

35 \

-1

$T_s =$ short span of slab / 35

. 10

Dead load = own weight of slab + cover
 = $t_s * 2.5 + 0.15$ Assume $t_s = 12$ cm

$$2 / 500 \quad 2 / 400 \quad 2 / 200 =$$

.2 / 1000

Live load = 300 kg./m².

Total load = $2.5 * 0.12 + 0.15 + 0.30 = 0.75$ t/m².

: ()

$$R = \frac{m_1 * L_x}{m * L_y}$$

Where :

R : Rectangularity Ratio.

m_1 and m could be 0.87 or 0.76

(m and/or m_1) = 0.87

(m and/or m_1) = 0.76

L_x and L_y are the slab dimensions in X and Y directions.

$2 > R$

$2 <$

R

:

:

2 / 300

-1

$$\alpha = 0.50R - 0.15$$

And
$$\beta = \frac{0.35}{R^2}$$

β

α

Grashoff

2 / 300 <

-2

$$\alpha = \frac{R}{R^4 + 1} \quad \text{And} \quad \beta = \frac{1}{R^4 + 1}$$

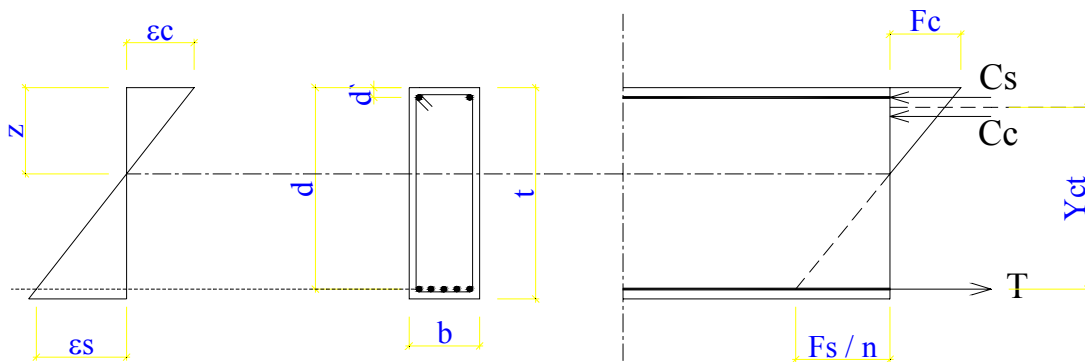
.Marcus

-3

R	1.0	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.0
α	0.395	0.473	0.543	0.606	0.660	0.705	0.745	0.778	0.806	0.830	0.849
β	0.395	0.323	0.262	0.212	0.172	0.140	0.113	0.093	0.077	0.63	0.055

1

Design of R.C. sections:



Notations:

b	Breadth of the rectangular section
t	Total depth of the cross section
d	Theoretical depth of cross section, from the center of tension steel to the outside fiber of the compression zone.
d'	Distance from compression steel to the outside fiber of the compression zone.
z	Distance from the Neutral Axis to the outside fiber of the compression zone.
Y_{CT}	Arm of resisting moment of internal force = distance between the compression and tension internal forces.
A_S	Cross sectional area of tension steel
$A_{S'}$	Cross sectional area of compression steel

Notations, Cont.

M	External moment acting on the section
C_C	Compression force on concrete
C_S	Compression force on compression steel
C	= $C_S + C_C$ = Total compression force on the section

T	Tensile force on tension steel
ϵ_C	Strain of outside fibers of concrete
ϵ_S	Strain of tension steel
F_C	Maximum compressive stress of concrete in compression
F_S	Tensile stress of steel in tension
F_S'	Compressive stress of steel in compression
n	Modular ratio = $\frac{E_S}{E_C}$, where E is the modulus of elasticity
ζ	(zeta) = $\frac{z}{d}$
β	(beta) = $\frac{d'}{d}$
r	= $\frac{F_S}{F_C}$
μ	Ratio of tension steel reinforcement in the cross section = $\frac{A_S}{b.d}$
μ'	Ratio of compression steel reinforcement in the cross section = $\frac{A_S'}{b.d}$

Assumptions:

-1

(Strain)

