\zeta = 0.53$ and

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 14.01\%$.

b. $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{75/2} = 0.107$ second.

c. Since system is Type 1, e_{ss} for $5u(t)$ is zero.

d. Since K_v is $\frac{5000}{75} = 66.67$, $e_{ss} = \frac{5}{K_v} = 0.075$.

e. $e_{ss} = \infty$, since system is Type 1.

10.

$$K_v = \lim_{s\to 0} sG(s) = \frac{10^5(3)(10)(20)}{(25)(\alpha)(30)} = 10^4$$

Thus, $\alpha = 8$.

11.

$$K_a = \lim_{s\to 0} s^2 G(s) = \frac{Kx2x4x6}{5x7} = 10{,}000$$. Therefore, K = 7291.667.

12.

a. $G_e(s) = \dfrac{\dfrac{5}{s(s+1)(s+2)}}{1+\dfrac{5(s+3)}{s(s+1)(s+2)}} = \dfrac{5}{s^3+3s^{2+}+7s+15}$

Therefore, $K_p = 1/3$; $K_v = 0$; and $K_a = 0$.

b. For 50u(t), $e(\infty) = \frac{50}{1+K_p} = 37.5$; For 50tu(t), $e(\infty) = \infty$; For $50t^2u(t)$, $e(\infty) = \infty$

c. Type 0

13.

$$E(s) = \frac{R(s)}{1+G(s)}\text{. Thus, } e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s\dfrac{6}{s^4}}{1+\dfrac{1000(s^2+4s+20)(s^2+20s+15)}{s^3(s+2)(s+10)}}$$

$= 4\text{x}10^{-4}$.

14.

Collapsing the inner loop and multiplying by 1000/*s* yields the equivalent forward-path transfer function as,

$$G_e(s) = \frac{10^5(s+2)}{s(s^2+1005s+2000)}$$

Hence, the system is Type 1.

15.

$$\dot{e}(\infty) = \lim_{s\to 0} s^2\,E(s) = \lim_{s\to 0} s^2 \frac{R(s)}{1+G(s)}\ .$$

For Type 0, step input: R(s) = $\frac{1}{s}$, and $\dot{e}(\infty)= \lim_{s\to 0}\frac{s}{1+G(s)} = 0$

For Type 0, ramp input: R(s) = $\frac{1}{s^2}$, and

$$\dot{e}(\infty)=\lim_{s\to 0}\frac{1}{1+G(s)} = \frac{1}{1+\lim_{s\to 0} G(s)} = \frac{1}{1+K_p}$$

For Type 0, parabolic input: R(s) = $\frac{1}{s^3}$, and $\dot{e}(\infty)=\lim_{s\to 0}\frac{1}{s+sG(s)} = \infty$

For Type 1, step input: R(s) = $\frac{1}{s}$, and $\dot{e}(\infty)= \lim_{s\to 0}\frac{s}{1+G(s)} = 0$

For Type 1, ramp input: R(s) = $\frac{1}{s^2}$, and $\dot{e}(\infty)=\lim_{s\to 0}\frac{1}{1+G(s)} = 0$

For Type 1, parabolic input: R(s) = $\frac{1}{s^3}$, and $\dot{e}(\infty)=\lim_{s\to 0}\frac{1}{s+sG(s)} = \frac{1}{K_v}$

For Type 2, step input: R(s) = $\frac{1}{s}$, and $\dot{e}(\infty)= \lim_{s\to 0}\frac{s}{1+G(s)} = 0$

For Type 2, ramp input: R(s) = $\frac{1}{s^2}$, and $\dot{e}(\infty)=\lim_{s\to 0}\frac{1}{1+G(s)} = 0$

For Type 2, parabolic input: R(s) = $\frac{1}{s^3}$, and $\dot{e}(\infty)=\lim_{s\to 0}\frac{1}{s+sG(s)} = 0$

		TYPE		
		0	**1**	**2**
INPUT	**Step**	0	0	0
	Ramp	$\frac{1}{1+K_p}$	0	0
	Parabola	∞	$\frac{1}{K_v}$	0

16.

a. $e(\infty)=\frac{1/10}{K_v} = 0.01;$ where $K_v = \frac{7K}{5x8x12} = 10$. Thus, $K = 685.71$.

b. $K_v = 10$.

c. The minimum error will occur for the maximum gain before instability. Using the Routh-Hurwitz Criterion along with $T(s)=\dfrac{K(s+7)}{s^4+25s^3+196s^2+(480+K)s+7K}$:

s^4	1	196	$7K$	For Stability
s^3	25	$480+K$		
s^2	$4420-K$	$175K$		$K < 4420$
s^1	$-K^2-435K+2121600$			$-1690.2 < K <$ 1255.2
s^0	$175K$			$K > 0$

Thus, for stability and minimum error $K = 1255.2$. Thus, $K_v=\dfrac{7K}{5x8x12}=18.3$ and

$e(\infty)=\dfrac{1/10}{K_v}=\dfrac{1/10}{18.3}=0.0055$.

17.

$e(\infty)=\dfrac{15}{K_v}=\dfrac{15}{Ka/10}=\dfrac{150}{Ka}=0.003$. Hence, Ka = 50,000.

18.

Find the equivalent G(s) for a unity feedback system. $G(s)=\dfrac{\dfrac{K}{s(s+1)}}{1+\dfrac{10}{s+1}}=\dfrac{K}{s(s+11)}$. Thus, $e(\infty)=$

$\dfrac{100}{K_v}=\dfrac{100}{K/11}=0.01$; from which K = 110,000.

19.

$K_a=\dfrac{2K}{4}$. $e(\infty)=\dfrac{20}{K_a}=0.01$. Hence, K = 4000.

20.

a. $e(\infty)=\dfrac{10}{K_v}=\dfrac{1}{6000}$. But, $K_v=\dfrac{30K}{5}=60{,}000$. Hence, K = 10,000. For finite error for a ramp input, n = 1.

b. $K_p=\lim\limits_{s\varnothing 0} G(s)=\lim\limits_{s\varnothing 0}\dfrac{10000(s^2+3s+30)}{s(s+5)}=\infty$

$K_v=\lim\limits_{s\varnothing 0} sG(s)=\lim\limits_{s\varnothing 0} s\dfrac{10000(s^2+3s+30)}{s(s+5)}=60{,}000$

$K_a=\lim\limits_{s\varnothing 0} s^2G(s)=\lim\limits_{s\varnothing 0} s^2\dfrac{10000(s^2+3s+30)}{s(s+5)}=0$

21.

a. Type 0

b. $E(s) = \frac{R(s)}{1 + G(s)}$. Thus, $e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \frac{10/s}{1 + \frac{K(s^2+2s+5)}{(s+2)^2(s+3)}} = \frac{120}{12+5K}$.

c. $e(\infty) = \infty$, since the system is Type 0.

22.

$$e(\infty) = \frac{25}{K_v} = \frac{25}{150K/420} = 0.1.$$ Thus, $K = 700$.

23.

$$e(\infty) = \frac{50}{1 + K_p} = \frac{50}{1 + \frac{4K}{26}} = 0.05.$$ Thus, $K = 6493.5$.

24.

The system is stable for $0 < K < 2000$. Since the maximum K_v is $K_v = \frac{K}{320} = \frac{2000}{320} = 6.25$, the minimum steady-state error is $\frac{1}{K_v} = \frac{1}{6.25} = 0.16$.

25.

To meet steady-state error characteristics:

$$E(s) = \frac{R(s)}{1 + G(s)} = \frac{1}{s\left(1 + \frac{K(s+\alpha)}{(s+\beta)^2}\right)}$$

$$e(t)\Big|_{t\to\infty} = sE(s)\Big|_{s\to 0} = \frac{1}{1 + \frac{K\alpha}{\beta^2}} = \frac{\beta^2}{\beta^2 + K\alpha} = 0.1$$

Therefore, $K\alpha = 9\beta^2$.

To meet the transient requirement: Since $T(s) = \frac{K(s+\alpha)}{s^2 + (K+2\beta)s + (\beta^2 + K\alpha)}$,

$\omega_n^2 = 10 = \beta^2 + K\alpha$; $2\zeta\omega_n = \sqrt{10} = K+2\beta$. Solving for β, $\beta = \pm 1$. For $\beta = +1$, $K = 1.16$ and $\alpha = 7.76$.

An alternate solution is $\beta = -1$, $K = 5.16$, and $\alpha = 1.74$.

