T E N

Frequency Response Techniques

SOLUTION TO CASE STUDY CHALLENGE

Antenna Control: Stability Design and Transient Performance

First find the forward transfer function, G(s).

Pot:

$$K_1 = \frac{10}{\pi} = 3.18$$

Preamp:

$$K$$

Power amp:

$$G_1(s) = \frac{100}{s(s+100)}$$

Motor and load:

$$J = 0.05 + 5\left(\frac{1}{5}\right)^2 = 0.25 \,;\; D = 0.01 + 3\left(\frac{1}{5}\right)^2 = 0.13;\; \frac{K_t}{Ra} = \frac{1}{5} \,;\; K_b = 1.$$

Therefore,

$$G_m(s) = \frac{\theta_m(s)}{E_a(s)} = \frac{\frac{K_t}{R_a J}}{s\left(s+\frac{1}{J}\left(D + \frac{K_t K_b}{R_a}\right)\right)} = \frac{0.8}{s(s+1.32)}\;.$$

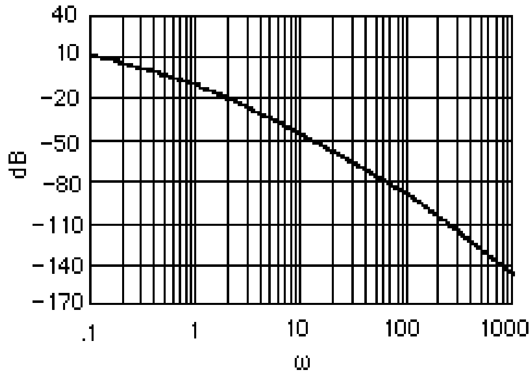
Gears:

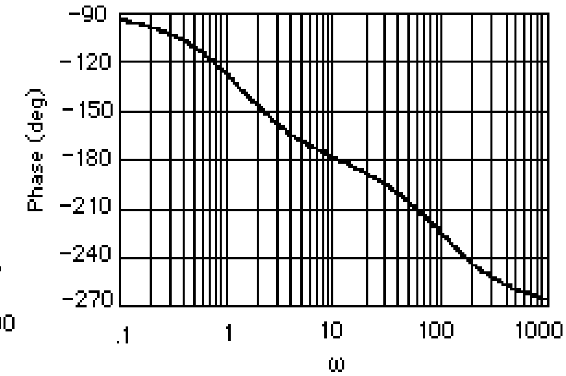
$$K_2 = \frac{50}{250} = \frac{1}{5}$$

Therefore,

$$G(s) = K_1 K G_1(s) G_m(s) K_2 = \frac{50.88K}{s(s+1.32)(s+100)}$$

Plotting the Bode plots for K = 1,





a. Phase is 180^o at $\omega = 11.5$ rad/s. At this frequency the gain is - 48.41 dB, or K = 263.36. Therefore, for stability, $0 < K < 263.36$.

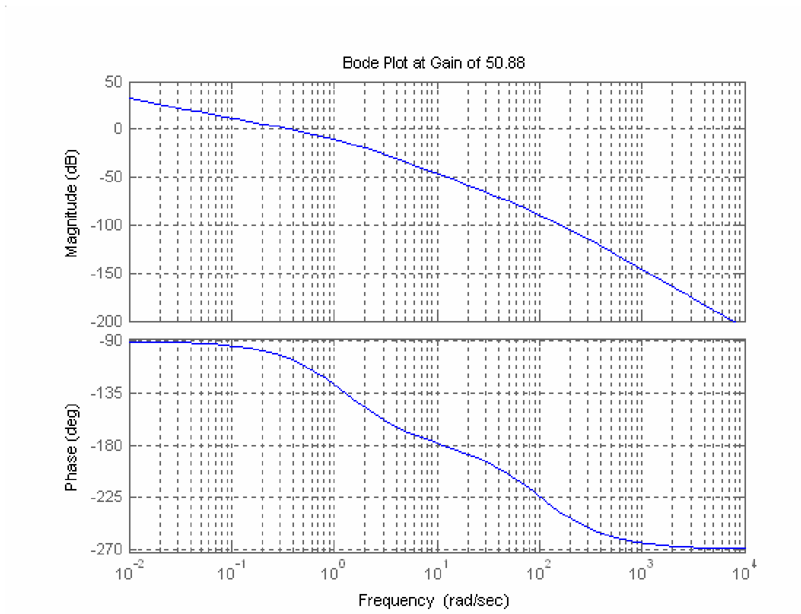
b. If K = 3, the magnitude curve will be 9.54 dB higher and go through zero dB at $\omega = 0.94$ rad/s. At this frequency, the phase response is -125.99^o. Thus, the phase margin is $180^o - 125.99^o = 54.01^o$. Using Eq. (10.73), $\zeta = 0.528$. Eq. (4.38) yields %OS = 14.18%.

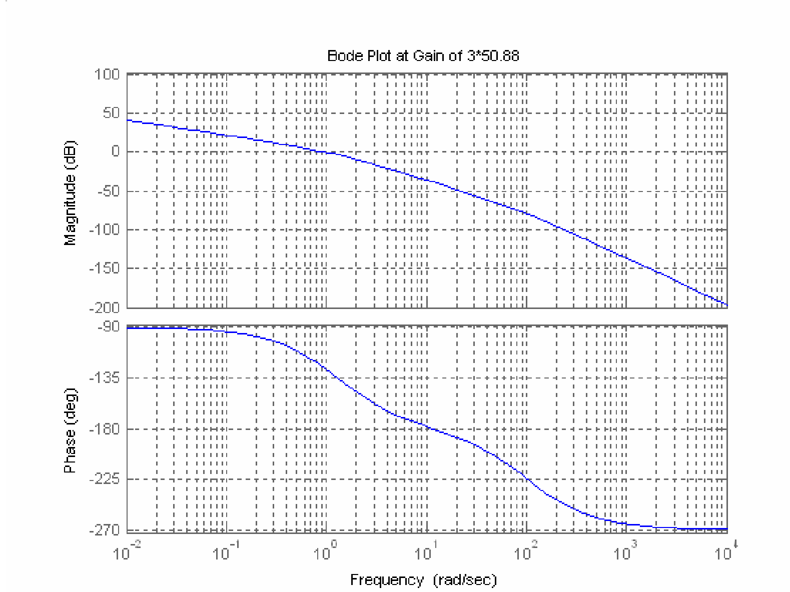
c.

Program:
```
numga=50.88;
denga=poly([0 -1.32 -100]);
'Ga(s)'
Ga=tf(numga,denga);
Gazpk=zpk(Ga)
'(a)'
bode(Ga)
title('Bode Plot at Gain of 50.88')
pause
[Gm,Pm,Wcp,Wcg]=margin(Ga);
'Gain for Stability'
Gm
pause
'(b)'
numgb=50.88*3;
dengb=denga;
'Gb(s)'
Gb=tf(numgb,dengb);
Gbzpk=zpk(Gb)
bode(Gb)
title('Bode Plot at Gain of 3*50.88')
[Gm,Pm,Wcp,Wcg]=margin(Gb);
'Phase Margin'
Pm
for z=0:.01:1
Pme=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi);
if Pm-Pme<=0;
break
end
end
z
percent=exp(-z*pi/sqrt(1-z^2))*100
```

Computer response:
```
ans =

Ga(s)
```

```
Zero/pole/gain:
      50.88
------------------
s (s+100) (s+1.32)

ans =

(a)

ans =

Gain for Stability

Gm =

  262.8585

ans =

(b)

ans =

Gb(s)

Zero/pole/gain:
      152.64
------------------
s (s+100) (s+1.32)

ans =

Phase Margin

Pm =

   53.9644

z =

    0.5300

percent =

   14.0366
```





ANSWERS TO REVIEW QUESTIONS

1. a. Transfer functions can be modeled easily from physical data; **b.** Steady-state error requirements can be considered easily along with the design for transient response; **c.** Settles ambiguities when sketching root locus; (d) Valuable tool for analysis and design of nonlinear systems.

2. A sinusoidal input is applied to a system. The sinusoidal output's magnitude and phase angle is measured in the steady-state. The ratio of the output magnitude divided by the input magnitude is the magnitude response at the applied frequency. The difference between the output phase angle and the input phase angle is the phase response at the applied frequency. If the magnitude and phase response are plotted over a range of different frequencies, the result would be the frequency response for the system.

3. Separate magnitude and phase curves; polar plot

4. If the transfer function of the system is G(s), let s=jω. The resulting complex number's magnitude is the magnitude response, while the resulting complex number's angle is the phase response.

5. Bode plots are asymptotic approximations to the frequency response displayed as separate magnitude and phase plots, where the magnitude and frequency are plotted in dB.

6. Negative 6 dB/octave which is the same as 20 dB/decade

7. Negative 24 dB/octave or 80 dB/decade

8. Negative 12 dB/octave or 40 dB/decade

9. Zero degrees until 0.2; a negative slope of 45^o/decade from a frequency of 0.2 until 20; a constant -90^o phase from a frequency of 20 until ∞

10. Second-order systems require a correction near the natural frequency due to the peaking of the curve for different values of damping ratio. Without the correction the accuracy is in question.

11. Each pole yields a maximum difference of 3.01 dB at the break frequency. Thus for a pole of multiplicity three, the difference would be 3x3.01 or 9.03 dB at the break frequency, - 4.

12. Z = P - N, where Z = # of closed-loop poles in the right-half plane, P = # of open-loop poles in the right-half plane, and N = # of counter-clockwise encirclements of -1 made by the mapping.

13. Whether a system is stable or not since the Nyquist criterion tells us how many rhp the system has

14. A Nyquist diagram, typically, is a mapping, through a function, of a semicircle that encloses the right half plane.

15. Part of the Nyquist diagram is a polar frequency response plot since the mapping includes the positive jω axis.

16. The contour must bypass them with a small semicircle.

17. We need only map the positive imaginary axis and then determine that the gain is less than unity when the phase angle is 180^o.

18. We need only map the positive imaginary axis and then determine that the gain is greater than unity when the phase angle is 180^o.

19. The amount of additional open-loop gain, expressed in dB and measured at 180^o of phase shift, required to make a closed-loop system unstable.

20. The phase margin is the amount of additional open-loop phase shift, Φ_M, required at unity gain to make the closed-loop system unstable.

21. Transient response can be obtained from (1) the closed-loop frequency response peak, (2) phase margin

22. a. Find T(jω)=G(jω)/[1+G(jω)H(jω)] and plot in polar form or separate magnitude and phase plots. **b.** Superimpose G(jω)H(jω) over the M and N circles and plot. **c.** Superimpose G(jω)H(jω) over the Nichols chart and plot.

23. For Type zero: K_p = low frequency gain; For Type 1: K_v = frequency value at the intersection of the initial slope with the frequency axis; For Type 2: K_a = square root of the frequency value at the intersection of the initial slope with the frequency axis.

24. No change at all

25. A straight line of negative slope, ωT, where T is the time delay

26. When the magnitude response is flat and the phase response is flat at 0^o.

SOLUTIONS TO PROBLEMS

1.

a.

$$G(s) = \frac{1}{s(s+2)(s+4)};\quad G(j\omega) = \frac{1}{-6\omega^2 + i(8\omega - \omega^3)}$$

$$M(\omega) = \frac{1}{\sqrt{(8\omega-\omega^3)^2 + (6\omega^2)^2}};\quad \Phi(\omega) = -\left(\pi + \arctan\left[\frac{8-\omega^2}{-6\omega}\right]\right)$$

b.

$$G(s) = \frac{(s+5)}{(s+2)(s+4)};\quad G(j\omega) = \frac{(\omega^2+40) - i(\omega^2+22)\omega}{\omega^4 + 20\omega^2 + 64}$$

$$M(\omega) = \frac{\sqrt{(\omega^2+40)^2 + \omega^2(\omega^2+22)^2}}{\omega^4 + 20\omega^2 + 64};\quad \Phi(\omega) = \arctan\left(\frac{-[\omega^2+22]\omega}{\omega^2+40}\right)$$

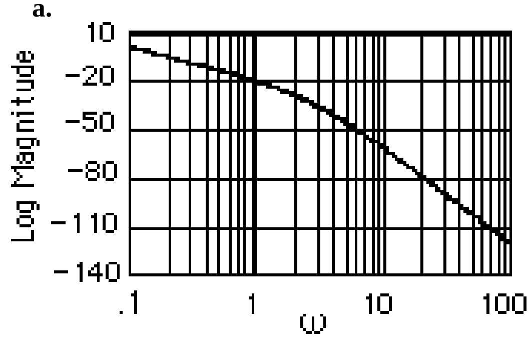
c.

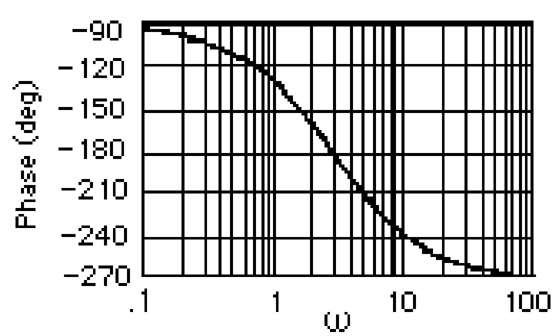
$$G(s) = \frac{(s+3)(s+5)}{s(s+4)(s+2)};\quad G(j\omega) = \frac{-2\omega(\omega^2+13) - i(\omega^4 + 25\omega^2 + 120)}{\omega^5 + 20\omega^3 + 64\omega}$$

$$M(\omega) = \frac{\sqrt{(2\omega[\omega^2+13])^2 + (\omega^4 + 25\omega^2 + 120)^2}}{\omega^5 + 20\omega^3 + 64\omega};\quad \Phi(\omega) = \pi + \arctan\left(\frac{\omega^4 + 25\omega^2 + 120}{2\omega[\omega^2+13]}\right)$$

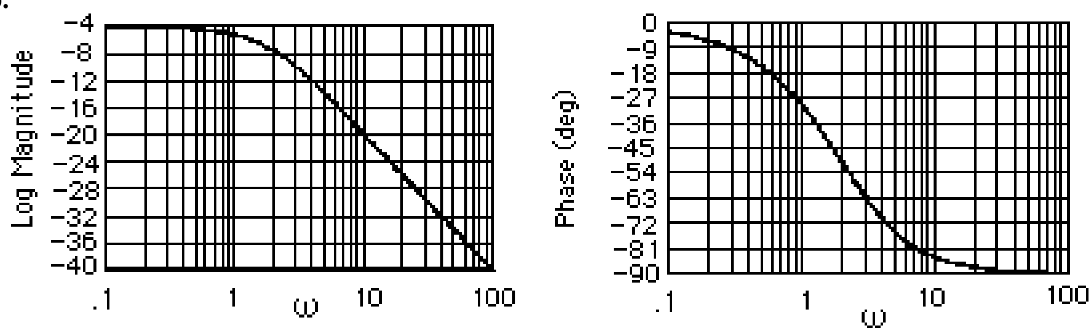
2.

a.

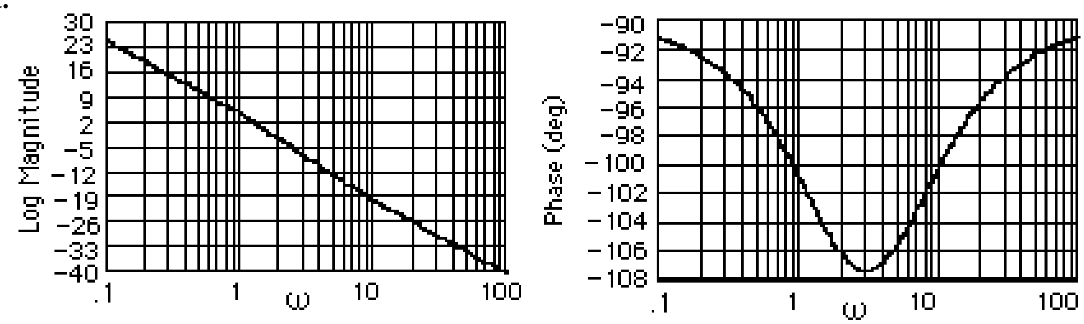




b.

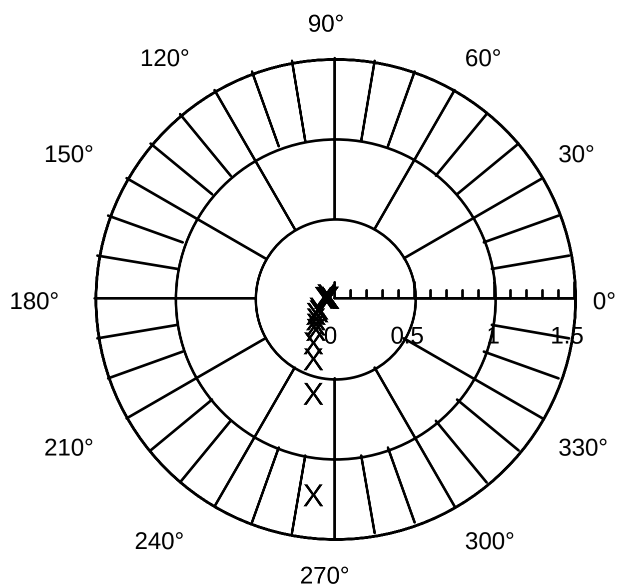

Log Magnitude
-4
-8
-12
-16
-20
-24
-28
-32
-36
-40
.1
1
10
100
ω
Phase (deg)
0
-9
-18
-27
-36
-45
-54
-63
-72
-81
-90
.1
1
10
100
ω

c.

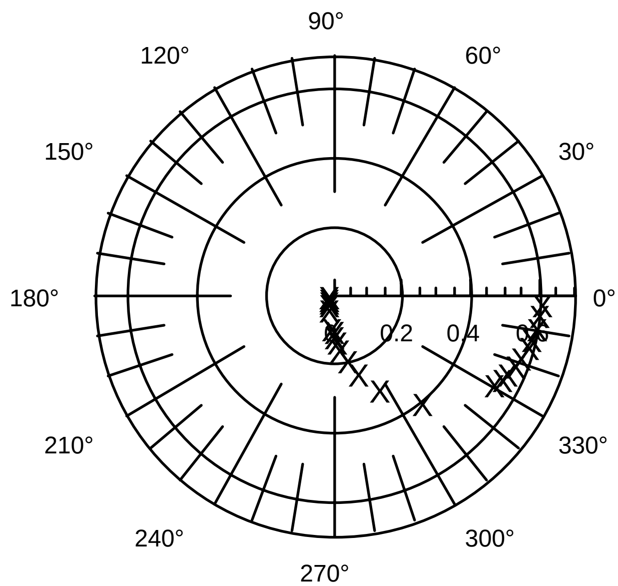

Log Magnitude
30
23
16
9
2
-5
-12
-19
-26
-33
-40
.1
1
10
100
ω
Phase (deg)
-90
-92
-94
-96
-98
-100
-102
-104
-106
-108
.1
1
10
100
ω

3.

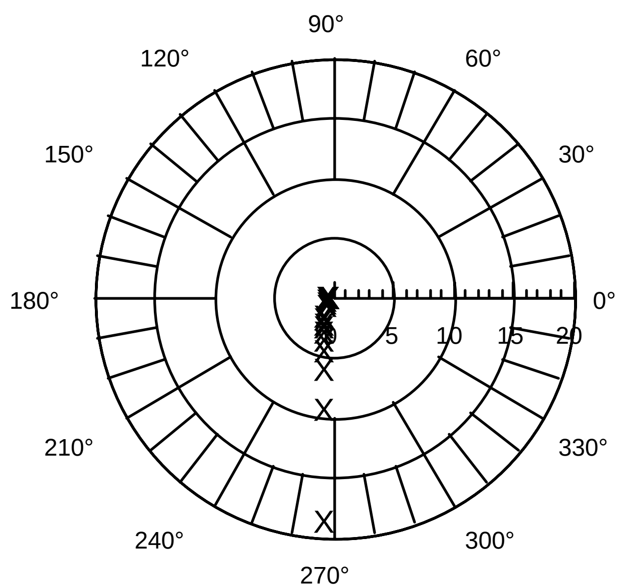
a.


90°
120°
60°
150°
30°
180°
0°
0
0.5
1
1.5
210°
330°
240°
300°
270°

b.

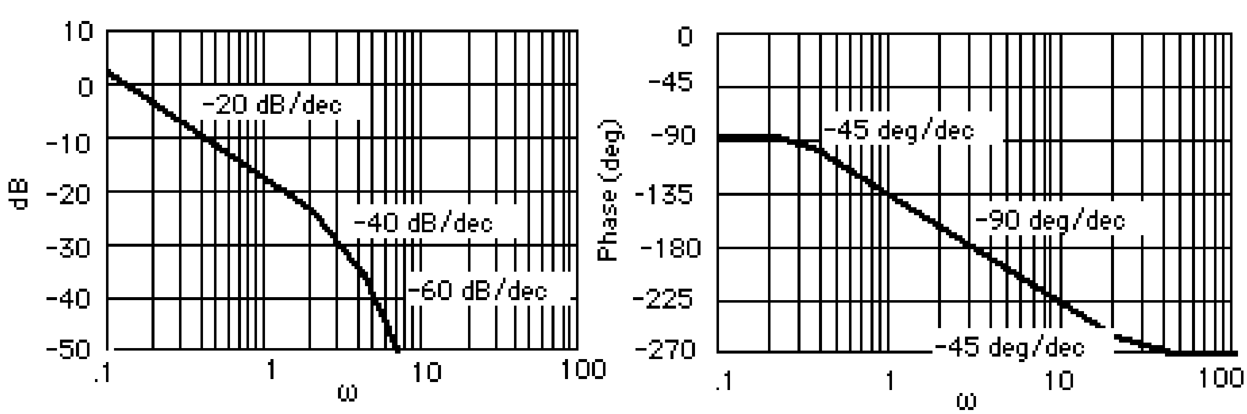

90°
120°
60°
150°
30°
180°
0°
0.2
0.4
210°
330°
240°
300°
270°

c.

