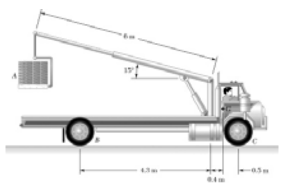
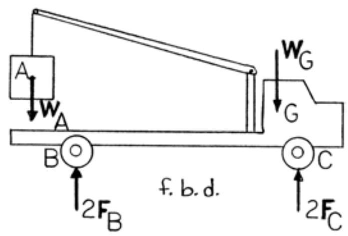
PROBLEM 4.1



The boom on a 4300-kg truck is used to unload a pallet of shingles of mass 1600 kg. Determine the reaction at each of the two (*a*) rear wheels *B*, (*b*) front wheels *C*.

SOLUTION



$$W_A = m_A g = (1600\text{ kg})(9.81\text{ m/s}^2)$$

$$= 15696\text{ N}$$

or $$\mathbf{W}_A = 15.696\text{ kN}\downarrow$$

$$W_G = m_G g = (4300\text{ kg})(9.81\text{ m/s}^2)$$

$$= 42\,183\text{ N}$$

or $$\mathbf{W}_G = 42.183\text{ kN}\downarrow$$

(*a*) From f.b.d. of truck with boom

$$+\circlearrowleft \Sigma M_C = 0: \quad (15.696\text{ kN})\left[(0.5 + 0.4 + 6\cos 15°)\text{ m}\right] - 2F_B\left[(0.5 + 0.4 + 4.3)\text{ m}\right]$$

$$+ (42.183\text{ kN})(0.5\text{ m}) = 0$$

$$\therefore\ 2F_B = \frac{126.185}{5.2} = 24.266\text{ kN}$$

or $\mathbf{F}_B = 12.13\text{ kN}\uparrow$ ◀

(*b*) From f.b.d. of truck with boom

$$+\circlearrowleft \Sigma M_B = 0: \quad (15.696\text{ kN})\left[(6\cos 15° - 4.3)\text{ m}\right] - (42.183\text{ kN})\left[(4.3 + 0.4)\text{ m}\right]$$

$$+ 2F_C\left[(4.3 + 0.9)\text{ m}\right] = 0$$

$$\therefore\ 2F_C = \frac{174.786}{5.2} = 33.613\text{ kN}$$

or $\mathbf{F}_C = 16.81\text{ kN}\uparrow$ ◀

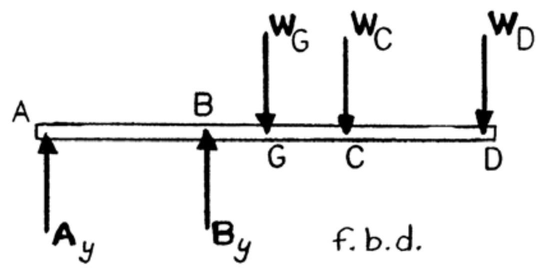
Check: $$+\uparrow \Sigma F_y = 0: \quad (33.613 - 42.183 + 24.266 - 15.696)\text{ kN} = 0?$$

$$(57.879 - 57.879)\text{ kN} = 0 \ \text{ ok}$$

PROBLEM 4.2

Two children are standing on a diving board of mass 65 kg. Knowing that the masses of the children at *C* and *D* are 28 kg and 40 kg, respectively, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.

SOLUTION



$$W_G = m_G g = (65\text{ kg})(9.81\text{ m/s}^2) = 637.65\text{ N}$$

$$W_C = m_C g = (28\text{ kg})(9.81\text{ m/s}^2) = 274.68\text{ N}$$

$$W_D = m_D g = (40\text{ kg})(9.81\text{ m/s}^2) = 392.4\text{ N}$$

(*a*) From f.b.d. of diving board

$$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(1.2\text{ m}) - (637.65\text{ N})(0.48\text{ m}) - (274.68\text{ N})(1.08\text{ m}) - (392.4\text{ N})(2.08\text{ m}) = 0$$

$$\therefore\ A_y = -\frac{1418.92}{1.2} = -1182.43\text{ N}$$

or $\mathbf{A}_y = 1.182\text{ kN} \downarrow$ ◀

(*b*) From f.b.d. of diving board

$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(1.2\text{ m}) - 637.65\text{ N}(1.68\text{ m}) - 274.68\text{ N}(2.28\text{ m}) - 392.4\text{ N}(3.28\text{ m}) = 0$$

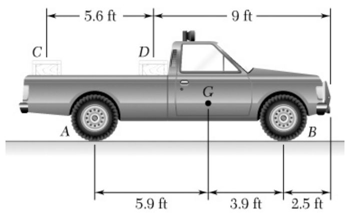
$$\therefore\ B_y = \frac{2984.6}{1.2} = 2487.2\text{ N}$$

or $\mathbf{B}_y = 2.49\text{ kN} \uparrow$ ◀

Check: $+\uparrow \Sigma F_y = 0: \quad (-1182.43 + 2487.2 - 637.65 - 274.68 - 392.4)\text{N} = 0?$

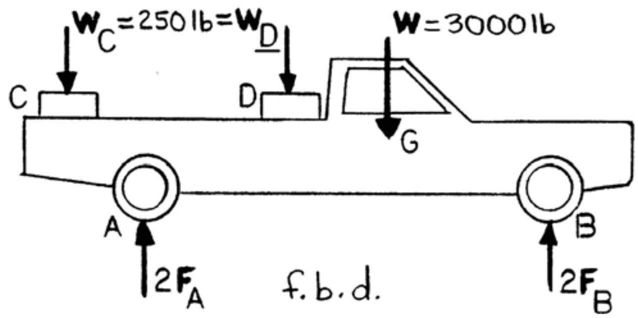
$$(2487.2 - 2487.2)\text{ N} = 0 \ \text{ ok}$$

PROBLEM 4.3



Two crates, each weighing 250 lb, are placed as shown in the bed of a 3000-lb pickup truck. Determine the reactions at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION



(*a*) From f.b.d. of truck

$$+\circlearrowleft \Sigma M_B = 0: \quad (250\text{ lb})(12.1\text{ ft}) + (250\text{ lb})(6.5\text{ ft}) + (3000\text{ lb})(3.9\text{ ft}) - (2F_A)(9.8\text{ ft}) = 0$$

$$\therefore\ 2F_A = \frac{16350}{9.8} = 1668.37\text{ lb}$$

$$\therefore\quad \mathbf{F}_A = 834\text{ lb}\uparrow \blacktriangleleft$$

(*b*) From f.b.d. of truck

$$+\circlearrowleft \Sigma M_A = 0: \quad (2F_B)(9.8\text{ ft}) - (3000\text{ lb})(5.9\text{ ft}) - (250\text{ lb})(3.3\text{ ft}) + (250\text{ lb})(2.3\text{ ft}) = 0$$

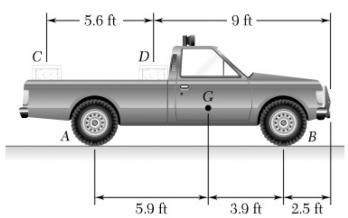
$$\therefore\ 2F_B = \frac{17950}{9.8} = 1831.63\text{ lb}$$

$$\therefore\quad \mathbf{F}_B = 916\text{ lb}\uparrow \blacktriangleleft$$

Check:

$$+\uparrow \Sigma F_y = 0: \quad (-250 + 1668.37 - 250 - 3000 + 1831.63)\text{ lb} = 0?$$

$$(3500 - 3500)\text{ lb} = 0 \quad \text{ok}$$

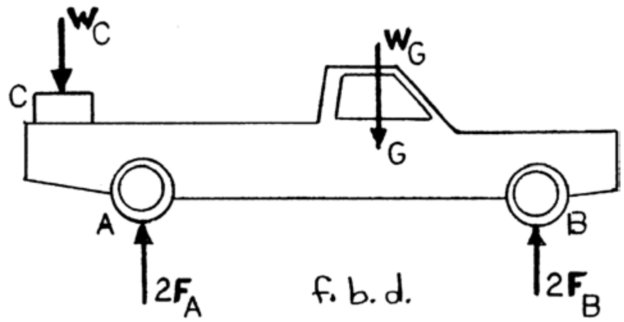


PROBLEM 4.4

Solve Problem 4.3 assuming that crate *D* is removed and that the position of crate *C* is unchanged.

P4.3 The boom on a 4300-kg truck is used to unload a pallet of shingles of mass 1600 kg. Determine the reaction at each of the two (*a*) rear wheels *B*, (*b*) front wheels *C*

SOLUTION



(*a*) From f.b.d. of truck

$$+\circlearrowleft \Sigma M_B = 0: \quad (3000\text{ lb})(3.9\text{ ft}) - (2F_A)(9.8\text{ ft}) + (250\text{ lb})(12.1\text{ ft}) = 0$$

$$\therefore \; 2F_A = \frac{14725}{9.8} = 1502.55\text{ lb}$$

or $\mathbf{F}_A = 751\text{ lb} \uparrow$ ◀

(*b*) From f.b.d. of truck

$$+\circlearrowleft \Sigma M_A = 0: \quad (2F_B)(9.8\text{ ft}) - (3000\text{ lb})(5.9\text{ ft}) + (250\text{ lb})(2.3\text{ ft}) = 0$$

$$\therefore \; 2F_B = \frac{17125}{9.8} = 1747.45\text{ lb}$$

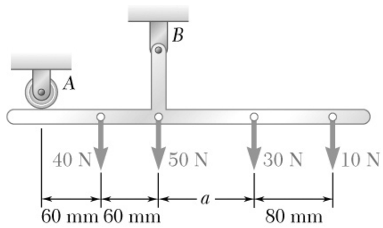
or $\mathbf{F}_B = 874\text{ lb} \uparrow$ ◀

Check: $+\uparrow \Sigma F_y = 0: \quad \left[2(751 + 874) - 3000 - 250\right]\text{ lb} = 0?$

$$(3250 - 3250)\text{ lb} = 0 \;\text{ ok}$$

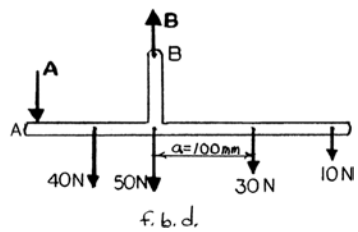
PROBLEM 4.5

A T-shaped bracket supports the four loads shown. Determine the reactions at A and B if (a) $a = 100$ mm, (b) $a = 70$ mm.



SOLUTION

(a)



From f.b.d. of bracket

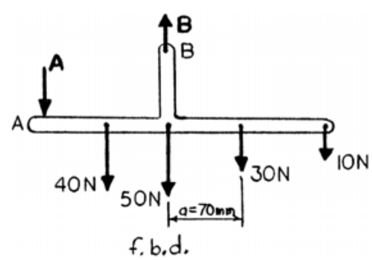
$$+\circlearrowleft \Sigma M_B = 0: \quad -(10\text{ N})(0.18\text{ m}) - (30\text{ N})(0.1\text{ m}) + (40\text{ N})(0.06\text{ m}) + A(0.12\text{ m}) = 0$$

$$\therefore\ A = \frac{2.400}{0.12} = 20\text{ N} \qquad \text{or } \mathbf{A} = 20.0\text{ N}\downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad B(0.12\text{ m}) - (40\text{ N})(0.06\text{ m}) - (50\text{ N})(0.12\text{ m}) - (30\text{ N})(0.22\text{ m}) - (10\text{ N})(0.3\text{ m}) = 0$$

$$\therefore\ B = \frac{18.000}{0.12} = 150\text{ N} \qquad \text{or } \mathbf{B} = 150.0\text{ N}\uparrow \blacktriangleleft$$

(b)



From f.b.d. of bracket

$$+\circlearrowleft \Sigma M_B = 0: \quad -(10\text{ N})(0.15\text{ m}) - (30\text{ N})(0.07\text{ m}) + (40\text{ N})(0.06\text{ m}) + A(0.12\text{ m}) = 0$$

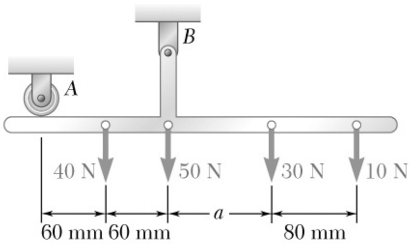
$$\therefore\ A = \frac{1.200}{0.12} = 10\text{ N} \qquad \text{or } \mathbf{A} = 10.00\text{ N}\downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad B(0.12\text{ m}) - (40\text{ N})(0.06\text{ m}) - (50\text{ N})(0.12\text{ m}) - (30\text{ N})(0.19\text{ m})$$

$$-(10\text{ N})(0.27\text{ m}) = 0$$

$$\therefore\ B = \frac{16.800}{0.12} = 140\text{ N} \qquad \text{or } \mathbf{B} = 140.0\text{ N}\uparrow \blacktriangleleft$$

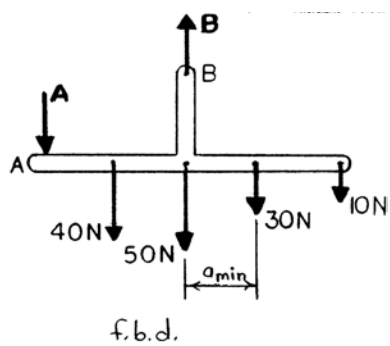
PROBLEM 4.6



For the bracket and loading of Problem 4.5, determine the smallest distance a if the bracket is not to move.

P4.5 A T-shaped bracket supports the four loads shown. Determine the reactions at A and B if (a) $a = 100$ mm, (b) $a = 70$ mm.

SOLUTION



The a_{min} value will be based on $\mathbf{A} = 0$

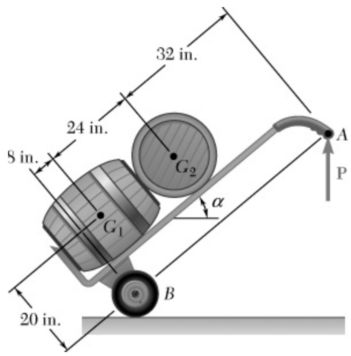
From f.b.d. of bracket

$$+\circlearrowright \Sigma M_B = 0: \quad (40\text{ N})(60\text{ mm}) - (30\text{ N})(a) - (10\text{ N})(a + 80\text{ mm}) = 0$$

$$\therefore\ a = \frac{1600}{40} = 40\text{ mm}$$

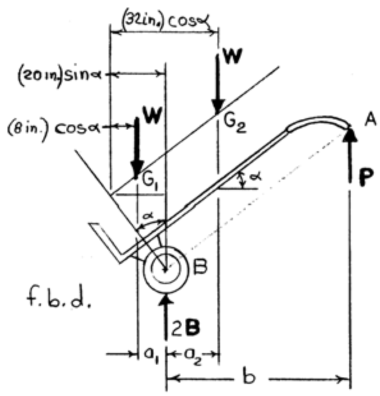
or $a_{min} = 40.0$ mm ◀

PROBLEM 4.7



A hand truck is used to move two barrels, each weighing 80 lb. Neglecting the weight of the hand truck, determine (*a*) the vertical force **P** which should be applied to the handle to maintain equilibrium when $\alpha = 35^{\circ}$, (*b*) the corresponding reaction at each of the two wheels.

SOLUTION



$$a_1 = (20 \text{ in.})\sin\alpha - (8 \text{ in.})\cos\alpha$$

$$a_2 = (32 \text{ in.})\cos\alpha - (20 \text{ in.})\sin\alpha$$

$$b = (64 \text{ in.})\cos\alpha$$

From f.b.d. of hand truck

$$+\circlearrowleft \Sigma M_B = 0: \quad P(b) - W(a_2) + W(a_1) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad P - 2w + 2B = 0 \qquad (2)$$

For $\alpha = 35^{\circ}$

$$a_1 = 20\sin 35^{\circ} - 8\cos 35^{\circ} = 4.9183 \text{ in.}$$

$$a_2 = 32\cos 35^{\circ} - 20\sin 35^{\circ} = 14.7413 \text{ in.}$$

$$b = 64\cos 35^{\circ} = 52.426 \text{ in.}$$

(*a*) From Equation (1)

$$P(52.426 \text{ in.}) - 80 \text{ lb}(14.7413 \text{ in.}) + 80 \text{ lb}(4.9183 \text{ in.}) = 0$$

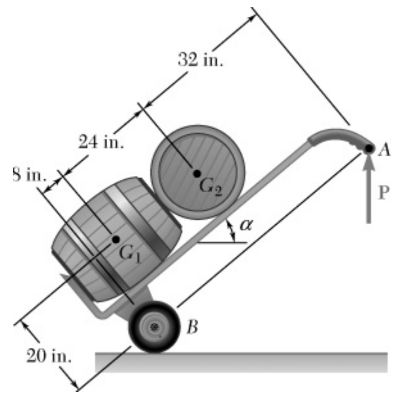
$$\therefore P = 14.9896 \text{ lb} \qquad \text{or } \mathbf{P} = 14.99 \text{ lb} \uparrow \blacktriangleleft$$

(*b*) From Equation (2)

$$14.9896 \text{ lb} - 2(80 \text{ lb}) + 2B = 0$$

$$\therefore B = 72.505 \text{ lb} \qquad \text{or } \mathbf{B} = 72.5 \text{ lb} \uparrow \blacktriangleleft$$

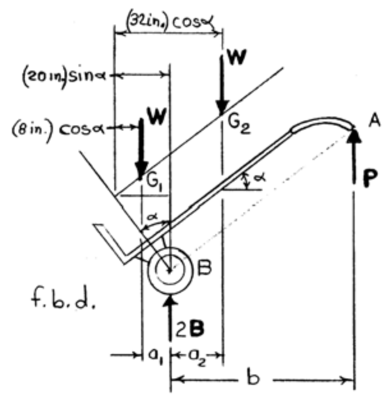
PROBLEM 4.8



Solve Problem 4.7 when $\alpha = 40^{\circ}$.

P4.7 A hand truck is used to move two barrels, each weighing 80 lb. Neglecting the weight of the hand truck, determine (*a*) the vertical force **P** which should be applied to the handle to maintain equilibrium when $\alpha = 35^{\circ}$, (*b*) the corresponding reaction at each of the two wheels.

SOLUTION



$$a_1 = (20\text{ in.})\sin\alpha - (8\text{ in.})\cos\alpha$$

$$a_2 = (32\text{ in.})\cos\alpha - (20\text{ in.})\sin\alpha$$

$$b = (64\text{ in.})\cos\alpha$$

From f.b.d. of hand truck

$$+\circlearrowleft \Sigma M_B = 0: \quad P(b) - W(a_2) + W(a_1) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad P - 2w + 2B = 0 \qquad (2)$$

For $\alpha = 40^{\circ}$

$$a_1 = 20\sin 40^{\circ} - 8\cos 40^{\circ} = 6.7274\text{ in.}$$

$$a_2 = 32\cos 40^{\circ} - 20\sin 40^{\circ} = 11.6577\text{ in.}$$

$$b = 64\cos 40^{\circ} = 49.027\text{ in.}$$

(*a*) From Equation (1)

$$P(49.027\text{ in.}) - 80\text{ lb}(11.6577\text{ in.}) + 80\text{ lb}(6.7274\text{ in.}) = 0$$

$$\therefore P = 8.0450\text{ lb}$$

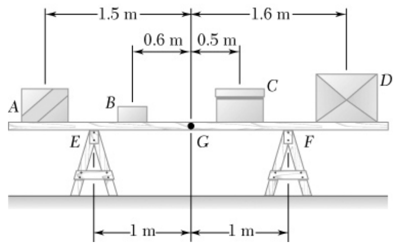
or $\mathbf{P} = 8.05\text{ lb} \uparrow$ ◀

(*b*) From Equation (2)

$$8.0450\text{ lb} - 2(80\text{ lb}) + 2B = 0$$

$$\therefore B = 75.9775\text{ lb}$$

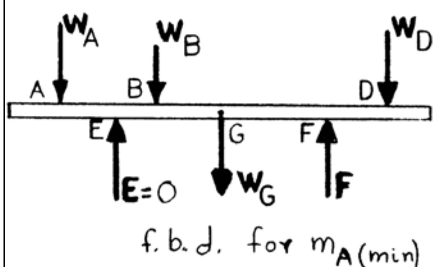
or $\mathbf{B} = 76.0\text{ lb} \uparrow$ ◀



PROBLEM 4.9

Four boxes are placed on a uniform 14-kg wooden plank which rests on two sawhorses. Knowing that the masses of boxes *B* and *D* are 4.5 kg and 45 kg, respectively, determine the range of values of the mass of box *A* so that the plank remains in equilibrium when box *C* is removed.

SOLUTION



$$W_A = m_A g \qquad W_D = m_D g = 45\, g$$

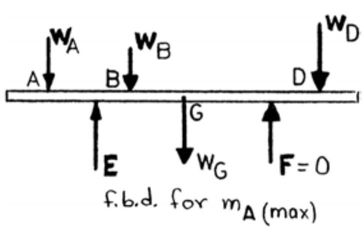
$$W_B = m_B g = 4.5g \qquad W_G = m_G g = 14g$$

For $(m_A)_{\text{min}}$, $E = 0$

$$+\circlearrowleft \Sigma M_F = 0: \quad (m_A g)(2.5\text{ m}) + (4.5g)(1.6\text{ m})$$

$$+(14g)(1\text{ m}) - (45g)(0.6\text{ m}) = 0$$

$$\therefore\ m_A = 2.32\text{ kg}$$



For $(m_A)_{\text{max}}$, $F = 0$:

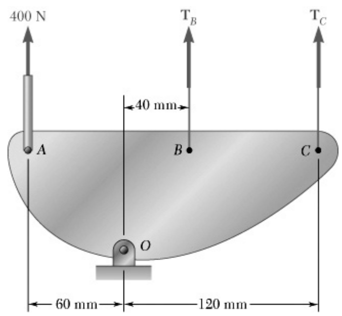
$$+\circlearrowleft \Sigma M_E = 0: \quad m_A g(0.5\text{ m}) - (4.5g)(0.4\text{ m}) - (14g)(1\text{ m})$$

$$-(45g)(2.6\text{ m}) = 0$$

$$\therefore\ m_A = 265.6\text{ kg}$$

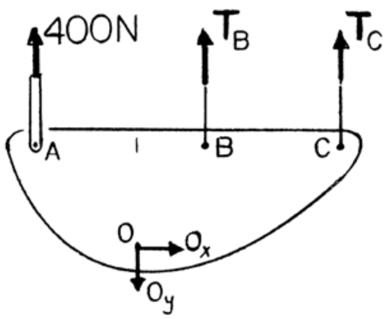
or $2.32\text{ kg} \le m_A \le 266\text{ kg}$ ◀

PROBLEM 4.10



A control rod is attached to a crank at A and cords are attached at B and C. For the given force in the rod, determine the range of values of the tension in the cord at C knowing that the cords must remain taut and that the maximum allowed tension in a cord is 180 N.

SOLUTION



f.b.d. of crank

For $\quad (T_C)_{max}, \quad T_B = 0$

$$+\circlearrowleft \Sigma M_O = 0: \quad (T_C)_{max}(0.120 \text{ m}) - (400 \text{ N})(0.060 \text{ m}) = 0$$

$$(T_C)_{max} = 200 \text{ N} > T_{max} = 180 \text{ N}$$

$$\therefore (T_C)_{max} = 180.0 \text{ N}$$

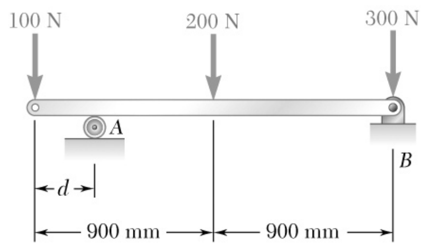
For $\quad (T_C)_{min}, \ T_B = T_{max} = 180 \text{ N}$

$$+\circlearrowleft \Sigma M_O = 0: \quad (T_C)_{min}(0.120 \text{ m}) + (180 \text{ N})(0.040 \text{ m})$$

$$-(400 \text{ N})(0.060 \text{ m}) = 0$$

$$\therefore (T_C)_{min} = 140.0 \text{ N}$$

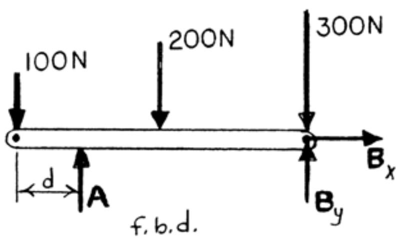
Therefore, $\quad 140.0 \text{ N} \le T_C \le 180.0 \text{ N}$ ◀



PROBLEM 4.11

The maximum allowable value of each of the reactions is 360 N. Neglecting the weight of the beam, determine the range of values of the distance d for which the beam is safe.

SOLUTION



From f.b.d. of beam

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0 \qquad \text{so that} \qquad B = B_y$$

$$+\uparrow \Sigma F_y = 0: \quad A + B - (100 + 200 + 300)\text{N} = 0$$

or

$$A + B = 600 \text{ N}$$

Therefore, if either **A** or **B** has a magnitude of the maximum of 360 N, the other support reaction will be $< 360 \text{ N}(600 \text{ N} - 360 \text{ N} = 240 \text{ N})$.

$$+\circlearrowleft \Sigma M_A = 0: \quad (100 \text{ N})(d) - (200 \text{ N})(0.9 - d) - (300 \text{ N})(1.8 - d) + B(1.8 - d) = 0$$

or

$$d = \frac{720 - 1.8B}{600 - B}$$

Since $B \le 360$ N,

$$d = \frac{720 - 1.8(360)}{600 - 360} = 0.300 \text{ m} \qquad \text{or} \qquad d \ge 300 \text{ mm}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (100 \text{ N})(1.8) - A(1.8 - d) + (200 \text{ N})(0.9) = 0$$

or

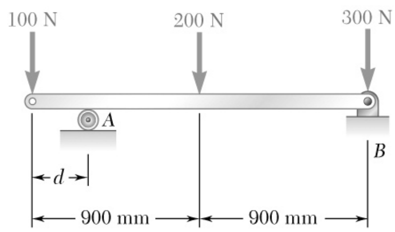
$$d = \frac{1.8A - 360}{A}$$

Since $A \le 360$ N,

$$d = \frac{1.8(360) - 360}{360} = 0.800 \text{ m} \qquad \text{or} \qquad d \le 800 \text{ mm}$$

or $300 \text{ mm} \le d \le 800 \text{ mm}$ ◀

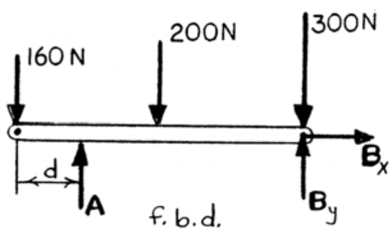
PROBLEM 4.12



Solve Problem 4.11 assuming that the 100-N load is replaced by a 160-N load.

P4.11 The maximum allowable value of each of the reactions is 360 N. Neglecting the weight of the beam, determine the range of values of the distance d for which the beam is safe.

SOLUTION



From f.b.d of beam

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0 \qquad \text{so that} \qquad B = B_y$$

$$+\uparrow \Sigma F_y = 0: \quad A + B - (160 + 200 + 300)\text{N} = 0$$

or

$$A + B = 660 \text{ N}$$

Therefore, if either **A** or **B** has a magnitude of the maximum of 360 N, the other support reaction will be $< 360 \text{ N}(660 - 360 = 300 \text{ N})$.

$$+\circlearrowleft \Sigma M_A = 0: \quad 160 \text{ N}(d) - 200 \text{ N}(0.9 - d) - 300 \text{ N}(1.8 - d) + B(1.8 - d) = 0$$

or

$$d = \frac{720 - 1.8B}{660 - B}$$

Since $B \leq 360$ N,

$$d = \frac{720 - 1.8(360)}{660 - 360} = 0.240 \text{ m} \qquad \text{or} \qquad d \geq 240 \text{ mm}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 160 \text{ N}(1.8) - A(1.8 - d) + 200 \text{ N}(0.9) = 0$$

or

$$d = \frac{1.8A - 468}{A}$$

Since $A \leq 360$ N,

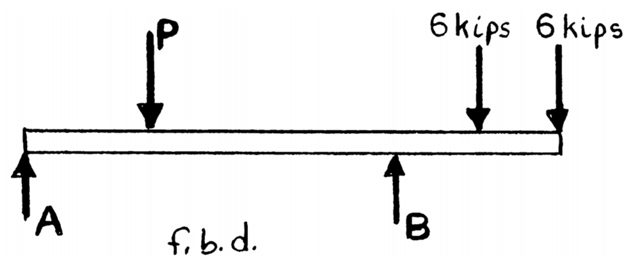
$$d = \frac{1.8(360) - 468}{360} = 0.500 \text{ m} \qquad \text{or} \qquad d \geq 500 \text{ mm}$$

or $240 \text{ mm} \leq d \leq 500 \text{ mm}$ ◀

PROBLEM 4.13

For the beam of Sample Problem 4.2, determine the range of values of P for which the beam will be safe knowing that the maximum allowable value of each of the reactions is 45 kips and that the reaction at A must be directed upward.

SOLUTION



For the force of **P** to be a minimum, **A** = 0.

With $A = 0$,

$$+\circlearrowleft \Sigma M_B = 0: \quad P_{\min}(6 \text{ ft}) - (6 \text{ kips})(2 \text{ ft}) - (6 \text{ kips})(4 \text{ ft}) = 0$$

$$\therefore \; P_{\min} = 6.00 \text{ kips}$$

For the force **P** to be a maximum, $\mathbf{A} = \mathbf{A}_{\max} = 45 \text{ kips} \uparrow$

With $A = 45$ kips,

$$+\circlearrowleft \Sigma M_B = 0: \quad -(45 \text{ kips})(9 \text{ ft}) + P_{\max}(6 \text{ ft}) - (6 \text{ kips})(2 \text{ ft}) - (6 \text{ kips})(4 \text{ ft}) = 0$$

$$\therefore \; P_{\max} = 73.5 \text{ kips}$$

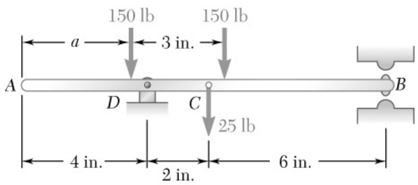
A check must be made to verify the assumption that the maximum value of **P** is based on the reaction force at A. This is done by making sure the corresponding value of B is < 45 kips.

$$+\uparrow \Sigma F_y = 0: \quad 45 \text{ kips} - 73.5 \text{ kips} + B - 6 \text{ kips} - 6 \text{ kips} = 0$$

$$\therefore \; B = 40.5 \text{ kips} < 45 \text{ kips} \qquad \therefore \text{ ok} \qquad \text{or } P_{\max} = 73.5 \text{ kips}$$

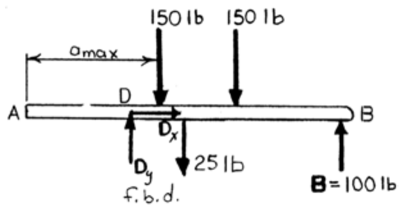
and $6.00 \text{ kips} \le P \le 73.5 \text{ kips}$ ◀

PROBLEM 4.14



For the beam and loading shown, determine the range of values of the distance a for which the reaction at B does not exceed 50 lb downward or 100 lb upward.

SOLUTION



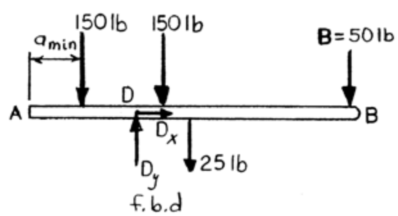
To determine a_{max} the two 150-lb forces need to be as close to B without having the vertical upward force at B exceed 100 lb.

From f.b.d. of beam with $\mathbf{B} = 100 \text{ lb} \uparrow$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(150 \text{ lb})(a_{max} - 4 \text{ in.}) - (150 \text{ lb})(a_{max} - 1 \text{ in.})$$

$$-(25 \text{ lb})(2 \text{ in.}) + (100 \text{ lb})(8 \text{ in.}) = 0$$

or $$a_{max} = 5.00 \text{ in.}$$



To determine a_{min} the two 150-lb forces need to be as close to A without having the vertical downward force at B exceed 50 lb.

From f.b.d. of beam with $\mathbf{B} = 50 \text{ lb} \downarrow$

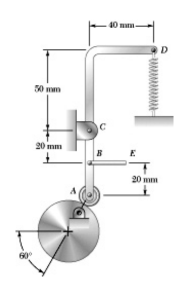
$$+\circlearrowleft \Sigma M_D = 0: \quad (150 \text{ lb})(4 \text{ in.} - a_{min}) - (150 \text{ lb})(a_{min} - 1 \text{ in.})$$

$$-(25 \text{ lb})(2 \text{ in.}) - (50 \text{ lb})(8 \text{ in.}) = 0$$

or $$a_{min} = 1.00 \text{ in.}$$

Therefore, or $1.00 \text{ in.} \le a \le 5.00 \text{ in.}$ ◀

PROBLEM 4.15



A follower *ABCD* is held against a circular cam by a stretched spring, which exerts a force of 21 N for the position shown. Knowing that the tension in rod *BE* is 14 N, determine (*a*) the force exerted on the roller at *A*, (*b*) the reaction at bearing *C*.

SOLUTION

Note: From f.b.d. of *ABCD*

$$A_x = A\cos 60° = \frac{A}{2}$$

$$A_y = A\sin 60° = A\frac{\sqrt{3}}{2}$$

(*a*) From f.b.d. of *ABCD*

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{A}{2}\right)(40\text{ mm}) - 21\text{ N}(40\text{ mm})$$

$$+ 14\text{ N}(20\text{ mm}) = 0$$

$$\therefore\ A = 28\text{ N}$$

or $\mathbf{A} = 28.0\text{ N} \measuredangle 60°$ ◀

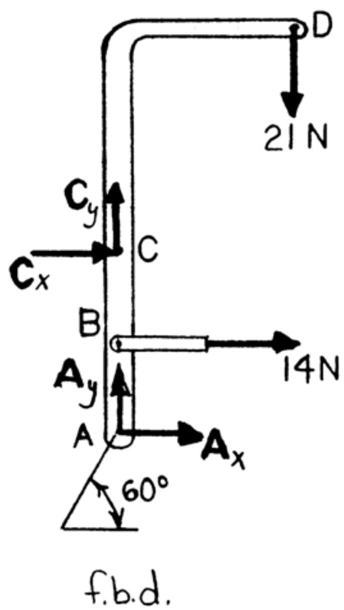
(*b*) From f.b.d. of *ABCD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 14\text{ N} + (28\text{ N})\cos 60° = 0$$

$$\therefore\ C_x = -28\text{ N} \qquad \text{or} \qquad \mathbf{C}_x = 28.0\text{ N} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 21\text{ N} + (28\text{ N})\sin 60° = 0$$

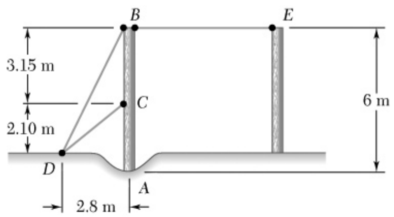
$$\therefore\ C_y = -3.2487\text{ N} \qquad \text{or} \qquad \mathbf{C}_y = 3.25\text{ N} \downarrow$$



Then $\qquad C = \sqrt{C_x^2 + C_y^2} = \sqrt{(28)^2 + (3.2487)^2} = 28.188\text{ N}$

and $\qquad \theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-3.2487}{-28}\right) = 6.6182°$

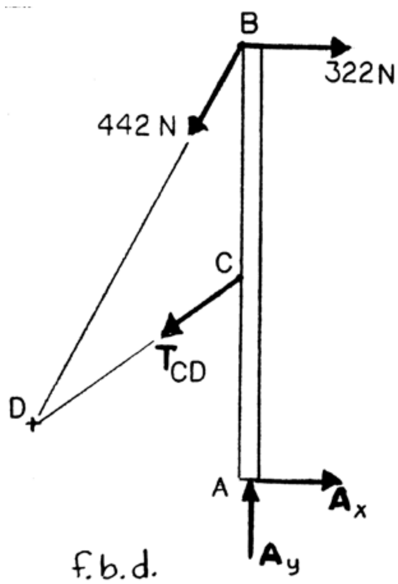
or $\mathbf{C} = 28.2\text{ N} \nearrow 6.62°$ ◀



PROBLEM 4.16

A 6-m-long pole *AB* is placed in a hole and is guyed by three cables. Knowing that the tensions in cables *BD* and *BE* are 442 N and 322 N, respectively, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *A*.

SOLUTION



Note:

$$\overline{DB} = \sqrt{(2.8)^2 + (5.25)^2} = 5.95 \text{ m}$$

$$\overline{DC} = \sqrt{(2.8)^2 + (2.10)^2} = 3.50 \text{ m}$$

(*a*) From f.b.d. of pole

$$+\circlearrowleft \Sigma M_A = 0: \quad -(322 \text{ N})(6 \text{ m}) + \left[\left(\frac{2.8 \text{ m}}{5.95 \text{ m}}\right)(442 \text{ N})\right](6 \text{ m})$$

$$+\left[\left(\frac{2.8 \text{ m}}{3.50 \text{ m}}\right)T_{CD}\right](2.85 \text{ m}) = 0$$

$$\therefore\ T_{CD} = 300 \text{ N}$$

or $T_{CD} = 300 \text{ N}$ ◀

(*b*) From f.b.d. of pole

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad 322 \text{ N} - \left(\frac{2.8 \text{ m}}{5.95 \text{ m}}\right)(442 \text{ N})$$

$$-\left(\frac{2.8 \text{ m}}{3.50 \text{ m}}\right)(300 \text{ N}) + A_x = 0$$

$$\therefore\ A_x = 126 \text{ N} \qquad \text{or} \qquad \mathbf{A}_x = 126 \text{ N} \longrightarrow$$

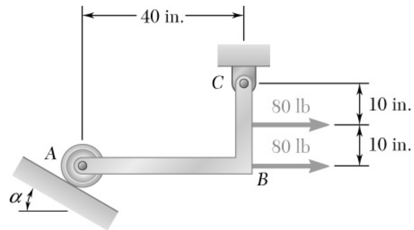
$$+\uparrow \Sigma F_y = 0: \quad A_y - \left(\frac{5.25 \text{ m}}{5.95 \text{ m}}\right)(442 \text{ N}) - \left(\frac{2.10 \text{ m}}{3.50 \text{ m}}\right)(300 \text{ N}) = 0$$

$$\therefore\ A_y = 570 \text{ N} \qquad \text{or} \qquad \mathbf{A}_y = 570 \text{ N} \uparrow$$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(126)^2 + (570)^2} = 583.76 \text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{570 \text{ N}}{126 \text{ N}}\right) = 77.535°$$

or $\mathbf{A} = 584 \text{ N} \angle 77.5°$ ◀

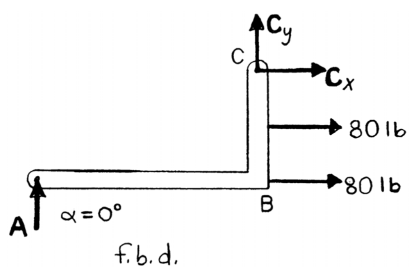


PROBLEM 4.17

Determine the reactions at A and C when (*a*) $\alpha = 0$, (*b*) $\alpha = 30^\circ$.

SOLUTION

(*a*)



(*a*) $\alpha = 0^\circ$

From f.b.d. of member ABC

$$+\circlearrowleft \Sigma M_C = 0: \quad (80 \text{ lb})(10 \text{ in.}) + (80 \text{ lb})(20 \text{ in.}) - A(40 \text{ in.}) = 0$$

$$\therefore \; A = 60 \text{ lb}$$

or $\mathbf{A} = 60.0 \text{ lb} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad C_y + 60 \text{ lb} = 0$$

$$\therefore \; C_y = -60 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = 60 \text{ lb} \downarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 80 \text{ lb} + 80 \text{ lb} + C_x = 0$$

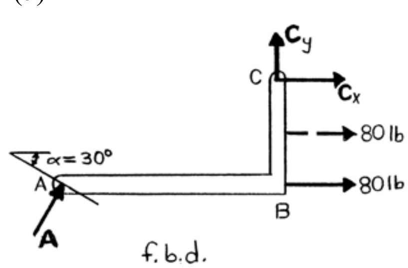
$$\therefore \; C_x = -160 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_x = 160 \text{ lb} \longleftarrow$$

Then $\quad C = \sqrt{C_x^2 + C_y^2} = \sqrt{(160)^2 + (60)^2} = 170.880 \text{ lb}$

and $\quad \theta = \tan^{-1}\left(\dfrac{C_y}{C_x}\right) = \tan^{-1}\left(\dfrac{-60}{-160}\right) = 20.556^\circ$

or $\mathbf{C} = 170.9 \text{ lb} \measuredangle 20.6^\circ$ ◀

(*b*)



(*b*) $\alpha = 30^\circ$

From f.b.d. of member ABC

$$+\circlearrowleft \Sigma M_C = 0: \quad (80 \text{ lb})(10 \text{ in.}) + (80 \text{ lb})(20 \text{ in.}) - (A\cos 30^\circ)(40 \text{ in.})$$

$$+ (A\sin 30^\circ)(20 \text{ in.}) = 0$$

$$\therefore \; A = 97.399 \text{ lb}$$

or $\mathbf{A} = 97.4 \text{ lb} \measuredangle 60^\circ$ ◀

PROBLEM 4.17 CONTINUED

$$\xrightarrow{+} \Sigma F_x = 0: \quad 80 \text{ lb} + 80 \text{ lb} + (97.399 \text{ lb})\sin 30° + C_x = 0$$

$$\therefore\ C_x = -208.70 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_x = 209 \text{ lb} \leftarrow$$

$$+\!\uparrow \Sigma F_y = 0: \quad C_y + (97.399 \text{ lb})\cos 30° = 0$$

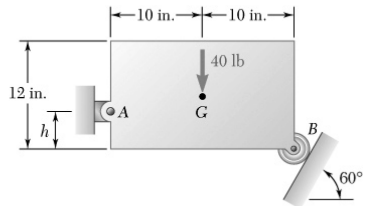
$$\therefore\ C_y = -84.350 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = 84.4 \text{ lb} \downarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(208.70)^2 + (84.350)^2} = 225.10 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-84.350}{-208.70}\right) = 22.007°$$

or $\mathbf{C} = 225 \text{ lb} \measuredangle 22.0°$ ◀

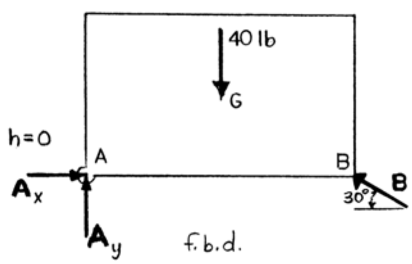
PROBLEM 4.18



Determine the reactions at A and B when (*a*) $h = 0$, (*b*) $h = 8$ in.

SOLUTION

(*a*)



(*a*) $h = 0$

From f.b.d. of plate

$$+\circlearrowleft \Sigma M_A = 0: \quad (B\sin 30°)(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

$$\therefore B = 40 \text{ lb}$$

or $\mathbf{B} = 40.0$ lb ⦪ 30° ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (40 \text{ lb})\cos 30° = 0$$

$\therefore A_x = 34.641$ lb or $\mathbf{A}_x = 34.6$ lb $\longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 40 \text{ lb} + (40 \text{ lb})\sin 30° = 0$$

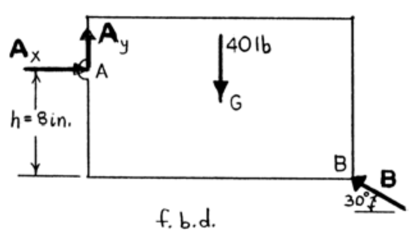
$\therefore A_y = 20$ lb or $\mathbf{A}_y = 20.0$ lb $\uparrow$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(34.641)^2 + (20)^2} = 39.999 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{20}{34.641}\right) = 30.001°$$

or $\mathbf{A} = 40.0$ lb ⦨ 30° ◀

(*b*)



(*b*) $h = 8$ in.

From f.b.d. of plate

$$+\circlearrowleft \Sigma M_A = 0: \quad (B\sin 30°)(20 \text{ in.}) - (B\cos 30°)(8 \text{ in.})$$

$$-(40 \text{ lb})(10 \text{ in.}) = 0$$

$$\therefore B = 130.217 \text{ lb}$$

or $\mathbf{B} = 130.2$ lb ⦪ 30.0° ◀

PROBLEM 4.18 CONTINUED

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (130.217 \text{ lb})\cos 30° = 0$$

$$\therefore \; A_x = 112.771 \text{ lb} \qquad \text{or} \qquad \mathbf{A}_x = 112.8 \text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 40 \text{ lb} + (130.217 \text{ lb})\sin 30° = 0$$

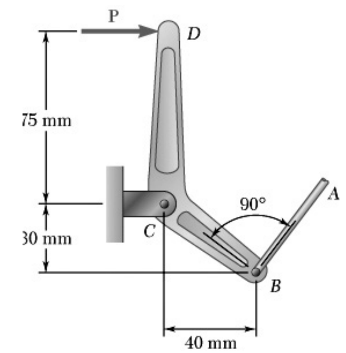
$$\therefore \; A_y = -25.108 \text{ lb} \qquad \text{or} \qquad \mathbf{A}_y = 25.1 \text{ lb} \downarrow$$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(112.771)^2 + (25.108)^2} = 115.532 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-25.108}{112.771}\right) = -12.5519°$$

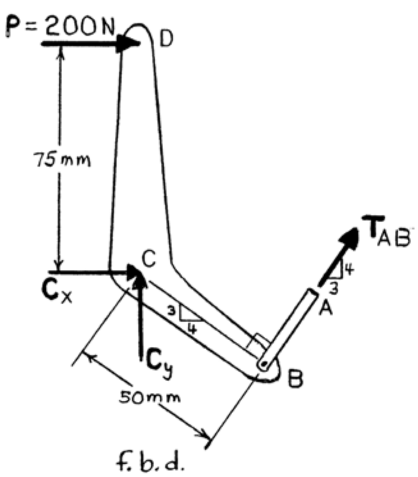
or $\mathbf{A} = 115.5$ lb ⦩ 12.55° ◀

PROBLEM 4.19



The lever *BCD* is hinged at *C* and is attached to a control rod at *B*. If $P = 200$ N, determine (*a*) the tension in rod *AB*, (*b*) the reaction at *C*.

SOLUTION



(*a*) From f.b.d. of lever *BCD*

$$+\circlearrowleft \Sigma M_C = 0: \quad T_{AB}(50 \text{ mm}) - 200 \text{ N}(75 \text{ mm}) = 0$$

$$\therefore T_{AB} = 300 \text{ N} \blacktriangleleft$$

(*b*) From f.b.d. of lever *BCD*

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad 200 \text{ N} + C_x + 0.6(300 \text{ N}) = 0$$

$$\therefore C_x = -380 \text{ N} \quad \text{or} \quad \mathbf{C}_x = 380 \text{ N} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 0.8(300 \text{ N}) = 0$$

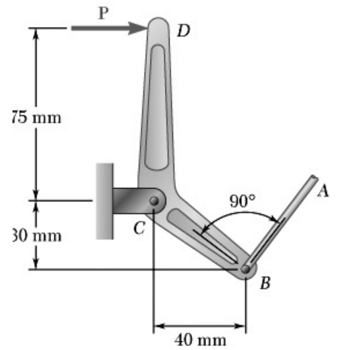
$$\therefore C_y = -240 \text{ N} \quad \text{or} \quad \mathbf{C}_y = 240 \text{ N} \downarrow$$

Then $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(380)^2 + (240)^2} = 449.44 \text{ N}$

and $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-240}{-380}\right) = 32.276°$

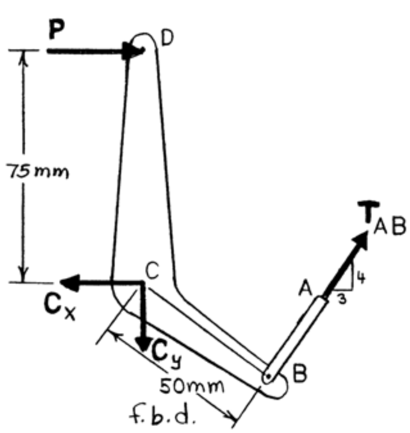
or $\mathbf{C} = 449 \text{ N} \measuredangle 32.3° \blacktriangleleft$

PROBLEM 4.20



The lever *BCD* is hinged at *C* and is attached to a control rod at *B*. Determine the maximum force **P** which can be safely applied at *D* if the maximum allowable value of the reaction at *C* is 500 N.

SOLUTION



From f.b.d. of lever *BCD*

$$+\circlearrowleft \Sigma M_C = 0: \quad T_{AB}(50 \text{ mm}) - P(75 \text{ mm}) = 0$$

$$\therefore T_{AB} = 1.5P \qquad (1)$$

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad 0.6T_{AB} + P - C_x = 0$$

$$\therefore C_x = P + 0.6T_{AB} \qquad (2)$$

From Equation (1) $\quad C_x = P + 0.6(1.5P) = 1.9P$

$$+\uparrow \Sigma F_y = 0: \quad 0.8T_{AB} - C_y = 0$$

$$\therefore C_y = 0.8T_{AB} \qquad (3)$$

From Equation (1) $\quad C_y = 0.8(1.5P) = 1.2P$

From Equations (2) and (3)

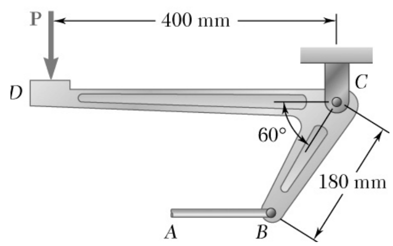
$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(1.9P)^2 + (1.2P)^2} = 2.2472P$$

Since $C_{max} = 500$ N,

$$\therefore 500 \text{ N} = 2.2472P_{max}$$

or $\quad P_{max} = 222.49$ lb

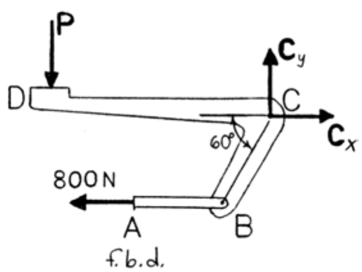
or **P** = 222 lb ⟶ ◀



PROBLEM 4.21

The required tension in cable AB is 800 N. Determine (a) the vertical force $\mathbf{P}$ which must be applied to the pedal, (b) the corresponding reaction at C.

SOLUTION



(a) From f.b.d. of pedal

$$+\circlearrowright \Sigma M_C = 0: \quad P(0.4\text{ m}) - (800\text{ N})\left[(0.18\text{ m})\sin 60^\circ\right] = 0$$

$$\therefore \quad P = 311.77\text{ N}$$

or $\mathbf{P} = 312\text{ N} \downarrow$ ◀

(b) From f.b.d. of pedal

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 800\text{ N} = 0$$

$$\therefore \quad C_x = 800\text{ N}$$

or $\mathbf{C}_x = 800\text{ N} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 311.77\text{ N} = 0$$

$$\therefore \quad C_y = 311.77\text{ N}$$

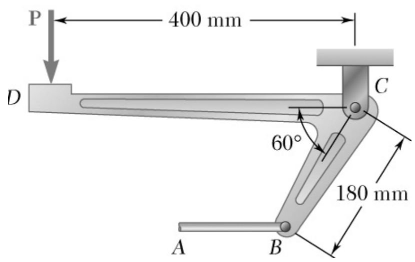
or $\mathbf{C}_y = 311.77\text{ N} \uparrow$

Then $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(800)^2 + (311.77)^2} = 858.60\text{ N}$

and $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{311.77}{800}\right) = 21.291^\circ$

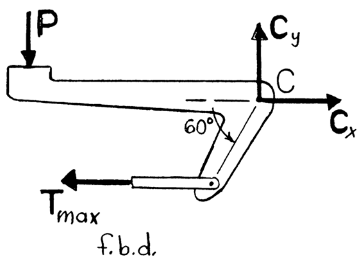
or $\mathbf{C} = 859\text{ N} \measuredangle 21.3^\circ$ ◀

PROBLEM 4.22



Determine the maximum tension which can be developed in cable AB if the maximum allowable value of the reaction at C is 1000 N.

SOLUTION



Have $$C_{max} = 1000 \text{ N}$$

Now $$C^2 = C_x^2 + C_y^2$$

$$\therefore \quad C_y = \sqrt{(1000)^2 - C_x^2} \qquad (1)$$

From f.b.d. of pedal

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T_{max} = 0$$

$$\therefore \quad C_x = T_{max} \qquad (2)$$

$$+\circlearrowright \Sigma M_D = 0: \quad C_y(0.4 \text{ m}) - T_{max}\left[(0.18 \text{ m})\sin 60°\right] = 0$$

$$\therefore \quad C_y = 0.38971 T_{max} \qquad (3)$$

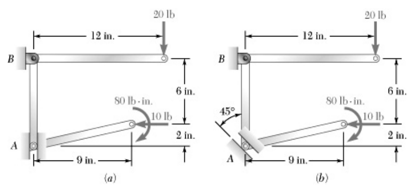
Equating the expressions for C_y in Equations (1) and (3), with $C_x = T_{max}$ from Equation (2)

$$\sqrt{(1000)^2 - T_{max}^2} = 0.389711 T_{max}$$

$$\therefore \quad T_{max}^2 = 868{,}150$$

and $$T_{max} = 931.75 \text{ N}$$

or $T_{max} = 932 \text{ N}$ ◀

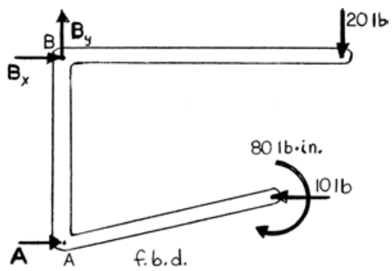


PROBLEM 4.23

A steel rod is bent to form a mounting bracket. For each of the mounting brackets and loadings shown, determine the reactions at A and B.

SOLUTION

(*a*)



(*a*) From f.b.d. of mounting bracket

$$+\circlearrowleft \Sigma M_E = 0: \quad A(8 \text{ in.}) - 80 \text{ lb·in.} - (10 \text{ lb})(6 \text{ in.})$$

$$-(20 \text{ lb})(12 \text{ in.}) = 0$$

$$\therefore \quad A = 47.5 \text{ lb}$$

or $\mathbf{A} = 47.5$ lb $\longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x - 10 \text{ lb} + 47.5 \text{ lb} = 0$$

$$\therefore \quad B_x = -37.5 \text{ lb}$$

or $\mathbf{B}_x = 37.5$ lb $\longleftarrow$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 20 \text{ lb} = 0$$

$$\therefore \quad B_y = 20 \text{ lb}$$

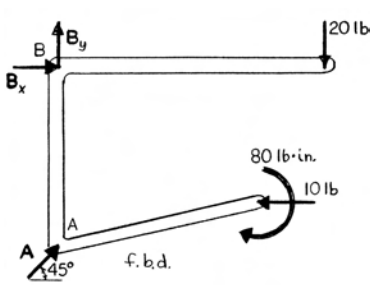
or $\mathbf{B}_y = 20.0$ lb $\uparrow$

Then $B = \sqrt{B_x^2 + B_y^2} = \sqrt{(37.5)^2 + (20.0)^2} = 42.5 \text{ lb}$

and $\theta = \tan^{-1}\left(\dfrac{B_y}{B_x}\right) = \tan^{-1}\left(\dfrac{20}{-37.5}\right) = -28.072°$

or $\mathbf{B} = 42.5$ lb ⦪ 28.1° ◀

(*b*)



(*b*) From f.b.d. of mounting bracket

$$+\circlearrowleft \Sigma M_B = 0: \quad (A\cos 45°)(8 \text{ in.}) - 80 \text{ lb·in.}$$

$$-(10 \text{ lb})(6 \text{ in.}) - (20 \text{ lb})(12 \text{ in.}) = 0$$

$$\therefore \quad A = 67.175 \text{ lb}$$

or $\mathbf{A} = 67.2$ lb ⦣ 45° ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x - 10 \text{ lb} + 67.175\cos 45° = 0$$

$$\therefore \quad B_x = -37.500 \text{ lb}$$

or $\mathbf{B}_x = 37.5$ lb $\longleftarrow$

PROBLEM 4.23 CONTINUED

$$+\uparrow \Sigma F_y = 0: \quad B_y - 20 \text{ lb} + 67.175 \sin 45° = 0$$

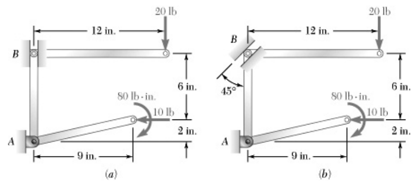
$$\therefore \quad B_y = -27.500 \text{ lb}$$

or $$\mathbf{B}_y = 27.5 \text{ lb} \downarrow$$

Then $$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(37.5)^2 + (27.5)^2} = 46.503 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{-27.5}{-37.5}\right) = 36.254°$$

or $\mathbf{B} = 46.5$ lb ↙ 36.3° ◀

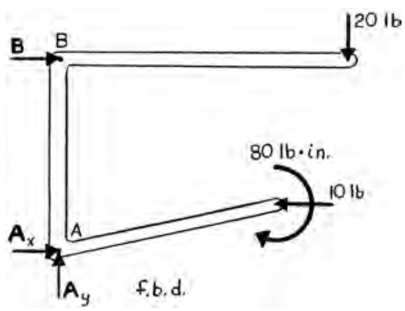


PROBLEM 4.24

A steel rod is bent to form a mounting bracket. For each of the mounting brackets and loadings shown, determine the reactions at A and B.

SOLUTION

(*a*)



(*a*) From f.b.d. of mounting bracket

$$+\circlearrowleft \Sigma M_A = 0: \quad -B(8\text{ in.}) - (20\text{ lb})(12\text{ in.}) + (10\text{ lb})(2\text{ in.}) - 80\text{ lb}\cdot\text{in.} = 0$$

$$\therefore \quad B = -37.5\text{ lb}$$

or $\mathbf{B} = 37.5\text{ lb} \longleftarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad -37.5\text{ lb} - 10\text{ lb} + A_x = 0$$

$$\therefore \quad A_x = 47.5\text{ lb}$$

or $\mathbf{A}_x = 47.5\text{ lb} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad -20\text{ lb} + A_y = 0$$

$$\therefore \quad A_y = 20\text{ lb}$$

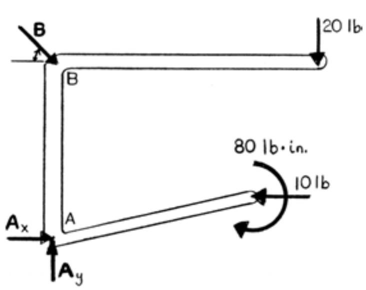
or $\mathbf{A}_y = 20.0\text{ lb} \uparrow$

Then $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(47.5)^2 + (20)^2} = 51.539\text{ lb}$

and $\theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right) = \tan^{-1}\left(\dfrac{20}{47.5}\right) = 22.834°$

or $\mathbf{A} = 51.5\text{ lb} \measuredangle 22.8°$ ◀

(*b*)



(*b*) From f.b.d. of mounting bracket

$$+\circlearrowleft \Sigma M_A = 0: \quad -(B\cos 45°)(8\text{ in.}) - (20\text{ lb})(2\text{ in.}) - 80\text{ lb}\cdot\text{in.} + (10\text{ lb})(2\text{ in.}) = 0$$

$$\therefore \quad B = -53.033\text{ lb}$$

or $\mathbf{B} = 53.0\text{ lb} \searrow 45°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + (-53.033\text{ lb})\cos 45° - 10 = 0$$

$$\therefore \quad A_x = 47.500\text{ lb}$$

or $\mathbf{A}_x = 47.5\text{ lb} \longrightarrow$

PROBLEM 4.24 CONTINUED

$$+\uparrow \Sigma F_y = 0: \quad A_y - (53.033 \text{ lb})\sin 45° - 20 = 0$$

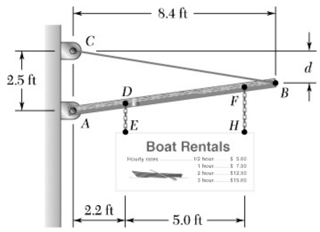
$$\therefore \quad A_y = -17.500 \text{ lb}$$

or $$\mathbf{A}_y = 17.50 \text{ lb} \downarrow$$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(47.5)^2 + (17.5)^2} = 50.621 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-17.5}{47.5}\right) = -20.225°$$

or $\mathbf{A} = 50.6 \text{ lb} \searrow 20.2°$ ◀

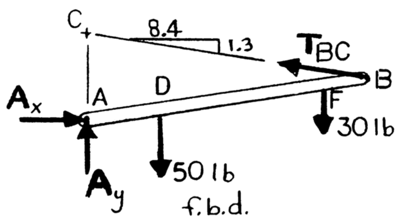


PROBLEM 4.25

A sign is hung by two chains from mast AB. The mast is hinged at A and is supported by cable BC. Knowing that the tensions in chains DE and FH are 50 lb and 30 lb, respectively, and that $d = 1.3$ ft, determine (*a*) the tension in cable BC, (*b*) the reaction at A.

SOLUTION

First note $$\overline{BC} = \sqrt{(8.4)^2 + (1.3)^2} = 8.5 \text{ ft}$$



(*a*) From f.b.d. of mast AB

$$+\circlearrowleft\ \Sigma M_A = 0: \quad \left[\left(\frac{8.4}{8.5}\right)T_{BC}\right](2.5 \text{ ft}) - (30 \text{ lb})(7.2 \text{ ft})$$

$$-50 \text{ lb}(2.2 \text{ ft}) = 0$$

$$\therefore \quad T_{BC} = 131.952 \text{ lb}$$

or $T_{BC} = 132.0$ lb ◀

(*b*) From f.b.d. of mast AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - \left(\frac{8.4}{8.5}\right)(131.952 \text{ lb}) = 0$$

$$\therefore \quad A_x = 130.400 \text{ lb}$$

or $$\mathbf{A}_x = 130.4 \text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + \left(\frac{1.3}{8.5}\right)(131.952 \text{ lb}) - 30 \text{ lb} - 50 \text{ lb} = 0$$

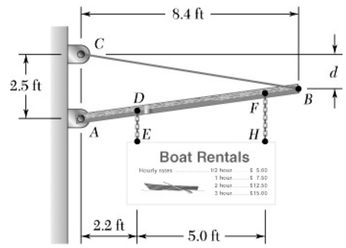
$$\therefore \quad A_y = 59.819 \text{ lb}$$

or $$\mathbf{A}_y = 59.819 \text{ lb} \uparrow$$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(130.4)^2 + (59.819)^2} = 143.466 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{59.819}{130.4}\right) = 24.643°$$

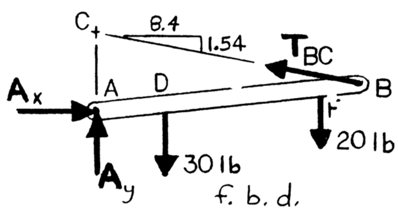
or $\mathbf{A} = 143.5$ lb $\measuredangle$ 24.6° ◀



PROBLEM 4.26

A sign is hung by two chains from mast AB. The mast is hinged at A and is supported by cable BC. Knowing that the tensions in chains DE and FH are 30 lb and 20 lb, respectively, and that $d = 1.54$ ft, determine (*a*) the tension in cable BC, (*b*) the reaction at A.

