E L E V E N

Design via Frequency Response

SOLUTIONS TO CASE STUDIES CHALLENGES

Antenna Control: Gain Design

a. The required phase margin for 25% overshoot (ζ = 0.404), found from Eq. (10.73), is 43.49°.

From the solution to the Case Study Challenge problem of Chapter 10, $G(s) = \frac{50.88K}{s(s+1.32)(s+100)}$.

Using the Bode plots for K = 1 from the solution to the Case Study Challenge problem of Chapter 10, we find the required phase margin at ω = 1.35 rad/s, where the magnitude response is -14 dB. Hence, K = 5.01 (14 dB).

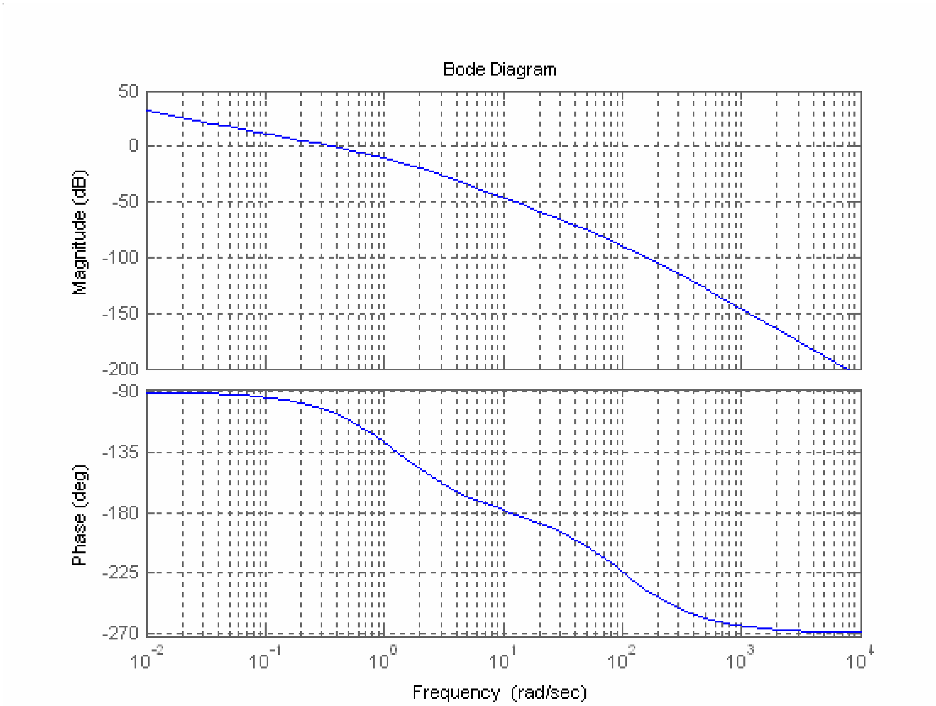
b.

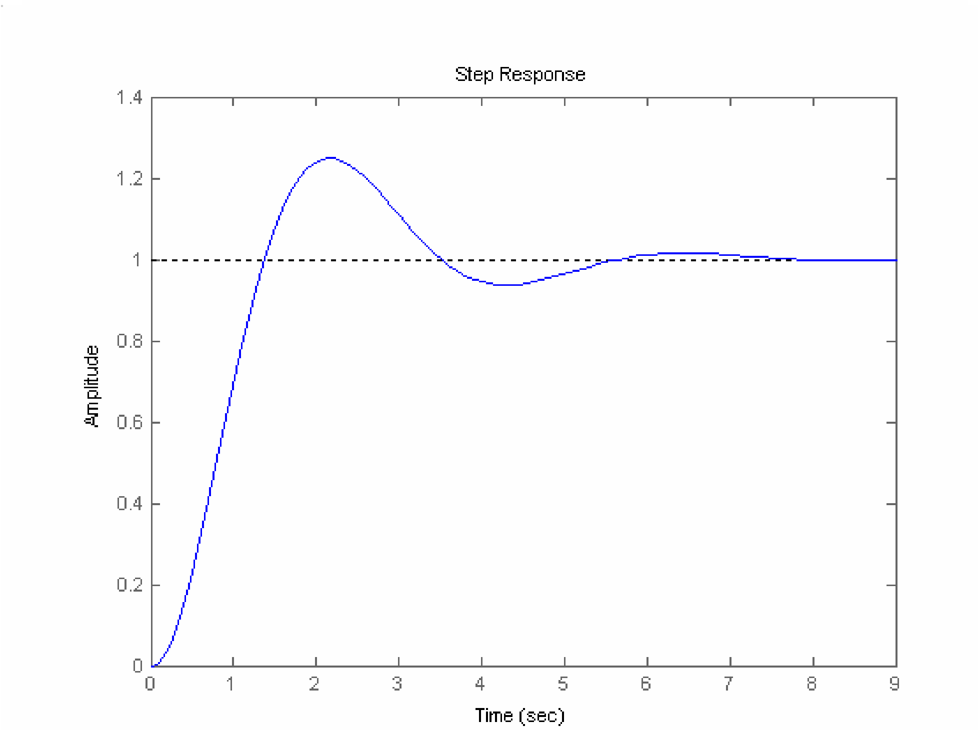
Program:

```
%Input system
numg=50.88;
deng=poly([0 -1.32 -100]);
G=tf(numg,deng);
%Percent Overshoot to Damping Ratio to Phase Margin
Po=input('Type %OS  ');
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
Pm=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi);
fprintf('\nPercent Overshoot = %g',Po)
fprintf(', Damping Ratio = %g',z)
fprintf(', Phase Margin = %g',Pm)
%Get Bode data
bode(G)
pause
w=0.01:0.05:1000;%Step size can be increased if memory low.
[M,P]=bode(G,w);
M=M(:,:);
P=P(:,:);
Ph=-180+Pm;
for i=1:1:length(P);
if P(i)-Ph<=0;
M=M(i);
K=1/M;
fprintf(', Frequency  = %g',w(i))
fprintf(', Phase  = %g',P(i))
fprintf(', Magnitude = %g',M)
fprintf(', Magnitude (dB) = %g',20*log10(M))
fprintf(', K = %g',K)
break
end
end
T=feedback(K*G,1);
step(T)
```

Computer response:

```
Type %OS  25

Percent Overshoot = 25, Damping Ratio = 0.403713, Phase Margin = 43.463,
Frequency  = 1.36, Phase  = -136.634, Magnitude = 0.197379, Magnitude (dB)
= -14.094, K = 5.06641
```





Antenna Control: Cascade Compensation Design

a. From the solution to the previous Case Study Challenge in this chapter, $G(s) = \frac{50.88K}{s(s+1.32)(s+100)}$.

For K_v = 20, K = 51.89. Hence, the gain compensated system is

$$G(s) = \frac{2640.16}{s(s+1.32)(s+100)}$$

Using Eq. (10.73), 15% overshoot (i.e. $\zeta = 0.517$) requires a phase margin of 53.18°. Using the Bode plots for K = 1 from the solution to the Case Study Challenge problem of Chapter 10, we find the required phase margin at $\omega = 0.97$ rad/s where the phase is -126.82°.

To speed up the system, we choose the compensated phase margin frequency to be 4.6 * 0.97 = 4.46 rad/s. Choose the lag compensator break a decade below this frequency, or $\omega = 0.446$ rad/s.

At the phase margin frequency, the phase angle is -166.067°, or a phase margin of 13.93°. Using 5° leeway, we need to add 53.18° - 13.93° + 5° = 44.25°. From Figure 11.8, $\beta = 0.15$, or $\gamma = \frac{1}{\beta}$ = 6.667. Using Eq. (11.15), the lag portion of the compensator is

$$G_{Lag}(s) = \frac{(s+0.446)}{(s+\frac{0.446}{6.667})} = \frac{s+0.446}{s+0.0669} .$$

Using Eqs. (11.9) and (11.15), $T_2 = \frac{1}{\omega_{max}\sqrt{\beta}}$ = 0.579. From Eq. (11.15), the lead portion of the compensator is

$$G_{Lead}(s) = \frac{s+1.727}{s+11.51}$$

The final forward path transfer function is

$$G(s)G_{Lag}(s)G_{Lead}(s) = \frac{2640.16(s+0.446)(s+1.727)}{s(s+1.32)(s+100)(s+0.0669)(s+11.51)}$$

b.

Program:

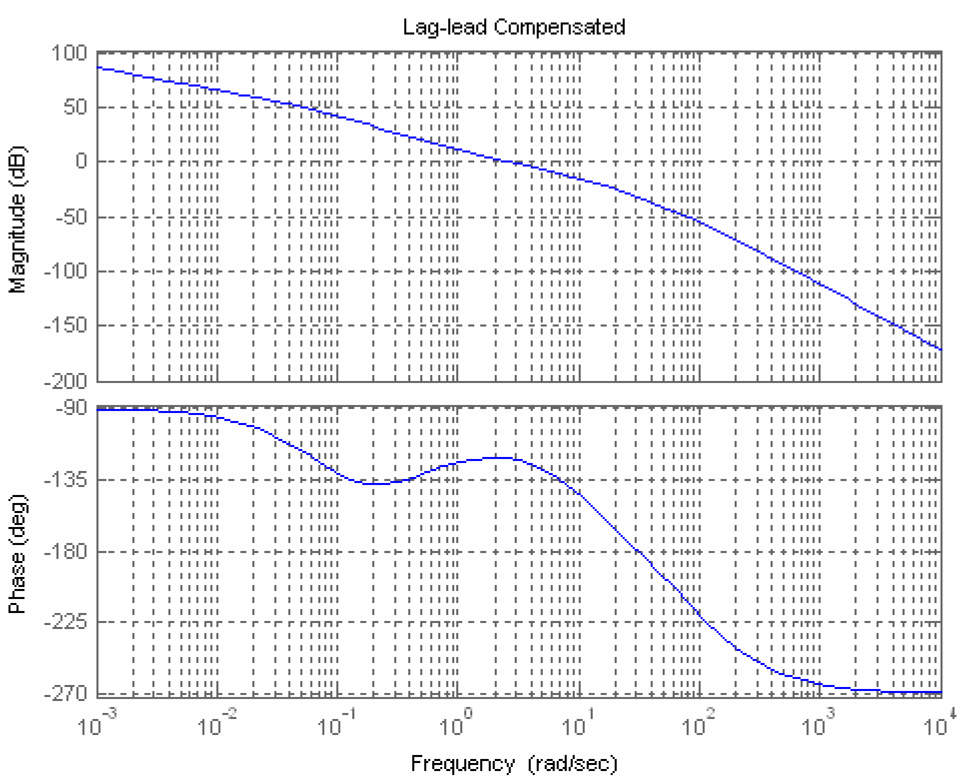
```
%Input system ******************************
K=51.89;
numg=50.88*K;
deng=poly([0 -1.32 -100]);
G=tf(numg,deng);
Po=15;
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
%Determine required phase margin**************
Pmreq=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi)
phreq=Pmreq-(180)%required phase
w=0.1:0.01:10;
[M,P]=bode(G,w);
for i=1:1:length(P);%search for phase angle
if P(i)-phreq<=0;
ph=P(i)
w(i)
break
end
end
wpm=4.6*w(i)
```

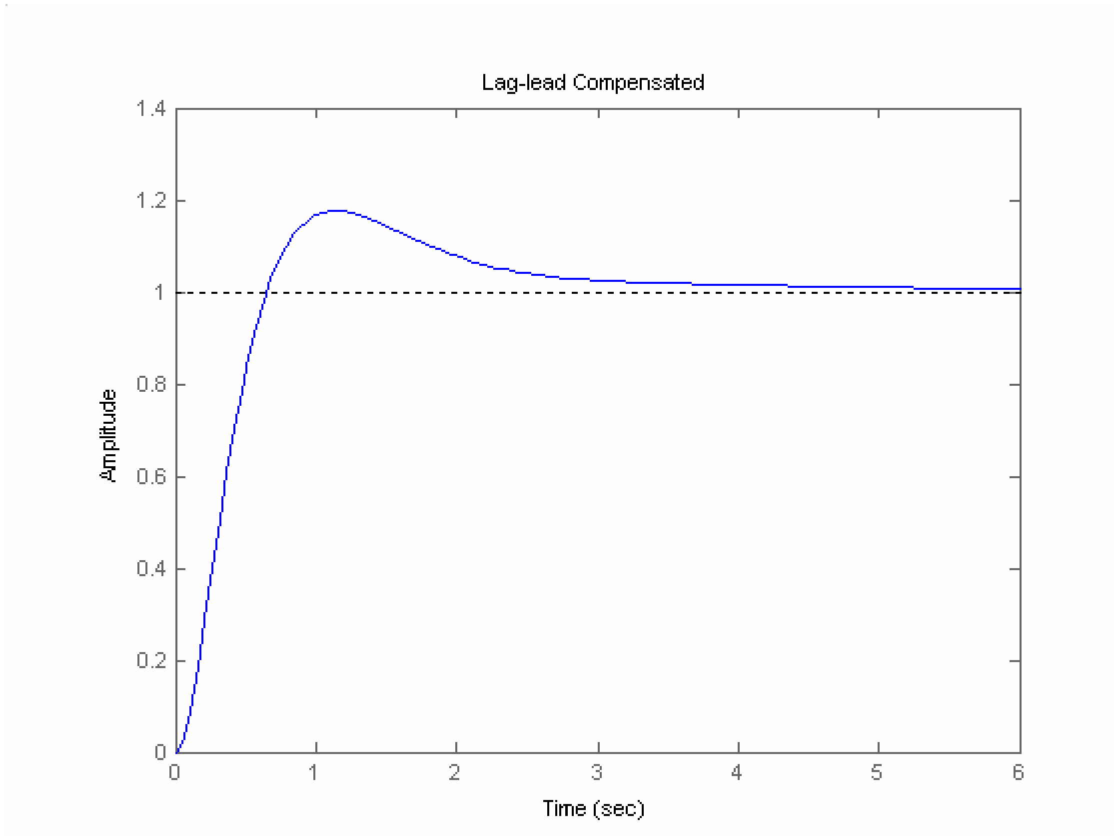
```
[M,P]=bode(G,wpm);%Find phase at wpm
Pmreqc=Pmreq-(180+P)+5%Find contribution required from compensator+5
beta=(1-sin(Pmreqc*pi/180))/(1+sin(Pmreqc*pi/180))
%Design lag compensator***************
zclag=wpm/10;
pclag=zclag*beta;
Kclag=beta;
%Design lead compensator**********
zclead=wpm*sqrt(beta);
pclead=zclead/beta;
Kclead=1/beta;
%Create compensated forward path*********
numgclag=Kclag*[1 zclag];
dengclag=[1 pclag];
'Gclag(s)'
Gclag=tf(numgclag,dengclag);
Gclagzpk=zpk(Gclag)
numgclead=Kclead*[1 zclead];
dengclead=[1 pclead];
'Gclead(s)'
Gclead=tf(numgclead,dengclead);
Gcleadzpk=zpk(Gclead)
Gc=Gclag*Gclead;
'Ge(s)=G(s)*Gclag(s)*Gclead(s)'
Ge=Gc*G;
Gezpk=zpk(Ge)
%Test lag-lead compensator****************
T=feedback(Ge,1);
bode(Ge)
title('Lag-lead Compensated')
[Gm,Pm,wcp,wcg]=margin(Ge);
'Compensated System Results'
fprintf('\nResulting Phase Margin = %g',Pm)
fprintf(', Resulting Phase Margin Frequency = %g',wcg)
pause
step(T)
title('Lag-lead Compensated')
```

Computer response:

```
Pmreq =

   53.1718

phreq =

 -126.8282

ph =

 -126.8660

ans =

    0.9700

wpm =
    4.4620
Pmreqc =

   44.2468

beta =

    0.1780

ans =
```

```
Gclag(s)

Zero/pole/gain:
0.17803 (s+0.4462)
------------------
   (s+0.07944)

ans =

Gclead(s)

Zero/pole/gain:
5.617 (s+1.883)
---------------
   (s+10.58)

ans =

Ge(s)=G(s)*Gclag(s)*Gclead(s)


Zero/pole/gain:
     2640.1632 (s+1.883) (s+0.4462)
----------------------------------------
s (s+100) (s+10.58) (s+1.32) (s+0.07944)

ans =

Compensated System Results

Resulting Phase Margin = 57.6157, Resulting Phase Margin Frequency =
2.68618»
```





Answers to Review Questions

1. Steady-state error requirements can be designed simultaneously with transient response requirements.

2. Via the phase margin

3. The lag compensator is a low pass filter. Thus, while the low frequency gain is increased, the high-frequency gain at 180 ^{o} is decreased to make the system stable.

4. The lag network affects the phase angle at low frequencies, but not at high frequencies. For the compensated system, the phase plot is about the same as that of the uncompensated system around and above the phase margin frequency yielding the same transient response.

5. To compensate for the slight negative angle that the lag compensator has near the phase margin frequency

6. Compensated system has higher low-frequency gain than the uncompensated system designed to yield the same transient response; compensated and uncompensated system have the same phase margin frequency; the compensated system has lower gain around the phase margin frequency; the compensated and uncompensated system's have approximately the same phase values around the phase margin frequency.

7. The lead network is a high pass filter. It raises the gain at high frequencies. The phase margin frequency is increased.

8. Not only is the magnitude curve increased at higher frequencies, but so is the phase curve. Thus the 180^{o} point moves up in frequency with the increase in gain.

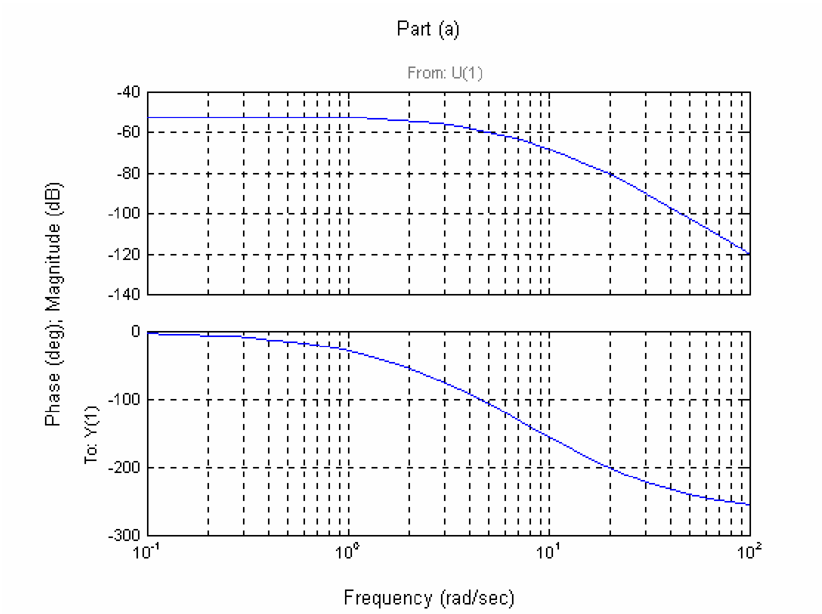
9. To correct for the negative phase angle of the uncompensated system

10. When designing the lag portion of a lag-lead compensator, we do not worry about the transient design. The transient response will be considered when designing the lead portion of a lag-lead compensator.

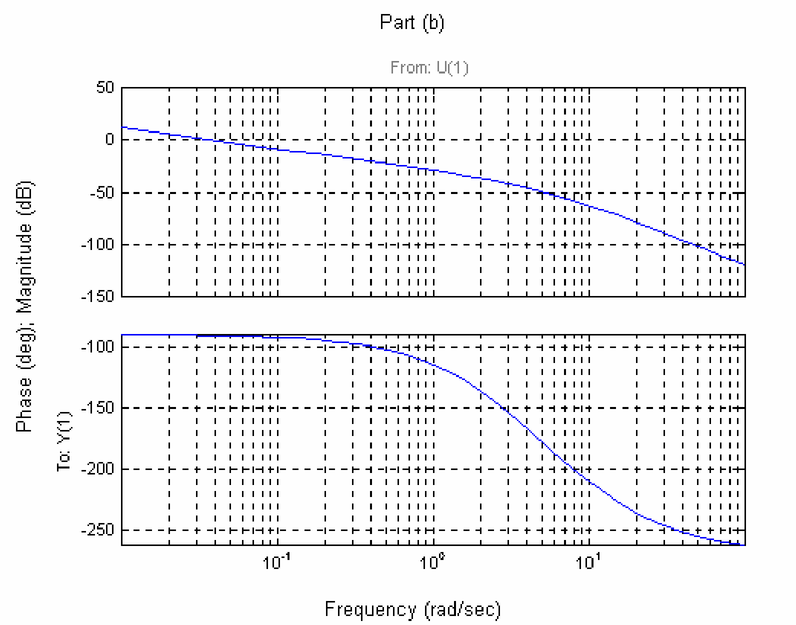
SOLUTIONS TO PROBLEMS

1.

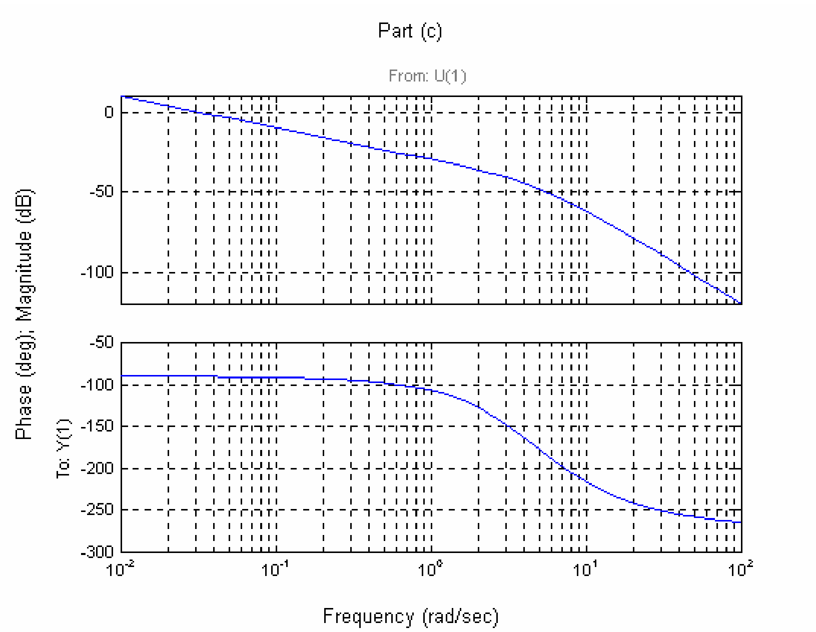
a. Plot Bode plots for K = 1; angle is 180^{o} at ω = 14.39 rad/s where the magnitude is –74.29 dB. Therefore a 64.29 dB (or K = 1639) increase will yield a 10 dB gain margin.



b. Plot Bode plots for K = 1; angle is 180^{o} at ω = 5.196 rad/s where the magnitude is –50.21 dB. Therefore a 40.21 dB (or K = 102.4) increase will yield a 10 dB gain margin.



c. Plot Bode plots for K = 1; angle is 180º at ω = 5.233 rad/s where the magnitude is –48.58 dB. Therefore a 38.58 dB (or K = 84.92) increase will yield a 10 dB gain margin.



