N I N E

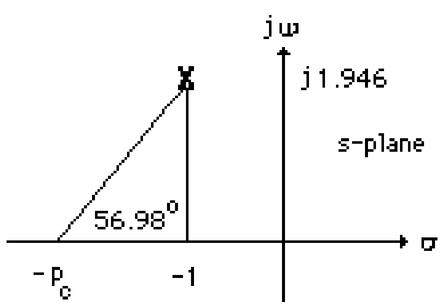
Design via Root Locus

SOLUTIONS TO CASE STUDIES CHALLENGES

Antenna Control: Lag-Lead Compensation

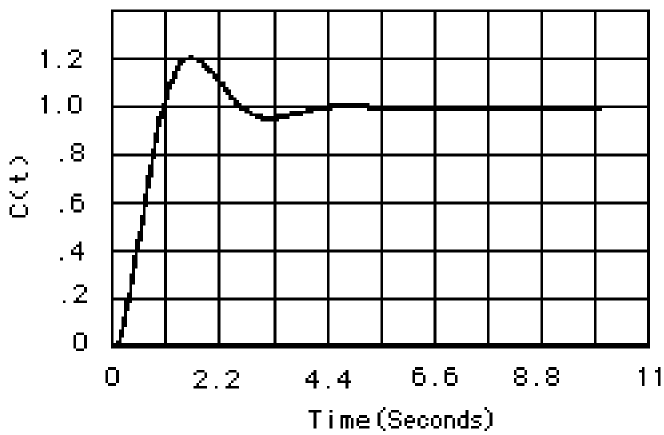
a. Uncompensated: From the Chapter 8 Case Study Challenge, $G(s) = \frac{76.39K}{s(s+150)(s+1.32)} = \frac{7194.23}{s(s+150)(s+1.32)}$ with the dominant poles at $-0.5 \pm j6.9$. Hence, $\zeta = \cos(\tan^{-1}\frac{6.9}{0.5}) = 0.0723$, or %OS = 79.63% and $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.5} = 8$ seconds. Also, $K_v = \frac{7194.23}{150 \times 1.32} = 36.33$.

b. Lead-Compensated: Reducing the percent overshoot by a factor of 4 yields, $\%OS = \frac{79.63}{4} = 19.91\%$, or $\zeta = 0.457$. Reducing the settling time by a factor of 2 yields, $T_s = \frac{8}{2} = 4$. Improving K_v by 2 yields $K_v = 72.66$. Using $T_s = \frac{4}{\zeta\omega_n} = 4$, $\zeta\omega_n = 1$, from which $\omega_n = 2.188$ rad/s. Thus, the design point equals $-\zeta\omega_n + j\,\omega_n\sqrt{1-\zeta^2} = -1 + j1.946$. Using the system's original poles and assuming a lead compensator zero at -1.5, the summation of the system's poles and the lead compensator zero to the design point is -123.017^{o} . Thus, the compensator pole must contribute $123.017^{o} - 180^{o} = -56.98^{o}$. Using the geometry below, $\frac{1.946}{p_c - 1} = \tan 56.98^{o}$, or $p_c = 2.26$.



Adding this pole to the system poles and the compensator zero yields 76.39K = 741.88 at -1+j1.946. Hence the lead-compensated open-loop transfer function is $G_{Lead\text{-}comp}(s) =$

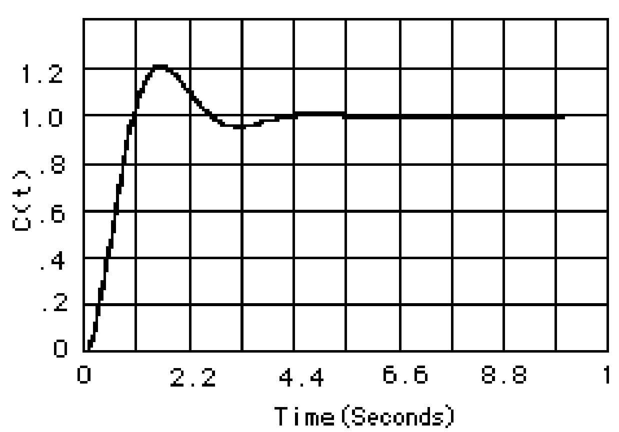
$\dfrac{741.88(s+1.5)}{s(s+150)(s+1.32)(s+2.26)}$. Searching the real axis segments of the root locus yields higher-order poles at greater than -150 and at -1.55. The response should be simulated since there may not be pole/zero cancellation. The lead-compensated step response is shown below.



Lead compensated

Since the settling time and percent overshoot meet the transient requirements, proceed with the lag compensator. The lead-compensated system has $K_v = \dfrac{741.88 \times 1.5}{150 \times 1.32 \times 2.26} = 2.487$. Since we want $K_v = 72.66$, an improvement of $\dfrac{72.66}{2.487} = 29.22$ is required. Select $G(s)_{Lag} = \dfrac{s+0.002922}{s+0.0001}$ to improve the steady-state error by 29.22. A simulation of the lag-lead compensated system,

$G_{Lag\text{-}lead\text{-}comp}(s) = \dfrac{741.88(s+1.5)(s+0.002922)}{s(s+150)(s+1.32)(s+2.26)(s+0.0001)}$ is shown below.



Lag-lead compensated

UFSS Vehicle: Lead and Feedback Compensation

Minor loop: Open-loop transfer function $G(s)H(s) = \dfrac{0.25K_2(s+0.437)}{(s+2)(s+1.29)(s+0.193)}$; Closed-loop transfer function: $T_{ML}(s) = \dfrac{0.25K_2(s+0.437)}{s(s^3+...)}$. Searching along the 126.87^o line ($\zeta = 0.6$), find the dominant second-order poles at $-1.554 \pm j2.072$ with $0.25K_2 = 4.7$. Thus $K_2 = 18.8$. Searching the real axis segment of the root locus for a gain of 4.7 yields a 3rd pole at -0.379.

Major loop: The unity feedback, open-loop transfer function found by using the minor-loop closed-loop poles is $G_{ML}(s) = \dfrac{-0.25K_1(s+0.437)}{s(s+0.379)(s+1.554+j2.072)(s+1.554-j2.072)}$. Searching along the 120^o line ($\zeta = 0.5$), find the dominant second-order poles at $-1.069 \pm j1.85$ with $0.25K_1 = 4.55$. Thus $K_1 = 18.2$. Searching the real axis segment of the root locus for a gain of 4.55 yields a 3rd pole at -0.53 and a 4th pole at -0.815.

ANSWERS TO REVIEW QUESTIONS

1. Chapter 8: Design via gain adjustment. Chapter 9: Design via cascaded or feedback filters

2. A. Permits design for transient responses not on original root locus and unattainable through simple gain adjustments. B. Transient response and steady-state error specifications can be met separately and independently without the need for tradeoffs

3. PI or lag compensation

4. PD or lead compensation

5. PID or lag-lead compensation

6. A pole is placed on or near the origin to increase or nearly increase the system type, and the zero is placed near the pole in order not to change the transient response.

7. The zero is placed closer to the imaginary axis than the pole. The total contribution of the pole and zero along with the previous poles and zeros must yield 180^0 at the design point. Placing the zero closer to the imaginary axis tends to speed up a slow response.

8. A PD controller yields a single zero, while a lead network yields a zero and a pole. The zero is closer to the imaginary axis.

9. Further out along the same radial line drawn from the origin to the uncompensated poles

10. The PI controller places a pole right at the origin, thus increasing the system type and driving the error to zero. A lag network places the pole only close to the origin yielding improvement but no zero error.

11. The transient response is approximately the same as the uncompensated system, except after the original settling time has passed. A slow movement toward the new final value is noticed.

12. 25 times; the improvement equals the ratio of the zero location to the pole location.

13. No; the feedback compensator's zero is not a zero of the closed-loop system.

14. A. Response of inner loops can be separately designed; B. Faster responses possible; C. Amplification may not be necessary since signal goes from high amplitude to low.

SOLUTIONS TO PROBLEMS

1.

Uncompensated system: Search along the ζ = 0.5 line and find the operating point is at -1.5356 ± j2.6598 with K = 73.09. Hence, $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}}\text{x}100$ = 16.3%; $T_s = \dfrac{4}{1.5356}$ = 2.6 seconds; $K_p = \dfrac{73.09}{30}$ =2.44. A higher-order pole is located at -10.9285.

Compensated: Add a pole at the origin and a zero at -0.1 to form a PI controller. Search along the ζ= 0.5 line and find the operating point is at -1.5072 ± j2.6106 with K = 72.23. Hence, the estimated performance specifications for the compensated system are: $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}}\text{x}100$ = 16.3%; $T_s = \dfrac{4}{1.5072}$ = 2.65 seconds; $K_p = \infty$. Higher-order poles are located at -0.0728 and -10.9125. The compensated system should be simulated to ensure effective pole/zero cancellation.

2.

a. Insert a cascade compensator, such as $G_c(s) = \dfrac{s+0.01}{s}$.

b.

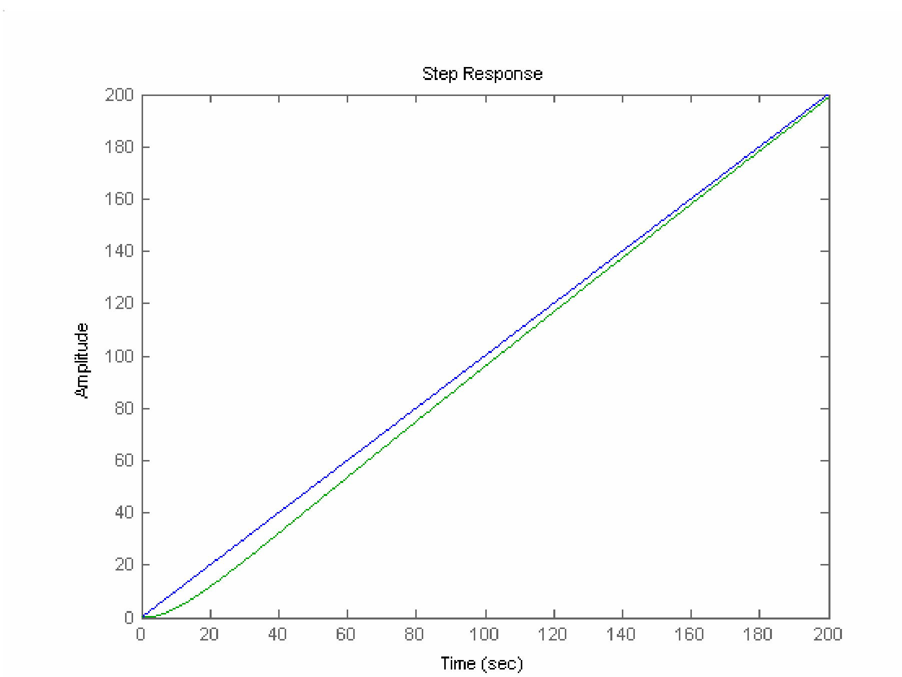
Program:

```
K=1
G1=zpk([],[0,-2,-5],K)  %G1=1/s(s+2)(s+5)
Gc=zpk([-0.01],[0],1)   %Gc=(s+0.01)/s
G=G1*Gc
rlocus(G)
T=feedback(G,1)
T1=tf(1,[1,0])          %Form 1/s to integrate step input
T2=T*T1
t=0:0.1:200;
step(T1,T2,t)           %Show input ramp and ramp response
```

Computer response:

```
K =

     1


Zero/pole/gain:
       1
-------------
s (s+2) (s+5)
```

```
Zero/pole/gain:
(s+0.01)
--------
   s


Zero/pole/gain:
   (s+0.01)
---------------
s^2 (s+2) (s+5)


Zero/pole/gain:

                 (s+0.01)
------------------------------------------
(s+5.064) (s+1.829) (s+0.09593)
                                 (s+0.01126)


Transfer function:
1
-
s


Zero/pole/gain:

                 (s+0.01)
------------------------------------------
s (s+5.064) (s+1.829) (s+0.09593)
                                 (s+0.01126)
```



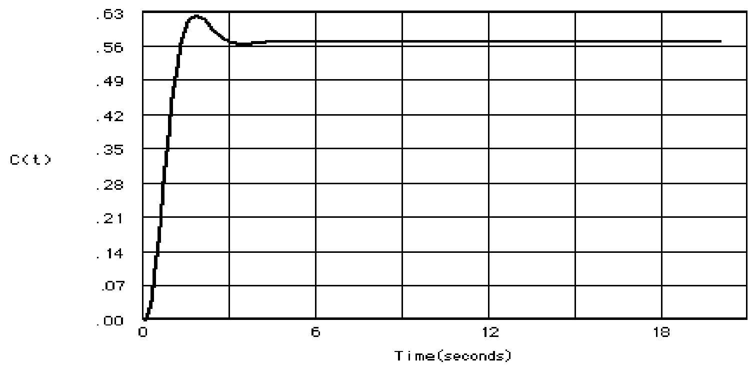
3.

a. Searching along the 126.16° line (10% overshoot, $\zeta = 0.59$), find the operating point at

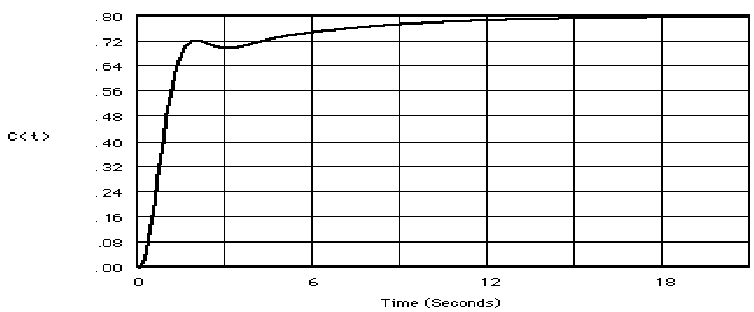
$-1.4 + j1.92$ with K = 20. Hence, $K_p = \frac{20}{1 \times 5 \times 3} = 1.333$.

b. A 3x improvement will yield $K_p = 4$. Use a lag compensator, $G_c(s) = \frac{s+0.3}{s+0.1}$.

c.



Uncompensated



Lag Compensated

4.

a. Searching along the 126.16° line (10% overshoot, $\zeta = 0.59$), find the operating point at

$-1.009 + j1.381$ with $K = 17.5$. Hence, $K_v = \frac{17.5}{5 \text{x} 3} = 1.1667$.

b. A 3.429x improvement will yield $K_v = 4$. Use a lag compensator, $G_c(s) = \frac{s+0.3429}{s+0.1}$.

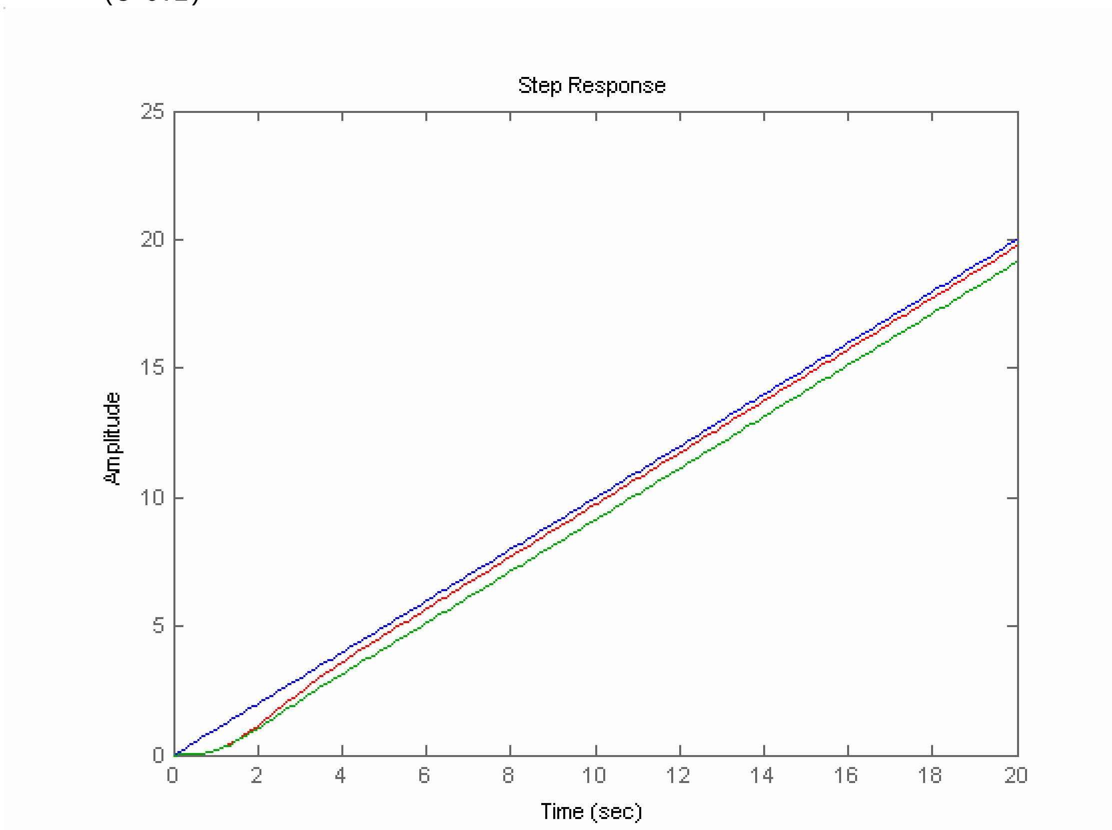
c.

Program:

```
K=17.5
G=zpk([],[0,-3,-5],K)
Gc=zpk([-0.3429],[-0.1],1)
Ge=G*Gc;
T1=feedback(G,1);
T2=feedback(Ge,1);
T3=tf(1,[1,0]);          %Form 1/s to integrate step input
T4=T1*T3;
T5=T2*T3;
t=0:0.1:20;
step(T3,T4,T5,t)         %Show input ramp and ramp responses
```

Computer response:

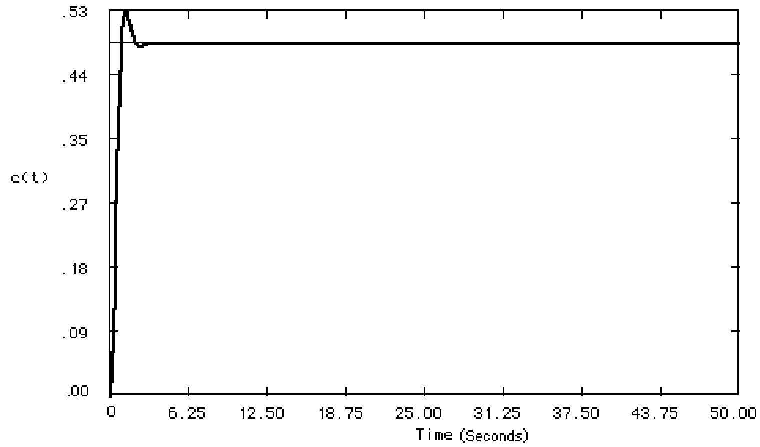
```
K =

   17.5000


Zero/pole/gain:
    17.5
-------------
s (s+3) (s+5)


Zero/pole/gain:
(s+0.3429)
----------
 (s+0.1)
```



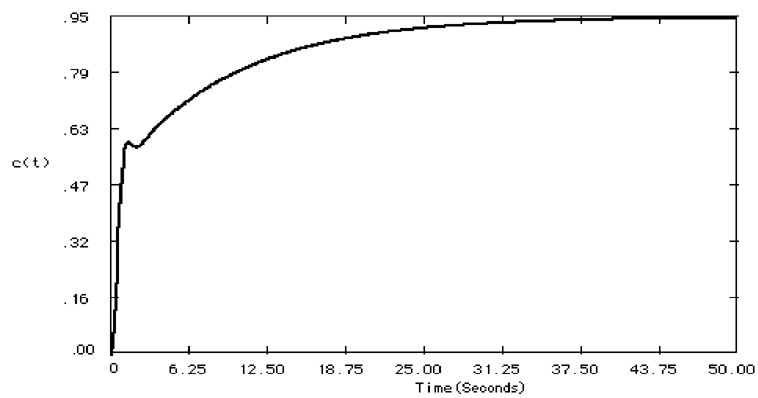
5.

a. Uncompensated: Searching along the 126.16^o line (10% overshoot, $\zeta = 0.59$), find the operating point at -2.03 + j2.77 with K = 45.72. Hence, $K_p = \frac{45.72}{2 \times 4 \times 6} = 0.9525$. An improvement of $\frac{20}{0.9525} = 20.1$ is required. Let $G_c(s) = \frac{0.201}{0.01}$. Compensated: Searching along the 126.16^o line (10% overshoot, $\zeta = 0.59$), find the operating point at - 1.99+j2.72 with K = 46.05. Hence, $K_p = \frac{46.05 \times 0.201}{2 \times 4 \times 6 \times 0.01} = 19.28$.

b.



Uncompensated



Lag Compensated

c. From (b), about 28 seconds

6.

Uncompensated: Searching along the 135° line (ζ = 0.707), find the operating point at

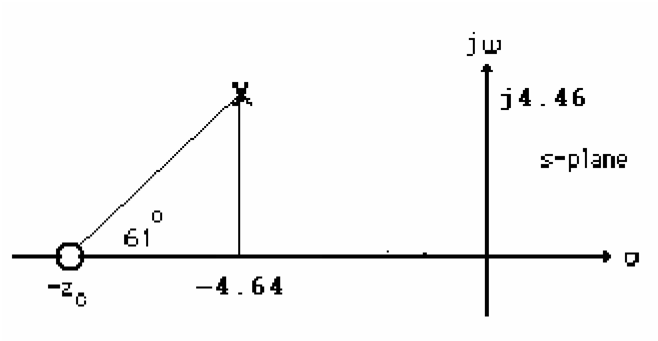
-2.32 + j2.32 with K = 4.6045. Hence, $K_p = \frac{4.6045}{30} = 0.153$; $T_s = \frac{4}{2.32} = 1.724$ seconds; $T_p =$

$\frac{\pi}{2.32} = 1.354$ seconds; $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 4.33\%$;

$\omega_n = \sqrt{2.32^2 + 2.32^2} = 3.28$ rad/s; higher-order pole at -5.366.

Compensated: To reduce the settling time by a factor of 2, the closed-loop poles should be – 4.64 ± j4.64. The summation of angles to this point is 119° . Hence, the contribution of the compensating zero should be 180° -119° =61° . Using the geometry shown below,

$\frac{4.64}{z_c - 4.64} = \tan(61°)$. Or, $z_c = 7.21$.



After adding the compensator zero, the gain at -4.64+j4.64 is K = 4.77. Hence,

$K_p = \frac{4.77x6x7.21}{2x3x5} = 6.88$. $T_s = \frac{4}{4.64} = 0.86$ second; $T_p = \frac{\pi}{4.64} = 0.677$ second;

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 4.33\%$; $\omega_n = \sqrt{4.64^2 + 4.64^2} = 6.56$ rad/s; higher-order pole at -5.49. The problem with the design is that there is steady-state error, and no effective pole/zero cancellation. The design should be simulated to be sure the transient requirements are met.

7.

Program:
```
clf
'Uncompensated System'
numg=[1 6];
deng=poly([-2 -3 -5]);
'G(s)'
G=tf(numg,deng);
Gzpk=zpk(G)
rlocus(G,0:1:100)
z=0.707;
pos=exp(-pi*z/sqrt(1-z^2))*100;
sgrid(z,0)
title(['Uncompensated Root Locus with ' , num2str(z), ' Damping Ratio
Line'])
[K,p]=rlocfind(G);  %Allows input by selecting point on graphic
'Closed-loop poles = '
p
i=input('Give pole number that is operating point   ');
'Summary of estimated specifications'
operatingpoint=p(i)
gain=K
estimated_settling_time=4/abs(real(p(i)))
estimated_peak_time=pi/abs(imag(p(i)))
estimated_percent_overshoot=pos
estimated_damping_ratio=z
estimated_natural_frequency=sqrt(real(p(i))^2+imag(p(i))^2)
Kp=dcgain(K*G)
'T(s)'
T=feedback(K*G,1)
'Press any key to continue and obtain the step response'
pause
step(T)
title(['Step Response for Uncompensated System with  ' , num2str(z),...
' Damping Ratio'])
'Press any key to go to PD compensation'
pause
'Compensated system'
```

```
done=1;
while done>0
a=input('Enter a Test PD Compensator, (s+a). a =     ')
numc=[1 a];
'Gc(s)'
GGc=tf(conv(numg,numc),deng);
GGczpk=zpk(GGc)
wn=4/[(estimated_settling_time/2)*z];
rlocus(GGc)
sgrid(z,wn)
title(['PD Compensated Root Locus with ' , num2str(z),...
' Damping Ratio Line', 'PD Zero at ', num2str(a), ', and Required Wn'])
done=input('Are you done? (y=0,n=1)  ');
end
[K,p]=rlocfind(GGc);  %Allows input by selecting point on graphic
'Closed-loop poles = '
p
i=input('Give pole number that is operating point   ');
'Summary of estimated specifications'
operatingpoint=p(i)
gain=K
estimated_settling_time=4/abs(real(p(i)))
estimated_peak_time=pi/abs(imag(p(i)))
estimated_percent_overshoot=pos
estimated_damping_ratio=z
estimated_natural_frequency=sqrt(real(p(i))^2+imag(p(i))^2)
Kp=dcgain(K*GGc)
'T(s)'
T=feedback(K*GGc,1)
'Press any key to continue and obtain the step response'
pause
step(T)
title(['Step Response for Compensated System with  ' , num2str(z),...
' Damping Ratio'])
```

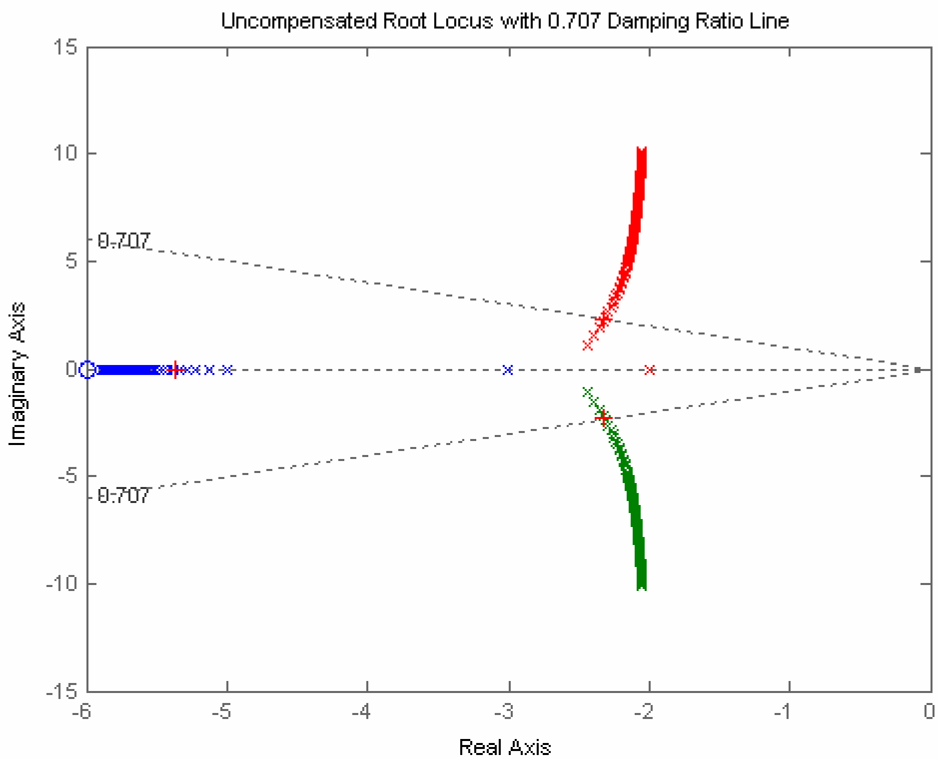
Computer response:

```
ans =

Uncompensated System


ans =

G(s)


Zero/pole/gain:
      (s+6)
-----------------
(s+5) (s+3) (s+2)

Select a point in the graphics window

selected_point =

  -2.3104 + 2.2826i


ans =

Closed-loop poles =


p =

  -5.3603
  -2.3199 + 2.2835i
```

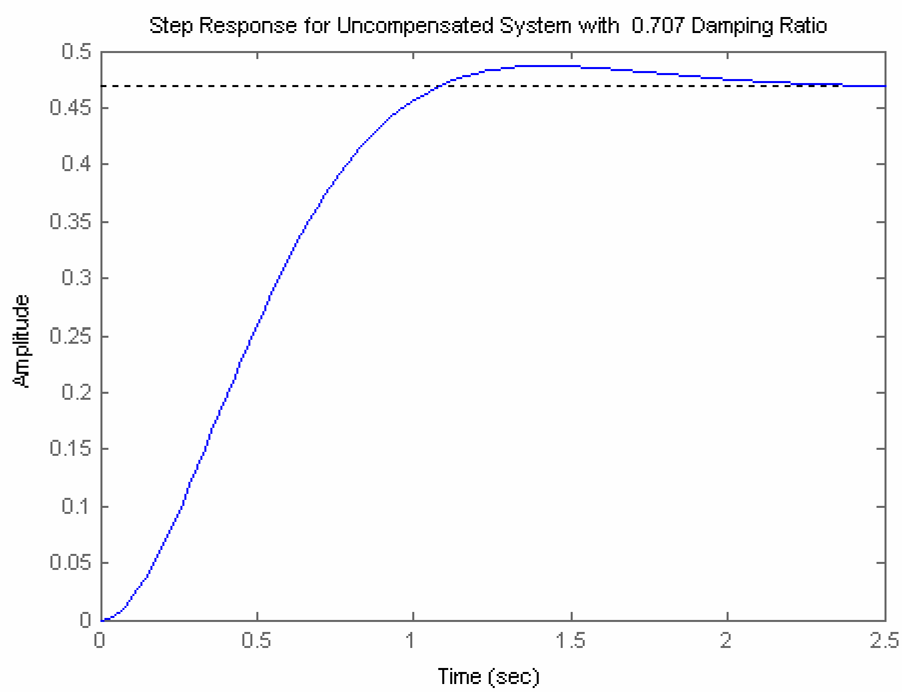
```
  -2.3199 - 2.2835i

Give pole number that is operating point   2

ans =

Summary of estimated specifications

operatingpoint =

  -2.3199 + 2.2835i

gain =

    4.4662

estimated_settling_time =

    1.7242

estimated_peak_time =

    1.3758

estimated_percent_overshoot =

    4.3255

estimated_damping_ratio =

    0.7070

estimated_natural_frequency =

    3.2552

Kp =

    0.8932

ans =

T(s)

Transfer function:
       4.466 s + 26.8
-----------------------------
s^3 + 10 s^2 + 35.47 s + 56.8

ans =

Press any key to continue and obtain the step response

ans =
```

```
Press any key to go to PD compensation


ans =

Compensated system

Enter a Test PD Compensator, (s+a). a =     6

a =

     6


ans =

Gc(s)


Zero/pole/gain:
     (s+6)^2
-----------------
(s+5) (s+3) (s+2)

Are you done? (y=0,n=1)  1
Enter a Test PD Compensator, (s+a). a =     7.1

a =

    7.1000


ans =

Gc(s)


Zero/pole/gain:
  (s+7.1) (s+6)
-----------------
(s+5) (s+3) (s+2)

Are you done? (y=0,n=1)  0
Select a point in the graphics window

selected_point =

  -4.6607 + 4.5423i


ans =

Closed-loop poles =


p =

  -4.6381 + 4.5755i
  -4.6381 - 4.5755i
  -5.4735

Give pole number that is operating point   1

ans =

Summary of estimated specifications
```