SOLUTION



First note $\qquad \overline{BC} = \sqrt{(8.4)^2 + (1.54)^2} = 8.54 \text{ ft}$

(*a*) From f.b.d. of mast AB

$$+\circlearrowleft \Sigma M_A = 0: \quad \left[\left(\frac{8.4}{8.54}\right)T_{BC}\right](2.5 \text{ ft}) - 20 \text{ lb}(7.2 \text{ ft})$$

$$- 30 \text{ lb}(2.2 \text{ ft}) = 0$$

$$\therefore \quad T_{BC} = 85.401 \text{ lb}$$

or $T_{BC} = 85.4$ lb ◀

(*b*) From f.b.d. of mast AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - \left(\frac{8.4}{8.54}\right)(85.401 \text{ lb}) = 0$$

$$\therefore \quad A_x = 84.001 \text{ lb}$$

or $\qquad \mathbf{A}_x = 84.001 \text{ lb} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y + \left(\frac{1.54}{8.54}\right)(85.401 \text{ lb}) - 20 \text{ lb} - 30 \text{ lb} = 0$$

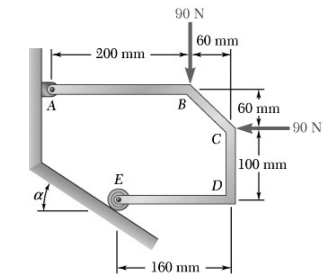
$$\therefore \quad A_y = 34.600 \text{ lb}$$

or $\qquad \mathbf{A}_y = 34.600 \text{ lb} \uparrow$

Then $\quad A = \sqrt{A_x^2 + A_y^2} = \sqrt{(84.001)^2 + (34.600)^2} = 90.848 \text{ lb}$

and $\qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{34.6}{84.001}\right) = 22.387°$

or $\mathbf{A} = 90.8$ lb $\angle$ 22.4° ◀

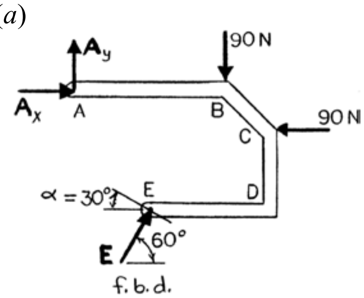


PROBLEM 4.27

For the frame and loading shown, determine the reactions at A and E when (a) $\alpha = 30°$, (b) $\alpha = 45°$.

SOLUTION

(a)



(a) Given $\alpha = 30°$

From f.b.d. of frame

$$+\circlearrowleft \Sigma M_A = 0: \quad -(90\text{ N})(0.2\text{ m}) - (90\text{ N})(0.06\text{ m})$$

$$+(E\cos 60°)(0.160\text{ m}) + (E\sin 60°)(0.100\text{ m}) = 0$$

$$\therefore \quad E = 140.454\text{ N}$$

or $\mathbf{E} = 140.5\text{ N}$ ∠ $60°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 90\text{ N} + (140.454\text{ N})\cos 60° = 0$$

$$\therefore \quad A_x = 19.7730\text{ N}$$

or $\mathbf{A}_x = 19.7730\text{ N} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 90\text{ N} + (140.454\text{ N})\sin 60° = 0$$

$$\therefore \quad A_y = -31.637\text{ N}$$

or $\mathbf{A}_y = 31.6\text{ N} \downarrow$

Then $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(19.7730)^2 + (31.637)^2}$

$= 37.308\text{ lb}$

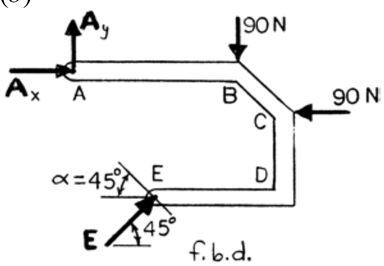
and $\theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right) = \tan^{-1}\left(\dfrac{-31.637}{19.7730}\right)$

$= -57.995°$

or $\mathbf{A} = 37.3\text{ N}$ ⦪ $58.0°$ ◀

PROBLEM 4.27 CONTINUED

(*b*)



(*b*) Given $\alpha = 45°$

From f.b.d. of frame

$$+\circlearrowright \Sigma M_A = 0: \quad -(90\text{ N})(0.2\text{ m}) - (90\text{ N})(0.06\text{ m})$$

$$+(E\cos 45°)(0.160\text{ m}) + (E\sin 45°)(0.100\text{ m}) = 0$$

$$\therefore E = 127.279\text{ N}$$

or $\mathbf{E} = 127.3\text{ N} \measuredangle 45°$ ◀

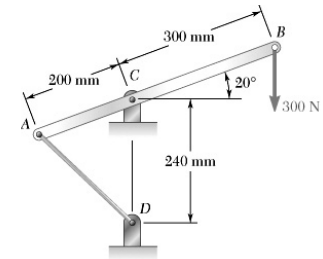
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 90 + (127.279\text{ N})\cos 45° = 0$$

$$\therefore \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 90 + (127.279\text{ N})\sin 45° = 0$$

$$\therefore \quad A_y = 0$$

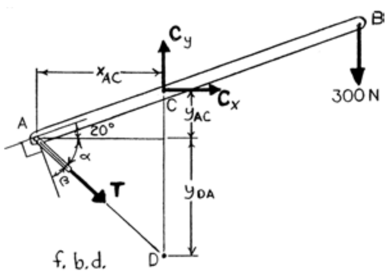
or $\mathbf{A} = 0$ ◀



PROBLEM 4.28

A lever AB is hinged at C and is attached to a control cable at A. If the lever is subjected to a 300-N vertical force at B, determine (a) the tension in the cable, (b) the reaction at C.

SOLUTION



First

$$x_{AC} = (0.200 \text{ m})\cos 20° = 0.187\,939 \text{ m}$$

$$y_{AC} = (0.200 \text{ m})\sin 20° = 0.068\,404 \text{ m}$$

Then

$$y_{DA} = 0.240 \text{ m} - y_{AC}$$

$$= 0.240 \text{ m} - 0.068404 \text{ m}$$

$$= 0.171596 \text{ m}$$

and

$$\tan\alpha = \frac{y_{DA}}{x_{AC}} = \frac{0.171\,596}{0.187\,939}$$

$$\therefore \quad \alpha = 42.397°$$

and

$$\beta = 90° - 20° - 42.397° = 27.603°$$

(a) From f.b.d. of lever AB

$$+\circlearrowleft \Sigma M_C = 0: \quad T\cos 27.603°(0.2 \text{ m})$$

$$- 300 \text{ N}\left[(0.3 \text{ m})\cos 20°\right] = 0$$

$$\therefore \quad T = 477.17 \text{ N}$$

or $T = 477$ N ◀

(b) From f.b.d. of lever AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (477.17 \text{ N})\cos 42.397° = 0$$

$$\therefore \quad C_x = -352.39 \text{ N}$$

or

$$\mathbf{C}_x = 352.39 \text{ N} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 300 \text{ N} - (477.17 \text{ N})\sin 42.397° = 0$$

$$\therefore \quad C_y = 621.74 \text{ N}$$

or

$$\mathbf{C}_y = 621.74 \text{ N} \uparrow$$

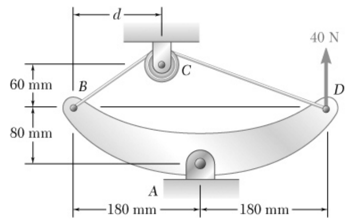
PROBLEM 4.28 CONTINUED

Then $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(352.39)^2 + (621.74)^2} = 714.66 \text{ N}$

and $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{621.74}{-352.39}\right) = -60.456°$

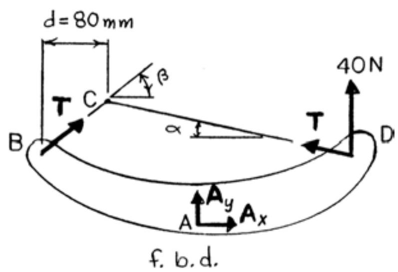
or $\mathbf{C} = 715 \text{ N} \measuredangle 60.5°$ ◀

PROBLEM 4.29



Neglecting friction and the radius of the pulley, determine the tension in cable *BCD* and the reaction at support *A* when $d = 80$ mm.

SOLUTION



First

$$\alpha = \tan^{-1}\left(\frac{60}{280}\right) = 12.0948°$$

$$\beta = \tan^{-1}\left(\frac{60}{80}\right) = 36.870°$$

From f.b.d. of object *BAD*

$$+\circlearrowleft \Sigma M_A = 0: \quad (40 \text{ N})(0.18 \text{ m}) + (T\cos\alpha)(0.08 \text{ m})$$

$$+(T\sin\alpha)(0.18 \text{ m}) - (T\cos\beta)(0.08 \text{ m})$$

$$-(T\sin\beta)(0.18 \text{ m}) = 0$$

$$\therefore \quad T = \left(\frac{7.2 \text{ N}\cdot\text{m}}{0.056061}\right) = 128.433 \text{ N}$$

or $T = 128.4$ N ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad (128.433 \text{ N})(\cos\beta - \cos\alpha) + A_x = 0$$

$$\therefore \quad A_x = 22.836 \text{ N}$$

or $\mathbf{A}_x = 22.836 \text{ N} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y + (128.433 \text{ N})(\sin\beta + \sin\alpha) + 40 \text{ N} = 0$$

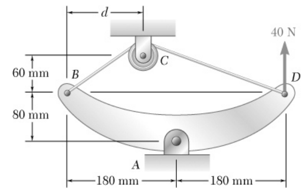
$$\therefore \quad A_y = -143.970 \text{ N}$$

or $\mathbf{A}_y = 143.970 \text{ N} \downarrow$

Then $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(22.836)^2 + (143.970)^2} = 145.770 \text{ N}$

and $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-143.970}{22.836}\right) = -80.987°$

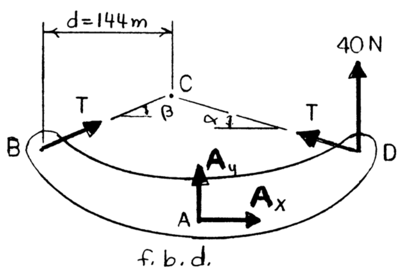
or $\mathbf{A} = 145.8$ N ⦪ 81.0° ◀



PROBLEM 4.30

Neglecting friction and the radius of the pulley, determine the tension in cable *BCD* and the reaction at support *A* when $d = 144$ mm.

SOLUTION



First note

$$\alpha = \tan^{-1}\left(\frac{60}{216}\right) = 15.5241°$$

$$\beta = \tan^{-1}\left(\frac{60}{144}\right) = 22.620°$$

From f.b.d. of member *BAD*

$$+\circlearrowleft \Sigma M_A = 0: \quad (40\text{ N})(0.18\text{ m}) + (T\cos\alpha)(0.08\text{ m})$$

$$+(T\sin\alpha)(0.18\text{ m}) - (T\cos\beta)(0.08\text{ m})$$

$$-(T\sin\beta)(0.18\text{ m}) = 0$$

$$\therefore \quad T = \left(\frac{7.2\text{ N}\cdot\text{m}}{0.0178199\text{m}}\right) = 404.04\text{ N}$$

or $T = 404$ N ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + (404.04\text{ N})(\cos\beta - \cos\alpha) = 0$$

$$\therefore \quad A_x = 16.3402\text{ N}$$

or $\mathbf{A}_x = 16.3402\text{ N} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y + (404.04\text{ N})(\sin\beta + \sin\alpha) + 40\text{ N} = 0$$

$$\therefore \quad A_y = -303.54\text{ N}$$

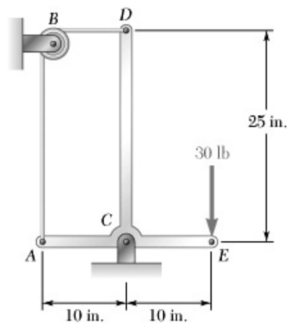
or $\mathbf{A}_y = 303.54\text{ N} \downarrow$

Then $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(16.3402)^2 + (303.54)^2} = 303.98\text{ N}$

and $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-303.54}{16.3402}\right) = -86.919°$

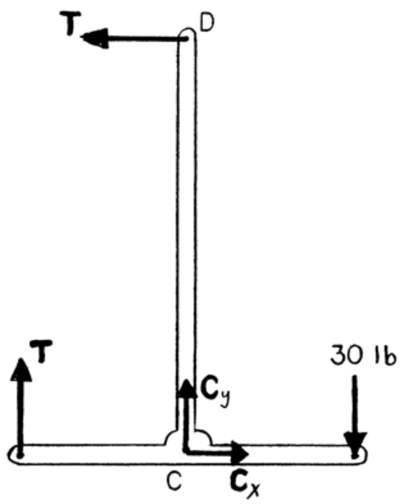
or $\mathbf{A} = 304$ N ⦣ 86.9° ◀

PROBLEM 4.31



Neglecting friction, determine the tension in cable ABD and the reaction at support C.

SOLUTION



From f.b.d. of inverted T-member

$$+\circlearrowleft \Sigma M_C = 0: \quad T(25 \text{ in.}) - T(10 \text{ in.}) - (30 \text{ lb})(10 \text{ in.}) = 0$$

$$\therefore \; T = 20 \text{ lb}$$

or $T = 20.0 \text{ lb}$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 20 \text{ lb} = 0$$

$$\therefore \; C_x = 20 \text{ lb}$$

or $\mathbf{C}_x = 20.0 \text{ lb} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 20 \text{ lb} - 30 \text{ lb} = 0$$

$$\therefore \; C_y = 10 \text{ lb}$$

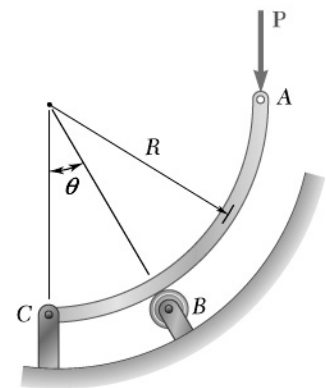
or $\mathbf{C}_y = 10.00 \text{ lb} \uparrow$

Then $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(20)^2 + (10)^2} = 22.361 \text{ lb}$

and $\theta = \tan^{-1}\left(\dfrac{C_y}{C_x}\right) = \tan^{-1}\left(\dfrac{10}{20}\right) = 26.565°$

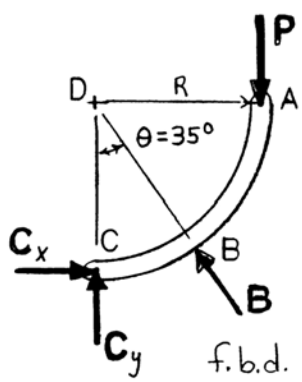
or $\mathbf{C} = 22.4 \text{ lb} \measuredangle 26.6°$ ◀

PROBLEM 4.32



Rod *ABC* is bent in the shape of a circular arc of radius *R*. Knowing that $\theta = 35°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION



For $\theta = 35°$

(*a*) From the f.b.d. of rod *ABC*

$$+\circlearrowleft \Sigma M_D = 0: \quad C_x(R) - P(R) = 0$$

$$\therefore \quad C_x = P$$

or $$\mathbf{C}_x = P \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - B\sin 35° = 0$$

$$\therefore \quad B = \frac{P}{\sin 35°} = 1.74345P$$

or $\mathbf{B} = 1.743P$ ⦪ 55.0° ◀

(*b*) From the f.b.d. of rod *ABC*

$$+\uparrow \Sigma F_y = 0: \quad C_y + (1.74345P)\cos 35° - P = 0$$

$$\therefore \quad C_y = -0.42815P$$

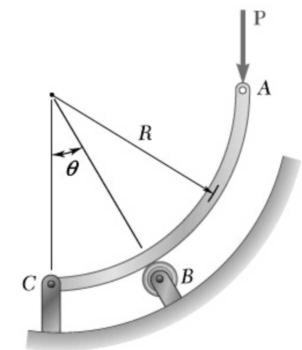
or $$\mathbf{C}_y = 0.42815P \downarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(P)^2 + (0.42815P)^2} = 1.08780P$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-0.42815P}{P}\right) = -23.178°$$

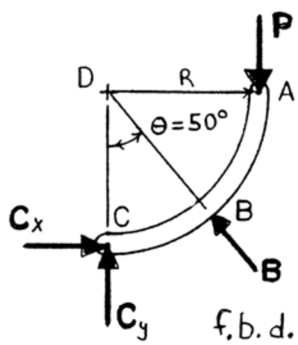
or $\mathbf{C} = 1.088P$ ⦩ 23.2° ◀

PROBLEM 4.33



Rod ABC is bent in the shape of a circular arc of radius R. Knowing that $\theta = 50^\circ$, determine the reaction (a) at B, (b) at C.

SOLUTION



For $\theta = 50^\circ$

(a) From the f.b.d. of rod ABC

$$+\circlearrowleft \Sigma M_D = 0: \quad C_x(R) - P(R) = 0$$

$$\therefore \quad C_x = P$$

or $$\mathbf{C}_x = P \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - B\sin 50^\circ = 0$$

$$\therefore \quad B = \frac{P}{\sin 50^\circ} = 1.30541P$$

or $\mathbf{B} = 1.305P$ ⦪ 40.0° ◀

(b) From the f.b.d. of rod ABC

$$+\uparrow \Sigma F_y = 0: \quad C_y - P + (1.30541P)\cos 50^\circ = 0$$

$$\therefore \quad C_y = 0.160900P$$

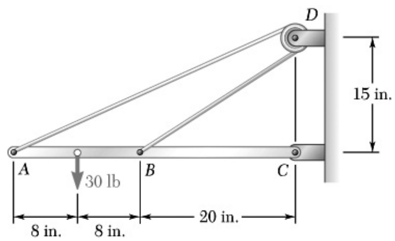
or $$\mathbf{C}_y = 0.1609P \uparrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(P)^2 + (0.1609P)^2} = 1.01286P$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{0.1609P}{P}\right) = 9.1405^\circ$$

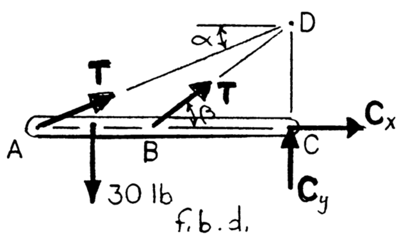
or $\mathbf{C} = 1.013P$ ⦩ 9.14° ◀

PROBLEM 4.34



Neglecting friction and the radius of the pulley, determine (*a*) the tension in cable *ABD*, (*b*) the reaction at *C*.

SOLUTION



First note

$$\alpha = \tan^{-1}\left(\frac{15}{36}\right) = 22.620°$$

$$\beta = \tan^{-1}\left(\frac{15}{20}\right) = 36.870°$$

(*a*) From f.b.d. of member *ABC*

$$+\circlearrowleft \Sigma M_C = 0: \quad (30 \text{ lb})(28 \text{ in.}) - (T \sin 22.620°)(36 \text{ in.})$$

$$-(T \sin 36.870°)(20 \text{ in.}) = 0$$

$$\therefore \quad T = 32.500 \text{ lb}$$

or $T = 32.5$ lb ◀

(*b*) From f.b.d. of member *ABC*

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (32.500 \text{ lb})(\cos 22.620° + \cos 36.870°) = 0$$

$$\therefore \quad C_x = -56.000 \text{ lb}$$

or $\mathbf{C}_x = 56.000 \text{ lb} \longleftarrow$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 30 \text{ lb} + (32.500 \text{ lb})(\sin 22.620° + \sin 36.870°) = 0$$

$$\therefore \quad C_y = -2.0001 \text{ lb}$$

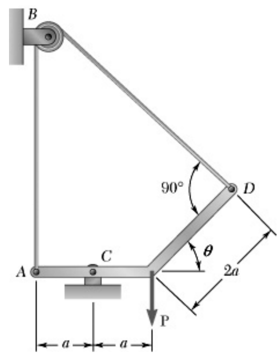
or $\mathbf{C}_y = 2.0001 \text{ lb} \downarrow$

Then $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(56.0)^2 + (2.001)^2} = 56.036 \text{ lb}$

and $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-2.0}{-56.0}\right) = 2.0454°$

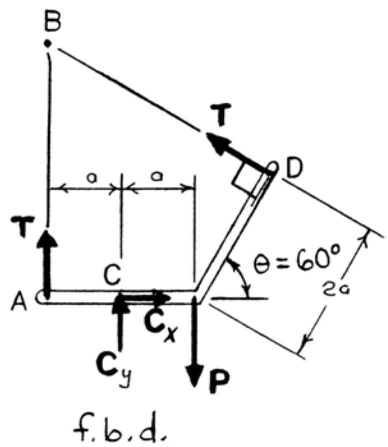
or $\mathbf{C} = 56.0$ lb ⦩ 2.05° ◀

PROBLEM 4.35



Neglecting friction, determine the tension in cable ABD and the reaction at C when $\theta = 60°$.

SOLUTION



From f.b.d. of bent ACD

$$+\circlearrowleft \Sigma M_C = 0: \quad (T\cos 30°)(2a\sin 60°) + (T\sin 30°)(a + 2a\cos 60°)$$

$$-T(a) - P(a) = 0$$

$$\therefore \quad T = \frac{P}{1.5}$$

$$\text{or } T = \frac{2P}{3} \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - \left(\frac{2P}{3}\right)\cos 30° = 0$$

$$\therefore \quad C_x = \frac{\sqrt{3}}{3}P = 0.57735P$$

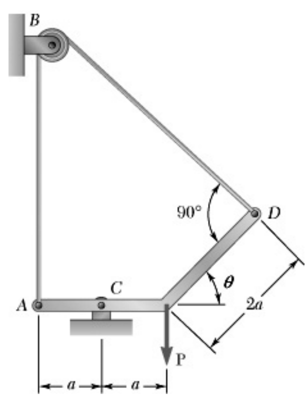
or $\mathbf{C}_x = 0.577P \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad C_y + \frac{2}{3}P - P + \left(\frac{2P}{3}\right)\cos 60° = 0$$

$$\therefore \quad C_y = 0$$

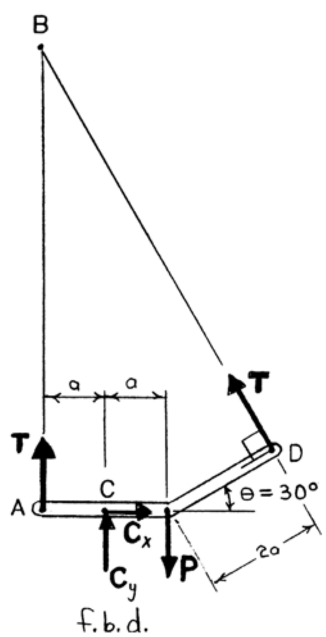
$$\text{or } \mathbf{C} = 0.577P \longrightarrow \blacktriangleleft$$

PROBLEM 4.36



Neglecting friction, determine the tension in cable *ABD* and the reaction at *C* when $\theta = 30^\circ$.

SOLUTION



From f.b.d. of bent *ACD*

$$+\circlearrowleft \Sigma M_C = 0: \quad (T\cos 60^\circ)(2a\sin 30^\circ) + T\sin 60^\circ(a + 2a\cos 30^\circ)$$

$$-P(a) - T(a) = 0$$

$$\therefore \quad T = \frac{P}{1.86603} = 0.53590P$$

or $T = 0.536P$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - (0.53590P)\cos 60^\circ = 0$$

$$\therefore \quad C_x = 0.26795P$$

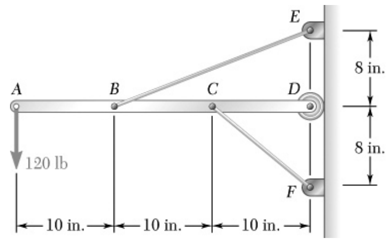
or $\mathbf{C}_x = 0.268P \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 0.53590P - P + (0.53590P)\sin 60^\circ = 0$$

$$\therefore \quad C_y = 0$$

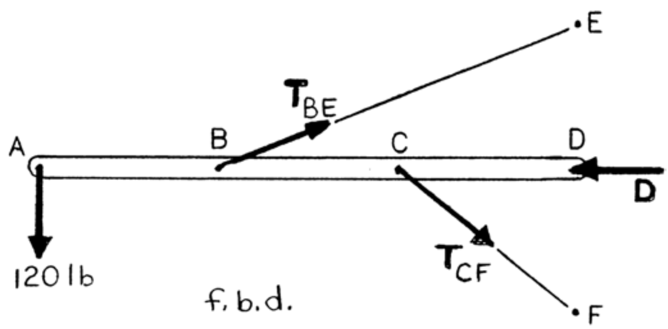
or $\mathbf{C} = 0.268P \longrightarrow$ ◀

PROBLEM 4.37



Determine the tension in each cable and the reaction at D.

SOLUTION



First note

$$\overline{BE} = \sqrt{(20)^2 + (8)^2} \text{ in.} = 21.541 \text{ in.}$$

$$\overline{CF} = \sqrt{(10)^2 + (8)^2} \text{ in.} = 12.8062 \text{ in.}$$

From f.b.d. of member $ABCD$

$$+\circlearrowleft \Sigma M_C = 0: \quad (120 \text{ lb})(20 \text{ in.}) - \left[\left(\frac{8}{21.541}\right)T_{BE}\right](10 \text{ in.}) = 0$$

$$\therefore \quad T_{BE} = 646.24 \text{ lb}$$

or $T_{BE} = 646$ lb ◀

$$+\uparrow \Sigma F_y = 0: \quad -120 \text{ lb} + \left(\frac{8}{21.541}\right)(646.24 \text{ lb}) - \left(\frac{8}{12.8062}\right)T_{CF} = 0$$

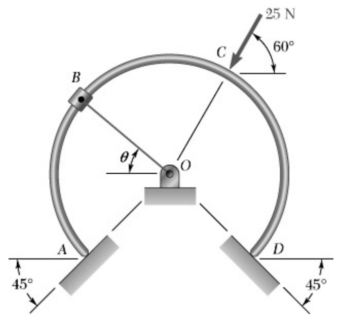
$$\therefore \quad T_{CF} = 192.099 \text{ lb}$$

or $T_{CF} = 192.1$ lb ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad \left(\frac{20}{21.541}\right)(646.24 \text{ lb}) + \left(\frac{10}{12.8062}\right)(192.099 \text{ lb}) - D = 0$$

$$\therefore \quad D = 750.01 \text{ lb}$$

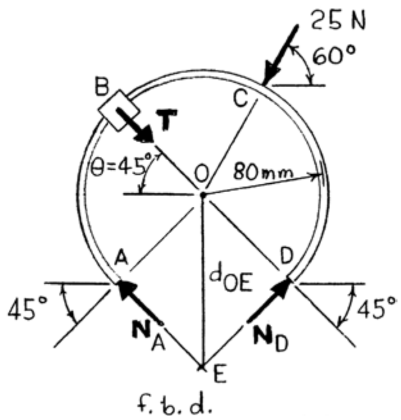
or $\mathbf{D} = 750$ lb ← ◀



PROBLEM 4.38

Rod *ABCD* is bent in the shape of a circular arc of radius 80 mm and rests against frictionless surfaces at *A* and *D*. Knowing that the collar at *B* can move freely on the rod and that $\theta = 45^\circ$. determine (*a*) the tension in cord *OB*, (*b*) the reactions at *A* and *D*.

SOLUTION



(*a*) From f.b.d. of rod *ABCD*

$$+\circlearrowleft \Sigma M_E = 0: \quad (25\text{ N})\cos 60°(d_{OE}) - (T\cos 45°)(d_{OE}) = 0$$

$$\therefore \quad T = 17.6777\text{ N}$$

or $T = 17.68$ N ◀

(*b*) From f.b.d. of rod *ABCD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(17.6777\text{ N})\cos 45° + (25\text{ N})\cos 60°$$

$$+ N_D\cos 45° - N_A\cos 45° = 0$$

$$\therefore \quad N_A - N_D = 0$$

or $$N_D = N_A \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_A\sin 45° + N_D\sin 45° - (17.6777\text{ N})\sin 45°$$

$$-(25\text{ N})\sin 60° = 0$$

$$\therefore \quad N_A + N_D = 48.296\text{ N} \qquad (2)$$

Substituting Equation (1) into Equation (2),

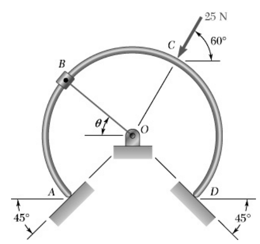
$$2N_A = 48.296\text{ N}$$

$$N_A = 24.148\text{ N}$$

or $\mathbf{N}_A = 24.1$ N ⦪ 45.0° ◀

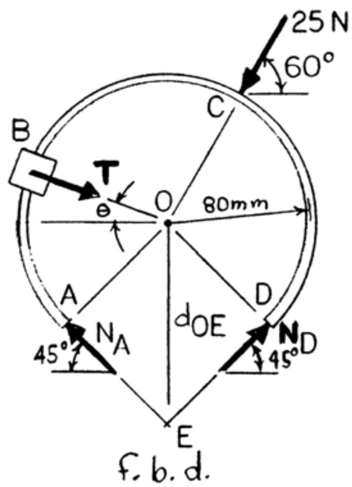
and $\mathbf{N}_D = 24.1$ N ∠ 45.0° ◀

PROBLEM 4.39



Rod *ABCD* is bent in the shape of a circular arc of radius 80 mm and rests against frictionless surfaces at *A* and *D*. Knowing that the collar at *B* can move freely on the rod, determine (*a*) the value of θ for which the tension in cord *OB* is as small as possible, (*b*) the corresponding value of the tension, (*c*) the reactions at *A* and *D*.

SOLUTION



(*a*) From f.b.d. of rod *ABCD*

$$+\circlearrowleft \Sigma M_E = 0: \quad (25\text{ N})\cos 60°(d_{OE}) - (T\cos\theta)(d_{OE}) = 0$$

or
$$T = \frac{12.5\text{ N}}{\cos\theta} \qquad (1)$$

$\therefore$ T is minimum when $\cos\theta$ is maximum,

or $\theta = 0°$ ◀

(*b*) From Equation (1)

$$T = \frac{12.5\text{ N}}{\cos 0} = 12.5\text{ N}$$

or $T_{\min} = 12.50\text{ N}$ ◀

(*c*) $\xrightarrow{+} \Sigma F_x = 0: \quad -N_A\cos 45° + N_D\cos 45° + 12.5\text{ N}$

$$-(25\text{ N})\cos 60° = 0$$

$$\therefore \quad N_D - N_A = 0$$

or
$$N_D = N_A \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad N_A\sin 45° + N_D\sin 45° - (25\text{ N})\sin 60° = 0$$

$$\therefore \quad N_D + N_A = 30.619\text{ N} \qquad (3)$$

Substituting Equation (2) into Equation (3),

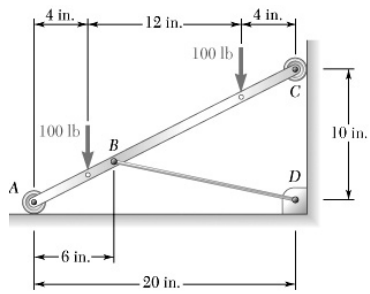
$$2N_A = 30.619$$

$$N_A = 15.3095\text{ N}$$

or $\mathbf{N}_A = 15.31\text{ N} \searrow 45.0°$ ◀

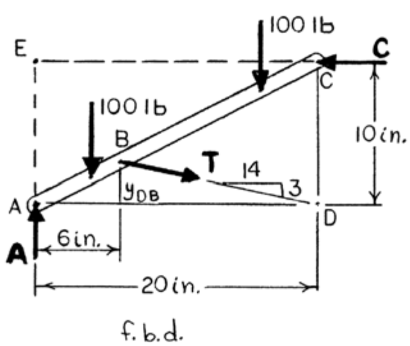
and $\mathbf{N}_D = 15.31\text{ N} \nearrow 45.0°$ ◀

PROBLEM 4.40



Bar AC supports two 100-lb loads as shown. Rollers A and C rest against frictionless surfaces and a cable BD is attached at B. Determine (a) the tension in cable BD, (b) the reaction at A, (c) the reaction at C.

SOLUTION



First note that from similar triangles

$$\frac{y_{DB}}{6} = \frac{10}{20} \qquad \therefore \; y_{DB} = 3 \text{ in.}$$

and

$$\overline{BD} = \sqrt{(3)^2 + (14)^2} \text{ in.} = 14.3178 \text{ in.}$$

$$T_x = \frac{14}{14.3178}T = 0.97780T$$

$$T_y = \frac{3}{14.3178}T = 0.20953T$$

(a) From f.b.d. of bar AC

$$+\circlearrowleft \Sigma M_E = 0: \quad (0.97780T)(7 \text{ in.}) - (0.20953T)(6 \text{ in.})$$

$$-(100 \text{ lb})(16 \text{ in.}) - (100 \text{ lb})(4 \text{ in.}) = 0$$

$$\therefore \; T = 357.95 \text{ lb}$$

or $T = 358$ lb ◀

(b) From f.b.d. of bar AC

$$+\uparrow \Sigma F_y = 0: \quad A - 100 - 0.20953(357.95) - 100 = 0$$

$$\therefore \; A = 275.00 \text{ lb}$$

or $\mathbf{A} = 275$ lb ↑ ◀

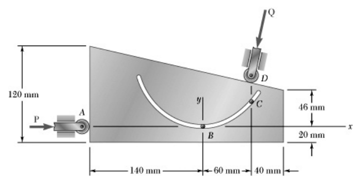
(c) From f.b.d of bar AC

$$\xrightarrow{+} \Sigma F_x = 0: \quad 0.97780(357.95) - C = 0$$

$$\therefore \; C = 350.00 \text{ lb}$$

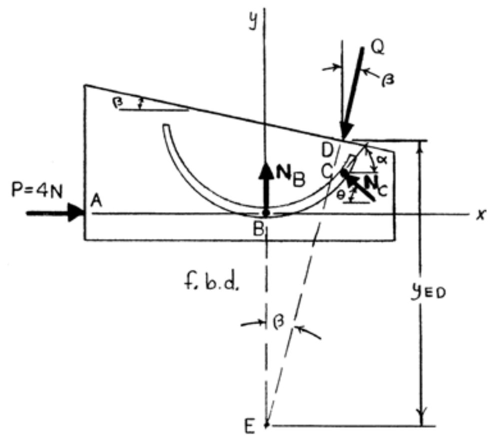
or $\mathbf{C} = 350$ lb ⟵ ◀

PROBLEM 4.41



A parabolic slot has been cut in plate *AD*, and the plate has been placed so that the slot fits two fixed, frictionless pins *B* and *C*. The equation of the slot is $y = x^2/100$, where *x* and *y* are expressed in mm. Knowing that the input force $P = 4$ N, determine (*a*) the force each pin exerts on the plate, (*b*) the output force **Q**.

SOLUTION



The equation of the slot is
$$y = \frac{x^2}{100}$$

Now
$$\left(\frac{dy}{dx}\right)_C = \text{slope of the slot at } C$$

$$= \left[\frac{2x}{100}\right]_{x=60 \text{ mm}} = 1.200$$

$$\therefore \quad \alpha = \tan^{-1}(1.200) = 50.194°$$

and
$$\theta = 90° - \alpha = 90° - 50.194° = 39.806°$$

Coordinates of *C* are

$$x_C = 60 \text{ mm}, \qquad y_C = \frac{(60)^2}{100} = 36 \text{ mm}$$

Also, the coordinates of *D* are
$$x_D = 60 \text{ mm}$$

$$y_D = 46 \text{ mm} + (40 \text{ mm})\sin\beta$$

where
$$\beta = \tan^{-1}\left(\frac{120 - 66}{240}\right) = 12.6804°$$

$$\therefore \quad y_D = 46 \text{ mm} + (40 \text{ mm})\tan 12.6804°$$

$$= 55.000 \text{ mm}$$

PROBLEM 4.41 CONTINUED

Also, $$y_{ED} = \frac{60\text{ mm}}{\tan\beta} = \frac{60\text{ mm}}{\tan 12.6804°}$$

$$= 266.67\text{ mm}$$

From f.b.d. of plate *AD*

$$+\circlearrowright \Sigma M_E = 0: \quad (N_C\cos\theta)\left[y_{ED} - (y_D - y_C)\right] + (N_C\sin\theta)(x_C) - (4\text{ N})(y_{ED} - y_D) = 0$$

$$(N_C\cos 39.806°)\left[266.67 - (55.0 - 36.0)\right]\text{mm} + N_C\sin(39.806°)(60\text{ mm}) - (4\text{ N})(266.67 - 55.0)\text{ mm} = 0$$

$$\therefore\ N_C = 3.7025\text{ N}$$

or $$\mathbf{N}_C = 3.70\text{ N} \searrow 39.8°$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -4\text{ N} + N_C\cos\theta + Q\sin\beta = 0$$

$$-4\text{ N} + (3.7025\text{ N})\cos 39.806° + Q\sin 12.6804° = 0$$

$$\therefore\ Q = 5.2649\text{ N}$$

or $$\mathbf{Q} = 5.26\text{ N} \nearrow 77.3°$$

$$+\uparrow \Sigma F_y = 0: \quad N_B + N_C\sin\theta - Q\cos\beta = 0$$

$$N_B + (3.7025\text{ N})\sin 39.806° - (5.2649\text{ N})\cos 12.6804° = 0$$

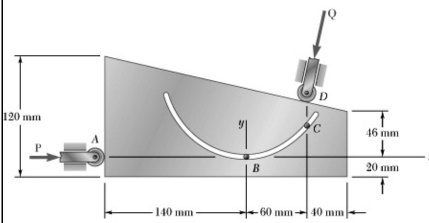
$$\therefore\ N_B = 2.7662\text{ N}$$

or $$\mathbf{N}_B = 2.77\text{ N} \uparrow$$

(*a*) $\mathbf{N}_B = 2.77\text{ N} \uparrow, \quad \mathbf{N}_C = 3.70\text{ N} \searrow 39.8°$ ◀

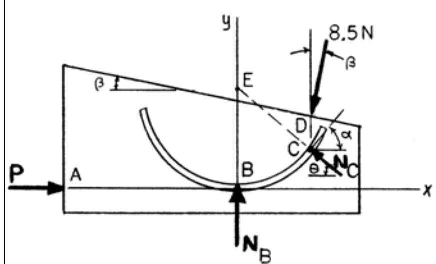
(*b*) $\mathbf{Q} = 5.26\text{ N} \nearrow 77.3°(\text{output})$ ◀

PROBLEM 4.42



A parabolic slot has been cut in plate *AD*, and the plate has been placed so that the slot fits two fixed, frictionless pins *B* and *C*. The equation of the slot is $y = x^2/100$, where *x* and *y* are expressed in mm. Knowing that the maximum allowable force exerted on the roller at *D* is 8.5 N, determine (*a*) the corresponding magnitude of the input force **P**, (*b*) the force each pin exerts on the plate.

SOLUTION



f.b.d.

The equation of the slot is,
$$y = \frac{x^2}{100}$$

Now
$$\left(\frac{dy}{dx}\right)_C = \text{slope of slot at } C$$
$$= \left[\frac{2x}{100}\right]_{x=60 \text{ mm}} = 1.200$$
$$\therefore \quad \alpha = \tan^{-1}(1.200) = 50.194°$$

and
$$\theta = 90° - \alpha = 90° - 50.194° = 39.806°$$

Coordinates of *C* are
$$x_C = 60 \text{ mm}, \quad y_C = \frac{(60)^2}{100} = 36 \text{ mm}$$

Also, the coordinates of *D* are
$$x_D = 60 \text{ mm}$$
$$y_D = 46 \text{ mm} + (40 \text{ mm})\sin\beta$$

where
$$\beta = \tan^{-1}\left(\frac{120 - 66}{240}\right) = 12.6804°$$
$$\therefore \quad y_D = 46 \text{ mm} + (40 \text{ mm})\tan 12.6804° = 55.000 \text{ mm}$$

Note:
$$x_E = 0$$
$$y_E = y_C + (60 \text{ mm})\tan\theta$$
$$= 36 \text{ mm} + (60 \text{ mm})\tan 39.806°$$
$$= 86.001 \text{ mm}$$

(*a*) From f.b.d. of plate *AD*

$$+\circlearrowleft \Sigma M_E = 0: \quad P(y_E) - [(8.5 \text{ N})\sin\beta](y_E - y_D)$$
$$-[(8.5 \text{ N})\cos\beta](60 \text{ mm}) = 0$$

PROBLEM 4.42 CONITNIUED

$$P(86.001\text{ mm}) - \left[(8.5\text{ N})\sin 12.6804^\circ\right](31.001\text{ mm})$$

$$-\left[(8.5\text{ N})\cos 12.6804^\circ\right](60\text{ mm}) = 0$$

$$\therefore \quad P = 6.4581\text{ N}$$

or $P = 6.46\text{ N}$ ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad P - (8.5\text{ N})\sin\beta - N_C\cos\theta = 0$

$$6.458\text{ N} - (8.5\text{ N})(\sin 12.6804^\circ) - N_C(\cos 39.806^\circ) = 0$$

$$\therefore \quad N_C = 5.9778\text{ N}$$

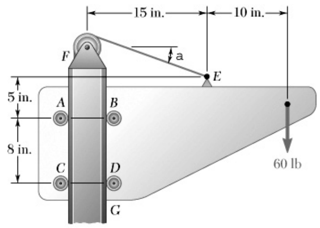
or $\mathbf{N}_C = 5.98\text{ N} \searrow 39.8^\circ$ ◀

$+\uparrow \Sigma F_y = 0: \quad N_B + N_C\sin\theta - (8.5\text{ N})\cos\beta = 0$

$$N_B + (5.9778\text{ N})\sin 39.806^\circ - (8.5\text{ N})\cos 12.6804^\circ = 0$$

$$\therefore \quad N_B = 4.4657\text{ N}$$

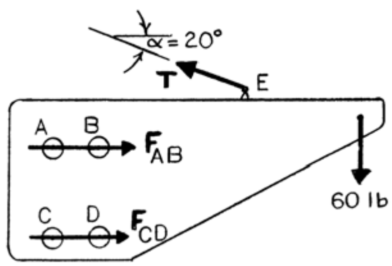
or $\mathbf{N}_B = 4.47\text{ N} \uparrow$ ◀



PROBLEM 4.43

A movable bracket is held at rest by a cable attached at *E* and by frictionless rollers. Knowing that the width of post *FG* is slightly less than the distance between the rollers, determine the force exerted on the post by each roller when $\alpha = 20^{\circ}$.

SOLUTION



From f.b.d. of bracket

$$+\uparrow \Sigma F_y = 0: \quad T \sin 20^{\circ} - 60 \text{ lb} = 0$$

$$\therefore \quad T = 175.428 \text{ lb}$$

$$T_x = (175.428 \text{ lb}) \cos 20^{\circ} = 164.849 \text{ lb}$$

$$T_y = (175.428 \text{ lb}) \sin 20^{\circ} = 60 \text{ lb}$$

Note: T_y and 60 lb force form a couple of

$$60 \text{ lb}(10 \text{ in.}) = 600 \text{ lb} \cdot \text{in.} \circlearrowleft$$

$$+\circlearrowright \Sigma M_B = 0: \quad 164.849 \text{ lb}(5 \text{ in.}) - 600 \text{ lb} \cdot \text{in.} + F_{CD}(8 \text{ in.}) = 0$$

$$\therefore \quad F_{CD} = -28.030 \text{ lb}$$

or $\mathbf{F}_{CD} = 28.0 \text{ lb} \longleftarrow$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CD} + F_{AB} - T_x = 0$$

$$-28.030 \text{ lb} + F_{AB} - 164.849 \text{ lb} = 0$$

$$\therefore \quad F_{AB} = 192.879 \text{ lb}$$

or $\mathbf{F}_{AB} = 192.9 \text{ lb} \longrightarrow$

Rollers *A* and *C* can only apply a horizontal force to the right onto the vertical post corresponding to the equal and opposite force to the left on the bracket. Since $\mathbf{F}_{AB}$ is directed to the right onto the bracket, roller *B* will react $\mathbf{F}_{AB}$. Also, since $\mathbf{F}_{CD}$ is acting to the left on the bracket, it will act to the right on the post at roller *C*.

PROBLEM 4.43 CONTINUED

$$\therefore \quad \mathbf{A} = \mathbf{D} = 0$$

$$\mathbf{B} = 192.9 \text{ lb} \longrightarrow$$

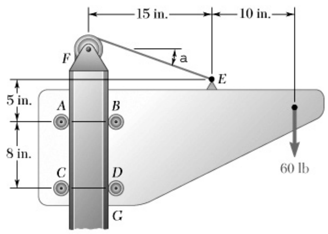
$$\mathbf{C} = 28.0 \text{ lb} \longleftarrow$$

Forces exerted on the post are

$$\mathbf{A} = \mathbf{D} = 0 \blacktriangleleft$$

$$\mathbf{B} = 192.9 \text{ lb} \longleftarrow \blacktriangleleft$$

$$\mathbf{C} = 28.0 \text{ lb} \longrightarrow \blacktriangleleft$$

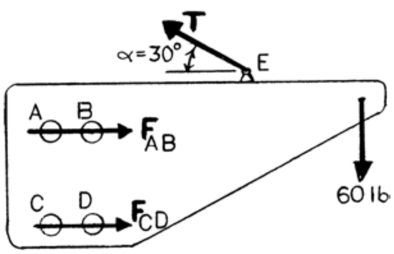


PROBLEM 4.44

Solve Problem 4.43 when $\alpha = 30^{\circ}$.

P4.43 A movable bracket is held at rest by a cable attached at *E* and by frictionless rollers. Knowing that the width of post *FG* is slightly less than the distance between the rollers, determine the force exerted on the post by each roller when $\alpha = 20^{\circ}$.

SOLUTION



f.b.d.

From f.b.d. of bracket

$$+\uparrow \Sigma F_y = 0: \quad T\sin 30^{\circ} - 60 \text{ lb} = 0$$

$$\therefore \quad T = 120 \text{ lb}$$

$$T_x = (120 \text{ lb})\cos 30^{\circ} = 103.923 \text{ lb}$$

$$T_y = (120 \text{ lb})\sin 30^{\circ} = 60 \text{ lb}$$

Note: T_y and 60 lb force form a couple of

$$(60 \text{ lb})(10 \text{ in.}) = 600 \text{ lb}\cdot\text{in.} \circlearrowleft$$

$$+\circlearrowright \Sigma M_B = 0: \quad (103.923 \text{ lb})(5 \text{ in.}) - 600 \text{ lb}\cdot\text{in.} + F_{CD}(8 \text{ in.}) = 0$$

$$\therefore \quad F_{CD} = 10.0481 \text{ lb}$$

or

$$\mathbf{F}_{CD} = 10.05 \text{ lb} \longrightarrow$$

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad F_{CD} + F_{AB} - T_x = 0$$

$$10.0481 \text{ lb} + F_{AB} - 103.923 \text{ lb} = 0$$

$$\therefore \quad F_{AB} = 93.875 \text{ lb}$$

or

$$\mathbf{F}_{AB} = 93.9 \text{ lb} \longrightarrow$$

Rollers *A* and *C* can only apply a horizontal force to the right on the vertical post corresponding to the equal and opposite force to the left on the bracket. The opposite direction apply to roller *B* and *D*. Since both $\mathbf{F}_{AB}$ and $\mathbf{F}_{CD}$ act to the right on the bracket, rollers *B* and *D* will react these forces.

$$\therefore \quad \mathbf{A} = \mathbf{C} = 0$$

$$\mathbf{B} = 93.9 \text{ lb} \longrightarrow$$

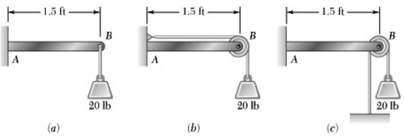
$$\mathbf{D} = 10.05 \text{ lb} \longrightarrow$$

Forces exerted on the post are

$$\mathbf{A} = \mathbf{C} = 0 \blacktriangleleft$$

$$\mathbf{B} = 93.9 \text{ lb} \longleftarrow \blacktriangleleft$$

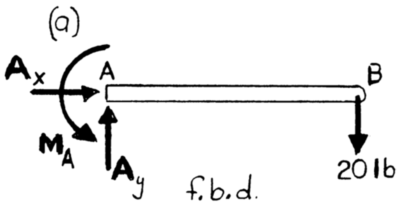
$$\mathbf{D} = 10.05 \text{ lb} \longleftarrow \blacktriangleleft$$



PROBLEM 4.45

A 20-lb weight can be supported in the three different ways shown. Knowing that the pulleys have a 4-in. radius, determine the reaction at A in each case.

SOLUTION



(*a*) From f.b.d. of AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 20 \text{ lb} = 0$$

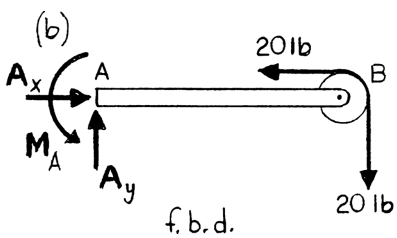
or $$A_y = 20.0 \text{ lb}$$

and $\mathbf{A} = 20.0 \text{ lb} \uparrow$ ◀

$$+\circlearrowright \Sigma M_A = 0: \quad M_A - (20 \text{ lb})(1.5 \text{ ft}) = 0$$

$$\therefore \; M_A = 30.0 \text{ lb}\cdot\text{ft}$$

or $\mathbf{M}_A = 30.0 \text{ lb}\cdot\text{ft} \circlearrowright$ ◀



(*b*) Note: $$4 \text{ in.}\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 0.33333 \text{ ft}$$

From f.b.d. of AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 20 \text{ lb} = 0$$

or $$A_x = 20.0 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 20 \text{ lb} = 0$$

or $$A_y = 20.0 \text{ lb}$$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(20.0)^2 + (20.0)^2} = 28.284 \text{ lb}$$

$\therefore \; \mathbf{A} = 28.3 \text{ lb} \measuredangle 45°$ ◀

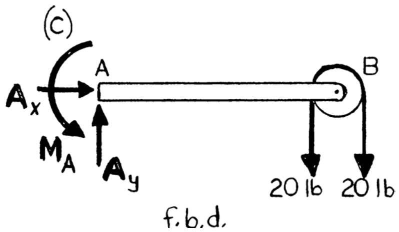
$$+\circlearrowright \Sigma M_A = 0: \quad M_A + (20 \text{ lb})(0.33333 \text{ ft})$$

$$-(20 \text{ lb})(1.5 \text{ ft} + 0.33333 \text{ ft}) = 0$$

$$\therefore \; M_A = 30.0 \text{ lb}\cdot\text{ft}$$

or $\mathbf{M}_A = 30.0 \text{ lb}\cdot\text{ft} \circlearrowright$ ◀

PROBLEM 4.45 CONTINUED



(*c*) From f.b.d. of *AB*

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 20\text{ lb} - 20\text{ lb} = 0$$

or

$$A_y = 40.0\text{ lb}$$

and $\mathbf{A} = 40.0\text{ lb}\uparrow$ ◀

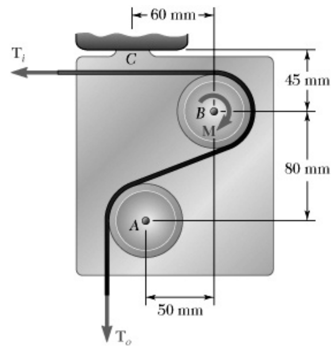
$$+\circlearrowright \Sigma M_A = 0: \quad M_A - (20\text{ lb})(1.5\text{ ft} - 0.33333\text{ ft})$$

$$-(20\text{ lb})(1.5\text{ ft} + 0.33333\text{ ft}) = 0$$

$$\therefore \quad M_A = 60.0\text{ lb}\cdot\text{ft}$$

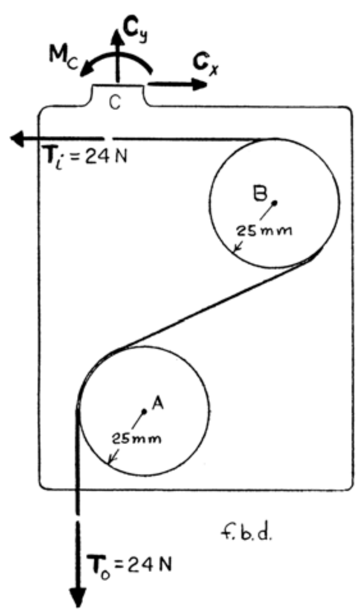
or $\mathbf{M}_A = 60.0\text{ lb}\cdot\text{ft} \circlearrowright$ ◀

PROBLEM 4.46



A belt passes over two 50-mm-diameter pulleys which are mounted on a bracket as shown. Knowing that $M = 0$ and $T_i = T_O = 24$ N, determine the reaction at C.

SOLUTION



From f.b.d. of bracket

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 24 \text{ N} = 0$$

$$\therefore \; C_x = 24 \text{ N}$$

$$+\!\uparrow \Sigma F_y = 0: \quad C_y - 24 \text{ N} = 0$$

$$\therefore \; C_y = 24 \text{ N}$$

Then

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(24)^2 + (24)^2} = 33.941 \text{ N}$$

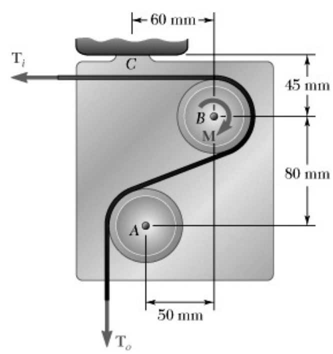
$$\therefore \; \mathbf{C} = 33.9 \text{ N} \measuredangle 45.0^\circ \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C - (24 \text{ N})[(45 - 25) \text{ mm}]$$

$$+ (24 \text{ N})[(25 + 50 - 60) \text{ mm}] = 0$$

$$\therefore \; M_C = 120 \text{ N}\cdot\text{mm}$$

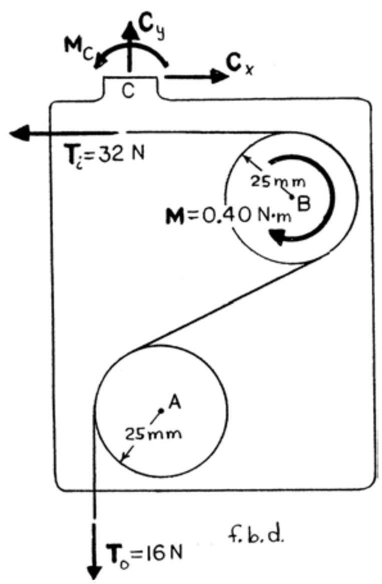
$$\text{or} \;\; \mathbf{M}_C = 0.120 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$



PROBLEM 4.47

A belt passes over two 50-mm-diameter pulleys which are mounted on a bracket as shown. Knowing that $M = 0.40\ \text{N}\cdot\text{m}$ m and that T_i and T_O are equal to 32 N and 16 N, respectively, determine the reaction at C.

SOLUTION



From f.b.d. of bracket

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 32\text{ N} = 0$$

$$\therefore \quad C_x = 32\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 16\text{ N} = 0$$

$$\therefore \quad C_y = 16\text{ N}$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(32)^2 + (16)^2} = 35.777\text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{16}{32}\right) = 26.565°$$

or $\mathbf{C} = 35.8\text{ N} \measuredangle 26.6°$ ◀

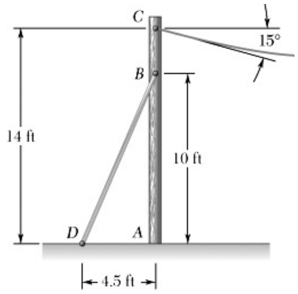
$$+\circlearrowleft \Sigma M_C = 0: \quad M_C - (32\text{ N})(45\text{ mm} - 25\text{ mm})$$

$$+ (16\text{ N})(25\text{ mm} + 50\text{ mm} - 60\text{ mm}) - 400\text{ N}\cdot\text{mm} = 0$$

$$\therefore \quad M_C = 800\text{ N}\cdot\text{mm}$$

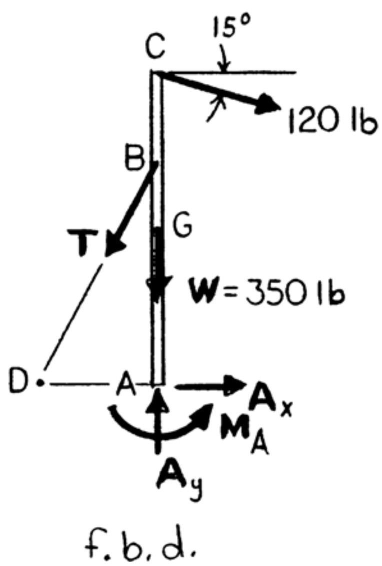
or $\mathbf{M}_C = 0.800\text{ N}\cdot\text{m} \circlearrowleft$ ◀

PROBLEM 4.48



A 350-lb utility pole is used to support at *C* the end of an electric wire. The tension in the wire is 120 lb, and the wire forms an angle of 15° with the horizontal at *C*. Determine the largest and smallest allowable tensions in the guy cable *BD* if the magnitude of the couple at *A* may not exceed 200 lb · ft.

SOLUTION



First note

$$L_{BD} = \sqrt{(4.5)^2 + (10)^2} = 10.9659 \text{ ft}$$

T_{max}: From f.b.d. of utility pole with $\mathbf{M}_A = 200 \text{ lb}\cdot\text{ft}$ ↻

$$+\circlearrowleft \Sigma M_A = 0: \quad -200 \text{ lb}\cdot\text{ft} - \big[(120 \text{ lb})\cos 15°\big](14 \text{ ft})$$

$$+\left[\left(\frac{4.5}{10.9659}\right)T_{max}\right](10 \text{ ft}) = 0$$

$$\therefore \quad T_{max} = 444.19 \text{ lb}$$

or $T_{max} = 444$ lb ◀

T_{min}: From f.b.d. of utility pole with $\mathbf{M}_A = 200 \text{ lb}\cdot\text{ft}$ ↺

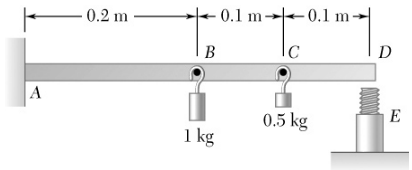
$$+\circlearrowleft \Sigma M_A = 0: \quad 200 \text{ lb}\cdot\text{ft} - \big[(120 \text{ lb})\cos 15°\big](14 \text{ ft})$$

$$+\left[\left(\frac{4.5}{10.9659}\right)T_{min}\right](10 \text{ ft}) = 0$$

$$\therefore \quad T_{min} = 346.71 \text{ lb}$$

or $T_{min} = 347$ lb ◀

PROBLEM 4.49

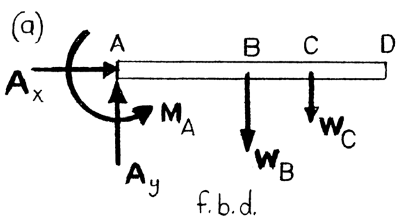


In a laboratory experiment, students hang the masses shown from a beam of negligible mass. (*a*) Determine the reaction at the fixed support *A* knowing that end *D* of the beam does not touch support *E*. (*b*) Determine the reaction at the fixed support *A* knowing that the adjustable support *E* exerts an upward force of 6 N on the beam.

SOLUTION

$$W_B = m_B g = (1\text{ kg})(9.81\text{ m/s}^2) = 9.81\text{ N}$$

$$W_C = m_C g = (0.5\text{ kg})(9.81\text{ m/s}^2) = 4.905\text{ N}$$



(*a*) From f.b.d. of beam *ABCD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - W_B - W_C = 0$$

$$A_y - 9.81\text{ N} - 4.905\text{ N} = 0$$

$$\therefore \quad A_y = 14.715\text{ N}$$

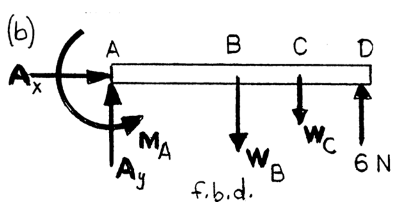
or $\mathbf{A} = 14.72\text{ N}\uparrow$ ◀

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - W_B(0.2\text{ m}) - W_C(0.3\text{ m}) = 0$$

$$M_A - (9.81\text{ N})(0.2\text{ m}) - (4.905\text{ N})(0.3\text{ m}) = 0$$

$$\therefore \quad M_A = 3.4335\text{ N}\cdot\text{m}$$

or $\mathbf{M}_A = 3.43\text{ N}\cdot\text{m}\circlearrowleft$ ◀



(*b*) From f.b.d. of beam *ABCD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - W_B - W_C + 6\text{ N} = 0$$

$$A_y - 9.81\text{ N} - 4.905\text{ N} + 6\text{ N} = 0$$

$$\therefore \quad A_y = 8.715\text{ N}$$

or $\mathbf{A} = 8.72\text{ N}\uparrow$ ◀

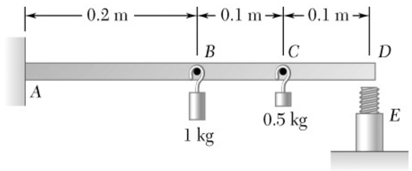
$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - W_B(0.2\text{ m}) - W_C(0.3\text{ m}) + (6\text{ N})(0.4\text{ m}) = 0$$

$$M_A - (9.81\text{ N})(0.2\text{ m}) - (4.905\text{ N})(0.3\text{ m}) + (6\text{ N})(0.4\text{ m}) = 0$$

$$\therefore \quad M_A = 1.03350\text{ N}\cdot\text{m}$$

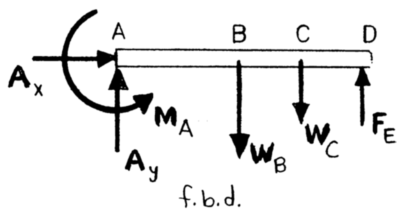
or $\mathbf{M}_A = 1.034\text{ N}\cdot\text{m}\circlearrowleft$ ◀

PROBLEM 4.50



In a laboratory experiment, students hang the masses shown from a beam of negligible mass. Determine the range of values of the force exerted on the beam by the adjustable support E for which the magnitude of the couple at A does not exceed $2.5\text{ N}\cdot\text{m}$.

SOLUTION



$$W_B = m_B g = 1\text{ kg}\left(9.81\text{ m/s}^2\right) = 9.81\text{ N}$$

$$W_C = m_C g = 0.5\text{ kg}\left(9.81\text{ m/s}^2\right) = 4.905\text{ N}$$

Maximum M_A value is $2.5\text{ N}\cdot\text{m}$

F_{min}: From f.b.d. of beam $ABCD$ with $\mathbf{M}_A = 2.5\text{ N}\cdot\text{m}$ ↻

$$+\circlearrowright \Sigma M_A = 0: \quad 2.5\text{ N}\cdot\text{m} - W_B(0.2\text{ m}) - W_C(0.3\text{ m}) + F_{\text{min}}(0.4\text{ m}) = 0$$

$$2.5\text{ N}\cdot\text{m} - (9.81\text{ N})(0.2\text{ m}) - (4.905\text{ N})(0.3\text{ m}) + F_{\text{min}}(0.4\text{ m}) = 0$$

$$\therefore \quad F_{\text{min}} = 2.3338\text{ N}$$

or $$F_{\text{min}} = 2.33\text{ N}$$

F_{max}: From f.b.d. of beam $ABCD$ with $\mathbf{M}_A = 2.5\text{ N}\cdot\text{m}$ ↺

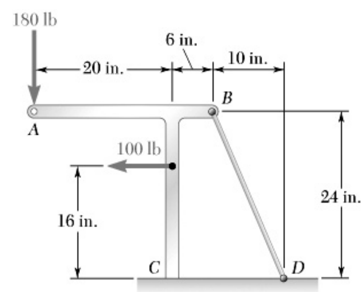
$$+\circlearrowright \Sigma M_A = 0: \quad -2.5\text{ N}\cdot\text{m} - W_B(0.2\text{ m}) - W_C(0.3\text{ m}) + F_{\text{max}}(0.4\text{ m}) = 0$$

$$-2.5\text{ N}\cdot\text{m} - (9.81\text{ N})(0.2\text{ m}) - (4.905\text{ N})(0.3\text{ m}) + F_{\text{max}}(0.4\text{ m}) = 0$$

$$\therefore \quad F_{\text{max}} = 14.8338\text{ N}$$

or $$F_{\text{max}} = 14.83\text{ N}$$

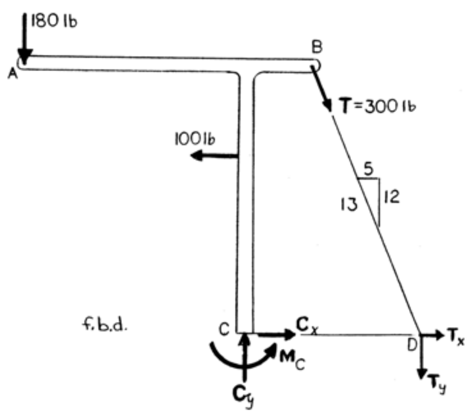
or $2.33\text{ N} \le F_E \le 14.83\text{ N}$ ◀



PROBLEM 4.51

Knowing that the tension in wire BD is 300 lb, determine the reaction at fixed support C for the frame shown.

