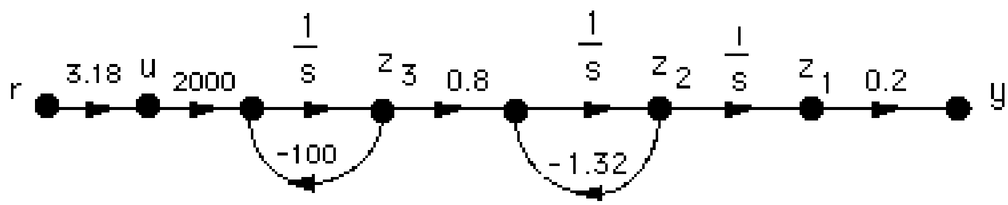
TWELVE

Design via State Space

SOLUTION TO CASE STUDY CHALLENGE

Antenna Control: Design of Controller and Observer

a. We first draw the signal-flow diagram of the plant using the physical variables of the system as state variables.



Writing the state equations for the physical variables shown in the signal-flow diagram, we obtain

$$\dot{\mathbf{z}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1.32 & 0.8 \\ 0 & 0 & -100 \end{bmatrix}\mathbf{z} + \begin{bmatrix} 0 \\ 0 \\ 2000 \end{bmatrix}u \; ; \quad y = \begin{bmatrix} 0.2 & 0 & 0 \end{bmatrix}\mathbf{z}$$

The characteristic polynomial for this system is $s^3 + 101.32s^2 + 132s + 0$. Hence, the **A** and **B** matrices of the phase-variable form are

Ax			Bx
0	1	0	0
0	0	1	0
0	-132	-101.32	1

Writing the controllability matrices and their determinants for both systems yields

CMz	Controllability Matrix of z		**CMx**	Controllability Matrix of x	
0	0	1600	0	0	1
0	1600	-162112	0	1	-101.32
2000	-200000	20000000	1	-101.32	10133.7424
Det(CMz)	-5.12E+09		**Det(CMx)**	-1	

where the system is controllable. Using Eq. (12.39), we find the transformation matrix and its inverse to be

P	Transformation Matrix z=Px		**PINV**		
1600	0	0	0.000625	0	0
0	1600	0	0	0.000625	0
0	2640	2000	0	-0.000825	0.0005

The characteristic polynomial of the phase-variable system with state feedback is

$$s^3 + (k_3 + 101.32)s^2 + (k_2 + 132)s + (k_1 + 0)$$

For 15% overshoot, $T_s = 2$ seconds, and a third pole 10 times further from the imaginary axis than the dominant poles, the characteristic polynomial is

$$(s + 20)(s^2 + 4s + 14.969) = s^3 + 24s^2 + 94.969s + 299.38$$

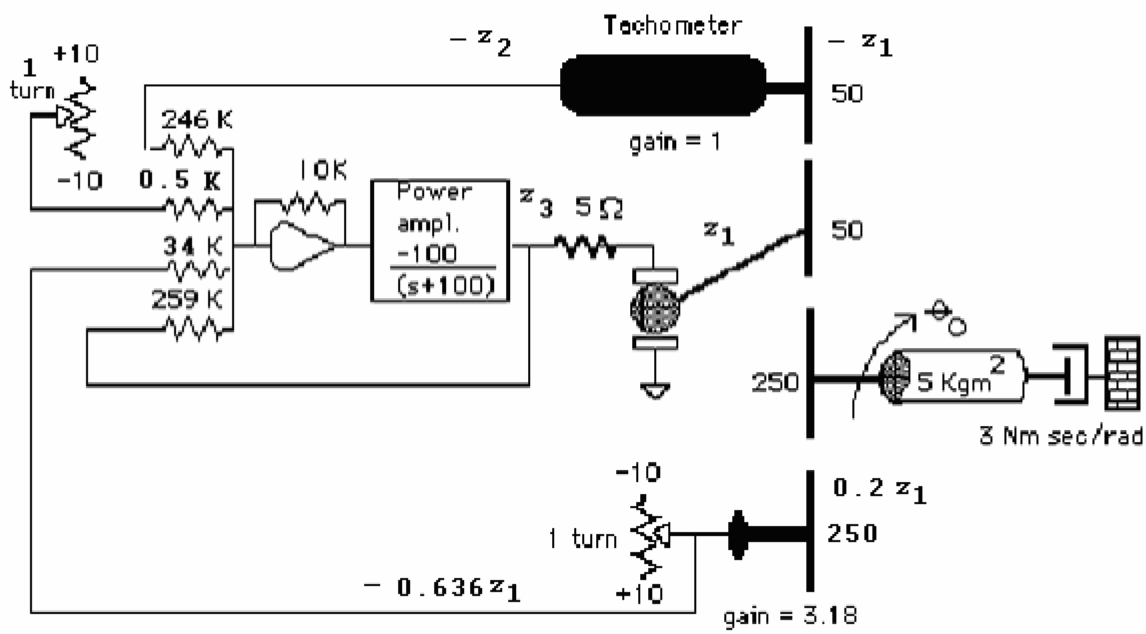
Equating coefficients, the controller for the phase-variable system is

Kx	Controller for x	
299.38	-37.031	-77.32

Using Eq. (12.42), the controller for the original system is

Kz	Controller for z	
0.1871125	0.04064463	-0.03866

b. Using $\mathbf{K_z}$, gain from θ_m = - 0.1871125 (including gear train, pot, and operational amplifier); gain from tachometer = - 0.04064463; and gain from power amplifier output = 0.03866.



c. Using the original system from part (a) and its characteristic polynomial, we find the observer canonical form which has the following **A** and **C** matrices:

Ax		
-101.32	1	0
-132	0	1
0	0	0
Cx		
1	0	0

Writing the observability matrices and their determinants for both systems yields

OMz	Observability Matrix of z		**OMx**	Observability Matrix of x	
0.2	0	0	1	0	0
0	0.2	0	-101.32	1	0
0	-0.264	0.16	10133.7424	-101.32	1
Det(OMz)	0.0064		**Det(OMx)**	1	

where the system is observable. Using Eq. (12.89), we find the transformation matrix and its inverse to be

P	Transformation Matrix z=Px		**PINV**		
5	0	0	0.20	0.00	0.00
-506.6	5	0	20.26	0.20	0.00
62500	-625	6.25	26.40	20.00	0.16

The characteristic polynomial of the dual phase-variable system with state feedback is

$$s^3 + (l_1 + 101.32)s^2 + (l_2 + 132)s + (l_3 + 0)$$

For 10% overshoot, $\omega_n = 10\sqrt{14.969} = 38.69$ rad/s, and a third pole 10 times further from the imaginary axis than the dominant observer poles, the characteristic polynomial is

$$(s + 228.72)(s^2 + 45.743s + 1496.916) = s^3 + 274.46s^2 + 11959s + 3.4237\text{x}10^5$$

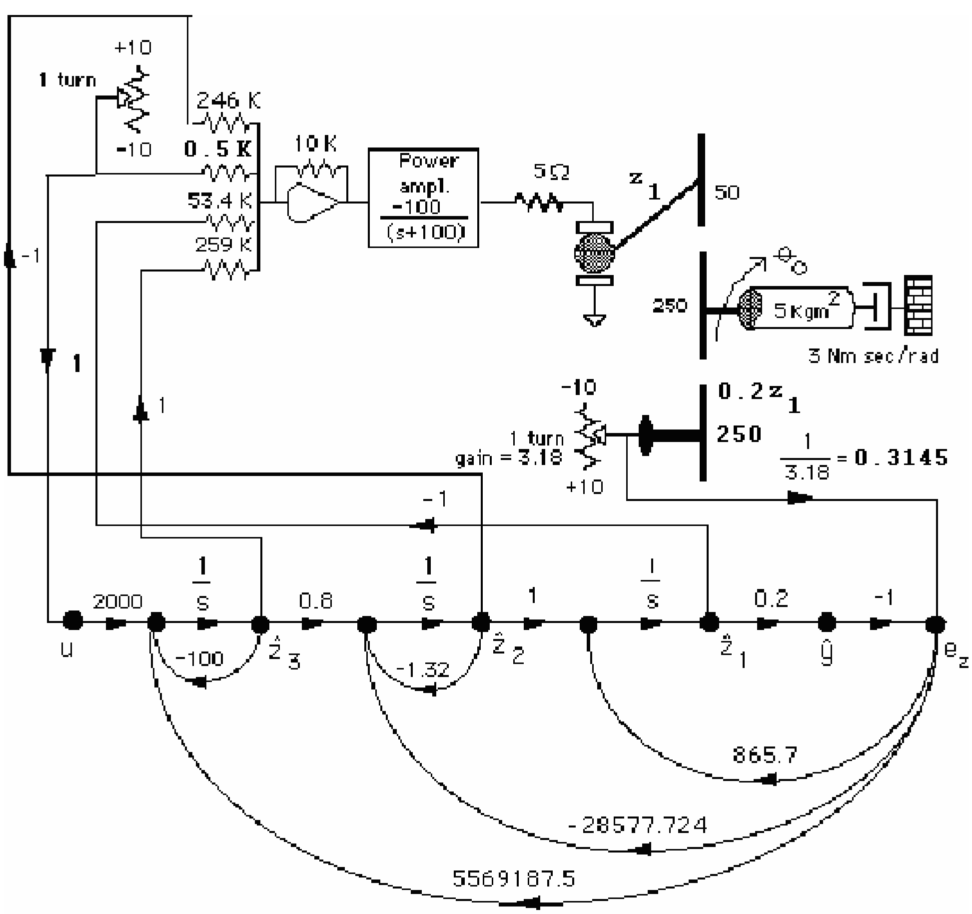
Equating coefficients, the observer for the observer canonical system is

Lx	Observer for x
173.14	
11827	
342370	

Using Eq. (12.92), the observer for the original system is

Lz	Observer for z
865.7	
-28577.724	
5569187.5	

d.



e.

Program:

```
'Controller'
A=[0 1 0;0 -1.32 0.8;0 0 -100];
B=[0;0;2000];
C=[0.2 0 0];
D=0;
pos=input('Type desired %OS  ');
Ts=input('Type desired settling time   ');
z=(-log(pos/100))/(sqrt(pi^2+log(pos/100)^2));
wn=4/(z*Ts);                   %Calculate required natural
                               %frequency.
[num,den]=ord2(wn,z);          %Produce a second-order system that
                               %meets the transient response
                               %requirements.
r=roots(den);                  %Use denominator to specify dominant
                               %poles.
poles=[r(1) r(2) 10*real(r(1))];
                               %Specify pole placement for all
                               %poles.
K=acker(A,B,poles)
'Observer'
pos=input('Type desired %OS  ');
z=(-log(pos/100))/(sqrt(pi^2+log(pos/100)^2));
wn=10*wn                        %Calculate required natural
                                %frequency.
[num,den]=ord2(wn,z);           %Produce a second-order system that
                                %meets the transient response
                                %requirements.
r=roots(den);                   %Use denominator to specify dominant
                                %poles.
poles=[r(1) r(2) 10*real(r(1))];%Specify pole placement for all
                                %poles.
l=acker(A',C',poles)'
```

Computer response:

```
ans =

Controller

Type desired %OS  15
Type desired settling time   2

K =

  0.1871    0.0406   -0.0387

ans =

Observer

Type desired %OS  10

wn =

  38.6899

l =

  1.0e+006 *

    0.0009
   -0.0286
    5.5691
```

ANSWERS TO REVIEW QUESTIONS

1. Both dominant and non-dominant poles can be specified with state-space design techniques.

2. Feedback all state variables to the plant's input through a variable gain for each. Decide upon a closed-loop characteristic equation that has a pole configuration to yield a desired response. Write the characteristic equation of the actual system. Match coefficients and solve for the values of the variable gains.

3. Phase-variable form

4. The control signal developed by the controller must be able to affect every state variable.

5. If the signal-flow diagram is in the parallel form, which leads to a diagonal system matrix, controllability can be determined by inspection by seeing that all state variables are fed by the control signal.

6. The system is controllable if the determinant of the controllability matrix is non-zero.

7. An observer is a system that estimates the state variables using information from the output of the actual plant.

8. If the plant's state-variables are not accessible, or too expensive to monitor

9. An observer is a copy of the plant. The difference between the plant's output and the observer's output is fed back to each of the derivatives of the observer's state variables through separate variable gains.

10. Dual phase-variable

11. The characteristic equation of the observer is derived and compared to a desired characteristic equation whose roots are poles that represent the desired transient response. The variable gains of each feedback path are evaluated to make the coefficients of the observer's characteristic equation equal the coefficients of the desired characteristic equation.

12. Typically, the transient response of the observer is designed to be much faster than that of the controller. Since the observer emulates the plant, we want the observer to estimate the plant's states rapidly.

13. Det[**A**-**BK**], where **A** is the system matrix, **B** is the input coupling matrix, and **K** is the controller.

14. Det[**A**-**LC**], where **A** is the system matrix, **C** is the output coupling matrix, and **L** is the observer.

15. The output signal of the system must be controlled by every state variable.

16. If the signal-flow diagram is in the parallel form, which leads to a diagonal system matrix, observability can be determined by inspection by seeing that all state variables feed the output.

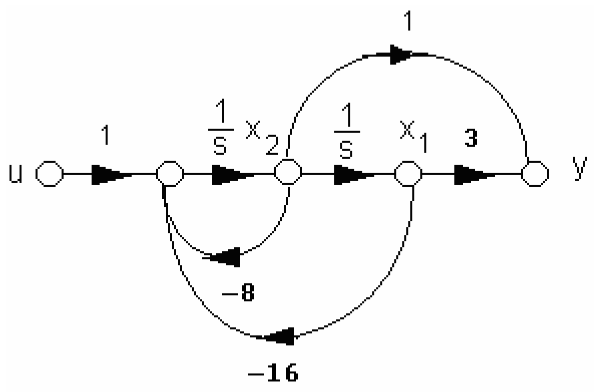
17. The system is observable if the determinant of the observability matrix is non-zero.

