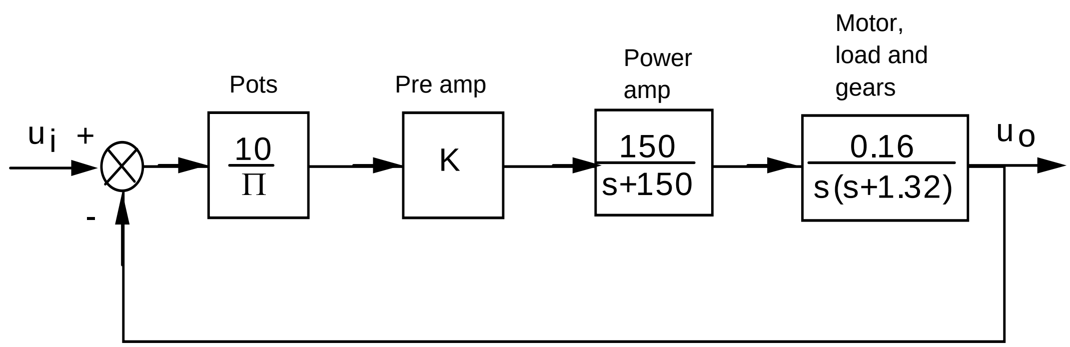
F I V E

Reduction of Multiple Subsystems

SOLUTIONS TO CASE STUDIES CHALLENGES

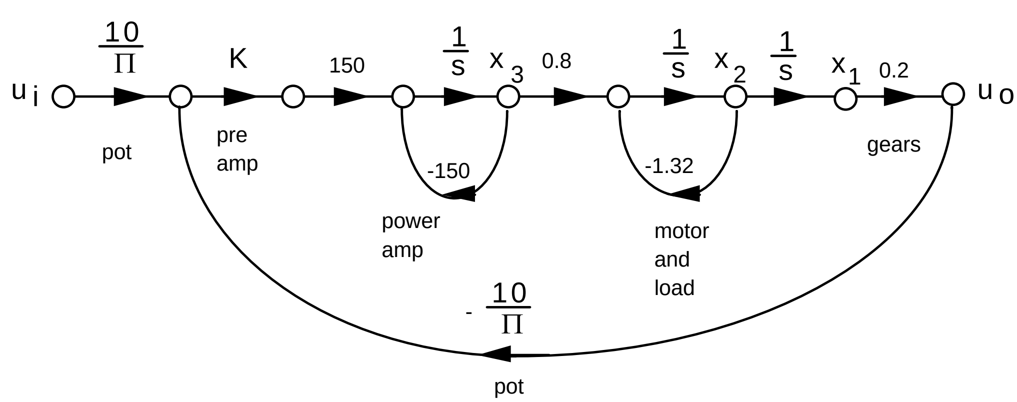
Antenna Control: Designing a Closed-Loop Response

a. Drawing the block diagram of the system:



Thus, $T(s) = \dfrac{76.39K}{s^3+151.32s^2+198s+76.39K}$

b. Drawing the signal flow-diagram for each subsystem and then interconnecting them yields:



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -1.32x_2 + 0.8x_3$$

$$\dot{x}_3 = -150x_3 + 150K\left(\frac{10}{\pi}(q_i - 0.2x_1)\right) = -95.49Kx_1 - 150x_3 + 477.46K\theta_i$$

$$\theta_o = 0.2x_1$$

In vector-matrix notation,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1.32 & 0.8 \\ -95.49K & 0 & -150 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 477.46K \end{bmatrix}\theta_i$$

$$\theta_o = \begin{bmatrix} 0.2 & 0 & 0 \end{bmatrix}\mathbf{x}$$

c. $T_1 = \left(\frac{10}{\pi}\right)(K)(150)\left(\frac{1}{s}\right)(0.8)\left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(0.2) = \frac{76.39}{s^3}$

$$G_{L1} = \frac{-150}{s};\ G_{L2} = \frac{-1.32}{s};\ G_{L3} = (K)(150)\left(\frac{1}{s}\right)(0.8)\left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(0.2)\left(\frac{-10}{\pi}\right) = \frac{-76.39K}{s^3}$$

Nontouching loops:

$$G_{L1}G_{L2} = \frac{198}{s^2}$$

$$\Delta = 1 - [G_{L1} + G_{L2} + G_{L3}] + [G_{L1}G_{L2}] = 1 + \frac{150}{s} + \frac{1.32}{s} + \frac{76.39K}{s^3} + \frac{198}{s^2}$$

$$\Delta_1 = 1$$

$$T(s) = \frac{T_1\Delta_1}{\Delta} = \frac{76.39K}{s^3+151.32s^2+198s+76.39K}$$

d. The equivalent forward path transfer function is $G(s) = \dfrac{\frac{10}{\pi}0.16K}{s(s+1.32)}$.

Therefore,

$$T(s) = \frac{2.55}{s^2+1.32s+2.55}$$

The poles are located at $-0.66 \pm j1.454$. $\omega_n = \sqrt{2.55} = 1.597$ rad/s; $2\zeta\omega_n = 1.32$, therefore, $\zeta = 0.413$.

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 24\%$; $T_s = \dfrac{4}{\zeta\omega_n} = \dfrac{4}{0.66} = 6.06$ seconds; $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \dfrac{\pi}{1.454} =$

2.16 seconds; Using Figure 4.16, the normalized rise time is 1.486. Dividing by the natural frequency, $T_r = \dfrac{1.486}{\sqrt{2.55}} = 0.93$ seconds.

e.

$$C(s) = \frac{2.55}{s(s^2 + 1.32s + 2.55)}$$

$$C(s) = \frac{1}{s} - \frac{1}{25}\frac{25s + 33}{s^2 + 1.32s + 2.55}$$

$$C(s) = \frac{1}{s} - \frac{1}{25}\frac{25(s + 0.66) + 11.347\sqrt{2.1144}}{(s + 0.66)^2 + \sqrt{2.1144}^2}$$

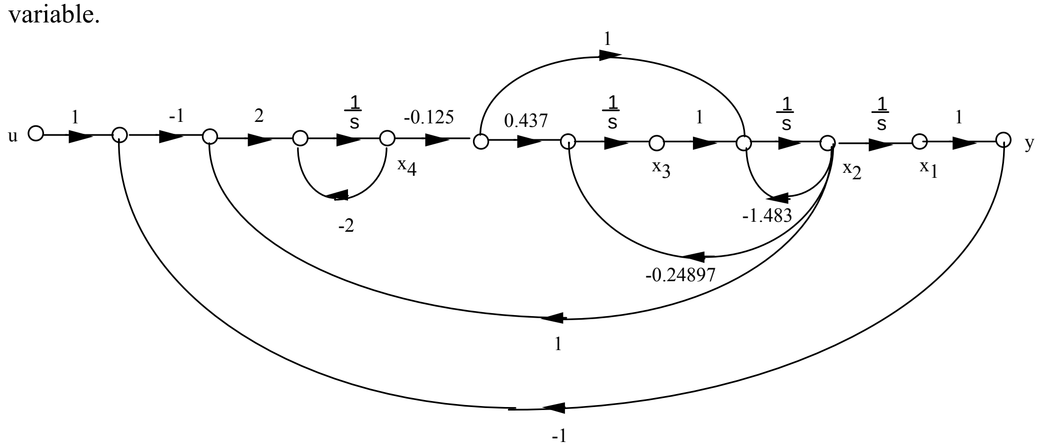
$$c(t) = 1 - e^{-0.66t}(\cos[1.454t] + 0.454\sin[1.454t])$$

f. Since $G(s) = \dfrac{0.51K}{s(s+1.32)}$, $T(s) = \dfrac{0.51K}{s^2+1.32s+0.51K}$. Also, $\zeta = \dfrac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = 0.517$ for 15% overshoot; $\omega_n = \sqrt{0.51K}$; and $2\zeta\omega_n = 1.32$. Therefore, $\omega_n = \dfrac{1.32}{2\zeta} = \dfrac{1.32}{2(0.5147)} = 1.277 = \sqrt{0.51K}$.

Solving for K, K=3.2.

UFSS Vehicle: Pitch-Angle Control Representation

a. Use the observer canonical form for the vehicle dynamics so that the output yaw rate is a state variable.



b. Using the signal flow graph to write the state equations:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -1.483x_2 + x_3 - 0.125x_4$$
$$\dot{x}_3 = -0.24897x_2 - (0.125 * 0.437)x_4$$
$$\dot{x}_4 = 2x_1 + 2x_2 - 2x_4 - 2u$$

In vector-matrix form:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1.483 & 1 & -0.125 \\ 0 & -0.24897 & 0 & -0.054625 \\ 2 & 2 & 0 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}\mathbf{x}$$

c.

Program:

```
numg1=-0.25*[1 0.437];
deng1=poly([-2 -1.29 -0.193 0]);
'G(s)'
G=tf(numg1,deng1)
numh1=[-1 0];
denh1=[0 1];
'H(s)'
H=tf(numh1,denh1)
'Ge(s)'
Ge=feedback(G,H)
'T(s)'
T=feedback(-1*Ge,1)
[numt,dent]=tfdata(T,'V');
[Acc,Bcc,Ccc,Dcc]=tf2ss(numt,dent)
```

Computer response:

```
ans =

G(s)

Transfer function:
           -0.25 s - 0.1093
-------------------------------------
s^4 + 3.483 s^3 + 3.215 s^2 + 0.4979 s

ans =

H(s)

Transfer function:
-s

ans =

Ge(s)

Transfer function:
           -0.25 s - 0.1093
-------------------------------------
s^4 + 3.483 s^3 + 3.465 s^2 + 0.6072 s
```

```
ans =

T(s)

Transfer function:
                0.25 s + 0.1093
-----------------------------------------------
s^4 + 3.483 s^3 + 3.465 s^2 + 0.8572 s + 0.1093

Acc =

   -3.4830   -3.4650   -0.8572   -0.1093
    1.0000         0         0         0
         0    1.0000         0         0
         0         0    1.0000         0

Bcc =

     1
     0
     0
     0

Ccc =

         0         0    0.2500    0.1093

Dcc =

     0
```

ANSWERS TO REVIEW QUESTIONS

1. Signals, systems, summing junctions, pickoff points

2. Cascade, parallel, feedback

3. Product of individual transfer functions, sum of individual transfer functions, forward gain divided by one plus the product of the forward gain times the feedback gain

4. Equivalent forms for moving blocks across summing junctions and pickoff points

5. As K is varied from 0 to ∞, the system goes from overdamped to critically damped to underdamped. When the system is underdamped, the settling time remains constant.

6. Since the real part remains constant and the imaginary part increases, the radial distance from the origin is increasing. Thus the angle θ is increasing. Since $\zeta = \cos\theta$ the damping ratio is decreasing.

7. Nodes (signals), branches (systems)

8. Signals flowing into a node are added together. Signals flowing out of a node are the sum of signals flowing into a node.

9. One

10. Phase-variable form, cascaded form, parallel form, Jordan canonical form, observer canonical form

11. The Jordan canonical form and the parallel form result from a partial fraction expansion.

12. Parallel form

13. The system poles, or eigenvalues

14. The system poles including all repetitions of the repeated roots

15. Solution of the state variables are achieved through decoupled equations. i.e. the equations are solvable individually and not simultaneously.

16. State variables can be identified with physical parameters; ease of solution of some representations

17. Systems with zeros

18. State-vector transformations are the transformation of the state vector from one basis system to another. i.e. the same vector represented in another basis.

19. A vector which under a matrix transformation is collinear with the original. In other words, the length of the vector has changed, but not its angle.

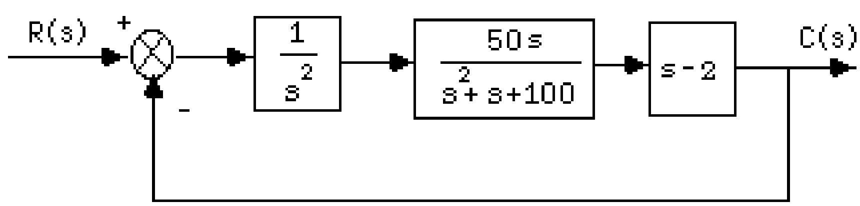
20. An eigenvalue is that multiple of the original vector that is the transformed vector.

21. Resulting system matrix is diagonal.

SOLUTIONS TO PROBLEMS

1.

a. Combine the inner feedback and the parallel pair.



Multiply the blocks in the forward path and apply the feedback formula to get,

$$T(s) = \frac{50(s-2)}{s^3+s^2+150s-100} .$$

b.

Program:

```
'G1(s)'
G1=tf(1,[1 0 0])
'G2(s)'
G2=tf(50,[1 1])
'G3(s)'
G3=tf(2,[1 0])
'G4(s)'
G4=tf([1 0],1)
'G5(s)'
G5=2
'Ge1(s)=G2(s)/(1+G2(s)G3(s))'
Ge1=G2/(1+G2*G3)
'Ge2(s)=G4(s)-G5(s)'
Ge2=G4-G5
'Ge3(s)=G1(s)Ge1(s)Ge2(s)'
Ge3=G1*Ge1*Ge2
```

```
'T(s)=Ge3(s)/(1+Ge3(s))'
T=feedback(Ge3,1);
T=minreal(T)
```

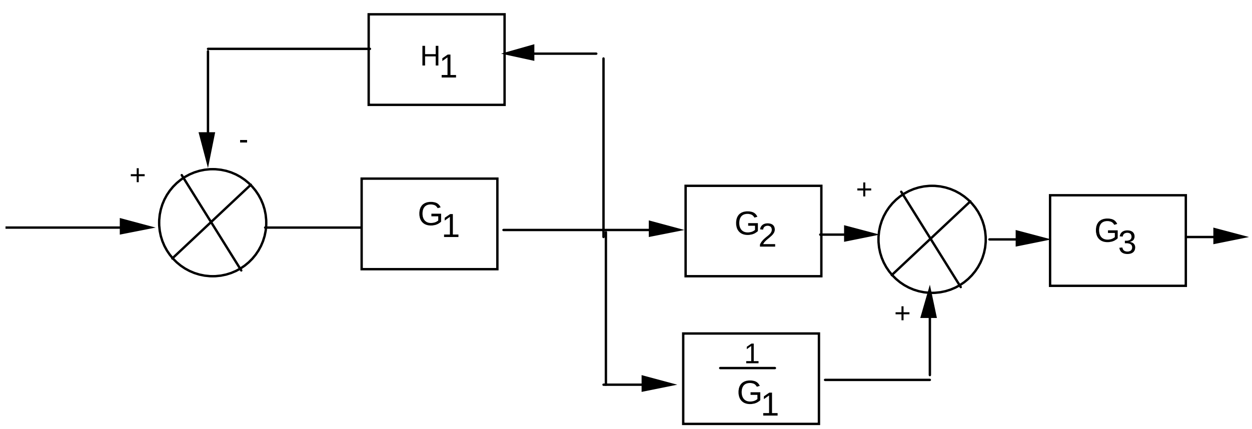
Computer response:

```
ans =

G1(s)

Transfer function:
 1
---
s^2

ans =

G2(s)

Transfer function:
 50
-----
s + 1

ans =

G3(s)

Transfer function:
2
-
s

ans =

G4(s)

Transfer function:
s

ans =

G5(s)

G5 =

     2

ans =

Ge1(s)=G2(s)/(1+G2(s)G3(s))


Transfer function:
      50 s^2 + 50 s
-------------------------
s^3 + 2 s^2 + 101 s + 100

ans =

Ge2(s)=G4(s)-G5(s)

Transfer function:
s - 2
```

```
ans =

Ge3(s)=G1(s)Ge1(s)Ge2(s)

Transfer function:
    50 s^3 - 50 s^2 - 100 s
-------------------------------
s^5 + 2 s^4 + 101 s^3 + 100 s^2

ans =

T(s)=Ge3(s)/(1+Ge3(s))

Transfer function:
      50 s - 100
-----------------------
s^3 + s^2 + 150 s - 100
```

2.

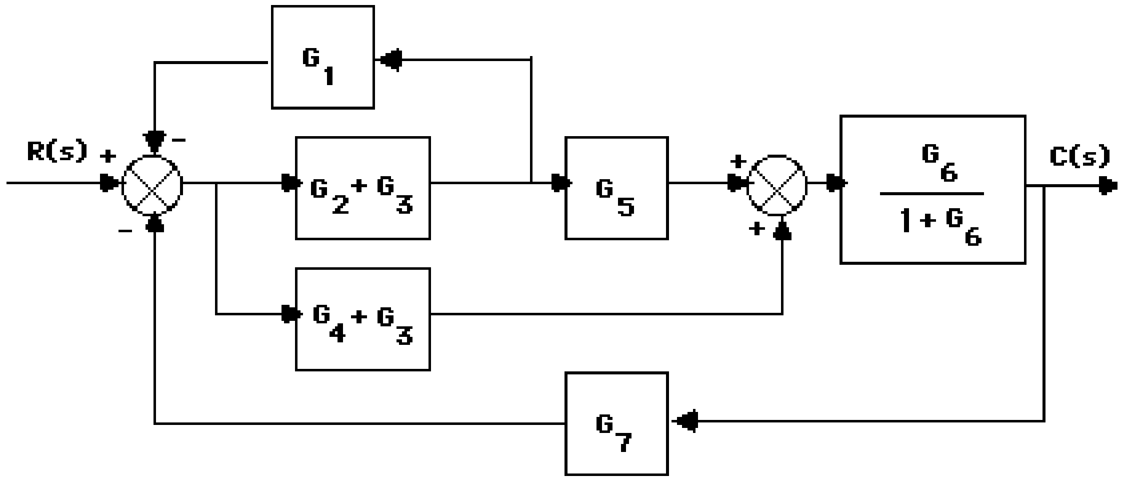
Push $G_1(s)$ to the left past the pickoff point.



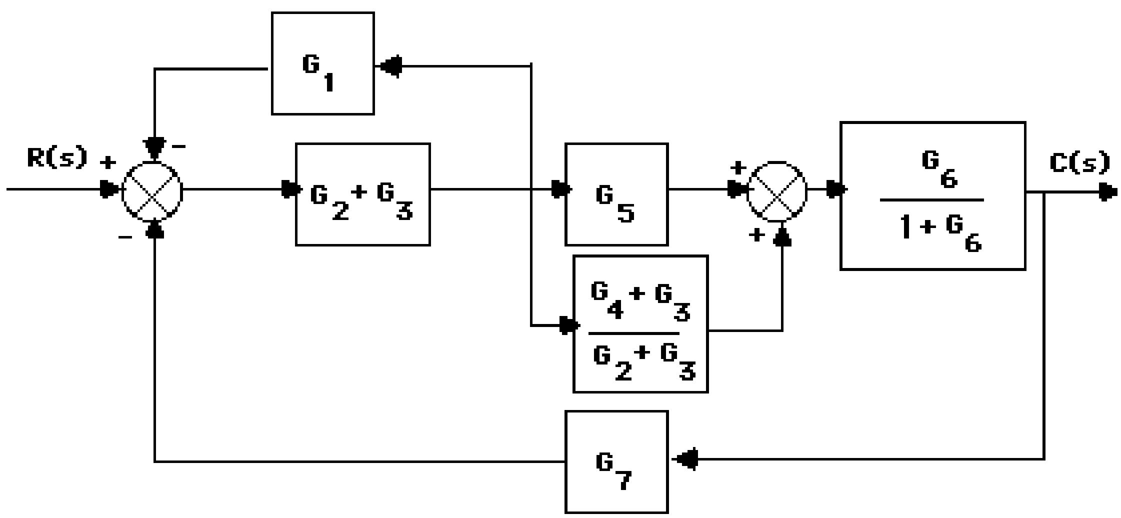
Thus, $T(s)=\left(\frac{G_1}{1+G_1H_1}\right)\left(G_2+\frac{1}{G_1}\right)G_3=\frac{(G_1G_2+1)G_3}{(1+G_1H_1)}$

3.

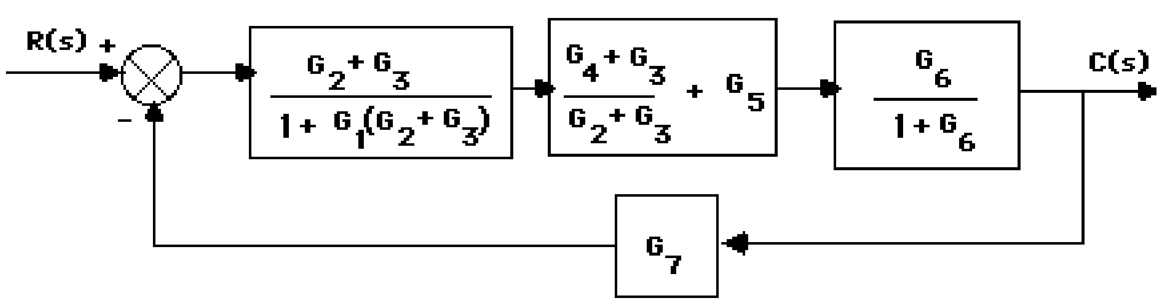
a. Split G_3 and combine with G_2 and G_4. Also use feedback formula on G_6 loop.



Push $G_2 + G_3$ to the left past the pickoff point.



Using the feedback formula and combining parallel blocks,

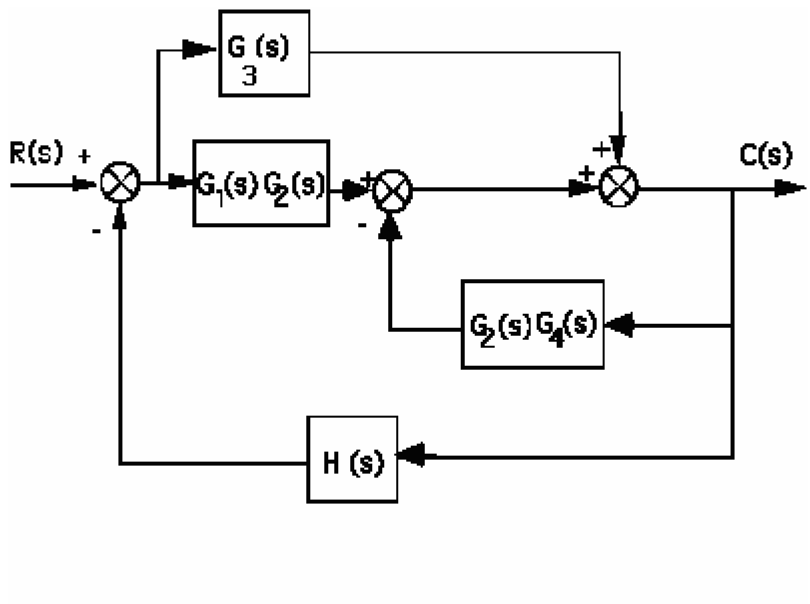


Multiplying the blocks of the forward path and applying the feedback formula,

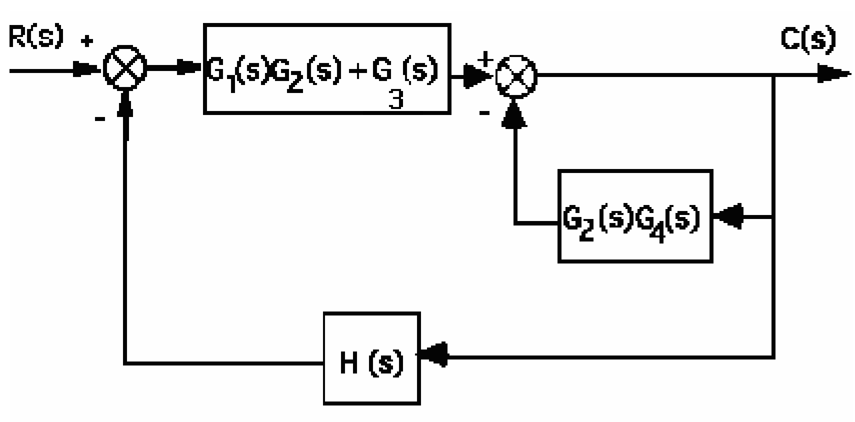
$$T(s) = \frac{G_6 G_4 + G_6 G_3 + G_6 G_5 G_3 + G_6 G_5 G_2}{1 + G_6 + G_3 G_1 + G_2 G_1 + G_7 G_6 G_4 + G_7 G_6 G_3 + G_7 G_6 G_5 G_3 + G_7 G_6 G_5 G_2 + G_6 G_3 G_1 + G_6 G_2 G_1}$$

4.

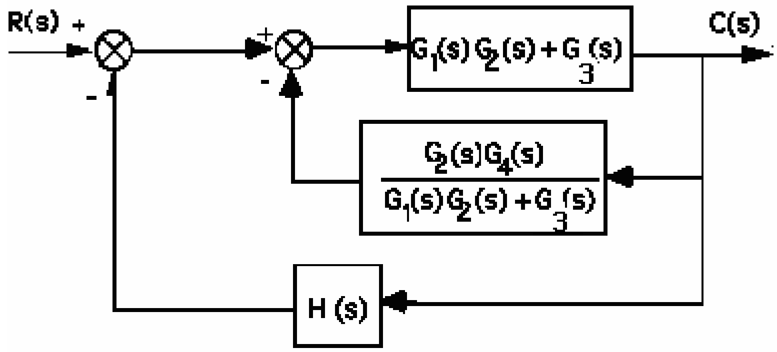
Push $G_2(s)$ to the left past the summing junction.



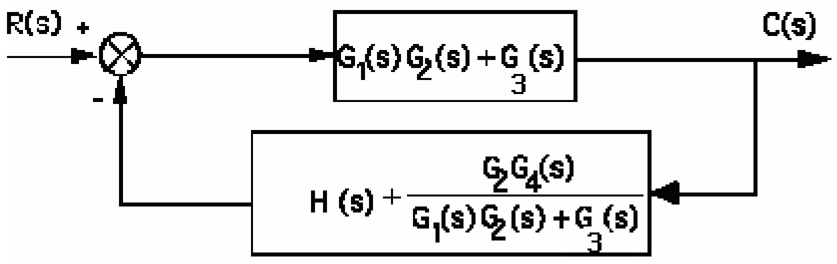
Collapse the summing junctions and add the parallel transfer functions.



Push $G_1(s)G_2(s) + G_5(s)$ to the right past the summing junction.



Collapse summing junctions and add feedback paths.



