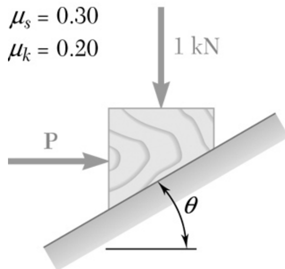
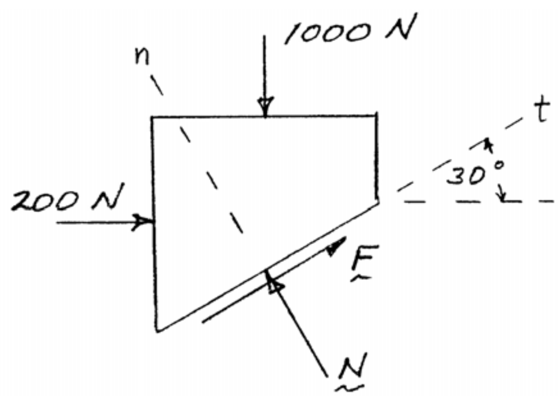


PROBLEM 8.1

Determine whether the block shown is in equilibrium, and find the magnitude and direction of the friction force when $\theta = 30^\circ$ and $P = 200\,\text{N}$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (1000\text{ N})\cos 30^\circ - (200\text{ N})\sin 30^\circ = 0$$

$$N = 966.03\text{ N}$$

Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad F + (200\text{ N})\cos 30^\circ - (1000\text{ N})\sin 30^\circ = 0$$

$$F = 326.8\text{ N} = F_{\text{eq.}}$$

But $\qquad F_{\max} = \mu_s N = (0.3)966\text{ N} = 290\text{ N}$

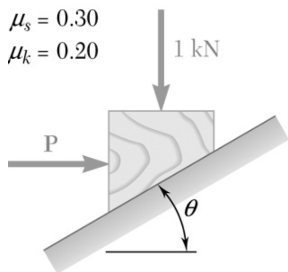
$F_{\text{eq.}} > F_{\max}$ $\qquad$ impossible $\Rightarrow$ Block moves ◀

and $\qquad F = \mu_k N$

$$= (0.2)(966.03\text{ N})$$

Block slides down $\qquad$ $\mathbf{F} = 193.2\text{ N} \nearrow$ ◀

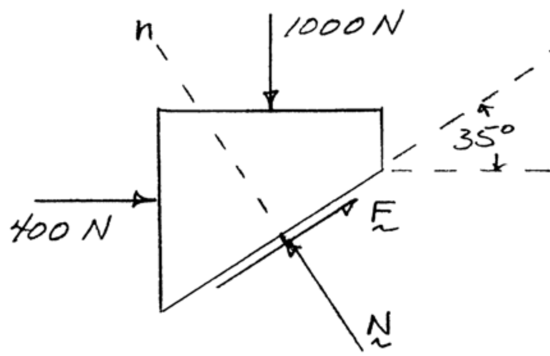
PROBLEM 8.2



Determine whether the block shown is in equilibrium, and find the magnitude and direction of the friction force when $\theta = 35^\circ$ and $P = 400$ N.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (1000\text{ N})\cos 35^\circ - (400\text{ N})\sin 35^\circ = 0$$

$$N = 1048.6\text{ N}$$

Assume equilibrium:

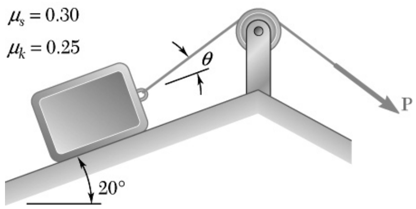
$$\nearrow \Sigma F_t = 0: \quad F - (1000\text{ N})\sin 35^\circ + (400\text{ N})\cos 35^\circ = 0$$

$$F = 246\text{ N} = F_{eq.}$$

$$F_{max} = \mu_s N = (0.3)(1048.6\text{ N}) = 314\text{ N}$$

$F_{eq.} < F_{max}$ OK equilibrium ◀

$\therefore$ $\mathbf{F} = 246$ N $\nearrow$ ◀

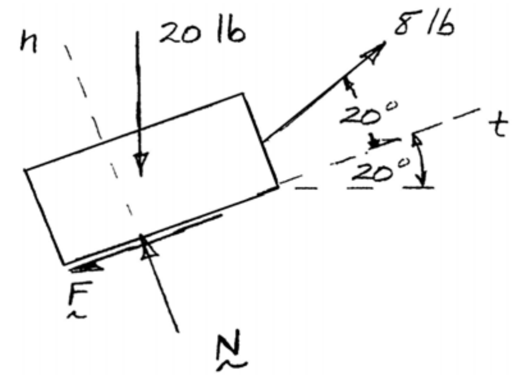


PROBLEM 8.3

Determine whether the 20-lb block shown is in equilibrium, and find the magnitude and direction of the friction force when $P = 8$ lb and $\theta = 20°$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (20 \text{ lb})\cos 20° + (8 \text{ lb})\sin 20° = 0$$

$$N = 16.0577 \text{ lb}$$

$$F_{max} = \mu_s N = (0.3)(16.0577 \text{ lb}) = 4.817 \text{ lb}$$

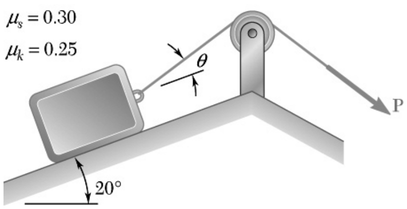
Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad (8 \text{ lb})\cos 20° - (20 \text{ lb})\sin 20° - F = 0$$

$$F = 0.6771 \text{ lb} = F_{eq.}$$

$F_{eq.} < F_{max}$ OK equilibrium ◀

and $\mathbf{F} = 0.677$ lb ↙ ◀

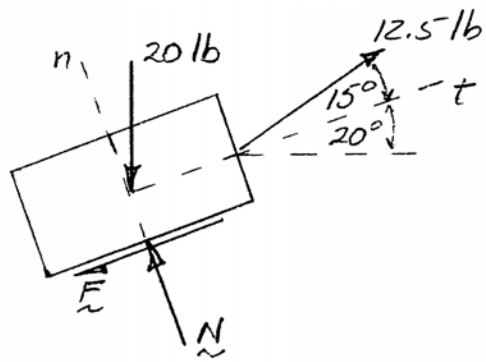


PROBLEM 8.4

Determine whether the 20-lb block shown is in equilibrium, and find the magnitude and direction of the friction force when $P = 12.5$ lb and $\theta = 15°$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (20 \text{ lb})\cos 20° + (12.5 \text{ lb})\sin 15° = 0$$

$$N = 15.559 \text{ lb}$$

$$F_{\max} = \mu_s N = (0.3)(15.559 \text{ lb}) = 4.668 \text{ lb}$$

Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad (12.5 \text{ lb})\cos 15° - (20 \text{ lb})\sin 20° - F = 0$$

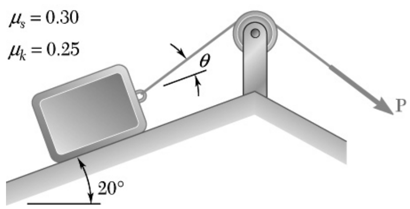
$$F = 5.23 \text{ lb} = F_{\text{eq.}}$$

but $F_{\text{eq.}} > F_{\max}$ impossible, so block slides up ◀

and $F = \mu_k N = (0.25)(15.559 \text{ lb})$

F = 3.89 lb ↙ ◀

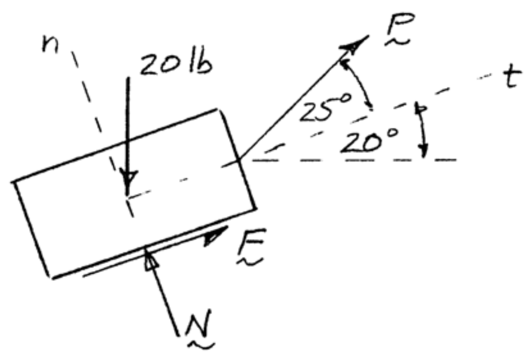
PROBLEM 8.5



Knowing that $\theta = 25°$, determine the range of values of P for which equilibrium is maintained.

SOLUTION

FBD block:



Block is in equilibrium:

$$\nwarrow \Sigma F_n = 0: \quad N - (20\text{ lb})\cos 20° + P\sin 25° = 0$$

$$N = 18.794\text{ lb} - P\sin 25°$$

$$\nearrow \Sigma F_t = 0: \quad F - (20\text{ lb})\sin 20° + P\cos 25° = 0$$

or

$$F = 6.840\text{ lb} - P\cos 25°$$

Impending motion up: $F = \mu_s N$; Impending motion down: $F = -\mu_s N$

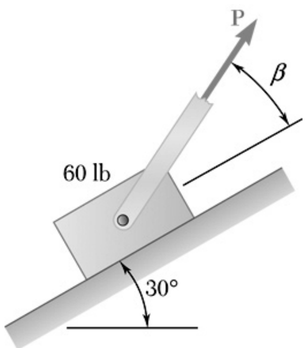
Therefore,

$$6.840\text{ lb} - P\cos 25° = \pm(0.3)(18.794\text{ lb} - P\sin 25°)$$

$$P_{\text{up}} = 12.08\text{ lb} \qquad P_{\text{down}} = 1.542\text{ lb}$$

$$1.542\text{ lb} \le P_{\text{eq.}} \le 12.08\text{ lb} ◀$$

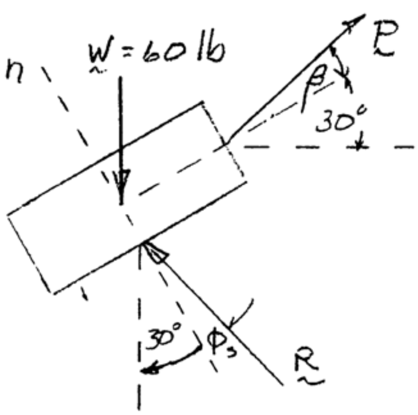
PROBLEM 8.6



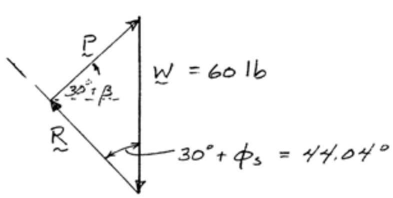
Knowing that the coefficient of friction between the 60-lb block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of P for which motion of the block up the incline is impending, (*b*) the corresponding value of β.

SOLUTION

FBD block (impending motion up)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.04°$$



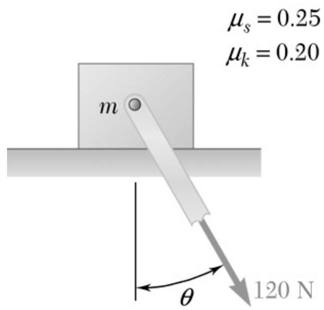
(*a*) Note: For minimum P, $\mathbf{P} \perp \mathbf{R}$ so $\beta = \phi_s$

Then $P = W\sin(30° + \phi_s)$

$$= (60 \text{ lb})\sin 44.04° = 41.71 \text{ lb}$$

$P_{min} = 41.7$ lb ◀

(*b*) Have $\beta = \phi_s$ $\beta = 14.04°$ ◀

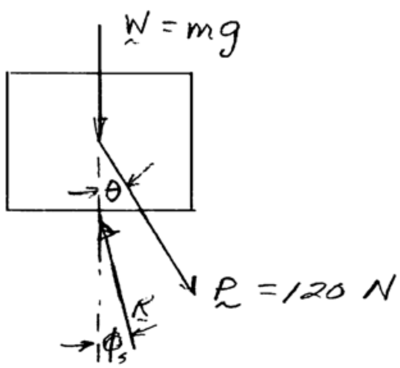


PROBLEM 8.7

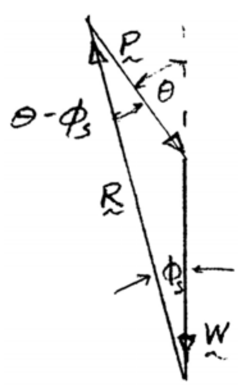
Considering only values of θ less than $90°$, determine the smallest value of θ for which motion of the block to the right is impending when (*a*) $m = 30$ kg, (*b*) $m = 40$ kg.

SOLUTION

FBD block (impending motion to the right)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$



$$\frac{P}{\sin\phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

$$\sin(\theta - \phi_s) = \frac{W}{P}\sin\phi_s \qquad W = mg$$

(*a*) $$m = 30\text{ kg:}\quad \theta - \phi_s = \sin^{-1}\left[\frac{(30\text{ kg})(9.81\text{ m/s}^2)}{120\text{ N}}\sin 14.036°\right]$$

$$= 36.499°$$

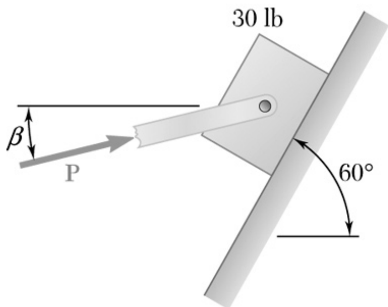
$$\therefore\quad \theta = 36.499° + 14.036° \qquad \text{or}\quad \theta = 50.5° \blacktriangleleft$$

(*b*) $$m = 40\text{ kg:}\quad \theta - \phi_s = \sin^{-1}\left[\frac{(40\text{ kg})(9.81\text{ m/s}^2)}{120\text{ N}}\sin 14.036°\right]$$

$$= 52.474°$$

$$\therefore\quad \theta = 52.474° + 14.036° \qquad \text{or}\quad \theta = 66.5° \blacktriangleleft$$

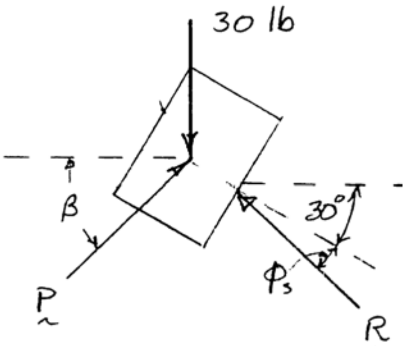
PROBLEM 8.8



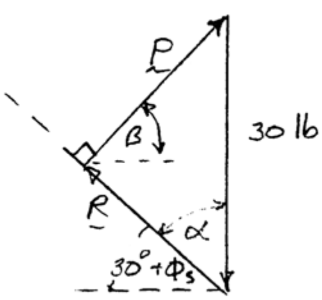
Knowing that the coefficient of friction between the 30-lb block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of P required to maintain the block in equilibrium, (*b*) the corresponding value of β.

SOLUTION

FBD block (impending motion downward)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$



(*a*) Note: For minimum P, **P** $\perp$ **R**

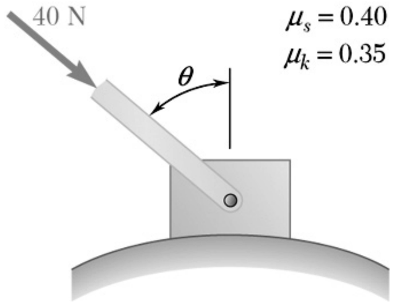
So $$\beta = \alpha = 90° - (30° + 14.036°) = 45.964°$$

and $$P = (30\text{ lb})\sin\alpha = (30\text{ lb})\sin(45.964°) = 21.567\text{ lb}$$

$P = 21.6$ lb ◀

(*b*) $\beta = 46.0°$ ◀

PROBLEM 8.9

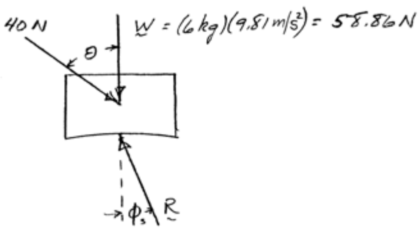


$\mu_s = 0.40$
$\mu_k = 0.35$

A 6-kg block is at rest as shown. Determine the positive range of values of θ for which the block is in equilibrium if (*a*) θ is less than 90°, (*b*) θ is between 90° and 180°.

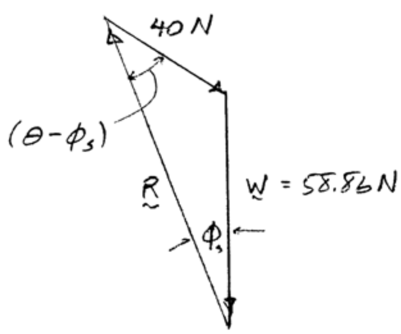
SOLUTION

FBD block (impending motion)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.4) = 21.801°$$

(*a*) $0° \le \theta \le 90°$:



$$\frac{58.86\text{ N}}{\sin(\theta - \phi_s)} = \frac{40\text{ N}}{\sin\phi_s}$$

$$\theta - \phi_s = \sin^{-1}\frac{58.86\text{ N}}{40\text{ N}}\sin(21.801°)$$

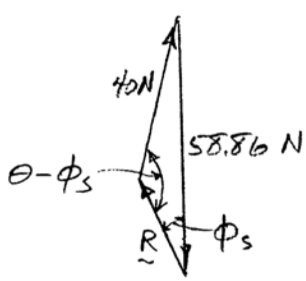
$$= 33.127°,\ 146.873°$$

$$\theta = 54.9° \quad \text{and} \quad \theta = 168.674°$$

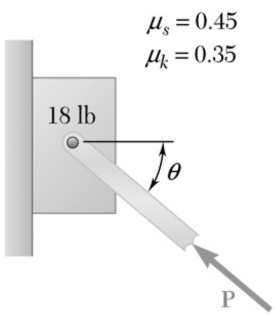
∴ (*a*) Equilibrium for $0 \le \theta \le 54.9°$ ◀

(*b*) and for $168.7° \le \theta \le 180.0°$ ◀

(*b*) $90° \le \theta \le 180°$:



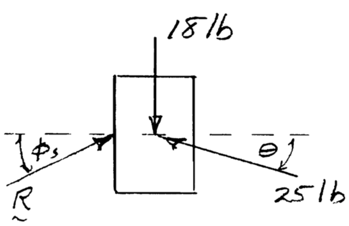
PROBLEM 8.10

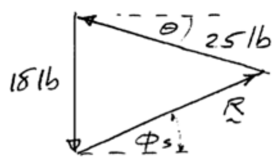


Knowing that $P = 25$ lb, determine the range of values of θ for which equilibrium of the 18-lb block is maintained.

SOLUTION

FBD block (impending motion down)





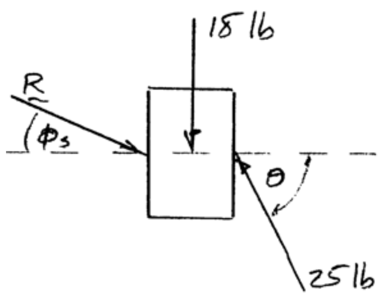
$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.45) = 24.228°$$

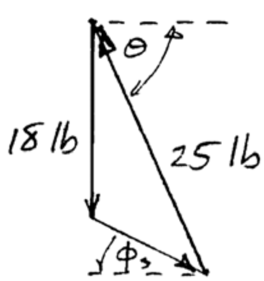
$$\frac{25 \text{ lb}}{\sin(90° - \phi_s)} = \frac{18 \text{ lb}}{\sin(\theta + \phi_s)}$$

$$\theta + \phi_s = \sin^{-1}\left[\frac{18 \text{ lb}}{25 \text{ lb}}\sin(90° - 24.228°)\right] = 41.04°$$

$$\theta = 16.81°$$

Impending motion up:





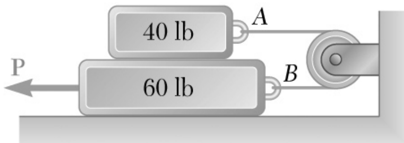
$$\frac{25 \text{ lb}}{\sin(90° + \phi_s)} = \frac{18 \text{ lb}}{\sin(\theta - \phi_s)}$$

$$\theta - \phi_s = \sin^{-1}\left[\frac{18 \text{ lb}}{25 \text{ lb}}\sin(90° + 24.228°)\right] = 41.04°$$

$$\theta = 65.27°$$

Equilibrium for $16.81° \le \theta \le 65.3°$ ◀

PROBLEM 8.11

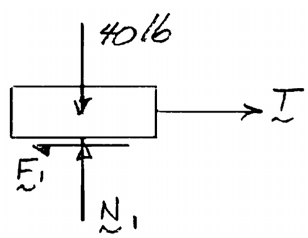


The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the force **P** for which motion of the 60-lb block is impending if cable AB (a) is attached as shown, (b) is removed.

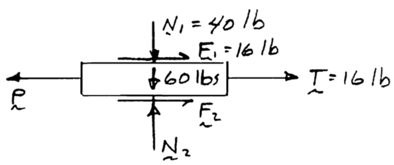
SOLUTION

FBDs

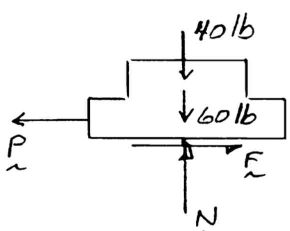
Top block:



Bottom block:



FBD blocks:



(a) Note: With the cable, motion must impend at both contact surfaces.

$$\uparrow \Sigma F_y = 0: \quad N_1 - 40 \text{ lb} = 0 \qquad N_1 = 40 \text{ lb}$$

Impending slip: $F_1 = \mu_s N_1 = 0.4(40 \text{ lb}) = 16 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad T - F_1 = 0 \qquad T - 16 \text{ lb} = 0 \qquad T = 16 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad N_2 - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N_2 = 100 \text{ lb}$$

Impending slip: $F_2 = \mu_s N_2 = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad -P + 16 \text{ lb} + 16 \text{ lb} + 40 \text{ lb} = 0$$

$\mathbf{P} = 72.0 \text{ lb} \longleftarrow$ ◀

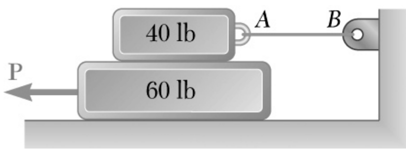
(b) Without the cable, both blocks will stay together and motion will impend only at the floor.

$$\uparrow \Sigma F_y = 0: \quad N - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N = 100 \text{ lb}$$

Impending slip: $F = \mu_s N = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad 40 \text{ lb} - P = 0$$

$\mathbf{P} = 40.0 \text{ lb} \longleftarrow$ ◀



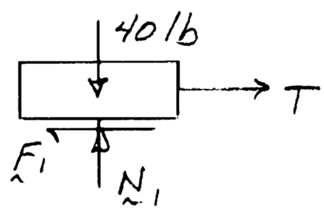
PROBLEM 8.12

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the force **P** for which motion of the 60-lb block is impending if cable AB (*a*) is attached as shown, (*b*) is removed.

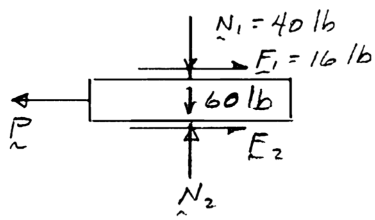
SOLUTION

FBDs

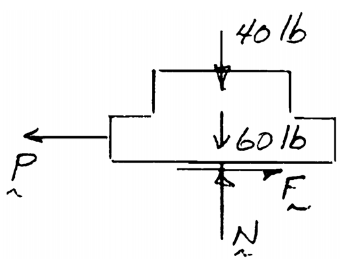
Top block:



Bottom block:



FBD blocks:



(*a*) With the cable, motion must impend at both surfaces.

$$\uparrow \Sigma F_y = 0: \quad N_1 - 40\text{ lb} = 0 \qquad N_1 = 40\text{ lb}$$

Impending slip: $F_1 = \mu_s N_1 = 0.4(40\text{ lb}) = 16\text{ lb}$

$$\uparrow \Sigma F_y = 0: \quad N_2 - 40\text{ lb} - 60\text{ lb} = 0 \qquad N_2 = 100\text{ lb}$$

Impending slip: $F_2 = \mu N_2 = 0.4(100\text{ lb}) = 40\text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad 16\text{ lb} + 40\text{ lb} - P = 0 \qquad P = 56\text{ lb}$$

$\mathbf{P} = 56.0\text{ lb} \longleftarrow$ ◀

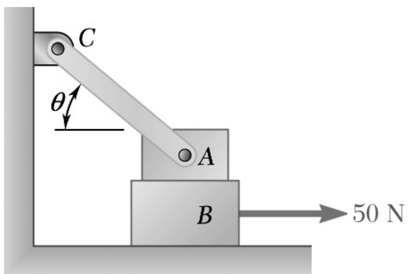
(*b*) Without the cable, both blocks stay together and motion will impend at the floor surface only.

$$\uparrow \Sigma F_y = 0: \quad N - 40\text{ lb} - 60\text{ lb} = 0 \qquad N = 100\text{ lb}$$

Impending slip: $F = \mu_s N = 0.4(100\text{ lb}) = 40\text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad -P + 40\text{ lb} = 0 \qquad P = 40\text{ lb}$$

$\mathbf{P} = 40.0\text{ lb} \longleftarrow$ ◀

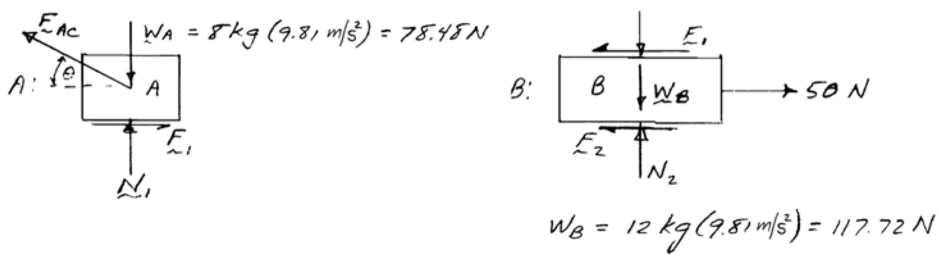


PROBLEM 8.13

The 8-kg block A is attached to link AC and rests on the 12-kg block B. Knowing that the coefficient of static friction is 0.20 between all surfaces of contact and neglecting the mass of the link, determine the value of θ for which motion of block B is impending.

SOLUTION

FBDs:



Motion must impend at both contact surfaces

Block A: $\uparrow \Sigma F_y = 0: \quad N_1 - W_A = 0 \qquad N_1 = W_A$

Block B: $\uparrow \Sigma F_y = 0: \quad N_2 - N_1 - W_B = 0$

$$N_2 = N_1 + W_B = W_A + W_B$$

Impending motion:

$$F_1 = \mu_s N_1 = \mu_s W_A$$

$$F_2 = \mu_s N_2 = \mu_s (N_1 + W_B)$$

Block B: $\rightarrow \Sigma F_x = 0: \quad 50\text{ N} - F_1 - F_2 = 0$

or

$$50\text{ N} = \mu_s (N_1 + N_1 + W_B) = 0.2(2N_1 + 117.72\text{ N})$$

$$N_1 = 66.14\text{ N} \qquad F_1 = 0.2(66.14\text{ N}) = 13.228\text{ N}$$

Block A: $\rightarrow \Sigma F_x = 0: \quad 13.228\text{ N} - F_{AC}\cos\theta = 0$

or

$$F_{AC}\cos\theta = 13.228\text{ N} \qquad (1)$$

$$\uparrow \Sigma F_y = 0: \quad 66.14\text{ N} - 78.48\text{ N} + F_{AC}\sin\theta = 0$$

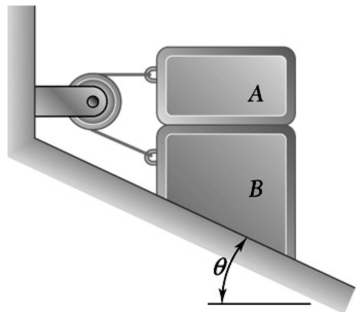
or

$$F_{AC}\sin\theta = 78.48\text{ N} - 66.14\text{ N} \qquad (2)$$

Then, $\dfrac{\text{Eq. (2)}}{\text{Eq. (1)}}$

$$\tan\theta = \frac{78.48\text{ N} - 66.14\text{ N}}{13.228\text{ N}}$$

$\theta = 43.0°$ ◀

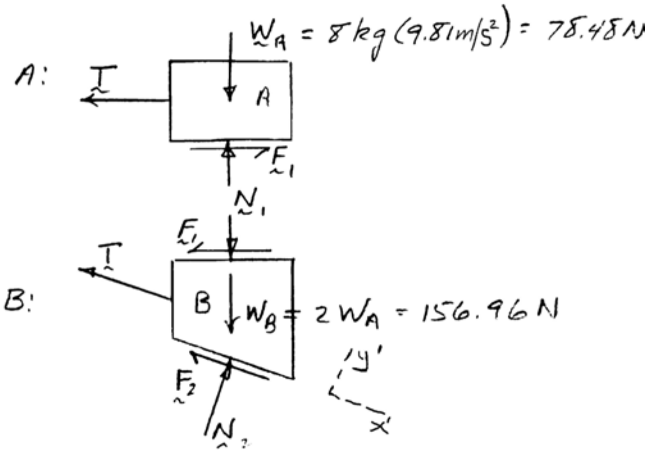


PROBLEM 8.14

The 8-kg block A and the 16-kg block B are at rest on an incline as shown. Knowing that the coefficient of static friction is 0.25 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

FBDs:



Block A: $\uparrow \Sigma F_y = 0$: $N_1 - W_A = 0 \qquad N_1 = W_A$

Impending motion: $F_1 = \mu_s N_1 = \mu_s W_A$

$\rightarrow \Sigma F_x = 0$: $F_1 - T = 0 \qquad T = F_1 = \mu_s W_A$

Block B: $\nearrow \Sigma F_{y'} = 0$: $N_2 - (N_1 + W_B)\cos\theta - F_1 \sin\theta = 0$

$$N_2 = 3W_A \cos\theta + \mu_s W_A \sin\theta$$

$$= W_A(3\cos\theta + 0.25\sin\theta)$$

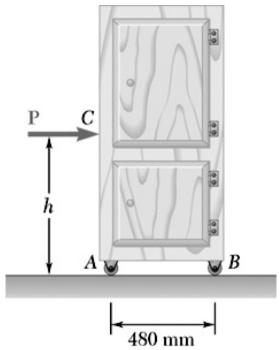
Impending motion: $F_2 = \mu_s N_2 = 0.25 W_A(3\cos\theta + 0.25\sin\theta)$

$\searrow \Sigma F_{x'} = 0$: $-T - F_2 - F_1\cos\theta + (N_1 + W_B)\sin\theta = 0$

$$\left[-0.25 - 0.25(3\cos\theta + 0.25\sin\theta) - 0.25\cos\theta + 3\sin\theta\right]W_A = 0$$

or $$47\sin\theta - 16\cos\theta - 4 = 0$$

Solving numerically $\theta = 23.4°$ ◀

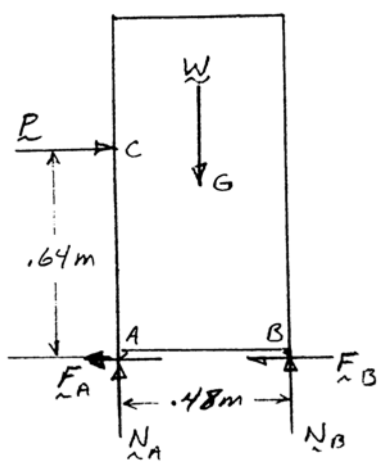


PROBLEM 8.15

A 48-kg cabinet is mounted on casters which can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Knowing that $h = 640$ mm, determine the magnitude of the force **P** required for impending motion of the cabinet to the right (*a*) if all casters are locked, (*b*) if the casters at *B* are locked and the casters at *A* are free to rotate, (*c*) if the casters at *A* are locked and the casters at *B* are free to rotate.

SOLUTION

FBD cabinet:



$$W = 48 \text{ kg}\left(9.81 \text{ m/s}^2\right)$$
$$= 470.88 \text{ N}$$
$$\mu_s = 0.3$$

Note: For tipping, $N_A = F_A = 0$

$$\circlearrowleft \Sigma M_B = 0: \quad (0.24 \text{ m})W - (0.64 \text{ m})P_{\text{tip}} = 0 \qquad P_{\text{tip}} = 0.375W$$

(*a*) All casters locked: Impending slip: $F_A = \mu_s N_A,\ F_B = \mu_s N_B$

$$\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0 \qquad N_A + N_B = W$$

So $F_A + F_B = \mu_s W$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0 \qquad P = F_A + F_B = \mu_s W$$

$$\therefore \quad P = 0.3(470.88 \text{ N}) \qquad \text{or} \qquad P = 141.3 \text{ N} \blacktriangleleft$$

$$\left(P = 0.3W < P_{\text{tip}} \qquad \text{OK}\right)$$

(*b*) Casters at *A* free, so $F_A = 0$

Impending slip: $F_B = \mu_s N_B$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_B = 0$$

$$P = F_B = \mu_s N_B \qquad N_B = \frac{P}{\mu_s}$$

$$\circlearrowleft \Sigma M_A = 0: \quad (0.64 \text{ m})P + (0.24 \text{ m})W - (0.48 \text{ m})N_B = 0$$

$$8P + 3W - 6\frac{P}{0.3} = 0 \qquad P = 0.25W$$

$$\left(P = 0.25W < P_{\text{tip}} \qquad \text{OK}\right)$$

$$\therefore \quad P = 0.25(470.88 \text{ N}) \qquad P = 117.7 \text{ N} \blacktriangleleft$$

PROBLEM 8.15 CONTINUED

(*c*) Casters at *B* free, so $F_B = 0$

Impending slip: $F_A = \mu_s N_A$

$\longrightarrow \Sigma F_x = 0\text{:} \quad P - F_A = 0 \qquad P = F_A = \mu_s N_A$

$$N_A = \frac{P}{\mu_s} = \frac{P}{0.3}$$

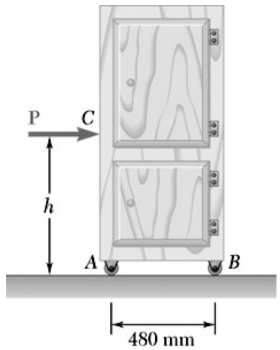
$\circlearrowright\, \Sigma M_B = 0\text{:} \quad (0.24\text{ m})W - (0.64\text{ m})P - (0.48\text{ m})N_A = 0$

$$3W - 8P - 6\frac{P}{0.3} = 0 \qquad P = 0.10714W = 50.45\text{ N}$$

$$\left(P < P_{\text{tip}} \qquad \text{OK}\right)$$

$P = 50.5\text{ N}$ ◀

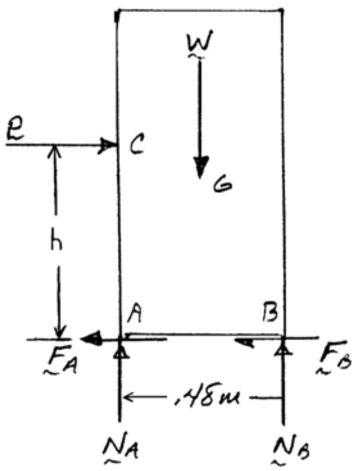
PROBLEM 8.16



A 48-kg cabinet is mounted on casters which can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Assuming that the casters at A and B are locked, determine (a) the force **P** required for impending motion of the cabinet to the right, (b) the largest allowable height h if the cabinet in not to tip over.

SOLUTION

FBD cabinet:



$$W = 48\text{ kg}\left(9.81\text{ m/s}^2\right)$$

$$= 470.88\text{ N}$$

(a) $\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0; \qquad N_A + N_B = W$

Impending slip: $\quad F_A = \mu_s N_A,\ F_B = \mu_s N_B$

So $\quad F_A + F_B = \mu_s W$

$\longrightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0 \qquad P = F_A + F_B = \mu_s W$

$$P = 0.3\left(470.88\text{ N}\right) = 141.26\text{ N}$$

$$\mathbf{P} = 141.3\text{ N} \longrightarrow \blacktriangleleft$$

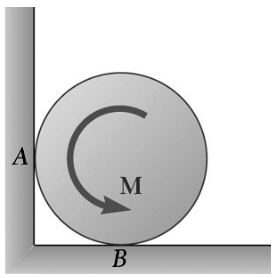
(b) For tipping, $\quad N_A = F_A = 0$

$\circlearrowleft \Sigma M_B = 0: \quad hP - \left(0.24\text{ m}\right)W = 0$

$$h_{\max} = \left(0.24\text{ m}\right)\frac{W}{P} = \left(0.24\text{ m}\right)\frac{1}{\mu_s} = \frac{0.24\text{ m}}{0.3}$$

$$h_{\max} = 0.800\text{ m} \blacktriangleleft$$

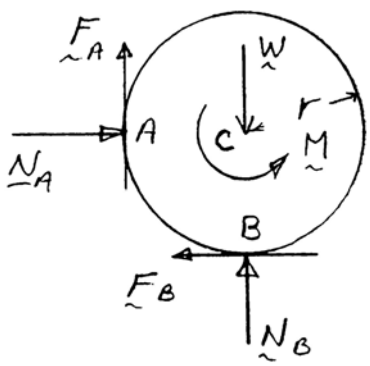
PROBLEM 8.17



The cylinder shown is of weight W and radius r, and the coefficient of static friction μ_s is the same at A and B. Determine the magnitude of the largest couple **M** which can be applied to the cylinder if it is not to rotate.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_s N_A, \; F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - F_B = 0 \qquad N_A = F_B = \mu_s N_B$$

$$F_A = \mu_s N_A = \mu_s^2 N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0 \qquad N_B + \mu_s^2 N_B = W$$

or

$$N_B = \frac{W}{1 + \mu_s^2}$$

and

$$F_B = \frac{\mu_s W}{1 + \mu_s^2}$$

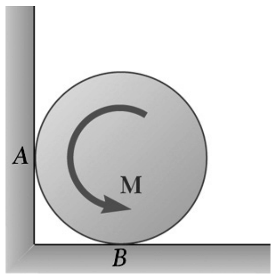
$$F_A = \frac{\mu_s^2 W}{1 + \mu^2}$$

$$\Sigma M_C = 0: \quad M - r\left(F_A + F_B\right) = 0$$

$$M = r\left(\mu_s + \mu_s^2\right)\frac{W}{1 + \mu_s^2}$$

$$M_{\max} = Wr\mu_s \frac{1 + \mu_s}{1 + \mu_s^2} \blacktriangleleft$$

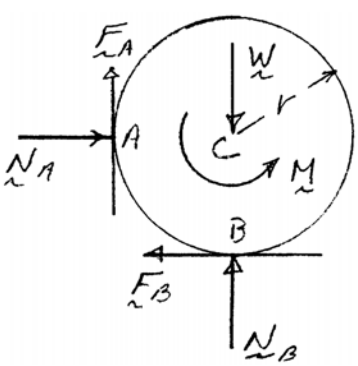
PROBLEM 8.18



The cylinder shown is of weight W and radius r. Express in terms of W and r the magnitude of the largest couple **M** which can be applied to the cylinder if it is not to rotate assuming that the coefficient of static friction is (a) zero at A and 0.36 at B, (b) 0.30 at A and 0.36 at B.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_A N_A; \qquad F_B = \mu_B N_B$$

$$\longrightarrow \Sigma F_x = 0: \quad N_A - F_B = 0 \qquad N_A = F_B = \mu_B N_B$$

$$F_A = \mu_A N_A = \mu_A \mu_B N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0 \qquad N_B\left(1 + \mu_A \mu_B\right) = W$$

or

$$N_B = \frac{1}{1 + \mu_A \mu_B} W$$

and

$$F_B = \mu_B N_B = \frac{\mu_B}{1 + \mu_A \mu_B} W$$

$$F_A = \mu_A \mu_B N_B = \frac{\mu_A \mu_B}{1 + \mu_A \mu_B} W$$

$$\circlearrowleft \Sigma M_C = 0: \quad M - r\left(F_A + F_B\right) = 0 \qquad M = Wr\mu_B \frac{1 + \mu_A}{1 + \mu_A \mu_B}$$

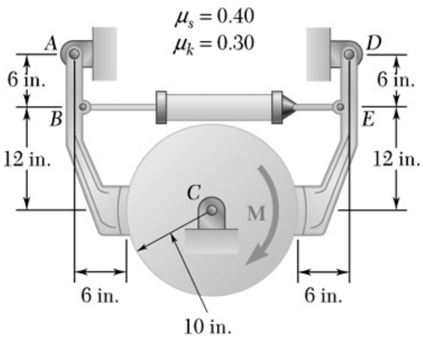
(a) For $\quad \mu_A = 0 \quad$ and $\quad \mu_B = 0.36$

$$M = 0.360Wr \blacktriangleleft$$

(b) For $\quad \mu_A = 0.30 \quad$ and $\quad \mu_B = 0.36$

$$M = 0.422Wr \blacktriangleleft$$

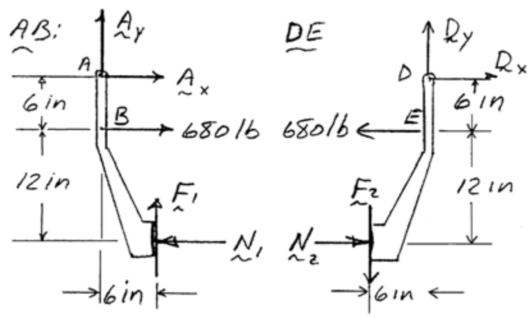
PROBLEM 8.19



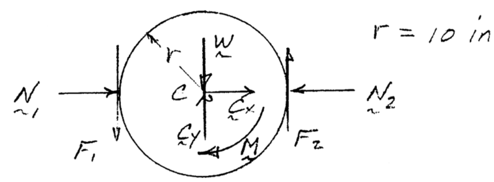
The hydraulic cylinder shown exerts a force of 680 lb directed to the right on point *B* and to the left on point *E*. Determine the magnitude of the couple **M** required to rotate the drum clockwise at a constant speed.

SOLUTION

FBDs



Drum:



Rotating drum ⇒ slip at both sides; constant speed ⇒ equilibrium

$$\therefore \quad F_1 = \mu_k N_1 = 0.3N_1; \qquad F_2 = \mu_k N_2 = 0.3N_2$$

AB: $\quad \circlearrowleft \Sigma M_A = 0\text{:} \quad (6\text{ in.})(680\text{ lb}) + (6\text{ in.})(F_1) - (18\text{ in.})N_1 = 0$

$$F_1\left(\frac{18\text{ in.}}{0.3} - 6\text{ in.}\right) = (6\text{ in.})(680\text{ lb}) \quad \text{or} \quad F_1 = 75.555\text{ lb}$$

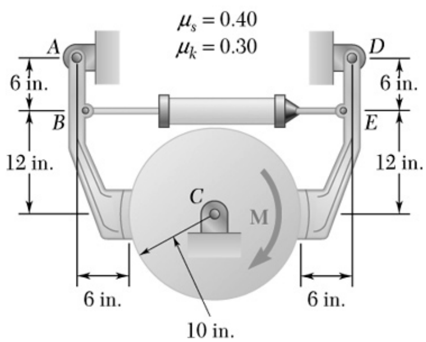
DE: $\quad \circlearrowleft \Sigma M_D = 0\text{:} \quad (6\text{ in.})F_2 + (18\text{ in.})N_2 - (6\text{ in.})(680\text{ lb}) = 0$

$$F_2\left(6\text{ in.} + \frac{18\text{ in.}}{0.3}\right) = (6\text{ in.})(680\text{ lb}) \quad \text{or} \quad F_2 = 61.818\text{ lb}$$

Drum: $\quad \circlearrowleft \Sigma M_C = 0\text{:} \quad r(F_1 + F_2) - M = 0$

$$M = (10\text{ in.})(75.555 + 61.818)\text{lb}$$

$M = 1374$ lb·in. ◀

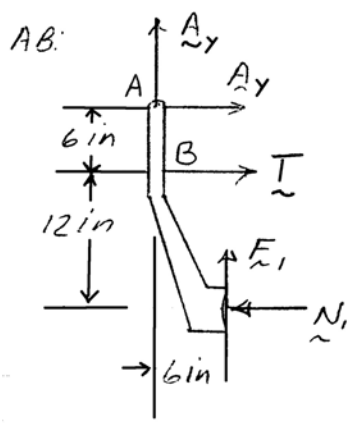


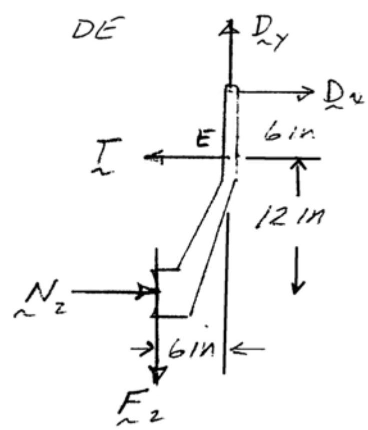
PROBLEM 8.20

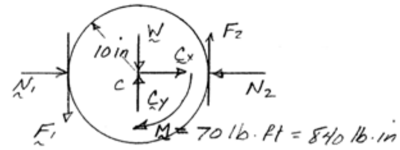
A couple **M** of magnitude 70 lb·ft is applied to the drum as shown. Determine the smallest force which must be exerted by the hydraulic cylinder on joints *B* and *E* if the drum is not to rotate.