2.

a. For a 40º phase margin, the phase must be -140º when the magnitude plot is zero dB. The phase is -140º at ω = 8.097 rad/s. At this frequency, the magnitude curve is –65.02 dB. Thus a 65.02 dB increase (K = 1782) will yield a 40º phase margin.

b. For a 40º phase margin, the phase must be -140º when the magnitude plot is zero dB. The phase is -140º at ω = 2.201 rad/s. At this frequency, the magnitude curve is – 37.60 dB. Thus a 37.60 dB increase (K = 75.86) will yield a 40º phase margin.

c. For a 40^o phase margin, the phase must be -140^o when the magnitude plot is zero dB. The phase is -140^o at $\omega = 2.653$ rad/s. At this frequency, the magnitude curve is – 38.78 dB. Thus a 38.78 dB increase (K = 86.9) will yield a 40^o phase margin.

3.

20% overshoot => $\zeta = 0.456$ => $\phi_M = 48.15^o$.

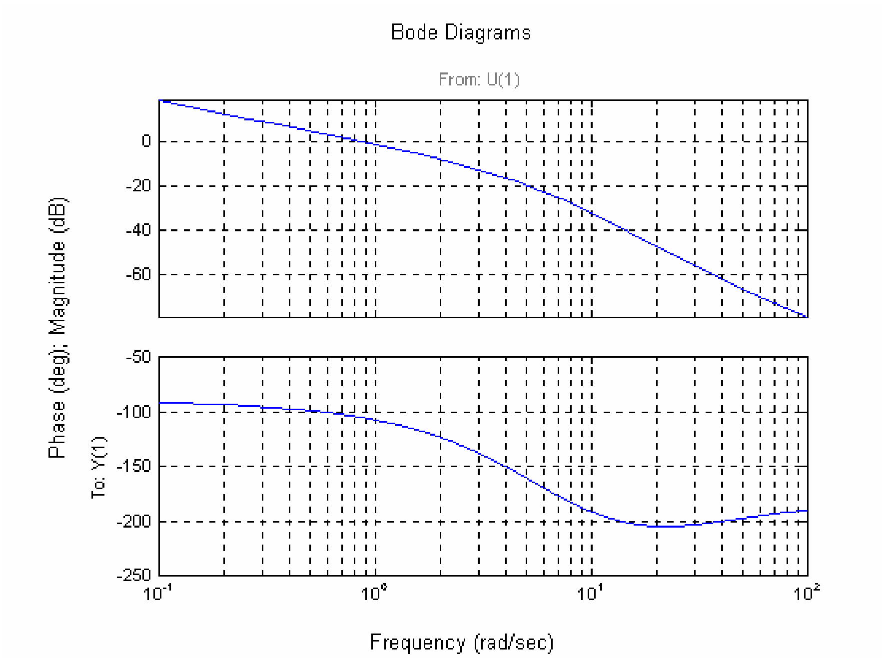
a. Looking at the phase diagram, where $\phi_M = 48.15^o$ (i.e. $\phi = -131.85^o$), the phase margin frequency = 3.105 rad/s. At this frequency, the magnitude curve is -48.3 dB. Thus the magnitude curve has to be raised by 48.3 dB (K = 260).

b. Looking at the phase diagram, where $\phi_M = 48.15^o$ (i.e. $\phi = -131.85^o$), the phase margin frequency = 6.462 rad/s. At this frequency, the magnitude curve is – 63.04 dB. Thus the magnitude curve has to be raised by 63.04 dB (K = 1419).

c. Looking at the phase diagram, where $\phi_M = 48.15^o$ (i.e. $\phi = -131.85^o$), the phase margin frequency = 6.939 rad/s. At this frequency, the magnitude curve is – 64.42 dB. Thus the magnitude curve has to be raised by 64.42 dB (K = 1663).

4.

a. Bode plots for K = 1:



Using Eqs. (4.39) and (10.73) a percent overshoot = 15 is equivalent to a $\zeta = 0.517$ and $\phi_M = 53.17^0$. The phase-margin frequency = 2.2 rad/s where the phase is $53.17^0 - 180^0 = -126.83^0$. The magnitude = -8.966 dB, or 0.0.3562. Hence K = 1/ 0.3562 = 2.807.

b.

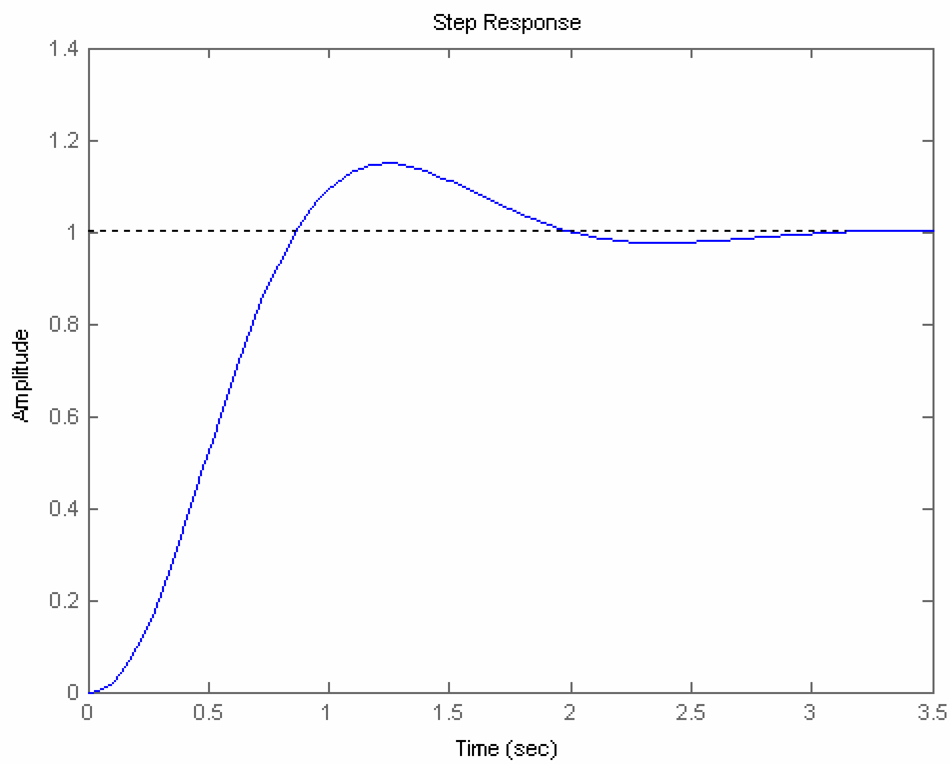
Program:

```
G=zpk([-20 -25],[0 -5 -8 -14],1)
K=2.807
T=feedback(K*G,1);
```

step(T)

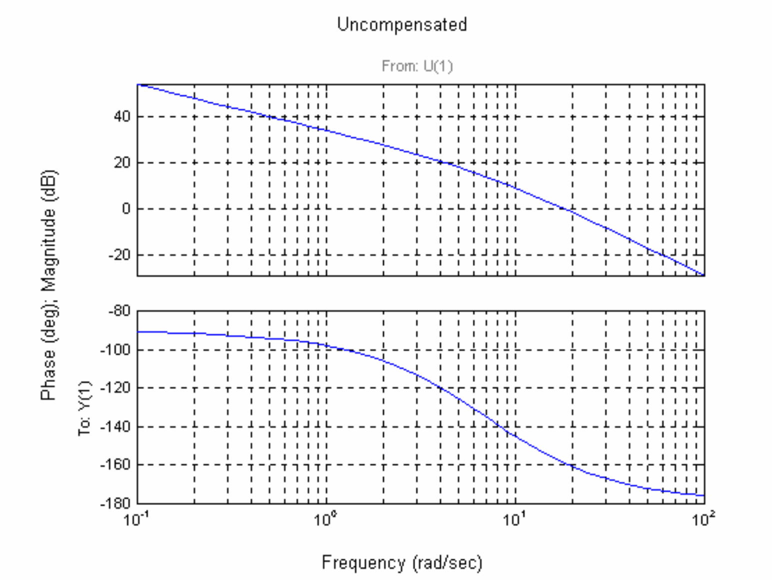
Computer response:

```
Zero/pole/gain:
   (s+20) (s+25)
--------------------
s (s+5) (s+8) (s+14)

K =

    2.8070
```

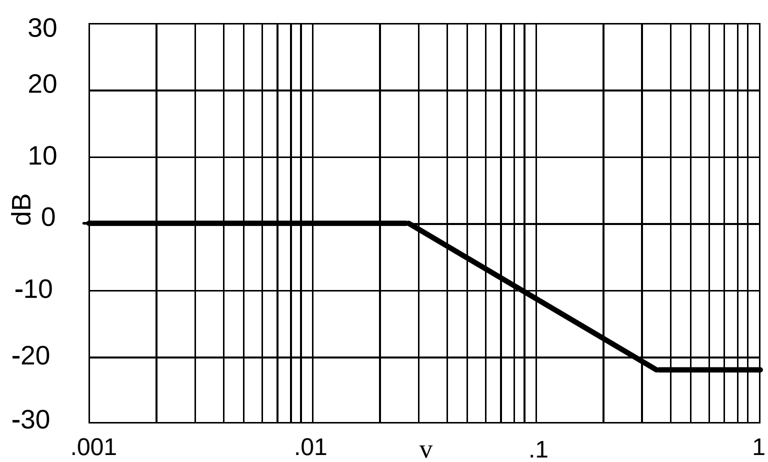


5.

For $K_V = 50$, $K = 350$. Plot the Bode plots for this gain.



Also, since %OS = 15%, $\zeta = 0.517$. Using Eq. (10.73), $\phi_M = 53.17^o$. Increasing ϕ_M by 10^o we will design for a phase margin of 63.17^o. The phase margin frequency is where the phase angle is $63.17 - 180^o = -116.83^o$, or $\omega_{\phi M} = 3.54$ rad/s. At this frequency, the magnitude is 22 dB. Start the magnitude of the compensator at - 22 dB and draw it to 1 decade below $\omega_{\phi M}$.



Then begin +20 dB/dec until zero dB is reached. Read the break frequencies as 0.028 rad/s and 0.354 rad/s from the Bode plot and form a lag transfer function that has unity dc gain:

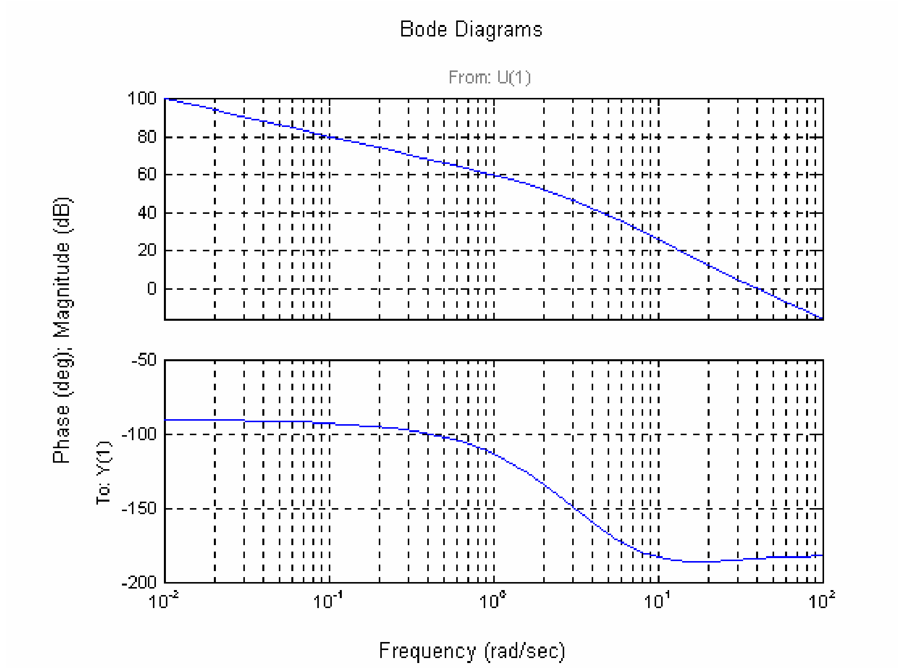
$$G_c(s) = 0.0791\,\frac{s+0.354}{s+0.028}$$

The compensated forward path is

$$G(s) = \frac{350 * 0.0791(s+0.354)}{s(s+7)(s+0.028)} = \frac{27.69(s+0.354)}{s(s+7)(s+0.028)}$$

6.

a. For K_V = 1000, K = 1473. Plotting the Bode for this value of K:



Using Eqs. (4.39) and (10.73) a percent overshoot = 15 is equivalent to a ζ = 0.517 and ϕ_M = 53.17.

Using an extra 10^o, the phase margin is 63.17^o. The phase-margin frequency = 1.21 rad/s. At this frequency, the magnitude = 57.55 dB = 754.2. Hence the lag compensator K = 1/754.2 = 0.001326. Following Steps 3 and 4 of the lag compensator design procedure in Section 11.3,

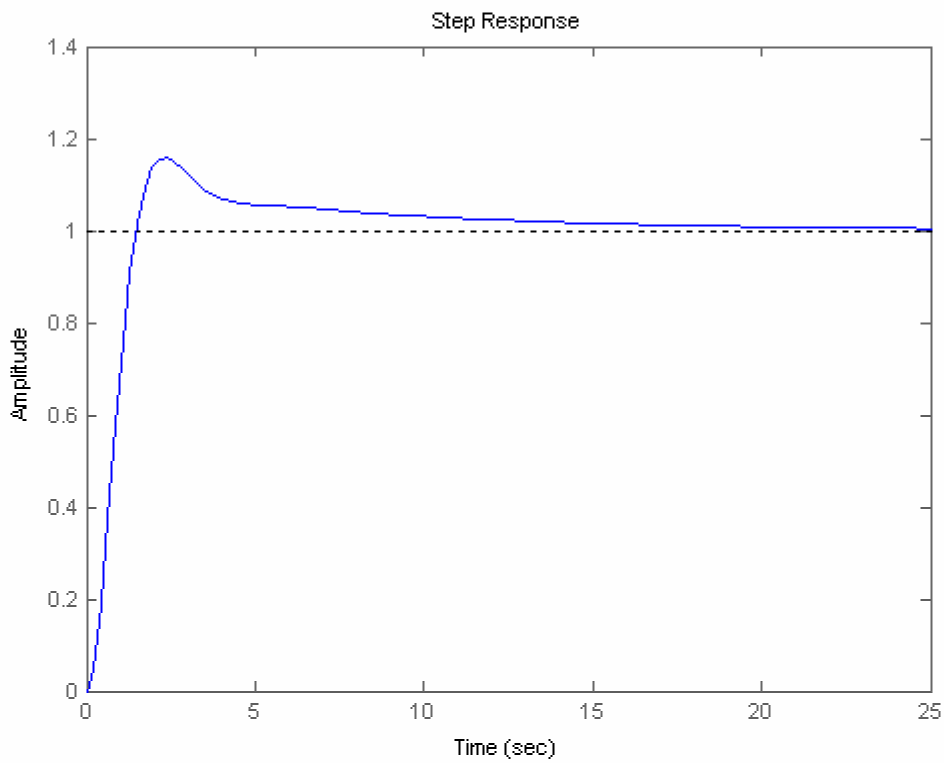
$$G_{lag}(s) = 0.001326 \frac{s + 0.121}{s + 0.0001604}$$

b.

Program:

```
%Input system
numg=1473*poly([-10 -11]);
deng=poly([0 -3 -6 -9]);
G=tf(numg,deng);
numc=0.001326*[1 0.121];
denc=[1 0.0001604];
Gc=tf(numc,denc);
Ge=G*Gc;
T=feedback(Ge,1);
step(T)
```

Computer response:



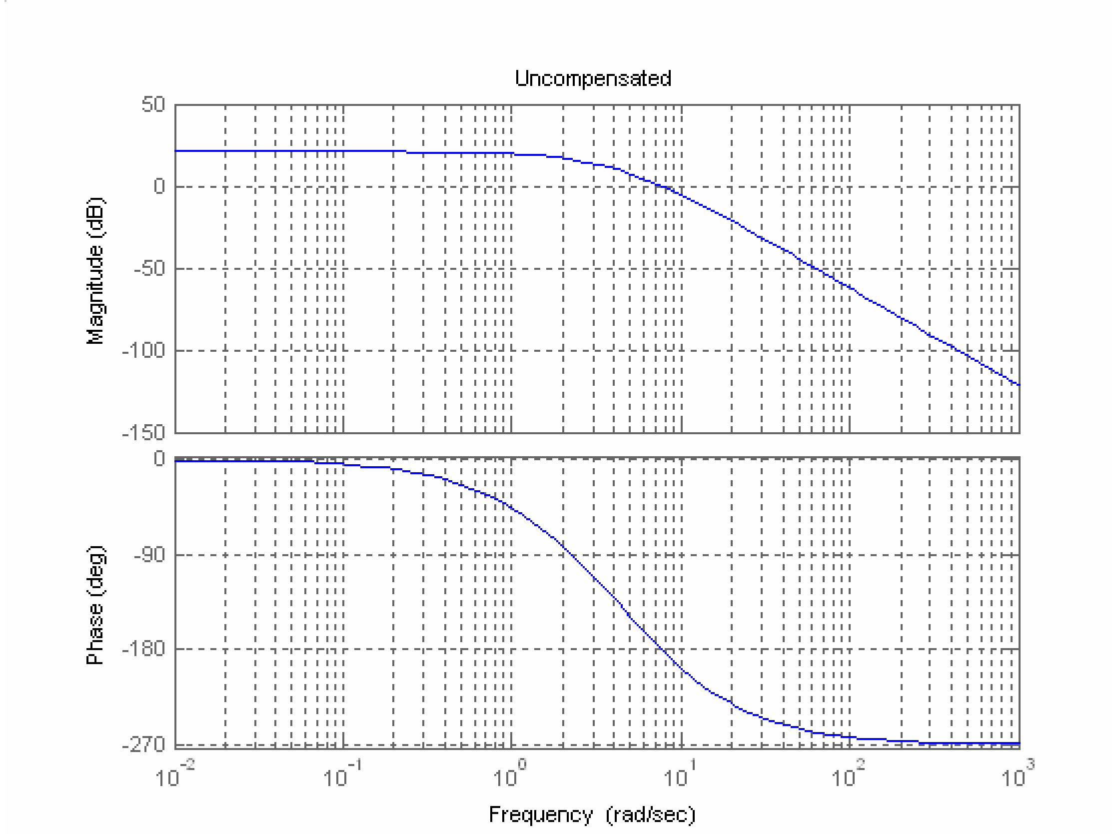
7.

Uncompensated system:

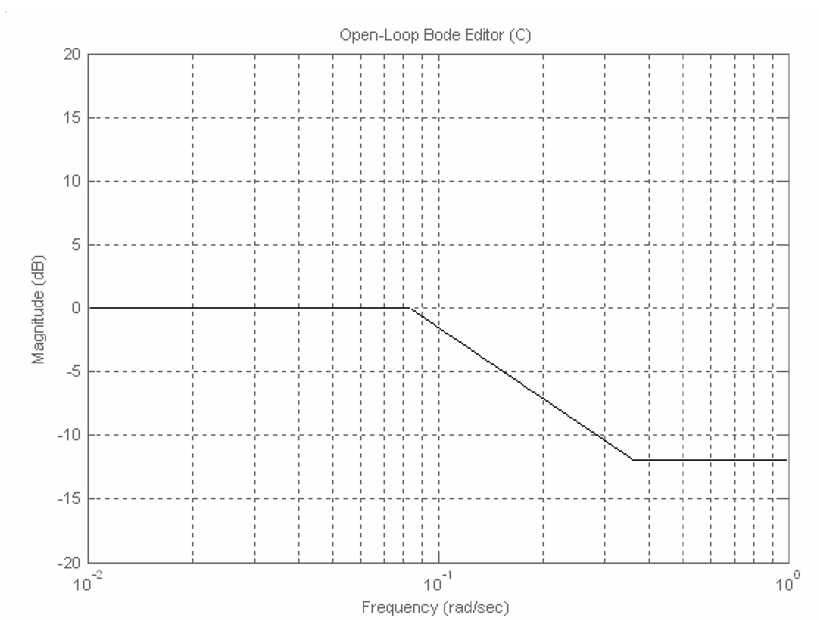
Searching along the 121.1° line (15% overshoot), find the dominant pole at -2.15 ± j3.56 with K = 97.7. Therefore, the uncompensated static error constant is $K_{vo} = \frac{97.7}{70} = 1.396$. On the frequency response curves, plotted for K = 97.7, unity gain occurs at ω = 1.64 rad/s with a phase angle of -71°. Therefore the uncompensated phase margin is 180° - 71° = 109°.

Compensated system:

The old steady-state error, $e_{step}(\infty) = \dfrac{1}{1+K_{po}} = \dfrac{1}{1+\dfrac{97.7}{70}} = 0.4174$. For a 5 times improvement in steady-state error, $e_{step}(\infty) = \dfrac{1}{1+K_{pn}} = 0.0835$, yielding , $K_{pn} = 10.98 = \dfrac{K}{70}$. Thus $K = 768.6$. Plotting the Bode plots at this gain,



Adding 5°, the desired phase margin for 15% overshoot is 58.17°, or a phase angle of -121.83°. This phase angle occurs at ω = 3.505 rad/s. At this frequency the magnitude plot is +12 dB. Start the magnitude of the compensator at - 12 dB and draw it to 1 decade below $\omega_{\Phi M}$.



Then, begin +20 dB/dec until zero dB is reached. Read the break frequencies as 0.08797 rad/s and 0.3505 rad/s from the Bode plot and form a lag transfer function that has unity dc gain,

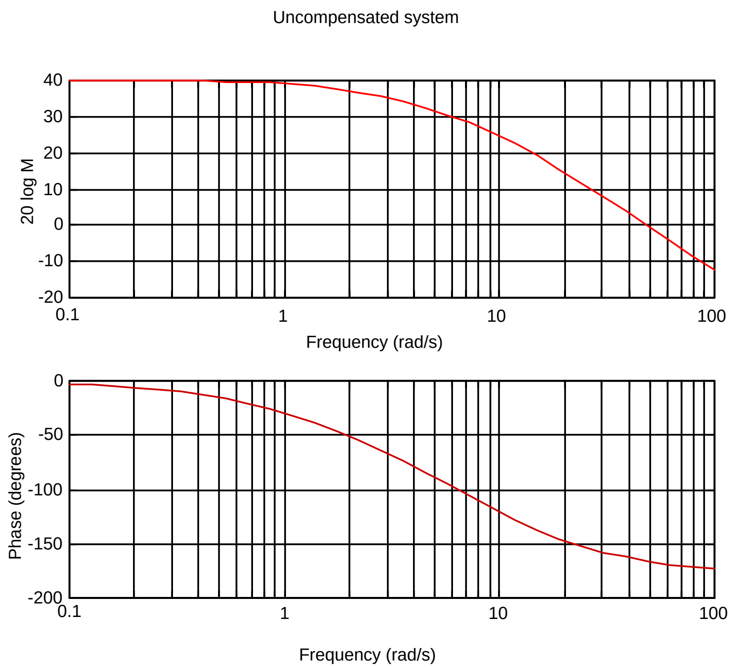
$$G_c(s) = 0.251\frac{s+0.3505}{s+0.08797}$$

The compensated forward path is

$$G(s) = 0.251*768.6\frac{(s+0.3505)}{(s+2)(s+5)(s+7)(s+0.08797)} = \frac{192.91(s+0.3505)}{(s+2)(s+5)(s+7)(s+0.08797)}.$$

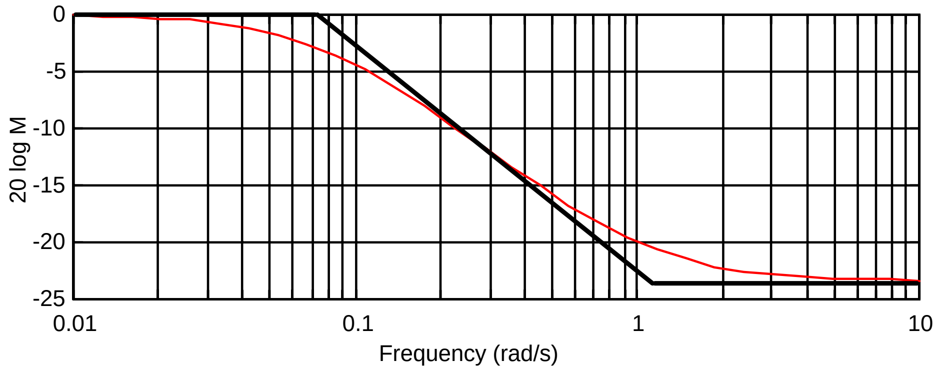
8.