SOLUTION



From f.b.d. of frame with $T = 300$ lb

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 100 \text{ lb} + \left(\frac{5}{13}\right)300 \text{ lb} = 0$$

$$\therefore \quad C_x = -15.3846 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_x = 15.3846 \text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 180 \text{ lb} - \left(\frac{12}{13}\right)300 \text{ lb} = 0$$

$$\therefore \quad C_y = 456.92 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = 456.92 \text{ lb} \uparrow$$

Then
$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(15.3846)^2 + (456.92)^2} = 457.18 \text{ lb}$$

and
$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{456.92}{-15.3846}\right) = -88.072°$$

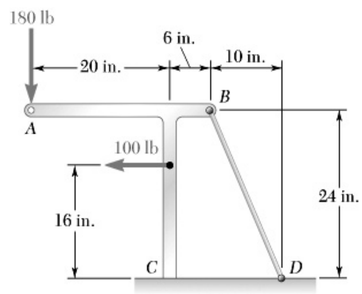
or $\mathbf{C} = 457$ lb ⦣ 88.1° ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (180 \text{ lb})(20 \text{ in.}) + (100 \text{ lb})(16 \text{ in.}) - \left[\left(\frac{12}{13}\right)300 \text{ lb}\right](16 \text{ in.}) = 0$$

$$\therefore M_C = -769.23 \text{ lb}\cdot\text{in.}$$

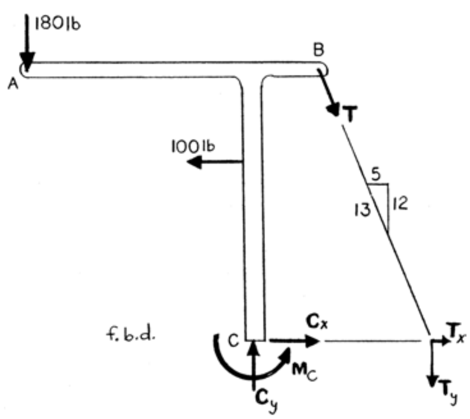
or $\mathbf{M}_C = 769$ lb·in. ↻ ◀

PROBLEM 4.52



Determine the range of allowable values of the tension in wire *BD* if the magnitude of the couple at the fixed support *C* is not to exceed $75 \text{ lb}\cdot\text{ft}$.

SOLUTION



T_{max} From f.b.d. of frame with $\mathbf{M}_C = 75 \text{ lb}\cdot\text{ft} \circlearrowleft = 900 \text{ lb}\cdot\text{in.} \circlearrowleft$

$$+\circlearrowleft \Sigma M_C = 0: \quad 900 \text{ lb}\cdot\text{in.} + (180 \text{ lb})(20 \text{ in.}) + (100 \text{ lb})(16 \text{ in.}) - \left[\left(\frac{12}{13}\right)T_{max}\right](16 \text{ in.}) = 0$$

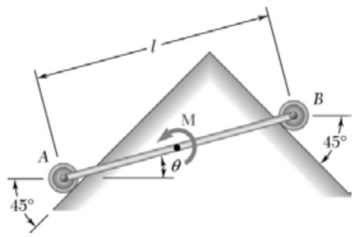
$$\therefore \quad T_{max} = 413.02 \text{ lb}$$

T_{min} From f.b.d. of frame with $\mathbf{M}_C = 75 \text{ lb}\cdot\text{ft} \circlearrowright = 900 \text{ lb}\cdot\text{in.} \circlearrowright$

$$+\circlearrowleft \Sigma M_C = 0: \quad -900 \text{ lb}\cdot\text{in.} + (180 \text{ lb})(20 \text{ in.}) + (100 \text{ lb})(16 \text{ in.}) - \left[\left(\frac{12}{13}\right)T_{min}\right](16 \text{ in.}) = 0$$

$$\therefore \quad T_{min} = 291.15 \text{ lb}$$

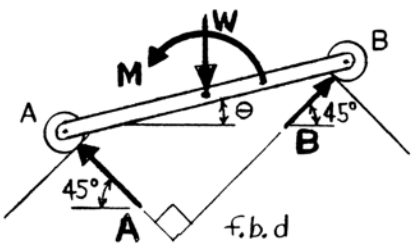
$$\therefore \quad 291 \text{ lb} \le T \le 413 \text{ lb} \blacktriangleleft$$



PROBLEM 4.53

Uniform rod AB of length l and weight W lies in a vertical plane and is acted upon by a couple **M**. The ends of the rod are connected to small rollers which rest against frictionless surfaces. (a) Express the angle θ corresponding to equilibrium in terms of M, W, and l. (b) Determine the value of θ corresponding to equilibrium when $M = 1.5\ \text{lb}\cdot\text{ft}$, $W = 4$ lb, and $l = 2$ ft.

SOLUTION



(a) From f.b.d. of uniform rod AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A\cos 45° + B\cos 45° = 0$$

$$\therefore\ -A + B = 0 \qquad \text{or} \qquad B = A \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad A\sin 45° + B\sin 45° - W = 0$$

$$\therefore\ A + B = \sqrt{2}W \tag{2}$$

From Equations (1) and (2)

$$2A = \sqrt{2}W$$

$$\therefore\ A = \frac{1}{\sqrt{2}}W$$

From f.b.d. of uniform rod AB

$$+\circlearrowleft \Sigma M_B = 0: \quad W\left[\left(\frac{l}{2}\right)\cos\theta\right] + M$$

$$-\left(\frac{1}{\sqrt{2}}W\right)\left[l\cos\left(45° - \theta\right)\right] = 0 \tag{3}$$

From trigonometric identity

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Equation (3) becomes

$$\left(\frac{Wl}{2}\right)\cos\theta + M - \left(\frac{Wl}{2}\right)\left(\cos\theta + \sin\theta\right) = 0$$

PROBLEM 4.53 CONTINUED

or $$\left(\frac{Wl}{2}\right)\cos\theta + M - \left(\frac{Wl}{2}\right)\cos\theta - \left(\frac{Wl}{2}\right)\sin\theta = 0$$

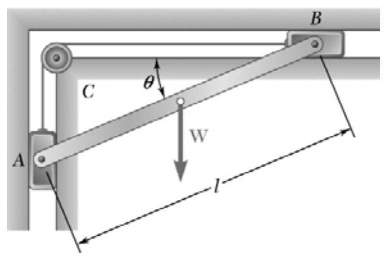
$$\therefore \quad \sin\theta = \frac{2M}{Wl}$$

or $\theta = \sin^{-1}\left(\frac{2M}{Wl}\right)$ ◀

(*b*) $$\theta = \sin^{-1}\left[\frac{2(1.5\text{ lb}\cdot\text{ft})}{(4\text{ lb})(2\text{ ft})}\right] = 22.024°$$

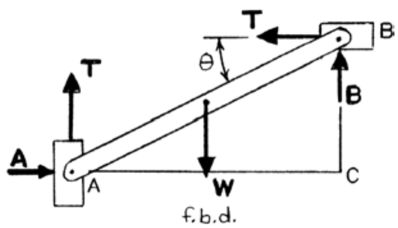
or $\theta = 22.0°$ ◀

PROBLEM 4.54



A slender rod AB, of weight W, is attached to blocks A and B, which move freely in the guides shown. The blocks are connected by an elastic cord which passes over a pulley at C. (*a*) Express the tension in the cord in terms of W and θ. (*b*) Determine the value of θ for which the tension in the cord is equal to $3W$.

SOLUTION



(*a*) From f.b.d. of rod AB

$$+\circlearrowleft \Sigma M_C = 0: \quad T(l\sin\theta) + W\left[\left(\frac{l}{2}\right)\cos\theta\right] - T(l\cos\theta) = 0$$

$$\therefore\ T = \frac{W\cos\theta}{2(\cos\theta - \sin\theta)}$$

Dividing both numerator and denominator by $\cos\theta$,

$$T = \frac{W}{2}\left(\frac{1}{1-\tan\theta}\right)$$

$$\text{or } T = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)} \blacktriangleleft$$

(*b*) For $T = 3W$,

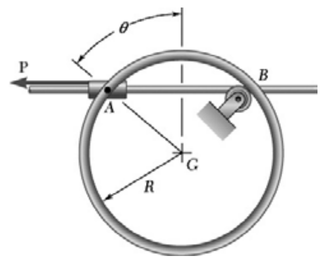
$$3W = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)}$$

$$\therefore\ 1 - \tan\theta = \frac{1}{6}$$

or $$\theta = \tan^{-1}\left(\frac{5}{6}\right) = 39.806°$$

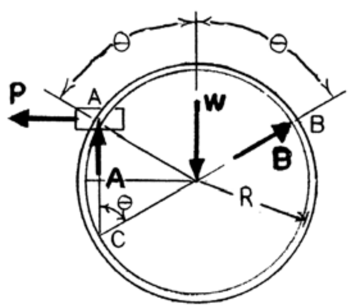
$$\text{or } \theta = 39.8° \blacktriangleleft$$

PROBLEM 4.55



A thin, uniform ring of mass m and radius R is attached by a frictionless pin to a collar at A and rests against a small roller at B. The ring lies in a vertical plane, and the collar can move freely on a horizontal rod and is acted upon by a horizontal force **P**. (*a*) Express the angle θ corresponding to equilibrium in terms of m and P. (*b*) Determine the value of θ corresponding to equilibrium when $m = 500$ g and $P = 5$ N.

SOLUTION



f.b.d.

(*a*) From f.b.d. of ring

$$+\circlearrowleft \Sigma M_C = 0: \quad P(R\cos\theta + R\cos\theta) - W(R\sin\theta) = 0$$

$$2P = W\tan\theta \quad \text{where} \quad W = mg$$

$$\therefore \quad \tan\theta = \frac{2P}{mg}$$

$$\text{or} \quad \theta = \tan^{-1}\left(\frac{2P}{mg}\right) \blacktriangleleft$$

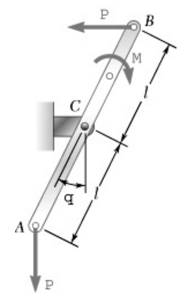
(*b*) Have $\quad m = 500 \text{ g} = 0.500 \text{ kg} \quad \text{and} \quad P = 5 \text{ N}$

$$\therefore \quad \theta = \tan^{-1}\left[\frac{2(5 \text{ N})}{(0.500 \text{ kg})(9.81 \text{ m/s}^2)}\right]$$

$$= 63.872^\circ$$

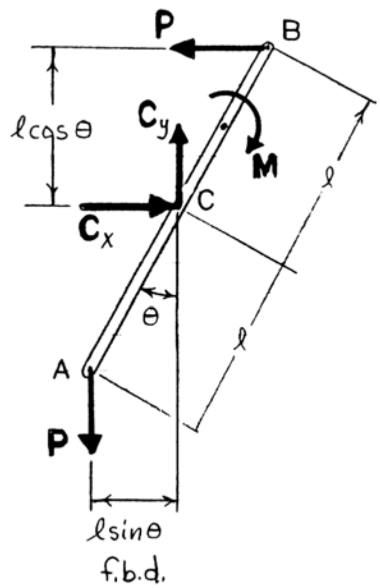
$$\text{or} \quad \theta = 63.9^\circ \blacktriangleleft$$

PROBLEM 4.56



Rod AB is acted upon by a couple **M** and two forces, each of magnitude P. (a) Derive an equation in θ, P, M, and l which must be satisfied when the rod is in equilibrium. (b) Determine the value of θ corresponding to equilibrium when $M = 150 \text{ lb} \cdot \text{in.}$, $P = 20 \text{ lb}$, and $l = 6 \text{ in.}$

SOLUTION



(a) From f.b.d. of rod AB

$$+\circlearrowleft \Sigma M_C = 0: \quad P(l\cos\theta) + P(l\sin\theta) - M = 0$$

$$\text{or} \quad \sin\theta + \cos\theta = \frac{M}{Pl} \blacktriangleleft$$

(b) For $M = 150 \text{ lb}\cdot\text{in.}$, $P = 20 \text{ lb}$, and $l = 6 \text{ in.}$

$$\sin\theta + \cos\theta = \frac{150 \text{ lb}\cdot\text{in.}}{(20 \text{ lb})(6 \text{ in.})} = \frac{5}{4} = 1.25$$

Using identity $\sin^2\theta + \cos^2\theta = 1$

$$\sin\theta + \left(1 - \sin^2\theta\right)^{\frac{1}{2}} = 1.25$$

$$\left(1 - \sin^2\theta\right)^{\frac{1}{2}} = 1.25 - \sin\theta$$

$$1 - \sin^2\theta = 1.5625 - 2.5\sin\theta + \sin^2\theta$$

$$2\sin^2\theta - 2.5\sin\theta + 0.5625 = 0$$

Using quadratic formula

$$\sin\theta = \frac{-(-2.5) \pm \sqrt{(6.25) - 4(2)(0.5625)}}{2(2)}$$

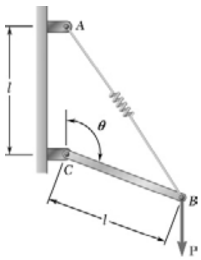
$$= \frac{2.5 \pm \sqrt{1.75}}{4}$$

or $\sin\theta = 0.95572$ and $\sin\theta = 0.29428$

$$\therefore \quad \theta = 72.886° \quad \text{and} \quad \theta = 17.1144°$$

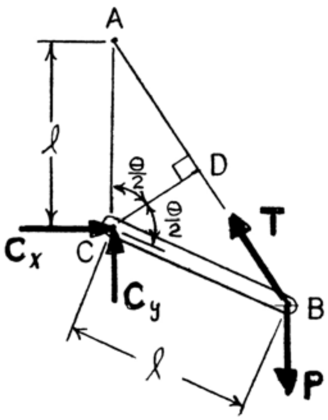
$$\text{or} \quad \theta = 17.11° \text{ and } \theta = 72.9° \blacktriangleleft$$

PROBLEM 4.57



A vertical load **P** is applied at end *B* of rod *BC*. The constant of the spring is *k*, and the spring is unstretched when $\theta = 90^\circ$. (*a*) Neglecting the weight of the rod, express the angle θ corresponding to equilibrium in terms of *P*, *k*, and *l*. (*b*) Determine the value of θ corresponding to equilibrium when $P = \frac{1}{4}kl$.

SOLUTION



First note

$$T = \text{tension in spring} = ks$$

where

$$\begin{aligned} s &= \text{elongation of spring} \\ &= \left(\overline{AB}\right)_\theta - \left(\overline{AB}\right)_{\theta=90^\circ} \\ &= 2l\sin\left(\frac{\theta}{2}\right) - 2l\sin\left(\frac{90^\circ}{2}\right) \\ &= 2l\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right] \end{aligned}$$

$$\therefore \quad T = 2kl\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right] \qquad (1)$$

(*a*) From f.b.d. of rod *BC*

$$+\circlearrowleft \Sigma M_C = 0: \quad T\left[l\cos\left(\frac{\theta}{2}\right)\right] - P(l\sin\theta) = 0$$

Substituting *T* From Equation (1)

$$2kl\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]\left[l\cos\left(\frac{\theta}{2}\right)\right] - P(l\sin\theta) = 0$$

$$2kl^2\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]\cos\left(\frac{\theta}{2}\right) - Pl\left[2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right] = 0$$

Factoring out $2l\cos\left(\frac{\theta}{2}\right)$, leaves

PROBLEM 4.57 CONTINUED

$$kl\left[\sin\left(\frac{\theta}{2}\right)-\left(\frac{1}{\sqrt{2}}\right)\right]-P\sin\left(\frac{\theta}{2}\right)=0$$

or

$$\sin\left(\frac{\theta}{2}\right)=\frac{1}{\sqrt{2}}\left(\frac{kl}{kl-P}\right)$$

$$\therefore\quad \theta=2\sin^{-1}\left[\frac{kl}{\sqrt{2}\left(kl-P\right)}\right] \blacktriangleleft$$

$(b)\quad P=\frac{kl}{4}$

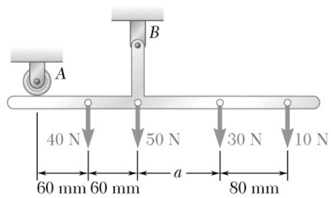
$$\theta=2\sin^{-1}\left[\frac{kl}{\sqrt{2}\left(kl-\frac{kl}{4}\right)}\right]=2\sin^{-1}\left[\frac{kl}{\sqrt{2}}\left(\frac{4}{3\text{ kl}}\right)\right]=2\sin^{-1}\left(\frac{4}{3\sqrt{2}}\right)$$

$$=2\sin^{-1}\left(0.94281\right)$$

$$=141.058^\circ$$

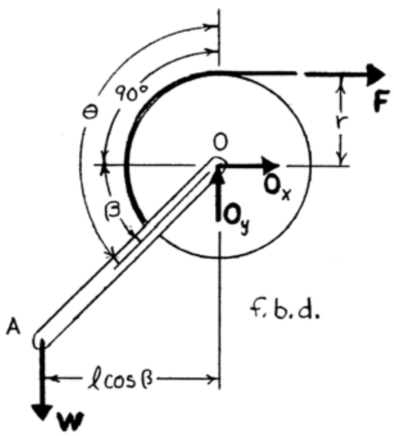
$$\text{or}\quad \theta=141.1^\circ \blacktriangleleft$$

PROBLEM 4.58



Solve Sample Problem 4.5 assuming that the spring is unstretched when $\theta = 90^\circ$.

SOLUTION



First note

$$T = \text{tension in spring} = ks$$

where

$$s = \text{deformation of spring}$$

$$= r\beta$$

$$\therefore \quad F = kr\beta$$

From f.b.d. of assembly

$$+\circlearrowleft \Sigma M_0 = 0: \quad W(l\cos\beta) - F(r) = 0$$

or

$$Wl\cos\beta - kr^2\beta = 0$$

$$\therefore \quad \cos\beta = \frac{kr^2}{Wl}\beta$$

For $\quad k = 250 \text{ lb/in.}, \ r = 3 \text{ in.}, \ l = 8 \text{ in.}, \ W = 400 \text{ lb}$

$$\cos\beta = \frac{(250 \text{ lb/in.})(3 \text{ in.})^2}{(400 \text{ lb})(8 \text{ in.})}\beta$$

or

$$\cos\beta = 0.703125\beta$$

Solving numerically,

$$\beta = 0.89245 \text{ rad}$$

or

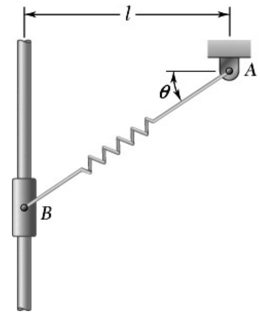
$$\beta = 51.134^\circ$$

Then

$$\theta = 90^\circ + 51.134^\circ = 141.134^\circ$$

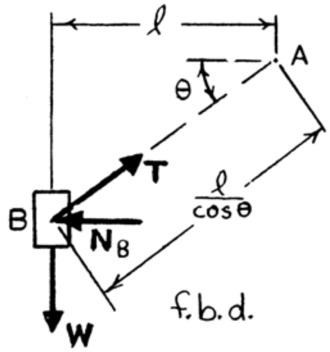
or $\theta = 141.1^\circ$ ◀

PROBLEM 4.59



A collar B of weight W can move freely along the vertical rod shown. The constant of the spring is k, and the spring is unstretched when $\theta = 0$. (a) Derive an equation in θ, W, k, and l which must be satisfied when the collar is in equilibrium. (b) Knowing that $W = 3$ lb, $l = 6$ in., and $k = 8$ lb/ft, determine the value of θ corresponding to equilibrium.

SOLUTION



First note $$T = ks$$

where $$k = \text{spring constant}$$

$$s = \text{elongation of spring}$$

$$= \frac{l}{\cos\theta} - l = \frac{l}{\cos\theta}(1 - \cos\theta)$$

$$\therefore \quad T = \frac{kl}{\cos\theta}(1 - \cos\theta)$$

(a) From f.b.d. of collar B

$$+\uparrow \Sigma F_y = 0: \quad T\sin\theta - W = 0$$

or $$\frac{kl}{\cos\theta}(1 - \cos\theta)\sin\theta - W = 0$$

or $\tan\theta - \sin\theta = \dfrac{W}{kl}$ ◀

(b) For $W = 3$ lb, $l = 6$ in., $k = 8$ lb/ft

$$l = \frac{6 \text{ in.}}{12 \text{ in./ft}} = 0.5 \text{ ft}$$

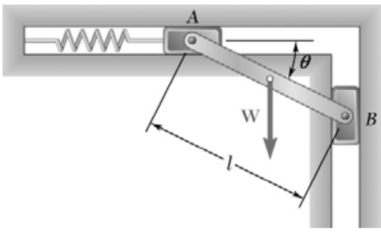
$$\tan\theta - \sin\theta = \frac{3 \text{ lb}}{(8 \text{ lb/ft})(0.5 \text{ ft})} = 0.75$$

Solving Numerically,

$$\theta = 57.957°$$

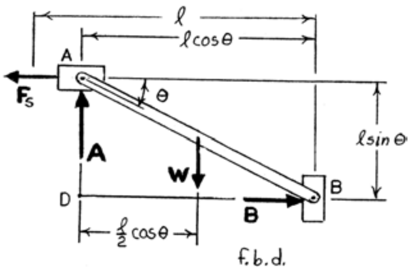
or $\theta = 58.0°$ ◀

PROBLEM 4.60



A slender rod AB, of mass m, is attached to blocks A and B which move freely in the guides shown. The constant of the spring is k, and the spring is unstretched when $\theta = 0$. (a) Neglecting the mass of the blocks, derive an equation in m, g, k, l, and θ which must be satisfied when the rod is in equilibrium. (b) Determine the value of θ when $m = 2$ kg, $l = 750$ mm, and $k = 30$ N/m.

SOLUTION



First note

$$F_s = \text{spring force} = ks$$

where $\quad k = \text{spring constant}$

$$s = \text{spring deformation}$$

$$= l - l\cos\theta$$

$$= l(1 - \cos\theta)$$

$$\therefore \quad F_s = kl(1 - \cos\theta)$$

(a) From f.b.d. of assembly

$$+\circlearrowleft \Sigma M_D = 0: \quad F_s(l\sin\theta) - W\left(\frac{l}{2}\cos\theta\right) = 0$$

$$kl(1 - \cos\theta)(l\sin\theta) - W\left(\frac{l}{2}\cos\theta\right) = 0$$

$$kl(\sin\theta - \cos\theta\sin\theta) - \left(\frac{W}{2}\right)\cos\theta = 0$$

Dividing by $\cos\theta$

$$kl(\tan\theta - \sin\theta) = \frac{W}{2}$$

$$\therefore \quad \tan\theta - \sin\theta = \frac{W}{2kl}$$

$$\text{or} \quad \tan\theta - \sin\theta = \frac{mg}{2kl} \blacktriangleleft$$

(b) For $m = 2$ kg, $l = 750$ mm, $k = 30$ N/m

$$l = 750 \text{ mm} = 0.750 \text{ m}$$

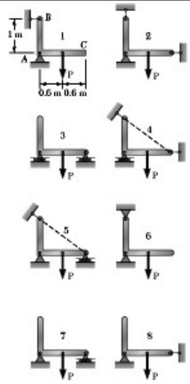
PROBLEM 4.60 CONTINUED

Then $$\tan\theta - \sin\theta = \frac{(2\text{ kg})(9.81\text{ m/s}^2)}{2(30\text{ N/m})(0.750\text{ m})} = 0.436$$

Solving Numerically,

$$\theta = 50.328°$$

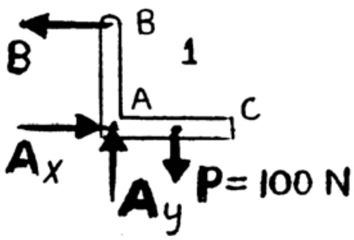
or $\theta = 50.3°$ ◀



PROBLEM 4.61

The bracket *ABC* can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions assuming that the magnitude of the force **P** is 100 N.

SOLUTION



1. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of bracket:

$$+\circlearrowleft \Sigma M_A = 0: \quad B(1\text{ m}) - (100\text{ N})(0.6\text{ m}) = 0$$

$$\therefore \quad \mathbf{B} = 60.0\text{ N} \longleftarrow$$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 60\text{ N} = 0$$

$$\therefore \quad \mathbf{A}_x = 60.0\text{ N} \longrightarrow$$

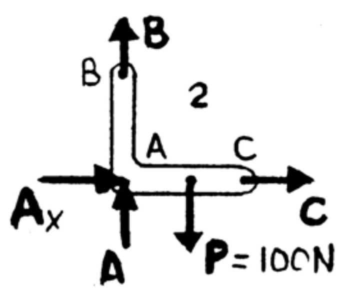
$$+\uparrow \Sigma F_y = 0: \quad A_y - 100\text{ N} = 0$$

$$\therefore \quad \mathbf{A}_y = 100\text{ N} \uparrow$$

Then $$A = \sqrt{(60.0)^2 + (100)^2} = 116.619\text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{100}{60.0}\right) = 59.036^\circ$$

$$\therefore \quad \mathbf{A} = 116.6\text{ N} \measuredangle 59.0^\circ$$ ◀

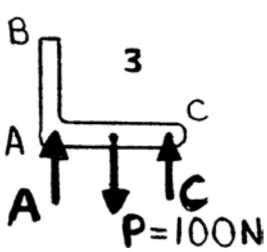


2. Four concurrent reactions through *A*

(*a*) Improperly constrained ◀

(*b*) Indeterminate ◀

(*c*) No equilibrium ◀



3. Two reactions

(*a*) Partially constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

PROBLEM 4.61 CONTINUED

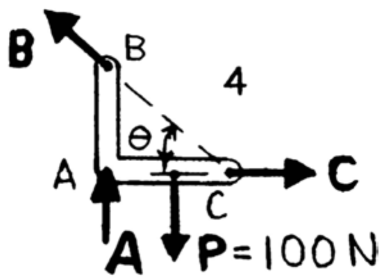
From f.b.d. of bracket

$$+\circlearrowleft \Sigma M_A = 0: \quad C(1.2 \text{ m}) - (100 \text{ N})(0.6 \text{ m}) = 0$$

$\therefore \quad \mathbf{C} = 50.0 \text{ N} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A - 100 \text{ N} + 50 \text{ N} = 0$$

$\therefore \quad \mathbf{A} = 50.0 \text{ N} \uparrow$ ◀



4. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of bracket

$$\theta = \tan^{-1}\left(\frac{1.0}{1.2}\right) = 39.8°$$

$$\overline{BC} = \sqrt{(1.2)^2 + (1.0)^2} = 1.56205 \text{ m}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad \left[\left(\frac{1.2}{1.56205}\right)B\right](1 \text{ m}) - (100 \text{ N})(0.6 \text{ m}) = 0$$

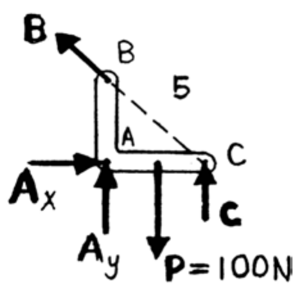
$\therefore \quad \mathbf{B} = 78.1 \text{ N} \searrow 39.8°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad C - (78.102 \text{ N})\cos 39.806° = 0$$

$\therefore \quad \mathbf{C} = 60.0 \text{ N} \longrightarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A + (78.102 \text{ N})\sin 39.806° - 100 \text{ N} = 0$$

$\therefore \quad \mathbf{A} = 50.0 \text{ N} \uparrow$ ◀



5. Four non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

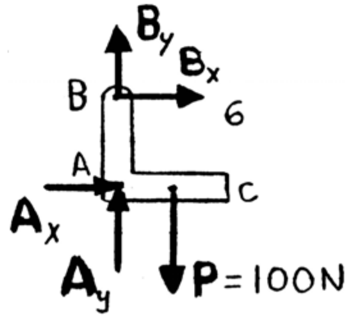
(*b*) Indeterminate ◀

(*c*) Equilibrium ◀

From f.b.d. of bracket

$$+\circlearrowleft \Sigma M_C = 0: \quad (100 \text{ N})(0.6 \text{ m}) - A_y(1.2 \text{ m}) = 0$$

$\therefore \quad A_y = 50 \text{ N}$ or $\mathbf{A}_y = 50.0 \text{ N} \uparrow$ ◀



6. Four non-concurrent non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Indeterminate ◀

(*c*) Equilibrium ◀

PROBLEM 4.61 CONTINUED

From f.b.d. of bracket

$$+\circlearrowright \Sigma M_A = 0: \quad -B_x(1 \text{ m}) - (100 \text{ N})(0.6 \text{ m}) = 0$$

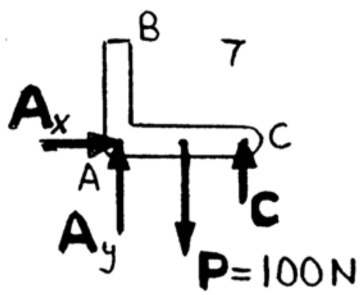
$$\therefore \quad B_x = -60.0 \text{ N}$$

or $\mathbf{B}_x = 60.0 \text{ N} \longleftarrow$ ◀

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad -60 + A_x = 0$$

$$\therefore \quad A_x = 60.0 \text{ N}$$

or $\mathbf{A}_x = 60.0 \text{ N} \longrightarrow$ ◀



7. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of bracket

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowright \Sigma M_A = 0: \quad C(1.2 \text{ m}) - (100 \text{ N})(0.6 \text{ m}) = 0$$

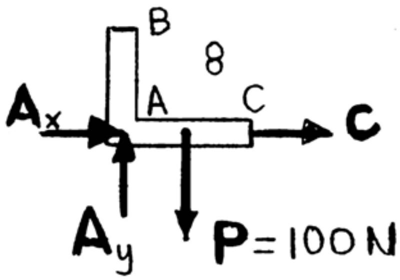
$$\therefore \quad C = 50.0 \text{ N}$$

or $\mathbf{C} = 50.0 \text{ N} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y - 100 \text{ N} + 50.0 \text{ N} = 0$$

$$\therefore \quad A_y = 50.0 \text{ N}$$

$\therefore \quad \mathbf{A} = 50.0 \text{ N} \uparrow$ ◀



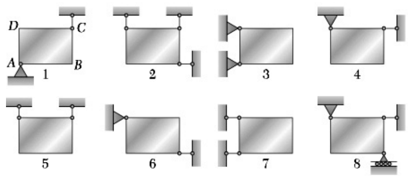
8. Three concurrent, non-parallel reactions

(*a*) Improperly constrained ◀

(*b*) Indeterminate ◀

(*c*) No equilibrium ◀

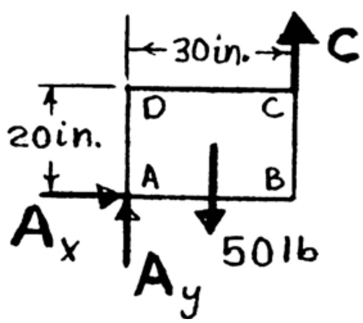
PROBLEM 4.62



Eight identical 20×30-in. rectangular plates, each weighing 50 lb, are held in a vertical plane as shown. All connections consist of frictionless pins, rollers, or short links. For each case, answer the questions listed in Problem 4.61, and, wherever possible, compute the reactions.

P6.1 The bracket *ABC* can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions assuming that the magnitude of the force **P** is 100 N.

SOLUTION



1. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of plate

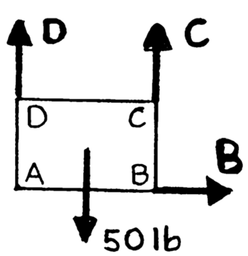
$$+\circlearrowleft \Sigma M_A = 0: \quad C(30 \text{ in.}) - 50 \text{ lb}(15 \text{ in.}) = 0$$

$$\mathbf{C} = 25.0 \text{ lb} \uparrow ◀$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 50 \text{ lb} + 25 \text{ lb} = 0$$

$$A_y = 25 \text{ lb} \qquad \mathbf{A} = 25.0 \text{ lb} \uparrow ◀$$



2. Three non-current, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of plate

$$\xrightarrow{+} \Sigma F_x = 0: \qquad \mathbf{B} = 0 ◀$$

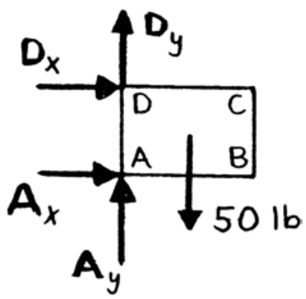
$$+\circlearrowleft \Sigma M_B = 0: \quad (50 \text{ lb})(15 \text{ in.}) - D(30 \text{ in.}) = 0$$

$$\mathbf{D} = 25.0 \text{ lb} \uparrow ◀$$

$$+\uparrow \Sigma F_y = 0: \quad 25.0 \text{ lb} - 50 \text{ lb} + C = 0$$

$$\mathbf{C} = 25.0 \text{ lb} \uparrow ◀$$

PROBLEM 4.62 CONTINUED



3. Four non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Indeterminate ◀

(*c*) Equilibrium ◀

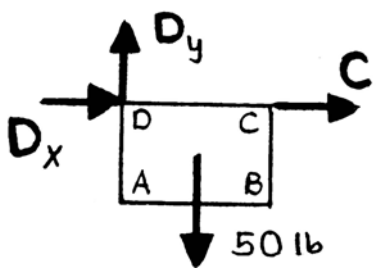
From f.b.d. of plate

$$+\circlearrowright \Sigma M_D = 0: \quad A_x(20 \text{ in.}) - (50 \text{ lb})(15 \text{ in.})$$

$$\therefore \quad \mathbf{A}_x = 37.5 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x + 37.5 \text{ lb} = 0$$

$$\therefore \quad \mathbf{D}_x = 37.5 \text{ lb} \longleftarrow \blacktriangleleft$$



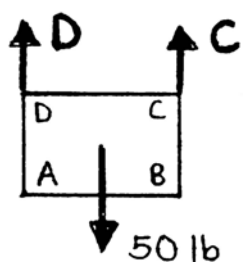
4. Three concurrent reactions

(*a*) Improperly constrained ◀

(*b*) Indeterminate ◀

(*c*) No equilibrium ◀

5. Two parallel reactions



(*a*) Partial constraint ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of plate

$$+\circlearrowright \Sigma M_D = 0: \quad C(30 \text{ in.}) - (50 \text{ lb})(15 \text{ in.}) = 0$$

$$\mathbf{C} = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad D - 50 \text{ lb} + 25 \text{ lb} = 0$$

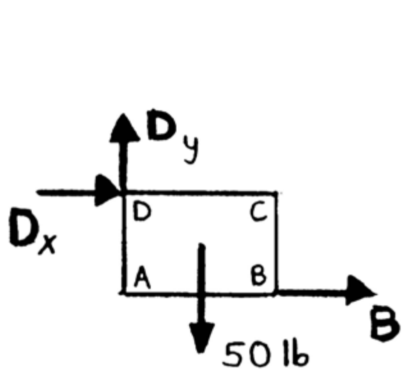
$$\mathbf{D} = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

6. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀



From f.b.d. of plate

$$+\circlearrowright \Sigma M_D = 0: \quad B(20 \text{ in.}) - (50 \text{ lb})(15 \text{ in.}) = 0$$

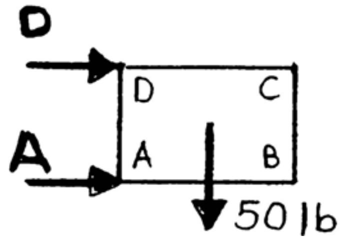
$$\mathbf{B} = 37.5 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x + 37.5 \text{ lb} = 0 \qquad \mathbf{D}_x = 37.5 \text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - 50 \text{ lb} = 0 \qquad \mathbf{D}_y = 50.0 \text{ lb} \uparrow$$

$$\text{or } \mathbf{D} = 62.5 \text{ lb} \searrow 53.1° \blacktriangleleft$$

PROBLEM 4.62 CONTINUED



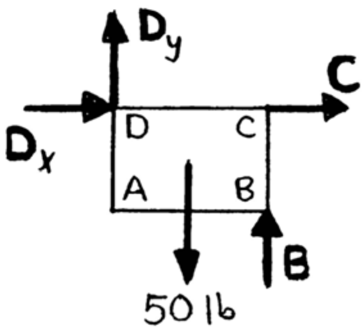
7. Two parallel reactions

(*a*) Improperly constrained ◀

(*b*) Reactions determined by dynamics ◀

(*c*) No equilibrium ◀

8. Four non-concurrent, non-parallel reactions



(*a*) Completely constrained ◀

(*b*) Indeterminate ◀

(*c*) Equilibrium ◀

From f.b.d. of plate

$$+\circlearrowleft \Sigma M_D = 0: \quad B(30 \text{ in.}) - (50 \text{ lb})(15 \text{ in.}) = 0$$

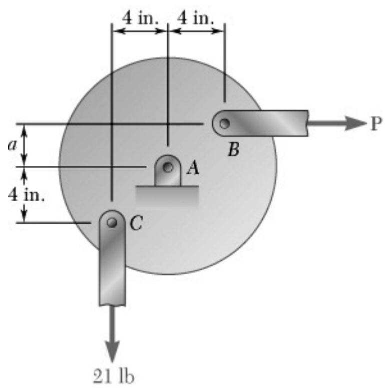
$\mathbf{B} = 25.0 \text{ lb} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad D_y - 50 \text{ lb} + 25.0 \text{ lb} = 0$$

$\mathbf{D}_y = 25.0 \text{ lb} \uparrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x + C = 0$$

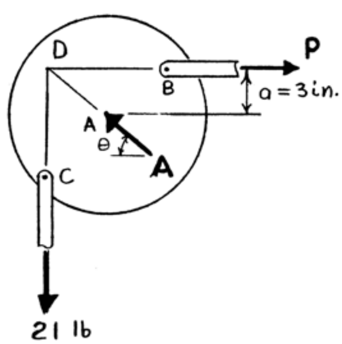
PROBLEM 4.63



Horizontal and vertical links are hinged to a wheel, and forces are applied to the links as shown. Knowing that $a = 3.0$ in., determine the value of P and the reaction at A.

SOLUTION

As shown on the f.b.d., the wheel is a three-force body. Let point D be the intersection of the three forces.

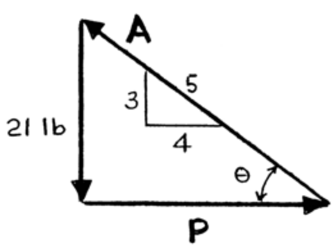


From force triangle

$$\frac{A}{5} = \frac{P}{4} = \frac{21 \text{ lb}}{3}$$

$$\therefore \quad P = \frac{4}{3}(21 \text{ lb}) = 28 \text{ lb}$$

or $P = 28.0$ lb ◀

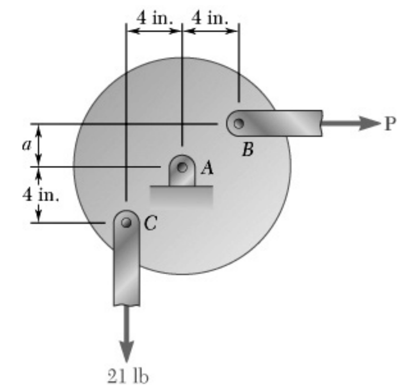


and $$A = \frac{5}{3}(21 \text{ lb}) = 35 \text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.870°$$

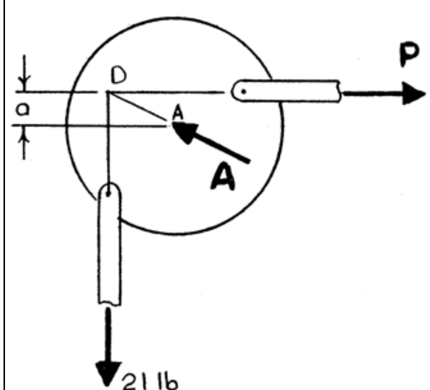
$\therefore$ **A** = 35.0 lb ⦪ 36.9° ◀

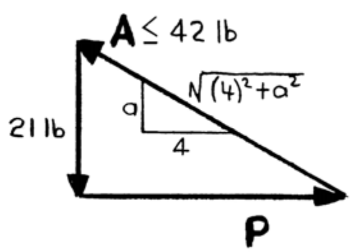
PROBLEM 4.64



Horizontal and vertical links are hinged to a wheel, and forces are applied to the links as shown. Determine the range of values of the distance a for which the magnitude of the reaction at A does not exceed 42 lb.

SOLUTION





Let D be the intersection of the three forces acting on the wheel.

From the force triangle

$$\frac{21\text{ lb}}{a} = \frac{A}{\sqrt{16 + a^2}}$$

or

$$A = 21\sqrt{\frac{16}{a^2} + 1}$$

For

$$A = 42\text{ lb}$$

$$\frac{21\text{ lb}}{a} = \frac{42\text{ lb}}{\sqrt{16 + a^2}}$$

or

$$a^2 = \frac{16 + a^2}{4}$$

or

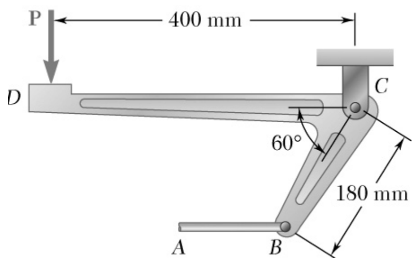
$$a = \sqrt{\frac{16}{3}} = 2.3094\text{ in.}$$

or $a \geq 2.31$ in. ◀

Since

$$A = 21\sqrt{\frac{16}{a^2} + 1}$$

as a increases, A decreases

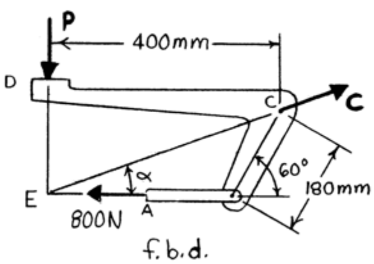


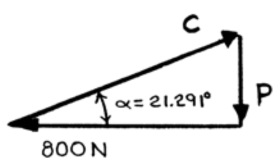
PROBLEM 4.65

Using the method of Section 4.7, solve Problem 4.21.

P4.21 The required tension in cable *AB* is 800 N. Determine (*a*) the vertical force **P** which must be applied to the pedal, (*b*) the corresponding reaction at *C*.

SOLUTION





Let *E* be the intersection of the three forces acting on the pedal device.

First note

$$\alpha = \tan^{-1}\left[\frac{(180\text{ mm})\sin 60^\circ}{400\text{ mm}}\right] = 21.291^\circ$$

From force triangle

(*a*)

$$P = (800\text{ N})\tan 21.291^\circ$$

$$= 311.76\text{ N}$$

or $\mathbf{P} = 312\text{ N}\downarrow$ ◀

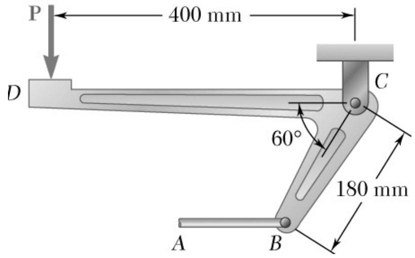
(*b*)

$$C = \frac{800\text{ N}}{\cos 21.291^\circ}$$

$$= 858.60\text{ N}$$

or $\mathbf{C} = 859\text{ N} \measuredangle 21.3^\circ$ ◀

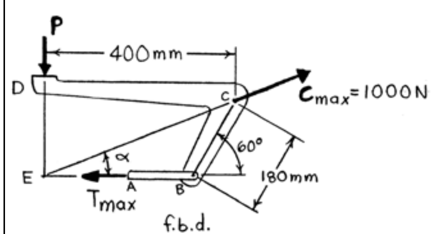
PROBLEM 4.66

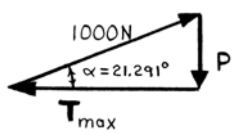


Using the method of Section 4.7, solve Problem 4.22.

P4.22 Determine the maximum tension which can be developed in cable *AB* if the maximum allowable value of the reaction at *C* is 1000 N.

SOLUTION





Let *E* be the intersection of the three forces acting on the pedal device.

First note

$$\alpha = \tan^{-1}\left[\frac{(180 \text{ mm})\sin 60^\circ}{400 \text{ mm}}\right] = 21.291^\circ$$

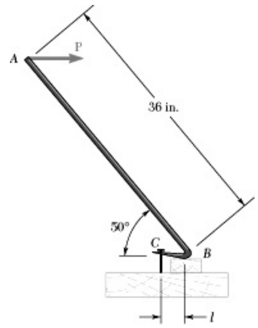
From force triangle

$$T_{\max} = (1000 \text{ N})\cos 21.291^\circ$$

$$= 931.75 \text{ N}$$

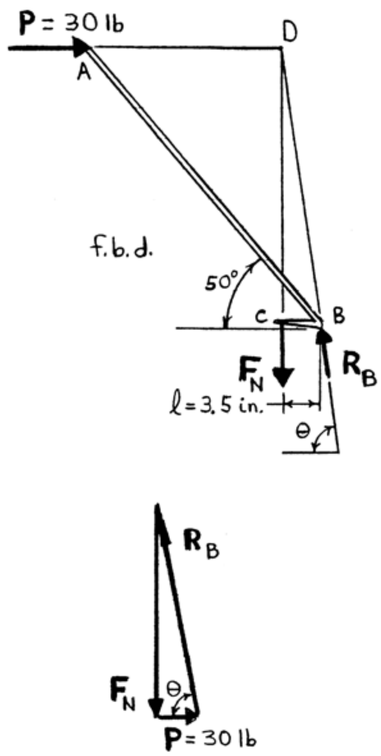
or $T_{\max} = 932 \text{ N}$ ◀

PROBLEM 4.67



To remove a nail, a small block of wood is placed under a crowbar, and a horizontal force **P** is applied as shown. Knowing that $l = 3.5$ in. and $P = 30$ lb, determine the vertical force exerted on the nail and the reaction at B.

SOLUTION



Let D be the intersection of the three forces acting on the crowbar.

First note

$$\theta = \tan^{-1}\left[\frac{(36 \text{ in.})\sin 50°}{3.5 \text{ in.}}\right] = 82.767°$$

From force triangle

$$F_N = P\tan\theta = (30 \text{ lb})\tan 82.767°$$

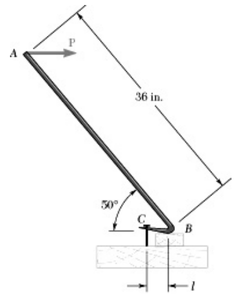
$$= 236.381 \text{ lb}$$

$\therefore$ on nail $\mathbf{F}_N = 236 \text{ lb} \uparrow$ ◀

$$R_B = \frac{P}{\cos\theta} = \frac{30 \text{ lb}}{\cos 82.767°} = 238.28 \text{ lb}$$

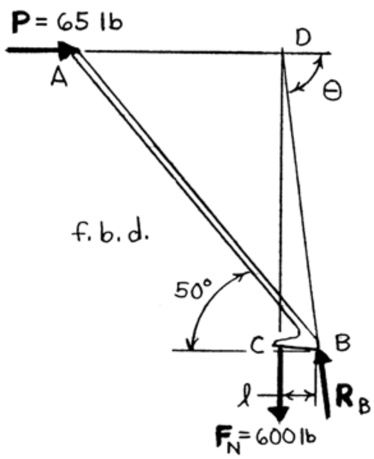
or $\mathbf{R}_B = 238 \text{ lb} \measuredangle 82.8°$ ◀

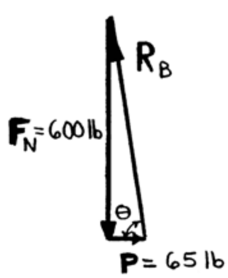
PROBLEM 4.68



To remove a nail, a small block of wood is placed under a crowbar, and a horizontal force **P** is applied as shown. Knowing that the maximum vertical force needed to extract the nail is 600 lb and that the horizontal force **P** is not to exceed 65 lb, determine the largest acceptable value of distance *l*.

SOLUTION





Let *D* be the intersection of the three forces acting on the crowbar.

From force diagram

$$\tan\theta = \frac{F_N}{P} = \frac{600\text{ lb}}{65\text{ lb}} = 9.2308$$

$$\therefore \quad \theta = 83.817°$$

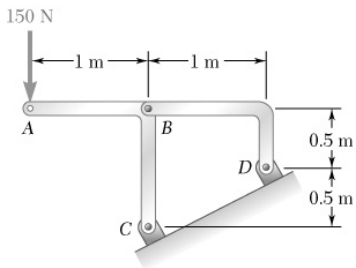
From f.b.d.

$$\tan\theta = \frac{(36\text{ in.})\sin 50°}{l}$$

$$\therefore \quad l = \frac{(36\text{ in.})\sin 50°}{\tan 83.817°} = 2.9876\text{ in.}$$

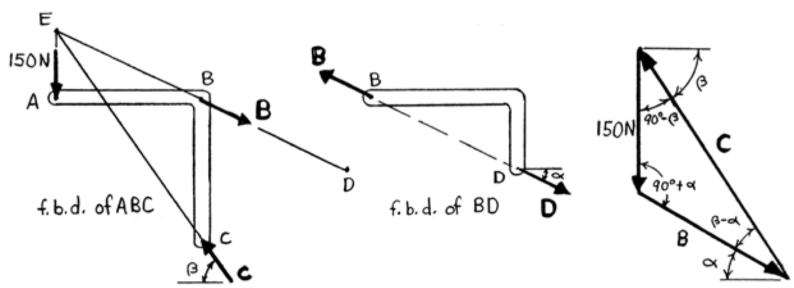
or $l = 2.99$ in. ◀

PROBLEM 4.69



For the frame and loading shown, determine the reactions at *C* and *D*.

SOLUTION



Since member *BD* is acted upon by two forces, **B** and **D**, they must be colinear, have the same magnitude, and be opposite in direction for *BD* to be in equilibrium. The force **B** acting at *B* of member *ABC* will be equal in magnitude but opposite in direction to force **B** acting on member *BD*. Member *ABC* is a three-force body with member forces intersecting at *E*. The f.b.d.'s of members *ABC* and *BD* illustrate the above conditions. The force triangle for member *ABC* is also shown. The angles α and β are found from the member dimensions:

$$\alpha = \tan^{-1}\left(\frac{0.5\text{ m}}{1.0\text{ m}}\right) = 26.565°$$

$$\beta = \tan^{-1}\left(\frac{1.5\text{ m}}{1.0\text{ m}}\right) = 56.310°$$

Applying the law of sines to the force triangle for member *ABC*,

$$\frac{150\text{ N}}{\sin(\beta - \alpha)} = \frac{C}{\sin(90° + \alpha)} = \frac{B}{\sin(90° - \beta)}$$

or

$$\frac{150\text{ N}}{\sin 29.745°} = \frac{C}{\sin 116.565°} = \frac{B}{\sin 33.690°}$$

$$\therefore\ C = \frac{(150\text{ N})\sin 116.565°}{\sin 29.745°} = 270.42\text{ N}$$

or **C** = 270 N ⦨ 56.3° ◀

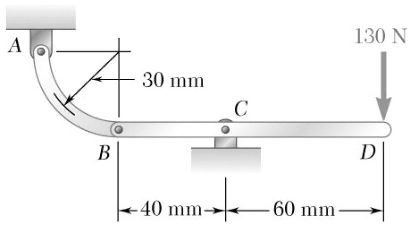
and

$$D = B = \frac{(150\text{ N})\sin 33.690°}{\sin 29.745°} = 167.704\text{ N}$$

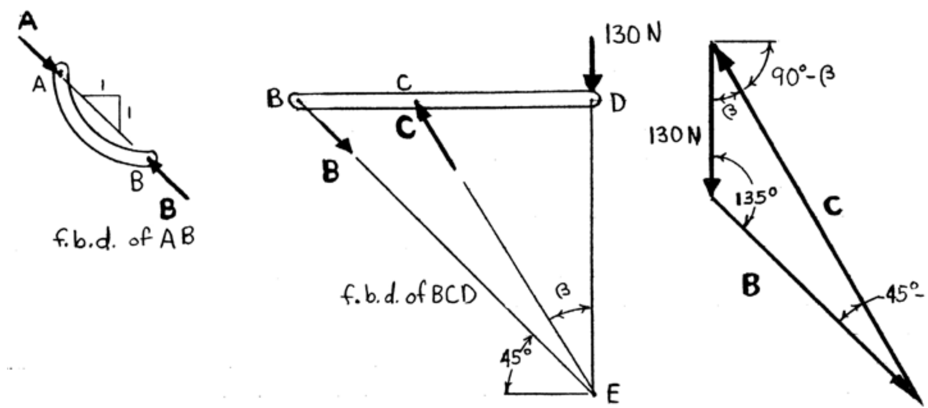
or **D** = 167.7 N ⦨ 26.6° ◀

PROBLEM 4.70

For the frame and loading shown, determine the reactions at *A* and *C*.



SOLUTION



Since member *AB* is acted upon by two forces, **A** and **B**, they must be colinear, have the same magnitude, and be opposite in direction for *AB* to be in equilibrium. The force **B** acting at *B* of member *BCD* will be equal in magnitude but opposite in direction to force **B** acting on member *AB*. Member *BCD* is a three-force body with member forces intersecting at *E*. The f.b.d.'s of members *AB* and *BCD* illustrate the above conditions. The force triangle for member *BCD* is also shown. The angle β is found from the member dimensions:

$$\beta = \tan^{-1}\left(\frac{60 \text{ m}}{100 \text{ m}}\right) = 30.964°$$

Applying of the law of sines to the force triangle for member *BCD*,

$$\frac{130 \text{ N}}{\sin(45° - \beta)} = \frac{B}{\sin\beta} = \frac{C}{\sin 135°}$$

or

$$\frac{130 \text{ N}}{\sin 14.036°} = \frac{B}{\sin 30.964°} = \frac{C}{\sin 135°}$$

$$\therefore \quad A = B = \frac{(130 \text{ N})\sin 30.964°}{\sin 14.036°} = 275.78 \text{ N}$$

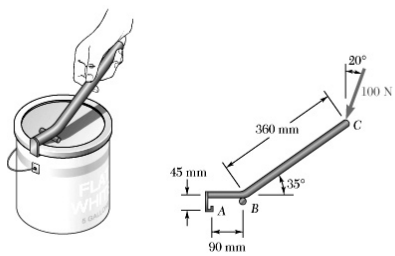
or $\mathbf{A} = 276$ N ↖ 45.0° ◀

and

$$C = \frac{(130 \text{ N})\sin 135°}{\sin 14.036°} = 379.02 \text{ N}$$

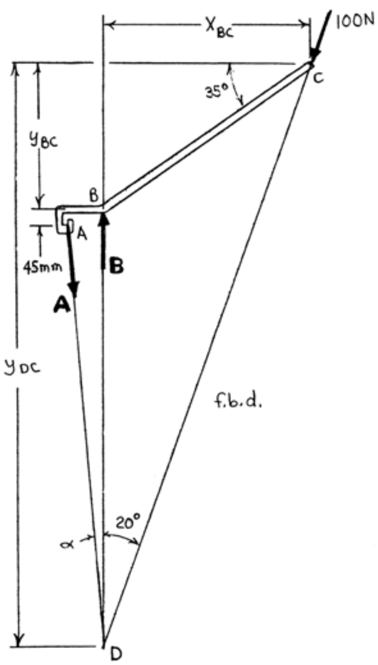
or $\mathbf{C} = 379$ N ↘ 59.0° ◀

PROBLEM 4.71



To remove the lid from a 5-gallon pail, the tool shown is used to apply an upward and radially outward force to the bottom inside rim of the lid. Assuming that the rim rests against the tool at *A* and that a 100-N force is applied as indicated to the handle, determine the force acting on the rim.

SOLUTION



The three-force member *ABC* has forces that intersect at *D,* where

$$\alpha = \tan^{-1}\left(\frac{90 \text{ mm}}{y_{DC} - y_{BC} - 45 \text{ mm}}\right)$$

and

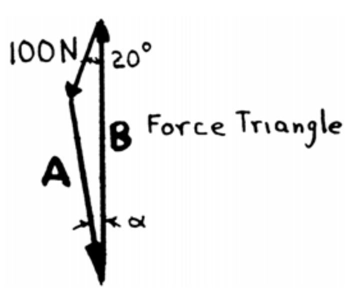
$$y_{DC} = \frac{x_{BC}}{\tan 20°} = \frac{(360 \text{ mm})\cos 35°}{\tan 20°}$$

$$= 810.22 \text{ mm}$$

$$y_{BC} = (360 \text{ mm})\sin 35°$$

$$= 206.49 \text{ mm}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{90}{558.73}\right) = 9.1506°$$



Based on the force triangle, the law of sines gives

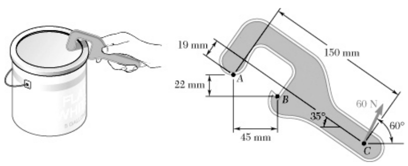
$$\frac{100 \text{ N}}{\sin\alpha} = \frac{A}{\sin 20°}$$

$$\therefore A = \frac{(100 \text{ N})\sin 20°}{\sin 9.1506°} = 215.07 \text{ N}$$

or $\mathbf{A} = 215 \text{ N}$ ↖ 80.8° on tool

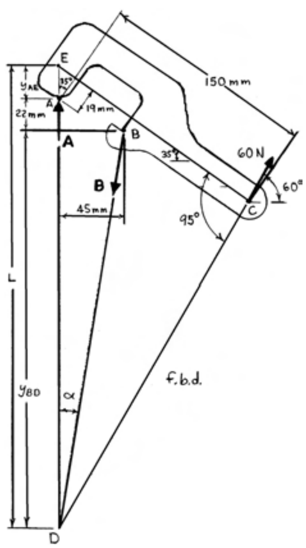
and $\mathbf{A} = 215 \text{ N}$ ↘ 80.8° on rim of can ◀

PROBLEM 4.72



To remove the lid from a 5-gallon pail, the tool shown is used to apply an upward and radially outward force to the bottom inside rim of the lid. Assuming that the top and the rim of the lid rest against the tool at *A* and *B*, respectively, and that a 60-N force is applied as indicated to the handle, determine the force acting on the rim.

SOLUTION



The three-force member *ABC* has forces that intersect at point *D*, where, from the law of sines (ΔCDE)

$$\frac{L}{\sin 95^\circ} = \frac{150\text{ mm} + (19\text{ mm})\tan 35^\circ}{\sin 30^\circ}$$

$$\therefore L = 325.37\text{ mm}$$

Then

$$\alpha = \tan^{-1}\left(\frac{45\text{ mm}}{y_{BD}}\right)$$

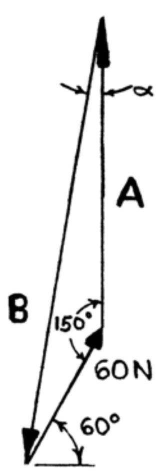
where

$$y_{BD} = L - y_{AE} - 22\text{ mm}$$

$$= 325.37\text{ mm} - \frac{19\text{ mm}}{\cos 35^\circ} - 22\text{ mm}$$

$$= 280.18\text{ mm}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{45\text{ mm}}{280.18\text{ mm}}\right) = 9.1246^\circ$$



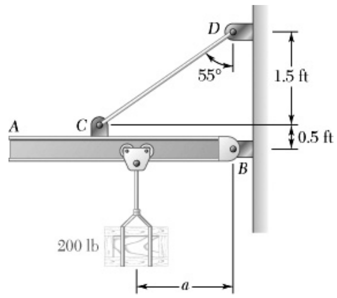
Applying the law of sines to the force triangle,

$$\frac{B}{\sin 150^\circ} = \frac{60\text{ N}}{\sin 9.1246^\circ}$$

$$\therefore B = 189.177\text{ N}$$

Or, on member **B** = 189.2 N ⦣ 80.9°

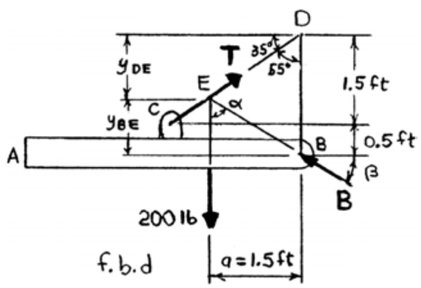
and, on lid **B** = 189.2 N ⦣ 80.9° ◀



PROBLEM 4.73

A 200-lb crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ ft, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

SOLUTION



From geometry of forces

$$\beta = \tan^{-1}\left(\frac{y_{BE}}{1.5 \text{ ft}}\right)$$

where

$$y_{BE} = 2.0 - y_{DE}$$

$$= 2.0 - 1.5\tan 35°$$

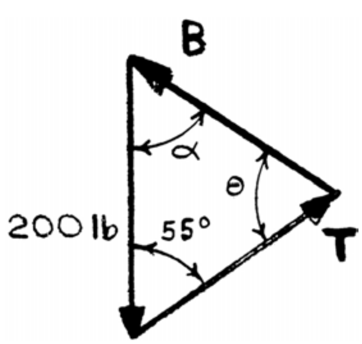
$$= 0.94969 \text{ ft}$$

$$\therefore \beta = \tan^{-1}\left(\frac{0.94969}{1.5}\right) = 32.339°$$

and $\alpha = 90° - \beta = 90° - 32.339° = 57.661°$

$$\theta = \beta + 35° = 32.339° + 35° = 67.339°$$

Applying the law of sines to the force triangle,



$$\frac{200 \text{ lb}}{\sin\theta} = \frac{T}{\sin\alpha} = \frac{B}{\sin 55°}$$

or

$$\frac{(200 \text{ lb})}{\sin 67.339°} = \frac{T}{\sin 57.661°} = \frac{B}{\sin 55°}$$

(*a*)

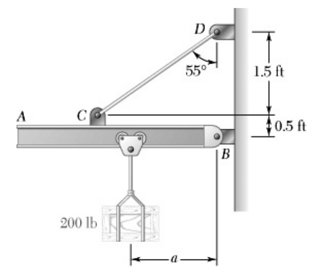
$$T = \frac{(200 \text{ lb})(\sin 57.661°)}{\sin 67.339°} = 183.116 \text{ lb}$$

or $T = 183.1$ lb ◀

(*b*)

$$B = \frac{(200 \text{ lb})(\sin 55°)}{\sin 67.339°} = 177.536 \text{ lb}$$

or **B** = 177.5 lb ⦨ 32.3° ◀

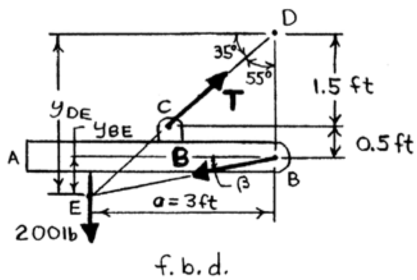


PROBLEM 4.74

Solve Problem 4.73 assuming that $a = 3$ ft.