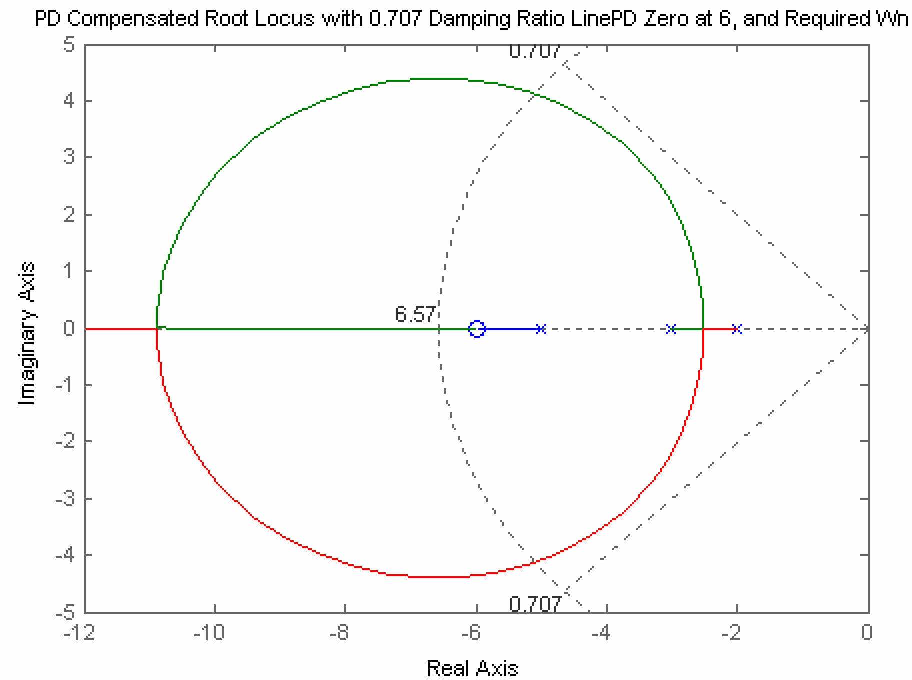
```
operatingpoint =

  -4.6381 + 4.5755i

gain =

    4.7496

estimated_settling_time =

    0.8624

estimated_peak_time =

    0.6866

estimated_percent_overshoot =

    4.3255

estimated_damping_ratio =

    0.7070

estimated_natural_frequency =

    6.5151

Kp =

    6.7444

ans =

T(s)

Transfer function:
   4.75 s^2 + 62.22 s + 202.3
---------------------------------
s^3 + 14.75 s^2 + 93.22 s + 232.3

ans =

Press any key to continue and obtain the step response
```

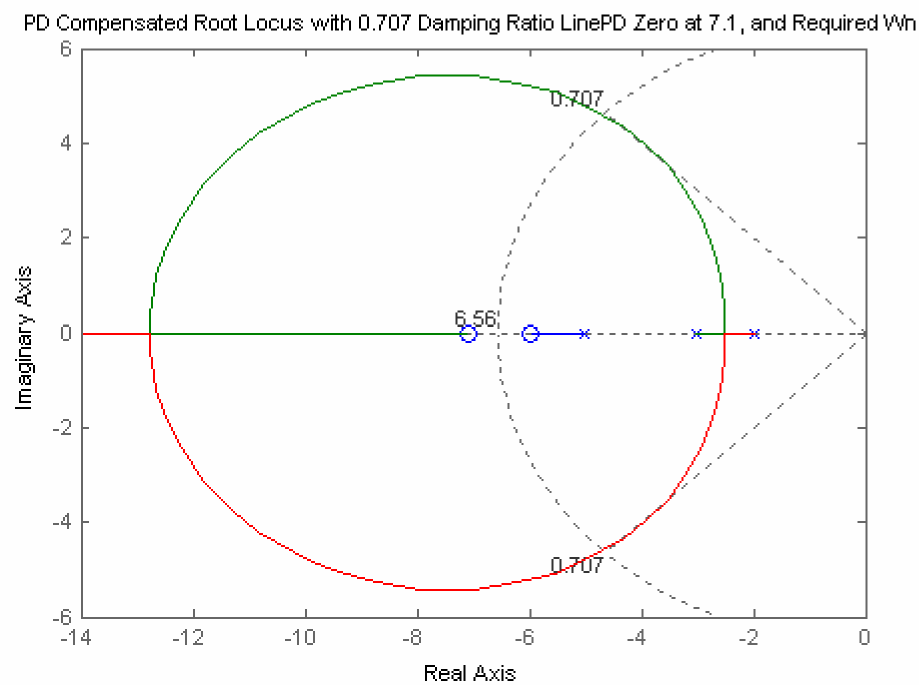


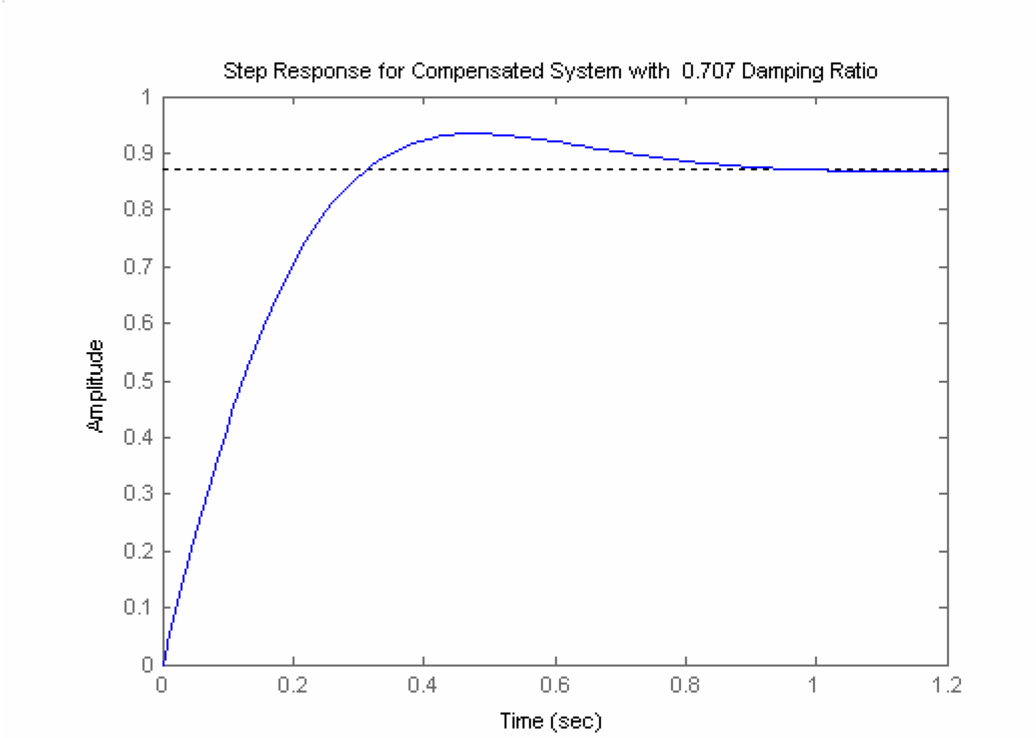
Uncompensated Root Locus with 0.707 Damping Ratio Line
Imaginary Axis
Real Axis
0.707
0.707
15
10
5
0
-5
-10
-15
-6
-5
-4
-3
-2
-1
0



Step Response for Uncompensated System with 0.707 Damping Ratio
Amplitude
Time (sec)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
2.5

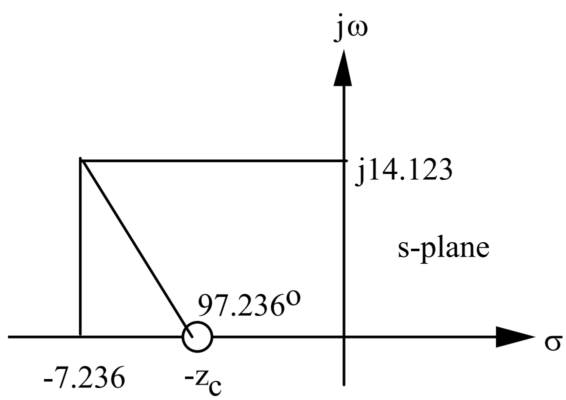

PD Compensated Root Locus with 0.707 Damping Ratio LinePD Zero at 6, and Required Wn
0.707
6.57
0.707
Imaginary Axis
Real Axis
5
4
3
2
1
0
-1
-2
-3
-4
-5
-12
-10
-8
-6
-4
-2
0


PD Compensated Root Locus with 0.707 Damping Ratio LinePD Zero at 7.1, and Required Wn
0.707
6.56
0.707
Imaginary Axis
Real Axis
6
4
2
0
-2
-4
-6
-14
-12
-10
-8
-6
-4
-2
0



8.

The uncompensated system performance is summarized in Table 9.8 in the text. To improve settling time by 4, the dominant poles need to be at $-7.236 \pm j14.123$. Summing the angles from the open-loop poles to the design point yields -277.326^{o}. Thus, the zero must contribute $277.326^{o} - 180^{o} = 97.326^{o}$. Using the geometry below,



$\frac{14.123}{7.236 - z_c}$ = tan(180-97.326). Thus, $z_c = 5.42$. Adding the zero and evaluating the gain at the design point yields K = 256.819. Summarizing results:

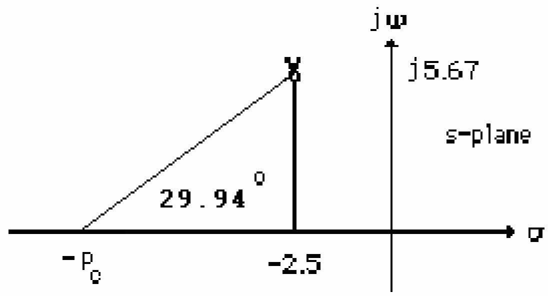
	Uncompensated	Compensated
Plant and compensator	$\frac{K}{s(s+5)(s+15)}$	$\frac{K(s+5.42)}{s(s+5)(s+15)}$
Feedback	1	1
Dominant poles	-1.809 ± j3.531	-7.236 ± j14.123
K	257.841	256.819
ζ	0.456	0.456
ω_n	3.97	15.88
%OS	20	20
T_s	2.21	0.55
T_p	0.89	0.22
K_v	3.44	18.559
e(∞) (ramp)	0.29	0.0539
Other poles	-16.4	-5.528
Zero	None	-5.42
Comments	Second-order approx. OK	Second-order approx. OK by assuming pole/zero cancellation.

9.

a. $\zeta\omega_n = \frac{4}{T_s} = 2.5$; $\zeta = \frac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = 0.404$. Thus, $\omega_n = 6.188$ rad/s and the operating point is $-2.5 \pm j5.67$.

b. Summation of angles including the compensating zero is -150.06^o. Therefore, the compensator pole must contribute $150.06^o - 180^o = -29.94^o$.

c. Using the geometry shown below, $\frac{5.67}{p_c - 2.5} = \tan 29.94^o$. Thus, $p_c = 12.34$.

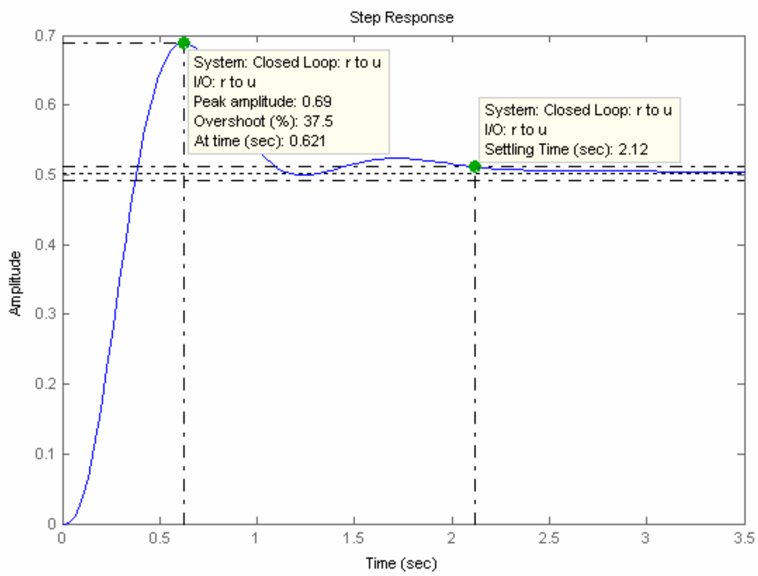


d. Adding the compensator pole and using -2.5 + j5.67 as the test point, K = 357.09.

e. Searching the real axis segments for K = 1049.41, we find higher-order poles at -15.15, and -1.186.

f. Pole at -15.15 is more than 5 times further from the imaginary axis than the dominant poles. Pole at -1.186 may not cancel the zero at -1

g.



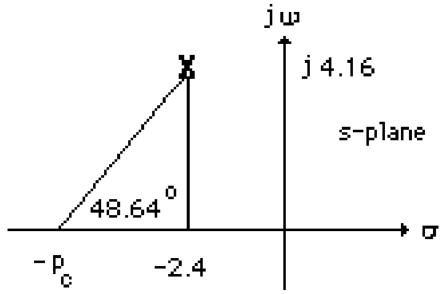
A simulation of the system shows a percent overshoot of 37.5% and a settling time of 2.12 seconds. Thus, the specifications were not met because pole-zero cancellation was not achieved. A redesign is required.

10.

a. $\zeta\omega_n = \frac{4}{T_s} = 2.4$; $\zeta = \frac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.5$. Thus, $\omega_n = 4.799$ rad/s and the operating point is -2.4 ± j4.16.

b. Summation of angles including the compensating zero is -131.36°. Therefore, the compensator pole must contribute 180° - 131.36° = -48.64°. Using the geometry shown below, $\frac{4.16}{p_c - 2.4} =$

tan 48.64°. Thus, p_c = 6.06.



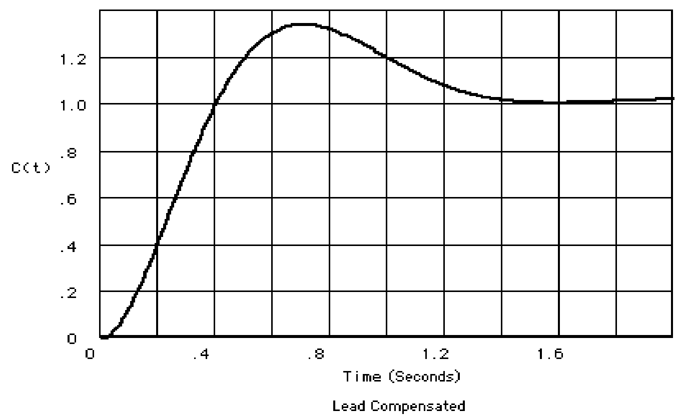
c. Adding the compensator pole and using -2.4 + j4.16 as the test point, K = 29.117.

d. Searching the real axis segments for K = 29.117, we find a higher-order pole at -1.263.

e. Pole at -1.263 is near the zero at -1. Simulate to ensure accuracy of results.

f. $K_a = \frac{29.117}{6.06} = 4.8$

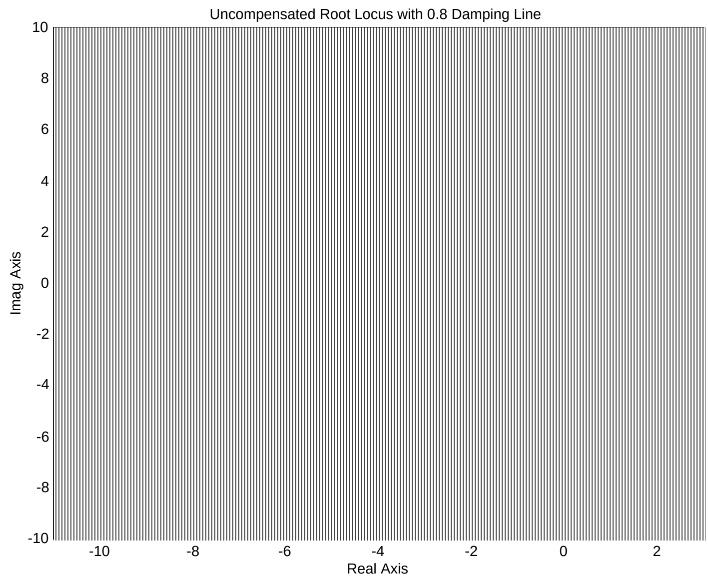
g.



From the plot, T_s = 1.4 seconds; T_p = 0.68 seconds; %OS = 35%.

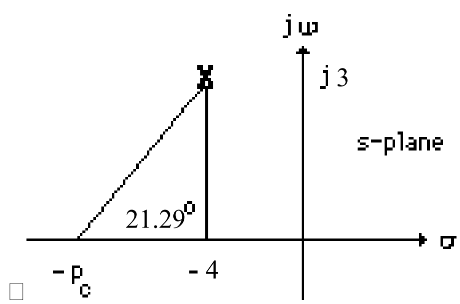
11.

a.



b. and **c.** Searching along the $\zeta = 0.8$ line (143.13^{o}), find the operating point at $-2.682 + j2.012$ with $K = 35.66$.

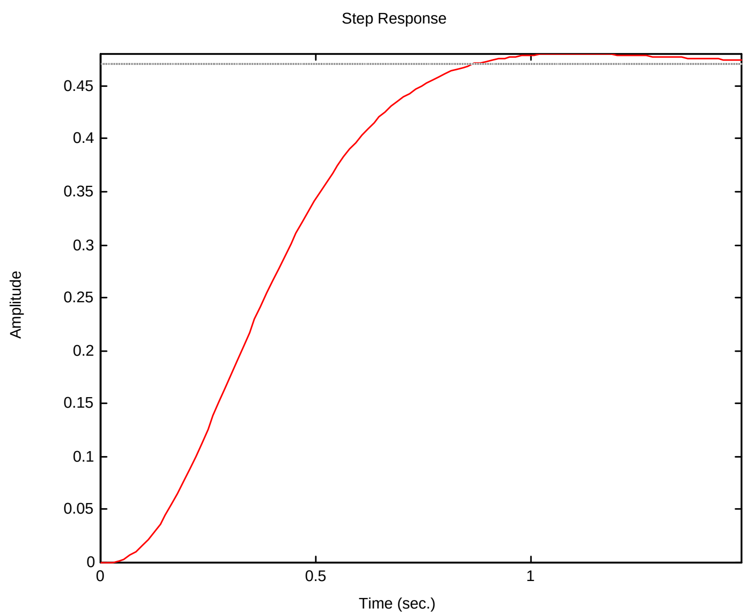
d. Since $\zeta\omega_n = \frac{4}{T_s}$, the real part of the compensated dominant pole is -4. The imaginary part is $4 \tan (180^{o}\text{-}143.13^{o}) = 3$. Using the uncompensated system's poles and zeros along with the compensator zero at - 4.5, the summation of angles to the design point, $-4 + j3$ is -158.71^{o}. Thus, the contribution of the compensator pole must be $158.71^{o} - 180^{o} = -21.29^{0}$. Using the following geometry, $\frac{3}{p_c - 4} = \tan 21.29^{0}$, or pc = 11.7.



Adding the compensator pole and using $-4 + j3$ as the test point, $K = 172.92$.

e. Compensated: Searching the real axis segments for $K = 172.92$, we find higher-order poles at 14.19, and approximately at $-5.26 \pm j0.553$. Since there is no pole/zero cancellation with the zeros at -6 and -4.5, the system should be simulated to check the settling time.

f.



The graph shows about 2% overshoot and a 0.8 second settling time compared to a desired 1.52% overshoot and a settling time of 1 second.

12.

Program:

```
clf
numg=[1 6];
deng=poly([-2 -4 -7 -8]);
'G(s)'
G=tf(numg,deng);
Gzpk=zpk(G)
rlocus(G)
z=0.8;
pos=exp(-pi*z/sqrt(1-z^2))*100;
sgrid(z,0)
title(['Uncompensated Root Locus with ' , num2str(z), ' Damping Ratio Line'])
[K,p]=rlocfind(G);
'Closed-loop poles = '
p
i=input('Give pole number that is operating point   ');
'Summary of estimated specifications'
operatingpoint=p(i)
gain=K
estimated_settling_time=4/abs(real(p(i)))
estimated_peak_time=pi/abs(imag(p(i)))
estimated_percent_overshoot=pos
estimated_damping_ratio=z
estimated_natural_frequency=sqrt(real(p(i))^2+imag(p(i))^2)
Kp=K*numg(max(size(numg)))/deng(max(size(deng)))
'T(s)'
T=feedback(K*G,1)
'Press any key to continue and obtain the step response'
pause
step(T)
title(['Step Response for Uncompensated System with  ' , num2str(z),...
' Damping Ratio'])
'Press any key to go to Lead compensation'
pause
```

```
'Compensated system'
b=4.5;
'Lead Zero at -4.5 '
done=1;
while done>0
a=input('Enter a Test Lead Compensator Pole, (s+a). a =     ');
'Gc(s)'
Gc=tf([1 b],[1 a])
GGc=G*Gc;
[numggc,denggc]=tfdata(GGc,'v');
'G(s)Gc(s)'
GGczpk=zpk(GGc)
wn=4/((1)*z);
rlocus(GGc);
sgrid(z,wn)
title(['Lead Compensated Root Locus with ' , num2str(z),...
' Damping Ratio Line, Lead Pole at  ', num2str(-a), ', and Required Wn'])
done=input('Are you done? (y=0,n=1)  ');
end
[K,p]=rlocfind(GGc);  %Allows input by selecting point on graphic
'Closed-loop poles = '
p
i=input('Give pole number that is operating point   ');
'Summary of estimated specifications'
operatingpoint=p(i)
gain=K
estimated_settling_time=4/abs(real(p(i)))
estimated_peak_time=pi/abs(imag(p(i)))
estimated_percent_overshoot=pos
estimated_damping_ratio=z
estimated_natural_frequency=sqrt(real(p(i))^2+imag(p(i))^2)
Kp=dcgain(K*GGc)
'T(s)'
T=feedback(K*GGc,1)
'Press any key to continue and obtain the step response'
pause
step(T)
title(['Step Response for Compensated System with  ' , num2str(z),...
' Damping Ratio'])
```

Computer response:

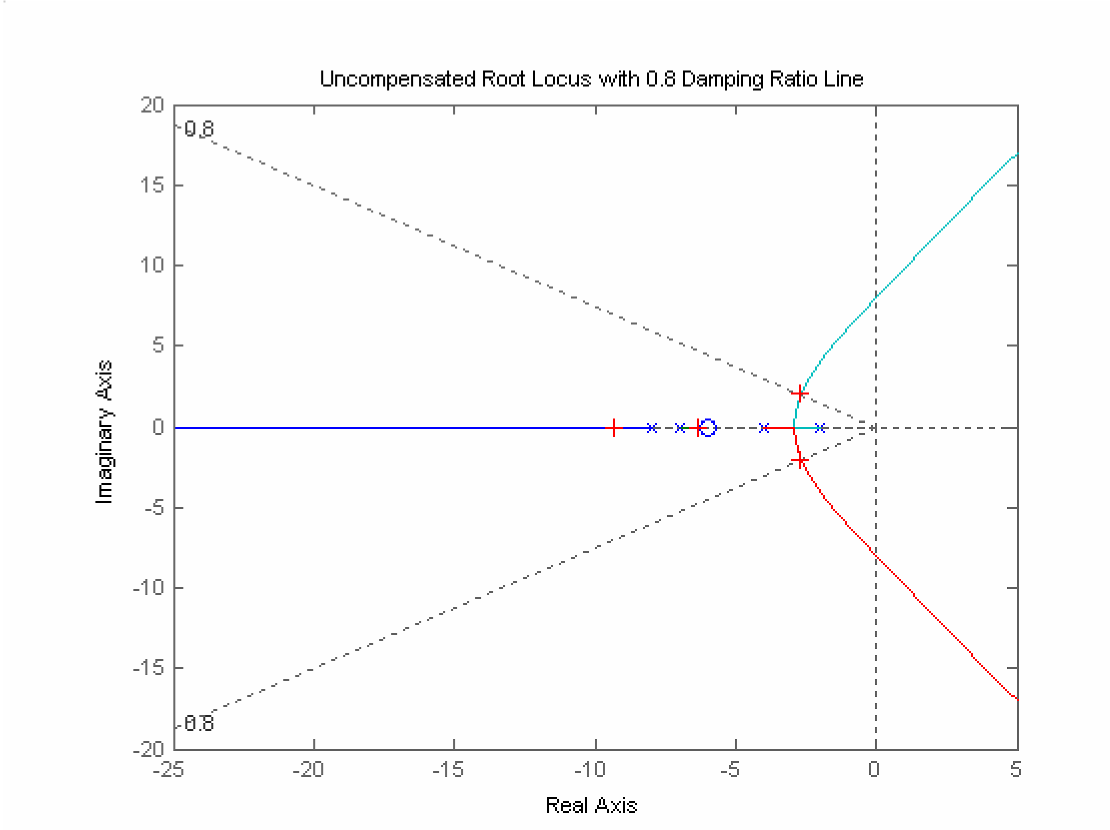
```
ans =

G(s)


Zero/pole/gain:
         (s+6)
-----------------------
(s+8) (s+7) (s+4) (s+2)

Select a point in the graphics window

selected_point =

  -2.7062 + 2.0053i


ans =

Closed-loop poles =


p =

  -9.3056
  -6.3230
```

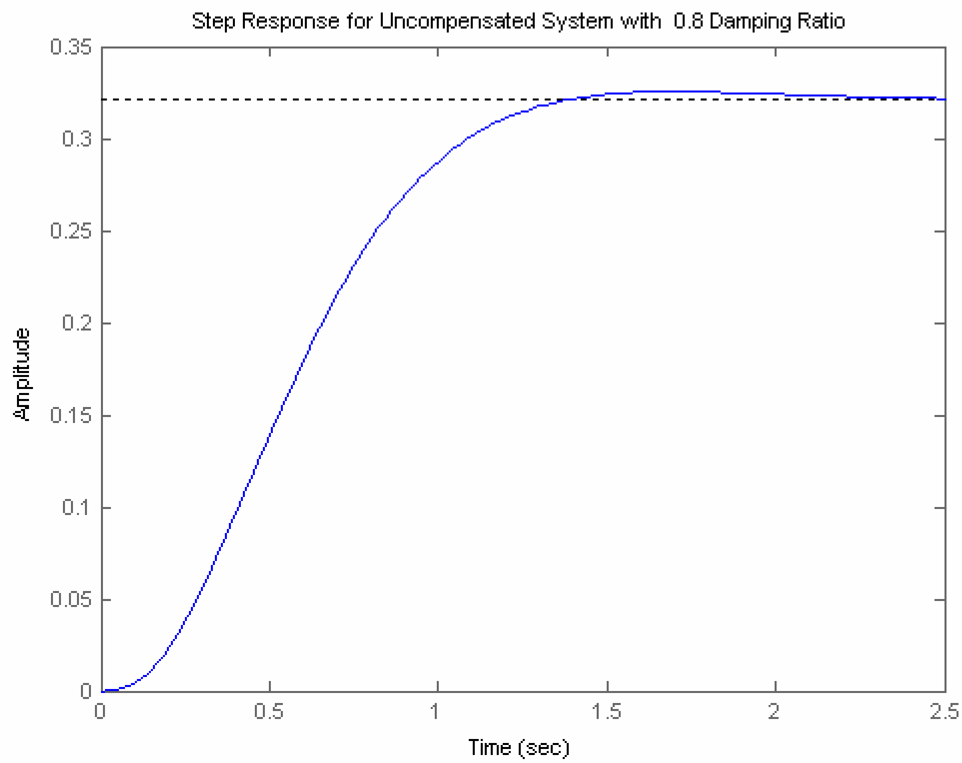
```
  -2.6857 + 2.0000i
  -2.6857 - 2.0000i

Give pole number that is operating point   3

ans =

Summary of estimated specifications


operatingpoint =

  -2.6857 + 2.0000i


gain =

   35.2956


estimated_settling_time =

    1.4894


estimated_peak_time =

    1.5708


estimated_percent_overshoot =

    1.5165


estimated_damping_ratio =

    0.8000


estimated_natural_frequency =

    3.3486


Kp =

    0.4727


ans =

T(s)


Transfer function:
             35.3 s + 211.8
----------------------------------------
s^4 + 21 s^3 + 154 s^2 + 491.3 s + 659.8


ans =

Press any key to continue and obtain the step response


ans =
```

