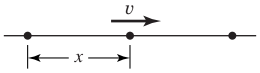
Chapter 1

Problems 1-1 through 1-4 are for student research.

1-5

(a) Point vehicles



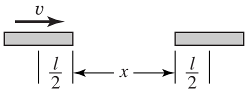
$$Q = \frac{\text{cars}}{\text{hour}} = \frac{v}{x} = \frac{42.1v - v^2}{0.324}$$

Seek stationary point maximum

$$\frac{dQ}{dv} = 0 = \frac{42.1 - 2v}{0.324} \quad \therefore v^* = 21.05\,\text{mph}$$

$$Q^* = \frac{42.1(21.05) - 21.05^2}{0.324} = 1368\,\text{cars/h} \quad \textit{Ans.}$$

(b)



$$Q = \frac{v}{x + l} = \left(\frac{0.324}{v(42.1) - v^2} + \frac{l}{v}\right)^{-1}$$

Maximize Q with $l = 10/5280$ mi

v	Q	
22.18	1221.431	
22.19	1221.433	
22.20	1221.435	←
22.21	1221.435	
22.22	1221.434	

$$\%\text{ loss of throughput} = \frac{1368 - 1221}{1221} = 12\% \quad \textit{Ans.}$$

(c) % increase in speed $\frac{22.2 - 21.05}{21.05} = 5.5\%$

Modest change in optimal speed *Ans.*

1-6 This and the following problem may be the student's first experience with a figure of merit.

- Formulate fom to reflect larger figure of merit for larger merit.
- Use a maximization optimization algorithm. When one gets into computer implementation and answers are not known, minimizing instead of maximizing is the largest error one can make.

$$\sum F_V = F_1 \sin\theta - W = 0$$

$$\sum F_H = -F_1 \cos\theta - F_2 = 0$$

From which

$$F_1 = W/\sin\theta$$

$$F_2 = -W\cos\theta/\sin\theta$$

$$\text{fom} = -\$ = -¢\gamma\ (\text{volume})$$

$$\doteq -¢\gamma(l_1A_1 + l_2A_2)$$

$$A_1 = \frac{F_1}{S} = \frac{W}{S\sin\theta}, \quad l_2 = \frac{l_1}{\cos\theta}$$

$$A_2 = \left|\frac{F_2}{S}\right| = \frac{W\cos\theta}{S\sin\theta}$$

$$\text{fom} = -¢\gamma\left(\frac{l_2}{\cos\theta}\,\frac{W}{S\sin\theta} + \frac{l_2 W\cos\theta}{S\sin\theta}\right)$$

$$= \frac{-¢\gamma W l_2}{S}\left(\frac{1+\cos^2\theta}{\cos\theta\sin\theta}\right)$$

Set leading constant to unity

$\theta°$	fom
0	$-\infty$
20	−5.86
30	−4.04
40	−3.22
45	−3.00
50	−2.87
54.736	−2.828
60	−2.886

$\theta^* = 54.736°$ *Ans.*

fom* $= -2.828$

Alternative:

$$\frac{d}{d\theta}\left(\frac{1+\cos^2\theta}{\cos\theta\sin\theta}\right) = 0$$

And solve resulting transcendental for θ^*.

Check second derivative to see if a maximum, minimum, or point of inflection has been found. Or, evaluate fom on either side of θ^*.

1-7

(a) $x_1 + x_2 = X_1 + e_1 + X_2 + e_2$

$$\begin{aligned} \text{error} = e &= (x_1 + x_2) - (X_1 + X_2) \\ &= e_1 + e_2 \quad \textit{Ans.} \end{aligned}$$

(b) $x_1 - x_2 = X_1 + e_1 - (X_2 + e_2)$

$$e = (x_1 - x_2) - (X_1 - X_2) = e_1 - e_2 \quad \textit{Ans.}$$

(c) $x_1 x_2 = (X_1 + e_1)(X_2 + e_2)$

$$\begin{aligned} e &= x_1 x_2 - X_1 X_2 = X_1 e_2 + X_2 e_1 + e_1 e_2 \\ &\doteq X_1 e_2 + X_2 e_1 = X_1 X_2 \left(\frac{e_1}{X_1} + \frac{e_2}{X_2} \right) \quad \textit{Ans.} \end{aligned}$$

(d) $\dfrac{x_1}{x_2} = \dfrac{X_1 + e_1}{X_2 + e_2} = \dfrac{X_1}{X_2} \left(\dfrac{1 + e_1/X_1}{1 + e_2/X_2} \right)$

$$\left(1 + \frac{e_2}{X_2}\right)^{-1} \doteq 1 - \frac{e_2}{X_2} \quad \text{and} \quad \left(1 + \frac{e_1}{X_1}\right)\left(1 - \frac{e_2}{X_2}\right) \doteq 1 + \frac{e_1}{X_1} - \frac{e_2}{X_2}$$

$$e = \frac{x_1}{x_2} - \frac{X_1}{X_2} \doteq \frac{X_1}{X_2} \left(\frac{e_1}{X_1} - \frac{e_2}{X_2} \right) \quad \textit{Ans.}$$

1-8

(a)

$$\begin{aligned} x_1 &= \sqrt{5} = 2.236\ 067\ 977\ 5 \\ X_1 &= 2.23 \quad \text{3-correct digits} \\ x_2 &= \sqrt{6} = 2.449\ 487\ 742\ 78 \\ X_2 &= 2.44 \quad \text{3-correct digits} \end{aligned}$$

$$x_1 + x_2 = \sqrt{5} + \sqrt{6} = 4.685\ 557\ 720\ 28$$

$$e_1 = x_1 - X_1 = \sqrt{5} - 2.23 = 0.006\ 067\ 977\ 5$$

$$e_2 = x_2 - X_2 = \sqrt{6} - 2.44 = 0.009\ 489\ 742\ 78$$

$$e = e_1 + e_2 = \sqrt{5} - 2.23 + \sqrt{6} - 2.44 = 0.015\ 557\ 720\ 28$$

$$\begin{aligned} \text{Sum} = x_1 + x_2 &= X_1 + X_2 + e \\ &= 2.23 + 2.44 + 0.015\ 557\ 720\ 28 \\ &= 4.685\ 557\ 720\ 28 \text{ (Checks)} \quad \textit{Ans.} \end{aligned}$$

(b) $X_1 = 2.24, \quad X_2 = 2.45$

$$\begin{aligned} e_1 &= \sqrt{5} - 2.24 = -0.003\ 932\ 022\ 50 \\ e_2 &= \sqrt{6} - 2.45 = -0.000\ 510\ 257\ 22 \\ e &= e_1 + e_2 = -0.004\ 442\ 279\ 72 \end{aligned}$$

$$\begin{aligned} \text{Sum} &= X_1 + X_2 + e \\ &= 2.24 + 2.45 + (-0.004\ 442\ 279\ 72) \\ &= 4.685\ 557\ 720\ 28 \quad \textit{Ans.} \end{aligned}$$

1-9

(a) $\sigma = 20(6.89) = 137.8\text{ MPa}$

(b) $F = 350(4.45) = 1558\text{ N} = 1.558\text{ kN}$

(c) $M = 1200\text{ lbf}\cdot\text{in }(0.113) = 135.6\text{ N}\cdot\text{m}$

(d) $A = 2.4(645) = 1548\text{ mm}^2$

(e) $I = 17.4\text{ in}^4\ (2.54)^4 = 724.2\text{ cm}^4$

(f) $A = 3.6(1.610)^2 = 9.332\text{ km}^2$

(g) $E = 21(1000)(6.89) = 144.69(10^3)\text{ MPa} = 144.7\text{ GPa}$

(h) $v = 45\text{ mi/h }(1.61) = 72.45\text{ km/h}$

(i) $V = 60\text{ in}^3\ (2.54)^3 = 983.2\text{ cm}^3 = 0.983\text{ liter}$

1-10

(a) $l = 1.5/0.305 = 4.918\text{ ft} = 59.02\text{ in}$

(b) $\sigma = 600/6.89 = 86.96\text{ kpsi}$

(c) $p = 160/6.89 = 23.22\text{ psi}$

(d) $Z = 1.84(10^5)/(25.4)^3 = 11.23\text{ in}^3$

(e) $w = 38.1/175 = 0.218\text{ lbf/in}$

(f) $\delta = 0.05/25.4 = 0.00197\text{ in}$

(g) $v = 6.12/0.0051 = 1200\text{ ft/min}$

(h) $\epsilon = 0.0021\text{ in/in}$

(i) $V = 30/(0.254)^3 = 1831\text{ in}^3$

1-11

(a) $\sigma = \dfrac{200}{15.3} = 13.1\text{ MPa}$

(b) $\sigma = \dfrac{42(10^3)}{6(10^{-2})^2} = 70(10^6)\text{ N/m}^2 = 70\text{ MPa}$

(c) $y = \dfrac{1200(800)^3(10^{-3})^3}{3(207)10^9(64)10^3(10^{-3})^4} = 1.546(10^{-2})\text{ m} = 15.5\text{ mm}$

(d) $\theta = \dfrac{1100(250)(10^{-3})}{79.3(10^9)(\pi/32)(25)^4(10^{-3})^4} = 9.043(10^{-2})\text{ rad} = 5.18^\circ$

1-12

(a) $\sigma = \dfrac{600}{20(6)} = 5\text{ MPa}$

(b) $I = \dfrac{1}{12}8(24)^3 = 9216\text{ mm}^4$

(c) $I = \dfrac{\pi}{64}32^4(10^{-1})^4 = 5.147\text{ cm}^4$

(d) $\tau = \dfrac{16(16)}{\pi(25^3)(10^{-3})^3} = 5.215(10^6)\text{ N/m}^2 = 5.215\text{ MPa}$

1-13

(a) $\tau = \dfrac{120(10^3)}{(\pi/4)(20^2)} = 382\ \text{MPa}$

(b) $\sigma = \dfrac{32(800)(800)(10^{-3})}{\pi(32)^3(10^{-3})^3} = 198.9(10^6)\ \text{N/m}^2 = 198.9\,\text{MPa}$

(c) $Z = \dfrac{\pi}{32(36)}(36^4 - 26^4) = 3334\ \text{mm}^3$

(d) $k = \dfrac{(1.6)^4\,(10^{-3})^4(79.3)(10^9)}{8(19.2)^3(10^{-3})^3(32)} = 286.8\,\text{N/m}$

Chapter 2

2-1 From Table A-20

$$S_{ut} = 470\,\text{MPa (68 kpsi)}, \quad S_y = 390\,\text{MPa (57 kpsi)} \quad \textit{Ans.}$$

2-2 From Table A-20

$$S_{ut} = 620\,\text{MPa (90 kpsi)}, \quad S_y = 340\,\text{MPa (49.5 kpsi)} \quad \textit{Ans.}$$

2-3 Comparison of yield strengths:

$$S_{ut} \text{ of G10 500 HR is } \frac{620}{470} = 1.32 \text{ times larger than SAE1020 CD} \quad \textit{Ans.}$$

$$S_{yt} \text{ of SAE1020 CD is } \frac{390}{340} = 1.15 \text{ times larger than G10500 HR} \quad \textit{Ans.}$$

From Table A-20, the ductilities (reduction in areas) show,

$$\text{SAE1020 CD is } \frac{40}{35} = 1.14 \text{ times larger than G10500} \quad \textit{Ans.}$$

The stiffness values of these materials are identical *Ans.*

	S_{ut} MPa (kpsi)	S_y MPa (kpsi)	Table A-20 Ductility $R\%$	Table A-5 Stiffness GPa (Mpsi)
SAE1020 CD	470(68)	390 (57)	40	207(30)
UNS10500 HR	620(90)	340(495)	35	207(30)

2-4 From Table A-21

$$\text{1040 Q\&T} \quad \bar{S}_y = 593\,(86) \text{ MPa (kpsi)} \quad \text{at } 205^\circ\text{C } (400^\circ\text{F}) \quad \textit{Ans.}$$

2-5 From Table A-21

$$\text{1040 Q\&T} \quad R = 65\% \quad \text{at } 650^\circ\text{C } (1200^\circ\text{F}) \quad \textit{Ans.}$$

2-6 Using Table A-5, the specific strengths are:

$$\text{UNS G10350 HR steel:} \quad \frac{S_y}{W} = \frac{39.5(10^3)}{0.282} = 1.40(10^5)\text{ in} \quad \textit{Ans.}$$

$$\text{2024 T4 aluminum:} \quad \frac{S_y}{W} = \frac{43(10^3)}{0.098} = 4.39(10^5)\text{ in} \quad \textit{Ans.}$$

$$\text{Ti-6Al-4V titanium:} \quad \frac{S_y}{W} = \frac{140(10^3)}{0.16} = 8.75(10^5)\text{ in} \quad \textit{Ans.}$$

ASTM 30 gray cast iron has no yield strength. *Ans.*

2-7 The specific moduli are:

$$\text{UNS G10350 HR steel:} \quad \frac{E}{W} = \frac{30(10^6)}{0.282} = 1.06(10^8)\text{ in} \quad \textit{Ans.}$$

$$\text{2024 T4 aluminum:} \quad \frac{E}{W} = \frac{10.3(10^6)}{0.098} = 1.05(10^8)\text{ in} \quad \textit{Ans.}$$

$$\text{Ti-6Al-4V titanium:} \quad \frac{E}{W} = \frac{16.5(10^6)}{0.16} = 1.03(10^8)\text{ in} \quad \textit{Ans.}$$

$$\text{Gray cast iron:} \quad \frac{E}{W} = \frac{14.5(10^6)}{0.26} = 5.58(10^7)\text{ in} \quad \textit{Ans.}$$

2-8

$$2G(1+\nu) = E \quad \Rightarrow \quad \nu = \frac{E-2G}{2G}$$

From Table A-5

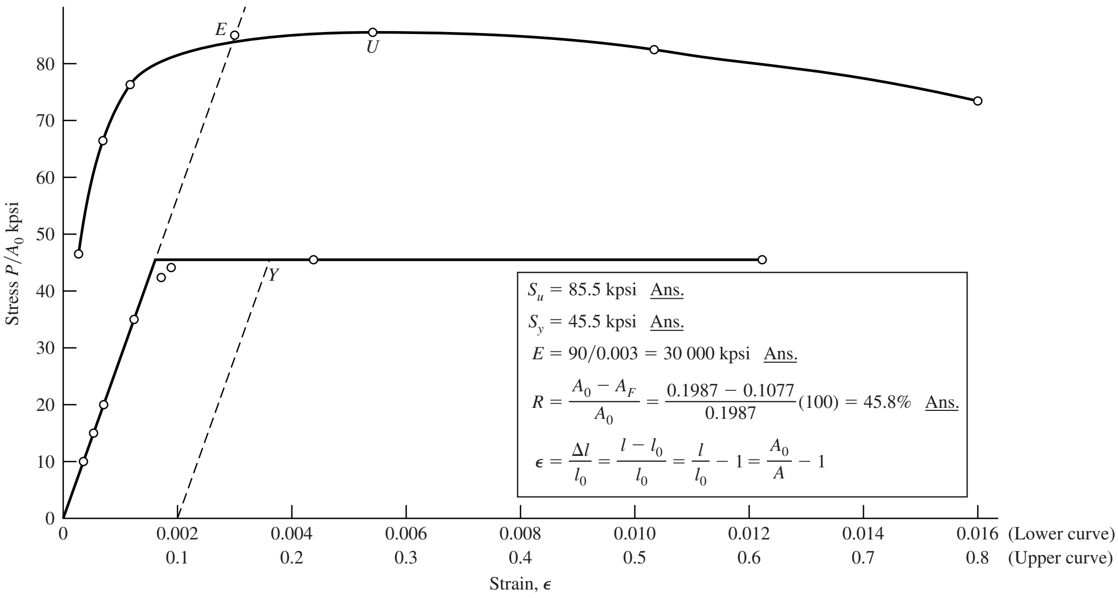
$$\text{Steel:} \quad \nu = \frac{30 - 2(11.5)}{2(11.5)} = 0.304 \quad \textit{Ans.}$$

$$\text{Aluminum:} \quad \nu = \frac{10.4 - 2(3.90)}{2(3.90)} = 0.333 \quad \textit{Ans.}$$

$$\text{Beryllium copper:} \quad \nu = \frac{18 - 2(7)}{2(7)} = 0.286 \quad \textit{Ans.}$$

$$\text{Gray cast iron:} \quad \nu = \frac{14.5 - 2(6)}{2(6)} = 0.208 \quad \textit{Ans.}$$

2-9



$S_u = 85.5$ kpsi <u>Ans.</u>

$S_y = 45.5$ kpsi <u>Ans.</u>

$E = 90/0.003 = 30\,000$ kpsi <u>Ans.</u>

$$R = \frac{A_0 - A_F}{A_0} = \frac{0.1987 - 0.1077}{0.1987}(100) = 45.8\% \quad \underline{\text{Ans.}}$$

$$\epsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1 = \frac{A_0}{A} - 1$$

2-10 To plot σ_{true} vs. ε, the following equations are applied to the data.

$$A_0 = \frac{\pi(0.503)^2}{4} = 0.1987\,\text{in}^2$$

Eq. (2-4)

$$\varepsilon = \ln\frac{l}{l_0} \quad \text{for} \quad 0 \le \Delta L \le 0.0028\,\text{in}$$

$$\varepsilon = \ln\frac{A_0}{A} \quad \text{for} \quad \Delta L > 0.0028\,\text{in}$$

$$\sigma_{\text{true}} = \frac{P}{A}$$

The results are summarized in the table below and plotted on the next page.
The last 5 points of data are used to plot $\log\sigma$ vs $\log\varepsilon$

The curve fit gives $m = 0.2306$

$$\log\sigma_0 = 5.1852 \quad \Rightarrow \quad \sigma_0 = 153.2\ \text{kpsi} \qquad \textit{Ans.}$$

For 20% cold work, Eq. (2-10) and Eq. (2-13) give,

$$A = A_0(1 - W) = 0.1987(1 - 0.2) = 0.1590\,\text{in}^2$$

$$\varepsilon = \ln\frac{A_0}{A} = \ln\frac{0.1987}{0.1590} = 0.2231$$

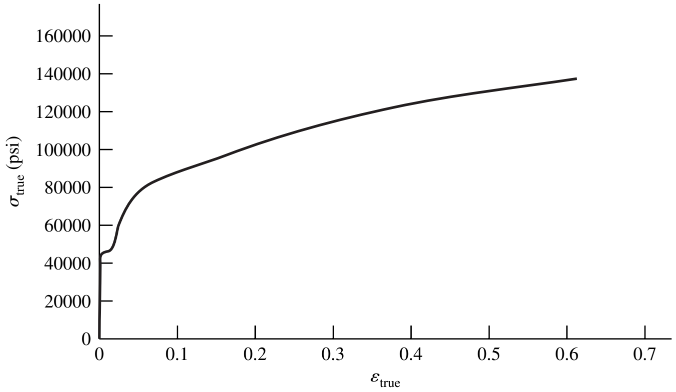
Eq. (2-14):

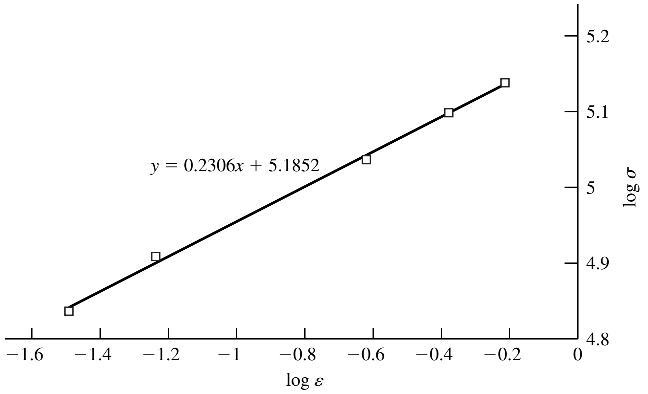
$$S'_y = \sigma_0\varepsilon^m = 153.2(0.2231)^{0.2306} = 108.4\,\text{kpsi} \quad \textit{Ans.}$$

Eq. (2-15), with $S_u = 85.5$ kpsi from Prob. 2-9,

$$S'_u = \frac{S_u}{1 - W} = \frac{85.5}{1 - 0.2} = 106.9\,\text{kpsi} \quad \textit{Ans.}$$

P	ΔL	A	ε	σ_{true}	$\log\varepsilon$	$\log\sigma_{\text{true}}$
0	0	0.198 713	0	0		
1 000	0.0004	0.198 713	0.000 2	5032.388	−3.699 01	3.701 774
2 000	0.0006	0.198 713	0.000 3	10 064.78	−3.522 94	4.002 804
3 000	0.0010	0.198 713	0.000 5	15 097.17	−3.301 14	4.178 895
4 000	0.0013	0.198 713	0.000 65	20 129.55	−3.187 23	4.303 834
7 000	0.0023	0.198 713	0.001 149	35 226.72	−2.939 55	4.546 872
8 400	0.0028	0.198 713	0.001 399	42 272.06	−2.854 18	4.626 053
8 800	0.0036	0.198 4	0.001 575	44 354.84	−2.802 61	4.646 941
9 200	0.0089	0.197 8	0.004 604	46 511.63	−2.336 85	4.667 562
9 100		0.196 3	0.012 216	46 357.62	−1.913 05	4.666 121
13 200		0.192 4	0.032 284	68 607.07	−1.491 01	4.836 369
15 200		0.187 5	0.058 082	81 066.67	−1.235 96	4.908 842
17 000		0.156 3	0.240 083	108 765.2	−0.619 64	5.036 49
16 400		0.130 7	0.418 956	125 478.2	−0.377 83	5.098 568
14 800		0.107 7	0.612 511	137 418.8	−0.212 89	5.138 046





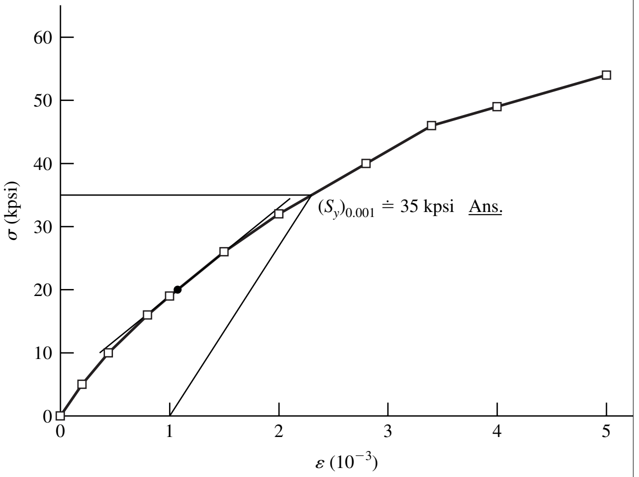
2-11 Tangent modulus at $\sigma = 0$ is

$$E_0 = \frac{\Delta\sigma}{\Delta\varepsilon} \doteq \frac{5000 - 0}{0.2(10^{-3}) - 0} = 25(10^6)\text{ psi}$$

At $\sigma = 20$ kpsi

$$E_{20} \doteq \frac{(26 - 19)(10^3)}{(1.5 - 1)(10^{-3})} = 14.0(10^6)\text{ psi} \quad \textit{Ans.}$$

$\varepsilon(10^{-3})$	σ (kpsi)
0	0
0.20	5
0.44	10
0.80	16
1.0	19
1.5	26
2.0	32
2.8	40
3.4	46
4.0	49
5.0	54



2-12 Since $|\varepsilon_o| = |\varepsilon_i|$

$$\left|\ln \frac{R+h}{R+N}\right| = \left|\ln \frac{R}{R+N}\right| = \left|-\ln \frac{R+N}{R}\right|$$

$$\frac{R+h}{R+N} = \frac{R+N}{R}$$

$$(R+N)^2 = R(R+h)$$

From which, $$N^2 + 2RN - Rh = 0$$

The roots are: $$N = R\left[-1 \pm \left(1 + \frac{h}{R}\right)^{1/2}\right]$$

The + sign being significant,

$$N = R\left[\left(1 + \frac{h}{R}\right)^{1/2} - 1\right] \quad Ans.$$

Substitute for N in

$$\varepsilon_o = \ln \frac{R+h}{R+N}$$

Gives $$\varepsilon_0 = \ln\left[\frac{R+h}{R + R\left(1 + \frac{h}{R}\right)^{1/2} - R}\right] = \ln\left(1 + \frac{h}{R}\right)^{1/2} \quad Ans.$$

These constitute a useful pair of equations in cold-forming situations, allowing the surface strains to be found so that cold-working strength enhancement can be estimated.

2-13 From Table A-22

AISI 1212 $$S_y = 28.0\ \text{kpsi}, \quad \sigma_f = 106\ \text{kpsi}, \quad S_{ut} = 61.5\ \text{kpsi}$$

$$\sigma_0 = 110\ \text{kpsi}, \quad m = 0.24, \quad \varepsilon_f = 0.85$$

From Eq. (2-12) $$\varepsilon_u = m = 0.24$$

Eq. (2-10) $$\frac{A_0}{A_i'} = \frac{1}{1-W} = \frac{1}{1-0.2} = 1.25$$

Eq. (2-13) $$\varepsilon_i = \ln 1.25 = 0.2231 \quad \Rightarrow \quad \varepsilon_i < \varepsilon_u$$

Eq. (2-14) $$S_y' = \sigma_0 \varepsilon_i^m = 110(0.2231)^{0.24} = 76.7\ \text{kpsi} \quad Ans.$$

Eq. (2-15) $$S_u' = \frac{S_u}{1-W} = \frac{61.5}{1-0.2} = 76.9\ \text{kpsi} \quad Ans.$$

2-14 For $H_B = 250$,

Eq. (2-17) $$\begin{aligned} S_u &= 0.495\ (250) = 124\ \text{kpsi} \\ &= 3.41\ (250) = 853\ \text{MPa} \end{aligned} \quad Ans.$$

2-15 For the data given,

$$\sum H_B = 2530 \quad \sum H_B^2 = 640\,226$$

$$\bar{H}_B = \frac{2530}{10} = 253 \quad \hat{\sigma}_{HB} = \sqrt{\frac{640\,226 - (2530)^2/10}{9}} = 3.887$$

Eq. (2-17)

$$\bar{S}_u = 0.495(253) = 125.2 \text{ kpsi} \quad \textit{Ans.}$$

$$\bar{\sigma}_{su} = 0.495(3.887) = 1.92 \text{ kpsi} \quad \textit{Ans.}$$

2-16 From Prob. 2-15, $\bar{H}_B = 253$ and $\hat{\sigma}_{HB} = 3.887$

Eq. (2-18)

$$\bar{S}_u = 0.23(253) - 12.5 = 45.7 \text{ kpsi} \quad \textit{Ans.}$$

$$\hat{\sigma}_{su} = 0.23(3.887) = 0.894 \text{ kpsi} \quad \textit{Ans.}$$

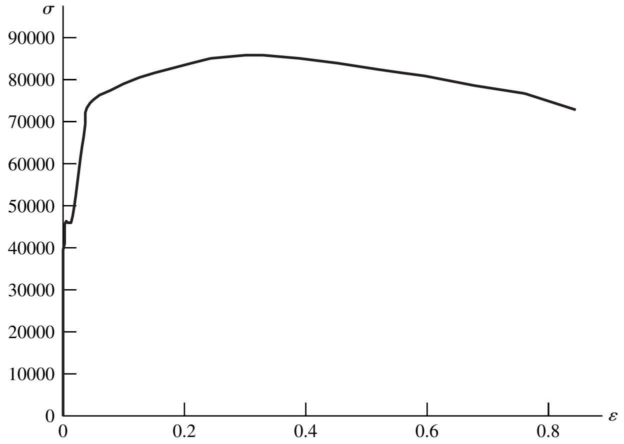
2-17

(a)

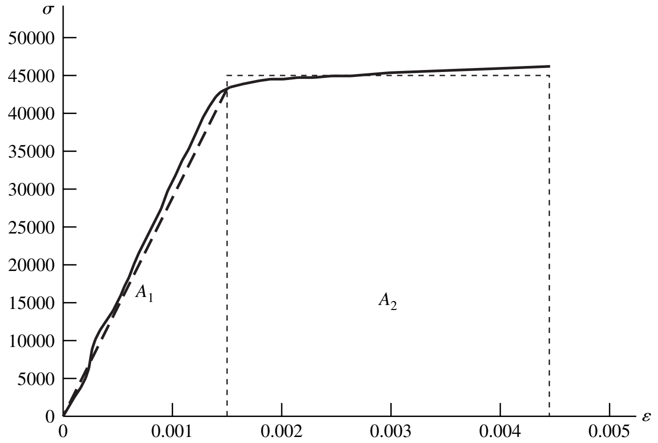
$$u_R \doteq \frac{45.5^2}{2(30)} = 34.5 \text{ in} \cdot \text{lbf/in}^3 \quad \textit{Ans.}$$

(b)

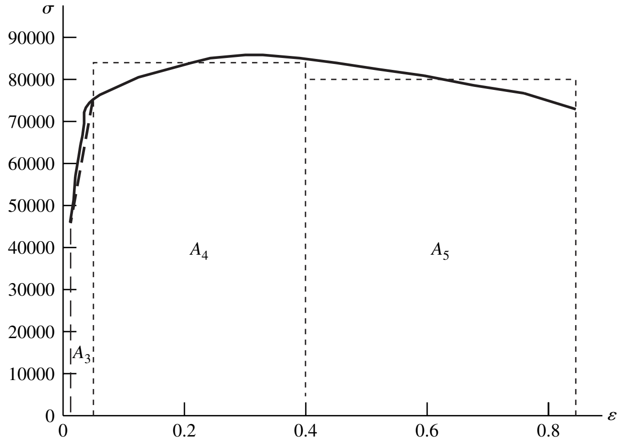
P	ΔL	A	$A_0/A - 1$	ε	$\sigma = P/A_0$
0	0			0	0
1 000	0.0004			0.0002	5 032.39
2 000	0.0006			0.0003	10 064.78
3 000	0.0010			0.0005	15 097.17
4 000	0.0013			0.000 65	20 129.55
7 000	0.0023			0.001 15	35 226.72
8 400	0.0028			0.0014	42 272.06
8 800	0.0036			0.0018	44 285.02
9 200	0.0089			0.004 45	46 297.97
9 100		0.1963	0.012 291	0.012 291	45 794.73
13 200		0.1924	0.032 811	0.032 811	66 427.53
15 200		0.1875	0.059 802	0.059 802	76 492.30
17 000		0.1563	0.271 355	0.271 355	85 550.60
16 400		0.1307	0.520 373	0.520 373	82 531.17
14 800		0.1077	0.845 059	0.845 059	74 479.35



All data points



First 9 data points



Last 6 data points

$$
\begin{aligned}
u_T \doteq \sum_{i=1}^{5} A_i &= \frac{1}{2}(43\,000)(0.001\,5) + 45\,000(0.004\,45 - 0.001\,5) \\
&\quad + \frac{1}{2}(45\,000 + 76\,500)(0.059\,8 - 0.004\,45) \\
&\quad + 81\,000(0.4 - 0.059\,8) + 80\,000(0.845 - 0.4)
\end{aligned}
$$

$$\doteq 66.7(10^3)\text{in} \cdot \text{lbf/in}^3 \quad \textit{Ans.}$$

2-18 $m = Al\rho$

For stiffness, $k = AE/l$, or, $A = kl/E$.

Thus, $m = kl^2\rho/E$, and, $M = E/\rho$. Therefore, $\beta = 1$

From Fig. 2-16, ductile materials include Steel, Titanium, Molybdenum, Aluminum, and Composites.

For strength, $S = F/A$, or, $A = F/S$.

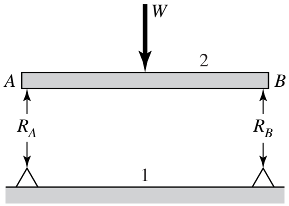
Thus, $m = Fl\,\rho/S$, and, $M = S/\rho$.

From Fig. 2-19, lines parallel to S/ρ give for ductile materials, Steel, Nickel, Titanium, and composites.

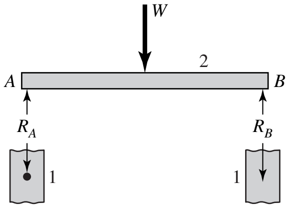
Common to both stiffness and strength are Steel, Titanium, Aluminum, and Composites. *Ans.*

Chapter 3

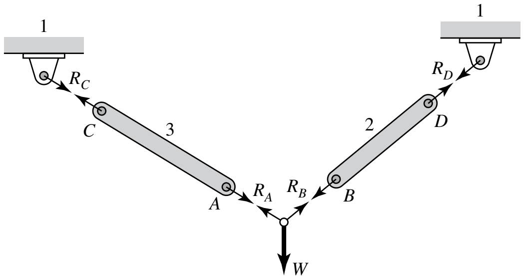
3-1



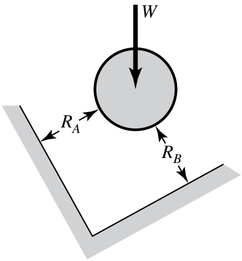
(a)



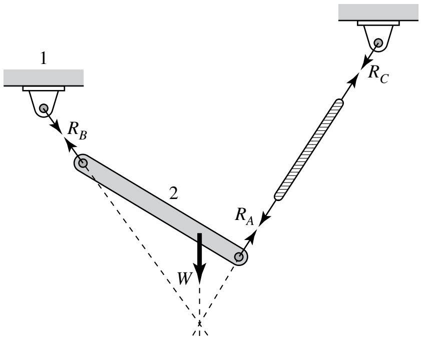
(b)



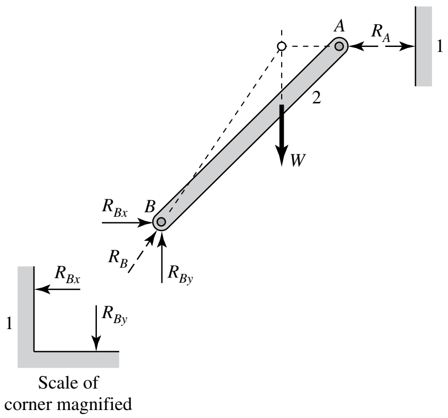
(c)



(d)



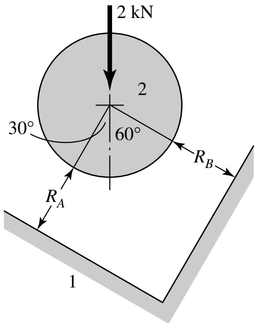
(e)

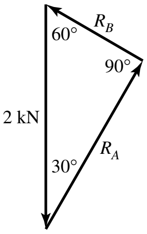


(f)

3-2

(a)

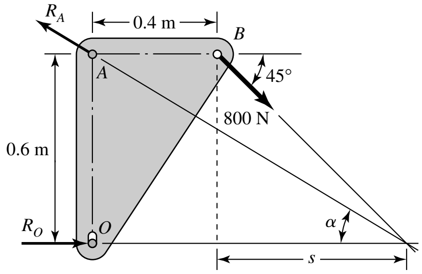




$$R_A = 2\sin 60 = 1.732\ \text{kN} \quad \textit{Ans.}$$

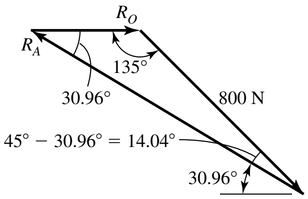
$$R_B = 2\sin 30 = 1\ \text{kN} \quad \textit{Ans.}$$

(b)



$$S = 0.6\ \text{m}$$

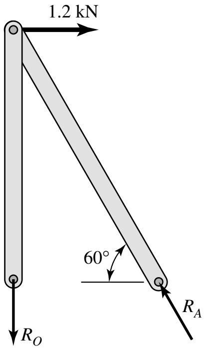
$$\alpha = \tan^{-1}\frac{0.6}{0.4+0.6} = 30.96^\circ$$

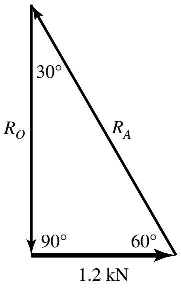


$$\frac{R_A}{\sin 135} = \frac{800}{\sin 30.96} \quad \Rightarrow \quad R_A = 1100\ \text{N} \quad \textit{Ans.}$$

$$\frac{R_O}{\sin 14.04} = \frac{800}{\sin 30.96} \quad \Rightarrow \quad R_O = 377\ \text{N} \quad \textit{Ans.}$$

(c)

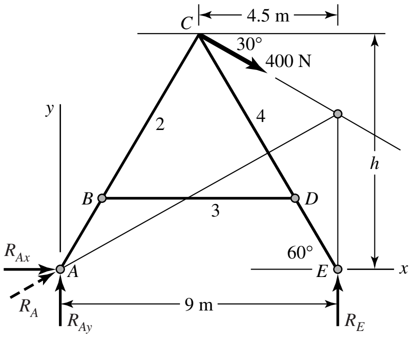




$$R_O = \frac{1.2}{\tan 30} = 2.078\ \text{kN} \quad \textit{Ans.}$$

$$R_A = \frac{1.2}{\sin 30} = 2.4\ \text{kN} \quad \textit{Ans.}$$

(d) Step 1: Find R_A and R_E



$$h = \frac{4.5}{\tan 30} = 7.794\,\text{m}$$

$$\circlearrowleft + \sum M_A = 0$$

$$9R_E - 7.794(400\cos 30) - 4.5(400\sin 30) = 0$$

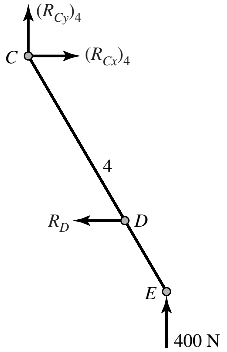
$$R_E = 400\,\text{N} \quad \textit{Ans.}$$

$$\sum F_x = 0 \quad R_{Ax} + 400\cos 30 = 0 \quad \Rightarrow \quad R_{Ax} = -346.4\,\text{N}$$

$$\sum F_y = 0 \quad R_{Ay} + 400 - 400\sin 30 = 0 \quad \Rightarrow \quad R_{Ay} = -200\,\text{N}$$

$$R_A = \sqrt{346.4^2 + 200^2} = 400\,\text{N} \quad \textit{Ans.}$$

Step 2: Find components of R_C on link 4 and R_D



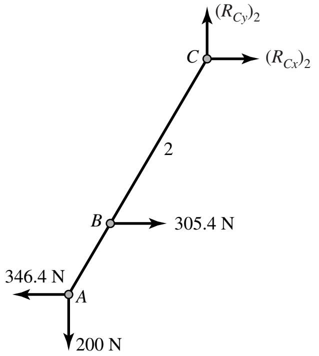
$$\circlearrowleft + \sum M_C = 0$$

$$400(4.5) - (7.794 - 1.9)R_D = 0 \quad \Rightarrow \quad R_D = 305.4\,\text{N} \quad \textit{Ans.}$$

$$\sum F_x = 0 \quad \Rightarrow \quad (R_{Cx})_4 = 305.4\,\text{N}$$

$$\sum F_y = 0 \quad \Rightarrow \quad (R_{Cy})_4 = -400\,\text{N}$$

Step 3: Find components of R_C on link 2

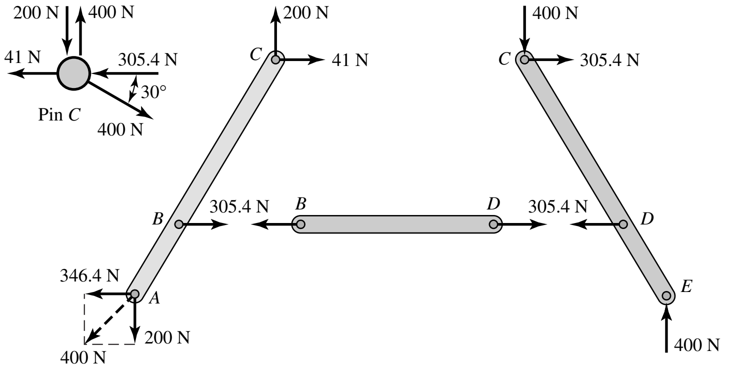


$$\sum F_x = 0$$

$$(R_{Cx})_2 + 305.4 - 346.4 = 0 \quad \Rightarrow \quad (R_{Cx})_2 = 41\,\text{N}$$

$$\sum F_y = 0$$

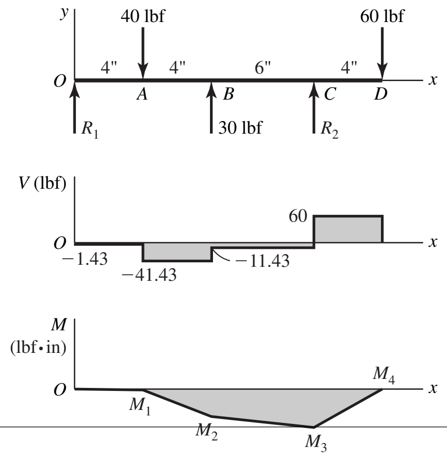
$$(R_{Cy})_2 = 200\,\text{N}$$



Ans.

3-3

(a)



$$\circlearrowleft + \sum M_0 = 0$$

$$-18(60) + 14R_2 + 8(30) - 4(40) = 0$$

$$R_2 = 71.43\text{ lbf}$$

$$\sum F_y = 0\text{: } R_1 - 40 + 30 + 71.43 - 60 = 0$$

$$R_1 = -1.43\text{ lbf}$$

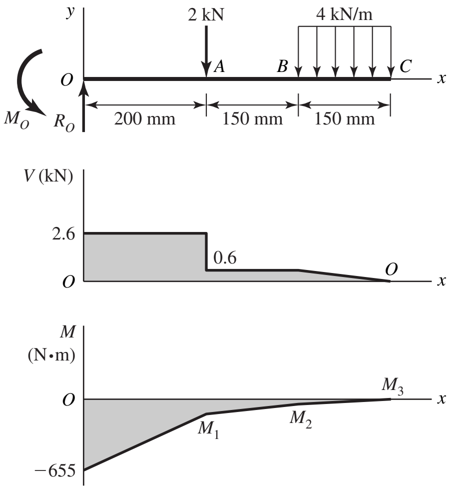
$$M_1 = -1.43(4) = -5.72\text{ lbf}\cdot\text{in}$$

$$M_2 = -5.72 - 41.43(4) = -171.44\text{ lbf}\cdot\text{in}$$

$$M_3 = -171.44 - 11.43(6) = -240\text{ lbf}\cdot\text{in}$$

$$M_4 = -240 + 60(4) = 0 \quad \text{checks!}$$

(b)



$$\sum F_y = 0$$

$$R_0 = 2 + 4(0.150) = 2.6\,\text{kN}$$

$$\sum M_0 = 0$$

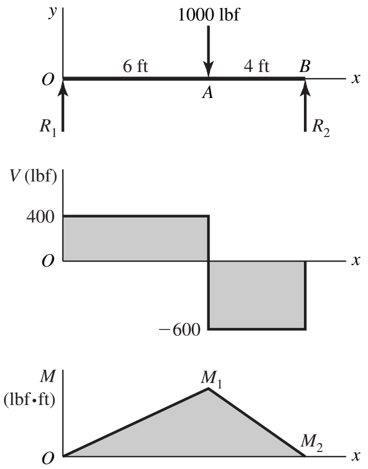
$$M_0 = 2000(0.2) + 4000(0.150)(0.425)$$
$$= 655\ \text{N} \cdot \text{m}$$

$$M_1 = -655 + 2600(0.2) = -135\ \text{N} \cdot \text{m}$$

$$M_2 = -135 + 600(0.150) = -45\ \text{N} \cdot \text{m}$$

$$M_3 = -45 + \frac{1}{2}600(0.150) = 0 \quad \text{checks!}$$

(c)



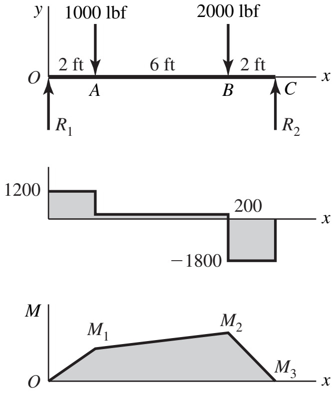
$$\sum M_0 = 0\text{: } 10R_2 - 6(1000) = 0 \quad \Rightarrow \quad R_2 = 600\ \text{lbf}$$

$$\sum F_y = 0\text{: } R_1 - 1000 + 600 = 0 \quad \Rightarrow \quad R_1 = 400\ \text{lbf}$$

$$M_1 = 400(6) = 2400\ \text{lbf} \cdot \text{ft}$$

$$M_2 = 2400 - 600(4) = 0 \quad \text{checks!}$$

(d)



$$\circlearrowleft + \sum M_C = 0$$

$$-10R_1 + 2(2000) + 8(1000) = 0$$

$$R_1 = 1200\ \text{lbf}$$

$$\sum F_y = 0\text{: } 1200 - 1000 - 2000 + R_2 = 0$$

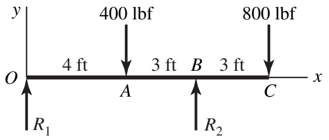
$$R_2 = 1800\ \text{lbf}$$

$$M_1 = 1200(2) = 2400\ \text{lbf} \cdot \text{ft}$$

$$M_2 = 2400 + 200(6) = 3600\ \text{lbf} \cdot \text{ft}$$

$$M_3 = 3600 - 1800(2) = 0 \quad \text{checks!}$$

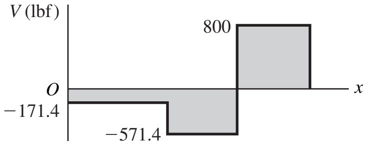
(e)



$$\circlearrowleft + \sum M_B = 0$$

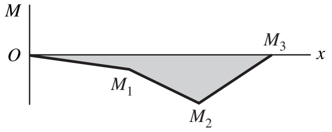
$$-7R_1 + 3(400) - 3(800) = 0$$

$$R_1 = -171.4 \text{ lbf}$$



$$\sum F_y = 0: -171.4 - 400 + R_2 - 800 = 0$$

$$R_2 = 1371.4 \text{ lbf}$$

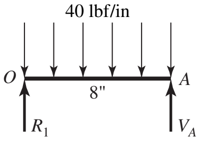


$$M_1 = -171.4(4) = -685.7 \text{ lbf} \cdot \text{ft}$$

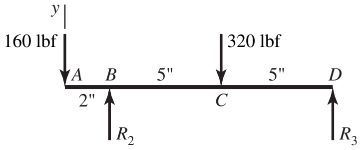
$$M_2 = -685.7 - 571.4(3) = -2400 \text{ lbf} \cdot \text{ft}$$

$$M_3 = -2400 + 800(3) = 0 \quad \text{checks!}$$

(f) Break at *A*



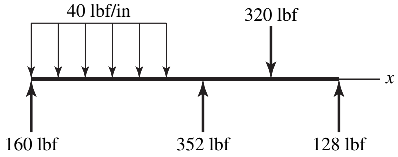
$$R_1 = V_A = \frac{1}{2}40(8) = 160 \text{ lbf}$$



$$\circlearrowleft + \sum M_D = 0$$

$$12(160) - 10R_2 + 320(5) = 0$$

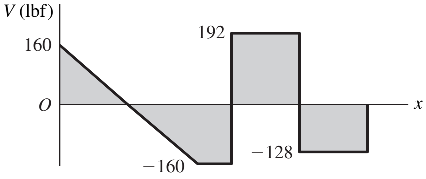
$$R_2 = 352 \text{ lbf}$$

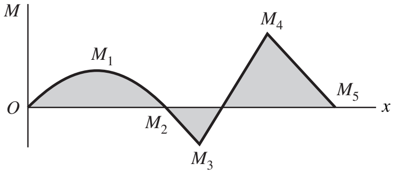


$$\sum F_y = 0$$

$$-160 + 352 - 320 + R_3 = 0$$

$$R_3 = 128 \text{ lbf}$$





$$M_1 = \frac{1}{2}160(4) = 320 \text{ lbf} \cdot \text{in}$$

$$M_2 = 320 - \frac{1}{2}160(4) = 0 \quad \text{checks! (hinge)}$$

$$M_3 = 0 - 160(2) = -320 \text{ lbf} \cdot \text{in}$$

$$M_4 = -320 + 192(5) = 640 \text{ lbf} \cdot \text{in}$$

$$M_5 = 640 - 128(5) = 0 \quad \text{checks!}$$

3-4

(a)
$$q = R_1\langle x\rangle^{-1} - 40\langle x-4\rangle^{-1} + 30\langle x-8\rangle^{-1} + R_2\langle x-14\rangle^{-1} - 60\langle x-18\rangle^{-1}$$
$$V = R_1 - 40\langle x-4\rangle^{0} + 30\langle x-8\rangle^{0} + R_2\langle x-14\rangle^{0} - 60\langle x-18\rangle^{0} \tag{1}$$
$$M = R_1 x - 40\langle x-4\rangle^{1} + 30\langle x-8\rangle^{1} + R_2\langle x-14\rangle^{1} - 60\langle x-18\rangle^{1} \tag{2}$$

for $x = 18^+$ $V = 0$ and $M = 0$ Eqs. (1) and (2) give

$$0 = R_1 - 40 + 30 + R_2 - 60 \quad\Rightarrow\quad R_1 + R_2 = 70 \tag{3}$$
$$0 = R_1(18) - 40(14) + 30(10) + 4R_2 \quad\Rightarrow\quad 9R_1 + 2R_2 = 130 \tag{4}$$

Solve (3) and (4) simultaneously to get $R_1 = -1.43$ lbf, $R_2 = 71.43$ lbf. *Ans.*

From Eqs. (1) and (2), at $x = 0^+$, $V = R_1 = -1.43$ lbf, $M = 0$

$x = 4^+$: $V = -1.43 - 40 = -41.43,\ M = -1.43x$

$x = 8^+$: $V = -1.43 - 40 + 30 = -11.43$
$M = -1.43(8) - 40(8-4)^1 = -171.44$

$x = 14^+$: $V = -1.43 - 40 + 30 + 71.43 = 60$
$M = -1.43(14) - 40(14-4) + 30(14-8) = -240.$

$x = 18^+$: $V = 0, M = 0$ See curves of V and M in Prob. 3-3 solution.

(b)
$$q = R_0\langle x\rangle^{-1} - M_0\langle x\rangle^{-2} - 2000\langle x-0.2\rangle^{-1} - 4000\langle x-0.35\rangle^{0} + 4000\langle x-0.5\rangle^{0}$$
$$V = R_0 - M_0\langle x\rangle^{-1} - 2000\langle x-0.2\rangle^{0} - 4000\langle x-0.35\rangle^{1} + 4000\langle x-0.5\rangle^{1} \tag{1}$$
$$M = R_0 x - M_0 - 2000\langle x-0.2\rangle^{1} - 2000\langle x-0.35\rangle^{2} + 2000\langle x-0.5\rangle^{2} \tag{2}$$

at $x = 0.5^+$ m, $V = M = 0$, Eqs. (1) and (2) give

$$R_0 - 2000 - 4000(0.5-0.35) = 0 \quad\Rightarrow\quad R_1 = 2600\,\text{N} = 2.6\ \text{kN} \quad \textit{Ans.}$$
$$R_0(0.5) - M_0 - 2000(0.5-0.2) - 2000(0.5-0.35)^2 = 0$$

with $R_0 = 2600$ N, $M_0 = 655$ N · m *Ans.*

With R_0 and M_0, Eqs. (1) and (2) give the same V and M curves as Prob. 3-3 (note for V, $M_0\langle x\rangle^{-1}$ has no physical meaning).

(c)
$$q = R_1\langle x\rangle^{-1} - 1000\langle x-6\rangle^{-1} + R_2\langle x-10\rangle^{-1}$$
$$V = R_1 - 1000\langle x-6\rangle^{0} + R_2\langle x-10\rangle^{0} \tag{1}$$
$$M = R_1 x - 1000\langle x-6\rangle^{1} + R_2\langle x-10\rangle^{1} \tag{2}$$

at $x = 10^+$ ft, $V = M = 0$, Eqs. (1) and (2) give

$$R_1 - 1000 + R_2 = 0 \quad\Rightarrow\quad R_1 + R_2 = 1000$$
$$10R_1 - 1000(10-6) = 0 \quad\Rightarrow\quad R_1 = 400\,\text{lbf}, \quad R_2 = 1000 - 400 = 600\,\text{lbf}$$

$0 \le x \le 6$: $V = 400$ lbf, $M = 400x$

$6 \le x \le 10$: $V = 400 - 1000(x-6)^0 = 600$ lbf
$M = 400x - 1000(x-6) = 6000 - 600x$

See curves of Prob. 3-3 solution.

(d)
$$q = R_1\langle x\rangle^{-1} - 1000\langle x-2\rangle^{-1} - 2000\langle x-8\rangle^{-1} + R_2\langle x-10\rangle^{-1}$$
$$V = R_1 - 1000\langle x-2\rangle^{0} - 2000\langle x-8\rangle^{0} + R_2\langle x-10\rangle^{0} \tag{1}$$
$$M = R_1 x - 1000\langle x-2\rangle^{1} - 2000\langle x-8\rangle^{1} + R_2\langle x-10\rangle^{1} \tag{2}$$

At $x = 10^+$, $V = M = 0$ from Eqs. (1) and (2)

$$R_1 - 1000 - 2000 + R_2 = 0 \quad \Rightarrow \quad R_1 + R_2 = 3000$$

$$10R_1 - 1000(10-2) - 2000(10-8) = 0 \quad \Rightarrow \quad R_1 = 1200\,\text{lbf},$$
$$R_2 = 3000 - 1200 = 1800\,\text{lbf}$$

$0 \le x \le 2$: $V = 1200\,\text{lbf}$, $M = 1200x \text{ lbf} \cdot \text{ft}$

$2 \le x \le 8$: $V = 1200 - 1000 = 200\,\text{lbf}$

$$M = 1200x - 1000(x-2) = 200x + 2000 \text{ lbf} \cdot \text{ft}$$

$8 \le x \le 10$: $V = 1200 - 1000 - 2000 = -1800\,\text{lbf}$

$$M = 1200x - 1000(x-2) - 2000(x-8) = -1800x + 18\,000 \text{ lbf} \cdot \text{ft}$$

Plots are the same as in Prob. 3-3.

(e)
$$q = R_1\langle x\rangle^{-1} - 400\langle x-4\rangle^{-1} + R_2\langle x-7\rangle^{-1} - 800\langle x-10\rangle^{-1}$$
$$V = R_1 - 400\langle x-4\rangle^{0} + R_2\langle x-7\rangle^{0} - 800\langle x-10\rangle^{0} \tag{1}$$
$$M = R_1 x - 400\langle x-4\rangle^{1} + R_2\langle x-7\rangle^{1} - 800\langle x-10\rangle^{1} \tag{2}$$

at $x = 10^+$, $V = M = 0$

$$R_1 - 400 + R_2 - 800 = 0 \quad \Rightarrow \quad R_1 + R_2 = 1200 \tag{3}$$
$$10R_1 - 400(6) + R_2(3) = 0 \quad \Rightarrow \quad 10R_1 + 3R_2 = 2400 \tag{4}$$

Solve Eqs. (3) and (4) simultaneously: $R_1 = -171.4$ lbf, $R_2 = 1371.4$ lbf

$0 \le x \le 4$: $V = -171.4\,\text{lbf}$, $M = -171.4x \text{ lbf} \cdot \text{ft}$

$4 \le x \le 7$: $V = -171.4 - 400 = -571.4$ lbf

$$M = -171.4x - 400(x-4) \text{ lbf} \cdot \text{ft} = -571.4x + 1600$$

$7 \le x \le 10$: $V = -171.4 - 400 + 1371.4 = 800\,\text{lbf}$

$$M = -171.4x - 400(x-4) + 1371.4(x-7) = 800x - 8000\,\text{lbf} \cdot \text{ft}$$

Plots are the same as in Prob. 3-3.

(f)
$$q = R_1\langle x\rangle^{-1} - 40\langle x\rangle^{0} + 40\langle x-8\rangle^{0} + R_2\langle x-10\rangle^{-1} - 320\langle x-15\rangle^{-1} + R_3\langle x-20\rangle$$
$$V = R_1 - 40x + 40\langle x-8\rangle^{1} + R_2\langle x-10\rangle^{0} - 320\langle x-15\rangle^{0} + R_3\langle x-20\rangle^{0} \tag{1}$$
$$M = R_1 x - 20x^2 + 20\langle x-8\rangle^{2} + R_2\langle x-10\rangle^{1} - 320\langle x-15\rangle^{1} + R_3\langle x-20\rangle^{1} \tag{2}$$

$M = 0$ at $x = 8$ in $\therefore 8R_1 - 20(8)^2 = 0 \quad \Rightarrow \quad R_1 = 160$ lbf

at $x = 20^+$, V and $M = 0$

$$160 - 40(20) + 40(12) + R_2 - 320 + R_3 = 0 \quad \Rightarrow \quad R_2 + R_3 = 480$$
$$160(20) - 20(20)^2 + 20(12)^2 + 10R_2 - 320(5) = 0 \quad \Rightarrow \quad R_2 = 352 \text{ lbf}$$
$$R_3 = 480 - 352 = 128 \text{ lbf}$$

$0 \le x \le 8$: $V = 160 - 40x$ lbf, $M = 160x - 20x^2 \text{ lbf} \cdot \text{in}$

$8 \le x \le 10$: $V = 160 - 40x + 40(x-8) = -160\,\text{lbf}$,

$$M = 160x - 20x^2 + 20(x-8)^2 = 1280 - 160x \text{ lbf} \cdot \text{in}$$

$10 \le x \le 15$: $V = 160 - 40x + 40(x-8) + 352 = 192\,\text{lbf}$

$$M = 160x - 20x^2 + 20(x-8) + 352(x-10) = 192x - 2240$$

$$15 \le x \le 20: \quad V = 160 - 40x + 40(x-8) + 352 - 320 = -128\,\text{lbf}$$

$$M = 160x - 20x^2 - 20(x-8) + 352(x-10) - 320(x-15)$$

$$= -128x + 2560$$

Plots of V and M are the same as in Prob. 3-3.

3-5 Solution depends upon the beam selected.

3-6

(a) Moment at center, $x_c = (l - 2a)/2$

$$M_c = \frac{w}{2}\left[\frac{l}{2}(l-2a) - \left(\frac{l}{2}\right)^2\right] = \frac{wl}{2}\left(\frac{l}{4} - a\right)$$

At reaction, $|M_r| = wa^2/2$

$a = 2.25$, $l = 10$ in, $w = 100$ lbf/in

$$M_c = \frac{100(10)}{2}\left(\frac{10}{4} - 2.25\right) = 125 \text{ lbf} \cdot \text{in}$$

$$M_r = \frac{100(2.25^2)}{2} = 253.1 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(b) Minimum occurs when $M_c = |M_r|$

$$\frac{wl}{2}\left(\frac{l}{4} - a\right) = \frac{wa^2}{2} \quad \Rightarrow \quad a^2 + al - 0.25l^2 = 0$$

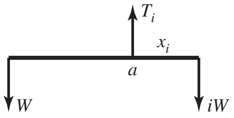
Taking the positive root

$$a = \frac{1}{2}\left[-l + \sqrt{l^2 + 4(0.25l^2)}\right] = \frac{l}{2}\left(\sqrt{2} - 1\right) = 0.2071l \quad \textit{Ans.}$$

for $l = 10$ in and $w = 100$ lbf, $\quad M_{\min} = (100/2)[(0.2071)(10)]^2 = 214 \text{ lbf} \cdot \text{in}$

3-7 For the ith wire from bottom, from summing forces vertically

(a)



$$T_i = (i+1)W$$

From summing moments about point a,

$$\sum M_a = W(l - x_i) - iWx_i = 0$$

Giving,

$$x_i = \frac{l}{i+1}$$

So

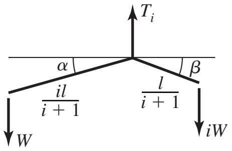
$$W = \frac{l}{1+1} = \frac{l}{2}$$

$$x = \frac{l}{2+1} = \frac{l}{3}$$

$$y = \frac{l}{3+1} = \frac{l}{4}$$

$$z = \frac{l}{4+1} = \frac{l}{5}$$

(b) With straight rigid wires, the mobile is not stable. Any perturbation can lead to all wires becoming collinear. Consider a wire of length l bent at its string support:



$$\sum M_a = 0$$

$$\sum M_a = \frac{iWl}{i+1}\cos\alpha - \frac{ilW}{i+1}\cos\beta = 0$$

$$\frac{iWl}{i+1}(\cos\alpha - \cos\beta) = 0$$

Moment vanishes when $\alpha = \beta$ for any wire. Consider a ccw rotation angle β, which makes $\alpha \rightarrow \alpha + \beta$ and $\beta \rightarrow \alpha - \beta$

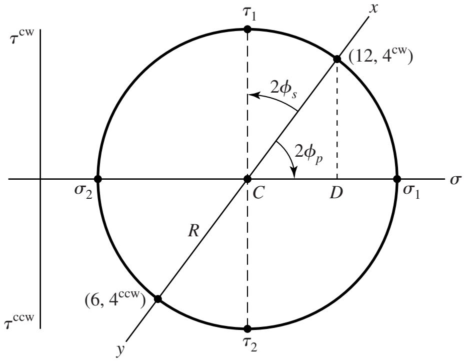
$$M_a = \frac{iWl}{i+1}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$$

$$= \frac{2iWl}{i+1}\sin\alpha\sin\beta \doteq \frac{2iWl\beta}{i+1}\sin\alpha$$

There exists a correcting moment of opposite sense to arbitrary rotation β. An equation for an upward bend can be found by changing the sign of W. The moment will no longer be correcting. A curved, convex-upward bend of wire will produce stable equilibrium too, but the equation would change somewhat.

3-8

(a)



$$C = \frac{12+6}{2} = 9$$

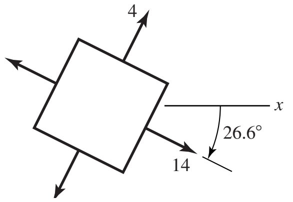
$$CD = \frac{12-6}{2} = 3$$

$$R = \sqrt{3^2 + 4^2} = 5$$

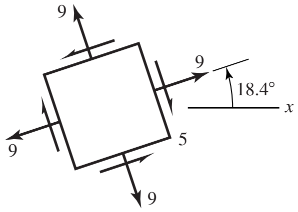
$$\sigma_1 = 5 + 9 = 14$$

$$\sigma_2 = 9 - 5 = 4$$

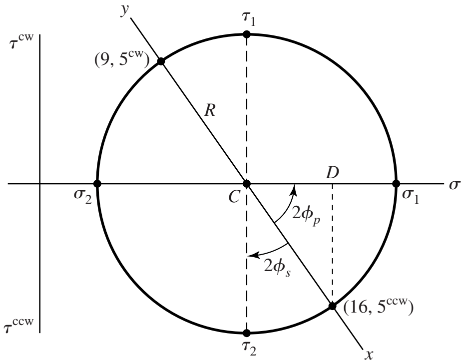
$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right) = 26.6^\circ \text{ cw}$$



$$\tau_1 = R = 5, \quad \phi_s = 45^\circ - 26.6^\circ = 18.4^\circ \text{ ccw}$$



(b)



$$C = \frac{9+16}{2} = 12.5$$

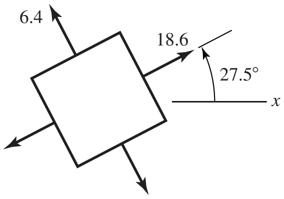
$$CD = \frac{16-9}{2} = 3.5$$

$$R = \sqrt{5^2 + 3.5^2} = 6.10$$

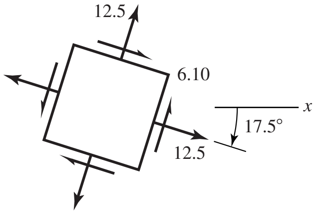
$$\sigma_1 = 6.1 + 12.5 = 18.6$$

$$\phi_p = \frac{1}{2}\tan^{-1}\frac{5}{3.5} = 27.5^\circ \text{ ccw}$$

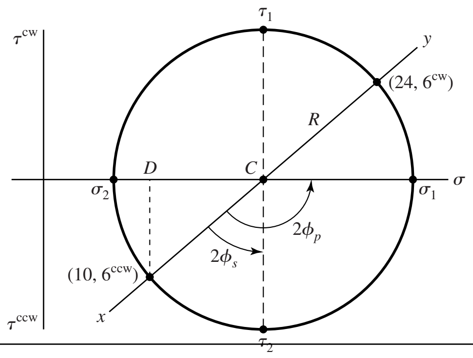
$$\sigma_2 = 12.5 - 6.1 = 6.4$$



$$\tau_1 = R = 6.10, \quad \phi_s = 45^\circ - 27.5^\circ = 17.5^\circ \text{ cw}$$



(c)



$$C = \frac{24+10}{2} = 17$$

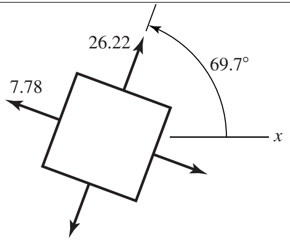
$$CD = \frac{24-10}{2} = 7$$

$$R = \sqrt{7^2 + 6^2} = 9.22$$

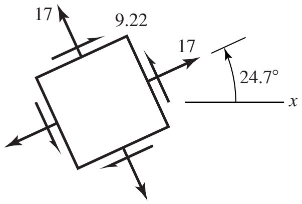
$$\sigma_1 = 17 + 9.22 = 26.22$$

$$\sigma_2 = 17 - 9.22 = 7.78$$

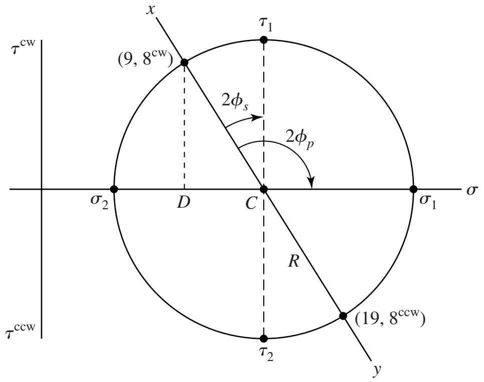
$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{7}{6}\right] = 69.7^\circ \text{ ccw}$$



$$\tau_1 = R = 9.22, \quad \phi_s = 69.7^\circ - 45^\circ = 24.7^\circ \text{ ccw}$$



(d)



$$C = \frac{9+19}{2} = 14$$

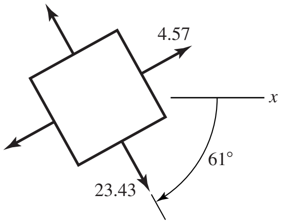
$$CD = \frac{19-9}{2} = 5$$

$$R = \sqrt{5^2 + 8^2} = 9.434$$

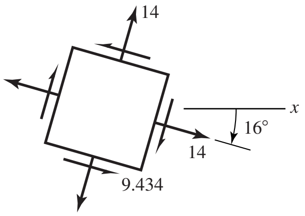
$$\sigma_1 = 14 + 9.43 = 23.43$$

$$\sigma_2 = 14 - 9.43 = 4.57$$

$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{5}{8}\right] = 61.0^\circ \text{ cw}$$

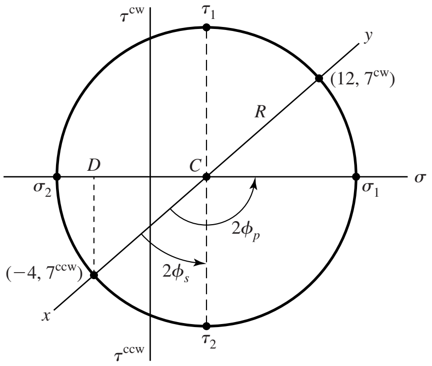


$$\tau_1 = R = 9.434, \quad \phi_s = 61^\circ - 45^\circ = 16^\circ \text{ cw}$$



3-9

(a)



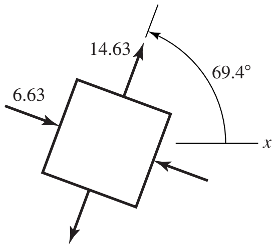
$$C = \frac{12 - 4}{2} = 4$$

$$CD = \frac{12 + 4}{2} = 8$$

$$R = \sqrt{8^2 + 7^2} = 10.63$$

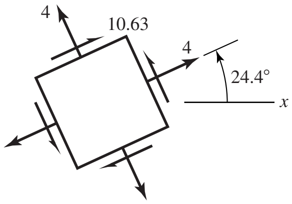
$$\sigma_1 = 4 + 10.63 = 14.63$$

$$\sigma_2 = 4 - 10.63 = -6.63$$

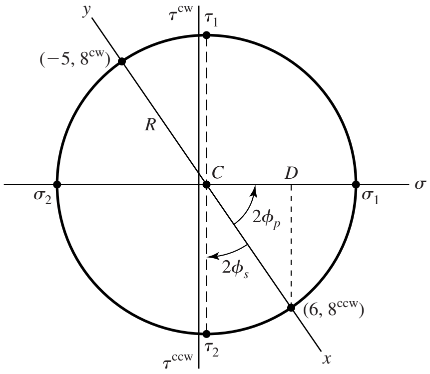


$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{8}{7}\right] = 69.4^\circ \text{ ccw}$$

$$\tau_1 = R = 10.63, \quad \phi_s = 69.4^\circ - 45^\circ = 24.4^\circ \text{ ccw}$$



(b)



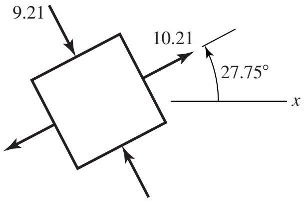
$$C = \frac{6 - 5}{2} = 0.5$$

$$CD = \frac{6 + 5}{2} = 5.5$$

$$R = \sqrt{5.5^2 + 8^2} = 9.71$$

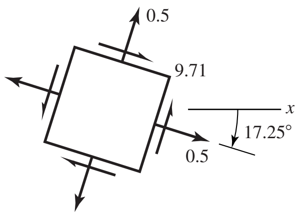
$$\sigma_1 = 0.5 + 9.71 = 10.21$$

$$\sigma_2 = 0.5 - 9.71 = -9.21$$

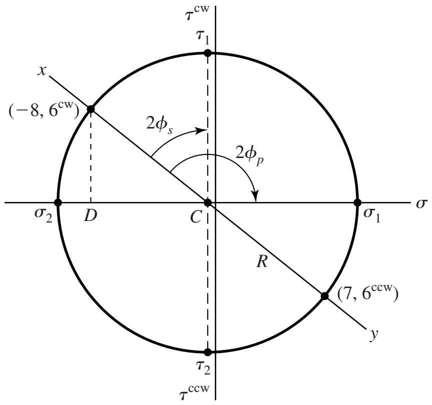


$$\phi_p = \frac{1}{2}\tan^{-1}\frac{8}{5.5} = 27.75^\circ \text{ ccw}$$

$$\tau_1 = R = 9.71, \quad \phi_s = 45^\circ - 27.75^\circ = 17.25^\circ \text{ cw}$$



(c)



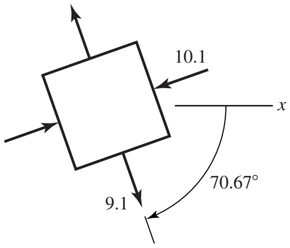
$$C = \frac{-8+7}{2} = -0.5$$

$$CD = \frac{8+7}{2} = 7.5$$

$$R = \sqrt{7.5^2 + 6^2} = 9.60$$

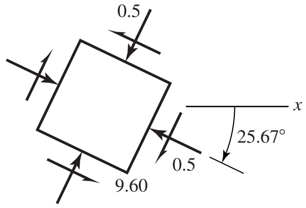
$$\sigma_1 = 9.60 - 0.5 = 9.10$$

$$\sigma_2 = -0.5 - 9.6 = -10.1$$

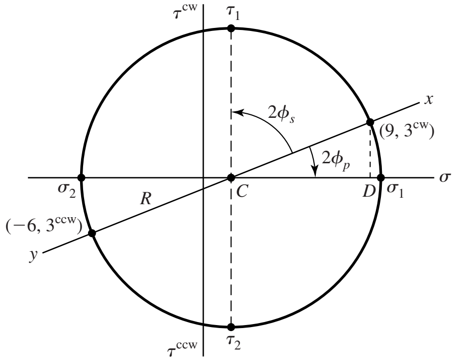


$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{7.5}{6}\right] = 70.67^\circ \text{ cw}$$

$$\tau_1 = R = 9.60, \quad \phi_s = 70.67^\circ - 45^\circ = 25.67^\circ \text{ cw}$$



(d)



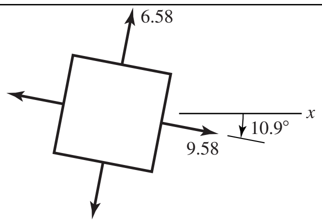
$$C = \frac{9-6}{2} = 1.5$$

$$CD = \frac{9+6}{2} = 7.5$$

$$R = \sqrt{7.5^2 + 3^2} = 8.078$$

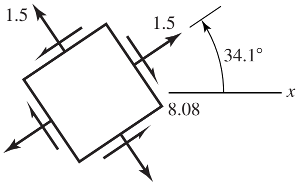
$$\sigma_1 = 1.5 + 8.078 = 9.58$$

$$\sigma_2 = 1.5 - 8.078 = -6.58$$



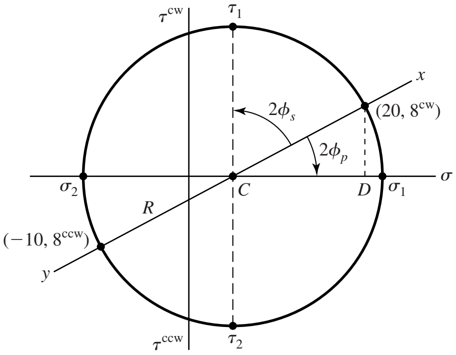
$$\phi_p = \frac{1}{2}\tan^{-1}\frac{3}{7.5} = 10.9^\circ \text{ cw}$$

$$\tau_1 = R = 8.078, \quad \phi_s = 45^\circ - 10.9^\circ = 34.1^\circ \text{ ccw}$$



3-10

(a)



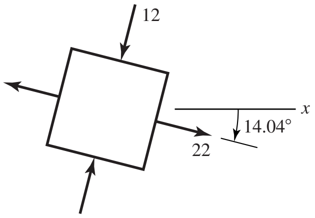
$$C = \frac{20-10}{2} = 5$$

$$CD = \frac{20+10}{2} = 15$$

$$R = \sqrt{15^2 + 8^2} = 17$$

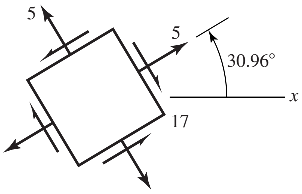
$$\sigma_1 = 5 + 17 = 22$$

$$\sigma_2 = 5 - 17 = -12$$

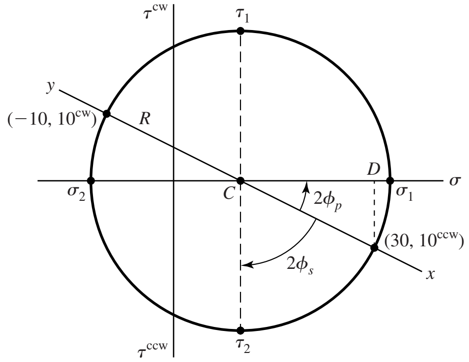


$$\phi_p = \frac{1}{2}\tan^{-1}\frac{8}{15} = 14.04^\circ \text{ cw}$$

$$\tau_1 = R = 17, \quad \phi_s = 45^\circ - 14.04^\circ = 30.96^\circ \text{ ccw}$$



(b)



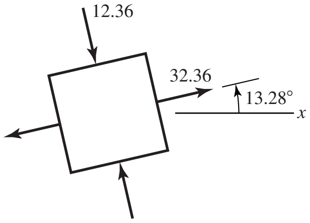
$$C = \frac{30 - 10}{2} = 10$$

$$CD = \frac{30 + 10}{2} = 20$$

$$R = \sqrt{20^2 + 10^2} = 22.36$$

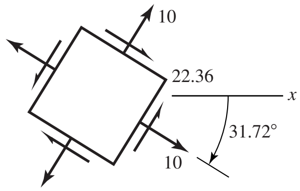
$$\sigma_1 = 10 + 22.36 = 32.36$$

$$\sigma_2 = 10 - 22.36 = -12.36$$

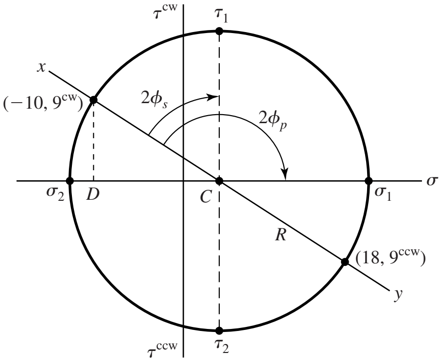


$$\phi_p = \frac{1}{2}\tan^{-1}\frac{10}{20} = 13.28^\circ \text{ ccw}$$

$$\tau_1 = R = 22.36, \quad \phi_s = 45^\circ - 13.28^\circ = 31.72^\circ \text{ cw}$$



(c)



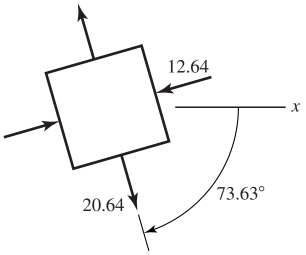
$$C = \frac{-10 + 18}{2} = 4$$

$$CD = \frac{10 + 18}{2} = 14$$

$$R = \sqrt{14^2 + 9^2} = 16.64$$

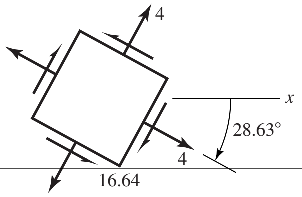
$$\sigma_1 = 4 + 16.64 = 20.64$$

$$\sigma_2 = 4 - 16.64 = -12.64$$

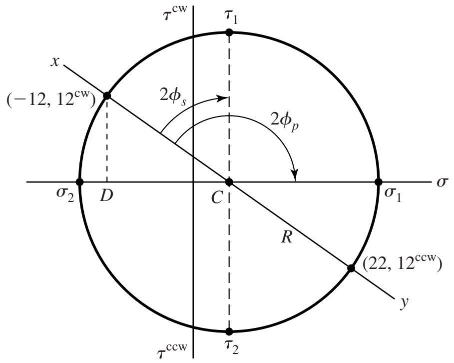


$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{14}{9}\right] = 73.63^\circ \text{ cw}$$

$$\tau_1 = R = 16.64, \quad \phi_s = 73.63^\circ - 45^\circ = 28.63^\circ \text{ cw}$$



(d)



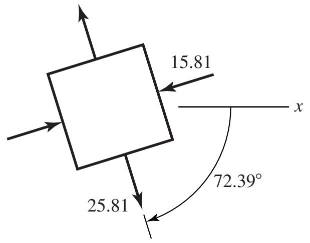
$$C = \frac{-12+22}{2} = 5$$

$$CD = \frac{12+22}{2} = 17$$

$$R = \sqrt{17^2 + 12^2} = 20.81$$

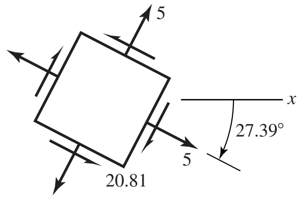
$$\sigma_1 = 5 + 20.81 = 25.81$$

$$\sigma_2 = 5 - 20.81 = -15.81$$



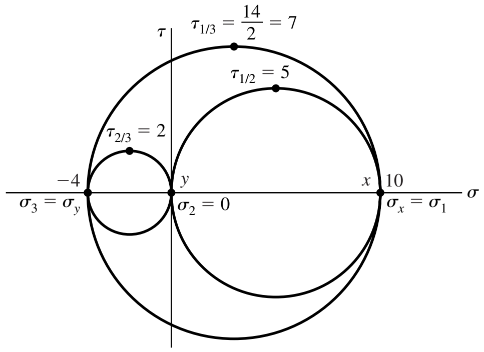
$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{17}{12}\right] = 72.39^\circ \text{ cw}$$

$$\tau_1 = R = 20.81, \quad \phi_s = 72.39^\circ - 45^\circ = 27.39^\circ \text{ cw}$$

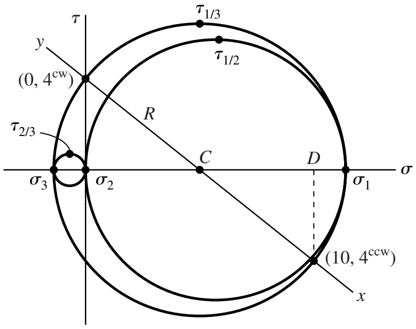


3-11

(a)



(b)



$$C = \frac{0+10}{2} = 5$$

$$CD = \frac{10-0}{2} = 5$$

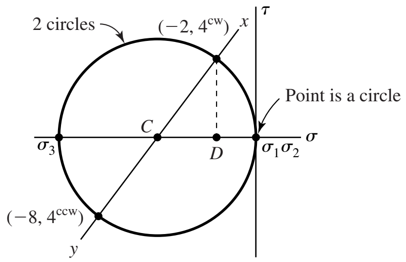
$$R = \sqrt{5^2 + 4^2} = 6.40$$

$$\sigma_1 = 5 + 6.40 = 11.40$$

$$\sigma_2 = 0, \quad \sigma_3 = 5 - 6.40 = -1.40$$

$$\tau_{1/3} = R = 6.40, \quad \tau_{1/2} = \frac{11.40}{2} = 5.70, \quad \tau_{2/3} = \frac{1.40}{2} = 0.70$$

(c)



$$C = \frac{-2-8}{2} = -5$$

$$CD = \frac{8-2}{2} = 3$$

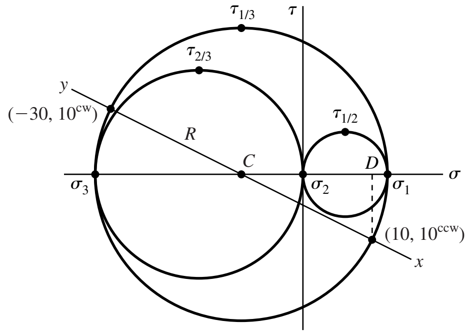
$$R = \sqrt{3^2 + 4^2} = 5$$

$$\sigma_1 = -5 + 5 = 0, \quad \sigma_2 = 0$$

$$\sigma_3 = -5 - 5 = -10$$

$$\tau_{1/3} = \frac{10}{2} = 5, \quad \tau_{1/2} = 0, \quad \tau_{2/3} = 5$$

(d)



$$C = \frac{10-30}{2} = -10$$

$$CD = \frac{10+30}{2} = 20$$

$$R = \sqrt{20^2 + 10^2} = 22.36$$

$$\sigma_1 = -10 + 22.36 = 12.36$$

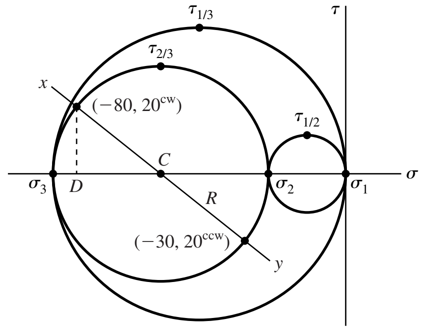
$$\sigma_2 = 0$$

$$\sigma_3 = -10 - 22.36 = -32.36$$

$$\tau_{1/3} = 22.36, \quad \tau_{1/2} = \frac{12.36}{2} = 6.18, \quad \tau_{2/3} = \frac{32.36}{2} = 16.18$$

3-12

(a)



$$C = \frac{-80-30}{2} = -55$$

$$CD = \frac{80-30}{2} = 25$$

$$R = \sqrt{25^2 + 20^2} = 32.02$$

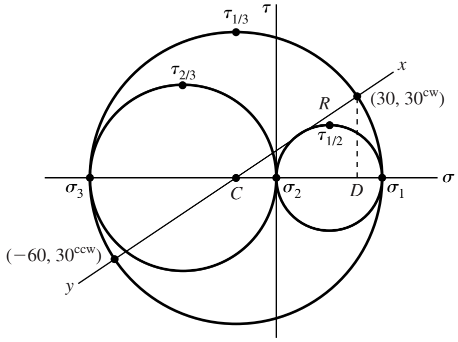
$$\sigma_1 = 0$$

$$\sigma_2 = -55 + 32.02 = -22.98 = -23.0$$

$$\sigma_3 = -55 - 32.0 = -87.0$$

$$\tau_{1/2} = \frac{23}{2} = 11.5, \quad \tau_{2/3} = 32.0, \quad \tau_{1/3} = \frac{87}{2} = 43.5$$

(b)



$$C = \frac{30 - 60}{2} = -15$$

$$CD = \frac{60 + 30}{2} = 45$$

$$R = \sqrt{45^2 + 30^2} = 54.1$$

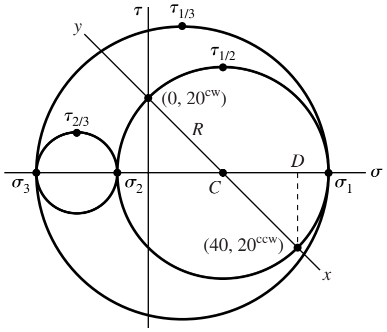
$$\sigma_1 = -15 + 54.1 = 39.1$$

$$\sigma_2 = 0$$

$$\sigma_3 = -15 - 54.1 = -69.1$$

$$\tau_{1/3} = \frac{39.1 + 69.1}{2} = 54.1, \quad \tau_{1/2} = \frac{39.1}{2} = 19.6, \quad \tau_{2/3} = \frac{69.1}{2} = 34.6$$

(c)



$$C = \frac{40 + 0}{2} = 20$$

$$CD = \frac{40 - 0}{2} = 20$$

$$R = \sqrt{20^2 + 20^2} = 28.3$$

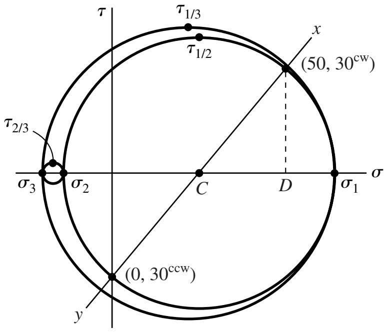
$$\sigma_1 = 20 + 28.3 = 48.3$$

$$\sigma_2 = 20 - 28.3 = -8.3$$

$$\sigma_3 = \sigma_z = -30$$

$$\tau_{1/3} = \frac{48.3 + 30}{2} = 39.1, \quad \tau_{1/2} = 28.3, \quad \tau_{2/3} = \frac{30 - 8.3}{2} = 10.9$$

(d)



$$C = \frac{50}{2} = 25$$

$$CD = \frac{50}{2} = 25$$

$$R = \sqrt{25^2 + 30^2} = 39.1$$

$$\sigma_1 = 25 + 39.1 = 64.1$$

$$\sigma_2 = 25 - 39.1 = -14.1$$

$$\sigma_3 = \sigma_z = -20$$

$$\tau_{1/3} = \frac{64.1 + 20}{2} = 42.1, \quad \tau_{1/2} = 39.1, \quad \tau_{2/3} = \frac{20 - 14.1}{2} = 2.95$$

3-13

$$\sigma = \frac{F}{A} = \frac{2000}{(\pi/4)(0.5^2)} = 10\,190 \text{ psi} = 10.19 \text{ kpsi} \quad \textit{Ans.}$$

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 10\,190\frac{72}{30(10^6)} = 0.024\,46 \text{ in} \quad \textit{Ans.}$$

$$\epsilon_1 = \frac{\delta}{L} = \frac{0.024\,46}{72} = 340(10^{-6}) = 340\mu \quad \textit{Ans.}$$

From Table A-5, $\nu = 0.292$

$$\epsilon_2 = -\nu\epsilon_1 = -0.292(340) = -99.3\mu \quad \textit{Ans.}$$

$$\Delta d = \epsilon_2 d = -99.3(10^{-6})(0.5) = -49.6(10^{-6}) \text{ in} \quad \textit{Ans.}$$

3-14 From Table A-5, $E = 71.7$ GPa

$$\delta = \sigma\frac{L}{E} = 135(10^6)\frac{3}{71.7(10^9)} = 5.65(10^{-3}) \text{ m} = 5.65 \text{ mm} \quad \textit{Ans.}$$

3-15 With $\sigma_z = 0$, solve the first two equations of Eq. (3-19) simultaneously. Place E on the left-hand side of both equations, and using Cramer's rule,

$$\sigma_x = \frac{\begin{vmatrix} E\epsilon_x & -\nu \\ E\epsilon_y & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\nu \\ -\nu & 1 \end{vmatrix}} = \frac{E\epsilon_x + \nu E\epsilon_y}{1-\nu^2} = \frac{E(\epsilon_x + \nu\epsilon_y)}{1-\nu^2}$$

Likewise,

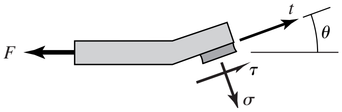
$$\sigma_y = \frac{E(\epsilon_y + \nu\epsilon_x)}{1-\nu^2}$$

From Table A-5, $E = 207$ GPa and $\nu = 0.292$. Thus,

$$\sigma_x = \frac{E(\epsilon_x + \nu\epsilon_y)}{1-\nu^2} = \frac{207(10^9)[0.0021 + 0.292(-0.000\,67)]}{1-0.292^2}(10^{-6}) = 431 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_y = \frac{207(10^9)[-0.000\,67 + 0.292(0.0021)]}{1-0.292^2}(10^{-6}) = -12.9 \text{ MPa} \quad \textit{Ans.}$$

3-16 The engineer has assumed the stress to be uniform. That is,



$$\sum F_t = -F\cos\theta + \tau A = 0 \quad \Rightarrow \quad \tau = \frac{F}{A}\cos\theta$$

When failure occurs in shear

$$S_{su} = \frac{F}{A}\cos\theta$$

The uniform stress assumption is common practice but is not exact. If interested in the details, see p. 570 of 6th edition.

