Resultant of forces system

The resultant is a representative force which has the same effect on the body as the group of forces it replaces.

A simplest force which can replace the original forces system without changing its external effect on a rigid body.

The symbol of resultant force is: ──►►

The unit of resultant force is : Newton (N)

Types of forces system

1- coplanar forces system :
   a- concurrent coplanar forces system
   b- non-concurrent coplanar forces system
2- Non coplanar forces system :
   a- concurrent non-coplanar forces system
   b- non-concurrent non-coplanar forces system

Resultant of concurrent coplanar forces system

We will find out the resultant force for many forces acting on a rigid body by using the following equations :

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2 \mp F_3.\cos\theta_3 \mp \ldots \mp F_n.\cos\theta_n$$

$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2 \mp F_3.\sin\theta_3 \mp \ldots \mp F_n.\sin\theta_n$$

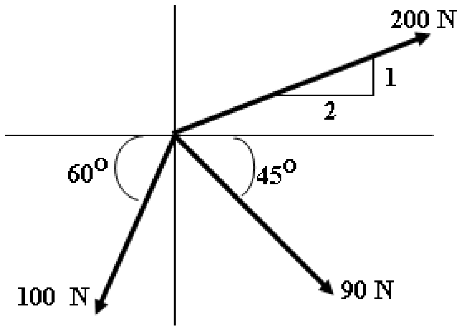
$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

The direction of resultant force may be determined as :

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Ex (1) :

Find the resultant force for the concurrent coplanar forces system, shown in fig.



Solution

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2 \mp F_3.\cos\theta_3$$

$$= 200 \bullet \tfrac{2}{\sqrt{5}} - 100\cos 60 + 90\cos 45 = +192.4N$$

$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2 \mp F_3.\sin\theta_3$$

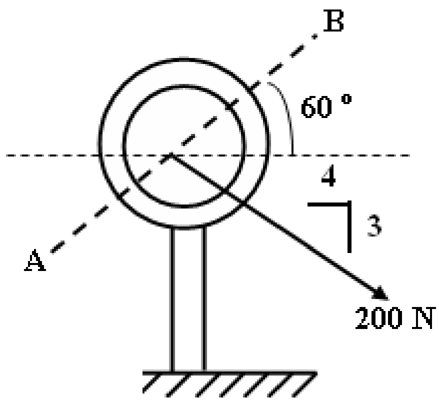
$$= 200 \bullet \tfrac{1}{\sqrt{5}} - 100\sin 60 - 90\sin 45 = -60.8N$$

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$= \sqrt{(192.4)^2 + (60.8)^2} = 202N$$

Ex (2) :

The **200 N** force is a resultant of two forces, one of the forces "**P**" has the direction along the tine **AB** and the other force "**Q**" is on the horizontal direction, determine them.



Solution

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2$$

$$R * \cos\theta_R = F_1.\cos\theta_1 \mp F_2.\cos\theta_2$$

$$200 * \tfrac{4}{5} = P * \cos 60 + Q\cos(0)$$

$$160 = 0.5P + Q$$

$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2$$

$$R * \sin\theta_R = F_1.\sin\theta_1 \mp F_2.\sin\theta_2$$

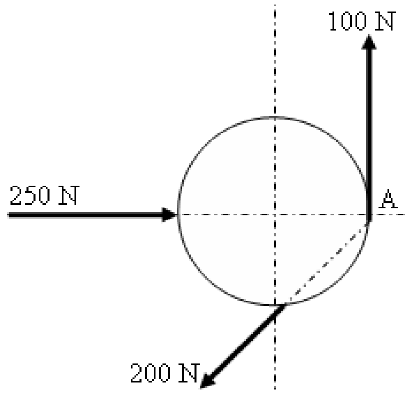
$$-200 * \tfrac{3}{5} = P * \sin 60 - Q\sin(0)$$

$$P = 138.5N \swarrow$$

$$\therefore Q = 229.25N$$

Ex (3) :

Determine the resultant force for the forces system shown in fig.