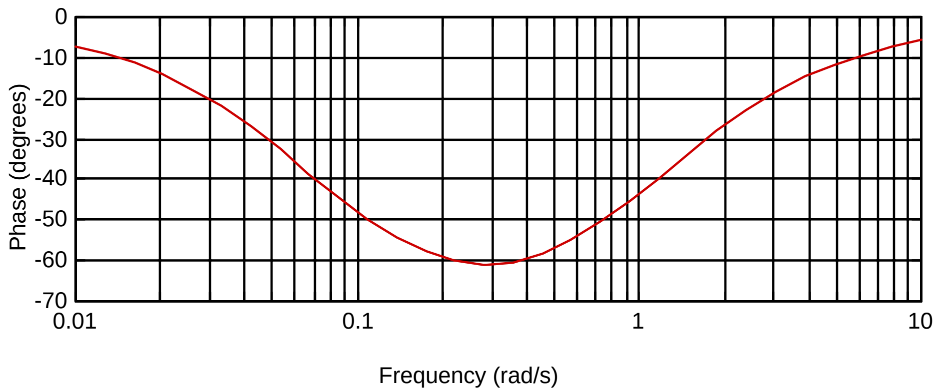
For $K_p = 100 = \dfrac{K(4)}{(2)(6)(8)}$, K = 2400. Plotting the Bode plot for this gain,



We will design the system for a phase margin 10^0 larger than the specification. Thus $\phi_m = 55^0$. The phase margin frequency is where the phase angle is $-180^0 + 55^0 = -125^0$. From the Bode plot this frequency is $\omega_{\phi_m} = 11$ rad/s. At this frequency the magnitude is 23.37 dB. Start the magnitude of the lag compensator at –23.37 dB and draw it to 1 decade below $\omega_{\phi_m} = 11$, or 1.1 rad/s. Then begin a +20 dB/dec climb until 0 dB is reached. Read the break frequencies as 0.0746 rad/s and 1.1 rad/s from the Bode plot as shown below.

Lag compensator





Ensuring unity dc gain, the transfer function of the lag is $G_{lag}(s) = 0.06782\dfrac{(s+1.1)}{(s+0.0746)}$. The compensated forward-path transfer function is thus the product of the plant and the compensator, or

$$G_e(s) = \frac{162.8(s+4)(s+1.1)}{(s+2)(s+6)(s+8)(s+0.0746)}$$

9.

From Example 11.1, K = 58251 yields 9.48% overshoot or a phase margin of 59.19^{o}. Also,

$$G(s) = \frac{58251}{s(s+36)(s+100)}$$

Allowing for a 10^{o} contribution from the PI controller, we want a phase margin of 69.19^{o}, or a phase angle of -180^{o} + 69.19^{o} = -110.81^{o}. This phase angle occurs at ω = 9.8 rad/s where the magnitude is 4 dB. Thus, the PI controller should contribute - 4 dB at ω = 9.8 rad/s. Selecting a break frequency a decade below the phase margin frequency,

$$G_c(s) = \frac{s+0.98}{s}$$

This function has a high-frequency gain of zero dB. Since we want a high-frequency gain of -4 dB (a gain of 0.631),

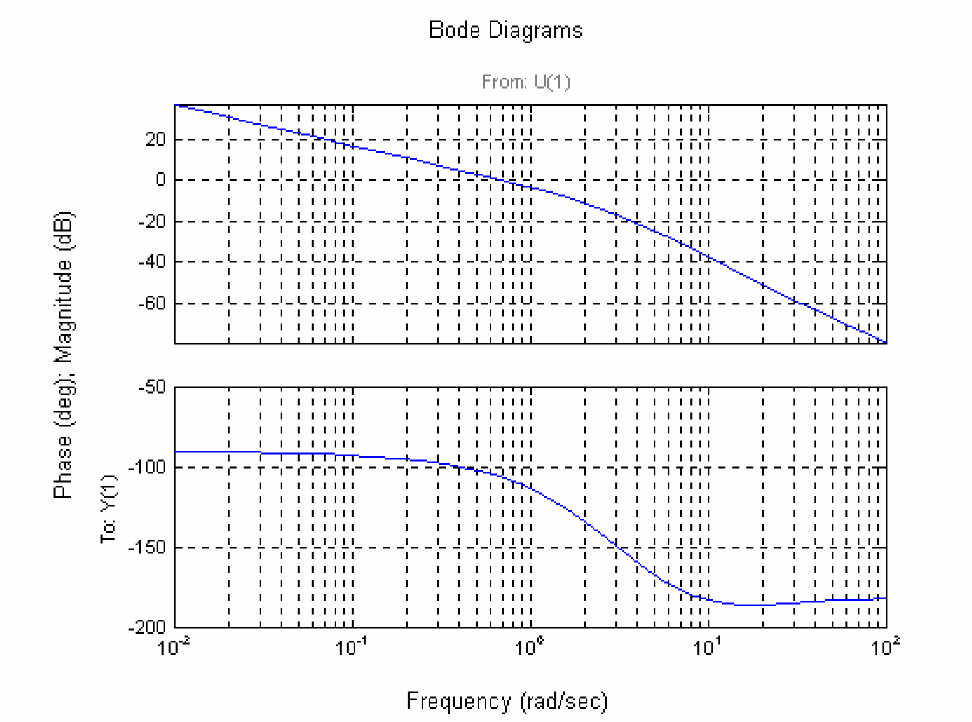
$$G_c(s) = 0.631 \frac{s+0.98}{s}$$

The compensated forward path is

$$G(s) = \frac{58251*0.631(s+0.98)}{s(s+36)(s+100)} = \frac{36756.38(s+0.98)}{s(s+36)(s+100)}$$

10.

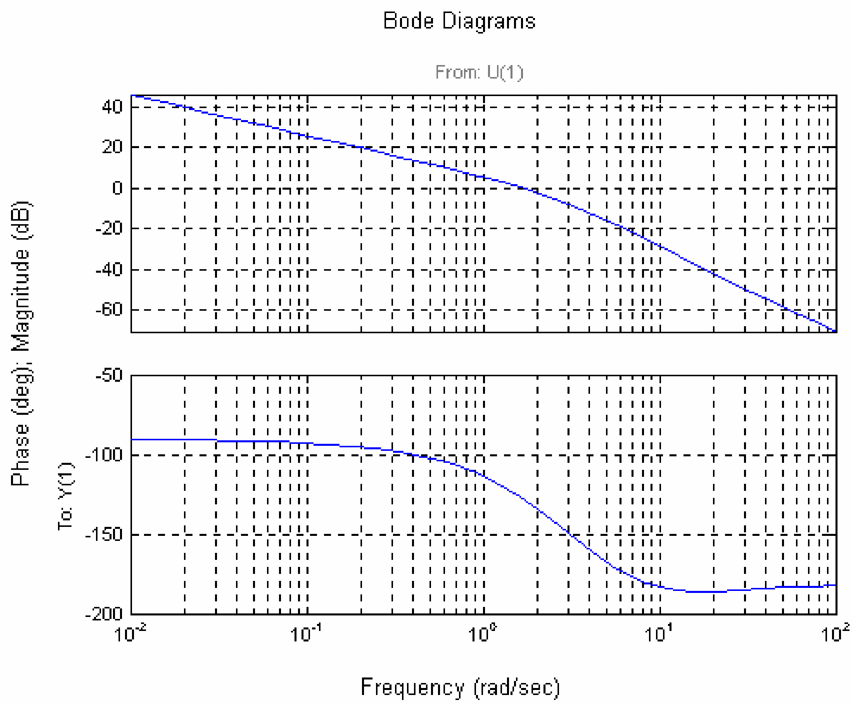
Bode plots for K = 1:



Using Eqs. (4.39) and (10.73) a percent overshoot = 15 is equivalent to a $\zeta = 0.517$ and $\phi_M = 53.17^o$. The phase-margin frequency = 1.66 rad/s. The magnitude = -9.174 dB = 0.3478. Hence K = 1/ 0.3478 = 2.876.

b.

Bode plots for K = 2.876.



Adding 10^o to the phase margin yields 63.17. Thus, the required phase is $-180^0 + 63.17^0 = -116.83^0$, which occurs at a frequency of 1.21 rad/s. The magnitude = 3.366 dB = 1.473. Hence, the lag compensator K = 1/ 1.473 = =0.6787. Selecting the break a decade below the phase-margin frequency,

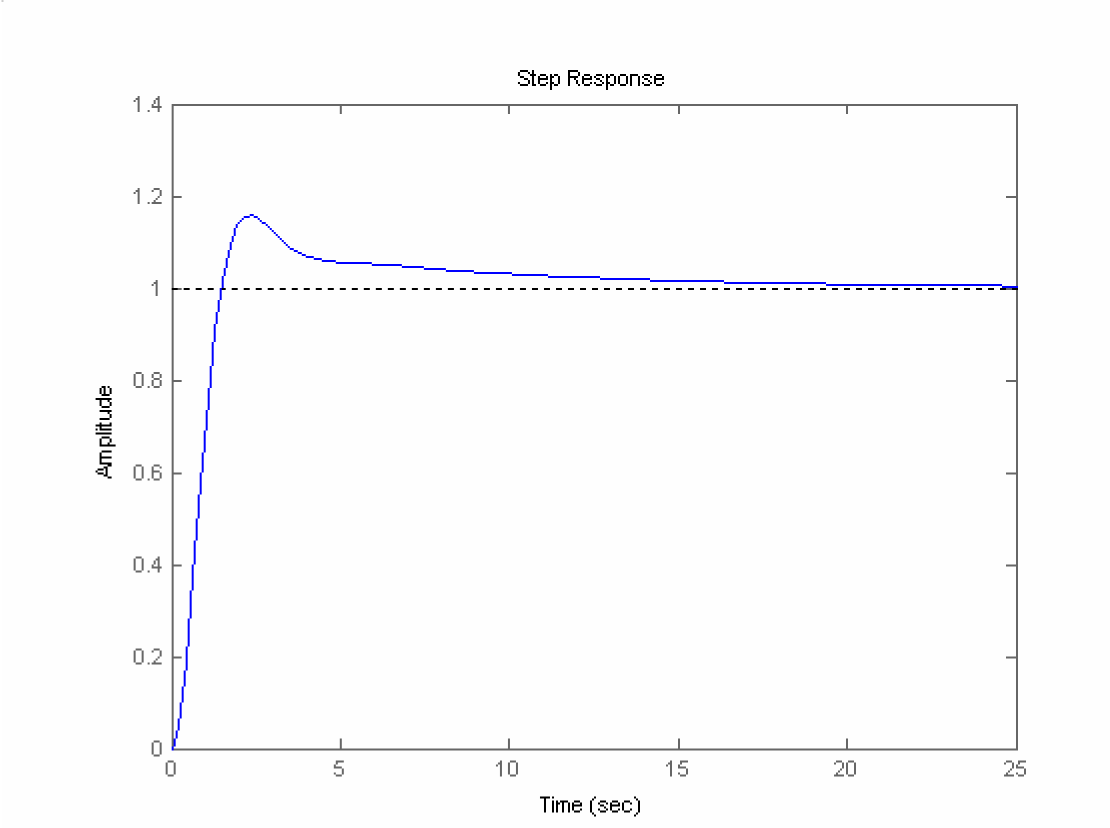
$$G_c(s) = 0.6787 \frac{s+0.121}{s}$$

c.

Program:

```
%Input system
numg=2.876*poly([-10 -11]);
deng=poly([0 -3 -6 -9]);
G=tf(numg,deng);
numc=0.6787*[1 0.121];
denc=[1 0];
Gc=tf(numc,denc);
Ge=G*Gc;
T=feedback(Ge,1);
step(T)
```

Computer response:



11.

Program:

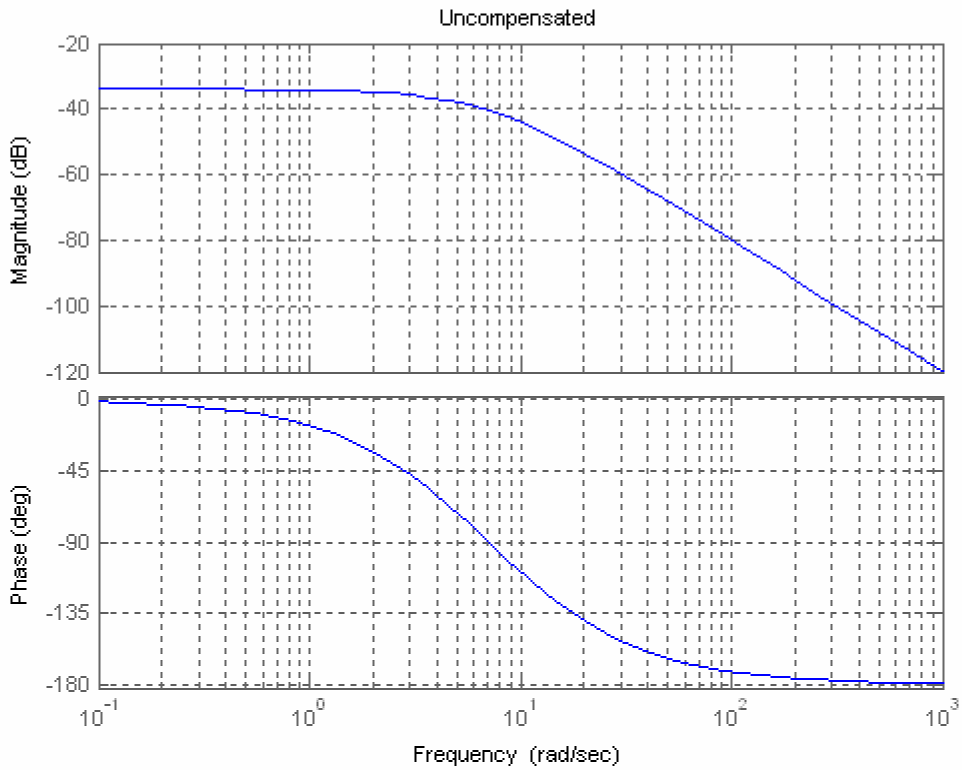
```
%PI Compensator Design via Frequency Response
%Input system
G=zpk([],[-5 -10],1);
G=tf(G);
%Percent Overshoot to Damping Ratio to Phase Margin
Po=input('Type %OS  ');
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
Pm=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi)+10;
fprintf('\nPercent Overshoot = %g',Po)
fprintf(', Damping Ratio = %g',z)
fprintf(', Phase Margin = %g',Pm)
%Get Bode data
bode(G)
title('Uncompensated')
pause
%Find frequency at desired phase margin and the gain at this frequency
w=logspace(-1,2,10000);
%w=.1:0.1:100;
[M,P,w]=bode(G,w);
Ph=-180+Pm
for i=1:1:length(P);
if P(i)-Ph<=0
Mag=M(i)
wf=w(i);
fprintf(', Frequency  = %g',wf)
fprintf(', Phase  = %g',P(i))
fprintf(', Magnitude = %g',Mag)
fprintf(', Magnitude (dB) = %g',20*log10(Mag))
break
end
end
```

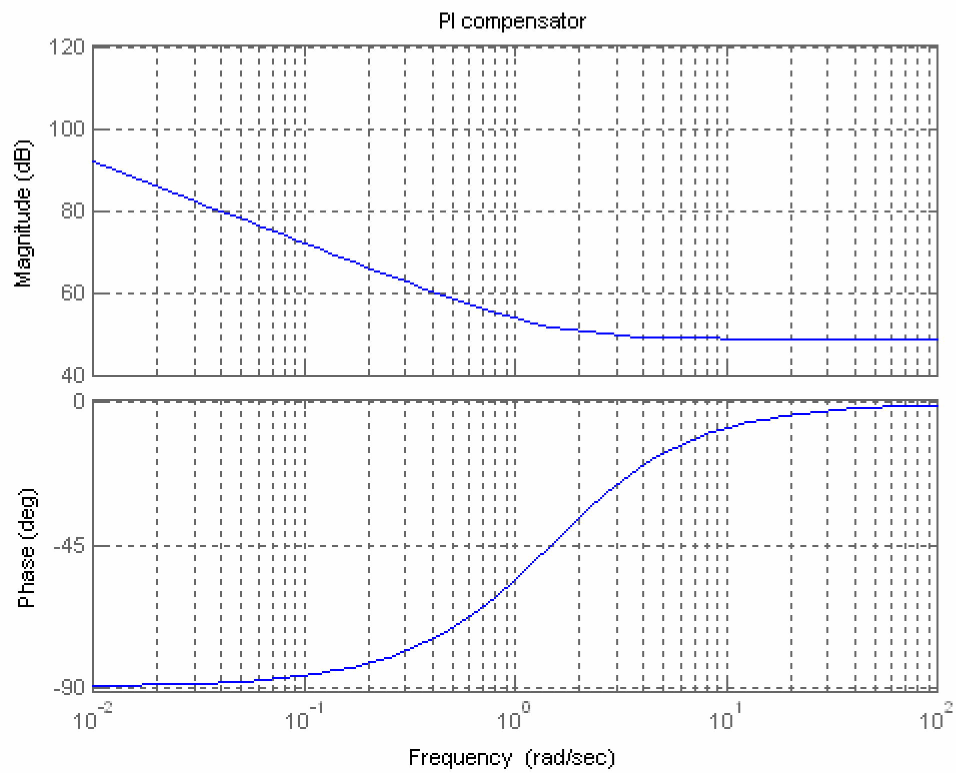
```
%Design PI compensator
%Break frequency is a decade below phase margin frequency
wh=wf/10;
%Magnitude is reciprocal of magnitude of G at the phase margin frequency
%so net magnitude is 0 dB at the phase margin frequency
Kc=1/Mag
'PI Compensator'
Gpi=tf(Kc*[1 wh],[1 0])
bode(Gpi)
title(['PI compensator'])
pause
'G(s)Gpi(s)'
Ge=series(G,Gpi);
Ge=zpk(Ge)
bode(Ge)
title('PI Compensated')
[Gm,Pm,Wcp,Wcg]=margin(Ge);
'Gain margin(dB); Phase margin(deg.); 0 dB freq. (r/s);'
'180 deg. freq. (r/s)'
margins=[20*log10(Gm),Pm,Wcg,Wcp]
pause
T=feedback(Ge,1);
step(T)
title('PI Compensated')
```

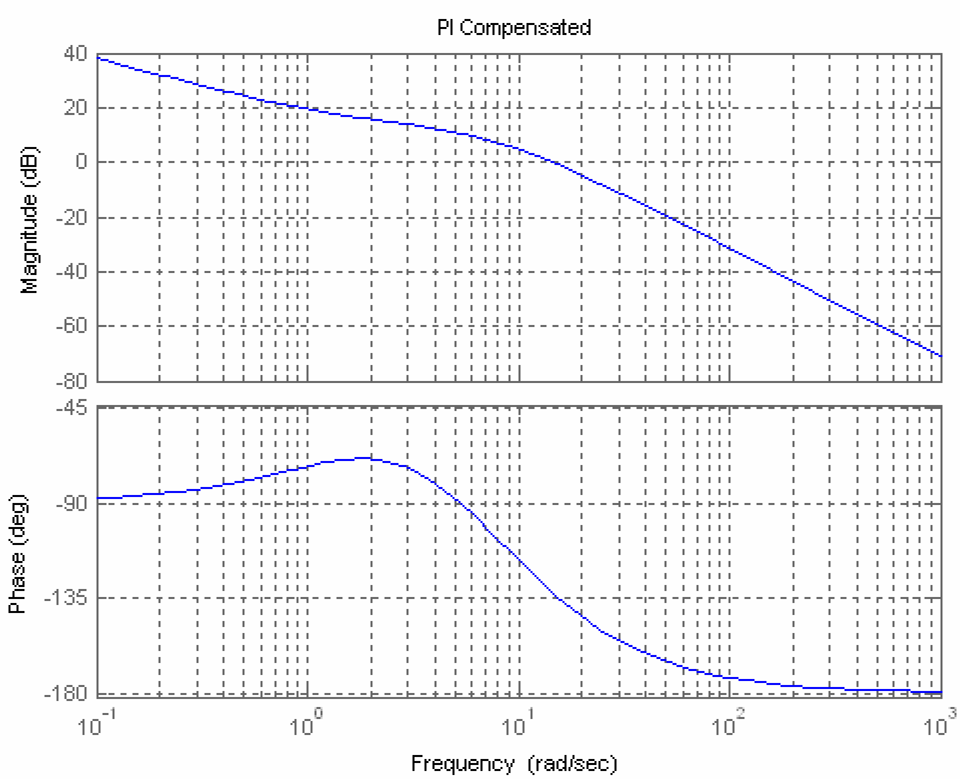
Computer response:

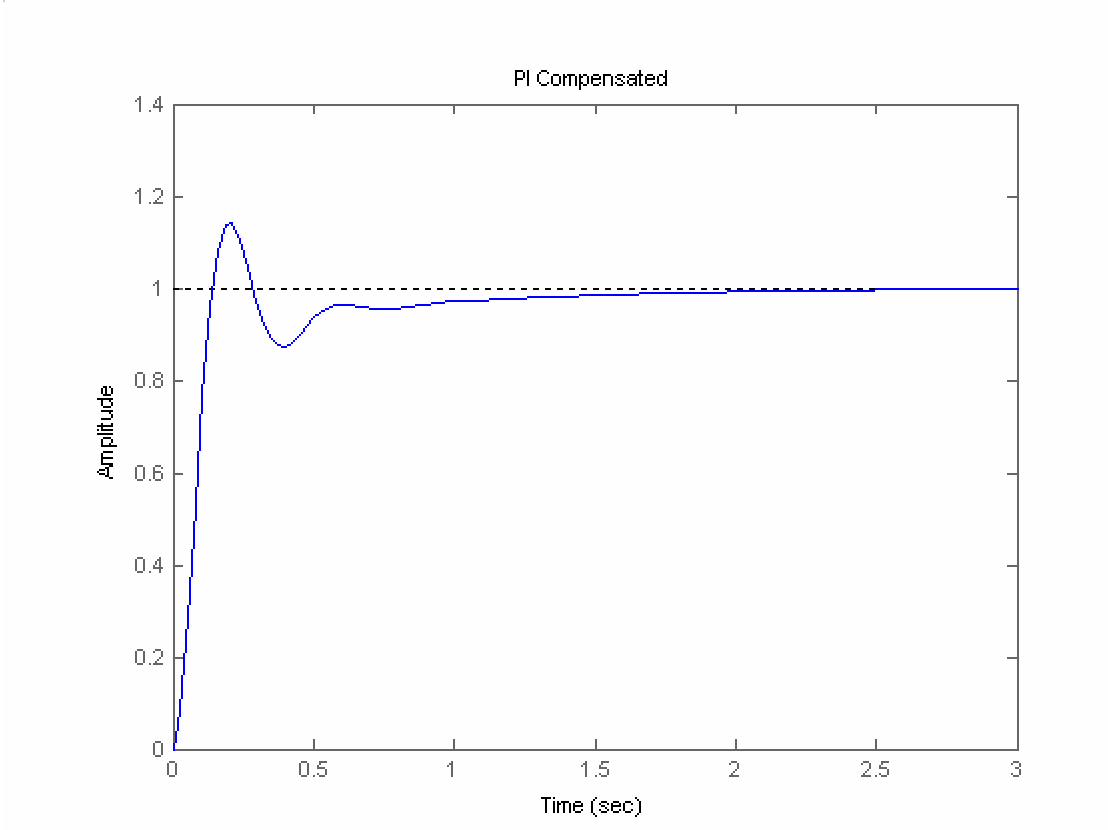
```
Type %OS  25

Percent Overshoot = 25, Damping Ratio = 0.403713, Phase Margin = 53.463
Ph =

 -126.5370


Mag =

    0.0037

, Frequency  = 14.5518, Phase  = -126.54, Magnitude = 0.00368082, Magnitude
(dB) = -48.6811
Kc =

  271.6786


ans =

PI Compensator


Transfer function:
271.7 s + 395.3
---------------
       s


ans =

G(s)Gpi(s)


Zero/pole/gain:
271.6786 (s+1.455)
------------------
  s (s+10) (s+5)
```

```
ans =

Gain margin(dB); Phase margin(deg.); 0 dB freq. (r/s);

ans =

180 deg. freq. (r/s)

margins =

       Inf   47.6277   14.5975       Inf
```