3-17 From Eq. (3-15)

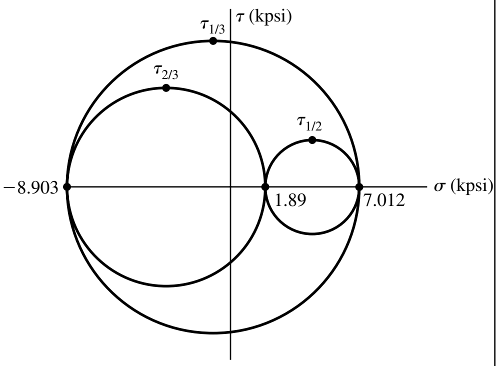
$$\sigma^3 - (-2+6-4)\sigma^2 + [-2(6)+(-2)(-4)+6(-4)-3^2-2^2-(-5)^2]\sigma$$
$$-[-2(6)(-4)+2(3)(2)(-5)-(-2)(2)^2-6(-5)^2-(-4)(3)^2]=0$$
$$\sigma^3 - 66\sigma + 118 = 0$$

Roots are: 7.012, 1.89, −8.903 kpsi *Ans.*

$$\tau_{1/2} = \frac{7.012-1.89}{2} = 2.56 \text{ kpsi}$$

$$\tau_{2/3} = \frac{8.903+1.89}{2} = 5.40\,\text{kpsi}$$

$$\tau_{\max} = \tau_{1/3} = \frac{8.903+7.012}{2} = 7.96 \text{ kpsi} \quad \textit{Ans.}$$



Note: For Probs. 3-17 to 3-19, one can also find the eigenvalues of the matrix

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z \end{bmatrix}$$

for the principal stresses

3-18 From Eq. (3-15)

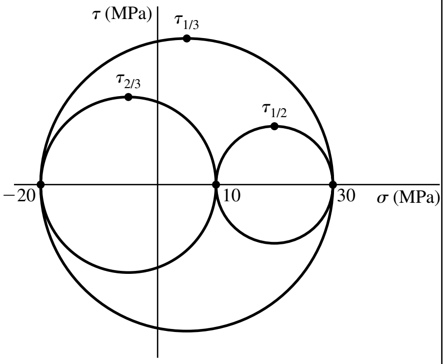
$$\sigma^3 - (10+0+10)\sigma^2 + \left[10(0)+10(10)+0(10)-20^2-\left(-10\sqrt{2}\right)^2-0^2\right]\sigma$$
$$-\left[10(0)(10)+2(20)\left(-10\sqrt{2}\right)(0)-10\left(-10\sqrt{2}\right)^2-0(0)^2-10(20)^2\right]=0$$
$$\sigma^3 - 20\sigma^2 - 500\sigma + 6000 = 0$$

Roots are: 30, 10, −20 MPa *Ans.*

$$\tau_{1/2} = \frac{30-10}{2} = 10\,\text{MPa}$$

$$\tau_{2/3} = \frac{10+20}{2} = 15\,\text{MPa}$$

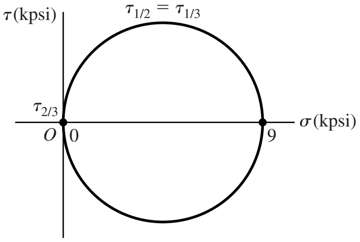
$$\tau_{\max} = \tau_{1/3} = \frac{30+20}{2} = 25 \text{ MPa} \quad \textit{Ans.}$$



3-19 From Eq. (3-15)

$$\sigma^3 - (1+4+4)\sigma^2 + [1(4)+1(4)+4(4)-2^2-(-4)^2-(-2)^2]\sigma$$
$$-[1(4)(4)+2(2)(-4)(-2)-1(-4)^2-4(-2)^2-4(2)^2]=0$$
$$\sigma^3 - 9\sigma^2 = 0$$

Roots are: 9, 0, 0 kpsi



$$\tau_{2/3} = 0, \quad \tau_{1/2} = \tau_{1/3} = \tau_{\max} = \frac{9}{2} = 4.5 \text{ kpsi} \quad \textit{Ans.}$$

3-20

(a) $R_1 = \dfrac{c}{l}F \quad M_{\max} = R_1 a = \dfrac{ac}{l}F$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{ac}{l}F \quad \Rightarrow \quad F = \frac{\sigma bh^2 l}{6ac} \quad \textit{Ans.}$$

(b) $\dfrac{F_m}{F} = \dfrac{(\sigma_m/\sigma)(b_m/b)\,(h_m/h)^2\,(l_m/l)}{(a_m/a)\,(c_m/c)} = \dfrac{1(s)(s)^2(s)}{(s)(s)} = s^2 \quad \textit{Ans.}$

For equal stress, the model load varies by the square of the scale factor.

3-21

$$R_1 = \frac{wl}{2}, \quad M_{\max}\big|_{x=l/2} = \frac{w}{2}\frac{l}{2}\left(l - \frac{l}{2}\right) = \frac{wl^2}{8}$$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{wl^2}{8} = \frac{3Wl}{4bh^2} \quad \Rightarrow \quad W = \frac{4}{3}\frac{\sigma bh^2}{l} \quad \textit{Ans.}$$

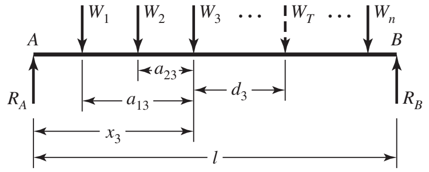
$$\frac{W_m}{W} = \frac{(\sigma_m/\sigma)(b_m/b)\,(h_m/h)^2}{l_m/l} = \frac{1(s)(s)^2}{s} = s^2 \quad \textit{Ans.}$$

$$\frac{w_m l_m}{wl} = s^2 \quad \Rightarrow \quad \frac{w_m}{w} = \frac{s^2}{s} = s \quad \textit{Ans.}$$

For equal stress, the model load w varies linearily with the scale factor.

3-22

(a) Can solve by iteration *or* derive equations for the general case.



Find maximum moment under wheel W_3

$W_T = \sum W$ at centroid of W's

$$R_A = \frac{l - x_3 - d_3}{l} W_T$$

Under wheel 3

$$M_3 = R_A x_3 - W_1 a_{13} - W_2 a_{23} = \frac{(l - x_3 - d_3)}{l} W_T x_3 - W_1 a_{13} - W_2 a_{23}$$

For maximum, $\frac{dM_3}{dx_3} = 0 = (l - d_3 - 2x_3)\frac{W_T}{l} \quad \Rightarrow \quad x_3 = \frac{l - d_3}{2}$

substitute into M, $\Rightarrow \quad M_3 = \frac{(l - d_3)^2}{4l} W_T - W_1 a_{13} - W_2 a_{23}$

This means the midpoint of d_3 intersects the midpoint of the beam

For wheel i $\quad x_i = \frac{l - d_i}{2}, \quad M_i = \frac{(l - d_i)^2}{4l} W_T - \sum_{j=1}^{i-1} W_j a_{ji}$

Note for wheel 1: $\Sigma W_j a_{ji} = 0$

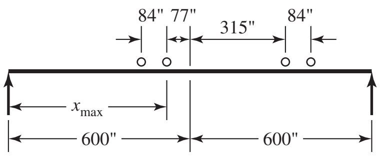
$$W_T = 104.4, \quad W_1 = W_2 = W_3 = W_4 = \frac{104.4}{4} = 26.1\ \text{kip}$$

Wheel 1: $\quad d_1 = \frac{476}{2} = 238\ \text{in}, \quad M_1 = \frac{(1200 - 238)^2}{4(1200)}(104.4) = 20\,128\ \text{kip}\cdot\text{in}$

Wheel 2: $\quad d_2 = 238 - 84 = 154\ \text{in}$

$$M_2 = \frac{(1200 - 154)^2}{4(1200)}(104.4) - 26.1(84) = 21\,605\ \text{kip}\cdot\text{in} = M_{\max}$$

Check if all of the wheels are on the rail



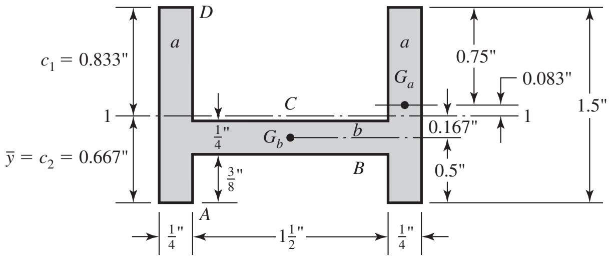
(b) $x_{\max} = 600 - 77 = 523\ \text{in}$

(c) See above sketch.

(d) inner axles

3-23

(a)



$$A_a = A_b = 0.25(1.5) = 0.375\ \text{in}^2$$

$$A = 3(0.375) = 1.125\ \text{in}^2$$

$$\bar{y} = \frac{2(0.375)(0.75) + 0.375(0.5)}{1.125} = 0.667\ \text{in}$$

$$I_a = \frac{0.25(1.5)^3}{12} = 0.0703\ \text{in}^4$$

$$I_b = \frac{1.5(0.25)^3}{12} = 0.001\,95\ \text{in}^4$$

$$I_1 = 2[0.0703 + 0.375(0.083)^2] + [0.001\ 95 + 0.375(0.167)^2] = 0.158\ \text{in}^4 \quad \textit{Ans.}$$

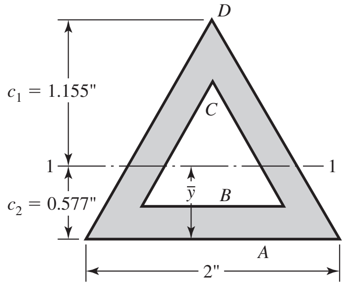
$$\sigma_A = \frac{10\,000(0.667)}{0.158} = 42(10)^3\ \text{psi} \quad \textit{Ans.}$$

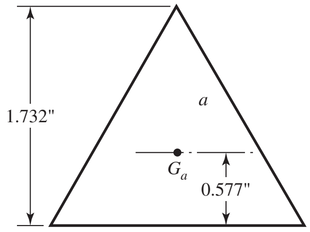
$$\sigma_B = \frac{10\,000(0.667 - 0.375)}{0.158} = 18.5(10)^3\ \text{psi} \quad \textit{Ans.}$$

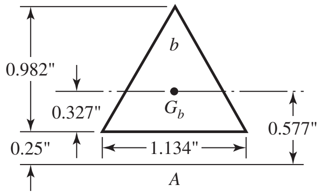
$$\sigma_C = \frac{10\,000(0.167 - 0.125)}{0.158} = 2.7(10)^3\ \text{psi} \quad \textit{Ans.}$$

$$\sigma_D = -\frac{10\,000(0.833)}{0.158} = -52.7(10)^3\ \text{psi} \quad \textit{Ans.}$$

(b)







Here we treat the hole as a negative area.

$$A_a = 1.732\ \text{in}^2$$

$$A_b = 1.134\left(\frac{0.982}{2}\right) = 0.557\ \text{in}^2$$

$$A = 1.732 - 0.557 = 1.175\,\text{in}^2$$

$$\bar{y} = \frac{1.732(0.577) - 0.557(0.577)}{1.175} = 0.577\ \text{in} \quad \textit{Ans.}$$

$$I_a = \frac{bh^3}{36} = \frac{2(1.732)^3}{36} = 0.289\ \text{in}^4$$

$$I_b = \frac{1.134(0.982)^3}{36} = 0.0298\,\text{in}^4$$

$$I_1 = I_a - I_b = 0.289 - 0.0298 = 0.259\,\text{in}^4 \quad \textit{Ans.}$$

because the centroids are coincident.

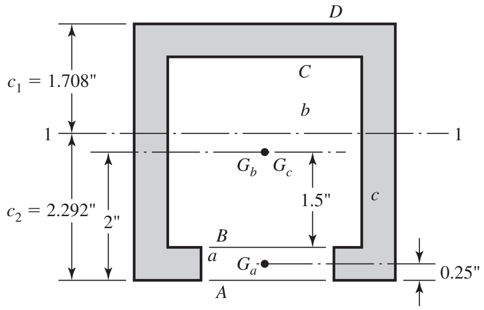
$$\sigma_A = \frac{10\,000(0.577)}{0.259} = 22.3(10)^3\ \text{psi} \quad \textit{Ans.}$$

$$\sigma_B = \frac{10\,000(0.327)}{0.259} = 12.6(10)^3\ \text{psi} \quad \textit{Ans.}$$

$$\sigma_C = -\frac{10\,000(0.982 - 0.327)}{0.259} = -25.3(10)^3\ \text{psi} \quad \textit{Ans.}$$

$$\sigma_D = -\frac{10\,000(1.155)}{0.259} = -44.6(10)^3\ \text{psi} \quad \textit{Ans.}$$

(c) Use two negative areas.



$$A_a = 1\,\text{in}^2, \qquad A_b = 9\ \text{in}^2, \quad A_c = 16\,\text{in}^2, \quad A = 16 - 9 - 1 = 6\,\text{in}^2;$$

$$\bar{y}_a = 0.25\,\text{in}, \quad \bar{y}_b = 2.0\,\text{in}, \quad \bar{y}_c = 2\,\text{in}$$

$$\bar{y} = \frac{16(2) - 9(2) - 1(0.25)}{6} = 2.292\,\text{in} \quad \textit{Ans.}$$

$$c_1 = 4 - 2.292 = 1.708\,\text{in}$$

$$I_a = \frac{2(0.5)^3}{12} = 0.020\,83\,\text{in}^4$$

$$I_b = \frac{3(3)^3}{12} = 6.75\,\text{in}^4$$

$$I_c = \frac{4(4)^3}{12} = 21.333\,\text{in}^4$$

$$
\begin{aligned}
I_1 &= [21.333 + 16(0.292)^2] - [6.75 + 9(0.292)^2] \\
&\quad - [0.020\,83 + 1(2.292 - 0.25)^2] \\
&= 10.99 \text{ in}^4 \quad \textit{Ans.}
\end{aligned}
$$

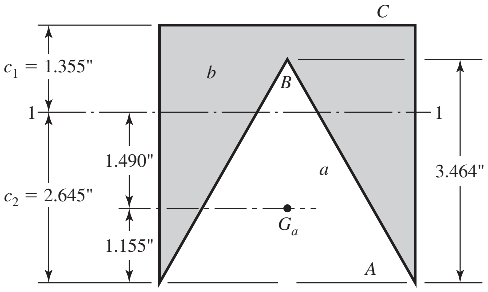
$$\sigma_A = \frac{10\,000(2.292)}{10.99} = 2086 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_B = \frac{10\,000(2.292 - 0.5)}{10.99} = 1631 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_C = -\frac{10\,000(1.708 - 0.5)}{10.99} = -1099 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_D = -\frac{10\,000(1.708)}{10.99} = -1554 \text{ psi} \quad \textit{Ans.}$$

(d) Use *a* as a negative area.



$$A_a = 6.928 \text{ in}^2, \quad A_b = 16 \text{ in}^2, \quad A = 9.072 \text{ in}^2;$$

$$\bar{y}_a = 1.155 \text{ in}, \quad \bar{y}_b = 2 \text{ in}$$

$$\bar{y} = \frac{2(16) - 1.155(6.928)}{9.072} = 2.645 \text{ in} \quad \textit{Ans.}$$

$$c_1 = 4 - 2.645 = 1.355 \text{ in}$$

$$I_a = \frac{bh^3}{36} = \frac{4(3.464)^3}{36} = 4.618 \text{ in}^4$$

$$I_b = \frac{4(4)^3}{12} = 21.33 \text{ in}^4$$

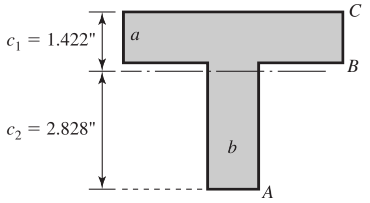
$$
\begin{aligned}
I_1 &= [21.33 + 16(0.645)^2] - [4.618 + 6.928(1.490)^2] \\
&= 7.99 \text{ in}^4 \quad \textit{Ans.}
\end{aligned}
$$

$$\sigma_A = \frac{10\,000(2.645)}{7.99} = 3310 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_B = -\frac{10\,000(3.464 - 2.645)}{7.99} = -1025 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_C = -\frac{10\,000(1.355)}{7.99} = -1696 \text{ psi} \quad \textit{Ans.}$$

(e)



$$A_a = 6(1.25) = 7.5\text{ in}^2$$

$$A_b = 3(1.5) = 4.5\text{ in}^2$$

$$A = A_c + A_b = 12\text{ in}^2$$

$$\bar{y} = \frac{3.625(7.5) + 1.5(4.5)}{12} = 2.828\text{ in} \quad \textit{Ans.}$$

$$I = \frac{1}{12}(6)(1.25)^3 + 7.5(3.625 - 2.828)^2 + \frac{1}{12}(1.5)(3)^3 + 4.5(2.828 - 1.5)^2$$

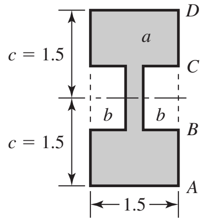
$$= 17.05\text{ in}^4 \quad \textit{Ans.}$$

$$\sigma_A = \frac{10\,000(2.828)}{17.05} = 1659\text{ psi} \quad \textit{Ans.}$$

$$\sigma_B = -\frac{10\,000(3 - 2.828)}{17.05} = -101\text{ psi} \quad \textit{Ans.}$$

$$\sigma_C = -\frac{10\,000(1.422)}{17.05} = -834\text{ psi} \quad \textit{Ans.}$$

(f)



Let a = total area

$$A = 1.5(3) - 1(1.25) = 3.25\text{ in}^2$$

$$I = I_a - 2I_b = \frac{1}{12}(1.5)(3)^3 - \frac{1}{12}(1.25)(1)^3$$

$$= 3.271\text{ in}^4 \quad \textit{Ans.}$$

$$\sigma_A = \frac{10\,000(1.5)}{3.271} = 4586\text{ psi}, \quad \sigma_D = -4586\text{ psi}$$

$$\textit{Ans.}$$

$$\sigma_B = \frac{10\,000(0.5)}{3.271} = 1529\text{ psi}, \quad \sigma_C = -1529\text{ psi}$$

3-24

(a) The moment is maximum and constant between A and B

$$M = -50(20) = -1000\text{ lbf}\cdot\text{in}, \quad I = \frac{1}{12}(0.5)(2)^3 = 0.3333\text{ in}^4$$

$$\rho = \left|\frac{EI}{M}\right| = \frac{1.6(10^6)(0.3333)}{1000} = 533.3\text{ in}$$

$$(x, y) = (30, -533.3)\text{ in} \quad \textit{Ans.}$$

(b) The moment is maximum and constant between A and B

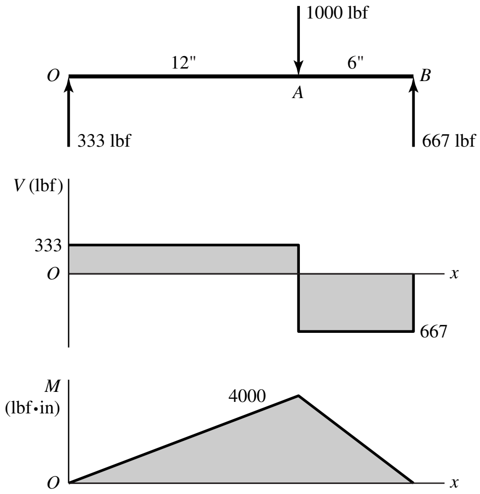
$$M = 50(5) = 250\text{ lbf}\cdot\text{in}, \quad I = 0.3333\text{ in}^4$$

$$\rho = \frac{1.6(10^6)(0.3333)}{250} = 2133\text{ in} \quad \textit{Ans.}$$

$$(x, y) = (20, 2133)\text{ in} \quad \textit{Ans.}$$

3-25

(a)



$$I = \frac{1}{12}(0.75)(1.5)^3 = 0.2109\,\text{in}^4$$

$$A = 0.75(1.5) = 1.125\,\text{in}$$

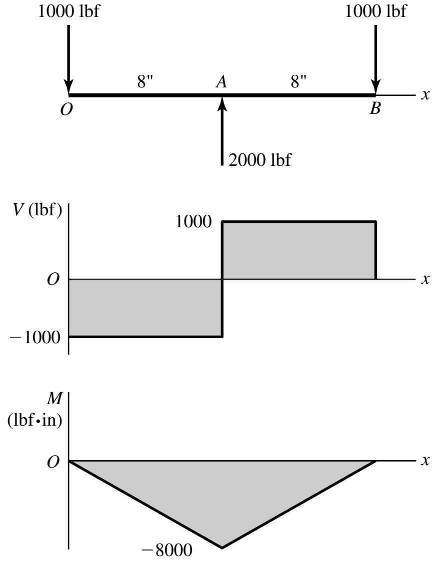
M_{max} is at A. At the bottom of the section,

$$\sigma_{\text{max}} = \frac{Mc}{I} = \frac{4000(0.75)}{0.2109} = 14\,225\,\text{psi} \quad \textit{Ans.}$$

Due to V, τ_{max} constant is between A and B at $y = 0$

$$\tau_{\text{max}} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{667}{1.125} = 889\,\text{psi} \quad \textit{Ans.}$$

(b)



$$I = \frac{1}{12}(1)(2)^3 = 0.6667\,\text{in}^4$$

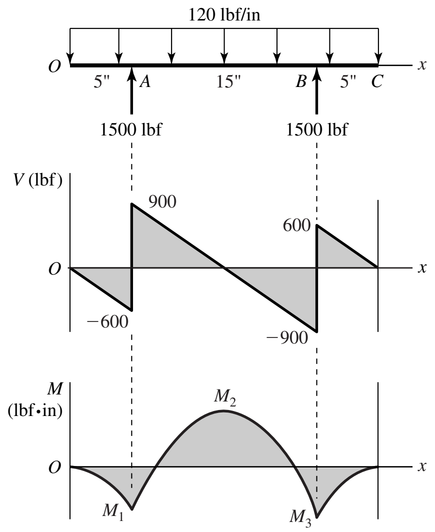
M_{max} is at A at the top of the beam

$$\sigma_{\text{max}} = \frac{8000(1)}{0.6667} = 12\,000\,\text{psi} \quad \textit{Ans.}$$

$$|V_{\text{max}}| = 1000\,\text{lbf} \quad \text{from } O \text{ to } B \text{ at } y = 0$$

$$\tau_{\text{max}} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{1000}{(2)(1)} = 750\,\text{psi} \quad \textit{Ans.}$$

(c)



$$I = \frac{1}{12}(0.75)(2)^3 = 0.5\,\text{in}^4$$

$$M_1 = -\frac{1}{2}600(5) = -1500\,\text{lbf}\cdot\text{in} = M_3$$

$$M_2 = -1500 + \frac{1}{2}(900)(7.5) = 1875\,\text{lbf}\cdot\text{in}$$

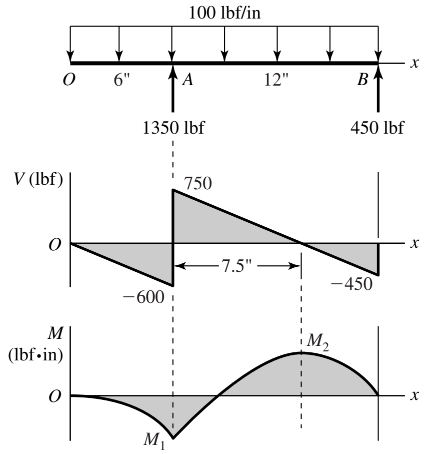
M_{max} is at span center. At the bottom of the beam,

$$\sigma_{\text{max}} = \frac{1875(1)}{0.5} = 3750\,\text{psi} \quad \textit{Ans.}$$

At A and B at $y = 0$

$$\tau_{\text{max}} = \frac{3}{2}\frac{900}{(0.75)(2)} = 900\,\text{psi} \quad \textit{Ans.}$$

(d)



$$I = \frac{1}{12}(1)(2)^3 = 0.6667\ \text{in}^4$$

$$M_1 = -\frac{600}{2}(6) = -1800\ \text{lbf}\cdot\text{in}$$

$$M_2 = -1800 + \frac{1}{2}750(7.5) = 1013\ \text{lbf}\cdot\text{in}$$

At A, top of beam

$$\sigma_{\max} = \frac{1800(1)}{0.6667} = 2700\ \text{psi} \quad \textit{Ans.}$$

At A, $y = 0$

$$\tau_{\max} = \frac{3}{2}\frac{750}{(2)(1)} = 563\ \text{psi} \quad \textit{Ans.}$$

3-26

$$M_{\max} = \frac{wl^2}{8} \quad \Rightarrow \quad \sigma_{\max} = \frac{wl^2 c}{8I} \quad \Rightarrow \quad w = \frac{8\sigma I}{cl^2}$$

(a) $l = 12(12) = 144\ \text{in}$, $I = (1/12)(1.5)(9.5)^3 = 107.2\ \text{in}^4$

$$w = \frac{8(1200)(107.2)}{4.75(144^2)} = 10.4\ \text{lbf/in} \quad \textit{Ans.}$$

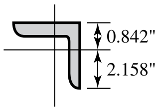
(b) $l = 48\ \text{in}$, $I = (\pi/64)(2^4 - 1.25^4) = 0.6656\ \text{in}^4$

$$w = \frac{8(12)(10^3)(0.6656)}{1(48)^2} = 27.7\ \text{lbf/in} \quad \textit{Ans.}$$

(c) $l = 48\ \text{in}$, $I \doteq (1/12)(2)(3^3) - (1/12)(1.625)(2.625^3) = 2.051\ \text{in}^4$

$$w = \frac{8(12)(10^3)(2.051)}{1.5(48)^2} = 57.0\ \text{lbf/in} \quad \textit{Ans.}$$

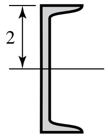
(d) $l = 72$ in; Table A-6, $I = 2(1.24) = 2.48\ \text{in}^4$



$$c_{\max} = 2.158"$$

$$w = \frac{8(12)(10^3)(2.48)}{2.158(72)^2} = 21.3\ \text{lbf/in} \quad \textit{Ans.}$$

(e) $l = 72$ in; Table A-7, $I = 3.85\ \text{in}^4$

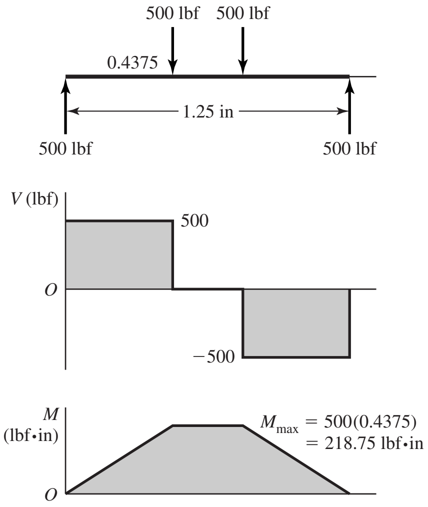


$$w = \frac{8(12)(10^3)(3.85)}{2(72^2)} = 35.6\ \text{lbf/in} \quad \textit{Ans.}$$

(f) $l = 72\ \text{in}$, $I = (1/12)(1)(4^3) = 5.333\ \text{in}^4$

$$w = \frac{8(12)(10^3)(5.333)}{(2)(72)^2} = 49.4\ \text{lbf/in} \quad \textit{Ans.}$$

3-27 **(a)** Model (c)



$$I = \frac{\pi}{64}(0.5^4) = 3.068(10^{-3})\ \text{in}^4$$

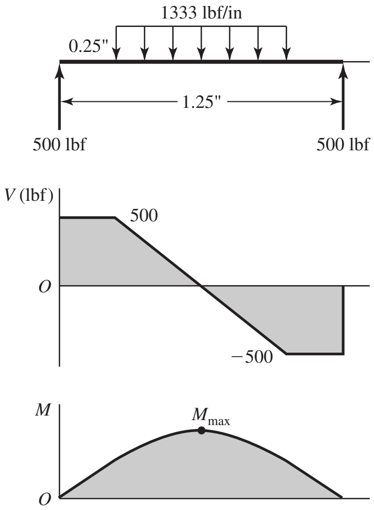
$$A = \frac{\pi}{4}(0.5^2) = 0.1963\ \text{in}^2$$

$$\sigma = \frac{Mc}{I} = \frac{218.75(0.25)}{3.068(10^{-3})}$$

$$= 17\,825\ \text{psi} = 17.8\ \text{kpsi} \quad \textit{Ans.}$$

$$\tau_{max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400\ \text{psi} \quad \textit{Ans.}$$

(b) Model (d)



$$M_{max} = 500(0.25) + \frac{1}{2}(500)(0.375)$$

$$= 218.75\ \text{lbf} \cdot \text{in}$$

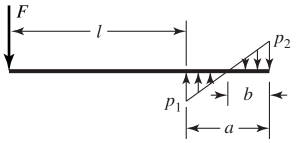
$$V_{max} = 500\ \text{lbf}$$

Same M and V

$\therefore \sigma = 17.8$ kpsi *Ans.*

$\tau_{max} = 3400$ psi *Ans.*

3-28



$$q = -F\langle x\rangle^{-1} + p_1\langle x-l\rangle^0 - \frac{p_1+p_2}{a}\langle x-l\rangle^1 + \text{ terms for } x > l+a$$

$$V = -F + p_1\langle x-l\rangle^1 - \frac{p_1+p_2}{2a}\langle x-l\rangle^2 + \text{ terms for } x > l+a$$

$$M = -Fx + \frac{p_1}{2}\langle x-l\rangle^2 - \frac{p_1+p_2}{6a}\langle x-l\rangle^3 + \text{ terms for } x > l+a$$

At $x = (l+a)^+$, $V = M = 0$, terms for $x > l + a = 0$

$$-F + p_1 a - \frac{p_1+p_2}{2a}a^2 = 0 \quad \Rightarrow \quad p_1 - p_2 = \frac{2F}{a} \qquad (1)$$

$$-F(l+a)+\frac{p_1a^2}{2}-\frac{p_1+p_2}{6a}a^3=0 \quad\Rightarrow\quad 2p_1-p_2=\frac{6F(l+a)}{a^2} \tag{2}$$

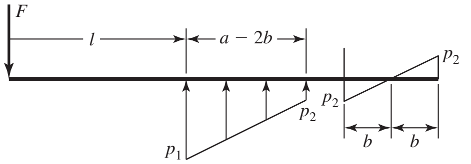
From (1) and (2)

$$p_1=\frac{2F}{a^2}(3l+2a),\quad p_2=\frac{2F}{a^2}(3l+a) \tag{3}$$

From similar triangles

$$\frac{b}{p_2}=\frac{a}{p_1+p_2} \quad\Rightarrow\quad b=\frac{ap_2}{p_1+p_2} \tag{4}$$

$M_{\max}$ occurs where $V=0$



$$x_{\max}=l+a-2b$$

$$\begin{aligned}M_{\max}&=-F(l+a-2b)+\frac{p_1}{2}(a-2b)^2-\frac{p_1+p_2}{6a}(a-2b)^3\\&=-Fl-F(a-2b)+\frac{p_1}{2}(a-2b)^2-\frac{p_1+p_2}{6a}(a-2b)^3\end{aligned}$$

Normally $M_{\max}=-Fl$
The fractional increase in the magnitude is

$$\Delta=\frac{F(a-2b)-(p_1/2)(a-2b)^2-[(p_1+p_2)/6a](a-2b)^3}{Fl} \tag{5}$$

For example, consider $F=1500$ lbf, $a=1.2$ in, $l=1.5$ in

(3)
$$p_1=\frac{2(1500)}{1.2^2}[3(1.5)+2(1.2)]=14\,375\ \text{lbf/in}$$

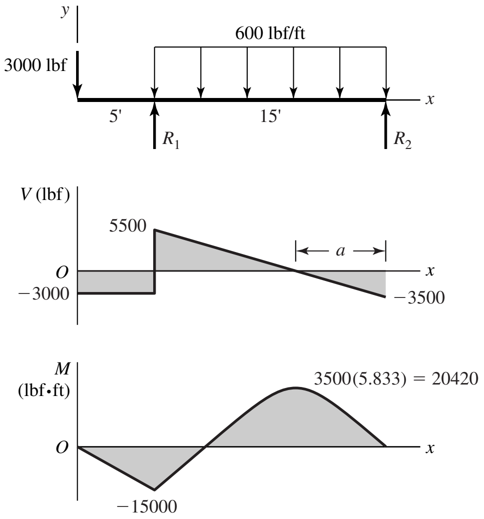
$$p_2=\frac{2(1500)}{1.2^2}[3(1.5)+1.2]=11\,875\ \text{lbf/in}$$

(4)
$$b=1.2(11\,875)/(14\,375+11\,875)=0.5429\ \text{in}$$

Substituting into (5) yields

$$\Delta=0.036\,89 \quad \text{or} \quad 3.7\%\ \text{higher than } -Fl$$

3-29

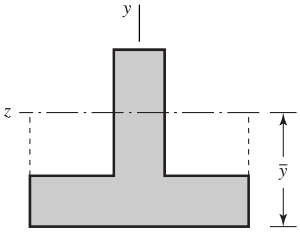


$$R_1 = \frac{600(15)}{2} + \frac{20}{15}3000 = 8500 \text{ lbf}$$

$$R_2 = \frac{600(15)}{2} - \frac{5}{15}3000 = 3500 \text{ lbf}$$

$$a = \frac{3500}{600} = 5.833 \text{ ft}$$

(a)



$$\bar{y} = \frac{1(12) + 5(12)}{24} = 3 \text{ in}$$

$$I_z = \frac{1}{3}[2(5^3) + 6(3^3) - 4(1^3)] = 136 \text{ in}^4$$

At $x = 5$ ft, $\quad y = -3$ in, $\quad \sigma_x = -\frac{-15000(12)(-3)}{136} = -3970$ psi

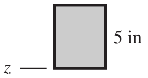
$y = 5$ in, $\quad \sigma_x = -\frac{-15000(12)5}{136} = 6620$ psi

At $x = 14.17$ ft, $\quad y = -3$ in, $\quad \sigma_x = -\frac{20420(12)(-3)}{136} = 5405$ psi

$y = 5$ in, $\quad \sigma_x = -\frac{20420(12)5}{136} = -9010$ psi

Max tension = 6620 psi *Ans.*
Max compression = −9010 psi *Ans.*

(b) $V_{\max} = 5500$ lbf

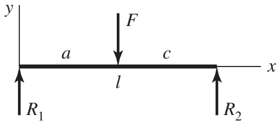


$$Q_{\text{n.a.}} = \bar{y}A = 2.5(5)(2) = 25 \text{ in}^3$$

$$\tau_{\underset{\text{v}}{\max}} = \frac{VQ}{Ib} = \frac{5500(25)}{136(2)} = 506 \text{ psi} \quad \textit{Ans.}$$

(c) $\tau_{\max} = \frac{|\sigma_{\max}|}{2} = \frac{9010}{2} = 4510$ psi *Ans.*

3-30



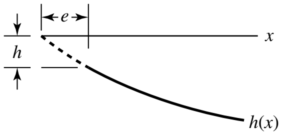
$$R_1 = \frac{c}{l}F$$

$$M = \frac{c}{l}Fx \quad 0 \le x \le a$$

$$\sigma = \frac{6M}{bh^2} = \frac{6(c/l)Fx}{bh^2} \quad \Rightarrow \quad h = \sqrt{\frac{6cFx}{bl\sigma_{\max}}} \quad 0 \le x \le a \quad \textit{Ans.}$$

3-31 From Prob. 3-30, $R_1 = \frac{c}{l}F = V, \quad 0 \le x \le a$

$$\tau_{\max} = \frac{3}{2}\frac{V}{bh} = \frac{3}{2}\frac{(c/l)F}{bh} \qquad \therefore\ h = \frac{3}{2}\frac{Fc}{lb\tau_{\max}} \quad \textit{Ans.}$$

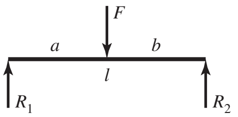


From Prob. 3-30 $= \sqrt{\dfrac{6Fcx}{lb\sigma_{\max}}}$ sub in $x = e$ and equate to h above

$$\frac{3}{2}\frac{Fc}{lb\tau_{\max}} = \sqrt{\frac{6Fce}{lb\sigma_{\max}}}$$

$$e = \frac{3}{8}\frac{Fc\sigma_{\max}}{lb\tau_{\max}^2} \quad \textit{Ans.}$$

3-32



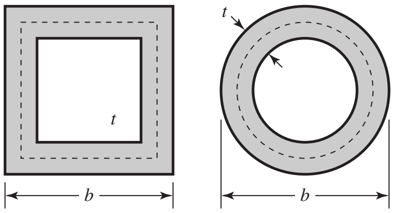
$$R_1 = \frac{b}{l}F$$

$$M = \frac{b}{l}Fx$$

$$\sigma_{\max} = \frac{32M}{\pi d^3} = \frac{32}{\pi d^3}\frac{b}{l}Fx$$

$$d = \left[\frac{32}{\pi}\frac{bFx}{l\sigma_{\max}}\right]^{1/3} \quad 0 \le x \le a \quad \textit{Ans.}$$

3-33



Square: $A_m = (b - t)^2$

$T_{sq} = 2A_m t\tau_{all} = 2(b - t)^2 t\tau_{all}$

Round: $A_m = \pi(b - t)^2/4$

$T_{rd} = 2\pi(b - t)^2 t\tau_{all}/4$

Ratio of torques

$$\frac{T_{\text{sq}}}{T_{\text{rd}}} = \frac{2(b-t)^2 t\tau_{\text{all}}}{\pi(b-t)^2 t\tau_{\text{all}}/2} = \frac{4}{\pi} = 1.27$$

Twist per unit length
square:

$$\theta_{\text{sq}} = \frac{2G\theta_1 t}{t\tau_{\text{all}}}\left(\frac{L}{A}\right)_m = C\left|\frac{L}{A}\right|_m = C\frac{4(b-t)}{(b-t)^2}$$

Round:

$$\theta_{\text{rd}} = C\left(\frac{L}{A}\right)_m = C\frac{\pi(b-t)}{\pi(b-t)^2/4} = C\frac{4(b-t)}{(b-t)^2}$$

Ratio equals 1, twists are the same.

Note the weight ratio is

$$\frac{W_{\text{sq}}}{W_{\text{rd}}} = \frac{\rho l(b-t)^2}{\rho l\pi(b-t)(t)} = \frac{b-t}{\pi t} \qquad \text{thin-walled assumes } b \ge 20t$$

$$= \frac{19}{\pi} = 6.04 \qquad \text{with } b = 20t$$

$$= 2.86 \qquad \text{with } b = 10t$$

3-34 $l = 40$ in, $\tau_{\text{all}} = 11\,500$ psi, $G = 11.5(10^6)$ psi, $t = 0.050$ in

$$r_m = r_i + t/2 = r_i + 0.025 \quad \text{for } r_i > 0$$

$$= 0 \quad \text{for } r_i = 0$$

$$A_m = (1-0.05)^2 - 4\left(r_m^2 - \frac{\pi}{4}r_m^2\right) = 0.95^2 - (4-\pi)r_m^2$$

$$L_m = 4(1 - 0.05 - 2r_m + 2\pi r_m/4) = 4[0.95 - (2-\pi/2)r_m]$$

Eq. (3-45): $T = 2A_m t\tau = 2(0.05)(11\,500)A_m = 1150A_m$

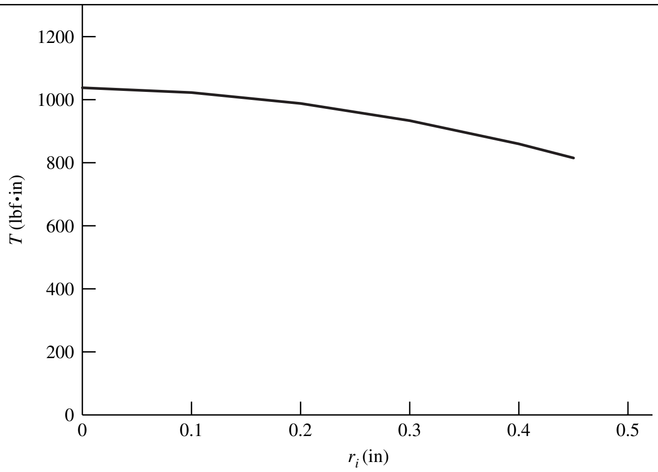
Eq. (3-46):

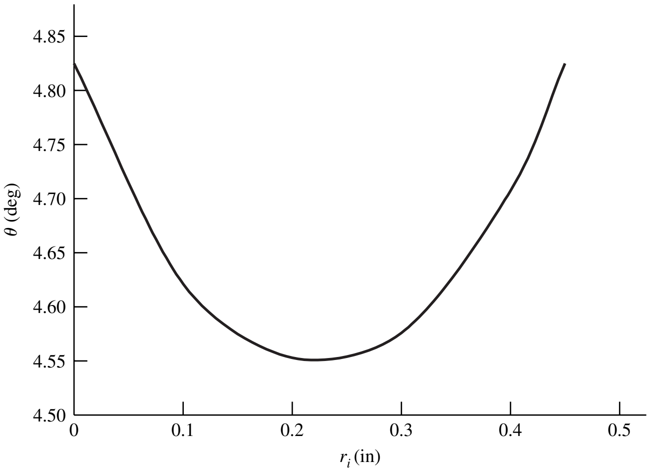
$$\theta(\text{deg}) = \theta_1 l\frac{180}{\pi} = \frac{TL_m l}{4GA_m^2 t}\frac{180}{\pi} = \frac{TL_m(40)}{4(11.5)(10^6)A_m^2(0.05)}\frac{180}{\pi}$$

$$= 9.9645(10^{-4})\frac{TL_m}{A_m^2}$$

Equations can then be put into a spreadsheet resulting in:

r_i	r_m	A_m	L_m	r_i	T(lbf · in)	r_i	θ(deg)
0	0	0.902 5	3.8	0	1037.9	0	4.825
0.10	0.125	0.889 087	3.585 398	0.10	1022.5	0.10	4.621
0.20	0.225	0.859 043	3.413 717	0.20	987.9	0.20	4.553
0.30	0.325	0.811 831	3.242 035	0.30	933.6	0.30	4.576
0.40	0.425	0.747 450	3.070 354	0.40	859.6	0.40	4.707
0.45	0.475	0.708 822	2.984 513	0.45	815.1	0.45	4.825





Torque carrying capacity reduces with r_i. However, this is based on an assumption of uniform stresses which is not the case for small r_i. Also note that weight also goes down with an increase in r_i.

3-35 From Eq. (3-47) where θ_1 is the same for each leg.

$$T_1 = \frac{1}{3}G\theta_1 L_1 c_1^3, \quad T_2 = \frac{1}{3}G\theta_1 L_2 c_2^3$$

$$T = T_1 + T_2 = \frac{1}{3}G\theta_1\left(L_1c_1^3 + L_2c_2^3\right) = \frac{1}{3}G\theta_1 \sum L_i c_i^3 \quad \textit{Ans.}$$

$$\tau_1 = G\theta_1 c_1, \quad \tau_2 = G\theta_1 c_2$$

$$\tau_{\max} = G\theta_1 c_{\max} \quad \textit{Ans.}$$

3-36

(a) $\tau_{\max} = G\theta_1 c_{\max}$

$$G\theta_1 = \frac{\tau_{\max}}{c_{\max}} = \frac{12\,000}{1/8} = 9.6(10^4) \text{ psi/in}$$

$$T_{1/16} = \frac{1}{3}G\theta_1(Lc^3)_{1/16} = \frac{1}{3}(9.6)(10^4)(5/8)(1/16)^3 = 4.88 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_{1/8} = \frac{1}{3}(9.6)(10^4)(5/8)(1/8)^3 = 39.06 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$\tau_{1/16} = 9.6(10^4)1/16 = 6000 \text{ psi}, \quad \tau_{1/8} = 9.6(10^4)1/8 = 12\,000 \text{ psi} \quad \textit{Ans.}$$

(b) $$\theta_1 = \frac{9.6(10^4)}{12(10^6)} = 87(10^{-3}) \text{ rad/in} = 0.458°\text{/in} \quad \textit{Ans.}$$

3-37 *Separate strips:* For each 1/16 in thick strip,

$$T = \frac{Lc^2\tau}{3} = \frac{(1)(1/16)^2(12\,000)}{3} = 15.625 \text{ lbf} \cdot \text{in}$$

$$\therefore T_{\max} = 2(15.625) = 31.25 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

For each strip,

$$\theta = \frac{3Tl}{Lc^3G} = \frac{3(15.625)(12)}{(1)(1/16)^3(12)(10^6)} = 0.192 \text{ rad} \quad \textit{Ans.}$$

$$k_t = T/\theta = 31.25/0.192 = 162.8 \text{ lbf} \cdot \text{in/rad} \quad \textit{Ans.}$$

Solid strip: From Eq. (3-47),

$$T_{\max} = \frac{Lc^2\tau}{3} = \frac{1(1/8)^2 12\,000}{3} = 62.5 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$\theta = \theta_1 l = \frac{\tau l}{Gc} = \frac{12\,000(12)}{12(10^6)(1/8)} = 0.0960 \text{ rad} \quad \textit{Ans.}$$

$$k_l = 62.5/0.0960 = 651 \text{ lbf} \cdot \text{in/rad} \quad \textit{Ans.}$$

3-38 $\tau_{\text{all}} = 60$ MPa, H = 35 kW

(a) $n = 2000$ rpm

Eq. (4-40) $$T = \frac{9.55H}{n} = \frac{9.55(35)10^3}{2000} = 167.1 \text{ N} \cdot \text{m}$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \quad \Rightarrow \quad d = \left(\frac{16T}{\pi \tau_{\max}}\right)^{1/3} = \left[\frac{16(167.1)}{\pi(60)10^6}\right]^{1/3} = 24.2(10^{-3}) \text{ m} = 24.2 \text{ mm} \quad \textit{Ans.}$$

(b) $n = 200$ rpm $\quad \therefore T = 1671 \text{ N} \cdot \text{m}$

$$d = \left[\frac{16(1671)}{\pi(60)10^6}\right]^{1/3} = 52.2(10^{-3}) \text{ m} = 52.2 \text{ mm} \quad \textit{Ans.}$$

3-39 $\tau_{\text{all}} = 110$ MPa, $\theta = 30°$, $d = 15$ mm, $l = ?$

$$\tau = \frac{16T}{\pi d^3} \quad \Rightarrow \quad T = \frac{\pi}{16}\tau d^3$$

$$\theta = \frac{Tl}{JG}\left(\frac{180}{\pi}\right)$$

$$l = \frac{\pi}{180}\frac{JG\theta}{T} = \frac{\pi}{180}\left[\frac{\pi}{32}\frac{d^4G\theta}{(\pi/16)\,\tau d^3}\right] = \frac{\pi}{360}\frac{dG\theta}{\tau}$$

$$= \frac{\pi}{360}\frac{(0.015)(79.3)(10^9)(30)}{110(10^6)} = 2.83 \text{ m} \quad \textit{Ans.}$$

3-40 $d = 3$ in, replaced by 3 in hollow with $t = 1/4$ in

(a) $$T_{\text{solid}} = \frac{\pi}{16}\tau(3^3) \quad T_{\text{hollow}} = \frac{\pi}{32}\tau\frac{(3^4 - 2.5^4)}{1.5}$$

$$\%\Delta T = \frac{(\pi/16)(3^3) - (\pi/32)\left[(3^4 - 2.5^4)/1.5\right]}{(\pi/16)(3^3)}(100) = 48.2\% \quad \textit{Ans.}$$

(b) $W_{\text{solid}} = kd^2 = k(3^2), \quad W_{\text{hollow}} = k(3^2 - 2.5^2)$

$$\%\Delta W = \frac{k(3^2) - k(3^2 - 2.5^2)}{k(3^2)}(100) = 69.4\% \quad \textit{Ans.}$$

3-41 $T = 5400$ N · m, $\tau_{\text{all}} = 150$ MPa

(a) $$\tau = \frac{Tc}{J} \quad \Rightarrow \quad 150(10^6) = \frac{5400(d/2)}{(\pi/32)[d^4 - (0.75d)^4]} = \frac{4.023(10^4)}{d^3}$$

$$d = \left(\frac{4.023(10^4)}{150(10^6)}\right)^{1/3} = 6.45(10^{-2})\,\text{m} = 64.5 \text{ mm}$$

From Table A-17, the next preferred size is $d = 80$ mm; $ID = 60$ mm *Ans.*

(b) $$J = \frac{\pi}{32}(0.08^4 - 0.06^4) = 2.749(10^{-6}) \text{ mm}^4$$

$$\tau_i = \frac{5400(0.030)}{2.749(10^{-6})} = 58.9(10^6) \text{ Pa} = 58.9 \text{ MPa} \quad \textit{Ans.}$$

3-42

(a) $$T = \frac{63\,025H}{n} = \frac{63\,025(1)}{5} = 12\,605 \text{ lbf} \cdot \text{in}$$

$$\tau = \frac{16T}{\pi d_C^3} \quad \Rightarrow \quad d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(12\,605)}{\pi(14\,000)}\right]^{1/3} = 1.66 \text{ in} \quad \textit{Ans.}$$

From Table A-17, select 1 3/4 in

$$\tau_{\text{start}} = \frac{16(2)(12\,605)}{\pi(1.75^3)} = 23.96(10^3) \text{ psi} = 23.96 \text{ kpsi}$$

(b) design activity

3-43 $\omega = 2\pi n/60 = 2\pi(8)/60 = 0.8378 \text{ rad/s}$

$$T = \frac{H}{\omega} = \frac{1000}{0.8378} = 1194 \text{ N} \cdot \text{m}$$

$$d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(1194)}{\pi(75)(10^6)}\right]^{1/3} = 4.328(10^{-2}) \text{ m} = 43.3 \text{ mm}$$

From Table A-17, select 45 mm *Ans.*

3-44 $s = \sqrt{A}, \quad d = \sqrt{4A/\pi}$

Square: Eq. (3-43) with $b = c$

$$\tau_{\max} = \frac{4.8T}{c^3}$$

$$(\tau_{\max})_{\text{sq}} = \frac{4.8T}{(A)^{3/2}}$$

Round: $$(\tau_{\max})_{\text{rd}} = \frac{16}{\pi}\frac{T}{d^3} = \frac{16T}{\pi(4A/\pi)^{3/2}} = \frac{3.545T}{(A)^{3/2}}$$

$$\frac{(\tau_{\max})_{\text{sq}}}{(\tau_{\max})_{\text{rd}}} = \frac{4.8}{3.545} = 1.354$$

Square stress is 1.354 times the round stress *Ans.*

3-45 $s = \sqrt{A}, \quad d = \sqrt{4A/\pi}$

Square: Eq. (3-44) with $b = c$, $\beta = 0.141$

$$\theta_{\text{sq}} = \frac{Tl}{0.141c^4G} = \frac{Tl}{0.141(A)^{4/2}G}$$

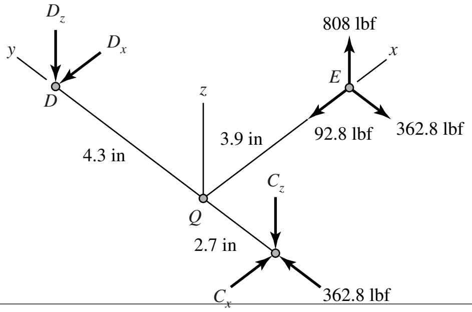
Round:

$$\theta_{\text{rd}} = \frac{Tl}{JG} = \frac{Tl}{(\pi/32)\,(4A/\pi)^{4/2}\,G} = \frac{6.2832Tl}{(A)^{4/2}G}$$

$$\frac{\theta_{\text{sq}}}{\theta_{\text{rd}}} = \frac{1/0.141}{6.2832} = 1.129$$

Square has greater θ by a factor of 1.13 *Ans.*

3-46



$$\left(\sum M_D\right)_z = 7C_x - 4.3(92.8) - 3.9(362.8) = 0$$

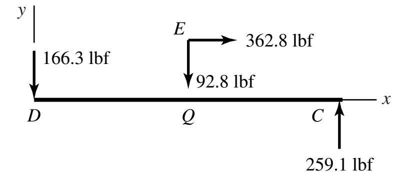
$$C_x = 259.1 \text{ lbf}$$

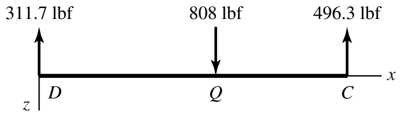
$$\left(\sum M_C\right)_z = -7D_x - 2.7(92.8) + 3.9(362.8) = 0$$

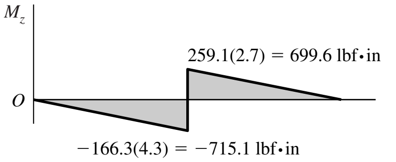
$$D_x = 166.3 \text{ lbf}$$

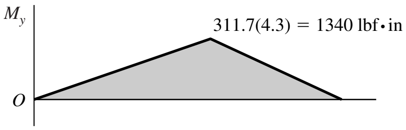
$$\left(\sum M_D\right)_x \quad \Rightarrow \quad C_z = \frac{4.3}{7}808 = 496.3 \text{ lbf}$$

$$\left(\sum M_C\right)_x \quad \Rightarrow \quad D_z = \frac{2.7}{7}808 = 311.7 \text{ lbf}$$









$$\text{Torque}: \underset{x=4.3^+_{\text{in}}}{T} = 808(3.9) = 3151 \text{ lbf}\cdot\text{in}$$

$$\text{Bending } Q: \underset{x=4.3^+_{\text{in}}}{M} = \sqrt{699.6^2 + 1340^2} = 1512 \text{ lbf}\cdot\text{in}$$

Torque:

$$\tau = \frac{16T}{\pi d^3} = \frac{16(3151)}{\pi(1.25^3)} = 8217 \text{ psi}$$

Bending:

$$\sigma_b = \pm\frac{32(1512)}{\pi(1.25^3)} = \pm 7885 \text{ psi}$$

Axial:

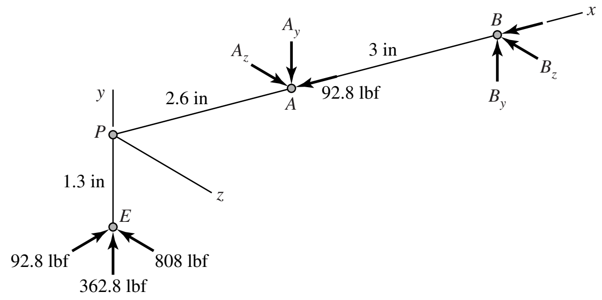
$$\sigma_a = -\frac{F}{A} = -\frac{362.8}{(\pi/4)(1.25^2)} = -296 \text{ psi}$$

$$|\sigma_{\max}| = 7885 + 296 = 8181 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{8181}{2}\right)^2 + 8217^2} = 9179 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_{\substack{\max \\ \text{tens.}}} = \frac{7885-296}{2} + \sqrt{\left(\frac{7885-296}{2}\right)^2 + 8217^2} = 12\,845 \text{ psi} \quad \textit{Ans.}$$

3-47



$$\left(\sum M_B\right)_z = -5.6(362.8) + 1.3(92.8) + 3A_y = 0$$

$$A_y = 637.0 \text{ lbf}$$

$$\left(\sum M_A\right)_z = -2.6(362.8) + 1.3(92.8) + 3B_y = 0$$

$$B_y = 274.2 \text{ lbf}$$

$$\left(\sum M_B\right)_y = 0 \quad \Rightarrow \quad A_z = \frac{5.6}{3}808 = 1508.3 \text{ lbf}$$

$$\left(\sum M_A\right)_y = 0 \quad \Rightarrow \quad B_z = \frac{2.6}{3}808 = 700.3 \text{ lbf}$$

Torsion: $T = 808(1.3) = 1050 \text{ lbf} \cdot \text{in}$

$$\tau = \frac{16(1050)}{\pi(1^3)} = 5348 \text{ psi}$$

Bending: $M_p = 92.8(1.3) = 120.6 \text{ lbf} \cdot \text{in}$

$$M_A = 3\sqrt{B_y^2 + B_z^2} = 3\sqrt{274.2^2 + 700.3^2}$$

$$= 2256 \text{ lbf} \cdot \text{in} = M_{\max}$$

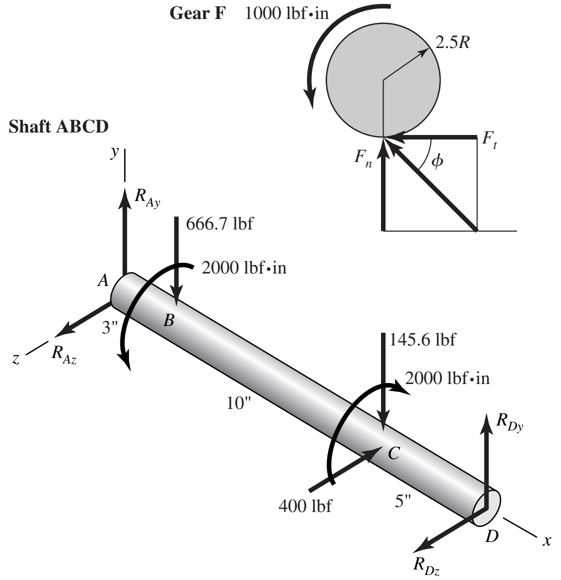
$$\sigma_b = \pm\frac{32(2256)}{\pi(1^3)} = \pm 22\,980 \text{ psi}$$

$$\text{Axial: } \underset{\text{inAP}}{\sigma} = -\frac{92.8}{(\pi/4)1^2} = -120 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{-22980-120}{2}\right)^2 + 5348^2} = 12\,730 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_{\substack{\max \\ \text{tens}}} = \frac{22980-120}{2} + \sqrt{\left(\frac{22980-120}{2}\right)^2 + 5348^2} = 24\,049 \text{ psi} \quad \textit{Ans.}$$

3-48



$$F_t = \frac{1000}{2.5} = 400\,\text{lbf}$$

$$F_n = 400 \tan 20 = 145.6 \text{ lbf}$$

Torque at C $\quad T_C = 400(5) = 2000 \text{ lbf} \cdot \text{in}$

$$P = \frac{2000}{3} = 666.7\,\text{lbf}$$

$$\sum (M_A)_z = 0 \quad \Rightarrow \quad 18R_{Dy} - 145.6(13) - 666.7(3) = 0 \quad \Rightarrow \quad R_{Dy} = 216.3 \text{ lbf}$$

$$\sum (M_A)_y = 0 \quad \Rightarrow \quad -18R_{Dz} + 400(13) = 0 \quad \Rightarrow \quad R_{Dz} = 288.9 \text{ lbf}$$

$$\sum F_y = 0 \quad \Rightarrow \quad R_{Ay} + 216.3 - 666.7 - 145.6 = 0 \quad \Rightarrow \quad R_{Ay} = 596.0 \text{ lbf}$$

$$\sum F_z = 0 \quad \Rightarrow \quad R_{Az} + 288.9 - 400 = 0 \quad \Rightarrow \quad R_{Az} = 111.1 \text{ lbf}$$

$$M_B = 3\sqrt{596^2 + 111.1^2} = 1819 \text{ lbf} \cdot \text{in}$$

$$M_C = 5\sqrt{216.3^2 + 288.9^2} = 1805 \text{ lbf} \cdot \text{in}$$

$\therefore$ Maximum stresses occur at B. *Ans.*

$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32(1819)}{\pi(1.25^3)} = 9486 \text{ psi}$$

$$\tau_B = \frac{16T_B}{\pi d^3} = \frac{16(2000)}{\pi(1.25^3)} = 5215 \text{ psi}$$

$$\sigma_{\max} = \frac{\sigma_B}{2} + \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \frac{9486}{2} + \sqrt{\left(\frac{9486}{2}\right)^2 + 5215^2} = 11\,792 \text{ psi} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = 7049 \text{ psi} \quad \textit{Ans.}$$

3-49 $r = d/2$

(a) For top, $\theta = 90^\circ$,

$$\sigma_r = \frac{\sigma}{2}[1 - 1 + (1 - 1)(1 - 3)\cos 180] = 0 \quad \textit{Ans.}$$

$$\sigma_\theta = \frac{\sigma}{2}[1 + 1 - (1 + 3)\cos 180] = 3\sigma \quad \textit{Ans.}$$

$$\tau_{r\theta} = -\frac{\sigma}{2}(1 - 1)(1 + 3)\sin 180 = 0 \quad \textit{Ans.}$$

For side, $\theta = 0^\circ$,

$$\sigma_r = \frac{\sigma}{2}[1 - 1 + (1 - 1)(1 - 3)\cos 0] = 0 \quad \textit{Ans.}$$

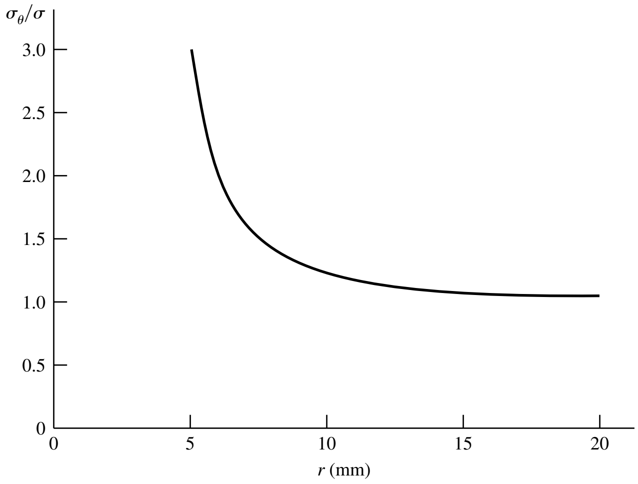
$$\sigma_\theta = \frac{\sigma}{2}[1 + 1 - (1 + 3)\cos 0] = -\sigma \quad \textit{Ans.}$$

$$\tau_{r\theta} = -\frac{\sigma}{2}(1 - 1)(1 + 3)\sin 0 = 0 \quad \textit{Ans.}$$

(b)

$$\sigma_\theta/\sigma = \frac{1}{2}\left[1 + \frac{100}{4r^2} - \left(1 + \frac{3}{16}\frac{10^4}{r^4}\right)\cos 180\right] = \frac{1}{2}\left(2 + \frac{25}{r^2} + \frac{3}{16}\frac{10^4}{r^4}\right)$$

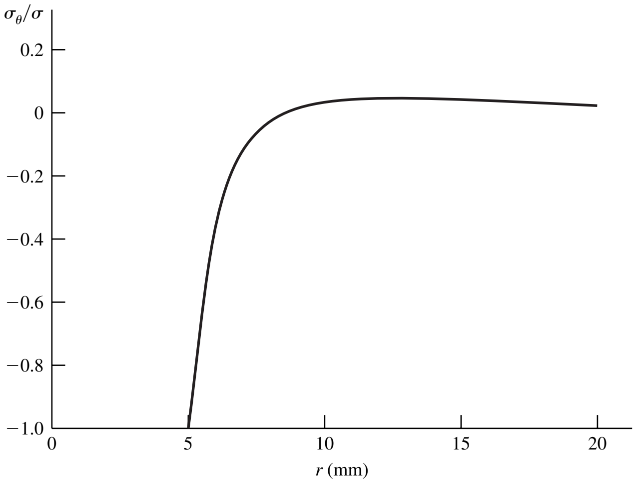
r	σ_θ/σ
5	3.000
6	2.071
7	1.646
8	1.424
9	1.297
10	1.219
11	1.167
12	1.132
13	1.107
14	1.088
15	1.074
16	1.063
17	1.054
18	1.048
19	1.042
20	1.037



(c)

$$\sigma_\theta/\sigma = \frac{1}{2}\left[1 + \frac{100}{4r^2} - \left(1 + \frac{3}{16}\frac{10^4}{r^4}\right)\cos 0\right] = \frac{1}{2}\left(\frac{25}{r^2} - \frac{3}{16}\frac{10^4}{r^4}\right)$$

r	σ_θ/σ
5	−1.000
6	−0.376
7	−0.135
8	−0.034
9	0.011
10	0.031
11	0.039
12	0.042
13	0.041
14	0.039
15	0.037
16	0.035
17	0.032
18	0.030
19	0.027
20	0.025



3-50

$$D/d = \frac{1.5}{1} = 1.5$$

$$r/d = \frac{1/8}{1} = 0.125$$

Fig. A-15-8: $K_{ts} \doteq 1.39$

Fig. A-15-9: $K_t \doteq 1.60$

$$\sigma_A = K_t\frac{Mc}{I} = \frac{32K_tM}{\pi d^3} = \frac{32(1.6)(200)(14)}{\pi(1^3)} = 45\,630 \text{ psi}$$

$$\tau_A = K_{ts}\frac{Tc}{J} = \frac{16K_{ts}T}{\pi d^3} = \frac{16(1.39)(200)(15)}{\pi(1^3)} = 21\,240 \text{ psi}$$

$$\sigma_{\max} = \frac{\sigma_A}{2} + \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_A^2} = \frac{45.63}{2} + \sqrt{\left(\frac{45.63}{2}\right)^2 + 21.24^2}$$

$$= 54.0 \text{ kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{45.63}{2}\right)^2 + 21.24^2} = 31.2 \text{ kpsi} \quad \textit{Ans.}$$

3-51 As shown in Fig. 3-32, the maximum stresses occur at the inside fiber where $r = r_i$. Therefore, from Eq. (3-50)

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right)$$

$$= p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \quad \textit{Ans.}$$

$$\sigma_{r,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right) = -p_i \quad \textit{Ans.}$$

3-52 If $p_i = 0$, Eq. (3-49) becomes

$$\sigma_t = \frac{-p_o r_o^2 - r_i^2 r_o^2 p_o/r^2}{r_o^2 - r_i^2}$$

$$= -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

The maximum tangential stress occurs at $r = r_i$. So

$$\sigma_{t,\max} = -\frac{2p_o r_o^2}{r_o^2 - r_i^2} \quad \textit{Ans.}$$

For σ_r, we have

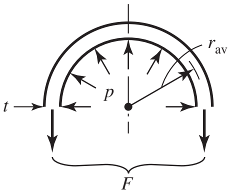
$$\sigma_r = \frac{-p_o r_o^2 + r_i^2 r_o^2 p_o/r^2}{r_o^2 - r_i^2}$$

$$= \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} - 1\right)$$

So $\sigma_r = 0$ at $r = r_i$. Thus at $r = r_o$

$$\sigma_{r,\max} = \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2 - r_o^2}{r_o^2}\right) = -p_o \quad \textit{Ans.}$$

3-53



$$F = pA = \pi r_{\text{av}}^2 p$$

$$\sigma_1 = \sigma_2 = \frac{F}{A_{\text{wall}}} = \frac{\pi r_{\text{av}}^2 p}{2\pi r_{\text{av}} t} = \frac{p r_{\text{av}}}{2t} \quad \textit{Ans.}$$

3-54 $\sigma_t > \sigma_l > \sigma_r$

$\tau_{\max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$ where σ_l is intermediate in value. From Prob. 4-50

$$\tau_{\max} = \frac{1}{2}(\sigma_{t,\,\max} - \sigma_{r,\,\max})$$

$$\tau_{\max} = \frac{p_i}{2}\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} + 1\right)$$

Now solve for p_i using $r_o = 75$ mm, $r_i = 69$ mm, and $\tau_{\max} = 25$ MPa. This gives $p_i = 3.84$ MPa *Ans.*

3-55 Given $r_o = 5$ in, $r_i = 4.625$ in and referring to the solution of Prob. 3-54,

$$\tau_{\max} = \frac{350}{2}\left[\frac{(5)^2 + (4.625)^2}{(5)^2 - (4.625)^2} + 1\right]$$

$$= 2\,424 \text{ psi} \quad \textit{Ans.}$$

3-56 From Table A-20, $S_y = 57$ kpsi; also, $r_o = 0.875$ in and $r_i = 0.625$ in

From Prob. 3-52

$$\sigma_{t,\max} = -\frac{2p_o r_o^2}{r_o^2 - r_i^2}$$

Rearranging

$$p_o = \frac{\left(r_o^2 - r_i^2\right)(0.8S_y)}{2r_o^2}$$

Solving, gives $p_o = 11\,200$ psi *Ans.*

3-57 From Table A-20, $S_y = 390$ MPa; also $r_o = 25$ mm, $r_i = 20$ mm.

From Prob. 3-51

$$\sigma_{t,\max} = p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \quad \text{therefore} \quad p_i = 0.8S_y\left(\frac{r_o^2 - r_i^2}{r_o^2 + r_i^2}\right)$$

solving gives $p_i = 68.5$ MPa *Ans.*

3-58 Since σ_t and σ_r are both positive and $\sigma_t > \sigma_r$

$$\tau_{\max} = (\sigma_t)_{\max}/2$$

where σ_t is max at r_i

Eq. (3-55) for $r = r_i = 0.375$ in

$$(\sigma_t)_{\max} = \frac{0.282}{386}\left[\frac{2\pi(7200)}{60}\right]^2\left(\frac{3+0.292}{8}\right)$$

$$\times\left[0.375^2+5^2+\frac{(0.375^2)(5^2)}{0.375^2}-\frac{1+3(0.292)}{3+0.292}(0.375^2)\right] = 8556 \text{ psi}$$

$$\tau_{\max} = \frac{8556}{2} = 4278 \text{ psi} \quad \textit{Ans.}$$

Radial stress: $$\sigma_r = k\left(r_i^2+r_o^2-\frac{r_i^2r_o^2}{r^2}-r^2\right)$$

Maxima: $$\frac{d\sigma_r}{dr} = k\left(2\frac{r_i^2r_o^2}{r^3}-2r\right) = 0 \quad\Rightarrow\quad r=\sqrt{r_ir_o}=\sqrt{0.375(5)} = 1.3693 \text{ in}$$

$$(\sigma_r)_{\max} = \frac{0.282}{386}\left[\frac{2\pi(7200)}{60}\right]^2\left(\frac{3+0.292}{8}\right)\left[0.375^2+5^2-\frac{0.375^2(5^2)}{1.3693^2}-1.3693^2\right]$$

$$= 3656 \text{ psi} \quad \textit{Ans.}$$

3-59

$$\omega = 2\pi(2069)/60 = 216.7 \text{ rad/s},$$
$$\rho = 3320\,\text{kg/m}^3,\ \nu = 0.24,\ r_i = 0.0125\,\text{m},\ r_o = 0.15\,\text{m};$$

use Eq. (3-55)

$$\sigma_t = 3320(216.7)^2\left(\frac{3+0.24}{8}\right)\left[(0.0125)^2+(0.15)^2+(0.15)^2 - \frac{1+3(0.24)}{3+0.24}(0.0125)^2\right](10)^{-6}$$

$$= 2.85 \text{ MPa} \quad \textit{Ans.}$$

3-60

$$\rho = \frac{(6/16)}{386(1/16)(\pi/4)(6^2-1^2)}$$
$$= 5.655(10^{-4})\ \text{lbf}\cdot\text{s}^2/\text{in}^4$$

$\tau_{\max}$ is at bore and equals $\dfrac{\sigma_t}{2}$

Eq. (3-55)

$$(\sigma_t)_{\max} = 5.655(10^{-4})\left[\frac{2\pi(10\,000)}{60}\right]^2\left(\frac{3+0.20}{8}\right)\left[0.5^2+3^2+3^2-\frac{1+3(0.20)}{3+0.20}(0.5)^2\right]$$

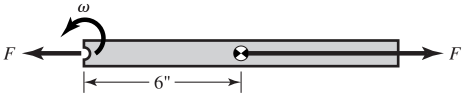
$$= 4496 \text{ psi}$$

$$\tau_{\max} = \frac{4496}{2} = 2248 \text{ psi} \quad \textit{Ans.}$$

3-61

$$\omega = 2\pi(3000)/60 = 314.2 \text{ rad/s}$$

$$m = \frac{0.282(1.25)(12)(0.125)}{386} = 1.370(10^{-3}) \text{ lbf}\cdot\text{s}^2/\text{in}$$



$$F = m\omega^2 r = 1.370(10^{-3})(314.2^2)(6) = 811.5 \text{ lbf}$$

$$A_{\text{nom}} = (1.25 - 0.5)(1/8) = 0.093\,75 \text{ in}^2$$

$$\sigma_{\text{nom}} = \frac{811.5}{0.093\,75} = 8656 \text{ psi} \quad \textit{Ans.}$$

Note: Stress concentration Fig. A-15-1 gives $K_t \doteq 2.25$ which increases $\sigma_{\max}$ and fatigue.

3-62 to 3-67

$$\nu = 0.292, \quad E = 30 \text{ Mpsi (207 GPa)}, \quad r_i = 0$$

$$R = 0.75 \text{ in (20 mm)}, \quad r_o = 1.5 \text{ in (40 mm)}$$

Eq. (3-57)

$$p_{\text{psi}} = \frac{30(10^6)\delta}{0.75^3}\left[\frac{(1.5^2 - 0.75^2)(0.75^2 - 0)}{2(1.5^2 - 0)}\right] = 1.5(10^7)\delta \tag{1}$$

$$p_{\text{Pa}} = \frac{207(10^9)\delta}{0.020^3}\left[\frac{(0.04^2 - 0.02^2)(0.02^2 - 0)}{2(0.04^2 - 0)}\right] = 3.881(10^{12})\delta \tag{2}$$

3-62

$$\delta_{\max} = \frac{1}{2}[40.042 - 40.000] = 0.021 \text{ mm} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}[40.026 - 40.025] = 0.0005 \text{ mm} \quad \textit{Ans.}$$

From (2)

$$p_{\max} = 81.5 \text{ MPa}, \quad p_{\min} = 1.94 \text{ MPa} \quad \textit{Ans.}$$

3-63

$$\delta_{\max} = \frac{1}{2}(1.5016 - 1.5000) = 0.0008 \text{ in} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}(1.5010 - 1.5010) = 0 \quad \textit{Ans.}$$

Eq. (1) $\quad p_{\max} = 12\,000 \text{ psi}, \quad p_{\min} = 0 \quad \textit{Ans.}$

3-64

$$\delta_{\max} = \frac{1}{2}(40.059 - 40.000) = 0.0295\,\text{mm} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}(40.043 - 40.025) = 0.009\ \text{mm} \quad \textit{Ans.}$$

Eq. (2) $p_{\max} = 114.5\,\text{MPa}, \quad p_{\min} = 34.9\ \text{MPa} \quad \textit{Ans.}$

3-65

$$\delta_{\max} = \frac{1}{2}(1.5023 - 1.5000) = 0.001\,15\,\text{in} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}(1.5017 - 1.5010) = 0.000\,35\ \text{in} \quad \textit{Ans.}$$

Eq. (1) $p_{\max} = 17\,250\,\text{psi} \quad p_{\min} = 5250\ \text{psi} \quad \textit{Ans.}$

3-66

$$\delta_{\max} = \frac{1}{2}(40.076 - 40.000) = 0.038\,\text{mm} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}(40.060 - 40.025) = 0.0175\ \text{mm} \quad \textit{Ans.}$$

Eq. (2) $p_{\max} = 147.5\,\text{MPa} \quad p_{\min} = 67.9\ \text{MPa} \quad \textit{Ans.}$

3-67

$$\delta_{\max} = \frac{1}{2}(1.5030 - 1.500) = 0.0015\,\text{in} \quad \textit{Ans.}$$

$$\delta_{\min} = \frac{1}{2}(1.5024 - 1.5010) = 0.0007\,\text{in} \quad \textit{Ans.}$$

Eq. (1) $p_{\max} = 22\,500\,\text{psi} \quad p_{\min} = 10\,500\,\text{psi} \quad \textit{Ans.}$

3-68

$$\delta = \frac{1}{2}(1.002 - 1.000) = 0.001\,\text{in} \quad r_i = 0, \quad R = 0.5\,\text{in}, \quad r_o = 1\,\text{in}$$

$$\nu = 0.292, \quad E = 30\,\text{Mpsi}$$

Eq. (3-57)

$$p = \frac{30(10^6)(0.001)}{0.5^3}\left[\frac{(1^2 - 0.5^2)(0.5^2 - 0)}{2(1^2 - 0)}\right] = 2.25(10^4)\ \text{psi} \quad \textit{Ans.}$$

Eq. (3-50) for outer member at $r_i = 0.5$ in

$$(\sigma_t)_o = \frac{0.5^2(2.25)(10^4)}{1^2 - 0.5^2}\left(1 + \frac{1^2}{0.5^2}\right) = 37\,500\ \text{psi} \quad \textit{Ans.}$$

Inner member, from Prob. 3-52

$$(\sigma_t)_i = -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right) = -\frac{2.25(10^4)(0.5^2)}{0.5^2 - 0}\left(1 + \frac{0}{0.5^2}\right) = -22\,500 \text{ psi} \quad Ans.$$

3-69

$$\nu_i = 0.292, \quad E_i = 30(10^6) \text{ psi}, \quad \nu_o = 0.211, \quad E_o = 14.5(10^6) \text{ psi}$$

$$\delta = \frac{1}{2}(1.002 - 1.000) = 0.001 \text{ in}, \quad r_i = 0, \quad R = 0.5, \quad r_o = 1$$

Eq. (3-56)

$$0.001 = \left[\frac{0.5}{14.5(10^6)}\left(\frac{1^2 + 0.5^2}{1^2 - 0.5^2} + 0.211\right) + \frac{0.5}{30(10^6)}\left(\frac{0.5^2 + 0}{0.5^2 - 0} - 0.292\right)\right] p$$

$$p = 13\,064 \text{ psi} \quad Ans.$$

Eq. (3-50) for outer member at $r_i = 0.5$ in

$$(\sigma_t)_o = \frac{0.5^2(13\,064)}{1^2 - 0.5^2}\left(1 + \frac{1^2}{0.5^2}\right) = 21\,770 \text{ psi} \quad Ans.$$

Inner member, from Prob. 3-52

$$(\sigma_t)_i = -\frac{13\,064(0.5^2)}{0.5^2 - 0}\left(1 + \frac{0}{0.5^2}\right) = -13\,064 \text{ psi} \quad Ans.$$

3-70

$$\delta_{\max} = \frac{1}{2}(1.003 - 1.000) = 0.0015 \text{ in} \quad r_i = 0, \quad R = 0.5 \text{ in}, \quad r_o = 1 \text{ in}$$

$$\delta_{\min} = \frac{1}{2}(1.002 - 1.001) = 0.0005 \text{ in}$$

Eq. (3-57)

$$p_{\max} = \frac{30(10^6)(0.0015)}{0.5^3}\left[\frac{(1^2 - 0.5^2)(0.5^2 - 0)}{2(1^2 - 0)}\right] = 33\,750 \text{ psi} \quad Ans.$$

Eq. (3-50) for outer member at $r = 0.5$ in

$$(\sigma_t)_o = \frac{0.5^2(33\,750)}{1^2 - 0.5^2}\left(1 + \frac{1^2}{0.5^2}\right) = 56\,250 \text{ psi} \quad Ans.$$

For inner member, from Prob. 3-52, with $r = 0.5$ in

$$(\sigma_t)_i = -33\,750 \text{ psi} \quad Ans.$$

For $\delta_{\min}$ all answers are $0.0005/0.0015 = 1/3$ of above answers *Ans.*

3-71

$$\nu_i = 0.292, \quad E_i = 30\text{ Mpsi}, \quad \nu_o = 0.334, \quad E_o = 10.4\text{ Mpsi}$$

$$\delta_{\max} = \frac{1}{2}(2.005 - 2.000) = 0.0025\text{ in}$$

$$\delta_{\min} = \frac{1}{2}(2.003 - 2.002) = 0.0005\text{ in}$$

$$0.0025 = \left[\frac{1.0}{10.4(10^6)}\left(\frac{2^2+1^2}{2^2-1^2} + 0.334\right) + \frac{1.0}{30(10^6)}\left(\frac{1^2+0}{1^2-0} - 0.292\right)\right] p_{\max}$$

$$p_{\max} = 11\,576\text{ psi} \quad \textit{Ans.}$$

Eq. (3-50) for outer member at $r = 1$ in

$$(\sigma_t)_o = \frac{1^2(11\,576)}{2^2 - 1^2}\left(1 + \frac{2^2}{1^2}\right) = 19\,293\text{ psi} \quad \textit{Ans.}$$

Inner member from Prob. 3-52 with $r = 1$ in

$$(\sigma_t)_i = -11\,576\text{ psi} \quad \textit{Ans.}$$

For $\delta_{\min}$ all above answers are $0.0005/0.0025 = 1/5$ *Ans.*

3-72

(a) Axial resistance

Normal force at fit interface

$$N = pA = p(2\pi Rl) = 2\pi pRl$$

Fully-developed friction force

$$F_{ax} = fN = 2\pi fpRl \quad \textit{Ans.}$$

(b) Torsional resistance at fully developed friction is

$$T = fRN = 2\pi fpR^2 l \quad \textit{Ans.}$$

3-73 $d = 1$ in, $r_i = 1.5$ in, $r_o = 2.5$ in.

From Table 3-4, for $R = 0.5$ in,

$$r_c = 1.5 + 0.5 = 2\text{ in}$$

$$r_n = \frac{0.5^2}{2\left(2 - \sqrt{2^2 - 0.5^2}\right)} = 1.968\,245\,8\text{ in}$$

$$e = r_c - r_n = 2.0 - 1.968\,245\,8 = 0.031\,754\text{ in}$$

$$c_i = r_n - r_i = 1.9682 - 1.5 = 0.4682\text{ in}$$

$$c_o = r_o - r_n = 2.5 - 1.9682 = 0.5318\text{ in}$$

$$A = \pi d^2/4 = \pi(1)^2/4 = 0.7854\text{ in}^2$$

$$M = Fr_c = 1000(2) = 2000\text{ lbf}\cdot\text{in}$$

Using Eq. (3-65)

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{1000}{0.7854} + \frac{2000(0.4682)}{0.7854(0.031\,754)(1.5)} = 26\,300 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{1000}{0.7854} - \frac{2000(0.5318)}{0.7854(0.031\,754)(2.5)} = -15\,800 \text{ psi} \quad \textit{Ans.}$$

3-74 Section AA:

$D = 0.75$ in, $r_i = 0.75/2 = 0.375$ in, $r_o = 0.75/2 + 0.25 = 0.625$ in

From Table 3-4, for $R = 0.125$ in,

$$r_c = (0.75 + 0.25)/2 = 0.500 \text{ in}$$

$$r_n = \frac{0.125^2}{2\left(0.5 - \sqrt{0.5^2 - 0.125^2}\right)} = 0.492\,061\,5 \text{ in}$$

$$e = 0.5 - r_n = 0.007\,939 \text{ in}$$

$$c_o = r_o - r_n = 0.625 - 0.492\,06 = 0.132\,94 \text{ in}$$

$$c_i = r_n - r_i = 0.492\,06 - 0.375 = 0.117\,06 \text{ in}$$

$$A = \pi(0.25)^2/4 = 0.049\,087$$

$$M = Fr_c = 100(0.5) = 50 \text{ lbf} \cdot \text{in}$$

$$\sigma_i = \frac{100}{0.049\,09} + \frac{50(0.117\,06)}{0.049\,09(0.007\,939)(0.375)} = 42\,100 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{100}{0.049\,09} - \frac{50(0.132\,94)}{0.049\,09(0.007\,939)(0.625)} = -25\,250 \text{ psi} \quad \textit{Ans.}$$

Section BB: Abscissa angle θ of line of radius centers is

$$\theta = \cos^{-1}\left(\frac{r_2 + d/2}{r_2 + d + D/2}\right)$$

$$= \cos^{-1}\left(\frac{0.375 + 0.25/2}{0.375 + 0.25 + 0.75/2}\right) = 60^\circ$$

$$M = F\frac{D + d}{2}\cos\theta = 100(0.5)\cos 60^\circ = 25 \text{ lbf} \cdot \text{in}$$

$$r_i = r_2 = 0.375 \text{ in}$$

$$r_o = r_2 + d = 0.375 + 0.25 = 0.625 \text{ in}$$

$$e = 0.007\,939 \text{ in} \quad \text{(as before)}$$

$$\sigma_i = \frac{F\cos\theta}{A} - \frac{Mc_i}{Aer_i}$$

$$= \frac{100\cos 60^\circ}{0.049\,09} - \frac{25(0.117\,06)}{0.049\,09(0.007\,939)0.375} = -19\,000 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{100\cos 60^\circ}{0.049\,09} + \frac{25(0.132\,94)}{0.049\,09(0.007\,939)0.625} = 14\,700 \text{ psi} \quad \textit{Ans.}$$

On section BB, the shear stress due to the shear force is zero at the surface.

3-75 $r_i = 0.125$ in, $r_o = 0.125 + 0.1094 = 0.2344$ in

From Table 3-4 for $h = 0.1094$

$$r_c = 0.125 + 0.1094/2 = 0.1797 \text{ in}$$

$$r_n = 0.1094/\ln(0.2344/0.125) = 0.174\,006 \text{ in}$$

$$e = r_c - r_n = 0.1797 - 0.174\,006 = 0.005\,694 \text{ in}$$

$$c_i = r_n - r_i = 0.174\,006 - 0.125 = 0.049\,006 \text{ in}$$

$$c_o = r_o - r_n = 0.2344 - 0.174\,006 = 0.060\,394 \text{ in}$$

$$A = 0.75(0.1094) = 0.082\,050 \text{ in}^2$$

$$M = F(4 + h/2) = 3(4 + 0.1094/2) = 12.16 \text{ lbf} \cdot \text{in}$$

$$\sigma_i = -\frac{3}{0.082\,05} - \frac{12.16(0.0490)}{0.082\,05(0.005\,694)(0.125)} = -10\,240 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{3}{0.082\,05} + \frac{12.16(0.0604)}{0.082\,05(0.005\,694)(0.2344)} = 6670 \text{ psi} \quad \textit{Ans.}$$

3-76 Find the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$.

$$\begin{aligned} F_x &= F_{1x} + F_{2x} = 250\cos 60^\circ + 333\cos 0^\circ \\ &= 458 \text{ lbf} \end{aligned}$$

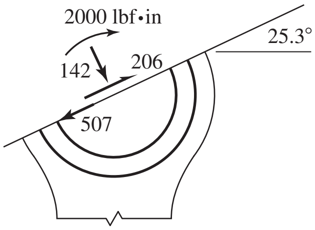
$$\begin{aligned} F_y &= F_{1y} + F_{2y} = 250\sin 60^\circ + 333\sin 0^\circ \\ &= 216.5 \text{ lbf} \end{aligned}$$

$$F = (458^2 + 216.5^2)^{1/2} = 506.6 \text{ lbf}$$

This is the pin force on the lever which acts in a direction

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{216.5}{458} = 25.3^\circ$$

On the 25.3° surface from $\mathbf{F}_1$



$$F_t = 250\cos(60^\circ - 25.3^\circ) = 206 \text{ lbf}$$

$$F_n = 250\sin(60^\circ - 25.3^\circ) = 142 \text{ lbf}$$

$$r_c = 1 + 3.5/2 = 2.75 \text{ in}$$

$$\begin{aligned} A &= 2[0.8125(0.375) + 1.25(0.375)] \\ &= 1.546\,875 \text{ in}^2 \end{aligned}$$

The denominator of Eq. (3-63), given below, has four additive parts.

$$r_n = \frac{A}{\int (dA/r)}$$

For $\int dA/r$, add the results of the following equation for each of the four rectangles.

$$\int_{r_i}^{r_o} \frac{b\,dr}{r} = b \ln \frac{r_o}{r_i}, \qquad b = \text{width}$$

$$\int \frac{dA}{r} = 0.375 \ln \frac{1.8125}{1} + 1.25 \ln \frac{2.1875}{1.8125} + 1.25 \ln \frac{3.6875}{3.3125} + 0.375 \ln \frac{4.5}{3.6875}$$
$$= 0.666\,810\,6$$

$$r_n = \frac{1.546\,875}{0.666\,810\,6} = 2.3198 \text{ in}$$

$$e = r_c - r_n = 2.75 - 2.3198 = 0.4302 \text{ in}$$
$$c_i = r_n - r_i = 2.320 - 1 = 1.320 \text{ in}$$
$$c_o = r_o - r_n = 4.5 - 2.320 = 2.180 \text{ in}$$

Shear stress due to 206 lbf force is zero at inner and outer surfaces.

$$\sigma_i = -\frac{142}{1.547} + \frac{2000(1.32)}{1.547(0.4302)(1)} = 3875 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{142}{1.547} - \frac{2000(2.18)}{1.547(0.4302)(4.5)} = -1548 \text{ psi} \quad \textit{Ans.}$$

3-77

$$A = (6 - 2 - 1)(0.75) = 2.25 \text{ in}^2$$
$$r_c = \frac{6+2}{2} = 4 \text{ in}$$

Similar to Prob. 3-76,

$$\int \frac{dA}{r} = 0.75 \ln \frac{3.5}{2} + 0.75 \ln \frac{6}{4.5} = 0.635\,473\,4 \text{ in}$$

$$r_n = \frac{A}{\int (dA/r)} = \frac{2.25}{0.635\,473\,4} = 3.5407 \text{ in}$$

$$e = 4 - 3.5407 = 0.4593 \text{ in}$$

$$\sigma_i = \frac{5000}{2.25} + \frac{20\,000(3.5407 - 2)}{2.25(0.4593)(2)} = 17\,130 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{5000}{2.25} - \frac{20\,000(6 - 3.5407)}{2.25(0.4593)(6)} = -5710 \text{ psi} \quad \textit{Ans.}$$

3-78

$$A = \int_{r_i}^{r_o} b\,dr = \int_2^6 \frac{2}{r}\,dr = 2 \ln \frac{6}{2}$$
$$= 2.197\,225 \text{ in}^2$$

$$r_c = \frac{1}{A}\int_{r_i}^{r_o} br\,dr = \frac{1}{2.197\,225}\int_2^6 \frac{2r}{r}\,dr$$

$$= \frac{2}{2.197\,225}(6-2) = 3.640\,957 \text{ in}$$

$$r_n = \frac{A}{\int_{r_i}^{r_o}(b/r)\,dr} = \frac{2.197\,225}{\int_2^6 (2/r^2)\,dr}$$

$$= \frac{2.197\,225}{2[1/2 - 1/6]} = 3.295\,837 \text{ in}$$

$$e = R - r_n = 3.640\,957 - 3.295\,837 = 0.345\,12$$

$$c_i = r_n - r_i = 3.2958 - 2 = 1.2958 \text{ in}$$

$$c_o = r_o - r_n = 6 - 3.2958 = 2.7042 \text{ in}$$

$$\sigma_i = \frac{20\,000}{2.197} + \frac{20\,000(3.641)(1.2958)}{2.197(0.345\,12)(2)} = 71\,330 \text{ psi} \quad Ans.$$

$$\sigma_o = \frac{20\,000}{2.197} - \frac{20\,000(3.641)(2.7042)}{2.197(0.345\,12)(6)} = -34\,180 \text{ psi} \quad Ans.$$

3-79 $r_c = 12$ in, $M = 20(2+2) = 80$ kip · in

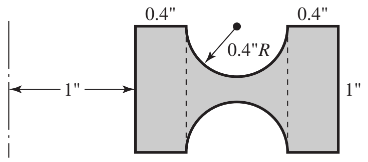
From statics book, $I = \frac{\pi}{4}a^3 b = \frac{\pi}{4}(2^3)1 = 2\pi \text{ in}^4$

Inside: $\sigma_i = \frac{F}{A} + \frac{My}{I}\frac{r_c}{r_i} = \frac{20}{2\pi} + \frac{80(2)}{2\pi}\frac{12}{10} = 33.7 \text{ kpsi}$ *Ans.*

Outside: $\sigma_o = \frac{F}{A} - \frac{My}{I}\frac{r_c}{r_o} = \frac{20}{2\pi} - \frac{80(2)}{2\pi}\frac{12}{14} = -18.6 \text{ kpsi}$ *Ans.*

Note: A much more accurate solution (see the 7th edition) yields $\sigma_i = 32.25$ kpsi and $\sigma_o = -19.40$ kpsi

3-80



For rectangle, $\int \frac{dA}{r} = b \ln r_o/r_i$

For circle, $\frac{A}{\int(dA/r)} = \frac{r^2}{2\left(r_c - \sqrt{r_c^2 - r^2}\right)}, \quad A_o = \pi r^2$

$$\therefore \int \frac{dA}{r} = 2\pi\left(r_c - \sqrt{r_c^2 - r^2}\right)$$

$$\sum \int \frac{dA}{r} = 1 \ln \frac{2.6}{1} - 2\pi \left(1.8 - \sqrt{1.8^2 - 0.4^2}\right) = 0.672\,723\,4$$

$$A = 1(1.6) - \pi(0.4^2) = 1.097\,345\,2 \text{ in}^2$$

$$r_n = \frac{1.097\,345\,2}{0.672\,723\,4} = 1.6312 \text{ in}$$

$$e = 1.8 - r_n = 0.1688 \text{ in}$$

$$c_i = 1.6312 - 1 = 0.6312 \text{ in}$$

$$c_o = 2.6 - 1.6312 = 0.9688 \text{ in}$$

$$M = 3000(5.8) = 17\,400 \text{ lbf} \cdot \text{in}$$

$$\sigma_i = \frac{3}{1.0973} + \frac{17.4(0.6312)}{1.0973(0.1688)(1)} = 62.03 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{3}{1.0973} - \frac{17.4(0.9688)}{1.0973(0.1688)(2.6)} = -32.27 \text{ kpsi} \quad \textit{Ans.}$$

3-81 From Eq. (3-68)

$$a = KF^{1/3} = F^{1/3} \left\{ \frac{3}{8} \frac{2[(1-\nu^2)/E]}{2(1/d)} \right\}^{1/3}$$

Use $\nu = 0.292$, F in newtons, E in N/mm^2 and d in mm, then

$$K = \left\{ \frac{3}{8} \frac{[(1-0.292^2)/207\,000]}{1/25} \right\}^{1/3} = 0.0346$$

$$\begin{aligned} p_{\max} &= \frac{3F}{2\pi a^2} = \frac{3F}{2\pi (KF^{1/3})^2} \\ &= \frac{3F^{1/3}}{2\pi K^2} = \frac{3F^{1/3}}{2\pi (0.0346)^2} \\ &= 399F^{1/3} \text{ MPa} = |\sigma_{\max}| \qquad \textit{Ans.} \\ \tau_{\max} &= 0.3 p_{\max} \\ &= 120F^{1/3} \text{ MPa} \qquad \textit{Ans.} \end{aligned}$$

3-82 From Prob. 3-81,

$$K = \left\{ \frac{3}{8} \frac{2[(1-0.292^2)/207\,000]}{1/25 + 0} \right\}^{1/3} = 0.0436$$

$$p_{\max} = \frac{3F^{1/3}}{2\pi K^2} = \frac{3F^{1/3}}{2\pi (0.0436)^2} = 251F^{1/3}$$

and so, $\sigma_z = -251F^{1/3}$ MPa *Ans.*

$$\tau_{\max} = 0.3(251)F^{1/3} = 75.3F^{1/3} \text{ MPa} \quad \textit{Ans.}$$

$$z = 0.48a = 0.48(0.0436)18^{1/3} = 0.055 \text{ mm} \quad \textit{Ans.}$$

3-83 $\nu_1 = 0.334$, $E_1 = 10.4$ Mpsi, $l = 2$ in, $d_1 = 1$ in, $\nu_2 = 0.211$, $E_2 = 14.5$ Mpsi, $d_2 = -8$ in.