26.

a. System Type = 1

b. Assume $G(s) = \frac{K}{s(s+\alpha)}$. Therefore, $e(\infty) = \frac{1}{K_v} = \frac{1}{K/\alpha} = 0.01$, or $\frac{K}{\alpha} = 100$.

But, $T(s) = \frac{G(s)}{1 + G(s)} = \frac{K}{s^2+\alpha s+K}$.

Since $\omega_n = 10$, $K = 100$, and $\alpha = 1$. Hence, $G(s) = \frac{100}{s(s+1)}$.

c. $2\zeta\omega_n = \alpha = 1$. Thus, $\zeta = \frac{1}{20}$.

27.

$T(s) = \frac{G(s)}{1 + G(s)} = \frac{K(s+\alpha)}{s^2+(K+\beta)\, s+\alpha K}$. Hence, $K+\beta = 2$, $K\alpha = \omega_n{}^2 = (1^2+1^2) = 2$.

Also, $e(\infty) = \frac{1}{K_v} = \frac{\beta}{K\alpha} = 0.1$. Therefore, $\beta = 0.1K\alpha = 0.2$, $K = 1.8$, and $\alpha = 1.111$.

28.

System Type = 1. $T(s) = \frac{G(s)}{1 + G(s)} = \frac{K}{s^2+as+K}$. From G(s), $K_v = \frac{K}{a} = 100$. For 10% overshoot, $\zeta =$ 0.6. Therefore, $2\zeta\omega_n = a$, and $\omega_n{}^2 = K$. Hence, $a = 1.2\sqrt{K}$.

Also, $a = \frac{K}{100}$ Solving simultaneously,

$K = 1.44 \times 10^4$, and $a = 1.44 \times 10^2$.

29.

a. For 20% overshoot, $\zeta = 0.456$. Also, $K_v = 1000 = \frac{K}{a}$. Since $T(s) = \frac{K}{s^2+as+K}$, $2\zeta\omega_n = a$, and $\omega_n = \sqrt{K}$. Hence, $a = 0.912\sqrt{K}$. Solving for a and K, K = 831,744, and a = 831.744.

b. For 10% overshoot, $\zeta = 0.591$. Also, $\frac{1}{K_v} = 0.01$. Thus, $K_v = 100 = \frac{K}{a}$. Since $T(s) = \frac{K}{s^2+as+K}$, $2\zeta\omega_n = a$, and $\omega_n = \sqrt{K}$. Hence, $a = 1.182\sqrt{K}$. Solving for a and K, K = 13971 and a = 139.71.

30.

a. For the inner loop:

$$G_1(s) = \frac{\frac{1}{s^2(s+1)}}{1 + \frac{1}{s^3(s+1)}} = \frac{s}{s^4+s^3+1}$$

$$G_e(s) = \frac{1}{s^2(s+3)}\ G_1(s) = \frac{1}{s(s^5+4s^4+3s^3+s+3)}$$

$$T(s) = \frac{G_e(s)}{1+G_e(s)} = \frac{1}{s^6+4s^5+3s^4+s^2+3s+1}$$

b. From $G_e(s)$, system is Type 1.

c. Since system is Type 1, $e_{ss} = 0$

d. ; From $G_e(s)$, $K_v = \lim_{s\to 0} sG_e(s) = \frac{1}{3}$. Therefore, $e_{ss} = \frac{5}{K_v} = 15$.

e. Poles of T(s) = -3.0190, -1.3166, 0.3426 ± j0.7762, -0.3495. Therefore, system is unstable and results of (c) and (d) are meaningless

31.

a. For the inner loop:

$$G_1(s) = \frac{\frac{10}{s(s+1)(s+3)(s+4)}}{1 + \frac{20}{(s+1)(s+3)(s+4)}} = \frac{10}{s(s^3+8s^2+19s+32)}$$

$$G_e(s) = \frac{20}{s(s^3+8s^2+19s+32)}$$

$$T(s) = \frac{G_e(s)}{1+G_e(s)} = \frac{20}{s^4+8s^3+19s^2+32s+20}$$

b. From $G_e(s)$, system is Type 1.

c. Since system is Type 1, $e_{ss} = 0$

d. From $G_e(s)$, $K_v = \lim_{s\to 0} sG_e(s) = \frac{20}{32} = \frac{5}{8}$. Therefore, $e_{ss} = \frac{5}{K_v} = 8$.

e. Poles of T(s) = -5.4755, -0.7622 ± j1.7526, -1. Therefore, system is stable and results of parts c and d are valid.

32.

Program:

```
numg1=[1 7];deng1=poly([0 -4 -8 -12]);
'G1(s)='
G1=tf(numg1,deng1)
numg2=5*poly([-9 -13]);deng2=poly([-10 -32 -64]);
'G2(s)='
G2=tf(numg2,deng2)
numh1=10;denh1=1;
'H1(s)='
H1=tf(numh1,denh1)
numh2=1;denh2=[1 3];
'H2(s)='
H2=tf(numh2,denh2)
%Close loop with H1 and form G3
'G3(s)=G2(s)/(1+G2(s)H1(s)'
G3=feedback(G2,H1)
%Form G4=G1G3
'G4(s)=G1(s)G3(s)'
G4=series(G1,G3)
%Form Ge=G4/1+G4H2
'Ge(s)=G4(s)/(1+G4(s)H2(s))'
Ge=feedback(G4,H2)
%Form T(s)=Ge(s)/(1+Ge(s)) to test stability
'T(s)=Ge(s)/(1+Ge(s))'
T=feedback(Ge,1)
'Poles of T(s)'
pole(T)
%Computer response shows that system is stable. Now find error specs.
Kp=dcgain(Ge)
'sGe(s)='
sGe=tf([1 0],1)*Ge;
'sGe(s)'
sGe=minreal(sGe)
Kv=dcgain(sGe)
's^2Ge(s)='
s2Ge=tf([1 0],1)*sGe;
's^2Ge(s)'
s2Ge=minreal(s2Ge)
Ka=dcgain(s2Ge)
essstep=30/(1+Kp)
essramp=30/Kv
essparabola=60/Ka
```

Computer response:

```
ans =
```

```
G1(s)=



Transfer function:

            s + 7

------------------------------

s^4 + 24 s^3 + 176 s^2 + 384 s

ans =

G2(s)=

Transfer function:

     5 s^2 + 110 s + 585

------------------------------

s^3 + 106 s^2 + 3008 s + 20480

ans =

H1(s)=

Transfer function:

10

ans =

H2(s)=

Transfer function:

  1

-----

s + 3

ans =

G3(s)=G2(s)/(1+G2(s)H1(s)

Transfer function:

     5 s^2 + 110 s + 585
```

```
------------------------------
s^3 + 156 s^2 + 4108 s + 26330


ans =

G4(s)=G1(s)G3(s)

Transfer function:

            5 s^3 + 145 s^2 + 1355 s + 4095

-------------------------------------------------------

s^7 + 180 s^6 + 8028 s^5 + 152762 s^4 + 1.415e006 s^3

                          + 6.212e006 s^2 + 1.011e007 s

ans =

Ge(s)=G4(s)/(1+G4(s)H2(s))

Transfer function:

      5 s^4 + 160 s^3 + 1790 s^2 + 8160 s + 12285

-------------------------------------------------------

s^8 + 183 s^7 + 8568 s^6 + 176846 s^5 + 1.873e006 s^4

        + 1.046e007 s^3 + 2.875e007 s^2 + 3.033e007 s

                                                 + 4095

ans =

T(s)=Ge(s)/(1+Ge(s))

Transfer function:

      5 s^4 + 160 s^3 + 1790 s^2 + 8160 s + 12285

-------------------------------------------------------

s^8 + 183 s^7 + 8568 s^6 + 176846 s^5 + 1.873e006 s^4

        + 1.046e007 s^3 + 2.875e007 s^2 + 3.034e007 s

                                                + 16380
```

```
ans =

Poles of T(s)


ans =

-124.7657

  -21.3495

  -12.0001

   -9.8847

   -7.9999

   -4.0000

   -2.9994

   -0.0005

Kp =

     3

ans =

sGe(s)=

ans =

sGe(s)

Transfer function:

   5 s^5 + 160 s^4 + 1790 s^3 + 8160 s^2 + 1.229e004 s

---------------------------------------------------------

s^8 + 183 s^7 + 8568 s^6 + 1.768e005 s^5 + 1.873e006 s^4

        + 1.046e007 s^3 + 2.875e007 s^2 + 3.033e007 s

                                                 + 4095

Kv =

     0

ans =
```

```
s^2Ge(s)=




ans =

s^2Ge(s)

Transfer function:

  5 s^6 + 160 s^5 + 1790 s^4 + 8160 s^3 + 1.229e004 s^2

---------------------------------------------------------

s^8 + 183 s^7 + 8568 s^6 + 1.768e005 s^5 + 1.873e006 s^4

        + 1.046e007 s^3 + 2.875e007 s^2 + 3.033e007 s

                                                  + 4095

Ka =

     0

essstep =

    7.5000

Warning: Divide by zero.

(Type "warning off MATLAB:divideByZero" to suppress this warning.)

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7 References\p7_32.m at line 40

essramp =

   Inf

Warning: Divide by zero.

(Type "warning off MATLAB:divideByZero" to suppress this warning.)

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7 References\p7_32.m at line 41

essparabola =

   Inf
```

33.