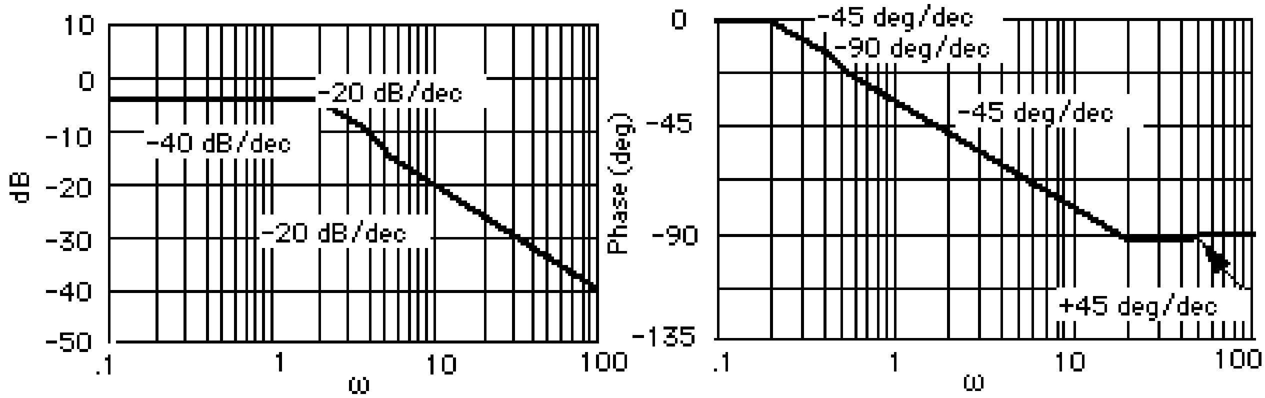

90°
120°
60°
150°
30°
180°
0°
5
10
15
20
210°
330°
240°
300°
270°

4.

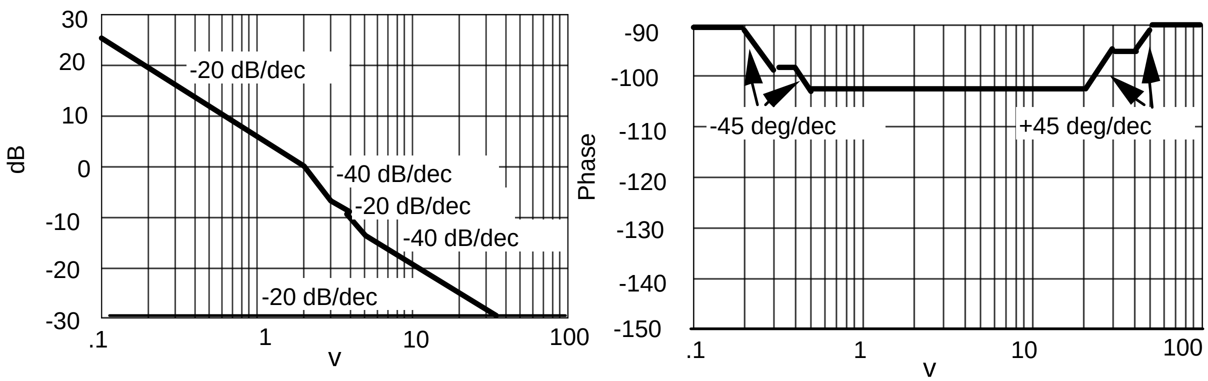
a.


dB
-20 dB/dec
-40 dB/dec
-60 dB/dec
ω
Phase (deg)
-45 deg/dec
-90 deg/dec
-45 deg/dec
ω

b.

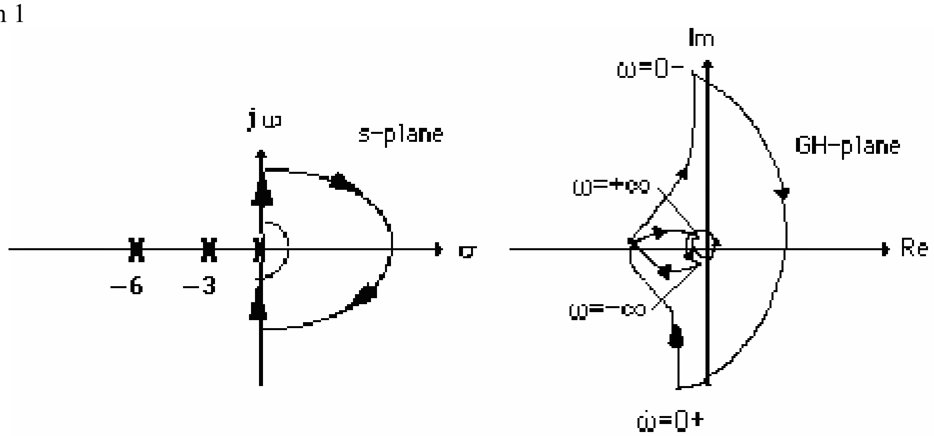


c.

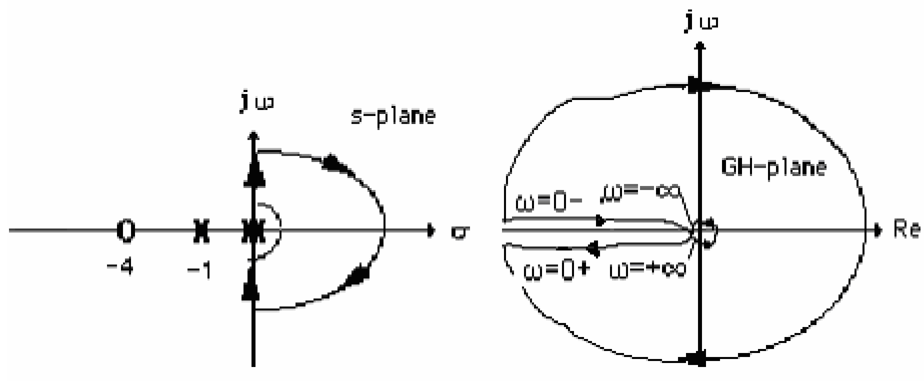


5.

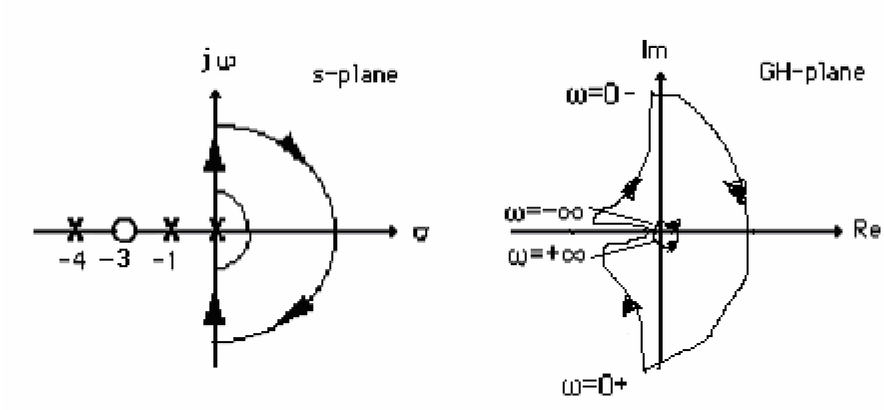
a. System 1



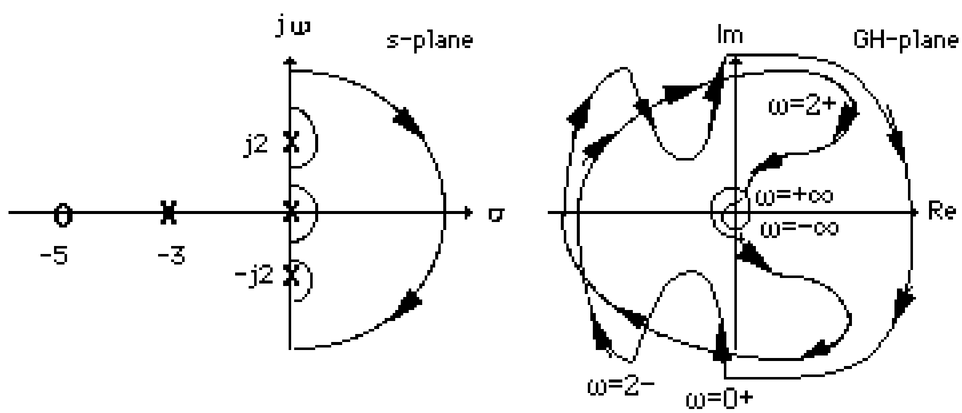
b. System 2



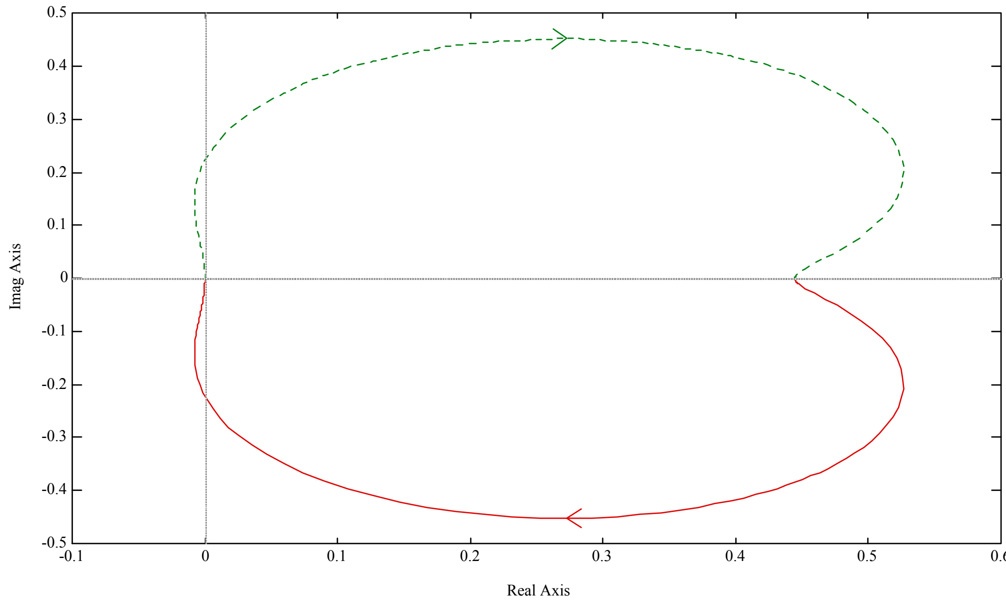
c. System 3



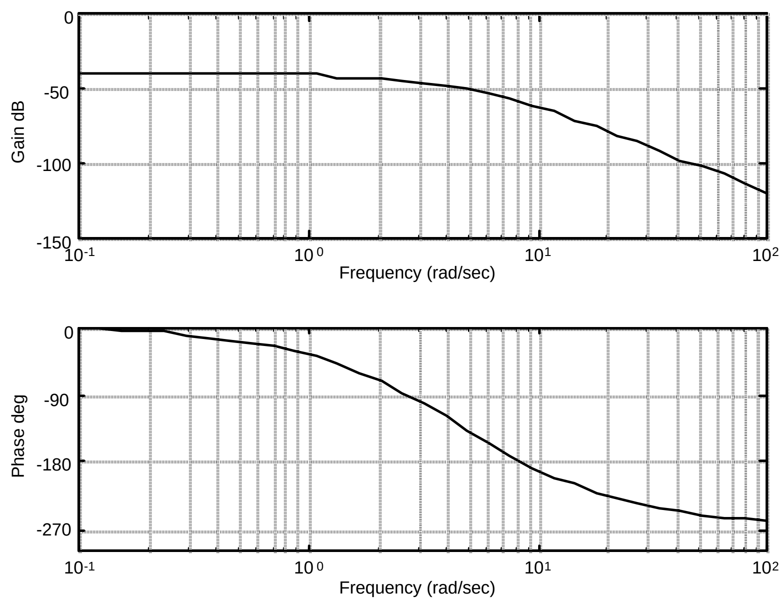
d.



6.



7.



8.

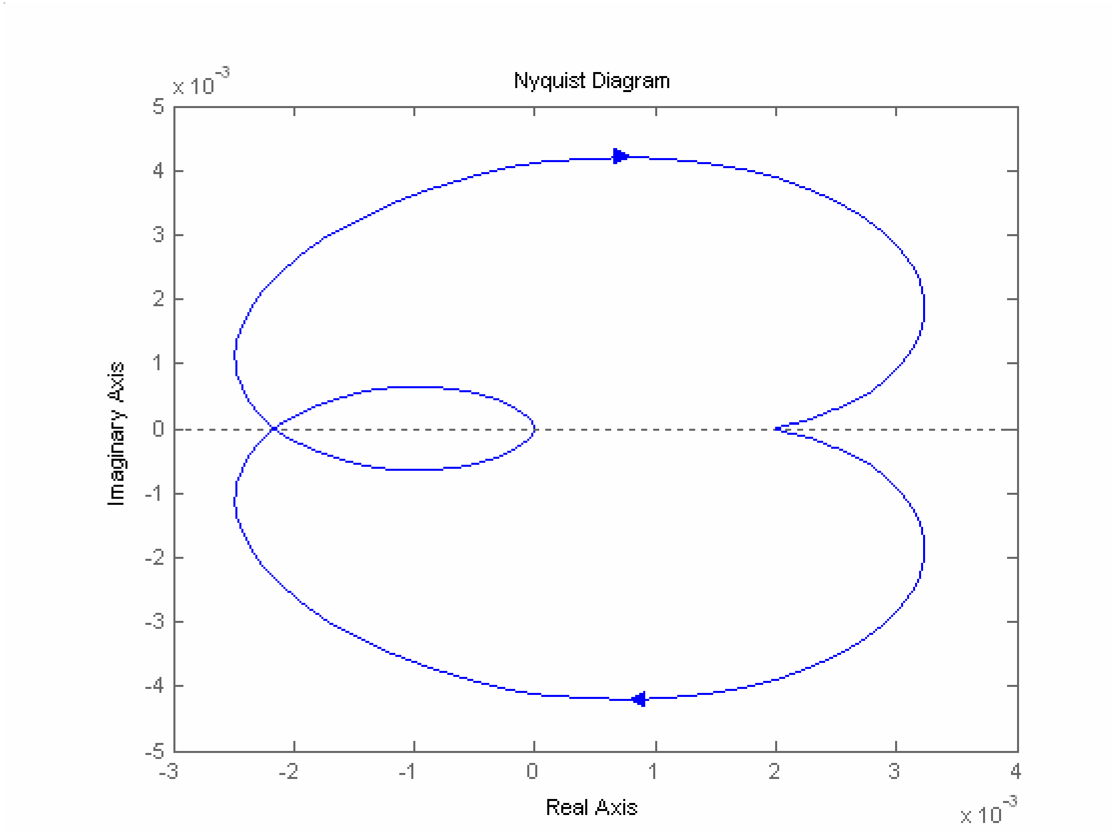
Program:

```
numg=[1 5];
deng=conv([1 6 100],[1 4 25]);
G=tf(numg,deng);
'G(s)'
Gzpk=zpk(G)
nyquist(G)
axis([-3e-3,4e-3,-5e-3,5e-3])
w=0:0.1:100;
[re,im]=nyquist(G,w);
for i=1:1:length(w)
```

```
M(i)=abs(re(i)+j*im(i));
A(i)=atan2(im(i),re(i))*(180/pi);
if 180-abs(A(i))<=1;
re(i);
im(i);
K=1/abs(re(i));
fprintf('\nw = %g',w(i))
fprintf(', Re = %g',re(i))
fprintf(', Im = %g',im(i))
fprintf(', M = %g',M(i))
fprintf(', Angle = %g',A(i))
fprintf(', K = %g',K)
Gm=20*log10(1/M(i));
fprintf(', Gm = %g',Gm)
break
end
end
```

Computer response:

```
ans =

G(s)


Zero/pole/gain:
              (s+5)
----------------------------------
(s^2  + 4s + 25) (s^2  + 6s + 100)


w = 10.1, Re = -0.00213722, Im = 2.07242e-005, M = 0.00213732, Angle =
179.444, K = 467.898, Gm = 53.4026

ans =

G(s)


Zero/pole/gain:
              (s+5)
----------------------------------
(s^2  + 4s + 25) (s^2  + 6s + 100)


w = 10.1, Re = -0.00213722, Im = 2.07242e-005, M = 0.00213732, Angle =
179.444, K = 467.898, Gm = 53.4026
```



9.

a. Since the real-axis crossing is at -0.3086, P = 0, N = 0. Therefore Z = P - N = 0. System is stable.

Derivation of real-axis crossing:

$$G(j\omega) = \left.\frac{50}{s(s+3)(s+6)}\right|_{s=j\omega} = \frac{50\left[-9\omega^2 - j\omega\left(18-\omega^2\right)\right]}{81\omega^4 + \left(18\omega-\omega^3\right)}.$$

Thus, the imaginary part = 0 at $\omega = \sqrt{18}$. Substituting this frequency into $G(j\omega)$, the real part is evaluated to be -0.3086.

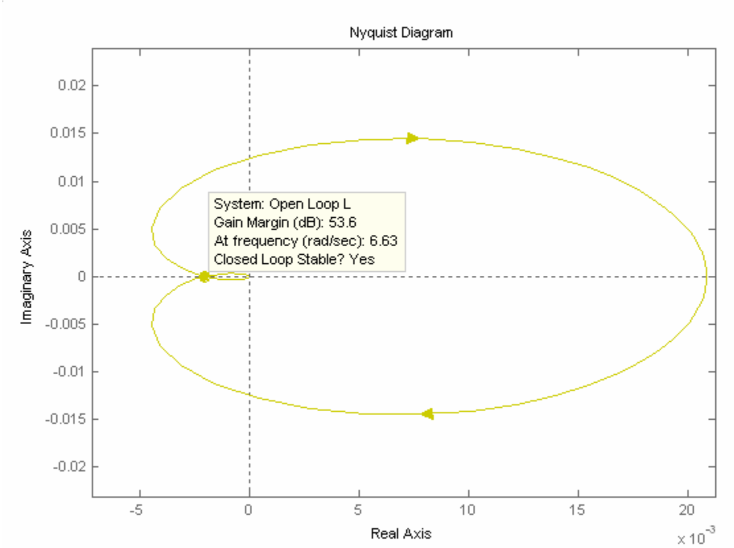
b. P = 0, N = -2. Therefore Z = P - N = 2. System is unstable.

c. P = 0, N = 0. Therefore Z = P - N = 0. System is stable

d. P = 0, N = -2. Therefore Z = P - N = 2. System is unstable.

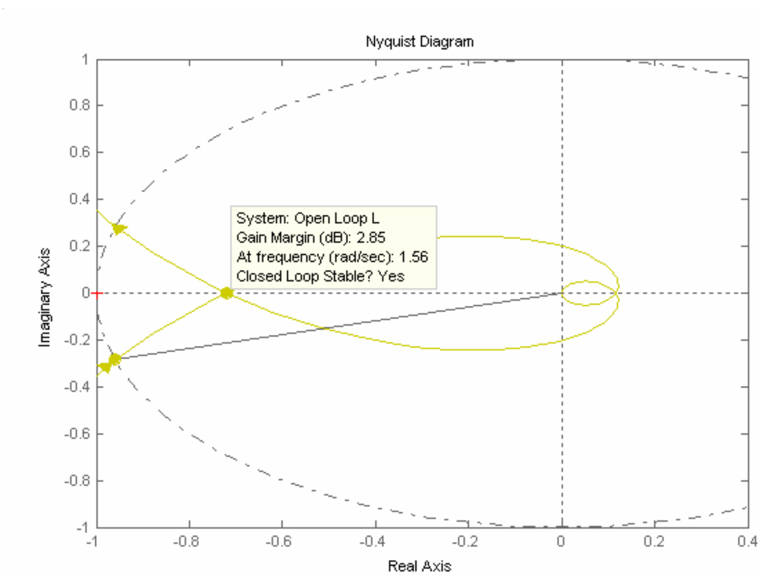
10.