Solution

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2 \mp F_3.\cos\theta_3$$

$$= 100\cos 90 + 250\cos(0) - 200\cos 45$$

$$= 192.5 \quad \text{N}$$

$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2 \mp F_3.\sin\theta_3$$

$$= 100\sin 90 + 250\sin(0) - 200\sin 45$$

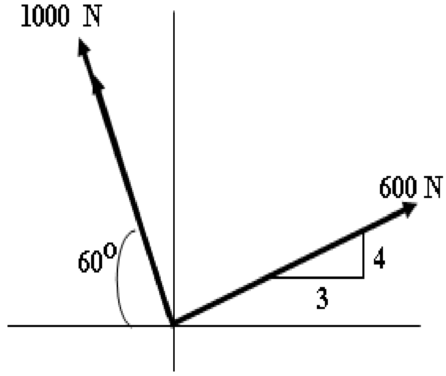
$$= -\ 60.78 \quad \text{N}$$

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$= \sqrt{(192.5)^2 + (-60.78)^2} = 201.8 \ \text{N}$$

Ex (4) :

The 1000 N force is a resultant of two forces, one of which is 600 N, Determine the other force?



Solution

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2$$

$$R * \cos\theta_R = F_1.\cos\theta_1 \mp F_2.\cos\theta_2$$

$$-1000 * \cos 60 = 600 * \tfrac{3}{5} + F_2 \cos\theta_2$$

$$-1000 * 0.5 = 360 + F_2 \cos\theta_2$$

$$F_2 \cos\theta_2 = -860N \quad \_\_\_\_\_\_\_\_ (1) \Rightarrow F_x$$

$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2$$

$$R * \sin\theta_R = F_1.\sin\theta_1 \mp F_2.\sin\theta_2$$

$$1000 * \sin 60 = 600 * \tfrac{4}{5} + F_2 \sin\theta_2$$

$$F_2 \sin\theta_2 = 386.02N \quad \_\_\_\_\_\_\_\_ (2) \Rightarrow F_y$$

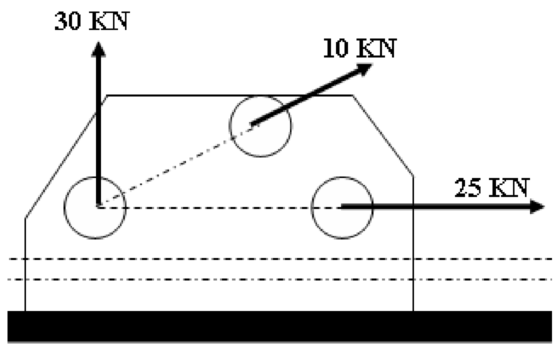
$$F = \sqrt{(F_x)^2 + (F_y)^2}$$

$$= \sqrt{(860)^2 + (386.02)^2}$$

$$= 942.62N$$

Ex (5) :

Determine the resultant force of the concurrent coplanar forces system shown in fig.



Solution

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2 \mp F_3.\cos\theta_3$$

$$= 25\cos 0 + 10\cos 45 + 30\cos 90$$

$$= 26.707N$$

$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2 \mp F_3.\sin\theta_3$$

$$= 25\sin 0 + 10\sin 45 + 30\sin 90$$

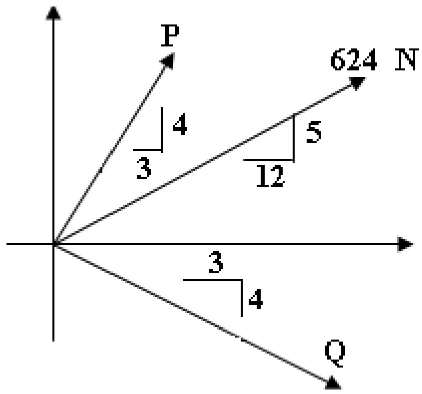
$$= 31.707$$

$$R = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(26.707)^2 + (31.707)^2}$$

$$= 41.455N$$

Ex (6) :

The 624 N force is a resultant of two force P & Q shown in fig. determine these forces .



