E I G H T

Root Locus Techniques

SOLUTIONS TO CASE STUDIES CHALLENGES

Antenna Control: Transient Design via Gain

a. From the Chapter 5 Case Study Challenge:

$$G(s) = \frac{76.39K}{s(s+150)(s+1.32)}$$

Since $T_s = 8$ seconds, we search along $-\frac{1}{2}$, the real part of poles with this settling time, for 180^o.

We find the point to be - 0.5+j6.9 with 76.39K = 7194.23, or K = 94.18. Second-order approximation is OK since third pole is much more than 5 times further from the imaginary axis than the dominant second-order pair.

b.

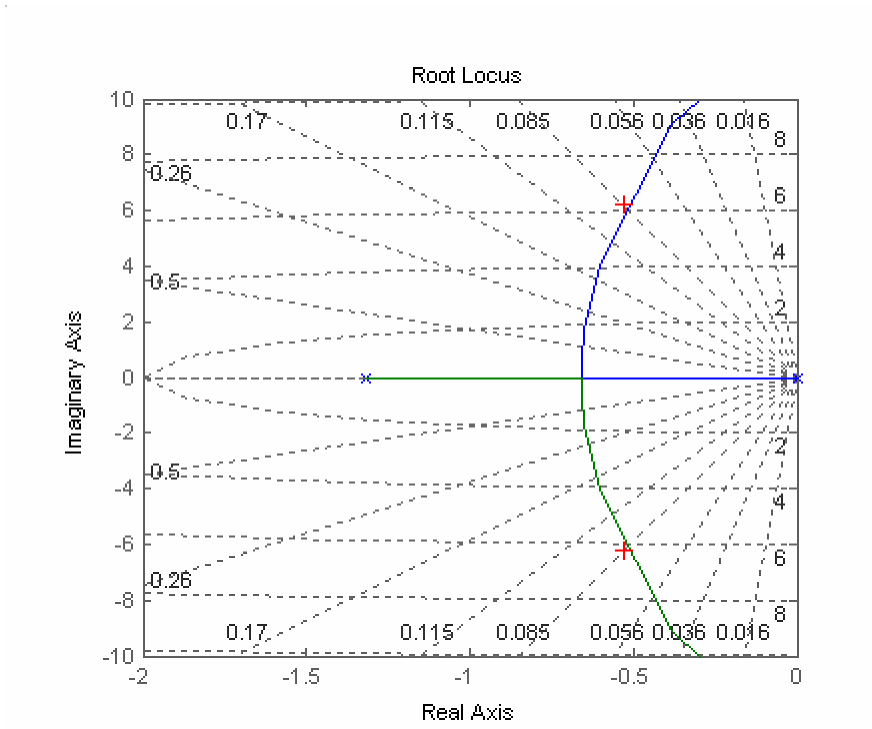
Program:

```
numg= 1;
deng=poly([0 -150 -1.32]);
'G(s)'
G=tf(numg,deng)
rlocus(G)
axis([-2,0,-10,10]);
title(['Root Locus'])
grid on
[K1,p]=rlocfind(G)
K=K1/76.39
```

Computer response:

```
ans =

G(s)


Transfer function:
            1
-----------------------
s^3 + 151.3 s^2 + 198 s

Select a point in the graphics window

selected_point =

  -0.5034 + 6.3325i
```

```
K1 =

  6.0690e+003


p =

  1.0e+002 *

  -1.5027
  -0.0052 + 0.0633i
  -0.0052 - 0.0633i


K =

   79.4469

>>
ans =

G(s)


Transfer function:
           1
-----------------------
s^3 + 151.3 s^2 + 198 s

Select a point in the graphics window

selected_point =

  -0.5000 + 6.2269i


K1 =

  5.8707e+003


p =

  1.0e+002 *

  -1.5026
  -0.0053 + 0.0623i
  -0.0053 - 0.0623i


K =

   76.8521
```



UFSS Vehicle: Transient Design via Gain

a. Push $-K_1$ to the right past the summing junction yielding $G(s) = \frac{K(s+0.437)}{s(s+2)(s+1.29)(s+0.193)}$, where $K = 0.25K_1$. Combine the parallel feedback paths and obtain $H(s) = (s+1)$. Hence, $G(s)H(s) = \frac{K(s+0.437)(s+1)}{s(s+2)(s+1.29)(s+0.193)}$. The root locus is shown below in (b). Searching the 10% overshoot line ($\zeta = 0.591$; $\theta = 126.24^{\text{o}}$), we find the operating point to be $-1.07 \pm j1.46$ where $K = 3.389$, or $K_1 = 13.556$.

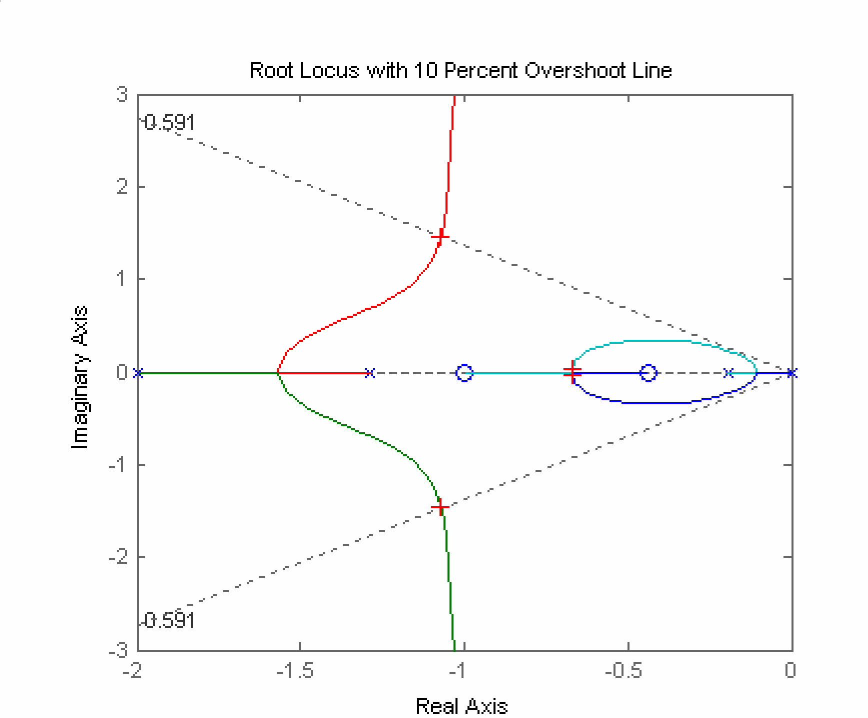
b.
Program:

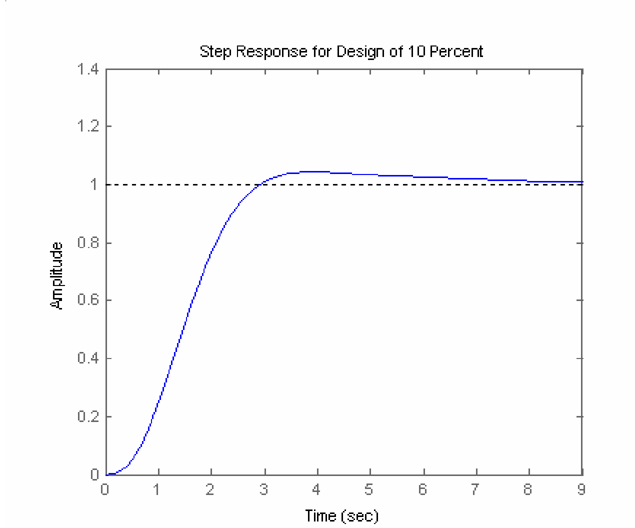
```
numg= [1 0.437];
deng=poly([0 -2 -1.29 -0.193]);
G=tf(numg,deng);
numh=[1 1];
denh=1;
H=tf(numh,denh);
GH=G*H;
rlocus(GH)
pos=(10);
z=-log(pos/100)/sqrt(pi^2+[log(pos/100)]^2);
sgrid(z,0)
title(['Root Locus with ' , num2str(pos), ' Percent Overshoot Line'])
[K,p]=rlocfind(GH);
pause
K1=K/0.25
T=feedback(K*G,H)
T=minreal(T)
step(T)
```

```
title(['Step Response for Design of ' , num2str(pos), ' Percent'])
```

Computer response:

```
Select a point in the graphics window

selected_point =

  -1.0704 + 1.4565i


K1 =

   13.5093


Transfer function:
               3.377 s + 1.476
---------------------------------------------
s^4 + 3.483 s^3 + 6.592 s^2 + 5.351 s + 1.476


Transfer function:
               3.377 s + 1.476
---------------------------------------------
s^4 + 3.483 s^3 + 6.592 s^2 + 5.351 s + 1.476
```





ANSWERS TO REVIEW QUESTIONS

1. The plot of a system's closed-loop poles as a function of gain

2. (1) Finding the closed-loop transfer function, substituting a range of gains into the denominator, and factoring the denominator for each value of gain. (2) Search on the s-plane for points that yield 180 degrees when using the open-loop poles and zeros.

3. K = 1/5

4. No

5. At the zeros of G(s) and the poles of H(s)

6. (1) Apply Routh-Hurwitz to the closed-loop transfer function's denominator. (2) Search along the imaginary axis for 180 degrees.

7. If any branch of the root locus is in the rhp, the system is unstable.

8.If the branch of the root locus is vertical, the settling time remains constant for that range of gain on the vertical section.

9. If the root locus is circular with origin at the center

10. Determine if there are any break-in or breakaway points

11. (1) Poles must be at least five times further from the imaginary axis than the dominant second order pair, (2) Zeros must be nearly canceled by higher order poles.

12. Number of branches, symmetry, starting and ending points

13. The zeros of the open loop system help determine the root locus. The root locus ends at the zeros. Thus, the zeros are the closed-loop poles for high gain.

SOLUTIONS TO PROBLEMS

1.

a. No: Not symmetric; On real axis to left of an even number of poles and zeros

b. No: On real axis to left of an even number of poles and zeros

c. No: On real axis to left of an even number of poles and zeros

d. Yes

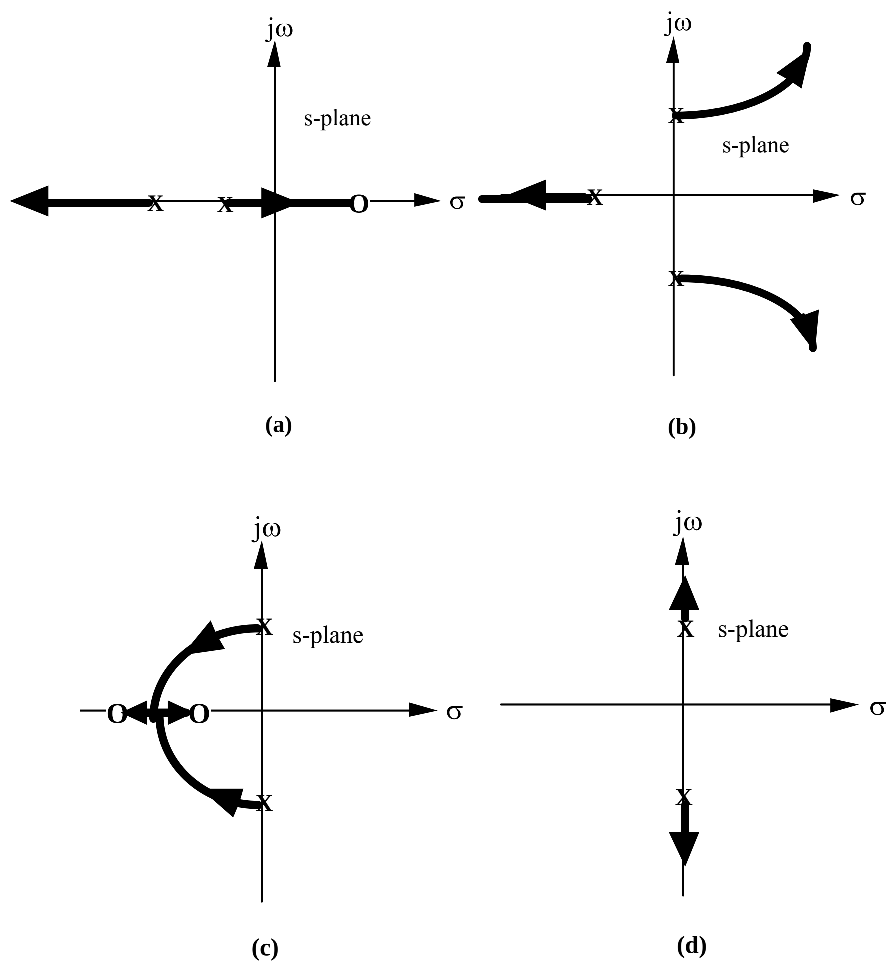
e. No: Not symmetric; Not on real axis to left of odd number of poles and/or zeros

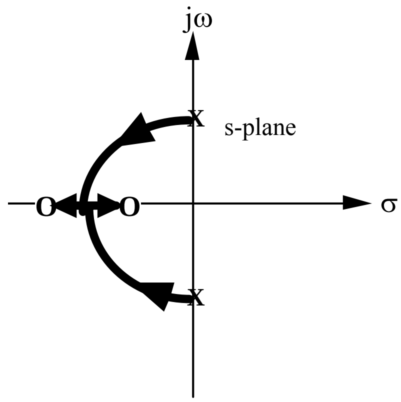
f. Yes

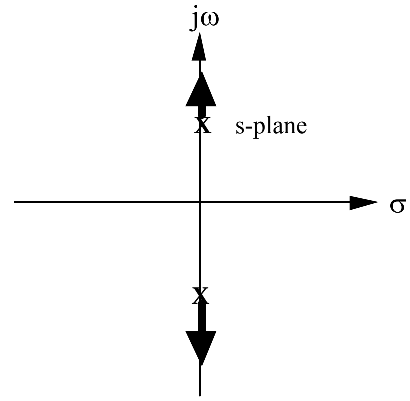
g. No: Not symmetric; real axis segment is not to the left of an odd number of poles

h. Yes

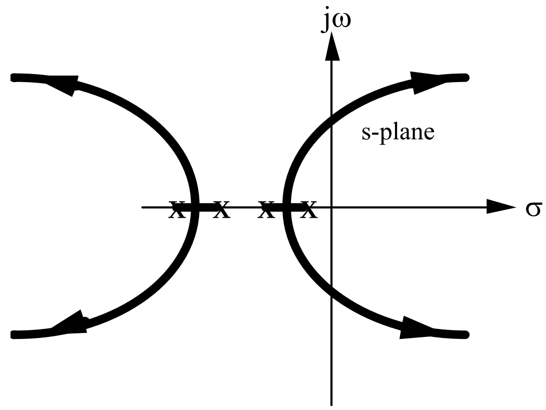
2.

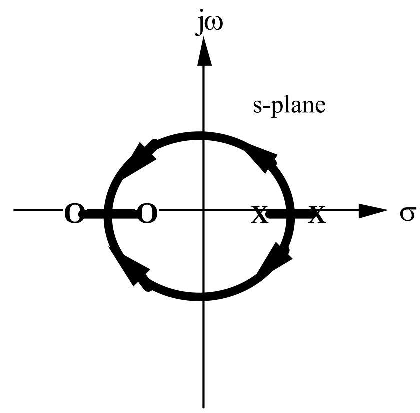



jω
s-plane
σ


jω
s-plane
σ

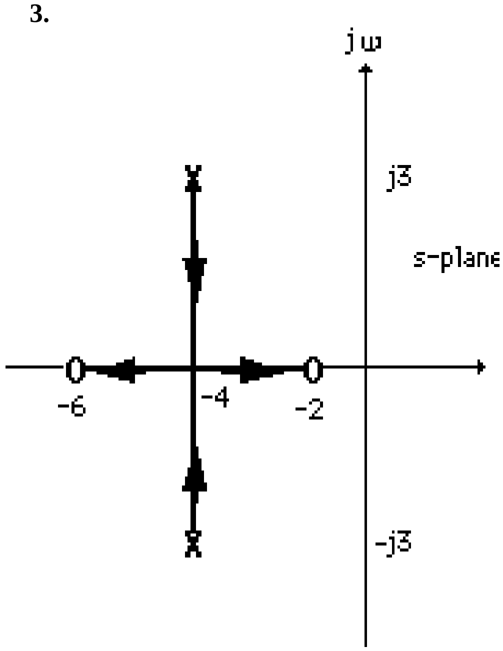
(c) **(d)**

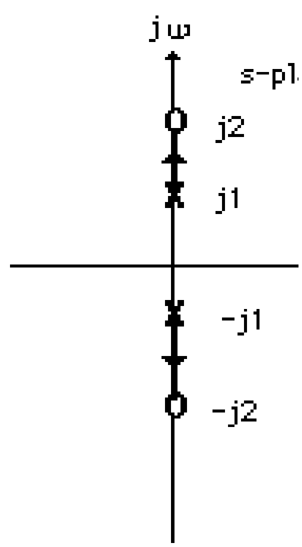

jω
s-plane
σ

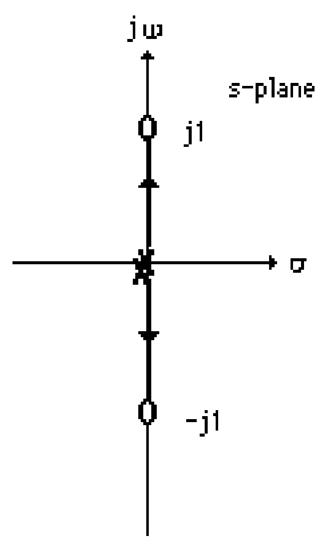

jω
s-plane
σ

(e) **(f)**

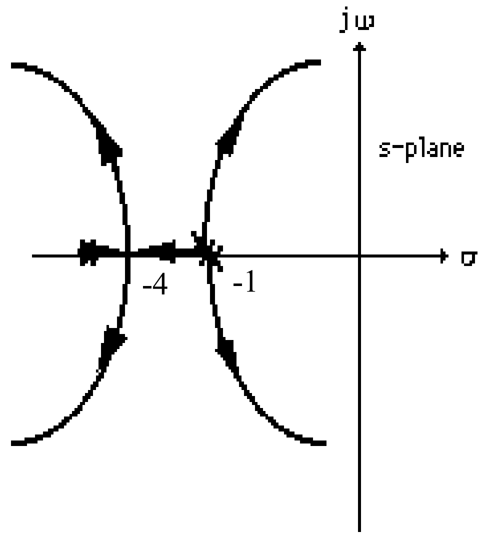
3.


jω
j3
s-plane
-6
-4
-2
-j3


jω
j2
j1
-j1
-j2

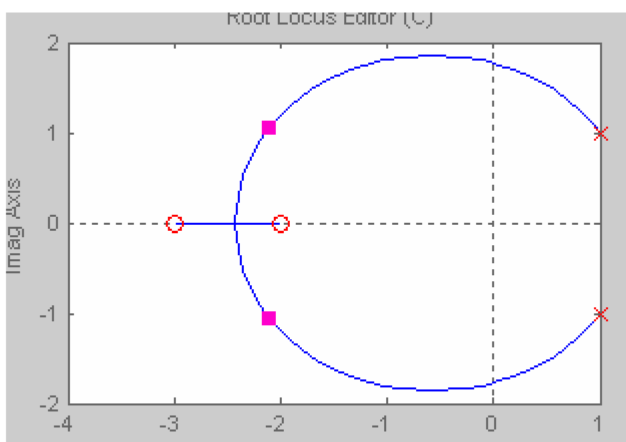

jω
s-plane
j1
σ
-j1

a. **b.** **c.**



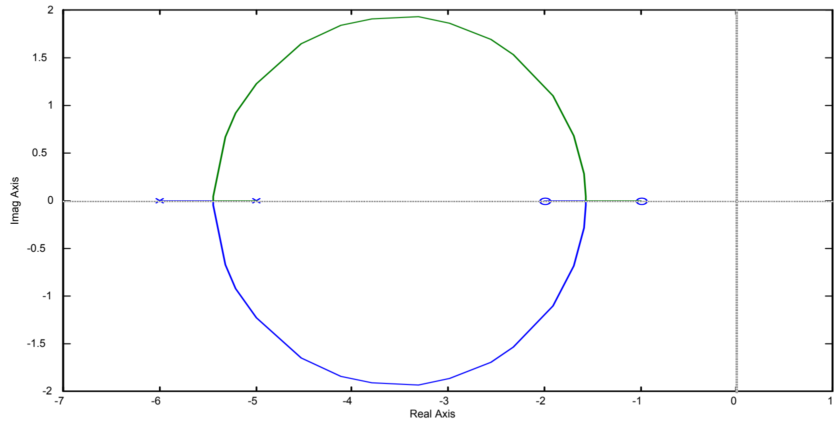
d.

