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Problem 12-1

A truck traveling along a straight road at speed v_1, increases its speed to v_2 in time t. If its acceleration is constant, determine the distance traveled.

Given:

$$v_1 = 20\ \frac{\text{km}}{\text{hr}} \qquad v_2 = 120\ \frac{\text{km}}{\text{hr}} \qquad t = 15\ \text{s}$$

Solution:

$$a = \frac{v_2 - v_1}{t} \qquad a = 1.852\ \frac{\text{m}}{\text{s}^2}$$

$$d = v_1 t + \frac{1}{2} a t^2 \qquad d = 291.67\ \text{m}$$

Problem 12-2

A car starts from rest and reaches a speed v after traveling a distance d along a straight road. Determine its constant acceleration and the time of travel.

Given: $v = 80\ \dfrac{\text{ft}}{\text{s}}$ $\quad d = 500\ \text{ft}$

Solution:

$$v^2 = 2ad \qquad a = \frac{v^2}{2d} \qquad a = 6.4\ \frac{\text{ft}}{\text{s}^2}$$

$$v = at \qquad t = \frac{v}{a} \qquad t = 12.5\ \text{s}$$

Problem 12-3

A baseball is thrown downward from a tower of height h with an initial speed v_0. Determine the speed at which it hits the ground and the time of travel.

Given:

$$h = 50\ \text{ft} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2} \qquad v_0 = 18\ \frac{\text{ft}}{\text{s}}$$

Solution:

$$v = \sqrt{v_0^2 + 2gh} \qquad v = 59.5\ \frac{\text{ft}}{\text{s}}$$

$$t = \frac{v - v_0}{g} \qquad t = 1.29 \text{ s}$$

***Problem 12–4**

Starting from rest, a particle moving in a straight line has an acceleration of $a = (bt + c)$. What is the particle's velocity at t_1 and what is its position at t_2?

Given: $b = 2\ \frac{\text{m}}{\text{s}^3}$ $\quad c = -6\ \frac{\text{m}}{\text{s}^2}$ $\quad t_1 = 6\ \text{s}$ $\quad t_2 = 11\ \text{s}$

Solution:

$$a(t) = b\,t + c \qquad v(t) = \int_0^t a(t)\ \mathrm{d}t \qquad d(t) = \int_0^t v(t)\ \mathrm{d}t$$

$$v(t_1) = 0\ \frac{\text{m}}{\text{s}} \qquad d(t_2) = 80.7\ \text{m}$$

Problem 12-5

Traveling with an initial speed v_0 a car accelerates at rate a along a straight road. How long will it take to reach a speed v_f? Also, through what distance does the car travel during this time?

Given: $v_0 = 70\ \frac{\text{km}}{\text{hr}}$ $\quad a = 6000\ \frac{\text{km}}{\text{hr}^2}$ $\quad v_f = 120\ \frac{\text{km}}{\text{hr}}$

Solution:

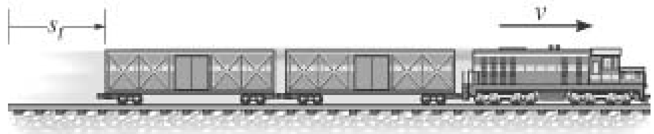
$$v_f = v_0 + a\,t \qquad t = \frac{v_f - v_0}{a} \qquad t = 30\ \text{s}$$

$$v_f^2 = v_0^2 + 2a\,s \qquad s = \frac{v_f^2 - v_0^2}{2a} \qquad s = 792\ \text{m}$$

Problem 12-6

A freight train travels at $v = v_0\left(1 - e^{-b\,t}\right)$ where t is the elapsed time. Determine the distance traveled in time t_1, and the acceleration at this time.

Given:



$$v_0 = 60\ \frac{\text{ft}}{\text{s}}$$

$$b = \frac{1}{\text{s}}$$

$$t_1 = 3\ \text{s}$$

Solution:

$$v(t) = v_0\left(1 - e^{-b\,t}\right) \qquad a(t) = \frac{d}{dt}v(t) \qquad d(t) = \int_0^t v(t)\ dt$$

$$d(t_1) = 123.0\ \text{ft} \qquad a(t_1) = 2.99\ \frac{\text{ft}}{\text{s}^2}$$

Problem 12-7

The position of a particle along a straight line is given by $s_p = at^3 + bt^2 + ct$. Determine its maximum acceleration and maximum velocity during the time interval $t_0 \le t \le t_f$.

Given: $a = 1\ \frac{\text{ft}}{\text{s}^3}$ $b = -9\ \frac{\text{ft}}{\text{s}^2}$ $c = 15\ \frac{\text{ft}}{\text{s}}$ $t_0 = 0\ \text{s}$ $t_f = 10\ \text{s}$

Solution:

$$s_p = a\,t^3 + b\,t^2 + c\,t$$

$$v_p = \frac{d}{dt}s_p = 3a\,t^2 + 2b\,t + c$$

$$a_p = \frac{d}{dt}v_p = \frac{d^2}{dt^2}s_p = 6a\,t + 2b$$

Since the acceleration is linear in time then the maximum will occur at the start or at the end. We check both possibilities.

$$a_{max} = \max\left(6a\,t_0 + b\,,6a\,t_f + 2b\right) \qquad a_{max} = 42\ \frac{\text{ft}}{\text{s}^2}$$

The maximum velocity can occur at the beginning, at the end, or where the acceleration is zero. We will check all three locations.

$$t_{cr} = \frac{-b}{3a} \qquad t_{cr} = 3\ \text{s}$$

$$v_{max} = \max\left(3a\,t_0^{\,2} + 2b\,t_0 + c\,,3a\,t_f^{\,2} + 2b\,t_f + c\,,3a\,t_{cr}^{\,2} + 2b\,t_{cr} + c\right) \qquad v_{max} = 135\,\frac{\text{ft}}{\text{s}}$$

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*__Problem 12-8__

From approximately what floor of a building must a car be dropped from an at-rest position so that it reaches a speed v_f when it hits the ground? Each floor is a distance h higher than the one below it. (Note: You may want to remember this when traveling at speed v_f)

Given: $v_f = 55\ \text{mph}$ $\quad h = 12\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$a_c = \text{g} \qquad v_f^{\,2} = 0 + 2a_c\,s \qquad H = \frac{v_f^{\,2}}{2a_c} \qquad H = 101.124\,\text{ft}$$

Number of floors $\quad N$

Height of one floor $\quad h = 12\,\text{ft}$

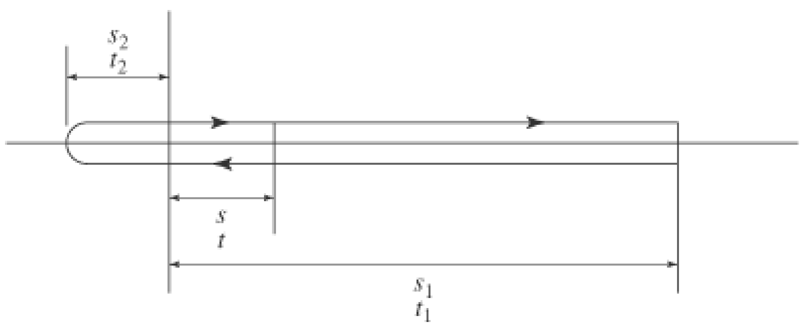
$$N = \frac{H}{h} \qquad N = 8.427 \qquad N = \text{ceil}(N)$$

The car must be dropped from floor number $N = 9$

Problem 12–9

A particle moves along a straight line such that its position is defined by $s_p = at^3 + bt^2 + c$. Determine the average velocity, the average speed, and the acceleration of the particle at time t_1.

Given: $a = 1\ \frac{\text{m}}{\text{s}^3}$ $\quad b = -3\ \frac{\text{m}}{\text{s}^2}$ $\quad c = 2\ \text{m}$ $\quad t_0 = 0\ \text{s}$ $\quad t_1 = 4\ \text{s}$



Solution:

$$s_p(t) = a\,t^3 + b\,t^2 + c \qquad v_p(t) = \frac{d}{dt}s_p(t) \qquad a_p(t) = \frac{d}{dt}v_p(t)$$

Find the critical velocity where $v_p = 0$.

$t_2 = 1.5\ \text{s}$ Given $v_p(t_2) = 0$ $t_2 = \text{Find}(t_2)$ $t_2 = 2\ \text{s}$

$$v_{ave} = \frac{s_p(t_1) - s_p(t_0)}{t_1} \qquad v_{ave} = 4\ \frac{\text{m}}{\text{s}}$$

$$v_{avespeed} = \frac{\left|s_p(t_2) - s_p(t_0)\right| + \left|s_p(t_1) - s_p(t_2)\right|}{t_1} \qquad v_{avespeed} = 6\ \frac{\text{m}}{\text{s}}$$

$$a_1 = a_p(t_1) \qquad a_1 = 18\ \frac{\text{m}}{\text{s}^2}$$

Problem 12–10

A particle is moving along a straight line such that its acceleration is defined as $a = -kv$. If $v = v_0$ when $d = 0$ and $t = 0$, determine the particle's velocity as a function of position and the distance the particle moves before it stops.

Given: $k = \frac{2}{\text{s}}$ $v_0 = 20\ \frac{\text{m}}{\text{s}}$

Solution: $a_p(v) = -kv$ $v\frac{\text{d}}{\text{d}s}v = -kv$ $\int_{v_0}^{v} 1\ \text{d}v = -k s_p$

Velocity as a function of position $v = v_0 - k s_p$

Distance it travels before it stops $0 = v_0 - k s_p$

$$s_p = \frac{v_0}{k} \qquad s_p = 10\ \text{m}$$

Problem 12-11

The acceleration of a particle as it moves along a straight line is given by $a = bt + c$. If $s = s_0$ and $v = v_0$ when $t = 0$, determine the particle's velocity and position when $t = t_1$. Also, determine the total distance the particle travels during this time period.

Given: $b = 2\ \frac{\text{m}}{\text{s}^3}$ $c = -1\ \frac{\text{m}}{\text{s}^2}$ $s_0 = 1\ \text{m}$ $v_0 = 2\ \frac{\text{m}}{\text{s}}$ $t_1 = 6\ \text{s}$

Solution:

$$\int_{v_0}^{v} 1\,dv = \int_0^t (b\,t + c)\,dt \qquad v = v_0 + \frac{b\,t^2}{2} + c\,t$$

$$\int_{s_0}^{s} 1\,ds = \int_0^t \left(v_0 + \frac{b\,t^2}{2} + c\,t\right) dt \qquad s = s_0 + v_0\,t + \frac{b}{6}t^3 + \frac{c}{2}t^2$$

When $t = t_1$ $\quad v_1 = v_0 + \frac{b\,t_1^{\,2}}{2} + c\,t_1 \qquad v_1 = 32\ \frac{\text{m}}{\text{s}}$

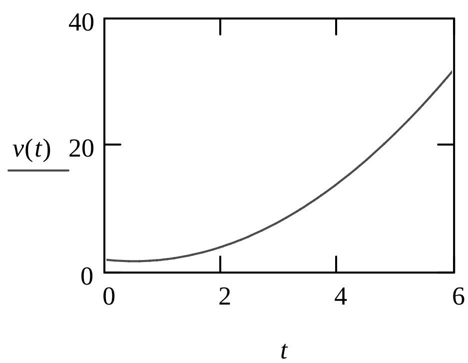
$$s_1 = s_0 + v_0\,t_1 + \frac{b}{6}t_1^{\,3} + \frac{c}{2}t_1^{\,2} \qquad s_1 = 67\ \text{m}$$

The total distance traveled depends on whether the particle turned around or not. To tell we will plot the velocity and see if it is zero at any point in the interval

$t = 0, 0.01t_1 .. t_1 \qquad v(t) = v_0 + \frac{b\,t^2}{2} + c\,t$

If v never goes to zero then

$d = s_1 - s_0 \qquad d = 66\ \text{m}$



***Problem 12–12**

A particle, initially at the origin, moves along a straight line through a fluid medium such that its velocity is defined as $v = b(1 - e^{-ct})$. Determine the displacement of the particle during the time $0 < t < t_1$.

Given: $\quad b = 1.8\ \frac{\text{m}}{\text{s}} \qquad c = \frac{0.3}{\text{s}} \qquad t_1 = 3\ \text{s}$

Solution:

$v(t) = b\left(1 - e^{-ct}\right)$ $\qquad s_p(t) = \int_0^t v(t)\, dt$ $\qquad s_p(t_1) = 1.839\ \text{m}$

Problem 12–13

The velocity of a particle traveling in a straight line is given $v = bt + ct^2$. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = t_1$. How far has the particle traveled during the time t_1, and what is its average speed?

Given: $b = 6\ \frac{\text{m}}{\text{s}^2}$ $\qquad c = -3\ \frac{\text{m}}{\text{s}^3}$ $\qquad t_0 = 0\ \text{s}$ $\qquad t_1 = 3\ \text{s}$

Solution: $v(t) = bt + ct^2$ $\qquad a(t) = \frac{d}{dt}v(t)$ $\qquad s_p(t) = \int_0^t v(t)\, dt$

Deceleration $\qquad a_1 = a(t_1)$ $\qquad a_1 = -12\ \frac{\text{m}}{\text{s}^2}$

Find the turning time t_2

$t_2 = 1.5\ \text{s}$ $\qquad$ Given $\qquad v(t_2) = 0$ $\qquad t_2 = \text{Find}(t_2)$ $\qquad t_2 = 2\ \text{s}$

Total distance traveled $\qquad d = \left|s_p(t_1) - s_p(t_2)\right| + \left|s_p(t_2) - s_p(t_0)\right|$ $\qquad d = 8\ \text{m}$

Average speed $\qquad v_{avespeed} = \frac{d}{t_1 - t_0}$ $\qquad v_{avespeed} = 2.667\ \frac{\text{m}}{\text{s}}$

Problem 12–14

A particle moves along a straight line such that its position is defined by $s = bt^2 + ct + d$. Determine the average velocity, the average speed, and the acceleration of the particle when $t = t_1$.

Given: $b = 1\ \frac{\text{m}}{\text{s}^2}$ $\qquad c = -6\ \frac{\text{m}}{\text{s}}$ $\qquad d = 5\ \text{m}$ $\qquad t_0 = 0\ \text{s}$ $\qquad t_1 = 6\ \text{s}$

Solution:

$s_p(t) = bt^2 + ct + d$ $\qquad v(t) = \frac{d}{dt}s_p(t)$ $\qquad a(t) = \frac{d}{dt}v(t)$

Find the critical time $\qquad t_2 = 2s$ $\qquad$ Given $\qquad v(t_2) = 0$ $\qquad t_2 = \text{Find}(t_2)$ $\qquad t_2 = 3\ \text{s}$

$v_{avevel} = \frac{s_p(t_1) - s_p(t_0)}{t_1}$ $\qquad v_{avevel} = 0\ \frac{\text{m}}{\text{s}}$

$$v_{avespeed} = \frac{\left|s_p(t_1) - s_p(t_2)\right| + \left|s_p(t_2) - s_p(t_0)\right|}{t_1} \qquad v_{avespeed} = 3\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$a_1 = a(t_1) \qquad a_1 = 2\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

Problem 12–15

A particle is moving along a straight line such that when it is at the origin it has a velocity v_0. If it begins to decelerate at the rate $a = bv^{1/2}$ determine the particle's position and velocity when $t = t_1$.

Given:

$$v_0 = 4\ \frac{\mathrm{m}}{\mathrm{s}} \qquad b = -1.5\ \sqrt{\frac{\mathrm{m}}{\mathrm{s}^3}} \qquad t_1 = 2\ \mathrm{s} \qquad a(v) = b\sqrt{v}$$

Solution:

$$a(v) = b\sqrt{v} = \frac{d}{dt}v \qquad \int_{v_0}^{v} \frac{1}{\sqrt{v}}\, dv = 2\left(\sqrt{v} - \sqrt{v_0}\right) = b\,t$$

$$v(t) = \left(\sqrt{v_0} + \frac{1}{2}b\,t\right)^2 \qquad v(t_1) = 0.25\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$s_p(t) = \int_0^t v(t)\, dt \qquad s_p(t_1) = 3.5\ \mathrm{m}$$

***Problem 12-16**

A particle travels to the right along a straight line with a velocity $v_p = a / (b + s_p)$. Determine its deceleration when $s_p = s_{p1}$.