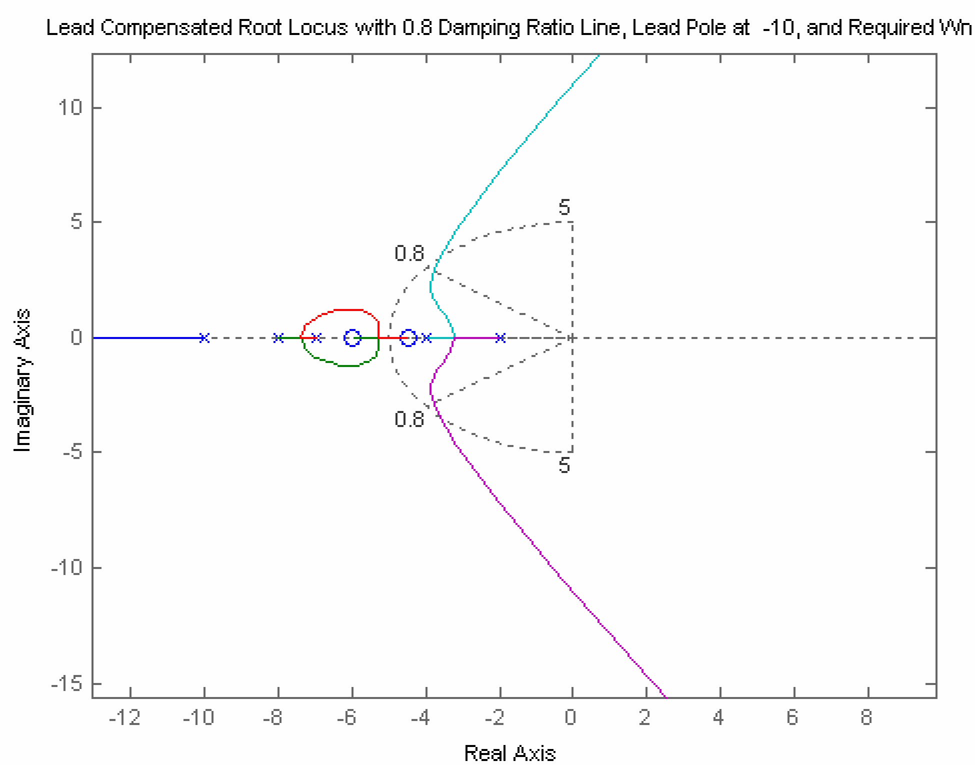
```
Press any key to go to Lead compensation


ans =

Compensated system


ans =

Lead Zero at -4.5

Enter a Test Lead Compensator Pole, (s+a). a =     10

ans =

Gc(s)


Transfer function:
s + 4.5
-------
s + 10


ans =

G(s)Gc(s)


Zero/pole/gain:
        (s+6) (s+4.5)
------------------------------
(s+10) (s+8) (s+7) (s+4) (s+2)

Are you done? (y=0,n=1)  1
Enter a Test Lead Compensator Pole, (s+a). a =     11.7

ans =

Gc(s)


Transfer function:
s + 4.5
--------
s + 11.7


ans =

G(s)Gc(s)


Zero/pole/gain:
         (s+6) (s+4.5)
--------------------------------
(s+11.7) (s+8) (s+7) (s+4) (s+2)

Are you done? (y=0,n=1)  0
Select a point in the graphics window

selected_point =

  -3.9885 + 3.0882i
```

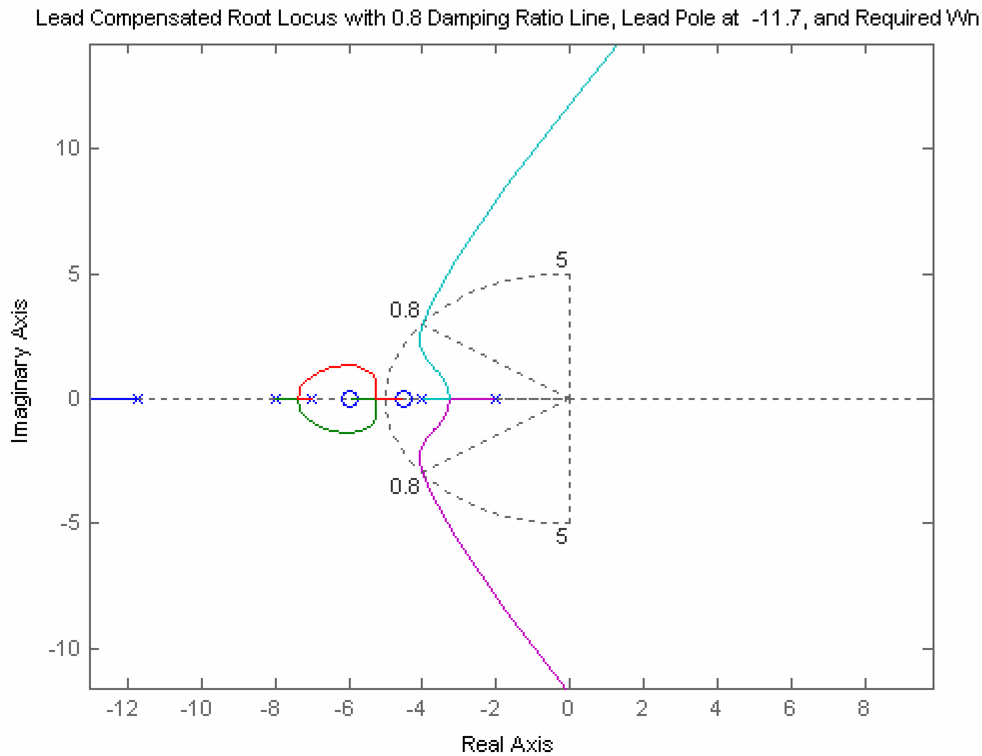
```
ans =

Closed-loop poles =


p =

 -14.2326
  -3.9797 + 3.0860i
  -3.9797 - 3.0860i
  -5.2540 + 0.5076i
  -5.2540 - 0.5076i

Give pole number that is operating point   2

ans =

Summary of estimated specifications


operatingpoint =

  -3.9797 + 3.0860i


gain =

  178.3530


estimated_settling_time =

    1.0051


estimated_peak_time =

    1.0180


estimated_percent_overshoot =

    1.5165


estimated_damping_ratio =

    0.8000


estimated_natural_frequency =

    5.0360


Kp =

    0.9187


ans =

T(s)


Transfer function:
```

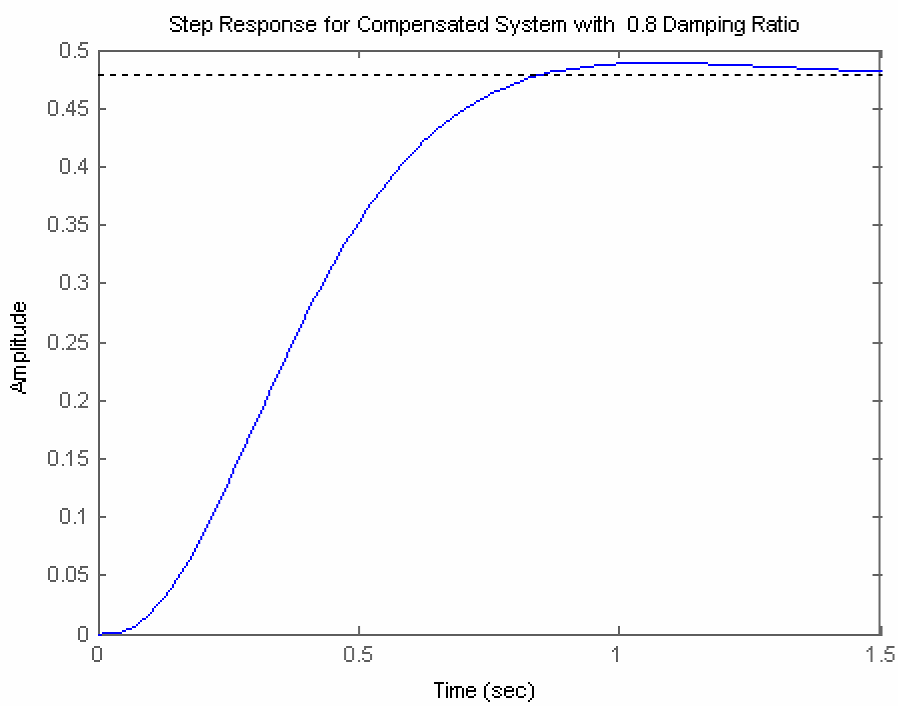
```
        178.4 s^2 + 1873 s + 4816
------------------------------------------
s^5 + 32.7 s^4 + 399.7 s^3 + 2436 s^2

                      + 7656 s + 1.006e004


ans =

Press any key to continue and obtain the step
response
```




Step Response for Uncompensated System with 0.8 Damping Ratio
Amplitude
Time (sec)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
2.5


Lead Compensated Root Locus with 0.8 Damping Ratio Line, Lead Pole at -10, and Required Wn
Imaginary Axis
Real Axis
10
5
0
-5
-10
-15
-12
-10
-8
-6
-4
-2
0
2
4
6
8
0.8
0.8
5
5





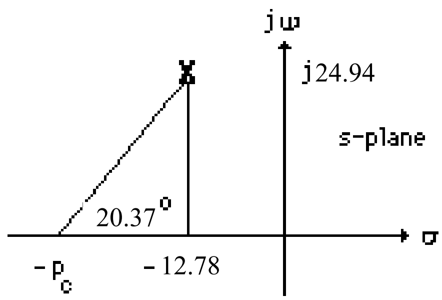
13.

a. Searching along the 117.13o line (%OS = 20%; ζ = 0.456), find the operating point at

-6.39 + j12.47 with K = 9273. Searching along the real axis for K = 9273, we find a higher-order pole at –47.22. Thus, $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{6.39} = 0.626$ second.

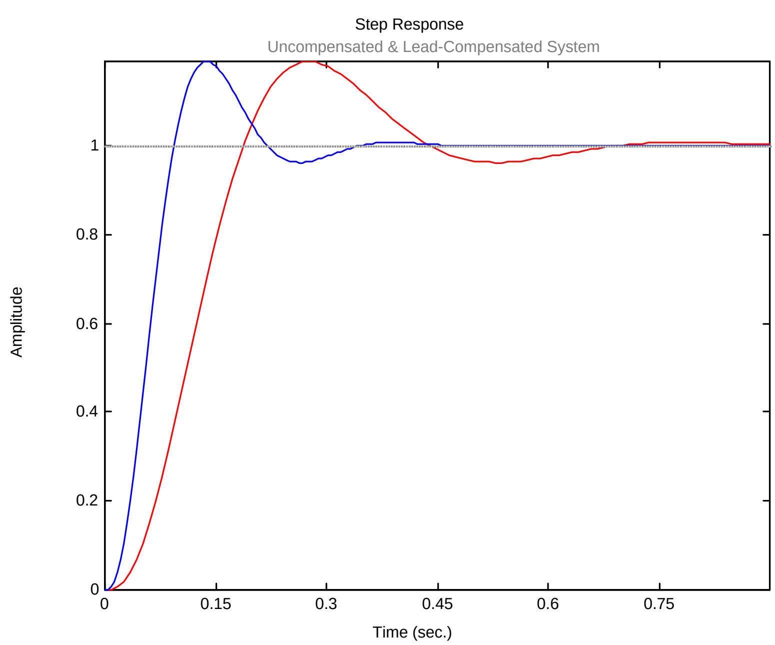
b. For the settling time to decrease by a factor of 2, Re = $-\zeta\omega_n$ = -6.39 x 2 = -12.78. The imaginary part is Im = -12.78 tan 117.13° = 24.94. Hence, the compensated closed-loop poles are -12.78 ± j24.94. A settling time of 0.313 second would result.

c. Assume a compensator zero at -20. Using the uncompensated system's poles along with the compensator zero, the summation of angles to the design point, -12.78 ± j24.94 is –159.63°. Thus, the contribution of the compensator pole must be 159.63°-180° = -20.37°. Using the following geometry, $\frac{24.94}{p_c - 12.78}$ = tan 20.37°, or p_c = 79.95.



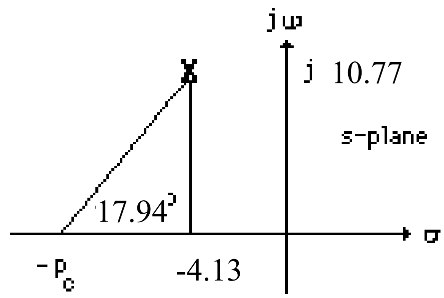
Adding the compensator pole and using --12.78 ± j24.94 as the test point, K = 74130.

d.



14.

a. Searching along the 110.97º line ($\%OS$ = 30%; ζ= 0.358), find the operating point at -2.065 + j5.388 with K = 366.8. Searching along the real axis for K = 366.8, we find a higher-order pole at –16.87. Thus, $T_s = \dfrac{4}{\zeta\omega_n} = \dfrac{4}{2.065} = 1.937$ seconds. For the settling time to decrease by a factor of 2, Re = $-\zeta\omega_n$ = -2.065 x 2 = - 4.13. The imaginary part is – 4.13 tan 110.97^0 = 10.77. Hence, the compensated dominant poles are – 4.13 ± j10.77. The compensator zero is at -7. Using the uncompensated system's poles along with the compensator zero, the summation of angles to the design point, – 4.13 ± j10.77 is –162.06º. Thus, the contribution of the compensator pole must be –162.06º - 180º = -17.94º. Using the following geometry, $\dfrac{10.77}{p_c - 4.13}$ = tan 17.94º, or p_c = 37.4.



Adding the compensator pole and using – 4.13 ± j10.77 as the test point, K = 5443.

b. Searching the real axis segments for K = 5443 yields higher-order poles at approximately –8.12 and –42.02. The pole at –42.02 can be neglected since it is more than five times further from the imaginary axis than the dominant pair. The pole at –8.12 may not be canceling the zero at -7. Hence, simulate to be sure the requirements are met.

c.

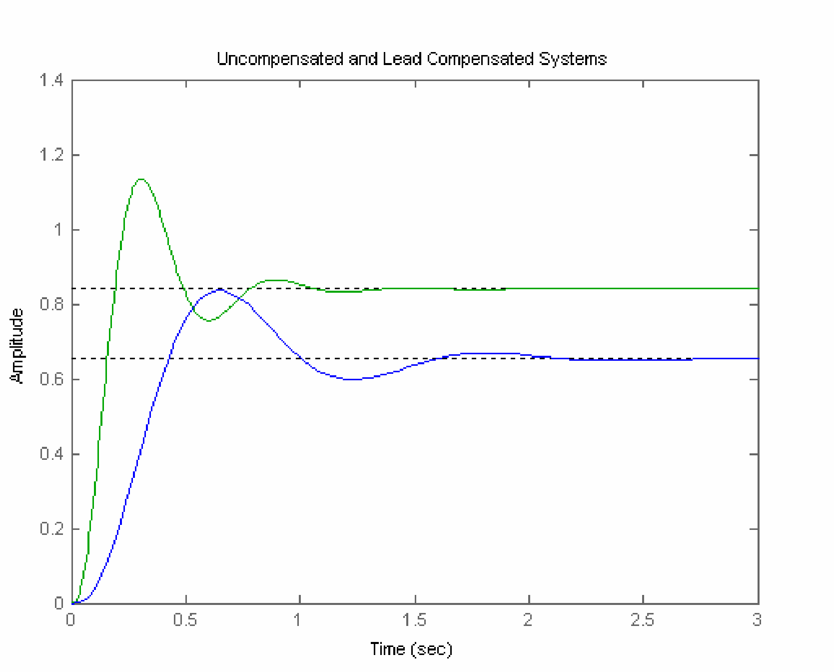
Program:

```
'Uncompensated System G1(s)'
numg1=1;
deng1=poly([-15 (-3+2*j) (-3-2*j)]);
G1=tf(numg1,deng1)
G1zpk=zpk(G1)
K1=366.8
'T1(s)'
T1=feedback(K1*G1,1);
T1zpk=zpk(T1)
'Compensator Gc(s)'
numc=[1 7];
denc=[1 37.4];
Gc=tf(numc,denc)
'Compensated System G2(s) = G1(s)Gc(s)'
K2=5443
G2=G1*Gc;
G2zpk=zpk(G2)
'T2(s)'
T2=feedback(K2*G2,1);
T2zpk=zpk(T2)
step(T1,T2)
title(['Uncompensated and Lead Compensated Systems'])
```

Computer response:

```
ans =

Uncompensated System G1(s)


Transfer function:
           1
--------------------------
s^3 + 21 s^2 + 103 s + 195


Zero/pole/gain:
          1
----------------------
(s+15) (s^2  + 6s + 13)


K1 =

  366.8000


ans =

T1(s)


Zero/pole/gain:
              366.8
--------------------------------
(s+16.87) (s^2  + 4.132s + 33.31)


ans =

Compensator Gc(s)


Transfer function:
 s + 7
--------
s + 37.4


ans =

Compensated System G2(s) = G1(s)Gc(s)


K2 =

        5443


Zero/pole/gain:
             (s+7)
-------------------------------
(s+37.4) (s+15) (s^2  + 6s + 13)


ans =

T2(s)


Zero/pole/gain:
```

```
                5443 (s+7)
-------------------------------------------
(s+42.02) (s+8.118) (s^2  + 8.261s + 133.1)
```

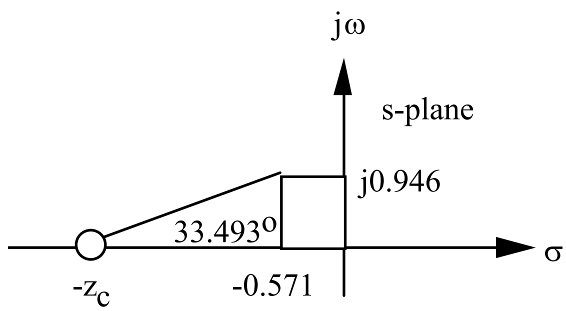


15.

a. Searching the 15% overshoot line (121.127^o) for 180^o yields $-0.372 + j0.615$. Hence, $T_s = \frac{4}{\sigma_d} = \frac{4}{0.372} = 10.75$ seconds.

b. For 7 seconds settling time, $\sigma_d = \frac{4}{T_s} = \frac{4}{7} = 0.571$. $\omega_d = 0.571 \tan(180^o - 121.127^o) = 0.946$.

Therefore, the design point is $-0.571 + j0.946$. Summing the angles of the uncompensated system's poles as well as the compensator pole at -15 yields -213.493^o. Therefore, the compensator zero must contribute $(213.493^o - 180^o) = 33.493^o$. Using the geometry below,



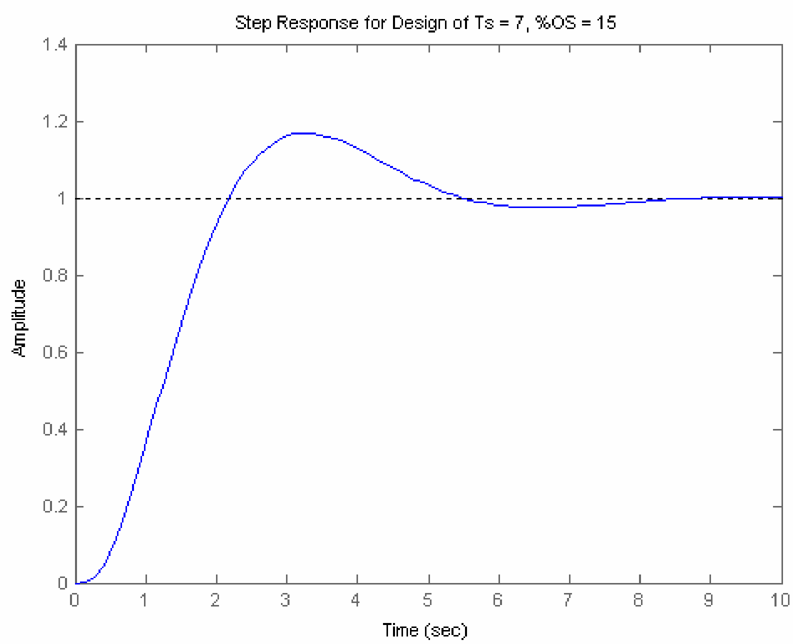
$\frac{0.946}{z_c - 0.571} = \tan(33.493^o)$. Hence, $z_c = 2$. The compensated open-loop transfer function is

$\frac{K(s+2)}{s(s+1)(s^2+10s+26)(s+15)}$. Evaluating the gain for this function at the point, -0.571 + j0.946 yields K = 207.512.

c.

Program:
```
numg= 207.512*[1 2];
r=roots([1,10,26]);
deng=poly([0 ,-1, r(1),r(2),-15]);
'G(s)'
G=tf(numg,deng);
Gzpk=zpk(G)
T=feedback(G,1);
step(T)
title(['Step Response for Design of Ts = 7, %OS = 15'])
```

Computer response:
```
ans =

G(s)

Zero/pole/gain:
                  207.512 (s+2)
          -------------------------------
          s (s+15) (s+1) (s^2 + 10s + 26)
```

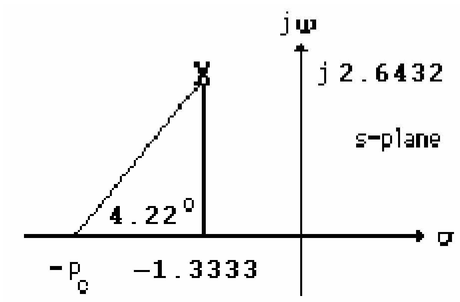


16.

a. From 20.5% overshoot evaluate $\zeta = 0.45$. Also, since $\zeta\omega_n = \frac{4}{T_s} = \frac{4}{3}$, $\omega_n = 2.963$. The compensated dominant poles are located at $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ = - 1.3333 ± j2.6432. Assuming the compensator zero at -0.02, the contribution of open-loop poles and the compensator zero to the design point, - 1.3333 ± j2.6432 is -175.78°. Hence, the compensator pole must contribute

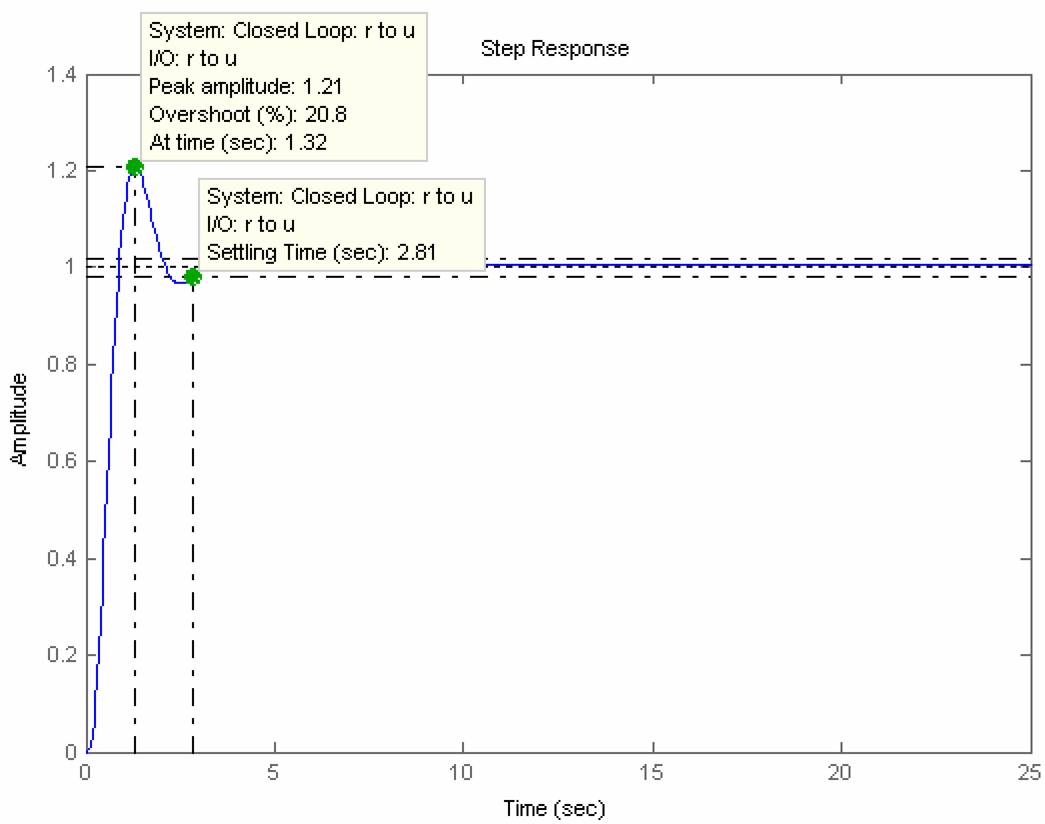
$175.78^o - 180^o = -4.22^o$. Using the following geometry, $\dfrac{2.6432}{p_c - 1.3333} = \tan 4.22^o$, or $p_c = 37.16$

Adding the pole to the system, $K = 4401.52$ at the design point..



b. Searching along the real axis segments of the root locus for $K = 4401.52$, we find higher-order poles at -0.0202, -13.46, and -37.02. There is pole/zero cancellation at -0.02. Also, the poles at , -13.46, and -37.02 are at least 5 times the design point's real part. Thus, the second-order approximation is valid.

c.

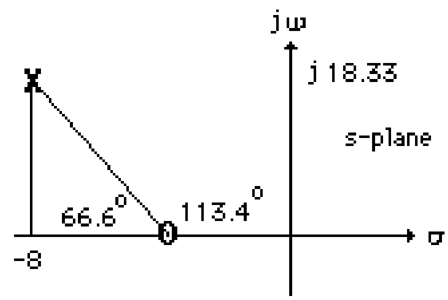


From the plot, T_s = 2.81 seconds, and %OS = 20.8%. Thus, the requirements are met.

17.

a. $\zeta\omega_n = \frac{4}{T_s} = \frac{4}{0.5}$ = 8. Since ζ = 0.4, ω_n = 20. Therefore the compensated closed-loop poles are located at $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ = -8 ± j18.33.

b. Using the system's poles along with the compensator's pole at -15, the sum of angles to the test point –8 ± j18.33 is -293.4° . Therefore, the compensator's zero must contribute 293.4° - 180° = 113.4° . Using the following geometry, $\frac{18.33}{8 - z_c}$ = tan 66.6°, or z_c = 0.0679.



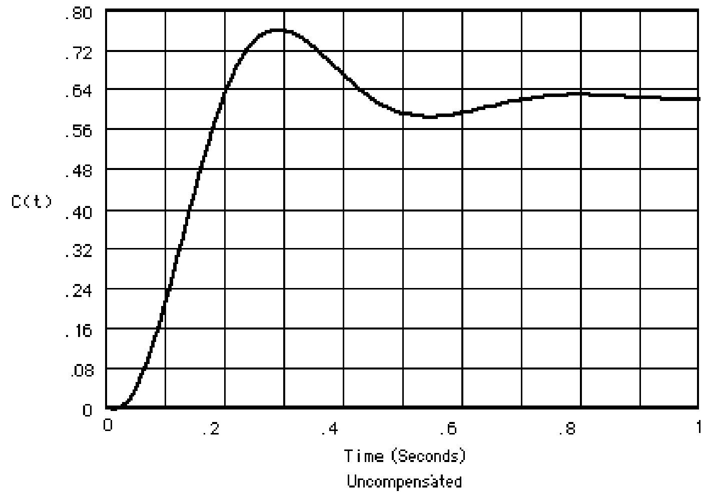
c. Adding the compensator zero and using –8 ± j18.33 as the test point, K = 7297.

d. Making a second-order assumption, the predicted performance is as follows:

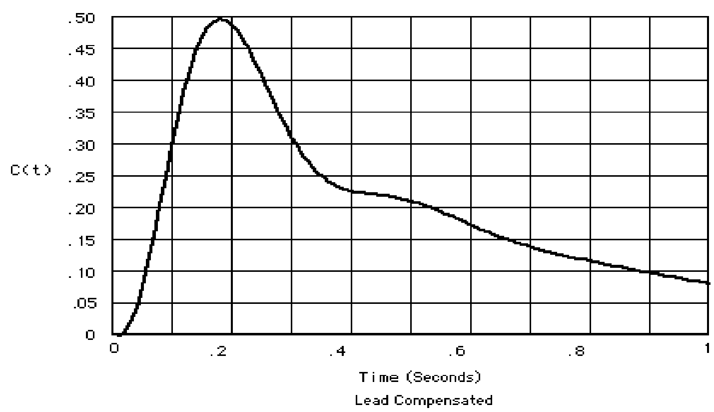
Uncompensated: Searching along the 133.58° line (ζ = 0.4), find the uncompensated closed-loop pole at -5.43 + j12.45 with K = 3353. Hence, $T_s = \frac{4}{\zeta\omega_n}$ = 0.74 seconds; $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \text{x}100 =$ 25.38%; $K_p = \frac{3353}{101\text{x}20}$ = 1.66. Checking the second-order assumption by searching the real axis segments of the root locus for K = 3353, we find a higher-order pole at -29.13. Since this pole is more than five times further from the imaginary axis than the dominant pair, the second order assumption is reasonable.

Compensated: Using the compensated dominant pole location, - 8 ± j18.33, $T_s = \frac{4}{\zeta\omega_n}$ = 0.5 seconds; $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \text{x}100 =$ 25.38%; $K_p = \frac{7297\text{x}0.0679}{101\text{x}20\text{x}15}$ = 0.016. Checking the second-order assumption by searching the real axis segments of the root locus for K = 7297, we find higher-order poles at -2.086 and -36.91. The poles are not five times further from the imaginary axis nor do they yield pole/zero cancellation. The second-order assumption is not valid.

e.



The uncompensated system exhibits a steady-state error of 0.38, a percent overshoot of 22.5%, and a settling time of 0.78 seconds.



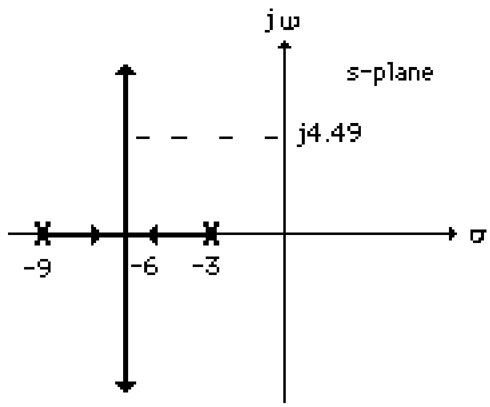
Since there is no pole/zero cancellation the closed-loop zero near the origin produces a large steady-state error. The student should be asked to find a viable design solution to this problem by choosing the compensator zero further from the origin. For example, placing the compensator zero at -20 yields a compensator pole at -90.75 and a gain of 28730. This design yields a valid second-order approximation.