4.



Breakaway: σ = -2.43 for K = 52.1

5.



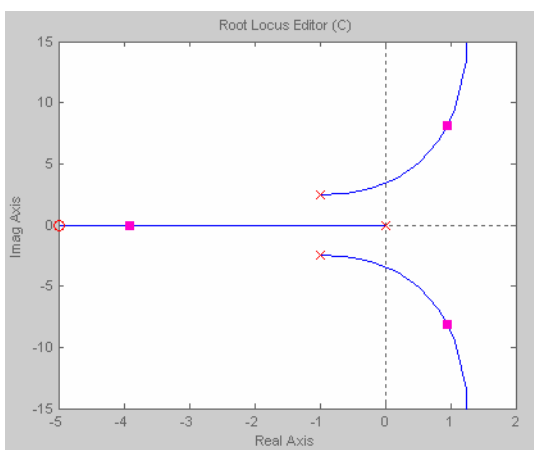
Break-in: σ = -1.5608 for K = 61.986; Breakaway: σ = -5.437 for K = 1.613.

6.

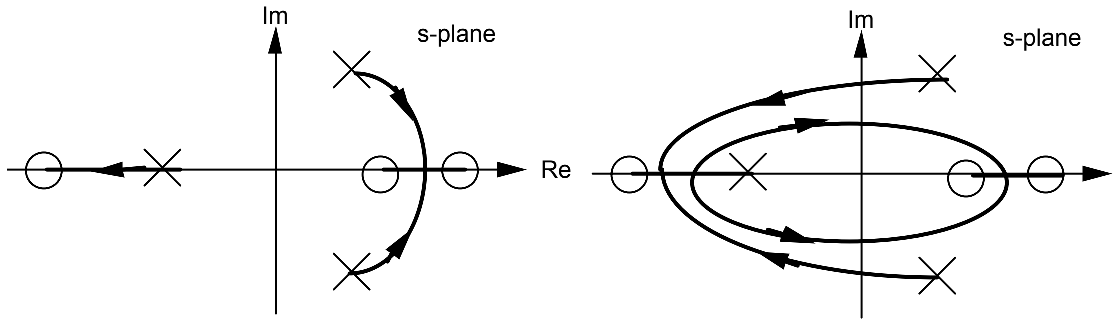
Convert the denominator to the following form: $D(s) = 1 + \dfrac{20K(s+5)}{s^3 + 2s^2 + 7s}$ and thus identify

$$G(s) = \frac{20K(s+5)}{s^3 + 2s^2 + 7s} = \frac{20K(s+5)}{s(s^2 + 2s + 7)}.$$

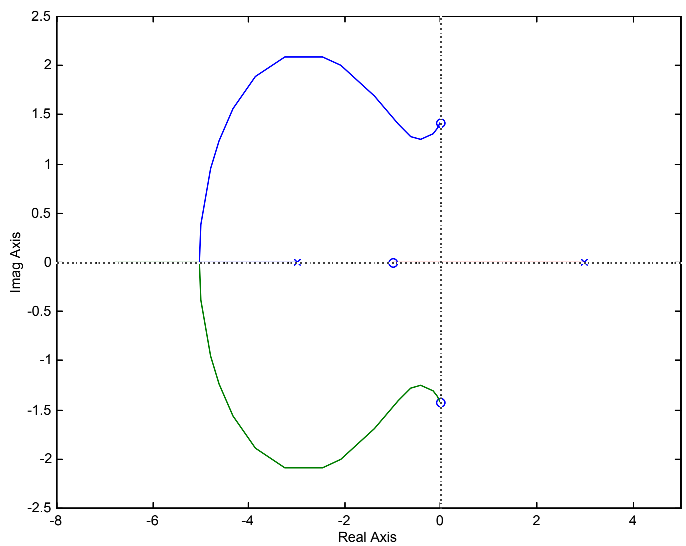
Plotting the root locus yields



7.



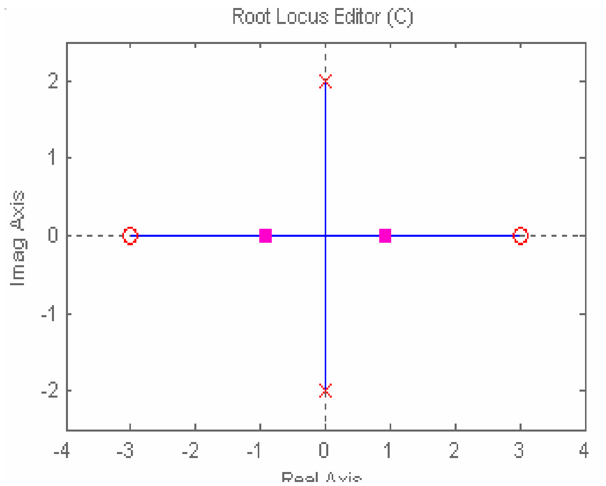
8.



Closed-loop poles will be in the left-half-plane when rhp pole reaches the origin,

or $K > \dfrac{(3)(3)}{\left(\sqrt{2}\right)\left(\sqrt{2}\right)(1)} = \dfrac{9}{2}$.

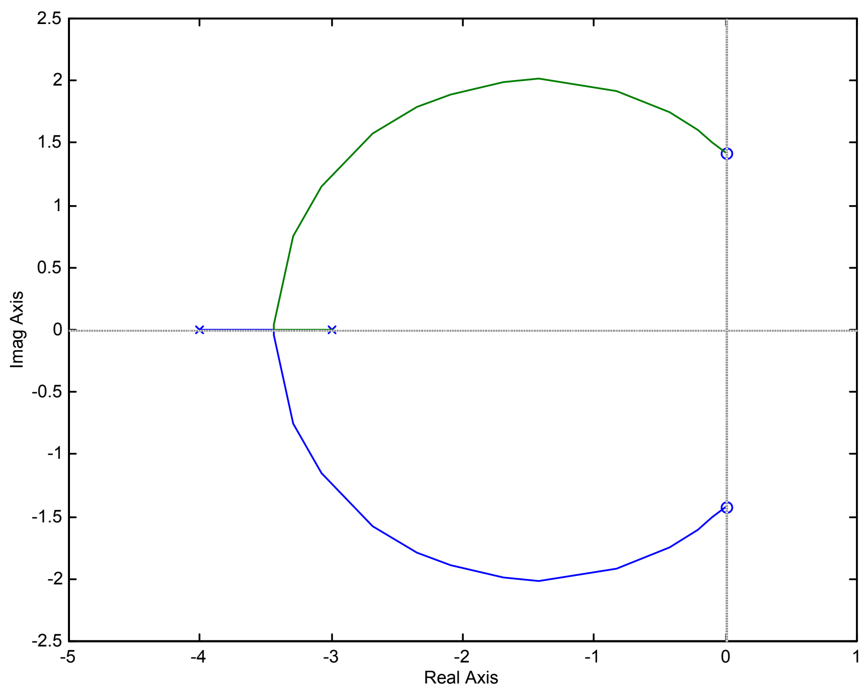
9.



Closed-loop poles will be in the right-half-plane for $K > \frac{(2)(2)}{(3)(3)} = \frac{4}{9}$ (gain at the origin).

Therefore, stable for $K < 4/9$; unstable for $K > 4/9$.

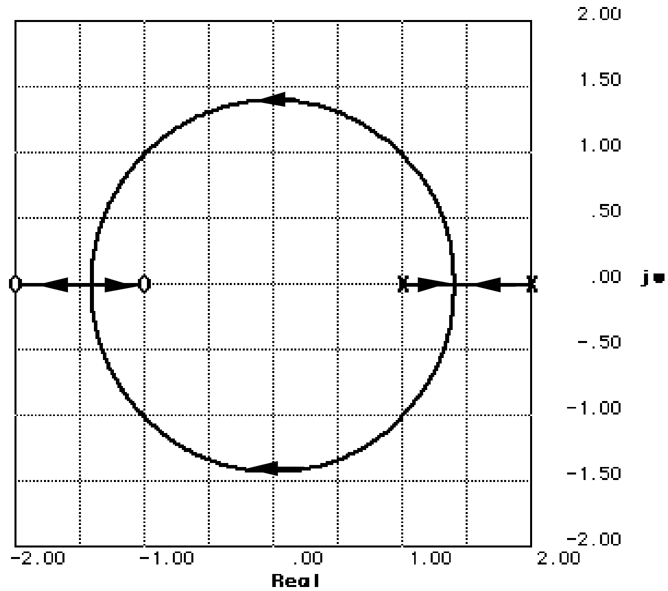
10.



Breakaway: σ = -3.436 for K = 1.781. System is never unstable. System is marginally stable for K = ∞.

11.

System 1:



(a)

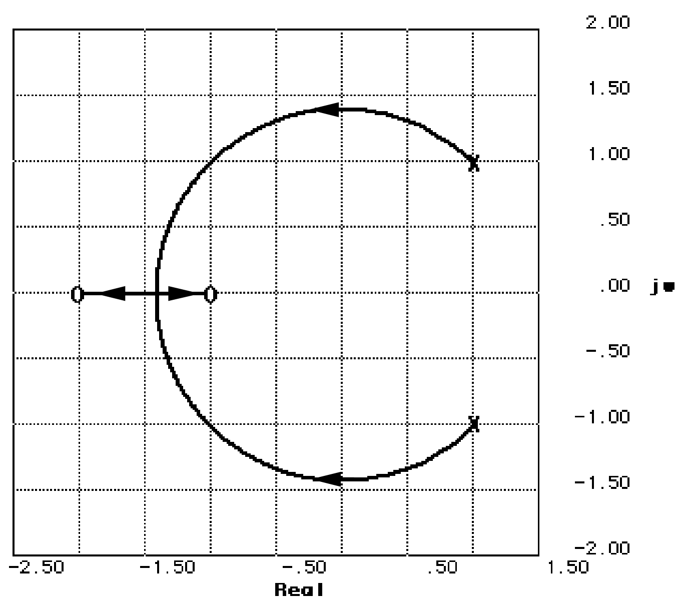
a. Breakaway: σ = 1.41 for K = 0.03; Break-in: σ = -1.41 for K = 33.97.

b. Imaginary axis crossing at j1.41 for K = 1. Thus stable for K > 1.

c. At break-in point, poles are multiple. Thus, K = 33.97.

d. Searching along 135^0 line for 180^0, K = 5 at 1.414 $\angle$ 135^0.

System 2:



(b)

a. Break-in: σ = -1.41 for K = 28.14.

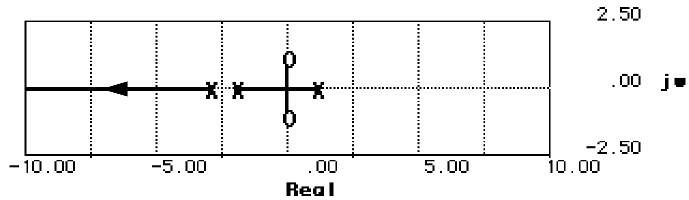
b. Imaginary axis crossing at j1.41 for K = 0.67. Thus stable for K > 0.67.

c. At break-in point, poles are multiple. Thus, K = 28.14.

d. Searching along 135^0 line for 180^0, K = 4 at 1.414 $\angle$ 135^0.

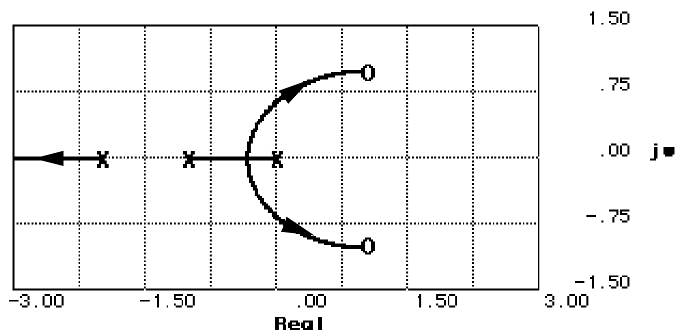
12.

a.



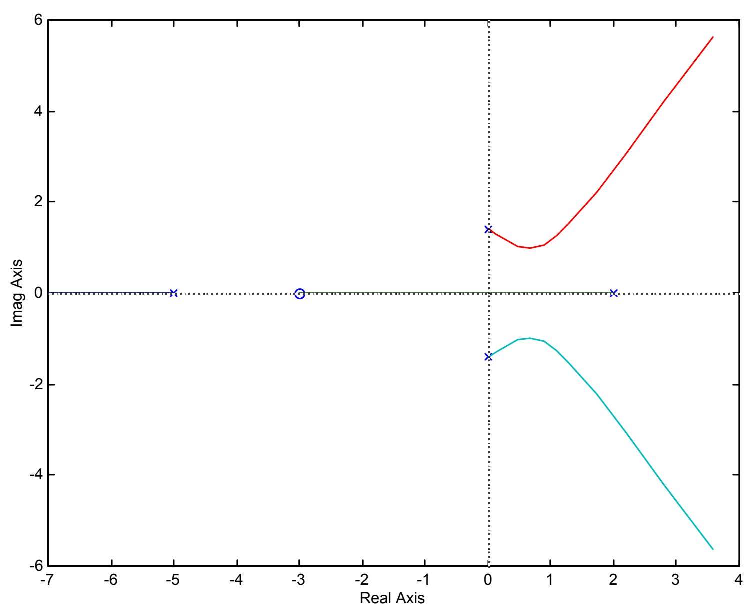
Root locus crosses the imaginary axis at the origin for K = 6. Thus the system is stable for K > 6.

b.



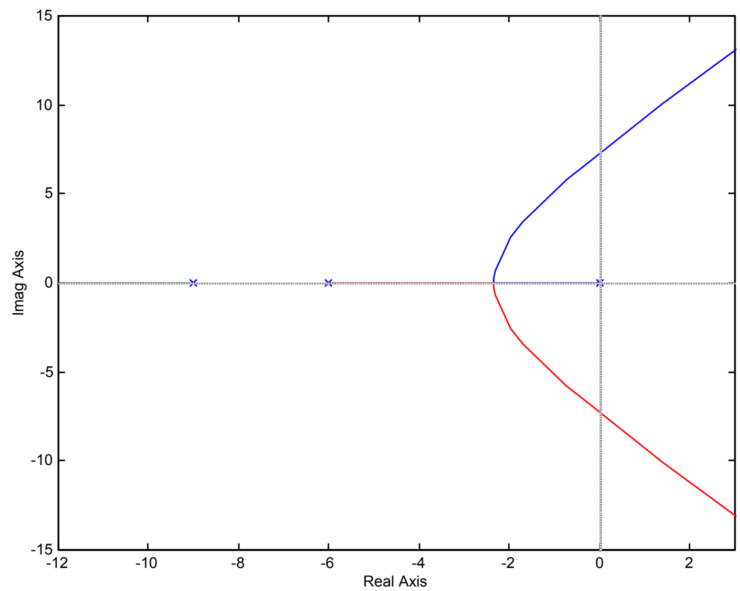
Root locus crosses the imaginary axis at j0.65 for K = 0.79. Thus, the system is stable for K < 0.79.

13.



There will be only two right-half-plane poles when pole at +2 moves into the left-half-plane at the origin. Thus $K = \dfrac{(5)(\sqrt{2})(\sqrt{2})(2)}{3} = 6.67$.

14.



Root locus crosses the imaginary axis at j7.348 with a gain of 810. Real axis breakaway is at –2.333 at a gain of 57.04. Real axis intercept for the asymptotes is $\dfrac{-15}{3} = -5$. The angle of the asymptotes is = $\dfrac{\pi}{3}$, π, $\dfrac{5\pi}{3}$. Some other points on the root locus are:

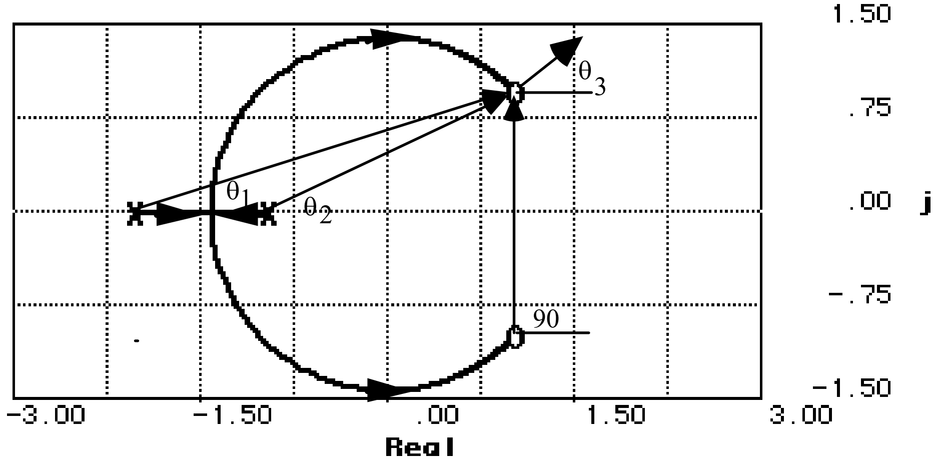
ζ = 0.4: -1.606 + j3.68, K = 190.1

ζ = 0.6: -1.956 + j2.6075, K = 117.8

ζ = 0.8: -2.189 + j1.642, K = 79.55

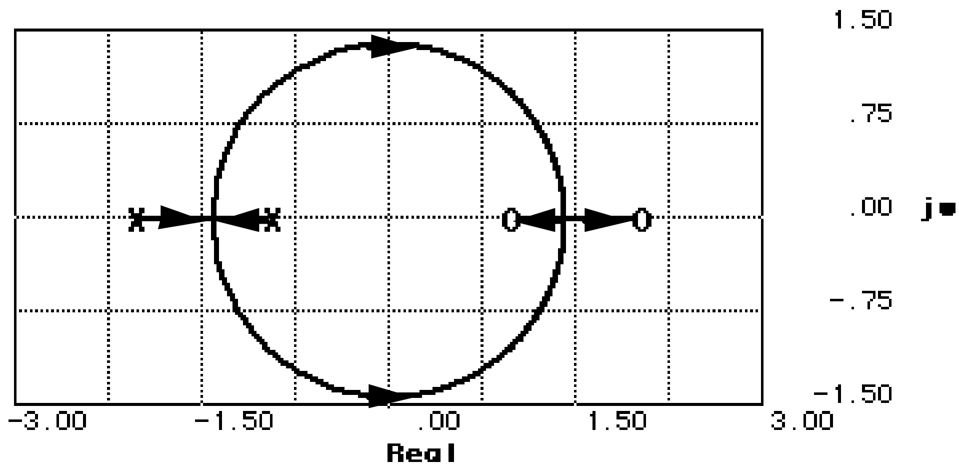
15.

a.