SOLUTION

FBDs **DE:** **Drum:**







For minimum *T*, slip impends at both sides, so

$$F_1 = \mu_s N_1 = 0.4N_1 \qquad F_2 = \mu_s N_2 = 0.4N_2$$

AB: $\circlearrowleft \Sigma M_A = 0\colon\ (6\text{ in.})T + (6\text{ in.})F_1 - (18\text{ in.})N_1 = 0$

$$F_1\left(\frac{18\text{ in.}}{0.4} - 6\text{ in.}\right) = (6\text{ in.})T \quad \text{or} \quad F_1 = \frac{T}{6.5}$$

DE: $\circlearrowleft \Sigma M_D = 0\colon\ (6\text{ in.})F_2 + (18\text{ in.})N_2 - (6\text{ in.})T = 0$

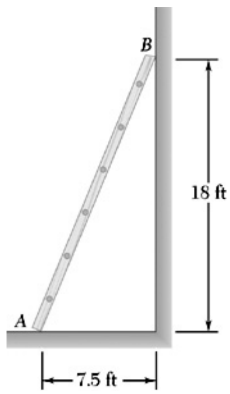
$$F_2\left(6\text{ in.} + \frac{18\text{ in.}}{0.4}\right) = (6\text{ in.})T \quad \text{or} \quad F_2 = \frac{T}{8.5}$$

Drum: $\circlearrowleft \Sigma M_C = 0\colon\ (10\text{ in.})(F_1 + F_2) - 840\text{ lb}\cdot\text{in.} = 0$

$$T\left(\frac{1}{6.5} + \frac{1}{8.5}\right) = 84\text{ lb}$$

$T = 309\text{ lb}$ ◀

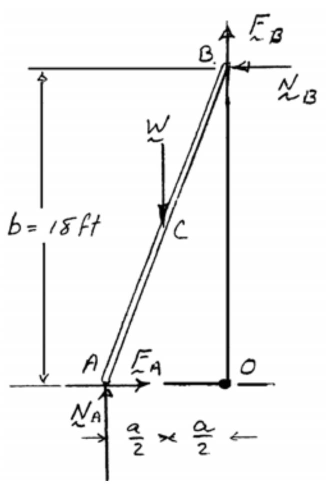
PROBLEM 8.21



A 19.5-ft ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

FBD ladder:



$a = 7.5\text{ ft}$

$b = 18\text{ ft}$

Motion impends at both A and B.

$$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad F_A - N_B = 0 \quad \text{or} \quad N_B = F_A = \mu_s N_A$$

Then $$F_B = \mu_s N_B = \mu_s^2 N_A$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W + F_B = 0 \quad \text{or} \quad N_A\left(1 + \mu_s^2\right) = W$$

$$\circlearrowleft \Sigma M_O = 0: \quad bN_B + \frac{a}{2}W - aN_A = 0$$

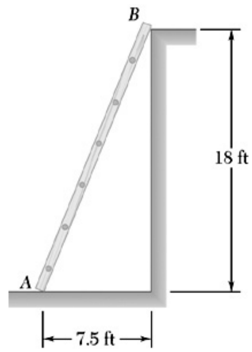
or $$aN_A - b\mu_s N_A = \frac{a}{2}W = \frac{a}{2}N_A\left(1 + \mu_s^2\right)$$

$$\mu_s^2 + \frac{2b}{a}\mu_s - 1 = 0$$

$$\mu_s = -\frac{b}{a} \pm \sqrt{\left(\frac{b}{a}\right)^2 + 1} = -2.4 \pm 2.6$$

The positive root is physically possible. Therefore, $\mu_s = 0.200$ ◀

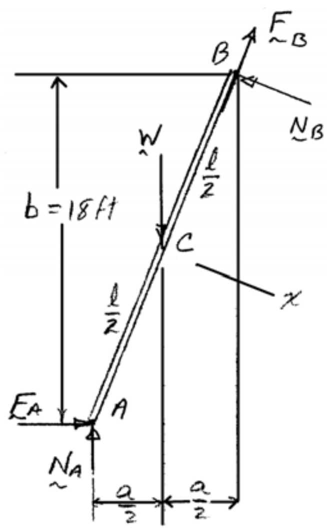
PROBLEM 8.22



A 19.5-ft ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

FBD ladder:



$a = 7.5$ ft

$l = 19.5$ ft

$$\frac{a}{l} = \frac{5}{13}$$

$$\frac{b}{l} = \frac{12}{13}$$

Motion impends at both A and B, so

$$F_A = \mu_s N_A \qquad \text{and} \qquad F_B = \mu_s N_B$$

$$\circlearrowleft \Sigma M_A = 0: \quad lN_B - \frac{a}{2}W = 0 \qquad \text{or} \qquad N_B = \frac{a}{2l}W = \frac{7.5 \text{ ft}}{39 \text{ ft}}W$$

or

$$N_B = \frac{2.5}{13}W$$

Then

$$F_B = \mu_s N_B = \mu_s \frac{2.5W}{13}$$

$$\longrightarrow \Sigma F_x = 0: \quad F_A + \frac{5}{13}F_B - \frac{12}{13}N_B = 0$$

$$\mu_s N_A + \frac{12.5}{(13)^2}\mu_s W - \frac{30}{(13)^2}W = 0$$

$$N_A - \frac{W}{(13)^2}\frac{(30 - 12.5\mu_s)}{\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W + \frac{12}{13}F_B + \frac{5}{13}N_B = 0$$

$$\left(\frac{30 - 12.5\mu_s}{\mu_s} + 30\mu_s + 12.5\right)\frac{W}{(13)^2} = W$$

or

$$\mu_s^2 - 5.6333\mu_s + 1 = 0$$

$$\mu_s = 2.8167 \pm 2.6332$$

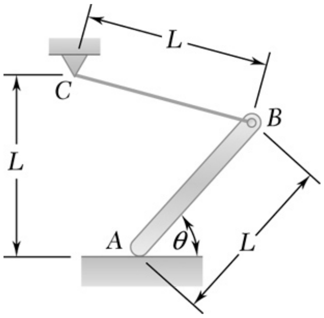
or

$$\mu_s = 0.1835 \qquad \text{and} \qquad \mu_s = 5.45$$

The larger value is very unlikely unless the surface is treated with some “non-skid” material.

In any event, the smallest value for equilibrium is $\mu_s = 0.1835$ ◀

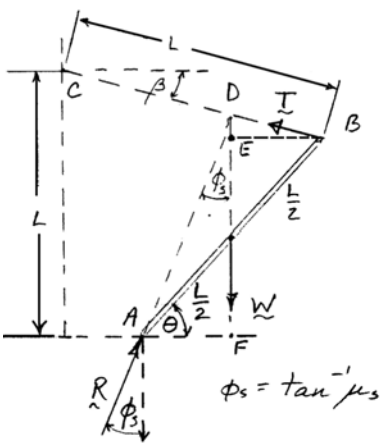
PROBLEM 8.23

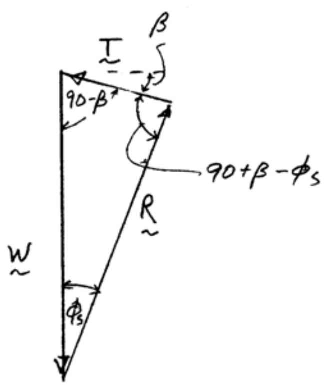


End A of a slender, uniform rod of weight W and length L bears on a horizontal surface as shown, while end B is supported by a cord BC of length L. Knowing that the coefficient of static friction is 0.40, determine (a) the value of θ for which motion is impending, (b) the corresponding value of the tension in the cord.

SOLUTION

FBD rod:





(a) Geometry: $BE = \dfrac{L}{2}\cos\theta \qquad DE = \left(\dfrac{L}{2}\cos\theta\right)\tan\beta$

$$EF = L\sin\theta \qquad DF = \frac{L}{2}\frac{\cos\theta}{\tan\phi_s}$$

So $$L\left(\frac{1}{2}\cos\theta\tan\beta + \sin\theta\right) = \frac{L}{2}\frac{\cos\theta}{\tan\phi_s}$$

or $$\tan\beta + 2\tan\theta = \frac{1}{\tan\phi_s} = \frac{1}{\mu_s} = \frac{1}{0.4} = 2.5 \qquad (1)$$

Also, $$L\sin\theta + L\sin\beta = L$$

or $$\sin\theta + \sin\beta = 1 \qquad (2)$$

Solving Eqs. (1) and (2) numerically $\theta_1 = 4.62° \qquad \beta_1 = 66.85°$

$\theta_2 = 48.20° \qquad \beta_2 = 14.75°$

Therefore, $\theta = 4.62°$ and $\theta = 48.2°$ ◀

(b) Now $$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$$

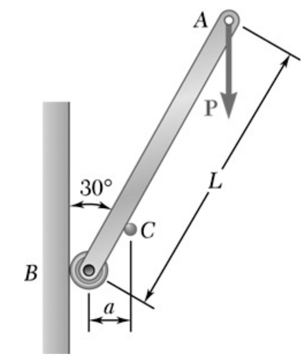
and $$\frac{T}{\sin\phi_s} = \frac{W}{\sin(90 + \beta - \phi_s)}$$

or $$T = W\frac{\sin\phi_s}{\sin(90 + \beta - \phi_s)}$$

For $\theta = 4.62° \qquad T = 0.526W$ ◀

$\theta = 48.2° \qquad T = 0.374W$ ◀

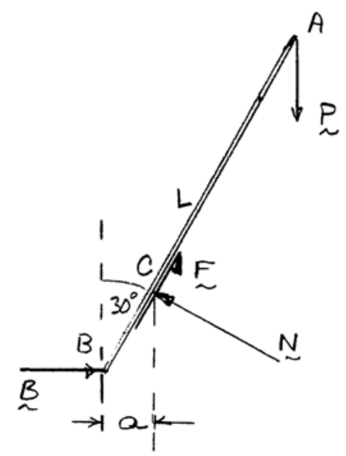
PROBLEM 8.24



A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction between the peg and the rod is 0.25 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_B = 0: \quad \frac{a}{\sin 30°} N - L \sin 30° P = 0$$

$$N = \frac{L}{a} \sin^2 30° P = \frac{L}{a} \frac{P}{4}$$

Impending motion at C: $\left.\begin{array}{l}\text{down} \rightarrow F = \mu_s N \\ \text{up} \rightarrow F = -\mu_s N\end{array}\right\} F = \pm \frac{N}{4}$

$$\uparrow \Sigma F_y = 0: \quad F \cos 30° + N \sin 30° - P = 0$$

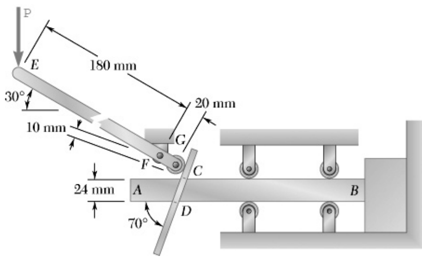
$$\pm \frac{L}{a} \frac{P}{16} \frac{\sqrt{3}}{2} + \frac{L}{a} \frac{P}{4} \frac{1}{2} = P$$

$$\frac{L}{a} \left[\frac{1}{8} \pm \frac{\sqrt{3}}{32} \right] = 1$$

$$\frac{L}{a} = \frac{32}{4 \pm \sqrt{3}}$$

or $\quad \frac{L}{a} = 5.583 \quad$ and $\quad \frac{L}{a} = 14.110$

For equilibrium: $\quad 5.58 \le \frac{L}{a} \le 14.11$ ◀

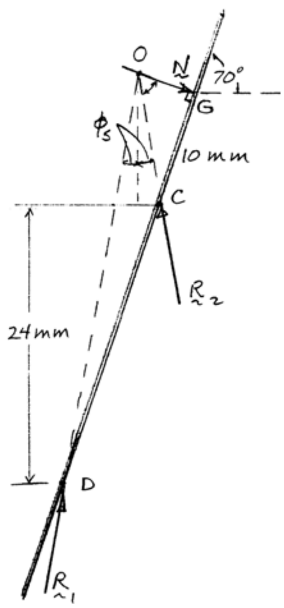


PROBLEM 8.25

The basic components of a clamping device are bar *AB*, locking plate *CD*, and lever *EFG*; the dimensions of the slot in *CD* are slightly larger than those of the cross section of *AB*. To engage the clamp, *AB* is pushed against the workpiece, and then force **P** is applied. Knowing that $P = 160$ N and neglecting the friction force between the lever and the plate, determine the smallest allowable value of the static coefficient of friction between the bar and the plate.

SOLUTION

FBD Plate:



DC is three-force member and motion impends at *C* and *D* (for minimum μ_s).

$$\measuredangle\, OCG = 20° + \phi_s \qquad \measuredangle\, ODG = 20° - \phi_s$$

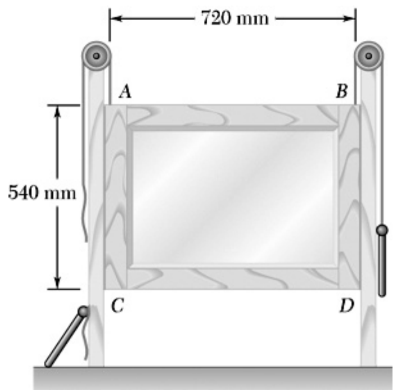
$$\overline{OG} = (10\text{ mm})\tan(20° + \phi_s) = \left(\frac{24\text{ mm}}{\sin 70°} + 10\text{ mm}\right)\tan(20° - \phi_s)$$

or $$\tan(20° + \phi_s) = 3.5540\tan(20° - \phi_s)$$

Solving numerically $$\phi_s = 10.565°$$

Now $$\mu_s = \tan\phi_s$$

so that $$\mu_s = 0.1865 \blacktriangleleft$$

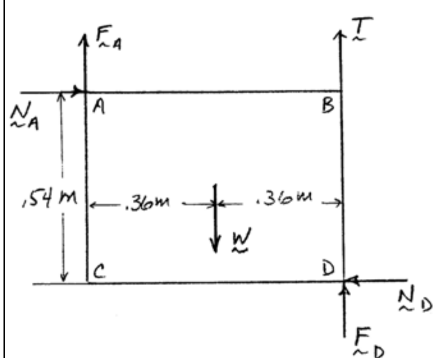


PROBLEM 8.26

A window sash having a mass of 4 kg is normally supported by two 2-kg sash weights. Knowing that the window remains open after one sash cord has broken, determine the smallest possible value of the coefficient of static friction. (Assume that the sash is slightly smaller that the frame and will bind only at points A and D.)

SOLUTION

FBD window:



$$W = (4\text{ kg})(9.81\text{ m/s}^2) = 39.24\text{ N}$$

$$T = (2\text{ kg})(9.81\text{ m/s}^2) = 19.62\text{ N} = \frac{W}{2}$$

$\rightarrow \Sigma F_x = 0$: $\quad N_A - N_D = 0 \qquad N_A = N_D$

Impending motion: $\quad F_A = \mu_s N_A \qquad F_D = \mu_s N_D$

$\circlearrowleft \Sigma M_D = 0$: $\quad (0.36\text{ m})W - (0.54\text{ m})N_A - (0.72\text{ m})F_A = 0$

$$W = \frac{3}{2}N_A + 2\mu_s N_A$$

$$N_A = \frac{2W}{3 + 4\mu_s}$$

$\uparrow \Sigma F_y = 0$: $\quad F_A - W + T + F_D = 0$

$$F_A + F_D = W - T$$

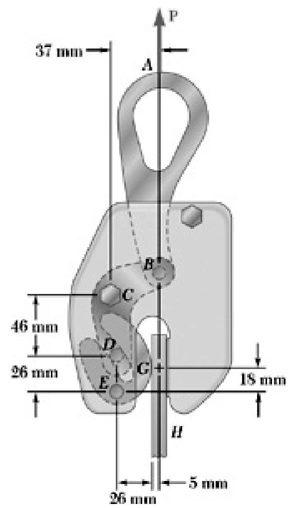
$$= \frac{W}{2}$$

Now $\quad F_A + F_D = \mu_s(N_A + N_D) = 2\mu_s N_A$

Then $\quad \dfrac{W}{2} = 2\mu_s \dfrac{2W}{3 + 4\mu_s}$

or $\quad \mu_s = 0.750$ ◀

PROBLEM 8.27



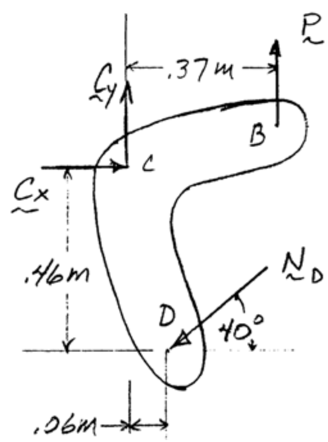
The steel-plate clamp shown is used to lift a steel plate H of mass 250 kg. Knowing that the normal force exerted on steel cam EG by pin D forms an angle of 40° with the horizontal and neglecting the friction force between the cam and the pin, determine the smallest allowable value of the coefficient of static friction.

SOLUTION

FBDs:

(Note: **P** is vertical as AB is two force member; also $P = W$ since clamp + plate is a two force FBD)

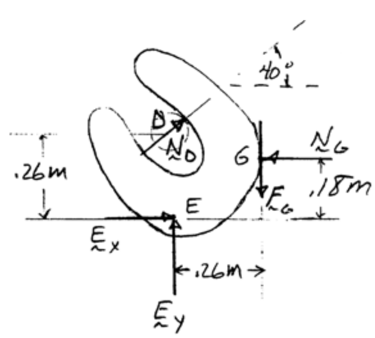
BCD:



$$\circlearrowleft \Sigma M_C = 0: \quad (0.37 \text{ m})P - (0.46 \text{ m})N_D \cos 40° - (0.06 \text{ m})N_D \sin 40° = 0$$

or $$N_D = 0.94642P = 0.94642W$$

EG:



$$\circlearrowleft \Sigma M_E = 0: \quad (0.18 \text{ m})N_G - (0.26 \text{ m})F_G - (0.26 \text{ m})N_D \cos 40° = 0$$

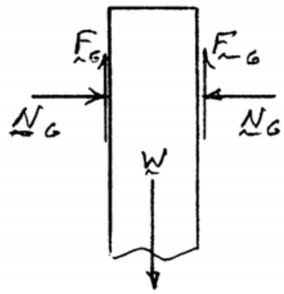
Impending motion: $$F_G = \mu_s N_G$$

Combining $$(18 + 26\mu_s)N_G = 19.9172N_D$$

$$= 18.850W$$

PROBLEM 8.27 CONTINUED

Plate:

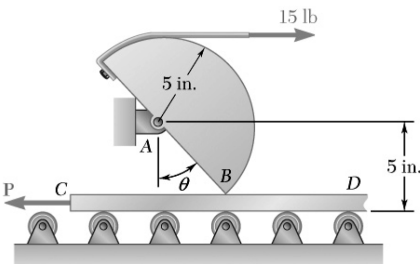


From plate: $F_G = \dfrac{W}{2}$ so that $N_G = \dfrac{W}{2\mu_s}$

Then $\left(18 + 26\mu_s\right)\dfrac{W}{2\mu_s} = 18.85W$

$\mu_s = 0.283$ ◀

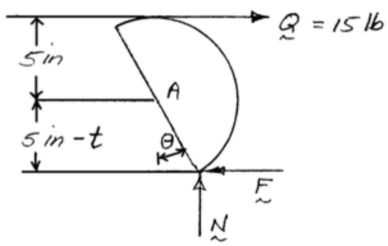
PROBLEM 8.28

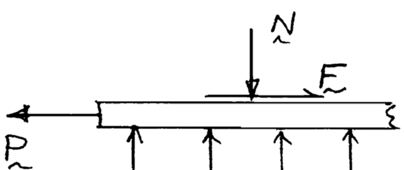


The 5-in.-radius cam shown is used to control the motion of the plate CD. Knowing that the coefficient of static friction between the cam and the plate is 0.45 and neglecting friction at the roller supports, determine (a) the force **P** for which motion of the plate is impending knowing that the plate is 1 in. thick, (b) the largest thickness of the plate for which the mechanism is self-locking, (that is, for which the plate cannot be moved however large the force **P** may be).

SOLUTION

FBDs:





From plate: $\xrightarrow{} \Sigma F_x = 0: \quad F - P = 0 \qquad F = P$

From cam geometry: $\cos\theta = \dfrac{5\text{ in.} - t}{5\text{ in.}}$

$$\circlearrowleft \Sigma M_A = 0: \quad \left[(5\text{ in.})\sin\theta\right]N - \left[(5\text{ in.})\cos\theta\right]F - (5\text{ in.})Q = 0$$

Impending motion: $F = \mu_s N$

So $N\sin\theta - \mu_s N\cos\theta = Q = 15\text{ lb}$

$$N = \frac{Q}{\sin\theta - \mu_s\cos\theta}$$

So $P = F = \mu_s N = \dfrac{\mu_s Q}{\sin\theta - \mu_s\cos\theta}$

(a) $t = 1\text{ in.} \Rightarrow \cos\theta = \dfrac{4\text{ in.}}{5\text{ in.}} = 0.8; \ \sin\theta = 0.6$

$$P = \frac{(0.45)(15\text{ lb})}{0.6 - (0.45)(0.8)} = 28.125\text{ lb}; \ \mathbf{P} = 28.1\text{ lb} \longleftarrow \blacktriangleleft$$

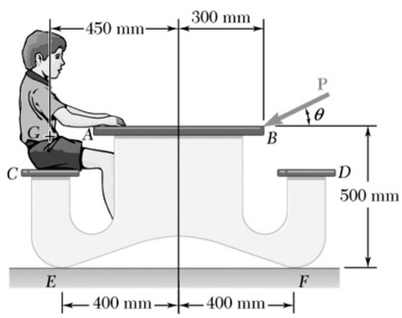
(b) $P \to \infty: \ \sin\theta - \mu_s\cos\theta = \dfrac{\mu_s Q}{P} \longrightarrow 0$

Thus $\tan\theta \to \mu_s = 0.45$ so that $\theta = 24.228°$

But $(5\text{ in.})\cos\theta = 5\text{ in.} - t$ or $t = (5\text{ in.})(1 - \cos\theta)$

$$t = 0.440\text{ in.} \blacktriangleleft$$

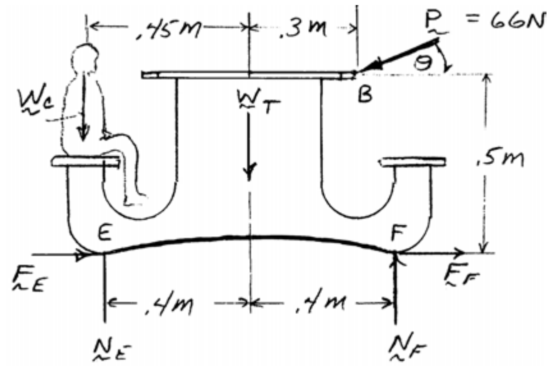
PROBLEM 8.29



A child having a mass of 18 kg is seated halfway between the ends of a small, 16-kg table as shown. The coefficient of static friction is 0.20 between the ends of the table and the floor. If a second child pushes on edge B of the table top at a point directly opposite to the first child with a force **P** lying in a vertical plane parallel to the ends of the table and having a magnitude of 66 N, determine the range of values of θ for which the table will (*a*) tip, (*b*) slide.

SOLUTION

FBD table + child:



$$W_C = 18\text{ kg}\left(9.81\text{ m/s}^2\right) = 176.58\text{ N}$$

$$W_T = 16\text{ kg}\left(9.81\text{ m/s}^2\right) = 156.96\text{ N}$$

(*a*) Impending tipping about E, $N_F = F_F = 0$, and

$$\circlearrowleft \Sigma M_E = 0: \quad (0.05\text{ m})(176.58\text{ N}) - (0.4\text{ m})(156.96\text{ N}) + (0.5\text{ m})P\cos\theta - (0.7\text{ m})P\sin\theta = 0$$

$$33\cos\theta - 46.2\sin\theta = 53.955$$

Solving numerically $\quad \theta = -36.3° \quad$ and $\quad \theta = -72.6°$

Therefore $\quad -72.6° \le \theta \le -36.3°$ ◀

Impending tipping about F is not possible

(*b*) For impending slip: $\quad F_E = \mu_s N_E = 0.2N_E \quad F_F = \mu_s N_F = 0.2N_F$

$$\longrightarrow \Sigma F_x = 0: \quad F_E + F_F - P\cos\theta = 0 \quad \text{or} \quad 0.2\left(N_E + N_F\right) = (66\text{ N})\cos\theta$$

$$\uparrow \Sigma F_y = 0: \quad N_E + N_F - 176.58\text{ N} - 156.96\text{ N} - P\sin\theta = 0$$

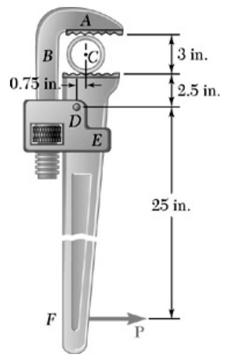
$$N_E + N_F = \left(66\sin\theta + 333.54\right)\text{ N}$$

So $\quad 330\cos\theta = 66\sin\theta + 333.54$

Solving numerically, $\quad \theta = -3.66° \quad$ and $\quad \theta = -18.96°$

Therefore, $\quad -18.96° \le \theta \le -3.66°$ ◀

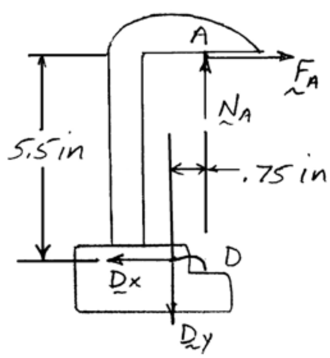
PROBLEM 8.30



A pipe of diameter 3 in. is gripped by the stillson wrench shown. Portions *AB* and *DE* of the wrench are rigidly attached to each other, and portion *CF* is connected by a pin at *D*. If the wrench is to grip the pipe and be self-locking, determine the required minimum coefficients of friction at *A* and *C*.

SOLUTION

FBD ABD:



$$\circlearrowleft \Sigma M_D = 0: \quad (0.75 \text{ in.})N_A - (5.5 \text{ in.})F_A = 0$$

Impending motion: $F_A = \mu_A N_A$

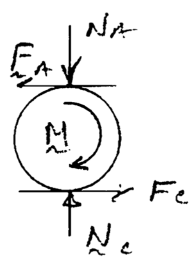
Then $0.75 - 5.5\mu_A = 0$

or $\mu_A = 0.13636$

$$\mu_A = 0.1364 \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad F_A - D_x = 0 \qquad D_x = F_A$$

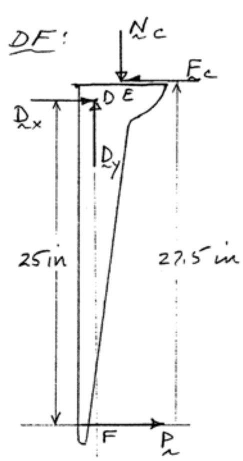
Pipe:



$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$

FBD DF:



$$\circlearrowleft \Sigma M_F = 0: \quad (27.5 \text{ in.})F_C - (0.75 \text{ in.})N_C - (25 \text{ in.})D_x = 0$$

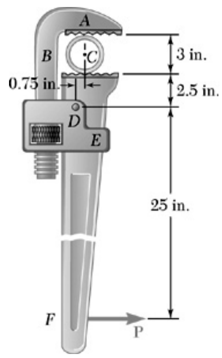
Impending motion: $F_C = \mu_C N_C$

Then $27.5\mu_C - 0.75 = 25\dfrac{F_A}{N_C}$

But $N_C = N_A$ and $\dfrac{F_A}{N_A} = \mu_A = 0.13636$

So $27.5\mu_C = 0.75 + 25(0.13636)$

$$\mu_C = 0.1512 \blacktriangleleft$$

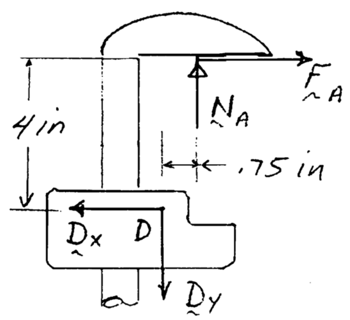


PROBLEM 8.31

Solve Problem 8.30 assuming that the diameter of the pipe is 1.5 in.

SOLUTION

FBD ABD:



$$\circlearrowleft \Sigma M_D = 0: \quad (0.75 \text{ in.})N_A - (4 \text{ in.})F_A = 0$$

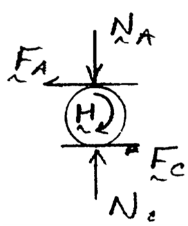
Impending motion: $F_A = \mu_A N_A$

Then $0.75 \text{ in.} - (4 \text{ in.})\mu_A = 0$ $\qquad \mu_A = 0.1875$ ◀

$$\longrightarrow \Sigma F_x = 0: \quad F_A - D_x = 0$$

so that $D_x = F_A = 0.1875 N_A$

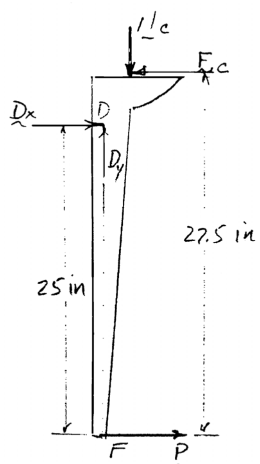
FBD Pipe:



$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$

FBD DF:



$$\circlearrowleft \Sigma M_F = 0: \quad (27.5 \text{ in.})F_C - (0.75 \text{ in.})N_C - (25 \text{ in.})D_x = 0$$

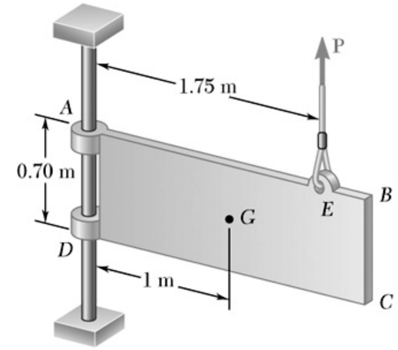
Impending motion: $F_C = \mu_C N_C$

$$27.5\mu_C - 0.75 = 25(0.1875)\frac{N_A}{N_C}$$

But $N_A = N_C$ (from pipe *FBD*) so $\dfrac{N_A}{N_C} = 1$

and $\mu_C = 0.1977$ ◀

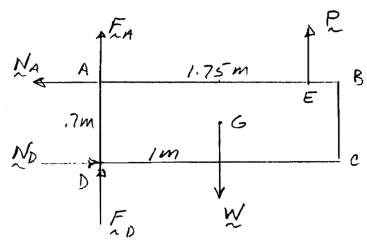
PROBLEM 8.32



The 25-kg plate *ABCD* is attached at *A* and *D* to collars which can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at *E* is (*a*) $P = 0$, (*b*) $P = 80$ N.

SOLUTION

FBD plate:



$$W = 25\text{ kg}(9.81\text{ N/kg})$$
$$= 245.25\text{ N}$$

(*a*) $P = 0$; assume equilibrium:

$$\circlearrowleft \Sigma M_A = 0: \quad (0.7\text{ m})N_D - (1\text{ m})W = 0 \qquad N_D = \frac{10W}{7}$$

$$\longrightarrow \Sigma F_x = 0: \quad N_D - N_A = 0 \qquad N_A = N_D = \frac{10W}{7}$$

$$(F_A)_{\max} = \mu_s N_A \qquad (F_D)_{\max} = \mu_s N_D$$

So $$(F_A + F_D)_{\max} = \mu_s(N_A + N_D) = \frac{20\mu_s W}{7} = 1.143W$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D - W = 0$$

$$\therefore \quad F_A + F_D = W < (F_A + F_D)_{\max} \qquad \text{OK.}$$

Plate is in equilibrium ◀

(*b*) $P = 80$ N; assume equilibrium:

$$\circlearrowleft \Sigma M_A = 0: \quad (1.75\text{ m})P + (0.7\text{ m})N_D - (1\text{ m})W = 0$$

or $$N_D = \frac{W - 1.75P}{0.7}$$

$$\longrightarrow \Sigma F_x = 0: \quad N_D - N_A = 0 \qquad N_D = N_A = \frac{W - 1.75P}{0.7}$$

$$(F_A)_{\max} = \mu_s N_A \qquad (F_B)_{\max} = \mu_s N_B$$

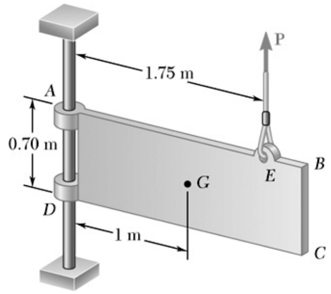
So $$(F_A + F_B)_{\max} = 0.4\frac{2W - 3.5P}{0.7} = 120.29\text{ N}$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D - W + P = 0$$

$$F_A + F_D = W - P = 165.25\text{ N}$$

$$(F_A + F_D)_{\text{equil}} > (F_A + F_D)_{\max}$$

Impossible, so plate slides downward ◀

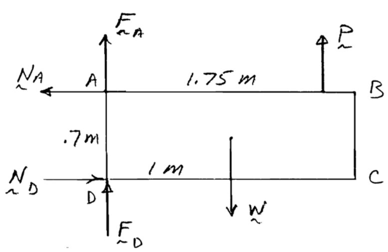


PROBLEM 8.33

In Problem 8.32, determine the range of values of the magnitude P of the vertical force applied at E for which the plate will move downward.

SOLUTION

FBD plate:



$$W = (25 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 245.25 \text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: \quad (0.7 \text{ m})N_D - (1 \text{ m})W + (1.75 \text{ m})P = 0$$

$$N_D = \frac{W - 1.75P}{0.7}$$

$$\longrightarrow \Sigma F_x = 0: \quad N_D - N_A = 0 \qquad \text{so that} \qquad N_A = N_D = \frac{W - 1.75P}{0.7}$$

Note: N_A and N_D will be > 0 if $P < \frac{4}{7}W$ and < 0 if $P > \frac{4}{7}W$.

Impending motion downward: F_A and F_B are both > 0, so

$$F_A = \mu_s |N_A| = \frac{0.4}{0.7}|W - 1.75P| = \left|\frac{4}{7}W - P\right|$$

$$F_D = \mu_S |N_D| = \left|\frac{4}{7}W - P\right|$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D - W + P = 0$$

$$2\left|\frac{4}{7}W - P\right| - W + P = 0$$

For $P < \frac{4}{7}W$; $\qquad P = \frac{W}{7} = 35.04 \text{ N}$

For $P > \frac{4}{7}W$; $\qquad P = \frac{5W}{7} = 175.2 \text{ N}$

Downward motion for $35.0 \text{ N} < P < 175.2 \text{ N}$ ◀

Alternative Solution

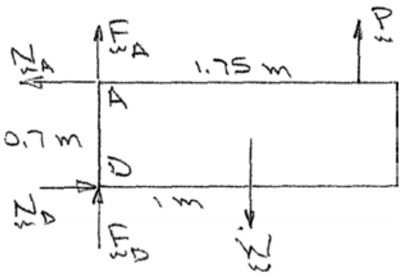
We first observe that for smaller values of the magnitude of **P** that (Case 1) the inner left-hand and right-hand surfaces of collars A and D, respectively, will contact the rod, whereas for larger values of the magnitude of **P** that (Case 2) the inner right-hand and left-hand surfaces of collars A and D, respectively, will contact the rod.

First note: $\qquad W = (25 \text{ kg})(9.81 \text{ m/s}^2)$

$$= 245.25 \text{ N}$$

PROBLEM 8.33 CONTINUED

Case 1



$$\circlearrowleft \Sigma M_D = 0: \quad (0.7\text{ m})N_A - (1\text{ m})(245.25\text{ N}) + (1.75\text{ m})P = 0$$

or $$N_A = \frac{10}{7}\left(245.25 - \frac{7}{4}P\right)\text{ N}$$

$$\rightarrow \Sigma F_x = 0: \quad -N_A + N_D = 0$$

or $$N_D = N_A$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 245.25\text{ N} = 0$$

or $$F_A + F_D = (245.25 - P)\text{ N}$$

Now $$(F_A)_{\max} = \mu_s N_A \qquad (F_D)_{\max} = \mu_s N_D$$

so that $$(F_A)_{\max} + (F_D)_{\max} = \mu_s(N_A + N_D)$$

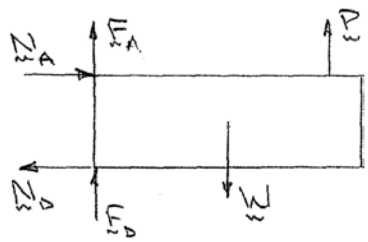
$$= 2(0.4)\left[\frac{10}{7}\left(245.25 - \frac{7}{4}P\right)\right]$$

For motion: $$F_A + F_D > (F_A)_{\max} + (F_D)_{\max}$$

Substituting $$245.25 - P > \frac{8}{7}\left(245.25 - \frac{7}{4}P\right)$$

or $$P > 35.0\text{ N}$$

Case 2



From Case 1: $$N_D = N_A$$

$$F_A + F_D = (245.25 - P)\text{ N}$$

$$(F_A)_{\max} + (F_D)_{\max} = 2\mu_s N_A$$

$$\circlearrowleft \Sigma M_D = 0: \quad -(0.7\text{ m})N_A - (1\text{ m})(245.25\text{ N}) + (1.75\text{ m})P = 0$$

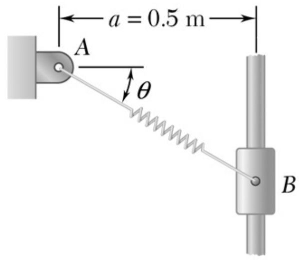
or $$N_A = \frac{10}{7}\left(\frac{7}{4}P - 245.25\right)\text{ N}$$

For motion: $$F_A + F_D > (F_A)_{\max} + (F_D)_{\max}$$

Substituting: $$245.25 - P > 2(0.4)\left[\frac{10}{7}\left(\frac{7}{4}P - 245.25\right)\right]$$

or $$P < 175.2\text{ N}$$

Therefore, have downward motion for $35.0\text{ N} < P < 175.2\text{ N}$ ◀



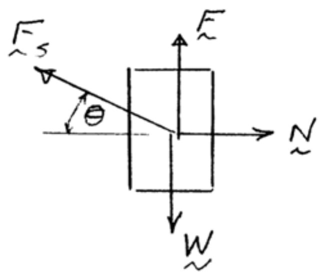
PROBLEM 8.34

A collar B of weight W is attached to the spring AB and can move along the rod shown. The constant of the spring is 1.5 kN/m and the spring is unstretched when $\theta = 0$. Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of W for which equilibrium is maintained when (*a*) $\theta = 20^\circ$, (*b*) $\theta = 30^\circ$.

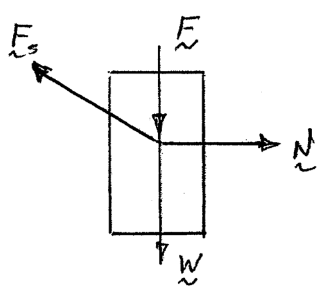
SOLUTION

FBD collar:

Impending motion down:



Impending motion up:



Stretch of spring $x = \overline{AB} - a = \dfrac{a}{\cos\theta} - a$

$$F_s = kx = k\left(\frac{a}{\cos\theta} - a\right) = (1.5\text{ kN/m})(0.5\text{ m})\left(\frac{1}{\cos\theta} - 1\right)$$

$$= (0.75\text{ kN})\left(\frac{1}{\cos\theta} - 1\right)$$

$\longrightarrow \Sigma F_x = 0$: $\quad N - F_s\cos\theta = 0$

$$N = F_s\cos\theta = (0.75\text{ kN})(1 - \cos\theta)$$

Impending slip: $\quad F = \mu_s N = (0.4)(0.75\text{ kN})(1 - \cos\theta)$

$$= (0.3\text{ kN})(1 - \cos\theta)$$

$+$ down, $-$ up

$\uparrow \Sigma F_y = 0$: $\quad F_s\sin\theta \pm F - W = 0$

$$(0.75\text{ kN})(\tan\theta - \sin\theta) \pm (0.3\text{ kN})(1 - \cos\theta) - W = 0$$

or $\quad W = (0.3\text{ kN})[2.5(\tan\theta - \sin\theta) \pm (1 - \cos\theta)]$

(*a*) $\theta = 20^\circ$: $\quad W_{\text{up}} = -0.00163\text{ kN}$ (impossible)

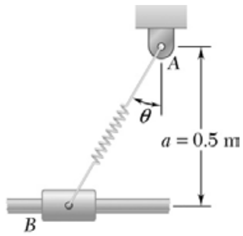
$W_{\text{down}} = 0.03455\text{ kN}$ (OK)

Equilibrium if $0 \le W \le 34.6\text{ N}$ ◀

(*b*) $\theta = 30^\circ$: $\quad W_{\text{up}} = 0.01782\text{ kN}$ (OK)

$W_{\text{down}} = 0.0982\text{ kN}$ (OK)

Equilibrium if $17.82\text{ N} \le W \le 98.2\text{ N}$ ◀

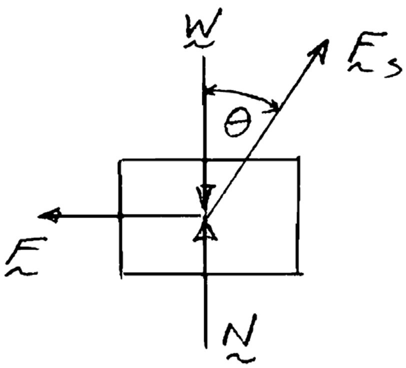


PROBLEM 8.35

A collar B of weight W is attached to the spring AB and can move along the rod shown. The constant of the spring is 1.5 kN/m and the spring is unstretched when $\theta = 0$. Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of W for which equilibrium is maintained when (*a*) $\theta = 20^{\circ}$, (*b*) $\theta = 30^{\circ}$.

SOLUTION

FBD collar:



Stretch of spring $x = \overline{AB} - a = \dfrac{a}{\cos\theta} - a$

$$F_s = k\left(\frac{a}{\cos\theta} - a\right) = (1.5\text{ kN/m})(0.5\text{ m})\left(\frac{1}{\cos\theta} - 1\right)$$

$$= (0.75\text{ kN})\left(\frac{1}{\cos\theta} - 1\right) = (750\text{ N})(\sec\theta - 1)$$

$$\uparrow \Sigma F_y = 0: \quad F_s\cos\theta - W + N = 0$$

or
$$W = N + (750\text{ N})(1 - \cos\theta)$$

Impending slip:

$$F = \mu_s|N| \quad (F\text{ must be } +,\text{ but } N \text{ may be positive or negative})$$

$$\longrightarrow \Sigma F_x = 0: \quad F_s\sin\theta - F = 0$$

or
$$F = F_s\sin\theta = (750\text{ N})(\tan\theta - \sin\theta)$$

(*a*) $\theta = 20^{\circ}$: $\quad F = (750\text{ N})(\tan 20^{\circ} - \sin 20^{\circ}) = 16.4626\text{ N}$

Impending motion: $\quad |N| = \dfrac{F}{\mu_s} = \dfrac{16.4626\text{ N}}{0.4} = 41.156\text{ N}$

(Note: for $|N| < 41.156\text{ N}$, motion will occur, equilibrium for $|N| > 41.156$)

But $\quad W = N + (750\text{ N})(1 - \cos 20^{\circ}) = N + 45.231\text{ N}$

So equilibrium for $W \leq 4.07\text{ N}$ and $W \geq 86.4\text{ N}$ ◀

(*b*) $\theta = 30^{\circ}$: $\quad F = (750\text{ N})(\tan 30^{\circ} - \sin 30^{\circ}) = 58.013\text{ N}$

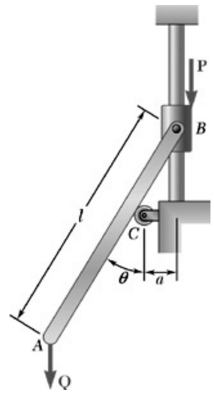
Impending motion: $\quad |N| = \dfrac{F}{\mu_s} = \dfrac{58.013}{0.4} = 145.032\text{ N}$

$$W = N + (750\text{ N})(1 - \cos 30^{\circ}) = N \pm 145.03\text{ N}$$

$$= -44.55\text{ N}(\text{impossible}),\ 245.51\text{ N}$$

Equilibrium for $W \geq 246\text{ N}$ ◀

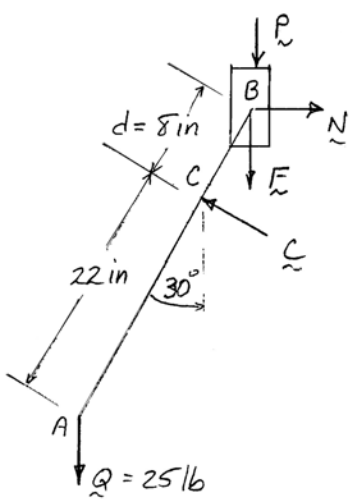
PROBLEM 8.36



The slender rod AB of length $l = 30$ in. is attached to a collar at B and rests on a small wheel located at a horizontal distance $a = 4$ in. from the vertical rod on which the collar slides. Knowing that the coefficient of static friction between the collar and the vertical rod is 0.25 and neglecting the radius of the wheel, determine the range of values of P for which equilibrium is maintained when $Q = 25$ lb and $\theta = 30^\circ$.