With $b = K_c F^{1/2}$, from Eq. (3-73),

$$K_c = \left(\frac{2}{\pi(2)} \, \frac{(1 - 0.334^2)/[10.4(10^6)] + (1 - 0.211^2)/[14.5(10^6)]}{1 - 0.125} \right)^{1/2}$$

$$= 0.000\,234\,6$$

Be sure to check σ_x for both ν_1 and ν_2. Shear stress is maximum in the aluminum roller. So,

$$\tau_{\max} = 0.3 p_{\max}$$

$$p_{\max} = \frac{4000}{0.3} = 13\,300 \text{ psi}$$

Since $p_{\max} = 2F/(\pi b l)$ we have

$$p_{\max} = \frac{2F}{\pi l K_c F^{1/2}} = \frac{2F^{1/2}}{\pi l K_c}$$

So,

$$F = \left(\frac{\pi l K_c p_{\max}}{2} \right)^2$$

$$= \left(\frac{\pi(2)(0.000\,234\,6)(13\,300)}{2} \right)^2$$

$$= 96.1 \text{ lbf} \quad \textit{Ans.}$$

3-84 Good class problem

3-85 From Table A-5, $\nu = 0.211$

$$\frac{\sigma_x}{p_{\max}} = (1 + \nu) - \frac{1}{2} = (1 + 0.211) - \frac{1}{2} = 0.711$$

$$\frac{\sigma_y}{p_{\max}} = 0.711$$

$$\frac{\sigma_z}{p_{\max}} = 1$$

These are principal stresses

$$\frac{\tau_{\max}}{p_{\max}} = \frac{1}{2}(\sigma_1 - \sigma_3) = \frac{1}{2}(1 - 0.711) = 0.1445$$

3-86 From Table A-5: $\nu_1 = 0.211$, $\nu_2 = 0.292$, $E_1 = 14.5(10^6)$ psi, $E_2 = 30(10^6)$ psi, $d_1 = 6$ in, $d_2 = \infty$, $l = 2$ in

(a) Eq. (3-73):
$$b = \sqrt{\frac{2(800)}{\pi(2)}\ \frac{(1-0.211^2)/14.5(10^6)+(1-0.292^2)/[30(10^6)]}{1/6+1/\infty}}$$
$$= 0.012\,135 \text{ in}$$
$$p_{\text{max}} = \frac{2(800)}{\pi(0.012\,135)(2)} = 20\,984 \text{ psi}$$

For $z = 0$ in,
$$\sigma_{x1} = -2\nu_1 p_{\text{max}} = -2(0.211)20\,984 = -8855 \text{ psi in wheel}$$
$$\sigma_{x2} = -2(0.292)20\,984 = -12\,254 \text{ psi}$$

In plate
$$\sigma_y = -p_{\text{max}} = -20\,984 \text{ psi}$$
$$\sigma_z = -20\,984 \text{ psi}$$

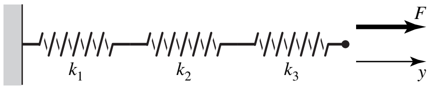
These are principal stresses.

(b) For $z = 0.010$ in,
$$\sigma_{x1} = -4177 \text{ psi} \quad \text{in wheel}$$
$$\sigma_{x2} = -5781 \text{ psi} \quad \text{in plate}$$
$$\sigma_y = -3604 \text{ psi}$$
$$\sigma_z = -16\,194 \text{ psi}$$

Chapter 4

4-1

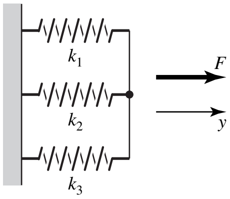
(a)



$$k = \frac{F}{y}; \quad y = \frac{F}{k_1} + \frac{F}{k_2} + \frac{F}{k_3}$$

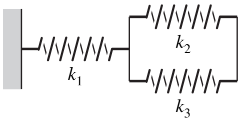
so
$$k = \frac{1}{(1/k_1) + (1/k_2) + (1/k_3)} \quad \textit{Ans.}$$

(b)



$$F = k_1 y + k_2 y + k_3 y$$
$$k = F/y = k_1 + k_2 + k_3 \quad \textit{Ans.}$$

(c)



$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2 + k_3} \qquad k = \left(\frac{1}{k_1} + \frac{1}{k_2 + k_3}\right)^{-1}$$

4-2 For a torsion bar, $k_T = T/\theta = Fl/\theta$, and so $\theta = Fl/k_T$. For a cantilever, $k_C = F/\delta$, $\delta = F/k_C$. For the assembly, $k = F/y$, $y = F/k = l\theta + \delta$

So
$$y = \frac{F}{k} = \frac{Fl^2}{k_T} + \frac{F}{k_C}$$

Or
$$k = \frac{1}{(l^2/k_T) + (1/k_C)} \quad \textit{Ans.}$$

4-3 For a torsion bar, $k = T/\theta = GJ/l$ where $J = \pi d^4/32$. So $k = \pi d^4 G/(32l) = Kd^4/l$. The springs, 1 and 2, are in parallel so

$$k = k_1 + k_2 = K\frac{d^4}{l_1} + K\frac{d^4}{l_2}$$
$$= Kd^4\left(\frac{1}{x} + \frac{1}{l - x}\right)$$

And
$$\theta = \frac{T}{k} = \frac{T}{Kd^4\left(\frac{1}{x} + \frac{1}{l - x}\right)}$$

Then
$$T = k\theta = \frac{Kd^4}{x}\theta + \frac{Kd^4\theta}{l - x}$$

Thus $$T_1 = \frac{Kd^4}{x}\theta; \qquad T_2 = \frac{Kd^4\theta}{l-x}$$

If $x = l/2$, then $T_1 = T_2$. If $x < l/2$, then $T_1 > T_2$

Using $\tau = 16T/\pi d^3$ and $\theta = 32Tl/(G\pi d^4)$ gives

$$T = \frac{\pi d^3 \tau}{16}$$

and so

$$\theta_{\text{all}} = \frac{32l}{G\pi d^4} \cdot \frac{\pi d^3 \tau}{16} = \frac{2l\tau_{\text{all}}}{Gd}$$

Thus, if $x < l/2$, the allowable twist is

$$\theta_{\text{all}} = \frac{2x\tau_{\text{all}}}{Gd} \quad \textit{Ans.}$$

Since $$k = Kd^4\left(\frac{1}{x} + \frac{1}{l-x}\right)$$

$$= \frac{\pi G d^4}{32}\left(\frac{1}{x} + \frac{1}{l-x}\right) \quad \textit{Ans.}$$

Then the maximum torque is found to be

$$T_{\max} = \frac{\pi d^3 x \tau_{\text{all}}}{16}\left(\frac{1}{x} + \frac{1}{l-x}\right) \quad \textit{Ans.}$$

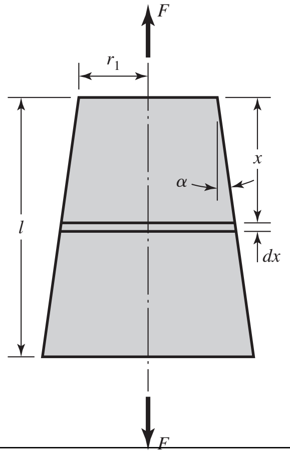
4-4 Both legs have the same twist angle. From Prob. 4-3, for equal shear, d is linear in x. Thus, $d_1 = 0.2d_2$ *Ans.*

$$k = \frac{\pi G}{32}\left[\frac{(0.2d_2)^4}{0.2l} + \frac{d_2^4}{0.8l}\right] = \frac{\pi G}{32l}\left(1.258d_2^4\right) \quad \textit{Ans.}$$

$$\theta_{\text{all}} = \frac{2(0.8l)\tau_{\text{all}}}{Gd_2} \quad \textit{Ans.}$$

$$T_{\max} = k\theta_{\text{all}} = 0.198d_2^3\tau_{\text{all}} \quad \textit{Ans.}$$

4-5



$$A = \pi r^2 = \pi(r_1 + x\tan\alpha)^2$$

$$d\delta = \frac{F\,dx}{AE} = \frac{F\,dx}{E\pi(r_1 + x\tan\alpha)^2}$$

$$\delta = \frac{F}{\pi E}\int_0^l \frac{dx}{(r_1 + x\tan\alpha)^2}$$

$$= \frac{F}{\pi E}\left(-\frac{1}{\tan\alpha(r_1 + x\tan\alpha)}\right)_0^l$$

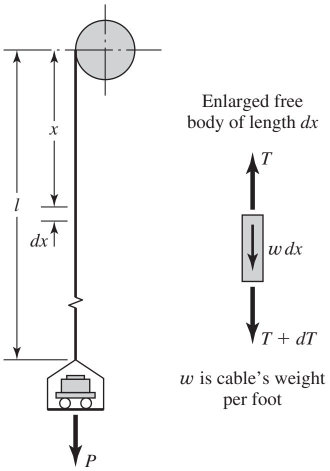
$$= \frac{F}{\pi E}\frac{1}{r_1(r_1 + l\tan\alpha)}$$

Then

$$k = \frac{F}{\delta} = \frac{\pi E r_1(r_1 + l\tan\alpha)}{l}$$

$$= \frac{EA_1}{l}\left(1 + \frac{2l}{d_1}\tan\alpha\right) \quad \textit{Ans.}$$

4-6



$$\sum F = (T + dT) + w\,dx - T = 0$$

$$\frac{dT}{dx} = -w$$

Solution is $T = -wx + c$

$$T|_{x=0} = P + wl = c$$

$$T = -wx + P + wl$$

$$T = P + w(l - x)$$

The infinitesmal stretch of the free body of original length dx is

$$d\delta = \frac{T\,dx}{AE}$$

$$= \frac{P + w(l - x)}{AE}\,dx$$

Integrating,

$$\delta = \int_0^l \frac{[P + w(l - x)]\,dx}{AE}$$

$$\delta = \frac{Pl}{AE} + \frac{wl^2}{2AE} \quad \textit{Ans.}$$

4-7

$$M = wlx - \frac{wl^2}{2} - \frac{wx^2}{2}$$

$$EI\frac{dy}{dx} = \frac{wlx^2}{2} - \frac{wl^2}{2}x - \frac{wx^3}{6} + C_1, \qquad \frac{dy}{dx} = 0 \text{ at } x = 0, \quad \therefore C_1 = 0$$

$$EIy = \frac{wlx^3}{6} - \frac{wl^2x^2}{4} - \frac{wx^4}{24} + C_2, \qquad y = 0 \text{ at } x = 0, \quad \therefore C_2 = 0$$

$$y = \frac{wx^2}{24EI}(4lx - 6l^2 - x^2) \quad \textit{Ans.}$$

4-8

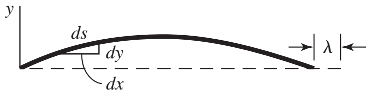
$$M = M_1 = M_B$$

$$EI\frac{dy}{dx} = M_B x + C_1, \qquad \frac{dy}{dx} = 0 \text{ at } x = 0, \quad \therefore C_1 = 0$$

$$EIy = \frac{M_B x^2}{2} + C_2, \qquad y = 0 \text{ at } x = 0, \quad \therefore C_2 = 0$$

$$y = \frac{M_B x^2}{2EI} \quad \textit{Ans.}$$

4-9



$$ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Expand right-hand term by Binomial theorem

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} = 1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2 + \cdots$$

Since dy/dx is small compared to 1, use only the first two terms,

$$\begin{aligned} d\lambda &= ds - dx \\ &= dx\left[1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2\right] - dx \\ &= \frac{1}{2}\left(\frac{dy}{dx}\right)^2 dx \\ \therefore \lambda &= \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx \quad \textit{Ans.} \end{aligned}$$

This contraction becomes important in a nonlinear, non-breaking extension spring.

4-10 $y = Cx^2(4lx - x^2 - 6l^2) \qquad \text{where } C = \dfrac{w}{24EI}$

$$\frac{dy}{dx} = Cx(12lx - 4x^2 - 12l^2) = 4Cx(3lx - x^2 - 3l^2)$$

$$\left(\frac{dy}{dx}\right)^2 = 16C^2(15l^2x^4 - 6lx^5 - 18x^3l^3 + x^6 + 9l^4x^2)$$

$$\lambda = \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx = 8C^2 \int_0^l (15l^2x^4 - 6lx^5 - 18x^3l^3 + x^6 + 9l^4x^2)\, dx$$

$$= 8C^2\left(\frac{9}{14}l^7\right) = 8\left(\frac{w}{24EI}\right)^2\left(\frac{9}{14}l^7\right) = \frac{1}{112}\left(\frac{w}{EI}\right)^2 l^7 \qquad \textit{Ans.}$$

4-11 $y = Cx(2lx^2 - x^3 - l^3)$ where $C = \dfrac{w}{24EI}$

$$\frac{dy}{dx} = C(6lx^2 - 4x^3 - l^3)$$

$$\left(\frac{dy}{dx}\right)^2 = C^2(36l^2x^4 - 48lx^5 - 12l^4x^2 + 16x^6 + 8x^3l^3 + l^6)$$

$$\lambda = \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx = \frac{1}{2}C^2\int_0^l (36l^2x^4 - 48lx^5 - 12l^4x^2 + 16x^6 + 8x^3l^3 + l^6)\,dx$$

$$= C^2\left(\frac{17}{70}l^7\right) = \left(\frac{w}{24EI}\right)^2\left(\frac{17}{70}l^7\right) = \frac{17}{40\,320}\left(\frac{w}{EI}\right)^2 l^7 \qquad \textit{Ans.}$$

4-12

$$I = 2(5.56) = 11.12 \text{ in}^4$$

$$y_{\max} = y_1 + y_2 = -\frac{wl^4}{8EI} + \frac{Fa^2}{6EI}(a - 3l)$$

Here $w = 50/12 = 4.167$ lbf/in, and $a = 7(12) = 84$ in, and $l = 10(12) = 120$ in.

$$y_1 = -\frac{4.167(120)^4}{8(30)(10^6)(11.12)} = -0.324 \text{ in}$$

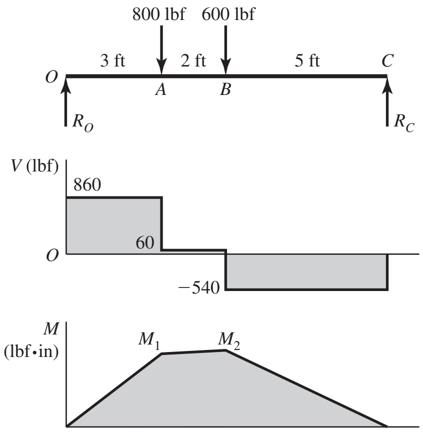
$$y_2 = -\frac{600(84)^2[3(120) - 84]}{6(30)(10^6)(11.12)} = -0.584 \text{ in}$$

So $$y_{\max} = -0.324 - 0.584 = -0.908 \text{ in} \quad \textit{Ans.}$$

$$\begin{aligned} M_0 &= -Fa - (wl^2/2) \\ &= -600(84) - [4.167(120)^2/2] \\ &= -80\,400 \text{ lbf} \cdot \text{in} \\ c &= 4 - 1.18 = 2.82 \text{ in} \\ \sigma_{\max} &= \frac{-My}{I} = -\frac{(-80\,400)(-2.82)}{11.12}(10^{-3}) \\ &= -20.4 \text{ kpsi} \quad \textit{Ans.} \end{aligned}$$

$\sigma_{\max}$ is at the bottom of the section.

4-13



$$R_O = \frac{7}{10}(800) + \frac{5}{10}(600) = 860\,\text{lbf}$$

$$R_C = \frac{3}{10}(800) + \frac{5}{10}(600) = 540\,\text{lbf}$$

$$M_1 = 860(3)(12) = 30.96(10^3)\,\text{lbf}\cdot\text{in}$$

$$M_2 = 30.96(10^3) + 60(2)(12)$$
$$= 32.40(10^3)\,\text{lbf}\cdot\text{in}$$

$$\sigma_{\max} = \frac{M_{\max}}{Z} \quad \Rightarrow \quad 6 = \frac{32.40}{Z} \qquad Z = 5.4\text{ in}^3$$

$$y|_{x=5\text{ft}} = \frac{F_1 a[l-(l/2)]}{6EIl}\left[\left(\frac{l}{2}\right)^2 + a^2 - 2l\frac{l}{2}\right] - \frac{F_2 l^3}{48EI}$$

$$-\frac{1}{16} = \frac{800(36)(60)}{6(30)(10^6)I(120)}[60^2 + 36^2 - 120^2] - \frac{600(120^3)}{48(30)(10^6)I}$$

$$I = 23.69\text{ in}^4 \quad \Rightarrow \quad I/2 = 11.84\text{ in}^4$$

Select two 6 in-8.2 lbf/ft channels; from Table A-7, $I = 2(13.1) = 26.2\text{ in}^4,\ Z = 2(4.38)\text{ in}^3$

$$y_{\max} = \frac{23.69}{26.2}\left(-\frac{1}{16}\right) = -0.0565\,\text{in}$$

$$\sigma_{\max} = \frac{32.40}{2(4.38)} = 3.70\,\text{kpsi}$$

4-14

$$I = \frac{\pi}{64}(40^4) = 125.66(10^3)\text{ mm}^4$$

Superpose beams A-9-6 and A-9-7,

$$y_A = \frac{1500(600)400}{6(207)10^9(125.66)10^3(1000)}(400^2 + 600^2 - 1000^2)(10^3)^2$$
$$+ \frac{2000(400)}{24(207)10^9(125.66)10^3}[2(1000)400^2 - 400^3 - 1000^3]10^3$$

$$y_A = -2.061\text{ mm} \quad \textit{Ans.}$$

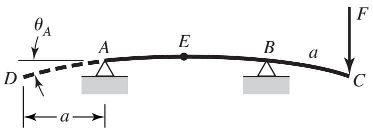
$$y|_{x=500} = \frac{1500(400)500}{24(207)10^9(125.66)10^3(1000)}[500^2 + 400^2 - 2(1000)500](10^3)^2$$
$$- \frac{5(2000)1000^4}{384(207)10^9(125.66)10^3}10^3 = -2.135\,\text{mm} \quad \textit{Ans.}$$

$$\%\text{ difference} = \frac{2.135 - 2.061}{2.061}(100) = 3.59\% \quad \textit{Ans.}$$

4-15

$$I = \frac{1}{12}(9)(35^3) = 32.\,156(10^3)\ \text{mm}^4$$

From Table A-9-10



$$y_C = -\frac{Fa^2}{3EI}(l + a)$$

$$\frac{dy_{AB}}{dx} = \frac{Fa}{6EIl}(l^2 - 3x^2)$$

Thus,

$$\theta_A = \frac{Fal^2}{6EIl} = \frac{Fal}{6EI}$$

$$y_D = -\theta_A a = -\frac{Fa^2 l}{6EI}$$

With both loads,

$$y_D = -\frac{Fa^2 l}{6EI} - \frac{Fa^2}{3EI}(l + a)$$

$$= -\frac{Fa^2}{6EI}(3l + 2a) = -\frac{500(250^2)}{6(207)(10^9)(32.156)(10^3)}[3(500) + 2(250)](10^3)^2$$

$$= -1.565\ \text{mm} \quad \textit{Ans.}$$

$$y_E = \frac{2Fa(l/2)}{6EIl}\left[l^2 - \left(\frac{l}{2}\right)^2\right] = \frac{Fal^2}{8EI}$$

$$= \frac{500(250)(500^2)(10^3)^2}{8(207)(10^9)(32.156)(10^3)} = 0.587\ \text{mm} \quad \textit{Ans.}$$

4-16 $a = 36$ in, $l = 72$ in, $I = 13\ \text{in}^4$, $E = 30$ Mpsi

$$y = \frac{F_1 a^2}{6EI}(a - 3l) - \frac{F_2 l^3}{3EI}$$

$$= \frac{400(36)^2(36 - 216)}{6(30)(10^6)(13)} - \frac{400(72)^3}{3(30)(10^6)(13)}$$

$$= -0.1675\ \text{in} \quad \textit{Ans.}$$

4-17

$$I = 2(1.85) = 3.7\ \text{in}^4$$

Adding the weight of the channels, $2(5)/12 = 0.833$ lbf/in,

$$y_A = -\frac{wl^4}{8EI} - \frac{Fl^3}{3EI} = -\frac{10.833(48^4)}{8(30)(10^6)(3.7)} - \frac{220(48^3)}{3(30)(10^6)(3.7)}$$

$$= -0.1378\ \text{in} \quad \textit{Ans.}$$

4-18

$$I = \pi d^4/64 = \pi(2)^4/64 = 0.7854\ \text{in}^4$$

Tables A-9-5 and A-9-9

$$y = -\frac{F_2 l^3}{48EI} + \frac{F_1 a}{24EI}(4a^2 - 3l^2)$$

$$= -\frac{120(40)^3}{48(30)(10^6)(0.7854)} + \frac{85(10)(400 - 4800)}{24(30)(10^6)(0.7854)} = -0.0134\ \text{in} \quad \textit{Ans.}$$

4-19

(a) Useful relations

$$k = \frac{F}{y} = \frac{48EI}{l^3}$$

$$I = \frac{kl^3}{48E} = \frac{2400(48)^3}{48(30)10^6} = 0.1843\ \text{in}^4$$

From $I = bh^3/12$

$$h = \sqrt[3]{\frac{12(0.1843)}{b}}$$

Form a table. First, Table A-17 gives likely available fractional sizes for b:

$$8\tfrac{1}{2},\ 9,\ 9\tfrac{1}{2},\ 10\ \text{in}$$

For h:

$$\frac{1}{2}, \frac{9}{16}, \frac{5}{8}, \frac{11}{16}, \frac{3}{4}$$

For available b what is necessary h for required I?

b	$\sqrt[3]{\frac{12(0.1843)}{b}}$	
8.5	0.638	
9.0	0.626 ←	choose 9" × $\frac{5"}{8}$ *Ans.*
9.5	0.615	
10.0	0.605	

(b)

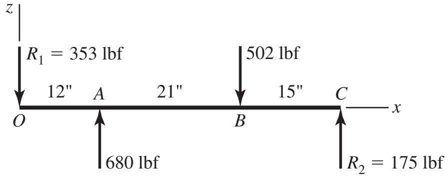
$$I = 9(0.625)^3/12 = 0.1831\ \text{in}^4$$

$$k = \frac{48EI}{l^3} = \frac{48(30)(10^6)(0.1831)}{48^3} = 2384\ \text{lbf/in}$$

$$F = \frac{4\sigma I}{cl} = \frac{4(90\,000)(0.1831)}{(0.625/2)(48)} = 4394\ \text{lbf}$$

$$y = \frac{F}{k} = \frac{4394}{2384} = 1.84\ \text{in} \quad \textit{Ans.}$$

4-20



$$\text{Torque} = (600 - 80)(9/2) = 2340\,\text{lbf}\cdot\text{in}$$

$$(T_2 - T_1)\frac{12}{2} = T_2(1 - 0.125)(6) = 2340$$

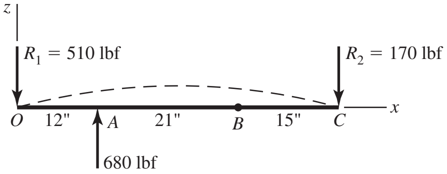
$$T_2 = \frac{2340}{6(0.875)} = 446\,\text{lbf}, \quad T_1 = 0.125(446) = 56\,\text{lbf}$$

$$\sum M_0 = 12(680) - 33(502) + 48R_2 = 0$$

$$R_2 = \frac{33(502) - 12(680)}{48} = 175\ \text{lbf}$$

$$R_1 = 680 - 502 + 175 = 353\ \text{lbf}$$

We will treat this as two separate problems and then sum the results.
First, consider the 680 lbf load as acting alone.



$$z_{OA} = -\frac{Fbx}{6EIl}(x^2 + b^2 - l^2); \quad \text{here } b = 36",$$

$$x = 12", \quad l = 48", \quad F = 680\ \text{lbf}$$

Also,

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.5)^4}{64} = 0.2485\ \text{in}^4$$

$$z_A = -\frac{680(36)(12)(144 + 1296 - 2304)}{6(30)(10^6)(0.2485)(48)}$$

$$= +0.1182\ \text{in}$$

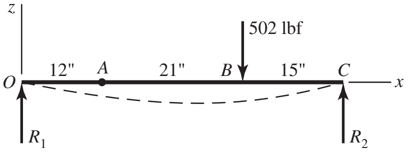
$$z_{AC} = -\frac{Fa(l - x)}{6EIl}(x^2 + a^2 - 2lx)$$

where $a = 12"$ and $x = 21 + 12 = 33"$

$$z_B = -\frac{680(12)(15)(1089 + 144 - 3168)}{6(30)(10^6)(0.2485)(48)}$$

$$= +0.1103\ \text{in}$$

Next, consider the 502 lbf load as acting alone.



$$z_{OB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2), \quad \text{where } b = 15\text{"},$$

$$x = 12\text{"}, \quad l = 48\text{"}, \quad I = 0.2485 \text{ in}^4$$

Then, $$z_A = \frac{502(15)(12)(144 + 225 - 2304)}{6(30)(10^6)(0.2485)(48)} = -0.081\,44 \text{ in}$$

For z_B use $x = 33$"

$$z_B = \frac{502(15)(33)(1089 + 225 - 2304)}{6(30)(10^6)(0.2485)(48)}$$
$$= -0.1146 \text{ in}$$

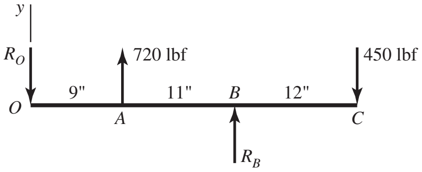
Therefore, by superposition

$$z_A = +0.1182 - 0.0814 = +0.0368 \text{ in} \quad \textit{Ans.}$$
$$z_B = +0.1103 - 0.1146 = -0.0043 \text{ in} \quad \textit{Ans.}$$

4-21

(a) Calculate torques and moment of inertia

$$T = (400 - 50)(16/2) = 2800 \text{ lbf} \cdot \text{in}$$
$$(8T_2 - T_2)(10/2) = 2800 \quad \Rightarrow \quad T_2 = 80 \text{ lbf}, \quad T_1 = 8(80) = 640 \text{ lbf}$$
$$I = \frac{\pi}{64}(1.25^4) = 0.1198 \text{ in}^4$$



Due to 720 lbf, flip beam A-9-6 such that $y_{AB} \rightarrow b = 9, x = 0, l = 20, F = -720$ lbf

$$\theta_B = \left.\frac{dy}{dx}\right|_{x=0} = -\frac{Fb}{6EIl}(3x^2 + b^2 - l^2)$$
$$= -\frac{-720(9)}{6(30)(10^6)(0.1198)(20)}(0 + 81 - 400) = -4.793(10^{-3}) \text{ rad}$$
$$y_C = -12\theta_B = -0.057\,52 \text{ in}$$

Due to 450 lbf, use beam A-9-10,

$$y_C = -\frac{Fa^2}{3EI}(l + a) = -\frac{450(144)(32)}{3(30)(10^6)(0.1198)} = -0.1923 \text{ in}$$

Adding the two deflections,

$$y_C = -0.057\,52 - 0.1923 = -0.2498 \text{ in} \quad \textit{Ans.}$$

(b) At O:

Due to 450 lbf:

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{Fa}{6EIl}(l^2 - 3x^2)\bigg|_{x=0} = \frac{Fal}{6EI}$$

$$\theta_O = -\frac{720(11)(0 + 11^2 - 400)}{6(30)(10^6)(0.1198)(20)} + \frac{450(12)(20)}{6(30)(10^6)(0.1198)} = 0.010\,13 \text{ rad} = 0.5805°$$

At B:

$$\theta_B = -4.793(10^{-3}) + \frac{450(12)}{6(30)(10^6)(0.1198)(20)}[20^2 - 3(20^2)]$$

$$= -0.014\,81 \text{ rad} = 0.8485°$$

$$I = 0.1198\left(\frac{0.8485°}{0.06°}\right) = 1.694 \text{ in}^4$$

$$d = \left(\frac{64I}{\pi}\right)^{1/4} = \left[\frac{64(1.694)}{\pi}\right]^{1/4} = 2.424 \text{ in}$$

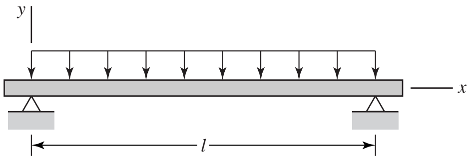
Use $d = 2.5$ in *Ans.*

$$I = \frac{\pi}{64}(2.5^4) = 1.917 \text{ in}^4$$

$$y_C = -0.2498\left(\frac{0.1198}{1.917}\right) = -0.015\,61 \text{ in} \quad \textit{Ans.}$$

4-22

(a) $l = 36(12) = 432$ in



$$y_{\max} = -\frac{5wl^4}{384EI} = -\frac{5(5000/12)(432)^4}{384(30)(10^6)(5450)}$$

$$= -1.16 \text{ in}$$

The frame is bowed up 1.16 in with respect to the bolsters. It is fabricated upside down and then inverted. *Ans.*

(b) The equation in *xy*-coordinates is for the center sill neutral surface

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3) \quad \textit{Ans.}$$

Differentiating this equation and solving for the slope at the left bolster gives

$$\frac{dy}{dx} = \frac{w}{24EI}(6lx^2 - 4x^3 - l^3)$$

Thus,
$$\left.\frac{dy}{dx}\right|_{x=0} = -\frac{wl^3}{24EI} = -\frac{(5000/12)(432)^3}{24(30)(10^6)(5450)}$$
$$= -0.008\,57$$

The slope at the right bolster is 0.008 57, so equation at left end is $y = -0.008\,57x$ and at the right end is $y = 0.008\,57(x - l)$. *Ans.*

4-23 From Table A-9-6,

$$y_L = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2)$$

$$y_L = \frac{Fb}{6EIl}(x^3 + b^2x - l^2x)$$

$$\frac{dy_L}{dx} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2)$$

$$\left.\frac{dy_L}{dx}\right|_{x=0} = \frac{Fb(b^2 - l^2)}{6EIl}$$

Let
$$\xi = \left|\frac{Fb(b^2 - l^2)}{6EIl}\right|$$

And set $I = \dfrac{\pi d_L^4}{64}$

And solve for d_L

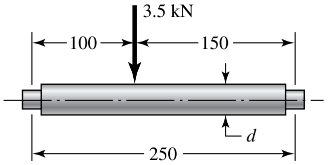
$$d_L = \left|\frac{32Fb(b^2 - l^2)}{3\pi El\xi}\right|^{1/4} \quad \textit{Ans.}$$

For the other end view, observe the figure of Table A-9-6 from the back of the page, noting that *a* and *b* interchange as do *x* and $-x$

$$d_R = \left|\frac{32Fa(l^2 - a^2)}{3\pi El\xi}\right|^{1/4} \quad \textit{Ans.}$$

For a uniform diameter shaft the necessary diameter is the larger of d_L and d_R.

4-24 Incorporating a design factor into the solution for d_L of Prob. 4-23,



$$d = \left[\frac{32n}{3\pi E l \xi} F b(l^2 - b^2)\right]^{1/4}$$

$$= \left|(\text{mm } 10^{-3})\frac{\text{kN mm}^3}{\text{GPa mm}}\frac{10^3(10^{-9})}{10^9(10^{-3})}\right|^{1/4}$$

$$d = \sqrt[4]{\frac{32(1.28)(3.5)(150)|(250^2 - 150^2)|}{3\pi(207)(250)(0.001)}10^{-12}}$$

$$= 36.4 \text{ mm} \quad \textit{Ans.}$$

4-25 The maximum occurs in the right section. Flip beam A-9-6 and use

$$y = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2) \quad \text{where } b = 100 \text{ mm}$$

$$\frac{dy}{dx} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2) = 0$$

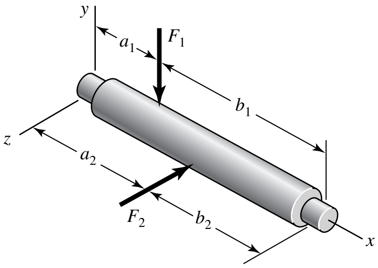
Solving for x,

$$x = \sqrt{\frac{l^2 - b^2}{3}} = \sqrt{\frac{250^2 - 100^2}{3}} = 132.29 \text{ mm} \quad \text{from right}$$

$$y = \frac{3.5(10^3)(0.1)(0.132\,29)}{6(207)(10^9)(\pi/64)(0.0364^4)(0.25)}[0.132\,29^2 + 0.1^2 - 0.25^2](10^3)$$

$$= -0.0606 \text{ mm} \quad \textit{Ans.}$$

4-26



The slope at $x = 0$ due to F_1 in the xy plane is

$$\theta_{xy} = \frac{F_1 b_1\left(b_1^2 - l^2\right)}{6EIl}$$

and in the xz plane due to F_2 is

$$\theta_{xz} = \frac{F_2 b_2\left(b_2^2 - l^2\right)}{6EIl}$$

For small angles, the slopes add as vectors. Thus

$$\theta_L = \left(\theta_{xy}^2 + \theta_{xz}^2\right)^{1/2}$$

$$= \left[\left(\frac{F_1 b_1\left(b_1^2 - l^2\right)}{6EIl}\right)^2 + \left(\frac{F_2 b_2\left(b_2^2 - l^2\right)}{6EIl}\right)^2\right]^{1/2}$$

Designating the slope constraint as ξ, we then have

$$\xi = |\theta_L| = \frac{1}{6EIl}\left\{\sum\left[F_i b_i\left(b_i^2 - l^2\right)\right]^2\right\}^{1/2}$$

Setting $I = \pi d^4/64$ and solving for d

$$d = \left|\frac{32}{3\pi El\xi}\left\{\sum\left[F_i b_i\left(b_i^2 - l^2\right)\right]^2\right\}^{1/2}\right|^{1/4}$$

For the LH bearing, $E = 30$ Mpsi, $\xi = 0.001$, $b_1 = 12$, $b_2 = 6$, and $l = 16$. The result is $d_L = 1.31$ in. Using a similar flip beam procedure, we get $d_R = 1.36$ in for the RH bearing. So use $d = 1\ 3/8$ in *Ans.*

4-27 $I = \dfrac{\pi}{64}(1.375^4) = 0.17546\ \text{in}^4$. For the xy plane, use y_{BC} of Table A-9-6

$$y = \frac{100(4)(16-8)}{6(30)(10^6)(0.17546)(16)}[8^2 + 4^2 - 2(16)8] = -1.115(10^{-3})\ \text{in}$$

For the xz plane use y_{AB}

$$z = \frac{300(6)(8)}{6(30)(10^6)(0.17546)(16)}[8^2 + 6^2 - 16^2] = -4.445(10^{-3})\ \text{in}$$

$$\delta = (-1.115\mathbf{j} - 4.445\mathbf{k})(10^{-3})\ \text{in}$$

$$|\delta| = 4.583(10^{-3})\ \text{in} \quad \textit{Ans.}$$

4-28

$$d_L = \left|\frac{32n}{3\pi El\xi}\left\{\sum\left[F_i b_i\left(b_i^2 - l^2\right)\right]^2\right\}^{1/2}\right|^{1/4}$$

$$= \left|\frac{32(1.5)}{3\pi(207)(10^9)(250)0.001}\left\{[3.5(150)(150^2 - 250^2)]^2 + [2.7(75)(75^2 - 250^2)]^2\right\}^{1/2}(10^3)^3\right|^{1/4}$$

$$= 39.2\ \text{mm}$$

$$d_R = \left|\frac{32(1.5)}{3\pi(207)10^9(250)0.001}\left\{[3.5(100)(100^2 - 250^2)]^2 + [2.7(175)(175^2 - 250^2)]^2\right\}^{1/2}(10^3)^3\right|^{1/4}$$

$$= 39.1\ \text{mm}$$

Choose $d \geq 39.2$ mm *Ans.*

4-29 From Table A-9-8 we have

$$y_L = \frac{M_B x}{6EIl}(x^2 + 3a^2 - 6al + 2l^2)$$

$$\frac{dy_L}{dx} = \frac{M_B}{6EIl}(3x^2 + 3a^2 - 6al + 2l^2)$$

At $x = 0$, the LH slope is

$$\theta_L = \frac{dy_L}{dx} = \frac{M_B}{6EIl}(3a^2 - 6al + 2l^2)$$

from which

$$\xi = |\theta_L| = \frac{M_B}{6EIl}(l^2 - 3b^2)$$

Setting $I = \pi d^4/64$ and solving for d

$$d = \left| \frac{32M_B(l^2 - 3b^2)}{3\pi El\xi} \right|^{1/4}$$

For a multiplicity of moments, the slopes add vectorially and

$$d_L = \left| \frac{32}{3\pi El\xi} \left\{ \sum \left[M_i \left(l^2 - 3b_i^2 \right) \right]^2 \right\}^{1/2} \right|^{1/4}$$

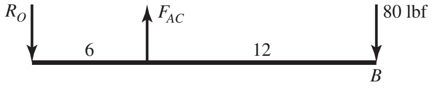
$$d_R = \left| \frac{32}{3\pi El\xi} \left\{ \sum \left[M_i \left(3a_i^2 - l^2 \right) \right]^2 \right\}^{1/2} \right|^{1/4}$$

The greatest slope is at the LH bearing. So

$$d = \left| \frac{32(1200)[9^2 - 3(4^2)]}{3\pi(30)(10^6)(9)(0.002)} \right|^{1/4} = 0.706 \text{ in}$$

So use $d = 3/4$ in *Ans.*

4-30



$$6F_{AC} = 18(80)$$

$$F_{AC} = 240 \text{ lbf}$$

$$R_O = 160 \text{ lbf}$$

$$I = \frac{1}{12}(0.25)(2^3) = 0.1667 \text{ in}^4$$

Initially, ignore the stretch of AC. From Table A-9-10

$$y_{B1} = -\frac{Fa^2}{3EI}(l + a) = -\frac{80(12^2)}{3(10)(10^6)(0.1667)}(6 + 12) = -0.041\,47 \text{ in}$$

Stretch of AC: $\delta = \left(\dfrac{FL}{AE}\right)_{AC} = \dfrac{240(12)}{(\pi/4)(1/2)^2(10)(10^6)} = 1.4668(10^{-3})$ in

Due to stretch of AC

$$y_{B2} = -3\delta = -4.400(10^{-3}) \text{ in}$$

By superposition, $y_B = -0.041\,47 - 0.0044 = -0.045\,87$ in *Ans.*

4-31

$$\theta = \frac{TL}{JG} = \frac{(0.1F)(1.5)}{(\pi/32)(0.012^4)(79.3)(10^9)} = 9.292(10^{-4})F$$

Due to twist

$$\delta_{B1} = 0.1(\theta) = 9.292(10^{-5})F$$

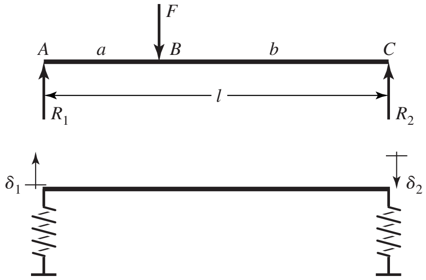
Due to bending

$$\delta_{B2} = \frac{FL^3}{3EI} = \frac{F(0.1^3)}{3(207)(10^9)(\pi/64)(0.012^4)} = 1.582(10^{-6})F$$

$$\delta_B = 1.582(10^{-6})F + 9.292(10^{-5})F = 9.450(10^{-5})F$$

$$k = \frac{1}{9.450(10^{-5})} = 10.58(10^3)\text{ N/m} = 10.58\text{ kN/m} \quad \textit{Ans.}$$

4-32



$$R_1 = \frac{Fb}{l} \qquad R_2 = \frac{Fa}{l}$$

$$\delta_1 = \frac{R_1}{k_1} \qquad \delta_2 = \frac{R_2}{k_2}$$

Spring deflection

$$y_S = -\delta_1 + \left(\frac{\delta_1 - \delta_2}{l}\right)x = -\frac{Fb}{k_1 l} + \left(\frac{Fb}{k_1 l^2} - \frac{Fa}{k_2 l^2}\right)x$$

$$y_{AB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2) + \frac{Fx}{l^2}\left(\frac{b}{k_1} - \frac{a}{k_2}\right) - \frac{Fb}{k_1 l} \quad \textit{Ans.}$$

$$y_{BC} = \frac{Fa(l-x)}{6EIl}(x^2 + a^2 - 2lx) + \frac{Fx}{l^2}\left(\frac{b}{k_1} - \frac{a}{k_2}\right) - \frac{Fb}{k_1 l} \quad \textit{Ans.}$$

4-33 See Prob. 4-32 for deflection due to springs. Replace Fb/l and Fa/l with $wl/2$

$$y_S = -\frac{wl}{2k_1} + \left(\frac{wl}{2k_1 l} - \frac{wl}{2k_2 l}\right)x = \frac{wx}{2}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) - \frac{wl}{2k_1}$$

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3) + \frac{wx}{2}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) - \frac{wl}{2k_1} \quad \textit{Ans.}$$

4-34 Let the load be at $x > l/2$. The maximum deflection will be in Section AB (Table A-9-10)

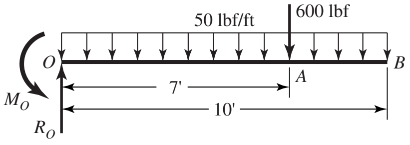
$$y_{AB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2)$$

$$\frac{dy_{AB}}{dx} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2) = 0 \quad \Rightarrow \quad 3x^2 + b^2 - l^2 = 0$$

$$x = \sqrt{\frac{l^2 - b^2}{3}}, \qquad x_{\max} = \sqrt{\frac{l^2}{3}} = 0.577l \quad Ans.$$

For $x < l/2 \quad x_{\min} = l - 0.577l = 0.423l \quad Ans.$

4-35



$$M_O = 50(10)(60) + 600(84)$$
$$= 80\,400 \text{ lbf} \cdot \text{in}$$
$$R_O = 50(10) + 600 = 1100 \text{ lbf}$$

$I = 11.12 \text{ in}^4$ from Prob. 4-12

$$M = -80\,400 + 1100x - \frac{4.167x^2}{2} - 600\langle x - 84\rangle^1$$

$$EI\frac{dy}{dx} = -80\,400x + 550x^2 - 0.6944x^3 - 300\langle x - 84\rangle^2 + C_1$$

$\dfrac{dy}{dx} = 0$ at $x = 0 \quad \therefore C_1 = 0$

$$EIy = -402\,00x^2 + 183.33x^3 - 0.1736x^4 - 100\langle x - 84\rangle^3 + C_2$$

$y = 0$ at $x = 0 \quad \therefore C_2 = 0$

$$y_B = \frac{1}{30(10^6)(11.12)}[-40\,200(120^2) + 183.33(120^3) - 0.1736(120^4) - 100(120 - 84)^3]$$
$$= -0.9075 \text{ in} \quad Ans.$$

4-36 See Prob. 4-13 for reactions: $R_O = 860$ lbf, $R_C = 540$ lbf

$$M = 860x - 800\langle x - 36\rangle^1 - 600\langle x - 60\rangle^1$$

$$EI\frac{dy}{dx} = 430x^2 - 400\langle x - 36\rangle^2 - 300\langle x - 60\rangle^2 + C_1$$

$$EIy = 143.33x^3 - 133.33\langle x - 36\rangle^3 - 100\langle x - 60\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$y = 0$ at $x = 120 \text{ in} \Rightarrow C_1 = -1.2254(10^6) \text{ lbf} \cdot \text{in}^2$

Substituting C_1 and C_2 and evaluating at $x = 60$,

$$EIy = 30(10^6)I\left(-\frac{1}{16}\right) = 143.33(60^3) - 133.33(60 - 36)^3 - 1.2254(10^6)(60)$$
$$I = 23.68 \text{ in}^4$$

Agrees with Prob. 4-13. The rest of the solution is the same.

4-37

$$I = \frac{\pi}{64}(40^4) = 125.66(10^3)\ \text{mm}^4$$

$$R_O = 2(500) + \frac{600}{1000}1500 = 1900\ \text{N}$$

$$M = 1900x - \frac{2000}{2}x^2 - 1500\langle x - 0.4\rangle^1 \quad \text{where } x \text{ is in meters}$$

$$EI\frac{dy}{dx} = 950x^2 - \frac{1000}{3}x^3 - 750\langle x - 0.4\rangle^2 + C_1$$

$$EIy = \frac{900}{3}x^3 - \frac{250}{3}x^4 - 250\langle x - 0.4\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$y = 0$ at $x = 1\ \text{m} \Rightarrow C_1 = -179.33\ \text{N}\cdot\text{m}^2$

Substituting C_1 and C_2 and evaluating y at $x = 0.4$ m,

$$y_A = \frac{1}{207(10^9)125.66(10^{-9})}\left[\frac{950}{3}(0.4^3) - \frac{250}{3}(0.4^4) - 179.33(0.4)\right]10^3$$

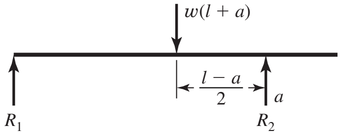
$$= -2.061\ \text{mm} \quad \textit{Ans.}$$

$$y|_{x=500} = \frac{1}{207(10^9)125.66(10^{-9})}\left[\frac{950}{3}(0.5^3) - \frac{250}{3}(0.5^4) - 250(0.5 - 0.4)^3 - 179.33(0.5)\right]10^3$$

$$= -2.135\ \text{mm} \quad \textit{Ans.}$$

$$\%\ \text{difference} = \frac{2.135 - 2.061}{2.061}(100) = 3.59\% \quad \textit{Ans.}$$

4-38



$$R_1 = \frac{w(l + a)[(l - a)/2)]}{l}$$

$$= \frac{w}{2l}(l^2 - a^2)$$

$$R_2 = w(l + a) - \frac{w}{2l}(l^2 - a^2) = \frac{w}{2l}(l + a)^2$$

$$M = \frac{w}{2l}(l^2 - a^2)x - \frac{wx^2}{2} + \frac{w}{2l}(l + a)^2\langle x - l\rangle^1$$

$$EI\frac{dy}{dx} = \frac{w}{4l}(l^2 - a^2)x^2 - \frac{w}{6}x^3 + \frac{w}{4l}(l + a)^2\langle x - l\rangle^2 + C_1$$

$$EIy = \frac{w}{12l}(l^2 - a^2)x^3 - \frac{w}{24}x^4 + \frac{w}{12l}(l + a)^2\langle x - l\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$y = 0$ at $x = l$

$$0 = \frac{w}{12l}(l^2 - a^2)l^3 - \frac{w}{24}l^4 + C_1 l \quad \Rightarrow \quad C_1 = \frac{wl}{24}(2a^2 - l^2)$$

$$y = \frac{w}{24EIl}[2(l^2 - a^2)x^3 - lx^4 + 2(l + a)^2\langle x - l\rangle^3 + l^2(2a^2 - l^2)x] \quad \textit{Ans.}$$

4-39 $R_A = R_B = 500\text{ N}$, and $I = \frac{1}{12}(9)35^3 = 32.156(10^3)\text{ mm}^4$

For first half of beam, $M = -500x + 500\langle x - 0.25\rangle^1$ where x is in meters

$$EI\frac{dy}{dx} = -250x^2 + 250\langle x - 0.25\rangle^2 + C_1$$

At $x = 0.5$ m, $dy/dx = 0 \Rightarrow 0 = -250(0.5^2) + 250(0.5 - 0.250)^2 + C_1 \Rightarrow C_1 = 46.875\text{ N}\cdot\text{m}^2$

$$EIy = -\frac{250}{3}x^3 + \frac{250}{3}\langle x - 0.25\rangle^3 + 46.875x + C_2$$

$$y = 0 \text{ at } x = 0.25\text{ m} \Rightarrow 0 = -\frac{250}{3}0.25^3 + 46.875(0.25) + C_2 \Rightarrow C_2 = -10.417\text{ N}\cdot\text{m}^3$$

$$\therefore EIy = -\frac{250}{3}x^3 + \frac{250}{3}\langle x - 0.25\rangle^3 + 46.875x - 10.42$$

Evaluating y at A and the center,

$$y_A = \frac{1}{207(10^9)32.156(10^{-9})}\left[-\frac{250}{3}(0^3) + \frac{250}{3}(0)^3 + 46.875(0) - 10.417\right]10^3$$

$$= -1.565\text{ mm} \quad \textit{Ans.}$$

$$y|_{x=0.5\text{m}} = \frac{1}{207(10^9)32.156(10^{-9})}\left[-\frac{250}{3}(0.5^3) + \frac{250}{3}(0.5 - 0.25)^3 + 46.875(0.5) - 10.417\right]10^3$$

$$= -2.135\text{ mm} \quad \textit{Ans.}$$

4-40 From Prob. 4-30, $R_O = 160$ lbf ↓, $F_{AC} = 240$ lbf $\quad I = 0.1667\text{ in}^4$

$$M = -160x + 240\langle x - 6\rangle^1$$

$$EI\frac{dy}{dx} = -80x^2 + 120\langle x - 6\rangle^2 + C_1$$

$$EIy = -26.67x^3 + 40\langle x - 6\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$$y_A = -\left(\frac{FL}{AE}\right)_{AC} = -\frac{240(12)}{(\pi/4)(1/2)^2(10)(10^6)} = -1.4668(10^{-3})\text{ in}$$

at $x = 6$

$$10(10^6)(0.1667)(-1.4668)(10^{-3}) = -26.67(6^3) + C_1(6)$$

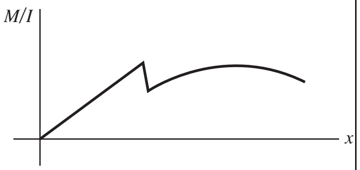
$$C_1 = 552.58\text{ lbf}\cdot\text{in}^2$$

$$y_B = \frac{1}{10(10^6)(0.1667)}[-26.67(18^3) + 40(18-6)^3 + 552.58(18)]$$

$$= -0.045\,87 \text{ in} \quad Ans.$$

4-41

$$I_1 = \frac{\pi}{64}(1.5^4) = 0.2485 \text{ in}^4 \qquad I_2 = \frac{\pi}{64}(2^4) = 0.7854 \text{ in}^4$$



$$R_1 = \frac{200}{2}(12) = 1200 \text{ lbf}$$

For $0 \le x \le 16$ in, $\quad M = 1200x - \frac{200}{2}\langle x-4\rangle^2$

$$\frac{M}{I} = \frac{1200x}{I_1} - 4800\left(\frac{1}{I_1} - \frac{1}{I_2}\right)\langle x-4\rangle^0 - 1200\left(\frac{1}{I_1} - \frac{1}{I_2}\right)\langle x-4\rangle^1 - \frac{100}{I_2}\langle x-4\rangle^2$$

$$= 4829x - 13\,204\langle x-4\rangle^0 - 3301.1\langle x-4\rangle^1 - 127.32\langle x-4\rangle^2$$

$$E\frac{dy}{dx} = 2414.5x^2 - 13\,204\langle x-4\rangle^1 - 1651\langle x-4\rangle^2 - 42.44\langle x-4\rangle^3 + C_1$$

Boundary Condition: $\frac{dy}{dx} = 0 \quad$ at $x = 10$ in

$$0 = 2414.5(10^2) - 13\,204(10-4)^1 - 1651(10-4)^2 - 42.44(10-4)^3 + C_1$$

$$C_1 = -9.362(10^4)$$

$$Ey = 804.83x^3 - 6602\langle x-4\rangle^2 - 550.3\langle x-4\rangle^3 - 10.61\langle x-4\rangle^4 - 9.362(10^4)x + C_2$$

$y = 0 \quad$ at $x = 0 \Rightarrow C_2 = 0$

For $0 \le x \le 16$ in

$$y = \frac{1}{30(10^6)}[804.83x^3 - 6602\langle x-4\rangle^2 - 550.3\langle x-4\rangle^3 - 10.61\langle x-4\rangle^4 - 9.362(10^4)x] \quad Ans.$$

at $x = 10$ in

$$y|_{x=10} = \frac{1}{30(10^6)}[804.83(10^3) - 6602(10-4)^2 - 550.3(10-4)^3 - 10.61(10-4)^4 - 9.362(10^4)(10)]$$

$$= -0.016\,72 \text{ in} \quad Ans.$$

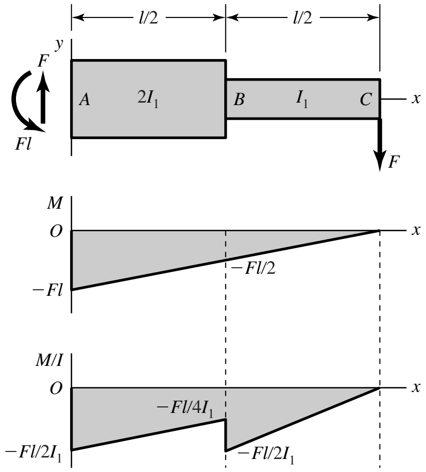
4-42 $\quad q = F\langle x\rangle^{-1} - Fl\langle x\rangle^{-2} - F\langle x-l\rangle^{-1}$

Integrations produce

$$V = F\langle x\rangle^0 - Fl\langle x\rangle^{-1} - F\langle x-l\rangle^0$$

$$M = F\langle x\rangle^1 - Fl\langle x\rangle^0 - F\langle x-l\rangle^1 = Fx - Fl$$

Plots for M and M/I are shown below



M/I can be expressed by singularity functions as

$$\frac{M}{I} = \frac{F}{2I_1}x - \frac{Fl}{2I_1} - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^0 + \frac{F}{2I_1}\left\langle x - \frac{l}{2}\right\rangle^1$$

where the step down and increase in slope at $x = l/2$ are given by the last two terms. Since $E\,d^2y/dx^2 = M/I$, two integrations yield

$$E\frac{dy}{dx} = \frac{F}{4I_1}x^2 - \frac{Fl}{2I_1}x - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^1 + \frac{F}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^2 + C_1$$

$$Ey = \frac{F}{12I_1}x^3 - \frac{Fl}{4I_1}x^2 - \frac{Fl}{8I_1}\left\langle x - \frac{l}{2}\right\rangle^2 + \frac{F}{12I_1}\left\langle x - \frac{l}{2}\right\rangle^3 + C_1x + C_2$$

At $x = 0$, $y = dy/dx = 0$. This gives $C_1 = C_2 = 0$, and

$$y = \frac{F}{24EI_1}\left(2x^3 - 6lx^2 - 3l\left\langle x - \frac{l}{2}\right\rangle^2 + 2\left\langle x - \frac{l}{2}\right\rangle^3\right)$$

At $x = l/2$ and l,

$$y|_{x=l/2} = \frac{F}{24EI_1}\left[2\left(\frac{l}{2}\right)^3 - 6l\left(\frac{l}{2}\right)^2 - 3l(0) + 2(0)\right] = -\frac{5Fl^3}{96EI_1} \qquad \textit{Ans.}$$

$$y|_{x=l} = \frac{F}{24EI_1}\left[2(l)^3 - 6l(l)^2 - 3l\left(l - \frac{l}{2}\right)^2 + 2\left(l - \frac{l}{2}\right)^3\right] = -\frac{3Fl^3}{16EI_1} \qquad \textit{Ans.}$$

The answers are identical to Ex. 4-11.

4-43 Define δ_{ij} as the deflection in the direction of the load at station i due to a unit load at station j. If U is the potential energy of strain for a body obeying Hooke's law, apply P_1 first. Then

$$U = \frac{1}{2}P_1(P_1\delta_{11})$$

When the second load is added, U becomes

$$U = \frac{1}{2}P_1(P_1\delta_{11}) + \frac{1}{2}P_2(P_2\,\delta_{22}) + P_1(P_2\,\delta_{12})$$

For loading in the reverse order

$$U' = \frac{1}{2}P_2(P_2\,\delta_{22}) + \frac{1}{2}P_1(P_1\delta_{11}) + P_2(P_1\,\delta_{21})$$

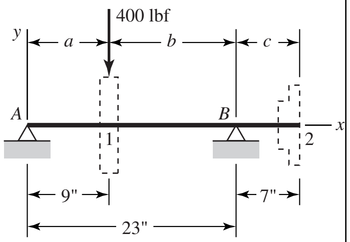
Since the order of loading is immaterial $U = U'$ and

$$P_1P_2\delta_{12} = P_2P_1\delta_{21} \qquad \text{when } P_1 = P_2,\ \delta_{12} = \delta_{21}$$

which states that the deflection at station 1 due to a unit load at station 2 is the same as the deflection at station 2 due to a unit load at 1. δ is sometimes called an *influence coefficient*.

4-44

(a) From Table A-9-10



$$y_{AB} = \frac{Fcx(l^2 - x^2)}{6EIl}$$

$$\delta_{12} = \frac{y}{F}\bigg|_{x=a} = \frac{ca(l^2 - a^2)}{6EIl}$$

$$y_2 = F\delta_{21} = F\delta_{12} = \frac{Fca(l^2 - a^2)}{6EIl}$$

Substituting $I = \dfrac{\pi d^4}{64}$

$$y_2 = \frac{400(7)(9)(23^2 - 9^2)(64)}{6(30)(10^6)(\pi)(2)^4(23)} = 0.00347 \text{ in} \quad \textit{Ans.}$$

(b) The slope of the shaft at *left* bearing at $x = 0$ is

$$\theta = \frac{Fb(b^2 - l^2)}{6EIl}$$

Viewing the illustration in Section 6 of Table A-9 from the back of the page provides the correct view of this problem. Noting that a is to be interchanged with b and $-x$ with x leads to

$$\theta = \frac{Fa(l^2 - a^2)}{6EIl} = \frac{Fa(l^2 - a^2)(64)}{6E\pi d^4 l}$$

$$\theta = \frac{400(9)(23^2 - 9^2)(64)}{6(30)(10^6)(\pi)(2)^4(23)} = 0.000\,496 \text{ in/in}$$

So $y_2 = 7\theta = 7(0.000\,496) = 0.00347$ in *Ans.*

4-45 Place a dummy load Q at the center. Then,

$$M = \frac{wx}{2}(l - x) + \frac{Qx}{2}$$

$$U = 2\int_0^{l/2} \frac{M^2 dx}{2EI}, \quad y_{\max} = \left.\frac{\partial U}{\partial Q}\right|_{Q=0}$$

$$y_{\max} = 2\left[\int_0^{l/2} \frac{2M}{2EI}\left(\frac{\partial M}{\partial Q}\right) dx\right]_{Q=0}$$

$$y_{\max} = \frac{2}{EI}\left\{\int_0^{l/2}\left[\frac{wx}{2}(l - x) + \frac{Qx}{2}\right]\frac{x}{2}\,dx\right\}_{Q=0}$$

Set $Q = 0$ and integrate

$$y_{\max} = \frac{w}{2EI}\left(\frac{lx^3}{3} - \frac{x^4}{4}\right)_0^{l/2}$$

$$y_{\max} = \frac{5wl^4}{384EI} \quad Ans.$$

4-46

$$I = 2(1.85) = 3.7 \text{ in}^4$$

Adding weight of channels of 0.833 lbf · in,

$$M = -Fx - \frac{10.833}{2}x^2 = -Fx - 5.417x^2 \qquad \frac{\partial M}{\partial F} = -x$$

$$\delta_B = \frac{1}{EI}\int_0^{48} M\frac{\partial M}{\partial F}\,dx = \frac{1}{EI}\int_0^{48}(Fx + 5.417x^2)(x)\,dx$$

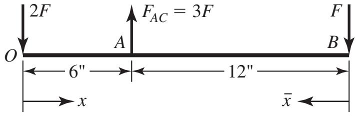
$$= \frac{(220/3)(48^3) + (5.417/4)(48^4)}{30(10^6)(3.7)} = 0.1378 \text{ in} \qquad \text{in direction of 220 lbf}$$

$$\therefore y_B = -0.1378 \text{ in} \quad Ans.$$

4-47

$$I_{OB} = \frac{1}{12}(0.25)(2^3) = 0.1667 \text{ in}^4, \quad A_{AC} = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.196\,35 \text{ in}^2$$

$$F_{AC} = 3F, \quad \frac{\partial F_{AC}}{\partial F} = 3$$



right left

$$M = -F\bar{x} \quad M = -2Fx$$

$$\frac{\partial M}{\partial F} = -\bar{x} \quad \frac{\partial M}{\partial F} = -2x$$

$$U = \frac{1}{2EI}\int_0^l M^2 dx + \frac{F_{AC}^2 L_{AC}}{2A_{AC}E}$$

$$\delta_B = \frac{\partial U}{\partial F} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial F}\,dx + \frac{F_{AC}(\partial F_{AC}/\partial F)L_{AC}}{A_{AC}E}$$

$$= \frac{1}{EI}\left[\int_0^{12} -F\bar{x}(-\bar{x})\,d\bar{x} + \int_0^6 (-2Fx)(-2x)\,dx\right] + \frac{3F(3)(12)}{A_{AC}E}$$

$$= \frac{1}{EI}\left[\frac{F}{3}(12^3) + 4F\left(\frac{6^3}{3}\right)\right] + \frac{108F}{A_{AC}E}$$

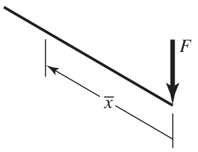
$$= \frac{864F}{EI} + \frac{108F}{A_{AC}E}$$

$$= \frac{864(80)}{10(10^6)(0.1667)} + \frac{108(80)}{0.196\,35(10)(10^6)} = 0.045\,86 \text{ in} \quad \textit{Ans.}$$

4-48

Torsion $\quad T = 0.1F \qquad \dfrac{\partial T}{\partial F} = 0.1$

Bending $\quad M = -F\bar{x} \qquad \dfrac{\partial M}{\partial F} = -\bar{x}$



$$U = \frac{1}{2EI}\int M^2\,dx + \frac{T^2L}{2JG}$$

$$\delta_B = \frac{\partial U}{\partial F} = \frac{1}{EI}\int M\frac{\partial M}{\partial F}\,dx + \frac{T(\partial T/\partial F)L}{JG}$$

$$= \frac{1}{EI}\int_0^{0.1} -F\bar{x}(-\bar{x})\,d\bar{x} + \frac{0.1F(0.1)(1.5)}{JG}$$

$$= \frac{F}{3EI}(0.1^3) + \frac{0.015F}{JG}$$

Where

$$I = \frac{\pi}{64}(0.012)^4 = 1.0179(10^{-9})\text{ m}^4$$

$$J = 2I = 2.0358(10^{-9})\text{ m}^4$$

$$\delta_B = F\left[\frac{0.001}{3(207)(10^9)(1.0179)(10^{-9})} + \frac{0.015}{2.0358(10^{-9})(79.3)(10^9)}\right] = 9.45(10^{-5})F$$

$$k = \frac{1}{9.45(10^{-5})} = 10.58(10^3)\text{ N/m} = 10.58\text{ kN/m} \quad \textit{Ans.}$$

4-49 From Prob. 4-41, $I_1 = 0.2485\text{ in}^4$, $I_2 = 0.7854\text{ in}^4$

For a dummy load ↑ Q at the center

$$0 \le x \le 10\text{ in} \qquad M = 1200x - \frac{Q}{2}x - \frac{200}{2}\langle x-4\rangle^2, \qquad \frac{\partial M}{\partial Q} = \frac{-x}{2}$$

$$y|_{x=10} = \left.\frac{\partial U}{\partial Q}\right|_{Q=0}$$

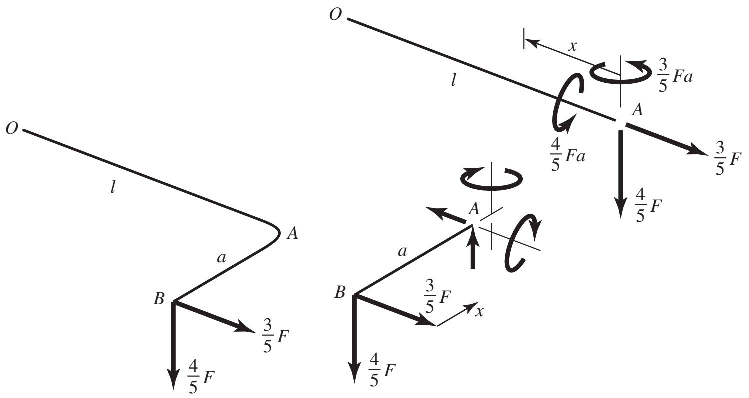
$$= \frac{2}{E}\left\{\frac{1}{I_1}\int_0^4 (1200x)\left(-\frac{x}{2}\right)dx + \frac{1}{I_2}\int_4^{10}[1200x - 100(x-4)^2]\left(-\frac{x}{2}\right)dx\right\}$$

$$= \frac{2}{E}\left[-\frac{200(4^3)}{I_1} - \frac{1.566(10^5)}{I_2}\right]$$

$$= -\frac{2}{30(10^6)}\left(\frac{1.28(10^4)}{0.2485} + \frac{1.566(10^5)}{0.7854}\right)$$

$$= -0.016\,73\text{ in} \quad \textit{Ans.}$$

4-50



AB

$$M = Fx \quad \frac{\partial M}{\partial F} = x$$

OA

$$N = \frac{3}{5}F \qquad \frac{\partial N}{\partial F} = \frac{3}{5}$$

$$T = \frac{4}{5}Fa \qquad \frac{\partial T}{\partial F} = \frac{4}{5}a$$

$$M_1 = \frac{4}{5}F\bar{x} \quad \frac{\partial M_1}{\partial F} = \frac{4}{5}\bar{x}$$

$$M_2 = \frac{3}{5}Fa \quad \frac{\partial M_2}{\partial F} = \frac{3}{5}a$$

$$\delta_B = \frac{\partial u}{\partial F} = \frac{1}{EI}\int_0^a Fx(x)\,dx + \frac{(3/5)F(3/5)l}{AE} + \frac{(4/5)Fa(4a/5)l}{JG}$$

$$+\frac{1}{EI}\int_0^l \frac{4}{5}F\bar{x}\left(\frac{4}{5}\bar{x}\right)d\bar{x} + \frac{1}{EI}\int_0^l \frac{3}{5}Fa\left(\frac{3}{5}a\right)d\bar{x}$$

$$= \frac{Fa^3}{3EI} + \frac{9}{25}\left(\frac{Fl}{AE}\right) + \frac{16}{25}\left(\frac{Fa^2 l}{JG}\right) + \frac{16}{75}\left(\frac{Fl^3}{EI}\right) + \frac{9}{25}\left(\frac{Fa^2 l}{EI}\right)$$

$$I = \frac{\pi}{64}d^4, \quad J = 2I, \quad A = \frac{\pi}{4}d^2$$

$$\delta_B = \frac{64Fa^3}{3E\pi d^4} + \frac{9}{25}\left(\frac{4Fl}{\pi d^2 E}\right) + \frac{16}{25}\left(\frac{32Fa^2 l}{\pi d^4 G}\right) + \frac{16}{75}\left(\frac{64Fl^3}{E\pi d^4}\right) + \frac{9}{25}\left(\frac{64Fa^2 l}{E\pi d^4}\right)$$

$$= \frac{4F}{75\pi Ed^4}\left(400a^3 + 27ld^2 + 384a^2 l\frac{E}{G} + 256l^3 + 432a^2 l\right) \quad \textit{Ans.}$$

4-51 The force applied to the copper and steel wire assembly is $F_c + F_s = 250$ lbf

Since $\delta_c = \delta_s$

$$\frac{F_c L}{3(\pi/4)(0.0801)^2(17.2)(10^6)} = \frac{F_s L}{(\pi/4)(0.0625)^2(30)(10^6)}$$

$$F_c = 2.825F_s$$

$$\therefore 3.825F_s = 250 \quad \Rightarrow \quad F_s = 65.36 \text{ lbf}, \quad F_c = 2.825F_s = 184.64 \text{ lbf}$$

$$\sigma_c = \frac{184.64}{3(\pi/4)(0.0801)^2} = 12\,200 \text{ psi} = 12.2 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_s = \frac{65.36}{(\pi/4)(0.0625^2)} = 21\,300 \text{ psi} = 21.3 \text{ kpsi} \quad \textit{Ans.}$$

4-52

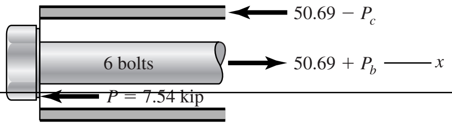
(a) Bolt stress $\sigma_b = 0.9(85) = 76.5$ kpsi *Ans.*

Bolt force $F_b = 6(76.5)\left(\frac{\pi}{4}\right)(0.375^2) = 50.69$ kips

Cylinder stress $\sigma_c = -\frac{F_b}{A_c} = -\frac{50.69}{(\pi/4)(4.5^2 - 4^2)} = -15.19$ kpsi *Ans.*

(b) Force from pressure

$$P = \frac{\pi D^2}{4}p = \frac{\pi(4^2)}{4}(600) = 7540 \text{ lbf} = 7.54 \text{ kip}$$



$$\sum F_x = 0$$

$$P_b + P_c = 7.54 \quad (1)$$

Since $\delta_c = \delta_b$, $$\frac{P_c L}{(\pi/4)(4.5^2 - 4^2)E} = \frac{P_b L}{6(\pi/4)(0.375^2)E}$$

$$P_c = 5.037 P_b \quad (2)$$

Substituting into Eq. (1)

$6.037 P_b = 7.54 \quad \Rightarrow \quad P_b = 1.249$ kip; and from Eq. (2), $P_c = 6.291$ kip

Using the results of (a) above, the total bolt and cylinder stresses are

$$\sigma_b = 76.5 + \frac{1.249}{6(\pi/4)(0.375^2)} = 78.4 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_c = -15.19 + \frac{6.291}{(\pi/4)(4.5^2 - 4^2)} = -13.3 \text{ kpsi} \quad \textit{Ans.}$$

4-53

$$T = T_c + T_s \quad \text{and} \quad \theta_c = \theta_s$$

Also, $$\frac{T_c L}{(GJ)_c} = \frac{T_s L}{(GJ)_s}$$

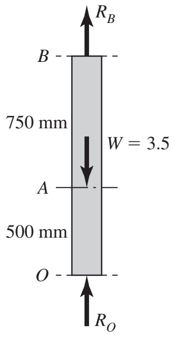
$$T_c = \frac{(GJ)_c}{(GJ)_s} T_s$$

Substituting into equation for T,

$$T = \left[1 + \frac{(GJ)_c}{(GJ)_s}\right] T_s$$

$$\%T_s = \frac{T_s}{T} = \frac{(GJ)_s}{(GJ)_s + (GJ)_c} \quad \textit{Ans.}$$

4-54



$$R_O + R_B = W \quad (1)$$

$$\delta_{OA} = \delta_{AB} \quad (2)$$

$$\frac{500 R_O}{AE} = \frac{750 R_B}{AE}, \quad R_O = \frac{3}{2} R_B$$

$$\frac{3}{2} R_B + R_B = 3.5$$

$$R_B = \frac{7}{5} = 1.4 \text{ kN} \quad \textit{Ans.}$$

$$R_O = 3.5 - 1.4 = 2.1 \text{ kN} \quad \textit{Ans.}$$

$$\sigma_O = -\frac{2100}{12(50)} = -3.50 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_B = \frac{1400}{12(50)} = 2.33 \text{ MPa} \quad \textit{Ans.}$$

4-55 Since $\theta_{OA} = \theta_{AB}$

$$\frac{T_{OA}(4)}{GJ} = \frac{T_{AB}(6)}{GJ}, \quad T_{OA} = \frac{3}{2}T_{AB}$$

Also $T_{OA} + T_{AB} = 50$

$$T_{AB}\left(\frac{3}{2}+1\right) = 50, \quad T_{AB} = \frac{50}{2.5} = 20\,\text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_{OA} = \frac{3}{2}T_{AB} = \frac{3}{2}(20) = 30\,\text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

4-56 Since $\theta_{OA} = \theta_{AB}$,

$$\frac{T_{OA}(4)}{G(\frac{\pi}{32}1.5^4)} = \frac{T_{AB}(6)}{G(\frac{\pi}{32}1.75^4)}, \quad T_{OA} = 0.80966T_{AB}$$

$$T_{OA} + T_{AB} = 50 \;\Rightarrow\; 0.80966\,T_{AB} + T_{AB} = 50 \;\Rightarrow\; T_{AB} = 27.63\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_{OA} = 0.80966T_{AB} = 0.80966(27.63) = 22.37\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

4-57

$$F_1 = F_2 \quad\Rightarrow\quad \frac{T_1}{r_1} = \frac{T_2}{r_2} \quad\Rightarrow\quad \frac{T_1}{1.25} = \frac{T_2}{3}$$

$$T_2 = \frac{3}{1.25}T_1$$

$$\therefore \theta_1 + \frac{3}{1.25}\theta_2 = \frac{4\pi}{180}\ \text{rad}$$

$$\frac{T_1(48)}{(\pi/32)(7/8)^4(11.5)(10^6)} + \frac{3}{1.25}\left[\frac{(3/1.25)T_1(48)}{(\pi/32)(1.25)^4(11.5)(10^6)}\right] = \frac{4\pi}{180}$$

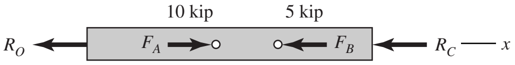
$$T_1 = 403.9\ \text{lbf}\cdot\text{in}$$

$$T_2 = \frac{3}{1.25}T_1 = 969.4\ \text{lbf}\cdot\text{in}$$

$$\tau_1 = \frac{16T_1}{\pi d^3} = \frac{16(403.9)}{\pi(7/8)^3} = 3071\ \text{psi} \quad \textit{Ans.}$$

$$\tau_2 = \frac{16(969.4)}{\pi(1.25)^3} = 2528\ \text{psi} \quad \textit{Ans.}$$

4-58



(1) Arbitrarily, choose R_C as redundant reaction

(2) $$\sum F_x = 0, \quad 10(10^3) - 5(10^3) - R_O - R_C = 0$$

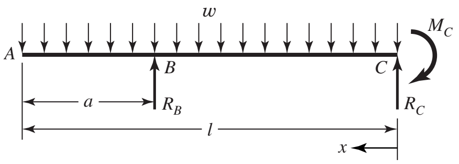
$$R_O + R_C = 5(10^3) \text{ lbf}$$

(3) $$\delta_C = \frac{[10(10^3) - 5(10^3) - R_C]20}{AE} - \frac{[5(10^3) + R_C]}{AE}(10) - \frac{R_C(15)}{AE} = 0$$

$$-45R_C + 5(10^4) = 0 \quad \Rightarrow \quad R_C = 1111 \text{ lbf} \quad \textit{Ans.}$$

$$R_O = 5000 - 1111 = 3889 \text{ lbf} \quad \textit{Ans.}$$

4-59



(1) Choose R_B as redundant reaction

(2) $$R_B + R_C = wl \quad (a) \qquad R_B(l-a) - \frac{wl^2}{2} + M_C = 0 \quad (b)$$

(3) $$y_B = \frac{R_B(l-a)^3}{3EI} + \frac{w(l-a)^2}{24EI}[4l(l-a) - (l-a)^2 - 6l^2] = 0$$

$$R_B = \frac{w}{8(l-a)}[6l^2 - 4l(l-a) + (l-a)^2]$$

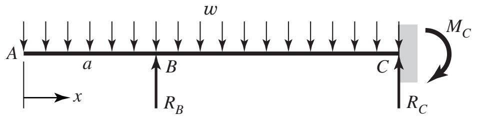
$$= \frac{w}{8(l-a)}(3l^2 + 2al + a^2) \quad \textit{Ans.}$$

Substituting,

Eq. (*a*) $$R_C = wl - R_B = \frac{w}{8(l-a)}(5l^2 - 10al - a^2) \quad \textit{Ans.}$$

Eq. (*b*) $$M_C = \frac{wl^2}{2} - R_B(l-a) = \frac{w}{8}(l^2 - 2al - a^2) \quad \textit{Ans.}$$

4-60



$$M = -\frac{wx^2}{2} + R_B\langle x-a\rangle^1, \qquad \frac{\partial M}{\partial R_B} = \langle x-a\rangle^1$$

$$\frac{\partial U}{\partial R_B} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial R_B}\,dx$$

$$= \frac{1}{EI}\int_0^a \frac{-wx^2}{2}(0)\,dx + \frac{1}{EI}\int_a^l \left[\frac{-wx^2}{2} + R_B(x-a)\right](x-a)\,dx = 0$$

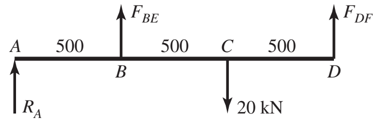
$$-\frac{w}{2}\left[\frac{1}{4}(l^4 - a^4) - \frac{a}{3}(l^3 - a^3)\right] + \frac{R_B}{3}\left[(l-a)^3 - (a-a)^3\right] = 0$$

$$R_B = \frac{w}{(l-a)^3}[3(L^4 - a^4) - 4a(l^3 - a^3)] = \frac{w}{8(l-a)}(3l^2 + 2al + a^2) \quad Ans.$$

$$R_C = wl - R_B = \frac{w}{8(l-a)}(5l^2 - 10al - a^2) \quad Ans.$$

$$M_C = \frac{wl^2}{2} - R_B(l-a) = \frac{w}{8}(l^2 - 2al - a^2) \quad Ans.$$

4-61



$$A = \frac{\pi}{4}(0.012^2) = 1.131(10^{-4})\text{ m}^2$$

(1)

$$R_A + F_{BE} + F_{DF} = 20\text{ kN} \qquad (a)$$

$$\sum M_A = 3F_{DF} - 2(20) + F_{BE} = 0$$

$$F_{BE} + 3F_{DF} = 40\text{ kN} \qquad (b)$$

(2)

$$M = R_A x + F_{BE}\langle x - 0.5\rangle^1 - 20(10^3)\langle x - 1\rangle^1$$

$$EI\frac{dy}{dx} = R_A\frac{x^2}{2} + \frac{F_{BE}}{2}\langle x - 0.5\rangle^2 - 10(10^3)\langle x - 1\rangle^2 + C_1$$

$$EIy = R_A\frac{x^3}{6} + \frac{F_{BE}}{6}\langle x - 0.5\rangle^3 - \frac{10}{3}(10^3)\langle x - 1\rangle^3 + C_1 x + C_2$$

(3) $y = 0$ at $x = 0$ $\quad \therefore C_2 = 0$

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(1)}{1.131(10^{-4})209(10^9)} = -4.2305(10^{-8})F_{BE}$$

Substituting and evaluating at $x = 0.5$ m

$$EIy_B = 209(10^9)(8)(10^{-7})(-4.2305)(10^{-8})F_{BE} = R_A\frac{0.5^3}{6} + C_1(0.5)$$

$$2.0833(10^{-2})R_A + 7.0734(10^{-3})F_{BE} + 0.5C_1 = 0 \qquad (c)$$

$$y_D = -\left(\frac{Fl}{AE}\right)_{DF} = -\frac{F_{DF}(1)}{1.131(10^{-4})(209)(10^9)} = -4.2305(10^{-8})F_{DF}$$

Substituting and evaluating at $x = 1.5$ m

$$EIy_D = -7.0734(10^{-3})F_{DF} = R_A\frac{1.5^3}{6} + \frac{F_{BE}}{6}(1.5 - 0.5)^3 - \frac{10}{3}(10^3)(1.5 - 1)^3 + 1.5C_1$$

$$0.5625R_A + 0.16667F_{BE} + 7.0734(10^{-3})F_{DF} + 1.5C_1 = 416.67 \qquad (d)$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 2.0833(10^{-2}) & 7.0734(10^{-3}) & 0 & 0.5 \\ 0.5625 & 0.16667 & 7.0734(10^{-3}) & 1.5 \end{bmatrix} \begin{Bmatrix} R_A \\ F_{BE} \\ F_{DF} \\ C_1 \end{Bmatrix} = \begin{Bmatrix} 20\,000 \\ 40\,000 \\ 0 \\ 416.67 \end{Bmatrix}$$

Solve simultaneously or use software

$$R_A = -3885\text{ N},\quad F_{BE} = 15\,830\text{ N},\quad F_{DF} = 8058\text{ N},\quad C_1 = -62.045\text{ N}\cdot\text{m}^2$$

$$\sigma_{BE} = \frac{15\,830}{(\pi/4)(12^2)} = 140\text{ MPa}\quad \textit{Ans.},\qquad \sigma_{DF} = \frac{8058}{(\pi/4)(12^2)} = 71.2\text{ MPa}\quad \textit{Ans.}$$

$$EI = 209(10^9)(8)(10^{-7}) = 167.2(10^3)\text{ N}\cdot\text{m}^2$$

$$y = \frac{1}{167.2(10^3)}\left[-\frac{3885}{6}x^3 + \frac{15\,830}{6}\langle x-0.5\rangle^3 - \frac{10}{3}(10^3)\langle x-1\rangle^3 - 62.045x\right]$$

$$B\text{: } x = 0.5\text{ m},\quad y_B = -6.70(10^{-4})\text{ m} = -0.670\text{ mm}\quad \textit{Ans.}$$

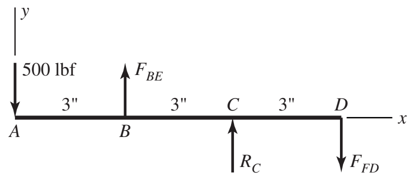
$$C\text{: } x = 1\text{ m},\quad y_C = \frac{1}{167.2(10^3)}\left[-\frac{3885}{6}(1^3) + \frac{15\,830}{6}(1-0.5)^3 - 62.045(1)\right]$$

$$= -2.27(10^{-3})\text{ m} = -2.27\text{ mm}\quad \textit{Ans.}$$

$$D\text{: } x = 1.5,\quad y_D = \frac{1}{167.2(10^3)}\left[-\frac{3885}{6}(1.5^3) + \frac{15\,830}{6}(1.5-0.5)^3 - \frac{10}{3}(10^3)(1.5-1)^3 - 62.045(1.5)\right]$$

$$= -3.39(10^{-4})\text{ m} = -0.339\text{ mm}\quad \textit{Ans.}$$

4-62



$$EI = 30(10^6)(0.050) = 1.5(10^6)\text{ lbf}\cdot\text{in}^2$$

(1)

$$R_C + F_{BE} - F_{FD} = 500 \tag{a}$$

$$3R_C + 6F_{BE} = 9(500) = 4500 \tag{b}$$

(2)

$$M = -500x + F_{BE}\langle x-3\rangle^1 + R_C\langle x-6\rangle^1$$

$$EI\frac{dy}{dx} = -250x^2 + \frac{F_{BE}}{2}\langle x-3\rangle^2 + \frac{R_C}{2}\langle x-6\rangle^2 + C_1$$

$$EIy = -\frac{250}{3}x^3 + \frac{F_{BE}}{6}\langle x-3\rangle^3 + \frac{R_C}{6}\langle x-6\rangle^3 + C_1x + C_2$$

$$y_B = \left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(2)}{(\pi/4)(5/16)^2(30)(10^6)} = -8.692(10^{-7})F_{BE}$$

Substituting and evaluating at $x = 3$ in

$$EIy_B = 1.5(10^6)[-8.692(10^{-7})F_{BE}] = -\frac{250}{3}(3^3) + 3C_1 + C_2$$

$$1.3038F_{BE} + 3C_1 + C_2 = 2250 \tag{c}$$

Since $y = 0$ at $x = 6$ in

$$EIy|_{=0} = -\frac{250}{3}(6^3) + \frac{F_{BE}}{6}(6-3)^3 + 6C_1 + C_2$$

$$4.5F_{BE} + 6C_1 + C_2 = 1.8(10^4) \qquad (d)$$

$$y_D = \left(\frac{Fl}{AE}\right)_{DF} = \frac{F_{DF}(2.5)}{(\pi/4)(5/16)^2(30)(10^6)} = 1.0865(10^{-6})F_{DF}$$

Substituting and evaluating at $x = 9$ in

$$EIy_D = 1.5(10^6)[1.0865(10^{-6})F_{DF}] = -\frac{250}{3}(9^3) + \frac{F_{BE}}{6}(9-3)^3 + \frac{R_C}{6}(9-6)^3 + 9C_1 + C_2$$

$$4.5R_C + 36F_{BE} - 1.6297F_{DF} + 9C_1 + C_2 = 6.075(10^4) \qquad (e)$$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 3 & 6 & 0 & 0 & 0 \\ 0 & 1.3038 & 0 & 3 & 1 \\ 0 & 4.5 & 0 & 6 & 1 \\ 4.5 & 36 & -1.6297 & 9 & 1 \end{bmatrix} \begin{Bmatrix} R_C \\ F_{BE} \\ F_{DF} \\ C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 500 \\ 4500 \\ 2250 \\ 1.8(10^4) \\ 6.075(10^4) \end{Bmatrix}$$

$$R_C = -590.4 \text{ lbf}, \quad F_{BE} = 1045.2 \text{ lbf}, \quad F_{DF} = -45.2 \text{ lbf}$$

$$C_1 = 4136.4 \text{ lbf} \cdot \text{in}^2, \quad C_2 = -11\,522 \text{ lbf} \cdot \text{in}^3$$

$$\sigma_{BE} = \frac{1045.2}{(\pi/4)(5/16)^2} = 13\,627 \text{ psi} = 13.6 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_{DF} = -\frac{45.2}{(\pi/4)(5/16)^2} = -589 \text{ psi} \quad \textit{Ans.}$$

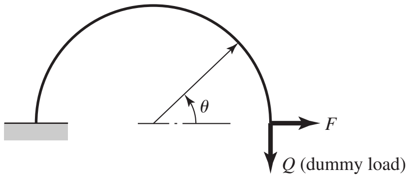
$$y_A = \frac{1}{1.5(10^6)}(-11\,522) = -0.007\,68 \text{ in} \quad \textit{Ans.}$$

$$y_B = \frac{1}{1.5(10^6)}\left[-\frac{250}{3}(3^3) + 4136.4(3) - 11\,522\right] = -0.000\,909 \text{ in} \quad \textit{Ans.}$$

$$y_D = \frac{1}{1.5(10^6)}\left[-\frac{250}{3}(9^3) + \frac{1045.2}{6}(9-3)^3 + \frac{-590.4}{6}(9-6)^3 + 4136.4(9) - 11\,522\right]$$

$$= -4.93(10^{-5}) \text{ in} \quad \textit{Ans.}$$

4-63



$$M = -PR\sin\theta + QR(1-\cos\theta) \qquad \frac{\partial M}{\partial Q} = R(1-\cos\theta)$$

$$\delta_Q = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \frac{1}{EI}\int_0^{\pi}(-PR\sin\theta)R(1-\cos\theta)R\,d\theta = -2\frac{PR^3}{EI}$$

Deflection is upward and equals $2(PR^3/EI)$ *Ans.*

4-64 Equation (4-28) becomes

$$U = 2\int_0^{\pi}\frac{M^2R\,d\theta}{2EI} \qquad R/h > 10$$

where $M = FR(1-\cos\theta)$ and $\dfrac{\partial M}{\partial F} = R(1-\cos\theta)$

$$\delta = \frac{\partial U}{\partial F} = \frac{2}{EI}\int_0^{\pi} M\frac{\partial M}{\partial F}R\,d\theta$$

$$= \frac{2}{EI}\int_0^{\pi} FR^3(1-\cos\theta)^2\,d\theta$$

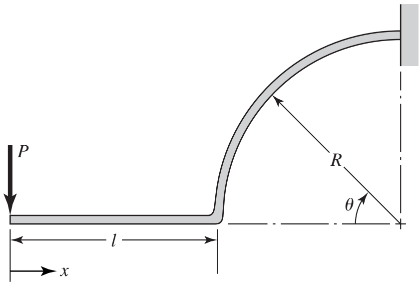
$$= \frac{3\pi FR^3}{EI}$$

Since $I = bh^3/12 = 4(6)^3/12 = 72\text{ mm}^4$ and $R = 81/2 = 40.5$ mm, we have

$$\delta = \frac{3\pi(40.5)^3F}{131(72)} = 66.4F\text{ mm} \quad \textit{Ans.}$$

where F is in kN.

4-65



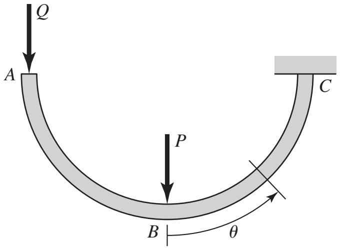
$$M = -Px, \quad \frac{\partial M}{\partial P} = -x \quad 0 \le x \le l$$

$$M = Pl + PR(1-\cos\theta), \quad \frac{\partial M}{\partial P} = l + R(1-\cos\theta) \quad 0 \le \theta \le l$$

$$\delta_P = \frac{1}{EI}\left\{\int_0^{l} -Px(-x)\,dx + \int_0^{\pi/2} P[l + R(1-\cos\theta)]^2R\,d\theta\right\}$$

$$= \frac{P}{12EI}\{4l^3 + 3R[2\pi l^2 + 4(\pi-2)lR + (3\pi-8)R^2]\} \quad \textit{Ans.}$$

4-66 *A*: Dummy load Q is applied at A. Bending in AB due only to Q which is zero.



$$M = PR\sin\theta + QR(1+\sin\theta), \quad \frac{\partial M}{\partial Q} = R(1+\sin\theta), \quad 0 \le \theta \le \frac{\pi}{2}$$

$$(\delta_A)_V = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \frac{1}{EI}\int_0^{\pi/2}(PR\sin\theta)[R(1+\sin\theta)]R\,d\theta$$

$$= \frac{PR^3}{EI}\left(-\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\Bigg|_0^{\pi/2} = \frac{PR^3}{EI}\left(1+\frac{\pi}{4}\right)$$

$$= \frac{\pi+4}{4}\,\frac{PR^3}{EI} \quad \textit{Ans.}$$

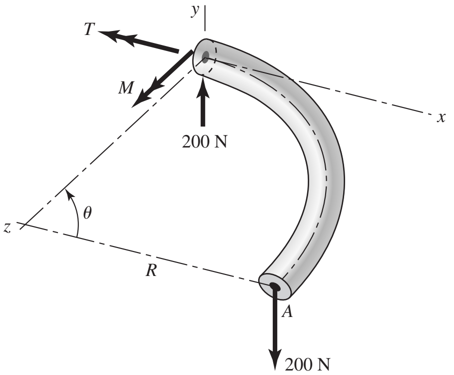
B:

$$M = PR\sin\theta, \quad \frac{\partial M}{\partial P} = R\sin\theta$$

$$(\delta_B)_V = \frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^{\pi/2}(PR\sin\theta)(R\sin\theta)R\,d\theta$$

$$= \frac{\pi}{4}\,\frac{PR^3}{EI} \quad \textit{Ans.}$$

4-67



$$M = PR\sin\theta, \qquad \frac{\partial M}{\partial P} = R\sin\theta \quad 0 < \theta < \frac{\pi}{2}$$

$$T = PR(1-\cos\theta), \quad \frac{\partial T}{\partial P} = R(1-\cos\theta)$$

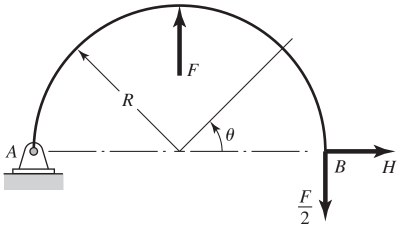
$$(\delta_A)_y = -\frac{\partial U}{\partial P} = -\left\{\frac{1}{EI}\int_0^{\pi/2} P(R\sin\theta)^2 R\,d\theta + \frac{1}{GJ}\int_0^{\pi/2} P[R(1-\cos\theta)]^2 R\,d\theta\right\}$$

Integrating and substituting $J = 2I$ and $G = E/[2(1+\nu)]$

$$(\delta_A)_y = -\frac{PR^3}{EI}\left[\frac{\pi}{4} + (1+\nu)\left(\frac{3\pi}{4} - 2\right)\right] = -[4\pi - 8 + (3\pi - 8)\nu]\frac{PR^3}{4EI}$$

$$= -[4\pi - 8 + (3\pi - 8)(0.29)]\frac{(200)(100)^3}{4(200)(10^3)(\pi/64)(5)^4} = -40.6 \text{ mm}$$

4-68 Consider the horizontal reaction, to be applied at B, subject to the constraint $(\delta_B)_H = 0$.