Imaginary axis crossing: j1.41 at K = 1.5. Stability: K < 1.5. Breakaway: -1.41 at K = 0.04. Points on root locus: -1.5 ± j0, K = 0.0345; -0.75 ± j1.199, K = 0.429; 0 ± j1.4142, K = 1.5; 0.75 ± j1.1989, K = 9. Finding angle of arrival: $90 - \theta_1 - \theta_2 + \theta_3 = 90^o - \tan^{-1}(1/3) - \tan^{-1}(1/2) + \theta_3 = 180^o$. Thus, $\theta_3 = 135^o$.

b.



Imaginary axis crossing: j1.41 at K = 1. Stability: K < 1. Breakaway: -1.41 at K = 0.03. Break-in: 1.41 at K = 33.97. Points on root locus: -1.5 ± j0, K = 0.02857; -0.75 ± j1.199, K = 0.33; 0 ± j1.4142, K = 1; 0.75 ± j1.1989, K = 3.

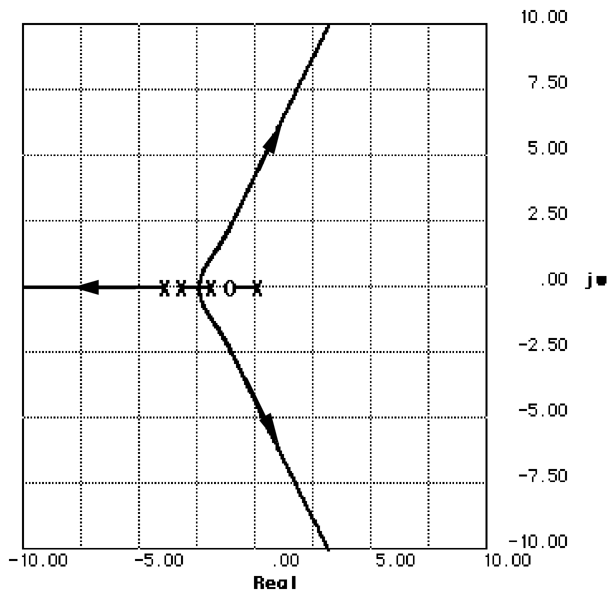
16.

a. Root locus crosses the imaginary axis at $\pm j3.162$ at $K = 52$.

b. Since the gain is the product of pole lengths to -5, $K = (1)\left(\sqrt{4^2+1^2}\right)\left(\sqrt{4^2+1^2}\right) = 17$.

17.

a.



b. $\sigma_a = \frac{(0 - 2 - 3 - 4) - (-1)}{3} = -\frac{8}{3}$; Angle $= \frac{(2k+1)\pi}{3} = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

c. Root locus crosses imaginary axis at j4.28 with K = 140.8.

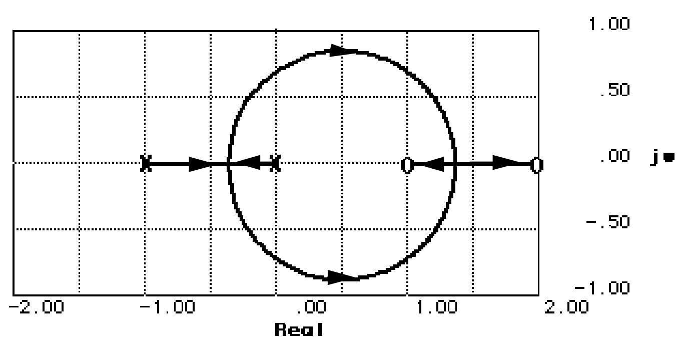
d. K = 13.125

18.

Assume that root locus is epsilon away from the asymptotes. Thus, $\sigma_a = \frac{(0 - 3 - 6) - (-\alpha)}{2} \approx -1$;

Angle $= \frac{(2k+1)\pi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}$. Hence $\alpha = 7$. Checking assumption at $-1 \pm j100$ yields -180° with K = 9997.02.

19.



a. Breakaway: -0.37 for K = 0.07. Break-in: 1.37 for K = 13.93

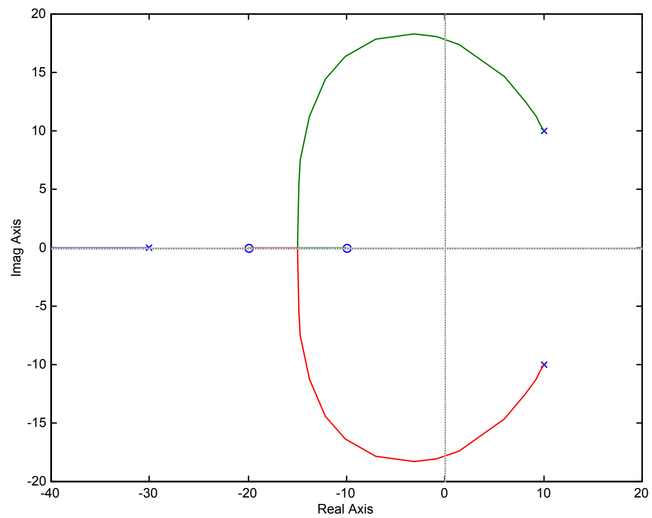
b. Imaginary axis crossing: $\pm j0.71$ for K = 0.33

c. System stable for K < 0.33

d. Searching 120° find point on root locus at $0.5\angle 120^{\circ} = -0.25 \pm j0.433$ for K = 0.1429

20.

a.

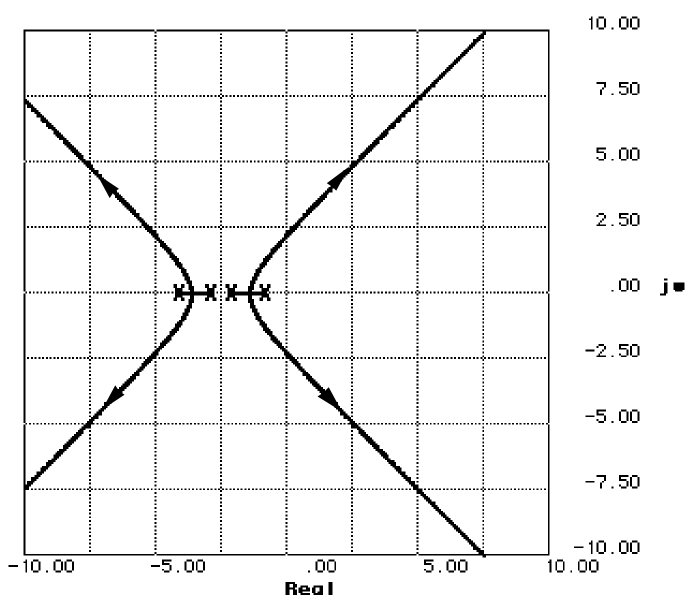


b. $0 < K < 23.93$

c. $K = 81.83$ @ $-13.04 \pm j13.04$

d. At the break-in point, $s = -14.965$, $K = 434.98$.

21.



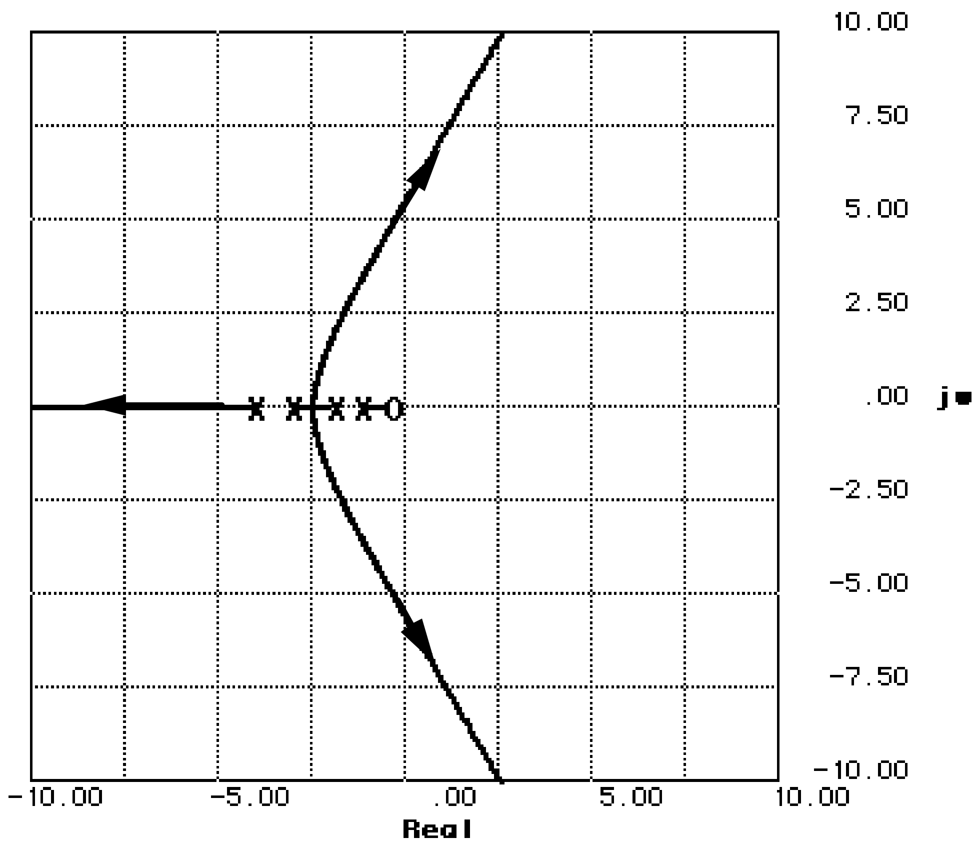
a. Asymptotes: $\sigma_{int} = \dfrac{(-1\ -2\ -3\ -4) - (0)}{4} = -\dfrac{5}{2}$; Angle $= \dfrac{(2k+1)\pi}{4} = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

b. Breakaway: -1.38 for K = 1 and -3.62 for K = 1

c. Root locus crosses the imaginary axis at $\pm j2.24$ for K = 126. Thus, stability for K < 126.

d. Search 0.7 damping ratio line (134.427 degrees) for 180^0. Point is $1.4171\angle 134.427^o =$
$-0.992 \pm j1.012$ for K = 10.32.

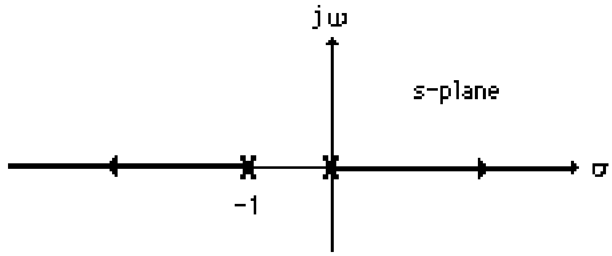
e. Without the zero, the angles to the point $\pm j5.5$ add up to -265.074^o. Therefore the contribution of the zero must be $265.074 - 180 = 85.074^o$. Hence, $\tan 85.074^o = \frac{5.5}{z_c}$, where $-z_c$ is the location of the zero. Thus, $z_c = 0.474$.



f. After adding the zero, the root locus crosses the imaginary axis at $\pm j5.5$ for K = 252.5. Thus, the system is stable for K < 252.5.

g. The new root locus crosses the 0.7 damping ratio line at $2.7318\angle 134.427^o$ for K = 11.075 compared to $1.4171\angle 134.427^o$ for K = 10.32 for the old root locus. Thus, the new system's settling time is shorter, but with the same percent overshoot.

22.

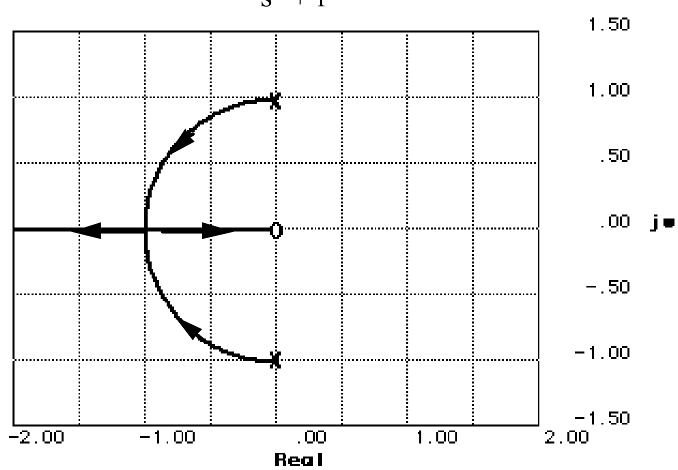


23.

$$T(s) = \frac{1}{s^2+\alpha s+1} = \frac{\dfrac{1}{s^2+1}}{1+\dfrac{\alpha s}{s^2+1}}$$

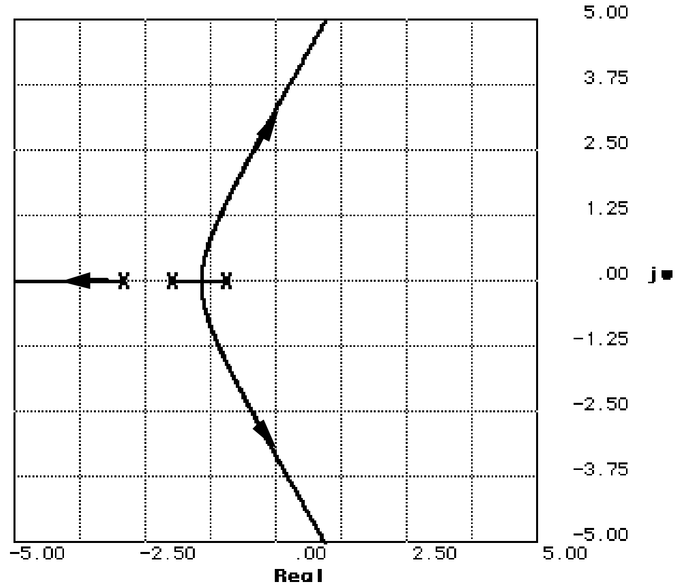
. Thus an equivalent system has $G(s) = \dfrac{1}{s^2+1}$ and $H(s) = \alpha s$.

Plotting a root locus for $G(s)H(s) = \dfrac{\alpha s}{s^2+1}$, we obtain,



24.

a.



b. Root locus crosses 20% overshoot line at $1.8994 \angle 117.126^{\circ} = -0.866 \pm j1.69$ for K = 9.398.

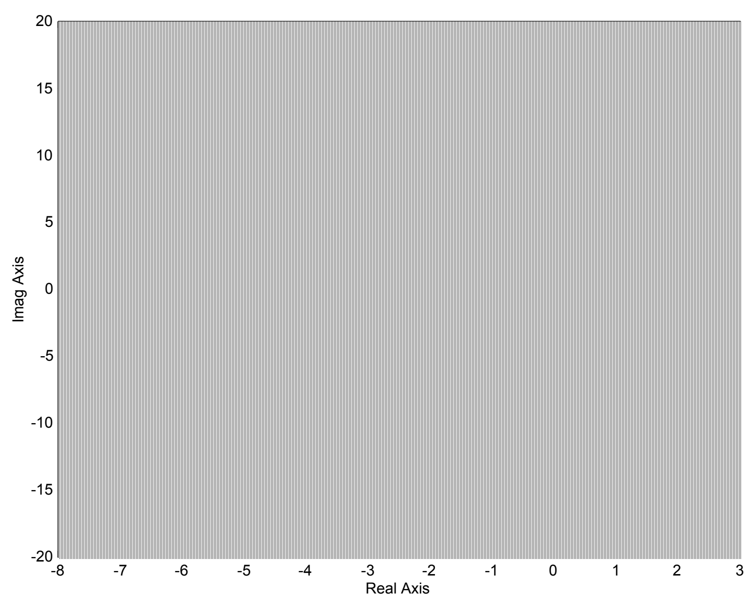
c. $T_s = \frac{4}{0.866} = 4.62$ seconds; $T_p = \frac{\pi}{1.69} = 1.859$ seconds

d. Root locus crosses imaginary axis at $\pm j3.32$ for K = 60. Therefore stability for K < 60.

e. Other poles with same gain as dominant poles: $\sigma = -4.27$

25.

a.



b.

$$\sigma_a = \frac{(-6-5-4-2)-(2)}{4-2} = -9.5$$

$$\theta_a = \frac{(2k+1)\pi}{4-2} = \frac{\pi}{2}, \ \frac{3\pi}{2}$$

c. At the $j\omega$ axis crossing, $K = 115.6$. Thus for stability, $0 < K < 115.6$.

d. Breakaway points at σ = -2.524 @ K = 0.496 and σ = -5.576 @ K = 0.031.

e. For 25% overshoot, Eq. (4.39) yields ζ = 0.404. Searching along this damping ratio line, we find the 180^0 point at $-0.6608 + j1.496$ where $K = 35.98$.

f. $-7.839 \pm j7.425$

g. Second-order approximation not valid because of the existence of closed-loop zeros in the rhp.