SOLUTION

FBD rod + collar:



Note: $d = \dfrac{a}{\sin\theta} = \dfrac{4\text{ in.}}{\sin 30^\circ} = 8\text{ in., so } AC = 22\text{ in.}$

Neglect weights of rod and collar.

$$\circlearrowright \Sigma M_B = 0: \quad (30\text{ in.})(\sin 30^\circ)(25\text{ lb}) - (8\text{ in.})C = 0$$

$$C = 46.875\text{ lb}$$

$$\rightarrow \Sigma F_x = 0: \quad N - C\cos 30^\circ = 0$$

$$N = (46.875\text{ lb})\cos 30^\circ = 40.595\text{ lb}$$

Impending motion up: $\quad F = \mu_s N = 0.25(40.595\text{ lb})$

$$= 10.149\text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -25\text{ lb} + (46.875\text{ lb})\sin 30^\circ - P - 10.149\text{ lb} = 0$$

or $\quad P = -1.563\text{ lb} - 10.149\text{ lb} = -11.71\text{ lb}$

Impending motion down: Direction of **F** is now upward, but still have

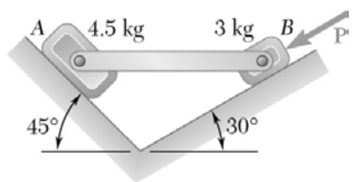
$$|F| = \mu_s N = 10.149\text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -25\text{ lb} + (46.875\text{ lb})\sin 30^\circ - P + 10.149\text{ lb} = 0$$

or $\quad P = -1.563\text{ lb} + 10.149\text{ lb} = 8.59\text{ lb}$

$\therefore \quad$ Equilibrium for $-11.71\text{ lb} \le P \le 8.59\text{ lb}$ ◀

PROBLEM 8.37

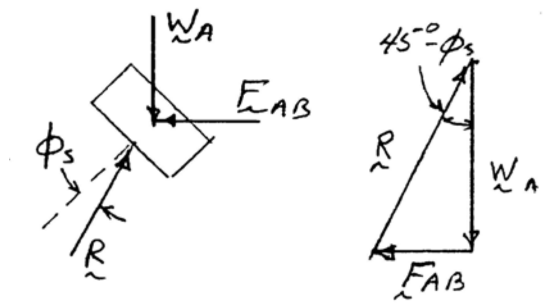


The 4.5-kg block A and the 3-kg block B are connected by a slender rod of negligible mass. The coefficient of static friction is 0.40 between all surfaces of contact. Knowing that for the position shown the rod is horizontal, determine the range of values of P for which equilibrium is maintained.

SOLUTION

FBDs: Note: $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$

(a) Block A impending slip ↘

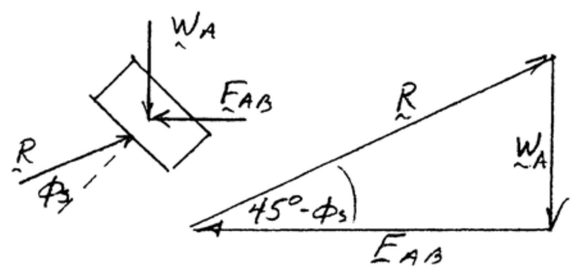


$$F_{AB} = W_A \tan(45° - \phi_s)$$

$$= (4.5\text{ kg})(9.81\text{ m/s}^2)\tan(23.199°)$$

$$= 18.9193\text{ N}$$

(b) Block A impending slip ↖

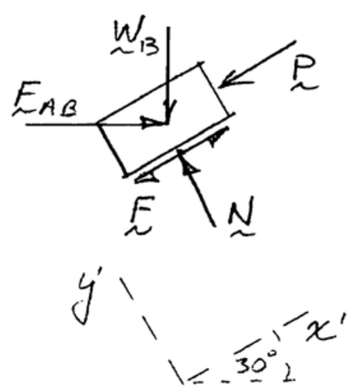


$$F_{AB} = W_A \operatorname{ctn}(45° - \phi_s)$$

$$= (4.5\text{ kg})(9.81\text{ m/s}^2)\operatorname{ctn}(23.199°)$$

$$= 103.005\text{ N}$$

Block B:



$$W_B = (3\text{ kg})(9.81\text{ m/s}^2)$$

$$= 29.43\text{ N}$$

From Block B: ↖ $\Sigma F_{y'} = 0\text{:}\quad N - W_B\cos 30° - F_{AB}\sin 30° = 0$

PROBLEM 8.37 CONTINUED

Case (a) $$N = (29.43\text{ N})\cos 30° + (18.9193\text{ N})\sin 30° = 34.947\text{ N}$$

Impending motion: $$F = \mu_s N = 0.4(34.947\text{ N}) = 13.979\text{ N}$$

$$\nearrow \Sigma F_{x'} = 0: \quad F_{AB}\cos 30° - W_B \sin 30° - 13.979\text{ N} - P = 0$$

$$P = (18.9193\text{ N})\cos 30° - (29.43\text{ N})\sin 30° - 13.979\text{ N}$$

$$= -12.31\text{ N}$$

Case (b) $$N = (29.43\text{ N})\cos 30° + (103.005\text{ N})\sin 30° = 76.9896\text{ N}$$

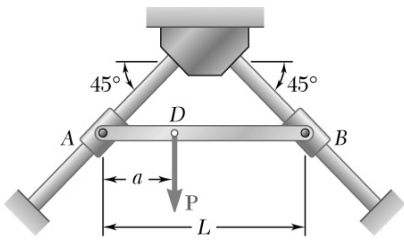
Impending motion: $$F = 0.4(76.9896\text{ N}) = 30.7958\text{ N}$$

$$\nearrow \Sigma F_{x'} = 0: \quad (103.005\text{ N})\cos 30° - (29.43\text{ N})\sin 30° + 30.7958\text{ N} - P = 0$$

$$P = 105.3\text{ N}$$

For equilibrium $-12.31\text{ N} \le P \le 105.3\text{ N}$ ◀

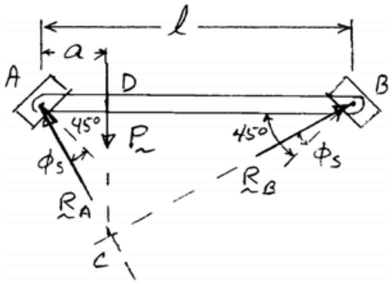
PROBLEM 8.38



Bar AB is attached to collars which can slide on the inclined rods shown. A force **P** is applied at point *D* located at a distance *a* from end *A*. Knowing that the coefficient of static friction μ_s between each collar and the rod upon which it slides is 0.30 and neglecting the weights of the bar and of the collars, determine the smallest value of the ratio *a/L* for which equilibrium is maintained.

SOLUTION

FBD bar + collars:



Impending motion

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.6992°$$

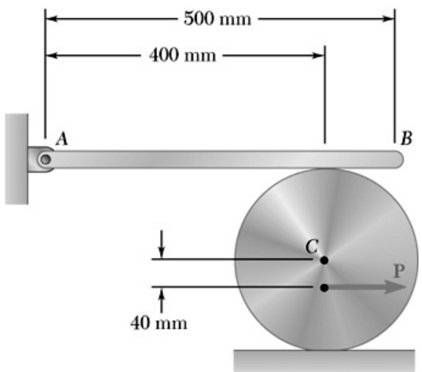
Neglect weights: 3-force *FBD* and $\measuredangle ACB = 90°$

So $$AC = \frac{a}{\cos(45° + \phi_s)} = l\sin(45° - \phi_s)$$

$$\frac{a}{l} = \sin(45° - 16.6992°)\cos(45° + 16.6992°)$$

$$\frac{a}{l} = 0.225 \blacktriangleleft$$

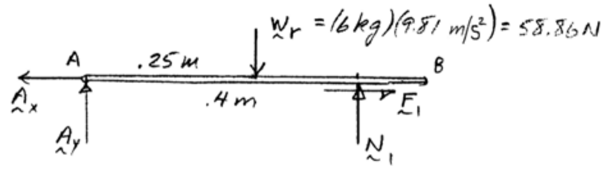
PROBLEM 8.39



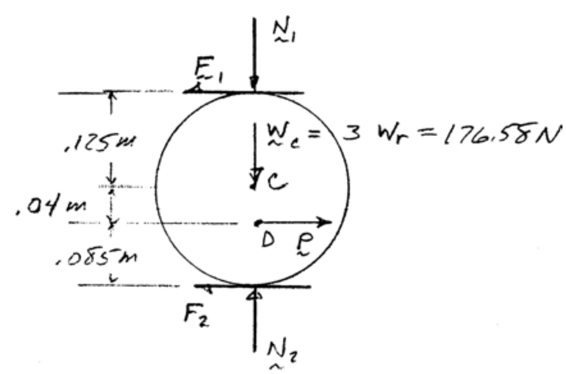
The 6-kg slender rod *AB* is pinned at *A* and rests on the 18-kg cylinder *C*. Knowing that the diameter of the cylinder is 250 mm and that the coefficient of static friction is 0.35 between all surfaces of contact, determine the largest magnitude of the force **P** for which equilibrium is maintained.

SOLUTION

FBD rod:



FBD cylinder:



$$\circlearrowleft \Sigma M_A = 0: \quad (0.4 \text{ m})N_1 - (0.25 \text{ m})W_r = 0$$

$$N_1 = 0.625W_r = 36.7875 \text{ N}$$

Cylinder:

$$\uparrow \Sigma F_y = 0: \quad N_2 - N_1 - W_C = 0 \quad \text{or} \quad N_2 = 0.625W_r + 3W_r = 3.625W_r = 5.8N_1$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.165 \text{ m})F_1 - (0.085 \text{ m})F_2 = 0 \quad \text{or} \quad F_2 = 1.941F_1$$

Since $\mu_{s1} = \mu_{s2}$, motion will impend first at top of the cylinder

So $$F_1 = \mu_s N_1 = 0.35(36.7875 \text{ N}) = 12.8756 \text{ N}$$

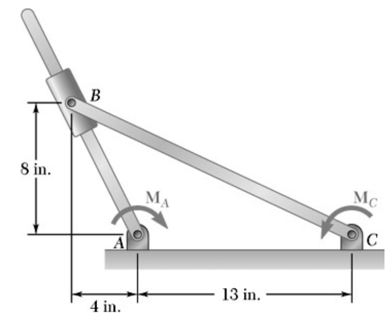
and $$F_2 = 1.941(12.8756 \text{ N}) = 24.992 \text{ N}$$

$$\left[\text{Check } F_2 = 25 \text{ N} < \mu_S N_2 = 74.7 \text{ N} \qquad \text{OK}\right]$$

$$\rightarrow \Sigma F_x = 0: \quad P - F_1 - F_2 = 0$$

or $$P = 12.8756 \text{ N} + 24.992 \text{ N}$$

or $P = 37.9 \text{ N}$ ◀

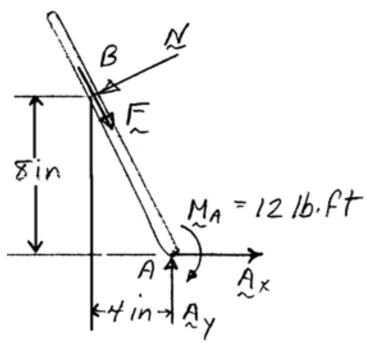


PROBLEM 8.40

Two rods are connected by a collar at B. A couple $\mathbf{M}_A$ of magnitude 12 lb·ft is applied to rod AB. Knowing that $\mu_s = 0.30$ between the collar and rod AB, determine the largest couple $\mathbf{M}_C$ for which equilibrium will be maintained.

SOLUTION

FBD AB:



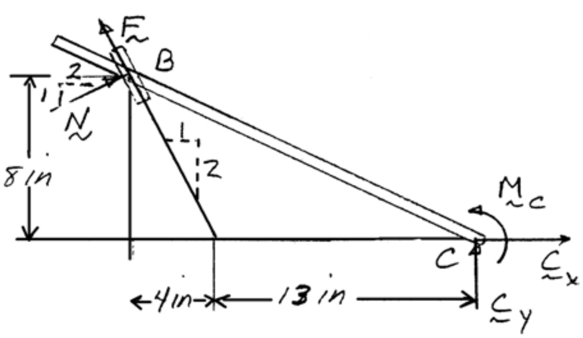
$$\circlearrowleft \Sigma M_A = 0: \quad \sqrt{8 \text{ in}^2 + 4 \text{ in}^2}\,(N) - M_A = 0$$

$$N = \frac{(12 \text{ lb}\cdot\text{ft})(12 \text{ in./ft})}{8.9443 \text{ in.}} = 16.100 \text{ lb}$$

Impending motion: $\quad F = \mu_s N = 0.3(16.100 \text{ lb}) = 4.83 \text{ lb}$

(Note: For max, M_C, need F in direction shown; see FBD BC.)

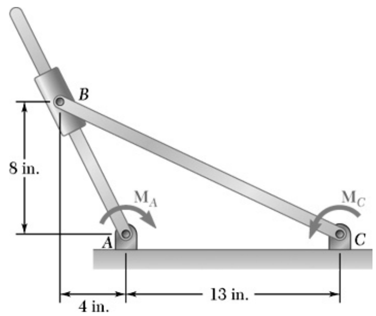
FBD BC + collar:



$$\circlearrowleft \Sigma M_C = 0: \quad M_C - (17 \text{ in.})\frac{1}{\sqrt{5}}N - (8 \text{ in.})\frac{2}{\sqrt{5}}N - (13 \text{ in.})\frac{2}{\sqrt{5}}F = 0$$

or $$M_C = \frac{17 \text{ in.}}{\sqrt{5}}(16.100 \text{ lb}) + \frac{16 \text{ in.}}{\sqrt{5}}(16.100 \text{ lb}) + \frac{26 \text{ in.}}{\sqrt{5}}(4.830 \text{ lb}) = 293.77 \text{ lb}\cdot\text{in.}$$

$$(\mathbf{M}_C)_{\max} = 24.5 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

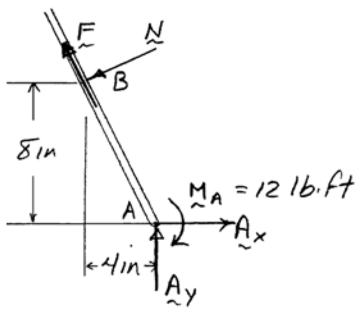


PROBLEM 8.41

In Problem 8.40, determine the smallest couple $\mathbf{M}_C$ for which equilibrium will be maintained.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad N\left(\sqrt{8\text{ in}^2 + 4\text{ in}^2}\right) - M_A = 0$$

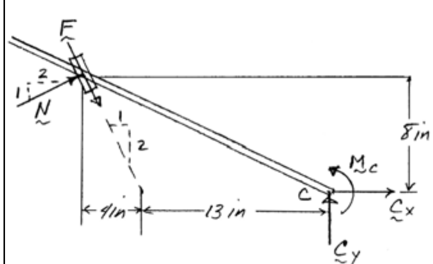
$$N = \frac{(12\text{ lb}\cdot\text{ft})(12\text{ in./ft})}{8.9443\text{ in.}} = 16.100\text{ lb}$$

Impending motion: $\quad F = \mu_s N = 0.3(16.100\text{ lb})$

$$= 4.830\text{ lb}$$

(Note: For min. M_C, need F in direction shown; see FBD BC.)

FBD BC + collar:



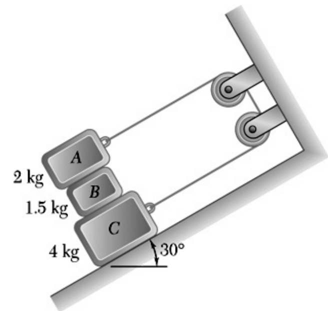
$$\circlearrowleft \Sigma M_C = 0: \quad M_C - (17\text{ in.})\frac{1}{\sqrt{5}}N - (8\text{ in.})\frac{2}{\sqrt{5}}N + (13\text{ in.})\frac{2}{\sqrt{5}}F = 0$$

$$M_C = \frac{1}{\sqrt{5}}\left[(17\text{ in.} + 16\text{ in.})(16.100\text{ lb}) - (26\text{ in.})(4.830\text{ lb})\right]$$

$$= 181.44\text{ lb}\cdot\text{in.}$$

$$(\mathbf{M}_C)_{\min} = 15.12\text{ lb}\cdot\text{ft} \circlearrowleft \blacktriangleleft$$

PROBLEM 8.42

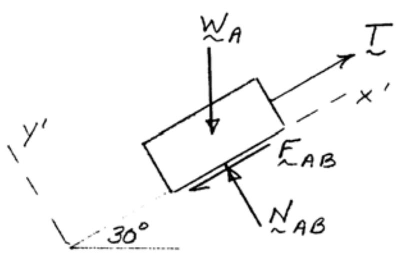


Blocks A, B, and C having the masses shown are at rest on an incline. Denoting by μ_s the coefficient of static friction between all surfaces of contact, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

For impending motion, C will start down and A will start up. Since, the normal force between B and C is larger than that between A and B, the corresponding friction force can be larger as well. Thus we assume that motion impends between A and B.

FBD A:



$$\nwarrow \Sigma F_{y'} = 0: \quad N_{AB} - W_A \cos 30° = 0; \quad N_{AB} = \frac{\sqrt{3}}{2} W_A$$

Impending motion:

$$F_{AB} = \mu_s N_{AB} = \frac{\sqrt{3}}{2} W_A \mu_s$$

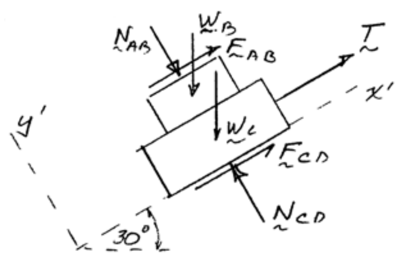
$$\nearrow \Sigma F_{x'} = 0: \quad T - F_{AB} - W_A \sin 30° = 0$$

or

$$T = \left(\sqrt{3}\mu_s + 1\right)\frac{W_A}{2}$$

$$\nwarrow \Sigma F_{y'} = 0: \quad N_{CD} - N_{AB} - \left(W_B + W_C\right)\cos 30° = 0$$

FBD B + C:



or

$$N_{CD} = \frac{\sqrt{3}}{2}\left(W_A + W_B + W_C\right)$$

Impending motion:

$$F_{CD} = \mu_s N_{CD} = \frac{\sqrt{3}}{2}\left(W_A + W_B + W_C\right)\mu_s$$

$$\nearrow \Sigma F_{x'} = 0: \quad T + F_{AB} + F_{CD} - \left(W_B + W_C\right)\sin 30° = 0$$

$$T = \frac{W_B + W_C}{2} - \frac{\sqrt{3}}{2}\mu_s\left(2W_A + W_B + W_C\right)$$

Equating T's:

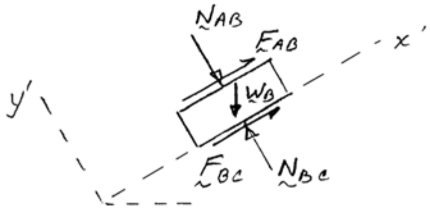
$$\sqrt{3}\mu_s\left(3W_A + W_B + W_C\right) = W_B + W_C - W_A$$

$$\mu_s = \frac{m_B + m_C - m_A}{\left(3m_A + m_B + m_C\right)\sqrt{3}} = \frac{1.5 \text{ kg} + 4 \text{ kg} - 2 \text{ kg}}{\left(6 \text{ kg} + 1.5 \text{ kg} + 4 \text{ kg}\right)\sqrt{3}}$$

$$\mu_s = 0.1757 \blacktriangleleft$$

PROBLEM 8.42 CONTINUED

FBD B:



$$\nwarrow \Sigma F_{y'} = 0: \quad N_{BC} - N_{AB} - W_B \cos 30° = 0$$

or $$N_{BC} = \frac{\sqrt{3}}{2}(W_A + W_B)$$

$$(F_{BC})_{\max} = \mu_s N_{BC} = 0.1757 \frac{\sqrt{3}}{2}(W_A + W_B)$$

$$= 0.1522(m_A + m_B)g = 0.1522(3.5 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 5.224 \text{ N}$$

$$\nearrow \Sigma F_{x'} = 0: \quad F_{AB} + F_{BC} - W_B \sin 30° = 0$$

or $$F_{BC} = -F_{AB} + \frac{1}{2}W_B = -\frac{\sqrt{3}}{2}W_A(0.1757) + \frac{W_B}{2}$$

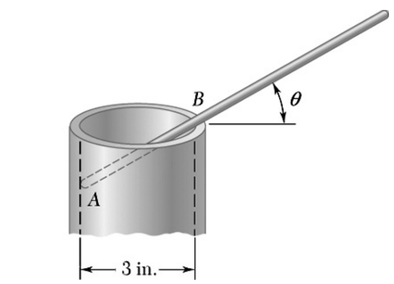
$$= (-0.1522m_A + 0.5m_B)g$$

$$= [-0.1522(2 \text{ kg}) + 0.5(1.5 \text{ kg})](9.81 \text{ m/s}^2)$$

$$= 4.37 \text{ N}$$

$$F_{BC} < F_{BC\max} \qquad \text{OK}$$

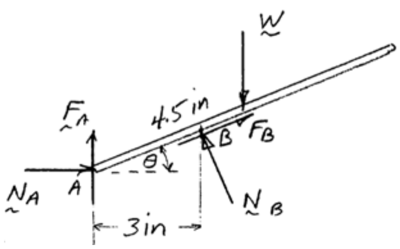
PROBLEM 8.43



A slender steel rod of length 9 in. is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad \frac{3\text{ in.}}{\cos\theta} N_B - \left[(4.5\text{ in.})\cos\theta\right] W = 0$$

or $$N_B = (1.5\cos^2\theta)W$$

Impending motion: $$F_B = \mu_s N_B = (1.5\mu_s \cos^2\theta)W$$

$$= (0.3\cos^2\theta)W$$

$$\longrightarrow \Sigma F_x = 0: \quad N_A - N_B \sin\theta + F_B \cos\theta = 0$$

or $$N_A = (1.5\cos^2\theta)W(\sin\theta - 0.2\cos\theta)$$

Impending motion: $$F_A = \mu_s N_A$$

$$= (0.3\cos^2\theta)W(\sin\theta - 0.2\cos\theta)$$

$$\uparrow \Sigma F_y = 0: \quad F_A + N_B\cos\theta + F_B\sin\theta - W = 0$$

or $$F_A = W\left(1 - 1.5\cos^3\theta - 0.3\cos^2\theta\sin\theta\right)$$

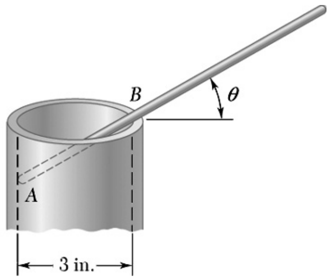
Equating F_A's

$$0.3\cos^2\theta(\sin\theta - 0.2\cos\theta) = 1 - 1.5\cos^3\theta - 0.3\cos^2\theta\sin\theta$$

$$0.6\cos^2\theta\sin\theta + 1.44\cos^3\theta = 1$$

Solving numerically $\theta = 35.8°$ ◀

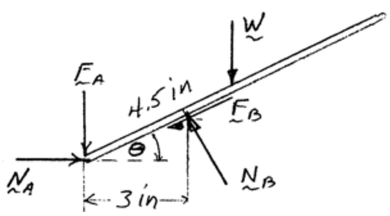
PROBLEM 8.44



In Problem 8.43, determine the smallest value of θ for which the rod will not fall out of the pipe.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad \frac{3 \text{ in.}}{\cos\theta} N_B - \left[(4.5 \text{ in.})\cos\theta\right] W = 0$$

or $$N_B = 1.5W\cos^2\theta$$

Impending motion: $$F_B = \mu_s N_B = 0.2\left(1.5W\cos^2\theta\right)$$

$$= 0.3W\cos^2\theta$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - N_B\sin\theta - F_B\cos\theta = 0$$

or $$N_A = W\cos^2\theta\left(1.5\sin\theta + 0.3\cos\theta\right)$$

Impending motion: $$F_A = \mu_s N_A$$

$$= W\cos^2\theta\left(0.3\sin\theta + 0.06\cos\theta\right)$$

$$\uparrow \Sigma F_y = 0: \quad N_B\cos\theta - F_B\sin\theta - W - F_A = 0$$

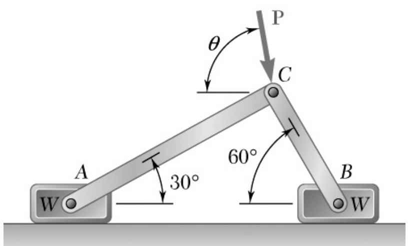
or $$F_A = W\left[\cos^2\theta\left(1.5\cos\theta - 0.3\sin\theta\right) - 1\right]$$

Equating F_A's:

$$\cos^2\theta\left(1.44\cos\theta - 0.6\sin\theta\right) = 1$$

Solving numerically $$\theta = 20.5° \blacktriangleleft$$

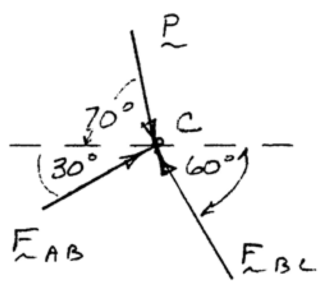
PROBLEM 8.45

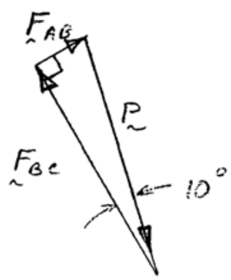


Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $\theta = 70^{\circ}$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD pin C:

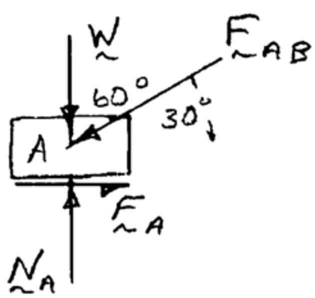




$$F_{AB} = P\sin 10^{\circ} = 0.173648P$$

$$F_{BC} = P\cos 10^{\circ} = 0.98481P$$

FBD block A:



$\uparrow \Sigma F_y = 0$: $N_A - W - F_{AB}\sin 30^{\circ} = 0$

or $N_A = W + 0.173648P\sin 30^{\circ} = W + 0.086824P$

$\longrightarrow \Sigma F_x = 0$: $F_A - F_{AB}\cos 30^{\circ} = 0$

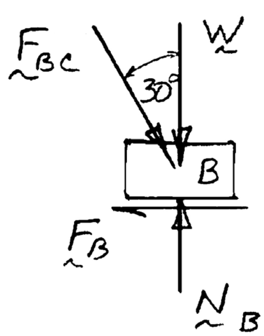
or $F_A = 0.173648P\cos 30^{\circ} = 0.150384P$

For impending motion at A: $F_A = \mu_s N_A$

Then $N_A = \dfrac{F_A}{\mu_s}$: $W + 0.086824P = \dfrac{0.150384}{0.3}P$

or $P = 2.413W$

FBD block B:



$\uparrow \Sigma F_y = 0$: $N_B - W - F_{BC}\cos 30^{\circ} = 0$

$N_B = W + 0.98481P\cos 30^{\circ} = W + 0.85287P$

$\longrightarrow \Sigma F_x = 0$: $F_{BC}\sin 30^{\circ} - F_B = 0$

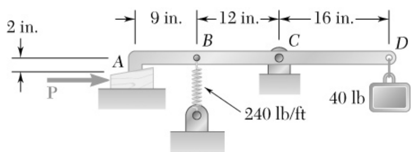
$F_B = 0.98481P\sin 30^{\circ} = 0.4924P$

For impending motion at B: $F_B = \mu_s N_B$

Then $N_B = \dfrac{F_B}{\mu_s}$: $W + 0.85287P = \dfrac{0.4924P}{0.3}$

or $P = 1.268W$

Thus, maximum P for equilibrium $P_{max} = 1.268W$ ◀



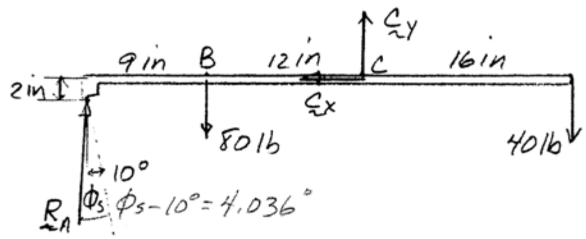
PROBLEM 8.46

A 40-lb weight is hung from a lever which rests against a 10° wedge at *A* and is supported by a frictionless hinge at *C*. Knowing that the coefficient of static friction is 0.25 at both surfaces of the wedge and that for the position shown the spring is stretched 4 in., determine (*a*) the magnitude of the force **P** for which motion of the wedge is impending, (*b*) the components of the corresponding reaction at *C*.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036° \qquad F_s = kx = (240 \text{ lb/ft})\left(\frac{4 \text{ in.}}{12 \text{ in./ft}}\right) = 80 \text{ lb}$$

FBD lever:



$$\circlearrowleft \Sigma M_C = 0: \quad (12 \text{ in.})(80 \text{ lb}) - (16 \text{ in.})(40 \text{ lb}) - (21 \text{ in.})R_A\cos(\phi_s - 10°)$$

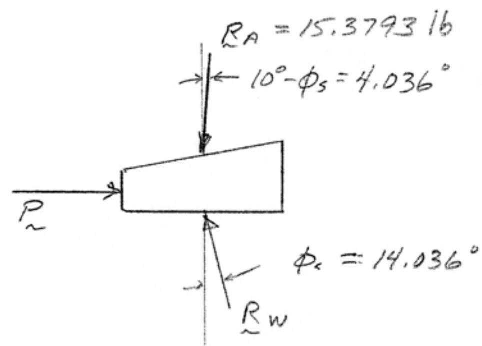
$$+ (2 \text{ in.})R_A\sin(\phi_s - 10°) = 0$$

or $\quad R_A = 15.3793 \text{ lb}$

(*b*) $\quad \longrightarrow \Sigma F_x = 0: \quad (15.379 \text{ lb})\sin(4.036°) - C_x = 0 \qquad \mathbf{C}_x = 1.082 \text{ lb} \longleftarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad (15.379 \text{ lb})\cos(4.036°) - 80 \text{ lb} - 40 \text{ lb} + C_y = 0 \qquad \mathbf{C}_y = 104.7 \text{ lb} \uparrow$ ◀

FBD wedge:

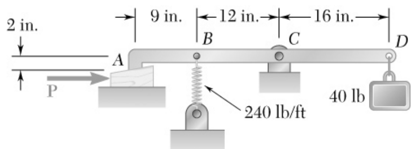


$$\uparrow \Sigma F_y = 0: \quad R_W\cos 14.036° - (15.3793 \text{ lb})\cos 4.036° = 0$$

or $\quad R_W = 15.8133 \text{ lb}$

(*a*) $\quad \longrightarrow \Sigma F_x = 0: \quad P - (15.3793 \text{ lb})\sin 4.036° - (15.8133 \text{ lb})\sin 14.036° = 0$

$P = 4.92 \text{ lb}$ ◀



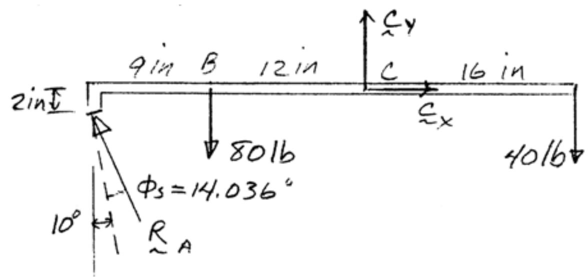
PROBLEM 8.47

Solve Problem 8.46 assuming that force **P** is directed to the left.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036° \qquad F_s = kx = (240 \text{ lb/ft})\left(\frac{4 \text{ in.}}{12 \text{ in./ft}}\right) = 80 \text{ lb}$$

FBD lever:



$$\circlearrowleft \Sigma M_C = 0: \quad (12 \text{ in.})(80 \text{ lb}) - (16 \text{ in.})(40 \text{ lb}) - (21 \text{ in.})R_A \cos 24.036°$$

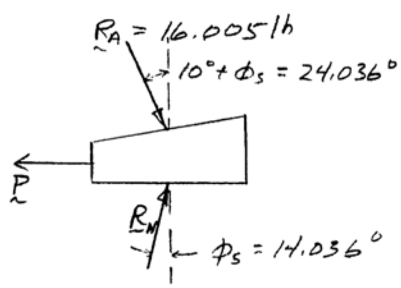
$$- (2 \text{ in.})R_A \sin 24.036° = 0$$

or $\quad R_A = 16.005$ lb

(*b*) $\quad \longrightarrow \Sigma F_x = 0: \quad C_x - (16.005 \text{ lb})\sin 24.036° = 0$ $\qquad \mathbf{C}_x = 6.52 \text{ lb} \longrightarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad C_y - 80 \text{ lb} - 40 \text{ lb} + (16.005 \text{ lb})\cos(24.036°) = 0$ $\qquad \mathbf{C}_y = 105.4 \text{ lb} \uparrow$ ◀

FBD wedge:

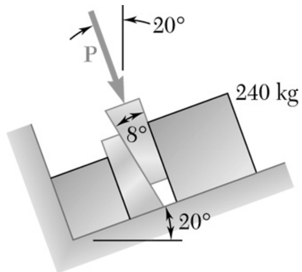


$\uparrow \Sigma F_y = 0: \quad R_W \cos 14.036° - (16.005 \text{ lb})\cos 24.036° = 0$

or $\quad R_W = 15.067$ lb

(*a*) $\quad \longrightarrow \Sigma F_x = 0: \quad (16.005 \text{ lb})\sin 24.036° + (15.067 \text{ lb})\sin 14.036° - P = 0$

$P = 10.17$ lb ◀



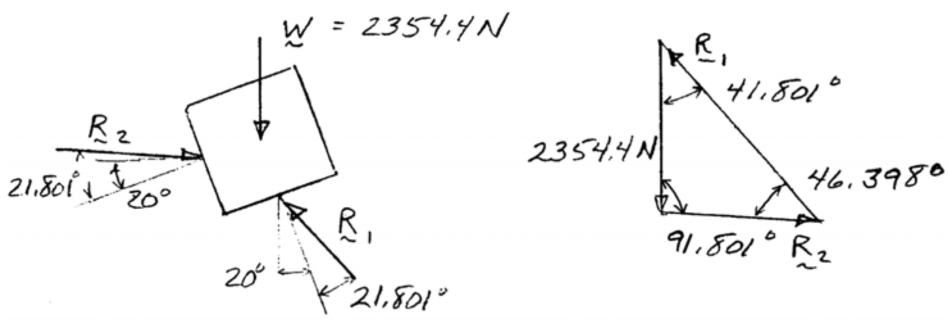
PROBLEM 8.48

Two 8° wedges of negligible mass are used to move and position a 240-kg block. Knowing that the coefficient of static friction is 0.40 at all surfaces of contact, determine the magnitude of the force **P** for which motion of the block is impending.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801° \qquad W = 240\text{ kg}\left(9.81\text{ m/s}^2\right) = 2354.4\text{ N}$$

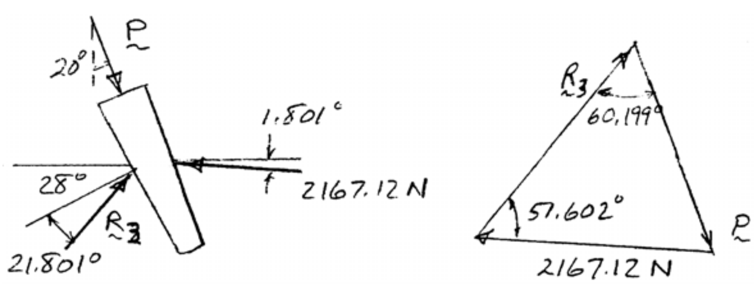
FBD block:



$$\frac{R_2}{\sin 41.801°} = \frac{2354.4\text{ N}}{\sin 46.398°}$$

$$R_2 = 2167.12\text{ N}$$

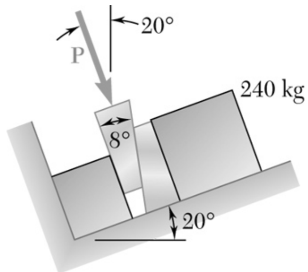
FBD wedge:



$$\frac{P}{\sin 51.602°} = \frac{2167.12\text{ N}}{\sin 60.199°}$$

$P = 1957$ N $\qquad\qquad$ $P = 1.957$ kN ◀

PROBLEM 8.49

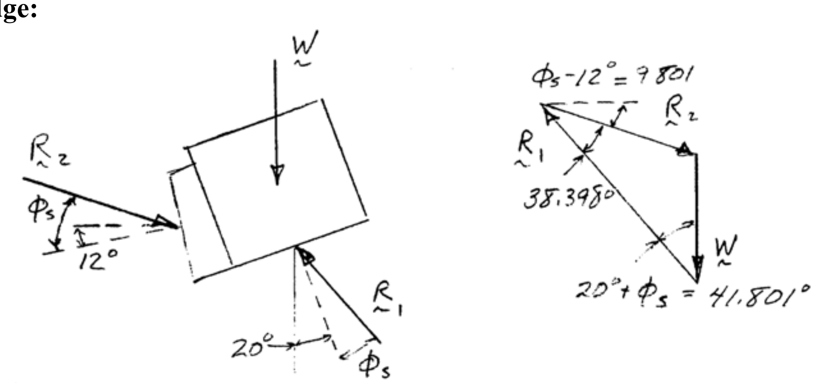


Two 8° wedges of negligible mass are used to move and position a 240-kg block. Knowing that the coefficient of static friction is 0.40 at all surfaces of contact, determine the magnitude of the force **P** for which motion of the block is impending.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801° \qquad W = 240\text{ kg}\left(9.81\text{ m/s}^2\right) = 2354.4\text{ N}$$

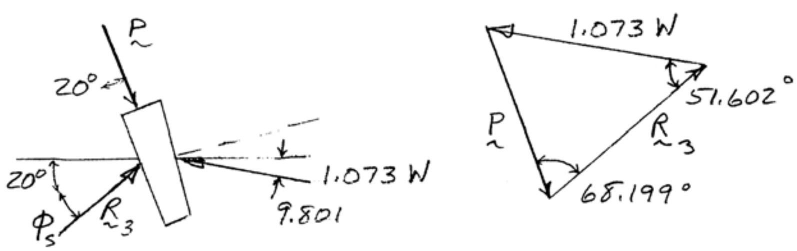
FBD block + wedge:



$$\frac{R_2}{\sin 41.801°} = \frac{2354.4\text{ N}}{\sin 38.398°}$$

$$R_2 = 2526.6\text{ N}$$

FBD wedge:

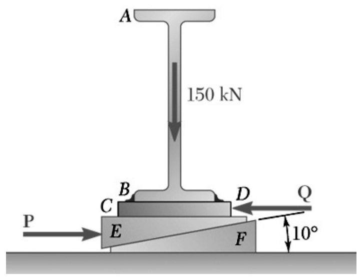


$$\frac{P}{\sin 51.602°} = \frac{2526.6\text{ N}}{\sin 68.199°}$$

$$P = 2132.7\text{ N}$$

$P = 2.13$ kN ◀

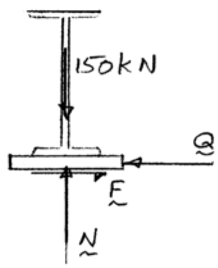
PROBLEM 8.50



The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 150 kN. The coefficient of static friction is 0.30 between the two steel surfaces and 0.60 between the steel and the concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

FBD AB + CD:



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70° \text{ for steel on steel}$$

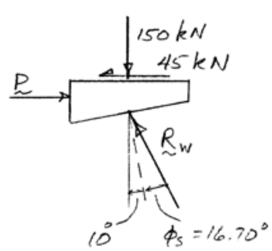
$$\uparrow \Sigma F_y = 0: \quad N - 150\text{ kN} = 0 \qquad N = 150\text{ kN}$$

Impending motion: $F = \mu_s N = 0.3(150\text{ kN}) = 45\text{ kN}$

$$\longrightarrow \Sigma F_x = 0: \quad F - Q = 0$$

(*b*) $\mathbf{Q} = 45.0\text{ kN} \longleftarrow$ ◀

FBD top wedge:



Assume bottom wedge doesn't move:

$$\uparrow \Sigma F_y = 0: \quad R_W \cos(10° + 16.70°) - 150\text{ kN} = 0$$

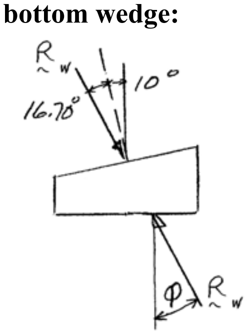
$$R_W = 167.9\text{ kN}$$

$$\longrightarrow \Sigma F_x = 0: \quad P - 45\text{ kN} - (167.9\text{ kN})\sin 26.70° = 0$$

$$P = 120.44\text{ kN}$$

(*a*) $\mathbf{P} = 120.4\text{ kN} \longrightarrow$ ◀

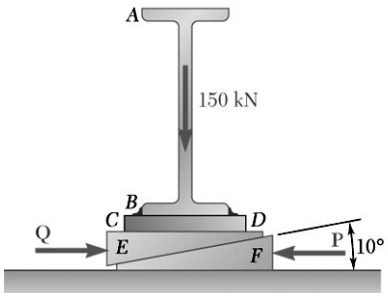
FBD bottom wedge:



Bottom wedge is two-force member, so $\phi = 26.70°$ for equilibrium, but

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.6 = 31.0° \text{ (steel on concrete)}$$

So $\phi < \phi_s$ OK.



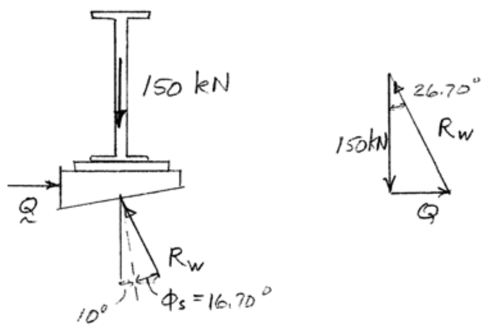
PROBLEM 8.51

The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 150 kN. The coefficient of static friction is 0.30 between the two steel surfaces and 0.60 between the steel and the concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70° \text{ for steel on steel}$$

FBD AB + CD + top wedge: Assume top wedge doesn't move

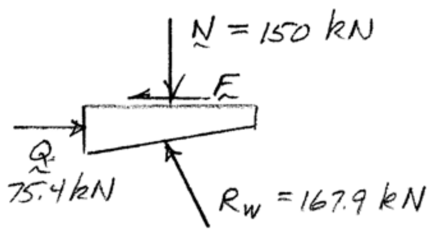


$$R_w = \frac{150 \text{ kN}}{\cos 26.70°} = 167.90 \text{ kN}$$

$$Q = (150 \text{ kN})\tan 26.70° = 75.44 \text{ kN}$$

(*b*) **Q** = 75.4 kN ⟶ ◀

FBD top wedge:

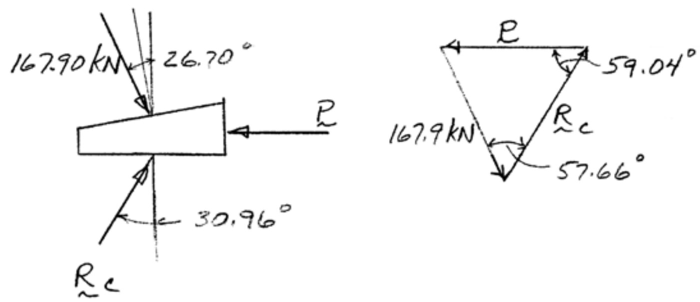


$$\longrightarrow \Sigma F_x = 0: \quad 75.44 \text{ kN} - 167.9 \text{ kN} \sin 26.70° - F = 0$$

$F = 0$ as expected.