18.

a. Since %OS = 1.5%, $\zeta = \dfrac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.8$. Since $T_s = \dfrac{4}{\zeta\omega_n} = \dfrac{2}{3}$ second,

$\omega_n = 7.49$ rad/s. Hence, the location of the closed-loop poles must be $-6 \pm j4.49$. The summation of angles from open-loop poles to $-6 \pm j4.49$ is -226.3°. Therefore, the design point is not on the root locus.

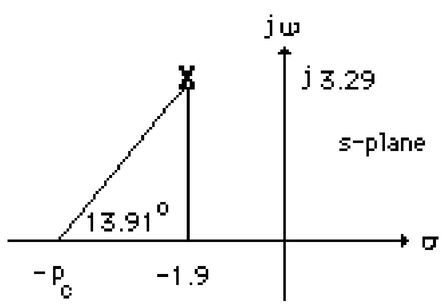
b. A compensator whose angular contribution is $226.3^o - 180^o = 46.3^o$ is required. Assume a compensator zero at -5 canceling the pole. Thus, the breakaway from the real axis will be at the required -6 if the compensator pole is at -9 as shown below.



Adding the compensator pole and zero to the system poles, the gain at the design point is found to be 29.16. Summarizing the results: $G_c(s) = \frac{s+5}{s+9}$ with K = 29.16.

19.

Lead compensator design: Searching along the 120^o line ($\zeta = 0.5$), find the operating point at $-1.531 + j2.652$ with K = 354.5. Thus, $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.531} = 2.61$ seconds. For the settling time to decrease by 0.5 second, $T_s = 2.11$ seconds, or $Re = -\zeta\omega_n = -\frac{4}{2.11} = -1.9$. The imaginary part is $-1.9 \tan 60^o = 3.29$. Hence, the compensated dominant poles are $-1.9 \pm j3.29$. The compensator zero is at -5. Using the uncompensated system's poles along with the compensator zero, the summation of angles to the design point, $-1.9 \pm j3.29$ is -166.09^o. Thus, the contribution of the compensator pole must be $166.09^o - 180^o = -13.91^o$. Using the following geometry, $\frac{3.29}{p_c - 1.9} = \tan 13.91^o$, or $p_c = 15.18$.



Adding the compensator pole and using $-1.9 \pm j3.29$ as the test point, K = 1417.

Computer simulations yield the following: Uncompensated: $T_s = 3$ seconds, %OS = 14.6%.

Compensated: T_s = 2.3 seconds, %OS = 15.3%.

Lag compensator design: The lead compensated open-loop transfer function is

$G_{LC}(s) = \dfrac{1417(s+5)}{(s+2)(s+4)(s+6)(s+8)(s+15.18)}$. The uncompensated

K_p = 354.5/(2 x 4 x 6 x 8) = 0.923. Hence, the uncompensated steady-state error is $\dfrac{1}{1+K_p}$ = 0.52.

Since we want 30 times improvement, the lag-lead compensated system must have a steady-state error of 0.52/30 = 0.017. The lead compensated K_p = 1417*5/(2*4*6*8*15.18) = 1.215. Hence, the lead-compensated error is $\dfrac{1}{1+K_p}$ = 0.451. Thus, the lag compensator must improve the lead-compensated error by 0.451/0.017 = 26.529 times. Thus 0.451/ ($\dfrac{1}{1+K_{pllc}}$) = 26.529, where K_{pllc} = 57.823 is the lead-lag compensated system's position constant. Thus, the improvement in K_p from the lead to the lag-lead compensated system is 57.823/1.215 = 47.59. Use a lag compensator, whose zero is 47.59 times farther than its pole, or $G_{lag} = \dfrac{(s+0.04759)}{(s+0.001)}$. Thus, the lead-lag compensated open-loop transfer function is $G_{LLC}(s) = \dfrac{1417(s+5)(s+0.04759)}{(s+2)(s+4)(s+6)(s+8)(s+15.18)(s+0.001)}$.

20.

Program:

```
numg=1;
deng=poly([-2 -4 -6 -8]);
'G(s)'
G=tf(numg,deng);
Gzpk=zpk(G)
rlocus(G,0:5:500)
z=0.5;
pos=exp(-pi*z/sqrt(1-z^2))*100;
sgrid(z,0)
title(['Uncompensated Root Locus with ' , num2str(z), ' Damping Ratio
Line'])
[K,p]=rlocfind(G);  %Allows input by selecting point on graphic
'Closed-loop poles = '
p
i=input('Give pole number that is operating point   ');
'Summary of estimated specifications for uncompensated system'
operatingpoint=p(i)
gain=K
estimated_settling_time=4/abs(real(p(i)))
estimated_peak_time=pi/abs(imag(p(i)))
estimated_percent_overshoot=pos
estimated_damping_ratio=z
estimated_natural_frequency=sqrt(real(p(i))^2+imag(p(i))^2)
Kpo=dcgain(K*G)
T=feedback(K*G,1);
'Press any key to continue and obtain the step response'
pause
step(T)

whitebg('w')
title(['Step Response for Uncompensated System with  ' , num2str(z),...
' Damping Ratio'],'color','black')
'Press any key to go to Lead compensation'
```

```
pause
'Compensated system'
b=5;
'Lead Zero at -b '
done=1;
while done>0
a=input('Enter a Test Lead Compensator Pole, (s+a). a =     ');
numgglead=[1 b];
dengglead=conv([1 a],poly([-2 -4 -6 -8]));
'G(s)Glead(s)'
GGlead=tf(numgglead,dengglead);
GGleadzpk=zpk(GGlead)
wn=4/((estimated_settling_time-0.5)*z);
clf
rlocus(GGlead,0:10:2000)
sgrid(z,wn)
axis([-10 0 -5 5])
title(['Lead Compensated Root Locus with ' , num2str(z),...
' Damping Ratio Line, Lead Pole at  ', num2str(-a), ', and Required Wn'])
done=input('Are you done? (y=0,n=1)  ');
end
[K,p]=rlocfind(GGlead);  %Allows input by selecting point on graphic
'Closed-loop poles = '
p
i=input('Give pole number that is operating point   ');
'Summary of estimated specifications for lead-compensated system'
operatingpoint=p(i)
gain=K
estimated_settling_time=4/abs(real(p(i)))
estimated_peak_time=pi/abs(imag(p(i)))
estimated_percent_overshoot=pos
estimated_damping_ratio=z
estimated_natural_frequency=sqrt(real(p(i))^2+imag(p(i))^2)
Kplead=dcgain(K*GGlead)
T=feedback(K*GGlead,1);
'Press any key to continue and obtain the step response'
pause
step(T)

whitebg('w')
title(['Step Response for Lead Compensated System with  ' , num2str(z),...
' Damping Ratio'],'color','black')
'Press any key to continue and design lag compensation'
pause
'Improvement in steady-state error with lead compensator is'
error_improvement=(1+Kplead)/(1+Kpo)
additional_error_improvement=30/error_improvement
Kpnn=additional_error_improvement*(1+Kplead)-1
pc=0.001
zc=pc*(Kpnn/Kplead)
numggleadlag=conv(numgglead,[1 zc]);
dengleadlag=conv(dengglead,[1 pc]);
'G(s)Glead(s)Glag(s)'
GGleadGlag=tf(numggleadlag,dengleadlag);
GGleadGlagzpk=zpk(GGleadGlag)
rlocus(GGleadGlag,0:10:2000)
z=0.5;
pos=exp(-pi*z/sqrt(1-z^2))*100;
sgrid(z,0)
title(['Lag-Lead Compensated Root Locus with ' , num2str(z), ...
' Damping Ratio Line and Lag Pole at ',num2str(-pc)])
[K,p]=rlocfind(GGleadGlag);  %Allows input by selecting point on graphic
'Closed-loop poles = '
p
i=input('Give pole number that is operating point   ');
'Summary of estimated specifications for lag-lead compensated system'
operatingpoint=p(i)
```

```
gain=K
estimated_settling_time=4/abs(real(p(i)))
estimated_peak_time=pi/abs(imag(p(i)))
estimated_percent_overshoot=pos
estimated_damping_ratio=z
estimated_natural_frequency=sqrt(real(p(i))^2+imag(p(i))^2)
Kpleadlag=dcgain(K*GGleadGlag)
T=feedback(K*GGleadGlag,1);
'Press any key to continue and obtain the step response'
pause
step(T)
whitebg('w')
title(['Step Response for Lag-Lead Compensated System with  ',
num2str(z),...
' Damping Ratio and Lag Pole at ',num2str(-pc)],'color','black')
```

Computer response:

```
ans =


G(s)



Zero/pole/gain:
           1
-----------------------
(s+8) (s+6) (s+4) (s+2)


Select a point in the graphics window


selected_point =


  -1.5036 + 2.6553i



ans =


Closed-loop poles =



p =


  -8.4807 + 2.6674i
  -8.4807 - 2.6674i
  -1.5193 + 2.6674i
  -1.5193 - 2.6674i


Give pole number that is operating point   3


ans =
```

```
Summary of estimated specifications for uncompensated system


operatingpoint =

  -1.5193 + 2.6674i


gain =

  360.8014


estimated_settling_time =

    2.6328


estimated_peak_time =

    1.1778


estimated_percent_overshoot =

   16.3034


estimated_damping_ratio =

    0.5000


estimated_natural_frequency =

    3.0698


Kpo =

    0.9396


ans =
```

```
Press any key to continue and obtain the step response


ans =

Press any key to go to Lead compensation


ans =

Compensated system


ans =

Lead Zero at -b

Enter a Test Lead Compensator Pole, (s+a). a =     10

ans =

G(s)Glead(s)


Zero/pole/gain:
            (s+5)
------------------------------
(s+10) (s+8) (s+6) (s+4) (s+2)

Are you done? (y=0,n=1)  1
Enter a Test Lead Compensator Pole, (s+a). a =     15

ans =

G(s)Glead(s)


Zero/pole/gain:
            (s+5)
------------------------------
(s+15) (s+8) (s+6) (s+4) (s+2)

Are you done? (y=0,n=1)  0
Select a point in the graphics window
```

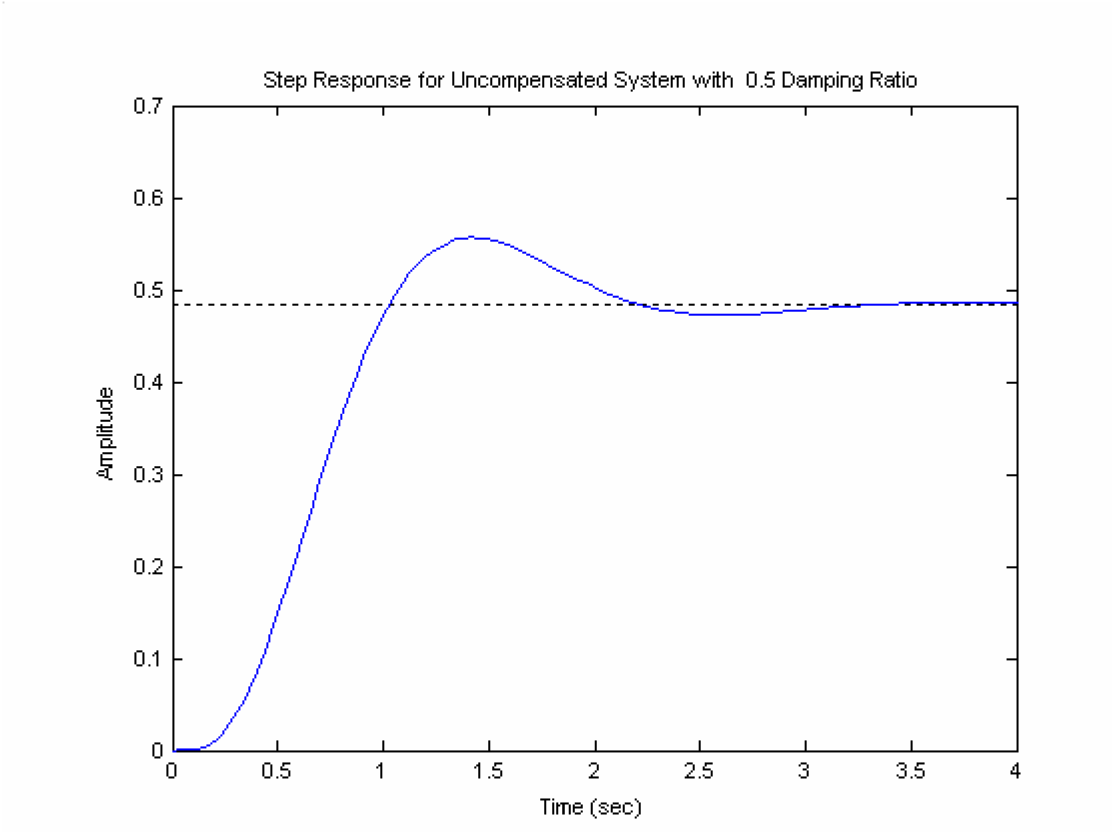
```
selected_point =

  -1.9076 + 3.2453i


ans =

Closed-loop poles =


p =

 -13.0497 + 1.9313i
 -13.0497 - 1.9313i
  -5.0654
  -1.9176 + 3.2514i
  -1.9176 - 3.2514i

Give pole number that is operating point   4

ans =

Summary of estimated specifications for lead-compensated system


operatingpoint =

  -1.9176 + 3.2514i


gain =

  1.3601e+003


estimated_settling_time =

    2.0860


estimated_peak_time =

    0.9662
```

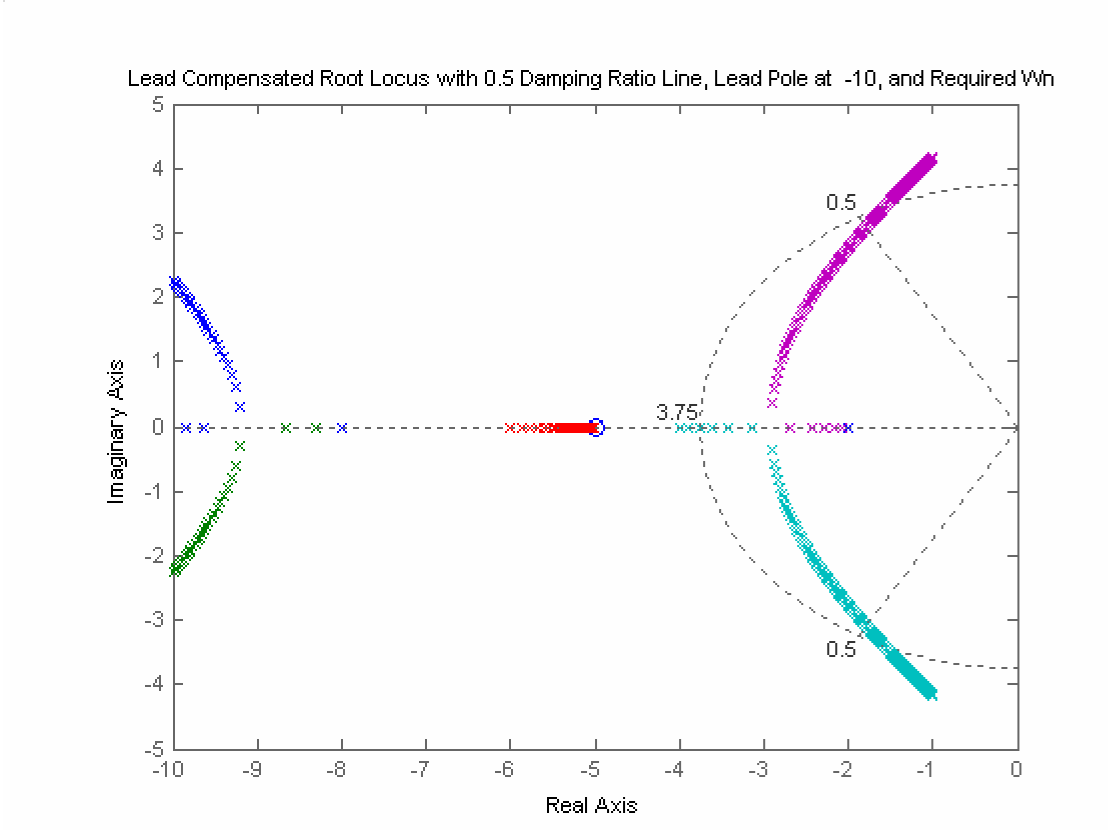
```
estimated_percent_overshoot =

   16.3034


estimated_damping_ratio =

    0.5000


estimated_natural_frequency =

    3.7747


Kplead =

    1.1806


ans =

Press any key to continue and obtain the step response


ans =

Press any key to continue and design lag compensation


ans =

Improvement in steady-state error with lead compensator is


error_improvement =

    1.1243


additional_error_improvement =

   26.6842
```

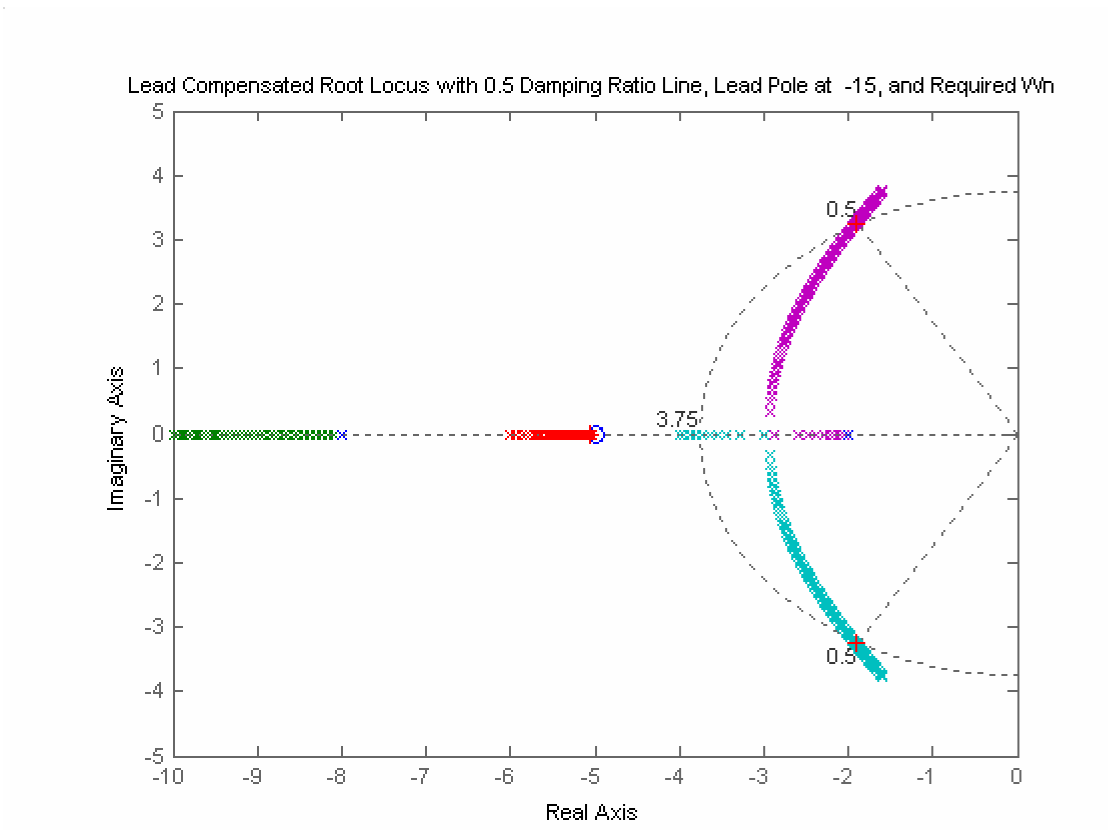
```
Kpnn =

   57.1876


pc =

    0.0010


zc =

    0.0484


ans =

G(s)Glead(s)Glag(s)


Zero/pole/gain:
           (s+5) (s+0.04844)
----------------------------------------
(s+15) (s+8) (s+6) (s+4) (s+2) (s+0.001)

Select a point in the graphics window

selected_point =

  -1.8306 + 3.2919i


ans =

Closed-loop poles =


p =

 -13.0938 + 2.0650i
 -13.0938 - 2.0650i
  -5.0623
  -1.8617 + 3.3112i
  -1.8617 - 3.3112i
  -0.0277
```

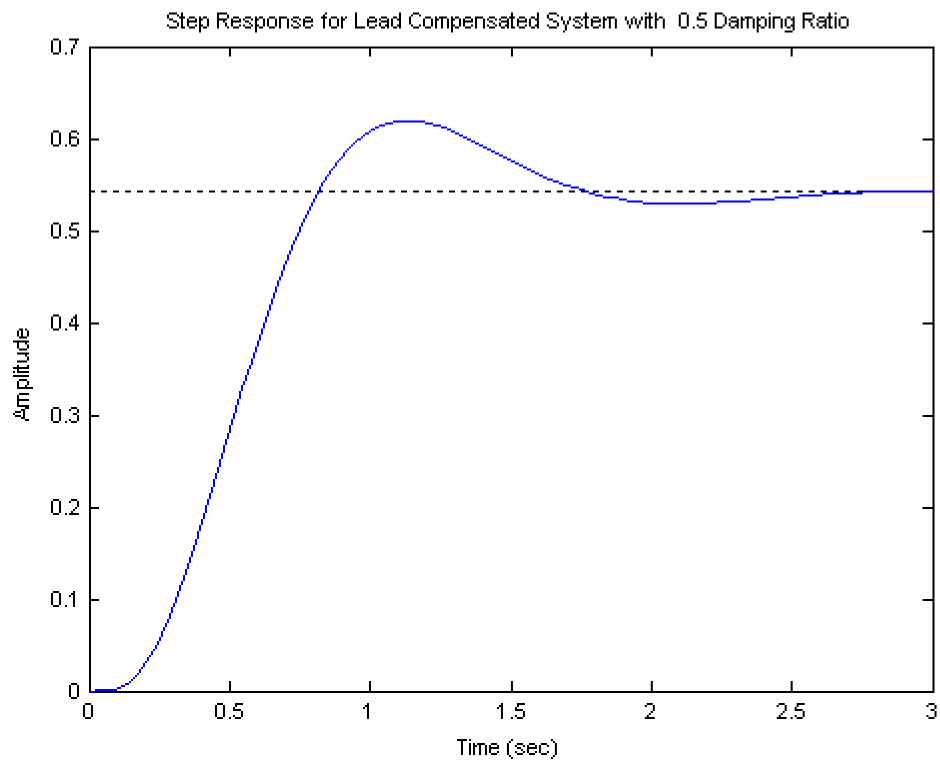
```
Give pole number that is operating point   4

ans =

Summary of estimated specifications for lag-lead compensated system


operatingpoint =

  -1.8617 + 3.3112i


gain =

  1.4428e+003


estimated_settling_time =

    2.1486


estimated_peak_time =

    0.9488


estimated_percent_overshoot =

   16.3034


estimated_damping_ratio =

    0.5000


estimated_natural_frequency =

    3.7987


Kpleadlag =
```

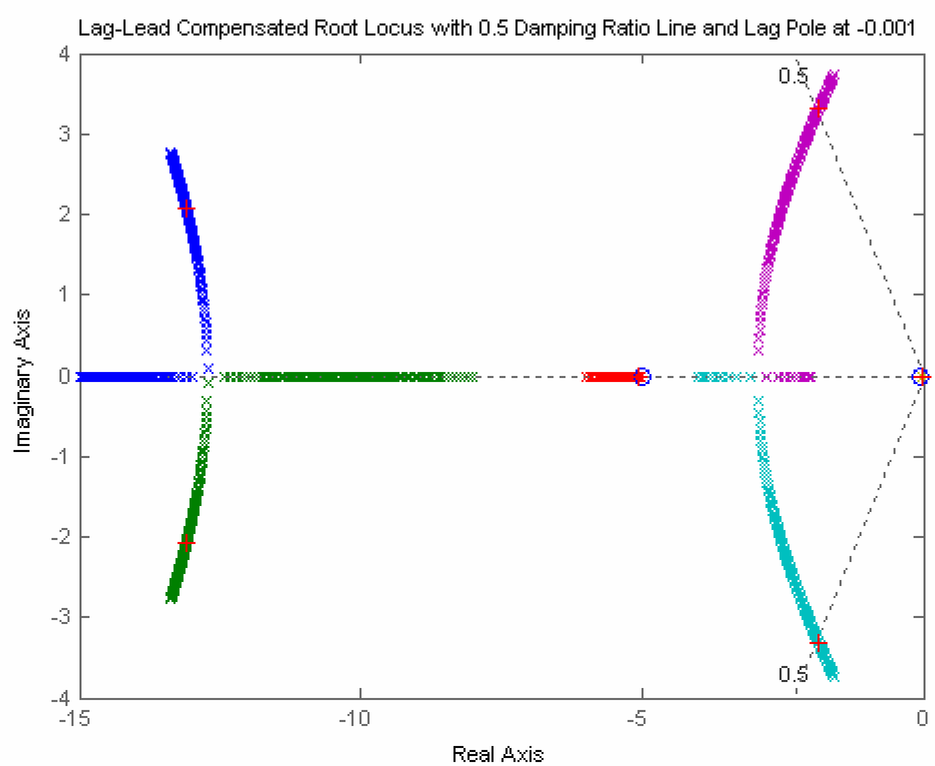
```
   60.6673


ans =

Press any key to continue and obtain the step response
```

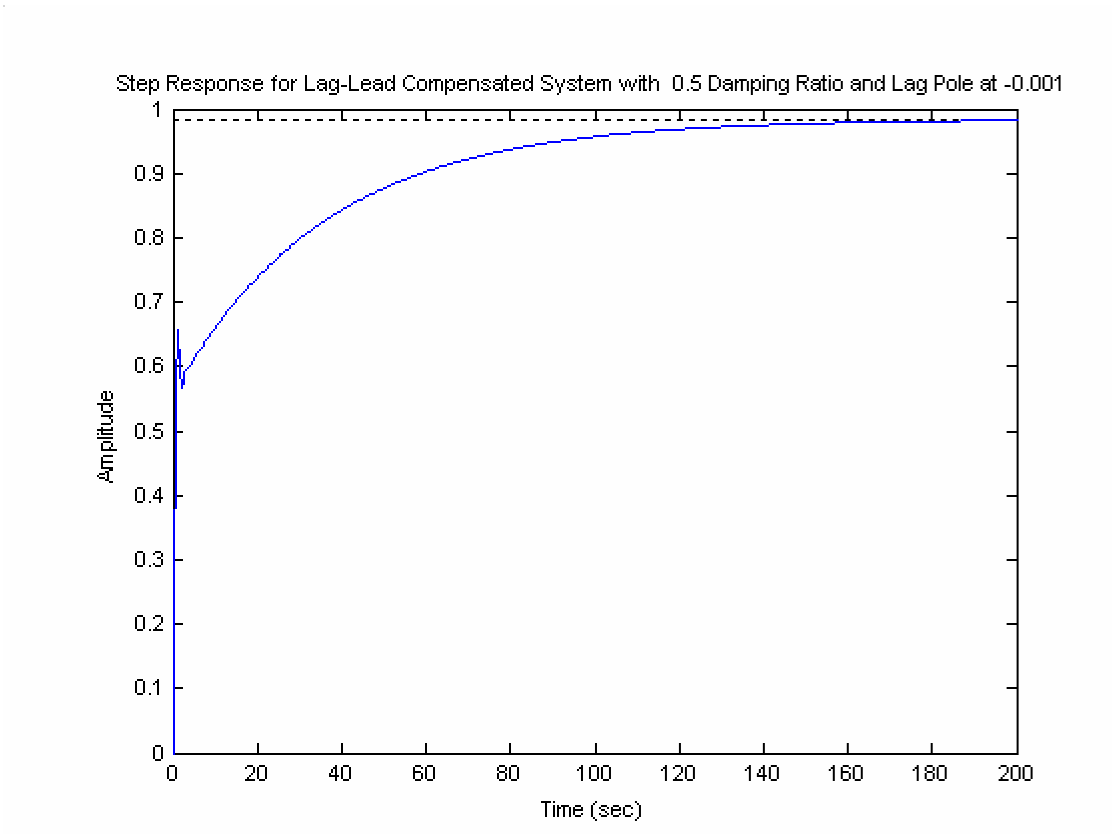



Lead Compensated Root Locus with 0.5 Damping Ratio Line, Lead Pole at -10, and Required Wn
Imaginary Axis
Real Axis
0.5
3.75
0.5
5
4
3
2
1
0
-1
-2
-3
-4
-5
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0


Lead Compensated Root Locus with 0.5 Damping Ratio Line, Lead Pole at -15, and Required Wn
Imaginary Axis
Real Axis
0.5
3.75
0.5
5
4
3
2
1
0
-1
-2
-3
-4
-5
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0

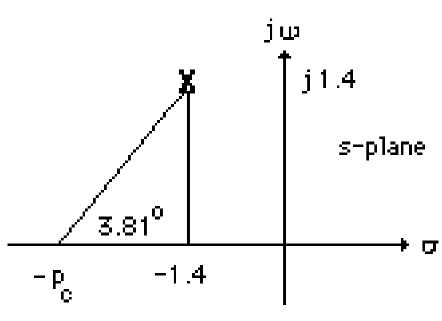

Step Response for Lead Compensated System with 0.5 Damping Ratio
Amplitude
Time (sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
0.5
1
1.5
2
2.5
3


Lag-Lead Compensated Root Locus with 0.5 Damping Ratio Line and Lag Pole at -0.001
Imaginary Axis
Real Axis
0.5
0.5
4
3
2
1
0
-1
-2
-3
-4
-15
-10
-5
0



21.

a. For the settling time to be 2.86 seconds with 4.32% overshoot, the real part of the compensated dominant poles must be $\frac{4}{T_s} = \frac{4}{2.86} = 1.4$. Hence the compensated dominant poles are $-1.4 \pm j1.4$. Assume the compensator zero to be at -1 canceling the system pole at -1. The summation of angles to the design point at $-1.4 \pm j1.4$ is -176.19^o. Thus the contribution of the compensator pole is $176.19^o - 180^o = 3.81^o$. Using the geometry below, $\frac{1.4}{p_c - 1.4} = \tan 3.81^o$, or $p_c = 22.42$.