System 1: For K = 1,



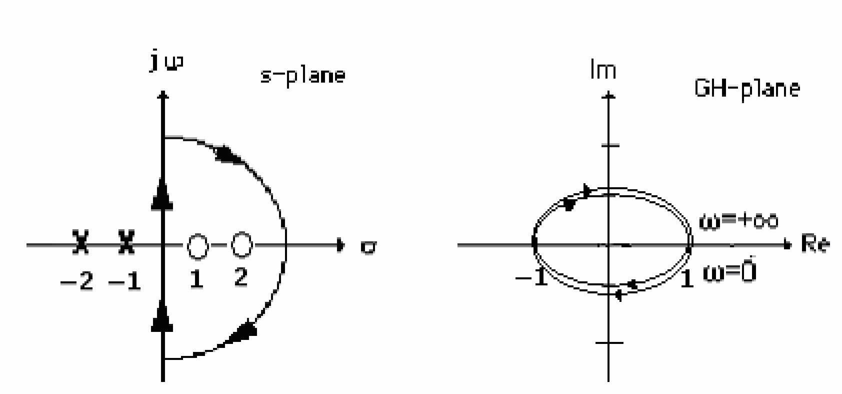
The Nyquist diagram intersects the real axis at -0.0021. Thus K can be increased to 478.63 before there are encirclements of -1. There are no poles encircles by the contour. Thus P = 0. Hence, Z = P - N, Z = 0 + 0 if K <478.63; Z = 0 –(-2) if K > 478.63. Therefore stability if 0 < K < 478.63.

System 2: For K = 1,



The Nyquist diagram intersects the real axis at -0.720. Thus K can be increased to 1.39 before there are encirclements of -1. There are no poles encircles by the contour. Thus P = 0. Hence, Z = P - N, Z = 0 + 0 if K <1.39; Z = 0 – (-2) if K > 1.39. Therefore stability if 0 < K < 1.39.

System 3: For K = 1,

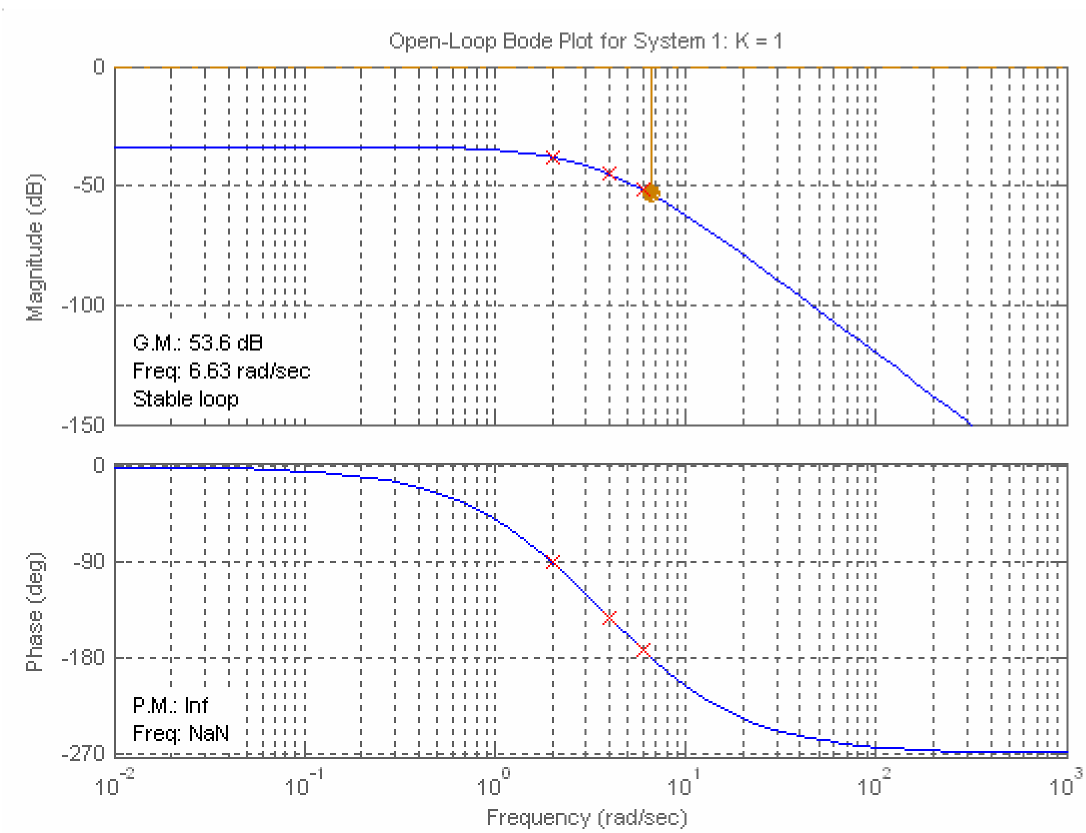


Stable if 0<K<1.

11.

Note: All results for this problem are based upon a non-asymptotic frequency response.

System 1: Plotting Bode plots for K = 1 yields the following Bode plot,



K = 1000:

For K = 1, phase response is 180^{o} at $\omega = 6.63$ rad/s. Magnitude response is -53.6 dB at this frequency.

For K = 1000, magnitude curve is raised by 60 dB yielding + 6.4 dB at 6.63 rad/s. Thus, the gain margin is

- 6.4 dB.

Phase margin: Raising the magnitude curve by 60 dB yields 0 dB at 9.07 rad/s, where the phase curve is 200.3°. Hence, the phase margin is 180°-200.3° = - 20.3°.

K = 100:

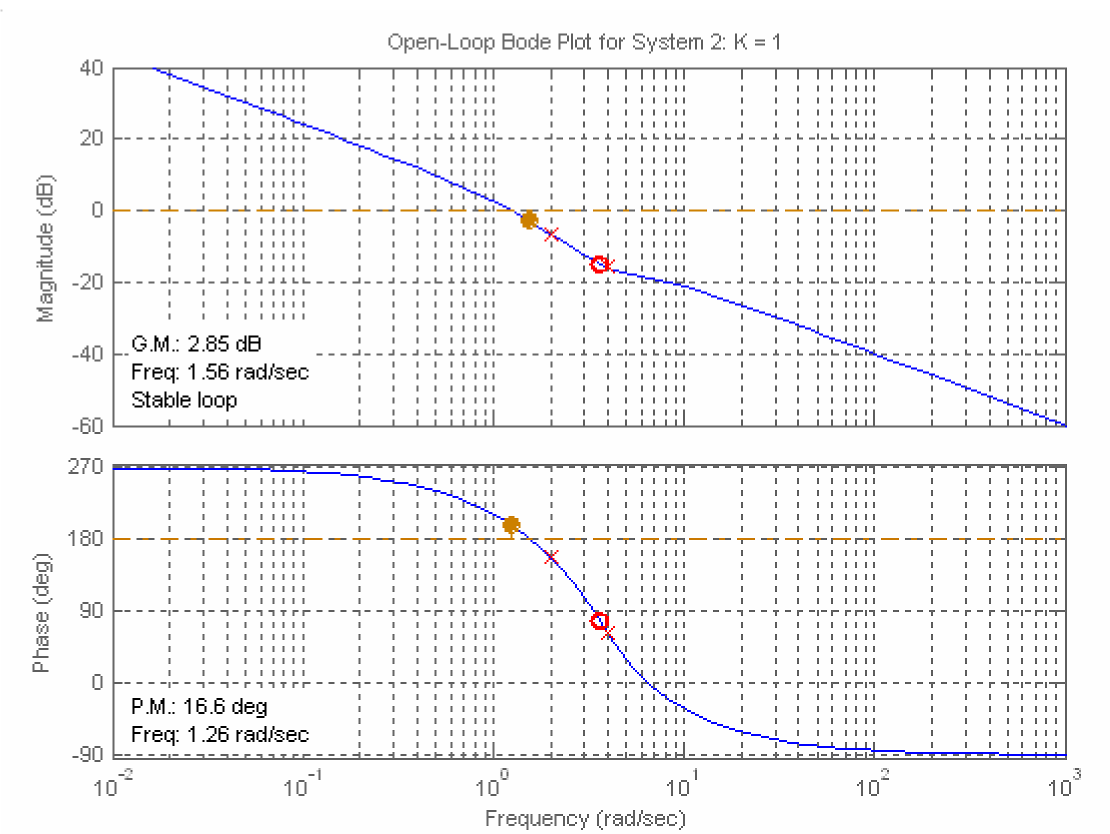
For K = 1, phase response is 180° at ω = 6.63 rad/s. Magnitude response is -53.6 dB at this frequency. For K = 100, magnitude curve is raised by 40 dB yielding – 13.6 dB at 6.63 rad/s. Thus, the gain margin is 13.6 dB.

Phase margin: Raising the magnitude curve by 40 dB yields 0 dB at 2.54 rad/s, where the phase curve is 107.3°. Hence, the phase margin is 180°-107.3° = 72.7°.

K = 0.1:

For K = 1, phase response is 180° at ω = 6.63 rad/s. Magnitude response is -53.6 dB at this frequency. For K = 0.1, magnitude curve is lowered by 20 dB yielding – 73.6 dB at 6.63 rad/s. Thus, the gain margin is 73.6 dB..

System 2: Plotting Bode plots for K = 1 yields



K = 1000:

For K = 1, phase response is 180° at ω = 1.56 rad/s. Magnitude response is -2.85 dB at this frequency. For K = 1000, magnitude curve is raised by 60 dB yielding + 57.15 dB at 1.56 rad/s. Thus, the gain margin is

– 57.15 dB.

Phase margin: Raising the magnitude curve by 54 dB yields 0 dB at 500 rad/s, where the phase curve is -91.03^{o}. Hence, the phase margin is $180^{o}-91.03^{o} = 88.97^{o}$.

K = 100:

For K = 1, phase response is 180^{o} at $\omega = 1.56$ rad/s. Magnitude response is -2.85 dB at this frequency. For K = 100, magnitude curve is raised by 40 dB yielding + 37.15 dB at 1.56 rad/s. Thus, the gain margin is

– 37.15 dB.

Phase margin: Raising the magnitude curve by 40 dB yields 0 dB at 99.8 rad/s, where the phase curve is -84.3^{o}. Hence, the phase margin is $180^{o}-84.3^{o} = 95.7^{o}$.

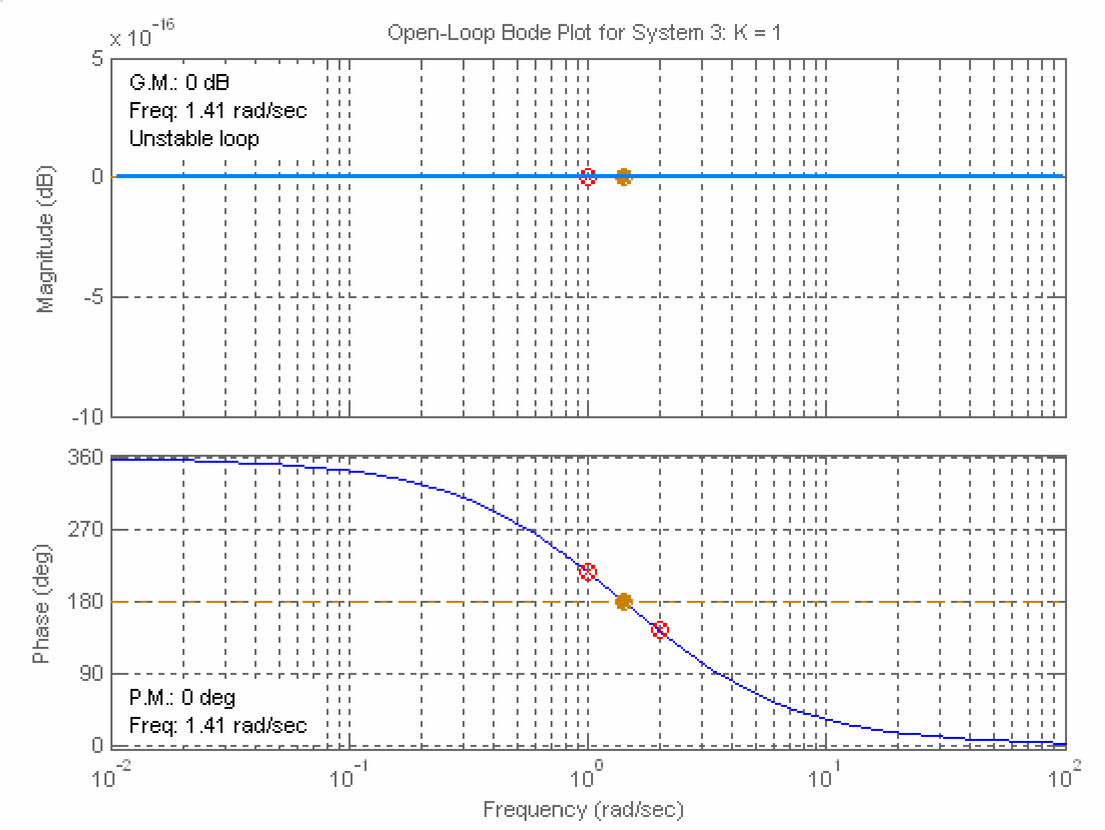
K = 0.1:

For K = 1, phase response is 180^{o} at $\omega = 1.56$ rad/s. Magnitude response is -2.85 dB at this frequency. For K = 0.1, magnitude curve is lowered by 20 dB yielding – 22.85 dB at 1.56 rad/s. Thus, the gain margin is

– 22.85 dB.

Phase margin: Lowering the magnitude curve by 20 dB yields 0 dB at 0.162 rad/s, where the phase curve is -99.8^{o}. Hence, the phase margin is $180^{o}-99.86^{o} = 80.2^{o}$.

System 3: Plotting Bode plots for K = 1 yields



K = 1000:

For K = 1, phase response is 180° at ω = 1.41 rad/s. Magnitude response is 0 dB at this frequency. For K = 1000, magnitude curve is raised by 60 dB yielding 60 dB at 1.41 rad/s. Thus, the gain margin is - 60 dB.

Phase margin: Raising the magnitude curve by 60 dB yields no frequency where the magnitude curve is 0 dB. Hence, the phase margin is infinite.

K = 100:

For K = 1, phase response is 180° at ω = 1.41 rad/s. Magnitude response is 0 dB at this frequency. For K = 100, magnitude curve is raised by 40 dB yielding 40 dB at 1.41 rad/s. Thus, the gain margin is - 40 dB.

Phase margin: Raising the magnitude curve by 40 dB yields no frequency where the magnitude curve is 0 dB. Hence, the phase margin is infinite.

K = 0.1:

For K = 1, phase response is 180° at ω = 1.41 rad/s. Magnitude response is 0 dB at this frequency. For K = 0.1, magnitude curve is lowered by 20 dB yielding -20 dB at 1.41 rad/s. Thus, the gain margin is 20 dB.

Phase margin: Lowering the magnitude curve by 20 dB yields no frequency where the magnitude curve is 0 dB. Hence, the phase margin is infinite.

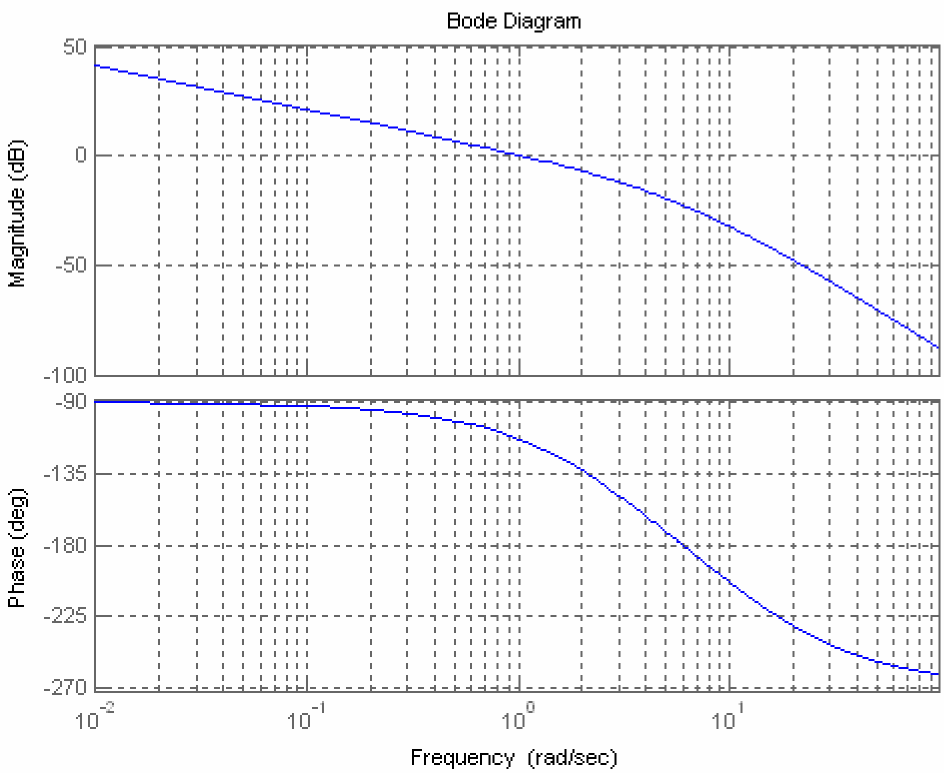
12.

Program:

```
%Enter G(s)************************
numg=1;
deng=poly([0 -3 -12]);
'G(s)'
G=tf(numg,deng)
w=0.01:0.1:100;
%Enter K **************************
K=input('Type gain, K ');
bode(K*G,w)
pause
[M,P]=bode(K*G,w);
%Calculate Gain Margin**************
for i=1:1:length(P);
if P(i)<=-180;
fprintf('\nGain K = %g',K)
fprintf(', Frequency(180 deg) = %g',w(i))
fprintf(', Magnitude = %g',M(i))
fprintf(', Magnitude (dB) = %g',20*log10(M(i)))
fprintf(', Phase = %g',P(i))
Gm=20*log10(1/M(i));
fprintf(', Gain Margin (dB) = %g',Gm)
break
end
end
%Calculate Phase Margin**************
for i=1:1:length(M);
if M(i)<=1;
fprintf('\nGain K = %g',K)
fprintf(', Frequency (0 dB) = %g',w(i))
fprintf(', Magnitude = %g',M(i))
fprintf(', Magnitude (dB) = %g',20*log10(M(i)))
fprintf(', Phase = %g',P(i))
Pm=180+P(i);;
fprintf(', Phase Margin = %g',Pm)
break
end
end

'Alternate program using MATLAB margin function:'

clear
clf
%Bode Plot and Find Points
%Enter G(s)************************
numg=1;
deng=poly([0 -3 -12]);
'G(s)'
G=tf(numg,deng)
w=0.01:0.1:100;
%Enter K **************************
K=input('Type gain, K ');
bode(K*G,w)
[Gm,Pm,Wcp,Wcg]=margin(K*G)
'Gm(dB)'
20*log10(Gm)
```

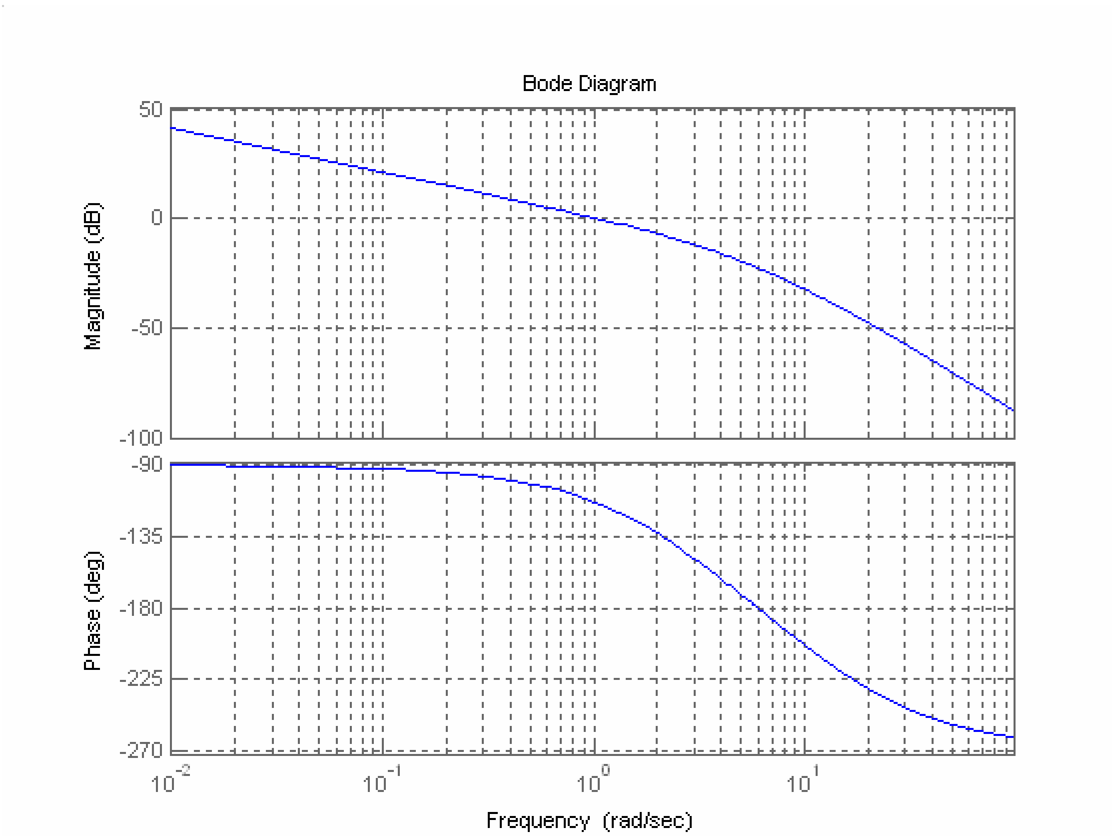
Computer response:

```
ans =

G(s)


Transfer function:
         1
-------------------
s^3 + 15 s^2 + 36 s

Type gain, K 40

Gain K = 40, Frequency(180 deg) = 6.01, Magnitude = 0.0738277, Magnitude
(dB) = -22.6356, Phase = -180.076, Gain Margin (dB) = 22.6356
Gain K = 40, Frequency (0 dB) = 1.11, Magnitude = 0.93481, Magnitude (dB) =
-0.585534, Phase = -115.589, Phase Margin = 64.4107
```




```
Alternate program using MATLAB margin function:


ans =

G(s)


Transfer function:
         1
-------------------
s^3 + 15 s^2 + 36 s

Type gain, K 40

Gm =
```

```
   13.5000

Pm =

   65.8119

Wcp =

     6

Wcg =

    1.0453

ans =

Gm(dB)

ans =

   22.6067
```