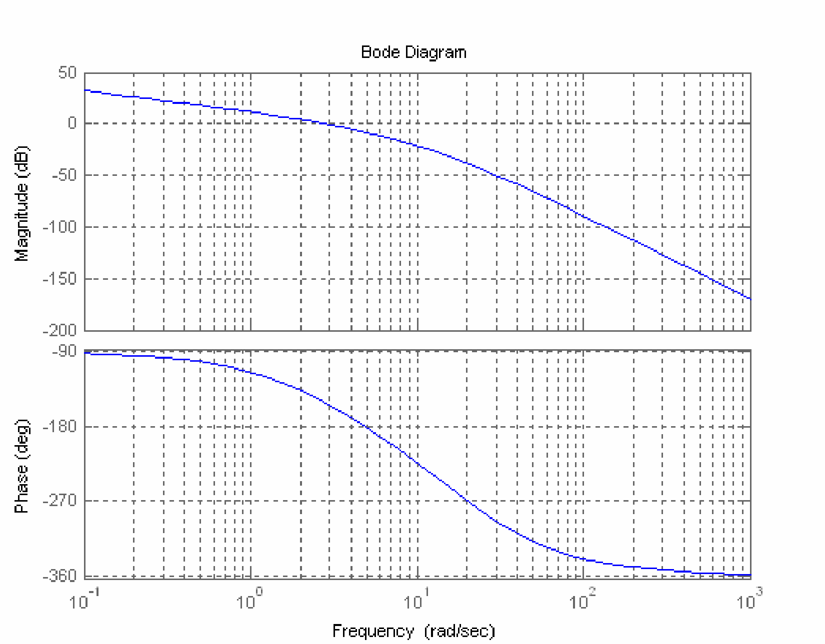

PI compensator
Magnitude (dB)
120
100
80
60
40
Phase (deg)
0
-45
-90
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/sec)


PI Compensated
Magnitude (dB)
40
20
0
-20
-40
-60
-80
Phase (deg)
-45
-90
-135
-180
10^-1
10^0
10^1
10^2
10^3
Frequency (rad/sec)



12.

For $K_v = 4$, $\frac{K}{900} = 4$, or $K = 3600$. Plot the Bode diagrams.



The magnitude curve crosses zero dB at ω = 2.83 rad/s. with a phase angle of 152.1°, which yields an uncompensated phase margin of 27.9°. Thus, we need an additional 12.1° plus an additional amount to compensate for the fact that the phase margin frequency will increase. Assume a lead network with a phase contribution of 22.1°. Using Eqs. (11.11), and (11.12),

The value of beta is:	0.453
The |G(jwmax)| for the compensator is:	1.485
or in db:	3.44

The magnitude curve has a gain of -3.44 dB at ω = 3.625 rad/s. Therefore, choose this frequency as the new phase margin frequency. Using Eqs. (11.9) and (11.6), the compensator transfer function has the following specifications:

T	0.41
zero	-2.44
pole	-5.38
gain	2.21

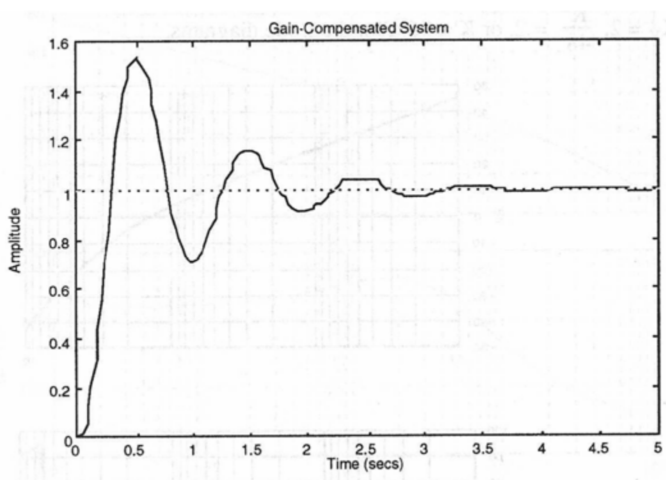
The compensated forward path is

$$G(s)=\frac{3600*2.21(s+2.44)}{s(s+3)(s+15)(s+20)(s+5.38)}=\frac{7956(s+2.44)}{s(s+3)(s+15)(s+20)(s+5.38)}$$

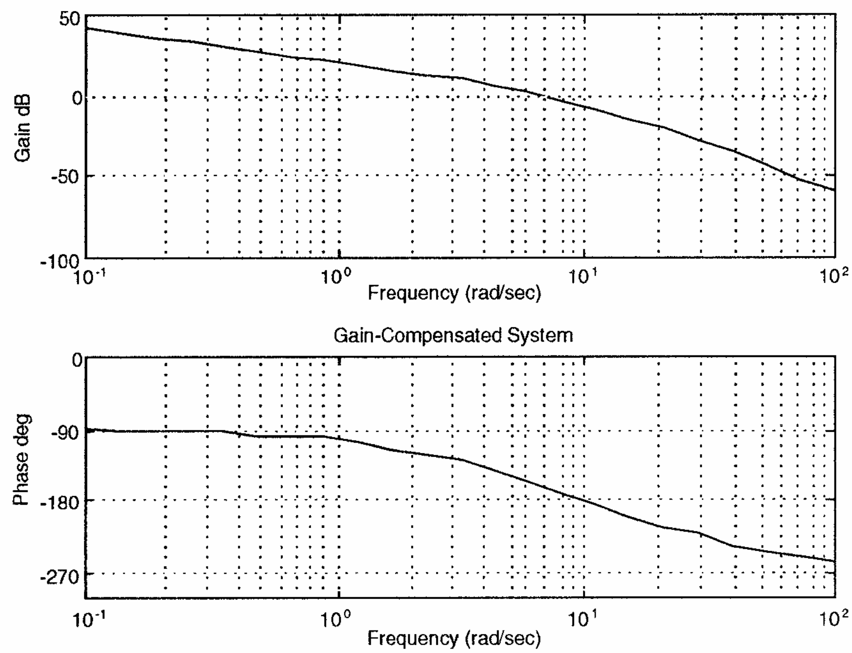
A Bode plot of G(s) shows a phase margin of 37.8°. Thus, a redesign is necessary to meet the exact requirement. This redesign can be done by adding a larger correction factor to the phase required from the lead compensator, See Control Solutions for the redesign.

13.

a. Gain-compensated time response:



Bode plots for K = 1000 (Kv = 10):

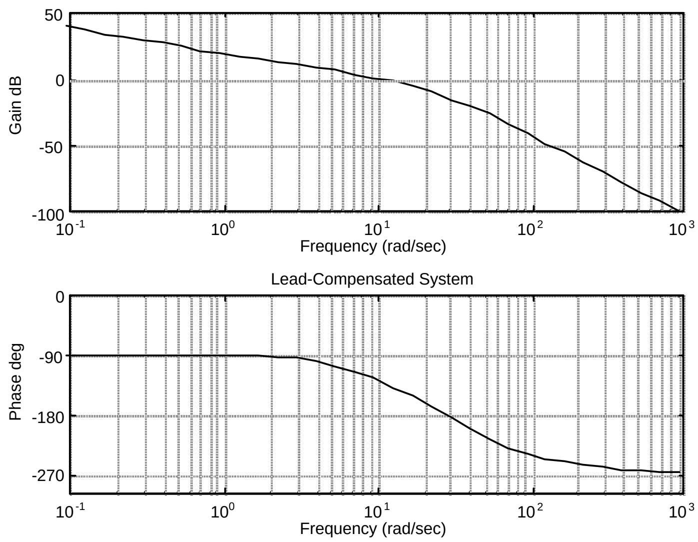


The specifications for the gain compensated system are: K = 1000, percent overshoot = 10, ζ = 0.591155, peak time = 0.5 s, current phase margin = 22.5362°.

To meet the requirements: required phase margin (Eq. 10.73) = 58.5931°, required phase margin with correction factor of 20° = 78.5931, required bandwidth (Eq. 10.56) = 9.03591, required phase contribution from compensator = 78.5931° - 22.5362° = 56.0569°, compensator beta (Eq. 11.11) = 0.0931398, new phase margin frequency (Eq. 11.12) = 11.51.

Now design the compensator: Compensator gain Kc = 1/β = 10.7366, compensator zero (Eq. 11.12) = -3.51272, compensator pole = z_c/β = -37.7144.

Lead-compensated Bode plots:



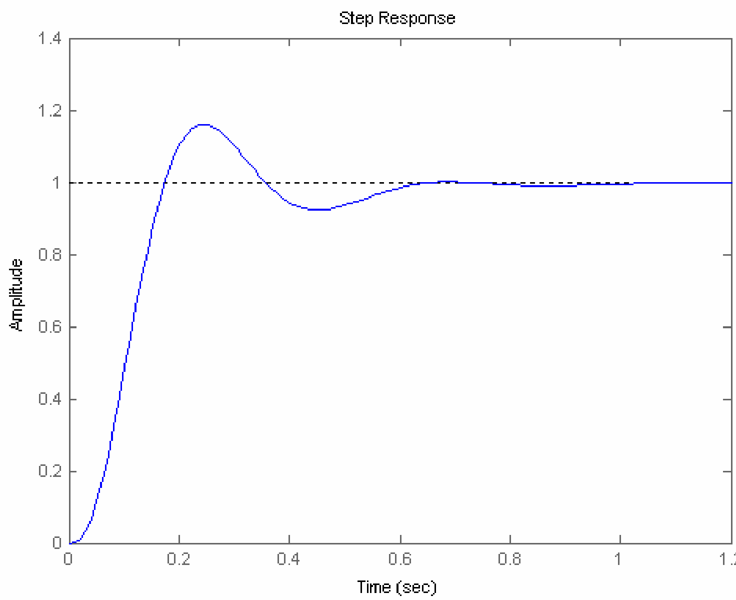
Lead-compensated phase margin = 50.2352.

b.

Program:

```
numg=1000;
deng=poly([0 -5 -20]);
G=tf(numg,deng);
numc=[1 3.51272];
denc=[1 37.7144];
Gc=tf(numc,denc);
Ge=G*10.7366*Gc;
T=feedback(Ge,1);
step(T)
```

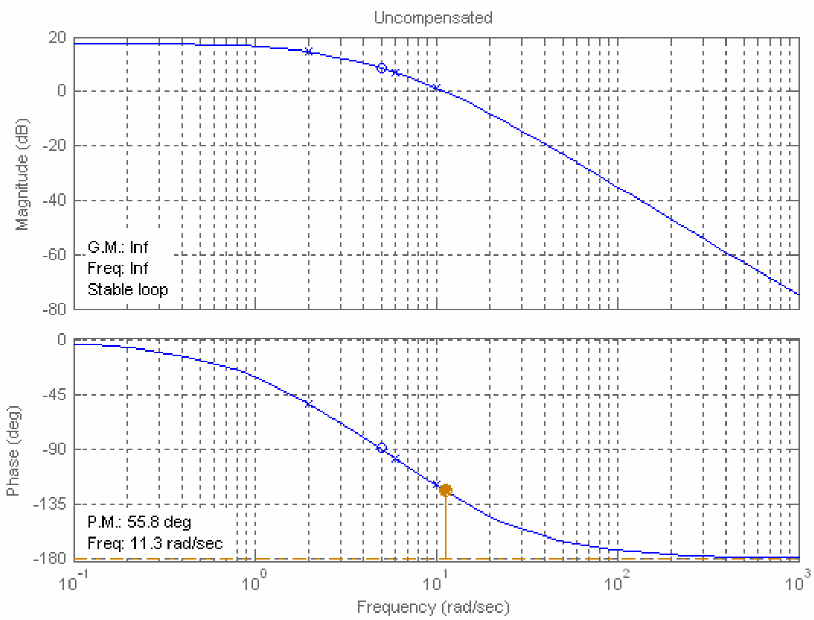
Computer response:



14.

Uncompensated system: Searching the $\zeta = 0.456$ line (20% overshoot), find the dominant poles Q = -6.544 ± j12.771 with a gain of 178.21. Hence, $T_s = \dfrac{4}{\zeta\omega_n}$ = 0.611 second,

$K_p = \dfrac{178.21*5}{2*6*10} = 7.425$. The Bode plot for the uncompensated system is:

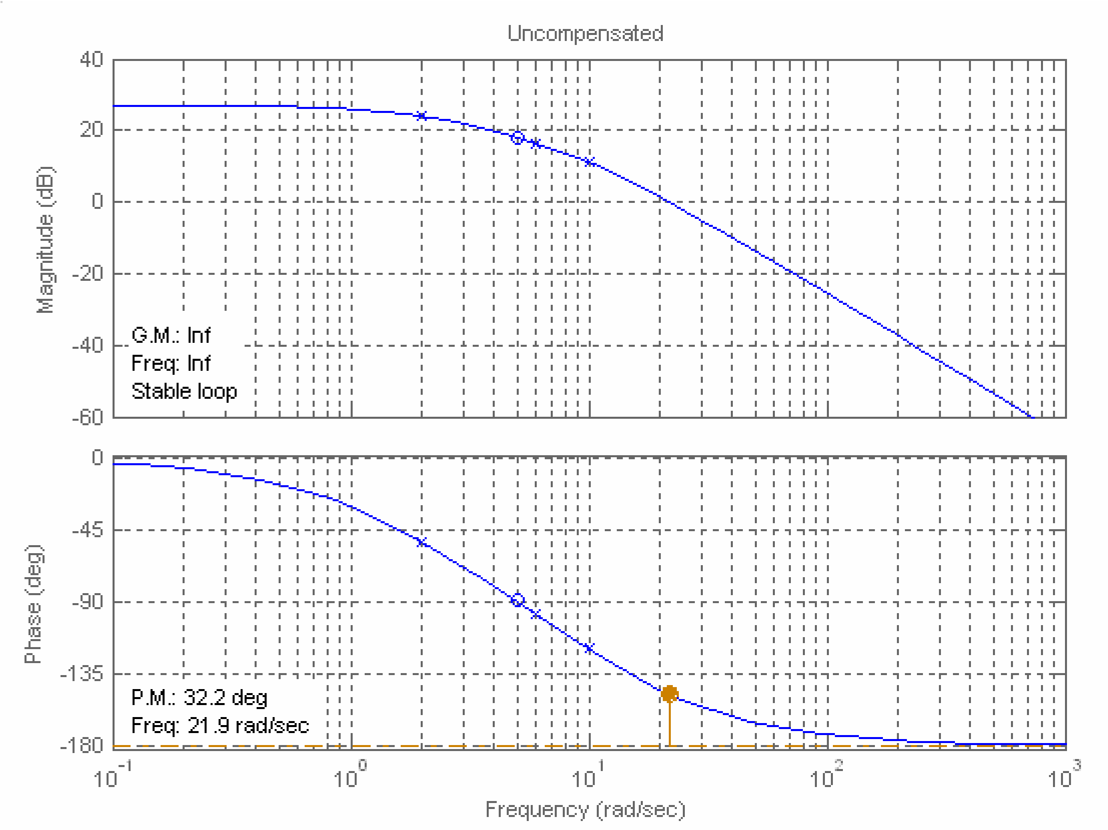


The uncompensated system has a phase margin of 55.8° and a phase margin frequency of 11.3 rad/s.

Compensated system: For a threefold improvement in K_p, K_{pn} = 22.28. Therefore, K = 3*178.21 = 534.63. For a twofold reduction in settling time, the new dominant poles are Q_n = 2Q = - 13.09 ± j25.54. The gain adjusted system is

$$G_c(s) = \frac{534.63(s+5)}{(s+2)(s+6)(s+10)}$$

Plotting the Bode diagrams for the gain compensated system,



At unity gain the phase is -147.8^o at $\omega = 21.9$ rad/s. Thus, the gain compensated phase margin is $180^o - 147.8^o = 32.2^o$. Using Eq. (10.73) with $\zeta = 0.456$ (i.e. 20% overshoot), the required $\Phi_M = 48.15^o$. We add 15.95^o plus a correction factor of 5^o to the phase margin of the gain compensated system for a total additional phase of 20.95^o. Using Eqs. (11.11), and (11.12),

The value of beta is:	0.473
The |G(jwmax)| for the compensator is:	1.45
or in db:	3.25

The magnitude curve has a gain of -3.25 dB at $\omega = 26.9$ rad/s. Therefore, choose this frequency as the new phase margin frequency. Using Eqs. (11.9) and (11.6), the compensator transfer function has the following specifications:

T	0.054
zero	-18.51
pole	-39.1
gain	2.11

The compensated forward path is

$$G(s)=\frac{534.63*2.11(s+5)(s+18.51)}{(s+2)(s+6)(s+10)(s+39.1)}=\frac{1128.1(s+5)(s+18.51)}{(s+2)(s+6)(s+10)(s+39.1)}$$

A simulation of the system shows percent overshoot = 23.2%, settling time = 0.263, phase margin = 48.4°, phase margin frequency = 26.7 r/s.

15.

If G(s) = $\frac{144000}{s(s+36)(s+100)}$, K_V = 40. Also, for a 0.1 second peak time, and ζ = 0.456 (20% overshoot), Eq. (10.56) yields a required bandwidth of 46.59 rad/s. Using Eq. (10.73), the required phase margin is 48.15°. Let us assume that we raise the phase margin frequency to 39 rad/s. The phase angle of the uncompensated system at this frequency is -158.6°. To obtain the required phase margin, the compensator must contribute 26.75° more at 39 rad/s. Assume the following form for the compensator: $G_C(s) = K'K_D(s+\frac{1}{K_D})$. The angle contributed by the compensator is

$\phi_C = \tan^{-1}\frac{\omega}{1/K_D}$ = 26.75°. Letting ω = 39 rad/s, K_D = 0.0129. Hence, the compensator is

$G_C(s)$ = 0.0129 (s+77.37). The compensated forward path is

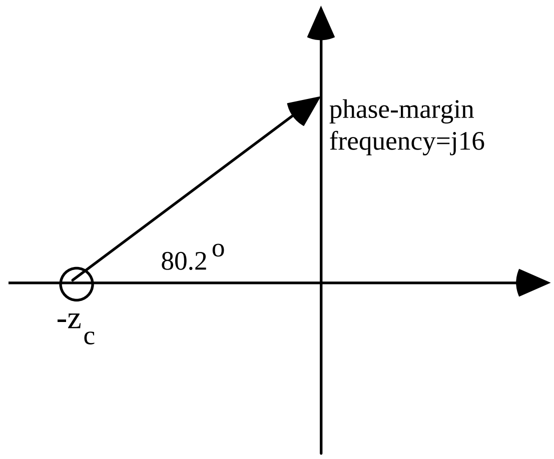
$$G(s)=\frac{144000*0.0129(s+77.37)}{s(s+36)(s+100)}=\frac{1857.6(s+77.37)}{s(s+36)(s+100)}$$

The closed-loop bandwidth is approximately 50 rad/s, which meets the requirements.
The lag compensated forward path is

$$G(s) = 7.759\frac{(s+0.058)}{s(s^2+2s+5)(s+3)(s+0.0015)}$$

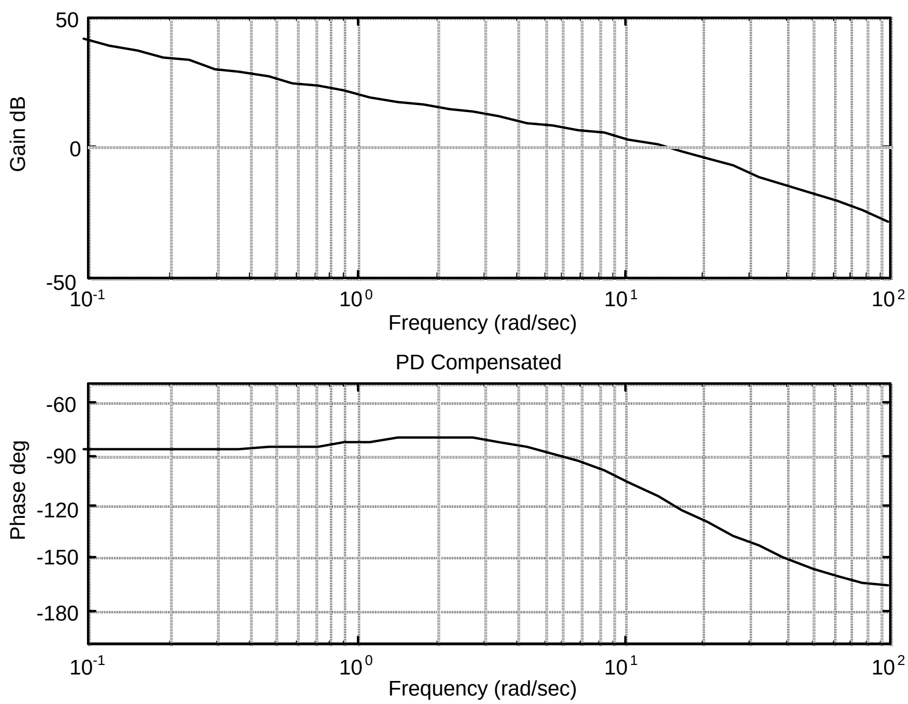
16.

a. Bode plots and specifications for gain compensated system are the same as Problem 13. Required phase margin and required bandwidth is the same as Problem 13. Select a phase margin frequency 7 rad/s higher than the bandwidth = 9 + 7 = 16 rad/s. The phase angle at the new phase-margin frequency is -201.6°. The phase contribution required from the compensator is –180° + 201.6° + 58.59° = 80.2° at the phase-margin frequency. Using the geometry below:



$\tan(80.2) = \frac{16}{z_c}$. Therefore, $z_c = 2.76$. Thus, $G_c(s) = \frac{1}{2.76}(s + 2.76)$.