-2

$$\frac{\epsilon_C}{\epsilon_S} = \frac{z}{d - z} \quad \dots \text{ for the elastic stage} \quad \dots \dots \dots (1)$$

$$\text{and } \frac{\text{Stress}}{\text{Strain}} = \text{const.} = E$$

$$\frac{F_S}{\epsilon_S} = E_S \quad \text{and} \quad \frac{F_C}{\epsilon_C} = E_C$$

$$(1) \quad \epsilon_S \quad \epsilon_C$$

$$\frac{F_C/E_C}{F_S/E_S} = \frac{z}{d-z} \quad \text{or} \quad \frac{F_C}{F_S} \cdot \frac{E_S}{E_C} = \frac{z}{d-z}$$

$$n \cdot \frac{F_C}{F_S} = \frac{z}{d-z} \quad \dots\dots\dots (2)$$

N.A **(2)**

$$F_C = \frac{F_S}{n} \cdot \frac{z}{d-z} \quad \dots\dots\dots (3)$$

(3) **T = F_S . A_S** **T**

$$T = n \cdot F_C \cdot \frac{d-z}{z} \cdot A_S \quad \text{or} \quad T = F_C \cdot \frac{d-z}{z} \cdot (n \cdot A_S) \quad \dots\dots\dots (4)$$

n.A_S **A_S**

$$z = \zeta \cdot d \quad \text{and} \quad \frac{F_S}{F_C} = r$$

(2)

$$\frac{n}{r} = \frac{\zeta \cdot d}{d - \zeta \cdot d} = \frac{\zeta}{1 - \zeta}$$

$$n(1 - \zeta) = \zeta \cdot r \quad \quad \quad n - n \cdot \zeta = \zeta \cdot r \quad \quad \quad n = \zeta(n + r)$$

$$\zeta = \frac{n}{n + r} \quad \dots\dots\dots (5)$$

1- Case 1

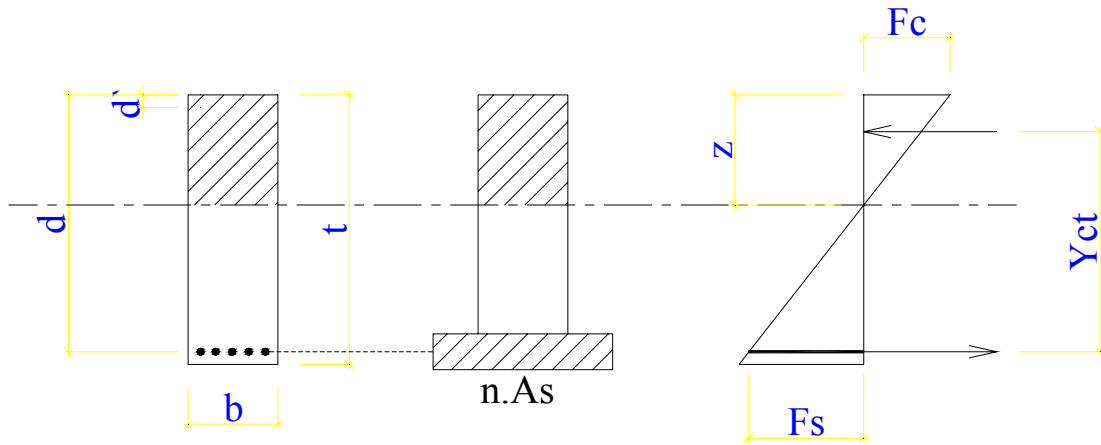
Rectangular section with tension steel only:

Given : M, b, d, and A_S.

Required : F_S, and F_C.

Transformed Section method	-1
Internal Forces method	-2

- 1-
- 2- **Transformed section method**



Equivalent area of concrete = $n.A_s$

The virtual area of the section (as if the section was made of one material) = $= b.t + n.A_s$.