Solution

$$R * \cos\theta_R = F_1.\cos\theta_1 \mp F_2.\cos\theta_2$$

$$624 * \frac{12}{13} = P * \frac{3}{5} + Q * \frac{3}{5}$$

$$960 = P + Q \quad \_\_\_\_\_\_\_\_(1)$$

$$R * \sin\theta_R = F_1.\sin\theta_1 \mp F_2.\sin\theta_2$$

$$624 * \frac{5}{13} = P * \frac{4}{5} - Q * \frac{4}{5}$$

$$1200 = 4(P - Q)$$

$$300 = P - Q \quad \_\_\_\_\_\_\_\_(2)$$

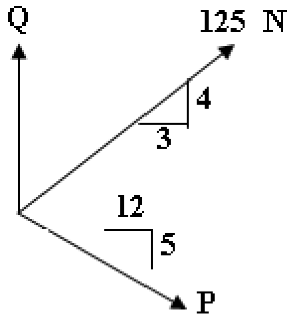
$$Subst\ (1)\ in\ (2):$$

$$P = 630\ N$$

$$Q = 330\ N$$

Ex (7) :

Determine the forces P and Q when the 125 N force is the resultant force of these forces



Solution

$$R * \cos\theta_R = F_1.\cos\theta_1 \mp F_2.\cos\theta_2$$

$$125 * \frac{3}{5} = P * \frac{12}{13} + Q \cos 90$$

$$\therefore P = 81.25$$

$$R * \sin\theta_R = F_1.\sin\theta_1 \mp F_2.\sin\theta_2$$

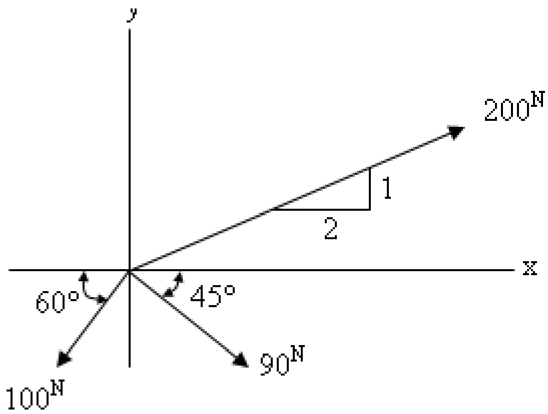
$$125 * \frac{4}{5} = Q - \frac{5}{13} P$$

$$100 = Q - \frac{5}{13} * 81.25$$

$$Q = 131.25\ N$$

Ex (8) :

Determine the resultant of concurrent coplanar forces system shown in fig.



Solution

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2 \mp F_3.\cos\theta_3$$

$$= 200 * \frac{2}{\sqrt{5}} - 100\cos 60 + 90\cos 45$$

$$= 192.5 \;\; \text{N}$$

$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2 \mp F_3.\sin\theta_3$$

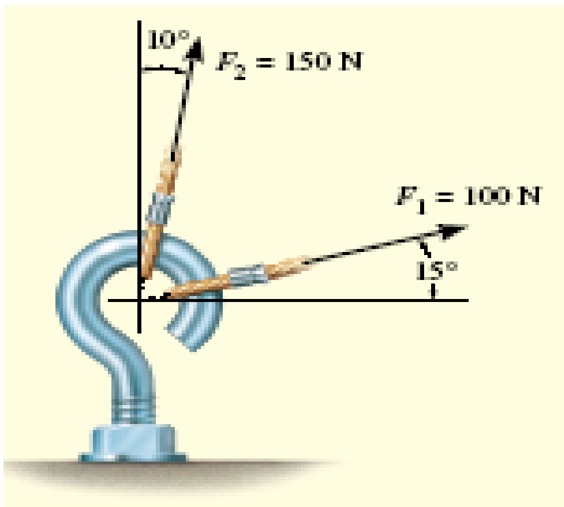
$$= 200 * \frac{1}{\sqrt{5}} - 100\sin 60 - 90\sin 45$$

$$= -\ 60.78 \;\; \text{N}$$

$$R = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(192.5)^2 + (-60.78)^2}$$

$$= 201.8 \text{ N}$$

Ex (9) : The screw eye in Fig. is subjected to two forces, and Determine the magnitude and direction of the resultant force.