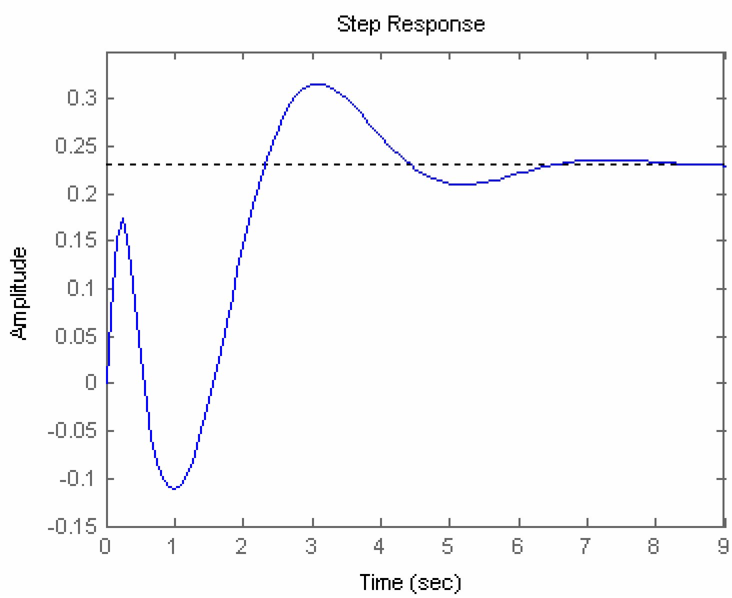
h.

Program:

```
numg=35.98*[1 -2 2];
deng=poly([-2 -4 -5 -6]);
G=tf(numg,deng);
T=feedback(G,1)
step(T)
```

Computer response:

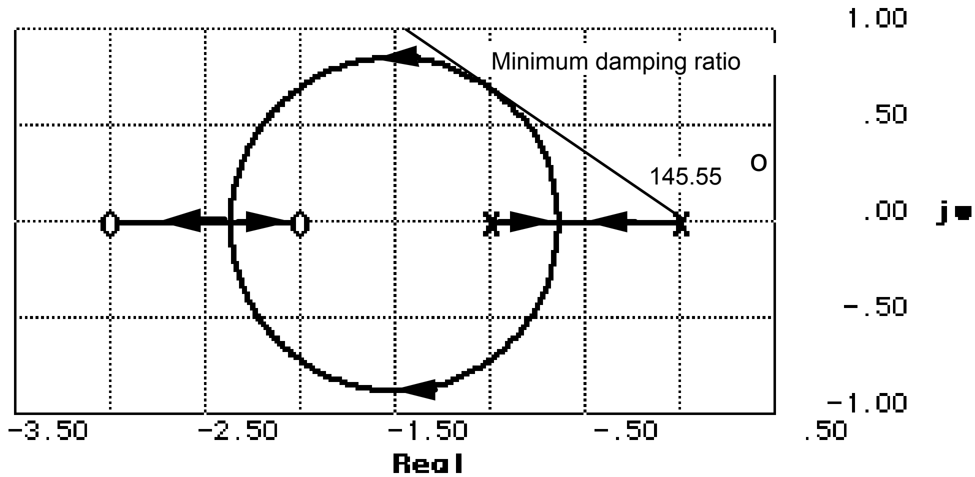
```
Transfer function:
    35.98 s^2 - 71.96 s + 71.96
------------------------------------
s^4 + 17 s^3 + 140 s^2 + 196 s + 312
```



Simulation shows over 30% overshoot and nonminimum-phase behavior. Second-order approximation not valid.

26.

a. Draw root locus and minimum damping ratio line.



Minimum damping ratio is $\zeta = \cos(180 - 145.55) = \cos 34.45^{o} = 0.825$. Coordinates at tangent point of $\zeta = 0.825$ line with the root locus is approximately $-1 + j0.686$. The gain at this point is 0.32.

b. Percent overshoot for $\zeta = 0.825$ is 1.019%.

c. $T_s = \frac{4}{1} = 4$ seconds; $T_p = \frac{\pi}{0.6875} = 4.57$ seconds

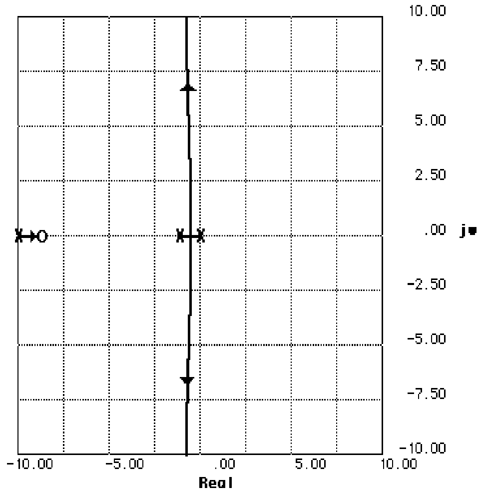
d. Second-order approximation is not valid because of the two zeros and no pole-zero cancellation.

27.

The root locus intersects the 0.55 damping ratio line at $-7.217 + j10.959$ with $K = 134.8$. A justification of a second-order approximation is not required. The problem stated the requirements in terms of damping ratio and not percent overshoot, settling time, or peak time. A second-order approximation is required to draw the equivalency between percent overshoot, settling time, and peak time and damping ratio and natural frequency.

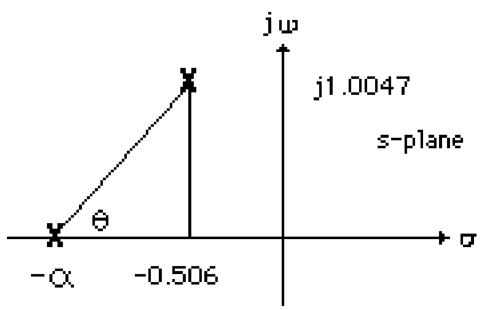
28.

Since the problem stated the settling time at large values of K, assume that the root locus is approximately close to the vertical asymptotes. Hence, $\sigma_{int} = \frac{-11 + \alpha}{2} = -\frac{4}{T_s}$. Since T_s is given as 4 seconds, $\sigma_{int} = -1$ and $\alpha = 9$. The root locus is shown below.



29.

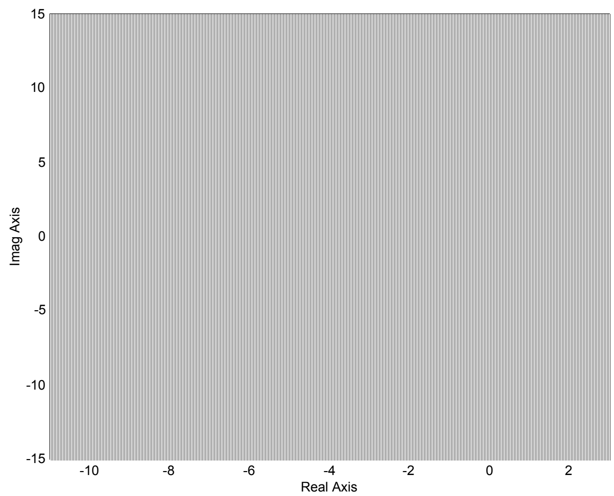
The design point is $-0.506 \pm j1.0047$. Excluding the pole at $-\alpha$, the sum of angles to the design point is -141.37^o. Thus, the contribution of the pole at $-\alpha$ is $141.37 - 180 = -38.63^o$. The following geometry applies:



Hence, $\tan\theta = \dfrac{1.0047}{\alpha - 0.506} = \tan 38.63 = 0.799$. Thus $\alpha = 1.763$. Adding this pole at -1.763 yields 180^o at $-0.506 \pm j1.0047$ with $K = 7.987$.

30.

a.



b. Searching along the 10% overshoot line (angle = 126.239°), the point - 0.7989 + j1.0898 yields 180° for K = 81.74.

c. Higher-order poles are located at approximately –6.318 and –7.084. Since these poles are more than 5 times further from the imaginary axis than the dominant pole found in (b), the second-order approximation is valid.

d. Searching along the imaginary axis yields 180° at j2.53, with K = 394.2.

Hence, for stability, $0 < K < 394.2$.

31.

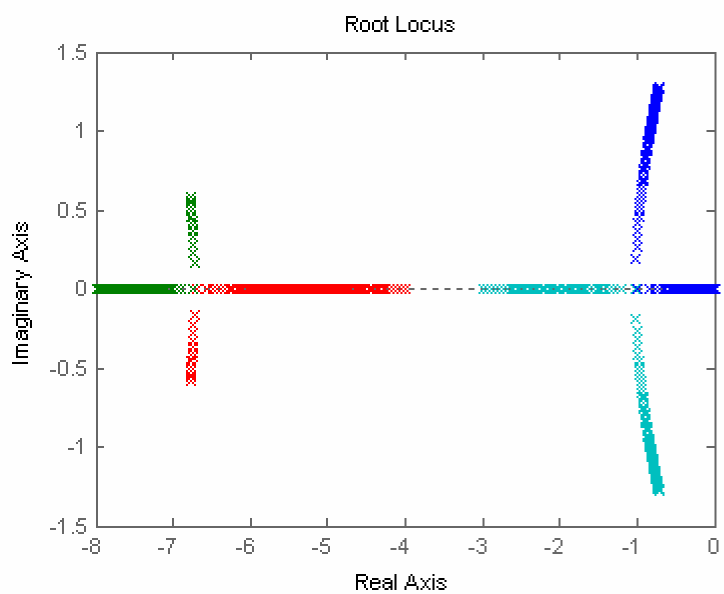
Program:

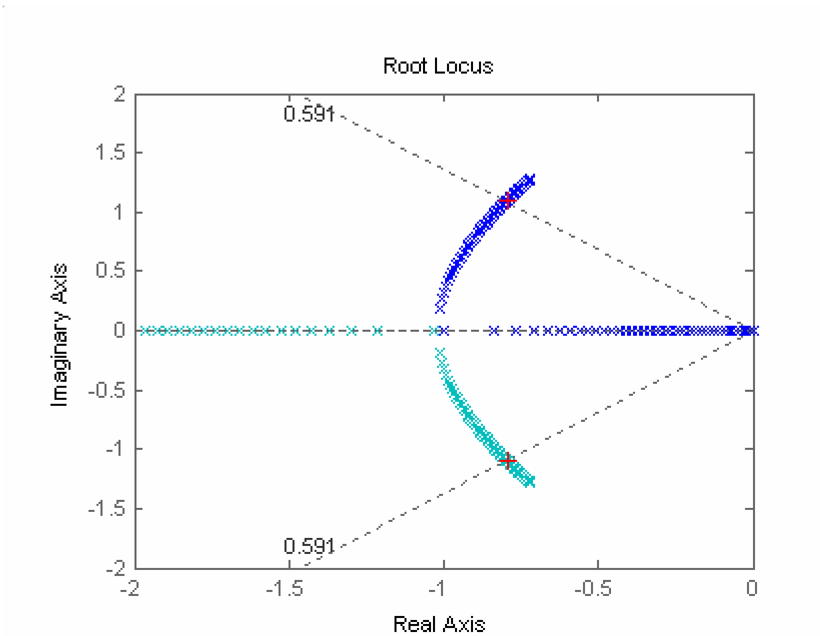
```
pos=10;
z=-log(pos/100)/sqrt(pi^2+[log(pos/100)]^2)
numg=1;
deng=poly([0 -3 -4 -8]);
G=tf(numg,deng)
Gzpk=zpk(G)
rlocus(G,0:1:100)
pause
axis([-2,0,-2,2])
sgrid(z,0)
pause
[K,P]=rlocfind(G)
T=feedback(K*G,1)
pause
step(T)
```

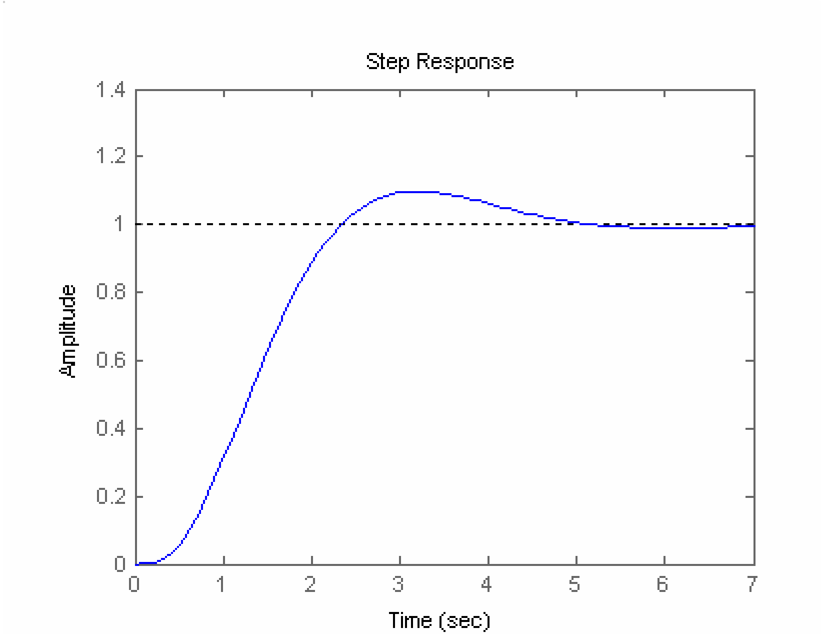
Computer response:

```
z =

    0.5912


Transfer function:
             1
```

```
-----------------------------
s^4 + 15 s^3 + 68 s^2 + 96 s


Zero/pole/gain:
         1
-------------------
s (s+8) (s+4) (s+3)

Select a point in the graphics window

selected_point =

  -0.7994 + 1.0802i


K =

   81.0240


P =

  -7.1058
  -6.2895
  -0.8023 + 1.0813i
  -0.8023 - 1.0813i


Transfer function:
               81.02
-------------------------------------
s^4 + 15 s^3 + 68 s^2 + 96 s + 81.02
```

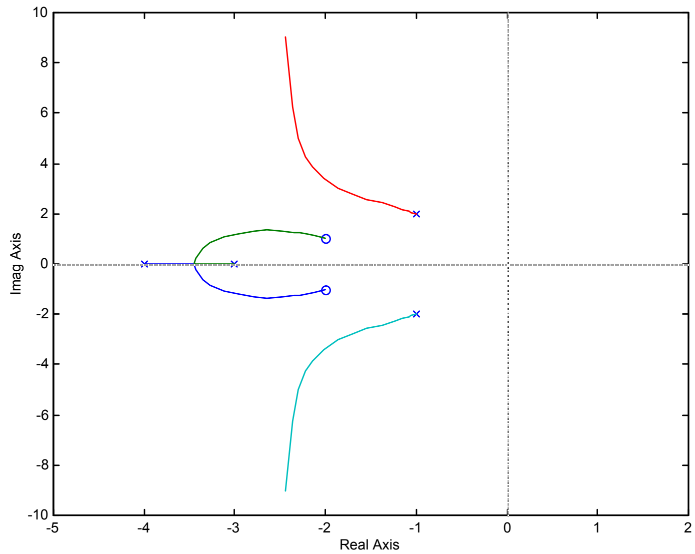






32.

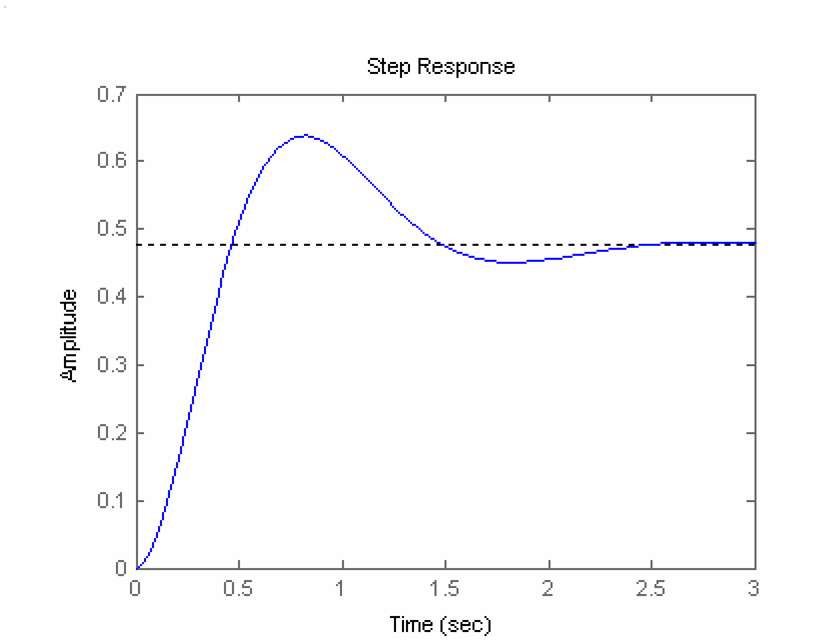
a. For a peak time of 1s, search along the horizontal line, Im = $\pi/ T_p = \pi$, to find the point of intersection with the root locus. The intersection occurs at $-2 \pm j\pi$ at a gain of 11.



b.

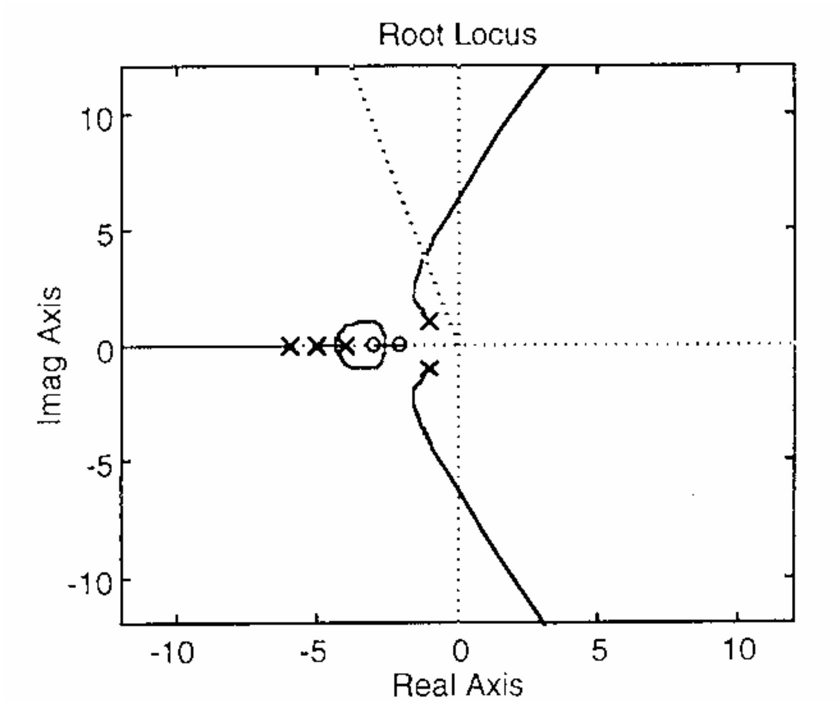
Program:

```
numg=11*[1 4 5];
deng=conv([1 2 5],poly([-3 -4]));
G=tf(numg,deng);
T=feedback(G,1);
step(T)
```



33.