Adding the compensator pole and using $-1.4 \pm j1.4$ as the test point, K = 88.68.

b. Uncompensated: Search the 135^o line (4.32% overshoot) and find the uncompensated dominant pole at $-0.419 + j0.419$ with K = 1.11. Thus $K_v = \frac{1.11}{3} = 0.37$. Hence, $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.419} = 9.55$

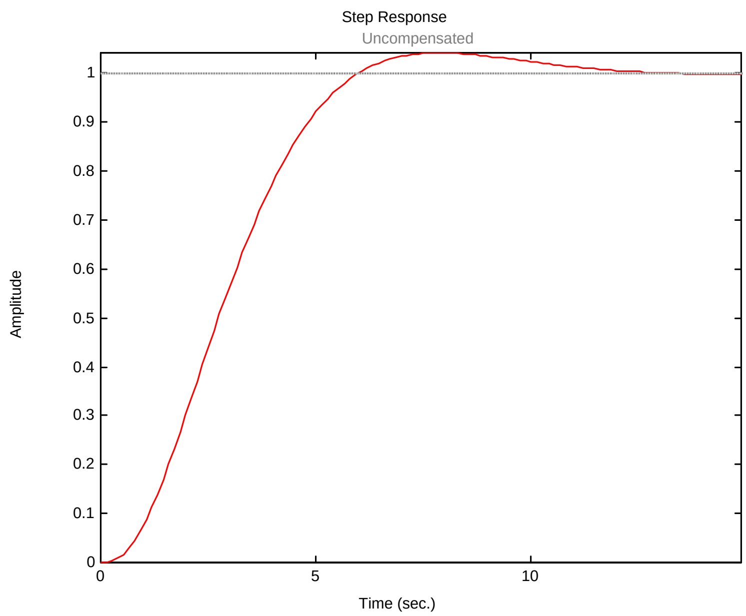
seconds and %OS = 4.32%. Compensated: $K_v = \frac{88.68}{22.42 \text{ x } 3} = 1.32$ (Note: steady-state error improvement is greater than 2). $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.4} = 2.86$ seconds and %OS = 4.32%.

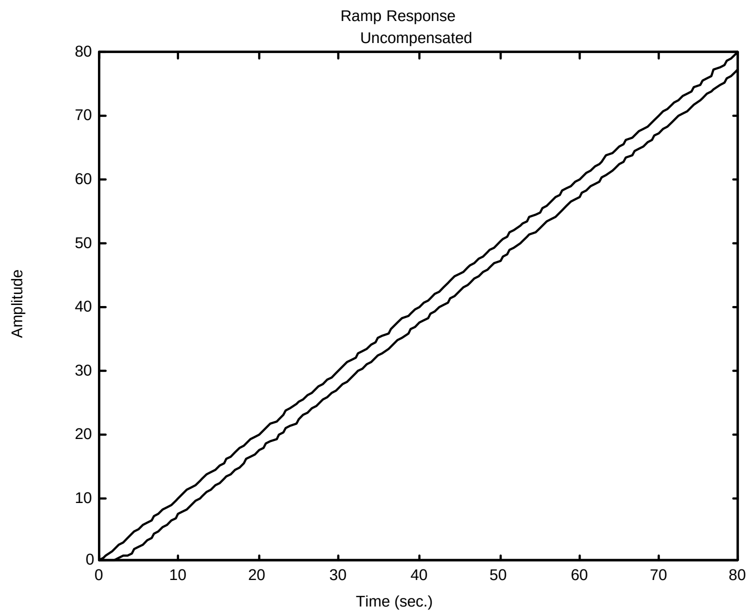
c. Uncompensated: K = 1.11; Compensated: K = 88.68.

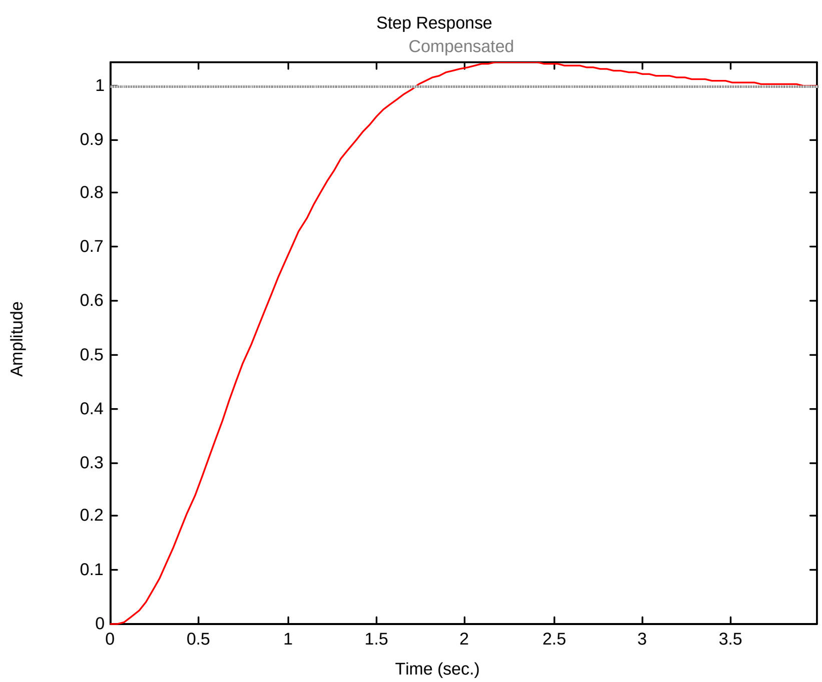
d. Uncompensated: Searching the real axis segments for K = 1.11 yields a higher-order pole at -3.16 which is more than five times the real part of the uncompensated dominant poles. Thus the second-order approximation for the uncompensated system is valid.

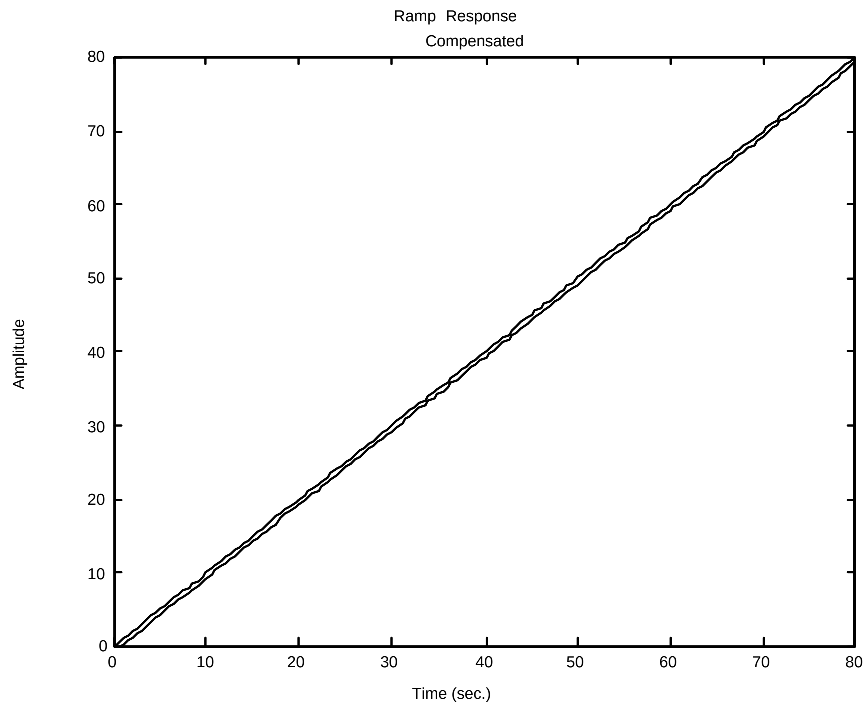
Compensated: Searching the real axis segments for K = 88.68 yields a higher-order pole at -22.62 which is more than five times the real part of the compensated dominant poles' real part. Thus the second order approximation is valid.

e.




Ramp Response
Uncompensated
Amplitude
Time (sec.)
0
10
20
30
40
50
60
70
80


Step Response
Compensated
Amplitude
Time (sec.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3
3.5

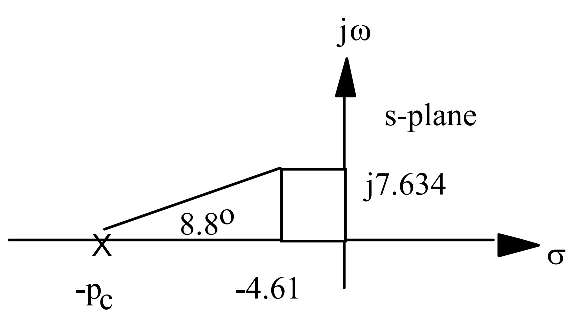


22.

a. Searching the 30% overshoot line ($\zeta = 0.358$; 110.97^o) for 180^o yields $-1.464 + j3.818$ with a gain, $K = 218.6$.

b. $T_p = \frac{\pi}{\omega_d} = \frac{\pi}{3.818} = 0.823$ second. $K_v = \frac{218.6}{(5)(11)} = 3.975$.

c. Lead design: From the requirements, the percent overshoot is 15% and the peak time is 0.4115 second. Thus, $\zeta = \frac{-\ln(\%/100)}{\sqrt{\pi^2+\ln^2(\%/100)}} = 0.517$; $\omega_d = \frac{\pi}{T_p} = 7.634 = \omega_n\sqrt{1-\zeta^2}$. Hence, $\omega_n = 8.919$. The design point is located at $-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} = -4.61 + j7.634$. Assume a lead compensator zero at -5. Summing the angles of the uncompensated system's poles as well as the compensator zero at -5 yields -171.2^o. Therefore, the compensator pole must contribute $(171.2^o - 180^o) = -8.8^o$. Using the geometry below,



$\frac{7.634}{p_c - 4.61}$ = tan (8.8°) . Hence, p_c = 53.92. The compensated open-loop transfer function is $\frac{K}{s(s+11)(s+53.92)}$. Evaluating the gain for this function at the point, -4.61 + j7.634 yields

K = 4430.

Lag design: The uncompensated $K_v = \frac{218.6}{(5)(11)} = 3.975$. The required K_v is 30*3.975 = 119.25.

The lead compensated $K_v = \frac{4430}{(11)(53.92)}$ = 7.469. Thus, we need an improvement over the lead

compensated system of 119.25/7.469 = 15.97. Thus, use a lag compensator

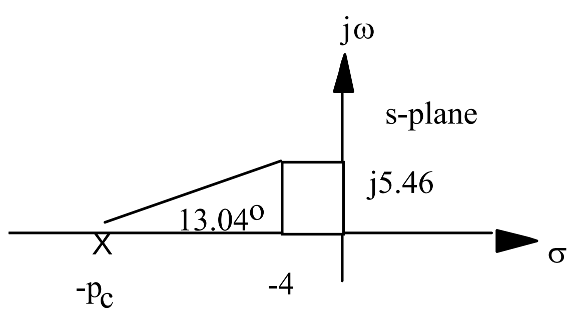
$G_{lag}(s) = \frac{s+0.01597}{s+0.001}$. The final open-loop function is $\frac{4430(s+0.01597)}{s(s+11)(s+53.92)(s+0.001)}$.

23.

a. Searching along the 10% overshoot line (ζ = 0.591) the operating point is found to be –1.85 + j2.53 with K = 21.27. A third pole is at –10.29. Thus, the estimated performance before compensation is: 10% overshoot, $T_s = \frac{4}{1.85} = 2.16$ seconds, and $K_p = \frac{21.27}{(8)(10)} = 0.266$.

b. Lead design: Place compensator zero at –3. The desired operating point is found from the desired specifications. $\zeta\omega_n = \frac{4}{T_s} = \frac{4}{1} = 4$ and $\omega_n = \frac{4}{\zeta} = \frac{4}{0.591} = 6.768$. Thus,

$\text{Im} = \omega_n\sqrt{1-\zeta^2} = 6.768\sqrt{1-0.591^2} = 5.46$. Hence the design point is –4 +j5.46. The angular contribution of the system poles and compensator zero at the design point is –166.96^0. Thus, the compensator pole must contribute –180^0 + 166.96^0 = -13.04^0. Using the geometry below,

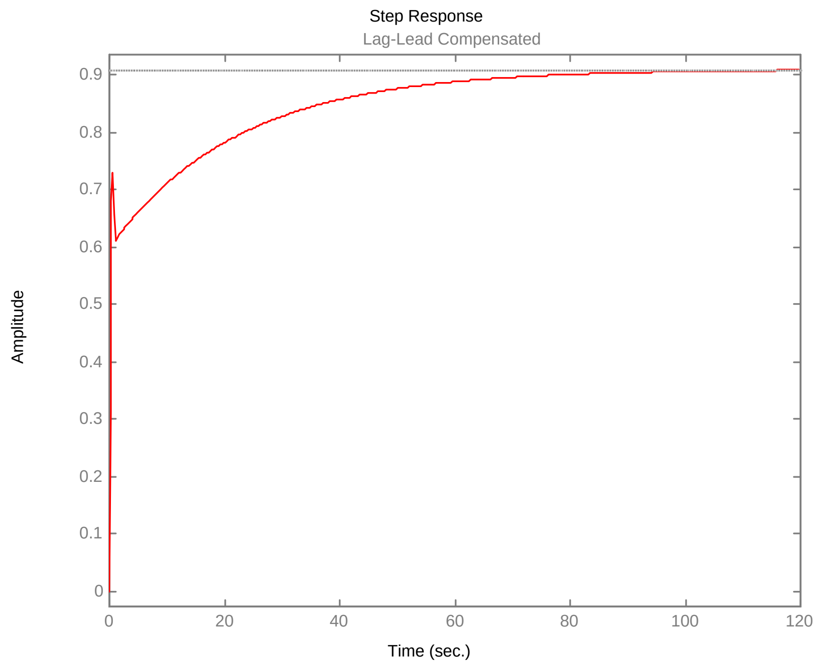


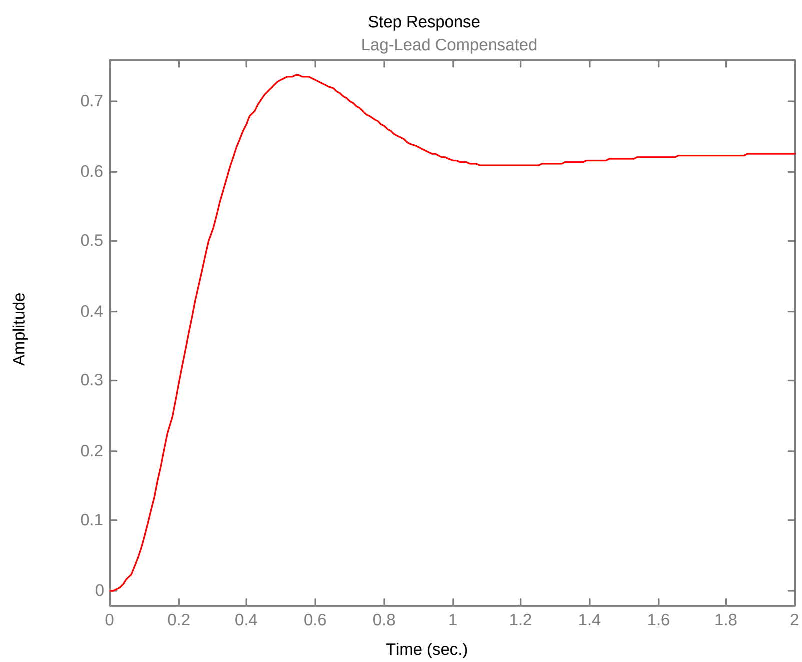
$\frac{5.46}{p_c - 4}$ = tan (13.04°) . Hence, p_c = 27.57. The compensated open-loop transfer function is $\frac{K(s+3)}{(s^2+4s+8)(s+10)(s+27.57)}$. Evaluating the gain for this function at the point

-4 + j5.46 yields K = 1092 with higher-order poles at –4.055 and –29.52.

Lag design: For the lead-compensated system, K_p = 1.485. Thus, we need an improvement of $\frac{10}{1.485} = 6.734$ times. Hence, $G_{lag}(s) = \frac{(s+0.06734)}{(s+0.01)}$. Finally, the equivalent forward-path transfer function is $G_e(s) = \frac{1092(s+3)(s+0.06734)}{(s^2+4s+8)(s+10)(s+27.57)(s+0.01)}$.

c.





24.

a. Uncompensated: Search the 135^o line (4.32% overshoot) for 180^o and find the dominant pole at –3 + j3 with K = 10.

Lag Compensated: Search the 135^o line (4.32% overshoot) for 180^o and find the dominant pole at -2.88 + j2.88 with K = 9.95.

b. Uncompensated: $K_p = \frac{10}{2 \text{ x } 4} = 1.25$

Lag compensated: $K_p = \frac{9.95 \text{ x } 0.5}{2 \text{ x } 4 \text{ x } 0.1} = 6.22$

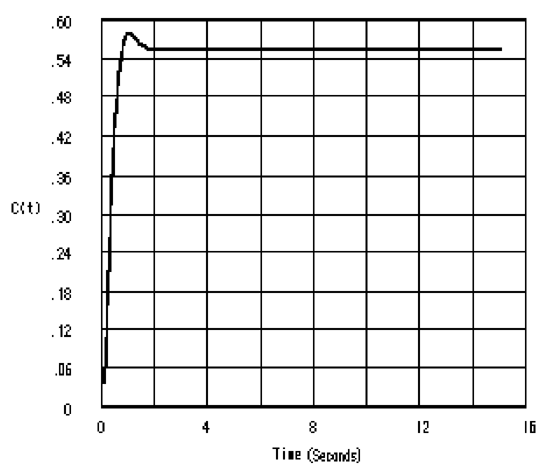
c. %OS = 4.32% both cases;

Uncompensated $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{3}$ 1.33 seconds; Compensated $T_s = \frac{4}{2.88} = 1.39$ seconds

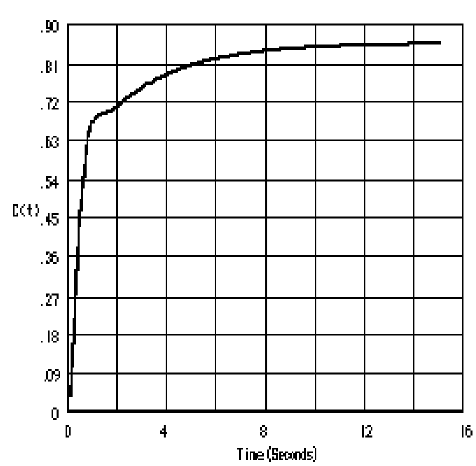
d. Uncompensated: Exact second-order system; approximation OK

Compensated: Search real axis segments of the root locus and find a higher-order pole at -0.3. System should be simulated to see if there is effective pole/zero cancellation with zero at - 0.5.

e.



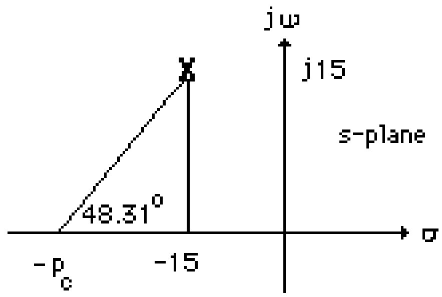
Uncompensated



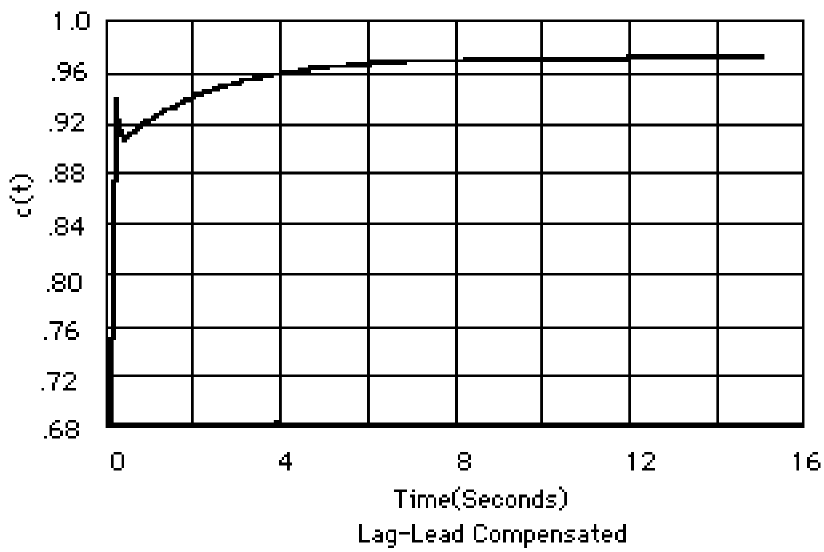
Lag Compensated

The compensated system's response takes a while to approach the final value.

f. We will design a lead compensator to speed up the system by a factor of 5. The lead-compensated dominant poles will thus be placed at $-15 \pm j15$. Assume a compensator zero at - 4 that cancels the open-loop pole at - 4. Using the system's poles and the compensator's zero, the sum of angles to the design point, $-15 \pm j15$ is 131.69^o. Thus, the angular contribution of the compensator pole must be $131.69^o - 180^o = -48.31^o$. Using the geometry below, $p_c = 28.36$.



Using the compensated open-loop transfer function, $G_e(s) = \frac{K(s+0.5)(s+4)}{(s+2)(s+4)(s+0.1)(s+28.36)}$ and using the design point $-15 \pm j15$, K = 404.1.The time response of the lag-lead compensated system is shown below.

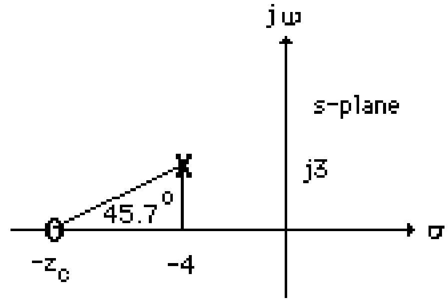


25.

Since $T_p = 1.047$, the imaginary part of the compensated closed-loop poles will be $\frac{\pi}{1.047} = 3$.

Since $\frac{Im}{Re} = \tan(\cos^{-1}\zeta)$, the magnitude of the real part will be $\frac{Im}{\tan(\cos^{-1}\zeta)} = 4$. Hence, the design point is $-4 + j3$. Assume an PI controller, $G_c(s) = \frac{s+0.1}{s}$, to reduce the steady-state error to zero.

Using the system's poles and the pole and zero of the ideal integral compensator, the summation of angles to the design point is -225.7°. Hence, the ideal derivative compensator must contribute 225.7°-180° = 45.7° . Using the geometry below, z_c = 6.93.



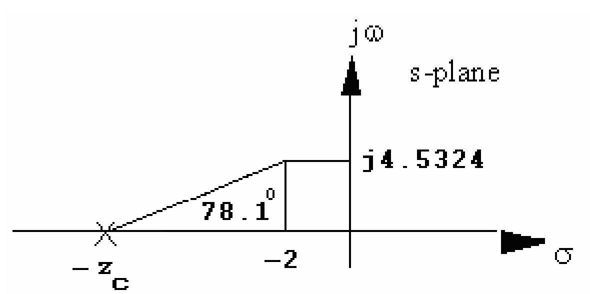
The PID controller is thus $\frac{(s+6.93)(s+0.1)}{s}$. Using all poles and zeros of the system and PID controller, the gain at the design point is K = 3.08. Searching the real axis segment, a higher-order pole is found at - 0.085. A simulation of the system shows the requirements are met.

26.

a. The desired operating point is found from the desired specifications. $\zeta\omega_n = \frac{4}{T_s} = \frac{4}{2} = 2$ and

$\omega_n = \frac{2}{\zeta} = \frac{2}{0.4037} = 4.954$. Thus, $\text{Im} = \omega_n\sqrt{1-\zeta^2} = 4.954\sqrt{1-0.4037^2} = 4.5324$. Hence the design point is –2 +j4.5324. Now, add a pole at the origin to increase system type and drive error to zero for step inputs.

Now design a PD controller. The angular contribution to the design point of the system poles and pole at the origin is 101.9°. Thus, the compensator zero must contribute 180° – 101.9° =78.1°. Using the geometry below,



$\frac{4.5324}{z_c - 2} = \tan(78.1^0)$. Hence, z_c = 2.955. The compensated open-loop transfer function with PD compensation is $\frac{K(s+2.955)}{s(s+4)(s+6)(s+10)}$. Adding the compensator zero to the system and evaluating the gain for this at the point $-2 + j4.5324$ yields K = 294.51 with a higher-order pole at -2.66 and -13.34.

PI design: Use $G_{PI}(s) = \frac{(s+0.01)}{s}$. Hence, the equivalent open-loop transfer function is

$$G_e(s) = \frac{K(s+2.955)(s+0.01)}{s^2(s+4)(s+6)(s+10)}$$ with K = 294.75.

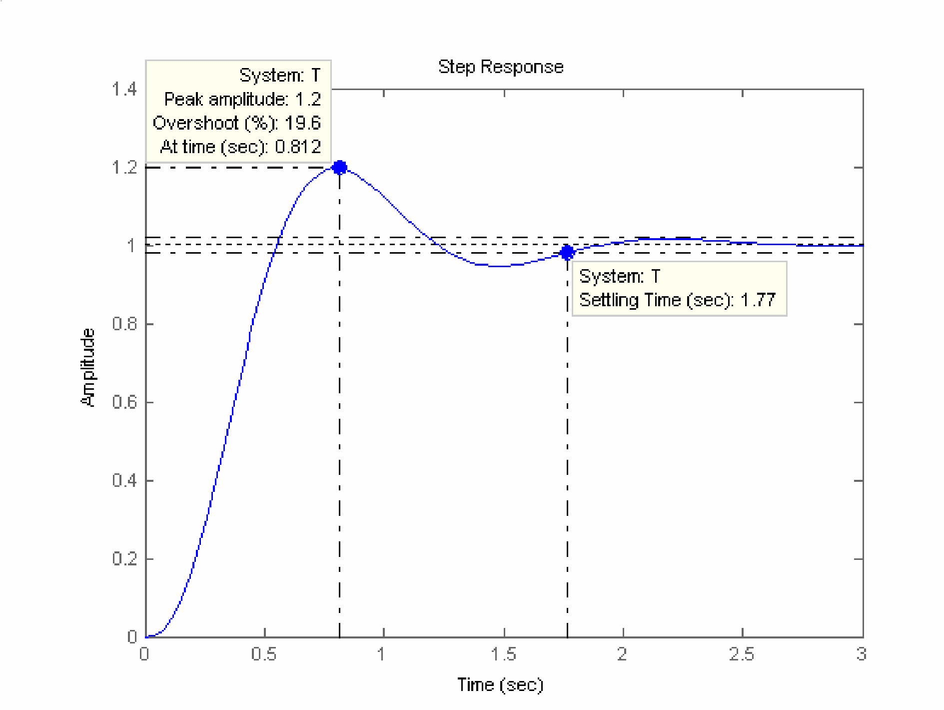
b.

Program (Step Response):

```
numg=[-2.995 -0.01];
deng=[0  0 -4 -6 -10];
K=294.75;
G=zpk(numg,deng,K)
T=feedback(G,1);
step(T)
```

Computer response:

```
Zero/pole/gain:
294.75 (s+2.995) (s+0.01)
-------------------------
 s^2 (s+4) (s+6) (s+10)
```

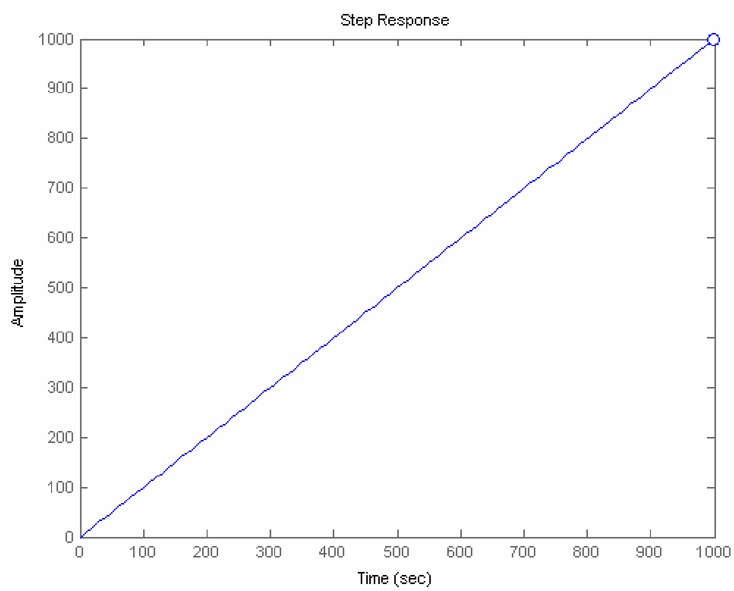


Program (Ramp Response):

```
numg=[-2.995 -0.01];
deng=[0 0 -4 -6 -10];
K=294.75;
G=zpk(numg,deng,K)
T=feedback(G,1);
Ta=tf([1],[1 0]);
step(T*Ta)
```

Computer response:

```
Zero/pole/gain:
294.75 (s+2.995) (s+0.01)
-------------------------
 s^2 (s+4) (s+6) (s+10)
```



27.