The PD compensated Bode plots:



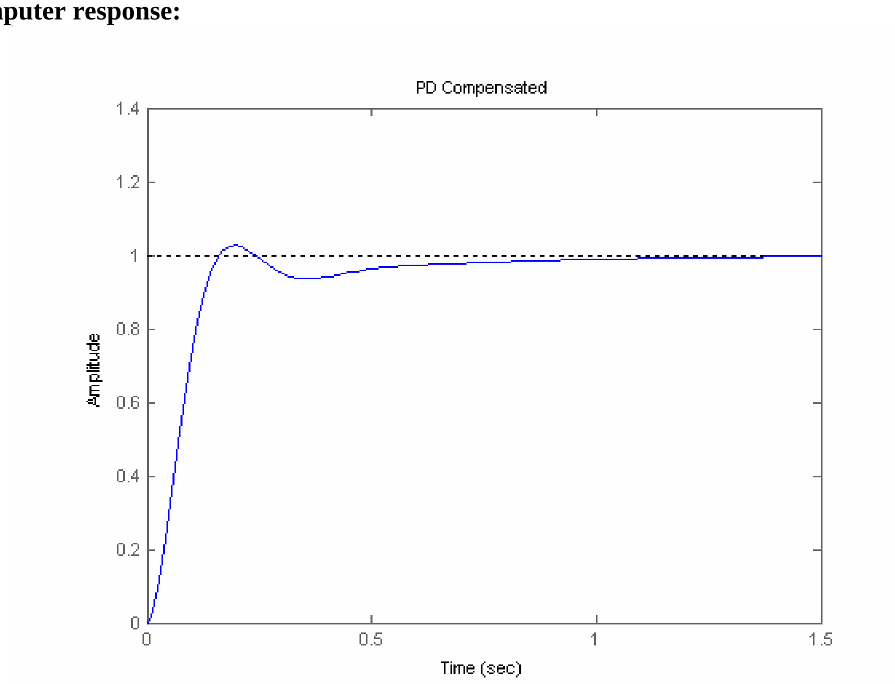
Compensated phase margin is 62.942^o.

b.

Program:

```
numg=1000;
deng=poly([0 -5 -20]);
G=tf(numg,deng);
numc=(1/2.76)*[1 2.76];
denc=1;
Gc=tf(numc,denc);
Ge=G*Gc;
T=feedback(Ge,1);
step(T)
title('PD Compensated')
```

Computer response:



17.

Program:

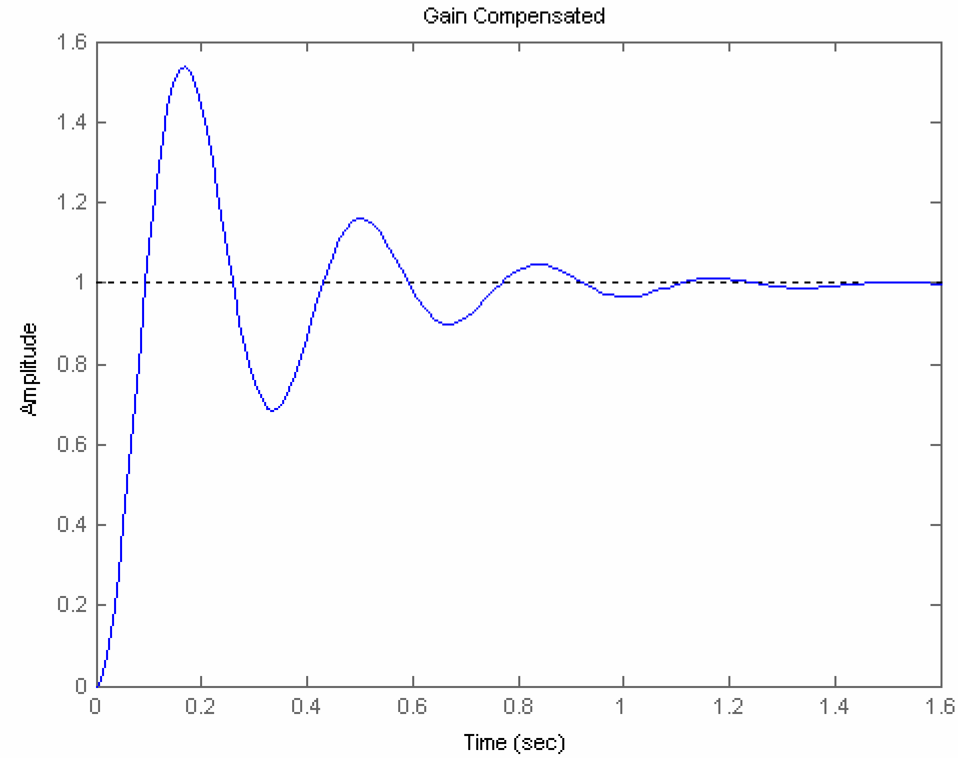
```
%Lead Compensator Design via Frequency Response
         %Input system
K=input('Type K to meet steady-state error  ');
numg=K*[1 1];
deng=poly([0 -2 -6]);
'Open-loop system'
'G(s)'
G=tf(numg,deng)
            %Generate uncompensated step response
T=feedback(G,1);
step(T)
title('Gain Compensated')

            %Input transient response specifications
Po=input('Type %OS  ');
%Ts=input('Type settling time   ');
Tp=input('Type peak time   ');
         %Determine required bandwidth
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
%wn=4/(z*Ts);
wn=pi/(Tp*sqrt(1-z^2));
wBW=wn*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2));

         %Make a Bode plot and get Bode data
%Get Bode data
bode(G)
title('Gain Compensated')

w=0.01:0.1:100;
[M,P]=bode(numg,deng,w);

         %Find current phase margin
[Gm,Pm,wcp,wcg]=margin(M,P,w);

         %Calculate required phase margin
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
Pmreq=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi);
```

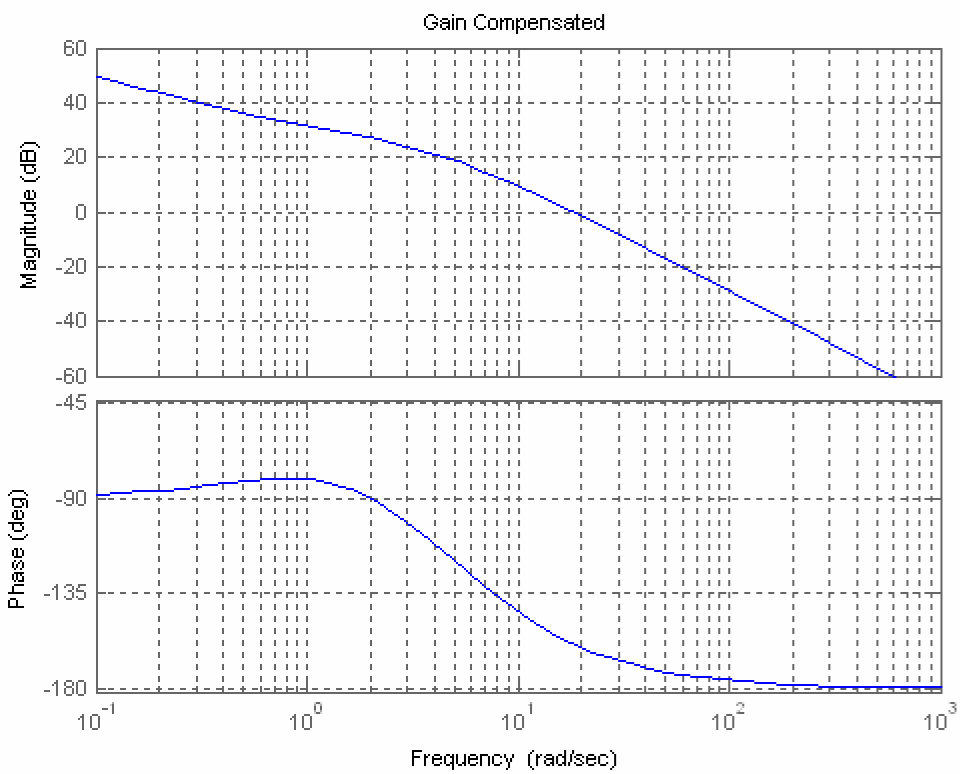
```
         %Add a correction factor of 10 degrees
Pmreqc=Pmreq+10;

         %Calculate phase required from compensator
Pc=Pmreqc-Pm;

         %Design lead compensator
%Find compensator beta and peak compensator magnitude
beta=(1-sin(Pc*pi/180))/(1+sin(Pc*pi/180));
magpc=1/sqrt(beta);
%Find frequency at which uncompensated system has a magnitude of 1/magpc
%This frequency will be the new phase margin frequency
for i=1:1:length(M);
if M(i)-(1/magpc)<=0;
wmax=w(i);
break
end
end
%Calculate the lead compensator's break frequencies
zc=wmax*sqrt(beta);
pc=zc/beta;
Kc=1/beta;
numc=[1 zc];
denc=[1 pc];
'Gc(s)'
Gc=tf(numc,denc)
           %Display data
fprintf('\nK = %g',K)
fprintf('  Percent Overshoot = %g',Po)
fprintf(', Damping Ratio = %g',z)
%fprintf(', Settling Time = %g',Ts)
fprintf(', Peak Time = %g',Tp)
fprintf(', Current Phase Margin = %g',Pm)
fprintf(', Required Phase Margin = %g',Pmreq)
fprintf(', Required Phase Margin with Correction Factor = %g',Pmreqc)
fprintf(', Required Bandwidth = %g',wBW)
fprintf(', Required Phase Contribution from Compensator = %g',Pc)
fprintf(', Compensator Beta = %g',beta)
fprintf(', New phase margin frequency = %g',wmax)
fprintf('  Compensator gain, Kc = %g',Kc)
fprintf('  Compensator zero,= %g',-zc)
fprintf('  Compensator pole,= %g',-pc)
'G(s)Gc(s)'
Ge=G*Kc*Gc
pause
            %Generate compensated Bode plots
%Make a Bode plot and get Bode data
%Get Bode data
bode(Ge)
title('Lead Compensated')

w=0.01:0.1:1000;
[M,P]=bode(Ge,w);
%Find compensated phase margin
[Gm,Pm,wcp,wcg]=margin(M,P,w);
fprintf('\nCompensated Phase Margin,= %g',Pm)
pause
            %Generate step response
T=feedback(Ge,1);
step(T)
title('Lead Compensated')
```

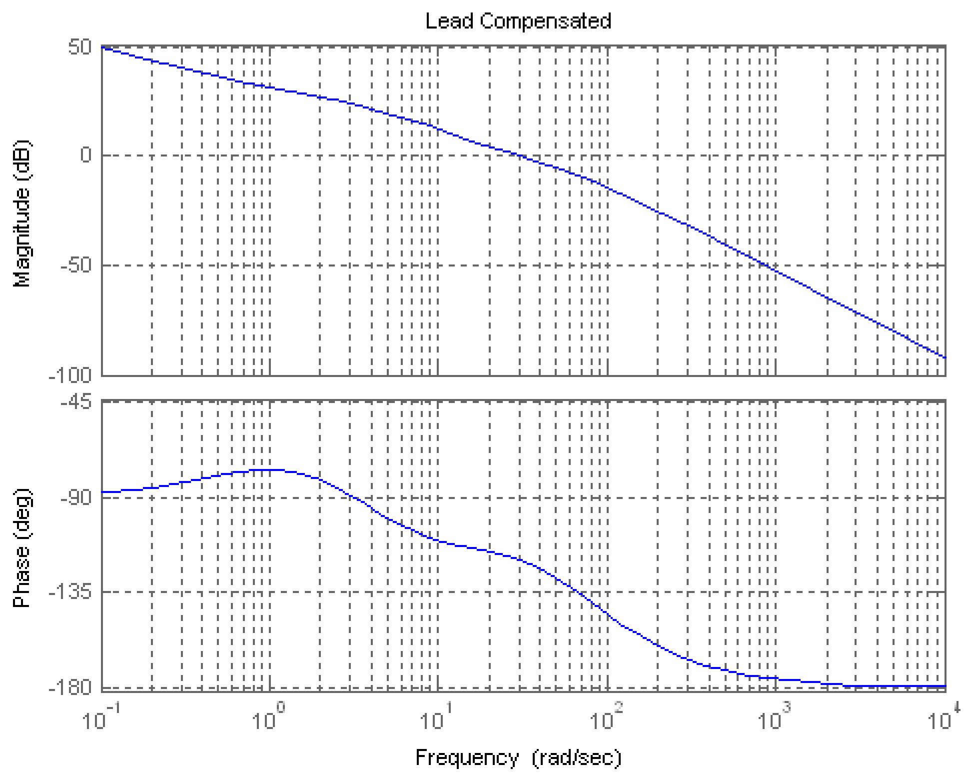
Computer response:

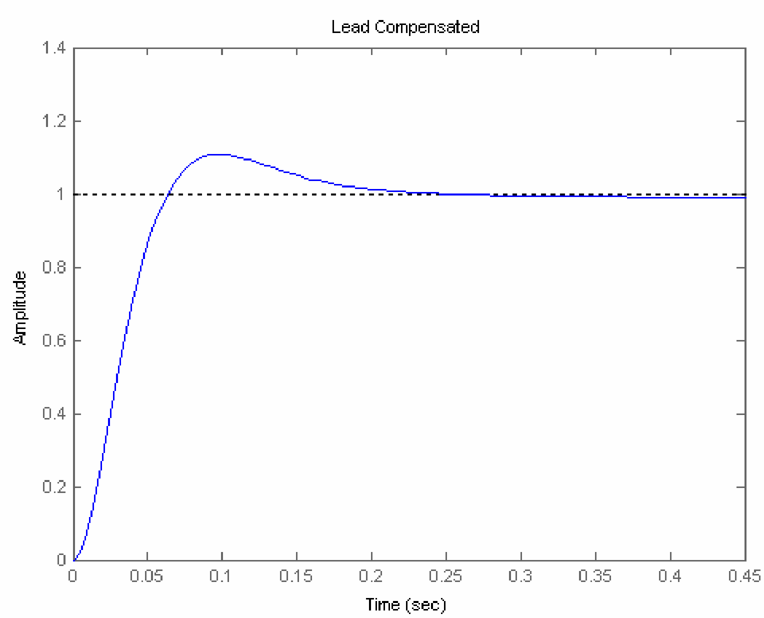
```
Type K to meet steady-state error  360

ans =
```

```
Open-loop system

ans =

G(s)

Transfer function:
   360 s + 360
------------------
s^3 + 8 s^2 + 12 s

Type %OS  10
Type peak time   0.1

ans =

Gc(s)

Transfer function:
s + 11.71
---------
s + 77.44

K = 360  Percent Overshoot = 10, Damping Ratio = 0.591155, Peak Time = 0.1,
Current Phase Margin = 21.0851, Required Phase Margin = 58.5931, Required
Phase Margin with Correction Factor = 68.5931, Required Bandwidth =
45.1795, Required Phase Contribution from Compensator = 47.508, Compensator
Beta = 0.151164, New phase margin frequency = 30.11  Compensator gain, Kc =
6.61532  Compensator zero,= -11.7067  Compensator pole,= -77.4437
ans =

G(s)Gc(s)

Transfer function:
 2382 s^2 + 3.026e004 s + 2.788e004
-------------------------------------
s^4 + 85.44 s^3 + 631.5 s^2 + 929.3 s

Compensated Phase Margin,= 60.676»
```


Gain Compensated
Amplitude
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6


Gain Compensated
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
60
40
20
0
-20
-40
-60
-45
-90
-135
-180


Lead Compensated
Magnitude (dB)
50
0
-50
-100
Phase (deg)
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Frequency (rad/sec)


Lead Compensated
Amplitude
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
Time (sec)

18.

Program:

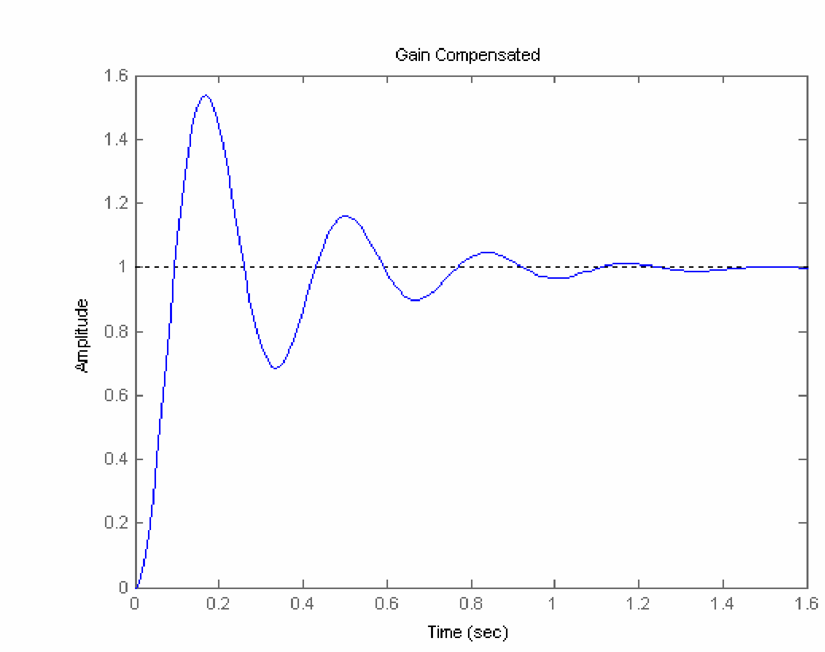
```
%PD Compensator Design via Frequency Response
         %Input system
%Uncompensated system
K=input('Type K to meet steady-state error  ');
numg=K*[1 1];
deng=poly([0 -2 -6]);
G=tf(numg,deng);
T=feedback(G,1);
step(T)
title('Gain Compensated')
'Open-loop system'
'G(s)'
Gzpk=zpk(G)

         %Input transient response specifications
Po=input('Type %OS  ');
%Ts=input('Type settling time   ');
Tp=input('Type peak time   ');

         %Determine required bandwidth
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
%wn=4/(z*Ts);
wn=pi/(Tp*sqrt(1-z^2));
wBW=wn*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2));

         %Make a Bode plot and get Bode data
%Get Bode data
bode(G)
title('Gain Compensated')
w=0.01:0.1:100;
[M,P]=bode(G,w);

         %Find current phase margin
[Gm,Pm,wcp,wcg]=margin(M,P,w);

         %Calculate required phase margin
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
Pmreq=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi)+20;

         %Determine a phase margin frequency
wpm=wBW+7;
         %Find phase angle at new phase margin frequency and

         %calculate phase required from the compensator
for i=1:1:length(w);
if w(i)-wpm>=0;
wpm=w(i);
Pwpm=P(i);
break
end
end
         %Design PD compensator
Pc=Pmreq-(180+Pwpm);
zc=wpm/tan(Pc*pi/180);
Kc=1/zc;
numc=Kc*[1 zc];
denc=1;
'Gc(s)'
Gc=tf(numc,denc);
Gczpk=zpk(Gc)
           %Display data
fprintf('\nK = %g',K)
fprintf('  Percent Overshoot = %g',Po)
fprintf(', Damping Ratio = %g',z)
%fprintf(', Settling Time = %g',Ts)
```

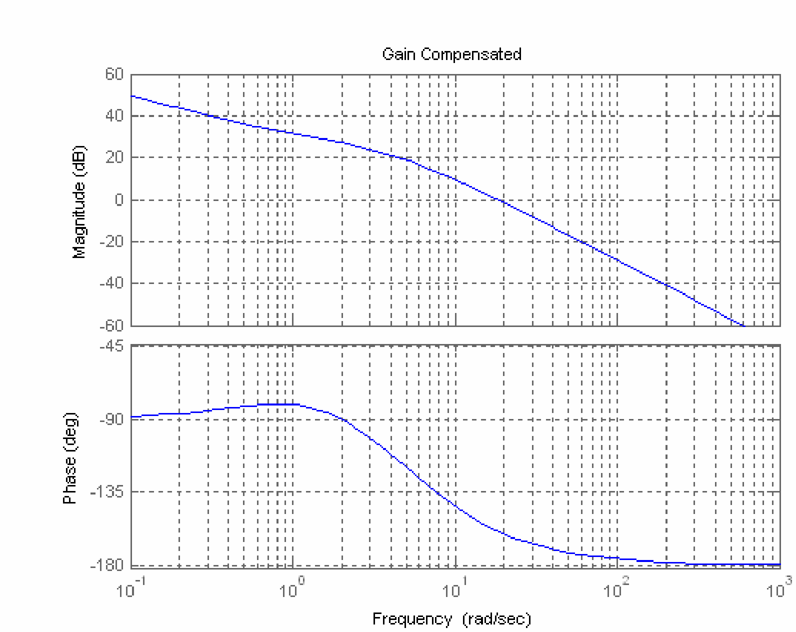
```
fprintf(', Peak Time = %g',Tp)
fprintf(', Current Phase Margin = %g',Pm)
fprintf(', Required Phase Margin = %g',Pmreq)
fprintf(', Required Bandwidth = %g',wBW)
fprintf(', New phase margin frequency = %g',wpm)
fprintf(', Phase angle at new phase margin frequency = %g',Pwpm)
fprintf(', Required Phase Contribution from Compensator = %g',Pc)
fprintf('  Compensator gain, Kc = %g',Kc)
fprintf('  Compensator zero,= %g',-zc)

pause
         %Generate compensated Bode plots
%Make a Bode plot and get Bode data
%Get Bode data
'G(s)Gc(s)'
Ge=G*Gc;
Gezpk=zpk(Ge)
bode(Ge)
title('PD Compensated')
w=0.01:0.1:100;
[M,P]=bode(Ge,w);
%Find compensated phase margin
[Gm,Pm,wcp,wcg]=margin(M,P,w);
fprintf('\nCompensated Phase Margin,= %g',Pm)
pause
            %Generate step response
T=feedback(Ge,1);
step(T)
title('PD Compensated')
```

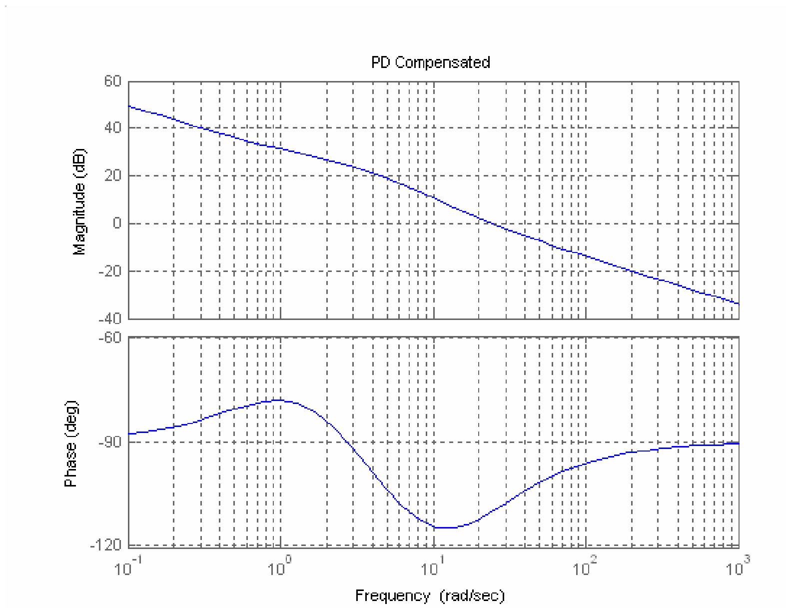
Computer response:

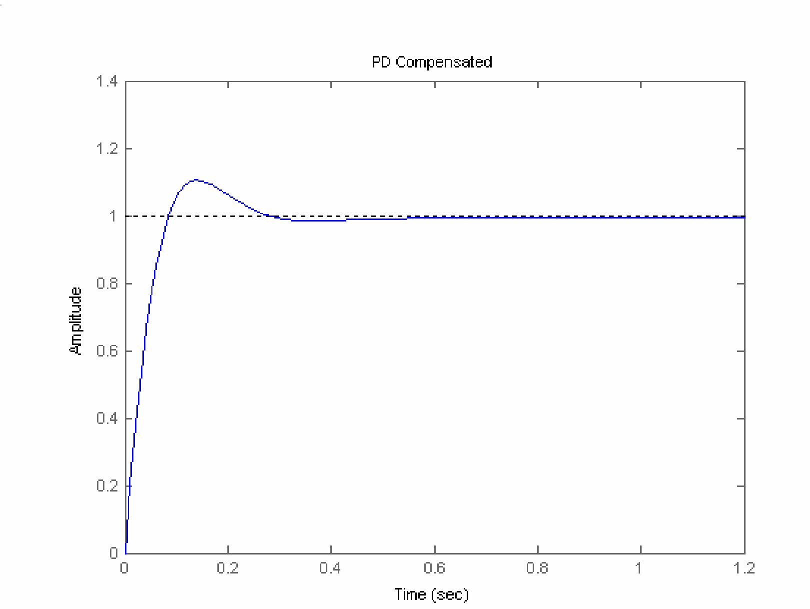
```
Type K to meet steady-state error  360

ans =

Open-loop system


ans =

G(s)


Zero/pole/gain:
  360 (s+1)
-------------
s (s+6) (s+2)

Type %OS  10
Type peak time   0.1

ans =

Gc(s)


Zero/pole/gain:
0.05544 (s+18.04)


K = 360  Percent Overshoot = 10, Damping Ratio = 0.591155, Peak Time = 0.1,
Current Phase Margin = 21.0851, Required Phase Margin = 78.5931, Required
Bandwidth = 45.1795, New phase margin frequency = 52.21, Phase angle at new
phase margin frequency = -172.348, Required Phase Contribution from
Compensator = 70.9409  Compensator gain, Kc = 0.0554397  Compensator zero,=
-18.0376
ans =
```

```
G(s)Gc(s)

Zero/pole/gain:
19.9583 (s+18.04) (s+1)
-----------------------
     s (s+6) (s+2)

Compensated Phase Margin,= 69.546
```



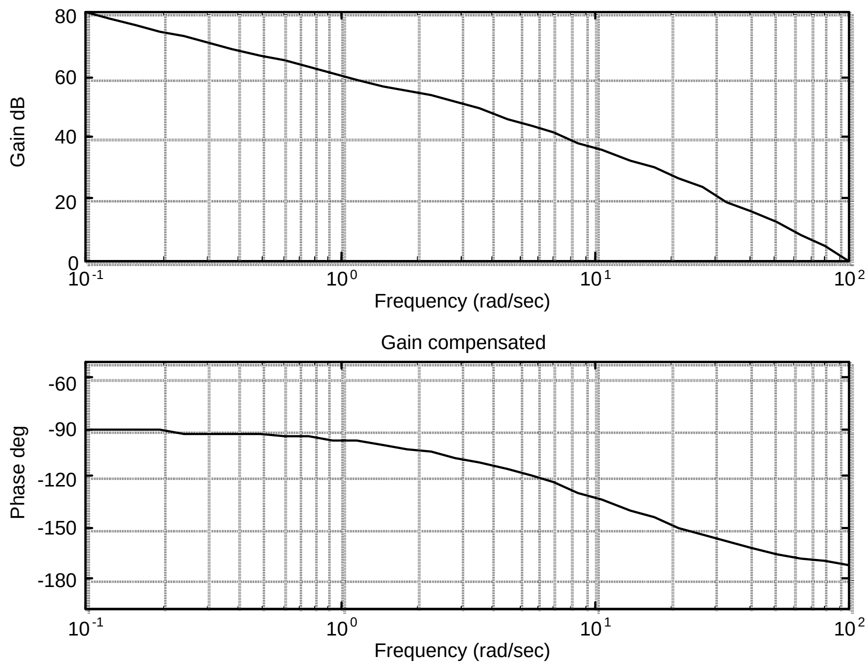






19.

K = 10714 for K_v = 1000. ζ = 0.517 for 15% overshoot using Eq. (4.39). Using Eq. (4.42), ω_n = 77.37. Using Eq. (10.54) the required bandwidth, ω_{BW} = 96.91. Using Eq. (10.73) with 5^o additional, Φ_m = 58.17^o. Choose the new phase-margin frequency ω_{Pm}= 0.8 w_{BW}= 77.53. Plotting the Bode plots for K = 10714,



At the new phase-margin frequency, the phase angle is -170.52. Thus, the contribution required from the lead is 58.17 - (180 -170.52) = 48.69°. Using Eq. (11.11), β = 0.142.

Lag compensator design: $z_{clag} = \omega_{Pm}/10 = 77.53/10 = 7.753$. $p_{clag} = z_{clag}*\beta = 1.102$. $K_{clag} = p_{clag}/z_{clag} = 0.1421$. Thus, $G_{lag}(s) = 0.1421\dfrac{s+7.753}{s+1.102}$.

Lead compensator design: Using Eqs. (11.6), (11.9), and (11.12) $z_{lead} = 1/T = \omega_{Pm}*\sqrt{\beta} = 29.22$. $p_{lead} = z_{lead}/\beta = 205.74$. $K_{lead} = p_{lead}/z_{lead} = 7.04$. Thus, $G_{lead}(s) = 7.04\dfrac{s+29.22}{s+205.74}$.

20.

Program:

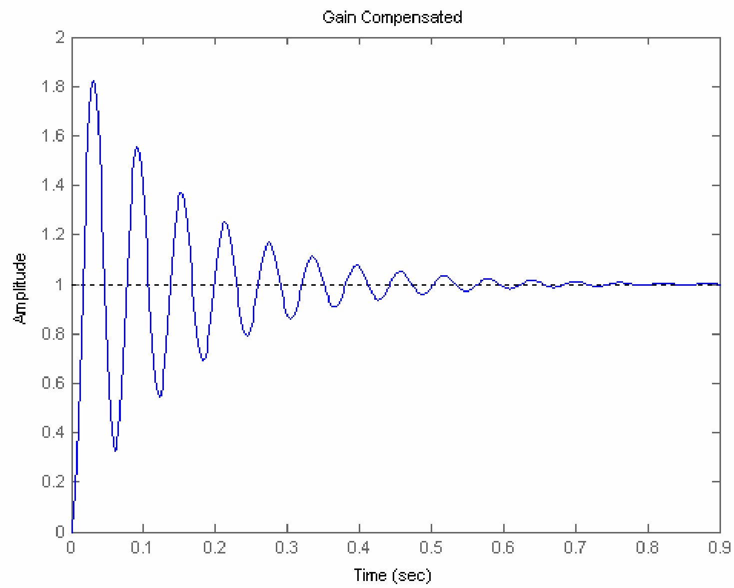
```
         %Lag-Lead Compensator Design via Frequency Response
         %Input system
K=input('Type K to meet steady-state error  ');
numg=K*[1 7];
deng=poly([0 -5 -15]);
G=tf(numg,deng);
'G(s)'
Gzpk=zpk(G)

         %Input transient response specifications
Po=input('Type %OS  ');
Ts=input('Type settling time   ');
%Tp=input('Type peak time   ');
T=feedback(G,1);
step(T)
title('Gain Compensated')
pause

        %Determine required bandwidth
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
wn=4/(z*Ts);
%wn=pi/(Tp*sqrt(1-z^2));
```

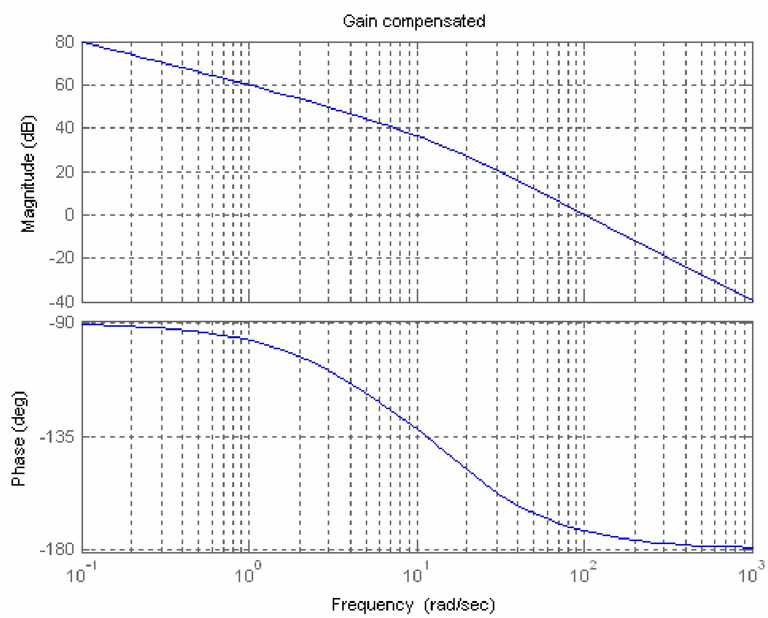
```
wBW=wn*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2));
%wBW=(4/(Ts*z))*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2));
%wBW=(pi/(Tp*sqrt(1-z^2)))*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2));

        %Determine required phase margin
Pmreq=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi)+5;

        %Choose new phase margin frequency
wpm=0.8*wBW;

        %Determine additional phase lead required at the
        %new phase margin frequency from the lead compensator
[M,P]=bode(G,wpm);
Pmreqc=Pmreq-(180+P);
beta=(1-sin(Pmreqc*pi/180))/(1+sin(Pmreqc*pi/180));
        %Display data
fprintf('\nPercent Overshoot = %g',Po)
fprintf(', Settling Time = %g',Ts)
%fprintf(', Peak Time = %g',Tp)
fprintf(', Damping Ratio = %g',z)
fprintf(', Required Phase Margin = %g',Pmreq)
fprintf(', Required Bandwidth = %g',wBW)
fprintf(', New Phase Margin Frequency = %g',wpm)
fprintf(', Required Phase from Lead Compensator = %g',Pmreqc)
fprintf(', Beta = %g',beta)
bode(numg,deng)
title('Gain compensated')
pause
          %Design lag compensator
zclag=wpm/10;
pclag=zclag*beta;
Kclag=beta;
'Lag compensator'
'Gclag'
Gclag=tf(Kclag*[1 zclag],[1 pclag]);
Gclagzpk=zpk(Gclag)
          %Design lead compensator
zclead=wpm*sqrt(beta);
pclead=zclead/beta;
Kclead=1/beta;
'Lead compensator'
'Gclead'
Gclead=tf(Kclead*[1 zclead],[1 pclead]);
Gcleadzpk=zpk(Gclead)
          %Create compensated forward path
'Gclag(s)Gclead(s)G(s)'
Ge=G*Gclag*Gclead;
Gezpk=zpk(Ge)
          %Test lag-lead compensator
T=feedback(Ge,1);
bode(Ge)
title('Lag-lead Compensated')
[M,P,w]=bode(Ge);
[Gm,Pm,wcp,wcg]=margin(M,P,w);
'Compensated System Results'
fprintf('\nResulting Phase Margin = %g',Pm)
fprintf(', Resulting Phase Margin Frequency = %g',wcg)
pause
step(T)
title('Lag-lead Compensated')
```

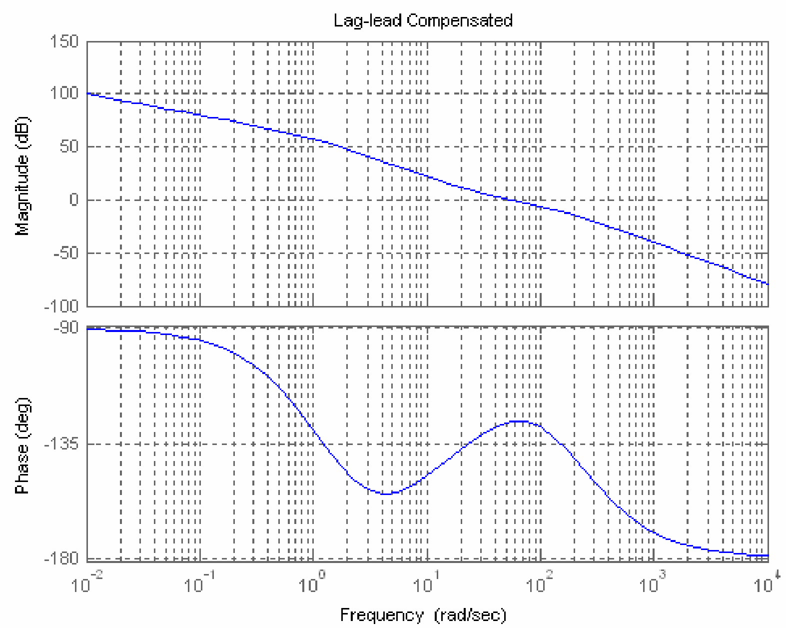
Computer response:

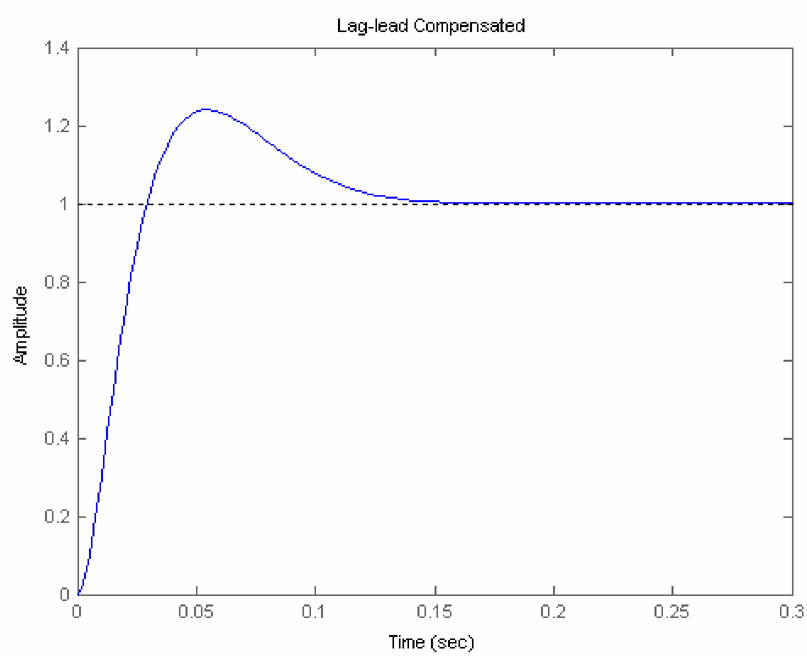
```
Type K to meet steady-state error  10714.29

ans =

G(s)
```

```
Zero/pole/gain:
10714.29 (s+7)
--------------
s (s+15) (s+5)

Type %OS  15
Type settling time   0.1

Percent Overshoot = 15, Settling Time = 0.1, Damping Ratio = 0.516931,
Required Phase Margin = 58.1718, Required Bandwidth = 96.9143, New Phase
Margin Frequency = 77.5314, Required Phase from Lead Compensator = 48.6912,
Beta = 0.142098
ans =

Lag compensator


ans =

Gclag


Zero/pole/gain:
0.1421 (s+7.753)
----------------
   (s+1.102)


ans =

Lead compensator


ans =

Gclead


Zero/pole/gain:
7.0374 (s+29.23)
----------------
   (s+205.7)


ans =

Gclag(s)Gclead(s)G(s)


Zero/pole/gain:
10714.29 (s+29.23) (s+7.753) (s+7)
----------------------------------
s (s+205.7) (s+15) (s+5) (s+1.102)


ans =

Compensated System Results


Resulting Phase Margin = 53.3994, Resulting Phase Margin Frequency =
55.5874
```


Gain Compensated
Amplitude
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0.1
0.3
0.5
0.7
0.9


Gain compensated
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
80
60
40
20
0
-20
-40
-90
-135
-180

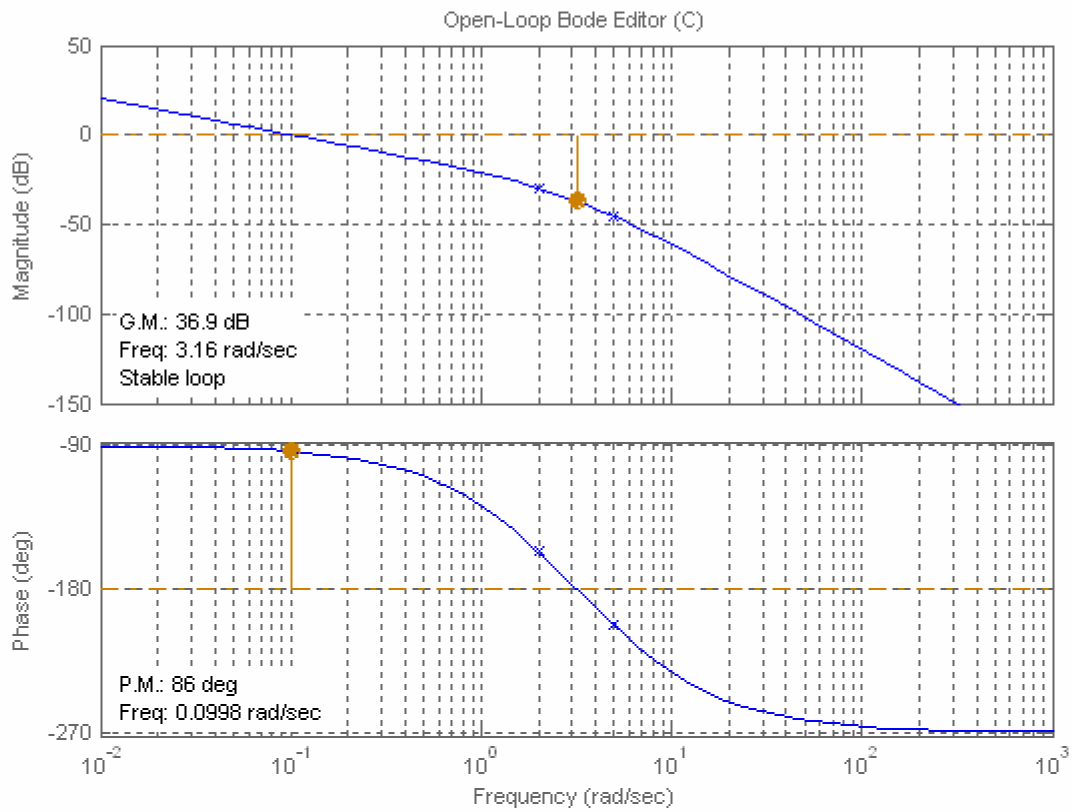




Percent overshoot exceeds requirements. Redesign if required.

21.

The required bandwidth for a peak time of 2 seconds and $\zeta = 0.456$ (i.e. 20% overshoot) is 2.3297 rad/s. Plotting the Bode diagrams for $K = 1$,



For 20% overshoot, Φ_M = 48.15°, or a phase angle of -180° + 48.15° = -131.85°. This angle occurs at 1.12 rad/s. If K = 13.1, the magnitude curve will intersect zero dB at 1.12 rad/s. Thus, the following function yields 20% overshoot: $G(s) = \dfrac{13.1}{s(s+2)(s+5)}$.

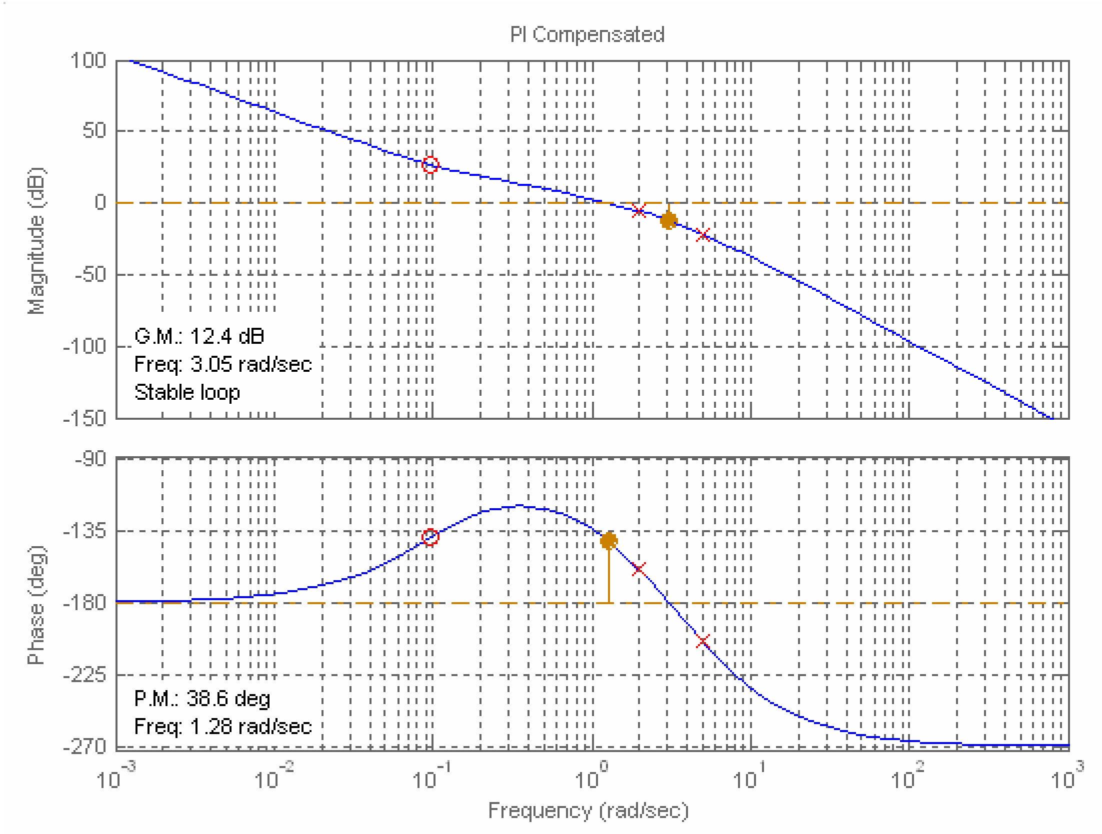
PI controller design: Allowing for a 5° margin, we want Φ_M = 48.15° + 5° = 53.15°, or a phase angle of -180° + 53.15° = -126.85°. This angle occurs at ω = 0.97 rad/s where the magnitude curve is 1.5321 dB. The controller should contribute - 1.5321 dB so that the magnitude curve passes through 0 dB at ω = 0.97 rad/s. Choosing the break frequency one decade below the phase margin frequency of 0.97 rad/s, and adjusting the controller's gain to yield -1.5321 dB at high frequencies, the ideal integral controller is

$$G_{cPI}(s) = \frac{1.198(s+0.097)}{s}$$

and the PI compensated forward path is

$$G_{PI}(s) = G(s)G_{cPI}(s) = \frac{15.694(s+0.097)}{s^2(s+2)(s+5)}$$

Plotting the Bode diagram for the PI compensated system yields,



This function is zero dB at ω = 1.28 rad/s. The phase at this frequency is – 141.4°. Thus, we have a phase margin of 38.6°.