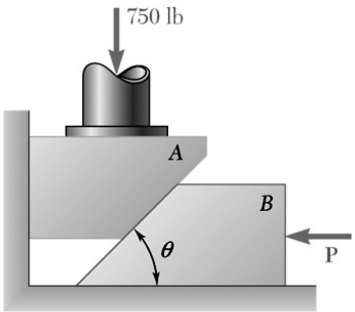
PROBLEM 8.51 CONTINUED

FBD bottom wedge: $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.6 = 30.96°$ steel on concrete



$$\frac{P}{\sin 57.66°} = \frac{167.90\text{ kN}}{\sin 59.04°}$$

(*a*) **P** = 165.4 kN ⟵ ◀



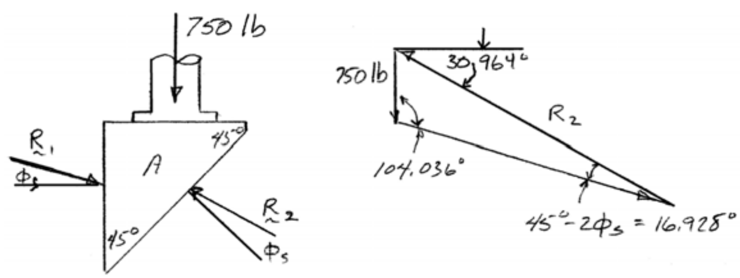
PROBLEM 8.52

Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** required to raise block *A*.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25 = 14.036°$$

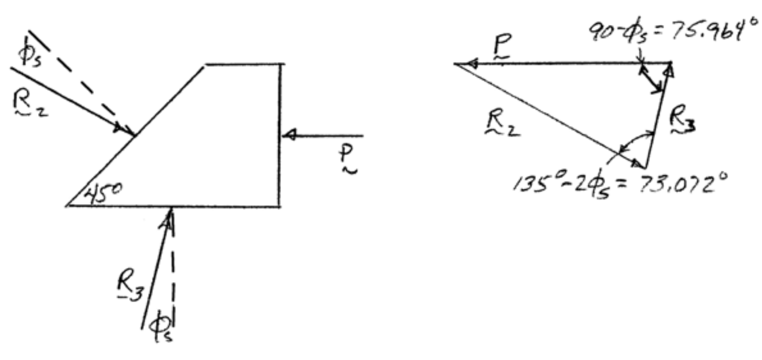
FBD block A:



$$\frac{R_2}{\sin 104.036°} = \frac{750 \text{ lb}}{\sin 16.928°}$$

$$R_2 = 2499.0 \text{ lb}$$

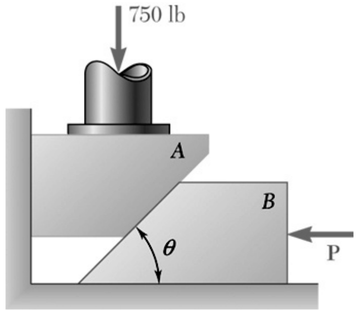
FBD wedge B:



$$\frac{P}{\sin 73.072°} = \frac{2499.0}{\sin 75.964°}$$

$$P = 2464 \text{ lb}$$

$\mathbf{P} = 2.46$ kips ← ◀



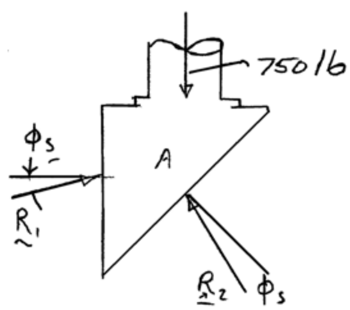
PROBLEM 8.53

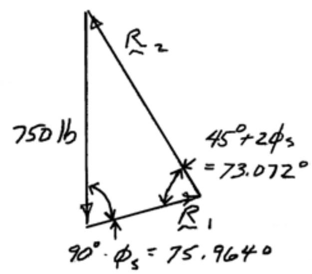
Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** for which equilibrium is maintained.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036°$$

FBD block A:

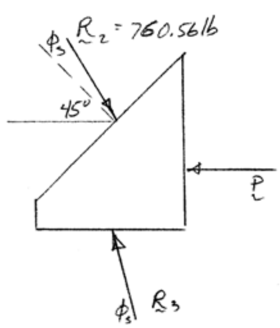


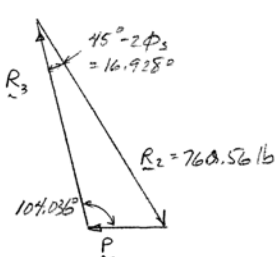


$$\frac{R_2}{\sin(75.964°)} = \frac{750 \text{ lb}}{\sin(73.072°)}$$

$$R_2 = 760.56 \text{ lb}$$

FBD wedge B:

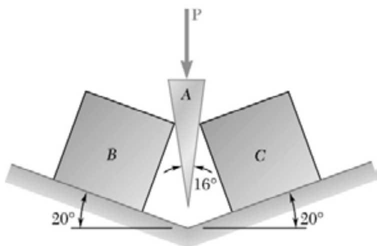




$$\frac{P}{\sin 16.928°} = \frac{760.56}{\sin 104.036°}$$

$$P = 228.3 \text{ lb}$$

P = 228 lb ⟵ ◀



PROBLEM 8.54

A 16° wedge A of negligible mass is placed between two 80-kg blocks B and C which are at rest on inclined surfaces as shown. The coefficient of static friction is 0.40 between both the wedge and the blocks and block C and the incline. Determine the magnitude of the force **P** for which motion of the wedge is impending when the coefficient of static friction between block B and the incline is (*a*) 0.40, (*b*) 0.60.

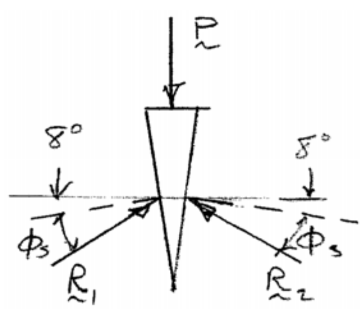
SOLUTION

(*a*)

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.8014°;$$

$$W = 80\text{ kg}\left(9.81\text{ m/s}^2\right) = 784.8\text{ N}$$

FBD wedge:



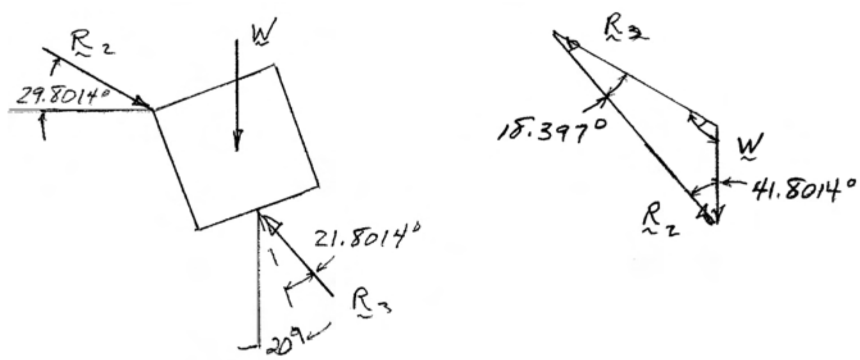
By symmetry:

$$\mathbf{R}_1 = \mathbf{R}_2$$

$$\uparrow \Sigma F_y = 0: \quad 2R_2\sin\left(8° + 21.8014°\right) - P = 0$$

$$P = 0.99400R_2$$

FBD block C:



$$\frac{R_2}{\sin 41.8014°} = \frac{W}{\sin 18.397°}$$

$$R_2 = 2.112\,W$$

PROBLEM 8.54 CONTINUED

$$P = 0.994R_2 = (0.994)(2.112W)$$

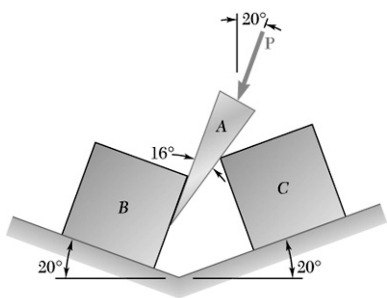
$$P = 2.099(784.8 \text{ N}) = 1647.5 \text{ N}$$

(*a*) $P = 1.648 \text{ kN}$ ◀

(*b*) Note that increasing the friction between block B and the incline has no effect on the above calculations. The physical effect is that slip of B will not impend.

(*b*) $P = 1.648 \text{ kN}$ ◀

PROBLEM 8.55

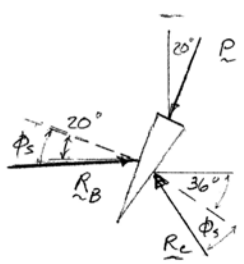


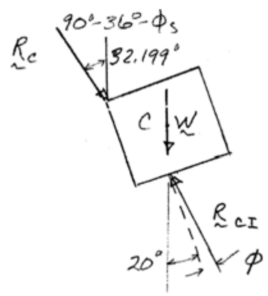
A 16° wedge *A* of negligible mass is placed between two 80-kg blocks *B* and *C* which are at rest on inclined surfaces as shown. The coefficient of static friction is 0.40 between both the wedge and the blocks and block *C* and the incline. Determine the magnitude of the force **P** for which motion of the wedge is impending when the coefficient of static friction between block *B* and the incline is (*a*) 0.40, (*b*) 0.60.

SOLUTION

(*a*) $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$ $\qquad W = 80\text{ kg}\left(9.81\text{ m/s}^2\right) = 784.8\text{ N}$

FBD wedge: **FBD block C:**



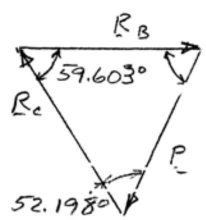


Note that, since $\left(R_{CI}\right)_y > \left(R_C\right)_y$, while the horizontal components are equal,

$$20° + \phi < 32.199°$$

$$\phi < 12.199° < \phi_s$$

Therefore, motion of *C* is *not* impending; thus, motion of *B* up the incline is impending.

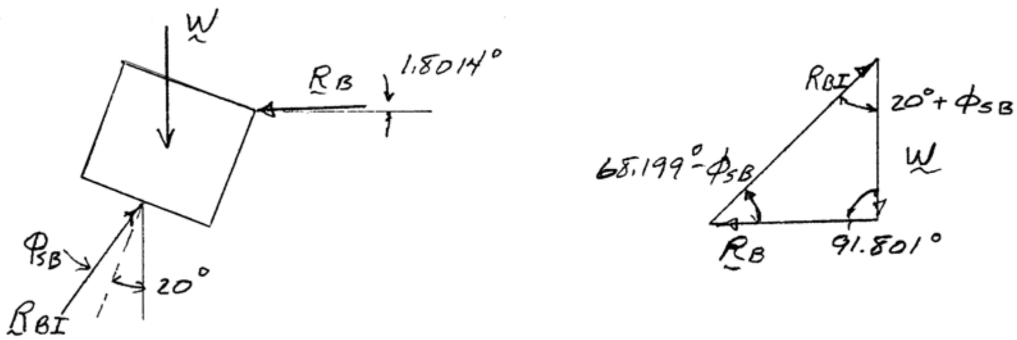


$$\frac{R_B}{\sin 52.198°} = \frac{P}{\sin 59.603°}$$

$$P = 1.0916R_B$$

PROBLEM 8.55 CONTINUED

FBD block B:



$$\frac{R_B}{\sin\left(20° + \phi_{sB}\right)} = \frac{W}{\sin\left(68.199° - \phi_{sB}\right)}$$

or

$$R_B = \frac{W\sin\left(20° + \phi_{sB}\right)}{\sin\left(68.199° - \phi_{sB}\right)}$$

(*a*) Have $\phi_{sB} = \phi_s = 21.801°$

Then

$$R_B = \frac{\left(784.8\text{ N}\right)\sin\left(20° + 21.801°\right)}{\sin\left(68.199° - 21.801°\right)} = 722.37\text{ N}$$

and

$$P = 1.0916\left(722.37\text{ N}\right)$$

or $P = 789\text{ N}$ ◀

(*b*) Have $\phi_{sB} = \tan^{-1}\mu_{sB} = \tan^{-1}0.6 = 30.964°$

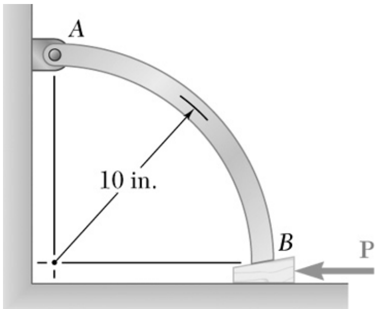
Then

$$R_B = \frac{\left(784.8\text{ N}\right)\sin\left(20° + 30.964°\right)}{\sin\left(68.199° - 30.964°\right)} = 1007.45\text{ N}$$

and

$$P = 1.0916\left(1007.45\text{ N}\right)$$

or $P = 1100\text{ N}$ ◀

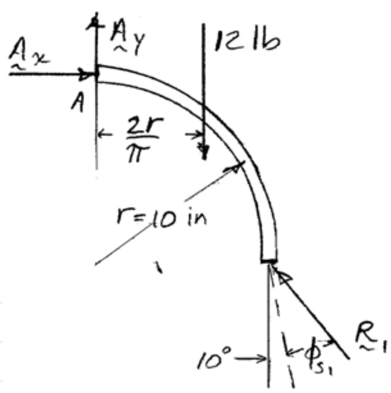


PROBLEM 8.56

A 10° wedge is to be forced under end B of the 12-lb rod AB. Knowing that the coefficient of static friction is 0.45 between the wedge and the rod and 0.25 between the wedge and the floor, determine the smallest force **P** required to raise end B of the rod.

SOLUTION

FBD AB:

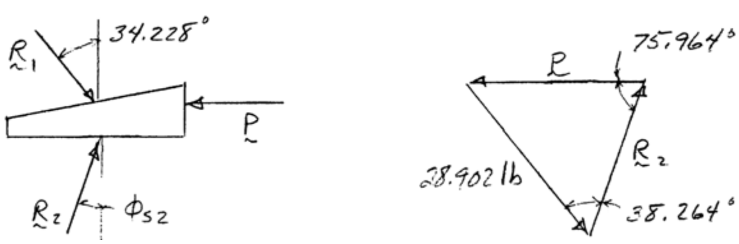


$$\phi_{s1} = \tan^{-1}(\mu_s)_1 = \tan^{-1} 0.45 = 24.228°$$

$$\circlearrowleft \Sigma M_A = 0: \quad rR_1 \cos(10° + 24.228°) - rR_1 \sin(10° + 24.228°) - \frac{2r}{\pi}(12\text{ lb}) = 0$$

$$R_1 = 28.902\text{ lb}$$

FBD wedge:

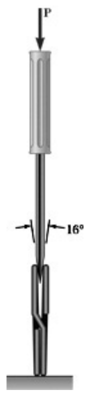


$$\phi_{s2} = \tan^{-1}(\mu_s)_2 = \tan^{-1} 0.25 = 14.036°$$

$$\frac{P}{\sin(38.264°)} = \frac{28.902\text{ lb}}{\sin 75.964°};$$

$\mathbf{P} = 22.2\text{ lb} \longleftarrow$ ◀

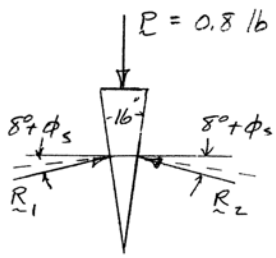
PROBLEM 8.57



A small screwdriver is used to pry apart the two coils of a circular key ring. The wedge angle of the screwdriver blade is 16° and the coefficient of static friction is 0.12 between the coils and the blade. Knowing that a force **P** of magnitude 0.8 lb was required to insert the screwdriver to the equilibrium position shown, determine the magnitude of the forces exerted on the ring by the screwdriver immediately after force **P** is removed.

SOLUTION

FBD wedge:



By symmetry: $R_1 = R_2$

$$\uparrow \Sigma F_y = 0: \quad 2R_1 \sin(8° + \phi_s) - P = 0$$

Have $\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.12 = 6.843°$ $\quad P = 0.8 \text{ lb}$

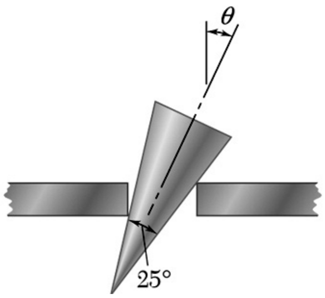
So $R_1 = R_2 = 1.5615 \text{ lb}$

When **P** is removed, the vertical components of R_1 and R_2 vanish, leaving the horizontal components, $R_1 \cos(14.843°)$, only

Therefore, side forces are 1.509 lb ◀

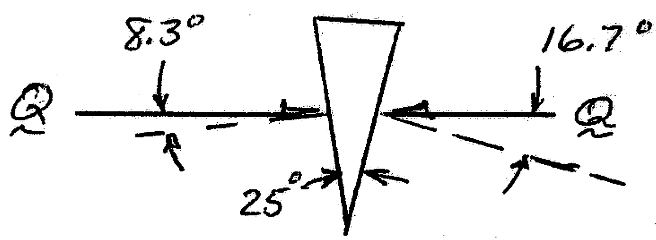
But these will occur only instantaneously as the angle between the force and the wedge normal is $8° > \phi_s = 6.84°$, so the screwdriver will slip out.

PROBLEM 8.58



A conical wedge is placed between two horizontal plates that are then slowly moved toward each other. Indicate what will happen to the wedge (a) if $\mu_s = 0.20$, (b) if $\mu_s = 0.30$.

SOLUTION

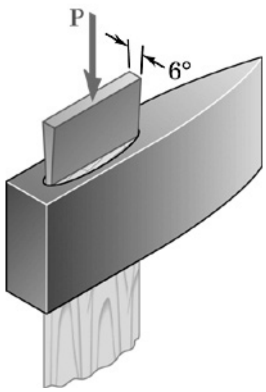


As the plates are moved, the angle θ will decrease.

(a) $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.2 = 11.31°$. As θ decreases, the minimum angle at the contact approaches $12.5° > \phi_s = 11.31°$, so the wedge will slide up and out from the slot. ◀

(b) $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.3 = 16.70°$. As θ decreases, the angle at one contact reaches $16.7°$. (At this time the angle at the other contact is $25° - 16.7° = 8.3° < \phi_s$) The wedge binds in the slot. ◀

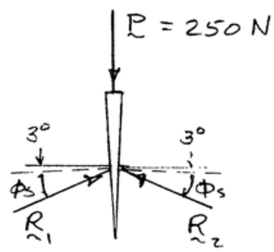
PROBLEM 8.59



A 6° steel wedge is driven into the end of an ax handle to lock the handle to the ax head. The coefficient of static friction between the wedge and the handle is 0.35. Knowing that a force **P** of magnitude 250 N was required to insert the wedge to the equilibrium position shown, determine the magnitude of the forces exerted on the handle by the wedge after force **P** is removed.

SOLUTION

FBD wedge:



By symmetry $R_1 = R_2$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.35 = 19.29°$$

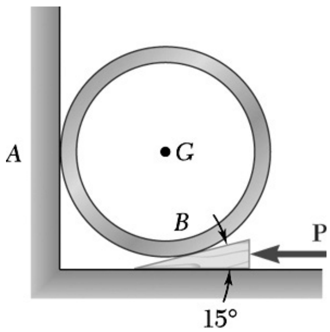
$$\uparrow \Sigma F_y = 0: \quad 2R\sin(19.29° + 3°) - P = 0$$

$$R_1 = R_2 = 329.56 \text{ N}$$

When force **P** is removed, the vertical components of R_1 and R_2 vanish, leaving only the horizontal components $H_1 = H_2 = R\cos(22.29°)$

$$H_1 = H_2 = 305 \text{ N} \blacktriangleleft$$

Since the wedge angle $3° < \phi_s = 19.3°$, the wedge is "self-locking" and will remain seated.

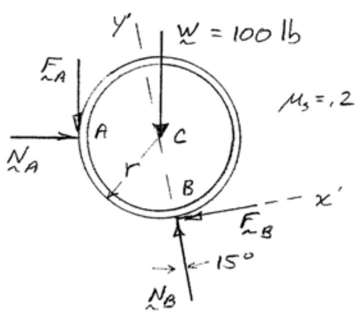


PROBLEM 8.60

A 15° wedge is forced under a 100-lb pipe as shown. The coefficient of static friction at all surfaces is 0.20. Determine (*a*) at which surface slipping of the pipe will first occur, (*b*) the force **P** for which motion of the wedge is impending.

SOLUTION

FBD pipe:



(*a*) $\circlearrowleft \Sigma M_C = 0\!: \quad rF_A - rF_B = 0$

or $$F_A = F_B$$

But it is apparent that $N_B > N_A$, so since $(\mu_s)_A = (\mu_s)_B$,
motion must first impend at A ◀

and $$F_B = F_A = \mu_s N_A = 0.2N_A$$

(*b*) $\circlearrowleft \Sigma M_B = 0\!: \quad (r\sin 15°)W + r(1+\sin 15°)F_A - (r\cos 15°)N_A = 0$

$$0.2588(100\text{ lb}) + 1.2588(0.2N_A) - 0.9659N_A = 0$$

or $$N_A = 36.24\text{ lb}$$

and $$F_A = 7.25\text{ lb}$$

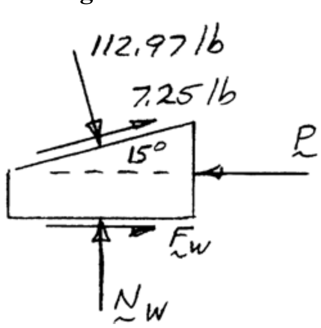
$\nwarrow \Sigma F_{y'} = 0\!: \quad N_B - N_A\sin 15° - F_A\cos 15° - W\cos 15° = 0$

$$N_B = (36.24\text{ lb})\sin 15° + (7.25\text{ lb} + 100\text{ lb})\cos 15°$$

$$= 112.97\text{ lb}$$

(note $N_B > N_A$ as stated, and $F_B < \mu_s N_B$)

FBD wedge:



$\uparrow \Sigma F_y = 0\!: \quad N_W + (7.25\text{ lb})\sin 15° - (112.97\text{ lb})\cos 15° = 0$

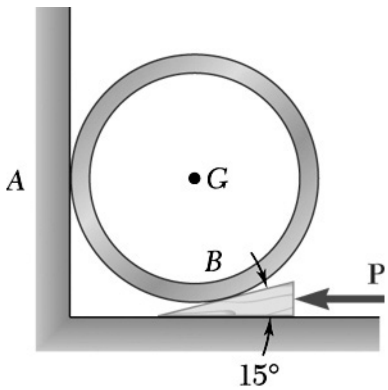
$$N_W = 107.24\text{ lb}$$

Impending slip: $$F_W = \mu_s N_W = 0.2(107.24) = 21.45\text{ lb}$$

$\rightarrow \Sigma F_x = 0\!: \quad 21.45\text{ lb} + (7.25\text{ lb})\cos 15° + (112.97\text{ lb})\sin 15° - P = 0$

P $= 57.7$ lb ⟵ ◀

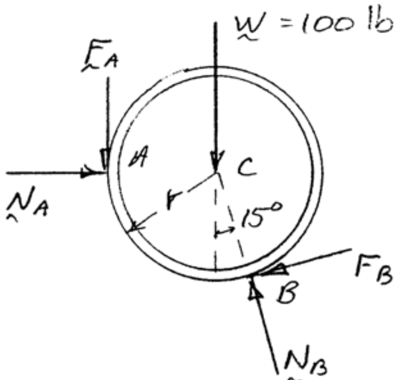
PROBLEM 8.61



A 15° wedge is forced under a 100-lb pipe as shown. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine the largest coefficient of static friction between the pipe and the vertical wall for which slipping is impending at *A*.

SOLUTION

FBD pipe:



$$\circlearrowleft \Sigma M_C = 0: \quad rF_A - rF_B = 0$$

or
$$F_A = F_B$$

It is apparent that $N_B > N_A$, so if $(\mu_s)_A = (\mu_s)_B$, motion must impend first at *A*. As $(\mu_s)_A$ is increased to some $(\mu_s^*)_A$, motion will impend simultaneously at *A* and *B*.

Then
$$F_A = F_B = \mu_{sB} N_B = 0.2 N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B \cos 15° - F_B \sin 15° - F_A - 100 \text{ lb} = 0$$

$$N_B \cos 15° - 0.2 N_B \sin 15° - 0.2 N_B = 100 \text{ lb}$$

or
$$N_B = 140.024 \text{ lb}$$

So
$$F_A = F_B = 0.2 N_B = 28.005 \text{ lb}$$

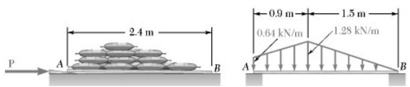
$$\longrightarrow \Sigma F_x = 0: \quad N_A - N_B \sin 15° - F_B \cos 15° = 0$$

$$N_A = 140.024 \sin 15° + 28.005 \cos 15° = 63.29 \text{ lb}$$

Then
$$(\mu_s^*)_A = \frac{F_A}{N_A} = \frac{28.005 \text{ lb}}{63.29 \text{ lb}}$$

or
$$(\mu_s^*)_A = 0.442 \blacktriangleleft$$

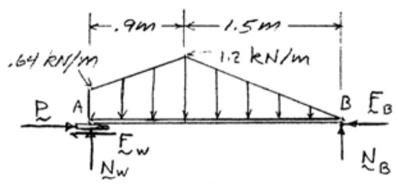
PROBLEM 8.62



Bags of grass seed are stored on a wooden plank as shown. To move the plank, a 9° wedge is driven under end A. Knowing that the weight of the grass seed can be represented by the distributed load shown and that the coefficient of static friction is 0.45 between all surfaces of contact, (a) determine the force **P** for which motion of the wedge is impending, (b) indicate whether the plank will slide on the floor.

SOLUTION

FBD plank + wedge:



(a) $\circlearrowleft \Sigma M_A = 0$: $(2.4\text{ m})N_B - (0.45\text{ m})(0.64\text{ kN/m})(0.9\text{ m})$

$$-(0.6\text{m})\frac{1}{2}(0.64\text{ kN/m})(0.9\text{ m})$$

$$-(1.4\text{ m})\frac{1}{2}(1.28\text{ kN/m})(1.5\text{ m}) = 0$$

or $$N_B = 0.740\text{ kN} = 740\text{ N}$$

$\uparrow \Sigma F_y = 0$: $N_W - (0.64\text{ kN/m})(0.9\text{ m}) - \frac{1}{2}(0.64\text{ kN/m})(0.9\text{ m})$

$$-\frac{1}{2}(1.28\text{ kN/m})(1.5\text{m}) = 0$$

or $$N_W = 1.084\text{ kN} = 1084\text{ N}$$

Assume impending motion of the wedge on the floor and the plank on the floor at B.

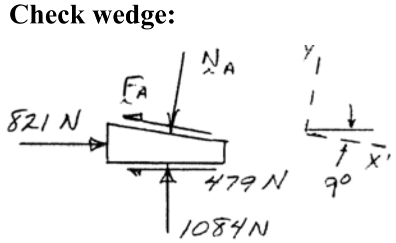
So $$F_W = \mu_s N_W = 0.45(1084\text{ N}) = 478.8\text{ N}$$

and $$F_B = \mu_s N_B = 0.45(740\text{ N}) = 333\text{ N}$$

$\longrightarrow \Sigma F_x = 0$: $P - F_W - F_B = 0$

or $$P = 478.8\text{ N} + 333\text{ N} \qquad P = 821\text{ N} \blacktriangleleft$$

Check wedge:



(b) $\uparrow \Sigma F_y = 0$: $(1084\text{ N})\cos 9° + (821\text{ N} - 479\text{ N})\sin 9° - N_A = 0$

or $$N_A = 1124\text{ N}$$

$\searrow \Sigma F_x = 0$: $(821\text{ N} - 479\text{ N})\cos 9° - (1084\text{ N})\sin 9° - F_A = 0$

or $$F_A = 168\text{ N}$$

$$F_A < \mu_s N_A = 0.45(1124\text{ N}) = 506\text{ N}$$

So, no impending motion at wedge/plank

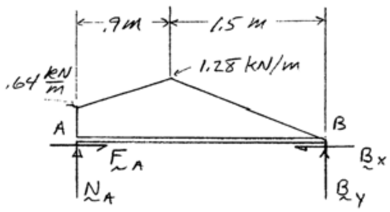
$\therefore$ Impending motion of plank on floor at B ◀

PROBLEM 8.63

Solve Problem 8.62 assuming that the wedge is driven under the plank at *B* instead of at *A*.

SOLUTION

FBD plank:



$\rightarrow \Sigma F_x = 0: \quad F_A - B_x = 0$

$F_A = B_x$

(*a*) $\circlearrowleft \Sigma M_A = 0: \quad (2.4\text{ m})B_y - (0.45\text{ m})(0.64\text{ kN/m})(0.9\text{ m})$

$$- (0.6\text{ m})\frac{1}{2}(0.64\text{ kN/m})(0.9\text{ m})$$

$$- (1.4\text{ m})\frac{1}{2}(1.28\text{ kN/m})(1.5\text{ m}) = 0$$

or $\quad B_y = 0.740\text{ kN} = 740\text{ N}$

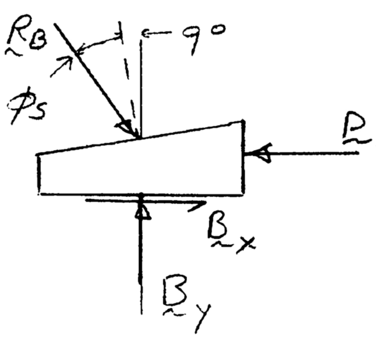
$\uparrow \Sigma F_y = 0: \quad N_A - (0.64\text{ kN/m})(0.9\text{ m}) - \frac{1}{2}(0.64\text{ kN/m})(0.9\text{ m})$

$$-\frac{1}{2}(1.28\text{ kN/m})(1.5\text{ m}) = 0$$

or $\quad N_A = 1.084\text{ kN} = 1084\text{ N}$

Since $B_y < N_A$, assume impending motion of the wedge under the plank at *B*.

FBD wedge:



$(R_B)_y = B_y = 740\text{ N} \quad \text{and} \quad B_x = \mu_s B_y = 0.45(740\text{ N}) = 333\text{ N}$

$$(R_B)_x = (R_B)_y \tan(9° + \phi_s)$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.45 = 24.228°$$

So $\quad (R_B)_x = (740\text{ N})\tan(9° + 24.228°) = 485\text{ N}$

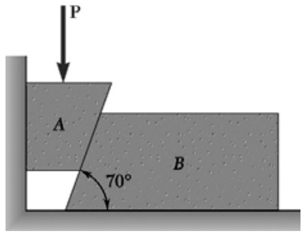
$\rightarrow \Sigma F_x = 0: \quad 485\text{ N} - 333\text{ N} - P = 0$

$\mathbf{P} = 818\text{ N} \leftarrow$ ◀

(*b*) Check:

$F_A = B_x = 333\text{ N} \quad \text{and} \quad \frac{F_A}{N_A} = \frac{333}{1084} = 0.307 < \mu_s \qquad \text{OK}$

No impending slip of plank at *A* ◀

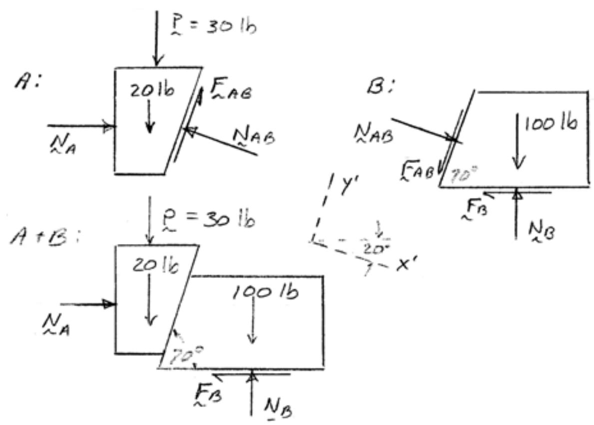


PROBLEM 8.64

The 20-lb block A is at rest against the 100-lb block B as shown. The coefficient of static friction μ_s is the same between blocks A and B and between block B and the floor, while friction between block A and the wall can be neglected. Knowing that $P = 30$ lb, determine the value of μ_s for which motion is impending.

SOLUTION

FBD's:



Impending motion at all surfaces $\qquad F_{AB} = \mu_s N_{AB}$

$$F_B = \mu_s N_B$$

$A + B$: $\qquad \uparrow \Sigma F_y = 0\!: \quad N_B - 30 \text{ lb} - 20 \text{ lb} - 100 \text{ lb} = 0$

or $\qquad N_B = 150 \text{ lb}$

and $\qquad F_B = \mu_s N_B = (150 \text{ lb})\mu_s$

$\longrightarrow \Sigma F_x = 0\!: \quad N_A - F_B = 0 \qquad$ so that $\qquad N_A = (150 \text{ lb})\mu_s$

A: $\qquad \searrow \Sigma F_{x'} = 0\!: \quad N_A \cos 20° + (30 \text{ lb} + 20 \text{ lb})\sin 20° - N_{AB} = 0$

or $\qquad N_{AB} = 17.1010 \text{ lb} + \mu_s(140.954 \text{ lb})$

$\nearrow \Sigma F_{y'} = 0\!: \quad F_{AB} + N_A \sin 20° - (30 \text{ lb} + 20 \text{ lb})\cos 20° = 0$

or $\qquad F_{AB} = 46.985 \text{ lb} - \mu_s(51.303 \text{ lb})$

But $\qquad F_{AB} = \mu_s N_{AB}\!: \quad 46.985 - 51.303\mu_s = 17.101\mu_s + 140.954\mu_s^2$

$$\mu_s^2 + 0.4853\mu_s - 0.3333 = 0$$

$$\mu_s = -0.2427 \pm 0.6263$$

$\mu_s > 0 \qquad$ so $\qquad \mu_s = 0.384$ ◀

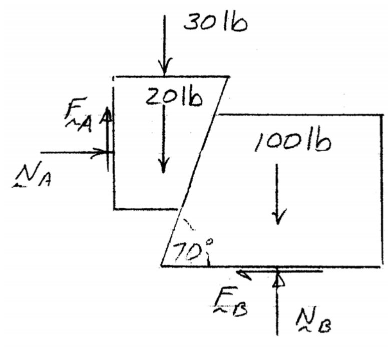
PROBLEM 8.65

Solve Problem 8.64 assuming that μ_s is the coefficient of static friction between all surfaces of contact.

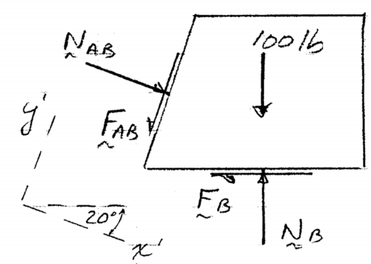
SOLUTION

FBD's:

$A + B$:



B:



Impending motion at all surfaces, so $\quad F_A = \mu_s N_A$

$$F_B = \mu_s N_B$$

$$F_{AB} = \mu_s N_{AB}$$

$A + B$: $\quad \longrightarrow \Sigma F_x = 0: \quad N_A - F_B = 0 \text{ or } N_A = F_B = \mu_s N_B$

$$\uparrow \Sigma F_y = 0: \quad F_A - 30 \text{ lb} - 20 \text{ lb} - 100 \text{ lb} + N_B = 0 \text{ or } \quad \mu_s N_A + N_B = 150 \text{ lb}$$

So $\quad N_B = \dfrac{150 \text{ lb}}{1 + \mu_s^2} \quad$ and $\quad F_B = \dfrac{\mu_s}{1 + \mu_s^2}(150 \text{ lb})$

B: $\quad \searrow \Sigma F_{x'} = 0: \quad N_{AB} + (100 \text{ lb} - N_B)\sin 20° - F_B \cos 20° = 0$

or $\quad N_{AB} = N_B \sin 20° + F_B \cos 20° - (100 \text{ lb})\sin 20°$

$$\nearrow \Sigma F_{y'} = 0: \quad -F_{AB} + (N_B - 100 \text{ lb})\cos 20° - F_B \sin 20° = 0$$

or $\quad F_{AB} = N_B \cos 20° - F_B \sin 20° - (100 \text{ lb})\cos 20°$

PROBLEM 8.65 CONTINUED

Now $\qquad F_{AB} = \mu_s N_{AB}: \quad \frac{150 \text{ lb}}{1+\mu_s^2}\cos 20° - \frac{\mu_s}{1+\mu_s^2}(150 \text{ lb})\sin 20° - (100 \text{ lb})\cos 20°$

$$= \frac{\mu_s}{1+\mu_s^2}(150 \text{ lb})\sin 20° + \frac{\mu_s^2}{1+\mu_s^2}(150 \text{ lb})\cos 20° - \mu_s(100 \text{ lb})\sin 20°$$

$$2\mu_s^3 - 5\mu_s^2 \text{ctn}\, 20° - 4\mu_s + \text{ctn}\, 20° = 0$$

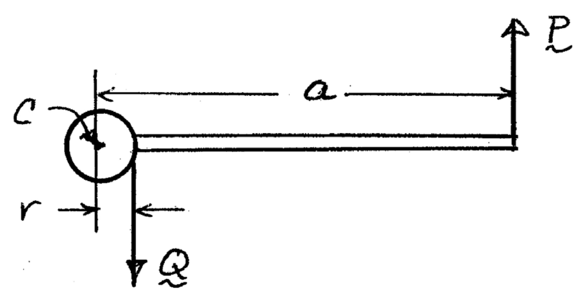
Solving numerically: $\mu_s = 0.330$ ◀

PROBLEM 8.66

Derive the following formulas relating the load **W** and the force **P** exerted on the handle of the jack discussed in Section 8.6. (*a*) $P = (Wr/a)\tan(\theta + \phi_s)$, to raise the load; (*b*) $P = (Wr/a)\tan(\phi_s - \theta)$, to lower the load if the screw is self-locking; (*c*) $P = (Wr/a)\tan(\theta - \phi_s)$, to hold the load if the screw is not self-locking.

SOLUTION

FBD jack handle:

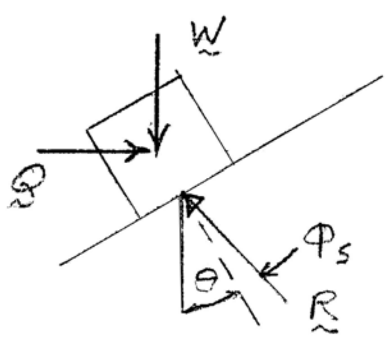


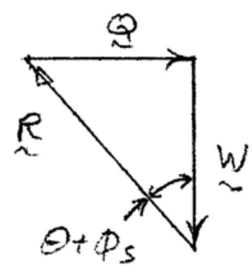
See Section 8.6

$$\circlearrowleft \Sigma M_C = 0: \quad aP - rQ = 0 \text{ or } P = \frac{r}{a}Q$$

FBD block on incline:

(*a*) Raising load



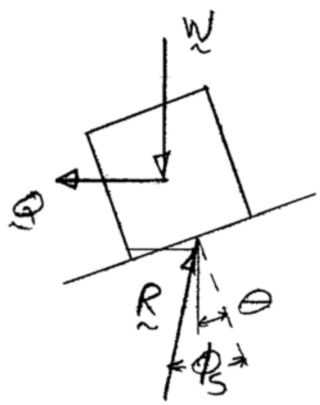


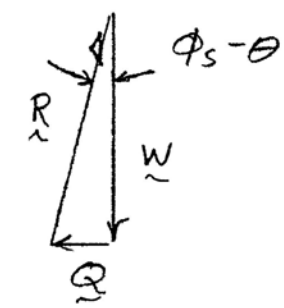
$$Q = W\tan(\theta + \phi_s)$$

$$P = \frac{r}{a}W\tan(\theta + \phi_s) \blacktriangleleft$$

PROBLEM 8.66 CONTINUED

(*b*) Lowering load if screw is self-locking (i.e.: if $\phi_s > \theta$)

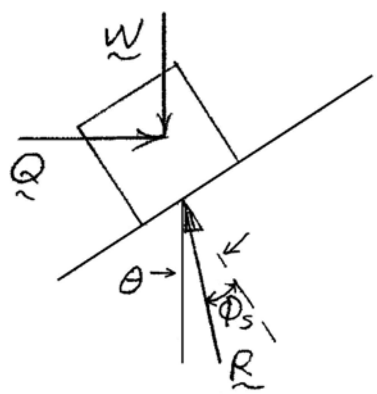


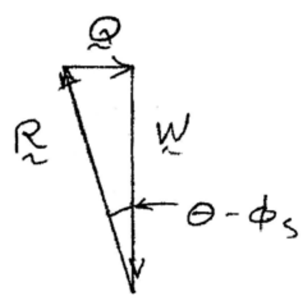


$$Q = W\tan(\phi_s - \theta)$$

$$P = \frac{r}{a}W\tan(\phi_s - \theta) \blacktriangleleft$$

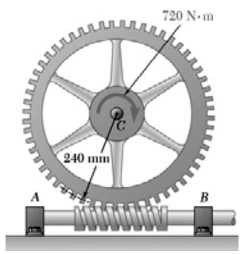
(*c*) Holding load is screw is not self-locking $\left(\text{i.e. if } \phi_s < \theta\right)$





$$Q = W\tan(\theta - \phi_s)$$

$$P = \frac{r}{a}W\tan(\theta - \phi_s) \blacktriangleleft$$

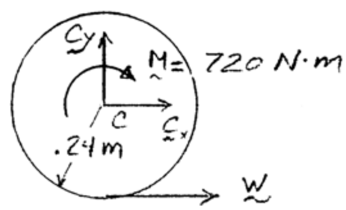


PROBLEM 8.67

The square-threaded worm gear shown has a mean radius of 30 mm and a lead of 7.5 mm. The larger gear is subjected to a constant clockwise couple of 720 N·m. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

SOLUTION

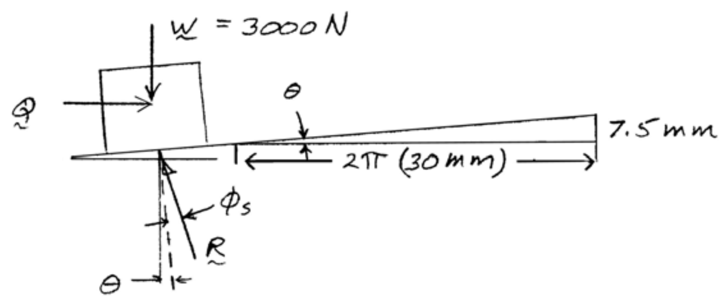
FBD large gear:



$$\circlearrowleft \Sigma M_C = 0: \quad (0.24 \text{ m})W - 720 \text{ N}\cdot\text{m} = 0$$

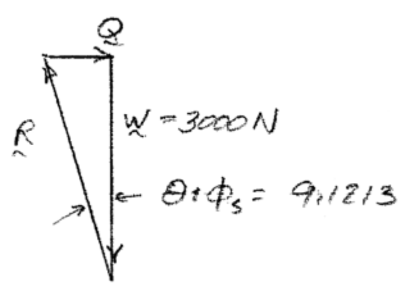
$$W = 3000 \text{ N}$$

Block on incline:



$$\theta = \tan^{-1}\frac{7.5 \text{ mm}}{2\pi(30 \text{ mm})} = 2.2785°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.12 = 6.8428°$$

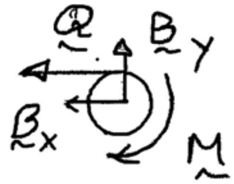


$$Q = (3000 \text{ N})\tan 9.1213°$$

$$= 481.7 \text{ N}$$

PROBLEM 8.67 CONTINUED

Worm gear:



$$r = 30 \text{ mm}$$

$$= 0.030 \text{ m}$$

$$\circlearrowright \Sigma M_B = 0: \quad rQ - M = 0$$

$$M = rQ = (0.030 \text{ m})(481.7 \text{ N})$$

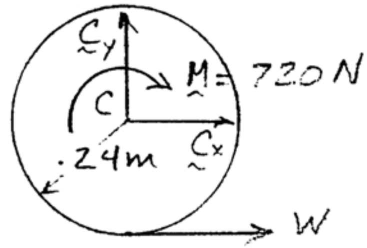
$M = 14.45 \text{ N}\cdot\text{m}$ ◀

PROBLEM 8.68

In Problem 8.67, determine the couple that must be applied to shaft *AB* in order to rotate the gear clockwise.

SOLUTION

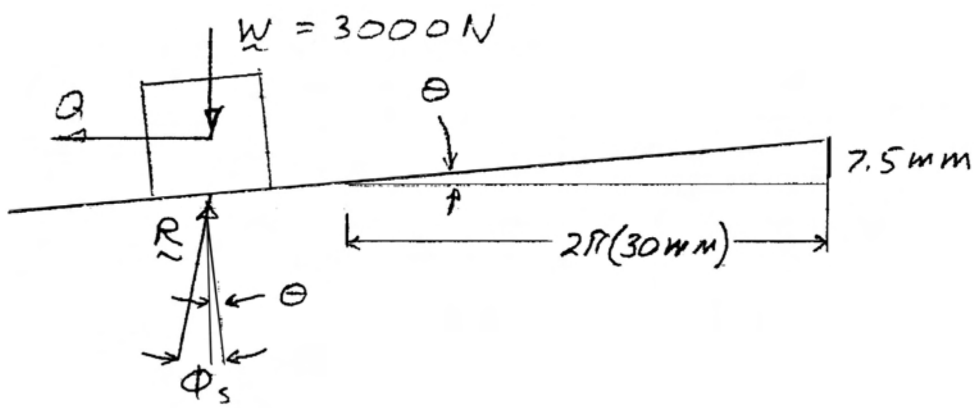
FBD large gear:



$$\circlearrowleft \Sigma M_C = 0: \quad (0.24 \text{ m})W - 720 \text{ N}\cdot\text{m} = 0$$

$$W = 3000 \text{ N}$$

Block on incline:



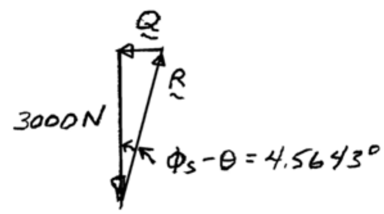
$$\theta = \tan^{-1}\frac{7.5 \text{ mm}}{2\pi(30 \text{ mm})} = 2.2785°$$

$$\phi_s = \tan^{-1}\mu = \tan^{-1}0.12$$

$$\phi_s = 6.8428°$$

$$\phi_s - \theta = 4.5643°$$

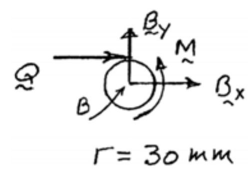
PROBLEM 8.68 CONTINUED



$$Q = (3000\text{ N})\tan 4.5643°$$

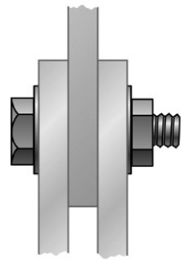
$$= 239.5\text{ N}$$

Worm gear:



$$\circlearrowleft \Sigma M_B = 0: \quad M - rQ = 0$$

$$M = rQ = (0.030\text{ m})(239.5\text{ N}) = 7.18\text{ N}\cdot\text{m} \blacktriangleleft$$

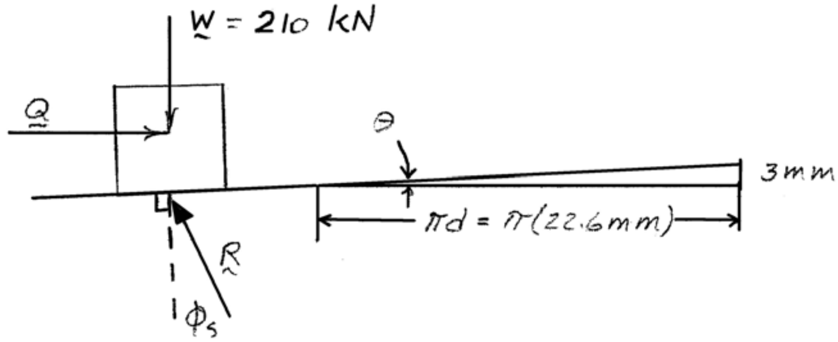


PROBLEM 8.69

High-strength bolts are used in the construction of many steel structures. For a 24-mm-nominal-diameter bolt the required minimum bolt tension is 210 kN. Assuming the coefficient of friction to be 0.40, determine the required couple that should be applied to the bolt and nut. The mean diameter of the thread is 22.6 mm, and the lead is 3 mm. Neglect friction between the nut and washer, and assume the bolt to be square-threaded.

