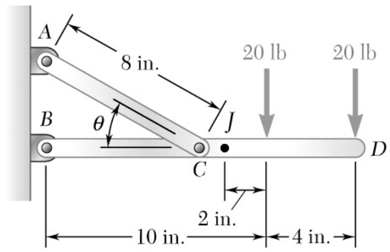
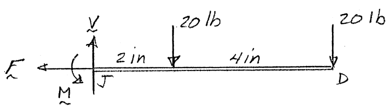


PROBLEM 7.1

Determine the internal forces (axial force, shearing force, and bending moment) at point J of the structure indicated:
Frame and loading of Prob. 6.77.

SOLUTION

FBD JD:



$$\rightarrow \Sigma F_x = 0: -F = 0$$

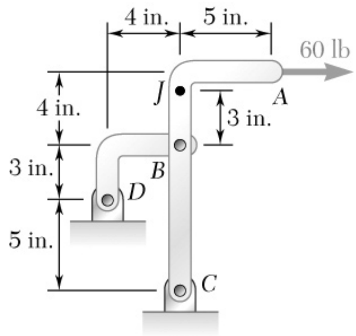
$\mathbf{F} = 0$ ◀

$$\uparrow \Sigma F_y = 0: V - 20\text{ lb} - 20\text{ lb} = 0$$

$\mathbf{V} = 40.0\text{ lb} \uparrow$ ◀

$$\circlearrowleft \Sigma M_J = 0: M - (2\text{ in.})(20\text{ lb}) - (6\text{ in.})(20\text{ lb}) = 0$$

$\mathbf{M} = 160.0\text{ lb}\cdot\text{in.} \circlearrowleft$ ◀

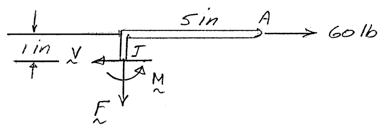


PROBLEM 7.2

Determine the internal forces (axial force, shearing force, and bending moment) at point *J* of the structure indicated:
Frame and loading of Prob. 6.76.

SOLUTION

FBD AJ:



$\xrightarrow{} \Sigma F_x = 0{:}\ 60\text{ lb} - V = 0$

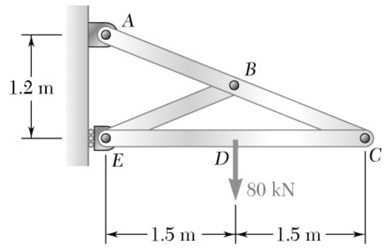
$\mathbf{V} = 60.0\text{ lb} \longleftarrow$ ◀

$\uparrow \Sigma F_y = 0{:}\ -F = 0$

$\mathbf{F} = 0$ ◀

$\circlearrowleft \Sigma M_J = 0{:}\ M - (1\text{ in.})(60\text{ lb}) = 0$

$\mathbf{M} = 60.0\text{ lb}\cdot\text{in.}$ ↻ ◀

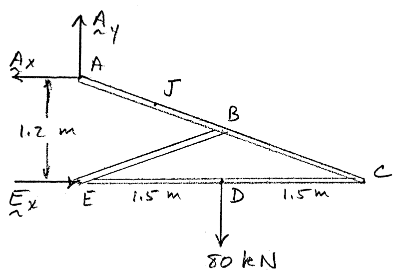


PROBLEM 7.3

For the frame and loading of Prob. 6.80, determine the internal forces at a point *J* located halfway between points *A* and *B*.

SOLUTION

FBD Frame:



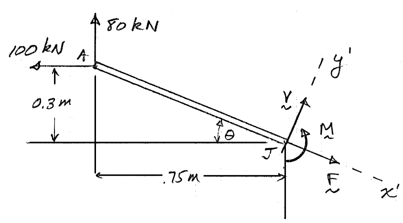
$$\longrightarrow \Sigma F_y = 0:\ A_y - 80\text{ kN} = 0 \qquad \mathbf{A}_y = 80\text{ kN} \uparrow$$

$$\circlearrowleft\ \Sigma M_E = 0:\ (1.2\text{ m})A_x - (1.5\text{ m})(80\text{ kN}) = 0$$

$$\mathbf{A}_x = 100\text{ kN} \longleftarrow$$

$$\theta = \tan^{-1}\left(\frac{0.3\text{ m}}{0.75\text{ m}}\right) = 21.801°$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0:\ F - (80\text{ kN})\sin 21.801° - (100\text{ kN})\cos 21.801° = 0$$

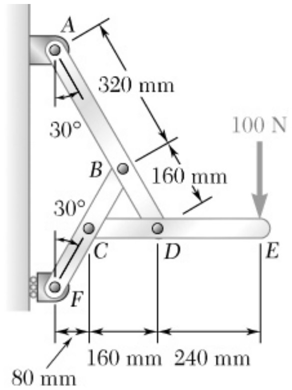
$$\mathbf{F} = 122.6\text{ kN} \searrow \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0:\ V + (80\text{ kN})\cos 21.801° - (100\text{ kN})\sin 21.801° = 0$$

$$\mathbf{V} = 37.1\text{ kN} \swarrow \blacktriangleleft$$

$$\circlearrowleft\ \Sigma M_J = 0:\ M + (.3\text{ m})(100\text{ kN}) - (.75\text{ m})(80\text{ kN}) = 0$$

$$\mathbf{M} = 30.0\text{ kN}\cdot\text{m} \circlearrowright \blacktriangleleft$$

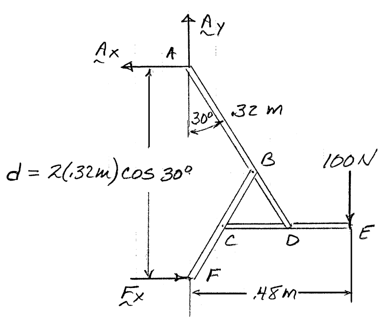


PROBLEM 7.4

For the frame and loading of Prob. 6.101, determine the internal forces at a point J located halfway between points A and B.

SOLUTION

FBD Frame:

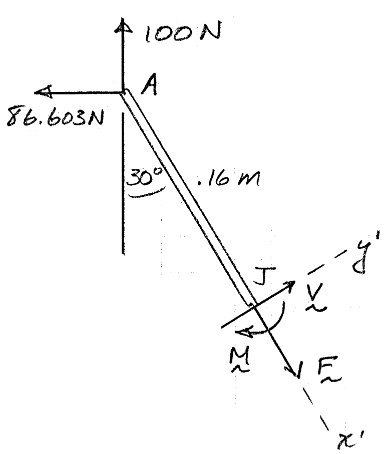


$\uparrow \Sigma F_y = 0: A_y - 100\text{ N} = 0 \qquad \mathbf{A}_y = 100\text{ N} \uparrow$

$\circlearrowleft \Sigma M_F = 0: \left[2(0.32\text{ m})\cos 30°\right]A_x - (0.48\text{ m})(100\text{ N}) = 0$

$\mathbf{A}_x = 86.603\text{ N} \longleftarrow$

FBD AJ:



$\searrow \Sigma F_{x'} = 0: F - (100\text{ N})\cos 30° - (86.603\text{ N})\sin 30° = 0$

$\mathbf{F} = 129.9\text{ N} \searrow$ ◀

$\nearrow \Sigma F_{y'} = 0: V + (100\text{ N})\sin 30° - (86.603\text{ N})\cos 30° = 0$

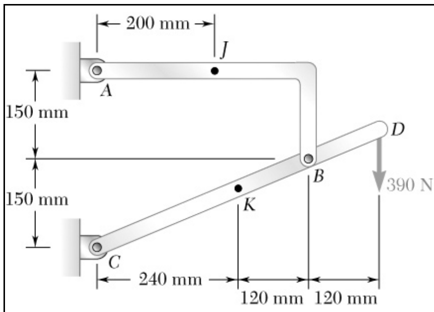
$\mathbf{V} = 25.0\text{ N} \nearrow$ ◀

$\circlearrowleft \Sigma M_J = 0: \left[(0.16\text{ m})\cos 30°\right](86.603\text{ N})$

$- \left[(0.16\text{ m})\sin 30°\right](100\text{ N}) - M = 0$

$\mathbf{M} = 4.00\text{ N}\cdot\text{m} \circlearrowright$ ◀

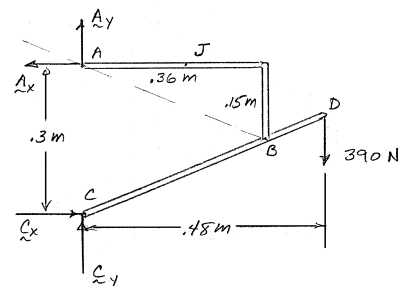
PROBLEM 7.5



Determine the internal forces at point J of the structure shown.

SOLUTION

FBD Frame:



AB is two-force member, so

$$\frac{A_x}{0.36\text{ m}} = \frac{A_y}{0.15\text{ m}} \qquad A_y = \frac{5}{12}A_x$$

$$\circlearrowleft \Sigma M_C = 0: (0.3\text{ m})A_x - (0.48\text{ m})(390\text{ N}) = 0$$

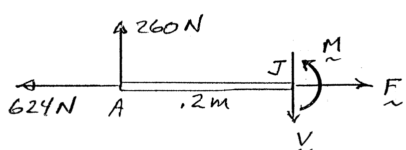
$$\mathbf{A}_x = 624\text{ N} \longleftarrow$$

$$A_y = \frac{5}{12}A_x = 260\text{ N or } \mathbf{A}_y = 260\text{ N} \uparrow$$

$$\longrightarrow \Sigma F_x = 0: F - 624\text{ N} = 0$$

$$\mathbf{F} = 624\text{ N} \longrightarrow \blacktriangleleft$$

FBD AJ:

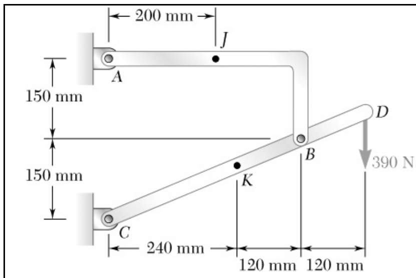


$$\uparrow \Sigma F_y = 0: 260\text{ N} - V = 0$$

$$\mathbf{V} = 260\text{ N} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - (0.2\text{ m})(260\text{ N}) = 0$$

$$\mathbf{M} = 52.0\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

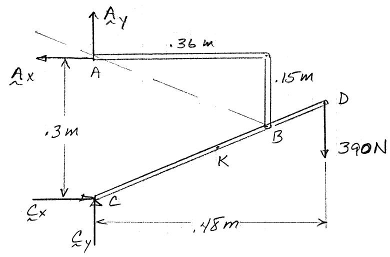


PROBLEM 7.6

Determine the internal forces at point K of the structure shown.

SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_C = 0: (0.3\text{ m})A_x - (0.48\text{ m})(390\text{ N}) = 0$$

$$\mathbf{A}_x = 624\text{ N} \longleftarrow$$

AB is two-force member, so

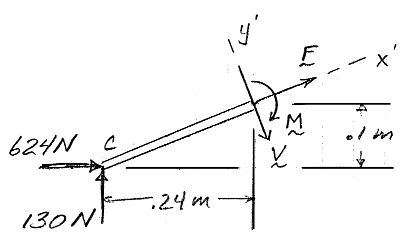
$$\frac{A_x}{0.36\text{ m}} = \frac{A_y}{0.15\text{ m}} \rightarrow A_y = \frac{5}{12}A_x \qquad \mathbf{A}_y = 260\text{ N} \uparrow$$

$$\longrightarrow \Sigma F_x = 0: -A_x + C_x = 0 \qquad \mathbf{C}_x = \mathbf{A}_x = 624\text{ N} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: A_y + C_y - 390\text{ N} = 0$$

$$C_y = 390\text{ N} - 260\text{ N} = 130\text{ N or } \mathbf{C}_y = 130\text{ N} \uparrow$$

FBD CK:



$$\nearrow \Sigma F_{x'} = 0: F + \frac{12}{13}(624\text{ N}) + \frac{5}{13}(130\text{ N}) = 0$$

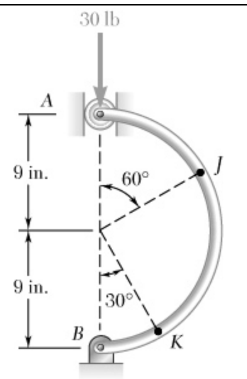
$$F = -626\text{ N} \qquad \mathbf{F} = 626\text{ N} \swarrow \blacktriangleleft$$

$$\nwarrow \Sigma F_{y'} = 0: \frac{12}{13}(130\text{ N}) - \frac{5}{13}(624\text{ N}) - V = 0$$

$$V = -120\text{ N} \qquad \mathbf{V} = 120.0\text{ N} \nwarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: (0.1\text{ m})(624\text{ N}) - (0.24\text{ m})(130\text{ N}) - M = 0$$

$$\mathbf{M} = 31.2\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

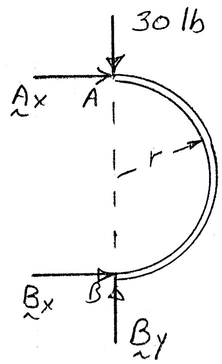


PROBLEM 7.7

A semicircular rod is loaded as shown. Determine the internal forces at point *J*.

SOLUTION

FBD Rod:



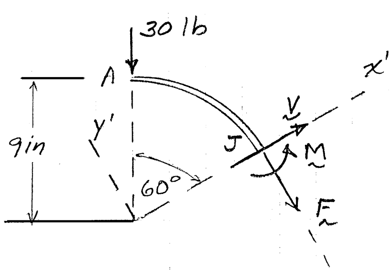
$$\circlearrowleft \Sigma M_B = 0: A_x(2r) = 0$$

$$\mathbf{A}_x = 0$$

$$\nearrow \Sigma F_{x'} = 0: V - (30 \text{ lb})\cos 60° = 0$$

$$\mathbf{V} = 15.00 \text{ lb} \nearrow \blacktriangleleft$$

FBD AJ:



$$\searrow \Sigma F_{y'} = 0: F + (30 \text{ lb})\sin 60° = 0$$

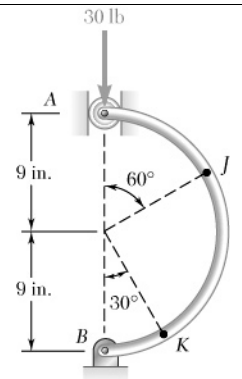
$$F = -25.98 \text{ lb}$$

$$\mathbf{F} = 26.0 \text{ lb} \nwarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - [(9 \text{ in.})\sin 60°](30 \text{ lb}) = 0$$

$$M = -233.8 \text{ lb} \cdot \text{in.}$$

$$\mathbf{M} = 234 \text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$

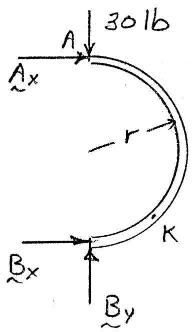


PROBLEM 7.8

A semicircular rod is loaded as shown. Determine the internal forces at point *K*.

SOLUTION

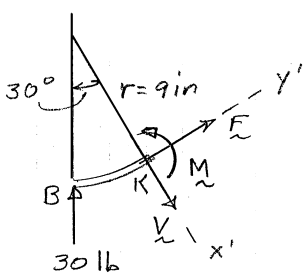
FBD Rod:



$\uparrow \Sigma F_y = 0: B_y - 30 \text{ lb} = 0 \qquad \mathbf{B}_y = 30 \text{ lb} \uparrow$

$\circlearrowleft \Sigma M_A = 0: 2rB_x = 0 \qquad \mathbf{B}_x = 0$

FBD BK:



$\searrow \Sigma F_{x'} = 0: V - (30 \text{ lb})\cos 30° = 0$

$V = 25.98 \text{ lb}$

$\mathbf{V} = 26.0 \text{ lb} \searrow$ ◀

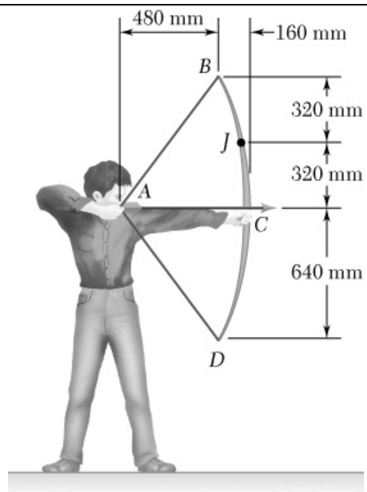
$\nearrow \Sigma F_{y'} = 0: F + (30 \text{ lb})\sin 30° = 0$

$F = -15 \text{ lb}$

$\mathbf{F} = 15.00 \text{ lb} \swarrow$ ◀

$\circlearrowleft \Sigma M_K = 0: M - [(9 \text{ in.})\sin 30°](30 \text{ lb}) = 0$

$\mathbf{M} = 135.0 \text{ lb}\cdot\text{in.} \circlearrowright$ ◀

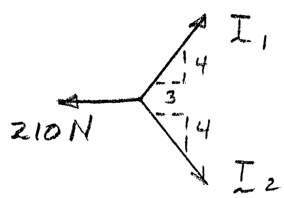


PROBLEM 7.9

An archer aiming at a target is pulling with a 210-N force on the bowstring. Assuming that the shape of the bow can be approximated by a parabola, determine the internal forces at point J.

SOLUTION

FBD Point A:

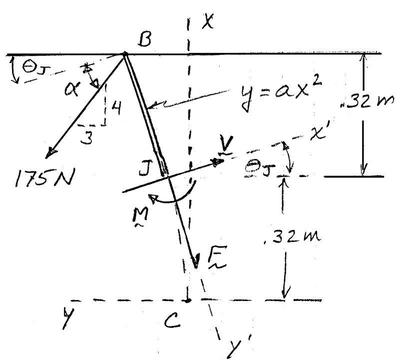


By symmetry $T_1 = T_2$

$$\rightarrow \Sigma F_x = 0: 2\left(\frac{3}{5}T_1\right) - 210 \text{ N} = 0 \qquad T_1 = T_2 = 175 \text{ N}$$

Curve CJB is parabolic: $y = ax^2$

FBD BJ:



At B: $x = 0.64$ m, $y = 0.16$ m $\qquad a = \dfrac{0.16 \text{ m}}{(0.64 \text{ m})^2} = \dfrac{1}{2.56 \text{ m}}$

So, at J: $y_J = \dfrac{1}{2.56 \text{ m}}(0.32 \text{ m})^2 = 0.04$ m

$$\text{Slope of parabola} = \tan\theta = \frac{dy}{dx} = 2ax$$

At J: $\theta_J = \tan^{-1}\left[\dfrac{2}{2.56 \text{ m}}(0.32 \text{ m})\right] = 14.036°$

So $\qquad \alpha = \tan^{-1}\dfrac{4}{3} - 14.036° = 39.094°$

$$\nearrow \Sigma F_{x'} = 0: V - (175 \text{ N})\cos(39.094°) = 0$$

$$\mathbf{V} = 135.8 \text{ N} \nearrow \blacktriangleleft$$

$$\searrow \Sigma F_{y'} = 0: F + (175 \text{ N})\sin(39.094°) = 0$$

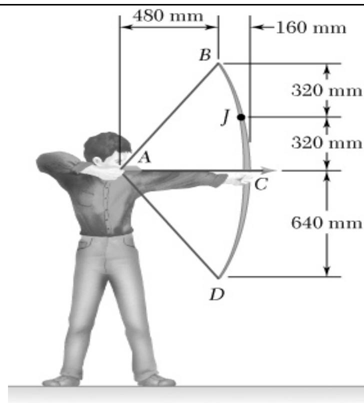
$$F = -110.35 \text{ N} \qquad \mathbf{F} = 110.4 \text{ N} \nwarrow \blacktriangleleft$$

PROBLEM 7.9 CONTINUED

$$\circlearrowleft \Sigma M_J = 0: M + (0.32 \text{ m})\left[\frac{3}{5}(175 \text{ N})\right]$$

$$+\left[(0.16 - 0.04)\text{m}\right]\left[\frac{4}{5}(175 \text{ N})\right] = 0$$

$$\mathbf{M} = 50.4 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



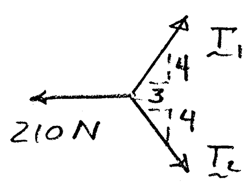
PROBLEM 7.10

For the bow of Prob. 7.9, determine the magnitude and location of the maximum (*a*) axial force, (*b*) shearing force, (*c*) bending moment.

SOLUTION

By symmetry $T_1 = T_2 = T$

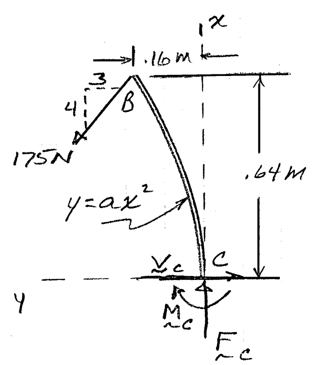
FBD Point A:



$$\rightarrow \Sigma F_x = 0: 2T_1\left(\frac{3}{5}\right) - 210 \text{ N} = 0 \qquad T_1 = 175 \text{ N}$$

$$\uparrow \Sigma F_y = 0: F_C - \frac{4}{5}(175 \text{ N}) = 0 \qquad F_C = 140 \text{ N} \uparrow$$

FBD BC:



$$\leftarrow \Sigma F_x = 0: \frac{3}{5}(175 \text{ N}) - V_C = 0 \qquad V_C = 105 \text{ N} \rightarrow$$

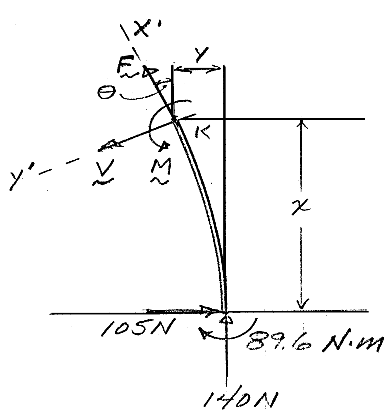
$$\circlearrowleft \Sigma M_C = 0: M_C - (0.64 \text{ m})\left[\frac{3}{5}(175 \text{ N})\right] - (0.16 \text{ m})\left[\frac{4}{5}(175 \text{ N})\right] = 0$$

$$M_C = 89.6 \text{ N}\cdot\text{m} \circlearrowright$$

Also: if $y = ax^2$ and, at B, $y = 0.16$ m, $x = 0.64$ m

Then $$a = \frac{0.16 \text{ m}}{(0.64 \text{ m})^2} = \frac{1}{2.56 \text{ m}};$$

FBD CK:



And $$\theta = \tan^{-1}\frac{dy}{dx} = \tan^{-1} 2ax$$

$$\nwarrow \Sigma F_{x'} = 0: (140 \text{ N})\cos\theta - (105 \text{ N})\sin\theta + F = 0$$

So $$F = (105 \text{ N})\sin\theta - (140 \text{ N})\cos\theta$$

$$\frac{dF}{d\theta} = (105 \text{ N})\cos\theta + (140 \text{ N})\sin\theta$$

$$\swarrow \Sigma F_{y'} = 0: V - (105 \text{ N})\cos\theta - (140 \text{ N})\sin\theta = 0$$

So $$V = (105 \text{ N})\cos\theta + (140 \text{ N})\sin\theta$$

PROBLEM 7.10 CONTINUED

And $$\frac{dV}{d\theta} = -(105\text{ N})\sin\theta + (140\text{ N})\cos\theta$$

$$\circlearrowleft \Sigma M_K = 0:\ M + x(105\text{ N}) + y(140\text{ N}) - 89.6\text{ N}\cdot\text{m} = 0$$

$$M = -(105\text{ N})x - \frac{(140\text{ N})x^2}{(2.56\text{ m})} + 89.6\text{ N}\cdot\text{m}$$

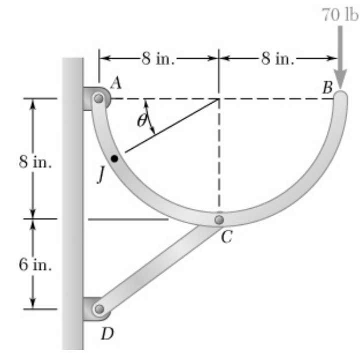
$$\frac{dM}{dx} = -(105\text{ N}) - (109.4\text{ N/m})x + 89.6\text{ N}\cdot\text{m}$$

Since none of the functions, F, V, or M has a vanishing derivative in the valid range of $0 \le x \le 0.64\text{ m}(0 \le \theta \le 26.6°)$, the maxima are at the limits $(x = 0,\ \text{or}\ x = 0.64\text{ m})$.

Therefore, (*a*) $\mathbf{F}_{max} = 140.0\text{ N}\uparrow$ at C ◀

(*b*) $\mathbf{V}_{max} = 156.5\text{ N}\swarrow$ at B ◀

(*c*) $\mathbf{M}_{max} = 89.6\text{ N}\cdot\text{m}\circlearrowright$ at C ◀

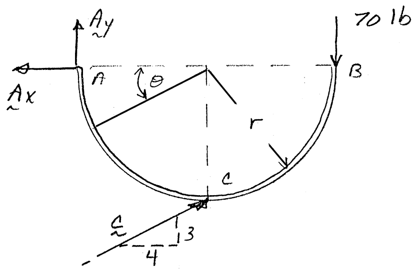


PROBLEM 7.11

A semicircular rod is loaded as shown. Determine the internal forces at point J knowing that $\theta = 30^{\circ}$.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(70 \text{ lb}) = 0$$

$$\mathbf{C} = 100 \text{ lb} \nearrow$$

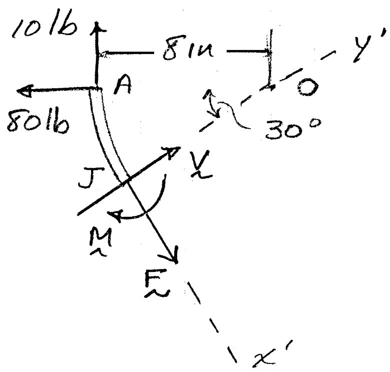
$$\longrightarrow \Sigma F_x = 0: -A_x + \frac{4}{5}(100 \text{ lb}) = 0$$

$$\mathbf{A}_x = 80 \text{ lb} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: A_y + \frac{3}{5}(100 \text{ lb}) - 70 \text{ lb} = 0$$

$$\mathbf{A}_y = 10 \text{ lb} \uparrow$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0: F - (80 \text{ lb})\sin 30° - (10 \text{ lb})\cos 30° = 0$$

$$F = 48.66 \text{ lb}$$

$$\mathbf{F} = 48.7 \text{ lb} \measuredangle 60° \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: V - (80 \text{ lb})\cos 30° + (10 \text{ lb})\sin 30° = 0$$

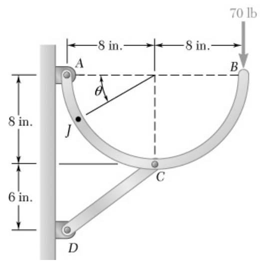
$$V = 64.28 \text{ lb}$$

$$\mathbf{V} = 64.3 \text{ lb} \measuredangle 30° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: (8 \text{ in.})(48.66 \text{ lb}) - (8 \text{ in.})(10 \text{ lb}) - M = 0$$

$$M = 309.28 \text{ lb} \cdot \text{in.}$$

$$\mathbf{M} = 309 \text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$

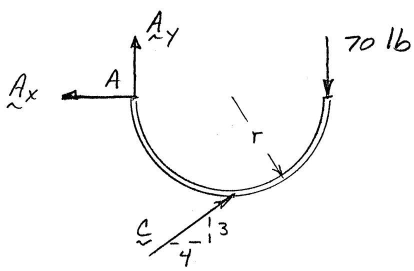


PROBLEM 7.12

A semicircular rod is loaded as shown. Determine the magnitude and location of the maximum bending moment in the rod.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(70 \text{ lb}) = 0$$

$$\mathbf{C} = 100 \text{ lb} \nearrow$$

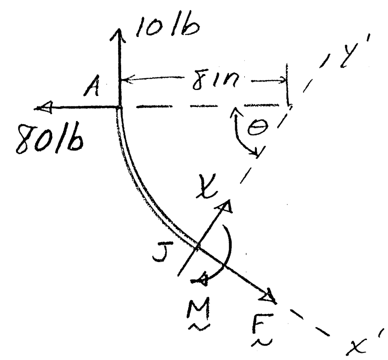
$$\longrightarrow \Sigma F_x = 0: -A_x + \frac{4}{5}(100 \text{ lb}) = 0$$

$$\mathbf{A}_x = 80 \text{ lb} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: A_y + \frac{3}{5}(100 \text{ lb}) - 70 \text{ lb} = 0$$

$$\mathbf{A}_y = 10 \text{ lb} \uparrow$$

FBD AJ:



$$\circlearrowleft \Sigma M_J = 0: M - (8 \text{ in.})(1 - \cos\theta)(10 \text{ lb}) - (8 \text{ in.})(\sin\theta)(80 \text{ lb}) = 0$$

$$M = (640 \text{ lb}\cdot\text{in.})\sin\theta + (80 \text{ lb}\cdot\text{in.})(\cos\theta - 1)$$

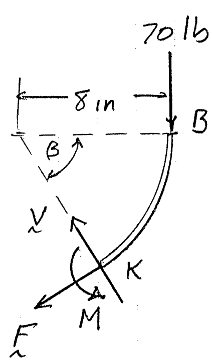
$$\frac{dM}{d\theta} = (640 \text{ lb}\cdot\text{in.})\cos\theta - (80 \text{ lb}\cdot\text{in.})\sin\theta = 0$$

for $\theta = \tan^{-1} 8 = 82.87°$,

where $\frac{d^2M}{d\theta^2} = -(640 \text{ lb}\cdot\text{in.})\sin\theta - (80 \text{ lb}\cdot\text{in.})\cos\theta < 0$

So $M = 565 \text{ lb}\cdot\text{in.}$ at $\theta = 82.9°$ is a *max* for *AC*

FBD BK:



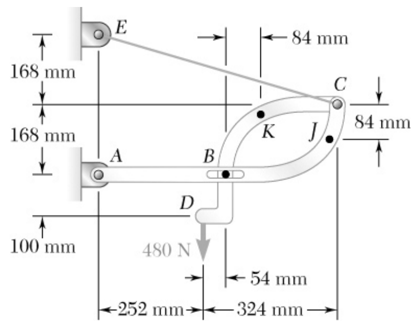
$$\circlearrowleft \Sigma M_K = 0: M - (8 \text{ in.})(1 - \cos\beta)(70 \text{ lb}) = 0$$

$$M = (560 \text{ lb}\cdot\text{in.})(1 - \cos\beta)$$

$$\frac{dM}{d\beta} = (560 \text{ lb}\cdot\text{in.})\sin\beta = 0 \quad \text{for } \beta = 0, \text{ where } M = 0$$

So, for $\beta = \frac{\pi}{2}$, $M = 560 \text{ lb}\cdot\text{in.}$ is max for *BC*

$$\therefore \ \mathbf{M}_{\max} = 565 \text{ lb}\cdot\text{in. at } \theta = 82.9° \blacktriangleleft$$

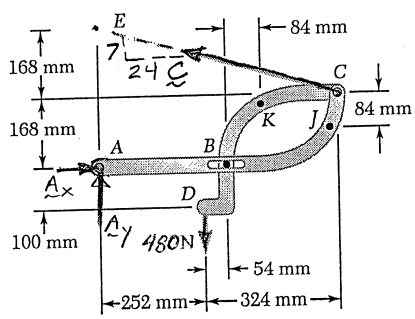


PROBLEM 7.13

Two members, each consisting of straight and 168-mm-radius quarter-circle portions, are connected as shown and support a 480-N load at D. Determine the internal forces at point J.

SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_A = 0: (0.336 \text{ m})\left(\frac{24}{25}C\right) - (0.252 \text{ m})(480 \text{ N}) = 0$$

$$C = 375 \text{ N}$$

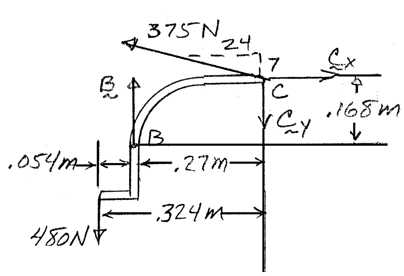
$$\rightarrow \Sigma F_y = 0: A_x - \frac{24}{25}C = 0 \qquad A_x = \frac{24}{25}(375 \text{ N}) = 360 \text{ N}$$

$$\mathbf{A}_x = 360 \text{ N} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: A_y - 480 \text{ N} + \frac{7}{24}(375 \text{ N}) = 0$$

$$\mathbf{A}_y = 375 \text{ N} \uparrow$$

FBD CD:



$$\circlearrowleft \Sigma M_C = 0: (0.324 \text{ m})(480 \text{ N}) - (0.27 \text{ m})B = 0$$

$$B = 576 \text{ N}$$

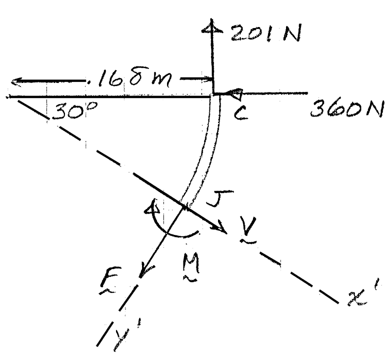
$$\rightarrow \Sigma F_x = 0: C_x - \frac{24}{25}(375 \text{ N}) = 0$$

$$\mathbf{C}_x = 360 \text{ N} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: -480 \text{ N} + \frac{7}{25}(375 \text{ N}) + (576 \text{ N}) - C_y = 0$$

$$\mathbf{C}_y = 201 \text{ N} \downarrow$$

FBD CJ:



$$\searrow \Sigma F_{x'} = 0: V - (360 \text{ N})\cos 30° - (201 \text{ N})\sin 30° = 0$$

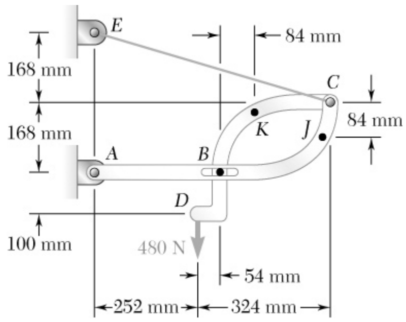
$$\mathbf{V} = 412 \text{ N} \searrow \blacktriangleleft$$

$$\swarrow \Sigma F_{y'} = 0: F + (360 \text{ N})\sin 30° - (201 \text{ N})\cos 30° = 0$$

$$F = -5.93 \text{ N} \qquad \mathbf{F} = 5.93 \text{ N} \nearrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: (0.168 \text{ m})(201 \text{ N} + 5.93 \text{ N}) - M = 0$$

$$M = 34.76 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 34.8 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

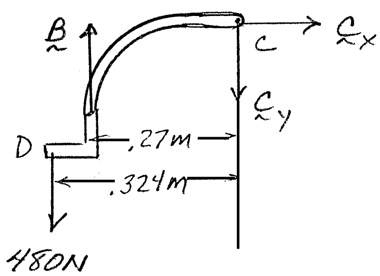


PROBLEM 7.14

Two members, each consisting of straight and 168-mm-radius quarter-circle portions, are connected as shown and support a 480-N load at D. Determine the internal forces at point K.

SOLUTION

FBD CD:



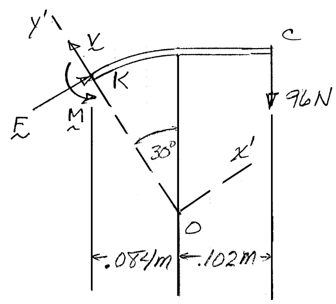
$$\longrightarrow \Sigma F_x = 0: \quad \mathbf{C}_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: (0.054 \text{ m})(480 \text{ N}) - (0.27 \text{ m})C_y = 0$$

$$\mathbf{C}_y = 96 \text{ N} \downarrow$$

$$\uparrow \Sigma F_y = 0: B - C_y = 0 \qquad \mathbf{B} = 96 \text{ N} \uparrow$$

FBD CK:



$$\nwarrow \Sigma F_{y'} = 0: V - (96 \text{ N})\cos 30° = 0$$

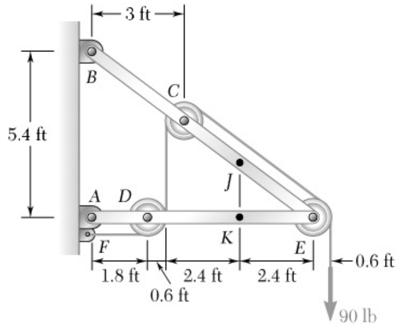
$$\mathbf{V} = 83.1 \text{ N} \nwarrow \blacktriangleleft$$

$$\nearrow \Sigma F_{x'} = 0: F - (96 \text{ N})\sin 30° = 0$$

$$\mathbf{F} = 48.0 \text{ N} \nearrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: M - (0.186 \text{ m})(96 \text{ N}) = 0$$

$$\mathbf{M} = 17.86 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

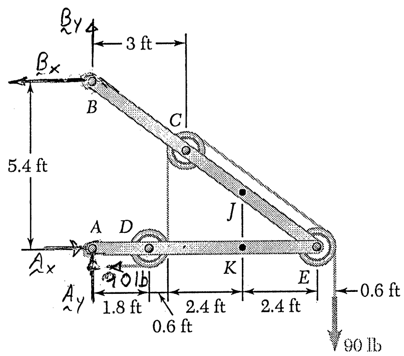


PROBLEM 7.15

Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point J of the frame shown.

SOLUTION

FBD Frame:



Note: Tension T in cord is 90 lb at any cut. All radii = 0.6 ft

$$\circlearrowleft \Sigma M_A = 0: (5.4 \text{ ft})B_x - (7.8 \text{ ft})(90 \text{ lb}) - (0.6 \text{ ft})(90 \text{ lb}) = 0$$

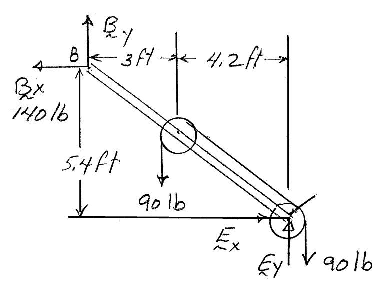
$$\mathbf{B}_x = 140 \text{ lb} \longleftarrow$$

$$\circlearrowleft \Sigma M_E = 0: (5.4 \text{ ft})(140 \text{ lb}) - (7.2 \text{ ft})B_y$$

$$+ (4.8 \text{ ft})90 \text{ lb} - (0.6 \text{ ft})90 \text{ lb} = 0$$

$$\mathbf{B}_y = 157.5 \text{ lb} \uparrow$$

FBD BCE with pulleys and cord:

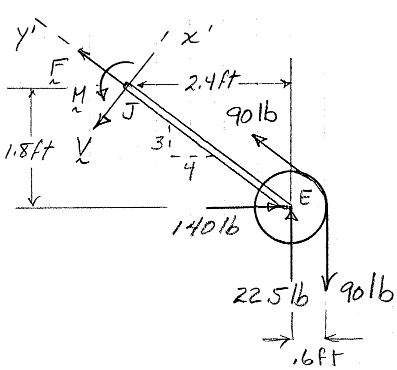


$$\rightarrow \Sigma F_x = 0: E_x - 140 \text{ lb} = 0 \qquad \mathbf{E}_x = 140 \text{ lb} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: 157.5 \text{ lb} - 90 \text{ lb} - 90 \text{ lb} + E_y = 0$$

$$\mathbf{E}_y = 22.5 \text{ lb} \uparrow$$

FBD EJ:



$$\nearrow \Sigma F_{x'} = 0: -V + \frac{3}{5}(140 \text{ lb}) + \frac{4}{5}(22.5 \text{ lb} - 90 \text{ lb}) = 0$$

$$V = 30 \text{ lb} \qquad \mathbf{V} = 30.0 \text{ lb} \swarrow \blacktriangleleft$$

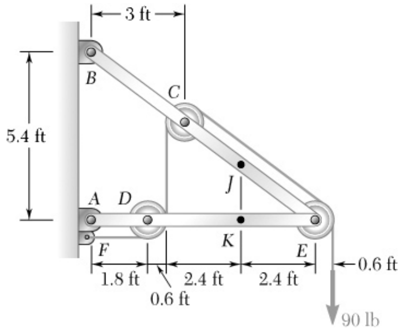
$$\nwarrow \Sigma F_{y'} = 0: F + 90 \text{ lb} - \frac{4}{5}(140 \text{ lb}) - \frac{3}{5}(90 \text{ lb} - 22.5 \text{ lb}) = 0$$

$$F = 62.5 \text{ lb} \qquad \mathbf{F} = 62.5 \text{ lb} \nwarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M + (1.8 \text{ ft})(140 \text{ lb}) + (0.6 \text{ ft})(90 \text{ lb})$$

$$+ (2.4 \text{ ft})(22.5 \text{ lb}) - (3.0 \text{ ft})(90 \text{ lb}) = 0$$

$$M = -90 \text{ lb}\cdot\text{ft} \qquad \mathbf{M} = 90.0 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

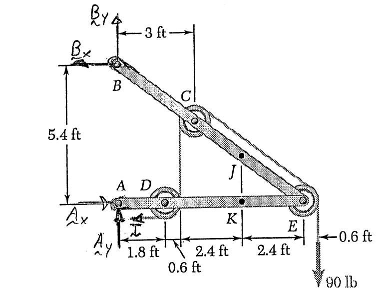


PROBLEM 7.16

Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point K of the frame shown.

SOLUTION

FBD Whole:

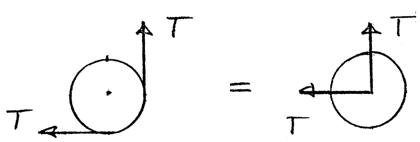


Note: $T = 90$ lb

$$\circlearrowleft \Sigma M_B = 0: (5.4 \text{ ft}) A_x - (6 \text{ ft})(90 \text{ lb}) - (7.8 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{A}_x = 2.30 \text{ lb} \longrightarrow$$

FBD AE:



Note: Cord tensions moved to point D as per Problem 6.91

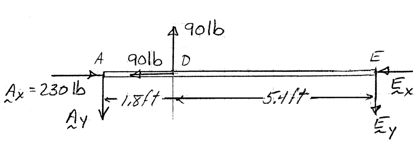
$$\longrightarrow \Sigma F_x = 0: 230 \text{ lb} - 90 \text{ lb} - E_x = 0$$

$$\mathbf{E}_x = 140 \text{ lb} \longleftarrow$$

$$\circlearrowleft \Sigma M_A = 0: (1.8 \text{ ft})(90 \text{ lb}) - (7.2 \text{ ft}) E_y = 0$$

$$\mathbf{E}_y = 22.5 \text{ lb} \downarrow$$

FBD KE:



$$\longrightarrow \Sigma F_x = 0: F - 140 \text{ lb} = 0$$

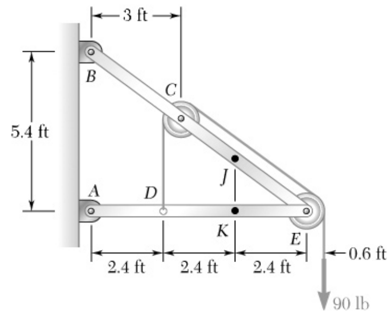
$$\mathbf{F} = 140.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: V - 22.5 \text{ lb} = 0$$

$$\mathbf{V} = 22.5 \text{ lb} \uparrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: M - (2.4 \text{ ft})(22.5 \text{ lb}) = 0$$

$$\mathbf{M} = 54.0 \text{ lb} \cdot \text{ft} \circlearrowright \blacktriangleleft$$

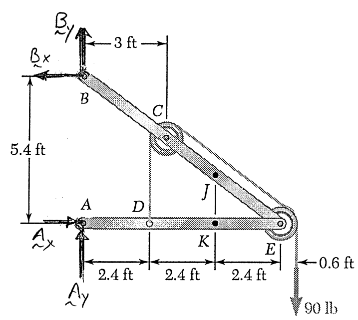


PROBLEM 7.17

Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point J of the frame shown.

SOLUTION

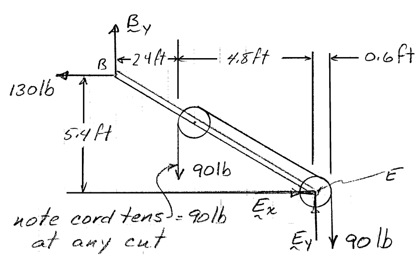
FBD Whole:



$$\circlearrowleft \Sigma M_A = 0\colon (5.4\text{ ft})B_x - (7.8\text{ ft})(90\text{ lb}) = 0$$

$$\mathbf{B}_x = 130\text{ lb} \longleftarrow$$

FBD BE with pulleys and cord:



$$\circlearrowleft \Sigma M_E = 0\colon (5.4\text{ ft})(130\text{ lb}) - (7.2\text{ ft})B_y$$

$$+ (4.8\text{ ft})(90\text{ lb}) - (0.6\text{ ft})(90\text{ lb}) = 0$$

$$\mathbf{B}_y = 150\text{ lb} \uparrow$$

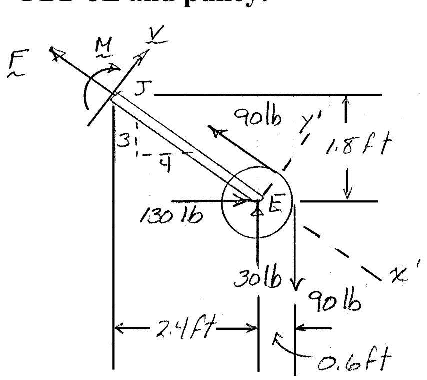
$$\longrightarrow \Sigma F_x = 0\colon E_x - 130\text{ lb} = 0$$

$$\mathbf{E}_x = 130\text{ lb} \longrightarrow$$

$$\uparrow \Sigma F_y = 0\colon E_y + 150\text{ lb} - 90\text{ lb} - 90\text{ lb} = 0$$

$$\mathbf{E}_y = 30\text{ lb} \uparrow$$

FBD JE and pulley:



$$\searrow \Sigma F_{x'} = 0\colon -F - 90\text{ lb} + \frac{4}{5}(130\text{ lb}) + \frac{3}{5}(90\text{ lb} - 30\text{ lb}) = 0$$

$$\mathbf{F} = 50.0\text{ lb} \nwarrow \blacktriangleleft$$

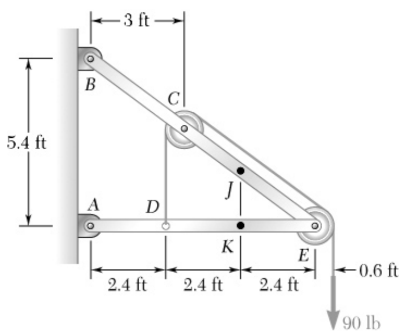
$$\nearrow \Sigma F_{y'} = 0\colon V + \frac{3}{5}(130\text{ lb}) + \frac{4}{5}(30\text{ lb} - 90\text{ lb}) = 0$$

$$V = -30\text{ lb} \qquad \mathbf{V} = 30.0\text{ lb} \swarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0\colon -M + (1.8\text{ ft})(130\text{ lb}) + (2.4\text{ ft})(30\text{ lb}) + (0.6\text{ ft})(90\text{ lb})$$

$$- (3.0\text{ ft})(90\text{ lb}) = 0$$

$$\mathbf{M} = 90.0\text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

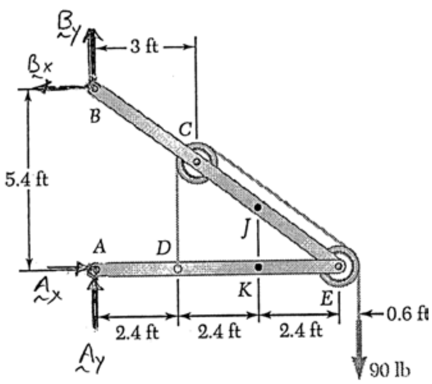


PROBLEM 7.18

Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point K of the frame shown.

SOLUTION

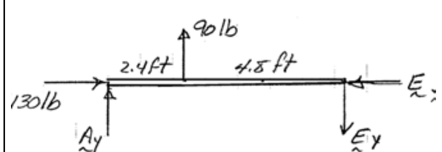
FBD Whole:



$$\circlearrowleft \Sigma M_B = 0: (5.4 \text{ ft})A_x - (7.8 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{A}_x = 130 \text{ lb} \longrightarrow$$

FBD AE:

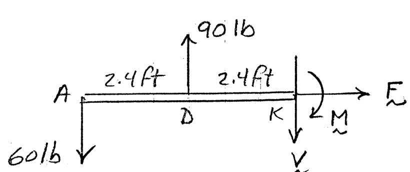


$$\circlearrowleft \Sigma M_E = 0: -(7.2 \text{ ft})A_y - (4.8 \text{ ft})(90 \text{ lb}) = 0$$

$$A_y = -60 \text{ lb} \qquad \mathbf{A}_y = 60 \text{ lb} \downarrow$$

$$\longrightarrow \Sigma F_x = 0: \qquad \mathbf{F} = 0 \blacktriangleleft$$

FBD AK:

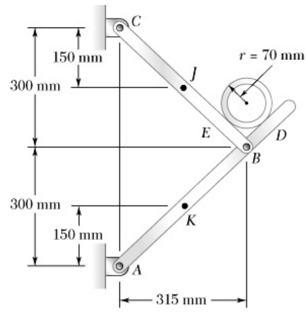


$$\uparrow \Sigma F_y = 0: -60 \text{ lb} + 90 \text{ lb} - V = 0$$

$$\mathbf{V} = 30.0 \text{ lb} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: (4.8 \text{ ft})(60 \text{ lb}) - (2.4 \text{ ft})(90 \text{ lb}) - M = 0$$

$$\mathbf{M} = 72.0 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

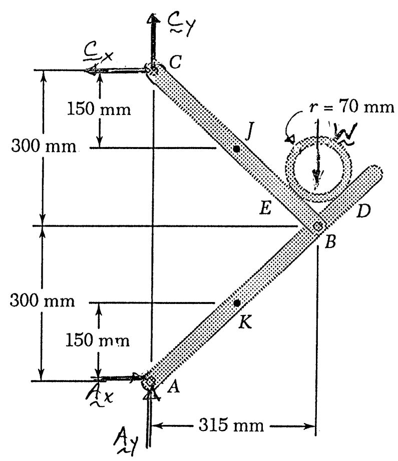


PROBLEM 7.19

A 140-mm-diameter pipe is supported every 3 m by a small frame consisting of two members as shown. Knowing that the combined mass per unit length of the pipe and its contents is 28 kg/m and neglecting the effect of friction, determine the internal forces at point *J*.

SOLUTION

FBD Whole:

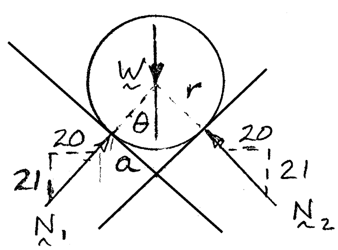


$$W = (3\text{ m})(28\text{ kg/m})(9.81\text{ m/s}^2) = 824.04\text{ N}$$

$$\circlearrowleft \Sigma M_A = (0.6\text{ m})C_x - (0.315\text{ m})(824.04\text{ N}) = 0$$

$$\mathbf{C}_x = 432.62\text{ N} \longleftarrow$$

FBD pipe:



By symmetry: $N_1 = N_2$

$$\uparrow \Sigma F_y = 0: 2\frac{21}{29}N_1 - W = 0$$

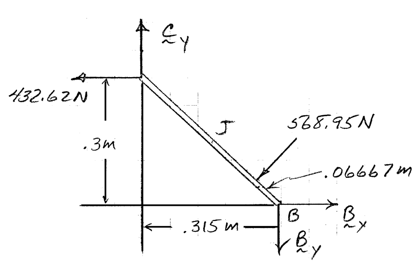
$$N_1 = \frac{29}{42}(824.04\text{ N})$$

$$= 568.98\text{ N}$$

Also note: $a = r\tan\theta = 70\text{ mm}\left(\frac{20}{21}\right)$

$$a = 66.67\text{ mm}$$

FBD BC:

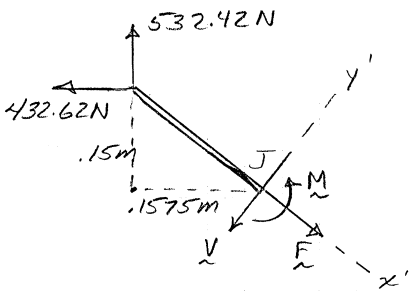


$$\circlearrowleft \Sigma M_B = 0: (0.3\text{ m})(432.62\text{ N}) - (0.315\text{ m})C_y + (0.06667\text{ m})(568.98\text{ N}) = 0$$

$$\mathbf{C}_y = 532.42\text{ N} \uparrow$$

PROBLEM 7.19 CONTINUED

FBD CJ:



$$\searrow \Sigma F_{x'} = 0: F - \frac{21}{29}(432.62\text{ N}) - \frac{20}{29}(532.42\text{ N}) = 0$$

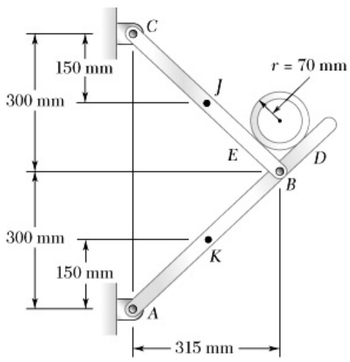
$\mathbf{F} = 680\text{ N} \searrow$ ◀

$$\nearrow \Sigma F_{y'} = 0: \frac{21}{29}(532.42\text{ N}) - \frac{20}{29}(432.62\text{ N}) - V = 0$$

$\mathbf{V} = 87.2\text{ N} \swarrow$ ◀

$$\circlearrowleft \Sigma M_J = 0: (0.15\text{ m})(432.62\text{ N}) - (0.1575\text{ m})(532.42\text{ N}) + M = 0$$

$\mathbf{M} = 18.96\text{ N}\cdot\text{m} \circlearrowright$ ◀

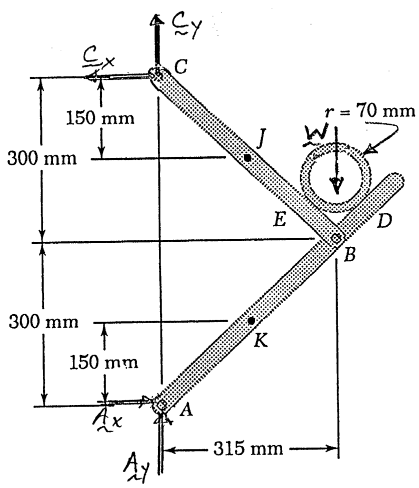


PROBLEM 7.20

A 140-mm-diameter pipe is supported every 3 m by a small frame consisting of two members as shown. Knowing that the combined mass per unit length of the pipe and its contents is 28 kg/m and neglecting the effect of friction, determine the internal forces at point *K*.