Program:

```
numg=[]
deng=[-4 -6 -10]
'G(s)'
G=zpk(numg,deng,1)
pos=input('Type desired percent overshoot ');
z=-log(pos/100)/sqrt(pi^2+[log(pos/100)]^2);
Ts=input('Type desired settling time ');
zci=input(...
'Type desired position of integral controller zero (absolute value) ');
wn=4/(Ts*z);
desired_pole=(-z*wn)+(wn*sqrt(1-z^2)*i)
angle_at_desired_pole=(180/pi)*angle(evalfr(G,desired_pole))
PD_angle=180-angle_at_desired_pole;
zcpd=((imag(desired_pole)/tan(PD_angle*pi/180))-real(desired_pole));
'PD Compensator'
numcpd=[1 zcpd];
dencpd=[0 1];
'Gcpd(s)'
Gcpd=tf(numcpd,dencpd)
Gcpi=zpk([-zci],[0],1)
Ge=G*Gcpd*Gcpi
rlocus(Ge)
sgrid(z,0)
title(['PID Compensated Root Locus with ' ,...
      num2str(pos), '% Damping Ratio Line'])
[K,p]=rlocfind(Ge);
'Closed-loop poles = '
p
f=input('Give pole number that is operating point   ');

'Summary of estimated specifications for selected point'
```

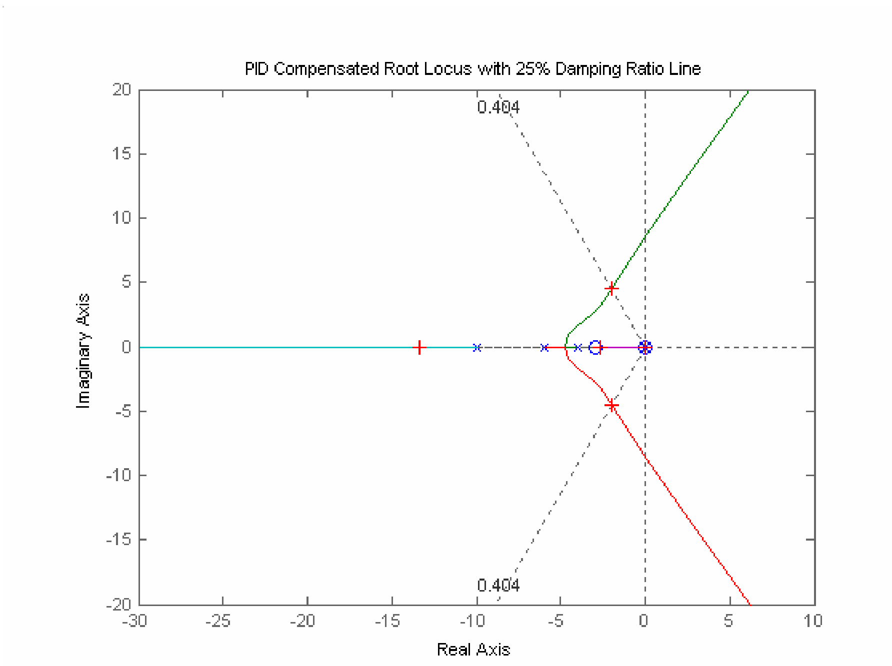
```
'on PID compensated root locus'

operatingpoint=p(f)
gain=K
estimated_settling_time=4/abs(real(p(f)))

estimated_peak_time=pi/abs(imag(p(f)))

estimated_percent_overshoot=pos

estimated_damping_ratio=z

estimated_natural_frequency=sqrt(real(p(f))^2+imag(p(f))^2)
T=feedback(K*Ge,1);
step(T)
title(['Step Response for PID Compensated System with  ' ,...
      num2str(pos),'% Damping Ratio Line'])
pause
one_over_s=tf(1,[1 0]);
Tr=T*one_over_s;
t=0:0.01:10;
step(one_over_s,Tr)
title('Ramp Response for PID Compensated System')
```

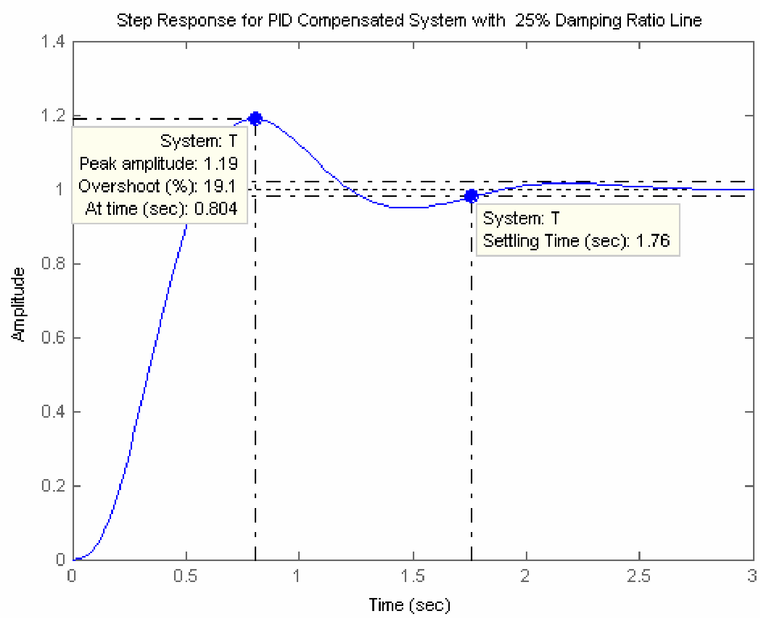
Computer response:

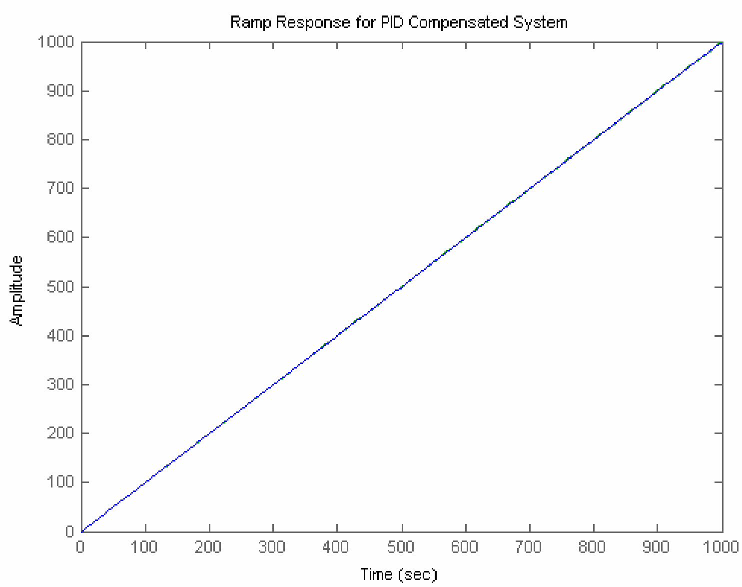
```
numg =

     []


deng =

     0    -4    -6   -10


ans =

G(s)


Zero/pole/gain:
         1
--------------------
s (s+4) (s+6) (s+10)

Type desired percent overshoot 25
Type desired settling time 2
Type desired position of integral controller zero (absolute value) 0.01

desired_pole =

  -2.0000 + 4.5324i


angle_at_desired_pole =

  101.8963


ans =

PD Compensator


ans =
```

```
Gcpd(s)


Transfer function:
s + 2.955


Zero/pole/gain:
(s+0.01)
--------
   s


Zero/pole/gain:
  (s+2.955) (s+0.01)
----------------------
s^2 (s+4) (s+6) (s+10)

Select a point in the graphics window

selected_point =

  -1.9931 + 4.5383i


ans =

Closed-loop poles =


p =

 -13.3485
  -1.9920 + 4.5377i
  -1.9920 - 4.5377i
  -2.6575
  -0.0100

Give pole number that is operating point   2

ans =

Summary of estimated specifications for selected point


ans =

on PID compensated root locus


operatingpoint =

  -1.9920 + 4.5377i


gain =

  295.6542


estimated_settling_time =

    2.0081


estimated_peak_time =
```

```
    0.6923

estimated_percent_overshoot =

    25

estimated_damping_ratio =

    0.4037

estimated_natural_frequency =

    4.9557
```







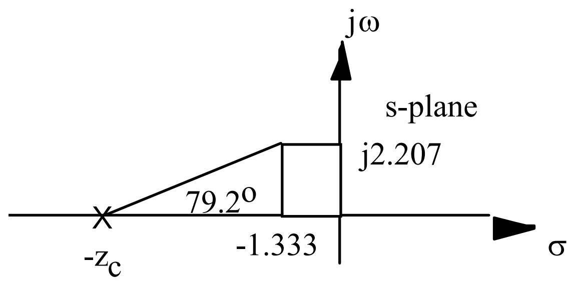
28.

Open-loop poles are at -2, - 0.134, and -1.87. An open-loop zero is at -3. Searching the 121.13° line (ζ = 0.517), find the closed-loop dominant poles at -0.747 + j1.237 with K = 1.58. Searching the real axis segments locates a higher-order pole at -2.51. Since the open-loop zero is a zero of H(s), it is not a closed-loop zero. Thus, there are no closed-loop zeros.

29.

a. The damping ratio for 15% overshoot is 0.517. The desired operating point is found from the desired specifications. $\zeta\omega_n = \frac{4}{T_s} = \frac{4}{3} = 1.333$ and $\omega_n = \frac{1.333}{\zeta} = \frac{1.333}{0.517} = 2.578$. Thus,

$\text{Im} = \omega_n\sqrt{1-\zeta^2} = 2.578\sqrt{1-0.517^2} = 2.207$. Hence the design point is –1.333 + j2.207. The angular contribution of the system poles and compensator zero at the design point is 100.8^0. Thus, the compensator zero must contribute $180^0 – 100.8^0 = 79.2^0$. Using the geometry below,

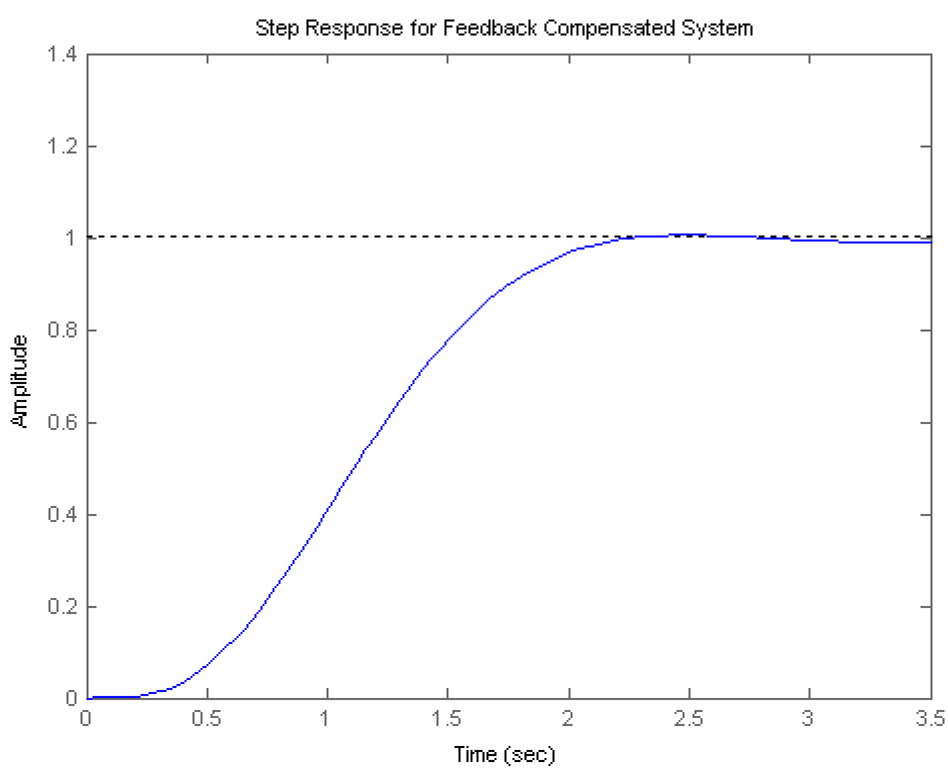


$\frac{2.207}{z_c - 1.333} = \tan(79.2^o)$. Hence, $z_c = 1.754$. The compensated open-loop transfer function with PD compensation is $\frac{K(s+1.754)}{s(s+2)(s+4)(s+6)}$. Evaluating the gain for this function at the point –1.333 + j2.207 yields K = 47.28 with higher-order poles at –1.617 and –7.718. Following Figure 9.49(c) in the text, $\frac{1}{K_f} = 1.754$. Therefore, $K_f = 0.5701$. Also, using the notation of Figure 9.49(c), $K_1K_f = 47.28$, from which $K_1 = 82.93$.

b.
Program:
```
K1=82.93;
numg=K1;
deng=poly([0 -2 -4 -6]);
'G(s)'
G=tf(numg,deng);
Gzpk=zpk(G)
Kf=0.5701
numh=Kf*[1 1.754];
denh=1
'H(s)'
H=tf(numh,denh);
Hzpk=zpk(H)
'T(s)'
T=feedback(G,H);
T=minreal(T)
step(T)
title('Step Response for Feedback Compensated System')
```

Computer response:
```
ans =

G(s)

Zero/pole/gain:
       82.93
-------------------
s (s+6) (s+4) (s+2)
```

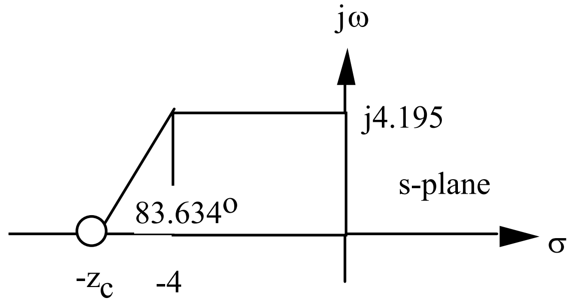
```
Kf =

    0.5701

denh =

     1

ans =

H(s)

Zero/pole/gain:
0.5701 (s+1.754)

ans =

T(s)

Transfer function:
                 82.93
---------------------------------------
s^4 + 12 s^3 + 44 s^2 + 95.28 s + 82.93
```



30.

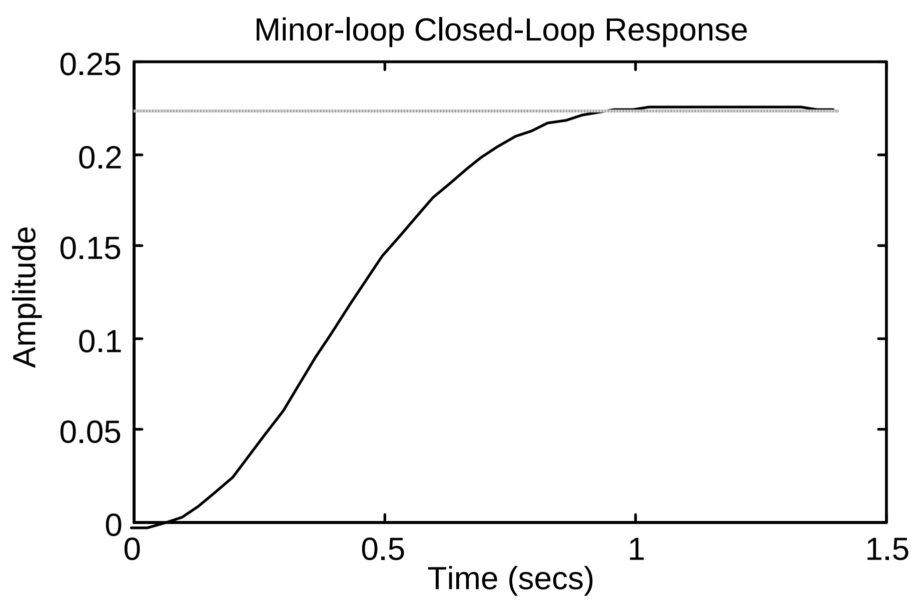
a. $\sigma_d = \zeta\omega_n = 4/T_s = 4/1 = 4$. 5% overshoot -> $\zeta = 0.69$. Since $\zeta\omega_n = 4$, $\omega_n = 5.8$.

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 4.195$. Thus, the design point is $-1 + j4.195$. The sum of angles from the minor-loop's open-loop poles to the design point is -263.634^o. Thus, the minor-loop's open-loop zero must contribute 83.634^{o} to yield 180^{o} at the design point. Hence, $\frac{4.195}{z_c - 4} = \tan 83.634^{o}$, or $z_c = a = 4.468$ from the geometry below.



Adding the zero and calculating the gain at the design point yields $K_1 = 38.33$. Therefore, the minor-loop open-loop transfer function is $K_1G(s)H(s) = \frac{38.33(s+4.468)}{s(s+4)(s+9)}$. The equivalent minor-loop closed-loop transfer function is $G_{ml}(s) = \frac{K_1G(s)}{1+K_1G(s)H(s)} = \frac{38.33}{s^3+13s^2+74.33s+171.258}$. A simulation of the step response of the minor loop is shown below.

Computer response:



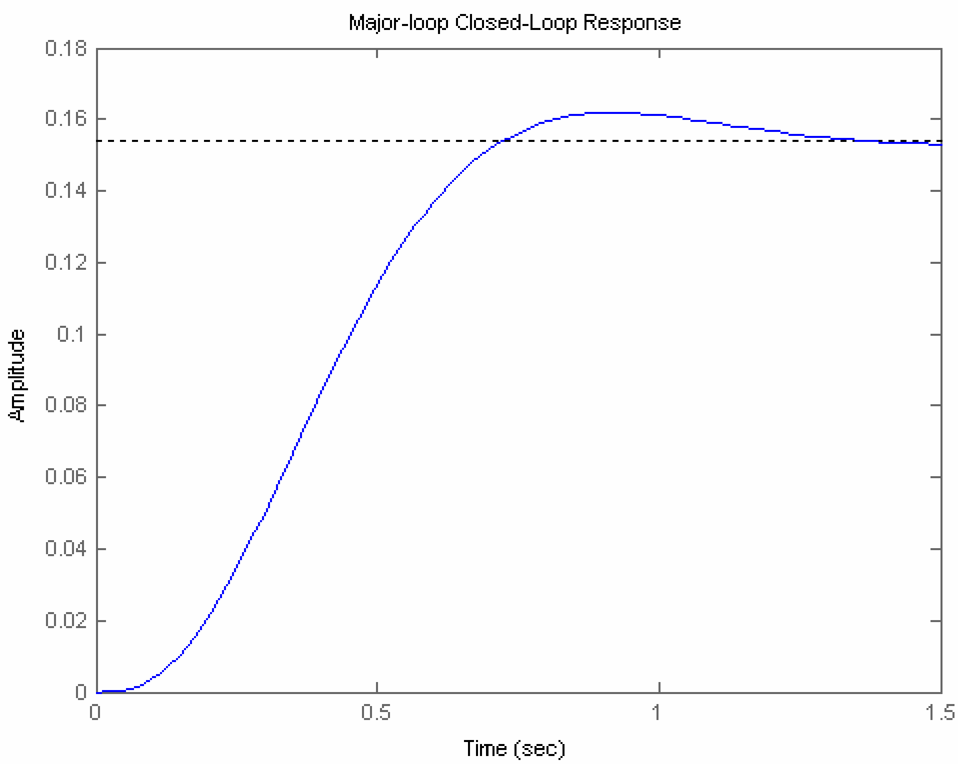
b. The major-loop open-loop transfer function is $G_e(s) = \frac{38.33K}{s^3 + 13s^2 + 74.33s + 171.258}$.

Drawing the root locus using $G_e(s)$ and searching along the 10% overshoot line ($\zeta = 0.591$) for 180^{o} yields the point $-3.349 + j4.572$ with a gain $38.33K = 31.131$, or $K = 0.812$.

c.

Program:
```
numg=31.131;
deng=[1 13 74.33 171.258];
'G(s)'
G=tf(numg,deng)
T=feedback(G,1);
step(T)
title('Major-loop Closed-Loop Response')
```

Computer response:
```
G(s)


Transfer function:
            31.13
------------------------------
s^3 + 13 s^2 + 74.33 s + 171.3
```



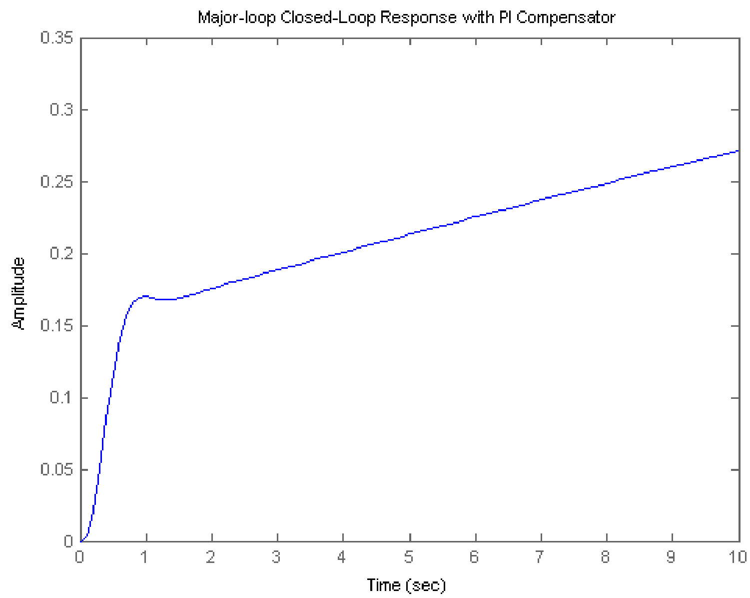
d. Adding the PI compensator, $G_e(s) = \dfrac{31.131(s+0.1)}{s(s^3+13s^2+74.33s+171.258)}$.

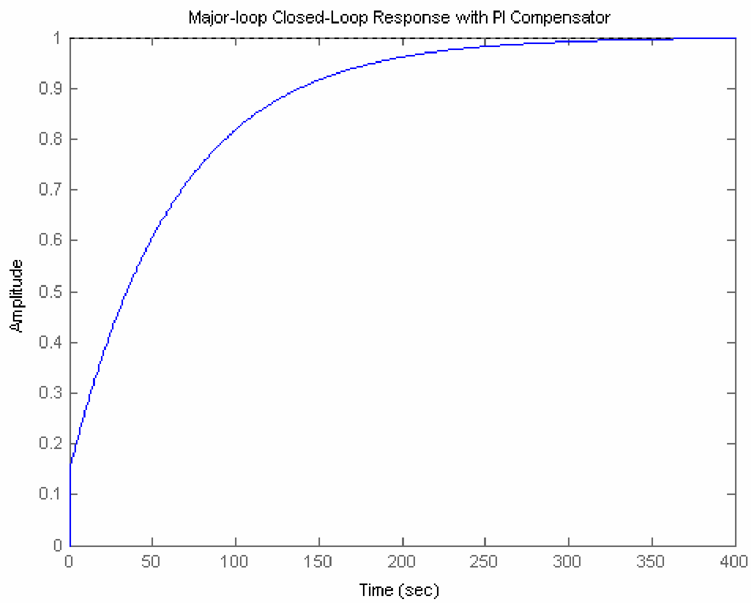
Program:
```
numge=31.131*[1 0.1];
denge=[1 13 74.33 171.258 0];
'Ge(s)'
Ge=tf(numge,denge)
T=feedback(Ge,1);
t=0:0.1:10;
step(T,t)
title('Major-loop Closed-Loop Response with PI Compensator')
pause
step(T)
```

```
title('Major-loop Closed-Loop Response with PI Compensator')
```

Computer response:

```
ans =

Ge(s)

Transfer function:
         31.13 s + 3.113
----------------------------------
s^4 + 13 s^3 + 74.33 s^2 + 171.3 s
```





31.

a. PI controller: Using Table 9.10, $\frac{R_2}{R_1}\frac{s+\frac{1}{R_2C}}{s} = \frac{s+0.01}{s}$, $R_2C = 100$. Let C = 25 μF. Therefore, $R_2 = 4$ MΩ. For unity gain, $R_1 = 4$ MΩ. Compensate elsewhere in the loop for the compensator negative sign.

b. **PD controller:** Using Table 9.10, $R_2C(s+\frac{1}{R_1C}) = s+2$. Hence, $R_1C = 0.5$. Let C = 1 μF. Therefore, $R_1 = 500$ KΩ. For unity gain, $R_2C = 1$, or $R_2 = 1$ MΩ. Compensate elsewhere in the loop for the compensator negative sign.

32.

a. Lag compensator: See Table 9.11. $\frac{s+\frac{1}{R_2C}}{s+\frac{1}{(R_1+R_2)C}} = \frac{s+0.1}{s+0.01}$. Thus, $R_2C = 10$, and $(R_1 + R_2)C = 100$. Letting C = 10 μ F, we find $R_2 = 1$ MΩ. Also $R_1C = 100 - R_2C = 90$, which yields $R_1 = 9$ MΩ. The loop gain also must be multiplied by $\frac{R_1+R_2}{R_2}$.

b. Lead compensator: See Table 9.11. $\frac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C}+\frac{1}{R_2C}} = \frac{s+2}{s+5}$. Thus, $R_1C = 0.5$, and

$\frac{1}{R_1C} + \frac{1}{R_2C} = 5$. Letting C = 1 μF, $R_2 = 333$ KΩ, and $R_1 = 500$ KΩ.

c. Lag-lead compensation: See Table 9.11.

$$\frac{(s+\frac{1}{R_1C_1})(s+\frac{1}{R_2C_2})}{s^2+(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1})s+\frac{1}{R_1R_2C_1C_2}} = \frac{(s+0.1)(s+1)}{s^2+10.01s+0.1}$$

. Thus, $R_1C_1 = 1$, and $R_2C_2 = 10$. Also, $\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1} = 1 + 0.1 + \frac{1}{R_2C_1} = 10.01$, or $R_2C_1 = 0.112$. Letting C_1 = 10 μF, we find $R_1 = 10$ MΩ , $R_2 = 1.12$ MΩ, and $C_2 = 8.9$ μF.

33.

a. Lag compensator: See Table 9.10 and Figure 9.58. $\frac{s+0.1}{s+0.01} = \frac{C_1}{C_2}\frac{(s+\frac{1}{R_1C_1})}{(s+\frac{1}{R_2C_2})}$. Therefore,

$R_1C_1 = 10$; $R_2C_2 = 100$. Letting $C_1 = C_2 = 20$ μF, we find $R_1 = 500$ KΩ and $R_2 = 5$ MΩ. Compensate elsewhere in the loop for the compensator negative sign.

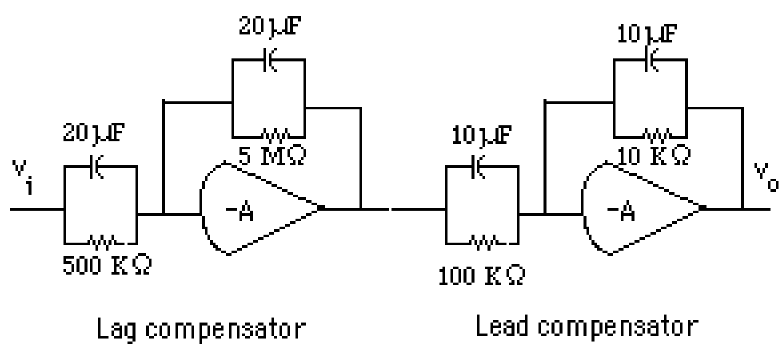
b. Lead compensator: See Table 9.10 and Figure 9.58. $\frac{s+2}{s+5} = \frac{C_1}{C_2}\frac{(s+\frac{1}{R_1C_1})}{(s+\frac{1}{R_2C_2})}$. Therefore,

$R_1C_1 = 0.5$ and $R_2C_2 = 0.2$. Letting $C_1 = C_2 = 20\ \mu F$, we find $R_1 = 25\ K\Omega$ and $R_2 = 10\ M\Omega$. Compensate elsewhere in the loop for the compensator negative sign.

c. Lag-lead compensator: See Table 9.10 and Figure 9.58. For lag portion, use (a). For lead:

$\frac{s+1}{s+10} = \frac{C_1}{C_2}\frac{(s+\frac{1}{R_1C_1})}{(s+\frac{1}{R_2C_2})}$. Therefore, $R_1C_1 = 1$ and $R_2C_2 = 0.1$. Letting $C_1 = C_2 = 10\ \mu F$, we find

$R_1 = 100\ K\Omega$ and $R_2 = 10\ K\Omega$. The following circuit can be used to implement the design.



SOLUTIONS TO DESIGN PROBLEMS

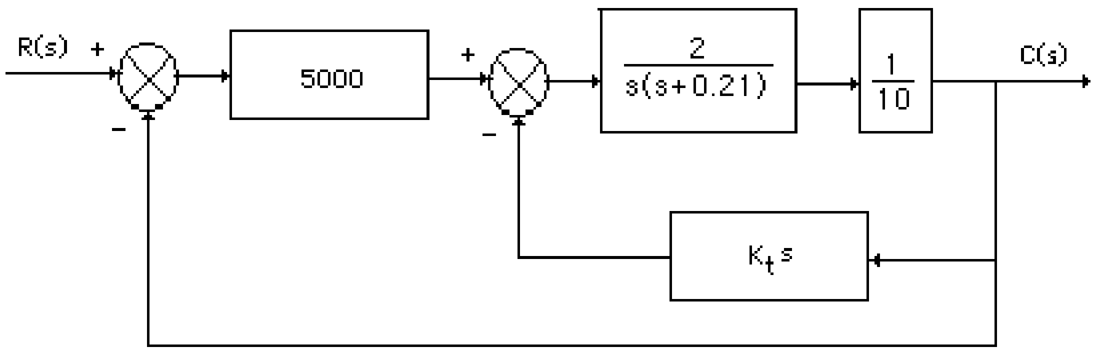
34.

a. $$\frac{\theta_m(s)}{E_a(s)} = \frac{\frac{K_t}{R_aJ}}{s(s+\frac{1}{J}(D+\frac{K_tK_b}{R_a}))}$$

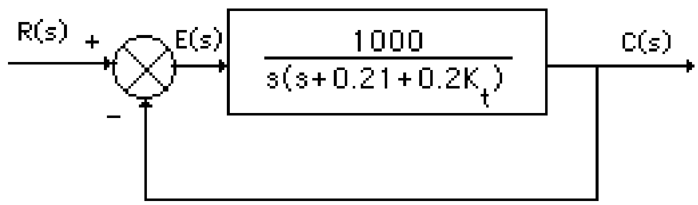
$K_b = \frac{E_a}{\omega} = \frac{5}{\frac{60000}{2\pi} \times \frac{1}{60} \times 2\pi} = 0.005$; $J_{eq} = 5\ (\frac{4}{10}\ \times \frac{1}{4})^2 = 0.05$; $D_{eq} = 1\ (\frac{1}{10}\ )^2 = 0.01$;

$\frac{K_t}{R_a} = \frac{T_s}{E_a} = \frac{0.5}{5} = 0.1$. Therefore, $\frac{\theta_m(s)}{E_a(s)} = \frac{2}{s(s+0.21)}$.

b. The block diagram of the system is shown below.



Forming an equivalent unity feedback system,



Now, $T(s) = \dfrac{1000}{s^2 + (0.21 + 0.2K_t)s + 1000}$. Thus, $\omega_n = \sqrt{1000}$; $2\zeta\omega_n = 0.21 + 0.2K_t$. Since $\zeta = 0.5$,

$K_t = 157.06$.

c. Uncompensated: $K_t = 0$; $T(s) = \dfrac{1000}{s^2 + 0.21s + 1000}$; $\omega_n = 31.62$ rad/s; $\zeta = 3.32 \times 10^{-3}$;

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 98.96\%$; $T_s = \dfrac{4}{\zeta\omega_n} = 38.09$ seconds;

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 9.93 \times 10^{-2}$ second; $K_v = \dfrac{1000}{0.21} = 4761.9$.