SOLUTION

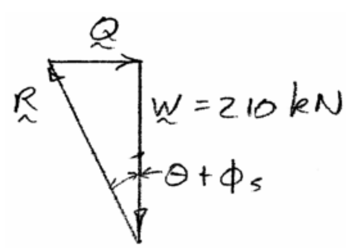
FBD block on incline:



$$\theta = \tan^{-1}\frac{3\text{ mm}}{(22.6\text{ mm})\pi} = 2.4195°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.40$$

$$\phi_s = 21.8014°$$



$$Q = (210\text{ kN})\tan(21.8014° + 2.4195°)$$

$$Q = 94.47\text{ kN}$$

$$\text{Torque} = \frac{d}{2}Q = \frac{22.6\text{ mm}}{2}(94.47\text{ kN})$$

$$= 1067.5\text{ N}\cdot\text{m}$$

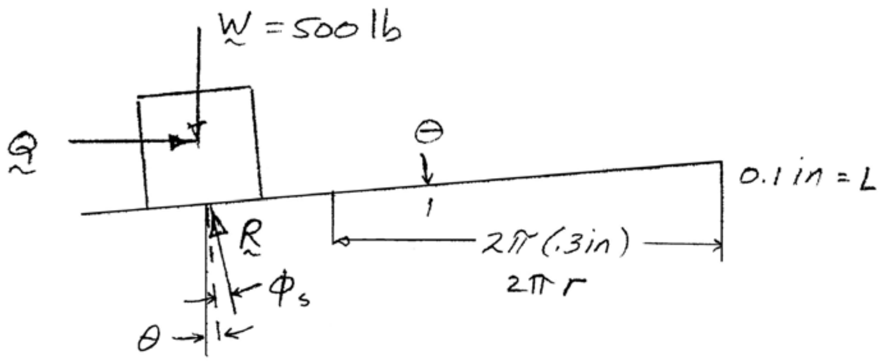
Torque = 1.068 kN·m ◀

PROBLEM 8.70

The ends of two fixed rods *A* and *B* are each made in the form of a single-threaded screw of mean radius 0.3 in. and pitch 0.1 in. Rod *A* has a right-handed thread and rod *B* a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.

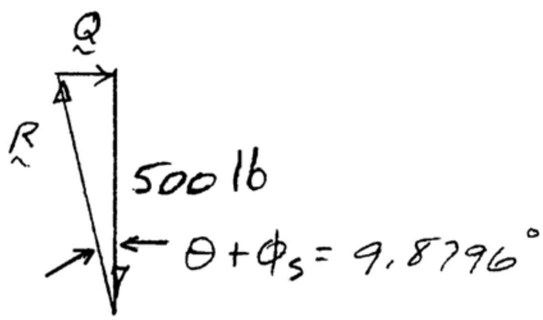
SOLUTION

Block on incline:



$$\theta = \tan^{-1}\frac{0.1 \text{ in.}}{2\pi(0.3 \text{ in.})} = 3.0368°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.12 = 6.8428°$$



$$Q = (500 \text{ lb})\tan 9.8796° = 87.08 \text{ lb}$$

Couple on each side

$$M = rQ = (0.3 \text{ in.})(87.08 \text{ lb}) = 26.12 \text{ lb}\cdot\text{in.}$$

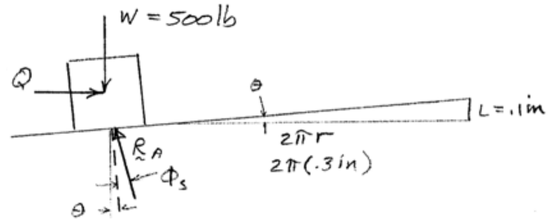
Couple to turn $= 2M = 52.2 \text{ lb}\cdot\text{in.}$ ◀

PROBLEM 8.71

Assuming that in Problem 8.70 a right-handed thread is used on *both* rods *A* and *B*, determine the magnitude of the couple that must be applied to the sleeve in order to rotate it.

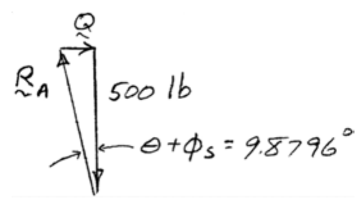
SOLUTION

Block on incline *A*:



$$\theta = \tan^{-1}\frac{0.1\text{ in.}}{2\pi(0.3\text{ in.})} = 3.0368°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.12 = 6.8428°$$



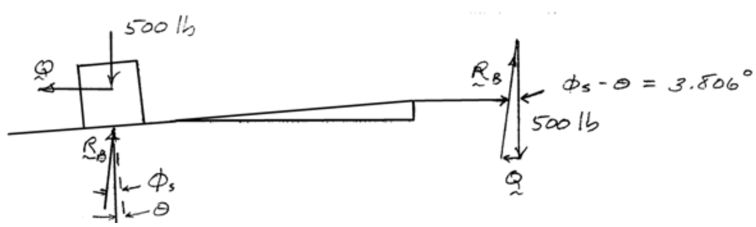
$$Q = (500\text{ lb})\tan 9.8796°$$

$$= 87.08\text{ lb}$$

$$\text{Couple at } A = (0.3\text{ in.})(87.08\text{ lb})$$

$$= 26.124\text{ lb}\cdot\text{in.}$$

Block on incline *B*:



$$Q = (500\text{ lb})\tan 3.806°$$

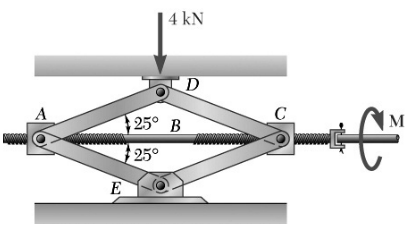
$$= 33.26\text{ lb}$$

$$\text{Couple at } B = (0.3\text{ in.})(33.26\text{ lb})$$

$$= 9.979\text{ lb}\cdot\text{in.}$$

$$\text{Total couple} = 26.124\text{ lb}\cdot\text{in.} + 9.979\text{ lb}\cdot\text{in.}$$

Couple to turn $= 36.1\text{ lb}\cdot\text{in.}$ ◀

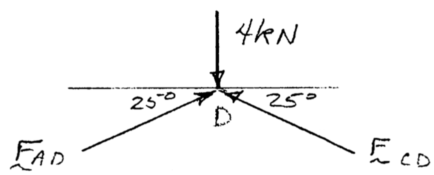


PROBLEM 8.72

The position of the automobile jack shown is controlled by a screw *ABC* that is single-threaded at each end (right-handed thread at *A*, left-handed thread at *C*). Each thread has a pitch of 2 mm and a mean diameter of 7.5 mm. If the coefficient of static friction is 0.15, determine the magnitude of the couple **M** that must be applied to raise the automobile.

SOLUTION

FBD joint *D*:



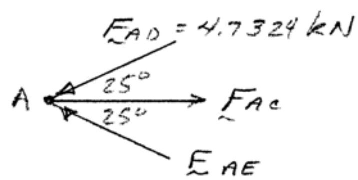
By symmetry:

$$F_{AD} = F_{CD}$$

$$\uparrow \Sigma F_y = 0: \quad 2F_{AD}\sin 25° - 4\text{ kN} = 0$$

$$F_{AD} = F_{CD} = 4.7324\text{ kN}$$

FBD joint *A*:



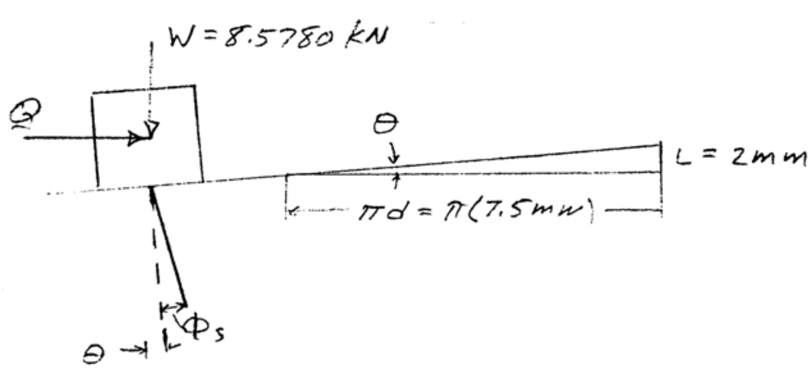
By symmetry:

$$F_{AE} = F_{AD}$$

$$\longrightarrow \Sigma F_x = 0: \quad F_{AC} - 2(4.7324\text{ kN})\cos 25° = 0$$

$$F_{AC} = 8.5780\text{ kN}$$

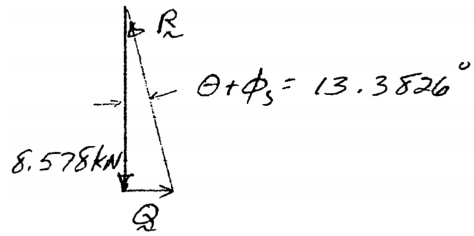
Block and incline *A*:



$$\theta = \tan^{-1}\frac{2\text{ mm}}{\pi(7.5\text{ mm})} = 4.8518°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.15 = 8.5308°$$

PROBLEM 8.72 CONTINUED



$$Q = (8.578 \text{ kN})\tan(13.3826°)$$

$$= 2.0408 \text{ kN}$$

Couple at *A*: $M_A = rQ$

$$= \left(\frac{7.5}{2} \text{ mm}\right)(2.0408 \text{ kN})$$

$$= 7.653 \text{ N}\cdot\text{m}$$

By symmetry: Couple at *C*: $M_C = 7.653 \text{ N}\cdot\text{m}$

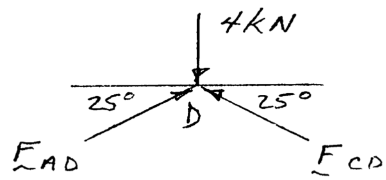
Total couple $M = 2(7.653 \text{ N}\cdot\text{m})$ $M = 15.31 \text{ N}\cdot\text{m}$ ◀

PROBLEM 8.73

For the jack of Problem 8.72, determine the magnitude of the couple **M** that must be applied to lower the automobile.

SOLUTION

FBD joint *D*:

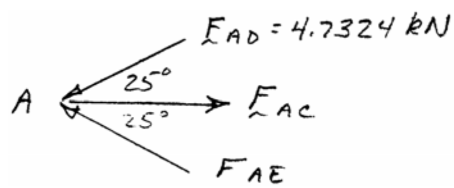


By symmetry: $F_{AD} = F_{CD}$

$$\uparrow \Sigma F_y = 0: \quad 2F_{AD}\sin 25° - 4\text{ kN} = 0$$

$$F_{AD} = F_{CD} = 4.7324\text{ kN}$$

FBD joint *A*:

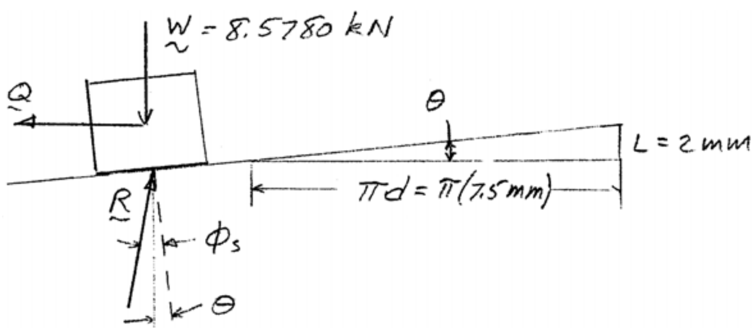


By symmetry: $F_{AE} = F_{AD}$

$$\longrightarrow \Sigma F_x = 0: \quad F_{AC} - 2(4.7324\text{ kN})\cos 25° = 0$$

$$F_{AC} = 8.5780\text{ kN}$$

Block and incline at *A*:

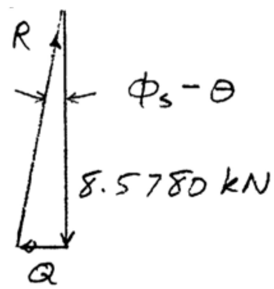


$$\theta = \tan^{-1}\frac{2\text{ mm}}{\pi(7.5\text{ mm})} = 4.8518°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.15$$

$$\phi_s = 8.5308°$$

PROBLEM 8.73 CONTINUED



$$\phi_s - \theta = 3.679°$$

$$Q = (8.5780 \text{ kN}) \tan 3.679°$$

$$Q = 0.55156 \text{ kN}$$

Couple at A: $M_A = Qr$

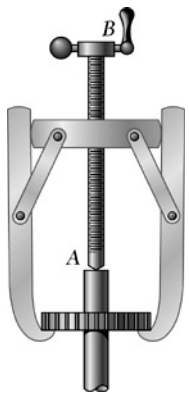
$$= (0.55156 \text{ kN})\left(\frac{7.5 \text{ mm}}{2}\right)$$

$$= 2.0683 \text{ N}\cdot\text{m}$$

By symmetry: Couple at C: $M_C = 2.0683 \text{ N}\cdot\text{m}$

Total couple $M = 2(2.0683 \text{ N}\cdot\text{m})$ $M = 4.14 \text{ N}\cdot\text{m}$ ◀

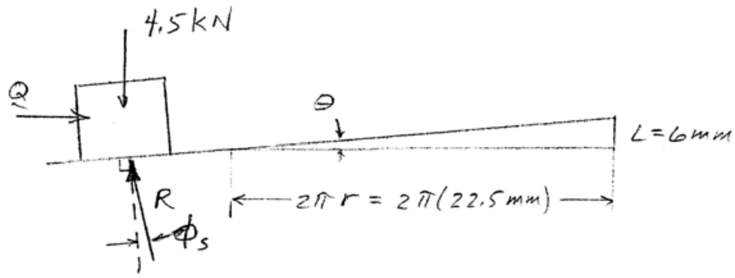
PROBLEM 8.74



In the gear-pulling assembly shown, the square-threaded screw *AB* has a mean radius of 22.5 mm and a lead of 6 mm. Knowing that the coefficient of static friction is 0.10, determine the couple which must be applied to the screw in order to produce a force of 4.5 kN on the gear. Neglect friction at end *A* of the screw.

SOLUTION

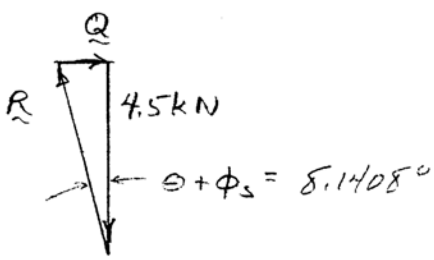
Block on incline:



$$\theta = \tan^{-1}\frac{6\text{ mm}}{2\pi(22.5\text{ mm})} = 2.4302°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.1$$

$$\phi_s = 5.7106°$$



$$Q = (4.5\text{ kN})\tan 8.1408°$$

$$= 0.6437\text{ kN}$$

Couple $M = rQ$

$$= (22.5\text{ mm})(0.6437\text{ kN})$$

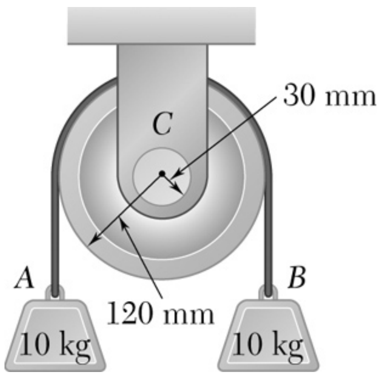
$$= 14.483\text{ N}\cdot\text{m}$$

$M = 14.48\text{ N}\cdot\text{m}$ ◀

NOTE FOR PROBLEMS 8.75–8.89

Note to instructors: In this manual, the singular sin $(\tan^{-1}\mu) \approx \mu$ is NOT used in the solution of journal bearing and axle friction problems. While this approximation may be valid for very small values of μ, there is little if any reason to use it, and the error may be significant. For example, in Problems 8.76–8.79, $\mu_s = 0.40$, and the error made by using the approximation is about 7.7%.

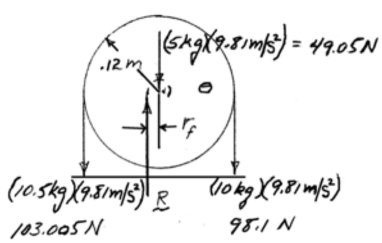
PROBLEM 8.75



A 120-mm-radius pulley of mass 5 kg is attached to a 30-mm-radius shaft which fits loosely in a fixed bearing. It is observed that the pulley will just start rotating if a 0.5-kg mass is added to block *A*. Determine the coefficient of static friction between the shaft and the bearing.

SOLUTION

FBD pulley:



$$\uparrow \Sigma F_y = 0: \quad R - 103.005\text{ N} - 49.05\text{ N} - 98.1\text{ N} = 0$$

$$R = 250.155\text{ N}$$

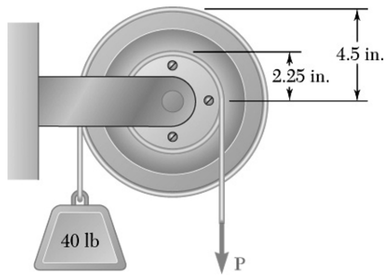
$$\circlearrowleft \Sigma M_O = 0: \quad (0.12\text{ m})(103.005\text{ N} - 98.1\text{ N}) - r_f(250.155\text{ N}) = 0$$

$$r_f = 0.0023529\text{ m} = 2.3529\text{ mm}$$

$$\phi_s = \sin^{-1}\frac{r_f}{r_s}$$

$$\mu_s = \tan\phi_s = \tan\left(\sin^{-1}\frac{r_f}{r_s}\right) = \tan\left(\sin^{-1}\frac{2.3529\text{ mm}}{30\text{ mm}}\right)$$

$$\mu_s = 0.0787 \blacktriangleleft$$

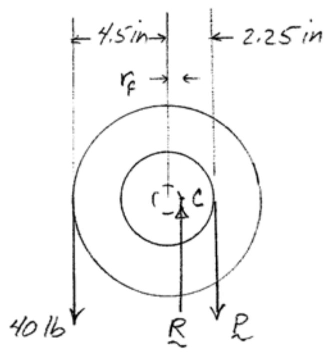


PROBLEM 8.76

The double pulley shown is attached to a 0.5-in.-radius shaft which fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION

FDB pulley:



$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^*$$

$$r_f = (0.5\text{ in.})\sin\left(\tan^{-1}0.40\right) = 0.185695\text{ in.}$$

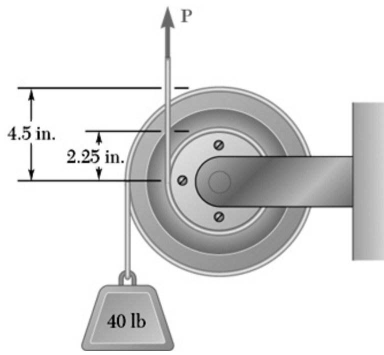
$$\circlearrowleft \Sigma M_C = 0: \quad (4.5\text{ in.} + 0.185695\text{ in.})(40\text{ lb})$$

$$-(2.25\text{ in.} - 0.185695\text{ in.})P = 0$$

$$P = 90.8\text{ lb} \blacktriangleleft$$

* See note before Problem 8.75.

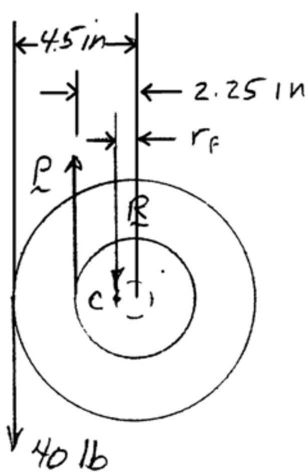
PROBLEM 8.77



The double pulley shown is attached to a 0.5-in.-radius shaft which fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION

FBD pulley:



$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right) = (0.5 \text{ in.})\sin\left(\tan^{-1} 0.4\right)^*$$

$$r_f = 0.185695 \text{ in.}$$

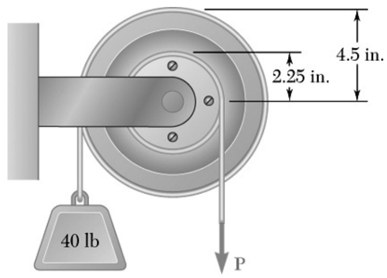
$$\circlearrowleft \Sigma M_C = 0: \quad (4.5 \text{ in.} - 0.185695 \text{ in.})(40 \text{ lb})$$

$$- (2.25 \text{ in.} - 0.185695 \text{ in.})P = 0$$

$$P = 83.6 \text{ lb} \blacktriangleleft$$

* See note before Problem 8.75.

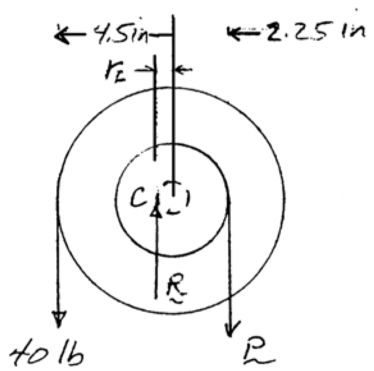
PROBLEM 8.78



The double pulley shown is attached to a 0.5-in.-radius shaft which fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

FBD pulley:



$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^*$$

$$r_f = (0.5\text{ in.})\sin\left(\tan^{-1} 0.40\right) = 0.185695\text{ in.}$$

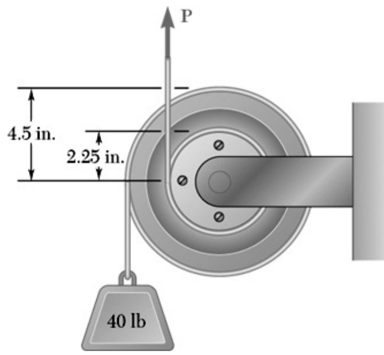
$$\circlearrowleft \Sigma M_C = 0: \quad (4.5\text{ in.} - 0.185695\text{ in.})(40\text{ lb})$$

$$-(2.25\text{ in.} + 0.185695\text{ in.})(P) = 0$$

$$P = 70.9\text{ lb} \blacktriangleleft$$

* See note before Problem 8.75.

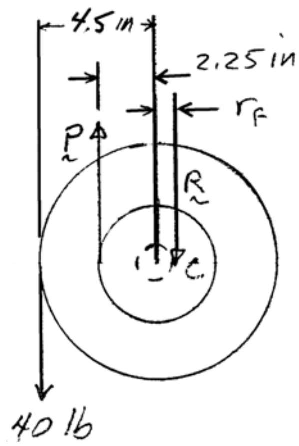
PROBLEM 8.79



The double pulley shown is attached to a 0.5-in.-radius shaft which fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

FBD pulley:



$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^*$$

$$r_f = (0.5 \text{ in.})\sin\left(\tan^{-1} 0.4\right) = 0.185695 \text{ in.}$$

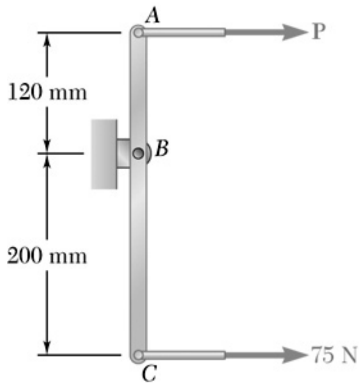
$$\circlearrowleft \Sigma M_C = 0: \quad (4.5 \text{ in.} + 0.185695 \text{ in.})(40 \text{ lb})$$

$$-(2.25 \text{ in.} + 0.185695 \text{ in.})P = 0$$

$$P = 77.0 \text{ lb} \blacktriangleleft$$

* See note before Problem 8.75.

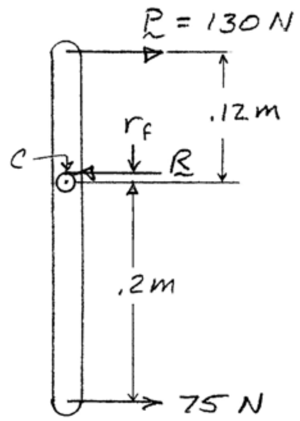
PROBLEM 8.80



Control lever *ABC* fits loosely on a 18-mm-diameter shaft at support *B*. Knowing that $P = 130$ N for impending clockwise rotation of the lever, determine (*a*) the coefficient of static friction between the pin and the lever, (*b*) the magnitude of the force **P** for which counterclockwise rotation of the lever is impending.

SOLUTION

(*a*) **FBD lever** (Impending *CW* rotation):



$$\circlearrowleft \Sigma M_C = 0: \quad (0.2 \text{ m} + r_f)(75 \text{ N}) - (0.12 \text{ m} - r_f)(130 \text{ N}) = 0$$

$$r_f = 0.0029268 \text{ m} = 2.9268 \text{ mm}$$

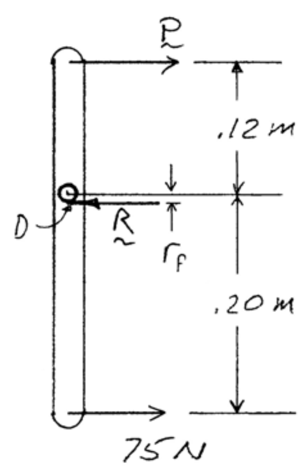
$$\sin\phi_s = \frac{r_f}{r_s}$$

$$\mu_s = \tan\phi_s = \tan\left(\sin^{-1}\frac{r_f}{r_s}\right) = \tan\left(\sin^{-1}\frac{2.9268 \text{ mm}}{18 \text{ mm}}\right)^*$$

$$= 0.34389$$

$$\mu_s = 0.344 \blacktriangleleft$$

(*b*) **FBD lever** (Impending *CCW* rotation):



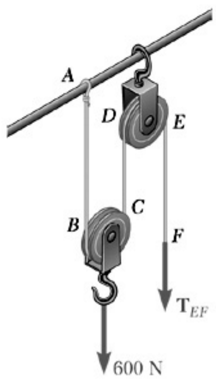
$$\circlearrowleft \Sigma M_D = 0: \quad (0.20 \text{ m} - 0.0029268 \text{ m})(75 \text{ N})$$

$$- (0.12 \text{ m} + 0.0029268 \text{ m})P = 0$$

$$P = 120.2 \text{ N} \blacktriangleleft$$

* See note before Problem 8.75.

PROBLEM 8.81



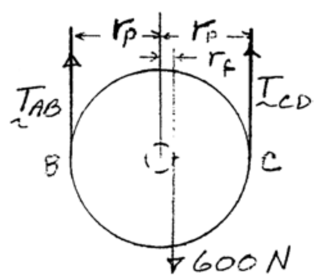
The block and tackle shown are used to raise a 600-N load. Each of the 60-mm-diameter pulleys rotates on a 10-mm-diameter axle. Knowing that the coefficient of kinetic friction is 0.20, determine the tension in each portion of the rope as the load is slowly raised.

SOLUTION

Pulley FBD's:

$r_p = 30 \text{ mm}$

Left:



$$r_f = r_{\text{axle}} \sin \phi_k = r_{\text{axle}} \sin\left(\tan^{-1} \mu_k\right)^*$$

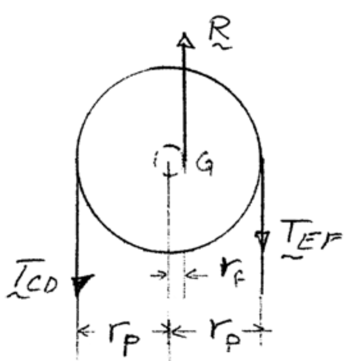
$$= (5 \text{ mm}) \sin\left(\tan^{-1} 0.2\right)$$

$$= 0.98058 \text{ mm}$$

Left:

$$\circlearrowleft \Sigma M_C = 0: \quad (r_p - r_f)(600 \text{ lb}) - 2r_p T_{AB} = 0$$

Right:



or $$T_{AB} = \frac{30 \text{ mm} - 0.98058 \text{ mm}}{2(30 \text{ mm})}(600 \text{ N}) = 290.19 \text{ N}$$

$T_{AB} = 290 \text{ N}$ ◀

$$\uparrow \Sigma F_y = 0: \quad 290.19 \text{ N} - 600 \text{ N} + T_{CD} = 0$$

or $T_{CD} = 309.81 \text{ N}$ $\qquad$ $T_{CD} = 310 \text{ N}$ ◀

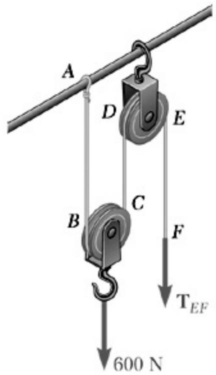
Right:

$$\circlearrowleft \Sigma M_G = 0: \quad (r_p + r_f)T_{CD} - (r_p - r_f)T_{EF} = 0$$

or $$T_{EF} = \frac{30 \text{ mm} + 0.98058 \text{ mm}}{30 \text{ mm} - 0.98058 \text{ mm}}(309.81 \text{ N}) = 330.75 \text{ N}$$

$T_{EF} = 331 \text{ N}$ ◀

* See note before Problem 8.75.



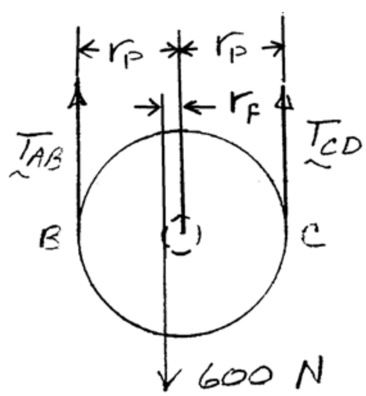
PROBLEM 8.82

The block and tackle shown are used to lower a 600-N load. Each of the 60-mm-diameter pulleys rotates on a 10-mm-diameter axle. Knowing that the coefficient of kinetic friction is 0.20, determine the tension in each portion of the rope as the load is slowly lowered.

SOLUTION

Pulley FBDs:

Left:



$$r_p = 30 \text{ mm}$$

$$r_f = r_{\text{axle}} \sin\phi_k = r_{\text{axle}} \sin\left(\tan^{-1}\mu_k\right)^*$$

$$= (5 \text{ mm})\sin\left(\tan^{-1} 0.2\right)$$

$$= 0.98058 \text{ mm}$$

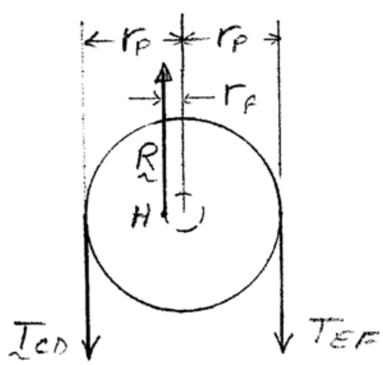
$$\circlearrowleft \Sigma M_C = 0: \quad (r_p + r_f)(600 \text{ N}) - 2r_p T_{AB} = 0$$

or $$T_{AB} = \frac{30 \text{ mm} + 0.98058 \text{ mm}}{2(30 \text{ mm})}(600 \text{ N}) = 309.81 \text{ N}$$

$$T_{AB} = 310 \text{ N} \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad T_{AB} - 600 \text{ N} + T_{CD} = 0$$

Right:



or $$T_{CD} = 600 \text{ N} - 309.81 \text{ N} = 290.19 \text{ N}$$

$$T_{CD} = 290 \text{ N} \blacktriangleleft$$

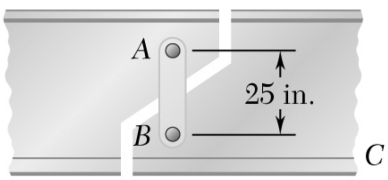
$$\circlearrowleft \Sigma M_H = 0: \quad (r_p - r_f)T_{CD} - (r_p + r_f)T_{EF} = 0$$

or $$T_{EF} = \frac{30 \text{ mm} - 0.98058 \text{ mm}}{30 \text{ mm} + 0.98058 \text{ mm}}(290.19 \text{ N})$$

$$T_{EF} = 272 \text{ N} \blacktriangleleft$$

* See note before Problem 8.75.

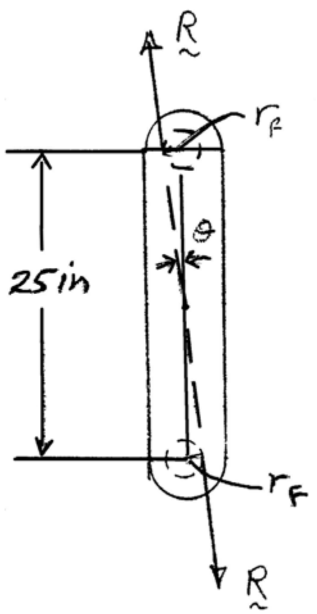
PROBLEM 8.83



The link arrangement shown is frequently used in highway bridge construction to allow for expansion due to changes in temperature. At each of the 3-in.-diameter pins *A* and *B* the coefficient of static friction is 0.20. Knowing that the vertical component of the force exerted by *BC* on the link is 50 kips, determine (*a*) the horizontal force which should be exerted on beam *BC* to just move the link, (*b*) the angle that the resulting force exerted by beam *BC* on the link will form with the vertical.

SOLUTION

FBD link AB:



Note that *AB* is a two force member. For impending motion, the pin forces are tangent to the friction circles.

$$\theta = \sin^{-1}\frac{r_f}{25\text{ in.}}$$

where $$r_f = r_p \sin\phi_s = r_p \sin\left(\tan^{-1}\mu_s\right)^*$$

$$= (1.5\text{ in.})\sin\left(\tan^{-1}0.2\right) = 0.29417\text{ in.}$$

Then $$\theta = \sin^{-1}\frac{0.29417\text{ in.}}{12.5\text{ in.}} = 1.3485°$$

(*b*) $\theta = 1.349°$ ◀

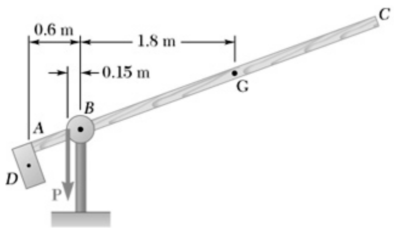
$$R_{\text{vert}} = R\cos\theta \qquad R_{\text{horiz}} = R\sin\theta$$

$$R_{\text{horiz}} = R_{\text{vert}}\tan\theta = (50\text{ kips})\tan 1.3485° = 1.177\text{ kips}$$

(*a*) $R_{\text{horiz}} = 1.177$ kips ◀

* See note before Problem 8.75.

PROBLEM 8.84

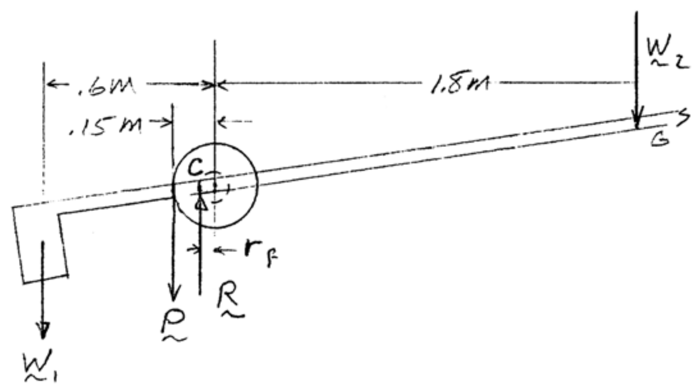


BEER • JOHNSTON Fig. P8-84 and P8-86
Vector Mechanics for Engineers: Statics & Dynamics, 7e
100% of size Fine Line Illustrations (516) 501-0400

A gate assembly consisting of a 24-kg gate *ABC* and a 66-kg counterweight *D* is attached to a 24-mm-diameter shaft *B* which fits loosely in a fixed bearing. Knowing that the coefficient of static friction is 0.20 between the shaft and the bearing, determine the magnitude of the force **P** for which counterclockwise rotation of the gate is impending.

SOLUTION

FBD gate:



$$W_1 = 66\text{ kg}\left(9.81\text{ m/s}^2\right) = 647.46\text{ N}$$

$$W_2 = 24\text{ kg}\left(9.81\text{ m/s}^2\right) = 235.44\text{ N}$$

$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)$$

$$= \left(0.012\text{ m}\right)\sin\left(\tan^{-1}0.2\right) = 0.0023534\text{ m}$$

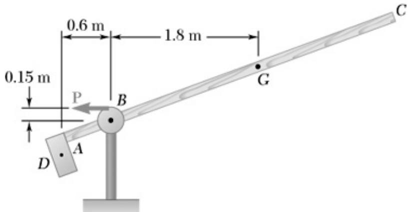
$$\circlearrowleft \Sigma M_C = 0: \quad \left(0.6\text{ m} - r_f\right)W_1 + \left(0.15\text{ m} - r_f\right)P - \left(1.8\text{ m} + r_f\right)W_2 = 0$$

$$P = \frac{\left(1.80235\text{ m}\right)\left(235.44\text{ N}\right) - \left(0.59765\text{ m}\right)\left(647.46\text{ N}\right)}{\left(0.14765\text{ m}\right)}$$

$$= 253.2\text{ N}$$

$P = 253\text{ N}$ ◀

PROBLEM 8.85



BEER • JOHNSTON Fig. P8-85 and P8-87
Vector Mechanics for Engineers: Statics & Dynamics, 7e
100% of size Fine Line Illustrations (516) 501-0400

A gate assembly consisting of a 24-kg gate *ABC* and a 66-kg counterweight *D* is attached to a 24-mm-diameter shaft *B* which fits loosely in a fixed bearing. Knowing that the coefficient of static friction is 0.20 between the shaft and the bearing, determine the magnitude of the force **P** for which counterclockwise rotation of the gate is impending.

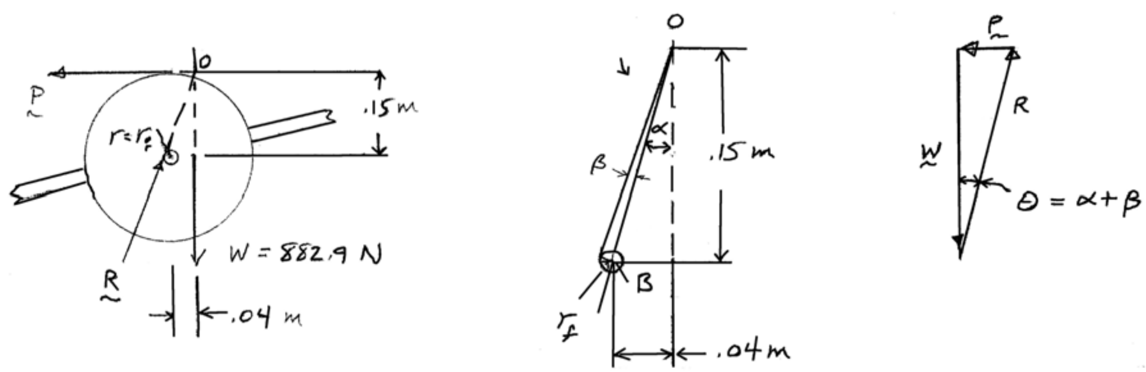
SOLUTION

It is convenient to replace the $(66\text{ kg})g$ and $(24\text{ kg})g$ weights with a single combined weight of $(90\text{ kg})(9.81\text{ m/s}^2) = 882.9\text{ N}$, located at a distance $x = \dfrac{(1.8\text{ m})(24\text{ kg}) - (0.6\text{ m})(24\text{ kg})}{90\text{ kg}} = 0.04\text{ m}$ to the right of *B*.

$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^* = (0.012\text{ m})\sin\left(\tan^{-1}0.2\right)$$

$$= 0.0023534\text{ m}$$

FBD pulley + gate:



$$\alpha = \tan^{-1}\frac{0.04\text{ m}}{0.15\text{ m}} = 14.931° \qquad OB = \frac{0.15}{\cos\alpha} = 0.15524\text{ m}$$

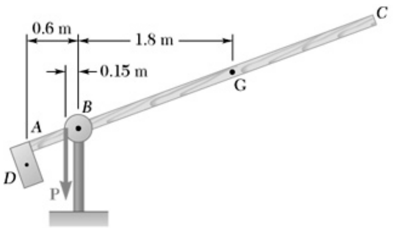
$$\beta = \sin^{-1}\frac{r_f}{OB} = \sin^{-1}\frac{0.0023534\text{ m}}{0.15524\text{ m}} = 0.8686° \qquad \text{then} \qquad \theta = \alpha + \beta = 15.800°$$

$$P = W\tan\theta = 248.9\text{ N}$$

$P = 250\text{ N}$ ◀

* See note before Problem 8.75.

PROBLEM 8.86

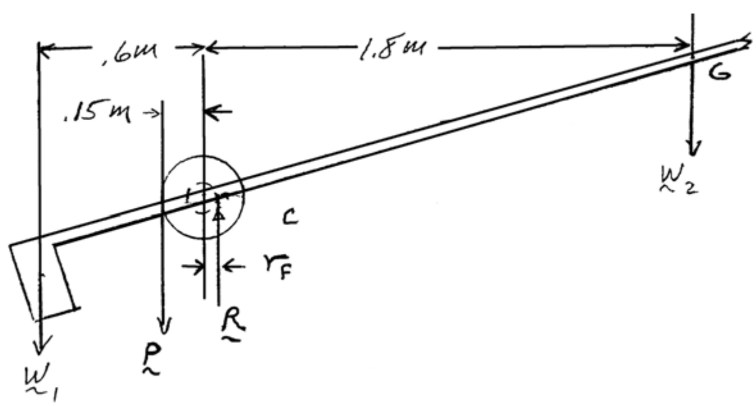


A gate assembly consisting of a 24-kg gate *ABC* and a 66-kg counterweight *D* is attached to a 24-mm-diameter shaft *B* which fits loosely in a fixed bearing. Knowing that the coefficient of static friction is 0.20 between the shaft and the bearing, determine the magnitude of the force **P** for which clockwise rotation of the gate is impending.

BEER • JOHNSTON Fig. P8-84 and P8-86
Vector Mechanics for Engineers: Statics & Dynamics, 7e
100% of size Fine Line Illustrations (516) 501-0400

SOLUTION

FBD gate:



$$W_1 = 66\text{ kg}\left(9.81\text{ m/s}^2\right) = 647.46\text{ N}$$

$$W_2 = 24\text{ kg}\left(9.81\text{ m/s}^2\right) = 235.44\text{ N}$$

$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^*$$

$$= \left(0.012\text{ m}\right)\sin\left(\tan^{-1}0.2\right) = 0.0023534\text{ m}$$

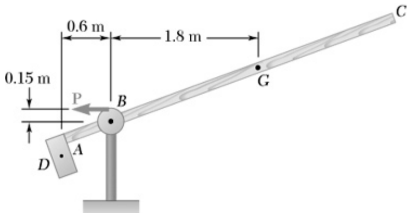
$$\circlearrowleft \Sigma M_C = 0: \quad \left(0.6\text{ m} + r_f\right)W_1 + \left(0.15\text{ m} + r_f\right)P - \left(1.8\text{ m} - r_f\right)W_2 = 0$$

$$P = \frac{\left(1.79765\text{ m}\right)\left(235.44\text{ N}\right) - \left(0.60235\text{ m}\right)\left(647.46\text{ N}\right)}{0.15235\text{ m}}$$

$$= 218.19\text{ N}$$

$P = 218\text{ N}$ ◀

* See note before Problem 8.75.



BEER • JOHNSTON Fig. P8-85 and P8-87
Vector Mechanics for Engineers: Statics & Dynamics, 7e
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PROBLEM 8.87

A gate assembly consisting of a 24-kg gate *ABC* and a 66-kg counterweight *D* is attached to a 24-mm-diameter shaft *B* which fits loosely in a fixed bearing. Knowing that the coefficient of static friction is 0.20 between the shaft and the bearing, determine the magnitude of the force **P** for which clockwise rotation of the gate is impending.

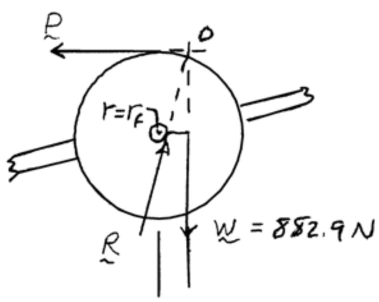
SOLUTION

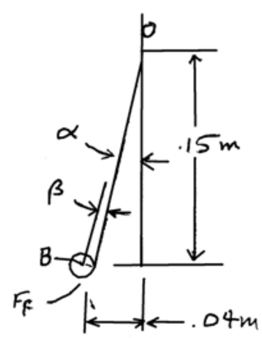
It is convenient to replace the $(66\text{ kg})g$ and $(24\text{ kg})g$ weights with a single weight of $(90\text{ kg})(9.81\text{ N/kg}) = 882.9\text{ N}$, located at a distance $x = \dfrac{(1.8\text{ m})(24\text{ kg}) - (0.15\text{ m})(66\text{ kg})}{90\text{ kg}} = 0.04\text{ m}$ to the right of *B*.

