S I X

Stability

SOLUTIONS TO CASE STUDIES CHALLENGES

Antenna Control: Stability Design via Gain

From the antenna control challenge of Chapter 5,

$$T(s) = \frac{76.39K}{s^3+151.32s^2+198s+76.39K}$$

Make a Routh table:

s^3	1	198
s^2	151.32	76.39K
s^1	$\frac{29961.36-76.39K}{151.32}$	0
s^0	76.39K	0

From the s^1 row, K<392.2. From the s^0 row, 0<K. Therefore, 0<K<392.2.

UFSS Vehicle: Stability Design via Gain

$$G_1 = \frac{-0.125(s+0.437)\cdot 2}{s(s+2)(s+1.29)(s+0.193)}$$

$$G_2 = \frac{G_1}{1+G_1(-s)}$$

$$G_2 = \frac{-0.25s-0.10925}{s^4+3.483s^3+3.465s^2+0.60719s}$$

$$G_3 = -K_1 G_2$$

$$G_3 = \frac{(0.25s+0.10925)K_1}{s^4+3.483s^3+3.465s^2+0.60719s}$$

$$T(s) = \frac{G_3(s)}{1+G_3(s)} = \frac{(0.25s+0.10925)K_1}{s^4+3.483s^3+3.465s^2+0.25(K_1+2.4288)s+0.10925K_1}$$

s^4	1	3.465	$0.10925K_1$
s^3	3.483	$0.25(K_1+2.4288)$	0
s^2	$\dfrac{-\frac{1}{4}(K_1 - 45.84)}{3.483}$	$0.10925K_1$	0
s^1	$0.25\,\dfrac{(K_1 + 4.2141)(K_1 - 26.42)}{K_1 - 45.84}$	0	0
s^0	$0.10925K_1$	0	0

For stability : $0 < K_1 < 26.42$

ANSWERS TO REVIEW QUESTIONS

1. Natural response

2. It grows without bound

3. It would destroy itself or hit limit stops

4. Sinusoidal inputs of the same frequency as the natural response yield unbounded responses even though the sinusoidal input is bounded.

5. Poles must be in the left-half-plane or on the jω axis.

6. The number of poles of the closed-loop transfer function that are in the left-half-plane, the right-half-plane, and on the jω axis.

7. If there is an even polynomial of second order and the original polynomial is of fourth order, the original polynomial can be easily factored.

8. Just the way the arithmetic works out

9. The presence of an even polynomial that is a factor of the original polynomial

10. For the ease of finding coefficients below that row

11. It would affect the number of sign changes

12. Seven

13. No; it could have quadrantal poles.

14. None; the even polynomial has 2 right-half-plane poles and two left-half-plane poles.

15. Yes

16. Det (s**I**-**A**) = 0

SOLUTIONS TO PROBLEMS

1.

s^5	1	5	1
s^4	3	4	3
s^3	3.667	0	0
s^2	4	3	0
s^1	-2.75	0	0
s^0	3	0	0

2 rhp; 3 lhp

2.

s^5	1	4	3
s^4	-1	-4	-2
s^3	ε	1	0
s^2	$\frac{1-4\varepsilon}{\varepsilon}$	-2	0
s^1	$\frac{2\varepsilon^2+1-4\varepsilon}{1-4\varepsilon}$	0	0
s^0	-2	0	0

3 rhp, 2 lhp

3.

s^5	1	3	2
s^4	-1	-3	-2
s^3	-2	-3	ROZ
s^2	-3	-4	
s^1	-1/3		
s^0	-4		

Even (4): 4 jω; Rest(1): 1 rhp; Total (5): 1 rhp; 4 jω

4.

s^4	1	8	15	
s^3	4	20	0	
s^2	3	15	0	
s^1	6	0	0	ROZ
s^0	15	0	0	

Even (2): 2 jω; Rest (2): 2 lhp; Total: 2 jω; 2 lhp

5.

s^6	1	-6	1	-6
s^5	1	0	1	
s^4	-6	0	-6	
s^3	-24	0	0	ROZ
s^2	ε	-6		
s^1	-144/ε	0		
s^0	-6			

Even (4): 2 rhp; 2 lhp; Rest (2): 1 rhp; 1 lhp; Total: 3 rhp; 3 lhp

6.

Program:

```
den=[1 1 -6 0 1 1 -6]
A=roots(den)
```

Computer response:

```
den =

     1     1    -6     0     1     1    -6


A =

  -3.0000
   2.0000
  -0.7071 + 0.7071i
  -0.7071 - 0.7071i
   0.7071 + 0.7071i
   0.7071 - 0.7071i
```

7.