(a) $(\delta_B)_H = \dfrac{\partial U}{\partial H} = 0$

Due to symmetry, consider half of the structure. F does not deflect horizontally.

$$M = \frac{FR}{2}(1 - \cos\theta) - HR\sin\theta, \qquad \frac{\partial M}{\partial H} = -R\sin\theta, \quad 0 < \theta < \frac{\pi}{2}$$

$$\frac{\partial U}{\partial H} = \frac{1}{EI}\int_0^{\pi/2}\left[\frac{FR}{2}(1-\cos\theta) - HR\sin\theta\right](-R\sin\theta)R\,d\theta = 0$$

$$-\frac{F}{2} + \frac{F}{4} + H\frac{\pi}{4} = 0 \quad \Rightarrow \quad H = \frac{F}{\pi} \quad \textit{Ans.}$$

Reaction at A is the same where H goes to the left

(b) For $0 < \theta < \dfrac{\pi}{2}$, $\quad M = \dfrac{FR}{2}(1 - \cos\theta) - \dfrac{FR}{\pi}\sin\theta$

$$M = \frac{FR}{2\pi}[\pi(1 - \cos\theta) - 2\sin\theta] \quad \textit{Ans.}$$

Due to symmetry, the solution for the left side is identical.

(c)

$$\frac{\partial M}{\partial F} = \frac{R}{2\pi}[\pi(1 - \cos\theta) - 2\sin\theta]$$

$$\delta_F = \frac{\partial U}{\partial F} = \frac{2}{EI}\int_0^{\pi/2}\frac{FR^2}{4\pi^2}[\pi(1-\cos\theta) - 2\sin\theta]^2 R\,d\theta$$

$$= \frac{FR^3}{2\pi^2 EI}\int_0^{\pi/2}(\pi^2 + \pi^2\cos^2\theta + 4\sin^2\theta - 2\pi^2\cos\theta - 4\pi\sin\theta + 4\pi\sin\theta\cos\theta)\,d\theta$$

$$= \frac{FR^3}{2\pi^2 EI}\left[\pi^2\left(\frac{\pi}{2}\right) + \pi^2\left(\frac{\pi}{4}\right) + 4\left(\frac{\pi}{4}\right) - 2\pi^2 - 4\pi + 2\pi\right]$$

$$= \frac{(3\pi^2 - 8\pi - 4)}{8\pi}\,\frac{FR^3}{EI} \quad \textit{Ans.}$$

4-69 Must use Eq. (4-33)

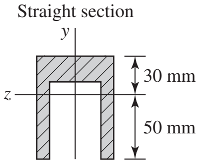
$$A = 80(60) - 40(60) = 2400 \text{ mm}^2$$

$$R = \frac{(25+40)(80)(60) - (25+20+30)(40)(60)}{2400} = 55 \text{ mm}$$

Section is equivalent to the "T" section of Table 3-4

$$r_n = \frac{60(20) + 20(60)}{60\ln[(25+20)/25] + 20\ln[(80+25)/(25+20)]} = 45.9654 \text{ mm}$$

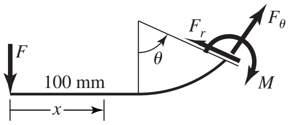
$$e = R - r_n = 9.035 \text{ mm}$$



$$I_z = \frac{1}{12}(60)(20^3) + 60(20)(30-10)^2$$

$$+ 2\left[\frac{1}{12}(10)(60^3) + 10(60)(50-30)^2\right]$$

$$= 1.36(10^6) \text{ mm}^4$$



For $0 \le x \le 100$ mm

$$M = -Fx, \quad \frac{\partial M}{\partial F} = -x; \quad V = F, \quad \frac{\partial V}{\partial F} = 1$$

For $\theta \le \pi/2$

$$F_r = F\cos\theta, \quad \frac{\partial F_r}{\partial F} = \cos\theta; \quad F_\theta = F\sin\theta, \quad \frac{\partial F_\theta}{\partial F} = \sin\theta$$

$$M = F(100 + 55\sin\theta), \quad \frac{\partial M}{\partial F} = (100 + 55\sin\theta)$$

Use Eq. (5-34), integrate from 0 to $\pi/2$, double the results and add straight part

$$\delta = \frac{2}{E}\left\{\frac{1}{I}\int_0^{100} Fx^2 dx + \int_0^{100}\frac{(1)F(1)\,dx}{2400(G/E)} + \int_0^{\pi/2} F\frac{(100+55\sin\theta)^2}{2400(9.035)}d\theta\right.$$

$$+ \int_0^{\pi/2}\frac{F\sin^2\theta(55)}{2400}d\theta - \int_0^{\pi/2}\frac{F(100+55\sin\theta)}{2400}\sin\theta d\theta$$

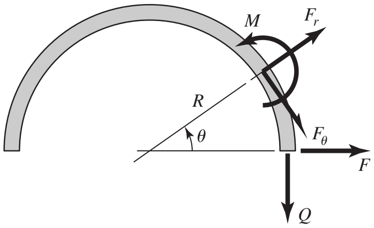
$$\left. - \int_0^{\pi/2}\frac{F\sin\theta(100+55\sin\theta)}{2400}d\theta + \int_0^{\pi/2}\frac{(1)F\cos^2\theta(55)}{2400(G/E)}d\theta\right\}$$

Substitute
$I = 1.36(10^3)$ mm^2, $F = 30(10^3)$ N, $E = 207(10^3)$ N/mm^2, $G = 79(10^3)$ N/mm^2

$$\delta = \frac{2}{207(10^3)}30(10^3)\left\{\frac{100^3}{3(1.36)(10^6)} + \frac{207}{79}\left(\frac{100}{2400}\right) + \frac{2.908(10^4)}{2400(9.035)} + \frac{55}{2400}\left(\frac{\pi}{4}\right)\right.$$

$$\left. - \frac{2}{2400}(143.197) + \frac{207}{79}\left(\frac{55}{2400}\right)\left(\frac{\pi}{4}\right)\right\} = 0.476 \text{ mm} \quad \textit{Ans.}$$

4-70



$$M = FR\sin\theta - QR(1-\cos\theta), \quad \frac{\partial M}{\partial Q} = -R(1-\cos\theta)$$

$$F_\theta = Q\cos\theta + F\sin\theta, \quad \frac{\partial F_\theta}{\partial Q} = \cos\theta$$

$$\frac{\partial}{\partial Q}(MF_\theta) = [FR\sin\theta - QR(1-\cos\theta)]\cos\theta + [-R(1-\cos\theta)][Q\cos\theta + F\sin\theta]$$

$$F_r = F\cos\theta - Q\sin\theta, \quad \frac{\partial F_r}{\partial Q} = -\sin\theta$$

From Eq. (4-33)

$$\delta = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \frac{1}{AeE}\int_0^\pi (FR\sin\theta)[-R(1-\cos\theta)]\,d\theta + \frac{R}{AE}\int_0^\pi F\sin\theta\cos\theta d\theta$$

$$- \frac{1}{AE}\int_0^\pi [FR\sin\theta\cos\theta - FR\sin\theta(1-\cos\theta)]\,d\theta$$

$$+ \frac{CR}{AG}\int_0^\pi -F\cos\theta\sin\theta d\theta$$

$$= -\frac{2FR^2}{AeE} + 0 + \frac{2FR}{AE} + 0 = -\left(\frac{R}{e} - 1\right)\frac{2FR}{AE} \quad \textit{Ans.}$$

4-71 The cross section at *A* does not rotate, thus for a single quadrant we have

$$\frac{\partial U}{\partial M_A} = 0$$

The bending moment at an angle θ to the x axis is

$$M = M_A - \frac{F}{2}(R - x) = M_A - \frac{FR}{2}(1-\cos\theta) \tag{1}$$

because $x = R\cos\theta$. Next,

$$U = \int \frac{M^2}{2EI}ds = \int_0^{\pi/2} \frac{M^2}{2EI}R\,d\theta$$

since $ds = R\,d\theta$. Then

$$\frac{\partial U}{\partial M_A} = \frac{R}{EI}\int_0^{\pi/2} M\frac{\partial M}{\partial M_A}\,d\theta = 0$$

But $\partial M/\partial M_A = 1$. Therefore

$$\int_0^{\pi/2} M\,d\theta = \int_0^{\pi/2} \left[M_A - \frac{FR}{2}(1-\cos\theta)\right] d\theta = 0$$

Since this term is zero, we have

$$M_A = \frac{FR}{2}\left(1 - \frac{2}{\pi}\right)$$

Substituting into Eq. (1)

$$M = \frac{FR}{2}\left(\cos\theta - \frac{2}{\pi}\right)$$

The maximum occurs at B where $\theta = \pi/2$. It is

$$M_B = -\frac{FR}{\pi} \quad \textit{Ans.}$$

4-72 For one quadrant

$$M = \frac{FR}{2}\left(\cos\theta - \frac{2}{\pi}\right); \quad \frac{\partial M}{\partial F} = \frac{R}{2}\left(\cos\theta - \frac{2}{\pi}\right)$$

$$\delta = \frac{\partial U}{\partial F} = 4\int_0^{\pi/2} \frac{M}{EI}\frac{\partial M}{\partial F} R\,d\theta$$

$$= \frac{FR^3}{EI}\int_0^{\pi/2}\left(\cos\theta - \frac{2}{\pi}\right)^2 d\theta$$

$$= \frac{FR^3}{EI}\left(\frac{\pi}{4} - \frac{2}{\pi}\right) \quad \textit{Ans.}$$

4-73

$$P_{\text{cr}} = \frac{C\pi^2 EI}{l^2}$$

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi D^4}{64}(1 - K^4)$$

$$P_{\text{cr}} = \frac{C\pi^2 E}{l^2}\left[\frac{\pi D^4}{64}(1 - K^4)\right]$$

$$D = \left[\frac{64 P_{\text{cr}} l^2}{\pi^3 CE(1 - K^4)}\right]^{1/4} \quad \textit{Ans.}$$

4-74

$$A = \frac{\pi}{4}D^2(1-K^2), \quad I = \frac{\pi}{64}D^4(1-K^4) = \frac{\pi}{64}D^4(1-K^2)(1+K^2),$$

$$k^2 = \frac{I}{A} = \frac{D^2}{16}(1+K^2)$$

From Eq. (4-43)

$$\frac{P_{\text{cr}}}{(\pi/4)D^2(1-K^2)} = S_y - \frac{S_y^2 l^2}{4\pi^2 k^2 CE} = S_y - \frac{S_y^2 l^2}{4\pi^2(D^2/16)(1+K^2)CE}$$

$$4P_{\text{cr}} = \pi D^2(1-K^2)S_y - \frac{4S_y^2 l^2 \pi D^2(1-K^2)}{\pi^2 D^2(1+K^2)CE}$$

$$\pi D^2(1-K^2)S_y = 4P_{\text{cr}} + \frac{4S_y^2 l^2(1-K^2)}{\pi(1+K^2)CE}$$

$$D = \left[\frac{4P_{\text{cr}}}{\pi S_y(1-K^2)} + \frac{4S_y^2 l^2(1-K^2)}{\pi(1+K^2)CE\pi(1-K^2)S_y}\right]^{1/2}$$

$$= 2\left[\frac{P_{\text{cr}}}{\pi S_y(1-K^2)} + \frac{S_y l^2}{\pi^2 CE(1+K^2)}\right]^{1/2} \quad \textit{Ans.}$$

4-75 **(a)**

$$\overset{+}{\curvearrowleft}\sum M_A = 0, \qquad 2.5(180) - \frac{3}{\sqrt{3^2+1.75^2}}F_{BO}(1.75) = 0 \quad \Rightarrow \quad F_{BO} = 297.7 \text{ lbf}$$

Using $n_d = 5$, design for $F_{\text{cr}} = n_d F_{BO} = 5(297.7) = 1488$ lbf, $l = \sqrt{3^2+1.75^2} =$ 3.473 ft, $S_y = 24$ kpsi

In plane: $k = 0.2887h = 0.2887"$, $C = 1.0$

Try 1" × 1/2" section

$$\frac{l}{k} = \frac{3.473(12)}{0.2887} = 144.4$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2(1)(30)(10^6)}{24(10^3)}\right)^{1/2} = 157.1$$

Since $(l/k)_1 > (l/k)$ use Johnson formula

$$P_{\text{cr}} = (1)\left(\frac{1}{2}\right)\left[24(10^3) - \left(\frac{24(10^3)}{2\pi}144.4\right)^2\left(\frac{1}{1(30)(10^6)}\right)\right] = 6930 \text{ lbf}$$

Try 1" × 1/4": $P_{\text{cr}} = 3465$ lbf

Out of plane: $k = 0.2887(0.5) = 0.1444$ in, $C = 1.2$

$$\frac{l}{k} = \frac{3.473(12)}{0.1444} = 289$$

Since $(l/k)_1 < (l/k)$ use Euler equation

$$P_{\text{cr}} = 1(0.5)\frac{1.2(\pi^2)(30)(10^6)}{289^2} = 2127 \text{ lbf}$$

1/4" increases l/k by 2, $\left(\frac{l}{k}\right)^2$ by 4, and A by 1/2

Try 1" × 3/8": $k = 0.2887(0.375) = 0.1083$ in

$$\frac{l}{k} = 385, \quad P_{\text{cr}} = 1(0.375)\frac{1.2(\pi^2)(30)(10^6)}{385^2} = 899 \text{ lbf} \quad \text{(too low)}$$

Use 1" × 1/2" *Ans.*

(b) $\sigma_b = -\frac{P}{\pi dl} = -\frac{298}{\pi(0.5)(0.5)} = -379$ psi *No,* bearing stress is not significant.

4-76 This is a design problem with no one distinct solution.

4-77

$$F = 800\left(\frac{\pi}{4}\right)(3^2) = 5655 \text{ lbf}, \quad S_y = 37.5 \text{ kpsi}$$

$$P_{\text{cr}} = n_d F = 3(5655) = 17\,000 \text{ lbf}$$

(a) Assume Euler with $C = 1$

$$I = \frac{\pi}{64}d^4 = \frac{P_{\text{cr}} l^2}{C\pi^2 E} \quad \Rightarrow \quad d = \left[\frac{64 P_{\text{cr}} l^2}{\pi^3 C E}\right]^{1/4} = \left[\frac{64(17)(10^3)(60^2)}{\pi^3(1)(30)(10^6)}\right]^{1/4} = 1.433 \text{ in}$$

Use $d = 1.5$ in; $k = d/4 = 0.375$

$$\frac{l}{k} = \frac{60}{0.375} = 160$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2(1)(30)(10^6)}{37.5(10^3)}\right)^{1/2} = 126 \quad \therefore \text{ use Euler}$$

$$P_{\text{cr}} = \frac{\pi^2(30)(10^6)(\pi/64)(1.5^4)}{60^2} = 20\,440 \text{ lbf}$$

$d = 1.5$ in is satisfactory. *Ans.*

(b) $d = \left[\frac{64(17)(10^3)(18^2)}{\pi^3(1)(30)(10^6)}\right]^{1/4} = 0.785$ in, so use 0.875 in

$$k = \frac{0.875}{4} = 0.2188 \text{ in}$$

$$l/k = \frac{18}{0.2188} = 82.3 \quad \text{try Johnson}$$

$$P_{\text{cr}} = \frac{\pi}{4}(0.875^2)\left[37.5(10^3) - \left(\frac{37.5(10^3)}{2\pi}82.3\right)^2 \frac{1}{1(30)(10^6)}\right] = 17\,714 \text{ lbf}$$

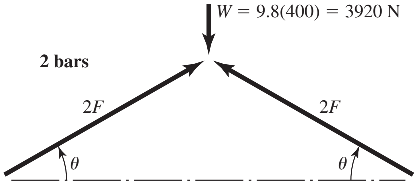
Use $d = 0.875$ in *Ans.*

(c)

$$n_{(a)} = \frac{20\,440}{5655} = 3.61 \quad \textit{Ans.}$$

$$n_{(b)} = \frac{17\,714}{5655} = 3.13 \quad \textit{Ans.}$$

4-78



$$4F\sin\theta = 3920$$

$$F = \frac{3920}{4\sin\theta}$$

In range of operation, F is maximum when $\theta = 15^\circ$

$$F_{\max} = \frac{3920}{4\sin 15} = 3786 \text{ N per bar}$$

$$P_{\text{cr}} = n_d F_{\max} = 2.5(3786) = 9465 \text{ N}$$

$l = 300$ mm, $h = 25$ mm

Try $b = 5$ mm: out of plane $k = (5/\sqrt{12}) = 1.443$ mm

$$\frac{l}{k} = \frac{300}{1.443} = 207.8$$

$$\left(\frac{l}{k}\right)_1 = \left[\frac{(2\pi^2)(1.4)(207)(10^9)}{380(10^6)}\right]^{1/2} = 123 \qquad \therefore \text{use Euler}$$

$$P_{\text{cr}} = (25)(5)\frac{(1.4\pi^2)(207)(10^3)}{(207.8)^2} = 8280 \text{ N}$$

Try: 5.5 mm: $k = 5.5/\sqrt{12} = 1.588$ mm

$$\frac{l}{k} = \frac{300}{1.588} = 189$$

$$P_{\text{cr}} = 25(5.5)\frac{(1.4\pi^2)(207)(10^3)}{189^2} = 11\,010 \text{ N}$$

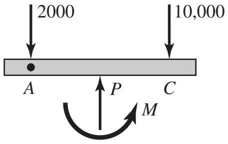
Use 25 × 5.5 mm bars *Ans.* The factor of safety is thus

$$n = \frac{11\,010}{3786} = 2.91 \quad \textit{Ans.}$$

4-79

$$\sum F = 0 = 2000 + 10\,000 - P \quad \Rightarrow \quad P = 12\,000 \text{ lbf} \quad \textit{Ans.}$$

$$\sum M_A = 12\,000\left(\frac{5.68}{2}\right) - 10\,000(5.68) + M = 0$$



$$M = 22\,720 \text{ lbf} \cdot \text{in}$$

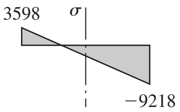
$$e = \frac{M}{P} = \frac{22}{12}\left(\frac{720}{000}\right) = 1.893 \text{ in} \quad \textit{Ans.}$$

From Table A-8, $A = 4.271 \text{ in}^2$, $I = 7.090 \text{ in}^4$

$$k^2 = \frac{I}{A} = \frac{7.090}{4.271} = 1.66 \text{ in}^2$$

$$\sigma_c = -\frac{12\,000}{4.271}\left[1 + \frac{1.893(2)}{1.66}\right] = -9218 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_t = -\frac{12\,000}{4.271}\left[1 - \frac{1.893(2)}{1.66}\right] = 3598 \text{ psi}$$

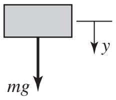


4-80 This is a design problem so the solutions will differ.

4-81 For free fall with $y \le h$

$$\sum F_y - m\ddot{y} = 0$$

$$mg - m\ddot{y} = 0, \quad \text{so } \ddot{y} = g$$

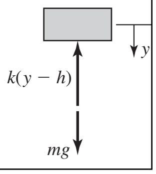


Using $y = a + bt + ct^2$, we have at $t = 0$, $y = 0$, and $\dot{y} = 0$, and so $a = 0$, $b = 0$, and $c = g/2$. Thus

$$y = \frac{1}{2}gt^2 \quad \text{and} \quad \dot{y} = gt \quad \text{for } y \le h$$

At impact, $y = h$, $t = (2h/g)^{1/2}$, and $v_0 = (2gh)^{1/2}$

After contact, the differential equatioin (D.E.) is



$$mg - k(y - h) - m\ddot{y} = 0 \qquad \text{for } y > h$$

Now let $x = y - h$; then $\dot{x} = \dot{y}$ and $\ddot{x} = \ddot{y}$. So the D.E. is $\ddot{x} + (k/m)x = g$ with solution $\omega = (k/m)^{1/2}$ and

$$x = A\cos\omega t' + B\sin\omega t' + \frac{mg}{k}$$

At contact, $t' = 0$, $x = 0$, and $\dot{x} = v_0$. Evaluating A and B then yields

$$x = -\frac{mg}{k}\cos\omega t' + \frac{v_0}{\omega}\sin\omega t' + \frac{mg}{k}$$

or

$$y = -\frac{W}{k}\cos\omega t' + \frac{v_0}{\omega}\sin\omega t' + \frac{W}{k} + h$$

and

$$\dot{y} = \frac{W\omega}{k}\sin\omega t' + v_0\cos\omega t'$$

To find $y_{\max}$ set $\dot{y} = 0$. Solving gives

$$\tan\omega t' = -\frac{v_0 k}{W\omega}$$

or

$$(\omega t')^* = \tan^{-1}\left(-\frac{v_0 k}{W\omega}\right)$$

The first value of $(\omega t')^*$ is a minimum and negative. So add π radians to it to find the maximum.

Numerical example: $h = 1$ in, $W = 30$ lbf, $k = 100$ lbf/in. Then

$$\omega = (k/m)^{1/2} = [100(386)/30]^{1/2} = 35.87 \text{ rad/s}$$

$$W/k = 30/100 = 0.3$$

$$v_0 = (2gh)^{1/2} = [2(386)(1)]^{1/2} = 27.78 \text{ in/s}$$

Then

$$y = -0.3\cos 35.87t' + \frac{27.78}{35.87}\sin 35.87t' + 0.3 + 1$$

For $y_{\max}$

$$\tan\omega t' = -\frac{v_0 k}{W\omega} = -\frac{27.78(100)}{30(35.87)} = -2.58$$

$$(\omega t')^* = -1.20 \text{ rad (minimum)}$$

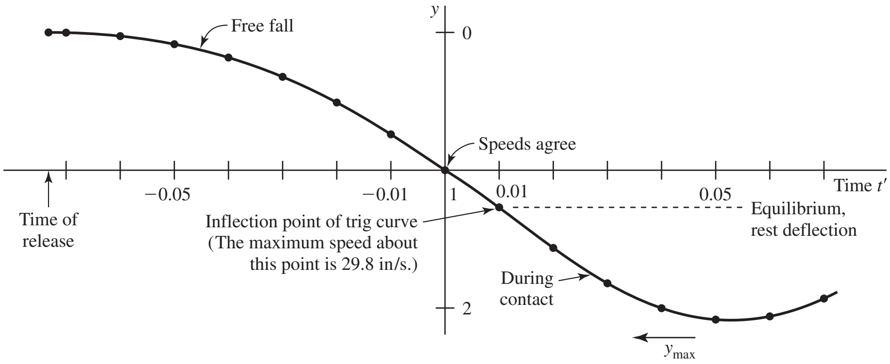
$$(\omega t')^* = -1.20 + \pi = 1.940 \text{ (maximum)}$$

Then $t'^* = 1.940/35.87 = 0.0541$ s. This means that the spring bottoms out at t'^* seconds. Then $(\omega t')^* = 35.87(0.0541) = 1.94$ rad

So $$y_{\max} = -0.3\cos 1.94 + \frac{27.78}{35.87}\sin 1.94 + 0.3 + 1 = 2.130 \text{ in} \quad \textit{Ans.}$$

The maximum spring force is $F_{\max} = k(y_{\max} - h) = 100(2.130 - 1) = 113$ lbf *Ans.*

The action is illustrated by the graph below. *Applications:* Impact, such as a dropped package or a pogo stick with a passive rider. The idea has also been used for a one-legged robotic walking machine.

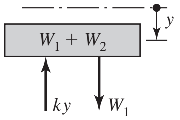


4-82 Choose $t' = 0$ at the instant of impact. At this instant, $v_1 = (2gh)^{1/2}$. Using momentum, $m_1 v_1 = m_2 v_2$. Thus

$$\frac{W_1}{g}(2gh)^{1/2} = \frac{W_1 + W_2}{g} v_2$$

$$v_2 = \frac{W_1 (2gh)^{1/2}}{W_1 + W_2}$$

Therefore at $t' = 0$, $y = 0$, and $\dot{y} = v_2$



Let $W = W_1 + W_2$

Because the spring force at $y = 0$ includes a reaction to W_2, the D.E. is

$$\frac{W}{g}\ddot{y} = -ky + W_1$$

With $\omega = (kg/W)^{1/2}$ the solution is

$$y = A\cos\omega t' + B\sin\omega t' + W_1/k$$

$$\dot{y} = -A\omega\sin\omega t' + B\omega\cos\omega t'$$

At $t' = 0$, $y = 0 \Rightarrow A = -W_1/k$

At $t' = 0$, $\dot{y} = v_2 \Rightarrow v_2 = B\omega$

Then

$$B = \frac{v_2}{\omega} = \frac{W_1(2gh)^{1/2}}{(W_1 + W_2)[kg/(W_1 + W_2)]^{1/2}}$$

We now have

$$y = -\frac{W_1}{k}\cos\omega t' + W_1\left[\frac{2h}{k(W_1 + W_2)}\right]^{1/2}\sin\omega t' + \frac{W_1}{k}$$

Transforming gives

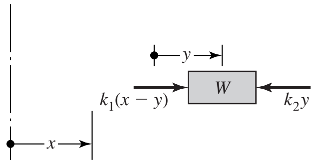
$$y = \frac{W_1}{k}\left(\frac{2hk}{W_1 + W_2} + 1\right)^{1/2}\cos(\omega t' - \phi) + \frac{W_1}{k}$$

where ϕ is a phase angle. The maximum deflection of W_2 and the maximum spring force are thus

$$y_{\max} = \frac{W_1}{k}\left(\frac{2hk}{W_1 + W_2} + 1\right)^{1/2} + \frac{W_1}{k} \quad Ans.$$

$$F_{\max} = ky_{\max} + W_2 = W_1\left(\frac{2hk}{W_1 + W_2} + 1\right)^{1/2} + W_1 + W_2 \quad Ans.$$

4-83 Assume $x > y$ to get a free-body diagram.



Then

$$\frac{W}{g}\ddot{y} = k_1(x - y) - k_2 y$$

A particular solution for $x = a$ is

$$y = \frac{k_1 a}{k_1 + k_2}$$

Then the complementary plus the particular solution is

$$y = A\cos\omega t + B\sin\omega t + \frac{k_1 a}{k_1 + k_2}$$

where

$$\omega = \left[\frac{(k_1 + k_2)g}{W}\right]^{1/2}$$

At $t = 0$, $y = 0$, and $\dot{y} = 0$. Therefore $B = 0$ and

$$A = -\frac{k_1 a}{k_1 + k_2}$$

Substituting,

$$y = \frac{k_1 a}{k_1 + k_2}(1 - \cos\omega t)$$

Since y is maximum when the cosine is -1

$$y_{\max} = \frac{2k_1 a}{k_1 + k_2} \quad Ans.$$

Chapter 5

5-1

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma_1 - \sigma_3}$

DE: $n = \dfrac{S_y}{\sigma'}$

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

(a) MSS: $\sigma_1 = 12,\ \sigma_2 = 6,\ \sigma_3 = 0$ kpsi

$$n = \frac{50}{12} = 4.17 \quad \textit{Ans.}$$

DE: $\sigma' = (12^2 - 6(12) + 6^2)^{1/2} = 10.39$ kpsi, $\quad n = \dfrac{50}{10.39} = 4.81 \quad$ *Ans.*

(b) $\sigma_A, \sigma_B = \dfrac{12}{2} \pm \sqrt{\left(\dfrac{12}{2}\right)^2 + (-8)^2} = 16, -4$ kpsi

$\sigma_1 = 16,\ \sigma_2 = 0,\ \sigma_3 = -4$ kpsi

MSS: $n = \dfrac{50}{16 - (-4)} = 2.5 \quad$ *Ans.*

DE: $\sigma' = (12^2 + 3(-8^2))^{1/2} = 18.33$ kpsi, $\quad n = \dfrac{50}{18.33} = 2.73 \quad$ *Ans.*

(c) $\sigma_A, \sigma_B = \dfrac{-6 - 10}{2} \pm \sqrt{\left(\dfrac{-6 + 10}{2}\right)^2 + (-5)^2} = -2.615, -13.385$ kpsi

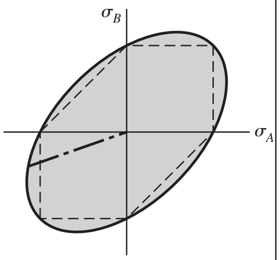
$\sigma_1 = 0,\ \sigma_2 = -2.615,\ \sigma_3 = -13.385$ kpsi

MSS: $n = \dfrac{50}{0 - (-13.385)} = 3.74 \quad$ *Ans.*

DE: $\sigma' = [(-6)^2 - (-6)(-10) + (-10)^2 + 3(-5)^2]^{1/2}$

$= 12.29$ kpsi

$$n = \frac{50}{12.29} = 4.07 \quad \textit{Ans.}$$



(d) $\sigma_A, \sigma_B = \dfrac{12 + 4}{2} \pm \sqrt{\left(\dfrac{12 - 4}{2}\right)^2 + 1^2} = 12.123, 3.877$ kpsi

$\sigma_1 = 12.123,\ \sigma_2 = 3.877,\ \sigma_3 = 0$ kpsi

MSS: $n = \dfrac{50}{12.123 - 0} = 4.12 \quad$ *Ans.*

DE: $\sigma' = [12^2 - 12(4) + 4^2 + 3(1^2)]^{1/2} = 10.72$ kpsi

$$n = \frac{50}{10.72} = 4.66 \quad \textit{Ans.}$$

5-2 $S_y = 50$ kpsi

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma_1 - \sigma_3}$

DE: $\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = S_y/n \quad \Rightarrow \quad n = S_y/\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$

(a) MSS: $\sigma_1 = 12$ kpsi, $\sigma_3 = 0$, $n = \dfrac{50}{12 - 0} = 4.17$ *Ans.*

DE: $n = \dfrac{50}{[12^2 - (12)(12) + 12^2]^{1/2}} = 4.17$ *Ans.*

(b) MSS: $\sigma_1 = 12$ kpsi, $\sigma_3 = 0$, $n = \dfrac{50}{12} = 4.17$ *Ans.*

DE: $n = \dfrac{50}{[12^2 - (12)(6) + 6^2]^{1/2}} = 4.81$ *Ans.*

(c) MSS: $\sigma_1 = 12$ kpsi, $\sigma_3 = -12$ kpsi, $n = \dfrac{50}{12 - (-12)} = 2.08$ *Ans.*

DE: $n = \dfrac{50}{[12^2 - (12)(-12) + (-12)^2]^{1/3}} = 2.41$ *Ans.*

(d) MSS: $\sigma_1 = 0$, $\sigma_3 = -12$ kpsi, $n = \dfrac{50}{-(-12)} = 4.17$ *Ans.*

DE: $n = \dfrac{50}{[(-6)^2 - (-6)(-12) + (-12)^2]^{1/2}} = 4.81$

5-3 $S_y = 390$ MPa

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma_1 - \sigma_3}$

DE: $\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = S_y/n \quad \Rightarrow \quad n = S_y/\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$

(a) MSS: $\sigma_1 = 180$ MPa, $\sigma_3 = 0$, $n = \dfrac{390}{180} = 2.17$ *Ans.*

DE: $n = \dfrac{390}{[180^2 - 180(100) + 100^2]^{1/2}} = 2.50$ *Ans.*

(b) $\sigma_A, \sigma_B = \dfrac{180}{2} \pm \sqrt{\left(\dfrac{180}{2}\right)^2 + 100^2} = 224.5, -44.5 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \dfrac{390}{224.5 - (-44.5)} = 1.45$ *Ans.*

DE: $n = \dfrac{390}{[180^2 + 3(100^2)]^{1/2}} = 1.56$ *Ans.*

(c) $\sigma_A, \sigma_B = -\dfrac{160}{2} \pm \sqrt{\left(-\dfrac{160}{2}\right)^2 + 100^2} = 48.06, -208.06 \text{ MPa} = \sigma_1, \sigma_3$

$$\text{MSS:} \quad n = \frac{390}{48.06 - (-208.06)} = 1.52 \quad \textit{Ans.}$$

$$\text{DE:} \quad n = \frac{390}{[-160^2 + 3(100^2)]^{1/2}} = 1.65 \quad \textit{Ans.}$$

(d) $\sigma_A, \sigma_B = 150, -150 \text{ MPa} = \sigma_1, \sigma_3$

$$\text{MSS:} \quad n = \frac{390}{150 - (-150)} = 1.30 \quad \textit{Ans.}$$

$$\text{DE:} \quad n = \frac{390}{[3(150)^2]^{1/2}} = 1.50 \quad \textit{Ans.}$$

5-4 $S_y = 220 \text{ MPa}$

(a) $\sigma_1 = 100, \sigma_2 = 80, \sigma_3 = 0 \text{ MPa}$

$$\text{MSS:} \quad n = \frac{220}{100 - 0} = 2.20 \quad \textit{Ans.}$$

$$\text{DET:} \quad \sigma' = [100^2 - 100(80) + 80^2]^{1/2} = 91.65 \text{ MPa}$$

$$n = \frac{220}{91.65} = 2.40 \quad \textit{Ans.}$$

(b) $\sigma_1 = 100, \sigma_2 = 10, \sigma_3 = 0 \text{ MPa}$

$$\text{MSS:} \quad n = \frac{220}{100} = 2.20 \quad \textit{Ans.}$$

$$\text{DET:} \quad \sigma' = [100^2 - 100(10) + 10^2]^{1/2} = 95.39 \text{ MPa}$$

$$n = \frac{220}{95.39} = 2.31 \quad \textit{Ans.}$$

(c) $\sigma_1 = 100, \sigma_2 = 0, \sigma_3 = -80 \text{ MPa}$

$$\text{MSS:} \quad n = \frac{220}{100 - (-80)} = 1.22 \quad \textit{Ans.}$$

$$\text{DE:} \quad \sigma' = [100^2 - 100(-80) + (-80)^2]^{1/2} = 156.2 \text{ MPa}$$

$$n = \frac{220}{156.2} = 1.41 \quad \textit{Ans.}$$

(d) $\sigma_1 = 0, \sigma_2 = -80, \sigma_3 = -100 \text{ MPa}$

$$\text{MSS:} \quad n = \frac{220}{0 - (-100)} = 2.20 \quad \textit{Ans.}$$

$$\text{DE:} \quad \sigma' = [(-80)^2 - (-80)(-100) + (-100)^2] = 91.65 \text{ MPa}$$

$$n = \frac{220}{91.65} = 2.40 \quad \textit{Ans.}$$

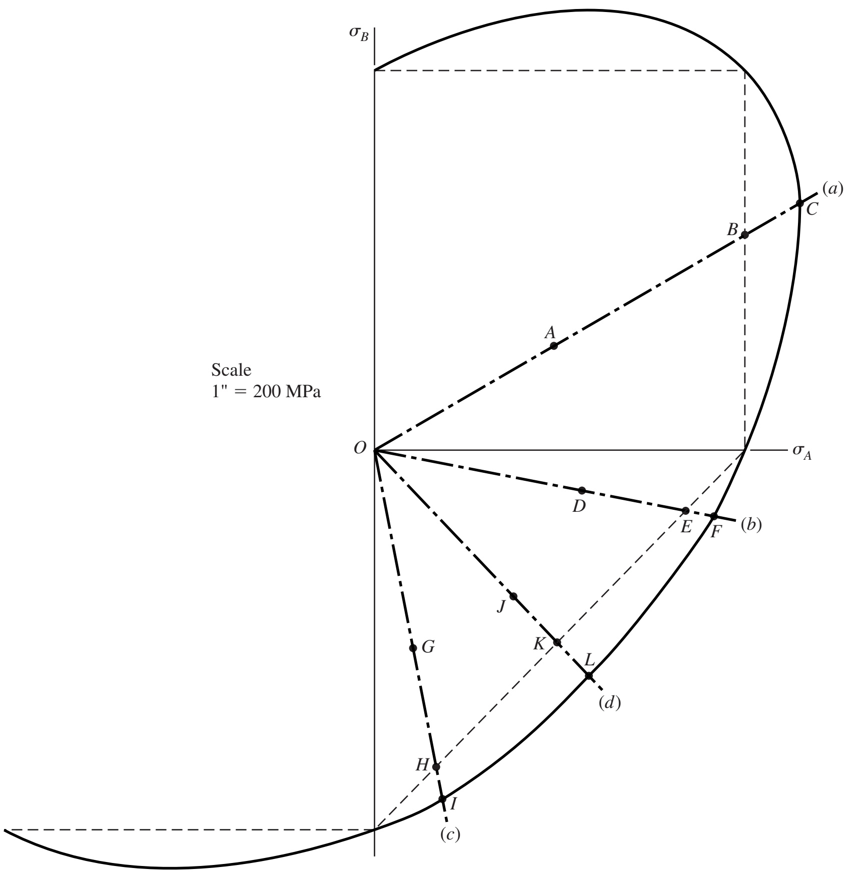
5-5

(a) MSS: $$n = \frac{OB}{OA} = \frac{2.23}{1.08} = 2.1$$

DE: $$n = \frac{OC}{OA} = \frac{2.56}{1.08} = 2.4$$

(b) MSS: $$n = \frac{OE}{OD} = \frac{1.65}{1.10} = 1.5$$

DE: $$n = \frac{OF}{OD} = \frac{1.8}{1.1} = 1.6$$



(c) MSS: $$n = \frac{OH}{OG} = \frac{1.68}{1.05} = 1.6$$

DE: $$n = \frac{OI}{OG} = \frac{1.85}{1.05} = 1.8$$

(d) MSS: $$n = \frac{OK}{OJ} = \frac{1.38}{1.05} = 1.3$$

DE: $$n = \frac{OL}{OJ} = \frac{1.62}{1.05} = 1.5$$

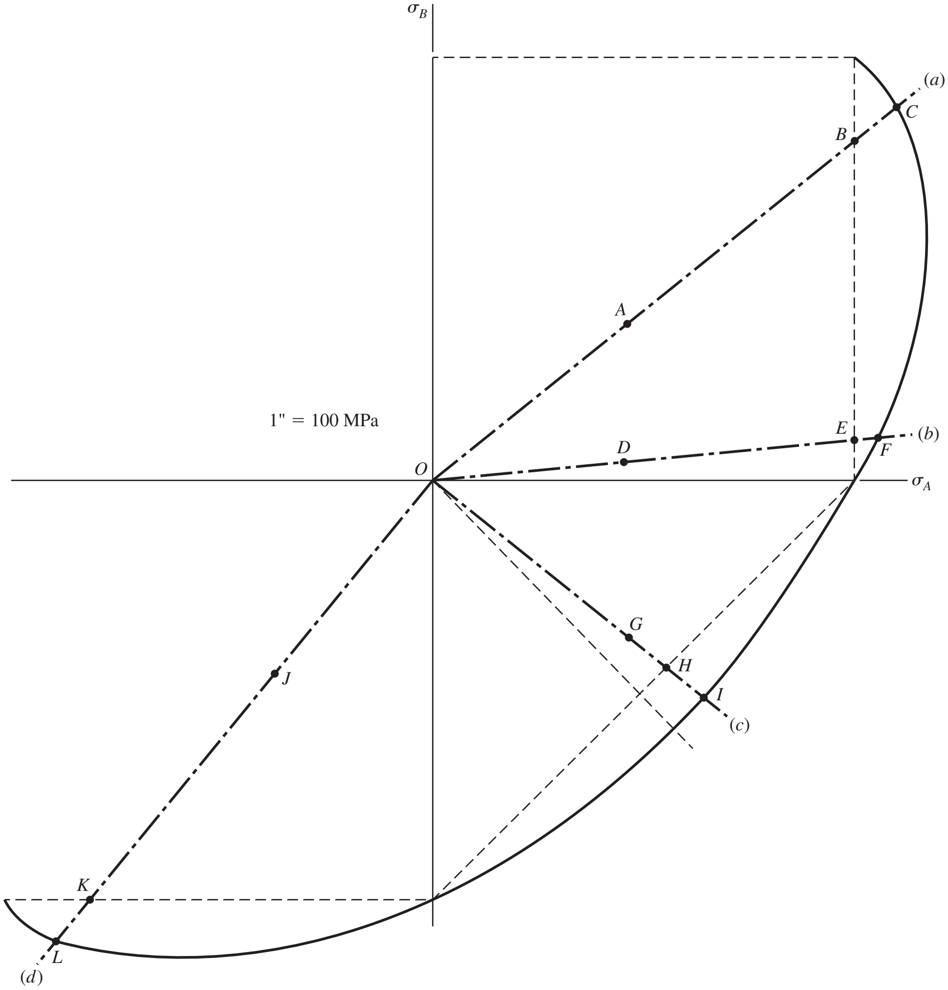
5-6 $S_y = 220$ MPa

(a) MSS: $$n = \frac{OB}{OA} = \frac{2.82}{1.3} = 2.2$$

DE: $$n = \frac{OC}{OA} = \frac{3.1}{1.3} = 2.4$$

(b) MSS: $$n = \frac{OE}{OD} = \frac{2.2}{1} = 2.2$$

DE: $$n = \frac{OF}{OD} = \frac{2.33}{1} = 2.3$$



(c) MSS: $$n = \frac{OH}{OG} = \frac{1.55}{1.3} = 1.2$$

DE: $$n = \frac{OI}{OG} = \frac{1.8}{1.3} = 1.4$$

(d) MSS: $$n = \frac{OK}{OJ} = \frac{2.82}{1.3} = 2.2$$

DE: $$n = \frac{OL}{OJ} = \frac{3.1}{1.3} = 2.4$$

5-7 $S_{ut} = 30$ kpsi, $S_{uc} = 100$ kpsi; $\sigma_A = 20$ kpsi, $\sigma_B = 6$ kpsi

(a) MNS: Eq. (5-30*a*) $\quad n = \dfrac{S_{ut}}{\sigma_x} = \dfrac{30}{20} = 1.5 \quad \textit{Ans.}$

BCM: Eq. (5-31*a*) $\quad n = \dfrac{30}{20} = 1.5 \quad \textit{Ans.}$

MM: Eq. (5-32*a*) $\quad n = \dfrac{30}{20} = 1.5 \quad \textit{Ans.}$

(b) $\sigma_x = 12$ kpsi, $\tau_{xy} = -8$ kpsi

$$\sigma_A, \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16, -4 \text{ kpsi}$$

MNS: Eq. (5-30*a*) $\quad n = \dfrac{30}{16} = 1.88 \quad \textit{Ans.}$

BCM: Eq. (5-31*b*) $\quad \dfrac{1}{n} = \dfrac{16}{30} - \dfrac{(-4)}{100} \quad \Rightarrow \quad n = 1.74 \quad \textit{Ans.}$

MM: Eq. (5-32*a*) $\quad n = \dfrac{30}{16} = 1.88 \quad \textit{Ans.}$

(c) $\sigma_x = -6$ kpsi, $\sigma_y = -10$ kpsi, $\tau_{xy} = -5$ kpsi

$$\sigma_A, \sigma_B = \frac{-6-10}{2} \pm \sqrt{\left(\frac{-6+10}{2}\right)^2 + (-5)^2} = -2.61, -13.39 \text{ kpsi}$$

MNS: Eq. (5-30*b*) $\quad n = -\dfrac{100}{-13.39} = 7.47 \quad \textit{Ans.}$

BCM: Eq. (5-31*c*) $\quad n = -\dfrac{100}{-13.39} = 7.47 \quad \textit{Ans.}$

MM: Eq. (5-32*c*) $\quad n = -\dfrac{100}{-13.39} = 7.47 \quad \textit{Ans.}$

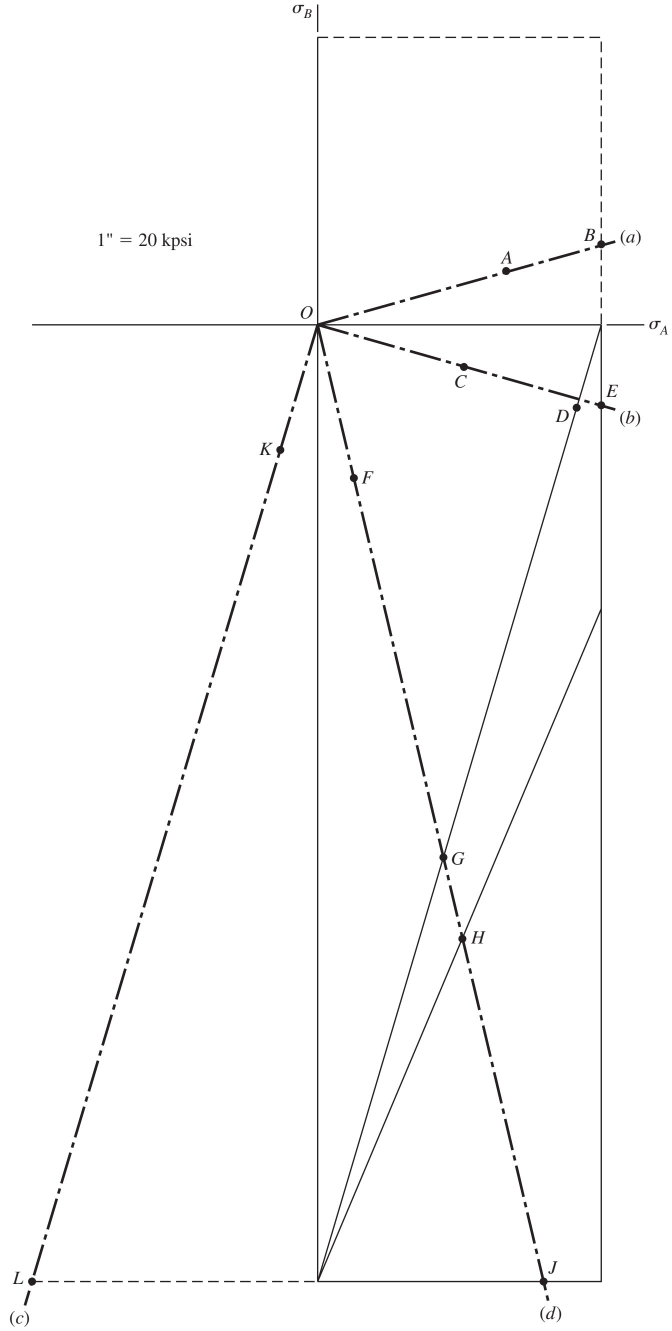
(d) $\sigma_x = -12$ kpsi, $\tau_{xy} = 8$ kpsi

$$\sigma_A, \sigma_B = -\frac{12}{2} \pm \sqrt{\left(-\frac{12}{2}\right)^2 + 8^2} = 4, -16 \text{ kpsi}$$

MNS: Eq. (5-30*b*) $\quad n = \dfrac{-100}{-16} = 6.25 \quad \textit{Ans.}$

BCM: Eq. (5-31b) $$\frac{1}{n} = \frac{4}{30} - \frac{(-16)}{100} \quad \Rightarrow \quad n = 3.41 \quad Ans.$$

MM: Eq. (5-32b) $$\frac{1}{n} = \frac{(100-30)4}{100(30)} - \frac{-16}{100} \quad \Rightarrow \quad n = 3.95 \quad Ans.$$



5-8 See Prob. 5-7 for plot.

(a) For all methods: $n = \frac{OB}{OA} = \frac{1.55}{1.03} = 1.5$

(b) BCM: $n = \frac{OD}{OC} = \frac{1.4}{0.8} = 1.75$

All other methods: $n = \frac{OE}{OC} = \frac{1.55}{0.8} = 1.9$

(c) For all methods: $n = \frac{OL}{OK} = \frac{5.2}{0.68} = 7.6$

(d) MNS: $n = \frac{OJ}{OF} = \frac{5.12}{0.82} = 6.2$

BCM: $n = \frac{OG}{OF} = \frac{2.85}{0.82} = 3.5$

MM: $n = \frac{OH}{OF} = \frac{3.3}{0.82} = 4.0$

5-9 Given: $S_y = 42$ kpsi, $S_{ut} = 66.2$ kpsi, $\varepsilon_f = 0.90$. Since $\varepsilon_f > 0.05$, the material is ductile and thus we may follow convention by setting $S_{yc} = S_{yt}$.

Use DE theory for analytical solution. For σ', use Eq. (5-13) or (5-15) for plane stress and Eq. (5-12) or (5-14) for general 3-D.

(a) $\sigma' = [9^2 - 9(-5) + (-5)^2]^{1/2} = 12.29$ kpsi

$$n = \frac{42}{12.29} = 3.42 \quad \textit{Ans.}$$

(b) $\sigma' = [12^2 + 3(3^2)]^{1/2} = 13.08$ kpsi

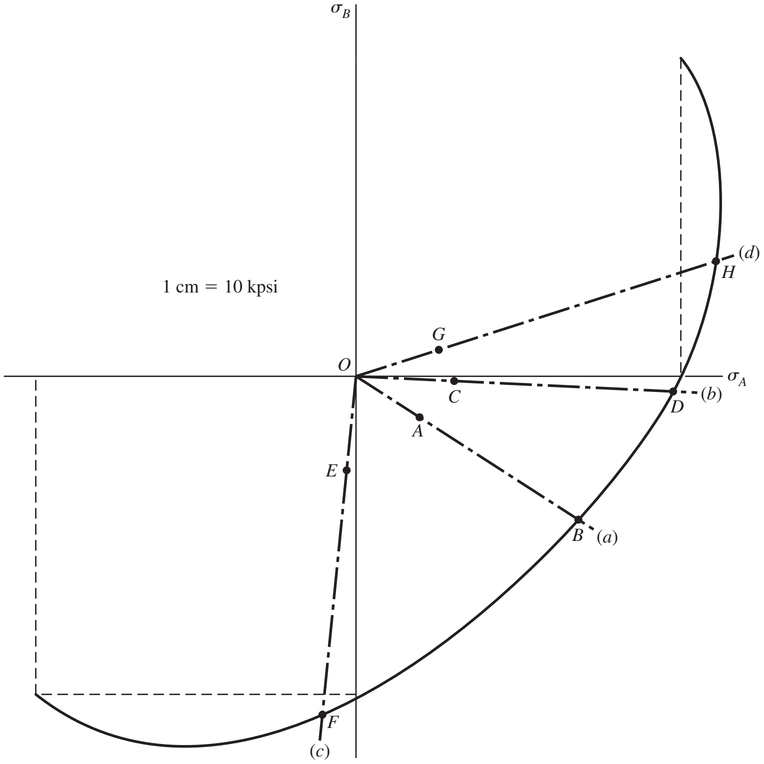
$$n = \frac{42}{13.08} = 3.21 \quad \textit{Ans.}$$

(c) $\sigma' = [(-4)^2 - (-4)(-9) + (-9)^2 + 3(5^2)]^{1/2} = 11.66$ kpsi

$$n = \frac{42}{11.66} = 3.60 \quad \textit{Ans.}$$

(d) $\sigma' = [11^2 - (11)(4) + 4^2 + 3(1^2)]^{1/2} = 9.798$

$$n = \frac{42}{9.798} = 4.29 \quad \textit{Ans.}$$



For graphical solution, plot load lines on DE envelope as shown.

(a) $\sigma_A = 9, \sigma_B = -5$ kpsi

$$n = \frac{OB}{OA} = \frac{3.5}{1} = 3.5 \quad Ans.$$

(b) $\sigma_A, \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + 3^2} = 12.7, -0.708$ kpsi

$$n = \frac{OD}{OC} = \frac{4.2}{1.3} = 3.23$$

(c) $\sigma_A, \sigma_B = \frac{-4-9}{2} \pm \sqrt{\left(\frac{4-9}{2}\right)^2 + 5^2} = -0.910, -12.09$ kpsi

$$n = \frac{OF}{OE} = \frac{4.5}{1.25} = 3.6 \quad Ans.$$

(d) $\sigma_A, \sigma_B = \frac{11+4}{2} \pm \sqrt{\left(\frac{11-4}{2}\right)^2 + 1^2} = 11.14, 3.86$ kpsi

$$n = \frac{OH}{OG} = \frac{5.0}{1.15} = 4.35 \quad Ans.$$

5-10 This heat-treated steel exhibits $S_{yt} = 235$ kpsi, $S_{yc} = 275$ kpsi and $\varepsilon_f = 0.06$. The steel is ductile ($\varepsilon_f > 0.05$) but of unequal yield strengths. The Ductile Coulomb-Mohr hypothesis (DCM) of Fig. 5-19 applies — confine its use to first and fourth quadrants.

(a) $\sigma_x = 90$ kpsi, $\sigma_y = -50$ kpsi, $\sigma_z = 0$ $\therefore$ $\sigma_A = 90$ kpsi and $\sigma_B = -50$ kpsi. For the fourth quadrant, from Eq. (5-31b)

$$n = \frac{1}{(\sigma_A/S_{yt}) - (\sigma_B/S_{uc})} = \frac{1}{(90/235) - (-50/275)} = 1.77 \quad \textit{Ans.}$$

(b) $\sigma_x = 120$ kpsi, $\tau_{xy} = -30$ kpsi ccw. $\sigma_A, \sigma_B = 127.1, -7.08$ kpsi. For the fourth quadrant

$$n = \frac{1}{(127.1/235) - (-7.08/275)} = 1.76 \quad \textit{Ans.}$$

(c) $\sigma_x = -40$ kpsi, $\sigma_y = -90$ kpsi, $\tau_{xy} = 50$ kpsi. σ_A, $\sigma_B = -9.10$, -120.9 kpsi. Although no solution exists for the third quadrant, use

$$n = -\frac{S_{yc}}{\sigma_y} = -\frac{275}{-120.9} = 2.27 \quad \textit{Ans.}$$

(d) $\sigma_x = 110$ kpsi, $\sigma_y = 40$ kpsi, $\tau_{xy} = 10$ kpsi cw. $\sigma_A, \sigma_B = 111.4, 38.6$ kpsi. For the first quadrant

$$n = \frac{S_{yt}}{\sigma_A} = \frac{235}{111.4} = 2.11 \quad \textit{Ans.}$$

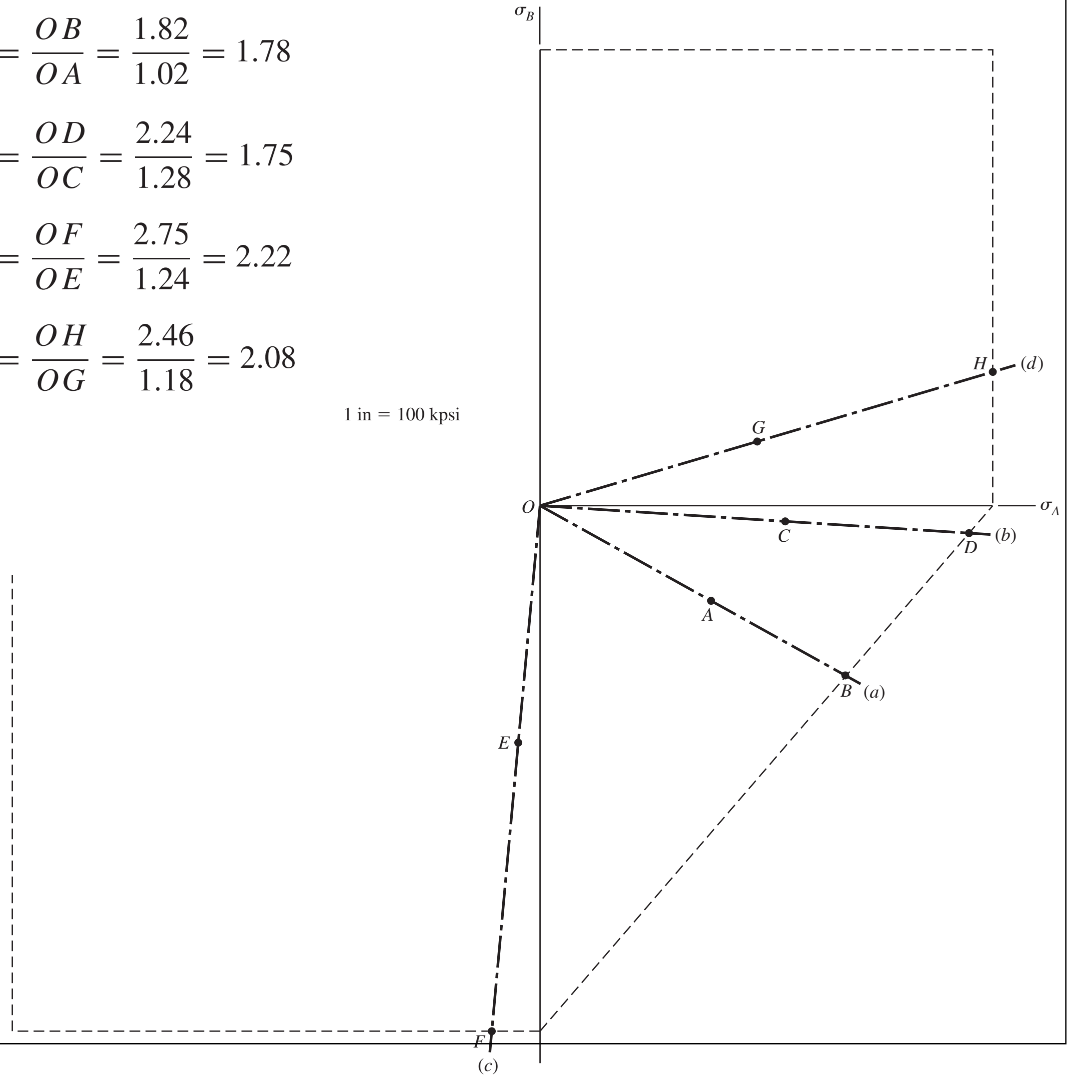
Graphical Solution:

(a) $n = \dfrac{OB}{OA} = \dfrac{1.82}{1.02} = 1.78$

(b) $n = \dfrac{OD}{OC} = \dfrac{2.24}{1.28} = 1.75$

(c) $n = \dfrac{OF}{OE} = \dfrac{2.75}{1.24} = 2.22$

(d) $n = \dfrac{OH}{OG} = \dfrac{2.46}{1.18} = 2.08$



5-11 The material is brittle and exhibits unequal tensile and compressive strengths. *Decision:* Use the Modified Mohr theory.

$$S_{ut} = 22 \text{ kpsi}, \quad S_{uc} = 83 \text{ kpsi}$$

(a) $\sigma_x = 9$ kpsi, $\sigma_y = -5$ kpsi. $\sigma_A, \sigma_B = 9, -5$ kpsi. For the fourth quadrant, $|\frac{\sigma_B}{\sigma_A}| = \frac{5}{9} < 1$, use Eq. (5-32*a*)

$$n = \frac{S_{ut}}{\sigma_A} = \frac{22}{9} = 2.44 \quad \textit{Ans.}$$

(b) $\sigma_x = 12$ kpsi, $\tau_{xy} = -3$ kpsi ccw. $\sigma_A, \sigma_B = 12.7, -0.708$ kpsi. For the fourth quadrant, $|\frac{\sigma_B}{\sigma_A}| = \frac{0.708}{12.7} < 1$,

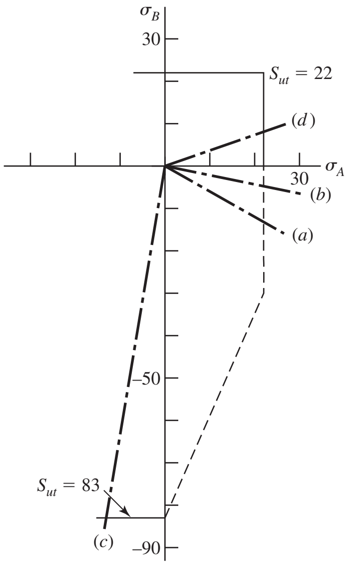
$$n = \frac{S_{ut}}{\sigma_A} = \frac{22}{12.7} = 1.73 \quad \textit{Ans.}$$

(c) $\sigma_x = -4$ kpsi, $\sigma_y = -9$ kpsi, $\tau_{xy} = 5$ kpsi. $\sigma_A, \sigma_B = -0.910, -12.09$ kpsi. For the third quadrant, no solution exists; however, use Eq. (6-32*c*)

$$n = \frac{-83}{-12.09} = 6.87 \quad \textit{Ans.}$$

(d) $\sigma_x = 11$ kpsi, $\sigma_y = 4$ kpsi, $\tau_{xy} = 1$ kpsi. $\sigma_A, \sigma_B = 11.14, 3.86$ kpsi. For the first quadrant

$$n = \frac{S_A}{\sigma_A} = \frac{S_{yt}}{\sigma_A} = \frac{22}{11.14} = 1.97 \quad \textit{Ans.}$$



5-12 Since $\varepsilon_f < 0.05$, the material is brittle. Thus, $S_{ut} \doteq S_{uc}$ and we may use MM which is basically the same as MNS.

(a) $\sigma_A, \sigma_B = 9, -5$ kpsi

$$n = \frac{35}{9} = 3.89 \quad \textit{Ans.}$$

(b) $\sigma_A, \sigma_B = 12.7, -0.708$ kpsi

$$n = \frac{35}{12.7} = 2.76 \quad \textit{Ans.}$$

(c) $\sigma_A, \sigma_B = -0.910, -12.09$ kpsi (3rd quadrant)

$$n = \frac{36}{12.09} = 2.98 \quad \textit{Ans.}$$

(d) $\sigma_A, \sigma_B = 11.14, 3.86$ kpsi

$$n = \frac{35}{11.14} = 3.14 \quad \textit{Ans.}$$

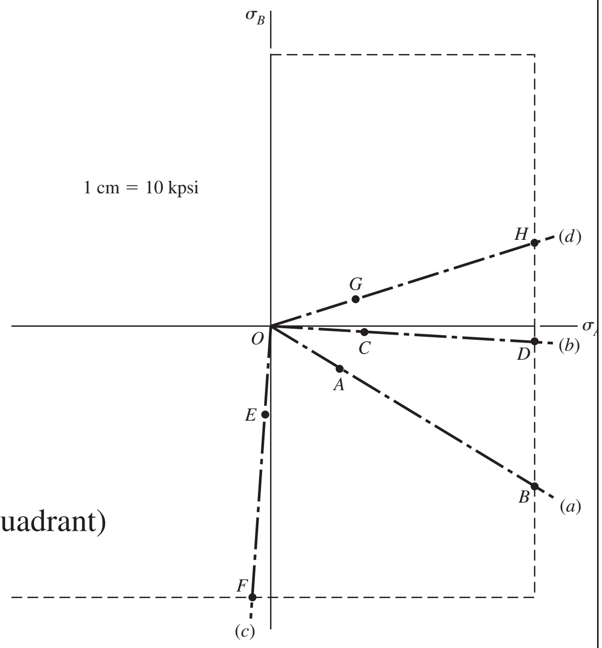
Graphical Solution:

(a) $n = \dfrac{OB}{OA} = \dfrac{4}{1} = 4.0 \quad \textit{Ans.}$

(b) $n = \dfrac{OD}{OC} = \dfrac{3.45}{1.28} = 2.70 \quad \textit{Ans.}$

(c) $n = \dfrac{OF}{OE} = \dfrac{3.7}{1.3} = 2.85 \quad \textit{Ans.}$ (3rd quadrant)

(d) $n = \dfrac{OH}{OG} = \dfrac{3.6}{1.15} = 3.13 \quad \textit{Ans.}$



5-13 $S_{ut} = 30$ kpsi, $S_{uc} = 109$ kpsi

Use MM:

(a) $\sigma_A, \sigma_B = 20, 20$ kpsi

Eq. (5-32*a*): $n = \dfrac{30}{20} = 1.5 \quad \textit{Ans.}$

(b) $\sigma_A, \sigma_B = \pm\sqrt{(15)^2} = 15, -15$ kpsi

Eq. (5-32*a*) $n = \dfrac{30}{15} = 2 \quad \textit{Ans.}$

(c) $\sigma_A, \sigma_B = -80, -80$ kpsi

For the 3rd quadrant, there is no solution but use Eq. (5-32*c*).

Eq. (5-32*c*): $n = -\dfrac{109}{-80} = 1.36 \quad \textit{Ans.}$

(d) $\sigma_A, \sigma_B = 15, -25$ kpsi, $|\sigma_B|\sigma_A| = 25/15 > 1$,

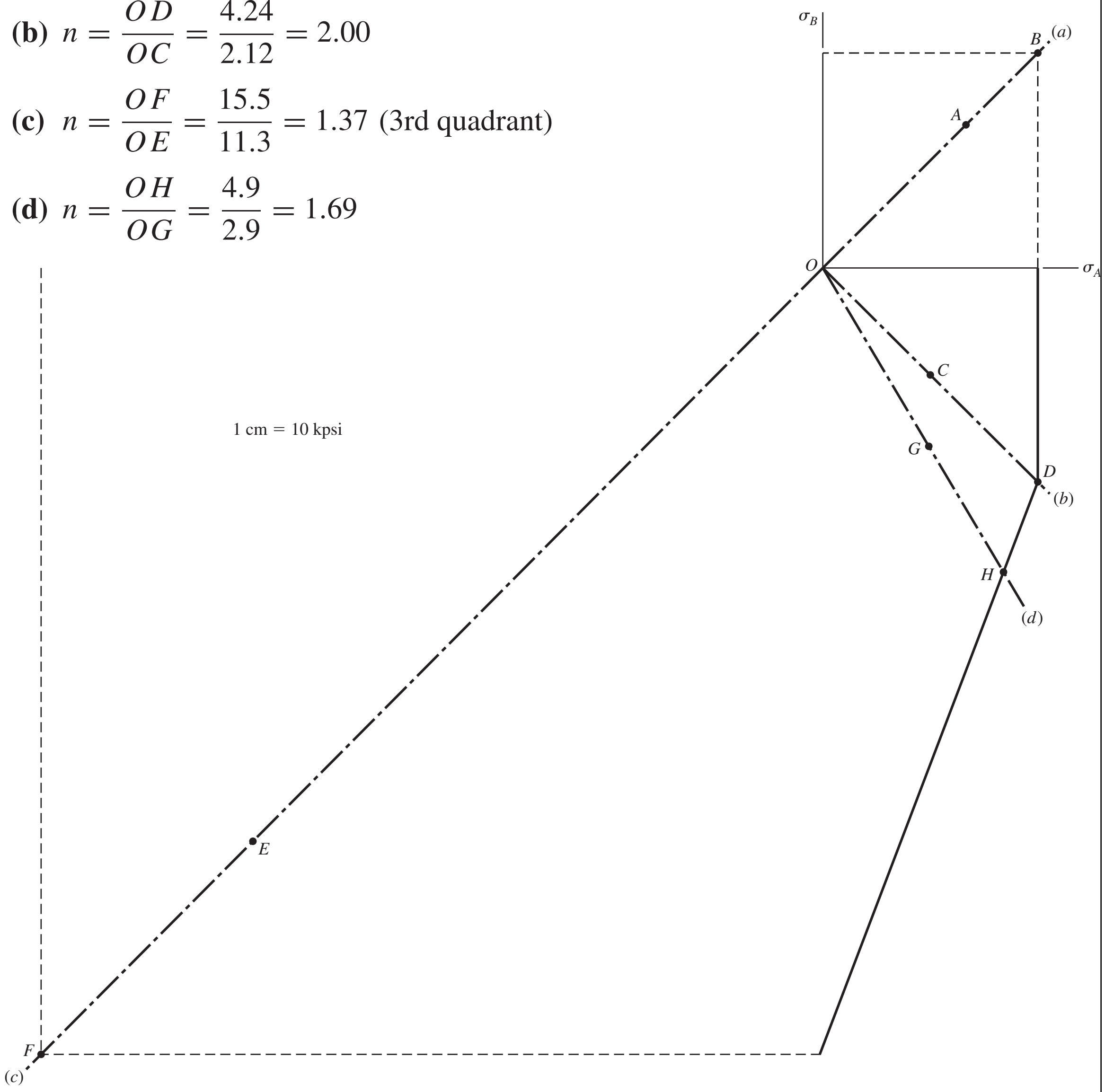
Eq. (5-32*b*):
$$\frac{(109-30)15}{109(30)} - \frac{-25}{109} = \frac{1}{n}$$
$$n = 1.69 \quad Ans.$$

(a) $n = \dfrac{OB}{OA} = \dfrac{4.25}{2.83} = 1.50$

(b) $n = \dfrac{OD}{OC} = \dfrac{4.24}{2.12} = 2.00$

(c) $n = \dfrac{OF}{OE} = \dfrac{15.5}{11.3} = 1.37$ (3rd quadrant)

(d) $n = \dfrac{OH}{OG} = \dfrac{4.9}{2.9} = 1.69$



5-14 Given: AISI 1006 CD steel, $F = 0.55$ N, $P = 8.0$ kN, and $T = 30$ N · m, applying the DE theory to stress elements A and B with $S_y = 280$ MPa

A:
$$\sigma_x = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32(0.55)(10^3)(0.1)}{\pi(0.020^3)} + \frac{4(8)(10^3)}{\pi(0.020^2)}$$
$$= 95.49(10^6)\text{ Pa} = 95.49\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(30)}{\pi(0.020^3)} = 19.10(10^6)\text{ Pa} = 19.10\text{ MPa}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = [95.49^2 + 3(19.1)^2]^{1/2} = 101.1\text{ MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{280}{101.1} = 2.77 \quad \textit{Ans.}$$

B:

$$\sigma_x = \frac{4P}{\pi d^3} = \frac{4(8)(10^3)}{\pi(0.020^2)} = 25.47(10^6)\text{ Pa} = 25.47\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} + \frac{4}{3}\frac{V}{A} = \frac{16(30)}{\pi(0.020^3)} + \frac{4}{3}\left[\frac{0.55(10^3)}{(\pi/4)(0.020^2)}\right]$$

$$= 21.43(10^6)\text{ Pa} = 21.43\text{ MPa}$$

$$\sigma' = [25.47^2 + 3(21.43^2)]^{1/2} = 45.02\text{ MPa}$$

$$n = \frac{280}{45.02} = 6.22 \quad \textit{Ans.}$$

5-15 $S_y = 32\text{ kpsi}$

At A, $M = 6(190) = 1\,140$ lbf·in, $T = 4(190) = 760$ lbf · in.

$$\sigma_x = \frac{32M}{\pi d^3} = \frac{32(1140)}{\pi(3/4)^3} = 27\,520\text{ psi}$$

$$\tau_{zx} = \frac{16T}{\pi d^3} = \frac{16(760)}{\pi(3/4)^3} = 9175\text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{27\,520}{2}\right)^2 + 9175^2} = 16\,540\text{ psi}$$

$$n = \frac{S_y}{2\tau_{\max}} = \frac{32}{2(16.54)} = 0.967 \quad \textit{Ans.}$$

MSS predicts yielding

5-16 From Prob. 4-15, $\sigma_x = 27.52$ kpsi, $\tau_{zx} = 9.175$ kpsi. For Eq. (5-15), adjusted for coordinates,

$$\sigma' = \left[27.52^2 + 3(9.175)^2\right]^{1/2} = 31.78\text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{31.78} = 1.01 \quad \textit{Ans.}$$

DE predicts no yielding, but it is extremely close. Shaft size should be increased.

5-17 Design decisions required:

- Material and condition
- Design factor
- Failure model
- Diameter of pin

Using $F = 416$ lbf from Ex. 5-3

$$\sigma_{\max} = \frac{32M}{\pi d^3}$$

$$d = \left(\frac{32M}{\pi\sigma_{\max}}\right)^{1/3}$$

Decision 1: Select the same material and condition of Ex. 5-3 (AISI 1035 steel, $S_y = 81\,000$).

Decision 2: Since we prefer the pin to yield, set n_d a little larger than 1. Further explanation will follow.

Decision 3: Use the Distortion Energy static failure theory.

Decision 4: Initially set $n_d = 1$

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{S_y}{1} = 81\,000 \text{ psi}$$

$$d = \left[\frac{32(416)(15)}{\pi(81\,000)}\right]^{1/3} = 0.922 \text{ in}$$

Choose preferred size of $d = 1.000$ in

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.274$$

Set design factor to $n_d = 1.274$

Adequacy Assessment:

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{81\,000}{1.274} = 63\,580 \text{ psi}$$

$$d = \left[\frac{32(416)(15)}{\pi(63\,580)}\right]^{1/3} = 1.000 \text{ in} \quad (OK)$$

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.274 \quad \text{(OK)}$$

5-18 For a thin walled cylinder made of AISI 1018 steel, $S_y = 54$ kpsi, $S_{ut} = 64$ kpsi.

The state of stress is

$$\sigma_t = \frac{pd}{4t} = \frac{p(8)}{4(0.05)} = 40p, \quad \sigma_l = \frac{pd}{8t} = 20p, \quad \sigma_r = -p$$

These three are all principal stresses. Therefore,

$$\sigma' = \frac{1}{\sqrt{2}}[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2]^{1/2}$$

$$= \frac{1}{\sqrt{2}}[(40p-20p)^2 + (20p+p)^2 + (-p-40p)^2]$$

$$= 35.51p = 54 \quad \Rightarrow \quad p = 1.52 \text{ kpsi} \quad \text{(for yield)} \quad \textit{Ans.}$$

For rupture, $35.51p \doteq 64 \Rightarrow p \doteq 1.80$ kpsi *Ans.*

5-19 For hot-forged AISI steel $w = 0.282$ lbf/in^3, $S_y = 30$ kpsi and $\nu = 0.292$. Then $\rho = w/g = 0.282/386$ lbf · s^2/in; $r_i = 3$ in; $r_o = 5$ in; $r_i^2 = 9$; $r_o^2 = 25$; $3+\nu = 3.292$; $1+3\nu = 1.876$.

Eq. (3-55) for $r = r_i$ becomes

$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left[2r_o^2 + r_i^2\left(1 - \frac{1+3\nu}{3+\nu}\right)\right]$$

Rearranging and substituting the above values:

$$\frac{S_y}{\omega^2} = \frac{0.282}{386}\left(\frac{3.292}{8}\right)\left[50 + 9\left(1 - \frac{1.876}{3.292}\right)\right]$$

$$= 0.016\,19$$

Setting the tangential stress equal to the yield stress,

$$\omega = \left(\frac{30\,000}{0.016\,19}\right)^{1/2} = 1361 \text{ rad/s}$$

or

$$n = 60\omega/2\pi = 60(1361)/(2\pi)$$

$$= 13\,000 \text{ rev/min}$$

Now check the stresses at $r = (r_o r_i)^{1/2}$, or $r = [5(3)]^{1/2} = 3.873$ in

$$\sigma_r = \rho\omega^2\left(\frac{3+\nu}{8}\right)(r_o - r_i)^2$$

$$= \frac{0.282\omega^2}{386}\left(\frac{3.292}{8}\right)(5-3)^2$$

$$= 0.001\,203\omega^2$$

Applying Eq. (3-55) for σ_t

$$\sigma_t = \omega^2\left(\frac{0.282}{386}\right)\left(\frac{3.292}{8}\right)\left[9 + 25 + \frac{9(25)}{15} - \frac{1.876(15)}{3.292}\right]$$

$$= 0.012\,16\omega^2$$

Using the Distortion-Energy theory

$$\sigma' = \left(\sigma_t^2 - \sigma_r\sigma_t + \sigma_r^2\right)^{1/2} = 0.011\,61\omega^2$$

Solving $$\omega = \left(\frac{30\,000}{0.011\,61}\right)^{1/2} = 1607\ \text{rad/s}$$

So the inner radius governs and $n = 13\,000$ rev/min *Ans.*

5-20 For a thin-walled pressure vessel,

$$d_i = 3.5 - 2(0.065) = 3.37\ \text{in}$$

$$\sigma_t = \frac{p(d_i + t)}{2t}$$

$$\sigma_t = \frac{500(3.37 + 0.065)}{2(0.065)} = 13\,212\ \text{psi}$$

$$\sigma_l = \frac{pd_i}{4t} = \frac{500(3.37)}{4(0.065)} = 6481\ \text{psi}$$

$$\sigma_r = -p_i = -500\ \text{psi}$$

These are all principal stresses, thus,

$$\sigma' = \frac{1}{\sqrt{2}}\{(13\,212 - 6481)^2 + [6481 - (-500)]^2 + (-500 - 13\,212)^2\}^{1/2}$$

$$\sigma' = 11\,876\ \text{psi}$$

$$n = \frac{S_y}{\sigma'} = \frac{46\,000}{\sigma'} = \frac{46\,000}{11\,876}$$

$$= 3.87$$ *Ans.*

5-21 Table A-20 gives S_y as 320 MPa. The maximum significant stress condition occurs at r_i where $\sigma_1 = \sigma_r = 0$, $\sigma_2 = 0$, and $\sigma_3 = \sigma_t$. From Eq. (3-49) for $r = r_i$, $p_i = 0$,

$$\sigma_t = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2} = -\frac{2(150^2)p_o}{150^2 - 100^2} = -3.6p_o$$

$$\sigma' = 3.6p_o = S_y = 320$$

$$p_o = \frac{320}{3.6} = 88.9\ \text{MPa}$$ *Ans.*

5-22 $S_{ut} = 30$ kpsi, $w = 0.260\ \text{lbf/in}^3$, $\nu = 0.211$, $3 + \nu = 3.211$, $1 + 3\nu = 1.633$. At the inner radius, from Prob. 5-19

$$\frac{\sigma_t}{\omega^2} = \rho\left(\frac{3+\nu}{8}\right)\left(2r_o^2 + r_i^2 - \frac{1+3\nu}{3+\nu}r_i^2\right)$$

Here $r_o^2 = 25$, $r_i^2 = 9$, and so

$$\frac{\sigma_t}{\omega^2} = \frac{0.260}{386}\left(\frac{3.211}{8}\right)\left(50 + 9 - \frac{1.633(9)}{3.211}\right) = 0.0147$$

Since σ_r is of the same sign, we use M2M failure criteria in the first quadrant. From Table A-24, $S_{ut} = 31$ kpsi, thus,

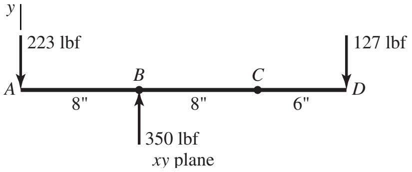
$$\omega = \left(\frac{31\,000}{0.0147}\right)^{1/2} = 1452\ \text{rad/s}$$

$$\text{rpm} = 60\omega/(2\pi) = 60(1452)/(2\pi)$$

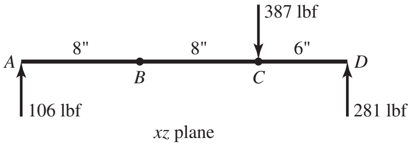
$$= 13\,866\ \text{rev/min}$$

Using the grade number of 30 for $S_{ut} = 30\,000$ kpsi gives a bursting speed of 13640 rev/min.

5-23 $T_C = (360 - 27)(3) = 1000\ \text{lbf} \cdot \text{in}$, $\quad T_B = (300 - 50)(4) = 1000\ \text{lbf} \cdot \text{in}$



In xy plane, $M_B = 223(8) = 1784\ \text{lbf} \cdot \text{in}$ and $M_C = 127(6) = 762\ \text{lbf} \cdot \text{in}$.



In the xz plane, $M_B = 848\ \text{lbf} \cdot \text{in}$ and $M_C = 1686\ \text{lbf} \cdot \text{in}$. The resultants are

$$M_B = [(1784)^2 + (848)^2]^{1/2} = 1975\ \text{lbf} \cdot \text{in}$$

$$M_C = [(1686)^2 + (762)^2]^{1/2} = 1850\ \text{lbf} \cdot \text{in}$$

So point B governs and the stresses are

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi d^3} = \frac{5093}{d^3}\ \text{psi}$$

$$\sigma_x = \frac{32M_B}{\pi d^3} = \frac{32(1975)}{\pi d^3} = \frac{20\,120}{d^3}\ \text{psi}$$

Then

$$\sigma_A, \sigma_B = \frac{\sigma_x}{2} \pm \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$\sigma_A, \sigma_B = \frac{1}{d^3}\left\{\frac{20.12}{2} \pm \left[\left(\frac{20.12}{2}\right)^2 + (5.09)^2\right]^{1/2}\right\}$$

$$= \frac{(10.06 \pm 11.27)}{d^3}\ \text{kpsi} \cdot \text{in}^3$$

Then

$$\sigma_A = \frac{10.06 + 11.27}{d^3} = \frac{21.33}{d^3}\text{ kpsi}$$

and

$$\sigma_B = \frac{10.06 - 11.27}{d^3} = -\frac{1.21}{d^3}\text{ kpsi}$$

For this state of stress, use the Brittle-Coulomb-Mohr theory for illustration. Here we use $S_{ut}(\text{min}) = 25\text{ kpsi}$, $S_{uc}(\text{min}) = 97\text{ kpsi}$, and Eq. (5-31*b*) to arrive at

$$\frac{21.33}{25d^3} - \frac{-1.21}{97d^3} = \frac{1}{2.8}$$

Solving gives $d = 1.34$ in. So use $d = 1\,3/8$ in *Ans.*

Note that this has been solved as a statics problem. Fatigue will be considered in the next chapter.

5-24 As in Prob. 5-23, we will assume this to be statics problem. Since the proportions are unchanged, the bearing reactions will be the same as in Prob. 5-23. Thus

xy plane: $M_B = 223(4) = 892\text{ lbf}\cdot\text{in}$

xz plane: $M_B = 106(4) = 424\text{ lbf}\cdot\text{in}$

So

$$M_{\max} = [(892)^2 + (424)^2]^{1/2} = 988\text{ lbf}\cdot\text{in}$$

$$\sigma_x = \frac{32M_B}{\pi d^3} = \frac{32(988)}{\pi d^3} = \frac{10\,060}{d^3}\text{ psi}$$

Since the torsional stress is unchanged,

$$\tau_{xz} = 5.09/d^3\text{ kpsi}$$

$$\sigma_A, \sigma_B = \frac{1}{d^3}\left\{\left(\frac{10.06}{2}\right) \pm \left[\left(\frac{10.06}{2}\right)^2 + (5.09)^2\right]^{1/2}\right\}$$

$$\sigma_A = 12.19/d^3 \quad\text{and}\quad \sigma_B = -2.13/d^3$$

Using the Brittle-Coulomb-Mohr, as was used in Prob. 5-23, gives

$$\frac{12.19}{25d^3} - \frac{-2.13}{97d^3} = \frac{1}{2.8}$$

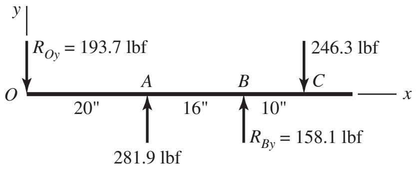
Solving gives $d = 1\,1/8$ in. *Ans.*

5-25

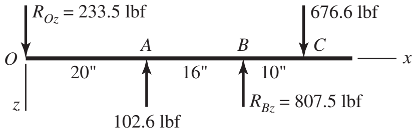
$$(F_A)_t = 300\cos 20 = 281.9\text{ lbf}, \quad (F_A)_r = 300\sin 20 = 102.6\text{ lbf}$$

$$T = 281.9(12) = 3383\text{ lbf}\cdot\text{in}, \quad (F_C)_t = \frac{3383}{5} = 676.6\text{ lbf}$$

$$(F_C)_r = 676.6\tan 20 = 246.3\text{ lbf}$$



xy plane



xz plane

$$M_A = 20\sqrt{193.7^2 + 233.5^2} = 6068\ \text{lbf}\cdot\text{in}$$

$$M_B = 10\sqrt{246.3^2 + 676.6^2} = 7200\ \text{lbf}\cdot\text{in}\quad \text{(maximum)}$$

$$\sigma_x = \frac{32(7200)}{\pi d^3} = \frac{73\,340}{d^3}$$

$$\tau_{xy} = \frac{16(3383)}{\pi d^3} = \frac{17\,230}{d^3}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \frac{S_y}{n}$$

$$\left[\left(\frac{73\,340}{d^3}\right)^2 + 3\left(\frac{17\,230}{d^3}\right)^2\right]^{1/2} = \frac{79\,180}{d^3} = \frac{60\,000}{3.5}$$

$d = 1.665$ in so use a standard diameter size of 1.75 in *Ans.*

5-26 From Prob. 5-25,

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2} = \frac{S_y}{2n}$$

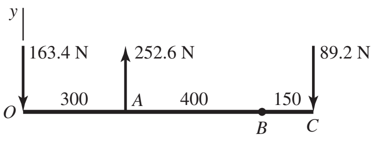
$$\left[\left(\frac{73\,340}{2d^3}\right)^2 + \left(\frac{17\,230}{d^3}\right)^2\right]^{1/2} = \frac{40\,516}{d^3} = \frac{60\,000}{2(3.5)}$$

$d = 1.678$ in so use 1.75 in *Ans.*

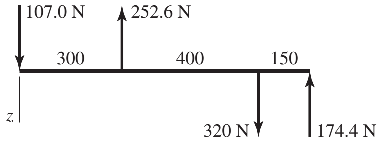
5-27 $T = (270 - 50)(0.150) = 33\ \text{N}\cdot\text{m},\ S_y = 370\ \text{MPa}$

$$(T_1 - 0.15T_1)(0.125) = 33 \quad\Rightarrow\quad T_1 = 310.6\ \text{N},\quad T_2 = 0.15(310.6) = 46.6\ \text{N}$$

$$(T_1 + T_2)\cos 45 = 252.6\ \text{N}$$



xy plane



xz plane

$$M_A = 0.3\sqrt{163.4^2 + 107^2} = 58.59\ \mathrm{N\cdot m} \qquad \text{(maximum)}$$

$$M_B = 0.15\sqrt{89.2^2 + 174.4^2} = 29.38\ \mathrm{N\cdot m}$$

$$\sigma_x = \frac{32(58.59)}{\pi d^3} = \frac{596.8}{d^3}$$

$$\tau_{xy} = \frac{16(33)}{\pi d^3} = \frac{168.1}{d^3}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \left[\left(\frac{596.8}{d^3}\right)^2 + 3\left(\frac{168.1}{d^3}\right)^2\right]^{1/2} = \frac{664.0}{d^3} = \frac{370(10^6)}{3.0}$$

$d = 17.5(10^{-3})$ m $= 17.5$ mm, so use 18 mm *Ans.*

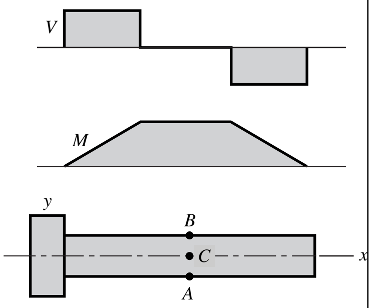
5-28 From Prob. 5-27,

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2} = \frac{S_y}{2n}$$

$$\left[\left(\frac{596.8}{2d^3}\right)^2 + \left(\frac{168.1}{d^3}\right)^2\right]^{1/2} = \frac{342.5}{d^3} = \frac{370(10^6)}{2(3.0)}$$

$d = 17.7(10^{-3})$ m $= 17.7$ mm, so use 18 mm *Ans.*

5-29 For the loading scheme shown in Figure (*c*),



$$M_{\max} = \frac{F}{2}\left(\frac{a}{2} + \frac{b}{4}\right) = \frac{4.4}{2}(6 + 4.5)$$
$$= 23.1\ \mathrm{N\cdot m}$$

For a stress element at *A*:

$$\sigma_x = \frac{32M}{\pi d^3} = \frac{32(23.1)(10^3)}{\pi(12)^3} = 136.2\ \mathrm{MPa}$$

The shear at *C* is

$$\tau_{xy} = \frac{4(F/2)}{3\pi d^2/4} = \frac{4(4.4/2)(10^3)}{3\pi(12)^2/4} = 25.94\ \mathrm{MPa}$$

$$\tau_{\max} = \left[\left(\frac{136.2}{2}\right)^2\right]^{1/2} = 68.1\ \mathrm{MPa}$$

Since $S_y = 220\ \mathrm{MPa}$, $S_{sy} = 220/2 = 110\ \mathrm{MPa}$, and

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{110}{68.1} = 1.62 \quad \textit{Ans.}$$

For the loading scheme depicted in Figure (*d*)

$$M_{\max} = \frac{F}{2}\left(\frac{a+b}{2}\right) - \frac{F}{2}\left(\frac{1}{2}\right)\left(\frac{b}{2}\right)^2 = \frac{F}{2}\left(\frac{a}{2}+\frac{b}{4}\right)$$

This result is the same as that obtained for Figure (*c*). At point *B*, we also have a surface compression of

$$\sigma_y = \frac{-F}{A} = \frac{-F}{bd} - \frac{-4.4(10^3)}{18(12)} = -20.4 \text{ MPa}$$

With $\sigma_x = -136.2$ MPa. From a Mohrs circle diagram, $\tau_{\max} = 136.2/2 = 68.1$ MPa.

$$n = \frac{110}{68.1} = 1.62 \text{ MPa} \quad \textit{Ans}.$$

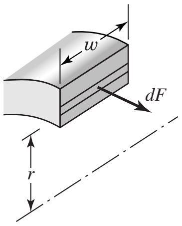
5-30 Based on Figure (*c*) and using Eq. (5-15)

$$\sigma' = \left(\sigma_x^2\right)^{1/2}$$
$$= (136.2^2)^{1/2} = 136.2 \text{ MPa}$$
$$n = \frac{S_y}{\sigma'} = \frac{220}{136.2} = 1.62 \quad \textit{Ans.}$$

Based on Figure (*d*) and using Eq. (5-15) and the solution of Prob. 5-29,

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2\right)^{1/2}$$
$$= [(-136.2)^2 - (-136.2)(-20.4) + (-20.4)^2]^{1/2}$$
$$= 127.2 \text{ MPa}$$
$$n = \frac{S_y}{\sigma'} = \frac{220}{127.2} = 1.73 \quad \textit{Ans.}$$

5-31



When the ring is set, the hoop tension in the ring is equal to the screw tension.