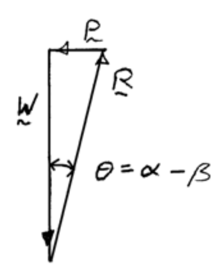
FBD pulley + gate:

$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^* = (0.012\text{ m})\sin\left(\tan^{-1}0.2\right)$$

$$r_f = 0.0023534\text{ m}$$







$$\alpha = \tan^{-1}\frac{0.04\text{ m}}{0.15\text{ m}} = 14.931° \qquad OB = \frac{0.15\text{ m}}{\cos\alpha} = 0.15524\text{ m}$$

$$\beta = \sin^{-1}\frac{r_f}{OB} = \sin^{-1}\frac{0.0023534\text{ m}}{0.15524\text{ m}} = 0.8686° \qquad \text{then} \qquad \theta = \alpha - \beta = 14.062°$$

$$P = W\tan\theta = 221.1\text{ N}$$

$P = 221\text{ N}$ ◀

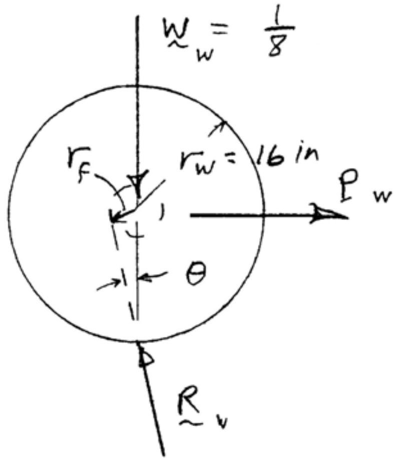
* See note before Problem 8.75.

PROBLEM 8.88

A loaded railroad car has a weight of 35 tons and is supported by eight 32-in.-diameter wheels with 5-in.-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) for impending motion of the car, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION

FBD wheel:



$$W_w = \frac{1}{8}W_c = \frac{1}{8}(35\text{ ton}) = \frac{1}{8}(70{,}000)\text{ lb}$$

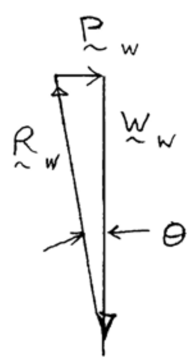
$$r_f = r_a \sin\phi = r_a \sin\left(\tan^{-1}\mu\right)^*$$

$$\theta = \sin^{-1}\frac{r_f}{r_w} = \sin^{-1}\left[\frac{(2.5\text{ in.})\sin\left(\tan^{-1}\mu\right)}{16\text{ in.}}\right]$$

$$= \sin^{-1}\left[0.15625\sin\left(\tan^{-1}\mu\right)\right]$$

(*a*) For impending motion use $\mu_s = 0.02$: then $\theta_s = 0.179014°$

(*b*) For steady motion use $\mu_k = 0.15$: then $\theta_k = 0.134272°$



$$P_w = W_w \tan\theta \qquad P_c = W_c \tan\theta = 8W_w \tan\theta$$

(*a*) $$P_c = (70{,}000\text{ lb})\tan(0.179014°)$$

$P_c = 219$ lb ◀

(*b*) $$P_c = (70{,}000\text{ lb})\tan(0.134272°)$$

$P_c = 164.0$ lb ◀

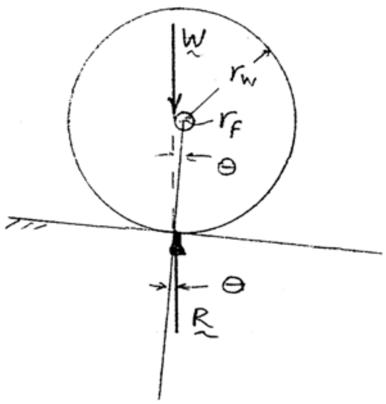
* See note before Problem 8.75.

PROBLEM 8.89

A scooter is designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 1-in.-diameter axles and the bearing is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION

FBD wheel:



Note: The wheel is a two-force member in equilibrium, so **R** and **W** must be collinear and tangent to friction circle.

$$2\% \text{ slope} \Rightarrow \tan\theta = 0.02$$

Also $$\sin\theta = \frac{r_f}{r_w}\sin\left(\tan^{-1}0.02\right) = 0.019996$$

But $$r_f = r_a \sin\phi_k = r_a \sin\left(\tan^{-1}\mu_k\right)^*$$

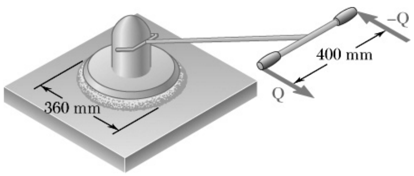
$$= (1\text{ in.})\sin\left(\tan^{-1}0.1\right) = 0.099504\text{ in.}$$

Then $$r_w = \frac{r_f}{\sin\theta} = \frac{0.099504}{0.019996} = 4.976\text{ in.}$$

and $$d_w = 2r_w \qquad d_w = 9.95\text{ in.} ◀$$

* See note before Problem 8.75.

PROBLEM 8.90



A 25-kg electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

Couple exerted on handle

$$M_H = dQ = (0.4\text{ m})Q$$

Couple exerted on floor

$$M_F = \frac{2}{3}\mu_k PR \qquad \text{(Equation 8.9)}$$

where

$$\mu_k = 0.25, \quad P = (25\text{ kg})(9.81\text{ m/s}^2) = 245.25\text{ N}, \quad R = 0.18\text{ m}$$

For equilibrium

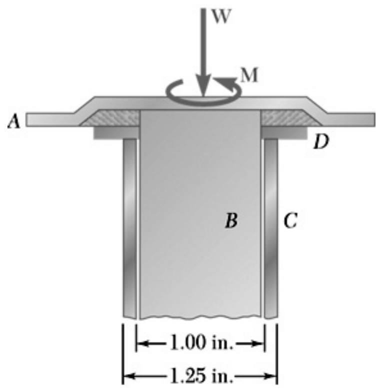
$$M_H = M_F,$$

so

$$Q = \frac{\frac{2}{3}(0.25)(245.25\text{ N})(0.18\text{ m})}{0.4\text{ m}}$$

$$Q = 18.39\text{ N} \blacktriangleleft$$

PROBLEM 8.91



The pivot for the seat of a desk chair consists of the steel plate *A*, which supports the seat, the solid steel shaft *B* which is welded to *A* and which turns freely in the tubular member *C*, and the nylon bearing *D*. If a person of weight $W = 180$ lb is seated directly above the pivot, determine the magnitude of the couple **M** for which rotation of the seat is impending knowing that the coefficient of static friction is 0.15 between the tubular member and the bearing.

SOLUTION

For an annular bearing area
$$M = \frac{2}{3}\mu_s P \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \quad \text{(Equation 8.8)}$$

Since $R = \dfrac{D}{2}$
$$M = \frac{1}{3}\mu_s P \frac{D_2^3 - D_1^3}{D_2^2 - D_1^2}$$

Now
$$\mu_s = 0.15, \; P = W = 180 \text{ lb}, \; D_1 = 1.00 \text{ in.}, \; D_2 = 1.25 \text{ in.}$$

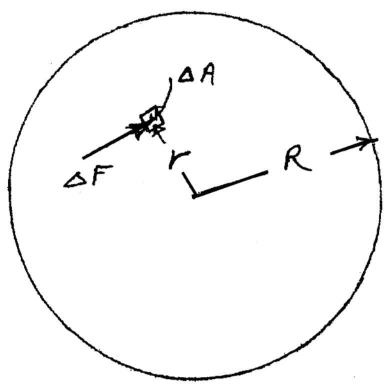
$$M = \frac{0.15}{3}(180 \text{ lb})\frac{(1.25 \text{ in.})^3 - (4 \text{ in.})^3}{(1.25 \text{ in.})^2 - (1 \text{ in.})^2}$$

$$M = 15.25 \text{ lb}\cdot\text{in.} \blacktriangleleft$$

PROBLEM 8.92

As the surfaces of a shaft and a bearing wear out, the frictional resistance of a thrust bearing decreases. It is generally assumed that the wear is directly proportional to the distance traveled by any given point of the shaft and thus to the distance r from the point to the axis of the shaft. Assuming, then, that the normal force per unit area is inversely proportional to r, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out end bearing (with contact over the full circular area) is equal to 75 percent of the value given by formula (8.9) for a new bearing.

SOLUTION



Let the normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = \dfrac{k}{r}$

As in the text $\qquad \Delta F = \mu \Delta N, \ \Delta M = r\Delta F$

The total normal force

$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi}\left(\int_0^R \frac{k}{r} r dr\right) d\theta$$

$$P = 2\pi\left(\int_0^R k dr\right) = 2\pi k R \qquad \text{or} \qquad k = \frac{P}{2\pi R}$$

The total couple $\qquad M_{\text{worn}} = \lim_{\Delta A \to 0} \Sigma \Delta M = \int_0^{2\pi}\left(\int_0^R r\mu \frac{k}{r} r dr\right) d\theta$

$$M_{\text{worn}} = 2\pi\mu k \int_0^R r dr = 2\pi\mu k \frac{R^2}{2} = 2\pi\mu \frac{P}{2\pi R}\frac{R^2}{2}$$

or $\qquad M_{\text{worn}} = \dfrac{1}{2}\mu P R$

Now $\qquad M_{\text{new}} = \dfrac{2}{3}\mu P R \qquad$ [Eq. (8.9)]

Thus $\qquad \dfrac{M_{\text{worn}}}{M_{\text{new}}} = \dfrac{\frac{1}{2}\mu P R}{\frac{2}{3}\mu P R} = \dfrac{3}{4} = 75\%$ ◀

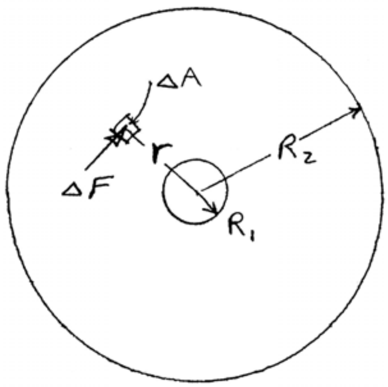
PROBLEM 8.93

Assuming that bearings wear out as indicated in Problem 8.92, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out collar bearing is $M = \frac{1}{2}\mu_k P(R_1 + R_2)$

where $P =$ magnitude of the total axial force

$R_1,\ R_2 =$ inner and outer radii of collar

SOLUTION



Let normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = \dfrac{k}{r}$

As in the text $\Delta F = \mu \Delta N,\ \Delta M = r\Delta F$

The total normal force P is

$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi}\left(\int_{R_1}^{R_2} \frac{k}{r} r dr\right) d\theta$$

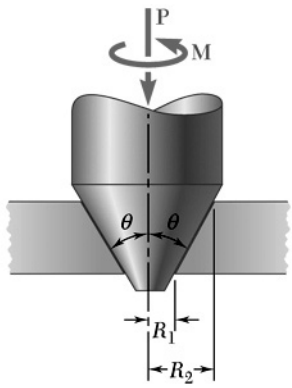
$$P = 2\pi \int_{R_1}^{R_2} k dr = 2\pi k\left(R_2 - R_1\right) \qquad \text{or} \qquad k = \frac{P}{2\pi\left(R_2 - R_1\right)}$$

The total couple is $$M_{\text{worn}} = \lim_{\Delta A \to 0} \Sigma \Delta M = \int_0^{2\pi}\left(\int_{R_1}^{R_2} r\mu \frac{k}{r} r dr\right) d\theta$$

$$M_{\text{worn}} = 2\pi\mu k \int_{R_1}^{R_2} \left(r dr\right) = \pi\mu k\left(R_2^2 - R_1^2\right) = \frac{\pi\mu P\left(R_2^2 - R_1^2\right)}{2\pi\left(R_2 - R_1\right)}$$

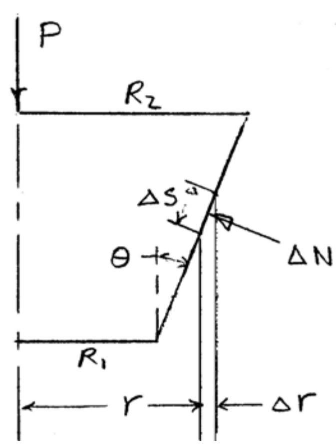
$$M_{\text{worn}} = \frac{1}{2}\mu P\left(R_2 + R_1\right) \blacktriangleleft$$

PROBLEM 8.94



Assuming that the pressure between the surfaces of contact is uniform, show that the magnitude M of the couple required to overcome frictional resistance for the conical bearing shown is $M = \frac{2}{3}\frac{\mu_k P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$

SOLUTION



Let normal force on ΔA be ΔN, and $\frac{\Delta N}{\Delta A} = k$,

so $\quad \Delta N = k\Delta A \qquad \Delta A = r\Delta s\Delta\phi \qquad \Delta s = \frac{\Delta r}{\sin\theta}$

where ϕ is the azimuthal angle around the symmetry axis of rotation

$$\Delta F_y = \Delta N \sin\theta = kr\Delta r\Delta\phi$$

Total vertical force $\quad P = \lim_{\Delta A\to 0} \Sigma\Delta F_y$

$$P = \int_0^{2\pi}\left(\int_{R_1}^{R_2} krdr\right)d\phi = 2\pi k\int_{R_1}^{R_2} rdr$$

$$P = \pi k\left(R_2^2 - R_1^2\right) \qquad \text{or} \qquad k = \frac{P}{\pi\left(R_2^2 - R_1^2\right)}$$

Friction force $\quad \Delta F = \mu\Delta N = \mu k\Delta A$

Moment $\quad \Delta M = r\Delta F = r\mu kr\frac{\Delta r}{\sin\theta}\Delta\phi$

Total couple $\quad M = \lim_{\Delta A\to 0}\Sigma\Delta M = \int_0^{2\pi}\left(\int_{R_1}^{R_2}\frac{\mu k}{\sin\theta}r^2dr\right)d\phi$

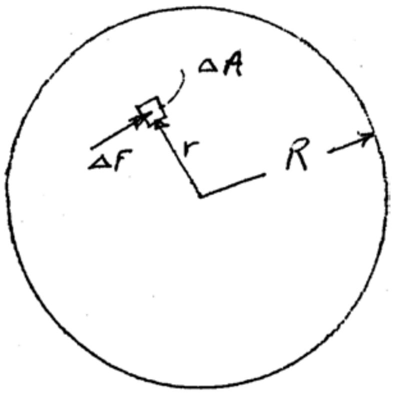
$$M = 2\pi\frac{\mu k}{\sin\theta}\int_{R_1}^{R_2} r^2dr = \frac{2}{3}\frac{\pi\mu}{\sin\theta}\frac{P}{\pi\left(R_2^2 - R_3^2\right)}\left(R_2^3 - R_3^3\right)$$

$$M = \frac{2}{3}\frac{\mu P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \blacktriangleleft$$

PROBLEM 8.95

Solve Problem 8.90 assuming that the normal force per unit area between the disk and the floor varies linearly from a maximum at the center to zero at the circumference of the disk.

SOLUTION



Let normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = k\left(1 - \dfrac{r}{R}\right)$

$$\Delta F = \mu \Delta N = \mu k\left(1 - \frac{r}{R}\right)\Delta A = \mu k\left(1 - \frac{r}{R}\right) r \Delta r \Delta \theta$$

$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi}\left[\int_0^R k\left(1 - \frac{r}{R}\right) r dr\right] d\theta$$

$$P = 2\pi k \int_0^R \left(1 - \frac{r}{R}\right) r dr = 2\pi k\left(\frac{R^2}{2} - \frac{R^3}{3R}\right)$$

$$P = \frac{1}{3}\pi k R^2 \qquad \text{or} \qquad k = \frac{3P}{\pi R^2}$$

$$M = \lim_{\Delta A \to 0} \Sigma r \Delta F = \int_0^{2\pi}\left[\int_0^R r \mu k\left(1 - \frac{r}{R}\right) r dr\right] d\theta$$

$$= 2\pi \mu k \int_0^R \left(r^2 - \frac{r^3}{R}\right) dr = 2\pi \mu k\left(\frac{R^3}{3} - \frac{R^4}{4R}\right) = \frac{1}{6}\pi \mu k R^3$$

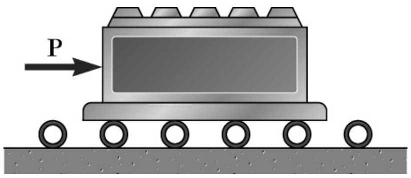
$$= \frac{\pi \mu}{6} \frac{3P}{\pi R^2} R^3 = \frac{1}{2}\mu P R$$

where $\qquad \mu = \mu_k = 0.25 \qquad R = 0.18 \text{ m}$

$$P = W = (25 \text{ kg})(9.81 \text{ m/s}^2) = 245.25 \text{ N}$$

Then $\qquad M = \dfrac{1}{2}(0.25)(245.25 \text{ N})(0.18 \text{ m}) = 5.5181 \text{ N}\cdot\text{m}$

Finally, $\qquad Q = \dfrac{M}{d} = \dfrac{5.5181 \text{ N}\cdot\text{m}}{0.4 \text{ m}} \qquad Q = 13.80 \text{ N}$ ◀

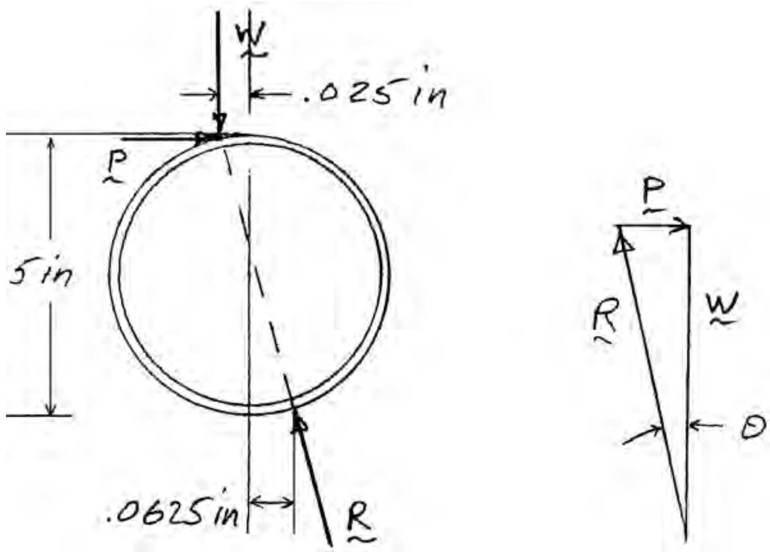


PROBLEM 8.96

A 1-ton machine base is rolled along a concrete floor using a series of steel pipes with outside diameters of 5 in. Knowing that the coefficient of rolling resistance is 0.025 in. between the pipes and the base and 0.0625 in. between the pipes and the concrete floor, determine the magnitude of the force **P** required to slowly move the base along the floor.

SOLUTION

FBD pipe:



$$\theta = \sin^{-1}\frac{0.025\text{ in.} + 0.0625\text{ in.}}{5\text{ in.}} = 1.00257°$$

$P = W\tan\theta$ for each pipe, so also for total

$$P = (2000\text{ lb})\tan(1.00257°)$$

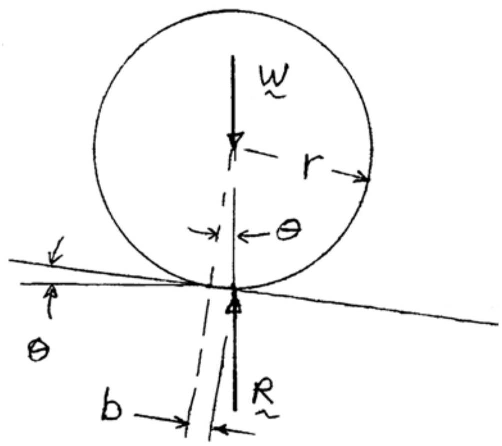
$P = 35.0$ lb ◀

PROBLEM 8.97

Knowing that a 120-mm-diameter disk rolls at a constant velocity down a 2 percent incline, determine the coefficient of rolling resistance between the disk and the incline.

SOLUTION

FBD disk:



$$\tan\theta = \text{slope} = 0.02$$

$$b = r\tan\theta = (60 \text{ mm})(0.02)$$

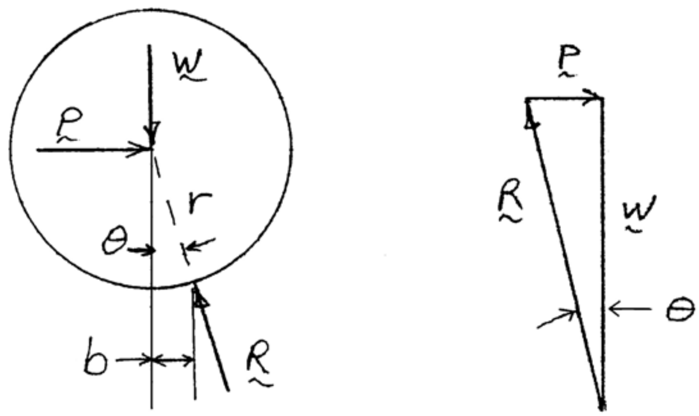
$b = 1.200$ mm ◀

PROBLEM 8.98

Determine the horizontal force required to move a 1-Mg automobile with 460-mm-diameter tires along a horizontal road at a constant speed. Neglect all forms of friction except rolling resistance, and assume the coefficient of rolling resistance to be 1 mm.

SOLUTION

FBD wheel:



$$r = 230 \text{ mm}$$

$$b = 1 \text{ mm}$$

$$\theta = \sin^{-1}\frac{b}{r}$$

$P = W\tan\theta = W\tan\left(\sin^{-1}\frac{b}{r}\right)$ for each wheel, so for total

$$P = (1000 \text{ kg})(9.81 \text{ m/s}^2)\tan\left(\sin^{-1}\frac{1}{230}\right)$$

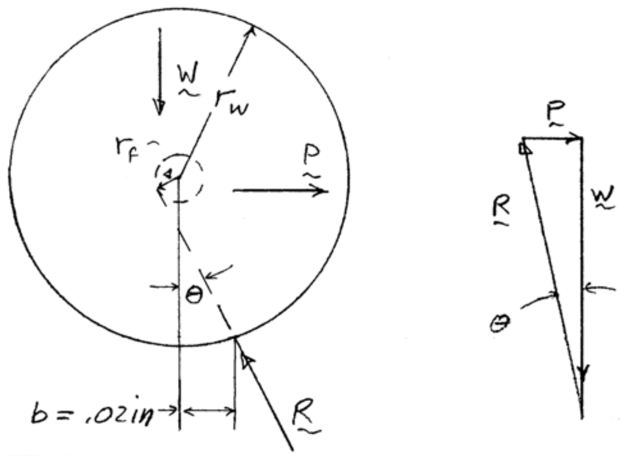
$P = 42.7$ N ◀

PROBLEM 8.99

Solve Problem 8.88 including the effect of a coefficient of rolling resistance of 0.02 in.

SOLUTION

FBD wheel:



$$r_f = r_a \sin\phi = r_a \sin\left(\tan^{-1}\mu\right)$$

$$= (2.5 \text{ in.})\sin\left(\tan^{-1}\mu\right)$$

$P = W\tan\theta$ for each wheel, so also for total $P = W\tan\theta$

$$\tan\theta \approx \frac{b + r_f}{r_w} \text{ for small } \theta$$

So $$P = (70{,}000 \text{ lb})\frac{(0.02 \text{ in.}) + r_f}{16 \text{ in.}}$$

(*a*) For impending motion, use $\mu_s = 0.02$:

Then $r_f = 0.04999$ in. and $P = 306$ lb ◀

(*b*) For steady motion, use $\mu_k = 0.015$:

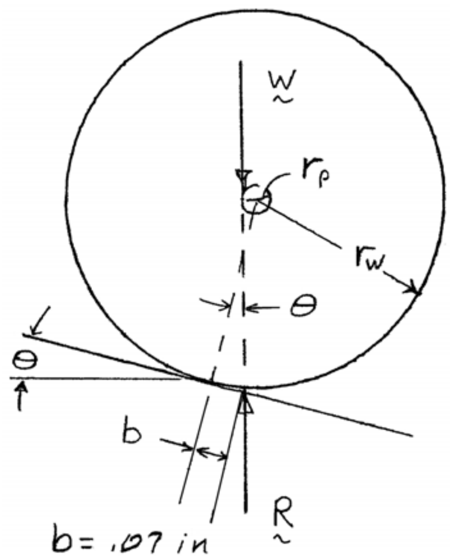
Then $r_f = 0.037496$ in. and $P = 252$ lb ◀

PROBLEM 8.100

Solve Problem 8.89 including the effect of a coefficient of rolling resistance of 0.07 in.

SOLUTION

FBD wheel:



The wheel is a two-force body, so **R** and **W** are colinear and tangent to the friction circle.

$$\tan\theta = \text{slope} = 0.02$$

$$\tan\theta \approx \frac{b + r_f}{r_w} \qquad \text{or} \qquad r_w \approx \frac{b + r_f}{\tan\theta}$$

Now

$$r_f = r_a \sin\phi_k = r_a \sin\left(\tan^{-1}\mu_k\right)$$

$$= (0.5\text{ in.})\sin\left(\tan^{-1}0.1\right)$$

$$= 0.049752$$

$$\therefore \quad r_w \approx \frac{0.07\text{ in.} + 0.049752\text{ in.}}{0.02} = 5.9876\text{ in.}$$

$$d = 2r_w$$

$d = 11.98$ in. ◀

PROBLEM 8.101

A hawser is wrapped two full turns around a bollard. By exerting a 320-N force on the free end of the hawser, a dockworker can resist a force of 20 kN on the other end of the hawser. Determine (*a*) the coefficient of static friction between the hawser and the bollard, (*b*) the number of times the hawser should be wrapped around the bollard if a 80-kN force is to be resisted by the same 320-N force.

SOLUTION

Two full turns of rope $\rightarrow$ $\qquad \beta = 4\pi \text{ rad}$

(*a*)

$$\mu_s \beta = \ln \frac{T_2}{T_1} \qquad \text{or} \qquad \mu_s = \frac{1}{\beta} \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{4\pi} \ln \frac{20\ 000 \text{ N}}{320 \text{ N}} = 0.329066$$

$$\mu_s = 0.329 \blacktriangleleft$$

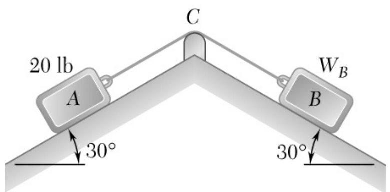
(*b*)

$$\beta = \frac{1}{\mu_s} \ln \frac{T_2}{T_1}$$

$$= \frac{1}{0.329066} \ln \frac{80\ 000 \text{ N}}{320 \text{ N}}$$

$$= 16.799 \text{ rad}$$

$$\beta = 2.67 \text{ turns} \blacktriangleleft$$

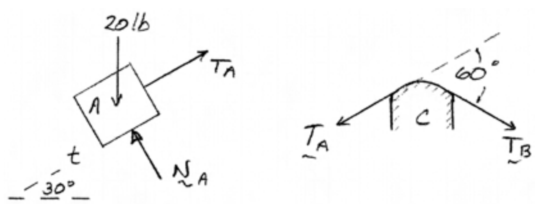


PROBLEM 8.102

Blocks A and B are connected by a cable that passes over support C. Friction between the blocks and the inclined surfaces can be neglected. Knowing that motion of block B up the incline is impending when $W_B = 16$ lb, determine (a) the coefficient of static friction between the rope and the support, (b) the largest value of W_B for which equilibrium is maintained. (*Hint:* See Problem 8.128.)

SOLUTION

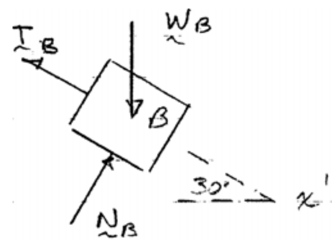
FBD A:



$$\nearrow \Sigma F_t = 0: \quad T_A - 20\text{ lb}\ \sin 30° = 0$$

$$T_A = 10\text{ lb}$$

FBD B:



$$\searrow \Sigma F_{x'} = 0: \quad W_B \sin 30° - T_B = 0$$

$$T_B = \frac{W_B}{2}$$

From hint, $\beta = 60° = \dfrac{\pi}{3}$ rad regardless of shape of support C

(a) For motion of B up incline when $W_B = 16$ lb, $\quad T_B = \dfrac{W_B}{2} = 8$ lb

and $\quad \mu_s \beta = \ln \dfrac{T_A}{T_B} \quad$ or $\quad \mu_s = \dfrac{1}{\beta} \ln \dfrac{T_A}{T_B} = \dfrac{3}{\pi} \ln \dfrac{10\text{ lb}}{8\text{ lb}} = 0.213086$

$$\mu_s = 0.213 \blacktriangleleft$$

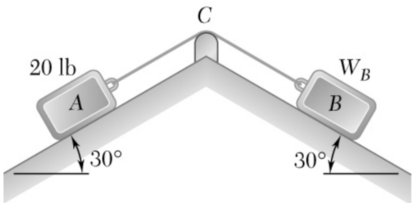
(b) For maximum W_B, motion of B impends down and $T_B > T_A$

So $\quad T_B = T_A e^{\mu_s \beta} = (10\text{ lb}) e^{0.213086\pi/3} = 12.500$ lb

Now $\quad W_B = 2T_B$

So that $\quad W_B = 25.0\text{ lb} \blacktriangleleft$

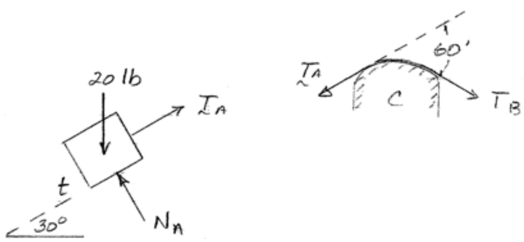
PROBLEM 8.103



Blocks A and B are connected by a cable that passes over support C. Friction between the blocks and the inclined surfaces can be neglected. Knowing that the coefficient of static friction between the rope and the support is 0.50, determine the range of values of W_B for which equilibrium is maintained. (*Hint:* See Problem 8.128.)

SOLUTION

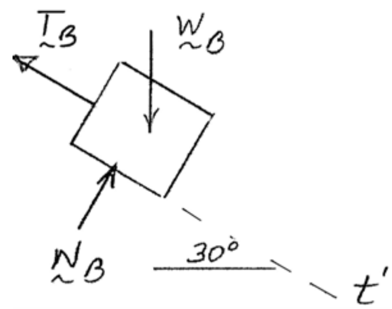
FBD A:



$\nearrow \Sigma F_t = 0\text{:} \quad T_A - 20 \text{ lb} \sin 30° = 0$

$$T_A = 10 \text{ lb}$$

FBD B:



$\searrow \Sigma F_{t'} = 0\text{:} \quad W_B \sin 30° - T_B = 0$

$$T_B = \frac{W_B}{2}$$

From hint, $\beta = 60° = \dfrac{\pi}{3}$ rad, regardless of shape of support C.

For impending motion of B up, $T_A > T_B$, so

$$T_A = T_B e^{\mu_s \beta} \qquad \text{or} \qquad T_B = T_A e^{-\mu_s \beta} = (10 \text{ lb}) e^{-0.5\pi/3} = 5.924 \text{ lb}$$

$$W_B = 2T_B = 11.85 \text{ lb}$$

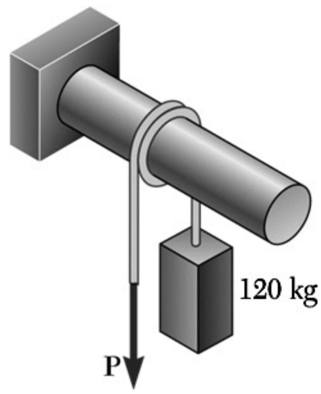
For impending motion of B down, $T_B > T_B$, so

$$T_B = T_A e^{\mu_s \beta} = (10 \text{ lb}) e^{0.5\pi/3} = 16.881 \text{ lb}$$

$$W_B = 2T_B = 33.76 \text{ lb}$$

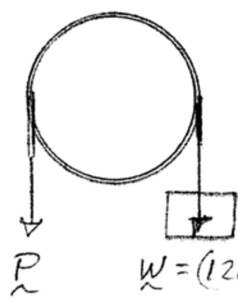
For equilibrium $\qquad 11.85 \text{ lb} \le W_B \le 33.8 \text{ lb}$ ◀

PROBLEM 8.104



A 120-kg block is supported by a rope which is wrapped $1\frac{1}{2}$ times around a horizontal rod. Knowing that the coefficient of static friction between the rope and the rod is 0.15, determine the range of values of P for which equilibrium is maintained.

SOLUTION



$$W = (120 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 1177.2 \text{ N}$$

$$\beta = 1.5 \text{ turns} = 3\pi \text{ rad}$$

For impending motion of W up

$$P = We^{\mu_s \beta} = (1177.2 \text{ N})e^{(0.15)3\pi}$$

$$= 4839.7 \text{ N}$$

For impending motion of W down

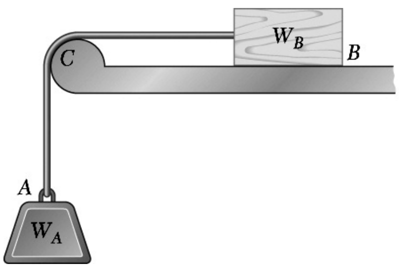
$$P = We^{-\mu_s \beta} = (1177.2 \text{ N})e^{-(0.15)3\pi}$$

$$= 286.3 \text{ N}$$

For equilibrium

$$286 \text{ N} \le P \le 4.84 \text{ kN} \blacktriangleleft$$

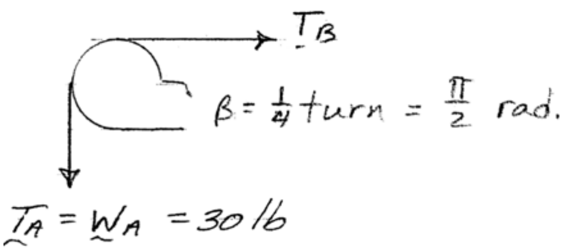
PROBLEM 8.105



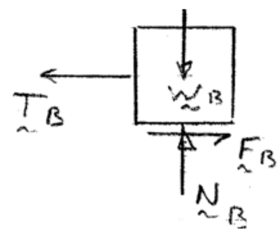
The coefficient of static friction between block B and the horizontal surface and between the rope and support C is 0.40. Knowing that $W_A = 30\text{ lb}$, determine the smallest weight of block B for which equilibrium is maintained.

SOLUTION

Support at C:



FBD block B:



$$\uparrow \Sigma F_y = 0: \quad N_B - W_B = 0 \qquad \text{or} \qquad N_B = W_B$$

Impending motion $\qquad F_B = \mu_s N_B = 0.4 N_B = 0.4 W_B$

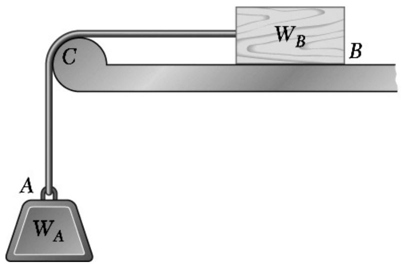
$$\longrightarrow \Sigma F_x = 0: \quad F_B - T_B = 0 \qquad \text{or} \qquad T_B = F_B = 0.4 W_B$$

At support, for impending motion of W_A down, $\qquad W_A = T_B e^{\mu_s \beta}$

so $\qquad T_B = W_A e^{-\mu_s \beta} = (30\text{ lb})e^{-(0.4)\pi/2} = 16.005\text{ lb}$

Now $\qquad W_B = \dfrac{T_B}{0.4}$

so that $\qquad W_B = 40.0\text{ lb}$ ◀

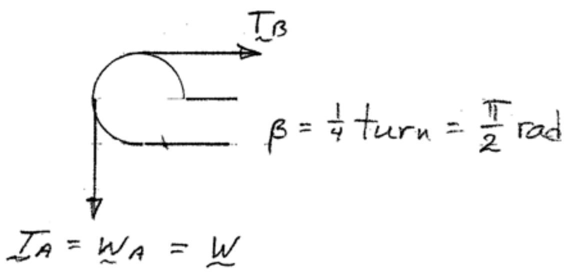


PROBLEM 8.106

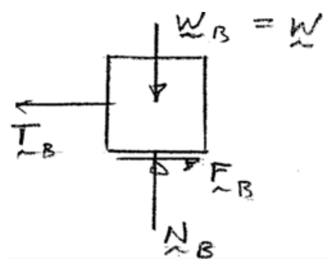
The coefficient of static friction μ_s is the same between block B and the horizontal surface and between the rope and support C. Knowing that $W_A = W_B$, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

Support at C



FBD B:



$$\uparrow \Sigma F_y = 0: \quad N_B - W = 0 \qquad \text{or} \qquad N_B = W$$

Impending motion: $\quad F_B = \mu_s N_B = \mu_s W$

$$\longrightarrow \Sigma F_x = 0: \quad F_B - T_B = 0 \qquad \text{or} \qquad T_B = F_B = \mu_s W$$

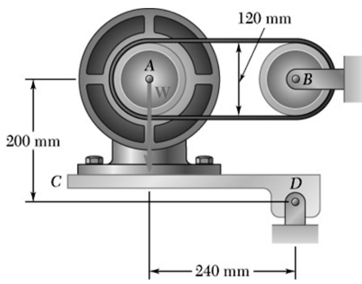
Impending motion of rope on support: $\quad W = T_B e^{\mu_s \beta} = \mu_s W e^{\mu_s \beta}$

or $\quad 1 = \mu_s e^{\mu_s \beta}$

or $\quad \mu_s e^{\frac{\pi}{2}\mu_s} = 1$

Solving numerically: $\quad \mu_s = 0.475$ ◀

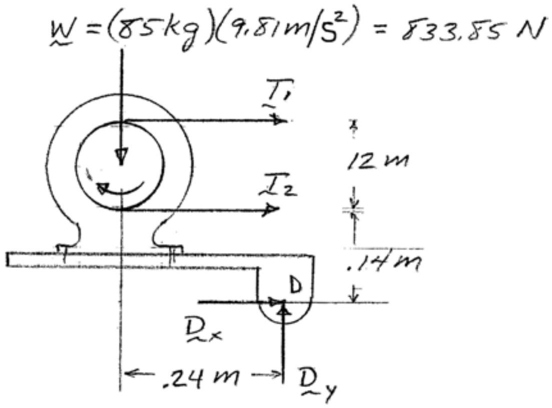
PROBLEM 8.107



In the pivoted motor mount shown, the weight **W** of the 85-kg motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums *A* and *B* is 0.40, and neglecting the weight of platform *CD*, determine the largest torque which can be transmitted to drum *B* when the drive drum *A* is rotating clockwise.

SOLUTION

FBD motor + mount:



For impending slipping of belt, $T_2 = T_1 e^{\mu_s \beta} = T_1 e^{0.4\pi} = 3.5136 T_1$

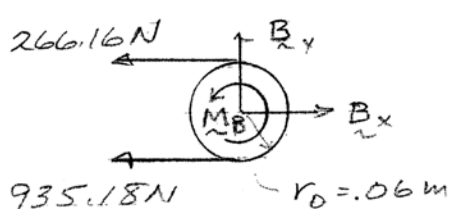
$$\circlearrowleft \Sigma M_D = 0: \quad (0.24\text{ m})(833.85\text{ N}) - (0.14\text{ m})T_2 - (0.26\text{ m})T_1 = 0$$

$$\left[(0.14\text{ m})(3.5136) + 0.26\text{ m}\right]T_1 = 200.124\text{ N}\cdot\text{m}$$

or $T_1 = 266.16\text{ N}$

and $T_2 = 3.5136 T_1 = 935.18\text{ N}$

FBD drum:

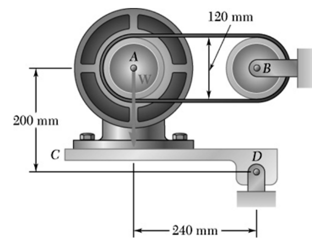


$$\circlearrowleft \Sigma M_B = 0: \quad M_B - (0.06\text{m})(266.16\text{ N} - 935.18\text{ N}) = 0$$

$M_B = 40.1\text{ N}\cdot\text{m}$ ◀

(Compare to $M_B = 81.7\text{ N}\cdot\text{m}$ using V-belt, Problem 8.130.)

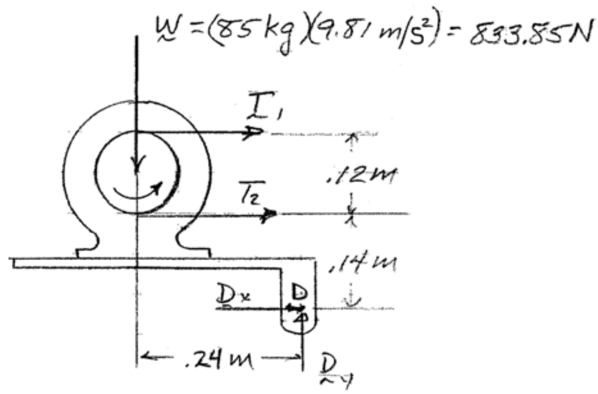
PROBLEM 8.108



Solve Problem 8.107 assuming that the drive drum A is rotating counterclockwise.

SOLUTION

FBD motor + mount:



Impending slipping of belt: $T_1 = T_2 e^{\mu_s \beta} = T_1 e^{0.4\pi} = 3.5136 T_2$

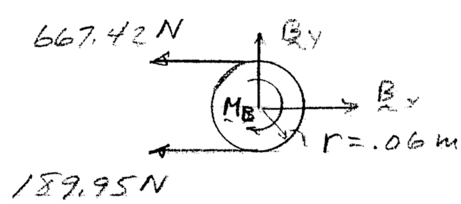
$$\circlearrowleft \Sigma M_D = 0: \quad (0.24 \text{ m})W - (0.26 \text{ m})T_1 - (0.14 \text{ m})T_2 = 0$$

$$\left[(0.26 \text{ m})(3.5136) + 0.14 \text{ m}\right]T_2 = (0.24 \text{ m})(833.85 \text{ N})$$

or $T_2 = 189.95 \text{ N}$

and $T_1 = 667.42 \text{ N}$

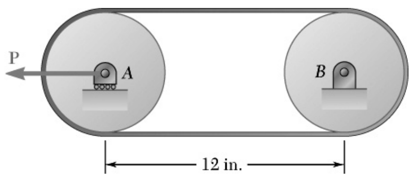
FBD drum:



$$\circlearrowleft \Sigma M_B = 0: \quad (0.06 \text{ m})(667.42 \text{ N} - 189.95 \text{ N}) - M_B = 0$$

$$M_B = 28.6 \text{ N}\cdot\text{m} \blacktriangleleft$$

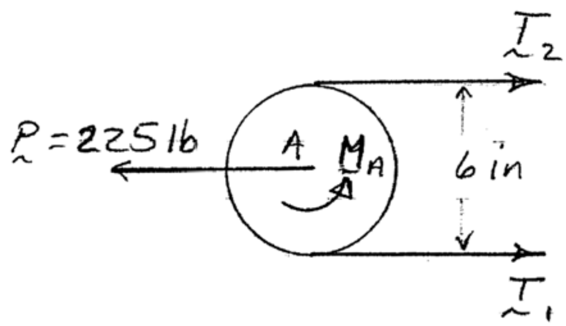
PROBLEM 8.109



A flat belt is used to transmit a torque from pulley A to pulley B. The radius of each pulley is 3 in., and a force of magnitude $P = 225\text{ lb}$ is applied as shown to the axle of pulley A. Knowing that the coefficient of static friction is 0.35, determine (a) the largest torque which can be transmitted, (b) the corresponding maximum value of the tension in the belt.

SOLUTION

FBD pulley A:



Impending slipping of belt: $T_2 = T_1 e^{\mu_s \beta}$

$$T_2 = T_1 e^{0.35\pi} = 3.0028 T_1$$

$\longrightarrow \Sigma F_x = 0{:}\quad T_1 + T_2 - 225\text{ lb} = 0$

$$T_1(1 + 3.0028) = 225\text{ lb} \qquad \text{or} \qquad T_1 = 56.211\text{ lb}$$

$$T_2 = 3.0028 T_1 \qquad \text{or} \qquad T_2 = 168.79\text{ lb}$$

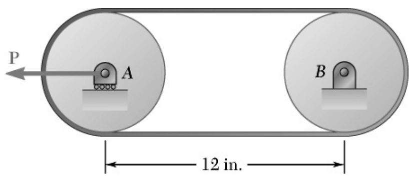
(a) $\circlearrowleft \Sigma M_A = 0{:}\quad M_A + (6\text{ in.})(T_1 - T_2) = 0 \qquad \text{or} \qquad M_A = (3\text{ in.})(168.79\text{ lb} - 56.21\text{ lb})$

$\therefore$ max. torque: $M_A = 338\text{ lb}\cdot\text{in.}$ ◀

(b) max. tension: $T_2 = 168.8\text{ lb}$ ◀

(Compare with $M_A = 638\text{ lb}\cdot\text{in.}$ with V-belt, Problem 8.131.)