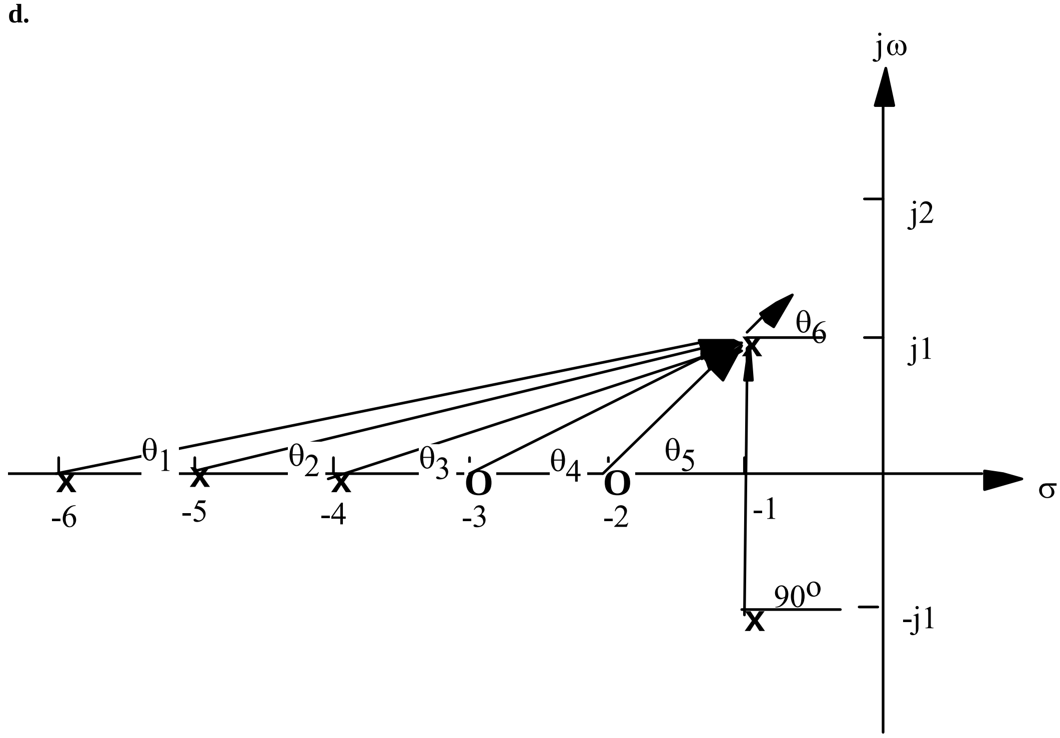
a.



b. Searching the jω axis for 180°, we locate the point j6.29 at a gain of 447.83.

c. Searching for maximum gain between -4 and -5 yields the breakaway point, -4.36. Searching for minimum gain between -2 and -3 yields the break-in point, -2.56.

d.



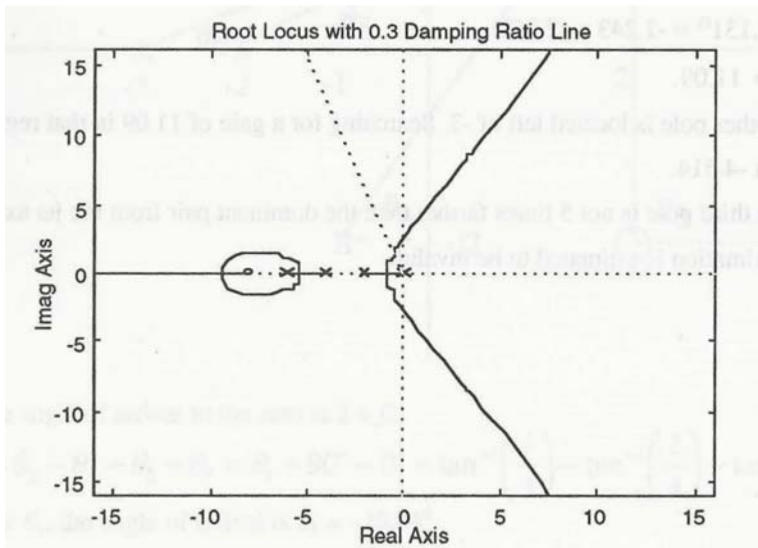
To find the angle of departure from the poles at -1±j1: $-\theta_1 - \theta_2 - \theta_3 + \theta_4 + \theta_5 - \theta_6 - 90^0$

$= -\tan^{-1}(1/5) - \tan^{-1}(1/4) - \tan^{-1}(1/3) + \tan^{-1}(1/2) + \tan^{-1}(1/1) - \theta_6 - 90^o = 180^0$. Thus, $\theta_6 = -242.22^o$

e. Searching along the $\zeta = 0.3$ line ($\theta = 180 - \cos^{-1}(\zeta) = 107.458^o$) for 180^o we locate the point $3.96 \angle 107.458^o = -1.188 \pm j3.777$. The gain is 127.133.

34.

a.

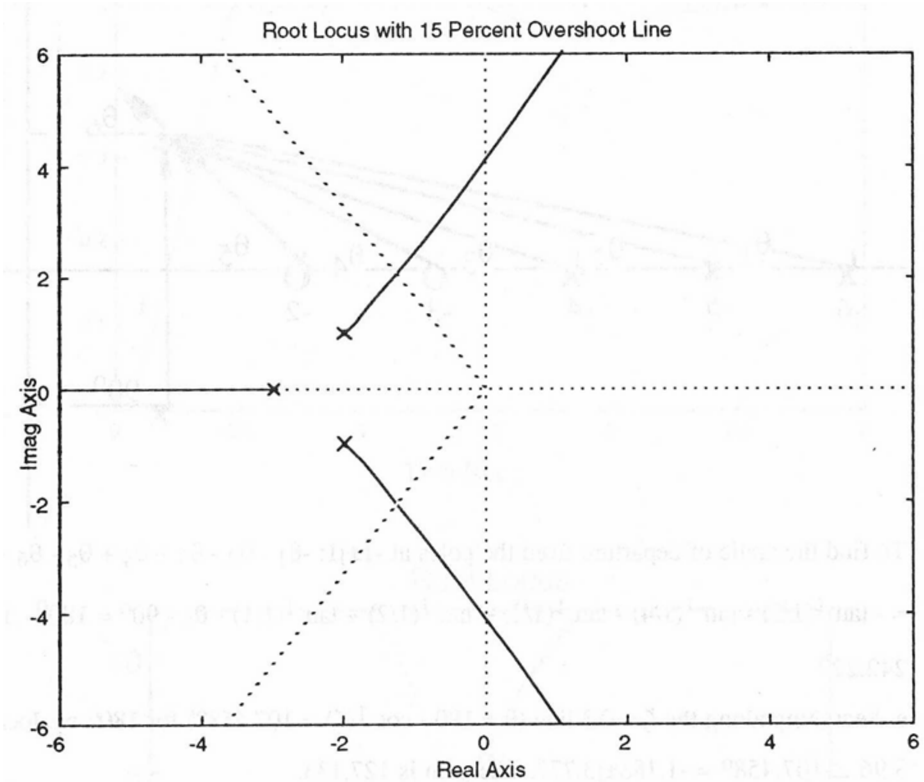


b. Searching the $j\omega$ axis for 180^o, we locate the point j2.56 at a gain of 30.686.

c. Searching for maximum gain between 0 and -2 yields the breakaway point, -0.823. Searching for maximum gain between -4 and -6 yields the breakaway point, -5.37. Searching for minimum gain beyond -8 yields the break-in point, -9.39.

e. Searching along the $\zeta = 0.3$ line ($\theta = 180 - \cos^{-1}(\zeta) = 107.458^o$) for 180^o we locate the point $1.6 \angle 107.458^o = -0.48 \pm j1.53$. The gain is 9.866.

35.



a. Searching the 15% overshoot line ($\zeta = 0.517$; $\theta = 121.131^{\circ}$) for 180°, we find the point $2.404 \angle 121.131^{\circ} = -1.243 + j2.058$.

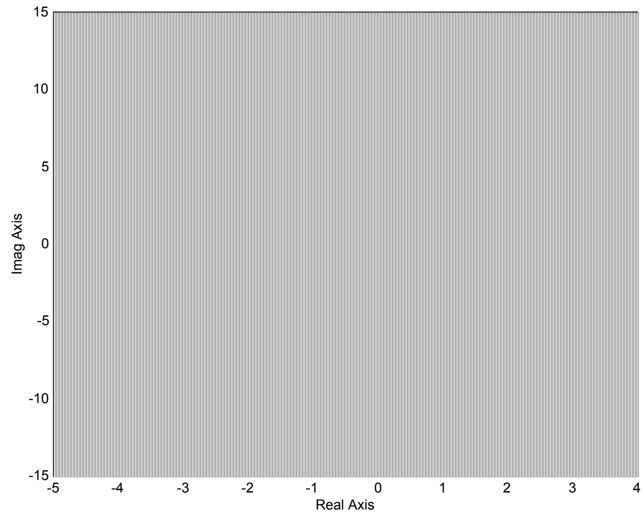
b. K = 11.09.

c. Another pole is located left of -3. Searching for a gain of 11.09 in that region, we find the third pole at -4.514.

d. The third pole is not 5 times farther than the dominant pair from the jω axis. the second-order approximation is estimated to be invalid.

36.

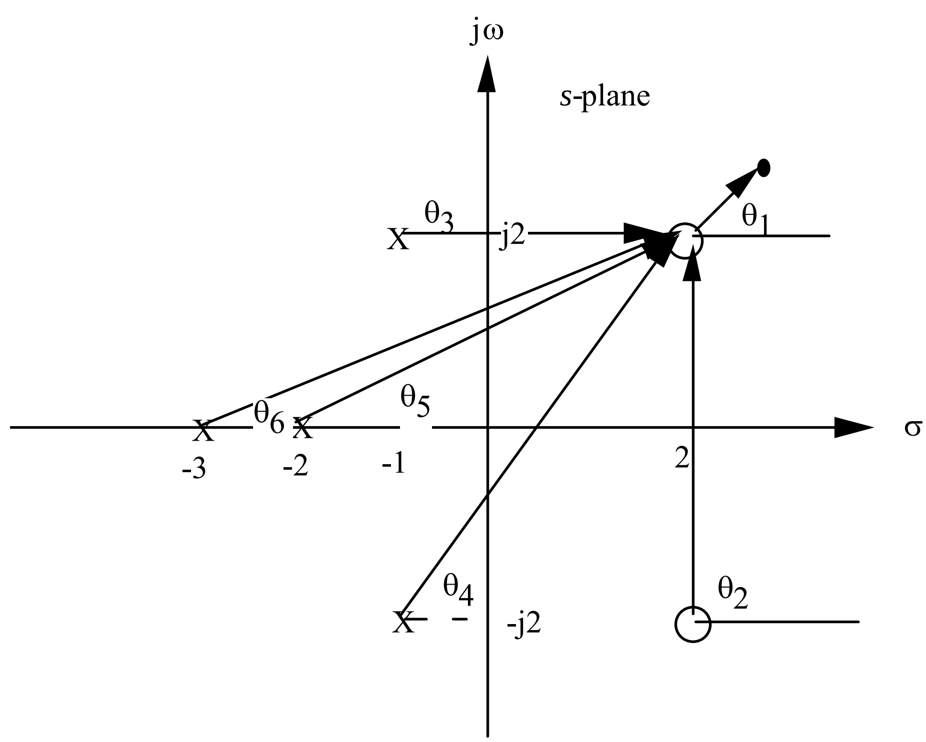
a.



b. Searching the $j\omega$ axis for 180°, we locate the point $j1.69$ at a gain of 4.249.

c. Searching between -2 and -3 for maximum gain, the breakaway is found at -2.512.

d.



To find the angle of arrival to the zero at $2 + j2$:

$$\theta_1 + \theta_2 - \theta_3 - \theta_4 - \theta_5 - \theta_6 = \theta_1 + 90^\circ - 0^\circ - \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{2}{4}\right) - \tan^{-1}\left(\frac{2}{5}\right) = 180^\circ$$

Solving for θ_1, the angle of arrival is $\theta_1 = -191.5^0$.

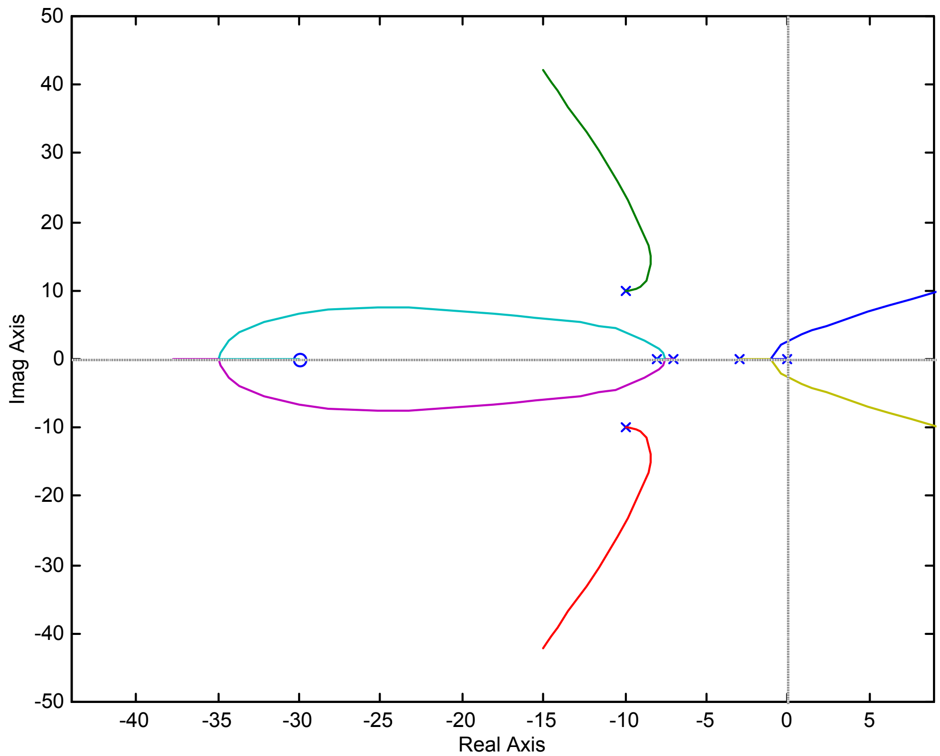
e. The closed-loop zeros are the poles of $H(s)$, or $-1 \pm j2$.

f. Searching the $\zeta = 0.358$; ($\theta = 110.97^o$) for 180^o, we find the point

= $-0.6537+j1.705$. The gain, $K = 0.8764$.

g. Higher-order poles are at $-2.846 \pm j1.731$. These are not 5 times further than the dominant poles. Further, there are closed-loop zeros at $-1 \pm j2$ that are not cancelled any higher-order poles. Thus, the second-order approximation is not valid.

37.



a. The root locus crosses the imaginary axis at $j2.621$ with $K = 4365$. Therefore, the system is stable for $0 < K < 4365$.

b. Search the 0.707 damping ratio line for 180^o and find $-0.949 + j0.949$ with $K = 827.2$.

c. Assume critical damping where root locus breaks away from the real axis. Locus breaks away at –1.104 with K = 527.6.

38.

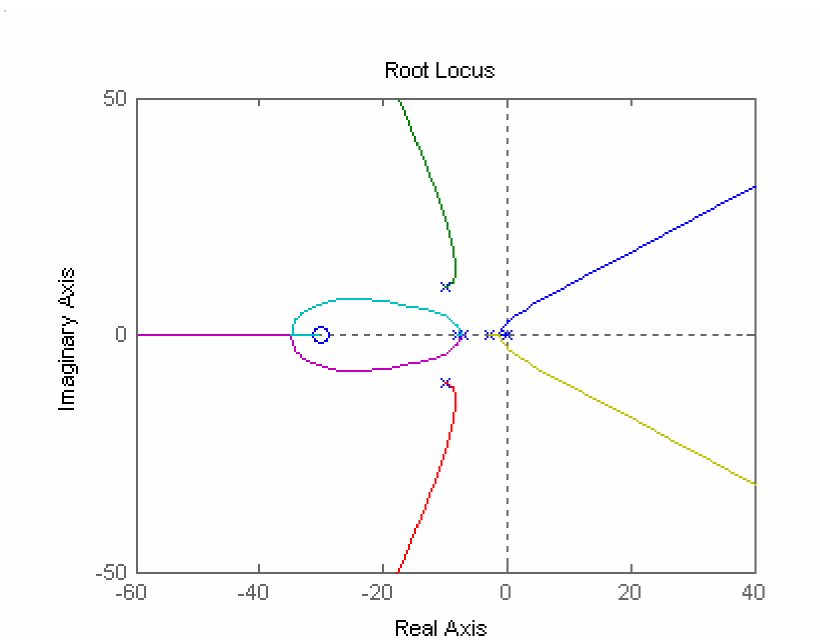
Program:
```
numg=1;
deng=poly([0 -3 -7 -8]);
numh=[1 30];
denh=[1 20 200];
G=tf(numg,deng)
Gzpk=zpk(G)
H=tf(numh,denh)
```

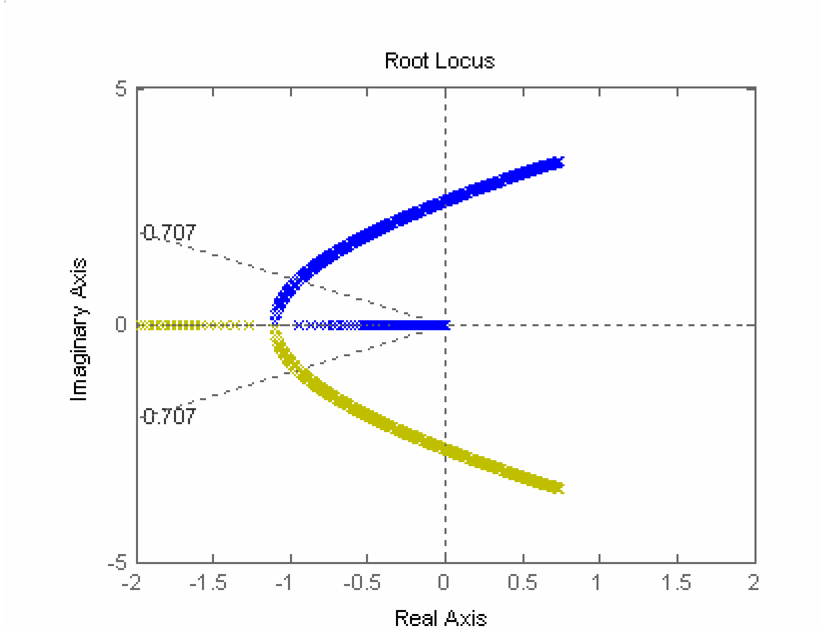
```
rlocus(G*H)
pause
K=0:10:1e4;
rlocus(G*H,K)
sgrid(0.707,0)
axis([-2,2,-5,5]);
pause
for i=1:1:3;
[K,P]=rlocfind(G*H)
end
T=feedback(K*G,H)
step(T)
```