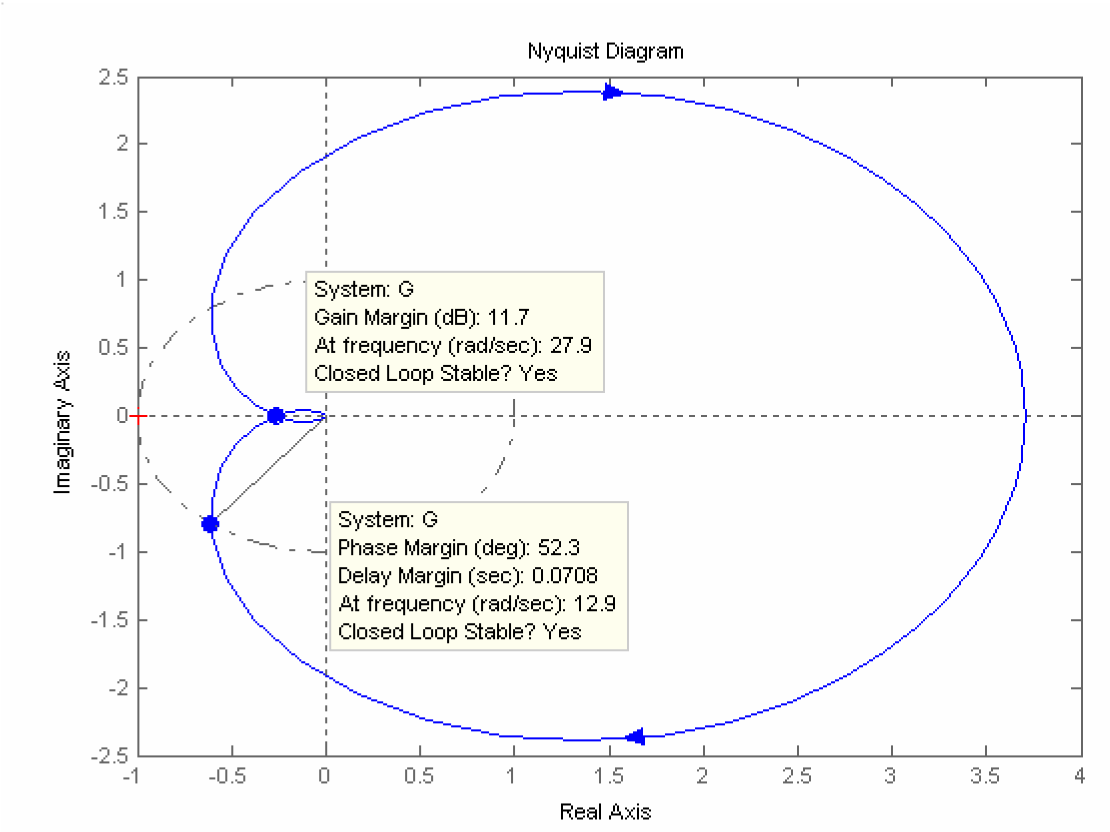
13.

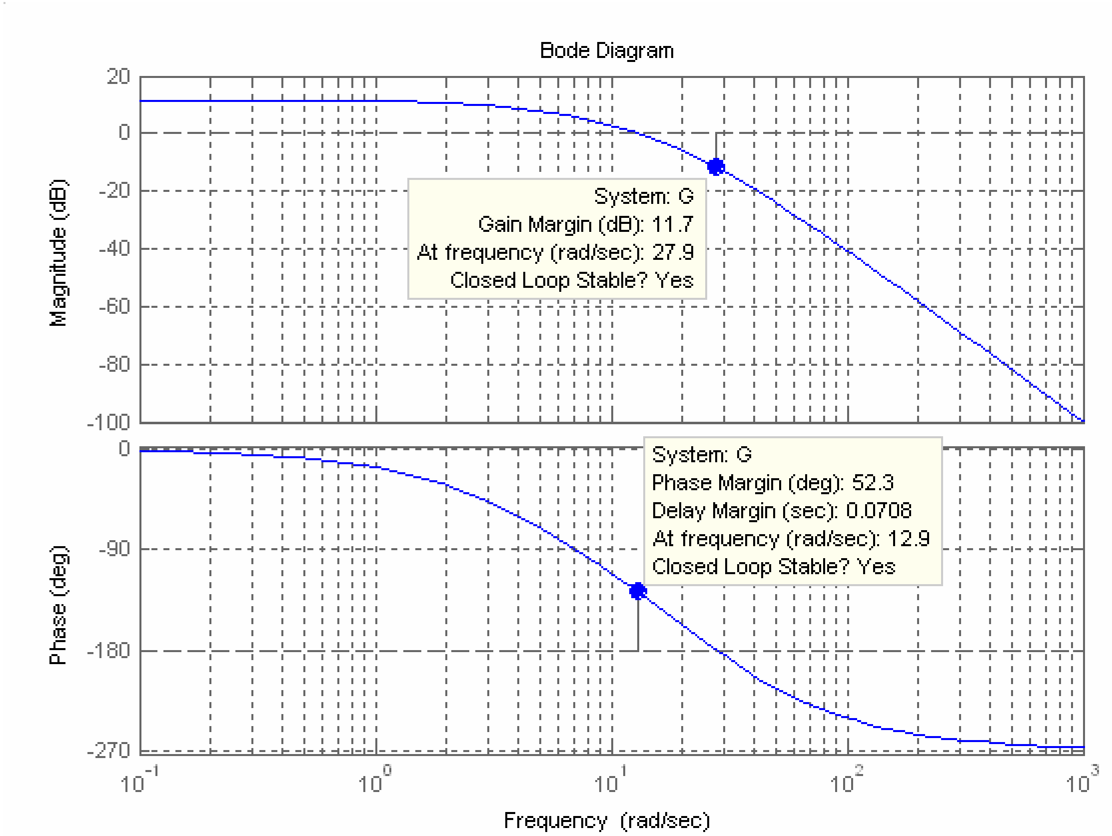
Program:

```
numg=10000;
deng=poly([-5 -18 -30]);
G=tf(numg,deng)
Ltiview
```

Computer response:

```
ans =

10.4N

Transfer function:
            10000
---------------------------
s^3 + 53 s^2 + 780 s + 2700
```





14.

Squaring Eq. (10.51) and setting it equal to $\left(\frac{1}{\sqrt{2}}\right)^2$ yields

$$\frac{\omega_n^4}{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2} = \frac{1}{2}$$

Simplifying,

$$\omega^4 + 2\omega_n^2(2\zeta^2 - 1)\omega^2 - \omega_n^4 = 0$$

Solving for ω^2 using the quadratic formula and simplifying yields,

$$\omega^2 = \omega_n^2\left[-(2\zeta^2 - 1) \pm \sqrt{4\zeta^4 - 4\zeta^2 + 2}\right]$$

Taking the square root and selecting the positive term,

$$\omega = \omega_n\sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

15.

a. Using Eq. (10.55), ω_{BW} = 10.06 rad/s.

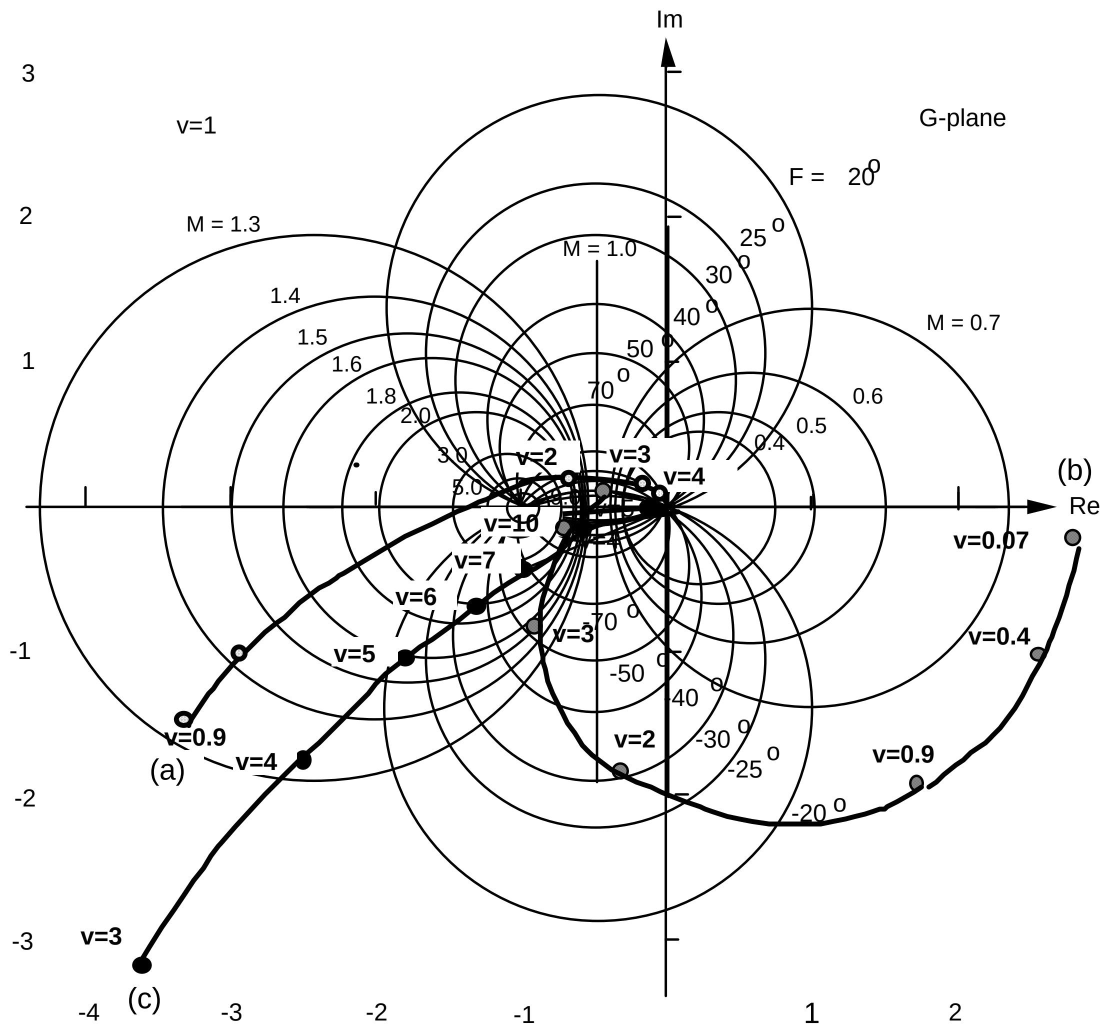
b. Using Eq. (10.56), ω_{BW} = 1.613 rad/s.

c. First find ζ. Since $T_s = \frac{4}{\zeta\omega_n}$ and $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$, $\frac{T_p}{T_s} = \frac{\zeta\pi}{4\sqrt{1-\zeta^2}}$. Solving for ζ with $\frac{T_p}{T_s}$ = 0.5 yields ζ = 0.537. Using either Eq. (10.55) or (10.56) yields ω_{BW} = 2.29 rad/s.

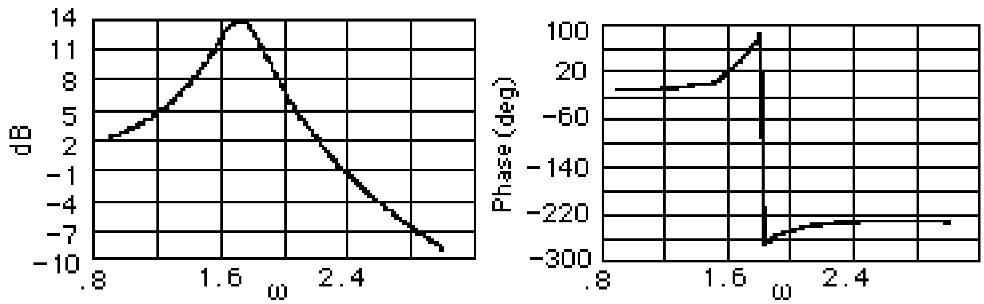
d. Using ζ = 0.3, $\omega_n T_r = 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1 = 1.3217$. Hence,

$$\omega_n = \frac{1.3217}{T_r} = \frac{1.3217}{4} = 0.3304 \text{ rad/s.}$$ Using Eq. (10.54) yields ω_{BW} = 0.4803 rad/s.

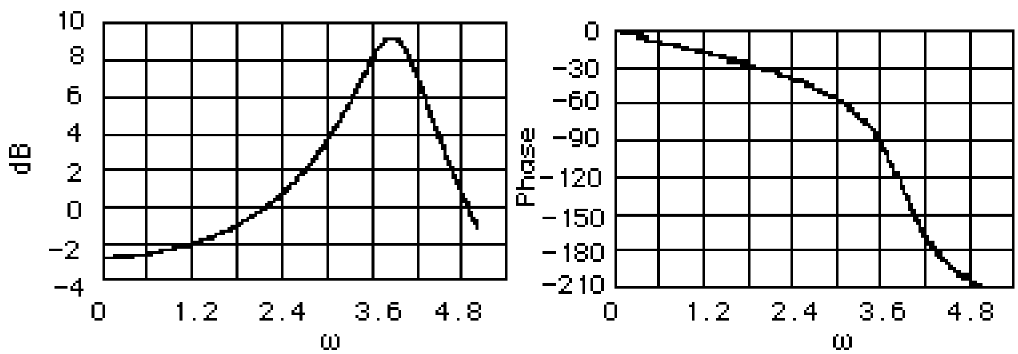
16.



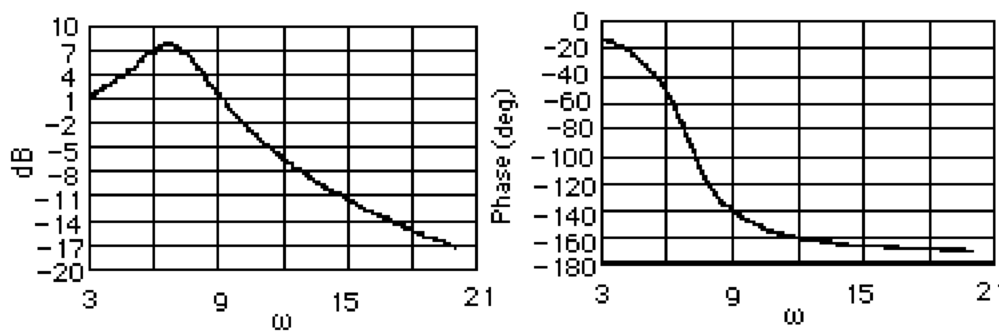
a.


14
11
8
5
2
−1
−4
−7
−10
dB
.8
1.6
2.4
ω
100
20
−60
−140
−220
−300
Phase (deg)
.8
1.6
2.4
ω

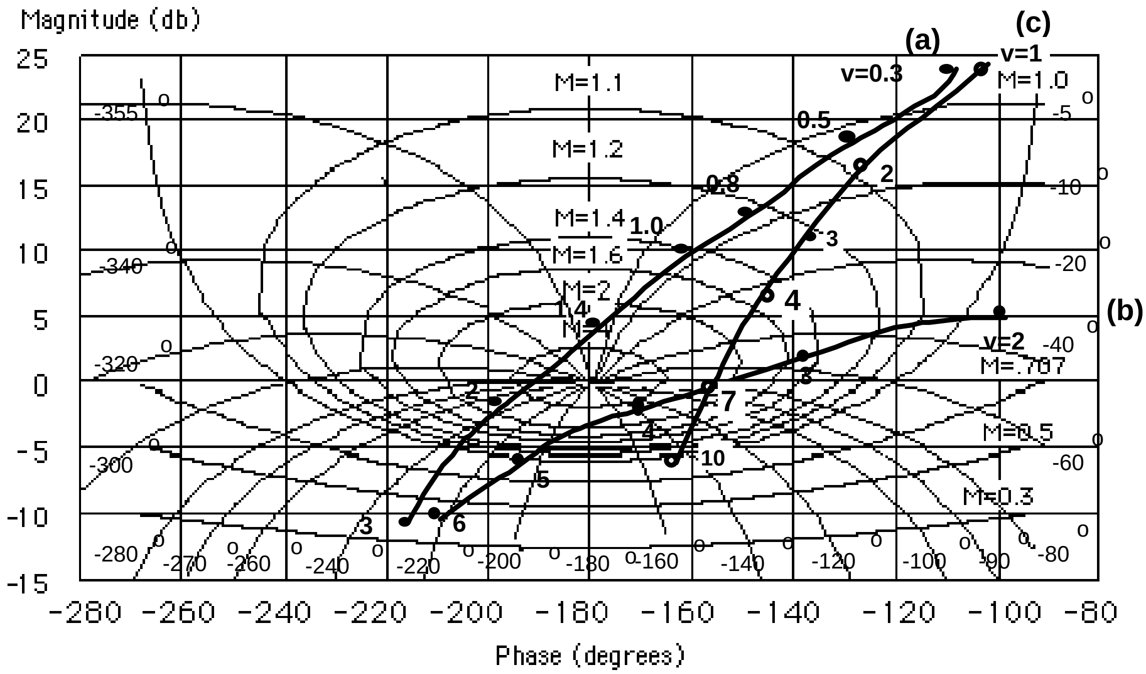
b.


10
8
6
4
2
0
−2
−4
dB
0
1.2
2.4
3.6
4.8
ω
0
−30
−60
−90
−120
−150
−180
−210
Phase
0
1.2
2.4
3.6
4.8
ω

c.


10
7
4
1
−2
−5
−8
−11
−14
−17
−20
dB
3
9
15
21
ω
0
−20
−40
−60
−80
−100
−120
−140
−160
−180
Phase (deg)
3
9
15
21
ω

17.



18.

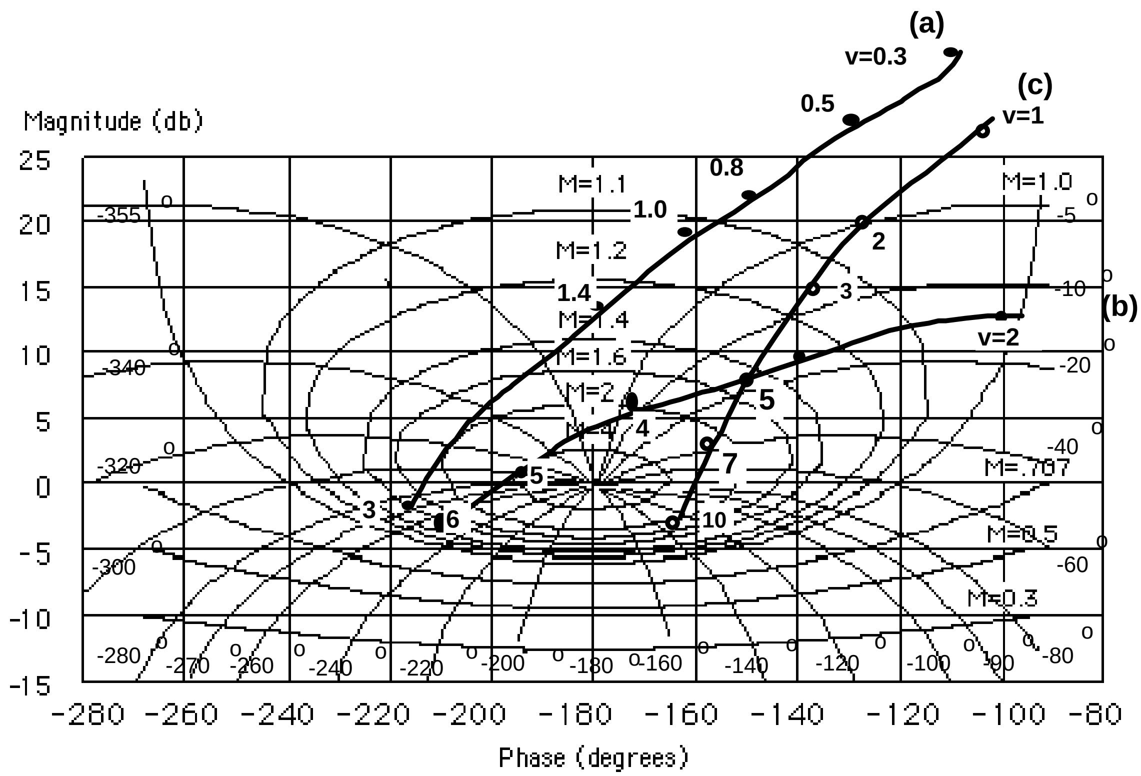
a. The polar plot is approximately tangent to M = 5. Using Figure 10.40, the student would estimate 72% overshoot. However, notice that the polar plot intersects the negative real axis at a magnitude greater than unity. Hence, the system is actually unstable and the estimated percent overshoot is not correct.

b. The polar plot is approximately tangent to M = 3. Using Figure 10.40, we estimate 58% overshoot.

c. The polar plot is approximately tangent to M = 2.5. Using Figure 10.40, we estimate 52% overshoot.