SOLUTION

FBD Whole:

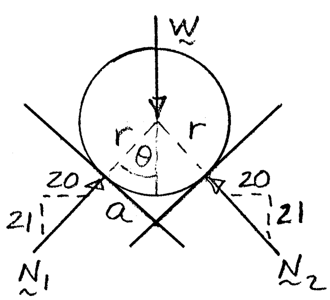


$$W = (3\text{ m})(28\text{ kg/m})(9.81\text{ m/s}^2) = 824.04\text{ N}$$

$$\circlearrowleft \Sigma M_C = 0: (.6\text{ m})A_x - (.315\text{ m})(824.04\text{ N}) = 0$$

$$\mathbf{A}_x = 432.62\text{ N} \longrightarrow$$

FBD pipe



By symmetry: $N_1 = N_2$

$$\uparrow \Sigma F_y = 0: 2\frac{21}{29}N_1 - W = 0$$

$$N_2 = \frac{29}{42}\,824.04\text{ N}$$

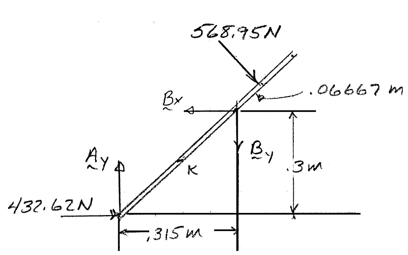
$$= 568.98\text{ N}$$

Also note:

$$a = r\tan\theta = (70\text{ mm})\frac{20}{21}$$

$$a = 66.67\text{ mm}$$

FBD AD:



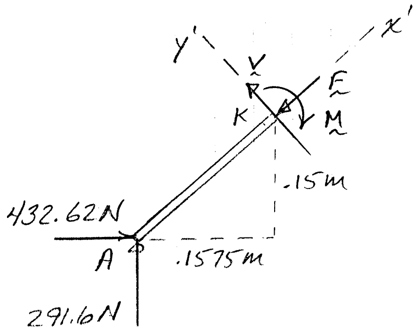
$$\circlearrowleft \Sigma M_B = 0: (0.3\text{ m})(432.62\text{ N}) - (0.315\text{ m})A_y$$

$$-(0.06667\text{ m})(568.98\text{ N}) = 0$$

$$\mathbf{A}_y = 291.6\text{ N} \uparrow$$

PROBLEM 7.20 CONTINUED

FBD AK:



$$\nearrow \Sigma F_{x'} = 0: \frac{21}{29}(432.62\text{ N}) + \frac{20}{29}(291.6\text{ N}) - F = 0$$

F = 514 N ↙ ◀

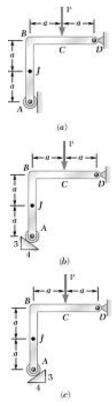
$$\nwarrow \Sigma F_{y'} = 0: \frac{21}{29}(291.6\text{ N}) - \frac{20}{29}(432.62\text{ N}) + V = 0$$

V = 87.2 N ↖ ◀

$$\circlearrowleft \Sigma M_K = 0: (0.15\text{ m})(432.62\text{ N}) - (0.1575\text{ m})(291.6\text{ N}) - M = 0$$

M = 18.97 N·m ↻ ◀

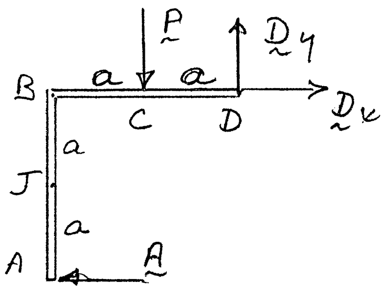
PROBLEM 7.21



A force **P** is applied to a bent rod which is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at point *J*.

SOLUTION

(*a*) **FBD Rod:**



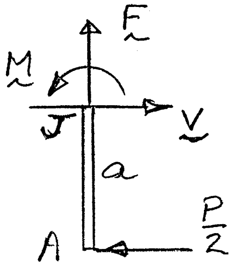
$$\circlearrowleft \Sigma M_D = 0: aP - 2aA = 0$$

$$\mathbf{A} = \frac{P}{2} \longleftarrow$$

$$\longrightarrow \Sigma F_x = 0: V - \frac{P}{2} = 0$$

$$\mathbf{V} = \frac{P}{2} \longrightarrow \blacktriangleleft$$

FBD AJ:

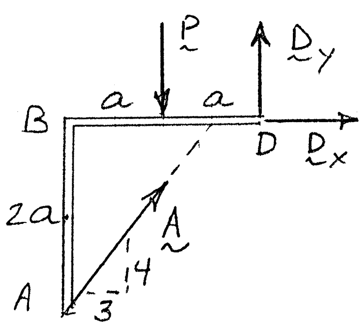


$$\uparrow \Sigma F_y = 0: \qquad \mathbf{F} = 0 \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - a\frac{P}{2} = 0$$

$$\mathbf{M} = \frac{aP}{2} \circlearrowright \blacktriangleleft$$

(*b*) **FBD Rod:**

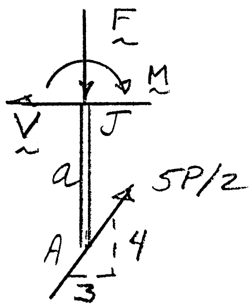


$$\circlearrowleft \Sigma M_D = 0: aP - \frac{a}{2}\left(\frac{4}{5}A\right) = 0$$

$$\mathbf{A} = \frac{5P}{2} \nearrow$$

PROBLEM 7.21 CONTINUED

FBD AJ:



$$\xrightarrow{} \Sigma F_x = 0: \frac{3}{5}\frac{5P}{2} - V = 0$$

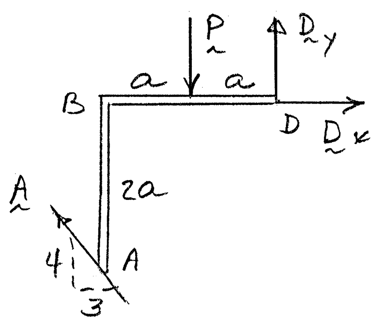
$$\mathbf{V} = \frac{3P}{2} \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{4}{5}\frac{5P}{2} - F = 0$$

$$\mathbf{F} = 2P \downarrow \blacktriangleleft$$

$$\mathbf{M} = \frac{3}{2}aP \circlearrowright \blacktriangleleft$$

(*c*) **FBD Rod:**



$$\circlearrowleft \Sigma M_D = 0: aP - 2a\left(\frac{3}{5}A\right) - 2a\left(\frac{4}{5}A\right) = 0$$

$$A = \frac{5P}{14}$$

$$\xrightarrow{} \Sigma F_x = 0: V - \left(\frac{3}{5}\frac{5P}{14}\right) = 0$$

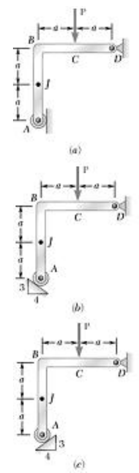
$$\mathbf{V} = \frac{3P}{14} \rightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{4}{5}\frac{5P}{14} - F = 0$$

$$\mathbf{F} = \frac{2P}{7} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - a\left(\frac{3}{5}\frac{5P}{14}\right) = 0$$

$$\mathbf{M} = \frac{3}{14}aP \circlearrowleft \blacktriangleleft$$

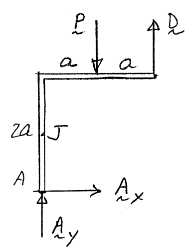


PROBLEM 7.22

A force **P** is applied to a bent rod which is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at point *J*.

SOLUTION

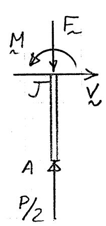
(*a*) **FBD Rod:**



$\rightarrow \Sigma F_x = 0\text{: } A_x = 0$

$\circlearrowleft \Sigma M_D = 0\text{: } aP - 2aA_y = 0 \qquad A_y = \dfrac{P}{2}$

FBD AJ:



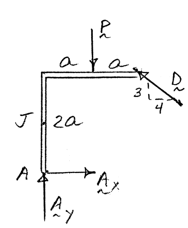
$\rightarrow \Sigma F_x = 0\text{: } \mathbf{V} = 0$ ◀

$\uparrow \Sigma F_y = 0\text{: } \dfrac{P}{2} - F = 0$

$\mathbf{F} = \dfrac{P}{2} \downarrow$ ◀

$\circlearrowleft \Sigma M_J = 0\text{: } \mathbf{M} = 0$ ◀

(*b*) **FBD Rod:**



$\circlearrowleft \Sigma M_A = 0$

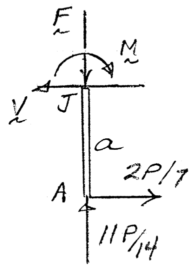
$2a\left(\dfrac{4}{5}D\right) + 2a\left(\dfrac{3}{5}D\right) - aP = 0 \qquad D = \dfrac{5P}{14}$

$\rightarrow \Sigma F_x = 0\text{: } A_x - \dfrac{4}{5}\dfrac{5}{14}P = 0 \qquad A_x = \dfrac{2P}{7}$

$\uparrow \Sigma F_y = 0\text{: } A_y - P + \dfrac{3}{5}\dfrac{5}{14}P = 0 \qquad A_y = \dfrac{11P}{14}$

PROBLEM 7.22 CONTINUED

FBD AJ:



$$\rightarrow \Sigma F_x = 0: \frac{2}{7}P - V = 0$$

$$\mathbf{V} = \frac{2P}{7} \leftarrow \blacktriangleleft$$

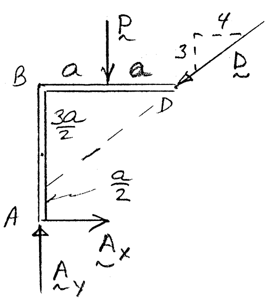
$$\uparrow \Sigma F_y = 0: \frac{11P}{14} - F = 0$$

$$\mathbf{F} = \frac{11P}{14} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: a\frac{2P}{7} - M = 0$$

$$\mathbf{M} = \frac{2}{7}aP \circlearrowright \blacktriangleleft$$

(c) **FBD Rod:**

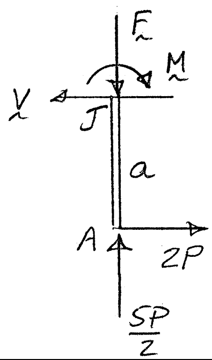


$$\circlearrowleft \Sigma M_A = 0: \frac{a}{2}\left(\frac{4D}{5}\right) - aP = 0 \qquad D = \frac{5P}{2}$$

$$\rightarrow \Sigma F_x = 0: A_x - \frac{4}{5}\frac{5P}{2} = 0 \qquad A_x = 2P$$

$$\uparrow \Sigma F_y = 0: A_y - P - \frac{3}{5}\frac{5P}{2} = 0 \qquad A_y = \frac{5P}{2}$$

FBD AJ:



$$\rightarrow \Sigma F_x = 0: 2P - V = 0$$

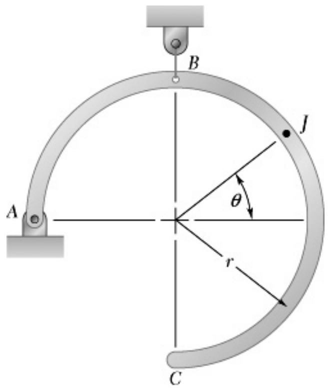
$$\mathbf{V} = 2P \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{5P}{2} - F = 0$$

$$\mathbf{F} = \frac{5P}{2} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: a(2P) - M = 0$$

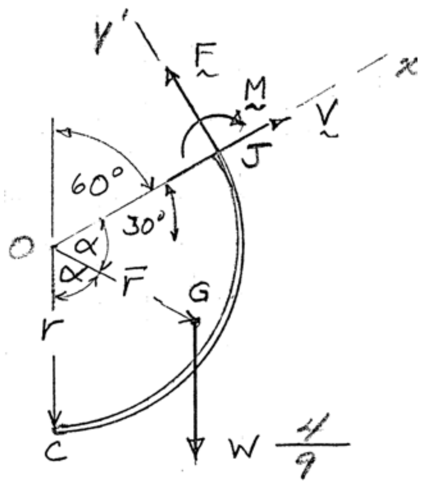
$$\mathbf{M} = 2aP \circlearrowright \blacktriangleleft$$



PROBLEM 7.23

A rod of weight W and uniform cross section is bent into the circular arc of radius r shown. Determine the bending moment at point J when $\theta = 30°$.

SOLUTION



Note $\alpha = \dfrac{180° - 60°}{2} = 60° = \dfrac{\pi}{3}$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha = \frac{3r}{\pi}\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2\pi}r$$

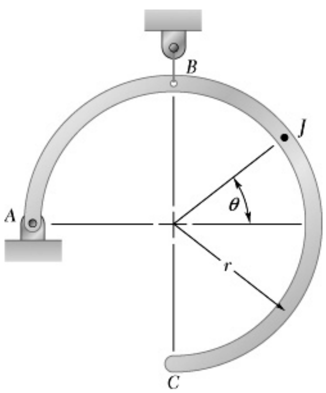
$$\text{Weight of section} = W\frac{120}{270} = \frac{4}{9}W$$

$$\nwarrow \Sigma F_{y'} = 0: F - \frac{4}{9}W\cos 30° = 0 \qquad F = \frac{2\sqrt{3}}{9}W$$

$$\circlearrowleft \Sigma M_0 = 0: rF - (\bar{r}\sin 60°)\frac{4W}{9} - M = 0$$

$$M = r\left[\frac{2\sqrt{3}}{9} - \frac{3\sqrt{3}}{2\pi}\frac{\sqrt{3}}{2}\frac{4}{9}\right]W = \left[\frac{2\sqrt{3}}{9} - \frac{1}{\pi}\right]Wr$$

$$\mathbf{M} = 0.0666Wr \circlearrowright \blacktriangleleft$$

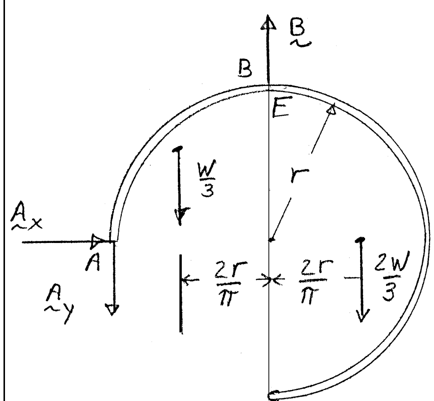


PROBLEM 7.24

A rod of weight W and uniform cross section is bent into the circular arc of radius r shown. Determine the bending moment at point J when $\theta = 120°$.

SOLUTION

(*a*) **FBD Rod:**

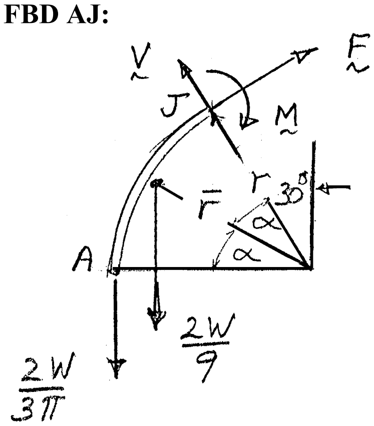


$$\rightarrow \Sigma F_x = 0: A_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: rA_y + \frac{2r}{\pi}\frac{W}{3} - \frac{2r}{\pi}\frac{2W}{3} = 0$$

$$A_y = \frac{2W}{3\pi}$$

FBD AJ:



Note:

$$\alpha = \frac{60°}{2} = 30° = \frac{\pi}{6}$$

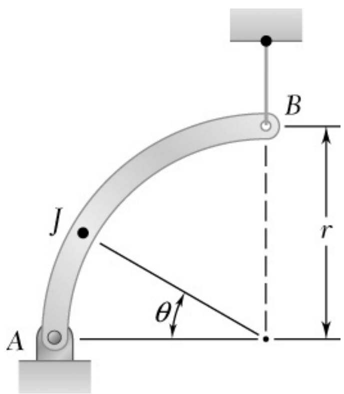
$$\text{Weight of segment} = W\frac{60}{270} = \frac{2W}{9}$$

$$F = \frac{r}{\alpha}\sin\alpha = \frac{r}{\pi/6}\sin 30° = \frac{3r}{\pi}$$

$$\circlearrowleft \Sigma M_J = 0: \left(\bar{r}\cos\alpha - r\sin 30°\right)\frac{2W}{9} + \left(r - r\sin 30°\right)\frac{2W}{3\pi} - M = 0$$

$$M = \frac{2W}{9}\left(\frac{3r}{\pi}\frac{\sqrt{3}}{2} - \frac{r}{2} + \frac{3r}{2\pi}\right) = Wr\left(\frac{\sqrt{3}}{3\pi} - \frac{1}{9} + \frac{1}{3\pi}\right)$$

$$\mathbf{M} = 0.1788Wr \circlearrowright \blacktriangleleft$$

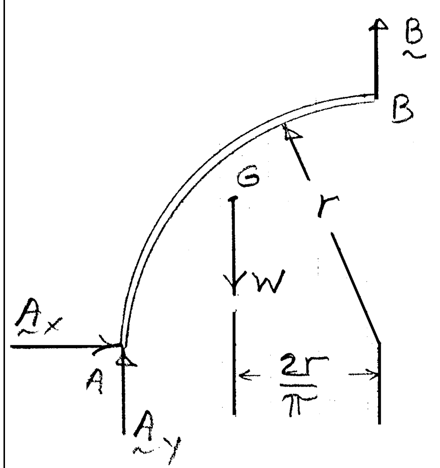


PROBLEM 7.25

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 30^\circ$.

SOLUTION

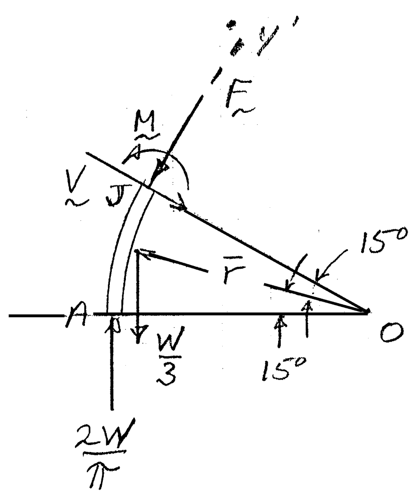
FBD Rod:



$$\rightarrow \Sigma F_x = 0: \mathbf{A}_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: \frac{2r}{\pi}W - rA_y = 0 \qquad \mathbf{A}_y = \frac{2W}{\pi}\uparrow$$

FBD AJ:



$$\alpha = 15^\circ, \text{ weight of segment } = W\frac{30^\circ}{90^\circ} = \frac{W}{3}$$

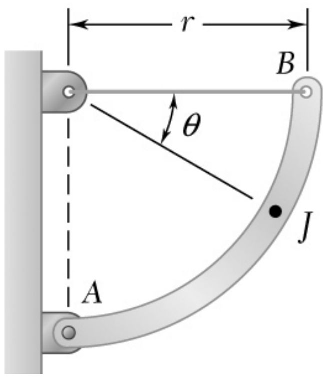
$$\bar{r} = \frac{r}{\alpha}\sin\alpha = \frac{r}{\pi/12}\sin 15^\circ = 0.9886r$$

$$\nearrow \Sigma F_{y'} = 0: \frac{2W}{\pi}\cos 30^\circ - \frac{W}{3}\cos 30^\circ - F = 0$$

$$\mathbf{F} = \frac{W\sqrt{3}}{2}\left(\frac{2}{\pi} - \frac{1}{3}\right)\swarrow$$

$$\circlearrowleft \Sigma M_0 = M + r\left(F - \frac{2W}{\pi}\right) + \bar{r}\cos 15^\circ\frac{W}{3} = 0$$

$$\mathbf{M} = 0.0557Wr \circlearrowright \blacktriangleleft$$

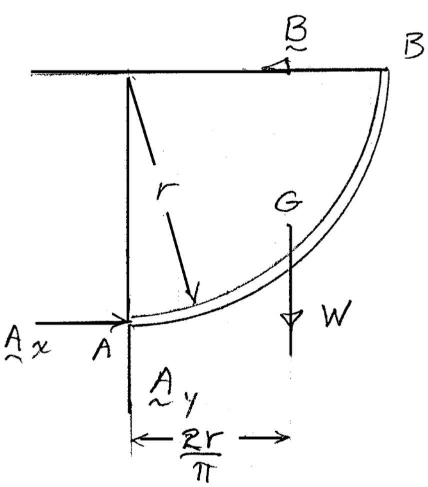


PROBLEM 7.26

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 30^\circ$.

SOLUTION

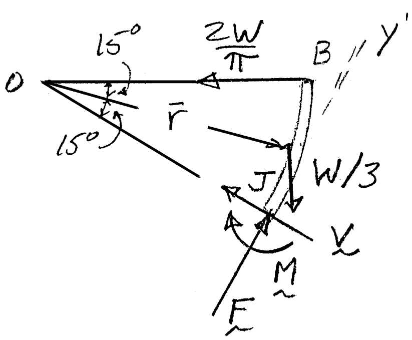
FBD Rod:



$$\circlearrowleft \Sigma M_A = 0: rB - \frac{2r}{\pi}W = 0$$

$$\mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

FBD BJ:



$$\alpha = 15^\circ = \frac{\pi}{12}$$

$$\bar{r} = \frac{r}{\pi/12}\sin 15^\circ = 0.98862r$$

$$\text{Weight of segment} = W\frac{30^\circ}{90^\circ} = \frac{W}{3}$$

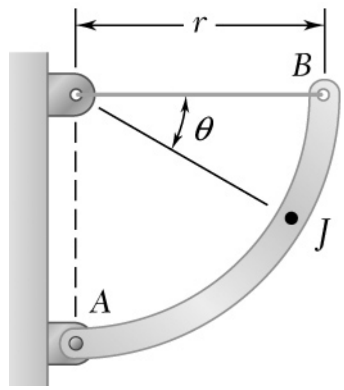
$$\nearrow \Sigma F_{y'} = 0: F - \frac{W}{3}\cos 30^\circ - \frac{2W}{\pi}\sin 30^\circ = 0$$

$$\mathbf{F} = \left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right)W \nearrow$$

$$\circlearrowleft \Sigma M_0 = 0: rF - (\bar{r}\cos 15^\circ)\frac{W}{3} - M = 0$$

$$M = rW\left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right) - \left(0.98862\frac{\cos 15^\circ}{3}\right)Wr$$

$$\mathbf{M} = 0.289Wr \circlearrowright \blacktriangleleft$$

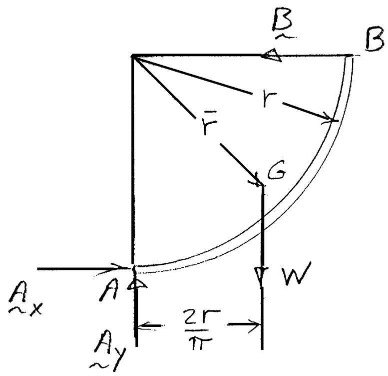


PROBLEM 7.27

For the rod of Prob.7.26, determine the magnitude and location of the maximum bending moment.

SOLUTION

FBD Bar:



$$\circlearrowleft \Sigma M_A = 0: rB - \frac{2r}{\pi}W = 0 \qquad \mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

$$\alpha = \frac{\theta}{2} \qquad \text{so} \qquad 0 \le \alpha \le \frac{\pi}{4}$$

$$\overline{r} = \frac{r}{\alpha}\sin\alpha,$$

$$\text{Weight of segment } = W\frac{2\alpha}{\pi/2}$$

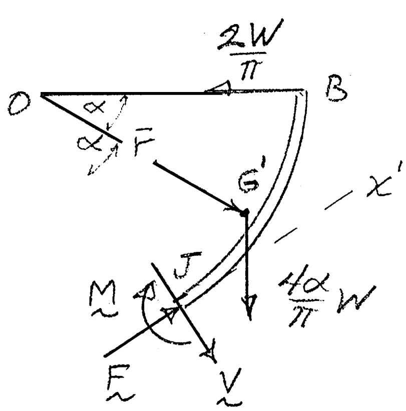
$$= \frac{4\alpha}{\pi}W$$

$$\nearrow \Sigma F_{x'} = 0: F - \frac{4\alpha}{\pi}W\cos 2\alpha - \frac{2W}{\pi}\sin 2\alpha = 0$$

$$F = \frac{2W}{\pi}\left(\sin 2\alpha + 2\alpha\cos 2\alpha\right)$$

$$= \frac{2W}{\pi}\left(\sin\theta + \theta\cos\theta\right)$$

FBD BJ:



$$\circlearrowleft \Sigma M_0 = 0: rF - \left(\overline{r}\cos\alpha\right)\frac{4\alpha}{\pi}W - M = 0$$

$$M = \frac{2}{\pi}Wr\left(\sin\theta + \theta\cos\theta\right) - \left(\frac{r}{\alpha}\sin\alpha\cos\alpha\right)\frac{4\alpha}{\pi}W$$

But, $\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

so $M = \frac{2Wr}{\pi}\left(\sin\theta + \theta\cos\theta - \sin\theta\right)$

or $M = \frac{2}{\pi}Wr\theta\cos\theta$

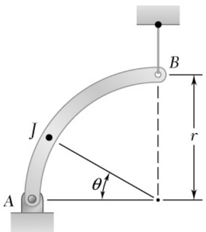
$$\frac{dM}{d\theta} = \frac{2}{\pi}Wr\left(\cos\theta - \theta\sin\theta\right) = 0 \quad \text{at } \theta\tan\theta = 1$$

PROBLEM 7.27 CONTINUED

Solving numerically $\theta = 0.8603$ rad and $\mathbf{M} = 0.357Wr$ ↻ ◀

at $\theta = 49.3°$ ◀

(Since $M = 0$ at both limits, this is the maximum)

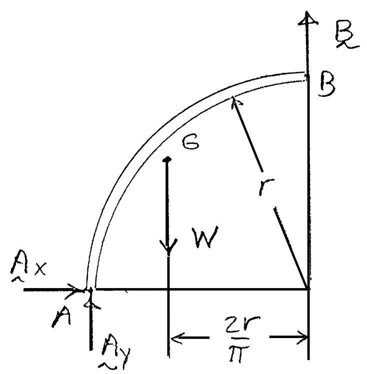


PROBLEM 7.28

For the rod of Prob.7.25, determine the magnitude and location of the maximum bending moment.

SOLUTION

FBD Rod:



$$\rightarrow \Sigma F_x = 0: A_x = 0$$

$$\Sigma M_B = 0: \frac{2r}{\pi}W - rA_y = 0 \qquad A_y = \frac{2W}{\pi}$$

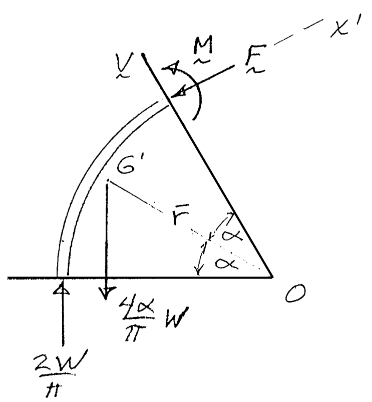
$$\alpha = \frac{\theta}{2}, \qquad \overline{r} = \frac{r}{\alpha}\sin\alpha$$

$$\text{Weight of segment} = W\frac{2\alpha}{\pi/2} = \frac{4\alpha}{\pi}W$$

$$\Sigma F_{x'} = 0: -F - \frac{4\alpha}{\pi}W\cos 2\alpha + \frac{2W}{\pi}\cos 2\alpha = 0$$

$$F = \frac{2W}{\pi}(1 - 2\alpha)\cos 2\alpha = \frac{2W}{\pi}(1 - \theta)\cos\theta$$

FBD AJ:



$$\Sigma M_0 = 0: M + \left(F - \frac{2W}{\pi}\right)r + (\overline{r}\cos\alpha)\frac{4\alpha}{\pi}W = 0$$

$$M = \frac{2W}{\pi}(1 + \theta\cos\theta - \cos\theta)r - \frac{4\alpha W}{\pi}\frac{r}{\alpha}\sin\alpha\cos\alpha$$

But, $\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

so $M = \frac{2r}{\pi}W(1 - \cos\theta + \theta\cos\theta - \sin\theta)$

$$\frac{dM}{d\theta} = \frac{2rW}{\pi}(\sin\theta - \theta\sin\theta + \cos\theta - \cos\theta) = 0$$

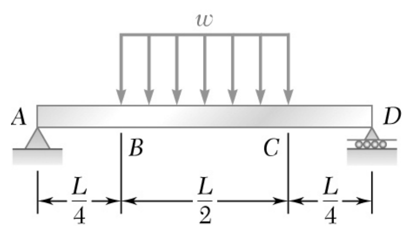
for $(1 - \theta)\sin\theta = 0$

$$\frac{dM}{d\theta} = 0 \qquad \text{for} \qquad \theta = 0, 1, n\pi \;\; (n = 1, 2, \cdots)$$

Only 0 and 1 in valid range

At $\theta = 0 \;\; M = 0$, at $\theta = 1\text{ rad}$

at $\theta = 57.3°$ $\quad M = M_{max} = 0.1009\,Wr$ ◀

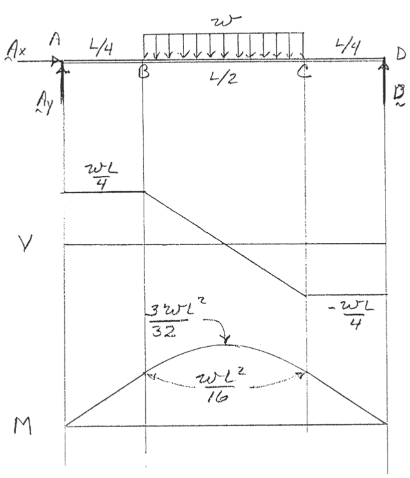


PROBLEM 7.29

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

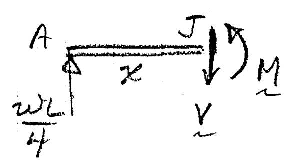
SOLUTION

FBD beam:



(*a*) By symmetry: $A_y = D = \frac{1}{2}(w)\frac{L}{2}$ $\quad \mathbf{A}_y = \mathbf{D} = \frac{wL}{4}\uparrow$

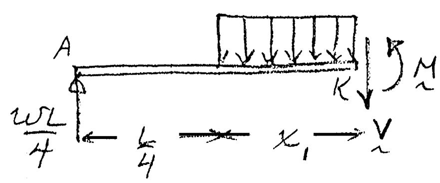
Along AB:



$$\uparrow \Sigma F_y = 0: \frac{wL}{4} - V = 0 \qquad V = \frac{wL}{4}$$

$$\circlearrowleft \Sigma M_J = 0: M - x\frac{wL}{4} = 0 \qquad M = \frac{wL}{4}x \text{ (straight)}$$

Along BC:



$$\uparrow \Sigma F_y = 0: \frac{wL}{4} - wx_1 - V = 0$$

$$V = \frac{wL}{4} - wx_1$$

straight with $\quad V = 0 \quad$ at $\quad x_1 = \frac{L}{4}$

$$\circlearrowleft \Sigma M_k = 0: M + \frac{x_1}{2}wx_1 - \left(\frac{L}{4} + x_1\right)\frac{wL}{4} = 0$$

$$M = \frac{w}{2}\left(\frac{L^2}{8} + \frac{L}{2}x_1 - x_1^2\right)$$

PROBLEM 7.29 CONTINUED

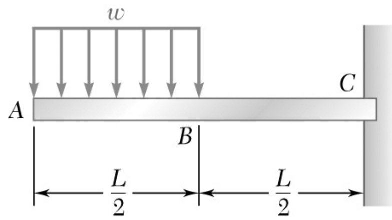
Parabola with $M = \frac{3}{32}wL^2$ at $x_1 = \frac{L}{4}$

Section *CD* by symmetry

(*b*) From diagrams:

$$|V|_{max} = \frac{wL}{4} \text{ on } AB \text{ and } CD \blacktriangleleft$$

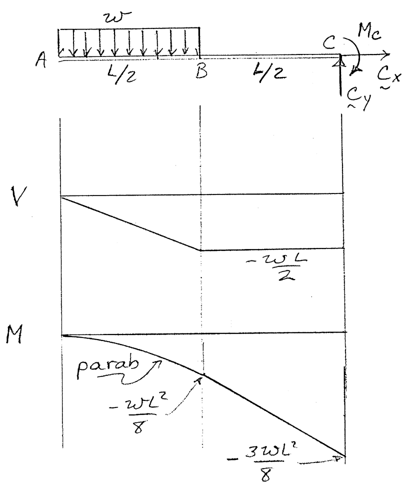
$$|M|_{max} = \frac{3wL^2}{32} \text{ at center} \blacktriangleleft$$



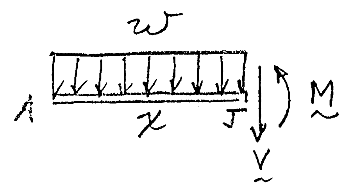
PROBLEM 7.30

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **Along AB:**



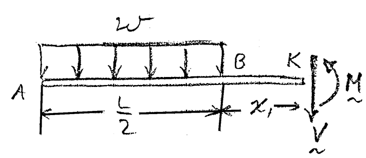
$$+\uparrow \Sigma F_y = 0: -wx - V = 0 \qquad V = -wx$$

straight with $V = -\dfrac{wL}{2}$ at $x = \dfrac{L}{2}$

$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}wx = 0 \qquad M = -\frac{1}{2}wx^2$$

parabola with $M = -\dfrac{wL^2}{8}$ at $x = \dfrac{L}{2}$

Along BC:



$$+\uparrow \Sigma F_y = 0: -w\frac{L}{2} - V = 0 \qquad V = -\frac{1}{2}wL$$

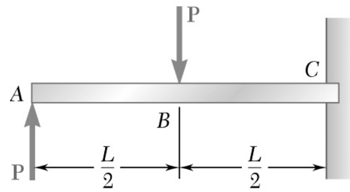
$$\circlearrowleft \Sigma M_k = 0: M + \left(x_1 + \frac{L}{4}\right)w\frac{L}{2} = 0$$

$$M = -\frac{wL}{2}\left(\frac{L}{4} + x_1\right)$$

straight with $M = -\dfrac{3}{8}wL^2$ at $x_1 = \dfrac{L}{2}$

(*b*) From diagrams: $|V|_{max} = \dfrac{wL}{2}$ on BC ◀

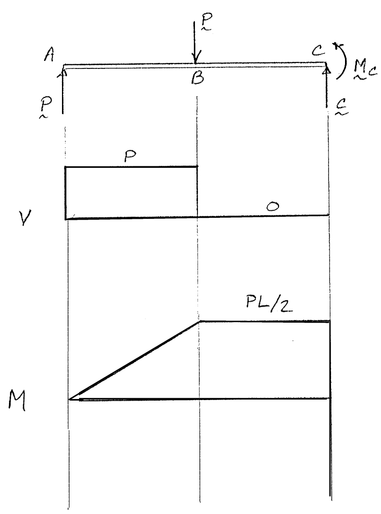
$|M|_{max} = \dfrac{3wL^2}{8}$ at C ◀



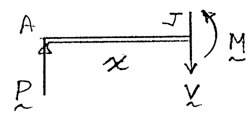
PROBLEM 7.31

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **Along AB:**

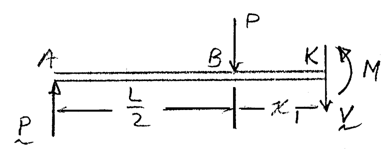


$+\uparrow \Sigma F_y = 0: \; P - V = 0 \qquad V = P$

$\Sigma M_J = 0: \; M - Px = 0 \qquad M = Px$

straight with $M = \dfrac{PL}{2}$ at B

Along BC:



$+\uparrow \Sigma F_y = 0: P - P - V = 0 \qquad V = 0$

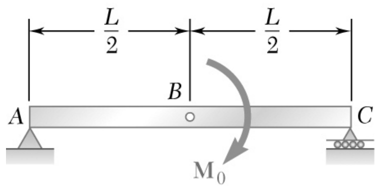
$\Sigma M_K = 0: M + Px_1 - P\left(\dfrac{L}{2} + x_1\right) = 0$

$$M = \frac{PL}{2} \quad \text{(constant)}$$

(*b*) From diagrams: $|V|_{max} = P$ along AB ◀

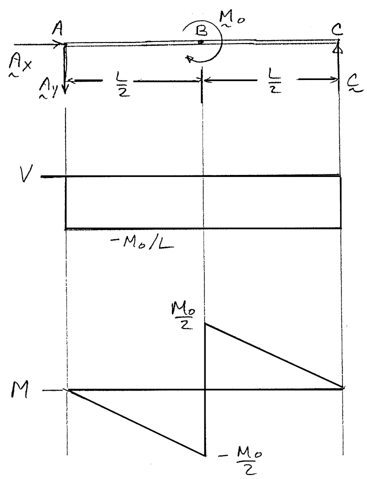
$|M|_{max} = \dfrac{PL}{2}$ along BC ◀

PROBLEM 7.32



For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



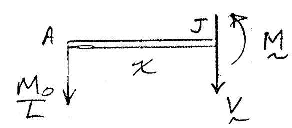
(a) **FBD Beam:**

$$\circlearrowleft \Sigma M_C = 0\colon\ LA_y - M_0 = 0$$

$$\mathbf{A}_y = \frac{M_0}{L} \downarrow$$

$$\uparrow \Sigma F_y = 0\colon -A_y + C = 0 \qquad \mathbf{C} = \frac{M_0}{L} \uparrow$$

Along AB:

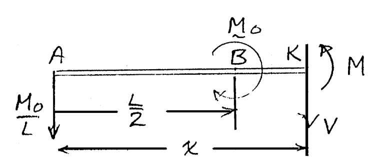


$$\uparrow \Sigma F_y = 0\colon -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_J = 0\colon\ x\frac{M_0}{L} + M = 0 \qquad M = -\frac{M_0}{L}x$$

straight with $M = -\dfrac{M_0}{2}$ at B

Along BC:



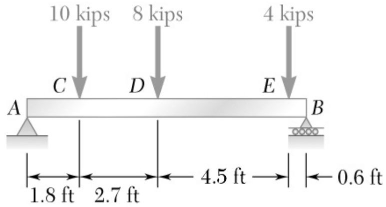
$$\uparrow \Sigma F_y = 0\colon\ -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_K = 0\colon\ M + x\frac{M_0}{L} - M_0 = 0 \qquad M = M_0\left(1 - \frac{x}{L}\right)$$

straight with $M = \dfrac{M_0}{2}$ at B $\qquad M = 0$ at C

(b) From diagrams: $\qquad |V|_{\max} = P$ everywhere ◀

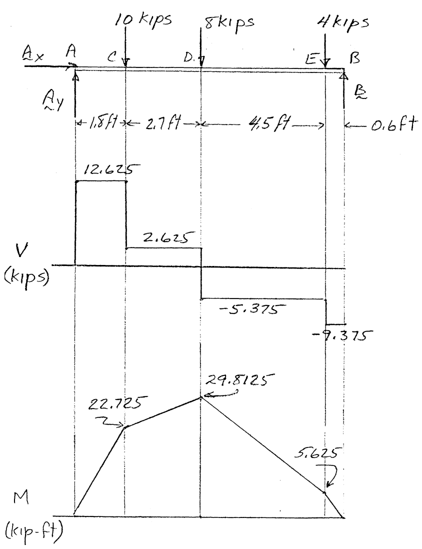
$$|M|_{\max} = \frac{M_0}{2} \text{ at } B \ ◀$$



PROBLEM 7.33

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:**

$$\circlearrowleft \Sigma M_B = 0:$$

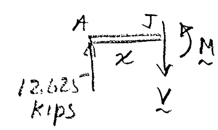
$$(.6\text{ ft})(4\text{ kips}) + (5.1\text{ ft})(8\text{ kips}) + (7.8\text{ ft})(10\text{ kips}) - (9.6\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 12.625\text{ kips} \uparrow$$

$$\uparrow \Sigma F_y = 0: \; 12.625\text{ kips} - 10\text{ kips} - 8\text{ kips} - 4\text{ kips} + B = 0$$

$$\mathbf{B} = 9.375\text{ kips} \uparrow$$

Along AC:



$$\uparrow \Sigma F_y = 0: \; 12.625\text{ kips} - V = 0$$

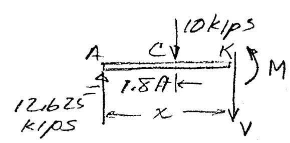
$$V = 12.625\text{ kips}$$

$$\circlearrowleft \Sigma M_J = 0: \; M - x(12.625\text{ kips}) = 0$$

$$M = (12.625\text{ kips})x$$

$$M = 22.725\text{ kip}\cdot\text{ft at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0: \; 12.625\text{ kips} - 10\text{ kips} - V = 0$$

$$V = 2.625\text{ kips}$$

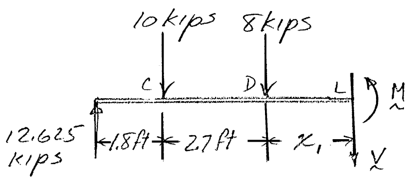
$$\circlearrowleft \Sigma M_K = 0: \; M + (x - 1.8\text{ ft})(10\text{ kips}) - x(12.625\text{ kips}) = 0$$

$$M = 18\text{ kip}\cdot\text{ft} + (2.625\text{ kips})\,x$$

$$M = 29.8125\text{ kip}\cdot\text{ft at } D \; (x = 4.5\text{ ft})$$

PROBLEM 7.33 CONTINUED

Along DE:



Along DE:

$$\uparrow \Sigma F_y = 0: (12.625 - 10 - 8) \text{ kips} - V = 0 \qquad V = -5.375 \text{ kips}$$

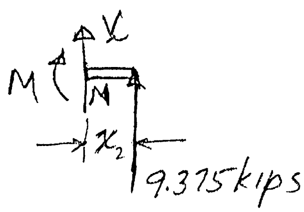
$$\circlearrowleft \Sigma M_L = 0: \; M + x_1(8 \text{ kips}) + (2.7 \text{ ft} + x_1)(10 \text{ kips})$$

$$- (4.5 \text{ ft} + x_1)(12.625 \text{ kips}) = 0$$

$$M = 29.8125 \text{ kip}\cdot\text{ft} - (5.375 \text{ kips})\, x_1$$

$$M = 5.625 \text{ kip}\cdot\text{ft at } E \;\; (x_1 = 4.5 \text{ ft})$$

Along EB:



Along EB:

$$\uparrow \Sigma F_y = 0: V + 9.375 \text{ kips} = 0 \qquad V = 9.375 \text{ kips}$$

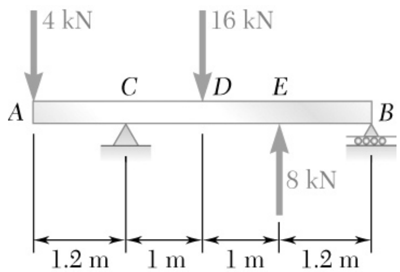
$$\circlearrowleft \Sigma M_N = 0: \; x_2(9.375 \text{ kip}) - M = 0$$

$$M = (9.375 \text{ kips})\, x_2$$

$$M = 5.625 \text{ kip}\cdot\text{ft at } E$$

(*b*) From diagrams: $|V|_{\max} = 12.63 \text{ kips on } AC$ ◀

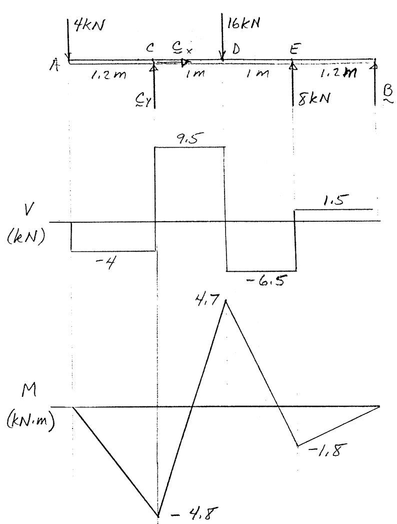
$|M|_{\max} = 29.8 \text{ kip}\cdot\text{ft at } D$ ◀



PROBLEM 7.34

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(a) **FBD Beam:**

$$\circlearrowleft \Sigma M_C = 0:$$

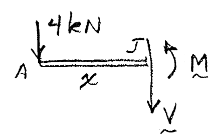
$$(1.2\text{ m})(4\text{ kN}) - (1\text{ m})(16\text{ kN}) + (2\text{ m})(8\text{ kN}) + (3.2\text{ m})B = 0$$

$$\mathbf{B} = -1.5\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: -4\text{ kN} + C_y - 16\text{ kN} + 8\text{ kN} - 1.5\text{ kN} = 0$$

$$\mathbf{C}_y = 13.5\text{ kN} \uparrow$$

Along AC:



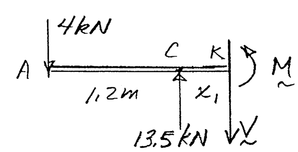
$$\uparrow \Sigma F_y = 0: -4\text{ kN} - V = 0$$

$$V = -4\text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0: M + x(4\text{ kN}) = 0 \qquad M = -4\text{ kN}\,x$$

$$M = -4.8\text{ kN}\cdot\text{m at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0: -4\text{ kN} + 13.5\text{ kN} - V = 0$$

$$V = 9.5\text{ kN}$$

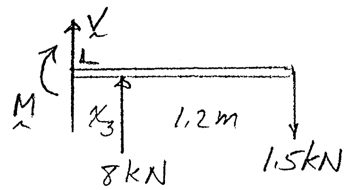
$$\circlearrowleft \Sigma M_K = 0: M + (1.2\text{ m} + x_1)(4\text{ kN}) - x_1(13.5\text{ kN}) = 0$$

$$M = -4.8\text{ kN}\cdot\text{m} + (9.5\text{ kN})x_1$$

$$M = 4.7\text{ kN}\cdot\text{m at } D \ (x_1 = 1\text{ m})$$

PROBLEM 7.34 CONTINUED

Along DE:



$$\uparrow \Sigma F_y = 0: V + 8 \text{ kN} - 1.5 \text{ kN} = 0$$

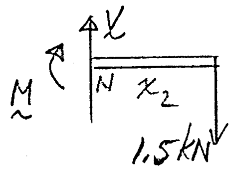
$$V = -6.5 \text{ kN}$$

$$\circlearrowleft \Sigma M_L = 0: M - x_3(8 \text{ kN}) + (x_3 + 1.2 \text{ m})(1.5 \text{ kN}) = 0$$

$$M = -1.8 \text{ kN}\cdot\text{m} + (6.5 \text{ kN})x_3$$

$$M = 4.7 \text{ kN}\cdot\text{m at } D \ (x_3 = 1 \text{ m})$$

Along EB:



$$\uparrow \Sigma F_y = 0: V - 1.5 \text{ kN} = 0$$

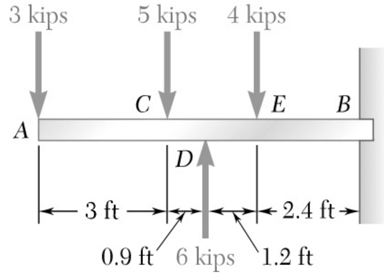
$$V = 1.5 \text{ kN}$$

$$\circlearrowleft \Sigma M_N = 0: \ x_2(1.5 \text{ kN}) + M = 0$$

$$M = -(1.5 \text{ kN})x_2 \qquad M = -1.8 \text{ kN}\cdot\text{m at } E$$

(*b*) From diagrams: $|V|_{\max} = 9.50 \text{ kN}\cdot on \ \ CD$ ◀

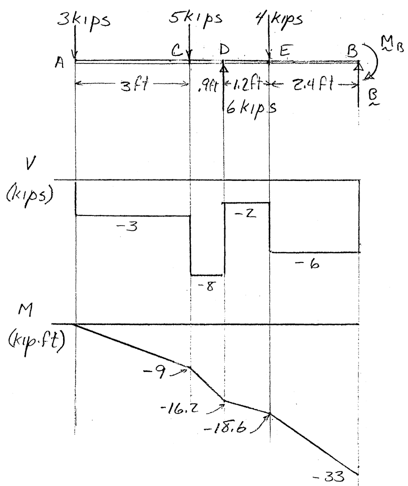
$|M|_{\max} = 4.80 \text{ kN}\cdot\text{m at } C$ ◀



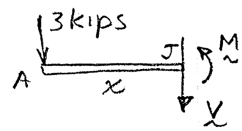
PROBLEM 7.35

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **Along AC:**

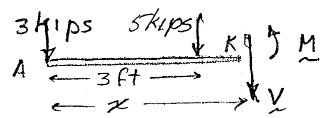


$$\uparrow \Sigma F_y = 0: -3 \text{ kip} - V = 0 \qquad V = -3 \text{ kips}$$

$$\circlearrowleft \Sigma M_J = 0: \; M + x(3 \text{ kips}) = 0 \qquad M = (3 \text{ kips})x$$

$$M = -9 \text{ kip} \cdot \text{ft at } C$$

Along CD:



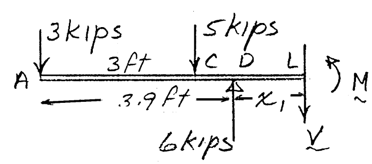
$$\uparrow \Sigma F_y = 0: -3 \text{ kips} - 5 \text{ kips} - V = 0 \qquad V = -8 \text{ kips}$$

$$\circlearrowleft \Sigma M_K = 0: \; M + (x - 3 \text{ ft})(5 \text{ kips}) + x(3 \text{ kips}) = 0$$

$$M = +15 \text{ kip} \cdot \text{ft} - (8 \text{ kips})x$$

$$M = -16.2 \text{ kip} \cdot \text{ft at } D \; (x = 3.9 \text{ ft})$$

Along DE:



$$\uparrow \Sigma F_y = 0: -3 \text{ kips} - 5 \text{ kips} + 6 \text{ kips} - V = 0$$

$$V = -2 \text{ kips}$$

$$\circlearrowleft \Sigma M_L = 0: \; M - x_1(6 \text{ kips}) + (.9 \text{ ft} + x_1)(5 \text{ kips})$$

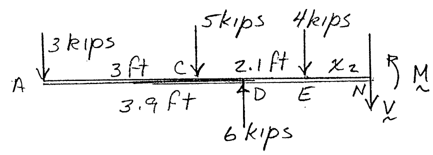
$$+ (3.9 \text{ ft} + x_1)(3 \text{ kips}) = 0$$

$$M = -16.2 \text{ kip} \cdot \text{ft} - (2 \text{ kips})x_1$$

$$M = -18.6 \text{ kip} \cdot \text{ft at } E \; (x_1 = 1.2 \text{ ft})$$

PROBLEM 7.35 CONTINUED

Along EB:



$$+\uparrow \Sigma F_y = 0: -3 \text{ kips} - 5 \text{ kips} + 6 \text{ kips} - 4 \text{ kips} - V = 0 \qquad V = -6 \text{ kips}$$

$$\Sigma M_N = 0: \; M + (4 \text{ kips})x_2 + (2.1 \text{ ft} + x_2)(5 \text{ kips})$$

$$+ (5.1 \text{ ft} + x_2)(3 \text{ kips}) - (1.2 \text{ ft} + x_2)(6 \text{ kips}) = 0$$

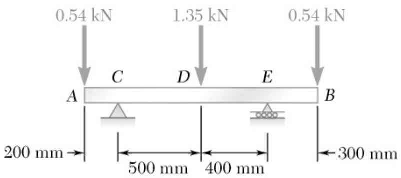
$$M = -18.6 \text{ kip}\cdot\text{ft} - (6 \text{ kips})x_2$$

$$M = -33 \text{ kip}\cdot\text{ft at } B \;\; (x_2 = 2.4 \text{ ft})$$

(b) From diagrams: $|V|_{max} = 8.00$ kips on CD ◀

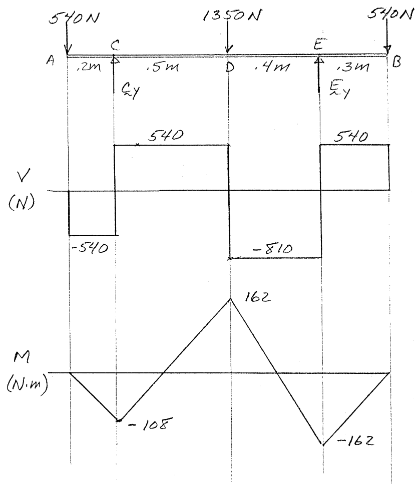
$|M|_{max} = 33.0$ kip·ft at B ◀

PROBLEM 7.36



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:** $\circlearrowleft \Sigma M_E = 0:$

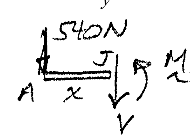
$$(1.1\text{ m})(540\text{ N}) - (0.9\text{ m})C_y + (0.4\text{ m})(1350\text{ N}) - (0.3\text{ m})(540\text{ N}) = 0$$

$$\mathbf{C}_y = 1080\text{ N}\uparrow$$

$$\uparrow \Sigma F_y = 0: -540\text{ N} + 1080\text{ N} - 1350\text{ N}$$

$$-540\text{ N} + E_y = 0 \qquad \mathbf{E}_y = 1350\text{ N}\uparrow$$

Along AC:

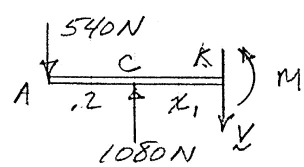


$$\uparrow \Sigma F_y = 0: -540\text{ N} - V = 0$$

$$V = -540\text{ N}$$

$$\circlearrowleft \Sigma M_J = 0: \; x(540\text{ N}) + M = 0 \qquad M = -(540\text{ N})x$$

Along CD:



$$\uparrow \Sigma F_y = 0: -540\text{ N} + 1080\text{ N} - V = 0 \qquad V = 540\text{ N}$$

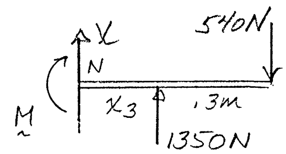
$$\circlearrowleft \Sigma M_K = 0: \; M + (0.2\text{ m} + x_1)(540\text{ N}) - x_1(1080\text{ N}) = 0$$

$$M = -108\text{ N}\cdot\text{m} + (540\text{ N})x_1$$

$$M = 162\text{ N}\cdot\text{m at } D \; (x_1 = 0.5\text{ m})$$

PROBLEM 7.36 CONTINUED

Along DE:



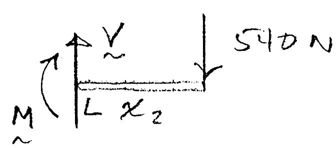
$\uparrow \Sigma F_y = 0: \quad V + 1350\text{ N} - 540\text{ N} = 0 \qquad V = -810\text{ N}$

$\circlearrowleft \Sigma M_N = 0: \; M + (x_3 + 0.3\text{ m})(540\text{ N}) - x_3(1350\text{ N}) = 0$

$$M = -162\text{ N}\cdot\text{m} + (810\text{ N})x_3$$

$$M = 162\text{ N}\cdot\text{m at } D \; (x_3 = 0.4)$$

Along EB:



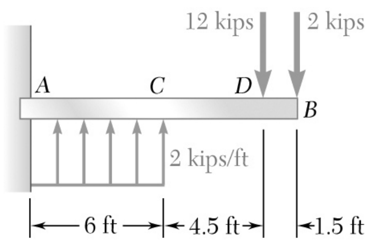
$\uparrow \Sigma F_y = 0: V - 540\text{ N} = 0 \qquad V = 540\text{ N}$