Program:

```
%-det([si() si();sj() sj()])/sj()
                              %Template for use in each cell.
syms e                        %Construct a symbolic object for
                              %epsilon.
%%%%%%%%%%%%%%%%%%%%%%%%%%$$$$$$$$$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
s5=[1  4   3   0   0]         %Create s^5 row of Routh table.
%%%%%%%%%%%%%%%%%%%%%%%%%%%$$$$$$$$$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
s4=[-1   -4   -2   0   0]     %Create s^4 row of Routh table.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if -det([s5(1) s5(2);s4(1) s4(2)])/s4(1)==0
                              s3=[e...
 -det([s5(1) s5(3);s4(1) s4(3)])/s4(1)   0   0];
                              %Create s^3 row of Routh table
                              %if 1st element is 0.
else
    s3=[-det([s5(1) s5(2);s4(1) s4(2)])/s4(1)...
 -det([s5(1) s5(3);s4(1) s4(3)])/s4(1)   0   0];
                              %Create s^3 row of Routh table
                              %if 1st element is not zero.
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if -det([s4(1) s4(2);s3(1) s3(2)])/s3(1)==0
                              s2=[e ...
 -det([s4(1) s4(3);s3(1) s3(3)])/s3(1)   0   0];
                              %Create s^2 row of Routh table
                              %if 1st element is 0.
else
    s2=[-det([s4(1) s4(2);s3(1) s3(2)])/s3(1) ...
 -det([s4(1) s4(3);s3(1) s3(3)])/s3(1)   0   0];
                             %Create s^2 row of Routh table
                             %if 1st element is not zero.
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if -det([s3(1) s3(2);s2(1) s2(2)])/s2(1)==0
                              s1=[e ...
 -det([s3(1) s3(3);s2(1) s2(3)])/s2(1)   0   0];
                              %Create s^1 row of Routh table
                              %if 1st element is 0.
else
s1=[-det([s3(1) s3(2);s2(1) s2(2)])/s2(1) ...
 -det([s3(1) s3(3);s2(1) s2(3)])/s2(1)   0   0];
                              %Create s^1 row of Routh table
                              %if 1st element is not zero
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

s0=[-det([s2(1) s2(2);s1(1) s1(2)])/s1(1) ...
 -det([s2(1) s2(3);s1(1) s1(3)])/s1(1)   0   0];
                              %Create s^0 row of Routh table.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

's3'                          %Display label.
s3=simplify(s3);              %Simplify terms in s^3 row.
pretty(s3)                    %Pretty print s^3 row.
's2'                          %Display label.
s2=simplify(s2);              %Simplify terms in s^2 row.
pretty(s2)                    %Pretty print s^2 row.
's1'                          %Display label.
s1=simplify(s1);              %Simplify terms in s^1 row.
pretty(s1)                    %Pretty print s^1 row.
's0'                          %Display label.
s0=simplify(s0);              %Simplify terms in s^0 row.
pretty(s0)                    %Pretty print s^0 row.
```

Computer response:

```
s5 =

     1     4     3     0     0

s4 =

    -1    -4    -2     0     0

ans =

s3


                              [e    1    0    0]
ans =

s2


                         [   -1 + 4 e            ]
                         [- --------    -2   0   0]
                         [     e                 ]

ans =

s1


                       [      2                     ]
                       [  2 e  + 1 - 4 e            ]
                       [- --------------   0   0   0]
                       [      -1 + 4 e              ]

ans =

s0


                              [-2    0    0    0]
```

8.

$$T(s) = \frac{240}{s^4 + 10s^3 + 35s^2 + 50s + 264}$$

s4	1	35	264
s3	10	50	0
s2	30	264	0
s1	-38	0	0
s0	264	0	0

2 rhp, 2 lhp

9.

Program:

```
numg=240;
deng=poly([-1 -2 -3 -4]);
'G(s)'
G=tf(numg,deng)
'Poles of G(s)'
pole(G)
'T(s)'
```

```
T=feedback(G,1)
'Poles of T(s)'
pole(T)
```

Computer response:

```
ans =

G(s)

Transfer function:
               240
---------------------------------
s^4 + 10 s^3 + 35 s^2 + 50 s + 24

ans =

Poles of G(s)

ans =

   -4.0000
   -3.0000
   -2.0000
   -1.0000

ans =

T(s)

Transfer function:
               240
----------------------------------
s^4 + 10 s^3 + 35 s^2 + 50 s + 264

ans =

Poles of T(s)

ans =

  -5.3948 + 2.6702i
  -5.3948 - 2.6702i
   0.3948 + 2.6702i
   0.3948 - 2.6702i
```

System is unstable, since two closed-loop poles are in the right half-plane.

10.

$$T(s) = \frac{1}{4s^4 + 4s^2 + 1}$$

s4	4	4	1	
s3	16	8	0	ROZ
s2	2	1	0	
s1	4	0	0	ROZ
s0	1	0	0	

Even (4): 4 jω

11.

$$T(s) = \frac{84}{s^8 + 5s^7 + 12s^6 + 25s^5 + 45s^4 + 50s^3 + 82s^2 + 60s + 84}$$

s^8	1	12	45	82	84
s^7	1	5	10	12	
s^6	1	5	10	12	
s^5	3	10	10		ROZ
s^4	5	20	36		
s^3	-5	-29			
s^2	-1	4			
s^1	-49				
s^0	4				