PID controller design: Let us increase the phase margin frequency to 4 rad/s. At this frequency the phase is -193.48°. To obtain the required phase margin of 48.15° the phase curve must be raised an additional 61.63°. Assume the following form for the compensator : $G_{cPD}(s) = K'K_D(s+\frac{1}{K_D})$. The angle contributed by the compensator is $\phi_c = \tan^{-1}\frac{\omega}{1/K_D}$ = 61.63°. Letting ω = 4 rad/s, K_D = 0.463. Hence, the compensator is $G_{cPD}(s) = 0.463K'(s+2.16)$. The final PID compensated forward path is

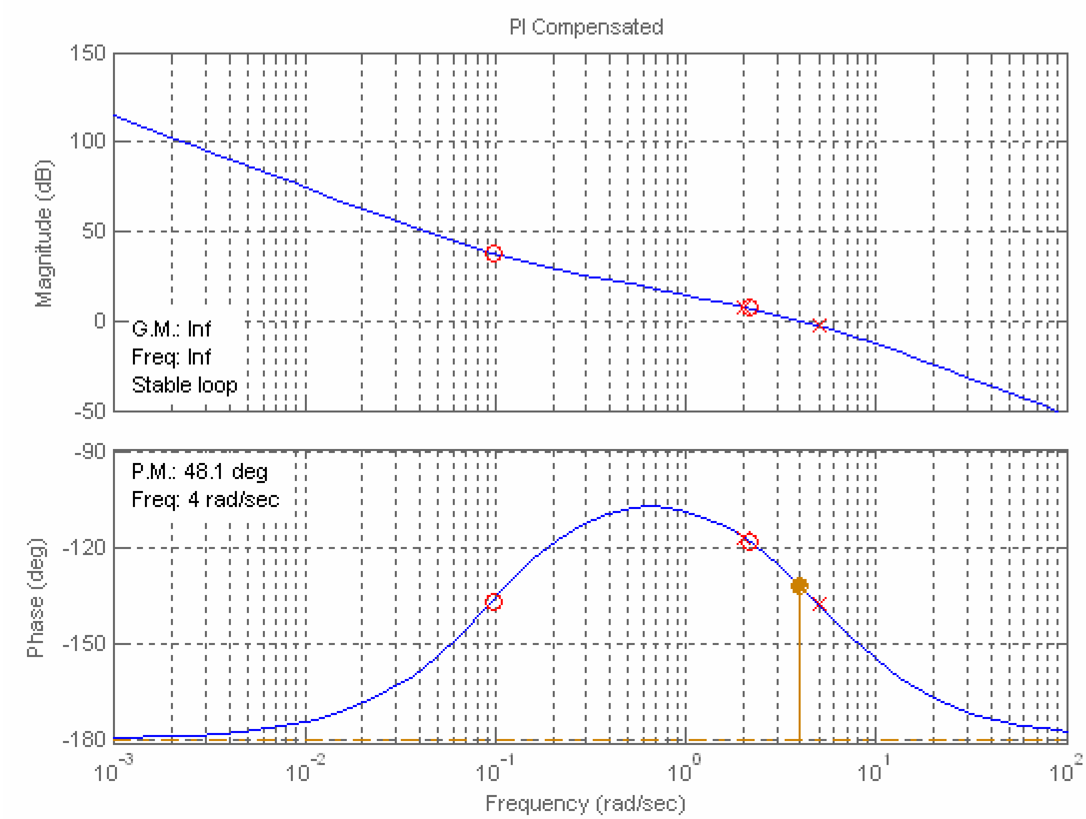
$$G_{PID}(s)=G_{PI}(s)G_{cPD}(s) = \frac{15.694(s+0.097)}{s^2(s+2)(s+5)} * 0.463K'(s+2.16) = \frac{2.266K'(s+0.097)(s+2.16)}{s^2(s+2)(s+5)}$$

Letting K' = 1 the magnitude of this function at 4 rad/s is -20.92 dB. Thus, K' must be adjusted to bring the magnitude to zero dB. Hence, K' = 11.12 (20.92 dB).

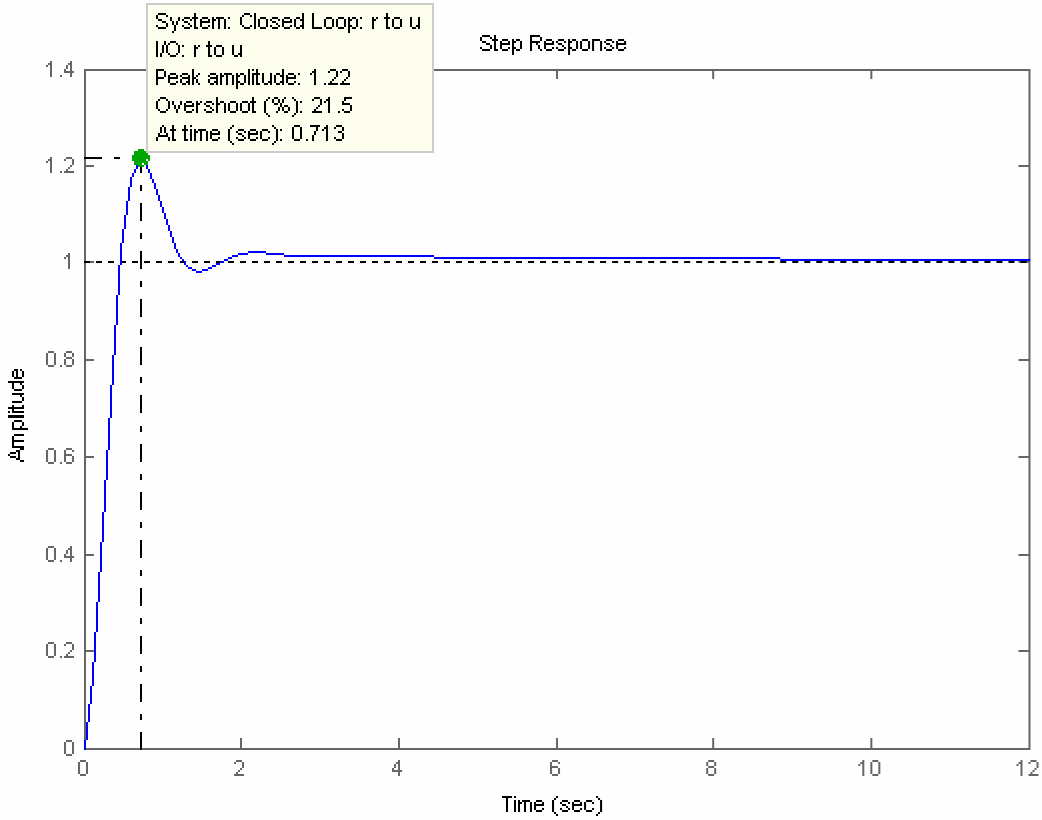
Thus,

$$G_{PID}(s) = \frac{25.2(s+0.097)(s+2.16)}{s^2(s+2)(s+5)}$$

The PID compensated Bode plot follows:



The PID compensated time response is shown below:



22.

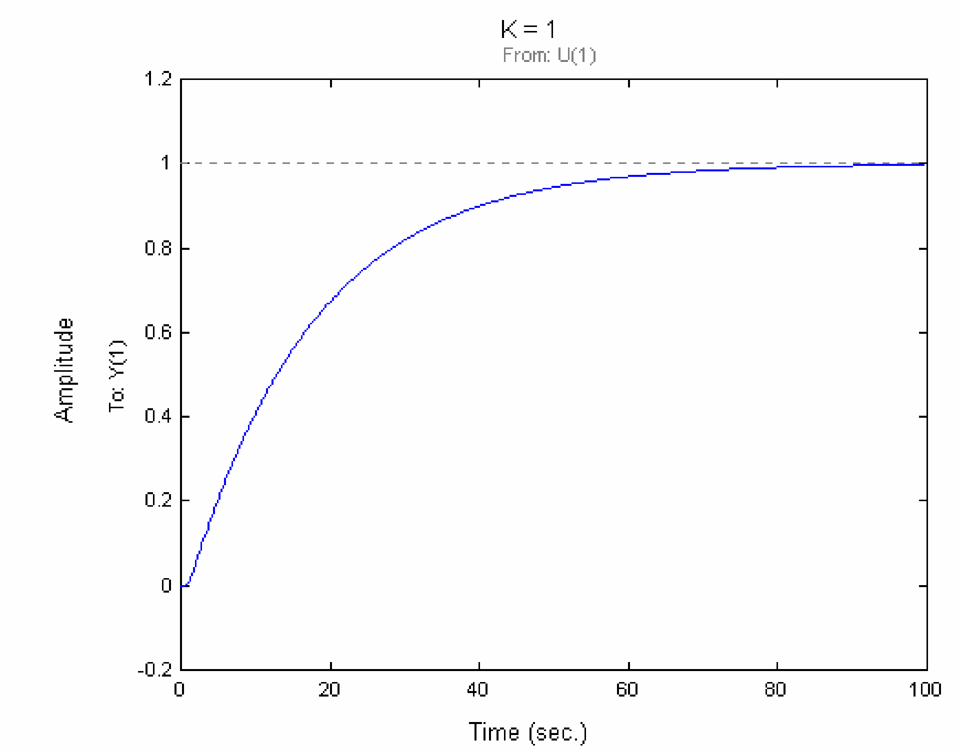
Program:

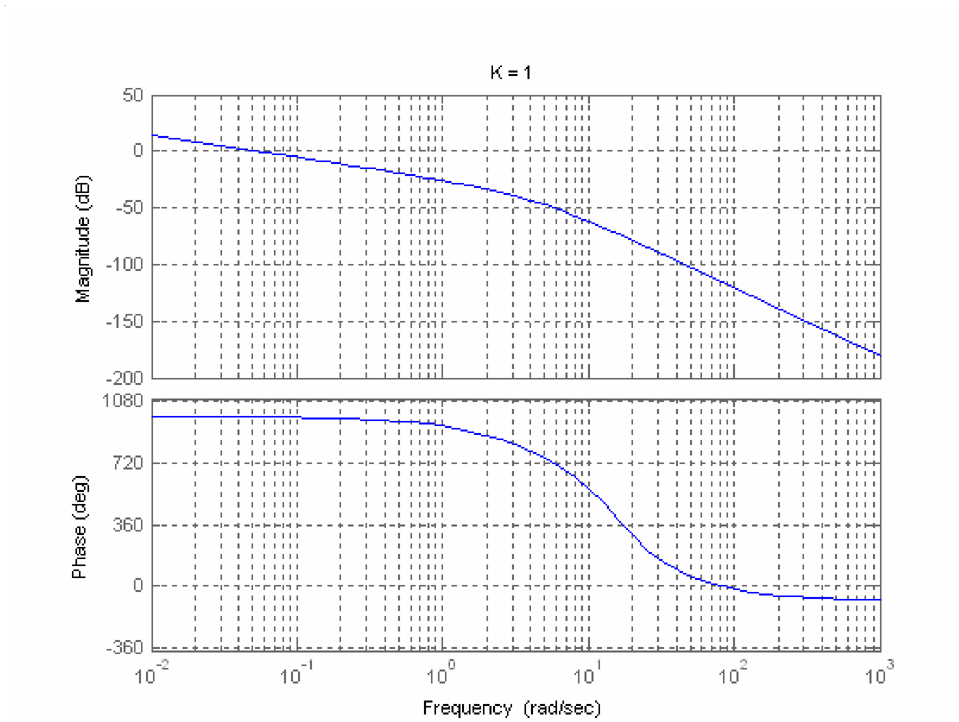
```
%Input system
numg1=1;
deng1=poly([0 -3 -6]);
G1=tf(numg1,deng1);
[numg2,deng2]=pade(0.5,5);
G2=tf(numg2,deng2);
'G(s)=G1(s)G2(s)'
G=G1*G2;
Gzpk=zpk(G)
Tu=feedback(G,1);
step(Tu)
title('K = 1')
%Percent Overshoot to Damping Ratio to Phase Margin
Po=input('Type %OS  ');
z=(-log(Po/100))/(sqrt(pi^2+log(Po/100)^2));
Pm=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi);
fprintf('\nPercent Overshoot = %g',Po)
fprintf(', Damping Ratio = %g',z)
fprintf(', Phase Margin = %g',Pm)
%Get Bode data
bode(G)
title('K = 1')
pause
w=0.1:0.01:100;
[M,P]=bode(G,w);
Ph=-180+Pm;
for i=1:1:length(P);
if P(i)-Ph<=0;
M=M(i);
K=1/M;
```

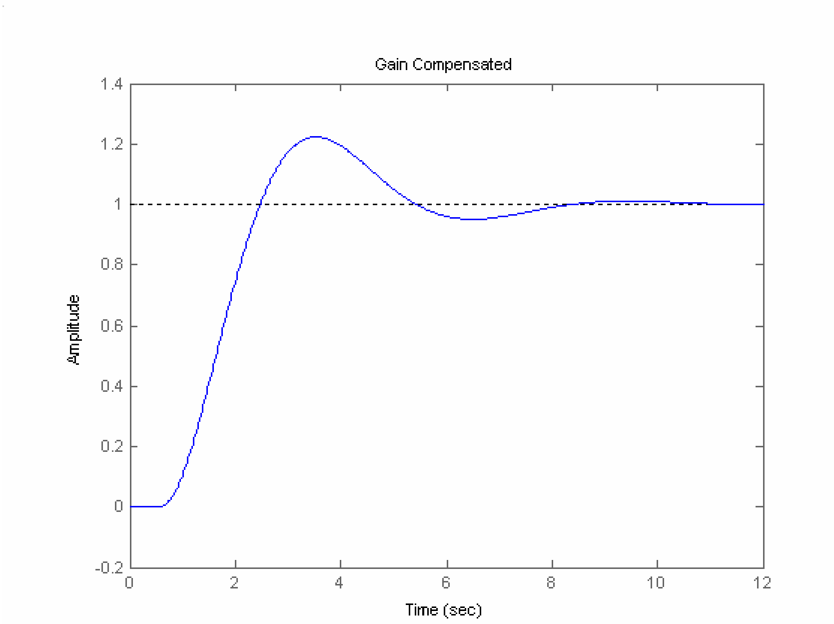
```
fprintf(', Frequency  = %g',w(i))
fprintf(', Phase  = %g',P(i))
fprintf(', Magnitude = %g',M)
fprintf(', Magnitude (dB) = %g',20*log10(M))
fprintf(', K = %g',K)
break
end
end
T=feedback(K*G,1);
step(T)
title('Gain Compensated')
```

Computer response:

```
ans =

G(s)=G1(s)G2(s)

Zero/pole/gain:
      - (s-14.59) (s^2 - 26.82s + 228.4) (s^2 - 18.6s + 290.5)
--------------------------------------------------------------------
s (s+14.59) (s+6) (s+3) (s^2 + 26.82s + 228.4) (s^2 + 18.6s + 290.5)

Type %OS  20

Percent Overshoot = 20, Damping Ratio = 0.45595, Phase Margin = 48.1477,
Frequency  = 0.74, Phase  = -132.087, Magnitude = 0.0723422, Magnitude (dB)
= -22.8122, K = 13.8232»
```





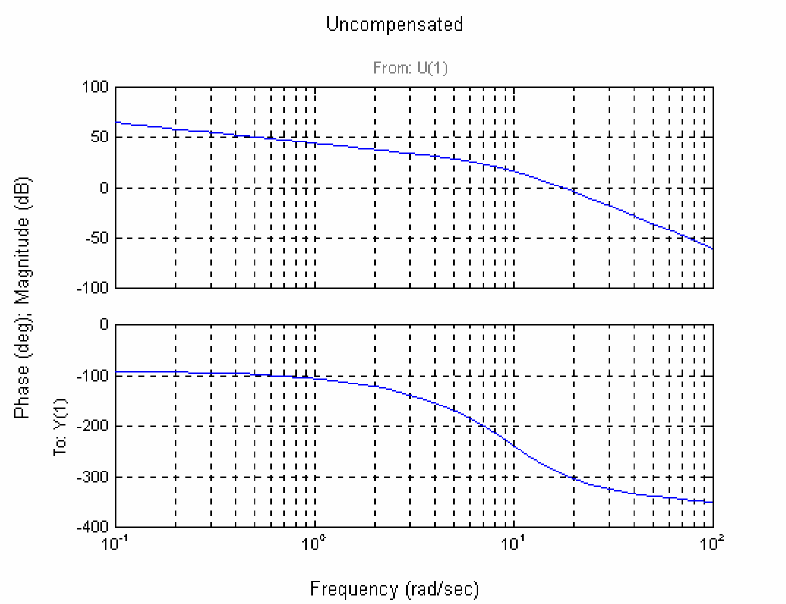


Second-order approximation not valid.

SOLUTIONS TO DESIGN PROBLEMS

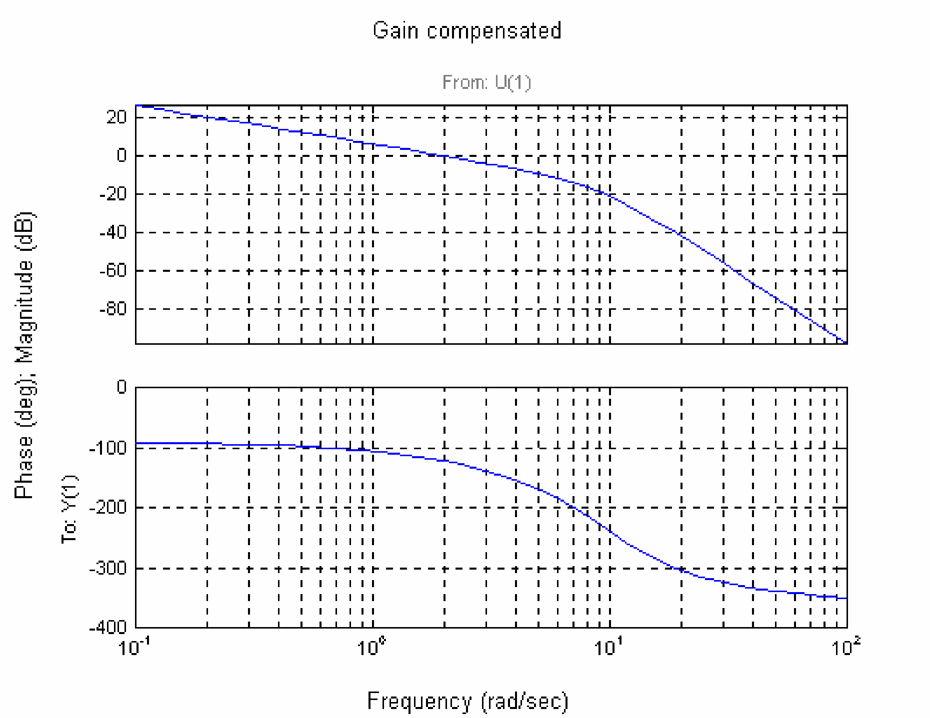
23.

a. Plot the Bode plot for $K = 1$.



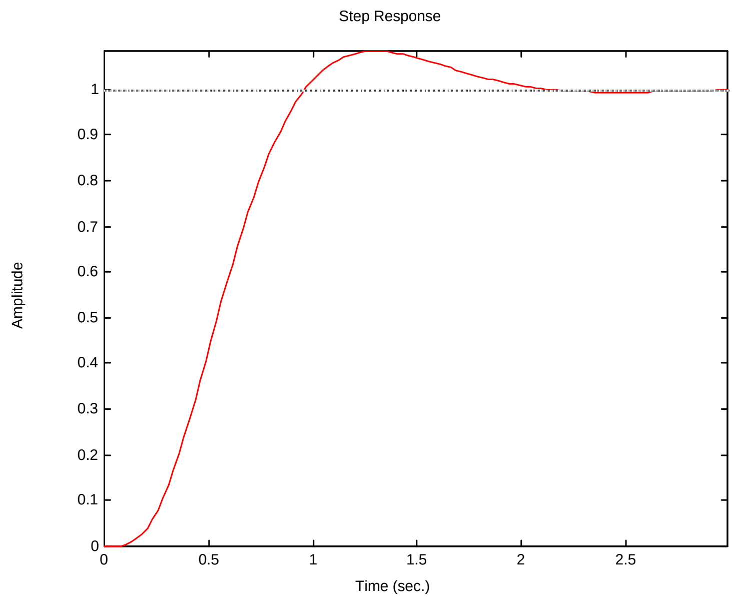
Using Eqs. (4.39) and (10.73) a percent overshoot = 10 is equivalent to a $\zeta = 0.591$ and $\phi_M = 58.59^0$. The phase-margin frequency = 1.933 rad/s where the phase is $58.59^0 - 180^0 = -121.41^0$. The magnitude = 38.37 dB, or 82.85. Hence K = 1/ 82.85 = 0.01207.

b. Plot the gain-compensated Bode plot (K = 0.01207).



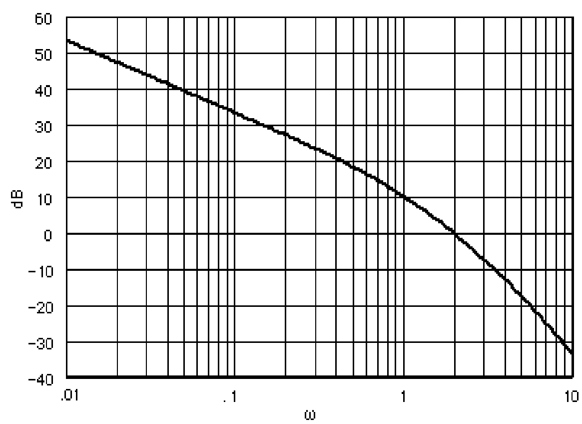
The bandwidth, ω_{BW}, is the frequency at which the magnitude is –7dB. From the compensated plots, this frequency is 3.9 rad/s. Eq. (10.55), $T_s = 2.01$ s. Using Eq. (10.57), $T_p = 1.16$ s.

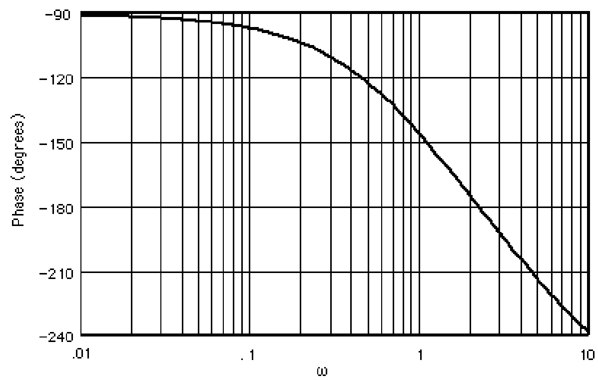
c.



24.

$G(s) = \dfrac{10K}{s(s+1)(s+5)}$. For $K_V = 5$, $K = 2.5$. Plot the Bode diagrams for this value of gain.