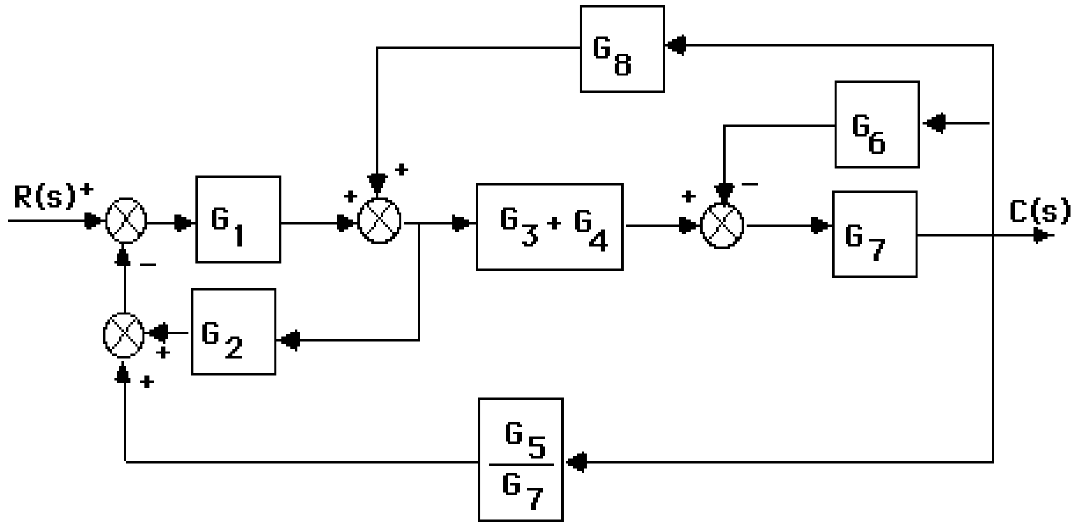
Applying the feedback formula,

$$T(s)=\frac{G_3(s)+G_1(s)G_2(s)}{1+[G_3(s)+G_1(s)G_2(s)]\left[H+\dfrac{G_2(s)G_4(s)}{G_3(s)+G_1(s)G_2(s)}\right]}$$

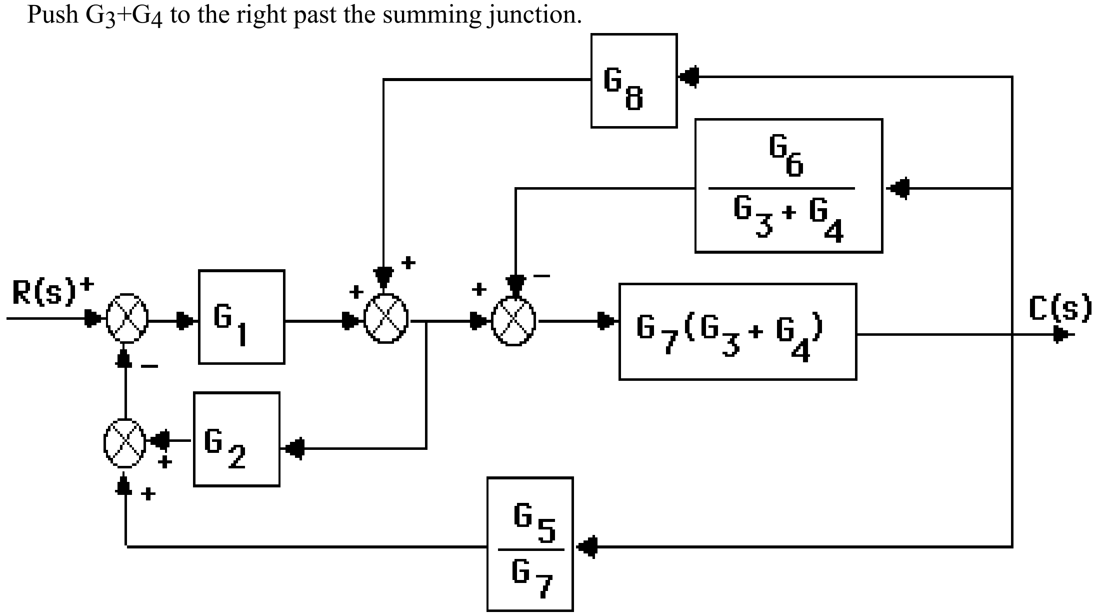
$$=\frac{G_3(s)+G_1(s)G_2(s)}{1+H[G_3(s)+G_1(s)G_2(s)]+G_2(s)G_4(s)}$$

5.

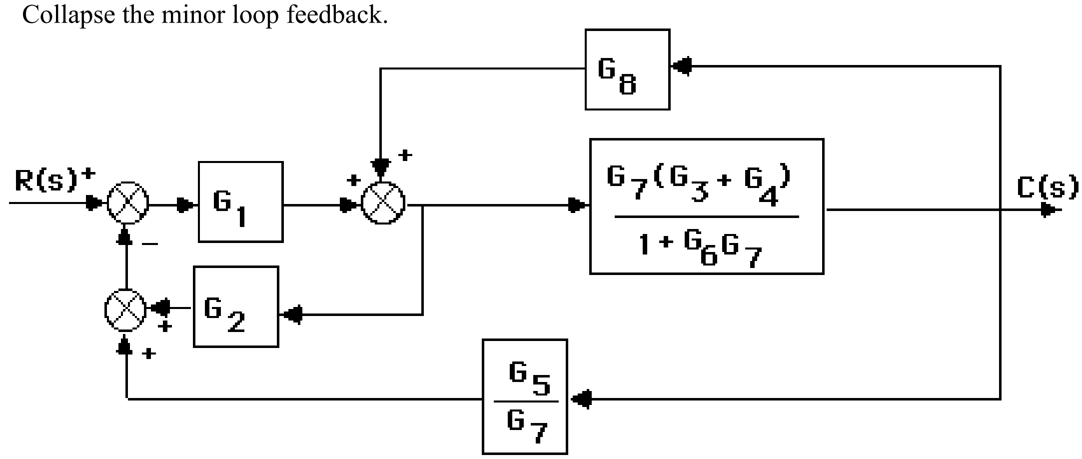
a. Push G_7 to the left past the pickoff point. Add the parallel blocks, G_3+G_4.



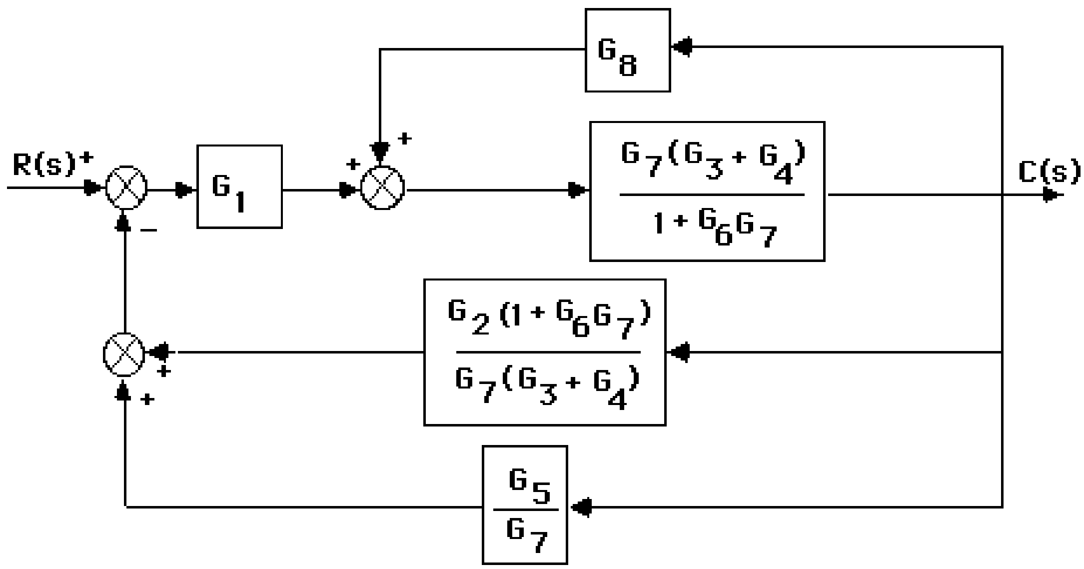
Push G_3+G_4 to the right past the summing junction.



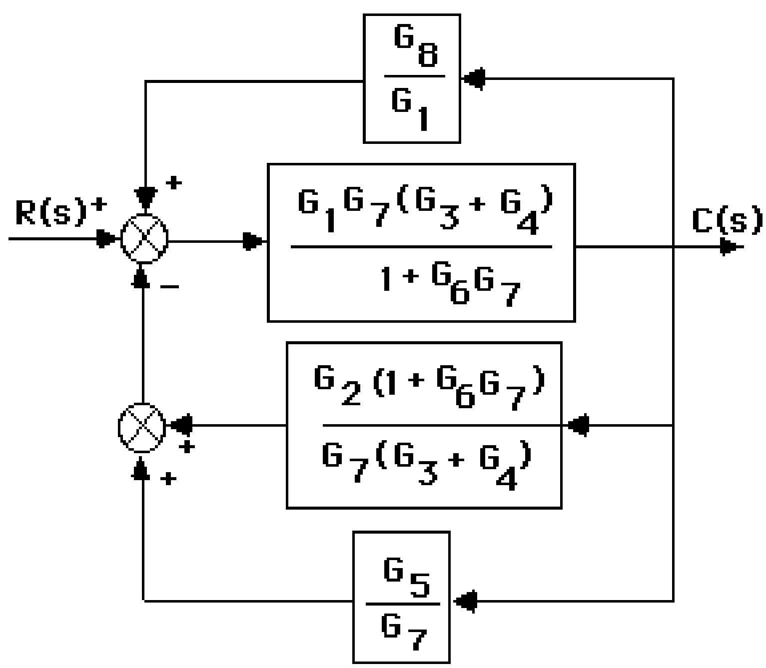
Collapse the minor loop feedback.



Push $\frac{G_7(G_3+G_4)}{1+G_6G_7}$ to the left past the pickoff point.



Push G_1 to the right past the summing junction.



Add the parallel feedback paths to get the single negative feedback,

$H(s) = \frac{G_5}{G_7} + \frac{G_2(1+G_6G_7)}{G_7(G_3+G_4)} - \frac{G_8}{G_1}$. Thus,

$$T(s) = \frac{G}{1+GH} = \frac{G_7 G_1 (G_4 + G_3)}{([G_7 G_6 + 1] G_2 G_1 + [G_4 + G_3] [G_5 G_1 - G_8 G_7]) + (G_7 G_6 + 1)}$$

b.

Program:

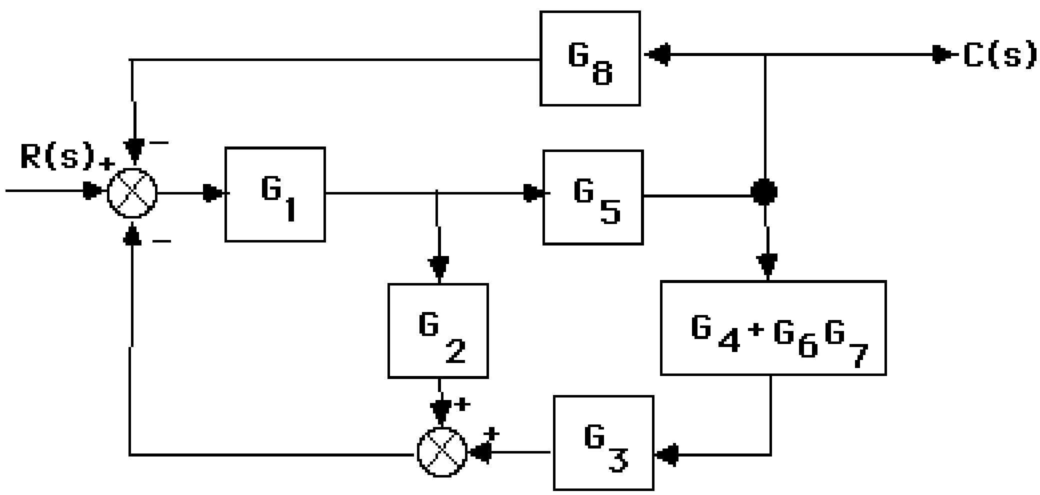
```
G1=tf([0 1],[1 7]);                  %G1=1/s+7 input transducer
G2=tf([0 0 1],[1 2 3]);              %G2=1/s^2+2s+3
G3=tf([0 1],[1 4]);                  %G3=1/s+4
G4=tf([0 1],[1 0]);                  %G4=1/s
G5=tf([0 5],[1 7]);                  %G5=5/s+7
G6=tf([0 0 1],[1 5 10]);             %G6=1/s^2+5s+10
G7=tf([0 3],[1 2]);                  %G7=3/s+2
G8=tf([0 1],[1 6]);                  %G8=1/s+6
G9=tf([1],[1]);                      %Add G9=1 transducer at the input
T1=append(G1,G2,G3,G4,G5,G6,G7,G8,G9);
Q=[1 -2 -5 9
2 1 8 0
3 1 8 0
4 1 8 0
5 3 4 -6
6 7 0 0
7 3 4 -6
8 7 0 0];
inputs=9;
outputs=7;
Ts=connect(T1,Q,inputs,outputs);
T=tf(Ts)
```

Computer response:

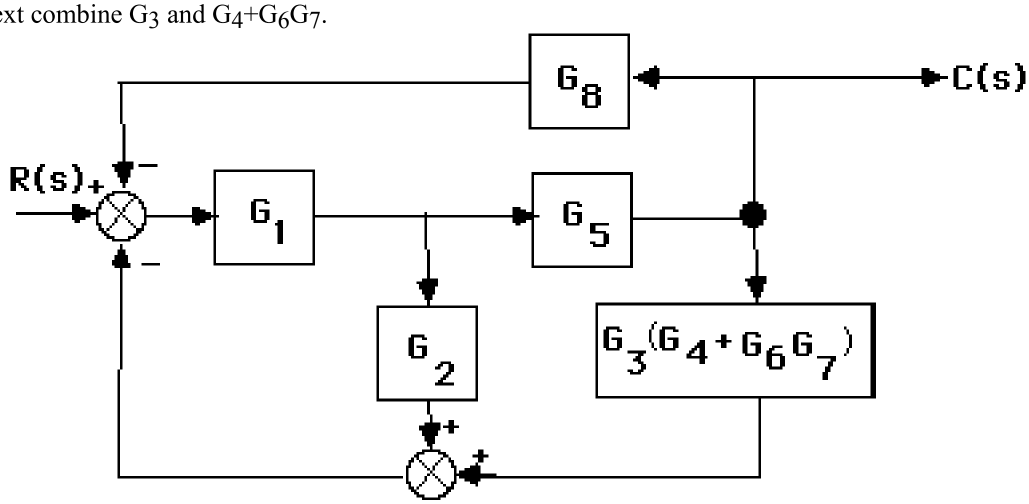
```
Transfer function:

6 s^7 + 132 s^6 + 1176 s^5 + 5640 s^4 + 1.624e004 s^3

                  + 2.857e004 s^2 + 2.988e004 s + 1.512e004

----------------------------------------------------------

s^10 + 33 s^9 + 466 s^8 + 3720 s^7 + 1.867e004 s^6

        + 6.182e004 s^5 + 1.369e005 s^4 + 1.981e005 s^3

                  + 1.729e005 s^2 + 6.737e004 s - 1.044e004
```

6.

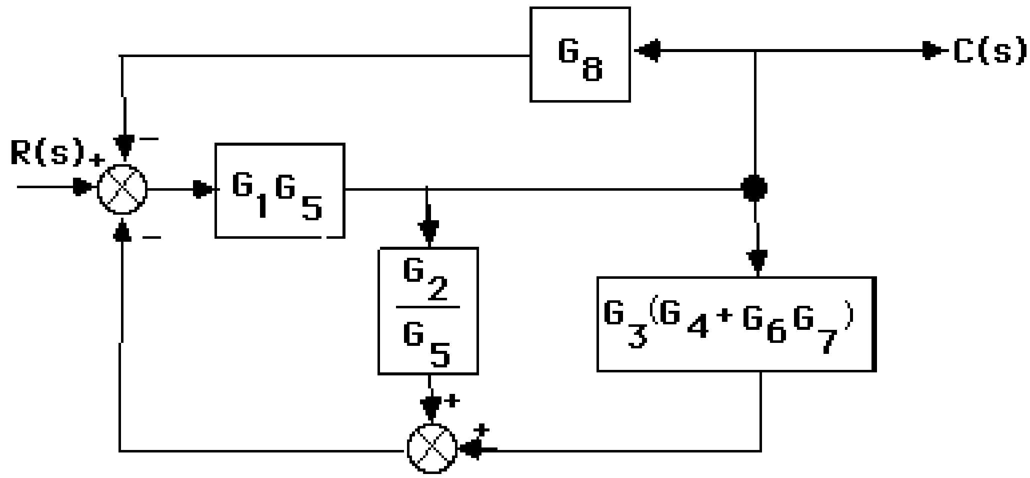
Combine G_6 and G_7 yielding G_6G_7. Add G_4 and obtain the following diagram:



Next combine G_3 and $G_4+G_6G_7$.



Push G_5 to the left past the pickoff point.



Notice that the feedback is in parallel form. Thus the equivalent feedback, $H_{eq}(s) = \frac{G_2}{G_5} + G_3(G_4+G_6G_7) + G_8$. Since the forward path transfer function is $G(s) = G_{eq}(s) = G_1G_5$, the closed-loop transfer function is

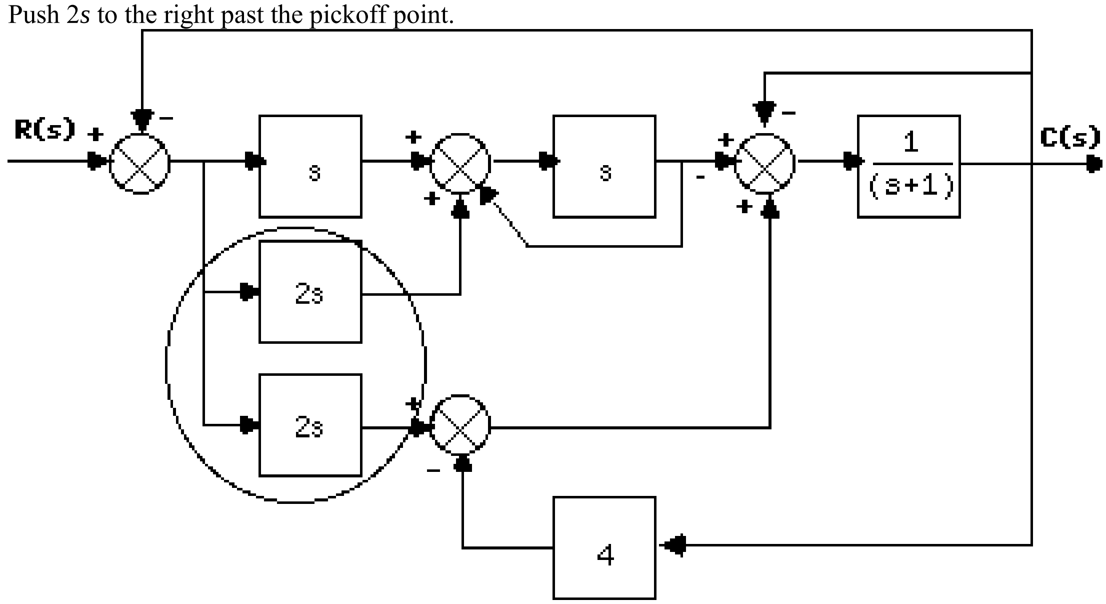
$$T(s) = \frac{G_{eq}(s)}{1+G_{eq}(s)H_{eq}(s)} .$$

Hence,

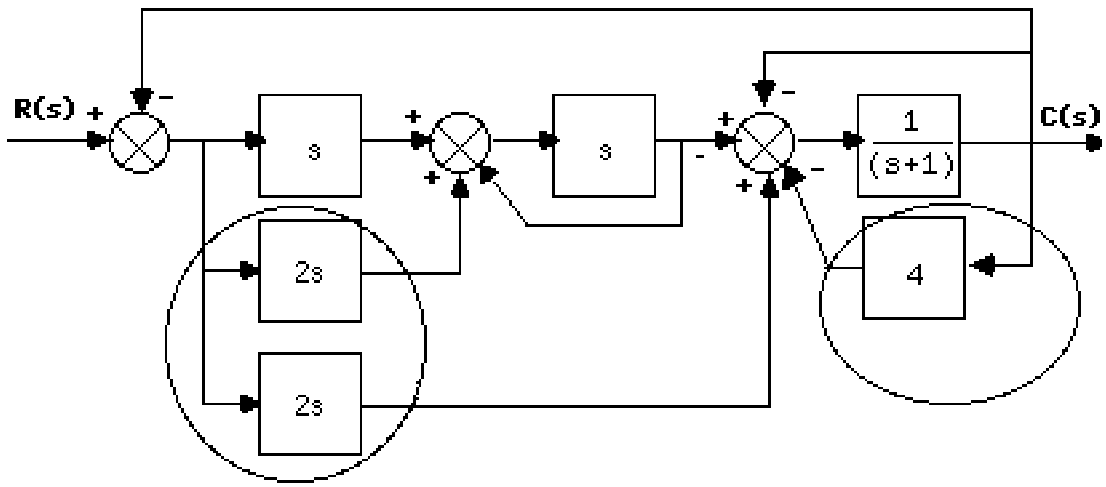
$$T(s) = \frac{G_5G_1}{1+G_1(G_8G_5+G_7G_6G_5G_3+G_5G_4G_3+G_2)}$$

7.

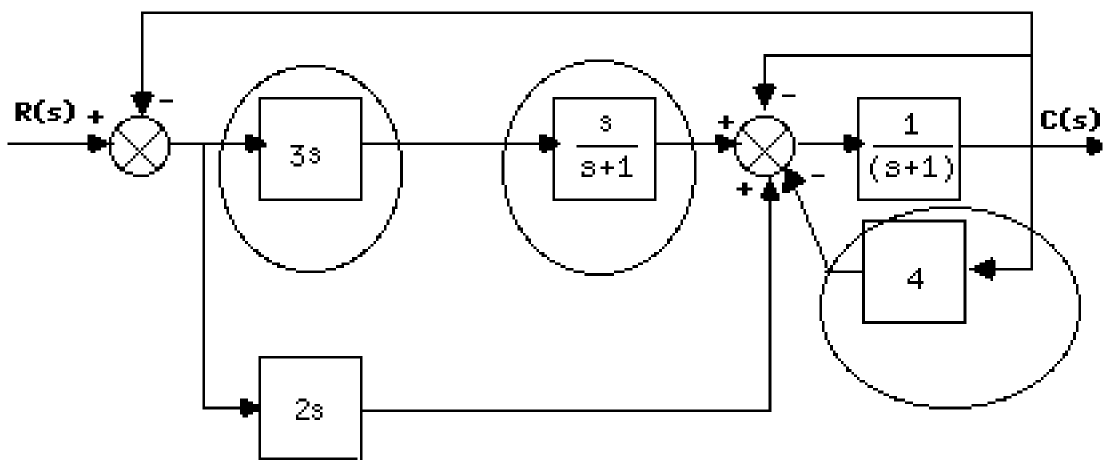
Push 2*s* to the right past the pickoff point.



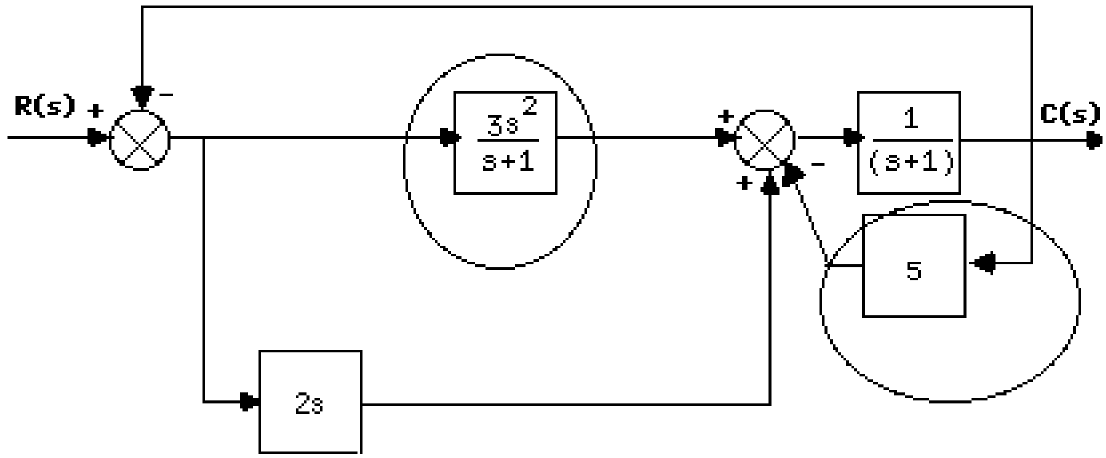
Combine summing junctions.



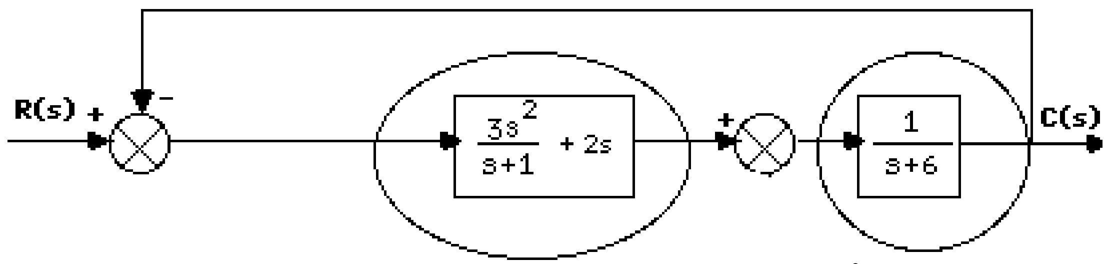
Combine parallel 2s and s. Apply feedback formula to unity feedback with G(s) = s.



Combine cascade pair and add feedback around 1/(s+1).