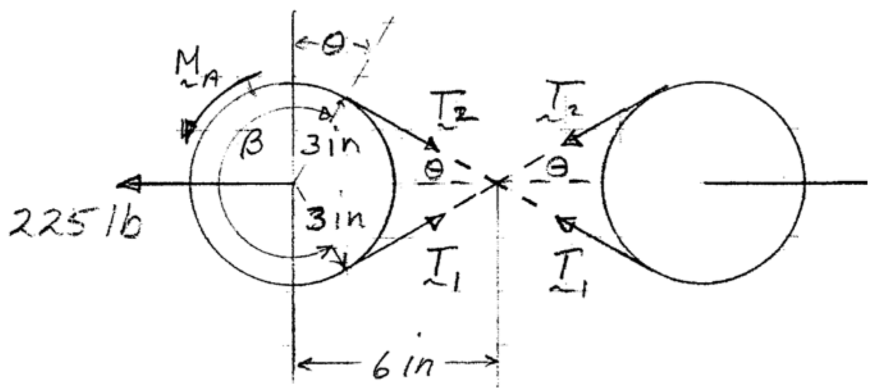
PROBLEM 8.110



Solve Problem 8.109 assuming that the belt is looped around the pulleys in a figure eight.

SOLUTION

FBDs pulleys:



$$\theta = \sin^{-1}\frac{3 \text{ in.}}{6 \text{ in.}} = 30° = \frac{\pi}{6}\text{rad.}$$

$$\beta = \pi + 2\frac{\pi}{6} = \frac{4\pi}{3}$$

Impending belt slipping: $T_2 = T_1 e^{\mu_s \beta}$

$$T_2 = T_1 e^{(0.35)4\pi/3} = 4.3322T_1$$

$$\xrightarrow{} \Sigma F_x = 0: \quad T_1 \cos 30° + T_2 \cos 30° - 225 \text{ lb} = 0$$

$$(T_1 + 4.3322T_1)\cos 30° = 225 \text{ lb} \qquad \text{or} \qquad T_1 = 48.7243 \text{ lb}$$

$$T_2 = 4.3322T_1 \qquad \text{so that} \qquad T_2 = 211.083 \text{ lb}$$

(*a*) $\circlearrowleft \Sigma M_A = 0: \quad M_A + (3 \text{ in.})(T_1 - T_2) = 0 \qquad \text{or} \qquad M_A = (3 \text{ in.})(211.083 \text{ lb} - 48.224 \text{ lb})$

$$M_{max} = M_A = 487 \text{ lb}\cdot\text{in.} ◀$$

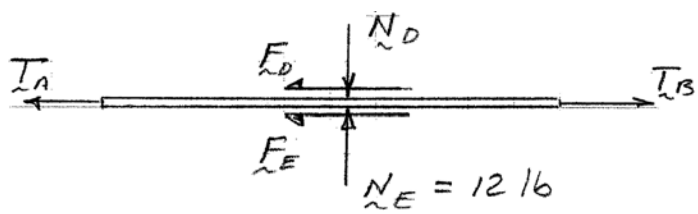
(*b*) $T_{max} = T_2 = 211 \text{ lb}$ ◀

PROBLEM 8.111

A couple $\mathbf{M}_B$ of magnitude $2\text{ lb}\cdot\text{ft}$ is applied to the drive drum B of a portable belt sander to maintain the sanding belt C at a constant speed. The total downward force exerted on the wooden workpiece E is 12 lb, and $\mu_k = 0.10$ between the belt and the sanding platen D. Knowing that $\mu_s = 0.35$ between the belt and the drive drum and that the radii of drums A and B are 1.00 in., determine (*a*) the minimum tension in the lower portion of the belt if no slipping is to occur between the belt and the drive drum, (*b*) the value of the coefficient of kinetic friction between the belt and the workpiece.

SOLUTION

FBD lower portion of belt:



$$+\uparrow \Sigma F_y = 0: \quad N_E - N_D = 0$$

or
$$N_D = N_E = 12 \text{ lb}$$

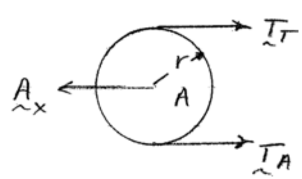
Slipping:
$$F_D = (\mu_k)_{\text{belt/platen}} N_D$$

$$F_D = 0.1(12 \text{ lb}) = 1.2 \text{ lb}$$

and
$$F_E = (\mu_k)_{\text{belt/wood}} N_E$$

$$F = (12 \text{ lb})(\mu_k)_{\text{belt/wood}} \qquad (1)$$

FBD drum A: (assumed free to rotate)

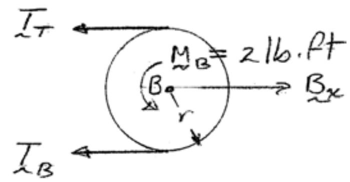


$$\rightarrow \Sigma F_x = 0: \quad T_B - T_A - F_D - F_E = 0 \qquad (2)$$

$$\Sigma M_A = 0: \quad r_A(T_A - T_T) = 0 \qquad \text{or} \qquad T_T = T_A$$

PROBLEM 8.111 CONTINUED

FBD drum B:



$$\circlearrowleft \Sigma M_B = 0: \quad M_B + r(T_T - T_B) = 0$$

or
$$T_B - T_T = \frac{M_B}{r} = \left(\frac{2 \text{ lb}\cdot\text{ft}}{1 \text{ in.}}\right)\left(\frac{12 \text{ in.}}{\text{ft}}\right) = 24 \text{ lb}$$

Impending slipping:
$$T_B = T_T e^{\mu_s \beta} = T_T e^{0.35\pi}$$

So
$$\left(e^{0.35\pi} - 1\right)T_T = 24 \text{ lb} \qquad \text{or} \qquad T_T = 11.983 \text{ lb}$$

Now
$$T_A = T_T = 11.983 \text{ lb then } T_B = (11.983 \text{ lb})e^{0.35\pi} = 35.983 \text{ lb}$$

From Equation (2):
$$35.983 \text{ lb} - 11.983 \text{ lb} - 1.2 \text{ lb} = F_E = 22.8 \text{ lb}$$

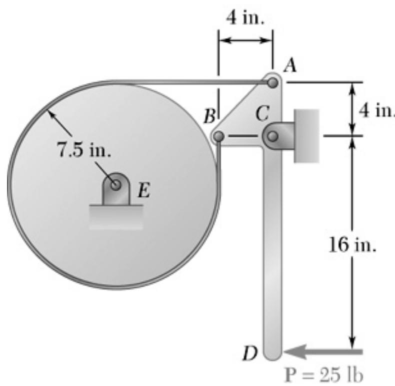
From Equation (1):
$$(\mu_k)_{\text{belt/wood}} = \frac{F_E}{12 \text{ lb}} = \frac{22.8 \text{ lb}}{12 \text{ lb}} = 1.900$$

Therefore

(*a*) $T_{\min} = T_A = 11.98 \text{ lb}$ ◀

(*b*) $(\mu_k)_{\text{belt/wood}} = 1.900$ ◀

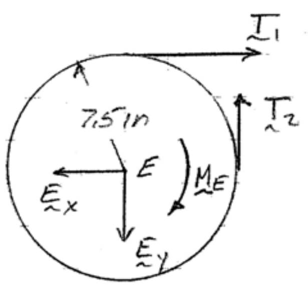
PROBLEM 8.112



A band belt is used to control the speed of a flywheel as shown. Determine the magnitude of the couple being applied to the flywheel knowing that the coefficient of kinetic friction between the belt and the flywheel is 0.25 and that the flywheel is rotating clockwise at a constant speed. Show that the same result is obtained if the flywheel rotates counterclockwise.

SOLUTION

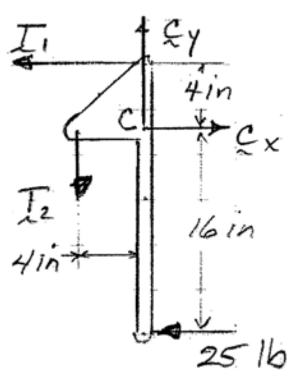
FBD wheel:



$$\circlearrowleft \Sigma M_E = 0: \quad -M_E + (7.5 \text{ in.})(T_2 - T_1) = 0$$

or $$M_E = (7.5 \text{ in.})(T_2 - T_1)$$

FBD lever:



$$\circlearrowleft \Sigma M_C = 0: \quad (4 \text{ in.})(T_1 + T_2) - (16 \text{ in.})(25 \text{ lb}) = 0$$

or $$T_1 + T_2 = 100 \text{ lb}$$

Impending slipping: $$T_2 = T_1 e^{\mu_s \beta}$$

or $$T_2 = T_1 e^{0.25\left(\frac{3\pi}{2}\right)} = 3.2482 T_1$$

So $$T_1(1 + 3.2482) = 100 \text{ lb}$$

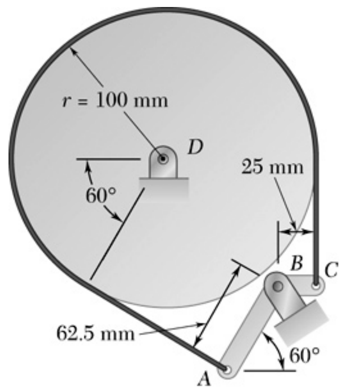
$$T_1 = 23.539 \text{ lb}$$

and $$M_E = (7.5 \text{ in.})(3.2482 - 1)(23.539 \text{ lb}) = 396.9 \text{ lb} \cdot \text{in.}$$

$$M_E = 397 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

Changing the direction of rotation will change the direction of M_E and will switch the magnitudes of T_1 and T_2.

The magnitude of the couple applied will not change. ◀

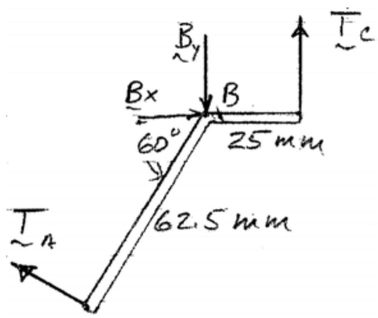


PROBLEM 8.113

The drum brake shown permits clockwise rotation of the drum but prevents rotation in the counterclockwise direction. Knowing that the maximum allowed tension in the belt is 7.2 kN, determine (*a*) the magnitude of the largest counterclockwise couple that can be applied to the drum, (*b*) the smallest value of the coefficient of static friction between the belt and the drum for which the drum will not rotate counterclockwise.

SOLUTION

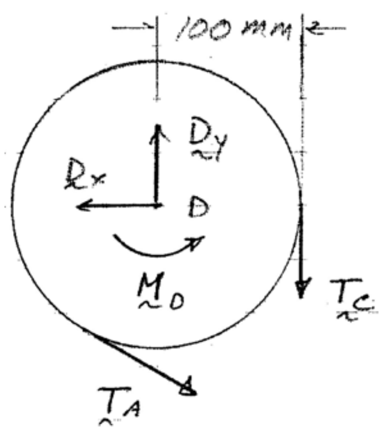
FBD lever:



$$\circlearrowleft \Sigma M_B = 0: \quad (25 \text{ mm})T_C - (62.5 \text{ mm})T_A = 0$$

$$T_C = 2.5T_A$$

FBD lever:



Impending ccw rotation:

(*a*) $$T_C = T_{\max} = 7.2 \text{ kN}$$

But $$T_C = 2.5T_A$$

So $$T_A = \frac{7.2 \text{ kN}}{2.5} = 2.88 \text{ kN}$$

$$\circlearrowleft \Sigma M_D = 0: \quad M_D + (100 \text{ mm})(T_A - T_C) = 0$$

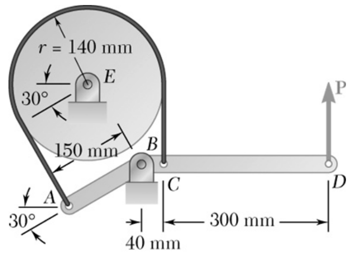
$$M_D = (100 \text{ mm})(7.2 - 2.88) \text{ kN}$$

$$M_D = 432 \text{ N}\cdot\text{m} \blacktriangleleft$$

(*b*) Also, impending slipping: $$\mu_s \beta = \ln \frac{T_C}{T_A}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_C}{T_A} = \frac{1}{\frac{4\pi}{3}} \ln 2.5 = 0.2187$$

Therefore, $$(\mu_s)_{\min} = 0.219 \blacktriangleleft$$

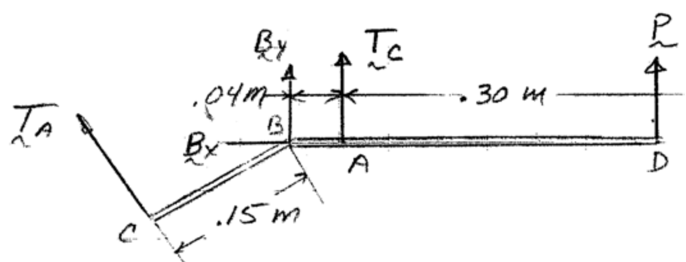


PROBLEM 8.114

A differential band brake is used to control the speed of a drum which rotates at a constant speed. Knowing that the coefficient of kinetic friction between the belt and the drum is 0.30 and that a couple of magnitude is $150\ \text{N}\cdot\text{m}$ applied to the drum, determine the corresponding magnitude of the force **P** that is exerted on end *D* of the lever when the drum is rotating (*a*) clockwise, (*b*) counterclockwise.

SOLUTION

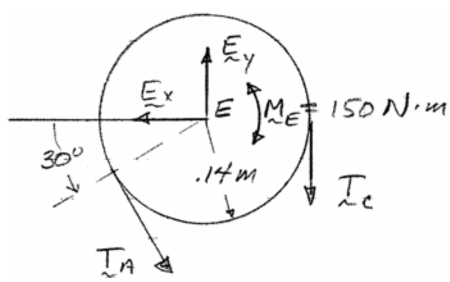
FBD lever:



$$\circlearrowleft \Sigma M_B = 0: \quad (0.34\ \text{m})P + (0.04\ \text{m})T_C - (0.15\ \text{m})T_A = 0$$

$$P = \frac{15T_A - 4T_C}{34} \qquad (1)$$

FBD drum:



(*a*) For cw rotation, $M_E \circlearrowright$

$$\circlearrowleft \Sigma M_E = 0: \quad (0.14\ \text{m})(T_A - T_C) - M_E = 0$$

$$T_A - T_C = \frac{150\ \text{N}\cdot\text{m}}{0.14\ \text{m}} = 1071.43\ \text{N}$$

Impending slipping: $\quad T_A = T_C e^{\mu_k \beta} = T_C e^{(0.3)\frac{7\pi}{6}}$

$$T_A = 3.00284T_C$$

So $\quad (3.00284 - 1)T_C = 1071.43\ \text{N} \quad$ or $\quad T_C = 534.96\ \text{N}$

and $\quad T_A = 1606.39\ \text{N}$

PROBLEM 8.114 CONTINUED

From Equation (1): $P = \dfrac{15(1606.39\text{ N}) - 4(534.96\text{ N})}{34}$

$P = 646\text{ N}$ ◀

(*b*) For ccw rotation, M_E ↻ and $\Sigma M_E = 0 \Rightarrow T_C - T_A = 1071.43\text{ N}$

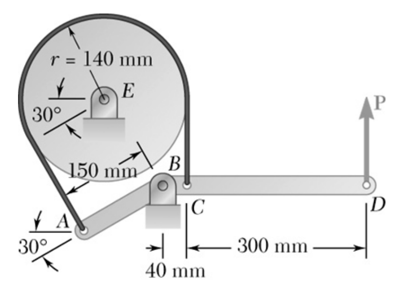
Also, impending slip $\Rightarrow$ $T_C = 3.00284T_A$, so $T_A = 534.96\text{ N}$

and $T_C = 1606.39\text{ N}$

And Equation (1) $\Rightarrow$ $P = \dfrac{15(534.96\text{ N}) - 4(1606.39\text{ N})}{34}$

$P = 47.0\text{ N}$ ◀

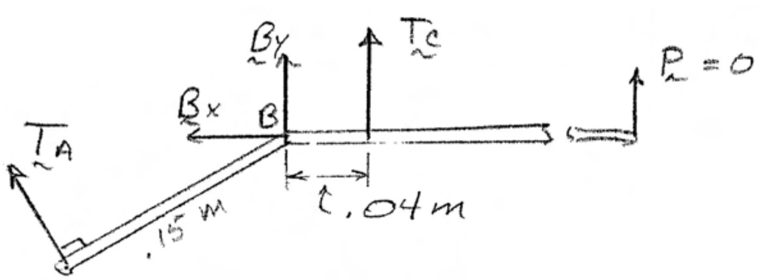
PROBLEM 8.115



A differential band brake is used to control the speed of a drum. Determine the minimum value of the coefficient of static friction for which the brake is self-locking when the drum rotates counterclockwise.

SOLUTION

FBD lever:

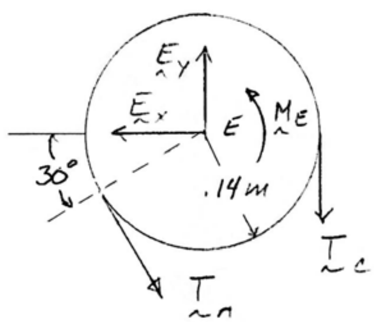


For self-locking **P** = 0

$$\circlearrowleft \Sigma M_B = 0: \quad (0.04\text{ m})T_C - (0.15\text{ m})T_A = 0$$

$$T_C = 3.75T_A$$

FBD drum:

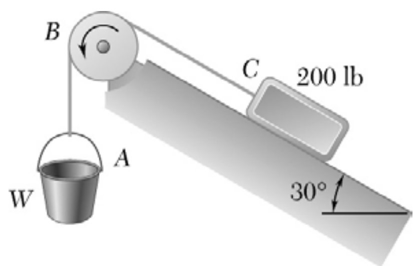


For impending slipping of belt $\quad T_C = T_A e^{\mu_s \beta}$

or $\quad \mu_s \beta = \ln \dfrac{T_C}{T_A}$

Then $\quad \mu_s = \dfrac{1}{\beta} \ln \dfrac{T_C}{T_A} = \dfrac{1}{\frac{7\pi}{6}} \ln 3.75 = 0.3606$

$(\mu_s)_{\text{req}} = 0.361$ ◀

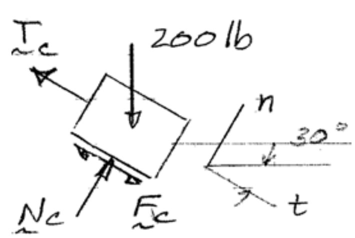


PROBLEM 8.116

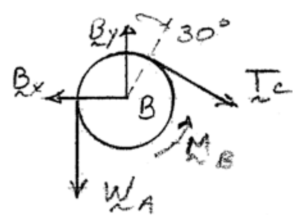
Bucket A and block C are connected by a cable that passes over drum B. Knowing that drum B rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined weight W of the bucket and its contents for which block C will (*a*) remain at rest, (*b*) be about to move up the incline, (*c*) continue moving up the incline at a constant speed.

SOLUTION

FBD block:



FBD drum:



↗ $\Sigma F_n = 0$: $N_C - (200 \text{ lb})\cos 30° = 0$; $N = 100\sqrt{3}$ lb

↖ $\Sigma F_t = 0$: $T_C - (200 \text{ lb})\sin 30° \mp F_C = 0$

$$T_C = 100 \text{ lb} \pm F_C \qquad (1)$$

where the upper signs apply when F_C acts ↘

(*a*) For impending motion of block ↘, F_C ↖, and

$$F_C = \mu_s N_C = 0.35\left(100\sqrt{3} \text{ lb}\right) = 35\sqrt{3} \text{ lb}$$

So, from Equation (1): $T_C = \left(100 - 35\sqrt{3}\right)$ lb

But belt slips on drum, so $T_C = W_A e^{\mu_k \beta}$

$$W_A = \left[\left(100 - 35\sqrt{3}\right)\text{lb}\right] e^{-0.25\left(\frac{2\pi}{3}\right)}$$

$W_A = 23.3$ lb ◀

(*b*) For impending motion of block ↖, F_C ↘ and $F_C = \mu_s N_C = 35\sqrt{3}$ lb

From Equation (1): $T_C = \left(100 + 35\sqrt{3}\right)$ lb

Belt still slips, so $W_A = T_C e^{-\mu_k \beta} = \left[\left(100 + 35\sqrt{3}\right) \text{lb}\right] e^{-0.25\left(\frac{2\pi}{3}\right)}$

$W_A = 95.1$ lb ◀

PROBLEM 8.116 CONTINUED

(*c*) For steady motion of block ↖, F_C ↘, and $F_C = \mu_k N_C = 25\sqrt{3}$ lb

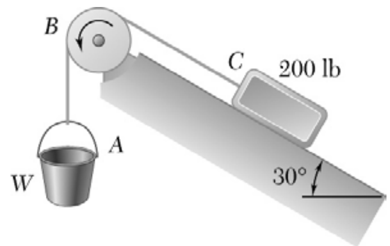
Then, from Equation (1): $T = \left(100 + 25\sqrt{3}\right)$ lb.

Also, belt is not slipping on drum, so

$$W_A = T_C e^{-\mu_s \beta} = \left[\left(100 + 25\sqrt{3}\right)\text{ lb}\right] e^{-0.35\left(\frac{2\pi}{3}\right)}$$

$W_A = 68.8$ lb ◀

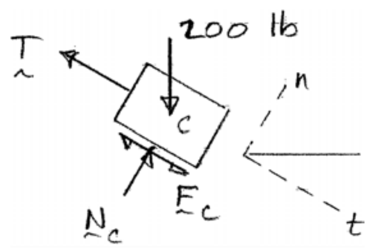
PROBLEM 8.117



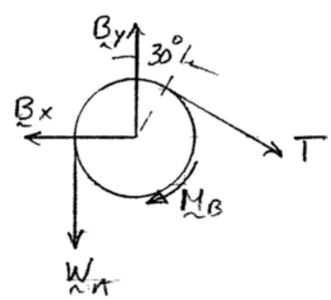
Solve Problem 8.116 assuming that drum B is frozen and cannot rotate.

SOLUTION

FBD block:



FBD drum:



$$\nearrow \Sigma F_n = 0: \quad N_C - (200\text{ lb})\cos 30° = 0; \; N_C = 100\sqrt{3}\text{ lb}$$

$$\searrow \Sigma F_t = 0: \quad \pm F_C + (200\text{ lb})\sin 30° - T = 0$$

$$T = 100\text{ lb} \pm F_C \qquad (1)$$

where the upper signs apply when F_C acts ↘

(*a*) For impending motion of block ↘, F_C ↖ and $F_C = \mu_s N_C$

So $\quad F_C = 0.35\left(100\sqrt{3}\text{ lb}\right) = 35\sqrt{3}\text{ lb}$

and $\quad T = 100\text{ lb} - 35\sqrt{3}\text{ lb} = 39.375\text{ lb}$

Also belt slipping is impending ↻ so $\quad T = W_A e^{\mu_s \beta}$

or $\quad W_A = Te^{-\mu_s \beta} = (39.378\text{ lb})e^{-0.35\left(\frac{2\pi}{3}\right)}$

$W_A = 18.92\text{ lb}$ ◀

(*b*) For impending motion of block ↖, F_C ↘, and

$F_C = \mu_s N_C = 35\sqrt{3}\text{ lb}$

But $\quad T = \left(100 + 35\sqrt{3}\right)\text{ lb} = 160.622\text{ lb}.$

Also belt slipping is impending ↻

So $\quad W_A = Te^{+\mu_s \beta} = (160.622\text{ lb})e^{0.35\left(\frac{2\pi}{3}\right)};$

$W_A = 334\text{ lb}$ ◀

(*c*) For steady motion of block ↖, F_C ↘, and $F_C = \mu_k N_C = 25\sqrt{3}\text{ lb}$

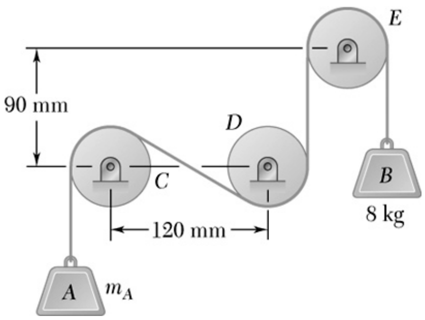
Then $\quad T = \left(100\text{ lb} + 25\sqrt{3}\text{ lb}\right) = 143.301\text{ lb}.$

Now belt is slipping ↻

So $\quad W_A = Te^{\mu_k \beta} = (143.301\text{ lb})e^{0.25\left(\frac{2\pi}{3}\right)}$

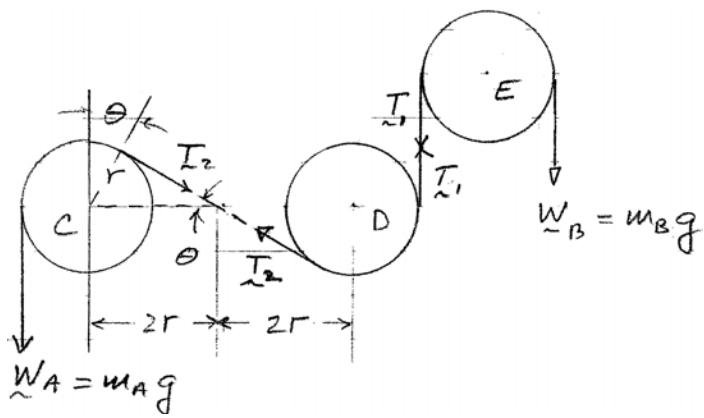
$W_A = 242\text{ lb}$ ◀

PROBLEM 8.118



A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Pulleys C and E are locked to prevent rotation, and the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$. Determine the range of values of the mass of block A for which equilibrium is maintained (a) if pulley D is locked, (b) if pulley D is free to rotate.

SOLUTION



Note: $$\theta = \sin^{-1}\frac{r}{2r} = 30° = \frac{\pi}{6} \text{ rad}$$

So $$\beta_C = \beta_D = \frac{2\pi}{3} \quad \text{and} \quad \beta_E = \pi$$

(a) All pulleys locked $\Rightarrow$ slipping impends at all surface simultaneously.

If A impends $\uparrow$, $$T_2 = W_A e^{\mu_s \beta_C}; \; T_1 = T_2 e^{\mu_s \beta_D}; \; W_B = T_1 e^{\mu_s \beta_E}$$

So $$W_B = W_A e^{\mu_s(\beta_C + \beta_D + \beta_E)} \quad \text{or} \quad W_A = W_B e^{-\mu_s(\beta_C + \beta_D + \beta_E)}$$

Then $$m_A = m_B e^{-\mu_s(\beta_C + \beta_D + \beta_E)} = (8 \text{ kg}) e^{-0.2\left(\frac{2\pi}{3} + \frac{2\pi}{3} + \pi\right)} = 1.847 \text{ kg}$$

If A impends $\downarrow$, $$W_A = T_2 e^{\mu_s \beta_C} = T_1 e^{\mu_s \beta_D} e^{\mu_s \beta_C} = W_B e^{\mu_s(\beta_E + \beta_D + \beta_C)}$$

So $$m_A = m_B e^{\mu_s(\beta_E + \beta_D + \beta_C)} = (8 \text{ kg}) e^{0.2\left(\pi + \frac{2\pi}{3} + \frac{2\pi}{3}\right)} = 34.7 \text{ kg}$$

Equilibrium for $1.847 \text{ kg} \le m_A \le 34.7 \text{ kg}$ ◀

PROBLEM 8.118 CONTINUED

(*b*) Pulleys *C* & *E* locked, pulley *D* free $\Rightarrow T_1 = T_2$, other relations remain the same.

If *A* impends ↑, $T_2 = W_A e^{\mu_s \beta_C} = T_1 \qquad W_B = T_1 e^{\mu_s \beta_E} = W_A e^{\mu_s(\beta_C + \beta_E)}$

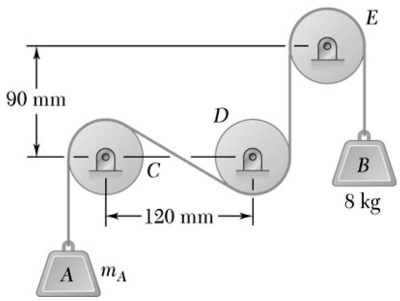
So $m_A = m_B e^{-\mu_s(\beta_C + \beta_E)} = (8\text{ kg})e^{-0.2\left(\frac{2\pi}{3} + \pi\right)} = 2.807\text{ kg}$

If *A* impends ↓ slipping is reversed, $W_A = W_B e^{+\mu_s(\beta_C + \beta_E)}$

Then $m_A = m_B e^{\mu_s(\beta_C + \beta_E)} = (8\text{ kg})e^{0.2\left(\frac{5\pi}{3}\right)} = 22.8\text{ kg}$

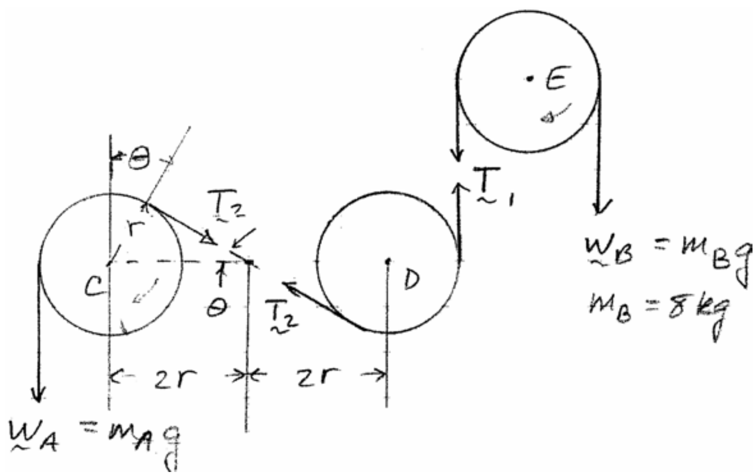
Equilibrium for $2.81\text{ kg} \le m_A \le 22.8\text{ kg}$ ◀

PROBLEM 8.119



A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Two of the pulleys are locked to prevent rotation, while the third pulley is rotated slowly at a constant speed. Knowing that the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$, determine the largest mass m_A which can be raised (*a*) if pulley *C* is rotated, (*b*) if pulley *E* is rotated.

SOLUTION



Note: $$\theta = \sin^{-1}\frac{r}{2r} = 30° = \frac{\pi}{6} \text{ rad}$$

$$\beta_C = \beta_D = \frac{2\pi}{3} \qquad \text{and} \qquad \beta_E = \pi$$

Mass *A* moves up

(*a*) *C* rotates ↻, for maximum W_A have no belt slipping on *C*, so

$$W_A = T_2 e^{\mu_s \beta_C}$$

D and *E* are fixed, so $$T_1 = T_2 e^{\mu_k \beta_D}$$

and $$W_B = T_1 e^{\mu_k \beta_E} = T_2 e^{\mu_k(\beta_D + \beta_E)} \Rightarrow T_2 = W_B e^{-\mu_k(\beta_D + \beta_E)}$$

Thus $$m_A g = m_B g e^{\mu_s \beta_C - \mu_k(\beta_D + \beta_E)} \qquad \text{or} \qquad m_A = (8 \text{ kg}) e^{\left(\frac{0.4\pi}{3} - 0.1\pi - 0.15\pi\right)}$$

$$m_A = 5.55 \text{ kg} \blacktriangleleft$$

PROBLEM 8.119 CONTINUED

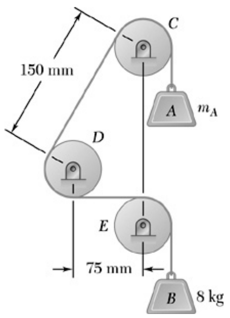
(*b*) *E* rotates ↻, no belt slip on *E*, so $$T_1 = W_B e^{\mu_s \beta_E}$$

C and *D* fixed, so $$T_1 = T_2 e^{\mu_k \beta_D} = W_A e^{\mu_k (\beta_C + \beta_D)}$$

or $$m_A g = T_1 e^{-\mu_k (\beta_C + \beta_D)} = m_B g e^{\mu_s \beta_E - \mu_k (\beta_C + \beta_D)}$$

Then $$m_A = (8 \text{ kg}) e^{(0.2\pi - 0.1\pi - 0.1\pi)} = 8.00 \text{ kg}$$

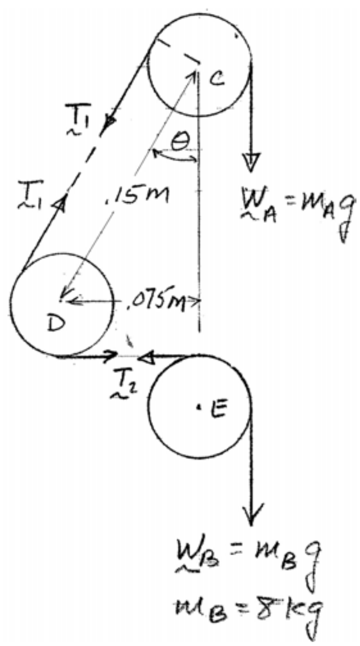
$m_A = 8.00 \text{ kg}$ ◀



PROBLEM 8.120

A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Pulleys C and E are locked to prevent rotation, and the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$. Determine the range of values of the mass of block A for which equilibrium is maintained (*a*) if pulley D is locked, (*b*) if pulley D is free to rotate.

SOLUTION



Note: $$\theta = \sin^{-1}\frac{0.075\text{ m}}{0.15\text{ m}} = 30° = \frac{\pi}{6}\text{ rad}$$

So $$\beta_C = \frac{5}{6}\pi,\ \beta_D = \frac{2}{3}\pi,\ \beta_E = \frac{1}{2}\pi$$

(*a*) All pulleys locked, slipping at all surfaces.

For m_A impending ↑, $$T_1 = W_A e^{\mu_s \beta_C},$$

$$T_2 = T_1 e^{\mu_s \beta_D}, \quad \text{and} \quad W_B = T_2 e^{\mu_k \beta_E},$$

So $$m_B g = m_A g e^{\mu_s(\beta_C + \beta_D + \beta_E)}$$

$$8\text{ kg} = m_A e^{0.2\left(\frac{5}{6}+\frac{2}{3}+\frac{1}{2}\right)\pi} \quad \text{or} \quad m_A = 2.28\text{ kg}$$

For m_A impending down, all tension ratios are inverted, so

$$m_A = (8\text{ kg})e^{0.2\left(\frac{5}{6}+\frac{2}{3}+\frac{1}{2}\right)\pi} = 28.1\text{ kg}$$

Equilibrium for $2.28\text{ kg} \le m_A \le 28.1\text{ kg}$ ◀

(*b*) Pulleys C and E locked, D free $\Rightarrow T_1 = T_2$, other ratios as in (*a*)

m_A impending ↑, $$T_1 = W_A e^{\mu_s \beta_C} = T_2$$

and $$W_B = T_2 e^{\mu_s \beta_E} = W_A e^{\mu_s(\beta_C + \beta_E)}$$

So $$m_B g = m_A g\, e^{\mu(\beta_C + \beta_E)} \quad \text{or} \quad 8\text{ kg} = m_A e^{0.2\left(\frac{5}{6}+\frac{1}{2}\right)\pi}$$

$$m_A = 3.46\text{ kg}$$

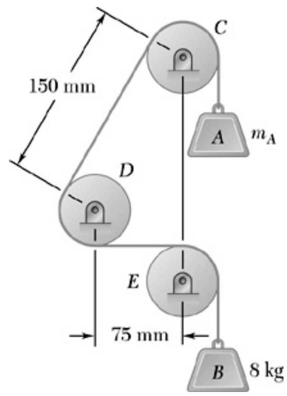
m_A impending ↓, all tension ratios are inverted, so

$$m_A = 8\text{ kg}\, e^{0.2\left(\frac{5}{6}+\frac{1}{2}\right)\pi}$$

$$= 18.49\text{ kg}$$

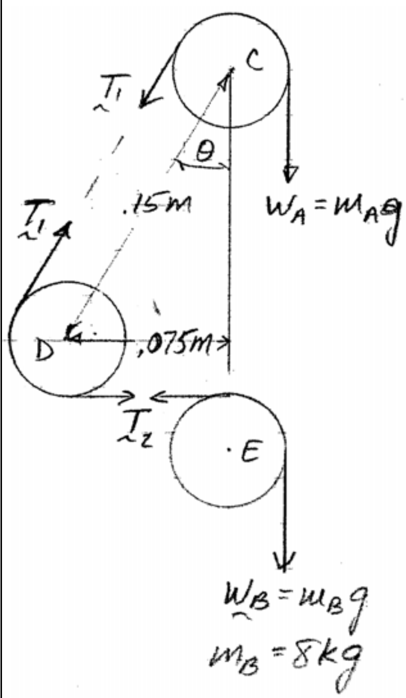
Equilibrium for $3.46\text{ kg} \le m_A \le 18.49\text{ kg}$ ◀

PROBLEM 8.121



A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Two of the pulleys are locked to prevent rotation, while the third pulley is rotated slowly at a constant speed. Knowing that the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$, determine the largest mass m_A which can be raised (*a*) if pulley *C* is rotated, (*b*) if pulley *E* is rotated.

SOLUTION



Note: $\theta = \sin^{-1}\dfrac{0.075 \text{ m}}{0.15 \text{ m}} = 30° = \dfrac{\pi}{6} \text{ rad}$

So $\beta_C = \dfrac{5}{6}\pi, \; \beta_D = \dfrac{2}{3}\pi, \; \beta_E = \dfrac{1}{2}\pi$

(*a*) To raise maximum m_A, with C rotating ↻ $W_A = T_1 e^{\mu_s \beta_C}$. If D and E are fixed, cable must slip there, so $T_2 = T_1 e^{\mu_k \beta_D}$

and $W_B = T_2 e^{\mu_k \beta_E} = T_1 e^{\mu_k(\beta_D + \beta_E)}$

$$= W_A e^{-\mu_s \beta_C} e^{\mu_k(\beta_D + \beta_E)}$$

$$(8 \text{ kg})g = m_A g e^{-0.2\left(\frac{5}{6}\pi\right)} e^{0.15\left(\frac{2}{3}+\frac{1}{2}\right)\pi}$$

$m_A = 7.79 \text{ kg}$ ◀

(*b*) With E rotating ↻, $T_2 = W_B e^{\mu_s \beta_E}$. With C and D fixed.

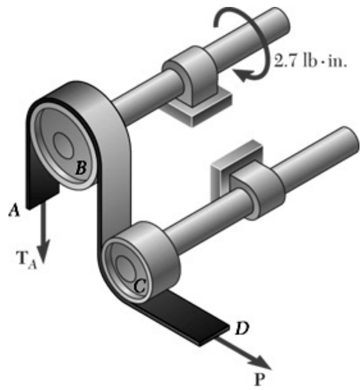
$T_1 = W_A e^{\mu_k \beta_C}$ and $T_2 = T_1 e^{\mu_k \beta_D} = W_A e^{\mu_k(\beta_C + \beta_D)}$

so $W_B = W_A e^{\mu_k(\beta_C + \beta_D)} e^{-\mu_s \beta_E}$

$$(8 \text{ kg})g = m_A g e^{0.15\left(\frac{5}{6}+\frac{2}{3}\right)\pi} e^{-0.2\left(\frac{1}{2}\pi\right)}$$

$m_A = 5.40 \text{ kg}$ ◀

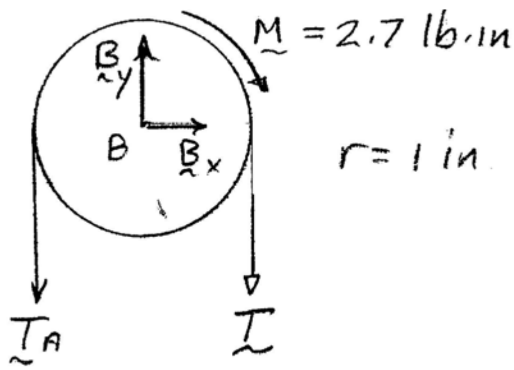
PROBLEM 8.122



A recording tape passes over the 1-in.-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

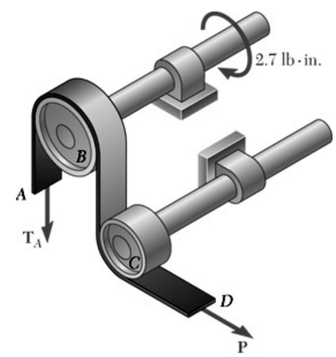
$$T_A - T = \frac{M}{r} = \frac{2.7 \text{ lb}\cdot\text{in.}}{1 \text{ in.}} = 2.7 \text{ lb}$$

Impending slipping: $T_A = Te^{\mu_S \beta} = Te^{0.4\pi}$

So $T\left(e^{0.4\pi} - 1\right) = 2.7 \text{ lb}$

or $T = 1.0742 \text{ lb}$

If C is free to rotate, $P = T$ $\quad P = 1.074 \text{ lb} \blacktriangleleft$

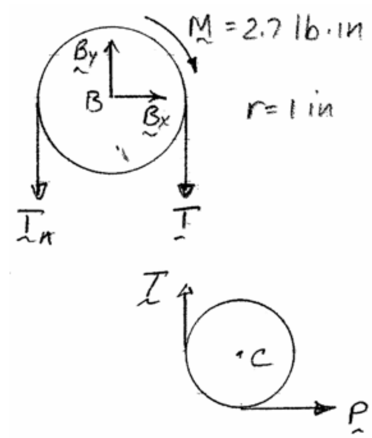


PROBLEM 8.123

Solve Problem 8.122 assuming that the idler drum C is frozen and cannot rotate.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

$$T_A - T = \frac{M}{r} = \frac{2.7 \text{ lb} \cdot \text{in.}}{1 \text{ in.}} = 2.7 \text{ lb}$$

Impending slipping: $T_A = Te^{\mu_s \beta} = Te^{0.4\pi}$

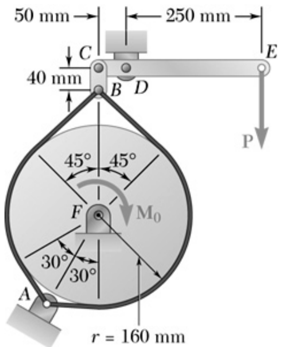
So $\left(e^{0.4\pi} - 1\right)T = 2.7 \text{ lb}$

or $T = 1.07416 \text{ lb}$

If C is fixed, the tape must slip $\circlearrowleft$

So $P = Te^{\mu_k \beta_C} = 1.07416 \text{ lb } e^{0.3\frac{\pi}{2}} = 1.7208 \text{ lb}$

$P = 1.721 \text{ lb}$ ◀

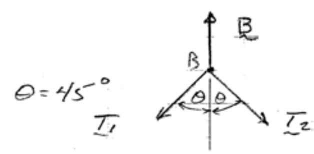


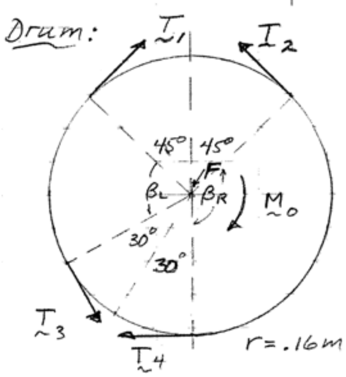
PROBLEM 8.124

For the band brake shown, the maximum allowed tension in either belt is 5.6 kN. Knowing that the coefficient of static friction between the belt and the 160-mm-radius drum is 0.25, determine (*a*) the largest clockwise moment $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force **P** exerted on end *E* of the lever.

SOLUTION

FBD pin B:





(*a*) By symmetry: $T_1 = T_2$

$$\uparrow \Sigma F_y = 0: \quad B - 2\left(\frac{\sqrt{2}}{2}T_1\right) = 0 \qquad \text{or} \qquad B = \sqrt{2}T_1 = \sqrt{2}T_2 \qquad (1)$$

For impending rotation ↻:

$$T_3 > T_1 = T_2 > T_4, \text{ so } T_3 = T_{\max} = 5.6 \text{ kN}$$

Then $$T_1 = T_3 e^{-\mu_s \beta_L} = (5.6 \text{ kN}) e^{-0.25\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}$$

or $$T_1 = 4.03706 \text{ kN} = T_2$$

and $$T_4 = T_2 e^{-\mu_s \beta_R} = (4.03706 \text{ kN}) e^{-0.25\left(\frac{3\pi}{4}\right)}$$

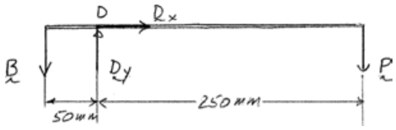
or $$T_4 = 2.23998 \text{ kN}$$

$$\circlearrowleft \Sigma M_F = 0: \quad M_0 + r(T_4 - T_3 + T_2 - T_1) = 0$$

or $$M_0 = (0.16 \text{ m})(5.6 \text{ kN} - 2.23998 \text{ kN}) = 0.5376 \text{ kN}\cdot\text{m}$$

$\mathbf{M}_0 = 538 \text{ N}\cdot\text{m}$ ↻ ◀

Lever:



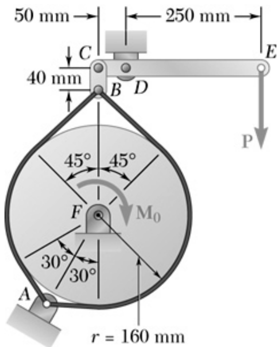
(*b*) Using Equation (1)

$$B = \sqrt{2}T_1 = \sqrt{2}(4.03706 \text{ kN})$$
$$= 5.70927 \text{ kN}$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.05 \text{ m})(5.70927 \text{ kN}) - (0.25 \text{ m})P = 0$$

$\mathbf{P} = 1.142 \text{ kN}\downarrow$ ◀

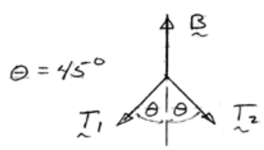
PROBLEM 8.125

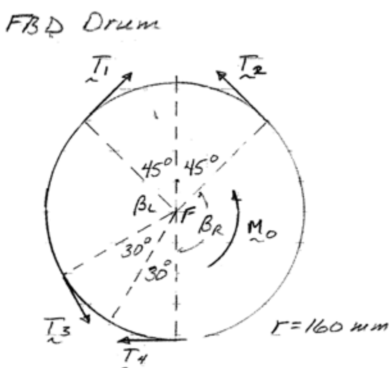


Solve Problem 8.124 assuming that a counterclockwise moment is applied to the drum.