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

We have the hoop tension at any radius. The differential hoop tension dF is

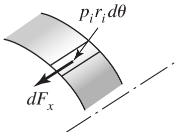
$$dF = w\sigma_t\, dr$$

$$F = \int_{r_i}^{r_o} w\sigma_t\, dr = \frac{wr_i^2 p_i}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(1 + \frac{r_o^2}{r^2}\right) dr = wr_i p_i \qquad (1)$$

The screw equation is

$$F_i = \frac{T}{0.2d} \tag{2}$$

From Eqs. (1) and (2)



$$p_i = \frac{F}{wr_i} = \frac{T}{0.2dwr_i}$$

$$dF_x = f p_i r_i \, d\theta$$

$$F_x = \int_o^{2\pi} f p_i w r_i \, d\theta = \frac{fTw}{0.2dwr_i} r_i \int_o^{2\pi} d\theta$$

$$= \frac{2\pi f T}{0.2d} \quad Ans.$$

5-32

(a) From Prob. 5-31, $T = 0.2F_i d$

$$F_i = \frac{T}{0.2d} = \frac{190}{0.2(0.25)} = 3800 \text{ lbf} \quad Ans.$$

(b) From Prob. 5-31, $F = wr_i p_i$

$$p_i = \frac{F}{wr_i} = \frac{F_i}{wr_i} = \frac{3800}{0.5(0.5)} = 15\,200 \text{ psi} \quad Ans.$$

(c)

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r}\right)_{r=r_i} = \frac{p_i\left(r_i^2 + r_o^2\right)}{r_o^2 - r_i^2}$$

$$= \frac{15\,200(0.5^2 + 1^2)}{1^2 - 0.5^2} = 25\,333 \text{ psi} \quad Ans.$$

$$\sigma_r = -p_i = -15\,200 \text{ psi}$$

(d)

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_t - \sigma_r}{2}$$

$$= \frac{25\,333 - (-15\,200)}{2} = 20\,267 \text{ psi} \quad Ans.$$

$$\sigma' = \left(\sigma_A^2 + \sigma_B^2 - \sigma_A\sigma_B\right)^{1/2}$$

$$= [25\,333^2 + (-15\,200)^2 - 25\,333(-15\,200)]^{1/2}$$

$$= 35\,466 \text{ psi} \quad Ans.$$

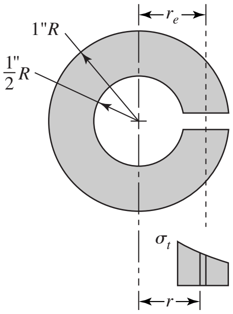
(e) Maximum Shear hypothesis

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{0.5S_y}{\tau_{\max}} = \frac{0.5(63)}{20.267} = 1.55 \quad Ans.$$

Distortion Energy theory

$$n = \frac{S_y}{\sigma'} = \frac{63}{35\,466} = 1.78 \quad Ans.$$

5-33



The moment about the center caused by force F is Fr_e where r_e is the effective radius. This is balanced by the moment about the center caused by the tangential (hoop) stress.

$$Fr_e = \int_{r_i}^{r_o} r\sigma_t w\,dr$$

$$= \frac{wp_i r_i^2}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(r + \frac{r_o^2}{r}\right)dr$$

$$r_e = \frac{wp_i r_i^2}{F\left(r_o^2 - r_i^2\right)}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)$$

From Prob. 5-31, $F = wr_i p_i$. Therefore,

$$r_e = \frac{r_i}{r_o^2 - r_i^2}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)$$

For the conditions of Prob. 5-31, $r_i = 0.5$ and $r_o = 1$ in

$$r_e = \frac{0.5}{1^2 - 0.5^2}\left(\frac{1^2 - 0.5^2}{2} + 1^2 \ln\frac{1}{0.5}\right) = 0.712 \text{ in}$$

5-34 $\delta_{\text{nom}} = 0.0005$ in

(a) From Eq. (3-57)

$$p = \frac{30(10^6)(0.0005)}{(1^3)}\left[\frac{(1.5^2 - 1^2)(1^2 - 0.5^2)}{2(1.5^2 - 0.5^2)}\right] = 3516 \text{ psi} \quad \textit{Ans.}$$

Inner member:

Eq. (3-58) $$(\sigma_t)_i = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -3516\left(\frac{1^2 + 0.5^2}{1^2 - 0.5^2}\right) = -5860 \text{ psi}$$

$$(\sigma_r)_i = -p = -3516 \text{ psi}$$

Eq. (5-13) $$\sigma_i' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$$

$$= [(-5860)^2 - (-5860)(-3516) + (-3516)^2]^{1/2}$$

$$= 5110 \text{ psi} \quad \textit{Ans.}$$

Outer member:

Eq. (3-59) $$(\sigma_t)_o = 3516\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 9142 \text{ psi}$$

$$(\sigma_r)_o = -p = -3516 \text{ psi}$$

Eq. (5-13) $$\sigma_o' = [9142^2 - 9142(-3516) + (-3516)^2]^{1/2}$$

$$= 11\,320 \text{ psi} \quad \textit{Ans.}$$

(b) For a solid inner tube,

$$p = \frac{30(10^6)(0.0005)}{1}\left[\frac{(1.5^2 - 1^2)(1^2)}{2(1^2)(1.5^2)}\right] = 4167 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_i = -p = -4167 \text{ psi}, \quad (\sigma_r)_i = -4167 \text{ psi}$$

$$\sigma_i' = [(-4167)^2 - (-4167)(-4167) + (-4167)^2]^{1/2} = 4167 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_o = 4167\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 10\,830 \text{ psi}, \quad (\sigma_r)_o = -4167 \text{ psi}$$

$$\sigma_o' = [10\,830^2 - 10\,830(-4167) + (-4167)^2]^{1/2} = 13\,410 \text{ psi} \quad \textit{Ans.}$$

5-35 Using Eq. (3-57) with diametral values,

$$p = \frac{207(10^3)(0.02)}{(50^3)}\left[\frac{(75^2 - 50^2)(50^2 - 25^2)}{2(75^2 - 25^2)}\right] = 19.41 \text{ MPa} \quad \textit{Ans.}$$

Eq. (3-58) $$(\sigma_t)_i = -19.41\left(\frac{50^2 + 25^2}{50^2 - 25^2}\right) = -32.35 \text{ MPa}$$

$$(\sigma_r)_i = -19.41 \text{ MPa}$$

Eq. (5-13) $$\sigma_i' = [(-32.35)^2 - (-32.35)(-19.41) + (-19.41)^2]^{1/2}$$
$$= 28.20 \text{ MPa} \quad \textit{Ans.}$$

Eq. (3-59) $$(\sigma_t)_o = 19.41\left(\frac{75^2 + 50^2}{75^2 - 50^2}\right) = 50.47 \text{ MPa},$$

$$(\sigma_r)_o = -19.41 \text{ MPa}$$

$$\sigma_o' = [50.47^2 - 50.47(-19.41) + (-19.41)^2]^{1/2} = 62.48 \text{ MPa} \quad \textit{Ans.}$$

5-36 Max. shrink-fit conditions: Diametral interference $\delta_d = 50.01 - 49.97 = 0.04$ mm. Equation (3-57) using diametral values:

$$p = \frac{207(10^3)0.04}{50^3}\left[\frac{(75^2 - 50^2)(50^2 - 25^2)}{2(75^2 - 25^2)}\right] = 38.81 \text{ MPa} \qquad \textit{Ans.}$$

Eq. (3-58): $$(\sigma_t)_i = -38.81\left(\frac{50^2 + 25^2}{50^2 - 25^2}\right) = -64.68 \text{ MPa}$$

$$(\sigma_r)_i = -38.81 \text{ MPa}$$

Eq. (5-13):

$$\sigma_i' = \left[(-64.68)^2 - (-64.68)(-38.81) + (-38.81)^2\right]^{1/2}$$

$$= 56.39 \text{ MPa} \qquad \textit{Ans.}$$

5-37

$$\delta = \frac{1.9998}{2} - \frac{1.999}{2} = 0.0004 \text{ in}$$

Eq. (3-56)

$$0.0004 = \frac{p(1)}{14.5(10^6)}\left[\frac{2^2+1^2}{2^2-1^2} + 0.211\right] + \frac{p(1)}{30(10^6)}\left[\frac{1^2+0}{1^2-0} - 0.292\right]$$

$$p = 2613 \text{ psi}$$

Applying Eq. (4-58) at R,

$$(\sigma_t)_o = 2613\left(\frac{2^2+1^2}{2^2-1^2}\right) = 4355 \text{ psi}$$

$$(\sigma_r)_o = -2613 \text{ psi}, \quad S_{ut} = 20 \text{ kpsi}, \quad S_{uc} = 83 \text{ kpsi}$$

$$\left|\frac{\sigma_o}{\sigma_A}\right| = \frac{2613}{4355} < 1, \ \therefore \text{ use Eq. (5-32a)}$$

$$h = S_{ut}/\sigma_A = 20/4.355 = 4.59 \quad \textit{Ans.}$$

5-38 $E = 30(10^6)$ psi, $\nu = 0.292$, $I = (\pi/64)(2^4 - 1.5^4) = 0.5369 \text{ in}^4$

Eq. (3-57) can be written in terms of diameters,

$$p = \frac{E\delta_d}{D}\left[\frac{\left(d_o^2 - D^2\right)\left(D^2 - d_i^2\right)}{2D^2\left(d_o^2 - d_i^2\right)}\right] = \frac{30(10^6)}{1.75}(0.002\,46)\left[\frac{(2^2-1.75^2)(1.75^2-1.5^2)}{2(1.75^2)(2^2-1.5^2)}\right]$$

$$= 2997 \text{ psi} = 2.997 \text{ kpsi}$$

Outer member:

Outer radius: $(\sigma_t)_o = \dfrac{1.75^2(2.997)}{2^2 - 1.75^2}(2) = 19.58 \text{ kpsi}, (\sigma_r)_o = 0$

Inner radius: $(\sigma_t)_i = \dfrac{1.75^2(2.997)}{2^2 - 1.75^2}\left(1 + \dfrac{2^2}{1.75^2}\right) = 22.58 \text{ kpsi}, (\sigma_r)_i = -2.997 \text{ kpsi}$

Bending:

r_o: $(\sigma_x)_o = \dfrac{6.000(2/2)}{0.5369} = 11.18 \text{ kpsi}$

r_i: $(\sigma_x)_i = \dfrac{6.000(1.75/2)}{0.5369} = 9.78 \text{ kpsi}$

Torsion: $J = 2I = 1.0738 \text{ in}^4$

r_o: $(\tau_{xy})_o = \dfrac{8.000(2/2)}{1.0738} = 7.45 \text{ kpsi}$

r_i: $(\tau_{xy})_i = \dfrac{8.000(1.75/2)}{1.0738} = 6.52 \text{ kpsi}$

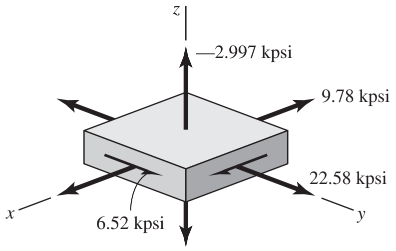
Outer radius is plane stress

$$\sigma_x = 11.18 \text{ kpsi}, \quad \sigma_y = 19.58 \text{ kpsi}, \quad \tau_{xy} = 7.45 \text{ kpsi}$$

Eq. (5-15) $\sigma' = [11.18^2 - (11.18)(19.58) + 19.58^2 + 3(7.45^2)]^{1/2} = \dfrac{S_y}{n_o} = \dfrac{60}{n_o}$

$$21.35 = \frac{60}{n_o} \quad \Rightarrow \quad n_o = 2.81 \quad \textit{Ans.}$$

Inner radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$

$$\sigma' = \frac{1}{\sqrt{2}}[(9.78 - 22.58)^2 + (22.58 + 2.997)^2 + (-2.997 - 9.78)^2 + 6(6.52)^2]^{1/2} = \frac{60}{n_i}$$

$$24.86 = \frac{60}{n_i} \quad \Rightarrow \quad n_i = 2.41 \quad \textit{Ans.}$$

5-39 From Prob. 5-38: $p = 2.997$ kpsi, $I = 0.5369$ in^4, $J = 1.0738$ in^4

Inner member:

Outer radius: $(\sigma_t)_o = -2.997 \left[\dfrac{(0.875^2 + 0.75^2)}{(0.875^2 - 0.75^2)}\right] = -19.60$ kpsi

$$(\sigma_r)_o = -2.997 \text{ kpsi}$$

Inner radius: $(\sigma_t)_i = -\dfrac{2(2.997)(0.875^2)}{0.875^2 - 0.75^2} = -22.59$ kpsi

$$(\sigma_r)_i = 0$$

Bending:

r_o: $(\sigma_x)_o = \dfrac{6(0.875)}{0.5369} = 9.78$ kpsi

r_i: $(\sigma_x)_i = \dfrac{6(0.75)}{0.5369} = 8.38$ kpsi

Torsion:

r_o: $(\tau_{xy})_o = \dfrac{8(0.875)}{1.0738} = 6.52$ kpsi

r_i: $(\tau_{xy})_i = \dfrac{8(0.75)}{1.0738} = 5.59$ kpsi

The inner radius is in plane stress: $\sigma_x = 8.38$ kpsi, $\sigma_y = -22.59$ kpsi, $\tau_{xy} = 5.59$ kpsi

$$\sigma_i' = [8.38^2 - (8.38)(-22.59) + (-22.59)^2 + 3(5.59^2)]^{1/2} = 29.4 \text{ kpsi}$$

$$n_i = \frac{S_y}{\sigma_i'} = \frac{60}{29.4} = 2.04 \quad \textit{Ans.}$$

Outer radius experiences a radial stress, σ_r

$$\sigma_o' = \frac{1}{\sqrt{2}}\left[(-19.60 + 2.997)^2 + (-2.997 - 9.78)^2 + (9.78 + 19.60)^2 + 6(6.52)^2\right]^{1/2}$$
$$= 27.9 \text{ kpsi}$$

$$n_o = \frac{60}{27.9} = 2.15 \quad \textit{Ans.}$$

5-40

$$\sigma_p = \frac{1}{2}\left(2\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\right) \pm \left[\left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\frac{3\theta}{2}\right)^2 + \left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)^2\right]^{1/2}$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left[\cos\frac{\theta}{2} \pm \left(\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\sin^2\frac{3\theta}{2} + \sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\cos^2\frac{3\theta}{2}\right)^{1/2}\right]$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left(\cos\frac{\theta}{2} \pm \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 \pm \sin\frac{\theta}{2}\right)$$

Plane stress: The third principal stress is zero and

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right), \quad \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right), \quad \sigma_3 = 0 \quad \textit{Ans.}$$

Plane strain: σ_1 and σ_2 equations still valid however,

$$\sigma_3 = \nu(\sigma_x + \sigma_y) = 2\nu\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2} \quad \textit{Ans.}$$

5-41 For $\theta = 0$ and plane strain, the principal stress equations of Prob. 5-40 give

$$\sigma_1 = \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}, \quad \sigma_3 = 2\nu\frac{K_I}{\sqrt{2\pi r}} = 2\nu\sigma_1$$

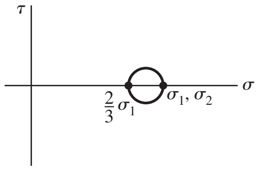
(a) DE: $$\frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_1)^2 + (\sigma_1 - 2\nu\sigma_1)^2 + (2\nu\sigma_1 - \sigma_1)^2]^{1/2} = S_y$$
$$\sigma_1 - 2\nu\sigma_1 = S_y$$

For $\nu = \frac{1}{3}$, $$\left[1 - 2\left(\frac{1}{3}\right)\right]\sigma_1 = S_y \quad \Rightarrow \quad \sigma_1 = 3S_y \quad \textit{Ans.}$$

(b) MSS: $\sigma_1 - \sigma_3 = S_y \quad \Rightarrow \quad \sigma_1 - 2\nu\sigma_1 = S_y$

$$\nu = \frac{1}{3} \quad \Rightarrow \quad \sigma_1 = 3S_y \quad \textit{Ans.}$$

$$\sigma_3 = \frac{2}{3}\sigma_1$$



Radius of largest circle

$$R = \frac{1}{2}\left[\sigma_1 - \frac{2}{3}\sigma_1\right] = \frac{\sigma_1}{6}$$

5-42 **(a)** Ignoring stress concentration

$$F = S_y A = 160(4)(0.5) = 320 \text{ kips} \quad \textit{Ans.}$$

(b) From Fig. 6-36: $h/b = 1$, $a/b = 0.625/4 = 0.1563$, $\beta = 1.3$

Eq. (6-51) $$70 = 1.3\frac{F}{4(0.5)}\sqrt{\pi(0.625)}$$

$$F = 76.9 \text{ kips} \quad \textit{Ans.}$$

5-43 Given: $a = 12.5$ mm, $K_{Ic} = 80 \text{ MPa} \cdot \sqrt{m}$, $S_y = 1200$ MPa, $S_{ut} = 1350$ MPa

$$r_o = \frac{350}{2} = 175 \text{ mm}, \quad r_i = \frac{350 - 50}{2} = 150 \text{ mm}$$

$$a/(r_o - r_i) = \frac{12.5}{175 - 150} = 0.5$$

$$r_i/r_o = \frac{150}{175} = 0.857$$

Fig. 5-30: $\beta \doteq 2.5$

Eq. (5-37): $K_{Ic} = \beta\sigma\sqrt{\pi a}$

$$80 = 2.5\sigma\sqrt{\pi(0.0125)}$$

$$\sigma = 161.5 \text{ MPa}$$

Eq. (3-50) at $r = r_o$:

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}(2)$$

$$161.5 = \frac{150^2 p_i(2)}{175^2 - 150^2}$$

$$p_i = 29.2 \text{ MPa} \quad \textit{Ans.}$$

5-44

(a) First convert the data to radial dimensions to agree with the formulations of Fig. 3-33. Thus

$$r_o = 0.5625 \pm 0.001\,\text{in}$$
$$r_i = 0.1875 \pm 0.001\,\text{in}$$
$$R_o = 0.375 \pm 0.0002\,\text{in}$$
$$R_i = 0.376 \pm 0.0002\,\text{in}$$

The stochastic nature of the dimensions affects the $\boldsymbol{\delta} = |\mathbf{R}_i| - |\mathbf{R}_o|$ relation in Eq. (3-57) but not the others. Set $R = (1/2)(R_i + R_o) = 0.3755$. From Eq. (3-57)

$$\mathbf{p} = \frac{E\boldsymbol{\delta}}{R}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{2R^2\left(r_o^2 - r_i^2\right)}\right]$$

Substituting and solving with $E = 30$ Mpsi gives

$$\mathbf{p} = 18.70(10^6)\,\boldsymbol{\delta}$$

Since $\boldsymbol{\delta} = \mathbf{R}_i - \mathbf{R}_o$

$$\bar{\delta} = \bar{R}_i - \bar{R}_o = 0.376 - 0.375 = 0.001 \text{ in}$$

and

$$\hat{\sigma}_\delta = \left[\left(\frac{0.0002}{4}\right)^2 + \left(\frac{0.0002}{4}\right)^2\right]^{1/2}$$
$$= 0.000\,070\,7 \text{ in}$$

Then

$$C_\delta = \frac{\hat{\sigma}_\delta}{\bar{\delta}} = \frac{0.000\,070\,7}{0.001} = 0.0707$$

The tangential inner-cylinder stress at the shrink-fit surface is given by

$$\boldsymbol{\sigma}_{it} = -\mathbf{p}\frac{\bar{R}^2 + \bar{r}_i^2}{\bar{R}^2 - \bar{r}_i^2}$$
$$= -18.70(10^6)\,\boldsymbol{\delta}\left(\frac{0.3755^2 + 0.1875^2}{0.3755^2 - 0.1875^2}\right)$$
$$= -31.1(10^6)\,\boldsymbol{\delta}$$
$$\bar{\sigma}_{it} = -31.1(10^6)\,\bar{\delta} = -31.1(10^6)(0.001)$$
$$= -31.1(10^3) \text{ psi}$$

Also

$$\hat{\sigma}_{\sigma_{it}} = |C_\delta \bar{\sigma}_{it}| = 0.0707(-31.1)10^3$$
$$= 2899 \text{ psi}$$
$$\boldsymbol{\sigma}_{it} = \mathbf{N}(-31\,100,\, 2899) \text{ psi} \quad \textit{Ans.}$$

(b) The tangential stress for the outer cylinder at the shrink-fit surface is given by

$$\begin{aligned}\boldsymbol{\sigma}_{ot} &= \mathbf{p}\left(\frac{\bar{r}_o^2+\bar{R}^2}{\bar{r}_o^2-\bar{R}^2}\right)\\ &= 18.70(10^6)\,\boldsymbol{\delta}\left(\frac{0.5625^2+0.3755^2}{0.5625^2-0.3755^2}\right)\\ &= 48.76(10^6)\,\boldsymbol{\delta}\ \text{psi}\\ \bar{\sigma}_{ot} &= 48.76(10^6)(0.001) = 48.76(10^3)\ \text{psi}\\ \hat{\sigma}_{\sigma_{ot}} &= C_\delta\bar{\sigma}_{ot} = 0.0707(48.76)(10^3) = 34.45\ \text{psi}\\ \therefore \boldsymbol{\sigma}_{ot} &= \mathbf{N}(48\,760,\ 3445)\ \text{psi}\quad \textit{Ans.}\end{aligned}$$

5-45 From Prob. 5-44, at the fit surface $\boldsymbol{\sigma}_{ot} = \mathbf{N}(48.8, 3.45)$ kpsi. The radial stress is the fit pressure which was found to be

$$\begin{aligned}\mathbf{p} &= 18.70(10^6)\,\boldsymbol{\delta}\\ \bar{p} &= 18.70(10^6)(0.001) = 18.7(10^3)\ \text{psi}\\ \hat{\sigma}_p &= C_\delta\bar{p} = 0.0707(18.70)(10^3)\\ &= 1322\ \text{psi}\end{aligned}$$

and so

$$\mathbf{p} = \mathbf{N}(18.7,\ 1.32)\ \text{kpsi}$$

and

$$\boldsymbol{\sigma}_{or} = -\mathbf{N}(18.7,\ 1.32)\ \text{kpsi}$$

These represent the principal stresses. The von Mises stress is next assessed.

$$\begin{aligned}\bar{\sigma}_A &= 48.8\ \text{kpsi},\quad \bar{\sigma}_B = -18.7\ \text{kpsi}\\ k &= \bar{\sigma}_B/\bar{\sigma}_A = -18.7/48.8 = -0.383\\ \bar{\sigma}' &= \bar{\sigma}_A(1-k+k^2)^{1/2}\\ &= 48.8[1-(-0.383)+(-0.383)^2]^{1/2}\\ &= 60.4\ \text{kpsi}\\ \hat{\sigma}_{\sigma'} &= C_p\bar{\sigma}' = 0.0707(60.4) = 4.27\ \text{kpsi}\end{aligned}$$

Using the interference equation

$$\begin{aligned}z &= -\frac{\bar{S}-\bar{\sigma}'}{\left(\hat{\sigma}_S^2+\hat{\sigma}_{\sigma'}^2\right)^{1/2}}\\ &= -\frac{95.5-60.4}{[(6.59)^2+(4.27)^2]^{1/2}} = -4.5\\ p_f &= \alpha = 0.000\,003\,40,\end{aligned}$$

or about 3 chances in a million. *Ans.*

5-46

$$\boldsymbol{\sigma}_t = \frac{\mathbf{p}d}{2t} = \frac{6000\mathbf{N}(1, 0.083\,33)(0.75)}{2(0.125)}$$

$$= 18\mathbf{N}(1, 0.083\,33) \text{ kpsi}$$

$$\boldsymbol{\sigma}_l = \frac{\mathbf{p}d}{4t} = \frac{6000\mathbf{N}(1, 0.083\,33)(0.75)}{4(0.125)}$$

$$= 9\mathbf{N}(1, 0.083\,33) \text{ kpsi}$$

$$\boldsymbol{\sigma}_r = -\mathbf{p} = -6000\mathbf{N}(1, 0.083\,33) \text{ kpsi}$$

These three stresses are principal stresses whose variability is due to the loading. From Eq. (5-12), we find the von Mises stress to be

$$\sigma' = \left\{\frac{(18-9)^2 + [9-(-6)]^2 + (-6-18)^2}{2}\right\}^{1/2}$$

$$= 21.0 \text{ kpsi}$$

$$\hat{\sigma}_{\sigma'} = C_p\bar{\sigma}' = 0.083\,33(21.0) = 1.75 \text{ kpsi}$$

$$z = -\frac{\bar{S} - \bar{\sigma}'}{\left(\hat{\sigma}_S^2 + \hat{\sigma}_{\sigma'}^2\right)^{1/2}}$$

$$= \frac{50 - 21.0}{(4.1^2 + 1.75^2)^{1/2}} = -6.5$$

The reliability is very high

$$R = 1 - \Phi(6.5) = 1 - 4.02(10^{-11}) \doteq 1 \quad Ans.$$

Chapter 6

Note to the instructor: Many of the problems in this chapter are carried over from the previous edition. The solutions have changed slightly due to some minor changes. First, the calculation of the endurance limit of a rotating-beam specimen S'_e is given by $S'_e = 0.5S_{ut}$ instead of $S'_e = 0.504S_{ut}$. Second, when the fatigue stress calculation is made for deterministic problems, only one approach is given, which uses the notch sensitivity factor, q, together with Eq. (6-32). *Neuber's* equation, Eq. (6-33), is simply another form of this. These changes were made to hopefully make the calculations less confusing, and diminish the idea that stress life calculations are precise.

6-1 $H_B = 490$

Eq. (2-17): $S_{ut} = 0.495(490) = 242.6\text{ kpsi} > 212\text{ kpsi}$

Eq. (6-8): $S'_e = 100\text{ kpsi}$

Table 6-2: $a = 1.34, \quad b = -0.085$

Eq. (6-19): $k_a = 1.34(242.6)^{-0.085} = 0.840$

Eq. (6-20): $k_b = \left(\dfrac{1/4}{0.3}\right)^{-0.107} = 1.02$

Eq. (6-18): $S_e = k_a k_b S'_e = 0.840(1.02)(100) = 85.7\text{ kpsi}$ *Ans.*

6-2

(a) $S_{ut} = 68\text{ kpsi}$, $S'_e = 0.5(68) = 34\text{ kpsi}$ *Ans.*

(b) $S_{ut} = 112\text{ kpsi}$, $S'_e = 0.5(112) = 56\text{ kpsi}$ *Ans.*

(c) 2024T3 has no endurance limit *Ans.*

(d) Eq. (6-8): $S'_e = 100\text{ kpsi}$ *Ans.*

6-3

Eq. (2-11): $\sigma'_F = \sigma_0\varepsilon^m = 115(0.90)^{0.22} = 112.4\text{ kpsi}$

Eq. (6-8): $S'_e = 0.5(66.2) = 33.1\text{ kpsi}$

Eq. (6-12): $b = -\dfrac{\log(112.4/33.1)}{\log(2\cdot 10^6)} = -0.084\,26$

Eq. (6-10): $f = \dfrac{112.4}{66.2}(2\cdot 10^3)^{-0.084\,26} = 0.8949$

Eq. (6-14): $a = \dfrac{[0.8949(66.2)]^2}{33.1} = 106.0\text{ kpsi}$

Eq. (6-13): $S_f = aN^b = 106.0(12\,500)^{-0.084\,26} = 47.9\text{ kpsi}$ *Ans.*

Eq. (6-16): $N = \left(\dfrac{\sigma_a}{a}\right)^{1/b} = \left(\dfrac{36}{106.0}\right)^{-1/0.084\,26} = 368\,250\text{ cycles}$ *Ans.*

6-4 From $S_f = aN^b$

$$\log S_f = \log a + b \log N$$

Substituting $(1, S_{ut})$

$$\log S_{ut} = \log a + b \log (1)$$

From which $\quad a = S_{ut}$

Substituting $(10^3, fS_{ut})$ and $a = S_{ut}$

$$\log fS_{ut} = \log S_{ut} + b \log 10^3$$

From which

$$b = \frac{1}{3} \log f$$

$$\therefore S_f = S_{ut} N^{(\log f)/3} \qquad 1 \le N \le 10^3$$

For 500 cycles as in Prob. 6-3

$$S_f \ge 66.2(500)^{(\log 0.8949)/3} = 59.9 \text{ kpsi} \quad \textit{Ans.}$$

6-5 Read from graph: $(10^3, 90)$ and $(10^6, 50)$. From $S = aN^b$

$$\log S_1 = \log a + b \log N_1$$

$$\log S_2 = \log a + b \log N_2$$

From which

$$\log a = \frac{\log S_1 \log N_2 - \log S_2 \log N_1}{\log N_2/N_1}$$

$$= \frac{\log 90 \log 10^6 - \log 50 \log 10^3}{\log 10^6/10^3}$$

$$= 2.2095$$

$$a = 10^{\log a} = 10^{2.2095} = 162.0$$

$$b = \frac{\log 50/90}{3} = -0.085\,09$$

$$(S_f)_{ax} = 162^{-0.085\,09} \qquad 10^3 \le N \le 10^6 \text{ in kpsi} \quad \textit{Ans.}$$

Check:

$$10^3(S_f)_{ax} = 162(10^3)^{-0.085\,09} = 90 \text{ kpsi}$$

$$10^6(S_f)_{ax} = 162(10^6)^{-0.085\,09} = 50 \text{ kpsi}$$

The end points agree.

6-6

Eq. (6-8): $\quad S'_e = 0.5(710) = 355$ MPa

Table 6-2: $\quad a = 4.51, \quad b = -0.265$

Eq. (6-19): $\quad k_a = 4.51(710)^{-0.265} = 0.792$

Eq. (6-20): $$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{32}{7.62}\right)^{-0.107} = 0.858$$

Eq. (6-18): $$S_e = k_a k_b S'_e = 0.792(0.858)(355) = 241 \text{ MPa} \quad \textit{Ans.}$$

6-7 For AISI 4340 as forged steel,

Eq. (6-8): $$S_e = 100 \text{ kpsi}$$

Table 6-2: $$a = 39.9, \quad b = -0.995$$

Eq. (6-19): $$k_a = 39.9(260)^{-0.995} = 0.158$$

Eq. (6-20): $$k_b = \left(\frac{0.75}{0.30}\right)^{-0.107} = 0.907$$

Each of the other Marin factors is unity.

$$S_e = 0.158(0.907)(100) = 14.3 \text{ kpsi}$$

For AISI 1040:

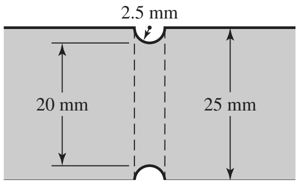
$$S'_e = 0.5(113) = 56.5 \text{ kpsi}$$
$$k_a = 39.9(113)^{-0.995} = 0.362$$
$$k_b = 0.907 \text{ (same as 4340)}$$

Each of the other Marin factors is unity.

$$S_e = 0.362(0.907)(56.5) = 18.6 \text{ kpsi}$$

Not only is AISI 1040 steel a contender, it has a superior endurance strength. Can you see why?

6-8



(a) For an AISI 1018 CD-machined steel, the strengths are

Eq. (2-17): $$S_{ut} = 440 \text{ MPa} \quad \Rightarrow \quad H_B = \frac{440}{3.41} = 129$$
$$S_y = 370 \text{ MPa}$$
$$S_{su} = 0.67(440) = 295 \text{ MPa}$$

Fig. A-15-15: $$\frac{r}{d} = \frac{2.5}{20} = 0.125, \quad \frac{D}{d} = \frac{25}{20} = 1.25, \quad K_{ts} = 1.4$$

Fig. 6-21: $$q_s = 0.94$$

Eq. (6-32): $$K_{fs} = 1 + 0.94(1.4 - 1) = 1.376$$

For a purely reversing torque of 200 N · m

$$\tau_{\max} = \frac{K_{fs} 16T}{\pi d^3} = \frac{1.376(16)(200 \times 10^3 \text{ N} \cdot \text{mm})}{\pi (20 \text{ mm})^3}$$
$$\tau_{\max} = 175.2 \text{ MPa} = \tau_a$$

$$S'_e = 0.5(440) = 220 \text{ MPa}$$

The Marin factors are

$$k_a = 4.51(440)^{-0.265} = 0.899$$

$$k_b = \left(\frac{20}{7.62}\right)^{-0.107} = 0.902$$

$$k_c = 0.59, \quad k_d = 1, \quad k_e = 1$$

Eq. (6-18): $$S_e = 0.899(0.902)(0.59)(220) = 105.3 \text{ MPa}$$

Eq. (6-14): $$a = \frac{[0.9(295)]^2}{105.3} = 669.4$$

Eq. (6-15): $$b = -\frac{1}{3}\log\frac{0.9(295)}{105.3} = -0.133\,88$$

Eq. (6-16): $$N = \left(\frac{175.2}{669.4}\right)^{1/-0.133\,88}$$

$$N = 22\,300 \text{ cycles} \quad \textit{Ans.}$$

(b) For an operating temperature of 450°C, the temperature modification factor, from Table 6-4, is

$$k_d = 0.843$$

Thus $$S_e = 0.899(0.902)(0.59)(0.843)(220) = 88.7 \text{ MPa}$$

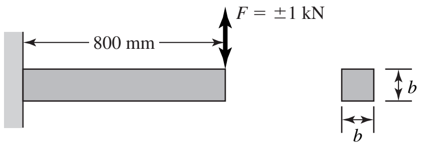
$$a = \frac{[0.9(295)]^2}{88.7} = 794.7$$

$$b = -\frac{1}{3}\log\frac{0.9(295)}{88.7} = -0.158\,71$$

$$N = \left(\frac{175.2}{794.7}\right)^{1/-0.15871}$$

$$N = 13\,700 \text{ cycles} \quad \textit{Ans.}$$

6-9



$$f = 0.9$$

$$n = 1.5$$

$$N = 10^4 \text{ cycles}$$

For AISI 1045 HR steel, $S_{ut} = 570$ MPa and $S_y = 310$ MPa

$$S'_e = 0.5(570 \text{ MPa}) = 285 \text{ MPa}$$

Find an initial guess based on yielding:

$$\sigma_a = \sigma_{\max} = \frac{Mc}{I} = \frac{M(b/2)}{b(b^3)/12} = \frac{6M}{b^3}$$

$$M_{\max} = (1 \text{ kN})(800 \text{ mm}) = 800 \text{ N} \cdot \text{m}$$

$$\sigma_{\max} = \frac{S_y}{n} \quad \Rightarrow \quad \frac{6(800 \times 10^3\ \mathrm{N \cdot mm})}{b^3} = \frac{310\ \mathrm{N/mm^2}}{1.5}$$

$$b = 28.5\ \mathrm{mm}$$

Eq. (6-25): $d_e = 0.808b$

Eq. (6-20): $k_b = \left(\dfrac{0.808b}{7.62}\right)^{-0.107} = 1.2714b^{-0.107}$

$$k_b = 0.888$$

The remaining Marin factors are

$$k_a = 57.7(570)^{-0.718} = 0.606$$

$$k_c = k_d = k_e = k_f = 1$$

Eq. (6-18): $S_e = 0.606(0.888)(285) = 153.4\ \mathrm{MPa}$

Eq. (6-14): $a = \dfrac{[0.9(570)]^2}{153.4} = 1715.6$

Eq. (6-15): $b = -\dfrac{1}{3}\log\dfrac{0.9(570)}{153.4} = -0.174\,76$

Eq. (6-13): $S_f = aN^b = 1715.6[(10^4)^{-0.174\,76}] = 343.1\ \mathrm{MPa}$

$$n = \frac{S_f}{\sigma_a} \quad \text{or} \quad \sigma_a = \frac{S_f}{n}$$

$$\frac{6(800 \times 10^3)}{b^3} = \frac{343.1}{1.5} \quad \Rightarrow \quad b = 27.6\ \mathrm{mm}$$

Check values for k_b, S_e, etc.

$$k_b = 1.2714(27.6)^{-0.107} = 0.891$$

$$S_e = 0.606(0.891)(285) = 153.9\ \mathrm{MPa}$$

$$a = \frac{[0.9(570)]^2}{153.9} = 1710$$

$$b = -\frac{1}{3}\log\frac{0.9(570)}{153.9} = -0.174\,29$$

$$S_f = 1710[(10^4)^{-0.174\,29}] = 343.4\ \mathrm{MPa}$$

$$\frac{6(800 \times 10^3)}{b^3} = \frac{343.4}{1.5}$$

$$b = 27.6\ \mathrm{mm} \quad \textit{Ans.}$$

6-10

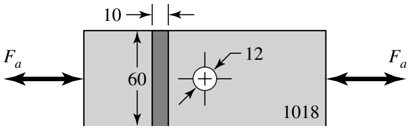


Table A-20: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

$$S'_e = 0.5(440) = 220 \text{ MPa}$$

Table 6-2: $k_a = 4.51(440)^{-0.265} = 0.899$

$$k_b = 1 \quad \text{(axial loading)}$$

Eq. (6-26): $k_c = 0.85$

$$S_e = 0.899(1)(0.85)(220) = 168.1 \text{ MPa}$$

Table A-15-1: $d/w = 12/60 = 0.2, \quad K_t = 2.5$

From Fig. 6-20, $q \doteq 0.82$

Eq. (6-32): $K_f = 1 + 0.82(2.5 - 1) = 2.23$

$$\sigma_a = K_f \frac{F_a}{A} \quad \Rightarrow \quad \frac{S_e}{n_f} = \frac{2.23F_a}{10(60-12)} = \frac{168.1}{1.8}$$

$F_a = 20\,100$ N $= 20.1$ kN *Ans.*

$$\frac{F_a}{A} = \frac{S_y}{n_y} \quad \Rightarrow \quad \frac{F_a}{10(60-12)} = \frac{370}{1.8}$$

$F_a = 98\,700$ N $= 98.7$ kN *Ans.*

Largest force amplitude is 20.1 kN. *Ans.*

6-11 A priori design decisions:

The design decision will be: d

Material and condition: 1095 HR and from Table A-20 $S_{ut} = 120$, $S_y = 66$ kpsi.

Design factor: $n_f = 1.6$ per problem statement.

Life: $(1150)(3) = 3450$ cycles

Function: carry 10 000 lbf load

Preliminaries to iterative solution:

$$S'_e = 0.5(120) = 60 \text{ kpsi}$$

$$k_a = 2.70(120)^{-0.265} = 0.759$$

$$\frac{I}{c} = \frac{\pi d^3}{32} = 0.098\,17d^3$$

$$M(\text{crit.}) = \left(\frac{6}{24}\right)(10\,000)(12) = 30\,000 \text{ lbf} \cdot \text{in}$$

The critical location is in the middle of the shaft at the shoulder. From Fig. A-15-9: $D/d = 1.5$, $r/d = 0.10$, and $K_t = 1.68$. With no direct information concerning f, use $f = 0.9$.

For an initial trial, set $d = 2.00$ in

$$k_b = \left(\frac{2.00}{0.30}\right)^{-0.107} = 0.816$$

$$S_e = 0.759(0.816)(60) = 37.2 \text{ kpsi}$$

$$a = \frac{[0.9(120)]^2}{37.2} = 313.5$$

$$b = -\frac{1}{3}\log\frac{0.9(120)}{37.2} = -0.15429$$

$$S_f = 313.5(3450)^{-0.15429} = 89.2 \text{ kpsi}$$

$$\sigma_0 = \frac{M}{I/c} = \frac{30}{0.098\,17d^3} = \frac{305.6}{d^3}$$

$$= \frac{305.6}{2^3} = 38.2 \text{ kpsi}$$

$$r = \frac{d}{10} = \frac{2}{10} = 0.2$$

Fig. 6-20: $q \doteq 0.87$

Eq. (6-32): $K_f \doteq 1 + 0.87(1.68 - 1) = 1.59$

$$\sigma_a = K_f\sigma_0 = 1.59(38.2) = 60.7 \text{ kpsi}$$

$$n_f = \frac{S_f}{\sigma_a} = \frac{89.2}{60.7} = 1.47$$

Design is adequate unless more uncertainty prevails.

Choose $d = 2.00$ in *Ans.*

6-12

Yield: $\sigma'_{\max} = [172^2 + 3(103^2)]^{1/2} = 247.8 \text{ kpsi}$

$$n_y = S_y/\sigma'_{\max} = 413/247.8 = 1.67 \quad \textit{Ans.}$$

$$\sigma'_a = 172 \text{ MPa} \quad \sigma'_m = \sqrt{3}\tau_m = \sqrt{3}(103) = 178.4 \text{ MPa}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(172/276) + (178.4/551)} = 1.06 \quad \textit{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{551}{178.4}\right)^2\left(\frac{172}{276}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(178.4)(276)}{551(172)}\right]^2}\right\} = 1.31 \quad \textit{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \left[\frac{1}{(172/276)^2 + (178.4/413)^2}\right]^{1/2} = 1.32 \quad \textit{Ans.}$$

6-13

Yield: $\sigma'_{\max} = [69^2 + 3(138)^2]^{1/2} = 248.8$ MPa

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{413}{248.8} = 1.66 \quad Ans.$$

$$\sigma'_a = 69 \text{ MPa}, \quad \sigma'_m = \sqrt{3}(138) = 239 \text{ MPa}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(69/276) + (239/551)} = 1.46 \quad Ans.$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{551}{239}\right)^2\left(\frac{69}{276}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(239)(276)}{551(69)}\right]^2}\right\} = 1.73 \quad Ans.$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \left[\frac{1}{(69/276)^2 + (239/413)^2}\right]^{1/2} = 1.59 \quad Ans.$$

6-14

Yield: $\sigma'_{\max} = [83^2 + 3(103 + 69)^2]^{1/2} = 309.2$ MPa

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{413}{309.3} = 1.34 \quad Ans.$$

$$\sigma'_a = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{83^2 + 3(69^2)} = 145.5 \text{ MPa}, \quad \sigma'_m = \sqrt{3}(103) = 178.4 \text{ MPa}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(145.5/276) + (178.4/551)} = 1.18 \quad Ans.$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{551}{178.4}\right)^2\left(\frac{145.5}{276}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(178.4)(276)}{551(145.5)}\right]^2}\right\} = 1.47 \quad Ans.$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \left[\frac{1}{(145.5/276)^2 + (178.4/413)^2}\right]^{1/2} = 1.47 \quad Ans.$$

6-15

$$\sigma'_{\max} = \sigma'_a = \sqrt{3}(207) = 358.5 \text{ MPa}, \qquad \sigma'_m = 0$$

Yield: $$358.5 = \frac{413}{n_y} \quad \Rightarrow \quad n_y = 1.15 \quad Ans.$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(358.5/276)} = 0.77 \quad Ans.$$

(b) Gerber criterion of Table 6-7 does not work; therefore use Eq. (6-47).

$$n_f \frac{\sigma_a}{S_e} = 1 \quad \Rightarrow \quad n_f = \frac{S_e}{\sigma_a} = \frac{276}{358.5} = 0.77 \quad Ans.$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\left(\frac{1}{358.5/276}\right)^2} = 0.77 \quad Ans.$$

Let $f = 0.9$ to assess the cycles to failure by fatigue

Eq. (6-14): $$a = \frac{[0.9(551)]^2}{276} = 891.0 \text{ MPa}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\frac{0.9(551)}{276} = -0.084\,828$$

Eq. (6-16): $$N = \left(\frac{358.5}{891.0}\right)^{-1/0.084\,828} = 45\,800 \text{ cycles} \quad Ans.$$

6-16

$$\sigma'_{\max} = [103^2 + 3(103)^2]^{1/2} = 206 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{413}{206} = 2.00 \quad Ans.$$

$$\sigma'_a = \sqrt{3}(103) = 178.4 \text{ MPa}, \quad \sigma'_m = 103 \text{ MPa}$$

(a) Modified Goodman, Table 7-9

$$n_f = \frac{1}{(178.4/276) + (103/551)} = 1.20 \quad Ans.$$

(b) Gerber, Table 7-10

$$n_f = \frac{1}{2}\left(\frac{551}{103}\right)^2\left(\frac{178.4}{276}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(103)(276)}{551(178.4)}\right]^2}\right\} = 1.44 \quad Ans.$$

(c) ASME-Elliptic, Table 7-11

$$n_f = \left[\frac{1}{(178.4/276)^2 + (103/413)^2}\right]^{1/2} = 1.44 \quad Ans.$$

6-17 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

$$A = 0.375(1 - 0.25) = 0.2813 \text{ in}^2$$

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{3000}{0.2813}(10^{-3}) = 10.67 \text{ kpsi}$$

$$n_y = \frac{54}{10.67} = 5.06 \quad Ans.$$

$$S_e' = 0.5(64) = 32 \text{ kpsi}$$

$$k_a = 2.70(64)^{-0.265} = 0.897$$

$$k_b = 1, \qquad k_c = 0.85$$

$$S_e = 0.897(1)(0.85)(32) = 24.4 \text{ kpsi}$$

Table A-15-1: $w = 1$ in, $d = 1/4$ in, $d/w = 0.25 \;\therefore K_t = 2.45$.

Fig. 6-20, with $r = 0.125$ in, $q \doteq 0.8$

Eq. (6-32): $K_f = 1 + 0.8(2.45 - 1) = 2.16$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right|$$

$$= 2.16 \left| \frac{3.000 - 0.800}{2(0.2813)} \right| = 8.45 \text{ kpsi}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A}$$

$$= 2.16 \left[\frac{3.000 + 0.800}{2(0.2813)} \right] = 14.6 \text{ kpsi}$$

(a) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{64}{14.6}\right)^2 \left(\frac{8.45}{24.4}\right)\left[-1 + \sqrt{1 + \left(\frac{2(14.6)(24.4)}{8.45(64)}\right)^2}\,\right]$$

$$= 2.17 \quad Ans.$$

(b) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(8.45/24.4)^2 + (14.6/54)^2}} = 2.28 \quad Ans.$$

6-18 Referring to the solution of Prob. 6-17, for load fluctuations of −800 to 3000 lbf

$$\sigma_a = 2.16 \left| \frac{3.000 - (-0.800)}{2(0.2813)} \right| = 14.59 \text{ kpsi}$$

$$\sigma_m = 2.16 \left| \frac{3.000 + (-0.800)}{2(0.2813)} \right| = 8.45 \text{ kpsi}$$

(a) Table 6-7, DE-Gerber

$$n_f = \frac{1}{2}\left(\frac{64}{8.45}\right)^2\left(\frac{14.59}{24.4}\right)\left[-1+\sqrt{1+\left(\frac{2(8.45)(24.4)}{64(14.59)}\right)^2}\,\right] = 1.60 \quad Ans.$$

(b) Table 6-8, DE-Elliptic

$$n_f = \sqrt{\frac{1}{(14.59/24.4)^2 + (8.45/54)^2}} = 1.62 \quad Ans.$$

6-19 Referring to the solution of Prob. 6-17, for load fluctuations of 800 to −3000 lbf

$$\sigma_a = 2.16\left|\frac{0.800-(-3.000)}{2(0.2813)}\right| = 14.59 \text{ kpsi}$$

$$\sigma_m = 2.16\left[\frac{0.800+(-3.000)}{2(0.2813)}\right] = -8.45 \text{ kpsi}$$

(a) We have a compressive midrange stress for which the failure locus is horizontal at the S_e level.

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{14.59} = 1.67 \quad Ans.$$

(b) Same as (a)

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{14.59} = 1.67 \quad Ans.$$

6-20

$$S_{ut} = 0.495(380) = 188.1 \text{ kpsi}$$

$$S_e' = 0.5(188.1) = 94.05 \text{ kpsi}$$

$$k_a = 14.4(188.1)^{-0.718} = 0.335$$

For a non-rotating round bar in bending, Eq. (6-24) gives: $d_e = 0.370d = 0.370(3/8) =$ 0.1388 in

$$k_b = \left(\frac{0.1388}{0.3}\right)^{-0.107} = 1.086$$

$$S_e = 0.335(1.086)(94.05) = 34.22 \text{ kpsi}$$

$$F_a = \frac{30-15}{2} = 7.5 \text{ lbf}, \qquad F_m = \frac{30+15}{2} = 22.5 \text{ lbf}$$

$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(22.5)(16)}{\pi(0.375^3)}(10^{-3}) = 69.54 \text{ kpsi}$$

$$\sigma_a = \frac{32(7.5)(16)}{\pi(0.375^3)}(10^{-3}) = 23.18 \text{ kpsi}$$

$$r = \frac{23.18}{69.54} = 0.333$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(23.18/34.22) + (69.54/188.1)} = 0.955$$

Since finite failure is predicted, proceed to calculate N

From Fig. 6-18, for $S_{ut} = 188.1$ kpsi, $f = 0.778$

Eq. (6-14): $$a = \frac{[0.7781(188.1)]^2}{34.22} = 625.8 \text{ kpsi}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\frac{0.778(188.1)}{34.22} = -0.210\,36$$

$$\frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_{ut}} = 1 \quad \Rightarrow \quad S_f = \frac{\sigma_a}{1-(\sigma_m/S_{ut})} = \frac{23.18}{1-(69.54/188.1)} = 36.78 \text{ kpsi}$$

Eq. (7-15) with $\sigma_a = S_f$

$$N = \left(\frac{36.78}{625.8}\right)^{1/-0.210\,36} = 710\,000 \text{ cycles} \quad \textit{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{188.1}{69.54}\right)^2\left(\frac{23.18}{34.22}\right)\left\{-1+\sqrt{1+\left[\frac{2(69.54)(34.22)}{188.1(23.18)}\right]^2}\right\}$$

$= 1.20$ Thus, infinite life is predicted ($N \geq 10^6$ cycles). *Ans.*

6-21

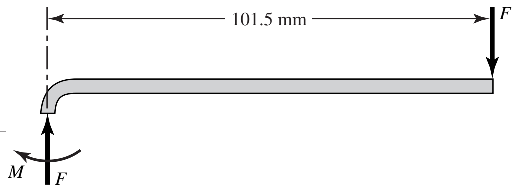
(a) $$I = \frac{1}{12}(18)(3^3) = 40.5 \text{ mm}^4$$

$$y = \frac{Fl^3}{3EI} \quad \Rightarrow \quad F = \frac{3EIy}{l^3}$$

$$F_{\min} = \frac{3(207)(10^9)(40.5)(10^{-12})(2)(10^{-3})}{(100^3)(10^{-9})} = 50.3 \text{ N} \quad \textit{Ans.}$$

$$F_{\max} = \frac{6}{2}(50.3) = 150.9 \text{ N} \quad \textit{Ans.}$$

(b)



$$M = 0.1015F \text{ N}\cdot\text{m}$$
$$A = 3(18) = 54 \text{ mm}^2$$

Curved beam: $r_n = \dfrac{h}{\ln(r_o/r_i)} = \dfrac{3}{\ln(6/3)} = 4.3281 \text{ mm}$

$$r_c = 4.5 \text{ mm}, \quad e = r_c - r_n = 4.5 - 4.3281 = 0.1719 \text{ mm}$$

$$\sigma_i = -\frac{Mc_i}{Aer_i} - \frac{F}{A} = -\frac{(0.1015F)(1.5 - 0.1719)}{54(0.1719)(3)(10^{-3})} - \frac{F}{54} = -4.859F \text{ MPa}$$

$$\sigma_o = \frac{Mc_o}{Aer_o} - \frac{F}{A} = \frac{(0.1015F)(1.5 + 0.1719)}{54(0.1719)(6)(10^{-3})} - \frac{F}{54} = 3.028F \text{ MPa}$$

$$(\sigma_i)_{\min} = -4.859(150.9) = -733.2 \text{ MPa}$$

$$(\sigma_i)_{\max} = -4.859(50.3) = -244.4 \text{ MPa}$$

$$(\sigma_o)_{\max} = 3.028(150.9) = 456.9 \text{ MPa}$$

$$(\sigma_o)_{\min} = 3.028(50.3) = 152.3 \text{ MPa}$$

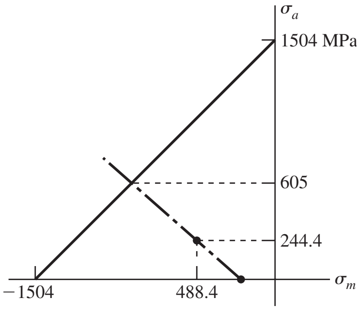
Eq. (2-17) $S_{ut} = 3.41(490) = 1671 \text{ MPa}$

Per the problem statement, estimate the yield as $S_y = 0.9S_{ut} = 0.9(1671) = 1504$ MPa. Then from Eq. (6-8), $S'_e = 700$ MPa; Eq. (6-19), $k_a = 1.58(1671)^{-0.085} = 0.841$; Eq. (6-25) $d_e = 0.808[18(3)]^{1/2} = 5.938$ mm; and Eq. (6-20), $k_b = (5.938/7.62)^{-0.107} = 1.027$.

$$S_e = 0.841(1.027)(700) = 605 \text{ MPa}$$

At Inner Radius $(\sigma_i)_a = \left| \dfrac{-733.2 + 244.4}{2} \right| = 244.4 \text{ MPa}$

$$(\sigma_i)_m = \frac{-733.2 - 244.4}{2} = -488.8 \text{ MPa}$$



Load line: $\sigma_m = -244.4 - \sigma_a$

Langer (yield) line: $\sigma_m = \sigma_a - 1504 = -244.4 - \sigma_a$

Intersection: $\sigma_a = 629.8 \text{ MPa}, \quad \sigma_m = -874.2 \text{ MPa}$

(Note that σ_a is more than 605 MPa)

Yield: $n_y = \dfrac{629.8}{244.4} = 2.58$

Fatigue: $n_f = \dfrac{605}{244.4} = 2.48$ Thus, the spring is likely to fail in fatigue at the inner radius. *Ans.*

At Outer Radius

$$(\sigma_o)_a = \frac{456.9 - 152.3}{2} = 152.3 \text{ MPa}$$

$$(\sigma_o)_m = \frac{456.9 + 152.3}{2} = 304.6 \text{ MPa}$$

Yield load line: $\sigma_m = 152.3 + \sigma_a$

Langer line: $\sigma_m = 1504 - \sigma_a = 152.3 + \sigma_a$

Intersection: $\sigma_a = 675.9 \text{ MPa}, \quad \sigma_m = 828.2 \text{ MPa}$

$$n_y = \frac{675.9}{152.3} = 4.44$$

Fatigue line: $\sigma_a = [1 - (\sigma_m/S_{ut})^2]S_e = \sigma_m - 152.3$

$$605\left[1 - \left(\frac{\sigma_m}{1671}\right)^2\right] = \sigma_m - 152.3$$

$$\sigma_m^2 + 4615.3\sigma_m - 3.4951(10^6) = 0$$

$$\sigma_m = \frac{-4615.3 + \sqrt{4615.3^2 + 4(3.4951)(10^6)}}{2} = 662.2 \text{ MPa}$$

$$\sigma_a = 662.2 - 152.3 = 509.9 \text{ MPa}$$

$$n_f = \frac{509.9}{152.3} = 3.35$$

Thus, the spring is not likely to fail in fatigue at the outer radius. *Ans.*

6-22 The solution at the inner radius is the same as in Prob. 6-21. At the outer radius, the yield solution is the same.

Fatigue line: $\sigma_a = \left(1 - \dfrac{\sigma_m}{S_{ut}}\right)S_e = \sigma_m - 152.3$

$$605\left(1 - \frac{\sigma_m}{1671}\right) = \sigma_m - 152.3$$

$$1.362\sigma_m = 757.3 \quad \Rightarrow \quad \sigma_m = 556.0 \text{ MPa}$$

$$\sigma_a = 556.0 - 152.3 = 403.7 \text{ MPa}$$

$$n_f = \frac{403.7}{152.3} = 2.65 \quad \textit{Ans.}$$

6-23 Preliminaries:

Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

$$S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$k_a = 2.70(64)^{-0.265} = 0.897$$

$$k_b = 1$$

$$k_c = 0.85$$

$$S_e = 0.897(1)(0.85)(32) = 24.4 \text{ kpsi}$$

Fillet:

Fig. A-15-5: $D = 3.75$ in, $d = 2.5$ in, $D/d = 3.75/2.5 = 1.5$, and $r/d = 0.25/2.5 = 0.10$
$\therefore K_t = 2.1$. Fig. 6-20 with $r = 0.25$ in, $q \doteq 0.82$

Eq. (6-32): $K_f = 1 + 0.82(2.1 - 1) = 1.90$

$$\sigma_{\max} = \frac{4}{2.5(0.5)} = 3.2 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-16}{2.5(0.5)} = -12.8 \text{ kpsi}$$

$$\sigma_a = 1.90 \left| \frac{3.2 - (-12.8)}{2} \right| = 15.2 \text{ kpsi}$$

$$\sigma_m = 1.90 \left[\frac{3.2 + (-12.8)}{2} \right] = -9.12 \text{ kpsi}$$

$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-12.8} \right| = 4.22$$

Since the midrange stress is negative,

$$S_a = S_e = 24.4 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{24.4}{15.2} = 1.61$$

Hole:

Fig. A-15-1: $d/w = 0.75/3.75 = 0.20$, $K_t = 2.5$. Fig. 6-20, with $r = 0.375$ in, $q \doteq 0.85$

Eq. (6-32): $K_f = 1 + 0.85(2.5 - 1) = 2.28$

$$\sigma_{\max} = \frac{4}{0.5(3.75 - 0.75)} = 2.67 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-16}{0.5(3.75 - 0.75)} = -10.67 \text{ kpsi}$$

$$\sigma_a = 2.28 \left| \frac{2.67 - (-10.67)}{2} \right| = 15.2 \text{ kpsi}$$

$$\sigma_m = 2.28\, \frac{2.67 + (-10.67)}{2} = -9.12 \text{ kpsi}$$

Since the midrange stress is negative,

$$n_y = \left|\frac{S_y}{\sigma_{\min}}\right| = \left|\frac{54}{-10.67}\right| = 5.06$$

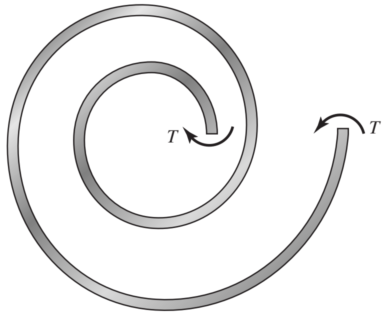
$$S_a = S_e = 24.4 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{24.4}{15.2} = 1.61$$

Thus the design is controlled by the threat of fatigue equally at the fillet and the hole; the minimum factor of safety is $n_f = 1.61$. *Ans.*

6-24

(a)



Curved beam in pure bending where $M = -T$ throughout. The maximum stress will occur at the inner fiber where $r_c = 20$ mm, but will be compressive. The maximum tensile stress will occur at the outer fiber where $r_c = 60$ mm. Why?

Inner fiber where $r_c = 20$ *mm*

$$r_n = \frac{h}{\ln(r_o/r_i)} = \frac{5}{\ln(22.5/17.5)} = 19.8954 \text{ mm}$$

$$e = 20 - 19.8954 = 0.1046 \text{ mm}$$

$$c_i = 19.8954 - 17.5 = 2.395 \text{ mm}$$

$$A = 25 \text{ mm}^2$$

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-T(2.395)10^{-3}}{25(10^{-6})0.1046(10^{-3})17.5(10^{-3})}(10^{-6}) = -52.34\,T \qquad (1)$$

where T is in N · m, and σ_i is in MPa.

$$\sigma_m = \frac{1}{2}(-52.34T) = -26.17T, \qquad \sigma_a = 26.17T$$

For the endurance limit, $S'_e = 0.5(770) = 385$ MPa

$$k_a = 4.51(770)^{-0.265} = 0.775$$

$$d_e = 0.808[5(5)]^{1/2} = 4.04 \text{ mm}$$

$$k_b = (4.04/7.62)^{-0.107} = 1.07$$

$$S_e = 0.775(1.07)385 = 319.3 \text{ MPa}$$

For a compressive midrange component, $\sigma_a = S_e/n_f$. Thus,

$$26.17T = 319.3/3 \Rightarrow T = 4.07 \text{ N} \cdot \text{m}$$

Outer fiber where $r_c = 60$ *mm*

$$r_n = \frac{5}{\ln(62.5/57.5)} = 59.96526 \text{ mm}$$

$$e = 60 - 59.96526 = 0.03474 \text{ mm}$$

$$c_o = 62.5 - 59.96526 = 2.535 \text{ mm}$$

$$\sigma_o = -\frac{Mc_i}{Aer_i} = -\frac{-T(2.535)10^{-3}}{25(10^{-6})0.03474(10^{-3})62.5(10^{-3})}(10^{-6}) = 46.7\ T$$

Comparing this with Eq. (1), we see that it is less in magnitude, but the midrange component is *tension.*

$$\sigma_a = \sigma_m = \frac{1}{2}(46.7T) = 23.35T$$

Using Eq. (6-46), for modified Goodman, we have

$$\frac{23.35T}{319.3} + \frac{23.35T}{770} = \frac{1}{3} \Rightarrow T = 3.22 \text{ N} \cdot \text{m} \quad Ans.$$

(b) Gerber, Eq. (6-47), at the outer fiber,

$$\frac{3(23.35T)}{319.3} + \left[\frac{3(23.35T)}{770}\right]^2 = 1$$

reduces to $T^2 + 26.51T - 120.83 = 0$

$$T = \frac{1}{2}\left(-26.51 + \sqrt{26.51^2 + 4(120.83)}\right) = 3.96 \text{ N} \cdot \text{m} \quad Ans.$$

(c) To guard against yield, use T of part (b) and the inner stress.

$$n_y = \frac{420}{52.34(3.96)} = 2.03 \quad Ans.$$

6-25 From Prob. 6-24, $S_e = 319.3$ MPa, $S_y = 420$ MPa, and $S_{ut} = 770$ MPa

(a) Assuming the beam is straight,

$$\sigma_{\max} = \frac{6M}{bh^2} = \frac{6T}{5^3[(10^{-3})^3]} = 48(10^6)T$$

Goodman: $$\frac{24T}{319.3} + \frac{24T}{770} = \frac{1}{3} \quad \Rightarrow \quad T = 3.13 \text{ N} \cdot \text{m} \quad Ans.$$

(b) Gerber: $$\frac{3(24)T}{319.3} + \left[\frac{3(24)T}{770}\right]^2 = 1$$

$$T^2 + 25.79T - 114.37 = 1$$

$$T = \frac{1}{2}\left[-25.79 + \sqrt{25.79^2 + 4(114.37)}\right] = 3.86 \text{ N} \cdot \text{m} \quad Ans.$$

(c) Using $\sigma_{\max} = 52.34(10^6)T$ from Prob. 6-24,

$$n_y = \frac{420}{52.34(3.86)} = 2.08 \quad Ans.$$

6-26

(a)

$$\tau_{\max} = \frac{16K_{fs}T_{\max}}{\pi d^3}$$

Fig. 6-21 for $H_B > 200, r = 3\text{ mm}, q_s \doteq 1$

$$K_{fs} = 1 + q_s(K_{ts} - 1)$$

$$K_{fs} = 1 + 1(1.6 - 1) = 1.6$$

$$T_{\max} = 2000(0.05) = 100\text{ N}\cdot\text{m}, \quad T_{\min} = \frac{500}{2000}(100) = 25\text{ N}\cdot\text{m}$$

$$\tau_{\max} = \frac{16(1.6)(100)(10^{-6})}{\pi(0.02)^3} = 101.9\text{ MPa}$$

$$\tau_{\min} = \frac{500}{2000}(101.9) = 25.46\text{ MPa}$$

$$\tau_m = \frac{1}{2}(101.9 + 25.46) = 63.68\text{ MPa}$$

$$\tau_a = \frac{1}{2}(101.9 - 25.46) = 38.22\text{ MPa}$$

$$S_{su} = 0.67S_{ut} = 0.67(320) = 214.4\text{ MPa}$$

$$S_{sy} = 0.577S_y = 0.577(180) = 103.9\text{ MPa}$$

$$S'_e = 0.5(320) = 160\text{ MPa}$$

$$k_a = 57.7(320)^{-0.718} = 0.917$$

$$d_e = 0.370(20) = 7.4\text{ mm}$$

$$k_b = \left(\frac{7.4}{7.62}\right)^{-0.107} = 1.003$$

$$k_c = 0.59$$

$$S_e = 0.917(1.003)(0.59)(160) = 86.8\text{ MPa}$$

Modified Goodman, Table 6-6

$$n_f = \frac{1}{(\tau_a/S_e) + (\tau_m/S_{su})} = \frac{1}{(38.22/86.8) + (63.68/214.4)} = 1.36 \quad Ans.$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_e}{S_{su}\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{214.4}{63.68}\right)^2 \frac{38.22}{86.8}\left\{-1 + \sqrt{1 + \left[\frac{2(63.68)(86.8)}{214.4(38.22)}\right]^2}\right\} = 1.70 \quad Ans.$$

6-27 $S_y = 800\text{ MPa}, S_{ut} = 1000\text{ MPa}$

(a) From Fig. 6-20, for a notch radius of 3 mm and $S_{ut} = 1$ GPa, $q \doteq 0.92$.

$$K_f = 1 + q(K_t - 1) = 1 + 0.92(3 - 1) = 2.84$$

$$\sigma_{\max} = -K_f \frac{4P}{\pi d^2} = -\frac{2.84(4)P}{\pi(0.030)^2} = -4018P$$

$$\sigma_m = -\sigma_a = \frac{1}{2}(-4018P) = -2009P$$

$$T = fP\left(\frac{D+d}{4}\right)$$

$$T_{\max} = 0.3P\left(\frac{0.150 + 0.03}{4}\right) = 0.0135P$$

From Fig. 6-21, $q_s \doteq 0.95$. Also, K_{ts} is given as 1.8. Thus,

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.95(1.8 - 1) = 1.76$$

$$\tau_{\max} = \frac{16K_{fs}T}{\pi d^3} = \frac{16(1.76)(0.0135P)}{\pi(0.03)^3} = 4482P$$

$$\tau_a = \tau_m = \frac{1}{2}(4482P) = 2241P$$

Eqs. (6-55) and (6-56):

$$\sigma'_a = \sigma'_m = \left[(\sigma_a/0.85)^2 + 3\tau_a^2\right]^{1/2} = \left[(-2009P/0.85)^2 + 3(2241P)^2\right]^{1/2} = 4545P$$

$$S'_e = 0.5(1000) = 500\text{ MPa}$$

$$k_a = 4.51(1000)^{-0.265} = 0.723$$

$$k_b = \left(\frac{30}{7.62}\right)^{-0.107} = 0.864$$

$$S_e = 0.723(0.864)(500) = 312.3\text{ MPa}$$

Modified Goodman: $$\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{n}$$

$$\frac{4545P}{312.3(10^6)} + \frac{4545P}{1000(10^6)} = \frac{1}{3} \quad \Rightarrow \quad P = 17.5(10^3)\text{ N} = 16.1\text{ kN} \quad \textit{Ans.}$$

Yield (conservative): $n_y = \dfrac{S_y}{\sigma'_a + \sigma'_m}$

$$n_y = \frac{800(10^6)}{2(4545)(17.5)(10^3)} = 5.03 \quad \textit{Ans.}$$

(actual): $$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(-4018P)^2 + 3(4482P)^2\right]^{1/2}$$

$$= 8741P$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{800(10^6)}{8741(17.5)10^3} = 5.22$$

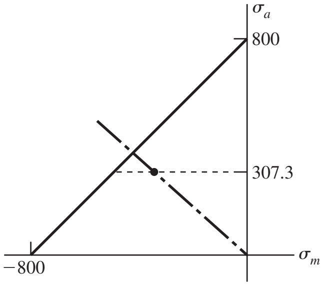
(b) If the shaft is not rotating, $\tau_m = \tau_a = 0$.

$$\sigma_m = \sigma_a = -2009P$$

$$k_b = 1 \qquad \text{(axial)}$$

$$k_c = 0.85 \qquad \text{(Since there is no tension, } k_c = 1 \text{ might be more appropriate.)}$$

$$S_e = 0.723(1)(0.85)(500) = 307.3 \text{ MPa}$$



$$n_f = \frac{307.3(10^6)}{2009P} \quad \Rightarrow \quad P = \frac{307.3(10^6)}{3(2009)} = 51.0(10^3) \text{ N}$$
$$= 51.0 \text{ kN} \quad \textit{Ans.}$$

Yield: $$n_y = \frac{800(10^6)}{2(2009)(51.0)(10^3)} = 3.90 \quad \textit{Ans.}$$

6-28 From Prob. 6-27, $K_f = 2.84$, $K_{fs} = 1.76$, $S_e = 312.3$ MPa

$$\sigma_{\max} = -K_f \frac{4P_{\max}}{\pi d^2} = -2.84\left[\frac{(4)(80)(10^{-3})}{\pi(0.030)^2}\right] = -321.4 \text{ MPa}$$

$$\sigma_{\min} = \frac{20}{80}(-321.4) = -80.4 \text{ MPa}$$

$$T_{\max} = fP_{\max}\left(\frac{D+d}{4}\right) = 0.3(80)(10^3)\left(\frac{0.150+0.03}{4}\right) = 1080 \text{ N}\cdot\text{m}$$

$$T_{\min} = \frac{20}{80}(1080) = 270 \text{ N}\cdot\text{m}$$

$$\tau_{\max} = K_{fs}\frac{16T_{\max}}{\pi d^3} = 1.76\left[\frac{16(1080)}{\pi(0.030)^3}(10^{-6})\right] = 358.5 \text{ MPa}$$

$$\tau_{\min} = \frac{20}{80}(358.5) = 89.6 \text{ MPa}$$

$$\sigma_a = \frac{321.4 - 80.4}{2} = 120.5 \text{ MPa}$$

$$\sigma_m = \frac{-321.4 - 80.4}{2} = -200.9 \text{ MPa}$$

$$\tau_a = \frac{358.5 - 89.6}{2} = 134.5 \text{ MPa}$$

$$\tau_m = \frac{358.5 + 89.6}{2} = 224.1 \text{ MPa}$$

Eqs. (6-55) and (6-56):

$$\sigma_a' = \left[(\sigma_a/0.85)^2 + 3\tau_a^2\right]^{1/2} = \left[(120.5/0.85)^2 + 3(134.5)^2\right]^{1/2} = 272.7 \text{ MPa}$$

$$\sigma_m' = \left[(-200.9/0.85)^2 + 3(224.1)^2\right]^{1/2} = 454.5 \text{ MPa}$$

Goodman:

$$(\sigma_a)_e = \frac{\sigma_a'}{1 - \sigma_m'/S_{ut}} = \frac{272.7}{1 - 454.5/1000} = 499.9 \text{ MPa}$$

Let $f = 0.9$

$$a = \frac{[0.9(1000)]^2}{312.3} = 2594 \text{ MPa}$$

$$b = -\frac{1}{3}\log\left[\frac{0.9(1000)}{312.3}\right] = -0.1532$$

$$N = \left[\frac{(\sigma_a)_e}{a}\right]^{1/b} = \left[\frac{499.9}{2594}\right]^{1/-0.1532} = 46\,520 \text{ cycles} \quad \textit{Ans.}$$

6-29

$$S_y = 490 \text{ MPa}, \quad S_{ut} = 590 \text{ MPa}, \quad S_e = 200 \text{ MPa}$$

$$\sigma_m = \frac{420 + 140}{2} = 280 \text{ MPa}, \quad \sigma_a = \frac{420 - 140}{2} = 140 \text{ MPa}$$

Goodman:

$$(\sigma_a)_e = \frac{\sigma_a}{1 - \sigma_m/S_{ut}} = \frac{140}{1 - (280/590)} = 266.5 \text{ MPa} > S_e \ \therefore \text{finite life}$$

$$a = \frac{[0.9(590)]^2}{200} = 1409.8 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9(590)}{200} = -0.141\,355$$

$$N = \left(\frac{266.5}{1409.8}\right)^{-1/0.143\,55} = 131\,200 \text{ cycles}$$

$$N_{\text{remaining}} = 131\,200 - 50\,000 = 81\,200 \text{ cycles}$$

Second loading:

$$(\sigma_m)_2 = \frac{350 + (-200)}{2} = 75 \text{ MPa}$$

$$(\sigma_a)_2 = \frac{350 - (-200)}{2} = 275 \text{ MPa}$$

$$(\sigma_a)_{e2} = \frac{275}{1 - (75/590)} = 315.0 \text{ MPa}$$

(a) Miner's method

$$N_2 = \left(\frac{315}{1409.8}\right)^{-1/0.141\,355} = 40\,200 \text{ cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \quad \Rightarrow \quad \frac{50\,000}{131\,200} + \frac{n_2}{40\,200} = 1$$

$$n_2 = 24\,880 \text{ cycles} \quad \textit{Ans.}$$

(b) Manson's method

Two data points: 0.9(590 MPa), 10^3 cycles
266.5 MPa, 81 200 cycles

$$\frac{0.9(590)}{266.5} = \frac{a_2(10^3)^{b_2}}{a_2(81\,200)^{b_2}}$$

$$1.9925 = (0.012\,315)^{b_2}$$

$$b_2 = \frac{\log 1.9925}{\log 0.012\,315} = -0.156\,789$$

$$a_2 = \frac{266.5}{(81\,200)^{-0.156\,789}} = 1568.4 \text{ MPa}$$

$$n_2 = \left(\frac{315}{1568.4}\right)^{1/-0.156\,789} = 27\,950 \text{ cycles} \quad \textit{Ans.}$$

6-30 **(a)** Miner's method

$$a = \frac{[0.9(76)]^2}{30} = 155.95 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.9(76)}{30} = -0.119\,31$$

$$\sigma_1 = 48 \text{ kpsi}, \quad N_1 = \left(\frac{48}{155.95}\right)^{1/-0.119\,31} = 19\,460 \text{ cycles}$$

$$\sigma_2 = 38 \text{ kpsi}, \quad N_2 = \left(\frac{38}{155.95}\right)^{1/-0.119\,31} = 137\,880 \text{ cycles}$$

$$\sigma_3 = 32 \text{ kpsi}, \quad N_3 = \left(\frac{32}{155.95}\right)^{1/-0.119\,31} = 582\,150 \text{ cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

$$\frac{4000}{19\,460} + \frac{60\,000}{137\,880} + \frac{n_3}{582\,150} = 1 \quad \Rightarrow \quad n_3 = 209\,160 \text{ cycles} \quad \textit{Ans.}$$

(b) Manson's method

The life remaining after the first cycle is $N_{R_1} = 19\,460 - 4000 = 15\,460$ cycles. The two data points required to define $S'_{e,1}$ are $[0.9(76), 10^3]$ and (48, 15 460).

$$\frac{0.9(76)}{48} = \frac{a_2(10^3)^{b_2}}{a_2(15\,460)} \quad \Rightarrow \quad 1.425 = (0.064\,683)^{b_2}$$

$$b_2 = \frac{\log(1.425)}{\log(0.064\,683)} = -0.129\,342$$

$$a_2 = \frac{48}{(15\,460)^{-0.129\,342}} = 167.14 \text{ kpsi}$$

$$N_2 = \left(\frac{38}{167.14}\right)^{-1/0.129\,342} = 94\,110 \text{ cycles}$$

$$N_{R_2} = 94\,110 - 60\,000 = 34\,110 \text{ cycles}$$

$$\frac{0.9(76)}{38} = \frac{a_3(10^3)^{b_3}}{a_3(34\,110)^{b_3}} \quad \Rightarrow \quad 1.8 = (0.029\,317)^{b_3}$$

$$b_3 = \frac{\log 1.8}{\log(0.029\,317)} = -0.166\,531, \quad a_3 = \frac{38}{(34\,110)^{-0.166\,531}} = 216.10 \text{ kpsi}$$

$$N_3 = \left(\frac{32}{216.1}\right)^{-1/0.166\,531} = 95\,740 \text{ cycles} \quad Ans.$$

6-31 Using Miner's method

$$a = \frac{[0.9(100)]^2}{50} = 162 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.9(100)}{50} = -0.085\,091$$

$$\sigma_1 = 70 \text{ kpsi}, \quad N_1 = \left(\frac{70}{162}\right)^{1/-0.085\,091} = 19\,170 \text{ cycles}$$

$$\sigma_2 = 55 \text{ kpsi}, \quad N_2 = \left(\frac{55}{162}\right)^{1/-0.085\,091} = 326\,250 \text{ cycles}$$

$$\sigma_3 = 40 \text{ kpsi}, \quad N_3 \to \infty$$

$$\frac{0.2N}{19\,170} + \frac{0.5N}{326\,250} + \frac{0.3N}{\infty} = 1$$

$$N = 83\,570 \text{ cycles} \quad Ans.$$

6-32 Given $\mathbf{S}_{ut} = 245\mathbf{LN}(1, 0.0508)$ kpsi

From Table 7-13: $a = 1.34, b = -0.086, C = 0.12$

$$\mathbf{k}_a = 1.34\bar{S}_{ut}^{-0.086}\mathbf{LN}(1, 0.120)$$
$$= 1.34(245)^{-0.086}\mathbf{LN}(1, 0.12)$$
$$= 0.835\mathbf{LN}(1, 0.12)$$
$$k_b = 1.02 \quad \text{(as in Prob. 6-1)}$$

Eq. (6-70) $$\mathbf{S}_e = 0.835(1.02)\mathbf{LN}(1, 0.12)[107\mathbf{LN}(1, 0.139)]$$

$$\bar{S}_e = 0.835(1.02)(107) = 91.1 \text{ kpsi}$$

Now

$$C_{Se} \doteq (0.12^2 + 0.139^2)^{1/2} = 0.184$$

$$\mathbf{S}_e = 91.1\mathbf{LN}(1, 0.184) \text{ kpsi} \quad \textit{Ans.}$$

6-33 A Priori Decisions:

- Material and condition: 1018 CD, $\mathbf{S}_{ut} = 440\mathbf{LN}(1, 0.03)$, and $\mathbf{S}_y = 370\mathbf{LN}(1, 0.061)$ MPa
- Reliability goal: $R = 0.999 \quad (z = -3.09)$
- Function:

  Critical location—hole

- Variabilities:

$$C_{ka} = 0.058$$

$$C_{kc} = 0.125$$

$$C_\phi = 0.138$$

$$C_{Se} = \left(C_{ka}^2 + C_{kc}^2 + C_\phi^2\right)^{1/2} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$

$$C_{kc} = 0.10$$

$$C_{Fa} = 0.20$$

$$C_{\sigma a} = (0.10^2 + 0.20^2)^{1/2} = 0.234$$

$$C_n = \sqrt{\frac{C_{Se}^2 + C_{\sigma a}^2}{1 + C_{\sigma a}^2}} = \sqrt{\frac{0.195^2 + 0.234^2}{1 + 0.234^2}} = 0.297$$

Resulting in a design factor n_f of,

Eq. (6-88): $n_f = \exp[-(-3.09)\sqrt{\ln(1 + 0.297^2)} + \ln\sqrt{1 + 0.297^2}] = 2.56$

- Decision: Set $n_f = 2.56$

Now proceed deterministically using the mean values:

Table 6-10: $\bar{k}_a = 4.45(440)^{-0.265} = 0.887$

$k_b = 1$

Table 6-11: $\bar{k}_c = 1.43(440)^{-0.0778} = 0.891$

Eq. (6-70): $\bar{S}'_e = 0.506(440) = 222.6$ MPa

Eq. (6-71): $\bar{S}_e = 0.887(1)0.891(222.6) = 175.9$ MPa

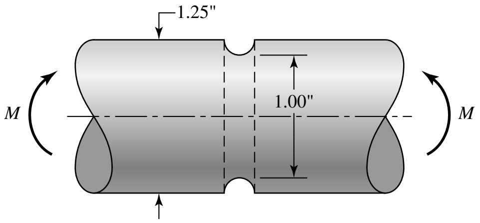
From Prob. 6-10, $K_f = 2.23$. Thus,

$$\bar{\sigma}_a = \bar{K}_f \frac{\bar{F}_a}{A} = \bar{K}_f \frac{\bar{F}_a}{t(60 - 12)} = \frac{\bar{S}_e}{\bar{n}_f}$$

$$\text{and, } t = \frac{\bar{n}_f \bar{K}_f \bar{F}_a}{48\bar{S}_e} = \frac{2.56(2.23)15(10^3)}{48(175.9)} = 10.14 \text{ mm}$$

Decision: Depending on availability, (1) select $t = 10$ mm, recalculate n_f and R, and determine whether the reduced reliability is acceptable, or, (2) select $t = 11$ mm or larger, and determine whether the increase in cost and weight is acceptable. *Ans*.

6-34



Rotation is presumed. M and S_{ut} are given as deterministic, but notice that $\boldsymbol{\sigma}$ is not; therefore, a reliability estimation can be made.

From Eq. (6-70):

$$\mathbf{S}'_e = 0.506(110)\mathbf{LN}(1, 0.138)$$
$$= 55.7\mathbf{LN}(1, 0.138) \text{ kpsi}$$

Table 6-10:

$$\mathbf{k}_a = 2.67(110)^{-0.265}\mathbf{LN}(1, 0.058)$$
$$= 0.768\mathbf{LN}(1, 0.058)$$

Based on $d = 1$ in, Eq. (6-20) gives

$$k_b = \left(\frac{1}{0.30}\right)^{-0.107} = 0.879$$

Conservatism is not necessary

$$\mathbf{S}_e = 0.768[\mathbf{LN}(1, 0.058)](0.879)(55.7)[\mathbf{LN}(1, 0.138)]$$
$$\bar{S}_e = 37.6 \text{ kpsi}$$
$$C_{Se} = (0.058^2 + 0.138^2)^{1/2} = 0.150$$
$$\mathbf{S}_e = 37.6\mathbf{LN}(1, 0.150)$$

Fig. A-15-14: $D/d = 1.25$, $r/d = 0.125$. Thus $K_t = 1.70$ and Eqs. (6-78), (6-79) and Table 6-15 give

$$\mathbf{K}_f = \frac{1.70\mathbf{LN}(1, 0.15)}{1 + \left(2/\sqrt{0.125}\right)[(1.70 - 1)/(1.70)](3/110)}$$
$$= 1.598\mathbf{LN}(1, 0.15)$$
$$\boldsymbol{\sigma} = \mathbf{K}_f \frac{32M}{\pi d^3} = 1.598[\mathbf{LN}(1 - 0.15)]\left[\frac{32(1400)}{\pi(1)^3}\right]$$
$$= 22.8\mathbf{LN}(1, 0.15) \text{ kpsi}$$

From Eq. (5-43), p. 242:

$$z = -\frac{\ln\left[(37.6/22.8)\sqrt{(1 + 0.15^2)/(1 + 0.15^2)}\right]}{\sqrt{\ln[(1 + 0.15^2)(1 + 0.15^2)]}} = -2.37$$

From Table A-10, $p_f = 0.008\,89$

$$\therefore R = 1 - 0.008\,89 = 0.991 \quad Ans.$$

Note: The correlation method uses only the mean of $\mathbf{S}_{ut}$; its variability is already included in the 0.138. When a deterministic load, in this case M, is used in a reliability estimate, engineers state, "For a *Design* Load of M, the reliability is 0.991." They are in fact referring to a Deterministic Design Load.

6-35 For completely reversed torsion, $\mathbf{k}_a$ and $\mathbf{k}_b$ of Prob. 6-34 apply, but $\mathbf{k}_c$ must also be considered.

Eq. 6-74:

$$\mathbf{k}_c = 0.328(110)^{0.125}\mathbf{LN}(1, 0.125)$$

$$= 0.590\mathbf{LN}(1, 0.125)$$

Note 0.590 is close to 0.577.

$$\mathbf{S}_{Se} = \mathbf{k}_a\, k_b\, \mathbf{k}_c\, \mathbf{S}'_e$$

$$= 0.768[\mathbf{LN}(1, 0.058)](0.878)[0.590\mathbf{LN}(1, 0.125)][55.7\mathbf{LN}(1, 0.138)]$$

$$\bar{S}_{Se} = 0.768(0.878)(0.590)(55.7) = 22.2 \text{ kpsi}$$

$$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$

$$\mathbf{S}_{Se} = 22.2\mathbf{LN}(1, 0.195) \text{ kpsi}$$

Fig. A-15-15: $D/d = 1.25$, $r/d = 0.125$, then $K_{ts} = 1.40$. From Eqs. (6-78), (6-79) and Table 6-15

$$\mathbf{K}_{ts} = \frac{1.40\mathbf{LN}(1, 0.15)}{1 + \left(2/\sqrt{0.125}\right)[(1.4-1)/1.4](3/110)} = 1.34\mathbf{LN}(1, 0.15)$$

$$\tau = \mathbf{K}_{ts}\,\frac{16T}{\pi d^3}$$

$$\tau = 1.34[\mathbf{LN}(1, 0.15)]\left[\frac{16(1.4)}{\pi(1)^3}\right]$$

$$= 9.55\mathbf{LN}(1, 0.15) \text{ kpsi}$$

From Eq. (5-43), p. 242:

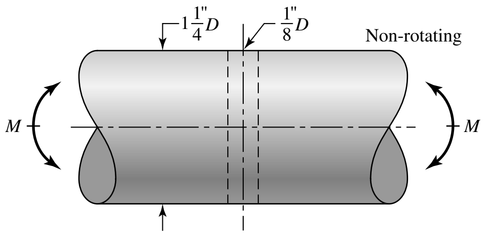
$$z = -\frac{\ln\left[(22.2/9.55)\sqrt{(1+0.15^2)/(1+0.195^2)}\right]}{\sqrt{\ln[(1+0.195^2)(1+0.15^2)]}} = -3.43$$

From Table A-10, $p_f = 0.0003$

$$R = 1 - p_f = 1 - 0.0003 = 0.9997 \quad Ans.$$

For a design with completely-reversed torsion of 1400 lbf · in, the reliability is 0.9997. The improvement comes from a smaller stress-concentration factor in torsion. See the note at the end of the solution of Prob. 6-34 for the reason for the phraseology.

6-36



$$S_{ut} = 58 \text{ kpsi}$$

$$\mathbf{S}'_e = 0.506(58)\mathbf{LN}(1, 0.138)$$

$$= 29.3\mathbf{LN}(1, 0.138) \text{ kpsi}$$

Table 6-10:
$$\mathbf{k}_a = 14.5(58)^{-0.719}\mathbf{LN}(1, 0.11)$$

$$= 0.782\mathbf{LN}(1, 0.11)$$

Eq. (6-24):

$$d_e = 0.37(1.25) = 0.463 \text{ in}$$

$$k_b = \left(\frac{0.463}{0.30}\right)^{-0.107} = 0.955$$

$$\mathbf{S}_e = 0.782[\mathbf{LN}(1, 0.11)](0.955)[29.3\mathbf{LN}(1, 0.138)]$$

$$\bar{S}_e = 0.782(0.955)(29.3) = 21.9 \text{ kpsi}$$

$$C_{Se} = (0.11^2 + 0.138^2)^{1/2} = 0.150$$

Table A-16: $d/D = 0, a/D = 0.1, A = 0.83 \therefore K_t = 2.27$.

From Eqs. (6-78) and (6-79) and Table 6-15

$$\mathbf{K}_f = \frac{2.27\mathbf{LN}(1, 0.10)}{1 + \left(2/\sqrt{0.125}\right)[(2.27-1)/2.27](5/58)} = 1.783\mathbf{LN}(1, 0.10)$$

Table A-16:

$$Z = \frac{\pi A D^3}{3^2} = \frac{\pi(0.83)(1.25^3)}{32} = 0.159 \text{ in}^3$$

$$\boldsymbol{\sigma} = \mathbf{K}_f \frac{M}{Z} = 1.783\mathbf{LN}(1, 0.10)\left(\frac{1.6}{0.159}\right)$$

$$= 17.95\mathbf{LN}(1, 0.10) \text{ kpsi}$$

$$\bar{\sigma} = 17.95 \text{ kpsi}$$

$$C_\sigma = 0.10$$

Eq. (5-43), p. 242:
$$z = -\frac{\ln\left[(21.9/17.95)\sqrt{(1+0.10^2)/(1+0.15^2)}\right]}{\sqrt{\ln[(1+0.15^2)(1+0.10^2)]}} = -1.07$$

Table A-10:
$$p_f = 0.1423$$

$$R = 1 - p_f = 1 - 0.1423 = 0.858 \quad Ans.$$

For a completely-reversed design load M_a of 1400 lbf · in, the reliability estimate is 0.858.

6-37 For a non-rotating bar subjected to completely reversed torsion of $T_a = 2400$ lbf · in

From Prob. 6-36:

$$S'_e = 29.3\mathbf{LN}(1, 0.138) \text{ kpsi}$$
$$\mathbf{k}_a = 0.782\mathbf{LN}(1, 0.11)$$
$$k_b = 0.955$$

For $\mathbf{k}_c$ use Eq. (6-74):

$$\begin{aligned}\mathbf{k}_c &= 0.328(58)^{0.125}\mathbf{LN}(1, 0.125)\\ &= 0.545\mathbf{LN}(1, 0.125)\\ \mathbf{S}_{Se} &= 0.782[\mathbf{LN}(1, 0.11)](0.955)[0.545\mathbf{LN}(1, 0.125)][29.3\mathbf{LN}(1, 0.138)]\\ \bar{S}_{Se} &= 0.782(0.955)(0.545)(29.3) = 11.9 \text{ kpsi}\\ C_{Se} &= (0.11^2 + 0.125^2 + 0.138^2)^{1/2} = 0.216\end{aligned}$$

Table A-16: $d/D = 0, a/D = 0.1, A = 0.92, K_{ts} = 1.68$

From Eqs. (6-78), (6-79), Table 6-15

$$\begin{aligned}\mathbf{K}_{fs} &= \frac{1.68\mathbf{LN}(1, 0.10)}{1 + \left(2/\sqrt{0.125}\right)[(1.68 - 1)/1.68](5/58)}\\ &= 1.403\mathbf{LN}(1, 0.10)\end{aligned}$$

Table A-16:

$$\begin{aligned}J_{\text{net}} &= \frac{\pi A D^4}{32} = \frac{\pi(0.92)(1.25^4)}{32} = 0.2201\\ \tau_a &= \mathbf{K}_{fs}\frac{T_a c}{J_{\text{net}}}\\ &= 1.403[\mathbf{LN}(1, 0.10)]\left[\frac{2.4(1.25/2)}{0.2201}\right]\\ &= 9.56\mathbf{LN}(1, 0.10) \text{ kpsi}\end{aligned}$$

From Eq. (5-43), p. 242:

$$z = -\frac{\ln\left[(11.9/9.56)\sqrt{(1 + 0.10^2)/(1 + 0.216^2)}\right]}{\sqrt{\ln[(1 + 0.10^2)(1 + 0.216^2)]}} = -0.85$$

Table A-10, $p_f = 0.1977$

$$R = 1 - p_f = 1 - 0.1977 = 0.80 \quad \textit{Ans.}$$

6-38 This is a very important task for the student to attempt before starting Part 3. It illustrates the drawback of the deterministic factor of safety method. It also identifies the a priori decisions and their consequences.

The range of force fluctuation in Prob. 6-23 is −16 to +4 kip, or 20 kip. Repeatedly-applied F_a is 10 kip. The stochastic properties of this heat of AISI 1018 CD are given.

Function	Consequences
Axial	$F_a = 10$ kip
Fatigue load	$C_{Fa} = 0$
	$C_{kc} = 0.125$
Overall reliability $R \geq 0.998$; with twin fillets $R \geq \sqrt{0.998} \geq 0.999$	$z = -3.09$ $C_{Kf} = 0.11$
Cold rolled or machined surfaces	$C_{ka} = 0.058$
Ambient temperature	$C_{kd} = 0$
Use correlation method	$C_{\phi} = 0.138$
Stress amplitude	$C_{Kf} = 0.11$
	$C_{\sigma a} = 0.11$
Significant strength S_e	$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2}$ $= 0.195$

Choose the mean design factor which will meet the reliability goal

$$C_n = \sqrt{\frac{0.195^2 + 0.11^2}{1 + 0.11^2}} = 0.223$$

$$\bar{n} = \exp\left[-(-3.09)\sqrt{\ln(1 + 0.223^2)} + \ln\sqrt{1 + 0.223^2}\right]$$

$$\bar{n} = 2.02$$

Review the number and quantitative consequences of the designer's a priori decisions to accomplish this. The operative equation is the definition of the design factor

$$\boldsymbol{\sigma}_a = \frac{\mathbf{S}_e}{\mathbf{n}}$$

$$\bar{\sigma}_a = \frac{\bar{S}_e}{\bar{n}} \Rightarrow \frac{\bar{K}_f F_a}{w_2 h} = \frac{\bar{S}_e}{\bar{n}}$$

Solve for thickness h. To do so we need

$$\bar{k}_a = 2.67\bar{S}_{ut}^{-0.265} = 2.67(64)^{-0.265} = 0.887$$

$$k_b = 1$$

$$\bar{k}_c = 1.23\bar{S}_{ut}^{-0.078} = 1.23(64)^{-0.078} = 0.889$$

$$\bar{k}_d = \bar{k}_e = 1$$

$$\bar{S}_e = 0.887(1)(0.889)(1)(1)(0.506)(64) = 25.5 \text{ kpsi}$$

Fig. A-15-5: $D = 3.75$ in, $d = 2.5$ in, $D/d = 3.75/2.5 = 1.5$, $r/d = 0.25/2.5 = 0.10$

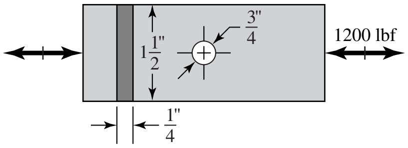
$\therefore K_t = 2.1$

$$\bar{K}_f = \frac{2.1}{1 + \left(2/\sqrt{0.25}\right)[(2.1-1)/(2.1)](4/64)} = 1.857$$

$$h = \frac{\bar{K}_f \bar{n} F_a}{w_2 \bar{S}_e} = \frac{1.857(2.02)(10)}{2.5(25.5)} = 0.667 \quad \textit{Ans.}$$

This thickness separates $\bar{S}_e$ and $\bar{\sigma}_a$ so as to realize the reliability goal of 0.999 at each shoulder. The design decision is to make *t* the next available thickness of 1018 CD steel strap from the same heat. This eliminates machining to the desired thickness and the extra cost of thicker work stock will be less than machining the fares. Ask your steel supplier what is available *in this heat*.