The equivalent forward transfer function is, $G(s) = \frac{10K_1}{s(s+1+10K_f)}$.

Also, $T(s) = \frac{G(s)}{1 + G(s)} = \frac{10K_1}{s^2+(10K_f+1)s+10K_1}$. From the problem statement, $K_v = \frac{10K_1}{1+10K_f} = 10$.

Also, $2\zeta\omega_n = 10K_f+1 = 2(0.5)\sqrt{10K_1} = \sqrt{10K_1}$. Solving for K_1 and K_f simultaneously, $K_1 = 10$ and $K_f = 0.9$.

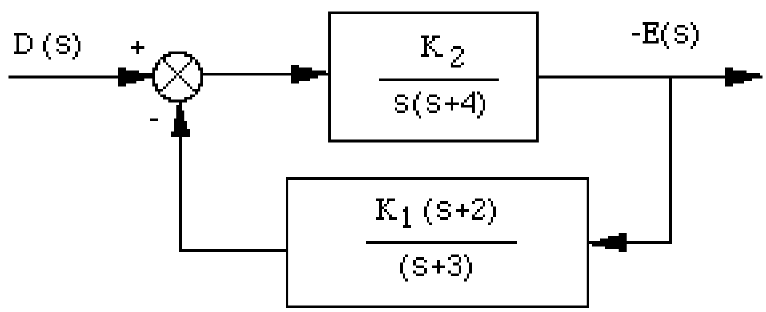
34.

$e(\infty) = \lim_{s \to 0} \frac{sR(s) - sD(s)G_2(s)}{1 + G_1(s)G_2(s)}$, where $G_1(s) = \frac{1}{s+5}$ and $G_2 = \frac{100}{s+2}$. From the problem statement,

$R(s) = D(s) = \frac{1}{s}$. Hence, $e(\infty) = \lim_{s \to 0} \frac{1 - \frac{100}{s+2}}{1 + \frac{1}{s+5}\frac{100}{s+2}} = -\frac{49}{11}$.

35.

Error due only to disturbance: Rearranging the block diagram to show D(s) as the input,



Therefore,

$$-E(s) = D(s)\frac{\frac{K_2}{s(s+4)}}{1 + \frac{K_1K_2(s+2)}{s(s+3)(s+4)}} = D(s)\frac{K_2(s+3)}{s(s+3)(s+4) + K_1K_2(s+2)}$$

For $D(s) = \frac{1}{s}$, $e_D(\infty) = \lim_{s \to 0} sE(s) = -\frac{3}{2K_1}$.

Error due only to input: $e_R(\infty) = \frac{1}{K_v} = \frac{1}{\frac{K_1K_2}{6}} = \frac{6}{K_1K_2}$.

Design:

$e_D(\infty) = -0.000012 = -\frac{3}{2K_1}$, or $K_1 = 125{,}000$.

$e_R(\infty) = 0.003 = \frac{6}{K_1K_2}$, or $K_2 = 0.016$

36.

$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_2(s)H_1(s)}; \ \therefore \frac{E_{a1}(s)}{R(s)} = \frac{G_1(s)}{1+G_2(s)H_1(s)}$$

$$e_{a1}(\infty) = \lim_{s\to 0}\frac{sR(s)G_1(s)}{1+G_2(s)H_1(s)}$$

37.

System 1:

Forming a unity-feedback path, the equivalent unity feedback system has a forward transfer function of

$$G_e(s) = \frac{\frac{10(s+10)}{s(s+2)}}{1+\frac{10(s+10)(s+3)}{s(s+2)}} = \frac{10(s+10)}{11s^2+132s+300}$$

a. Type 0 System; **b.** K_p = $K_p = \lim_{s\to 0} G_e(s) = 1/3$; **c.** step input; **d.** $e(\infty) = \frac{1}{1+K_p} = 3/4$;

e. $e_{a-step}(\infty) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)H(s)} = \lim_{s\to 0}\frac{s\left(\frac{1}{s}\right)}{1+\frac{10(s+10)(s+4)}{s(s+2)}} = 0$.

System 2:

Forming a unity-feedback path, the equivalent unity feedback system has a forward transfer function of

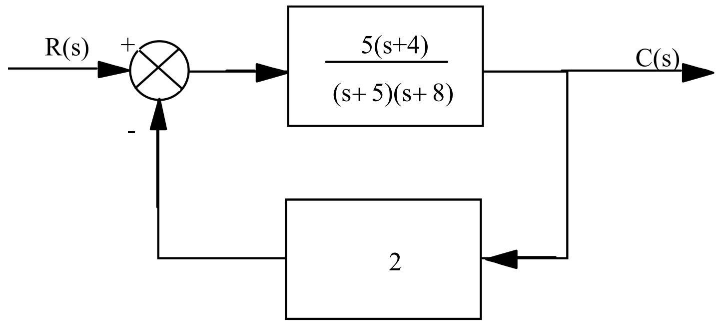
$$G_e(s) = \frac{\frac{10(s+10)}{s(s+2)}}{1+\frac{10(s+10)s}{s(s+2)}} = \frac{10(s+10)}{s(11s+102)}$$

a. Type 1 System; **b.** $K_v = \lim_{s\to 0} sG_e(s) = 0.98$; **c.** ramp input; **d.** $e(\infty) = \frac{1}{K_v} = 1.02$;

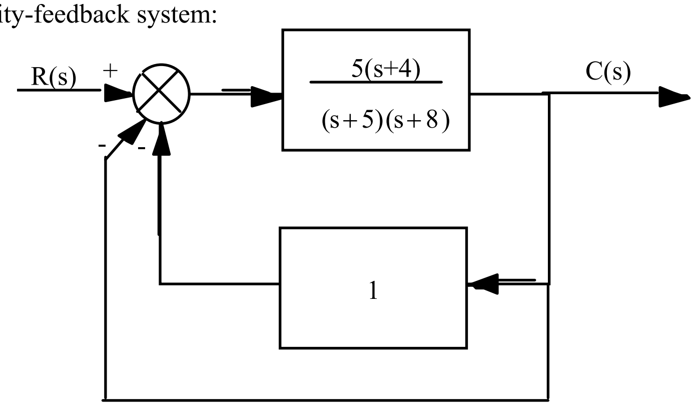
e. $e_{a-ramp}(\infty) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)H(s)} = \lim_{s\to 0}\frac{s\left(\frac{1}{s^2}\right)}{1+\frac{10(s+10)(s+1)}{s(s+2)}} = \frac{1}{50}$.

38.