SOLUTION

FBD pin B:





(*a*) By symmetry: $T_1 = T_2$

$$\uparrow \Sigma F_y = 0: \quad B - 2\left(\frac{\sqrt{2}}{2}T_1\right) = 0 \qquad \text{or} \qquad B = \sqrt{2}T_1 \qquad (1)$$

For impending rotation ↺:

$$T_4 > T_2 = T_1 > T_3, \text{ so } T_4 = T_{\max} = 5.6 \text{ kN}$$

Then $\quad T_2 = T_4 e^{-\mu_s \beta_R} = (5.6 \text{ kN})e^{-0.25\left(\frac{3\pi}{4}\right)}$

or $\quad T_2 = 3.10719 \text{ kN} = T_1$

and $\quad T_3 = T_1 e^{-\mu_s \beta_L} = (3.10719 \text{ kN})e^{-0.25\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}$

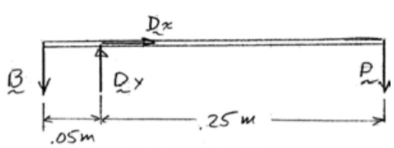
or $\quad T_3 = 2.23999 \text{ kN}$

$$\circlearrowleft \Sigma M_F = 0: \quad M_0 + r(T_2 - T_1 + T_3 - T_4) = 0$$

$$M_0 = (160 \text{ mm})(5.6 \text{ kN} - 2.23999 \text{ kN}) = 537.6 \text{ N}\cdot\text{m}$$

$$\mathbf{M}_0 = 538 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

FBD Lever:



(*b*) Using Equation (1)

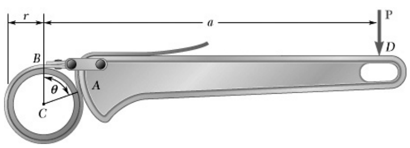
$$B = \sqrt{2}T_1 = \sqrt{2}(3.10719 \text{ kN})$$

$$B = 4.3942 \text{ kN}$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.05 \text{ m})(4.3942 \text{ kN}) - (0.25 \text{ m})P = 0$$

$$\mathbf{P} = 879 \text{ N} \downarrow \blacktriangleleft$$

PROBLEM 8.126

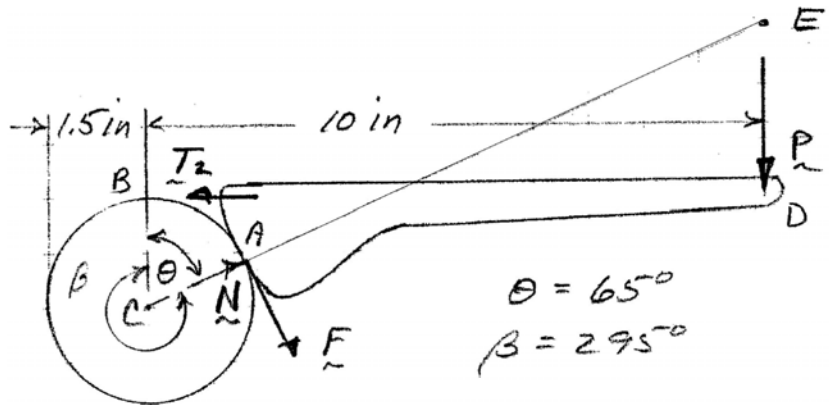


The strap wrench shown is used to grip the pipe firmly without marring the surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 10$ in., $r = 1.5$ in., and $\theta = 65^\circ$.

SOLUTION

For the wrench to be self-locking, friction must be sufficient to maintain equilibrium as P is increased from zero to $P_{\max}$, as well as to prevent slipping of the belt on the pipe.

FBD wrench:



$$\circlearrowleft \Sigma M_E = 0: \quad \left(\frac{10\text{ in.}}{\sin 65^\circ} - 1.5\text{ in.}\right)F - \left(\frac{10\text{ in.}}{\tan 65^\circ} - 1.5\text{ in.}\right)T_2 = 0$$

$$9.5338F = 3.1631T_2 \qquad \text{or} \qquad 3.01406 = \frac{T_2}{F} \tag{1}$$

$$\longrightarrow \Sigma F_x = 0: \quad -T_2 + N\sin 65^\circ + F\cos 65^\circ = 0$$

Impending slipping: $\qquad N = F/\mu_s$

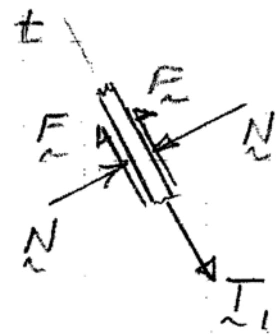
So $\qquad F\left(\frac{\sin 65^\circ}{\mu_s} + \cos 65^\circ\right) = T_2$

or $\qquad \frac{0.90631}{\mu_s} + 0.42262 = \frac{T_2}{F} \qquad (2)$

Solving Equations (1) and (2) yields $\mu_s = 0.3497;$ must still check belt on pipe.

PROBLEM 8.126 CONTINUED

Small portion of belt at A:



$$\nwarrow \Sigma F_t = 0: \quad 2F - T_1 = 0$$

or $$T_1 = 2F$$

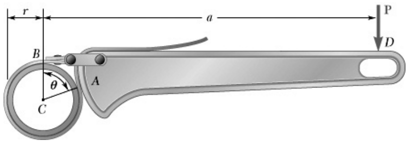
Belt impending slipping: $$\ln \frac{T_2}{T_1} = \mu_s \beta$$

So $$\mu_s = \frac{1}{\beta} \ln \frac{T_2}{T_1} = \frac{1}{\beta} \ln \frac{T_2}{2F}$$

Using Equation (1) $$\mu_s = \frac{180}{295\pi} \ln 1.50703$$

$$= 0.0797$$

$\therefore$ for self-locking, need $\mu_s = 0.350$ ◀



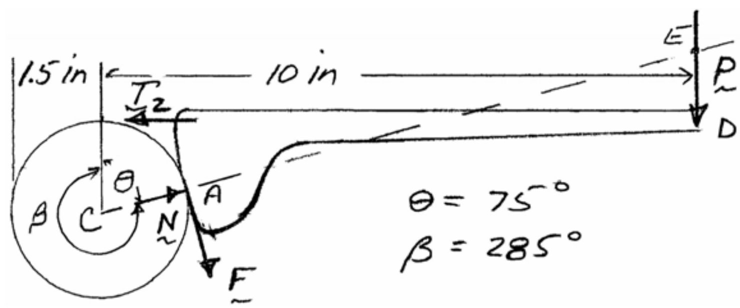
PROBLEM 8.127

Solve Problem 8.126 assuming that $\theta = 75^{\circ}$.

SOLUTION

For the wrench to be self-locking, friction must be sufficient to maintain equilibrium as P is increased from zero to $P_{\max}$, as well as to prevent slipping of the belt on the pipe.

FBD wrench:



$$\circlearrowleft \Sigma M_E = 0: \quad \left(\frac{10 \text{ in.}}{\sin 75^{\circ}} - 1.5 \text{ in.}\right)F - \left(\frac{10 \text{ in.}}{\tan 75^{\circ}} - 1.5 \text{ in.}\right)T_2 = 0$$

or $$\frac{T_2}{F} = 7.5056 \qquad (1)$$

$$\rightarrow \Sigma F_x = 0: \quad -T_2 + N\sin 75^{\circ} + F\cos 75^{\circ} = 0$$

Impending slipping: $$N = F/\mu_s$$

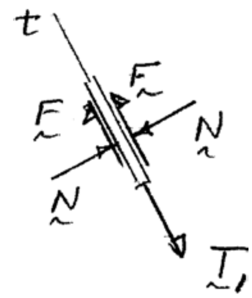
So $$F\left(\frac{\sin 75^{\circ}}{\mu_s} + \cos 75^{\circ}\right) = T_2$$

or $$\frac{T_2}{F} = \frac{0.96593}{\mu_s} + 0.25882 \qquad (2)$$

Solving Equations (1) and (2): $\mu_s = 0.13329$; must still check belt on pipe.

PROBLEM 8.127 CONTINUED

Small portion of belt at A:



$$\nwarrow \Sigma F_t = 0: \quad 2F - T_1 = 0$$

or $$T_1 = 2F$$

Impending belt slipping: $$\ln \frac{T_2}{T_1} = \mu_s \beta$$

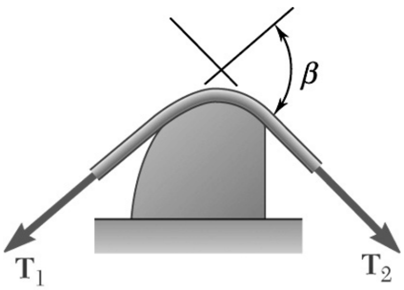
So $$\mu_s = \frac{1}{\beta} \ln \frac{T_2}{T_1} = \frac{1}{\beta} \ln \frac{T_2}{2F}$$

Using Equation (1): $$\mu_s = \frac{180}{285\pi} \ln \frac{7.5056}{2}$$

$$= 0.2659$$

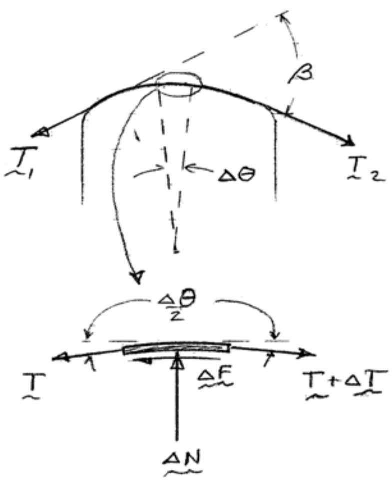
$\therefore$ for self-locking, $\mu_s = 0.266$ ◀

PROBLEM 8.128



Prove that Equatins (8.13) and (8.14) are valid for any shape of surface provided that the coefficient of friction is the same at all points of contact.

SOLUTION



$$\uparrow \Sigma F_n = 0: \quad \Delta N - \left[T + \left(T + \Delta T\right)\right]\sin\frac{\Delta\theta}{2} = 0$$

or $$\Delta N = \left(2T + \Delta T\right)\sin\frac{\Delta\theta}{2}$$

$$\longrightarrow \Sigma F_t = 0: \quad \left[\left(T + \Delta T\right) - T\right]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

or $$\Delta F = \Delta T\cos\frac{\Delta\theta}{2}$$

Impending slipping: $$\Delta F = \mu_s \Delta N$$

So $$\Delta T\cos\frac{\Delta\theta}{2} = \mu_s 2T\sin\frac{\Delta\theta}{2} + \mu_s\Delta T\frac{\sin\Delta\theta}{2}$$

In limit as $\Delta\theta \to 0: \quad dT = \mu_s T d\theta, \qquad$ or $\qquad \dfrac{dT}{T} = \mu_s d\theta$

So $$\int_{T_1}^{T_2}\frac{dT}{T} = \int_0^\beta \mu_s d\theta;$$

and $$\ln\frac{T_2}{T_1} = \mu_s\beta$$

or $T_2 = T_1 e^{\mu_s\beta}$ ◀

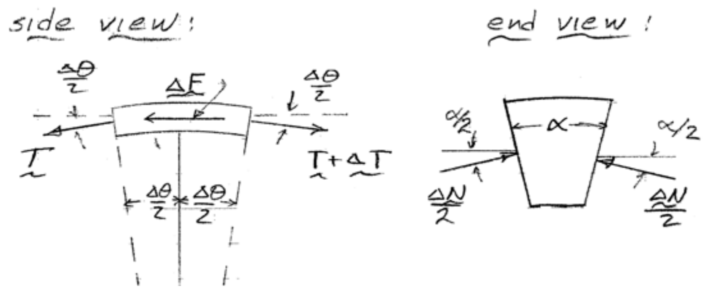
Note: Nothing above depends on the shape of the surface, except it is assumed smooth.

PROBLEM 8.129

Complete the derivation of Equation (8.15), which relates the tension in both parts of a V belt.

SOLUTION

Small belt section:



$$\uparrow \Sigma F_y = 0: \quad 2\frac{\Delta N}{2}\sin\frac{\alpha}{2} - \left[T + (T + \Delta T)\right]\sin\frac{\Delta\theta}{2} = 0$$

$$\longrightarrow \Sigma F_x = 0: \quad \left[(T + \Delta T) - T\right]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

Impending slipping:

$$\Delta F = \mu_s \Delta N \Rightarrow \Delta T \cos\frac{\Delta\theta}{2} = \mu_s \frac{2T + \Delta T}{\sin\frac{\alpha}{2}} \sin\frac{\Delta\theta}{2}$$

In limit as $\Delta\theta \to 0$:

$$dT = \frac{\mu_s T d\theta}{\sin\frac{\alpha}{2}} \qquad \text{or} \qquad \frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}} d\theta$$

So

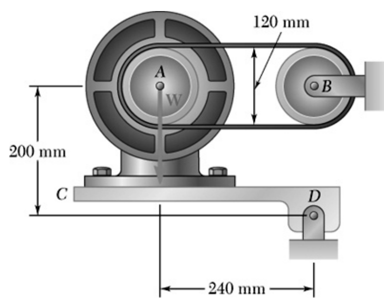
$$\int_{T_1}^{T_2} \frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}} \int_0^{\beta} d\theta$$

or

$$\ln\frac{T_2}{T_1} = \frac{\mu_s \beta}{\sin\frac{\alpha}{2}}$$

or

$$T_2 = T_1 e^{\mu_s \beta / \sin\frac{\alpha}{2}} \blacktriangleleft$$



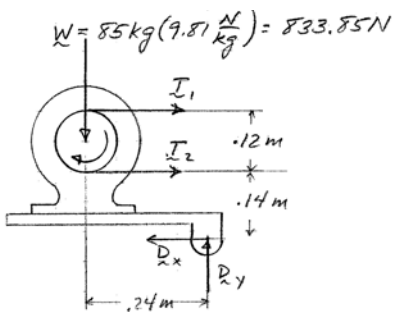
PROBLEM 8.130

Solve Problem 8.107 assuming that the flat belt and drums are replaced by a V belt and V pulleys with $\alpha = 36^\circ$. (The angle α is as shown in Figure 8.15*a*.)

SOLUTION

FBD motor + mount:

$$\circlearrowleft \Sigma M_D = 0: \quad (0.24\text{ m})W - (0.26\text{ m})T_1 - (0.14\text{ m})T_2 = 0$$



Impending slipping: $\quad T_2 = T_1 e^{\mu_S \beta / \sin\frac{\alpha}{2}}$

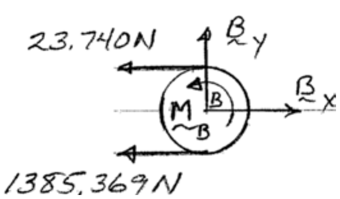
$$T_2 = T_1 e^{\frac{0.4\pi}{\sin 18^\circ}} = 58.356 T_1$$

Thus $\quad (0.24\text{ m})(833.85\text{ N}) - \left[0.26\text{ m} + (0.14\text{ m})(58.356)\right]T_1 = 0$

$$T_1 = 23.740\text{ N}$$

$$T_2 = 1385.369\text{ N}$$

FBD Drum:

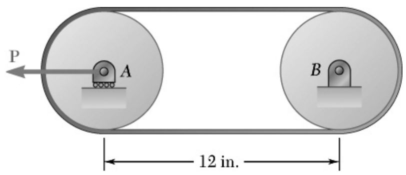


$$\circlearrowleft \Sigma M_B = 0: \quad M_B + (0.06\text{ m})(23.740\text{ N} - 1385.369\text{ N}) = 0$$

$$M_B = 81.7\text{ N}\cdot\text{m} \blacktriangleleft$$

(Compare to $M_B = 40.1\text{ N}\cdot\text{m}$ using flat belt, Problem 8.107.)

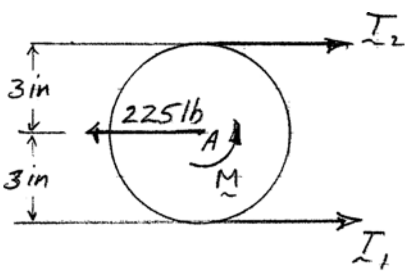
PROBLEM 8.131



Solve Problem 8.109 assuming that the flat belt and drums are replaced by a V belt and V pulleys with $\alpha = 36^{\circ}$. (The angle α is as shown in Figure 8.15*a*.)

SOLUTION

FBD pulley A:



Impending slipping: $T_2 = T_1 e^{\mu_s \beta / \sin\frac{\alpha}{2}}$

$$T_2 = T_1 e^{0.35\pi/\sin 18^{\circ}} = 35.1015 T_1$$

$$\longrightarrow \Sigma F_x = 0: \quad T_1 + T_2 - 225 \text{ lb} = 0$$

$$T_1(1 + 35.1015) = 225 \text{ lb}$$

So $T_1 = 6.2324 \text{ lb}$

$$T_2 = 218.768 \text{ lb} = T_{\max}$$

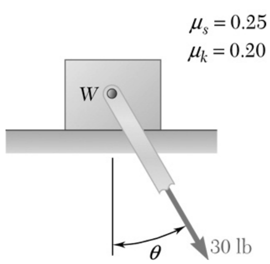
$$\circlearrowleft \Sigma M_A = 0: \quad M + (3 \text{ in.})(T_1 - T_2) = 0$$

$$M = (3 \text{ in.})(218.768 \text{ lb} - 6.232 \text{ lb})$$

(*a*) $M = 638 \text{ lb}\cdot\text{in.}$ ◀

(Compare to $338 \text{ lb}\cdot\text{in.}$ with flat belt, Problem 8.109.)

(*b*) $T_{\max} = 219 \text{ lb}$ ◀

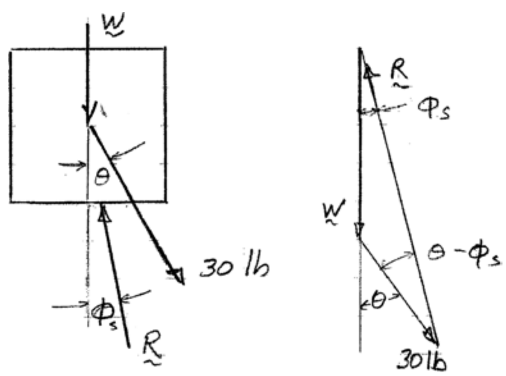


PROBLEM 8.132

Considering only values of θ less than $90°$, determine the smallest value of θ required to start the block moving to the right when (*a*) $W = 75$ lb, (*b*) $W = 100$ lb.

SOLUTION

FBD block: (motion impending)



$$\phi_s = \tan^{-1}\mu_s = 14.036°$$

$$\frac{30\text{ lb}}{\sin\phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

$$\sin(\theta - \phi_s) = \frac{W \sin 14.036°}{30\text{ lb}}$$

or

$$\sin(\theta - 14.036°) = \frac{W}{123.695\text{ lb}}$$

(*a*)

$$W = 75\text{ lb:}\quad \theta = 14.036° + \sin^{-1}\frac{75\text{ lb}}{123.695\text{ lb}}$$

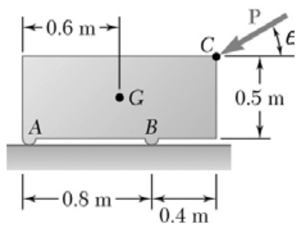
$\theta = 51.4°$ ◀

(*b*)

$$W = 100\text{ lb:}\quad \theta = 14.036° + \sin^{-1}\frac{100\text{ lb}}{123.695\text{ lb}}$$

$\theta = 68.0°$ ◀

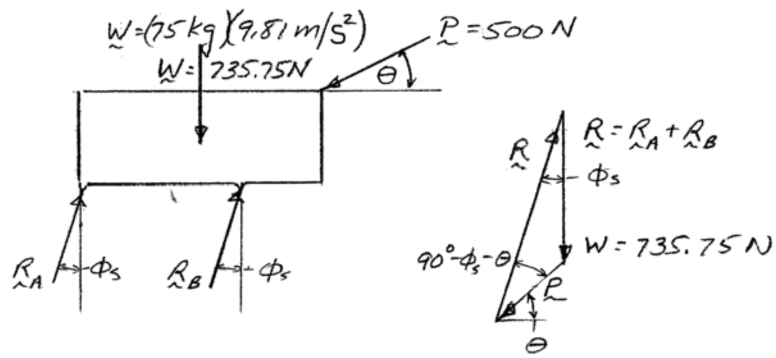
PROBLEM 8.133



The machine base shown has a mass of 75 kg and is fitted with skids at A and B. The coefficient of static friction between the skids and the floor is 0.30. If a force **P** of magnitude 500 N is applied at corner C, determine the range of values of θ for which the base will not move.

SOLUTION

FBD machine base (slip impending):



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.699°$$

$$\frac{W}{\sin(90° - \phi_s - \theta)} = \frac{P}{\sin\phi_s}$$

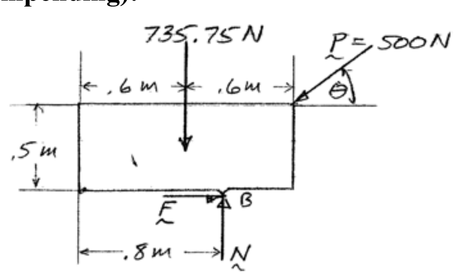
$$\sin(90° - \phi_s - \theta) = \frac{W\sin 16.699°}{P}$$

$$90° - 16.699° - \theta = \sin^{-1}\left[\frac{735.75 \text{ lb}}{500 \text{ lb}}(0.28734)\right]$$

$$\theta = 73.301° - 25.013°$$

$$\theta = 48.3°$$

FBD machine base (tip about B impending):



$$\circlearrowleft \Sigma M_B = 0: \quad (0.2 \text{ m})(735.75 \text{ N}) + (0.5 \text{ m})\left[(500 \text{ N})\cos\theta\right]$$

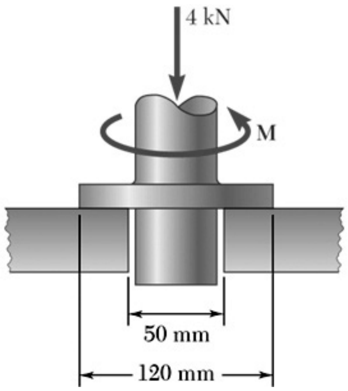
$$-(0.4 \text{ m})\left[(500 \text{ N})\sin\theta\right] = 0$$

$$0.8\sin\theta - \cos\theta = 0.5886$$

Solving numerically $\quad \theta = 78.7°$

So, for equilibrium $\quad 48.3° \le \theta \le 78.7°$ ◀

PROBLEM 8.134



Knowing that a couple of magnitude $30\ \text{N}\cdot\text{m}$ is required to start the vertical shaft rotating, determine the coefficient of static friction between the annular surfaces of contact.

SOLUTION

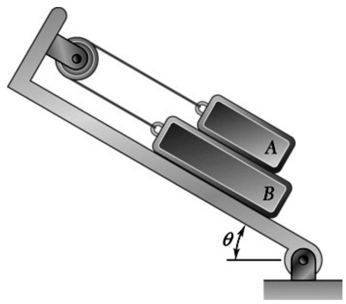
For annular contact regions, use Equation 8.8 with impending slipping:

$$M = \frac{2}{3}\mu_s N \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

So,

$$30\ \text{N}\cdot\text{m} = \frac{2}{3}\mu_s(4000\ \text{N})\frac{(0.06\ \text{m})^3 - (0.025\ \text{m})^3}{(0.06\ \text{m})^2 - (0.025\ \text{m})^2}$$

$\mu_s = 0.1670$ ◀



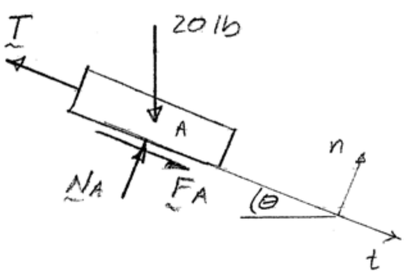
PROBLEM 8.135

The 20-lb block A and the 30-lb block B are supported by an incline which is held in the position shown. Knowing that the coefficient of static friction is 0.15 between the two blocks and zero between block B and the incline, determine the value of θ for which motion is impending.

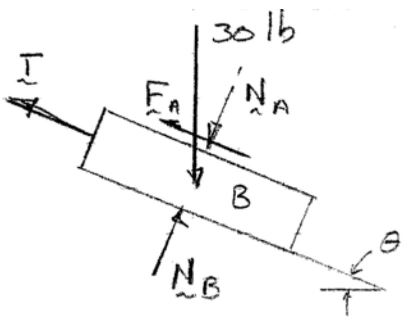
SOLUTION

FBD's

Block A:



Block B:



A: $\nearrow \Sigma F_n = 0$: $N_A - (20\text{ lb})\cos\theta = 0$ or $N_A = (20\text{ lb})\cos\theta$

B: $\nearrow \Sigma F_n = 0$: $N_B - N_A - (30\text{ lb})\cos\theta = 0$

or $$N_B = N_A + (30\text{ lb})\cos\theta = (50\text{ lb})\cos\theta$$

Impending motion at all surfaces:

$$F_A = \mu_s N_A$$

$$= 0.15(20\text{ lb})\cos\theta$$

$$= (3\text{ lb})\cos\theta$$

A: $\searrow \Sigma F_t = 0$: $F_A + (20\text{ lb})\sin\theta - T = 0$

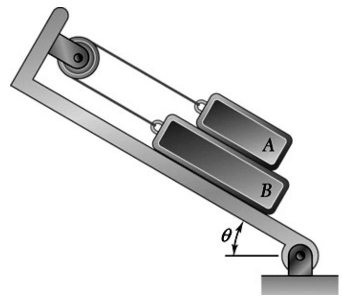
B: $\searrow \Sigma F_t = 0$: $-F_A + (30\text{ lb})\sin\theta - T = 0$

So $$(10\text{ lb})\sin\theta - 2F_A = 0$$

$$(10\text{ lb})\sin\theta = 2(3\text{ lb})\cos\theta$$

$$\theta = \tan^{-1}\frac{6\text{ lb}}{10\text{ lb}} = 30.96°$$

$\theta = 31.0°$ ◀

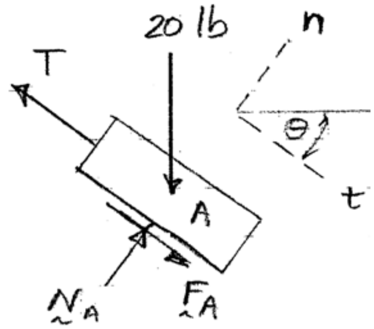


PROBLEM 8.136

The 20-lb block *A* and the 30-lb block *B* are supported by an incline which is held in the position shown. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

FBD's
Block A:



A: ↗ $\Sigma F_n = 0$: $N_A - (20\text{ lb})\cos\theta = 0$ or $N_A = (20\text{ lb})\cos\theta$

B: ↗ $\Sigma F_n = 0$: $N_B - N_A - (30\text{ lb})\cos\theta = 0$

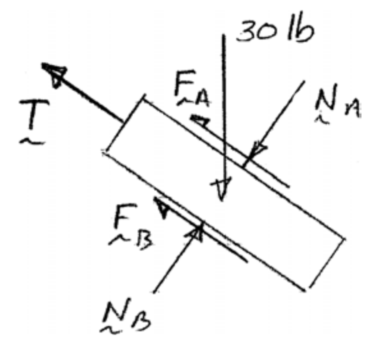
or $$N_B = N_A + (30\text{ lb})\cos\theta = (50\text{ lb})\cos\theta$$

Impending motion at all surfaces; *B* impends ↘:

$$F_A = \mu_s N_A = (0.15)(20\text{ lb})\cos\theta = (3\text{ lb})\cos\theta$$

$$F_B = \mu_s N_B = (0.15)(50\text{ lb})\cos\theta = (7.5\text{ lb})\cos\theta$$

Block B:



A: ↘ $\Sigma F_t = 0$: $(20\text{ lb})\sin\theta + F_A - T = 0$

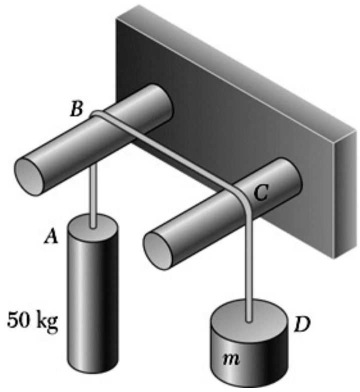
B: ↘ $\Sigma F_t = 0$: $(30\text{ lb})\sin\theta - F_A - F_B - T = 0$

So $$(10\text{ lb})\sin\theta - 2F_A - F_B = 0$$

$$(10\text{ lb})\sin\theta = 2(3\text{ lb})\cos\theta + (7.5\text{ lb})\cos\theta$$

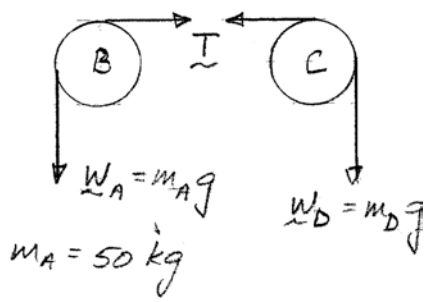
$$\tan\theta = \frac{13.5\text{ lb}}{10\text{ lb}} = 1.35;$$ $\theta = 53.5°$ ◀

PROBLEM 8.137



Two cylinders are connected by a rope that passes over two fixed rods as shown. Knowing that the coefficient of static friction between the rope and the rods is 0.40, determine the range of values of the mass *m* of cylinder *D* for which equilibrium is maintained.

SOLUTION



For impending motion of *A* up: $$T = W_A e^{\mu_s \beta_B}$$

and $$W_D = Te^{\mu_s \beta_C} = W_A e^{\mu_s(\beta_B + \beta_C)}$$

or $$m_D g = (50 \text{ kg}) g e^{0.4\left(\frac{\pi}{2} + \frac{\pi}{2}\right)}$$

$$m_D = 175.7 \text{ kg}$$

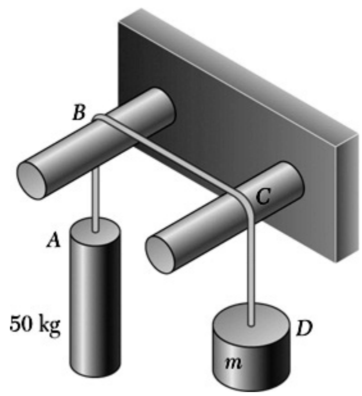
For impending motion of *A* down, the tension ratios are inverted, so

$$W_A = W_D e^{\mu_s(\beta_C + \beta_B)}$$

$$(50 \text{ kg}) g = m_D g e^{0.4\left(\frac{\pi}{2} + \frac{\pi}{2}\right)}$$

$$m_D = 14.23 \text{ kg}$$

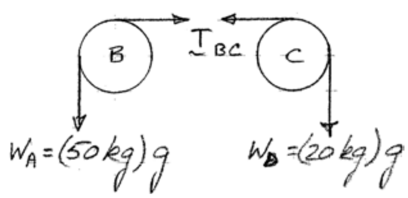
For equilibrium: $14.23 \text{ kg} \le m_D \le 175.7 \text{ kg}$ ◀



PROBLEM 8.138

Two cylinders are connected by a rope that passes over two fixed rods as shown. Knowing that for cylinder D upward motion impends when $m = 20\text{ kg}$, determine (*a*) the coefficient of static friction between the rope and the rods, (*b*) the corresponding tension in portion BC of the rope.

SOLUTION



(*a*) Motion of D impends upward, so

$$T_{BC} = W_D e^{\mu_s \beta_C} \qquad (1)$$

$$W_A = T_{BC} e^{\mu_s \beta_B} = W_D e^{\mu_s (\beta_C + \beta_B)}$$

So

$$\mu_s \left(\frac{\pi}{2} + \frac{\pi}{2} \right) = \ln \frac{W_A}{W_D} = \ln \left(\frac{50\text{ kg}}{20\text{ kg}} \right)$$

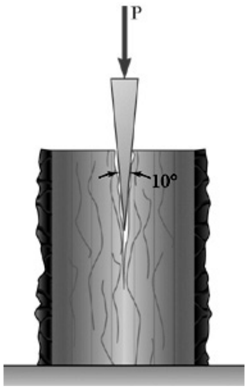
$$\mu_s = 0.29166$$

$$\mu_s = 0.292 \blacktriangleleft$$

(*b*) From Equation (1):

$$T_{BC} = (20\text{ kg})\left(9.81\text{ m/s}^2\right) e^{0.29166\,\pi/2}$$

$$T_{BC} = 310\text{ N} \blacktriangleleft$$

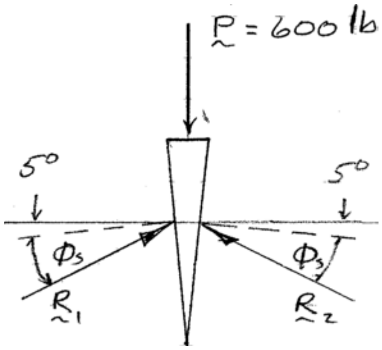


PROBLEM 8.139

A 10° wedge is used to split a section of a log. The coefficient of static friction between the wedge and the log is 0.35. Knowing that a force **P** of magnitude 600 lb was required to insert the wedge, determine the magnitude of the forces exerted on the wood by the wedge after insertion.

SOLUTION

FBD wedge (impending motion ↓):



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.35 = 19.29°$$

By symmetry: $R_1 = R_2$

$$\uparrow \Sigma F_y = 0: \quad 2R_1\sin(5° + \phi_s) - 600 \text{ lb} = 0$$

or
$$R_1 = R_2 = \frac{300 \text{ lb}}{\sin(5° + 19.29°)} = 729.30 \text{ lb}$$

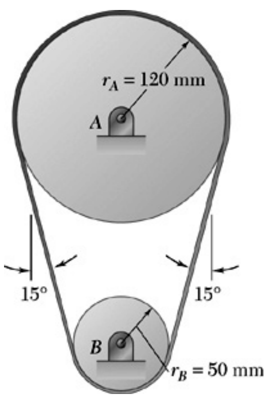
When P is removed, the vertical components of R_1 and R_2 vanish, leaving the horizontal components

$$R_{1x} = R_{2x} = R_1\cos(5° + \phi_s)$$

$$= (729.30 \text{ lb})\cos(5° + 19.29°)$$

$$R_{1x} = R_{2x} = 665 \text{ lb} ◀$$

(Note that $\phi_s > 5°$, so wedge is self-locking.)

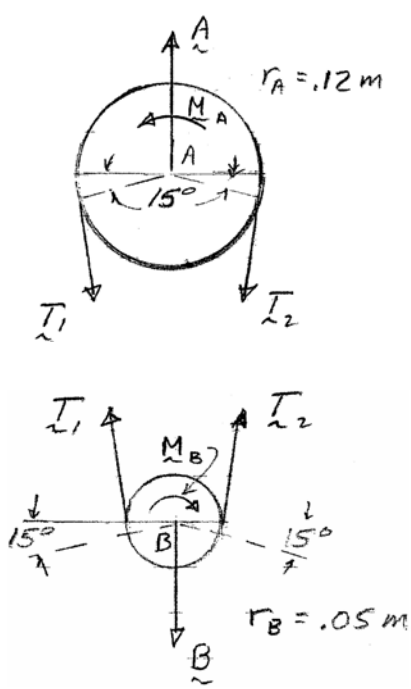


PROBLEM 8.140

A flat belt is used to transmit a torque from drum B to drum A. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest torque that can be exerted on drum A.

SOLUTION

FBD's drums:



$$\beta_A = 180° + 30° = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

$$\beta_B = 180° - 30° = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Since $\beta_B < \beta_A$, slipping will impend first on B (friction coefficients being equal)

So $$T_2 = T_{\max} = T_1 e^{\mu_s \beta_B}$$

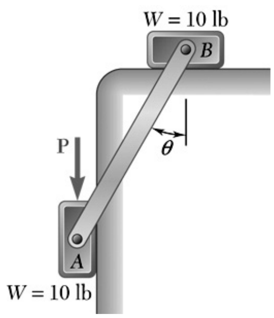
$$450\text{ N} = T_1 e^{(0.4)5\pi/6} \qquad \text{or} \qquad T_1 = 157.914\text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: \quad M_A + (0.12\text{ m})(T_1 - T_2) = 0$$

$$M_A = (0.12\text{ m})(450\text{ N} - 157.914\text{ N}) = 35.05\text{ N}\cdot\text{m}$$

$$M_A = 35.1\text{ N}\cdot\text{m} \blacktriangleleft$$

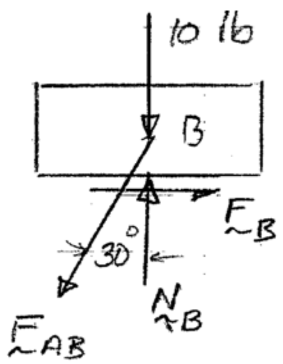
PROBLEM 8.141



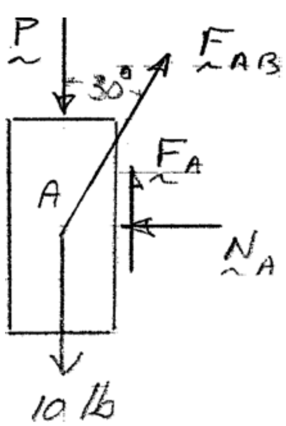
Two 10-lb blocks A and B are connected by a slender rod of negligible weight. The coefficient of static friction is 0.30 between all surfaces of contact, and the rod forms an angle $\theta = 30°$ with the vertical. (a) Show that the system is in equilibrium when $P = 0$. (b) Determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD block B:



FBD block A:



(b) For P_{max}, motion impends at both surfaces

B: $\uparrow \Sigma F_y = 0$: $N_B - 10 \text{ lb} - F_{AB}\cos 30° = 0$

$$N_B = 10 \text{ lb} + \frac{\sqrt{3}}{2}F_{AB} \qquad (1)$$

Impending motion: $F_B = \mu_s N_B = 0.3N_B$

$\longrightarrow \Sigma F_x = 0$: $F_B - F_{AB}\sin 30° = 0$

$$F_{AB} = 2F_B = 0.6N_B \qquad (2)$$

Solving (1) and (2)

$$N_B = 10 \text{ lb} + \frac{\sqrt{3}}{2}(0.6N_B)$$

$$= 20.8166 \text{ lb}$$

Then $F_{AB} = 0.6N_B = 12.4900 \text{ lb}$

A: $\longrightarrow \Sigma F_x = 0$: $F_{AB}\sin 30° - N_A = 0$

$$N_A = \frac{1}{2}F_{AB} = \frac{1}{2}(12.4900 \text{ lb}) = 6.2450 \text{ lb}$$

Impending motion: $F_A = \mu_s N_A = 0.3(6.2450 \text{ lb}) = 1.8735 \text{ lb}$

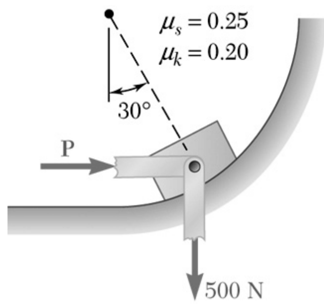
$\uparrow \Sigma F_y = 0$: $F_A + F_{AB}\cos 30° - P - 10 \text{ lb} = 0$

$$P = F_A + \frac{\sqrt{3}}{2}F_{AB} - 10 \text{ lb}$$

$$= 1.8735 \text{ lb} + \frac{\sqrt{3}}{2}(12.4900 \text{ lb}) - 10 \text{ lb} = 2.69 \text{ lb}$$

$P = 2.69$ lb ◀

(a) Since $P = 2.69$ lb to initiate motion,
equilibrium exists with $P = 0$ ◀

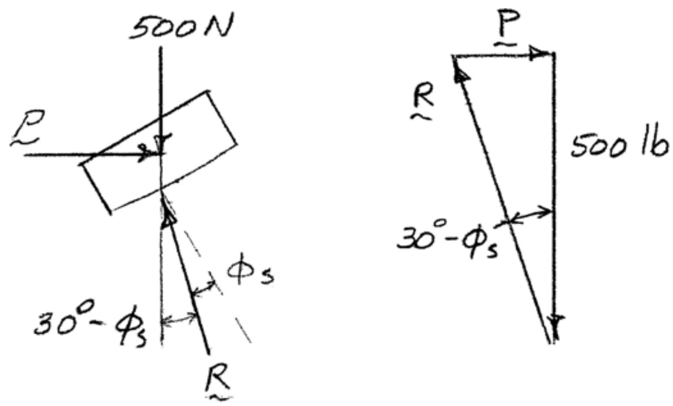


PROBLEM 8.142

Determine the range of values of P for which equilibrium of the block shown is maintained.

SOLUTION

FBD block (Impending motion down):

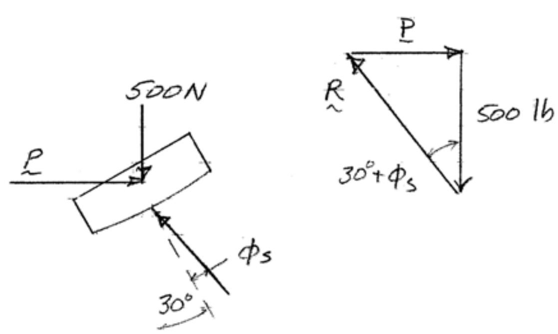


$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25$$

$$P = (500\text{ lb})\tan\left(30° - \tan^{-1}0.25\right)$$

$$= 143.03\text{ lb}$$

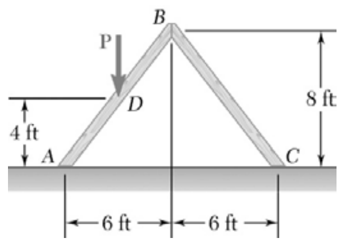
(Impending motion up):



$$P = (500\text{ lb})\tan\left(30° + \tan^{-1}0.25\right)$$

$$= 483.46\text{ lb}$$

Equilibrium for 143.0 lb $\le P \le$ 483 lb ◀

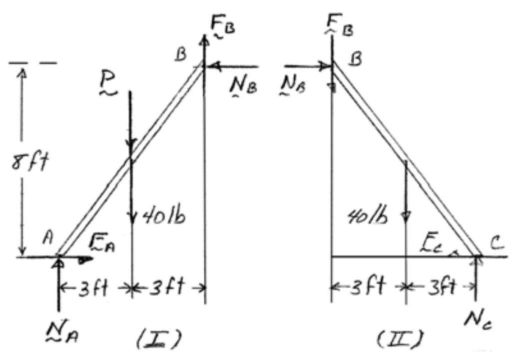


PROBLEM 8.143

Two identical uniform boards, each of weight 40 lb, are temporarily leaned against each other as shown. Knowing that the coefficient of static friction between all surfaces is 0.40, determine (*a*) the largest magnitude of the force **P** for which equilibrium will be maintained, (*b*) the surface at which motion will impend.

SOLUTION

Board FBDs:



Assume impending motion at *C*, so $F_C = \mu_s N_C$

$$= 0.4N_C$$

FBD II: $\circlearrowleft \Sigma M_B = 0: \quad (6 \text{ ft})N_C - (8 \text{ ft})F_C - (3 \text{ ft})(40 \text{ lb}) = 0$

$$\left[6 \text{ ft} - 0.4(8 \text{ ft})\right]N_C = (3 \text{ ft})(40 \text{ lb})$$

or $N_C = 42.857 \text{ lb}$

and $F_C = 0.4N_C = 17.143 \text{ lb}$

$$\rightarrow \Sigma F_x = 0: \quad N_B - F_C = 0$$

$$N_B = F_C = 17.143 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -F_B - 40 \text{ lb} + N_C = 0$$

$$F_B = N_C - 40 \text{ lb} = 2.857 \text{ lb}$$

Check for motion at *B*: $\dfrac{F_B}{N_B} = \dfrac{2.857 \text{ lb}}{17.143 \text{ lb}} = 0.167 < \mu_s,$ OK, no motion.

PROBLEM 8.143 CONTINUED

FBD I: $\circlearrowleft \Sigma M_A = 0: \quad (8 \text{ ft})N_B + (6 \text{ ft})F_B - (3 \text{ ft})(P + 40 \text{ lb}) = 0$

$$P = \frac{(8 \text{ ft})(17.143 \text{ lb}) + (6 \text{ ft})(2.857 \text{ lb})}{3 \text{ ft}} - 40 \text{ lb} = 11.429 \text{ lb}$$

Check for slip at A (unlikely because of P)

$$\longrightarrow \Sigma F_x = 0: \quad F_A - N_B = 0 \qquad \text{or} \qquad F_A = N_B = 17.143 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad N_A - P - 40 \text{ lb} + F_B = 0 \qquad \text{or} \qquad N_A = 11.429 \text{ lb} + 40 \text{ lb} - 2.857 \text{ lb}$$

$$= 48.572 \text{ lb}$$

Then $\dfrac{F_A}{N_A} = \dfrac{17.143 \text{ lb}}{48.572 \text{ lb}} = 0.353 < \mu_s,$ OK, no slip $\Rightarrow$ assumption is correct

Therefore, (*a*) $P_{max} = 11.43$ lb ◀

(*b*) Motion impends at C ◀