P4.73 A 200-lb crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ ft, determine (*a*) the tension in cable *CD*, (*b*) the reaction
at *B*.

SOLUTION



From geometry of forces

$$\beta = \tan^{-1}\left(\frac{y_{BE}}{3 \text{ ft}}\right)$$

where

$$y_{BE} = y_{DE} - 2.0 \text{ ft}$$

$$= 3\tan 35° - 2.0$$

$$= 0.100623 \text{ ft}$$

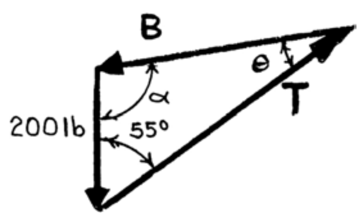
$$\therefore \beta = \tan^{-1}\left(\frac{0.100623}{3}\right) = 1.92103°$$

and

$$\alpha = 90° + \beta = 90° + 1.92103° = 91.921°$$

$$\theta = 35° - \beta = 35° - 1.92103° = 33.079°$$

Applying the law of sines to the force triangle,



$$\frac{200 \text{ lb}}{\sin\theta} = \frac{T}{\sin\alpha} = \frac{B}{\sin 55°}$$

or

$$\frac{200 \text{ lb}}{\sin 33.079°} = \frac{T}{\sin 91.921°} = \frac{\text{B}}{\sin 55°}$$

(*a*)

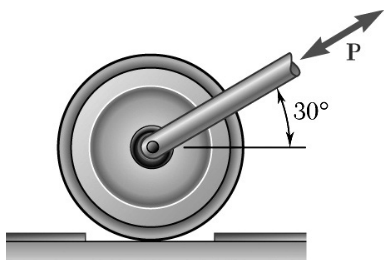
$$T = \frac{(200 \text{ lb})(\sin 91.921°)}{\sin 33.079°} = 366.23 \text{ lb}$$

or $T = 366$ lb ◀

(*b*)

$$B = \frac{(200 \text{ lb})(\sin 55°)}{\sin 33.079°} = 300.17 \text{ lb}$$

or **B** = 300 lb ↗ 1.921° ◀

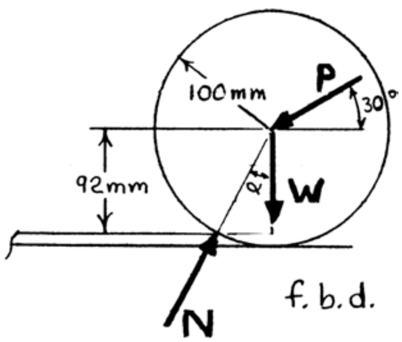


PROBLEM 4.75

A 20-kg roller, of diameter 200 mm, which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 8 mm, determine the force **P** required to move the roller onto the tiles if the roller is pushed to the left.

SOLUTION

Based on the roller having impending motion to the left, the only contact between the roller and floor will be at the edge of the tile.



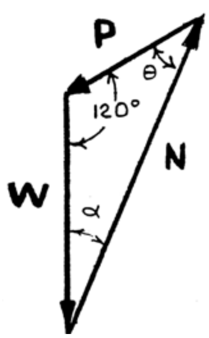
First note $W = mg = (20\text{ kg})(9.81\text{ m/s}^2) = 196.2\text{ N}$

From the geometry of the three forces acting on the roller

$$\alpha = \cos^{-1}\left(\frac{92\text{ mm}}{100\text{ mm}}\right) = 23.074°$$

and

$$\theta = 90° - 30° - \alpha$$
$$= 60° - 23.074$$
$$= 36.926°$$



Applying the law of sines to the force triangle,

$$\frac{W}{\sin\theta} = \frac{P}{\sin\alpha}$$

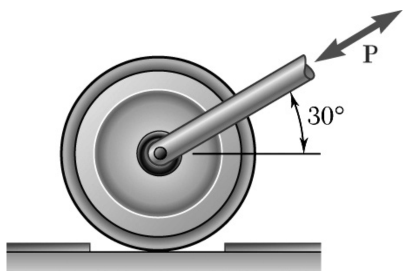
or

$$\frac{196.2\text{ N}}{\sin 36.926°} = \frac{P}{\sin 23.074°}$$

$$\therefore P = 127.991\text{ N}$$

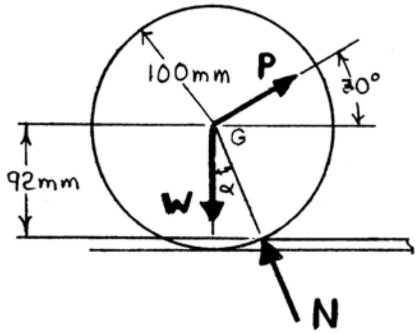
or **P** = 128.0 N ↗ 30° ◀

PROBLEM 4.76



A 20-kg roller, of diameter 200 mm, which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 8 mm, determine the force **P** required to move the roller onto the tiles if the roller is pulled to the right.

SOLUTION



Based on the roller having impending motion to the right, the only contact between the roller and floor will be at the edge of the tile.

First note $W = mg = (20 \text{ kg})(9.81 \text{ m/s}^2)$

$$= 196.2 \text{ N}$$

From the geometry of the three forces acting on the roller

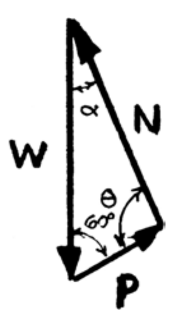
$$\alpha = \cos^{-1}\left(\frac{92 \text{ mm}}{100 \text{ mm}}\right) = 23.074°$$

and

$$\theta = 90° + 30° - \alpha$$

$$= 120° - 23.074°$$

$$= 96.926°$$



Applying the law of sines to the force triangle,

$$\frac{W}{\sin\theta} = \frac{P}{\sin\alpha}$$

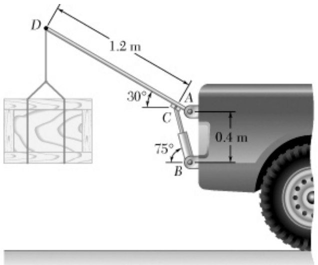
or

$$\frac{196.2 \text{ N}}{\sin 96.926°} = \frac{P}{\sin 23.074}$$

$$\therefore P = 77.460 \text{ N}$$

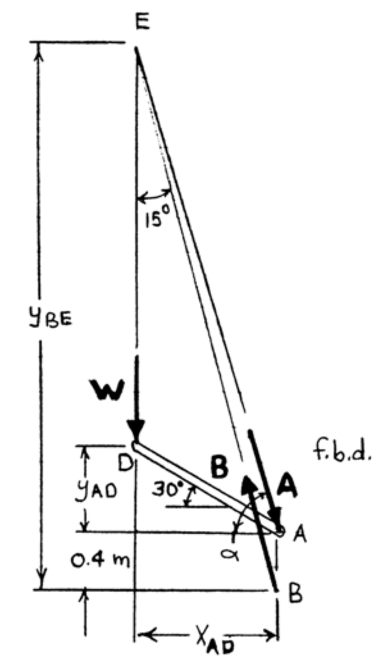
or $\mathbf{P} = 77.5 \text{ N} \measuredangle 30°$ ◀

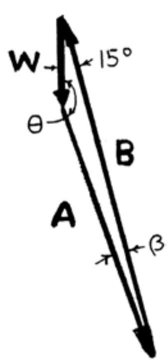
PROBLEM 4.77



A small hoist is mounted on the back of a pickup truck and is used to lift a 120-kg crate. Determine (*a*) the force exerted on the hoist by the hydraulic cylinder *BC*, (*b*) the reaction at *A*.

SOLUTION





First note $W = mg = (120 \text{ kg})(9.81 \text{ m/s}^2) = 1177.2 \text{ N}$

From the geometry of the three forces acting on the small hoist

$$x_{AD} = (1.2 \text{ m})\cos 30° = 1.03923 \text{ m}$$

$$y_{AD} = (1.2 \text{ m})\sin 30° = 0.6 \text{ m}$$

and

$$y_{BE} = x_{AD}\tan 75° = (1.03923 \text{ m})\tan 75° = 3.8785 \text{ m}$$

Then

$$\alpha = \tan^{-1}\left(\frac{y_{BE} - 0.4 \text{ m}}{x_{AD}}\right) = \tan^{-1}\left(\frac{3.4785}{1.03923}\right) = 73.366°$$

$$\beta = 75° - \alpha = 75° - 73.366° = 1.63412°$$

$$\theta = 180° - 15° - \beta = 165° - 1.63412° = 163.366°$$

Applying the law of sines to the force triangle,

$$\frac{W}{\sin\beta} = \frac{B}{\sin\theta} = \frac{A}{\sin 15°}$$

or

$$\frac{1177.2 \text{ N}}{\sin 1.63412°} = \frac{B}{\sin 163.366°} = \frac{A}{\sin 15°}$$

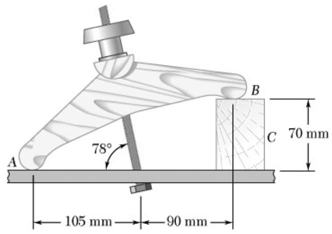
(*a*) $B = 11\,816.9 \text{ N}$

or $\mathbf{B} = 11.82 \text{ kN} \measuredangle\ 75.0°$ ◀

(*b*) $A = 10\,684.2 \text{ N}$

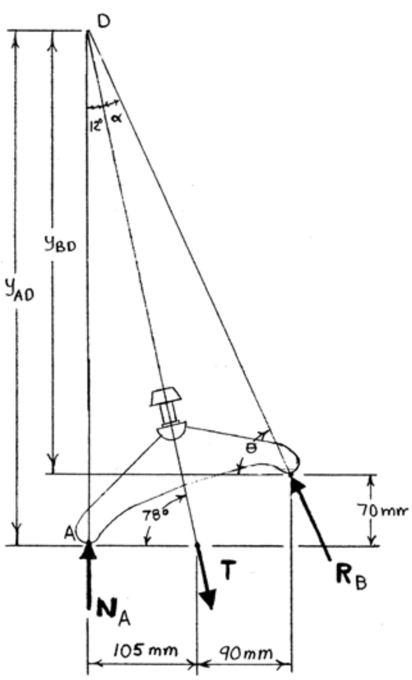
or $\mathbf{A} = 10.68 \text{ kN} \measuredangle\ 73.4°$ ◀

PROBLEM 4.78



The clamp shown is used to hold the rough workpiece *C*. Knowing that the maximum allowable compressive force on the workpiece is 200 N and neglecting the effect of friction at *A*, determine the corresponding (*a*) reaction at *B*, (*b*) reaction at *A*, (*c*) tension in the bolt.

SOLUTION



From the geometry of the three forces acting on the clamp

$$y_{AD} = (105 \text{ mm})\tan 78° = 493.99 \text{ mm}$$

$$y_{BD} = y_{AD} - 70 \text{ mm} = (493.99 - 70) \text{ mm} = 423.99 \text{ mm}$$

Then $$\theta = \tan^{-1}\left(\frac{y_{BD}}{195 \text{ mm}}\right) = \tan^{-1}\left(\frac{423.99}{195}\right) = 65.301°$$

$$\alpha = 90° - \theta - 12° = 78° - 65.301° = 12.6987°$$

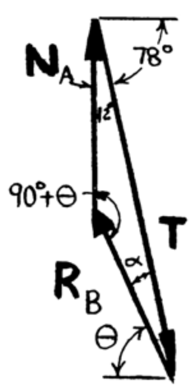
(*a*) Based on the maximum allowable compressive force on the workpiece of 200 N,

$$(R_B)_y = 200 \text{ N}$$

or $$R_B \sin\theta = 200 \text{ N}$$

$$\therefore R_B = \frac{200 \text{ N}}{\sin 65.301°} = 220.14 \text{ N}$$

or $\mathbf{R}_B = 220$ N ⦪ 65.3° ◀



Applying the law of sines to the force triangle,

$$\frac{R_B}{\sin 12°} = \frac{N_A}{\sin\alpha} = \frac{T}{\sin(90° + \theta)}$$

or $$\frac{220.14 \text{ N}}{\sin 12°} = \frac{N_A}{\sin 12.6987°} = \frac{T}{\sin 155.301°}$$

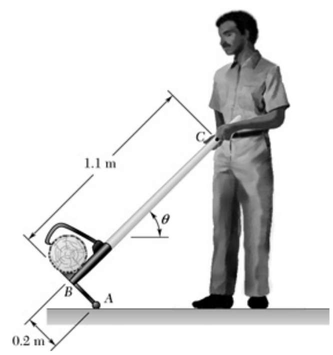
(*b*) $$N_A = 232.75 \text{ N}$$

or $\mathbf{N}_A = 233$ N ↑ ◀

(*c*) $$T = 442.43 \text{ N}$$

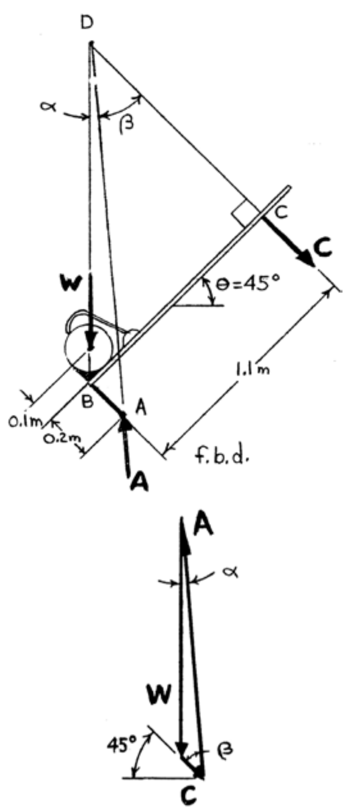
or $T = 442$ N ◀

PROBLEM 4.79



A modified peavey is used to lift a 0.2-m-diameter log of mass 36 kg. Knowing that $\theta = 45°$ and that the force exerted at C by the worker is perpendicular to the handle of the peavey, determine (*a*) the force exerted at C, (*b*) the reaction at A.

SOLUTION



First note $W = mg = (36 \text{ kg})(9.81 \text{ m/s}^2) = 353.16 \text{ N}$

From the geometry of the three forces acting on the modified peavey

$$\beta = \tan^{-1}\left(\frac{1.1 \text{ m}}{1.1 \text{ m} + 0.2 \text{ m}}\right) = 40.236°$$

$$\alpha = 45° - \beta = 45° - 40.236° = 4.7636°$$

Applying the law of sines to the force triangle,

$$\frac{W}{\sin\beta} = \frac{C}{\sin\alpha} = \frac{A}{\sin 135°}$$

or

$$\frac{353.16 \text{ N}}{\sin 40.236°} = \frac{C}{\sin 4.7636} = \frac{A}{\sin 135°}$$

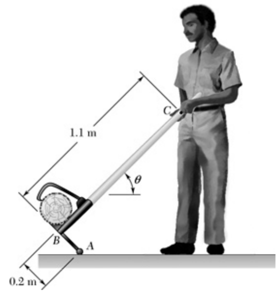
(*a*) $C = 45.404 \text{ N}$

or $\mathbf{C} = 45.4 \text{ N} \measuredangle 45.0°$ ◀

(*b*) $A = 386.60 \text{ N}$

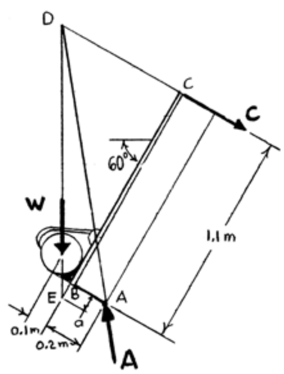
or $\mathbf{A} = 387 \text{ N} \measuredangle 85.2°$ ◀

PROBLEM 4.80



A modified peavey is used to lift a 0.2-m-diameter log of mass 36 kg. Knowing that $\theta = 60°$ and that the force exerted at C by the worker is perpendicular to the handle of the peavey, determine (*a*) the force exerted at C, (*b*) the reaction at A.

SOLUTION



First note $\quad W = mg = (36\text{ kg})(9.81\text{ m/s}^2) = 353.16\text{ N}$

From the geometry of the three forces acting on the modified peavey

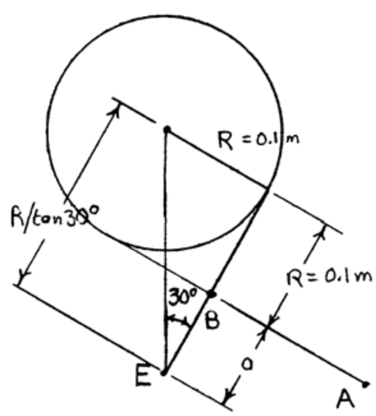
$$\beta = \tan^{-1}\left(\frac{1.1\text{ m}}{DC + 0.2\text{ m}}\right)$$

where $\quad DC = (1.1\text{ m} + a)\tan 30°$

$$a = \left(\frac{R}{\tan 30°}\right) - R$$

$$= \left(\frac{0.1\text{ m}}{\tan 30°}\right) - 0.1\text{ m}$$

$$= 0.073205\text{ m}$$



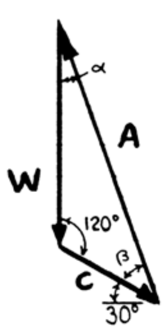
$$\therefore\ DC = (1.173205)\tan 30°$$

$$= 0.67735\text{ m}$$

and $\quad \beta = \tan^{-1}\left(\frac{1.1}{0.87735}\right) = 51.424°$

$$\alpha = 60° - \beta = 60° - 51.424° = 8.5756°$$

Applying the law of sines to the force triangle,



$$\frac{W}{\sin\beta} = \frac{C}{\sin\alpha} = \frac{A}{\sin 120°}$$

or

$$\frac{353.16\text{ N}}{\sin 51.424°} = \frac{C}{\sin 8.5756°} = \frac{A}{\sin 120°}$$

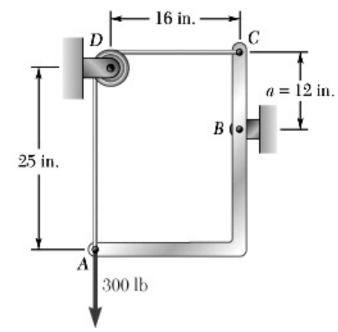
(*a*) $\quad C = 67.360\text{ N}$

or $\mathbf{C} = 67.4\text{ N}$ ⦨ $30°$ ◀

(*b*) $\quad A = 391.22\text{ N}$

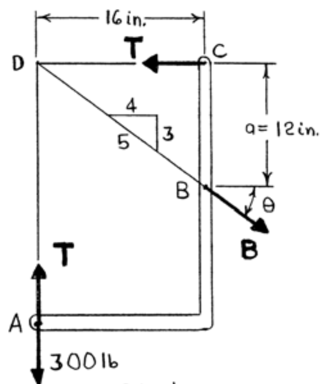
or $\mathbf{A} = 391\text{ N}$ ⦩ $81.4°$ ◀

PROBLEM 4.81

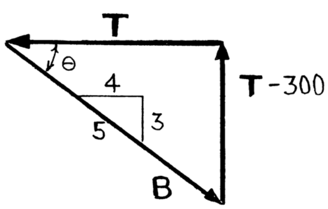


Member *ABC* is supported by a pin and bracket at *B* and by an inextensible cord at *A* and *C* and passing over a frictionless pulley at *D*. The tension may be assumed to be the same in portion *AD* and *CD* of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at *B*.

SOLUTION



f.b.d.



From the f.b.d. of member *ABC*, it is seen that the member can be treated as a three-force body.

From the force triangle

$$\frac{T - 300}{T} = \frac{3}{4}$$

$$3T = 4T - 1200$$

$$\therefore \quad T = 1200 \text{ lb} \blacktriangleleft$$

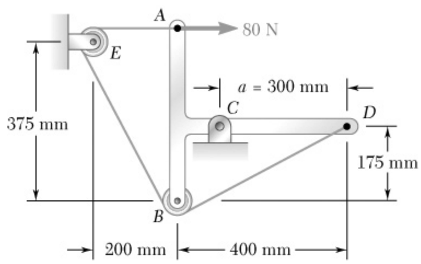
Also,

$$\frac{B}{T} = \frac{5}{4}$$

$$\therefore \quad B = \frac{5}{4}T = \frac{5}{4}(1200 \text{ lb}) = 1500 \text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.870°$$

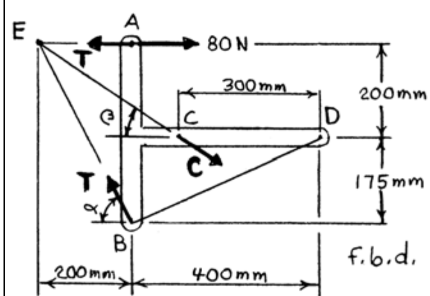
and $\mathbf{B} = 1500 \text{ lb}$ ⦪ $36.9° \blacktriangleleft$



PROBLEM 4.82

Member *ABCD* is supported by a pin and bracket at *C* and by an inextensible cord attached at *A* and *D* and passing over frictionless pulleys at *B* and *E*. Neglecting the size of the pulleys, determine the tension in the cord and the reaction at *C*.

SOLUTION



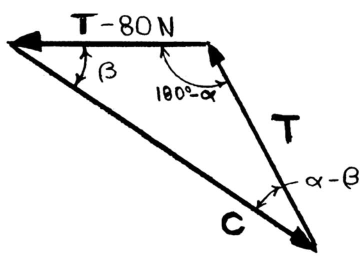
From the geometry of the forces acting on member *ABCD*

$$\beta = \tan^{-1}\left(\frac{200}{300}\right) = 33.690°$$

$$\alpha = \tan^{-1}\left(\frac{375}{200}\right) = 61.928°$$

$$\alpha - \beta = 61.928° - 33.690° = 28.237°$$

$$180° - \alpha = 180° - 61.928° = 118.072°$$



Applying the law of sines to the force triangle,

$$\frac{T - 80\text{ N}}{\sin(\alpha - \beta)} = \frac{T}{\sin\beta} = \frac{C}{\sin(180° - \alpha)}$$

or
$$\frac{T - 80\text{ N}}{\sin 28.237°} = \frac{T}{\sin 33.690°} = \frac{C}{\sin 118.072°}$$

Then
$$(T - 80\text{ N})\sin 33.690° = T\sin 28.237°$$

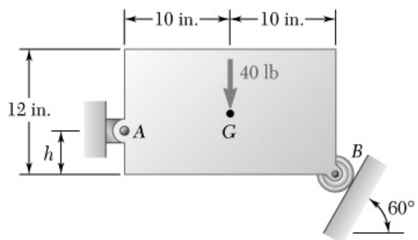
$$\therefore \quad T = 543.96\text{ N}$$

or T = 544 N ◀

and
$$(543.96\text{ N})\sin 118.072 = C\sin 33.690°$$

$$\therefore \quad C = 865.27\text{ N}$$

or **C** = 865 N ⦪ 33.7° ◀

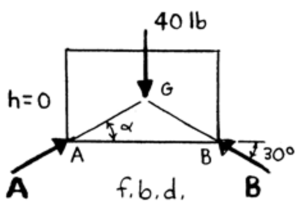


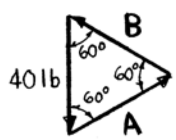
PROBLEM 4.83

Using the method of Section 4.7, solve Problem 4.18.

P4.18 Determine the reactions at A and B when (a) $h = 0$, (b) $h = 8$ in.

SOLUTION





(*a*) Based on symmetry

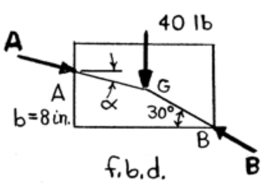
$$\alpha = 30°$$

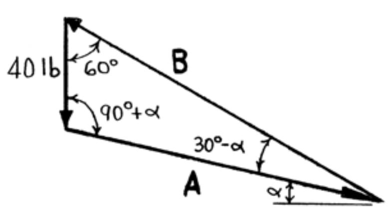
From force triangle

$$A = B = 40 \text{ lb}$$

or **A** = 40.0 lb ∠ 30° ◀

and **B** = 40.0 lb ⦪ 30° ◀





(*b*) From geometry of forces

$$\alpha = \tan^{-1}\left(\frac{8 \text{ in.} - (10 \text{ in.})\tan 30°}{10 \text{ in.}}\right) = 12.5521°$$

Also,

$$30° - \alpha = 30° - 12.5521° = 17.4479°$$

$$90° + \alpha = 90° + 12.5521° = 102.5521°$$

Applying law of sines to the force triangle,

$$\frac{40 \text{ lb}}{\sin(30° - \alpha)} = \frac{A}{\sin 60°} = \frac{B}{\sin(90° + \alpha)}$$

or

$$\frac{40 \text{ lb}}{\sin 17.4479°} = \frac{A}{\sin 60°} = \frac{B}{\sin 102.5521}$$

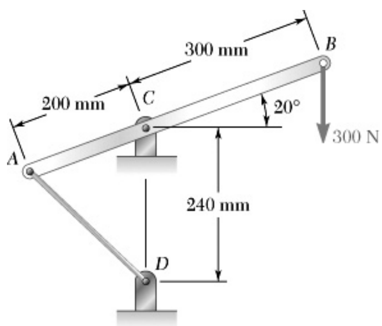
$$A = 115.533 \text{ lb}$$

or **A** = 115.5 lb ⦬ 12.55° ◀

$$B = 130.217 \text{ lb}$$

or **B** = 130.2 lb ⦪ 30.0° ◀

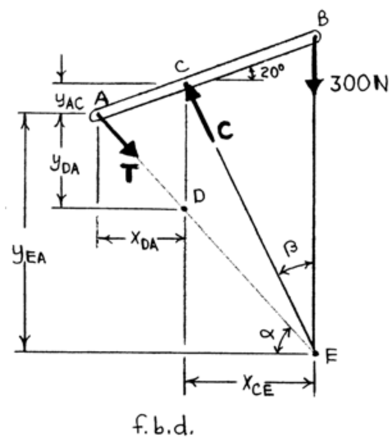
PROBLEM 4.84



Using the method of Section 4.7, solve Problem 4.28.

P4.28 A lever is hinged at *C* and is attached to a control cable at *A*. If the lever is subjected to a 300-N vertical force at *B*, determine (a) the tension in the cable, (b) the reaction at *C*.

SOLUTION



From geometry of forces acting on lever

$$\alpha = \tan^{-1}\left(\frac{y_{DA}}{x_{DA}}\right)$$

where

$$y_{DA} = 0.24 \text{ m} - y_{AC} = 0.24 \text{ m} - (0.2 \text{ m})\sin 20°$$

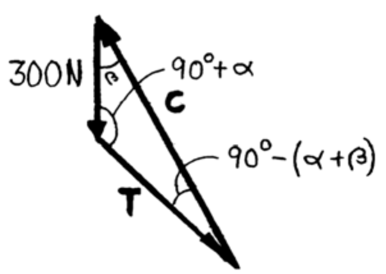
$$= 0.171596 \text{ m}$$

$$x_{DA} = (0.2 \text{ m})\cos 20°$$

$$= 0.187939 \text{ m}$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{0.171596}{0.187939}\right) = 42.397°$$

$$\beta = 90° - \tan^{-1}\left(\frac{y_{AC} + y_{EA}}{x_{CE}}\right)$$



where $x_{CE} = (0.3 \text{ m})\cos 20° = 0.28191 \text{ m}$

$$y_{AC} = (0.2 \text{ m})\sin 20° = 0.068404 \text{ m}$$

$$y_{EA} = (x_{DA} + x_{CE})\tan\alpha$$

$$= (0.187939 + 0.28191)\tan 42.397°$$

$$= 0.42898 \text{ m}$$

$$\therefore \quad \beta = 90° - \tan^{-1}\left(\frac{0.49739}{0.28191}\right) = 29.544°$$

Also, $90° - (\alpha + \beta) = 90° - 71.941° = 18.0593°$

$$90° + \alpha = 90° + 42.397° = 132.397°$$

PROBLEM 4.84 CONTINUED

Applying the law of sines to the force triangle,

$$\frac{300\text{ N}}{\sin\left[90^\circ - (\alpha + \beta)\right]} = \frac{T}{\sin\beta} = \frac{C}{\sin(90^\circ + \alpha)}$$

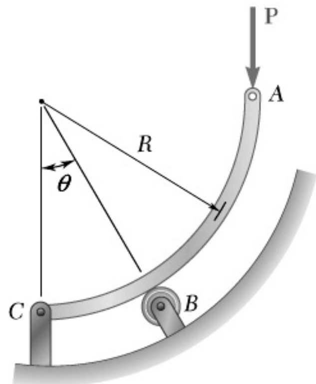
or

$$\frac{300\text{ N}}{\sin 18.0593^\circ} = \frac{T}{\sin 29.544^\circ} = \frac{C}{\sin 132.397^\circ}$$

(*a*) $T = 477.18\text{ N}$ or $T = 477\text{ N}$ ◀

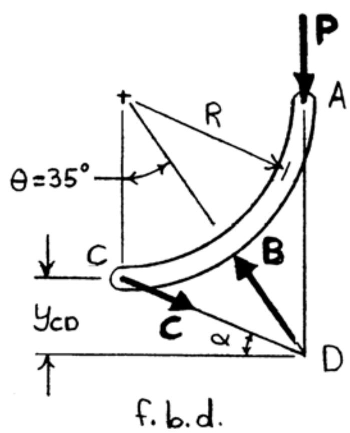
(*b*) $C = 714.67\text{ N}$ or $\mathbf{C} = 715\text{ N}$ ⦬ 60.5° ◀

PROBLEM 4.85



Knowing that $\theta = 35^\circ$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION



From the geometry of the three forces applied to the member *ABC*

$$\alpha = \tan^{-1}\left(\frac{y_{CD}}{R}\right)$$

where

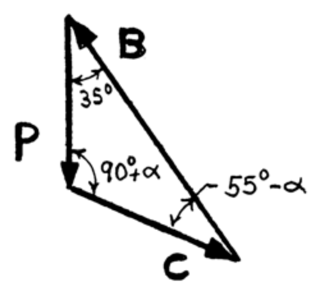
$$y_{CD} = R\tan 55^\circ - R = 0.42815R$$

$$\therefore \quad \alpha = \tan^{-1}(0.42815) = 23.178^\circ$$

Then $\quad 55^\circ - \alpha = 55^\circ - 23.178^\circ = 31.822^\circ$

$$90^\circ + \alpha = 90^\circ + 23.178^\circ = 113.178^\circ$$

Applying the law of sines to the force triangle,



$$\frac{P}{\sin(55^\circ - \alpha)} = \frac{B}{\sin(90^\circ + \alpha)} = \frac{C}{\sin 35^\circ}$$

or

$$\frac{P}{\sin 31.822^\circ} = \frac{B}{\sin 113.178^\circ} = \frac{C}{\sin 35^\circ}$$

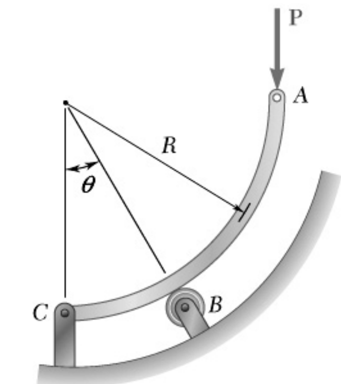
(*a*) $\quad B = 1.74344P$

or $\mathbf{B} = 1.743P$ ⦪ 55.0° ◀

(*b*) $\quad C = 1.08780P$

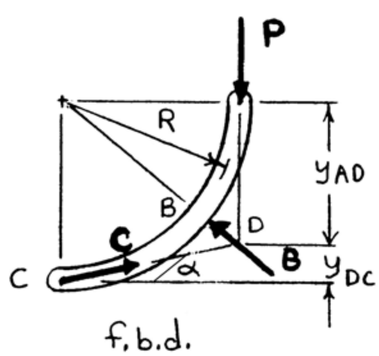
or $\mathbf{C} = 1.088P$ ⦩ 23.2° ◀

PROBLEM 4.86



Knowing that $\theta = 50^\circ$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION



From the geometry of the three forces acting on member *ABC*

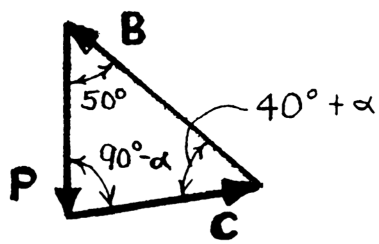
$$\alpha = \tan^{-1}\left(\frac{y_{DC}}{R}\right)$$

where

$$y_{DC} = R - y_{AD} = R\left[1 - \tan\left(90^\circ - 50^\circ\right)\right]$$

$$= 0.160900R$$

$$\therefore \quad \alpha = \tan^{-1}\left(0.160900\right) = 9.1406^\circ$$



Then $\qquad 90^\circ - \alpha = 90^\circ - 9.1406^\circ = 80.859^\circ$

$$40^\circ + \alpha = 40^\circ + 9.1406^\circ = 49.141^\circ$$

Applying the law of sines to the force triangle,

$$\frac{P}{\sin\left(40^\circ + \alpha\right)} = \frac{B}{\sin\left(90^\circ - \alpha\right)} = \frac{C}{\sin 50^\circ}$$

or

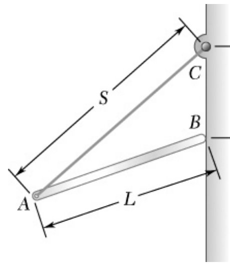
$$\frac{P}{\sin 49.141^\circ} = \frac{B}{\sin\left(80.859^\circ\right)} = \frac{C}{\sin 50^\circ}$$

(*a*) $\qquad B = 1.30540P$

or $\mathbf{B} = 1.305P$ ⦨ 40.0° ◀

(*b*) $\qquad C = 1.01286P$

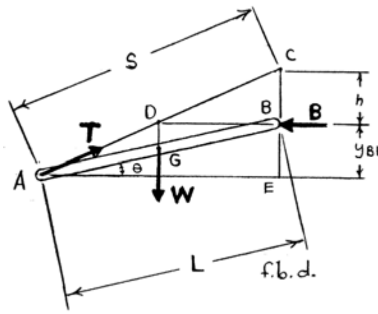
or $\mathbf{C} = 1.013P$ ⦨ 9.14° ◀



PROBLEM 4.87

A slender rod of length L and weight W is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length S. Derive an expression for the distance h in terms of L and S. Show that this position of equilibrium does not exist if $S > 2L$.

SOLUTION



From the f.b.d of the three-force member AB, forces must intersect at D. Since the force T intersects point D, directly above G,

$$y_{BE} = h$$

For triangle ACE:

$$S^2 = (AE)^2 + (2h)^2 \tag{1}$$

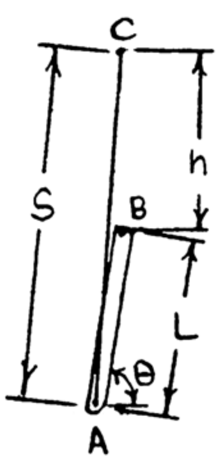
For triangle ABE:

$$L^2 = (AE)^2 + (h)^2 \tag{2}$$

Subtracting Equation (2) from Equation (1)

$$S^2 - L^2 = 3h^2 \tag{3}$$

$$\text{or} \quad h = \sqrt{\frac{S^2 - L^2}{3}} \blacktriangleleft$$



As length S increases relative to length L, angle θ increases until rod AB is vertical. At this vertical position:

$$h + L = S \qquad \text{or} \qquad h = S - L$$

Therefore, for all positions of AB $\quad h \geq S - L \quad$ (4)

or

$$\sqrt{\frac{S^2 - L^2}{3}} \geq S - L$$

or

$$S^2 - L^2 \geq 3(S - L)^2 = 3(S^2 - 2SL + L^2) = 3S^2 - 6SL + 3L^2$$

or

$$0 \geq 2S^2 - 6SL + 4L^2$$

and

$$0 \geq S^2 - 3SL + 2L^2 = (S - L)(S - 2L)$$

For

$$S - L = 0 \qquad S = L$$

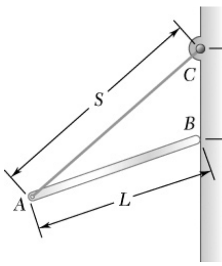
$\therefore$ Minimum value of S is L

For

$$S - 2L = 0 \qquad S = 2L$$

$\therefore$ Maximum value of S is $2L$

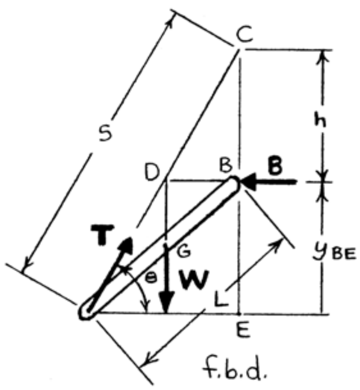
Therefore, equilibrium does not exist if $S > 2L$ ◀



PROBLEM 4.88

A slender rod of length $L = 200$ mm is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length $S = 300$ mm. Knowing that the mass of the rod is 1.5 kg, determine (*a*) the distance *h*, (*b*) the tension in the cord, (*c*) the reaction at *B*.

SOLUTION



From the f.b.d of the three-force member *AB*, forces must intersect at *D*. Since the force *T* intersects point *D*, directly above *G*,

$$y_{BE} = h$$

For triangle *ACE*:

$$S^2 = (AE)^2 + (2h)^2 \qquad (1)$$

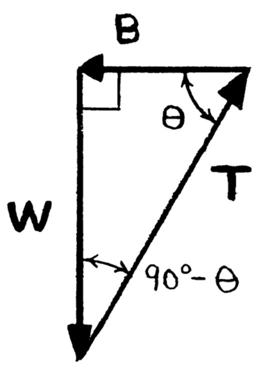
For triangle *ABE*:

$$L^2 = (AE)^2 + (h)^2 \qquad (2)$$

Subtracting Equation (2) from Equation (1)

$$S^2 - L^2 = 3h^2$$

$$\text{or} \quad h = \sqrt{\frac{S^2 - L^2}{3}}$$



(*a*) For $L = 200$ mm and $S = 300$ mm

$$h = \sqrt{\frac{(300)^2 - (200)^2}{3}} = 129.099 \text{ mm}$$

or $h = 129.1$ mm ◀

(*b*) Have $\quad W = mg = (1.5 \text{ kg})(9.81 \text{ m/s}^2) = 14.715 \text{ N}$

and

$$\theta = \sin^{-1}\left(\frac{2h}{s}\right) = \sin^{-1}\left[\frac{2(129.099)}{300}\right]$$

$$\theta = 59.391°$$

From the force triangle

$$T = \frac{W}{\sin\theta} = \frac{14.715 \text{ N}}{\sin 59.391°} = 17.0973 \text{ N}$$

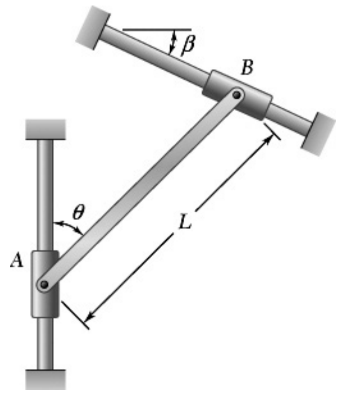
or $T = 17.10$ N ◀

(*c*)

$$B = \frac{W}{\tan\theta} = \frac{14.715 \text{ N}}{\tan 59.391°} = 8.7055 \text{ N}$$

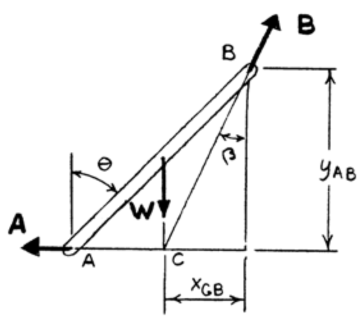
or $\mathbf{B} = 8.71$ N ⟵ ◀

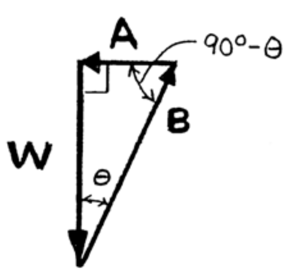
PROBLEM 4.89



A slender rod of length L and weight W is attached to collars which can slide freely along the guides shown. Knowing that the rod is in equilibrium, derive an expression for the angle θ in terms of the angle β.

SOLUTION





As shown in the f.b.d of the slender rod AB, the three forces intersect at C. From the force geometry

$$\tan\beta = \frac{x_{GB}}{y_{AB}}$$

where

$$y_{AB} = L\cos\theta$$

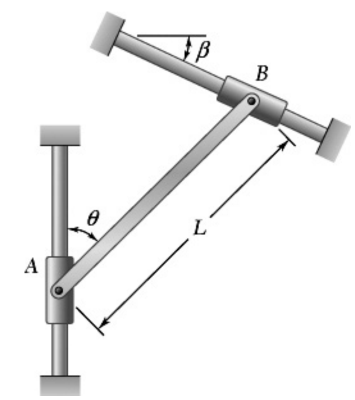
and

$$x_{GB} = \frac{1}{2}L\sin\theta$$

$$\therefore \quad \tan\beta = \frac{\frac{1}{2}L\sin\theta}{L\cos\theta} = \frac{1}{2}\tan\theta$$

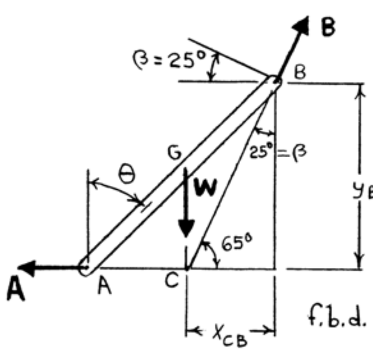
or $\tan\theta = 2\tan\beta$ ◀

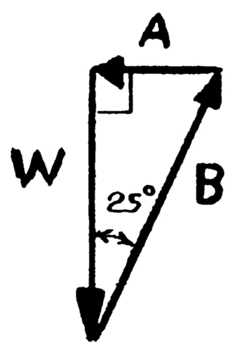
PROBLEM 4.90



A 10-kg slender rod of length L is attached to collars which can slide freely along the guides shown. Knowing that the rod is in equilibrium and that $\beta = 25°$, determine (*a*) the angle θ that the rod forms with the vertical, (*b*) the reactions at A and B.

SOLUTION





(*a*) As shown in the f.b.d. of the slender rod AB, the three forces intersect at C. From the geometry of the forces

$$\tan\beta = \frac{x_{CB}}{y_{BC}}$$

where

$$x_{CB} = \frac{1}{2}L\sin\theta$$

and

$$y_{BC} = L\cos\theta$$

$$\therefore \quad \tan\beta = \frac{1}{2}\tan\theta$$

or

$$\tan\theta = 2\tan\beta$$

For

$$\beta = 25°$$

$$\tan\theta = 2\tan 25° = 0.93262$$

$$\therefore \quad \theta = 43.003°$$

or $\theta = 43.0°$ ◀

(*b*)

$$W = mg = (10\text{ kg})\left(9.81\text{ m/s}^2\right) = 98.1\text{ N}$$

From force triangle

$$A = W\tan\beta$$

$$= (98.1\text{ N})\tan 25°$$

$$= 45.745\text{ N}$$

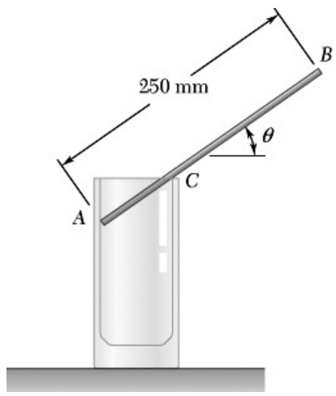
or $\mathbf{A} = 45.7\text{ N} \longleftarrow$ ◀

and

$$B = \frac{W}{\cos\beta} = \frac{98.1\text{ N}}{\cos 25°} = 108.241\text{ N}$$

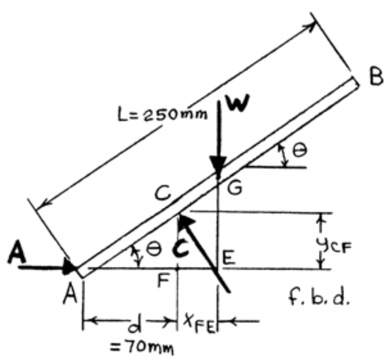
or $\mathbf{B} = 108.2\text{ N} \measuredangle 65.0°$ ◀

PROBLEM 4.91



A uniform slender rod of mass 5 g and length 250 mm is balanced on a glass of inner diameter 70 mm. Neglecting friction, determine the angle θ corresponding to equilibrium.

SOLUTION



From the geometry of the forces acting on the three-force member *AB*

Triangle *ACF*

$$y_{CF} = d\tan\theta$$

Triangle *CEF* $\quad x_{FE} = y_{CF}\tan\theta = d\tan^2\theta$

Triangle *AGE*

$$\cos\theta = \frac{d + x_{FE}}{\left(\frac{L}{2}\right)} = \frac{d + d\tan^2\theta}{\left(\frac{L}{2}\right)}$$

$$= \frac{2d}{L}\left(1 + \tan^2\theta\right)$$

Now $\quad \left(1 + \tan^2\theta\right) = \sec^2\theta \quad$ and $\quad \sec\theta = \dfrac{1}{\cos\theta}$

Then $\quad \cos\theta = \dfrac{2d}{L}\sec^2\theta = \dfrac{2d}{L}\left(\dfrac{1}{\cos^2\theta}\right)$

$$\therefore \quad \cos^3\theta = \frac{2d}{L}$$

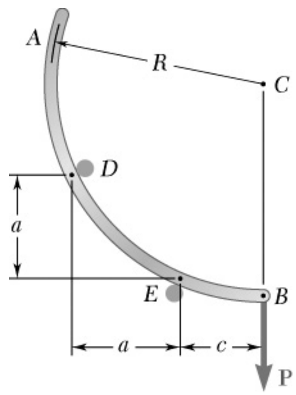
For $\quad d = 70 \text{ mm} \quad$ and $\quad L = 250 \text{ mm}$

$$\cos^3\theta = \frac{2(70)}{250} = 0.56$$

$$\therefore \quad \cos\theta = 0.82426$$

and $\quad \theta = 34.487°$

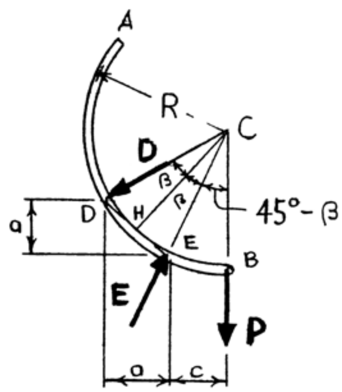
or $\theta = 34.5°$ ◀



PROBLEM 4.92

Rod AB is bent into the shape of a circular arc and is lodged between two pegs D and E. It supports a load **P** at end B. Neglecting friction and the weight of the rod, determine the distance c corresponding to equilibrium when $a = 1$ in. and $R = 5$ in.

SOLUTION



f.b.d.

Since $$y_{ED} = x_{ED} = a,$$

Slope of ED is ⦣ 45°

$\therefore$ slope of HC is ∡ 45°

Also $$DE = \sqrt{2}a$$

and $$DH = HE = \left(\frac{1}{2}\right)DE = \frac{a}{\sqrt{2}}$$

For triangles DHC and EHC

$$\sin\beta = \frac{a/\sqrt{2}}{R} = \frac{a}{\sqrt{2}R}$$

Now $$c = R\sin(45° - \beta)$$

For $$a = 1\text{ in.} \quad \text{and} \quad R = 5\text{ in.}$$

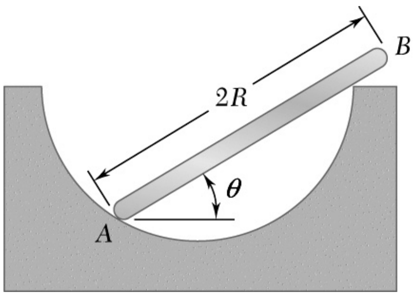
$$\sin\beta = \frac{1\text{ in.}}{\sqrt{2}(5\text{ in.})} = 0.141421$$

$$\therefore\ \beta = 8.1301°$$ or $\beta = 8.13°$ ◀

and $$c = (5\text{ in.})\sin(45° - 8.1301°) = 3.00\text{ in.}$$

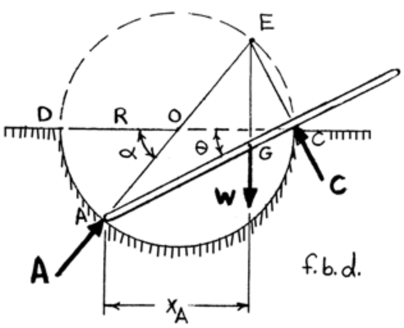
or $c = 3.00$ in. ◀

PROBLEM 4.93



A uniform rod AB of weight W and length $2R$ rests inside a hemispherical bowl of radius R as shown. Neglecting friction determine the angle θ corresponding to equilibrium.

SOLUTION



Based on the f.b.d., the uniform rod AB is a three-force body. Point E is the point of intersection of the three forces. Since force A passes through O, the center of the circle, and since force **C** is perpendicular to the rod, triangle ACE is a right triangle inscribed in the circle. Thus, E is a point on the circle.

Note that the angle α of triangle DOA is the central angle corresponding to the inscribed angle θ of triangle DCA.

$$\therefore\ \alpha = 2\theta$$

The horizontal projections of $AE, (x_{AE})$, and $AG, (x_{AG})$, are equal.

$$\therefore\ x_{AE} = x_{AG} = x_A$$

or
$$(AE)\cos 2\theta = (AG)\cos\theta$$

and
$$(2R)\cos 2\theta = R\cos\theta$$

Now
$$\cos 2\theta = 2\cos^2\theta - 1$$

then
$$4\cos^2\theta - 2 = \cos\theta$$

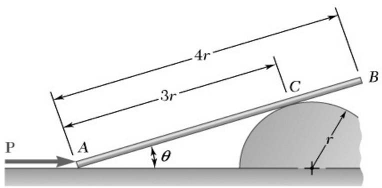
or
$$4\cos^2\theta - \cos\theta - 2 = 0$$

Applying the quadratic equation

$$\cos\theta = 0.84307 \quad \text{and} \quad \cos\theta = -0.59307$$

$$\therefore\ \theta = 32.534° \quad \text{and} \quad \theta = 126.375° \text{(Discard)}$$

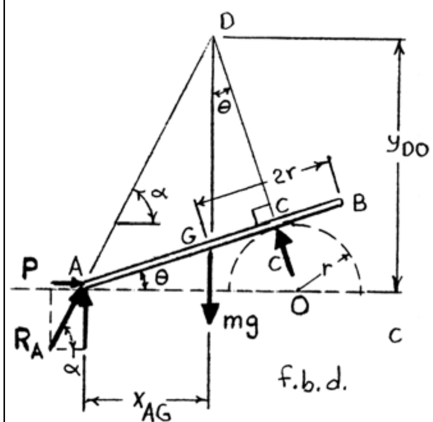
or $\theta = 32.5°$ ◀

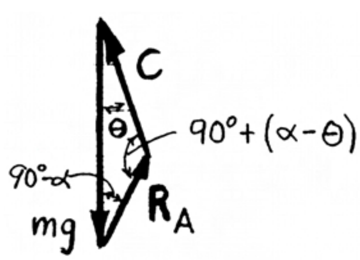


PROBLEM 4.94

A uniform slender rod of mass m and length $4r$ rests on the surface shown and is held in the given equilibrium position by the force **P**. Neglecting the effect of friction at A and C, (a) determine the angle θ, (b) derive an expression for P in terms of m.

SOLUTION





The forces acting on the three-force member intersect at D.

(a) From triangle ACO

$$\theta = \tan^{-1}\left(\frac{r}{3r}\right) = \tan^{-1}\left(\frac{1}{3}\right) = 18.4349° \qquad \text{or } \theta = 18.43° ◀$$

(b) From triangle DCG

$$\tan\theta = \frac{r}{DC}$$

$$\therefore \quad DC = \frac{r}{\tan\theta} = \frac{r}{\tan 18.4349°} = 3r$$

and

$$DO = DC + r = 3r + r = 4r$$

$$\alpha = \tan^{-1}\left(\frac{y_{DO}}{x_{AG}}\right)$$

where

$$y_{DO} = (DO)\cos\theta = (4r)\cos 18.4349°$$
$$= 3.4947r$$

and

$$x_{AG} = (2r)\cos\theta = (2r)\cos 18.4349°$$
$$= 1.89737r$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{3.4947r}{1.89737r}\right) = 63.435°$$

where

$$90° + (\alpha - \theta) = 90° + 45° = 135.00°$$

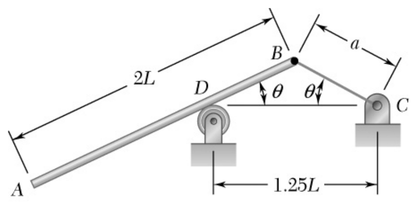
Applying the law of sines to the force triangle,

$$\frac{mg}{\sin[90° + (\alpha - \theta)]} = \frac{R_A}{\sin\theta}$$

$$\therefore \quad R_A = (0.44721)mg$$

Finally,

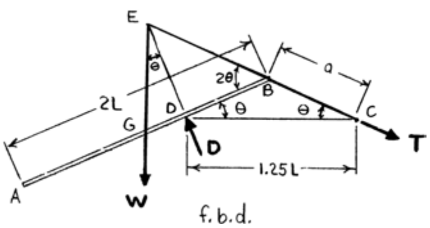
$$P = R_A\cos\alpha$$
$$= (0.44721mg)\cos 63.435°$$
$$= 0.20000mg \qquad \text{or } P = \frac{mg}{5} ◀$$



PROBLEM 4.95

A uniform slender rod of length $2L$ and mass m rests against a roller at D and is held in the equilibrium position shown by a cord of length a. Knowing that $L = 200$ mm, determine (*a*) the angle θ, (*b*) the length a.

SOLUTION



(*a*) The forces acting on the three-force member AB intersect at E. Since triangle DBC is isosceles, $DB = a$.

From triangle BDE

$$ED = DB\tan 2\theta = a\tan 2\theta$$

From triangle GED

$$ED = \frac{(L - a)}{\tan\theta}$$

$$\therefore \quad a\tan 2\theta = \frac{L - a}{\tan\theta} \quad \text{or} \quad a(\tan\theta\tan 2\theta + 1) = L \qquad (1)$$

From triangle BCD $\quad a = \dfrac{\frac{1}{2}(1.25L)}{\cos\theta} \quad$ or $\quad \dfrac{L}{a} = 1.6\cos\theta \qquad (2)$

Substituting Equation (2) into Equation (1) yields

$$1.6\cos\theta = 1 + \tan\theta\tan 2\theta$$

Now

$$\tan\theta\tan 2\theta = \frac{\sin\theta}{\cos\theta}\frac{\sin 2\theta}{\cos 2\theta}$$

$$= \frac{\sin\theta}{\cos\theta}\frac{2\sin\theta\cos\theta}{2\cos^2\theta - 1}$$

$$= \frac{2(1 - \cos^2\theta)}{2\cos^2\theta - 1}$$

Then

$$1.6\cos\theta = 1 + \frac{2(1 - \cos^2\theta)}{2\cos^2\theta - 1}$$

or

$$3.2\cos^3\theta - 1.6\cos\theta - 1 = 0$$

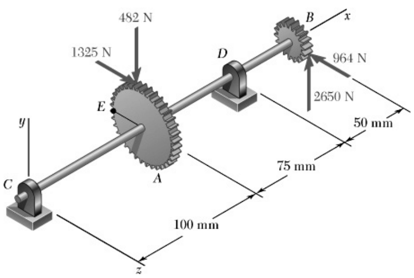
Solving numerically $\quad \theta = 23.515° \quad$ or $\quad \theta = 23.5°$ ◀

(*b*) From Equation (2) for $L = 200$ mm and $\theta = 23.5°$

$$a = \frac{5}{8}\frac{(200\text{ mm})}{\cos 23.515°} = 136.321\text{ mm}$$

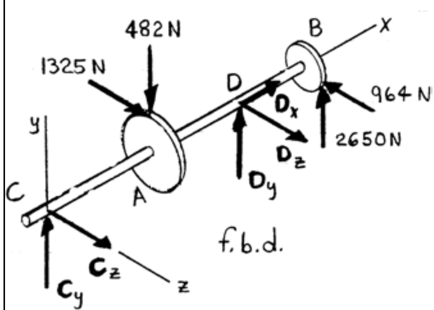
or $a = 136.3$ mm ◀

PROBLEM 4.96



Gears *A* and *B* are attached to a shaft supported by bearings at *C* and *D*. The diameters of gears *A* and *B* are 150 mm and 75 mm, respectively, and the tangential and radial forces acting on the gears are as shown. Knowing that the system rotates at a constant rate, determine the reactions at *C* and *D*. Assume that the bearing at *C* does not exert any axial force, and neglect the weights of the gears and the shaft.

SOLUTION



Assume moment reactions at the bearing supports are zero. From f.b.d. of shaft

$$\Sigma F_x = 0: \quad \therefore \quad D_x = 0$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad -C_y(175\text{ mm}) + (482\text{ N})(75\text{ mm})$$

$$+ (2650\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_y = 963.71\text{ N}$$

or $\qquad \mathbf{C}_y = (964\text{ N})\mathbf{j}$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(175\text{ mm}) + (1325\text{ N})(75\text{ mm})$$

$$+ (964\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_z = -843.29\text{ N}$$

or $\qquad \mathbf{C}_z = (843\text{ N})\mathbf{k}$

and $\mathbf{C} = (964\text{ N})\mathbf{j} - (843\text{ N})\mathbf{k}$ ◀

$$\Sigma M_{C(z\text{-axis})} = 0: \quad -(482\text{ N})(100\text{ mm}) + D_y(175\text{ mm})$$

$$+ (2650\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_y = -3131.7\text{ N}$$

or $\qquad \mathbf{D}_y = -(3130\text{ N})\mathbf{j}$

$$\Sigma M_{C(y\text{-axis})} = 0: \quad -(1325\text{ N})(100\text{ mm}) - D_z(175\text{ mm})$$

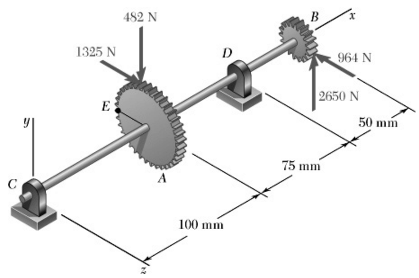
$$+ (964\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_z = 482.29\text{ N}$$

or $\qquad \mathbf{D}_z = (482\text{ N})\mathbf{k}$

and $\mathbf{D} = -(3130\text{ N})\mathbf{j} + (482\text{ N})\mathbf{k}$ ◀

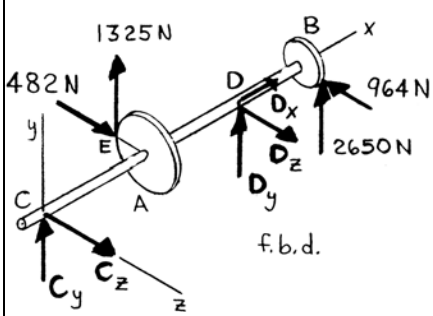
PROBLEM 4.97



Solve Problem 4.96 assuming that for gear A the tangential and radial forces are acting at E, so that $\mathbf{F}_A = (1325\text{ N})\mathbf{j} + (482\text{ N})\mathbf{k}$.

P4.96 Gears A and B are attached to a shaft supported by bearings at C and D. The diameters of gears A and B are 150 mm and 75 mm, respectively, and the tangential and radial forces acting on the gears are as shown. Knowing that the system rotates at a constant rate, determine the reactions at C and D. Assume that the bearing at C does not exert any axial force, and neglect the weights of the gears and the shaft.

SOLUTION



Assume moment reactions at the bearing supports are zero. From f.b.d. of shaft

$$\Sigma F_x = 0: \quad \therefore \quad D_x = 0$$

$$\Sigma M_{D(z\text{-}axis)} = 0: \quad -C_y(175\text{ mm}) - (1325\text{ N})(75\text{ mm})$$

$$+ (2650\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_y = 189.286\text{ N}$$

or $$\mathbf{C}_y = (189.3\text{ N})\mathbf{j}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(175\text{ mm}) + (482\text{ N})(75\text{ mm})$$

$$+ (964\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_z = -482.00\text{ N}$$

or $$\mathbf{C}_z = -(482\text{ N})\mathbf{k}$$

and $\mathbf{C} = (189.3\text{ N})\mathbf{j} - (482\text{ N})\mathbf{k}$ ◀

$$\Sigma M_{C(z\text{-axis})} = 0: \quad (1325\text{ N})(100\text{ mm}) + D_y(175\text{ mm})$$

$$+ (2650\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_y = -4164.3\text{ N}$$

or $$\mathbf{D}_y = -(4160\text{ N})\mathbf{j}$$

$$\Sigma M_{C(y\text{-axis})} = 0: \quad -(482\text{ N})(100\text{ mm}) - D_z(175\text{ mm})$$

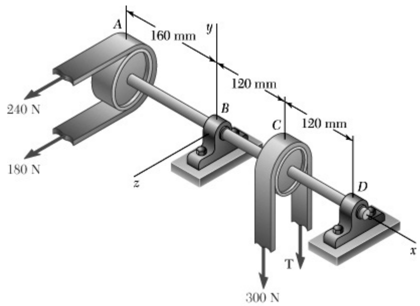
$$+(964\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_z = 964.00\text{ N}$$

or $$\mathbf{D}_z = (964\text{ N})\mathbf{k}$$

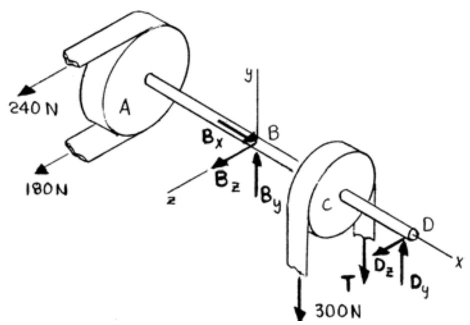
and $\mathbf{D} = -(4160\text{ N})\mathbf{j} + (964\text{ N})\mathbf{k}$ ◀

PROBLEM 4.98



Two transmission belts pass over sheaves welded to an axle supported by bearings at *B* and *D*. The sheave at *A* has a radius of 50 mm, and the sheave at *C* has a radius of 40 mm. Knowing that the system rotates with a constant rate, determine (*a*) the tension *T*, (*b*) the reactions at *B* and *D*. Assume that the bearing at *D* does not exert any axial thrust and neglect the weights of the sheaves and the axle.