Even (6): 2 rhp, 2 lhp, 2 jω; Rest (2): 0 rhp, 2 lhp, 0 jω ; Total: 2 rhp, 4 lhp, 2 jω

12.

$$T(s) = \frac{1}{2s^4 + 5s^3 + s^2 + 2s + 1}$$

s^4	2	1	1
s^3	5	2	0
s^2	1	5	
s^1	-23	0	
s^0	5		

Total: 2 lhp, 2 rhp

13.

$$T(s) = \frac{8}{s^7 - 2s^6 - s^5 + 2s^4 + 4s^3 - 8s^2 - 4s + 8}$$

s^7	1	-1	4	-4	
s^6	-2	2	-8	8	
s^5	-12	8	-16	0	ROZ
s^4	0.6667	-5.333	8	0	
s^3	-88	128	0	0	
s^2	-4.364	8	0	0	
s^1	-33.33	0	0	0	
s^0	8	0	0	0	

Even (6): 3 rhp, 3 lhp; Rest (1): 1 rhp; Total: 4 rhp, 3 lhp

14.

Program:

```
numg=8;
deng=[1 -2 -1 2 4 -8 -4 0];
'G(s)'
G=tf(numg,deng)
'T(s)'
T=feedback(G,1)
'Poles of T(s)'
pole(T)
```

Computer response:

```
ans =

G(s)


Transfer function:
                       8
-----------------------------------------------
s^7 - 2 s^6 - s^5 + 2 s^4 + 4 s^3 - 8 s^2 - 4 s


ans =

T(s)





Transfer function:
                         8
---------------------------------------------------
s^7 - 2 s^6 - s^5 + 2 s^4 + 4 s^3 - 8 s^2 - 4 s + 8


ans =
```

```
Poles of T(s)

ans =

  -1.0000 + 1.0000i
  -1.0000 - 1.0000i
  -1.0000
   2.0000
   1.0000 + 1.0000i
   1.0000 - 1.0000i
   1.0000
```

Thus, there are 4 rhp poles and 3 lhp poles.

15.

Even (6): 1 rhp, 1 lhp, 4 jω; Rest (1): 1 lhp; Total: 1 rhp, 2 lhp, 4 jω

16.

$$T(s) = \frac{18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s - 18}$$

s^5	1	-7	-18	
s^4	1	-7	-18	
s^3	4	-14	0	ROZ
s^2	-3.5	-18	0	
s^1	-34.57	0	0	
s^0	-18	0	0	

Even (4): 1 rhp, 1 lhp, 2 jω; Rest (1): 1 lhp; Total: 1 rhp, 2 lhp, 2 jω

17.

$G(s) = \dfrac{507}{s^4 + 3s^3 + 10s^2 + 30s + 169}$; $H(s) = \dfrac{1}{s}$. Therefore,

$$T(s) = \frac{G}{1 + GH} = \frac{507s}{s^5 + 3s^4 + 10s^3 + 30s^2 + 169s + 507}$$

s^5	1	10	169	
s^4	3	30	507	
s^3	12	60	0	ROZ
s^2	15	507	0	
s^1	-345.6	0	0	

s^0	507	0	0	

Even (4): 2 rhp, 2 lhp, 0 jω; Rest (1): 0 rhp, 1 lhp, 0 jω; Total (5): 2 rhp, 3 lhp, 0 jω

18.

$T(s) = \dfrac{K(s^2+1)}{(1+K)s^2 + 3s + (2+K)}$. For a second-order system, if all coefficients are positive, the roots will be in the lhp. Thus, $K > -1$.

19.

$$T(s) = \frac{K(s+6)}{s^3 + 4s^2 + (K+3)s + 6K}$$

s^3	1	$3 + K$
s^2	4	$6K$
s^1	$3 - \frac{1}{2}K$	0
s^0	$6K$	0

Stable for $0 < K < 6$

20.

$$T(s) = \frac{K(s+3)(s+5)}{(1+K)s^2 + (8K-6)s + (8+15K)}$$

			For 1st column negative	For 1st column positive
s^2	$1+K$	$8+15K$	$K < -1$	$K > -1$
s^1	$8K-6$	0	$K < 6/8$	$K > 6/8$
s^0	$8+15K$	0	$K < -8/15$	$K > -8/15$

Stable for $K > 6/8$

21.

Program:
```
K=[-6:0.00005:0];
for i=1:length(K);
dent=[(1+K(i)) (8*K(i)-6) (8+15*K(i))];
R=roots(dent);
A=real(R);
B=max(A);
if B>0
R
K=K(i)
break
end
end
K=[6:-0.00005:0];
for i=1:length(K);
dent=[(1+K(i)) (8*K(i)-6) (8+15*K(i))];
R=roots(dent);
```

```
A=real(R);
B=max(A);
if B>0
R
K=K(i)
break
end
end
```

Computer response:

```
R =

  1.0e+005 *

    2.7999
   -0.0000


K =

   -1.0000


R =

   0.0001 + 3.3166i
   0.0001 - 3.3166i


K =

    0.7500
```

22.