19.

Raise each curve in Problem 17 by (a) 9.54 dB, (b) 7.96 dB, and (c) 3.52 dB, respectively.



Systems (a) and (b) are both unstable since the open-loop magnitude is greater than unity when the open-loop phase is 180^o. System (c) is tangent to approximately M = 3. Using Figure 10.40, we estimate 58% overshoot.

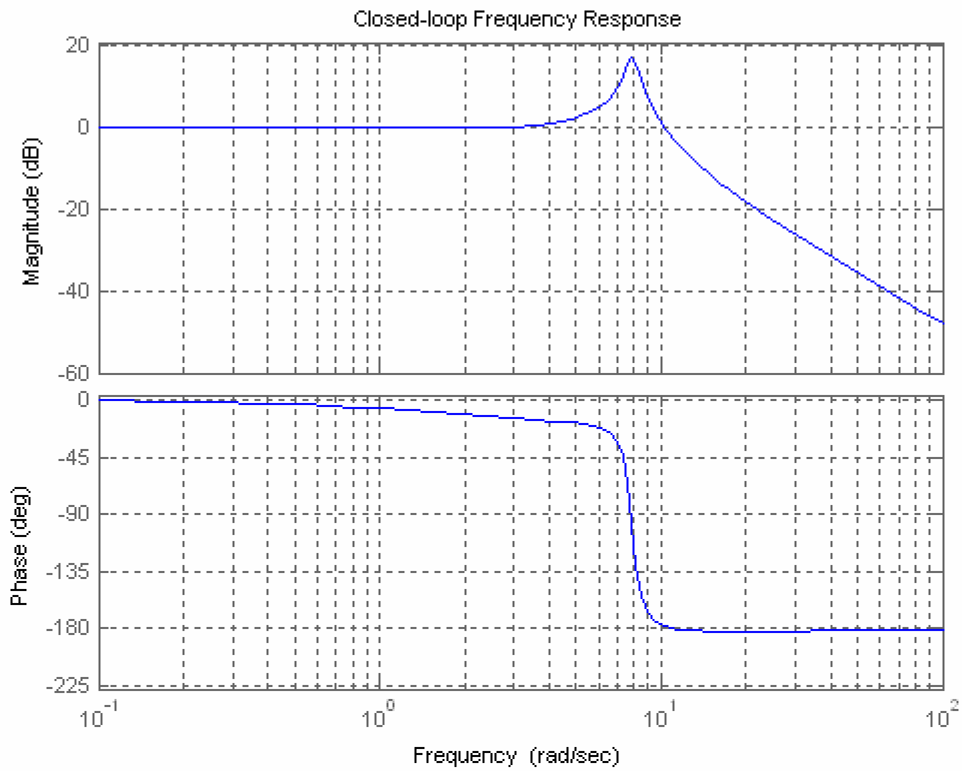
20.

Program:

```
%Enter G(s)***********************
numg=[1 5];
deng=[1 4 25 0];
'G(s)'
G=tf(numg,deng)
%Enter K ************************
K=input('Type gain, K ');
'T(s)'
T=feedback(K*G,1)
bode(T)
title('Closed-loop Frequency Response')
[M,P,w]=bode(T);
[Mp i]=max(M);
Mp
MpdB=20*log10(Mp)
wp=w(i)
for i=1:1:length(M);
if M(i)<=0.707;
fprintf('Bandwidth = %g',w(i))
break
end
end
```

Computer response:

```
ans =

G(s)

Transfer function:

      s + 5

------------------

s^3 + 4 s^2 + 25 s


Type gain, K 40

ans =

T(s)

Transfer function:

       40 s + 200

------------------------

s^3 + 4 s^2 + 65 s + 200


Mp =

    6.9745

MpdB =

   16.8702

wp =

    7.8822

Bandwidth = 11.4655
```



21.

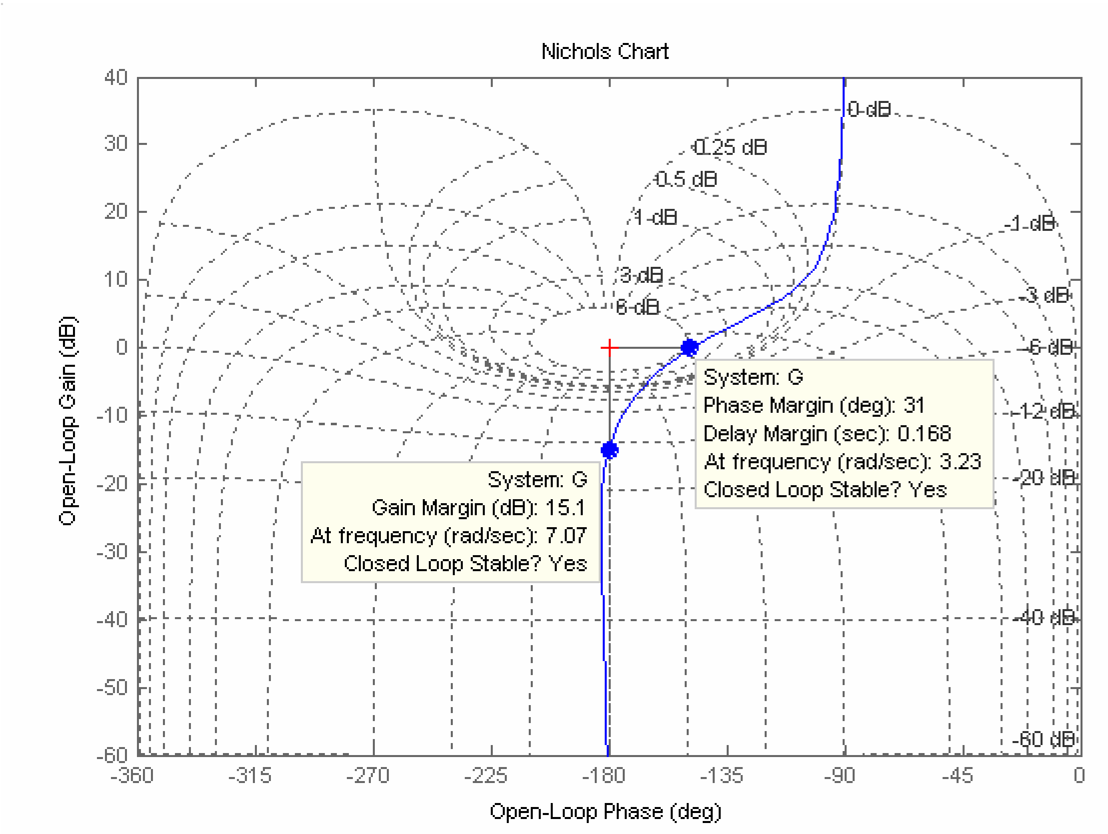
Program:

```
numg=[7 35];
deng=[1 4 10 0];
G=tf(numg,deng)
bode(G)                                %Make a Bode plot.
title('Open-Loop Frequency Response')
                                       %Add a title to the Bode plot.
[Gm,Pm,Wcp,Wcg]=margin(G);             %Find margins and margin
                                       %frequencies.
'Gain margin(dB); Phase margin(deg.); 0 dB freq. (r/s);'
'180 deg. freq. (r/s)'                 %Display label.
margins=[20*log10(Gm),Pm,Wcg,Wcp]      %Display margin data.
Ltiview
```

Computer response:

```
Transfer function:
     7 s + 35
------------------
s^3 + 4 s^2 + 10 s


ans =

Gain margin(dB); Phase margin(deg.); 0 dB freq. (r/s);


ans =

180 deg. freq. (r/s)
```

```
margins =

   15.1403   31.0397    3.2252    7.0715
```



22.

Program:

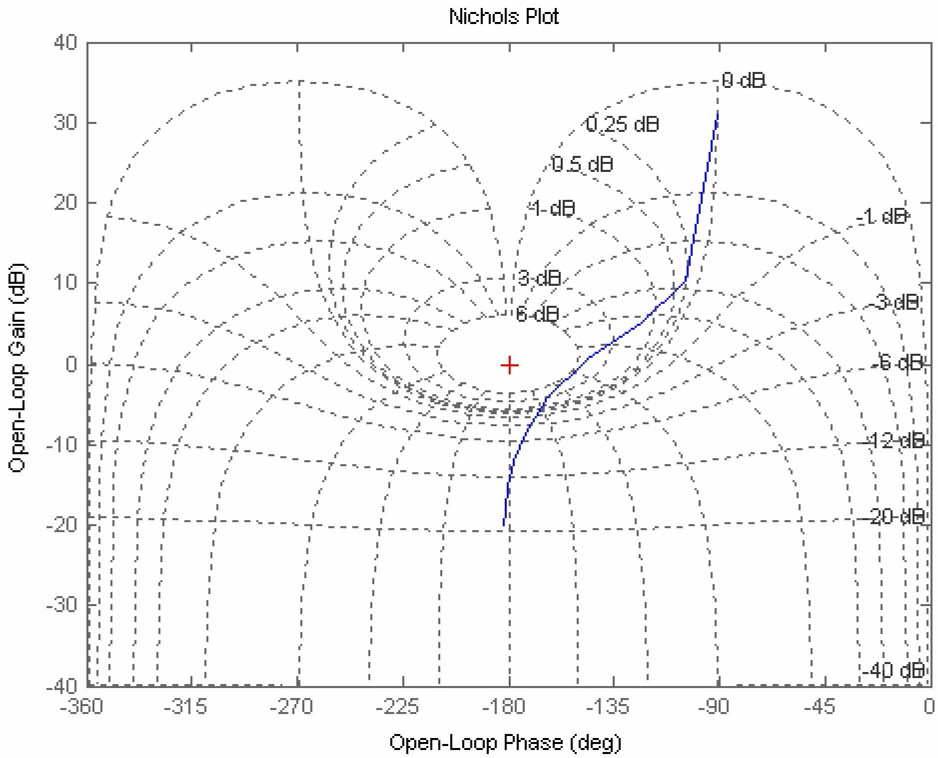
```
%Enter G(s)************************************
numg=7*[1 5];
deng=[1 4 10 0];
'Open-Loop System'
'G(s)'
G=tf(numg,deng)
clf
w=.10:1:10;
nichols(G,w)
ngrid
title('Nichols Plot')
[M,P]=nichols(G,w);
for i=1:1:length(M);
if M(i)<=0.45;
BW=w(i);
break
end
end
pause
MpdB=input('Enter Mp in dB from Nichols Plot  ');
Mp=10^(MpdB/20);
z2=roots([4,-4,(1/Mp^2)]);%Since Mp=1/sqrt(4z^2(1-z^2))
z1=sqrt(z2);
z=min(z1);
Pos=exp(-z*pi/(sqrt(1-z^2)));
```

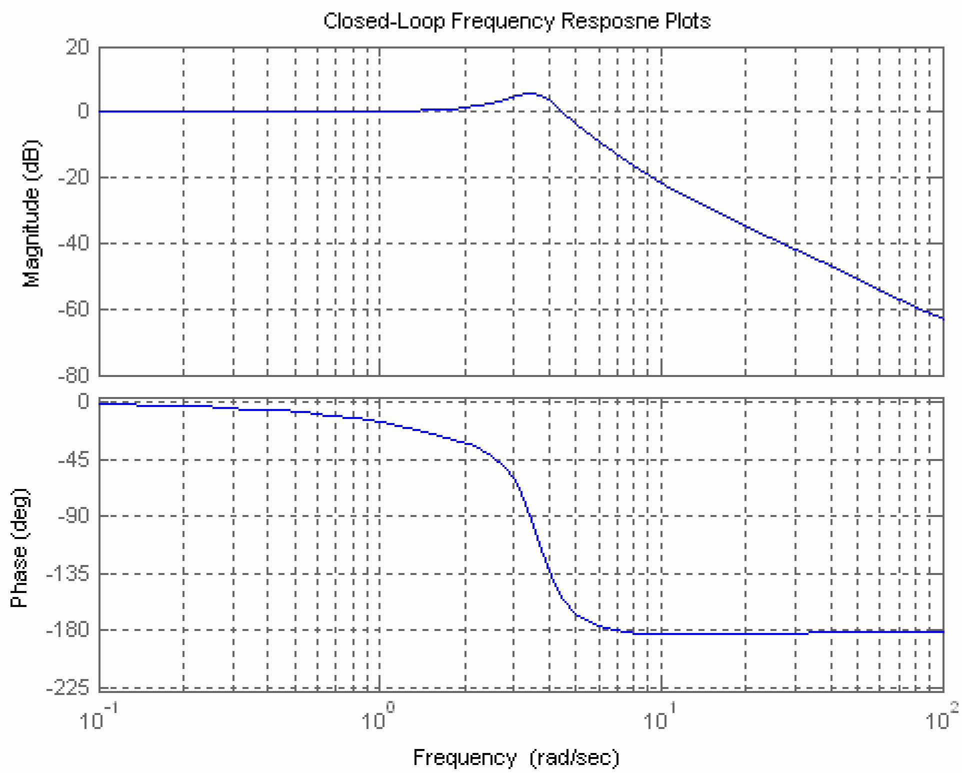
```
Ts=(4/(BW*z))*sqrt((1-z^2)+sqrt(4*z^4-4*z^2+2));
Tp=(pi/(BW*sqrt(1-z^2)))*sqrt((1-z^2)+sqrt(4*z^4-4*z^2+2));
'Closed-Loop System'
'T(s)'
T=feedback(G,1)
bode(T)
title('Closed-Loop Frequency Resposne Plots')
fprintf('\nDamping Ratio = %g',z)
fprintf(', Percent Overshoot = %g',Pos*100)
fprintf(', Bandwidth = %g',BW)
fprintf(', Mp (dB) = %g',MpdB)
fprintf(', Mp = %g',Mp)
fprintf(', Settling Time = %g',Ts)
fprintf(', Peak Time = %g',Tp)
pause
step(T)
title('Closed-Loop Step Response')
```

Computer response:

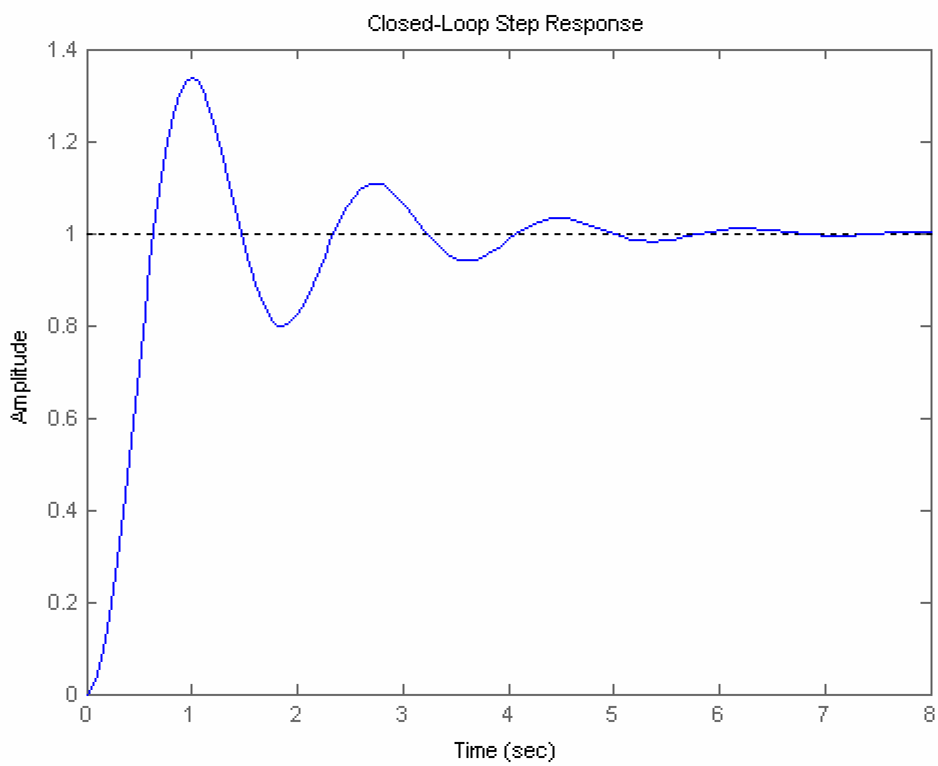
```
ans =

Open-Loop System

ans =

G(s)

Transfer function:
     7 s + 35
------------------
s^3 + 4 s^2 + 10 s

Enter Mp in dB from Nichols Plot  6

ans =

Closed-Loop System

ans =

T(s)


Transfer function:
       7 s + 35
-----------------------
s^3 + 4 s^2 + 17 s + 35
```

```
Damping Ratio = 0.259481, Percent Overshoot = 42.9946, Bandwidth = 5.1, Mp
0.957852
```





Closed-Loop Frequency Resposne Plots
Magnitude (dB)
20
0
-20
-40
-60
-80
Phase (deg)
0
-45
-90
-135
-180
-225
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)



Closed-Loop Step Response
Amplitude
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
Time (sec)

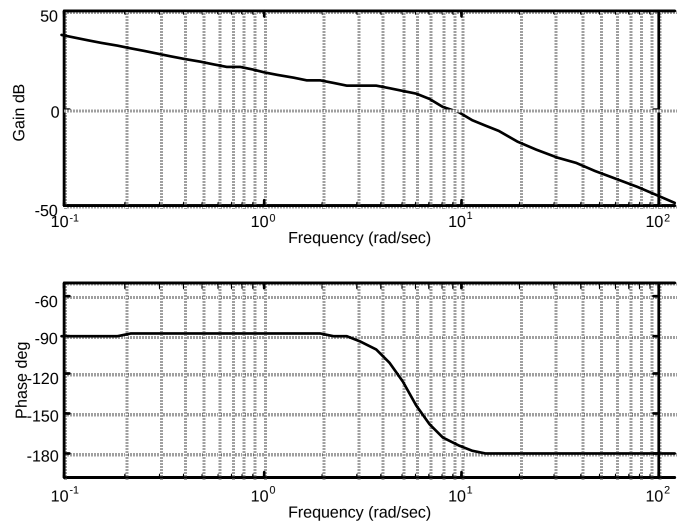
23.

System 1: Using non-asymptotic frequency response plots, the zero dB crossing is at 9.7 rad/s at a phase of -163.2^o. Therefore the phase margin is $180^o - 163.2^o = 16.8^o$. $|G(j\omega)|$ is down 7 dB at 14.75 rad/s. Therefore the bandwidth is 14.75 rad/s. Using Eq. (10.73), $\zeta = 0.15$. Using Eq. (4.38), %OS = 62.09%. Eq. (10.55) yields $T_s = 2.76$ s, and Eq. (10.56) yields $T_p = 0.329$ s.

System 2: Using non-asymptotic frequency response plots, the zero dB crossing is at 6.44 rad/s at a phase of -150.73^o. Therefore the phase margin is $180^o - 150.73^o = 29.27^o$. $|G(j\omega)|$ is down 7 dB at 10.1 rad/s. Therefore the bandwidth is 10.1 rad/s. Using Eq. (10.73), $\zeta = 0.262$. Using Eq. (4.38), %OS = 42.62%. Eq. (10.55) yields $T_s = 2.23$ s, and Eq. (10.56) yields $T_p = 0.476$ s.

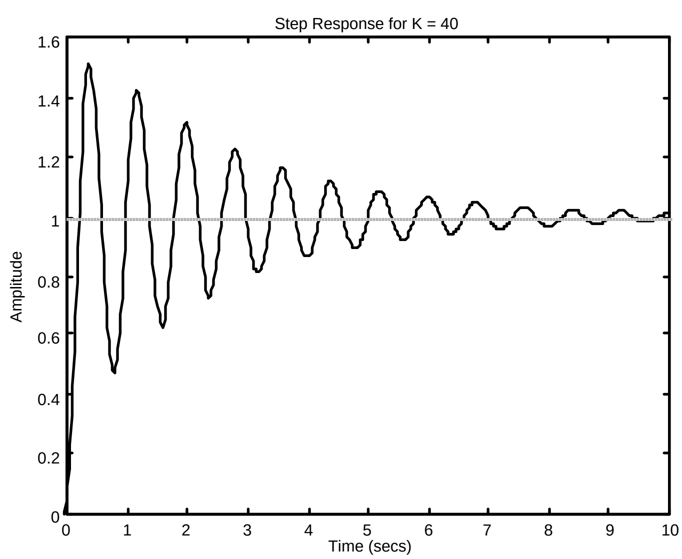
24.

a.



b. Zero dB frequency = 7.8023; Looking at the phase diagram at this frequency, the phase margin is 8.777 degrees. Using Eq. (10.73) or Figure 10.48, $\zeta = 0.08$. Thus, %OS = 77.7.

c.



25.