Compensated: $K_t = 157.06$; $T(s) = \dfrac{1000}{s^2 + 31.62s + 1000}$; $\omega_n = 31.62$ rad/s; $\zeta = 0.5$;

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 16.3\%$; $T_s = \dfrac{4}{\zeta\omega_n} = 0.253$ second; $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.115$ second;

$K_v = \dfrac{1000}{31.62} = 31.63$.

35.

a. $T(s) = \dfrac{25}{s^2 + s + 25}$; Therefore, $\omega_n = 5$; $2\zeta\omega_n = 1$; $\zeta = 0.1$;

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 73\%$; $T_s = \dfrac{4}{\zeta\omega_n} = 8$ seconds.

b. From Figure P9.6(b), $T(s) = \dfrac{25K_1}{s^2 + (1 + 25K_f)s + 25K_1}$. Thus,

$\omega_n = \sqrt{25K_1}$; $2\zeta\omega_n = 1 + 25K_f$. For 25% overshoot, $\zeta = 0.404$. For $T_s = 0.2 = \dfrac{4}{\zeta\omega_n}$, $\zeta\omega_n = 20$.

Therefore $1 + 25K_f = 2\zeta\omega_n = 40$, or $K_f = 1.56$. Also, $\omega_n = \dfrac{20}{\zeta} = 49.5$.

Hence $K_1 = \dfrac{\omega_n^2}{25} = \dfrac{49.5^2}{25} = 98.01$.

c. Uncompensated: $G(s) = \dfrac{25}{s(s+1)}$; Therefore, $K_v = 25$, and $e(\infty) = \dfrac{1}{K_v} = 0.04$.

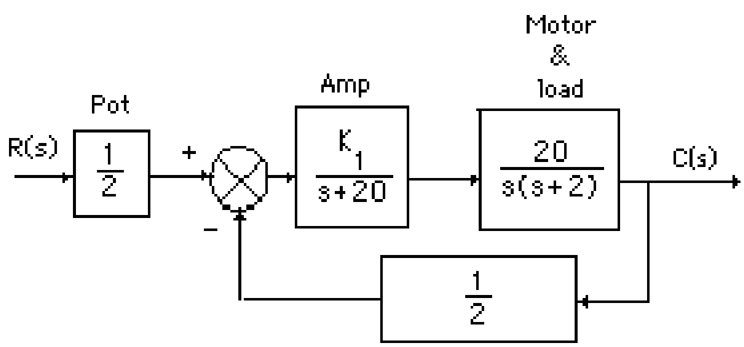
Compensated: $G(s) = \dfrac{25K_1}{s(s + 1 + 25K_f)}$; Therefore, $K_v = \dfrac{25 \text{ x } 98.01}{1+25 \text{ x } 1.56} = 61.26$, and

$e(\infty) = \dfrac{1}{K_v} = 0.0163$.

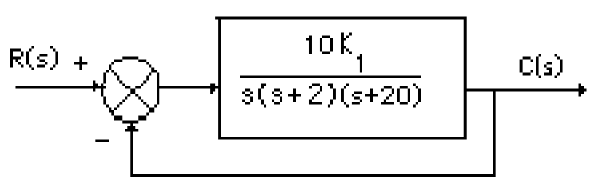
36.

a. The transfer functions of the subsystems are as follows:

Pot: $G_p(s) = \dfrac{5\pi}{10\pi} = \dfrac{1}{2}$; Amplifier: $G_a(s) = \dfrac{K_1}{s+20}$; Motor and load: Since the time to rise to 63% of the final value is 0.5 second, the pole is at -2. Thus, the motor transfer function is of the form, $G_m(s) = \dfrac{K}{s(s+2)}$. But, from the problem statement, $\dfrac{K}{2} = \dfrac{100}{10}$, or K = 20. The block diagram of the system is shown below.



Block diagram



Equivalent system

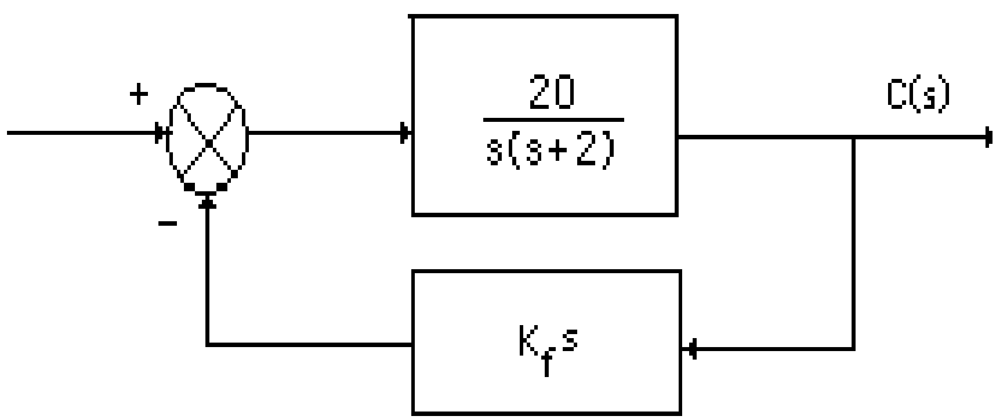
Using the equivalent system, search along the 117.126^o line (20% overshoot) and find the dominant second-order pole at $-0.89 + j1.74$ with $K = 10K_1 = 77.4$. Hence, $K_1 = 7.74$.

b. $K_v = \frac{77.4}{2 \times 20} = 1.935$. Therefore, $e(\infty) = \frac{1}{K_v} = 0.517$.

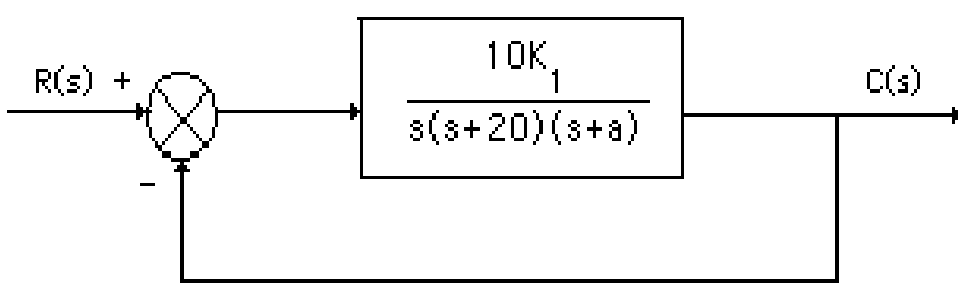
c. %OS = 20%; $\zeta = \frac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = 0.456$; $\omega_n = \sqrt{0.89^2 + 1.74^2} = 1.95$ rad/s;

$T_s = \frac{4}{\zeta\omega_n} = 4.49$ seconds; $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.81$ seconds.

d. The block diagram of the minor loop is shown below.

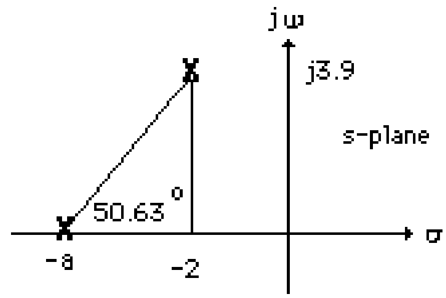


The transfer function of the minor loop is $G_{ML}(s) = \frac{20}{s(s+2+20K_f)}$. Hence, the block diagram of the equivalent system is



where $a = 2 + 20K_f$. The design point is now found. Since %OS = 20%, $\zeta = \frac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} =$ 0.456. Also, since $T_s = \frac{4}{\zeta\omega_n} = 2$ seconds, $\omega_n = 4.386$ rad/s. Hence, the design point is $-2 + j3.9$.

Using just the open-loop poles at the origin and at -20, the summation of angles to the design point is -129.37^{o}. The pole at -a must then be contributing $129.37^{o} - 180^{o} = -50.63^{o}$. Using the geometry below, a = 5.2, or $K_f = 0.16$.



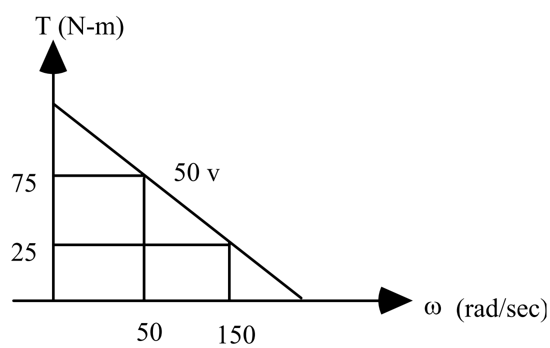
Adding the pole at -5.2 and using the design point, we find $10K_1 = 407.23$, or $K_1 = 40.723$. Summarizing the compensated transient characteristics: $\zeta = 0.456$; $\omega_n = 4.386$; %OS = 20%; $T_s = \frac{4}{\zeta\omega_n} = 2$ seconds; $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.81$ seconds; $K_v = \frac{407.23}{20 \times 5.2} = 3.92$.

37.

Block diagram

Preamplifier/Power amplifier: $\frac{K_1}{(s+40)}$; Pots: $\frac{20\pi \text{ volts}}{5(2\pi) \text{ rad.}} = 2$.

Torque-speed curve:



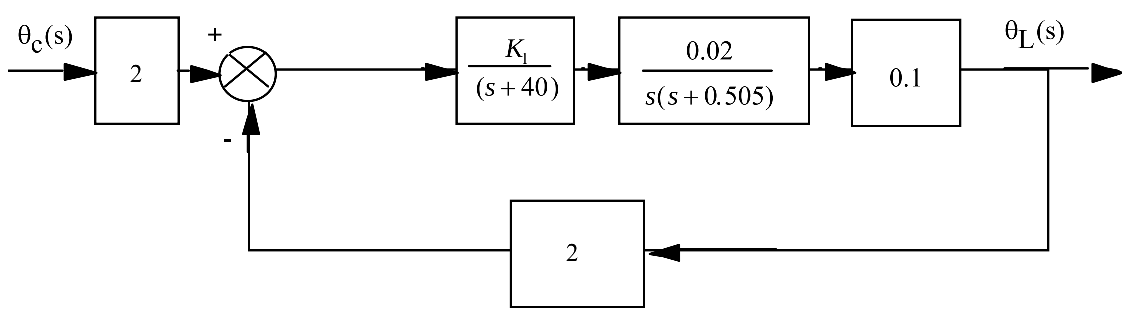
where $1432.35 \frac{\text{rev}}{\text{min}} \times \frac{1}{60}\frac{\text{min}}{\text{sec}} \times 2\pi \frac{\text{rad}}{\text{rev}} = 150$ rad/sec; $477.45 \frac{\text{rev}}{\text{min}} \times \frac{1}{60}\frac{\text{min}}{\text{sec}} \times 2\pi \frac{\text{rad}}{\text{rev}} = 50$ rad/sec.

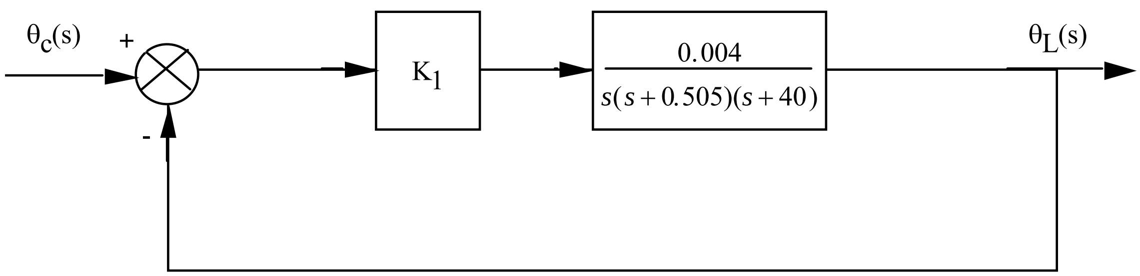
The slope of the line is $-\frac{50}{100} = -0.5$. Thus, its equation is $y = -0.5x + b$. Substituting one of the points, find b = 100. Thus $T_{stall} = 100$, and $\omega_{no\ load} = 200$. $\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} = \frac{100}{50} = 2$; $K_b = \frac{e_a}{\omega_{no\ load}} = \frac{50}{200} = 0.25$.

Motor: $\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_aJ)}{s(s+\frac{1}{J}(D+\frac{K_tK_b}{R_a}))} = \frac{0.02}{s(s+0.505)}$, where J = 100, D = 50.

Gears: 0.1

Drawing block diagram:

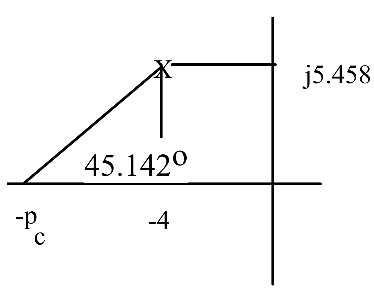




b. Compensator design - Lead

10% overshoot and T_s = 1 sec yield a design point of - 4 + j5.458. Sum of angles of uncompensated system poles to this point is -257.491°. If we place the lead compensator zero over the uncompensated system pole at -0.505, the angle at the design point is -134.858°. Thus, the lead compensator pole must contribute 134.858° - 180° = -45.142°. Using the geometry below

$\frac{5.458}{p_c - 4} = \tan(45.142^\circ)$, or $p_c = 9.431$.



Using the uncompensated poles and the lead compensator, the gain at the design point is $0.004K_1 = 1897.125$.

Compensator design - Lag

With lead compensation, $K_v = \frac{1897.125}{(40)(9.431)} = 5.0295.029$. Since we want $K_v = 1000$, $\frac{z_{lag}}{p_{lag}} = \frac{1000}{5.029} =$ 198.85. Use $p_{lag} = 0.001$. Hence $z_{lag} = 0.1988$. The lag compensated

$$G_e(s) = \frac{1897.125(s+0.1988)}{s(s+40)(s+9.431)(s+0.001)}\ .$$

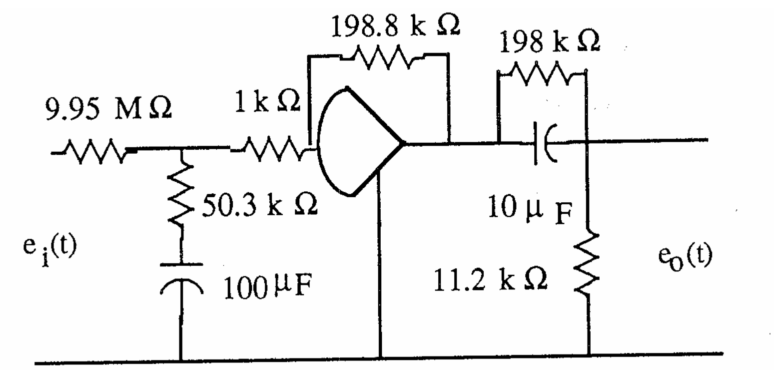
c. Compensator schematic

lag: $\frac{1}{R_2C} = 0.1988$. Let C = 100 μF. Then $R_2 = 50.3$ kΩ. Now, $\frac{1}{(R_1+R_2)C} = 0.001$.

Thus, $R_1 = 9.95$ MΩ. Buffer gain = reciprocal of lag compensator's $\frac{R_2}{R_1 + R_2}$. Hence buffer gain = $\frac{R_1 + R_2}{R_2} = 198.8$.

lead: $\frac{1}{R_1C} = 0.505$. Let C = 10 μF. Then $R_1 = 198$ kΩ. Now, $\frac{1}{R_1C} + \frac{1}{R_2C} = 9.431$.

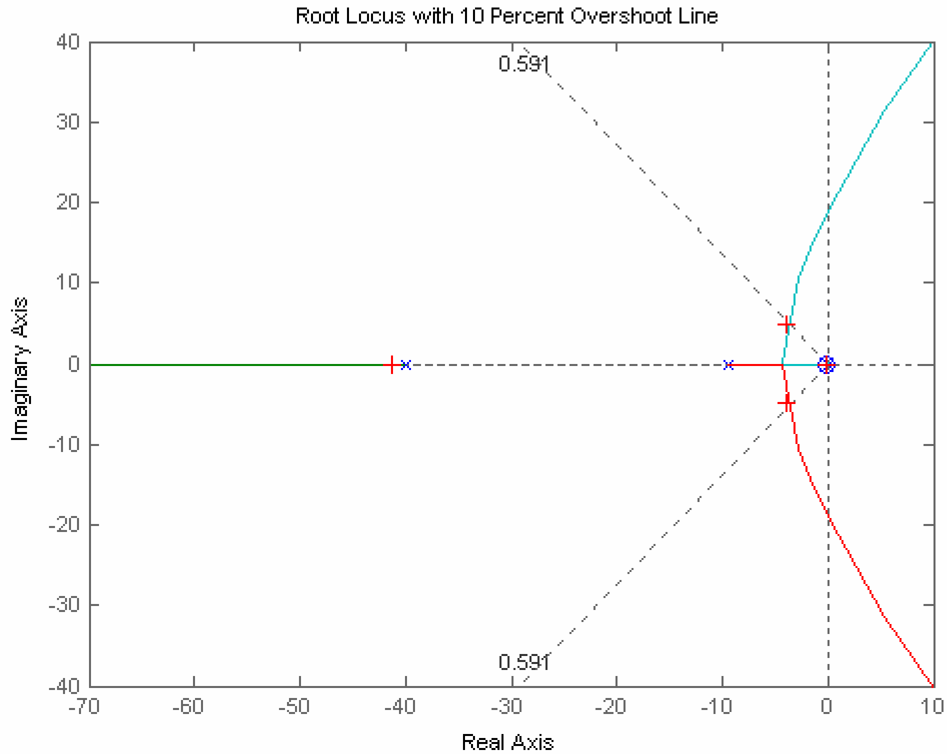
Thus, $R_2 = 11.2$ k□Ω.

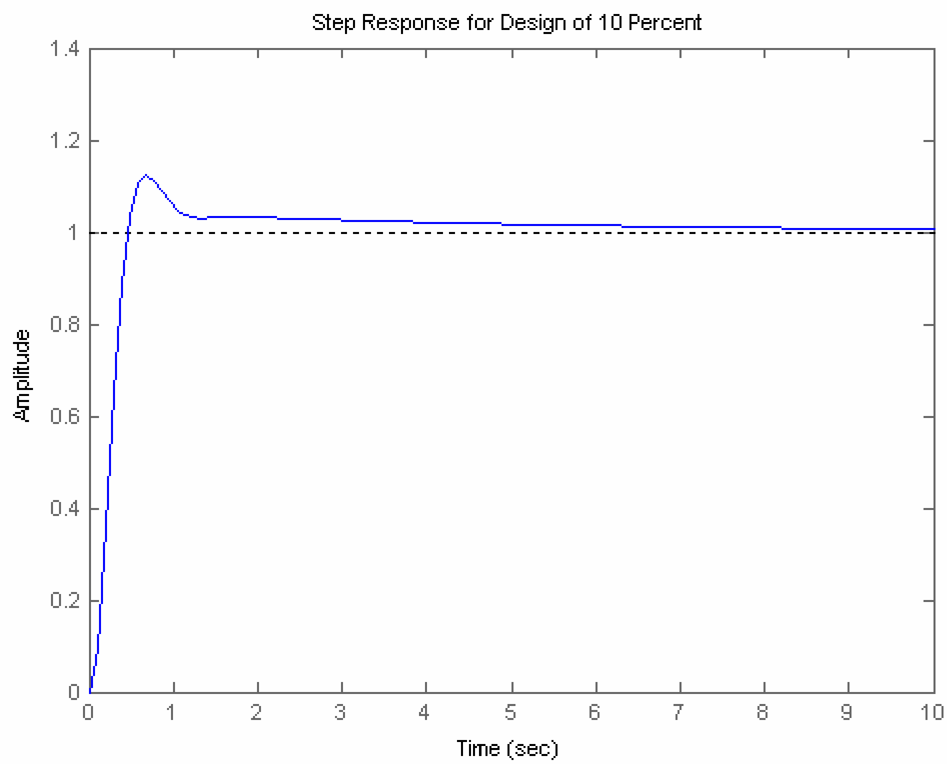


d.
Program:
```
numg= 1897.125*[1 0.1988];
deng=poly([0 -40 -9.431 -.001]);
'G(s)'
G=tf(numg,deng);
Gzpk=zpk(G)
rlocus(G)
pos=10
z=-log(pos/100)/sqrt(pi^2+[log(pos/100)]^2)
sgrid(z,0)
title(['Root Locus with ' , num2str(pos), ' Percent Overshoot Line'])
[K,p]=rlocfind(G)  %Allows input by selecting point on graphic
pause
T=feedback(K*G,1);
step(T)
title(['Step Response for Design of ' , num2str(pos), ' Percent'])
```

Computer response:

```
ans =

G(s)


Zero/pole/gain:
    1897.125 (s+0.1988)
----------------------------
s (s+40) (s+9.431) (s+0.001)
pos =

    10


z =

    0.5912

Select a point in the graphics window

selected_point =

  -3.3649 + 4.8447i


K =

    0.9090


p =

 -41.3037
  -3.9602 + 4.9225i
  -3.9602 - 4.9225i
  -0.2080
```


Root Locus with 10 Percent Overshoot Line
0.591
0.591
Imaginary Axis
Real Axis
40
30
20
10
0
-10
-20
-30
-40
-70
-60
-50
-40
-30
-20
-10
0
10


Step Response for Design of 10 Percent
Amplitude
Time (sec)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10

38.

Consider only the minor loop. Searching along the 143.13° line (ζ = 0.8), locate the minor-loop dominant poles at -3.36 ± j2.52 with K_f = 8.53. Searching the real axis segments for K_f = 8.53 locates a higher-order pole at - 0.28. Using the minor-loop poles as the open-loop poles for the entire system, search along the 120° line (ζ = 0.5) and find the dominant second-order poles at -1.39 + j2.41 with K = 27.79. Searching the real axis segment locates a higher-order pole at - 4.2.

39.

Consider only the minor loop. Searching along the 143.13° line (ζ = 0.8), locate the minor-loop dominant poles at -7.74 ± j5.8 with K_f = 36.71. Searching the real axis segments for K_f = 36.71 locates a higher-order pole at - 0.535. Using the minor-loop poles at -7.74 ± j5.8 and - 0.535 as the open-loop poles (the open-loop zero at the origin is not a closed-loop zero) for the entire system, search along the 135° line (ζ = 0.707; 4.32% overshoot) and find the dominant second-order poles at - 4.38 + j4 .38 with K = 227.91. Searching the real axis segment locates a higher-order pole at -7.26.

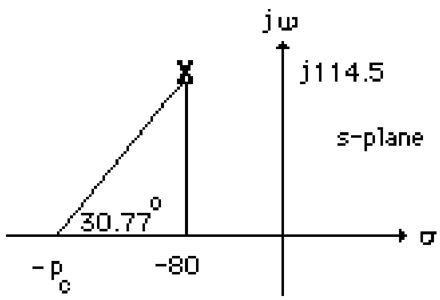
Uncompensated system performance: Setting K_f = 0 and searching along the 135° line (4.32% overshoot) yields -2.39 + j2.39 as the point on the root locus with K = 78.05. Searching the real axis segments of the root locus for K = 78.05 locates a higher-order pole at -11.2. The following table compares the predicted uncompensated characteristics with the predicted compensated characteristics.

Uncompensated	**Compensated**
$G(s)=\dfrac{78.05}{(s+1)(s+5)(s+10)}$	$G(s)=\dfrac{227.91}{s^3+16s^2+101.71s+50}$
Dominant poles: -2.39 + j2.39	Dominant poles: - 4.38 + j4 .38
$\zeta = 0.707$	$\zeta = 0.707$
$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 4.32\%$	$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 4.32\%$
$\omega_n = \sqrt{2.39^2+2.39^2} = 3.38$ rad/s	$\omega_n = \sqrt{4.38^2+4.38^2} = 6.19$ rad/s
$T_s = \dfrac{4}{\zeta\omega_n} = 1.67$ seconds	$T_s = \dfrac{4}{\zeta\omega_n} = 0.91$ second
$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.31$ seconds	$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.72$ second
$K_p = \dfrac{78.05}{1 \times 5 \times 10} = 1.56$	$K_p = \dfrac{227.91}{50} = 4.56$
Higher-order pole: -11.22	Higher-order pole: -7.26
Second-order approximation OK	Higher-order pole not 5x further from imaginary axis than dominant poles. Simulate to be sure of the performance.

40.

In Problem 46, Chapter 8 , the dominant poles, - 40 ± j57.25, yielded T_s = 0.1 second and 11.14% overshoot. The unity feedback system consisted of a gain adjusted forward transfer function of

$G(s) = \frac{20000K}{s(s+100)(s+500)(s+800)}$, where K = 102,300. To reduce the settling time by a factor of 2 to 0.05 seconds and keep the percent overshoot the same, we double the coordinates of the dominant poles to $-80 \pm j114.5$. Assume a lead compensator with a zero at -100 that cancels the plant's pole at -100. The summation of angles of the remaining plant poles to the design point is 149.23^o. Thus, the angular contribution of the compensator pole must be $149.23^o - 180^o = 30.77^o$. Using the geometry below, $\frac{114.5}{p_c - 80} = \tan 30.77^o$, or $p_c = 272.3$.



Adding this pole to the poles at the origin, -500, and -800 yields $K = 9.92 \times 10^9$ at the design point, $-80 \pm j114.5$. Any higher-order poles will have a real part greater than 5 times that of the dominant pair. Thus, the second-order approximation is OK.

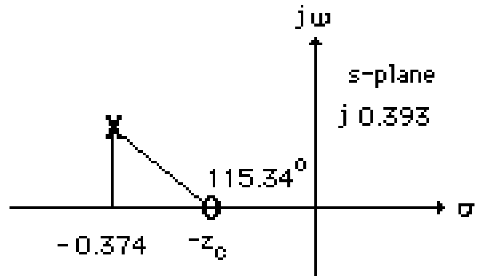
41.

Uncompensated: $G(s)H(s) = \frac{0.35K}{(s+0.4)(s+0.5)(s+0.163)(s+1.537)}$. Searching the 133.639^o line (%OS = 5%), find the dominant poles at $-0.187 \pm j0.196$ with gain, $0.35K = 2.88 \times 10^{-2}$. Hence, the estimated values are: %OS = 5%; $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.187} = 21.39$ seconds; $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{0.196} =$ 16.03 seconds; $K_p = 0.575$.

PD compensated: Design for 8 seconds peak time and 5% overshoot.

$$\zeta = \frac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.69.$$ Since $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 8$ seconds and $\omega_n\sqrt{1-\zeta^2} = 0.393$,

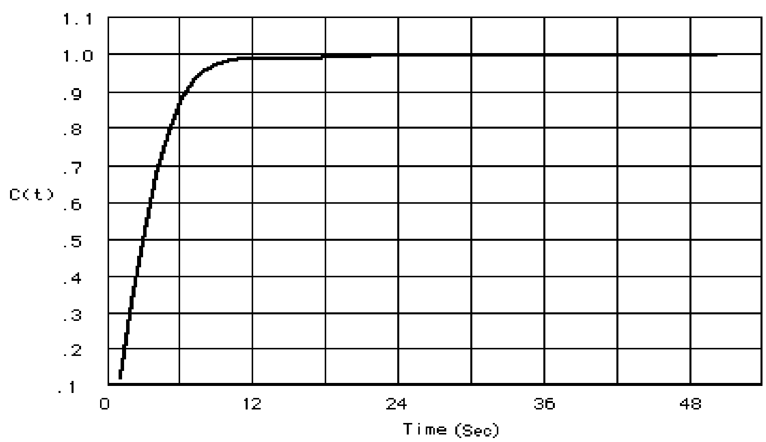
$\omega_n = 0.5426$. Hence, $\zeta\omega_n = 0.374$. Thus, the design point is $-0.374 + j0.393$. The summation of angles from the system's poles to the design point is -295.34^o. Thus, the angular contribution of the controller zero must be $295.34^o - 180^o = 115.34^o$. Using the geometry below,



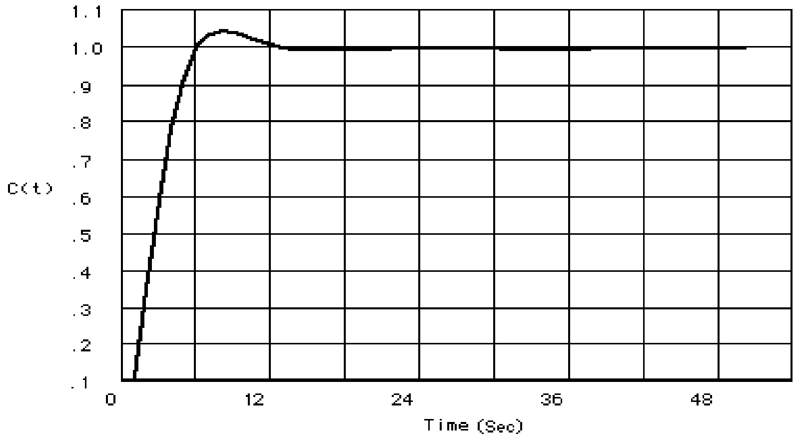
$\frac{0.393}{0.374 - z_c}$ = tan (180° - 115.34°), from which z_c = 0.19. Adding this zero to the system's poles and using the design point, - 0.374 + j0.393, the gain, 0.35K = 0.205.

PID compensated: Assume the integral controller, $G_c(s) = \frac{s+0.01}{s}$. The total open-loop transfer function is $G_{PID}(s)H(s) = \frac{0.35K(s+0.19)(s+0.01)}{s(s+0.4)(s+0.5)(s+0.163)(s+1.537)}$.

Check: The PID compensated system yields a very slow rise time due to the lag zero at 0.01. The rise time can be sped up by moving the zero further from the imaginary axis with resultant changes in the transient response. The plots below show the step response with the PI zero at - 0.24.



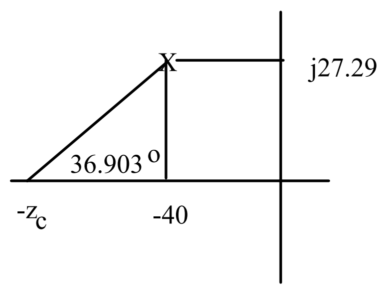
The response compares favorably with a two-pole system step response that yields 5% overshoot and a peak time of 8 seconds as shown below.