6-39



$$F_a = 1200 \text{ lbf}$$
$$S_{ut} = 80 \text{ kpsi}$$

(a) *Strength*

$$\mathbf{k}_a = 2.67(80)^{-0.265}\mathbf{LN}(1, 0.058)$$
$$= 0.836\mathbf{LN}(1, 0.058)$$
$$k_b = 1$$
$$\mathbf{k}_c = 1.23(80)^{-0.078}\mathbf{LN}(1, 0.125)$$
$$= 0.874\mathbf{LN}(1, 0.125)$$
$$\mathbf{S}'_a = 0.506(80)\mathbf{LN}(1, 0.138)$$
$$= 40.5\mathbf{LN}(1, 0.138) \text{ kpsi}$$
$$\mathbf{S}_e = 0.836[\mathbf{LN}(1, 0.058)](1)[0.874\mathbf{LN}(1, 0.125)][40.5\mathbf{LN}(1, 0.138)]$$
$$\bar{S}_e = 0.836(1)(0.874)(40.5) = 29.6 \text{ kpsi}$$
$$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$

Stress: Fig. A-15-1; $d/w = 0.75/1.5 = 0.5$, $K_t = 2.17$. From Eqs. (6-78), (6-79) and Table 6-15

$$\mathbf{K}_f = \frac{2.17\mathbf{LN}(1, 0.10)}{1 + \left(2/\sqrt{0.375}\right)[(2.17-1)/2.17](5/80)}$$
$$= 1.95\mathbf{LN}(1, 0.10)$$
$$\boldsymbol{\sigma}_a = \frac{\mathbf{K}_f F_a}{(w-d)t}, \quad C_\sigma = 0.10$$
$$\bar{\sigma}_a = \frac{\bar{K}_f F_a}{(w-d)t} = \frac{1.95(1.2)}{(1.5-0.75)(0.25)} = 12.48 \text{ kpsi}$$

$$\bar{S}_a = \bar{S}_e = 29.6 \text{ kpsi}$$

$$z = -\frac{\ln(\bar{S}_a/\bar{\sigma}_a)\sqrt{\left(1+C_\sigma^2\right)/\left(1+C_S^2\right)}}{\sqrt{\ln\left(1+C_\sigma^2\right)\left(1+C_S^2\right)}}$$

$$= -\frac{\ln\left[(29.6/12.48)\sqrt{(1+0.10^2)/(1+0.195^2)}\right]}{\sqrt{\ln(1+0.10^2)(1+0.195^2)}} = -3.9$$

From Table A-20

$$p_f = 4.481(10^{-5})$$

$$R = 1 - 4.481(10^{-5}) = 0.999\,955 \quad \textit{Ans.}$$

(b) All computer programs will differ in detail.

6-40 Each computer program will differ in detail. When the programs are working, the experience should reinforce that the decision regarding $\bar{n}_f$ is independent of mean values of strength, stress or associated geometry. The reliability goal can be realized by noting the impact of all those a priori decisions.

6-41 Such subprograms allow a simple call when the information is needed. The calling program is often named an executive routine (executives tend to delegate chores to others and only want the answers).

6-42 This task is similar to Prob. 6-41.

6-43 Again, a similar task.

6-44 The results of Probs. 6-41 to 6-44 will be the basis of a class computer aid for fatigue problems. The codes should be made available to the class through the library of the computer network or main frame available to your students.

6-45 Peterson's notch sensitivity q has very little statistical basis. This subroutine can be used to show the variation in $\mathbf{q}$, which is not apparent to those who embrace a deterministic q.

6-46 An additional program which is useful.

Chapter 7

7-1 **(a)** DE-Gerber, Eq. (7-10):

$$A = \left\{4[2.2(600)]^2 + 3[1.8(400)]^2\right\}^{1/2} = 2920 \text{ lbf} \cdot \text{in}$$

$$B = \left\{4[2.2(500)]^2 + 3[1.8(300)]^2\right\}^{1/2} = 2391 \text{ lbf} \cdot \text{in}$$

$$d = \left\{ \frac{8(2)(2920)}{\pi(30\,000)} \left[1 + \left(1 + \left[\frac{2(2391)(30\,000)}{2920(100\,000)} \right]^2 \right)^{1/2} \right] \right\}^{1/3}$$

$$= 1.016 \text{ in} \quad \textit{Ans.}$$

(b) DE-elliptic, Eq. (7-12) can be shown to be

$$d = \left(\frac{16n}{\pi} \sqrt{\frac{A^2}{S_e^2} + \frac{B^2}{S_y^2}} \right)^{1/3}$$

$$= \left[\frac{16(2)}{\pi} \sqrt{\left(\frac{2920}{30\,000}\right)^2 + \left(\frac{2391}{80\,000}\right)^2} \right]^{1/3} = 1.012 \text{ in} \quad \textit{Ans.}$$

(c) DE-Soderberg, Eq. (7-14) can be shown to be

$$d = \left[\frac{16n}{\pi} \left(\frac{A}{S_e} + \frac{B}{S_y} \right) \right]^{1/3}$$

$$= \left[\frac{16(2)}{\pi} \left(\frac{2920}{30\,000} + \frac{2391}{80\,000} \right) \right]^{1/3}$$

$$= 1.090 \text{ in} \quad \textit{Ans.}$$

(d) DE-Goodman: Eq. (7-8) can be shown to be

$$d = \left[\frac{16n}{\pi} \left(\frac{A}{S_e} + \frac{B}{S_{ut}} \right) \right]^{1/3}$$

$$= \left[\frac{16(2)}{\pi} \left(\frac{2920}{30\,000} + \frac{2391}{100\,000} \right) \right]^{1/3} = 1.073 \text{ in} \quad \textit{Ans.}$$

Criterion	d (in)	Compared to DE-Gerber	
DE-Gerber	1.016		
DE-elliptic	1.012	0.4% lower	less conservative
DE-Soderberg	1.090	7.3% higher	more conservative
DE-Goodman	1.073	5.6% higher	more conservative

7-2 This problem has to be done by successive trials, since S_e is a function of shaft size. The material is SAE 2340 for which $S_{ut} = 1226$ MPa, $S_y = 1130$ MPa, and $H_B \geq 368$.

Eq. (6-19): $k_a = 4.51(1226)^{-0.265} = 0.685$

Trial #1: Choose $d_r = 22$ mm

Eq. (6-20): $k_b = \left(\frac{22}{7.62}\right)^{-0.107} = 0.893$

Eq. (6-18): $S_e = 0.685(0.893)(0.5)(1226) = 375$ MPa

$$d_r = d - 2r = 0.75D - 2D/20 = 0.65D$$

$$D = \frac{d_r}{0.65} = \frac{22}{0.65} = 33.8 \text{ mm}$$

$$r = \frac{D}{20} = \frac{33.8}{20} = 1.69 \text{ mm}$$

Fig. A-15-14:

$$d = d_r + 2r = 22 + 2(1.69) = 25.4 \text{ mm}$$

$$\frac{d}{d_r} = \frac{25.4}{22} = 1.15$$

$$\frac{r}{d_r} = \frac{1.69}{22} = 0.077$$

$$K_t = 1.9$$

Fig. A-15-15: $K_{ts} = 1.5$

Fig. 6-20: $r = 1.69$ mm, $q = 0.90$

Fig. 6-21: $r = 1.69$ mm, $q_s = 0.97$

Eq. (6-32): $K_f = 1 + 0.90(1.9 - 1) = 1.81$

$$K_{fs} = 1 + 0.97(1.5 - 1) = 1.49$$

We select the DE-ASME Elliptic failure criteria.

Eq. (7-12) with d as d_r, and $M_m = T_a = 0$,

$$d_r = \left\{\frac{16(2.5)}{\pi}\left[4\left(\frac{1.81(70)(10^3)}{375}\right)^2 + 3\left(\frac{1.49(45)(10^3)}{1130}\right)^2\right]^{1/2}\right\}^{1/3}$$

$$= 20.6 \text{ mm}$$

Trial #2: Choose $d_r = 20.6$ mm

$$k_b = \left(\frac{20.6}{7.62}\right)^{-0.107} = 0.899$$

$$S_e = 0.685(0.899)(0.5)(1226) = 377.5 \text{ MPa}$$

$$D = \frac{d_r}{0.65} = \frac{20.6}{0.65} = 31.7 \text{ mm}$$

$$r = \frac{D}{20} = \frac{31.7}{20} = 1.59 \text{ mm}$$

Figs. A-15-14 and A-15-15:

$$d = d_r + 2r = 20.6 + 2(1.59) = 23.8 \text{ mm}$$

$$\frac{d}{d_r} = \frac{23.8}{20.6} = 1.16$$

$$\frac{r}{d_r} = \frac{1.59}{20.6} = 0.077$$

We are at the limit of readability of the figures so

$$K_t = 1.9, \quad K_{ts} = 1.5 \qquad q = 0.9, \quad q_s = 0.97$$

$$\therefore K_f = 1.81 \qquad K_{fs} = 1.49$$

Using Eq. (7-12) produces $d_r = 20.5$ mm. Further iteration produces no change.

Decisions:

$$d_r = 20.5 \text{ mm}$$

$$D = \frac{20.5}{0.65} = 31.5 \text{ mm}, \quad d = 0.75(31.5) = 23.6 \text{ mm}$$

Use $D = 32$ mm, $d = 24$ mm, $r = 1.6$ mm *Ans.*

7-3 $F \cos 20°(d/2) = T, \quad F = 2T/(d \cos 20°) = 2(3000)/(6 \cos 20°) = 1064 \text{ lbf}$

$$M_C = 1064(4) = 4257 \text{ lbf} \cdot \text{in}$$

For sharp fillet radii at the shoulders, from Table 7-1, $K_t = 2.7$, and $K_{ts} = 2.2$. Examining Figs. 6-20 and 6-21, with $S_{ut} = 80$ kpsi, conservatively estimate $q = 0.8$ and $q_s = 0.9$. These estimates can be checked once a specific fillet radius is determined.

Eq. (6-32): $K_f = 1 + (0.8)(2.7 - 1) = 2.4$

$$K_{fs} = 1 + (0.9)(2.2 - 1) = 2.1$$

(a) Static analysis using fatigue stress concentration factors:

From Eq. (7-15) with $M = M_m$, $T = T_m$, and $M_a = T_a = 0$,

$$\sigma'_{\max} = \left[\left(\frac{32K_f M}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T}{\pi d^3}\right)^2\right]^{1/2}$$

Eq. (7-16):
$$n = \frac{S_y}{\sigma'_{\max}} = \frac{S_y}{\left[\left(\dfrac{32K_f M}{\pi d^3}\right)^2 + 3\left(\dfrac{16K_{fs}T}{\pi d^3}\right)^2\right]^{1/2}}$$

Solving for d,

$$\begin{aligned} d &= \left\{\frac{16n}{\pi S_y}\left[4(K_f M)^2 + 3(K_{fs}T)^2\right]^{1/2}\right\}^{1/3} \\ &= \left\{\frac{16(2.5)}{\pi(60\,000)}\left[4(2.4)(4257)^2 + 3(2.1)(3000)^2\right]^{1/2}\right\}^{1/3} \\ &= 1.700 \text{ in} \quad Ans. \end{aligned}$$

(b)
$$k_a = 2.70(80)^{-0.265} = 0.845$$

Assume $d = 2.00$ in to estimate the size factor,

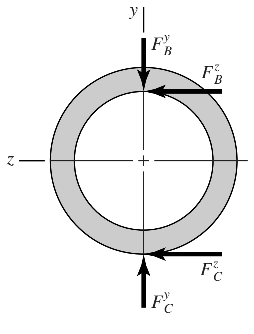
$$k_b = \left(\frac{2}{0.3}\right)^{-0.107} = 0.816$$
$$S_e = 0.845(0.816)(0.5)(80) = 27.6 \text{ kpsi}$$

Selecting the DE-ASME Elliptic criteria, use Eq. (7-12) with $M_m = T_a = 0$.

$$d = \left\{\frac{16(2.5)}{\pi}\left[4\left(\frac{2.4(4257)}{27\,600}\right)^2 + 3\left(\frac{2.1(3000)}{60\,000}\right)^2\right]^{1/2}\right\}^{1/3} = 2.133 \text{ in}$$

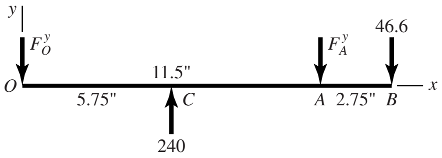
Revising k_b results in $d = 2.138$ in *Ans.*

7-4 We have a design task of identifying bending moment and torsion diagrams which are preliminary to an industrial roller shaft design.



$$\begin{aligned} F_C^y &= 30(8) = 240 \text{ lbf} \\ F_C^z &= 0.4(240) = 96 \text{ lbf} \\ T &= F_C^z(2) = 96(2) = 192 \text{ lbf}\cdot\text{in} \\ F_B^z &= \frac{T}{1.5} = \frac{192}{1.5} = 128 \text{ lbf} \\ F_B^y &= F_B^z \tan 20^\circ = 128 \tan 20^\circ = 46.6 \text{ lbf} \end{aligned}$$

(a) *xy-plane*



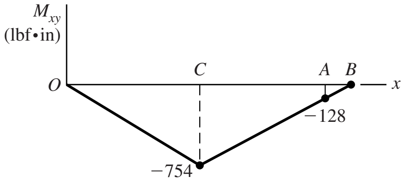
$$\sum M_O = 240(5.75) - F_A^y(11.5) - 46.6(14.25) = 0$$

$$F_A^y = \frac{240(5.75) - 46.6(14.25)}{11.5} = 62.3\ \text{lbf}$$

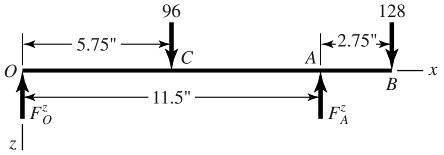
$$\sum M_A = F_O^y(11.5) - 46.6(2.75) - 240(5.75) = 0$$

$$F_O^y = \frac{240(5.75) + 46.6(2.75)}{11.5} = 131.1\ \text{lbf}$$

Bending moment diagram



xz-plane



$$\sum M_O = 0$$

$$= 96(5.75) - F_A^z(11.5) + 128(14.25)$$

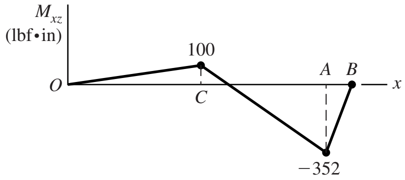
$$F_A^z = \frac{96(5.75) + 128(14.25)}{11.5} = 206.6\ \text{lbf}$$

$$\sum M_A = 0$$

$$= F_O^z(11.5) + 128(2.75) - 96(5.75)$$

$$F_O^z = \frac{96(5.75) - 128(2.75)}{11.5} = 17.4\ \text{lbf}$$

Bending moment diagram:

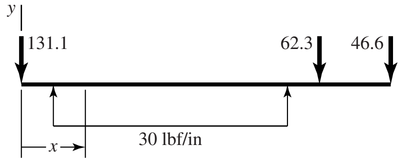


$$M_C = \sqrt{100^2 + (-754)^2} = 761\ \text{lbf} \cdot \text{in}$$

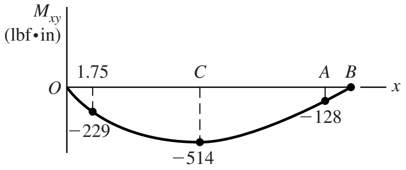
$$M_A = \sqrt{(-128)^2 + (-352)^2} = 375\ \text{lbf} \cdot \text{in}$$

This approach over-estimates the bending moment at *C*, but not at *A*.

(b) *xy-plane*



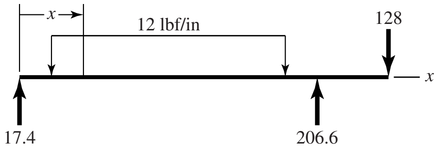
$$M_{xy} = -131.1x + 15\langle x - 1.75\rangle^2 - 15\langle x - 9.75\rangle^2 - 62.3\langle x - 11.5\rangle^1$$



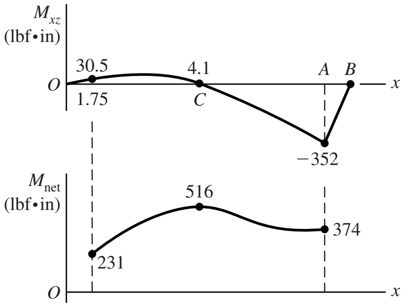
$$M_{\text{max}} \text{ occurs at } 6.12 \text{ in}$$
$$M_{\text{max}} = -516 \text{ lbf} \cdot \text{in}$$
$$M_C = 131.1(5.75) - 15(5.75 - 1.75)^2 = 514$$

Reduced from 754 lbf · in. The maximum occurs at $x = 6.12$ in rather than C, but it is close enough.

xz-plane



$$M_{xz} = 17.4x - 6\langle x - 1.75\rangle^2 + 6\langle x - 9.75\rangle^2 + 206.6\langle x - 11.5\rangle^1$$



Let $M_{\text{net}} = \sqrt{M_{xy}^2 + M_{xz}^2}$

Plot $M_{\text{net}}(x)$

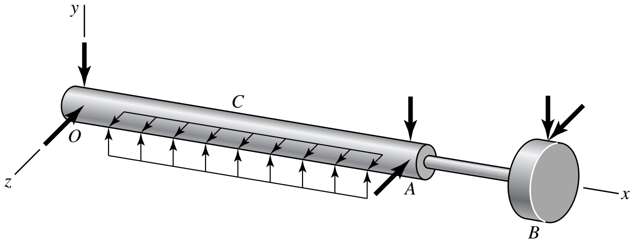
$1.75 \le x \le 11.5$ in

$M_{\text{max}} = 516$ lbf · in
at $x = 6.25$ in

Torque: In both cases the torque rises from 0 to 192 lbf · in linearly across the roller and is steady until the coupling keyway is encountered; then it falls linearly to 0 across the key. *Ans.*

7-5 This is a design problem, which can have many acceptable designs. See the solution for Problem 7-7 for an example of the design process.

7-6 If students have access to finite element or beam analysis software, have them model the shaft to check deflections. If not, solve a simpler version of shaft. The 1" diameter sections will not affect the results much, so model the 1" diameter as 1.25". Also, ignore the step in *AB*.



From Prob. 18-10, integrate M_{xy} and M_{xz}

xy plane, with $dy/dx = y'$

$$EIy' = -\frac{131.1}{2}(x^2) + 5\langle x - 1.75\rangle^3 - 5\langle x - 9.75\rangle^3 - \frac{62.3}{2}\langle x - 11.5\rangle^2 + C_1 \quad (1)$$

$$EIy = -\frac{131.1}{6}(x^3) + \frac{5}{4}\langle x - 1.75\rangle^4 - \frac{5}{4}\langle x - 9.75\rangle^4 - \frac{62.3}{6}\langle x - 11.5\rangle^3 + C_1 x + C_2$$

$$y = 0 \text{ at } x = 0 \quad \Rightarrow \quad C_2 = 0$$

$$y = 0 \text{ at } x = 11.5 \quad \Rightarrow \quad C_1 = 1908.4 \text{ lbf}\cdot\text{in}^3$$

From (1) $\quad x = 0$: $\quad EIy' = 1908.4$

$x = 11.5$: $\quad EIy' = -2153.1$

xz plane (treating $z \uparrow +$)

$$EIz' = \frac{17.4}{2}(x^2) - 2\langle x - 1.75\rangle^3 + 2\langle x - 9.75\rangle^3 + \frac{206.6}{2}\langle x - 11.5\rangle^2 + C_3 \quad (2)$$

$$EIz = \frac{17.4}{6}(x^3) - \frac{1}{2}\langle x - 1.75\rangle^4 + \frac{1}{2}\langle x - 9.75\rangle^4 + \frac{206.6}{6}\langle x - 11.5\rangle^3 + C_3 x + C_4$$

$$z = 0 \text{ at } x = 0 \quad \Rightarrow \quad C_4 = 0$$

$$z = 0 \text{ at } x = 11.5 \quad \Rightarrow \quad C_3 = 8.975 \text{ lbf}\cdot\text{in}^3$$

From (2)

$x = 0$: $\quad EIz' = 8.975$

$x = 11.5$: $\quad EIz' = -683.5$

At *O*: $\quad EI\theta = \sqrt{1908.4^2 + 8.975^2} = 1908.4 \text{ lbf}\cdot\text{in}^3$

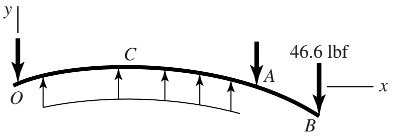
A: $\quad EI\theta = \sqrt{(-2153.1)^2 + (-683.5)^2} = 2259 \text{ lbf}\cdot\text{in}^3$ (dictates size)

$$\theta = \frac{2259}{30(10^6)(\pi/64)(1.25^4)} = 0.000\,628 \text{ rad}$$

$$n = \frac{0.001}{0.000\,628} = 1.59$$

At gear mesh, B

xy plane



With $I = I_1$ in section *OCA*,

$$y'_A = -2153.1/EI_1$$

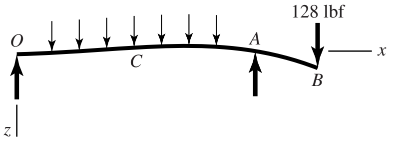
Since $y'_{B/A}$ is a cantilever, from Table A-9-1, with $I = I_2$ in section *AB*

$$y'_{B/A} = \frac{Fx(x-2l)}{2EI_2} = \frac{46.6}{2EI_2}(2.75)[2.75 - 2(2.75)] = -176.2/EI_2$$

$$\therefore y'_B = y'_A + y'_{B/A} = -\frac{2153.1}{30(10^6)(\pi/64)(1.25^4)} - \frac{176.2}{30(10^6)(\pi/64)(0.875^4)}$$

$$= -0.000\,803 \text{ rad} \quad \text{(magnitude greater than 0.0005 rad)}$$

xz plane



$$z'_A = -\frac{683.5}{EI_1}, \quad z'_{B/A} = -\frac{128(2.75^2)}{2EI_2} = -\frac{484}{EI_2}$$

$$z'_B = -\frac{683.5}{30(10^6)(\pi/64)(1.25^4)} - \frac{484}{30(10^6)(\pi/64)(0.875^4)} = -0.000\,751 \text{ rad}$$

$$\theta_B = \sqrt{(-0.000\,803)^2 + (0.000\,751)^2} = 0.001\,10 \text{ rad}$$

Crowned teeth must be used.

Finite element results:	Error in simplified model
$\theta_O = 5.47(10^{-4})$ rad	3.0%
$\theta_A = 7.09(10^{-4})$ rad	11.4%
$\theta_B = 1.10(10^{-3})$ rad	0.0%

The simplified model yielded reasonable results.

Strength $\quad S_{ut} = 72$ kpsi, $\quad S_y = 39.5$ kpsi

At the shoulder at A, $x = 10.75$ in. From Prob. 7-4,

$$M_{xy} = -209.3 \text{ lbf} \cdot \text{in}, \quad M_{xz} = -293.0 \text{ lbf} \cdot \text{in}, \quad T = 192 \text{ lbf} \cdot \text{in}$$

$$M = \sqrt{(-209.3)^2 + (-293)^2} = 360.0 \text{ lbf} \cdot \text{in}$$

$$S'_e = 0.5(72) = 36 \text{ kpsi}$$

$$k_a = 2.70(72)^{-0.265} = 0.869$$

$$k_b = \left(\frac{1}{0.3}\right)^{-0.107} = 0.879$$

$$k_c = k_d = k_e = k_f = 1$$

$$S_e = 0.869(0.879)(36) = 27.5 \text{ kpsi}$$

From Fig. A-15-8 with $D/d = 1.25$ and $r/d = 0.03,\ K_{ts} = 1.8.$

From Fig. A-15-9 with $D/d = 1.25$ and $r/d = 0.03,\ K_t = 2.3$

From Fig. 6-20 with $r = 0.03$ in, $q = 0.65.$

From Fig. 6-21 with $r = 0.03$ in, $q_s = 0.83$

Eq. (6-31):
$$K_f = 1 + 0.65(2.3 - 1) = 1.85$$
$$K_{fs} = 1 + 0.83(1.8 - 1) = 1.66$$

Using DE-elliptic, Eq. (7-11) with $M_m = T_a = 0,$

$$\frac{1}{n} = \frac{16}{\pi(1^3)}\left\{4\left[\frac{1.85(360)}{27\,500}\right]^2 + 3\left[\frac{1.66(192)}{39\,500}\right]^2\right\}^{1/2}$$

$$n = 3.89$$

Perform a similar analysis at the profile keyway under the gear.

The main problem with the design is the undersized shaft overhang with excessive slope at the gear. The use of crowned-teeth in the gears will eliminate this problem.

7-7 **(a)** One possible shaft layout is shown. Both bearings and the gear will be located against shoulders. The gear and the motor will transmit the torque through keys. The bearings can be lightly pressed onto the shaft. The left bearing will locate the shaft in the housing, while the right bearing will float in the housing.

(b) From summing moments around the shaft axis, the tangential transmitted load through the gear will be

$$W_t = T/(d/2) = 2500/(4/2) = 1250 \text{ lbf}$$

The radial component of gear force is related by the pressure angle.

$$W_r = W_t \tan\phi = 1250 \tan 20^\circ = 455 \text{ lbf}$$

$$W = [W_r^2 + W_t^2]^{1/2} = (455^2 + 1250^2)^{1/2} = 1330 \text{ lbf}$$

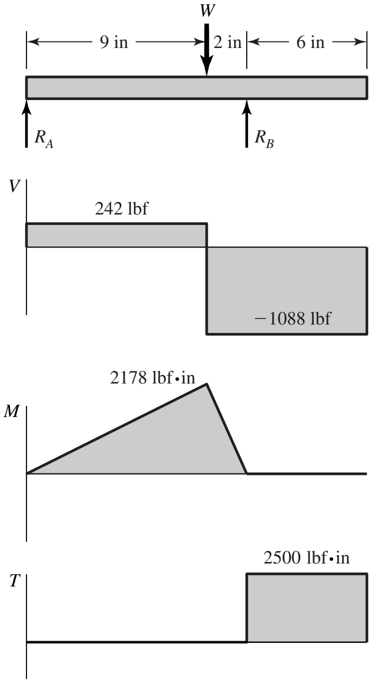
Reactions R_A and R_B, and the load W are all in the same plane. From force and moment balance,

$$R_A = 1330(2/11) = 242 \text{ lbf}$$

$$R_B = 1330(9/11) = 1088 \text{ lbf}$$

$$M_{\max} = R_A(9) = (242)(9) = 2178 \text{ lbf} \cdot \text{in}$$

Shear force, bending moment, and torque diagrams can now be obtained.



Ans.

(c) Potential critical locations occur at each stress concentration (shoulders and keyways). To be thorough, the stress at each potentially critical location should be evaluated. For now, we will choose the most likely critical location, by observation of the loading situation, to be in the keyway for the gear. At this point there is a large stress concentration, a large bending moment, and the torque is present. The other locations either have small bending moments, or no torque. The stress concentration for the keyway is highest at the ends. For simplicity, and to be conservative, we will use the maximum bending moment, even though it will have dropped off a little at the end of the keyway.

(d) At the gear keyway, approximately 9 in from the left end of the shaft, the bending is completely reversed and the torque is steady.

$$M_a = 2178 \text{ lbf} \cdot \text{in} \qquad T_m = 2500 \text{ lbf} \cdot \text{in} \qquad M_m = T_a = 0$$

From Table 7-1, estimate stress concentrations for the end-milled keyseat to be $K_t = 2.2$ and $K_{ts} = 3.0$. For the relatively low strength steel specified (AISI 1020 CD), estimate notch sensitivities of $q = 0.75$ and $q_s = 0.9$, obtained by observation of Figs. 6-20 and 6-21. Assuming a typical radius at the bottom of the keyseat of $r/d = 0.02$ (p. 361), these estimates for notch sensitivity are good for up to about 3 in shaft diameter.

Eq. (6-32): $K_f = 1 + 0.75(2.2 - 1) = 1.9$

$K_{fs} = 1 + 0.9(3.0 - 1) = 2.8$

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

For estimating k_b, guess $d = 2$ in.

$k_b = (2/0.3)^{-0.107} = 0.816$

$S_e = (0.883)(0.816)(0.5)(68) = 24.5 \text{ kpsi}$

Selecting the DE-Goodman criteria for a conservative first design,

Eq. (7-8): $$d = \left[\frac{16n}{\pi} \left\{ \frac{\left[4(K_f M_a)^2\right]^{1/2}}{S_e} + \frac{\left[3(K_{fs} T_m)^2\right]^{1/2}}{S_{ut}} \right\} \right]^{1/3}$$

$$d = \left[\frac{16n}{\pi} \left\{ \frac{\left[4(1.9 \cdot 2178)^2\right]^{1/2}}{24\,500} + \frac{\left[3(2.8 \cdot 2500)^2\right]^{1/2}}{68\,000} \right\} \right]^{1/3}$$

$$d = 1.58 \text{ in} \quad \textit{Ans.}$$

With this diameter, the estimates for notch sensitivity and size factor were conservative, but close enough for a first iteration until deflections are checked.

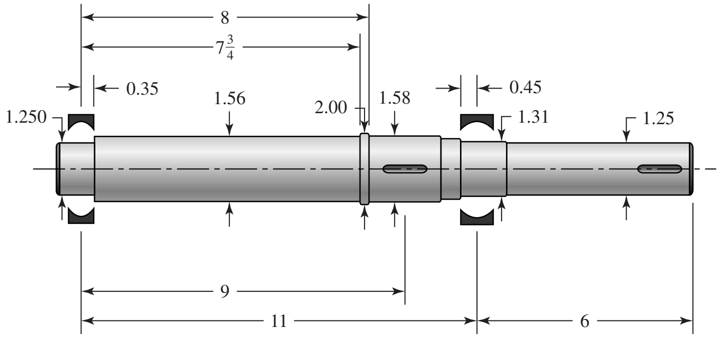
Check for static failure.

Eq. (7-15): $$\sigma'_{\max} = \left[\left(\frac{32 K_f M_a}{\pi d^3} \right)^2 + 3 \left(\frac{16 K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2}$$

$$\sigma'_{\max} = \left[\left(\frac{32(1.9)(2178)}{\pi (1.58)^3} \right)^2 + 3 \left(\frac{16(2.8)(2500)}{\pi (1.58)^3} \right)^2 \right]^{1/2} = 19.0 \text{ kpsi}$$

$$n_y = S_y / \sigma'_{\max} = 57/19.0 = 3.0 \quad \textit{Ans.}$$

(e) Now estimate other diameters to provide typical shoulder supports for the gear and bearings (p. 360). Also, estimate the gear and bearing widths.



(f) Entering this shaft geometry into beam analysis software (or Finite Element software), the following deflections are determined:

Left bearing slope:	0.000532 rad
Right bearing slope:	−0.000850 rad
Gear slope:	−0.000545 rad
Right end of shaft slope:	−0.000850 rad
Gear deflection:	−0.00145 in
Right end of shaft deflection:	0.00510 in

Comparing these deflections to the recommendations in Table 7-2, everything is within typical range except the gear slope is a little high for an uncrowned gear.

(g) To use a non-crowned gear, the gear slope is recommended to be less than 0.0005 rad. Since all other deflections are acceptable, we will target an increase in diameter only for the long section between the left bearing and the gear. Increasing this diameter from the proposed 1.56 in to 1.75 in, produces a gear slope of –0.000401 rad. All other deflections are improved as well.

7-8 **(a)** Use the distortion-energy elliptic failure locus. The torque and moment loadings on the shaft are shown in the solution to Prob. 7-7.

Candidate critical locations for strength:

- Pinion seat keyway
- Right bearing shoulder
- Coupling keyway

Table A-20 for 1030 HR: $S_{ut} = 68$ kpsi, $S_y = 37.5$ kpsi, $H_B = 137$

Eq. (6-8): $S'_e = 0.5(68) = 34.0$ kpsi

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

$$k_c = k_d = k_e = 1$$

Pinion seat keyway

See Table 7-1 for keyway stress concentration factors

$$\left.\begin{aligned} K_t &= 2.2 \\ K_{ts} &= 3.0 \end{aligned}\right\} \text{Profile keyway}$$

For an end-mill profile keyway cutter of 0.010 in radius,

From Fig. 6-20: $q = 0.50$

From Fig. 6-21: $q_s = 0.65$

Eq. (6-32):

$$\begin{aligned} K_{fs} &= 1 + q_s(K_{ts} - 1) \\ &= 1 + 0.65(3.0 - 1) = 2.3 \\ K_f &= 1 + 0.50(2.2 - 1) = 1.6 \end{aligned}$$

Eq. (6-20): $k_b = \left(\dfrac{1.875}{0.30}\right)^{-0.107} = 0.822$

Eq. (6-18): $S_e = 0.883(0.822)(34.0) = 24.7$ kpsi

Eq. (7-11):

$$\frac{1}{n} = \frac{16}{\pi(1.875^3)}\left\{4\left[\frac{1.6(2178)}{24\,700}\right]^2 + 3\left[\frac{2.3(2500)}{37\,500}\right]^2\right\}^{1/2}$$

$$= 0.353, \quad \text{from which } n = 2.83$$

Right-hand bearing shoulder

The text does not give minimum and maximum shoulder diameters for 03-series bearings (roller). Use $D = 1.75$ in.

$$\frac{r}{d} = \frac{0.030}{1.574} = 0.019, \qquad \frac{D}{d} = \frac{1.75}{1.574} = 1.11$$

From Fig. A-15-9,

$$K_t = 2.4$$

From Fig. A-15-8,

$$K_{ts} = 1.6$$

From Fig. 6-20, $q = 0.65$

From Fig. 6-21, $q_s = 0.83$

$$K_f = 1 + 0.65(2.4 - 1) = 1.91$$
$$K_{fs} = 1 + 0.83(1.6 - 1) = 1.50$$
$$M = 2178\left(\frac{0.453}{2}\right) = 493 \text{ lbf} \cdot \text{in}$$

Eq. (7-11):

$$\frac{1}{n} = \frac{16}{\pi(1.574^3)}\left[4\left(\frac{1.91(493)}{24\,700}\right)^2 + 3\left(\frac{1.50(2500)}{37\,500}\right)^2\right]^{1/2}$$
$$= 0.247, \quad \text{from which } n = 4.05$$

Overhanging coupling keyway

There is no bending moment, thus Eq. (7-11) reduces to:

$$\frac{1}{n} = \frac{16\sqrt{3}K_{fs}T_m}{\pi d^3 S_y} = \frac{16\sqrt{3}(1.50)(2500)}{\pi(1.5^3)(37\,500)}$$
$$= 0.261 \quad \text{from which } n = 3.83$$

(b) One could take pains to model this shaft exactly, using say finite element software. However, for the bearings and the gear, the shaft is basically of uniform diameter, 1.875 in. The reductions in diameter at the bearings will change the results insignificantly. Use $E = 30(10^6)$ psi.

To the left of the load:

$$\theta_{AB} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2)$$
$$= \frac{1449(2)(3x^2 + 2^2 - 11^2)}{6(30)(10^6)(\pi/64)(1.825^4)(11)}$$
$$= 2.4124(10^{-6})(3x^2 - 117)$$

At $x = 0$: $\theta = -2.823(10^{-4})$ rad

At $x = 9$ in: $\theta = 3.040(10^{-4})$ rad

At $x = 11$ in:
$$\theta = \frac{1449(9)(11^2 - 9^2)}{6(30)(10^6)(\pi/64)(1.875^4)(11)} = 4.342(10^{-4})\text{ rad}$$

Obtain allowable slopes from Table 7-2.

Left bearing:

$$n_{fs} = \frac{\text{Allowable slope}}{\text{Actual slope}} = \frac{0.001}{0.000\,282\,3} = 3.54$$

Right bearing:

$$n_{fs} = \frac{0.0008}{0.000\,434\,2} = 1.84$$

Gear mesh slope:

Table 7-2 recommends a minimum relative slope of 0.0005 rad. While we don't know the slope on the next shaft, we know that it will need to have a larger diameter and be stiffer. At the moment we can say

$$n_{fs} < \frac{0.0005}{0.000\,304} = 1.64$$

7-9 The solution to Problem 7-8 may be used as an example of the analysis process for a similar situation.

7-10 If you have a finite element program available, it is highly recommended. Beam deflection programs can be implemented but this is time consuming and the programs have narrow applications. Here we will demonstrate how the problem can be simplified and solved using singularity functions.

Deflection: First we will ignore the steps near the bearings where the bending moments are low. Thus let the 30 mm dia. be 35 mm. Secondly, the 55 mm dia. is very thin, 10 mm. The full bending stresses will not develop at the outer fibers so full stiffness will not develop either. Thus, ignore this step and let the diameter be 45 mm.

Statics: Left support: $R_1 = 7(315 - 140)/315 = 3.889$ kN

Right support: $R_2 = 7(140)/315 = 3.111$ kN

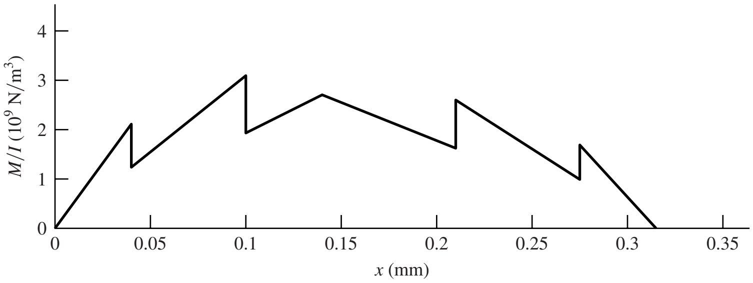
Determine the bending moment at each step.

x(mm)	0	40	100	140	210	275	315
M(N · m)	0	155.56	388.89	544.44	326.67	124.44	0

$I_{35} = (\pi/64)(0.035^4) = 7.366(10^{-8})\text{ m}^4,\ I_{40} = 1.257(10^{-7})\text{ m}^4,\ I_{45} = 2.013(10^{-7})\text{ m}^4$

Plot M/I as a function of x.

x (m)	M/I (10^9 N/m^3)	Step	Slope	Δ Slope
0	0		52.8	
0.04	2.112			
0.04	1.2375	−0.8745	30.942	−21.86
0.1	3.094			
0.1	1.932	−1.162	19.325	−11.617
0.14	2.705			
0.14	2.705	0	−15.457	−34.78
0.21	1.623			
0.21	2.6	0.977	−24.769	−9.312
0.275	0.99			
0.275	1.6894	0.6994	−42.235	−17.47
0.315	0			



The steps and the change of slopes are evaluated in the table. From these, the function M/I can be generated:

$$\begin{aligned} M/I = \big[&52.8x - 0.8745\langle x - 0.04\rangle^0 - 21.86\langle x - 0.04\rangle^1 - 1.162\langle x - 0.1\rangle^0 \\ &- 11.617\langle x - 0.1\rangle^1 - 34.78\langle x - 0.14\rangle^1 + 0.977\langle x - 0.21\rangle^0 \\ &- 9.312\langle x - 0.21\rangle^1 + 0.6994\langle x - 0.275\rangle^0 - 17.47\langle x - 0.275\rangle^1\big]\,10^9 \end{aligned}$$

Integrate twice:

$$\begin{aligned} E\frac{dy}{dx} = \big[&26.4x^2 - 0.8745\langle x - 0.04\rangle^1 - 10.93\langle x - 0.04\rangle^2 - 1.162\langle x - 0.1\rangle^1 \\ &- 5.81\langle x - 0.1\rangle^2 - 17.39\langle x - 0.14\rangle^2 + 0.977\langle x - 0.21\rangle^1 \\ &- 4.655\langle x - 0.21\rangle^2 + 0.6994\langle x - 0.275\rangle^1 - 8.735\langle x - 0.275\rangle^2 + C_1\big]\,10^9 \qquad (1) \end{aligned}$$

$$\begin{aligned} Ey = \big[&8.8x^3 - 0.4373\langle x - 0.04\rangle^2 - 3.643\langle x - 0.04\rangle^3 - 0.581\langle x - 0.1\rangle^2 \\ &- 1.937\langle x - 0.1\rangle^3 - 5.797\langle x - 0.14\rangle^3 + 0.4885\langle x - 0.21\rangle^2 \\ &- 1.552\langle x - 0.21\rangle^3 + 0.3497\langle x - 0.275\rangle^2 - 2.912\langle x - 0.275\rangle^3 + C_1x + C_2\big]\,10^9 \end{aligned}$$

Boundary conditions: $y = 0$ at $x = 0$ yields $C_2 = 0$;

$y = 0$ at $x = 0.315$ m yields $C_1 = -0.295\,25$ N/m^2.

Equation (1) with $C_1 = -0.295\,25$ provides the slopes at the bearings and gear. The following table gives the results in the second column. The third column gives the results from a similar finite element model. The fourth column gives the result of a full model which models the 35 and 55 mm diameter steps.

x (mm)	θ (rad)	F.E. Model	Full F.E. Model
0	−0.001 4260	−0.001 4270	−0.001 4160
140	−0.000 1466	−0.000 1467	−0.000 1646
315	0.001 3120	0.001 3280	0.001 3150

The main discrepancy between the results is at the gear location $(x = 140 \text{ mm})$. The larger value in the full model is caused by the stiffer 55 mm diameter step. As was stated earlier, this step is not as stiff as modeling implicates, so the exact answer is somewhere between the full model and the simplified model which in any event is a small value. As expected, modeling the 30 mm dia. as 35 mm does not affect the results much.

It can be seen that the allowable slopes at the bearings are exceeded. Thus, either the load has to be reduced or the shaft "beefed" up. If the allowable slope is 0.001 rad, then the maximum load should be $F_{\max} = (0.001/0.001\,46)7 = 4.79$ kN. With a design factor this would be reduced further.

To increase the stiffness of the shaft, increase the diameters by $(0.001\,46/0.001)^{1/4} = 1.097$, from Eq. (7-18). Form a table:

Old d, mm	20.00	30.00	35.00	40.00	45.00	55.00
New ideal d, mm	21.95	32.92	38.41	43.89	49.38	60.35
Rounded up d, mm	22.00	34.00	40.00	44.00	50.00	62.00

Repeating the full finite element model results in

$$x = 0: \quad \theta = -9.30 \times 10^{-4} \text{ rad}$$

$$x = 140 \text{ mm}: \quad \theta = -1.09 \times 10^{-4} \text{ rad}$$

$$x = 315 \text{ mm}: \quad \theta = 8.65 \times 10^{-4} \text{ rad}$$

Well within our goal. Have the students try a goal of 0.0005 rad at the bearings.

Strength: Due to stress concentrations and reduced shaft diameters, there are a number of locations to look at. A table of nominal stresses is given below. Note that torsion is only to the right of the 7 kN load. Using $\sigma = 32M/(\pi d^3)$ and $\tau = 16T/(\pi d^3)$,

x (mm)	0	15	40	100	110	140	210	275	300	330
σ (MPa)	0	22.0	37.0	61.9	47.8	60.9	52.0	39.6	17.6	0
τ (MPa)	0	0	0	0	0	6	8.5	12.7	20.2	68.1
σ' (MPa)	0	22.0	37.0	61.9	47.8	61.8	53.1	45.3	39.2	118.0

Table A-20 for AISI 1020 CD steel: $S_{ut} = 470$ MPa, $S_y = 390$ MPa

At $x = 210$ mm:

$$k_a = 4.51(470)^{-0.265} = 0.883, \quad k_b = (40/7.62)^{-0.107} = 0.837$$

$$S_e = 0.883(0.837)(0.5)(470) = 174 \text{ MPa}$$

$$D/d = 45/40 = 1.125, \quad r/d = 2/40 = 0.05.$$

From Figs. A-15-8 and A-15-9, $K_t = 1.9$ and $K_{ts} = 1.32$.

From Figs. 6-20 and 6-21, $q = 0.75$ and $q_s = 0.92$,

$$K_f = 1 + 0.75(1.9 - 1) = 1.68, \text{ and } K_{fs} = 1 + 0.92(1.32 - 1) = 1.29.$$

From Eq. (7-11), with $M_m = T_a = 0$,

$$\frac{1}{n} = \frac{16}{\pi(0.04)^3}\left\{4\left[\frac{1.68(326.67)}{174(10^6)}\right]^2 + 3\left[\frac{1.29(107)}{390(10^6)}\right]^2\right\}^{1/2}$$

$$n = 1.98$$

At $x = 330$ mm: The von Mises stress is the highest but it comes from the steady torque only.

$$D/d = 30/20 = 1.5, \quad r/d = 2/20 = 0.1 \quad \Rightarrow \quad K_{ts} = 1.42,$$

$$q_s = 0.92 \quad \Rightarrow \quad K_{fs} = 1.39$$

$$\frac{1}{n} = \frac{16}{\pi(0.02)^3}\left(\sqrt{3}\right)\left[\frac{1.39(107)}{390(10^6)}\right]$$

$$n = 2.38$$

Check the other locations.

If worse-case is at $x = 210$ mm, the changes discussed for the slope criterion will improve the strength issue.

7-11 and 7-12 With these design tasks each student will travel different paths and almost all details will differ. The important points are

- The student gets a blank piece of paper, a statement of function, and some constraints–explicit and implied. At this point in the course, this is a good experience.
- It is a good preparation for the capstone design course.
- The adequacy of their design must be demonstrated and possibly include a designer's notebook.
- Many of the fundaments of the course, based on this text and this course, are useful. The student will find them useful and notice that he/she is doing it.
- Don't let the students create a time sink for themselves. Tell them how far you want them to go.

7-13 I used this task as a final exam when all of the students in the course had consistent test scores going into the final examination; it was my expectation that they would not change things much by taking the examination.

This problem is a learning experience. Following the task statement, the following guidance was added.

- Take the first half hour, resisting the temptation of putting pencil to paper, and decide what the problem really is.
- Take another twenty minutes to list several possible remedies.
- Pick one, and show your instructor how you would implement it.

The students' initial reaction is that he/she does not know much from the problem statement. Then, slowly the realization sets in that they do know some important things that the designer did not. They knew how it failed, where it failed, and that the design wasn't good enough; it was close, though.

Also, a fix at the bearing seat lead-in could transfer the problem to the shoulder fillet, and the problem may not be solved.

To many students' credit, they chose to keep the shaft geometry, and selected a new material to realize about twice the Brinell hardness.

7-14 In Eq. (7-24) set

$$I = \frac{\pi d^4}{64}, \quad A = \frac{\pi d^2}{4}$$

to obtain

$$\omega = \left(\frac{\pi}{l}\right)^2 \left(\frac{d}{4}\right) \sqrt{\frac{gE}{\gamma}} \tag{1}$$

or

$$d = \frac{4l^2\omega}{\pi^2} \sqrt{\frac{\gamma}{gE}} \tag{2}$$

(a) From Eq. (1) and Table A-5,

$$\omega = \left(\frac{\pi}{24}\right)^2 \left(\frac{1}{4}\right) \sqrt{\frac{386(30)(10^6)}{0.282}} = 868 \text{ rad/s} \quad \textit{Ans.}$$

(b) From Eq. (2),

$$d = \frac{4(24)^2(2)(868)}{\pi^2} \sqrt{\frac{0.282}{386(30)(10^6)}} = 2 \text{ in} \quad \textit{Ans.}$$

(c) From Eq. (2),

$$l\omega = \frac{\pi^2}{4} \frac{d}{l} \sqrt{\frac{gE}{\gamma}}$$

Since d/l is the same regardless of the scale.

$$l\omega = \text{constant} = 24(868) = 20\,832$$

$$\omega = \frac{20\,832}{12} = 1736 \text{ rad/s} \quad \textit{Ans.}$$

Thus the first critical speed doubles.

7-15 From Prob. 7-14, $\omega = 868$ rad/s

$$A = 0.7854 \text{ in}^2, \quad I = 0.04909 \text{ in}^4, \quad \gamma = 0.282 \text{ lbf/in}^3,$$
$$E = 30(10^6) \text{ psi}, \quad w = A\gamma l = 0.7854(0.282)(24) = 5.316 \text{ lbf}$$

One element:

Eq. (7-24) $$\delta_{11} = \frac{12(12)(24^2 - 12^2 - 12^2)}{6(30)(10^6)(0.049\,09)(24)} = 1.956(10^{-4}) \text{ in/lbf}$$

$$y_1 = w_1\delta_{11} = 5.316(1.956)(10^{-4}) = 1.0398(10^{-3}) \text{ in}$$

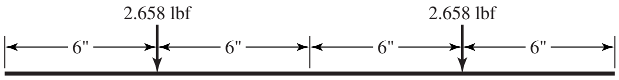
$$y_1^2 = 1.0812(10^{-6})$$

$$\sum wy = 5.316(1.0398)(10^{-3}) = 5.528(10^{-3})$$

$$\sum wy^2 = 5.316(1.0812)(10^{-6}) = 5.748(10^{-6})$$

$$\omega_1 = \sqrt{g\frac{\sum wy}{\sum wy^2}} = \sqrt{386\left[\frac{5.528(10^{-3})}{5.748(10^{-6})}\right]} = 609 \text{ rad/s} \quad (30\% \text{ low})$$

Two elements:



$$\delta_{11} = \delta_{22} = \frac{18(6)(24^2 - 18^2 - 6^2)}{6(30)(10^6)(0.049\,09)(24)} = 1.100(10^{-4}) \text{ in/lbf}$$

$$\delta_{12} = \delta_{21} = \frac{6(6)(24^2 - 6^2 - 6^2)}{6(30)(10^6)(0.049\,09)(24)} = 8.556(10^{-5}) \text{ in/lbf}$$

$$y_1 = w_1\delta_{11} + w_2\delta_{12} = 2.658(1.100)(10^{-4}) + 2.658(8.556)(10^{-5})$$
$$= 5.198(10^{-4}) \text{ in} = y_2,$$

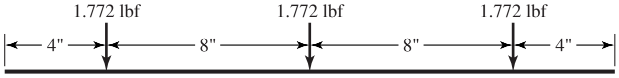
$$y_1^2 = y_2^2 = 2.702(10^{-7}) \text{ in}^2$$

$$\sum wy = 2(2.658)(5.198)(10^{-4}) = 2.763(10^{-3})$$

$$\sum wy^2 = 2(2.658)(2.702)(10^{-7}) = 1.436(10^{-6})$$

$$\omega_1 = \sqrt{386\left[\frac{2.763(10^{-3})}{1.436(10^{-6})}\right]} = 862 \text{ rad/s} \quad (0.7\% \text{ low})$$

Three elements:



$$\delta_{11} = \delta_{33} = \frac{20(4)(24^2 - 20^2 - 4^2)}{6(30)(10^6)(0.049\,09)(24)} = 6.036(10^{-5}) \text{ in/lbf}$$

$$\delta_{22} = \frac{12(12)(24^2 - 12^2 - 12^2)}{6(30)(10^6)(0.049\,09)(24)} = 1.956(10^{-4}) \text{ in/lbf}$$

$$\delta_{12} = \delta_{32} = \frac{12(4)(24^2 - 12^2 - 4^2)}{6(30)(10^6)(0.049\,09)(24)} = 9.416(10^{-5}) \text{ in/lbf}$$

$$\delta_{13} = \frac{4(4)(24^2 - 4^2 - 4^2)}{6(30)(10^6)(0.049\,09)(24)} = 4.104(10^{-5}) \text{ in/lbf}$$

$$y_1 = 1.772[6.036(10^{-5}) + 9.416(10^{-5}) + 4.104(10^{-5})] = 3.465(10^{-4}) \text{ in}$$
$$y_2 = 1.772[9.416(10^{-5}) + 1.956(10^{-4}) + 9.416(10^{-5})] = 6.803(10^{-4}) \text{ in}$$
$$y_3 = 1.772[4.104(10^{-5}) + 9.416(10^{-5}) + 6.036(10^{-5})] = 3.465(10^{-4}) \text{ in}$$
$$\sum wy = 2.433(10^{-3}), \quad \sum wy^2 = 1.246(10^{-6})$$
$$\omega_1 = \sqrt{386\left[\frac{2.433(10^{-3})}{1.246(10^{-6})}\right]} = 868 \text{ rad/s} \qquad \text{(same as in Prob. 7-14)}$$

The point was to show that convergence is rapid using a static deflection beam equation. The method works because:

- If a deflection curve is chosen which meets the boundary conditions of moment-free and deflection-free ends, and in this problem, of symmetry, the strain energy is not very sensitive to the equation used.
- Since the static bending equation is available, and meets the moment-free and deflection-free ends, it works.

7-16 **(a)** For two bodies, Eq. (7-26) is

$$\begin{vmatrix} (m_1\delta_{11} - 1/\omega^2) & m_2\delta_{12} \\ m_1\delta_{21} & (m_2\delta_{22} - 1/\omega^2) \end{vmatrix} = 0$$

Expanding the determinant yields,

$$\left(\frac{1}{\omega^2}\right)^2 - (m_1\delta_{11} + m_2\delta_{22})\left(\frac{1}{\omega_1^2}\right) + m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21}) = 0 \qquad (1)$$

Eq. (1) has two roots $1/\omega_1^2$ and $1/\omega_2^2$. Thus

$$\left(\frac{1}{\omega^2} - \frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega^2} - \frac{1}{\omega_2^2}\right) = 0$$

or,

$$\left(\frac{1}{\omega^2}\right)^2 + \left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}\right)\left(\frac{1}{\omega}\right)^2 + \left(\frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega_2^2}\right) = 0 \qquad (2)$$

Equate the third terms of Eqs. (1) and (2), which must be identical.

$$\frac{1}{\omega_1^2}\frac{1}{\omega_2^2} = m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21}) \quad \Rightarrow \quad \frac{1}{\omega_2^2} = \omega_1^2 m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21})$$

and it follows that

$$\omega_2 = \frac{1}{\omega_1}\sqrt{\frac{g^2}{w_1w_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21})}} \quad Ans.$$

(b) In Ex. 7-5, Part (b) the first critical speed of the two-disk shaft ($w_1 = 35$ lbf, $w_2 = 55$ lbf) is $\omega_1 = 124.7$ rad/s. From part (a), using influence coefficients

$$\omega_2 = \frac{1}{124.7}\sqrt{\frac{386^2}{35(55)[2.061(3.534) - 2.234^2](10^{-8})}} = 466 \text{ rad/s} \quad Ans.$$

7-17 In Eq. (7-22) the term $\sqrt{I/A}$ appears. For a hollow unform diameter shaft,

$$\sqrt{\frac{I}{A}} = \sqrt{\frac{\pi\left(d_o^4 - d_i^4\right)/64}{\pi\left(d_o^2 - d_i^2\right)/4}} = \sqrt{\frac{1}{16}\frac{\left(d_o^2 + d_i^2\right)\left(d_o^2 - d_i^2\right)}{d_o^2 - d_i^2}} = \frac{1}{4}\sqrt{d_o^2 + d_i^2}$$

This means that when a solid shaft is hollowed out, the critical speed increases beyond that of the solid shaft. By how much?

$$\frac{\frac{1}{4}\sqrt{d_o^2 + d_i^2}}{\frac{1}{4}\sqrt{d_o^2}} = \sqrt{1 + \left(\frac{d_i}{d_o}\right)^2}$$

The possible values of d_i are $0 \le d_i \le d_o$, so the range of critical speeds is

$$\omega_s\sqrt{1+0} \text{ to about } \omega_s\sqrt{1+1}$$

or from ω_s to $\sqrt{2}\omega_s$. *Ans.*

7-18 All steps will be modeled using singularity functions with a spreadsheet. Programming both loads will enable the user to first set the left load to 1, the right load to 0 and calculate δ_{11} and δ_{21}. Then setting left load to 0 and the right to 1 to get δ_{12} and δ_{22}. The spreadsheet shown on the next page shows the δ_{11} and δ_{21} calculation. Table for M/I vs x is easy to make. The equation for M/I is:

$$\begin{aligned} M/I = D13x &+ C15\langle x-1\rangle^0 + E15\langle x-1\rangle^1 + E17\langle x-2\rangle^1 \\ &+ C19\langle x-9\rangle^0 + E19\langle x-9\rangle^1 + E21\langle x-14\rangle^1 \\ &+ C23\langle x-15\rangle^0 + E23\langle x-15\rangle^1 \end{aligned}$$

Integrating twice gives the equation for Ey. Boundary conditions $y = 0$ at $x = 0$ and at $x = 16$ inches provide integration constants ($C_2 = 0$). Substitution back into the deflection equation at $x = 2$, 14 inches provides the δ's. The results are: $\delta_{11} - 2.917(10^{-7})$, $\delta_{12} = \delta_{21} = 1.627(10^{-7})$, $\delta_{22} = 2.231(10^{-7})$. This can be verified by finite element analysis.

$$y_1 = 20(2.917)(10^{-7}) + 35(1.627)(10^{-7}) = 1.153(10^{-5})$$

$$y_2 = 20(1.627)(10^{-7}) + 35(2.231)(10^{-7}) = 1.106(10^{-5})$$

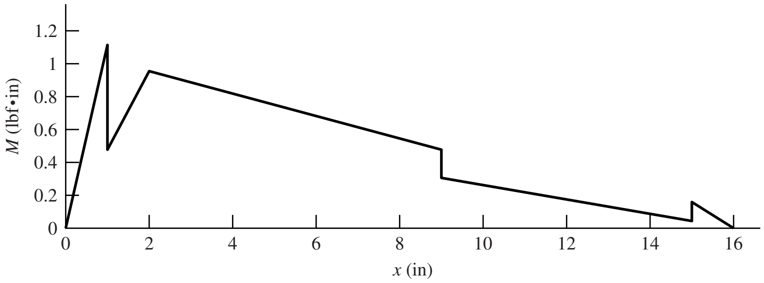
$$y_1^2 = 1.329(10^{-10}), \quad y_2^2 = 1.224(10^{-10})$$

$$\sum wy = 6.177(10^{-4}), \quad \sum wy^2 = 6.942(10^{-9})$$

Neglecting the shaft, Eq. (7-23) gives

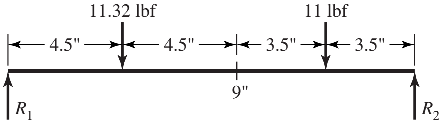
$$\omega_1 = \sqrt{386\left[\frac{6.177(10^{-4})}{6.942(10^{-9})}\right]} = 5860 \text{ rad/s} \quad \text{or} \quad 55\,970 \text{ rev/min} \quad \textit{Ans.}$$

	A	B	C	D	E	F	G	H	I
1		$F_1 = 1$		$F_2 = 0$	$R_1 = 0.875$ (left reaction)				
2									
3	x	M		$I_1 = I_4 = 0.7854$					
4	0	0		$I_2 = 1.833$					
5	1	0.875		$I_3 = 2.861$					
6	2	1.75							
7	9	0.875							
8	14	0.25							
9	15	0.125							
10	16	0							
11									
12	x	M/I	step	slope	Δ slope				
13	0	0		1.114 082					
14	1	1.114 082							
15	1	0.477 36	−0.636 722 477	0.477 36	−0.636 72				
16	2	0.954 719							
17	2	0.954 719	0	−0.068 19	−0.545 55				
18	9	0.477 36							
19	9	0.305 837	−0.171 522 4	−0.043 69	0.024 503				
20	14	0.087 382							
21	14	0.087 382	0	−0.043 69	0				
22	15	0.043 691							
23	15	0.159 155	0.115 463 554	−0.159 15	−0.115 46				
24	16	0							
25									
26		$C_1 = -4.906\ 001\ 093$							
27									
28									
29		$\delta_{11} = $ 2.91701E-07							
30		$\delta_{21} = $ 1.6266E-07							



Repeat for $F_1 = 0$ and $F_2 = 1$.

Modeling the shaft separately using 2 elements gives approximately



The spreadsheet can be easily modified to give

$$\delta_{11} = 9.605(10^{-7}), \quad \delta_{12} = \delta_{21} = 5.718(10^{-7}), \quad \delta_{22} = 5.472(10^{-7})$$

$$y_1 = 1.716(10^{-5}), \quad y_2 = 1.249(10^{-5}), \quad y_1^2 = 2.946(10^{-10}),$$

$$y_2^2 = 1.561(10^{-10}), \quad \sum wy = 3.316(10^{-4}), \quad \sum wy^2 = 5.052(10^{-9})$$

$$\omega_1 = \sqrt{386\left[\frac{3.316(10^{-4})}{5.052(10^{-9})}\right]} = 5034 \text{ rad/s} \quad \textit{Ans.}$$

A finite element model of the exact shaft gives $\omega_1 = 5340$ rad/s. The simple model is 5.7% low.

Combination Using Dunkerley's equation, Eq. (7-32):

$$\frac{1}{\omega_1^2} = \frac{1}{5860^2} + \frac{1}{5034^2} \quad \Rightarrow \quad 3819 \text{ rad/s} \quad \textit{Ans.}$$

7-19 We must not let the basis of the stress concentration factor, as presented, impose a viewpoint on the designer. Table A-16 shows K_{ts} as a decreasing monotonic as a function of a/D. All is not what it seems.

Let us change the basis for data presentation to the full section rather than the net section.

$$\tau = K_{ts}\tau_0 = K'_{ts}\tau'_0$$

$$K_{ts} = \frac{32T}{\pi A D^3} = K'_{ts}\left(\frac{32T}{\pi D^3}\right)$$

Therefore

$$K'_{ts} = \frac{K_{ts}}{A}$$

Form a table:

(a/D)	A	K_{ts}	K'_{ts}	
0.050	0.95	1.77	1.86	
0.075	0.93	1.71	1.84	
0.100	0.92	1.68	1.83	← minimum
0.125	0.89	1.64	1.84	
0.150	0.87	1.62	1.86	
0.175	0.85	1.60	1.88	
0.200	0.83	1.58	1.90	

K'_{ts} has the following attributes:

- It exhibits a minimum;
- It changes little over a wide range;
- Its minimum is a stationary point minimum at $a/D \doteq 0.100$;
- Our knowledge of the minima location is

$$0.075 \le (a/D) \le 0.125$$

We can form a design rule: in torsion, the pin diameter should be about 1/10 of the shaft diameter, for greatest shaft capacity. However, it is not catastrophic if one forgets the rule.

7-20 Choose 15 mm as basic size, D, d. Table 7-9: fit is designated as 15H7/h6. From Table A-11, the tolerance grades are $\Delta D = 0.018$ mm and $\Delta d = 0.011$ mm.

Hole: Eq. (7-36)

$$D_{\max} = D + \Delta D = 15 + 0.018 = 15.018 \text{ mm} \quad \textit{Ans.}$$

$$D_{\min} = D = 15.000 \text{ mm} \quad \textit{Ans.}$$

Shaft: From Table A-12, fundamental deviation $\delta_F = 0$. From Eq. (2-39)

$$d_{\max} = d + \delta_F = 15.000 + 0 = 15.000 \text{ mm} \quad \textit{Ans.}$$

$$d_{\min} = d + \delta_R - \Delta d = 15.000 + 0 - 0.011 = 14.989 \text{ mm} \quad \textit{Ans.}$$

7-21 Choose 45 mm as basic size. Table 7-9 designates fit as 45H7/s6. From Table A-11, the tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm

Hole: Eq. (7-36)

$$D_{\max} = D + \Delta D = 45.000 + 0.025 = 45.025 \text{ mm} \quad \textit{Ans.}$$

$$D_{\min} = D = 45.000 \text{ mm} \quad \textit{Ans.}$$

Shaft: From Table A-12, fundamental deviation $\delta_F = +0.043$ mm. From Eq. (7-38)

$$d_{\min} = d + \delta_F = 45.000 + 0.043 = 45.043 \text{ mm} \quad \textit{Ans.}$$

$$d_{\max} = d + \delta_F + \Delta d = 45.000 + 0.043 + 0.016 = 45.059 \text{ mm} \quad \textit{Ans.}$$

7-22 Choose 50 mm as basic size. From Table 7-9 fit is 50H7/g6. From Table A-11, the tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm.

Hole:

$$D_{\max} = D + \Delta D = 50 + 0.025 = 50.025 \text{ mm} \quad \textit{Ans.}$$

$$D_{\min} = D = 50.000 \text{ mm} \quad \textit{Ans.}$$

Shaft: From Table A-12 fundamental deviation $= -0.009$ mm

$$d_{\max} = d + \delta_F = 50.000 + (-0.009) = 49.991 \text{ mm} \quad \textit{Ans.}$$

$$\begin{aligned} d_{\min} &= d + \delta_F - \Delta d \\ &= 50.000 + (-0.009) - 0.016 \\ &= 49.975 \text{ mm} \end{aligned}$$

7-23 Choose the basic size as 1.000 in. From Table 7-9, for 1.0 in, the fit is H8/f7. From Table A-13, the tolerance grades are $\Delta D = 0.0013$ in and $\Delta d = 0.0008$ in.

Hole:

$$D_{\max} = D + (\Delta D)_{\text{hole}} = 1.000 + 0.0013 = 1.0013 \text{ in} \quad \textit{Ans.}$$
$$D_{\min} = D = 1.0000 \text{ in} \quad \textit{Ans.}$$

Shaft: From Table A-14: Fundamental deviation $= -0.0008$ in

$$d_{\max} = d + \delta_F = 1.0000 + (-0.0008) = 0.9992 \text{ in} \quad \textit{Ans.}$$
$$d_{\min} = d + \delta_F - \Delta d = 1.0000 + (-0.0008) - 0.0008 = 0.9984 \text{ in} \quad \textit{Ans.}$$

Alternatively,

$$d_{\min} = d_{\max} - \Delta d = 0.9992 - 0.0008 = 0.9984 \text{ in.} \quad \textit{Ans.}$$

7-24 **(a)** Basic size is $D = d = 1.5$ in.

Table 7-9: H7/s6 is specified for medium drive fit.

Table A-13: Tolerance grades are $\Delta D = 0.001$ in and $\Delta d = 0.0006$ in.

Table A-14: Fundamental deviation is $\delta_F = 0.0017$ in.

Eq. (7-36): $D_{\max} = D + \Delta D = 1.501$ in *Ans.*

$D_{\min} = D = 1.500$ in *Ans.*

Eq. (7-37): $d_{\max} = d + \delta_F + \Delta d = 1.5 + 0.0017 + 0.0006 = 1.5023$ in *Ans.*

Eq. (7-38): $d_{\min} = d + \delta_F = 1.5 + 0.0017 + 1.5017$ in *Ans.*

(b) Eq. (7-42): $\delta_{\min} = d_{\min} - D_{\max} = 1.5017 - 1.501 = 0.0007$ in

Eq. (7-43): $\delta_{\max} = d_{\max} - D_{\min} = 1.5023 - 1.500 = 0.0023$ in

Eq. (7-40):
$$p_{\max} = \frac{E\delta_{\max}}{2d^3}\left[\frac{(d_o^2 - d^2)(d^2 - d_i^2)}{d_o^2 - d_i^2}\right]$$
$$= \frac{(30)(10^6)(0.0023)}{2(1.5)^3}\left[\frac{(2.5^2 - 1.5^2)(1.5^2 - 0)}{2.5^2 - 0}\right] = 14\,720 \text{ psi} \quad \textit{Ans.}$$
$$p_{\min} = \frac{E\delta_{\min}}{2d^3}\left[\frac{\left(d_o^2 - d^2\right)\left(d^2 - d_i^2\right)}{d_o^2 - d_i^2}\right]$$
$$= \frac{(30)(10^6)(0.0007)}{2(1.5)^3}\left[\frac{(2.5^2 - 1.5^2)(1.5^2 - 0)}{2.5^2 - 0}\right] = 4480 \text{ psi} \quad \textit{Ans.}$$

(c) For the shaft:

Eq. (7-44): $\sigma_{t,\text{shaft}} = -p = -14\,720$ psi

Eq. (7-46): $\sigma_{r,\text{shaft}} = -p = -14\,720$ psi

Eq. (5-13):
$$\sigma' = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)^{1/2}$$
$$= [(-14\,720)^2 - (-14\,720)(-14\,720) + (-14\,720)^2]^{1/2}$$
$$= 14\,720 \text{ psi}$$
$$n = S_y/\sigma' = 57\,000/14\,720 = 3.9 \quad \textit{Ans.}$$

For the hub:

Eq. (7-45): $$\sigma_{t,\text{hub}} = p\frac{d_o^2 + d^2}{d_o^2 - d^2} = (14\,720)\left(\frac{2.5^2 + 1.5^2}{2.5^2 - 1.5^2}\right) = 31\,280 \text{ psi}$$

Eq. (7-46): $$\sigma_{r,\text{hub}} = -p = -14\,720 \text{ psi}$$

Eq. (5-13): $$\sigma' = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)^{1/2}$$

$$= [(31\,280)^2 - (31\,280)(-14\,720) + (-14\,720)^2]^{1/2} = 40\,689 \text{ psi}$$

$$n = S_y/\sigma' = 85\,000/40\,689 = 2.1 \quad \textit{Ans.}$$

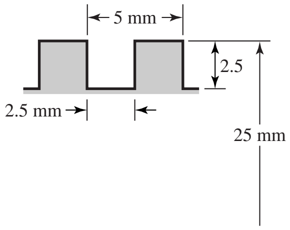
(d) Eq. (7-49) $$T = (\pi/2) f p_{\min} l d^2$$

$$= (\pi/2)(0.3)(4480)(2)(1.5)^2 = 9500 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

Chapter 8

8-1

(a)



Thread depth = 2.5 mm *Ans.*

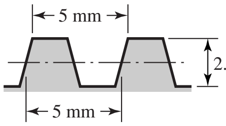
Width = 2.5 mm *Ans.*

$$d_m = 25 - 1.25 - 1.25 = 22.5 \text{ mm}$$

$$d_r = 25 - 5 = 20 \text{ mm}$$

$$l = p = 5 \text{ mm} \quad \textit{Ans.}$$

(b)



Thread depth = 2.5 mm *Ans.*

Width at pitch line = 2.5 mm *Ans.*

$$d_m = 22.5 \text{ mm}$$

$$d_r = 20 \text{ mm}$$

$$l = p = 5 \text{ mm} \quad \textit{Ans.}$$

8-2 From Table 8-1,

$$d_r = d - 1.226\,869p$$

$$d_m = d - 0.649\,519p$$

$$\bar{d} = \frac{d - 1.226\,869p + d - 0.649\,519p}{2} = d - 0.938\,194p$$

$$A_t = \frac{\pi \bar{d}^2}{4} = \frac{\pi}{4}(d - 0.938\,194p)^2 \quad \textit{Ans.}$$

8-3 From Eq. (*c*) of Sec. 8-2,

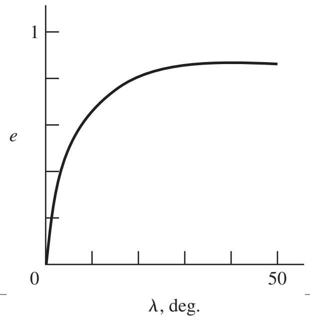
$$P = F\frac{\tan\lambda + f}{1 - f\tan\lambda}$$

$$T = \frac{Pd_m}{2} = \frac{Fd_m}{2}\,\frac{\tan\lambda + f}{1 - f\tan\lambda}$$

$$e = \frac{T_0}{T} = \frac{Fl/(2\pi)}{Fd_m/2}\,\frac{1 - f\tan\lambda}{\tan\lambda + f} = \tan\lambda\frac{1 - f\tan\lambda}{\tan\lambda + f} \quad \textit{Ans.}$$

Using $f = 0.08$, form a table and plot the efficiency curve.

λ, deg.	e
0	0
10	0.678
20	0.796
30	0.838
40	0.8517
45	0.8519



8-4 Given $F = 6$ kN, $l = 5$ mm, and $d_m = 22.5$ mm, the torque required to raise the load is found using Eqs. (8-1) and (8-6)

$$T_R = \frac{6(22.5)}{2}\left[\frac{5 + \pi(0.08)(22.5)}{\pi(22.5) - 0.08(5)}\right] + \frac{6(0.05)(40)}{2}$$

$$= 10.23 + 6 = 16.23 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

The torque required to lower the load, from Eqs. (8-2) and (8-6) is

$$T_L = \frac{6(22.5)}{2}\left[\frac{\pi(0.08)22.5 - 5}{\pi(22.5) + 0.08(5)}\right] + \frac{6(0.05)(40)}{2}$$

$$= 0.622 + 6 = 6.622 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

Since T_L is positive, the thread is self-locking. The efficiency is

Eq. (8-4):

$$e = \frac{6(5)}{2\pi(16.23)} = 0.294 \quad \textit{Ans.}$$

8-5 Collar (thrust) bearings, at the bottom of the screws, must bear on the collars. The bottom segment of the screws must be in compression. Where as tension specimens and their grips must be in tension. Both screws must be of the same-hand threads.

8-6 Screws rotate at an angular rate of

$$n = \frac{1720}{75} = 22.9 \text{ rev/min}$$

(a) The lead is 0.5 in, so the linear speed of the press head is

$$V = 22.9(0.5) = 11.5 \text{ in/min} \quad \textit{Ans.}$$

(b) $F = 2500$ lbf/screw

$$d_m = 3 - 0.25 = 2.75 \text{ in}$$

$$\sec\alpha = 1/\cos(29/2) = 1.033$$

Eq. (8-5):

$$T_R = \frac{2500(2.75)}{2}\left(\frac{0.5 + \pi(0.05)(2.75)(1.033)}{\pi(2.75) - 0.5(0.05)(1.033)}\right) = 377.6 \text{ lbf} \cdot \text{in}$$

Eq. (8-6):

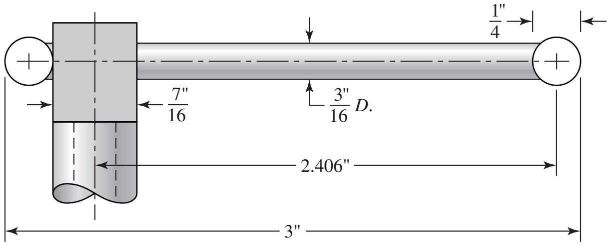
$$T_c = 2500(0.06)(5/2) = 375 \text{ lbf} \cdot \text{in}$$

$$T_{\text{total}} = 377.6 + 375 = 753 \text{ lbf} \cdot \text{in/screw}$$

$$T_{\text{motor}} = \frac{753(2)}{75(0.95)} = 21.1 \text{ lbf} \cdot \text{in}$$

$$H = \frac{Tn}{63\,025} = \frac{21.1(1720)}{63\,025} = 0.58 \text{ hp} \quad \textit{Ans.}$$

8-7 The force F is perpendicular to the paper.



$$L = 3 - \frac{1}{8} - \frac{1}{4} - \frac{7}{32} = 2.406 \text{ in}$$

$$T = 2.406F$$

$$M = \left(L - \frac{7}{32}\right)F = \left(2.406 - \frac{7}{32}\right)F = 2.188F$$

$$S_y = 41 \text{ kpsi}$$

$$\sigma = S_y = \frac{32M}{\pi d^3} = \frac{32(2.188)F}{\pi(0.1875)^3} = 41\,000$$

$$F = 12.13 \text{ lbf}$$

$$T = 2.406(12.13) = 29.2 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(b) Eq. (8-5), $2\alpha = 60^\circ$, $l = 1/14 = 0.0714$ in, $f = 0.075$, $\sec\alpha = 1.155$, $p = 1/14$ in

$$d_m = \frac{7}{16} - 0.649\,519\left(\frac{1}{14}\right) = 0.3911 \text{ in}$$

$$T_R = \frac{F_{\text{clamp}}(0.3911)}{2}\left(\frac{\text{Num}}{\text{Den}}\right)$$

$$\text{Num} = 0.0714 + \pi(0.075)(0.3911)(1.155)$$

$$\text{Den} = \pi(0.3911) - 0.075(0.0714)(1.155)$$

$$T = 0.028\,45F_{\text{clamp}}$$

$$F_{\text{clamp}} = \frac{T}{0.028\,45} = \frac{29.2}{0.028\,45} = 1030 \text{ lbf} \quad \textit{Ans.}$$

(c) The column has one end fixed and the other end pivoted. Base decision on the mean diameter column. Input: $C = 1.2$, $D = 0.391$ in, $S_y = 41$ kpsi, $E = 30(10^6)$ psi, $L = 4.1875$ in, $k = D/4 = 0.097\,75$ in, $L/k = 42.8$.

For this J. B. Johnson column, the critical load represents the limiting clamping force for bucking. Thus, $F_{\text{clamp}} = P_{\text{cr}} = 4663$ lbf.

(d) This is a subject for class discussion.

8-8

$$T = 6(2.75) = 16.5 \text{ lbf} \cdot \text{in}$$

$$d_m = \frac{5}{8} - \frac{1}{12} = 0.5417 \text{ in}$$

$$l = \frac{1}{6} = 0.1667 \text{ in}, \quad \alpha = \frac{29^\circ}{2} = 14.5^\circ, \quad \sec 14.5^\circ = 1.033$$

Eq. (8-5): $$T = 0.5417(F/2)\left[\frac{0.1667 + \pi(0.15)(0.5417)(1.033)}{\pi(0.5417) - 0.15(0.1667)(1.033)}\right] = 0.0696F$$

Eq. (8-6): $$T_c = 0.15(7/16)(F/2) = 0.032\,81F$$

$$T_{\text{total}} = (0.0696 + 0.0328)F = 0.1024F$$

$$F = \frac{16.5}{0.1024} = 161 \text{ lbf} \quad \textit{Ans.}$$

8-9 $d_m = 40 - 3 = 37$ mm, $l = 2(6) = 12$ mm

From Eq. (8-1) and Eq. (8-6)

$$T_R = \frac{10(37)}{2}\left[\frac{12 + \pi(0.10)(37)}{\pi(37) - 0.10(12)}\right] + \frac{10(0.15)(60)}{2}$$
$$= 38.0 + 45 = 83.0 \text{ N} \cdot \text{m}$$

Since $n = V/l = 48/12 = 4$ rev/s

$$\omega = 2\pi n = 2\pi(4) = 8\pi \text{ rad/s}$$

so the power is

$$H = T\omega = 83.0(8\pi) = 2086\ W \quad \textit{Ans.}$$

8-10

(a) $d_m = 36 - 3 = 33$ mm, $l = p = 6$ mm

From Eqs. (8-1) and (8-6)

$$T = \frac{33F}{2}\left[\frac{6 + \pi(0.14)(33)}{\pi(33) - 0.14(6)}\right] + \frac{0.09(90)F}{2}$$
$$= (3.292 + 4.050)F = 7.34F \text{ N} \cdot \text{m}$$
$$\omega = 2\pi n = 2\pi(1) = 2\pi \text{ rad/s}$$
$$H = T\omega$$
$$T = \frac{H}{\omega} = \frac{3000}{2\pi} = 477 \text{ N} \cdot \text{m}$$
$$F = \frac{477}{7.34} = 65.0 \text{ kN} \quad \textit{Ans.}$$

(b) $e = \dfrac{Fl}{2\pi T} = \dfrac{65.0(6)}{2\pi(477)} = 0.130$ *Ans.*

8-11

(a) $L_T = 2D + \dfrac{1}{4} = 2(0.5) + 0.25 = 1.25$ in *Ans.*

(b) From Table A-32 the washer thickness is 0.109 in. Thus,

$$l = 0.5 + 0.5 + 0.109 = 1.109 \text{ in} \quad \textit{Ans.}$$

(c) From Table A-31, $H = \dfrac{7}{16} = 0.4375$ in *Ans.*

(d) $l + H = 1.109 + 0.4375 = 1.5465$ in
This would be rounded to 1.75 in per Table A-17. The bolt is long enough. *Ans.*

(e) $l_d = L - L_T = 1.75 - 1.25 = 0.500$ in *Ans.*

$l_t = l - l_d = 1.109 - 0.500 = 0.609$ in *Ans.*

These lengths are needed to estimate bolt spring rate k_b.

Note: In an analysis problem, you need not know the fastener's length at the outset, although you can certainly check, if appropriate.

8-12

(a) $L_T = 2D + 6 = 2(14) + 6 = 34$ mm *Ans.*

(b) From Table A-33, the maximum washer thickness is 3.5 mm. Thus, the grip is, $l = 14 + 14 + 3.5 = 31.5$ mm *Ans.*

(c) From Table A-31, $H = 12.8$ mm

(d) $l + H = 31.5 + 12.8 = 44.3$ mm
Adding one or two threads and rounding up to $L = 50$ mm. The bolt is long enough. *Ans.*

(e) $l_d = L - L_T = 50 - 34 = 16$ mm *Ans.*

$l_t = l - l_d = 31.5 - 16 = 15.5$ mm *Ans.*
These lengths are needed to estimate the bolt spring rate k_b.

8-13

(a) $L_T = 2D + \frac{1}{4} = 2(0.5) + 0.25 = 1.25$ in *Ans.*

(b) $l' > h + \frac{d}{2} = t_1 + \frac{d}{2} = 0.875 + \frac{0.5}{2} = 1.125$ in *Ans.*

(c) $L > h + 1.5d = t_1 + 1.5d = 0.875 + 1.5(0.5) = 1.625$ in
From Table A-17, this rounds to 1.75 in. The cap screw is long enough. *Ans.*

(d) $l_d = L - L_T = 1.75 - 1.25 = 0.500$ in *Ans.*

$l_t = l' - l_d = 1.125 - 0.5 = 0.625$ in *Ans.*

8-14

(a) $L_T = 2(12) + 6 = 30$ mm *Ans.*

(b) $l' = h + \frac{d}{2} = t_1 + \frac{d}{2} = 20 + \frac{12}{2} = 26$ mm *Ans.*

(c) $L > h + 1.5d = t_1 + 1.5d = 20 + 1.5(12) = 38$ mm
This rounds to 40 mm (Table A-17). The fastener is long enough. *Ans.*

(d) $l_d = L - L_T = 40 - 30 = 10$ mm *Ans.*

$l_T = l' - l_d = 26 - 10 = 16$ mm *Ans.*

8-15

(a)

$$A_d = 0.7854(0.75)^2 = 0.442\ \text{in}^2$$

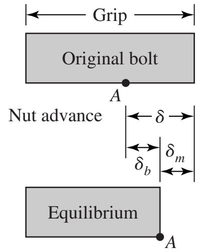
$$A_{\text{tube}} = 0.7854(1.125^2 - 0.75^2) = 0.552\ \text{in}^2$$

$$k_b = \frac{A_d E}{\text{grip}} = \frac{0.442(30)(10^6)}{13} = 1.02(10^6)\ \text{lbf/in} \quad \textit{Ans.}$$

$$k_m = \frac{A_{\text{tube}} E}{13} = \frac{0.552(30)(10^6)}{13} = 1.27(10^6)\ \text{lbf/in} \quad \textit{Ans.}$$

$$C = \frac{1.02}{1.02 + 1.27} = 0.445 \quad \textit{Ans.}$$

(b)



$$\delta = \frac{1}{16} \cdot \frac{1}{3} = \frac{1}{48} = 0.020\,83\ \text{in}$$

$$|\delta_b| = \left(\frac{|P|l}{AE}\right)_b = \frac{(13 - 0.020\,83)}{0.442(30)(10^6)}|P| = 9.79(10^{-7})|P|\ \text{in}$$

$$|\delta_m| = \left(\frac{|P|l}{AE}\right)_m = \frac{|P|(13)}{0.552(30)(10^6)} = 7.85(10^{-7})|P|\ \text{in}$$

$$|\delta_b| + |\delta_m| = \delta = 0.020\,83$$

$$9.79(10^{-7})|P| + 7.85(10^{-7})|P| = 0.020\,83$$

$$F_i = |P| = \frac{0.020\,83}{9.79(10^{-7}) + 7.85(10^{-7})} = 11\,810\ \text{lbf} \quad \textit{Ans.}$$

(c) At opening load P_0

$$9.79(10^{-7})P_0 = 0.020\,83$$

$$P_0 = \frac{0.020\,83}{9.79(10^{-7})} = 21\,280\ \text{lbf} \quad \textit{Ans.}$$

As a check use $F_i = (1 - C)P_0$

$$P_0 = \frac{F_i}{1 - C} = \frac{11\,810}{1 - 0.445} = 21\,280\ \text{lbf}$$

8-16 The movement is known at one location when the nut is free to turn

$$\delta = pt = t/N$$

Letting N_t represent the turn of the nut from snug tight, $N_t = \theta/360^\circ$ and $\delta = N_t/N$. The elongation of the bolt δ_b is

$$\delta_b = \frac{F_i}{k_b}$$

The advance of the nut along the bolt is the algebraic sum of $|\delta_b|$ and $|\delta_m|$

$$|\delta_b| + |\delta_m| = \frac{N_t}{N}$$

$$\frac{F_i}{k_b} + \frac{F_i}{k_m} = \frac{N_t}{N}$$

$$N_t = NF_i\left[\frac{1}{k_b} + \frac{1}{k_m}\right] = \left(\frac{k_b + k_m}{k_b k_m}\right)F_i N = \frac{\theta}{360^\circ} \quad \textit{Ans.}$$

As a check invert Prob. 8-15. What Turn-of-Nut will induce $F_i = 11\,808$ lbf?

$$N_t = 16(11\,808)\left(\frac{1}{1.02(10^6)} + \frac{1}{1.27(10^6)}\right)$$
$$= 0.334 \text{ turns} \doteq 1/3 \text{ turn} \quad \text{(checks)}$$

The relationship between the Turn-of-Nut method and the Torque Wrench method is as follows.

$$N_t = \left(\frac{k_b + k_m}{k_b k_m}\right)F_i N \qquad \text{(Turn-of-Nut)}$$

$$T = KF_i d \qquad \text{(Torque Wrench)}$$

Eliminate F_i

$$N_t = \left(\frac{k_b + k_m}{k_b k_m}\right)\frac{NT}{Kd} = \frac{\theta}{360^\circ} \quad \textit{Ans.}$$

8-17

(a) From Ex. 8-4, $F_i = 14.4$ kip, $k_b = 5.21(10^6)$ lbf/in, $k_m = 8.95(10^6)$ lbf/in

Eq. (8-27): $T = kF_i d = 0.2(14.4)(10^3)(5/8) = 1800 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$

From Prob. 8-16,

$$t = NF_i\left(\frac{1}{k_b} + \frac{1}{k_m}\right) = 16(14.4)(10^3)\left[\frac{1}{5.21(10^6)} + \frac{1}{8.95(10^6)}\right]$$
$$= 0.132 \text{ turns} = 47.5^\circ \quad \textit{Ans.}$$

Bolt group is $(1.5)/(5/8) = 2.4$ diameters. Answer is lower than RB&W recommendations.

(b) From Ex. 8-5, $F_i = 14.4$ kip, $k_b = 6.78$ Mlbf/in, and $k_m = 17.4$ Mlbf/in

$$T = 0.2(14.4)(10^3)(5/8) = 1800 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$t = 11(14.4)(10^3)\left[\frac{1}{6.78(10^6)} + \frac{1}{17.4(10^6)}\right]$$
$$= 0.0325 = 11.7^\circ \quad \textit{Ans.} \quad \text{Again lower than RB\&W.}$$

8-18 From Eq. (8-22) for the conical frusta, with $d/l = 0.5$

$$\left.\frac{k_m}{Ed}\right|_{(d/l)=0.5} = \frac{0.5774\pi}{2\ln\{5[0.5774 + 0.5(0.5)]/[0.5774 + 2.5(0.5)]\}} = 1.11$$

Eq. (8-23), from the Wileman *et al.* finite element study, using the general expression,

$$\left.\frac{k_m}{Ed}\right|_{(d/l)=0.5} = 0.789\,52\exp[0.629\,14(0.5)] = 1.08$$

8-19 For cast iron, from Table 8-8: $A = 0.778\,71,\ B = 0.616\,16,\ E = 14.5$ Mpsi

$$k_m = 14.5(10^6)(0.625)(0.778\,71)\exp\left(0.616\,16\frac{0.625}{1.5}\right) = 9.12(10^6)\text{ lbf/in}$$

This member's spring rate applies to both members. We need k_m for the upper member which represents half of the joint.

$$k_{ci} = 2k_m = 2[9.12(10^6)] = 18.24(10^6)\text{ lbf/in}$$

For steel from Table 8-8: $A = 0.787\,15,\ B = 0.628\,73,\ E = 30$ Mpsi

$$k_m = 30(10^6)(0.625)(0.787\,15)\exp\left(0.628\,73\frac{0.625}{1.5}\right) = 19.18(10^6)\text{ lbf/in}$$

$$k_{\text{steel}} = 2k_m = 2(19.18)(10^6) = 38.36(10^6)\text{ lbf/in}$$

For springs in series

$$\frac{1}{k_m} = \frac{1}{k_{ci}} + \frac{1}{k_{\text{steel}}} = \frac{1}{18.24(10^6)} + \frac{1}{38.36(10^6)}$$

$$k_m = 12.4(10^6)\text{ lbf/in} \quad \textit{Ans.}$$

8-20 The external tensile load per bolt is

$$P = \frac{1}{10}\left(\frac{\pi}{4}\right)(150)^2(6)(10^{-3}) = 10.6\text{ kN}$$

Also, $l = 40$ mm and from Table A-31, for $d = 12$ mm, $H = 10.8$ mm. No washer is specified.

$$L_T = 2D + 6 = 2(12) + 6 = 30\text{ mm}$$

$$l + H = 40 + 10.8 = 50.8\text{ mm}$$

Table A-17: $$L = 60\text{ mm}$$

$$l_d = 60 - 30 = 30\text{ mm}$$

$$l_t = 45 - 30 = 15\text{ mm}$$

$$A_d = \frac{\pi(12)^2}{4} = 113\text{ mm}^2$$

Table 8-1: $$A_t = 84.3\text{ mm}^2$$

Eq. (8-17):

$$k_b = \frac{113(84.3)(207)}{113(15) + 84.3(30)} = 466.8\text{ MN/m}$$

Steel: Using Eq. (8-23) for $A = 0.787\,15,\ B = 0.628\,73$ and $E = 207$ GPa

Eq. (8-23): $k_m = 207(12)(0.787\,15)\exp[(0.628\,73)(12/40)] = 2361$ MN/m

$$k_s = 2k_m = 4722 \text{ MN/m}$$

Cast iron: $A = 0.778\,71$, $B = 0.616\,16$, $E = 100$ GPa

$$k_m = 100(12)(0.778\,71)\exp[(0.616\,16)(12/40)] = 1124 \text{ MN/m}$$

$$k_{ci} = 2k_m = 2248 \text{ MN/m}$$

$$\frac{1}{k_m} = \frac{1}{k_s} + \frac{1}{k_{ci}} \quad \Rightarrow \quad k_m = 1523 \text{ MN/m}$$

$$C = \frac{466.8}{466.8 + 1523} = 0.2346$$

Table 8-1: $A_t = 84.3 \text{ mm}^2$, Table 8-11, $S_p = 600$ MPa

Eqs. (8-30) and (8-31): $F_i = 0.75(84.3)(600)(10^{-3}) = 37.9$ kN

Eq. (8-28):

$$n = \frac{S_p A_t - F_i}{CP} = \frac{600(10^{-3})(84.3) - 37.9}{0.2346(10.6)} = 5.1 \quad Ans.$$

8-21 Computer programs will vary.