Given: $a = 5\ \frac{\mathrm{m}^2}{\mathrm{s}}$ $\quad b = 4\ \mathrm{m}$ $\quad s_{p1} = 2\ \mathrm{m}$

Solution: $$v_p = \frac{a}{b + s_p} \qquad a_p = v_p \frac{dv_p}{ds_p} = \frac{a}{b + s_p}\ \frac{-a}{\left(b + s_p\right)^2} = \frac{-a^2}{\left(b + s_p\right)^3}$$

$$a_{p1} = \frac{-a^2}{\left(b + s_{p1}\right)^3} \qquad a_{p1} = -0.116\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

Problem 12–17

Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (at - b)$ and $a_B = (ct_2 - d)$, where t is in seconds. Determine the distance between them at t and the total distance each has traveled in time t.

Given:

$$a = 6\ \frac{\text{ft}}{\text{s}^3} \qquad b = 3\ \frac{\text{ft}}{\text{s}^2} \qquad c = 12\ \frac{\text{ft}}{\text{s}^3} \qquad d = 8\ \frac{\text{ft}}{\text{s}^2} \qquad t = 4\ \text{s}$$

Solution:

$$\frac{dv_A}{dt} = a\,t - b \qquad v_A = \left(\frac{a\,t^2}{2} - b\,t\right)$$

$$s_A = \left(\frac{a\,t^3}{6} - \frac{b\,t^2}{2}\right)$$

$$\frac{dv_B}{dt} = c\,t^2 - d \qquad v_B = \left(\frac{c\,t^3}{3\ \text{s}} - d\,t\right) \qquad s_B = \left(\frac{c\,t^4}{12\ \text{s}} - \frac{d\,t^2}{2}\right)$$

Distance between A and B

$$d_{AB} = \left|\frac{a\,t^3}{6} - \frac{b\,t^2}{2} - \frac{c\,t^4}{12\ \text{s}} + \frac{d\,t^2}{2}\right| \qquad d_{AB} = 46.33\ \text{m}$$

Total distance A and B has travelled.

$$D = \frac{a\,t^3}{6} - \frac{b\,t^2}{2} + \frac{c\,t^4}{12\ \text{s}} - \frac{d\,t^2}{2} \qquad D = 70.714\ \text{m}$$

Problem 12–18

A car is to be hoisted by elevator to the fourth floor of a parking garage, which is at a height h above the ground. If the elevator can accelerate at a_1, decelerate at a_2, and reach a maximum speed v, determine the shortest time to make the lift, starting from rest and ending at rest.

Given: $h = 48\ \text{ft}$ $\quad a_1 = 0.6\ \frac{\text{ft}}{\text{s}^2}$ $\quad a_2 = 0.3\ \frac{\text{ft}}{\text{s}^2}$ $\quad v = 8\ \frac{\text{ft}}{\text{s}}$

Solution: Assume that the elevator never reaches its maximum speed.

Guesses $t_1 = 1\ \text{s}$ $\quad t_2 = 2\ \text{s}$ $\quad v_{max} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad h_1 = 1\ \text{ft}$

Given $v_{max} = a_1\,t_1$

$$h_1 = \frac{1}{2}a_1 t_1^{\,2}$$

$$0 = v_{max} - a_2(t_2 - t_1)$$

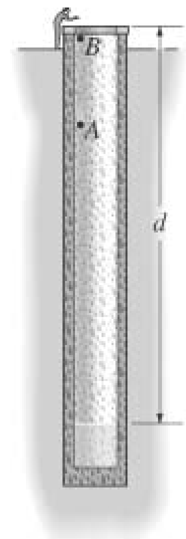
$$h = h_1 + v_{max}(t_2 - t_1) - \frac{1}{2}a_2(t_2 - t_1)^2$$

$$\begin{pmatrix} t_1 \\ t_2 \\ v_{max} \\ h_1 \end{pmatrix} = \text{Find}(t_1, t_2, v_{max}, h_1) \qquad t_2 = 21.909 \text{ s}$$

Since $v_{max} = 4.382 \frac{\text{ft}}{\text{s}} < v = 8 \frac{\text{ft}}{\text{s}}$ then our original assumption is correct.

Problem 12-19

A stone A is dropped from rest down a well, and at time t_1 another stone B is dropped from rest. Determine the distance between the stones at a later time t_2.



Given: $d = 80 \text{ ft}$ $\quad t_1 = 1 \text{ s}$ $\quad t_2 = 2 \text{ s}$ $\quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution:

$a_A = g \qquad v_A = g t \qquad s_A = \frac{g}{2}t^2$

$a_B = g \qquad v_B = g(t - t_1) \qquad s_B = \frac{g}{2}(t - t_1)^2$

At time t_2

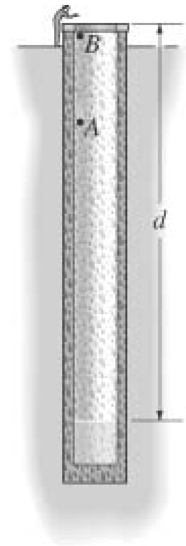
$s_{A2} = \frac{g}{2}t_2^{\,2} \qquad s_{A2} = 64.4 \text{ ft}$

$s_{B2} = \frac{g}{2}(t_2 - t_1)^2 \qquad s_{B2} = 16.1 \text{ ft}$

$d = s_{A2} - s_{B2} \qquad d = 48.3 \text{ ft}$

***Problem 12-20**

A stone A is dropped from rest down a well, and at time t_1 another stone B is dropped from rest. Determine the time interval between the instant A strikes the water and the instant B strikes the water. Also, at what speed do they strike the water?



Given: $d = 80 \text{ ft}$ $t_1 = 1 \text{ s}$ $g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution:

$a_A = g$ $v_A = g t$ $s_A = \frac{g}{2} t^2$

$a_B = g$ $v_B = g(t - t_1)$ $s_B = \frac{g}{2}(t - t_1)^2$

Time to hit for each particle

$t_A = \sqrt{\frac{2d}{g}}$ $t_A = 2.229 \text{ s}$

$t_B = \sqrt{\frac{2d}{g}} + t_1$ $t_B = 3.229 \text{ s}$

$\Delta t = t_B - t_A$ $\Delta t = 1 \text{ s}$

Speed

$v_A = g t_A$ $v_B = v_A$ $v_A = 71.777 \frac{\text{ft}}{\text{s}}$ $v_B = 71.777 \frac{\text{ft}}{\text{s}}$

Problem 12–21

A particle has an initial speed v_0. If it experiences a deceleration $a = bt$, determine the distance traveled before it stops.

Given: $v_0 = 27 \frac{\text{m}}{\text{s}}$ $b = -6 \frac{\text{m}}{\text{s}^3}$

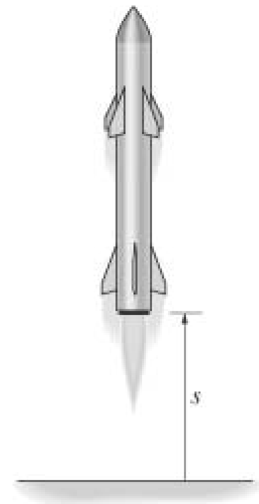
Solution:

$a(t) = b t$ $v(t) = b \frac{t^2}{2} + v_0$ $s_p(t) = b \frac{t^3}{6} + v_0 t$

$t = \sqrt{\frac{2v_0}{-b}}$ $t = 3 \text{ s}$ $s_p(t) = 54 \text{ m}$

Problem 12-22

The acceleration of a rocket traveling upward is given by $a_p = b + c\, s_p$. Determine the rocket's velocity when $s_p = s_{p1}$ and the time needed to reach this altitude. Initially, $v_p = 0$ and $s_p = 0$ when $t = 0$.



Given: $b = 6\ \frac{m}{s^2}$ $c = 0.02\ \frac{1}{s^2}$ $s_{p1} = 2000\ m$

Solution:

$$a_p = b + c\,s_p = v_p \frac{dv_p}{ds_p}$$

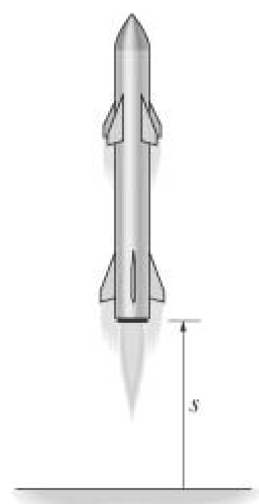
$$\int_0^{v_p} v_p\,dv_p = \int_0^{s_p} (b + c\,s_p)\,ds_p$$

$$\frac{v_p^2}{2} = b\,s_p + \frac{c}{2} s_p^2$$

$$v_p = \frac{ds_p}{dt} = \sqrt{2b\,s_p + c\,s_p^2} \qquad v_{p1} = \sqrt{2b\,s_{p1} + c\,s_{p1}^2} \qquad v_{p1} = 322.49\ \frac{m}{s}$$

$$t = \int_0^{s_p} \frac{1}{\sqrt{2b\,s_p + c\,s_p^2}}\,ds_p \qquad t_1 = \int_0^{s_{p1}} \frac{1}{\sqrt{2b\,s_p + c\,s_p^2}}\,ds_p \qquad t_1 = 19.274\ s$$

Problem 12-23



The acceleration of a rocket traveling upward is given by $a_p = b + c\ s_p$. Determine the time needed for the rocket to reach an altitute s_{p1}. Initially, $v_p = 0$ and $s_p = 0$ when $t = 0$.

Given: $b = 6\ \frac{m}{s^2}$ $c = 0.02\ \frac{1}{s^2}$ $s_{p1} = 100\ m$

Solution:

$$a_p = b + c\,s_p = v_p \frac{dv_p}{ds_p}$$

$$\int_0^{v_p} v_p\,dv_p = \int_0^{s_p} (b + c\,s_p)\,ds_p$$

$$\frac{v_p^2}{2} = b\,s_p + \frac{c}{2} s_p^2$$

$$v_p = \frac{ds_p}{dt} = \sqrt{2b\,s_p + c\,s_p^2} \qquad v_{p1} = \sqrt{2b\,s_{p1} + c\,s_{p1}^2} \qquad v_{p1} = 37.417\ \frac{m}{s}$$

$$t = \int_0^{s_p} \frac{1}{\sqrt{2b\,s_p + c\,s_p^{\,2}}}\,ds_p \qquad t_1 = \int_0^{s_{p1}} \frac{1}{\sqrt{2b\,s_p + c\,s_p^{\,2}}}\,ds_p \qquad t_1 = 5.624\ \text{s}$$

***Problem 12–24**

A particle is moving with velocity v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -k\,v^3$, where k is a constant, determine its velocity and position as functions of time.

Solution:

$$a = \frac{dv}{dt} = -kv^3 \qquad \int_{v_0}^{v} v^{-3}\,dv = \int_0^t -k\,dt \qquad \frac{-1}{2}\left(v^{-2} - v_0^{\,-2}\right) = -kt$$

$$v(t) = \frac{1}{\sqrt{2kt + \dfrac{1}{v_0^{\,2}}}}$$

$$ds = v\,dt \qquad \int_0^s 1\,ds = \int_0^t \frac{1}{\sqrt{2kt + \left(\dfrac{1}{v_0^{\,2}}\right)}}\,dt$$

$$s(t) = \frac{1}{k}\left[\sqrt{2kt + \left(\frac{1}{v_0^{\,2}}\right)} - \frac{1}{v_0}\right]$$

Problem 12–25

A particle has an initial speed v_0. If it experiences a deceleration $a = bt$, determine its velocity when it travels a distance s_1. How much time does this take?

Given: $v_0 = 27\ \dfrac{\text{m}}{\text{s}}$ $\qquad b = -6\ \dfrac{\text{m}}{\text{s}^3}$ $\qquad s_1 = 10\ \text{m}$

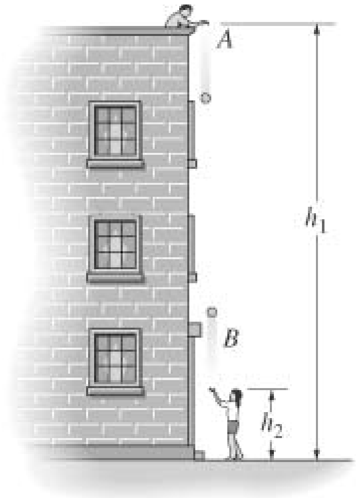
Solution:

$$a(t) = b\,t \qquad v(t) = b\frac{t^2}{2} + v_0 \qquad s_p(t) = b\frac{t^3}{6} + v_0\,t$$

Guess $t_1 = 1\ \text{s}$ Given $s_p(t_1) = s_1$ $\qquad t_1 = \text{Find}(t_1)$ $\qquad t_1 = 0.372\ \text{s}$

$$v(t_1) = 26.6\ \frac{\text{m}}{\text{s}}$$

Problem 12-26



Ball A is released from rest at height h_1 at the same time that a second ball B is thrown upward from a distance h_2 above the ground. If the balls pass one another at a height h_3 determine the speed at which ball B was thrown upward.

Given:

$h_1 = 40\ \text{ft}$

$h_2 = 5\ \text{ft}$

$h_3 = 20\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

For ball A:

$a_A = -g$

$v_A = -g\,t$

$s_A = \left(\dfrac{-g}{2}\right)t^2 + h_1$

For ball B:

$a_B = -g$

$v_B = -g\,t + v_{B0}$

$s_B = \left(\dfrac{-g}{2}\right)t^2 + v_{B0}\,t + h_2$

Guesses $t = 1\ \text{s}$ $v_{B0} = 2\ \dfrac{\text{ft}}{\text{s}}$

Given $h_3 = \left(\dfrac{-g}{2}\right)t^2 + h_1$ $h_3 = \left(\dfrac{-g}{2}\right)t^2 + v_{B0}\,t + h_2$

$\begin{pmatrix} t \\ v_{B0} \end{pmatrix} = \text{Find}(t, v_{B0})$ $t = 1.115\ \text{s}$ $v_{B0} = 31.403\ \dfrac{\text{ft}}{\text{s}}$

Problem 12–27

A car starts from rest and moves along a straight line with an acceleration $a = k\,s^{-1/3}$. Determine the car's velocity and position at $t = t_1$.

Given: $k = 3\left(\dfrac{\text{m}^4}{\text{s}^6}\right)^{\frac{1}{3}}$ $t_1 = 6\ \text{s}$

Solution:

$$a = v\frac{d}{ds_p}v = k s_p^{\frac{-1}{3}} \qquad \int_0^v v\,dv = \frac{v^2}{2} = \int_0^{s_p} k s_p^{\frac{-1}{3}}\,ds = \frac{3}{2}k s_p^{\frac{2}{3}}$$

$$v = \sqrt{3k}\,s_p^{\frac{1}{3}} = \frac{d}{dt}s_p \qquad \sqrt{3k}\,t = \int_0^{s_p} s_p^{\frac{-1}{3}}\,ds_p = \frac{3}{2}s_p^{\frac{2}{3}}$$

$$s_p(t) = \left(\frac{2\sqrt{3k}t}{3}\right)^{\frac{3}{2}} \qquad s_p(t_1) = 41.6 \text{ m}$$

$$v(t) = \frac{d}{dt}s_p(t) \qquad v(t_1) = 10.39 \frac{\text{m}}{\text{s}}$$

***Problem 12-28**

The acceleration of a particle along a straight line is defined by $a_p = b\,t + c$. At $t = 0$, $s_p = s_{p0}$ and $v_p = v_{p0}$. When $t = t_1$ determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

Given: $b = 2 \frac{\text{m}}{\text{s}^3}$ $\quad c = -9 \frac{\text{m}}{\text{s}^2}$ $\quad s_{p0} = 1 \text{ m}$ $\quad v_{p0} = 10 \frac{\text{m}}{\text{s}}$ $\quad t_1 = 9 \text{ s}$

Solution:

$$a_p = b\,t + c$$

$$v_p = \left(\frac{b}{2}\right)t^2 + c\,t + v_{p0}$$

$$s_p = \left(\frac{b}{6}\right)t^3 + \left(\frac{c}{2}\right)t^2 + v_{p0}t + s_{p0}$$

a) The position $\quad s_{p1} = \left(\frac{b}{6}\right)t_1^3 + \left(\frac{c}{2}\right)t_1^2 + v_{p0}t_1 + s_{p0} \qquad s_{p1} = -30.5 \text{ m}$

b) The total distance traveled - find the turning times $\quad v_p = \left(\frac{b}{2}\right)t^2 + c\,t + v_{p0} = 0$

$$t_2 = \frac{-c - \sqrt{c^2 - 2b\,v_{p0}}}{b} \qquad t_2 = 1.298 \text{ s}$$

$$t_3 = \frac{-c + \sqrt{c^2 - 2b\,v_{p0}}}{b} \qquad t_3 = 7.702\ \text{s}$$

$$s_{p2} = \left(\frac{b}{6}\right)t_2^{\,3} + \left(\frac{c}{2}\right)t_2^{\,2} + v_{p0}t_2 + s_{p0} \qquad s_{p2} = 7.127\ \text{m}$$

$$s_{p3} = \left(\frac{b}{6}\right)t_3^{\,3} + \frac{c}{2}t_3^{\,2} + v_{p0}t_3 + s_{p0} \qquad s_{p3} = -36.627\ \text{m}$$

$$d = \left|s_{p2} - s_{p0}\right| + \left|s_{p2} - s_{p3}\right| + \left|s_{p1} - s_{p3}\right| \qquad d = 56.009\ \text{m}$$

c) The velocity $v_{p1} = \left(\frac{b}{2}\right)t_1^{\,2} + c\,t_1 + v_{p0}$ $\qquad v_{p1} = 10\ \frac{\text{m}}{\text{s}}$

Problem 12–29

A particle is moving along a straight line such that its acceleration is defined as $a = k\,s^2$. If $v = v_0$ when $s = s_{p0}$ and $t = 0$, determine the particle's velocity as a function of position.