Solution

Solution

Parallelogram Law. The parallelogram law of addition is shown in Fig. 2–10*b*. The two unknowns are the magnitude of $\mathbf{F}_R$ and the angle θ (theta).

Trigonometry. From Fig. 2–10*b*, the vector triangle, Fig. 2–10*c*, is constructed. F_R is determined by using the law of cosines:

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115°}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N} \qquad \textit{Ans.}$$

The angle θ is determined by applying the law of sines, using the computed value of F_R.

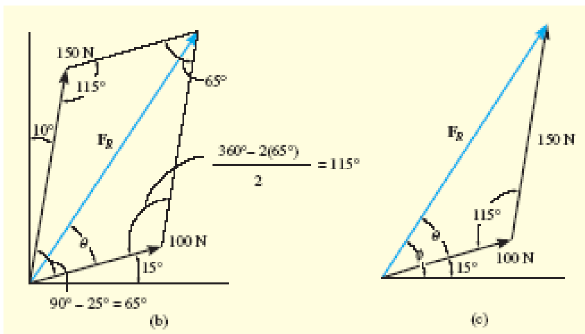
$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115°}$$

$$\sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(0.9063)$$

$$\theta = 39.8°$$

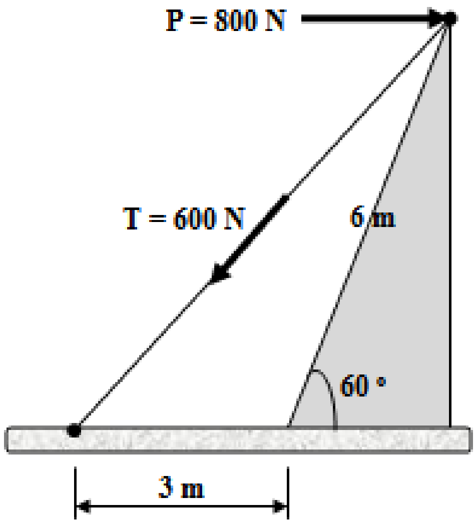
Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8° + 15.0° = 54.8° \qquad \textit{Ans.}$$



Ex (10) :

Determine the resultant (**R**) of the two forces system shown in fig.