SOLUTION



Assume moment reactions at the bearing supports are zero. From f.b.d. of shaft

(*a*) $$\Sigma M_{x\text{-axis}} = 0: \quad (240 \text{ N} - 180 \text{ N})(50 \text{ mm}) + (300 \text{ N} - T)(40 \text{ mm}) = 0$$

$$\therefore \; T = 375 \text{ N} \blacktriangleleft$$

(*b*) $$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad (300 \text{ N} + 375 \text{ N})(120 \text{ mm}) - B_y(240 \text{ mm}) = 0$$

$$\therefore \; B_y = 337.5 \text{ N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad (240 \text{ N} + 180 \text{ N})(400 \text{ mm}) + B_z(240 \text{ mm}) = 0$$

$$\therefore \; B_z = -700 \text{ N}$$

or $\mathbf{B} = (338 \text{ N})\mathbf{j} - (700 \text{ N})\mathbf{k} \blacktriangleleft$

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -(300 \text{ N} + 375 \text{ N})(120 \text{ mm}) + D_y(240 \text{ mm}) = 0$$

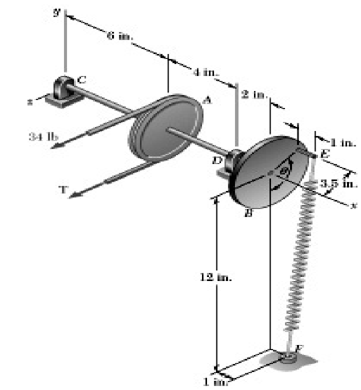
$$\therefore \; D_y = 337.5 \text{ N}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad (240 \text{ N} + 180 \text{ N})(160 \text{ mm}) + D_z(240 \text{ mm}) = 0$$

$$\therefore \; D_z = -280 \text{ N}$$

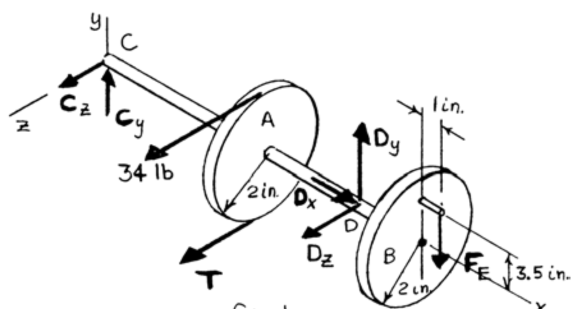
or $\mathbf{D} = (338 \text{ N})\mathbf{j} - (280 \text{ N})\mathbf{k} \blacktriangleleft$

PROBLEM 4.99



For the portion of a machine shown, the 4-in.-diameter pulley *A* and wheel *B* are fixed to a shaft supported by bearings at *C* and *D*. The spring of constant 2 lb/in. is unstretched when $\theta = 0$, and the bearing at *C* does not exert any axial force. Knowing that $\theta = 180°$ and that the machine is at rest and in equilibrium, determine (*a*) the tension *T*, (*b*) the reactions at *C* and *D*. Neglect the weights of the shaft, pulley, and wheel.

SOLUTION



First, determine the spring force, $\mathbf{F}_E$, at $\theta = 180°$.

$$F_E = k_s x$$

where $$k_s = 2 \text{ lb/in.}$$

$$x = (y_E)_{\text{final}} - (y_E)_{\text{initial}} = (12 \text{ in.} + 3.5 \text{ in.}) - (12 \text{ in.} - 3.5 \text{ in.}) = 7.0 \text{ in.}$$

$$\therefore \quad F_E = (2 \text{ lb/in.})(7.0 \text{ in.}) = 14.0 \text{ lb}$$

(*a*) From f.b.d. of machine part

$$\Sigma M_x = 0: \quad (34 \text{ lb})(2 \text{ in.}) - T(2 \text{ in.}) = 0$$

$$\therefore \quad T = 34 \text{ lb}$$

or $T = 34.0$ lb ◀

(*b*)

$$\Sigma M_{D(z\text{-axis})} = 0: \quad -C_y(10 \text{ in.}) - F_E(2 \text{ in.} + 1 \text{ in.}) = 0$$

$$-C_y(10 \text{ in.}) - 14.0 \text{ lb}(3 \text{ in.}) = 0$$

$$\therefore \quad C_y = -4.2 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = -(4.20 \text{ lb})\mathbf{j}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(10 \text{ in.}) + 34 \text{ lb}(4 \text{ in.}) + 34 \text{ lb}(4 \text{ in.}) = 0$$

$$\therefore \quad C_z = -27.2 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_z = -(27.2 \text{ lb})\mathbf{k}$$

and $\mathbf{C} = -(4.20 \text{ lb})\mathbf{j} - (27.2 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.99 CONTINUED

$$\Sigma F_x = 0: \quad D_x = 0$$

$$\Sigma M_{C(z\text{-axis})} = 0: \quad D_y(10\text{ in.}) - F_E(12\text{ in.} + 1\text{ in.}) = 0$$

or

$$D_y(10\text{ in.}) - 14.0(13\text{ in.}) = 0$$

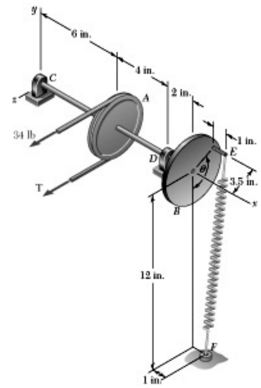
$$\therefore\ D_y = 18.2\text{ lb} \qquad \text{or} \qquad \mathbf{D}_y = (18.20\text{ lb})\mathbf{j}$$

$$\Sigma M_{C(y\text{-axis})} = 0: \quad -2(34\text{ lb})(6\text{ in.}) - D_z(10\text{ in.}) = 0$$

$$\therefore\ D_z = -40.8\text{ lb} \qquad \text{or} \qquad \mathbf{D}_z = -(40.8\text{ lb})\mathbf{k}$$

and $\mathbf{D} = (18.20\text{ lb})\mathbf{j} - (40.8\text{ lb})\mathbf{k}$ ◀

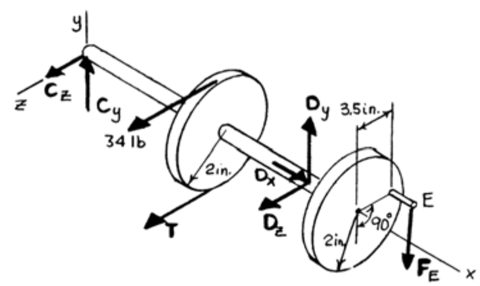
PROBLEM 4.100



Solve Problem 4.99 for $\theta = 90°$.

P4.99 For the portion of a machine shown, the 4-in.-diameter pulley *A* and wheel *B* are fixed to a shaft supported by bearings at *C* and *D*. The spring of constant 2 lb/in. is unstretched when $\theta = 0$, and the bearing at *C* does not exert any axial force. Knowing that $\theta = 180°$ and that the machine is at rest and in equilibrium, determine (*a*) the tension *T*, (*b*) the reactions at *C* and *D*. Neglect the weights of the shaft, pulley, and wheel.

SOLUTION



First, determine the spring force, $\mathbf{F}_E$, at $\theta = 90°$.

$$F_E = k_s x$$

where $$k_s = 2 \text{ lb/in.}$$

and $$x = L_{\text{final}} - L_{\text{initial}} = \left(\sqrt{(3.5)^2 + (12)^2}\right) - (12 - 3.5) = 12.5 - 8.5 = 4.0 \text{ in.}$$

$$\therefore\ F_E = (2 \text{ lb/in.})(4.0 \text{ in.}) = 8.0 \text{ lb}$$

Then $$\mathbf{F}_E = \frac{-12.0}{12.5}(8.0 \text{ lb})\mathbf{j} + \frac{3.5}{12.5}(8.0 \text{ lb})\mathbf{k} = -(7.68 \text{ lb})\mathbf{j} + (2.24 \text{ lb})\mathbf{k}$$

(*a*) From f.b.d. of machine part

$$\Sigma M_x = 0\text{:}\quad (34 \text{ lb})(2 \text{ in.}) - T(2 \text{ in.}) - (7.68 \text{ lb})(3.5 \text{ in.}) = 0$$

$$\therefore\ T = 20.56 \text{ lb}$$

or $T = 20.6$ lb ◀

(*b*) $$\Sigma M_{D(z\text{-axis})} = 0\text{:}\quad -C_y(10 \text{ in.}) - (7.68 \text{ lb})(3.0 \text{ in.}) = 0$$

$$\therefore\ C_y = -2.304 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = -(2.30 \text{ lb})\mathbf{j}$$

$$\Sigma M_{D(y\text{-axis})} = 0\text{:}\quad C_z(10 \text{ in.}) + (34 \text{ lb})(4.0 \text{ in.}) + (20.56 \text{ lb})(4.0 \text{ in.}) - (2.24 \text{ lb})(3 \text{ in.}) = 0$$

$$\therefore\ C_z = -21.152 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_z = -(21.2 \text{ lb})\mathbf{k}$$

and $\mathbf{C} = -(2.30 \text{ lb})\mathbf{j} - (21.2 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.100 CONTINUED

$\Sigma F_x = 0: \quad D_x = 0$

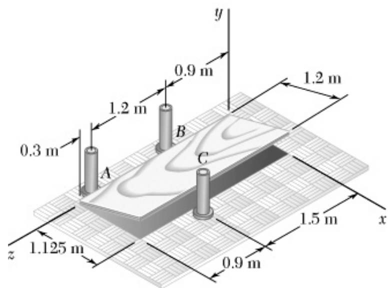
$\Sigma M_{C(z\text{-axis})} = 0: \quad D_y(10 \text{ in.}) - (7.68 \text{ lb})(13 \text{ in.}) = 0$

$\therefore \quad D_y = 9.984 \text{ lb} \qquad \text{or} \qquad \mathbf{D}_y = (9.98 \text{ lb})\mathbf{j}$

$\Sigma M_{C(y\text{-axis})} = 0: \quad -(34 \text{ lb})(6 \text{ in.}) - (20.56 \text{ lb})(6 \text{ in.}) - D_z(10 \text{ in.}) - (2.24 \text{ lb})(13 \text{ in.}) = 0$

$\therefore \quad D_z = -35.648 \text{ lb} \qquad \text{or} \qquad \mathbf{D}_z = -(35.6 \text{ lb})\mathbf{k}$

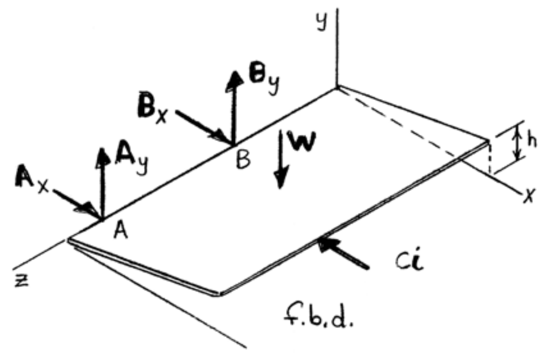
and $\mathbf{D} = (9.98 \text{ lb})\mathbf{j} - (35.6 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.101

A 1.2 × 2.4-m sheet of plywood having a mass of 17 kg has been temporarily placed among three pipe supports. The lower edge of the sheet rests on small collars *A* and *B* and its upper edge leans against pipe *C*. Neglecting friction at all surfaces, determine the reactions at *A*, *B*, and *C*.

SOLUTION



First note $$W = mg = (17\text{ kg})(9.81\text{ m/s}^2) = 166.77\text{ N}$$

$$h = \sqrt{(1.2)^2 - (1.125)^2} = 0.41758\text{ m}$$

From f.b.d. of plywood sheet

$$\Sigma M_z = 0: \quad C(h) - W\left[\frac{(1.125\text{ m})}{2}\right] = 0$$

$$C(0.41758\text{ m}) - (166.77\text{ N})(0.5625\text{ m}) = 0$$

$$\therefore \quad C = 224.65\text{ N} \qquad \text{or} \qquad \mathbf{C} = -(225\text{ N})\mathbf{i}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad -(224.65\text{ N})(0.6\text{ m}) + A_x(1.2\text{ m}) = 0$$

$$\therefore \quad A_x = 112.324\text{ N} \qquad \text{or} \qquad \mathbf{A}_x = (112.3\text{ N})\mathbf{i}$$

$$\Sigma M_{B(x\text{-axis})} = 0: \quad (166.77\text{ N})(0.3\text{ m}) - A_y(1.2\text{ m}) = 0$$

$$\therefore \quad A_y = 41.693\text{ N} \qquad \text{or} \qquad \mathbf{A}_y = (41.7\text{ N})\mathbf{j}$$

$$\Sigma M_{A(y\text{-axis})} = 0: \quad (224.65\text{ N})(0.6\text{ m}) - B_x(1.2\text{ m}) = 0$$

$$\therefore \quad B_x = 112.325\text{ N} \qquad \text{or} \qquad \mathbf{B}_x = (112.3\text{ N})\mathbf{i}$$

PROBLEM 4.101 CONTINUED

$$\Sigma M_{A(x\text{-axis})} = 0: \quad B_y(1.2 \text{ m}) - (166.77 \text{ N})(0.9 \text{ m}) = 0$$

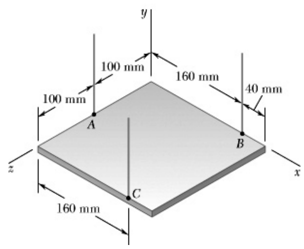
$$\therefore \quad B_y = 125.078 \text{ N} \qquad \text{or} \qquad \mathbf{B}_y = (125.1 \text{ N})\mathbf{j}$$

$$\therefore \quad \mathbf{A} = (112.3 \text{ N})\mathbf{i} + (41.7 \text{ N})\mathbf{j} \blacktriangleleft$$

$$\mathbf{B} = (112.3 \text{ N})\mathbf{i} + (125.1 \text{ N})\mathbf{j} \blacktriangleleft$$

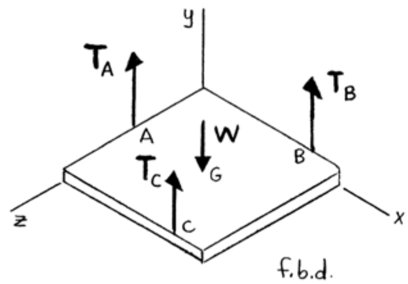
$$\mathbf{C} = -(225 \text{ N})\mathbf{i} \blacktriangleleft$$

PROBLEM 4.102



The 200×200-mm square plate shown has a mass of 25 kg and is supported by three vertical wires. Determine the tension in each wire.

SOLUTION



First note $$W = mg = (25\text{ kg})(9.81\text{ m/s}^2) = 245.25\text{ N}$$

From f.b.d. of plate

$$\Sigma M_x = 0: \quad (245.25\text{ N})(100\text{ mm}) - T_A(100\text{ mm}) - T_C(200\text{ mm}) = 0$$

$$\therefore \quad T_A + 2T_C = 245.25\text{ N} \tag{1}$$

$$\Sigma M_z = 0: \quad T_B(160\text{ mm}) + T_C(160\text{ mm}) - (245.25\text{ N})(100\text{ mm}) = 0$$

$$\therefore \quad T_B + T_C = 153.281\text{ N} \tag{2}$$

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - 245.25\text{ N} = 0$$

$$\therefore \quad T_B + T_C = 245.25 - T_A \tag{3}$$

Equating Equations (2) and (3) yields

$$T_A = 245.25\text{ N} - 153.281\text{ N} = 91.969\text{ N} \tag{4}$$

or $$T_A = 92.0\text{ N}$$

Substituting the value of T_A into Equation (1)

$$T_C = \frac{(245.25\text{ N} - 91.969\text{ N})}{2} = 76.641\text{ N} \tag{5}$$

or $$T_C = 76.6\text{ N}$$

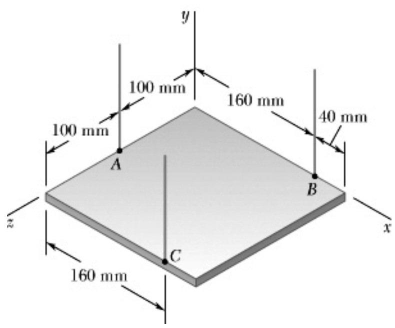
Substituting the value of T_C into Equation (2)

$$T_B = 153.281\text{ N} - 76.641\text{ N} = 76.639\text{ N} \qquad \text{or} \qquad T_B = 76.6\text{ N}$$

$T_A = 92.0\text{ N}$ ◀

$T_B = 76.6\text{ N}$ ◀

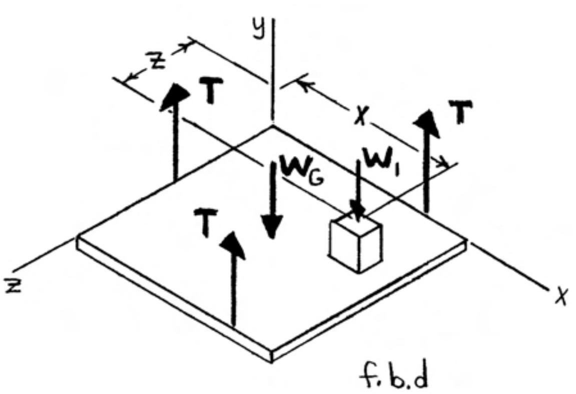
$T_C = 76.6\text{ N}$ ◀



PROBLEM 4.103

The 200×200-mm square plate shown has a mass of 25 kg and is supported by three vertical wires. Determine the mass and location of the lightest block which should be placed on the plate if the tensions in the three cables are to be equal.

SOLUTION



First note $$W_G = m_{p1}g = (25 \text{ kg})(9.81 \text{ m/s}^2) = 245.25 \text{ N}$$

$$W_1 = mg = m(9.81 \text{ m/s}^2) = (9.81m)\text{N}$$

From f.b.d. of plate

$$\Sigma F_y = 0: \quad 3T - W_G - W_1 = 0 \, (1)$$

$$\Sigma M_x = 0: \quad W_G(100 \text{ mm}) + W_1(z) - T(100 \text{ mm}) - T(200 \text{ mm}) = 0$$

$$\text{or} \quad -300T + 100W_G + W_1 z = 0 \qquad (2)$$

$$\Sigma M_z = 0: \quad 2T(160 \text{ mm}) - W_G(100 \text{ mm}) - W_1(x) = 0$$

$$\text{or} \quad 320T - 100W_G - W_1 x = 0 \qquad (3)$$

Eliminate T by forming $100 \times [\text{Eq. } (1) + \text{Eq. } (2)]$

$$-100W_1 + W_1 z = 0$$

$$\therefore \quad z = 100 \text{ mm} \qquad 0 \le z \le 200 \text{ mm}, \quad \therefore \quad \text{okay}$$

Now, $3 \times [\text{Eq. } (3)] - 320 \times [\text{Eq. } (1)]$ yields

$$3(320T) - 3(100)W_G - 3W_1 x - 320(3T) + 320W_G + 320W_1 = 0$$

PROBLEM 4.103 CONTINUED

or
$$20W_G + (320 - 3x)W_1 = 0$$

or
$$\frac{W_1}{W_G} = \frac{20}{(3x - 320)}$$

The smallest value of $\frac{W_1}{W_G}$ will result in the smallest value of W_1 since W_G is given.

$$\therefore \quad \text{Use } x = x_{\text{max}} = 200 \text{ mm}$$

and then
$$\frac{W_1}{W_G} = \frac{20}{3(200) - 320} = \frac{1}{14}$$

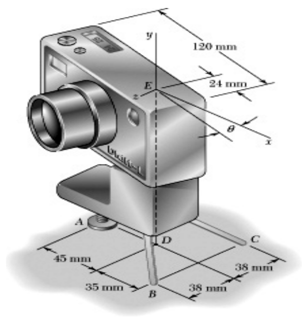
$$\therefore \quad W_1 = \frac{W_G}{14} = \frac{245.25 \text{ N}}{14} = 17.5179 \text{ N}(\text{minimum})$$

and
$$m = \frac{W_1}{g} = \frac{17.5179 \text{ N}}{9.81 \text{ m/s}^2} = 1.78571 \text{ kg}$$

or $m = 1.786$ kg ◀

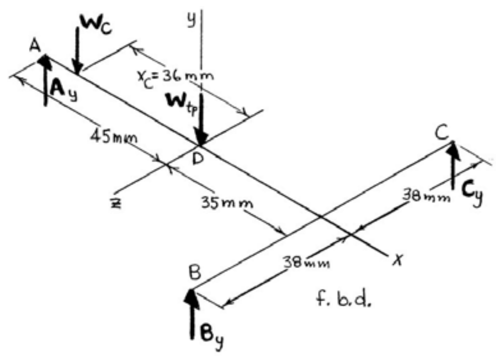
at $x = 200$ mm, $z = 100$ mm ◀

PROBLEM 4.104



A camera of mass 240 g is mounted on a small tripod of mass 200 g. Assuming that the mass of the camera is uniformly distributed and that the line of action of the weight of the tripod passes through D, determine (a) the vertical components of the reactions at A, B, and C when $\theta = 0$, (b) the maximum value of θ if the tripod is not to tip over.

SOLUTION



First note
$$W_C = m_C g = (0.24 \text{ kg})(9.81 \text{ m/s}^2) = 2.3544 \text{ N}$$

$$W_{\text{tp}} = m_{\text{tp}} g = (0.20 \text{ kg})(9.81 \text{ m/s}^2) = 1.9620 \text{ N}$$

For $\theta = 0$
$$x_C = -(60 \text{ mm} - 24 \text{ mm}) = -36 \text{ mm}$$

$$z_C = 0$$

(a) From f.b.d. of camera and tripod as projected onto plane $ABCD$

$$\Sigma F_y = 0: \quad A_y + B_y + C_y - W_C - W_{\text{tp}} = 0$$

$$\therefore \quad A_y + B_y + C_y = 2.3544 \text{ N} + 1.9620 \text{ N} = 4.3164 \text{ N} \qquad (1)$$

$$\Sigma M_x = 0: \quad C_y(38 \text{ mm}) - B_y(38 \text{ mm}) = 0 \qquad \therefore \quad C_y = B_y \qquad (2)$$

$$\Sigma M_z = 0: \quad B_y(35 \text{ mm}) + C_y(35 \text{ mm}) + (2.3544 \text{ N})(36 \text{ mm}) - A_y(45 \text{ mm}) = 0$$

$$\therefore \quad 9A_y - 7B_y - 7C_y = 16.9517 \qquad (3)$$

Substitute C_y with B_y from Equation (2) into Equations (1) and (3), and solve by elimination

$$7(A_y + 2B_y = 4.3164)$$

$$\underline{9A_y - 14B_y = 16.9517}$$

$$16A_y \qquad = 47.166$$

PROBLEM 4.104 CONTINUED

$$\therefore \quad A_y = 2.9479 \text{ N}$$

or $\mathbf{A}_y = 2.95 \text{ N} \uparrow$ ◀

Substituting $A_y = 2.9479 \text{ N}$ into Equation (1)

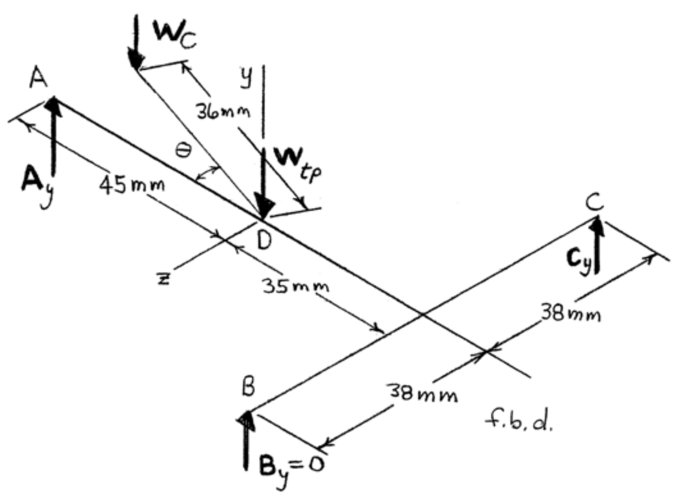
$$2.9479 \text{ N} + 2B_y = 4.3164$$

$$\therefore \quad B_y = 0.68425 \text{ N}$$

$$C_y = 0.68425 \text{ N}$$

or $\mathbf{B}_y = \mathbf{C}_y = 0.684 \text{ N} \uparrow$ ◀

(*b*) $B_y = 0$ for impending tipping



From f.b.d. of camera and tripod as projected onto plane *ABCD*

$$\Sigma F_y = 0: \quad A_y + C_y - W_C - W_{\text{tp}} = 0$$

$$\therefore \quad A_y + C_y = 4.3164 \text{ N} \tag{1}$$

$$\Sigma M_x = 0: \quad C_y(38 \text{ mm}) - (2.3544 \text{ N})\left[(36 \text{ mm})\sin\theta\right] = 0$$

$$\therefore \quad C_y = 2.2305\sin\theta \tag{2}$$

$$\Sigma M_z = 0: \quad C_y(35 \text{ mm}) - A_y(45 \text{ mm}) + (2.3544 \text{ N})\left[(36 \text{ mm})\cos\theta\right] = 0$$

$$\therefore \quad 9A_y - 7C_y = (16.9517 \text{ N})\cos\theta \tag{3}$$

Forming $7 \times [\text{Eq. } (1)] + [\text{Eq. } (3)]$ yields

$$16A_y = 30.215 \text{ N} + (16.9517 \text{ N})\cos\theta \tag{4}$$

PROBLEM 4.104 CONTINUED

Substituting Equation (2) into Equation (3)

$$9A_y - (15.6134\text{ N})\sin\theta = (16.9517\text{ N})\cos\theta \tag{5}$$

Forming $9 \times [\text{Eq. }(4)] - 16 \times [\text{Eq. }(5)]$ yields

$$(249.81\text{ N})\sin\theta = 271.93\text{ N} - (118.662\text{ N})\cos\theta$$

or

$$\cos^2\theta = \left[2.2916\text{ N} - (2.1053\text{ N})\sin\theta\right]^2$$

Now

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\therefore \quad 5.4323\sin^2\theta - 9.6490\sin\theta + 4.2514 = 0$$

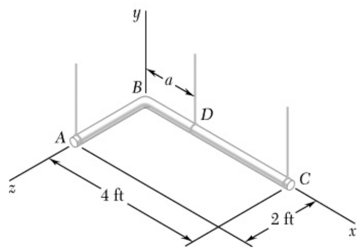
Using quadratic formula to solve,

$$\sin\theta = 0.80981 \quad \text{and} \quad \sin\theta = 0.96641$$

$$\therefore \quad \theta = 54.078° \quad \text{and} \quad \theta = 75.108°$$

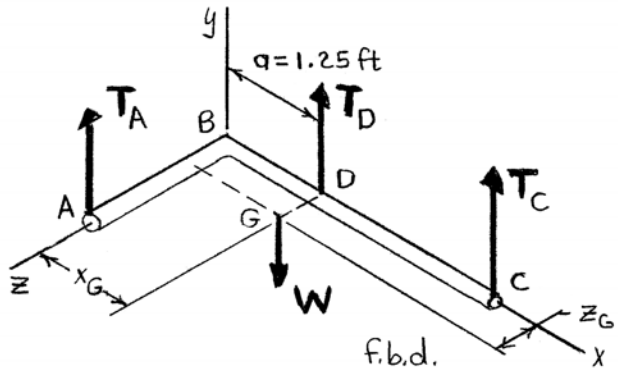
or $\theta_{max} = 54.1°$ before tipping ◀

PROBLEM 4.105



Two steel pipes AB and BC, each having a weight per unit length of 5 lb/ft, are welded together at B and are supported by three wires. Knowing that $a = 1.25$ ft, determine the tension in each wire.

SOLUTION



First note $\quad W_{AB} = (5 \text{ lb/ft})(2 \text{ ft}) = 10 \text{ lb}$

$$W_{BC} = (5 \text{ lb/ft})(4 \text{ ft}) = 20 \text{ lb}$$

$$W = W_{AB} + W_{BC} = 30 \text{ lb}$$

To locate the equivalent force of the pipe assembly weight

$$\mathbf{r}_{G/B} \times \mathbf{W} = \Sigma(\mathbf{r_i} \times \mathbf{W_i}) = \mathbf{r}_{G(AB)} \times \mathbf{W}_{AB} + \mathbf{r}_{G(BC)} \times \mathbf{W}_{BC}$$

or

$$(x_G\mathbf{i} + z_G\mathbf{k}) \times (-30 \text{ lb})\mathbf{j} = (1 \text{ ft})\mathbf{k} \times (-10 \text{ lb})\mathbf{j} + (2 \text{ ft})\mathbf{i} \times (-20 \text{ lb})\mathbf{j}$$

$$\therefore \quad -(30 \text{ lb})x_G\mathbf{k} + (30 \text{ lb})z_G\mathbf{i} = (10 \text{ lb}\cdot\text{ft})\mathbf{i} - (40 \text{ lb}\cdot\text{ft})\mathbf{k}$$

From **i**-coefficient $\quad z_G = \dfrac{10 \text{ lb}\cdot\text{ft}}{30 \text{ lb}} = \dfrac{1}{3}\text{ft}$

k-coefficient $\quad x_G = \dfrac{40 \text{ lb}\cdot\text{ft}}{30 \text{ lb}} = 1\dfrac{1}{3}\text{ft}$

From f.b.d. of piping

$$\Sigma M_x = 0: \quad W(z_G) - T_A(2 \text{ ft}) = 0$$

$$\therefore \quad T_A = \left(\frac{1}{2} \text{ ft}\right)30 \text{ lb}\left(\frac{1}{3}\text{ft}\right) = 5 \text{ lb} \qquad \text{or} \qquad T_A = 5.00 \text{ lb}$$

$$\Sigma F_y = 0: \quad 5 \text{ lb} + T_D + T_C - 30 \text{ lb} = 0$$

$$\therefore \quad T_D + T_C = 25 \text{ lb} \qquad (1)$$

PROBLEM 4.105 CONTINUED

$$\Sigma M_z = 0: \quad T_D(1.25 \text{ ft}) + T_C(4 \text{ ft}) - 30 \text{ lb}\left(\frac{4}{3}\text{ft}\right) = 0$$

$$\therefore \quad 1.25T_D + 4T_C = 40 \text{ lb}\cdot\text{ft} \qquad (2)$$

$-4[\text{Equation } (1)]$ $$-4T_D - 4T_C = -100 \qquad (3)$$

Equation (2) + Equation (3) $$-2.75T_D = -60$$

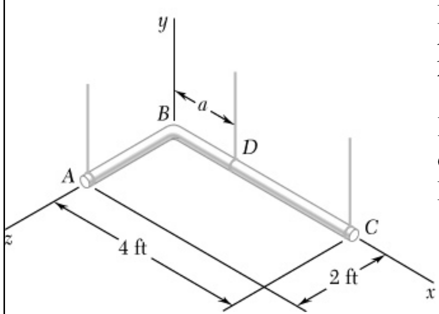
$$\therefore \quad T_D = 21.818 \text{ lb} \qquad \text{or} \qquad T_D = 21.8 \text{ lb}$$

From Equation (1) $$T_C = 25 - 21.818 = 3.1818 \text{ lb} \qquad \text{or} \qquad T_C = 3.18 \text{ lb}$$

Results: $T_A = 5.00$ lb ◀

$T_C = 3.18$ lb ◀

$T_D = 21.8$ lb ◀

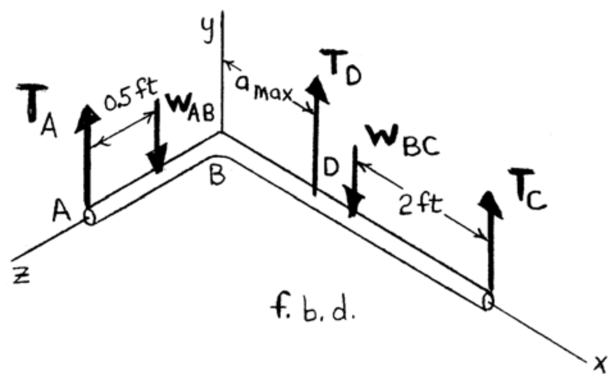


PROBLEM 4.106

For the pile assembly of Problem 4.105, determine (*a*) the largest permissible value of *a* if the assembly is not to tip, (*b*) the corresponding tension in each wire.

P4.105 Two steel pipes *AB* and *BC*, each having a weight per unit length of 5 lb/ft, are welded together at *B* and are supported by three wires. Knowing that $a = 1.25$ ft, determine the tension in each wire.

SOLUTION



First note $$W_{AB} = (5 \text{ lb/ft})(2 \text{ ft}) = 10 \text{ lb}$$

$$W_{BC} = (5 \text{ lb/ft})(4 \text{ ft}) = 20 \text{ lb}$$

From f.b.d. of pipe assembly

$$\Sigma F_y = 0: \quad T_A + T_C + T_D - 10 \text{ lb} - 20 \text{ lb} = 0$$

$$\therefore \quad T_A + T_C + T_D = 30 \text{ lb} \quad (1)$$

$$\Sigma M_x = 0: \quad (10 \text{ lb})(1 \text{ ft}) - T_A(2 \text{ ft}) = 0$$

or $$T_A = 5.00 \text{ lb} \quad (2)$$

From Equations (1) and (2) $$T_C + T_D = 25 \text{ lb} \quad (3)$$

$$\Sigma M_z = 0: \quad T_C(4 \text{ ft}) + T_D(a_{\max}) - 20 \text{ lb}(2 \text{ ft}) = 0$$

or $$(4 \text{ ft})T_C + T_D a_{\max} = 40 \text{ lb}\cdot\text{ft} \quad (4)$$

PROBLEM 4.106 CONTINUED

Using Equation (3) to eliminate T_C

$$4(25 - T_D) + T_D a_{max} = 40$$

or

$$a_{max} = 4 - \frac{60}{T_D}$$

By observation, a is maximum when T_D is maximum. From Equation (3), $(T_D)_{max}$ occurs when $T_C = 0$. Therefore, $(T_D)_{max} = 25$ lb and

$$a_{max} = 4 - \frac{60}{25}$$

$$= 1.600 \text{ ft}$$

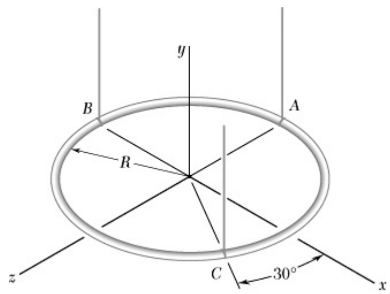
Results: (*a*) $a_{max} = 1.600$ ft ◀

(*b*) $T_A = 5.00$ lb ◀

$T_C = 0$ ◀

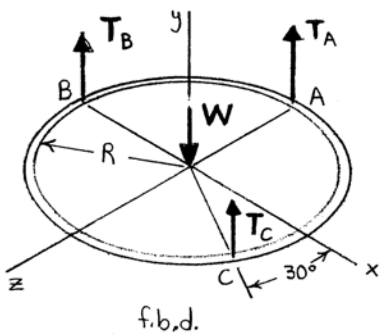
$T_D = 25.0$ lb ◀

PROBLEM 4.107



A uniform aluminum rod of weight W is bent into a circular ring of radius R and is supported by three wires as shown. Determine the tension in each wire.

SOLUTION



From f.b.d. of ring

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - W = 0$$

$$\therefore \quad T_A + T_B + T_C = W \qquad (1)$$

$$\Sigma M_x = 0: \quad T_A(R) - T_C(R\sin 30°) = 0$$

$$\therefore \quad T_A = 0.5T_C \qquad (2)$$

$$\Sigma M_z = 0: \quad T_C(R\cos 30°) - T_B(R) = 0$$

$$\therefore \quad T_B = 0.86603T_C \qquad (3)$$

Substituting T_A and T_B from Equations (2) and (3) into Equation (1)

$$0.5T_C + 0.86603T_C + T_C = W$$

$$\therefore \quad T_C = 0.42265W$$

From Equation (2)

$$T_A = 0.5(0.42265W) = 0.21132W$$

From Equation (3)

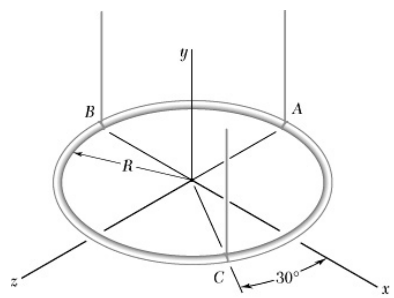
$$T_B = 0.86603(0.42265W) = 0.36603W$$

or $T_A = 0.211W$ ◀

$T_B = 0.366W$ ◀

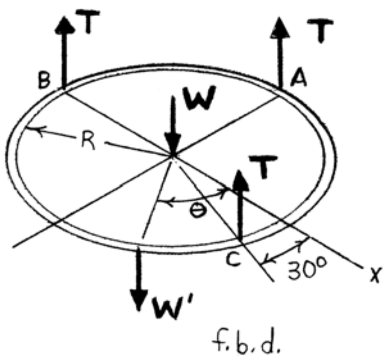
$T_C = 0.423W$ ◀

PROBLEM 4.108



A uniform aluminum rod of weight W is bent into a circular ring of radius R and is supported by three wires as shown. A small collar of weight W' is then placed on the ring and positioned so that the tensions in the three wires are equal. Determine (*a*) the position of the collar, (*b*) the value of W', (*c*) the tension in the wires.

SOLUTION



Let θ = angle from x-axis to small collar of weight W'

From f.b.d. of ring

$$\Sigma F_y = 0: \quad 3T - W - W' = 0 \tag{1}$$

$$\Sigma M_x = 0: \quad T(R) - T(R\sin 30°) + W'(R\sin\theta) = 0$$

or

$$W'\sin\theta = -\frac{1}{2}T \tag{2}$$

$$\Sigma M_z = 0: \quad T(R\cos 30°) - W'(R\cos\theta) - T(R) = 0$$

or

$$W'\cos\theta = -\left(1 - \frac{\sqrt{3}}{2}\right)T \tag{3}$$

Dividing Equation (2) by Equation (3)

$$\tan\theta = \left(\frac{1}{2}\right)\left[1 - \left(\frac{\sqrt{3}}{2}\right)\right]^{-1} = 3.7321$$

$$\therefore \quad \theta = 75.000° \quad \text{and} \quad \theta = 255.00°$$

Based on Equations (2) and (3), $\theta = 75.000°$ will give a negative value for W', which is not acceptable.

(*a*) $\therefore$ W' is located at $\theta = 255°$ from the x-axis or $15°$ from A towards B. ◀

(*b*) From Equation (1) and Equation (2)

$$W' = 3(-2W')(\sin 255°) - W$$

$$\therefore \quad W' = 0.20853W$$

or $W' = 0.209W$ ◀

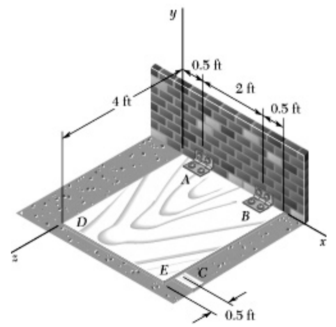
(*c*) From Equation (1)

$$T = -2(0.20853W)\sin 255°$$

$$= 0.40285W$$

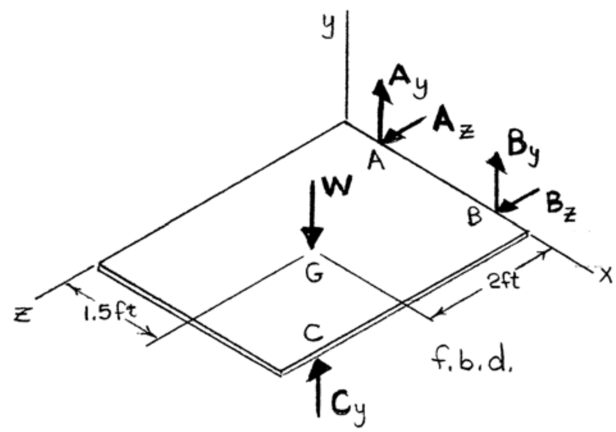
or $T = 0.403W$ ◀

PROBLEM 4.109



An opening in a floor is covered by a 3×4-ft sheet of plywood weighing 12 lb. The sheet is hinged at A and B and is maintained in a position slightly above the floor by a small block C. Determine the vertical component of the reaction (a) at A, (b) at B, (c) at C.

SOLUTION



From f.b.d. of plywood sheet

$$\Sigma M_x = 0: \quad (12 \text{ lb})(2 \text{ ft}) - C_y(3.5 \text{ ft}) = 0$$

$$\therefore \quad C_y = 6.8571 \text{ lb} \qquad \text{or} \qquad C_y = 6.86 \text{ lb}$$

$$\Sigma M_{B(z\text{-axis})} = 0: \quad (12 \text{ lb})(1 \text{ ft}) + (6.8571 \text{ lb})(0.5 \text{ ft}) - A_y(2 \text{ ft}) = 0$$

$$\therefore \quad A_y = 7.7143 \text{ lb} \qquad \text{or} \qquad A_y = 7.71 \text{ lb}$$

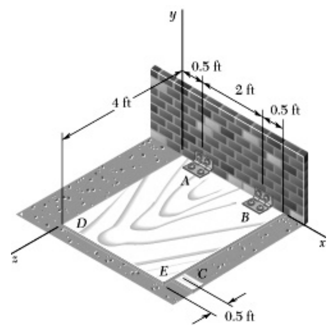
$$\Sigma M_{A(z\text{-axis})} = 0: \quad -(12 \text{ lb})(1 \text{ ft}) + B_y(2 \text{ ft}) + (6.8571 \text{ lb})(2.5 \text{ ft}) = 0$$

$$\therefore \quad B_y = 2.5714 \text{ lb} \qquad \text{or} \qquad B_y = 2.57 \text{ lb}$$

(a) $A_y = 7.71$ lb ◀

(b) $B_y = 2.57$ lb ◀

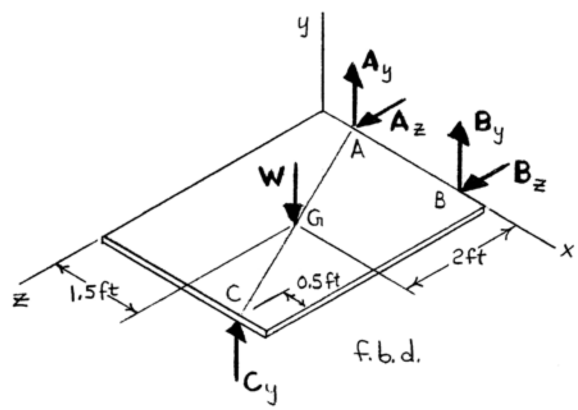
(c) $C_y = 6.86$ lb ◀



PROBLEM 4.110

Solve Problem 4.109 assuming that the small block *C* is moved and placed under edge *DE* at a point 0.5 ft from corner *E*.

SOLUTION



First,

$$\mathbf{r}_{B/A} = (2 \text{ ft})\mathbf{i}$$

$$\mathbf{r}_{C/A} = (2 \text{ ft})\mathbf{i} + (4 \text{ ft})\mathbf{k}$$

$$\mathbf{r}_{G/A} = (1 \text{ ft})\mathbf{i} + (2 \text{ ft})\mathbf{k}$$

From f.b.d. of plywood sheet

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{B/A} \times \left(B_y\mathbf{j} + B_z\mathbf{k}\right) + \mathbf{r}_{C/A} \times C_y\mathbf{j} + \mathbf{r}_{G/A} \times \left(-W\mathbf{j}\right) = 0$$

$$(2 \text{ ft})\mathbf{i} \times B_y\mathbf{j} + (2 \text{ ft})\mathbf{i} \times B_z\mathbf{k} + \left[(2 \text{ ft})\mathbf{i} + (4 \text{ ft})\mathbf{k}\right] \times C_y\mathbf{j}$$

$$+ \left[(1 \text{ ft})\mathbf{i} + (2 \text{ ft})\mathbf{k}\right] \times (-12 \text{ lb})\mathbf{j} = 0$$

$$2B_y\mathbf{k} - 2B_z\mathbf{j} + 2C_y\mathbf{k} - 4C_y\mathbf{i} - 12\mathbf{k} + 24\mathbf{i} = 0$$

i-coeff. $\quad -4C_y + 24 = 0 \qquad \therefore \; C_y = 6.00 \text{ lb}$

j-coeff. $\quad -2B_z = 0 \qquad \therefore \; B_z = 0$

k-coeff. $\quad 2B_y + 2C_y - 12 = 0$

or $\quad 2B_y + 2(6) - 12 = 0 \qquad \therefore \; B_y = 0$

PROBLEM 4.110 CONTINUED

$$\Sigma \mathbf{F} = 0: \quad A_y\mathbf{j} + A_z\mathbf{k} + B_y\mathbf{j} + B_z\mathbf{k} + C_y\mathbf{j} - W\mathbf{j} = 0$$

$$A_y\mathbf{j} + A_z\mathbf{k} + 0\mathbf{j} + 0\mathbf{k} + 6\mathbf{j} - 12\mathbf{j} = 0$$

j-coeff. $\quad A_y + 6 - 12 = 0 \qquad \therefore \ A_y = 6.00 \text{ lb}$

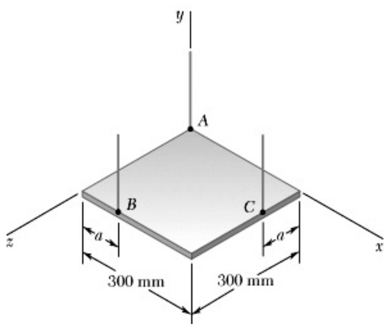
k-coeff. $\quad A_z = 0 \qquad A_z = 0$

$\therefore$ *a*) $A_y = 6.00 \text{ lb}$ ◀

b) $B_y = 0$ ◀

c) $C_y = 6.00 \text{ lb}$ ◀

PROBLEM 4.111

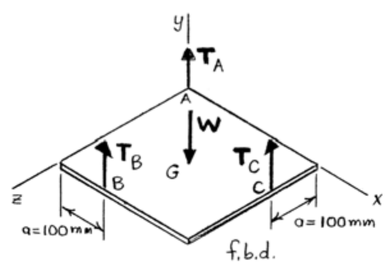


The 10-kg square plate shown is supported by three vertical wires. Determine (*a*) the tension in each wire when $a = 100$ mm, (*b*) the value of a for which tensions in the three wires are equal.

SOLUTION

First note $\quad W = mg = (10\text{ kg})(9.81\text{ m/s}^2) = 98.1\text{ N}$

(*a*)



(*a*) From f.b.d. of plate

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - W = 0$$

$$\therefore \quad T_A + T_B + T_C = 98.1\text{ N} \qquad (1)$$

$$\Sigma M_x = 0: \quad W(150\text{ mm}) - T_B(300\text{ mm}) - T_C(100\text{ mm}) = 0$$

$$\therefore \quad 6T_B + 2T_C = 294.3 \qquad (2)$$

$$\Sigma M_z = 0: \quad T_B(100\text{ mm}) + T_C(300\text{ mm}) - (98.1\text{ N})(150\text{ mm}) = 0$$

$$\therefore \quad -6T_B - 18T_C = -882.9 \qquad (3)$$

Equation (2) + Equation (3)

$$-16T_C = -588.6$$

$$\therefore \quad T_C = 36.788\text{ N}$$

or $\quad T_C = 36.8\text{ N}$ ◀

Substitution into Equation (2)

$$6T_B + 2(36.788\text{ N}) = 294.3\text{ N}$$

$$\therefore \quad T_B = 36.788\text{ N} \quad \text{or} \quad T_B = 36.8\text{ N}$$ ◀

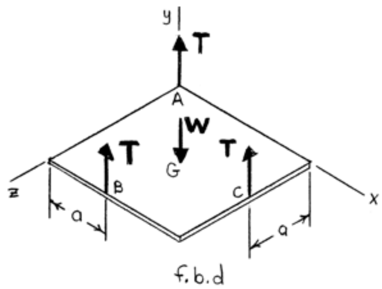
From Equation (1)

$$T_A + 36.788 + 36.788 = 98.1\text{ N}$$

$$\therefore \quad T_A = 24.525\text{ N} \quad \text{or} \quad T_A = 24.5\text{ N}$$ ◀

PROBLEM 4.111 CONTINUED

(*b*)



(*b*) From f.b.d. of plate

$$\Sigma F_y = 0: \quad 3T - W = 0$$

$$\therefore \quad T = \frac{1}{3}W \qquad (1)$$

$$\Sigma M_x = 0: \quad W(150 \text{ mm}) - T(a) - T(300 \text{ mm}) = 0$$

$$\therefore \quad T = \frac{150W}{a + 300} \qquad (2)$$

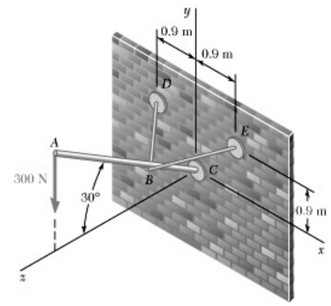
Equating Equation (1) to Equation (2)

$$\frac{1}{3}W = \frac{150W}{a + 300}$$

or

$$a + 300 = 3(150)$$

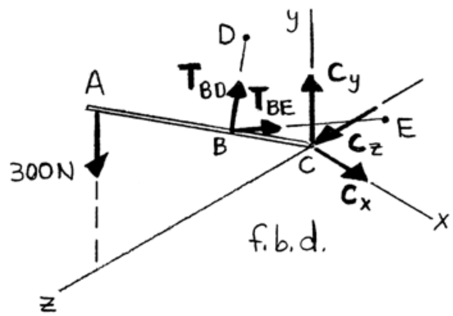
or $a = 150.0$ mm ◀



PROBLEM 4.112

The 3-m flagpole AC forms an angle of 30° with the z axis. It is held by a ball-and-socket joint at C and by two thin braces BD and BE. Knowing that the distance BC is 0.9 m, determine the tension in each brace and the reaction at C.

SOLUTION



T_{BE} can be found from ΣM about line CE

From f.b.d. of flagpole

$$\Sigma M_{CE} = 0: \quad \lambda_{CE} \cdot \left(\mathbf{r}_{B/C} \times \mathbf{T}_{BD}\right) + \lambda_{CE} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{F}_{A}\right) = 0$$

where $$\lambda_{CE} = \frac{(0.9\text{ m})\mathbf{i} + (0.9\text{ m})\mathbf{j}}{\sqrt{(0.9)^2 + (0.9)^2}\text{ m}} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$$

$$\mathbf{r}_{B/C} = \left[(0.9\text{ m})\sin 30^{\circ}\right]\mathbf{j} + \left[(0.9\text{ m})\cos 30^{\circ}\right]\mathbf{k}$$

$$= (0.45\text{ m})\mathbf{j} + (0.77942\text{ m})\mathbf{k}$$

$$\mathbf{T}_{BD} = \lambda_{BD}T_{BD} = \left\{\frac{-(0.9\text{ m})\mathbf{i} + \left[0.9\text{ m} - (0.9\text{ m})\sin 30^{\circ}\right]\mathbf{j} - \left[(0.9\text{ m})\cos 30^{\circ}\right]\mathbf{k}}{\sqrt{(0.9)^2 + (0.45)^2 + (0.77942)^2}\text{ m}}\right\}T_{BD}$$

$$= \left[-(0.9\text{ m})\mathbf{i} + (0.45\text{ m})\mathbf{j} - (0.77942\text{ m})\mathbf{k}\right]\frac{T_{BD}}{\sqrt{1.62}}$$

$$= (-0.70711\mathbf{i} + 0.35355\mathbf{j} - 0.61237\mathbf{k})T_{BD}$$

$$\mathbf{r}_{A/C} = (3\text{ m})\sin 30^{\circ}\mathbf{j} + (3\text{ m})\cos 30^{\circ}\mathbf{k} = (1.5\text{ m})\mathbf{j} + (2.5981\text{ m})\mathbf{k}$$

$$\mathbf{F}_{A} = -(300\text{ N})\mathbf{j}$$

$$\therefore \quad \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0.45 & 0.77942 \\ -0.70711 & 0.35355 & -0.61237 \end{vmatrix}\left(\frac{T_{BD}}{\sqrt{2}}\right) + \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1.5 & 2.5981 \\ 0 & -300 & 0 \end{vmatrix}\left(\frac{1}{\sqrt{2}}\right) = 0$$

PROBLEM 4.112 CONTINUED

or $$-1.10227T_{BD} + 779.43 = 0$$

$$\therefore \quad T_{BD} = 707.12 \text{ N}$$ or $T_{BD} = 707 \text{ N}$ ◀

Based on symmetry with *yz*-plane, $T_{BE} = T_{BD} = 707.12 \text{ N}$ or $T_{BE} = 707 \text{ N}$ ◀

The reaction forces at C are found from $\Sigma \mathbf{F} = 0$

$$\Sigma F_x = 0: \quad -(T_{BD})_x + (T_{BE})_x + C_x = 0 \qquad \text{or} \qquad C_x = 0$$

$$\Sigma F_y = 0: \quad (T_{BD})_y + (T_{BE})_y + C_y - 300 \text{ N} = 0$$

$$C_y = 300 \text{ N} - 2(0.35355)(707.12 \text{ N})$$

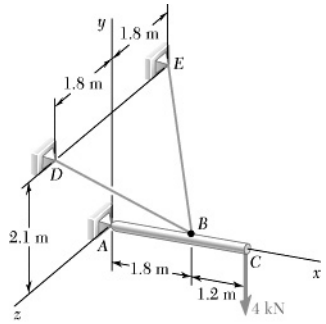
$$\therefore \quad C_y = -200.00 \text{ N}$$

$$\Sigma F_z = 0: \quad C_z - (T_{BD})_z - (T_{BE})_z = 0$$

$$C_z = 2(0.61237)(707.12 \text{ N})$$

$$\therefore \quad C_z = 866.04 \text{ N}$$

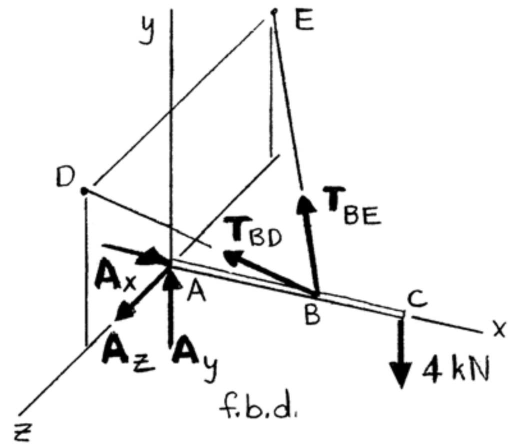
or $\mathbf{C} = -(200 \text{ N})\mathbf{j} + (866 \text{ N})\mathbf{k}$ ◀



PROBLEM 4.113

A 3-m boom is acted upon by the 4-kN force shown. Determine the tension in each cable and the reaction at the ball-and-socket joint at A.

SOLUTION



From f.b.d. of boom

$$\Sigma M_{AE} = 0: \quad \boldsymbol{\lambda}_{AE} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{T}_{BD}\right) + \boldsymbol{\lambda}_{AE} \cdot \left(\mathbf{r}_{C/A} \times \mathbf{F}_C\right) = 0$$

where

$$\boldsymbol{\lambda}_{AE} = \frac{(2.1\text{ m})\mathbf{j} - (1.8\text{ m})\mathbf{k}}{\sqrt{(2.1)^2 + (1.8)^2}\text{ m}}$$

$$= 0.27451\mathbf{j} - 0.23529\mathbf{k}$$

$$\mathbf{r}_{B/A} = (1.8\text{ m})\mathbf{i}$$

$$\mathbf{T}_{BD} = \boldsymbol{\lambda}_{BD} T_{BD} = \frac{(-1.8\text{ m})\mathbf{i} + (2.1\text{ m})\mathbf{j} + (1.8\text{ m})\mathbf{k}}{\sqrt{(1.8)^2 + (2.1)^2 + (1.8)^2}\text{ m}} T_{BD}$$

$$= (-0.54545\mathbf{i} + 0.63636\mathbf{j} + 0.54545\mathbf{k}) T_{BD}$$

$$\mathbf{r}_{C/A} = (3.0\text{ m})\mathbf{i}$$

$$\mathbf{F}_C = -(4\text{ kN})\mathbf{j}$$

PROBLEM 4.113 CONTINUED

$$\therefore \begin{vmatrix} 0 & 0.27451 & -0.23529 \\ 1.8 & 0 & 0 \\ -0.54545 & 0.63636 & 0.54545 \end{vmatrix} T_{BD} + \begin{vmatrix} 0 & 0.27451 & -0.23529 \\ 3 & 0 & 0 \\ 0 & -4 & 0 \end{vmatrix} = 0$$

$$\left(-0.149731 - 0.149729\right)1.8T_{BD} + 2.82348 = 0$$

$$\therefore \quad T_{BD} = 5.2381 \text{ kN}$$ or $T_{BD} = 5.24$ kN ◀

Based on symmetry, $$T_{BE} = T_{BD} = 5.2381 \text{ kN}$$ or $T_{BE} = 5.24$ kN ◀

$$\Sigma F_z = 0: \quad A_z + \left(T_{BD}\right)_z - \left(T_{BE}\right)_z = 0 \qquad A_z = 0$$

$$\Sigma F_y = 0: \quad A_y + \left(T_{BD}\right)_y + \left(T_{BD}\right)_y - 4 \text{ kN} = 0$$

$$A_y + 2\left(0.63636\right)\left(5.2381 \text{ kN}\right) - 4 \text{ kN} = 0$$

$$\therefore \quad A_y = -2.6666 \text{ kN}$$

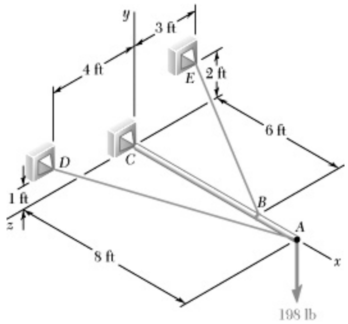
$$\Sigma F_x = 0: \quad A_x - \left(T_{BD}\right)_x - \left(T_{BE}\right)_x = 0$$

$$A_x - 2\left(0.54545\right)\left(5.2381 \text{ kN}\right) = 0$$

$$\therefore \quad A_x = 5.7142 \text{ kN}$$

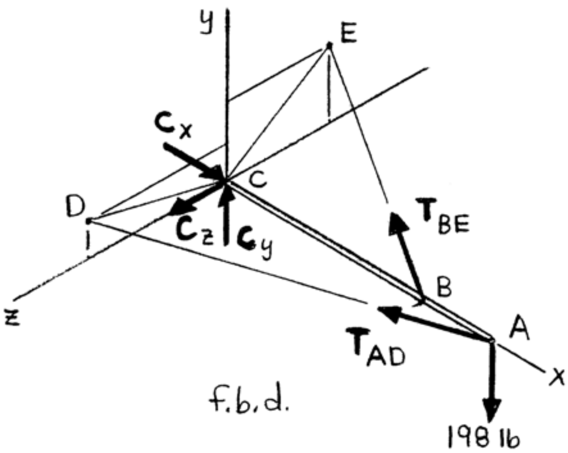
and $\mathbf{A} = \left(5.71 \text{ N}\right)\mathbf{i} - \left(2.67 \text{ N}\right)\mathbf{j}$ ◀

PROBLEM 4.114



An 8-ft-long boom is held by a ball-and-socket joint at *C* and by two cables *AD* and *BE*. Determine the tension in each cable and the reaction at *C*.

SOLUTION



From f.b.d. of boom

$$\Sigma M_{CE} = 0: \quad \boldsymbol{\lambda}_{CE} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{T}_{AD}\right) + \boldsymbol{\lambda}_{CE} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{F}_{A}\right) = 0$$

where

$$\boldsymbol{\lambda}_{CE} = \frac{(2\text{ ft})\mathbf{j} - (3\text{ ft})\mathbf{k}}{\sqrt{(2)^2 + (3)^2}\text{ ft}} = \frac{1}{\sqrt{13}}(2\mathbf{j} - 3\mathbf{k})$$

$$\mathbf{r}_{A/C} = (8\text{ ft})\mathbf{i}$$

$$\mathbf{T}_{AD} = \boldsymbol{\lambda}_{AD} T_{AD} = \frac{-(8\text{ ft})\mathbf{i} + (1\text{ ft})\mathbf{j} + (4\text{ ft})\mathbf{k}}{\sqrt{(8)^2 + (1)^2 + (4)^2}\text{ ft}} T_{AD}$$

$$= \left(\frac{1}{9}\right) T_{AD}(-8\mathbf{i} + \mathbf{j} + 4\mathbf{k})$$

$$\mathbf{F}_A = -(198\text{ lb})\mathbf{j}$$

$$\therefore \quad \begin{vmatrix} 0 & 2 & -3 \\ 8 & 0 & 0 \\ -8 & 1 & 4 \end{vmatrix} \left(\frac{T_{AD}}{9\sqrt{13}}\right) + \begin{vmatrix} 0 & 2 & -3 \\ 8 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \left(\frac{198}{\sqrt{13}}\right) = 0$$

PROBLEM 4.114 CONTINUED

$$\left(-64 - 24\right)\frac{T_{AD}}{9\sqrt{13}} + \left(24\right)\frac{198}{\sqrt{13}} = 0$$

$$\therefore \quad T_{AD} = 486.00 \text{ lb}$$

or $T_{AD} = 486 \text{ lb}$ ◀

$$\Sigma M_{CD} = 0: \quad \boldsymbol{\lambda}_{CD} \cdot \left(\mathbf{r}_{B/C} \times \mathbf{T}_{BE}\right) + \boldsymbol{\lambda}_{CD} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{F}_A\right)$$

where $\boldsymbol{\lambda}_{CD} = \dfrac{(1 \text{ ft})\mathbf{j} + (4 \text{ ft})\mathbf{k}}{\sqrt{17} \text{ ft}} = \dfrac{1}{\sqrt{17}}(1\mathbf{j} + 4\mathbf{k})$

$$\mathbf{r}_{B/C} = (6 \text{ ft})\mathbf{i}$$

$$\mathbf{T}_{BE} = \boldsymbol{\lambda}_{BE} T_{BE} = \frac{-(6 \text{ ft})\mathbf{i} + (2 \text{ ft})\mathbf{j} - (3 \text{ ft})\mathbf{k}}{\sqrt{(6)^2 + (2)^2 + (3)^2} \text{ ft}} T_{BE} = \left(\frac{1}{7}\right) T_{BE}\left(-6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\right)$$

$$\therefore \quad \begin{vmatrix} 0 & 1 & 4 \\ 6 & 0 & 0 \\ -6 & 2 & -3 \end{vmatrix} \frac{T_{BE}}{7\sqrt{17}} + \begin{vmatrix} 0 & 1 & 4 \\ 8 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \frac{198}{\sqrt{17}} = 0$$

$$\left(18 + 48\right)\frac{T_{BE}}{7} + \left(-32\right)198 = 0$$

$$\therefore \quad T_{BE} = 672.00 \text{ lb}$$

or $T_{BE} = 672 \text{ lb}$ ◀

$$\Sigma F_x = 0: \quad C_x - \left(T_{AD}\right)_x - \left(T_{BE}\right)_x = 0$$

$$C_x - \left(\frac{8}{9}\right)486 - \left(\frac{6}{7}\right)672 = 0$$

$$\therefore \quad C_x = 1008 \text{ lb}$$

$$\Sigma F_y = 0: \quad C_y + \left(T_{AD}\right)_y + \left(T_{BE}\right)_y - 198 \text{ lb} = 0$$

$$C_y + \left(\frac{1}{9}\right)486 + \left(\frac{2}{7}\right)672 - 198 \text{ lb} = 0$$

$$\therefore \quad C_y = -48.0 \text{ lb}$$

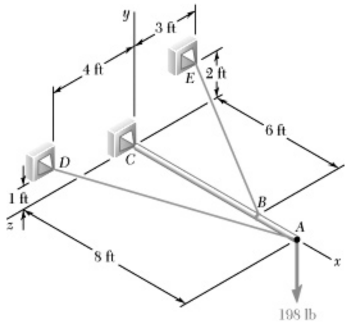
$$\Sigma F_z = 0: \quad C_z + \left(T_{AD}\right)_z - \left(T_{BE}\right)_z = 0$$

$$C_z + \left(\frac{4}{9}\right)486 - \left(\frac{3}{7}\right)\left(672\right) = 0$$

$$\therefore \quad C_z = 72.0 \text{ lb}$$

or $\mathbf{C} = (1008 \text{ lb})\mathbf{i} - (48.0 \text{ lb})\mathbf{j} + (72.0 \text{ lb})\mathbf{k}$ ◀

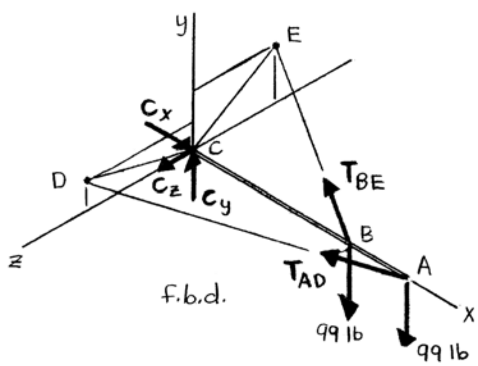
PROBLEM 4.115



Solve Problem 4.114 assuming that the given 198-lb load is replaced with two 99-lb loads applied at *A* and *B*.