$\circlearrowleft \Sigma M_L = 0: M + x_2(540\text{ N}) = 0 \qquad M = -540\text{ N}\,x_2$

$$M = -162\text{ N}\cdot\text{m at } E \quad (x_2 = 0.3\text{ m})$$

(*b*) From diagrams $\qquad |V|_{max} = 810\text{ N on } DE$ ◀

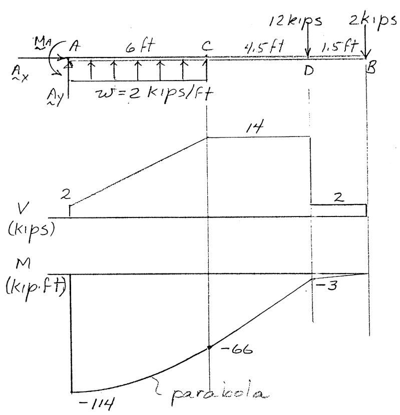
$|M|_{max} = 162.0\text{ N}\cdot\text{m at } D \text{ and } E$ ◀



PROBLEM 7.37

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:**

$$\uparrow \Sigma F_y = 0: A_y + (6 \text{ ft})(2 \text{ kips/ft}) - 12 \text{ kips} - 2 \text{ kips} = 0$$

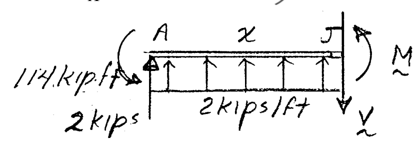
$$\mathbf{A}_y = 2 \text{ kips} \uparrow$$

$$\circlearrowleft \Sigma M_A = 0: M_A + (3 \text{ ft})(6 \text{ ft})(2 \text{ kips/ft})$$

$$- (10.5 \text{ ft})(12 \text{ kips}) - (12 \text{ ft})(2 \text{ kips}) = 0$$

$$\mathbf{M}_A = 114 \text{ kip}\cdot\text{ft} \circlearrowright$$

Along AC:



$$\uparrow \Sigma F_y = 0: 2 \text{ kips} + x(2 \text{ kips/ft}) - V = 0$$

$$V = 2 \text{ kips} + (2 \text{ kips/ft})\, x$$

$$V = 14 \text{ kips at } C \ (x = 6 \text{ ft})$$

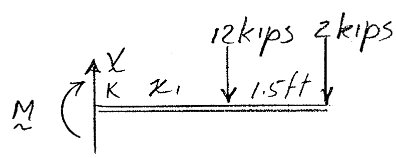
$$\circlearrowleft \Sigma M_J = 0: 114 \text{ kip}\cdot\text{ft} - x(2 \text{ kips})$$

$$- \frac{x}{2} x(2 \text{ kips/ft}) + M = 0$$

$$M = (1 \text{ kip/ft})x^2 + (2 \text{ kips})x - 114 \text{ kip}\cdot\text{ft}$$

$$M = -66 \text{ kip}\cdot\text{ft at } C \ (x = 6 \text{ ft})$$

Along CD:



$$\uparrow \Sigma F_y = 0: V - 12 \text{ kips} - 2 \text{ kips} = 0 \qquad V = 14 \text{ kips}$$

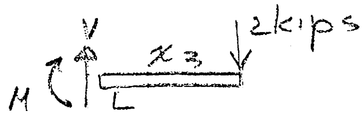
$$\circlearrowleft \Sigma M_k = 0: -M - x_1(12 \text{ kips}) - (1.5 \text{ ft} + x_1)(2 \text{ kips}) = 0$$

PROBLEM 7.37 CONTINUED

$$M = -3 \text{ kip}\cdot\text{ft} - (14 \text{ kips})x_1$$

$$M = -66 \text{ kip}\cdot\text{ft at } C \quad (x_1 = 4.5 \text{ ft})$$

Along DB:



$$\uparrow \Sigma F_y = 0: \quad V - 2 \text{ kips} = 0 \qquad V = +2 \text{ kips}$$

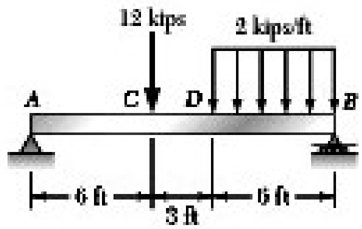
$$\circlearrowleft \Sigma M_L = 0: -M - 2 \text{ kip } x_3 = 0$$

$$M = -(2 \text{ kips})x_3$$

$$M = -3 \text{ kip}\cdot\text{ft at } D \ (x = 1.5 \text{ ft})$$

(*b*) From diagrams: $|V|_{max} = 14.00$ kips on CD ◀

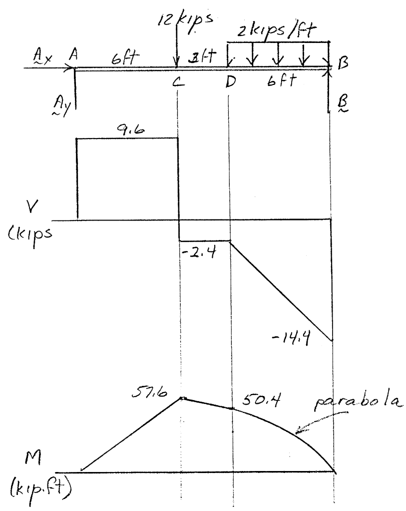
$|M|_{max} = 114.0$ kip·ft at A ◀



PROBLEM 7.38

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:**

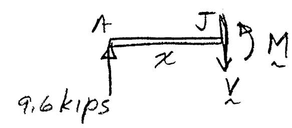
$$\Sigma M_A = (15 \text{ ft})B - (12 \text{ ft})(2 \text{ kips/ft})(6 \text{ ft}) - (6 \text{ ft})(12 \text{ kips}) = 0$$

$$\mathbf{B} = 14.4 \text{ kips} \uparrow$$

$$\uparrow \Sigma F_y = 0\text{: } A_y - 12 \text{ kips} - (2 \text{ kips/ft})(6 \text{ ft}) + 14.4 \text{ kips}$$

$$\mathbf{A}_y = 9.6 \text{ kips} \uparrow$$

Along AC:



$$\uparrow \Sigma F_y = 0\text{: } 9.6 \text{ kips} - V = 0$$

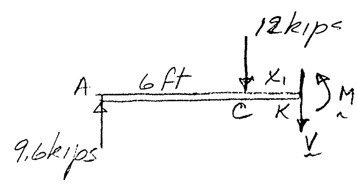
$$V = 9.6 \text{ kips}$$

$$\circlearrowleft \Sigma M_J = 0\text{: } M - x(9.6 \text{ kips}) = 0$$

$$M = (9.6 \text{ kips})x$$

$$M = 57.6 \text{ kip}\cdot\text{ft at } C \ (x = 6 \text{ ft})$$

Along CD:



$$\uparrow \Sigma F_y = 0\text{: } 9.6 \text{ kips} - 12 \text{ kips} - V = 0$$

$$V = -2.4 \text{ kips}$$

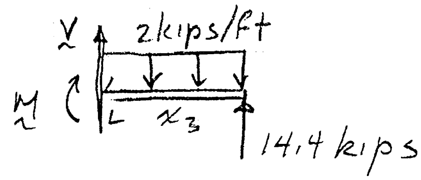
$$\circlearrowleft \Sigma M_K = 0\text{: } M + x_1(12 \text{ kips}) - (6 \text{ ft} + x_1)(9.6 \text{ kips}) = 0$$

$$M = 57.6 \text{ kip}\cdot\text{ft} - (2.4 \text{ kips})x_1$$

$$M = 50.4 \text{ kip}\cdot\text{ft at } D$$

PROBLEM 7.38 CONTINUED

Along DB:



$$\Sigma F_y = 0: \; V - x_3(2 \text{ kips/ft}) + 14.4 \text{ kips} = 0$$

$$V = -14.4 \text{ kips} + (2 \text{ kips/ft})x_3$$

$$V = -2.4 \text{ kips at } D$$

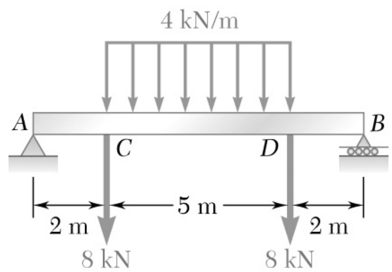
$$\circlearrowleft \Sigma M_L = 0: M + \frac{x_3}{2}(2 \text{ kips/ft})(x_3) - x_3(14.4 \text{ kips}) = 0$$

$$M = (14.4 \text{ kips})x_3 - (1 \text{ kip/ft})x_3^{\,2}$$

$$M = 50.4 \text{ kip}\cdot\text{ft at } D \; (x_3 = 6 \text{ ft})$$

(b) From diagrams: $|V|_{max} = 14.40 \text{ kips at } B$ ◀

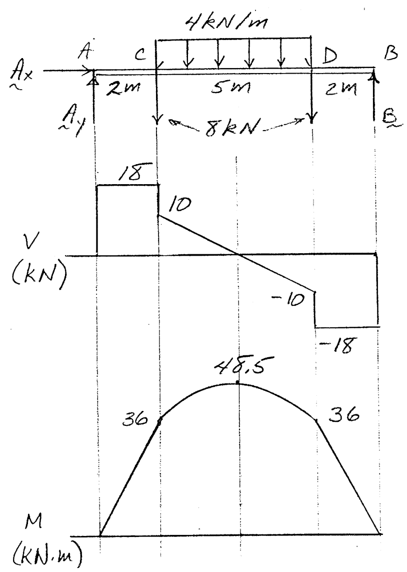
$|M|_{max} = 57.6 \text{ kip}\cdot\text{ft at } C$ ◀



PROBLEM 7.39

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

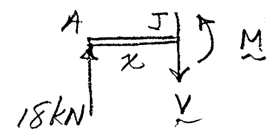
SOLUTION



(a) By symmetry:

$$A_y = B = 8\text{ kN} + \frac{1}{2}(4\text{ kN/m})(5\text{ m}) \qquad \mathbf{A}_y = \mathbf{B} = 18\text{ kN}\uparrow$$

Along AC:

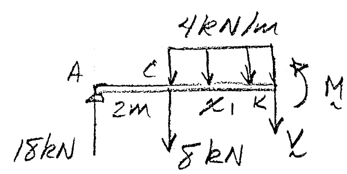


$$\uparrow \Sigma F_y = 0{:}\ 18\text{ kN} - V = 0 \qquad V = 18\text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0{:}\ M - x(18\text{ kN}) \qquad M = (18\text{ kN})x$$

$$M = 36\text{ kN}\cdot\text{m at } C\ (x = 2\text{ m})$$

Along CD:



$$\uparrow \Sigma F_y = 0{:}\ 18\text{ kN} - 8\text{ kN} - (4\text{ kN/m})x_1 - V = 0$$

$$V = 10\text{ kN} - (4\text{ kN/m})x_1$$

$$V = 0 \text{ at } x_1 = 2.5\text{ m (at center)}$$

$$\circlearrowleft \Sigma M_K = 0{:}\ M + \frac{x_1}{2}(4\text{ kN/m})x_1 + (8\text{ kN})x_1 - (2\text{ m} + x_1)(18\text{ kN}) = 0$$

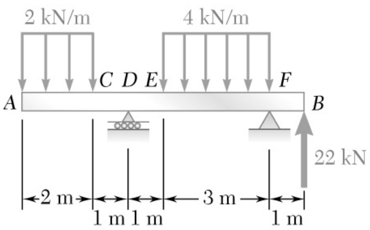
$$M = 36\text{ kN}\cdot\text{m} + (10\text{ kN/m})x_1 - (2\text{ kN/m})x_1^2$$

$$M = 48.5\text{ kN}\cdot\text{m at } x_1 = 2.5\text{ m}$$

Complete diagram by symmetry

(b) From diagrams: $|V|_{max} = 18.00\text{ kN on } AC \text{ and } DB$ ◀

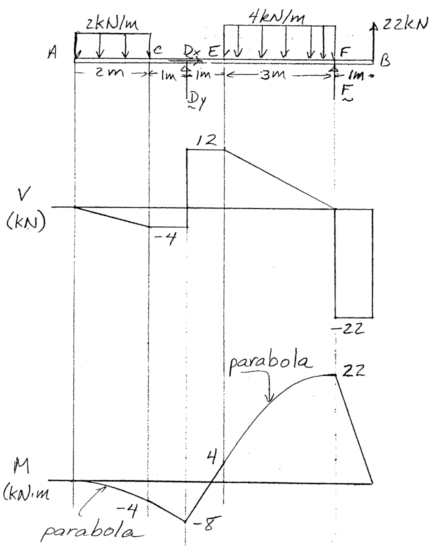
$|M|_{max} = 48.5\text{ kN}\cdot\text{m at center}$ ◀



PROBLEM 7.40

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) $\circlearrowleft \Sigma M_D = 0: (2\text{ m})(2\text{ kN/m})(2\text{ m}) - (2.5\text{ m})(4\text{ kN/m})(3\text{ m})$

$$-(4\text{ m})F - (5\text{ m})(22\text{ kN}) = 0$$

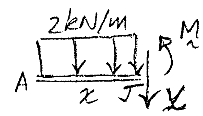
$$\mathbf{F} = 22\text{ kN} \downarrow$$

$\uparrow \Sigma F_y = 0: -(2\text{ m})(2\text{ kN/m}) + D_y$

$$-(3\text{ m})(4\text{ kN/m}) - 22\text{ kN} + 22\text{ kN} = 0$$

$$\mathbf{D}_y = 16\text{ kN} \uparrow$$

Along AC:



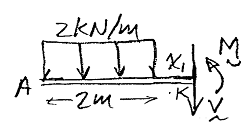
$$\uparrow \Sigma F_y = 0: -x(2\text{ kN/m}) - V = 0$$

$$V = -(2\text{ kN/m})x \qquad V = -4\text{ kN at } C$$

$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}(2\text{ kN/m})(x) \neq 0$$

$$M = -(1\text{ kN/m})x^2 \qquad M = -4\text{ kN}\cdot\text{m at } C$$

Along CD:



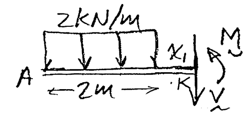
$$\uparrow \Sigma F_y = 0: -(2\text{ m})(2\text{ kN/m}) - V = 0 \qquad V = -4\text{ kN}$$

$$\circlearrowleft \Sigma M_K = 0: (1\text{ m} + x_1)(2\text{ kN/m})(2\text{ m}) = 0$$

$$M = -4\text{ kN}\cdot\text{m} - (4\text{ kN/m})x_1 \qquad M = -8\text{ kN}\cdot\text{m at } D$$

PROBLEM 7.40 CONTINUED

Along DE:

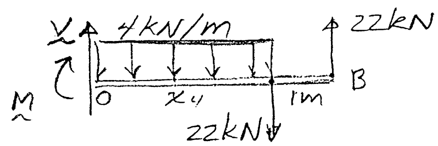


$$\uparrow \Sigma F_y = 0: -(2\ \text{kN/m})(2\ \text{m}) + 16\ \text{kN} - V = 0 \qquad V = 12\ \text{kN}$$

$$\circlearrowleft \Sigma M_L = 0: M - x_2(16\ \text{kN}) + (x_2 + 2\ \text{m})(2\ \text{kN/m})(2\ \text{m}) = 0$$

$$M = -8\ \text{kN}\cdot\text{m} + (12\ \text{kN})x_2 \qquad M = 4\ \text{kN}\cdot\text{m at } E$$

Along EF:



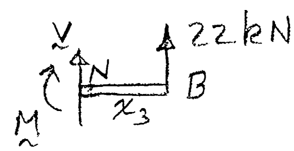
$$\uparrow \Sigma F_y = 0: V - x_4(4\ \text{kN/m}) - 22\ \text{kN} + 22\ \text{kN} = 0$$

$$V = (4\ \text{kN/m})x_4 \qquad V = 12\ \text{kN at } E$$

$$\circlearrowleft \Sigma M_0 = 0: M + \frac{x_4}{2}(4\ \text{kN/m})x_4 - (1\ \text{m})(22\ \text{kN}) = 0$$

$$M = 22\ \text{kN}\cdot\text{m} - (2\ \text{kN/m})x_4^2 \qquad M = 4\ \text{kN}\cdot\text{m at } E$$

Along FB:



$$\uparrow \Sigma F_y = 0: V + 22\ \text{kN} = 0 \qquad V = 22\ \text{kN}$$

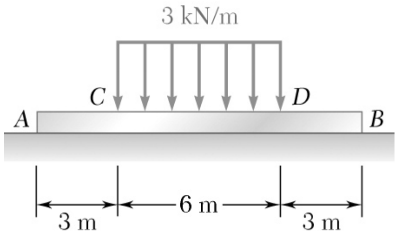
$$\circlearrowleft \Sigma M_N = 0: M - x_3(22\ \text{kN}) = 0$$

$$M = (22\ \text{kN})x_3$$

$$M = 22\ \text{kN}\cdot\text{m at } F$$

(*b*) From diagrams: $|V|_{max} = 22.0\ \text{kN on } FB$ ◀

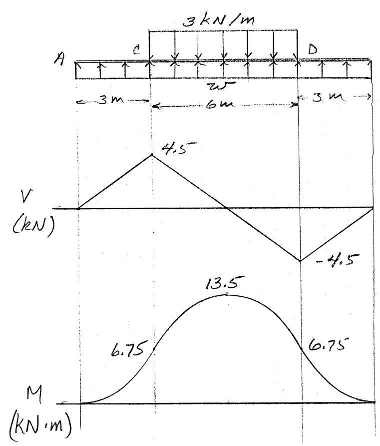
$|M|_{max} = 22.0\ \text{kN}\cdot\text{m at } F$ ◀



PROBLEM 7.41

Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

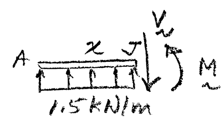
SOLUTION



(*a*) $+\uparrow \Sigma F_y = 0\text{: } (12\text{ m})w - (6\text{ m})(3\text{ kN/m}) = 0$

$$w = 1.5\text{ kN/m}$$

Along AC:



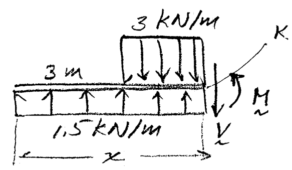
$$+\uparrow \Sigma F_y = 0\text{: } x(1.5\text{ kN/m}) - V = 0 \qquad V = (1.5\text{ kN/m})x$$

$$V = 4.5\text{ kN at } C$$

$$\circlearrowleft \Sigma M_J = 0\text{: } M - \frac{x}{2}(1.5\text{ kN/m})(x) = 0$$

$$M = (0.75\text{ kN/m})x^2 \qquad M = 6.75\text{ N}\cdot\text{m at } C$$

Along CD:



$$+\uparrow \Sigma F_y = 0\text{: } x(1.5\text{ kN/m}) - (x - 3\text{ m})(3\text{ kN/m}) - V = 0$$

$$V = 9\text{ kN} - (1.5\text{ kN/m})x \qquad V = 0 \text{ at } x = 6\text{ m}$$

$$\circlearrowleft \Sigma M_K = 0\text{: } M + \left(\frac{x - 3\text{ m}}{2}\right)(3\text{ kN/m})(x - 3\text{ m}) - \frac{x}{2}(1.5\text{ kN/m})x = 0$$

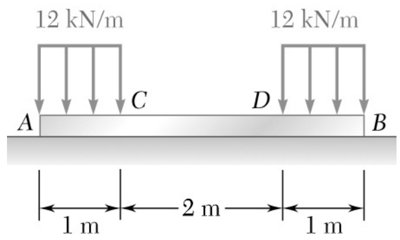
$$M = -13.5\text{ kN}\cdot\text{m} + (9\text{ kN})x - (0.75\text{ kN/m})x^2$$

$$M = 13.5\text{ kN}\cdot\text{m at center } (x = 6\text{ m})$$

Finish by symmetry

(*b*) From diagrams: $|V|_{max} = 4.50\text{ kN at } C \text{ and } D$ ◀

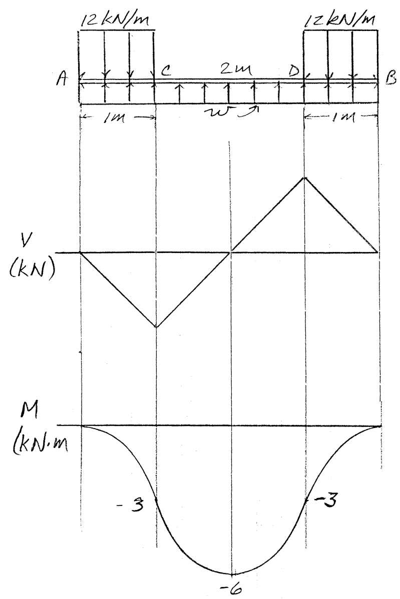
$|M|_{max} = 13.50\text{ kN}\cdot\text{m at center}$ ◀



PROBLEM 7.42

Assuming the upward reaction of the ground on beam AB to be uniformly distributed, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

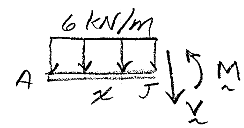


(a) **FBD Beam:**

$$\uparrow \Sigma F_y = 0: (4\text{ m})(w) - (2\text{ m})(12\text{ kN/m}) = 0$$

$$w = 6\text{ kN/m}$$

Along AC:



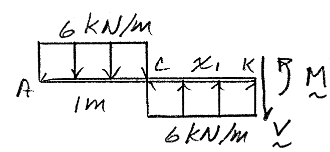
$$\uparrow \Sigma F_y = 0: -x(6\text{ kN/m}) - V = 0 \qquad V = -(6\text{ kN/m})x$$

$$V = -6\text{ kN at } C(x = 1\text{ m})$$

$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}(6\text{ kN/m})(x) = 0$$

$$M = -(3\text{ kN/m})x^2 \qquad M = -3\text{ kN}\cdot\text{m at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0: -(1\text{ m})(6\text{ kN/m}) + x_1(6\text{ kN/m}) - v = 0$$

$$V = (6\text{ kN/m})(1\text{ m} - x_1) \qquad V = 0 \text{ at } x_1 = 1\text{ m}$$

$$\circlearrowleft \Sigma M_K = 0: M + (0.5\text{ m} + x_1)(6\text{ kN/m})(1\text{ m}) - \frac{x_1}{2}(6\text{ kN/m})x_1 = 0$$

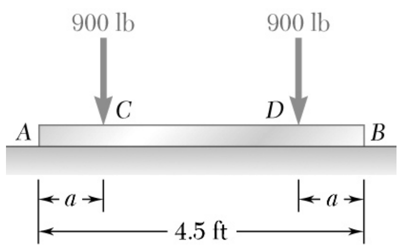
$$M = -3\text{ kN}\cdot\text{m} - (6\text{ kN})x_1 + (3\text{ kN/m})x_1^2$$

$$M = -6\text{ kN}\cdot\text{m at center } (x_1 = 1\text{ m})$$

Finish by symmetry

(b) From diagrams: $|V|_{max} = 6.00\text{ kN at } C \text{ and } D$ ◀

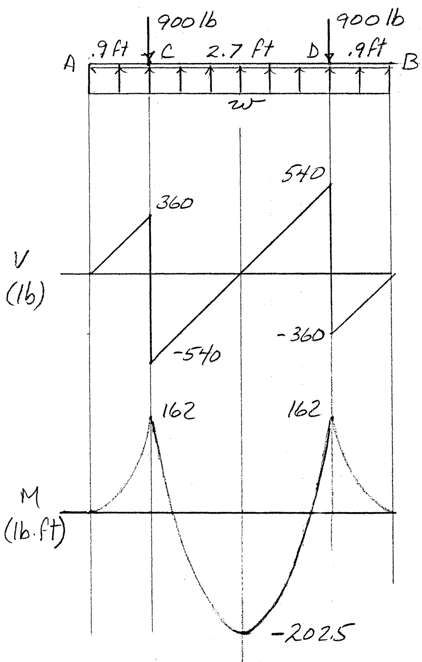
$|M|_{max} = 6.00\text{ kN at center}$ ◀



PROBLEM 7.43

Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $a = 0.9$ ft, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

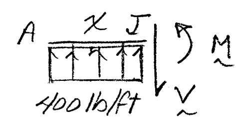


(*a*) **FBD Beam:**

$$+\uparrow \Sigma F_y = 0: (4.5 \text{ ft})w - 900 \text{ lb} - 900 \text{ lb} = 0$$

$$w = 400 \text{ lb/ft}$$

Along AC:



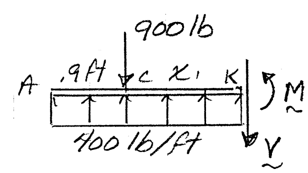
$$+\uparrow \Sigma F_y = 0: x(400 \text{ lb}) - V = 0 \qquad V = (400 \text{ lb})x$$

$$V = 360 \text{ lb at } C \quad (x = 0.9 \text{ ft})$$

$$\Sigma M_J = 0: M - \frac{x}{2}(400 \text{ lb/ft})x = 0$$

$$M = (200 \text{ lb/ft})x^2 \qquad M = 162 \text{ lb}\cdot\text{ft at } C$$

Along CD:



$$+\uparrow \Sigma F_y = 0: (0.9 \text{ ft} + x_1)(400 \text{ lb/ft}) - 900 \text{ lb} - V = 0$$

$$V = -540 \text{ lb} + (400 \text{ lb/ft})x_1 \qquad V = 0 \text{ at } x_1 = 1.35 \text{ ft}$$

$$\Sigma M_K = 0: \; M + x_1(900 \text{ lb}) - \frac{0.9 \text{ ft} + x_1}{2}(400 \text{ lb/ft})(0.9 \text{ ft} + x_1) = 0$$

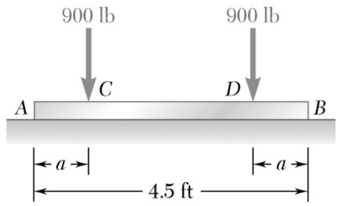
$$M = 162 \text{ lb}\cdot\text{ft} - (540 \text{ lb})x_1 + (200 \text{ lb/ft})x_1^2$$

$$M = -202.5 \text{ lb}\cdot\text{ft at center} \quad (x_1 = 1.35 \text{ ft})$$

Finish by symmetry

(*b*) From diagrams: $|V|_{max} = 540 \text{ lb at } C^+ \text{ and } D^-$ ◀

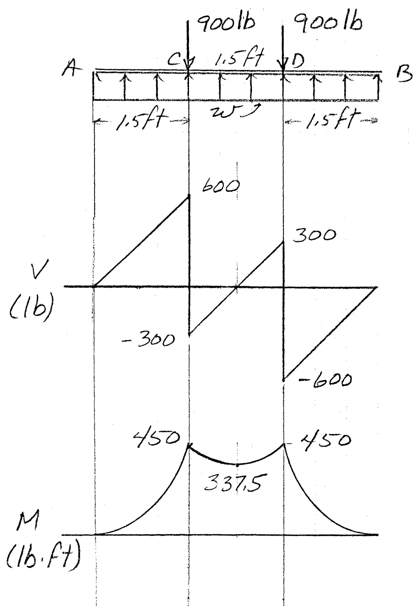
$$|M|_{max} = 203 \text{ lb}\cdot\text{ft at center}$$ ◀



PROBLEM 7.44

Solve Prob. 7.43 assuming that $a = 1.5$ ft.

SOLUTION

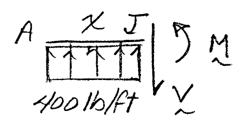


(*a*) **FBD Beam:**

$$\uparrow \Sigma F_y = 0: (4.5 \text{ ft})w - 900 \text{ lb} - 900 \text{ lb} = 0$$

$$w = 400 \text{ lb/ft}$$

Along AC:



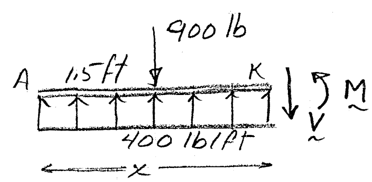
$$\uparrow \Sigma F_y = 0: \qquad x(400 \text{ lb/ft}) - V = 0$$

$$V = (400 \text{ lb/ft})x \qquad V = 600 \text{ lb at } C \quad (x = 1.5 \text{ ft})$$

$$\circlearrowleft \Sigma M_J = 0: M - \frac{x}{2}(400 \text{ lb/ft})x = 0$$

$$M = (200 \text{ lb/ft})x^2 \qquad M = 450 \text{ lb}\cdot\text{ft at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0: x(400 \text{ lb/ft}) - 900 \text{ lb} - V = 0$$

$$V = -900 \text{ lb} + (400 \text{ lb/ft})x \qquad V = -300 \text{ at } x = 1.5 \text{ ft}$$

$$V = 0 \text{ at } x = 2.25 \text{ ft}$$

$$\circlearrowleft \Sigma M_K = 0: M + (x - 1.5 \text{ ft})(900 \text{ lb}) - \frac{x}{2}(400 \text{ lb/ft})x = 0$$

$$M = 1350 \text{ lb}\cdot\text{ft} - (900 \text{ lb})x + (200 \text{ lb/ft})x^2$$

$$M = 450 \text{ lb}\cdot\text{ft at } x = 1.5 \text{ ft}$$

$$M = 337.5 \text{ lb}\cdot\text{ft at } x = 2.25 \text{ ft (center)}$$

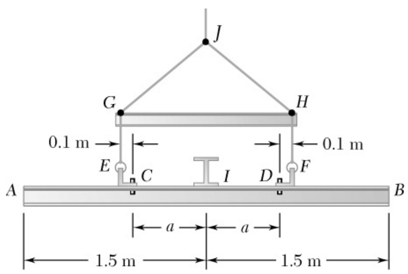
Finish by symmetry

(*b*) From diagrams:

$$|V|_{max} = 600 \text{ lb at } C^- \text{ and } D^+ \blacktriangleleft$$

$$|M|_{max} = 450 \text{ lb}\cdot\text{ft at } C \text{ and } D \blacktriangleleft$$

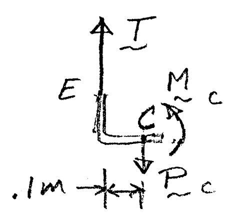
PROBLEM 7.45

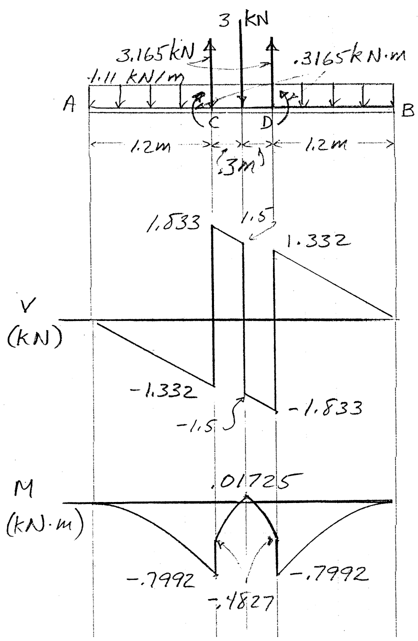


Two short angle sections *CE* and *DF* are bolted to the uniform beam *AB* of weight 3.33 kN, and the assembly is temporarily supported by the vertical cables *EG* and *FH* as shown. A second beam resting on beam *AB* at *I* exerts a downward force of 3 kN on *AB*. Knowing that $a = 0.3$ m and neglecting the weight of the ngle sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

FBD angle CE:





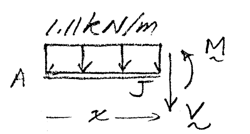
(*a*) By symmetry: $T = \dfrac{3.33\text{ kN} + 3\text{ kN}}{2} = 3.165\text{ kN}$

$$\uparrow \Sigma F_y = 0: T - P_C = 0 \qquad P_C = T = 3.165\text{ kN}$$

$$\circlearrowleft \Sigma M_C = 0: M_C - (0.1\text{ m})(3.165\text{ kN}) = 0 \qquad M_C = 0.3165\text{ kN}\cdot\text{m}$$

By symmetry: $P_D = 3.165\text{ kN}; M_D = 0.3165\text{ kN}\cdot\text{m}$

Along AC:



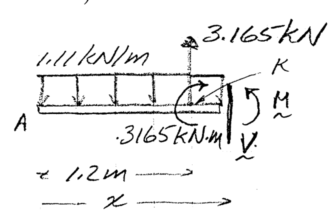
$$\uparrow \Sigma F_y = 0: -x(1.11\text{ kN/m}) - V = 0$$

$$V = -(1.11\text{ kN/m})x \qquad V = -1.332\text{ kN at } C \quad (x = 1.2\text{ m})$$

$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}(1.11\text{ kN/m})x = 0$$

$$M = (0.555\text{ kN/m})x^2 \qquad M = -0.7992\text{ kN}\cdot\text{m at } C$$

Along CI:



$$\uparrow \Sigma F_y = 0: -(1.11\text{ kN/m})x + 3.165\text{ kN} - V = 0$$

$$V = 3.165\text{ kN} - (1.11\text{ kN/m})x \qquad V = 1.5\text{ kN at } I \quad (x = 1.5\text{ m})$$

$\circlearrowleft \Sigma M_k = 0:$

$$M + (1.11\text{ kN/m})x - (x - 1.2\text{ m})(3.165\text{ kN}) - (0.3165\text{ kN}\cdot\text{m}) = 0$$

PROBLEM 7.45 CONTINUED

$$M = 3.4815 \text{ kN}\cdot\text{m} + (3.165 \text{ kN})x - (0.555 \text{ kN/m})x^2$$

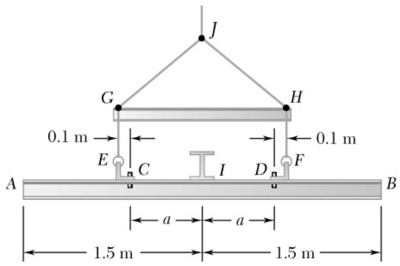
$$M = -0.4827 \text{ kN}\cdot\text{m at } C \qquad M = 0.01725 \text{ kN}\cdot\text{m at } I$$

Note: At I, the downward 3 kN force will reduce the shear V by 3 kN, from +1.5 kN to –1.5 kN, with no change in M. From I to B, the diagram can be completed by symmetry.

(b) From diagrams:

$|V|_{max} = 1.833$ kN at C and D ◀

$|M|_{max} = 799$ N·m at C and D ◀

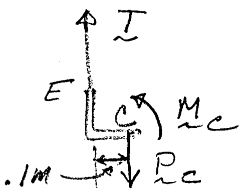


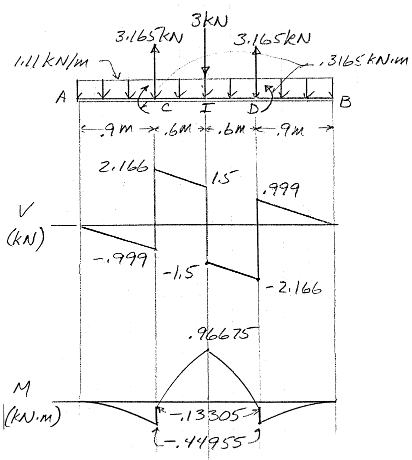
PROBLEM 7.46

Solve Prob. 7.45 when $a = 0.6$ m.

SOLUTION

FBD angle CE:





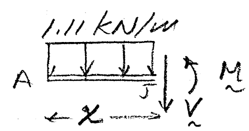
(*a*) By symmetry: $T = \dfrac{3.33 \text{ kN} + 3 \text{ kN}}{2} = 3.165 \text{ kN}$

$$\uparrow \Sigma F_y = 0: T - P_C = 0 \qquad P_C = T = 3.165 \text{ kN}$$

$$\circlearrowleft \Sigma M_C = 0: M_C - (0.1 \text{ m})(3.165 \text{ kN}) = 0 \qquad M_C = 0.3165 \text{ kN}\cdot\text{m}$$

By symmetry: $P_D = 3.165$ kN $\qquad M_D = 0.3165 \text{ kN}\cdot\text{m}$

Along AC:



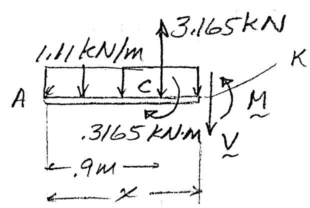
$$\uparrow \Sigma F_y = 0: -(1.11 \text{ kN/m})x - V = 0$$

$$V = -(1.11 \text{ kN/m})x \qquad V = -0.999 \text{ kN at } C \ \ (x = 0.9 \text{ m})$$

$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}(1.11 \text{ kN/m})x = 0$$

$$M = -(0.555 \text{ kN/m})x^2 \qquad M = -0.44955 \text{ kN}\cdot\text{m at } C$$

Along CI:



$$\uparrow \Sigma F_y = 0: -x(1.11 \text{ kN/m}) + 3.165 \text{ kN} - V = 0$$

$$V = 3.165 \text{ kN} - (1.11 \text{ kN/m})x \qquad V = 2.166 \text{ kN at } C$$

$$V = 1.5 \text{ kN at } I \ \ (x = 1.5 \text{ m})$$

$\circlearrowleft \Sigma M_K = 0$:

$$M - 0.3165 \text{ kN}\cdot\text{m} + (x - 0.9 \text{ m})(3.165 \text{ kN}) + \frac{x}{2}(1.11 \text{ kN/m})x = 0$$

PROBLEM 7.46 CONTINUED

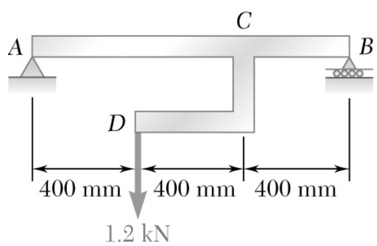
$$M = -2.532 \text{ kN}\cdot\text{m} + (3.165 \text{ kN})x - (0.555 \text{ kN/m})x^2$$

$$M = -0.13305 \text{ kN}\cdot\text{m at } C \qquad M = 0.96675 \text{ kN}\cdot\text{m at } I$$

Note: At I, the downward 3 kN force will reduce the shear V by 3 kN, from +1.5 kN to –1.5 kN, with no change in M. From I to B, the diagram can be completed by symmetry.

(*b*) From diagrams: $|V|_{max} = 2.17$ kN at C and D ◀

$|M|_{max} = 967$ N·m at I ◀

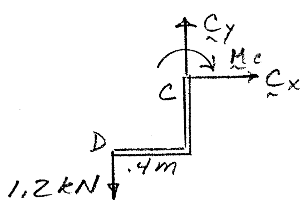


PROBLEM 7.47

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the shear and bending moment (*a*) just to the left of *C*, (*b*) just to the right of *C*.

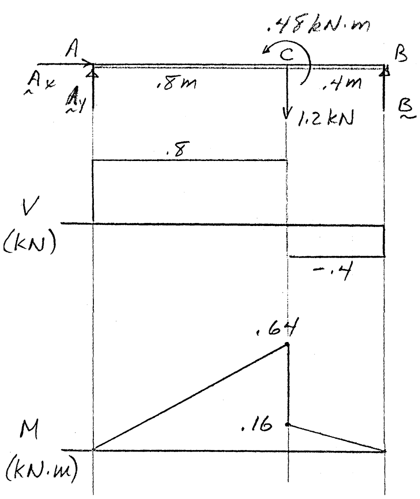
SOLUTION

FBD CD:



$$\uparrow \Sigma F_y = 0: -1.2 \text{ kN} + C_y = 0 \qquad \mathbf{C}_y = 1.2 \text{ kN}\uparrow$$

$$\circlearrowleft \Sigma M_C = 0: (0.4 \text{ m})(1.2 \text{ kN}) - M_C = 0 \qquad M_C = 0.48 \text{ kN}\cdot\text{m}$$



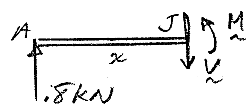
FBD Beam:

$$\circlearrowleft \Sigma M_A = 0: (1.2 \text{ m})B + 0.48 \text{ kN}\cdot\text{m} - (0.8 \text{ m})(1.2 \text{ kN}) = 0$$

$$\mathbf{B} = 0.4 \text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 1.2 \text{ kN} + 0.4 \text{ kN} = 0 \qquad \mathbf{A}_y = 0.8 \text{ kN}\uparrow$$

Along AC:

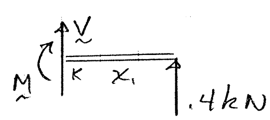


$$\uparrow \Sigma F_y = 0: 0.8 \text{ kN} - V = 0 \qquad V = 0.8 \text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0: M - x(0.8 \text{ kN}) = 0 \qquad M = (0.8 \text{ kN})x$$

$$M = 0.64 \text{ kN}\cdot\text{m at } x = 0.8 \text{ m}$$

Along CB:



$$\uparrow \Sigma F_y = 0: \quad V + 0.4 \text{ kN} = 0 \qquad V = -0.4 \text{ kN}$$

$$\circlearrowleft \Sigma M_K = 0: x_1(0.4 \text{ kN}) - M = 0 \qquad M = (0.4 \text{ kN})x_1$$

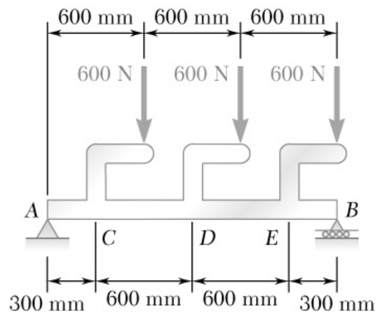
$$M = 0.16 \text{ kN}\cdot\text{m at } x_1 = 0.4 \text{ m}$$

(*a*) Just left of *C*: $V = 800 \text{ N}$ ◀

$M = 640 \text{ N}\cdot\text{m}$ ◀

(*b*) Just right of *C*: $V = -400 \text{ N}$ ◀

$M = 160.0 \text{ N}\cdot\text{m}$ ◀

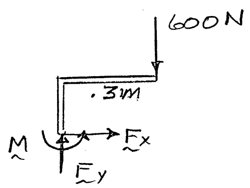


PROBLEM 7.48

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

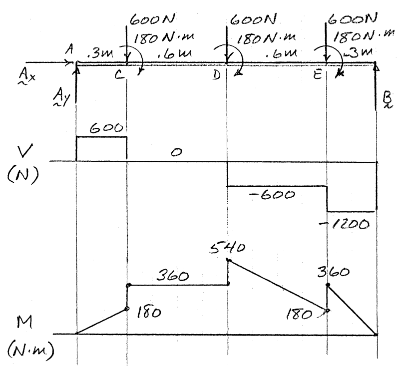
FBD angle:



$$+\uparrow \Sigma F_y = 0:\ F_y - 600\text{ N} = 0 \qquad F_y = 600\text{ N}$$

$$\circlearrowleft \Sigma M_{\text{Base}} = 0:\ \ M - (0.3\text{ m})(600\text{ N}) = 0 \qquad M = 180\text{ N}\cdot\text{m}$$

All three angles are the same.



FBD Beam:

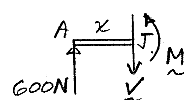
$$\circlearrowleft \Sigma M_A = 0:\ (1.8\text{ m})B - 3(180\text{ N}\cdot\text{m})$$

$$-(0.3\text{ m} + 0.9\text{ m} + 1.5\text{ m})(600\text{ N}) = 0$$

$$\mathbf{B} = 1200\text{ N}\uparrow$$

$$+\uparrow \Sigma F_y = 0:\ A_y - 3(600\text{ N}) + 1200\text{ N} = 0 \qquad \mathbf{A}_y = 600\text{ N}\uparrow$$

Along AC:

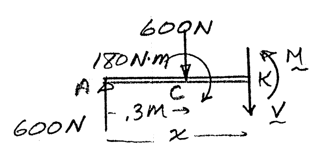


$$+\uparrow \Sigma F_y = 0:\ 600\text{ N} - V = 0 \qquad V = 600\text{ N}$$

$$\circlearrowleft \Sigma M_J = 0:\ M - x(600\text{ N}) = 0$$

$$M = (600\text{ N})x = 180\text{ N}\cdot\text{m at } x = .3\text{ m}$$

Along CD:



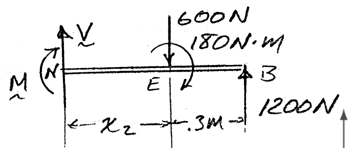
$$+\uparrow \Sigma F_y = 0:\ 600\text{ N} - 600\text{ N} - V = 0 \qquad V = 0$$

$$\circlearrowleft \Sigma M_K = 0:\ M + (x - 0.3\text{ m})(600\text{ N}) - 180\text{ N}\cdot\text{m} - x(600\text{ N}) = 0$$

$$M = 360\text{ N}\cdot\text{m}$$

PROBLEM 7.48 CONTINUED

Along DE:

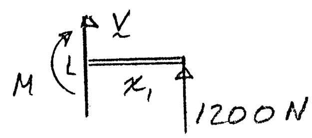


$$\Sigma F_y = 0: V - 600\text{ N} + 1200\text{ N} = 0 \qquad V = -600\text{ N}$$

$$\Sigma M_N = 0: -M - 180\text{ N}\cdot\text{m} - x_2(600\text{ N}) + (x_2 + 0.3\text{ m})(1200\text{ N}) = 0$$

$$M = 180\text{ N}\cdot\text{m} + (600\text{ N})x_2 = 540\text{ N}\cdot\text{m at D}, x_2 = 0.6\text{ m}$$
$$M = 180\text{ N}\cdot\text{m at E } (x_2 = 0)$$

Along EB:



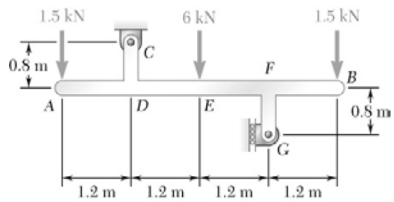
$$\Sigma F_y = 0: V + 1200\text{ N} = 0 \qquad V = -1200\text{ N}$$

$$\Sigma M_L = 0: x_1(1200\text{ N}) - M = 0 \qquad M = (1200\text{ N})x_1$$

$$M = 360\text{ N}\cdot\text{m at } x_1 = 0.3\text{ m}$$

From diagrams: $|V|_{max} = 1200\text{ N on } EB$ ◀

$|M|_{max} = 540\text{ N}\cdot\text{m at } D^+$ ◀

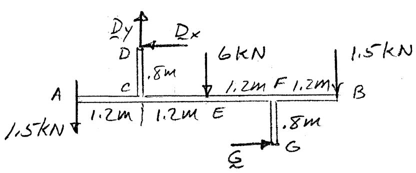


PROBLEM 7.49

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Whole:



$$\circlearrowleft \Sigma M_D = 0: (1.2\text{ m})(1.5\text{ kN}) - (1.2\text{ m})(6\text{ kN})$$

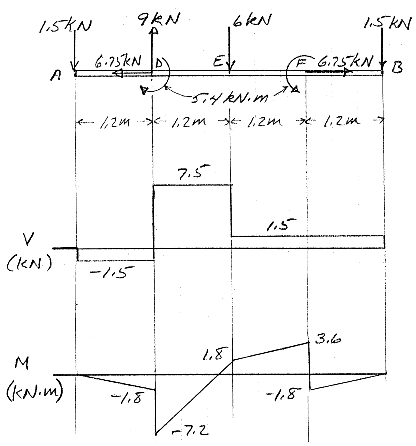
$$-(3.6\text{ m})(1.5\text{ kN}) + (1.6\text{ m})G = 0$$

$$\mathbf{G} = 6.75\text{ kN} \longrightarrow$$

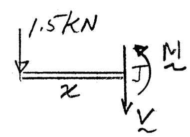
$$\longrightarrow \Sigma F_x = 0: -D_x + G = 0 \qquad \mathbf{D}_x = 6.75\text{ kN} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: D_y - 1.5\text{ kN} - 6\text{ kN} - 1.5\text{ kN} = 0 \qquad \mathbf{D}_y = 9\text{ kN} \uparrow$$

Beam AB, with forces **D** and **G** replaced by equivalent force/couples at *C* and *F*



Along AD:

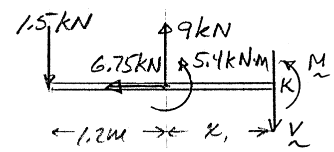


$$\uparrow \Sigma F_y = 0: -1.5\text{ kN} - V = 0 \qquad V = -1.5\text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0: x(1.5\text{ kN}) + M = 0 \qquad M = -(1.5\text{ kN})x$$

$$M = -1.8\text{ kN at } x = 1.2\text{ m}$$

Along DE:



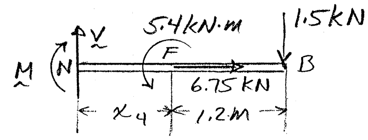
$$\uparrow \Sigma F_y = 0: -1.5\text{ kN} + 9\text{ kN} - V = 0 \qquad V = 7.5\text{ kN}$$

$$\circlearrowleft \Sigma M_K = 0: M + 5.4\text{ kN}\cdot\text{m} - x_1(9\text{ kN}) + (1.2\text{ m} + x_1)(1.5\text{ kN}) = 0$$

$$M = 7.2\text{ kN}\cdot\text{m} + (7.5\text{ kN})x_1 \qquad M = 1.8\text{ kN}\cdot\text{m at } x_1 = 1.2\text{ m}$$

PROBLEM 7.49 CONTINUED

Along EF:



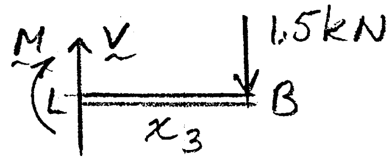
$$\uparrow \Sigma F_y = 0: V - 1.5 \text{ kN} = 0 \qquad V = 1.5 \text{ kN}$$

$$\circlearrowleft \Sigma M_N = 0: -M + 5.4 \text{ kN}\cdot\text{m} - (x_4 + 1.2 \text{ m})(1.5 \text{ kN})$$

$$M = 3.6 \text{ kN}\cdot\text{ m} - (1.5 \text{ kN})x_4$$

$M = 1.8$ kN·m at $x_4 = 1.2$ m; M = 3.6 kN·m at $x_4 = 0$

Along FB:



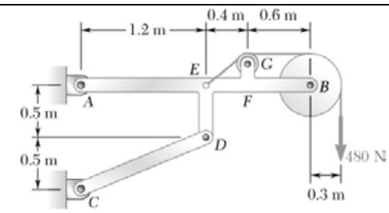
$$\uparrow \Sigma F_y = 0: V - 1.5 \text{ kN} = 0 \qquad V = 1.5 \text{ kN}$$

$$\circlearrowleft \Sigma M_L = 0: -M - x_3(1.5 \text{ kN}) = 0 \qquad M = (-1.5 \text{ kN})x_3$$

$$M = -1.8 \text{ kN}\cdot\text{m at } x_3 = 1.2 \text{ m}$$

From diagrams: $|V|_{max} = 7.50$ kN on DE ◀

$|M|_{max} = 7.20$ kN·m at D^+ ◀

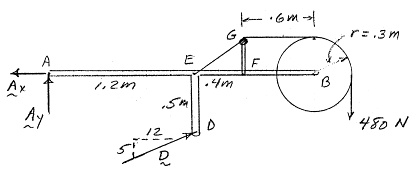


PROBLEM 7.50

Neglecting the size of the pulley at G, (a) draw the shear and bending-moment diagrams for the beam AB, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Whole:



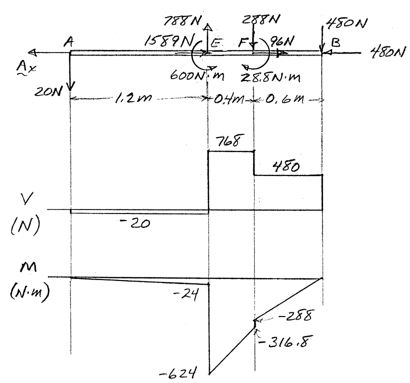
(a) $$\circlearrowleft \Sigma M_A = 0: (0.5\text{ m})\frac{12}{13}D + (1.2\text{ m})\frac{5}{13}D - (2.5\text{ m})(480\text{ N}) = 0$$

$$D = 1300\text{ N}$$

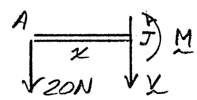
$$\uparrow \Sigma F_y = 0: A_y + \frac{5}{13}(1300\text{ N}) - 480\text{ N} = 0$$

$$A_y = -20\text{ N} \qquad \mathbf{A}_y = 20\text{ N}\downarrow$$

Beam AB with pulley forces and force at D replaced by equivalent force-couples at B, F, E



Along AE:

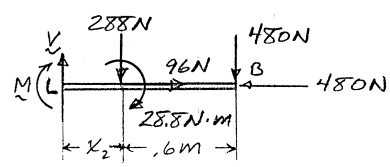


$$\uparrow \Sigma F_y = 0: -20\text{ N} - V = 0 \qquad V = -20\text{ N}$$

$$\circlearrowleft \Sigma M_J = 0: M + x(20\text{ N}) \qquad M = -(20\text{ N})x$$

$$M = -24\text{ N}\cdot\text{m at } x = 1.2\text{ m}$$

Along EF:



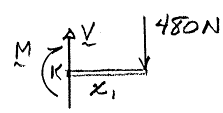
$$\uparrow \Sigma F_y = 0: V - 288\text{ N} - 480\text{ N} = 0 \qquad V = 768\text{ N}$$

$$\circlearrowleft \Sigma M_L = 0: -M - x_2(288\text{ N}) - (28.8\text{ N}\cdot\text{m}) - (x_2 + 0.6\text{ m})(480\text{ N}) = 0$$

$$M = -316.8\text{ N}\cdot\text{m} - (768\text{ N})x_2$$

$$M = -316.8\text{ N}\cdot\text{m at } x_2 = 0; \quad M = -624\text{ N}\cdot\text{m at } x_2 = 0.4\text{ m}$$

Along FB:



$$\uparrow \Sigma F_y = 0: V - 480\text{ N} = 0 \qquad V = 480\text{ N}$$

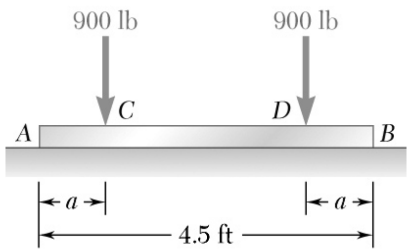
$$\circlearrowleft \Sigma M_K = 0: -M - x_1(480\text{ N}) = 0 \qquad M = -(480\text{ N})x_1$$

$$M = -288\text{ N}\cdot\text{m at } x_1 = 0.6\text{ m}$$

PROBLEM 7.50 CONTINUED

(*b*) From diagrams: $|V|_{max} = 768$ N along EF ◀

$|M|_{max} = 624$ N·m at E^+ ◀



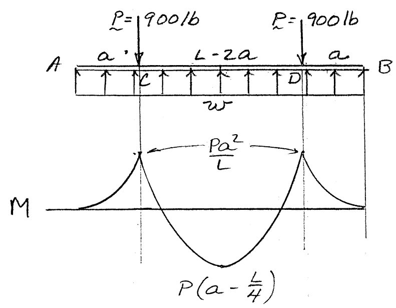
PROBLEM 7.51

For the beam of Prob. 7.43, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{\max}$.