Given: $k = 4\ \frac{1}{\text{m}\,\text{s}^2}$ $\qquad v_0 = -100\ \frac{\text{m}}{\text{s}}$ $\qquad s_{p0} = 10\ \text{m}$

Solution:

$$a = v\frac{d}{ds_p}v = k\,s_p^{\,2} \qquad \int_{v_0}^{v} v\ dv = \int_{s_{p0}}^{s_p} k\,s_p^{\,2}\ ds_p$$

$$\frac{1}{2}\left(v^2 - v_0^{\,2}\right) = \frac{1}{3}k\left(s_p^{\,3} - s_{p0}^{\,3}\right) \qquad v = \sqrt{v_0^{\,2} + \frac{2}{3}k\left(s_p^{\,3} - s_{p0}^{\,3}\right)}$$

Problem 12–30

A car can have an acceleration and a deceleration a. If it starts from rest, and can have a maximum speed v, determine the shortest time it can travel a distance d at which point it stops.

Given: $a = 5\ \frac{\text{m}}{\text{s}^2}$ $\qquad v = 60\ \frac{\text{m}}{\text{s}}$ $\qquad d = 1200\ \text{m}$

Solution: Assume that it can reach maximum speed

Guesses $t_1 = 1\ \text{s}$ $\quad t_2 = 2\ \text{s}$ $\quad t_3 = 3\ \text{s}$ $\quad d_1 = 1\ \text{m}$ $\quad d_2 = 2\ \text{m}$

Given $a\,t_1 = v$ $\qquad \frac{1}{2}a\,t_1^{\,2} = d_1$ $\qquad d_2 = d_1 + v\left(t_2 - t_1\right)$

$$d = d_2 + v(t_3 - t_2) - \frac{1}{2}a(t_3 - t_2)^2 \qquad 0 = v - a(t_3 - t_2)$$

$$\begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ d_1 \\ d_2 \end{pmatrix} = \text{Find}(t_1, t_2, t_3, d_1, d_2) \qquad \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 20 \\ 32 \end{pmatrix} \text{s} \qquad \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 360 \\ 840 \end{pmatrix} \text{m}$$

$$t_3 = 32 \text{ s}$$

Problem 12-31

Determine the time required for a car to travel a distance d along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at a_1 and decelerate at a_2.

Given: $d = 1 \text{ km}$ $\quad a_1 = 1.5 \frac{\text{m}}{\text{s}^2}$ $\quad a_2 = 2 \frac{\text{m}}{\text{s}^2}$

Let t_1 be the time at which it stops accelerating and t the total time.

Solution: Guesses $t_1 = 1 \text{ s}$ $\quad d_1 = 1 \text{ m}$ $\quad t = 3 \text{ s}$ $\quad v_1 = 1 \frac{\text{m}}{\text{s}}$

Given $\quad d_1 = \frac{a_1}{2}t_1^2 \qquad v_1 = a_1 t_1 \qquad v_1 = a_2(t - t_1)$

$$d = d_1 + v_1(t - t_1) - \frac{1}{2}a_2(t - t_1)^2$$

$$\begin{pmatrix} t_1 \\ t \\ v_1 \\ d_1 \end{pmatrix} = \text{Find}(t_1, t, v_1, d_1) \qquad t_1 = 27.603 \text{ s} \quad v_1 = 41.404 \frac{\text{m}}{\text{s}} \quad d_1 = 571.429 \text{ m}$$

$$t = 48.305 \text{ s}$$

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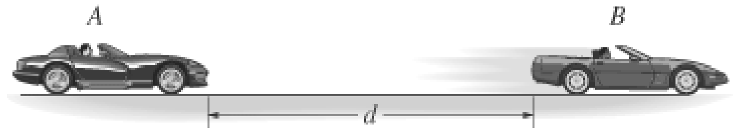
*__Problem 12-32__

When two cars A and B are next to one another, they are traveling in the same direction with speeds v_{A0} and v_{B0} respectively. If B maintains its constant speed, while A begins to decelerate at the rate a_A, determine the distance d between the cars at the instant A stops.



Solution:

Motion of car A:

$$-a_A = \text{constant} \qquad 0 = v_{A0} - a_A t \qquad s_A = v_{A0} t - \frac{1}{2} a_A t^2$$

$$t = \frac{v_{A0}}{a_A} \qquad s_A = \frac{v_{A0}^2}{2a_A}$$

Motion of car B:

$$a_B = 0 \qquad v_B = v_{B0} \qquad s_B = v_{B0} t \qquad s_B = \frac{v_{B0} v_{A0}}{a_A}$$

The distance between cars A and B is

$$d = \left| s_B - s_A \right| = \left| \frac{v_{B0} v_{A0}}{a_A} - \frac{v_{A0}^2}{2a_A} \right| = \left| \frac{2v_{B0} v_{A0} - v_{A0}^2}{2a_A} \right|$$

$$d = \left| \frac{2v_{B0} v_{A0} - v_{A0}^2}{2a_A} \right|$$

Problem 12-33

If the effects of atmospheric resistance are accounted for, a freely falling body has an acceleration defined by the equation $a = g\left(1 - c v^2\right)$, where the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity at time t_1 and (b) the body's terminal or maximum attainable velocity as $t \rightarrow \infty$.

Given: $\quad t_1 = 5 \text{ s} \qquad g = 9.81 \dfrac{\text{m}}{\text{s}^2} \qquad c = 10^{-4} \dfrac{\text{s}^2}{\text{m}^2}$

Solution:

(a) $\quad a = \dfrac{dv}{dt} = g\left(1 - c v^2\right)$

Guess $\quad v_1 = 1 \dfrac{\text{m}}{\text{s}}$

Given $\quad \displaystyle\int_0^{v_1} \frac{1}{1 - c v^2}\, dv = \int_0^{t_1} g\, dt \qquad v_1 = \text{Find}(v_1) \qquad v_1 = 45.461 \frac{\text{m}}{\text{s}}$

(b) Terminal velocity means $a = 0$

$$0 = g\left(1 - c\,v_{term}^{2}\right) \qquad v_{term} = \sqrt{\frac{1}{c}} \qquad v_{term} = 100\ \frac{\text{m}}{\text{s}}$$

Problem 12-34

As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g[R^2/(R+y)^2]$, where g is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g = 9.81$ m/s^2 and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \to \infty$.

Solution: $g = 9.81\ \frac{\text{m}}{\text{s}^2}$ $\quad R = 6356\ \text{km}$

$$v\,\mathrm{d}v = a\,\mathrm{d}y = \frac{-g R^2}{(R+y)^2}\mathrm{d}y$$

$$\int_v^0 v\ \mathrm{d}v = -g R^2 \int_0^\infty \frac{1}{(R+y)^2}\ \mathrm{d}y \qquad \frac{-v^2}{2} = -g R$$

$$v = \sqrt{2 g R} \qquad v = 11.2\ \frac{\text{km}}{\text{s}}$$

Problem 12-35

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12-34), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude y_0. Use the numerical data in Prob. 12-34.

Solution: $g = 9.81\ \frac{\text{m}}{\text{s}^2}$ $\quad R = 6356\ \text{km}$ $\quad y_0 = 500\ \text{km}$

$$v\,\mathrm{d}v = a\,\mathrm{d}y = \frac{-g R^2}{(R+y)^2}\mathrm{d}y$$

$$\int_0^v v\,dv = -gR^2 \int_{y_0}^{y} \frac{1}{(R+y)^2}\,dy$$

$$\frac{v^2}{2} = gR^2\left(\frac{1}{R+y} - \frac{1}{R+y_0}\right) = \frac{gR^2(y_0 - y)}{(R+y)(R+y_0)} \qquad v = \sqrt{\frac{2gR^2(y_0-y)}{(R+y)(R+y_0)}}$$

When it hits, $y = 0$ $\qquad v_{earth} = \sqrt{\frac{2gRy_0}{R+y_0}} \qquad v_{earth} = 3.016\ \frac{\text{km}}{\text{s}}$

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*__Problem 12-36__

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v_f^2)(v_f^2 - v^2)$, determine the time needed for the velocity to become $v < v_f$. Initially the particle falls from rest.

Solution:

$$\frac{dv}{dt} = a = \frac{g}{v_f^2}\left(v_f^2 - v^2\right) \qquad \int_0^v \frac{1}{v_f^2 - v^2}\,dv = \frac{g}{v_f^2}\int_0^t 1\,dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f+v}{v_f-v}\right) = \left(\frac{g}{v_f^2}\right)t \qquad t = \frac{v_f}{2g}\ln\left(\frac{v_f+v}{v_f-v}\right)$$

Problem 12-37

An airplane starts from rest, travels a distance d down a runway, and after uniform acceleration, takes off with a speed v_r. It then climbs in a straight line with a uniform acceleration a_a until it reaches a constant speed v_a. Draw the *s-t*, *v-t*, and *a-t* graphs that describe the motion.

Given: $\quad d = 5000\ \text{ft} \qquad v_r = 162\ \frac{\text{mi}}{\text{hr}}$

$\qquad a_a = 3\ \frac{\text{ft}}{\text{s}^2} \qquad v_a = 220\ \frac{\text{mi}}{\text{hr}}$

Solution: First find the acceleration and time on the runway and the time in the air

$$a_r = \frac{v_r^2}{2d} \qquad a_r = 5.645\ \frac{\text{ft}}{\text{s}^2} \qquad t_r = \frac{v_r}{a_r} \qquad t_r = 42.088\ \text{s}$$

$$t_a = \frac{v_a - v_r}{a_a} \qquad t_a = 28.356 \text{ s}$$

The equations of motion

$$t_1 = 0, 0.01t_r .. t_r$$

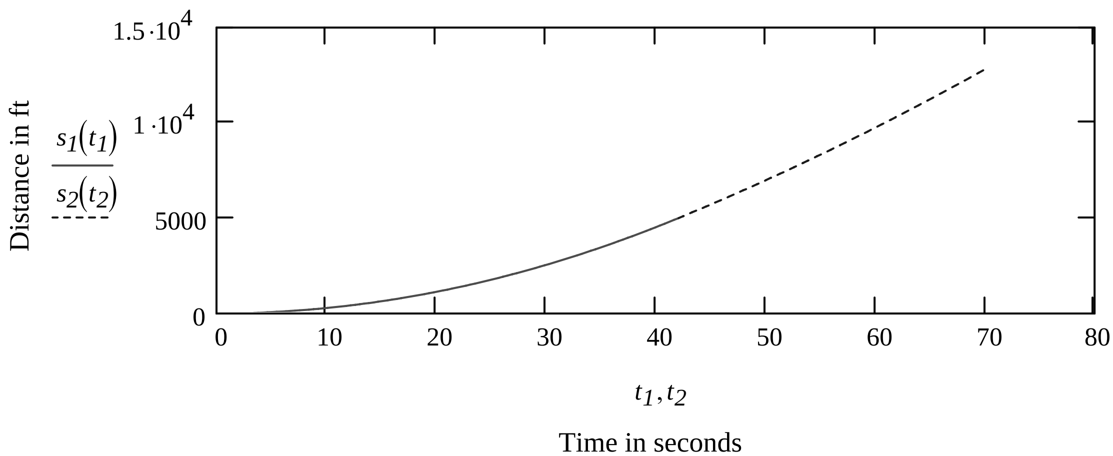
$$a_1(t_1) = a_r\frac{s^2}{ft} \qquad v_1(t_1) = a_r t_1 \frac{s}{ft} \qquad s_1(t_1) = \frac{1}{2}a_r t_1{}^2 \frac{1}{ft}$$

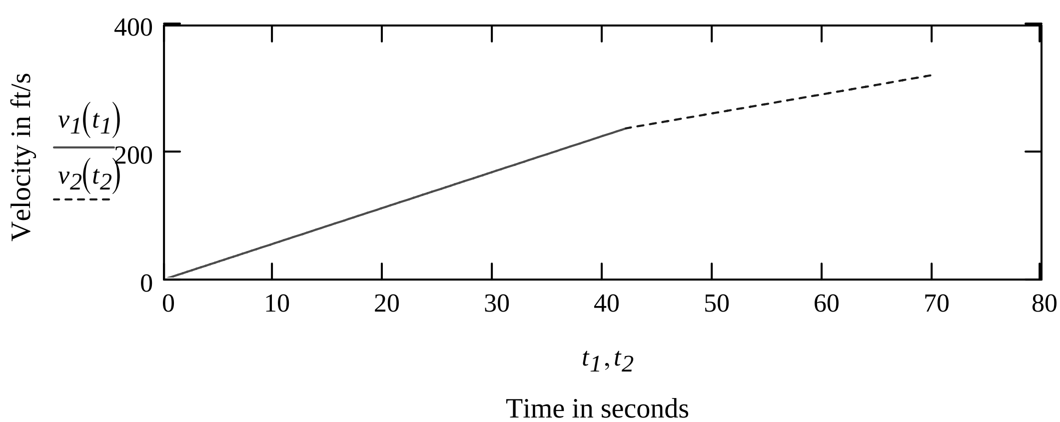
$$t_2 = t_r, 1.01t_r .. t_r + t_a$$

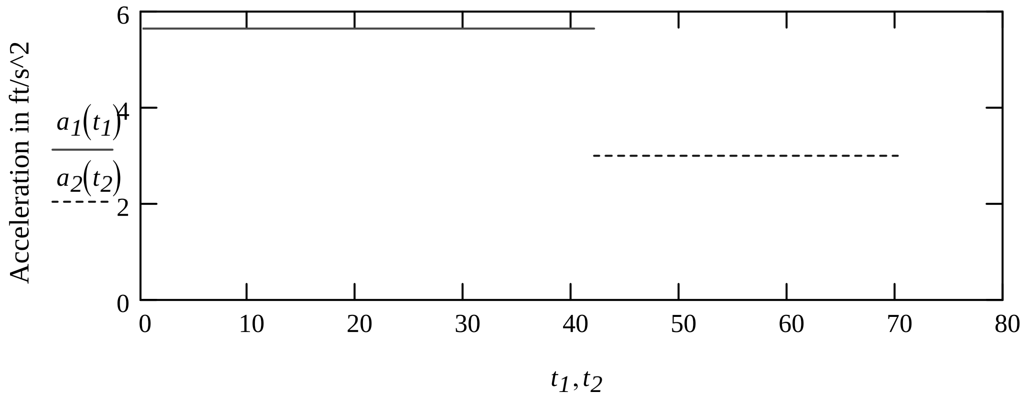
$$a_2(t_2) = a_a\frac{s^2}{ft} \qquad v_2(t_2) = \left[a_r t_r + a_a(t_2 - t_r)\right]\frac{s}{ft}$$

$$s_2(t_2) = \left[\frac{1}{2}a_r t_r{}^2 + a_r t_r(t_2 - t_r) + \frac{1}{2}a_a(t_2 - t_r)^2\right]\frac{1}{ft}$$

The plots

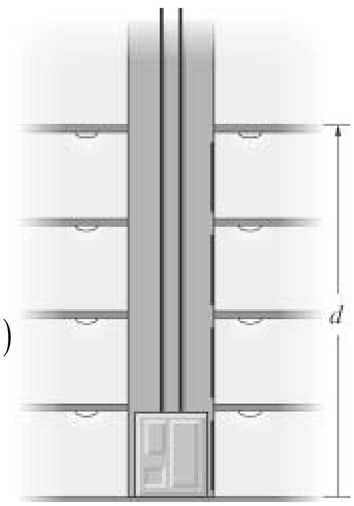






Problem 12-38

The elevator starts from rest at the first floor of the building. It can accelerate at rate a_1 and then decelerate at rate a_2. Determine the shortest time it takes to reach a floor a distance d above the ground. The elevator starts from rest and then stops. Draw the *a-t*, *v-t*, and *s-t* graphs for the motion.