P4.114 An 8-ft-long boom is held by a ball-and-socket joint at *C* and by two cables *AD* and *BE*. Determine the tension in each cable and the reaction at *C*.

SOLUTION



From f.b.d. of boom

$$\Sigma M_{CE} = 0: \quad \boldsymbol{\lambda}_{CE} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{T}_{AD}\right) + \boldsymbol{\lambda}_{CE} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{F}_A\right) + \boldsymbol{\lambda}_{CE} \cdot \left(\mathbf{r}_{B/C} \times \mathbf{F}_B\right) = 0$$

where

$$\boldsymbol{\lambda}_{CE} = \frac{(2 \text{ ft})\mathbf{j} - (3 \text{ ft})\mathbf{k}}{\sqrt{(2)^2 + (3)^2} \text{ ft}} = \frac{1}{\sqrt{13}}(2\mathbf{j} - 3\mathbf{k})$$

$$\mathbf{r}_{A/C} = (8 \text{ ft})\mathbf{i}$$

$$\mathbf{r}_{B/C} = (6 \text{ ft})\mathbf{i}$$

$$\mathbf{T}_{AD} = \boldsymbol{\lambda}_{AD} T_{AD} = \frac{-(8 \text{ ft})\mathbf{i} + (1 \text{ ft})\mathbf{j} + (4 \text{ ft})\mathbf{k}}{\sqrt{(8)^2 + (1)^2 + (4)^2} \text{ ft}} T_{AD}$$

$$= \left(\frac{1}{9}\right) T_{AD}(-8\mathbf{i} + \mathbf{j} + 4\mathbf{k})$$

$$\mathbf{F}_A = -(99 \text{ lb})\mathbf{j}$$

$$\mathbf{F}_B = -(99 \text{ lb})\mathbf{j}$$

$$\therefore \quad \begin{vmatrix} 0 & 2 & -3 \\ 8 & 0 & 0 \\ -8 & 1 & 4 \end{vmatrix} \frac{T_{AD}}{9\sqrt{13}} + \begin{vmatrix} 0 & 2 & -3 \\ 8 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \frac{99}{\sqrt{13}} + \begin{vmatrix} 0 & 2 & -3 \\ 6 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \frac{99}{\sqrt{13}} = 0$$

PROBLEM 4.115 CONTINUED

$$(-64 - 24)\frac{T_{AD}}{9\sqrt{13}} + (24 + 18)\frac{99}{\sqrt{13}} = 0$$

or $$T_{AD} = 425.25 \text{ lb}$$

or $T_{AD} = 425 \text{ lb}$ ◀

$$\Sigma M_{CD} = 0: \quad \lambda_{CD} \cdot (\mathbf{r}_{B/C} \times \mathbf{T}_{BE}) + \lambda_{CD} \cdot (\mathbf{r}_{A/C} \times \mathbf{F}_A) + \lambda_{CD} \cdot (\mathbf{r}_{B/C} \times \mathbf{F}_B) = 0$$

where $$\lambda_{CD} = \frac{(1 \text{ ft})\mathbf{j} + (4 \text{ ft})\mathbf{k}}{\sqrt{17}} = \frac{1}{\sqrt{17}}(\mathbf{j} + 4\mathbf{k})$$

$$\mathbf{r}_{B/C} = (6 \text{ ft})\mathbf{i}$$

$$\mathbf{r}_{A/C} = (8 \text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BE} = \lambda_{BE}T_{BE} = \frac{-(6 \text{ ft})\mathbf{i} + (2 \text{ ft})\mathbf{j} - (3 \text{ ft})\mathbf{k}}{\sqrt{(6)^2 + (2)^2 + (3)^2} \text{ ft}}T_{BE} = \frac{T_{BE}}{7}(-6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

$$\therefore \begin{vmatrix} 0 & 1 & 4 \\ 6 & 0 & 0 \\ -6 & 2 & -3 \end{vmatrix}\left(\frac{T_{BE}}{7\sqrt{17}}\right) + \begin{vmatrix} 0 & 1 & 4 \\ 8 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{99}{\sqrt{17}}\right) + \begin{vmatrix} 0 & 1 & 4 \\ 6 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{99}{\sqrt{17}}\right) = 0$$

$$(18 + 48)\left(\frac{T_{BE}}{7\sqrt{17}}\right) + (-32 - 24)\left(\frac{99}{\sqrt{17}}\right) = 0$$

or $$T_{BE} = 588.00 \text{ lb}$$

or $T_{BE} = 588 \text{ lb}$ ◀

$$\Sigma F_x = 0: \quad C_x - (T_{AD})_x - (T_{BE})_x = 0$$

$$C_x - \left(\frac{8}{9}\right)425.25 - \left(\frac{6}{7}\right)588.00 = 0$$

$$\therefore \quad C_x = 882 \text{ lb}$$

$$\Sigma F_y = 0: \quad C_y + (T_{AD})_y + (T_{BE})_y - 99 - 99 = 0$$

$$C_y + \left(\frac{1}{9}\right)425.25 + \left(\frac{2}{7}\right)588.00 - 198 = 0$$

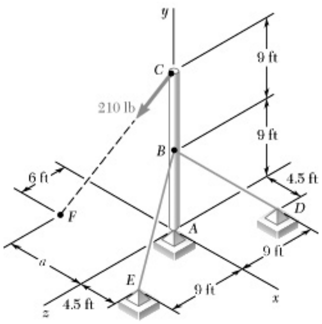
$$\therefore \quad C_y = -17.25 \text{ lb}$$

$$\Sigma F_z = 0: \quad C_z + (T_{AD})_z - (T_{BE})_z = 0$$

$$C_z + \left(\frac{4}{9}\right)425.25 - \left(\frac{3}{7}\right)588.00 = 0$$

$$\therefore \quad C_z = 63.0 \text{ lb}$$

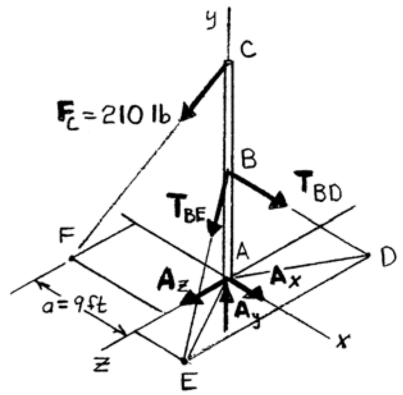
or $\mathbf{C} = (882 \text{ lb})\mathbf{i} - (17.25 \text{ lb})\mathbf{j} + (63.0 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.116

The 18-ft pole *ABC* is acted upon by a 210-lb force as shown. The pole is held by a ball-and-socket joint at *A* and by two cables *BD* and *BE*. For $a = 9$ ft, determine the tension in each cable and the reaction at *A*.

SOLUTION



f.b.d.

From f.b.d. of pole *ABC*

$$\Sigma M_{AE} = 0: \quad \lambda_{AE} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{T}_{BD}\right) + \lambda_{AE} \cdot \left(\mathbf{r}_{C/A} \times \mathbf{F}_C\right) = 0$$

where
$$\lambda_{AE} = \frac{(4.5\text{ ft})\mathbf{i} + (9\text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2}\text{ ft}} = \frac{1}{\sqrt{101.25}}(4.5\mathbf{i} + 9\mathbf{k})$$

$$\mathbf{r}_{B/A} = (9\text{ ft})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (18\text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BD} = \lambda_{BD}T_{BD} = \frac{(4.5\text{ ft})\mathbf{i} - (9\text{ ft})\mathbf{j} - (9\text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2 + (9)^2}\text{ ft}}T_{BD}$$

$$= \left(\frac{T_{BD}}{13.5}\right)(4.5\mathbf{i} - 9\mathbf{j} - 9\mathbf{k})$$

$$\mathbf{F}_C = \lambda_{CF}(210\text{ lb}) = \frac{-9\mathbf{i} - 18\mathbf{j} + 6\mathbf{k}}{\sqrt{(9)^2 + (18)^2 + (6)^2}}(210\text{ lb}) = 10\text{ lb}(-9\mathbf{i} - 18\mathbf{j} + 6\mathbf{k})$$

$$\therefore \begin{vmatrix} 4.5 & 0 & 9 \\ 0 & 9 & 0 \\ 4.5 & -9 & -9 \end{vmatrix}\left(\frac{T_{BD}}{13.5\sqrt{101.25}}\right) + \begin{vmatrix} 4.5 & 0 & 9 \\ 0 & 18 & 0 \\ -9 & -18 & 6 \end{vmatrix}\left(\frac{10\text{ lb}}{\sqrt{101.25}}\right) = 0$$

PROBLEM 4.116 CONTINUED

$$\frac{(-364.5 - 364.5)}{13.5\sqrt{101.25}}T_{BD} + \frac{(486 + 1458)}{\sqrt{101.25}}(10 \text{ lb}) = 0$$

and $$T_{BD} = 360.00 \text{ lb}$$

or $T_{BD} = 360 \text{ lb}$ ◀

$$\Sigma M_{AD} = 0: \quad \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BE}) + \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{C/A} \times \mathbf{F}_C) = 0$$

where $$\boldsymbol{\lambda}_{AD} = \frac{(4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2} \text{ ft}} = \frac{1}{\sqrt{101.25}}(4.5\mathbf{i} - 9\mathbf{k})$$

$$\mathbf{r}_{B/A} = (9 \text{ ft})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (18 \text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BE} = \boldsymbol{\lambda}_{BE}T_{BE} = \frac{(4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{j} + (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2 + (9)^2} \text{ ft}}T_{BE} = \frac{T_{BE}}{13.5}(4.5\mathbf{i} - 9\mathbf{j} + 9\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 4.5 & 0 & -9 \\ 0 & 9 & 0 \\ 4.5 & -9 & 9 \end{vmatrix} \left(\frac{T_{BE}}{13.5\sqrt{101.25}} \right) + \begin{vmatrix} 4.5 & 0 & -9 \\ 0 & 18 & 0 \\ -9 & -18 & 6 \end{vmatrix} \left(\frac{10 \text{ lb}}{\sqrt{101.25}} \right) = 0$$

$$\frac{(364.5 + 364.5)}{13.5\sqrt{101.25}}T_{BE} + \frac{(486 - 1458)10 \text{ lb}}{\sqrt{101.25}} = 0$$

or $$T_{BE} = 180.0 \text{ lb}$$

or $T_{BE} = 180.0 \text{ lb}$ ◀

$$\Sigma F_x = 0: \quad A_x + (T_{BD})_x + (T_{BE})_x - (F_C)_x = 0$$

$$A_x + \left(\frac{4.5}{13.5}\right)360 + \left(\frac{4.5}{13.5}\right)180 - \left(\frac{9}{21}\right)210 = 0$$

$$\therefore \quad A_x = -90.0 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y - (T_{BD})_y - (T_{BE})_y - (F_C)_y = 0$$

$$A_y - \left(\frac{9}{13.5}\right)360 - \left(\frac{9}{13.5}\right)180 - \left(\frac{18}{21}\right)210 = 0$$

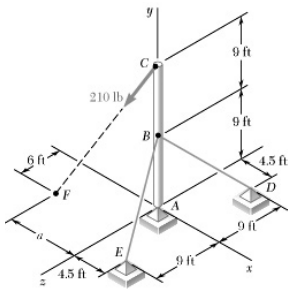
$$\therefore \quad A_y = 540 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z - (T_{BD})_z + (T_{BE})_z + (F_C)_z = 0$$

$$A_z - \left(\frac{9}{13.5}\right)360 + \left(\frac{9}{13.5}\right)180 + \left(\frac{6}{21}\right)210 = 0$$

$$\therefore \quad A_z = 60.0 \text{ lb}$$

or $\mathbf{A} = -(90.0 \text{ lb})\mathbf{i} + (540 \text{ lb})\mathbf{j} + (60.0 \text{ lb})\mathbf{k}$ ◀

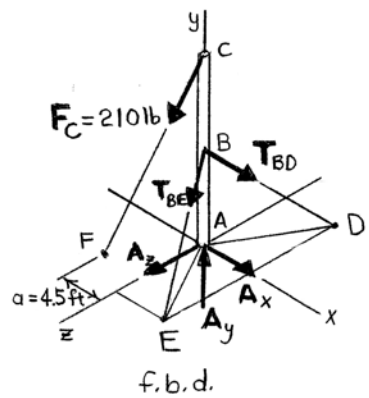


PROBLEM 4.117

Solve Problem 4.116 for $a = 4.5$ ft.

P4.116 The 18-ft pole *ABC* is acted upon by a 210-lb force as shown. The pole is held by a ball-and-socket joint at *A* and by two cables *BD* and *BE*. For $a = 9$ ft, determine the tension in each cable and the reaction at *A*.

SOLUTION



From f.b.d. of pole *ABC*

$$\Sigma M_{AE} = 0: \quad \lambda_{AE} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{T}_{BD}\right) + \lambda_{AE} \cdot \left(\mathbf{r}_{C/A} \times \mathbf{F}_C\right) = 0$$

where

$$\lambda_{AE} = \frac{(4.5\text{ ft})\mathbf{i} + (9\text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2}\text{ ft}} = \frac{1}{\sqrt{101.25}}(4.5\mathbf{i} + 9\mathbf{k})$$

$$\mathbf{r}_{B/A} = (9\text{ ft})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (18\text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BD} = \lambda_{BD} T_{BD} = \frac{(4.5\text{ ft})\mathbf{i} - (9\text{ ft})\mathbf{j} - (9\text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2 + (9)^2}\text{ ft}} T_{BD}$$

$$= \left(\frac{T_{BD}}{13.5}\right)(4.5\mathbf{i} - 9\mathbf{j} - 9\mathbf{k})$$

$$\mathbf{F}_C = \lambda_{CF}(210\text{ lb}) = \frac{-4.5\mathbf{i} - 18\mathbf{j} + 6\mathbf{k}}{\sqrt{(4.5)^2 + (18)^2 + (6)^2}}(210\text{ lb})$$

$$= \left(\frac{210\text{ lb}}{19.5}\right)(-4.5\mathbf{i} - 18\mathbf{j} + 6\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 4.5 & 0 & 9 \\ 0 & 9 & 0 \\ 4.5 & -9 & -9 \end{vmatrix}\left(\frac{T_{BD}}{13.5\sqrt{101.25}}\right) + \begin{vmatrix} 4.5 & 0 & 9 \\ 0 & 18 & 0 \\ -4.5 & -18 & 6 \end{vmatrix}\left(\frac{210\text{ lb}}{19.5\sqrt{101.25}}\right) = 0$$

PROBLEM 4.117 CONTINUED

$$\frac{(-364.5 - 364.5)}{13.5\sqrt{101.25}}T_{BD} + \frac{(486 + 729)}{19.5\sqrt{101.25}}(210 \text{ lb}) = 0$$

or $$T_{BD} = 242.31 \text{ lb}$$

or $T_{BD} = 242 \text{ lb}$ ◀

$$\Sigma M_{AD} = 0: \quad \lambda_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BE}) + \lambda_{AD} \cdot (\mathbf{r}_{C/A} \times \mathbf{F}_C) = 0$$

where $$\lambda_{AD} = \frac{(4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2} \text{ ft}} = \frac{1}{\sqrt{101.25}}(4.5\mathbf{i} - 9\mathbf{k}),$$

$$\mathbf{r}_{B/A} = (9 \text{ ft})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (18 \text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BE} = \lambda_{BE}T_{BE} = \frac{(4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{j} + (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2 + (9)^2} \text{ ft}}T_{BE} = \frac{T_{BE}}{13.5}(4.5\mathbf{i} - 9\mathbf{j} + 9\mathbf{k})$$

$$\therefore \begin{vmatrix} 4.5 & 0 & -9 \\ 0 & 9 & 0 \\ 4.5 & -9 & 9 \end{vmatrix}\left(\frac{T_{BE}}{13.5\sqrt{101.25}}\right) + \begin{vmatrix} 4.5 & 0 & -9 \\ 0 & 18 & 0 \\ -4.5 & -18 & 6 \end{vmatrix}\left(\frac{210 \text{ lb}}{19.5\sqrt{101.25}}\right) = 0$$

$$\frac{(364.5 + 364.5)}{13.5\sqrt{101.25}}T_{BE} + \frac{(486 - 729)(210 \text{ lb})}{19.5\sqrt{101.25}} = 0$$

or $$T_{BE} = 48.462 \text{ lb}$$

or $T_{BE} = 48.5 \text{ lb}$ ◀

$$\Sigma F_x = 0: \quad A_x + (T_{BD})_x + (T_{BE})_x - (F_C)_x = 0$$

$$A_x + \left(\frac{4.5}{13.5}\right)242.31 + \left(\frac{4.5}{13.5}\right)48.462 - \left(\frac{4.5}{19.5}\right)210 = 0$$

$$\therefore \quad A_x = -48.459 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y - (T_{BD})_y - (T_{BE})_y - (F_C)_y = 0$$

$$A_y - \left(\frac{9}{13.5}\right)242.31 - \left(\frac{9}{13.5}\right)48.462 - \left(\frac{18}{19.5}\right)210 =$$

$$\therefore \quad A_y = 387.69 \text{ lb}$$

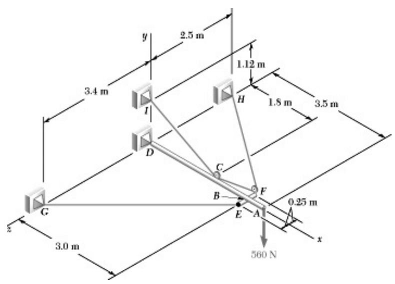
$$\Sigma F_z = 0: \quad A_z - (T_{BD})_z + (T_{BE})_z + (F_C)_z = 0$$

$$A_z - \left(\frac{9}{13.5}\right)242.31 + \left(\frac{9}{13.5}\right)48.462 + \left(\frac{6}{19.5}\right)2$$

$$\therefore \quad A_z = 64.591 \text{ lb}$$

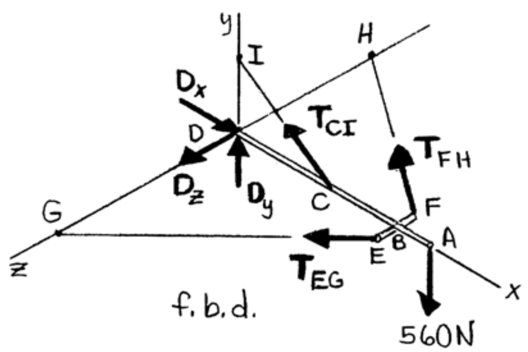
or $\mathbf{A} = -(48.5 \text{ lb})\mathbf{i} + (388 \text{ lb})\mathbf{j} + (64.6 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.118



Two steel pipes *ABCD* and *EBF* are welded together at *B* to form the boom shown. The boom is held by a ball-and-socket joint at *D* and by two cables *EG* and *ICFH*; cable *ICFH* passes around frictionless pulleys at *C* and *F*. For the loading shown, determine the tension in each cable and the reaction at *D*.

SOLUTION



From f.b.d. of boom

$$\Sigma M_z = 0: \quad \mathbf{k}\cdot\left(\mathbf{r}_{C/D}\times\mathbf{T}_{CI}\right)+\mathbf{k}\cdot\left(\mathbf{r}_{A/D}\times\mathbf{F}_A\right)=0$$

where

$$\mathbf{r}_{C/D} = (1.8\text{ m})\mathbf{i}$$

$$\mathbf{T}_{CI} = \lambda_{CI}T_{CI} = \frac{-(1.8\text{ m})\mathbf{i}+(1.12\text{ m})\mathbf{j}}{\sqrt{(1.8)^2+(1.12)^2}\text{ m}}T_{CI}$$

$$= \left(\frac{T_{CI}}{2.12}\right)\left(-1.8\mathbf{i}+1.12\mathbf{j}\right)$$

$$\mathbf{r}_{A/D} = (3.5\text{ m})\mathbf{i}$$

$$\mathbf{F}_A = -(560\text{ N})\mathbf{j}$$

$$\therefore\ \Sigma M_z = \begin{vmatrix} 0 & 0 & 1 \\ 1.8 & 0 & 0 \\ -1.8 & 1.12 & 0 \end{vmatrix}\left(\frac{T_{CI}}{2.12}\right)+\begin{vmatrix} 0 & 0 & 1 \\ 3.5 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}(560\text{ N})=0$$

$$(2.016)\frac{T_{CI}}{2.12}+(-3.5)560=0$$

or

$$T_{CI} = T_{FH} = 2061.1\text{ N}$$

$$T_{ICFH} = 2.06\text{ kN} \blacktriangleleft$$

PROBLEM 4.118 CONTINUED

$$\Sigma M_y = 0: \quad \mathbf{j}\cdot\left(\mathbf{r}_{G/D}\times\mathbf{T}_{EG}\right)+\mathbf{j}\cdot\left(\mathbf{r}_{H/D}\times\mathbf{T}_{FH}\right)=0$$

where $\mathbf{r}_{G/D} = (3.4\text{ m})\mathbf{k}$

$$\mathbf{r}_{H/D} = -(2.5\text{ m})\mathbf{k}$$

$$\mathbf{T}_{EG} = \frac{-(3.0\text{ m})\mathbf{i}+(3.15\text{ m})\mathbf{k}}{\sqrt{(3)^2+(3.15)^2}\text{ m}}T_{EG} = \left(\frac{T_{EG}}{4.35}\right)(-3\mathbf{i}+3.15\mathbf{k})$$

$$\mathbf{T}_{FH} = \lambda_{FH}T_{FH} = \frac{-(3.0\text{ m})\mathbf{i}-(2.25\text{ m})\mathbf{k}}{\sqrt{(3)^2+(2.25)^2}\text{ m}}(2061.1\text{ N}) = \frac{2061.1\text{ N}}{3.75}(-3\mathbf{i}-2.25\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 3.4 \\ -3 & 0 & 3.15 \end{vmatrix}\left(\frac{T_{EG}}{4.35}\right)+\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & -2.5 \\ -3 & 0 & -2.25 \end{vmatrix}\left(\frac{2061.1\text{ N}}{3.75}\right)=0$$

$$-(10.2)\frac{T_{EG}}{4.35}+(7.5)\frac{2061.1\text{ N}}{3.75}=0$$

or

$$T_{EG} = 1758.00\text{ N}$$

$T_{EG} = 1.758$ kN ◀

$$\Sigma F_x = 0: \quad D_x-(T_{CI})_x-(T_{FH})_x-(T_{EG})_x=0$$

$$D_x-\left(\frac{1.8}{2.12}\right)(2061.1\text{ N})-\left(\frac{3.0}{3.75}\right)(2061.1\text{ N})-\left(\frac{3}{4.35}\right)(1758\text{ N})=0$$

$$\therefore \quad D_x = 4611.3\text{ N}$$

$$\Sigma F_y = 0: \quad D_y+(T_{CI})_y-560\text{ N}=0$$

$$D_y+\left(\frac{1.12}{2.12}\right)(2061.1\text{ N})-560\text{ N}=0$$

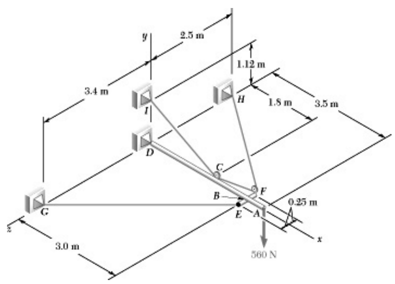
$$\therefore \quad D_y = -528.88\text{ N}$$

$$\Sigma F_z = 0: \quad D_z+(T_{EG})_z-(T_{FH})_z=0$$

$$D_z+\left(\frac{3.15}{4.35}\right)(1758\text{ N})-\left(\frac{2.25}{3.75}\right)(2061.1\text{ N})=0$$

$$\therefore \quad D_z = -36.374\text{ N}$$

and $\mathbf{D} = (4610\text{ N})\mathbf{i}-(529\text{ N})\mathbf{j}-(36.4\text{ N})\mathbf{k}$ ◀

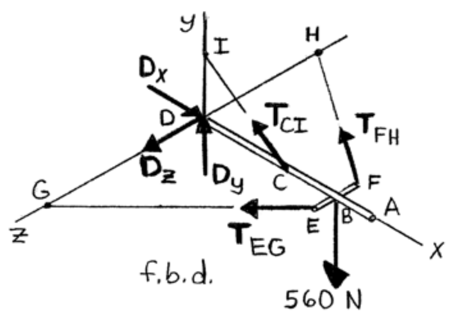


PROBLEM 4.119

Solve Problem 4.118 assuming that the 560-N load is applied at *B*.

P4.118 Two steel pipes *ABCD* and *EBF* are welded together at *B* to form the boom shown. The boom is held by a ball-and-socket joint at *D* and by two cables *EG* and *ICFH*; cable *ICFH* passes around frictionless pulleys at *C* and *F*. For the loading shown, determine the tension in each cable and the reaction at *D*.

SOLUTION



From f.b.d. of boom

$$\Sigma M_z = 0: \quad \mathbf{k} \cdot \left(\mathbf{r}_{C/D} \times \mathbf{T}_{CI}\right) + \mathbf{k} \cdot \left(\mathbf{r}_{B/D} \times \mathbf{F}_B\right) = 0$$

where $\mathbf{r}_{C/D} = (1.8 \text{ m})\mathbf{i}$

$$\mathbf{T}_{CI} = \lambda_{CI} T_{CI} = \frac{-(1.8 \text{ m})\mathbf{i} + (1.12 \text{ m})\mathbf{j}}{\sqrt{(1.8)^2 + (1.12)^2} \text{ m}} T_{CI}$$

$$= \left(\frac{T_{CI}}{2.12}\right)(-1.8\mathbf{i} + 1.12\mathbf{j})$$

$$\mathbf{r}_{B/D} = (3.0 \text{ m})\mathbf{i}$$

$$\mathbf{F}_B = -(560 \text{ N})\mathbf{j}$$

$$\therefore \quad \begin{vmatrix} 0 & 0 & 1 \\ 1.8 & 0 & 0 \\ -1.8 & 1.12 & 0 \end{vmatrix} \left(\frac{T_{CI}}{2.12}\right) + \begin{vmatrix} 0 & 0 & 1 \\ 3 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} (560 \text{ N}) = 0$$

$$(2.016)\frac{T_{CI}}{2.12} + (-3)560 = 0$$

or $T_{CI} = T_{FH} = 1766.67 \text{ N}$

$T_{ICFH} = 1.767 \text{ kN}$ ◀

PROBLEM 4.119 CONTINUED

$$\Sigma M_y = 0: \quad \mathbf{j}\cdot\left(\mathbf{r}_{G/D} \times \mathbf{T}_{EG}\right) + \mathbf{j}\cdot\left(\mathbf{r}_{H/D} \times \mathbf{T}_{FH}\right) = 0$$

where

$$\mathbf{r}_{G/D} = (3.4 \text{ m})\mathbf{k}$$

$$\mathbf{r}_{H/D} = -(2.5 \text{ m})\mathbf{k}$$

$$\mathbf{T}_{EG} = \lambda_{EG}T_{EG} = \frac{-(3.0 \text{ m})\mathbf{i} + (3.15 \text{ m})\mathbf{k}}{\sqrt{(3)^2 + (3.15)^2} \text{ m}}T_{EG} = \frac{T_{EG}}{4.35}(-3\mathbf{i} + 3.15\mathbf{k})$$

$$\mathbf{T}_{FH} = \lambda_{FH}T_{FH} = \frac{-(3.0 \text{ m})\mathbf{i} - (2.25 \text{ m})\mathbf{k}}{\sqrt{(3)^2 + (2.25)^2} \text{ m}}T_{FH} = \frac{1766.67 \text{ N}}{3.75}(-3\mathbf{i} - 2.25\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 3.4 \\ -3 & 0 & 3.15 \end{vmatrix}\left(\frac{T_{EG}}{4.35}\right) + \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & -2.5 \\ -3 & 0 & -2.25 \end{vmatrix}\left(\frac{1766.67}{3.75}\right) = 0$$

$$-(10.2)\frac{T_{EG}}{4.35} + (7.5)\frac{1766.67}{3.75} = 0$$

or

$$T_{EG} = 1506.86 \text{ N}$$

$$T_{EG} = 1.507 \text{ kN} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad D_x - (T_{CI})_x - (T_{FH})_x - (T_{EG})_x = 0$$

$$D_x - \left(\frac{1.8}{2.12}\right)(1766.67 \text{ N}) - \left(\frac{3}{3.75}\right)(1766.67 \text{ N}) - \left(\frac{3}{4.35}\right)(1506.86 \text{ N}) = 0$$

$$\therefore \quad D_x = 3952.5 \text{ N}$$

$$\Sigma F_y = 0: \quad D_y + (T_{CI})_y - 560 \text{ N} = 0$$

$$D_y + \left(\frac{1.12}{2.12}\right)(1766.67 \text{ N}) - 560 \text{ N} = 0$$

$$\therefore \quad D_y = -373.34 \text{ N}$$

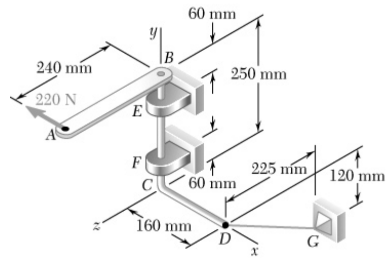
$$\Sigma F_z = 0: \quad D_z + (T_{EG})_z - (T_{FH})_z = 0$$

$$D_z + \left(\frac{3.15}{4.35}\right)(1506.86 \text{ N}) - \left(\frac{2.25}{3.75}\right)(1766.67 \text{ N}) = 0$$

$$\therefore \quad D_z = -31.172 \text{ N}$$

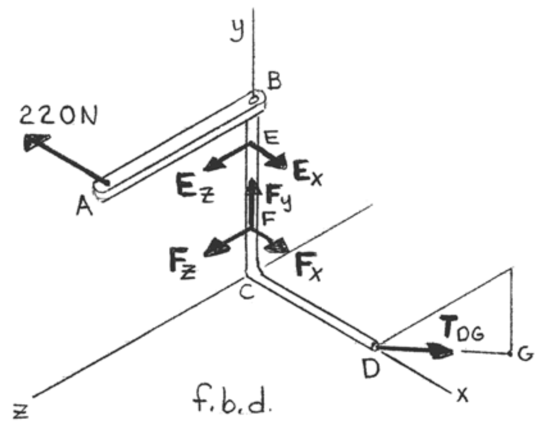
$$\mathbf{D} = (3950 \text{ N})\mathbf{i} - (373 \text{ N})\mathbf{j} - (31.2 \text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.120



The lever AB is welded to the bent rod BCD which is supported by bearings at E and F and by cable DG. Knowing that the bearing at E does not exert any axial thrust, determine (a) the tension in cable DG, (b) the reactions at E and F.

SOLUTION



(a) From f.b.d. of assembly

$$\mathbf{T}_{DG} = \lambda_{DG} T_{DG} = \left[\frac{-(0.12\text{ m})\mathbf{j} - (0.225\text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.225)^2}\text{ m}}\right] = \frac{T_{DG}}{0.255}\left[-(0.12)\mathbf{j} - (0.225)\mathbf{k}\right]$$

$$\Sigma M_y = 0: \quad -(220\text{ N})(0.24\text{ m}) + \left[T_{DG}\left(\frac{0.225}{0.255}\right)\right](0.16\text{ m}) = 0$$

$$\therefore \quad T_{DG} = 374.00\text{ N}$$

or $T_{DG} = 374$ N ◀

(b) From f.b.d. of assembly

$$\Sigma M_{F(z\text{-axis})} = 0: \quad (220\text{ N})(0.19\text{ m}) - E_x(0.13\text{ m}) - \left[374\text{ N}\left(\frac{0.120}{0.255}\right)\right](0.16\text{ m}) = 0$$

$$\therefore \quad E_x = 104.923\text{ N}$$

$$\Sigma F_x = 0: \quad F_x + 104.923\text{ N} - 220\text{ N} = 0$$

$$\therefore \quad F_x = 115.077\text{ N}$$

$$\Sigma M_{F(x\text{-axis})} = 0: \quad E_z(0.13\text{ m}) + \left[374\text{ N}\left(\frac{0.225}{0.255}\right)\right](0.06\text{ m}) = 0$$

$$\therefore \quad E_z = -152.308\text{ N}$$

PROBLEM 4.120 CONTINUED

$$\Sigma F_z = 0: \quad F_z - 152.308 \text{ N} - (374 \text{ N})\left(\frac{0.225}{0.255}\right) = 0$$

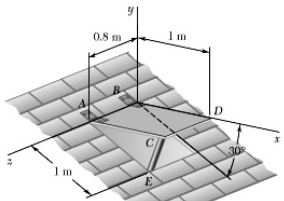
$$\therefore \quad F_z = 482.31 \text{ N}$$

$$\Sigma F_y = 0: \quad F_y - (374 \text{ N})\left(\frac{0.12}{0.255}\right) = 0$$

$$\therefore \quad F_y = 176.0 \text{ N}$$

$$\mathbf{E} = (104.9 \text{ N})\mathbf{i} - (152.3 \text{ N})\mathbf{k} \blacktriangleleft$$

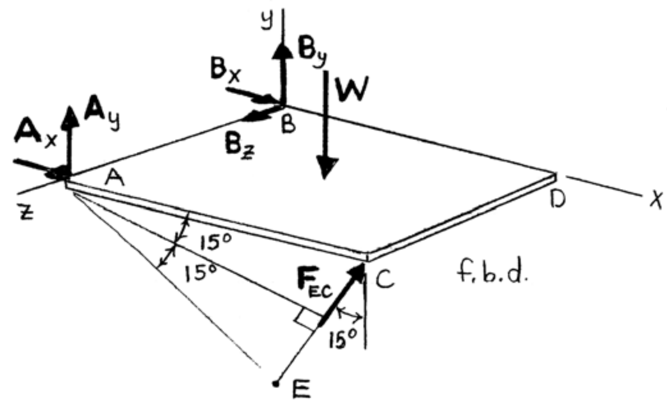
$$\mathbf{F} = (115.1 \text{ N})\mathbf{i} + (176.0 \text{ N})\mathbf{j} + (482 \text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.121

A 30-kg cover for a roof opening is hinged at corners A and B. The roof forms an angle of 30^{o} with the horizontal, and the cover is maintained in a horizontal position by the brace CE. Determine (a) the magnitude of the force exerted by the brace, (b) the reactions at the hinges. Assume that the hinge at A does not exert any axial thrust.

SOLUTION



First note $$W = mg = (30\text{ kg})\left(9.81\text{ m/s}^2\right) = 294.3\text{ N}$$

$$\mathbf{F}_{EC} = \lambda_{EC}F_{EC} = \left[(\sin 15°)\mathbf{i} + (\cos 15°)\mathbf{j}\right]F_{EC}$$

From f.b.d. of cover

(a) $$\Sigma M_z = 0: \quad \left(F_{EC}\cos 15°\right)(1.0\text{ m}) - W(0.5\text{ m}) = 0$$

or $$F_{EC}\cos 15°(1.0\text{ m}) - (294.3\text{ N})(0.5\text{ m}) = 0$$

$$\therefore \quad F_{EC} = 152.341\text{ N}$$

or $F_{EC} = 152.3\text{ N}$ ◀

(b) $$\Sigma M_x = 0: \quad W(0.4\text{ m}) - A_y(0.8\text{ m}) - \left(F_{EC}\cos 15°\right)(0.8\text{ m}) = 0$$

or $$(294.3\text{ N})(0.4\text{ m}) - A_y(0.8\text{ m}) - \left[(152.341\text{ N})\cos 15°\right](0.8\text{ m}) = 0$$

$$\therefore \quad A_y = 0$$

$$\Sigma M_y = 0: \quad A_x(0.8\text{ m}) + \left(F_{EC}\sin 15°\right)(0.8\text{ m}) = 0$$

or $$A_x(0.8\text{ m}) + \left[(152.341\text{ N})\sin 15°\right](0.8\text{ m}) = 0$$

$$\therefore \quad A_x = -39.429\text{ N}$$

$$\Sigma F_x = 0: \quad A_x + B_x + F_{EC}\sin 15° = 0$$

$$-39.429\text{ N} + B_x + (152.341\text{ N})\sin 15° = 0$$

$$\therefore \quad B_x = 0$$

PROBLEM 4.121 CONTINUED

$$\Sigma F_y = 0: \quad F_{EC}\cos 15° - W + B_y = 0$$

or

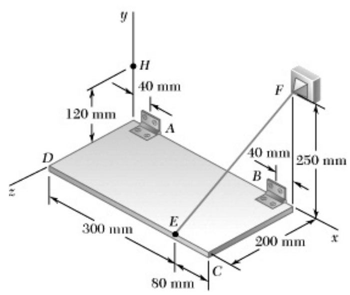
$$(152.341 \text{ N})\cos 15° - 294.3 \text{ N} + B_y = 0$$

$$\therefore \quad B_y = 147.180 \text{ N}$$

or $\mathbf{A} = -(39.4 \text{ N})\mathbf{i}$ ◀

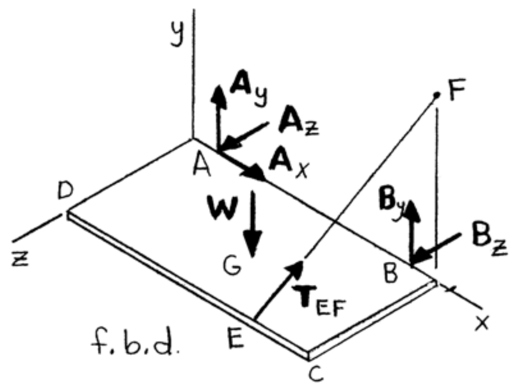
$\mathbf{B} = (147.2 \text{ N})\mathbf{j}$ ◀

PROBLEM 4.122



The rectangular plate shown has a mass of 15 kg and is held in the position shown by hinges *A* and *B* and cable *EF*. Assuming that the hinge at *B* does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*.

SOLUTION



First note $$W = mg = (15 \text{ kg})(9.81 \text{ m/s}^2) = 147.15 \text{ N}$$

$$\mathbf{T}_{EF} = \lambda_{EF} T_{EF} = \left[\frac{(0.08 \text{ m})\mathbf{i} + (0.25 \text{ m})\mathbf{j} - (0.2 \text{ m})\mathbf{k}}{\sqrt{(0.08)^2 + (0.25)^2 + (0.2)^2} \text{ m}}\right] T_{EF} = \frac{T_{EF}}{0.33}(0.08\mathbf{i} + 0.25\mathbf{j} - 0.2\mathbf{k})$$

From f.b.d. of rectangular plate

$$\Sigma M_x = 0: \quad (147.15 \text{ N})(0.1 \text{ m}) - (T_{EF})_y (0.2 \text{ m}) = 0$$

or $$14.715 \text{ N}\cdot\text{m} - \left[\left(\frac{0.25}{0.33}\right) T_{EF}\right](0.2 \text{ m}) = 0$$

or $$T_{EF} = 97.119 \text{ N}$$

or $T_{EF} = 97.1 \text{ N}$ ◀

$$\Sigma F_x = 0: \quad A_x + (T_{EF})_x = 0$$

$$A_x + \left(\frac{0.08}{0.33}\right)(97.119 \text{ N}) = 0$$

$$\therefore \quad A_x = -23.544 \text{ N}$$

PROBLEM 4.122 CONTINUED

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(0.3 \text{ m}) - (T_{EF})_y(0.04 \text{ m}) + W(0.15 \text{ m}) = 0$$

or

$$-A_y(0.3 \text{ m}) - \left[\left(\frac{0.25}{0.33}\right)97.119 \text{ N}\right](0.04 \text{ m}) + 147.15 \text{ N}(0.15 \text{ m}) = 0$$

$$\therefore \quad A_y = 63.765 \text{ N}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(0.3 \text{ m}) + (T_{EF})_x(0.2 \text{ m}) + (T_{EF})_z(0.04 \text{ m}) = 0$$

$$A_z(0.3 \text{ m}) + \left[\left(\frac{0.08}{0.33}\right)T_{EF}\right](0.2 \text{ m}) - \left[\left(\frac{0.2}{0.33}\right)T_{EF}\right](0.04 \text{ m}) = 0$$

$$\therefore \quad A_z = -7.848 \text{ N}$$

and $\mathbf{A} = -(23.5 \text{ N})\mathbf{i} + (63.8 \text{ N})\mathbf{j} - (7.85 \text{ N})\mathbf{k}$ ◀

$$\Sigma F_y = 0: \quad A_y - W + (T_{EF})_y + B_y = 0$$

$$63.765 \text{ N} - 147.15 \text{ N} + \left(\frac{0.25}{0.33}\right)(97.119 \text{ N}) + B_y = 0$$

$$\therefore \quad B_y = 9.81 \text{ N}$$

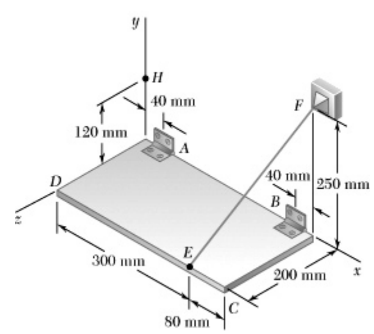
$$\Sigma F_z = 0: \quad A_z - (T_{EF})_z + B_z = 0$$

$$-7.848 \text{ N} - \left(\frac{0.2}{0.33}\right)(97.119 \text{ N}) + B_z = 0$$

$$\therefore \quad B_z = 66.708 \text{ N}$$

and $\mathbf{B} = (9.81 \text{ N})\mathbf{j} + (66.7 \text{ N})\mathbf{k}$ ◀

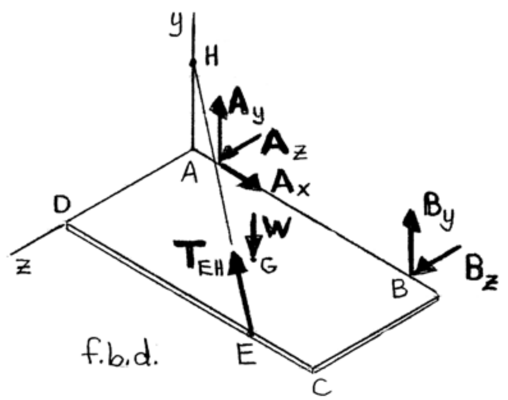
PROBLEM 4.123



Solve Problem 4.122 assuming that cable *EF* is replaced by a cable attached at points *E* and *H*.

P4.122 The rectangular plate shown has a mass of 15 kg and is held in the position shown by hinges *A* and *B* and cable *EF*. Assuming that the hinge at *B* does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*.

SOLUTION



First note $$W = mg = (15\text{ kg})(9.81\text{ m/s}^2) = 147.15\text{ N}$$

$$\mathbf{T}_{EH} = \lambda_{EH}T_{EH} = \left[\frac{-(0.3\text{ m})\mathbf{i} + (0.12\text{ m})\mathbf{j} - (0.2\text{ m})\mathbf{k}}{\sqrt{(0.3)^2 + (0.12)^2 + (0.2)^2}\text{ m}}\right]T_{EH} = \frac{T_{EH}}{0.38}\left[-(0.3)\mathbf{i} + (0.12)\mathbf{j} - (0.2)\mathbf{k}\right]$$

From f.b.d. of rectangular plate

$$\Sigma M_x = 0: \quad (147.15\text{ N})(0.1\text{ m}) - (T_{EH})_y(0.2\text{ m}) = 0$$

or $$(147.15\text{ N})(0.1\text{ m}) - \left[\left(\frac{0.12}{0.38}\right)T_{EH}\right](0.2\text{ m}) = 0$$

or $$T_{EH} = 232.99\text{ N}$$

or $T_{EH} = 233\text{ N}$ ◀

$$\Sigma F_x = 0: \quad A_x + (T_{EH})_x = 0$$

$$A_x - \left(\frac{0.3}{0.38}\right)(232.99\text{ N}) = 0$$

$$\therefore \quad A_x = 183.938\text{ N}$$

PROBLEM 4.123 CONTINUED

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(0.3\text{ m}) - (T_{EH})_y(0.04\text{ m}) + W(0.15\text{ m}) = 0$$

or

$$-A_y(0.3\text{ m}) - \left[\frac{0.12}{0.38}(232.99\text{ N})\right](0.04\text{ m}) + (147.15\text{ N})(0.15\text{ m}) = 0$$

$$\therefore \quad A_y = 63.765\text{ N}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(0.3\text{ m}) + (T_{EH})_x(0.2\text{ m}) + (T_{EH})_z(0.04\text{ m}) = 0$$

or

$$A_z(0.3\text{ m}) - \left[\left(\frac{0.3}{0.38}\right)(232.99\text{ N})\right](0.2\text{ m}) - \left[\left(\frac{0.2}{0.38}\right)(232.99)\right](0.04\text{ m}) = 0$$

$$\therefore \quad A_z = 138.976\text{ N}$$

and $\mathbf{A} = (183.9\text{ N})\mathbf{i} + (63.8\text{ N})\mathbf{j} + (139.0\text{ N})\mathbf{k}$ ◀

$$\Sigma F_y = 0: \quad A_y + B_y - W + (T_{EH})_y = 0$$

$$63.765\text{ N} + B_y - 147.15\text{ N} + \left(\frac{0.12}{0.38}\right)(232.99\text{ N}) = 0$$

$$\therefore \quad B_y = 9.8092\text{ N}$$

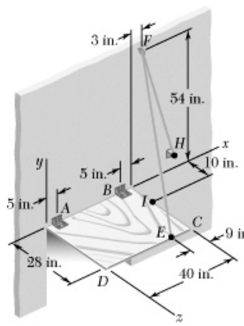
$$\Sigma F_z = 0: \quad A_z + B_z - (T_{EH})_z = 0$$

$$138.976\text{ N} + B_z - \left(\frac{0.2}{0.38}\right)(232.99\text{ N}) = 0$$

$$\therefore \quad B_z = -16.3497\text{ N}$$

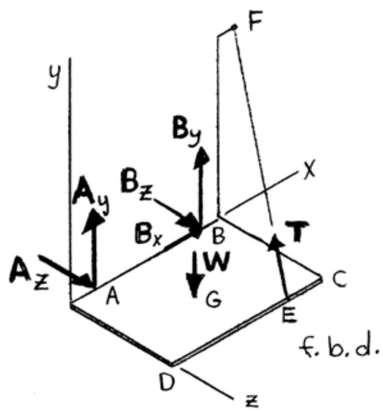
and $\mathbf{B} = (9.81\text{ N})\mathbf{j} - (16.35\text{ N})\mathbf{k}$ ◀

PROBLEM 4.124



A small door weighing 16 lb is attached by hinges A and B to a wall and is held in the horizontal position shown by rope EFH. The rope passes around a small, frictionless pulley at F and is tied to a fixed cleat at H. Assuming that the hinge at A does not exert any axial thrust, determine (a) the tension in the rope, (b) the reactions at A and B.

SOLUTION



First note

$$\mathbf{T} = \lambda_{EF}T = \frac{(12\text{ in.})\mathbf{i} + (54\text{ in.})\mathbf{j} - (28\text{ in.})\mathbf{k}}{\sqrt{(12)^2 + (54)^2 + (28)^2}\text{ in.}}T$$

$$= \frac{T}{62}(12\mathbf{i} + 54\mathbf{j} - 28\mathbf{k}) = \frac{T}{31}(6\mathbf{i} + 27\mathbf{j} - 14\mathbf{k})$$

$$\mathbf{W} = -(16\text{ lb})\mathbf{j} \quad \text{at } G$$

From f.b.d. of door $ABCD$

(a) $\quad \Sigma M_x = 0\text{:} \quad T_y(28\text{ in.}) - W(14\text{ in.}) = 0$

$$\left[T\left(\frac{27}{31}\right)\right](28\text{ in.}) - (16\text{ lb})(14\text{ in.}) = 0$$

$$\therefore \quad T = 9.1852\text{ lb}$$

or $T = 9.19$ lb ◀

(b) $\quad \Sigma M_{B(z\text{-axis})} = 0\text{:} \quad -A_y(30\text{ in.}) + W(15\text{ in.}) - T_y(4\text{ in.}) = 0$

$$-A_y(30\text{ in.}) + (16\text{ lb})(15\text{ in.}) - \left[(9.1852\text{ lb})\left(\frac{27}{31}\right)\right](4\text{ in.}) = 0$$

$$\therefore \quad A_y = 6.9333\text{ lb}$$

PROBLEM 4.124 CONTINUED

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(30 \text{ in.}) + T_x(28 \text{ in.}) - T_z(4 \text{ in.}) = 0$$

$$A_z(30 \text{ in.}) + \left[(9.1852 \text{ lb})\left(\frac{6}{31}\right)\right](28 \text{ in.}) - \left[(9.1852 \text{ lb})\left(\frac{14}{31}\right)\right](4 \text{ in.}) = 0$$

$$\therefore \quad A_z = -1.10617 \text{ lb}$$

or $\mathbf{A} = (6.93 \text{ lb})\mathbf{j} - (1.106 \text{ lb})\mathbf{k}$ ◀

$$\Sigma F_x = 0: \quad B_x + T_x = B_x + (9.1852 \text{ lb})\left(\frac{6}{31}\right) = 0$$

$$\therefore \quad B_x = -1.77778 \text{ lb}$$

$$\Sigma F_y = 0: \quad B_y + T_y - W + A_y = 0$$

$$B_y + (9.1852 \text{ lb})\left(\frac{27}{31}\right) - 16 \text{ lb} + 6.9333 \text{ lb} = 0$$

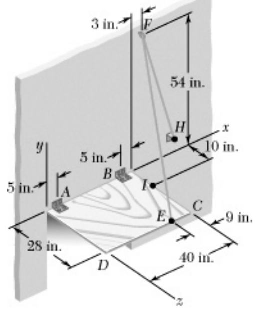
$$\therefore \quad B_y = 1.06666 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z - T_z + B_z = 0$$

$$-1.10617 \text{ lb} - (9.1852 \text{ lb})\left(\frac{14}{31}\right) + B_z = 0$$

$$\therefore \quad B_z = 5.2543 \text{ lb}$$

or $\mathbf{B} = -(1.778 \text{ lb})\mathbf{i} + (1.067 \text{ lb})\mathbf{j} + (5.25 \text{ lb})\mathbf{k}$ ◀

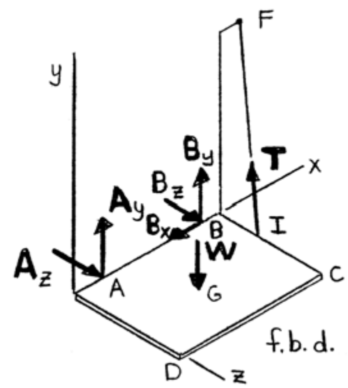


PROBLEM 4.125

Solve Problem 4.124 assuming that the rope is attached to the door at *I*.

P4.124 A small door weighing 16 lb is attached by hinges *A* and *B* to a wall and is held in the horizontal position shown by rope *EFH*. The rope passes around a small, frictionless pulley at *F* and is tied to a fixed cleat at *H*. Assuming that the hinge at *A* does not exert any axial thrust, determine (*a*) the tension in the rope, (*b*) the reactions at *A* and *B*.

SOLUTION



First note
$$\mathbf{T} = \lambda_{IF}T = \frac{(3\text{ in.})\mathbf{i} + (54\text{ in.})\mathbf{j} - (10\text{ in.})\mathbf{k}}{\sqrt{(3)^2 + (54)^2 + (10)^2}\text{ in.}}T$$

$$= \frac{T}{55}(3\mathbf{i} + 54\mathbf{j} - 10\mathbf{k})$$

$$\mathbf{W} = -(16\text{ lb})\mathbf{j}$$

From f.b.d. of door *ABCD*

(*a*) $$\Sigma M_x = 0: \quad W(14\text{ in.}) - T_y(10\text{ in.}) = 0$$

$$(16\text{ lb})(14\text{ in.}) - \left(\frac{54}{55}\right)T(10\text{ in.}) = 0$$

$$\therefore \quad T = 22.815\text{ lb}$$

or $T = 22.8$ lb ◀

(*b*) $$\Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(30\text{ in.}) + W(15\text{ in.}) + T_y(5\text{ in.}) = 0$$

$$-A_y(30\text{ in.}) + (16\text{ lb})(15\text{ in.}) + (22.815\text{ lb})\left(\frac{54}{55}\right)(5\text{ in.}) = 0$$

$$\therefore A_y = 11.7334\text{ lb}$$

PROBLEM 4.125 CONTINUED

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(30\text{ in.}) + T_x(10\text{ in.}) + T_z(5\text{ in.}) = 0$$

$$A_z(30\text{ in.}) + \left[(22.815\text{ lb})\left(\frac{3}{55}\right)\right](10\text{ in.}) + \left[(22.815\text{ lb})\left(\frac{10}{55}\right)\right](5\text{ in.}) = 0$$

$$\therefore\ A_z = -1.10618\text{ lb}$$

or $\mathbf{A} = (11.73\text{ lb})\mathbf{j} - (1.106\text{ lb})\mathbf{k}$ ◀

$$\Sigma F_x = 0: \quad B_x + T_x = 0$$

$$B_x + \left(\frac{3}{55}\right)(22.815\text{ lb}) = 0$$

$$\therefore\ B_x = -1.24444\text{ lb}$$

$$\Sigma F_y = 0: \quad A_y - W + T_y + B_y = 0$$

$$11.7334\text{ lb} - 16\text{ lb} + (22.815\text{ lb})\left(\frac{54}{55}\right) + B_y = 0$$

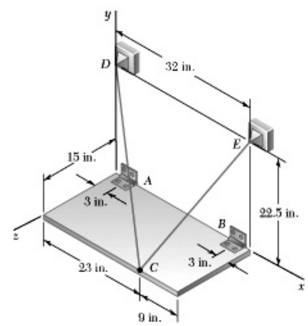
$$\therefore\ B_y = -18.1336\text{ lb}$$

$$\Sigma F_z = 0: \quad A_z - T_z + B_z = 0$$

$$-1.10618\text{ lb} - (22.815\text{ lb})\left(\frac{10}{55}\right) + B_z = 0$$

$$\therefore\ B_z = 5.2544\text{ lb}$$

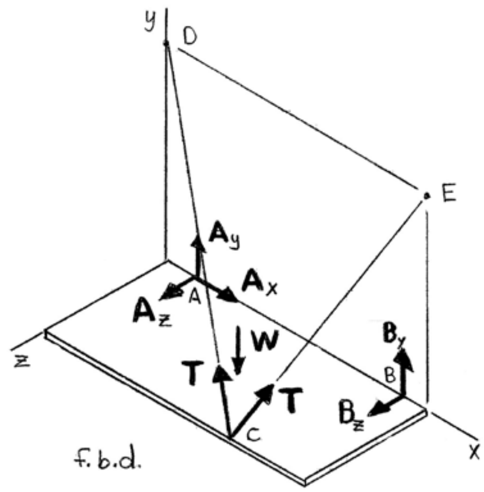
or $\mathbf{B} = -(1.244\text{ lb})\mathbf{i} - (18.13\text{ lb})\mathbf{j} + (5.25\text{ lb})\mathbf{k}$ ◀



PROBLEM 4.126

A 285-lb uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE*, which passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION



First note

$$\lambda_{CD} = \frac{-(23 \text{ in.})\mathbf{i} + (22.5 \text{ in.})\mathbf{j} - (15 \text{ in.})\mathbf{k}}{35.5 \text{ in.}}$$

$$= \frac{1}{35.5}(-23\mathbf{i} + 22.5\mathbf{j} - 15\mathbf{k})$$

$$\lambda_{CE} = \frac{(9 \text{ in.})\mathbf{i} + (22.5 \text{ in.})\mathbf{j} - (15 \text{ in.})\mathbf{k}}{28.5 \text{ in.}}$$

$$= \frac{1}{28.5}(9\mathbf{i} + 22.5\mathbf{j} - 15\mathbf{k})$$

$$\mathbf{W} = -(285 \text{ lb})\mathbf{j}$$

From f.b.d. of plate

(*a*) $$\Sigma M_x = 0: \quad (285 \text{ lb})(7.5 \text{ in.}) - \left[\left(\frac{22.5}{35.5}\right)T\right](15 \text{ in.}) - \left[\left(\frac{22.5}{28.5}\right)T\right](15 \text{ in.}) = 0$$

$$\therefore \quad T = 100.121 \text{ lb}$$

or $T = 100.1$ lb ◀

PROBLEM 4.126 CONTINUED

(*b*) $$\Sigma F_x = 0: \quad A_x - T\left(\frac{23}{35.5}\right) + T\left(\frac{9}{28.5}\right) = 0$$

$$A_x - (100.121 \text{ lb})\left(\frac{23}{35.5}\right) + (100.121 \text{ lb})\left(\frac{9}{28.5}\right) = 0$$

$$\therefore \quad A_x = 33.250 \text{ lb}$$

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(26 \text{ in.}) + W(13 \text{ in.}) - \left[T\left(\frac{22.5}{35.5}\right)\right](6 \text{ in.}) - \left[T\left(\frac{22.5}{28.5}\right)\right](6 \text{ in.}) = 0$$

or $$-A_y(26 \text{ in.}) + (285 \text{ lb})(13 \text{ in.}) - \left[(100.121 \text{ lb})\left(\frac{22.5}{35.5}\right)\right](6 \text{ in.})$$

$$-\left[(100.121 \text{ lb})\left(\frac{22.5}{28.5}\right)\right](6 \text{ in.}) = 0$$

$$\therefore \quad A_y = 109.615 \text{ lb}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(26 \text{ in.}) - \left[T\left(\frac{15}{35.5}\right)\right](6 \text{ in.}) - \left[T\left(\frac{23}{35.5}\right)\right](15 \text{ in.})$$

$$-\left[T\left(\frac{15}{28.5}\right)\right](6 \text{ in.}) + \left[T\left(\frac{9}{28.5}\right)\right](15 \text{ in.}) = 0$$

or $$A_z(26 \text{ in.}) + \left[\frac{-1}{35.5}(90 + 345) - \frac{1}{28.5}(90 - 135)\right](100.121 \text{ lb}) = 0$$

$$\therefore \quad A_z = 41.106 \text{ lb}$$

or $\mathbf{A} = (33.3 \text{ lb})\mathbf{i} + (109.6 \text{ lb})\mathbf{j} + (41.1 \text{ lb})\mathbf{k}$ ◀

$$\Sigma F_y = 0: \quad B_y - W + T\left(\frac{22.5}{35.5}\right) + T\left(\frac{22.5}{28.5}\right) + A_y = 0$$

$$B_y - 285 \text{ lb} + (100.121 \text{ lb})\left(\frac{22.5}{35.5} + \frac{22.5}{28.5}\right) + 109.615 \text{ lb} = 0$$

$$\therefore \quad B_y = 32.885 \text{ lb}$$

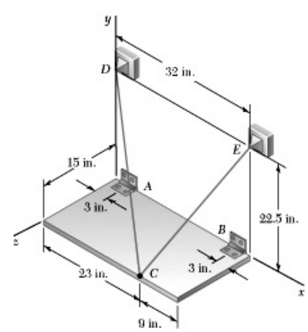
$$\Sigma F_z = 0: \quad B_z + A_z - T\left(\frac{15}{35.5}\right) - T\left(\frac{15}{28.5}\right) = 0$$

$$B_z + 41.106 \text{ lb} - (100.121 \text{ lb})\left(\frac{15}{35.5} + \frac{15}{28.5}\right) = 0$$

$$\therefore \quad B_z = 53.894 \text{ lb}$$

or $\mathbf{B} = (32.9 \text{ lb})\mathbf{j} + (53.9 \text{ lb})\mathbf{k}$ ◀

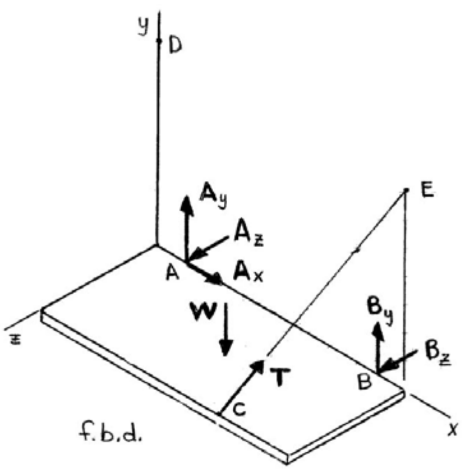
PROBLEM 4.127



Solve Problem 4.126 assuming that cable *DCE* is replaced by a cable attached to point *E* and hook *C*.

P4.126 A 285-lb uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE*, which passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION



First note

$$\lambda_{CE} = \frac{(9\text{ in.})\mathbf{i} + (22.5\text{ in.})\mathbf{j} - (15\text{ in.})\mathbf{k}}{28.5\text{ in.}}$$

$$= \frac{1}{28.5}(9\mathbf{i} + 22.5\mathbf{j} - 15\mathbf{k})$$

$$\mathbf{W} = -(285\text{ lb})\mathbf{j}$$

From f.b.d. of plate

(*a*)

$$\Sigma M_x = 0: \quad (285\text{ lb})(7.5\text{ in.}) - \left[\left(\frac{22.5}{28.5}\right)T\right](15\text{ in.}) = 0$$

$$\therefore \quad T = 180.500\text{ lb}$$

or $T = 180.5\text{ lb}$ ◀

(*b*)

$$\Sigma F_x = 0: \quad A_x + T\left(\frac{9}{28.5}\right) = 0$$

$$A_x + 180.5\text{ lb}\left(\frac{9}{28.5}\right) = 0$$

$$\therefore \quad A_x = -57.000\text{ lb}$$

PROBLEM 4.127 CONTINUED

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(26 \text{ in.}) + W(13 \text{ in.}) - \left[T\left(\frac{22.5}{28.5}\right)\right](6 \text{ in.}) = 0$$

$$-A_y(26 \text{ in.}) + (285 \text{ lb})(13 \text{ in.}) - \left[(180.5 \text{ lb})\left(\frac{22.5}{28.5}\right)\right](6 \text{ in.}) = 0$$

$$\therefore \quad A_y = 109.615 \text{ lb}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(26 \text{ in.}) - \left[T\left(\frac{15}{28.5}\right)\right](6 \text{ in.}) + \left[T\left(\frac{9}{28.5}\right)\right](15 \text{ in.}) = 0$$

$$A_z(26 \text{ in.}) + (180.5 \text{ lb})\left(\frac{45}{28.5}\right) = 0$$

$$\therefore \quad A_z = -10.9615 \text{ lb}$$

or $\mathbf{A} = -(57.0 \text{ lb})\mathbf{i} + (109.6 \text{ lb})\mathbf{j} - (10.96 \text{ lb})\mathbf{k}$ ◀

$$\Sigma F_y = 0: \quad B_y - W + T\left(\frac{22.5}{28.5}\right) + A_y = 0$$

$$B_y - 285 \text{ lb} + (180.5 \text{ lb})\left(\frac{22.5}{28.5}\right) - 109.615 \text{ lb} = 0$$

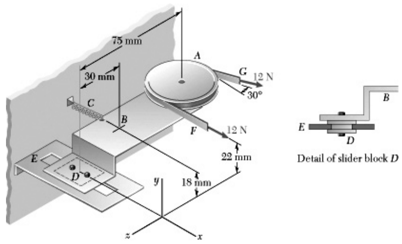
$$\therefore \quad B_y = 32.885 \text{ lb}$$

$$\Sigma F_z = 0: \quad B_z + A_z - T\left(\frac{15}{28.5}\right) = 0$$

$$B_z - 10.9615 \text{ lb} - 180.5 \text{ lb}\left(\frac{15}{28.5}\right) = 0$$

$$\therefore \quad B_z = 105.962 \text{ lb}$$

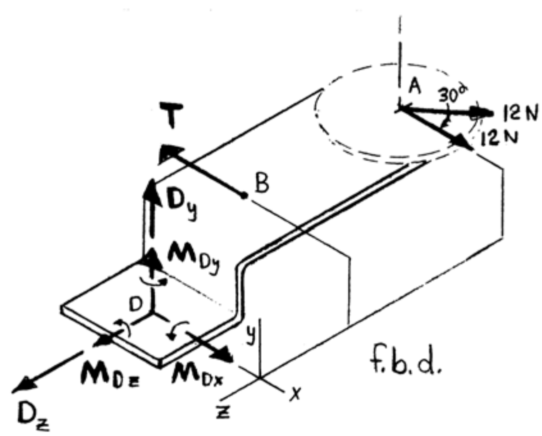
or $\mathbf{B} = (32.9 \text{ lb})\mathbf{j} + (106.0 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.128

The tensioning mechanism of a belt drive consists of frictionless pulley *A*, mounting plate *B*, and spring *C*. Attached below the mounting plate is slider block *D* which is free to move in the frictionless slot of bracket *E*. Knowing that the pulley and the belt lie in a horizontal plane, with portion *F* of the belt parallel to the *x* axis and portion *G* forming an angle of 30° with the *x* axis, determine (*a*) the force in the spring, (*b*) the reaction at *D*.