(*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

SOLUTION

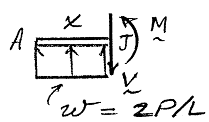
FBD Beam:



$$+\uparrow \Sigma F_y = 0:\ Lw - 2P = 0$$

$$w = 2\frac{P}{L}$$

Along AC:



$$\Sigma M_J = 0:\ M - \frac{x}{2}\left(\frac{2P}{L}x\right) = 0 \qquad M = \frac{P}{L}x^2$$

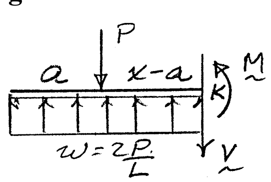
$$M = \frac{P}{L}a^2 \quad \text{at} \quad x = a$$

$$\Sigma M_K = 0:\ M + (x-a)P - \frac{x}{2}\left(\frac{2P}{L}x\right) = 0$$

$$M = P(a - x) + \frac{P}{L}x^2 = \frac{Pa^2}{L} \qquad \text{at} \qquad x = a$$

$$M = P\left(a - \frac{L}{4}\right) \text{ at } x = \frac{L}{2}$$

Along CD:



This is *M* min by symmetry–see moment diagram completed by symmetry.

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$:

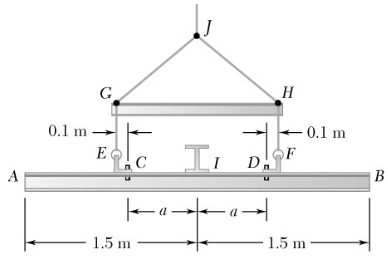
$$P\frac{a^2}{L} = -P\left(a - \frac{L}{4}\right)$$

or $$a^2 + La - \frac{L^2}{4} = 0$$

Solving: $$a = \frac{-1 \pm \sqrt{2}}{2}L$$

Positive answer (*a*) $a = 0.20711L = 0.932$ ft ◀

(*b*) $|M|_{\max} = 0.04289PL = 173.7$ lb·ft ◀

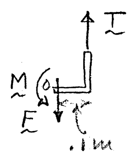


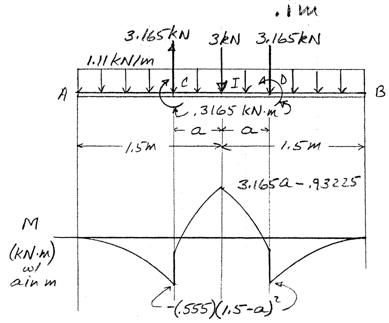
PROBLEM 7.52

For the assembly of Prob. 7.45, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Prob. 7.51.)

SOLUTION

FBD Angle:





By symmetry of whole arrangement:

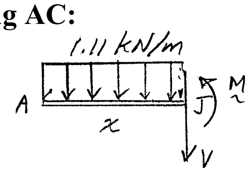
$$T = \frac{3.33 \text{ kN} + 3 \text{ kN}}{2} = 3.165 \text{ kN}$$

$$\uparrow \Sigma F_y = 0: T - F = 0 \qquad F = 3.165 \text{ kN}$$

$$\Sigma M_0 = 0: M - (0.1 \text{ m})(3.165 \text{ kN}) = 0 \qquad M = 0.3165 \text{ kN}\cdot\text{m}$$

$$\Sigma M_J = 0: M + \frac{x}{2}(1.11 \text{ kN/m})x = 0$$

Along AC:

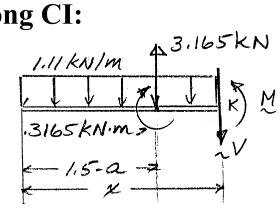


$$M = -(0.555 \text{ kN/m})x^2 = -(0.555 \text{ kN/m})(1.5 \text{ m} - a)^2$$

at C (this is $M_{\min}$)

$$\Sigma M_K = 0: M - 0.3165 \text{ kN}\cdot\text{m} + \frac{x}{2}(1.11 \text{ kN/m})x$$
$$- \left[x - (1.5 \text{ m} - a)\right](3.165 \text{ kN}) = 0$$

Along CI:



$$M = -4.431 \text{ kN}\cdot\text{m} + (3.165 \text{ kN})(x + a) - (0.555 \text{ kN/m})x^2$$

$$M_{\max}(\text{at } x = 1.5 \text{ m}) = -0.93225 \text{ kN}\cdot\text{m} + (3.165 \text{ kN})a$$

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$:

$$-0.93225 \text{ kN}\cdot\text{m} + (3.165 \text{ kN})a = (0.555 \text{ kN/m})(1.5 \text{ m} - a)^2$$

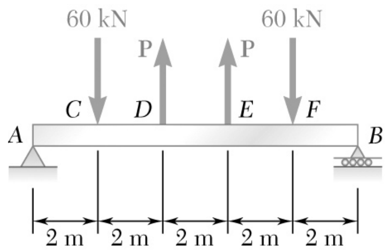
Yielding: $a^2 - (8.7027 \text{ m})a + 3.92973 \text{ m}^2 = 0$

Solving: $a = 4.3514 \pm \sqrt{13.864} = 0.4778 \text{ m},\ 8.075 \text{ m}$

Second solution out of range, so (*a*) $a = 0.478 \text{ m}$ ◀

$M_{\max} = 0.5801 \text{ kN}\cdot\text{m}$ ◀

(*b*) $M_{\max} = 580 \text{ N}\cdot\text{m}$ ◀

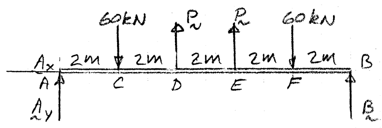


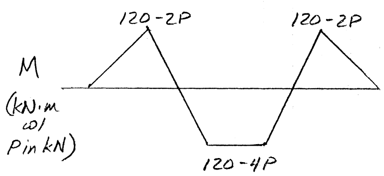
PROBLEM 7.53

For the beam shown, determine (*a*) the magnitude P of the two upward forces for which the maximum value of the bending moment is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Prob. 7.51.)

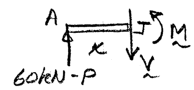
SOLUTION

By symmetry: $A_y = B = 60\text{ kN} - P$





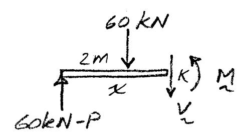
Along AC:



$$\circlearrowleft \Sigma M_J = 0: M - x(60\text{ kN} - P) = 0 \qquad M = (60\text{ kN} - P)x$$

$$M = 120\text{ kN}\cdot\text{m} - (2\text{ m})P \quad \text{at} \quad x = 2\text{ m}$$

Along CD:

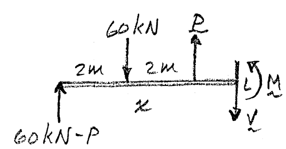


$$\circlearrowleft \Sigma M_K = 0: M + (x - 2\text{ m})(60\text{ kN}) - x(60\text{ kN} - P) = 0$$

$$M = 120\text{ kN}\cdot\text{m} - Px$$

$$M = 120\text{ kN}\cdot\text{m} - (4\text{ m})P \quad \text{at} \quad x = 4\text{ m}$$

Along DE:



$$\circlearrowleft \Sigma M_L = 0: M - (x - 4\text{ m})P + (x - 2\text{ m})(60\text{ kN}) - x(60\text{ kN} - P) = 0$$

$$M = 120\text{ kN}\cdot\text{m} - (4\text{ m})P \quad \text{(const)}$$

Complete diagram by symmetry

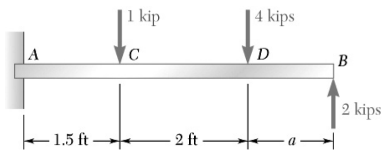
For minimum $|M|_{max}$, set $M_{max} = -M_{min}$

$$120\text{ kN}\cdot\text{m} - (2\text{ m})P = -[120\text{ kN}\cdot\text{m} - (4\text{ m})P]$$

(*a*) $P = 40.0\text{ kN}$ ◀

$M_{min} = 120\text{ kN}\cdot\text{m} - (4\text{ m})P$ (*b*) $|M|_{max} = 40.0\text{ kN}\cdot\text{m}$ ◀

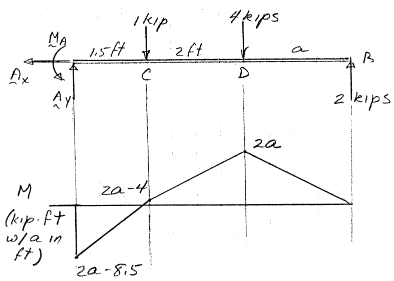
PROBLEM 7.54



For the beam and loading shown, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Prob. 7.51.)

SOLUTION

FBD Beam:



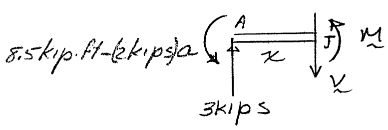
$$\Sigma M_A = 0: M_A - (1.5\text{ ft})(1\text{ kip}) - (3.5\text{ ft})(4\text{ kips}) + (3.5\text{ ft} + a)(2\text{ kips}) = 0$$

$$\mathbf{M}_A = [8.5\text{ kip}\cdot\text{ft} - (2\text{ kips})a] \circlearrowleft$$

$$\uparrow \Sigma F_y = 0: A_y - 1\text{ kip} - 4\text{ kips} + 2\text{ kips} = 0$$

$$\mathbf{A}_y = 3\text{ kips} \uparrow$$

Along AC:



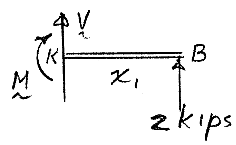
$$\circlearrowleft \Sigma M_J = 0: M - x(3\text{ kips}) + 8.5\text{ kip}\cdot\text{ft} - (2\text{ kips})a = 0$$

$$M = (3\text{ kips})x + (2\text{ kips})a - 8.5\text{ kip}\cdot\text{ft}$$

$$M = (2\text{ kips})a - 4\text{ kip}\cdot\text{ft} \quad \text{at } C(x = 1.5\text{ ft})$$

$$M = (2\text{ kips})a - 8.5\text{ kip}\cdot\text{ft} \quad \text{at } A(M_{min})$$

Along DB:

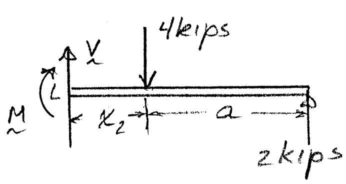


$$\circlearrowleft \Sigma M_K = 0: -M + x_1(2\text{ kips}) = 0 \qquad M = (2\text{ kips})x_1$$

$$M = (2\text{ kips})a \quad \text{at } D$$

$$\circlearrowleft \Sigma M_L = 0: (x_2 + a)(2\text{ kips}) - x_2(4\text{ kips}) - M = 0$$

Along CD:



$$M = (2\text{ kips})a - (2\text{ kips})x_2$$

$$M = (2\text{ kips})a - 4\text{ kip}\cdot\text{ft} \quad \text{at } C \quad (\text{see above})$$

For minimum $|M|_{max}$, set $M_{max}(\text{at } D) = -M_{min}(\text{at } A)$

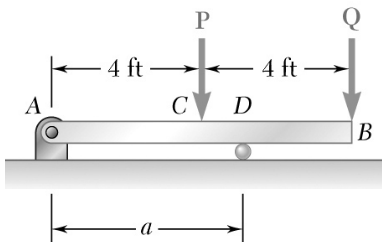
$$(2\text{ kips})a = -[(2\text{ kips})a - 8.5\text{ kip}\cdot\text{ft}]$$

$$4a = 8.5\text{ ft} \qquad a = 2.125\text{ ft}$$

(*a*) $a = 2.13\text{ ft}$ ◀

So $M_{max} = (2\text{ kips})a = 4.25\text{ kip}\cdot\text{ft}$

(*b*) $|M|_{max} = 4.25\text{ kip}\cdot\text{ft}$ ◀



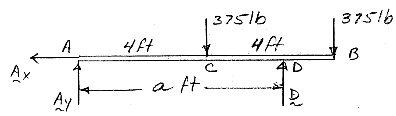
PROBLEM 7.55

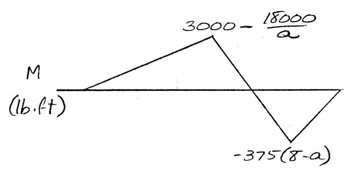
Knowing that $P = Q = 375$ lb, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Prob. 7.51.)

SOLUTION

$$\circlearrowleft \Sigma M_A = 0: (a \text{ ft})D - (4 \text{ ft})(375 \text{ lb}) - (8 \text{ ft})(375 \text{ lb}) = 0$$

FBD Beam:





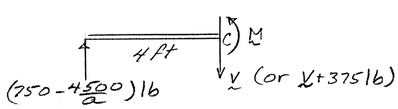
$$\mathbf{D} = \frac{4500}{a} \text{ lb} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 2(375 \text{ lb}) + \frac{4500}{a} \text{ lb} = 0$$

$$\mathbf{A}_y = \left(750 - \frac{4500}{a}\right) \text{lb} \uparrow$$

It is apparent that $M = 0$ at *A* and *B*, and that all segments of the *M* diagram are straight, so the max and min values of *M* must occur at *C* and *D*

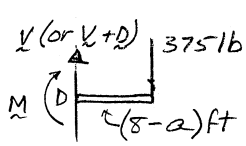
Segment AC:



$$\circlearrowleft \Sigma M_C = 0: M - (4 \text{ ft})\left(750 - \frac{4500}{a}\right)\text{lb} = 0$$

$$M = \left(3000 - \frac{18000}{a}\right)\text{lb}\cdot\text{ft}$$

Segment DB:



$$\circlearrowleft \Sigma M_D = 0: -[(8 - a)\text{ft}](375 \text{ lb}) - M = 0$$

$$M = -375(8 - a)\text{lb}\cdot\text{ft}$$

For minimum $|M|_{max}$, set $M_{max} = -M_{min}$

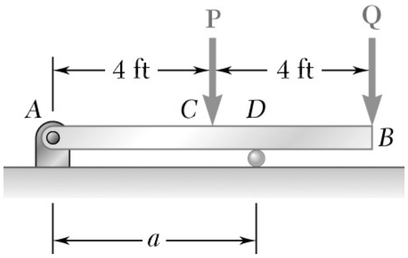
So $$3000 - \frac{18000}{a} = 375(8 - a)$$

$$a^2 = 48 \qquad a = 6.9282 \text{ ft}$$

(*a*) $a = 6.93$ ft ◀

$$M_{max} = 375(8 - a) = 401.92 \text{ lb}\cdot\text{ft}$$

(*b*) $|M|_{max} = 402$ lb·ft ◀

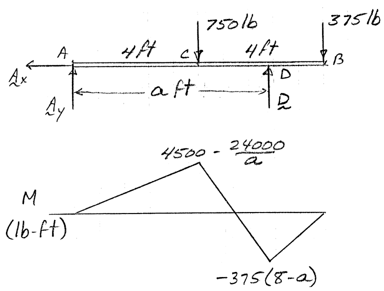


PROBLEM 7.56

Solve Prob. 7.55 assuming that $P = 750$ lb and $Q = 375$ lb.

SOLUTION

FBD Beam:



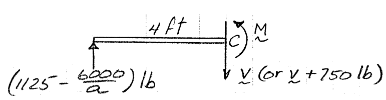
$$\circlearrowleft \Sigma M_D = 0: -(a \text{ ft})A_y + \left[(a-4)\text{ft}\right](750 \text{ lb})$$

$$-\left[(8-a)\text{ft}\right](375 \text{ lb}) = 0$$

$$\mathbf{A}_y = \left(1125 - \frac{6000}{a}\right)\text{lb} \uparrow$$

It is apparent that $M = 0$ at A and B, and that all segments of the M-diagram are straight, so $M_{\max}$ and $M_{\min}$ occur at C and D.

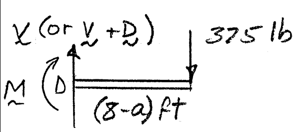
Segment AC:



$$\circlearrowleft \Sigma M_C = 0: M - (4 \text{ ft})\left(1125 - \frac{6000}{a}\right)\text{lb} = 0$$

$$M = \left(4500 - \frac{24000}{a}\right)\text{lb}\cdot\text{ft}$$

Segment DB:



$$\circlearrowleft \Sigma M_D = 0: \; -M - \left[(8-a)\text{ft}\right](375 \text{ lb}) = 0$$

$$M = -375(8-a)\text{lb}\cdot\text{ft}$$

For minimum $M_{\max}$, set $M_{\max} = -M_{\min}$

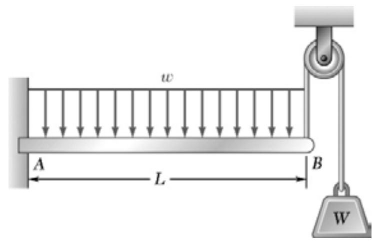
$$4500 - \frac{24000}{a} = 375(8-a)$$

$$a^2 + 4a - 64 = 0 \qquad a = -2 \pm \sqrt{68}\,(\text{need } +)$$

$$a = 6.2462 \text{ ft} \qquad (a) \qquad a = 6.25 \text{ ft} \blacktriangleleft$$

Then $\quad M_{\max} = 375(8-a) = 657.7 \text{ lb}\cdot\text{ft}$

$$(b) \qquad |M|_{\max} = 658 \text{ lb}\cdot\text{ft} \blacktriangleleft$$

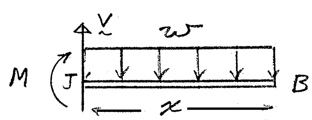


PROBLEM 7.57

In order to reduce the bending moment in the cantilever beam AB, a cable and counterweight are permanently attached at end B. Determine the magnitude of the counterweight for which the maximum absolute value of the bending moment in the beam is as small as possible and the corresponding value of $|M|_{max}$. Consider (*a*) the case when the distributed load is permanently applied to the beam, (*b*) the more general case when the distributed load may either be applied or removed.

SOLUTION

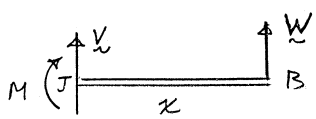
M due to distributed load:



$$\circlearrowleft \Sigma M_J = 0: -M - \frac{x}{2}wx = 0$$

$$M = -\frac{1}{2}wx^2$$

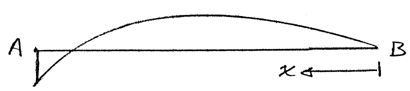
M due to counter weight:



$$\circlearrowleft \Sigma M_J = 0: -M + xw = 0$$

$$M = wx$$

(*a*) **Both applied:**



$$M = W_x - \frac{w}{2}x^2 \qquad \frac{dM}{dx} = W - wx = 0 \text{ at } x = \frac{W}{w}$$

And here $M = \dfrac{W^2}{2w} > 0$ so M_{max}; M_{min} must be at $x = L$

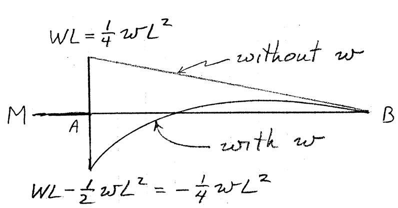
So $M_{min} = WL - \dfrac{1}{2}wL^2$. For minimum $|M|_{max}$ set $M_{max} = -M_{min}$, so

$$\frac{W^2}{2w} = -WL + \frac{1}{2}wL^2 \text{ or } W^2 + 2wLW - w^2L^2 = 0$$

$$W = -wL \pm \sqrt{2w^2L^2}\,(\text{need}+) \qquad W = \left(\sqrt{2}-1\right)wL = 0.414wL \blacktriangleleft$$

$$M_{max} = \frac{W^2}{2w} = \frac{\left(\sqrt{2}-1\right)^2}{2}wL^2 \qquad M_{max} = 0.858wL^2 \blacktriangleleft$$

(*b*) ***w* may be removed:**



Without w, $\qquad M = Wx,\ M_{max} = WL$ at A

With w (see part *a*) $\qquad M = Wx - \dfrac{w}{2}x^2, \qquad M_{max} = \dfrac{W^2}{2w}$ at $x = \dfrac{W}{w}$

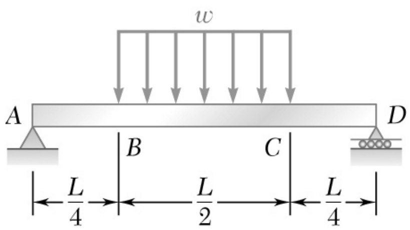
$$M_{min} = WL - \frac{1}{2}wL^2 \text{ at } x = L$$

PROBLEM 7.57 CONTINUED

For minimum $M_{\max}$, set $M_{\max}(\text{no } w) = -M_{\min}(\text{with } w)$

$$WL = -WL + \frac{1}{2}wL^2 \rightarrow W = \frac{1}{4}wL \rightarrow \qquad M_{\max} = \frac{1}{4}wL^2 \blacktriangleleft$$

With $$W = \frac{1}{4}wL \blacktriangleleft$$



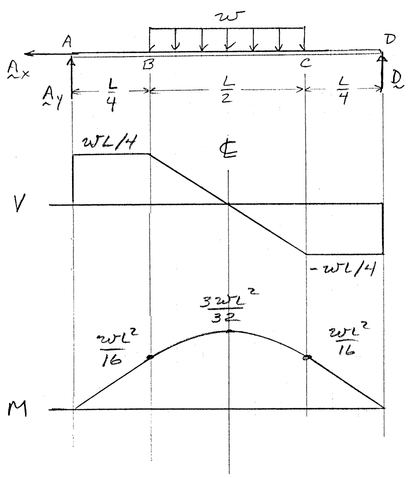
PROBLEM 7.58

Using the method of Sec. 7.6, solve Prob. 7.29.

SOLUTION

(*a*) and (*b*)

By symmetry: $A_y = D = \frac{1}{2}\left(w\frac{L}{2}\right) = \frac{wL}{4}$ or $\mathbf{A}_y = \mathbf{D} = \frac{wL}{4} \uparrow$



Shear Diag: V jumps to $A_y = \frac{wL}{4}$ at A,

and stays constant (no load) to B. From B to C, V is linear $\left(\frac{dV}{dx} = -w\right)$, and it becomes $\frac{wL}{4} - w\frac{L}{2} = -\frac{wL}{4}$ at C.

(Note: $V = 0$ at center of beam. From C to D, V is again constant.)

Moment Diag: M starts at zero at A

and increases linearly $\left(\frac{dM}{dV} = \frac{wL}{4}\right)$ to B.

$$M_B = 0 + \frac{L}{4}\left(\frac{wL}{4}\right) = \frac{wL^2}{16}.$$

From B to C M is parabolic

$\left(\frac{dM}{dx} = V, \text{ which decreases to zero at center and } -\frac{wL}{4} \text{ at } C\right),$

M is maximum at center. $M_{\max} = \frac{wL^2}{16} + \frac{1}{2}\left(\frac{L}{4}\right)\left(\frac{wL}{4}\right)$

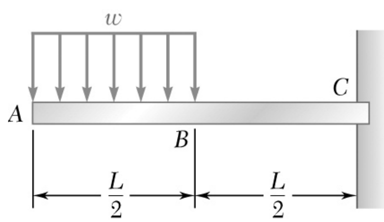
Then, M is linear with $\frac{dM}{dy} = -\frac{wL}{4}$ to D

$$M_D = 0$$

$$|V|_{\max} = \frac{wL}{4} \blacktriangleleft$$

$$|M|_{\max} = \frac{3wL^2}{32} \blacktriangleleft$$

Notes: Symmetry could have been invoked to draw second half.
Smooth transitions in M at B and C, as no discontinuities in V.



PROBLEM 7.59

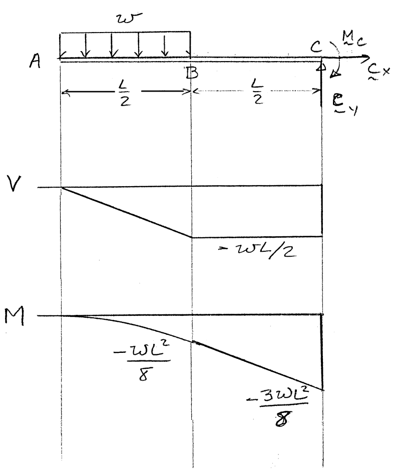
Using the method of Sec. 7.6, solve Prob. 7.30.

SOLUTION

(a) and (b)

Shear Diag: $V = 0$ at A and is linear

$\left(\frac{dV}{dx} = -w\right)$ to $-w\left(\frac{L}{2}\right) = -\frac{wL}{2}$ at B. V is constant $\left(\frac{dV}{dx} = 0\right)$ from B to C.



$$|V|_{max} = \frac{wL}{2} \blacktriangleleft$$

Moment Diag: $M = 0$ at A and is

parabolic $\left(\frac{dM}{dx} \text{ decreasing with } V\right)$ to B.

$$M_B = \frac{1}{2}\left(\frac{L}{2}\right)\left(-\frac{wL}{2}\right) = -\frac{wL^2}{8}$$

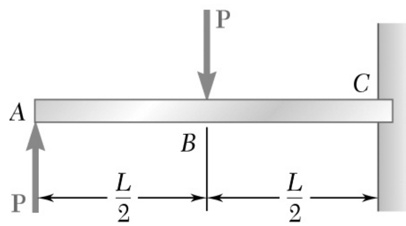
From B to C, M is linear $\left(\frac{dM}{dx} = -\frac{wL}{2}\right)$

$$M_C = -\frac{wL^2}{8} - \left(\frac{L}{2}\right)\left(\frac{wL}{2}\right) = -\frac{3wL^2}{8}$$

$$|M|_{max} = \frac{3wL^2}{8} \blacktriangleleft$$

Notes: Smooth transition in M at B, as no discontinuity in V.

It was not necessary to predetermine reactions at C.
In fact they are given by $-V_C$ and $-M_C$.

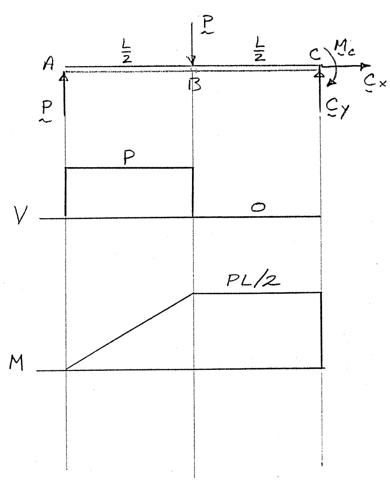


PROBLEM 7.60

Using the method of Sec. 7.6, solve Prob. 7.31.

SOLUTION

(a) and (b)



Shear Diag:

V jumps to P at A, then is constant $\left(\frac{dV}{dx} = 0\right)$ to B. V jumps down P to zero at B, and is constant (zero) to C.

$$|V|_{max} = P \blacktriangleleft$$

Moment Diag:

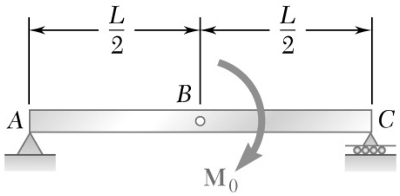
M is linear $\left(\frac{dM}{dy} = V = P\right)$ to B.

$$M_B = 0 + \left(\frac{L}{2}\right)(P) = \frac{PL}{2}.$$

M is constant $\left(\frac{dM}{dx} = 0\right)$ at $\frac{PL}{2}$ to C

$$|M|_{max} = \frac{PL}{2} \blacktriangleleft$$

Note: It was not necessary to predetermine reactions at C. In fact they are given by $-V_C$ and $-M_C$.



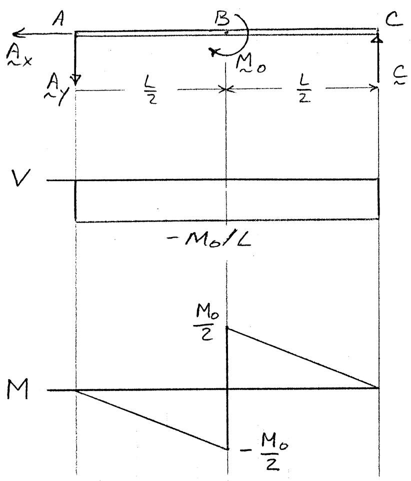
PROBLEM 7.61

Using the method of Sec. 7.6, solve Prob. 7.32.

SOLUTION

(*a*) and (*b*)

$$\circlearrowleft \Sigma M_C = 0\colon LA_y - M_0 = 0 \qquad \mathbf{A}_y = \frac{M_0}{L} \downarrow$$



Shear Diag:

V jumps to $-\dfrac{M_0}{L}$ at A and is constant $\left(\dfrac{dV}{dx} = 0\right)$ all the way to C

$$|V|_{\max} = \frac{M_0}{L} \blacktriangleleft$$

Moment Diag:

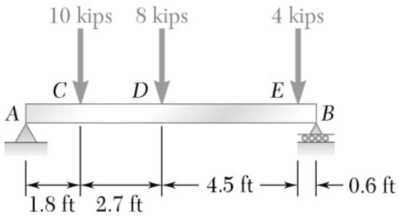
M is zero at A and linear $\left(\dfrac{dM}{dx} = V = -\dfrac{M_0}{L}\right)$ throughout.

$$M_{B^-} = -\frac{L}{2}\left(\frac{M_0}{L}\right) = -\frac{M_0}{2},$$

but M jumps by $+M_0$ to $+\dfrac{M_0}{2}$ at B.

$$M_C = \frac{M_0}{2} - \frac{L}{2}\left(\frac{M_0}{L}\right) = 0$$

$$|M|_{\max} = \frac{M_0}{2} \blacktriangleleft$$

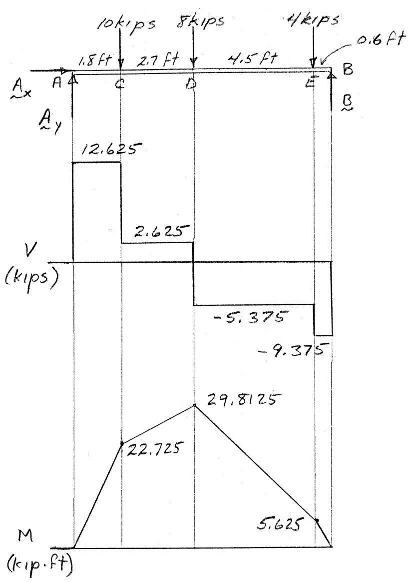


PROBLEM 7.62

Using the method of Sec. 7.6, solve Prob. 7.33.

SOLUTION

(*a*) and (*b*)



$$\circlearrowleft \Sigma M_B = 0: (0.6\text{ ft})(4\text{ kips}) + (5.1\text{ ft})(8\text{ kips}) + (7.8\text{ ft})(10\text{ kips}) - (9.6\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 12.625\text{ kips} \uparrow$$

Shear Diag:

V is piecewise constant, $\left(\dfrac{dV}{dx} = 0\right)$ with discontinuities at each concentrated force. (equal to force)

$$|V|_{\max} = 12.63\text{ kips} \blacktriangleleft$$

Moment Diag:

M is zero at A, and piecewise linear $\left(\dfrac{dM}{dx} = V\right)$ throughout.

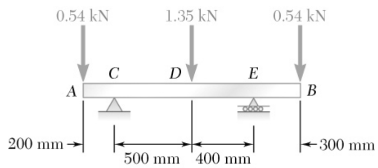
$$M_C = (1.8\text{ ft})(12.625\text{ kips}) = 22.725\text{ kip}\cdot\text{ft}$$

$$M_D = 22.725\text{ kip}\cdot\text{ft} + (2.7\text{ ft})(2.625\text{ kips}) = 29.8125\text{ kip}\cdot\text{ft}$$

$$M_E = 29.8125\text{ kip}\cdot\text{ft} - (4.5\text{ ft})(5.375\text{ kips}) = 5.625\text{ kip}\cdot\text{ft}$$

$$M_B = 5.625\text{ kip}\cdot\text{ft} - (0.6\text{ ft})(9.375\text{ kips}) = 0$$

$$|M|_{\max} = 29.8\text{ kip}\cdot\text{ft} \blacktriangleleft$$

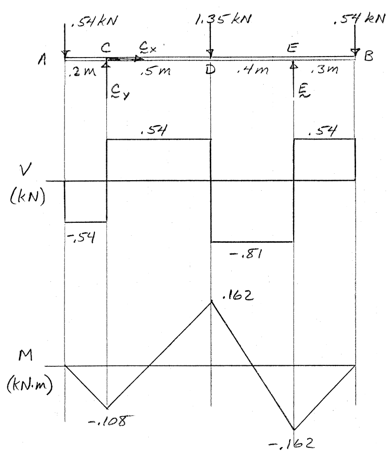


PROBLEM 7.63

Using the method of Sec. 7.6, solve Prob. 7.36.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\circlearrowleft \Sigma M_E = 0: (1.1\text{ m})(0.54\text{ kN}) - (0.9\text{ m})C_y + (0.4\text{ m})(1.35\text{ kN}) - (0.3\text{ m})(0.54\text{ kN}) = 0$$

$$\mathbf{C}_y = 1.08\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: -0.54\text{ kN} + 1.08\text{ kN} - 1.35\text{ kN} + E - 0.54\text{ kN} = 0$$

$$\mathbf{E} = 1.35\text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant, $\left(\dfrac{dV}{dx} = 0 \text{ everywhere}\right)$ with discontinuities at each concentrated force. (equal to the force)

$$|V|_{max} = 810\text{ N} \blacktriangleleft$$

Moment Diag:

M is piecewise linear starting with $M_A = 0$

$$M_C = 0 - 0.2\text{ m}(0.54\text{ kN}) = 0.108\text{ kN}\cdot\text{m}$$

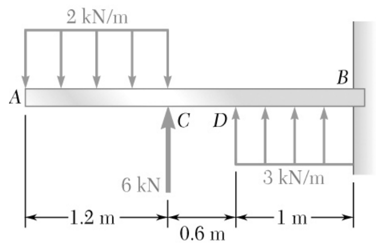
$$M_D = 0.108\text{ kN}\cdot\text{m} + (0.5\text{ m})(0.54\text{ kN}) = 0.162\text{ kN}\cdot\text{m}$$

$$M_E = 0.162\text{ kN}\cdot\text{m} - (0.4\text{ m})(0.81\text{ kN}) = -0.162\text{ kN}\cdot\text{m}$$

$$M_B = 0.162\text{ kN}\cdot\text{m} + (0.3\text{ m})(0.54\text{ kN}) = 0$$

$$|M|_{max} = 0.162\text{ kN}\cdot\text{m} = 162.0\text{ N}\cdot\text{m} \blacktriangleleft$$

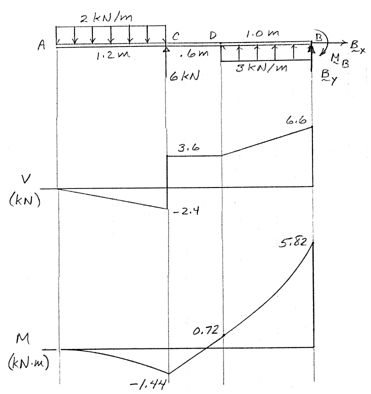
PROBLEM 7.64



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) and (*b*)



Shear Diag:

$V = 0$ at A and linear $\left(\dfrac{dV}{dx} = -2 \text{ kN/m}\right)$ to C

$$V_C = -1.2 \text{ m}(2 \text{ kN/m}) = -2.4 \text{ kN}.$$

At C, V jumps 6 kN to 3.6 kN, and is constant to D. From there, V is linear $\left(\dfrac{dV}{dx} = +3 \text{ kN/m}\right)$ to B

$$V_B = 3.6 \text{ kN} + (1 \text{ m})(3 \text{ kN/m}) = 6.6 \text{ kN}$$

$$|V|_{max} = 6.60 \text{ kN} \blacktriangleleft$$

Moment Diag: $M_A = 0.$

From A to C, M is parabolic, $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$.

$$M_C = -\frac{1}{2}(1.2 \text{ m})(2.4 \text{ kN}) = -1.44 \text{ kN}\cdot\text{m}$$

From C to D, M is linear $\left(\dfrac{dM}{dx} = 3.6 \text{ kN}\right)$

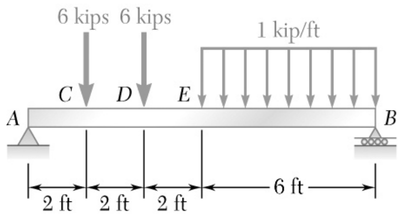
$$M_D = -1.44 \text{ kN}\cdot\text{m} + (0.6 \text{ m})(3.6 \text{ kN})$$
$$= 0.72 \text{ kN}\cdot\text{m}.$$

From D to B, M is parabolic $\left(\dfrac{dM}{dx} \text{ increasing with } V\right)$

$$M_B = 0.72 \text{ kN}\cdot\text{m} + \frac{1}{2}(1 \text{ m})(3.6 + 6.6) \text{ kN}$$
$$= 5.82 \text{ kN}\cdot\text{m}$$

$$|M|_{max} = 5.82 \text{ kN}\cdot\text{m} \blacktriangleleft$$

Notes: Smooth transition in M at D. It was unnecessary to predetermine reactions at B, but they are given by $-V_B$ and $-M_B$

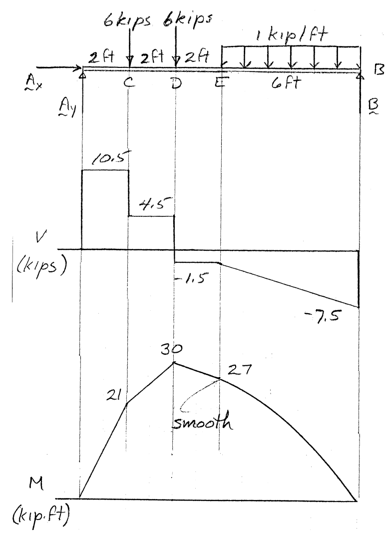


PROBLEM 7.65

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) and (*b*)



$$\circlearrowleft \Sigma M_B = 0{:}\ (3\text{ ft})(1\text{ kip/ft})(6\text{ ft}) + (8\text{ ft})(6\text{ kips}) + (10\text{ ft})(6\text{ kips}) - (12\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 10.5\text{ kips} \uparrow$$

Shear Diag:

V is piecewise constant from A to E, with discontinuities at A, C, and E equal to the forces. $V_E = -1.5$ kips. From E to B, V is linear

$$\left(\frac{dV}{dx} = -1\text{ kip/ft}\right),$$

so

$$V_B = -1.5\text{ kips} - (6\text{ ft})(1\text{ kip/ft}) = -7.5\text{ kips}$$

$$|V|_{\max} = 10.50\text{ kips} \blacktriangleleft$$

Moment Diag: $M_A = 0$, then M is piecewise linear to E

$$M_C = 0 + 2\text{ ft}(10.5\text{ kips}) = 21\text{ kip}\cdot\text{ft}$$

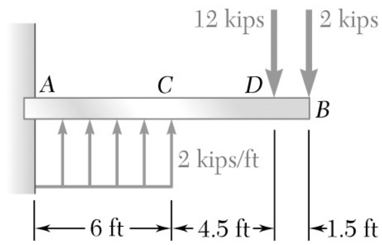
$$M_D = 21\text{ kip}\cdot\text{ft} + (2\text{ ft})(4.5\text{ kips}) = 30\text{ kip}\cdot\text{ft}$$

$$M_E = 30\text{ kip}\cdot\text{ft} - (2\text{ ft})(1.5\text{ kips}) = 27\text{ kip}\cdot\text{ft}$$

From E to B, M is parabolic $\left(\frac{dM}{dx}\text{ decreasing with } V\right)$, and

$$M_B = 27\text{ kip}\cdot\text{ft} - \frac{1}{2}(6\text{ ft})(1.5\text{ kips} + 7.5\text{ kips}) = 0$$

$$|M|_{\max} = 30.0\text{ kip}\cdot\text{ft} \blacktriangleleft$$

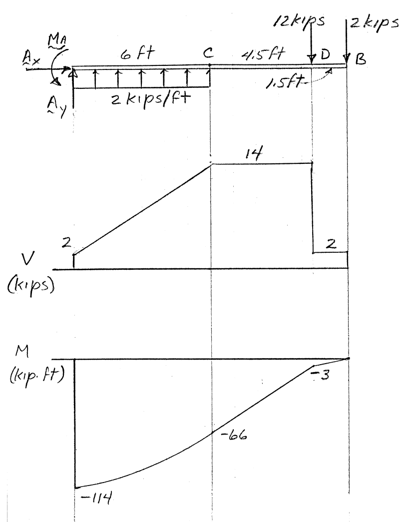


PROBLEM 7.66

Using the method of Sec. 7.6, solve Prob. 7.37.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\uparrow \Sigma F_y = 0:\ A_y + (6\text{ ft})(2\text{ kips/ft}) - 12\text{ kips} - 2\text{ kips} = 0$$

$$\mathbf{A}_y = 2\text{ kips} \uparrow$$

$$\circlearrowleft \Sigma M_A = 0:\ M_A + (3\text{ ft})(2\text{ kips/ft})(6\text{ ft}) - (10.5\text{ ft})(12\text{ kips}) - (12\text{ ft})(2\text{ kips}) = 0$$

$$\mathbf{M}_A = 114\text{ kip}\cdot\text{ft} \circlearrowleft$$

Shear Diag:

$V_A = A_y = 2$ kips. Then V is linear $\left(\dfrac{dV}{dx} = 2\text{ kips/ft}\right)$ to C, where

$$V_C = 2\text{ kips} + (6\text{ ft})(2\text{ kips/ft}) = 14\text{ kips.}$$

V is constant at 14 kips to D, then jumps down 12 kips to 2 kips and is constant to B

$$|V|_{max} = 14.00\text{ kips} \blacktriangleleft$$

Moment Diag: $M_A = -114\text{ kip}\cdot\text{ft.}$

From A to C, M is parabolic $\left(\dfrac{dM}{dx}\text{ increasing with } V\right)$ and

$$M_C = -114\text{ kip}\cdot\text{ft} + \frac{1}{2}(2\text{ kips} + 14\text{ kips})(6\text{ ft})$$

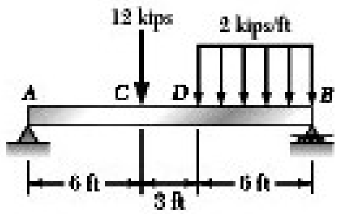
$$M_C = -66\text{ kip}\cdot\text{ft.}$$

Then M is piecewise linear.

$$M_D = -66\text{ kip}\cdot\text{ft} + (14\text{ kips})(4.5\text{ ft}) = -3\text{ kip}\cdot\text{ft}$$

$$M_B = -3\text{ kip}\cdot\text{ft} + (2\text{ kips})(1.5\text{ ft}) = 0$$

$$|M|_{max} = 114.0\text{ kip}\cdot\text{ft} \blacktriangleleft$$

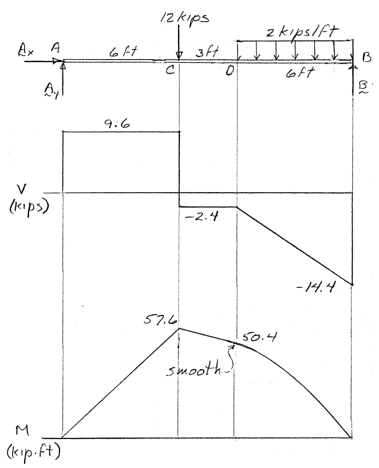


PROBLEM 7.67

Using the method of Sec. 7.6, solve Prob. 7.38.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (3\text{ ft})\left(2\frac{\text{kips}}{\text{ft}}\right)(6\text{ ft}) + (9\text{ ft})(12\text{ kips}) - (15\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 9.6\text{ kips}\uparrow$$

Shear Diag:

V jumps to $A_y = 9.6$ kips at *A*, is constant to *C*, jumps down 12 kips to -2.4 kips at *C*, is constant to *D*, and then is linear $\left(\frac{dV}{dx} = -2\text{ kips/ft}\right)$ to *B*

$$V_B = -2.4\text{ kips} - (2\text{ kips/ft})(6\text{ ft})$$
$$= -14.4\text{ kips}$$

$$|V|_{max} = 14.40\text{ kips} \blacktriangleleft$$

Moment Diag:

M is linear from *A* to *C* $\left(\frac{dM}{dx} = 9.6\text{ kips/ft}\right)$

$$M_C = 9.6\text{ kips}(6\text{ ft}) = 57.6\text{ kip}\cdot\text{ft},$$

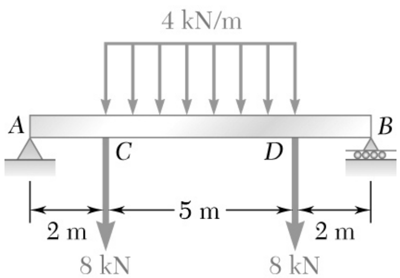
M is linear from *C* to *D* $\left(\frac{dM}{dx} = -2.4\text{ kips/ft}\right)$

$$M_D = 57.6\text{ kip}\cdot\text{ft} - 2.4\text{ kips}(3\text{ ft})$$
$$M_D = 50.4\text{ kip}\cdot\text{ft}.$$

M is parabolic $\left(\frac{dM}{dx}\text{ decreasing with } V\right)$ to *B*.

$$M_B = 50.4\text{ kip}\cdot\text{ft} - \frac{1}{2}(2.4\text{ kips} + 14.4\text{ kips})(6\text{ ft}) = 0$$
$$= 0$$

$$|M|_{max} = 57.6\text{ kip}\cdot\text{ft} \blacktriangleleft$$

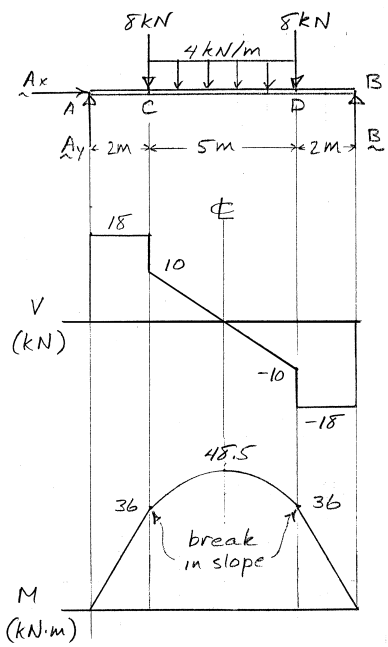


PROBLEM 7.68

Using the method of Sec. 7.6, solve Prob. 7.39.

SOLUTION

(*a*) and (*b*)



FBD Beam:

By symmetry: $A_y = B = \frac{1}{2}(5\text{ m})(4\text{ kN/m}) + 8\text{ kN}$

or $\mathbf{A}_y = \mathbf{B} = 18\text{ kN}\uparrow$

Shear Diag:

V jumps to 18 kN at A, and is constant to C, then drops 8 kN to 10 kN.

After C, V is linear $\left(\frac{dV}{dx} = -4\text{ kN/m}\right)$, reaching -10 kN at $D\left[V_D = 10\text{ kN} - (4\text{ kN/m})(5\text{ m})\right]$ passing through zero at the beam center. At D, V drops 8 kN to -18 kN and is then constant to B

$|V|_{max} = 18.00\text{ kN}$ ◀

Moment Diag:

$M_A = 0$. Then M is linear $\left(\frac{dM}{dx} = 18\text{ kN/m}\right)$ to C

$M_C = (18\text{ kN})(2\text{ m}) = 36\text{ kN}\cdot\text{m}$, M is parabolic to D

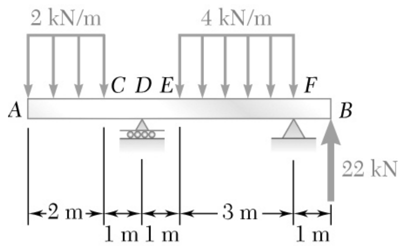
$\left(\frac{dM}{dx}\text{ decreases with } V \text{ to zero at center}\right)$

$$M_{center} = 36\text{ kN}\cdot\text{m} + \frac{1}{2}(10\text{ kN})(2.5\text{ m}) = 48.5\text{ kN}\cdot\text{m} = M_{max}$$

$|M|_{max} = 48.5\text{ kN}\cdot\text{m}$ ◀

Complete by invoking symmetry.

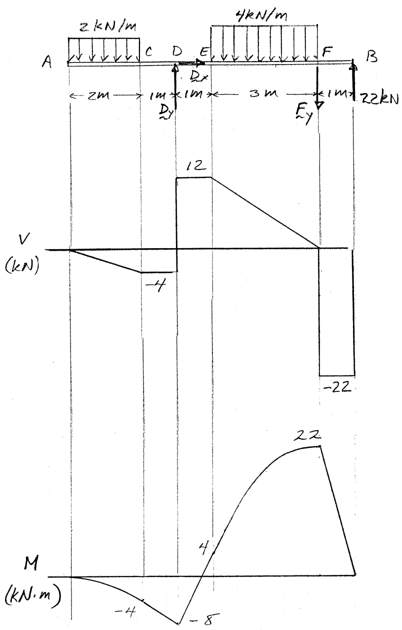
PROBLEM 7.69



Using the method of Sec. 7.6, solve Prob. 7.40.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\circlearrowleft \Sigma M_F = 0: (1\text{ m})(22\text{ kN}) + (1.5\text{ m})(4\text{ kN/m})(3\text{ m})$$

$$-(4\text{ m})D_y + (6\text{ m})(2\text{ kN/m})(2\text{ m}) = 0$$

$$\mathbf{D}_y = 16\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: 16\text{ kN} + 22\text{ kN} - F_y - (2\text{ kN/m})(2\text{ m})$$

$$-(4\text{ kN/m})(3\text{ m}) = 0$$

$$\mathbf{F}_y = 22\text{ kN}\downarrow$$

Shear Diag:

$V_A = 0$, then V is linear $\left(\dfrac{dV}{dx} = -2\text{ kN/m}\right)$ to C;

$$V_C = -2\text{ kN/m}(4\text{ m}) = -4\text{ kN}$$

V is constant to D, jumps 16 kN to 12 kN and is constant to E.

Then V is linear $\left(\dfrac{dV}{dx} = -4\text{ kN/m}\right)$ to F.

$$V_F = 12\text{ kN} - (4\text{ kN/m})(3\text{ m}) = 0.$$

V jumps to -22 kN at F, is constant to B, and returns to zero.

$$|V|_{\max} = 22.0\text{ kN} \blacktriangleleft$$

Moment Diag:

$M_A = 0$, M is parabolic $\left(\dfrac{dM}{dx} \text{ decreases with } V\right)$ to C.

$$M_C = -\frac{1}{2}(4\text{ kN})(2\text{ m}) = -4\text{ kN}\cdot\text{m}.$$

PROBLEM 7.69 CONTINUED

Then M is linear $\left(\frac{dM}{dx} = -4\text{ kN}\right)$ to D.

$$M_D = -4\text{ kN}\cdot\text{m} - (4\text{ kN})(1\text{ m}) = -8\text{ kN}\cdot\text{m}$$

From D to E, M is linear $\left(\frac{dM}{dx} = 12\text{ kN}\right)$, and

$$M_E = -8\text{ kN}\cdot\text{m} + (12\text{ kN})(1\text{m})$$

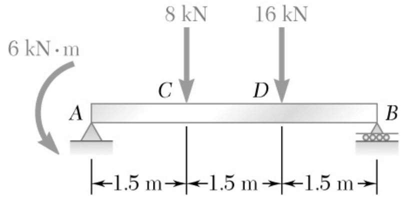
$$M_E = 4\text{ kN}\cdot\text{m}$$

M is parabolic $\left(\frac{dM}{dx}\text{ decreasing with } V\right)$ to F.

$$M_F = 4\text{ kN}\cdot\text{m} + \frac{1}{2}(12\text{ kN})(3\text{ m}) = 22\text{ kN}\cdot\text{m}.$$

Finally, M is linear $\left(\frac{dM}{dx} = -22\text{ kN}\right)$, back to zero at B.

$$|M|_{max} = 22.0\text{ kN}\cdot\text{m} \blacktriangleleft$$

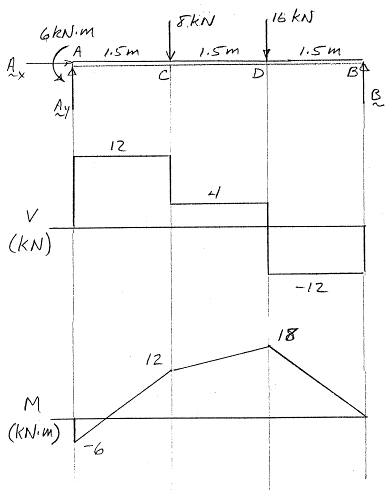


PROBLEM 7.70

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (1.5\text{ m})(16\text{ kN})$$

$$+ (3\text{ m})(8\text{ kN}) + 6\text{ kN}\cdot\text{m} - (4.5\text{ m})A_y = 0$$

$$\mathbf{A}_y = 12\text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant with discontinuities equal to the concentrated forces at A, C, D, B

$$|V|_{max} = 12.00\text{ kN} \blacktriangleleft$$

Moment Diag:

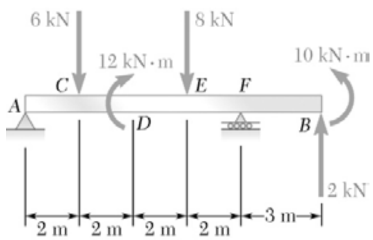
After a jump of $-6\text{ kN}\cdot\text{m}$ at A, M is piecewise linear $\left(\frac{dM}{dx} = V\right)$

So $$M_C = -6\text{ kN}\cdot\text{m} + (12\text{ kN})(1.5\text{ m}) = 12\text{ kN}\cdot\text{m}$$

$$M_D = 12\text{ kN}\cdot\text{m} + (4\text{ kN})(1.5\text{ m}) = 18\text{ kN}\cdot\text{m}$$

$$M_B = 18\text{ kN}\cdot\text{m} - (12\text{ kN})(1.5\text{ m}) = 0$$

$$|M|_{max} = 18.00\text{ kN}\cdot\text{m} \blacktriangleleft$$

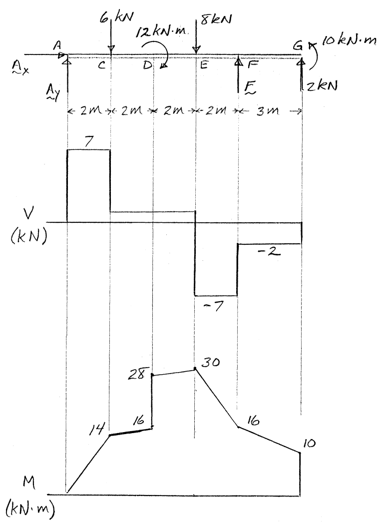


PROBLEM 7.71

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_A = 0: (8\text{ m})F + (11\text{ m})(2\text{ kN}) + 10\text{ kN}\cdot\text{m} - (6\text{ m})(8\text{ kN})$$

$$-12\text{ kN}\cdot\text{m} - (2\text{ m})(6\text{ kN}) = 0 \quad \mathbf{F} = 5\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 6\text{ kN} - 8\text{ kN} + 5\text{ kN} + 2\text{ kN} = 0$$

$$\mathbf{A}_y = 7\text{ kN}\uparrow$$

Shear Diag:

V is piecewise constant with discontinuities equal to the concentrated forces at A, C, E, F, G

Moment Diag:

M is piecewise linear with a discontinuity equal to the couple at D.

$$M_C = (7\text{ kN})(2\text{ m}) = 14\text{ kN}\cdot\text{m}$$

$$M_{D^-} = 14\text{ kN}\cdot\text{m} + (1\text{ kN})(2\text{ m}) = 16\text{ kN}\cdot\text{m}$$

$$M_{D^+} = 16\text{ kN}\cdot\text{m} + 12\text{ kN}\cdot\text{m} = 28\text{ kN}\cdot\text{m}$$

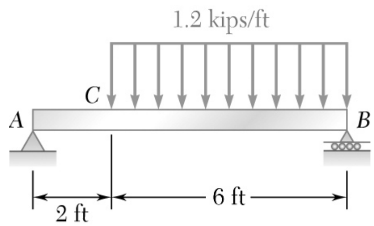
$$M_E = 28\text{ kN}\cdot\text{m} + (1\text{ kN})(2\text{ m}) = 30\text{ kN}\cdot\text{m}$$

$$M_F = 30\text{ kN}\cdot\text{m} - (7\text{ kN})(2\text{ m}) = 16\text{ kN}\cdot\text{m}$$

$$M_G = 16\text{ kN}\cdot\text{m} - (2\text{ kN})(3\text{ m}) = 10\text{ kN}\cdot\text{m}$$

(*b*) $|V|_{max} = 7.00\text{ kN}$ ◀

$|M|_{max} = 30.0\text{ kN}$ ◀



PROBLEM 7.72

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum bending moment.