Given: $a_1 = 5\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad a_2 = 2\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad d = 40\ \text{ft}$

Solution: Guesses $t_1 = 1\ \text{s}$ $\quad t = 2\ \text{s}$

$d_1 = 20\ \text{ft}$ $\quad v_{max} = 1\ \dfrac{\text{ft}}{\text{s}}$

Given $v_{max} = a_1 t_1$ $\quad d_1 = \dfrac{1}{2} a_1 t_1^2$ $\quad v_{max} = a_2(t - t_1)$

$$d = d_1 + v_{max}(t - t_1) - \frac{1}{2} a_2 (t - t_1)^2$$

$$\begin{pmatrix} t_1 \\ t \\ d_1 \\ v_{max} \end{pmatrix} = \text{Find}(t_1, t, d_1, v_{max})$$

$d_1 = 11.429\ \text{ft}$ $\quad t_1 = 2.138\ \text{s}$ $\quad v_{max} = 10.69\ \dfrac{\text{ft}}{\text{s}}$

$t = 7.483\ \text{s}$

The equations of motion

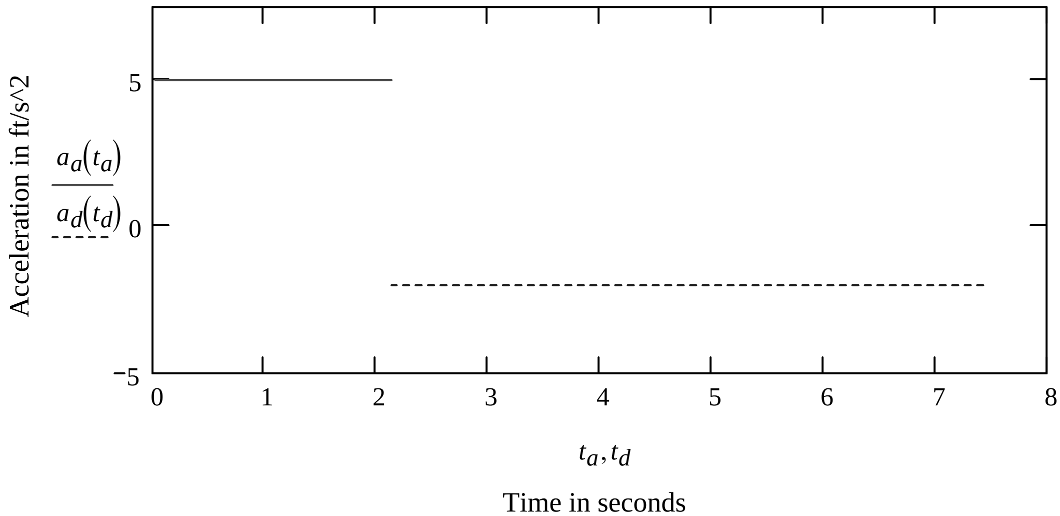
$t_a = 0, 0.01t_1 .. t_1$ $\quad t_d = t_1, 1.01t_1 .. t$

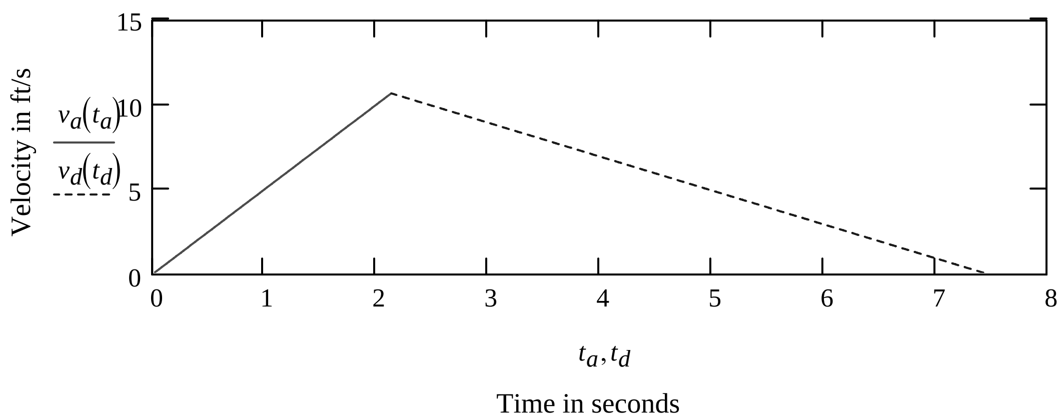
$a_a(t_a) = a_1 \dfrac{\text{s}^2}{\text{ft}}$ $\quad a_d(t_d) = -a_2 \dfrac{\text{s}^2}{\text{ft}}$

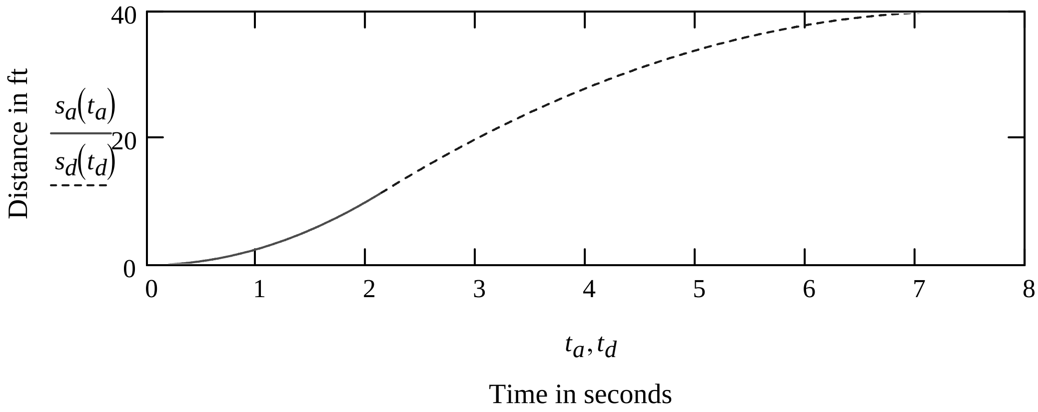
$$v_a(t_a) = a_1 t_a \frac{s}{ft} \qquad v_d(t_d) = \left[v_{max} - a_2(t_d - t_1)\right]\frac{s}{ft}$$

$$s_a(t_a) = \frac{1}{2}a_1 t_a^{\,2}\frac{1}{ft} \qquad s_d(t_d) = \left[d_1 + v_{max}(t_d - t_1) - \frac{1}{2}a_2(t_d - t_1)^2\right]\frac{1}{ft}$$

The plots





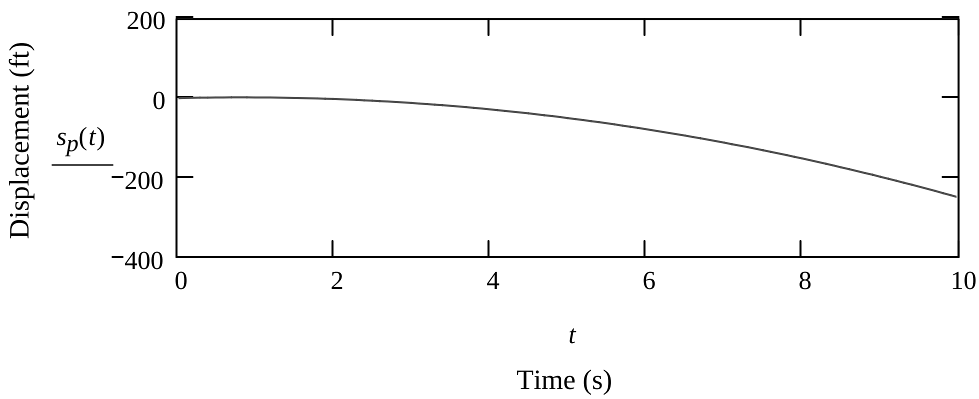


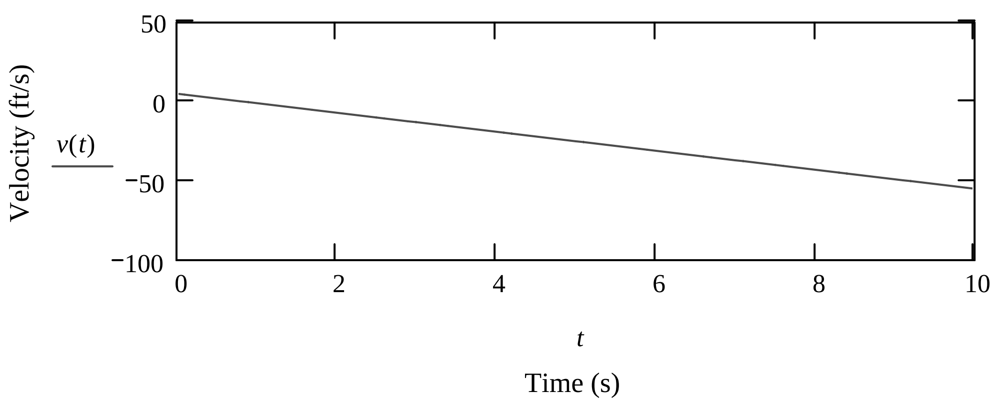
Problem 12–39

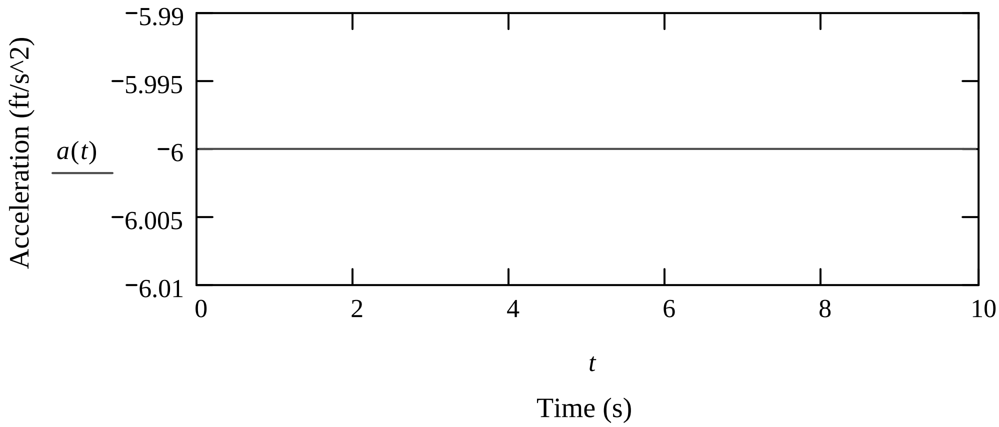
If the position of a particle is defined as $s = bt + ct^2$, construct the *s–t*, *v–t*, and *a–t* graphs for $0 \le t \le T$.

Given: $b = 5\ \text{ft}$ $c = -3\ \text{ft}$ $T = 10\ \text{s}$ $t = 0, 0.01T..T$

Solution: $s_p(t) = \left(b\,t + c\,t^2\right)\dfrac{1}{\text{ft}}$ $v(t) = (b + 2c\,t)\dfrac{\text{s}}{\text{ft}}$ $a(t) = (2c)\dfrac{\text{s}^2}{\text{ft}}$







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*__Problem 12-40__

If the position of a particle is defined by $s_p = b\,\sin(ct) + d$, construct the *s-t*, *v-t*, and *a-t* graphs for $0 \le t \le T$.

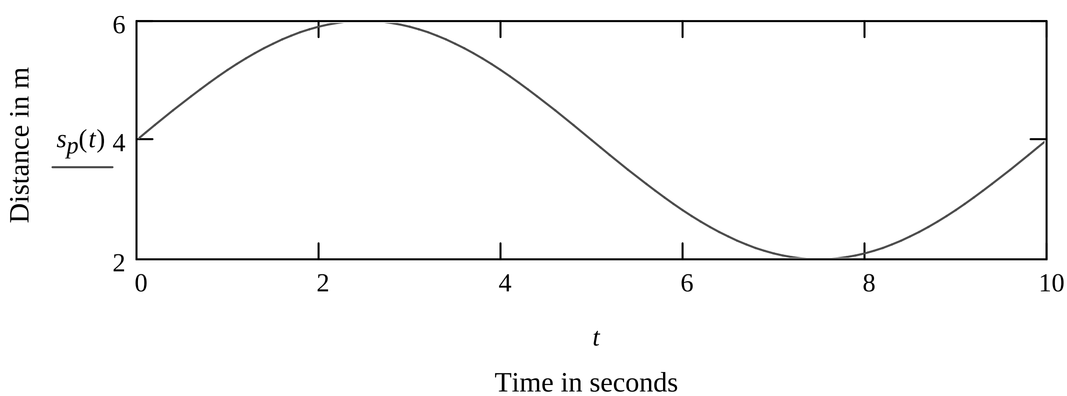
Given: $b = 2\text{ m}$ $c = \frac{\pi}{5}\frac{1}{\text{s}}$ $d = 4\text{ m}$ $T = 10\text{ s}$ $t = 0, 0.01T .. T$

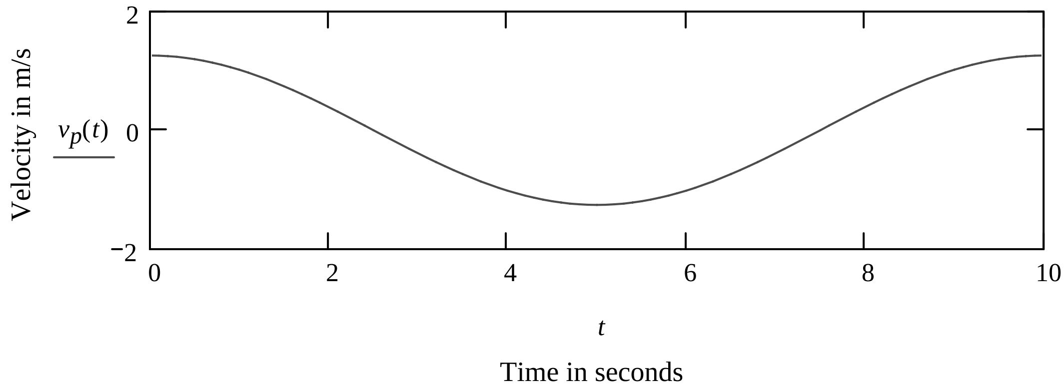
Solution:

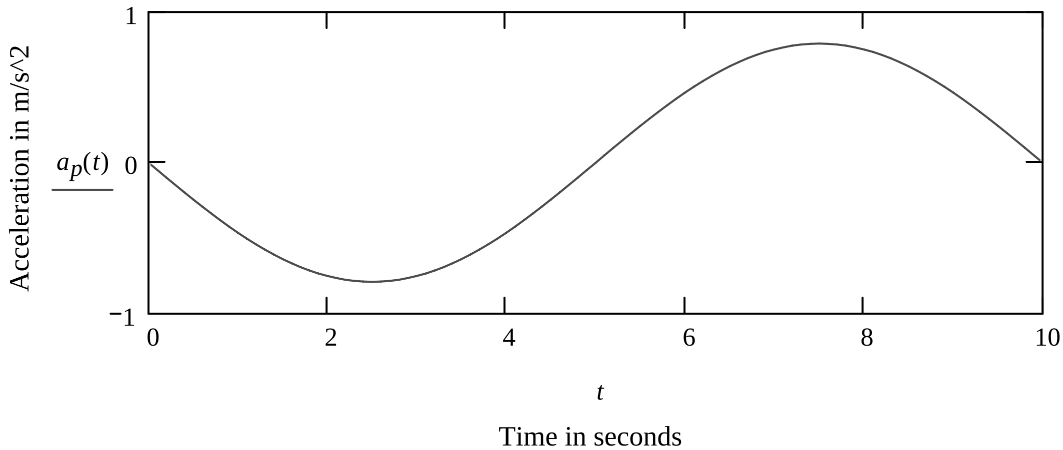
$$s_p(t) = (b\sin(ct) + d)\frac{1}{\text{m}}$$

$$v_p(t) = bc\cos(ct)\frac{\text{s}}{\text{m}}$$

$$a_p(t) = -bc^2\sin(ct)\frac{\text{s}}{\text{m}^2}$$







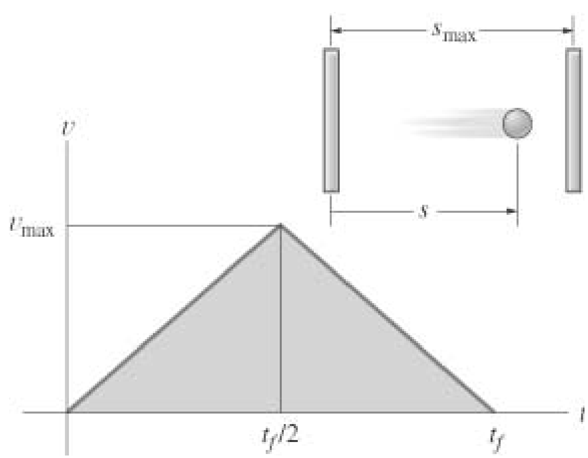
Problem 12-41

The v-t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude a. If the plates are spaced s_{max} apart, determine the maximum velocity v_{max} and the time t_f for the particle to travel from one plate to the other. Also draw the s-t graph. When $t = t_f/2$ the particle is at $s = s_{max}/2$.