SOLUTION



From f.b.d. of plate *B*

(*a*) $\Sigma F_x = 0$: $12\text{ N} + (12\text{ N})\cos 30° - T = 0$

$\therefore\ T = 22.392\text{ N}$ or $T = 22.4\text{ N}$ ◀

(*b*) $\Sigma F_y = 0$: $D_y = 0$

$\Sigma F_z = 0$: $D_z - (12\text{ N})\sin 30° = 0$

$\therefore\ D_z = 6\text{ N}$ or $\mathbf{D} = (6.00\text{ N})\mathbf{k}$ ◀

$\Sigma M_x = 0$: $M_{D_x} - [(12\text{ N})\sin 30°](22\text{ mm}) = 0$

$\therefore\ M_{D_x} = 132.0\text{ N}\cdot\text{mm}$

$\Sigma M_{D(y\text{-axis})} = 0$: $M_{D_y} + (22.392\text{ N})(30\text{ mm}) - (12\text{ N})(75\text{ mm}) - [(12\text{ N})\cos 30°](75\text{ mm}) = 0$

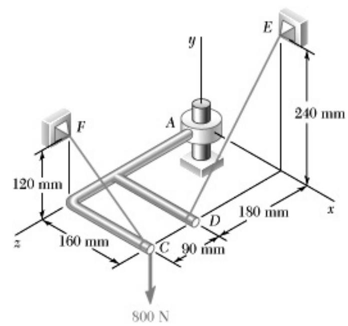
$\therefore\ M_{D_y} = 1007.66\text{ N}\cdot\text{mm}$

$\Sigma M_{D(z\text{-axis})} = 0$: $M_{D_z} + (22.392\text{ N})(18\text{ mm}) - (12\text{ N})(22\text{ mm}) - [(12\text{ N})\cos 30°](22\text{ mm}) = 0$

$\therefore\ M_{D_z} = 89.575\text{ N}\cdot\text{mm}$

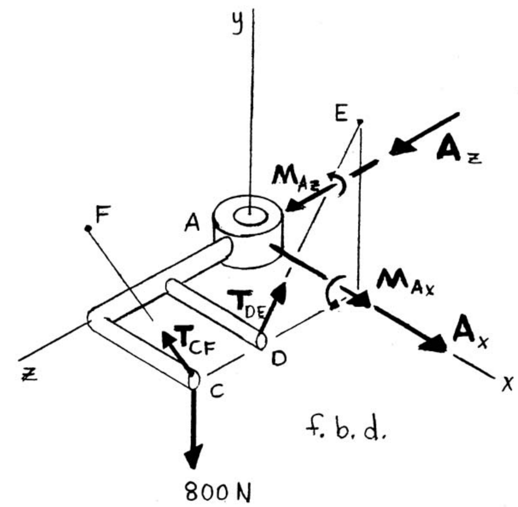
or $\mathbf{M}_D = (0.1320\text{ N}\cdot\text{m})\mathbf{i} + (1.008\text{ N}\cdot\text{m})\mathbf{j} + (0.0896\text{ N}\cdot\text{m})\mathbf{k}$ ◀

PROBLEM 4.129



The assembly shown is welded to collar A which fits on the vertical pin shown. The pin can exert couples about the x and z axes but does not prevent motion about or along the y axis. For the loading shown, determine the tension in each cable and the reaction at A.

SOLUTION



First note

$$\mathbf{T}_{CF} = \lambda_{CF} T_{CF} = \frac{-(0.16\text{ m})\mathbf{i} + (0.12\text{ m})\mathbf{j}}{\sqrt{(0.16)^2 + (0.12)^2}\text{ m}} T_{CF}$$

$$= T_{CF}(-0.8\mathbf{i} + 0.6\mathbf{j})$$

$$\mathbf{T}_{DE} = \lambda_{DE} T_{DE} = \frac{(0.24\text{ m})\mathbf{j} - (0.18\text{ m})\mathbf{k}}{\sqrt{(0.24)^2 + (0.18)^2}\text{ m}} T_{DE}$$

$$= T_{DE}(0.8\mathbf{j} - 0.6\mathbf{k})$$

(a) From f.b.d. of assembly

$$\Sigma F_y = 0: \quad 0.6T_{CF} + 0.8T_{DE} - 800\text{ N} = 0$$

or $$0.6T_{CF} + 0.8T_{DE} = 800\text{ N} \qquad (1)$$

$$\Sigma M_y = 0: \quad -(0.8T_{CF})(0.27\text{ m}) + (0.6T_{DE})(0.16\text{ m}) = 0$$

or $$T_{DE} = 2.25T_{CF} \qquad (2)$$

PROBLEM 4.129 CONTINUED

Substituting Equation (2) into Equation (1)

$$0.6T_{CF} + 0.8\left[(2.25)T_{CF}\right] = 800 \text{ N}$$

$$\therefore \quad T_{CF} = 333.33 \text{ N}$$

or $T_{CF} = 333 \text{ N}$ ◀

and from Equation (2) $\quad T_{DE} = 2.25(333.33 \text{ N}) = 750.00 \text{ N}$

or $T_{DE} = 750 \text{ N}$ ◀

(*b*) From f.b.d. of assembly

$$\Sigma F_z = 0: \quad A_z - (0.6)(750.00 \text{ N}) = 0 \qquad \therefore A_z = 450.00 \text{ N}$$

$$\Sigma F_x = 0: \quad A_x - (0.8)(333.33 \text{ N}) = 0 \qquad \therefore A_x = 266.67 \text{ N}$$

or $\mathbf{A} = (267 \text{ N})\mathbf{i} + (450 \text{ N})\mathbf{k}$ ◀

$$\Sigma M_x = 0: \quad M_{A_x} + (800 \text{ N})(0.27 \text{ m}) - \left[(333.33 \text{ N})(0.6)\right](0.27 \text{ m}) - \left[(750 \text{ N})(0.8)\right](0.18 \text{ m}) = 0$$

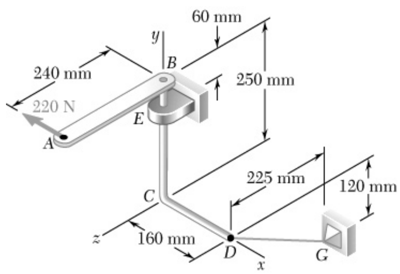
$$\therefore \quad M_{A_x} = -54.001 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: \quad M_{A_z} - (800 \text{ N})(0.16 \text{ m}) + \left[(333.33 \text{ N})(0.6)\right](0.16 \text{ m}) + \left[(750 \text{ N})(0.8)\right](0.16 \text{ m}) = 0$$

$$\therefore \quad M_{A_z} = 0$$

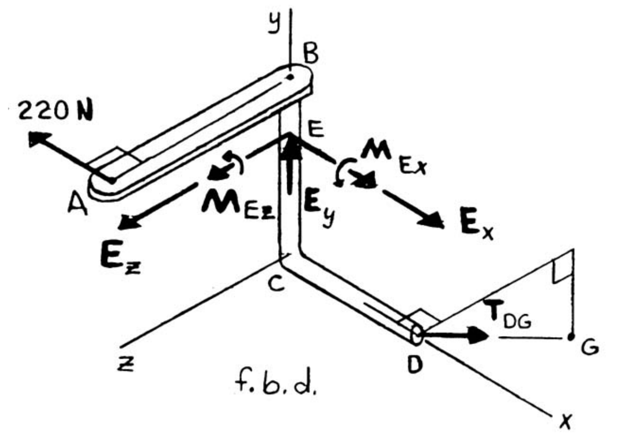
or $\mathbf{M}_A = -(54.0 \text{ N}\cdot\text{m})\mathbf{i}$ ◀

PROBLEM 4.130



The lever *AB* is welded to the bent rod *BCD* which is supported by bearing *E* and by cable *DG*. Assuming that the bearing can exert an axial thrust and couples about axes parallel to the *x* and *z* axes, determine (*a*) the tension in cable *DG*, (*b*) the reaction at *E*.

SOLUTION



First note

$$\mathbf{T}_{DG} = \lambda_{DG}T_{DG} = \frac{-(0.12\text{ m})\mathbf{j} - (0.225\text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.225)^2}\text{ m}}T_{DG}$$

$$= \frac{T_{DG}}{0.255}(-0.12\mathbf{j} - 0.225\mathbf{k})$$

(*a*) From f.b.d. of weldment

$$\Sigma M_y = 0: \quad \left[\left(\frac{0.225}{0.255}\right)T_{DG}\right](0.16\text{ m}) - (220\text{ N})(0.24\text{ m}) = 0$$

$$\therefore \quad T_{DG} = 374.00\text{ N}$$

or $T_{DG} = 374\text{ N}$ ◀

(*b*) From f.b.d. of weldment

$$\Sigma F_x = 0: \quad E_x - 220\text{ N} = 0$$

$$\therefore \quad E_x = 220.00\text{ N}$$

$$\Sigma F_y = 0: \quad E_y - (374.00\text{ N})\left(\frac{0.12}{0.255}\right) = 0$$

$$\therefore \quad E_y = 176.000\text{ N}$$

PROBLEM 4.130 CONTINUED

$$\Sigma F_z = 0: \quad E_z - (374.00 \text{ N})\left(\frac{0.225}{0.255}\right) = 0$$

$$\therefore \quad E_z = 330.00 \text{ N}$$

$$\text{or } \mathbf{E} = (220 \text{ N})\mathbf{i} + (176.0 \text{ N})\mathbf{j} + (330 \text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_x = 0: \quad M_{E_x} + (330.00 \text{ N})(0.19 \text{ m}) = 0$$

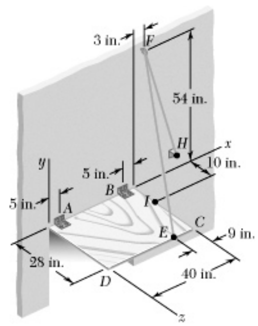
$$\therefore \quad M_{E_x} = -62.700 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: \quad (220 \text{ N})(0.06 \text{ m}) + M_{E_z} - \left[(374.00 \text{ N})\left(\frac{0.12}{0.255}\right)\right](0.16 \text{ m}) = 0$$

$$\therefore \quad M_{E_z} = -14.9600 \text{ N}\cdot\text{m}$$

$$\text{or } \mathbf{M}_E = -(62.7 \text{ N}\cdot\text{m})\mathbf{i} - (14.96 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

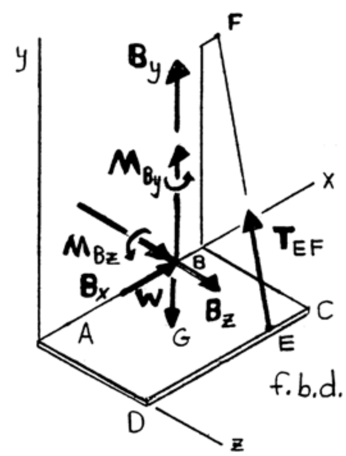
PROBLEM 4.131



Solve Problem 4.124 assuming that the hinge at A is removed and that the hinge at B can exert couples about the y and z axes.

P4.124 A small door weighing 16 lb is attached by hinges A and B to a wall and is held in the horizontal position shown by rope EFH. The rope passes around a small, frictionless pulley at F and is tied to a fixed cleat at H. Assuming that the hinge at A does not exert any axial thrust, determine (a) the tension in the rope, (b) the reactions at A and B.

SOLUTION



From f.b.d. of door

(a) $$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{G/B} \times \mathbf{W} + \mathbf{r}_{E/B} \times \mathbf{T}_{EF} + \mathbf{M}_B = 0$$

where

$$\mathbf{W} = -(16 \text{ lb})\mathbf{j}$$

$$\mathbf{M}_B = M_{B_y}\mathbf{j} + M_{B_z}\mathbf{k}$$

$$\mathbf{T}_{EF} = \lambda_{EF} T_{EF} = \frac{(12 \text{ in.})\mathbf{i} + (54 \text{ in.})\mathbf{j} - (28 \text{ in.})\mathbf{k}}{\sqrt{(12)^2 + (54)^2 + (28)^2} \text{ in.}} T_{EF}$$

$$= \frac{T_{EF}}{31}(6\mathbf{i} + 27\mathbf{j} - 14\mathbf{k})$$

$$\mathbf{r}_{G/B} = -(15 \text{ in.})\mathbf{i} + (14 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{E/B} = -(4 \text{ in.})\mathbf{i} + (28 \text{ in.})\mathbf{k}$$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -15 & 0 & 14 \\ 0 & -1 & 0 \end{vmatrix} (16 \text{ lb}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 0 & 28 \\ 6 & 27 & -14 \end{vmatrix} \left(\frac{T_{EF}}{31}\right) + \left(M_{B_y}\mathbf{j} + M_{B_z}\mathbf{k}\right) = 0$$

or $$(224 - 24.387T_{EF})\mathbf{i} + \left(3.6129T_{EF} + M_{B_y}\right)\mathbf{j}$$

$$+ \left(240 - 3.4839T_{EF} + M_{B_z}\right)\mathbf{k} = 0$$

From **i**-coefficient $$224 - 24.387T_{EF} = 0$$

$$\therefore \quad T_{EF} = 9.1852 \text{ lb}$$

or $T_{EF} = 9.19 \text{ lb}$ ◀

(b) From **j**-coefficient $$3.6129(9.1852) + M_{B_y} = 0$$

$$\therefore \quad M_{B_y} = -33.185 \text{ lb}\cdot\text{in.}$$

PROBLEM 4.131 CONTINUED

From **k**-coefficient $240 - 3.4839(9.1852) + M_{B_z} = 0$

$$\therefore \quad M_{B_z} = -208.00 \text{ lb}\cdot\text{in.}$$

or $\mathbf{M}_B = -(33.2 \text{ lb}\cdot\text{in.})\mathbf{j} - (208 \text{ lb}\cdot\text{in.})\mathbf{k}$ ◀

$$\Sigma F_x = 0: \quad B_x + \frac{6}{31}(9.1852 \text{ lb}) = 0$$

$$\therefore \quad B_x = -1.77778 \text{ lb}$$

$$\Sigma F_y = 0: \quad B_y - 16 \text{ lb} + \frac{27}{31}(9.1852 \text{ lb}) = 0$$

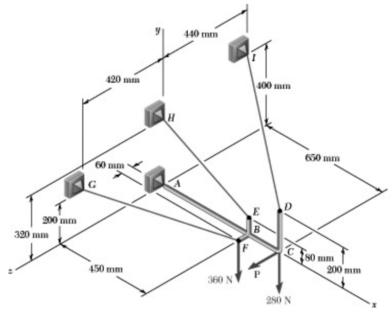
$$\therefore \quad B_y = 8.0000 \text{ lb}$$

$$\Sigma F_z = 0: \quad B_z - \frac{14}{31}(9.1852 \text{ lb}) = 0$$

$$\therefore \quad B_z = 4.1482 \text{ lb}$$

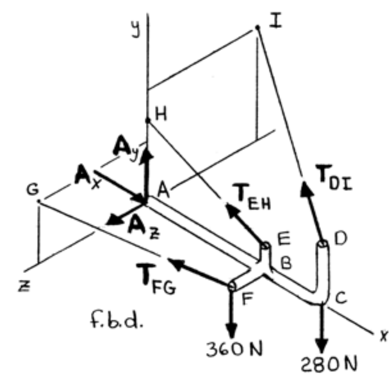
or $\mathbf{B} = -(1.778 \text{ lb})\mathbf{i} + (8.00 \text{ lb})\mathbf{j} + (4.15 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.132



The frame shown is supported by three cables and a ball-and-socket joint at A. For $\mathbf{P} = 0$, determine the tension in each cable and the reaction at A.

SOLUTION



First note

$$\mathbf{T}_{DI} = \lambda_{DI} T_{DI} = \frac{-(0.65\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} - (0.44\text{ m})\mathbf{k}}{\sqrt{(0.65)^2 + (0.2)^2 + (0.44)^2}\text{ m}} T_{DI}$$

$$= \frac{T_{DI}}{0.81}(-0.65\mathbf{i} + 0.2\mathbf{j} - 0.44\mathbf{k})$$

$$\mathbf{T}_{EH} = \lambda_{EH} T_{EH} = \frac{-(0.45\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j}}{\sqrt{(0.45)^2 + (0.24)^2}\text{ m}} T_{EH}$$

$$= \frac{T_{EH}}{0.51}(-0.45\mathbf{i}) + (0.24\,\mathbf{j})$$

$$\mathbf{T}_{FG} = \lambda_{FG} T_{FG} = \frac{-(0.45\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} + (0.36\text{ m})\mathbf{k}}{\sqrt{(0.45\ )^2 + (0.2)^2 + (0.36)^2}\text{ m}} T_{FG}$$

$$= \frac{T_{FG}}{0.61}(-0.45\mathbf{i} + 0.2\mathbf{j} + 0.36\mathbf{k})$$

From f.b.d. of frame

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{D/A} \times \mathbf{T}_{DI} + \mathbf{r}_{C/A} \times (-280\text{ N})\mathbf{j} + \mathbf{r}_{H/A} \times \mathbf{T}_{EH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FG} + \mathbf{r}_{F/A} \times (-360\text{ N})\mathbf{j} = 0$$

or

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0.2 & 0 \\ -0.65 & 0.2 & -0.44 \end{vmatrix}\left(\frac{T_{DI}}{0.81}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}(280\text{ N}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.32 & 0 \\ -0.45 & 0.24 & 0 \end{vmatrix}\left(\frac{T_{EH}}{0.51}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ -0.45 & 0.2 & 0.36 \end{vmatrix}\left(\frac{T_{FG}}{0.61}\right)$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ 0 & -1 & 0 \end{vmatrix}(360\text{ N}) = 0$$

or

$$(-0.088\mathbf{i} + 0.286\mathbf{j} + 0.26\mathbf{k})\frac{T_{DI}}{0.81} + (-0.65\mathbf{k})280\text{ N} + (0.144\mathbf{k})\frac{T_{EH}}{0.51}$$

$$+ (-0.012\mathbf{i} - 0.189\mathbf{j} + 0.09\mathbf{k})\frac{T_{FG}}{0.61} + (0.06\mathbf{i} - 0.45\mathbf{k})(360\text{ N}) = 0$$

PROBLEM 4.132 CONTINUED

From **i**-coefficient $$-0.088\left(\frac{T_{DI}}{0.81}\right) - 0.012\left(\frac{T_{FG}}{0.61}\right) + 0.06\left(360\text{ N}\right) = 0$$

$$\therefore \quad 0.108642T_{DI} + 0.0196721T_{FG} = 21.6 \qquad (1)$$

From **j**-coefficient $$0.286\left(\frac{T_{DI}}{0.81}\right) - 0.189\left(\frac{T_{FG}}{0.61}\right) = 0$$

$$\therefore \quad T_{FG} = 1.13959T_{DI} \qquad (2)$$

From **k**-coefficient

$$0.26\left(\frac{T_{DI}}{0.81}\right) - 0.65\left(280\text{ N}\right) + 0.144\left(\frac{T_{EH}}{0.51}\right) + 0.09\left(\frac{T_{FG}}{0.61}\right)$$

$$-0.45\left(360\text{ N}\right) = 0$$

$$\therefore \quad 0.32099T_{DI} + 0.28235T_{EH} + 0.147541T_{FG} = 344\text{ N} \qquad (3)$$

Substitution of Equation (2) into Equation (1)

$$0.108642T_{DI} + 0.0196721\left(1.13959T_{DI}\right) = 21.6$$

$$\therefore \quad T_{DI} = 164.810\text{ N}$$

or $$T_{DI} = 164.8\text{ N} \blacktriangleleft$$

Then from Equation (2)

$$T_{FG} = 1.13959\left(164.810\text{ N}\right) = 187.816\text{ N}$$

or $$T_{FG} = 187.8\text{ N} \blacktriangleleft$$

And from Equation (3)

$$0.32099\left(164.810\text{ N}\right) + 0.28235T_{EH} + 0.147541\left(187.816\text{ N}\right) = 344\text{ N}$$

$$\therefore \quad T_{EH} = 932.84\text{ N}$$

or $$T_{EH} = 933\text{ N} \blacktriangleleft$$

The vector forms of the cable forces are:

$$\mathbf{T}_{DI} = \frac{164.810\text{ N}}{0.81}\left(-0.65\mathbf{i} + 0.2\mathbf{j} - 0.44\mathbf{k}\right)$$

$$= -\left(132.25\text{ N}\right)\mathbf{i} + \left(40.694\text{ N}\right)\mathbf{j} - \left(89.526\text{ N}\right)\mathbf{k}$$

$$\mathbf{T}_{EH} = \frac{932.84\text{ N}}{0.51}\left(-0.45\mathbf{i} + 0.24\mathbf{j}\right) = -\left(823.09\text{ N}\right)\mathbf{i} + \left(438.98\text{ N}\right)\mathbf{j}$$

$$\mathbf{T}_{FG} = \frac{187.816\text{ N}}{0.61}\left(-0.45\mathbf{i} + 0.2\mathbf{j} + 0.36\mathbf{k}\right)$$

$$= -\left(138.553\text{ N}\right)\mathbf{i} + \left(61.579\text{ N}\right)\mathbf{j} + \left(110.842\text{ N}\right)\mathbf{k}$$

PROBLEM 4.132 CONTINUED

Then, from f.b.d. of frame

$$\Sigma F_x = 0: \quad A_x - 132.25 - 823.09 - 138.553 = 0$$

$$\therefore \quad A_x = 1093.89 \text{ N}$$

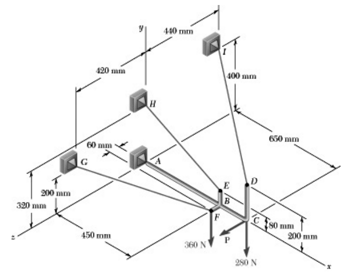
$$\Sigma F_y = 0: \quad A_y + 40.694 + 438.98 + 61.579 - 360 - 280 = 0$$

$$\therefore \quad A_y = 98.747 \text{ N}$$

$$\Sigma F_z = 0: \quad A_z - 89.526 + 110.842 = 0$$

$$\therefore \quad A_z = -21.316 \text{ N}$$

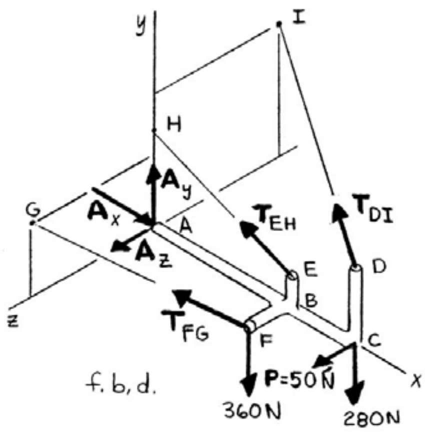
or $$\mathbf{A} = (1094 \text{ N})\mathbf{i} + (98.7 \text{ N})\mathbf{j} - (21.3 \text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.133

The frame shown is supported by three cables and a ball-and-socket joint at A. For $P = 50$ N, determine the tension in each cable and the reaction at A.

SOLUTION



First note

$$\mathbf{T}_{DI} = \lambda_{DI} T_{DI} = \frac{-(0.65\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} - (0.44\text{ m})\mathbf{k}}{\sqrt{(0.65)^2 + (0.2)^2 + (0.44)^2}\text{ m}} T_{DI}$$

$$= \frac{T_{DI}}{81}(-65\mathbf{i} + 20\mathbf{j} - 44\mathbf{k})$$

$$\mathbf{T}_{EH} = \lambda_{EH} T_{EH} = \frac{-(0.45\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j}}{\sqrt{(0.45)^2 + (0.24)^2}\text{ m}} T_{EH}$$

$$= \frac{T_{EH}}{17}(-15\mathbf{i} + 8\mathbf{j})$$

$$\mathbf{T}_{FG} = \lambda_{FG} T_{FG} = \frac{-(0.45\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} + (0.36\text{ m})\mathbf{k}}{\sqrt{(0.45)^2 + (0.2)^2 + (0.36)^2}\text{ m}} T_{FG}$$

$$= \frac{T_{FG}}{61}(-45\mathbf{i} + 20\mathbf{j} + 36\mathbf{k})$$

From f.b.d. of frame

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{D/A} \times \mathbf{T}_{DI} + \mathbf{r}_{C/A} \times \left[-(280\text{ N})\mathbf{j} + (50\text{ N})\mathbf{k}\right]$$

$$+ \mathbf{r}_{H/A} \times \mathbf{T}_{EH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FG} + \mathbf{r}_{F/A} \times (-360\text{ N})\mathbf{j}$$

or

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0.2 & 0 \\ -65 & 20 & -44 \end{vmatrix} \left(\frac{T_{DI}}{81}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0 & 0 \\ 0 & -280 & 50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.32 & 0 \\ -15 & 8 & 0 \end{vmatrix} \left(\frac{T_{EH}}{17}\right)$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ -45 & 20 & 36 \end{vmatrix} \left(\frac{T_{FG}}{61}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ 0 & -1 & 0 \end{vmatrix} (360\text{ N}) = 0$$

and

$$(-8.8\mathbf{i} + 28.6\mathbf{j} + 26\mathbf{k})\left(\frac{T_{DI}}{81}\right) + (-32.5\mathbf{j} - 182\mathbf{k}) + (4.8\mathbf{k})\left(\frac{T_{EH}}{17}\right)$$

$$+(-1.2\mathbf{i} - 18.9\mathbf{j} + 9.0\mathbf{k})\left(\frac{T_{FG}}{61}\right) + (0.06\mathbf{i} - 0.45\mathbf{k})(360) = 0$$

PROBLEM 4.133 CONTINUED

From **i**-coefficient $-8.8\left(\frac{T_{DI}}{81}\right) - 1.2\left(\frac{T_{FG}}{61}\right) + 0.06(360) = 0$

$$\therefore \quad 0.108642T_{DI} + 0.0196721T_{FG} = 21.6 \qquad (1)$$

From **j**-coefficient $28.6\left(\frac{T_{DI}}{81}\right) - 32.5 - 18.9\left(\frac{T_{FG}}{61}\right) = 0$

$$\therefore \quad 0.35309T_{DI} - 0.30984T_{FG} = 32.5 \qquad (2)$$

From **k**-coefficient

$$26\left(\frac{T_{DI}}{81}\right) - 182 + 4.8\left(\frac{T_{EH}}{17}\right) + 9.0\left(\frac{T_{FG}}{61}\right) - 0.45(360) = 0$$

$$\therefore \quad 0.32099T_{DI} + 0.28235T_{EH} + 0.147541T_{FG} = 344 \qquad (3)$$

$-3.25 \times$ Equation (1) $\quad -0.35309T_{DI} - 0.063935T_{FG} = -70.201$

Add Equation (2) $\quad \underline{0.35309T_{DI} - 0.30984T_{FG} = 32.5}$

$$-0.37378T_{FG} = -37.701$$

$$\therefore \quad T_{FG} = 100.864 \text{ N}$$

or $\qquad T_{FG} = 100.9 \text{ N}$ ◀

Then from Equation (1)

$$0.108642T_{DI} + 0.0196721(100.864) = 21.6$$

$$\therefore \quad T_{DI} = 180.554 \text{ N}$$

or $\qquad T_{DI} = 180.6 \text{ N}$ ◀

and from Equation (3)

$$0.32099(180.554) + 0.28235T_{EH} + 0.147541(100.864) = 344$$

$$\therefore \quad T_{EH} = 960.38 \text{ N}$$

or $\qquad T_{EH} = 960 \text{ N}$ ◀

The vector forms of the cable forces are:

$$\mathbf{T}_{DI} = \frac{180.554 \text{ N}}{81}(-65\mathbf{i} + 20\mathbf{j} - 44\mathbf{k})$$

$$= -(144.889 \text{ N})\mathbf{i} + (44.581 \text{ N})\mathbf{j} - (98.079 \text{ N})\mathbf{k}$$

$$\mathbf{T}_{EH} = \frac{960.38 \text{ N}}{17}(-15\mathbf{i} + 8\mathbf{j}) = -(847.39 \text{ N})\mathbf{i} + (451.94 \text{ N})\mathbf{j}$$

$$\mathbf{T}_{FG} = \frac{100.864 \text{ N}}{61}(-45\mathbf{i} + 20\mathbf{j} + 36\mathbf{k})$$

$$= -(74.409 \text{ N})\mathbf{i} + (33.070 \text{ N})\mathbf{j} + (59.527 \text{ N})\mathbf{k}$$

PROBLEM 4.133 CONTINUED

Then from f.b.d. of frame

$$\Sigma F_x = 0: \quad A_x - 144.889 - 847.39 - 74.409 = 0$$

$$\therefore \quad A_x = 1066.69 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + 44.581 + 451.94 + 33.070 - 360 - 280 = 0$$

$$\therefore \quad A_y = 110.409 \text{ N}$$

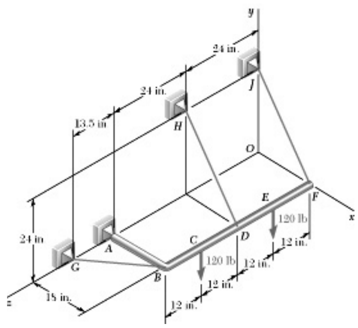
$$\Sigma F_z = 0: \quad A_z - 98.079 + 59.527 + 50 = 0$$

$$\therefore \quad A_z = -11.448 \text{ N}$$

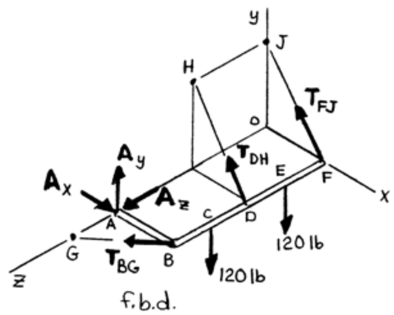
Therefore, $\mathbf{A} = (1067 \text{ N})\mathbf{i} + (110.4 \text{ N})\mathbf{j} - (11.45 \text{ N})\mathbf{k}$ ◀

PROBLEM 4.134

The rigid L-shaped member *ABF* is supported by a ball-and-socket joint at *A* and by three cables. For the loading shown, determine the tension in each cable and the reaction at *A*.



SOLUTION



First note

$$\mathbf{T}_{BG} = \lambda_{BG}T_{BG} = \frac{-(18\text{ in.})\mathbf{i} + (13.5\text{ in.})\mathbf{k}}{\sqrt{(18)^2 + (13.5)^2}\text{ in.}}T_{BG}$$

$$= T_{BG}(-0.8\mathbf{i} + 0.6\mathbf{k})$$

$$\mathbf{T}_{DH} = \lambda_{DH}T_{DH} = \frac{-(18\text{ in.})\mathbf{i} + (24\text{ in.})\mathbf{j}}{\sqrt{(18)^2 + (24)^2}\text{ in.}}T_{DH}$$

$$= T_{DH}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

Since $\lambda_{FJ} = \lambda_{DH}$,

$$\mathbf{T}_{FJ} = T_{FJ}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

From f.b.d. of member *ABF*

$$\Sigma M_{A(x\text{-}axis)} = 0: \quad (0.8T_{FJ})(48\text{ in.}) + (0.8T_{DH})(24\text{ in.}) - (120\text{ lb})(36\text{ in.}) - (120\text{ lb})(12\text{ in.}) = 0$$

$$\therefore \quad 3.2T_{FJ} + 1.6T_{DH} = 480 \qquad (1)$$

$$\Sigma M_{A(z\text{-}axis)} = 0: \quad (0.8T_{FJ})(18\text{ in.}) + (0.8T_{DH})(18\text{ in.}) - (120\text{ lb})(18\text{ in.}) - (120\text{ lb})(18\text{ in.}) = 0$$

$$\therefore \quad -3.2T_{FJ} - 3.2T_{DH} = -960 \qquad (2)$$

Equation (1) + Equation (2) $\qquad T_{DH} = 300\text{ lb}$ ◀

Substituting in Equation (1) $\qquad T_{FJ} = 0$ ◀

$$\Sigma M_{A(y\text{-}axis)} = 0: \quad (0.6T_{FJ})(48\text{ in.}) + [0.6(300\text{ lb})](24\text{ in.}) - (0.6T_{BG})(18\text{ in.}) = 0$$

$\therefore \quad T_{BG} = 400\text{ lb}$ ◀

PROBLEM 4.134 CONTINUED

$$\Sigma F_x = 0: \quad -0.6T_{FJ} - 0.6T_{DH} - 0.8T_{BG} + A_x = 0$$

$$-0.6(300 \text{ lb}) - 0.8(400 \text{ lb}) + A_x = 0$$

$$\therefore \quad A_x = 500 \text{ lb}$$

$$\Sigma F_y = 0: \quad 0.8T_{FJ} + 0.8T_{DH} - 240 \text{ lb} + A_y = 0$$

$$0.8(300 \text{ lb}) - 240 + A_y = 0$$

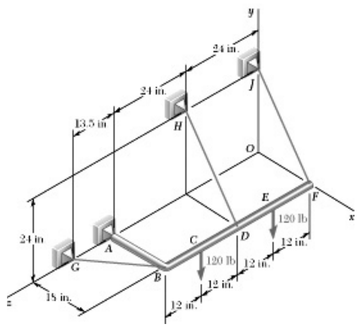
$$\therefore \quad A_y = 0$$

$$\Sigma F_z = 0: \quad 0.6T_{BG} + A_z = 0$$

$$0.6(400 \text{ lb}) + A_z = 0$$

$$\therefore \quad A_z = -240 \text{ lb}$$

Therefore, $\mathbf{A} = (500 \text{ lb})\mathbf{i} - (240 \text{ lb})\mathbf{k}$ ◀

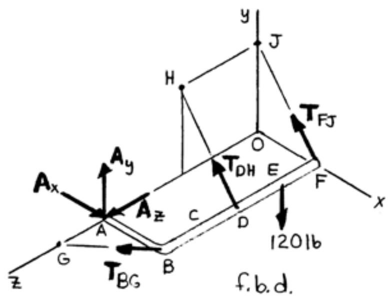


PROBLEM 4.135

Solve Problem 4.134 assuming that the load at C has been removed.

P4.134 The rigid L-shaped member ABF is supported by a ball-and-socket joint at A and by three cables. For the loading shown, determine the tension in each cable and the reaction at A.

SOLUTION



First

$$\mathbf{T}_{BG} = \lambda_{BG} T_{BG} = \frac{-(18\text{ in.})\mathbf{i} + (13.5\text{ in.})\mathbf{k}}{\sqrt{(18)^2 + (13.5)^2}\text{ in.}} T_{BG}$$

$$= T_{BG}(-0.8\mathbf{i} + 0.6\mathbf{k})$$

$$\mathbf{T}_{DH} = \lambda_{DH} T_{DH} = \frac{-(18\text{ in.})\mathbf{i} + (24\text{ in.})\mathbf{j}}{\sqrt{(18)^2 + (24)^2}\text{ in.}} T_{DH}$$

$$= T_{DH}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

Since $\lambda_{FJ} = \lambda_{DH}$

$$\mathbf{T}_{FJ} = T_{FJ}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

From f.b.d. of member ABF

$$\Sigma M_{A(x\text{-axis})} = 0: \quad (0.8T_{FJ})(48\text{ in.}) + (0.8T_{DH})(24\text{ in.}) - (120\text{ lb})(36\text{ in.}) = 0$$

$$\therefore \quad 3.2T_{FJ} + 1.6T_{DH} = 360 \qquad (1)$$

$$\Sigma M_{A(z\text{-axis})} = 0: \quad (0.8T_{FJ})(18\text{ in.}) + (0.8T_{DH})(18\text{ in.}) - (120\text{ lb})(18\text{ in.}) = 0$$

$$\therefore \quad -3.2T_{FJ} - 3.2T_{DH} = -480 \qquad (2)$$

Equation (1) + Equation (2) $\qquad T_{DH} = 75.0\text{ lb}$ ◀

Substituting into Equation (2) $\qquad T_{FJ} = 75.0\text{ lb}$ ◀

$$\Sigma M_{A(y\text{-axis})} = 0: \quad (0.6T_{FJ})(48\text{ in.}) + (0.6T_{DH})(24\text{ in.}) - (0.6T_{BG})(18\text{ in.}) = 0$$

or

$$(75.0\text{ lb})(48\text{ in.}) + (75.0\text{ lb})(24\text{ in.}) = T_{BG}(18\text{ in.})$$

$T_{BG} = 300\text{ lb}$ ◀

PROBLEM 4.135 CONTINUED

$$\Sigma F_x = 0: \quad -0.6T_{FJ} - 0.6T_{DH} - 0.8T_{BG} + A_x = 0$$

$$-0.6(75.0 + 75.0) - 0.8(300) + A_x = 0$$

$$\therefore \quad A_x = 330 \text{ lb}$$

$$\Sigma F_y = 0: \quad 0.8T_{FJ} + 0.8T_{DH} - 120 \text{ lb} + A_y = 0$$

$$0.8(150 \text{ lb}) - 120 \text{ lb} + A_y = 0$$

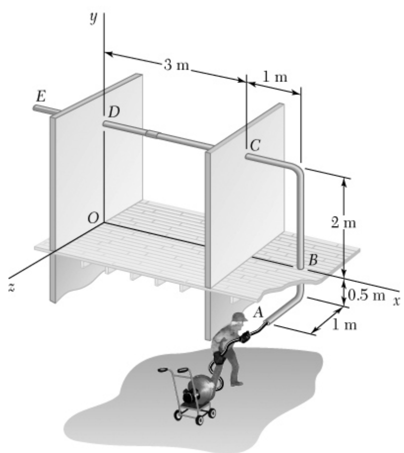
$$\therefore \quad A_y = 0$$

$$\Sigma F_z = 0: \quad 0.6T_{BG} + A_z = 0$$

$$0.6(300 \text{ lb}) + A_z = 0$$

$$\therefore \quad A_z = -180 \text{ lb}$$

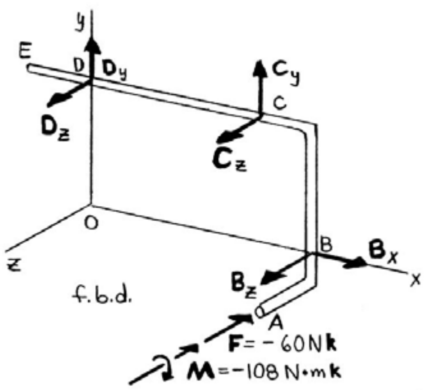
Therefore $\mathbf{A} = (330 \text{ lb})\mathbf{i} - (180 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.136

In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor which rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(60\ \text{N})\mathbf{k}$, $\mathbf{M} = -(108\ \text{N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION



From f.b.d. of pipe assembly *ABCD*

$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (60\ \text{N})(2.5\ \text{m}) - B_z(2\ \text{m}) = 0$$

$$\therefore \quad B_z = 75.0\ \text{N}$$

and $\mathbf{B} = (75.0\ \text{N})\mathbf{k}$ ◀

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\ \text{m}) - 108\ \text{N}\cdot\text{m} = 0$$

$$\therefore \quad C_y = 36.0\ \text{N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad -C_z(3\ \text{m}) - (75\ \text{N})(4\ \text{m}) + (60\ \text{N})(4\ \text{m}) = 0$$

$$\therefore \quad C_z = -20.0\ \text{N}$$

and $\mathbf{C} = (36.0\ \text{N})\mathbf{j} - (20.0\ \text{N})\mathbf{k}$ ◀

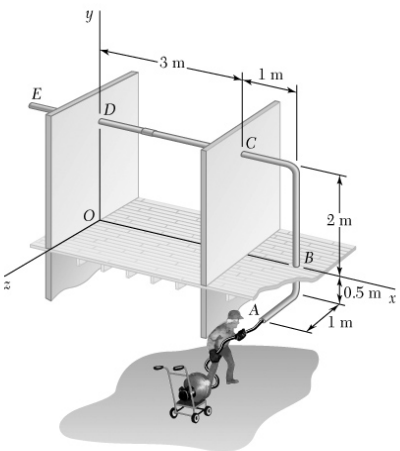
$$\Sigma F_y = 0: \quad D_y + 36.0 = 0$$

$$\therefore \quad D_y = -36.0\ \text{N}$$

$$\Sigma F_z = 0: \quad D_z - 20.0\ \text{N} + 75.0\ \text{N} - 60\ \text{N} = 0$$

$$\therefore \quad D_z = 5.00\ \text{N}$$

and $\mathbf{D} = -(36.0\ \text{N})\mathbf{j} + (5.00\ \text{N})\mathbf{k}$ ◀

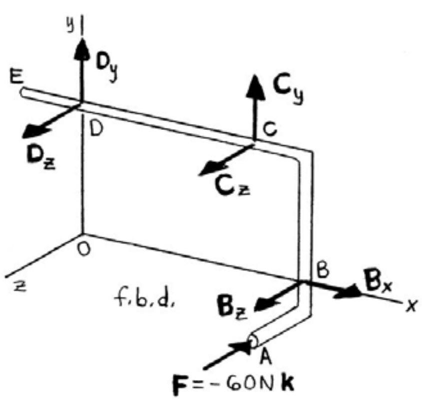


PROBLEM 4.137

Solve Problem 4.136 assuming that the plumber exerts a force $\mathbf{F} = -(60\ \text{N})\mathbf{k}$ and that the motor is turned off $(\mathbf{M} = 0)$.

P4.136 In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor which rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(60\ \text{N})\mathbf{k}$, $\mathbf{M} = -(108\ \text{N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION



From f.b.d. of pipe assembly *ABCD*

$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (60\ \text{N})(2.5\ \text{m}) - B_z(2\ \text{m}) = 0$$

$$\therefore \quad B_z = 75.0\ \text{N}$$

and $\mathbf{B} = (75.0\ \text{N})\mathbf{k}$ ◀

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\ \text{m}) - B_x(2\ \text{m}) = 0$$

$$\therefore \quad C_y = 0$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(3\ \text{m}) - (75.0\ \text{N})(4\ \text{m}) + (60\ \text{N})(4\ \text{m}) = 0$$

$$\therefore \quad C_z = -20\ \text{N}$$

and $\mathbf{C} = -(20.0\ \text{N})\mathbf{k}$ ◀

$$\Sigma F_y = 0: \quad D_y + C_y = 0$$

$$\therefore \quad D_y = 0$$

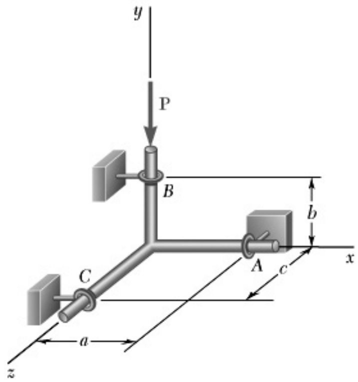
$$\Sigma F_z = 0: \quad D_z + B_z + C_z - F = 0$$

$$D_z + 75\ \text{N} - 20\ \text{N} - 60\ \text{N} = 0$$

$$\therefore \quad D_z = 5.00\ \text{N}$$

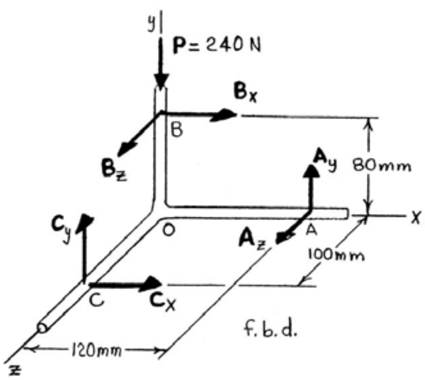
and $\mathbf{D} = (5.00\ \text{N})\mathbf{k}$ ◀

PROBLEM 4.138



Three rods are welded together to form a "corner" which is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ N, $a = 120$ mm, $b = 80$ mm, and $c = 100$ mm.

SOLUTION



From f.b.d. of weldment

$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 120 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 100 \\ C_x & C_y & 0 \end{vmatrix} = 0$$

$$\left(-120A_z\mathbf{j} + 120A_y\mathbf{k}\right) + \left(80B_z\mathbf{i} - 80B_x\mathbf{k}\right) + \left(-100C_y\mathbf{i} + 100C_x\mathbf{j}\right) = 0$$

From **i**-coefficient $\quad 80B_z - 100C_y = 0$

or $\quad B_z = 1.25C_y \quad (1)$

j-coefficient $\quad -120A_z + 100C_x = 0$

or $\quad C_x = 1.2A_z \quad (2)$

k-coefficient $\quad 120A_y - 80B_x = 0$

or $\quad B_x = 1.5A_y \quad (3)$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} - \mathbf{P} = 0$$

or $\quad \left(B_x + C_x\right)\mathbf{i} + \left(A_y + C_y - 240\text{ N}\right)\mathbf{j} + \left(A_z + B_z\right)\mathbf{k} = 0$

From **i**-coefficient $\quad B_x + C_x = 0$

or $\quad C_x = -B_x \quad (4)$

j-coefficient $\quad A_y + C_y - 240\text{ N} = 0$

or $\quad A_y + C_y = 240\text{ N} \quad (5)$

k-coefficient $\quad A_z + B_z = 0$

or $\quad A_z = -B_z \quad (6)$

PROBLEM 4.138 CONTINUED

Substituting C_x from Equation (4) into Equation (2)

$$-B_z = 1.2A_z \tag{7}$$

Using Equations (1), (6), and (7)

$$C_y = \frac{B_z}{1.25} = \frac{-A_z}{1.25} = \frac{1}{1.25}\left(\frac{B_x}{1.2}\right) = \frac{B_x}{1.5} \tag{8}$$

From Equations (3) and (8)

$$C_y = \frac{1.5A_y}{1.5} \qquad \text{or} \qquad C_y = A_y$$

and substituting into Equation (5)

$$2A_y = 240 \text{ N}$$

$$\therefore \quad A_y = C_y = 120 \text{ N} \tag{9}$$

Using Equation (1) and Equation (9)

$$B_z = 1.25(120 \text{ N}) = 150.0 \text{ N}$$

Using Equation (3) and Equation (9)

$$B_x = 1.5(120 \text{ N}) = 180.0 \text{ N}$$

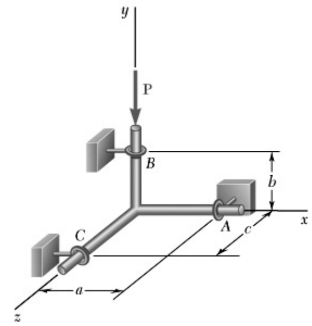
From Equation (4) $\qquad C_x = -180.0 \text{ N}$

From Equation (6) $\qquad A_z = -150.0 \text{ N}$

Therefore $\qquad \mathbf{A} = (120.0 \text{ N})\mathbf{j} - (150.0 \text{ N})\mathbf{k}$ ◀

$\mathbf{B} = (180.0 \text{ N})\mathbf{i} + (150.0 \text{ N})\mathbf{k}$ ◀

$\mathbf{C} = -(180.0 \text{ N})\mathbf{i} + (120.0 \text{ N})\mathbf{j}$ ◀

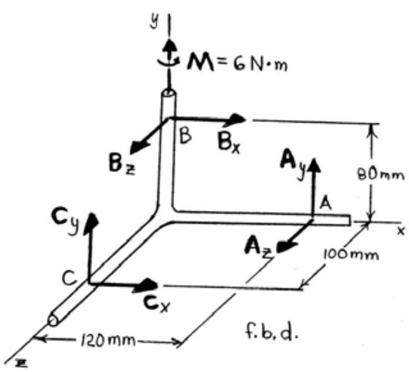


PROBLEM 4.139

Solve Problem 4.138 assuming that the force **P** is removed and is replaced by a couple $\mathbf{M} = +(6\ \text{N}\cdot\text{m})\mathbf{j}$ acting at B.

P4.138 Three rods are welded together to form a "corner" which is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ N, $a = 120$ mm, $b = 80$ mm, and $c = 100$ mm.

SOLUTION



From f.b.d. of weldment

$$\Sigma \mathbf{M}_O = 0:\quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} + \mathbf{M} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.08 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.1 \\ C_x & C_y & 0 \end{vmatrix} + (6\ \text{N}\cdot\text{m})\mathbf{j} = 0$$

$$\left(-0.12A_z\mathbf{j} + 0.12A_y\mathbf{k}\right) + \left(0.08B_z\mathbf{j} - 0.08B_x\mathbf{k}\right)$$

$$+\left(-0.1C_y\mathbf{i} + 0.1C_x\mathbf{j}\right) + (6\ \text{N}\cdot\text{m})\mathbf{j} = 0$$

From **i**-coefficient $\qquad 0.08B_z - 0.1C_y = 0$

or $\qquad C_y = 0.8B_z \qquad (1)$

j-coefficient $\qquad -0.12A_z + 0.1C_x + 6 = 0$

or $\qquad C_x = 1.2A_z - 60 \qquad (2)$

k-coefficient $\qquad 0.12A_y - 0.08B_x = 0$

or $\qquad B_x = 1.5A_y \qquad (3)$

$$\Sigma \mathbf{F} = 0:\quad \mathbf{A} + \mathbf{B} + \mathbf{C} = 0$$

$$\left(B_x + C_x\right)\mathbf{i} + \left(A_y + C_y\right)\mathbf{j} + \left(A_z + B_z\right)\mathbf{k} = 0$$

From **i**-coefficient $\qquad C_x = -B_x \qquad (4)$

j-coefficient $\qquad C_y = -A_y \qquad (5)$

k-coefficient $\qquad A_z = -B_z \qquad (6)$

Substituting C_x from Equation (4) into Equation (2)

$$A_z = 50 - \left(\frac{B_x}{1.2}\right) \qquad (7)$$

PROBLEM 4.139 CONTINUED

Using Equations (1), (6), and (7)

$$C_y = 0.8B_z = -0.8A_z = \left(\frac{2}{3}\right)B_x - 40 \tag{8}$$

From Equations (3) and (8)

$$C_y = A_y - 40$$

Substituting into Equation (5) $\quad 2A_y = 40$

$$\therefore \quad A_y = 20.0 \text{ N}$$

From Equation (5) $\quad C_y = -20.0 \text{ N}$

Equation (1) $\quad B_z = -25.0 \text{ N}$

Equation (3) $\quad B_x = 30.0 \text{ N}$

Equation (4) $\quad C_x = -30.0 \text{ N}$

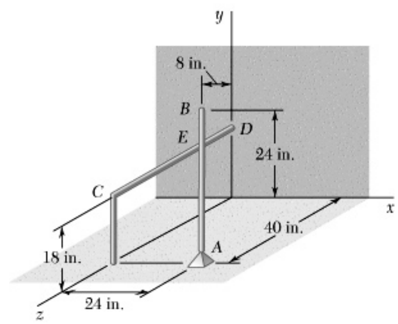
Equation (6) $\quad A_z = 25.0 \text{ N}$

Therefore $\quad \mathbf{A} = (20.0 \text{ N})\mathbf{j} + (25.0 \text{ N})\mathbf{k}$ ◀

$\mathbf{B} = (30.0 \text{ N})\mathbf{i} - (25.0 \text{ N})\mathbf{k}$ ◀

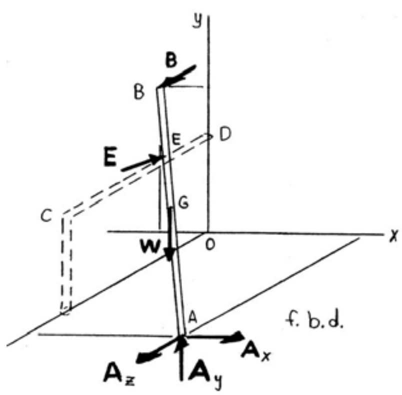
$\mathbf{C} = -(30.0 \text{ N})\mathbf{i} - (20.0 \text{ N})\mathbf{j}$ ◀

PROBLEM 4.140



The uniform 10-lb rod *AB* is supported by a ball-and-socket joint at *A* and leans against both the rod *CD* and the vertical wall. Neglecting the effects of friction, determine (*a*) the force which rod *CD* exerts on *AB*, (*b*) the reactions at *A* and *B*. (*Hint:* The force exerted by *CD* on *AB* must be perpendicular to both rods.)

SOLUTION



(*a*) The force acting at *E* on the f.b.d. of rod *AB* is perpendicular to *AB* and *CD*. Letting $\lambda_E =$ direction cosines for force **E**,

$$\lambda_E = \frac{\mathbf{r}_{B/A} \times \mathbf{k}}{\left|\mathbf{r}_{B/A} \times \mathbf{k}\right|}$$

$$= \frac{\left[-(32 \text{ in.})\mathbf{i} + (24 \text{ in.})\mathbf{j} - (40 \text{ in.})\mathbf{k}\right] \times \mathbf{k}}{\sqrt{(32)^2 + (24)^2} \text{ in.}}$$

$$= 0.6\mathbf{i} + 0.8\mathbf{j}$$

Also, $\mathbf{W} = -(10 \text{ lb})\mathbf{j}$

$$\mathbf{B} = B\mathbf{k}$$

$$\mathbf{E} = E(0.6\mathbf{i} + 0.8\mathbf{j})$$

From f.b.d. of rod *AB*

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{G/A} \times \mathbf{W} + \mathbf{r}_{E/A} \times \mathbf{E} + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -16 & 12 & -20 \\ 0 & -1 & 0 \end{vmatrix}(10 \text{ lb}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -24 & 18 & -30 \\ 0.6 & 0.8 & 0 \end{vmatrix} E + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -32 & 24 & -40 \\ 0 & 0 & 1 \end{vmatrix} B = 0$$

$$(-20\mathbf{i} + 16\mathbf{k})(10 \text{ lb}) + (24\mathbf{i} - 18\mathbf{j} - 30\mathbf{k})E + (24\mathbf{i} + 32\mathbf{j})B = 0$$

From **k**-coefficient $\quad 160 - 30E = 0$

$$\therefore \; E = 5.3333 \text{ lb}$$

and $\quad \mathbf{E} = 5.3333 \text{ lb}(0.6\mathbf{i} + 0.8\mathbf{j})$

or $\quad \mathbf{E} = (3.20 \text{ lb})\mathbf{i} + (4.27 \text{ lb})\mathbf{j}$ ◀

(*b*) From **j**-coefficient $\quad -18(5.3333 \text{ lb}) + 32B = 0$

$$\therefore \; B = 3.00 \text{ lb}$$

or $\quad \mathbf{B} = (3.00 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.140 CONTINUED

From f.b.d. of rod AB

$$\Sigma\mathbf{F} = 0: \quad \mathbf{A} + \mathbf{W} + \mathbf{E} + \mathbf{B} = 0$$

$$A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} - (10 \text{ lb})\mathbf{j} + \left[(3.20 \text{ lb})\mathbf{i} + (4.27 \text{ lb})\mathbf{j}\right] + (3.00 \text{ lb})\mathbf{k} = 0$$

From **i**-coefficient $\quad A_x + 3.20 \text{ lb} = 0$

$$\therefore \quad A_x = -3.20 \text{ lb}$$

j-coefficient $\quad A_y - 10 \text{ lb} + 4.27 \text{ lb} = 0$

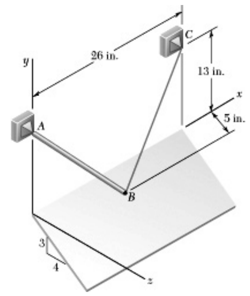
$$\therefore \quad A_y = 5.73 \text{ lb}$$

k-coefficient $\quad A_z + 3.00 \text{ lb} = 0$

$$\therefore \quad A_z = -3.00 \text{ lb}$$

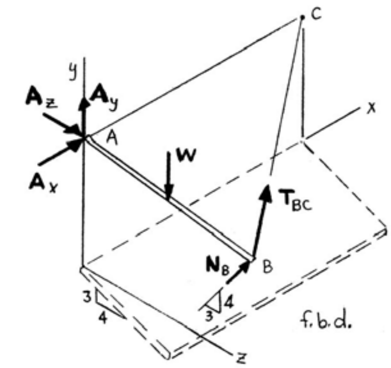
Therefore $\quad \mathbf{A} = -(3.20 \text{ lb})\mathbf{i} + (5.73 \text{ lb})\mathbf{j} - (3.00 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.141



A 21-in.-long uniform rod AB weighs 6.4 lb and is attached to a ball-and-socket joint at A. The rod rests against an inclined frictionless surface and is held in the position shown by cord BC. Knowing that the cord is 21 in. long, determine (a) the tension in the cord, (b) the reactions at A and B.

SOLUTION



First note

$$\mathbf{W} = -(6.4 \text{ lb})\mathbf{j}$$

$$\mathbf{N}_B = N_B(0.8\mathbf{j} + 0.6\mathbf{k})$$

$$L_{AB} = 21 \text{ in.}$$

$$= \sqrt{(x_B)^2 + (13+3)^2 + (4)^2} = \sqrt{x_B^2 + (16)^2 + (4)^2}$$

$$\therefore\ x_B = 13 \text{ in.}$$

$$\mathbf{T}_{BC} = \boldsymbol{\lambda}_{BC}T_{BC} = \frac{(13 \text{ in.})\mathbf{i} + (16 \text{ in.})\mathbf{j} - (4 \text{ in.})\mathbf{k}}{21 \text{ in.}}T_{BC}$$

$$= \frac{T_{BC}}{21}(13\mathbf{i} + 16\mathbf{j} - 4\mathbf{k})$$

From f.b.d. of rod AB

$$\Sigma\mathbf{M}_A = 0\text{:}\quad \mathbf{r}_{G/A} \times \mathbf{W} + \mathbf{r}_{B/A} \times \mathbf{N}_B + \mathbf{r}_{C/A} \times \mathbf{T}_{BC} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6.5 & -8 & 2 \\ 0 & -6.4 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 13 & -16 & 4 \\ 0 & 0.8 & 0.6 \end{vmatrix} N_B + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 13 & 16 & -4 \end{vmatrix} \frac{26T_{BC}}{21} = 0$$

$$(12.8\mathbf{i} - 41.6\mathbf{k}) + (-12.8\mathbf{i} - 7.8\mathbf{j} + 10.4\mathbf{k})N_B + (4\mathbf{j} + 16\mathbf{k})\frac{26T_{BC}}{21} = 0$$

PROBLEM 4.141 CONTINUED

From **i**-coeff. $12.8 - 12.8N_B = 0 \quad \therefore \quad N_B = 1.00 \text{ lb}$

or $$\mathbf{N}_B = (0.800 \text{ lb})\mathbf{j} + (0.600 \text{ lb})\mathbf{k}$$

From **j**-coeff. $-7.8N_B + 4\left(\frac{26}{21}\right)T_{BC} = 0 \quad \therefore \quad T_{BC} = 1.575 \text{ lb}$

From f.b.d. of rod AB

$\Sigma\mathbf{F} = 0: \quad \mathbf{A} + \mathbf{W} + \mathbf{N}_B + \mathbf{T}_{BC} = 0$

$$\left(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\right) - (6.4 \text{ lb})\mathbf{j} + (0.800 \text{ lb})\mathbf{j} + (0.600 \text{ lb})\mathbf{k} + \left(\frac{1.575}{21}\right)(13\mathbf{i} + 16\mathbf{j} - 4\mathbf{k}) = 0$$

From **i**-coefficient $A_x = -0.975 \text{ lb}$

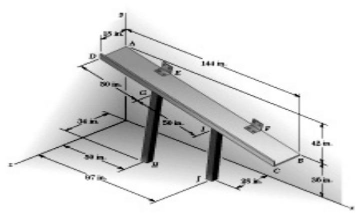
j-coefficient $A_y = 4.40 \text{ lb}$

k-coefficient $A_z = -0.3 \text{ lb}$

$\therefore$ (*a*) $T_{BC} = 1.575 \text{ lb}$ ◀

(*b*) $\mathbf{A} = -(0.975 \text{ lb})\mathbf{i} + (4.40 \text{ lb})\mathbf{j} - (0.300 \text{ lb})\mathbf{k}$ ◀

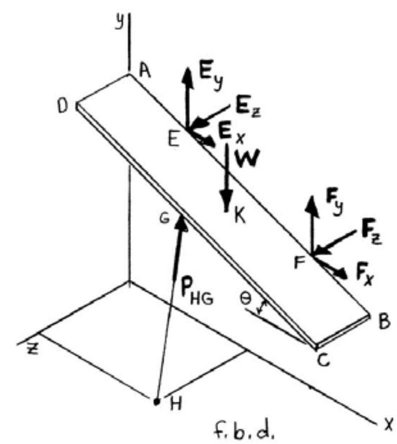
$\mathbf{N}_B = (0.800 \text{ lb})\mathbf{j} + (0.600 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.142

While being installed, the 56-lb chute *ABCD* is attached to a wall with brackets *E* and *F* and is braced with props *GH* and *IJ*. Assuming that the weight of the chute is uniformly distributed, determine the magnitude of the force exerted on the chute by prop *GH* if prop *IJ* is removed.