From the Bode plots: Gain margin = 14.96 dB; phase margin = 49.57^0; 0 dB frequency = 2.152 rad/s; 180^0 frequency = 6.325 rad/s; bandwidth(@-7 dB point) = 3.8 rad/s. From Eq. (10.73) ζ = 0.48; from Eq. (4.38) %OS = 17.93; from Eq. (10.55) T_s = 2.84 s; from Eq. (10.56) T_p = 1.22 s.

26.

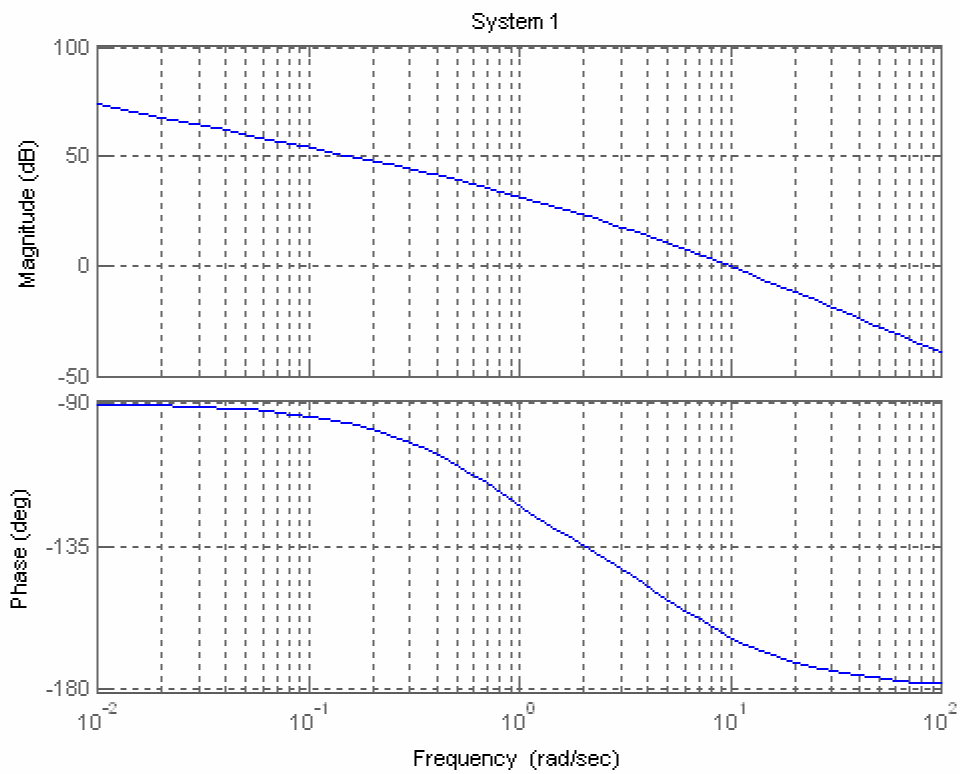
Program:

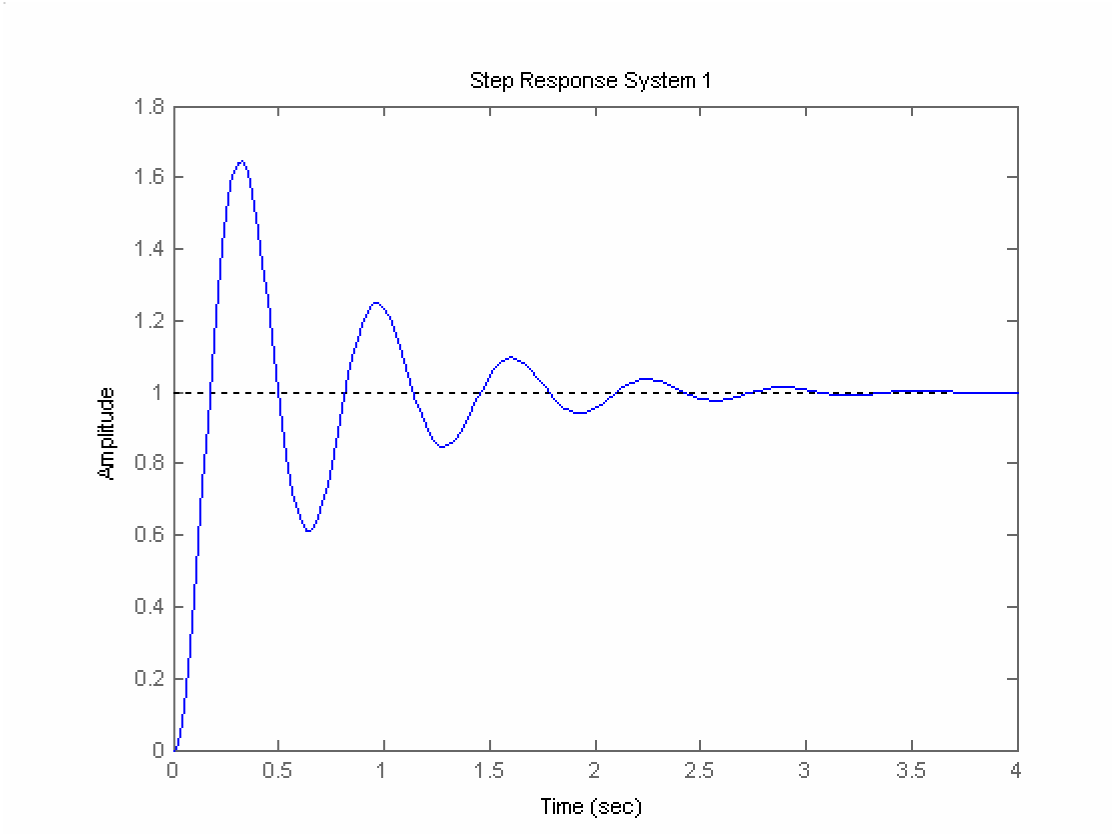
```
G=zpk([-2],[0 -1 -4],100)
%G=zpk([-3 -5],[0 -2 -4 -6],50)
G=tf(G)
bode(G)
title('System 1')
%title('System 2')
pause
%Find Phase Margin
[Gm,Pm,Wcg,Wcp]=margin(G);
w=1:.01:20;
[M,P,w]=bode(G,w);
%Find Bandwidth
for k=1:1:length(M);
   if 20*log10(M(k))+7<=0;
   'Mag'
   20*log10(M(k))
   'BW'
   wBW=w(k)
   break
end
end
%Find Damping Ratio,Percent Overshoot, Settling Time, and Peak Time
for z= 0:.01:10
Pt=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi);
if (Pm-Pt)<=0
   z;
   Po=exp(-z*pi/sqrt(1-z^2));
   Ts=(4/(wBW*z))*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2));
   Tp=(pi/(wBW*sqrt(1-z^2)))*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2));
fprintf('Bandwidth = %g ',wBW)
fprintf('Phase Margin = %g',Pm)
fprintf(', Damping Ratio = %g',z)
fprintf(', Percent Overshoot = %g',Po*100)
fprintf(',Settling Time = %g',Ts)
fprintf(', Peak Time = %g',Tp)
   break
end
end
T=feedback(G,1);
step(T)
title('Step Response System 1')
%title('Step Response System 2')
```

Computer response:

```
Zero/pole/gain:
  100 (s+2)
-------------
s (s+1) (s+4)


Transfer function:
   100 s + 200
-----------------
s^3 + 5 s^2 + 4 s


ans =
```

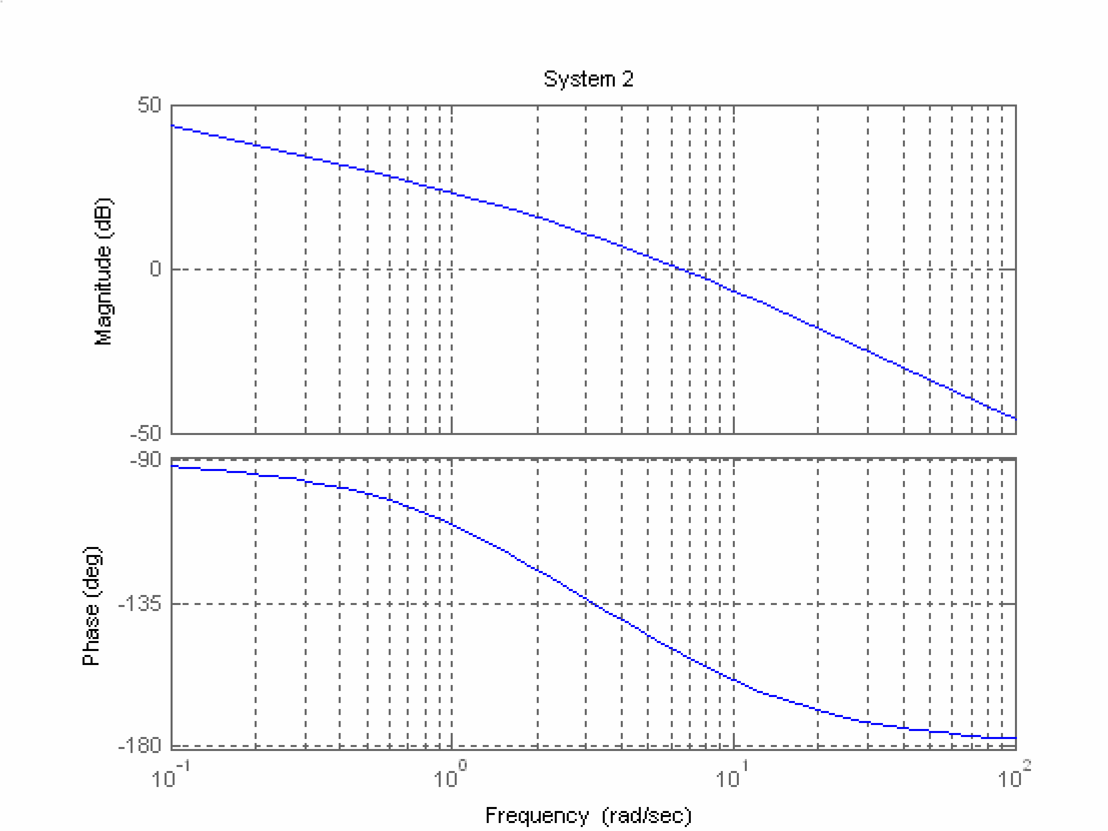
```
Mag

ans =

   -7.0007


ans =

BW


wBW =

   14.7500

Bandwidth = 14.75 Phase Margin = 16.6617, Damping Ratio = 0.15, Percent
Overshoot = 62.0871,Settling Time = 2.76425, Peak Time = 0.329382
```

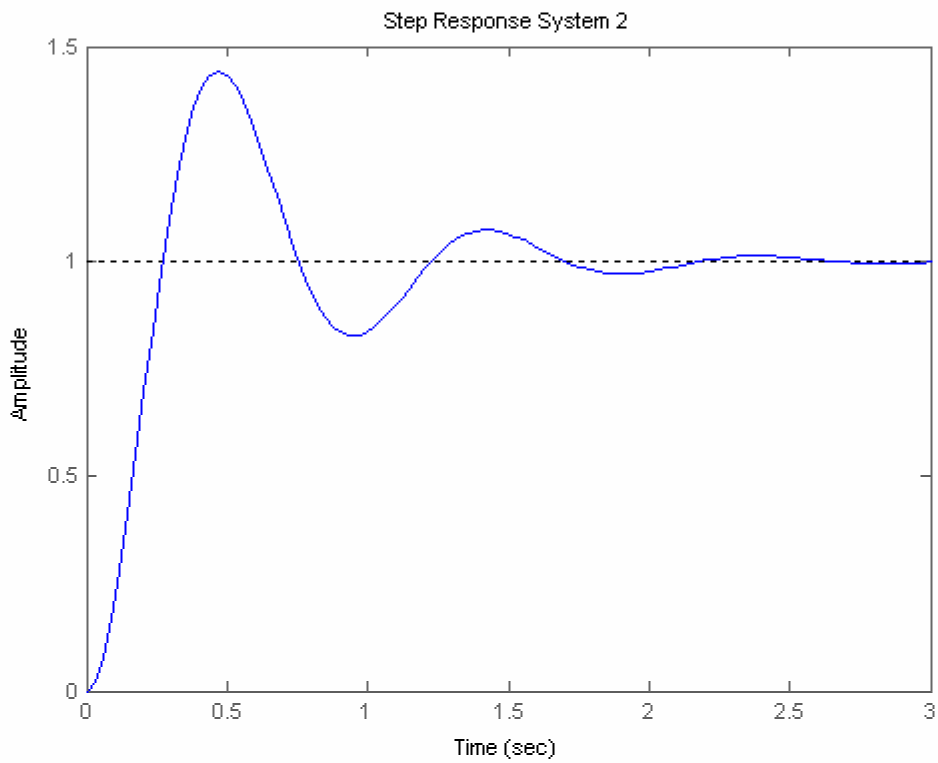





```
Zero/pole/gain:

  50 (s+3) (s+5)

-------------------

s (s+2) (s+4) (s+6)

Transfer function:

    50 s^2 + 400 s + 750

----------------------------

s^4 + 12 s^3 + 44 s^2 + 48 s

ans =

Mag

ans =

   -7.0026

ans =
```

```
BW

wBW =

   10.1100

Bandwidth = 10.11 Phase Margin = 29.2756, Damping Ratio = 0.27, Percent Overshoot

= 41.439,Settling Time = 2.1583, Peak Time =

0.475337
```





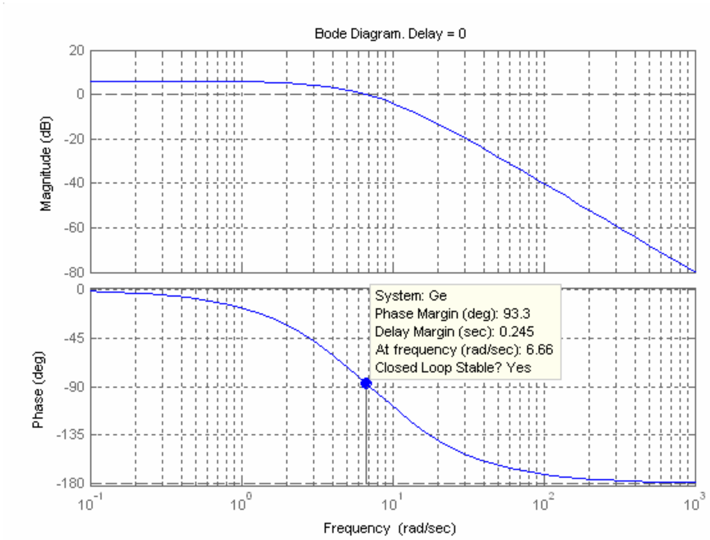
27.

The phase margin of the given system is 20^o. Using Eq. (10.73), $\zeta = 0.176$. Eq. (4.38) yields 57% overshoot. The system is Type 1 since the initial slope is - 20 dB/dec. Continuing the initial slope down to the 0 dB line yields $K_v = 4$. Thus, steady-state error for a unit step input is zero; steady state error for a unit ramp input is $\dfrac{1}{K_v} = 0.25$; steady-state error for a parabolic input is infinite.

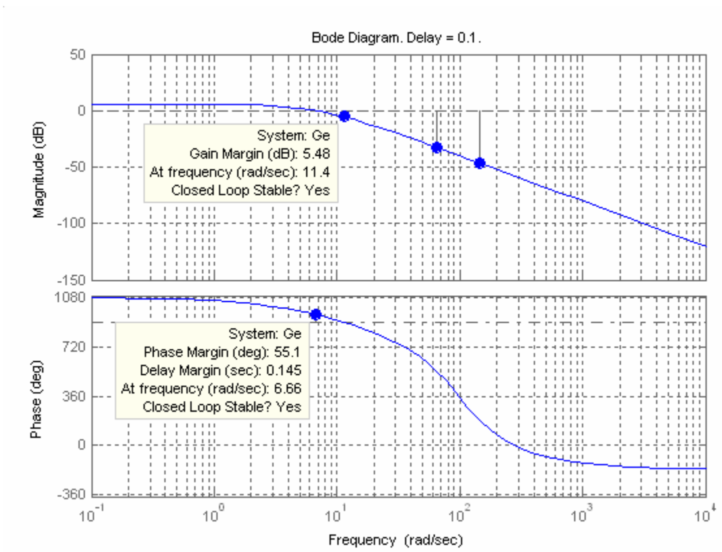
28.

The magnitude response is the same for all time delays and crosses zero dB at 0.5 rad/s. The following is a plot of the magnitude and phase responses for the given time delays:

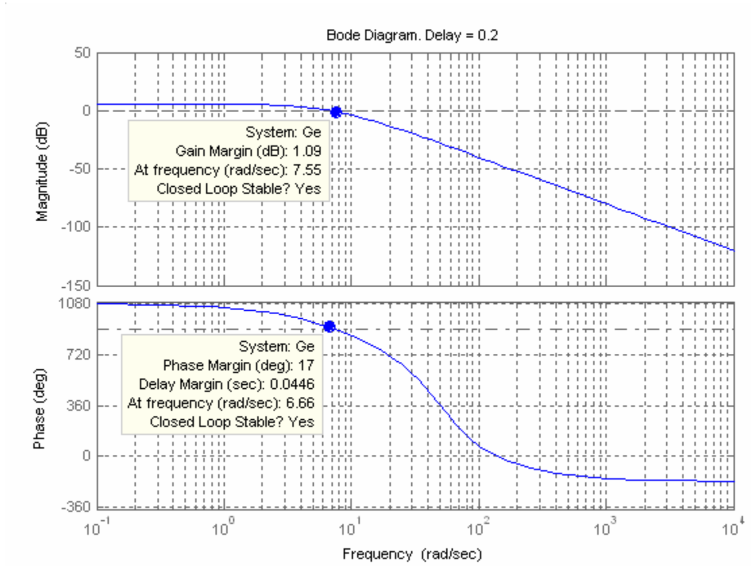
a.



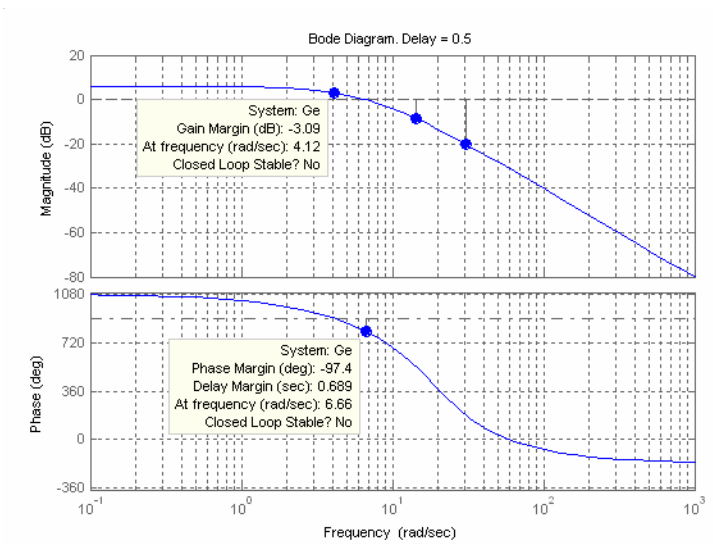
For T = 0, $\Phi_M = 93.3^o$; System is stable.



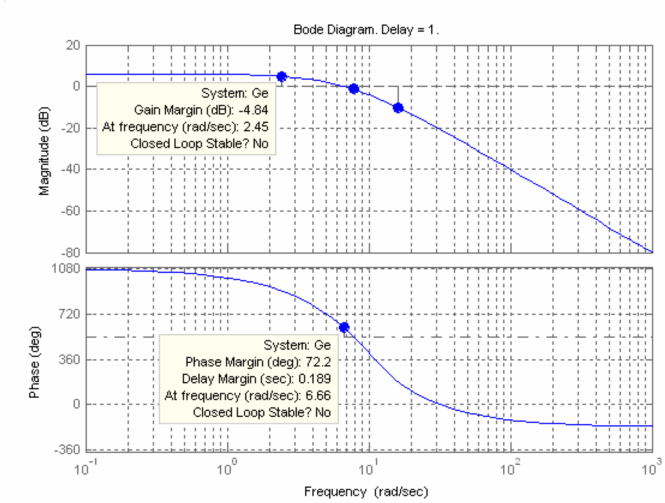
For T = 0.1, $\Phi_M = 55.1^o$; System is stable.



For T = 0.2, $\Phi_M = 17^o$; System is stable.



For T = 0.5, $\Phi_M = -97^o$; System is unstable.



For T = 1, $\Phi_M = 72.2^o$; System is unstable because the gain margin is -4.84 dB.

b.

For T = 0, the phase response reaches 180^o at infinite frequency. Therefore the gain margin is infinite. The system is stable.

For T = 0.1, the phase response is -180^o at 11.4 rad/s. The magnitude response is -5.48 dB at 11.4 rad/s. Therefore, the gain margin is 5.48 dB. The system is stable.

For T = 0.2, the phase response is -180^o at 7.55 rad/s. The magnitude response is -1.09 dB at 7.55 rad/s. Therefore, the gain margin is 1.09 dB and the system is stable.

For T = .5, the phase response is -180^o at 4.12 rad/s. The magnitude response is +3.09 dB at 4.12 rad/s. Therefore, the gain margin is – 3.09 dB and the system is unstable.