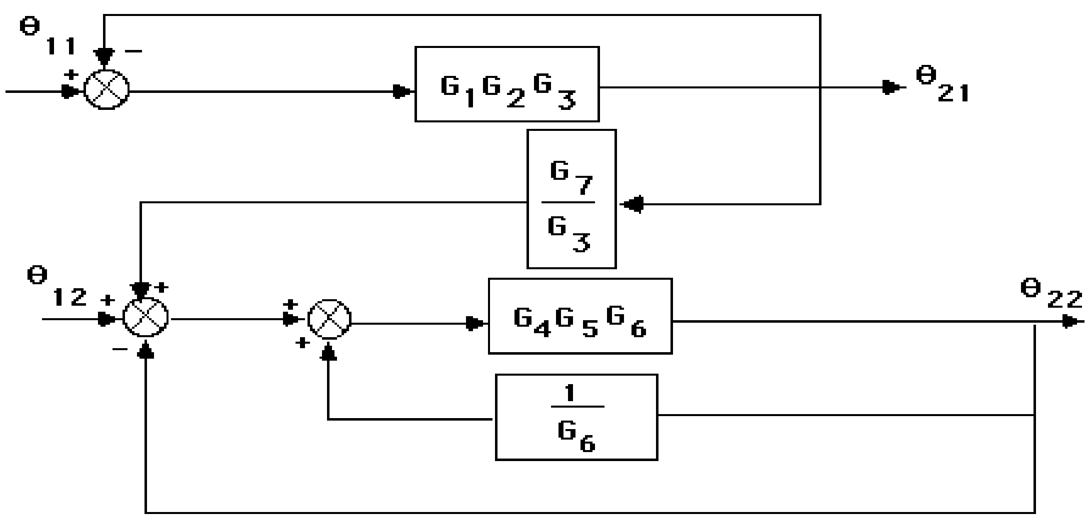
Combine parallel pair and feedback in forward path.



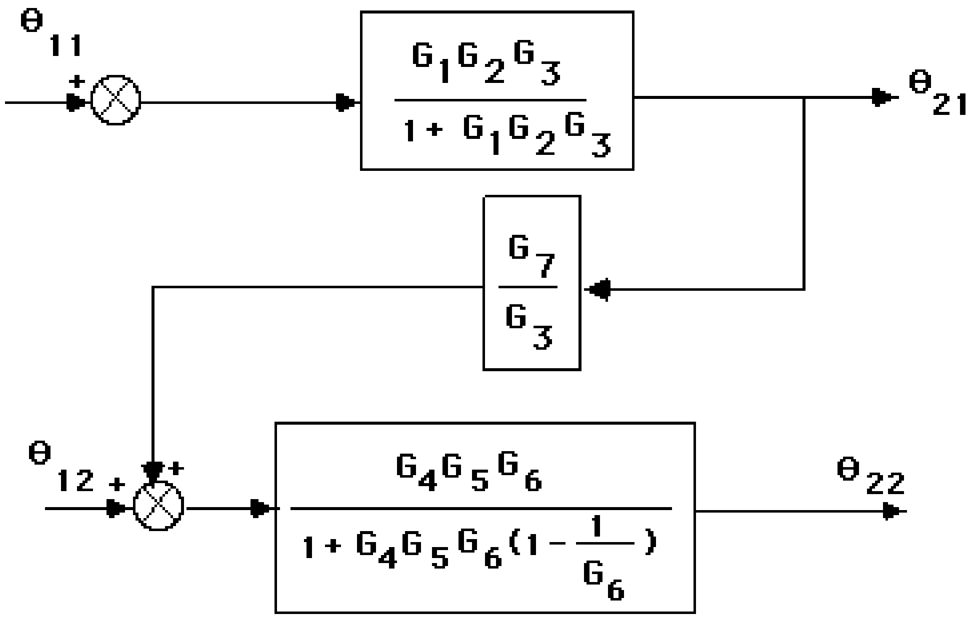
Combine cascade pair and apply final feedback formula yielding $T(s)=\dfrac{5s^2+2s}{6s^2+9s+6}$.

8.

Push G_3 to the left past the pickoff point. Push G_6 to the left past the pickoff point.



Hence,

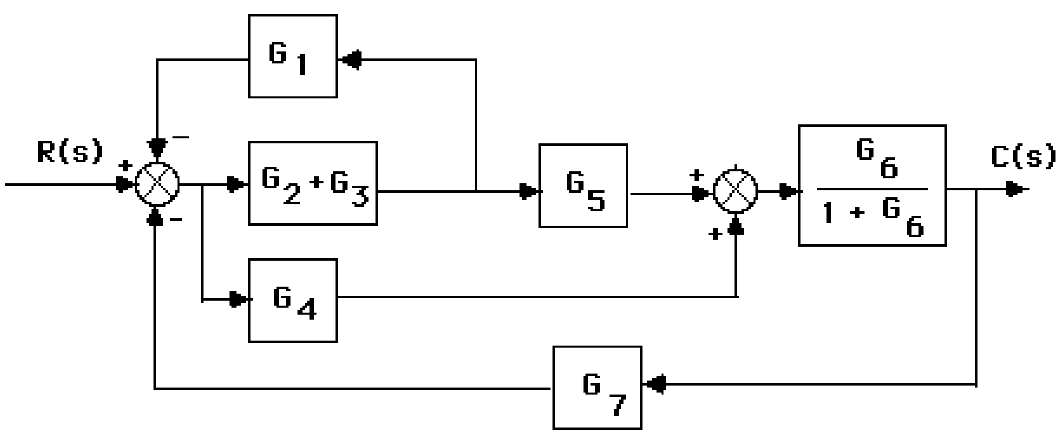


Thus the transfer function is the product of the functions, or

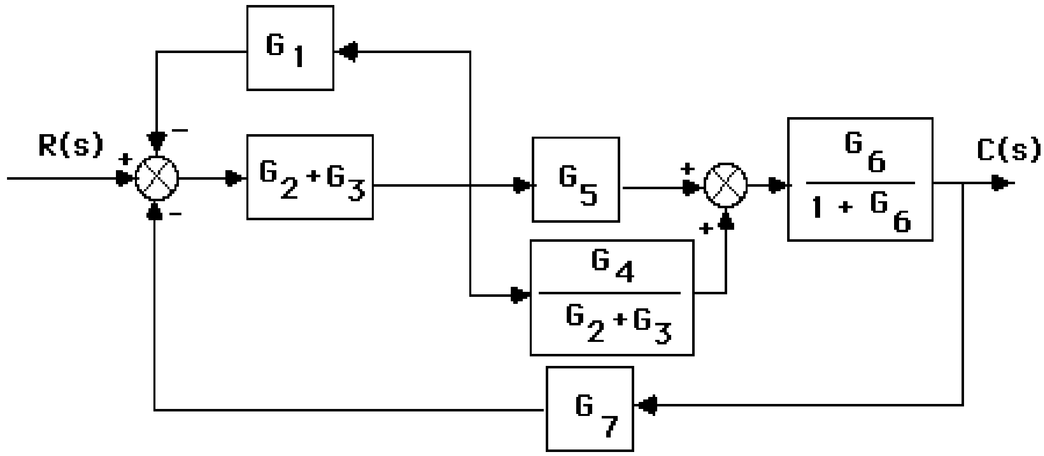
$$\frac{\theta_{22}(s)}{\theta_{11}(s)} = \frac{G_1G_2G_4G_5G_6G_7}{1 - G_4G_5 + G_4G_5G_6 + G_1G_2G_3 - G_1G_2G_3G_4G_5 + G_1G_2G_3G_4G_5G_6}$$

9.

Combine the feedback with G_6 and combine the parallel G_2 and G_3.



Move G_2+G_3 to the left past the pickoff point.



Combine feedback and parallel pair in the forward path yielding an equivalent forward-path transfer function of

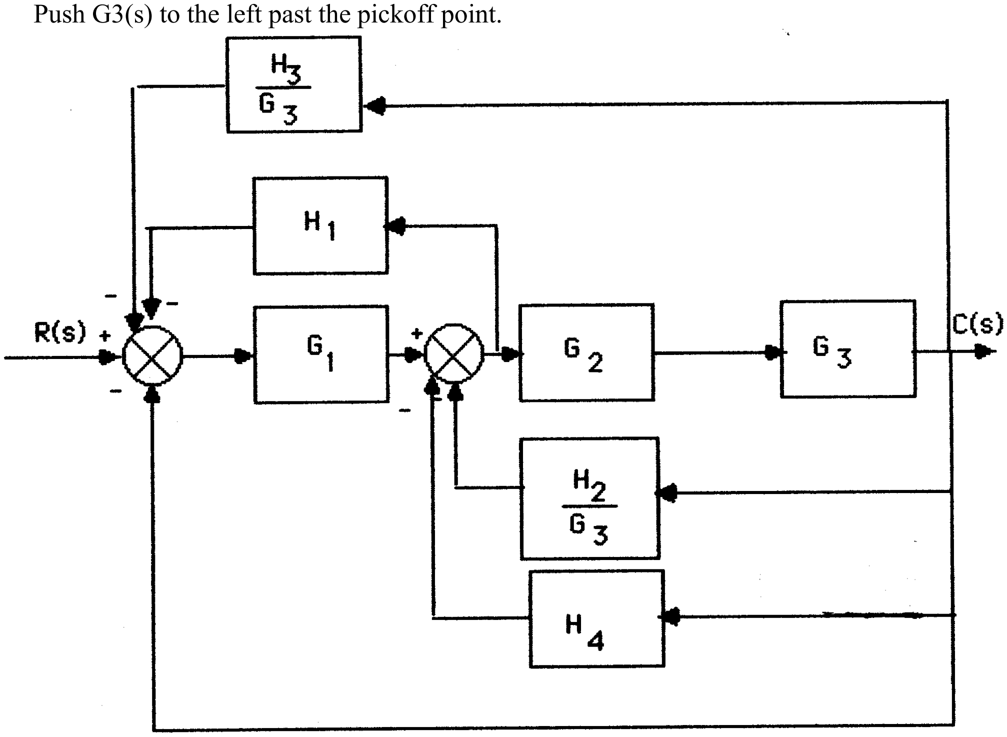
$$G_e(s) = \left(\frac{G_2+G_3}{1+G_1(G_2+G_3)}\right)\left(G_5+\frac{G_4}{G_2+G_3}\right)\left(\frac{G_6}{1+G_6}\right)$$

But, $T(s) = \dfrac{G_e(s)}{1+G_e(s)G_7(s)}$. Thus,

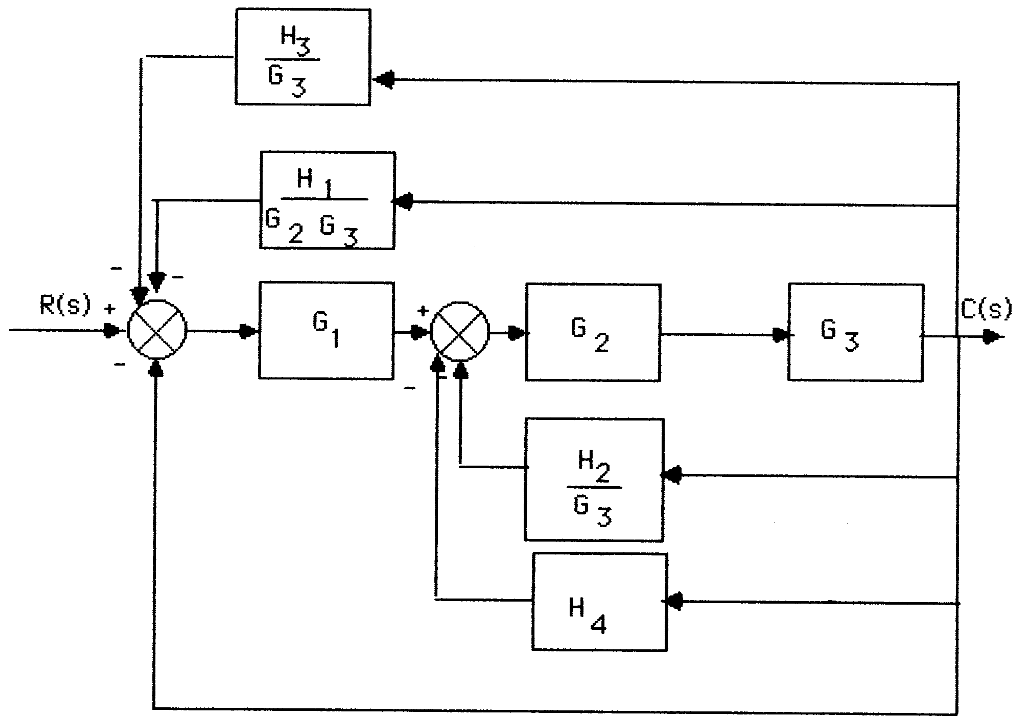
$$T(s) = \frac{G_6\,(G_4+G_5\,G_3+G_5\,G_2)}{G_6\,(G_7\,G_4+G_7\,G_5\,G_3+G_7\,G_5\,G_2+G_3\,G_1+G_2\,G_1+1)+G_1\,(G_3+G_2)+1}$$

10.

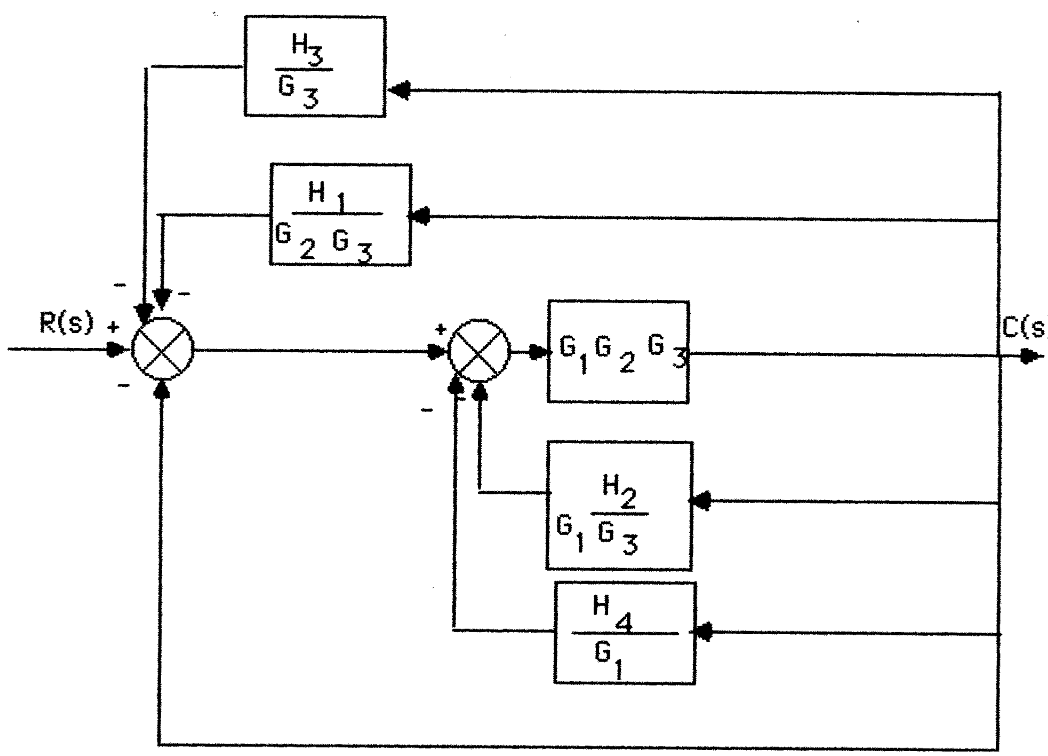
Push G3(s) to the left past the pickoff point.



Push G2(s)G3(s) to the left past the pickoff point.



Push G1(s) to the right past the summing junction.



Collapsing the summing junctions and adding the feedback transfer functions,

$$T(s) = \frac{G_1(s)G_2(s)G_3(s)}{1+G_1(s)G_2(s)G_3(s)H_{eq}(s)}$$

where

$$H_{eq}(s) = \frac{H_3(s)}{G_3(s)} + \frac{H_1(s)}{G_2(s)G_3(s)} + \frac{H_2(s)}{G_1(s)G_3(s)} + \frac{H_4(s)}{G_1(s)} + 1$$

11.

$T(s) = \dfrac{225}{s^2+12s+225}$. Therefore, $2\zeta\omega_n = 12$, and $\omega_n = 15$. Hence, $\zeta = 0.4$.

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 16.3\%$; $T_s = \dfrac{4}{\zeta\omega_n} = 0.667$; $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.229$.

12.

$$C(s) = \frac{5}{s(s^2+3s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+3s+5}$$

$$A = 1$$

$$5 = s^2+3s+5+Bs^2+Cs$$

$$\therefore B = -1,\ C = -3$$

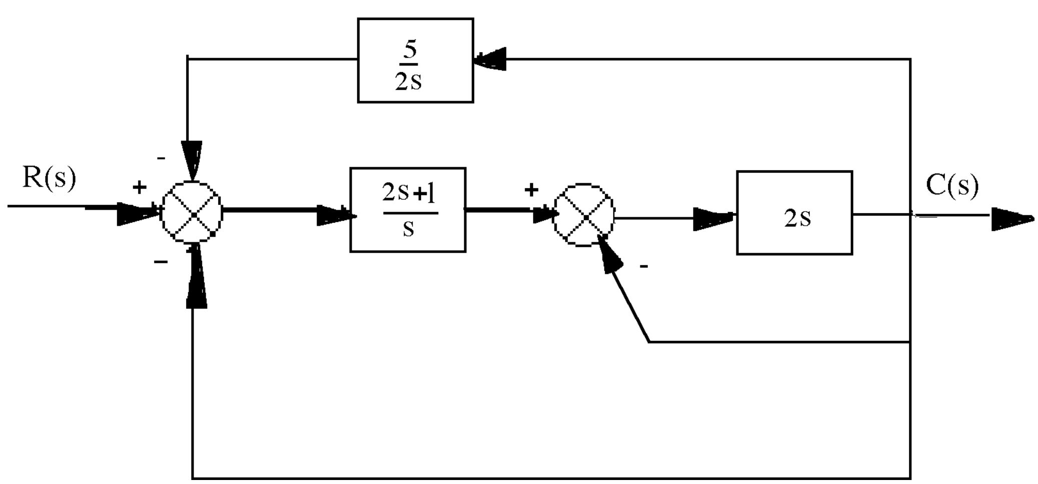
$$C(s) = \frac{1}{s} - \frac{s+3}{s^2+3s+5} = \frac{1}{s} - \frac{s+3}{(s+1.5)^2+2.75}$$

$$= \frac{1}{s} - \frac{(s+1.5)+1.5}{(s+1.5)^2+2.75} = \frac{1}{s} - \frac{(s+1.5)+\frac{1.5}{\sqrt{2.75}}\sqrt{2.75}}{(s+1.5)^2+2.75}$$

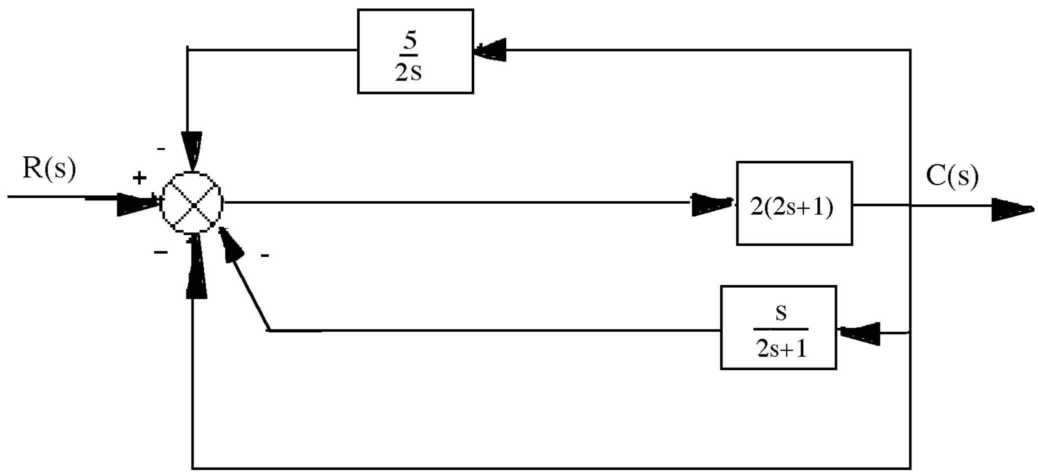
$$c(t) = 1 - e^{-1.5t}\left(\cos\sqrt{2.75}t + \frac{1.5}{\sqrt{2.75}}\sin\sqrt{2.75}t\right)$$

13.

Push 2s to the left past the pickoff point and combine the parallel combination of 2 and 1/s.



Push (2s+1)/s to the right past the summing junction and combine summing junctions.



Hence, $T(s)=\dfrac{2(2s+1)}{1+2(2s+1)H_{eq}(s)}$, where $H_{eq}(s)=1+\dfrac{s}{2s+1}+\dfrac{5}{2s}$.

14.

Since $G(s)=\dfrac{K}{s(s+30)}$, $T(s)=\dfrac{G(s)}{1+G(s)}=\dfrac{K}{s^2+30s+K}$. Therefore, $2\zeta\omega_n = 30$. Thus, ζ = $15/\omega_n$ = 0.456 (i.e. 20% overshoot). Hence, $\omega_n = 32.89 = \sqrt{K}$. Therefore K = 1082.

15.

$$T(s)=\frac{K}{s^2+\alpha s+K};\ \zeta=\frac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2+\ln^2(\frac{\%OS}{100})}}=0.358\,;T_s=\frac{4}{\zeta\omega_n}=0.2.$$ Therefore, ω_n = 55.89. K = ω_n^2 = 3124. $\alpha = 2\zeta\omega_n$ = 40.

16.

The equivalent forward-path transfer function is $G(s)=\dfrac{10K_1}{s[s+(10K_2+2)]}$. Hence,

$T(s)=\dfrac{G(s)}{1+G(s)}=\dfrac{10K_1}{s^2+(10K_2+2)s+10K_1}$. Since

$T_s=\dfrac{4}{\mathrm{Re}}=2,\ \therefore \mathrm{Re}=2;$ and $\mathrm{T_p}=\dfrac{\pi}{\mathrm{Im}}=1,\ \therefore \mathrm{Im}=\pi$. The poles are thus at $-2+j\pi$. Hence,

$\omega_n=\sqrt{2^2+\pi^2}=\sqrt{10K_1}$. Thus, K_1 = 1.387. Also, $(10K_2 + 2)/2$ = Re = 2. Hence, K_2 = 1/5.

17.

a. For the inner loop, $G_e(s)=\dfrac{20}{s(s+12)}$, and $H_e(s)$ = 0.2s. Therefore, $T_e(s)=\dfrac{G_e(s)}{1+G_e(s)H_e(s)}$ = $\dfrac{20}{s(s+16)}$. Combining with the equivalent transfer function of the parallel pair, $G_p(s)$ = 20, the system is reduced to an equivalent unity feedback system with $G(s) = G_p(s)\,T_e(s) = \frac{400}{s(s+16)}$. Hence, $T(s) = \frac{G(s)}{1+G(s)} = \frac{400}{s^2+16s+400}$.

b. $\omega_n^2 = 400$; $2\zeta\omega_n = 16$. Therefore, $\omega_n = 20$, and $\zeta = 0.4$. $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}}x100 = 25.38$;

$T_s = \frac{4}{\zeta\omega_n} = 0.5$; $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.171$. From Figure 4.16, $\omega_n T_r = 1.463$. Hence, $T_r = 0.0732$.

$\omega_d = \mathrm{Im} = \omega_n\sqrt{1-\zeta^2} = 18.33$.

18.

$T(s) = \frac{28900}{s^2+200s+28900}$; from which, $2\zeta\omega_n = 200$ and $\omega_n = \sqrt{28900} = 170$. Hence,

$\zeta = 0.588$. $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}}x100 = 10.18\%$; $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.0229$ s.

Also, $T_s = \frac{4}{\zeta\omega_n} = 0.04$ s.

19.

For the generator, $E_g(s) = K_f I_f(s)$. But, $I_f(s) = \frac{E_i(s)}{R_f+L_f s}$. Therefore, $\frac{E_g(s)}{E_i(s)} = \frac{2}{s+1}$. For the motor,

consider $R_a = 2\ \Omega$, the sum of both resistors. Also, $J_e = J_a+J_L(\frac{1}{2})^2 = 0.75+\frac{1}{4} = 1$; $D_e = D_L(\frac{1}{2})^2 = 1$.

Therefore,

$$\frac{\theta_m(s)}{E_g(s)} = \frac{\frac{K_t}{R_a J_e}}{s\left(s+\frac{1}{J_e}\left(D_e+\frac{K_t K_a}{R_a}\right)\right)} = \frac{0.5}{s(s+1.5)}.$$

But, $\frac{\theta_o(s)}{\theta_m(s)} = \frac{1}{2}$. Thus, $\frac{\theta_o(s)}{E_g(s)} = \frac{0.25}{s(s+1.5)}$. Finally,

$$\frac{\theta_o(s)}{E_i(s)} = \frac{E_g(s)}{E_i(s)}\frac{\theta_o(s)}{E_g(s)} = \frac{0.5}{s(s+1)(s+1.5)}.$$

20.

For the mechanical system, $J\left(\frac{N_2}{N_1}\right)^2 s^2\theta_2(s) = T\left(\frac{N_2}{N_1}\right)$. For the potentiometer, $E_i(s) = 10\frac{\theta_2(s)}{2\pi}$, or

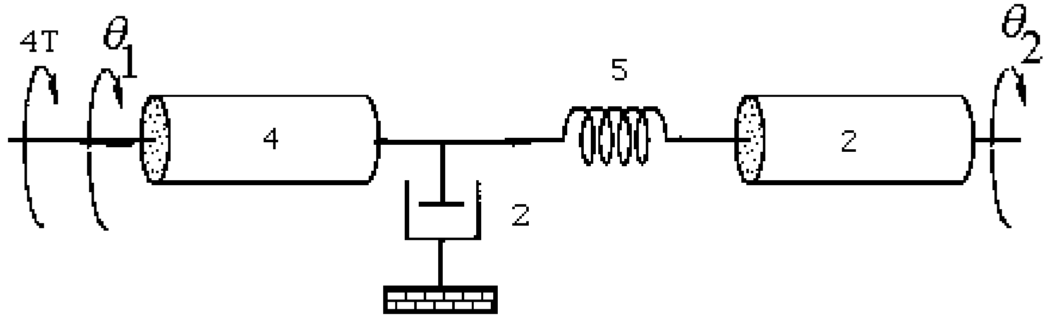
$\theta_2(s) = \frac{\pi}{5}E_i(s)$. For the network, $E_o(s) = E_i(s)\frac{R}{R+\frac{1}{Cs}} = E_i(s)\frac{s}{s+\frac{1}{RC}}$, or $E_i(s) = E_o(s)\frac{s+\frac{1}{RC}}{s}$.

Therefore, $\theta_2(s) = \frac{\pi}{5} E_o(s) \frac{s + \frac{1}{RC}}{s}$. Substitute into mechanical equation and obtain,

$$\frac{E_o(s)}{T(s)} = \frac{\frac{5N_1}{J\pi N_2}}{s\left(s + \frac{1}{RC}\right)}.$$

21.