SOLUTION



First note

$$\theta = \tan^{-1}\left(\frac{42 \text{ in.}}{144 \text{ in.}}\right) = 16.2602°$$

$$x_G = (50 \text{ in.})\cos 16.2602° = 48 \text{ in.}$$

$$y_G = 78 \text{ in.} - (50 \text{ in.})\sin 16.2602° = 64 \text{ in.}$$

$$\boldsymbol{\lambda}_{BA} = \frac{-(144 \text{ in.})\mathbf{i} + (42 \text{ in.})\mathbf{j}}{\sqrt{(144)^2 + (42)^2} \text{ in.}} = \frac{1}{25}(-24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{r}_{K/A} = (72 \text{ in.})\mathbf{i} - (21 \text{ in.})\mathbf{j} + (9 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{G/A} = (48 \text{ in.})\mathbf{i} - (78 \text{ in.} - 64 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k} = (48 \text{ in.})\mathbf{i} - (14 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k}$$

$$\mathbf{W} = -(56 \text{ lb})\mathbf{j}$$

$$\mathbf{P}_{HG} = \boldsymbol{\lambda}_{HG} P_{HG}$$

$$= \frac{-(2 \text{ in.})\mathbf{i} + (64 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}}{\sqrt{(2)^2 + (64)^2 + (16)^2} \text{ in.}} P_{HG}$$

$$= \frac{P_{HG}}{33}(-\mathbf{i} + 32\mathbf{j} - 8\mathbf{k})$$

PROBLEM 4.142 CONTINUED

From the f.b.d. of the chute

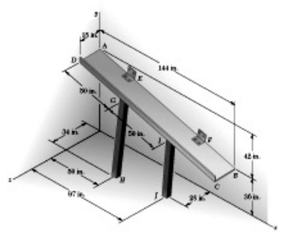
$$\Sigma M_{BA} = 0: \quad \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{K/A} \times \mathbf{W}\right) + \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{P}_{HG}\right) = 0$$

$$\begin{vmatrix} -24 & 7 & 0 \\ 72 & -21 & 9 \\ 0 & -1 & 0 \end{vmatrix} \left(\frac{56}{25}\right) + \begin{vmatrix} -24 & 7 & 0 \\ 48 & -14 & 18 \\ -1 & 32 & -8 \end{vmatrix} \left[\frac{P_{HG}}{33(25)}\right] = 0$$

$$\frac{-216(56)}{25} + \left[-24(-14)(-8) - (-24)(18)(32) + 7(18)(-1) - (7)(48)(-8)\right]\frac{P_{HG}}{33(25)} = 0$$

$$\therefore \quad P_{HG} = 29.141 \text{ lb}$$

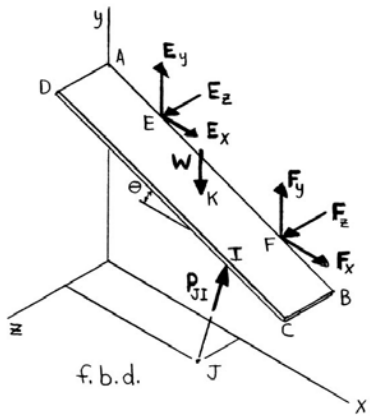
or $P_{HG} = 29.1$ lb ◀



PROBLEM 4.143

While being installed, the 56-lb chute *ABCD* is attached to a wall with brackets *E* and *F* and is braced with props *GH* and *IJ*. Assuming that the weight of the chute is uniformly distributed, determine the magnitude of the force exerted on the chute by prop *IJ* if prop *GH* is removed.

SOLUTION



First note

$$\theta = \tan^{-1}\left(\frac{42 \text{ in.}}{144 \text{ in.}}\right) = 16.2602°$$

$$x_I = (100 \text{ in.})\cos 16.2602° = 96 \text{ in.}$$

$$y_I = 78 \text{ in.} - (100 \text{ in.})\sin 16.2602° = 50 \text{ in.}$$

$$\lambda_{BA} = \frac{-(144 \text{ in.})\mathbf{i} + (42 \text{ in.})\mathbf{j}}{\sqrt{(144)^2 + (42)^2} \text{ in.}} = \frac{1}{25}(-24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{r}_{K/A} = (72 \text{ in.})\mathbf{i} - (21 \text{ in.})\mathbf{j} + (9 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{I/A} = (96 \text{ in.})\mathbf{i} - (78 \text{ in.} - 50 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k} = (96 \text{ in.})\mathbf{i} - (28 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k}$$

$$\mathbf{W} = -(56 \text{ lb})\mathbf{j}$$

$$\mathbf{P}_{JI} = \lambda_{JI} P_{JI}$$

$$= \frac{-(1 \text{ in.})\mathbf{i} + (50 \text{ in.})\mathbf{j} - (10 \text{ in.})\mathbf{k}}{\sqrt{(1)^2 + (50)^2 + (10)^2} \text{ in.}} P_{JI}$$

$$= \frac{P_{JI}}{51}(-\mathbf{i} + 50\mathbf{j} - 10\mathbf{k})$$

PROBLEM 4.143 CONTINUED

From the f.b.d. of the chute

$$\Sigma M_{BA} = 0: \quad \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{K/A} \times \mathbf{W}\right) + \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{I/A} \times \mathbf{P}_{JI}\right) = 0$$

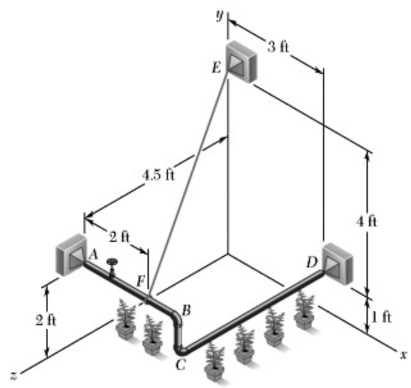
$$\begin{vmatrix} -24 & 7 & 0 \\ 72 & -21 & 9 \\ 0 & -1 & 0 \end{vmatrix} \left(\frac{56}{25}\right) + \begin{vmatrix} -24 & 7 & 0 \\ 96 & -28 & 18 \\ -1 & 50 & -10 \end{vmatrix} \left[\frac{P_{JI}}{51(25)}\right] = 0$$

$$\frac{-216(56)}{25} + \left[-24(-28)(-10) - (-24)(18)(50) + 7(18)(-1) - (7)(96)(-10)\right]\frac{P_{JI}}{51(25)} = 0$$

$$\therefore \quad P_{JI} = 28.728 \text{ lb}$$

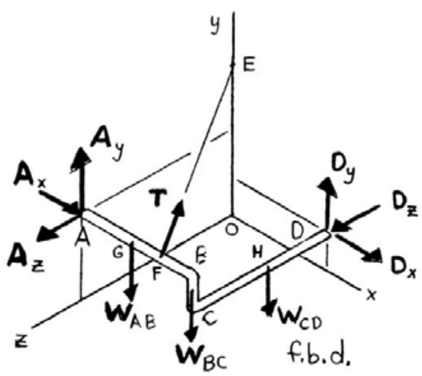
or $P_{JI} = 28.7 \text{ lb}$ ◀

PROBLEM 4.144



To water seedlings, a gardener joins three lengths of pipe, *AB*, *BC*, and *CD*, fitted with spray nozzles and suspends the assembly using hinged supports at *A* and *D* and cable *EF*. Knowing that the pipe weighs 0.85 lb/ft, determine the tension in the cable.

SOLUTION



First note $\mathbf{r}_{G/A} = (1.5 \text{ ft})\mathbf{i}$

$$\mathbf{W}_{AB} = -(0.85 \text{ lb/ft})(3 \text{ ft})\mathbf{j} = -(2.55 \text{ lb})\mathbf{j}$$

$$\mathbf{r}_{F/A} = (2 \text{ ft})\mathbf{i}$$

$$\mathbf{T} = \lambda_{FE}T = \frac{-(2 \text{ ft})\mathbf{i} + (3 \text{ ft})\mathbf{j} - (4.5 \text{ ft})\mathbf{k}}{\sqrt{(2)^2 + (3)^2 + (4.5)^2} \text{ ft}} T$$

$$= \left(\frac{T}{\sqrt{33.25}}\right)(-2\mathbf{i} + 3\mathbf{j} - 4.5\mathbf{k})$$

$$\mathbf{r}_{B/A} = (3 \text{ ft})\mathbf{i}$$

$$\mathbf{W}_{BC} = -(0.85 \text{ lb/ft})(1 \text{ ft})\mathbf{j} = -(0.85 \text{ lb})\mathbf{j}$$

$$\mathbf{r}_{H/A} = (3 \text{ ft})\mathbf{i} - (2.25 \text{ ft})\mathbf{k}$$

$$\mathbf{W}_{CD} = -(0.85 \text{ lb/ft})(4.5 \text{ ft})\mathbf{j} = -(3.825 \text{ lb})\mathbf{j}$$

$$\boldsymbol{\lambda}_{AD} = \frac{(3 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} - (4.5 \text{ ft})\mathbf{k}}{\sqrt{(3)^2 + (1)^2 + (4.5)^2} \text{ ft}} = \frac{1}{5.5}(3\mathbf{i} - \mathbf{j} - 4.5\mathbf{k})$$

PROBLEM 4.144 CONTINUED

From f.b.d. of the pipe assembly

$$\Sigma M_{AD} = 0: \quad \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{W}_{AB}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{F/A} \times \mathbf{T}\right)$$

$$+ \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{W}_{BC}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{H/A} \times \mathbf{W}_{CD}\right) = 0$$

$$\therefore \quad \begin{vmatrix} 3 & -1 & -4.5 \\ 1.5 & 0 & 0 \\ 0 & -2.55 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 2 & 0 & 0 \\ -2 & 3 & -4.5 \end{vmatrix} \left(\frac{T}{5.5\sqrt{33.25}}\right)$$

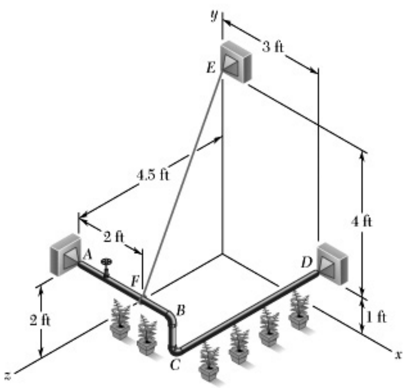
$$+ \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & 0 \\ 0 & -0.85 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & -2.25 \\ 0 & -3.825 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) = 0$$

$$(17.2125) + (-36)\left(\frac{T}{\sqrt{33.25}}\right) + (11.475) + (25.819) = 0$$

$$\therefore \quad T = 8.7306 \text{ lb}$$

or $T = 8.73$ lb ◀

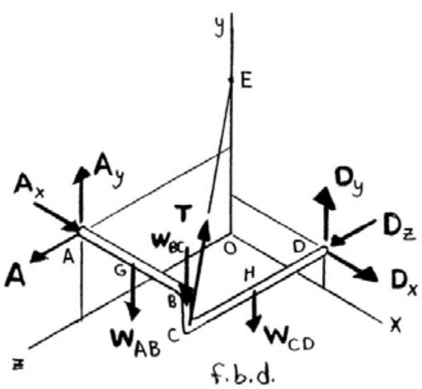
PROBLEM 4.145



Solve Problem 4.144 assuming that cable *EF* is replaced by a cable connecting *E* and *C*.

P4.144 To water seedlings, a gardener joins three lengths of pipe, *AB*, *BC*, and *CD*, fitted with spray nozzles and suspends the assembly using hinged supports at *A* and *D* and cable *EF*. Knowing that the pipe weighs 0.85 lb/ft, determine the tension in the cable.

SOLUTION



First note $\mathbf{r}_{G/A} = (1.5 \text{ ft})\mathbf{i}$

$$\mathbf{W}_{AB} = -(0.85 \text{ lb/ft})(3 \text{ ft})\mathbf{j} = -(2.55 \text{ lb})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (3 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j}$$

$$\mathbf{T} = \boldsymbol{\lambda}_{CE}T = \frac{-(3 \text{ ft})\mathbf{i} + (4 \text{ ft})\mathbf{j} - (4.5 \text{ ft})\mathbf{k}}{\sqrt{(3)^2 + (4)^2 + (4.5)^2} \text{ ft}}T$$

$$= \left(\frac{T}{\sqrt{45.25}}\right)(-3\mathbf{i} + 4\mathbf{j} - 4.5\mathbf{k})$$

$$\mathbf{r}_{B/A} = (3 \text{ ft})\mathbf{i}$$

$$\mathbf{W}_{BC} = -(0.85 \text{ lb/ft})(1 \text{ ft})\mathbf{j} = -(0.85 \text{ lb})\mathbf{j}$$

$$\mathbf{r}_{H/A} = (3 \text{ ft})\mathbf{i} - (2.25 \text{ ft})\mathbf{k}$$

$$\mathbf{W}_{CD} = -(0.85 \text{ lb/ft})(1 \text{ ft})\mathbf{j} = -(3.825 \text{ lb})\mathbf{j}$$

$$\boldsymbol{\lambda}_{AD} = \frac{(3 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} - (4.5 \text{ ft})\mathbf{k}}{\sqrt{(3)^2 + (1)^2 + (4.5)^2} \text{ ft}} = \frac{1}{5.5}(3\mathbf{i} - \mathbf{j} - 4.5\mathbf{k})$$

PROBLEM 4.145 CONTINUED

From f.b.d. of the pipe assembly

$$\Sigma M_{AD} = 0: \quad \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{W}_{AB}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{C/A} \times \mathbf{T}\right)$$

$$+ \ \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{W}_{BC}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{H/A} \times \mathbf{W}_{CD}\right) = 0$$

$$\therefore \begin{vmatrix} 3 & -1 & -4.5 \\ 1.5 & 0 & 0 \\ 0 & -2.55 & 0 \end{vmatrix}\left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & -1 & 0 \\ -3 & 4 & -4.5 \end{vmatrix}\left(\frac{T}{5.5\sqrt{45.25}}\right)$$

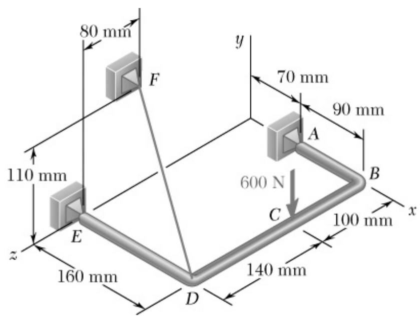
$$+\begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & 0 \\ 0 & -0.85 & 0 \end{vmatrix}\left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & -2.25 \\ 0 & -3.825 & 0 \end{vmatrix}\left(\frac{1}{5.5}\right) = 0$$

$$\left(17.2125\right) + \left(-40.5\right)\left(\frac{T}{\sqrt{45.25}}\right) + \left(11.475\right) + \left(25.819\right) = 0$$

$$\therefore \quad T = 9.0536 \text{ lb}$$

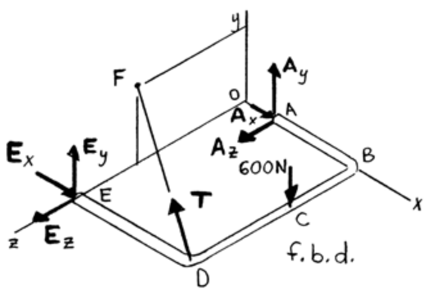
or $T = 9.05$ lb ◀

PROBLEM 4.146



The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 600-N load is applied at *C* as shown, determine the tension in the cable.

SOLUTION



First note

$$\lambda_{AE} = \frac{-(70 \text{ mm})\mathbf{i} + (240 \text{ mm})\mathbf{k}}{\sqrt{(70)^2 + (240)^2} \text{ mm}} = \frac{1}{25}(-7\mathbf{i} + 24\mathbf{k})$$

$$\mathbf{r}_{C/A} = (90 \text{ mm})\mathbf{i} + (100 \text{ mm})\mathbf{k}$$

$$\mathbf{F}_C = -(600 \text{ N})\mathbf{j}$$

$$\mathbf{r}_{D/A} = (90 \text{ mm})\mathbf{i} + (240 \text{ mm})\mathbf{k}$$

$$\mathbf{T} = \lambda_{DF}T = \frac{-(160 \text{ mm})\mathbf{i} + (110 \text{ mm})\mathbf{j} - (80 \text{ mm})\mathbf{k}}{\sqrt{(160)^2 + (110)^2 + (80)^2} \text{ mm}}T$$

$$= \frac{T}{21}(-16\,\mathbf{i} + 11\mathbf{j} - 8\mathbf{k})$$

From the f.b.d. of the bend rod

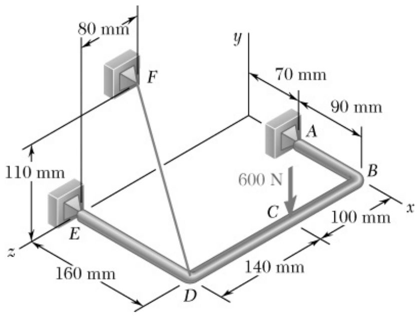
$$\Sigma M_{AE} = 0: \quad \lambda_{AE} \cdot (\mathbf{r}_{C/A} \times \mathbf{F}_C) + \lambda_{AE} \cdot (\mathbf{r}_{D/A} \times \mathbf{T}) = 0$$

$$\therefore \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 100 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{600}{25}\right) + \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 240 \\ -16 & 11 & -8 \end{vmatrix}\left[\frac{T}{25(21)}\right] = 0$$

$$(-700 - 2160)\left(\frac{600}{25}\right) + (18\,480 + 23\,760)\left[\frac{T}{25(21)}\right] = 0$$

$$\therefore \quad T = 853.13 \text{ N}$$

or $T = 853$ N ◀

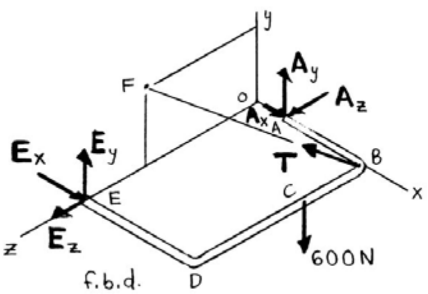


PROBLEM 4.147

Solve Problem 4.146 assuming that cable *DF* is replaced by a cable connecting *B* and *F*.

P4.146 The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 600-N load is applied at *C* as shown, determine the tension in the cable.

SOLUTION



First note

$$\lambda_{AE} = \frac{-(70 \text{ mm})\mathbf{i} + (240 \text{ mm})\mathbf{k}}{\sqrt{(70)^2 + (240)^2} \text{ mm}} = \frac{1}{25}(-7\mathbf{i} + 24\mathbf{k})$$

$$\mathbf{r}_{C/A} = (90 \text{ mm})\mathbf{i} + (100 \text{ mm})\mathbf{k}$$

$$\mathbf{F}_C = -(600 \text{ N})\mathbf{j}$$

$$\mathbf{r}_{B/A} = (90 \text{ mm})\mathbf{i}$$

$$\mathbf{T} = \lambda_{BF}T = \frac{-(160 \text{ mm})\mathbf{i} + (110 \text{ mm})\mathbf{j} + (160 \text{ mm})\mathbf{k}}{\sqrt{(160)^2 + (110)^2 + (160)^2} \text{ mm}}T$$

$$= \frac{1}{251.59}(-160\,\mathbf{i} + 110\mathbf{j} + 160\mathbf{k})$$

From the f.b.d. of the bend rod

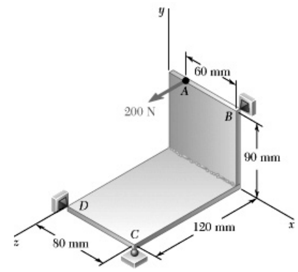
$$\Sigma M_{AE} = 0: \quad \lambda_{AE} \cdot (\mathbf{r}_{C/A} \times \mathbf{F}_C) + \lambda_{AE} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\therefore \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 100 \\ 0 & -1 & 0 \end{vmatrix} \left(\frac{600}{25}\right) + \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 0 \\ -160 & 110 & 160 \end{vmatrix} \left[\frac{T}{25(251.59)}\right] = 0$$

$$(-700 - 2160)\left(\frac{600}{25}\right) + (237\ 600)\left[\frac{T}{25(251.59)}\right] = 0$$

$$\therefore \quad T = 1817.04 \text{ N}$$

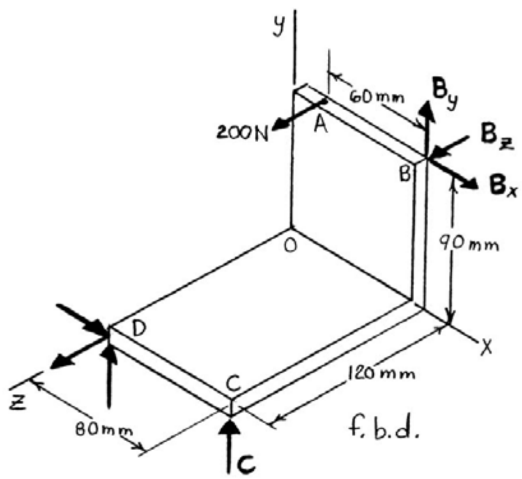
or $T = 1817$ N ◀



PROBLEM 4.148

Two rectangular plates are welded together to form the assembly shown. The assembly is supported by ball-and-socket joints at B and D and by a ball on a horizontal surface at C. For the loading shown, determine the reaction at C.

SOLUTION



First note

$$\lambda_{BD} = \frac{-(80 \text{ mm})\mathbf{i} - (90 \text{ mm})\mathbf{j} + (120 \text{ mm})\mathbf{k}}{\sqrt{(80)^2 + (90)^2 + (120)^2} \text{ mm}}$$

$$= \frac{1}{17}(-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k})$$

$$\mathbf{r}_{A/B} = -(60 \text{ mm})\mathbf{i}$$

$$\mathbf{P} = (200 \text{ N})\mathbf{k}$$

$$\mathbf{r}_{C/D} = (80 \text{ mm})\mathbf{i}$$

$$\mathbf{C} = (C)\mathbf{j}$$

From the f.b.d. of the plates

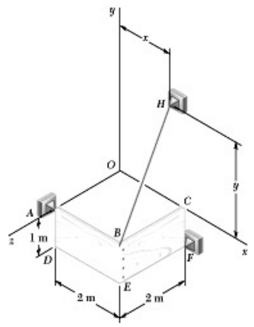
$$\Sigma M_{BD} = 0: \quad \lambda_{BD} \cdot (\mathbf{r}_{A/B} \times \mathbf{P}) + \lambda_{BD} \cdot (\mathbf{r}_{C/D} \times \mathbf{C}) = 0$$

$$\therefore \quad \begin{vmatrix} -8 & -9 & 12 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \left[\frac{60(200)}{17}\right] + \begin{vmatrix} -8 & -9 & 12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \left[\frac{C(80)}{17}\right] = 0$$

$$(-9)(60)(200) + (12)(80)C = 0$$

$$\therefore \quad C = 112.5 \text{ N}$$

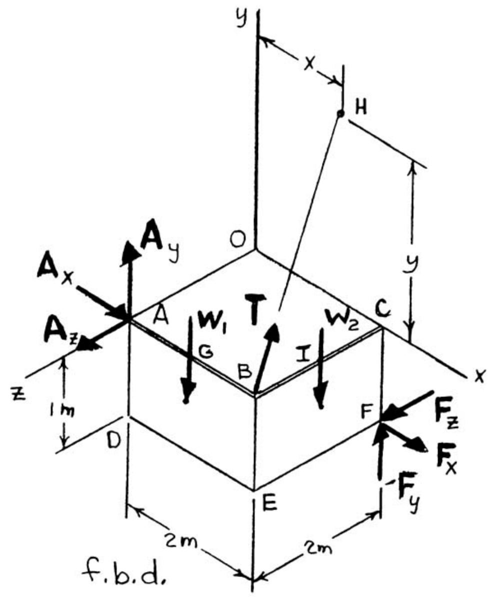
or $\mathbf{C} = (112.5 \text{ N})\mathbf{j}$ ◀



PROBLEM 4.149

Two 1×2-m plywood panels, each of mass 15 kg, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (*a*) the location of H in the xy plane if the tension in the wire is to be minimum, (*b*) the corresponding minimum tension.

SOLUTION



Let

$$\mathbf{W}_1 = \mathbf{W}_2 = -(mg)\mathbf{j} = -(15\text{ kg})\left(9.81\text{ m/s}^2\right)\mathbf{j}$$

$$= -(147.15\text{ N})\mathbf{j}$$

From the f.b.d. of the panels

$$\Sigma M_{AF} = 0: \quad \boldsymbol{\lambda}_{AF} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{W}_1\right) + \boldsymbol{\lambda}_{AF} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{T}\right) + \boldsymbol{\lambda}_{AF} \cdot \left(\mathbf{r}_{I/A} \times \mathbf{W}_2\right) = 0$$

where

$$\boldsymbol{\lambda}_{AF} = \frac{(2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{j} - (2\text{ m})\mathbf{k}}{\sqrt{(2)^2 + (1)^2 + (2)^2}\text{ m}} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G/A} = (1\text{ m})\mathbf{i}$$

$$\mathbf{r}_{B/A} = (2\text{ m})\mathbf{i}$$

$$\mathbf{r}_{I/A} = (2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{k}$$

PROBLEM 4.149 CONTINUED

$$\lambda_{BH} = \frac{(x-2)\mathbf{i} + (y)\mathbf{j} - (2)\mathbf{k}}{\sqrt{(x-2)^2 + y^2 + (2)^2}}$$

$$\mathbf{T} = \lambda_{BH}T = \frac{(x-2)\mathbf{i} + (y)\mathbf{j} - (2)\mathbf{k}}{\sqrt{(x-2)^2 + y^2 + (2)^2}}$$

$$\therefore \quad \begin{vmatrix} 2 & -1 & -2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & 0 \\ x-2 & y & -2 \end{vmatrix}\left(\frac{T}{3\sqrt{(x-2)^2 + y^2 + (2)^2}}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) = 0$$

$$\frac{2(147.15)}{3} + (-4 - 4y)\frac{T}{3\sqrt{(x-2)^2 + y^2 + (2)^2}} + (-2 + 4)\frac{147.15}{3} = 0$$

or

$$T = \frac{147.15}{1 + y}\sqrt{(x-2)^2 + y^2 + (2)^2}$$

For $x - 2$ m, $T = T_{\min}$

$$\therefore \quad T_{\min} = \frac{147.15}{(1 + y)}\left(y^2 + 4\right)^{\frac{1}{2}}$$

The y-value for $T_{\min}$ is found from

$$\left(\frac{dT}{dy}\right) = 0: \quad \frac{(1 + y)\frac{1}{2}\left(y^2 + 4\right)^{-\frac{1}{2}}(2y) - \left(y^2 + 4\right)^{\frac{1}{2}}(1)}{(1 + y)^2} = 0$$

Setting the numerator equal to zero,

$$(1 + y)y = y^2 + 4$$

$$y = 4 \text{ m}$$

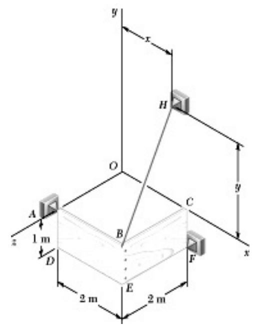
Then

$$T\min = \frac{147.15}{(1 + 4)}\sqrt{(2-2)^2 + (4)^2 + (2)^2} = 131.615 \text{ N}$$

$\therefore$ (*a*) $x = 2.00$ m, $y = 4.00$ m ◀

(*b*) $T_{\min} = 131.6$ N ◀

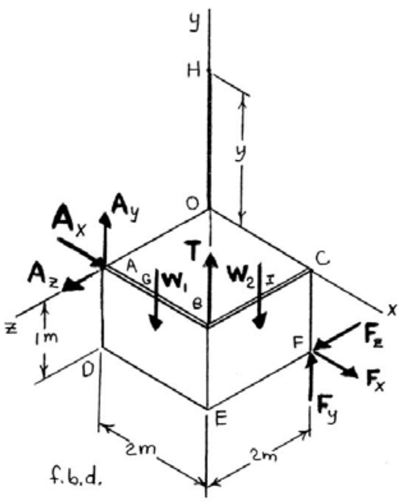
PROBLEM 4.150



Solve Problem 4.149 subject to the restriction that H must lie on the y axis.

P4.149 Two 1×2-m plywood panels, each of mass 15 kg, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (*a*) the location of H in the xy plane if the tension in the wire is to be minimum, (*b*) the corresponding minimum tension.

SOLUTION



Let $$\mathbf{W}_1 = \mathbf{W}_2 = -(mg)\mathbf{j} = -(15\text{ kg})(9.81\text{ m/s}^2)\mathbf{j} = -(147.15\text{ N})\mathbf{j}$$

From the f.b.d. of the panels

$$\Sigma M_{AF} = 0:\quad \lambda_{AF}\cdot(\mathbf{r}_{G/A}\times\mathbf{W}_1) + \lambda_{AF}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}) + \lambda_{AF}\cdot(\mathbf{r}_{I/A}\times\mathbf{W}_2) = 0$$

where $$\lambda_{AF} = \frac{(2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{j} - (2\text{ m})\mathbf{k}}{\sqrt{(2)^2 + (1)^2 + (2)^2}\text{ m}} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G/A} = (1\text{ m})\mathbf{i}$$

$$\mathbf{r}_{B/A} = (2\text{ m})\mathbf{i}$$

$$\mathbf{r}_{I/A} = (2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{k}$$

$$\mathbf{T} = \lambda_{BH}T = \frac{-(2\text{ m})\mathbf{i} + (y)\mathbf{j} - (2\text{ m})\mathbf{k}}{\sqrt{(2)^2 + (y)^2 + (2)^2}\text{ m}}T$$

$$= \frac{T}{\sqrt{8 + y^2}}(-2\mathbf{i} + y\mathbf{j} - 2\mathbf{k})$$

PROBLEM 4.150 CONTINUED

$$\therefore \quad \begin{vmatrix} 2 & -1 & -2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & 0 \\ -2 & y & -2 \end{vmatrix}\left(\frac{T}{3\sqrt{8 + y^2}}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) = 0$$

$$2(147.15) + (-4 - 4y)\left(T\sqrt{8 + y^2}\right) + (2)147.15 = 0$$

$$\therefore \quad T = \frac{(147.15)\sqrt{8 + y^2}}{(1 + y)}$$

For T_{min}, $\quad \left(\frac{dT}{dy}\right) = 0 \quad \therefore \quad \dfrac{(1 + y)\frac{1}{2}\left(8 + y^2\right)^{-\frac{1}{2}}(2y) - \left(8 + y^2\right)^{\frac{1}{2}}(1)}{(1 + y)^2} = 0$

Setting the numerator equal to zero,

$$(1 + y)y = 8 + y^2$$

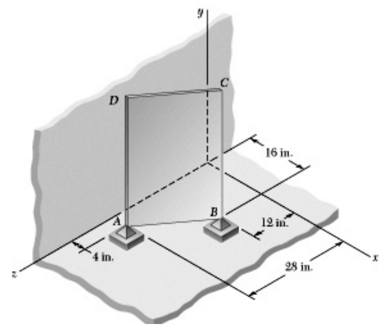
$$\therefore \quad y = 8.00 \text{ m}$$

and $\quad T_{min} = \dfrac{(147.15)\sqrt{8 + (8)^2}}{(1 + 8)} = 138.734 \text{ N}$

$\therefore$ (*a*) $\quad x = 0, \; y = 8.00 \text{ m}$ ◀

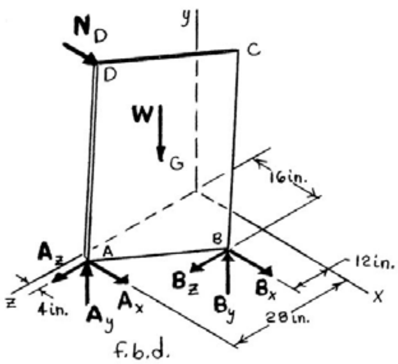
(*b*) $\quad T_{min} = 138.7 \text{ N}$ ◀

PROBLEM 4.151



A uniform 20 × 30-in. steel plate *ABCD* weighs 85 lb and is attached to ball-and-socket joints at *A* and *B*. Knowing that the plate leans against a frictionless vertical wall at *D*, determine (*a*) the location of *D*, (*b*) the reaction at *D*.

SOLUTION



(*a*) Since $\mathbf{r}_{D/A}$ is perpendicular to $\mathbf{r}_{B/A}$,

$$\mathbf{r}_{D/A} \cdot \mathbf{r}_{B/A} = 0$$

where coordinates of *D* are $(0, y, z)$, and

$$\mathbf{r}_{D/A} = -(4\text{ in.})\mathbf{i} + (y)\mathbf{j} + (z - 28\text{ in.})\mathbf{k}$$

$$\mathbf{r}_{B/A} = (12\text{ in.})\mathbf{i} - (16\text{ in.})\mathbf{k}$$

$$\therefore\ \mathbf{r}_{D/A} \cdot \mathbf{r}_{B/A} = -48 - 16z + 448 = 0$$

or $$z = 25\text{ in.}$$

Since $$L_{AD} = 30\text{ in.}$$

$$30 = \sqrt{(4)^2 + (y)^2 + (25 - 28)^2}$$

$$900 = 16 + y^2 + 9$$

or $$y = \sqrt{875}\text{ in.} = 29.580\text{ in.}$$

$\therefore$ Coordinates of *D*: $x = 0,\ y = 29.6\text{ in.},\ z = 25.0\text{ in.}$ ◀

(*b*) From f.b.d. of steel plate *ABCD*

$$\Sigma M_{AB} = 0:\quad \boldsymbol{\lambda}_{AB} \cdot (\mathbf{r}_{D/A} \times \mathbf{N}_D) + \boldsymbol{\lambda}_{AB} \cdot (\mathbf{r}_{G/B} \times \mathbf{W}) = 0$$

where $$\boldsymbol{\lambda}_{AB} = \frac{(12\text{ in.})\mathbf{i} - (16\text{ in.})\mathbf{k}}{\sqrt{(12)^2 + (16)^2}\text{ in.}} = \frac{1}{5}(3\mathbf{i} - 4\mathbf{k})$$

$$\mathbf{r}_{D/A} = -(4\text{ in.})\mathbf{i} + (29.580\text{ in.})\mathbf{j} - (3\text{ in.})\mathbf{k}$$

$$\mathbf{N}_D = N_D\mathbf{i}$$

PROBLEM 4.151 CONTINUED

$$\mathbf{r}_{G/B} = \frac{1}{2}\mathbf{r}_{D/B} = \frac{1}{2}\left[-(16\text{ in.})\mathbf{i} + (29.580\text{ in.})\mathbf{j} + (25\text{ in.} - 12\text{ in.})\mathbf{k}\right]$$

$$\mathbf{W} = -(85\text{ lb})\mathbf{j}$$

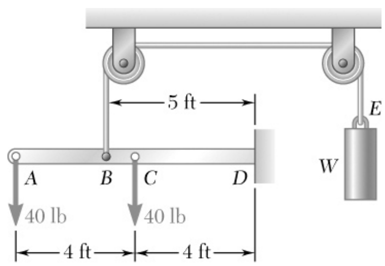
$$\therefore \begin{vmatrix} 3 & 0 & -4 \\ -4 & 29.580 & -3 \\ 1 & 0 & 0 \end{vmatrix}\left(\frac{N_D}{5}\right) + \begin{vmatrix} 3 & 0 & -4 \\ -16 & 29.580 & 13 \\ 0 & -1 & 0 \end{vmatrix}\left[\frac{85}{2(5)}\right] = 0$$

$$118.32N_D + (39 - 64)42.5 = 0$$

$$\therefore\ N_D = 8.9799\text{ lb}$$

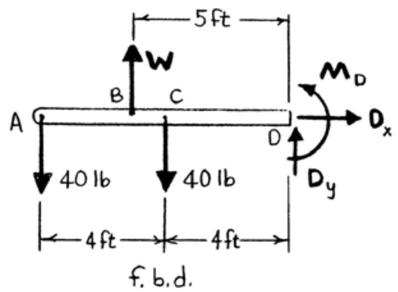
or $\mathbf{N}_D = (8.98\text{ lb})\mathbf{i}$ ◀

PROBLEM 4.152



Beam *AD* carries the two 40-lb loads shown. The beam is held by a fixed support at *D* and by the cable *BE* which is attached to the counter-weight *W*. Determine the reaction at *D* when (*a*) $W = 100$ lb, (*b*) $W = 90$ lb.

SOLUTION



(*a*) $W = 100 \text{ lb}$

From f.b.d. of beam *AD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x = 0$$

$$+\!\uparrow \Sigma F_y = 0: \quad D_y - 40 \text{ lb} - 40 \text{ lb} + 100 \text{ lb} = 0$$

$$\therefore D_y = -20.0 \text{ lb}$$

$$\text{or } \mathbf{D} = 20.0 \text{ lb} \downarrow \blacktriangleleft$$

$$+\circlearrowright \Sigma M_D = 0: \quad M_D - (100 \text{ lb})(5 \text{ ft}) + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) = 0$$

$$\therefore M_D = 20.0 \text{ lb}\cdot\text{ft}$$

$$\text{or } \mathbf{M}_D = 20.0 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

(*b*) $W = 90 \text{ lb}$

From f.b.d. of beam *AD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x = 0$$

$$+\!\uparrow \Sigma F_y = 0: \quad D_y + 90 \text{ lb} - 40 \text{ lb} - 40 \text{ lb} = 0$$

$$\therefore D_y = -10.00 \text{ lb}$$

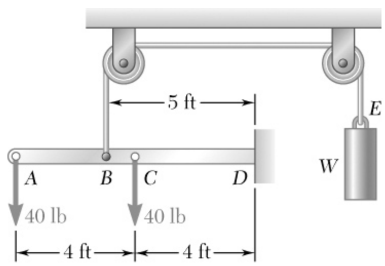
$$\text{or } \mathbf{D} = 10.00 \text{ lb} \downarrow \blacktriangleleft$$

$$+\circlearrowright \Sigma M_D = 0: \quad M_D - (90 \text{ lb})(5 \text{ ft}) + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) = 0$$

$$\therefore M_D = -30.0 \text{ lb}\cdot\text{ft}$$

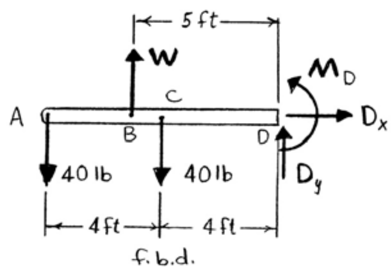
$$\text{or } \mathbf{M}_D = 30.0 \text{ lb}\cdot\text{ft} \circlearrowleft \blacktriangleleft$$

PROBLEM 4.153



For the beam and loading shown, determine the range of values of W for which the magnitude of the couple at D does not exceed $40\text{ lb}\cdot\text{ft}$.

SOLUTION



For $W_{\min}$, $\quad M_D = -40\text{ lb}\cdot\text{ft}$

From f.b.d. of beam AD

$$+\circlearrowleft\ \Sigma M_D = 0: \quad (40\text{ lb})(8\text{ ft}) - W_{\min}(5\text{ ft}) + (40\text{ lb})(4\text{ ft}) - 40\text{ lb}\cdot\text{ft} = 0$$

$$\therefore\ W_{\min} = 88.0\text{ lb}$$

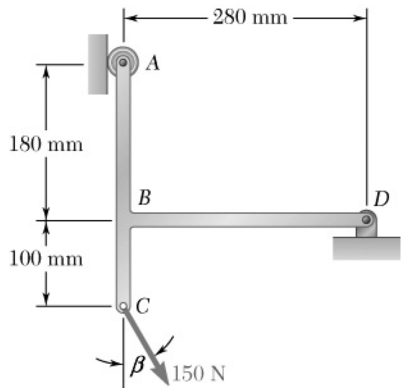
For $W_{\max}$, $\quad M_D = 40\text{ lb}\cdot\text{ft}$

From f.b.d. of beam AD

$$+\circlearrowleft\ \Sigma M_D = 0: \quad (40\text{ lb})(8\text{ ft}) - W_{\max}(5\text{ ft}) + (40\text{ lb})(4\text{ ft}) + 40\text{ lb}\cdot\text{ft} = 0$$

$$\therefore\ W_{\max} = 104.0\text{ lb}$$

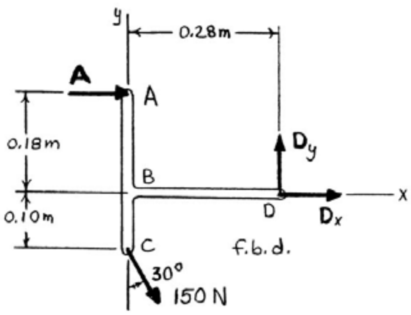
or $88.0\text{ lb} \le W \le 104.0\text{ lb}$ ◀



PROBLEM 4.154

Determine the reactions at A and D when $\beta = 30°$.

SOLUTION



From f.b.d. of frame $ABCD$

$$+\circlearrowleft \Sigma M_D = 0: \quad -A(0.18\text{ m}) + \left[(150\text{ N})\sin 30°\right](0.10\text{ m})$$

$$+\left[(150\text{ N})\cos 30°\right](0.28\text{ m}) = 0$$

$$\therefore\ A = 243.74\text{ N}$$

or $\mathbf{A} = 244\text{ N} \longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad (243.74\text{ N}) + (150\text{ N})\sin 30° + D_x = 0$$

$$\therefore\ D_x = -318.74\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - (150\text{ N})\cos 30° = 0$$

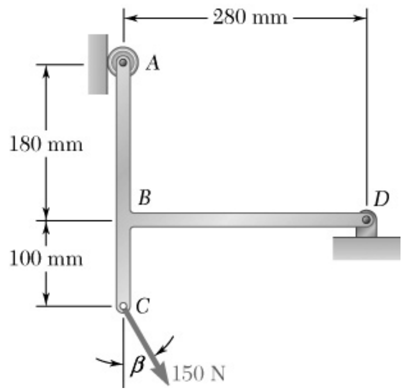
$$\therefore\ D_y = 129.904\text{ N}$$

Then $\quad D = \sqrt{(D_x)^2 + D_x^2} = \sqrt{(318.74)^2 + (129.904)^2} = 344.19\text{ N}$

and $\quad \theta = \tan^{-1}\left(\dfrac{D_y}{D_x}\right) = \tan^{-1}\left(\dfrac{129.904}{-318.74}\right) = -22.174°$

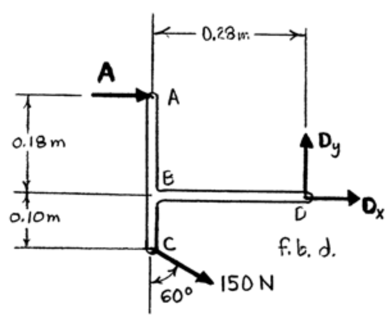
or $\mathbf{D} = 344\text{ N}$ ⦥ $22.2°$ ◀

PROBLEM 4.155



Determine the reactions at A and D when $\beta = 60°$.

SOLUTION



From f.b.d. of frame $ABCD$

$$+\circlearrowleft \Sigma M_D = 0: \quad -A(0.18\text{ m}) + \left[(150\text{ N})\sin 60°\right](0.10\text{ m})$$

$$+\left[(150\text{ N})\cos 60°\right](0.28\text{ m}) = 0$$

$$\therefore \; A = 188.835\text{ N}$$

or $\mathbf{A} = 188.8\text{ N} \longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad (188.835\text{ N}) + (150\text{ N})\sin 60° + D_x = 0$$

$$\therefore \; D_x = -318.74\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - (150\text{ N})\cos 60° = 0$$

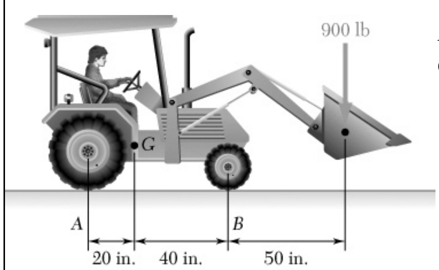
$$\therefore \; D_y = 75.0\text{ N}$$

Then $$D = \sqrt{(D_x)^2 + (D_y)^2} = \sqrt{(318.74)^2 + (75.0)^2} = 327.44\text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}\left(\frac{75.0}{-318.74}\right) = -13.2409°$$

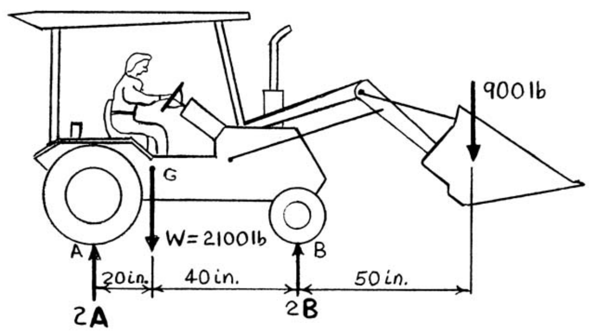
or $\mathbf{D} = 327\text{ N} \measuredangle 13.24°$ ◀

PROBLEM 4.156



A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION



(*a*) From f.b.d. of tractor

$$+\circlearrowleft \Sigma M_B = 0: \quad (2100 \text{ lb})(40 \text{ in.}) - (2A)(60 \text{ in.}) - (900 \text{ lb})(50 \text{ in.}) = 0$$

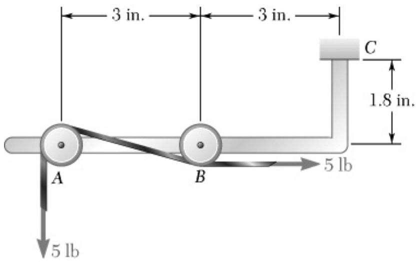
$\therefore\ A = 325 \text{ lb}$ or $\mathbf{A} = 325 \text{ lb} \uparrow$ ◀

(*b*) From f.b.d. of tractor

$$+\circlearrowleft \Sigma M_A = 0: \quad (2B)(60 \text{ in.}) - (2100 \text{ lb})(20 \text{ in.}) - (900 \text{ lb})(110 \text{ in.}) = 0$$

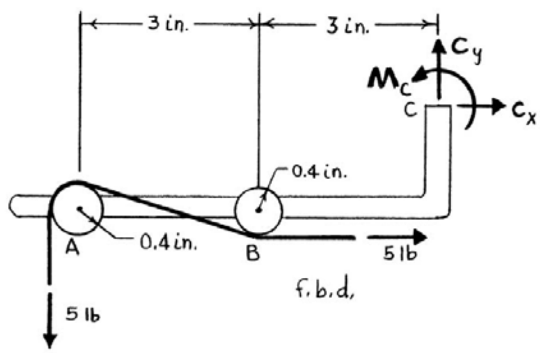
$\therefore\ B = 1175 \text{ lb}$ or $\mathbf{B} = 1175 \text{ lb} \uparrow$ ◀

PROBLEM 4.157



A tension of 5 lb is maintained in a tape as it passes the support system shown. Knowing that the radius of each pulley is 0.4 in., determine the reaction at *C*.

SOLUTION



From f.b.d. of system

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (5 \text{ lb}) = 0$$

$$\therefore \; C_x = -5 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - (5 \text{ lb}) = 0$$

$$\therefore \; C_y = 5 \text{ lb}$$

Then $$C = \sqrt{(C_x)^2 + (C_y)^2} = \sqrt{(5)^2 + (5)^2} = 7.0711 \text{ lb}$$

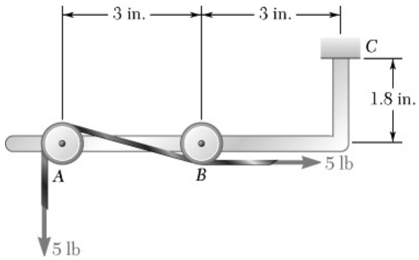
and $$\theta = \tan^{-1}\left(\frac{+5}{-5}\right) = -45°$$

or $\mathbf{C} = 7.07$ lb ⦪ 45.0° ◀

$$+\circlearrowright \Sigma M_C = 0: \quad M_C + (5 \text{ lb})(6.4 \text{ in.}) + (5 \text{ lb})(2.2 \text{ in.}) = 0$$

$$\therefore \; M_C = -43.0 \text{ lb}\cdot\text{in}$$

or $\mathbf{M}_C = 43.0 \text{ lb}\cdot\text{in.}$ ↻ ◀

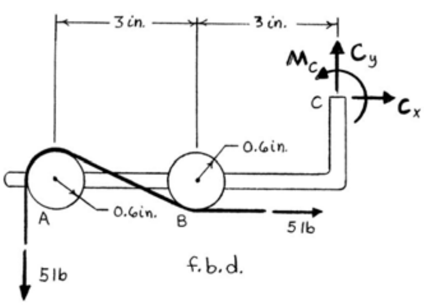


PROBLEM 4.158

Solve Problem 4.157 assuming that 0.6-in.-radius pulleys are used.

P4.157 A tension of 5 lb is maintained in a tape as it passes the support system shown. Knowing that the radius of each pulley is 0.4 in., determine the reaction at *C*.

SOLUTION



From f.b.d of system

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (5\text{ lb}) = 0$$

$$\therefore \; C_x = -5\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - (5\text{ lb}) = 0$$

$$\therefore \; C_y = 5\text{ lb}$$

Then $$C = \sqrt{(C_x)^2 + (C_y)^2} = \sqrt{(5)^2 + (5)^2} = 7.0711\text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{5}{-5}\right) = -45.0°$$

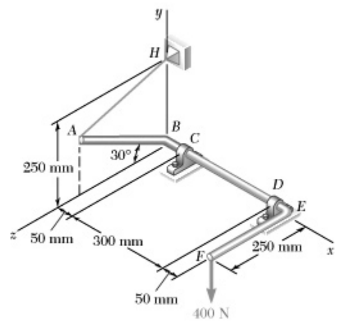
or $\mathbf{C} = 7.07\text{ lb} \; \measuredangle \; 45.0°$ ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (5\text{ lb})(6.6\text{ in.}) + (5\text{ lb})(2.4\text{ in.}) = 0$$

$$\therefore \; M_C = -45.0\text{ lb}\cdot\text{in.}$$

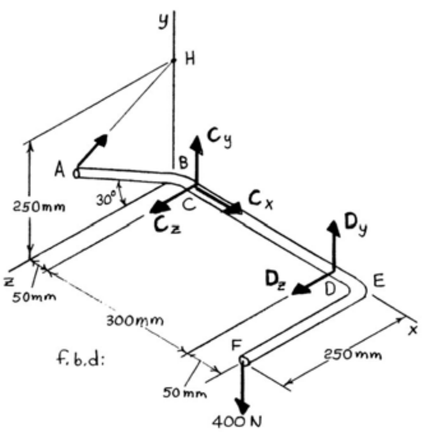
or $\mathbf{M}_C = 45.0\text{ lb}\cdot\text{in.} \circlearrowright$ ◀

PROBLEM 4.159



The bent rod *ABEF* is supported by bearings at *C* and *D* and by wire *AH*. Knowing that portion *AB* of the rod is 250 mm long, determine (*a*) the tension in wire *AH*, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION



(*a*) From f.b.d. of bent rod

$$\Sigma M_{CD} = 0: \quad \boldsymbol{\lambda}_{CD} \cdot \left(\mathbf{r}_{H/B} \times \mathbf{T}\right) + \boldsymbol{\lambda}_{CD} \cdot \left(\mathbf{r}_{F/E} \times \mathbf{F}\right) = 0$$

where $\quad \boldsymbol{\lambda}_{CD} = \mathbf{i}$

$$\mathbf{r}_{H/B} = (0.25\text{ m})\mathbf{j}$$

$$\mathbf{T} = \boldsymbol{\lambda}_{AH} T$$

$$= \frac{(y_{AH})\mathbf{j} - (z_{AH})\mathbf{k}}{\sqrt{(y_{AH})^2 + (z_{AH})^2}} T$$

$$y_{AH} = (0.25\text{ m}) - (0.25\text{ m})\sin 30°$$

$$= 0.125\text{ m}$$

$$z_{AH} = (0.25\text{ m})\cos 30°$$

$$= 0.21651\text{ m}$$

$$\therefore \mathbf{T} = \frac{T}{0.25}(0.125\mathbf{j} - 0.21651\mathbf{k})$$

$$\mathbf{r}_{F/E} = (0.25\text{ m})\mathbf{k}$$

$$\mathbf{F} = -400\text{ N }\mathbf{j}$$

$$\therefore \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.125 & -0.21651 \end{vmatrix} (0.25)\left(\frac{T}{0.25}\right) + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix} (0.25)(400\text{ N}) = 0$$

$$-0.21651T + 0.25(400\text{ N}) = 0$$

$$\therefore T = 461.88\text{ N}$$

or $T = 462$ N ◀

PROBLEM 4.159 CONTINUED

(b) From f.b.d. of bent rod

$$\Sigma F_x = 0: \quad C_x = 0$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad -\left[(461.88\text{ N})\sin 30^\circ\right](0.35\text{ m}) - C_y(0.3\text{ m})$$

$$-(400\text{ N})(0.05\text{ m}) = 0$$

$$\therefore\ C_y = -336.10\text{ N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(0.3\text{ m}) - \left[(461.88\text{ N})\cos 30^\circ\right](0.35\text{ m}) = 0$$

$$\therefore\ C_z = 466.67\text{ N}$$

or $\mathbf{C} = -(336\text{ N})\mathbf{j} + (467\text{ N})\mathbf{k}$ ◀

$$\Sigma F_y = 0: \quad D_y - 336.10\text{ N} + (461.88\text{ N})\sin 30^\circ - 400\text{ N} = 0$$

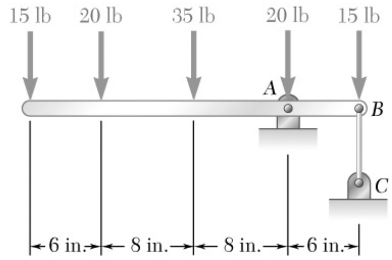
$$\therefore\ D_y = 505.16\text{ N}$$

$$\Sigma F_z = 0: \quad D_z + 466.67\text{ N} - (461.88\text{ N})\cos 30^\circ = 0$$

$$\therefore\ D_z = -66.670\text{ N}$$

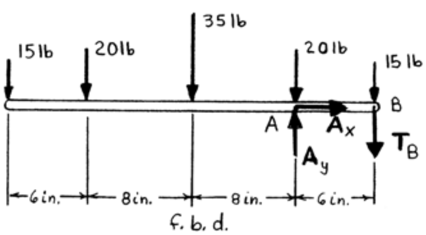
or $\mathbf{D} = (505\text{ N})\mathbf{j} - (66.7\text{ N})\mathbf{k}$ ◀

PROBLEM 4.160



For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the tension in cable *BC*.

SOLUTION



(*a*) From f.b.d of beam

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowright \Sigma M_B = 0: \quad (15 \text{ lb})(28 \text{ in.}) + (20 \text{ lb})(22 \text{ in.}) + (35 \text{ lb})(14 \text{ in.})$$

$$+ (20 \text{ lb})(6 \text{ in.}) - A_y(6 \text{ in.}) = 0$$

$$\therefore \; A_y = 245 \text{ lb}$$

or $\mathbf{A} = 245 \text{ lb} \uparrow$ ◀

(*b*) From f.b.d of beam

$$+\circlearrowright \Sigma M_A = 0: \quad (15 \text{ lb})(22 \text{ in.}) + (20 \text{ lb})(16 \text{ in.}) + (35 \text{ lb})(8 \text{ in.})$$

$$- (15 \text{ lb})(6 \text{ in.}) - T_B(6 \text{ in.}) = 0$$

$$\therefore \; T_B = 140.0 \text{ lb}$$

or $T_B = 140.0 \text{ lb}$ ◀

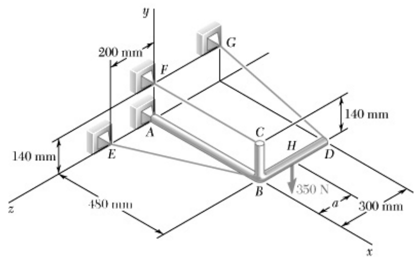
Check:

$$+\uparrow \Sigma F_y = 0: \quad -15 \text{ lb} - 20 \text{ lb} - 35 \text{ lb} - 20 \text{ lb}$$

$$- 15 \text{ lb} - 140 \text{ lb} + 245 \text{ lb} = 0?$$

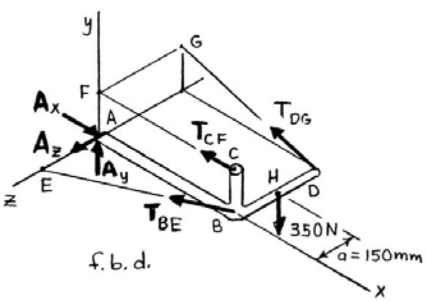
$$245 \text{ lb} - 245 \text{ lb} = 0 \;\; \text{ok}$$

PROBLEM 4.161



Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. For a = 150 mm, determine the tension in each cable and the reaction at *A*.

SOLUTION



First note $\quad \mathbf{T}_{DG} = \boldsymbol{\lambda}_{DG}T_{DG} = \dfrac{-(0.48\text{ m})\mathbf{i} + (0.14\text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2}\text{ m}}T_{DG}$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50}T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{T}_{BE} = \boldsymbol{\lambda}_{BE}T_{BE} = \frac{-(0.48\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2}\text{ m}}T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52}T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{j} + 5\mathbf{k})$$

From f.b.d. of frame *ABCD*

$$\Sigma M_x = 0: \quad \left(\frac{7}{25}T_{DG}\right)(0.3\text{ m}) - (350\text{ N})(0.15\text{ m}) = 0$$

or $\quad T_{DG} = 625\text{ N}$ ◀

$$\Sigma M_y = 0: \quad \left(\frac{24}{25} \times 625\text{ N}\right)(0.3\text{ m}) - \left(\frac{5}{13}T_{BE}\right)(0.48\text{ m}) = 0$$

or $\quad T_{BE} = 975\text{ N}$ ◀

$$\Sigma M_z = 0: \quad T_{CF}(0.14\text{ m}) + \left(\frac{7}{25} \times 625\text{ N}\right)(0.48\text{ m})$$

$$-(350\text{ N})(0.48\text{ m}) = 0$$

or $\quad T_{CF} = 600\text{ N}$ ◀

PROBLEM 4.161 CONTINUED

$$\Sigma F_x = 0: \quad A_x + T_{CF} + (T_{BE})_x + (T_{DG})_x = 0$$

$$A_x - 600 \text{ N} - \left(\frac{12}{13} \times 975 \text{ N}\right) - \left(\frac{24}{25} \times 625 \text{ N}\right) = 0$$

$$\therefore \; A_x = 2100 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + (T_{DG})_y - 350 \text{ N} = 0$$

$$A_y + \left(\frac{7}{25} \times 625 \text{ N}\right) - 350 \text{ N} = 0$$

$$\therefore \; A_y = 175.0 \text{ N}$$

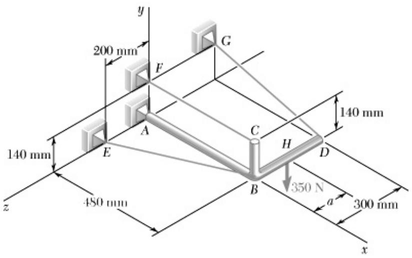
$$\Sigma F_z = 0: \quad A_z + (T_{BE})_z = 0$$

$$A_z + \left(\frac{5}{13} \times 975 \text{ N}\right) = 0$$

$$\therefore \; A_z = -375 \text{ N}$$

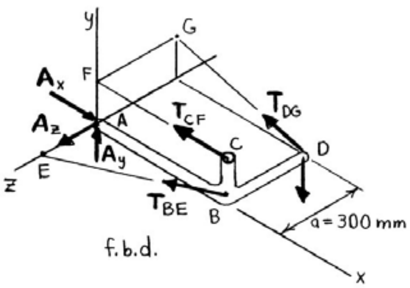
Therefore $\mathbf{A} = (2100 \text{ N})\mathbf{i} + (175.0 \text{ N})\mathbf{j} - (375 \text{ N})\mathbf{k}$ ◀

PROBLEM 4.162



Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. Knowing that the 350-N load is applied at *D* (a = 300 mm), determine the tension in each cable and the reaction at *A*.

SOLUTION



First note

$$\mathbf{T}_{DG} = \lambda_{DG}T_{DG} = \frac{-(0.48\text{ m})\mathbf{i} + (0.14\text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2}\text{ m}}T_{DG}$$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50}T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{T}_{BE} = \lambda_{BE}T_{BE} = \frac{-(0.48\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2}\text{ m}}T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52}T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{i} + 5\mathbf{k})$$

From f.b.d of frame *ABCD*

$$\Sigma M_x = 0: \quad \left(\frac{7}{25}T_{DG}\right)(0.3\text{ m}) - (350\text{ N})(0.3\text{ m}) = 0$$

or $T_{DG} = 1250\text{ N}$ ◀

$$\Sigma M_y = 0: \quad \left(\frac{24}{25} \times 1250\text{ N}\right)(0.3\text{ m}) - \left(\frac{5}{13}T_{BE}\right)(0.48\text{ m}) = 0$$

or $T_{BE} = 1950\text{ N}$ ◀

$$\Sigma M_z = 0: \quad T_{CF}(0.14\text{ m}) + \left(\frac{7}{25} \times 1250\text{ N}\right)(0.48\text{ m})$$

$$-(350\text{ N})(0.48\text{ m}) = 0$$

or $T_{CF} = 0$ ◀

PROBLEM 4.162 CONTINUED

$$\Sigma F_x = 0: \quad A_x + T_{CF} + \left(T_{BE}\right)_x + \left(T_{DG}\right)_x = 0$$

$$A_x + 0 - \left(\frac{12}{13} \times 1950 \text{ N}\right) - \left(\frac{24}{25} \times 1250 \text{ N}\right) = 0$$

$$\therefore \ A_x = 3000 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + \left(T_{DG}\right)_y - 350 \text{ N} = 0$$

$$A_y + \left(\frac{7}{25} \times 1250 \text{ N}\right) - 350 \text{ N} = 0$$

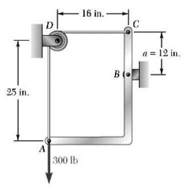
$$\therefore \ A_y = 0$$

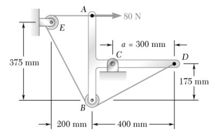
$$\Sigma F_z = 0: \quad A_z + \left(T_{BE}\right)_z = 0$$

$$A_z + \left(\frac{5}{13} \times 1950 \text{ N}\right) = 0$$

$$\therefore \ A_z = -750 \text{ N}$$

Therefore $\mathbf{A} = (3000 \text{ N})\mathbf{i} - (750 \text{ N})\mathbf{k}$ ◀



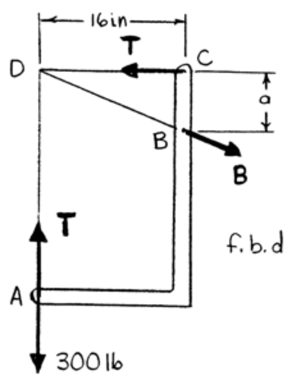


PROBLEM 4.163

In the problems listed below, the rigid bodies considered were completely constrained and the reactions were statically determinate. For each of these rigid bodies it is possible to create an improper set of constraints by changing a dimension of the body. In each of the following problems determine the value of a which results in improper constraints. (*a*) Problem 4.81, (*b*) Problem 4.82.

SOLUTION

(*a*)



(*a*) $+\circlearrowleft \Sigma M_B = 0: \quad (300\text{ lb})(16\text{ in.}) - T(16\text{ in.}) + T(a) = 0$

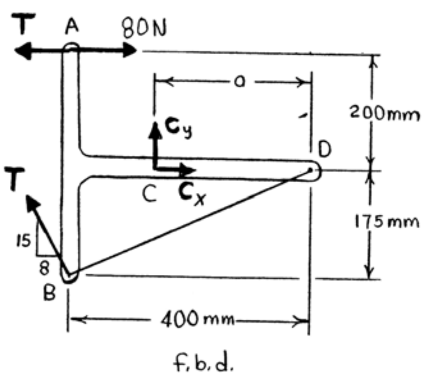
or $$T = \frac{(300\text{ lb})(16\text{ in.})}{(16 - a)\text{in.}}$$

$\therefore$ T becomes infinite when

$$16 - a = 0$$

or $a = 16.00$ in. ◀

(*b*)



(*b*) $+\circlearrowleft \Sigma M_C = 0: \quad (T - 80\text{ N})(0.2\text{ m}) - \left(\frac{8}{17}T\right)(0.175\text{ m})$

$$-\left(\frac{15}{17}T\right)(0.4\text{ m} - a) = 0$$

$$0.2T - 16.0 - 0.82353T - 0.35294T + 0.88235Ta = 0$$

or $$T = \frac{16.0}{0.88235a - 0.23529}$$

$\therefore$ T becomes infinite when

$$0.88235a - 0.23529 = 0$$

$$a = 0.26666\text{ m}$$

or $a = 267$ mm ◀