System 1. Push 5 to the right past the summing junction:



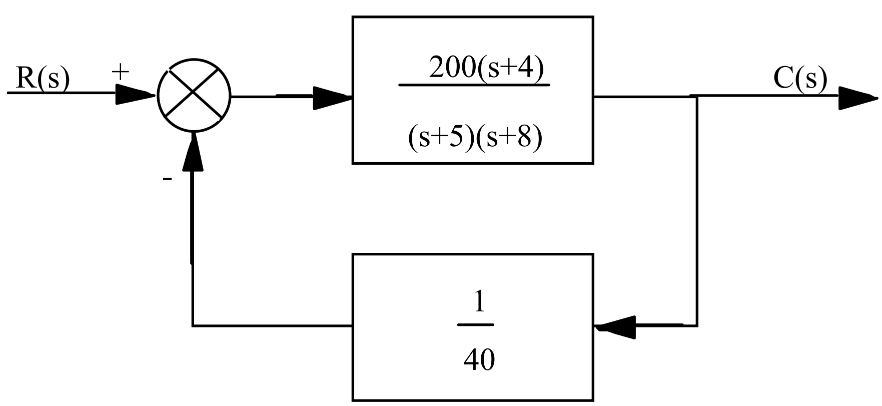
Produce a unity-feedback system:



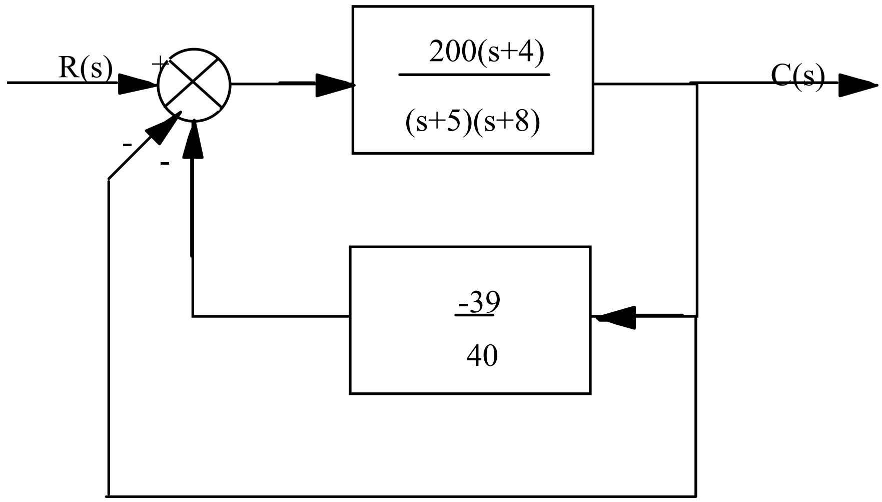
Thus, $G_e(s) = \dfrac{\dfrac{5(s+4)}{(s+5)(s+8)}}{1+\dfrac{5(s+4)}{(s+5)(s+8)}} = \dfrac{5(s+4)}{s^2+18s+60}$. $K_p = \dfrac{1}{3}$. $e_{step} = \dfrac{1}{1+K_p} = 0.75$, $e_{ramp} = \infty$, $e_{parabola} = \infty$.

Checking for stability, from first block diagram above, T(s) = $\dfrac{5(s+4)}{s^2+23s+80}$. The system is stable.

System 2. Push 20 to the right past the summing junction and push 10 to the left past the pickoff point:



Produce a unity-feedback system:



Thus, $G_e(s) = \dfrac{\dfrac{200(s+4)}{(s+5)(s+8)}}{1 - \dfrac{200(s+4)}{(s+5)(s+8)}\left(\dfrac{39}{40}\right)} = \dfrac{200(s+4)}{s^2 - 182s - 740}$. $K_p = \dfrac{200(4)}{-740} = -1.08$.

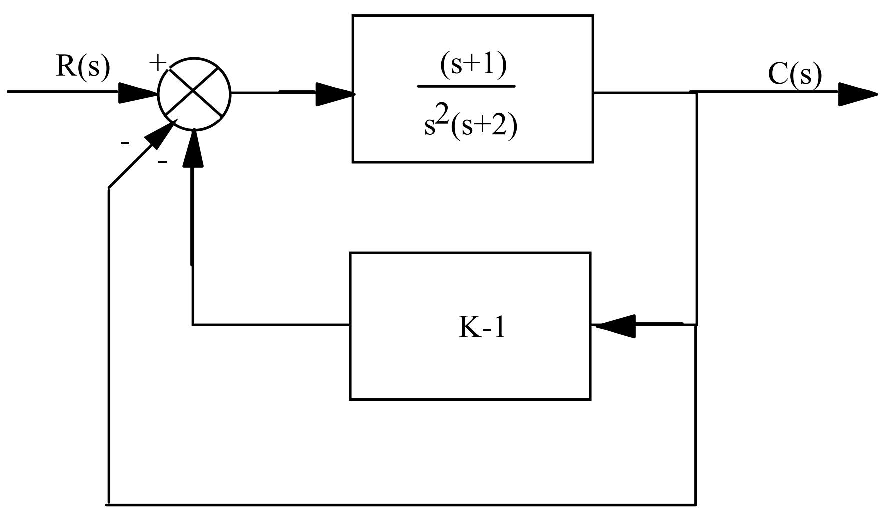
$e_{step} = \dfrac{1}{1+K_p} = -12.5$, $e_{ramp} = \infty$, $e_{parabola} = \infty$.

Checking for stability, from first block diagram above, $T(s) = \dfrac{G_e(s)}{1+G_e(s)} = \dfrac{200(s+4)}{s^2+18s+60}$.

Therefore, system is stable and steady-state error calculations are valid.

39.

Produce a unity-feedback system:



Thus, $G_e(s) = \dfrac{\dfrac{(s+1)}{s^2(s+2)}}{1+\dfrac{(s+1)(K-1)}{s^2(s+2)}} = \dfrac{s+1}{s^3+2s^2+(K-1)s+(K-1)}$. Error = 0.001 = $\dfrac{1}{1+K_p}$.

Therefore, $K_p = 999 = \dfrac{1}{K-1}$. Hence, K = 1.001001.

Check stability: Using original block diagram, $T(s) = \dfrac{\dfrac{(s+1)}{s^2(s+2)}}{1+\dfrac{K(s+1)}{s^2(s+2)}} = \dfrac{s+1}{s^3+2s^2+Ks+K}$.

Making a Routh table:

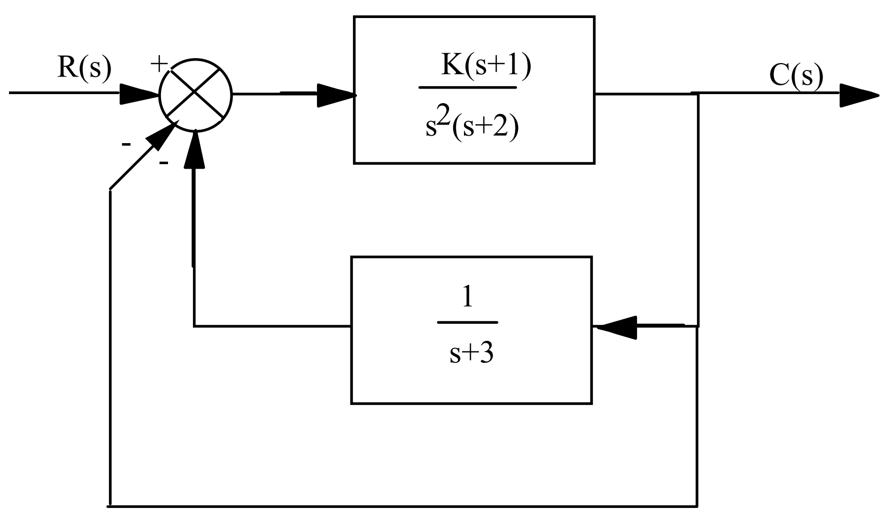
s^3	1	K
s^2	2	K
s^1	$\frac{K}{2}$	0
s^0	K	0

Therefore, system is stable and steady-state error calculations are valid.

40.

a. Produce a unity-feedback system:

$$H_1(s) = \frac{s+4}{s+3} - 1 = \frac{1}{s+3}$$



Thus, $G_e(s) = \dfrac{\dfrac{K(s+1)}{s^2(s+2)}}{1+\dfrac{\frac{1}{s+3}K(s+1)}{s^2(s+2)}} = \dfrac{K(s+1)(s+3)}{s^4+5s^3+6s^2+Ks+K}$. System is Type 0.

b. Since Type 0, appropriate static error constant is K_p.

c. $K_p = \frac{3K}{K} = 3$

d. $e_{step} = \frac{1}{1+K_p} = \frac{1}{4}$

Check stability: Using original block diagram, $T(s) = \dfrac{\dfrac{K(s+1)}{s^2(s+2)}}{1+\dfrac{(s+4)K(s+1)}{(s+3)s^2(s+2)}} = \dfrac{K(s+1)(s+3)}{s^4+5s^3+(K+6)s^2+5Ks+4K}$.

Making a Routh table:

s^4	1	K+6	4K
s^3	5	5K	0
s^2	6	4K	0
s^1	$\frac{5}{3}K$	0	0
s^0	4K	0	0

Therefore, system is stable for $0 < K$ and steady-state error calculations are valid.