SOLUTIONS TO PROBLEMS

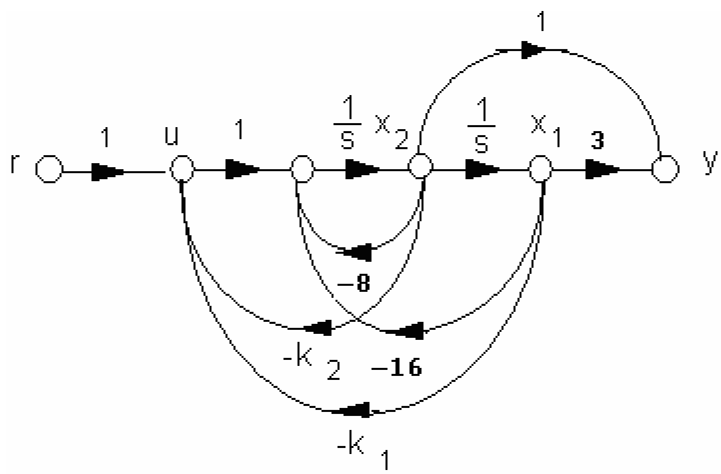
1.

i. $G(s) = \dfrac{(s+3)}{(s+4)^2} = \dfrac{1}{s^2+8s+16} * (s+3)$

a.



b.



c.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -(k_1+16) & -(k_2+8) \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \ ; \ y = \begin{bmatrix} 3 & 1 \end{bmatrix}\mathbf{x}$$

d.

$$T(s) = \frac{s+3}{s^2 + (k_2+8)s + (k_1+16)}$$

$$\dot{\mathbf{x}}_1 = (-20 - 71.25k_1)x_1 - 71.25k_2x_2 - 71.25k_3x_3 + 71.25r$$

$$\dot{\mathbf{x}}_2 = 27.5k_1x_1 + (-10x_2 + 27.5k_2)x_2 + 27.5k_3x_3 - 27.5r$$

$$\dot{\mathbf{x}}_3 = -6.25k_1x_1 - 6.25k_2x_2 - 6.25k_3x_3 + 6.25r$$

$$\mathbf{A} = \begin{bmatrix} (-20 - 71.25k_1) & -71.25k_2 & -71.25k_3 \\ 27.5k_1 & (-10x_2 + 27.5k_2) & 27.5k_3 \\ -6.25k_1 & -6.25k_2 & -6.25k_3 \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 71.25 \\ -27.5 \\ 6.25 \end{bmatrix}; \mathbf{C} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

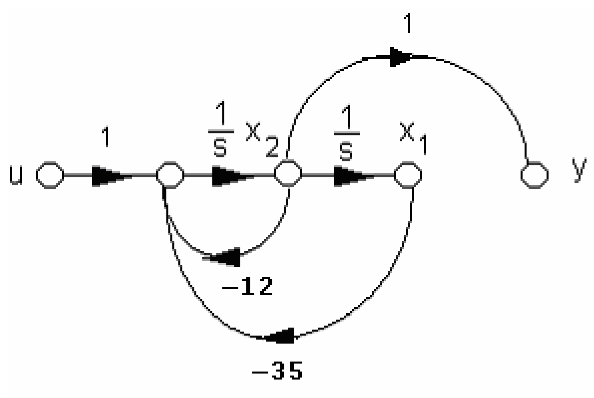
$$T(s) = \frac{200(s^2 + 7s + 25)}{4s^3 + (120 + 285k_1 - 110k_2 + 25k_3)s^2 + (800 + 2850k_1 - 2200k_2 + 750k_3)s + 5000k_3}$$

e.

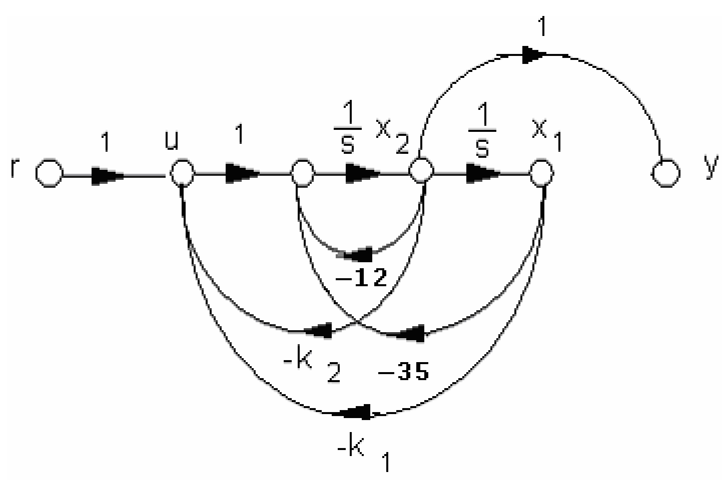
Part d. yields same result as i(d).

ii. $G(s) = \dfrac{s}{(s+5)(s+7)} = \dfrac{1}{s^2 + 12s + 35} * s$

a.



b.



c.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -(k_1+35) & -(k_2+12) \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \ ; \ y = \begin{bmatrix} 0 & 1 \end{bmatrix}\mathbf{x}$$

d.

$$T(s) = \frac{s}{s^2 + (k_2+12)s + (k_1+35)}$$

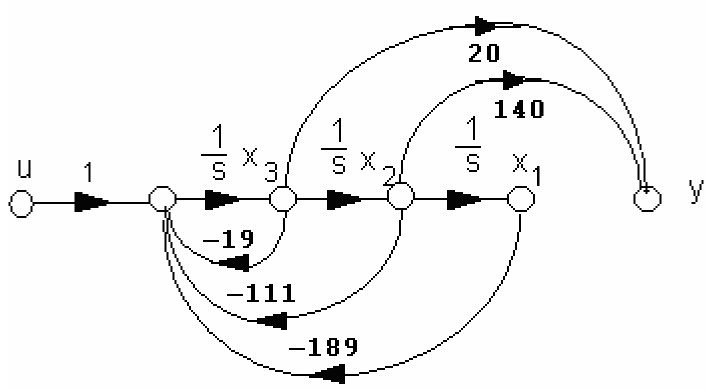
e.

$$T(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B};\ \mathbf{A} = \begin{bmatrix} 0 & 1 \\ -(k_1+35) & -(k_2+12) \end{bmatrix};\ \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ \mathbf{C} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

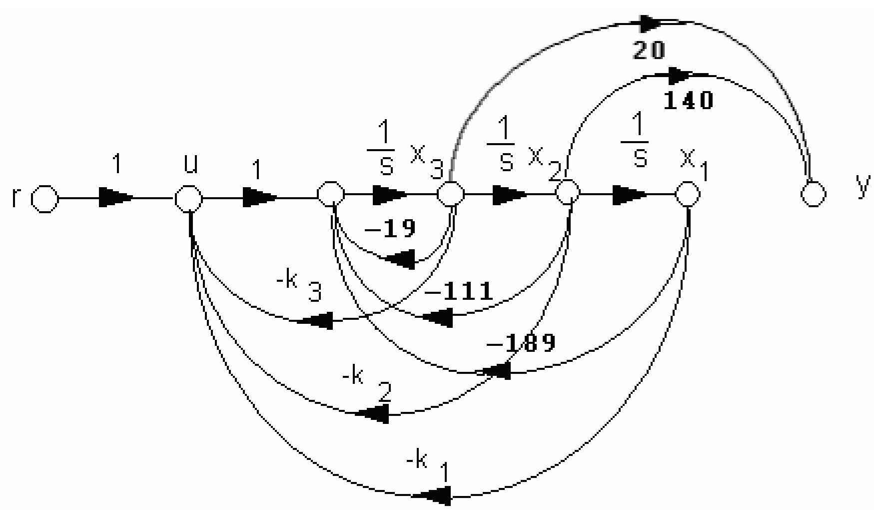
which yields the same result as ii(d).

iii. $G(s) = \dfrac{20s(s+7)}{(s+3)(s+7)(s+9)} = \dfrac{1}{s^3 + 19s^2 + 111s + 189} *\ (20s^2+140s)$

a.



b.



c.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(k_1+189) & -(k_2+111) & -(k_3+19) \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r \ ; \ y = \begin{bmatrix} 0 & 140 & 20 \end{bmatrix}\mathbf{x}$$

d.

$$T(s) = \frac{20s(s+7)}{s^3 + (k_3+19)s^2 + (k_2+111)s + (k_1+189)}$$

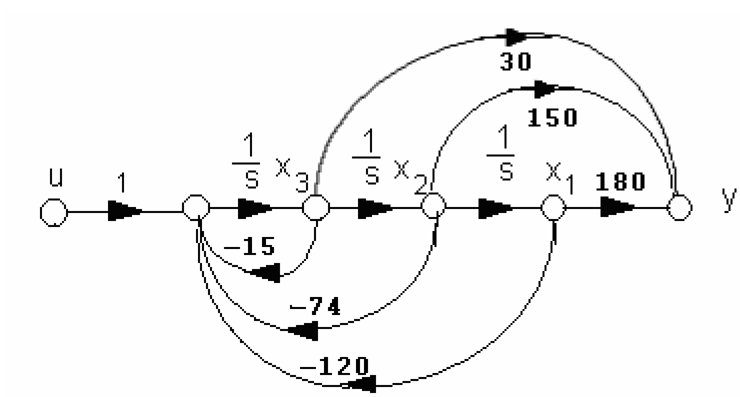
e.

$$T(s) = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B};\ \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(k_1+189) & -(k_2+111) & -(k_3+19) \end{bmatrix};\ \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};\ \mathbf{C} = \begin{bmatrix} 0 & 140 & 20 \end{bmatrix}$$

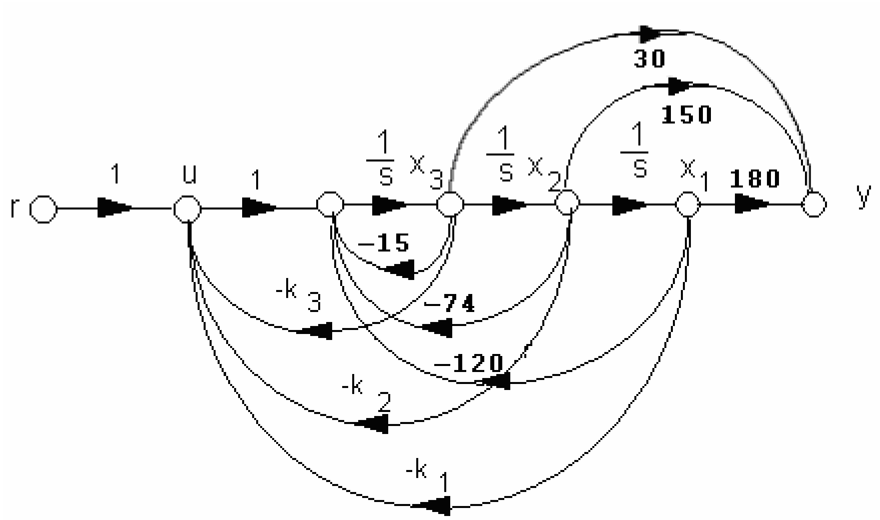
which yields the same result as iii(d).

iv. $G(s) = \dfrac{30(s+2)(s+3)}{(s+4)(s+5)(s+6)} = \dfrac{1}{s^3 + 15s^2 + 74s + 120} * \ (30s^2 + 150s + 180)$

a.



b.



c.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(k_1+120) & -(k_2+74) & -(k_3+15) \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r \;\; ; \; y = \begin{bmatrix} 180 & 150 & 30 \end{bmatrix}\mathbf{x}$$

d.

$$T(s) = \frac{30s^2 + 150s + 180}{s^3 + (k_3+15)s^2 + (k_2+74)s + (k_1+120)}$$

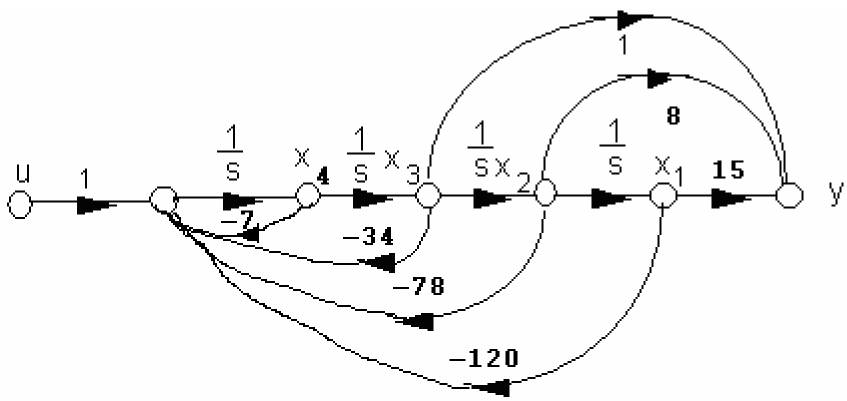
e.

$$T(s) = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B};\; \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(k_1+120) & -(k_2+74) & -(k_3+15) \end{bmatrix};\; \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};\; \mathbf{C} = \begin{bmatrix} 180 & 150 & 30 \end{bmatrix}$$

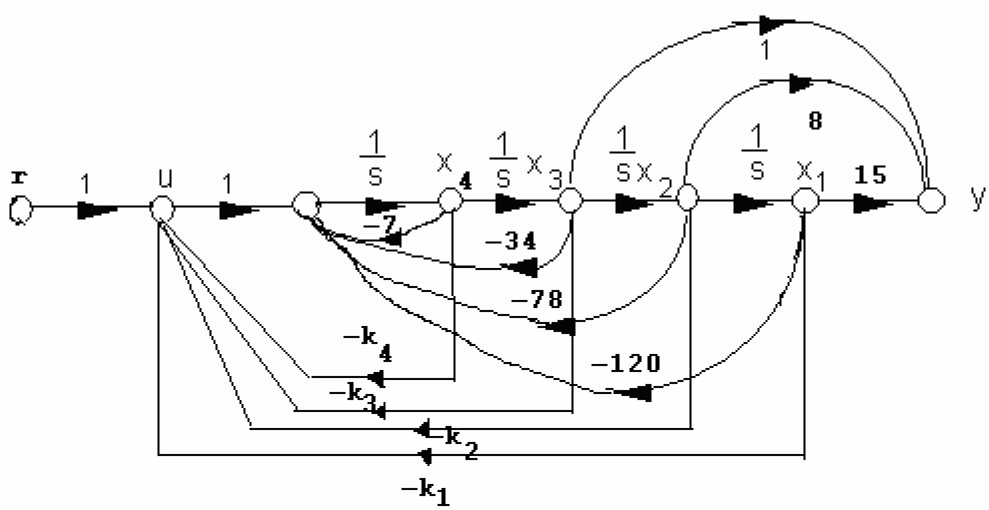
which yields the same result as iv(d).

v. $$G(s) = \frac{s^2+8s+15}{(s^2+4s+10)(s^2+3s+12)} = \frac{1}{s^4+7s^3+34s^2+78s+120} * \;(s^2+8s+15)$$

a.



b.



c.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -(k_1+120) & -(k_2+78) & -(k_3+34) & -(k_4+7) \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} r \;\; ; \;\; y = \begin{bmatrix} 15 & 8 & 1 & 0 \end{bmatrix}\mathbf{x}$$

d.

$$T(s) = \frac{s^2+8s+15}{s^4+(k_4+7)s^3+(k_3+34)s^2+(k_2+78)s+(k_1+120)}$$

e.