Given:

$$a = 4\ \frac{\text{m}}{\text{s}^2}$$

$$s_{max} = 200\ \text{mm}$$



Solution:

$$s_{max} = 2\left[\frac{1}{2}a\left(\frac{t_f}{2}\right)^2\right]$$

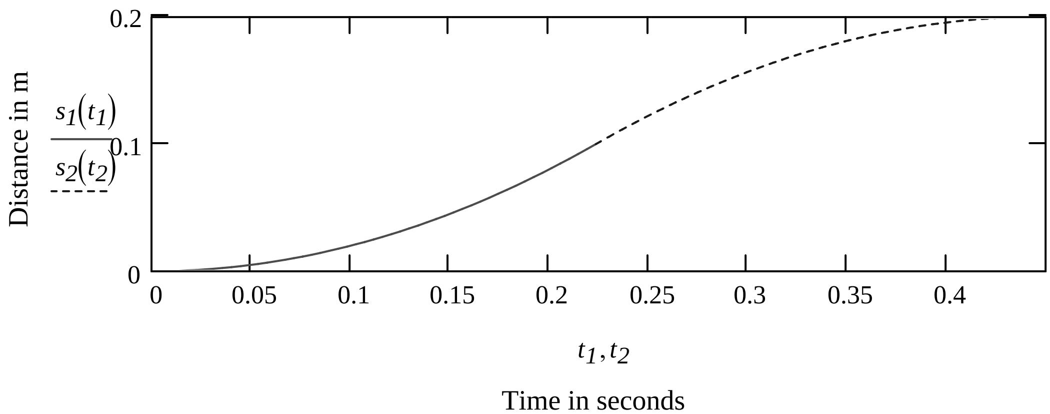
$$t_f = \sqrt{\frac{4s_{max}}{a}} \qquad t_f = 0.447\ \text{s}$$

$$v_{max} = a\frac{t_f}{2} \qquad v_{max} = 0.894\ \frac{\text{m}}{\text{s}}$$

The plots

$$t_1 = 0, 0.01t_f .. \frac{t_f}{2} \qquad s_1(t_1) = \frac{1}{2}a\,t_1^{\,2}\,\frac{1}{\text{m}}$$

$$t_2 = \frac{t_f}{2}, 1.01\frac{t_f}{2} .. t_f \qquad s_2(t_2) = \left[\frac{1}{2}a\left(\frac{t_f}{2}\right)^2 + a\frac{t_f}{2}\left(t_2 - \frac{t_f}{2}\right) - \frac{1}{2}a\left(t_2 - \frac{t_f}{2}\right)^2\right]\frac{1}{\text{m}}$$



Problem 12-42

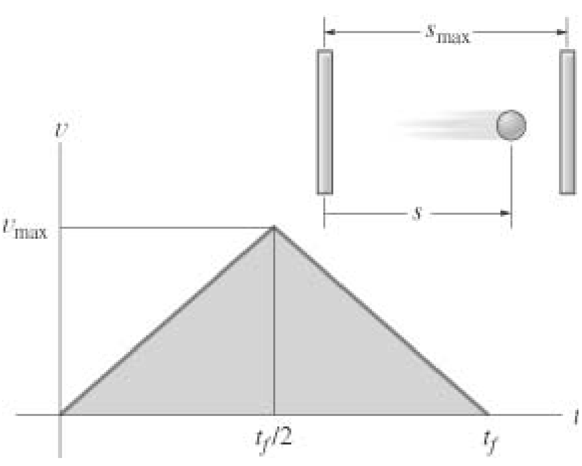
The *v*-*t* graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where t_f and v_{max} are given. Draw the *s*-*t* and *a*-*t* graphs for the particle. When $t = t_f/2$ the particle is at $s = s_c$.

Given:

$$t_f = 0.2\ \text{s}$$

$$v_{max} = 10\ \frac{\text{m}}{\text{s}}$$

$$s_c = 0.5\ \text{m}$$



Solution:

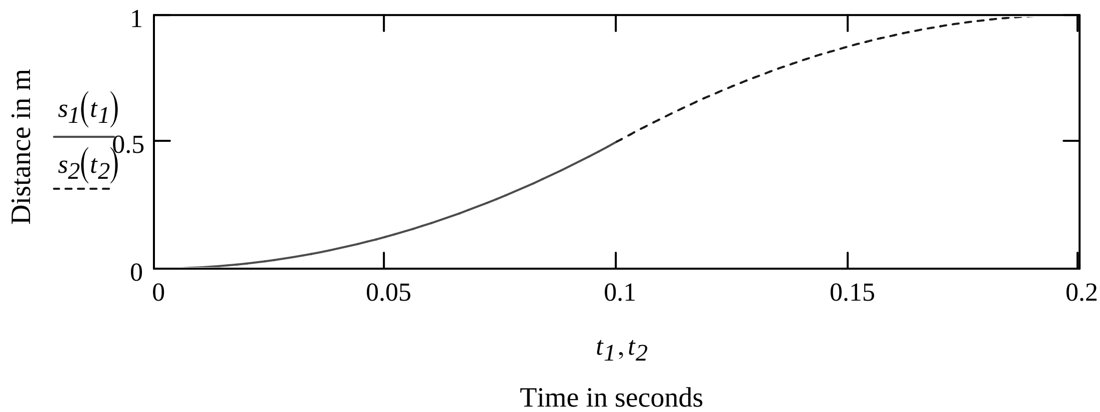
$$a = \frac{2v_{max}}{t_f} \qquad a = 100\ \frac{\text{m}}{\text{s}^2}$$

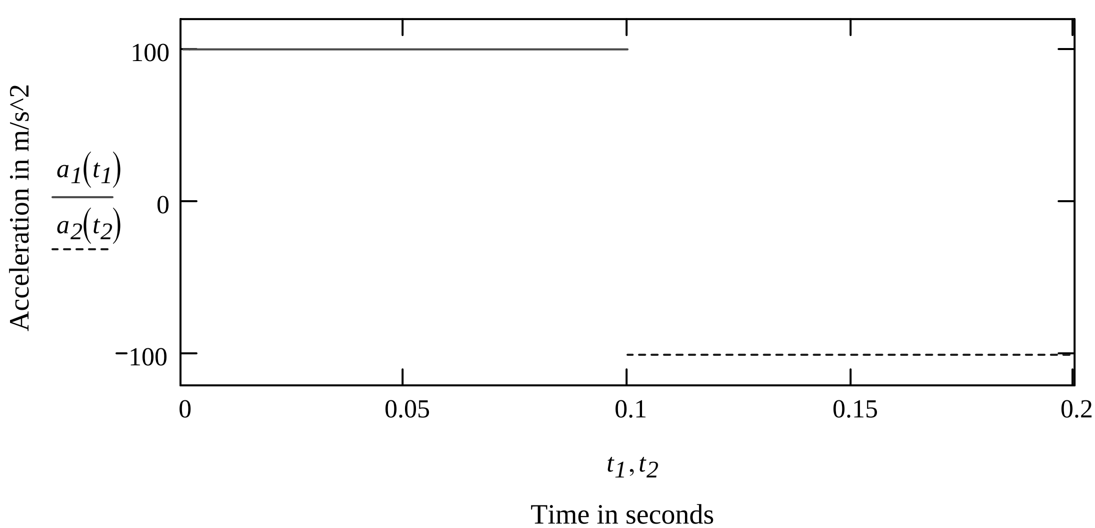
The plots

$$t_1 = 0, 0.01t_f .. \frac{t_f}{2} \qquad s_1(t_1) = \frac{1}{2} a t_1^2 \frac{1}{\text{m}} \qquad a_1(t_1) = a\frac{\text{s}^2}{\text{m}}$$

$$t_2 = \frac{t_f}{2}, 1.01\frac{t_f}{2} .. t_f \qquad s_2(t_2) = \left[\frac{1}{2}a\left(\frac{t_f}{2}\right)^2 + a\frac{t_f}{2}\left(t_2 - \frac{t_f}{2}\right) - \frac{1}{2}a\left(t_2 - \frac{t_f}{2}\right)^2\right]\frac{1}{\text{m}}$$

$$a_2(t_2) = -a\frac{\text{s}^2}{\text{m}}$$





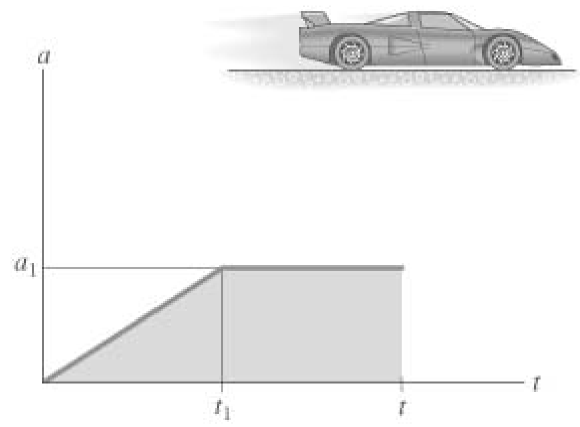
Problem 12–43

A car starting from rest moves along a straight track with an acceleration as shown. Determine the time t for the car to reach speed v.

Given: $v = 50\ \frac{\text{m}}{\text{s}}$

$a_1 = 8\ \frac{\text{m}}{\text{s}^2}$

$t_1 = 10\ \text{s}$



Solution:

Assume that $t > t_1$

Guess $t = 12\ \text{s}$

Given $v = \frac{a_1}{t_1}\frac{t_1^2}{2} + a_1(t - t_1)$ $t = \text{Find}(t)$ $t = 11.25\ \text{s}$

***Problem 12-44**

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v-t graph. Determine the motorcycle's acceleration and position when $t = t_4$ and $t = t_5$.

Given:

$v_0 = 5\ \frac{\text{m}}{\text{s}}$

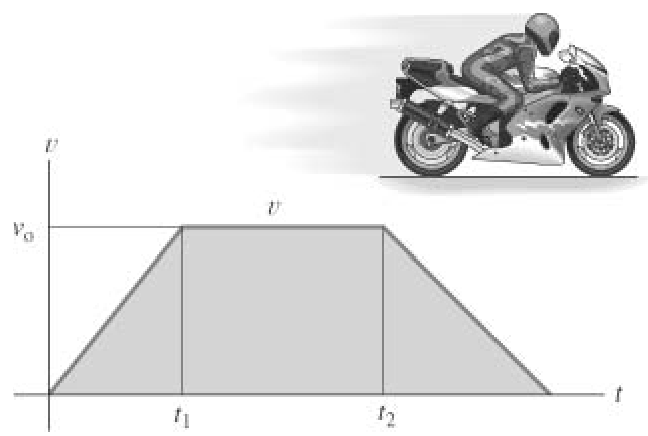
$t_1 = 4\ \text{s}$

$t_2 = 10\ \text{s}$

$t_3 = 15\ \text{s}$

$t_4 = 8\ \text{s}$

$t_5 = 12\ \text{s}$



Solution: At $t = t_4$ Because $t_1 < t_4 < t_2$ then $a_4 = \frac{dv}{dt} = 0$

$$s_4 = \frac{1}{2}v_0 t_1 + (t_4 - t_1)v_0$$ $s_4 = 30\ \text{m}$

At $t = t_5$ Because $t_2 < t_5 < t_3$ then

$$a_5 = \frac{-v_0}{t_3 - t_2}$$ $a_5 = -1\ \frac{\text{m}}{\text{s}^2}$

$$s_5 = \frac{1}{2}t_1 v_0 + v_0(t_2 - t_1) + \frac{1}{2}v_0(t_3 - t_2) - \frac{1}{2}\frac{t_3 - t_5}{t_3 - t_2}v_0(t_3 - t_5)$$

$$s_5 = 48 \text{ m}$$

Problem 12–45

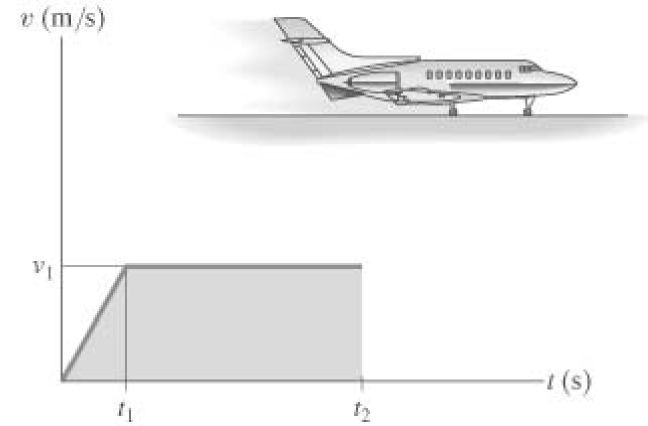
From experimental data, the motion of a jet plane while traveling along a runway is defined by the *v*–*t* graph shown. Construct the *s*-*t* and *a*-*t* graphs for the motion.

Given:

$v_1 = 80 \frac{\text{m}}{\text{s}}$

$t_1 = 10 \text{ s}$

$t_2 = 40 \text{ s}$

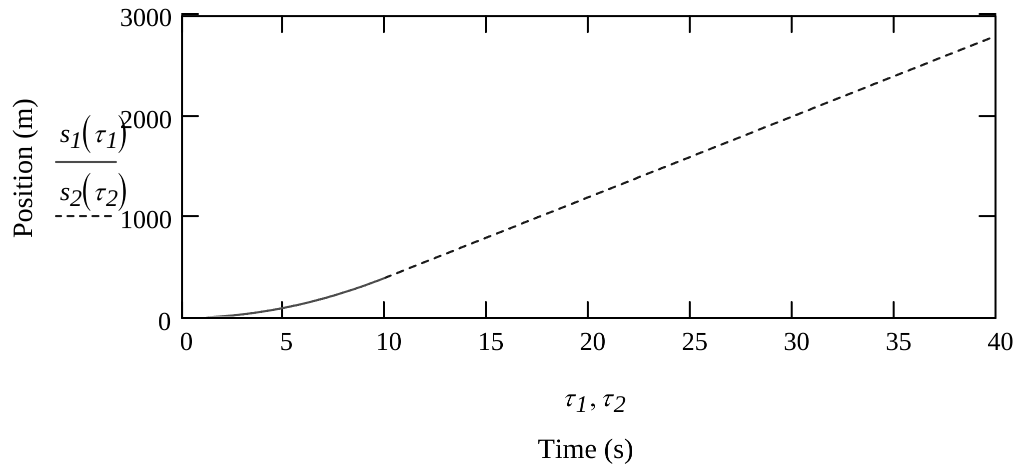


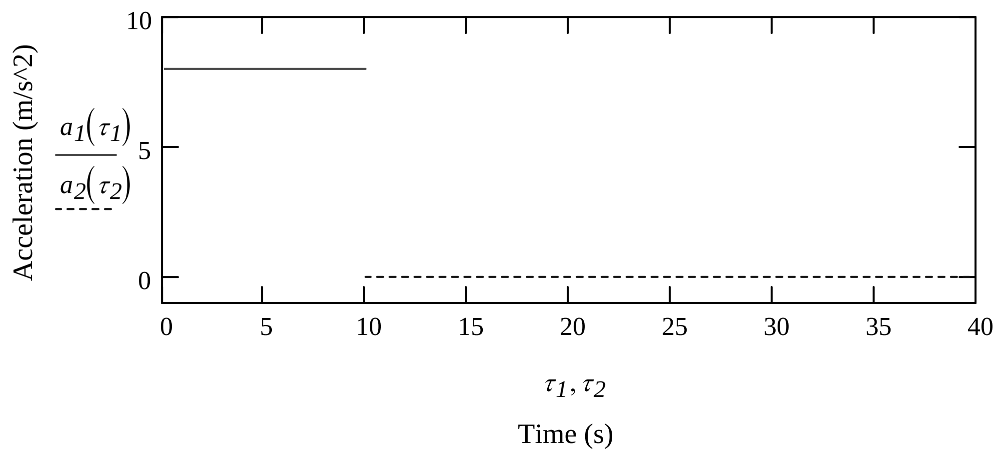
Solution:

$k_1 = \frac{v_1}{t_1}$ $\quad k_2 = 0 \frac{\text{m}}{\text{s}^2}$

$\tau_1 = 0, 0.01t_1 .. t_1$ $\quad s_1(\tau_1) = \left(\frac{1}{2}k_1 \tau_1^2\right) \text{m}$ $\quad a_1(\tau_1) = k_1 \frac{\text{s}^2}{\text{m}}$

$\tau_2 = t_1, 1.01t_1 .. t_2$ $\quad s_2(\tau_2) = \left(v_1 \tau_2 - \frac{1}{2}k_1 t_1^2\right) \text{m}$ $\quad a_2(\tau_2) = k_2 \frac{\text{s}^2}{\text{m}}$





Problem 12–46

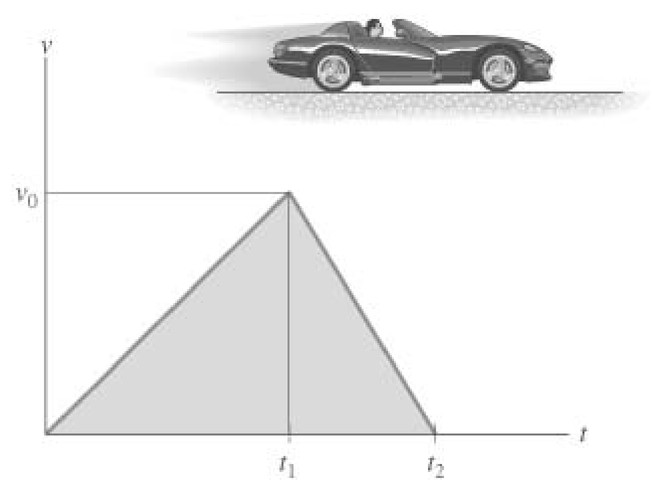
A car travels along a straight road with the speed shown by the v–t graph. Determine the total distance the car travels until it stops at t_2. Also plot the s–t and a–t graphs.