The equivalent mechanical system is found by reflecting mechanical impedances to the spring.



Writing the equations of motion:

$$\left(4s^2 + 2s + 5\right)\theta_1(s) - 5\theta_2(s) = 4T(s)$$

$$-5\theta_1(s) + \left(2s^2 + 5\right)\theta_2(s) = 0$$

Solving for $\theta_2(s)$,

$$\theta_2(s) = \frac{\begin{vmatrix} \left(4s^2 + 2s + 5\right) & 4T(s) \\ -5 & 0 \end{vmatrix}}{\begin{vmatrix} \left(4s^2 + 2s + 5\right) & -5 \\ -5 & \left(2s^2 + 5\right) \end{vmatrix}} = \frac{20T(s)}{8s^4 + 4s^3 + 30s^2 + 10s}$$

The angular rotation of the pot is 0.2 that of θ_2, or

$$\frac{\theta_p(s)}{T(s)} = \frac{2}{s\left(4s^3 + 2s^2 + 15s + 5\right)}$$

For the pot:

$$\frac{E_p(s)}{\theta_p(s)} = \frac{50}{5(2\pi)} = \frac{5}{\pi}$$

For the electrical network: Using voltage division,

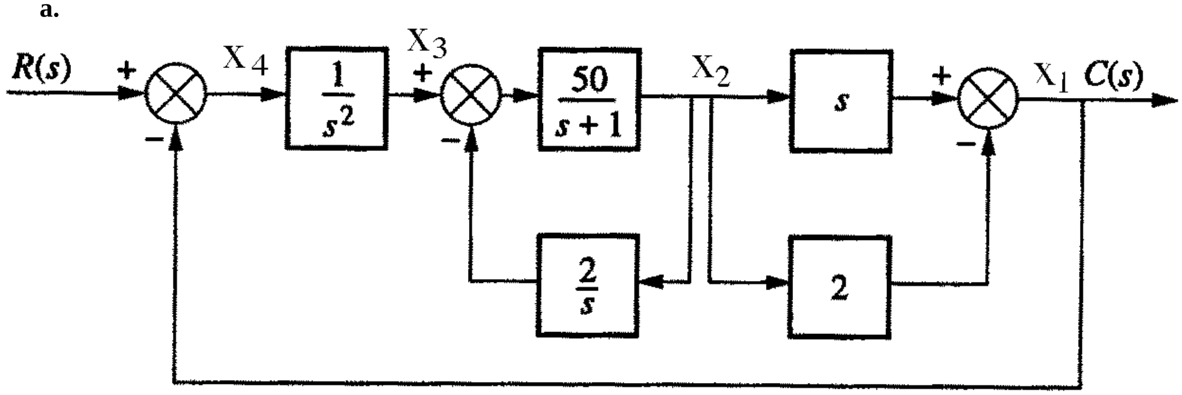
$$\frac{E_o(s)}{E_p(s)} = \frac{200{,}000}{\frac{1}{10^{-5}s} + 200{,}000} = \frac{s}{s + \frac{1}{2}}$$

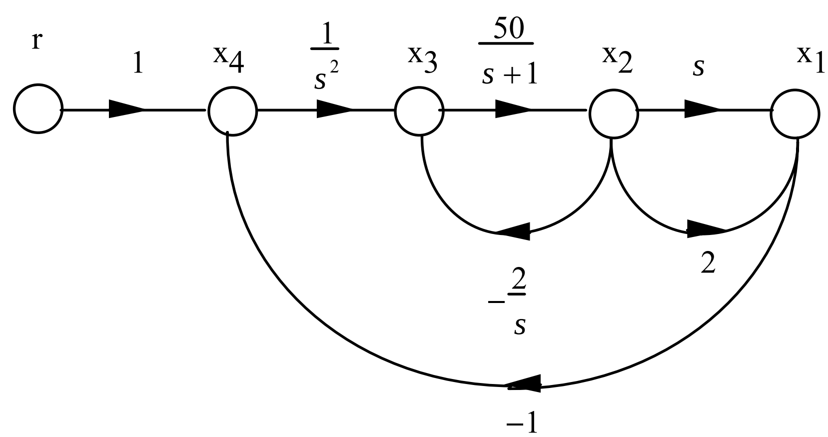
Substituting the previously obtained values,

$$\frac{E_o(s)}{T(s)} = \left(\frac{\theta_p(s)}{T(s)}\right)\left(\frac{E_p(s)}{\theta_p(s)}\right)\left(\frac{E_o(s)}{E_p(s)}\right) = \frac{\frac{10}{\pi}s}{s\left(s+\frac{1}{2}\right)\left(4s^3 + 2s^2 + 15s + 5\right)}$$

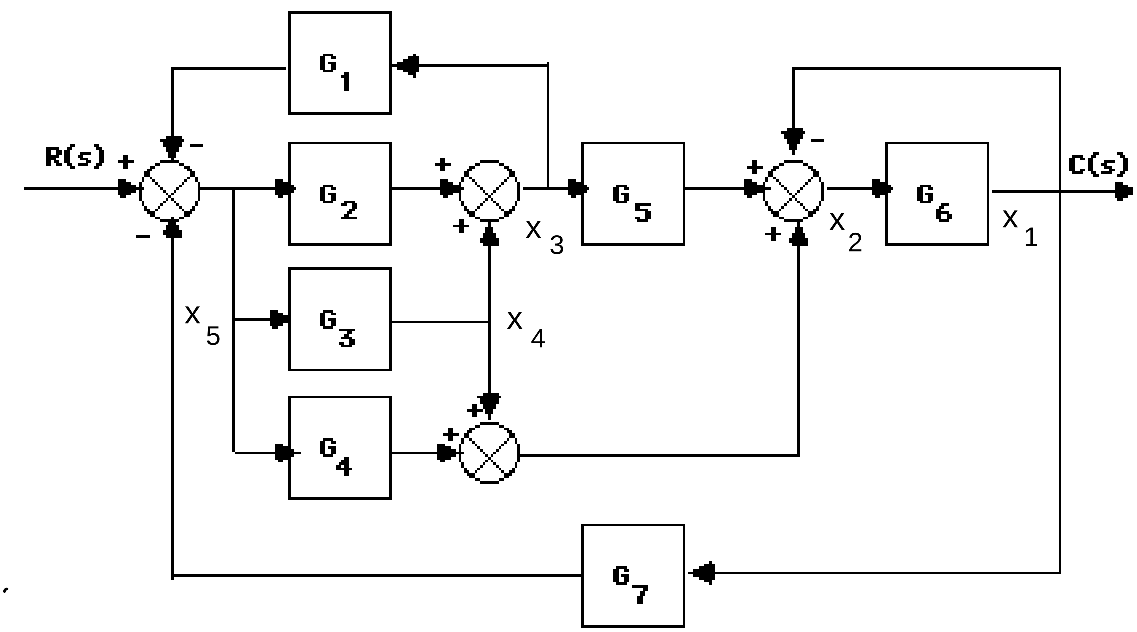
22.

a.

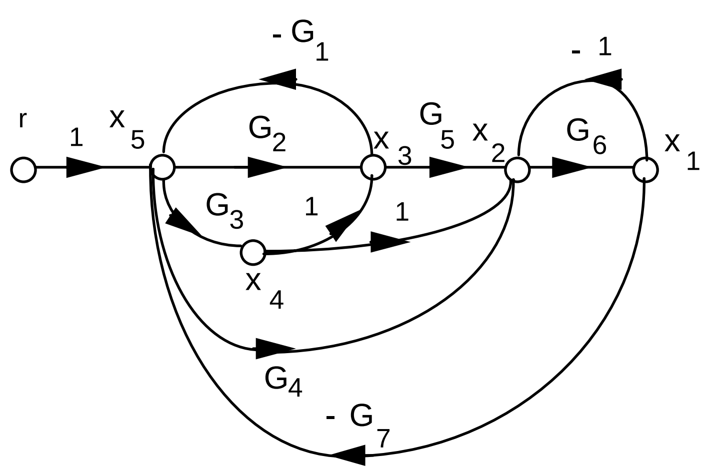




b.

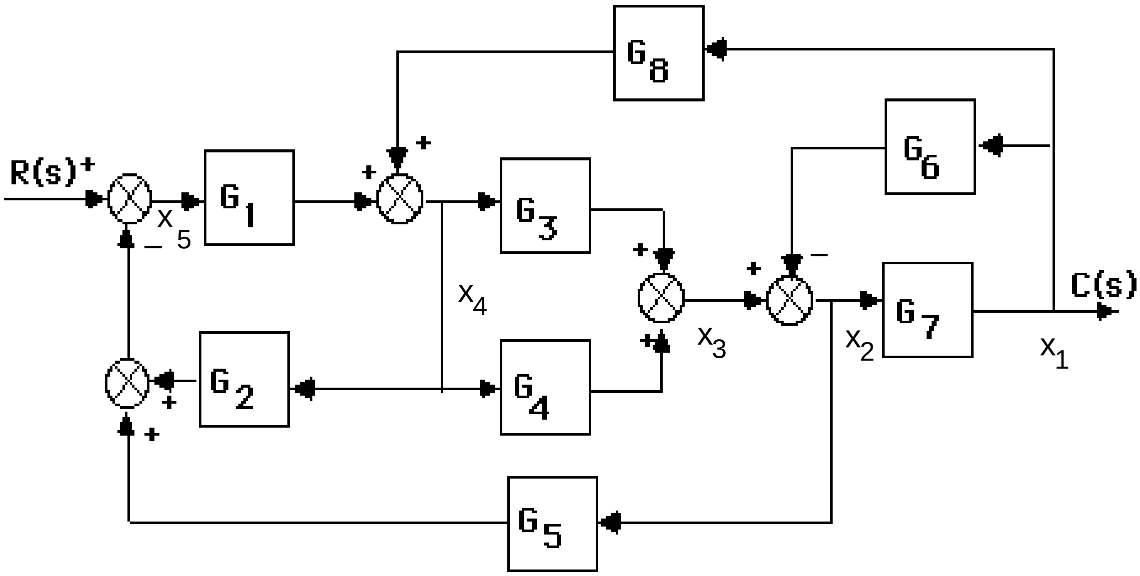


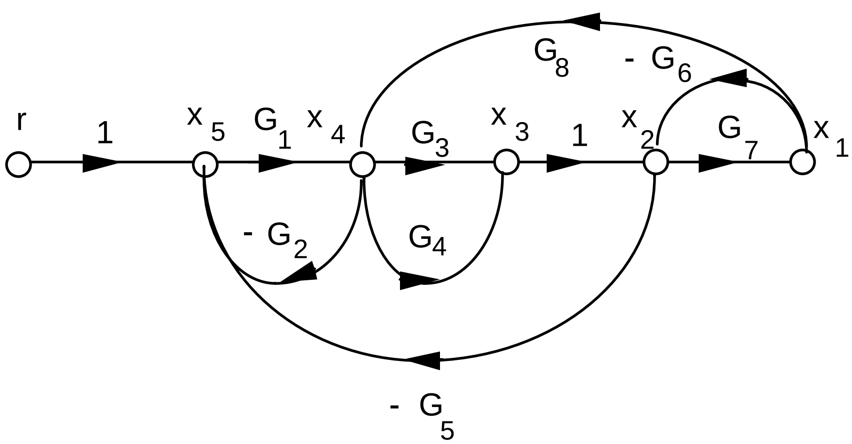
G1
R(s)
G2
G5
G6
C(s)
x3
x2
x1
x5
G3
x4
G4
G7



- G1
- 1
r
1
x5
G2
x3
G5
x2
G6
x1
G3
1
1
x4
G4
- G7

c.

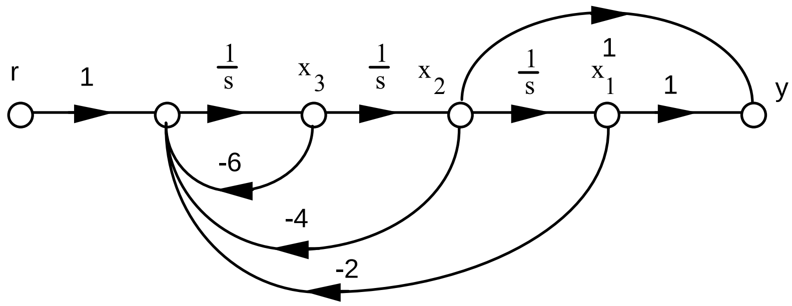




23.

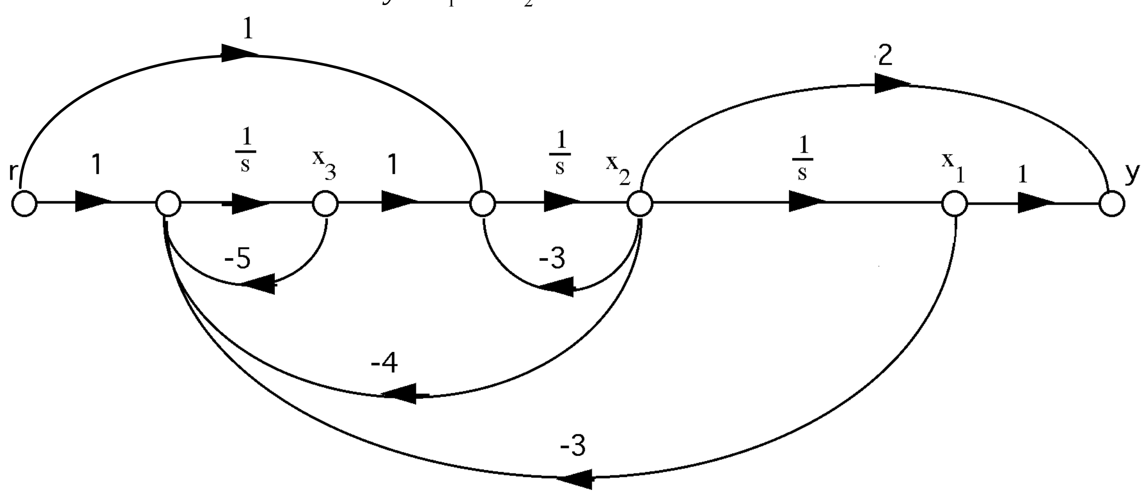
a.

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -2x_1 - 4x_2 - 6x_3 + r$$
$$y = x_1 + x_2$$



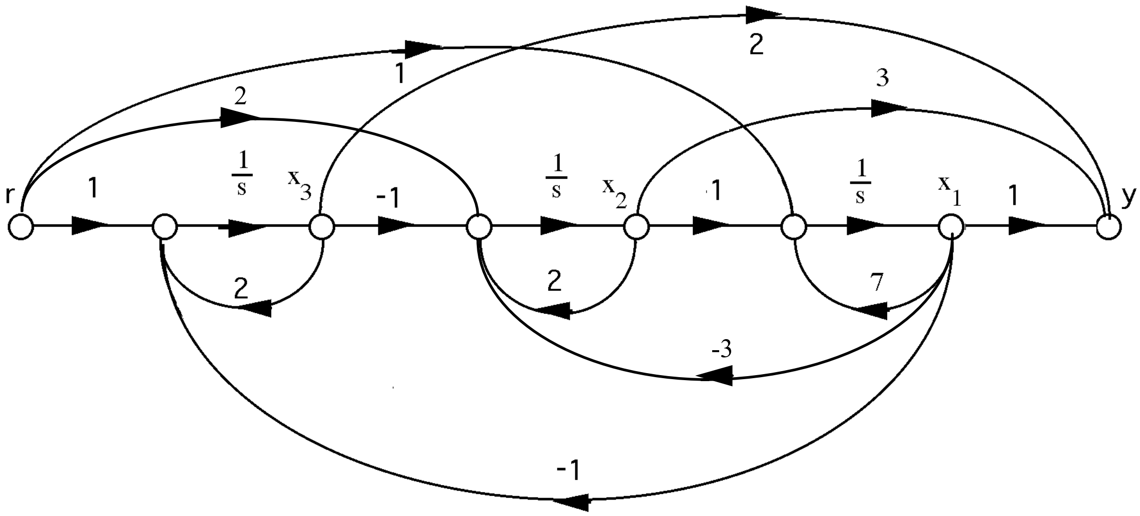
b.

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -3x_2 + x_3 + r$$
$$\dot{x}_3 = -3x_1 - 4x_2 - 5x_3 + r$$
$$y = x_1 + 2x_2$$



c.

$$\dot{x}_1 = 7x_1 + x_2 + r$$
$$\dot{x}_2 = -3x_1 + 2x_2 - x_3 + 2r$$
$$\dot{x}_3 = -x_1 + 2x_3 + r$$
$$y = x_1 + 3x_2 + 2x_3$$



24.

a. Since $G(s) = \dfrac{10}{s^3 + 24s^2 + 191s + 504} = \dfrac{C(s)}{R(s)}$,

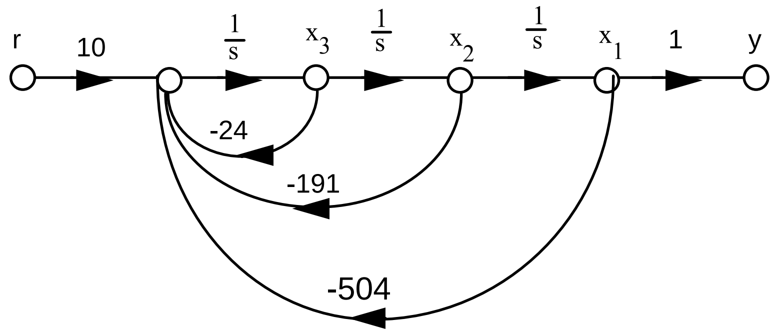
$$\dddot{c} + 24\ddot{c} + 191\dot{c} + 504c = 10r$$

Let,

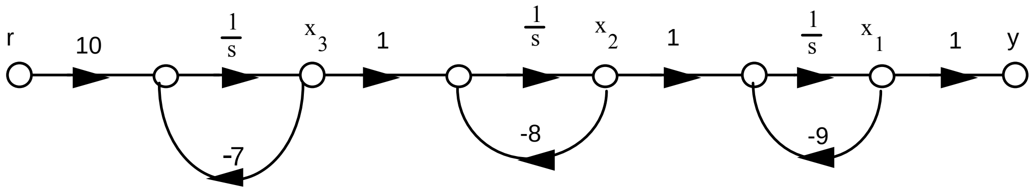
$$c = x_1$$
$$\dot{c} = x_2$$
$$\ddot{c} = x_3$$

Therefore,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -504x_1 - 191x_2 - 24x_3 + 10r$$
$$y = x_1$$



b. G(s) = $\left(\frac{10}{s+7}\right)\left(\frac{1}{s+8}\right)\left(\frac{1}{s+9}\right)$



Therefore,

$$\dot{x}_1 = -9x_1 + x_2$$
$$\dot{x}_2 = -8x_2 + x_3$$
$$\dot{x}_3 = -7x_3 + 10r$$
$$y = x_1$$

25.

a. Since G(s) = $\frac{20}{s^4 + 15s^3 + 66s^2 + 80s} = \frac{C(s)}{R(s)}$,

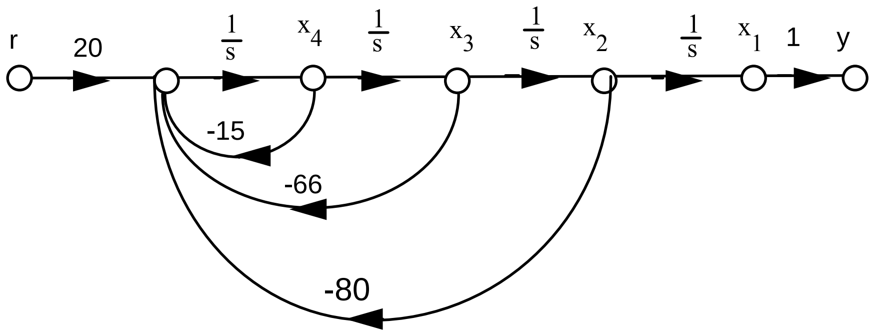
$$\ddddot{c} + 15\dddot{c} + 66\ddot{c} + 80\dot{c} = 20r$$

Let,

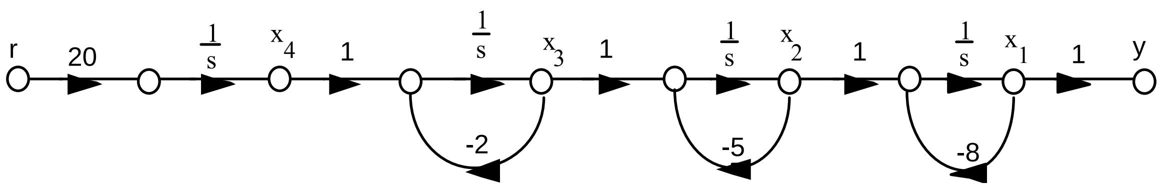
$$c = x_1$$
$$\dot{c} = x_2$$
$$\ddot{c} = x_3$$
$$\dddot{c} = x_4$$

Therefore,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = -80x_2 - 66x_3 - 15x_4 + 20r$$
$$y = x_1$$



b. $G(s) = (\frac{20}{s})(\frac{1}{s+2})(\frac{1}{s+5})(\frac{1}{s+8})$. Hence,



From which,

$$\dot{x}_1 = -8x_1 + x_2$$
$$\dot{x}_2 = -5x_2 + x_3$$
$$\dot{x}_3 = -2x_3 + x_4$$
$$\dot{x}_4 = 20r$$
$$y = x_1$$

26.

$\Delta = 1 + [G_2G_3G_4 + G_3G_4 + G_4 + 1] + [G_3G_4 + G_4]$; $T_1 = G_1G_2G_3G_4$; $\Delta_1 = 1$. Therefore,

$$T(s) = \frac{T_1\Delta_1}{\Delta} = \frac{G_1G_2G_3G_4}{2 + G_2G_3G_4 + 2G_3G_4 + 2G_4}$$

27.

Closed-loop gains: $G_2G_4G_6G_7H_3$; $G_2G_5G_6G_7H_3$; $G_3G_4G_6G_7H_3$; $G_3G_5G_6G_7H_3$; G_6H_1; G_7H_2

Forward-path gains: $T_1 = G_1G_2G_4G_6G_7$; $T_2 = G_1G_2G_5G_6G_7$; $T_3 = G_1G_3G_4G_6G_7$; $T_4 =$

$G_1G_3G_5G_6G_7$

Nontouching loops 2 at a time: $G_6H_1G_7H_2$

$\Delta = 1 - [H_3G_6G_7(G_2G_4 + G_2G_5 + G_3G_4 + G_3G_5) + G_6H_1 + G_7H_2] + [G_6H_1G_7H_2]$

$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$

$$T(s) = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4}{\Delta}$$

$$= \frac{G_1G_2G_4G_6G_7 + G_1G_2G_5G_6G_7 + G_1G_3G_4G_6G_7 + G_1G_3G_5G_6G_7}{1 - H_3G_6G_7(G_2G_4 + G_2G_5 + G_3G_4 + G_3G_5) - G_6H_1 - G_7H_2 + G_6H_1G_7H_2}$$

28.