41.

Program:

```
K=10
numg1=K*poly([-1 -2]);deng1=poly([0 0 -3 -4 -5]);
'G1(s)='
G1=tf(numg1,deng1)
numh1=[1 6];denh1=poly([-7 -8]);
'H1(s)='
H1=tf(numh1,denh1)
'H2(s)=H1-1'
H2=H1-1
%Form Ge(s)=G1(s)/(1+G1(s)H2(s)
'Ge(s)=G1(s)/(1+G1(s)H2(s))'
Ge=feedback(G1,H2)
%Test system stability
'T(s)=Ge(s)/(1+Ge(s))'
T=feedback(Ge,1)
pole(T)
Kp=dcgain(Ge)
'sGe(s)'
sGe=tf([1 0],1)*Ge;
sGe=minreal(sGe)
Kv=dcgain(sGe)
's^2Ge(s)'
s2Ge=tf([1 0],1)*sGe;
s2Ge=minreal(s2Ge)
Ka=dcgain(s2Ge)
essstep=30/(1+Kp)
essramp=30/Kv
essparabola=60/Ka

K=1E6
numg1=K*poly([-1 -2]);deng1=poly([0 0 -3 -4 -5]);
'G1(s)='
G1=tf(numg1,deng1)
numh1=[1 6];denh1=poly([-7 -8]);
```

```
'H1(s)='
H1=tf(numh1,denh1)
'H2(s)=H1-1'
H2=H1-1
%Form Ge(s)=G1(s)/(1+G1(s)H2(s)
'Ge(s)=G1(s)/(1+G1(s)H2(s))'
Ge=feedback(G1,H2)
%Test system stability
'T(s)=Ge(s)/(1+Ge(s))'
T=feedback(Ge,1)
pole(T)
Kp=dcgain(Ge)
'sGe(s)'
sGe=tf([1 0],1)*Ge;
sGe=minreal(sGe)
Kv=dcgain(sGe)
's^2Ge(s)'
s2Ge=tf([1 0],1)*sGe;
s2Ge=minreal(s2Ge)
Ka=dcgain(s2Ge)
essstep=30/(1+Kp)
essramp=30/Kv
essparabola=60/Ka
```

Computer response:

```
K =

    10


ans =

G1(s)=


Transfer function:
      10 s^2 + 30 s + 20
------------------------------
s^5 + 12 s^4 + 47 s^3 + 60 s^2


ans =

H1(s)=


Transfer function:
     s + 6
---------------
s^2 + 15 s + 56


ans =

H2(s)=H1-1


Transfer function:
-s^2 - 14 s - 50
----------------
s^2 + 15 s + 56


ans =

Ge(s)=G1(s)/(1+G1(s)H2(s))
```

```
Transfer function:

             10 s^4 + 180 s^3 + 1030 s^2 + 1980 s + 1120
---------------------------------------------------------------------
s^7 + 27 s^6 + 283 s^5 + 1427 s^4 + 3362 s^3 + 2420 s^2 - 1780 s

                                                               - 1000

ans =

T(s)=Ge(s)/(1+Ge(s))


Transfer function:
             10 s^4 + 180 s^3 + 1030 s^2 + 1980 s + 1120
---------------------------------------------------------------------
s^7 + 27 s^6 + 283 s^5 + 1437 s^4 + 3542 s^3 + 3450 s^2 + 200 s + 120


ans =

  -7.6131
  -7.4291
  -5.2697
  -3.3330 + 0.1827i
  -3.3330 - 0.1827i
  -0.0111 + 0.1898i
  -0.0111 - 0.1898i


Kp =

   -1.1200


ans =

sGe(s)


Transfer function:

           10 s^5 + 180 s^4 + 1030 s^3 + 1980 s^2 + 1120 s
---------------------------------------------------------------------
s^7 + 27 s^6 + 283 s^5 + 1427 s^4 + 3362 s^3 + 2420 s^2 - 1780 s

                                                               - 1000


Kv =

     0


ans =

s^2Ge(s)


Transfer function:

          10 s^6 + 180 s^5 + 1030 s^4 + 1980 s^3 + 1120 s^2
---------------------------------------------------------------------
s^7 + 27 s^6 + 283 s^5 + 1427 s^4 + 3362 s^3 + 2420 s^2 - 1780 s

                                                               - 1000
```

```
Ka =

     0


essstep =

 -250.0000

Warning: Divide by zero.
(Type "warning off MATLAB:divideByZero" to suppress this warning.)
> In D:\My Documents\Control Systems Engineering Book\CSE 4th ed\Solutions Manual\Chap 7
References\p7_41.m at line 27

essramp =

   Inf

Warning: Divide by zero.
(Type "warning off MATLAB:divideByZero" to suppress this warning.)
> In D:\My Documents\Control Systems Engineering Book\CSE 4th ed\Solutions Manual\Chap 7
References\p7_41.m at line 28

essparabola =

   Inf


K =

     1000000


ans =

G1(s)=


Transfer function:
 1e006 s^2 + 3e006 s + 2e006
------------------------------
s^5 + 12 s^4 + 47 s^3 + 60 s^2


ans =

H1(s)=


Transfer function:
     s + 6
---------------
s^2 + 15 s + 56


ans =

H2(s)=H1-1


Transfer function:
-s^2 - 14 s - 50
----------------
s^2 + 15 s + 56
```

```
ans =

Ge(s)=G1(s)/(1+G1(s)H2(s))


Transfer function:

   1e006 s^4 + 1.8e007 s^3 + 1.03e008 s^2 + 1.98e008 s + 1.12e008
---------------------------------------------------------------------
s^7 + 27 s^6 + 283 s^5 - 998563 s^4 - 1.7e007 s^3 - 9.4e007 s^2

                                                 - 1.78e008 s - 1e008

ans =

T(s)=Ge(s)/(1+Ge(s))


Transfer function:

   1e006 s^4 + 1.8e007 s^3 + 1.03e008 s^2 + 1.98e008 s + 1.12e008
---------------------------------------------------------------------
s^7 + 27 s^6 + 283 s^5 + 1437 s^4 + 1.004e006 s^3 + 9.003e006 s^2

                                                  + 2e007 s + 1.2e007


ans =

 -26.9750 +22.2518i
 -26.9750 -22.2518i
  17.9750 +22.2398i
  17.9750 -22.2398i
  -6.0000
  -1.9998
  -1.0002


Kp =

   -1.1200


ans =

sGe(s)


Transfer function:

 1e006 s^5 + 1.8e007 s^4 + 1.03e008 s^3 + 1.98e008 s^2 + 1.12e008 s
---------------------------------------------------------------------
s^7 + 27 s^6 + 283 s^5 - 9.986e005 s^4 - 1.7e007 s^3 - 9.4e007 s^2

                                                 - 1.78e008 s - 1e008

Kv =

     0


ans =

s^2Ge(s)
```

```
Transfer function:

1e006 s^6 + 1.8e007 s^5 + 1.03e008 s^4 + 1.98e008 s^3 + 1.12e008 s^2
--------------------------------------------------------------------
s^7 + 27 s^6 + 283 s^5 - 9.986e005 s^4 - 1.7e007 s^3 - 9.4e007 s^2

                                                 - 1.78e008 s - 1e008

Ka =

     0


essstep =

 -250.0000

Warning: Divide by zero.
(Type "warning off MATLAB:divideByZero" to suppress this warning.)
> In D:\My Documents\Control Systems Engineering Book\CSE 4th ed\Solutions Manual\Chap 7
References\p7_41.m at line 56

essramp =

   Inf

Warning: Divide by zero.
(Type "warning off MATLAB:divideByZero" to suppress this warning.)
> In D:\My Documents\Control Systems Engineering Book\CSE 4th ed\Solutions Manual\Chap 7
References\p7_41.m at line 57

essparabola =

   Inf
```

42.

$$Y(s) = R(s)\frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)} + \frac{D(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

$$E(s) = R(s) - Y(s) = R(s) - \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}\ R(s) - \frac{D(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

$$= \left[1 - \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}\right] R(s) - \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}\ D(s)$$

Thus,

$$e(\infty) = \lim_{s\varnothing 0} sE(s) = \lim_{s\varnothing 0} \left\{ \left[1 - \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}\right] R(s) - \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} D(s)\right\}$$

43.

a. $E(s) = R(s) - C(s)$. But, $C(s) = [R(s) - C(s)H(s)]G_1(s)G_2(s) + D(s)$. Solving for $C(s)$,

$$C(s) = \frac{R(s)G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)} + \frac{D(s)}{1 + G_1(s)G_2(s)H(s)}$$

Substituting into $E(s)$,

$$E(s) = \left[1 - \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}\right] R(s) - \frac{1}{1 + G_1(s)G_2(s)H(s)}\ D(s)$$

b. For $R(s) = D(s) = \frac{1}{s}$,

$$e(\infty) = \lim_{s\to 0} sE(s) = 1 - \frac{\lim_{s\to 0} G_1(s)G_2(s)}{1 + \lim_{s\to 0} G_1(s)G_2(s)H(s)} - \frac{1}{1 + \lim_{s\to 0} G_1(s)G_2(s)H(s)}$$

c. Zero error if $G_1(s)$ and/or $G_2(s)$ is Type 1. Also, $H(s)$ is Type 0 with unity dc gain.