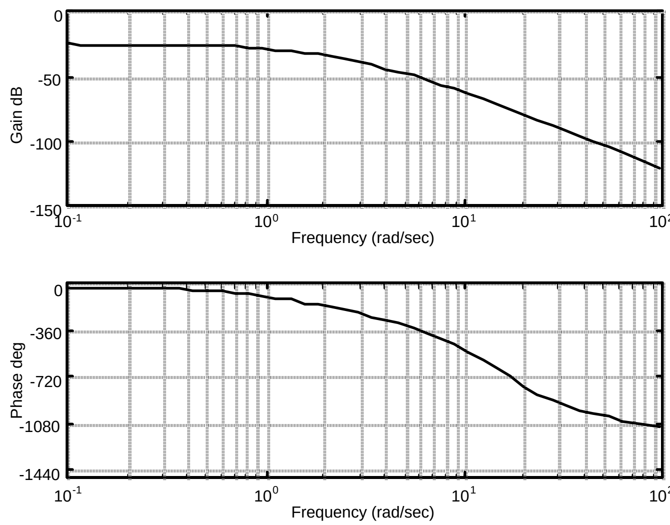
For T = 1, the phase response is -180^o at 2.45 rad/s. The magnitude response is +4.84 dB at 2.45 rad/s. Therefore, the gain margin is -4.84 dB and the system is unstable.

c.
T = 0; T = 0.1; T = 0.2

d.
T = 0.5, -3.09 dB; T = 1, - 4.84 dB;

29.

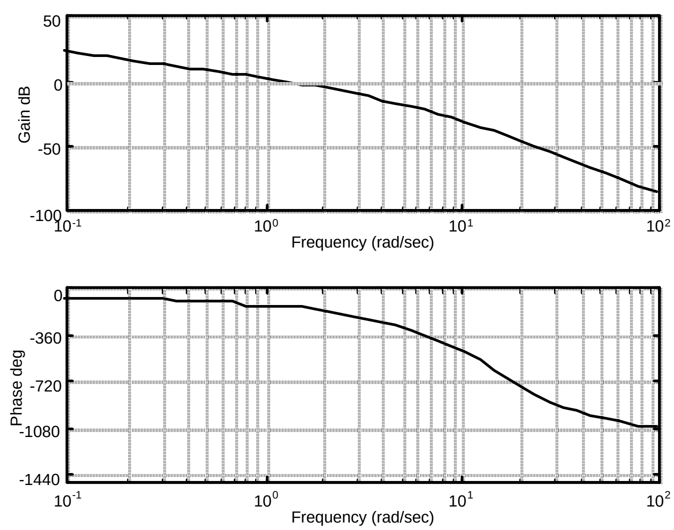
The Bode plots for K = 1 and 0.5 second delay is:



The phase is -180° at 2.12 rad/s. At this frequency, the gain is -34.76 dB. Thus the gain can be raised by 34.76 dB = 54.71. Hence for stability, $0<K<54.71$.

30.

The Bode plots for K = 40 and a delay of 0.5 second is shown below.



The magnitude curve crosses zero dB at a frequency of 1.0447 rad/s. At this frequency, the phase plot shows a phase margin of 35.74 degrees. Using Eq. (10.73) or Figure 10.48, $\zeta = 0.33$. Thus, %OS = 33.3.

31.

Program:

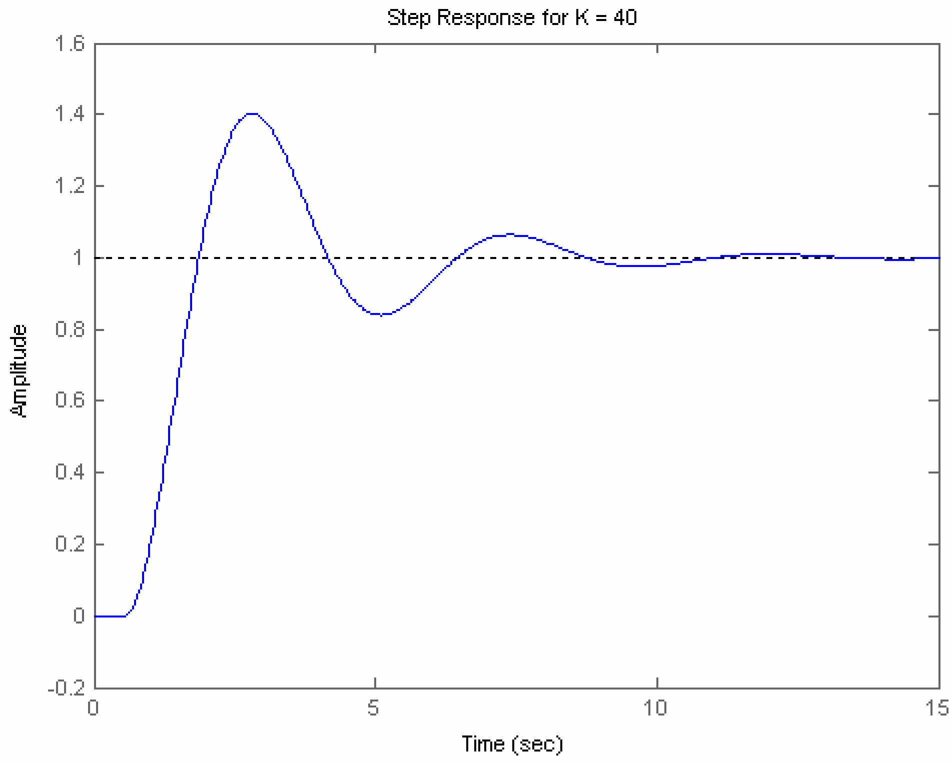
```
%Enter G(s)************************
numg1=1;
deng1=poly([0 -3 -12]);
'G1(s)'
G1=tf(numg1,deng1)
[numg2,deng2]=pade(0.5,5);
'G2(s) (delay)'
```

```
G2=tf(numg2,deng2)
'G(s)=G1(s)G2(s)'
G=G1*G2
%Enter K ***************************
K=input('Type gain, K ');
T=feedback(K*G,1);
step(T)
title(['Step Response for K = ',num2str(K)])
```

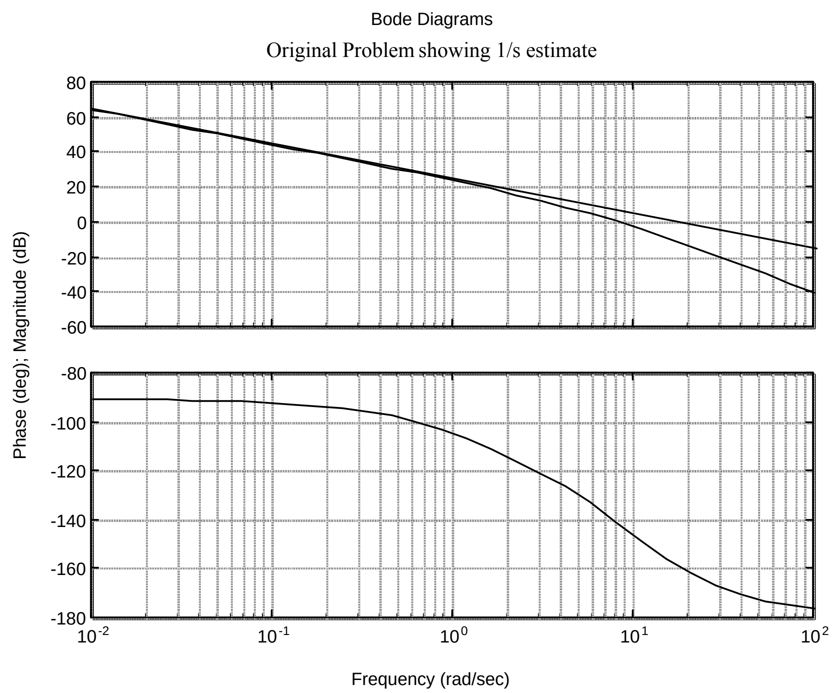
Computer response:

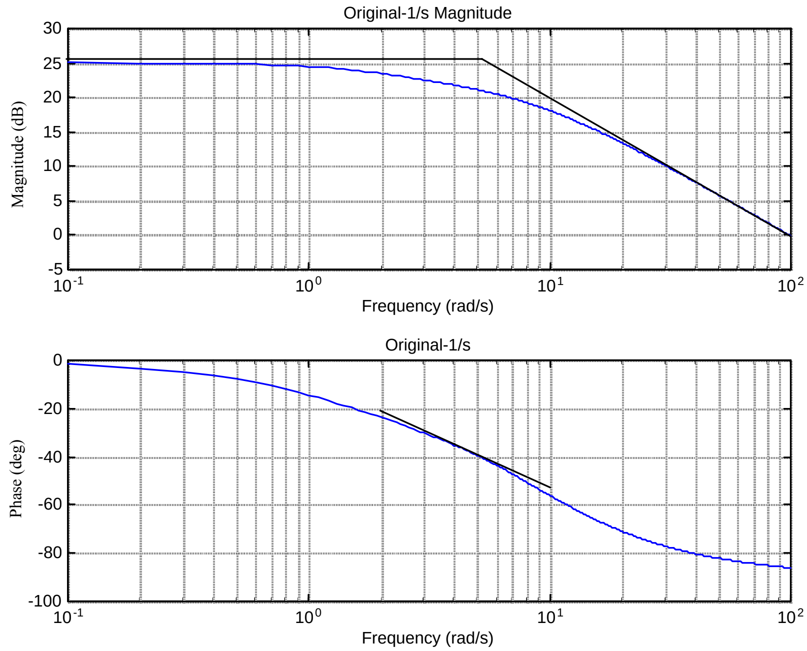
```
ans =

G1(s)

Transfer function:

         1

-------------------

s^3 + 15 s^2 + 36 s

ans =

G2(s) (delay)

Transfer function:

-s^5 + 60 s^4 - 1680 s^3 + 2.688e004 s^2 - 2.419e005 s + 9.677e005

-----------------------------------------------------------------

s^5 + 60 s^4 + 1680 s^3 + 2.688e004 s^2 + 2.419e005 s + 9.677e005

ans =

G(s)=G1(s)G2(s)

Transfer function:

    -s^5 + 60 s^4 - 1680 s^3 + 2.688e004 s^2 - 2.419e005 s + 9.677e005

-------------------------------------------------------------------------

s^8 + 75 s^7 + 2616 s^6 + 5.424e004 s^5 + 7.056e005 s^4 + 5.564e006 s^3

                                             + 2.322e007 s^2 + 3.484e007 s
```

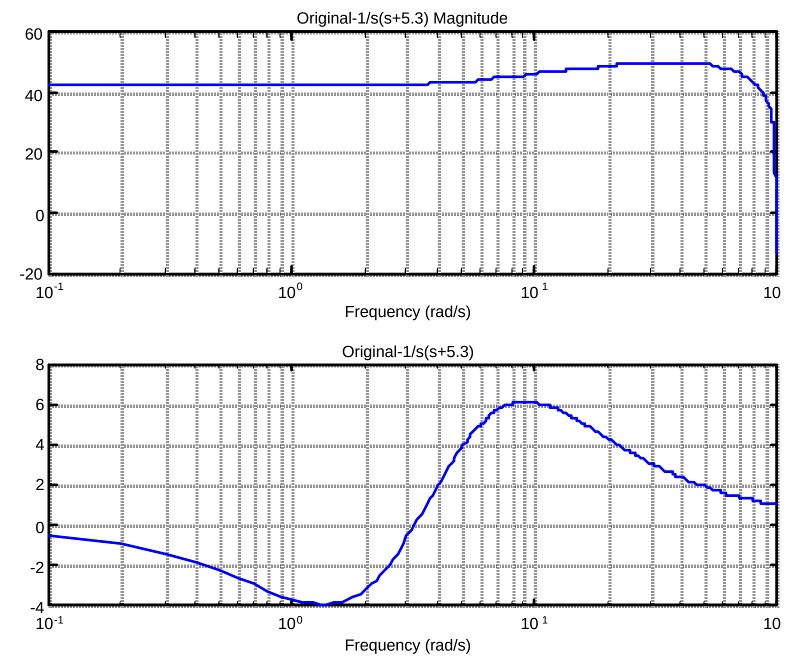
```
Type gain, K 40
```



32.







Estimated $K = 41$ dB $= 112$. Therefore, final estimate is $G(s) = \dfrac{112}{s(s+5.3)}$.

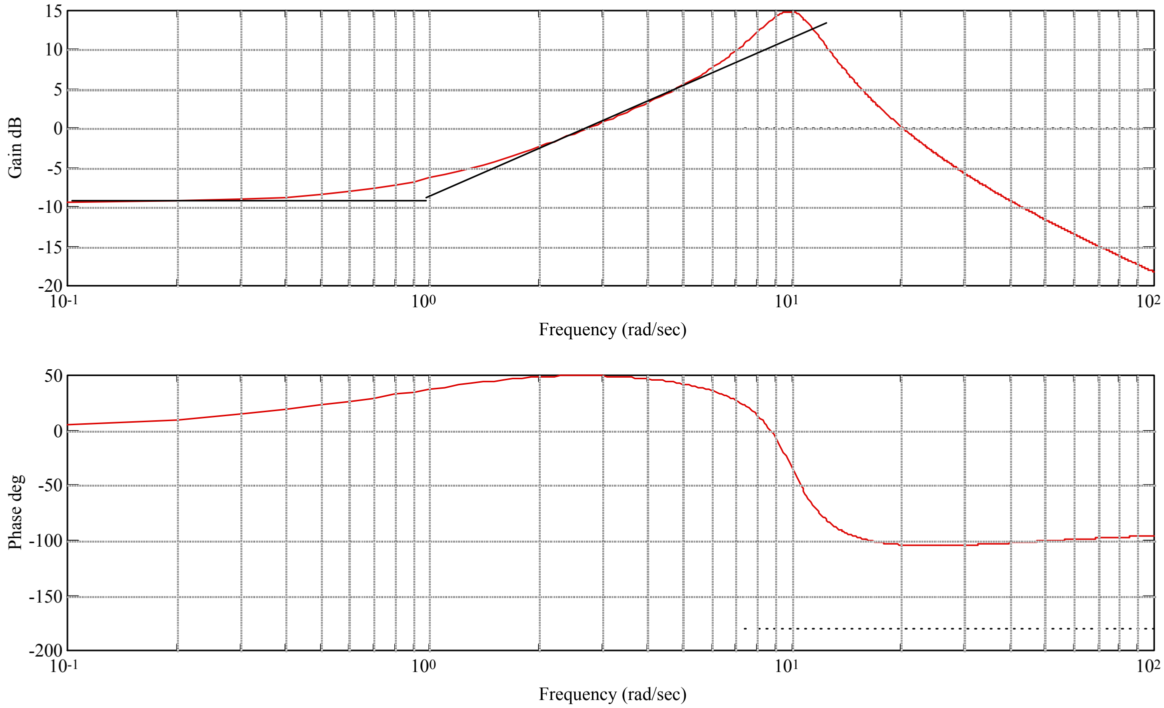
33.

Program:

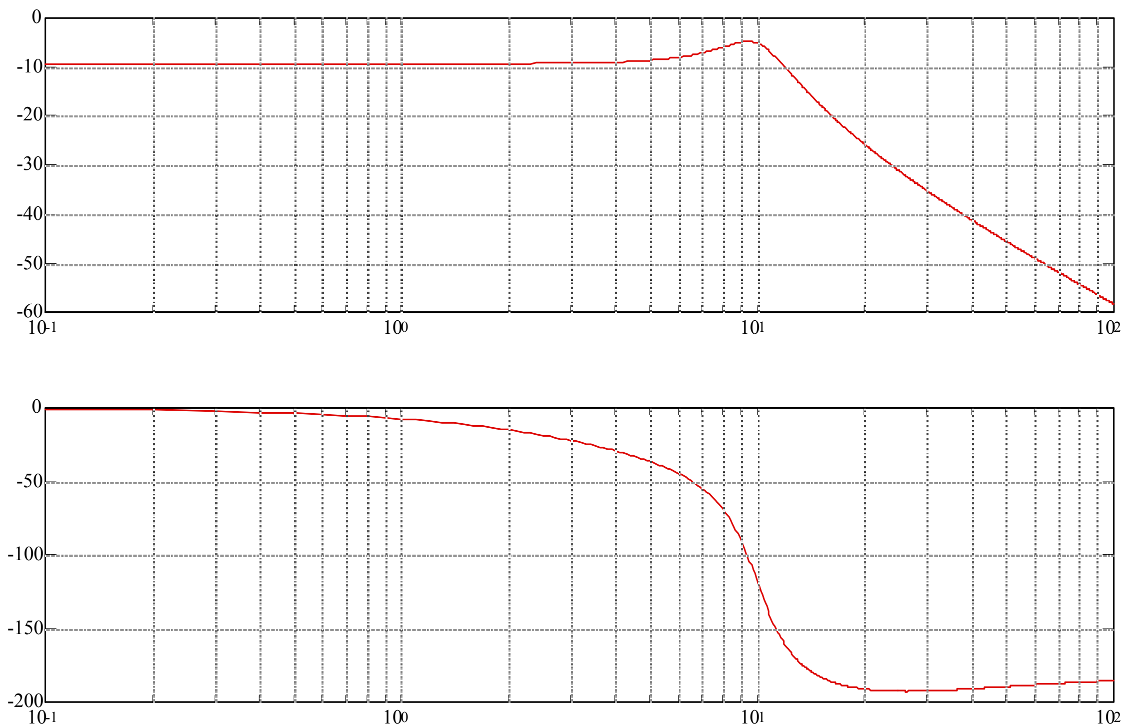
```
%Generate total system Bode plots - numg0,deng0 - M0,P0
clf
numg0=12*poly([-1 -20]);
deng0=conv([1 7],[1 4 100]);
G0=tf(numg0,deng0);
w=0.1:0.1:100;
```

```
[M0,P0]=bode(G0,w);
M0=M0(:,:);
P0=P0(:,:);
[20*log10(M0),P0,w];
bode(G0,w)
pause
%Subtract (s+1) [numg1,deng1] and generate Bode plot-M2,P2
numg1=[1 1];
deng1=1;
G1=tf(numg1,deng1);
[M1,P1]=bode(G1,w);
M1=M1(:,:);
P1=P1(:,:);
M2=20*log10(M0)-20*log10(M1);
P2=P0-P1;
clf
subplot(2,1,1)
semilogx(w,M2)
grid
subplot(2,1,2)
semilogx(w,P2)
grid
pause
%Subtract10^2/(s^2+2*0.3*10s+10^2) [numg2,deng2] and generate Bode plot-
M4,P4
numg2=100;
deng2=[1 2*0.3*10 10^2];
G2=tf(numg2,deng2);
[M3,P3]=bode(G2,w);
M3=M3(:,:);
P3=P3(:,:);
M4=M2-20*log10(M3);
P4=P2-P3;
clf
subplot(2,1,1)
semilogx(w,M4)
grid
subplot(2,1,2)
semilogx(w,P4)
grid
pause
%Subtract(8.5/23)(s+23)/(s+8.5) [numg3,deng3] and generate Bode plot-M6,P6
numg3=(8.5/23)*[1 23];
deng3=[1 8.5];
G3=tf(numg3,deng3);
[M5,P5]=bode(G3,w);
M5=M5(:,:);
P5=P5(:,:);
M6=M4-20*log10(M5);
P6=P4-P5;
clf
subplot(211)
semilogx(w,M6)
grid
subplot(212)
semilogx(w,P6)
grid
```

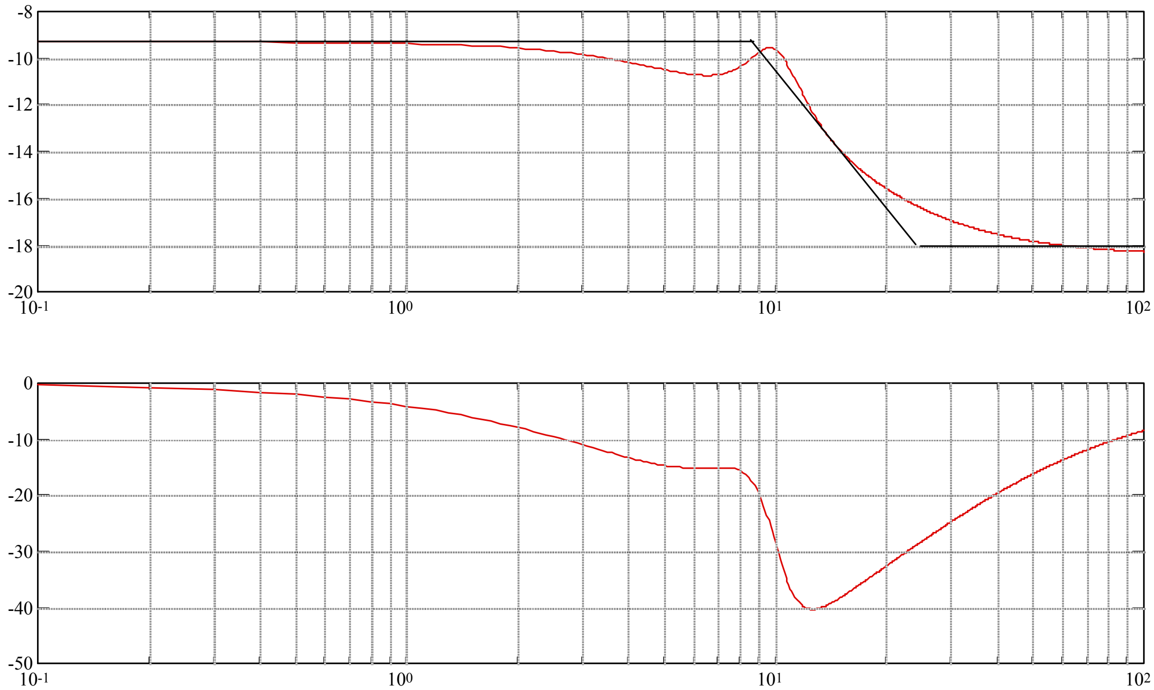
Computer responses and analysis:



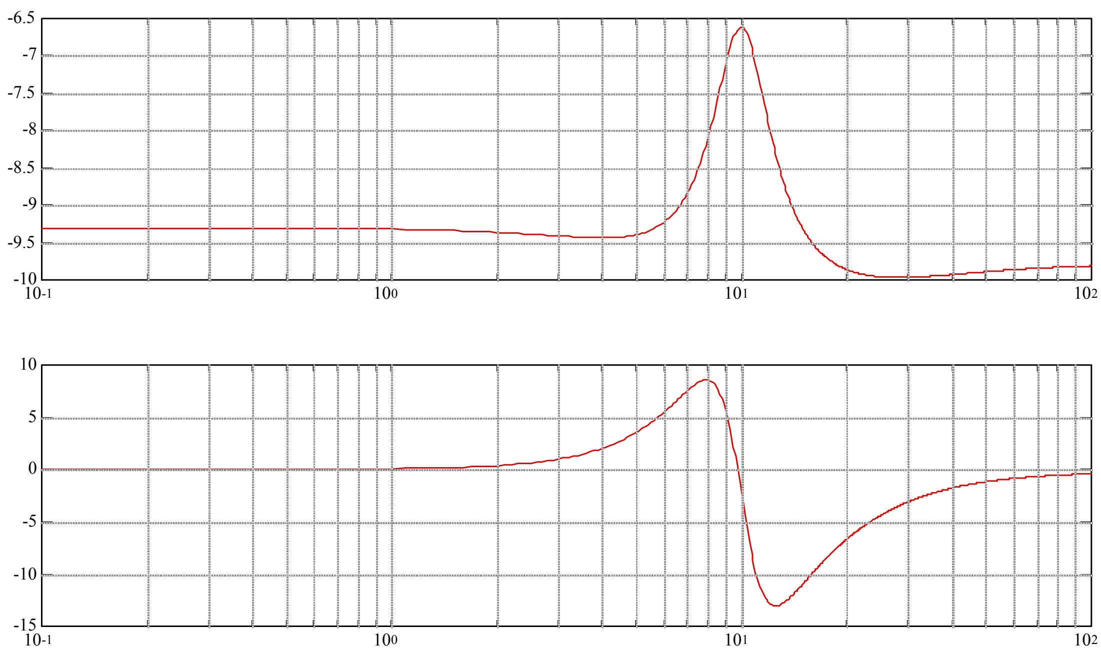
Original data showing estimate of a component, (s+1)



Original data minus (s+1) showing estimate of $(10^2/(s^2+2*0.3*10s+10^2))$



Original data minus $(s+1)(10^2/(s^2+2*0.3*10s+10^2))$ showing estimate of $(8.5/23)(s+23)/(s+8.5)$

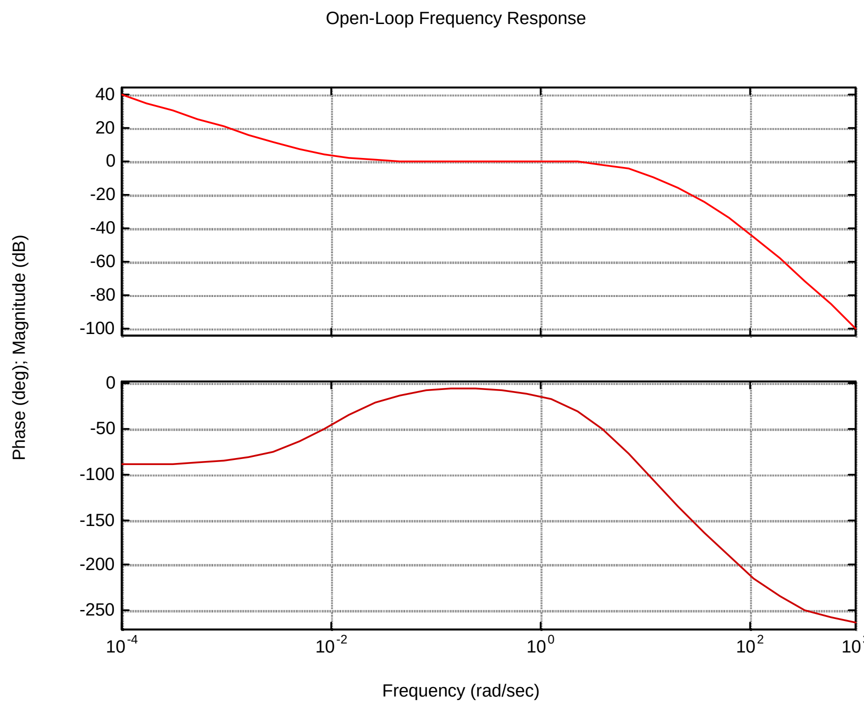


Original data minus final estimate of $G(s) = (s+1)*\dfrac{100}{s^2+6s+100}*\dfrac{8.5}{23}\dfrac{s+23}{s+8.5}$

Thus the final estimate is $G(s) = (s+1)*\frac{100}{s^2+6s+100}*\frac{8.5}{23}\frac{s+23}{s+8.5}*K$. Since the original plot starts from -10 dB, 20 log K = -10, or K = 0.32.

34.

a.

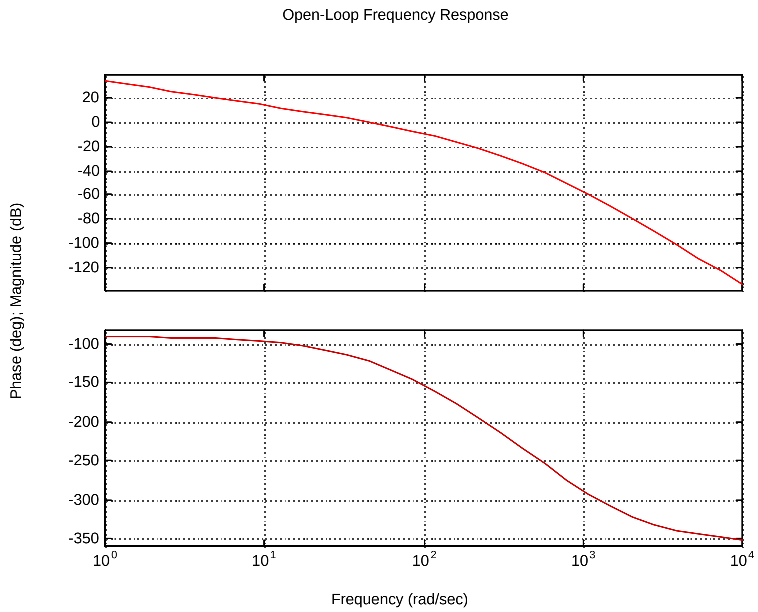


From the Bode plot: Gain margin = 29 52 dB; phase margin = 157.5^0; 0 dB frequency = 1.63 rad/s; 180^0 frequency = 49.8 rad/s.

b. System is stable since it has 180^0 of phase with a magnitude less than 0 dB.

35.

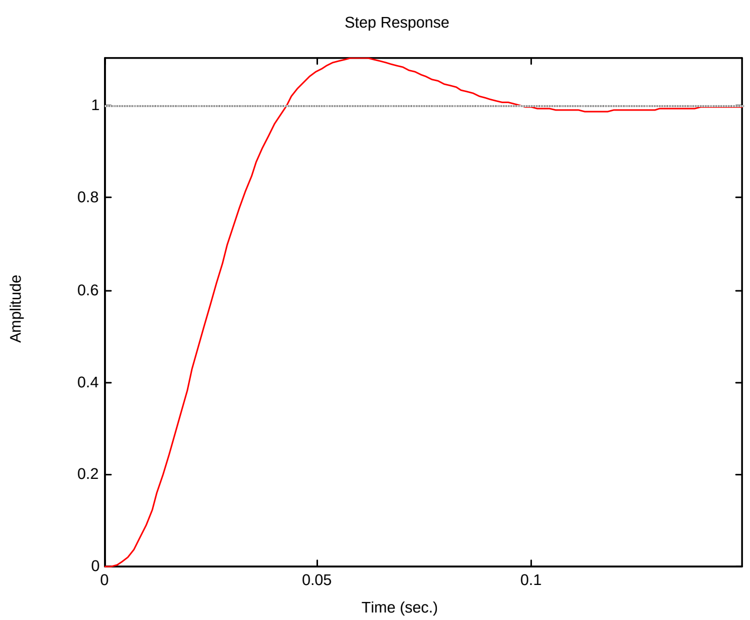
a.



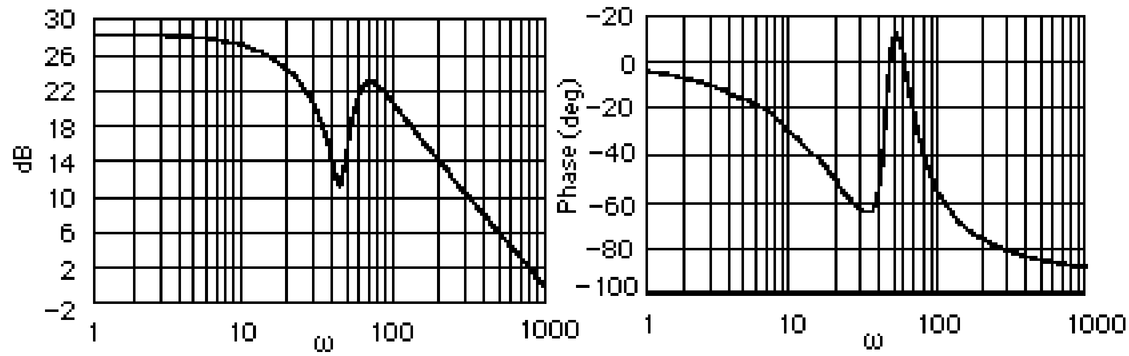
From the Bode plot: Gain margin = 17.1 dB; phase margin = 57.22^0; 0 dB frequency = 45.29 rad/s; 180^0 frequency = 169.03 rad/s; bandwidth(@-7 dB open-loop) = 85.32 rad/s.

b. From Eq. (10.73) $\zeta = 0.58$; from Eq. (4.38) %OS = 10.68; from Eq. (10.55) $T_s = 0.0949$ s; from Eq. (10.56) $T_p = 0.0531$ s.

c.



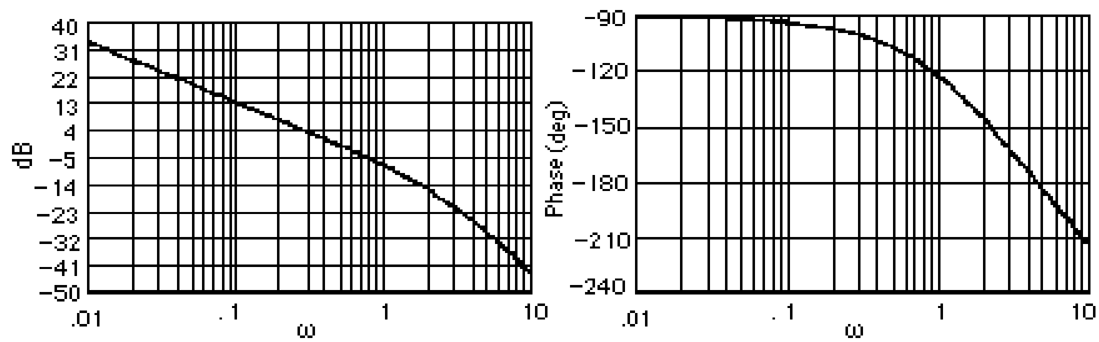
36.



Resonance at 70 rad/s.

37.

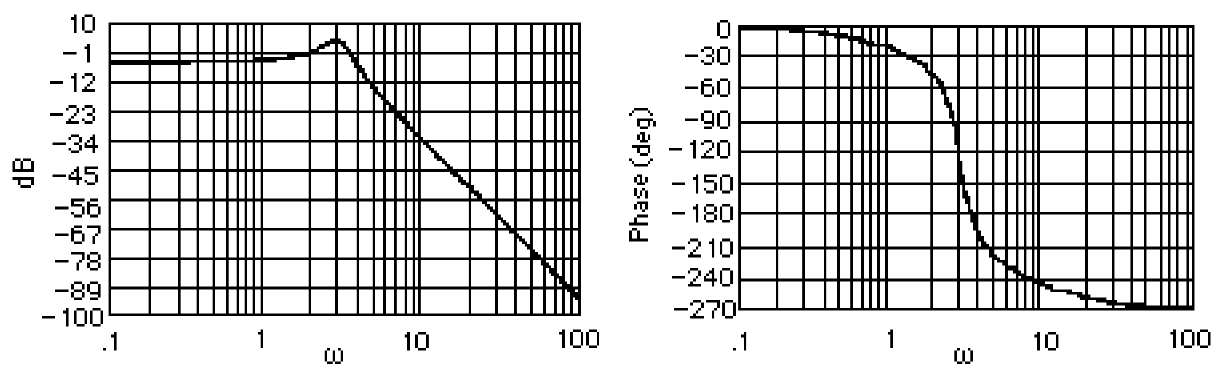
$G(s) = \dfrac{10}{s(s+2)(s+10)}$. Plotting the Bode plots,



The gain is zero dB at 0.486 rad/s and the phase angle is -106.44. Thus, the phase margin is 180^o - $106.44^o = 73.56^o$. Using Eq. (10.73), $\zeta = 0.9$. Using Eq. (4.38), %OS = 0.15%.

38.

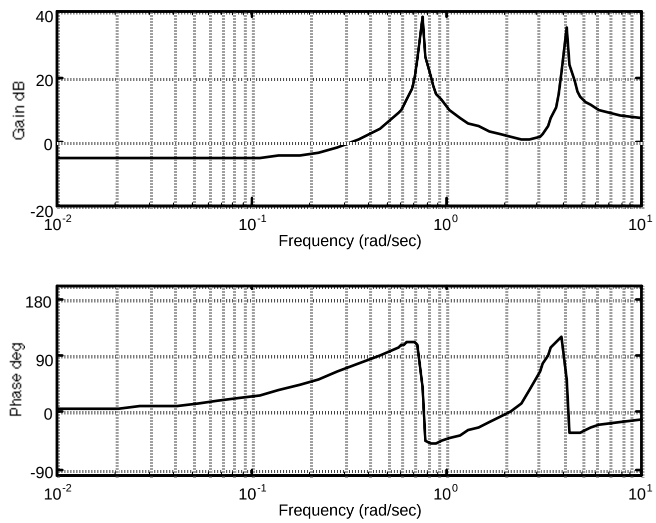
$G(s) = \dfrac{22.5}{(s+4)(s^2+0.9s+9)}$. Plotting the Bode plots,



The phase response is 180^o at $\omega = 3.55$ rad/s, where the gain is -1.17 dB. Thus, the gain margin is 1.17 dB. Unity gain is at $\omega = 2.094$ rad/s, where the phase is - 49.85^o and at $\omega = 3.452$ rad/s, where the phase is -173.99°. Hence the phase margin is measured at $\omega = 3.452$ rad/s and is 180°- 173.99° = 6.01°. Using Eq. (10.73), $\zeta = 0.0525$. Eq. (4.38) yields %OS = 84.78%.

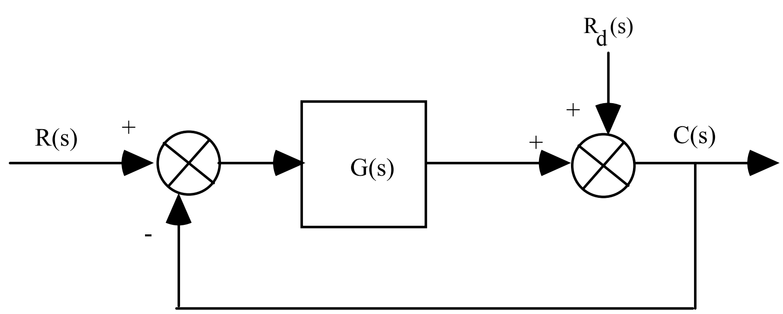
39.

a.



The frequencies that will be reduced occur at the peaks of the magnitude plot. The frequencies at the peaks are 4.14 rad/s and 0.754 rad/s.

b. Consider a system with a disturbance, R_d at the output of a system:



The transfer function relating C(s) to R_d(s) is $\dfrac{C(s)}{R_d(s} = \dfrac{1}{1+G(s)}$. Therefore,

$$C(s) = \frac{1}{1+\dfrac{N_G}{D_G}} * \frac{N_{Rd}}{D_{Rd}} = \frac{D_G}{D_G + N_G} * \frac{N_{Rd}}{D_{Rd}}$$

Thus, if the poles of G(s) match the poles of R_d ($D_G = D_{Rd}$) there will be cancellation and the dynamics of the disturbance will be reduced. Thus, if the dynamics of R_d is oscillation, add poles in cascade with G(s) that have the same dynamics. Since the poles yield large gain at these bending frequencies a zero is placed near the poles so that the filter will have minimal effect on the transient response (similar to placing a zero near a pole for a lag compensator). This arrangement of poles and zeros is called a dipole. Also note that a high gain at the bending frequency yields negative feedback for the output to subtract from R_d. Care should be exercised through analysis and simulation to be sure that the system's response to an input, other than the disturbance, is not adversely affected by the additional poles.

40.

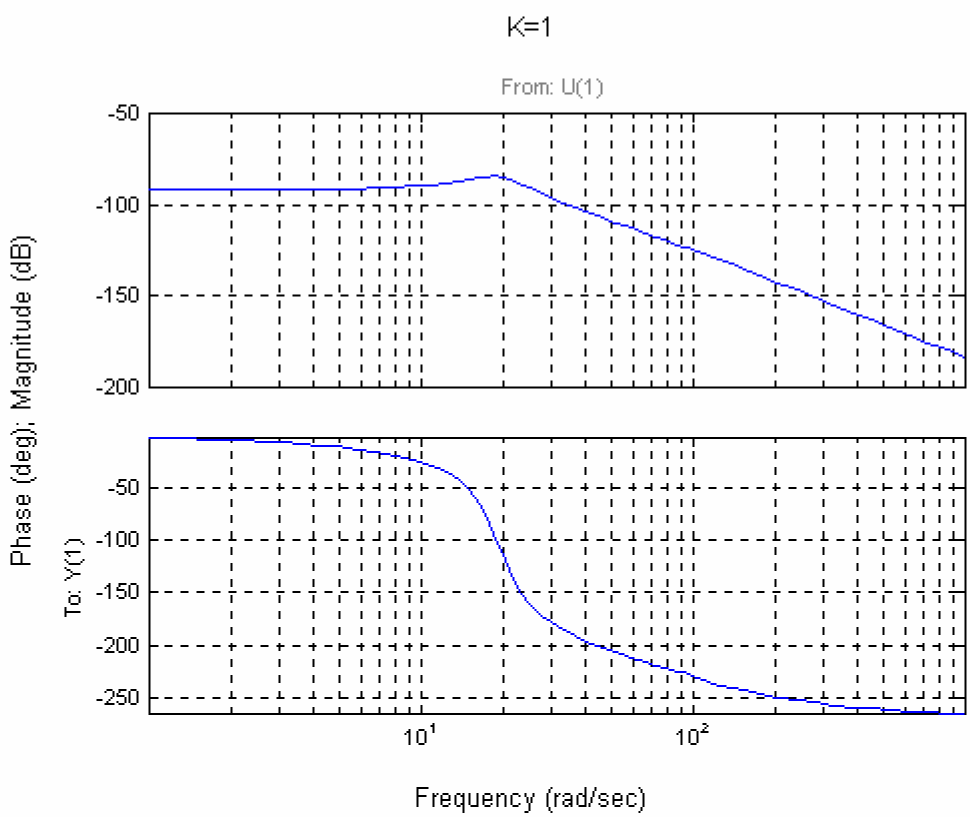
a. From Chapter 8,

$$G_e(s) = \frac{0.6488K\,(s+53.85)}{(s^2 + 8.119s + 376.3)\,(s\text{^}2 + 15.47s + 9283)}$$

Cascading the notch filter,

$$G_{et}(s) = \frac{0.6488K\,(s+53.85)(s^2 + 16s + 9200)}{(s^2 + 8.119s + 376.3)\,(s\text{^}2 + 15.47s + 9283)(s+60)^2}$$

Plotting the Bode plot,



From the Bode plot: Gain margin = 96.74 dB; phase margin = ∞; 0 dB frequency = N/A; 180^0 frequency = 30.44 rad/s.

b. K = 96.74 dB = 68732

c. In Chapter 6 K = 188444. The difference is due to the notch filter.