Closed-loop gains: $-s^2; -\frac{1}{s}\ ;\ -\frac{1}{s}\ ; -s^2$

Forward-path gains: $T_1 = s; T_2 = \frac{1}{s^2}$

Nontouching loops: None

$\Delta = 1 - (-s^2 - \frac{1}{s} - \frac{1}{s} - s^2)$

$\Delta_1 = \Delta_2 = 1$

$$G(s) = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} = \frac{s + \frac{1}{s^2}}{1 + (s^2 + \frac{1}{s} + \frac{1}{s} + s^2)} = \frac{s^3+1}{2s^4+s^2+2s}$$

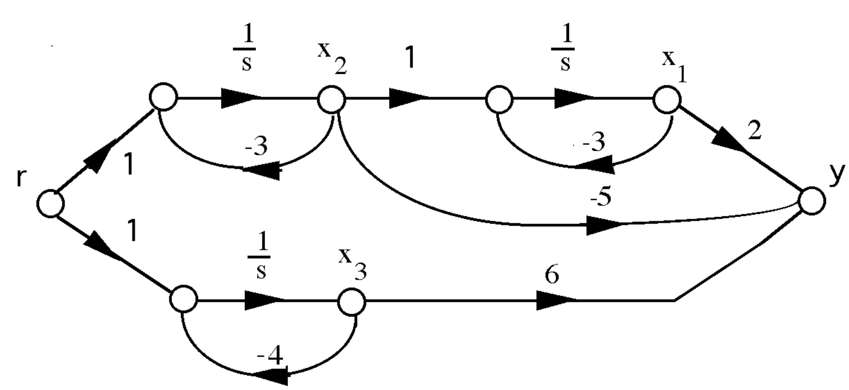
29.

$$T(s) = \frac{G_1\left(\frac{G_2G_3G_4G_5}{(1-G_2H_1)(1-G_4H_2)}\right)}{1 - \frac{G_2G_3G_4G_5G_6G_7G_8}{(1-G_2H_1)(1-G_4H_2)(1-G_7H_4)}} =$$

$$\frac{G_1G_2G_3G_4G_5(1-G_7H_4)}{1-G_2H_1-G_4H_2+G_2G_4H_1H_2-G_7H_4+G_2G_7H_1H_4+G_4G_7H_2H_4-G_2G_4G_7H_1H_2H_4-G_2G_3G_4G_5G_6G_7G_8}$$

30.

a. $G(s) = \dfrac{(s+1)(s+2)}{(s+3)^2(s+4)} = \dfrac{2}{(s+3)^2} - \dfrac{5}{s+3} + \dfrac{6}{s+4}$



Writing the state and output equations,

$$\dot{x}_1 = -3x_1 + x_2$$

$$\dot{x}_2 = -3x_2 + r$$

$$\dot{x}_3 = -4x_3 + r$$

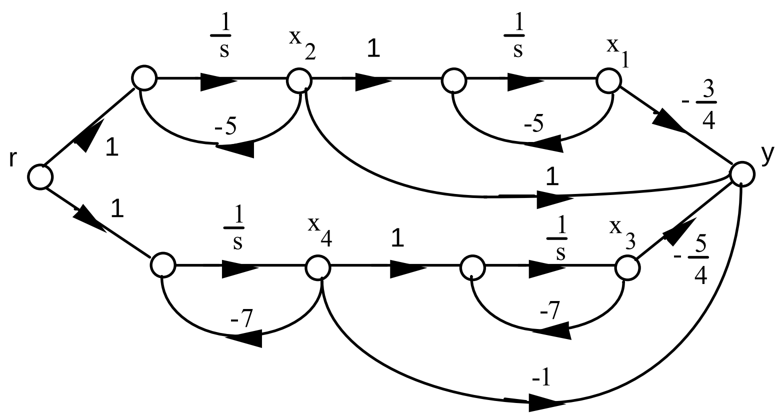
$$y = 2x_1 - 5x_2 + 6x_3$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 2 & -5 & 6 \end{bmatrix}$$

b. G(s) = $G(s) = \dfrac{(s+2)}{(s+5)^2(s+7)^2} = -\dfrac{3/4}{(s+5)^2} + \dfrac{1}{s+5} - \dfrac{5/4}{(s+7)^2} - \dfrac{1}{s+7}$



Writing the state and output equations,

$$\dot{x}_1 = -5x_1 + x_2$$

$$\dot{x}_2 = -5x_2 + r$$

$$\dot{x}_3 = -7x_3 + x_4$$

$$\dot{x}_4 = -7x_4 + r$$

$$y = -\frac{3}{4}x_1 + x_2 - \frac{5}{4}x_3 - x_4$$

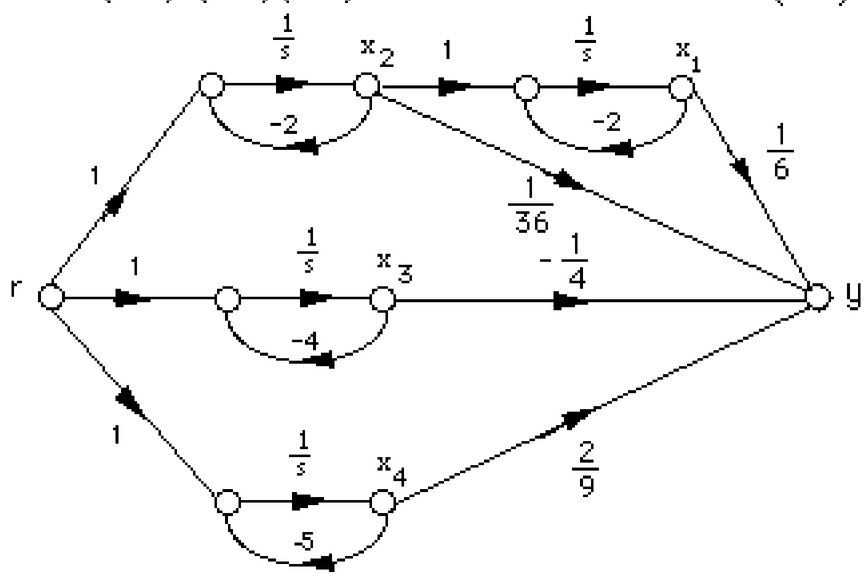
In vector matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & 1 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & -7 & 1 \\ 0 & 0 & 0 & -7 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} -\frac{3}{4} & 1 & -\frac{5}{4} & -1 \end{bmatrix} \mathbf{x}$$

c.

$$G(s) = \frac{s+3}{(s+2)^2(s+4)(s+5)} = \frac{2}{9}\frac{1}{s+5} - \frac{1}{4}\frac{1}{s+4} + \frac{1}{36}\frac{1}{s+2} + \frac{1}{6}\frac{1}{(s+2)^2}$$



Writing the state and output equations,

$$\dot{x}_1 = -2x_1 + x_2$$

$$\dot{x}_2 = -2x_2 + r$$

$$\dot{x}_3 = -4x_3 + r$$

$$\dot{x}_4 = -5x_4 + r$$

$$y = \frac{1}{6}x_1 + \frac{1}{36}x_2 - \frac{1}{4}x_3 + \frac{2}{9}x_4$$

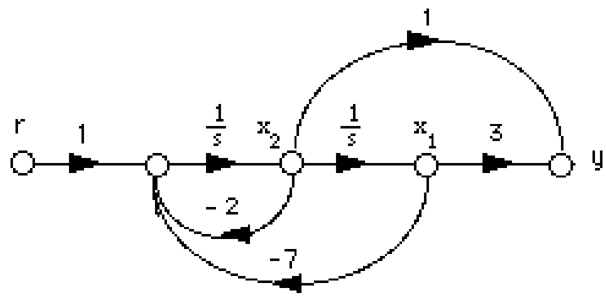
In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} \frac{1}{6} & \frac{1}{36} & -\frac{1}{4} & \frac{2}{9} \end{bmatrix} \mathbf{x}$$

31.

a.



Writing the state equations,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -7x_1 - 2x_2 + r$$

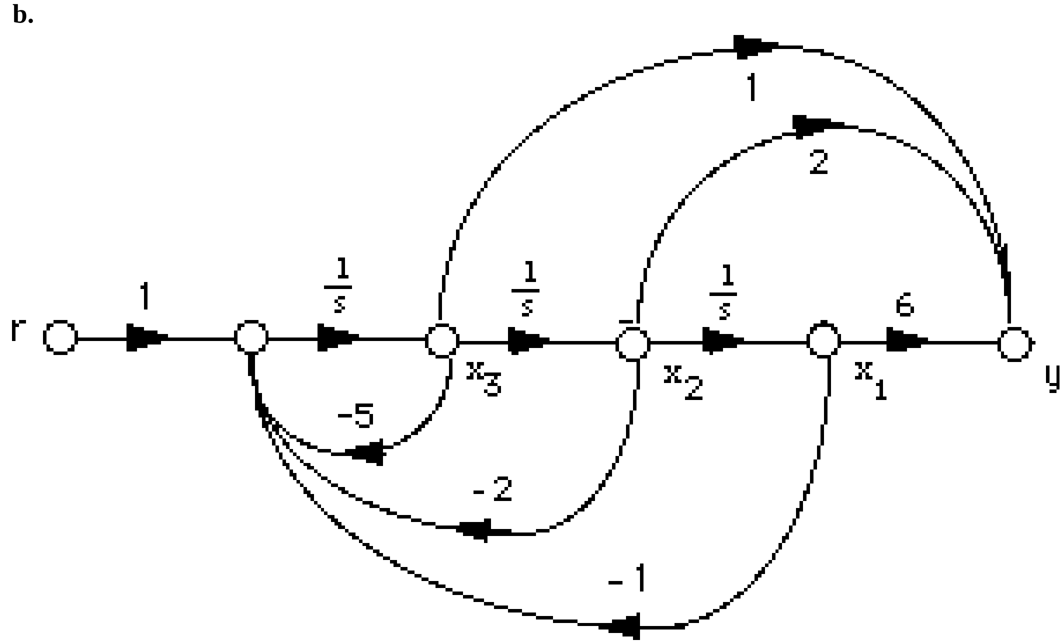
$$y = 3x_1 + x_2$$

In vector matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -7 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 3 & 1 \end{bmatrix}\mathbf{x}$$

b.



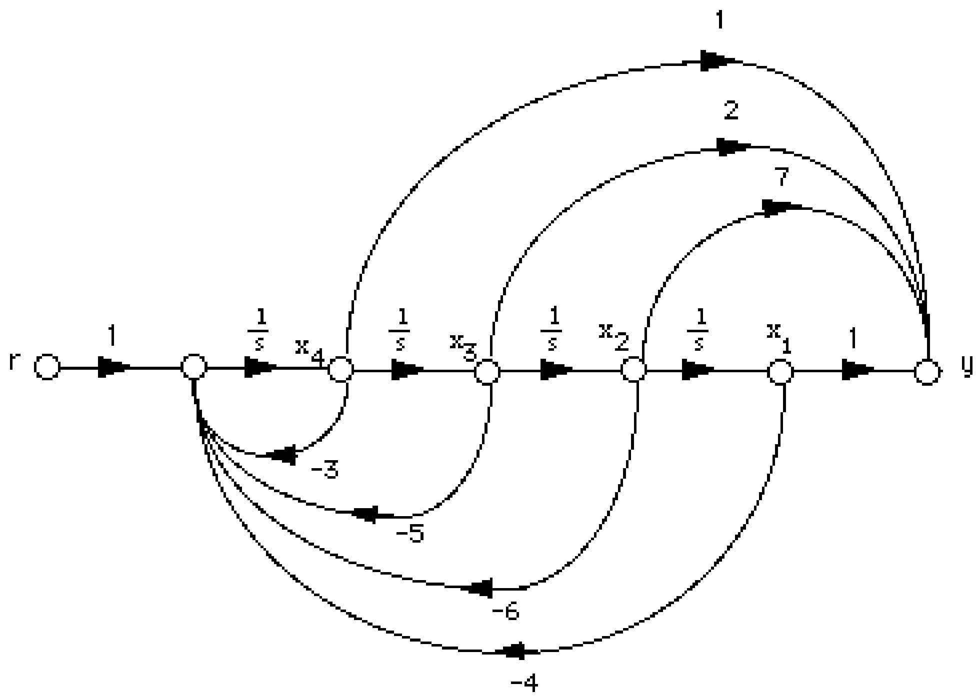
Writing the state equations**,**

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -x_1 - 2x_2 - 5x_3 + r$$
$$y = 6x_1 + 2x_2 + x_3$$

In vector matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -5 \end{bmatrix}\mathbf{X} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$
$$y = \begin{bmatrix} 6 & 2 & 1 \end{bmatrix}\mathbf{x}$$

c.



$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = -4x_1 - 6x_2 - 5x_3 - 3x_4 + r$$
$$y = x_1 + 7x_2 + 2x_3 + x_4$$

In vector matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -6 & -5 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}r$$

$$y = \begin{bmatrix} 1 & 7 & 2 & 1 \end{bmatrix}\mathbf{x}$$

32.

a. Controller canonical form:

From the phase-variable form in Problem 5.31(a), reverse the order of the state variables and obtain,

$$\dot{x}_2 = x_1$$

$$\dot{x}_1 = -7x_2 - 2x_1 + r$$

$$y = 3x_2 + x_1$$

Putting the equations in order,

$$\dot{x}_1 = -2x_1 - 7x_2 + r$$

$$\dot{x}_2 = x_1$$

$$y = x_1 + 3x_2$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & -7 \\ 1 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}r$$

$$y = \begin{bmatrix} 1 & 3 \end{bmatrix}\mathbf{x}$$

Observer canonical form:

$G(s) = \dfrac{s+3}{s^2+2s+7}$. Divide each term by $\dfrac{1}{s^2}$ and get

$$G(s) = \frac{\frac{1}{s} + \frac{3}{s^2}}{1 + \frac{2}{s} + \frac{7}{s^2}} = \frac{C(s)}{R(s)}$$

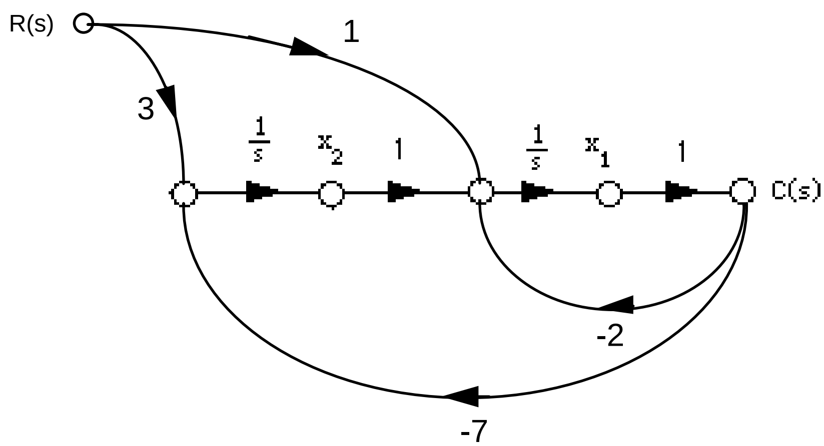
Cross multiplying,

$$\left(\frac{1}{s} + \frac{3}{s^2}\right) R(s) = \left(1 + \frac{2}{s} + \frac{7}{s^2}\right) C(s)$$

Thus,

$$\frac{1}{s}(R(s) - 2C(s)) + \frac{1}{s^2}(3R(s) - 7C(s)) = C(s)$$

Drawing the signal-flow graph,



Writing the state and output equations,

$$\dot{x}_1 = -2x_1 + x_2 + r$$

$$\dot{x}_2 = -7x_1 + 3r$$

$$y = x_1$$

In vector matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 1 \\ -7 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

b. Controller canonical form:

From the phase-variable form in Problem 5.31(b), reverse the order of the state variables and obtain,

$$\dot{x}_3 = x_2$$

$$\dot{x}_2 = x_1$$

$$\dot{x}_1 = -x_3 - 2x_2 - 5x_1$$

$$y = 6x_3 + 2x_2 + x_1$$

Putting the equations in order,

$$\dot{x}_1 = -5x_1 - 2x_2 - x_3$$

$$\dot{x}_2 = x_1$$

$$\dot{x}_3 = x_2$$

$$y = x_1 + 2x_2 + 6x_3$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & -2 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}r$$

$$y = \begin{bmatrix} 1 & 2 & 6 \end{bmatrix}\mathbf{x}$$

Observer canonical form:

$G(s) = \dfrac{s^2 + 2s + 6}{s^3 + 5s^2 + 2s + 1}$. Divide each term by $\dfrac{1}{s^3}$ and get

$$G(s) = \frac{\frac{1}{s} + \frac{2}{s^2} + \frac{6}{s^3}}{1 + \frac{5}{s} + \frac{2}{s^2} + \frac{1}{s^3}} = \frac{C(s)}{R(s)}$$

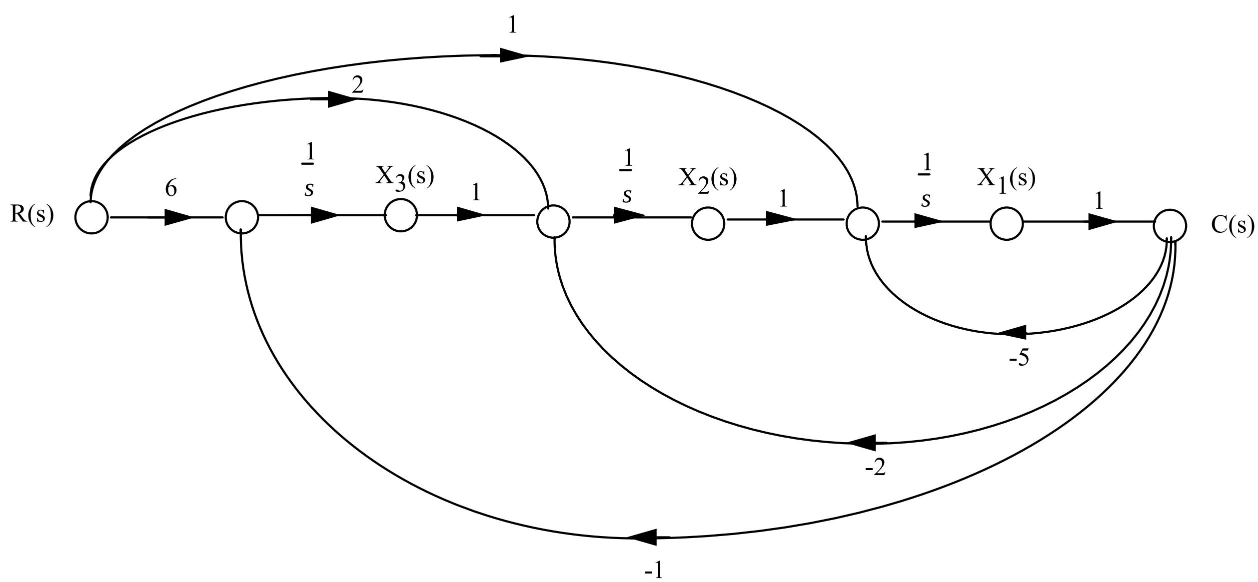
Cross-multiplying,

$$\left(\frac{1}{s} + \frac{2}{s^2} + \frac{6}{s^3}\right)R(s) = \left(1 + \frac{5}{s} + \frac{2}{s^2} + \frac{1}{s^3}\right)C(s)$$

Thus,

$$\frac{1}{s}(R(s) - 5c(s)) + \frac{1}{s^2}(2R(s) - 2C(s)) + \frac{1}{s^3}(6R(s) - C(s)) = C(s)$$

Drawing the signal-flow graph,



Writing the state and output equations,

$$\dot{x}_1 = -5x_1 + x_2 + r$$

$$\dot{x}_2 = -2x_1 + x_3 + 2r$$

$$\dot{x}_3 = -x_1 + 6r$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & 1 & 0 \\ -2 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}$$

c. Controller canonical form:

From the phase-variable form in Problem 5.31(c), reverse the order of the state variables and obtain,

$$\dot{x}_4 = x_3$$

$$\dot{x}_3 = x_2$$

$$\dot{x}_2 = x_1$$

$$\dot{x}_1 = -4x_4 - 6x_3 - 5x_2 - 3x_1 + r$$

$$y = x_4 + 7x_3 + 2x_2 + x_1$$

Putting the equations in order,

$$\dot{x}_1 = -3x_1 - 5x_2 - 6x_3 - 4x_4 + r$$

$$\dot{x}_2 = x_1$$

$$\dot{x}_3 = x_2$$

$$\dot{x}_4 = x_3$$

$$y = x_1 + 2x_2 + 7x_3 + x_4$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & -5 & -6 & -4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\mathbf{X} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 2 & 7 & 1 \end{bmatrix}\mathbf{x}$$

Observer canonical form:

$G(s) = \dfrac{s^3+2s^2+7s+1}{s^4+3s^3+5s^2+6s+4}$. Divide each term by $\dfrac{1}{s^2}$ and get

$$G(s) = \frac{\dfrac{1}{s} + \dfrac{2}{s^2} + \dfrac{7}{s^3} + \dfrac{1}{s^4}}{1 + \dfrac{3}{s} + \dfrac{5}{s^2} + \dfrac{6}{s^3} + \dfrac{4}{s^4}} = \frac{C(s)}{R(s)}$$

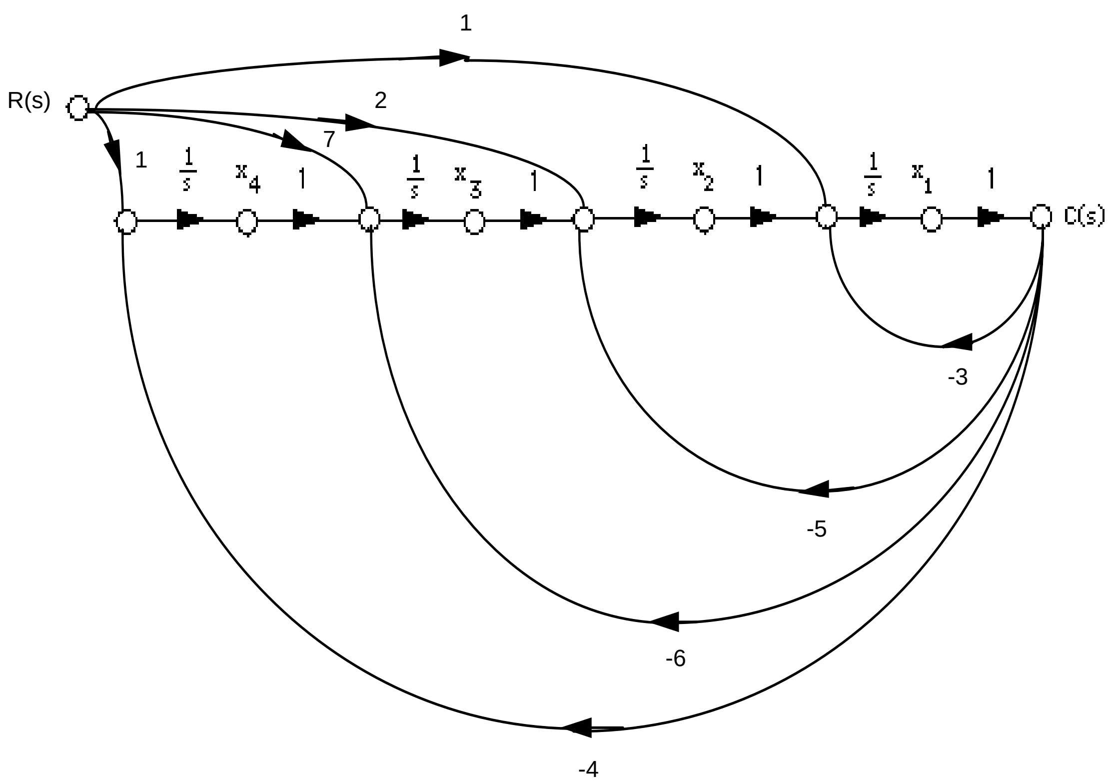
Cross multiplying,

$$\left(\frac{1}{s}+\frac{2}{s^2}+\frac{7}{s^3}+\frac{1}{s^4}\right)R(s) = \left(1+\frac{3}{s}+\frac{5}{s^2}+\frac{6}{s^3}+\frac{4}{s^4}\right)C(s)$$

Thus,

$$\frac{1}{s}(R(s) - 3C(s)) + \frac{1}{s^2}(2R(s) - 5C(s)) + \frac{1}{s^3}(7R(s) - 6C(s)) + \frac{1}{s^4}(R(s) - 4C(s)) = C(s)$$

Drawing the signal-flow graph,



Writing the state and output equations,

$$\dot{x}_1 = -3x_1 + x_2 + r$$

$$\dot{x}_2 = -5x_1 + x_3 + 2r$$

$$\dot{x}_3 = -6x_1 + x_4 + 7r$$

$$\dot{x}_4 = -4x_1 + r$$

$$y = x_1$$

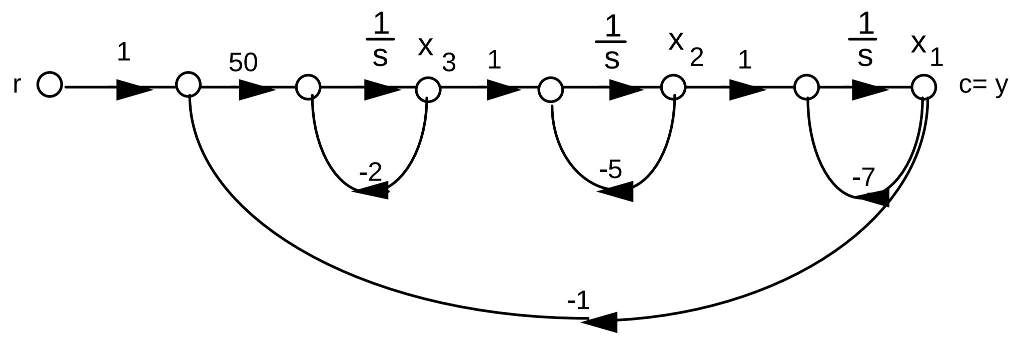
In vector matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ -6 & 0 & 0 & 1 \\ -4 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 2 \\ 7 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}$$

33.

a.



Writing the state equations,

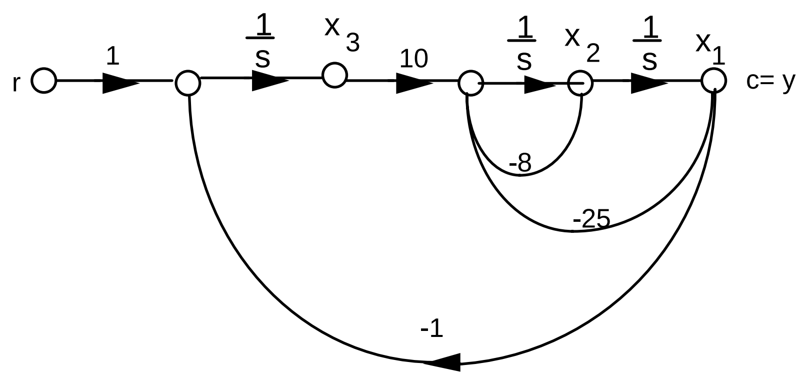
$$\dot{x}_1 = -7x_1 + x_2$$
$$\dot{x}_2 = -5x_2 + x_3$$
$$\dot{x}_3 = -50x_1 - 2x_3 + 50r$$
$$y = x_1$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -7 & 1 & 0 \\ 0 & -5 & 1 \\ -50 & 0 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 50 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}$$

b.



Writing the state equations,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -25x_1 - 8x_2 + 10x_3$$

$$\dot{x}_3 = -x_1 + r$$

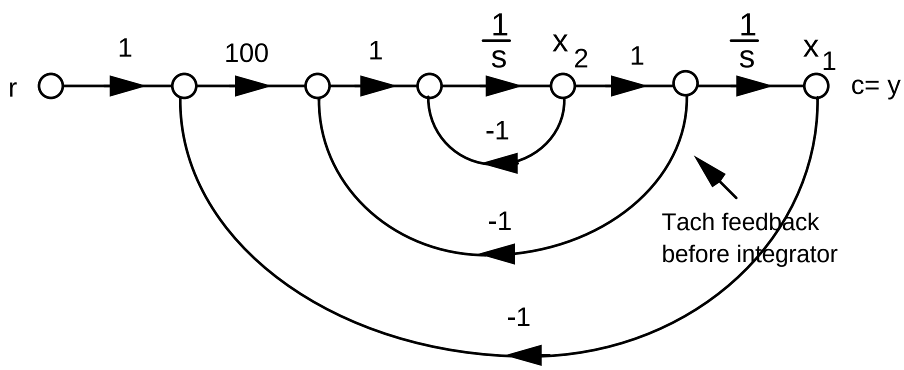
$$y = x_1$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -25 & -8 & 10 \\ -1 & 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}$$

c.