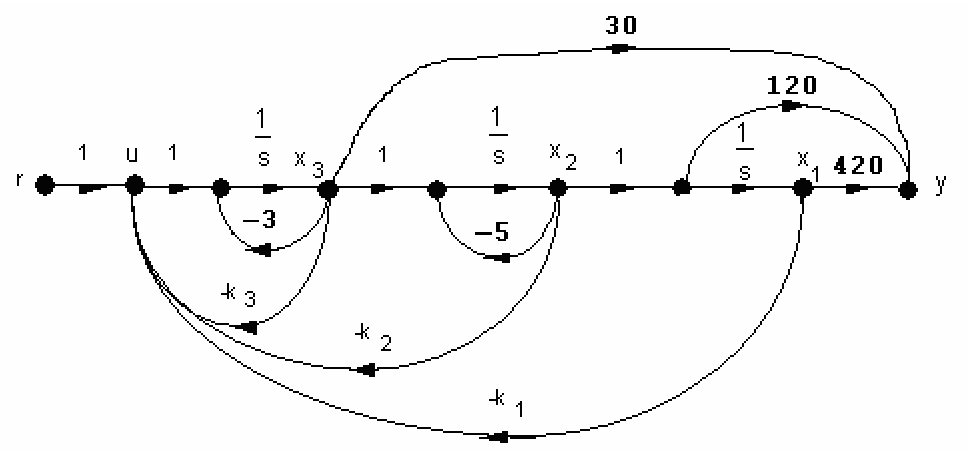
$$T(s)=\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B};\ \mathbf{A}=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\-(k_1+120)&-(k_2+78)&-(k_3+34)&-(k_4+7)\end{bmatrix};\ \mathbf{B}=\begin{bmatrix}0\\0\\0\\1\end{bmatrix};\ \mathbf{C}=\begin{bmatrix}15&8&1&0\end{bmatrix}$$

which yields the same result as v(d).

2.

i

a. The output is



Since,

$$y=(30s^2+270s+420)x_1=30\ddot{x}_1+270\dot{x}_1+420x_1=30\dot{x}_2+270x_2+420x_1$$
$$=30(-5x_2+x_3)+270x_2+420x_1=420x_1+120x_2+30x_3$$

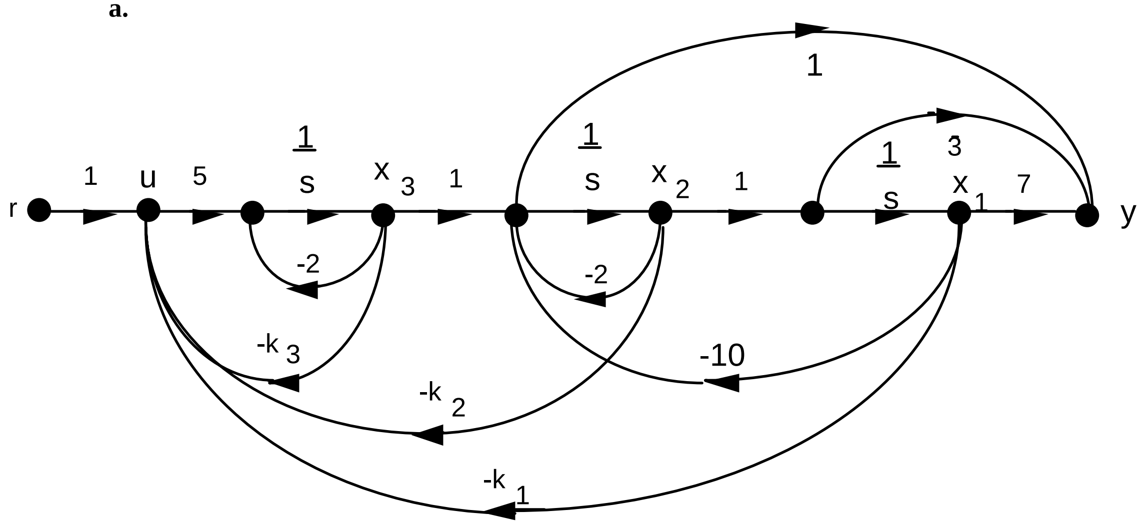
b.

$$T(s)=\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B};\ \mathbf{A}=\begin{bmatrix}0&1&0\\0&-5&1\\-k_1&-k_2&-(k_3+3)\end{bmatrix};\ \mathbf{B}=\begin{bmatrix}0\\0\\1\end{bmatrix};\ \mathbf{C}=\begin{bmatrix}420&120&30\end{bmatrix}$$

$$T(s)=\frac{30(s+2)(s+7)}{s^3(k_3+8)s^2+(5k_3+k_2+15)s+k_1}$$

ii

a.



b.

$$T(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B};\ \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -k_2 & -(5k_3+2) \end{bmatrix};\ \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix};\ \mathbf{C} = \begin{bmatrix} -3 & 1 & 1 \end{bmatrix}$$

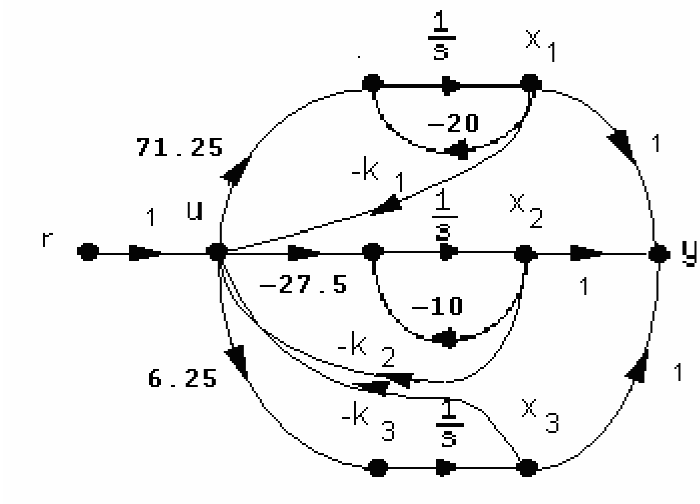
$$= \frac{5(s^2+3s+7)}{s^3+(5k_3+4)s^2+(10k_3+k_2+14)s+(50k_3+k_1+20)}$$

3.

i

a.

$$G(s) = \frac{50(s^2+7s+25)}{s(s+10)(s+20)} = \frac{6.25}{s} - \frac{27.5}{s+10} + \frac{71.25}{s+20}$$



b. Writing the state equations:

$$\dot{\mathbf{x}}_1 = -20x_1 + 71.25u$$

$$\dot{\mathbf{x}}_2 = -10x_2 - 27.5u$$

$$\dot{\mathbf{x}}_3 = 6.25u$$

But, $u = -k_1x_1 - k_2x_2 - k_3x_3 + r$. Substituting into the state equations,

$$\dot{\mathbf{x}}_1 = (-20 - 71.25k_1)x_1 - 71.25k_2x_2 - 71.25k_3x_3 + 71.25r$$

$$\dot{\mathbf{x}}_2 = 27.5k_1x_1 + (-10x_2 + 27.5k_2)x_2 + 27.5k_3x_3 - 27.5r$$

$$\dot{\mathbf{x}}_3 = -6.25k_1x_1 - 6.25k_2x_2 - 6.25k_3x_3 + 6.25r$$

Therefore, $T(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$, where

$$\mathbf{A} = \begin{bmatrix} (-20 - 71.25k_1) & -71.25k_2 & -71.25k_3 \\ 27.5k_1 & (-10x_2 + 27.5k_2) & 27.5k_3 \\ -6.25k_1 & -6.25k_2 & -6.25k_3 \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 71.25 \\ -27.5 \\ 6.25 \end{bmatrix}; \mathbf{C} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

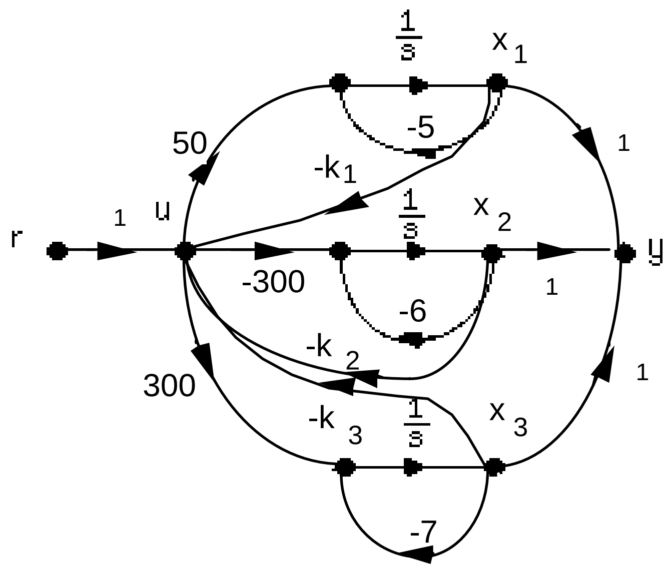
Hence,

$$T(s) = \frac{200(s^2 + 7s + 25)}{4s^3 + (120 + 285k_1 - 110k_2 + 25k_3)s^2 + (800 + 2850k_1 - 2200k_2 + 750k_3)s + 5000k_3}$$

ii

a.

$$G(s)=\frac{50(s+3)(s+4)}{(s+5)(s+6)(s+7)}=\frac{50}{s+5}-\frac{300}{s+6}+\frac{300}{s+7}$$



b. Writing the state equations:

$$\dot{x}_1=-5x_1+50u$$

$$\dot{x}_2=-6x_2-300u$$

$$\dot{x}_3=-7x_3+300u$$

But,

$$u=-k_1x_1-k_2x_2-k_3x_3+r$$

Substituting into the state equations, collecting terms, and converting to vector-matrix form yields

$$\dot{\mathbf{x}}=\begin{bmatrix}-(5+k_1) & -50k_2 & -50k_3\\ 300k_1 & (300k_2-6) & 300k_3\\ -(300k_1+7) & -300k_2 & 300k_3\end{bmatrix}\mathbf{x}+\begin{bmatrix}50\\ -300\\ 300\end{bmatrix}r$$

$$y=\begin{bmatrix}1 & 1 & 1\end{bmatrix}\mathbf{x}$$

Therefore, $T(s)=\boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}$, or

$$T(s)=\frac{50s^2+1750s+(6900-88200k_1)}{s^3+(300k_3-300k_2+k_1+11)s^2+2(1475k_3-750k_2+3k_1-7350k_3k_1+7350k_2k_1+15)s+300k_3(23-294k_1)}$$

4.

The plant is given by

$$G(s)=\frac{20}{(s+1)(s+3)(s+7)}=\frac{20}{s^3+11s^2+31s+21}$$

The characteristic polynomial for the plant with phase-variable state feedback is

$$s^3+(k_3+11)s^2+(k_2+31)s+(k_1+21)=0$$

The desired characteristic equation is

$$(s+53.33)(s^2+10.67s+106.45)=s^3+64s^2+675.48s+5676.98$$

based upon 10% overshoot, T_S = 0.5 second, and a third pole ten times further from the imaginary axis than the dominant poles. Comparing the two characteristic equations,

$$k_1=5655.98,\ k_2=644.48, \text{ and } k_3=53.$$

5.

a. The system in controller canonical form is:

$$\mathbf{A}=\begin{bmatrix}-a_{n-1} & -a_{n-2} & \cdots & -a_1 & -a_0\\ 1 & 0 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & 1 & 0\end{bmatrix};\mathbf{B}=\begin{bmatrix}1\\0\\0\\0\end{bmatrix};\mathbf{C}=\begin{bmatrix}c_1 & c_2 & c_3 & c_4\end{bmatrix}$$

The characteristic equation of the plant is:

$$s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 = 0$$

Forming the closed-loop system by feeding back each state variable and the input to u forming

$$u = -\mathbf{Kx} + r$$

where

$$\mathbf{K} = [k_1 \quad k_2 \quad \ldots \quad k_n]$$

and substituting u into the state equation, we obtain

$$\dot{\mathbf{x}} = \mathbf{Ax}+\mathbf{B}u=(\mathbf{A}-\mathbf{BK})\mathbf{x}+\mathbf{B}r$$

Forming **A - BK**:

$$\mathbf{A}-\mathbf{BK}=\begin{bmatrix}-(a_{n-1}+k_1) & -(a_{n-2}+k_2) & \cdots & -(a_1+k_{n-1}) & -(a_0+k_n)\\ 1 & 0 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & 1 & 0\end{bmatrix}$$

The characteristic equation is:

$$s^n + (a_{n-1}+k_1)s^{n-1} + (a_{n-2}+k_2)\ldots + (a_1+k_{n-1})s + (a_0+k_n) = 0$$

Assuming a desired characteristic equation,

$$s_n + d_{n-1}s^{n-1} + d_{n-2}s^{n-2} + \ldots + d_2 s^2 + d_1 s + d_0 = 0$$

Equating coefficients,

$$d_i = a_i + k_{n-i} \text{ ; } i = 0, 1, 2, \ldots n-1$$

from which

$$k_{n-i} = d_i - a_i \tag{1}$$

b. The desired characteristic equation is

$$s^3 + 15.9s^2 + 136.08s + 413.1 = 0$$

the characteristic equation of the plant is

$$s^3 + 5s^2 + 4s + 0 = 0$$

Using Eq. (1) above, $k_{3-i} = d_i - a_i$. Therefore, $k_3 = d_0 - a_0 = 413.1 - 0 = 413.1$; $k_2 = d_1 - a_1 = 136.08 - 4 = 132.08$; $k_1 = d_2 - a_2 = 15.9 - 5 = 10.9$. Hence,

$$\mathbf{K} = \begin{bmatrix} 10.9 & 132.08 & 413.1 \end{bmatrix}$$

6.

Using Eqs. (4.39) and (4.34) to find $\zeta = 0.5169$ and $\omega_n = 7.3399$, respectively. Factoring the denominator of Eq. (4.22), the required poles are $-3.7942 \pm j6.2832$. We place the third pole at -2 to cancel the open loop zero. Multiplying the three closed-loop pole terms yields the desired characteristic equation:

$s^3 + 9.5885s^2 + 69.0516s + 107.7493 = 0$. Since $G(s) = \dfrac{100s^2 + 2200s + 4000}{s^3 + 8s^2 + 19s + 12}$, the controller canonical form is $\mathbf{A} = \begin{bmatrix} -8 & -19 & -12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$; $\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; $\mathbf{C} = [100 \quad 2200 \quad 4000]$. The first row of **A** contains the coefficients of the characteristic equation. Thus comparing the first row of **A** to the desired characteristic equation and using the results of Problem 5, $k_1 = -(9.5885 - 8) = 1.5885$; $k_2 = -(69.0516 - 19) = 50.0516$; and $k_3 = -(107.7493 - 12) = 95.7493$.

7.

The plant is given by

$$G(s) = \frac{20(s+2)}{s(s+4)(s+6)} = \frac{20s + 40}{s^3 + 10s^2 + 24s + 0}$$

The characteristic polynomial for the plant with phase-variable state feedback is

$$s^3 + (k_3 + 10)s^2 + (k_2 + 24)s + (k_3 + 0)$$

The desired characteristic equation is

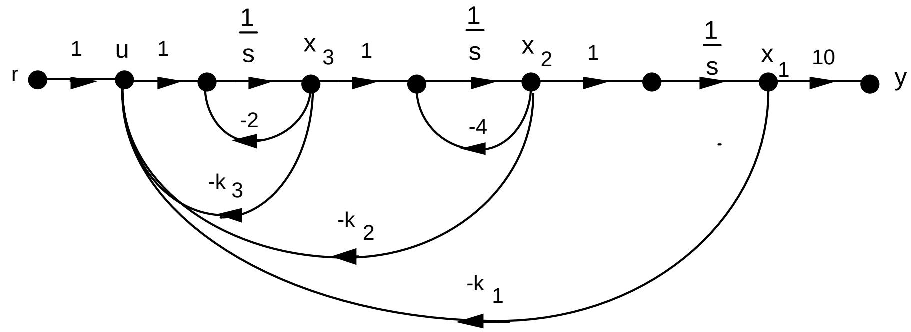
$$(s + 20)(s^2 + 4s + 11.45) = s^3 + 24s^2 + 91.45s + 229$$

based upon 10% overshoot, $T_s = 2$ seconds, and a third pole ten times further from the imaginary axis than the dominant poles. Comparing the two characteristic equations,

$$k_1 = 229,\ k_2 = 67.45, \text{ and } k_3 = 14 .$$

8.