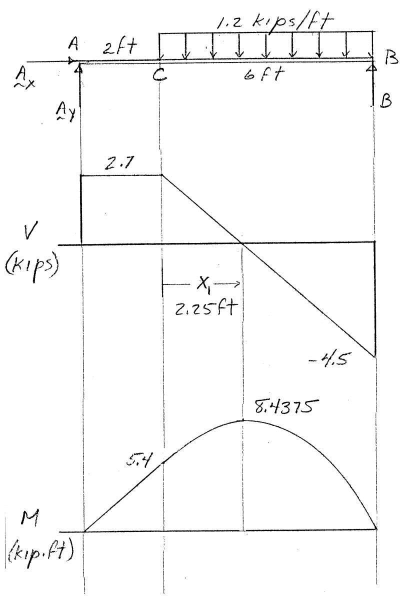
SOLUTION

(*a*)

FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (3\text{ ft})(1.2\text{ kips/ft})(6\text{ ft}) - (8\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 2.7\text{ kips} \uparrow$$



Shear Diag:

$$V = A_y = 2.7\text{ kips at } A, \text{ is constant to } C, \text{ then linear}$$

$$\left(\frac{dV}{dx} = -1.2\text{ kips/ft}\right) \text{ to B.} \qquad V_B = 2.7\text{ kips} - (1.2\text{ kips/ft})(6\text{ ft})$$

$$V_B = -4.5\text{ kips}$$

$$V = 0 = 2.7\text{ kips} - (1.2\text{ kips/ft})x_1 \text{ at } x_1 = 2.25\text{ ft}$$

Moment Diag:

$$M_A = 0, \ M \text{ is linear } \left(\frac{dM}{dx} = 2.7\text{ kips}\right) \text{ to } C.$$

$$M_C = (2.7\text{ kips})(2\text{ ft}) = 5.4\text{ kip}\cdot\text{ft}$$

Then M is parabolic $\left(\frac{dM}{dx} \text{ decreasing with } V\right)$

(*b*)

$M_{\max}$ occurs where $\frac{dM}{dx} = V = 0$

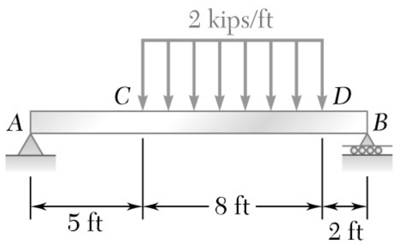
$$M_{\max} = 5.4\text{ kip}\cdot\text{ft} + \frac{1}{2}(2.7\text{ kips})x_1; \qquad x_1 = 2.25\text{ m}$$

$$M_{\max} = 8.4375\text{ kip}\cdot\text{ft}$$

$$M_{\max} = 8.44\text{ kip}\cdot\text{ft, 2.25 m right of } C \blacktriangleleft$$

Check: $M_B = 8.4375\text{ kip}\cdot\text{ft} - \frac{1}{2}(4.5\text{ kips})(3.75\text{ ft}) = 0$

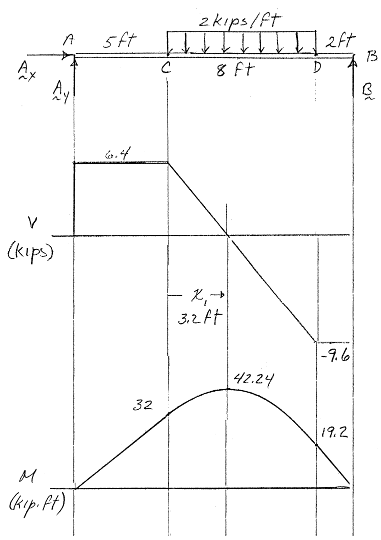
PROBLEM 7.73



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (6 \text{ ft})(2 \text{ kips/ft})(8 \text{ ft}) - (15 \text{ ft})A_y = 0$$

$$\mathbf{A}_y = 6.4 \text{ kips} \uparrow$$

Shear Diag:

$V = A_y = 6.4$ kips at A, and is constant to C, then linear

$\left(\dfrac{dV}{dx} = -2 \text{ kips/ft}\right)$ to D,

$$V_D = 6.4 \text{ kips} - (2 \text{ kips/ft})(8 \text{ ft}) = -9.6 \text{ kips}$$

$$V = 0 = 6.4 \text{ kips} - (2 \text{ kips/ft})x_1 \text{ at } x_1 = 3.2 \text{ ft}$$

Moment Diag:

$M_A = 0$, then M is linear $\left(\dfrac{dM}{dx} = 6.4 \text{ kips}\right)$ to $M_C = (6.4 \text{ kips})(5 \text{ ft})$.

$M_C = 32$ kip·ft. M is then parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$.

(*b*)

$$M_{\max} \text{ occurs where } \frac{dM}{dx} = V = 0.$$

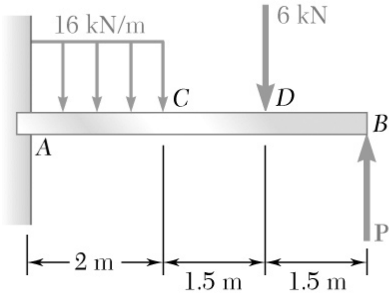
$$M_{\max} = 32 \text{ kip}\cdot\text{ft} + \frac{1}{2}(6.4 \text{ kips})x_1; \qquad x_1 = 3.2 \text{ ft}$$

$$M_{\max} = 42.24 \text{ kip}\cdot\text{ft}$$

$$M_{\max} = 42.2 \text{ kip}\cdot\text{ft, 3.2 ft right of } C \blacktriangleleft$$

$$M_D = 42.24 \text{ kip}\cdot\text{ft} - \frac{1}{2}(9.6 \text{ kips})(4.8 \text{ ft}) = 19.2 \text{ kip}\cdot\text{ft}$$

M is linear from D to zero at B.

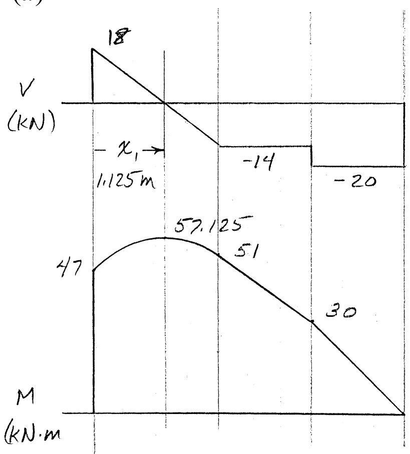


PROBLEM 7.74

For the beam shown, draw the shear and bending-moment diagrams and determine the maximum absolute value of the bending moment knowing that (*a*) $P = 14$ kN, (*b*) $P = 20$ kN.

SOLUTION

(*a*)



FBD Beam:

$$\uparrow \Sigma F_y = 0\text{: } A_y - (16 \text{ kN/m})(2 \text{ m}) - 6 \text{ kN} + P = 0$$

$$A_y = 38 \text{ kN} - P$$

(*a*) $\mathbf{A}_y = 24 \text{ kN} \uparrow$

(*b*) $\mathbf{A}_y = 18 \text{ kN} \uparrow$

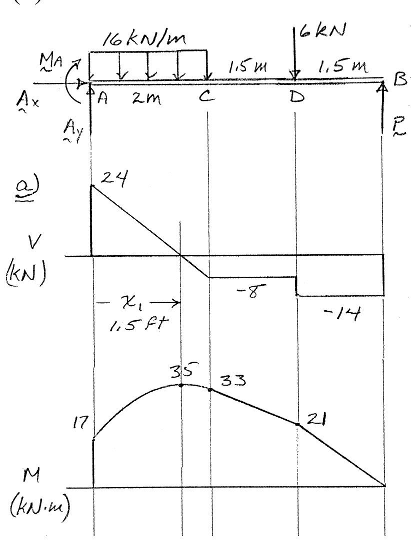
$$\circlearrowleft \Sigma M_A = 0\text{: } (5 \text{ m})P - (3.5 \text{ m})(6 \text{ kN}) - 1 \text{ m}(16 \text{ kN/m})(2 \text{ m}) - M_A = 0$$

$$M_A = (5 \text{ m})P - 53 \text{ kN}\cdot\text{m}$$

(*a*) $\mathbf{M}_A = 17 \text{ kN}\cdot\text{m} \circlearrowright$

(*b*) $\mathbf{M}_A = 47 \text{ kN}\cdot\text{m} \circlearrowright$

(*b*)



Shear Diags:

$V_A = A_y$. Then V is linear $\left(\dfrac{dV}{dx} = -16 \text{ kN/m}\right)$ to C.

$$V_C = V_A - (16 \text{ kN/m})(2 \text{ m}) = V_A - 32 \text{ kN}$$

(*a*) $V_C = -8$ kN

(*b*) $V_C = -14$ kN

$$V = 0 = V_A - (16 \text{ kN/m})x_1$$

(*a*) $x_1 = 1.5$ m

(*b*) $x_1 = 1.125$ m

V is constant from C to D, decreases by 6 kN at D and is constant to B (at $-P$)

PROBLEM 7.74 CONTINUED

Moment Diags:

$M_A = M_A$ reaction. Then M is parabolic $\left(\frac{dM}{dx} \text{ decreasing with } V\right)$.

The maximum occurs where $V = 0$. $M_{\max} = M_A + \frac{1}{2}V_A x_1$.

(*a*) $$M_{\max} = 17 \text{ kN}\cdot\text{m} + \frac{1}{2}(24 \text{ kN})(1.5 \text{ m}) = 35.0 \text{ kN}\cdot\text{m} \blacktriangleleft$$

1.5 ft from A ◀

(*b*) $$M_{\max} = 47 \text{ kN}\cdot\text{m} + \frac{1}{2}(18 \text{ kN})(1.125 \text{ m}) = 57.125 \text{ kN}\cdot\text{m}$$

$M_{\max} = 57.1 \text{ kN}\cdot\text{m}$ 1.125 ft from A ◀

$$M_C = M_{\max} + \frac{1}{2}V_C(2 \text{ m} - x_1)$$

(*a*) $$M_C = 35 \text{ kN}\cdot\text{m} - \frac{1}{2}(8 \text{ kN})(0.5 \text{ m}) = 33 \text{ kN}\cdot\text{m}$$

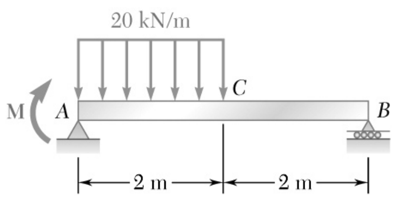
(*b*) $$M_C = 57.125 \text{ kN}\cdot\text{m} - \frac{1}{2}(14 \text{ kN})(0.875 \text{ m}) = 51 \text{ kN}\cdot\text{m}$$

M is piecewise linear along C, D, B, with $M_B = 0$ and $M_D = (1.5 \text{ m})P$

(*a*) $$M_D = 21 \text{ kN}\cdot\text{m}$$

(*b*) $$M_D = 30 \text{ kN}\cdot\text{m}$$

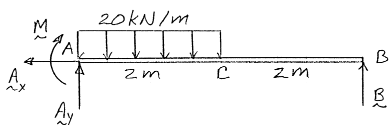
PROBLEM 7.75



For the beam shown, draw the shear and bending-moment diagrams, and determine the magnitude and location of the maximum absolute value of the bending moment knowing that (*a*) $M = 0$, (*b*) $M = 12\ \text{kN}\cdot\text{m}$.

SOLUTION

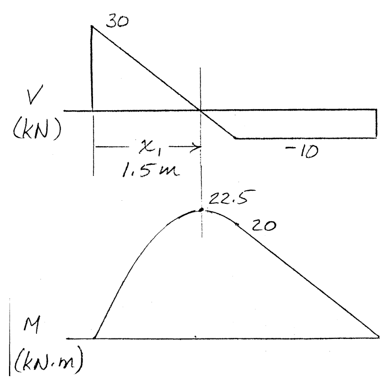
FBD Beam:



$$\circlearrowleft \Sigma M_A = 0\colon (4\text{ m})B - (1\text{ m})(20\text{ kN/m})(2\text{ m}) - M = 0$$

$$B = 10\text{ kN} + \frac{M}{4\text{ m}}$$

(*a*)



(*a*) $\mathbf{B} = 10\text{ kN}\uparrow$

(*b*) $\mathbf{B} = 13\text{ kN}\uparrow$

$$\uparrow \Sigma F_y = 0\colon A_y - (20\text{ kN/m})(2\text{ m}) + B = 0$$

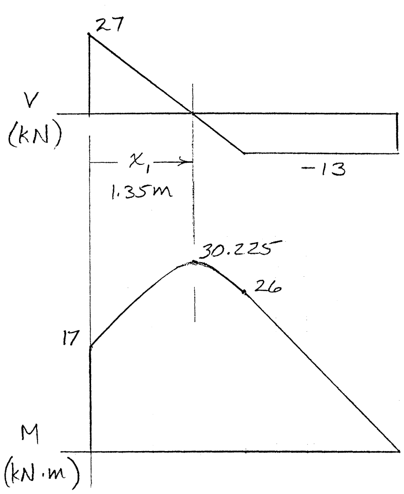
$$A_y = 40\text{ kN} - B$$

(*a*) $\mathbf{A}_y = 30\text{ kN}\uparrow$

(*b*) $\mathbf{A}_y = 27\text{ kN}\uparrow$

Shear Diags:

(*b*)



$V_A = A_y$, then V is linear $\left(\dfrac{dV}{dx} = -20\text{ kN/m}\right)$ to C.

$$V_C = A_y - (20\text{ kN/m})(2\text{ m}) = A_y - 40\text{ kN}$$

(*a*) $V_C = -10\text{ kN}$

(*b*) $V_C = -13\text{ kN}$

$$V = 0 = A_y - (20\text{ kN/m})x_1 \text{ at } x_1 = \frac{A_y\text{ m}}{20\text{ kN}}$$

(*a*) $x_1 = 1.5\text{ m}$

(*b*) $x_1 = 1.35\text{ m}$

V is constant from C to B.

PROBLEM 7.75 CONTINUED

Moment Diags:

M_A = applied M. Then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreases with } V\right)$

M is max where $V = 0$. $M_{\max} = M + \dfrac{1}{2}A_y x_1$.

(*a*) $$|M|_{\max} = \frac{1}{2}(30 \text{ kN})(1.5 \text{ m}) = 22.5 \text{ kN}\cdot\text{m} \blacktriangleleft$$

1.500 m from A ◀

(*b*) $$M_{\max} = 12 \text{ kN}\cdot\text{m} + \frac{1}{2}(27 \text{ kN})(1.35 \text{ m}) = 30.225 \text{ kN}\cdot\text{m} \blacktriangleleft$$

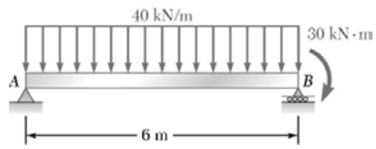
$|M|_{\max} = 30.2$ kN   1.350 m from A ◀

$$M_C = M_{\max} - \frac{1}{2}V_C(2 \text{ m} - x_1)$$

(*a*) $M_C = 20$ kN·m

(*b*) $M_C = 26$ kN·m

Finally, M is linear $\left(\dfrac{dM}{dx} = V_C\right)$ to zero at B.

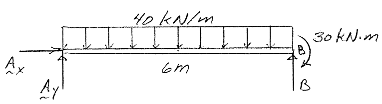


PROBLEM 7.76

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (3\text{ m})(40\text{ kN/m})(6\text{ m}) - (30\text{ kN}\cdot\text{m}) - (6\text{ m})A_y = 0$$

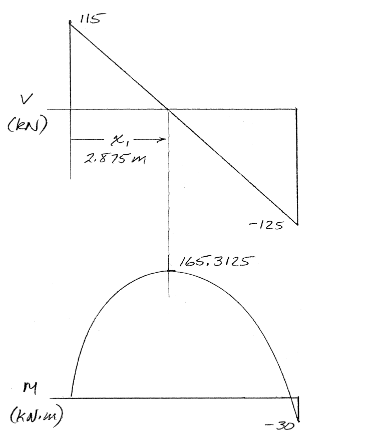
$$\mathbf{A}_y = 115\text{ kN} \uparrow$$

Shear Diag:

$V_A = A_y = 115$ kN, then V is linear $\left(\dfrac{dM}{dx} = -40\text{ kN/m}\right)$ to B.

$$V_B = 115\text{ kN} - (40\text{ kN/m})(6\text{ m}) = -125\text{ kN.}$$

$$V = 0 = 115\text{ kN} - (40\text{ kN/m})x_1 \text{ at } x_1 = 2.875\text{ m}$$



Moment Diag:

$M_A = 0$. Then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$. Max M occurs where $V = 0$,

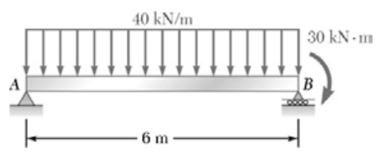
$$M_{\max} = \frac{1}{2}(115\text{ kN/m})(2.875\text{ m}) = 165.3125\text{ kN}\cdot\text{m}$$

$$M_B = M_{\max} - \frac{1}{2}(125\text{ kN})(6\text{ m} - x_1)$$

$$= 165.3125\text{ kN}\cdot\text{m} - \frac{1}{2}(125\text{ kN})(6 - 2.875)\text{m}$$

$$= -30\text{ kN}\cdot\text{m as expected.}$$

(*b*) $|M|_{\max} = 165.3\text{ kN}\cdot\text{m}(2.88\text{ m from }A)$ ◀

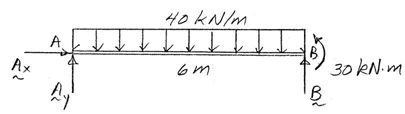


PROBLEM 7.77

Solve Prob. 7.76 assuming that the $30\ \text{kN}\cdot\text{m}$ couple applied at B is counterclockwise

SOLUTION

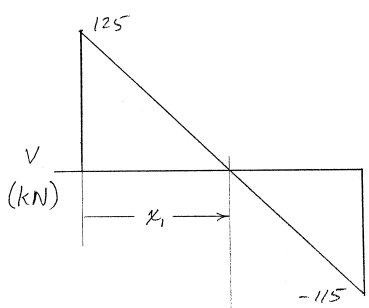
(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0\text{: } 30\ \text{kN}\cdot\text{m} + (3\ \text{m})(40\ \text{kN/m})(6\ \text{m}) - (6\ \text{m})A_y = 0$$

$$\mathbf{A}_y = 125\ \text{kN} \uparrow$$

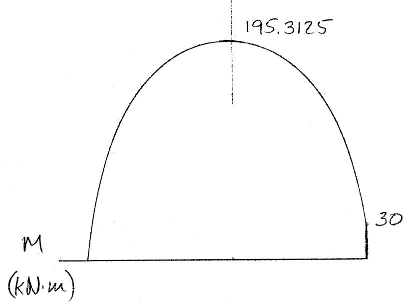


Shear Diag:

$V_A = A_y = 125\ \text{kN}$, V is linear $\left(\dfrac{dV}{dx} = -40\ \text{kN/m}\right)$ to B.

$$V_B = 125\ \text{kN} - (40\ \text{kN/m})(6\ \text{m}) = -115\ \text{kN}$$

$$V = 0 = 115\ \text{kN} - (40\ \text{kN/m})x_1 \text{ at } x_1 = 3.125\ \text{m}$$



Moment Diag:

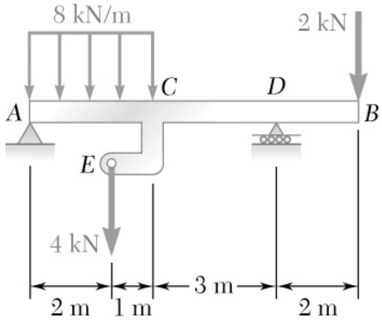
$M_A = 0$. Then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreases with } V\right)$. Max M occurs where $V = 0$,

$$M_{\max} = \frac{1}{2}(125\ \text{kN})(3.125\ \text{m}) = 195.3125\ \text{kN}\cdot\text{m}$$

(*b*) $\quad |M|_{\max} = 195.3\ \text{kN}\cdot\text{m}\,(3.125\ \text{m from } A)$ ◀

$$M_B = 195.3125\ \text{kN}\cdot\text{m} - \frac{1}{2}(115\ \text{kN})(6 - 3.125)\text{m}$$

$M_B = 30\ \text{kN}\cdot\text{m}$ as expected.

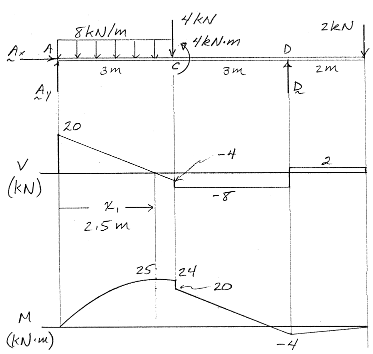


PROBLEM 7.78

For beam AB, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

(*a*)
Replacing the load at E with equivalent force-couple at C:



$$\circlearrowleft \Sigma M_A = 0: (6\text{ m})D - (8\text{ m})(2\text{ kN}) - (3\text{ m})(4\text{ kN}) - (1.5\text{ m})(8\text{ kN/m})(3\text{ m}) - 4\text{ kN}\cdot\text{m} = 0$$

$$\mathbf{D} = 10\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: A_y + 10\text{ kN} - 2\text{ kN} - 4\text{ kN} - (8\text{ kN/m})(3\text{ m}) = 0$$

$$\mathbf{A}_y = 20\text{ kN}\uparrow$$

Shear Diag:

$V_A = A_y = 20\text{ kN}$, then V is linear $\left(\dfrac{dV}{dx} = -8\text{ kN/m}\right)$ to C.

$$V_C = 20\text{ kN} - (8\text{ kN/m})(3\text{ m}) = -4\text{ kN}$$

$$V = 0 = 20\text{ kN} - (8\text{ kN/m})x_1 \text{ at } x_1 = 2.5\text{ m}$$

At C, V decreases by 4 kN to -8 kN.

At D, V increases by 10 kN to 2 kN.

Moment Diag:

$M_A = 0$, then M is parabolic $\left(\dfrac{dM}{dx}\text{ decreasing with }V\right)$. Max M occurs where $V = 0$.

$$M_{max} = \frac{1}{2}(20\text{ kN})(2.5\text{ m}) = 25\text{ kN}\cdot\text{m}$$

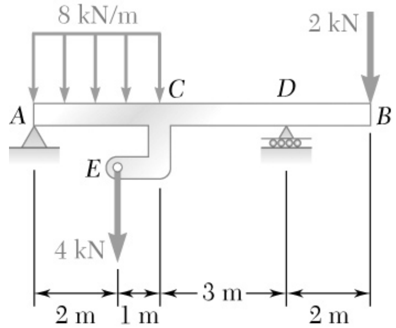
(*b*) $M_{max} = 25.0\text{ kN}\cdot\text{m}$, 2.50 m from A ◀

PROBLEM 7.78 CONTINUED

$$M_C = 25 \text{ kN}\cdot\text{m} - \frac{1}{2}(4 \text{ kN})(0.5 \text{ m}) = 24 \text{ kN}\cdot\text{m}.$$

At C, M decreases by $4 \text{ kN}\cdot\text{m}$ to $20 \text{ kN}\cdot\text{m}$. From C to B, M is piecewise linear with $\frac{dM}{dx} = -8 \text{ kN}$ to D, then $\frac{dM}{dx} = +2 \text{ kN}$ to B.

$$M_D = 20 \text{ kN}\cdot\text{m} - (8 \text{ kN})(3 \text{ m}) = -4 \text{ kN}\cdot\text{m}. \quad M_B = 0$$

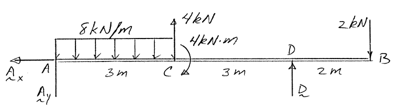


PROBLEM 7.79

Solve Prob. 7.78 assuming that the 4-kN force applied at E is directed upward.

SOLUTION

(a)
Replacing the load at E with equivalent force-couple at C.

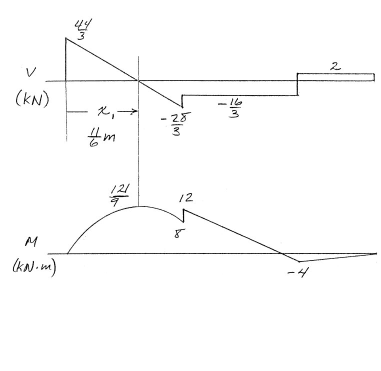


$$\circlearrowleft \Sigma M_A = 0: (6\text{ m})D - (8\text{ m})(2\text{ kN}) + (3\text{ m})(4\text{ kN}) - 4\text{ kN}\cdot\text{m} - (1.5\text{ m})(8\text{ kN/m})(3\text{ m}) = 0$$

$$\mathbf{D} = \frac{22}{3}\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y + \frac{22}{3}\text{ kN} - (8\text{ kN/m})(3\text{ m}) + 4\text{ kN} - 2\text{ kN} = 0$$

$$\mathbf{A}_y = \frac{44}{3}\text{ kN} \uparrow$$



Shear Diag:

$$V_A = A_y = \frac{44}{3}\text{ kN, then } V \text{ is linear } \left(\frac{dV}{dx} = -8\text{ kN/m}\right) \text{ to } C.$$

$$V_C = \frac{44}{3}\text{ kN} - (8\text{ kN/m})(3\text{ m}) = -\frac{28}{3}\text{ kN}$$

$$V = 0 = \frac{44}{3}\text{ kN} - (8\text{ kN/m})x_1 \text{ at } x_1 = \frac{11}{6}\text{ m.}$$

At C, V increases 4 kN to $-\frac{16}{3}$ kN.

At D, V increases $\frac{22}{3}$ kN to 2 kN.

PROBLEM 7.79 CONTINUED

Moment Diag:

$M_A = 0$. Then M is parabolic $\left(\dfrac{dM}{dx}\text{ decreasing with } V\right)$. Max M occurs where $V = 0$.

$$M_{\max} = \frac{1}{2}\left(\frac{44}{3}\text{ kN}\right)\left(\frac{11}{6}\text{ m}\right) = \frac{121}{9}\text{ kN}\cdot\text{m}$$

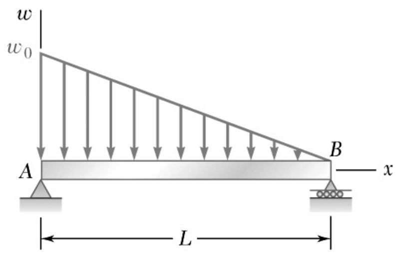
(*b*) $M_{\max} = 13.44\text{ kN}\cdot\text{m at } 1.833\text{ m from } A$ ◀

$$M_C = \frac{121}{9}\text{ kN}\cdot\text{m} - \frac{1}{2}\left(\frac{28}{3}\text{ kN}\right)\left(\frac{7}{6}\text{ m}\right) = 8\text{ kN}\cdot\text{m}.$$

At C, M increases by $4\text{ kN}\cdot\text{m}$ to $12\text{ kN}\cdot\text{m}$. Then M is linear $\left(\dfrac{dM}{dx} = -\dfrac{16}{3}\text{ kN}\right)$ to D.

$M_D = 12\text{ kN}\cdot\text{m} - \left(\dfrac{16}{3}\text{ kN}\right)(3\text{ m}) = -4\text{ kN}\cdot\text{m}$. M is again linear

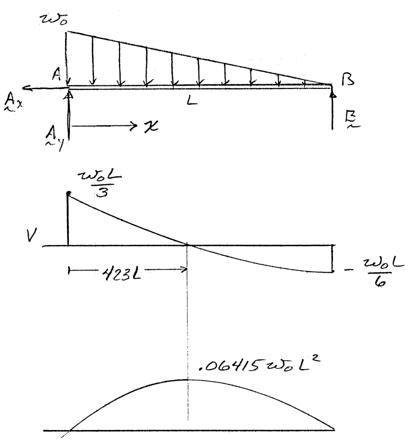
$\left(\dfrac{dM}{dx} = 2\text{ kN}\right)$ to zero at B.



PROBLEM 7.80

For the beam and loading shown, (*a*) derive the equations of the shear and bending-moment curves, (*b*) draw the shear and bending-moment diagrams, (*c*) determine the magnitude and location of the maximum bending moment.

SOLUTION



Distributed load $w = w_0\left(1 - \frac{x}{L}\right) \qquad \left(\text{total} = \frac{1}{2}w_0 L\right)$

$$\circlearrowleft \Sigma M_A = 0: \frac{L}{3}\left(\frac{1}{2}w_0 L\right) - LB = 0 \qquad \mathbf{B} = \frac{w_0 L}{6}\uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - \frac{1}{2}w_0 L + \frac{w_0 L}{6} = 0 \qquad \mathbf{A}_y = \frac{w_0 L}{3}\uparrow$$

Shear:

$$V_A = A_y = \frac{w_0 L}{3},$$

Then
$$\frac{dV}{dx} = -w \to V = V_A - \int_0^x w_0\left(1 - \frac{x}{L}\right)dx$$

$$V = \left(\frac{w_0 L}{3}\right) - w_0 x + \frac{1}{2}\frac{w_0}{L}x^2 = w_0 L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]$$

Note: At $x = L,\ V = -\frac{w_0 L}{6};$

$$V = 0 \quad \text{at} \quad \left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right) + \frac{2}{3} = 0 \to \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}$$

Moment:

$$M_A = 0,$$

Then
$$\left(\frac{dM}{dx}\right) = V \to M = \int_0^x V dx = L\int_0^{x/L} V\left(\frac{x}{L}\right)d\left(\frac{x}{L}\right)$$

$$M = w_0 L^2 \int_0^{x/L}\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]d\left(\frac{x}{L}\right)$$

$$M = w_0 L^2\left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right]$$

PROBLEM 7.80 CONTINUED

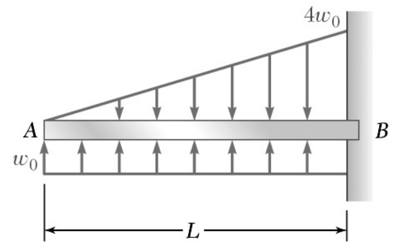
$$M_{\max}\left(\text{at } \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}\right) = 0.06415 w_0 L^2$$

(*a*) $$V = w_0 L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right] \blacktriangleleft$$

$$M = w_0 L^2\left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right] \blacktriangleleft$$

(*c*) $$M_{\max} = 0.0642\ w_0 L^2 \blacktriangleleft$$

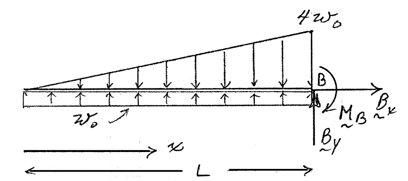
$$\text{at } x = 0.423L \blacktriangleleft$$



PROBLEM 7.81

For the beam and loading shown, (*a*) derive the equations of the shear and bending-moment curves, (*b*) draw the shear and bending-moment diagrams, (*c*) determine the magnitude and location of the maximum bending moment.

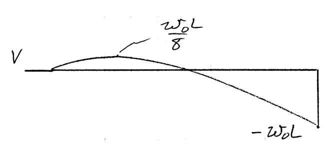
SOLUTION



Distributed load $\qquad w = w_0\left[4\left(\frac{x}{L}\right) - 1\right]$

Shear: $\qquad \frac{dV}{dx} = -w, \text{ and } V(0) = 0, \text{ so}$

$$V = \int_0^x -w\,dx = -\int_0^{x/L} Lw\,d\left(\frac{x}{L}\right)$$

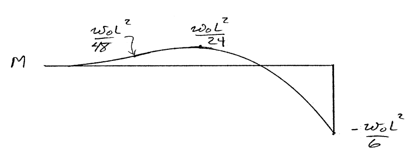


$$V = \int_0^{x/L} w_0 L\left[1 - 4\left(\frac{x}{L}\right)\right] d\left(\frac{x}{L}\right) = w_0 L\left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right]$$

Notes: At $x = L, V = -w_0 L$

And $V = 0$ at $\qquad \left(\frac{x}{L}\right) = 2\left(\frac{x}{L}\right)^2 \quad \text{or} \quad \frac{x}{L} = \frac{1}{2}$

Also V is max where $w = 0 \; \left(\frac{x}{L} = \frac{1}{4}\right)$



$$V_{max} = \frac{1}{8} w_0 L$$

Moment: $\qquad M(0) = 0, \frac{dM}{dx} = V$

$$M = \int_0^x v\,dx = L\int_0^{x/L} V\left(\frac{x}{L}\right) d\left(\frac{x}{L}\right)$$

$$M = w_0 L^2 \int_0^{x/L} \left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right] d\left(\frac{x}{L}\right)$$

(*a*) $\qquad V = w_0 L\left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right]$ ◀

$$M = w_0 L^2\left[\frac{1}{2}\left(\frac{x}{L}\right)^2 - \frac{2}{3}\left(\frac{x}{L}\right)^3\right] \blacktriangleleft$$

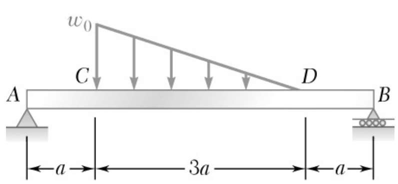
PROBLEM 7.81 CONTINUED

$$M_{\max} = \frac{1}{24} w_0 L^2 \text{ at } \quad x = \frac{L}{2}$$

$$M_{\min} = -\frac{1}{6} w_0 L^2 \text{ at } x = L$$

$$M_{\max} = \frac{w_0 L^2}{24} \text{ at } x = \frac{L}{2}$$

(*c*) $$|M|_{\max} = -M_{\min} = \frac{w_0 L^2}{6} \text{ at } B \blacktriangleleft$$

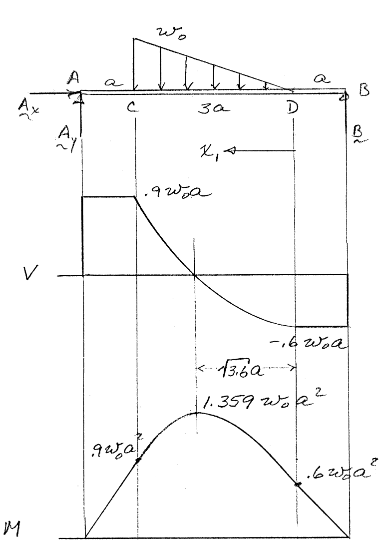


PROBLEM 7.82

For the beam shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum bending moment. (*Hint:* Derive the equations of the shear and bending-moment curves for portion *CD* of the beam.)

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: \ (3a)\left[\frac{1}{2}w_0(3a)\right] - 5aA_y = 0 \qquad \mathbf{A}_y = 0.9w_0a \uparrow$$

$$\uparrow \Sigma F_y = 0: \ 0.9w_0a - \frac{1}{2}w_0(3a) + B = 0$$

$$\mathbf{B} = 0.6w_0a \uparrow$$

Shear Diag:

$V = A_y = 0.9w_0a$ from A to C and $V = B = -0.6w_0a$ from B to D.

Then from D to C, $w = w_0\dfrac{x_1}{3a}$. If x_1 is measured right to left, $\dfrac{dV}{dx_1} = +w$ and $\dfrac{dM}{dx_1} = -V$. So, from D, $V = -0.6w_0a + \int_0^{x_1}\dfrac{w_0}{3a}x_1dx_1$,

$$V = w_0a\left[-0.6 + \frac{1}{6}\left(\frac{x_1}{a}\right)^2\right]$$

$$\text{Note: } V = 0 \text{ at } \left(\frac{x_1}{a}\right)^2 = 3.6, \ x_1 = \sqrt{3.6}a$$

Moment Diag:

$M = 0$ at A, increasing linearly $\left(\dfrac{dM}{dx_1} = 0.9w_0a\right)$ to $M_C = 0.9w_0a^2$.

Similarly $M = 0$ at B increasing linearly $\left(\dfrac{dM}{dx} = 0.6w_0a\right)$ to $M_D = 0.6w_0a^2$. Between C and D

$$M = 0.6w_0a^2 + w_0a\int_0^{x_1}\left[0.6 - \frac{1}{6}\left(\frac{x_1}{a}\right)^2\right]dx_1,$$

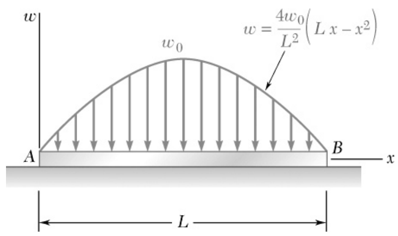
$$M = w_0a^2\left[0.6 + 0.6\left(\frac{x_1}{a}\right) - \frac{1}{18}\left(\frac{x_1}{a}\right)^3\right]$$

PROBLEM 7.82 CONTINUED

(*b*)

At $\frac{x_1}{a} = \sqrt{3.6},\ M = M_{max} = 1.359 w_0 a^2$ ◀

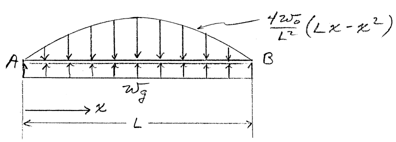
$x_1 = 1.897a$ left of D ◀



PROBLEM 7.83

Beam AB, which lies on the ground, supports the parabolic load shown. Assuming the upward reaction of the ground to be uniformly distributed, (a) write the equations of the shear and bending-moment curves, (b) determine the maximum bending moment.

SOLUTION



(a) $$\uparrow \Sigma F_y = 0\colon\ w_g L - \int_0^L \frac{4w_0}{L^2}\left(Lx - x^2\right)dx = 0$$

$$w_g L = \frac{4w_0}{L^2}\left(\frac{1}{2}LL^2 - \frac{1}{3}L^3\right) = \frac{2}{3}w_0 L \qquad w_g = \frac{2w_0}{3}$$

Define $\xi = \dfrac{x}{L}$ so $d\xi = \dfrac{dx}{L}$ $\rightarrow$ net load $w = 4w_0\left[\dfrac{x}{L} - \left(\dfrac{x}{L}\right)^2\right] - \dfrac{2}{3}w_0$

or $$w = 4w_0\left(-\frac{1}{6} + \xi - \xi^2\right)$$

$$V = V(0) - \int_0^{\xi} 4w_0 L\left(-\frac{1}{6} + \xi - \xi^2\right)d\xi =$$

$$0 + 4w_0 L\left(\frac{1}{6}\xi + \frac{1}{2}\xi^2 - \frac{1}{3}\xi^3\right)$$

$$V = \frac{2}{3}w_0 L\left(\xi - 3\xi^2 + 2\xi^3\right) \blacktriangleleft$$

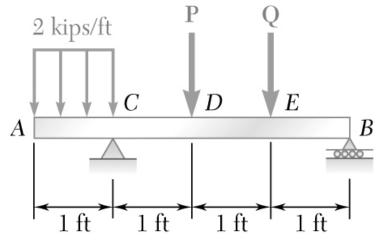
$$M = M_0 + \int_0^x V dx = 0 + \frac{2}{3}w_0 L^2 \int_0^{\xi}\left(\xi - 3\xi^2 + 2\xi^3\right)d\xi$$

$$= \frac{2}{3}w_0 L^2\left(\frac{1}{2}\xi^2 - \xi^3 + \frac{1}{2}\xi^4\right) = \frac{1}{3}w_0 L^2\left(\xi^2 - 2\xi^3 + \xi^4\right) \blacktriangleleft$$

(b) Max M occurs where $V = 0 \rightarrow 1 - 3\xi + 2\xi^2 = 0 \rightarrow \xi = \dfrac{1}{2}$

$$M\left(\xi = \frac{1}{2}\right) = \frac{1}{3}w_0 L^2\left(\frac{1}{4} - \frac{2}{8} + \frac{1}{16}\right) = \frac{w_0 L^2}{48}$$

$$M_{\max} = \frac{w_0 L^2}{48} \text{ at center of beam} \blacktriangleleft$$

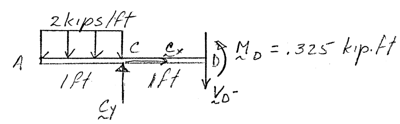


PROBLEM 7.84

The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is +325 lb ft at D and +800 lb ft at E, (a) determine **P** and **Q**, (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

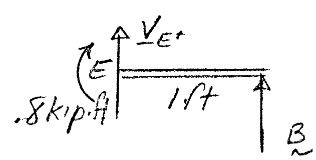
FBD ACD:



$$(a) \quad \circlearrowleft \Sigma M_{D^-} = 0: \; 0.325 \text{ kip}\cdot\text{ft} - (1 \text{ ft})C_y + (1.5 \text{ ft})(2 \text{ kips/ft})(1 \text{ ft}) = 0$$

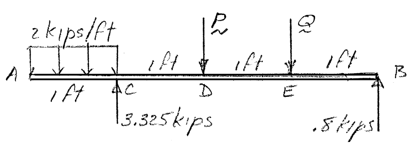
$$\mathbf{C}_y = 3.325 \text{ kips} \uparrow$$

FBD EB:



$$\circlearrowleft \Sigma M_E = 0: \; (1 \text{ ft})B - 0.8 \text{ kip}\cdot\text{ft} = 0 \qquad \mathbf{B} = 0.8 \text{ kip} \uparrow$$

FBD Beam:



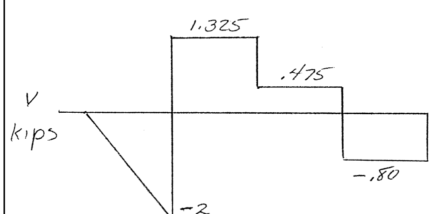
$$\circlearrowleft \Sigma M_D = 0: \; (1.5 \text{ ft})(2 \text{ kips/ft})(1 \text{ ft}) - (1 \text{ ft})(3.325 \text{ kips})$$
$$- (1 \text{ ft})Q + 2 \text{ ft}(0.8 \text{ kips}) = 0$$

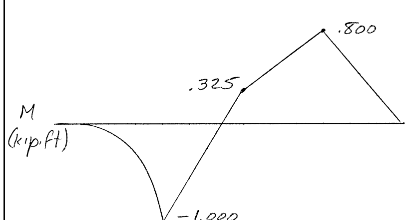
$$Q = 1.275 \text{ kips}$$

$$\uparrow \Sigma F_y = 0: \; 3.325 \text{ kips} + 0.8 \text{ kips} - 1.275 \text{ kips}$$
$$- (2 \text{ kips/ft})(1 \text{ ft}) - P = 0 \qquad P = 0.85 \text{ kip} \downarrow$$

(a) $\mathbf{P} = 850 \text{ lb} \downarrow$ ◀

$\mathbf{Q} = 1.275 \text{ kips} \downarrow$ ◀





(b) **Shear Diag:**

V is linear $\left(\dfrac{dV}{dx} = -2 \text{ kips/ft}\right)$ from 0 at A to

$-(2 \text{ kips/ft})(1 \text{ ft}) = -2 \text{ kips}$ at C. Then V is piecewise constant with discontinuities equal to forces at C, D, E, B

Moment Diag:

M is parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$ from 0 at A to

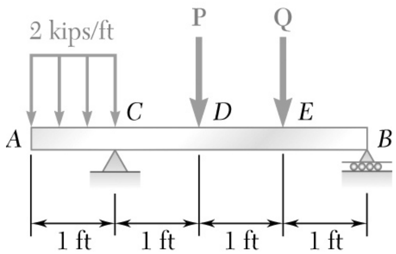
$-\dfrac{1}{2}(2 \text{ kips})(1 \text{ ft}) = -1 \text{ kip}\cdot\text{ft}$ at C. Then M is piecewise linear with

PROBLEM 7.84 CONTINUED

$$M_D = -1\text{ kip}\cdot\text{ft} + (1.325\text{ kips})(1\text{ ft}) = 0.325\text{ kip}\cdot\text{ft}$$

$$M_E = 0.325\text{ kip}\cdot\text{ft} + (0.475\text{ kips})(1\text{ ft}) = 0.800\text{ kip}\cdot\text{ft}$$

$$M_B = 0.8\text{ kip}\cdot\text{ft} - (0.8\text{ kip})(1\text{ ft}) = 0$$

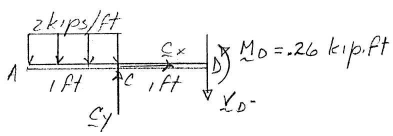


PROBLEM 7.85

Solve Prob. 7.84 assuming that the bending moment was found to be +260 lb ft at D and +860 lb ft at E.

SOLUTION

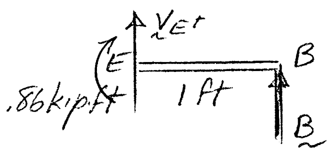
FBD ACD:



(a) $\circlearrowleft \Sigma M_D = 0\text{: } 0.26\,\text{kip}\cdot\text{ft} - (1\text{ ft})C_y + (1.5\text{ ft})(2\text{ kips/ft})(1\text{ ft}) = 0$

$\mathbf{C}_y = 3.26\text{ kips} \uparrow$

FBD DB:



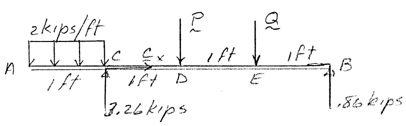
$\circlearrowleft \Sigma M_E = 0\text{: } (1\text{ ft})B - 0.86\text{ kip}\cdot\text{ft}$ $\qquad \mathbf{B} = 0.86\text{ kip} \uparrow$

FBD Beam:

$\circlearrowleft \Sigma M_D = 0\text{: } (1.5\text{ ft})(2\text{ kips/ft})(1\text{ ft})$

$- (1\text{ ft})(3.26\text{ kips}) + (1\text{ ft})Q + (2\text{ ft})(0.86\text{ kips}) = 0$

$Q = 1.460\text{ kips} \qquad \mathbf{Q} = 1.460\text{ kips} \downarrow$ ◀



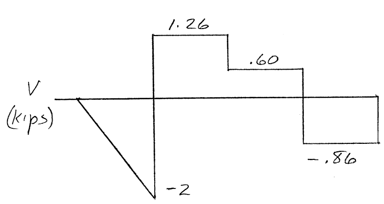
$\uparrow \Sigma F_y = 0\text{: } 3.26\text{ kips} + 0.86\text{ kips} - 1.460\text{ kips}$

$-P - (2\text{ kips/ft})(1\text{ ft}) = 0$

$P = 0.66\text{ kips}$

$\mathbf{P} = 660\text{ lb} \downarrow$ ◀

(b) **Shear Diag:**

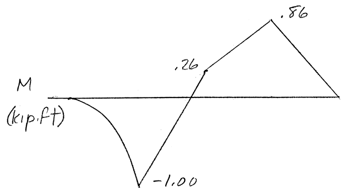


V is linear $\left(\dfrac{dV}{dx} = -2\text{ kips/ft}\right)$ from 0 at A to $-(2\text{ kips/ft})(1\text{ ft}) = -2\text{ kips}$ at C. Then V is piecewise constant with discontinuities equal to forces at C, D, E, B.

Moment Diag:

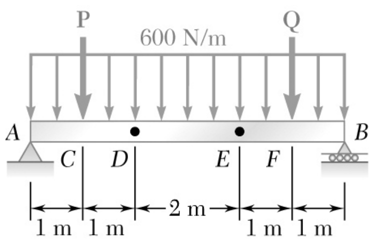
M is parabolic $\left(\dfrac{dM}{dx}\text{ decreasing with } V\right)$ from 0 at A to $-\dfrac{1}{2}(2\text{ kips/ft})(1\text{ ft}) = -1\text{ kip}\cdot\text{ft}$ at C. Then M is piecewise linear with

PROBLEM 7.85 CONTINUED



$M_0 = 0.26 \text{ kip}\cdot\text{ft}$

$M_E = 0.86 \text{ kip}\cdot\text{ft}, \quad M_B = 0$

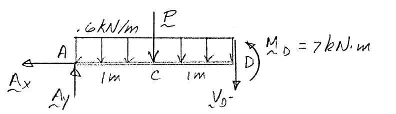


PROBLEM 7.86

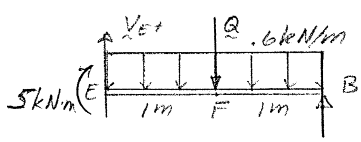
The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+7\text{ kN}\cdot\text{m}$ at D and $+5\text{ kN}\cdot\text{m}$ at E, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

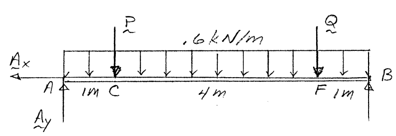
SOLUTION

FBD AD:



FBD EB:





(*a*)

$$\circlearrowleft \Sigma M_D = 0:\quad 7\text{ kN}\cdot\text{m} + (1\text{ m})(0.6\text{ kN/m})(2\text{ m}) - (2\text{ m})A_y = 0$$

$$2A_y - P = 8.2\text{ kN} \qquad (1)$$

$$\circlearrowleft \Sigma M_E = 0: (2\text{ m})B - (1\text{ m})Q - (1\text{ m})(0.6\text{ kN/m})(2\text{ m}) - 5\text{ kN}\cdot\text{m} = 0$$

$$2B - Q = 6.2\text{ kN} \qquad (2)$$

$$\circlearrowleft \Sigma M_A = 0:\quad (6\text{ m})B - (1\text{ m})P - (5\text{ m})Q - (3\text{ m})(0.6\text{ kN/m})(6\text{ m}) = 0$$

$$6B - P - 5Q = 10.8\text{ kN} \qquad (3)$$

$$\circlearrowleft \Sigma M_B = 0: (1\text{ m})Q + (5\text{ m})P + (3\text{ m})(0.6\text{ kN/m})(6\text{ m}) - (6\text{ m})A = 0$$

$$6A - Q - 5P = 10.8\text{ kN} \qquad (4)$$

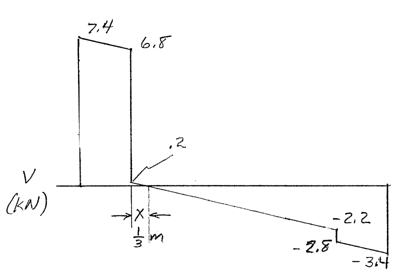
Solving (1)–(4): $\mathbf{P} = 6.60\text{ kN}\downarrow$, $\mathbf{Q} = 600\text{ N}\downarrow$ ◀

$\mathbf{A}_y = 7.4\text{ kN}\uparrow$, $\quad \mathbf{B} = 3.4\text{ kN}\uparrow$

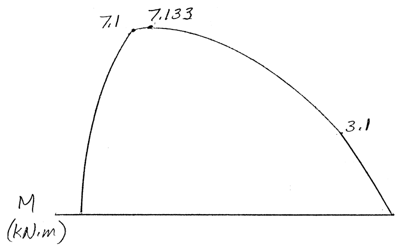
(*b*) **Shear Diag:**

V is piecewise linear with $\dfrac{dV}{dx} = -0.6\text{ kN/m}$ throughout, and discontinuities equal to forces at A, C, F, B.

Note $\quad V = 0 = 0.2\text{ kN} - (0.6\text{ kN/m})x \;\text{ at }\; x = \dfrac{1}{3}\text{ m}$



PROBLEM 7.86 CONTINUED



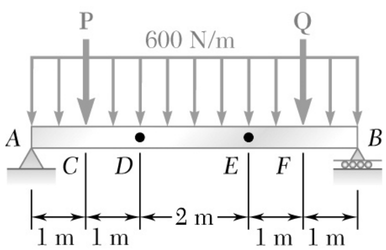
Moment Diag:

M is piecewise parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$ with "breaks" in slope at C and F.

$$M_C = \frac{1}{2}(7.4 + 6.8)\text{kN}(1\text{ m}) = 7.1\text{ kN}\cdot\text{m}$$

$$M_{\max} = 7.1\text{ kN}\cdot\text{m} + \frac{1}{2}(0.2\text{ kN})\left(\frac{1}{3}\text{ m}\right) = 7.13\dot{3}\text{ kN}\cdot\text{m}$$

$$M_F = 7.13\dot{3}\text{ kN}\cdot\text{m} - \frac{1}{2}(2.2\text{ kN})\left(3\frac{2}{3}\text{ m}\right) = 3.1\text{ kN}\cdot\text{m}$$

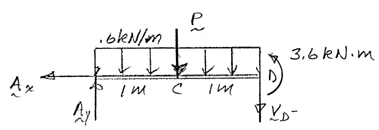


PROBLEM 7.87

Solve Prob. 7.86 assuming that the bending moment was found to be $+3.6\ \text{kN}\cdot\text{m}$ at D and $+4.14\ \text{kN}\cdot\text{m}$ at E.

SOLUTION

FBD AD:

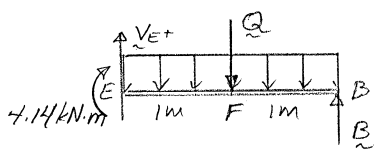


(*a*)

$$\circlearrowleft \Sigma M_D = 0\text{: } 3.6\,\text{kN}\cdot\text{m} + (1\text{ m})P + (1\text{ m})(0.6\text{ kN/m})(2\text{ m}) - (2\text{ m})A_y = 0$$

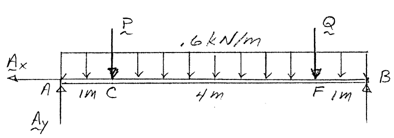
$$2A_y - P = 4.8\text{ kN} \qquad (1)$$

FBD EB:



$$\circlearrowleft \Sigma M_E = 0\text{: } (2\text{ m})B - (1\text{ m})Q - (1\text{ m})(0.6\text{ kN/m})(2\text{ m}) - 4.14\,\text{kN}\cdot\text{m} = 0$$

$$2B - Q = 5.34\text{ kN} \qquad (2)$$



$$\circlearrowleft \Sigma M_A = 0\text{: } (6\text{ m})B - (5\text{ m})Q - (1\text{ m})P - (3\text{ m})(0.6\text{ kN/m})(6\text{ m}) = 0$$

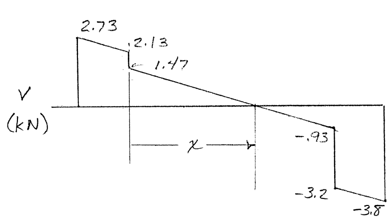
$$6B - P - 5Q = 10.8\text{ kN} \qquad (3)$$

By symmetry:

$$6A - Q - 5P = 10.8\text{ kN} \qquad (4)$$

Solving (1)–(4) $\qquad \mathbf{P} = 660\text{ N}\downarrow,\ \mathbf{Q} = 2.28\text{ kN}\downarrow$ ◀

$$A_y = 2.73\text{ kN}\uparrow,\ B = 3.81\text{ kN}\uparrow$$



(*b*) **Shear Diag:**

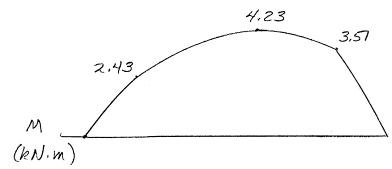
V is piecewise linear with $\left(\dfrac{dV}{dx} = -0.6\text{ kN/m}\right)$ throughout, and discontinuities equal to forces at A, C, F, B.