Program:

```
%-det([si() si();sj() sj()])/sj()
                              %Template for use in each cell.
syms K                        %Construct a symbolic object for
                              %gain, K.
s2=[(1+K)   (8+15*K) 0];      %Create s^2 row of Routh table.
s1=[(8*K-6)  0 0];            %Create s^1 row of Routh table.
s0=[-det([s2(1) s2(2);s1(1) s1(2)])/s1(1)...
 -det([s2(1) s2(3);s1(1) s1(3)])/s1(1)   0   0];
                              %Create s^0 row of Routh table.
's2'                          %Display label.
s2=simplify(s2);              %Simplify terms in s^1 row.
pretty(s2)                    %Pretty print s^1 row.
's1'                          %Display label.
s1=simplify(s1);              %Simplify terms in s^1 row.
pretty(s1)                    %Pretty print s^1 row.
's0'                          %Display label.
s0=simplify(s0);              %Simplify terms in s^0 row.
pretty(s0)                    %Pretty print s^0 row.
```

Computer response:

```
ans =

s2


                           [1 + K    8 + 15 K    0]

ans =
```

```
s1

                        [8 K - 6    0    0]

ans =

s0

                    [8 + 15 K    0    0    0]
```

23.

$T(s)=\dfrac{K(s+2)(s-2)}{(K+1)s^2+(3-4K)}$. For positive coefficients in the denominator, $-1<K<\dfrac{3}{4}$. Hence

marginal stability only for this range of K.

24.

$T(s)=\dfrac{K(s+1)}{s^5+2s^4+Ks+K}$. Always unstable since s^3 and s^2 terms are missing.

25.

$$T(s)=\frac{K(s-2)(s+4)(s+5)}{Ks^3+(7K+1)s^2+2Ks+(3-40K)}$$

s^3	K	2K
s^1	$\dfrac{54K^2-K}{7K+1}$	0
s^0	3-40K	0

For stability, $\dfrac{1}{54}<K<\dfrac{3}{40}$

26.

$T(s)=\dfrac{K(s+2)}{s^4+3s^3-3s^2+(K+3)s+(2K-4)}$

s^4	1	- 3	2K - 4
s^3	3	K+ 3	0
s^2	$\dfrac{-(K+12)}{3}$	2K - 4	0
s^1	$\dfrac{K(K+33)}{K+12}$	0	0
s^0	2K - 4	0	0

Conditions state that K < -12, K > 2, and K > -33. These conditions cannot be met simultaneously. System is not stable for any value of K.

27.

$$T(s)=\frac{K}{s^3+80s^2+2001s+(K+15390)}$$

s^3	1	2001
s^2	80	K+15390
s^1	$-\frac{1}{80}K+\frac{14469}{8}$	0
s^0	K+15390	0

There will be a row of zeros at s^1 row if K = 144690. The previous row, s^2, yields the auxiliary equation, $80s^2+(144690+15390)=0$. Thus, s = ± j44.73. Hence, K = 144690 yields an oscillation of 44.73 rad/s.

28.

$$T(s)=\frac{Ks^4-Ks^2+2Ks+2K}{(K+1)s^2+2(1-K)s+(2K+1)}$$

Since all coefficients must be positive for stability in a second-order polynomial, $-1<K<\infty$; $-\infty<K<1$; $-1<2K<\infty$. Hence, $-\frac{1}{2}<K<1$.

29.

$$T(s)=\frac{(s+2)(s+7)}{s^4+11s^3+(K+31)s^2+(8K+21)s+12K}$$

Making a Routh table,

s^4	1	$K+31$	$12K$
s^3	11	$8K+21$	0
s^2	$\frac{3K+320}{11}$	$12K$	0
s^1	$\frac{24K^2+1171K+6720}{3K+320}$	0	0
s^0	$12K$	0	0

s^2 row says $-106.7<K$. s^1 row says $K<-42.15$ and $-6.64<K$. s^0 row says $0<K$.

30.

$$T(s)=\frac{K(s+4)}{s^3+3s^2+(2+K)s+4K}$$

Making a Routh table,

s^3	1	$2+K$
s^2	3	$4K$
s^1	$6 - K$	0
s^0	$4K$	0

a. For stability, $0 < K < 6$.

b. Oscillation for K = 6.

c. From previous row with $K = 6$, $3s^2 + 24 = 0$. Thus $s = \pm j\sqrt{8}$, or $\omega = \sqrt{8}$ rad/s.

31.

a. $G(s)=\dfrac{K(s-1)(s-2)}{(s+2)(s^2+2s+2)}$. Therefore, $T(s)=\dfrac{(s-2)(s-1)K}{s^3+(K+4)s^2+(6-3K)s+2(K+2)}$.

Making a Routh table,

s^3	1	6-3K
s^2	4+K	4+2K
s^1	$\dfrac{-(3K^2+8K-20)}{K+4}$	0
s^0	4+2K	0

From s^1 row: K = 1.57, -4.24; From s^2 row: $-4 < K$; From s^0 row: $-2 < K$. Therefore, $-2 < K < 1.57$.

b. If K = 1.57, the previous row is $5.57s^2 + 7.14$. Thus, $s = \pm j1.13$.

c. From part b, $\omega = 1.13$ rad/s.