Given:

$t_1 = 30\ \text{s}$

$t_2 = 48\ \text{s}$

$v_0 = 6\ \dfrac{\text{m}}{\text{s}}$



Solution:

$$k_1 = \frac{v_0}{t_1}$$

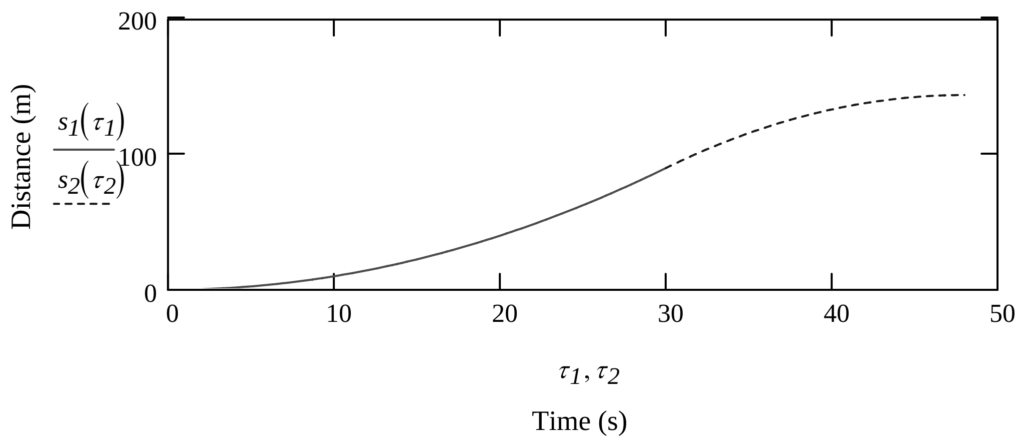
$$k_2 = \frac{v_0}{t_2 - t_1}$$

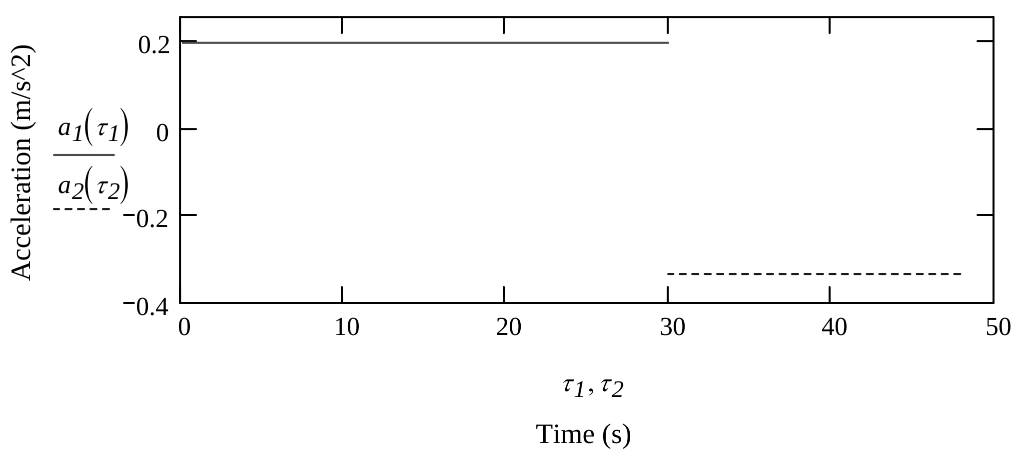
$\tau_1 = 0, 0.01t_1 .. t_1$ $\qquad s_1(t) = \left(\frac{1}{2}k_1 t^2\right)$

$a_1(t) = k_1 \qquad a_2(t) = -k_2$

$\tau_2 = t_1, 1.01t_1 .. t_2$ $\qquad s_2(t) = \left[s_1(t_1) + (v_0 + k_2 t_1)(t - t_1) - \frac{k_2}{2}\left(t^2 - t_1^{\,2}\right)\right]$

$d = s_2(t_2)$ $\qquad d = 144\ \text{m}$





Problem 12–47

The *v–t* graph for the motion of a train as it moves from station *A* to station *B* is shown. Draw the *a–t* graph and determine the average speed and the distance between the stations.

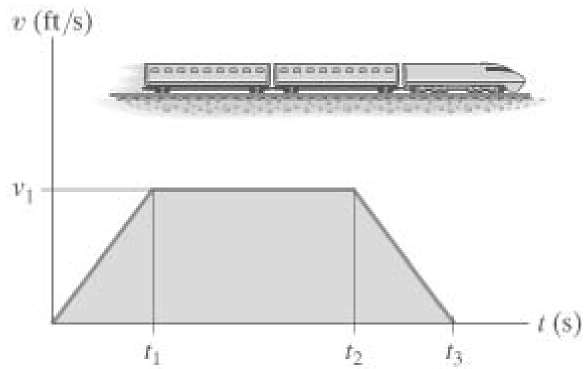
Given:

$t_1 = 30\ \text{s}$

$t_2 = 90\ \text{s}$

$t_3 = 120\ \text{s}$

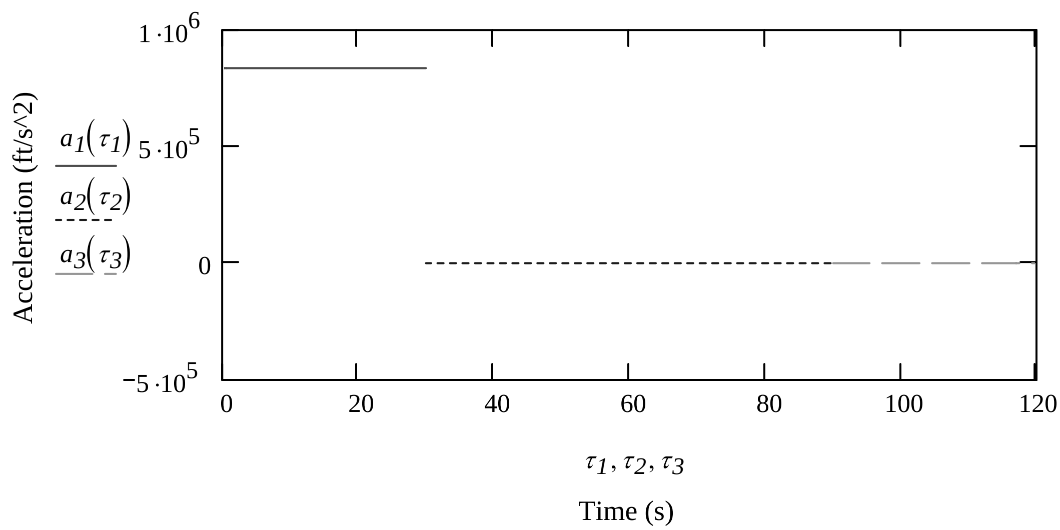
$v_1 = 40\ \dfrac{\text{ft}}{\text{s}}$



Solution:

$\tau_1 = 0, 0.01t_1 .. t_1 \qquad \tau_2 = t_1, 1.01t_1 .. t_2 \qquad \tau_3 = t_2, 1.01t_2 .. t_3$

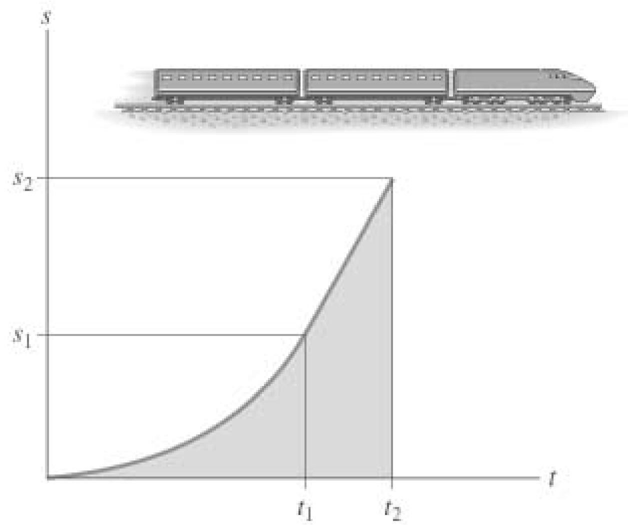
$$a_1(t) = \frac{v_1}{t_1}\frac{\text{s}^2}{\text{ft}} \qquad a_2(t) = 0 \qquad a_3(t) = \frac{-v_1}{t_3 - t_2}\frac{\text{s}^2}{\text{ft}}$$



$$d = \frac{1}{2}v_1 t_1 + v_1(t_2 - t_1) + \frac{1}{2}v_1(t_3 - t_2) \qquad d = 3600\,\text{ft}$$

$$speed = \frac{d}{t_3} \qquad speed = 30\,\frac{\text{ft}}{\text{s}}$$

*__Problem 12–48__



The *s–t* graph for a train has been experimentally determined. From the data, construct the *v–t* and *a–t* graphs for the motion; $0 \le t \le t_2$. For $0 \le t \le t_1$, the curve is a parabola, and then it becomes straight for $t \ge t_1$.

Given:

$t_1 = 30 \text{ s}$

$t_2 = 40 \text{ s}$

$s_1 = 360 \text{ m}$

$s_2 = 600 \text{ m}$

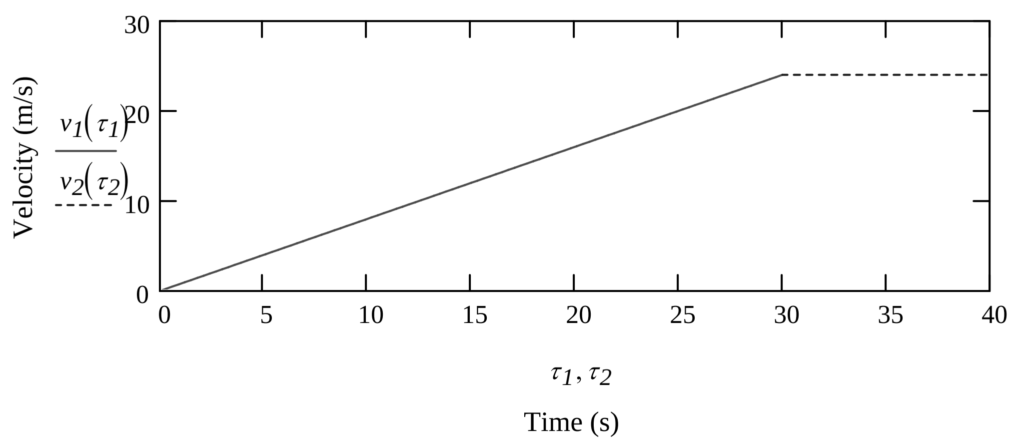
Solution:

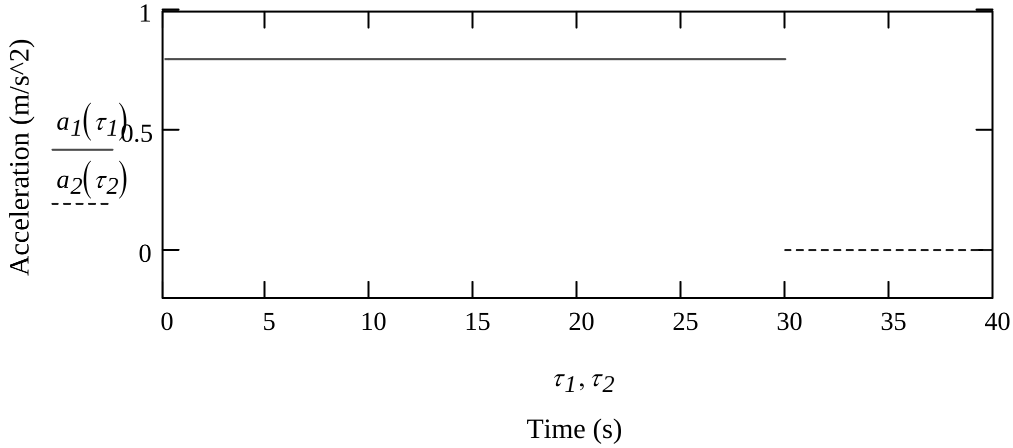
$$k_1 = \frac{s_1}{t_1^{\,2}} \qquad k_2 = \frac{s_2 - s_1}{t_2 - t_1}$$

$$\tau_1 = 0, 0.01t_1 .. t_1 \qquad \tau_2 = t_1, 1.01t_1 .. t_2$$

$$s_{p1}(t) = k_1 t^2 \qquad v_1(t) = 2k_1 t \qquad a_1(t) = 2k_1$$

$$s_{p2}(t) = s_{p1}(t_1) + k_2(t - t_1) \qquad v_2(t) = k_2 \qquad a_2(t) = 0$$





Problem 12-49

The *v*-*t* graph for the motion of a car as if moves along a straight road is shown. Draw the *a*-*t* graph and determine the maximum acceleration during the time interval $0 < t < t_2$. The car starts from rest at $s = 0$.

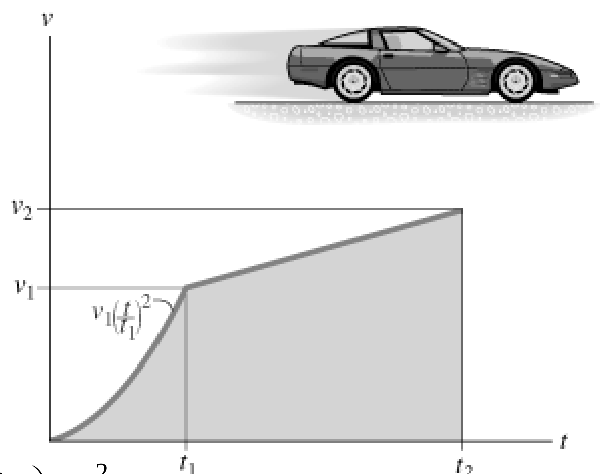
Given:

$t_1 = 10 \text{ s}$

$t_2 = 30 \text{ s}$

$v_1 = 40 \dfrac{\text{ft}}{\text{s}}$

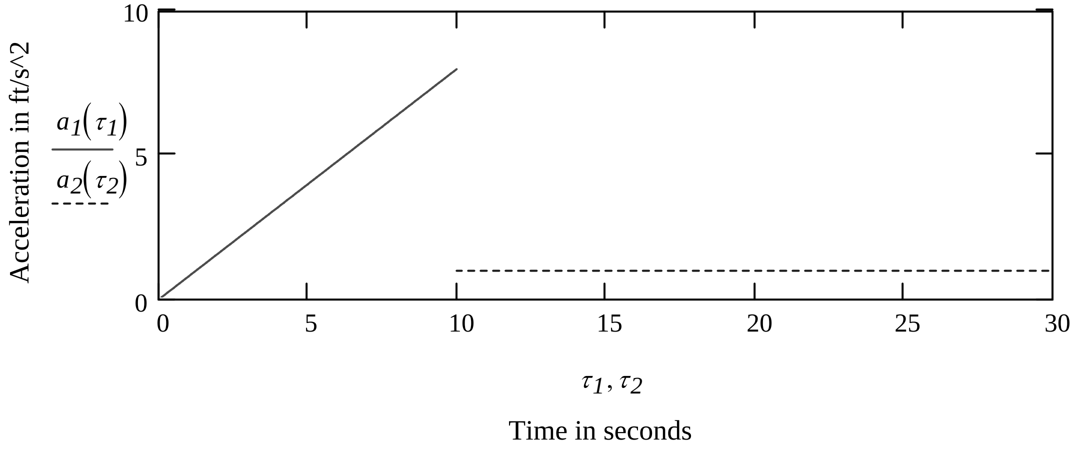
$v_2 = 60 \dfrac{\text{ft}}{\text{s}}$



Solution:

$\tau_1 = 0, 0.01t_1 .. t_1 \qquad a_1(\tau_1) = \left(\dfrac{2v_1}{t_1^2}\right)\tau_1 \dfrac{\text{s}^2}{\text{ft}}$

$\tau_2 = t_1, 1.01t_1 .. t_2 \qquad a_2(\tau_2) = \dfrac{v_2 - v_1}{t_2 - t_1} \dfrac{\text{s}^2}{\text{ft}}$



$$a_{max} = 2\left(\frac{v_1}{t_1^2}\right)t_1 \qquad a_{max} = 8\frac{\text{ft}}{\text{s}^2}$$

Problem 12-50

The v-t graph for the motion of a car as it moves along a straight road is shown. Draw the s-t graph and determine the average speed and the distance traveled for the time interval $0 < t < t_2$. The car starts from rest at $s = 0$.