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_2 - x_2 + 100(r-x_1) = -100x_1 - 2x_2 + 100r$$

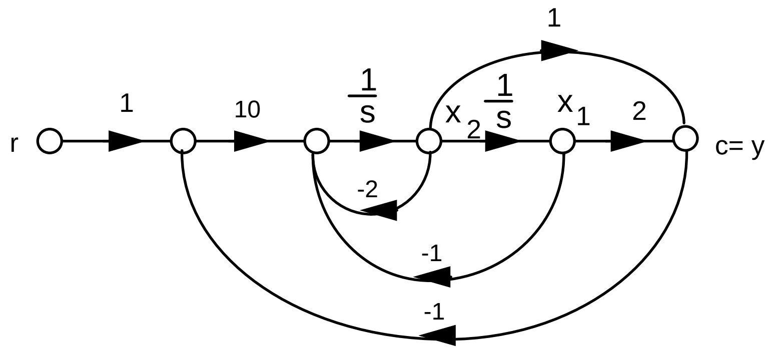
$$y = x_1$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -100 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 100 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$

d. Since $\frac{1}{(s+1)^2} = \frac{1}{s^2+2s+1}$, we draw the signal-flow as follows:



Writing the state equations,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - 2x_2 + 10(r-c) = -x_1 - 2x_2 + 10(r - (2x_1+x_2)) = -21x_1 - 12x_2 + 10r$$

$$y = 2x_1 + x_2$$

In vector-matrix form,

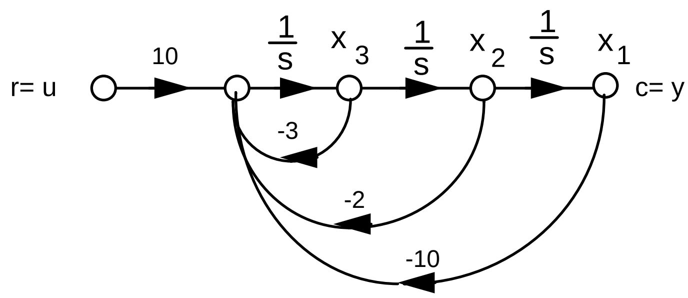
$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -21 & -12 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} r$$

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix}\mathbf{x}$$

34.

a. Phase-variable form:

$$T(s) = \frac{10}{s^3+3s^2+2s+10}$$



Writing the state equations,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -10x_1 - 2x_2 - 3x_3 + 10u$$

$$y = x_1$$

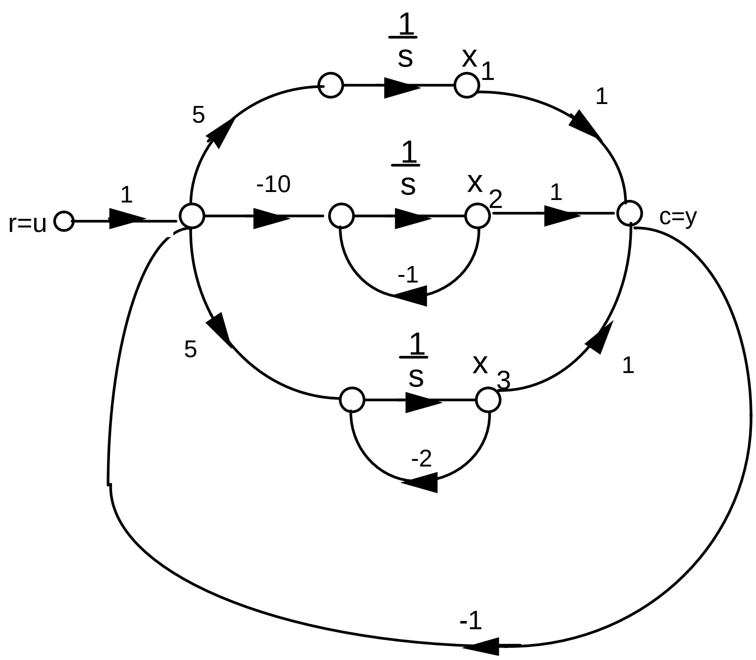
In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -2 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}$$

b. Parallel form:

$$G(s) = \frac{5}{s} + \frac{-10}{s+1} + \frac{5}{s+2}$$



Writing the state equations,

$$\dot{x}_1 = 5(u - x_1 - x_2 - x_3) = -5x_1 - 5x_2 - 5x_3 + 5u$$

$$\dot{x}_2 = -10(u - x_1 - x_2 - x_3) - x_2 = 10x_1 + 9x_2 + 10x_3 - 10u$$

$$\dot{x}_3 = 5(u - x_1 - x_2 - x_3) - 2x_3 = -5x_1 - 5x_2 - 7x_3 + 5u$$

$$y = x_1 + x_2 + x_3$$

In vector-matrix form,

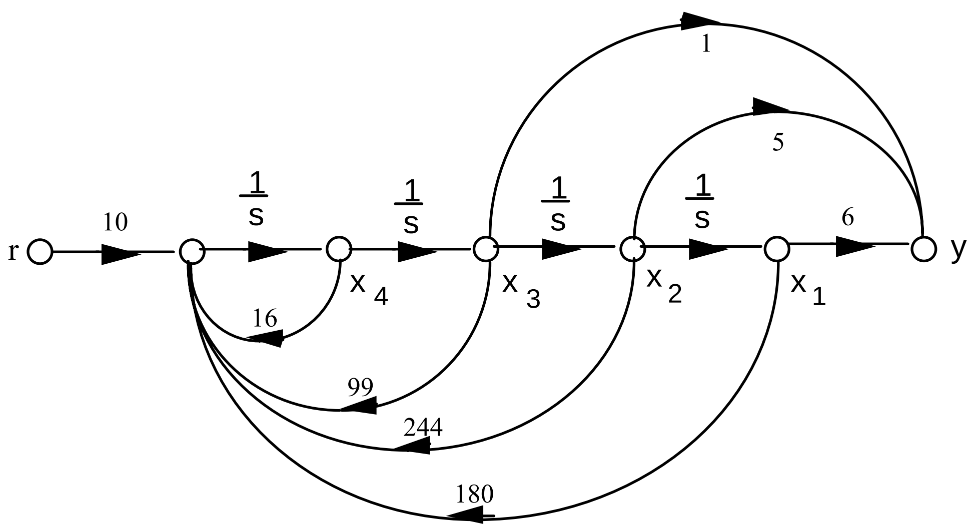
$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & -5 & -5 \\ 10 & 9 & 10 \\ -5 & -5 & -7 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 5 \\ -10 \\ 5 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\mathbf{x}$$

35.

a. $T(s) = \dfrac{10(s^2 + 5s + 6)}{s^4 + 16s^3 + 99s^2 + 244s + 180}$

Drawing the signal-flow diagram,



Writing the state and output equations,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_1 = -180x_1 - 244x_2 - 99x_3 - 16x_4 + 10r$$
$$y = 6x_1 + 5x_2 + x_3$$

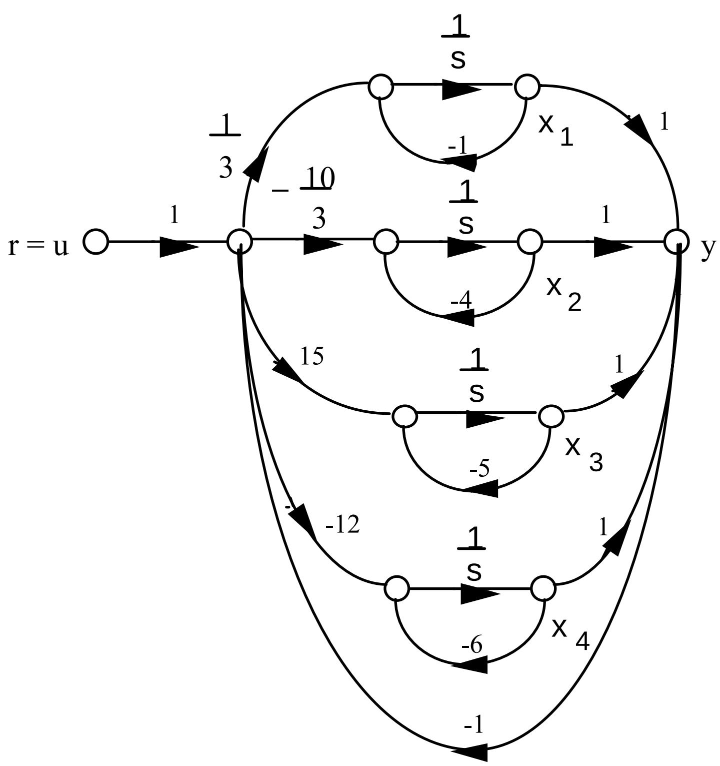
In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -180 & -244 & -99 & -16 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix}r$$

$$y = \begin{bmatrix} 6 & 5 & 1 & 0 \end{bmatrix}\mathbf{x}$$

b. $G(s)=\dfrac{10(s+2)(s+3)}{(s+1)(s+4)(s+5)(s+6)}=\dfrac{1/3}{s+1}-\dfrac{10/3}{s+4}+\dfrac{15}{s+5}-\dfrac{12}{s+6}$

Drawing the signal-flow diagram and including the unity-feedback path,



Writing the state and output equations,

$$\dot{x}_1=\frac{1}{3}(u-x_1-x_2-x_3-x_4)-x_1$$

$$\dot{x}_2=\frac{-10}{3}(u-x_1-x_2-x_3-x_4)-4x_2$$

$$\dot{x}_3=15(u-x_1-x_2-x_3-x_4)-5x_3$$

$$\dot{x}_4=-12(u-x_1-x_2-x_3-x_4)-12x_4$$

$$y=x_1+x_2+x_3+x_4$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{4}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \frac{10}{3} & -\frac{2}{3} & \frac{10}{3} & \frac{10}{3} \\ -15 & -15 & -20 & -15 \\ 12 & 12 & 12 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} \frac{1}{3} \\ -\frac{10}{3} \\ 15 \\ -12 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}\mathbf{x}$$

36.

Program:

```
'(a)'
'G(s)'
G=zpk([-2 -3],[-1 -4 -5 -6],10)
'T(s)'
T=feedback(G,1,-1)
[numt,dent]=tfdata(T,'v');
'Find controller canonical form'
[Acc,Bcc,Ccc,Dcc]=tf2ss(numt,dent)
A1=flipud(Acc);
'Transform to phase-variable form'
Apv=fliplr(A1)
Bpv=flipud(Bcc)
Cpv=fliplr(Ccc)
'(b)'
'G(s)'
G=zpk([-2 -3],[-1 -4 -5 -6],10)
'T(s)'
T=feedback(G,1,-1)
[numt,dent]=tfdata(T,'v');
'Find controller canonical form'
[Acc,Bcc,Ccc,Dcc]=tf2ss(numt,dent)
'Transform to modal form'
[A,B,C,D]=canon(Acc,Bcc,Ccc,Dcc,'modal')
```

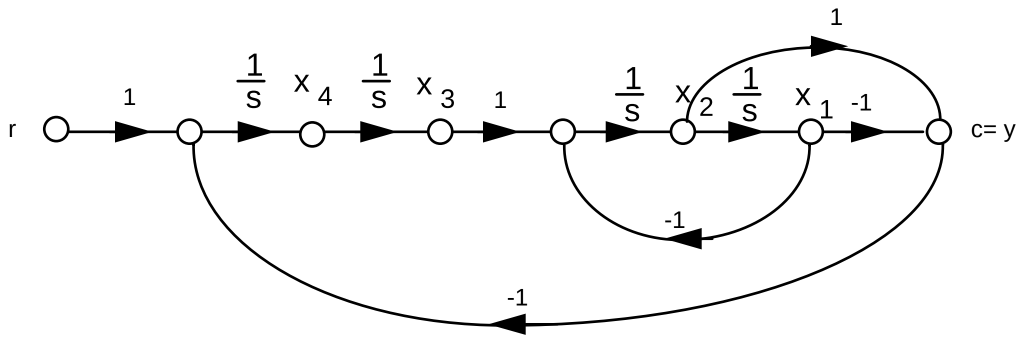
Computer response:

```
ans =

(a)

ans =

G(s)

Zero/pole/gain:
    10 (s+2) (s+3)
-----------------------
(s+1) (s+4) (s+5) (s+6)

ans =

T(s)

Zero/pole/gain:

              10 (s+2) (s+3)
------------------------------------------
(s+1.264) (s+3.412) (s^2 + 11.32s + 41.73)

ans =
```

```
Find controller canonical form

Acc =

  -16.0000  -99.0000 -244.0000 -180.0000
    1.0000         0         0         0
         0    1.0000         0         0
         0         0    1.0000         0
Bcc =

     1
     0
     0
     0

Ccc =

         0   10.0000   50.0000   60.0000

Dcc =

     0

ans =

Transform to phase-variable form

Apv =

         0    1.0000         0         0
         0         0    1.0000         0
         0         0         0    1.0000
 -180.0000 -244.0000  -99.0000  -16.0000

Bpv =

     0
     0
     0
     1

Cpv =

   60.0000   50.0000   10.0000         0

ans =

(b)

ans =

G(s)

Zero/pole/gain:
    10 (s+2) (s+3)
-----------------------
(s+1) (s+4) (s+5) (s+6)

ans =

T(s)

Zero/pole/gain:
```

```
              10 (s+2) (s+3)
------------------------------------------
(s+1.264) (s+3.412) (s^2 + 11.32s + 41.73)

ans =

Find controller canonical form

Acc =

  -16.0000  -99.0000 -244.0000 -180.0000
    1.0000         0         0         0
         0    1.0000         0         0
         0         0    1.0000         0

Bcc =

     1
     0
     0
     0

Ccc =

         0   10.0000   50.0000   60.0000

Dcc =

     0

ans =

Transform to modal form

A =

   -5.6618    3.1109         0         0
   -3.1109   -5.6618         0         0
         0         0   -3.4124         0
         0         0         0   -1.2639

B =

   -4.1108
    1.0468
    1.3125
    0.0487
C =

    0.1827    0.6973   -0.1401    4.2067

D =

     0
```

37.



Writing the state equations,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 + x_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = x_1 - x_2 + r$$

$$y = -x_1 + x_2$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{r}$$

$$y = c = [-1 \quad 1 \quad 0 \quad 0]\,\mathbf{x}$$

38.

a.

$$\ddot{\theta}_1 + 5\dot{\theta}_1 + 6\theta_1 - 3\dot{\theta}_2 - 4\theta_2 = 0$$

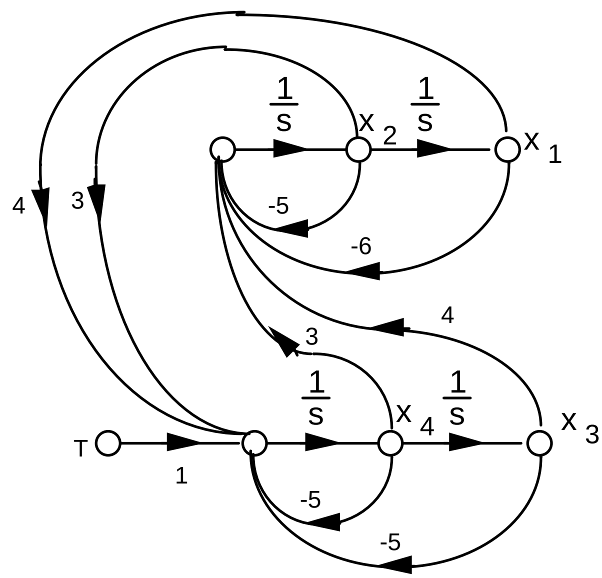
$$-3\dot{\theta}_1 - 4\theta_1 + \ddot{\theta}_2 + 5\dot{\theta}_2 + 5\theta_2 = T$$

or

$$\ddot{\theta}_1 = -5\dot{\theta}_1 - 6\theta_1 + 3\dot{\theta}_2 + 4\theta_2$$

$$\ddot{\theta}_2 = 3\dot{\theta}_1 + 4\theta_1 - 5\dot{\theta}_2 - 5\theta_2 + T$$

Letting, $\theta_1 = x_1$; $\dot{\theta}_1 = x_2$; $\theta_2 = x_3$; $\dot{\theta}_2 = x_4$,



where x = θ.

b. Using the signal-flow diagram,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -6x_1 - 5x_2 + 4x_3 + 3x_4$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = 4x_1 + 3x_2 - 5x_3 - 5x_4 + T$$

$$y = x_3$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -6 & -5 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 4 & 3 & -5 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}T$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}\mathbf{x}$$

39.

Program:

```
numg=7;
deng=poly([0 -9 -12]);
G=tf(numg,deng);
T=feedback(G,1)
[numt,dent]=tfdata(T,'v')
[A,B,C,D]=tf2ss(numt,dent); %Obtain controller canonical form
'(a)'                       %Display label
A=flipud(A);                %Convert to phase-variable form
A=fliplr(A)                 %Convert to phase-variable form
B=flipud(B)                 %Convert to phase-variable form
C=fliplr(C)                 %Convert to phase-variable form
```

```
'(b)'                        %Display label
[a,b,c,d]=canon(A,B,C,D)     %Convert to parallel form
```

Computer response:

```
Transfer function:
           7
------------------------
s^3 + 21 s^2 + 108 s + 7

numt =

     0     0     0     7

dent =

     1    21   108     7

ans =

(a)

A =

     0     1     0
     0     0     1
    -7  -108   -21

B =

     0
     0
     1

C =

     7     0     0

ans =
```

```
(b)

a =

   -0.0657         0         0
         0  -12.1807         0
         0         0   -8.7537


b =

   -0.0095
   -3.5857
    2.5906


c =

   -6.9849   -0.0470   -0.0908


d =

     0
```

40.

$$\dot{\mathbf{x}}_1 = \mathbf{A_1 x_1} + \mathbf{B_1}r \tag{1}$$

$$y_1 = \mathbf{C_1 x_1} \tag{2}$$

$$\dot{\mathbf{x}}_2 = \mathbf{A_2 x_2} + \mathbf{B_2}y_1 \tag{3}$$

$$y_2 = \mathbf{C_2 x_2} \tag{4}$$

Substituting Eq. (2) into Eq. (3),

$$\dot{\mathbf{x}}_1 = \mathbf{A_1 x_1} + \mathbf{B_1}r$$

$$\dot{\mathbf{x}}_2 = \mathbf{B_2 C_1 x_1} + \mathbf{A_2 x_2}$$

$$y_2 = \mathbf{C_2 x_2}$$

In vector-matrix notation,

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \left[\begin{array}{c:c} \mathbf{A_1} & \mathbf{O} \\ \mathbf{B_2 C_1} & \mathbf{A_2} \end{array}\right] \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{O} \end{bmatrix} r$$

$$\mathbf{y}_2 = \left[\begin{array}{c:c} \mathbf{O} & \mathbf{C}_2 \end{array}\right] \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix}$$

41.

$$\dot{\mathbf{x}}_1 = \mathbf{A_1 x_1} + \mathbf{B_1}r \tag{1}$$

$$y_1 = \mathbf{C_1 x_1} \tag{2}$$

$$\dot{\mathbf{x}}_2 = \mathbf{A_2 x_2} + \mathbf{B_2}r \qquad (3)$$

$$y_2 = \mathbf{C_2 x_2} \qquad (4)$$

In vector-matrix form,

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{A_2} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B_2} \end{bmatrix} r$$

$$\mathbf{y} = \mathbf{y}_1 + \mathbf{y}_2 = [\mathbf{C}_1 \;\; \mathbf{C}_2] \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix}$$

42.

$$\dot{\mathbf{x}}_1 = \mathbf{A_1 x_1} + \mathbf{B_1}e \qquad (1)$$

$$y = \mathbf{C_1 x_1} \qquad (2)$$

$$\dot{\mathbf{x}}_2 = \mathbf{A_2 x_2} + \mathbf{B_2}y \qquad (3)$$

$$p = \mathbf{C_2 x_2} \qquad (4)$$

Substituting e = r - p into Eq. (1) and substituting Eq. (2) into (3), we obtain,

$$\dot{\mathbf{x}}_1 = \mathbf{A_1 x_1} + \mathbf{B_1}(r - p) \qquad (5)$$

$$y = \mathbf{C_1 x_1} \qquad (6)$$

$$\dot{\mathbf{x}}_2 = \mathbf{A_2 x_2} + \mathbf{B_2 C_1 x_1} \qquad (7)$$

$$p = \mathbf{C_2 x_2} \qquad (8)$$

Substituting Eq. (8) into Eq. (5),

$$\dot{\mathbf{x}}_1 = \mathbf{A_1 x_1} - \mathbf{B_1 C_2 x_2} + \mathbf{B_1 r}$$

$$\dot{\mathbf{x}}_2 = \mathbf{B_2 C_1 x_1} + \mathbf{A_2 x_2}$$

$$\mathbf{y} = \mathbf{C_1 x_1}$$

In vector-matrix form,

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A_1} & -\mathbf{B_1 C_2} \\ \mathbf{B_2 C_1} & \mathbf{A_2} \end{bmatrix} \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix} + \begin{bmatrix} \mathbf{B_1} \\ \mathbf{0} \end{bmatrix} \mathbf{r}$$

$$y = [\mathbf{C}_1 \;\; \mathbf{0}] \begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix}$$

43.

$$\dot{\mathbf{z}} = \mathbf{P}^{-1}\mathbf{APz} + \mathbf{P}^{-1}\mathbf{B}u$$

$$y = \mathbf{CPz}$$

$$\mathbf{P}^{-1} = \begin{bmatrix} 2 & 1 & -4 \\ 1 & -2 & 0 \\ 4 & 6 & 2 \end{bmatrix}; \ \therefore \ \mathbf{P} = \begin{bmatrix} 0.0606 & 0.3939 & 0.1212 \\ 0.0303 & -0.3030 & 0.0606 \\ -0.2121 & 0.1212 & 0.0758 \end{bmatrix}$$

$$\mathbf{P}^{-1}\mathbf{AP} = \begin{bmatrix} -1.6 & 1.23 & 3.81 \\ 3.33 & 1.33 & -2.33 \\ 1.63 & -1.79 & 1.26 \end{bmatrix}; \ \mathbf{P}^{-1}\mathbf{B} = \begin{bmatrix} -3 \\ 4 \\ 4 \end{bmatrix}; \ \mathbf{CP} = \begin{bmatrix} -0.544 & -0.0702 & 0.912 \end{bmatrix}$$

44.

$$\dot{z} = P^{-1}APz + P^{-1}Bu$$

$$y = CPz$$

$$P^{-1} = \begin{pmatrix} 4 & -1 & 0 \\ 2 & 3 & -2 \\ 8 & 5 & 1 \end{pmatrix} \qquad P = \begin{pmatrix} \frac{13}{70} & \frac{1}{70} & \frac{1}{35} \\ -\frac{9}{35} & \frac{2}{35} & \frac{4}{35} \\ -\frac{1}{5} & -\frac{2}{5} & \frac{1}{5} \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} 1.46 & -0.657 & -0.314 \\ -1.94 & -6.46 & 4.09 \\ 0 & -8 & 4 \end{pmatrix} \qquad P^{-1}B = \begin{pmatrix} 27 \\ 21 \\ 59 \end{pmatrix} \qquad CP = \left(\frac{11}{70}, -\frac{123}{70}, \frac{17}{35}\right)$$

45.

Eigenvalues are -1, -2, and -3 since,

$$|\lambda I - A| = (\lambda + 3)(\lambda + 2)(\lambda + 1)$$

Solving for the eigenvectors, $\mathbf{Ax} = \lambda\mathbf{x}$

or,

$$4x_3 - 5x_2 - (\lambda + 5)x_1 = 0$$
$$-2x_3 - x_2\lambda + 2x_1 = 0$$
$$-2x_2 - (\lambda + 1)x_3 = 0$$

For $\lambda = -1$, $x_2 = 0$, $x_1 = x_3$. For $\lambda = -2$, $x_1 = x_2 = \frac{x_3}{2}$. For $\lambda = -3$, $x_1 = -\frac{x_2}{2}$, $x_2 = x_3$. Thus,

$\dot{\mathbf{z}} = \mathbf{P}^{-1}\mathbf{APz} + \mathbf{P}^{-1}\mathbf{B}u$; $y = \mathbf{CPz}$, where

$$P^{-1}AP = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix}; \ P^{-1}B = \begin{pmatrix} -12 \\ 8 \\ -3 \end{pmatrix}; \ CP = (1, 4, 7)$$

46.

Eigenvalues are 1, -2, and 3 since,

$$|\lambda \mathbf{I} - \mathbf{A}| = (\lambda - 3)(\lambda + 2)(\lambda - 1)$$

Solving for the eigenvectors, $\mathbf{Ax} = \lambda\mathbf{x}$

or,

$$(-\lambda - 10)x_1 + 7x_3 - 3x_2 = 0$$
$$\frac{73}{4}x_1 + \left(-\lambda + \frac{25}{4}\right)x_2 - \frac{47}{4}x_3 = 0$$
$$-\frac{29}{4}x_1 - \frac{9}{4}x_2 + \left(-\lambda + \frac{23}{4}\right)x_3 = 0$$

For $\lambda = 1$, $x_1 = x_2 = \frac{x_3}{2}$. For $\lambda = -2$, $x_1 = 2x_3$, $x_2 = -3x_3$. For $\lambda = 3$, $x_1 = x_3$, $x_2 = -2x_3$. Thus,

$\dot{\mathbf{z}} = \mathbf{P}^{-1}\mathbf{APz} + \mathbf{P}^{-1}\mathbf{B}u$; $y = \mathbf{CPz}$, where

$$\mathbf{P}^{-1}\mathbf{AP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix} ; \quad \mathbf{P}^{-1}\mathbf{B} = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \\ -\frac{3}{2} \end{pmatrix} ; \quad \mathbf{CP} = (7, 12, 9)$$

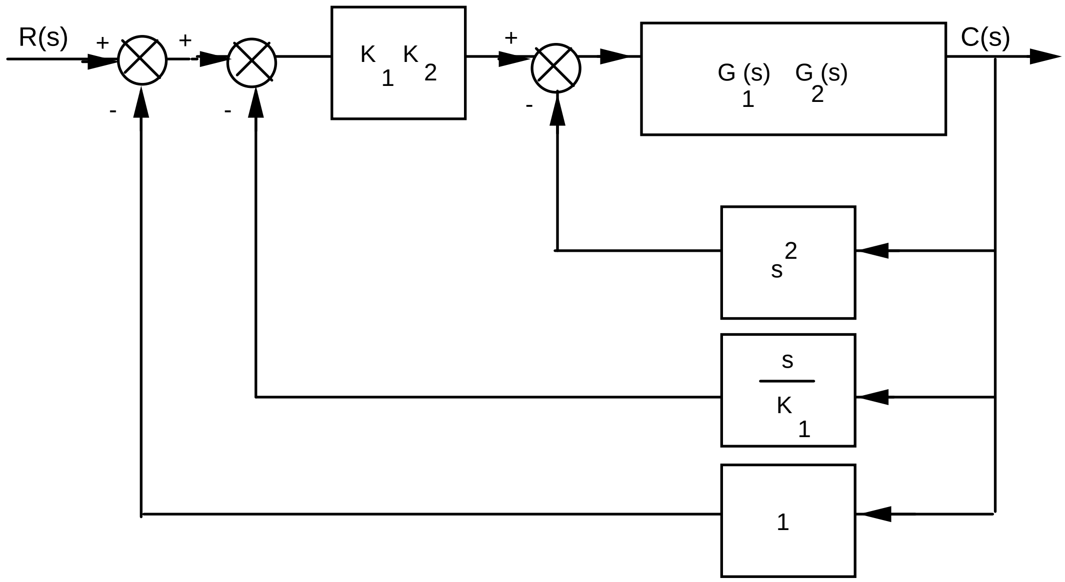
47.