The uncompensated system has unity gain at $\omega = 2.04$ rad/s. The phase is -176.08^o at this frequency yielding a phase margin of 3.92^o. We want a 60^o phase margin plus, after trial and error, a correction factor of 20^o, or a total of 80^o. Thus, the lead compensator must contribute $80^o - 3.92^o = 76.08^o$. Using Eqs. (11.11), and (11.12),

The value of beta is:	0.01490254
The |G(jwmax)| for the compensator is:	8.1916207
or in db:	18.2673967

The magnitude curve has a gain of - 18.27 dB at $\omega = 5.27$ rad/s. Therefore, choose this frequency as the new phase margin frequency. Using Eqs. (11.9) and (11.6), the compensator transfer function has the following specifications:

T	1.55438723
zero	-0.6433403
pole	-43.169841
gain	67.1026497

The compensated forward path is

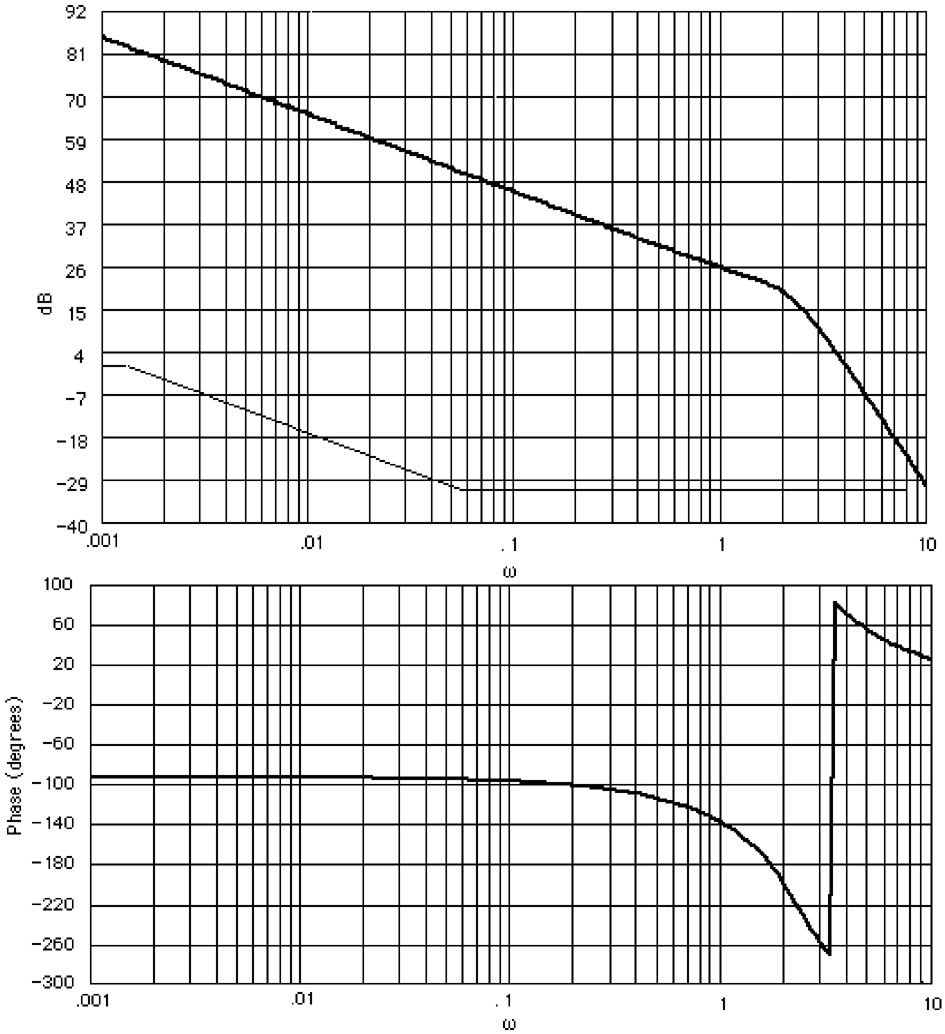
$$G(s) = \frac{25*67.1(s+0.64)}{s(s+1)(s+5)(s+43.17)} = \frac{1677.5(s+0.64)}{s(s+1)(s+5)(s+43.17)}$$

25.

$G(s) = \dfrac{10}{s(s^2+2s+5)(s+3)}$. Therefore, $K_{vo} = \dfrac{2}{3}$. We want $K_{vn} = 30K_{vo} = 20$. Increasing K by 30 times yields

$$G(s) = \frac{300}{s(s^2+2s+5)(s+3)}$$

Plotting the Bode diagrams,



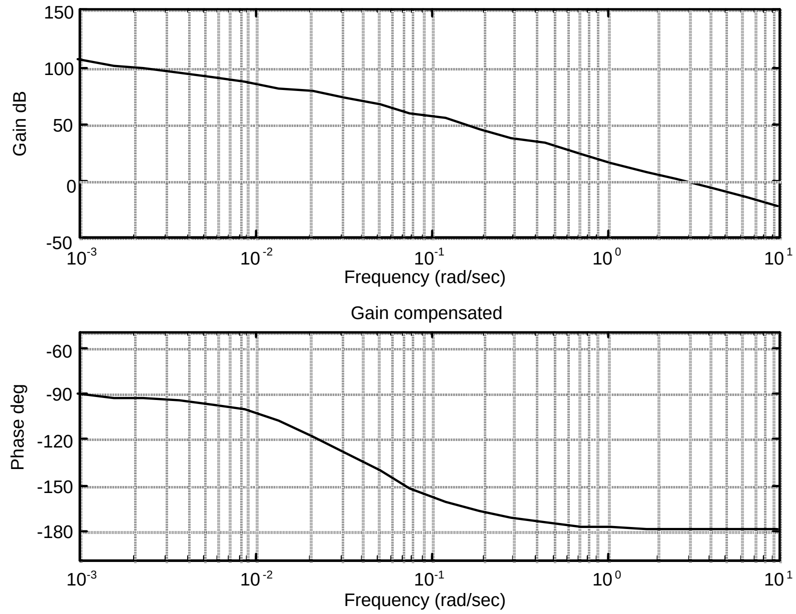
For 11% overshoot, the phase margin should be 57.48°. Adding a correction, we will use a 65° phase margin, or a phase angle of 115°, which occurs at ω = 0.58 rad/s. The magnitude curve is 30.93 dB. Thus the high-frequency asymptote of the lag compensator is - 30.93 dB. Drawing the lag-compensator curve as shown on the magnitude curve, the break frequencies are found and the compensator's transfer function is evaluated as

$$G_c(s) = 25.86 \times 10^{-3} \frac{s+0.058}{s+0.0015}$$

26.

a. The equivalent forward transfer function is $G_e(s) = \dfrac{4.514e\text{-}06K}{s(s+0.04348)}$.

$K_v = 200 = \dfrac{4.514e\text{-}06K}{0.04348}$ or K = 1926500. Using Eq. (4.39), $\zeta = 0.456$. Using Eq. (10.55), ω_{BW} = 1.16. Using Eq. (10.73) with 15° additional, the required phase margin, $\phi_{req} = 63.15°$. Select a new phase-margin frequency, $\omega_{Pm} = 0.8\ \omega_{BW} = 0.93$. Plot the Bode plots for K = 1926500.



At $\omega_{Pm} = 0.93$, the phase, $\phi = -177.3^{o}$. Hence, the phase required from the compensator, $\phi_C = \phi_{req} - (180+\phi) = 63.15 - (180 - 177.3) = 60.45^{o}$. Using Eq. (11.11), $\beta = 0.07$.

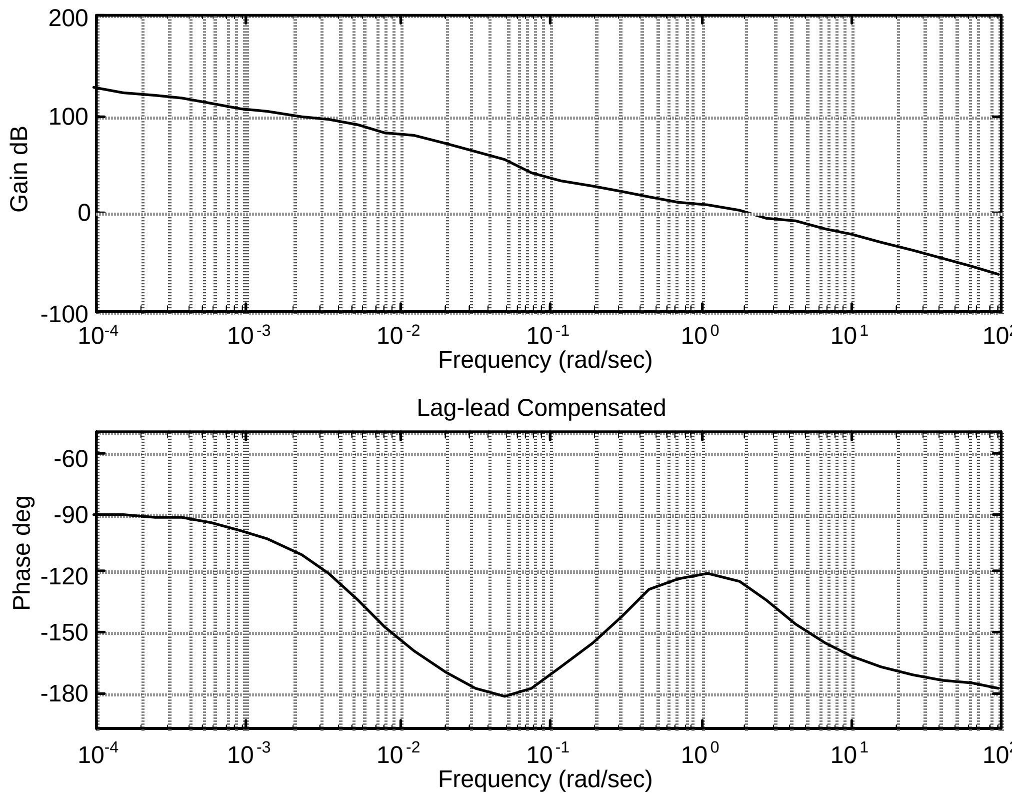
Design lag: $z_{clag} = \omega_{pm}/10 = 0.093$; $p_{clag} = z_{clag} * \beta = 0.0065$; $K_{clag} = \beta = 0.07$. Thus,

$$G_{clag}(s) = 0.07 \frac{s+0.093}{s+0.0065}.$$

Design lead compensator: $z_{clead} = \omega_{Pm} * \sqrt{\beta} = 0.25$; $p_{clead} = z_{clead}/\beta = 3.57$; $K_{clead} = 1/\beta = 14.29$. Thus,

$$G_{clead}(s) = 14.29 \frac{s+0.25}{s+3.57}.$$

The lag-lead compensated Bode plot:

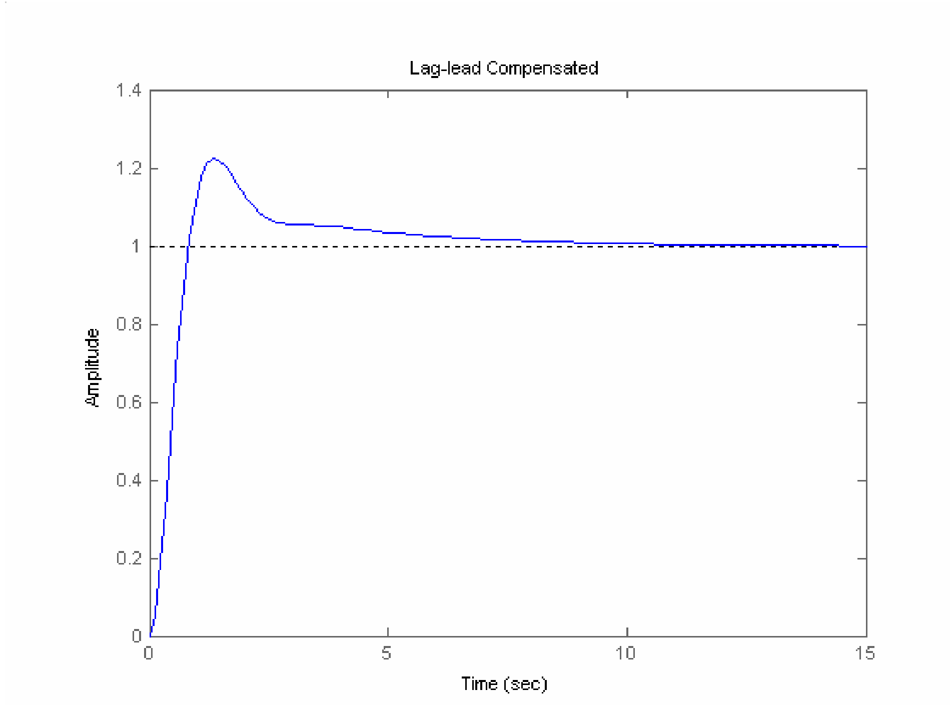


b.

Program:

```
K=1926500;
numg=4.514e-6;
deng=[1 0.04348 0];
G=tf(numg,deng);
numgclag=0.07*[1 0.093];
dengclag=[1 0.0065];
Gclag=tf(numgclag,dengclag);
numgclead=14.29*[1  0.25];
dengclead=[1 3.57];
Gclead=tf(numgclead,dengclead);
Ge=K*G*Gclag*Gclead;
T=feedback(Ge,1);
step(T)
title('Lag-lead Compensated')
```

Computer response:



27.

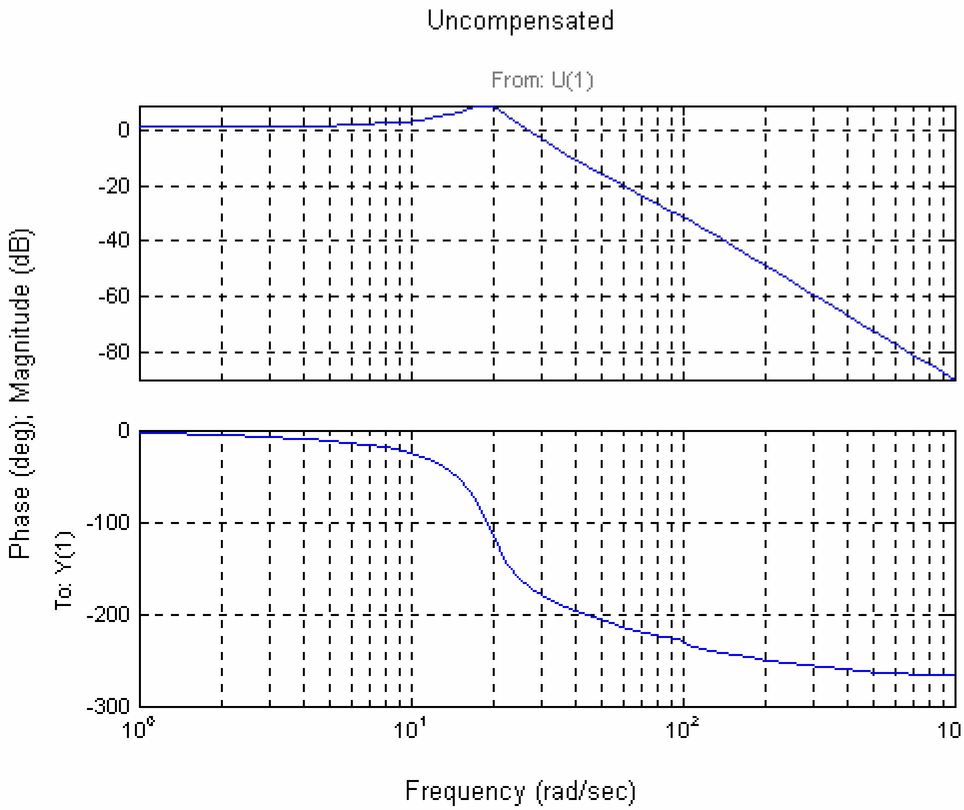
From Chapter 8,

$$G_e(s) = \frac{0.6488K\,(s+53.85)}{(s^2 + 8.119s + 376.3)\,(s\text{^}2 + 15.47s + 9283)}$$

Cascading the notch filter,

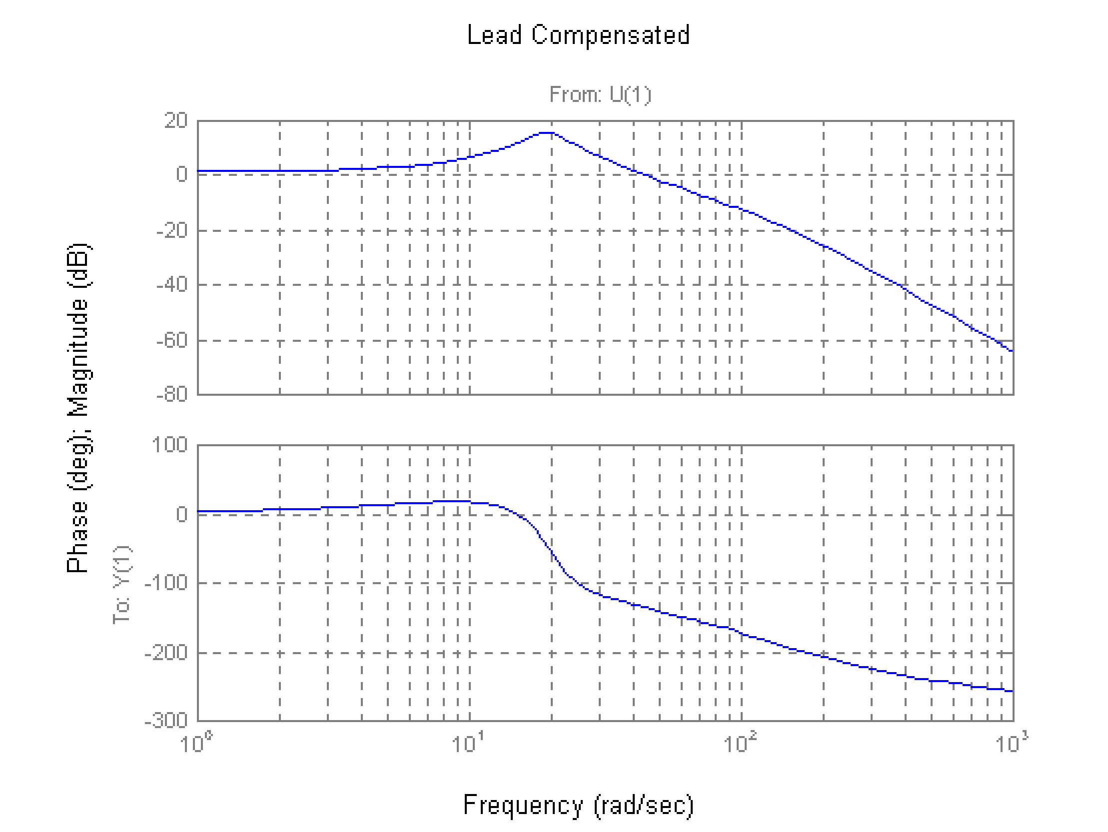
$$G_{et}(s) = \frac{0.6488K\,(s+53.85)(s^2 + 16s + 9200)}{(s^2 + 8.119s + 376.3)\,(s\text{^}2 + 15.47s + 9283)(s+60)^2}$$

Since $e_{step}(\infty) = \frac{1}{1+K_p}$, $K_p = 9$ yields 10% error. Thus, $K_p = \frac{K_e * 53.85 * 9200}{376.3 * 9283 * 60^2} = 9$. Thus, $K_e = 0.6488K = 228452$. Let us use $K_e = 30{,}000$ in designing the lead portion and we'll make up the rest with the lag. Plotting the Bode plot for $K_e = 30{,}000$,



Design lead: The uncompensated phase margin = 10.29^0. Assume a required phase margin of 45^0. The required phase margin, assuming a 30^0 correction is 75^0. The phase contribution required from the compensator is $75^0 – 10.29^0 = 64.71^0$. Using the inverse of Eq. (11.11), the compensator's β = 0.05033. Using Eq. (11.12), $|G_c(j\omega_{\max})| = \frac{1}{\sqrt{\beta}} = 4.457 = 12.98\text{ dB}$. The new phase margin frequency is where the uncompensated system has a magnitude of –12.98 dB, or $\omega_{\max}$ = 44.65 rad/s. Using Eqs. (11.6) and (11.9), the compensator is $G_{lead}(s) = \frac{19.87(s+10.02)}{(s+199)}$. The plant is

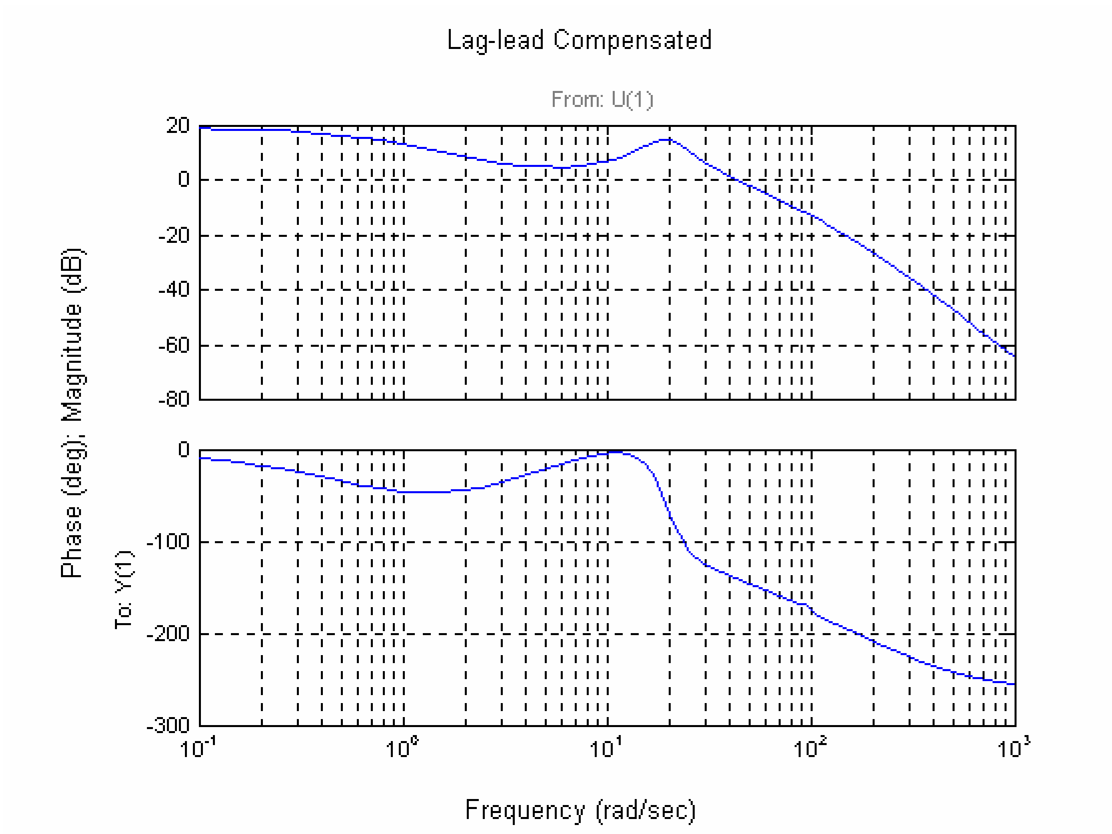
$$G(s) = \frac{228452(s+53.85)(s^2+16s+9200)}{(s+60)^2(s^2+8.119s+376.3)(s^2+15.47s+9283)}$$. Draw the lead-compensated Bode plot.



Design lag: The phase-margin frequency occurs where the phase is –135^0, or at the required 45^0 phase margin. From the lead-compensated Bode plots, this phase margin occurs at 43.64 rad/s. Let the upper break of the lag compensator be one decade lower, or 4.364. Since the magnitude is 17.97 dB at the new phase-margin frequency, set the high-frequency asymptote of the lag compensator at –17.97 dB. Draw a -20 dB/dec slope starting at 0.4364 rad/s and –17.96 dB and moving toward 0 dB. At 0 dB locate the lag compensator's low-frequency break, or 0.551. Thus,

$$G_{lag}(s) = \frac{0.551}{4.364}\frac{(s+4.364)}{(s+0.551)} = 0.126\frac{(s+4.364)}{(s+0.551)}$$

Check bandwidth: Draw the lag-lead compensated Bode diagram for $G(s)G_{lag}(s)G_{lead}(s)$.



From the open-loop plot, the magnitude is at –7 dB at 70 rad/s. Hence, the bandwidth is sufficient. Also, the lag-lead compensated Bode plot shows a phase margin of 40^0 . The transfer function, $G(s) = G(s)G_{lag}(s)G_{lead}(s)$ shows $K_p = 9$, or an error of 0.1. Thus all requirements have been met.