Drawing the signal-flow diagram,



Writing the state equations yields the following **A** matrix:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -4 & 1 \\ -k_1 & -k_2 & -[2+k_3] \end{pmatrix}$$

from which,

$$|s\mathbf{I} - \mathbf{A}| = s^3 + (k_3 + 6)s^2 + (4k_3 + k_2 + 8)s + k_1$$

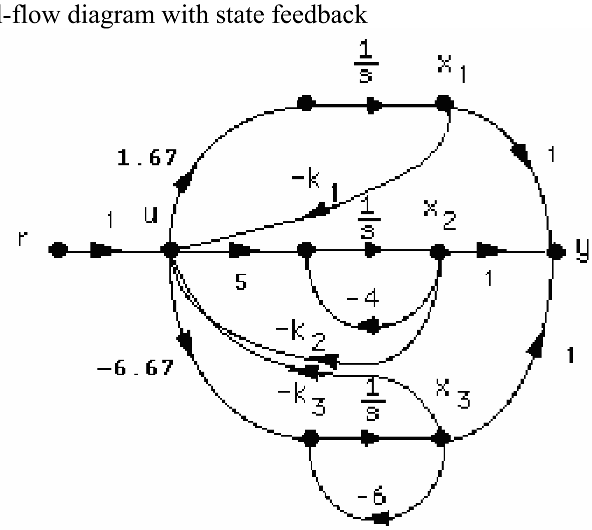
The desired characteristic equation is $(s + 80)(s^2 + 16s + 183.137) = s^3 + 96s^2 + 1463.1s + 14651$ based upon 10% overshoot, $T_s = 0.5$ second, and a third pole ten times further from the imaginary axis than the dominant poles. Comparing the two characteristic equations, , $k_1 = 14651$, $k_2 = 1095.1$, and $k_3 = 90$.

9.

Expand G(s) by partial fractions and obtain

$$G(s) = \frac{20}{s(s+4)(s+6)} = \frac{1.67}{s} + \frac{5}{s+4} - \frac{6.67}{s+6}$$

Drawing the signal-flow diagram with state feedback



Writing the state equations yields the following system matrix:

$$\mathbf{A}=\begin{bmatrix} -1.67k_1 & -1.67k_2 & -1.67k_3 \\ -5k_1 & -(5k_2+4) & -5k_3 \\ 6.67k_1 & 6.67k_2 & (6.67k_3-6) \end{bmatrix}$$

Evaluating the characteristic polynomial yields,

$$|s\mathbf{I}-\mathbf{A}| = (-6.67k_3+5k_2+1.67k_1+10)s^2+(-26.68k_3+30k_2+16.7k_1+24)s+40.08k_1$$

From Problem 7, the desired characteristic polynomial is

$$s^3+24s^2+91.45s+229\,.$$

Equating coefficients and solving simultaneously yields

$$k_1=5.71,\ k_2=-4.58,\text{ and } \mathrm{k}_3=-4.10\,.$$

10.

Writing the state equation and the controllability matrix for the system yields

$$\dot{\mathbf{x}}=\begin{bmatrix} -5 & 1 \\ -1 & -3 \end{bmatrix}\mathbf{x}+\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}u\ ;\ \mathbf{C_M}=\begin{bmatrix}\mathbf{B} & \mathbf{AB}\end{bmatrix}=\begin{bmatrix} b_1 & -5b_1+b_2 \\ b_2 & -b_1-3b_2 \end{bmatrix}$$

The controllability matrix has a zero determinant if $b_2 = b_1$.

11.

The controllability matrix is given by Eq. (12.26) for each of the following solutions:

a.

$$\mathbf{A}=\begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix};\ \mathbf{B}=\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix};\ \mathbf{C_M}=\begin{bmatrix} 0 & 1 & -5 \\ 1 & -2 & 4 \\ 1 & -3 & 9 \end{bmatrix};\ \det \mathbf{C_M} = 0\ ;\ \text{system is uncontrollable}$$

b.

$$\mathbf{A}=\begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix};\ \mathbf{B}=\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix};\ \mathbf{C_M}=\begin{bmatrix} 0 & 1 & -4 \\ 1 & -2 & 4 \\ 1 & -3 & 9 \end{bmatrix};\ \det \mathbf{C_M} = -1;\ \text{system is controllable}$$

c.

$$\mathbf{A}=\begin{bmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{bmatrix};\ \mathbf{B}=\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix};\ \mathbf{C_M}=\begin{bmatrix} 0 & 2 & -7 \\ 2 & 1 & -3 \\ 1 & -3 & 9 \end{bmatrix};\ \det \mathbf{C_M} = 7;\ \text{system is controllable}$$

d.

$$\mathbf{A} = \begin{bmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & 0 & -3 \end{bmatrix}; \ \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}; \ \mathbf{C_M} = \begin{bmatrix} 1 & -4 & 17 \\ 0 & 1 & -8 \\ 1 & -8 & 44 \end{bmatrix}; \ \det \ \mathbf{C_M} = -5; \ \text{system is controllable}$$

e.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}; \ \mathbf{B} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}; \ \mathbf{C_M} = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}; \ \det \ \mathbf{C_M} = 0; \ \text{system is uncontrollable}$$

f.

$$\mathbf{A} = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -6 \end{bmatrix}; \ \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}; \ \mathbf{C_M} = \begin{bmatrix} 1 & -4 & 16 \\ 0 & 0 & 0 \\ 1 & -6 & 36 \end{bmatrix}; \ \det \ \mathbf{C_M} = 0; \ \text{system is uncontrollable}$$

This system can also be determined uncontrollable by inspection.

12.

Program:

```
'(d)'
A=[-4 1 0;0 0 1;-5 0 -3]
B=[1;0;1]
Cm=ctrb(A,B)
Rank=rank(Cm)
pause
'(f)'
A=[-4 0 0;0 -5 0;0 0 -6]
B=[1;0;1]
Cm=ctrb(A,B)
Rank=rank(Cm)
```

Computer response:

```
ans =

(d)

A =

    -4     1     0
     0     0     1
    -5     0    -3

B =

     1
     0
     1

Cm =

     1    -4    17
     0     1    -8
     1    -8    44

Rank =

     3
```

```
ans =

(f)

A =

    -4     0     0
     0    -5     0
     0     0    -6
B =

     1
     0
     1

Cm =

     1    -4    16
     0     0     0
     1    -6    36

Rank =

     2
```

13.

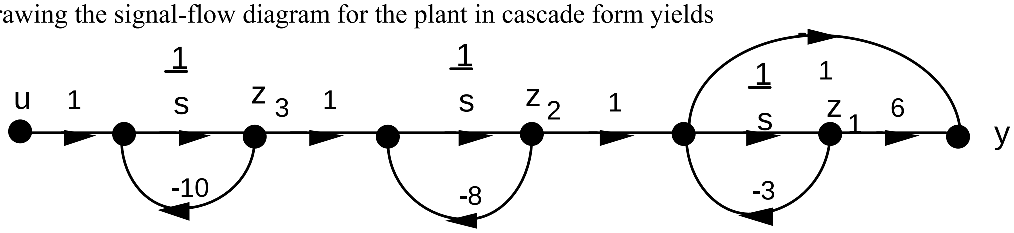
From Eq. (12.46) we write the controller canonical form: $\mathbf{A_{cc}} = \begin{bmatrix} -8 & -17 & -10 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$; $\mathbf{B_{cc}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

The controllability matrices are found using Eq. (12.35). For the original system of Eq. (12.44),

$\mathbf{C_{Mz}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -3 \\ 1 & -1 & 1 \end{bmatrix}$. For the controller canonical form, $\mathbf{C_{Mcc}} = \begin{bmatrix} 1 & -8 & 47 \\ 0 & 1 & -8 \\ 0 & 0 & 1 \end{bmatrix}$. The transformation matrix is, $\mathbf{P = C_{Mz}C_{Mcc}}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 5 \\ 1 & 7 & 10 \end{bmatrix}$. Comparing the first row of $\mathbf{A_{cc}}$ with the desired characteristic equation, Eq. (12.50), $\mathbf{K_{cc}}$ = [-2 -4 10]. Transforming back to the original system, $\mathbf{K_z} = \mathbf{K_{cc}P}^{-1}$ = [-20 10 -2].

14.

Drawing the signal-flow diagram for the plant in cascade form yields



Writing the **A** and **B** matrices for the z system,

Az			Bz
-3	1	0	0

0	-8	1	0
0	0	-10	1

Writing the **A** and **B** matrices for the x (phase-variable) system,

Ax			**Bx**	
0	1	0	0	
0	0	1	0	Phase-Variable Form
-240	-134	-21	1	

From the phase variable from, the characteristic polynomial is $s^3 + 21s^2 + 134s + 240$.

Finding the controllability matrices and their determinants for the z and x systems shows that there is controllability,

CMz	Controllability Matrix of z		**CMx**	Controllability Matrix of x	
0	0	1	0	0	1
0	1	-18	0	1	-21
1	-10	100	1	-21	307
Det(CMz)	-1		**Det(CMx)**	-1	

Using Eq. (12.39), the transformation matrix **P** and its inverse are found to be

P	Transformation Matrix z=Px		**PINV**		
1	0	0	1.00	0.00	0.00
3	1	0	-3.00	1.00	0.00
24	11	1	9.00	-11.00	1.00

Using the given transient requirements, and placing the third closed-loop pole over the zero at -6 yields the following desired closed-loop characteristic polynomial:

$$(s^2 + 8s + 45.78)(s + 6) = s^3 + 14s^2 + 93.78s + 274.68$$

Using the phase-variable system with state feedback the characteristic polynomial is

$$s^3 + (k_3 + 21)s^2 + (k_2 + 134)s + (k_1 + 240)$$

Equating the two characteristic polynomials yields the state feedback vector for the x system as

Kx	Controller for x	
34.68	-40.22	-7

Using Eq. (12.42),

Kz	Controller for z	
92.34	36.78	-7

15.

Program:

```
A=[-3 1 0;0 -8 1;0 0 -10];        %Generate system matrix A
B=[0;0;1];                        %Generate input coupling matrix B
```

```
C=[3 1 0];                       %Generate output coupling matrix C
D=0;                             %Generate matrix D
Po=10;                           %Input desired percent overshoot
Ts=1;                            %Input desired settling time
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
                                 %Calculate required damping ratio
wn=4/(z*Ts);                     %Calculate required natural
                                 %frequency
[num,den]=ord2(wn,z);            %Produce a second-order system that
                                 %meets transient requirements
r=roots(den);                    %Use denominator to specify
                                 %dominant poles
poles=[r(1) r(2) -6];            %Specify pole placement for all
                                 %poles.
                                 %A few tries at the the third-pole
                                 %value shows T(s) with a closed-
                                 %loop zero at -7.
                                 %Thus, choose the third pole to
                                 %cancel this zero.
K=acker(A,B,poles)               %Calculate controller gains in z-
                                 %system
Anew=A-B*K;                      %Form compensated A matrix
Bnew=B;                          %Form compensated B matrix
Cnew=C;                          %Form compensated C matrix
Dnew=D;                          %Form compensated D matrix
[numt,dent]=ss2tf(Anew,Bnew,Cnew,Dnew);
                                 %Form T(s)
'T(s)'                           %Display label
T=tf(numt,dent)                  %Display T(s)
poles=pole(T)                    %Display poles of T(s)
```

Computer response:

```
K =

   92.3531   36.7844   -7.0000

ans =

T(s)

Transfer function:
   -3.553e-015 s^2 + s + 6
------------------------------
s^3 + 14 s^2 + 93.78 s + 274.7

poles =

  -4.0000 + 5.4575i
  -4.0000 - 5.4575i
  -6.0000
```

16.

Expanding by partial fractions,

$$G(s) = \frac{(s+6)}{(s+3)(s+8)(s+10)} = \frac{0.085714}{(s+3)} - \frac{0.2}{(s+8)} - \frac{0.28571}{(s+10)}$$

Writing the **A** and **B** matrices for the z system with k_i's set to zero,

Az			**Bz**
-3	0	0	0.085714

0	-8	0	0.2
0	0	-10	-0.28571

Writing the **A** and **B** matrices for the x (phase-variable) system,

Ax			**Bx**	
0	1	0	0	
0	0	1	0	Phase-Variable Form
-240	-134	-21	1	

From the phase variable from, the characteristic polynomial is $s^3 + 21s^2 + 134s + 240$.

Finding the controllability matrices and their determinants for the z and x systems shows that there is controllability,

CMz	Controllability Matrix of z		**CMx**	Controllability Matrix of x	
0.085714	-0.257142	0.771426	0	0	1
0.2	-1.6	12.8	0	1	-21
-0.28571	2.8571	-28.571	1	-21	307

Det(CMz)	0.342850857	**Det(CMx)**	-1

Using Eq. (12.39), the transformation matrix **P** and its inverse are found to be

P	Transformation Matrix z=Px		**PINV**		
6.85712	1.542852	0.085714	0.33	-0.50	-0.25
6	2.6	0.2	-1.00	4.00	2.50
-6.85704	-3.14281	-0.28571	3.00	-32.00	-25.00

Using the given transient requirements, and placing the third closed-loop pole over the zero at -6 yields the following desired closed-loop characteristic polynomial:

$$(s^2 + 8s + 45.78)(s + 6) = s^3 + 14s^2 + 93.78s + 274.68$$

Using the phase-variable system with state feedback the characteristic polynomial is

$$s^3 + (k_3 + 21)s^2 + (k_2 + 134)s + (k_1 + 240)$$

Equating the two characteristic polynomials yields the state feedback vector for the x system as

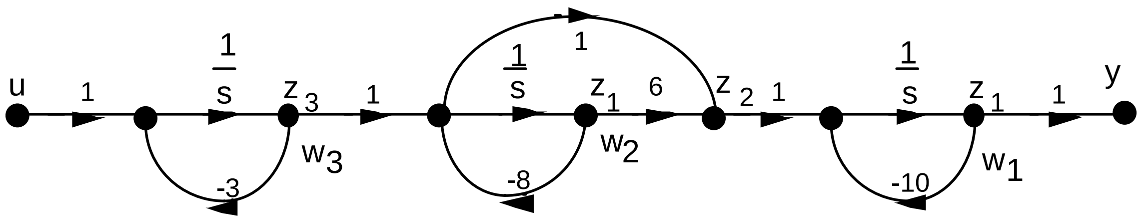
Kx	Controller for x	
34.7062	-40.2156	-7

Using Eq. (12.42),

Kz	Controller for z	
30.78443595	45.7845	65.78543678

17.

Draw signal-flow diagram showing state variables, **z**, at the output of each subsystem and the state variables, **w**, at the output of the integrators.