z

ΣM of areas about the N.A = 0.0

$$b.z(z/2) = n.A_s(d-z)$$

$$(b/2)z^2 + (n.A_s)z - n.A_s d = 0.0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = -\frac{n.A_s}{b} \pm \sqrt{\frac{n^2.A_s^2 + 2.b.n.A_s.d}{b^2}}$$

$$z = -\frac{n.A_s}{b} + \sqrt{\frac{n^2.A_s^2}{b^2} \left(1 + \frac{2.b.d}{n.A_s}\right)}$$

$$z = -\frac{n.A_s}{b} + \frac{n.A_s}{b} \sqrt{1 + \frac{2.b.d}{n.A_s}}$$

$$z = \frac{n.A_s}{b} \left(-1 + \sqrt{1 + \frac{2.b.d}{n.A_s}} \right) \quad \dots\dots\dots (1-1)$$

$$(A_s / b.d) = \mu \quad z = \zeta.d$$

$$\zeta.d = \frac{n.\mu.b.d}{b} \left(-1 + \sqrt{1 + \frac{2}{n.\mu}} \right)$$

$$\zeta = n.\mu.d \left(-1 + \sqrt{1 + \frac{2}{n.\mu}} \right) \quad \dots\dots\dots (1-2)$$

$$I_x = \frac{b.z^3}{12} + b.z \left(\frac{z}{2} \right)^2 + n.A_s.(d - z)^2$$

$$F_C = M \cdot \frac{z}{I_x} \quad \text{and} \quad F_S = M \cdot \frac{(d - z)}{I_x} \quad \dots\dots\dots (1-3)$$

3- Internal forces method

Internal moment = external applied moment

.T

$$C.Y_{CT} = M$$

but the force = (stress * area)

$$C = \int F_C . dA$$

$$C = \int F_C . b . dy \quad (\quad \quad \quad)$$

$$C = b . \int F_C . dy = b . \text{area of stress diagram}$$

the compression force (C) = width of the section * area of stress diagram

$$C = (0.5 F_C.z) . b$$

$$M = (0.5 F_C.z) . b . Y_{CT} \quad \text{and} \quad Y_{CT} = d - z/3$$

$$F_C = \frac{2M}{bz(d - z/3)} \quad \dots\dots\dots (1-4)$$

substituting for z with $\zeta.d$

$$F_c = \frac{2.M}{b.\zeta.d(d - \zeta.d/3)} = \frac{2}{\zeta.(1 - \zeta/3)} \cdot \frac{M}{b.d^2}$$

putting $\frac{2}{\zeta.(1 - \zeta/3)} = C_1$

$$F_c = C_1 \cdot \frac{M}{b.d^2} \quad \dots\dots\dots (1-5)$$

where $C_1 = \frac{2}{\zeta.\eta}$

taking the moment about the line of action of compression force:

$$T \cdot Y_{CT} = M$$

$$A_s \cdot F_s \left(d - \frac{z}{3}\right) = M$$

$$F_s = \frac{M}{A_s \left(d - \frac{z}{3}\right)} \quad \dots\dots\dots (1-6)$$

Putting $A_s = \mu \cdot b \cdot d$ & $z = \zeta \cdot d$

$$F_s = \frac{M}{\mu.b.d^2 \left(1 - \frac{\zeta}{3}\right)} = C_2 \cdot \frac{M}{b.d^2} \quad \dots\dots\dots (1-7)$$

Where $C_2 = \frac{1}{\mu \cdot \left(1 - \frac{\zeta}{3}\right)}$ & $\eta = 1 - \frac{\zeta}{3}$

then $C_2 = \frac{1}{\mu \cdot \eta}$

2- Case (2)

Given : M, b, d, A_s & A_s'
 required : F_s , F_c

Σ Moment of areas about the N.A = 0.0

$$\frac{b}{2} \cdot z^2 + n.A_s' \cdot (z - d) = n.A_s \cdot (d - z)$$

$$\frac{b}{2}.z^2 + n.(A_{S'} + A_S).z - n.(A_S.d + A_{S'}.d') = 0.0$$

z

$$z = \frac{-n(A_{S'} + A_S)}{b} + \sqrt{\left(\frac{n(A_{S'} + A_S)}{b}\right)^2 + \frac{2n}{b}(A_S.d + A_{S'}.d')} \quad \dots\dots\dots (2-1)$$

$$z = \zeta . d \quad , \quad A_S = \mu . b . d \quad , \quad A_{S'} = \mu' . b . d$$

$$\alpha = \mu' / \mu \quad , \quad \beta = d' / d$$

$$\zeta . d = \frac{-n.b.d(\mu' + \mu)}{b} + \sqrt{\left(\frac{n.b.d(\mu' + \mu)}{b}\right)^2 + \frac{2n}{b}(\mu.b.d^2 + \mu'.b.d.d')}$$