8-22 $D_3 = 150$ mm, $A = 100$ mm, $B = 200$ mm, $C = 300$ mm, $D = 20$ mm, $E = 25$ mm. ISO 8.8 bolts: $d = 12$ mm, $p = 1.75$ mm, coarse pitch of $p = 6$ MPa.

$$P = \frac{1}{10}\left(\frac{\pi}{4}\right)(150^2)(6)(10^{-3}) = 10.6 \text{ kN/bolt}$$

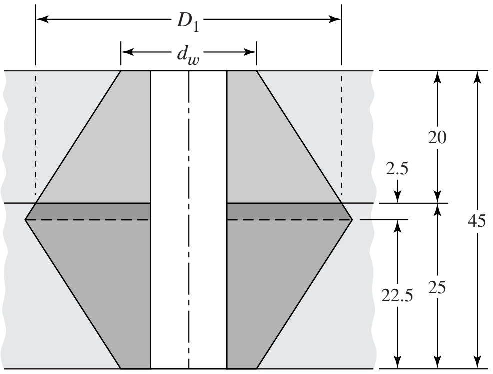
$$l = D + E = 20 + 25 = 45 \text{ mm}$$

$$L_T = 2D + 6 = 2(12) + 6 = 30 \text{ mm}$$

Table A-31: $H = 10.8$ mm

$$l + H = 45 + 10.8 = 55.8 \text{ mm}$$

Table A-17: $L = 60$ mm



$$l_d = 60 - 30 = 30 \text{ mm}, \quad l_t = 45 - 30 = 15 \text{ mm}, \quad A_d = \pi(12^2/4) = 113 \text{ mm}^2$$

Table 8-1: $A_t = 84.3 \text{ mm}^2$

Eq. (8-17):

$$k_b = \frac{113(84.3)(207)}{113(15) + 84.3(30)} = 466.8\ \text{MN/m}$$

There are three frusta: $d_m = 1.5(12) = 18$ mm

$$D_1 = (20 \tan 30°)2 + d_w = (20 \tan 30°)2 + 18 = 41.09\ \text{mm}$$

Upper Frustum: $t = 20$ mm, $E = 207$ GPa, $D = 1.5(12) = 18$ mm

Eq. (8-20): $k_1 = 4470$ MN/m

Central Frustum: $t = 2.5$ mm, $D = 41.09$ mm, $E = 100$ GPa (Table A-5) $\Rightarrow$ $k_2 =$ 52 230 MN/m

Lower Frustum: $t = 22.5$ mm, $E = 100$ GPa, $D = 18$ mm $\Rightarrow$ $k_3 = 2074$ MN/m

From Eq. (8-18): $k_m = [(1/4470) + (1/52\,230) + (1/2074)]^{-1} = 1379$ MN/m

Eq. (*e*), p. 421: $$C = \frac{466.8}{466.8 + 1379} = 0.253$$

Eqs. (8-30) and (8-31):

$$F_i = KF_p = KA_tS_p = 0.75(84.3)(600)(10^{-3}) = 37.9\ \text{kN}$$

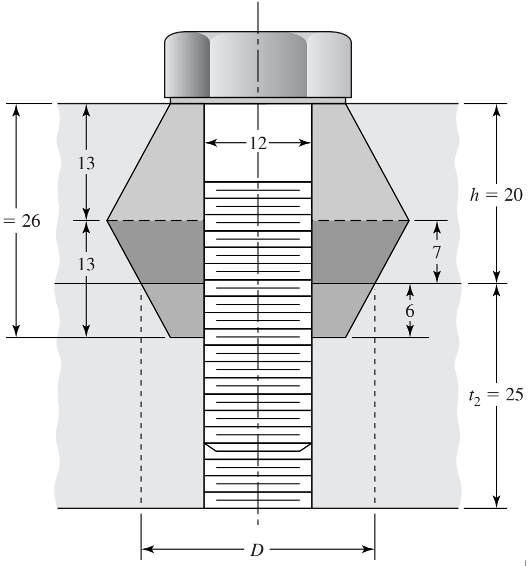
Eq. (8-28): $$n = \frac{S_pA_t - F_i}{CP} = \frac{600(10^{-3})(84.3) - 37.9}{0.253(10.6)} = 4.73 \quad \textit{Ans.}$$

8-23 $P = \dfrac{1}{8}\left(\dfrac{\pi}{4}\right)(120^2)(6)(10^{-3}) = 8.48$ kN

From Fig. 8-21, $t_1 = h = 20$ mm and $t_2 = 25$ mm

$$l = 20 + 12/2 = 26\ \text{mm}$$
$$t = 0 \quad \text{(no washer)}, \qquad L_T = 2(12) + 6 = 30\ \text{mm}$$
$$L > h + 1.5d = 20 + 1.5(12) = 38\ \text{mm}$$

Use 40 mm cap screws.



$$l_d = 40 - 30 = 10\ \text{mm}$$
$$l_t = l - l_d = 26 - 10 = 16\ \text{mm}$$
$$A_d = 113\ \text{mm}^2, \quad A_t = 84.3\ \text{mm}^2$$

Eq. (8-17):

$$k_b = \frac{113(84.3)(207)}{113(16) + 84.3(10)}$$
$$= 744\ \text{MN/m} \quad \textit{Ans.}$$

$$d_w = 1.5(12) = 18\ \text{mm}$$

$$D = 18 + 2(6)(\tan 30) = 24.9\ \text{mm}$$

From Eq. (8-20):

Top frustum: $D = 18, t = 13, E = 207$ GPa $\Rightarrow k_1 = 5316$ MN/m

Mid-frustum: $t = 7, E = 207$ GPa, $D = 24.9$ mm $\Rightarrow k_2 = 15\,620$ MN/m

Bottom frustum: $D = 18, t = 6, E = 100$ GPa $\Rightarrow k_3 = 3887$ MN/m

$$k_m = \frac{1}{(1/5316) + (1/55\,620) + (1/3887)} = 2158 \text{ MN/m} \quad \textit{Ans.}$$

$$C = \frac{744}{744 + 2158} = 0.256 \quad \textit{Ans.}$$

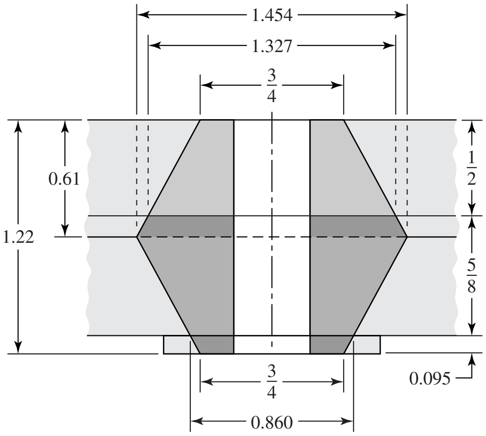
From Prob. 8-22, $F_i = 37.9$ kN

$$n = \frac{S_p A_t - F_i}{CP} = \frac{600(0.0843) - 37.9}{0.256(8.48)} = 5.84 \quad \textit{Ans.}$$

8-24 Calculation of bolt stiffness:

$$H = 7/16 \text{ in}$$
$$L_T = 2(1/2) + 1/4 = 1\,1/4 \text{ in}$$
$$l = 1/2 + 5/8 + 0.095 = 1.22 \text{ in}$$
$$L > 1.125 + 7/16 + 0.095 = 1.66 \text{ in}$$

Use $L = 1.75$ in



$$l_d = L - L_T = 1.75 - 1.25 = 0.500 \text{ in}$$
$$l_t = 1.125 + 0.095 - 0.500 = 0.72 \text{ in}$$
$$A_d = \pi(0.50^2)/4 = 0.1963 \text{ in}^2$$
$$A_t = 0.1419 \text{ in}^2 \text{ (UNC)}$$
$$k_t = \frac{A_t E}{l_t} = \frac{0.1419(30)}{0.72} = 5.9125 \text{ Mlbf/in}$$
$$k_d = \frac{A_d E}{l_d} = \frac{0.1963(30)}{0.500} = 11.778 \text{ Mlbf/in}$$
$$k_b = \frac{1}{(1/5.9125) + (1/11.778)} = 3.936 \text{ Mlbf/in} \quad \textit{Ans.}$$

Member stiffness for four frusta and joint constant C using Eqs. (8-20) and (e).

Top frustum: $D = 0.75, t = 0.5, d = 0.5, E = 30 \quad \Rightarrow \quad k_1 = 33.30$ Mlbf/in

2nd frustum: $D = 1.327, t = 0.11, d = 0.5, E = 14.5 \quad \Rightarrow \quad k_2 = 173.8$ Mlbf/in

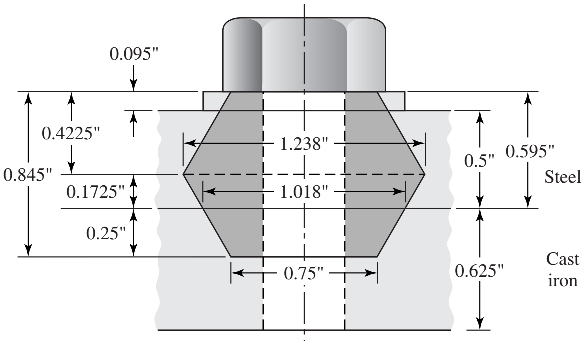
3rd frustum: $D = 0.860, t = 0.515, E = 14.5 \quad \Rightarrow \quad k_3 = 21.47$ Mlbf/in

Fourth frustum: $D = 0.75, t = 0.095, d = 0.5, E = 30 \quad \Rightarrow \quad k_4 = 97.27$ Mlbf/in

$$k_m = \left(\sum_{i=1}^{4} 1/k_i \right)^{-1} = 10.79 \text{ Mlbf/in} \quad \textit{Ans.}$$

$$C = 3.94/(3.94 + 10.79) = 0.267 \quad \textit{Ans.}$$

8-25



$$k_b = \frac{A_t E}{l} = \frac{0.1419(30)}{0.845} = 5.04 \text{ Mlbf/in} \quad \textit{Ans.}$$

From Fig. 8-21,

$$h = \frac{1}{2} + 0.095 = 0.595 \text{ in}$$

$$l = h + \frac{d}{2} = 0.595 + \frac{0.5}{2} = 0.845$$

$$D_1 = 0.75 + 0.845 \tan 30^\circ = 1.238 \text{ in}$$

$$l/2 = 0.845/2 = 0.4225 \text{ in}$$

From Eq. (8-20):

Frustum 1: $D = 0.75, t = 0.4225$ in, $d = 0.5$ in, $E = 30$ Mpsi $\Rightarrow k_1 = 36.14$ Mlbf/in

Frustum 2: $D = 1.018$ in, $t = 0.1725$ in, $E = 70$ Mpsi, $d = 0.5$ in $\Rightarrow k_2 = 134.6$ Mlbf/in

Frustum 3: $D = 0.75, t = 0.25$ in, $d = 0.5$ in, $E = 14.5$ Mpsi $\Rightarrow k_3 = 23.49$ Mlbf/in

$$k_m = \frac{1}{(1/36.14) + (1/134.6) + (1/23.49)} = 12.87 \text{ Mlbf/in} \quad \textit{Ans.}$$

$$C = \frac{5.04}{5.04 + 12.87} = 0.281 \quad \textit{Ans.}$$

8-26 Refer to Prob. 8-24 and its solution. Additional information: $A = 3.5$ in, $D_s = 4.25$ in, static pressure 1500 psi, $D_b = 6$ in, C (joint constant) $= 0.267$, ten SAE grade 5 bolts.

$$P = \frac{1}{10}\,\frac{\pi(4.25^2)}{4}(1500) = 2128 \text{ lbf}$$

From Tables 8-2 and 8-9,

$$A_t = 0.1419 \text{ in}^2$$

$$S_p = 85\,000 \text{ psi}$$

$$F_i = 0.75(0.1419)(85) = 9.046 \text{ kip}$$

From Eq. (8-28),

$$n = \frac{S_p A_t - F_i}{CP} = \frac{85(0.1419) - 9.046}{0.267(2.128)} = 5.31 \quad \textit{Ans.}$$

8-27 From Fig. 8-21, $t_1 = 0.25$ in

$$h = 0.25 + 0.065 = 0.315 \text{ in}$$

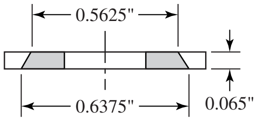
$$l = h + (d/2) = 0.315 + (3/16) = 0.5025 \text{ in}$$

$$D_1 = 1.5(0.375) + 0.577(0.5025) = 0.8524 \text{ in}$$

$$D_2 = 1.5(0.375) = 0.5625 \text{ in}$$

$$l/2 = 0.5025/2 = 0.251\,25 \text{ in}$$

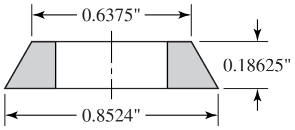
Frustum 1: Washer



$$E = 30 \text{ Mpsi}, \quad t = 0.065 \text{ in}, \quad D = 0.5625 \text{ in}$$

$$k = 78.57 \text{ Mlbf/in} \qquad \text{(by computer)}$$

Frustum 2: Cap portion

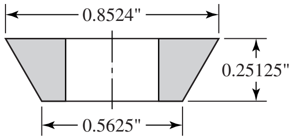


$$E = 14 \text{ Mpsi}, \quad t = 0.186\,25 \text{ in}$$

$$D = 0.5625 + 2(0.065)(0.577) = 0.6375 \text{ in}$$

$$k = 23.46 \text{ Mlbf/in} \qquad \text{(by computer)}$$

Frustum 3: Frame and Cap



$$E = 14 \text{ Mpsi}, \quad t = 0.251\,25 \text{ in}, \quad D = 0.5625 \text{ in}$$

$$k = 14.31 \text{ Mlbf/in} \qquad \text{(by computer)}$$

$$k_m = \frac{1}{(1/78.57) + (1/23.46) + (1/14.31)} = 7.99 \text{ Mlbf/in} \quad \textit{Ans.}$$

For the bolt, $L_T = 2(3/8) + (1/4) = 1$ in. So the bolt is threaded all the way. Since $A_t = 0.0775\ \text{in}^2$

$$k_b = \frac{0.0775(30)}{0.5025} = 4.63\ \text{Mlbf/in} \quad \textit{Ans.}$$

8-28

(a) $F'_b = RF'_{b,\max} \sin\theta$

Half of the external moment is contributed by the line load in the interval $0 \le \theta \le \pi$.

$$\frac{M}{2} = \int_0^\pi F'_b R^2 \sin\theta\, d\theta = \int_0^\pi F'_{b,\,\max} R^2 \sin^2\theta\, d\theta$$

$$\frac{M}{2} = \frac{\pi}{2} F'_{b,\,\max} R^2$$

from which $F'_{b,\max} = \dfrac{M}{\pi R^2}$

$$F_{\max} = \int_{\phi_1}^{\phi_2} F'_b R \sin\theta\, d\theta = \frac{M}{\pi R^2} \int_{\phi_1}^{\phi_2} R \sin\theta\, d\theta = \frac{M}{\pi R}(\cos\phi_1 - \cos\phi_2)$$

Noting $\phi_1 = 75^\circ, \phi_2 = 105^\circ$

$$F_{\max} = \frac{12\,000}{\pi(8/2)}(\cos 75^\circ - \cos 105^\circ) = 494\ \text{lbf} \quad \textit{Ans.}$$

(b)

$$F_{\max} = F'_{b,\,\max} R\Delta\phi = \frac{M}{\pi R^2}(R)\left(\frac{2\pi}{N}\right) = \frac{2M}{RN}$$

$$F_{\max} = \frac{2(12\,000)}{(8/2)(12)} = 500\ \text{lbf} \quad \textit{Ans.}$$

(c) $F = F_{\max} \sin\theta$

$$M = 2F_{\max} R[(1)\sin^2 90^\circ + 2\sin^2 60^\circ + 2\sin^2 30^\circ + (1)\sin^2(0)] = 6F_{\max} R$$

from which

$$F_{\max} = \frac{M}{6R} = \frac{12\,000}{6(8/2)} = 500\ \text{lbf} \quad \textit{Ans.}$$

The simple general equation resulted from part (*b*)

$$F_{\max} = \frac{2M}{RN}$$

8-29 **(a)** Table 8-11: $S_p = 600$ MPa

Eq. (8-30): $F_i = 0.9A_t S_p = 0.9(245)(600)(10^{-3}) = 132.3$ kN

Table (8-15): $K = 0.18$

Eq. (8-27) $T = 0.18(132.3)(20) = 476\ \text{N}\cdot\text{m}$ *Ans.*

(b) Washers: $t = 3.4$ mm, $d = 20$ mm, $D = 30$ mm, $E = 207$ GPa $\Rightarrow$ $k_1 = 42\,175$ MN/m

Cast iron: $t = 20$ mm, $d = 20$ mm, $D = 30 + 2(3.4)\tan 30^\circ = 33.93$ mm,

$E = 135$ GPa $\quad\Rightarrow\quad k_2 = 7885$ MN/m

Steel: $t = 20$ mm, $d = 20$ mm, $D = 33.93$ mm, $E = 207$ GPa $\Rightarrow k_3 = 12\,090$ MN/m

$$k_m = (2/42\,175 + 1/7885 + 1/12\,090)^{-1} = 3892 \text{ MN/m}$$

Bolt: $l = 46.8$ mm. Nut: $H = 18$ mm. $L > 46.8 + 18 = 64.8$ mm. Use $L = 80$ mm.

$$L_T = 2(20) + 6 = 46 \text{ mm}, \; l_d = 80 - 46 = 34 \text{ mm}, \; l_t = 46.8 - 34 = 12.8 \text{ mm},$$

$$A_t = 245 \text{ mm}^2, \quad A_d = \pi 20^2/4 = 314.2 \text{ mm}^2$$

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{314.2(245)(207)}{314.2(12.8) + 245(34)} = 1290 \text{ MN/m}$$

$$C = 1290/(1290 + 3892) = 0.2489, \quad S_p = 600 \text{ MPa}, \quad F_i = 132.3 \text{ kN}$$

$$n = \frac{S_p A_t - F_i}{C(P/N)} = \frac{600(0.245) - 132.3}{0.2489(15/4)} = 15.7 \quad \textit{Ans.}$$

Bolts are a bit oversized for the load.

8-30 **(a)** ISO M 20 × 2.5 grade 8.8 coarse pitch bolts, lubricated.

Table 8-2 $\quad A_t = 245 \text{ mm}^2$

Table 8-11 $\quad S_p = 600 \text{ MPa}$

$$A_d = \pi(20)^2/4 = 314.2 \text{ mm}^2$$

$$F_p = 245(0.600) = 147 \text{ kN}$$

$$F_i = 0.90F_p = 0.90(147) = 132.3 \text{ kN}$$

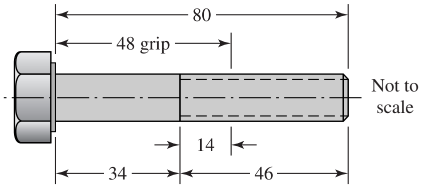
$$T = 0.18(132.3)(20) = 476 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

(b) $L \geq l + H = 48 + 18 = 66$ mm. Therefore, set $L = 80$ mm per Table A-17.

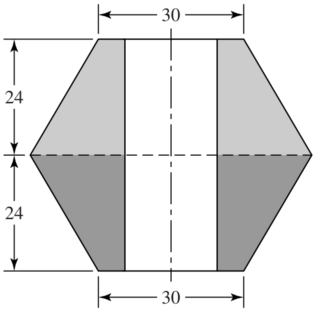
$$L_T = 2D + 6 = 2(20) + 6 = 46 \text{ mm}$$

$$l_d = L - L_T = 80 - 46 = 34 \text{ mm}$$

$$l_t = l - l_d = 48 - 34 = 14 \text{ mm}$$



$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{314.2(245)(207)}{314.2(14) + 245(34)} = 1251.9 \text{ MN/m}$$



Use Wileman *et al.*

Eq. (8-23)

$$A = 0.787\,15, \quad B = 0.628\,73$$

$$\frac{k_m}{Ed} = A\exp\left(\frac{Bd}{L_G}\right) = 0.787\,15\exp\left[0.628\,73\left(\frac{20}{48}\right)\right] = 1.0229$$

$$k_m = 1.0229(207)(20) = 4235 \text{ MN/m}$$

$$C = \frac{1251.9}{1251.9 + 4235} = 0.228$$

Bolts carry 0.228 of the external load; members carry 0.772 of the external load. *Ans.* Thus, the actual loads are

$$F_b = CP + F_i = 0.228(20) + 132.3 = 136.9 \text{ kN}$$

$$F_m = (1 - C)P - F_i = (1 - 0.228)20 - 132.3 = -116.9 \text{ kN}$$

8-31 Given $p_{\max} = 6$ MPa, $p_{\min} = 0$ and from Prob. 8-20 solution, $C = 0.2346$, $F_i = 37.9$ kN, $A_t = 84.3 \text{ mm}^2$.

For 6 MPa, $P = 10.6$ kN per bolt

$$\sigma_i = \frac{F_i}{A_t} = \frac{37.9(10^3)}{84.3} = 450 \text{ MPa}$$

Eq. (8-35):

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.2346(10.6)(10^3)}{2(84.3)} = 14.75 \text{ MPa}$$

$$\sigma_m = \sigma_a + \sigma_i = 14.75 + 450 = 464.8 \text{ MPa}$$

(a) Goodman Eq. (8-40) for 8.8 bolts with $S_e = 129$ MPa, $S_{ut} = 830$ MPa

$$S_a = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut} + S_e} = \frac{129(830 - 450)}{830 + 129} = 51.12 \text{ MPa}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{51.12}{14.75} = 3.47 \quad \textit{Ans.}$$

(b) Gerber Eq. (8-42)

$$S_a = \frac{1}{2S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(129)}\left[830\sqrt{830^2 + 4(129)(129 + 450)} - 830^2 - 2(450)(129)\right]$$

$$= 76.99 \text{ MPa}$$

$$n_f = \frac{76.99}{14.75} = 5.22 \quad \textit{Ans.}$$

(c) ASME-elliptic Eq. (8-43) with $S_p = 600$ MPa

$$S_a = \frac{S_e}{S_p^2 + S_e^2}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{129}{600^2 + 129^2}\left[600\sqrt{600^2 + 129^2 - 450^2} - 450(129)\right] = 65.87 \text{ MPa}$$

$$n_f = \frac{65.87}{14.75} = 4.47 \quad \textit{Ans.}$$

8-32

$$P = \frac{pA}{N} = \frac{\pi D^2 p}{4N} = \frac{\pi(0.9^2)(550)}{4(36)} = 9.72 \text{ kN/bolt}$$

Table 8-11: $S_p = 830$ MPa, $S_{ut} = 1040$ MPa, $S_y = 940$ MPa

Table 8-1:

$$A_t = 58 \text{ mm}^2$$

$$A_d = \pi(10^2)/4 = 78.5 \text{ mm}^2$$

$$l = D + E = 20 + 25 = 45 \text{ mm}$$

$$L_T = 2(10) + 6 = 26 \text{ mm}$$

Table A-31: $H = 8.4$ mm

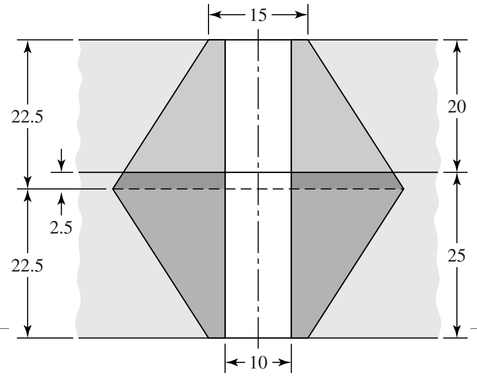
$$L \ge l + H = 45 + 8.4 = 53.4 \text{ mm}$$

Choose $L = 60$ mm from Table A-17

$$l_d = L - L_T = 60 - 26 = 34 \text{ mm}$$

$$l_t = l - l_d = 45 - 34 = 11 \text{ mm}$$

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.5(58)(207)}{78.5(11) + 58(34)} = 332.4 \text{ MN/m}$$



Frustum 1: Top, $E = 207, t = 20$ mm, $d = 10$ mm, $D = 15$ mm

$$k_1 = \frac{0.5774\pi(207)(10)}{\ln\left\{\left[\dfrac{1.155(20) + 15 - 10}{1.155(20) + 15 + 10}\right]\left(\dfrac{15 + 10}{15 - 10}\right)\right\}}$$

$$= 3503 \text{ MN/m}$$

Frustum 2: Middle, $E = 96$ GPa, $D = 38.09$ mm, $t = 2.5$ mm, $d = 10$ mm

$$k_2 = \frac{0.5774\pi(96)(10)}{\ln\left\{\left[\dfrac{1.155(2.5) + 38.09 - 10}{1.155(2.5) + 38.09 + 10}\right]\left(\dfrac{38.09 + 10}{38.09 - 10}\right)\right\}}$$

$$= 44\,044 \text{ MN/m}$$

could be neglected due to its small influence on k_m.

Frustum 3: Bottom, $E = 96$ GPa, $t = 22.5$ mm, $d = 10$ mm, $D = 15$ mm

$$k_3 = \frac{0.5774\pi(96)(10)}{\ln\left\{\left[\dfrac{1.155(22.5) + 15 - 10}{1.155(22.5) + 15 + 10}\right]\left(\dfrac{15 + 10}{15 - 10}\right)\right\}}$$

$$= 1567 \text{ MN/m}$$

$$k_m = \frac{1}{(1/3503) + (1/44\,044) + (1/1567)} = 1057 \text{ MN/m}$$

$$C = \frac{332.4}{332.4 + 1057} = 0.239$$

$$F_i = 0.75A_t S_p = 0.75(58)(830)(10^{-3}) = 36.1 \text{ kN}$$

Table 8-17: $S_e = 162$ MPa

$$\sigma_i = \frac{F_i}{A_t} = \frac{36.1(10^3)}{58} = 622 \text{ MPa}$$

(a) Goodman Eq. (8-40)

$$S_a = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut} + S_e} = \frac{162(1040 - 622)}{1040 + 162} = 56.34 \text{ MPa}$$

$$n_f = \frac{56.34}{20} = 2.82 \quad \textit{Ans.}$$

(b) Gerber Eq. (8-42)

$$S_a = \frac{1}{2S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(162)}\left[1040\sqrt{1040^2 + 4(162)(162 + 622)} - 1040^2 - 2(622)(162)\right]$$

$$= 86.8 \text{ MPa}$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.239(9.72)(10^3)}{2(58)} = 20 \text{ MPa}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{86.8}{20} = 4.34 \quad \textit{Ans.}$$

(c) ASME elliptic

$$S_a = \frac{S_e}{S_p^2 + S_e^2}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{162}{830^2 + 162^2}\left[830\sqrt{830^2 + 162^2 - 622^2} - 622(162)\right] = 84.90 \text{ MPa}$$

$$n_f = \frac{84.90}{20} = 4.24 \quad \textit{Ans.}$$

8-33 Let the repeatedly-applied load be designated as P. From Table A-22, $S_{ut} =$ 93.7 kpsi. Referring to the Figure of Prob. 3-74, the following notation will be used for the radii of Section AA.

$$r_i = 1 \text{ in}, \quad r_o = 2 \text{ in}, \quad r_c = 1.5 \text{ in}$$

From Table 4-5, with $R = 0.5$ in

$$r_n = \frac{0.5^2}{2\left(1.5 - \sqrt{1.5^2 - 0.5^2}\right)} = 1.457\,107 \text{ in}$$

$$e = r_c - r_n = 1.5 - 1.457\,107 = 0.042\,893 \text{ in}$$

$$c_o = r_o - r_n = 2 - 1.457\,109 = 0.542\,893 \text{ in}$$

$$c_i = r_n - r_i = 1.457\,107 - 1 = 0.457\,107 \text{ in}$$

$$A = \pi(1^2)/4 = 0.7854 \text{ in}^2$$

If P is the maximum load

$$M = Pr_c = 1.5P$$

$$\sigma_i = \frac{P}{A}\left(1 + \frac{r_c c_i}{e r_i}\right) = \frac{P}{0.7854}\left(1 + \frac{1.5(0.457)}{0.0429(1)}\right) = 21.62P$$

$$\sigma_a = \sigma_m = \frac{\sigma_i}{2} = \frac{21.62P}{2} = 10.81P$$

(a) *Eye:* Section AA

$$k_a = 14.4(93.7)^{-0.718} = 0.553$$

$$d_e = 0.37d = 0.37(1) = 0.37 \text{ in}$$

$$k_b = \left(\frac{0.37}{0.30}\right)^{-0.107} = 0.978$$

$$k_c = 0.85$$

$$S'_e = 0.5(93.7) = 46.85 \text{ kpsi}$$

$$S_e = 0.553(0.978)(0.85)(46.85) = 21.5 \text{ kpsi}$$

Since no stress concentration exists, use a load line slope of 1. From Table 7-10 for Gerber

$$S_a = \frac{93.7^2}{2(21.5)}\left[-1+\sqrt{1+\left(\frac{2(21.5)}{93.7}\right)^2}\right] = 20.47 \text{ kpsi}$$

Note the mere 5 percent degrading of S_e in S_a

$$n_f = \frac{S_a}{\sigma_a} = \frac{20.47(10^3)}{10.81P} = \frac{1894}{P}$$

Thread: Die cut. Table 8-17 gives 18.6 kpsi for rolled threads. Use Table 8-16 to find S_e for die cut threads

$$S_e = 18.6(3.0/3.8) = 14.7 \text{ kpsi}$$

Table 8-2:

$$A_t = 0.663 \text{ in}^2$$

$$\sigma = P/A_t = P/0.663 = 1.51P$$

$$\sigma_a = \sigma_m = \sigma/2 = 1.51P/2 = 0.755P$$

From Table 7-10, Gerber

$$S_a = \frac{120^2}{2(14.7)}\left[-1+\sqrt{1+\left(\frac{2(14.7)}{120}\right)^2}\right] = 14.5 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{14\,500}{0.755P} = \frac{19\,200}{P}$$

Comparing $1894/P$ with $19\,200/P$, we conclude that the *eye* is weaker in fatigue. *Ans.*

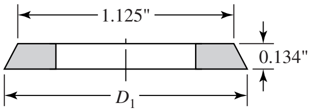
(b) Strengthening steps can include heat treatment, cold forming, cross section change (a round is a poor cross section for a curved bar in bending because the bulk of the material is located where the stress is small). *Ans.*

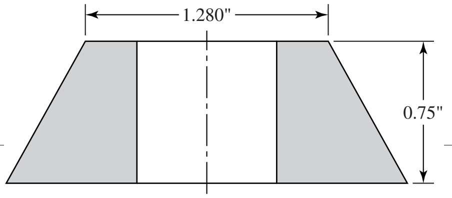
(c) For $n_f = 2$

$$P = \frac{1894}{2} = 947 \text{ lbf, max. load} \quad \textit{Ans.}$$

8-34 **(a)** $L \geq 1.5 + 2(0.134) + \frac{41}{64} = 2.41 \text{ in}$. Use $L = 2\frac{1}{2}$ in *Ans.*

(b) Four frusta: Two washers and two members





Washer: $E = 30$ Mpsi, $t = 0.134$ in, $D = 1.125$ in, $d = 0.75$ in

Eq. (8-20): $k_1 = 153.3$ Mlbf/in

Member: $E = 16$ Mpsi, $t = 0.75$ in, $D = 1.280$ in, $d = 0.75$ in

Eq. (8-20): $k_2 = 35.5$ Mlbf/in

$$k_m = \frac{1}{(2/153.3) + (2/35.5)} = 14.41 \text{ Mlbf/in} \quad Ans.$$

Bolt:

$$L_T = 2(3/4) + 1/4 = 1^3\!/_4 \text{ in}$$

$$l = 2(0.134) + 2(0.75) = 1.768 \text{ in}$$

$$l_d = L - L_T = 2.50 - 1.75 = 0.75 \text{ in}$$

$$l_t = l - l_d = 1.768 - 0.75 = 1.018 \text{ in}$$

$$A_t = 0.373 \text{ in}^2 \quad \text{(Table 8-2)}$$

$$A_d = \pi(0.75)^2/4 = 0.442 \text{ in}^2$$

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.442(0.373)(30)}{0.442(1.018) + 0.373(0.75)} = 6.78 \text{ Mlbf/in} \quad Ans.$$

$$C = \frac{6.78}{6.78 + 14.41} = 0.320 \quad Ans.$$

(c) From Eq. (8-40), Goodman with $S_e = 18.6$ kpsi, $S_{ut} = 120$ kpsi

$$S_a = \frac{18.6[120 - (25/0.373)]}{120 + 18.6} = 7.11 \text{ kpsi}$$

The stress components are

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.320(6)}{2(0.373)} = 2.574 \text{ kpsi}$$

$$\sigma_m = \sigma_a + \frac{F_i}{A_t} = 2.574 + \frac{25}{0.373} = 69.6 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{7.11}{2.574} = 2.76 \quad Ans.$$

(d) Eq. (8-42) for Gerber

$$S_a = \frac{1}{2(18.6)}\left[120\sqrt{120^2 + 4(18.6)\left(18.6 + \frac{25}{0.373}\right)} - 120^2 - 2\left(\frac{25}{0.373}\right)18.6\right]$$

$$= 10.78 \text{ kpsi}$$

$$n_f = \frac{10.78}{2.574} = 4.19 \quad Ans.$$

(e) $n_{\text{proof}} = \dfrac{85}{2.654 + 69.8} = 1.17 \quad Ans.$

8-35

(a) Table 8-2: $A_t = 0.1419 \text{ in}^2$

Table 8-9: $S_p = 85 \text{ kpsi}, \quad S_{ut} = 120 \text{ kpsi}$

Table 8-17: $S_e = 18.6 \text{ kpsi}$

$$F_i = 0.75 A_t S_p = 0.75(0.1419)(85) = 9.046 \text{ kip}$$

$$C = \frac{4.94}{4.94 + 15.97} = 0.236$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.236P}{2(0.1419)} = 0.832P \text{ kpsi}$$

Eq. (8-40) for Goodman criterion

$$S_a = \frac{18.6(120 - 9.046/0.1419)}{120 + 18.6} = 7.55 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{7.55}{0.832P} = 2 \quad \Rightarrow \quad P = 4.54 \text{ kip} \quad \textit{Ans.}$$

(b) Eq. (8-42) for Gerber criterion

$$S_a = \frac{1}{2(18.6)}\left[120\sqrt{120^2 + 4(18.6)\left(18.6 + \frac{9.046}{0.1419}\right)} - 120^2 - 2\left(\frac{9.046}{0.1419}\right)18.6\right]$$

$$= 11.32 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{11.32}{0.832P} = 2$$

From which

$$P = \frac{11.32}{2(0.832)} = 6.80 \text{ kip} \quad \textit{Ans.}$$

(c) $\sigma_a = 0.832P = 0.832(6.80) = 5.66 \text{ kpsi}$

$\sigma_m = S_a + \sigma_a = 11.32 + 63.75 = 75.07 \text{ kpsi}$

Load factor, Eq. (8-28)

$$n = \frac{S_p A_t - F_i}{CP} = \frac{85(0.1419) - 9.046}{0.236(6.80)} = 1.88 \quad \textit{Ans.}$$

Separation load factor, Eq. (8-29)

$$n = \frac{F_i}{(1-C)P} = \frac{9.046}{6.80(1 - 0.236)} = 1.74 \quad \textit{Ans.}$$

8-36 Table 8-2: $A_t = 0.969 \text{ in}^2$ (coarse)

$A_t = 1.073 \text{ in}^2$ (fine)

Table 8-9: $S_p = 74 \text{ kpsi}, \quad S_{ut} = 105 \text{ kpsi}$

Table 8-17: $S_e = 16.3 \text{ kpsi}$

Coarse thread, UNC

$$F_i = 0.75(0.969)(74) = 53.78 \text{ kip}$$

$$\sigma_i = \frac{F_i}{A_t} = \frac{53.78}{0.969} = 55.5 \text{ kpsi}$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.30P}{2(0.969)} = 0.155P \text{ kpsi}$$

Eq. (8-42):

$$S_a = \frac{1}{2(16.3)}\left[105\sqrt{105^2 + 4(16.3)(16.3 + 55.5)} - 105^2 - 2(55.5)(16.3)\right] = 9.96 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{9.96}{0.155P} = 2$$

From which

$$P = \frac{9.96}{0.155(2)} = 32.13 \text{ kip} \quad \textit{Ans.}$$

Fine thread, UNF

$$F_i = 0.75(1.073)(74) = 59.55 \text{ kip}$$

$$\sigma_i = \frac{59.55}{1.073} = 55.5 \text{ kpsi}$$

$$\sigma_a = \frac{0.32P}{2(1.073)} = 0.149P \text{ kpsi}$$

$$S_a = 9.96 \quad \text{(as before)}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{9.96}{0.149P} = 2$$

From which

$$P = \frac{9.96}{0.149(2)} = 33.42 \text{ kip} \quad \textit{Ans.}$$

Percent improvement

$$\frac{33.42 - 32.13}{32.13}(100) \doteq 4\% \quad \textit{Ans.}$$

8-37 For a M 30 × 3.5 ISO 8.8 bolt with $P = 80$ kN/bolt and $C = 0.33$

Table 8-1: $A_t = 561 \text{ mm}^2$

Table 8-11: $S_p = 600$ MPa

$S_{ut} = 830$ MPa

Table 8-17: $S_e = 129$ MPa

$$F_i = 0.75(561)(10^{-3})(600) = 252.45 \text{ kN}$$

$$\sigma_i = \frac{252.45(10^{-3})}{561} = 450 \text{ MPa}$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.33(80)(10^3)}{2(561)} = 23.53 \text{ MPa}$$

Eq. (8-42):

$$S_a = \frac{1}{2(129)}\left[830\sqrt{830^2 + 4(129)(129+450)} - 830^2 - 2(450)(129)\right] = 77.0 \text{ MPa}$$

Fatigue factor of safety

$$n_f = \frac{S_a}{\sigma_a} = \frac{77.0}{23.53} = 3.27 \quad Ans.$$

Load factor from Eq. (8-28),

$$n = \frac{S_p A_t - F_i}{CP} = \frac{600(10^{-3})(561) - 252.45}{0.33(80)} = 3.19 \quad Ans.$$

Separation load factor from Eq. (8-29),

$$n = \frac{F_i}{(1-C)P} = \frac{252.45}{(1-0.33)(80)} = 4.71 \quad Ans.$$

8-38

(a) Table 8-2: $A_t = 0.0775 \text{ in}^2$

Table 8-9: $S_p = 85 \text{ kpsi}, \quad S_{ut} = 120 \text{ kpsi}$

Table 8-17: $S_e = 18.6 \text{ kpsi}$

Unthreaded grip

$$k_b = \frac{A_d E}{l} = \frac{\pi(0.375)^2(30)}{4(13.5)} = 0.245 \text{ Mlbf/in per bolt} \quad Ans.$$

$$A_m = \frac{\pi}{4}[(D+2t)^2 - D^2] = \frac{\pi}{4}(4.75^2 - 4^2) = 5.154 \text{ in}^2$$

$$k_m = \frac{A_m E}{l} = \frac{5.154(30)}{12}\left(\frac{1}{6}\right) = 2.148 \text{ Mlbf/in/bolt.} \quad Ans.$$

(b)

$$F_i = 0.75(0.0775)(85) = 4.94 \text{ kip}$$

$$\sigma_i = 0.75(85) = 63.75 \text{ kpsi}$$

$$P = pA = \frac{2000}{6}\left[\frac{\pi}{4}(4)^2\right] = 4189 \text{ lbf/bolt}$$

$$C = \frac{0.245}{0.245 + 2.148} = 0.102$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.102(4.189)}{2(0.0775)} = 2.77 \text{ kpsi}$$

Eq. (8-40) for Goodman

$$S_a = \frac{18.6(120 - 63.75)}{120 + 18.6} = 7.55 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{7.55}{2.77} = 2.73 \quad \textit{Ans.}$$

(c) From Eq. (8-42) for Gerber fatigue criterion,

$$S_a = \frac{1}{2(18.6)}\left[120\sqrt{120^2 + 4(18.6)(18.6 + 63.75)} - 120^2 - 2(63.75)(18.6)\right]$$

$$= 11.32 \text{ kpsi}$$

$$n_f = \frac{S_a}{\sigma_a} = \frac{11.32}{2.77} = 4.09 \quad \textit{Ans.}$$

(d) Pressure causing joint separation from Eq. (8-29)

$$n = \frac{F_i}{(1 - C)P} = 1$$

$$P = \frac{F_i}{1 - C} = \frac{4.94}{1 - 0.102} = 5.50 \text{ kip}$$

$$p = \frac{P}{A} = \frac{5500}{\pi(4^2)/4} 6 = 2626 \text{ psi} \quad \textit{Ans.}$$

8-39 This analysis is important should the initial bolt tension fail. Members: $S_y = 71$ kpsi, $S_{sy} = 0.577(71) = 41.0$ kpsi. Bolts: SAE grade 8, $S_y = 130$ kpsi, $S_{sy} = 0.577(130) =$ 75.01 kpsi

Shear in bolts

$$A_s = 2\left[\frac{\pi(0.375^2)}{4}\right] = 0.221 \text{ in}^2$$

$$F_s = \frac{A_s S_{sy}}{n} = \frac{0.221(75.01)}{3} = 5.53 \text{ kip}$$

Bearing on bolts

$$A_b = 2(0.375)(0.25) = 0.188 \text{ in}^2$$

$$F_b = \frac{A_b S_{yc}}{n} = \frac{0.188(130)}{2} = 12.2 \text{ kip}$$

Bearing on member

$$F_b = \frac{0.188(71)}{2.5} = 5.34 \text{ kip}$$

Tension of members

$$A_t = (1.25 - 0.375)(0.25) = 0.219 \text{ in}^2$$

$$F_t = \frac{0.219(71)}{3} = 5.18 \text{ kip}$$

$$F = \min(5.53, 12.2, 5.34, 5.18) = 5.18 \text{ kip} \quad \textit{Ans.}$$

The tension in the members controls the design.

8-40 Members: $S_y = 32$ kpsi

Bolts: $S_y = 92$ kpsi, $S_{sy} = (0.577)92 = 53.08$ kpsi

Shear of bolts

$$A_s = 2\left[\frac{\pi(0.375)^2}{4}\right] = 0.221 \text{ in}^2$$

$$\tau = \frac{F_s}{A_s} = \frac{4}{0.221} = 18.1 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau} = \frac{53.08}{18.1} = 2.93 \quad Ans.$$

Bearing on bolts

$$A_b = 2(0.25)(0.375) = 0.188 \text{ in}^2$$

$$\sigma_b = \frac{-4}{0.188} = -21.3 \text{ kpsi}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{92}{|-21.3|} = 4.32 \quad Ans.$$

Bearing on members

$$n = \frac{S_{yc}}{|\sigma_b|} = \frac{32}{|-21.3|} = 1.50 \quad Ans.$$

Tension of members

$$A_t = (2.375 - 0.75)(1/4) = 0.406 \text{ in}^2$$

$$\sigma_t = \frac{4}{0.406} = 9.85 \text{ kpsi}$$

$$n = \frac{S_y}{A_t} = \frac{32}{9.85} = 3.25 \quad Ans.$$

8-41 Members: $S_y = 71$ kpsi

Bolts: $S_y = 92$ kpsi, $S_{sy} = 0.577(92) = 53.08$ kpsi

Shear of bolts

$$F = S_{sy}A/n$$

$$F_s = \frac{53.08(2)(\pi/4)(7/8)^2}{1.8} = 35.46 \text{ kip}$$

Bearing on bolts

$$F_b = \frac{2(7/8)(3/4)(92)}{2.2} = 54.89 \text{ kip}$$

Bearing on members

$$F_b = \frac{2(7/8)(3/4)(71)}{2.4} = 38.83 \text{ kip}$$

Tension in members

$$F_t = \frac{(3 - 0.875)(3/4)(71)}{2.6} = 43.52 \text{ kip}$$

$$F = \min(35.46, 54.89, 38.83, 43.52) = 35.46 \text{ kip} \quad Ans.$$

8-42 Members: $S_y = 47$ kpsi

Bolts: $S_y = 92$ kpsi, $S_{sy} = 0.577(92) = 53.08$ kpsi

Shear of bolts

$$A_d = \frac{\pi(0.75)^2}{4} = 0.442 \text{ in}^2$$

$$\tau_s = \frac{20}{3(0.442)} = 15.08 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau_s} = \frac{53.08}{15.08} = 3.52 \quad Ans.$$

Bearing on bolt

$$\sigma_b = -\frac{20}{3(3/4)(5/8)} = -14.22 \text{ kpsi}$$

$$n = -\frac{S_y}{\sigma_b} = -\left(\frac{92}{-14.22}\right) = 6.47 \quad Ans.$$

Bearing on members

$$\sigma_b = -\frac{F}{A_b} = -\frac{20}{3(3/4)(5/8)} = -14.22 \text{ kpsi}$$

$$n = -\frac{S_y}{\sigma_b} = -\frac{47}{14.22} = 3.31 \quad Ans.$$

Tension on members

$$\sigma_t = \frac{F}{A} = \frac{20}{(5/8)[7.5 - 3(3/4)]} = 6.10 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma_t} = \frac{47}{6.10} = 7.71 \quad Ans.$$

8-43 Members: $S_y = 57$ kpsi

Bolts: $S_y = 92$ kpsi, $S_{sy} = 0.577(92) = 53.08$ kpsi

Shear of bolts

$$A_s = 3\left[\frac{\pi(3/8)^2}{4}\right] = 0.3313 \text{ in}^2$$

$$\tau_s = \frac{F}{A} = \frac{5.4}{0.3313} = 16.3 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau_s} = \frac{53.08}{16.3} = 3.26 \quad Ans.$$

Bearing on bolt

$$A_b = 3\left(\frac{3}{8}\right)\left(\frac{5}{16}\right) = 0.3516 \text{ in}^2$$

$$\sigma_b = -\frac{F}{A_b} = -\frac{5.4}{0.3516} = -15.36 \text{ kpsi}$$

$$n = -\frac{S_y}{\sigma_b} = -\left(\frac{92}{-15.36}\right) = 5.99 \quad \textit{Ans.}$$

Bearing on members

$$A_b = 0.3516 \text{ in}^2 \text{ (From bearing on bolt calculations)}$$

$$\sigma_b = -15.36 \text{ kpsi (From bearing on bolt calculations)}$$

$$n = -\frac{S_y}{\sigma_b} = -\left(\frac{57}{-15.36}\right) = 3.71 \quad \textit{Ans.}$$

Tension in members

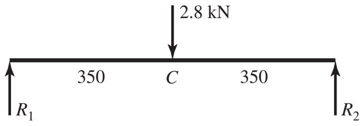
Failure across two bolts

$$A = \frac{5}{16}\left[2\frac{3}{8} - 2\left(\frac{3}{8}\right)\right] = 0.5078 \text{ in}^2$$

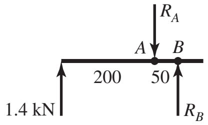
$$\sigma = \frac{F}{A} = \frac{5.4}{0.5078} = 10.63 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma_t} = \frac{57}{10.63} = 5.36 \quad \textit{Ans.}$$

8-44



By symmetry, $R_1 = R_2 = 1.4$ kN



$$\sum M_B = 0 \quad 1.4(250) - 50R_A = 0 \quad \Rightarrow \quad R_A = 7 \text{ kN}$$

$$\sum M_A = 0 \quad 200(1.4) - 50R_B = 0 \quad \Rightarrow \quad R_B = 5.6 \text{ kN}$$

Members: $S_y = 370$ MPa

Bolts: $S_y = 420$ MPa, $S_{sy} = 0.577(420) = 242.3$ MPa

Bolt shear:

$$A_s = \frac{\pi}{4}(10^2) = 78.54 \text{ mm}^2$$

$$\tau = \frac{7(10^3)}{78.54} = 89.13 \text{ MPa}$$

$$n = \frac{S_{sy}}{\tau} = \frac{242.3}{89.13} = 2.72$$

Bearing on member: $A_b = td = 10(10) = 100\ \text{mm}^2$

$$\sigma_b = \frac{-7(10^3)}{100} = -70\ \text{MPa}$$

$$n = -\frac{S_y}{\sigma} = \frac{-370}{-70} = 5.29$$

Strength of member

At A, $M = 1.4(200) = 280\ \text{N}\cdot\text{m}$

$$I_A = \frac{1}{12}[10(50^3) - 10(10^3)] = 103.3(10^3)\ \text{mm}^4$$

$$\sigma_A = \frac{Mc}{I_A} = \frac{280(25)}{103.3(10^3)}(10^3) = 67.76\ \text{MPa}$$

$$n = \frac{S_y}{\sigma_A} = \frac{370}{67.76} = 5.46$$

At C, $M = 1.4(350) = 490\ \text{N}\cdot\text{m}$

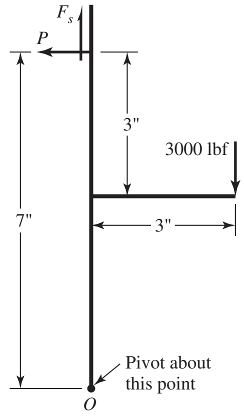
$$I_C = \frac{1}{12}(10)(50^3) = 104.2(10^3)\ \text{mm}^4$$

$$\sigma_C = \frac{490(25)}{104.2(10^3)}(10^3) = 117.56\ \text{MPa}$$

$$n = \frac{S_y}{\sigma_C} = \frac{370}{117.56} = 3.15 < 5.46 \qquad C \text{ more critical}$$

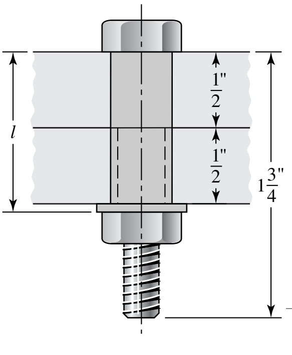
$$n = \min(2.72, 5.29, 3.15) = 2.72 \quad \textit{Ans.}$$

8-45



$$F_s = 3000\ \text{lbf}$$

$$P = \frac{3000(3)}{7} = 1286\ \text{lbf}$$



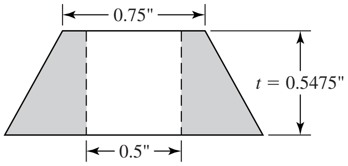
$$H = \frac{7}{16}\ \text{in}$$

$$l = \frac{1}{2} + \frac{1}{2} + 0.095 = 1.095\ \text{in}$$

$$L \geq l + H = 1.095 + (7/16) = 1.532\ \text{in}$$

Use $1\frac{3}{4}$" bolts

$$
\begin{aligned}
L_T &= 2D + \frac{1}{4} = 2(0.5) + 0.25 = 1.25 \text{ in} \\
l_d &= 1.75 - 1.25 = 0.5 \\
l_t &= 1.095 - 0.5 = 0.595 \\
A_d &= \frac{\pi(0.5)^2}{4} = 0.1963 \text{ in}^2 \\
A_t &= 0.1419 \text{ in} \\
k_b &= \frac{A_d A_t E}{A_d l_t + A_t l_d} \\
&= \frac{0.1963(0.1419)(30)}{0.1963(0.595) + 0.1419(0.5)} \\
&= 4.451 \text{ Mlbf/in}
\end{aligned}
$$



Two identical frusta

$$
\begin{aligned}
A &= 0.787\,15,\ B = 0.628\,73 \\
k_m &= EdA \exp\left(0.628\,73 \frac{d}{L_G}\right) \\
&= 30(0.5)(0.787\,15)\left[\exp\left(0.628\,73 \frac{0.5}{1.095}\right)\right] \\
k_m &= 15.733 \text{ Mlbf/in} \\
C &= \frac{4.451}{4.451 + 15.733} = 0.2205 \\
S_p &= 85 \text{ kpsi} \\
F_i &= 0.75(0.1419)(85) = 9.046 \text{ kip} \\
\sigma_i &= 0.75(85) = 63.75 \text{ kpsi} \\
\sigma_b &= \frac{CP + F_i}{A_t} = \frac{0.2205(1.286) + 9.046}{0.1419} = 65.75 \text{ kpsi} \\
\tau_s &= \frac{F_s}{A_s} = \frac{3}{0.1963} = 15.28 \text{ kpsi}
\end{aligned}
$$

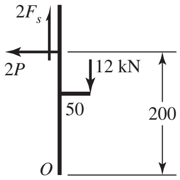
von Mises stress

$$\sigma' = \left(\sigma_b^2 + 3\tau_s^2\right)^{1/2} = [65.74^2 + 3(15.28^2)]^{1/2} = 70.87 \text{ kpsi}$$

Stress margin

$$m = S_p - \sigma' = 85 - 70.87 = 14.1 \text{ kpsi} \quad \textit{Ans.}$$

8-46



$$2P(200) = 12(50)$$

$$P = \frac{12(50)}{2(200)} = 1.5 \text{ kN per bolt}$$

$$F_s = 6 \text{ kN/bolt}$$

$$S_p = 380 \text{ MPa}$$

$$A_t = 245 \text{ mm}^2,\ A_d = \frac{\pi}{4}(20^2) = 314.2 \text{ mm}^2$$

$$F_i = 0.75(245)(380)(10^{-3}) = 69.83 \text{ kN}$$

$$\sigma_i = \frac{69.83(10^3)}{245} = 285 \text{ MPa}$$

$$\sigma_b = \frac{CP + F_i}{A_t} = \left(\frac{0.30(1.5) + 69.83}{245}\right)(10^3) = 287 \text{ MPa}$$

$$\tau = \frac{F_s}{A_d} = \frac{6(10^3)}{314.2} = 19.1 \text{ MPa}$$

$$\sigma' = [287^2 + 3(19.1^2)]^{1/2} = 289 \text{ MPa}$$

$$m = S_p - \sigma' = 380 - 289 = 91 \text{ MPa}$$

Thus the bolt will *not* exceed the proof stress. *Ans.*

8-47 Using the result of Prob. 5-31 for lubricated assembly

$$F_x = \frac{2\pi f T}{0.18d}$$

With a design factor of n_d gives

$$T = \frac{0.18 n_d F_x d}{2\pi f} = \frac{0.18(3)(1000)d}{2\pi(0.12)} = 716d$$

or $T/d = 716$. Also

$$\frac{T}{d} = K(0.75 S_p A_t)$$

$$= 0.18(0.75)(85\,000)A_t$$

$$= 11\,475 A_t$$

Form a table

Size	A_t	$T/d = 11\,475 A_t$	n
$\frac{1}{4} - 28$	0.0364	417.7	1.75
$\frac{5}{16} - 24$	0.058	665.55	2.8
$\frac{3}{8} - 24$	0.0878	1007.5	4.23

The factor of safety in the last column of the table comes from

$$n = \frac{2\pi f(T/d)}{0.18F_x} = \frac{2\pi(0.12)(T/d)}{0.18(1000)} = 0.0042(T/d)$$

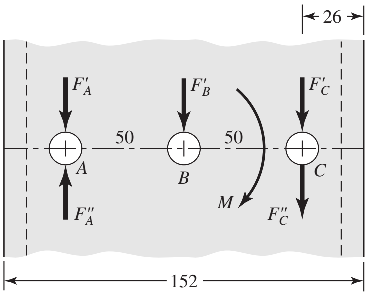
Select a $\frac{3}{8}$" $- 24$ UNF capscrew. The setting is given by

$$T = (11\,475A_t)d = 1007.5(0.375) = 378 \text{ lbf} \cdot \text{in}$$

Given the coarse scale on a torque wrench, specify a torque wrench setting of 400 lbf · in. Check the factor of safety

$$n = \frac{2\pi f T}{0.18F_x d} = \frac{2\pi(0.12)(400)}{0.18(1000)(0.375)} = 4.47$$

8-48



Bolts: $S_p = 380$ MPa, $S_y = 420$ MPa
Channel: $t = 6.4$ mm, $S_y = 170$ MPa
Cantilever: $S_y = 190$ MPa
Nut: $H = 10.8$ mm

$$F'_A = F'_B = F'_C = F/3$$

$$M = (50 + 26 + 125)F = 201F$$

$$F''_A = F''_C = \frac{201F}{2(50)} = 2.01F$$

$$F_C = F'_C + F''_C = \left(\frac{1}{3} + 2.01\right) F = 2.343F$$

Bolts:
The shear bolt area is $A = \pi(12^2)/4 = 113.1 \text{ mm}^2$

$$S_{sy} = 0.577(420) = 242.3 \text{ MPa}$$

$$F = \frac{S_{sy}}{n}\left(\frac{A}{2.343}\right) = \frac{242.3(113.1)(10^{-3})}{2.8(2.343)} = 4.18 \text{ kN}$$

Bearing on bolt: For a 12-mm bolt, at the channel,

$$A_b = td = (6.4)(12) = 76.8 \text{ mm}^2$$

$$F = \frac{S_y}{n}\left(\frac{A_b}{2.343}\right) = \frac{420}{2.8}\left[\frac{76.8(10^{-3})}{2.343}\right] = 4.92 \text{ kN}$$

Bearing on channel: $A_b = 76.8 \text{ mm}^2$, $S_y = 170\,\text{MPa}$

$$F = \frac{170}{2.8}\left[\frac{76.8(10^{-3})}{2.343}\right] = 1.99 \text{ kN}$$

Bearing on cantilever:

$$A_b = 12(12) = 144 \text{ mm}^2$$

$$F = \frac{190}{2.8}\left[\frac{(144)(10^{-3})}{2.343}\right] = 4.17 \text{ kN}$$

Bending of cantilever:

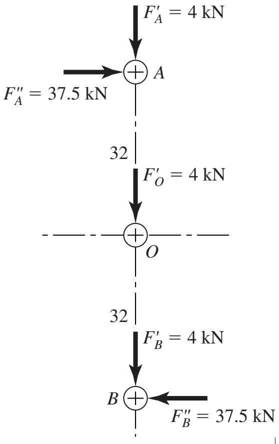
$$I = \frac{1}{12}(12)(50^3 - 12^3) = 1.233(10^5) \text{ mm}^4$$

$$\frac{I}{c} = \frac{1.233(10^5)}{25} = 4932$$

$$F = \frac{M}{151} = \frac{4932(190)}{2.8(151)(10^3)} = 2.22 \text{ kN}$$

So $F = 1.99$ kN based on bearing on channel *Ans.*

8-49 $F' = 4$ kN; $M = 12(200) = 2400$ N · m



$$F''_A = F''_B = \frac{2400}{64} = 37.5 \text{ kN}$$

$$F_A = F_B = \sqrt{(4)^2 + (37.5)^2} = 37.7 \text{ kN} \quad \textit{Ans.}$$

$$F_O = 4 \text{ kN} \quad \textit{Ans.}$$

Bolt shear:

$$A_s = \frac{\pi(12)^2}{4} = 113 \text{ mm}^2$$

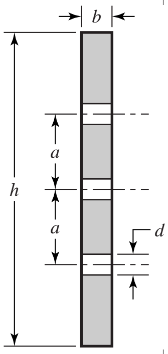
$$\tau = \frac{37.7(10)^3}{113} = 334 \text{ MPa} \quad \textit{Ans.}$$

Bearing on member:

$$A_b = 12(8) = 96 \text{ mm}^2$$

$$\sigma = -\frac{37.7(10)^3}{96} = -393 \text{ MPa} \quad \textit{Ans.}$$

Bending stress in plate:



$$I = \frac{bh^3}{12} - \frac{bd^3}{12} - 2\left(\frac{bd^3}{12} + a^2bd\right)$$

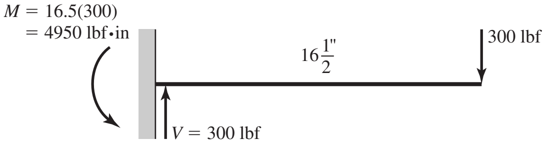
$$= \frac{8(136)^3}{12} - \frac{8(12)^3}{12} - 2\left[\frac{8(12)^3}{12} + (32)^2(8)(12)\right]$$

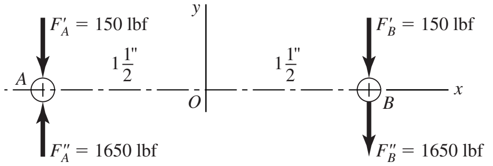
$$= 1.48(10)^6 \text{ mm}^4 \quad \textit{Ans.}$$

$$M = 12(200) = 2400 \text{ N} \cdot \text{m}$$

$$\sigma = \frac{Mc}{I} = \frac{2400(68)}{1.48(10)^6}(10)^3 = 110 \text{ MPa} \quad \textit{Ans.}$$

8-50





Shear of bolt:

$$A_s = \frac{\pi}{4}(0.5)^2 = 0.1963 \text{ in}^2$$

$$\tau = \frac{F}{A} = \frac{1800}{0.1963} = 9170 \text{ psi}$$

$$S_{sy} = 0.577(92) = 53.08 \text{ kpsi}$$

$$n = \frac{53.08}{9.17} = 5.79 \quad \textit{Ans.}$$

$$F''_A = F''_B = \frac{4950}{3} = 1650 \text{ lbf}$$

$$F_A = 1500 \text{ lbf}, \qquad F_B = 1800 \text{ lbf}$$

Bearing on bolt:

$$A_b = \frac{1}{2}\left(\frac{3}{8}\right) = 0.1875 \text{ in}^2$$

$$\sigma = -\frac{F}{A} = -\frac{1800}{0.1875} = -9600 \text{ psi}$$

$$n = \frac{92}{9.6} = 9.58 \quad \textit{Ans.}$$

Bearing on members: $S_y = 54$ kpsi, $n = \dfrac{54}{9.6} = 5.63$ *Ans.*

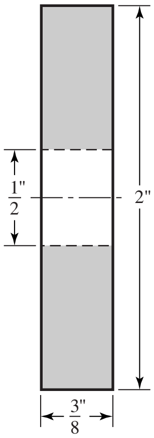
Bending of members: Considering the right-hand bolt

$$M = 300(15) = 4500 \text{ lbf} \cdot \text{in}$$

$$I = \frac{0.375(2)^3}{12} - \frac{0.375(0.5)^3}{12} = 0.246 \text{ in}^4$$

$$\sigma = \frac{Mc}{I} = \frac{4500(1)}{0.246} = 18\,300 \text{ psi}$$

$$n = \frac{54(10)^3}{18\,300} = 2.95 \quad \textit{Ans.}$$



8-51 The direct shear load per bolt is $F' = 2500/6 = 417$ lbf. The moment is taken only by the four outside bolts. This moment is $M = 2500(5) = 12\,500$ lbf · in.

Thus $F'' = \dfrac{12\,500}{2(5)} = 1250$ lbf and the resultant bolt load is

$$F = \sqrt{(417)^2 + (1250)^2} = 1318 \text{ lbf}$$

Bolt strength, $S_y = 57$ kpsi; Channel strength, $S_y = 46$ kpsi; Plate strength, $S_y = 45.5$ kpsi

Shear of bolt: $A_s = \pi(0.625)^2/4 = 0.3068 \text{ in}^2$

$$n = \frac{S_{sy}}{\tau} = \frac{(0.577)(57\,000)}{1318/0.3068} = 7.66 \quad \textit{Ans.}$$

Bearing on bolt: Channel thickness is $t = 3/16$ in;

$$A_b = (0.625)(3/16) = 0.117\ \text{in}^2;\ n = \frac{57\,000}{1318/0.117} = 5.07 \quad \textit{Ans.}$$

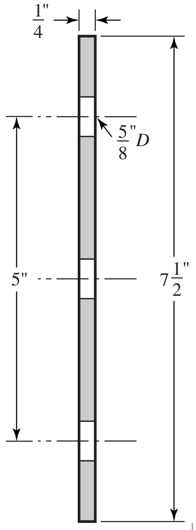
Bearing on channel:

$$n = \frac{46\,000}{1318/0.117} = 4.08 \quad \textit{Ans.}$$

Bearing on plate:

$$A_b = 0.625(1/4) = 0.1563\ \text{in}^2$$

$$n = \frac{45\,500}{1318/0.1563} = 5.40 \quad \textit{Ans.}$$



Bending of plate:

$$I = \frac{0.25(7.5)^3}{12} - \frac{0.25(0.625)^3}{12}$$

$$-2\left[\frac{0.25(0.625)^3}{12} + \left(\frac{1}{4}\right)\left(\frac{5}{8}\right)(2.5)^2\right] = 6.821\ \text{in}^4$$

$$M = 6250\ \text{lbf} \cdot \text{in per plate}$$

$$\sigma = \frac{Mc}{I} = \frac{6250(3.75)}{6.821} = 3436\ \text{psi}$$

$$n = \frac{45\,500}{3436} = 13.2 \quad \textit{Ans.}$$

8-52 Specifying bolts, screws, dowels and rivets is the way a student learns about such components. However, choosing an array a priori is based on experience. Here is a chance for students to build some experience.

8-53 Now that the student can put an a priori decision of an array together with the specification of fasteners.

8-54 A computer program will vary with computer language or software application.

Chapter 9

9-1 Eq. (9-3):

$$F = 0.707hl\tau = 0.707(5/16)(4)(20) = 17.7 \text{ kip} \quad \textit{Ans.}$$

9-2 Table 9-6: $\tau_{\text{all}} = 21.0$ kpsi

$$\begin{aligned} f &= 14.85h \text{ kip/in} \\ &= 14.85(5/16) = 4.64 \text{ kip/in} \\ F &= fl = 4.64(4) = 18.56 \text{ kip} \quad \textit{Ans.} \end{aligned}$$

9-3 Table A-20:

1018 HR: $S_{ut} = 58$ kpsi, $S_y = 32$ kpsi
1018 CR: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned} \tau_{\text{all}} &= \min(0.30S_{ut}, 0.40S_y) \\ &= \min[0.30(58), 0.40(32)] \\ &= \min(17.4, 12.8) = 12.8 \text{ kpsi} \end{aligned}$$

for both materials.

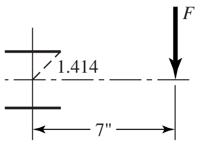
Eq. (9-3):

$$\begin{aligned} F &= 0.707hl\tau_{\text{all}} \\ F &= 0.707(5/16)(4)(12.8) = 11.3 \text{ kip} \quad \textit{Ans.} \end{aligned}$$

9-4 Eq. (9-3)

$$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(32)}{(5/16)(4)(2)} = 18.1 \text{ kpsi} \quad \textit{Ans.}$$

9-5 $b = d = 2$ in



(a) *Primary shear* Table 9-1

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(2)} = 1.13F \text{ kpsi}$$

Secondary shear Table 9-1

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{2[(3)(2^2) + 2^2]}{6} = 5.333 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/16)(5.333) = 1.18 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{7F(1)}{1.18} = 5.93F \text{ kpsi}$$

Maximum shear

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau'_y + \tau''_y)^2} = F\sqrt{5.93^2 + (1.13 + 5.93)^2} = 9.22F \text{ kpsi}$$

$$F = \frac{\tau_{\text{all}}}{9.22} = \frac{20}{9.22} = 2.17 \text{ kip} \quad \textit{Ans.} \tag{1}$$

(b) For E7010 from Table 9-6, $\tau_{\text{all}} = 21$ kpsi

Table A-20:

HR 1020 Bar: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

HR 1015 Support: $S_{ut} = 50$ kpsi, $S_y = 27.5$ kpsi

Table 9-5, E7010 Electrode: $S_{ut} = 70$ kpsi, $S_y = 57$ kpsi

The support controls the design.

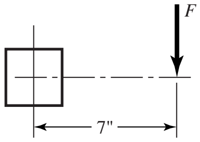
Table 9-4:

$$\tau_{\text{all}} = \min[0.30(50), 0.40(27.5)] = \min[15, 11] = 11 \text{ kpsi}$$

The allowable load from Eq. (1) is

$$F = \frac{\tau_{\text{all}}}{9.22} = \frac{11}{9.22} = 1.19 \text{ kip} \quad \textit{Ans.}$$

9-6 $b = d = 2$ in



Primary shear

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(2+2)} = 0.566F$$

Secondary shear

Table 9-1: $$J_u = \frac{(b+d)^3}{6} = \frac{(2+2)^3}{6} = 10.67 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/16)(10.67) = 2.36 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{(7F)(1)}{2.36} = 2.97F$$

Maximum shear

$$\tau_{\max} = \sqrt{\tau_x''^2 + (\tau_y' + \tau_y'')^2} = F\sqrt{2.97^2 + (0.556 + 2.97)^2} = 4.61F \text{ kpsi}$$

$$F = \frac{\tau_{\text{all}}}{4.61} \quad \textit{Ans.}$$

which is twice $\tau_{\max}/9.22$ of Prob. 9-5.

9-7 Weldment, subjected to alternating fatigue, has throat area of

$$A = 0.707(6)(60 + 50 + 60) = 721 \text{ mm}^2$$

Members' endurance limit: AISI 1010 steel

$$S_{ut} = 320 \text{ MPa}, \qquad S_e' = 0.5(320) = 160 \text{ MPa}$$

$$k_a = 272(320)^{-0.995} = 0.875$$

$$k_b = 1 \quad \text{(direct shear)}$$

$$k_c = 0.59 \quad \text{(shear)}$$

$$k_d = 1$$

$$k_f = \frac{1}{K_{fs}} = \frac{1}{2.7} = 0.370$$

$$S_{se} = 0.875(1)(0.59)(0.37)(160) = 30.56 \text{ MPa}$$

Electrode's endurance: 6010

$$S_{ut} = 62(6.89) = 427 \text{ MPa}$$

$$S_e' = 0.5(427) = 213.5 \text{ MPa}$$

$$k_a = 272(427)^{-0.995} = 0.657$$

$$k_b = 1 \quad \text{(direct shear)}$$

$$k_c = 0.59 \quad \text{(shear)}$$

$$k_d = 1$$

$$k_f = 1/K_{fs} = 1/2.7 = 0.370$$

$$S_{se} = 0.657(1)(0.59)(0.37)(213.5) = 30.62 \text{ MPa} \doteq 30.56$$

Thus, the members and the electrode are of equal strength. For a factor of safety of 1,

$$F_a = \tau_a A = 30.6(721)(10^{-3}) = 22.1 \text{ kN} \quad \textit{Ans.}$$

9-8 *Primary shear* $\tau' = 0$ (why?)

Secondary shear

Table 9-1:

$$J_u = 2\pi r^3 = 2\pi(4)^3 = 402 \text{ cm}^3$$

$$J = 0.707hJ_u = 0.707(0.5)(402) = 142 \text{ cm}^4$$

$$M = 200F \text{ N}\cdot\text{m} \quad (F \text{ in kN})$$

$$\tau'' = \frac{Mr}{2J} = \frac{(200F)(4)}{2(142)} = 2.82F \quad \text{(2 welds)}$$

$$F = \frac{\tau_{\text{all}}}{\tau''} = \frac{140}{2.82} = 49.2 \text{ kN} \quad \textit{Ans.}$$

9-9

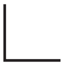
Rank

$$\text{fom}' = \frac{J_u}{lh} = \frac{a^3/12}{ah} = \frac{a^2}{12h} = 0.0833\left(\frac{a^2}{h}\right)$$

⑤

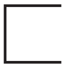
$$\text{fom}' = \frac{a(3a^2+a^2)}{6(2a)h} = \frac{a^2}{3h} = 0.3333\left(\frac{a^2}{h}\right)$$

①



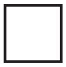
$$\text{fom}' = \frac{(2a)^4 - 6a^2a^2}{12(a+a)2ah} = \frac{5a^2}{24h} = 0.2083\left(\frac{a^2}{h}\right)$$

④



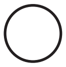
$$\text{fom}' = \frac{1}{3ah}\left[\frac{8a^3+6a^3+a^3}{12} - \frac{a^4}{2a+a}\right] = \frac{11}{36}\frac{a^2}{h} = 0.3056\left(\frac{a^2}{h}\right)$$

②



$$\text{fom}' = \frac{(2a)^3}{6h}\frac{1}{4a} = \frac{8a^3}{24ah} = \frac{a^2}{3h} = 0.3333\left(\frac{a^2}{h}\right)$$

①



$$\text{fom}' = \frac{2\pi(a/2)^3}{\pi ah} = \frac{a^3}{4ah} = \frac{a^2}{4h} = 0.25\left(\frac{a^2}{h}\right)$$

③

These rankings apply to fillet weld patterns in torsion that have a square area $a \times a$ in which to place weld metal. The object is to place as much metal as possible to the border. If your area is rectangular, your goal is the same but the rankings may change.

Students will be surprised that the circular weld bead does not rank first.

9-10

$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{a}\left(\frac{a^3}{12}\right)\left(\frac{1}{h}\right) = \frac{1}{12}\left(\frac{a^2}{h}\right) = 0.0833\left(\frac{a^2}{h}\right)$$

⑤

$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{2ah}\left(\frac{a^3}{6}\right) = 0.0833\left(\frac{a^2}{h}\right)$$

⑤

$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{2ah}\left(\frac{a^2}{2}\right) = \frac{1}{4}\left(\frac{a^2}{h}\right) = 0.25\left(\frac{a^2}{h}\right)$$

①

□

$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{[2(2a)]h}\left(\frac{a^2}{6}\right)(3a + a) = \frac{1}{6}\left(\frac{a^2}{h}\right) = 0.1667\left(\frac{a^2}{h}\right) \qquad ②$$

⊓

$$\bar{x} = \frac{b}{2} = \frac{a}{2}, \quad \bar{y} = \frac{d^2}{b + 2d} = \frac{a^2}{3a} = \frac{a}{3}$$

$$I_u = \frac{2d^3}{3} - 2d^2\left(\frac{a}{3}\right) + (b + 2d)\left(\frac{a^2}{9}\right) = \frac{2a^3}{3} - \frac{2a^3}{3} + 3a\left(\frac{a^2}{9}\right) = \frac{a^3}{3}$$

$$\text{fom}' = \frac{I_u}{lh} = \frac{a^3/3}{3ah} = \frac{1}{9}\left(\frac{a^2}{h}\right) = 0.1111\left(\frac{a^2}{h}\right) \qquad ④$$

○

$$I_u = \pi r^3 = \frac{\pi a^3}{8}$$

$$\text{fom}' = \frac{I_u}{lh} = \frac{\pi a^3/8}{\pi a h} = \frac{a^2}{8h} = 0.125\left(\frac{a^2}{h}\right) \qquad ③$$

The CEE-section pattern was not ranked because the deflection of the beam is out-of-plane.

If you have a square area in which to place a fillet weldment pattern under bending, your objective is to place as much material as possible away from the *x*-axis. If your area is rectangular, your goal is the same, but the rankings may change.

9-11 Materials:

Attachment (1018 HR) $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi
Member (A36) $S_y = 36$ kpsi, S_{ut} ranges from 58 to 80 kpsi, use 58.

The member and attachment are weak compared to the E60XX electrode.

Decision Specify E6010 electrode

Controlling property: $\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = \min(16.6, 12.8) = 12.8$ kpsi

For a static load the parallel and transverse fillets are the same. If *n* is the number of beads,

$$\tau = \frac{F}{n(0.707)hl} = \tau_{\text{all}}$$

$$nh = \frac{F}{0.707 l \tau_{\text{all}}} = \frac{25}{0.707(3)(12.8)} = 0.921$$

Make a table.

Number of beads *n*	Leg size *h*
1	0.921
2	0.460 → 1/2"
3	0.307 → 5/16"
4	0.230 → 1/4"

Decision: Specify 1/4" leg size

Decision: Weld all-around

Weldment Specifications:

Pattern: All-around square
Electrode: E6010
Type: Two parallel fillets *Ans.*
Two transverse fillets
Length of bead: 12 in
Leg: 1/4 in

For a figure of merit of, in terms of weldbead volume, is this design optimal?

9-12 *Decision:* Choose a parallel fillet weldment pattern. By so-doing, we've chosen an optimal pattern (see Prob. 9-9) and have thus reduced a synthesis problem to an analysis problem:

Table 9-1: $A = 1.414hd = 1.414(h)(3) = 4.24h \text{ in}^3$

Primary shear

$$\tau'_y = \frac{V}{A} = \frac{3000}{4.24h} = \frac{707}{h}$$

Secondary shear

Table 9-1: $J_u = \dfrac{d(3b^2 + d^2)}{6} = \dfrac{3[3(3^2) + 3^2]}{6} = 18 \text{ in}^3$

$$J = 0.707(h)(18) = 12.7h \text{ in}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{3000(7.5)(1.5)}{12.7h} = \frac{2657}{h} = \tau''_y$$

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau'_y + \tau''_y)^2} = \frac{1}{h}\sqrt{2657^2 + (707 + 2657)^2} = \frac{4287}{h}$$

Attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Member (A36): $S_y = 36$ kpsi

The attachment is weaker

Decision: Use E60XX electrode

$$\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = 12.8 \text{ kpsi}$$

$$\tau_{\max} = \tau_{\text{all}} = \frac{4287}{h} = 12\,800 \text{ psi}$$

$$h = \frac{4287}{12\,800} = 0.335 \text{ in}$$

Decision: Specify 3/8" leg size

Weldment Specifications:
Pattern: Parallel fillet welds
Electrode: E6010
Type: Fillet *Ans.*
Length of bead: 6 in
Leg size: 3/8 in

9-13 An optimal square space (3" × 3") weldment pattern is $||$ or $\overline{\underline{\quad}}$ or □. In Prob. 9-12, there was roundup of leg size to 3/8 in. Consider the member material to be structural A36 steel.

Decision: Use a parallel horizontal weld bead pattern for welding optimization and convenience.

Materials:

Attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Member (A36): $S_y = 36$ kpsi, S_{ut} 58–80 kpsi; use 58 kpsi

From Table 9-4 AISC welding code,

$$\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = \min(16.6, 12.8) = 12.8 \text{ kpsi}$$

Select a stronger electrode material from Table 9-3.

Decision: Specify E6010

Throat area and other properties:

$$A = 1.414hd = 1.414(h)(3) = 4.24h \text{ in}^2$$

$$\bar{x} = b/2 = 3/2 = 1.5 \text{ in}$$

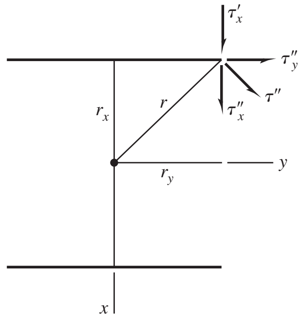
$$\bar{y} = d/2 = 3/2 = 1.5 \text{ in}$$

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{3[3(3^2) + 3^2]}{6} = 18 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(h)(18) = 12.73h \text{ in}^4$$

Primary shear:

$$\tau'_x = \frac{V}{A} = \frac{3000}{4.24h} = \frac{707.5}{h}$$



Secondary shear:

$$\tau'' = \frac{Mr}{J}$$

$$\tau''_x = \tau'' \cos 45^\circ = \frac{Mr}{J} \cos 45^\circ = \frac{Mr_x}{J}$$

$$\tau''_x = \frac{3000(6 + 1.5)(1.5)}{12.73h} = \frac{2651}{h}$$

$$\tau''_y = \tau''_x = \frac{2651}{h}$$

$$\tau_{\max} = \sqrt{(\tau_x'' + \tau_x')^2 + \tau_y''^2}$$

$$= \frac{1}{h}\sqrt{(2651 + 707.5)^2 + 2651^2}$$

$$= \frac{4279}{h} \text{ psi}$$

Relate stress and strength:

$$\tau_{\max} = \tau_{\text{all}}$$

$$\frac{4279}{h} = 12\,800$$

$$h = \frac{4279}{12\,800} = 0.334 \text{ in} \rightarrow 3/8 \text{ in}$$

Weldment Specifications:

Pattern: Horizontal parallel weld tracks
Electrode: E6010
Type of weld: Two parallel fillet welds
Length of bead: 6 in
Leg size: 3/8 in

Additional thoughts:

Since the round-up in leg size was substantial, why not investigate a backward C ⊐ weld pattern. One might then expect shorter horizontal weld beads which will have the advantage of allowing a shorter member (assuming the member has not yet been designed). This will show the inter-relationship between attachment design and supporting members.

9-14 *Materials:*

Member (A36): $S_y = 36$ kpsi, $S_{ut} = 58$ to 80 kpsi; use $S_{ut} = 58$ kpsi

Attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

$$\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = 12.8 \text{ kpsi}$$

Decision: Use E6010 electrode. From Table 9-3: $S_y = 50$ kpsi, $S_{ut} = 62$ kpsi,
$\tau_{\text{all}} = \min[0.3(62), 0.4(50)] = 20$ kpsi

Decision: Since A36 and 1018 HR are weld metals to an unknown extent, use
$\tau_{\text{all}} = 12.8$ kpsi

Decision: Use the most efficient weld pattern–square, weld-all-around. Choose 6" × 6" size.

Attachment length:

$$l_1 = 6 + a = 6 + 6.25 = 12.25 \text{ in}$$

Throat area and other properties:

$$A = 1.414h(b + d) = 1.414(h)(6 + 6) = 17.0h$$

$$\bar{x} = \frac{b}{2} = \frac{6}{2} = 3 \text{ in}, \quad \bar{y} = \frac{d}{2} = \frac{6}{2} = 3 \text{ in}$$

Primary shear

$$\tau_y' = \frac{V}{A} = \frac{F}{A} = \frac{20\,000}{17h} = \frac{1176}{h} \text{ psi}$$

Secondary shear

$$J_u = \frac{(b+d)^3}{6} = \frac{(6+6)^3}{6} = 288 \text{ in}^3$$

$$J = 0.707h(288) = 203.6h \text{ in}^4$$

$$\tau_x'' = \tau_y'' = \frac{Mr_y}{J} = \frac{20\,000(6.25+3)(3)}{203.6h} = \frac{2726}{h} \text{ psi}$$

$$\tau_{\max} = \sqrt{\tau_x''^2 + (\tau_y'' + \tau_y')^2} = \frac{1}{h}\sqrt{2726^2 + (2726+1176)^2} = \frac{4760}{h} \text{ psi}$$

Relate stress to strength

$$\tau_{\max} = \tau_{\text{all}}$$

$$\frac{4760}{h} = 12\,800$$

$$h = \frac{4760}{12\,800} = 0.372 \text{ in}$$

Decision:
Specify 3/8 in leg size

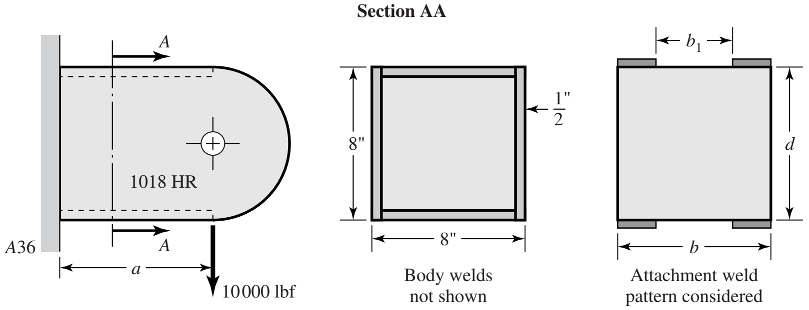
Specifications:
Pattern: All-around square weld bead track
Electrode: E6010
Type of weld: Fillet
Weld bead length: 24 in
Leg size: 3/8 in
Attachment length: 12.25 in

9-15 This is a good analysis task to test the students' understanding

(1) Solicit information related to a priori decisions.
(2) Solicit design variables b and d.
(3) Find h and round and output all parameters on a single screen. Allow return to Step 1 or Step 2.
(4) When the iteration is complete, the final display can be the bulk of your adequacy assessment.

Such a program can teach too.

9-16 The objective of this design task is to have the students teach themselves that the weld patterns of Table 9-3 can be added or subtracted to obtain the properties of a comtemplated weld pattern. The instructor can control the level of complication. I have left the presentation of the drawing to you. Here is one possibility. Study the problem's opportunities, then present this (or your sketch) with the problem assignment.



Use b_1 as the design variable. Express properties as a function of b_1. From Table 9-3, category 3:

$$A = 1.414h(b - b_1)$$

$$\bar{x} = b/2, \quad \bar{y} = d/2$$

$$I_u = \frac{bd^2}{2} - \frac{b_1 d^2}{2} = \frac{(b - b_1)d^2}{2}$$

$$I = 0.707hI_u$$

$$\tau' = \frac{V}{A} = \frac{F}{1.414h(b - b_1)}$$

$$\tau'' = \frac{Mc}{I} = \frac{Fa(d/2)}{0.707hI_u}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2}$$

Parametric study

Let $a = 10$ in, $b = 8$ in, $d = 8$ in, $b_1 = 2$ in, $\tau_{\text{all}} = 12.8$ kpsi, $l = 2(8 - 2) = 12$ in

$$A = 1.414h(8 - 2) = 8.48h \text{ in}^2$$

$$I_u = (8 - 2)(8^2/2) = 192 \text{ in}^3$$

$$I = 0.707(h)(192) = 135.7h \text{ in}^4$$

$$\tau' = \frac{10\,000}{8.48h} = \frac{1179}{h} \text{ psi}$$

$$\tau'' = \frac{10\,000(10)(8/2)}{135.7h} = \frac{2948}{h} \text{ psi}$$

$$\tau_{\max} = \frac{1}{h}\sqrt{1179^2 + 2948^2} = \frac{3175}{h} = 12\,800$$

from which $h = 0.248$ in. Do not round off the leg size – something to learn.

$$\text{fom}' = \frac{I_u}{hl} = \frac{192}{0.248(12)} = 64.5$$

$$A = 8.48(0.248) = 2.10 \text{ in}^2$$

$$I = 135.7(0.248) = 33.65 \text{ in}^4$$

$$\text{vol} = \frac{h^2}{2}l = \frac{0.248^2}{2}12 = 0.369\,\text{in}^3$$

$$\frac{I}{\text{vol}} = \frac{33.65}{0.369} = 91.2 = \text{eff}$$

$$\tau' = \frac{1179}{0.248} = 4754\,\text{psi}$$

$$\tau'' = \frac{2948}{0.248} = 11\,887\,\text{psi}$$

$$\tau_{\text{max}} = \frac{4127}{0.248} \doteq 12\,800\,\text{psi}$$

Now consider the case of uninterrupted welds,

$$b_1 = 0$$

$$A = 1.414(h)(8-0) = 11.31h$$

$$I_u = (8-0)(8^2/2) = 256\,\text{in}^3$$

$$I = 0.707(256)h = 181h\,\text{in}^4$$

$$\tau' = \frac{10\,000}{11.31h} = \frac{884}{h}$$

$$\tau'' = \frac{10\,000(10)(8/2)}{181h} = \frac{2210}{h}$$

$$\tau_{\text{max}} = \frac{1}{h}\sqrt{884^2 + 2210^2} = \frac{2380}{h} = \tau_{\text{all}}$$

$$h = \frac{\tau_{\text{max}}}{\tau_{\text{all}}} = \frac{2380}{12\,800} = 0.186\,\text{in}$$

Do not round off h.

$$A = 11.31(0.186) = 2.10\,\text{in}^2$$

$$I = 181(0.186) = 33.67$$

$$\tau' = \frac{884}{0.186} = 4753\,\text{psi}, \quad \text{vol} = \frac{0.186^2}{2}16 = 0.277\,\text{in}^3$$

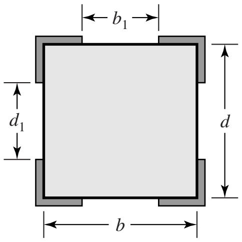
$$\tau'' = \frac{2210}{0.186} = 11\,882\,\text{psi}$$

$$\text{fom}' = \frac{I_u}{hl} = \frac{256}{0.186(16)} = 86.0$$

$$\text{eff} = \frac{I}{(h^2/2)l} = \frac{33.67}{(0.186^2/2)16} = 121.7$$

Conclusions: To meet allowable stress limitations, I and A do not change, nor do τ and σ. To meet the shortened bead length, h is increased proportionately. However, volume of bead laid down increases as h^2. The uninterrupted bead is superior. In this example, we did not round h and as a result we learned something. Our measures of merit are also sensitive to rounding. When the design decision is made, rounding to the next larger standard weld fillet size will decrease the merit.

Had the weld bead gone around the corners, the situation would change. Here is a followup task analyzing an alternative weld pattern.



9-17 From Table 9-2

For the box $A = 1.414h(b + d)$

Subtracting b_1 from b and d_1 from d

$$A = 1.414\,h(b - b_1 + d - d_1)$$

$$I_u = \frac{d^2}{6}(3b + d) - \frac{d_1^3}{6} - \frac{b_1 d^2}{2}$$

$$= \frac{1}{2}(b - b_1)d^2 + \frac{1}{6}\left(d^3 - d_1^3\right)$$

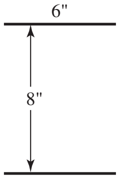
length of bead $l = 2(b - b_1 + d - d_1)$

$$\text{fom} = I_u/hl$$

9-18 Computer programs will vary.

9-19 $\tau_{\text{all}} = 12\,800$ psi. Use Fig. 9-17(a) for general geometry, but employ = beads and then || beads.

Horizontal parallel weld bead pattern



$$b = 6\text{ in}$$
$$d = 8\text{ in}$$

From Table 9-2, category 3

$$A = 1.414\,hb = 1.414(h)(6) = 8.48\,h\text{ in}^2$$

$$\bar{x} = b/2 = 6/2 = 3\text{ in}, \qquad \bar{y} = d/2 = 8/2 = 4\text{ in}$$

$$I_u = \frac{bd^2}{2} = \frac{6(8)^2}{2} = 192\text{ in}^3$$

$$I = 0.707hI_u = 0.707(h)(192) = 135.7h\text{ in}^4$$

$$\tau' = \frac{10\,000}{8.48h} = \frac{1179}{h}\text{ psi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10\,000(10)(8/2)}{135.7h} = \frac{2948}{h}\text{ psi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}(1179^2 + 2948^2)^{1/2} = \frac{3175}{h}\text{ psi}$$

Equate the maximum and allowable shear stresses.

$$\tau_{\max} = \tau_{\text{all}} = \frac{3175}{h} = 12\,800$$

from which $h = 0.248$ in. It follows that

$$I = 135.7(0.248) = 33.65\text{ in}^4$$

The volume of the weld metal is

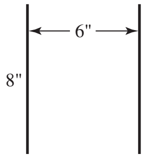
$$\text{vol} = \frac{h^2 l}{2} = \frac{0.248^2(6+6)}{2} = 0.369\text{ in}^3$$

The effectiveness, $(\text{eff})_{\text{H}}$, is

$$(\text{eff})_{\text{H}} = \frac{I}{\text{vol}} = \frac{33.65}{0.369} = 91.2\text{ in}$$

$$(\text{fom}')_{\text{H}} = \frac{I_u}{hl} = \frac{192}{0.248(6+6)} = 64.5\text{ in}$$

Vertical parallel weld beads



$b = 6$ in
$d = 8$ in

From Table 9-2, category 2

$$A = 1.414hd = 1.414(h)(8) = 11.31h\text{ in}^2$$

$$\bar{x} = b/2 = 6/2 = 3\text{ in}, \qquad \bar{y} = d/2 = 8/2 = 4\text{ in}$$

$$I_u = \frac{d^3}{6} = \frac{8^3}{6} = 85.33\text{ in}^3$$

$$I = 0.707hI_u = 0.707(h)(85.33) = 60.3h$$

$$\tau' = \frac{10\,000}{11.31h} = \frac{884}{h}\text{ psi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10\,000(10)(8/2)}{60.3\,h} = \frac{6633}{h}\text{ psi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}(884^2 + 6633^2)^{1/2}$$

$$= \frac{6692}{h}\text{ psi}$$

Equating τ_{max} to τ_{all} gives $h = 0.523$ in. It follows that

$$I = 60.3(0.523) = 31.5\ \text{in}^4$$

$$\text{vol} = \frac{h^2 l}{2} = \frac{0.523^2}{2}(8+8) = 2.19\ \text{in}^3$$

$$(\text{eff})_{\text{V}} = \frac{I}{\text{vol}} = \frac{31.6}{2.19} = 14.4\ \text{in}$$

$$(\text{fom}')_{\text{V}} = \frac{I_u}{hl} = \frac{85.33}{0.523(8+8)} = 10.2\ \text{in}$$

The ratio of $(\text{eff})_{\text{V}}/(\text{eff})_{\text{H}}$ is $14.4/91.2 = 0.158$. The ratio $(\text{fom}')_{\text{V}}/(\text{fom}')_{\text{H}}$ is $10.2/64.5 = 0.158$. This is not surprising since

$$\text{eff} = \frac{I}{\text{vol}} = \frac{I}{(h^2/2)l} = \frac{0.707\,hI_u}{(h^2/2)l} = 1.414\,\frac{I_u}{hl} = 1.414\ \text{fom}'$$

The ratios $(\text{eff})_{\text{V}}/(\text{eff})_{\text{H}}$ and $(\text{fom}')_{\text{V}}/(\text{fom}')_{\text{H}}$ give the same information.