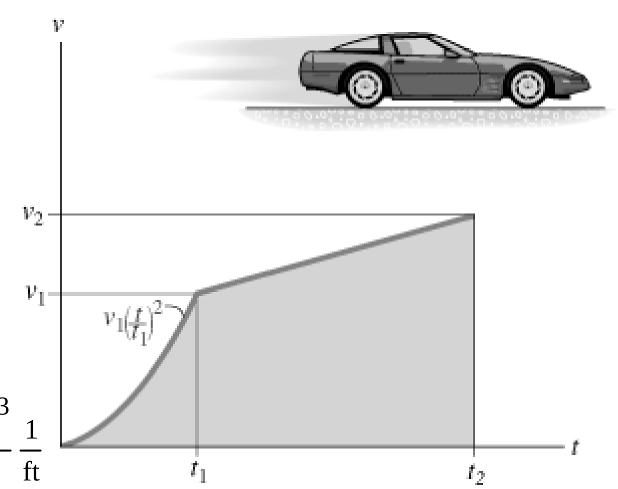
Given:

$$t_1 = 10 \text{ s}$$

$$t_2 = 30 \text{ s}$$

$$v_1 = 40 \frac{\text{ft}}{\text{s}}$$

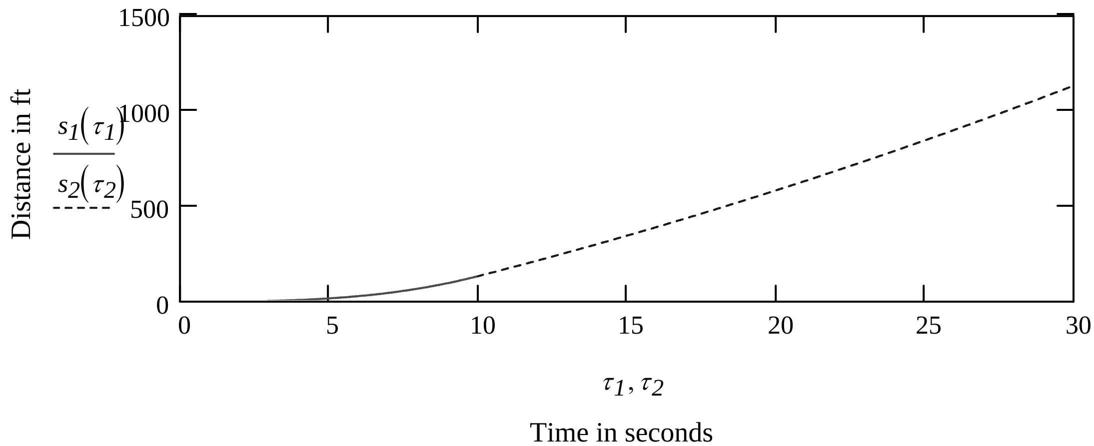
$$v_2 = 60 \frac{\text{ft}}{\text{s}}$$



Solution: The graph

$$\tau_1 = 0, 0.01t_1 .. t_1 \qquad s_1(\tau_1) = \frac{v_1}{t_1^2}\frac{\tau_1^3}{3}\frac{1}{\text{ft}}$$

$$\tau_2 = t_1, 1.01t_1 .. t_2 \qquad s_2(\tau_2) = \left[\frac{v_1 t_1}{3} + v_1(\tau_2 - t_1) + \frac{v_2 - v_1}{t_2 - t_1}\frac{(\tau_2 - t_1)^2}{2}\right]\frac{1}{\text{ft}}$$



Distance traveled $\quad d = \dfrac{v_1 t_1}{3} + v_1(t_2 - t_1) + \dfrac{v_2 - v_1}{t_2 - t_1}\dfrac{(t_2 - t_1)^2}{2} \qquad d = 1.133 \times 10^3\ \text{ft}$

Average speed $\quad v_{ave} = \dfrac{d}{t_2} \qquad v_{ave} = 37.778\ \dfrac{\text{ft}}{\text{s}}$

Problem 12-51

The a–s graph for a boat moving along a straight path is given. If the boat starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = s_2$, and s_3, respectively. Use Simpson's rule with n to evaluate v at $s = s_3$.

Given:

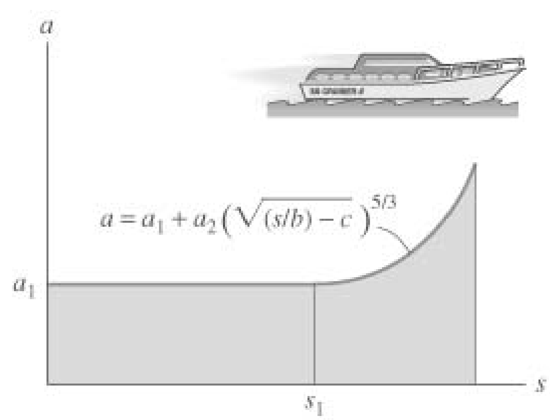
$a_1 = 5\ \dfrac{\text{ft}}{\text{s}^2} \qquad b = 1\ \text{ft}$

$a_2 = 6\ \dfrac{\text{ft}}{\text{s}^2} \qquad c = 10$

$s_1 = 100\ \text{ft}$

$s_2 = 75\ \text{ft}$

$s_3 = 125\ \text{ft}$



Solution:

Since $s_2 = 75\,\text{ft} < s_1 = 100\,\text{ft}$

$$a = v\frac{d}{ds}v \qquad \frac{v_2^{\,2}}{2} = \int_0^{s_2} a\, ds \qquad v_2 = \sqrt{2\int_0^{s_2} a_1\, ds} \qquad v_2 = 27.386\,\frac{ft}{s}$$

Since $s_3 = 125\,ft > s_1 = 100\,ft$

$$v_3 = \sqrt{2\int_0^{s_1} a_1\, ds + 2\int_{s_1}^{s_3} a_1 + a_2\left(\sqrt{\frac{s}{b}} - c\right)^{\frac{5}{3}} ds} \qquad v_3 = 37.444\,\frac{ft}{s}$$

---

*__Problem 12-52__

A man riding upward in a freight elevator accidentally drops a package off the elevator when it is a height h from the ground. If the elevator maintains a constant upward speed v_0, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v-t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

Given: $h = 100\ ft \qquad v_0 = 4\ \frac{ft}{s} \qquad g = 32.2\ \frac{ft}{s^2}$

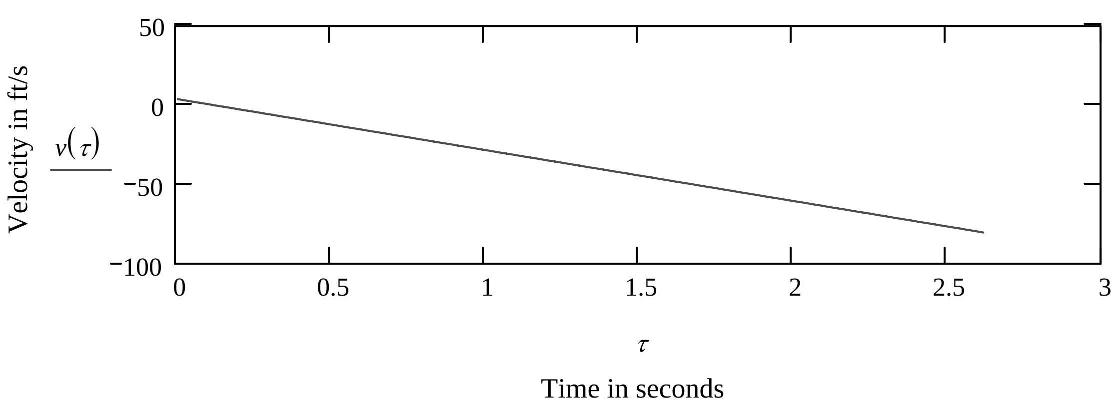
For the package $\quad a = -g \qquad v = v_0 - g\,t \qquad s = h + v_0\,t - \frac{1}{2}g\,t^2$

When it hits the ground we have

$$0 = h + v_0\,t - \frac{1}{2}g\,t^2 \qquad t = \frac{v_0 + \sqrt{v_0^{\,2} + 2g\,h}}{g} \qquad t = 2.62\ s$$

For the elevator $\quad s_y = v_0\,t + h \qquad s_y = 110.5\,ft$

The plot $\quad \tau = 0, 0.01t\,..\,t \qquad v(\tau) = (v_0 - g\tau)\frac{s}{ft}$



Problem 12-53

Two cars start from rest side by side and travel along a straight road. Car *A* accelerates at the rate a_A for a time t_1, and then maintains a constant speed. Car *B* accelerates at the rate a_B until reaching a constant speed v_B and then maintains this speed. Construct the *a-t*, *v-t*, and *s-t* graphs for each car until $t = t_2$. What is the distance between the two cars when $t = t_2$?

Given: $a_A = 4\ \frac{\text{m}}{\text{s}^2}$ $\quad t_1 = 10\ \text{s}$ $\quad a_B = 5\ \frac{\text{m}}{\text{s}^2}$ $\quad v_B = 25\ \frac{\text{m}}{\text{s}}$ $\quad t_2 = 15\ \text{s}$

Solution:

Car *A*:

$\tau_1 = 0, 0.01t_1 .. t_1$ $\quad a_1(t) = a_A\frac{\text{s}^2}{\text{m}}$ $\quad v_1(t) = a_A t\frac{\text{s}}{\text{m}}$ $\quad s_1(t) = \frac{1}{2}a_A t^2\frac{1}{\text{m}}$

$\tau_2 = t_1, 1.01t_1 .. t_2$ $\quad a_2(t) = 0\frac{\text{s}^2}{\text{m}}$ $\quad v_2(t) = v_1(t_1)\frac{\text{s}}{\text{m}}$

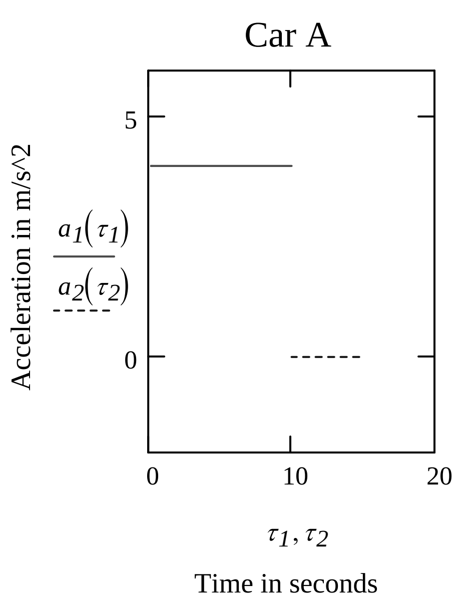
$$s_2(t) = \left[\frac{1}{2}a_A t_1^{\,2} + a_A t_1(t - t_1)\right]\frac{1}{\text{m}}$$

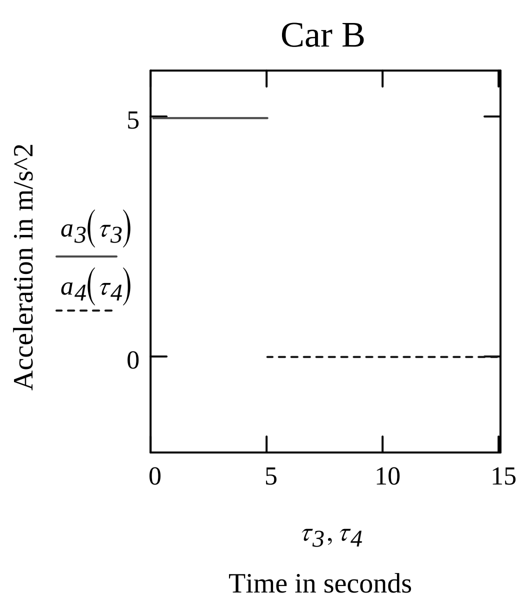
Car *B*: $\quad t_3 = \frac{v_B}{a_B}$

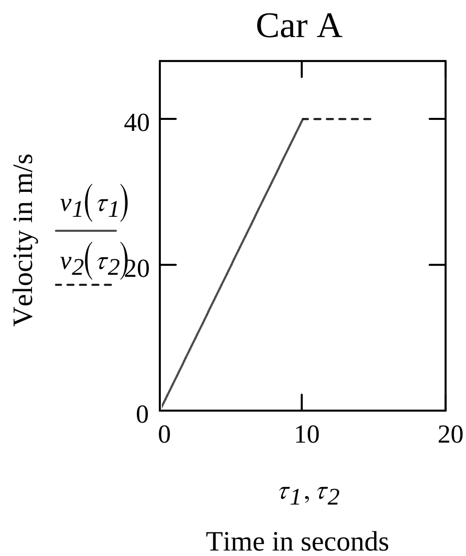
$\tau_3 = 0, 0.01t_3 .. t_3$ $\quad a_3(t) = a_B\frac{\text{s}^2}{\text{m}}$ $\quad v_3(t) = a_B t\frac{\text{s}}{\text{m}}$ $\quad s_3(t) = \frac{1}{2}a_B t^2\frac{1}{\text{m}}$

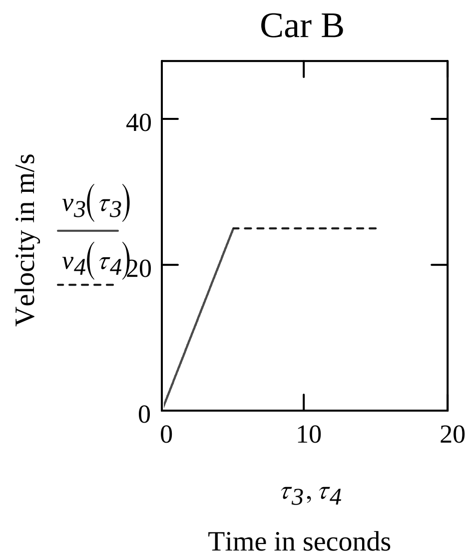
$\tau_4 = t_3, 1.01t_3 .. t_2$ $\quad a_4(t) = 0$ $\quad v_4(t) = a_B t_3\frac{\text{s}}{\text{m}}$

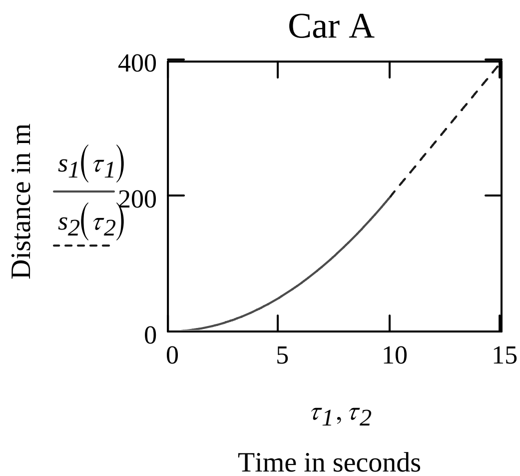
$$s_4(t) = \left[\frac{1}{2}a_B t_3^{\,2} + a_B t_3(t - t_3)\right]\frac{1}{\text{m}}$$

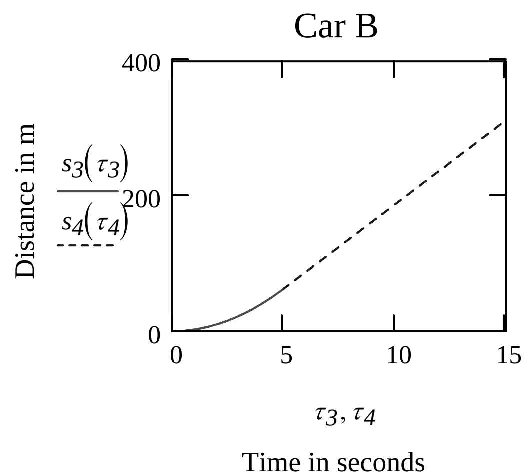












When $t = t_2$ $$d = \left|\frac{1}{2}a_A t_1^{\,2} + a_A t_1(t_2 - t_1) - \left[\frac{1}{2}a_B t_3^{\,2} + a_B t_3(t_2 - t_3)\right]\right|$$

$$d = 87.5\,\mathrm{m}$$

Problem 12-54

A two-stage rocket is fired vertically from rest at $s = 0$ with an acceleration as shown. After time t_1 the first stage A burns out and the second stage B ignites. Plot the v-t and s-t graphs which describe the motion of the second stage for $0 < t < t_2$.