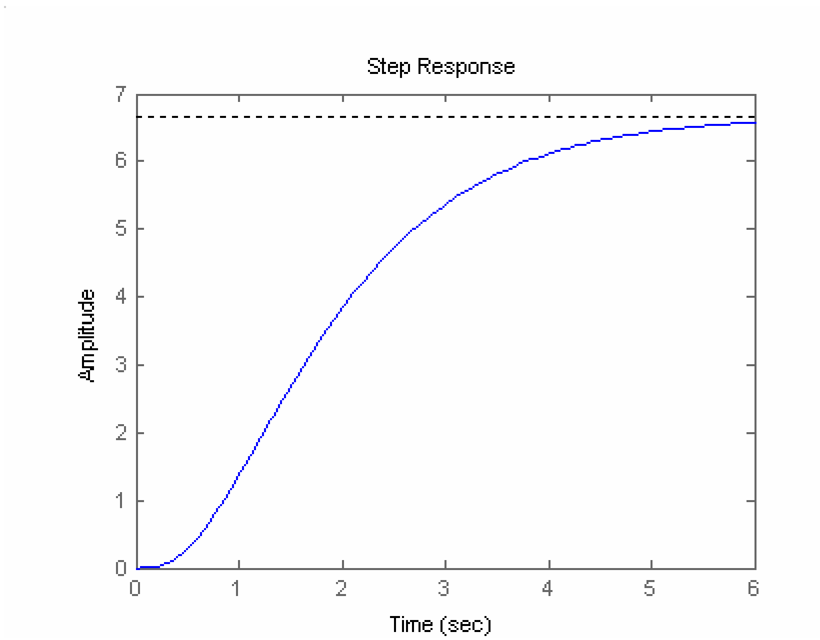
Computer response:

```
Transfer function:
              1
------------------------------
s^4 + 18 s^3 + 101 s^2 + 168 s


Zero/pole/gain:
         1
-------------------
s (s+8) (s+7) (s+3)


Transfer function:
     s + 30
----------------
s^2 + 20 s + 200

Select a point in the graphics window

selected_point =

  -0.9450 + 0.9499i


K =

  828.1474


P =

  -9.9500 +10.0085i
  -9.9500 -10.0085i
  -8.1007 + 1.8579i
  -8.1007 - 1.8579i
  -0.9492 + 0.9512i
  -0.9492 - 0.9512i

Select a point in the graphics window

selected_point =

   0.0103 + 2.6385i


K =

  4.4369e+003


P =

  -9.7320 +10.0691i
```

```
  -9.7320 -10.0691i
  -9.2805 + 3.3915i
  -9.2805 - 3.3915i
   0.0126 + 2.6367i
   0.0126 - 2.6367i

Select a point in the graphics window

selected_point =

  -1.0962 - 0.0000i


K =

  527.5969


P =

  -9.9682 +10.0052i
  -9.9682 -10.0052i
  -7.9286 + 1.5303i
  -7.9286 - 1.5303i
  -1.1101
  -1.0962


Transfer function:

       527.6 s^2 + 1.055e004 s + 1.055e005
-------------------------------------------------
s^6 + 38 s^5 + 661 s^4 + 5788 s^3 + 23560 s^2

                        + 3.413e004 s + 1.583e004
```







39.

a. Search $j\omega = j10$ line for 180^o and find $-4.533 + j10$ with K = 219.676.

b. $K_a = \dfrac{219.676 \text{ x } 6}{20}$

c. A settling time of 0.4 seconds yields a real part of -10. Thus if the zero is at the origin, G(s)

$\dfrac{K}{s(s+20)}$, which yields complex poles with -10 as the real part. At the design point, -10 + j10, K = 200.

40.

a. Searching along $\zeta\omega_n = -1$ for 180^o, find $-1 + j2.04$ with K = 170.13.

b. Assume critical damping when root locus breaks away form the real axis. Searching for maximum gain, the breakaway point is at -1.78 with K = 16.946.

41.

$T(s) = \frac{K}{s^3 + 6s^2 + 5s + K}$. Differentiating the characteristic equation, $s^3 + 6s^2 + 5s + K = 0$, yields,

$$3s^2 \frac{\delta s}{\delta K} + 12s \frac{\delta s}{\delta K} + 5 \frac{\delta s}{\delta K} + 1 = 0.$$

Solving for $\frac{\delta s}{\delta K}$,

$$\frac{\delta s}{\delta K} = \frac{-1}{3s^2 + 12s + 5}$$

The sensitivity of s to K is

$$S_{s:K} = \frac{K}{s}\frac{\delta s}{\delta K} = \frac{K}{s}\frac{-1}{3s^2 + 12s + 5}$$

a. Search along the $\zeta = 0.591$ line and find the root locus intersects at $s = 0.7353\angle 126.228^o$ = $-0.435 + j0.593$ with K = 2.7741. Substituting s and K into $S_{s:K}$ yields

$$S_{s:K} = 0.487 - j0.463 = 0.672\angle -43.553^o$$

b. Search along the $\zeta = 0.456$ line and find the root locus intersects at $s = 0.8894\angle 117.129^o$ = $-0.406 + j0.792$ with K = 4.105. Substituting s and K into $S_{s:K}$ yields

$$S_{s:K} = 0.482 - j0.358 = 0.6\angle -36.603^o$$

c. Least sensitive: $\zeta = 0.456$.

42.

The sum of the feedback paths is $H_e(s) = 1 + 0.02s + \frac{0.00076s^3}{s+0.06}$. Thus,

$$H_e(s) = \frac{0.00076\left(s^3 + 26.316s^2 + 1317.4s + 78.947\right)}{s + 0.06}$$

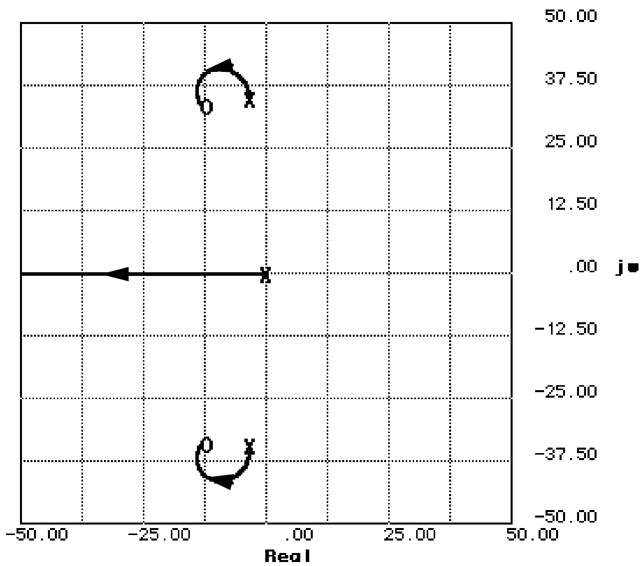
and

$$G(s)H_e(s) = 0.00076\frac{K\left(s^3 + 26.316s^2 + 1317.4s + 78.947\right)}{s(s+0.06)\left(s^2 + 7s + 1220\right)}$$

$$G(s)H_e(s) = 0.00076\frac{K([s + 0.06]\,[(s + 13.128 + 33.815i)\,(s + 13.128 - 33.815i)])}{s(s+0.06)\left(s^2 + 7s + 1220\right)}$$

$$G(s)H_e(s) = 0.00076\frac{K([s + 13.128 + 33.815i]\,[s + 13.128 - 33.815i])}{s([s + 3.5 + 34.753i]\,[s + 3.5 - 34.753i])}$$

Plotting the root locus,



Searching vertical lines to calibrate the root locus, we find that 0.00076K is approximately 49.03 at

-10 ± j41.085. Searching the real axis for 0.00076K = 49.03, we find the third pole at -36.09.

a. $\zeta = \cos(\tan^{-1}(\frac{41.085}{10})) = 0.236$

b. $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 46.63\%$

c. $\omega_n = \sqrt{10^2 + 41.085^2} = 42.28$ rad/s

d. $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{10} = 0.4$ seconds

e. $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{41.085} = 0.076$ seconds

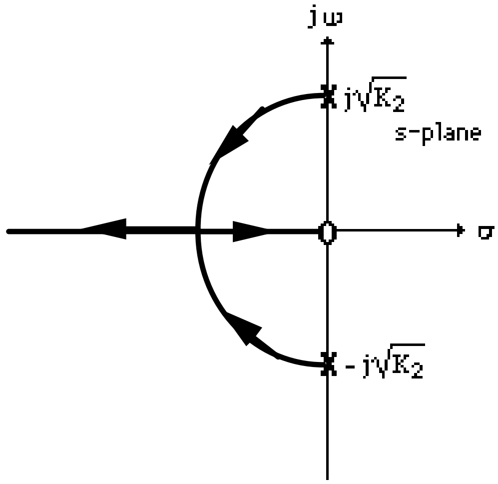
43.

Push K_2 to the right past the summing junction and find, $T(s) = (1 + \frac{K_1 s}{K_2})(\frac{K_2}{s^2 + K_3 s + K_2})$

$= \frac{K_1 (s + \frac{K_2}{K_1})}{s^2 + K_3 s + K_2}$. Changing form, $T(s) = \frac{\frac{K_1(s+\frac{K_2}{K_1})}{s^2 + K_2}}{1 + \frac{K_3 s}{s^2 + K_2}}$. Thus, $G(s)H(s) = \frac{K_3 s}{s^2 + K_2}$. Sketching the

root locus,

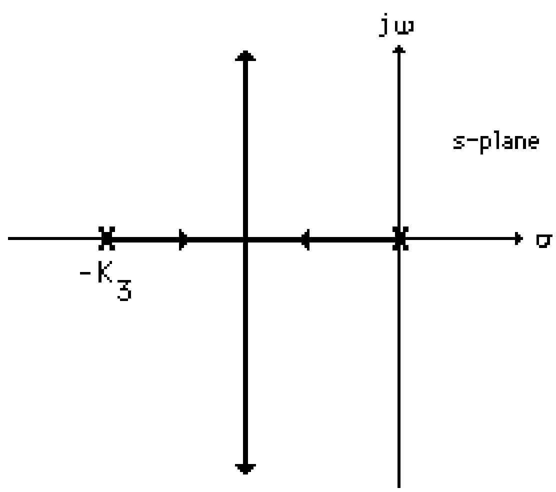
a.



b. $T(s) = \dfrac{\dfrac{K_1(s+\frac{K_2}{K_1})}{s^2 + K_2}}{1 + \dfrac{K_3 s}{s^2 + K_2}} = \dfrac{K_1(s+\frac{K_2}{K_1})}{s^2 + K_3 s + K_2}$. Therefore closed-loop zero at $-\dfrac{K_2}{K_1}$. Notice that the zero at the origin of the root locus is not a closed-loop zero.

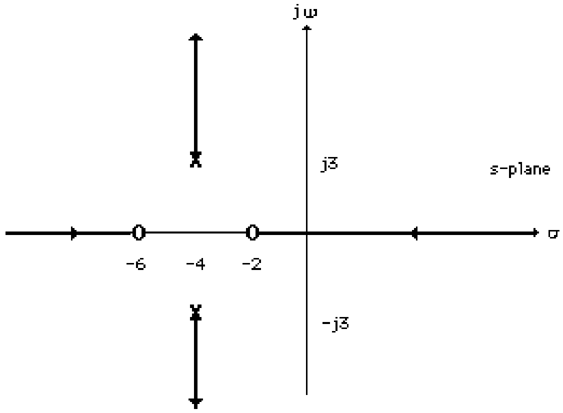
c. Push K_2 to the right past the summing junction and find, $T(s) = (1 + \dfrac{K_1 s}{K_2})\left(\dfrac{K_2}{s^2 + K_3 s + K_2}\right)$

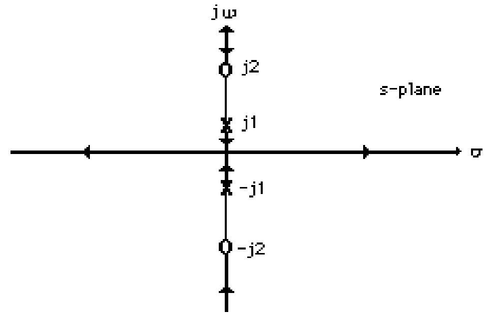
$= \dfrac{K_1\,(s + \frac{K_2}{K_1})}{s^2 + K_3 s + K_2}$. Changing form, $T(s) = \dfrac{\dfrac{K_1(s+\frac{K_2}{K_1})}{s^2 + K_3 s}}{1 + \dfrac{K_2}{s^2 + K_3 s}}$. Thus, $G(s)H(s) = \dfrac{K_2}{s^2 + K_3 s}$. Sketching the root locus,



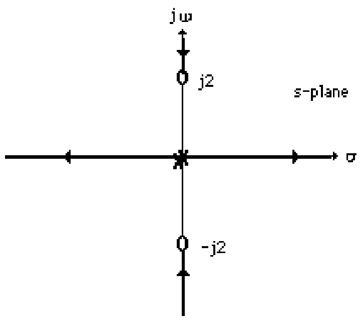
The closed-loop zero is at $-\dfrac{K_2}{K_1}$.

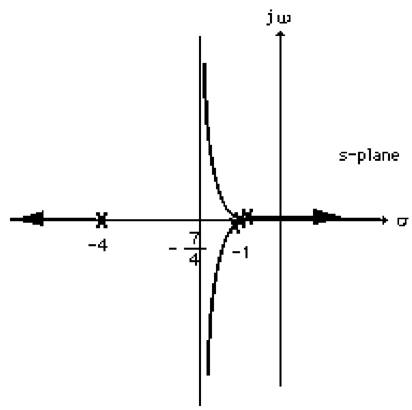
44.





a **b**





c **d**

45.

a. Using Figure P8.15(a),

$$[Ms^2+(D+D_c)s+(K+K_c)]X(s) - [D_cs+K_c]X_a(s) = 0$$

Rearranging,

$$[Ms^2+Ds+K]X(s) = -[D_cs+K_c](X(s)-X_a(s)) \tag{1}$$

where $[D_cs+K_c](X(s)-X_a(s))$ can be thought of as the input to the plant.

For the active absorber,

$$(M_cs^2+D_cs+K_c)X_a(s) - (D_cs+K_c)X(s) = 0$$

or

$$M_cs^2X_a(s)+D_cs(X_a(s)-X(s))+K_c(X_a(s)-X(s)) = 0$$

Adding $-M_cs^2X(s)$ to both sides,

$$M_cs^2(X_a(s)-X(s))+D_cs(X_a(s)-X(s))+K_c(X_a(s)-X(s)) = -M_cs^2X(s)$$

Let $X_a(s)-X(s) = X_c(s)$ and $s^2X(s) = C(s)$ = plant output acceleration. Therefore,

$$M_c s^2 X_c(s)+D_c s X_c(s)+K_c X_c(s) = -M_c C(s)$$

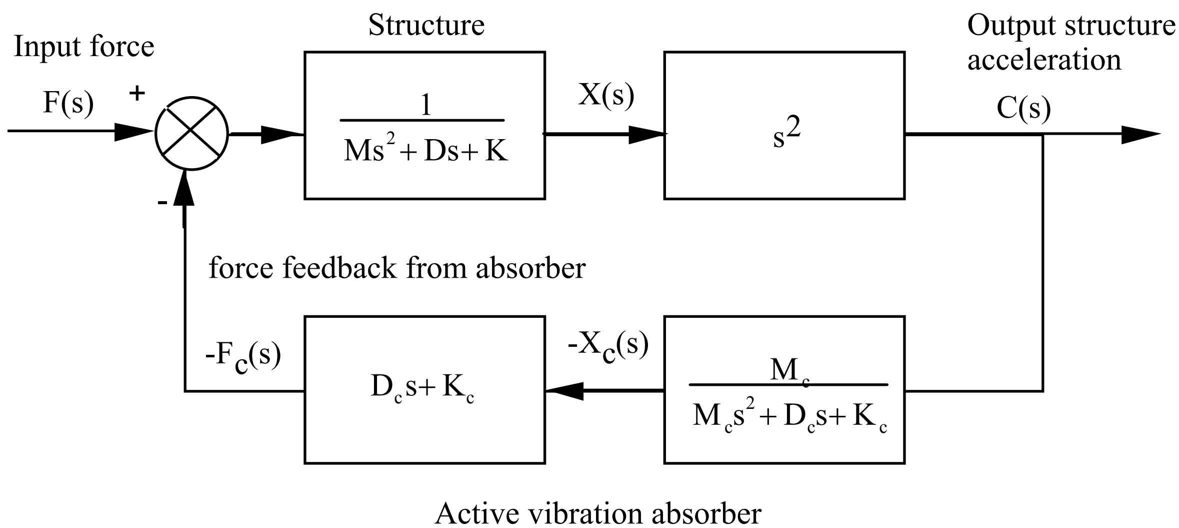
or

$$(M_c s^2+D_c s+K_c)X_c(s) = -M_c C(s) \qquad (2)$$

Using Eqs. (1) and (2), and $X_a(s)-X(s) = X_c(s)$,

$$\frac{X_c(s)}{C(s)} = \frac{-M_c}{M_c s^2+D_c s+K_c} \; ; \; \frac{X(s)}{X_c(s)} = \frac{D_c s+K_c}{Ms^2+Ds+K}$$

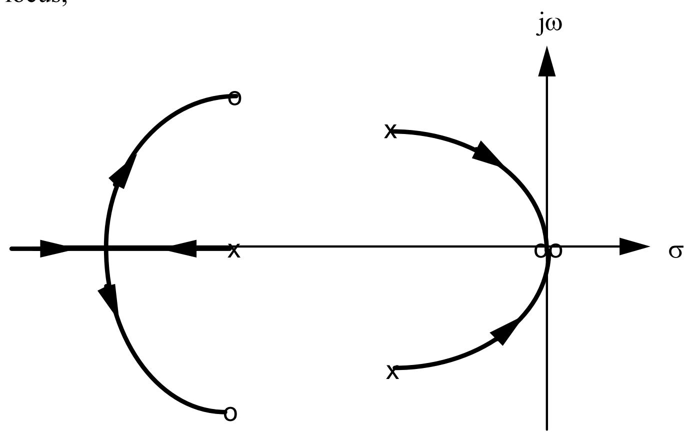
which suggests the following block diagram:



b. Substituting $M = D = K = D_c = K_c = 1$ and redrawing the block diagram above to show X(s) as the output yields a block diagram with $G(s) = \frac{1}{s^2+s+1}$ and $H(s) = \frac{M_c s^2 (s+1)}{M_c s^2+s+1}$. To study the steady-state error, we create a unity-feedback system by subtracting unity from H(s). Thus $H_e(s) = H(s)-1 = (M_c s^3-s-1)\frac{1}{M_c s^2+s+1}$. The equivalent G(s) for this unity-feedback system is $G_e(s) = \frac{G}{1+G H_e}$ $= \frac{M_c s^2+s+1}{M_c s^4+2 M_c s^3+s^3+M_c s^2+2 s^2+s}$. Hence the equivalent unity-feedback system is Type 1 and will respond with zero steady-state error for a step force input.

c. Using $G_e(s)$ in part b, we find $T(s) = \frac{G_e}{1+G_e} = \frac{M_c s^2+s+1}{(s^2+2s+2)M_c s^2+s^3+2s^2+2s+1}$. Dividing numerator and denominator by s^3+2s^2+2s+1, $T(s) = \frac{\frac{M_c s^2+s+1}{s^3+2s^2+2s+1}}{\frac{(s^2+2s+2)M_c s^2}{s^3+2s^2+2s+1}+1}$. Thus, the system has the same root locus as a system with $G(s)H(s) = \frac{(s^2+2s+2)M_c s^2}{s^3+2s^2+2s+1} = \frac{(s^2+2s+2)M_c s^2}{(s+1)(s^2+s+1)}$.