Program:
```
A=[-10 -3 7;18.25 6.25 -11.75;-7.25 -2.25 5.75];
B=[1;3;2];
C=[1 -2 4];
[P,d]=eig(A);
Ad=inv(P)*A*P
Bd=inv(P)*B
Cd=C*P
```

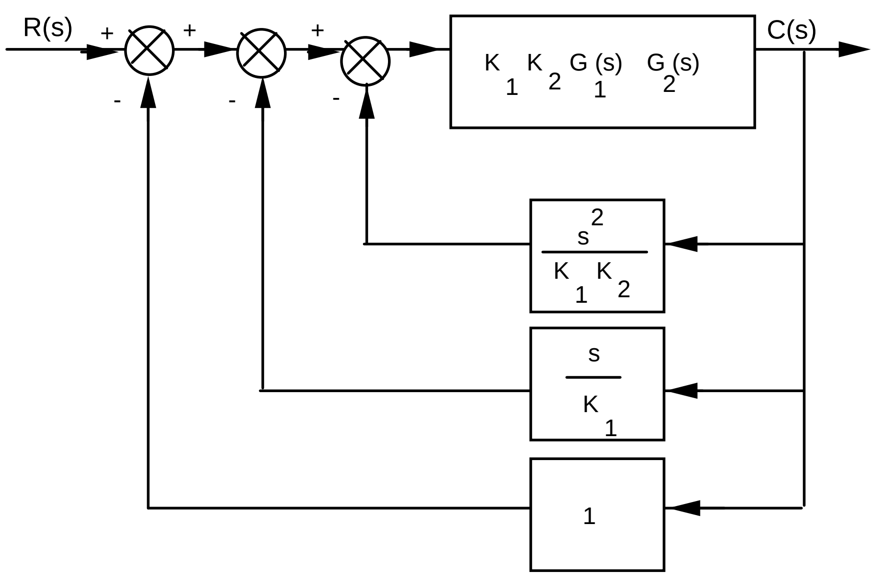
Computer response:
```
Ad =

   -2.0000    0.0000    0.0000
   -0.0000    3.0000   -0.0000
    0.0000    0.0000    1.0000


Bd =

    1.8708
   -3.6742
    3.6742


Cd =

    3.2071    3.6742    2.8577
```

48.

a. Combine $G_1(s)$ and $G_2(s)$. Then push K_1 to the right past the summing junction:

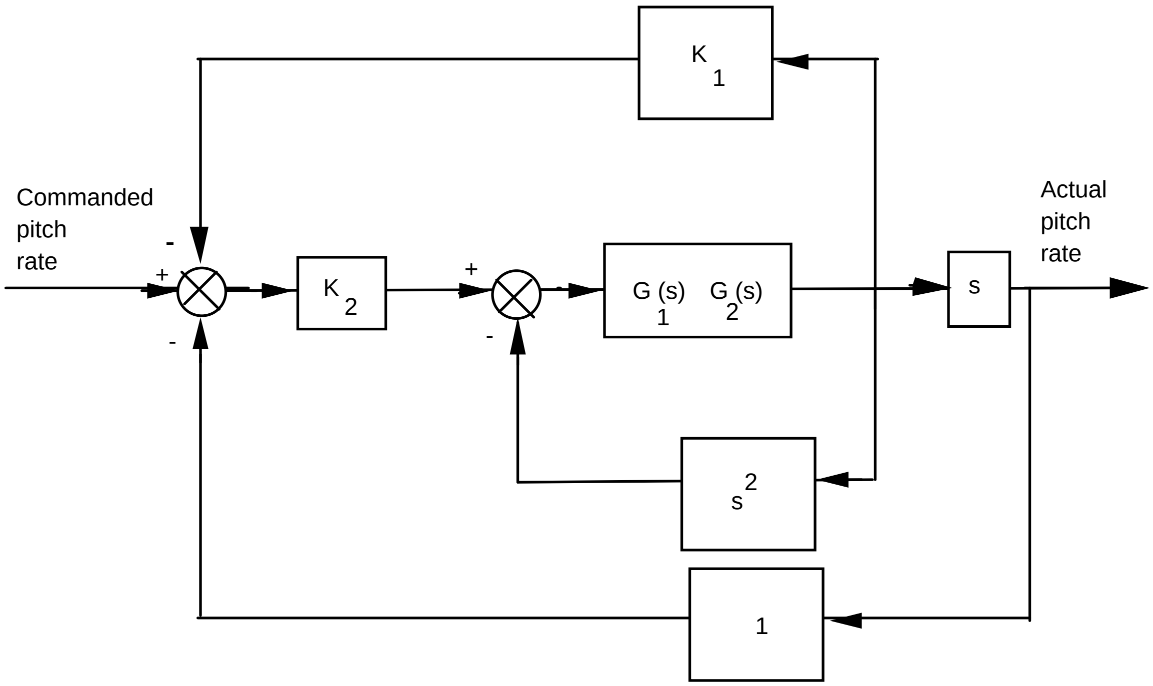


Push K_1K_2 to the right past the summing junction:

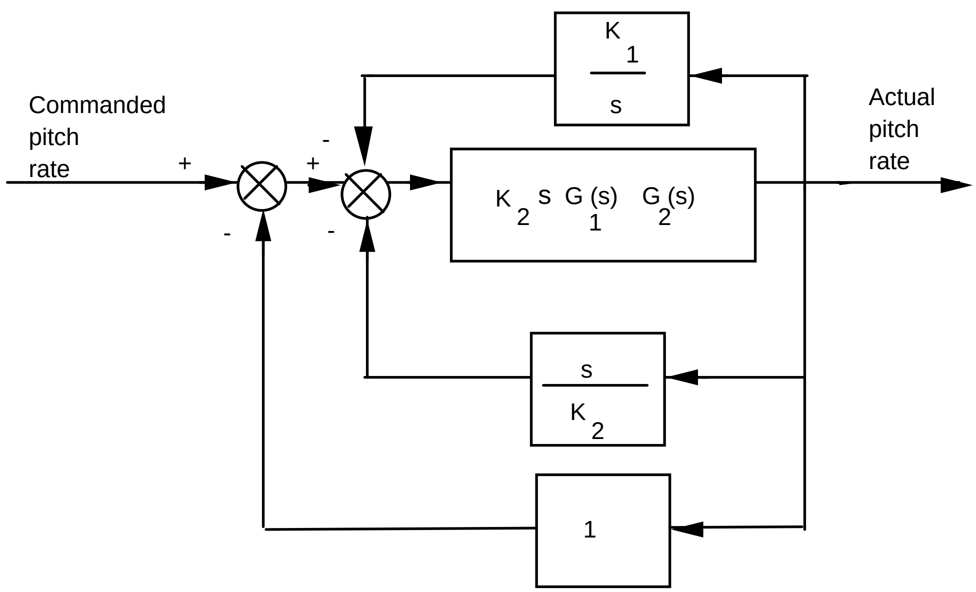


Hence, $T(s) = \dfrac{K_1K_2G_1(s)G_2(s)}{1 + K_1K_2G_1(s)G_2(s)\left(1 + \dfrac{s}{K_1} + \dfrac{s^2}{K_1K_2}\right)}$

b. Rearranging the block diagram to show commanded pitch rate as the input and actual pitch rate as the output:



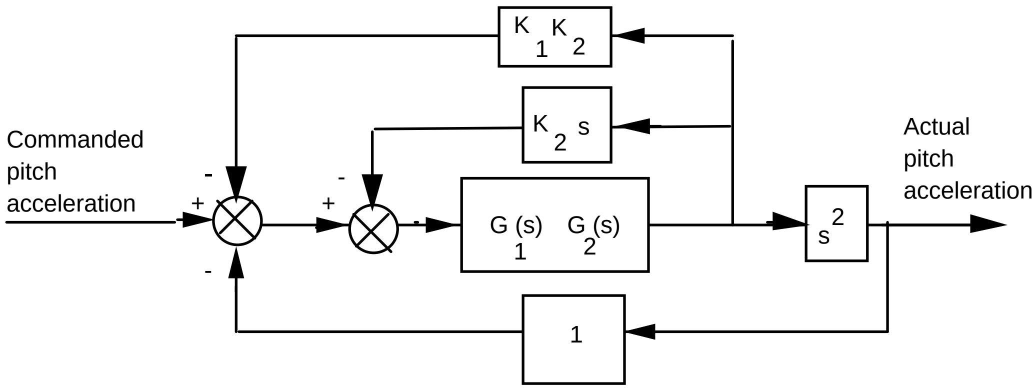
Pushing K_2 to the right past the summing junction; and pushing s to the left past the pick-off point yields,



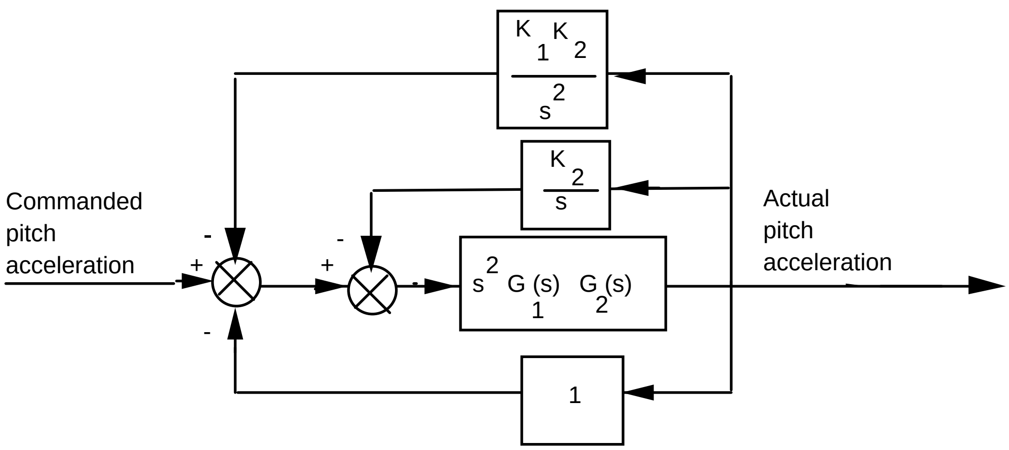
Finding the closed-loop transfer function:

$$T(s) = \frac{K_2 s G_1(s) G_2(s)}{1 + K_2 s G_1(s) G_2(s)\left(1 + \frac{s}{K_2} + \frac{K_1}{s}\right)} = \frac{K_2 s G_1(s) G_2(s)}{1 + G_1(s) G_2(s)(s^2 + K_2 s + K_1 K_2)}$$

c. Rearranging the block diagram to show commanded pitch acceleration as the input and actual pitch acceleration as the output:



Pushing s^2 to the left past the pick-off point yields,

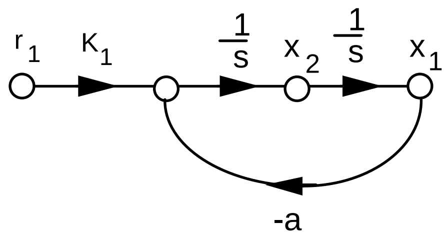


Finding the closed-loop transfer function:

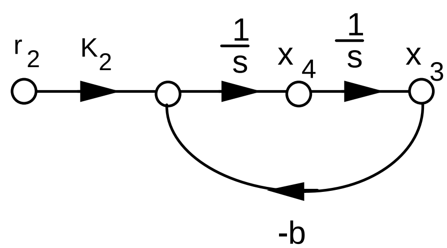
$$T(s) = \frac{s^2G_1(s)G_2(s)}{1 + s^2G_1(s)G_2(s)\left(1 + \frac{K_1K_2}{s^2} + \frac{K_2}{s}\right)} = \frac{s^2G_1(s)G_2(s)}{1 + G_1(s)G_2(s)(s^2 + K_2s + K_1K_2)}$$

49.

Establish a sinusoidal model for the carrier: $T(s) = \frac{K_1}{s^2+a^2}$



Establish a sinusoidal model for the message: $T(s) = \frac{K_2}{s^2+b^2}$



Writing the state equations,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -a^2x_1 + K_1r$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -b^2x_3 + K_2r$$

$$y = x_1x_3$$

50.

The equivalent forward transfer function is $G(s) = \frac{K_1K_2}{s(s+a_1)}$. The equivalent feedback transfer function is

$H(s) = K_3 + \frac{K_4s}{s+a_2}$. Hence, the closed-loop transfer function is

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{K_1K_2(s+a_2)}{s^3 + (a_1+a_2)s^2 + (a_1a_2+K_1K_2K_3+K_1K_2K_4)s + K_1K_2K_3a_2}$$

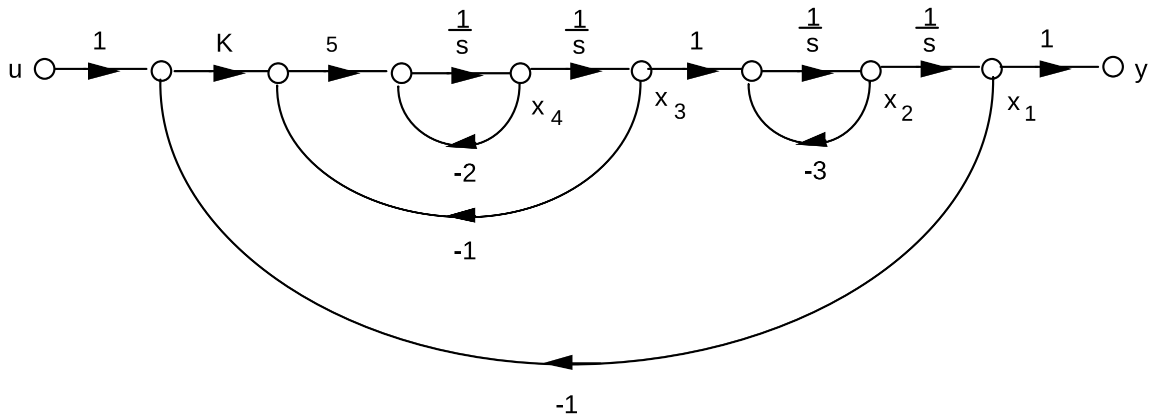
51.

a. The equivalent forward transfer function is

$$G_e(s) = K\frac{\frac{5}{s(s+2)}}{1+\frac{5}{s(s+2)}}\frac{1}{s(s+3)} = \frac{5K}{s(s+3)(s^2+2s+5)}$$

$$T(s) = \frac{G_e}{1+G_e} = \frac{5K}{s^4+5s^3+11s^2+15s+5K}$$

b. Draw the signal-flow diagram:



Writing the state and output equations from the signal-flow diagram:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -3x_2 + x_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -5Kx_1 - 5x_3 - 2x_4 + 5Ku$$

$$y = x_1$$

In vector-matrix form:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5K & 0 & -5 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5\mathrm{K} \end{bmatrix}u$$

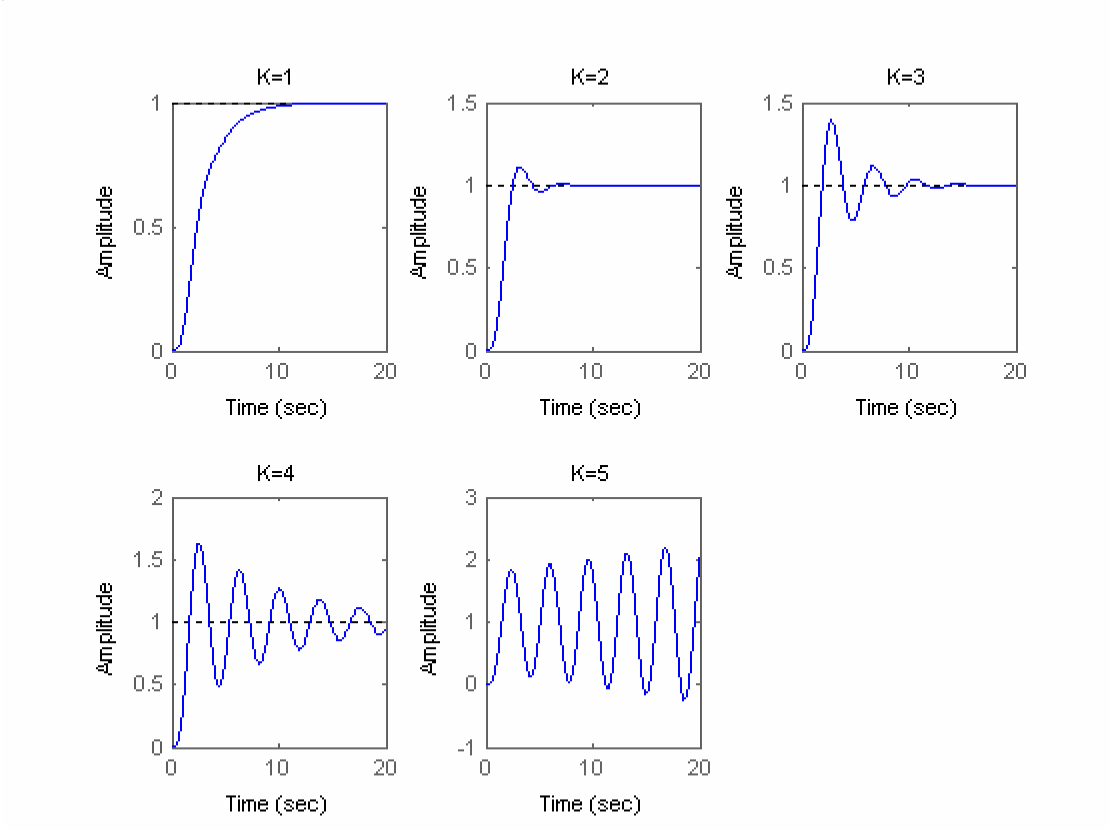
$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}\mathbf{x}$$

c.

Program:

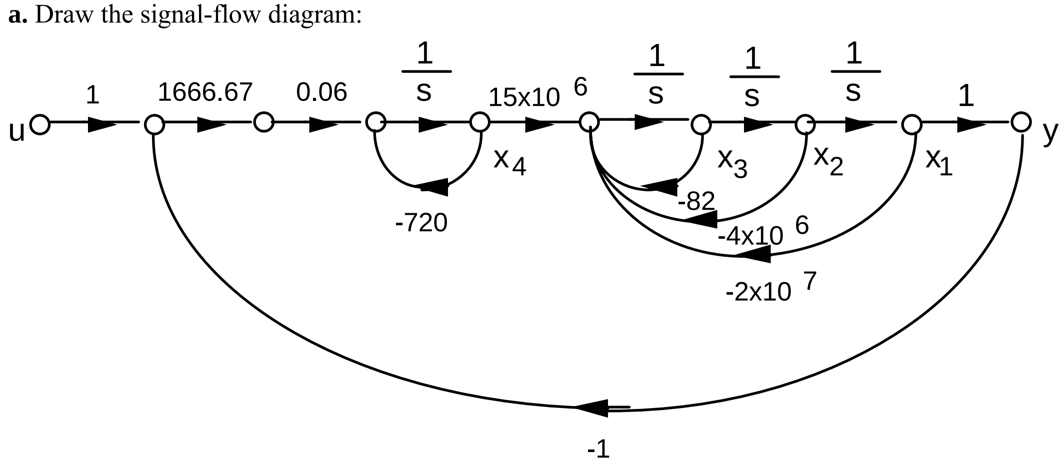
```
for K=1:1:5
numt=5*K;
dent=[1 5 11 15 5*K];
T=tf(numt,dent);
hold on;
subplot(2,3,K);
step(T,0:0.01:20)
title(['K=',int2str(K)])
end
```

Computer response:



52.

a. Draw the signal-flow diagram:



Write state and output equations from the signal-flow diagram:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -2*10^7 x_1 - 4*10^6 x_2 - 82x_3 + 15*10^6 x_4$$

$$\dot{x}_4 = -100x_1 - 720x_4 + 100u$$

$$y = x_1$$

In vector-matrix form:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2*10^7 & -4*10^6 & -82 & 15*10^6 \\ -100 & 0 & 0 & -720 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 100 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}\mathbf{x}$$

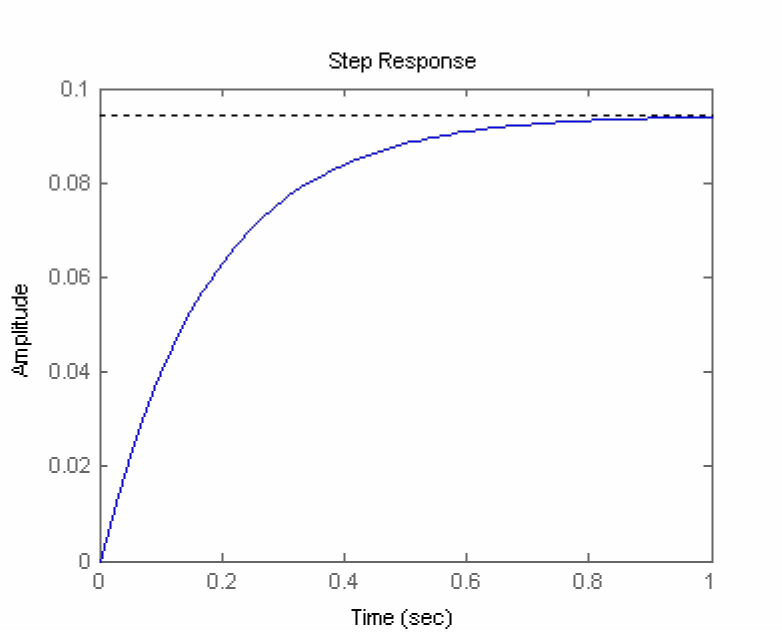
b.

Program:

```
numg=1666.67*0.06*15e6;
deng=conv([1 720],[1 82 4e6 2e7]);
'G(s)'
G=tf(numg,deng)
'T(s)'
T=feedback(G,1)
step(T)
```

Computer response:

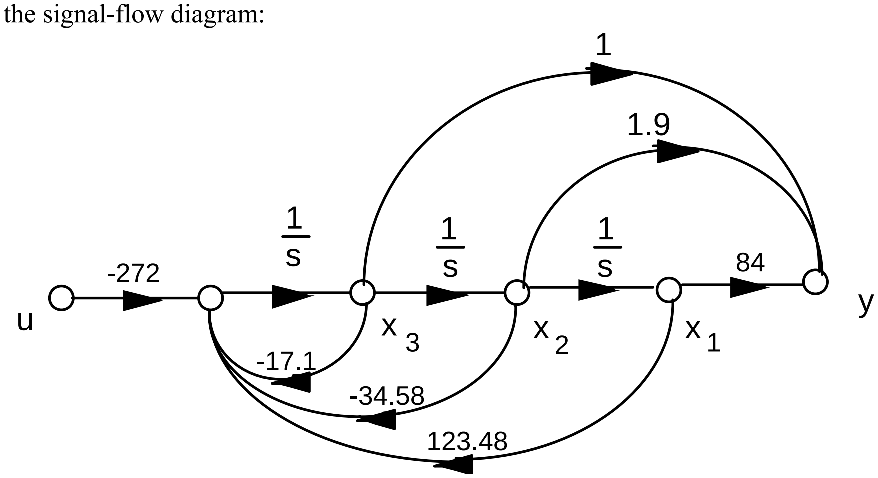
```
ans =

G(s)

Transfer function:
                      1.5e009
----------------------------------------------------
s^4 + 802 s^3 + 4.059e006 s^2 + 2.9e009 s + 1.44e010

ans =

T(s)

Transfer function:
                      1.5e009
----------------------------------------------------
s^4 + 802 s^3 + 4.059e006 s^2 + 2.9e009 s + 1.59e010
```



53.

a. Phase-variable from: $G(s) = \dfrac{-272(s^2+1.9s+84)}{s^3+17.1s^2+34.58s-123.48}$

Drawing the signal-flow diagram:



Writing the state and output equations:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = 123.48x_1 - 34.58x_2 - 17.1x_3 - 272u$$
$$y = 84x_1 + 1.9x_2 + x_3$$

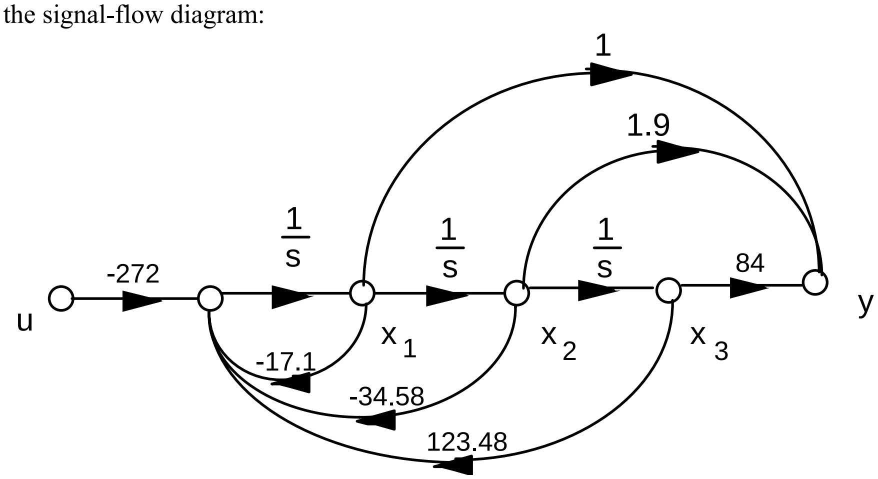
In vector-matrix form:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 123.48 & -34.58 & -17.1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ -272 \end{bmatrix}u$$

$$y = \begin{bmatrix} 84 & 1.9 & 1 \end{bmatrix}\mathbf{x}$$

b. Controller canonical form: $G(s) = \frac{-272(s^2+1.9s+84)}{s^3+17.1s^2+34.58s-123.48}$

Drawing the signal-flow diagram:



Writing the state and output equations:

$$\dot{x}_1 = -17.1x_1 - 34.58x_2 + 123.48x_3 - 272u$$

$$\dot{x}_2 = x_1$$

$$\dot{x}_3 = x_2$$

$$y = x_1 + 1.9x_2 + 84x_3$$

In vector-matrix form:

$$\dot{\mathbf{x}} = \begin{bmatrix} -17.1 & -34.58 & 123.48 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -272 \\ 0 \\ 0 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 1.9 & 84 \end{bmatrix}\mathbf{x}$$

c. Observer canonical form: Divide by highest power of s and obtain

$$G(s) = \frac{\frac{-272}{s} - \frac{516.8}{s^2} - \frac{22848}{s^3}}{1 + \frac{17.1}{s} + \frac{34.58}{s^2} - \frac{123.48}{s^3}}$$