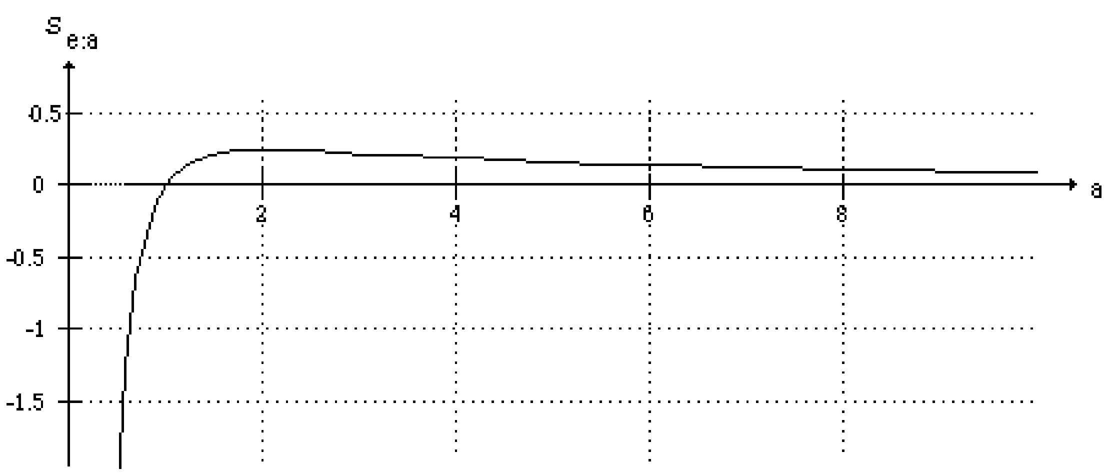
44.

First find the forward transfer function of an equivalent unity feedback system.

$$G_e(s) = \frac{\dfrac{K}{s(s+1)(s+4)}}{1+\dfrac{K(s+a-1)}{s(s+1)(s+4)}} = \frac{K}{s^3+5s^2+(K+4)s+K(a-1)}$$

Thus, $e(\infty) = e(\infty) = \dfrac{1}{1+K_p} = \dfrac{1}{1+\dfrac{K}{K(a-1)}} = \dfrac{a-1}{a}$

Finding the sensitivity of $e(\infty)$, $S_{e:a} = \dfrac{a}{e}\dfrac{\delta e}{\delta a} = \dfrac{a}{\dfrac{a}{a-1}}\left(\dfrac{a-(a-1)}{a^2}\right) = \dfrac{a-1}{a^2}$.



45.

From Eq. (7.70),

$$e(\infty) = 1 - \lim_{s\to 0}\left(\frac{\dfrac{K_1K_2}{(s+2)}}{1+\dfrac{K_1K_2(s+1)}{(s+2)}}\right) - \lim_{s\to 0}\left(\frac{\dfrac{K_2}{(s+2)}}{1+\dfrac{K_1K_2(s+1)}{(s+2)}}\right) = \frac{2-K_2}{2+K_1K_2}$$

Sensitivity to K_1:

$$S_{e:K_1} = \frac{K_1}{e}\frac{\delta e}{\delta K_1} = -\frac{K_1K_2}{2+K_1K_2} = -\frac{(100)(0.1)}{2+(100)(0.1)} = -0.833$$

Sensitivity to K_2:

$$S_{e:K_2} = \frac{K_2}{e}\frac{\delta e}{\delta K_2} = \frac{2K_2(1+K_1)}{(K_2-2)(2+K_1K_2)} = \frac{2(0.1)(1+100)}{(0.1-2)(2+(100)(0.1))} = -0.89$$

46.

a. Using Eq. (7.89) with

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{s^3+20s^2+111s+164}\begin{bmatrix} s^2+15s+50 & -(4s+22) & -(2s+20) \\ -(3s+15) & s^2+10s+23 & 6 \\ -(s+13) & s+9 & s^2+15s+38 \end{bmatrix}$$

yields $e(\infty) = 1.09756$ for a step input and $e(\infty) = \infty$ for a ramp input. The same results are obtained using

$$A^{-1} = -\frac{1}{164}\begin{pmatrix} 50 & -22 & -20 \\ -15 & 23 & 6 \\ -13 & 9 & 38 \end{pmatrix}$$

and Eq. (7.96) for a step input and Eq. (7.103) for a ramp input.

b. Using Eq. (7.89) with

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{s^3+9s^2+5s+7}\begin{bmatrix} s^2+9s & s & 7 \\ -(5s+7) & s^2 & 7s \\ -(s+9) & -1 & s^2+9s+5 \end{bmatrix}$$

yields $e(\infty) = 0$ for a step input and $e(\infty) = \frac{5}{7}$ for a ramp input. The same results are obtained using

$$A^{-1} = -\frac{1}{7}\begin{pmatrix} 0 & 0 & 7 \\ -7 & 0 & 0 \\ -9 & -1 & 5 \end{pmatrix}$$

and Eq. (12.123) for a step input and Eq. (12.130) for a ramp input.

c. Using Eq. (7.89) with

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{s^3+14s^2+43s+17}\begin{pmatrix} s^2+5s-4 & -5s-23 & -s+10 \\ s+11 & s^2+14s+42 & -2s-19 \\ -3s-2 & -2s-3 & s^2+9s+5 \end{pmatrix}$$

yields $e(\infty) = 6$ for a step input and $e(\infty) = \infty$ for a ramp input. The same results are obtained using

$$A^{-1} = -\frac{1}{17}\begin{pmatrix} -4 & -23 & 10 \\ 11 & 42 & -19 \\ -2 & -3 & 5 \end{pmatrix}$$

and Eq. (7.96) for a step input and Eq. (7.103) for a ramp input.

47.

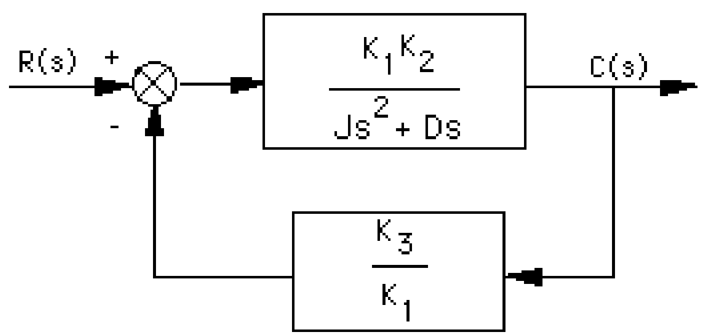
Find $G_4(s)$: Since 100 mi/hr = 146.67 ft/sec, the velocity response of $G_4(s)$ to a step displacement of the accelerator is $v(t) = 146.67(1 - e^{-\alpha t})$. Since 60 mi/hr = 88 ft/sec, the velocity equation at 10 seconds becomes $88 = 146.67\,(1 - e^{-\alpha 10})$. Solving for α yields $\alpha = 0.092$. Thus, $G_4(s) = \frac{K_1}{s+0.092}$. But, from the velocity equation, the dc value of $G_4(s)$ is $\frac{K_1}{0.092} = 146.67$. Solving for K_1, $G_4(s) = \frac{13.49}{s+0.092}$.

Find error: The forward transfer function of the velocity control loop is

$G_3(s)G_4(s) = \dfrac{13.49K}{s(s+1)(s+0.092)}$. Therefore, $K_v = \dfrac{13.49K}{0.092}$. $e(\infty) = \dfrac{1}{K_v} = 6.82 \times 10^{-3}K$.