Recognizing that $z_2 = 6w_2 - 8w_2 + w_3 = -2w_2 + w_3$, we can write the state equations for **w** as

$$\dot{\mathbf{w}} = \begin{bmatrix} -10 & -2 & 1 \\ 0 & -8 & 1 \\ 0 & 0 & -3 \end{bmatrix}\mathbf{w} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{w}$$

Writing the relationship between **z** and **w** yileds

$$\mathbf{z} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{w} = \mathbf{P}^{-1}\mathbf{w}$$

Thus

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Converting the state equations in **w** to state equations in **z**, we use Eqs. (5.87) and obtain the **A** matrix and **B** vector as

Az			**Bz**
-10	1	0	0
0	-8	3	1
0	0	-3	1

Writing the **A** and **B** matrices for the x (phase-variable) system,

Ax			**Bx**	
0	1	0	0	
0	0	1	0	Phase-Variable Form
-240	-134	-21	1	

From the phase variable from, the characteristic polynomial is $s^3 + 21s^2 + 134s + 240$

Finding the controllability matrices and their determinants for the z and x systems shows that there is controllability,

CMz	Controllability Matrix of z		CMx	Controllability Matrix of x	
0	1	-15	0	0	1
1	-5	31	0	1	-21
1	-3	9	1	-21	307
Det(CMz)	-8		**Det(CMx)**	-1	

Using Eq. (12.39), the transformation matrix **P** and its inverse are found to be

P	Transformation Matrix z=Px		PINV		
6	1	0	-0.25	-0.13	0.13
60	16	1	2.50	0.75	-0.75
80	18	1	-25.00	-3.50	4.50

Using the given transient requirements, and placing the third closed-loop pole over the zero at -6 yields the following desired closed-loop characteristic polynomial:

$$(s^2 + 8s + 45.78)(s + 6) = s^3 + 14s^2 + 93.78s + 274.68$$

Using the phase-variable system with state feedback the characteristic polynomial is

$$s^3 + (k_3 + 21)s^2 + (k_2 + 134)s + (k_1 + 240)$$

Equating the two characteristic polynomials yields the state feedback vector for the x system as

Kx	Controller for x	
34.68	-40.22	-7

Using Eq. (12.42),

Kz	Controller for z	
65.78	-10	3

18.

Using Eqs. (4.39) and (4.34) to find $\zeta = 0.5169$ and $\omega_n = 18.3498$ respectively. Factoring the denominator of Eq. (4.22), the required poles are $-9.4856 \pm j15.708$. We place the third pole 10 times further at -94.856. Multiplying the three closed-loop pole terms yields the desired characteristic equation: $s^3 + 114s^2 + 2136s + 31940 = 0$. Representing the plant in parallel form:

$\mathbf{A}_{\mathbf{par}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -8 \end{bmatrix}$; $\mathbf{B}_{\mathbf{par}} = \begin{bmatrix} 3.125 \\ -6.25 \\ 3.125 \end{bmatrix}$; $\mathbf{C}_{\mathbf{par}} = [1 \quad 1 \quad 1]$. Using Eq. (12.26),

$\mathbf{C}_{\mathbf{Mpar}} = \begin{bmatrix} 3.125 & 0 & 0 \\ -6.25 & 25 & -100 \\ 3.125 & -25 & 200 \end{bmatrix}$, which is controllable since the determinant is 7812.5. Since

$G(s) = \dfrac{100}{s^3 + 12s^2 + 32s}$, the controller canonical form is $\mathbf{A}_{\mathbf{cc}} = \begin{bmatrix} -12 & -32 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$; $\mathbf{B}_{\mathbf{cc}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$;

C = [0 0 100]. Using Eq. (12.26), $\mathbf{C_{Mcc}} = \begin{bmatrix} 1 & -12 & 112 \\ 0 & 1 & -12 \\ 0 & 0 & 1 \end{bmatrix}$, which is controllable since the determinant is 1. The first row of $\mathbf{A_{cc}}$ contains the coefficients of the characteristic equation. Comparing the first row of $\mathbf{A_{cc}}$ to the desired characteristic equation and using the results of Problem 5, $(12 + k_1) = 114$; $(32 + k_2)= 2136$; and $(0 + k_3)= 31940$. Hence $\mathbf{K_{cc}}$ =[31940 2104 102]. The transformation matrix is, $\mathbf{P} = \mathbf{C_{Mpar}C_{Mcc}}^{-1} = \begin{bmatrix} 100 & 37.5 & 3.125 \\ 0 & -50 & -6.25 \\ 0 & 12.5 & 3.125 \end{bmatrix}$. Transforming back to the original system, $\mathbf{K_{par}} = \mathbf{K_{cc}P}^{-1}$ = [319.396 251.5184 216.2255].

19.

$$G(s) = \frac{1}{s(s+3)(s+7)} = \frac{1}{s^3 + 10s^2 + 21s + 0}$$

Writing the **A** and **C** matrices for the observer canonical system,

Az

-10	1	0
-21	0	1
0	0	0

Cz

1	0	0

The characteristic polynomial is $s^3 + 10s^2 + 21s + 0$.

Now check observability by calculating the observability matrix and its determinant.

OMz Observability Matrix of z

1	0	0
-10	1	0
79	-10	1

Det(OMz) 1

Using the given transient requirements, and placing the third closed-loop pole 10 times further from the imaginary axis than the dominant poles yields the following desired characteristic polynomial:

$$(s+300)(s^2 + 60s + 5625) = s^3 + 360s^2 + 23625s + 1687500$$

Equating this polynomial to Eq. (12.67), yields the observer gains as:

Lz Observer for z

350

23604

1687500

20.

Using Eqs. (4.39) and (4.34) to find $\zeta = 0.5912$ and $\omega_n = 19.4753$ respectively. Factoring the denominator of Eq. (4.22), the required poles are $-11.513 \pm j15.708$. We place the third pole 20 times further at –230.26. Multiplying the three closed-loop pole terms yields the desired characteristic equation: $s^3 + 253.28s^2 + 5681.19s + 87334.19 = 0$.

Representing the plant in observer canonical form: $\mathbf{A_{oc}} = \begin{bmatrix} -20 & 1 & 0 \\ -108 & 0 & 1 \\ -144 & 0 & 0 \end{bmatrix}$; $\mathbf{B_{oc}} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$;

$\mathbf{C_{oc}} = [1 \quad 0 \quad 0]$. The first column of $\mathbf{A_{oc}}$ contains the coefficients of the characteristic equation. Comparing the first column of $\mathbf{A_{oc}}$ to the desired characteristic equation and using Eq. (12.67), $l_1 = 253.28 - 20 = 233.28$; $l_2 = 5681.19 - 108 = 5573.19$; and $l_3 = 87334.19 - 144 = 87190.19$. Hence, $\mathbf{L_{oc}} = [233.28 \;\; 5573.19 \;\; 87190.19]^T$.

21.

The **A**, **L**, and **C** matrices for the phase-variable system are:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -21 & -10 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$$

Hence,

$$|\lambda - (\mathbf{A} - \mathbf{LC})| = \left|\begin{bmatrix} \lambda + l_1 & -1 & 0 \\ l_2 & \lambda & -1 \\ l_3 & 21 & \lambda + 10 \end{bmatrix}\right|$$

or

$$|\lambda - (\mathbf{A} - \mathbf{LC})| = \lambda^3 + (10 + l_1)\lambda^2 + (21 + 10l_1 + l_2)\lambda + (21l_1 + 10l_2 + l_3)$$

From Problem 19, the desired characteristic polynomial is $\lambda^3 + 360\lambda^2 + 23625\lambda + 1687500$. Equating coefficients yields:

$$10 + l_1 = 360;\; (21 + 10l_1 + l_2) = 23625;\; (21l_1 + 10l_2 + l_3) = 1687500$$

Solving successively,

$$l_1 = 350;\; l_2 = 20104;\; l_3 = 1479110$$

22.

The **A**, **L**, and **C** matrices for the phase-variable system are:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -45 & -14 \end{bmatrix}; \ \mathbf{C} = \begin{bmatrix} 2 & 1 \end{bmatrix}; \ \mathbf{L} = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$$

Hence,

$$|\lambda - (\mathbf{A} - \mathbf{LC})| = \left| \begin{bmatrix} \lambda + 2l_1 & l_1 - 1 \\ 2l_2 + 45 & l_2 + \lambda + 14 \end{bmatrix} \right|$$

or

$$\lambda^2 + (2l_1 + l_2 + 14)\lambda + (2l_2 - 17l_1 + 45)$$

From the problem statement, the desired characteristic polynomial is $\lambda^2 + 144\lambda + 14400$.

Equating coefficients yields,

$$(2l_1 + l_2 + 14) = 144; \ (2l_2 - 17l_1 + 45) = 14400$$

Solving simultaneously,

$$l_1 = -671.2; \ l_2 = 1472.4$$

23.

The **A** matrix for each part is given in the solution to Problem 11. Each observability matrix is calculated from Eq. (12.79).

a.

$$A = \begin{pmatrix} -2 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix}; \ C = (5, 5, 5); \ O_M = \begin{pmatrix} 5 & 5 & 5 \\ -10 & -10 & -10 \\ 20 & 20 & 20 \end{pmatrix}; \ |O_M| = 0; \text{ unobservable}$$

b.

$$A = \begin{pmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix}; \ C = (5, 0, 5); \ O_M = \begin{pmatrix} 5 & 0 & 5 \\ -10 & 5 & -15 \\ 20 & -20 & 45 \end{pmatrix}; \ |O_M| = 125; \text{ observable}$$

c.

$$A = \begin{pmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{pmatrix}; \ C = (1, 0, 0); \ O_M = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 16 & -4 & 1 \end{pmatrix}; \ |O_M| = 1; \text{ observable}$$

d.

$$A = \begin{pmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & 0 & -3 \end{pmatrix}; \ C = (1, 0, 0); \ O_M = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 16 & -4 & 1 \end{pmatrix}; \ |O_M| = 1; \text{ observable}$$

e.

$$A = \begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix}; \ C = (1, 0); \ O_M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \ |O_M| = 1; \text{ observable}$$

f.

$$A = \begin{pmatrix} -4 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -6 \end{pmatrix}; \ C = (1, 1, 1); \ O_M = \begin{pmatrix} 1 & 1 & 1 \\ -4 & -5 & -6 \\ 16 & 25 & 36 \end{pmatrix}; \ |O_M| = -2; \text{ observable}$$

24.

Program:

```
'(a)'
A=[-2 0 1;0 -2 0;0 0 -3]           %Form compensated A matrix
C=[5 5 5]                       %Form compensated C matrix
Om=obsv(A,C)                    %Form observability matrix
Rank=rank(Om)                   %Find rank of observability
                                %matrix
'(f)'
A=[-4 0 0;0 -5 0;0 0 -6]        %Form compensated A matrix
```

```
C=[1 1 1]                         %Form compensated C matrix
Om=obsv(A,C)                      %Form observability matrix
Rank=rank(Om)                     %Find rank of observability
```

Computer response:

```
ans =

(a)

A =

    -2     0     1
     0    -2     0
     0     0    -3

C =

     5     5     5

Om =

     5     5     5
   -10   -10   -10
    20    20    20

Rank =

     1

ans =

(f)

A =

    -4     0     0
     0    -5     0
     0     0    -6

C =

     1     1     1

Om =

     1     1     1
    -4    -5    -6
    16    25    36

Rank =

     3
```

25.

Representing the system in state space yields

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \;;\quad y = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \mathbf{x}$$

Using Eq. (12.79),

$$\mathbf{O_M} = \begin{bmatrix} c_1 & c_2 \\ -c_2 & (c_1 - 2c_2) \end{bmatrix} \text{ and } \det \mathbf{O_M} = c_1^2 - 2c_1c_2 + c_2^2$$

Thus, the system is unobservable if $c_1 = c_2$.

26.

The **A** and **C** matrices for the system represented in cascade form is

Az		
-20	1	0
0	-13	1
0	0	-5

Cz		
1	0	0

The characteristic polynomial found from the transfer function of the plant is

$$s^3 + 38s^2 + 425s + 1300$$

From this characteristic polynomial, we can write observer canonical form of the state equations. The **A** and **C** matrices of the observer canonical form are given below as

Ax		
-38	1	0
-425	0	1
-1300	0	0

Cx		
1	0	0

To test observability, we write the observability matrices for both systems and show that both observability matrices have non zero determinants. Using Eq. (12.79),

OMz	Observability Matrix of z		**OMx**	Observability Matrix of x	
1	0	0	1	0	0
-20	1	0	-38	1	0
400	-33	1	1019	-38	1
Det(OMz)	1		**Det(OMx)**	1	

Using Eq. (12.89), we obtain the transformation matrix, **P**, and its inverse as

P	Transformation Matrix z=Px		**PINV**		
1	0	0	1.00	0.00	0.00
-18	1	0	18.00	1.00	0.00
25	-5	1	65.00	5.00	1.00

Using the characteristic polynomial given in the problem statement, the plant's characteristic equation, and Eq. (12.67), the observer for the observer canonical system is

Lx	Observer for x
562	
39575	
1498700	

Using Eq. (12.92), the observer for the cascade system is found to be

Lz	Observer for z
562	
29459	
1314875	

27.

Program:

```
A=[-20 1 0;0 -13 1;0 0 -5]
B=[0;0;1]
C=[1 0 0]
D=0
poles=roots([1 600 40000 1500000])
L=acker(A',C',poles);
'L'
L'
```

Computer response:

```
A =

   -20     1     0
     0   -13     1
     0     0    -5

B =

     0
     0
     1

C =

     1     0     0

D =

     0

poles =

  1.0e+002 *

  -5.2985
  -0.3508 + 0.4001i
  -0.3508 - 0.4001i


ans =

L


ans =

  1.0e+006 *

    0.0006
    0.0295
    1.3149
```

28.

Expanding the plant by partial fractions, we obtain

$$G(s)=\frac{1}{(s+5)(s+13)(s+20)}=\frac{0.008333}{(s+5)}-\frac{0.017857}{(s+13)}+\frac{0.0095238}{(s+20)}$$

The **A** and **C** matrices for the system represented in parallel form is

Az		
-5	0	0
0	-13	0
0	0	-20

Cz		
1	1	1

The characteristic polynomial found from the transfer function of the plant is

$$s^3 + 38s^2 + 425s + 1300$$

From this characteristic polynomial, we can write the observer canonical form of the state equations. The **A** and **C** matrices of the observer canonical form are given below as

Ax		
-38	1	0
-425	0	1
-1300	0	0

Cx		
1	0	0

To test observability, we write the observability matrices for both systems and show that both observability matrices have non zero determinants. Using Eq. (12.79),

OMz	Observability Matrix of z		**OMx**	Observability Matrix of x	
1	1	1	1	0	0
-5	-13	-20	-38	1	0
25	169	400	1019	-38	1
Det(OMz)	-840		**Det(OMx)**	1	

Using Eq. (12.89), we obtain the transformation matrix, **P**, and its inverse as

P	Transformation Matrix z=Px		**PINV**		
0.2083333	-0.04166667	0.008333333	1.00	1.00	1.00
-3.017857	0.232142857	-0.01785714	33.00	25.00	18.00
3.8095238	-0.19047619	0.00952381	260.00	100.00	65.00

Using the characteristic polynomial given in the problem statement, the plant's characteristic equation, and Eq. (12.67), the observer for the observer canonical system is

Lx	Observer for x
562	

39575
1498700

Using Eq. (12.92), the observer for the parallel system is found to be

Lz	Observer for z
10957.29167	
-19271.4821	
8876.190476	

29.