32.

Applying the feedback formula on the inner loop and multiplying by K yields

$$G_e(s)=\frac{K}{s(s^2+5s+7)}$$

Thus,

$$T(s)=\frac{K}{s^3+5s^2+7s+K}$$

Making a Routh table:

s^3	1	7
s^2	5	K
s^1	$\frac{35 - K}{5}$	0
s^0	K	0

For oscillation, the s^1 row must be a row of zeros. Thus, K = 35 will make the system oscillate. The previous row now becomes, $5s^2 + 35$. Thus, $s^2 + 7 = 0$, or $s = \pm j\sqrt{7}$. Hence, the frequency of oscillation is $\sqrt{7}$ rad/s.

33.

$$T(s) = \frac{Ks^2 + 2Ks}{s^3 + (K-1)s^2 + (2K-4)s + 24}$$

s^3	1	2K-4
s^2	K-1	24
s^1	$\frac{2K^2 - 6K - 20}{K - 1}$	0
s^0	24	0

For stability, K > 5; Row of zeros if K = 5. Therefore, $4s^2 + 24 = 0$. Hence, $\omega = \sqrt{6}$ for oscillation.

34.

Program:

```
K=[0:0.001:200];
for i=1:length(K);
deng=conv([1 -4 8],[1 3]);
numg=[0 K(i) 2*K(i) 0];
dent=numg+deng;
R=roots(dent);
A=real(R);
B=max(A);
if B<0
R
K=K(i)
break
end
end
```

Computer response:

```
R =

  -4.0000
  -0.0000 + 2.4495i
```

```
  -0.0000 - 2.4495i


K =

     5
```

a. From the computer response, (a) the range of K for stability is $0 < K < 5$.

b. The system oscillates at K = 5 at a frequency of 2.4494 rad/s as seen from R, the poles of the closed-loop system.

35.

$$T(s) = \frac{K(s+2)}{s^4 + 3s^3 - 3s^2 + (K+3)s + (2K-4)}$$

s^4	1	- 3	2K-4
s^3	3	K+3	0
s^2	$-\frac{K+12}{3}$	2K-4	0
s^1	$\frac{K(K+33)}{K+12}$	0	0
s^0	2K-4	0	0

For $K < -33$: 1 sign change; For $-33 < K < -12$: 1 sign change; For $-12 < K < 0$: 1 sign change; For $0 < K < 2$: 3 sign changes; For $K > 2$: 2 sign changes. Therefore, $K > 2$ yields two right-half-plane poles.

36.

$$T(s) = \frac{K}{s^4 + 7s^3 + 15s^2 + 13s + (4+K)}$$

s^4	1	15	K+4
s^3	7	13	0
s^2	$\frac{92}{7}$	K+4	0
s^1	$\frac{1000-49K}{92}$	0	0
s^0	K+4	0	0

a. System is stable for $-4 < K < 20.41$.

b. Row of zeros when K = 20.41. Therefore, $\frac{92}{7}s^2 + 24.41$. Thus, $s = \pm j1.3628$, or $\omega = 1.3628$ rad/s.

37.

$$T(s) = \frac{K}{s^3 + 14s^2 + 45s + (K+50)}$$

s^3	1	45
s^2	14	K+50
s^1	$\frac{580-K}{14}$	0
s^0	K+50	0

a. System is stable for $-50 < K < 580$.

b. Row of zeros when K = 580. Therefore, $14s^2 + 630$. Thus, $s = \pm j\sqrt{45}$, or $\omega = 6.71$ rad/s.

38.

$$T(s) = \frac{K}{s^4 + 8s^3 + 17s^2 + 10s + K}$$

s^4	1	17	K
s^3	8	10	0
s^2	$\frac{126}{8}$	K	0
s^1	$-\frac{32}{63}K + 10$	0	0
s^0	K	0	0

a. For stability $0 < K < 19.69$.

b. Row of zeros when K = 19.69. Therefore, $\frac{126}{8}s^2 + 19.69$. Thus, $s = \pm j\sqrt{1.25}$, or

$\omega = 1.118$ rad/s.

c. Denominator of closed-loop transfer function is $s^4 + 8s^3 + 17s^2 + 10s + K$. Substituting K = 19.69 and solving for the roots yield $s = \pm j1.118$, -4.5, and -3.5.

39.

$$T(s) = \frac{K(s^2 + 2s + 1)}{s^3 + 2s^2 + (K+1)s - K}$$

s^3	1	K+1
s^2	2	- K
s^1	$\frac{3K+2}{2}$	0
s^0	- K	0

Stability if $-\frac{2}{3} < K < 0$.

40.

$$T(s) = \frac{2s^4 + (K+2)s^3 + Ks^2}{s^3 + s^2 + 2s + K}$$

s^3	1	2
s^2	1	K
s^1	2 - K	0
s^0	K	0

Row of zeros when K = 2. Therefore $s^2 + 2$ and $s = \pm j\sqrt{2}$, or ω = 1.414 rad/s. Thus K = 2 will yield the even polynomial with 2 jω roots and no sign changes.