Solution

h = 6 sin 60 = 5.2 m

b = 6 cos 60 = 3 m

$$\tan\alpha = \frac{h}{b+3} = \frac{5.2}{3+3} = 0.866$$

$$\alpha = \tan^{-1}(0.866) = 40.9$$

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2$$

$$= 800 - 600\cos 40.9$$

$$= 346.5 \text{ N}$$

$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2$$

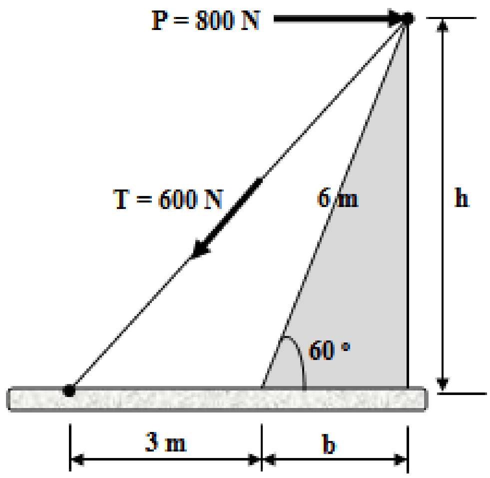
$$= 600\sin 40.9$$

$$= 392.84 \text{ N}$$

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

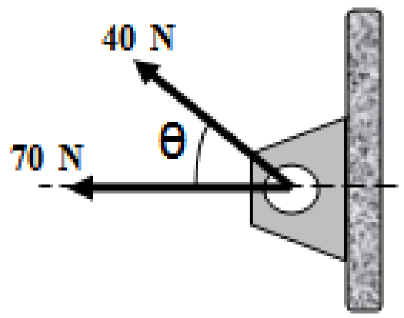
$$= \sqrt{(346.58)^2 + (392.84)^2}$$

$$= 523.8 \text{ N}$$



Ex (11) :

Determine the angle (**θ**) so that the resultant (**R**) of the two forces system shown in fig. is 100 N



Solution

$$R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2\cos\theta}$$

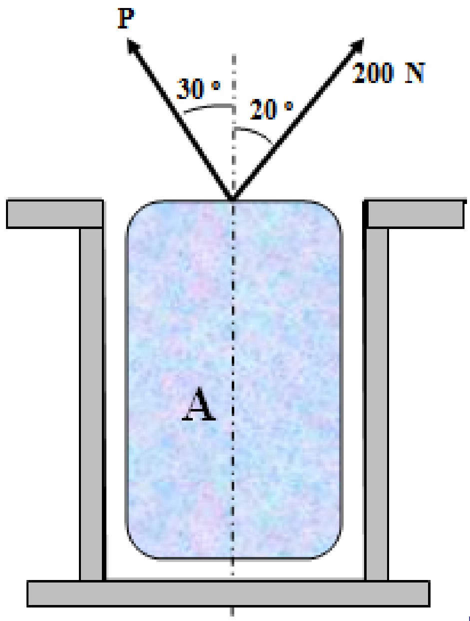
$$100 = \sqrt{(70)^2 + (40)^2 + 2*70*40*\cos\theta}$$

$$\cos\theta = 0.625$$

$$\theta = 51.3^o$$

Ex (12) :

Find out the magnitude of the force (**P**) and the resultant (**R**) of the two forces to rise the cylinder up without touch the vertical wall .



Solution

$$R * \cos\theta_R = F_1.\cos\theta_1 \mp F_2.\cos\theta_2$$

$$R * \cos 90 = 200\cos 70 - P\cos 60$$

$$0 = 68.4 - 0.5P$$

$$\therefore P = 136.8N$$

$$R * \sin\theta_R = F_1.\sin\theta_1 \mp F_2.\sin\theta_2$$

$$R\sin 90 = 200\sin\theta + P\sin\theta$$

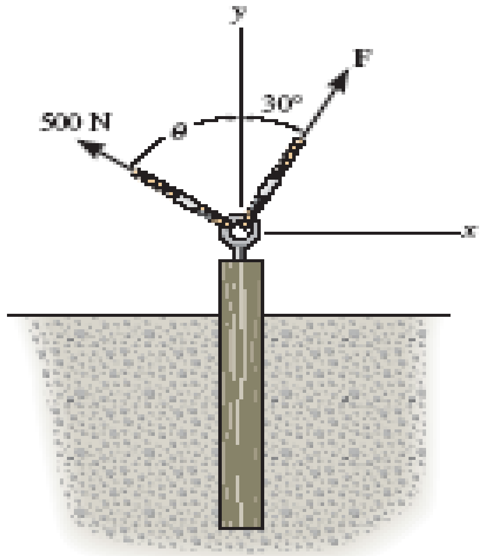
$$R\sin 90 = 200\sin\theta + 136.8\sin 60$$