Note that $V = 0 = 1.47\text{ kN} - (0.6\text{ kN/m})x$ at $x = 2.45\text{ m}$

Moment Diag:

M is piecewise parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$, with "breaks" in slope at C and F.

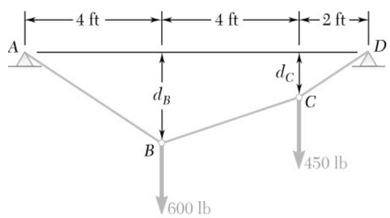
PROBLEM 7.87 CONTINUED



$$M_C = \frac{1}{2}(2.73 + 2.13)\text{kN}(1\text{ m}) = 2.43\text{ kN}\cdot\text{m}$$

$$M_{\max} = 2.43\text{ kN}\cdot\text{m} + \frac{1}{2}(1.47\text{ kN})(2.45\text{ m}) = 4.231\text{ kN}\cdot\text{m}$$

$$M_F = 4.231\text{ kN}\cdot\text{m} - \frac{1}{2}(0.93\text{ kN})(1.55\text{ m}) = 3.51\text{ kN}\cdot\text{m}$$

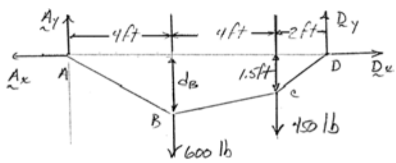


PROBLEM 7.88

Two loads are suspended as shown from cable *ABCD*. Knowing that $d_C = 1.5$ ft, determine (*a*) the distance d_B, (*b*) the components of the reaction at *A*, (*c*) the maximum tension in the cable.

SOLUTION

FBD cable:



$$\circlearrowleft \Sigma M_A = 0: (10 \text{ ft})D_y - 8 \text{ ft}(450 \text{ lb}) - 4 \text{ ft}(600 \text{ lb}) = 0$$

$$D_y = 600 \text{ lb} \uparrow$$

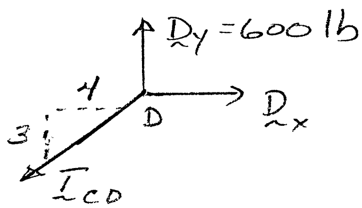
$$\uparrow \Sigma F_y = 0: A_y + 600 \text{ lb} - 600 \text{ lb} - 450 \text{ lb} = 0$$

$$A_y = 450 \text{ lb}$$

$$\leftarrow \Sigma F_x = 0: \quad A_x - D_x = 0 \qquad (1)$$

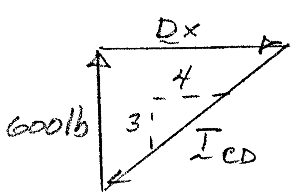
FBD pt D:

$$\frac{600 \text{ lb}}{3} = \frac{D_x}{4} = \frac{T_{CD}}{5}: D_x = 800 \text{ lb} \longrightarrow = A_x$$



So

And $\qquad T_{CD} = 1000 \text{ lb}$



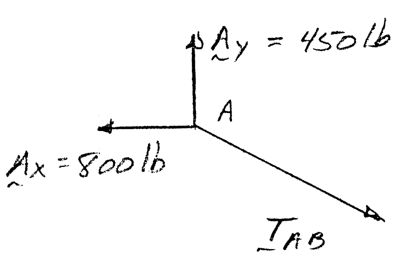
$$\frac{800 \text{ lb}}{4 \text{ ft}} = \frac{450 \text{ lb}}{d_B}$$

FBD pt A:

(*a*) $\qquad d_B = 2.25 \text{ ft}$ ◀

(b) $\qquad \mathbf{A}_x = 800 \text{ lb} \longleftarrow$ ◀

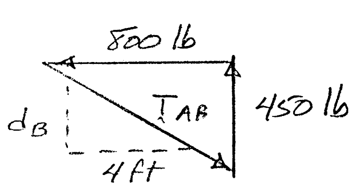
$\qquad \mathbf{A}_y = 450 \text{ lb} \uparrow$ ◀

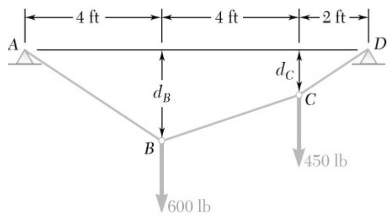


$$T_{AB} = \sqrt{(800 \text{ lb})^2 + (450 \text{ lb})^2} = 918 \text{ lb}$$

So $\qquad$ (*c*) $\qquad T_{max} = T_{CD} = 1000 \text{ lb}$ ◀

Note: T_{CD} is T_{max} as cable slope is largest in section *CD*.



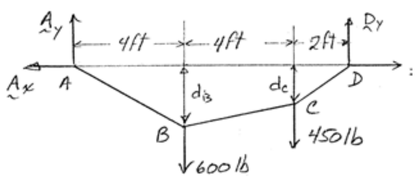


PROBLEM 7.89

Two loads are suspended as shown from cable *ABCD*. Knowing that the maximum tension in the cable is 720 lb, determine (*a*) the distance d_B, (*b*) the distance d_C.

SOLUTION

FBD cable:



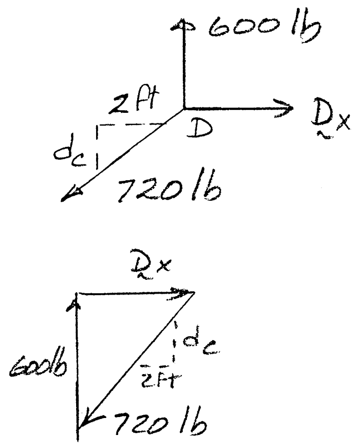
$$\circlearrowleft \Sigma M_A = 0: (10\text{ ft})D_y - (8\text{ ft})(450\text{ lb}) - (4\text{ ft})(600\text{ lb}) = 0$$

$$\mathbf{D}_y = 600\text{ lb} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y + 600\text{ lb} - 600\text{ lb} - 450\text{ lb} = 0$$

$$\mathbf{A}_y = 450\text{ lb} \uparrow$$

FBD pt D:



$$\leftarrow \Sigma F_x = 0: A_x - B_x = 0$$

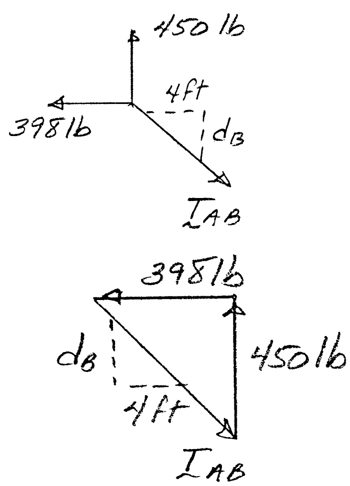
Since $A_x = B_x$; And $D_y > A_y$, Tension $T_{CD} > T_{AB}$

So $T_{CD} = T_{\max} = 720\text{ lb}$

$$D_x = \sqrt{(720\text{ lb})^2 - (600\text{ lb})^2} = 398\text{ lb} = A_x$$

$$\frac{d_C}{600\text{ lb}} = \frac{2\text{ ft}}{398\text{ lb}} \qquad d_C = 3.015\text{ ft}$$

FBD pt. A:

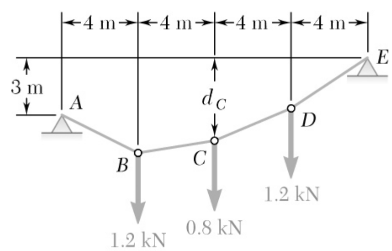


$$\frac{d_B}{450\text{ lb}} = \frac{4\text{ ft}}{398\text{ lb}}$$

(*a*) $d_B = 4.52\text{ ft}$ ◀

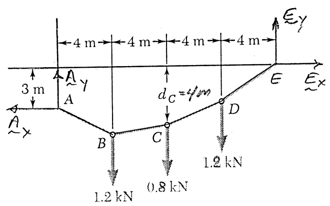
(*b*) $d_C = 3.02\text{ ft}$ ◀

PROBLEM 7.90



Knowing that $d_C = 4$ m, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

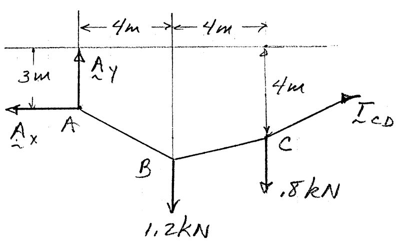
SOLUTION



(*a*) **FBD cable:**

$$\circlearrowleft \Sigma M_E = 0: \quad (4\text{ m})(1.2\text{ kN}) + (8\text{ m})(0.8\text{ kN}) + (12\text{ m})(1.2\text{ kN}) - (3\text{ m})A_x - (16\text{ m})A_y = 0$$

$$3A_x + 16A_y = 25.6\text{ kN} \qquad (1)$$



FBD ABC:

$$\circlearrowleft \Sigma M_C = 0: (4\text{ m})(1.2\text{ kN}) + (1\text{ m})A_x - (8\text{ m})A_y = 0$$

$$A_x - 8A_y = -4.8\text{ kN} \qquad (2)$$

Solving (1) and (2) $\qquad A_x = 3.2\text{ kN} \qquad A_y = 1\text{ kN}$

So $\mathbf{A} = 3.35\text{ kN} \searrow 17.35°$ ◀

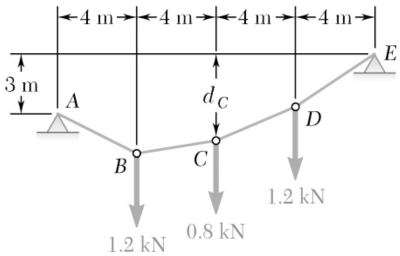
(*b*) cable: $\quad \rightarrow \Sigma F_x = 0: -A_x + E_x = 0$

$$E_x = A_x = 3.2\text{ kN}$$

$$\uparrow \Sigma F_y = 0: A_y - (1.2 + 0.8 + 1.2)\text{ kN} + E_y = 0$$

$$E_y = 3.2\text{ kN} - A_y = (3.2 - 1)\text{ kN} = 2.2\text{ kN}$$

So $\mathbf{E} = 3.88\text{ kN} \angle 34.5°$ ◀

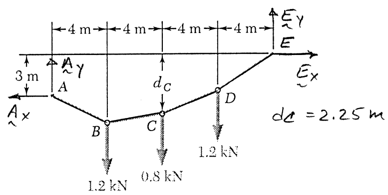


PROBLEM 7.91

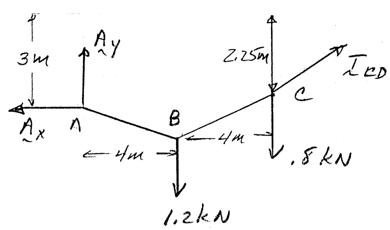
Knowing that $d_C = 2.25$ m, determine (*a*) the reaction at A, (*b*) the reaction at E.

SOLUTION

FBD Cable:



FBD ABC:



(*a*)

$$\circlearrowleft \Sigma M_E = 0: (4\text{ m})(1.2\text{ kN}) + (8\text{ m})(0.8\text{ kN}) + (12\text{ m})(1.2\text{ kN}) - (3\text{ m})A_x - (16\text{ m})A_y = 0$$

$$3A_x + 16A_y = 25.6\text{ kN} \qquad (1)$$

$$\circlearrowleft \Sigma M_C = 0: (4\text{ m})(1.2\text{ kN}) - (0.75\text{ m})A_x - (8\text{ m})A_y = 0$$

$$0.75A_x + 8A_y = 4.8\text{ kN} \qquad (2)$$

Solving (1) and (2) $\qquad A_x = \dfrac{32}{3}$ kN, $\qquad A_y = -0.4$ kN

So $\qquad \mathbf{A} = 10.67$ kN ↗ 2.15° ◀

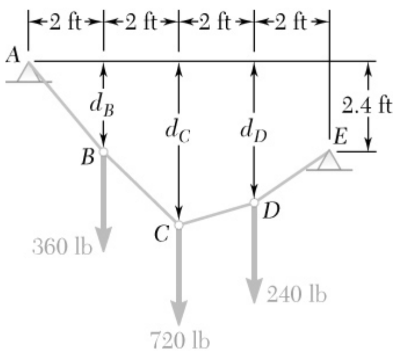
Note: this implies $d_B < 3$ m (in fact $d_B = 2.85$ m)

(*b*) FBD cable: $\qquad \rightarrow \Sigma F_x = 0: -\dfrac{32}{3}\text{ kN} + E_x = 0 \qquad E_x = \dfrac{32}{3}$ kN

$$\uparrow \Sigma F_y = 0: \quad -0.4\text{ kN} - 1.2\text{ kN} - 0.8\text{ kN} - 1.2\text{ kN} + E_y = 0$$

$$E_y = 3.6\text{ kN}$$

$\mathbf{E} = 11.26$ kN ∠ 18.65° ◀

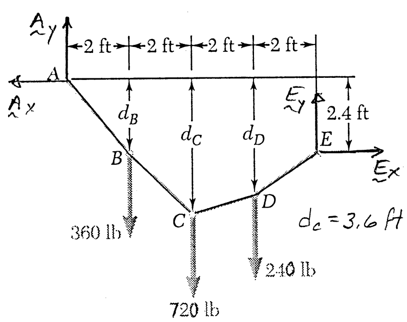


PROBLEM 7.92

Cable *ABCDE* supports three loads as shown. Knowing that $d_C = 3.6$ ft, determine (*a*) the reaction at *E*, (*b*) the distances d_B and d_D.

SOLUTION

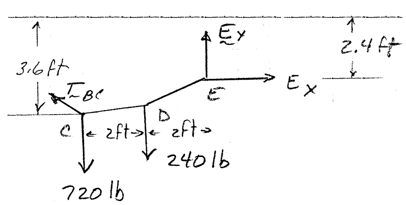
FBD Cable:



(*a*) $\circlearrowleft \Sigma M_A = 0: (2.4 \text{ ft})E_x + (8 \text{ ft})E_y - (2 \text{ ft})(360) - (4 \text{ ft})(720 \text{ lb}) - (6 \text{ ft})(240 \text{ lb}) = 0$

$$0.3E_x + E_y = 630 \text{ lb} \qquad (1)$$

FBD CDE:



$\circlearrowleft \Sigma M_C = 0: -(1.2 \text{ ft})E_x + (4 \text{ ft})E_y - (2 \text{ ft})(240 \text{ lb}) = 0$

$$-0.3E_x + E_y = +120 \text{ lb} \qquad (2)$$

Solving (1) and (2) $\qquad E_x = 850 \text{ lb} \qquad E_y = 375 \text{ lb}$

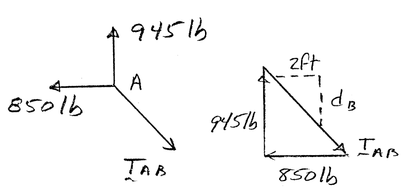
(*a*) $\qquad \mathbf{E} = 929 \text{ lb} \measuredangle 23.8°$ ◀

(*b*) cable: $\quad \rightarrow \Sigma F_x = 0: -A_x + E_x = 0 \qquad A_x = E_x = 850 \text{ lb}$

$\uparrow \Sigma F_y = 0: A_y - 360 \text{ lb} - 720 \text{ lb} - 240 \text{ lb} + 375 \text{ lb} = 0$

$$A_y = 945 \text{ lb}$$

Point A:

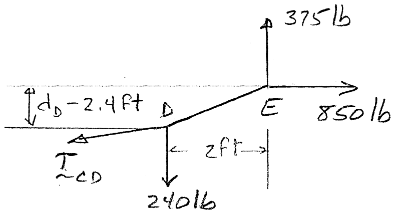


$$\frac{d_B}{2 \text{ ft}} = \frac{945 \text{ lb}}{850 \text{ lb}} \qquad d_B = 2.22 \text{ ft}$$ ◀

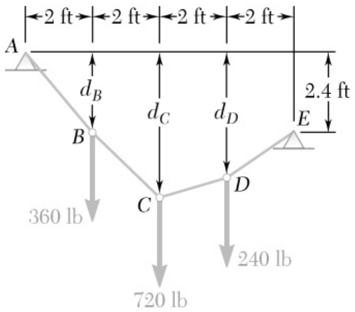
PROBLEM 7.92 CONTINUED

$$\circlearrowleft \Sigma M_D = 0: (2\text{ ft})(375\text{ lb}) - (d_D - 2.4\text{ ft})(850\text{ lb}) = 0$$

Segment DE:



$$d_D = 3.28\text{ ft} ◀$$

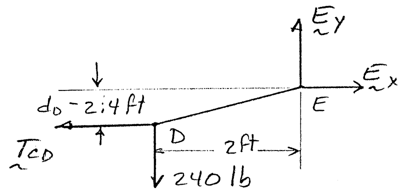


PROBLEM 7.93

Cable *ABCDE* supports three loads as shown. Determine (*a*) the distance d_C for which portion *CD* of the cable is horizontal, (*b*) the corresponding reactions at the supports.

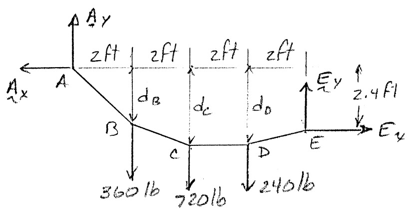
SOLUTION

Segment DE:



$$\uparrow \Sigma F_y = 0: E_y - 240 \text{ lb} = 0 \qquad \mathbf{E}_y = 240 \text{ lb} \uparrow$$

FBD Cable:



$$\circlearrowleft \Sigma M_A = (2.4 \text{ ft})E_x + (8 \text{ ft})(240 \text{ lb}) - (6 \text{ ft})(240 \text{ lb}) - (4 \text{ ft})(720 \text{ lb}) - (2 \text{ ft})(360 \text{ lb}) = 0$$

So

$$\mathbf{E}_x = 1300 \text{ lb} \longrightarrow$$

From Segment DE:

$$\circlearrowleft \Sigma M_D = 0: (2 \text{ ft})E_y - (d_C - 2.4 \text{ ft})E_x = 0$$

$$d_C = 2.4 \text{ ft} + \frac{E_y}{E_x}(2 \text{ ft}) = (2.4 \text{ ft}) + \frac{240 \text{ lb}}{1300 \text{ lb}}(2 \text{ ft}) = 2.7692 \text{ ft}$$

(*a*) $d_C = 2.77$ ft ◀

From FBD Cable:

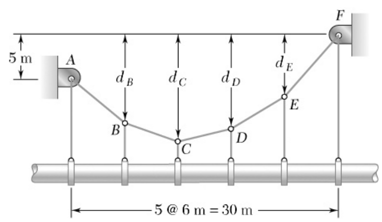
$$\longrightarrow \Sigma F_x = 0: \; -A_x + E_x = 0 \qquad \mathbf{A}_x = 1300 \text{ lb} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: A_y - 360 \text{ lb} - 720 \text{ lb} - 240 \text{ lb} + E_y = 0$$

$$\mathbf{A}_y = 1080 \text{ lb} \uparrow$$

(*b*) $\mathbf{A} = 1.690$ kips ⦪ 39.7° ◀

$\mathbf{E} = 1.322$ kips ⦩ 10.46° ◀

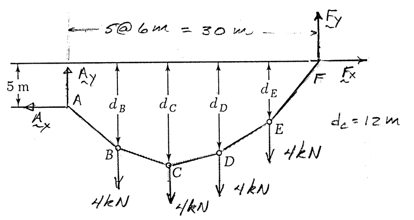


PROBLEM 7.94

An oil pipeline is supported at 6-m intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents, the tension in each hanger is 4 kN. Knowing that $d_C = 12$ m, determine (*a*) the maximum tension in the cable, (*b*) the distance d_D.

SOLUTION

FBD Cable:

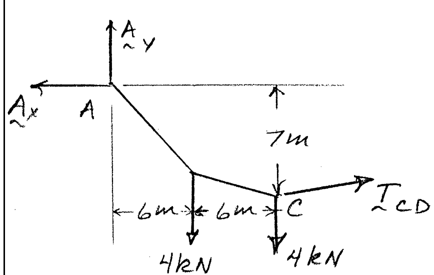


Note: $\mathbf{A}_y$ and $\mathbf{F}_y$ shown are forces on cable, assuming the 4 kN loads at *A* and *F* act on supports.

$$\circlearrowleft \Sigma M_F = 0: (6\text{ m})\left[1(4\text{ kN}) + 2(4\text{ kN}) + 3(4\text{ kN}) + 4(4\text{ kN})\right] - (30\text{ m})A_y - (5\text{ m})A_x = 0$$

$$A_x + 6A_y = 48\text{ kN} \qquad (1)$$

FBD ABC:



$$\circlearrowleft \Sigma M_C = 0: (6\text{ m})(4\text{ kN}) + (7\text{ m})A_x - (12\text{ m})A_y = 0$$

$$7A_x - 12A_y = -24\text{ kN} \qquad (2)$$

Solving (1) and (2) $\qquad \mathbf{A}_x = 8\text{ kN} \longrightarrow \qquad \mathbf{A}_y = \dfrac{20}{3}\text{ kN} \uparrow$

From FBD Cable:

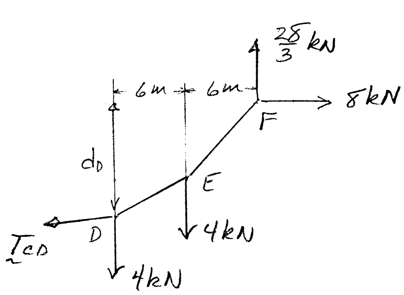
$$\longrightarrow \Sigma F_x = 0: -A_x + F_x = 0 \qquad F_x = A_x = 8\text{ kN}$$

$$\uparrow \Sigma F_y = 0: A_y - 4(4\text{ kN}) + F_y = 0$$

$$F_y = 16\text{ kN} - A_y = \left(16 - \frac{20}{3}\right)\text{kN} = \frac{28}{3}\text{ kN} > A_y$$

So $\qquad T_{EF} > T_{AB} \qquad T_{\max} = T_{EF} = \sqrt{F_x^2 + F_y^2}$

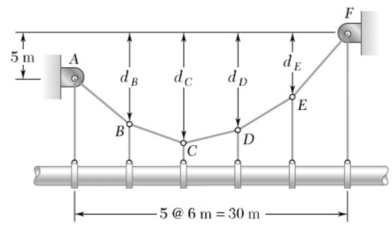
FBD DEF:



(*a*) $$T_{\max} = \sqrt{(18\text{ kN})^2 + \left(\frac{28}{3}\text{ kN}\right)^2} = 12.29\text{ kN} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_D = 0: (12\text{ m})\left(\frac{28}{3}\text{ kN}\right) - d_D(8\text{ kN}) - (6\text{ m})(4\text{ kN}) = 0$$

(*b*) $$d_D = 11.00\text{ m} \blacktriangleleft$$



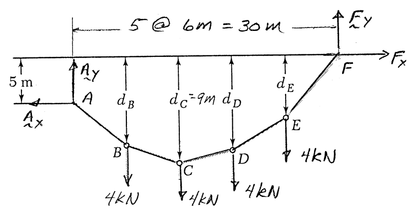
PROBLEM 7.95

Solve Prob. 7.94 assuming that $d_C = 9$ m.

SOLUTION

Note: 4 kN loads at A and F act directly on supports, not on cable.

FBD Cable:



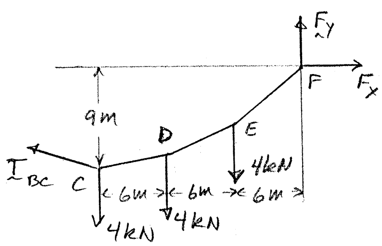
$$\circlearrowleft \Sigma M_A = 0: (30 \text{ m})F_y - (5 \text{ m})F_x$$

$$-(6 \text{ m})\left[1(4 \text{ kN}) + 2(4 \text{ kN}) + 3(4 \text{ kN}) + 4(4 \text{ kN})\right] = 0$$

$$F_x - 6F_y = -48 \text{ kN} \qquad (1)$$

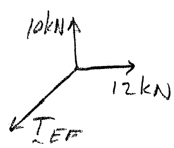
$$\circlearrowleft \Sigma M_C = 0: (18)F_y - (9 \text{ m})F_x - (12 \text{ m})(4 \text{ kN}) - (6 \text{ m})(4 \text{ kN}) = 0$$

FBD CDEF:



$$F_x - 2F_y = -8 \text{ kN} \qquad (2)$$

Solving (1) and (2) $\qquad \mathbf{F}_x = 12 \text{ kN} \longrightarrow \qquad \mathbf{F}_y = 10 \text{ kN} \uparrow$



$$T_{EF} = \sqrt{(10 \text{ kN})^2 + (12 \text{ kN})^2} = 15.62 \text{ kN}$$

Since slope $EF >$ slope AB this is T_{max}

(*a*) $\qquad T_{max} = 15.62 \text{ kN}$ ◀

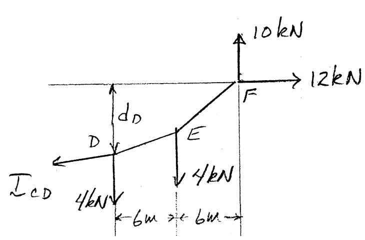
Also could note from FBD cable

$$\uparrow \Sigma F_y = 0: A_y + F_y - 4(4 \text{ kN}) = 0$$

$$A_y = 16 \text{ kN} - 12 \text{ kN} = 4 \text{ kN}$$

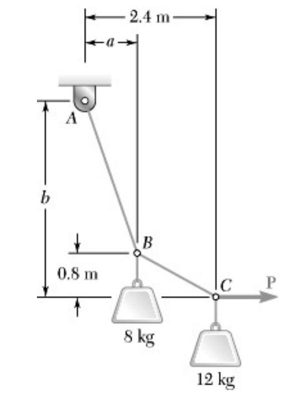
Thus $\qquad A_y < F_y \qquad$ and $\qquad T_{AB} < T_{EF}$

FBD DEF:



(*b*) $\qquad \circlearrowleft \Sigma M_D = 0: (12 \text{ m})(10 \text{ kN}) - d_D(12 \text{ kN}) - (6 \text{ m})(4 \text{ kN}) = 0$

$$d_D = 8.00 \text{ m} \blacktriangleleft$$

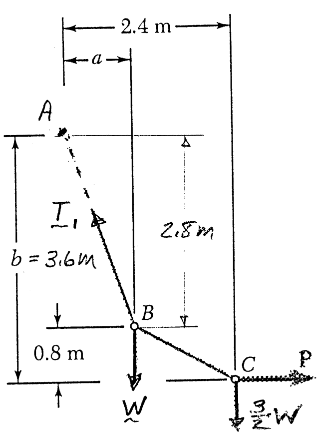


PROBLEM 7.96

Cable *ABC* supports two boxes as shown. Knowing that $b = 3.6$ m, determine (*a*) the required magnitude of the horizontal force **P**, (*b*) the corresponding distance *a*.

SOLUTION

FBD BC:



$$W = (8\text{ kg})(9.81\text{ m/s}^2) = 78.48\text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: (3.6\text{ m})P - (2.4\text{ m})\frac{3W}{2} - aW = 0$$

$$P = W\left(1 + \frac{a}{3.6\text{ m}}\right) \qquad (1)$$

$$\longrightarrow \Sigma F_x = 0: -T_{1x} + P = 0 \qquad T_{1x} = P$$

$$\uparrow \Sigma F_y = 0: T_{1y} - W - \frac{3}{2}W = 0 \qquad T_{1y} = \frac{5W}{2}$$

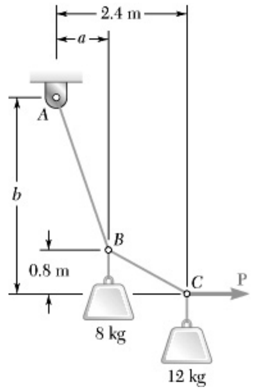
But $\dfrac{T_{1y}}{T_{1x}} = \dfrac{2.8\text{ m}}{a}$ so $\dfrac{5W}{2P} = \dfrac{2.8\text{ m}}{a}$

$$P = \frac{5Wa}{5.6\text{ m}} \qquad (2)$$

Solving (1) and (2): $a = 1.6258$ m, $P = 1.4516W$

So (*a*) $P = 1.4516(78.48) = 113.9\text{ N}$ ◀

(*b*) $a = 1.626\text{ m}$ ◀

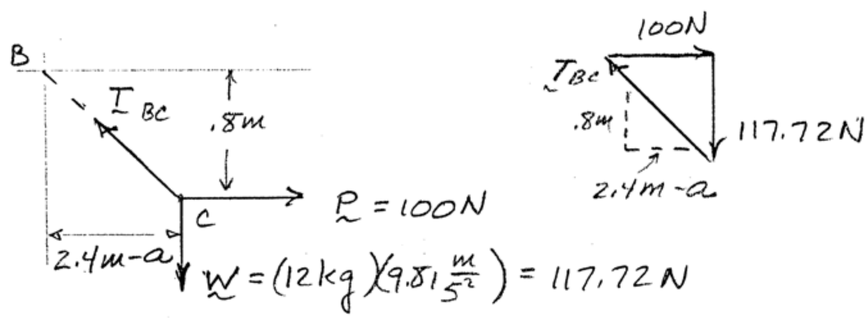


PROBLEM 7.97

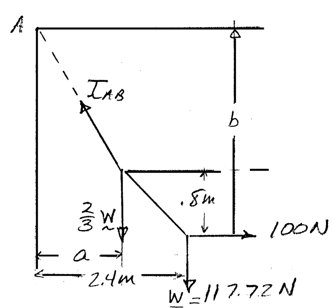
Cable *ABC* supports two boxes as shown. Determine the distances *a* and *b* when a horizontal force **P** of magnitude 100 N is applied at *C*.

SOLUTION

FBD pt C:



Segment BC:



$$\frac{2.4\text{ m} - a}{100\text{ N}} = \frac{0.8\text{ m}}{117.72\text{ N}}$$

$$a = 1.7204\text{ m}$$

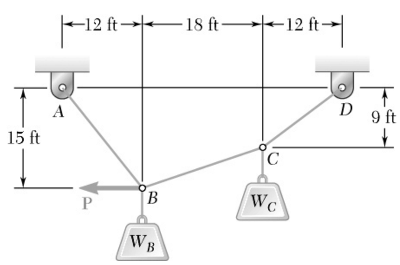
$a = 1.720\text{ m}$ ◀

$$\circlearrowleft \Sigma M_A = 0: b(100\text{ N}) - (2.4\text{ m})(117.72\text{ N}) - (1.7204\text{ m})\left(\frac{2}{3}117.72\text{ N}\right) = 0$$

$$b = 4.1754\text{ m}$$

$b = 4.18\text{ m}$ ◀

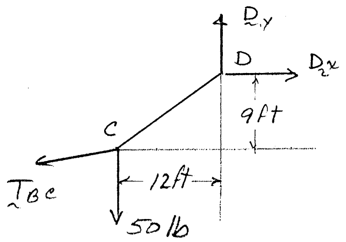
PROBLEM 7.98



Knowing that $W_B = 150$ lb and $W_C = 50$ lb, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

FBD CD:



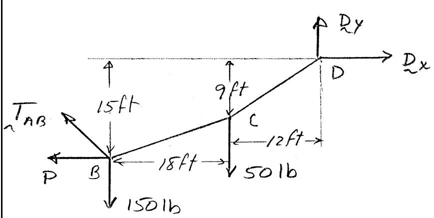
$$\circlearrowleft \Sigma M_C = 0: (12 \text{ ft})D_y - (9 \text{ ft})D_x = 0$$

$$3D_x = 4D_y \qquad (1)$$

$$\circlearrowleft \Sigma M_B = 0: (30 \text{ ft})D_y - (15 \text{ ft})D_x - (18 \text{ ft})(50 \text{ lb}) = 0$$

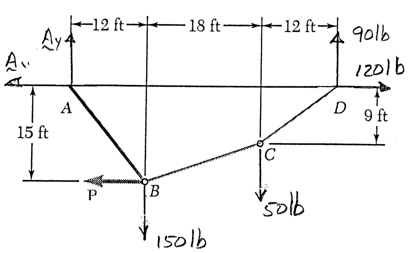
$$2D_y - D_x = 60 \text{ lb} \qquad (2)$$

FBD BCD:



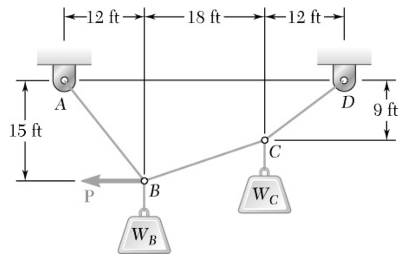
Solving (1) and (2) $\qquad \mathbf{D}_x = 120 \text{ lb} \longrightarrow \qquad \mathbf{D}_y = 90 \text{ lb} \uparrow$

FBD Cable:



$$\circlearrowleft \Sigma M_A = 0: (42 \text{ ft})(90 \text{ lb}) - (30 \text{ ft})(50 \text{ lb}) - (12 \text{ ft})(150 \text{ lb}) - (15 \text{ ft})P = 0$$

$$P = 32.0 \text{ lb} \blacktriangleleft$$

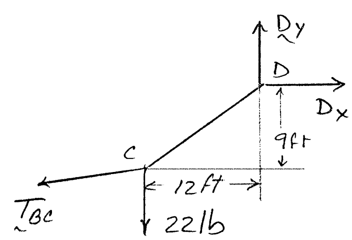


PROBLEM 7.99

Knowing that $W_B = 40$ lb and $W_C = 22$ lb, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

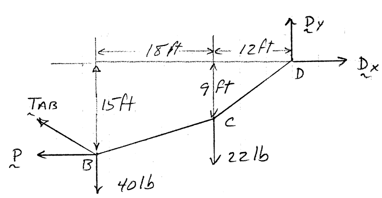
FBD CD:



$$\circlearrowleft \Sigma M_C = 0: (12 \text{ ft})D_y - (9 \text{ ft})D_x = 0$$

$$4D_y = 3D_x \qquad (1)$$

FBD BCD:

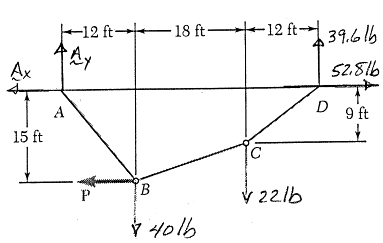


$$\circlearrowleft \Sigma M_B = 0: (30 \text{ ft})D_y - (15 \text{ ft})D_x - (18 \text{ ft})(22 \text{ lb}) = 0$$

$$10D_y - 5D_x = 132 \text{ lb} \qquad (2)$$

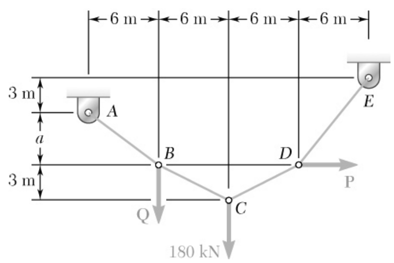
Solving (1) and (2) $\quad \mathbf{D}_x = 52.8 \text{ lb} \longrightarrow \quad \mathbf{D}_y = 39.6 \text{ lb} \uparrow$

FBD Whole:



$$\circlearrowleft \Sigma M_A = 0: (42 \text{ ft})(39.6 \text{ lb}) - (30 \text{ ft})(22 \text{ lb}) - (12 \text{ ft})(40 \text{ lb}) - (15 \text{ ft})P = 0$$

$$P = 34.9 \text{ lb} \blacktriangleleft$$

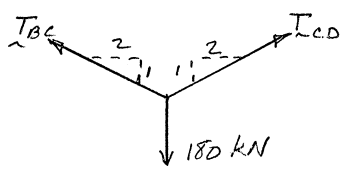


PROBLEM 7.100

If $a = 4.5$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

FBD pt C:

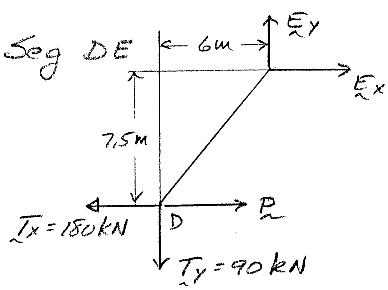


By symmetry: $T_{BC} = T_{CD} = T$

$$\uparrow \Sigma F_y = 0: 2\left(\frac{1}{\sqrt{5}}T\right) - 180 \text{ kN} = 0 \qquad T = 90\sqrt{5} \text{ kN}$$

$$T_x = 180 \text{ kN} \qquad T_y = 90 \text{ kN}$$

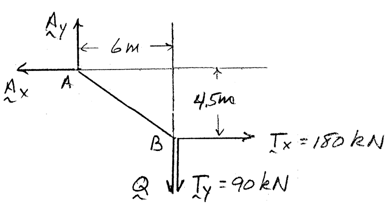
Segment DE:



$$\circlearrowleft \Sigma M_E = 0: (7.5 \text{ m})(P - 180 \text{ kN}) + (6 \text{ m})(90 \text{ kN}) = 0$$

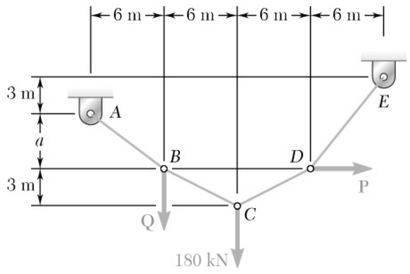
$$P = 108.0 \text{ kN} \blacktriangleleft$$

Segment AB:



$$\circlearrowleft \Sigma M_A = 0: (4.5 \text{ m})(180 \text{ kN}) - (6 \text{ m})(Q + 90 \text{ kN}) = 0$$

$$Q = 45.0 \text{ kN} \blacktriangleleft$$

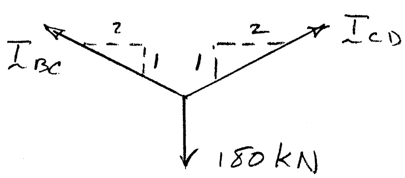


PROBLEM 7.101

If $a = 6$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

FBD pt C:

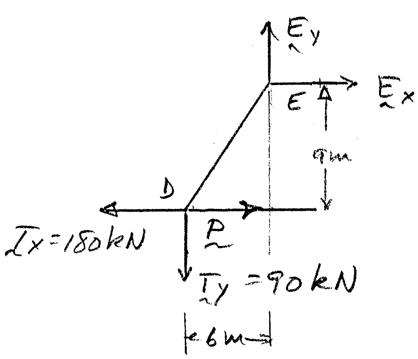


By symmetry: $T_{BC} = T_{CD} = T$

$$\uparrow \Sigma F_y = 0: 2\left(\frac{1}{\sqrt{5}}T\right) - 180\text{ kN} = 0 \qquad T = 90\sqrt{5}\text{ kN}$$

$$T_x = 180\text{ kN} \qquad T_y = 90\text{ kN}$$

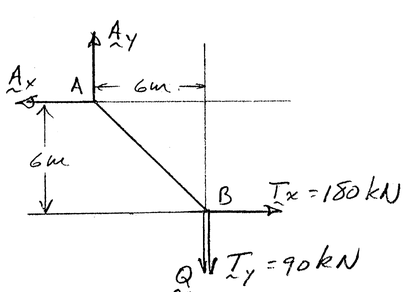
FBD DE:



$$\circlearrowleft \Sigma M_E = 0: (9\text{ m})(P - 180\text{ kN}) + (6\text{ m})(90\text{ kN}) = 0$$

$$P = 120.0\text{ kN} \blacktriangleleft$$

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: (6\text{ m})(180\text{ kN}) - (6\text{ m})(Q + 90\text{ kN}) = 0$$

$$Q = 90.0\text{ kN} \blacktriangleleft$$

PROBLEM 7.102

A transmission cable having a mass per unit length of 1 kg/m is strung between two insulators at the same elevation that are 60 m apart. Knowing that the sag of the cable is 1.2 m, determine (*a*) the maximum tension in the cable, (*b*) the length of the cable.

SOLUTION

(*a*) Since $h = 1.2\text{ m} \ll L = 30\text{ m}$ we can approximate the load as evenly distributed in the horizontal direction.

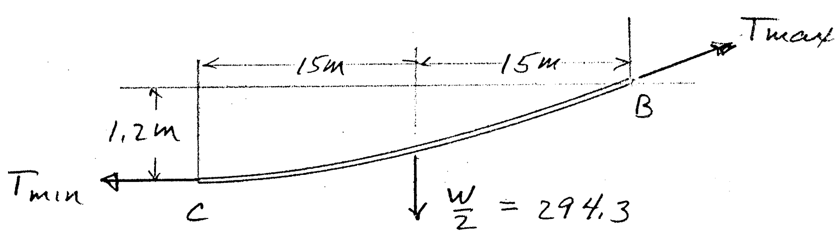
$$w = 1\text{ kg/m}\left(9.81\text{ m/s}^2\right) = 9.81\text{ N/m.}$$

$$w = (60\text{ m})(9.81\text{ N/m})$$

$$w = 588.6\text{ N}$$

Also we can assume that the weight of half the cable acts at the $\frac{1}{4}$ chord point.

FBD half-cable:



$$\circlearrowleft \Sigma M_B = 0: (15\text{ m})(294.3\text{ N}) - (1.2\text{ m})T_{\min} = 0$$

$$T_{\min} = 3678.75\text{ N} = T_{\max\,x}$$

$$\uparrow \Sigma F_y = 0: T_{\max\,y} - 294.3\text{ N} = 0$$

$$T_{\max\,y} = 294.3\text{ N}$$

$$T_{\max} = 3690.5\text{ N}$$

$T_{\max} = 3.69\text{ kN}$ ◀

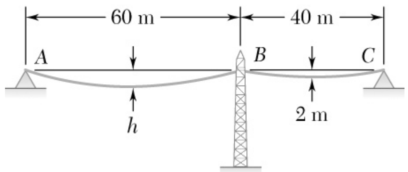
(*b*)

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \cdots\right]$$

$$= (30\text{ m})\left[1 + \frac{2}{3}\left(\frac{1.2}{30}\right)^2 - \frac{2}{5}\left(\frac{1.2}{30}\right)^4 + \cdots\right] = 30.048\text{ m} \qquad \text{so} \qquad s = 2s_B = 60.096\text{ m}$$

$s = 60.1\text{ m}$ ◀

Note: The more accurate methods of section 7.10, which assume the load is evenly distributed along the length instead of horizontally, yield $T_{\max} = 3690.5\text{ N}$ and $s = 60.06$ m. Answers agree to 3 digits at least.

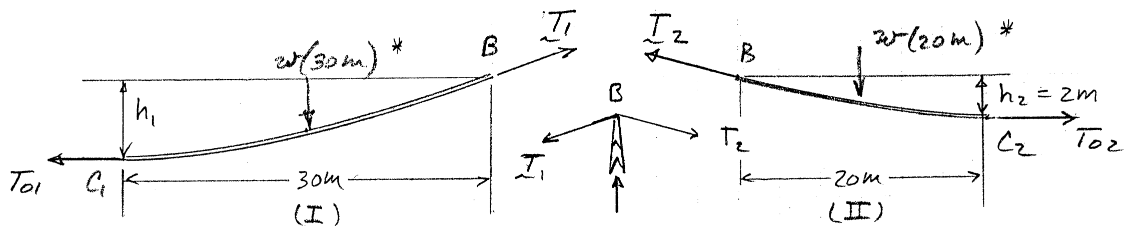


PROBLEM 7.103

Two cables of the same gauge are attached to a transmission tower at B. Since the tower is slender, the horizontal component of the resultant of the forces exerted by the cables at B is to be zero. Knowing that the mass per unit length of the cables is 0.4 kg/m, determine (*a*) the required sag h, (*b*) the maximum tension in each cable.

SOLUTION

Half-cable FBDs:



$T_{1x} = T_{2x}$ to create zero horizontal force on tower $\rightarrow$ thus $T_{01} = T_{02}$

FBD I: $\qquad \circlearrowleft \Sigma M_B = 0\colon (15\text{ m})\left[w(30\text{ m})\right] - h_1 T_0 = 0$

$$h_1 = \frac{\left(450\text{ m}^2\right)w}{T_0}$$

FBD II: $\qquad \circlearrowleft \Sigma M_B = 0\colon (2\text{ m})T_0 - (10\text{ m})\left[w(20\text{ m})\right] = 0$

$$T_0 = (100\text{ m})w$$

(*a*) $\qquad h_1 = \dfrac{\left(450\text{ m}^2\right)w}{(100\text{ m})w} = 4.50\text{ m}$

FBD I: $\qquad \rightarrow \Sigma F_x = 0\colon T_{1x} - T_0 = 0$

$$T_{1x} = (100\text{ m})w$$

$$\uparrow \Sigma F_y = 0\colon T_{1y} - (30\text{ m})w = 0$$

$$T_{1y} = (30\text{ m})w$$

$$T_1 = \sqrt{(100\text{ m})^2 + (30\text{ m})^2}\,w$$

$$= (104.4\text{ m})(0.4\text{ kg/m})\left(9.81\text{ m/s}^2\right)$$

$$= 409.7\text{ N}$$

PROBLEM 7.103 CONTINUED

FBD II:

$$\uparrow \Sigma F_y = 0: T_{2y} - (20 \text{ m})w = 0$$

$$T_{2y} = (20 \text{ m})w$$

$$T_{2x} = T_{1x} = (100 \text{ m})w$$

$$T_2 = \sqrt{(100 \text{ m})^2 + (20 \text{ m})^2}\,w = 400.17 \text{ N}$$

(*b*) $T_1 = 410 \text{ N}$ ◀

$T_2 = 400 \text{ N}$ ◀

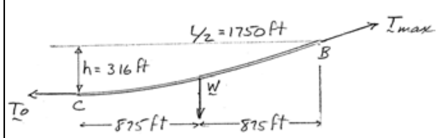
* Since $h \ll L$ it is reasonable to approximate the cable weight as being distributed uniformly along the horizontal. The methods of section 7.10 are more accurate for cables sagging under their own weight.

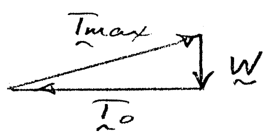
PROBLEM 7.104

The center span of the George Washington Bridge, as originally constructed, consisted of a uniform roadway suspended from four cables. The uniform load supported by each cable was $w = 9.75$ kips/ft along the horizontal. Knowing that the span L is 3500 ft and that the sag h is 316 ft, determine for the original configuration (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION

FBD half-span:





$$W = (9.75 \text{ kips/ft})(1750 \text{ ft}) = 17{,}062.5 \text{ kips}$$

$$\circlearrowleft \Sigma M_B = 0\text{: } (875 \text{ ft})(17{,}065 \text{ kips}) - (316 \text{ ft})T_0 = 0$$

$$T_0 = 47{,}246 \text{ kips}$$

$$T_{\max} = \sqrt{T_0^2 + W^2} = \sqrt{(47{,}246 \text{ kips})^2 + (17{,}063 \text{ kips})^2}$$

(*a*) $T_{\max} = 50{,}200 \text{ kips}$ ◀

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

$$s_B = (1750 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{316 \text{ ft}}{1750 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{316 \text{ ft}}{1750 \text{ ft}}\right)^4 + \cdots\right]$$

$$= 1787.3 \text{ ft}$$

(*b*) $l = 2s_B = 3575 \text{ ft}$ ◀

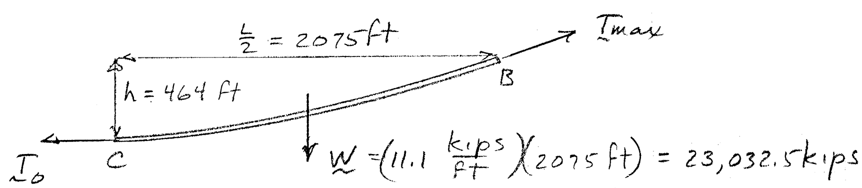
* To get 3-digit accuracy, only two terms are needed.

PROBLEM 7.105

Each cable of the Golden Gate Bridge supports a load $w = 11.1$ kips/ft along the horizontal. Knowing that the span L is 4150 ft and that the sag h is 464 ft, determine (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION

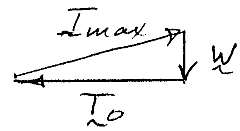
FBD half-span:



(*a*)

$$\circlearrowleft \Sigma M_B = 0: \left(\frac{2075 \text{ ft}}{2}\right)(23032.5 \text{ kips}) - (464 \text{ ft})T_0 = 0$$

$$T_0 = 47,246 \text{ kips}$$



$$T_{max} = \sqrt{T_0^2 + W^2} = \sqrt{(47,246 \text{ kips})^2 + (23,033 \text{ kips})^2} = 56,400 \text{ kips} \blacktriangleleft$$

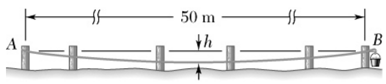
(*b*)

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

$$s_B = (2075 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{464 \text{ ft}}{2075 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{464 \text{ ft}}{2075 \text{ ft}}\right)^4 + \cdots\right]$$

$$s_B = 2142 \text{ ft} \qquad l = 2s_B \qquad l = 4284 \text{ ft} \blacktriangleleft$$

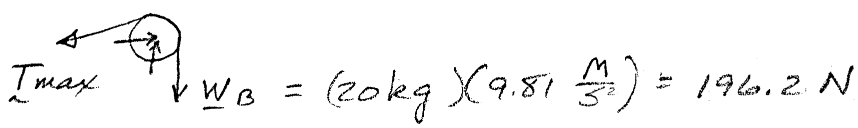
PROBLEM 7.106



To mark the positions of the rails on the posts of a fence, a homeowner ties a cord to the post at A, passes the cord over a short piece of pipe attached to the post at B, and ties the free end of the cord to a bucket filled with bricks having a total mass of 20 kg. Knowing that the mass per unit length of the rope is 0.02 kg/m and assuming that A and B are at the same elevation, determine (a) the sag h, (b) the slope of the cable at B. Neglect the effect of friction.

SOLUTION

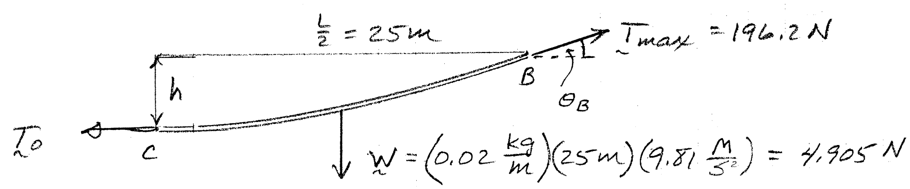
FBD pulley:

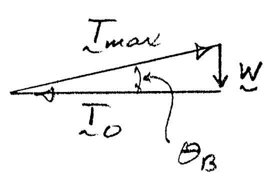


$$\circlearrowleft \Sigma M_P = 0: \; (T_{max} - W_B)r = 0$$

$$T_{max} = W_B = 196.2 \text{ N}$$

FBD half-span:*





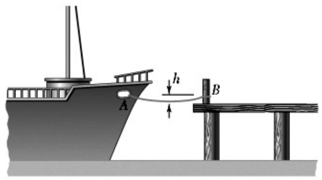
$$T_0 = \sqrt{T_{max}^2 - W^2} = \sqrt{(196.2 \text{ N})^2 - (4.91 \text{ N})^2} = 196.139 \text{ N}$$

$$\circlearrowleft \Sigma M_B = 0: \left(\frac{25 \text{ m}}{2}\right)(4.905 \text{ N}) - h(196.139 \text{ N}) = 0$$

(a) $\quad h = 0.3126 \text{ m} = 313 \text{ mm}$ ◀

(b) $\quad \theta_B = \sin^{-1}\dfrac{W}{T_{max}} = \sin^{-1}\left(\dfrac{4.905 \text{ N}}{196.2 \text{ N}}\right) = 1.433°$ ◀

*See note Prob. 7.103

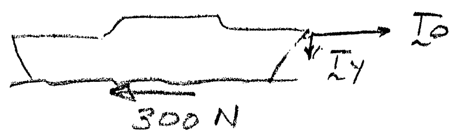


PROBLEM 7.107

A small ship is tied to a pier with a 5-m length of rope as shown. Knowing that the current exerts on the hull of the ship a 300-N force directed from the bow to the stern and that the mass per unit length of the rope is 2.2 kg/m, determine (*a*) the maximum tension in the rope, (*b*) the sag *h*. [*Hint:* Use only the first two terms of Eq. (7.10).]

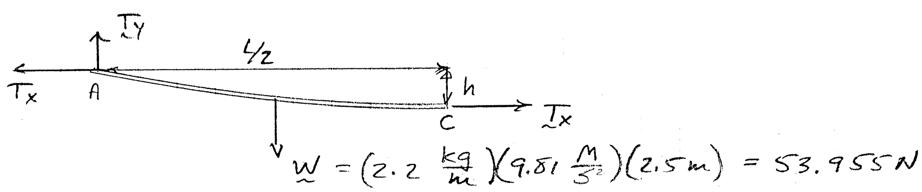
SOLUTION

(*a*) **FBD ship:**



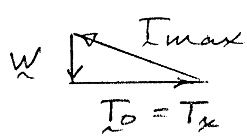
$$\rightarrow \Sigma F_x = 0: T_0 - 300\text{ N} = 0 \qquad T_0 = 300\text{ N}$$

FBD half-span:*



$$T_{max} = \sqrt{T_0^2 + W^2} = \sqrt{(300\text{ N})^2 = (54\text{ N})^2} = 305\text{ N} \blacktriangleleft$$

(*b*) $$\circlearrowleft \Sigma M_A = 0: hT_x - \frac{L}{4}W = 0 \qquad h = \frac{LW}{4T_x}$$



$$s = x\left[1 + \frac{2}{3}\left(\frac{4}{x}\right)^2 + \cdots\right] \qquad \text{but} \qquad y_A = h = \frac{LW}{4T_x} \qquad \text{so} \qquad \frac{y_A}{x_A} = \frac{W}{2T_x}$$

$$(2.5\text{ m}) = \frac{L}{2}\left[1 + \frac{2}{3}\left(\frac{53.955\text{ N}}{600\text{ N}}\right)^2 - \cdots\right] \rightarrow L = 4.9732\text{ m}$$

$$\text{So} \quad h = \frac{LW}{4T_x} = 0.2236\text{ m} \qquad\qquad h = 224\text{ mm} \blacktriangleleft$$

*See note Prob. 7.103

PROBLEM 7.108

The center span of the Verrazano-Narrows Bridge consists of two uniform roadways suspended from four cables. The design of the bridge allowed for the effect of extreme temperature changes which cause the sag of the center span to vary from $h_w = 386$ ft in winter to $h_s = 394$ ft in summer. Knowing that the span is $L = 4260$ ft, determine the change in length of the cables due to extreme temperature changes.