41.

	1	K_2	1
s^3	K_1	5	0
s^2	$\frac{K_1K_2 - 5}{K_1}$	1	0
s^1	$\frac{K_1^2 - 5K_1K_2 + 25}{5 - K_1K_2}$	0	0
s^0	1	0	0

For stability, $K_1K_2 > 5$; $K_1^2 + 25 < 5K_1K_2$; and $K_1 > 0$. Thus $0 < K_1^2 < 5K_1K_2 - 25$, or $0 < K_1 < \sqrt{5K_1K_2 - 25}$.

42.

s^4	1	1	1
s^3	K_1	K_2	0
s^2	$\frac{K_1 - K_2}{K_1}$	1	0
s^1	$\frac{K_1^2 - K_1K_2 + K_2^2}{K_2 - K_1}$	0	0
s^0	1	0	0

For two jω poles, $K_1^2 - K_1K_2 + K_2^2 = 0$. However, there are no real roots. Therefore, there is no relationship between K_1 and K_2 that will yield just two jω poles.

43.

s^8	1	1.18E+03	2.15E+03	-1.06E+04	-415
s^7	103	4.04E+03	-8.96E+03	-1.55E+03	0
s^6	1140.7767	2236.99029	-10584.951	-415	0
s^5	3838.02357	-8004.2915	-1512.5299	0	0
s^4	4616.10784	-10135.382	-415	0	0
s^3	422.685462	-1167.4817	0	0	0
s^2	2614.57505	-415	0	0	0
s^1	-1100.3907	0	0	0	0
s^0	-415	0	0	0	0

a. From the first column, 1 rhp, 7 lhp, 0 jω.

b. G(s) is not stable because of 1 rhp.

44.

Eigenvalues are the roots of the following equation:

$$|s\mathbf{I}-\mathbf{A}| = \left\| \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 3 \\ 2 & 2 & -4 \\ 1 & -4 & 3 \end{bmatrix} \right\| = \left\| \begin{bmatrix} s & -1 & -3 \\ -2 & s-2 & 4 \\ -1 & 4 & s-3 \end{bmatrix} \right\| = s^3 - 5s^2 - 15s + 40$$

Hence, eigenvalues are -3.2824, 1.9133, 6.3691. Therefore, 1 rhp, 2 lhp, 0 jω.

45.

Program:

```
A=[0 1 0;0 1 -4;-1 1 3];
eig(A)
```

Computer response:

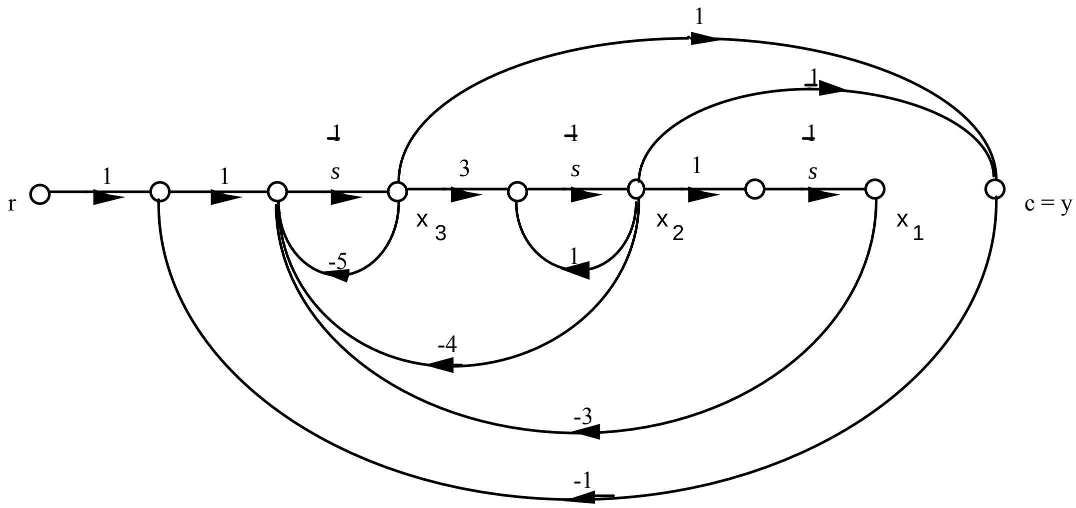
```
ans =

   1.0000
   1.5000 + 1.3229i
   1.5000 - 1.3229I
```

46.