Sketching the root locus,



SOLUTIONS TO DESIGN PROBLEMS

46.

a. For a settling time of 0.1 seconds, the real part of the dominant pole is $-\frac{4}{0.1} = -40$. Searching along the $\sigma = -40$ line for 180°, we find the point –40 + j57.25 with 20,000K = 2.046 x 10^9, or K = 102,300.

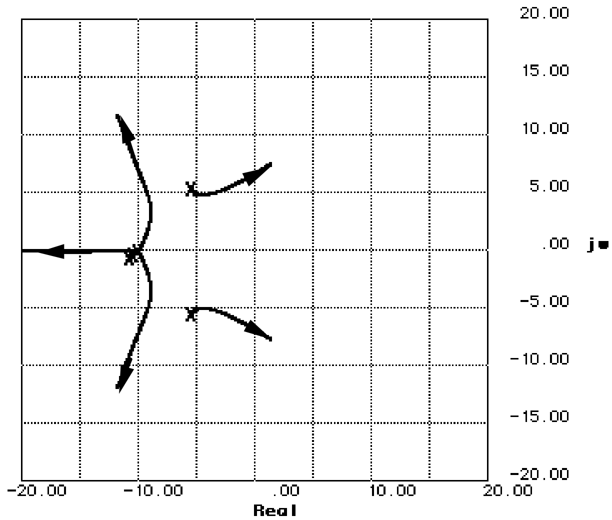
b. Since, for the dominant pole, $\tan^{-1}\left(\frac{57.25}{40}\right) = 55.058^\circ$, $\zeta = \cos(55.058^\circ) = 0.573$. Thus,

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 11.14\%$.

c. Searching the imaginary axis for 180°, we find ω = 169.03 rad/s for 20,000K = 1.43 x 10^{10}. Hence, K = 715,000. Therefore, for stability, K < 715,000.

47.

$$G(s) = \frac{61.73K}{(s+10)^3(s^2+11.11s+61.73)}$$



a. Root locus crosses the imaginary axis at ±j6.755 with 61.73K equal to 134892.8. Thus for oscillations, K = 2185.21.

b. From (a) the frequency of the oscillations is 6.755 rad/s.

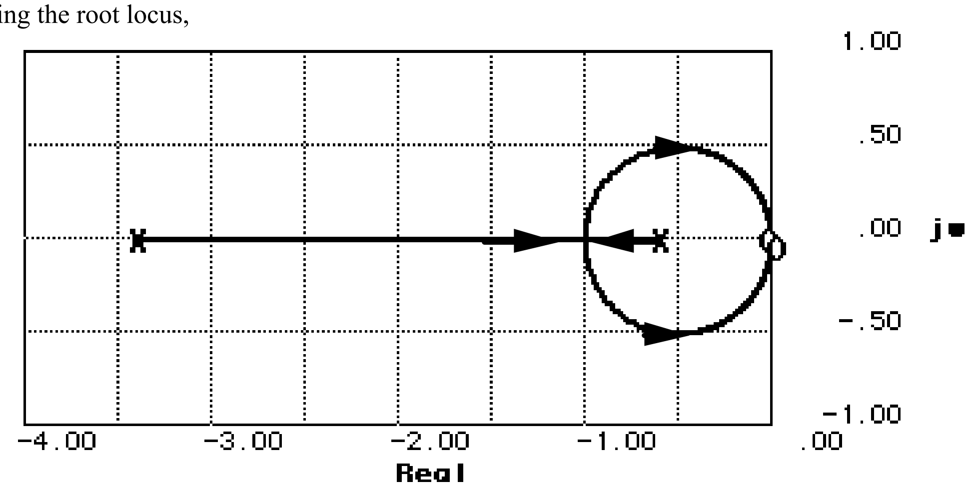
c. The root locus crosses the 20% overshoot line at $6\angle 117.126^{o} = -2.736 + j5.34$ with 61.73K = 23323.61. Thus, K = 377.83 and $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{2.736} = 1.462$ seconds.

48.

a. Finding the transfer function with C_a as a parameter,

$$\frac{\ddot{Y}_m(s)}{Y_G(s)} = \frac{s^2(2s+2)}{(C_a+1)s^2+4s+2} = \frac{\frac{2s^2(s+1)}{s^2+4s+2}}{1+\frac{C_a s^2}{s^2+4s+2}}$$

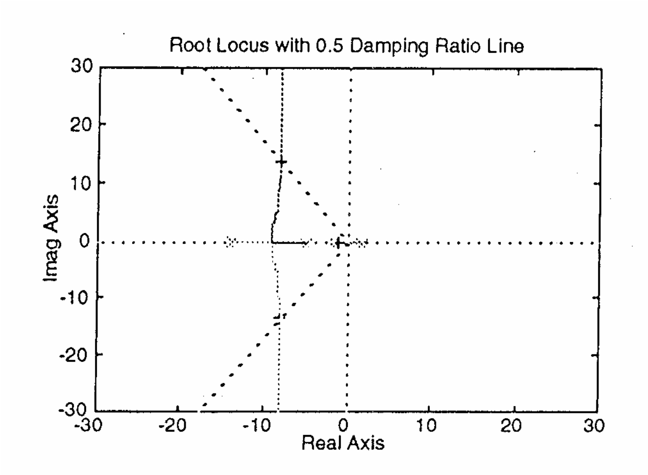
Plotting the root locus,



b. Since $2\zeta\omega_n = \frac{4}{C_a+1}$; $\omega_n^2 = \frac{2}{C_a+1}$, $\zeta^2 = \frac{2}{C_a+1} = 0.69^2$. Hence, $C_a = 3.2$.

49.

a.



b. The pole at 1.8 moves left and crosses the origin at a gain of 77.18. Hence, the system is stable for $K > 77.18$, where $K = -508K_2$. Hence, $K_2 < -0.152$.

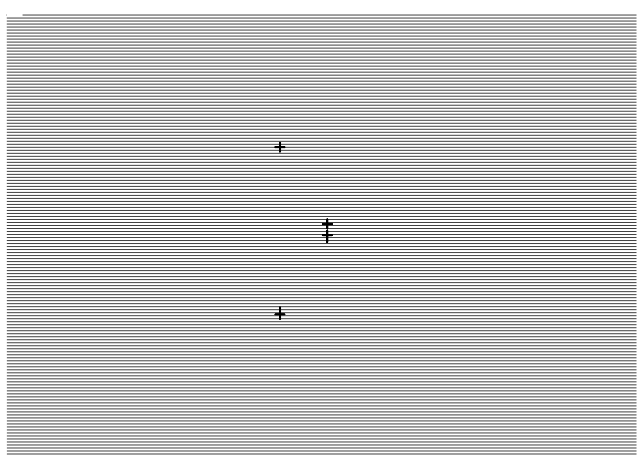
c. Search the $\zeta = 0.5$ ($\theta = 120^o$) damping ratio line for 180^o and find the point, $-8.044 + j13.932$ $= 16.087 \angle 120^o$ with a $K = -508K_2 = 240.497$. Thus, $K_2 = -0.473$.

d. Search the real axis between 1.8 and -1.6 for $K = 240.497$ and find the point -1.01.

Thus $G_e(s) = \dfrac{240.497K_1(s+1.6)}{s(s+1.01)(s+8.044+j13.932)(s+8.044-j13.932)} = \dfrac{240.497K_1(s+1.6)}{s(s+1.01)(s^2+16.088s+258.8066)}$.

Plotting the root locus and searching the $j\omega$ axis for 180^o we find $j15.792$ with $240.497K_1 = 4002.6$, or $K_1 = 16.643$.

e.

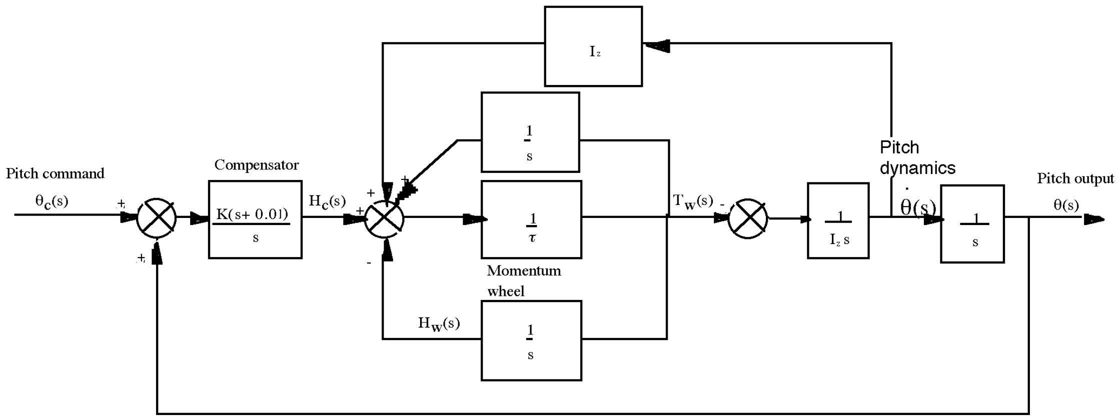


Search the $\zeta = 0.45$ ($\theta = 116.744^o$) damping ratio line for 180^o and find the point, $-6.685 + j13.267$

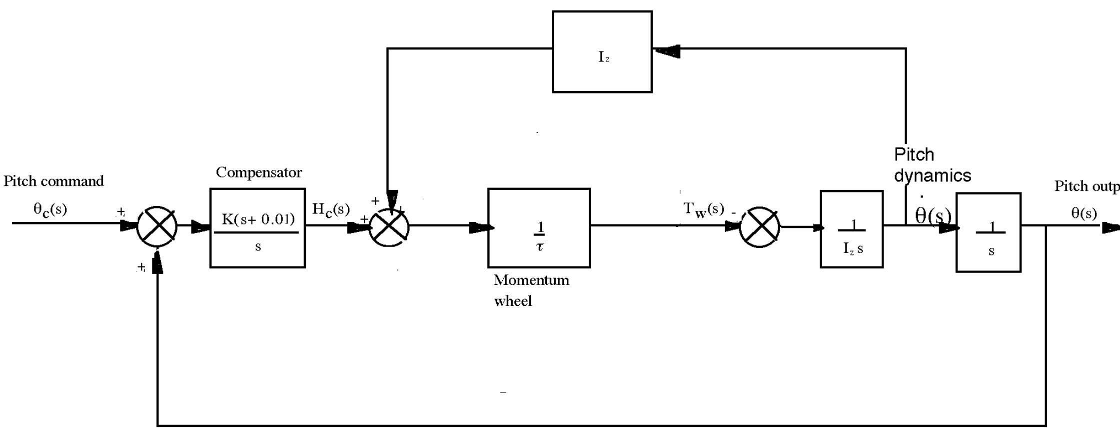
$= 14.856 \angle 116.744^o$ with a $K = 240.497K_1 = 621.546$. Thus, $K_1 = 2.584$.

50.

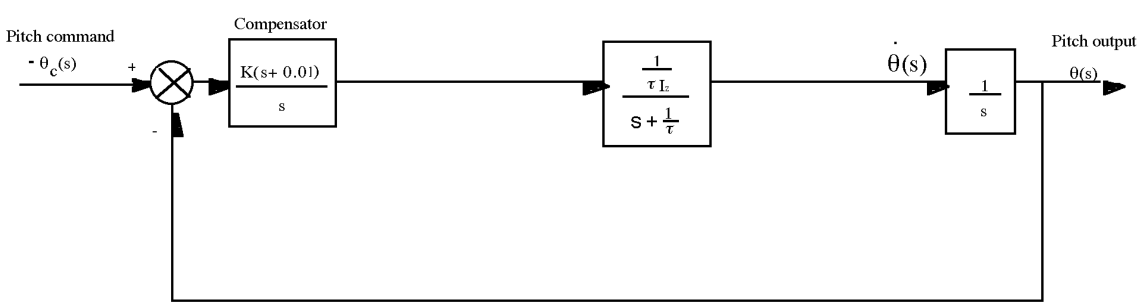
a. Update the block diagram to show the signals that form $H_{sys}(s)$.



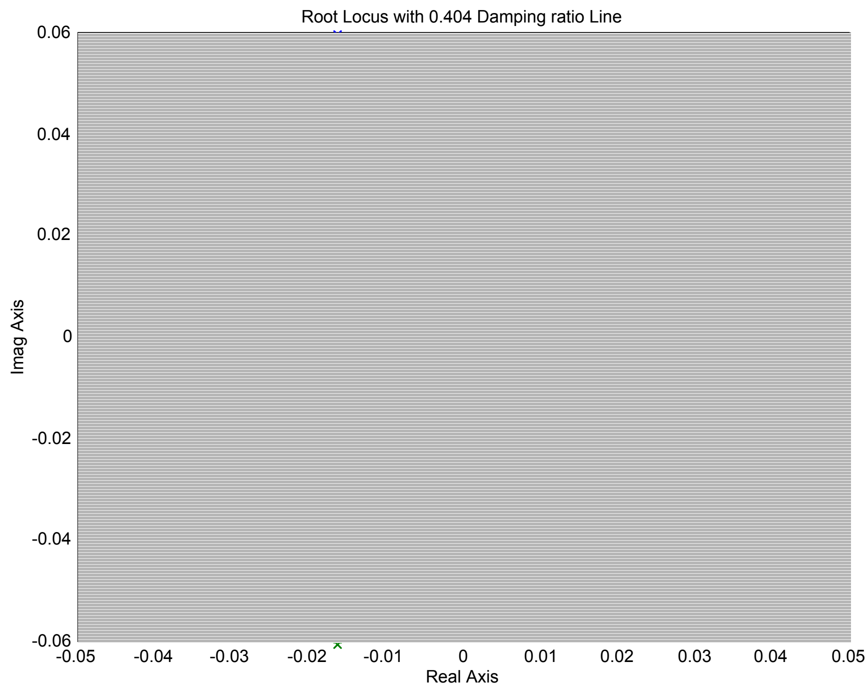
Perform block diagram reduction of the parallel paths from $T_W(s)$.



Reduce the momentum wheel assembly to a single block.



Substitute values and find $G_e(s) = \dfrac{4.514\text{x}10^{-6}K(s+0.01)}{s^2(s+0.043)}$. Plotting the root locus yields

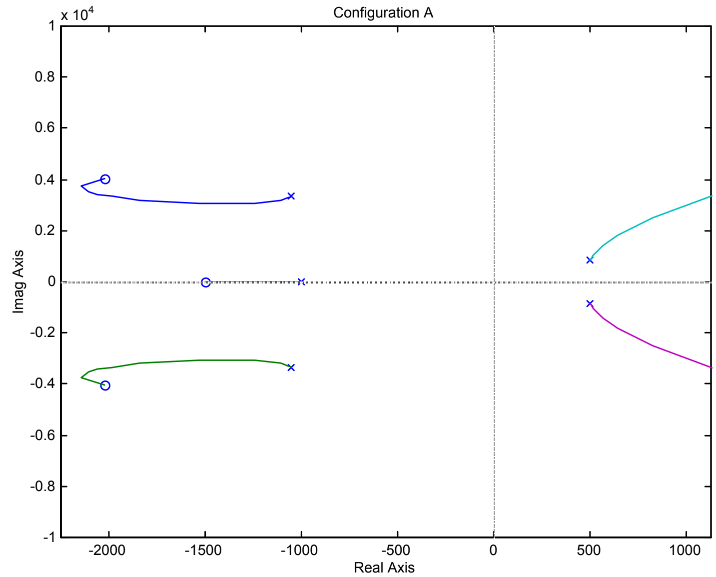


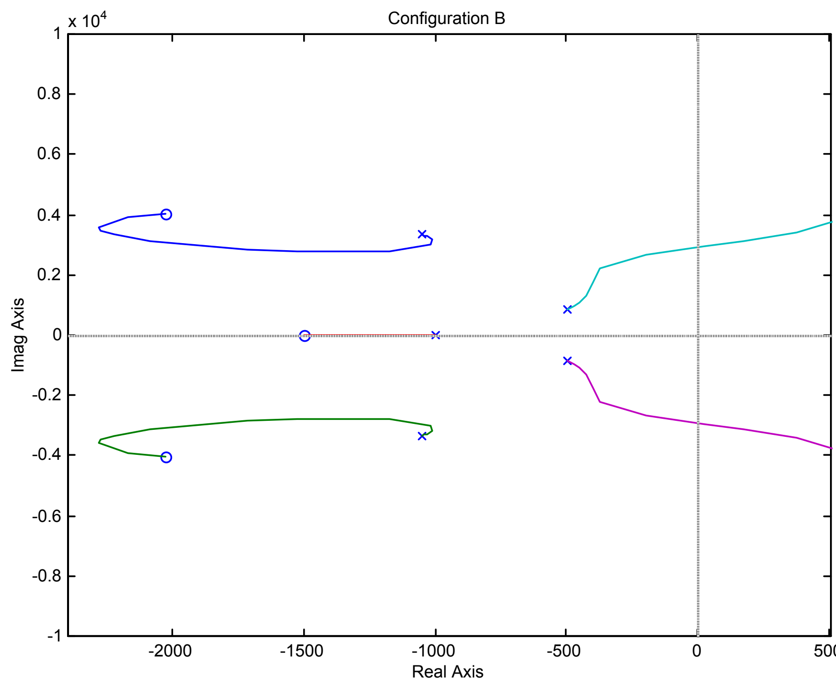
b. Searching the 25% overshoot line ($\zeta = 0.404$; $\theta = 113.8^{\circ}$) for 180° yields -0.0153 + j0.0355 with a gain = 4.514E-6K = 0.0019. Thus, K = 420.9.