9-20 Because the loading is pure torsion, there is no primary shear. From Table 9-1, category 6:

$$J_u = 2\pi r^3 = 2\pi(1)^3 = 6.28\ \text{in}^3$$

$$J = 0.707\,hJ_u = 0.707(0.25)(6.28)$$

$$= 1.11\ \text{in}^4$$

$$\tau = \frac{Tr}{J} = \frac{20(1)}{1.11} = 18.0\ \text{kpsi} \quad \textit{Ans.}$$

9-21

$$h = 0.375\ \text{in}, \quad d = 8\ \text{in}, \quad b = 1\ \text{in}$$

From Table 9-2, category 2:

$$A = 1.414(0.375)(8) = 4.24\ \text{in}^2$$

$$I_u = \frac{d^3}{6} = \frac{8^3}{6} = 85.3\ \text{in}^3$$

$$I = 0.707hI_u = 0.707(0.375)(85.3) = 22.6\ \text{in}^4$$

$$\tau' = \frac{F}{A} = \frac{5}{4.24} = 1.18\ \text{kpsi}$$

$$M = 5(6) = 30\ \text{kip}\cdot\text{in}$$

$$c = (1+8+1-2)/2 = 4\ \text{in}$$

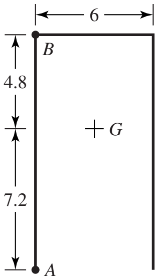
$$\tau'' = \frac{Mc}{I} = \frac{30(4)}{22.6} = 5.31\ \text{kpsi}$$

$$\tau_{\text{max}} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{1.18^2 + 5.31^2}$$

$$= 5.44\ \text{kpsi} \quad \textit{Ans.}$$

9-22 $h = 0.6\,\text{cm}, \quad b = 6\,\text{cm}, \quad d = 12\,\text{cm}.$

Table 9-3, category 5:



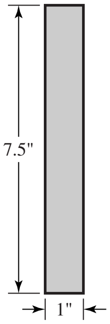
$$A = 0.707h(b + 2d)$$
$$= 0.707(0.6)[6 + 2(12)] = 12.7\,\text{cm}^2$$
$$\bar{y} = \frac{d^2}{b + 2d} = \frac{12^2}{6 + 2(12)} = 4.8\,\text{cm}$$
$$I_u = \frac{2d^3}{3} - 2d^2\bar{y} + (b + 2d)\bar{y}^2$$
$$= \frac{2(12)^3}{3} - 2(12^2)(4.8) + [6 + 2(12)]4.8^2$$
$$= 461\,\text{cm}^3$$
$$I = 0.707hI_u = 0.707(0.6)(461) = 196\,\text{cm}^4$$
$$\tau' = \frac{F}{A} = \frac{7.5(10^3)}{12.7(10^2)} = 5.91\text{ MPa}$$
$$M = 7.5(120) = 900\,\text{N}\cdot\text{m}$$
$$c_A = 7.2\,\text{cm}, \qquad c_B = 4.8\,\text{cm}$$

The critical location is at *A*.

$$\tau''_A = \frac{Mc_A}{I} = \frac{900(7.2)}{196} = 33.1\,\text{MPa}$$
$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = (5.91^2 + 33.1^2)^{1/2} = 33.6\,\text{MPa}$$
$$n = \frac{\tau_{\text{all}}}{\tau_{\max}} = \frac{120}{33.6} = 3.57 \quad \textit{Ans.}$$

9-23 The largest possible weld size is 1/16 in. This is a small weld and thus difficult to accomplish. The bracket's load-carrying capability is not known. There are geometry problems associated with sheet metal folding, load-placement and location of the center of twist. This is not available to us. We will identify the strongest possible weldment.

Use a rectangular, weld-all-around pattern – Table 9-2, category 6:



$$A = 1.414\,h(b + d)$$
$$= 1.414(1/16)(1 + 7.5)$$
$$= 0.751\,\text{in}^2$$
$$\bar{x} = b/2 = 0.5\,\text{in}$$
$$\bar{y} = \frac{d}{2} = \frac{7.5}{2} = 3.75\,\text{in}$$

$$I_u = \frac{d^2}{6}(3b + d) = \frac{7.5^2}{6}[3(1) + 7.5] = 98.4\,\text{in}^3$$

$$I = 0.707hI_u = 0.707(1/16)(98.4) = 4.35\,\text{in}^4$$

$$M = (3.75 + 0.5)W = 4.25W$$

$$\tau' = \frac{V}{A} = \frac{W}{0.751} = 1.332W$$

$$\tau'' = \frac{Mc}{I} = \frac{4.25W(7.5/2)}{4.35} = 3.664W$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = W\sqrt{1.332^2 + 3.664^2} = 3.90W$$

Material properties: The allowable stress given is low. Let's demonstrate that.

For the A36 structural steel member, $S_y = 36\,\text{kpsi}$ and $S_{ut} = 58\,\text{kpsi}$. For the 1020 CD attachment, use HR properties of $S_y = 30\,\text{kpsi}$ and $S_{ut} = 55$. The E6010 electrode has strengths of $S_y = 50$ and $S_{ut} = 62\,\text{kpsi}$.

Allowable stresses:

A36: $\tau_{\text{all}} = \min[0.3(58), 0.4(36)]$

$= \min(17.4, 14.4) = 14.4\,\text{kpsi}$

1020: $\tau_{\text{all}} = \min[0.3(55), 0.4(30)]$

$\tau_{\text{all}} = \min(16.5, 12) = 12\,\text{kpsi}$

E6010: $\tau_{\text{all}} = \min[0.3(62), 0.4(50)]$

$= \min(18.6, 20) = 18.6\,\text{kpsi}$

Since Table 9-6 gives 18.0 kpsi as the allowable shear stress, use this lower value.

Therefore, the allowable shear stress is

$$\tau_{\text{all}} = \min(14.4, 12, 18.0) = 12\,\text{kpsi}$$

However, the allowable stress in the problem statement is 0.9 kpsi which is low from the weldment perspective. The load associated with this strength is

$$\tau_{\max} = \tau_{\text{all}} = 3.90W = 900$$

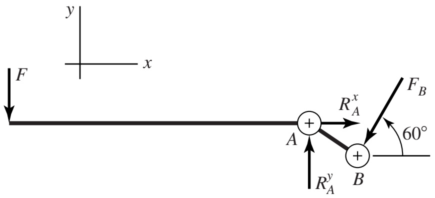
$$W = \frac{900}{3.90} = 231\,\text{lbf}$$

If the welding can be accomplished (1/16 leg size is a small weld), the weld strength is 12 000 psi and the load $W = 3047\,\text{lbf}$. Can the bracket carry such a load?

There are geometry problems associated with sheet metal folding. Load placement is important and the center of twist has not been identified. Also, the load-carrying capability of the top bend is unknown.

These uncertainties may require the use of a different weld pattern. Our solution provides the best weldment and thus insight for comparing a welded joint to one which employs screw fasteners.

9-24



$$F = 100\,\text{lbf}, \qquad \tau_{\text{all}} = 3\,\text{kpsi}$$

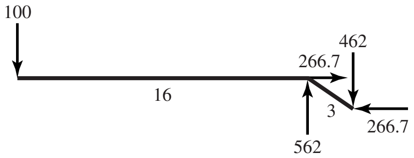
$$F_B = 100(16/3) = 533.3\,\text{lbf}$$

$$F_B^x = -533.3\cos 60^\circ = -266.7\,\text{lbf}$$

$$F_B^y = -533.3\cos 30^\circ = -462\,\text{lbf}$$

It follows that $R_A^y = 562\,\text{lbf}$ and $R_A^x = 266.7\,\text{lbf}$, $R_A = 622\,\text{lbf}$

$$M = 100(16) = 1600\,\text{lbf}\cdot\text{in}$$



The OD of the tubes is 1 in. From Table 9-1, category 6:

$$A = 1.414(\pi h r)(2)$$

$$= 2(1.414)(\pi h)(1/2) = 4.44h\,\text{in}^2$$

$$J_u = 2\pi r^3 = 2\pi(1/2)^3 = 0.785\,\text{in}^3$$

$$J = 2(0.707)hJ_u = 1.414(0.785)h = 1.11h\,\text{in}^4$$

$$\tau' = \frac{V}{A} = \frac{622}{4.44h} = \frac{140}{h}$$

$$\tau'' = \frac{Tc}{J} = \frac{Mc}{J} = \frac{1600(0.5)}{1.11h} = \frac{720.7}{h}$$

The shear stresses, τ' and τ'', are additive algebraically

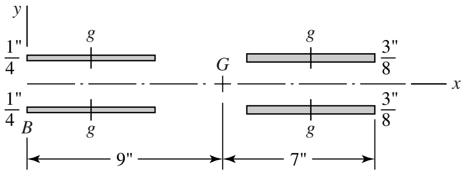
$$\tau_{\max} = \frac{1}{h}(140 + 720.7) = \frac{861}{h}\,\text{psi}$$

$$\tau_{\max} = \tau_{\text{all}} = \frac{861}{h} = 3000$$

$$h = \frac{861}{3000} = 0.287 \rightarrow 5/16"$$

Decision: Use 5/16 in fillet welds *Ans.*

9-25



For the pattern in bending shown, find the centroid G of the weld group.

$$\bar{x} = \frac{6(0.707)(1/4)(3) + 6(0.707)(3/8)(13)}{6(0.707)(1/4) + 6(0.707)(3/8)}$$

$$= 9\text{ in}$$

$$I_{1/4} = 2\left(I_G + A_{\bar{x}}^{2}\right)$$

$$= 2\left[\frac{0.707(1/4)(6^3)}{12} + 0.707(1/4)(6)(6^2)\right]$$

$$= 82.7\text{ in}^4$$

$$I_{3/8} = 2\left[\frac{0.707(3/8)(6^3)}{12} + 0.707(3/8)(6)(4^2)\right]$$

$$= 60.4\text{ in}^4$$

$$I = I_{1/4} + I_{3/8} = 82.7 + 60.4 = 143.1\text{ in}^4$$

The critical location is at B. From Eq. (9-3),

$$\tau' = \frac{F}{2[6(0.707)(3/8 + 1/4)]} = 0.189F$$

$$\tau'' = \frac{Mc}{I} = \frac{(8F)(9)}{143.1} = 0.503F$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = F\sqrt{0.189^2 + 0.503^2} = 0.537F$$

Materials:

A36 Member: $S_y = 36$ kpsi

1015 HR Attachment: $S_y = 27.5$ kpsi

E6010 Electrode: $S_y = 50$ kpsi

$$\tau_{\text{all}} = 0.577\min(36, 27.5, 50) = 15.9\text{ kpsi}$$

$$F = \frac{\tau_{\text{all}}/n}{0.537} = \frac{15.9/2}{0.537} = 14.8\text{ kip} \quad \textit{Ans.}$$

9-26 Figure P9-26*b* is a free-body diagram of the bracket. Forces and moments that act on the welds are equal, but of opposite sense.

(a) $M = 1200(0.366) = 439\text{ lbf}\cdot\text{in}$ *Ans.*

(b) $F_y = 1200\sin 30^\circ = 600\text{ lbf}$ *Ans.*

(c) $F_x = 1200\cos 30^\circ = 1039\text{ lbf}$ *Ans.*

(d) From Table 9-2, category 6:

$$A = 1.414(0.25)(0.25 + 2.5) = 0.972 \text{ in}^2$$

$$I_u = \frac{d^2}{6}(3b + d) = \frac{2.5^2}{6}[3(0.25) + 2.5] = 3.39 \text{ in}^3$$

The second area moment about an axis through G and parallel to z is

$$I = 0.707hI_u = 0.707(0.25)(3.39) = 0.599 \text{ in}^4 \quad \textit{Ans.}$$

(e) Refer to Fig. P.9-26*b*. The shear stress due to F_y is

$$\tau_1 = \frac{F_y}{A} = \frac{600}{0.972} = 617 \text{ psi}$$

The shear stress along the throat due to F_x is

$$\tau_2 = \frac{F_x}{A} = \frac{1039}{0.972} = 1069 \text{ psi}$$

The resultant of τ_1 and τ_2 is in the throat plane

$$\tau' = \left(\tau_1^2 + \tau_2^2\right)^{1/2} = (617^2 + 1069^2)^{1/2} = 1234 \text{ psi}$$

The bending of the throat gives

$$\tau'' = \frac{Mc}{I} = \frac{439(1.25)}{0.599} = 916 \text{ psi}$$

The maximum shear stress is

$$\tau_{\max} = (\tau'^2 + \tau''^2)^{1/2} = (1234^2 + 916^2)^{1/2} = 1537 \text{ psi} \quad \textit{Ans.}$$

(f) *Materials:*

1018 HR Member: $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi (Table A-20)

E6010 Electrode: $S_y = 50$ kpsi (Table 9-3)

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{0.577S_y}{\tau_{\max}} = \frac{0.577(32)}{1.537} = 12.0 \quad \textit{Ans.}$$

(g) Bending in the attachment near the base. The cross-sectional area is approximately equal to *bh*.

$$A_1 \doteq bh = 0.25(2.5) = 0.625 \text{ in}^2$$

$$\tau_{xy} = \frac{F_x}{A_1} = \frac{1039}{0.625} = 1662 \text{ psi}$$

$$\frac{I}{c} = \frac{bd^2}{6} = \frac{0.25(2.5)^2}{6} = 0.260 \text{ in}^3$$

At location A

$$\sigma_y = \frac{F_y}{A_1} + \frac{M}{I/c}$$

$$\sigma_y = \frac{600}{0.625} + \frac{439}{0.260} = 2648 \text{ psi}$$

The von Mises stress σ' is

$$\sigma' = \left(\sigma_y^2 + 3\tau_{xy}^2\right)^{1/2} = [2648^2 + 3(1662)^2]^{1/2} = 3912 \text{ psi}$$

Thus, the factor of safety is,

$$n = \frac{S_y}{\sigma'} = \frac{32}{3.912} = 8.18 \quad \textit{Ans.}$$

The clip on the mooring line bears against the side of the 1/2-in hole. If the clip fills the hole

$$\sigma = \frac{F}{td} = \frac{-1200}{0.25(0.50)} = -9600 \text{ psi}$$

$$n = -\frac{S_y}{\sigma'} = -\frac{32(10^3)}{-9600} = 3.33 \quad \textit{Ans.}$$

Further investigation of this situation requires more detail than is included in the task statement.

(h) In shear fatigue, the weakest constituent of the weld melt is 1018 with $S_{ut} = 58$ kpsi

$$S_e' = 0.5S_{ut} = 0.5(58) = 29 \text{ kpsi}$$

Table 7-4:

$$k_a = 14.4(58)^{-0.718} = 0.780$$

For the size factor estimate, we first employ Eq. (7-24) for the equivalent diameter.

$$d_e = 0.808\sqrt{0.707hb} = 0.808\sqrt{0.707(2.5)(0.25)} = 0.537 \text{ in}$$

Eq. (7-19) is used next to find k_b

$$k_b = \left(\frac{d_e}{0.30}\right)^{-0.107} = \left(\frac{0.537}{0.30}\right)^{-0.107} = 0.940$$

The load factor for shear k_c, is

$$k_c = 0.59$$

The endurance strength in shear is

$$S_{se} = 0.780(0.940)(0.59)(29) = 12.5 \text{ kpsi}$$

From Table 9-5, the shear stress-concentration factor is $K_{fs} = 2.7$. The loading is repeatedly-applied.

$$\tau_a = \tau_m = K_{fs}\frac{\tau_{\max}}{2} = 2.7\frac{1.537}{2} = 2.07 \text{ kpsi}$$

Table 7-10: Gerber factor of safety n_f, adjusted for shear, with $S_{su} = 0.67S_{ut}$

$$n_f = \frac{1}{2}\left[\frac{0.67(58)}{2.07}\right]^2\left(\frac{2.07}{12.5}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(2.07)(12.5)}{0.67(58)(2.07)}\right]^2}\right\} = 5.52 \quad \textit{Ans.}$$

Attachment metal should be checked for bending fatigue.

9-27 Use $b = d = 4$ in. Since $h = 5/8$ in, the primary shear is

$$\tau' = \frac{F}{1.414(5/8)(4)} = 0.283F$$

The secondary shear calculations, for a moment arm of 14 in give

$$J_u = \frac{4[3(4^2) + 4^2]}{6} = 42.67 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/8)42.67 = 18.9 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{14F(2)}{18.9} = 1.48F$$

Thus, the maximum shear and allowable load are:

$$\tau_{\max} = F\sqrt{1.48^2 + (0.283 + 1.48)^2} = 2.30F$$

$$F = \frac{\tau_{\text{all}}}{2.30} = \frac{20}{2.30} = 8.70 \text{ kip} \quad Ans.$$

From Prob. 9-5*b*, $\tau_{\text{all}} = 11$ kpsi

$$F_{\text{all}} = \frac{\tau_{\text{all}}}{2.30} = \frac{11}{2.30} = 4.78 \text{ kip}$$

The allowable load has thus increased by a factor of 1.8 *Ans.*

9-28 Purchase the hook having the design shown in Fig. P9-28*b*. Referring to text Fig. 9-32*a*, this design reduces peel stresses.

9-29 **(a)**

$$\bar{\tau} = \frac{1}{l}\int_{-l/2}^{l/2} \frac{P\omega\cosh(\omega x)}{4b\sinh(\omega l/2)}\,dx$$

$$= A_1\int_{-l/2}^{l/2} \cosh(\omega x)\,dx$$

$$= \frac{A_1}{\omega}\sinh(\omega x)\Big|_{-l/2}^{l/2}$$

$$= \frac{A_1}{\omega}[\sinh(\omega l/2) - \sinh(-\omega l/2)]$$

$$= \frac{A_1}{\omega}[\sinh(\omega l/2) - (-\sinh(\omega l/2))]$$

$$= \frac{2A_1\sinh(\omega l/2)}{\omega}$$

$$= \frac{P\omega}{4bl\sinh(\omega l/2)}[2\sinh(\omega l/2)]$$

$$\bar{\tau} = \frac{P}{2bl} \quad Ans.$$

(b)

$$\tau(l/2) = \frac{P\omega\cosh(\omega l/2)}{4b\sinh(\omega l/2)} = \frac{P\omega}{4b\tanh(\omega l/2)} \quad \textit{Ans.}$$

(c)

$$K = \frac{\tau(l/2)}{\bar{\tau}} = \frac{P\omega}{4b\sinh(\omega l/2)}\left(\frac{2bl}{P}\right)$$

$$K = \frac{\omega l/2}{\tanh(\omega l/2)} \quad \textit{Ans.}$$

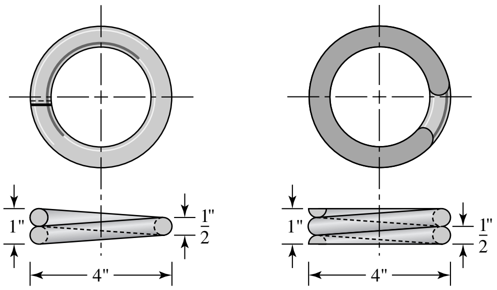
For computer programming, it can be useful to express the hyperbolic tangent in terms of exponentials:

$$K = \frac{\omega l}{2}\,\frac{\exp(\omega l/2) - \exp(-\omega l/2)}{\exp(\omega l/2) + \exp(-\omega l/2)} \quad \textit{Ans.}$$

9-30 This is a computer programming exercise. All programs will vary.

Chapter 10

10-1



10-2 $A = Sd^m$

$$\dim(A_{\text{uscu}}) = \dim(S)\,\dim(d^m) = \text{kpsi}\cdot\text{in}^m$$

$$\dim(A_{\text{SI}}) = \dim(S_1)\,\dim\left(d_1^m\right) = \text{MPa}\cdot\text{mm}^m$$

$$A_{\text{SI}} = \frac{\text{MPa}}{\text{kpsi}}\cdot\frac{\text{mm}^m}{\text{in}^m}A_{\text{uscu}} = 6.894\,757(25.40)^m A_{\text{uscu}} \doteq 6.895(25.4)^m A_{\text{uscu}} \quad \textit{Ans.}$$

For music wire, from Table 10-4:

$$A_{\text{uscu}} = 201, \quad m = 0.145; \quad \text{what is } A_{\text{SI}}?$$

$$A_{\text{SI}} = 6.89(25.4)^{0.145}(201) = 2214\ \text{MPa}\cdot\text{mm}^m \quad \textit{Ans.}$$

10-3 Given: Music wire, $d = 0.105$ in, OD $= 1.225$ in, plain ground ends, $N_t = 12$ coils.

Table 10-1: $N_a = N_t - 1 = 12 - 1 = 11$

$$L_s = dN_t = 0.105(12) = 1.26\ \text{in}$$

Table 10-4: $A = 201,\quad m = 0.145$

(a) Eq. (10-14): $$S_{ut} = \frac{201}{(0.105)^{0.145}} = 278.7\ \text{kpsi}$$

Table 10-6: $$S_{sy} = 0.45(278.7) = 125.4\ \text{kpsi}$$

$$D = 1.225 - 0.105 = 1.120\ \text{in}$$

$$C = \frac{D}{d} = \frac{1.120}{0.105} = 10.67$$

Eq. (10-6): $$K_B = \frac{4(10.67)+2}{4(10.67)-3} = 1.126$$

Eq. (10-3): $$F|_{S_{sy}} = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi(0.105)^3(125.4)(10^3)}{8(1.126)(1.120)} = 45.2\ \text{lbf}$$

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.105)^4(11.75)(10^6)}{8(1.120)^3(11)} = 11.55\ \text{lbf/in}$$

$$L_0 = \frac{F|_{S_{sy}}}{k} + L_s = \frac{45.2}{11.55} + 1.26 = 5.17\ \text{in} \quad \textit{Ans.}$$

(b) $F|_{S_{sy}} = 45.2$ lbf *Ans.*

(c) $k = 11.55$ lbf/in *Ans.*

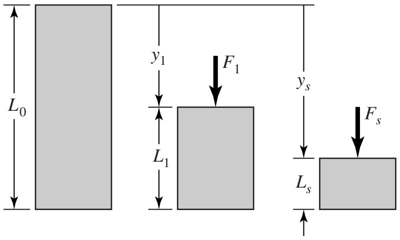
(d) $$(L_0)_{\text{cr}} = \frac{2.63D}{\alpha} = \frac{2.63(1.120)}{0.5} = 5.89 \text{ in}$$

Many designers provide $(L_0)_{\text{cr}}/L_0 \geq 5$ or more; therefore, plain ground ends are not often used in machinery due to buckling uncertainty.

10-4 Referring to Prob. 10-3 solution, $C = 10.67$, $N_a = 11$, $S_{sy} = 125.4$ kpsi, $(L_0)_{\text{cr}} = 5.89$ in and $F = 45.2$ lbf (at yield).

Eq. (10-18): $4 \leq C \leq 12$ $C = 10.67$ *O.K.*

Eq. (10-19): $3 \leq N_a \leq 15$ $N_a = 11$ *O.K.*



$$L_0 = 5.17 \text{ in}, \quad L_s = 1.26 \text{ in}$$

$$y_1 = \frac{F_1}{k} = \frac{30}{11.55} = 2.60 \text{ in}$$

$$L_1 = L_0 - y_1 = 5.17 - 2.60 = 2.57 \text{ in}$$

$$\xi = \frac{y_s}{y_1} - 1 = \frac{5.17 - 1.26}{2.60} - 1 = 0.50$$

Eq. (10-20): $\xi \geq 0.15$, $\xi = 0.50$ *O.K.*

From Eq. (10-3) for static service

$$\tau_1 = K_B\left(\frac{8F_1D}{\pi d^3}\right) = 1.126\left[\frac{8(30)(1.120)}{\pi(0.105)^3}\right] = 83\,224 \text{ psi}$$

$$n_s = \frac{S_{sy}}{\tau_1} = \frac{125.4(10^3)}{83\,224} = 1.51$$

Eq. (10-21): $n_s \geq 1.2$, $n_s = 1.51$ *O.K.*

$$\tau_s = \tau_1\left(\frac{45.2}{30}\right) = 83\,224\left(\frac{45.2}{30}\right) = 125\,391 \text{ psi}$$

$$S_{sy}/\tau_s = 125.4(10^3)/125\,391 \doteq 1$$

$S_{sy}/\tau_s \geq (n_s)_d$: Not solid-safe. *Not O.K.*

$L_0 \leq (L_0)_{\text{cr}}$: $5.17 \leq 5.89$ Margin could be higher, *Not O.K.*

Design is unsatisfactory. Operate over a rod? *Ans.*

10-5 Static service spring with: HD steel wire, $d = 2$ mm, OD $= 22$ mm, $N_t = 8.5$ turns plain and ground ends.

Preliminaries

Table 10-5: $A = 1783 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.190$

Eq. (10-14): $S_{ut} = \dfrac{1783}{(2)^{0.190}} = 1563 \text{ MPa}$

Table 10-6: $S_{sy} = 0.45(1563) = 703.4 \text{ MPa}$

Then,

$$D = \text{OD} - d = 22 - 2 = 20 \text{ mm}$$

$$C = 20/2 = 10$$

$$K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$$

$$N_a = 8.5 - 1 = 7.5 \text{ turns}$$

$$L_s = 2(8.5) = 17 \text{ mm}$$

Eq. (10-21): Use $n_s = 1.2$ for solid-safe property.

$$F_s = \frac{\pi d^3 S_{sy}/n_s}{8K_B D} = \frac{\pi(2)^3(703.4/1.2)}{8(1.135)(20)}\left[\frac{(10^{-3})^3(10^6)}{10^{-3}}\right] = 81.12 \text{ N}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(2)^4(79.3)}{8(20)^3(7.5)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.002\,643(10^6) = 2643 \text{ N/m}$$

$$y_s = \frac{F_s}{k} = \frac{81.12}{2643(10^{-3})} = 30.69 \text{ mm}$$

(a) $L_0 = y + L_s = 30.69 + 17 = 47.7$ mm *Ans.*

(b) Table 10-1: $p = \dfrac{L_0}{N_t} = \dfrac{47.7}{8.5} = 5.61$ mm *Ans.*

(c) $F_s = 81.12$ N (from above) *Ans.*

(d) $k = 2643$ N/m (from above) *Ans.*

(e) Table 10-2 and Eq. (10-13):

$$(L_0)_{\text{cr}} = \frac{2.63D}{\alpha} = \frac{2.63(20)}{0.5} = 105.2 \text{ mm}$$

$$(L_0)_{\text{cr}}/L_0 = 105.2/47.7 = 2.21$$

This is less than 5. Operate over a rod?

Plain and ground ends have a poor eccentric footprint. *Ans.*

10-6 Referring to Prob. 10-5 solution: $C = 10$, $N_a = 7.5$, $k = 2643$ N/m, $d = 2$ mm, $D = 20$ mm, $F_s = 81.12$ N and $N_t = 8.5$ turns.

Eq. (10-18): $4 \le C \le 12, \quad C = 10 \quad$ *O.K.*

Eq. (10-19): $3 \le N_a \le 15, \quad N_a = 7.5 \quad O.K.$

$$y_1 = \frac{F_1}{k} = \frac{75}{2643(10^{-3})} = 28.4 \text{ mm}$$

$$(y)_{\text{for yield}} = \frac{81.12(1.2)}{2643(10^{-3})} = 36.8 \text{ mm}$$

$$y_s = \frac{81.12}{2643(10^{-3})} = 30.69 \text{ mm}$$

$$\xi = \frac{(y)_{\text{for yield}}}{y_1} - 1 = \frac{36.8}{28.4} - 1 = 0.296$$

Eq. (10-20): $\xi \ge 0.15, \quad \xi = 0.296 \quad O.K.$

Table 10-6: $S_{sy} = 0.45 S_{ut} \quad O.K.$

As-wound

$$\tau_s = K_B\left(\frac{8F_sD}{\pi d^3}\right) = 1.135\left[\frac{8(81.12)(20)}{\pi(2)^3}\right]\left[\frac{10^{-3}}{(10^{-3})^3(10^6)}\right] = 586 \text{ MPa}$$

Eq. (10-21): $\dfrac{S_{sy}}{\tau_s} = \dfrac{703.4}{586} = 1.2 \quad O.K.$ (Basis for Prob. 10-5 solution)

Table 10-1: $L_s = N_t d = 8.5(2) = 17$ mm

$$L_0 = \frac{F_s}{k} + L_s = \frac{81.12}{2.643} + 17 = 47.7 \text{ mm}$$

$$\frac{2.63D}{\alpha} = \frac{2.63(20)}{0.5} = 105.2 \text{ mm}$$

$$\frac{(L_0)_{\text{cr}}}{L_0} = \frac{105.2}{47.7} = 2.21$$

which is less than 5. Operate over a rod? *Not O.K.*

Plain and ground ends have a poor eccentric footprint. *Ans.*

10-7 Given: A228 (music wire), SQ&GRD ends, $d = 0.006$ in, OD $= 0.036$ in, $L_0 = 0.63$ in, $N_t = 40$ turns.

Table 10-4: $A = 201 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.145$

$$D = \text{OD} - d = 0.036 - 0.006 = 0.030 \text{ in}$$

$$C = D/d = 0.030/0.006 = 5$$

$$K_B = \frac{4(5)+2}{4(5)-3} = 1.294$$

Table 10-1: $N_a = N_t - 2 = 40 - 2 = 38$ turns

$$S_{ut} = \frac{201}{(0.006)^{0.145}} = 422.1 \text{ kpsi}$$

$$S_{sy} = 0.45(422.1) = 189.9 \text{ kpsi}$$

$$k = \frac{Gd^4}{8D^3N_a} = \frac{12(10^6)(0.006)^4}{8(0.030)^3(38)} = 1.895 \text{ lbf/in}$$

Table 10-1: $L_s = N_t d = 40(0.006) = 0.240$ in

Now $F_s = k y_s$ where $y_s = L_0 - L_s = 0.390$ in. Thus,

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.294\left[\frac{8(1.895)(0.39)(0.030)}{\pi(0.006)^3}\right](10^{-3}) = 338.2 \text{ kpsi} \quad (1)$$

$\tau_s > S_{sy}$, that is, $338.2 > 189.9$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y_s' = \frac{(\tau_s/n_s)(\pi d^3)}{8K_B kD} = \frac{(189\,900/1.2)(\pi)(0.006)^3}{8(1.294)(1.895)(0.030)} = 0.182 \text{ in}$$

Using a design factor of 1.2,

$$L_0' = L_s + y_s' = 0.240 + 0.182 = 0.422 \text{ in}$$

The spring should be wound to a free length of 0.422 in. *Ans.*

10-8 Given: B159 (phosphor bronze), SQ&GRD ends, $d = 0.012$ in, OD $= 0.120$ in, $L_0 = 0.81$ in, $N_t = 15.1$ turns.

Table 10-4: $A = 145 \text{ kpsi}\cdot\text{in}^m, \quad m = 0$

Table 10-5: $G = 6$ Mpsi

$$D = \text{OD} - d = 0.120 - 0.012 = 0.108 \text{ in}$$

$$C = D/d = 0.108/0.012 = 9$$

$$K_B = \frac{4(9)+2}{4(9)-3} = 1.152$$

Table 10-1: $N_a = N_t - 2 = 15.1 - 2 = 13.1$ turns

$$S_{ut} = \frac{145}{0.012^0} = 145 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.35(145) = 50.8$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{6(10^6)(0.012)^4}{8(0.108)^3(13.1)} = 0.942 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.012(15.1) = 0.181$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 0.81 - 0.181 = 0.629$ in

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.152\left[\frac{8(0.942)(0.6)(0.108)}{\pi(0.012)^3}\right](10^{-3}) = 108.6 \text{ kpsi} \quad (1)$$

$\tau_s > S_{sy}$, that is, $108.6 > 50.8$ kpsi; the spring is not solid safe. Solving Eq. (1) for y_s' gives

$$y_s' = \frac{(S_{sy}/n)\pi d^3}{8K_B kD} = \frac{(50.8/1.2)(\pi)(0.012)^3(10^3)}{8(1.152)(0.942)(0.108)} = 0.245 \text{ in}$$

$$L_0' = L_s + y_s' = 0.181 + 0.245 = 0.426 \text{ in}$$

Wind the spring to a free length of 0.426 in. *Ans.*

10-9 Given: A313 (stainless steel), SQ&GRD ends, $d = 0.040$ in, OD $= 0.240$ in, $L_0 = 0.75$ in, $N_t = 10.4$ turns.

Table 10-4: $A = 169\text{ kpsi}\cdot\text{in}^m,\quad m = 0.146$

Table 10-5: $G = 10(10^6)$ psi

$$D = \text{OD} - d = 0.240 - 0.040 = 0.200\text{ in}$$

$$C = D/d = 0.200/0.040 = 5$$

$$K_B = \frac{4(5)+2}{4(5)-3} = 1.294$$

Table 10-6: $N_a = N_t - 2 = 10.4 - 2 = 8.4$ turns

$$S_{ut} = \frac{169}{(0.040)^{0.146}} = 270.4\text{ kpsi}$$

Table 10-13: $S_{sy} = 0.35(270.4) = 94.6$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{10(10^6)(0.040)^4}{8(0.2)^3(8.4)} = 47.62\text{ lbf/in}$$

Table 10-6: $L_s = dN_t = 0.040(10.4) = 0.416$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 0.75 - 0.416 = 0.334$ in

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.294\left[\frac{8(47.62)(0.334)(0.2)}{\pi(0.040)^3}\right](10^{-3}) = 163.8\text{ kpsi} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $163.8 > 94.6$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y'_s = \frac{(S_{sy}/n)(\pi d^3)}{8K_BkD} = \frac{(94\,600/1.2)(\pi)(0.040)^3}{8(1.294)(47.62)(0.2)} = 0.161\text{ in}$$

$$L'_0 = L_s + y'_s = 0.416 + 0.161 = 0.577\text{ in}$$

Wind the spring to a free length 0.577 in. *Ans.*

10-10 Given: A227 (hard drawn steel), $d = 0.135$ in, OD $= 2.0$ in, $L_0 = 2.94$ in, $N_t = 5.25$ turns.

Table 10-4: $A = 140\text{ kpsi}\cdot\text{in}^m,\quad m = 0.190$

Table 10-5: $G = 11.4(10^6)$ psi

$$D = \text{OD} - d = 2 - 0.135 = 1.865\text{ in}$$

$$C = D/d = 1.865/0.135 = 13.81$$

$$K_B = \frac{4(13.81)+2}{4(13.81)-3} = 1.096$$

$$N_a = N_t - 2 = 5.25 - 2 = 3.25\text{ turns}$$

$$S_{ut} = \frac{140}{(0.135)^{0.190}} = 204.8\text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45(204.8) = 92.2$ kpsi

$$k = \frac{Gd^4}{8D^3 N_a} = \frac{11.4(10^6)(0.135)^4}{8(1.865)^3(3.25)} = 22.45 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.135(5.25) = 0.709$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 2.94 - 0.709 = 2.231$ in

$$\tau_s = K_B \left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.096 \left[\frac{8(22.45)(2.231)(1.865)}{\pi(0.135)^3}\right](10^{-3}) = 106.0 \text{ kpsi} \quad (1)$$

$\tau_s > S_{sy}$, that is, $106 > 92.2$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y'_s = \frac{(S_{sy}/n)(\pi d^3)}{8K_B kD} = \frac{(92\,200/1.2)(\pi)(0.135)^3}{8(1.096)(22.45)(1.865)} = 1.612 \text{ in}$$

$$L'_0 = L_s + y'_s = 0.709 + 1.612 = 2.321 \text{ in}$$

Wind the spring to a free length of 2.32 in. *Ans.*

10-11 Given: A229 (OQ&T steel), SQ&GRD ends, $d = 0.144$ in, OD $= 1.0$ in, $L_0 = 3.75$ in, $N_t = 13$ turns.

Table 10-4: $A = 147 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.187$

Table 10-5: $G = 11.4(10^6)$ psi

$$D = \text{OD} - d = 1.0 - 0.144 = 0.856 \text{ in}$$

$$C = D/d = 0.856/0.144 = 5.944$$

$$K_B = \frac{4(5.944) + 2}{4(5.944) - 3} = 1.241$$

Table 10-1: $N_a = N_t - 2 = 13 - 2 = 11$ turns

$$S_{ut} = \frac{147}{(0.144)^{0.187}} = 211.2 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50(211.2) = 105.6$ kpsi

$$k = \frac{Gd^4}{8D^3 N_a} = \frac{11.4(10^6)(0.144)^4}{8(0.856)^3(11)} = 88.8 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.144(13) = 1.872$ in

Now $F_s = ky_s$, $y_s = L_0 - L_s = 3.75 - 1.872 = 1.878$ in

$$\tau_s = K_B \left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.241 \left[\frac{8(88.8)(1.878)(0.856)}{\pi(0.144)^3}\right](10^{-3}) = 151.1 \text{ kpsi} \quad (1)$$

$\tau_s > S_{sy}$, that is, $151.1 > 105.6$ kpsi; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y'_s = \frac{(S_{sy}/n)(\pi d^3)}{8K_B kD} = \frac{(105\,600/1.2)(\pi)(0.144)^3}{8(1.241)(88.8)(0.856)} = 1.094 \text{ in}$$

$$L'_0 = L_s + y'_s = 1.878 + 1.094 = 2.972 \text{ in}$$

Wind the spring to a free length 2.972 in. *Ans.*

10-12 Given: A232 (Cr-V steel), SQ&GRD ends, $d = 0.192$ in, OD $= 3$ in, $L_0 = 9$ in, $N_t = 8$ turns.

Table 10-4: $A = 169 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.168$

Table 10-5: $G = 11.2(10^6)$ psi

$$D = \text{OD} - d = 3 - 0.192 = 2.808 \text{ in}$$

$$C = D/d = 2.808/0.192 = 14.625 \quad \text{(large)}$$

$$K_B = \frac{4(14.625) + 2}{4(14.625) - 3} = 1.090$$

Table 10-1: $N_a = N_t - 2 = 8 - 2 = 6$ turns

$$S_{ut} = \frac{169}{(0.192)^{0.168}} = 223.0 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50(223.0) = 111.5$ kpsi

$$k = \frac{Gd^4}{8D^3N_a} = \frac{11.2(10^6)(0.192)^4}{8(2.808)^3(6)} = 14.32 \text{ lbf/in}$$

Table 10-1: $L_s = dN_t = 0.192(8) = 1.536$ in

Now $F_s = ky_s,\ y_s = L_0 - L_s = 9 - 1.536 = 7.464$ in

$$\tau_s = K_B \left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.090 \left[\frac{8(14.32)(7.464)(2.808)}{\pi(0.192)^3}\right](10^{-3}) = 117.7 \text{ kpsi} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $117.7 > 111.5$ kpsi; the spring is not solid safe. Solving Eq. (1) for y_s gives

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_B kD} = \frac{(111\,500/1.2)(\pi)(0.192)^3}{8(1.090)(14.32)(2.808)} = 5.892 \text{ in}$$

$$L_0' = L_s + y_s' = 1.536 + 5.892 = 7.428 \text{ in}$$

Wind the spring to a free length of 7.428 in. *Ans.*

10-13 Given: A313 (stainless steel) SQ&GRD ends, $d = 0.2$ mm, OD $= 0.91$ mm, $L_0 = 15.9$ mm, $N_t = 40$ turns.

Table 10-4: $A = 1867 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.146$

Table 10-5: $G = 69.0$ GPa

$$D = \text{OD} - d = 0.91 - 0.2 = 0.71 \text{ mm}$$

$$C = D/d = 0.71/0.2 = 3.55 \quad \text{(small)}$$

$$K_B = \frac{4(3.55) + 2}{4(3.55) - 3} = 1.446$$

$$N_a = N_t - 2 = 40 - 2 = 38 \text{ turns}$$

$$S_{ut} = \frac{1867}{(0.2)^{0.146}} = 2361.5 \text{ MPa}$$

Table 10-6:

$$S_{sy} = 0.35(2361.5) = 826.5 \text{ MPa}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.2)^4(69.0)}{8(0.71)^3(38)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right]$$

$$= 1.0147(10^{-3})(10^6) = 1014.7 \text{ N/m} \quad \text{or} \quad 1.0147 \text{ N/mm}$$

$$L_s = dN_t = 0.2(40) = 8 \text{ mm}$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 15.9 - 8 = 7.9$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.446\left[\frac{8(1.0147)(7.9)(0.71)}{\pi(0.2)^3}\right]\left[\frac{10^{-3}(10^{-3})(10^{-3})}{(10^{-3})^3}\right]$$

$$= 2620(1) = 2620 \text{ MPa} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $2620 > 826.5$ MPa; the spring is not solid safe. Solve Eq. (1) for y_s giving

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_B kD} = \frac{(826.5/1.2)(\pi)(0.2)^3}{8(1.446)(1.0147)(0.71)} = 2.08 \text{ mm}$$

$$L_0' = L_s + y_s' = 8.0 + 2.08 = 10.08 \text{ mm}$$

Wind the spring to a free length of 10.08 mm. This only addresses the solid-safe criteria. There are additional problems. *Ans.*

10-14 Given: A228 (music wire), SQ&GRD ends, $d = 1$ mm, OD $= 6.10$ mm, $L_0 = 19.1$ mm, $N_t = 10.4$ turns.

Table 10-4: $A = 2211 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.145$

Table 10-5: $G = 81.7$ GPa

$$D = \text{OD} - d = 6.10 - 1 = 5.1 \text{ mm}$$

$$C = D/d = 5.1/1 = 5.1$$

$$N_a = N_t - 2 = 10.4 - 2 = 8.4 \text{ turns}$$

$$K_B = \frac{4(5.1)+2}{4(5.1)-3} = 1.287$$

$$S_{ut} = \frac{2211}{(1)^{0.145}} = 2211 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.45(2211) = 995$ MPa

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(1)^4(81.7)}{8(5.1)^3(8.4)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.009\,165(10^6)$$

$$= 9165 \text{ N/m} \quad \text{or} \quad 9.165 \text{ N/mm}$$

$$L_s = dN_t = 1(10.4) = 10.4 \text{ mm}$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 19.1 - 10.4 = 8.7 \text{ mm}$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.287\left[\frac{8(9.165)(8.7)(5.1)}{\pi(1)^3}\right] = 1333 \text{ MPa} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $1333 > 995$ MPa; the spring is not solid safe. Solve Eq. (1) for y_s giving

$$y'_s = \frac{(S_{sy}/n)(\pi d^3)}{8K_B kD} = \frac{(995/1.2)(\pi)(1)^3}{8(1.287)(9.165)(5.1)} = 5.43 \text{ mm}$$

$$L'_0 = L_s + y'_s = 10.4 + 5.43 = 15.83 \text{ mm}$$

Wind the spring to a free length of 15.83 mm. *Ans.*

10-15 Given: A229 (OQ&T spring steel), SQ&GRD ends, $d = 3.4$ mm, OD $= 50.8$ mm, $L_0 = 74.6$ mm, $N_t = 5.25$.

Table 10-4: $A = 1855 \text{ MPa}\cdot\text{mm}^m, \quad m = 0.187$

Table 10-5: $G = 77.2$ GPa

$$D = \text{OD} - d = 50.8 - 3.4 = 47.4 \text{ mm}$$

$$C = D/d = 47.4/3.4 = 13.94 \quad \text{(large)}$$

$$N_a = N_t - 2 = 5.25 - 2 = 3.25 \text{ turns}$$

$$K_B = \frac{4(13.94)+2}{4(13.94)-3} = 1.095$$

$$S_{ut} = \frac{1855}{(3.4)^{0.187}} = 1476 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.50(1476) = 737.8$ MPa

$$k = \frac{d^4G}{8D^3N_a} = \frac{(3.4)^4(77.2)}{8(47.4)^3(3.25)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.003\,75(10^6)$$

$$= 3750 \text{ N/m} \quad \text{or} \quad 3.750 \text{ N/mm}$$

$$L_s = dN_t = 3.4(5.25) = 17.85$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 74.6 - 17.85 = 56.75 \text{ mm}$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right]$$

$$= 1.095\left[\frac{8(3.750)(56.75)(47.4)}{\pi(3.4)^3}\right] = 720.2 \text{ MPa} \qquad (1)$$

$\tau_s < S_{sy}$, that is, $720.2 < 737.8$ MPa

$\therefore$ The spring is solid safe. With $n_s = 1.2$,

$$y'_s = \frac{(S_{sy}/n)(\pi d^3)}{8K_B k D} = \frac{(737.8/1.2)(\pi)(3.4)^3}{8(1.095)(3.75)(47.4)} = 48.76 \text{ mm}$$

$$L'_0 = L_s + y'_s = 17.85 + 48.76 = 66.61 \text{ mm}$$

Wind the spring to a free length of 66.61 mm. *Ans.*

10-16 Given: B159 (phosphor bronze), SQ&GRD ends, $d = 3.7$ mm, OD $= 25.4$ mm, $L_0 = 95.3$ mm, $N_t = 13$ turns.

Table 10-4: $A = 932 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.064$

Table 10-5: $G = 41.4$ GPa

$$D = \text{OD} - d = 25.4 - 3.7 = 21.7 \text{ mm}$$

$$C = D/d = 21.7/3.7 = 5.865$$

$$K_B = \frac{4(5.865) + 2}{4(5.865) - 3} = 1.244$$

$$N_a = N_t - 2 = 13 - 2 = 11 \text{ turns}$$

$$S_{ut} = \frac{932}{(3.7)^{0.064}} = 857.1 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.35(857.1) = 300$ MPa

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(3.7)^4(41.4)}{8(21.7)^3(11)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.008\,629(10^6)$$

$$= 8629 \text{ N/m} \quad \text{or} \quad 8.629 \text{ N/mm}$$

$$L_s = dN_t = 3.7(13) = 48.1 \text{ mm}$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 95.3 - 48.1 = 47.2 \text{ mm}$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right]$$

$$= 1.244\left[\frac{8(8.629)(47.2)(21.7)}{\pi(3.7)^3}\right] = 553 \text{ MPa} \qquad (1)$$

$\tau_s > S_{sy}$, that is, $553 > 300$ MPa; the spring is not solid-safe. Solving Eq. (1) for y_s gives

$$y'_s = \frac{(S_{sy}/n)(\pi d^3)}{8K_B k D} = \frac{(300/1.2)(\pi)(3.7)^3}{8(1.244)(8.629)(21.7)} = 21.35 \text{ mm}$$

$$L'_0 = L_s + y'_s = 48.1 + 21.35 = 69.45 \text{ mm}$$

Wind the spring to a free length of 69.45 mm. *Ans.*

10-17 Given: A232 (Cr-V steel), SQ&GRD ends, $d = 4.3$ mm, OD = 76.2 mm, $L_0 = 228.6$ mm, $N_t = 8$ turns.

Table 10-4: $$A = 2005\ \text{MPa}\cdot\text{mm}^m, \quad m = 0.168$$

Table 10-5: $$G = 77.2\ \text{GPa}$$

$$D = \text{OD} - d = 76.2 - 4.3 = 71.9\ \text{mm}$$

$$C = D/d = 71.9/4.3 = 16.72 \quad \text{(large)}$$

$$K_B = \frac{4(16.72)+2}{4(16.72)-3} = 1.078$$

$$N_a = N_t - 2 = 8 - 2 = 6\ \text{turns}$$

$$S_{ut} = \frac{2005}{(4.3)^{0.168}} = 1569\ \text{MPa}$$

Table 10-6:

$$S_{sy} = 0.50(1569) = 784.5\ \text{MPa}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(4.3)^4(77.2)}{8(71.9)^3(6)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right] = 0.001\,479(10^6)$$

$$= 1479\ \text{N/m} \quad \text{or} \quad 1.479\ \text{N/mm}$$

$$L_s = dN_t = 4.3(8) = 34.4\ \text{mm}$$

$$F_s = ky_s$$

$$y_s = L_0 - L_s = 228.6 - 34.4 = 194.2\ \text{mm}$$

$$\tau_s = K_B\left[\frac{8(ky_s)D}{\pi d^3}\right] = 1.078\left[\frac{8(1.479)(194.2)(71.9)}{\pi(4.3)^3}\right] = 713.0\ \text{MPa} \qquad (1)$$

$\tau_s < S_{sy}$, that is, $713.0 < 784.5$; the spring is solid safe. With $n_s = 1.2$

Eq. (1) becomes

$$y_s' = \frac{(S_{sy}/n)(\pi d^3)}{8K_B k D} = \frac{(784.5/1.2)(\pi)(4.3)^3}{8(1.078)(1.479)(71.9)} = 178.1\ \text{mm}$$

$$L_0' = L_s + y_s' = 34.4 + 178.1 = 212.5\ \text{mm}$$

Wind the spring to a free length of $L_0' = 212.5$ mm. *Ans.*

10-18 For the wire diameter analyzed, $G = 11.75$ Mpsi per Table 10-5. Use squared and ground ends. The following is a spread-sheet study using Fig. 10-3 for parts (*a*) and (*b*). For N_a, $k = 20/2 = 10$ lbf/in.

(a) Spring over a Rod					(b) Spring in a Hole				
Source	Parameter Values				Source	Parameter Values			
	d	0.075	0.08	0.085		d	0.075	0.08	0.085
	D	0.875	0.88	0.885		D	0.875	0.870	0.865
	ID	0.800	0.800	0.800		ID	0.800	0.790	0.780
	OD	0.950	0.960	0.970		OD	0.950	0.950	0.950
Eq. (10-2)	C	11.667	11.000	10.412	Eq. (10-2)	C	11.667	10.875	10.176
Eq. (10-9)	N_a	6.937	8.828	11.061	Eq. (10-9)	N_a	6.937	9.136	11.846
Table 10-1	N_t	8.937	10.828	13.061	Table 10-1	N_t	8.937	11.136	13.846
Table 10-1	L_s	0.670	0.866	1.110	Table 10-1	L_s	0.670	0.891	1.177
$1.15y + L_s$	L_0	2.970	3.166	3.410	$1.15y + L_s$	L_0	2.970	3.191	3.477
Eq. (10-13)	$(L_0)_{\text{cr}}$	4.603	4.629	4.655	Eq. (10-13)	$(L_0)_{\text{cr}}$	4.603	4.576	4.550
Table 10-4	A	201.000	201.000	201.000	Table 10-4	A	201.000	201.000	201.000
Table 10-4	m	0.145	0.145	0.145	Table 10-4	m	0.145	0.145	0.145
Eq. (10-14)	S_{ut}	292.626	289.900	287.363	Eq. (10-14)	S_{ut}	292.626	289.900	287.363
Table 10-6	S_{sy}	131.681	130.455	129.313	Table 10-6	S_{sy}	131.681	130.455	129.313
Eq. (10-6)	K_B	1.115	1.122	1.129	Eq. (10-6)	K_B	1.115	1.123	1.133
Eq. (10-3)	n_s	0.973	1.155	1.357	Eq. (10-3)	n_s	0.973	1.167	1.384
Eq. (10-22)	fom	−0.282	−0.391	−0.536	Eq. (10-22)	fom	−0.282	−0.398	−0.555

For $n_s \geq 1.2$, the optimal size is $d = 0.085$ in for both cases.

10-19 From the figure: $L_0 = 120$ mm, OD $= 50$ mm, and $d = 3.4$ mm. Thus

$$D = \text{OD} - d = 50 - 3.4 = 46.6 \text{ mm}$$

(a) By counting, $N_t = 12.5$ turns. Since the ends are squared along 1/4 turn on each end,

$$N_a = 12.5 - 0.5 = 12 \text{ turns} \quad Ans.$$

$$p = 120/12 = 10 \text{ mm} \quad Ans.$$

The solid stack is 13 diameters across the top and 12 across the bottom.

$$L_s = 13(3.4) = 44.2 \text{ mm} \quad Ans.$$

(b) $d = 3.4/25.4 = 0.1339$ in and from Table 10-5, $G = 78.6$ GPa

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(3.4)^4(78.6)(10^9)}{8(46.6)^3(12)}(10^{-3}) = 1080 \text{ N/m} \quad Ans.$$

(c) $F_s = k(L_0 - L_s) = 1080(120 - 44.2)(10^{-3}) = 81.9 \text{ N} \quad Ans.$

(d) $C = D/d = 46.6/3.4 = 13.71$

$$K_B = \frac{4(13.71) + 2}{4(13.71) - 3} = 1.096$$

$$\tau_s = \frac{8K_B F_s D}{\pi d^3} = \frac{8(1.096)(81.9)(46.6)}{\pi(3.4)^3} = 271 \text{ MPa} \quad Ans.$$

10-20 One approach is to select A227-47 HD steel for its low cost. Then, for $y_1 \leq 3/8$ at $F_1 = 10$ lbf, $k \geq 10/0.375 = 26.67$ lbf/in. Try $d = 0.080$ in #14 gauge

For a clearance of 0.05 in: ID $= (7/16) + 0.05 = 0.4875$ in; OD $= 0.4875 + 0.16 = 0.6475$ in

$$D = 0.4875 + 0.080 = 0.5675 \text{ in}$$

$$C = 0.5675/0.08 = 7.094$$

$$G = 11.5 \text{ Mpsi}$$

$$N_a = \frac{d^4G}{8kD^3} = \frac{(0.08)^4(11.5)(10^6)}{8(26.67)(0.5675)^3} = 12.0 \text{ turns}$$

$$N_t = 12 + 2 = 14 \text{ turns}, \quad L_s = dN_t = 0.08(14) = 1.12 \text{ in} \quad O.K.$$

$$L_0 = 1.875 \text{ in}, \quad y_s = 1.875 - 1.12 = 0.755 \text{ in}$$

$$F_s = ky_s = 26.67(0.755) = 20.14 \text{ lbf}$$

$$K_B = \frac{4(7.094) + 2}{4(7.094) - 3} = 1.197$$

$$\tau_s = K_B\left(\frac{8F_sD}{\pi d^3}\right) = 1.197\left[\frac{8(20.14)(0.5675)}{\pi(0.08)^3}\right] = 68\,046 \text{ psi}$$

Table 10-4: $\quad A = 140 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.190$

$$S_{sy} = 0.45\frac{140}{(0.080)^{0.190}} = 101.8 \text{ kpsi}$$

$$n = \frac{101.8}{68.05} = 1.50 > 1.2 \quad O.K.$$

$$\tau_1 = \frac{F_1}{F_s}\tau_s = \frac{10}{20.14}(68.05) = 33.79 \text{ kpsi},$$

$$n_1 = \frac{101.8}{33.79} = 3.01 > 1.5 \quad O.K.$$

There is much latitude for reducing the amount of material. Iterate on y_1 using a spread sheet. The final results are: $y_1 = 0.32$ in, $k = 31.25$ lbf/in, $N_a = 10.3$ turns, $N_t = 12.3$ turns, $L_s = 0.985$ in, $L_0 = 1.820$ in, $y_s = 0.835$ in, $F_s = 26.1$ lbf, $K_B = 1.197$, $\tau_s = 88\,190$ kpsi, $n_s = 1.15$, and $n_1 = 3.01$.

$$\text{ID} = 0.4875 \text{ in}, \quad \text{OD} = 0.6475 \text{ in}, \quad d = 0.080 \text{ in}$$

Try other sizes and/or materials.

10-21 A stock spring catalog may have over two hundred pages of compression springs with up to 80 springs per page listed.

- Students should be aware that such catalogs exist.
- Many springs are selected from catalogs rather than designed.
- The wire size you want may not be listed.
- Catalogs may also be available on disk or the web through search routines. For example, disks are available from Century Spring at

  1 − (800) − 237 − 5225
  www.centuryspring.com

- It is better to familiarize yourself with vendor resources rather than invent them yourself.
- Sample catalog pages can be given to students for study.

10-22 For a coil radius given by:

$$R = R_1 + \frac{R_2 - R_1}{2\pi N}\theta$$

The torsion of a section is $T = PR$ where $dL = R\,d\theta$

$$\delta_p = \frac{\partial U}{\partial P} = \frac{1}{GJ}\int T\frac{\partial T}{\partial P}\,dL = \frac{1}{GJ}\int_0^{2\pi N} PR^3\,d\theta$$

$$= \frac{P}{GJ}\int_0^{2\pi N}\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^3 d\theta$$

$$= \frac{P}{GJ}\left(\frac{1}{4}\right)\left(\frac{2\pi N}{R_2 - R_1}\right)\left[\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^4\right]\Bigg|_0^{2\pi N}$$

$$= \frac{\pi PN}{2GJ(R_2 - R_1)}\left(R_2^4 - R_1^4\right) = \frac{\pi PN}{2GJ}(R_1 + R_2)\left(R_1^2 + R_2^2\right)$$

$$J = \frac{\pi}{32}d^4 \quad \therefore \delta_p = \frac{16PN}{Gd^4}(R_1 + R_2)\left(R_1^2 + R_2^2\right)$$

$$k = \frac{P}{\delta_p} = \frac{d^4G}{16N(R_1 + R_2)\left(R_1^2 + R_2^2\right)} \quad Ans.$$

10-23 For a food service machinery application select A313 Stainless wire.

$$G = 10(10^6)\text{ psi}$$

Note that for $0.013 \le d \le 0.10$ in $\quad A = 169, \quad m = 0.146$

$0.10 < d \le 0.20$ in $\quad A = 128, \quad m = 0.263$

$$F_a = \frac{18 - 4}{2} = 7\text{ lbf}, \quad F_m = \frac{18 + 4}{2} = 11\text{ lbf}, \quad r = 7/11$$

Try $\quad d = 0.080$ in, $\quad S_{ut} = \dfrac{169}{(0.08)^{0.146}} = 244.4$ kpsi

$$S_{su} = 0.67S_{ut} = 163.7\text{ kpsi}, \qquad S_{sy} = 0.35S_{ut} = 85.5\text{ kpsi}$$

Try unpeened using Zimmerli's endurance data: $S_{sa} = 35$ kpsi, $S_{sm} = 55$ kpsi

Gerber: $\quad S_{se} = \dfrac{S_{sa}}{1 - (S_{sm}/S_{su})^2} = \dfrac{35}{1 - (55/163.7)^2} = 39.5$ kpsi

$$S_{sa} = \frac{(7/11)^2(163.7)^2}{2(39.5)}\left\{-1 + \sqrt{1 + \left[\frac{2(39.5)}{(7/11)(163.7)}\right]^2}\right\} = 35.0\text{ kpsi}$$

$$\alpha = S_{sa}/n_f = 35.0/1.5 = 23.3\text{ kpsi}$$

$$\beta = \frac{8F_a}{\pi d^2}(10^{-3}) = \left[\frac{8(7)}{\pi(0.08^2)}\right](10^{-3}) = 2.785\text{ kpsi}$$

$$C = \frac{2(23.3) - 2.785}{4(2.785)} + \sqrt{\left[\frac{2(23.3) - 2.785}{4(2.785)}\right]^2 - \frac{3(23.3)}{4(2.785)}} = 6.97$$

$$D = Cd = 6.97(0.08) = 0.558\text{ in}$$

$$K_B = \frac{4(6.97)+2}{4(6.97)-3} = 1.201$$

$$\tau_a = K_B\left(\frac{8F_aD}{\pi d^3}\right) = 1.201\left[\frac{8(7)(0.558)}{\pi(0.08^3)}(10^{-3})\right] = 23.3 \text{ kpsi}$$

$$n_f = 35/23.3 = 1.50 \quad \text{checks}$$

$$N_a = \frac{Gd^4}{8kD^3} = \frac{10(10^6)(0.08)^4}{8(9.5)(0.558)^3} = 31.02 \text{ turns}$$

$$N_t = 31+2 = 33 \text{ turns}, \quad L_s = dN_t = 0.08(33) = 2.64 \text{ in}$$

$$y_{\max} = F_{\max}/k = 18/9.5 = 1.895 \text{ in},$$

$$y_s = (1+\xi)y_{\max} = (1+0.15)(1.895) = 2.179 \text{ in}$$

$$L_0 = 2.64 + 2.179 = 4.819 \text{ in}$$

$$(L_0)_{\text{cr}} = 2.63\frac{D}{\alpha} = \frac{2.63(0.558)}{0.5} = 2.935 \text{ in}$$

$$\tau_s = 1.15(18/7)\tau_a = 1.15(18/7)(23.3) = 68.9 \text{ kpsi}$$

$$n_s = S_{sy}/\tau_s = 85.5/68.9 = 1.24$$

$$f = \sqrt{\frac{kg}{\pi^2 d^2 D N_a \gamma}} = \sqrt{\frac{9.5(386)}{\pi^2(0.08^2)(0.558)(31.02)(0.283)}} = 109 \text{ Hz}$$

These steps are easily implemented on a spreadsheet, as shown below, for different diameters.

	d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263
A	169.000	169.000	128.000	128.000
S_{ut}	244.363	239.618	231.257	223.311
S_{su}	163.723	160.544	154.942	149.618
S_{sy}	85.527	83.866	80.940	78.159
S_{se}	39.452	39.654	40.046	40.469
S_{sa}	35.000	35.000	35.000	35.000
α	23.333	23.333	23.333	23.333
β	2.785	2.129	1.602	1.228
C	6.977	9.603	13.244	17.702
D	0.558	0.879	1.397	2.133
K_B	1.201	1.141	1.100	1.074
τ_a	23.333	23.333	23.333	23.333
n_f	1.500	1.500	1.500	1.500
N_a	30.893	13.594	5.975	2.858
N_t	32.993	15.594	7.975	4.858
L_s	2.639	1.427	0.841	0.585
y_s	2.179	2.179	2.179	2.179
L_0	4.818	3.606	3.020	2.764
$(L_0)_{\text{cr}}$	2.936	4.622	7.350	11.220
τ_s	69.000	69.000	69.000	69.000
n_s	1.240	1.215	1.173	1.133
f (Hz)	108.895	114.578	118.863	121.775

The shaded areas depict conditions outside the recommended design conditions. Thus, one spring is satisfactory–A313, as wound, unpeened, squared and ground,

$$d = 0.0915\text{ in},\quad \text{OD} = 0.879 + 0.092 = 0.971\text{ in},\quad N_t = 15.59\text{ turns}$$

10-24 The steps are the same as in Prob. 10-23 except that the Gerber-Zimmerli criterion is replaced with Goodman-Zimmerli:

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})}$$

The problem then proceeds as in Prob. 10-23. The results for the wire sizes are shown below (see solution to Prob. 10-23 for additional details).

Iteration of d for the first trial

	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.151	1.108	1.078	1.058
A	169.000	169.000	128.000	128.000	τ_a	29.008	29.040	29.090	29.127
S_{ut}	244.363	239.618	231.257	223.311	n_f	1.500	1.500	1.500	1.500
S_{su}	163.723	160.544	154.942	149.618	N_a	14.191	6.456	2.899	1.404
S_{sy}	85.527	83.866	80.940	78.159	N_t	16.191	8.456	4.899	3.404
S_{se}	52.706	53.239	54.261	55.345	L_s	1.295	0.774	0.517	0.410
S_{sa}	43.513	43.560	43.634	43.691	y_s	2.179	2.179	2.179	2.179
α	29.008	29.040	29.090	29.127	L_0	3.474	2.953	2.696	2.589
β	2.785	2.129	1.602	1.228	$(L_0)_{\text{cr}}$	3.809	5.924	9.354	14.219
C	9.052	12.309	16.856	22.433	τ_s	85.782	85.876	86.022	86.133
D	0.724	1.126	1.778	2.703	n_s	0.997	0.977	0.941	0.907
					f (Hz)	141.284	146.853	151.271	154.326

Without checking all of the design conditions, it is obvious that none of the wire sizes satisfy $n_s \geq 1.2$. Also, the Gerber line is closer to the yield line than the Goodman. Setting $n_f = 1.5$ for Goodman makes it impossible to reach the yield line ($n_s < 1$). The table below uses $n_f = 2$.

Iteration of d for the second trial

	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.221	1.154	1.108	1.079
A	169.000	169.000	128.000	128.000	τ_a	21.756	21.780	21.817	21.845
S_{ut}	244.363	239.618	231.257	223.311	n_f	2.000	2.000	2.000	2.000
S_{su}	163.723	160.544	154.942	149.618	N_a	40.243	17.286	7.475	3.539
S_{sy}	85.527	83.866	80.940	78.159	N_t	42.243	19.286	9.475	5.539
S_{se}	52.706	53.239	54.261	55.345	L_s	3.379	1.765	1.000	0.667
S_{sa}	43.513	43.560	43.634	43.691	y_s	2.179	2.179	2.179	2.179
α	21.756	21.780	21.817	21.845	L_0	5.558	3.944	3.179	2.846
β	2.785	2.129	1.602	1.228	$(L_0)_{\text{cr}}$	2.691	4.266	6.821	10.449
C	6.395	8.864	12.292	16.485	τ_s	64.336	64.407	64.517	64.600
D	0.512	0.811	1.297	1.986	n_s	1.329	1.302	1.255	1.210
					f (Hz)	99.816	105.759	110.312	113.408

The satisfactory spring has design specifications of: A313, as wound, unpeened, squared and ground, $d = 0.0915$ in, $\text{OD} = 0.811 + 0.092 = 0.903$ in, $N_t = 19.3$ turns.

10-25 This is the same as Prob. 10-23 since $S_{se} = S_{sa} = 35$ kpsi. Therefore, design the spring using: A313, as wound, un-peened, squared and ground, $d = 0.915$ in, OD $= 0.971$ in, $N_t = 15.59$ turns.

10-26 For the Gerber fatigue-failure criterion, $S_{su} = 0.67S_{ut}$,

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})^2}, \qquad S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2S_{se}}{rS_{su}}\right)^2}\,\right]$$

The equation for S_{sa} is the basic difference. The last 2 columns of diameters of Ex. 10-5 are presented below with additional calculations.

	$d = 0.105$	$d = 0.112$		$d = 0.105$	$d = 0.112$
S_{ut}	278.691	276.096	N_a	8.915	6.190
S_{su}	186.723	184.984	L_s	1.146	0.917
S_{se}	38.325	38.394	L_0	3.446	3.217
S_{sy}	125.411	124.243	$(L_0)_{\mathrm{cr}}$	6.630	8.160
S_{sa}	34.658	34.652	K_B	1.111	1.095
α	23.105	23.101	τ_a	23.105	23.101
β	1.732	1.523	n_f	1.500	1.500
C	12.004	13.851	τ_s	70.855	70.844
D	1.260	1.551	n_s	1.770	1.754
ID	1.155	1.439	f_n	105.433	106.922
OD	1.365	1.663	fom	−0.973	−1.022

There are only slight changes in the results.

10-27 As in Prob. 10-26, the basic change is S_{sa}.

For Goodman,
$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})}$$

Recalculate S_{sa} with
$$S_{sa} = \frac{rS_{se}S_{su}}{rS_{su} + S_{se}}$$

Calculations for the last 2 diameters of Ex. 10-5 are given below.

	$d = 0.105$	$d = 0.112$		$d = 0.105$	$d = 0.112$
S_{ut}	278.691	276.096	N_a	9.153	6.353
S_{su}	186.723	184.984	L_s	1.171	0.936
S_{se}	49.614	49.810	L_0	3.471	3.236
S_{sy}	125.411	124.243	$(L_0)_{\mathrm{cr}}$	6.572	8.090
S_{sa}	34.386	34.380	K_B	1.112	1.096
α	22.924	22.920	τ_a	22.924	22.920
β	1.732	1.523	n_f	1.500	1.500
C	11.899	13.732	τ_s	70.301	70.289
D	1.249	1.538	n_s	1.784	1.768
ID	1.144	1.426	f_n	104.509	106.000
OD	1.354	1.650	fom	−0.986	−1.034

There are only slight differences in the results.

10-28 Use: $E = 28.6$ Mpsi, $G = 11.5$ Mpsi, $A = 140$ kpsi $\cdot$ in^m, $m = 0.190$, rel cost $= 1$.

Try $\quad d = 0.067$ in, $\quad S_{ut} = \dfrac{140}{(0.067)^{0.190}} = 234.0$ kpsi

Table 10-6: $\quad S_{sy} = 0.45 S_{ut} = 105.3$ kpsi

Table 10-7: $\quad S_y = 0.75 S_{ut} = 175.5$ kpsi

Eq. (10-34) with $D/d = C$ and $C_1 = C$

$$\sigma_A = \frac{F_{\max}}{\pi d^2}[(K)_A(16C) + 4] = \frac{S_y}{n_y}$$

$$\frac{4C^2 - C - 1}{4C(C-1)}(16C) + 4 = \frac{\pi d^2 S_y}{n_y F_{\max}}$$

$$4C^2 - C - 1 = (C-1)\left(\frac{\pi d^2 S_y}{4 n_y F_{\max}} - 1\right)$$

$$C^2 - \frac{1}{4}\left(1 + \frac{\pi d^2 S_y}{4 n_y F_{\max}} - 1\right)C + \frac{1}{4}\left(\frac{\pi d^2 S_y}{4 n_y F_{\max}} - 2\right) = 0$$

$$C = \frac{1}{2}\left[\frac{\pi d^2 S_y}{16 n_y F_{\max}} \pm \sqrt{\left(\frac{\pi d^2 S_y}{16 n_y F_{\max}}\right)^2 - \frac{\pi d^2 S_y}{4 n_y F_{\max}} + 2}\right] \quad \text{take positive root}$$

$$= \frac{1}{2}\left\{\frac{\pi(0.067^2)(175.5)(10^3)}{16(1.5)(18)} + \sqrt{\left[\frac{\pi(0.067)^2(175.5)(10^3)}{16(1.5)(18)}\right]^2 - \frac{\pi(0.067)^2(175.5)(10^3)}{4(1.5)(18)} + 2}\right\} = 4.590$$

$$D = Cd = 0.3075 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range. This results in the best fom.

$$F_i = \frac{\pi(0.067)^3}{8(0.3075)}\left\{\frac{33\,500}{\exp[0.105(4.590)]} - 1000\left(4 - \frac{4.590-3}{6.5}\right)\right\} = 6.505 \text{ lbf}$$

For simplicity, we will round up to the next integer or half integer;
therefore, use $F_i = 7$ lbf

$$k = \frac{18-7}{0.5} = 22 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.067)^4(11.5)(10^6)}{8(22)(0.3075)^3} = 45.28 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 45.28 - \frac{11.5}{28.6} = 44.88 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.590) - 1 + 44.88](0.067) = 3.555 \text{ in}$$

$$L_{18\,\text{lbf}} = 3.555 + 0.5 = 4.055 \text{ in}$$

Body: $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(4.590)+2}{4(4.590)-3} = 1.326$

$$\tau_{\max} = \frac{8K_B F_{\max} D}{\pi d^3} = \frac{8(1.326)(18)(0.3075)}{\pi(0.067)^3}(10^{-3}) = 62.1 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{S_{sy}}{\tau_{\max}} = \frac{105.3}{62.1} = 1.70$$

$$r_2 = 2d = 2(0.067) = 0.134 \text{ in}, \quad C_2 = \frac{2r_2}{d} = \frac{2(0.134)}{0.067} = 4$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4)-1}{4(4)-4} = 1.25$$

$$\tau_B = (K)_B \left[\frac{8F_{\max} D}{\pi d^3}\right] = 1.25\left[\frac{8(18)(0.3075)}{\pi(0.067)^3}\right](10^{-3}) = 58.58 \text{ kpsi}$$

$$(n_y)_B = \frac{S_{sy}}{\tau_B} = \frac{105.3}{58.58} = 1.80$$

$$\text{fom} = -(1)\frac{\pi^2 d^2 (N_b + 2) D}{4} = -\frac{\pi^2 (0.067)^2 (44.88 + 2)(0.3075)}{4} = -0.160$$

Several diameters, evaluated using a spreadsheet, are shown below.

d:	0.067	0.072	0.076	0.081	0.085	0.09	0.095	0.104
S_{ut}	233.977	230.799	228.441	225.692	223.634	221.219	218.958	215.224
S_{sy}	105.290	103.860	102.798	101.561	100.635	99.548	98.531	96.851
S_y	175.483	173.100	171.331	169.269	167.726	165.914	164.218	161.418
C	4.589	5.412	6.099	6.993	7.738	8.708	9.721	11.650
D	0.307	0.390	0.463	0.566	0.658	0.784	0.923	1.212
F_i (calc)	6.505	5.773	5.257	4.675	4.251	3.764	3.320	2.621
F_i (rd)	7.0	6.0	5.5	5.0	4.5	4.0	3.5	3.0
k	22.000	24.000	25.000	26.000	27.000	28.000	29.000	30.000
N_a	45.29	27.20	19.27	13.10	9.77	7.00	5.13	3.15
N_b	44.89	26.80	18.86	12.69	9.36	6.59	4.72	2.75
L_0	3.556	2.637	2.285	2.080	2.026	2.071	2.201	2.605
$L_{18\text{ lbf}}$	4.056	3.137	2.785	2.580	2.526	2.571	2.701	3.105
K_B	1.326	1.268	1.234	1.200	1.179	1.157	1.139	1.115
$\tau_{\max}$	62.118	60.686	59.707	58.636	57.875	57.019	56.249	55.031
$(n_y)_{\text{body}}$	1.695	1.711	1.722	1.732	1.739	1.746	1.752	1.760
τ_B	58.576	59.820	60.495	61.067	61.367	61.598	61.712	61.712
$(n_y)_B$	1.797	1.736	1.699	1.663	1.640	1.616	1.597	1.569
$(n_y)_A$	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500
fom	−0.160	−0.144	−0.138	−0.135	−0.133	−0.135	−0.138	−0.154

Except for the 0.067 in wire, all springs satisfy the requirements of length and number of coils. The 0.085 in wire has the highest fom.

10-29 Given: $N_b = 84$ coils, $F_i = 16$ lbf, OQ&T steel, OD $= 1.5$ in, $d = 0.162$ in.

$$D = 1.5 - 0.162 = 1.338 \text{ in}$$

(a) Eq. (10-39):

$$L_0 = 2(D - d) + (N_b + 1)d$$

$$= 2(1.338 - 0.162) + (84 + 1)(0.162) = 16.12 \text{ in} \quad \textit{Ans.}$$

or

$$2d + L_0 = 2(0.162) + 16.12 = 16.45 \text{ in overall.}$$

(b)

$$C = \frac{D}{d} = \frac{1.338}{0.162} = 8.26$$

$$K_B = \frac{4(8.26) + 2}{4(8.26) - 3} = 1.166$$

$$\tau_i = K_B \left[\frac{8F_i D}{\pi d^3}\right] = 1.166 \left[\frac{8(16)(1.338)}{\pi (0.162)^3}\right] = 14\,950 \text{ psi} \quad \textit{Ans.}$$

(c) From Table 10-5 use: $G = 11.4(10^6)$ psi and $E = 28.5(10^6)$ psi

$$N_a = N_b + \frac{G}{E} = 84 + \frac{11.4}{28.5} = 84.4 \text{ turns}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.162)^4(11.4)(10^6)}{8(1.338)^3(84.4)} = 4.855 \text{ lbf/in} \quad \textit{Ans.}$$

(d) Table 10-4: $A = 147 \text{ psi} \cdot \text{in}^m, \quad m = 0.187$

$$S_{ut} = \frac{147}{(0.162)^{0.187}} = 207.1 \text{ kpsi}$$

$$S_y = 0.75(207.1) = 155.3 \text{ kpsi}$$

$$S_{sy} = 0.50(207.1) = 103.5 \text{ kpsi}$$

Body

$$F = \frac{\pi d^3 S_{sy}}{\pi K_B D}$$

$$= \frac{\pi (0.162)^3 (103.5)(10^3)}{8(1.166)(1.338)} = 110.8 \text{ lbf}$$

Torsional stress on hook point B

$$C_2 = \frac{2r_2}{d} = \frac{2(0.25 + 0.162/2)}{0.162} = 4.086$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4.086) - 1}{4(4.086) - 4} = 1.243$$

$$F = \frac{\pi (0.162)^3 (103.5)(10^3)}{8(1.243)(1.338)} = 103.9 \text{ lbf}$$

Normal stress on hook point A

$$C_1 = \frac{2r_1}{d} = \frac{1.338}{0.162} = 8.26$$

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} = \frac{4(8.26)^2 - 8.26 - 1}{4(8.26)(8.26 - 1)} = 1.099$$

$$S_{yt} = \sigma = F\left[\frac{16(K)_A D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$

$$F = \frac{155.3(10^3)}{[16(1.099)(1.338)]/[\pi(0.162)^3] + \{4/[\pi(0.162)^2]\}} = 85.8 \text{ lbf}$$

$$= \min(110.8, 103.9, 85.8) = 85.8 \text{ lbf} \quad \textit{Ans.}$$

(e) Eq. (10-48):

$$y = \frac{F - F_i}{k} = \frac{85.8 - 16}{4.855} = 14.4 \text{ in} \quad \textit{Ans.}$$

10-30 $F_{\min} = 9$ lbf, $F_{\max} = 18$ lbf

$$F_a = \frac{18 - 9}{2} = 4.5 \text{ lbf}, \qquad F_m = \frac{18 + 9}{2} = 13.5 \text{ lbf}$$

A313 stainless: $0.013 \le d \le 0.1 \qquad A = 169 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.146$

$0.1 \le d \le 0.2 \qquad A = 128 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.263$

$E = 28$ Mpsi, $\quad G = 10$ Gpsi

Try $d = 0.081$ in and refer to the discussion following Ex. 10-7

$$S_{ut} = \frac{169}{(0.081)^{0.146}} = 243.9 \text{ kpsi}$$

$$S_{su} = 0.67 S_{ut} = 163.4 \text{ kpsi}$$

$$S_{sy} = 0.35 S_{ut} = 85.4 \text{ kpsi}$$

$$S_y = 0.55 S_{ut} = 134.2 \text{ kpsi}$$

Table 10-8: $\quad S_r = 0.45 S_{ut} = 109.8$ kpsi

$$S_e = \frac{S_r/2}{1 - [S_r/(2S_{ut})]^2} = \frac{109.8/2}{1 - [(109.8/2)/243.9]^2} = 57.8 \text{ kpsi}$$

$$r = F_a/F_m = 4.5/13.5 = 0.333$$

Table 6-7: $\quad S_a = \dfrac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\dfrac{2S_e}{rS_{ut}}\right)^2}\,\right]$

$$S_a = \frac{(0.333)^2(243.9^2)}{2(57.8)}\left[-1 + \sqrt{1 + \left[\frac{2(57.8)}{0.333(243.9)}\right]^2}\,\right] = 42.2 \text{ kpsi}$$

Hook bending

$$(\sigma_a)_A = F_a\left[(K)_A \frac{16C}{\pi d^2} + \frac{4}{\pi d^2}\right] = \frac{S_a}{(n_f)_A} = \frac{S_a}{2}$$

$$\frac{4.5}{\pi d^2}\left[\frac{(4C^2 - C - 1)16C}{4C(C - 1)} + 4\right] = \frac{S_a}{2}$$

This equation reduces to a quadratic in C—see Prob. 10-28

The useable root for C is

$$C = 0.5\left[\frac{\pi d^2 S_a}{144} + \sqrt{\left(\frac{\pi d^2 S_a}{144}\right)^2 - \frac{\pi d^2 S_a}{36} + 2}\right]$$

$$= 0.5\left\{\frac{\pi(0.081)^2(42.2)(10^3)}{144} + \sqrt{\left[\frac{\pi(0.081)^2(42.2)(10^3)}{144}\right]^2 - \frac{\pi(0.081)^2(42.2)(10^3)}{36} + 2}\right\}$$

$$= 4.91$$

$$D = Cd = 0.398 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range.

$$F_i = \frac{\pi(0.081)^3}{8(0.398)}\left\{\frac{33\,500}{\exp[0.105(4.91)]} - 1000\left(4 - \frac{4.91-3}{6.5}\right)\right\}$$

$$= 8.55 \text{ lbf}$$

For simplicity we will round up to next $1/4$ integer.

$$F_i = 8.75 \text{ lbf}$$

$$k = \frac{18-9}{0.25} = 36 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.081)^4(10)(10^6)}{8(36)(0.398)^3} = 23.7 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 23.7 - \frac{10}{28} = 23.3 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.91) - 1 + 23.3](0.081) = 2.602 \text{ in}$$

$$L_{\max} = L_0 + (F_{\max} - F_i)/k = 2.602 + (18 - 8.75)/36 = 2.859 \text{ in}$$

$$(\sigma_a)_A = \frac{4.5(4)}{\pi d^2}\left(\frac{4C^2 - C - 1}{C-1} + 1\right)$$

$$= \frac{18(10^{-3})}{\pi(0.081^2)}\left[\frac{4(4.91^2) - 4.91 - 1}{4.91 - 1} + 1\right] = 21.1 \text{ kpsi}$$

$$(n_f)_A = \frac{S_a}{(\sigma_a)_A} = \frac{42.2}{21.1} = 2 \quad \text{checks}$$

Body: $$K_B = \frac{4C+2}{4C-3} = \frac{4(4.91)+2}{4(4.91)-3} = 1.300$$

$$\tau_a = \frac{8(1.300)(4.5)(0.398)}{\pi(0.081)^3}(10^{-3}) = 11.16 \text{ kpsi}$$

$$\tau_m = \frac{F_m}{F_a}\tau_a = \frac{13.5}{4.5}(11.16) = 33.47 \text{ kpsi}$$

The repeating allowable stress from Table 7-8 is

$$S_{sr} = 0.30 S_{ut} = 0.30(243.9) = 73.17 \text{ kpsi}$$

The Gerber intercept is

$$S_{se} = \frac{73.17/2}{1 - [(73.17/2)/163.4]^2} = 38.5 \text{ kpsi}$$

From Table 6-7,

$$(n_f)_{\text{body}} = \frac{1}{2}\left(\frac{163.4}{33.47}\right)^2 \left(\frac{11.16}{38.5}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(33.47)(38.5)}{163.4(11.16)}\right]^2}\right\} = 2.53$$

Let $r_2 = 2d = 2(0.081) = 0.162$

$$C_2 = \frac{2r_2}{d} = 4, \qquad (K)_B = \frac{4(4) - 1}{4(4) - 4} = 1.25$$

$$(\tau_a)_B = \frac{(K)_B}{K_B}\tau_a = \frac{1.25}{1.30}(11.16) = 10.73 \text{ kpsi}$$

$$(\tau_m)_B = \frac{(K)_B}{K_B}\tau_m = \frac{1.25}{1.30}(33.47) = 32.18 \text{ kpsi}$$

Table 10-8: $(S_{sr})_B = 0.28 S_{ut} = 0.28(243.9) = 68.3$ kpsi

$$(S_{se})_B = \frac{68.3/2}{1 - [(68.3/2)/163.4]^2} = 35.7 \text{ kpsi}$$

$$(n_f)_B = \frac{1}{2}\left(\frac{163.4}{32.18}\right)^2 \left(\frac{10.73}{35.7}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(32.18)(35.7)}{163.4(10.73)}\right]^2}\right\} = 2.51$$

Yield

Bending:

$$(\sigma_A)_{\max} = \frac{4F_{\max}}{\pi d^2}\left[\frac{(4C^2 - C - 1)}{C - 1} + 1\right]$$

$$= \frac{4(18)}{\pi(0.081^2)}\left[\frac{4(4.91)^2 - 4.91 - 1}{4.91 - 1} + 1\right](10^{-3}) = 84.4 \text{ kpsi}$$

$$(n_y)_A = \frac{134.2}{84.4} = 1.59$$

Body:

$$\tau_i = (F_i/F_a)\tau_a = (8.75/4.5)(11.16) = 21.7 \text{ kpsi}$$

$$r = \tau_a/(\tau_m - \tau_i) = 11.16/(33.47 - 21.7) = 0.948$$

$$(S_{sa})_y = \frac{r}{r+1}(S_{sy} - \tau_i) = \frac{0.948}{0.948 + 1}(85.4 - 21.7) = 31.0 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{(S_{sa})_y}{\tau_a} = \frac{31.0}{11.16} = 2.78$$

Hook shear:

$$S_{sy} = 0.3 S_{ut} = 0.3(243.9) = 73.2 \text{ kpsi}$$

$$\tau_{\max} = (\tau_a)_B + (\tau_m)_B = 10.73 + 32.18 = 42.9 \text{ kpsi}$$

$$(n_y)_B = \frac{73.2}{42.9} = 1.71$$

$$\text{fom} = -\frac{7.6\pi^2 d^2 (N_b + 2) D}{4} = -\frac{7.6\pi^2 (0.081)^2 (23.3 + 2)(0.398)}{4} = -1.239$$

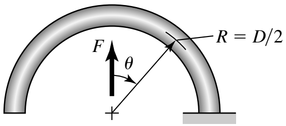
A tabulation of several wire sizes follow

d	0.081	0.085	0.092	0.098	0.105	0.12
S_{ut}	243.920	242.210	239.427	237.229	234.851	230.317
S_{su}	163.427	162.281	160.416	158.943	157.350	154.312
S_r	109.764	108.994	107.742	106.753	105.683	103.643
S_e	57.809	57.403	56.744	56.223	55.659	54.585
S_a	42.136	41.841	41.360	40.980	40.570	39.786
C	4.903	5.484	6.547	7.510	8.693	11.451
D	0.397	0.466	0.602	0.736	0.913	1.374
OD	0.478	0.551	0.694	0.834	1.018	1.494
F_i (calc)	8.572	7.874	6.798	5.987	5.141	3.637
F_i (rd)	8.75	9.75	10.75	11.75	12.75	13.75
k	36.000	36.000	36.000	36.000	36.000	36.000
N_a	23.86	17.90	11.38	8.03	5.55	2.77
N_b	23.50	17.54	11.02	7.68	5.19	2.42
L_0	2.617	2.338	2.127	2.126	2.266	2.918
$L_{18\,\text{lbf}}$	2.874	2.567	2.328	2.300	2.412	3.036
$(\sigma_a)_A$	21.068	20.920	20.680	20.490	20.285	19.893
$(n_f)_A$	2.000	2.000	2.000	2.000	2.000	2.000
K_B	1.301	1.264	1.216	1.185	1.157	1.117
$(\tau_a)_{\text{body}}$	11.141	10.994	10.775	10.617	10.457	10.177
$(\tau_m)_{\text{body}}$	33.424	32.982	32.326	31.852	31.372	30.532
S_{sr}	73.176	72.663	71.828	71.169	70.455	69.095
S_{se}	38.519	38.249	37.809	37.462	37.087	36.371
$(n_f)_{\text{body}}$	2.531	2.547	2.569	2.583	2.596	2.616
$(K)_B$	1.250	1.250	1.250	1.250	1.250	1.250
$(\tau_a)_B$	10.705	10.872	11.080	11.200	11.294	11.391
$(\tau_m)_B$	32.114	32.615	33.240	33.601	33.883	34.173
$(S_{sr})_B$	68.298	67.819	67.040	66.424	65.758	64.489
$(S_{se})_B$	35.708	35.458	35.050	34.728	34.380	33.717
$(n_f)_B$	2.519	2.463	2.388	2.341	2.298	2.235
S_y	134.156	133.215	131.685	130.476	129.168	126.674
$(\sigma_A)_{\text{max}}$	84.273	83.682	82.720	81.961	81.139	79.573
$(n_y)_A$	1.592	1.592	1.592	1.592	1.592	1.592
τ_i	21.663	23.820	25.741	27.723	29.629	31.097
r	0.945	1.157	1.444	1.942	2.906	4.703
$(S_{sy})_{\text{body}}$	85.372	84.773	83.800	83.030	82.198	80.611
$(S_{sa})_y$	30.958	32.688	34.302	36.507	39.109	40.832
$(n_y)_{\text{body}}$	2.779	2.973	3.183	3.438	3.740	4.012
$(S_{sy})_B$	73.176	72.663	71.828	71.169	70.455	69.095
$(\tau_B)_{\text{max}}$	42.819	43.486	44.321	44.801	45.177	45.564
$(n_y)_B$	1.709	1.671	1.621	1.589	1.560	1.516
fom	−1.246	−1.234	−1.245	−1.283	−1.357	−1.639

optimal fom

The shaded areas show the conditions not satisfied.

10-31 For the hook,



$$M = FR\sin\theta, \quad \partial M/\partial F = R\sin\theta$$

$$\delta_F = \frac{1}{EI}\int_0^{\pi/2} FR^2 \sin^2 R\, d\theta = \frac{\pi}{2}\frac{PR^3}{EI}$$

The total deflection of the body and the two hooks

$$\delta = \frac{8FD^3N_b}{d^4G} + 2\frac{\pi}{2}\frac{FR^3}{EI} = \frac{8FD^3N_b}{d^4G} + \frac{\pi F(D/2)^3}{E(\pi/64)(d^4)}$$

$$= \frac{8FD^3}{d^4G}\left(N_b + \frac{G}{E}\right) = \frac{8FD^3N_a}{d^4G}$$

$$\therefore \quad N_a = N_b + \frac{G}{E} \quad \text{QED}$$

10-32 Table 10-4 for A227:

$$A = 140 \text{ kpsi} \cdot \text{in}^m, \quad m = 0.190$$

Table 10-5:

$$E = 28.5(10^6) \text{ psi}$$

$$S_{ut} = \frac{140}{(0.162)^{0.190}} = 197.8 \text{ kpsi}$$

Eq. (10-57):

$$S_y = \sigma_{\text{all}} = 0.78(197.8) = 154.3 \text{ kpsi}$$

$$D = 1.25 - 0.162 = 1.088 \text{ in}$$

$$C = D/d = 1.088/0.162 = 6.72$$

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4(6.72)^2 - 6.72 - 1}{4(6.72)(6.72 - 1)} = 1.125$$

From

$$\sigma = K_i \frac{32M}{\pi d^3}$$

Solving for *M* for the yield condition,

$$M_y = \frac{\pi d^3 S_y}{32K_i} = \frac{\pi(0.162)^3(154\,300)}{32(1.125)} = 57.2 \text{ lbf} \cdot \text{in}$$

Count the turns when $M = 0$

$$N = 2.5 - \frac{M_y}{d^4E/(10.8DN)}$$

from which

$$N = \frac{2.5}{1 + [10.8DM_y/(d^4E)]}$$

$$= \frac{2.5}{1 + \{[10.8(1.088)(57.2)]/[(0.162)^4(28.5)(10^6)]\}} = 2.417 \text{ turns}$$

This means $(2.5 - 2.417)(360^\circ)$ or 29.9° from closed. Treating the hand force as in the middle of the grip

$$r = 1 + \frac{3.5}{2} = 2.75 \text{ in}$$

$$F = \frac{M_y}{r} = \frac{57.2}{2.75} = 20.8 \text{ lbf} \quad \textit{Ans.}$$

10-33 The spring material and condition are unknown. Given $d = 0.081$ in and $\text{OD} = 0.500$,

(a) $D = 0.500 - 0.081 = 0.419$ in

Using $E = 28.6$ Mpsi for an estimate

$$k' = \frac{d^4 E}{10.8 D N} = \frac{(0.081)^4 (28.6)(10^6)}{10.8(0.419)(11)} = 24.7 \text{ lbf} \cdot \text{in/turn}$$

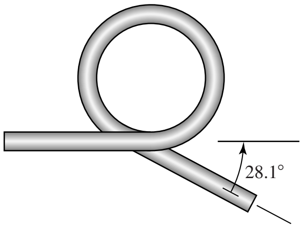
for each spring. The moment corresponding to a force of 8 lbf

$$Fr = (8/2)(3.3125) = 13.25 \text{ lbf} \cdot \text{in/spring}$$

The fraction windup turn is

$$n = \frac{Fr}{k'} = \frac{13.25}{24.7} = 0.536 \text{ turns}$$

The arm swings through an arc of slightly less than 180°, say 165°. This uses up $165/360$ or 0.458 turns. So $n = 0.536 - 0.458 = 0.078$ turns are left (or $0.078(360^\circ) = 28.1^\circ$). The original configuration of the spring was



Ans.

(b)

$$C = \frac{0.419}{0.081} = 5.17$$

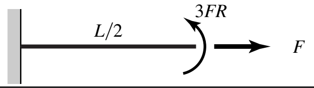
$$K_i = \frac{4(5.17)^2 - 5.17 - 1}{4(5.17)(5.17 - 1)} = 1.168$$

$$\sigma = K_i \frac{32M}{\pi d^3}$$

$$= 1.168 \left[\frac{32(13.25)}{\pi (0.081)^3} \right] = 296\,623 \text{ psi} \quad \textit{Ans.}$$

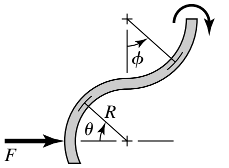
To achieve this stress level, the spring had to have set removed.

10-34 Consider half and double results

Straight section: 

$M = 3FR, \quad \frac{\partial M}{\partial P} = 3R$

Upper 180° section:



$$M = F[R + R(1 - \cos\phi)]$$

$$= FR(2 - \cos\phi), \qquad \frac{\partial M}{\partial P} = R(2 - \cos\phi)$$

Lower section:

$$M = FR\sin\theta$$

$$\frac{\partial M}{\partial P} = R\sin\theta$$

Considering bending only:

$$\delta = \frac{2}{EI}\left[\int_0^{L/2} 9FR^2\,dx + \int_0^{\pi} FR^2(2 - \cos\phi)^2 R\,d\phi + \int_0^{\pi/2} F(R\sin\theta)^2 R\,d\theta\right]$$

$$= \frac{2F}{EI}\left[\frac{9}{2}R^2L + R^3\left(4\pi - 4\sin\phi\Big|_0^{\pi} + \frac{\pi}{2}\right) + R^3\left(\frac{\pi}{4}\right)\right]$$

$$= \frac{2FR^2}{EI}\left(\frac{19\pi}{4}R + \frac{9}{2}L\right) = \frac{FR^2}{2EI}(19\pi R + 18L) \quad \textit{Ans.}$$

10-35 Computer programs will vary.

10-36 Computer programs will vary.