Writing the open-loop state and output equations we get,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_2 + 3x_3$$
$$\dot{x}_3 = -3x_1 - 4x_2 - 5x_3 + u$$
$$y = x_2 + x_3$$

Drawing the signal-flow diagram and including the unity feedback path yields,



Writing the closed-loop state and output equations from the signal-flow diagram,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_2 + 3x_3$$
$$\begin{aligned}\dot{x}_3 &= -3x_1 - 4x_2 - 5x_3 + r - c \\ &= -3x_1 - 4x_2 - 5x_3 + r - (x_2 + x_3) \\ &= -3x_1 - 5x_2 - 6x_3 + r\end{aligned}$$
$$y = x_2 + x_3$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 3 \\ -3 & -5 & -6 \end{bmatrix}\mathbf{X} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}\mathbf{x}$$

Now, find the characteristic equation.

$$|s\mathbf{I}-A| = \left|\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 3 \\ -3 & -5 & -6 \end{bmatrix}\right| = \begin{bmatrix} s & -1 & 0 \\ 0 & (s-1) & -3 \\ 3 & 5` & (s+6) \end{bmatrix}$$

$$= s^3 + 5s^2 + 9s + 9$$

Forming a Routh table to determine stability

s^3	1	9
s^2	5	9
s^1	$\frac{36}{5}$	0
s^0	9	0

Since there are no sign changes, the closed-loop system is stable.

47.

Program:

```
A=[0,1,0;0,1,3;-3,-4,-5];
B=[0;0;1];
C=[0,1,1];
D=0;
'G'
G=ss(A,B,C,D)
'T'
T=feedback(G,1)
'Eigenvalues of T'
ssdata(T);
eig(T)
```

Computer response:

```
ans =

G


a =
        x1  x2  x3
   x1    0   1   0
   x2    0   1   3
   x3   -3  -4  -5


b =
        u1
   x1    0
   x2    0
   x3    1


c =
        x1  x2  x3
   y1    0   1   1


d =
        u1
```

```
   y1   0

Continuous-time model.

ans =

T


a =
         x1  x2  x3
   x1    0   1   0
   x2    0   1   3
   x3   -3  -5  -6


b =
         u1
   x1    0
   x2    0
   x3    1


c =
         x1  x2  x3
   y1    0   1   1


d =
         u1
   y1    0

Continuous-time model.

ans =

Eigenvalues of T


ans =

  -1.0000 + 1.4142i
  -1.0000 - 1.4142i
  -3.0000
```

SOLUTIONS TO DESIGN PROBLEMS

48.

$$T(s) = \frac{K(s+1)(s+10)}{s^3 + (5.45+K)s^2 + (11.91+11K)s + (43.65+10K)}$$

s^3	1	11.91+11K
s^2	5.45+K	43.65+10K
s^1	$\frac{11K^2 + 61.86K + 21.26}{5.45+K}$	0
s^0	43.65+10K	0

For stability, - 0.36772 < K < ∞. Stable for all positive K.

49.

$$T(s) = \frac{0.7K(s+0.1)}{s^4 + 2.2s^3 + 1.14s^2 + 0.193s + (0.07K+0.01)}$$

s^4	1	1.14	0.07K+0.01
s^3	2.2	0.193	0
s^2	1.0523	0.07K+0.01	0
s^1	0.17209 - 0.14635K	0	0
s^0	0.07K+0.01	0	0

For stability, - 0.1429 < K < 1.1759

50.

$$T(s) = \frac{0.6K + 10Ks^2 + 60.1Ks}{s^5 + 130s^4 + 3229s^3 + 10(K+2348)s^2 + (60.1K + 58000)s + 0.6K}$$

s^5	1	3229	60.1K+58000
s^4	130	10K+23480	0.6K
s^3	-10K+396290	7812.4K+7540000	0
s2	$\frac{-100K^2+2712488K+8.3247E9}{-10K+396290}$	0.6K	0
s^1	$\frac{7813E3K^4-5.1401E11K^3+7.2469E15K^2+3.3213E19K+2.4874E22}{1000K^3-66753880K^2+9.9168E11K+3.299E15}$	0	0
s^0	0.6K	0	0

Note: s^3 row was multiplied by 130

From s^1 row after canceling common roots:

$$\frac{7813000(K-39629)(K+967.31586571671)(K+2776.9294183336)(K-29908.070615165)}{1000(K-39629)(K+2783.405672635)(K-29908.285672635)}$$

From s^0 row: K > 0

From s^3 row: K < 39629

From s^2 row: K < 29908.29; 39629 < K

From s^1 row: 29908.29 < K, or K < 29908.07;

Therefore, for stability, 0 < K < 29908.07

51.

s^5	1	1311.2	1000(100K+1)
s^4	112.1	10130	60000K
s^3	1220.8	99465K+1000	0
s^2	10038-9133.4K	60000K	0
s^1	$99465\dfrac{(K+0.010841)(K-1.0192)}{(K-1.0991)}$	0	0
s^0	60000K	0	0

From s^2 row: K < 1.099

From s^1 row: -0.010841 < K < 1.0192; K > 1.0991

From s^0 row: 0 < K

Therefore, 0 < K < 1.0192

52.

Find the closed-loop transfer function.

$$G(s) = \frac{63\text{x}10^6 K}{(s+30)(s+140)(s+2.5)}$$

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{63\text{x}10^6 K}{s^3+172.5s^2+4625s+(10500+63\text{x}10^6 K)}$$

Make a Routh table.

s^3	1	4625
s^2	172.5	$10500+63\text{x}10^6K$
s^1	$4564.13-365217.39K$	0
s^0	$10500+63\text{x}10^6K$	0

The s^1 line says $K < 1.25\text{x}10^{-2}$ for stability. The s^0 line says $K > -1.67\text{x}10^{-4}$ for stability.