$$R = 187.93 + 118.47$$

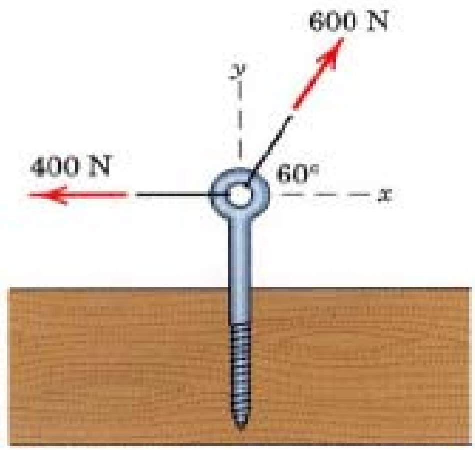
$$R = 306.4N$$

PROBLEMS

1 - Two forces are applied at the end of a screw eye in order to remove the post. Determine the angle and the magnitude of force **F** so that the resultant force acting on the post is directed vertically upward and has a magnitude of 750 N.

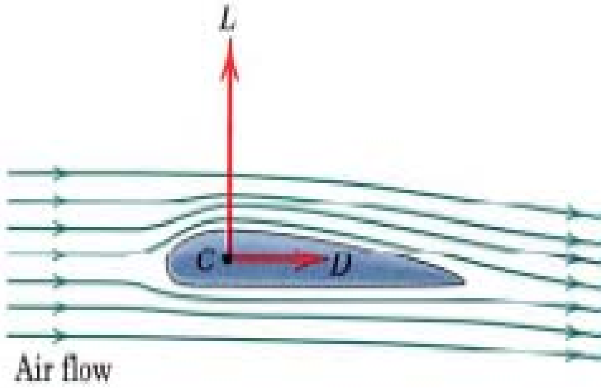


2 – Determine the resultant (**R**) of the two forces shown in fig.

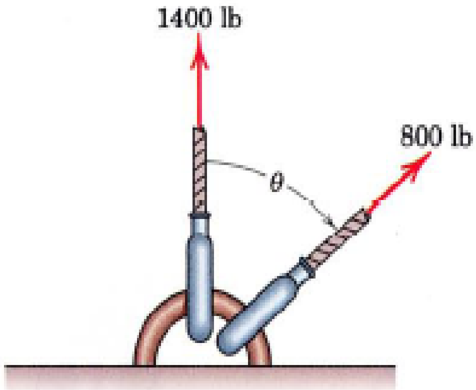


3 - The ratio of the lift force (**L**) to the drag force (**D**) for the simple airfoil is (**L / D = 10**) . If the left force on a short section of the airfoil is (**50 N**) , Compute the magnitude of the resultant force (**R**) and the angle (**θ**) which it makes with the horizontal .

Ans. **R** = 50.2 N , **θ** = 84.3°

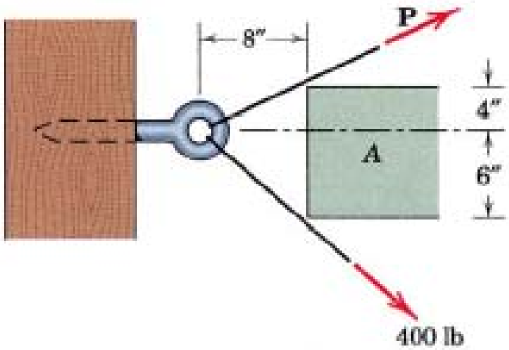


4 - At what angle θ must the 800-lb force be applied in order that the resultant **R** of the two forces has a magnitude of 2000 lb? For this condition, determine the angle β between **R** and the vertical.



5 – It is desired to remove the spike from the timber by applying force along its horizontal axis. An obstruction *A* prevents direct access, so that two forces, one 400 lb and the other **P**, are applied by cables as shown. Compute the magnitude of **P** necessary to ensure a resultant **T** directed along the spike. Also find *T*.

Ans. P = 537 lb
T = 800 lb



6 - Determine the magnitude (**Fs**) of the tensile spring force in order that the resultant of (**Fs**) and (**F**) is a vertical force . Determine the magnitude (**R**) of this vertical resultant force .