$$\zeta . d = -n.d.\mu(1 + \alpha) + \sqrt{n^2.d^2.\mu^2(1 + \alpha)^2 + 2.n.\mu.d^2(1 + \alpha.\beta)}$$

$$\zeta . d = -n.d.\mu(1 + \alpha) \sqrt{n^2.d^2.\mu^2(1 + \alpha)^2 \left(1 + \frac{2(1 + \alpha.\beta)}{n.\mu(1 + \alpha)^2}\right)}$$

$$\zeta = n.\mu(1 + \alpha) \left(-1 + \sqrt{1 + \frac{2(1 + \alpha.\beta)}{n.\mu(1 + \alpha)^2}} \right) \quad \dots\dots\dots (2-2)$$

Stresses in concrete and steel:

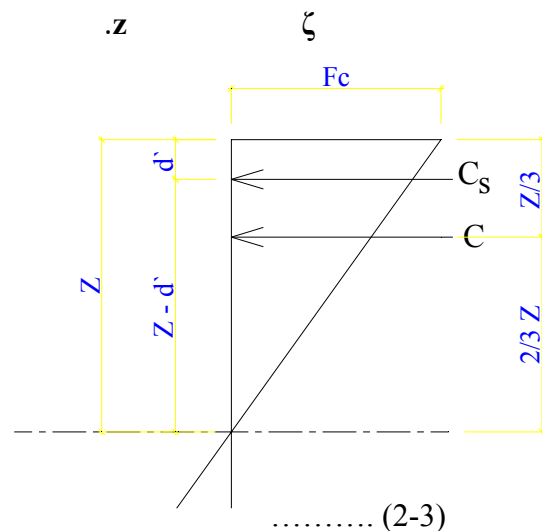
$$C = C_S + C_C$$

$$\Sigma M \text{ about tension steel} = 0.0$$

$$M = C_C \left(d - \frac{z}{3} \right) + C_S (d - d')$$

$$M = \frac{1}{2}.F_C.z.b \left(d - \frac{z}{3} \right) + A_{S'}.F_{S'}(d - d')$$

but $\frac{F_C}{F_{S'}/n} = \frac{z}{z - d'}$



$$F_s = \frac{n.F_c.(z-d)}{z} \quad \text{or} \quad F_s = \frac{n.F_c.(\zeta.d - \beta.d)}{\zeta.d}$$

$$F_s = \frac{n.F_c.(\zeta - \beta)}{\zeta} \quad \dots\dots\dots (2-4)$$

(3-2) (4-2) F_s

$$M = \frac{1}{2}.F_c.z.b\left(d - \frac{z}{3}\right) + A_s.(d-d').\frac{n.F_c.(z-d)}{z}$$

$$F_c = \frac{M}{\frac{b.z}{2}\left(d - \frac{z}{3}\right) + n.A_s.\frac{z-d}{z}(d-d')} \quad \dots\dots\dots (2-5)$$

$$z = \zeta . d \quad , \quad A_s = \mu . b . d \quad , \quad A_s = \mu' . b . d$$

$$\alpha = \mu' / \mu \quad , \quad \beta = d' / d$$

$$F_c = \frac{M}{\frac{b.\zeta.d}{2}\left(d - \frac{\zeta.d}{3}\right) + n.\mu'.b.d.\frac{\zeta.d-d'}{\zeta.d}(d - \beta.d)}$$

$$F_c = \frac{M}{\frac{1}{2}.\zeta\left(1 - \frac{\zeta}{3}\right).b.d^2 + n.\alpha.\mu\left(\frac{\zeta - \beta}{\zeta}\right).(1 - \beta).b.d^2}$$

$$F_c = \frac{M}{b.d^2} \cdot \frac{1}{\frac{1}{2}.\zeta\left(1 - \frac{\zeta}{3}\right) + n.\alpha.\mu\left(\frac{\zeta - \beta}{\zeta}\right).(1 - \beta)} \quad \dots\dots\dots (2-6)$$

$$F_c = C_1 \cdot \frac{M}{b.d^2} \quad \dots\dots\dots (2-7)$$

$$\text{where : } C_1 = \frac{1}{\frac{1}{2}.\zeta\left(1 - \frac{\zeta}{3}\right) + n.\alpha.\mu\left(\frac{\zeta - \beta}{\zeta}\right).(1 - \beta)} \quad \dots\dots\dots (2-8)$$