Use Eqs. (4.39) and (4.42) to find $\zeta = 0.5912$ and $\omega_n = 135.328$ respectively. Factoring the denominator of Eq. (4.22), the required poles are $-80 \pm j109.15$. We place the third pole 10 times further at -800. Multiplying the three closed-loop pole terms yields the desired characteristic equation:

$s^3 + 960s^2 + 146313.746s + 14650996.915 = 0$.

Since $G(s) = \dfrac{50}{s^3 + 18s^2 + 99s + 162}$, the plant in observer canonical form is: $\mathbf{A_{oc}} = \begin{bmatrix} -18 & 1 & 0 \\ -99 & 0 & 1 \\ -162 & 0 & 0 \end{bmatrix}$;

$\mathbf{B_{oc}} = \begin{bmatrix} 0 \\ 0 \\ 50 \end{bmatrix}$; $\mathbf{C_{oc}} = [1 \quad 0 \quad 0]$. Using Eq. (12.79), $\mathbf{O_{Moc}} = \begin{bmatrix} 1 & 0 & 0 \\ -18 & 1 & 0 \\ 225 & -18 & 1 \end{bmatrix}$, which is observable since the determinant is 1. Since $G(s) = \dfrac{50}{s^3 + 18s^2 + 99s + 162}$, the phase-variable form is

$\mathbf{A_{pv}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -162 & -99 & -18 \end{bmatrix}$; $\mathbf{B_{pv}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; $\mathbf{C} = [50 \quad 0 \quad 0]$. Using Eq. (12.79),

$\mathbf{O_{Mpv}} = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 50 \end{bmatrix}$, which is observable since the determinant is 125000. The first column of $\mathbf{A_{oc}}$ contains the negatives values of the coefficients of the characteristic equation. Comparing the first column of $\mathbf{A_{oc}}$ to the desired characteristic equation and using Eq. (12.67), $l_1 = 960\text{-}18 = 942$; $l_2 = 146313.746\text{-}99 = 146214.746$; and $l_3 = 14650996.915\text{-}162 = 14650834.915$. Hence, $\mathbf{L_{oc}} = [942 \quad 146214.746 \quad 14650834.915]$. The transformation matrix is,

$$\mathbf{P} = \mathbf{O_{Mpv}}^{-1}\mathbf{O_{Moc}} = \begin{bmatrix} 0.02 & 0 & 0 \\ -0.36 & 0.02 & 0 \\ 4.5 & -0.36 & 0.02 \end{bmatrix}$$

Transforming back to the original system, $\mathbf{L_{pv}} = \mathbf{PL_{oc}} = [18.84 \quad 2585.175 \quad 244618.39]^T$.

30.

The open-loop transfer function of the plant is $T(s) = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} = \frac{s+2}{s^2-s-2}$.

Using Eqs. (12.115), the closed-loop state equations with integral control is

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_N \end{pmatrix} = \begin{pmatrix} -1 & 1 & 0 \\ -k_1 & -k_2+2 & k_e \\ -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_N \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r \quad ; \quad y = (1, 1, 0) \begin{pmatrix} x_1 \\ x_2 \\ x_N \end{pmatrix}$$

The characteristic polynomial is

$$s^3 + (k_2-1)s^2 + (k_2 + k_1 + k_e - 2)s + 2k_e$$

The desired characteristic polynomial is calculated from the desired transient response stated in the problem. Also, the third pole will be placed to cancel the zero at -2. Hence, the desired characteristic polynomial is

$$(s + 2)(s^2 + 16s + 183.137) = s^3 + 18s^2 + 215.14s + 366.27$$

Equating coefficients of the characteristic polynomials yields,

$$k_e = 183.135,\ k_2 = 19,\ k_1 = 15.005$$

31.

The open-loop transfer function of the plant is $T(s) = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} = \frac{s+3}{s^2+7s+10}$.

Using Eqs. (12.115), the closed-loop state equations with integral control is

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_N \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ -k_1 & -(5+k_2) & k_e \\ -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_N \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r \quad ; \quad y = (1, 1, 0) \begin{pmatrix} x_1 \\ x_2 \\ x_N \end{pmatrix}$$

The characteristic polynomial is

$$s^3 + (k_2 + 7)s^2 + (2k_2 + k_1 + k_e + 10)s + 3k_e$$

The desired characteristic polynomial is calculated from the desired transient response stated in the problem. Also, the third pole will be placed to cancel the zero at -3. Hence, the desired characteristic polynomial is

$$(s + 3)(s^2 + 16s + 183.14) = s^3 + 19s^2 + 231.14s + 549.41$$

Equating coefficients of the characteristic polynomials yields,

$$k_e = 183.137,\ k_2 = 12,\ k_1 = 14.003$$

SOLUTIONS TO DESIGN PROBLEMS

32.

Writing the **A** and **B** matrices for (G(s) represented in phase-variables form,

A			B
0	1	0	0
0	0	1	0
1.30E+06	4551	-286	10

From the phase-variable from, the characteristic polynomial is $s^3 + 286s^2 - 4551s - 1301586$.

Finding the controllability matrix and it's determinant shows that there is controllability,

CM		
0	0	10
0	10	-2860
10	-2860	863470
Det(CM)	-1000	

Using the given transient requirements, and arbitrarily placing the third closed-loop pole more than 5 times further than the dominant pair at -50 yields the following desired closed-loop characteristic polynomial:

$$(s^2 + 16s + 134.384)(s + 50) = s^3 + 66s^2 + 934.4s + 6719.2$$

Using the phase-variable system with state feedback the characteristic polynomial is

$$s^3 + (k_3 + 286)s^2 + (k_2 - 4551)s + (k_1 - 1301586)$$

Equating the two characteristic polynomials yields the state feedback vector for the phase-variable system as

K		
1308305.2	5485.4	-220

33.

Controller design:

The transfer function for the plant is

$$G(s) = \frac{5}{(s+0.4)(s+0.8)(s+5)} = \frac{5}{s^3+6.2s^2+6.32s+1.6}$$

The characteristic polynomial for the plant with phase-variable state feedback is

$$s^3 + (6.2 + k_3)s^2 + (6.32 + k_2)s + (1.6 + k_1)$$

Using the given transient response of 5% overshoot and T_s = 10 minutes, and placing the third pole ten times further from the imaginary axis than the dominant pair, the desired characteristic equation is

$$(s + 4)(s^2 + 0.8s + 0.336) = s^3 + 4.8s^2 + 3.536s + 1.344.$$

Comparing the two characteristic equations, k_1 = - 0.256, k_2 = -2.784, and k_3 = -1.4.

Observer design:

The **A** and **C** matrices for the system represented in phase-variable form is

Az

0	1	0
0	0	1
-1.6	-6.32	-6.2

Cz

5	0	0

The characteristic polynomial found from the transfer function of the plant is

$$s^3 + 6.2s^2 + 6.32s + 1.6$$

From this characteristic polynomial, we can write the dual phase-variable form of the state equations. The **A** and **C** matrices of the dual phase-variable form are given below as

Ax

-6.2	1	0
-6.32	0	1
-1.6	0	0

Cx

1	0	0

To test observability, we write the observability matrices for both systems and show that both observability matrices have nonzero determinants. Using Eq. (12.79),

OMz	Observability Matrix of z		**OMx**	Observability Matrix of x	
5	0	0	1	0	0
0	5	0	-6.2	1	0
0	0	5	32.12	-6.2	1
Det(OMz)	125		**Det(OMx)**	1	

Using Eq. (12.89), we obtain the transformation matrix, **P**, and its inverse as

P	Transformation Matrix z=Px		**PINV**		
0.2	0	0	5.00	0.00	0.00
-1.24	0.2	0	31.00	5.00	0.00
6.424	-1.24	0.2	31.60	31.00	5.00

Using the characteristic polynomial given in the problem statement, the observer for the dual phase-variable system is

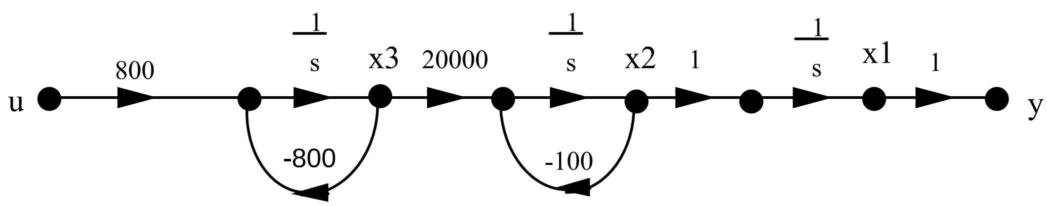
Lx
41.8
347.28
1342.4

Using Eq. (12.92), the observer for the cascade system is found to be

Lz
8.36
17.624
106.376

34.

a. Using the following signal-flow graph,



the plant is represented in state space with

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -100 & 20000 \\ 0 & 0 & -800 \end{bmatrix};\ \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 800 \end{bmatrix};\text{ and } \mathbf{C} = [1 \quad 0 \quad 0].$$

Using Eq. (12.26),

$$\mathbf{C_M} = \begin{bmatrix} 0 & 0 & 1.6E07 \\ 0 & 1.6E07 & -1.44E10 \\ 800 & -6.4E05 & 5.12E08 \end{bmatrix}$$

The system is controllable since the determinant of $\mathbf{C_M}$ = -2.04e^{17}. Use Eqs. (4.39) and (4.42) to find ζ = 0.5912 and ω_n = 135.3283 respectively. Factoring the denominator of Eq. (4.22), the required poles are -80 ± j109.15. Place the third pole 10 times farther at = 800. Multiplying the three closed-loop pole terms yields the desired characteristic equation

$$s^3 + 960s^2 + \quad 1.463E05s + 1.4651E07 = 0.$$

Since the plant's characteristic equation is $s^3 + 900s^2 + 80000s$, we write the plant in controller canonical form as

$$\mathbf{A_{cc}} = \begin{bmatrix} -900 & -80000 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix};\ \mathbf{B_{cc}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};\text{ and } \mathbf{C_{cc}} = [0 \quad 0 \quad 1.6E07]$$

The controllability matrix for controllable canonical form is

$$\mathbf{C_{Mcc}} = \begin{bmatrix} 1 & -900 & 730000 \\ 0 & 1 & -900 \\ 0 & 0 & 1 \end{bmatrix}$$

Comparing the first row of $\mathbf{A_{cc}}$ to the desired characteristic equation and using the results of Problem 5, k_1 = -(900 - 960) = 60; k_2 = -(80000 - 1.463E05) = 66300; and k_3 = -(0 - 1.465E07) = 1.465E07. Hence.

$$\mathbf{K_{cc}} = [60 \quad 66300 \quad 1.465\text{E}07]$$

The transformation matrix is,

$$\mathbf{P} = \mathbf{C_M C_{Mcc}}^{-1} = \begin{bmatrix} 0 & 150 & 1.6\text{E}07 \\ 0 & 1.6\text{E}07 & 0 \\ 800 & 8\text{E}04 & 0 \end{bmatrix}$$

Transforming back to the original system,

$$\mathbf{K} = \mathbf{K_{cc}P}^{-1} = [9.1569\text{E-}01 \quad 3.7696\text{E-}03 \quad 7.5\text{E-}02]$$

The controller compensated system is

$$\mathbf{A - BK} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -100 & 20000 \\ -732.55 & -3.0157 & -860 \end{bmatrix}; \ \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 800 \end{bmatrix}; \ \mathbf{C} = [1 \quad 0 \quad 0]$$

b. To evaluate the steady-state error, use Eq. (7.89) where

$$\mathbf{A - BK} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -100 & 20000 \\ -732.55 & -3.0157 & -860 \end{bmatrix}$$

is the system matrix. Thus,

$$(sI - [A - BK])^{-1} =$$

$$\frac{1}{s^3 + 960s^2 + 1.4631\times10^5 s + 14651040}\begin{pmatrix} s^2 + 960s + 1.4631\times10^5 & s + 860 & 20000 \\ -14651040 & s^2 + 860s & 20000s \\ -732.55s - 73255 & -3.0157s - 732.55 & s^2 + 100s \end{pmatrix}$$

The steady-state error is given by

$$sR(s)[1 - C(sI - [A - BK])^{-1}B] \text{ as s->0}$$

For a step input, R(s) = 1/s. Since

$$1 - C(sI - [A - BK])^{-1}B = 1 - \frac{1}{s^3 + 960s^2 + 1.4631\times10^5 s + 14651040}\cdot 16000000$$

for a step input $e(\infty)$ = -0.092073. Using Eqs. 12.115, the system with integral control is:

$$A_I = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -100 & 20000 & 0 \\ -800K_1 & -800K_2 & -800K_3 - 800 & 800K_e \\ -1 & 0 & 0 & 0 \end{pmatrix}; \ B_I = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix};$$

$$C_I = (1, 0, 0, 0)$$

Assume the following desired characteristic equation:

$$(s^3 + 960s^2 + 1.463\text{E}05s + 1.4651\text{E}07)(s + 1000) =$$

$$s^4 + 1960s^3 + 1.1063\times10^6 s^2 + 1.6096\times10^8 s + 1.4651\times10^{10} = 0,$$

which is the desired characteristic equation from part (a) plus an additional pole at -1000. But the integral controlled system characteristic equation is

$|s\mathbf{I} - \mathbf{A_I}| = s^4 + 100\,(8K_3+9)\,s^3 + 80000\,(K_3+200K_2+1)\,s^2 + 16000000\,K_1\,s + 16000000\,K_e$

Equating coefficients to the desired characteristic equation

$100\,(8K_3+9) = 1960$; $80000\,(K_3+200K_2+1) = 1.1063\times10^6$; $16000000K_1 = 1.6096\times10^8$; $16000000\,K_e = 1.4651\times10^{10}$

Solving for the controller gains: $K_e = 915.69$; $K_1 = 10.06$; $K_2 = 0.05752$; and $K_3 = 1.325$.

Substituting into $\mathbf{A_I}$ yields the integral controlled system.

$$A_I = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -100 & 20000 & 0 \\ -8048.2 & -46.016 & -1860 & 7.3255\times10^5 \\ -1 & 0 & 0 & 0 \end{pmatrix};\ B_I = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix};\ C_I = (1,0,0,0)$$

Finding the characteristic equation as a check yields

$$s^4 + 1960s^3 + 1.1063\times10^6 s^2 + 1.6096\times10^8 s + 1.4651\times10^{10}$$

which checks with the desired characteristic equation. Now check the steady-state error using Eq. (7.89) using the integral controlled system. We find the error is zero.

c.