48.

First, reduce the system to an equivalent unity feedback system. Push K_1 to the right past the summing junction.



Convert to a unity feedback system by adding a unity feedback path and subtracting unity from $\frac{K_1}{K_3}$. The equivalent forward transfer function is,

$$G_e(s) = \frac{\dfrac{K_1K_2}{Js^2+Ds}}{1 + \dfrac{K_1K_2}{Js^2+Ds}\left(\dfrac{K_3}{K_1} - 1\right)} = \frac{K_1K_2}{Js^2+Ds+K_2(K_3-K_1)}$$

The system is Type 0 with $K_p = \dfrac{K_1}{K_3 - K_1}$. Assuming the input concentration is R_o,

$e(\infty) = \dfrac{R_o}{1+K_p} = \dfrac{R_o(K_3 - K_1)}{K_3}$. The error can be reduced if $K_3 = K_1$.

49.

a. For the inner loop, $G_{1e}(s) = \dfrac{K\dfrac{(s+0.01)}{s^2}}{1+K\dfrac{(s+0.01)}{s^2}} = \dfrac{K\,(s+0.01)}{s^2+Ks+0.01K}$, where $K = \dfrac{K_c}{J}$.

Form $G_e(s) = G_{1e}(s)\,\dfrac{(s+0.01)}{s^2} = K\,\dfrac{(s+0.01)^2}{s^2(s^2+Ks+0.01K)}$.

System is Type 2. Therefore, $e_{step} = 0$,

b. $e_{ramp} = 0$,

c. $e_{parabola} = \dfrac{1}{K_a} = \dfrac{1}{0.01} = 100$

d. $T(s) = \dfrac{G_e(s)}{1+G_e(s)} = \dfrac{K(s+0.01)^2}{s^4+Ks^3+1.01Ks^2+0.02Ks+10^{-4}K}$

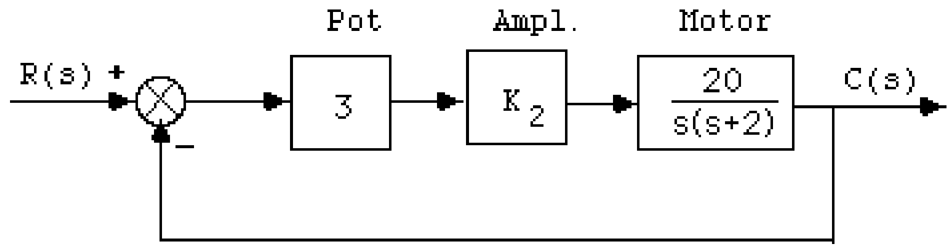
s^4	1	1.01K	10^{-4}K	
s^3	K	0.02K	0	$0 < K$
s^2	1.01K-0.02	10^{-4}K	0	$0.0198 < K$
s^1	$\dfrac{0.0201\,K^2-0.0004\,K}{1.01\,K-0.02}$	0	0	$0.0199 < K$
s^0	10^{-4}K			$0 < K$

Thus, for stability $K = \dfrac{K_c}{J} > 0.0199$

SOLUTIONS TO DESIGN PROBLEMS

50.

Pot gains: $K_1 = \dfrac{3\pi}{\pi} = 3$; Amplifier gain: K_2 ; Motor transfer function: Since time constant = 0.5, α = 2. Also, $\dfrac{K}{\alpha} = \dfrac{100}{10} = 10$. Hence, K = 20. The motor transfer function is now computed as $\dfrac{C(s)}{E_a(s)} = \dfrac{20}{s(s+2)}$. The following block diagram results after pushing the potentiometers to the right past the summing junction:



Finally, since $K_v = 10 = \frac{60K_2}{2}$, from which $K_2 = \frac{1}{3}$.

51.

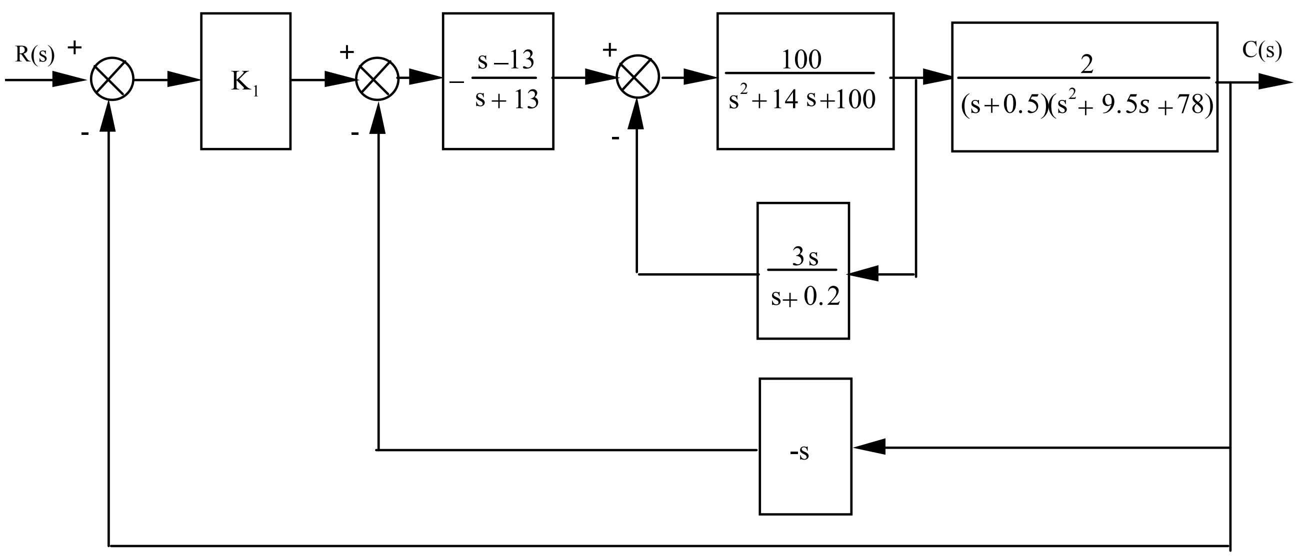
First find K_V: Circumference = 2π nautical miles. Therefore, boat makes 1 revolution in $\frac{2\pi}{20} = 0.314$ hr.

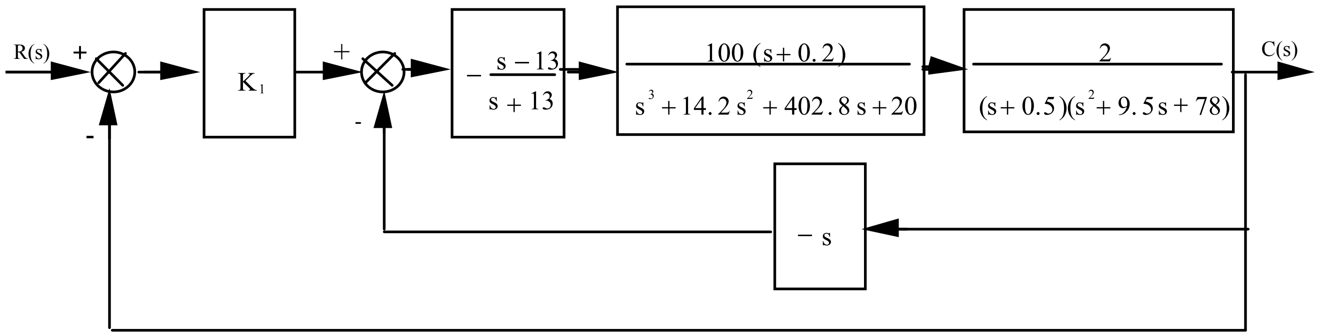
Angular velocity is thus, $\frac{1}{0.314}\frac{\text{rev}}{\text{hr}} = \frac{2\pi}{3600 \text{ x } 0.314}\frac{\text{rad}}{\text{sec}} = 5.56 \text{ x } 10^{-3}\frac{\text{rad}}{\text{sec}}$.