.F_C

(4-2) (7-2) F_C

$$F_s = \frac{n.(\zeta - \beta)}{\zeta} . C_1 \cdot \frac{M}{b.d^2} \quad \dots\dots\dots (2-9)$$

to find F_s

$$\frac{F_c}{F_s/n} = \frac{z}{d-z} \quad (z = \zeta \cdot d)$$

$$F_s = \frac{n \cdot (1 - \zeta)}{\zeta} \cdot F_c \quad \dots\dots\dots (2-10)$$

(10-2) (6-2) F_c

$$F_s = \frac{M}{b \cdot d^2} \cdot \frac{1}{\frac{1}{2} \cdot \zeta \left(1 - \frac{\zeta}{3}\right) + n \cdot \alpha \cdot \mu \left(\frac{\zeta - \beta}{\zeta}\right) \cdot (1 - \beta)} \cdot \frac{n(1 - \zeta)}{\zeta}$$

$$F_s = \frac{M}{b \cdot d^2} \cdot \frac{n(1 - \zeta)}{\frac{1}{2} \cdot \zeta^2 \left(1 - \frac{\zeta}{3}\right) + n \cdot \alpha \cdot \mu \left(\frac{\zeta - \beta}{\zeta}\right) \cdot (1 - \beta)}$$

$$F_s = C_2 \cdot \frac{M}{b \cdot d^2} \quad \dots\dots\dots (2-11)$$

where:

$$C_2 = \frac{n(1 - \zeta)}{\frac{1}{2} \cdot \zeta^2 \left(1 - \frac{\zeta}{3}\right) + n \cdot \alpha \cdot \mu \left(\frac{\zeta - \beta}{\zeta}\right) \cdot (1 - \beta)} \quad \dots\dots\dots (2-12)$$

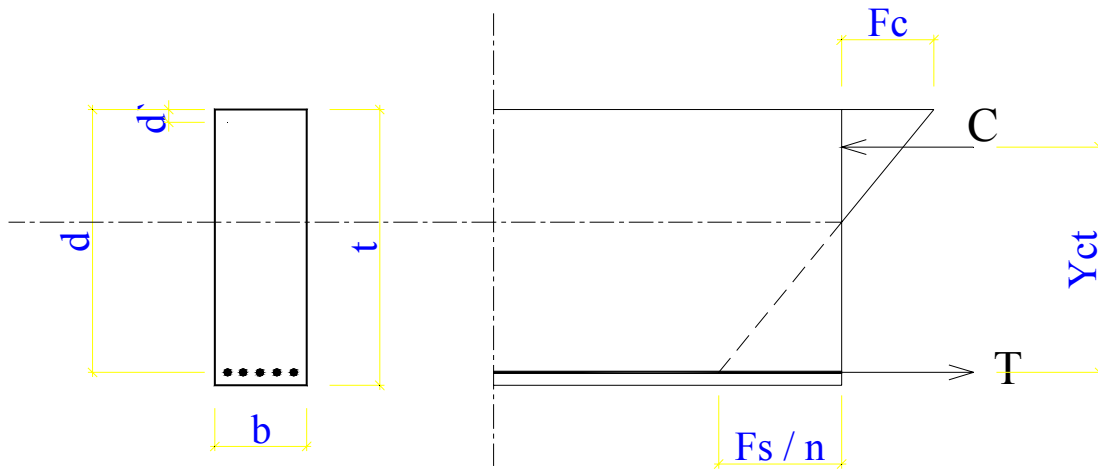
Case 3

Design of Rectangular section with tension steel only:

3-1 Free design (the depth is unlimited)