Program:

```
 'Controller Compensated'
A=[0 1 0;0 -100 20000;-732.55 -3.0157 -860];
B=[0;0;800];
C=[1 0 0];
D=0;
S=ss(A,B,C,D)
step(S)
title('Controller Compensated')
pause
'Integral Controller'
A=[0 1 0 0;0 -100 20000 0;-8048.2 -46.016 -1860 7.3255e05;-1 0 0 0];
B=[0;0;0;1];
C=[1 0 0 0];
D=0;
S=ss(A,B,C,D)
step(S)
title('Integral Controller')
```

Computer response:

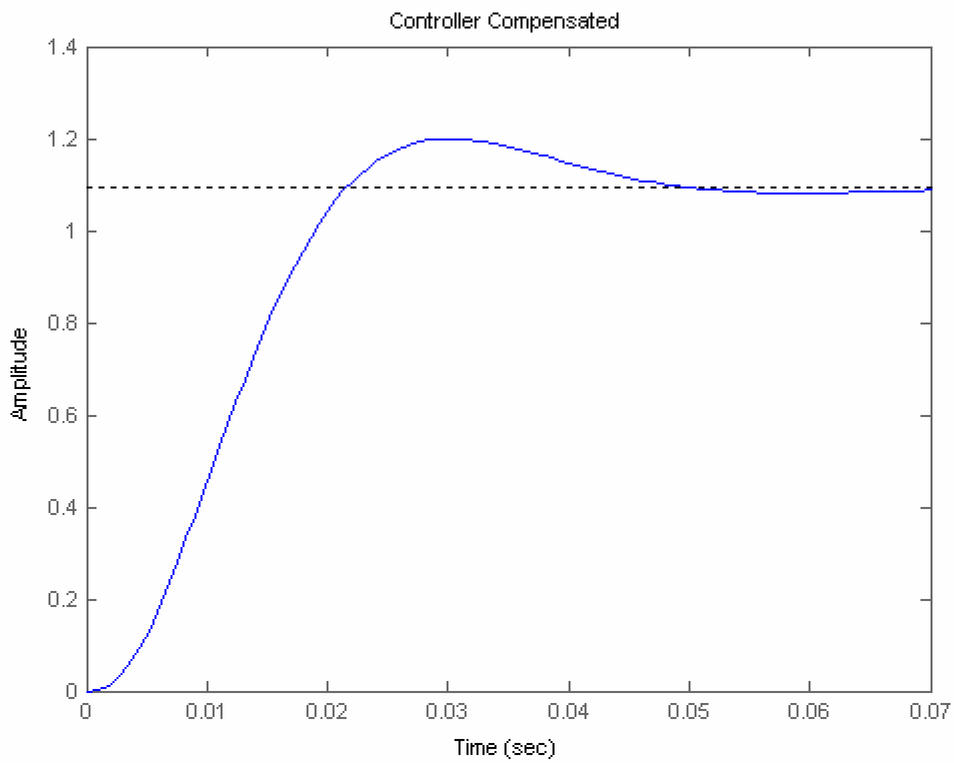
ans =

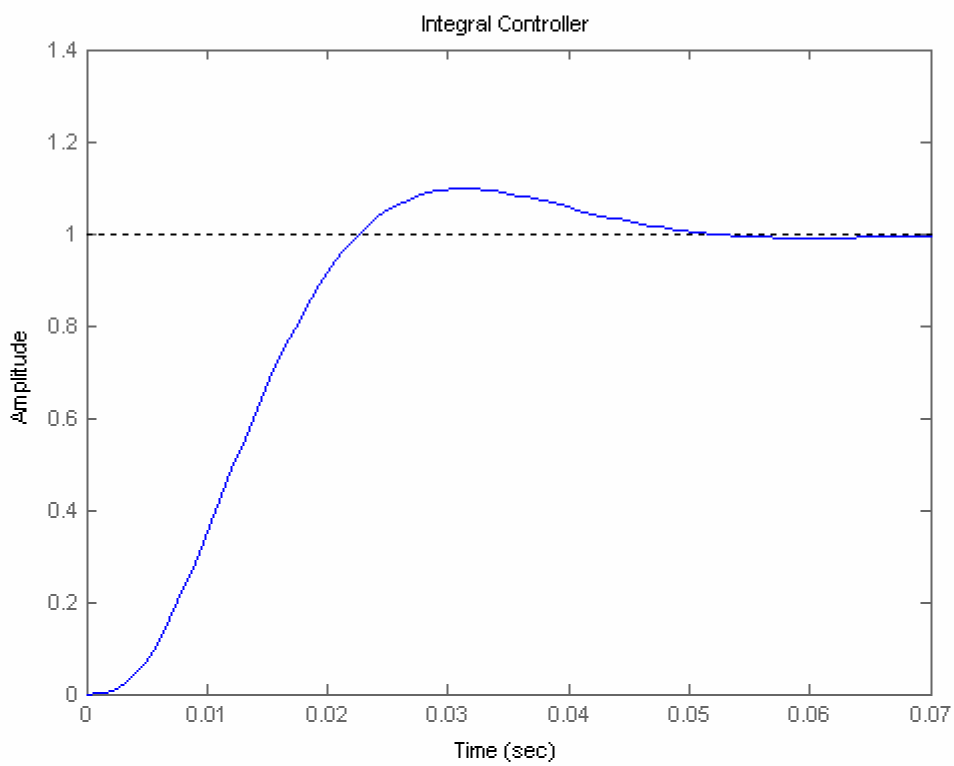
Controller Compensated

a =

	x1	x2	x3
x1	0	1	0
x2	0	-100	2e+004
x3	-732.5	-3.016	-860

```
b =
        u1
   x1    0
   x2    0
   x3  800


c =
       x1  x2  x3
   y1   1   0   0


d =
       u1
   y1   0

Continuous-time model.

ans =

Integral Controller


a =
               x1          x2          x3          x4
   x1           0           1           0           0
   x2           0        -100     2e+004           0
   x3       -8048      -46.02       -1860  7.326e+005
   x4          -1           0           0           0


b =
       u1
   x1   0
   x2   0
   x3   0
   x4   1


c =
       x1  x2  x3  x4
   y1   1   0   0   0


d =
       u1
   y1   0

Continuous-time model.
```


Controller Compensated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
Time (sec)


Integral Controller
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
Time (sec)

35.

Program:

```
%Enter G(s)
numg=0.072*conv([1 23],[1 0.05 0.04]);
deng=conv([1 0.08 0.04],poly([0.7 -1.7]));
'G(s)'
G=tf(numg,deng)
'Plant Zeros'
plantzeros=roots(numg)
%Input transient response specifications
Po=input('Type %OS  ');
Ts=input('Type settling time   ');

%Determine pole location
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
wn=4/(z*Ts);
%wn=pi/(Tp*sqrt(1-z^2));
[num,den]=ord2(wn,z);
r=roots(den);
poles=[r(1) r(2) plantzeros(2) plantzeros(3)]
characteristiceqdesired=poly(poles)


%Find controller canonical form of state-space representation of G(s)
'Controller Canonical Form'
[Ac Bc Cc Dc]=tf2ss(numg,deng)

%Design controller gains
Kc=acker(Ac,Bc,poles)
Acnew=Ac-Bc*Kc
Bcnew=Bc
Ccnew=Cc
Dcnew=Dc
characteristiceqcontroller=poly(eig(Acnew))

%Transform to phase-variable form
P=[0 0 0 1;0 0 1 0;0 1 0 0;1 0 0 0];
'Phase-variable form'
Ap=inv(P)*Ac*P
Bp=inv(P)*Bc
Cp=Cc*P
Dp=Dc
Kp=acker(Ap,Bp,poles)
Apnew=Ap-Bp*Kp
Bpnew=Bp
Cpnew=Cp
Dpnew=Dp
characteristiceqphase=poly(eig(Apnew))
[numt,dent]=ss2tf(Apnew,Bpnew,Cpnew,Dpnew);
T=tf(numt,dent);
'T(s)'
T=minreal(T)
step(T)
'T(s)'
Tzpk=zpk(T)
'Poles of T(s)'
p=pole(T
```

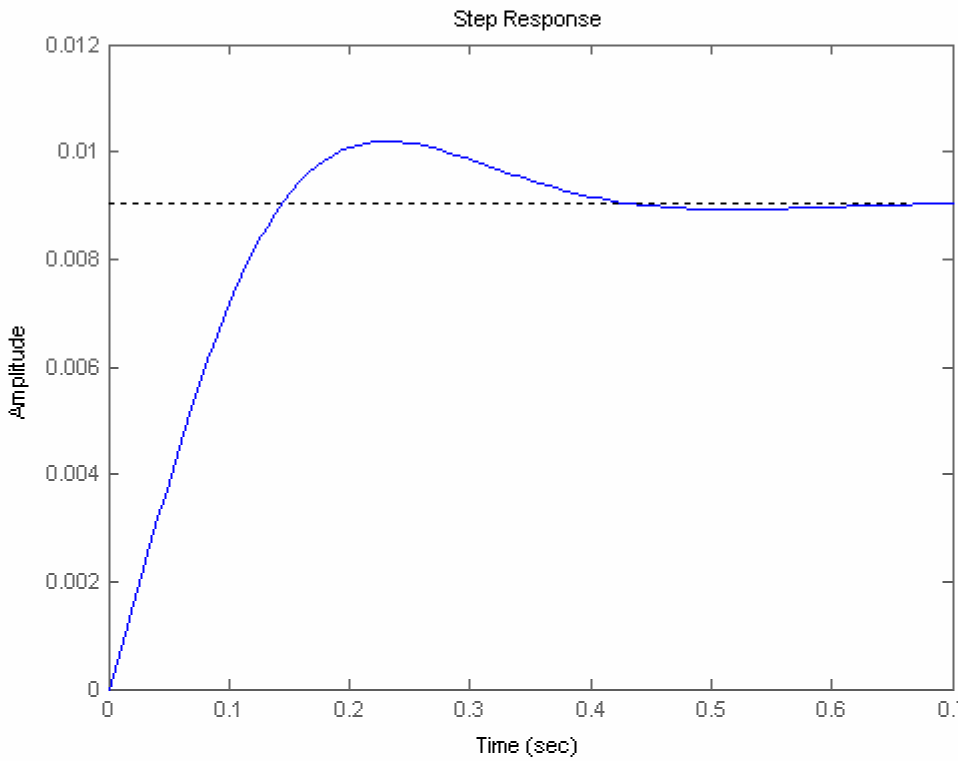
Computer response:

```
ans =
```

```
G(s)

Transfer function:
 0.072 s^3 + 1.66 s^2 + 0.08568 s + 0.06624
---------------------------------------------
s^4 + 1.08 s^3 - 1.07 s^2 - 0.0552 s - 0.0476


ans =

Plant Zeros


plantzeros =

 -23.0000
  -0.0250 + 0.1984i
  -0.0250 - 0.1984i

Type %OS  10
Type settling time   0.5

poles =

  -8.0000 +10.9150i  -8.0000 -10.9150i  -0.0250 + 0.1984i  -0.0250 -
0.1984i


characteristiceqdesired =

    1.0000   16.0500  183.9775    9.7969    7.3255


ans =

Controller Canonical Form


Ac =

   -1.0800    1.0700    0.0552    0.0476
    1.0000         0         0         0
         0    1.0000         0         0
         0         0    1.0000         0


Bc =

     1
     0
     0
     0


Cc =

    0.0720    1.6596    0.0857    0.0662


Dc =

     0


Kc =
```

```
   14.9700  185.0475    9.8521    7.3731


Acnew =

  -16.0500 -183.9775   -9.7969   -7.3255
    1.0000         0         0         0
         0    1.0000         0         0
         0         0    1.0000         0


Bcnew =

     1
     0
     0
     0


Ccnew =

    0.0720    1.6596    0.0857    0.0662


Dcnew =

     0


characteristiceqcontroller =

    1.0000   16.0500  183.9775    9.7969    7.3255


ans =

Phase-variable form


Ap =

         0    1.0000         0         0
         0         0    1.0000         0
         0         0         0    1.0000
    0.0476    0.0552    1.0700   -1.0800


Bp =

     0
     0
     0
     1


Cp =

    0.0662    0.0857    1.6596    0.0720


Dp =

     0

Kp =
```

```
    7.3731    9.8521  185.0475   14.9700


Apnew =

         0    1.0000         0         0
         0         0    1.0000         0
         0         0         0    1.0000
   -7.3255   -9.7969 -183.9775  -16.0500


Bpnew =

     0
     0
     0
     1


Cpnew =

    0.0662    0.0857    1.6596    0.0720


Dpnew =

     0


characteristiceqphase =

    1.0000   16.0500  183.9775    9.7969    7.3255


ans =

T(s)


Transfer function:
 0.072 s + 1.656
------------------
s^2 + 16 s + 183.1


ans =

T(s)


Zero/pole/gain:
    0.072 (s+23)
--------------------
(s^2  + 16s + 183.1)


ans =

Poles of T(s)


p =

  -8.0000 +10.9150i
  -8.0000 -10.9150i
```



36.

Program:

```
%Enter G(s)
numg=0.072*conv([1 23],[1 0.05 0.04]);
deng=conv([1 0.08 0.04],poly([0.7 -1.7]));
'Uncompensated Plant Transfer Function'
'G(s)'
G=tf(numg,deng)
'Uncompensated Plant Zeros'
plantzeros=roots(numg)
%Input transient response specifications
Po=input('Type %OS  ');
Ts=input('Type settling time   ');

%Determine pole location
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
wn=4/(z*Ts);
%wn=pi/(Tp*sqrt(1-z^2));
[num,den]=ord2(wn,z);
r=roots(den);
'Desired Observer Poles'
poles=[r(1) r(2) plantzeros(2) plantzeros(3)]'
'Desired Characteristic Equation of Observer'
poly(poles)


%Find phase variable form of state-space representation of Estimated Plant
%Find controller canonical form
[Ac Bc Cc Dc]=tf2ss(numg,deng);

%Transform to phase-variable form of Uncompensated Plant
P=[0 0 0 1;0 0 1 0;0 1 0 0;1 0 0 0];
'Phase-variable form of Estimated Plant'
Ap=inv(P)*Ac*P
```

```
Bp=inv(P)*Bc
Cp=Cc*P
Dp=Dc

%Design observer gains for phase variables
'Observer gains'
Lp=acker(Ap',Cp',poles)'
 'Error System Matrix'
Aep=Ap-Lp*Cp
'Error System Eigenvalues'
eig(Aep)
'Error Characteristic Polynomial'
poly(eig(Aep))
```