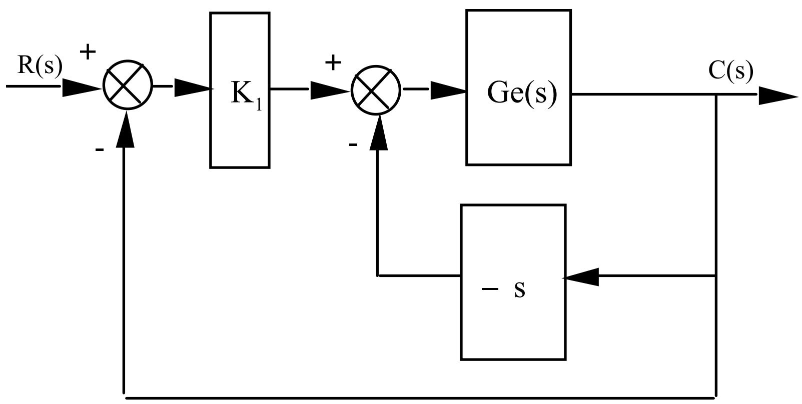
For 0.1^o error, $e(\infty) = \frac{1/10^o}{360^o} \text{ x } 2\pi \text{ rad} = \frac{5.56 \text{ x } 10^{-3}}{K_v}$. Thus $K_v = 3.19 = \frac{K}{4}$; from which, K = 12.76.

52.

a. Performing block diagram reduction:







$$G_e(s) = (-200)\frac{s^2-12.8s-2.6}{s^7+37.2s^6+942.15s^5+13420s^4+1.0249\times10^5 s^3+4.6048\times10^5 s^2+2.2651\times10^5 s+10140}$$

System is unity feedback with a forward transfer function, $G_t(s)$, where

$$G_t(s) = -200K_1\frac{s^2-12.8s-2.6}{s^7+37.2s^6+942.15s^5+13420s^4+1.0269\times10^5 s^3+4.5792\times10^5 s^2+2.2599\times10^5 s+10140}$$

Thus, system is Type 0.

b. From $G_t(s)$, $K_p = \frac{520K_1}{10140} = 700$. Thus, $K_1 = 13650$.

c. $T(s) = \frac{G_t}{1+G_t}$

For $K_1 = 13650$,

$$T(s) = -2730000\frac{s^2-12.8s-2.6}{s^7+37.2s^6+942.15s^5+13420s^4+1.0269\times10^5 s^3-2.2721\times10^6 s^2+3.517\times10^7 s+7108140}$$

Because of the negative coefficient in the denominator the system is unstable and the pilot would not be hired.

53.

The force error is the actuating signal. The equivalent forward-path transfer function is

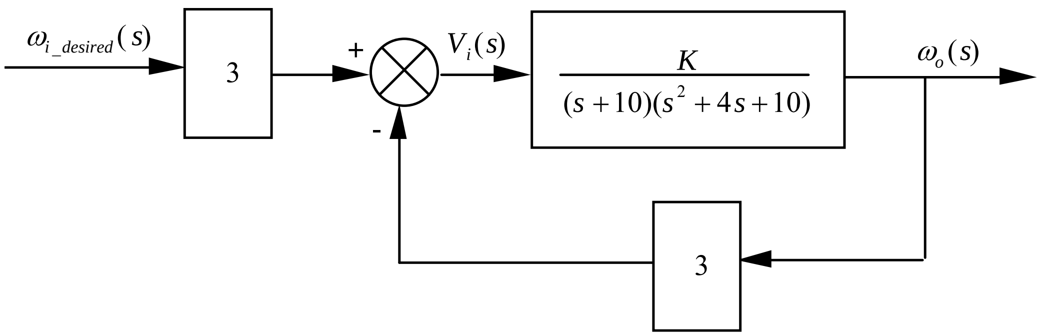
$G_e(s) = \dfrac{K_1}{s(s+K_1K_2)}$. The feedback is $H(s) = D_e s + K_e$. Using Eq. (7.72)

$E_a(s) = \dfrac{R(s)}{1+G_e(s)H(s)}$. Applying the final value theorem,

$e_{a\_ramp}(\infty) = \lim\limits_{s\to 0} \dfrac{s\left(\dfrac{1}{s^2}\right)}{1+\dfrac{K_1(D_e s+K_e)}{s(s+K_1K_2)}} = \dfrac{K_2}{K_e} < 0.1$. Thus, $K_2 < 0.1K_e$. Since the closed-loop system is second-order with positive coefficients, the system is always stable.

54.

a. The minimum steady-state error occurs for a maximum setting of gain, *K*. The maximum *K* possible is determined by the maximum gain for stability. The block diagram for the system is shown below.



Pushing the input transducer to the right past the summing junction and finding the closed-loop transfer function, we get

$$T(s) = \frac{\dfrac{3K}{(s+10)(s^2+4s+10)}}{1+\dfrac{3K}{(s+10)(s^2+4s+10)}} = \frac{3K}{s^3+14s^2+50s+(3K+100)}$$

Forming a Routh table,

s^3	1	50
s^2	14	$3K+100$
s^1	$\dfrac{-3K+600}{14}$	0
s^0	$3K+100$	0

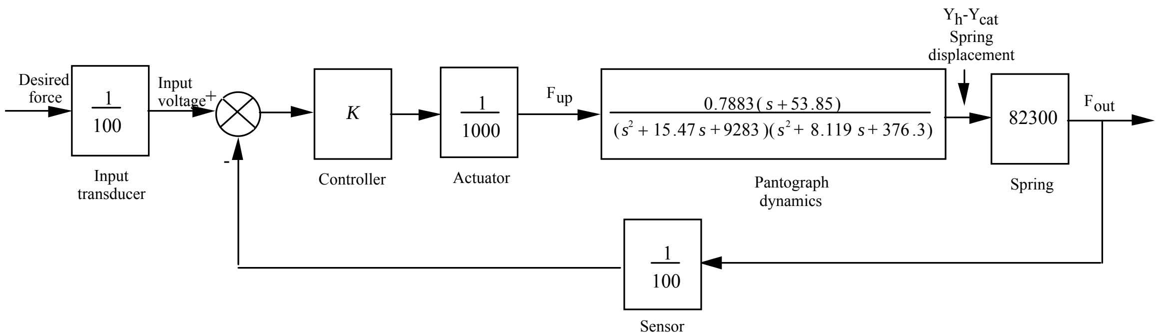
The s^1 row says $-\infty < K < 200$. The s^0 row says $-\frac{100}{3} < K$. Thus for stability, $-\frac{100}{3} < K < 200$. Hence, the maximum value of K is 200.

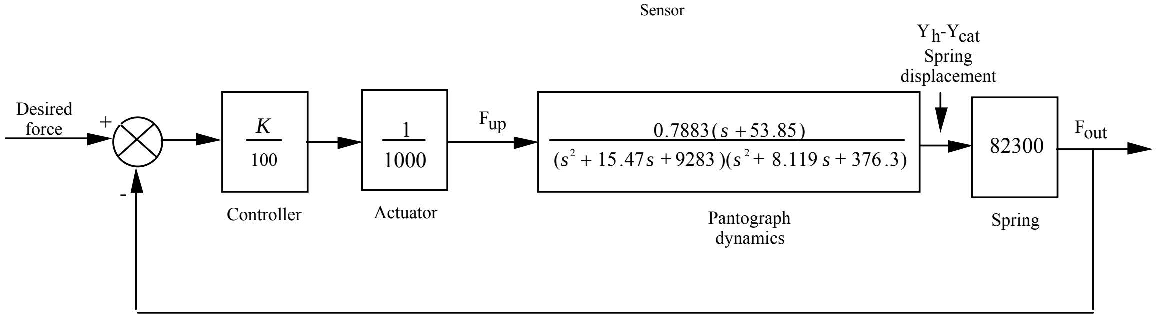
b. $K_p = \frac{3K}{100} = 6$. Hence, $e_{step}(\infty) = \frac{1}{1+K_p} = \frac{1}{7}$.

c. Step input

55.

a.





b.

$$G(s) = \frac{Y_h(s) - Y_{cat}(s)}{F_{up}(s)} = \frac{0.7883(s+53.85)}{(s^2+15.47s+9283)(s^2+8.119s+376.3)}$$

$$Ge(s) = (K/100)*(1/1000)*G(s)*82.3e3$$

$$G_e(s) = \frac{0.6488K\,(s+53.85)}{(s^2+8.119s+376.3)\,(s^2+15.47s+9283)}$$

$$K_p = 0.6488K*53.85/[(376.3)(9283)] = K*1.0002E\text{-}5$$

Maximum K minimizes the steady-state error. Maximum K possible is that which yields stability. From Chapter 6 maximum K for stability is $K = 1.88444 \times 10^5$. Therefore, $K_p = 1.8848$.

c. $e_{ss} = 1/(1+K_p) = 0.348$.