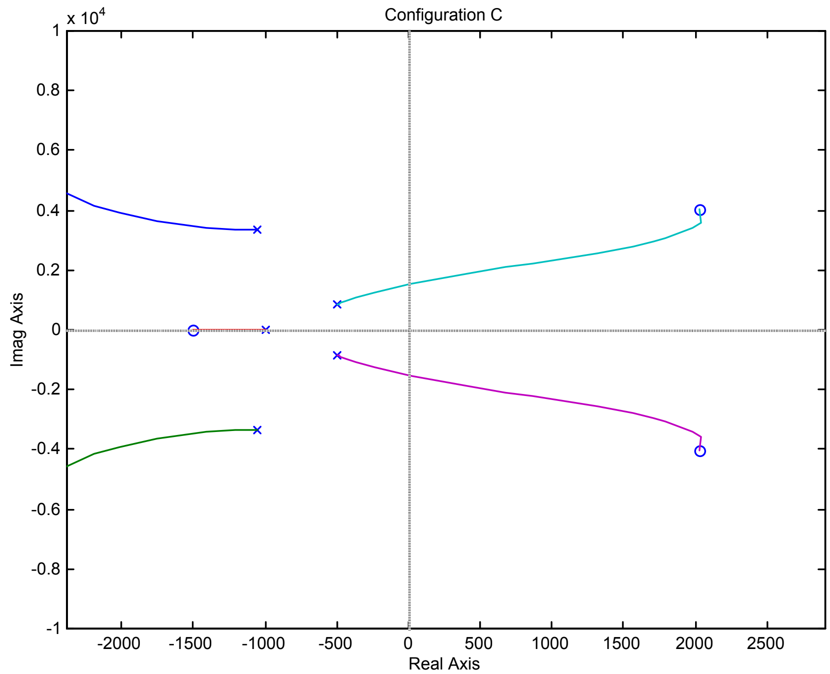
c. Searching the real axis between –0.025 and –0.043 for a gain of 0.0019. we find the third pole at -0.0125. Simulate the system. There is no pole-zero cancellation. A simulation shows approximately 95% overshoot. Thus, even though the compensator yields zero steady-state error, a system redesign for transient response is necessary using methods discussed in Chapter 9.

51.

a.


Configuration A
x 10^4
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Imag Axis
-2000
-1500
-1000
-500
0
500
1000
Real Axis


Configuration B
x 10^4
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Imag Axis
-2000
-1500
-1000
-500
0
500
Real Axis



b.

Configuration A: System is always unstable.

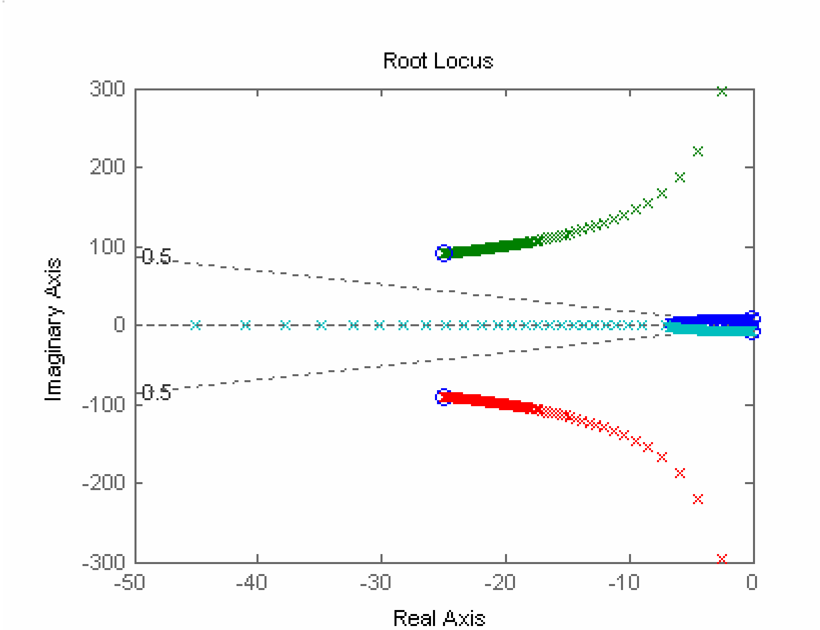
Configuration B: root locus crosses $j\omega$ axis at $j2897$ with a gain of 3.22 x 10^6. Thus, for stability, K < 3.22 x 10^6.

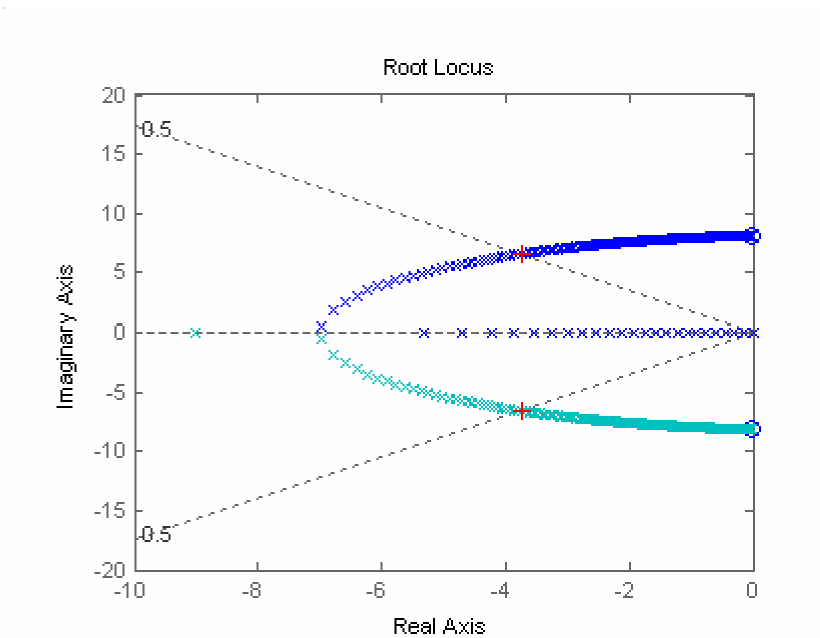
Configuration C: root locus crosses $j\omega$ axis at $j1531$ with a gain of 9.56 x 10^5. System is unstable at high gains. Thus, for stability, 9.56 x $10^5 > K$.

52.

a. Using MATLAB and the Symbolic Math Toolbox, the open-loop expression that yields a root locus as a function of N^2 is

$$G_{dt}(s) = \frac{0.2284\text{x}10^7 N^2 (s^2 + 3.772\text{e-}05s + 66.27)(s^2 + 49.99s + 8789)}{s(s+45.12)(s^2 + 4.893s + 8.777\text{e}04)}$$





Program:

```
syms s N KLSS KHSS KG JR JG tel s
numGdt=3.92*N^2*KLSS*KHSS*KG*s;
denGdt=(N^2*KHSS*(JR*s^2+KLSS)*(JG*s^2*[tel*s+1]+KG*s)+JR*s^2*KLSS*[(JG*s^2
+KHSS)*(tel*s+1)+KG*s]);
Gdt=numGdt/denGdt;
'Gdt in General Terms'
pretty(Gdt)
'Values to Substitute'
KLSS=12.6e6
KHSS=301e3
KG=668
JR=190120
JG=3.8
tel=20e-3
numGdt=3.92*N^2*KLSS*KHSS*KG*s;
```

```
numGdt=vpa(numGdt,4);
denGdt=(N^2*KHSS*(JR*s^2+KLSS)*(JG*s^2*[tel*s+1]+KG*s)+JR*s^2*KLSS*[(JG*s^2
+KHSS)*(tel*s+1)+KG*s]);
denGdt=vpa(denGdt,4);
'Gdt with Values Substituted'
Gdt=numGdt/denGdt;
pretty(Gdt)
Gdt=expand(Gdt);
Gdt=vpa(Gdt,4);
'Gdt Different Form 1'
pretty(Gdt);
denGdt=collect(denGdt,N^2);
'Gdt Different Form 2'
Gdt=collect(Gdt,N^2);
pretty(Gdt)
[numGdt,denGdt]=numden(Gdt);
numGdt=numGdt/0.4349e10;
denGdt=denGdt/0.4349e10;
denGdt=expand(denGdt);
denGdt=collect(denGdt,N^2);
Gdt=vpa(numGdt/denGdt,4);
'Gdt Different Form 3'
pretty(Gdt)
'Putting into Form for RL as a Function of N^2 using previous results'
numGH=[1 49.99 8855 3313 582400];
denGH=[41.87 2094 0.3684e7 0.1658e9 0];
denGH=denGH/denGH(1)
GH=tf(numGH,denGH)
GHzpk=zpk(GH)
'Zeros of GH'
rootsnumGH=roots(numGH)
'Poles of GH'
rootsdenGH=roots(denGH)
K=0:1:10000;
rlocus(GH,K)
sgrid(0.5,0)
pause
axis([-10,0,-20,20])
[K,P]=rlocfind(GH)
```

Computer response:

```
ans =

Gdt in General Terms


  98  2                   /   2           2              2
  -- N  KLSS KHSS KG s   /  (N  KHSS (JR s  + KLSS) (JG s  (tel s + 1) + KG s)
  25                    /

             2            2
        + JR s  KLSS ((JG s  + KHSS) (tel s + 1) + KG s))

ans =

Values to Substitute


KLSS =

    12600000


KHSS =

      301000
```

```
KG =

   668

JR =

      190120

JG =

    3.8000

tel =

    0.0200

ans =

Gdt with Values Substituted

          16  2     /
  .9931 10   N  s  /  (301000.
                  /

        2            2            8          2
       N  (190100. s  + .1260 10 ) (3.800 s  (.02000 s + 1.) + 668. s)

                   13  2             2
         + .2396 10   s  ((3.800 s  + 301000.) (.02000 s + 1.) + 668. s))

ans =

Gdt Different Form 1

          16  2     /            10  2  5             12  2  4             14  2  3
  .9931 10   N  s  /  (.4349 10   N  s  + .2174 10   N  s  + .3851 10   N  s
                  /

                   14  2  2             16  2              12  5              13  4
         + .1441 10   N  s  + .2533 10   N  s + .1821 10   s  + .9105 10   s

                   17  3             18  2
         + .1602 10   s  + .7212 10   s )

ans =

Gdt Different Form 2

          16  2     /             10  5             12  4             14  3
  .9931 10   N  s  /  ((.4349 10   s  + .2174 10   s  + .3851 10   s
                  /

                   14  2             16      2             18  2              12  5
         + .1441 10   s  + .2533 10   s) N  + .7212 10   s  + .1821 10   s

                   13  4             17  3
         + .9105 10   s  + .1602 10   s )

ans =
```

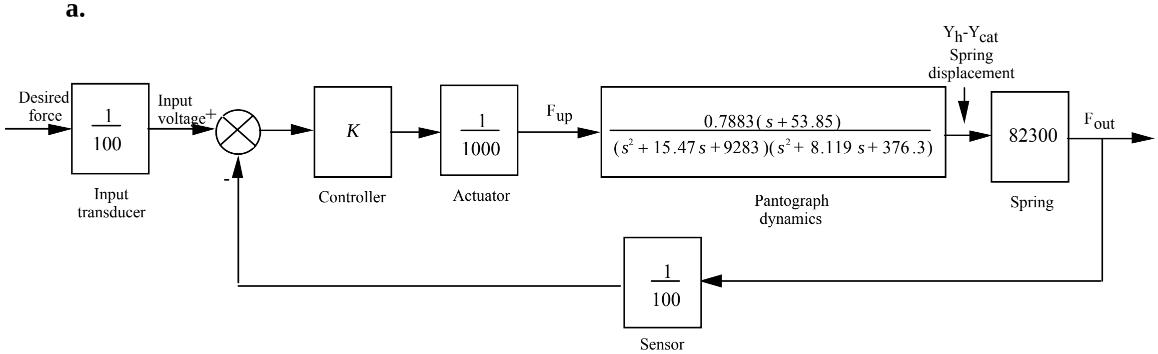
```
Gdt Different Form 3

          7  2      /
  .2284 10  N  s  /  (
                 /

                5          4          3          2               2
        (1.000 s  + 49.99 s  + 8855. s  + 3313. s  + 582400. s) N

                   9  2           5          4           7  3
         + .1658 10  s  + 41.87 s  + 2094. s  + .3684 10  s )

ans =

Putting into Form for RL as a Function of N^2 using previous results


denGH =

  1.0e+006 *

  Columns 1 through 4

    0.0000    0.0001    0.0880    3.9599

  Column 5

         0


Transfer function:
s^4 + 49.99 s^3 + 8855 s^2 + 3313 s + 582400
--------------------------------------------
s^4 + 50.01 s^3 + 8.799e004 s^2 + 3.96e006 s


Zero/pole/gain:
 (s^2  + 66.27) (s^2  + 49.99s + 8789)
---------------------------------------
s (s+45.12) (s^2  + 4.893s + 8.777e004)


ans =

Zeros of GH


rootsnumGH =

 -24.9950 +90.3548i
 -24.9950 -90.3548i
  -0.0000 + 8.1404i
  -0.0000 - 8.1404i


ans =

Poles of GH


rootsdenGH =

  1.0e+002 *

        0
```

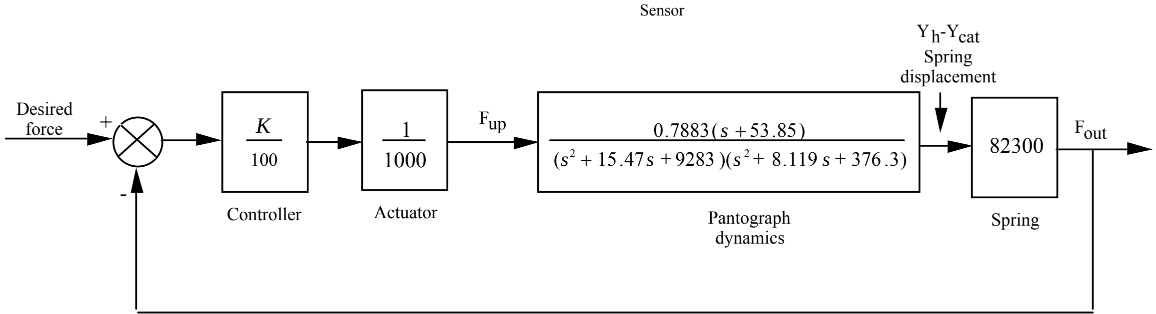
```
  -0.0245 + 2.9624i
  -0.0245 - 2.9624i
  -0.4512

Select a point in the graphics window

selected_point =

  -3.8230 + 6.5435i


K =

   51.5672


P =

 -21.1798 +97.6282i
 -21.1798 -97.6282i
  -3.8154 + 6.5338i
  -3.8154 - 6.5338i
```

b. From the computer response, $K = 0.2284\text{x}10^7 N^2 = 49.6$. Therefore, N is approximately 5/1000.

53.

a.

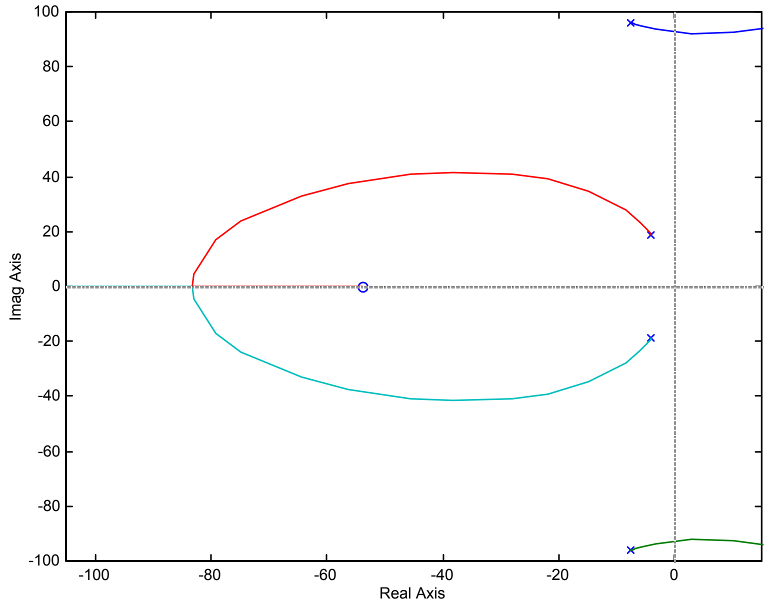




$$G(s) = \frac{Y_h(s) - Y_{cat}(s)}{F_{up}(s)} = \frac{0.7883(s+53.85)}{(s^2+15.47s+9283)(s^2+8.119s+376.3)}$$

Ge(s)=(K/100)*(1/1000)*G(s)*82.3e3

$$G_e(s) = \frac{0.6488\text{K (s+53.85)}}{(s^2 + 8.119s + 376.3)\ (s\text{^}2 + 15.47s + 9283)}$$



b. 38% overshoot yields $\zeta = 0.294$. The $\zeta = 0.294$ line intersects the root locus at $-9 + j27.16$. Here, $K_e = 7.179 \times 10^4$. Thus $K = K_e/0.6488$, or $K = 1.107 \times 10^5$.

c. $T_s = 4/Re = 4/9 = 0.44$ s; $T_p = \pi/Im = \pi/27.16 = 0.116$ s

d. Nondominant closed-loop poles are located at $-3.4 \pm j93.94$. Thus poles are closer to the imaginary axis than the dominant poles. Second order approximation not valid.

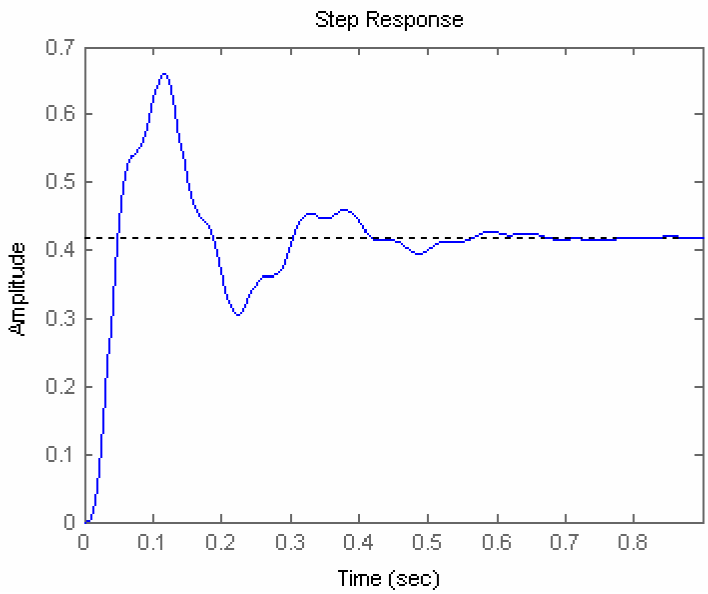
e.
Program:

```
syms s
numg=(s+53.85);
deng=(s^2+15.47*s+9283)*(s^2+8.119*s+376.3);
numg=sym2poly(numg);
deng=sym2poly(deng);
G=tf(numg,deng)
K=7.179e4
Ke=0.6488*K
T=feedback(Ke*G,1)
step(T)
```

Computer response:

```
Transfer function:

                    s + 53.85
-------------------------------------------------
s^4 + 23.59 s^3 + 9785 s^2 + 8.119e004 s

                                      + 3.493e006


K =

       71790


Ke =
```

```
  4.6577e+004

Transfer function:
             4.658e004 s + 2.508e006
-------------------------------------------------
s^4 + 23.59 s^3 + 9785 s^2 + 1.278e005 s

                                      + 6.001e006
```



$T_p = 0.12$ s, $T_s = 0.6$ s, $\%OS = \dfrac{0.66 - 0.42}{0.42} = 57.1\%$.