Computer response:

```
 ans =

Uncompensated Plant Transfer Function


ans =

G(s)


Transfer function:
 0.072 s^3 + 1.66 s^2 + 0.08568 s + 0.06624
---------------------------------------------
s^4 + 1.08 s^3 - 1.07 s^2 - 0.0552 s - 0.0476


ans =

Uncompensated Plant Zeros


plantzeros =

 -23.0000
  -0.0250 + 0.1984i
  -0.0250 - 0.1984i

Type %OS  10
Type settling time   0.5/15

ans =

Desired Observer Poles


poles =

  1.0e+002 *

  -1.2000 - 1.6373i
  -1.2000 + 1.6373i
  -0.0003 - 0.0020i
  -0.0003 + 0.0020i


ans =

Desired Characteristic Equation of Observer
```

```
ans =

  1.0e+004 *

    0.0001    0.0240    4.1218    0.2070    0.1648


ans =

Phase-variable form of Estimated Plant


Ap =

         0    1.0000         0         0
         0         0    1.0000         0
         0         0         0    1.0000
    0.0476    0.0552    1.0700   -1.0800


Bp =

     0
     0
     0
     1


Cp =

    0.0662    0.0857    1.6596    0.0720


Dp =

     0


ans =

Observer gains


Lp =

  1.0e+004 *

   -0.0002
    0.0043
   -0.0986
    2.5994


ans =

Error System Matrix


Aep =

  1.0e+004 *

    0.0000    0.0001    0.0003    0.0000
   -0.0003   -0.0004   -0.0071   -0.0003
    0.0065    0.0084    0.1636    0.0072
   -0.1722   -0.2227   -4.3139   -0.1873
```

```
ans =

Error System Eigenvalues


ans =

  1.0e+002 *

  -1.2000 + 1.6373i
  -1.2000 - 1.6373i
  -0.0003 + 0.0020i
  -0.0003 - 0.0020i


ans =

Error Characteristic Polynomial


ans =

  1.0e+004 *

    0.0001    0.0240    4.1218    0.2070    0.1648
```

37.

a. Using Eqs. (12.115), the system with integral control is:

$$A_I = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -K_1+0.0476 & -K_2+0.0552 & -K_3+1.07 & -K_4-1.08 & K_e \\ -0.06624 & -0.08568 & -1.6596 & -0.072 & 0 \end{pmatrix}; B_I = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix};$$

$$C_I = (1, 0, 0, 0, 0)$$

Assume the following desired characteristic equation,

$$(s+8+10.915i)(s+8-10.915i)(s+0.025+0.1984i)(s+0.025-0.1984i)(s+23) =$$
$$s^5+39.05s^4+553.13s^3+4241.3s^2+232.65s+168.43$$

which is the desired characteristic equation from Problem 35 plus an additional pole at -23, the closed-loop zero. But the integral controlled system characteristic equation is $|s\mathbf{I} - \mathbf{A_I}| =$

$$s^5+(K_4+1.08)s^4+(K_3+0.072K_e-1.07)s^3+(K_2+1.6596K_e-0.0552)s^2+(K_1+0.08568K_e-0.0476)s+0.06624K_e$$

Equating coefficients to the desired characteristic equation

$$K_4+1.08 = 39.05;\ K_3+0.072K_e-1.07 = 553.13;\ K_2+1.6596K_e-0.0552 = 4241.3;$$
$$K_1+0.08568K_e-0.0476 = 232.65;\text{ and } 0.06624K_e = 168.43$$

Solving for the controller gains

$$K_1 = 14.829;\ K_2 = 21.328;\ K_3 = 371.12;\ K_4 = 37.97 \text{ and } K_e = 2542.8$$

Substituting into $\mathbf{A_I}$ yields the integral controlled system,

$$A_I = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -14.781 & -21.272 & -370.05 & -39.05 & 2542.8 \\ -0.06624 & -0.08568 & -1.6596 & -0.072 & 0 \end{pmatrix}; B_I = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix};$$

$$C_I = (0.06624, 0.08568, 1.6596, 0.072, 0)$$

Finding the characteristic equation as a check yields

$$s^5 + 39.05s^4 + 553.13s^3 + 4241.3s^2 + 232.65s + 168.43$$

which checks with the desired. Now check the steady-state error using Eq. (7.89) using the integral controlled system. We find the error is zero.

b.

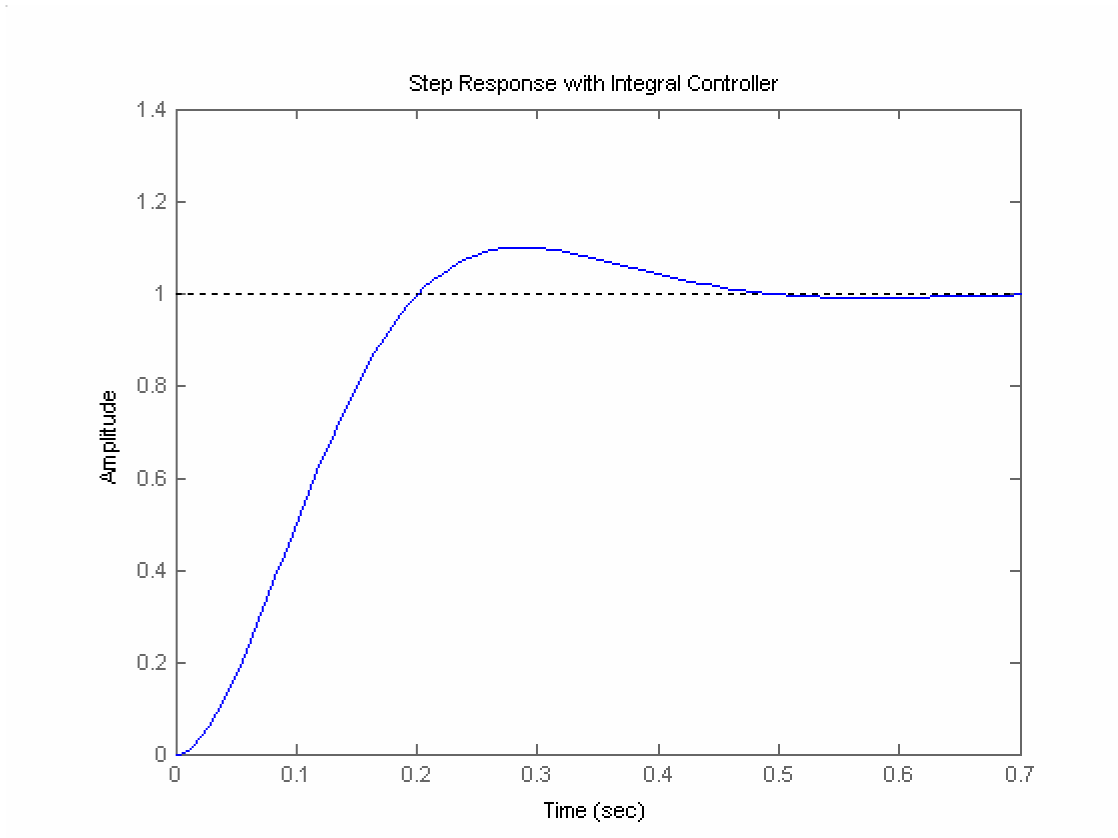
Program:

```
%Design with Integral Control
'State-Space Representation of System with Integral Control'
AI=[0 1  0 0 0;0 0 1 0 0;0 0 0 1 0;...
-14.781 -21.272 -370.05 -39.05 2542.8;...
-0.06624 -0.08568 -1.6596 -0.072 0]
BI=[0;0;0;0;1]
CI=[0.06624 0.08568 1.6596 0.072 0]
DI=0

%Convert to transfer function
[numt,dent]=ss2tf(AI,BI,CI,DI);
'Integral Control Transfer Function'
'T(s)'
T=tf(numt,dent)
'Integral Control Transfer Function Zeros'
roots(numt)
'Integral Control Transfer Function Poles'
roots(dent)
step(T)
title('Step Response with Integral Controller')
```

Computer response:

```
 ans =

State-Space Representation of System with Integral Control


AI =

  1.0e+003 *

         0    0.0010         0         0         0
         0         0    0.0010         0         0
         0         0         0    0.0010         0
   -0.0148   -0.0213   -0.3700   -0.0390    2.5428
   -0.0001   -0.0001   -0.0017   -0.0001         0


BI =

     0
     0
     0
     0
```

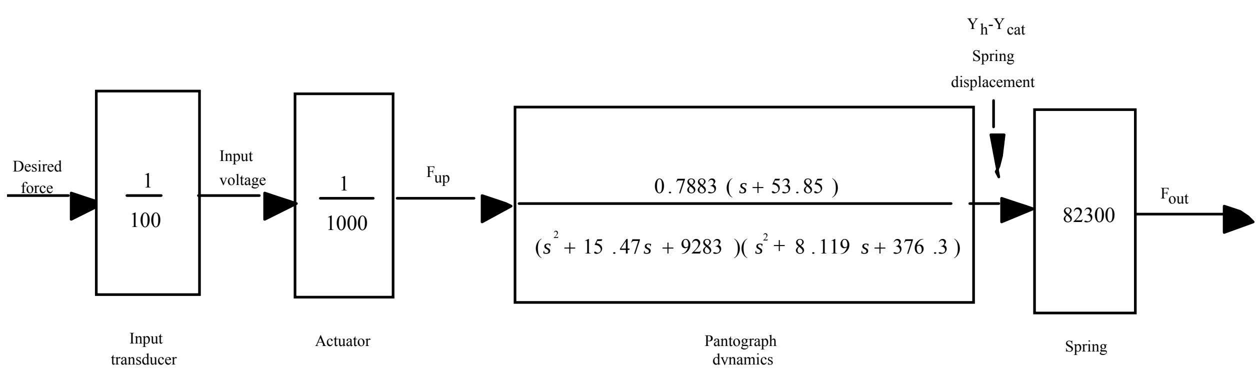
```
     1


CI =

    0.0662    0.0857    1.6596    0.0720         0


DI =

     0


ans =

Integral Control Transfer Function


ans =

T(s)


Transfer function:
-1.421e-014 s^4 + 183.1 s^3 + 4220 s^2 + 217.9 s + 168.4
--------------------------------------------------------
s^5 + 39.05 s^4 + 553.1 s^3 + 4241 s^2 + 232.6 s + 168.4


ans =

Integral Control Transfer Function Zeros


ans =

  1.0e+016 *

   1.2883
  -0.0000
  -0.0000 + 0.0000i
  -0.0000 - 0.0000i


ans =

Integral Control Transfer Function Poles


ans =

 -22.9998
  -8.0001 +10.9151i
  -8.0001 -10.9151i
  -0.0250 + 0.1984i
  -0.0250 - 0.1984i
```



38.

a.

The open-loop block diagram is



From Chapter 3, the state-space representation for $[Y_h(s) – Y_{cat}(s)]/F_{up}(s)$ is

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -9353 & -14.286 & 769.23 & 14.286 \\ 0 & 0 & 0 & 1 \\ 406.98 & 7.5581 & -406.98 & -9.3023 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0581 \end{bmatrix} f_{up}$$

$$y = \begin{bmatrix} 0.94911 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}$$

where $y = y_h - y_{cat}$ and $\mathbf{x} = \begin{bmatrix} y_h \\ \dot{y}_h \\ y_f \\ \dot{y}_f \end{bmatrix}$

Let v_i represent the input voltage shown on the diagram. Thus,

$$f_{up} = v_i/1000.$$

Also, $f_{out} = 82300(y_h - y_{cat})$.

Thus,

$$f_{out} = 82300y$$

Substituting f_{up} and f_{out} into the state-equations above yields

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -9353 & -14.286 & 769.23 & 14.286 \\ 0 & 0 & 0 & 1 \\ 406.98 & 7.5581 & -406.98 & -9.3023 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0581\text{x}10^{-3} \end{bmatrix} v_i$$

$$f_{out} = \begin{bmatrix} 78{,}112 & 0 & 0 & 0 \end{bmatrix}\mathbf{x}$$

Thus,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -9353 & -14.286 & 769.23 & 14.286 \\ 0 & 0 & 0 & 1 \\ 406.98 & 7.5581 & -406.98 & -9.3023 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0581\text{x}10^{-3} \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}$$

Hence,

A-BK =

[0 , 1 , 0 , 0

[-9350 , -14.3 , 769 , 14.3

[0 , 0 , 0 , 1

[407 - 0.0000581 k1 , 7.56 - .0000581 k2 , -407 - 0.0000581 k3 , -9.30 - 0.0000581 k4] and

$$|\mathbf{A\text{-}BK}| = s^4 + (0.0000581\ k4 + 23.60)\ s^3$$
$$+ (0.00083083\ k4 + 0.00083083\ k2 + 9781.882 + 0.0000581\ k3)\ s^2$$
$$+ (0.00083083\ k1 + 81141.36 + 0.543235\ k4 + 0.00083083\ k3 + 0.0446789\ k2)\ s$$
$$+(0.0446789\ k1 + 0.3492467\ 10^7 + 0.543235\ k3)$$

Input transient response specifications,

Po = 20

Ts = 1

yields poles at

-4.0000 + 7.8079i, -4.0000 - 7.8079i, -53.8500, -50.0000

Thus, the desired characteristic equation is

$$s^4 + 112s^3 + 3600s^2 + 29500s + 207000 = 0$$

We now equate the coefficients of |**A-BK**| to the coefficients of the desired characteristic equation. For compactness we solve for the coefficients, **K**, using the form **FK** = **G**, where

F =

0	0	0	0.0000581
0	0.00083083	0.0000581	0.00083083
0.00083083	0.0446789	0.00083083	0.543235
0 .0446789	0	0.543235	0

and

G =
88.4
–6181.882
-51641.36
-3285467

Solving for K using $K = F^{-1}G$

K =
-4.8225e8
-0.1131e8
0.3361e8
0.0152e8

b.

Integral Control Design

A =
1.0e+03 *

0	0.0010	0	0
-9.3530	-0.0143	0.7692	0.0143
0	0	0	0.0010
0.4070	0.0076	-0.4070	-0.0093

B =
1.0e-04 *
0
0
0
0.5810

C =

78112 0 0 0

Aaug = A-BK =

[0 , 1. , 0 , 0 , 0

[-9350. , -14.3 , 769. , 14.3 , 0

[0 , 0 , 0 , 1. , 0

[407. - 0.0000581 k1 , 7.56 - 0.0000581 k2 , -407. - 0.0000581 k3 ,

-9.30 - 0.0000581 k4 , 0.0000581 Ke

[-78100. , 0 , 0 , 0 , 0]

Desired poles

Po = 20

Ts = 1

Determine pole location

poles = -4.0000 + 7.8079i, -4.0000 - 7.8079i, -53.85, -50 , -50

Desired characteristic equation

$$s^5 + 162s^4 + 0.919e4s^3 + 0.210e6s^2 + 0.168e7s + 0.104e8$$

System characteristic equation

|sI-Aaug| =

s^5 + (23.60 + 0.0000581 k4) s^4

+ (0.00083083 k4 + 0.00083083 k2 + 0.0000581 k3 + 9781.882) s^3

+ (0.00083083 k1 + 81141.36 + 0.0446789 k2 + 0.543235 k4 + 0.00083083 k3) s^2

+ (0.0446789 k1 + 64.887823 Ke + 0.543235 k3 + 0.3492467 10^7) s

+ 3489.42209 Ke

Solving for Coefficients, **K**, using **FK** = **G** as in (a), where

F

0	0	0	5.8100e-05	0
0	8.3083e-04	5.8100e-05	8.3083e-04	0
8.3083e-04	4.4679e-02	8.3083e-04	5.4324e-01	0
4.4679e-02	0	5.4324e-01	0	6.4888e+01
0	0	0	0	3.4894e+03

G =

1.3840e+02
-5.9188e+02
1.2886e+05
-1.8125e+06
1.0800e+07

Thus,

K =

-1.0157e+09
-8.6768e+06
7.9827e+07
2.3821e+06

3.0951e+03