SOLUTION

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

Knowing

$$l = 2s_{\text{TOT}} = L\left[1 + \frac{2}{3}\left(\frac{h}{L/2}\right)^2 - \frac{2}{5}\left(\frac{h}{L/2}\right)^2 + \cdots\right]$$

Winter:

$$l_w = (4260\text{ ft})\left[1 + \frac{2}{3}\left(\frac{386\text{ ft}}{2130\text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{386\text{ ft}}{2130\text{ ft}}\right)^4 + \cdots\right] = 4351.43\text{ ft}$$

Summer:

$$l_s = (4260\text{ ft})\left[1 + \frac{2}{3}\left(\frac{394\text{ ft}}{2130\text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{394\text{ ft}}{2130\text{ ft}}\right)^4 + \cdots\right] = 4355.18\text{ ft}$$

$$\Delta l = l_s - l_w = 3.75\text{ ft} \blacktriangleleft$$

PROBLEM 7.109

A cable of length $L+\Delta$ is suspended between two points which are at the same elevation and a distance L apart. (*a*) Assuming that Δ is small compared to L and that the cable is parabolic, determine the approximate sag in terms of L and Δ. (*b*) If $L = 30$ m and $\Delta = 1.2$ m, determine the approximate sag. [*Hint:* Use only the first two terms of Eq. (7.10).]

SOLUTION

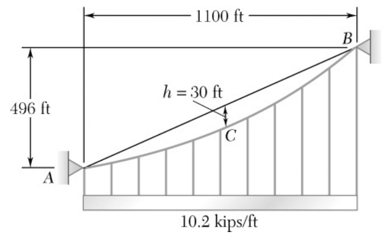
(*a*)

$$s = x\left[1+\frac{2}{3}\left(\frac{y}{x}\right)^2 - \cdots\right]$$

$$L + \Delta = 2s_{\text{TOT}} = L\left[1+\frac{2}{3}\left(\frac{h}{L/2}\right)^2 - \cdots\right]$$

$$\frac{\Delta}{L} = \frac{2}{3}\left(\frac{2h}{L}\right)^2 = \frac{8}{3}\left(\frac{h}{L}\right)^2 \rightarrow h = \sqrt{\frac{3}{8}L\Delta} \blacktriangleleft$$

(*b*) For $L = 30$ m, $\Delta = 1.2$ m $\qquad h = 3.67$ m ◀

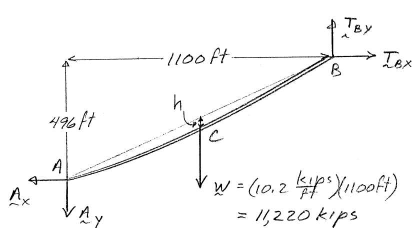


PROBLEM 7.110

Each cable of the side spans of the Golden Gate Bridge supports a load $w = 10.2$ kips/ft along the horizontal. Knowing that for the side spans the maximum vertical distance h from each cable to the chord AB is 30 ft and occurs at midspan, determine (*a*) the maximum tension in each cable, (*b*) the slope at B.

SOLUTION

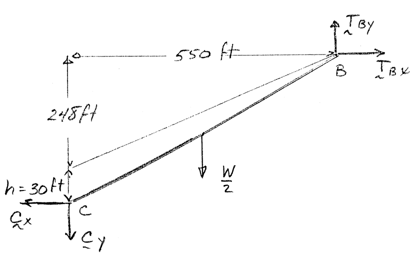
FBD AB:



$$\circlearrowleft \Sigma M_A = 0: (1100 \text{ ft})T_{By} - (496 \text{ ft})T_{Bx} - (550 \text{ ft})W = 0$$

$$11T_{By} - 4.96T_{Bx} = 5.5W \qquad (1)$$

FBD CB:



$$\circlearrowleft \Sigma M_C = 0: (550 \text{ ft})T_{By} - (278 \text{ ft})T_{Bx} - (275 \text{ ft})\frac{W}{2} = 0$$

$$11T_{By} - 5.56T_{Bx} = 2.75W \qquad (2)$$

Solving (1) and (2)

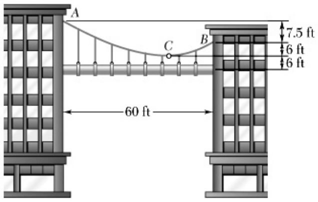
$$T_{By} = 28,798 \text{ kips}$$

$$T_{Bx} = 51,425 \text{ kips}$$

$$T_{max} = T_B = \sqrt{T_{B_x}^2 + T_{B_y}^2} \qquad \tan\theta_B = \frac{T_{B_y}}{T_{B_x}}$$

So that (*a*) $T_{max} = 58,900 \text{ kips}$ ◀

(*b*) $\theta_B = 29.2°$ ◀

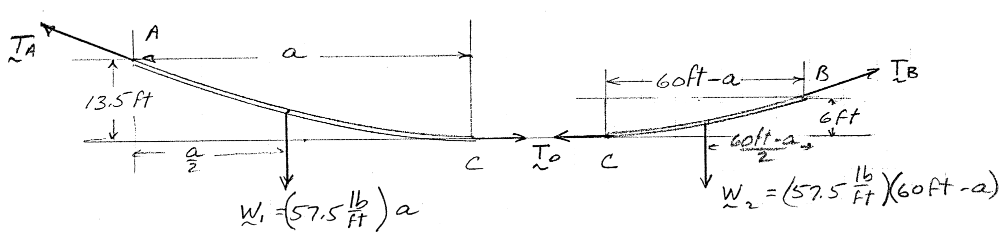


PROBLEM 7.111

A steam pipe weighting 50 lb/ft that passes between two buildings 60 ft apart is supported by a system of cables as shown. Assuming that the weight of the cable is equivalent to a uniformly distributed loading of 7.5 lb/ft, determine (*a*) the location of the lowest point *C* of the cable, (*b*) the maximum tension in the cable.

SOLUTION

FBD AC: **FBD CB:**



$$\circlearrowleft \Sigma M_A = 0: (13.5\text{ ft})T_0 - \frac{a}{2}(57.5\text{ lb/ft})a = 0$$

$$T_0 = \left(2.12963\text{ lb/ft}^2\right)a^2 \qquad (1)$$

$$\circlearrowleft \Sigma M_B = 0: \frac{60\text{ ft} - a}{2}(57.5\text{ lb/ft})(60\text{ ft} - a) - (6\text{ ft})T_0 = 0$$

$$6T_0 = \left(28.75\text{ lb/ft}^2\right)\left[3600\text{ ft}^2 - (120\text{ ft})a + a^2\right] \qquad (2)$$

Using (1) in (2) $\quad 0.55a^2 - (120\text{ ft})a + 3600\text{ ft}^2 = 0$

Solving: $\quad a = (108 \pm 72)\text{ ft} \qquad a = 36\text{ ft}$ (180 ft out of range)

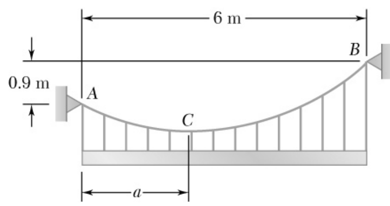
So *C* is 36 ft from *A*

(*a*) *C* is 6 ft below and 24 ft left of *B* ◀

$$T_0 = 2.1296\text{ lb/ft}^2(36\text{ ft})^2 = 2760\text{ lb}$$

$$W_1 = (57.5\text{ lb/ft})(36\text{ ft}) = 2070\text{ lb}$$

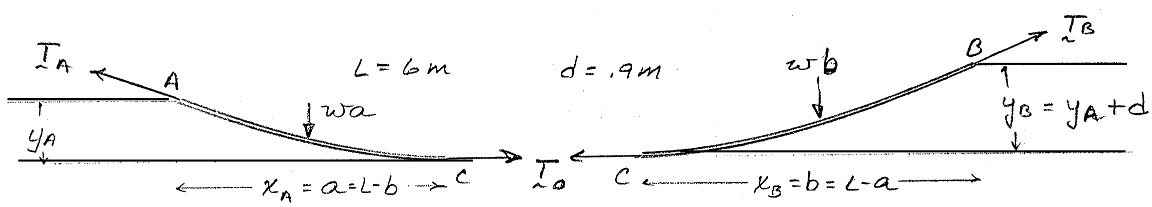
(*b*) $\quad T_{max} = T_A = \sqrt{T_0^2 + W_1^2} = \sqrt{(2760\text{ lb})^2 + (2070\text{ lb})^2} = 3450\text{ lb}$ ◀



PROBLEM 7.112

Chain AB supports a horizontal, uniform steel beam having a mass per unit length of 85 kg/m. If the maximum tension in the cable is not to exceed 8 kN, determine (*a*) the horizontal distance a from A to the lowest point C of the chain, (*b*) the approximate length of the chain.

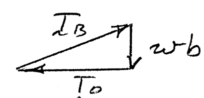
SOLUTION



$$\circlearrowleft \Sigma M_A = 0: y_A T_0 - \frac{a}{2} wa = 0 \qquad \circlearrowleft \Sigma M_B = 0: -y_B T_0 + \frac{b}{2} wb = 0$$

$$y_A = \frac{wa^2}{2T_0} \qquad y_B = \frac{wb^2}{2T_0}$$

$$d = (y_B - y_B) = \frac{w}{2T_0}\left(b^2 - a^2\right)$$



But $$T_0 = \sqrt{T_B^2 - (wb)^2} = \sqrt{T_{max}^2 - (wb)^2}$$

$$\therefore \quad (2d)^2\left[T_{max}^2 - (wb)^2\right] = w^2\left(b^2 - a^2\right)^2 = L^2w^2\left(4b^2 - 4Lb + L^2\right)$$

or $$4\left(L^2 + d^2\right)b^2 - 4L^3b + \left(L^4 - 4d^2\frac{T_{max}^2}{w^2}\right) = 0$$

Using $L = 6$ m, $d = 0.9$ m, $T_{max} = 8$ kN, $w = (85 \text{ kg/m})(9.81 \text{ m/s}^2) = 833.85$ N/m

yields $b = (2.934 \pm 1.353)$ m $\qquad b = 4.287$ m $\qquad$ (since $b > 3$ m)

(*a*) $\qquad a = 6 \text{ m} - b = 1.713 \text{ m}$ ◀

PROBLEM 7.112 CONTINUED

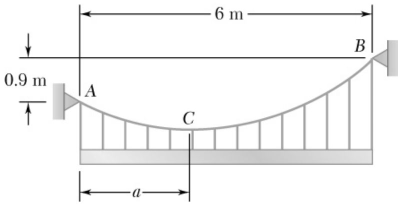
$$T_0 = \sqrt{T_{\max}^2 - (wb)^2} = 7156.9 \text{ N}$$

$$\frac{y_A}{x_A} = \frac{wa}{2T_0} = 0.09979 \qquad \frac{y_B}{x_B} = \frac{wb}{2T_0} = 0.24974$$

$$l = s_A + s_B = a\left[1 + \frac{2}{3}\left(\frac{y_A}{x_A}\right)^2 + \cdots\right] + b\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 + \cdots\right]$$

$$= (1.713 \text{ m})\left[1 + \frac{2}{3}(0.09979)^2\right] + (4.287 \text{ m})\left[1 + \frac{2}{3}(0.24974)^2\right] = 6.19 \text{ m}$$

(*b*) $l = 6.19$ m ◀

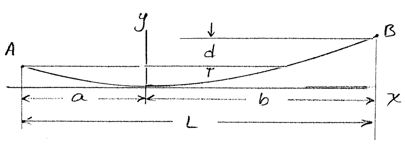


PROBLEM 7.113

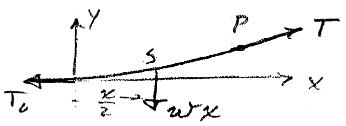
Chain AB of length 6.4 m supports a horizontal, uniform steel beam having a mass per unit length of 85 kg/m. Determine (a) the horizontal distance a from A to the lowest point C of the chain, (b) the maximum tension in the chain.

SOLUTION

Geometry:



FBD Segment:



$$\circlearrowleft \Sigma M_P = 0: \frac{x}{2}wx - yT_0 = 0$$

$$y = \frac{wx^2}{2T_0} \qquad \text{so} \qquad \frac{y}{x} = \frac{wx}{2T_0}$$

$$\text{and } d = y_B - y_A = \frac{w}{2T_0}\left(b^2 - a^2\right)$$

Also
$$l = s_A + s_B = a\left[1 + \frac{2}{3}\left(\frac{y_A}{a}\right)^2\right] + b\left[1 + \frac{2}{3}\left(\frac{y_B}{b}\right)^2\right]$$

$$l - L = \frac{2}{3}\left[\left(\frac{y_A}{a}\right)^2 + \left(\frac{y_B}{b}\right)^2\right] = \frac{w^2}{6T_0^2}\left(a^3 + b^3\right)$$

$$= \frac{1}{6}\frac{4d^2}{\left(b^2 - a^2\right)^2}\left(a^3 + b^3\right) = \frac{2}{3}\frac{d^2\left(a^3 + b^3\right)}{\left(b^2 - a^2\right)^2}$$

Using $l = 6.4$ m, $L = 6$ m, $d = 0.9$ m, $b = 6 \text{ m} - a$, and solving for a, knowing that $a < 3$ ft

$$a = 2.2196 \text{ m} \qquad (a) \qquad a = 2.22 \text{ m} \blacktriangleleft$$

Then
$$T_0 = \frac{w}{2d}\left(b^2 - a^2\right)$$

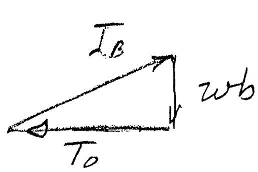
And with
$$w = (85 \text{ kg/m})\left(9.81 \text{ m/s}^2\right) = 833.85 \text{ N/m}$$

And
$$b = 6 \text{ m} - a = 3.7804 \text{ m} \qquad T_0 = 4338 \text{ N}$$

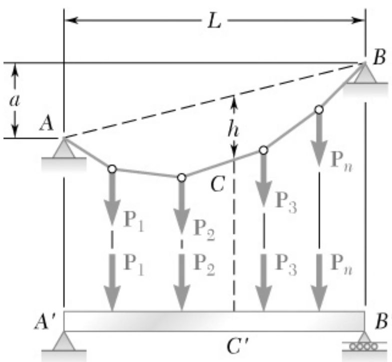
$$T_{max} = T_B = \sqrt{T_0^2 + (wb)^2}$$

$$= \sqrt{(4338 \text{ N})^2 + (833.85 \text{ N/m})^2 (3.7804 \text{ m})^2}$$

$$T_{max} = 5362 \text{ N} \qquad (b) \qquad T_{max} = 5.36 \text{ kN} \blacktriangleleft$$



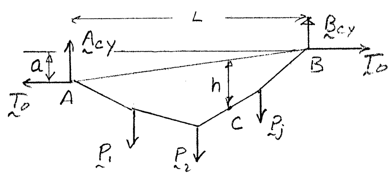
PROBLEM 7.114



A cable AB of span L and a simple beam $A'B'$ of the same span are subjected to identical vertical loadings as shown. Show that the magnitude of the bending moment at a point C' in the beam is equal to the product, T_0h where T_0 is the magnitude of the horizontal component of the tension force in the cable and h is the vertical distance between point C and the chord joining the points of support A and B.

SOLUTION

FBD Cable:

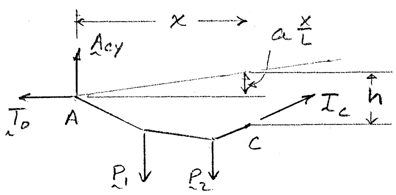


$$\circlearrowleft \Sigma M_B = 0:\ LA_{Cy} + aT_0 - \Sigma M_{B\text{ loads}} = 0 \tag{1}$$

(Where $\Sigma M_{B\text{ loads}}$ includes all applied loads)

$$\circlearrowleft \Sigma M_C = 0:\ xA_{Cy} - \left(h - a\frac{x}{L}\right)T_0 - \Sigma M_{C\text{ left}} = 0 \tag{2}$$

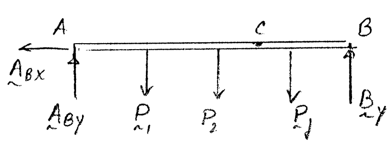
FBD AC:



(Where $\Sigma M_{C\text{ left}}$ includes all loads left of C)

$$\frac{x}{L}(1) - (2):\quad hT_0 - \frac{x}{L}\Sigma M_{B\text{ loads}} + \Sigma M_{C\text{ left}} = 0 \tag{3}$$

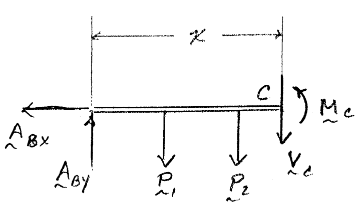
FBD Beam:



$$\circlearrowleft \Sigma M_B = 0:\ LA_{By} - \Sigma M_{B\text{ loads}} = 0 \tag{4}$$

$$\circlearrowleft \Sigma M_C = 0:\ xA_{By} - \Sigma M_{C\text{ left}} - M_C = 0 \tag{5}$$

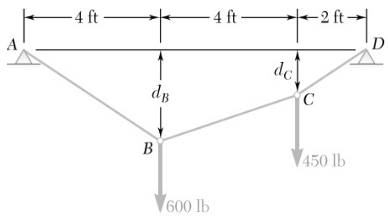
FBD AC:



$$\frac{x}{L}(4) - (5):\quad -\frac{x}{L}\Sigma M_{B\text{ loads}} + \Sigma M_{C\text{ left}} + M_C = 0 \tag{6}$$

Comparing (3) and (6) $\qquad M_C = hT_0 \qquad$ Q.E.D.

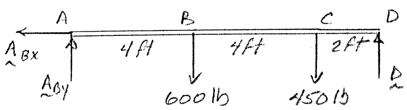
PROBLEM 7.115



Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem. Prob. 7.89*a*.

SOLUTION

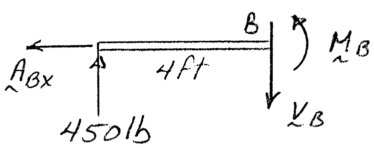
FBD Beam:



$$\circlearrowleft \Sigma M_D = 0: (2\text{ ft})(450\text{ lb}) + (6\text{ ft})(600\text{ lb}) - (10\text{ ft})\mathbf{A}_{By} = 0$$

$$\mathbf{A}_{By} = 450\text{ lb} \uparrow$$

Section AB:



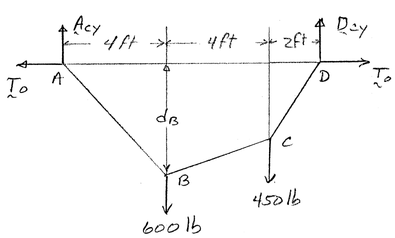
$$\circlearrowleft \Sigma M_B = 0: M_B - (4\text{ ft})(450\text{ lb}) = 0$$

$$M_B = 1800\text{ lb}\cdot\text{ft}$$

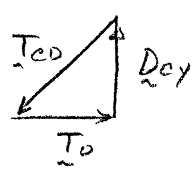
$$\circlearrowleft \Sigma M_A = 0: (10\text{ ft})D_{Cy} - (8\text{ ft})(450\text{ lb}) - (4\text{ ft})(600\text{ lb}) = 0$$

$$D_{Cy} = 600\text{ lb}$$

Cable:



(Note: $D_y > A_y$ so $T_{max} = T_{CD}$)



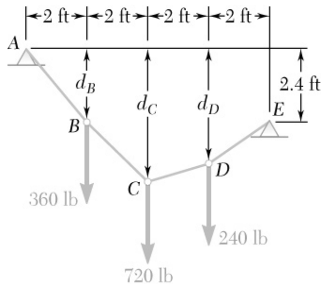
$$T_0 = \sqrt{T_{max}^2 - D_{Cy}^2}$$

$$T_0 = \sqrt{(720\text{ lb})^2 - (600\text{ lb})^2}$$

$$T_0 = 398\text{ lb}$$

$$d_B = \frac{M_B}{T_0} = \frac{1800\text{ lb}\cdot\text{ft}}{398\text{ lb}} = 4.523\text{ ft}$$

$$d_B = 4.52\text{ ft} \blacktriangleleft$$

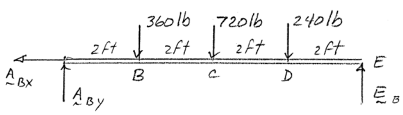


PROBLEM 7.116

Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem. Prob. 7.92*b*.

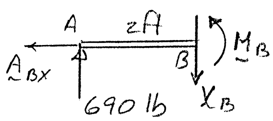
SOLUTION

FBD Beam:



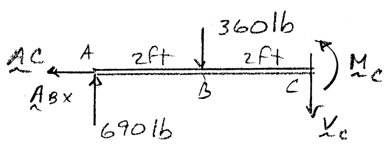
$$\circlearrowleft \Sigma M_E = 0: (2\text{ ft})(240\text{ lb}) + (4\text{ ft})(720\text{ lb}) + (6\text{ ft})(360\text{ lb}) - (8\text{ ft})A_{By} = 0$$

$$\mathbf{A}_{By} = 690\text{ lb}\uparrow$$



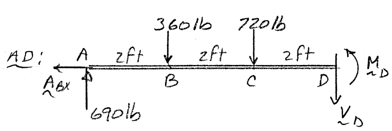
$$\circlearrowleft \Sigma M_B = 0: M_B - (2\text{ ft})(690\text{ lb}) = 0$$

$$M_B = 1380\text{ lb}\cdot\text{ft}$$



$$\circlearrowleft \Sigma M_B = 0: M_C + (2\text{ ft})(360\text{ lb}) - (4\text{ ft})(690\text{ lb}) = 0$$

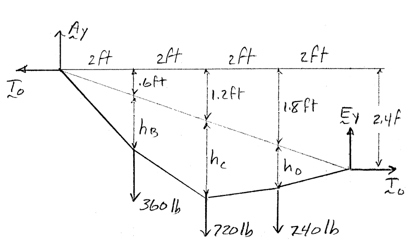
$$M_C = 2040\text{ lb}\cdot\text{ft}$$



$$\circlearrowleft \Sigma M_D = 0: M_D + (2\text{ ft})(720\text{ lb}) + (4\text{ ft})(360\text{ lb}) - (6\text{ ft})(690\text{ lb}) = 0$$

$$M_D = 1260\text{ lb}\cdot\text{ft}$$

Cable:



$$h_C = d_C - 1.2\text{ ft} = 3.6\text{ ft} - 1.2\text{ ft} = 2.4\text{ ft}$$

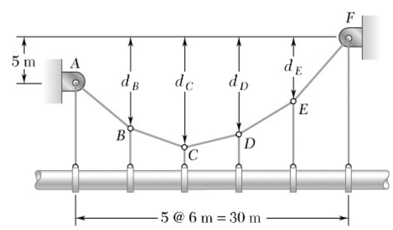
$$T_0 = \frac{M_C}{h_C} = \frac{2040\text{ lb}\cdot\text{ft}}{2.4\text{ ft}} = 850\text{ lb}$$

$$h_B = \frac{M_B}{T_0} = \frac{1380\text{ lb}\cdot\text{ft}}{850\text{ lb}} = 1.6235\text{ ft}$$

$$d_B = h_B + 0.6\text{ ft} \qquad d_B = 2.22\text{ ft} \blacktriangleleft$$

$$h_0 = \frac{M_D}{T_0} = \frac{1260\text{ lb}\cdot\text{ft}}{850\text{ lb}} = 1.482\text{ ft}$$

$$d_B = h_0 + 1.8\text{ ft} \qquad d_D = 3.28\text{ ft} \blacktriangleleft$$

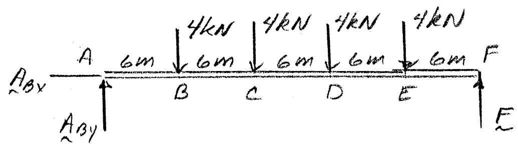


PROBLEM 7.117

Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem. Prob. 7.94*b*.

SOLUTION

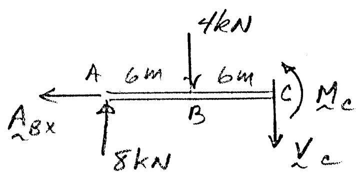
FBD Beam:



By symmetry: $\mathbf{A}_{By} = \mathbf{F} = 8\ kN$

$$M_B = M_E; \qquad M_C = M_D$$

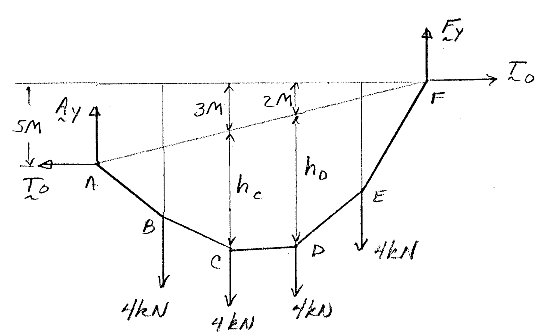
AC:



$$\circlearrowleft \Sigma M_C = 0: M_C + (6\text{ m})(4\text{ kN}) - (12\text{ m})(8\text{ kN}) = 0$$

$$M_C = 72\text{ kN}\cdot\text{m} \qquad \text{so} \qquad M_D = 72\text{ kN}\cdot\text{m}$$

Cable:

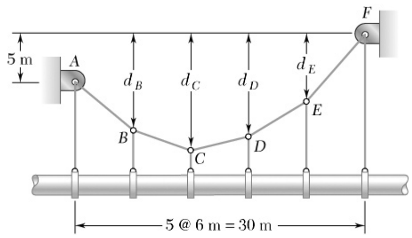


Since $M_D = M_C$

$$h_D = h_C = 12\text{ m} - 3\text{ m} = 9\text{ m}$$

$$d_D = h_D + 2\text{ m} = 11\text{ m}$$

$d_D = 11.00\text{ m}$ ◀

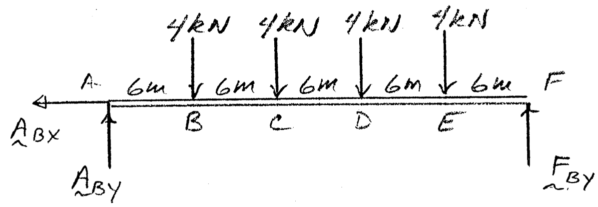


PROBLEM 7.118

Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem. Prob. 7.95*b*.

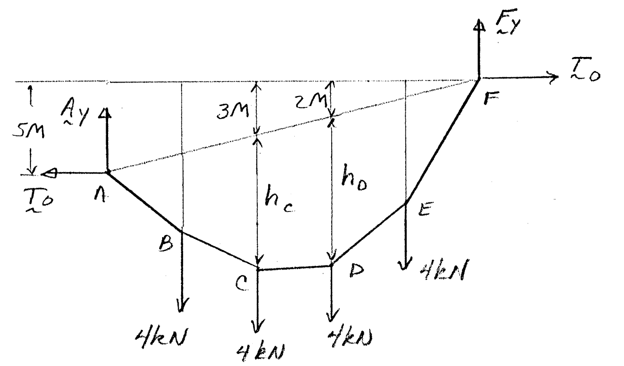
SOLUTION

FBD Beam:



By symmetry: $M_B = M_E$ and $M_C = M_D$

Cable:



Since $M_D = M_C$, $h_D = h_C$

$$h_D = h_C = d_C - 3\text{ m} = 9\text{ m} - 3\text{ m} = 6\text{ m}$$

Then $$d_D = h_D + 2\text{ m} = 6\text{ m} + 2\text{ m} = 8\text{ m}$$

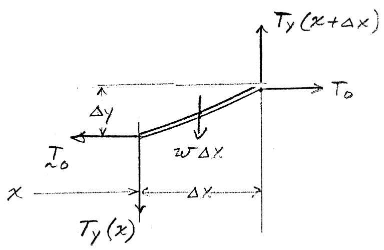
$d_D = 8.00\text{ m}$ ◀

PROBLEM 7.119

Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

FBD Elemental segment:



$$\uparrow \Sigma F_y = 0: T_y(x+\Delta x) - T_y(x) - w(x)\Delta x = 0$$

So
$$\frac{T_y(x+\Delta x)}{T_0} - \frac{T_y(x)}{T_0} = \frac{w(x)}{T_0}\Delta x$$

But
$$\frac{T_y}{T_0} = \frac{dy}{dx}$$

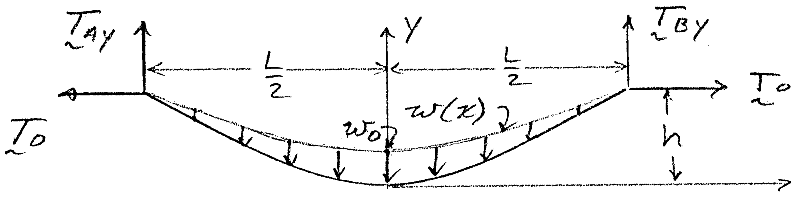
So
$$\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x} = \frac{w(x)}{T_0}$$

In $\lim\limits_{\Delta x \to 0}$:
$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} \qquad \text{Q.E.D.}$$

PROBLEM 7.120

Using the property indicated in Prob. 7.119, determine the curve assumed by a cable of span L and sag h carrying a distributed load $w = w_0 \cos(\pi x / L)$, where x is measured from mid-span. Also determine the maximum and minimum values of the tension in the cable.

SOLUTION



$$w(x) = w_0 \cos\frac{\pi x}{L}$$

From Problem 7.119

$$\frac{d^2 y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0}\cos\frac{\pi x}{L}$$

So $$\frac{dy}{dx} = \frac{W_0 L}{T_0 \pi}\sin\frac{\pi x}{L} \qquad \left(\text{using } \left.\frac{dy}{dx}\right|_0 = 0\right)$$

$$y = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi x}{L}\right) \quad \left[\text{using } y(0) = 0\right] \blacktriangleleft$$

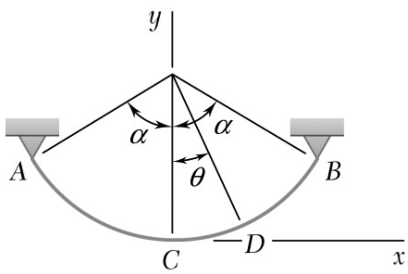
But $$y\left(\frac{L}{2}\right) = h = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi}{2}\right) \qquad \text{so} \qquad T_0 = \frac{w_0 L^2}{\pi^2 h}$$

And $$T_0 = T_{\min} \qquad \text{so} \qquad T_{\min} = \frac{w_0 L^2}{\pi^2 h} \blacktriangleleft$$

$$T_{\max} = T_A = T_B: \quad \frac{T_{By}}{T_0} = \left.\frac{dy}{dx}\right|_{x=\frac{L}{2}} = \frac{w_0 L}{T_0 \pi}$$

$$T_{By} = \frac{w_0 L}{\pi}$$

$$T_B = \sqrt{T_{By}^2 + T_0^2} = \frac{w_0 L}{\pi}\sqrt{1 + \left(\frac{L}{\pi h}\right)^2} \blacktriangleleft$$

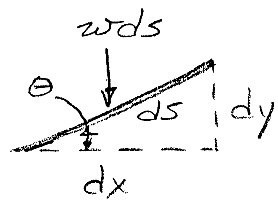


PROBLEM 7.121

If the weight per unit length of the cable AB is $w_0 / \cos^2 \theta$, prove that the curve formed by the cable is a circular arc. (*Hint:* Use the property indicated in Prob. 7.119.)

SOLUTION

Elemental Segment:



Load on segment* $\quad w(x)dx = \dfrac{w_0}{\cos^2 \theta} ds$

But $\quad dx = \cos\theta ds, \quad$ so $\quad w(x) = \dfrac{w_0}{\cos^3 \theta}$

From Problem 7.119
$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0 \cos^3 \theta}$$

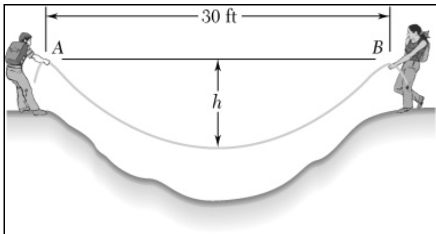
In general
$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan\theta) = \sec^2\theta \frac{d\theta}{dx}$$

So
$$\frac{d\theta}{dx} = \frac{w_0}{T_0 \cos^3 \theta \sec^2 \theta} = \frac{w_0}{T_0 \cos\theta}$$

or
$$\frac{T_0}{w_0} \cos\theta d\theta = dx = r d\theta \cos\theta$$

Giving $r = \dfrac{T_0}{w_0}$ = constant. So curve is circular arc Q.E.D.

*For large sag, it is not appropriate to approximate *ds* by *dx*.

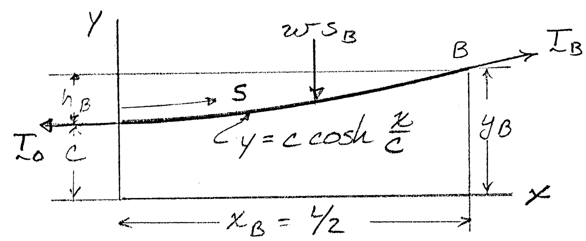


PROBLEM 7.122

Two hikers are standing 30-ft apart and are holding the ends of a 35-ft length of rope as shown. Knowing that the weight per unit length of the rope is 0.05 lb/ft, determine (*a*) the sag *h*, (*b*) the magnitude of the force exerted on the hand of a hiker.

SOLUTION

Half-span:



$$w = 0.05\text{ lb/ft},\qquad L = 30\text{ ft},\qquad s_B = \frac{35}{2}\text{ ft}$$

$$s_B = c\ \sinh\frac{y_B}{x_B}$$

$$17.5\text{ ft} = c\sinh\left(\frac{15\text{ ft}}{c}\right)$$

Solving numerically, $c = 15.36$ ft

Then $$y_B = c\cosh\frac{x_B}{c} = (15.36\text{ ft})\cosh\frac{15\text{ ft}}{15.36\text{ ft}} = 23.28\text{ ft}$$

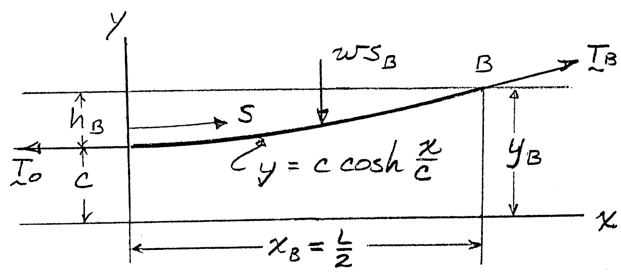
(*a*) $h_B = y_B - c = 23.28\text{ ft} - 15.36\text{ ft} = 7.92\text{ ft}$ ◀

(*b*) $T_B = wy_B = (0.05\text{ lb/ft})(23.28\text{ ft}) = 1.164\text{ lb}$ ◀

PROBLEM 7.123

A 60-ft chain weighing 120 lb is suspended between two points at the same elevation. Knowing that the sag is 24 ft, determine (*a*) the distance between the supports, (*b*) the maximum tension in the chain.

SOLUTION



$$s_B = 30 \text{ ft}, \qquad w = \frac{120 \text{ lb}}{60 \text{ ft}} = 2 \text{ lb/ft}$$

$$h_B = 24 \text{ ft}, \qquad x_B = \frac{L}{2}$$

$$y_B^2 = c^2 + s_B^2 = (h_B + c)^2$$
$$= h_B^2 + 2ch_B + c^2$$

$$c = \frac{s_B^2 - h_B^2}{2h_B} = \frac{(30 \text{ ft})^2 - (24 \text{ ft})^2}{2(24 \text{ ft})}$$

$$c = 6.75 \text{ ft}$$

Then

$$s_B = c \sinh \frac{x_B}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c}$$

$$x_B = (6.75 \text{ ft}) \sinh^{-1}\left(\frac{30 \text{ ft}}{6.75 \text{ ft}}\right) = 14.83 \text{ ft}$$

(*a*) $L = 2x_B = 29.7 \text{ ft}$ ◀

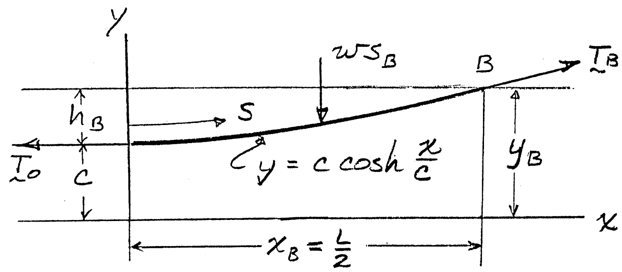
$$T_{max} = T_B = wy_B = w(c + h_B) = (2 \text{ lb/ft})(6.75 \text{ ft} + 24 \text{ ft}) = 61.5 \text{ lb}$$

(*b*) $T_{max} = 61.5 \text{ lb}$ ◀

PROBLEM 7.124

A 200-ft steel surveying tape weighs 4 lb. If the tape is stretched between two points at the same elevation and pulled until the tension at each end is 16 lb, determine the horizontal distance between the ends of the tape. Neglect the elongation of the tape due to the tension.

SOLUTION



$$s_B = 100 \text{ ft}, \qquad w = \frac{4 \text{ lb}}{200 \text{ ft}} = 0.02 \text{ lb/ft}$$

$$T_{\max} = 16 \text{ lb}$$

$$T_{\max} = T_B = wy_B$$

$$y_B = \frac{T_B}{w} = \frac{16 \text{ lb}}{0.02 \text{ lb/ft}} = 800 \text{ ft}$$

$$c^2 = y_B^2 - s_B^2$$

$$c = \sqrt{(800 \text{ ft})^2 - (100 \text{ ft})^2} = 793.73 \text{ ft}$$

But $$y_B = x_B \cosh\frac{x_B}{c} \rightarrow x_B = c\cosh^{-1}\frac{y_B}{c}$$

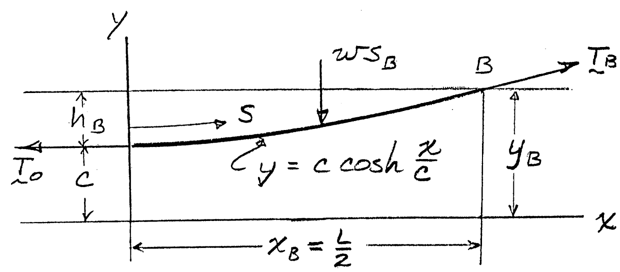
$$= (793.73 \text{ ft}) \cosh^{-1}\left(\frac{800 \text{ ft}}{793.73 \text{ ft}}\right) = 99.74 \text{ ft}$$

$$L = 2x_B = 2(99.74 \text{ ft}) = 199.5 \text{ ft} \blacktriangleleft$$

PROBLEM 7.125

An electric transmission cable of length 130 m and mass per unit length of 3.4 kg/m is suspended between two points at the same elevation. Knowing that the sag is 30 m, determine the horizontal distance between the supports and the maximum tension.

SOLUTION



$$s_B = 65 \text{ m}, \qquad h_B = 30 \text{ m}$$

$$w = (3.4 \text{ kg/m})(9.81 \text{ m/s}^2) = 33.35 \text{ N/m}$$

$$y_B^2 = c^2 + s_B^2$$

$$(c + h_B)^2 = c^2 + s_B^2$$

$$c = \frac{s_B^2 - h_B^2}{2h_B} = \frac{(65 \text{ m})^2 - (30 \text{ m})^2}{2(30 \text{ m})}$$

$$= 55.417 \text{ m}$$

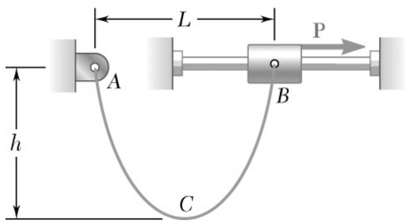
Now $\quad s_B = c \sinh \frac{x_B}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c} = (55.417 \text{ m}) \sinh^{-1}\left(\frac{65 \text{ m}}{55.417 \text{ m}}\right)$

$$= 55.335 \text{ m}$$

$$L = 2x_B = 2(55.335 \text{ m}) = 110.7 \text{ m} \blacktriangleleft$$

$$T_{max} = w y_B = w(c + h_B) = (33.35 \text{ N/m})(55.417 \text{ m} + 30 \text{ m}) = 2846 \text{ N}$$

$$T_{max} = 2.85 \text{ kN} \blacktriangleleft$$

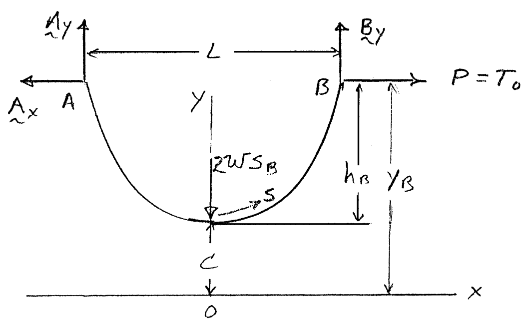


PROBLEM 7.126

A 30-m length of wire having a mass per unit length of 0.3 kg/m is attached to a fixed support at *A* and to a collar at *B*. Neglecting the effect of friction, determine (*a*) the force **P** for which $h = 12$ m, (*b*) the corresponding span *L*.

SOLUTION

FBD Cable:



$$s = 30 \text{ m} \qquad \left(\text{so } s_B = \frac{30 \text{ m}}{2} = 15 \text{ m}\right)$$

$$w = (0.3 \text{ kg/m})(9.81 \text{ m/s}^2) = 2.943 \text{ N/m}$$

$$h_B = 12 \text{ m}$$

$$y_B^2 = (c + h_B)^2 = c^2 + s_B^2$$

So
$$c = \frac{s_B^2 - h_B^2}{2h_B}$$

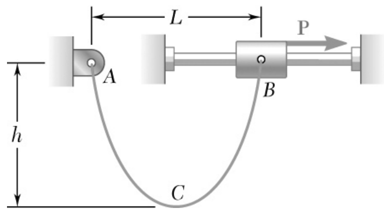
$$c = \frac{(15 \text{ m})^2 - (12 \text{ m})^2}{2(12 \text{ m})} = 3.375 \text{ m}$$

Now
$$s_B = c \sinh \frac{x_B}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c} = (3.375 \text{ m}) \sinh^{-1}\left(\frac{15 \text{ m}}{3.375 \text{ m}}\right)$$

$$x_B = 7.4156 \text{ m}$$

$$P = T_0 = wc = (2.943 \text{ N/m})(3.375 \text{ m}) \qquad (a) \qquad \mathbf{P} = 9.93 \text{ N} \longrightarrow \blacktriangleleft$$

$$L = 2x_B = 2(7.4156 \text{ m}) \qquad (b) \qquad L = 14.83 \text{ m} \blacktriangleleft$$

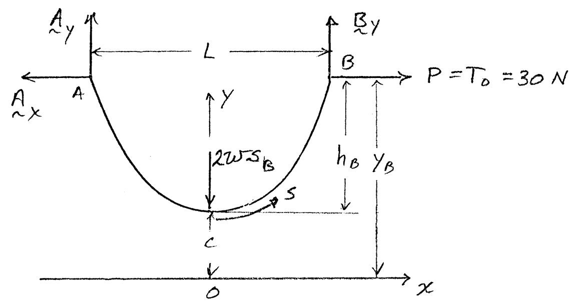


PROBLEM 7.127

A 30-m length of wire having a mass per unit length of 0.3 kg/m is attached to a fixed support at *A* and to a collar at *B*. Knowing that the magnitude of the horizontal force applied to the collar is $P = 30$ N, determine (*a*) the sag *h*, (*b*) the corresponding span *L*.

SOLUTION

FBD Cable:



$$s_T = 30 \text{ m}, \qquad w = (0.3 \text{ kg/m})(9.81 \text{ m/s}^2) = 2.943 \text{ N/m}$$

$$P = T_0 = wc \qquad c = \frac{P}{w}$$

$$c = \frac{30 \text{ N}}{2.943 \text{ N/m}} = 10.1937 \text{ m}$$

$$y_B^2 = (h_B + c)^2 = c^2 + s_B^2$$

$$h^2 + 2ch - s_B^2 = 0 \qquad s_B = \frac{30 \text{ m}}{2} = 15 \text{ m}$$

$$h^2 + 2(10.1937 \text{ m})h - 225 \text{ m}^2 = 0$$

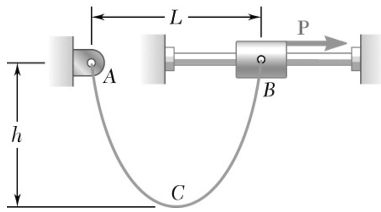
$$h = 7.9422 \text{ m} \qquad (a) \qquad h = 7.94 \text{ m} \blacktriangleleft$$

$$s_B = c\sinh\frac{x_A}{c} \rightarrow x_B = c\sinh^{-1}\frac{s_B}{c} = (10.1937 \text{ m})\sinh^{-1}\left(\frac{15 \text{ m}}{10.1937 \text{ m}}\right)$$

$$= 12.017 \text{ m}$$

$$L = 2x_B = 2(12.017 \text{ m}) \qquad (b) \qquad L = 24.0 \text{ m} \blacktriangleleft$$

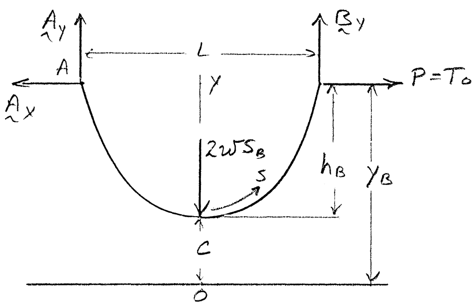
PROBLEM 7.128



A 30-m length of wire having a mass per unit length of 0.3 kg/m is attached to a fixed support at A and to a collar at B. Neglecting the effect of friction, determine (a) the sag h for which $L = 22.5$ m, (b) the corresponding force **P**.

SOLUTION

FBD Cable:



$$s_T = 30 \text{ m} \rightarrow s_B = \frac{30 \text{ m}}{2} = 15 \text{ m}$$

$$w = (0.3 \text{ kg/m})(9.81 \text{ m/s}^2) = 2.943 \text{ N/m}$$

$$L = 22.5 \text{ m}$$

$$s_B = c \sinh \frac{x_B}{c} = c \sinh \frac{L/2}{c}$$

$$15 \text{ m} = c \sinh \frac{11.25 \text{ m}}{c}$$

Solving numerically: $c = 8.328$ m

$$y_B^2 = c^2 + s_B^2 = (8.328 \text{ m})^2 + (15 \text{ m})^2 = 294.36 \text{ m}^2 \qquad y_B = 17.157 \text{ m}$$

$$h_B = y_B - c = 17.157 \text{ m} - 8.328 \text{ m}$$

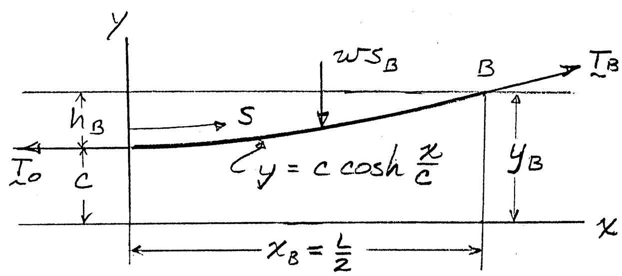
(a) $h_B = 8.83$ m ◀

$P = wc = (2.943 \text{ N/m})(8.328 \text{ m})$ (b) **P** $= 24.5$ N ⟶ ◀

PROBLEM 7.129

A 30-ft wire is suspended from two points at the same elevation that are 20 ft apart. Knowing that the maximum tension is 80 lb, determine (*a*) the sag of the wire, (*b*) the total weight of the wire.

SOLUTION



$$L = 20 \text{ ft} \qquad x_B = \frac{20 \text{ ft}}{2} = 10 \text{ ft}$$

$$s_B = \frac{30 \text{ ft}}{2} = 15 \text{ ft}$$

$$s_B = c \sinh \frac{x_B}{c} = c \sinh \frac{10 \text{ ft}}{c}$$

Solving numerically: $c = 6.1647 \text{ ft}$

$$y_B = c \cosh \frac{x_B}{c} = (6.1647 \text{ ft}) \cosh\left(\frac{10 \text{ ft}}{1.1647 \text{ ft}}\right)$$

$$y_B = 16.217 \text{ ft}$$

$$h_B = y_B - c = 16.217 \text{ ft} - 6.165 \text{ ft}$$

(*a*) $h_B = 10.05 \text{ ft}$ ◀

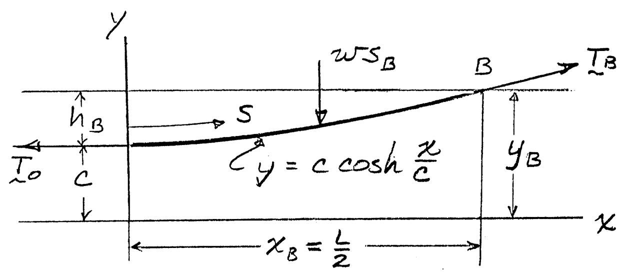
$$T_{\max} = w y_B \qquad \text{and} \qquad W = w(2s_B)$$

So $W = \frac{T_{\max}}{y_B}(2s_B) = \frac{80 \text{ lb}}{16.217 \text{ ft}}(30 \text{ ft})$ (*b*) $\mathbf{W}_m = 148.0 \text{ lb}$ ◀

PROBLEM 7.130

Determine the sag of a 45-ft chain which is attached to two points at the same elevation that are 20 ft apart.

SOLUTION



$$s_B = \frac{45\text{ ft}}{2} = 22.5\text{ ft} \qquad L = 20\text{ ft}$$

$$x_B = \frac{L}{2} = 10\text{ ft}$$

$$s_B = c\sinh\frac{x_B}{c}$$

$$22.5\text{ ft} = c\sinh\frac{10\text{ ft}}{c}$$

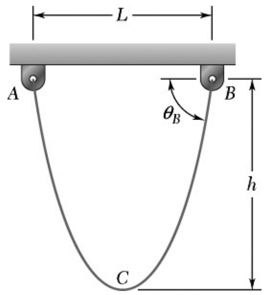
Solving numerically: $c = 4.2023\text{ ft}$

$$y_B = c\cosh\frac{x_B}{c}$$

$$= (4.2023\text{ ft})\cosh\frac{10\text{ ft}}{4.2023\text{ ft}} = 22.889\text{ ft}$$

$$h_B = y_B - c = 22.889\text{ ft} - 4.202\text{ ft}$$

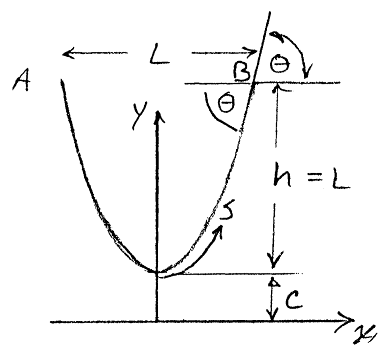
$h_B = 18.69\text{ ft}$ ◀



PROBLEM 7.131

A 10-m rope is attached to two supports A and B as shown. Determine (a) the span of the rope for which the span is equal to the sag, (b) the corresponding angle θ_B.

SOLUTION



We know $$y = c\cosh\frac{x}{c}$$

At B, $$y_B = c + h = c\cosh\frac{h}{2c}$$

or $$1 = \cosh\frac{h}{2c} - \frac{h}{c}$$

Solving numerically $$\frac{h}{c} = 4.933$$

$$s_B = c\sinh\frac{x_B}{c} \rightarrow \frac{s_T}{2} = c\sinh\frac{h}{2c}$$

So $$c = \frac{s_T}{2\sinh\left(\frac{h}{2c}\right)} = \frac{10\text{ m}}{2\sinh\left(\frac{4.933}{2}\right)} = 0.8550\text{ m}$$

$$h = 4.933c = 4.933(0.8550)\text{ m} = 4.218\text{ m} \qquad h = 4.22\text{ m}$$

(a) $L = h = 4.22\text{ m}$ ◀

From $$y = c\cosh\frac{x}{c}, \qquad \frac{dy}{dx} = \sinh\frac{x}{c}$$

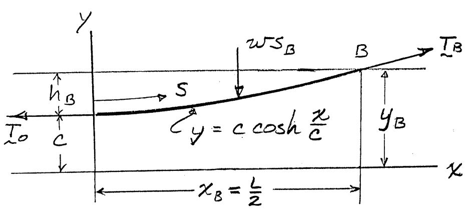
At B, $$\tan\theta = \left.\frac{dy}{dx}\right|_B = \sinh\frac{L}{2c} = \sinh\frac{4.933}{2} = 5.848$$

$$\theta = \tan^{-1}5.848$$ (b) $\theta = 80.3°$ ◀

PROBLEM 7.132

A cable having a mass per unit length of 3 kg/m is suspended between two points at the same elevation that are 48 m apart. Determine the smallest allowable sag of the cable if the maximum tension is not to exceed 1800 N.

SOLUTION



$$w = (3\text{ kg/m})(9.81\text{ m/s}^2) = 29.43\text{ N/m}$$

$$L = 48\text{ m}, \qquad T_{\max} \leq 1800\text{ N}$$

$$T_{\max} = wy_B \rightarrow y_B = \frac{T_{\max}}{w}$$

$$y_B \leq \frac{1800\text{ N}}{29.43\text{ N/m}} = 61.162\text{ m}$$

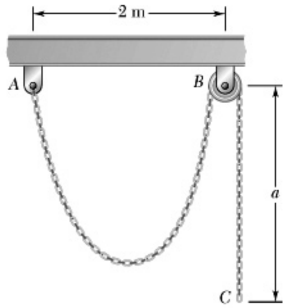
$$y_B = c\cosh\frac{x_B}{c} \qquad 61.162\text{ m} = c\cosh\frac{48\text{ m}/2}{c} \;*$$

Solving numerically $\quad c = 55.935\text{ m}$

$$h = y_B - c = 61.162\text{ m} - 55.935\text{ m}$$

$h = 5.23\text{ m}$ ◀

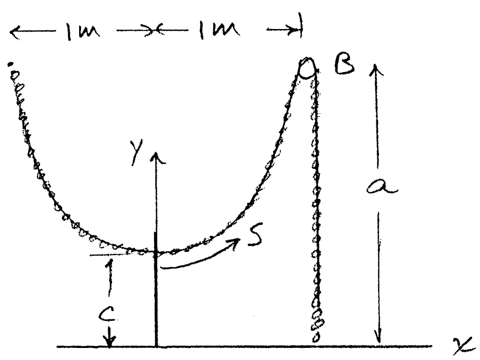
*Note: There is another value of c which will satisfy this equation. It is much smaller, thus corresponding to a much larger h.