Given : M, b, F_c, and F_s

Required : d, and A_s



:

$$\frac{F_C}{F_s/n} = \frac{z}{d-z} \quad \dots\dots\dots (3-1)$$

Σ of moment about tension steel

$$C.Y_{CT} = M$$

$$\frac{1}{2} \cdot F_C \cdot z \cdot b \cdot \left(d - \frac{z}{3}\right) = M \quad \dots\dots\dots (3-2)$$

and from equation 3-1 $z = \frac{n}{n + F_s/F_C} \cdot d$ and $\frac{F_s}{F_C} = r$

$$z = \frac{n}{n+r} \cdot d \quad \dots\dots\dots (3-2) \quad z$$

$$\frac{1}{2} \cdot F_C \cdot b \cdot \frac{n}{n+r} \cdot d \cdot \left(d - \frac{1}{3} \frac{n}{n+r} d\right) = M \quad \text{and} \quad \frac{n}{n+r} = \zeta$$

$$\frac{1}{2} \cdot F_C \cdot b \cdot \zeta \cdot d^2 \cdot \left(1 - \frac{\zeta}{3}\right) = M$$

$$d = \sqrt{\frac{2}{F_C \cdot \zeta \cdot \left(1 - \frac{\zeta}{3}\right)}} \cdot \sqrt{\frac{M}{b}} \quad \text{putting} \quad \sqrt{\frac{2}{F_C \cdot \zeta \cdot \left(1 - \frac{\zeta}{3}\right)}} = K_1$$

$$d = K_1 \sqrt{\frac{M}{b}} \quad \dots\dots\dots (3-3)$$

(Balanced Depth)

(3-3)

$$.F_s \quad F_c \quad K_1$$

$$\Sigma M \text{ about } C = 0.0$$

$$M = T \cdot Y_{CT}$$

$$M = A_s \cdot F_s \cdot \left(d - \frac{z}{3} \right) \quad z = \zeta \cdot d$$

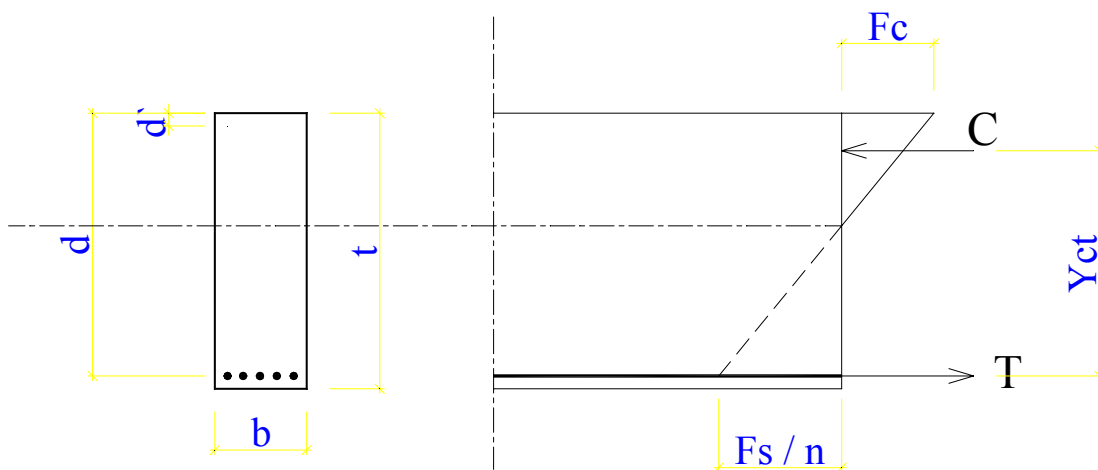
$$A_s = \frac{M}{F_s \cdot d \cdot \left(1 - \frac{\zeta}{3} \right)} \quad 1 - \frac{\zeta}{3} = \eta$$

$$A_s = \frac{M}{\eta \cdot F_s \cdot d} \quad \eta \cdot F_s = K_2$$

$$A_s = \frac{M}{K_2 \cdot d} \quad \dots\dots\dots (3-4)$$

3-2 Fixed design (the depth is given)

3-2-1 The given depth > the balanced depth



Given : M, b, d, F_C, and F_S

Required : A_S

(Balanced Depth)

$$.A_s \quad F_s$$

$$K_1 = \frac{d}{\sqrt{M/b}} \quad \text{so we can know "K}_1\text{"}$$

1- from the curves between "K₁" on Y-Axis, and "F_C" on X-Axis we can get F_C.

2- from knowing "F_C" we can determine if it is allowable or not (in this case of design F_C must be < F_{C all.}).