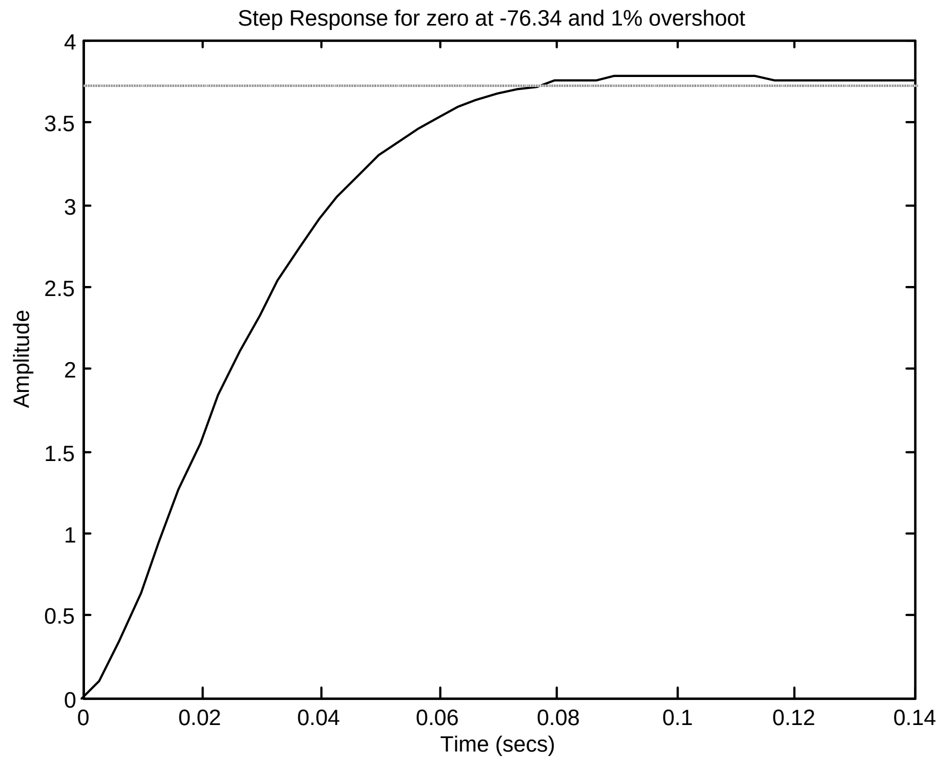
42.

a. PD compensator design: Pushing the gain, 10, to the right past the summing junction, the system can be represented as an equivalent unity feedback system with $G_e(s) = \frac{10^6}{(s^2 - 4551)(s+286)}$.

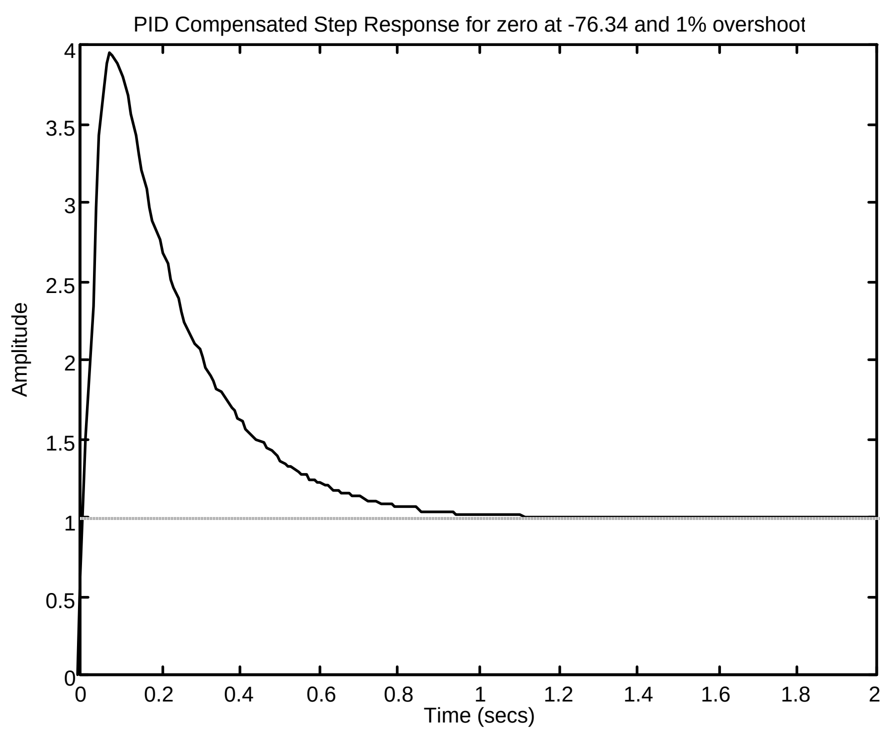
This system is unstable at any gain. For 1% overshoot and $T_s = 0.1$, the design point is $-40 + j27.29$. The summation of angles from the poles of $G_e(s)$ to this point is -216.903^o. Therefore, the compensator zero must contribute $216.903^o - 180^o = 36.903^o$. Using the following geometry:



$\frac{27.29}{z_c - 40} = \tan(36.903)$. Thus, $z_c = 76.34$. Adding this zero to the poles of $G_e(s)$, the gain at the design point is $10^6K = 23377$. The PD compensated response is shown below.



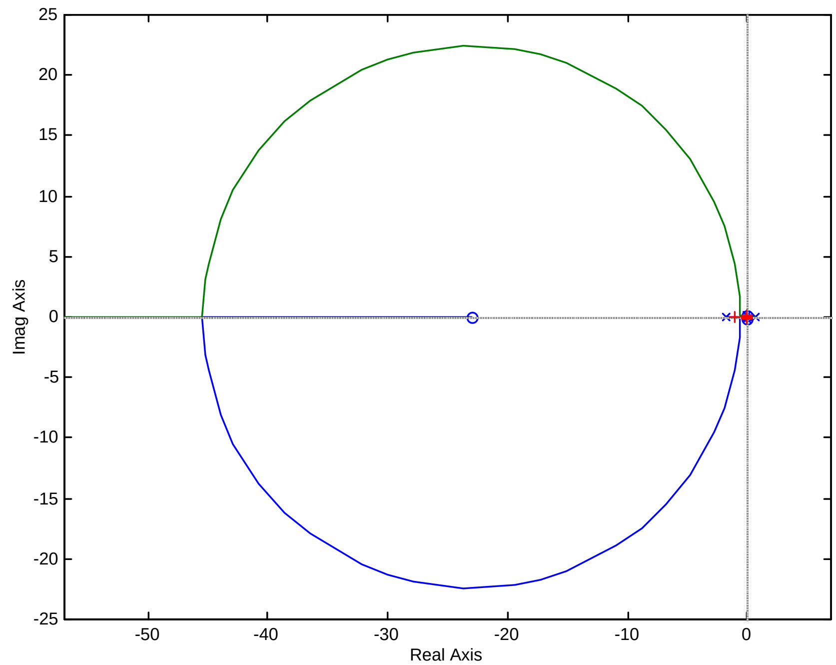
b. PI compensator design: To reduce the steady-state error to zero, we add a PI controller of the form $\frac{s+1}{s}$. The PID compensated step response is shown below.



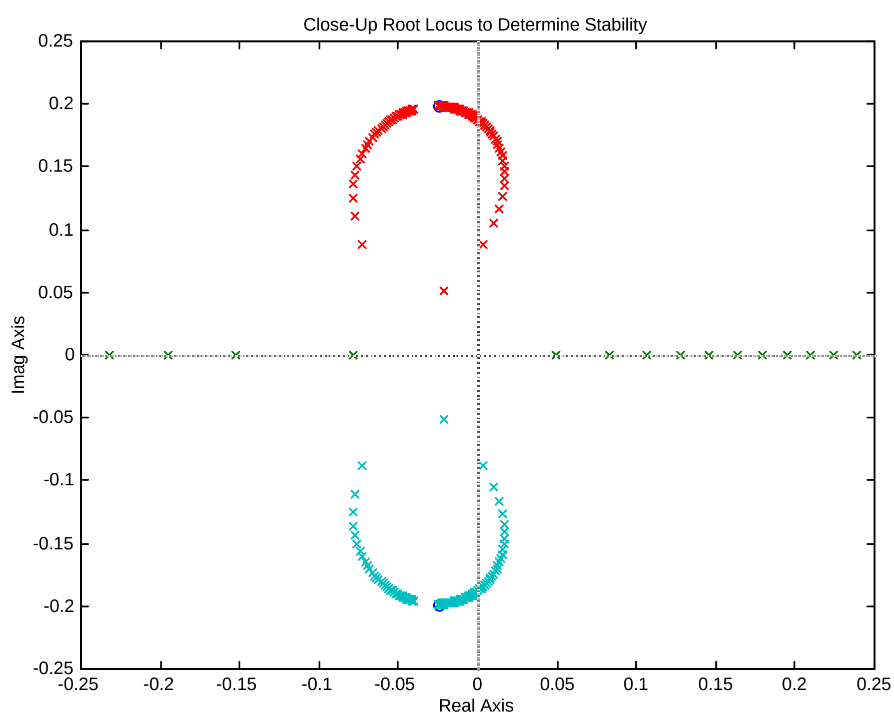
We can see the 1% overshoot at about 0.1 second as in the PD compensated system above. But the system now corrects to zero error.

43.

a. Root locus sketch yields;



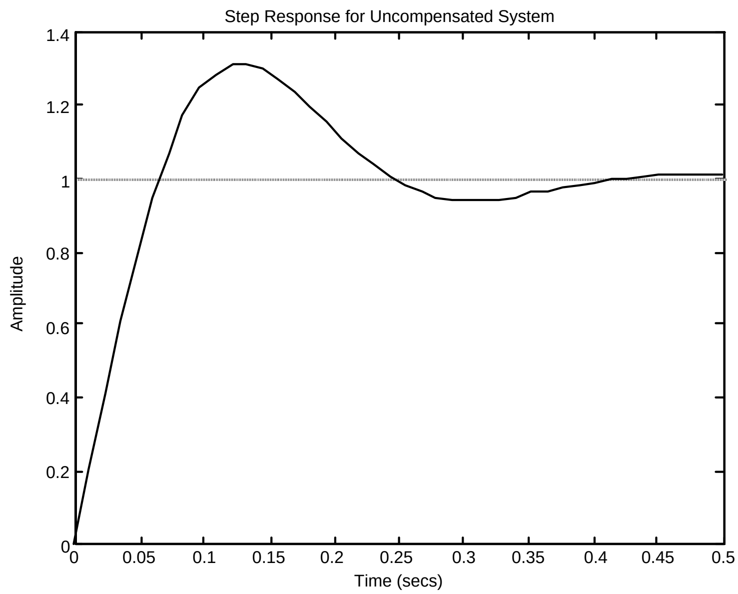
Root locus sketch near imaginary axis yields;



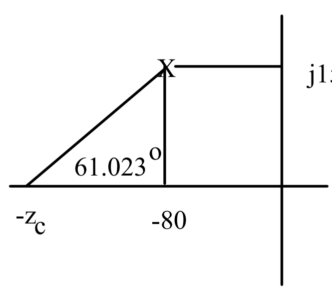
Searching imaginary axis for 180^o yields: j0.083 at a gain of 0.072K = 0.0528 and j0.188 at a gain of 0.072K = 0.081. Also, the gain at the origin is 0.0517. Thus, the system is stable for 0.0517 < 0.072K < 0.0528; 0.072K > 0.081. Equivalently, for 0.7181 < K < 0.7333; 0.072K > 1.125.

b. See (a)

c. Uncompensated system: Searching the 20% overshoot line, we find the operating point at $-8.987 + j17.4542 = 19.71\angle 117.126^o$ at 0.072K = 16.94 for the uncompensated system. Simulating the response at this gain yields,

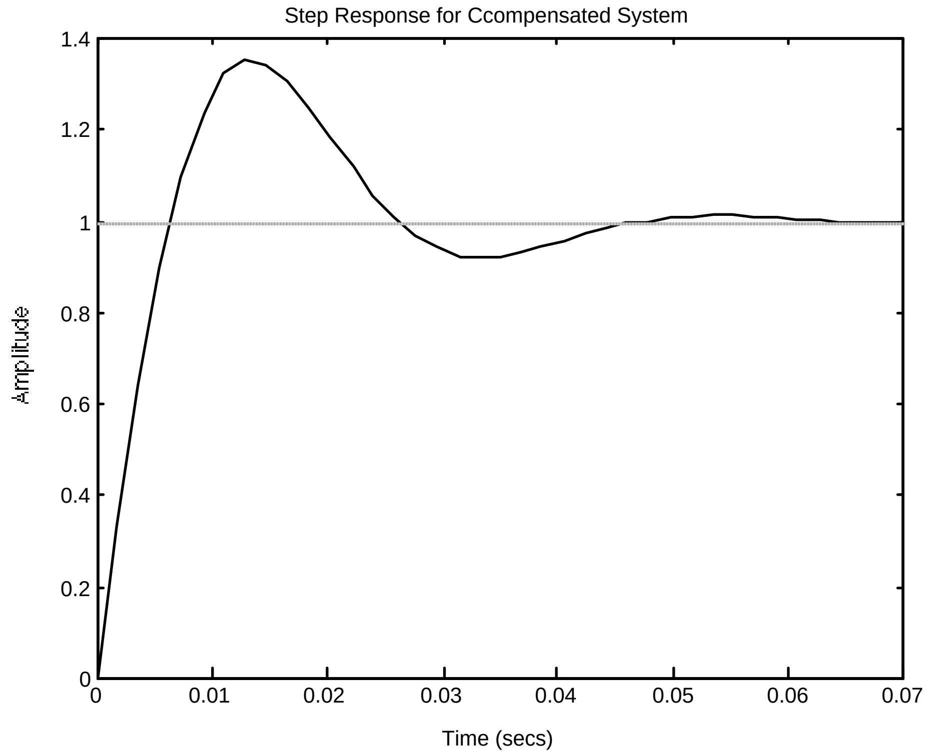


For 20% overshoot and $T_s = 0.05$ s, a design point of $-80 + j156.159$ is required. The sum of angles to the design point is -123.897^o. To meet the requirements at the design point, a zero would have to contribute $+303.897^o$, which is too high for a single zero. Let us first add the pole at the origin to drive the steady-state error to zero to reduce the angle required from the zero. Summing angles with this pole at the origin yields -241.023. Thus a zero contributing 61.023^o is required. Using the geometry below with $\frac{156.159}{z_c - 80} = \tan(61.023)$, $z_c = 166.478$.

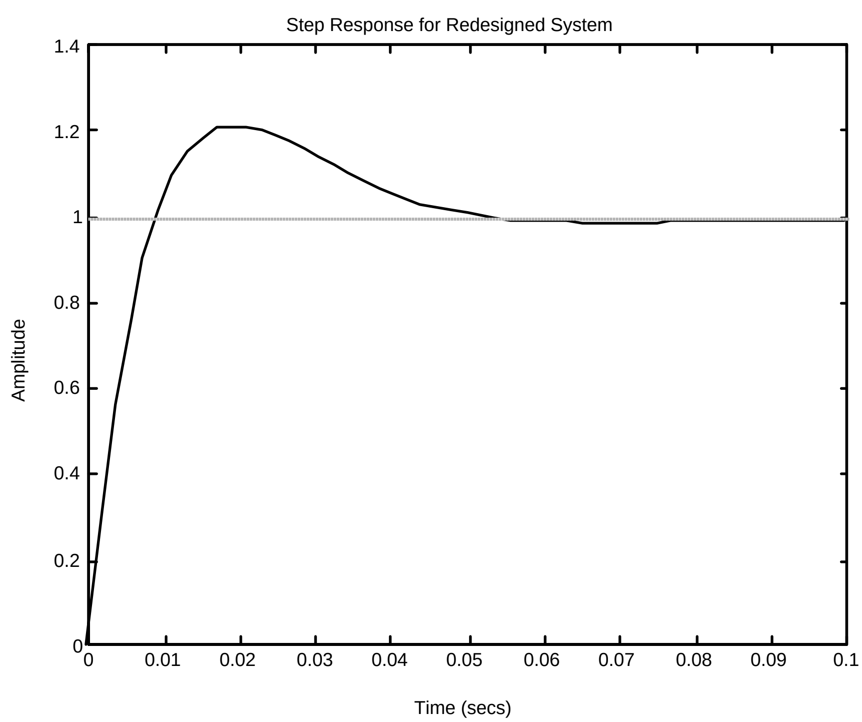


The gain at the design point is 0.072K = 181.55.

d.



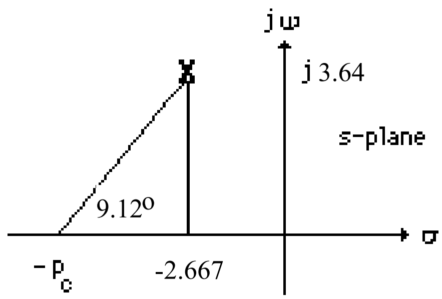
The settling time requirement has been met, but the percent overshoot has not. Repeating the design for 1% overshoot and a $T_S = 0.05$ s yields a design point of $-80 + j54.575$. The compensator zero is found to be at -47.855 at a gain 0.072K = 180.107.



44.

$$\zeta\omega_n = \frac{4}{T_s} = 2.667; \;\; \zeta = \frac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.591. \text{ Thus, } \omega_n = 4.512 \text{ rad/s.}$$

$\mathrm{Im} = \omega_n\sqrt{1-\zeta^2} = 4.512\sqrt{1-0.591^2} = 3.64$. Thus, and the operating point is $-2.667 \pm j3.64$. Summation of angles, assuming the compensating zero is at –5 (to cancel the open-loop pole at –5, is -170.88°. Therefore, the compensator pole must contribute $180^{\circ} - 170.88^{\circ} = -9.12^{\circ}$. Using the geometry shown below,



$\frac{3.64}{p_c - 2.667} = \tan 9.12^{\circ}$. Thus, $p_c = 25.34$. Adding the compensator pole and using $-2.667 \pm j3.64$ as the test point, 50K = 2504, or K = 50.08. Thus the compensated open-loop transfer function is

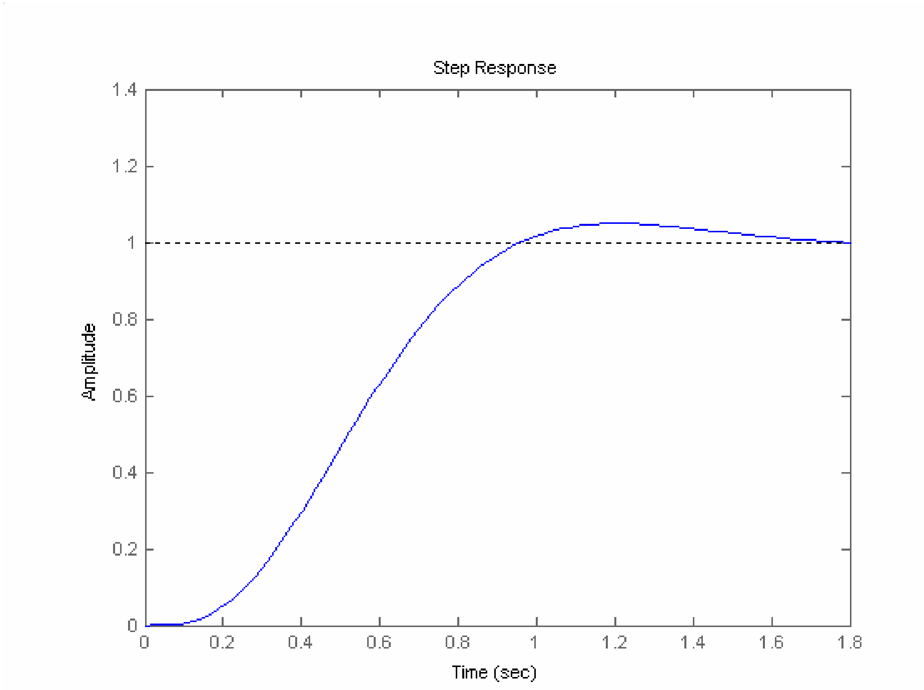
$G_e(s) = \frac{2504(s+5)}{s(s+5)(s^2+10s+50)(s+25.34)}$. Higher-order pole are at –25.12, -5, and-4.898. The pole at –5 is cancelled by the closed-loop zero at -5. The pole at –4.898 is not far enough away from the dominant second-order pair. Thus, the system should be simulated to determine if the response meets the requirements.

Program:

```
syms s
numg=2504;
deng=expand(s*(s^2+10*s+50)*(s+25.34));
deng=sym2poly(deng);
G=tf(numg,deng);
Gzpk=zpk(G)
T=feedback(G,1);
step(T)
```

Computer response:

```
Zero/pole/gain:
            2504
----------------------------
s (s+25.34) (s^2 + 10s + 50)
```



45.

a. From Chapter 8,

$$G_e(s) = \frac{0.6488K\,(s+53.85)}{(s^2 + 8.119s + 376.3)\,(s^2 + 15.47s + 9283)}$$

Cascading the notch filter,

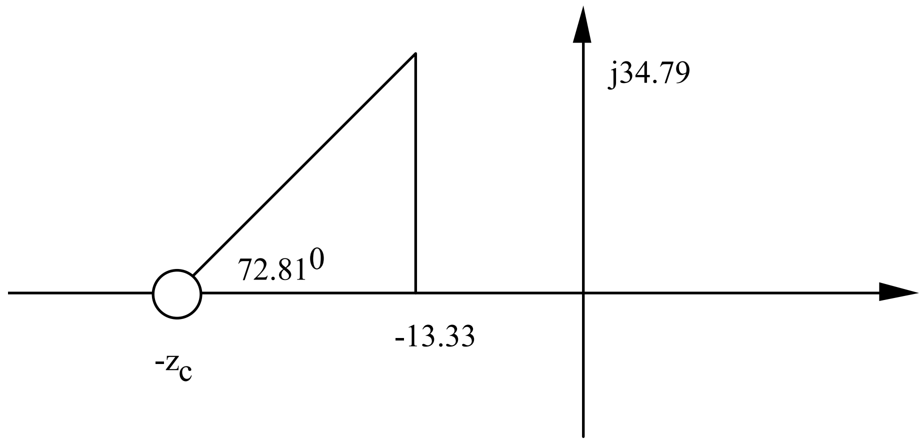
$$G_{et}(s) = \frac{0.6488K\,(s+53.85)(s^2 + 16s + 9200)}{(s^2 + 8.119s + 376.3)\,(s^2 + 15.47s + 9283)(s+60)^2}$$

Arbitrarily design for %OS = 30% ($\zeta = 0.358$) and $T_s = 0.3$ s. This places desired poles at

$-13.33 \pm j34.79$. At the design point, the sum of the angles without the PD controller is 107.19^0.

Thus,

$$\frac{34.79}{z_c - 13.33} = \tan 72.81$$



From which, z_c = 24.09. Putting this into the forward path,

$$G_{et}(s) = \frac{0.6488K\,(s+53.85)(s^2 + 16s + 9200)(s+24.09)}{(s^2 + 8.119s + 376.3)\,(s^2 + 15.47s + 9283)(s+60)^2}$$

Using root locus, the gain 0.6488K = 1637, or K = 2523.

b. Add a PI controller

$$G_{PI}(s) = \frac{(s+0.1)}{s}$$

Thus,

$$G_{et}(s) = \frac{0.6488K\,(s+53.85)(s^2 + 16s + 9200)(s+24.09)(s+0.1)}{s\,(s^2 + 8.119s + 376.3)\,(s^2 + 15.47s + 9283)(s+60)^2}$$

Using root locus, the gain 0.6488K = 1740, or K = 2682.

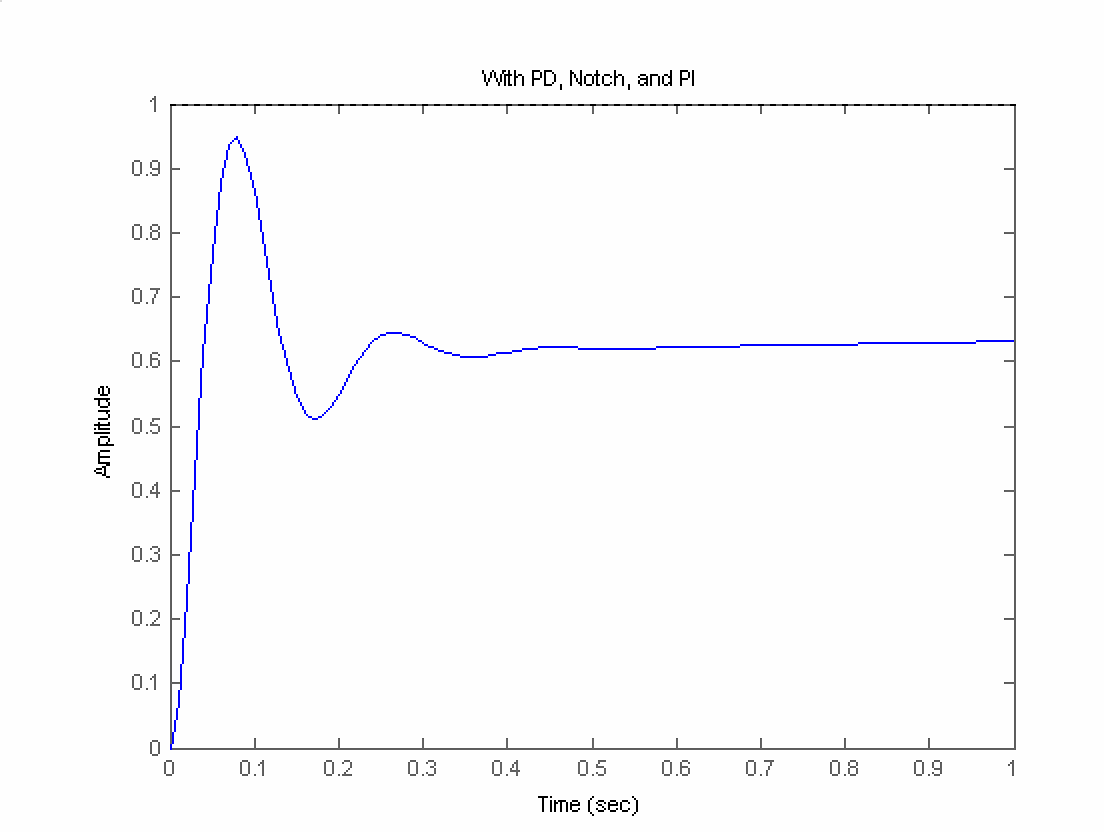
c.

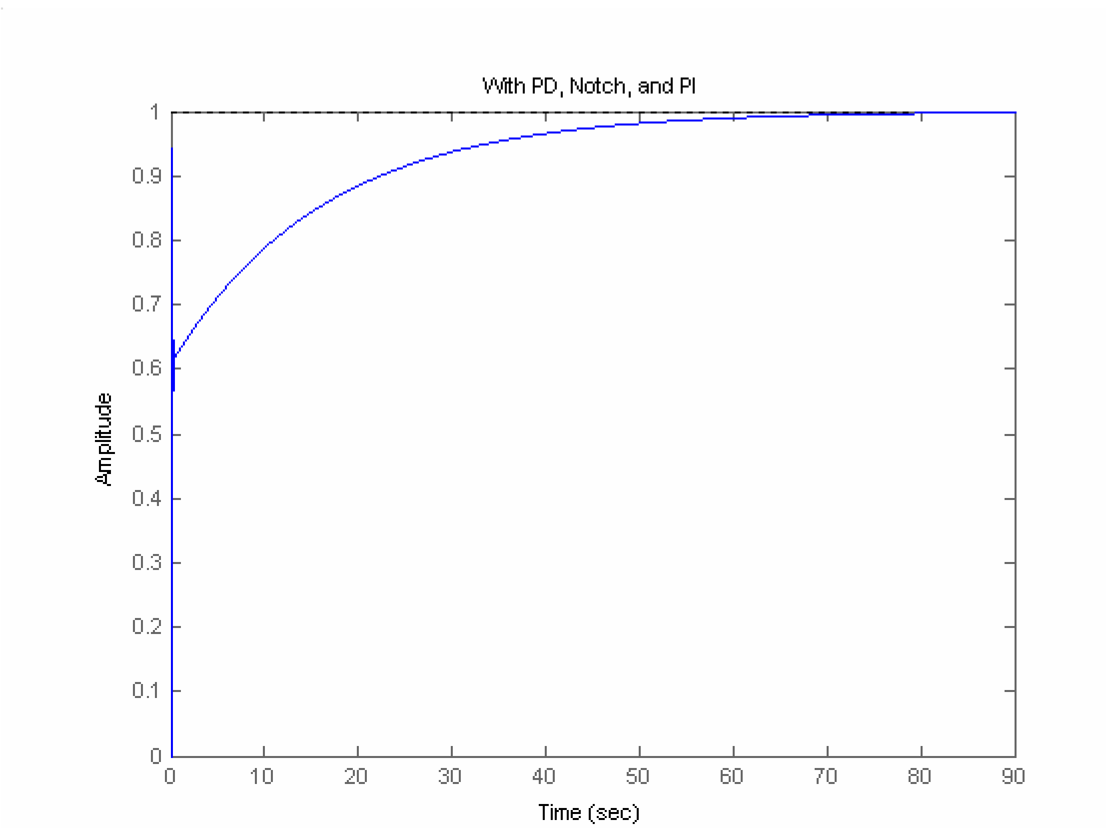
Program:

```
syms s
numg=1637*(s+53.85)*(s^2+16*s+9200)*(s+24.09)*(s+0.1);
deng=s*(s^2+15.47*s+9283)*(s^2+8.119*s+376.3)*(s+60)^2;
numg=sym2poly(numg);
deng=sym2poly(deng);
G=tf(numg,deng);
Gzpk=zpk(G)
T=feedback(G,1);
step(T,0:0.01:1)
title(['With PD, Notch, and PI'])
pause
step(T)
title(['With PD, Notch, and PI'])
```

Computer response:

```
Zero/pole/gain:
  1637 (s+53.85) (s+24.09) (s+0.1) (s^2 + 16s + 9200)
-------------------------------------------------------
s (s+60)^2 (s^2 + 8.119s + 376.3) (s^2 + 15.47s + 9283)
```


With PD, Notch, and PI
Amplitude
Time (sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


With PD, Notch, and PI
Amplitude
Time (sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10
20
30
40
50
60
70
80
90