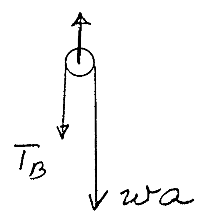


PROBLEM 7.133

An 8-m length of chain having a mass per unit length of 3.72 kg/m is attached to a beam at *A* and passes over a small pulley at *B* as shown. Neglecting the effect of friction, determine the values of distance *a* for which the chain is in equilibrium.

SOLUTION





Neglect pulley size and friction $T_B = wa$

But $T_B = wy_B$ so $y_B = a$

$$y_B = c\cosh\frac{x_B}{c}$$

$$c\cosh\frac{1\text{ m}}{c} = a$$

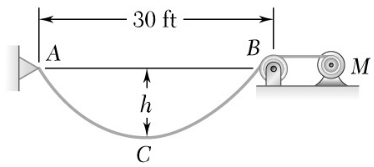
But $s_B = c\sinh\dfrac{x_B}{c}$ $\quad \dfrac{8\text{ m} - a}{2} = c\sinh\dfrac{1\text{ m}}{c}$

So $$4\text{ m} = c\sinh\frac{1\text{ m}}{c} + \frac{c}{2}\cosh\frac{1\text{ m}}{c}$$

$$16\text{ m} = c\left(3e^{1/c} - e^{-1/c}\right)$$

Solving numerically $c = 0.3773\text{ m},\ 5.906\text{ m}$

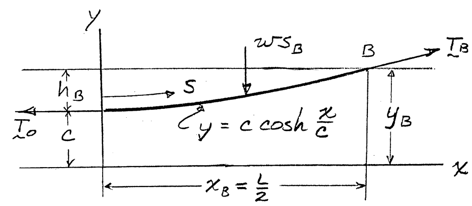
$$a = c\cosh\frac{1\text{ m}}{c} = \begin{cases} (0.3773\text{ m})\cosh\dfrac{1\text{ m}}{0.3773\text{ m}} = 2.68\text{ m} \blacktriangleleft \\ (5.906\text{ m})\cosh\dfrac{1\text{ m}}{5.906\text{ m}} = 5.99\text{ m} \blacktriangleleft \end{cases}$$



PROBLEM 7.134

A motor M is used to slowly reel in the cable shown. Knowing that the weight per unit length of the cable is 0.5 lb/ft, determine the maximum tension in the cable when $h = 15$ ft.

SOLUTION



$$w = 0.5 \text{ lb/ft} \qquad L = 30 \text{ ft} \qquad h_B = 15 \text{ ft}$$

$$y_B = c\cosh\frac{x_B}{c}$$

$$h_B + c = c\cosh\frac{L}{2c}$$

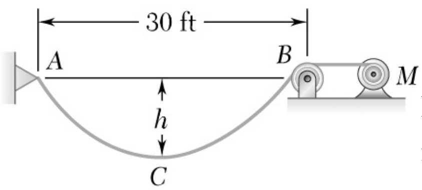
$$15 \text{ ft} = c\left(\cosh\frac{15 \text{ ft}}{c} - 1\right)$$

Solving numerically $\quad c = 9.281 \text{ ft}$

$$y_B = (9.281 \text{ ft})\cosh\frac{15 \text{ ft}}{9.281 \text{ ft}} = 24.281 \text{ ft}$$

$$T_{\max} = T_B = wy_B = (0.5 \text{ lb/ft})(24.281 \text{ ft})$$

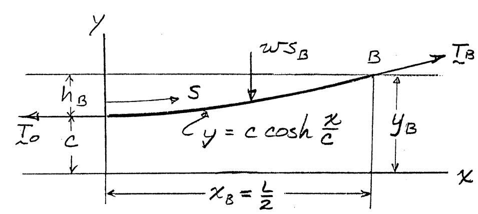
$T_{\max} = 12.14$ lb ◀



PROBLEM 7.135

A motor M is used to slowly reel in the cable shown. Knowing that the weight per unit length of the cable is 0.5 lb/ft, determine the maximum tension in the cable when $h = 9$ ft.

SOLUTION



$$w = 0.5 \text{ lb/ft}, \qquad L = 30 \text{ ft}, \qquad h_B = 9 \text{ ft}$$

$$y_B = h_B + c = c\cosh\frac{x_B}{c} = c\cosh\frac{L}{2c}$$

$$9 \text{ ft} = c\left(\cosh\frac{15 \text{ ft}}{c} - 1\right)$$

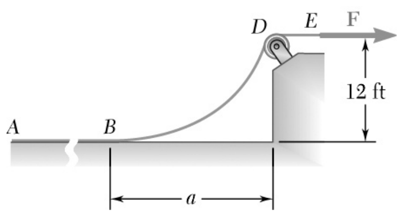
Solving numerically $c = 13.783$ ft

$$y_B = h_B + c = 9 \text{ ft} + 13.783 \text{ ft} = 21.783 \text{ ft}$$

$$T_{\max} = T_B = wy_B = (0.5 \text{ lb/ft})(21.78 \text{ ft})$$

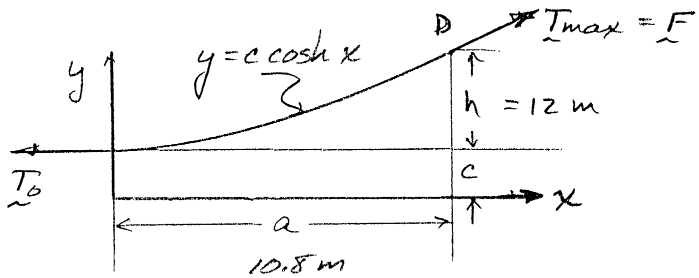
$T_{\max} = 11.39$ lb ◀

PROBLEM 7.136



To the left of point B the long cable $ABDE$ rests on the rough horizontal surface shown. Knowing that the weight per unit length of the cable is 1.5 lb/ft, determine the force **F** when $a = 10.8$ ft.

SOLUTION



$$y_D = c\cosh\frac{x_D}{c}$$

$$h + c = c\cosh\frac{a}{c}$$

$$12\text{ m} = c\left(\cosh\frac{10.8\text{ m}}{c} - 1\right)$$

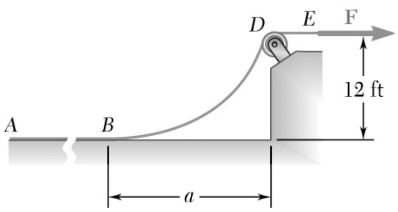
Solving numerically $\quad c = 6.2136$ m

Then $\quad y_B = (6.2136\text{ m})\cosh\dfrac{10.8\text{ m}}{6.2136\text{ m}} = 18.2136\text{ m}$

$$F = T_{\max} = wy_B = (1.5\text{ lb/ft})(18.2136\text{ m})$$

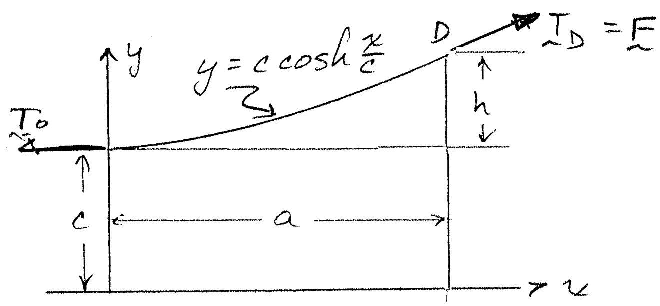
F = 27.3 lb ⟶ ◀

PROBLEM 7.137



To the left of point *B* the long cable *ABDE* rests on the rough horizontal surface shown. Knowing that the weight per unit length of the cable is 1.5 lb/ft, determine the force **F** when $a = 18$ ft.

SOLUTION



$$y_D = c\cosh\frac{x_D}{c}$$

$$c + h = c\cosh\frac{a}{c}$$

$$h = c\left(\cosh\frac{a}{c} - 1\right)$$

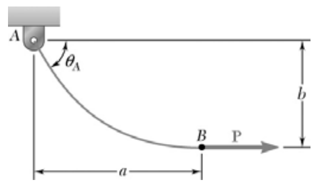
$$12\text{ ft} = c\left(\cosh\frac{18\text{ ft}}{c} - 1\right)$$

Solving numerically $\qquad c = 15.162$ ft

$$y_B = h + c = 12\text{ ft} + 15.162\text{ ft} = 27.162\text{ ft}$$

$$F = T_D = wy_D = (1.5\text{ lb/ft})(27.162\text{ ft}) = 40.74\text{ lb}$$

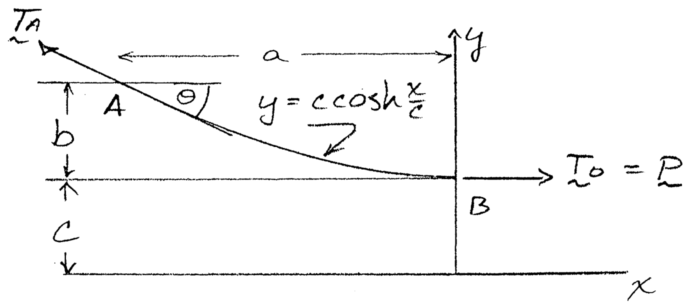
$\mathbf{F} = 40.7$ lb ⟶ ◀



PROBLEM 7.138

A uniform cable has a mass per unit length of 4 kg/m and is held in the position shown by a horizontal force **P** applied at B. Knowing that $P = 800$ N and $\theta_A = 60°$, determine (a) the location of point B, (b) the length of the cable.

SOLUTION



$$w = 4\text{ kg/m}\left(9.81\text{ m/s}^2\right) = 39.24\text{ N/m}$$

$$P = T_0 = wc \qquad c = \frac{P}{w} = \frac{800\text{ N}}{39.24\text{ N/m}}$$

$$c = 20.387\text{ m}$$

$$y = c\cosh\frac{x}{c}$$

$$\frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = -\frac{dy}{dx}\bigg|_{-a} = -\sinh\frac{-a}{c} = \sinh\frac{a}{c}$$

$$a = c\sinh^{-1}(\tan\theta) = (20.387\text{ m})\sinh^{-1}(\tan 60°)$$

$$a = 26.849\text{ m}$$

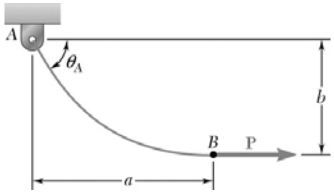
$$y_A = c\cosh\frac{a}{c} = (20.387\text{ m})\cosh\frac{26.849\text{ m}}{20.387\text{ m}} = 40.774\text{ m}$$

$$b = y_A - c = 40.774\text{ m} - 20.387\text{ m} = 20.387\text{ m}$$

So (a) B is 26.8 m right and 20.4 m down from A ◀

$$s = c\sinh\frac{a}{c} = (20.387\text{ m})\sinh\frac{26.849\text{ m}}{20.387\text{ m}} = 35.31\text{ m} \qquad (b)$$

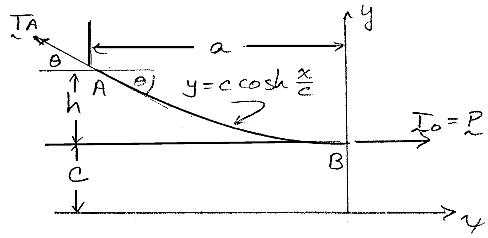
$s = 35.3$ m ◀



PROBLEM 7.139

A uniform cable having a mass per unit length of 4 kg/m is held in the position shown by a horizontal force **P** applied at *B*. Knowing that $P = 600$ N and $\theta_A = 60°$, determine (*a*) the location of point *B*, (*b*) the length of the cable.

SOLUTION



$$w = (4 \text{ kg/m})(9.81 \text{ m/s}^2) = 39.24 \text{ N/m}$$

$$P = T_0 = wc \qquad c = \frac{P}{w} = \frac{600 \text{ N}}{39.24 \text{ N/m}}$$

$$c = 15.2905 \text{ m}$$

$$y = c\cosh\frac{x}{c} \qquad \frac{dy}{dx} = \sinh\frac{x}{c}$$

At *A*: $\tan\theta = -\left.\frac{dy}{dx}\right|_{-a} = -\sinh\frac{-a}{c} = \sinh\frac{a}{c}$

So $\qquad a = c\sinh^{-1}(\tan\theta) = (15.2905 \text{ m})\sinh^{-1}(\tan 60°) = 20.137 \text{ m}$

$$y_B = h + c = c\cosh\frac{a}{c}$$

$$h = c\left(\cosh\frac{a}{c} - 1\right)$$

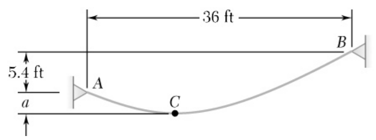
$$= (15.2905 \text{ m})\left(\cosh\frac{20.137 \text{ m}}{15.2905 \text{ m}} - 1\right)$$

$$= 15.291 \text{ m}$$

So $\qquad$ (*a*) $\qquad$ *B* is 20.1 m right and 15.29 m down from *A* ◀

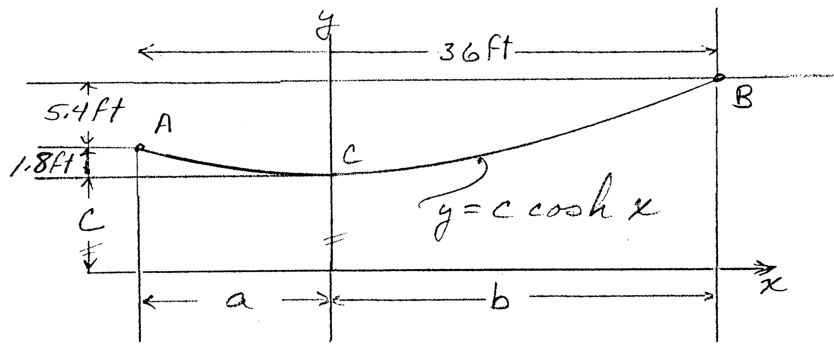
$s = c\sinh\frac{a}{c} = (15.291 \text{ m})\sinh\frac{20.137 \text{ m}}{15.291 \text{ m}} = 26.49 \text{ m}$ $\qquad$ (*b*) $\qquad$ $s = 26.5 \text{ m}$ ◀

PROBLEM 7.140



The cable *ACB* weighs 0.3 lb/ft. Knowing that the lowest point of the cable is located at a distance $a = 1.8$ ft below the support *A*, determine (*a*) the location of the lowest point *C*, (*b*) the maximum tension in the cable.

SOLUTION



$$y_A = c\cosh\frac{-a}{c} = c + 1.8\text{ ft}$$

$$a = c\cosh^{-1}\left(1 + \frac{1.8\text{ ft}}{c}\right)$$

$$y_B = c\cosh\frac{b}{c} = c + 7.2\text{ ft}$$

$$b = c\cosh^{-1}\left(1 + \frac{7.2\text{ ft}}{c}\right)$$

But $$a + b = 36\text{ ft} = c\left[\cosh^{-1}\left(1 + \frac{1.8\text{ ft}}{c}\right) + \cosh^{-1}\left(1 + \frac{7.2\text{ ft}}{c}\right)\right]$$

Solving numerically $c = 40.864$ ft

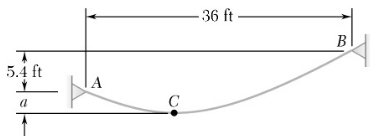
Then $$b = (40.864\text{ ft})\cosh^{-1}\left(1 + \frac{7.2\text{ ft}}{40.864\text{ ft}}\right) = 23.92\text{ ft}$$

(*a*) *C* is 23.9 ft left of and 7.20 ft below *B* ◀

$$T_{max} = wy_B = (0.3\text{ lb/ft})(40.864\text{ ft} + 7.2\text{ ft})$$

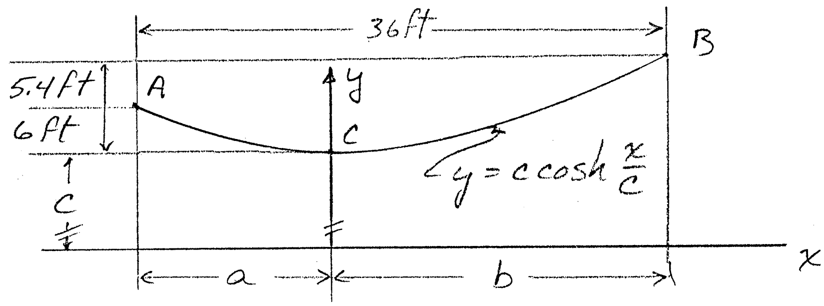
(*b*) $T_{max} = 14.42$ lb ◀

PROBLEM 7.141



The cable *ACB* weighs 0.3 lb/ft. Knowing that the lowest point of the cable is located at a distance *a* = 6 ft below the support *A*, determine (*a*) the location of the lowest point *C*, (*b*) the maximum tension in the cable.

SOLUTION



$$y_A = c\cosh\frac{-a}{c} = c + 6 \text{ ft}$$

$$a = c\cosh^{-1}\left(1 + \frac{6 \text{ ft}}{c}\right)$$

$$y_B = c\cosh\frac{b}{c} = c + 11.4 \text{ ft}$$

$$b = c\cosh^{-1}\left(1 + \frac{11.4 \text{ ft}}{c}\right)$$

So
$$a + b = c\left[\cosh^{-1}\left(1 + \frac{6 \text{ ft}}{c}\right) + \cosh^{-1}\left(1 + \frac{11.4 \text{ ft}}{\text{c}}\right)\right] = 36 \text{ ft}$$

Solving numerically
$$c = 20.446 \text{ ft}$$

$$b = (20.446 \text{ ft})\cosh^{-1}\left(1 + \frac{11.4 \text{ ft}}{20.446 \text{ ft}}\right) = 20.696 \text{ ft}$$

(a) *C* is 20.7 ft left of and 11.4 ft below *B* ◀

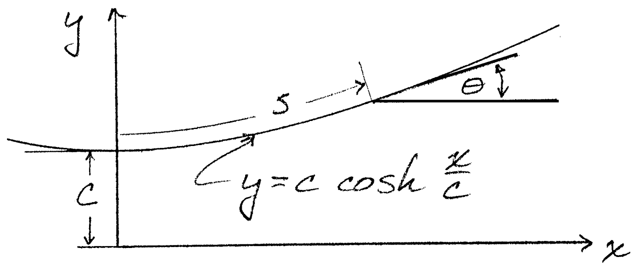
$$T_{\max} = wy_B = (0.3 \text{ lb/ft})(20.446 \text{ ft})\cosh\left(\frac{20.696 \text{ ft}}{20.446 \text{ ft}}\right) = 9.554 \text{ lb}$$

(*b*) $T_{\max} = 9.55$ lb ◀

PROBLEM 7.142

Denoting by θ the angle formed by a uniform cable and the horizontal, show that at any point (*a*) $s = c \tan\theta$, (*b*) $y = c\sec\theta$.

SOLUTION



(*a*) $$\tan\theta = \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$s = c\sinh\frac{x}{c} = c\tan\theta \quad \text{Q.E.D.}$$

(*b*) Also $$y^2 = s^2 + c^2\left(\cosh^2 x = \sinh^2 x + 1\right)$$

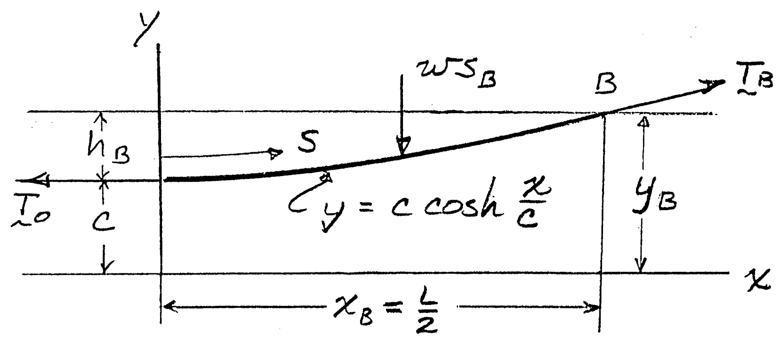
So $$y^2 = c^2\left(\tan^2\theta + 1\right) = c^2\sec^2\theta$$

And $$y = c\sec\theta \quad \text{Q.E.D.}$$

PROBLEM 7.143

(*a*) Determine the maximum allowable horizontal span for a uniform cable of mass per unit length m' if the tension in the cable is not to exceed a given value T_m. (*b*) Using the result of part *a*, determine the maximum span of a steel wire for which $m' = 0.34$ kg/m and $T_m = 32$ kN.

SOLUTION



$$T_B = T_{max} = wy_B$$

$$= wc\cosh\frac{x_B}{c} = w\frac{L}{2}\left(\frac{2c}{L}\right)\cosh\frac{L}{2c}$$

Let $\xi = \dfrac{L}{2c}$ so $T_{max} = \dfrac{wL}{2\xi}\cosh\xi$

$$\frac{dT_{max}}{d\xi} = \frac{wL}{2\xi}\left(\sinh\xi - \frac{1}{\xi}\cosh\xi\right)$$

For $\min T_{max}$, $\tanh\xi - \dfrac{1}{\xi} = 0$

Solving numerically $\xi = 1.1997$

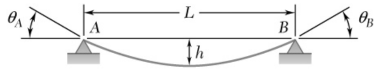
$$\left(T_{max}\right)_{min} = \frac{wL}{2(1.9997)}\cosh(1.1997) = 0.75444wL$$

(*a*) $L_{max} = \dfrac{T_{max}}{0.75444w} = 1.3255\dfrac{T_{max}}{w}$ ◀

If $T_{max} = 32$ kN and $w = (0.34\text{ kg/m})(9.81\text{ m/s}^2) = 3.3354$ N/m

$$L_{max} = 1.3255\frac{32.000\text{ N}}{3.3354\text{ N/m}} = 12\,717\text{ m}$$

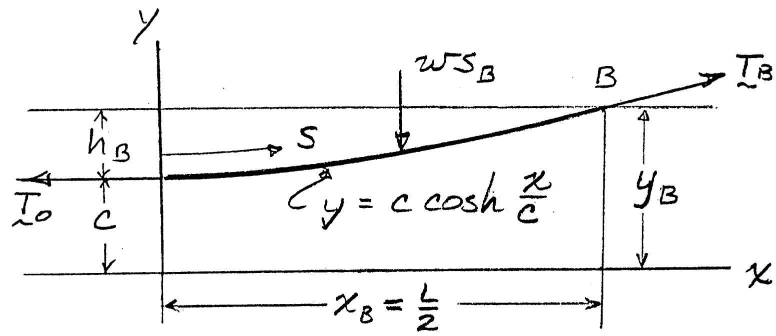
(*b*) $L_{max} = 12.72$ km ◀



PROBLEM 7.144

A cable has a weight per unit length of 2 lb/ft and is supported as shown. Knowing that the span L is 18 ft, determine the two values of the sag h for which the maximum tension is 80 lb.

SOLUTION



$$y_{max} = c\cosh\frac{L}{2c} = h + c$$

$$T_{max} = wy_{max} \qquad y_{max} = \frac{T_{max}}{w}$$

$$y_{max} = \frac{80 \text{ lb}}{2 \text{ lb/ft}} = 40 \text{ ft}$$

$$c\cosh\frac{9 \text{ ft}}{c} = 40 \text{ ft}$$

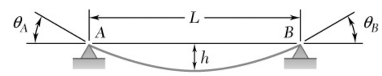
Solving numerically $\quad c_1 = 2.6388$ ft

$$c_2 = 38.958 \text{ ft}$$

$$h = y_{max} - c$$

$h_1 = 40$ ft $- 2.6388$ ft $\qquad h_1 = 37.4$ ft ◀

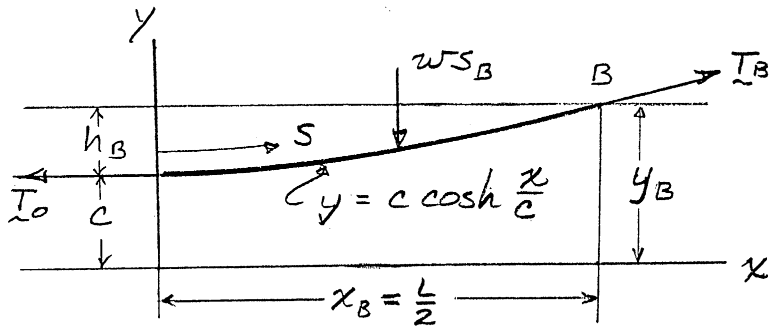
$h_2 = 40$ ft $- 38.958$ ft $\qquad h_2 = 1.042$ ft ◀



PROBLEM 7.145

Determine the sag-to-span ratio for which the maximum tension in the cable is equal to the total weight of the entire cable *AB*.

SOLUTION



$$T_{\max} = wy_B = 2ws_B$$

$$y_B = 2s_B$$

$$c \cosh \frac{L}{2c} = 2c \sinh \frac{L}{2c}$$

$$\tanh \frac{L}{2c} = \frac{1}{2}$$

$$\frac{L}{2c} = \tanh^{-1} \frac{1}{2} = 0.549306$$

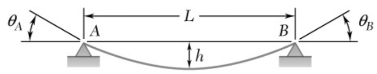
$$\frac{h_B}{c} = \frac{y_B - c}{c} = \cosh \frac{L}{2c} - 1$$

$$= 0.154701$$

$$\frac{h_B}{L} = \frac{h_B / c}{2(L/2c)}$$

$$= \frac{0.5(0.154701)}{0.549306} = 0.14081$$

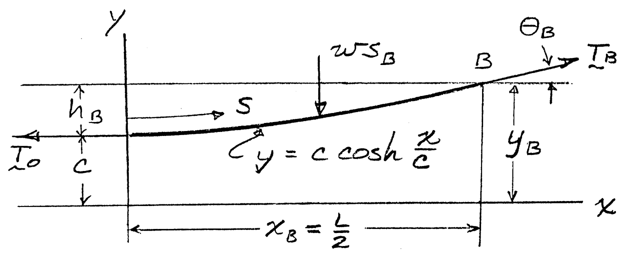
$$\frac{h_B}{L} = 0.1408 \blacktriangleleft$$



PROBLEM 7.146

A cable of weight w per unit length is suspended between two points at the same elevation that are a distance L apart. Determine (a) the sag-to-span ratio for which the maximum tension is as small as possible, (b) the corresponding values of θ_B and T_m.

SOLUTION



(a)

$$T_{max} = wy_B = wc\cosh\frac{L}{2c}$$

$$\frac{dT_{max}}{dc} = w\left(\cosh\frac{L}{2c} - \frac{L}{2c}\sinh\frac{L}{2c}\right)$$

For $\min T_{max}$, $\dfrac{dT_{max}}{dc} = 0$

$$\tanh\frac{L}{2c} = \frac{2c}{L} \rightarrow \frac{L}{2c} = 1.1997$$

$$\frac{y_B}{c} = \cosh\frac{L}{2c} = 1.8102$$

$$\frac{h}{c} = \frac{y_B}{c} - 1 = 0.8102$$

$$\frac{h}{L} = \left[\frac{1}{2}\frac{h}{c}\left(\frac{2c}{L}\right)\right] = \frac{0.8102}{2(1.1997)} = 0.3375$$

$$\frac{h}{L} = 0.338 \blacktriangleleft$$

(b)

$$T_0 = wc \qquad T_{max} = wc\cosh\frac{L}{2c} \qquad \frac{T_{max}}{T_0} = \cosh\frac{L}{2c} = \frac{y_B}{c}$$

But $T_0 = T_{max}\cos\theta_B \qquad \dfrac{T_{max}}{T_0} = \sec\theta_B$

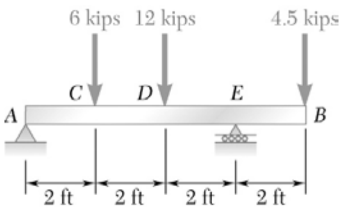
So $\theta_B = \sec^{-1}\left(\dfrac{y_B}{c}\right) = \sec^{-1}(1.8102)$

$$= 56.46°$$

$$\theta_B = 56.5° \blacktriangleleft$$

$$T_{max} = wy_B = w\frac{y_B}{c}\left(\frac{2c}{L}\right)\left(\frac{L}{2}\right) = w(1.8102)\frac{L}{2(1.1997)}$$

$$T_{max} = 0.755wL \blacktriangleleft$$

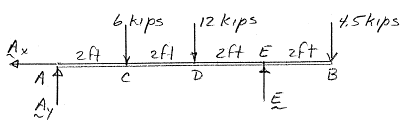


PROBLEM 7.147

For the beam and loading shown, (*a*) draw the shear and bending moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Beam:



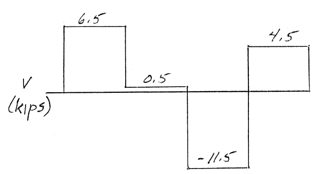
(*a*)

$$\circlearrowleft \Sigma M_A = 0: (6 \text{ ft})E - (8 \text{ ft})(4.5 \text{ kips})$$
$$- (4 \text{ ft})(12 \text{ kips}) - (2 \text{ ft})(6 \text{ kips}) = 0$$

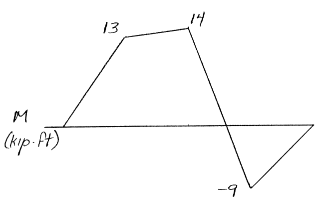
$$\mathbf{E} = 16 \text{ kips}\uparrow$$

$$\circlearrowleft \Sigma M_E = 0: -(6 \text{ ft})A_y + (4 \text{ ft})(6 \text{ kips})$$
$$+ (2 \text{ ft})(12 \text{ kips}) - (2 \text{ ft})(4.5 \text{ kips}) = 0$$

$$\mathbf{A}_y = 6.5 \text{ kips}\uparrow$$



Shear Diag: V is piece wise constant with discontinuities equal to the forces at A, C, D, E, B



Moment Diag: M is piecewise linear with slope changes at C, D, E

$$M_A = 0$$

$$M_C = (6.5 \text{ kips})(2 \text{ ft}) = 13 \text{ kip}\cdot\text{ft}$$

$$M_C = 13 \text{ kip}\cdot\text{ft} + (0.5 \text{ kips})(2 \text{ ft}) = 14 \text{ kip}\cdot\text{ft}$$

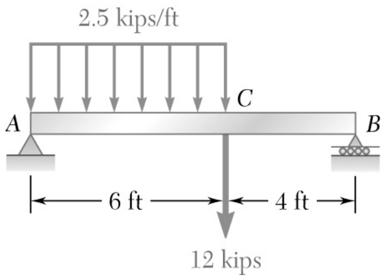
$$M_D = 14 \text{ kip}\cdot\text{ft} - (11.5 \text{ kips})(2 \text{ ft}) = -9 \text{ kip}\cdot\text{ft}$$

$$M_B = -9 \text{ kip}\cdot\text{ft} + (4.5 \text{ kips})(2 \text{ ft}) = 0$$

(*b*) $|V|_{\max} = 11.50$ kips on DE ◀

$|M|_{\max} = 14.00$ kip·ft at D ◀

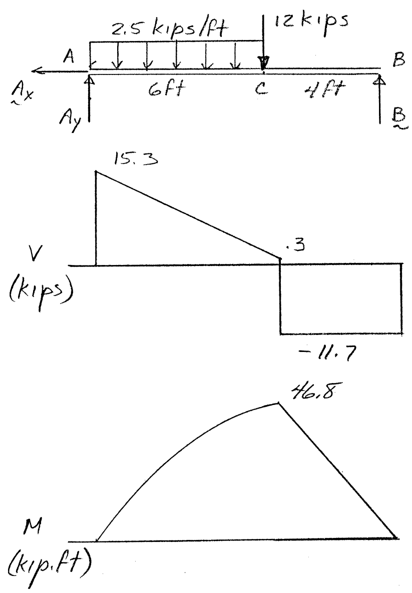
PROBLEM 7.148



For the beam and loading shown, (*a*) draw the shear and bending moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Beam:



(*a*) $\circlearrowleft \Sigma M_B = 0\text{: } (4 \text{ ft})(12 \text{ kips}) + (7 \text{ ft})(2.5 \text{ kips/ft})(6 \text{ ft}) - (10 \text{ ft})A_y = 0$

$$\mathbf{A}_y = 15.3 \text{ kips} \uparrow$$

Shear Diag: $V_A = A_y = 15.3$ kips, then V is linear

$\left(\frac{dV}{dx} = -2.5 \text{ kips/ft}\right)$ to C.

$$V_C = 15.3 \text{ kips} - (2.5 \text{ kips/ft})(6 \text{ ft}) = 0.3 \text{ kips}$$

At C, V decreases by 12 kips to -11.7 kips and is constant to B.

Moment Diag: $M_A = 0$ and M is parabolic

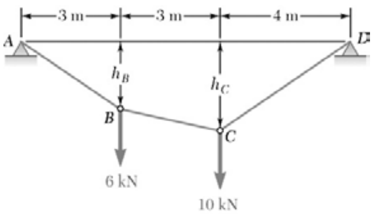
$\left(\frac{dM}{dx} \text{ decreasing with } V\right)$ to C

$$M_C = \frac{1}{2}(15.3 \text{ kips} + 0.3 \text{ kip})(6 \text{ ft}) = 46.8 \text{ kip}\cdot\text{ft}$$

$$M_B = 46.8 \text{ kip}\cdot\text{ft} - (11.7 \text{ kips})(4 \text{ ft}) = 0$$

(*b*) $|V|_{max} = 15.3$ kips ◀

$|M|_{max} = 46.8$ kip·ft ◀

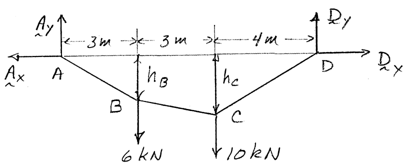


PROBLEM 7.149

Two loads are suspended as shown from the cable *ABCD*. Knowing that $h_B = 1.8$ m, determine (*a*) the distance h_C, (*b*) the components of the reaction at *D*, (*c*) the maximum tension in the cable.

SOLUTION

FBD Cable:



$$\rightarrow \Sigma F_x = 0: -A_x + D_x = 0 \qquad A_x = D_x$$

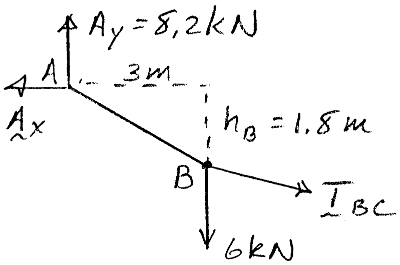
$$\circlearrowleft \Sigma M_A = 0: (10\text{ m})D_y - (6\text{ m})(10\text{ kN}) - (3\text{ m})(6\text{ kN}) = 0$$

$$\mathbf{D}_y = 7.8\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 6\text{ kN} - 10\text{ kN} + 7.8\text{ kN} = 0$$

$$\mathbf{A}_y = 8.2\text{ kN} \uparrow$$

FBD AB:

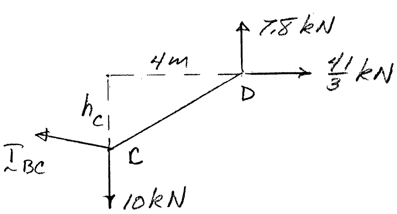


$$\circlearrowleft \Sigma M_B = 0: (1.8\text{ m})A_x - (3\text{ m})(8.2\text{ kN}) = 0$$

$$\mathbf{A}_x = \frac{41}{3}\text{ kN} \leftarrow$$

From above $\quad D_x = A_x = \dfrac{41}{3}$ kN

FBD CD:



$$\circlearrowleft \Sigma M_C = 0: (4\text{ m})(7.8\text{ kN}) - h_C\left(\frac{41}{3}\text{ kN}\right) = 0$$

$$h_C = 2.283\text{ m}$$

(*a*) $\qquad h_C = 2.28\text{ m}$ ◀

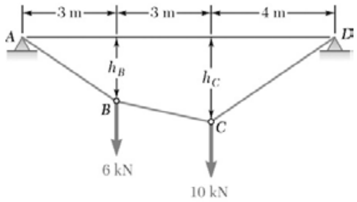
(*b*) $\qquad D_x = 13.67\text{ kN} \rightarrow$ ◀

$D_y = 7.80\text{ kN} \uparrow$ ◀

Since $A_x = B_x$ and $A_y > B_y$, max T is T_{AB}

$$T_{AB} = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(\frac{41}{3}\text{ kN}\right)^2 + (8.2\text{ kN})^2}$$

(*c*) $\qquad T_{\max} = 15.94\text{ kN}$ ◀

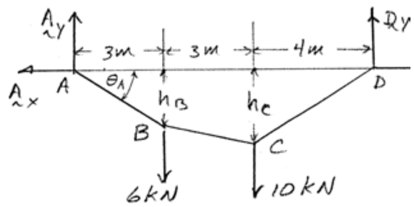


PROBLEM 7.150

Knowing that the maximum tension in cable *ABCD* is 15 kN, determine (*a*) the distance h_B, (*b*) the distance h_C.

SOLUTION

FBD Cable:



$$\rightarrow \Sigma F_x = 0: -A_x + D_x = 0 \qquad A_x = D_x$$

$$\circlearrowleft \Sigma M_A = 0: (10\text{ m})D_y - (6\text{ m})(10\text{ kN}) - (3\text{ m})(6\text{ kN}) = 0$$

$$\mathbf{D}_y = 7.8\text{ kN}\uparrow$$

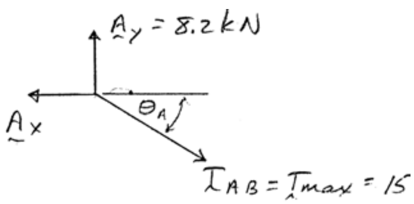
$$\uparrow \Sigma F_y = 0: A_y - 6\text{ kN} - 10\text{ kN} + 7.8\text{ kN} = 0$$

$$\mathbf{A}_y = 8.2\text{ kN}\uparrow$$

Since $\qquad A_x = D_x \quad$ and $\quad A_y > D_y, \qquad T_{\max} = T_{AB}$

$$\uparrow \Sigma F_y = 0: 8.2\text{ kN} - (15\text{ kN})\sin\theta_A = 0$$

FBD pt A:

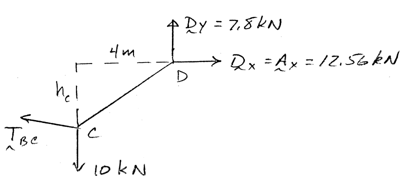


$$\theta_A = \sin^{-1}\frac{8.2\text{ kN}}{15\text{ kN}} = 33.139°$$

$$\rightarrow \Sigma F_x = 0: -A_x + (15\text{ kN})\cos\theta_A = 0$$

$$A_x = (15\text{ kN})\cos(33.139°) = 12.56\text{ kN}$$

FBD CD:

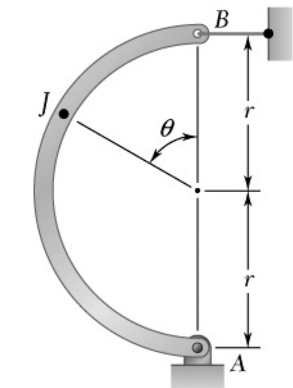


From FBD cable: $\qquad h_B = (3\text{ m})\tan\theta_A = (3\text{ m})\tan(33.139°)$

(*a*) $\qquad h_B = 1.959\text{ m}$ ◀

$$\circlearrowleft \Sigma M_C = 0: (4\text{ m})(7.8\text{ kN}) - h_C(12.56\text{ kN}) = 0$$

(*b*) $\qquad h_C = 2.48\text{ m}$ ◀

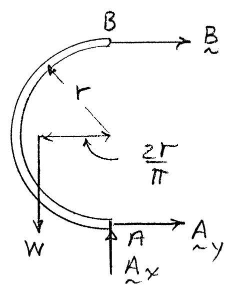


PROBLEM 7.151

A semicircular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 60°$.

SOLUTION

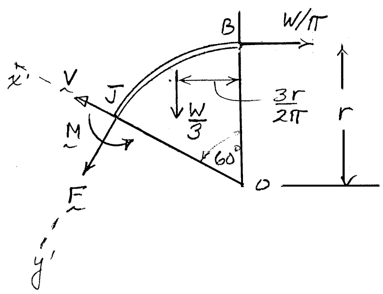
FBD Rod:



$$\circlearrowleft \Sigma M_A = 0: \frac{2r}{\pi}W - 2rB = 0$$

$$\mathbf{B} = \frac{W}{\pi} \longrightarrow$$

FBD BJ:



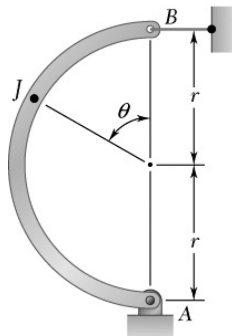
$$\swarrow \Sigma F_{y'} = 0: F + \frac{W}{3}\sin 60° - \frac{W}{\pi}\cos 60° = 0$$

$$F = -0.12952W$$

$$\circlearrowleft \Sigma M_0 = 0: r\left(F - \frac{W}{\pi}\right) + \frac{3r}{2\pi}\left(\frac{W}{3}\right) + M = 0$$

$$M = Wr\left(0.12952 + \frac{1}{\pi} - \frac{1}{2\pi}\right) = 0.28868Wr$$

On BJ $\qquad \mathbf{M}_J = 0.289Wr \circlearrowright$ ◀

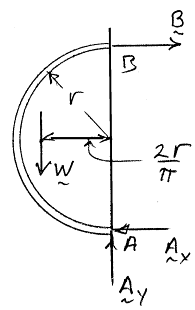


PROBLEM 7.152

A semicircular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 150°$.

SOLUTION

FBD rod:

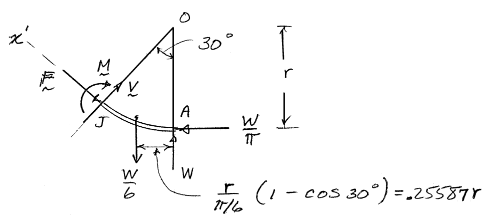


$$\uparrow \Sigma F_y = 0: A_y - W = 0 \qquad \mathbf{A}_y = W \uparrow$$

$$\Sigma M_B = 0: \frac{2r}{\pi}W - 2rA_x = 0$$

$$\mathbf{A}_x = \frac{W}{\pi} \longleftarrow$$

FBD AJ:



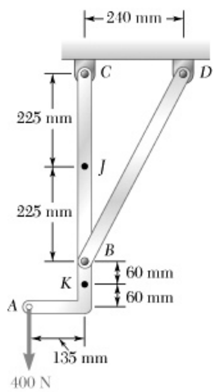
$$\nwarrow \Sigma F_{x'} = 0: \frac{W}{\pi}\cos 30° + \frac{5W}{6}\sin 30° - F = 0 \quad F = 0.69233W \searrow$$

$$\circlearrowleft \Sigma M_0 = 0: 0.25587r\left(\frac{W}{6}\right) + r\left(F - \frac{W}{\pi}\right) - M = 0$$

$$M = Wr\left[\frac{0.25587}{6} + 0.69233 - \frac{1}{\pi}\right]$$

$$M = Wr(0.4166)$$

On AJ $\qquad \mathbf{M} = 0.417Wr \circlearrowright$ ◀

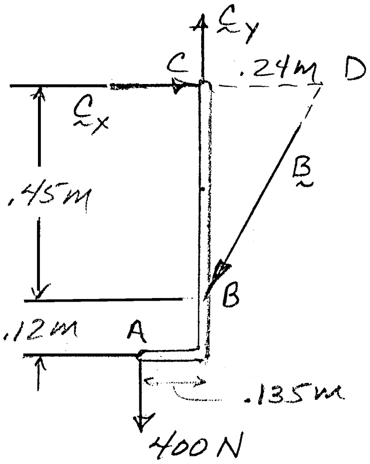


PROBLEM 7.153

Determine the internal forces at point J of the structure shown.

SOLUTION

FBD ABC:



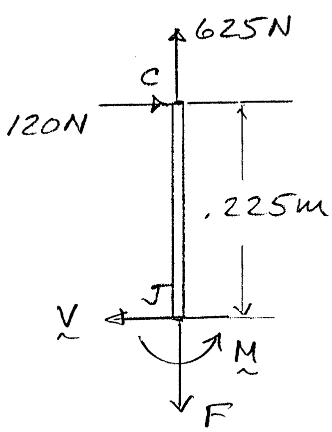
$$\circlearrowleft \Sigma M_D = 0: (0.375\text{ m})(400\text{ N}) - (0.24\text{ m})C_y = 0$$

$$\mathbf{C}_y = 625\text{ N} \uparrow$$

$$\circlearrowleft \Sigma M_B = 0: -(0.45\text{ m})C_x + (0.135\text{ m})(400\text{ N}) = 0$$

$$\mathbf{C}_x = 120\text{ N} \longrightarrow$$

FBD CJ:



$$\uparrow \Sigma F_y = 0: 625\text{ N} - F = 0$$

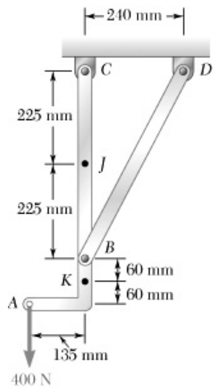
$$\mathbf{F} = 625\text{ N} \downarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: 120\text{ N} - V = 0$$

$$\mathbf{V} = 120.0\text{ N} \longleftarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - (0.225\text{ m})(120\text{ N}) = 0$$

$$\mathbf{M} = 27.0\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

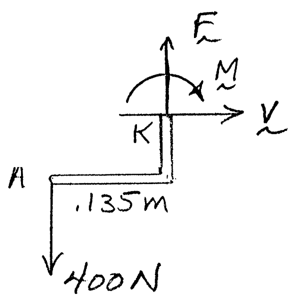


PROBLEM 7.154

Determine the internal forces at point K of the structure shown.

SOLUTION

FBD AK:



$$\rightarrow \Sigma F_x = 0: V = 0$$

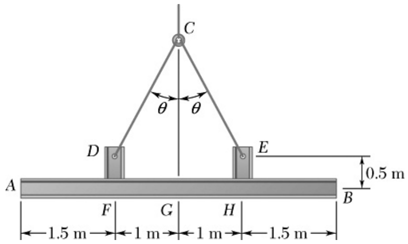
$\mathbf{V} = 0$ ◀

$$\uparrow \Sigma F_y = 0: F - 400 \text{ N} = 0$$

$\mathbf{F} = 400 \text{ N} \uparrow$ ◀

$$\circlearrowleft \Sigma M_K = 0: (0.135 \text{ m})(400 \text{ N}) - M = 0$$

$\mathbf{M} = 54.0 \text{ N}\cdot\text{m} \circlearrowright$ ◀

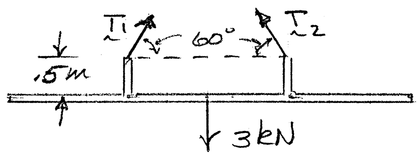


PROBLEM 7.155

Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing the $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

FBD Beam + channels:



(*a*) By symmetry: $T_1 = T_2 = T$

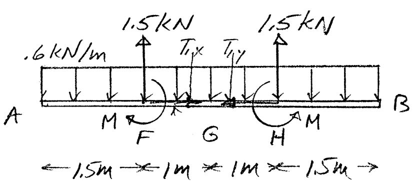
$$\uparrow \Sigma F_y = 0:\ 2T\sin 60° - 3 \text{ kN} = 0$$

$$T = \frac{3}{\sqrt{3}} \text{ kN} \qquad T_{1x} = \frac{3}{2\sqrt{3}} \qquad T_{1y} = \frac{3}{2} \text{ kN}$$

FBD Beam:

$$M = (0.5 \text{ m})\frac{3}{2\sqrt{3}} \text{ kN} = 0.433 \text{ kN}\cdot\text{m}$$

With cable force replaced by equivalent force-couple system at *F* and *G*



Shear Diagram: *V* is piecewise linear

$$\left(\frac{dV}{dx} = -0.6 \text{ kN/m}\right) \text{ with } 1.5 \text{ kN}$$

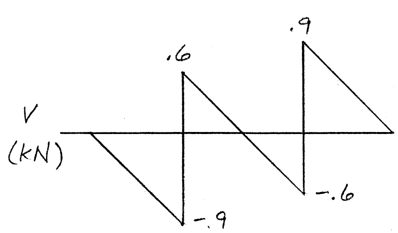
discontinuities at *F* and *H*.

$$V_{F^-} = -(0.6 \text{ kN/m})(1.5 \text{ m}) = 0.9 \text{ kN}$$

V increases by 1.5 kN to $+\ 0.6$ kN at F^+

$$V_G = 0.6 \text{ kN} - (0.6 \text{ kN/m})(1 \text{ m}) = 0$$

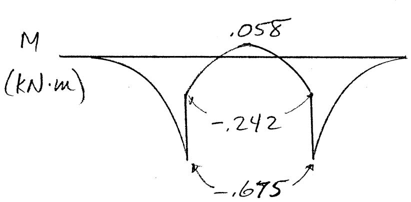
Finish by invoking symmetry



Moment Diagram: *M* is piecewise parabolic

$$\left(\frac{dM}{dx} \text{ decreasing with } V\right) \text{ with discontinuities of .433 kN at } F \text{ and } H.$$

$$M_{F^-} = -\frac{1}{2}(0.9 \text{ kN})(1.5 \text{ m}) = -0.675 \text{ kN}\cdot\text{m}$$



M increases by $0.433 \text{ kN}\cdot\text{m}$ to $-0.242 \text{ kN}\cdot\text{m}$ at F^+

$$M_G = -0.242 \text{ kN}\cdot\text{m} + \frac{1}{2}(0.6 \text{ kN})(1 \text{ m}) = 0.058 \text{ kN}\cdot\text{m}$$

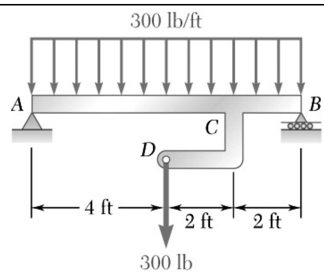
Finish by invoking symmetry

(*b*) $|V|_{max} = 900$ N ◀

at F^- and G^+

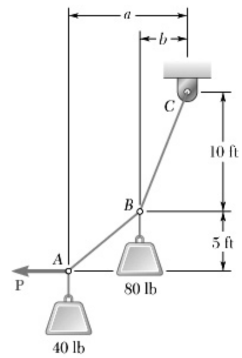
$|M|_{max} = 675$ N·m ◀

at *F* and *G*



PROBLEM 7.156

(*a*) Draw the shear and bending moment diagrams for beam *AB*,
(*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

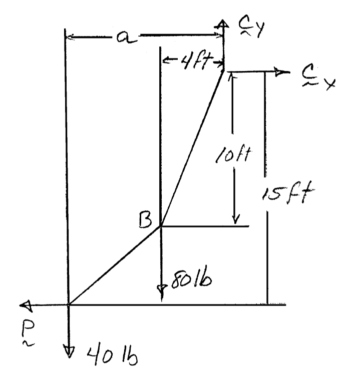


PROBLEM 7.157

Cable *ABC* supports two loads as shown. Knowing that $b = 4$ ft, determine (*a*) the required magnitude of the horizontal force **P**, (*b*) the corresponding distance *a*.

SOLUTION

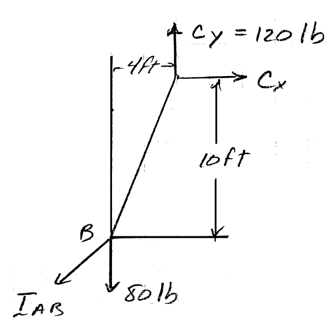
FBD ABC:



$$\uparrow \Sigma F_y = 0: -40 \text{ lb} - 80 \text{ lb} + C_y = 0$$

$$\mathbf{C}_y = 120 \text{ lb} \uparrow$$

FBD BC:



$$\circlearrowleft \Sigma M_B = 0: (4 \text{ ft})(120 \text{ lb}) - (10 \text{ ft})C_x = 0$$

$$\mathbf{C}_x = 48 \text{ lb} \longrightarrow$$

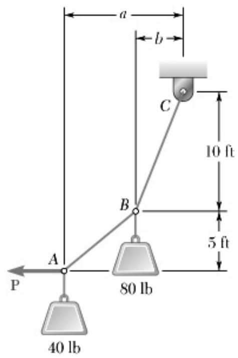
From ABC: $\longrightarrow \Sigma F_x = 0: -P + C_x = 0$

$$P = C_x = 48 \text{ lb}$$

(*a*) $P = 48.0$ lb ◀

$$\circlearrowleft \Sigma M_C = 0: (4 \text{ ft})(80 \text{ lb}) + a(40 \text{ lb}) - (15 \text{ ft})(48 \text{ lb}) = 0$$

(*b*) $a = 10.00$ ft ◀

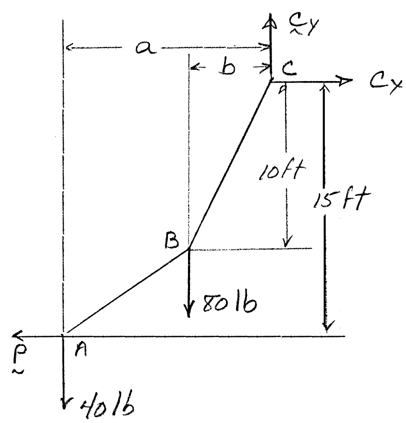


PROBLEM 7.158

Cable *ABC* supports two loads as shown. Determine the distances *a* and *b* when a horizontal force **P** of magnitude 60 lb is applied at *A*.

SOLUTION

FBD ABC:

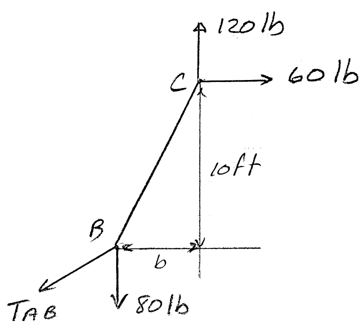


$$\rightarrow \Sigma F_x = 0: C_x - P = 0 \qquad C_x = 60 \text{ lb} \rightarrow$$

$$\uparrow \Sigma F_y = 0: C_y - 40 \text{ lb} - 80 \text{ lb} = 0$$

$$\mathbf{C}_y = 120 \text{ lb} \uparrow$$

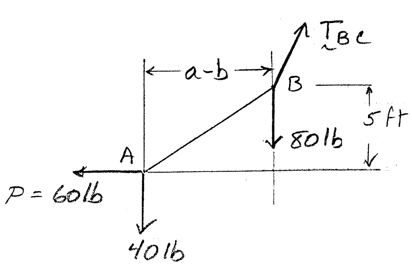
FBD BC:



$$\circlearrowleft \Sigma M_B = 0: b(120 \text{ lb}) - (10 \text{ ft})(60 \text{ lb}) = 0$$

$$b = 5.00 \text{ ft} \blacktriangleleft$$

FBD AB:



$$\Sigma M_B = 0: (a - b)(40 \text{ lb}) - (5 \text{ ft})60 \text{ lb} = 0$$

$$a - b = 7.5 \text{ ft}$$

$$a = b + 7.5 \text{ ft}$$

$$= 5 \text{ ft} + 7.5 \text{ ft}$$

$$a = 12.50 \text{ ft} \blacktriangleleft$$