4- from the curves between F_C, and K₂, we can find K₂.

$$-5 \quad A_s = \frac{M}{K_2 \cdot d}$$

(Balanced Depth)

F_S F_C

$$K_1 = \frac{d}{\sqrt{M/b}} \quad \text{so we can know "K}_1\text{"}$$

from the maximum allowable F_C on X-Axis, and the value of K₁ on Y-Axis, draw two horizontal and vertical lines respectively, and the two lines will intersect on a curve with the maximum F_S of certain "α", so we can get K₂.

Design of T-Sections

effective width of T-Sec =
or =

F_C

T

B

-1

(T)

-2

()

()

) (T)

-3

(

F_{C (all.)} [in case of T-Sec.] = 0.5 ~ 0.75 of F_{C (all.)} [in case of R-Sec.]

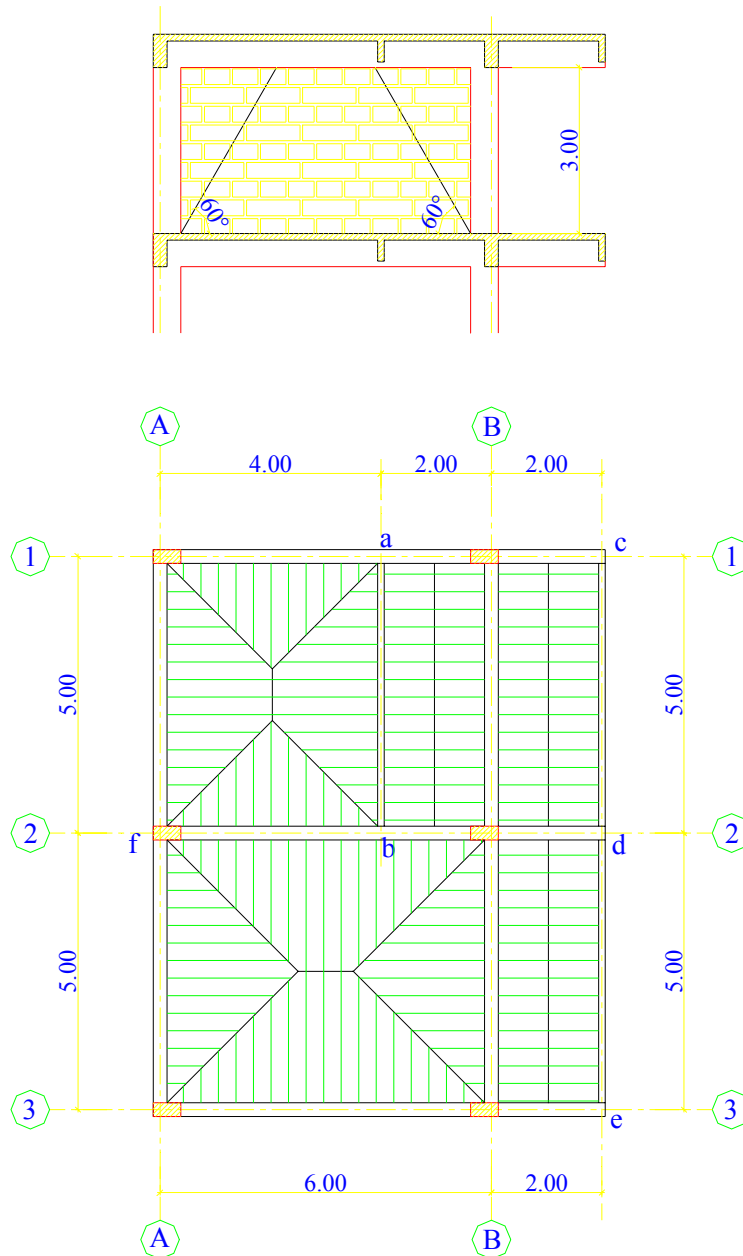
F_C

Examples:

For the plan shown in the following figure, find the loads on all beams knowing that:

- 1- Slab thickness = 12.0 cm
- 2- Weight of cover = 150.0 kg/m²
- 3- Live load = 200.0 kg/m²
- 4- Wall height = 3.0 m
- 5- Depth of all beams = 60.0 cm
- 6- Breadth of beams ab, cde = 12.0 cm
- 7- Breadth of the remaining beams = 25.0 cm

And the draw the absolute shearing force and bending moment of the main beam (fbd), and design the critical sections, the draw to scale 1:20 the details of reinforcement of a longitudinal section of the beam (fbd).



Absolute bending moment of a continues beam