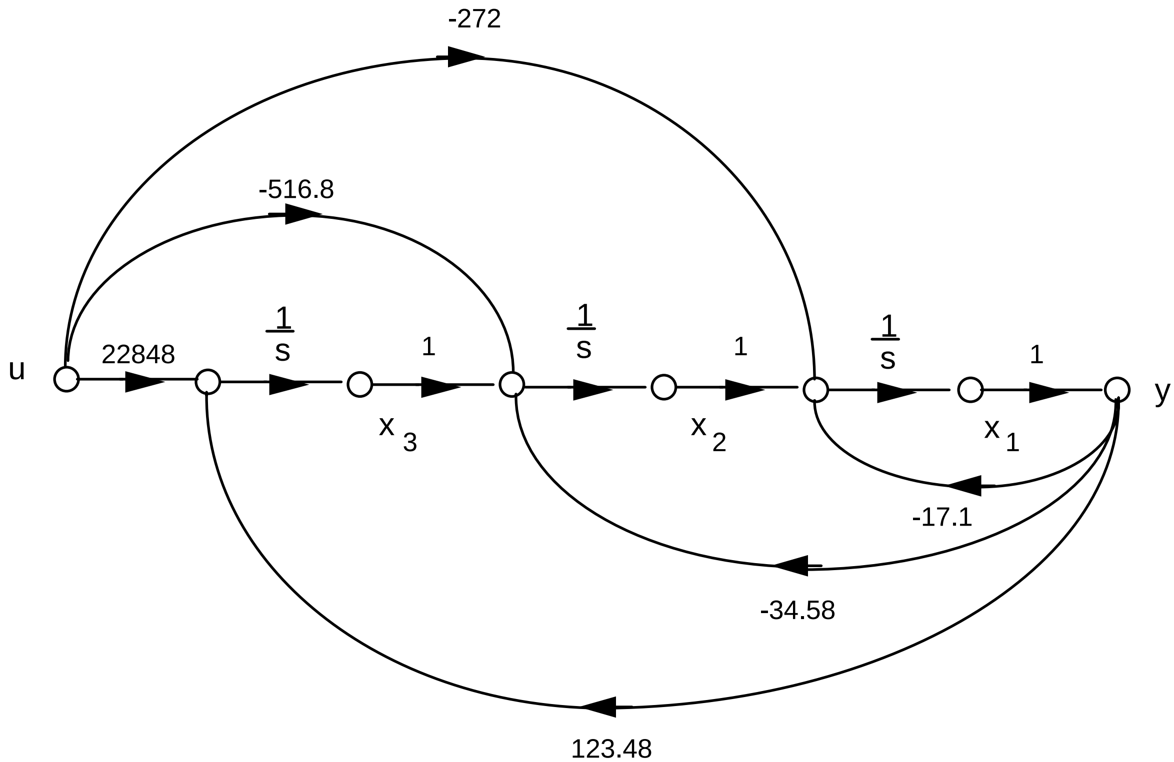
Cross multiplying,

$$\left[\frac{-272}{s} - \frac{516.8}{s^2} - \frac{22848}{s^3}\right]R(s) = \left[1 + \frac{17.1}{s} + \frac{34.58}{s^2} - \frac{123.48}{s^3}\right]C(s)$$

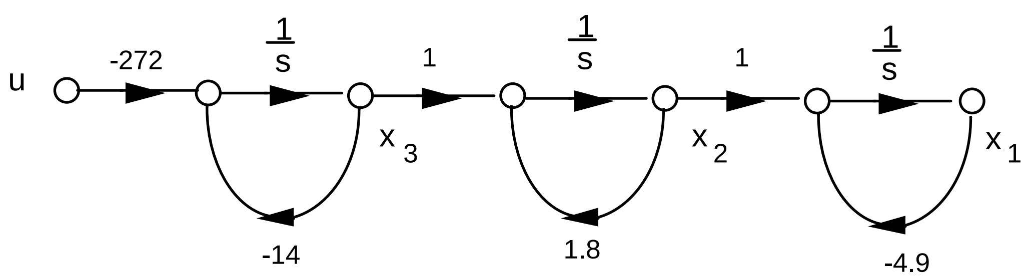
Rearranging,

$$C(s) = \frac{1}{s}\,[-272R(s) - 17.1C(s)] + \frac{1}{s^2}\,[-516.8R(s) - 34.58C(s)] + \frac{1}{s^3}\,[-22848R(s) + 123.48C(s)]$$

Drawing the signal-flow diagram, where r = u and y = c:



d. Draw signal-flow ignoring the polynomial in the numerator:



Write the state equations:

$$\dot{x}_1 = -4.9x_1 + x_2$$

$$\dot{x}_2 = 1.8x_2 + x_3$$

$$\dot{x}_3 = -14x_3 - 272u$$

The output equation is

$$y = \ddot{x}_1 + 1.9\dot{x}_1 + 84x_1 \tag{1}$$

But,

$$\dot{x}_1 = -4.9x_1 + x_2 \tag{2}$$

and

$$\ddot{x}_1 = -4.9\dot{x}_1 + \dot{x}_2 = -4.9(-4.9x_1 + x_2) + 1.8x_2 + x_3 \tag{3}$$

Substituting Eqs. (2) and (3) into (1) yields,

$$y = 98.7x_1 - 1.2x_2 + x_3$$

In vector-matrix form:

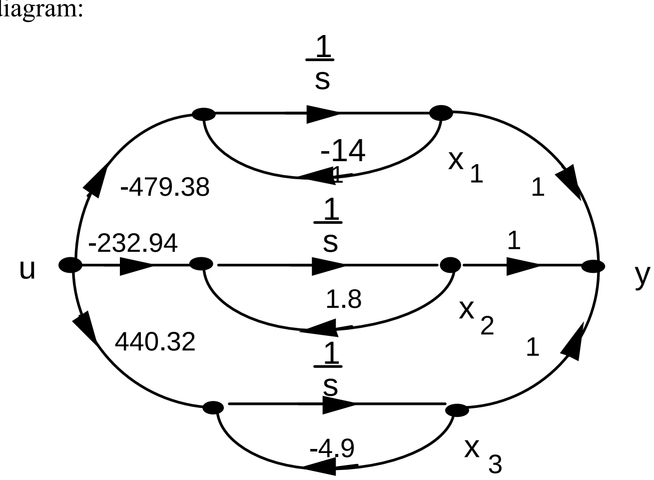
$$\dot{\mathbf{x}} = \begin{bmatrix} -4.9 & 1 & 0 \\ 0 & 1.8 & 1 \\ 0 & 0 & -14 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ -272 \end{bmatrix}u$$

$$y = \begin{bmatrix} 98.7 & -1.2 & 1 \end{bmatrix}\mathbf{x}$$

e. Expand as partial fractions:

$$G(s) = -479.38\frac{1}{s+14} - 232.94\frac{1}{s-1.8} + 440.32\frac{1}{s+4.9}$$

Draw signal-flow diagram:



Write state and output equations:

$$\dot{x}_1 = -14x_1 + -479.38u$$

$$\dot{x}_2 = 1.8x_2 - 232.94u$$

$$\dot{x}_3 = -4.9x_3 + 440.32u$$

$$y = x_1 + x_2 + x_3$$

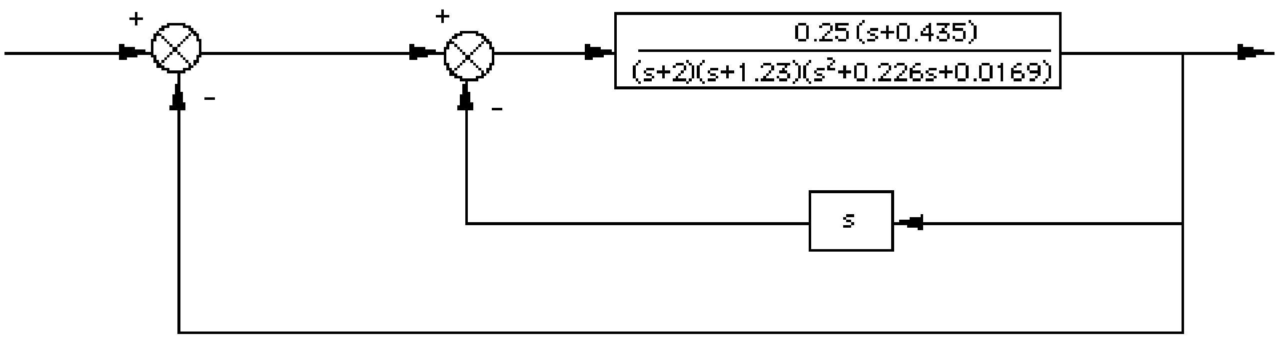
In vector-matrix form:

$$\dot{\mathbf{x}} = \begin{bmatrix} -14 & 0 & 0 \\ 0 & 1.8 & 0 \\ 0 & 0 & -4.9 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -479.38 \\ -232.94 \\ 440.32 \end{bmatrix}u$$

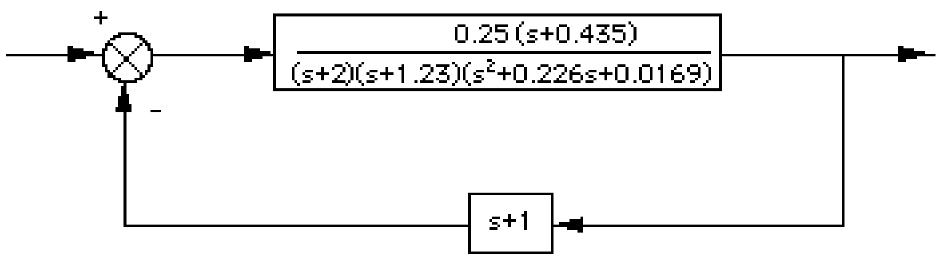
$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\mathbf{x}$$

54.

Push Pitch Gain to the right past the pickoff point.



Collapse the summing junctions and add the feedback transfer functions.



Apply the feedback formula and obtain,

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{0.25(s+0.435)}{s^4 + 3.4586s^3 + 3.4569s^2 + 0.9693s + 0.15032}$$

55.

Program:

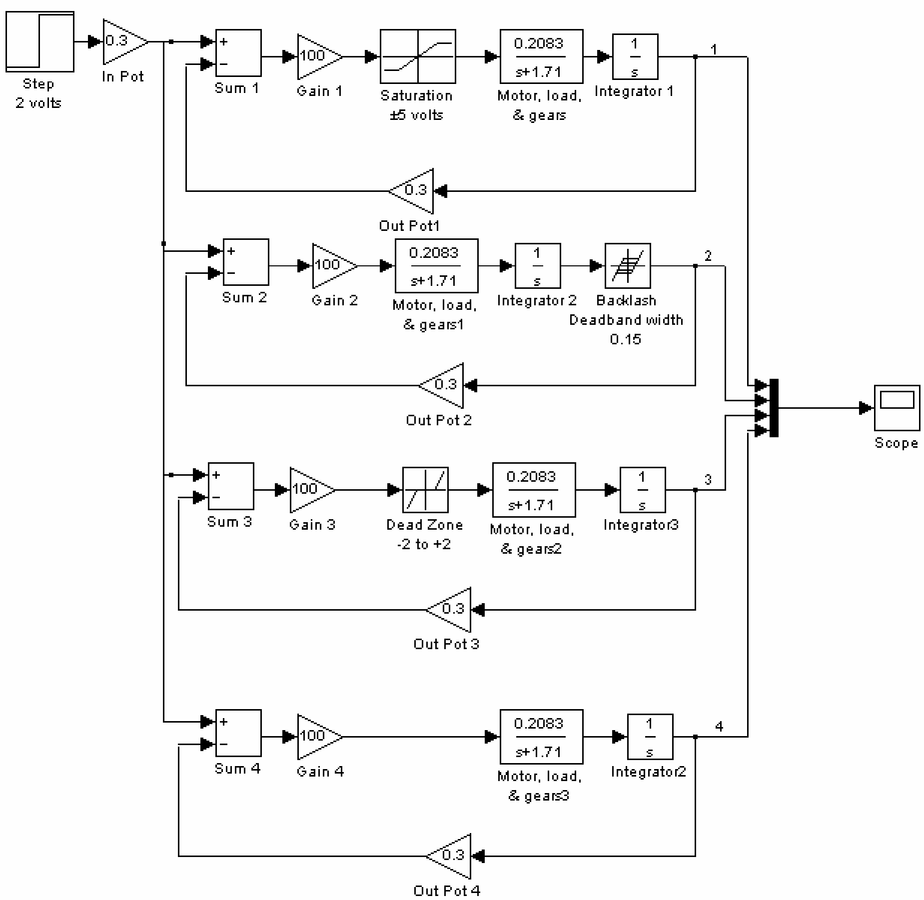
```
numg1=-0.125*[1 0.435]
deng1=conv([1 1.23],[1 0.226 0.0169])
'G1'
G1=tf(numg1,deng1)
'G2'
G2=tf(2,[1 2])
G3=-1
'H1'
H1=tf([-1 0],1)
'Inner Loop'
Ge=feedback(G1*G2,H1)
'Closed-Loop'
T=feedback(G3*Ge,1)
```

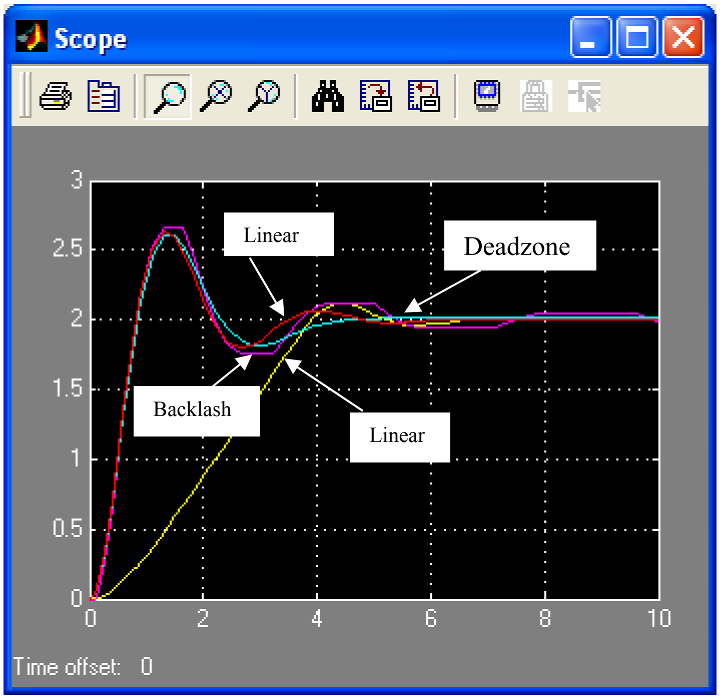
Computer response:

```
numg1 =

   -0.1250   -0.0544


deng1 =

    1.0000    1.4560    0.2949    0.0208


ans =

G1


Transfer function:
         -0.125 s - 0.05438
------------------------------------
s^3 + 1.456 s^2 + 0.2949 s + 0.02079


ans =

G2


Transfer function:
  2
-----
s + 2


G3 =

    -1


ans =

H1


Transfer function:
-s


ans =

Inner Loop


Transfer function:
                -0.25 s - 0.1088
------------------------------------------------
s^4 + 3.456 s^3 + 3.457 s^2 + 0.7193 s + 0.04157


ans =

Closed-Loop
```

```
Transfer function:
                0.25 s + 0.1088
-----------------------------------------------
s^4 + 3.456 s^3 + 3.457 s^2 + 0.9693 s + 0.1503
```

56.





57.

a. Since $V_L(s) = V_g(s) - V_R(s)$, the summing junction has $V_g(s)$ as the positive input and $V_R(s)$ as the negative input, and $V_L(s)$ as the error. Since $I(s) = V_L(s)\,(1/(Ls))$, $G(s) = 1/(Ls)$. Also, since $V_R(s) = I(s)R$, the feedback is $H(s) = R$. Summarizing, the circuit can be modeled as a negative feedback system, where $G(s) = 1/(Ls)$, $H(s) = R$, input $= V_g(s)$, output $= I(s)$, and error $= V_L(s)$, where the negative input to the summing junction is $V_R(s)$.

b. $T(s) = \dfrac{I(s)}{V_g(s)} = \dfrac{G(s)}{1+G(s)H(s)} = \dfrac{\frac{1}{Ls}}{1+\frac{1}{Ls}R} = \dfrac{1}{Ls+R}$. Hence, $I(s) = V_g(s)\dfrac{1}{Ls+R}$.

c. Using circuit analysis, $I(s) = \dfrac{V_g(s)}{Ls+R}$.

SOLUTIONS TO DESIGN PROBLEMS

58.

$J_e = J_a + J_L(\frac{1}{20})^2 = 2+2 = 4$; $D_e = D_a + D_L(\frac{1}{20})^2 = 2 + D_L(\frac{1}{20})^2$. Therefore, the forward-path transfer function is,

$G(s) = (1000)\left(\dfrac{\frac{1}{4}}{s(s+\frac{1}{4}(D_e+2))}\right)\left(\dfrac{1}{20}\right)$. Thus, $T(s) = \dfrac{G}{1+G} = \dfrac{\frac{25}{2}}{s^2+\frac{1}{4}(D_e+2)s+\frac{25}{2}}$.

Hence, $\zeta = \dfrac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = 0.456$; $\omega_n = \sqrt{\dfrac{25}{2}}$; $2\zeta\omega_n = \dfrac{D_e+2}{4}$. Therefore $D_e = 10.9$; from

which $D_L = 3560$.

59.

a. $T(s) = \dfrac{25}{s^2+s+25}$; from which, $2\zeta\omega_n = 1$ and $\omega_n = 5$. Hence, $\zeta = 0.1$. Therefore,

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}}x100 = 72.92\%$; $T_s = \dfrac{4}{\zeta\omega_n} = 8$.

b. $T(s) = \dfrac{25K_1}{s^2+(1+25K_2)s+25K_1}$; from which, $2\zeta\omega_n = 1+25K_2$ and $\omega_n = 5\sqrt{K_1}$. Hence,

$\zeta = \dfrac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = 0.404$. Also, $T_s = \dfrac{4}{\zeta\omega_n} = 0.2$, Thus, $\zeta\omega_n = 20$; from which $K_2 = \dfrac{39}{25}$ and

$\omega_n = 49.5$. Hence, $K_1 = 98.01$.

60.

The equivalent forward path transfer function is $G_e(s) = \dfrac{K}{s(1+(1+K_2))}$. Thus, $T(s) = \dfrac{G_e(s)}{1+G_e(s)} =$

$\dfrac{K}{s^2+(1+K_2)s+K}$. Prior to tachometer compensation ($K_2 = 0$), $T(s) = \dfrac{K}{s^2+s+K}$. Therefore $K = \omega_n^2 =$

100. Thus, after tachometer compensation, $T(s) = \dfrac{100}{s^2+(1+K_2)s+100}$. Hence, $\omega_n = 10$; $2\zeta\omega_n = 1+K_2$.

Therefore, $K_2 = 2\zeta\omega_n - 1 = 2(0.5)(10) - 1 = 9$.

61.

At the N_2 shaft, with rotation, $\theta_L(s)$

$$(J_{eq}s^2 + D_{eq}s)\theta_L(s) + F(s)r = T_{eq}(s)$$
$$F(s) = (Ms^2 + f_v s)X(s)$$

Thus,

$$(J_{eq}s^2 + D_{eq}s)\theta_L(s) + (Ms^2 + f_v s)X(s)r = T_{eq}(s)$$

But, $X(s) = r\theta_L(s)$. Hence,

$$\left[(J_{eq} + Mr^2)s^2 + (D_{eq} + f_v r^2)s\right]\theta_L(s) = T_{eq}(s)$$

where

$$J_{eq} = J_a(2)^2 + J = 5$$
$$D_{eq} = D_a(2)^2 + D = 4 + D$$
$$r = 2$$

Thus, the total load inertia and load damping is

$$J_L = J_{eq} + Mr^2 = 5 + 4M$$
$$D_L = D_{eq} + f_v r^2 = 4 + D + (1)(2)^2 = 8 + D$$

Reflecting J_L and D_L to the motor yields,

$$J_m = \frac{(5+4M)}{4};\ D_m = \frac{(8+D)}{4}$$

Thus, the motor transfer function is

$$\frac{\theta_m(s)}{E_a(s)} \frac{\frac{K_t}{R_a J_m}}{s(s+\frac{1}{J_m}(D_m + \frac{K_t K_a}{R_a}))} = \frac{\frac{1}{J_m}}{s(s+\frac{1}{J_m}(D_m+1))}$$

The gears are (10/20)(1) = 1/2. Thus, the forward-path transfer function is

$$G_e(s) = (500)\left(\frac{\frac{1}{J_m}}{s(s+\frac{1}{J_m}(D_m+1))}\right)\frac{1}{2}$$

Finding the closed-loop transfer function yields,

$$T(s) = \frac{G_e(s)}{1+G_e(s)} = \frac{250/J_m}{s^2 + \frac{D_m+1}{J_m}s + \frac{250}{J_m}}$$

For $T_s = 2$, $\frac{D_m+1}{J_m} = 4$. For 20% overshoot, $\zeta = 0.456$. Thus,

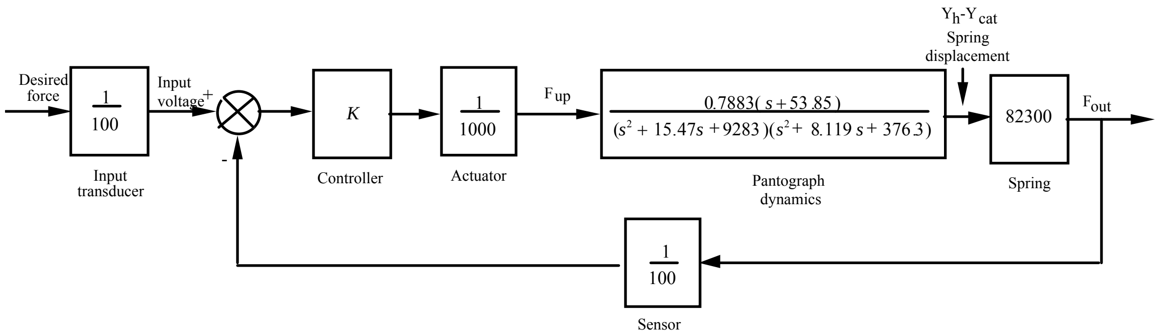
$$2\zeta\omega_n = 2(0.456)\omega_n = \frac{D_m+1}{J_m} = 4$$

Or, $\omega_n = 4.386 = \sqrt{\frac{250}{J_m}}$; from which $J_m = 13$ and hence, $D_m = 51$. But,

$J_m = \frac{(5+4M)}{4};\ D_m = \frac{(8+D)}{4}$. Thus, M = 11.75 and D = 196.

62.

a.



b. G(s) = $\dfrac{Y_h(s) - Y_{cat}(s)}{F_{up}(s)} = \dfrac{0.7883(s+53.85)}{(s^2+15.47s+9283)(s^2+8.119s+376.3)}$

$G_e(s)$ = (K/100)*(1/1000)*G(s)*82.3e3 = $\dfrac{648.7709\ (s+53.85)}{(s^2 + 8.119s + 376.3)\ (s\text{^}2 + 15.47s + 9283)}$

T(s) = Ge/(1+Ge) = $\dfrac{648.7709\ (s+53.85)}{(s\text{^}2 + 8.189s + 380.2)\ (s^2 + 15.4s + 9279)}$

$= \dfrac{648.8\ s + 3.494e04}{s^4 + 23.59\ s^3 + 9785\ s^2 + 8.184e04\ s + 3.528e06}$

c.

For G(s) = (yh-ycat)/Fup

Phase-variable form

Ap =

0	1	0	0
0	0	1	0
0	0	0	1
-3.493e6	-81190	- 9785	-23.59

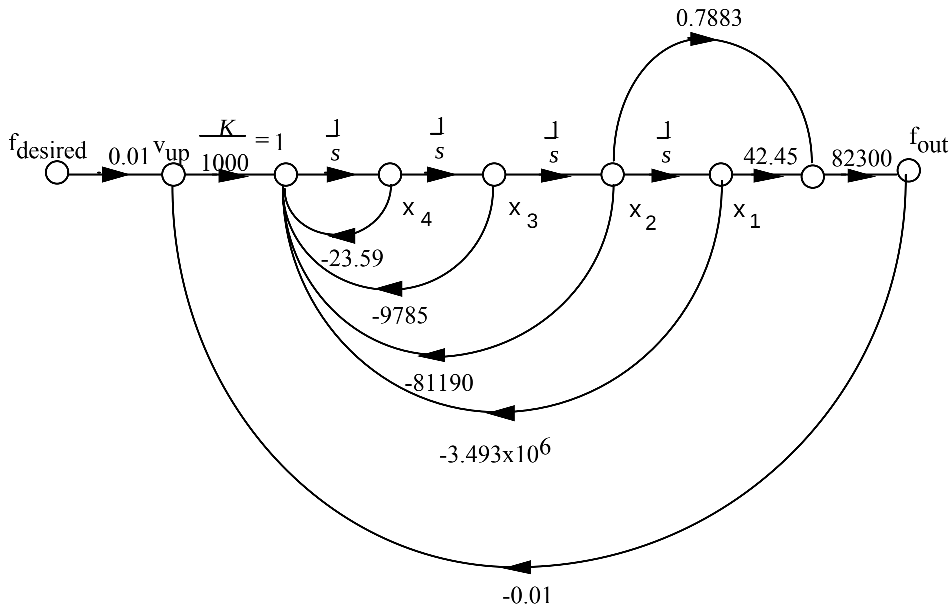
Bp =

0
0
0
1

Cp =

42.45 0.7883 0 0

Using this result to draw the signal-flow diagram,



Writing the state and output equations

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -23.59x_4 - 9785x_3 - 81190x_2 - 3493000x_1 + 0.01 f_{desired} - 0.01 f_{out}$$

But,

$$f_{out} = 42.45 * 82300x_1 + 0.7883 * 82300x_2$$

Substituting f_{out} into the state equations yields

$$\dot{x}_4 = -3527936.35x_1 - 81838.7709x_2 - 9785x_3 - 23.59x_4 + 0.01 f_{desired}$$

Putting the state and output equations into vector-matrix form.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3.528x10^6 & -81840 & -9785 & -23.59 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.01 \end{bmatrix} f_{desired}$$

$$y = f_{out} = \begin{bmatrix} 3494000 & 64880 & 0 & 0 \end{bmatrix} \mathbf{x}$$