Hence, $-1.67\text{x}10^{-4} < K < 1.25\text{x}10^{-2}$ for stability.

53.

Find the closed-loop transfer function.

$$G(s) = \frac{7570Kp(s+103)(s+0.8)}{s(s+62.61)(s-62.61)}$$

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{7570Kp(s+103)(s+0.8)}{s^3 + 7570K_p s^2 + (785766K_p - 3918.76)s + 623768K_p}$$

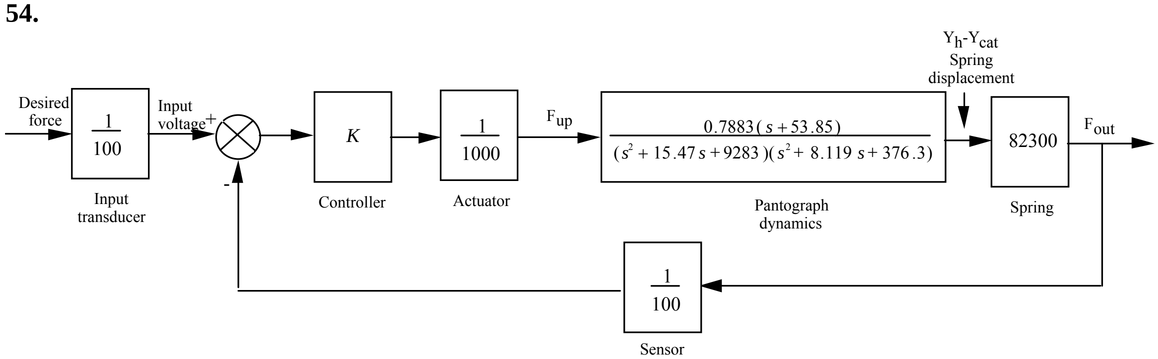
Make a Routh table.

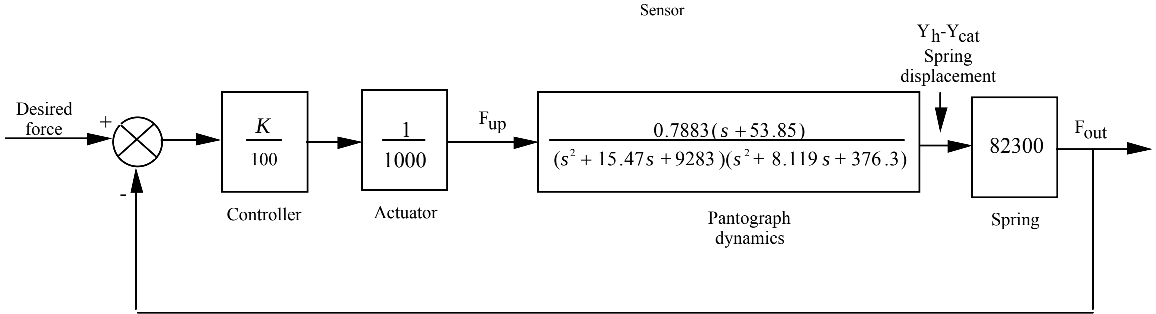
s^3	1	$785766K_p - 3918.76$
s^2	7570	$623768K_p$
s^1	$785766K_p - 4001.16$	0
s^0	$623768K_p$	0

The s^1 line says $K_p > 5.09\text{x}10^{-3}$ for stability. The s^0 line says $K_p > 0$ for stability.

Hence, $K_p > 5.09\text{x}10^{-3}$ for stability.

54.





$$\text{G(s)} = \frac{Y_h(s) - Y_{cat}(s)}{F_{up}(s)} = \frac{0.7883(s+53.85)}{(s^2 + 15.47s + 9283)(s^2 + 8.119s + 376.3)}$$

Ge(s)=(K/100)*(1/1000)*G(s)*82.3e3

$$G_e(s) = \frac{0.6488K\,(s+53.85)}{(s^2 + 8.119s + 376.3)\,(s^2 + 15.47s + 9283)}$$

$$T(s) = \frac{0.6488K\,(s+53.85)}{s^4 + 23.589\,s^3 + 9784.90093\,s^2 + (0.6488\,K + 81190.038)s + (34.94\,K + 0.34931929\;10^7)}$$

s^4	1	9785	(0.3493e7+34.94K)	+
s^3	23.59	(0.6488K+81190)	0	+
s^2	(-0.0275K+6343)	(0.3493e7+34.94K)	0	K<230654
s^1	$\frac{-0.0178K^2+10587K+432.59e6}{-.0275K+6343}$		0	-128966<K<188444
s^0	(0.3493e7+34.94K)		0	-99971<K

The last column evaluates the range of *K* for stability for each row. Therefore –99971 < K < 188444.