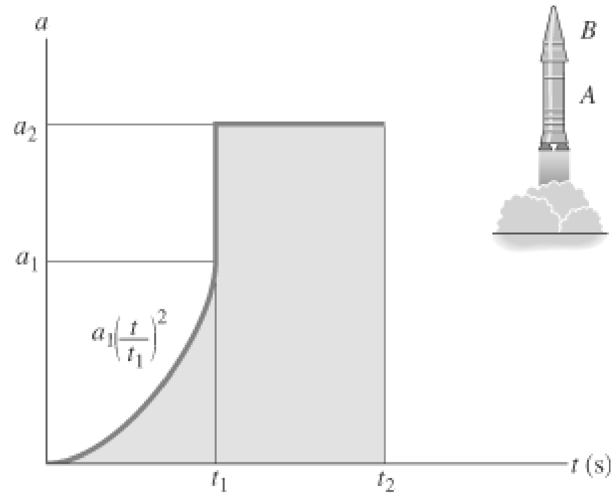
Given:

$t_1 = 30\ \text{s}$

$t_2 = 60\text{s}$

$a_1 = 9\ \dfrac{\text{m}}{\text{s}^2}$

$a_2 = 15\ \dfrac{\text{m}}{\text{s}^2}$

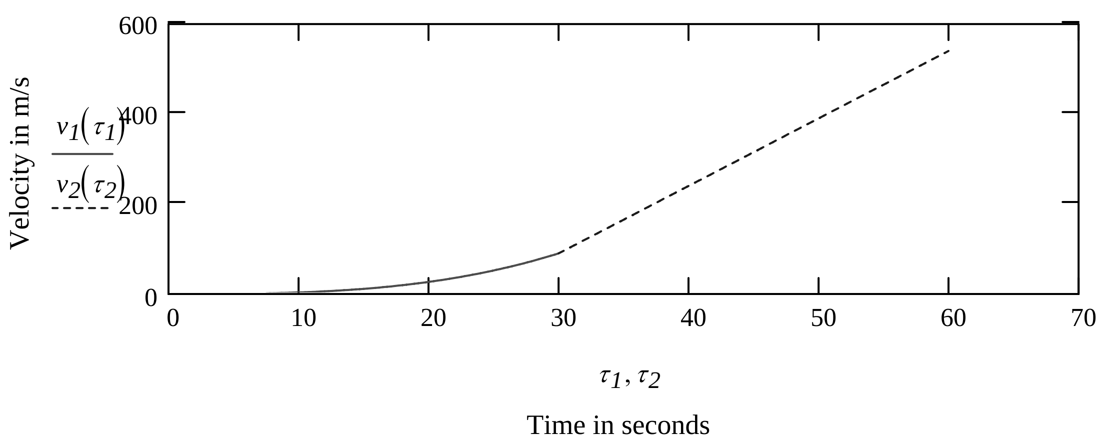


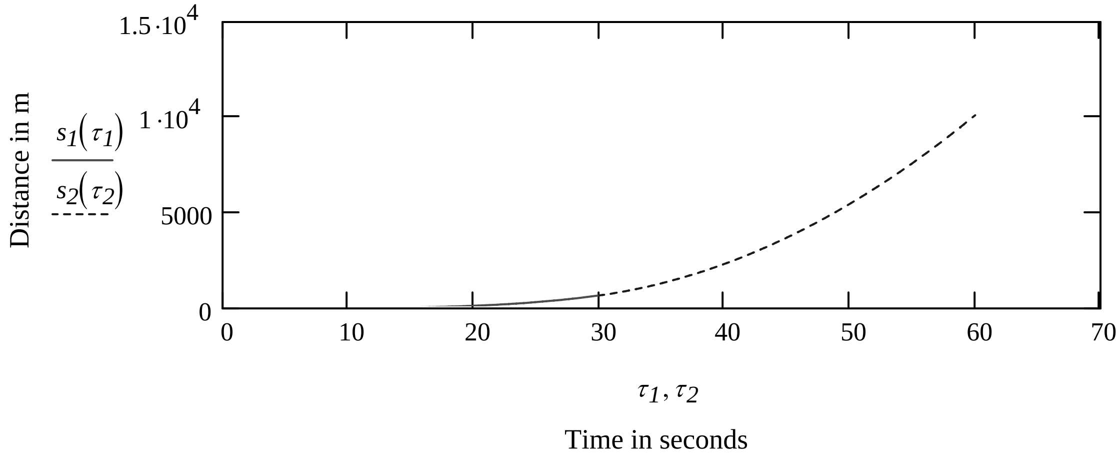
Solution:

$\tau_1 = 0, 0.01t_1 .. t_1 \qquad v_1(\tau_1) = \dfrac{a_1}{t_1^2}\dfrac{\tau_1^3}{3}\dfrac{\text{s}}{\text{m}} \qquad s_1(\tau_1) = \dfrac{a_1}{t_1^2}\dfrac{\tau_1^4}{12}\dfrac{1}{\text{m}}$

$\tau_2 = t_1, 1.01t_1 .. t_2 \qquad v_2(\tau_2) = \left[\dfrac{a_1 t_1}{3} + a_2(\tau_2 - t_1)\right]\dfrac{\text{s}}{\text{m}}$

$$s_2(\tau_2) = \left[\frac{a_1 t_1^2}{12} + \frac{a_1 t_1}{3}(\tau_2 - t_1) + a_2\frac{(\tau_2 - t_1)^2}{2}\right]\frac{\text{s}^2}{\text{m}}$$





Problem 12–55

The *a–t* graph for a motorcycle traveling along a straight road has been estimated as shown. Determine the time needed for the motorcycle to reach a maximum speed v_{max} and the distance traveled in this time. Draw the *v–t* and *s–t* graphs.The motorcycle starts from rest at $s = 0$.

Given:

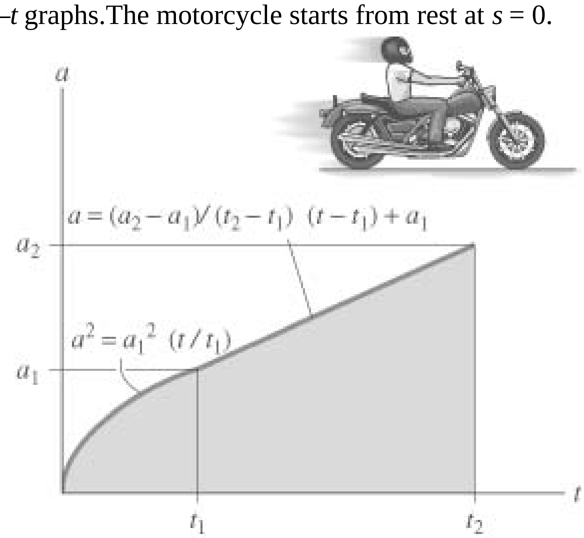
$t_1 = 10\ \text{s}$

$t_2 = 30\ \text{s}$

$a_1 = 10\ \frac{\text{ft}}{\text{s}^2}$

$a_2 = 20\ \frac{\text{ft}}{\text{s}^2}$

$v_{max} = 100\ \frac{\text{ft}}{\text{s}}$



Solution: Assume that $t_1 < t < t_2$

$\tau_1 = 0, 0.01t_1 .. t_1$ $\tau_2 = t_1, 1.01t_1 .. t_2$

$$a_{p1}(t) = a_1\sqrt{\frac{t}{t_1}} \qquad v_{p1}(t) = \left(\frac{2a_1}{3\sqrt{t_1}}\right)\sqrt{t^3} \qquad s_{p1}(t) = \left(\frac{4a_1}{15\sqrt{t_1}}\right)\sqrt{t^5}$$

$$a_{p2}(t) = (a_2 - a_1)\frac{t - t_1}{t_2 - t_1} + a_1$$

$$v_{p2}(t) = \frac{a_2 - a_1}{2}\frac{(t - t_1)^2}{t_2 - t_1} + a_1(t - t_1) + v_{p1}(t_1)$$

$$s_{p2}(t) = \frac{a_2 - a_1}{6}\frac{(t - t_1)^3}{t_2 - t_1} + \frac{a_1}{2}(t - t_1)^2 + v_{p1}(t_1)(t - t_1) + s_{p1}(t_1)$$

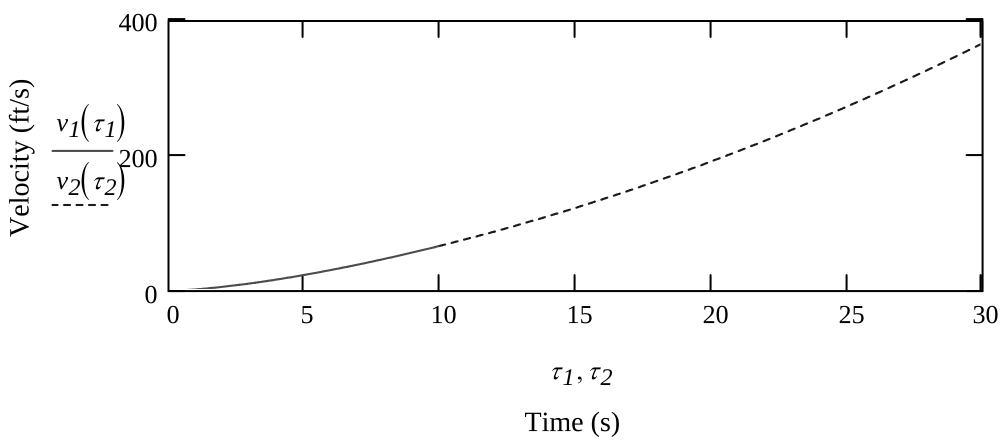
Guess $t = 1\ \text{s}$ Given $v_{p2}(t) = v_{max}$

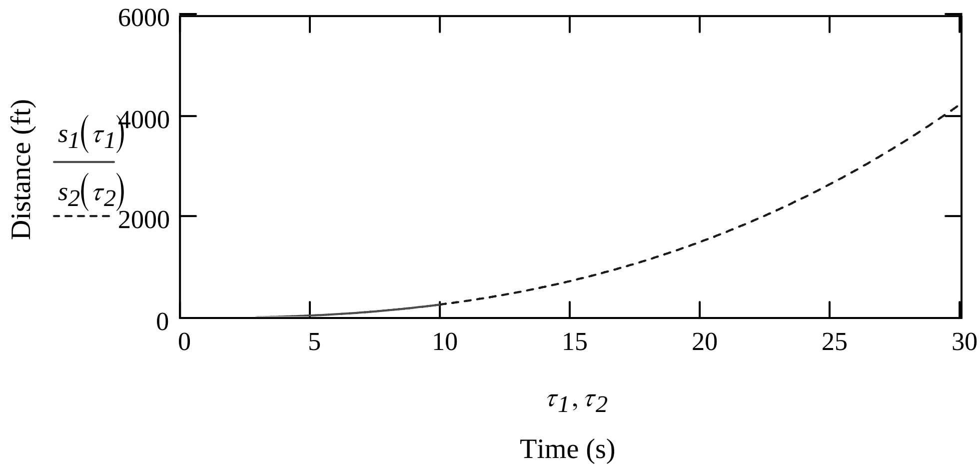
$t = \text{Find}(t)$ $t = 13.09\ \text{s}$

$d = s_{p2}(t)$ $d = 523\,\text{ft}$

$v_1(t) = v_{p1}(t)\frac{\text{s}}{\text{ft}}$ $v_2(t) = v_{p2}(t)\frac{\text{s}}{\text{ft}}$

$s_1(t) = s_{p1}(t)\frac{1}{\text{ft}}$ $s_2(t) = s_{p2}(t)\frac{1}{\text{ft}}$





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*__Problem 12-56__

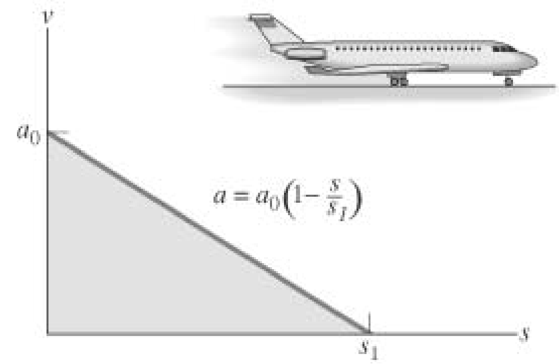
The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled a distance d. Also, how much time is required for it to travel the distance d?

Given:

$d = 200\ \text{ft}$

$a_0 = 75\ \dfrac{\text{ft}}{\text{s}^2}$

$s_1 = 500\ \text{ft}$



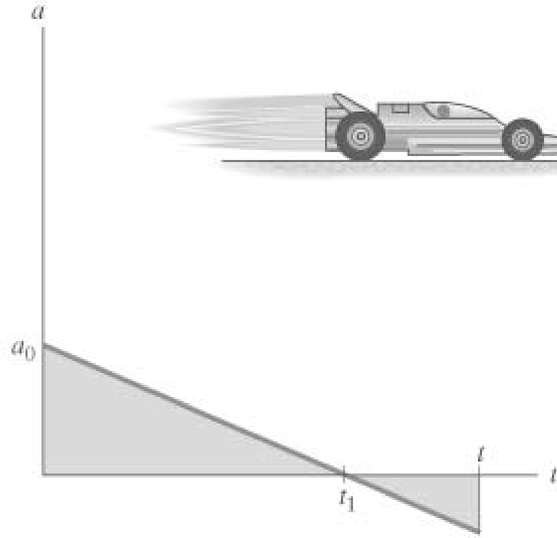
Solution:

$$a = a_0\left(1 - \frac{s}{s_1}\right) \qquad \int_0^{v_2} v\,dv = \int_0^{s} a_0\left(1 - \frac{s}{s_1}\right) ds \qquad \frac{v^2}{2} = a_0\left(s - \frac{s^2}{2s_1}\right)$$

$$v_d = \sqrt{2a_0\left(d - \frac{d^2}{2s_1}\right)} \qquad v_d = 155\ \frac{\text{ft}}{\text{s}}$$

$$v = \frac{ds}{dt} \qquad \int_0^t 1\,dt = \int_0^d \frac{1}{v}\,ds \qquad t = \int_0^d \frac{1}{\sqrt{2a_0\left(s - \dfrac{s^2}{2s_1}\right)}}\,ds \qquad t = 2.394\ \text{s}$$

Problem 12–57



The jet car is originally traveling at speed v_0 when it is subjected to the acceleration shown in the graph. Determine the car's maximum speed and the time t when it stops.

Given:

$$v_0 = 20\ \frac{m}{s}$$

$$a_0 = 10\ \frac{m}{s^2}$$

$$t_1 = 20\ s$$

Solution:

$$a(t) = a_0\left(1 - \frac{t}{t_1}\right) \qquad v(t) = v_0 + \int_0^t a(t)\ dt \qquad s_p(t) = \int_0^t v(t)\ dt$$

Guess $\quad t_{stop} = 30\ s \quad$ Given $\quad v(t_{stop}) = 0 \qquad t_{stop} = \text{Find}(t_{stop})$

$$v_{max} = v(t_1) \qquad v_{max} = 120\ \frac{m}{s} \qquad t_{stop} = 41.909\ s$$

Problem 12-58

A motorcyclist at A is traveling at speed v_1 when he wishes to pass the truck T which is traveling at a constant speed v_2. To do so the motorcyclist accelerates at rate a until reaching a maximum speed v_3. If he then maintains this speed, determine the time needed for him to reach a point located a distance d_3 in front of the truck. Draw the v-t and s-t graphs for the motorcycle during this time.

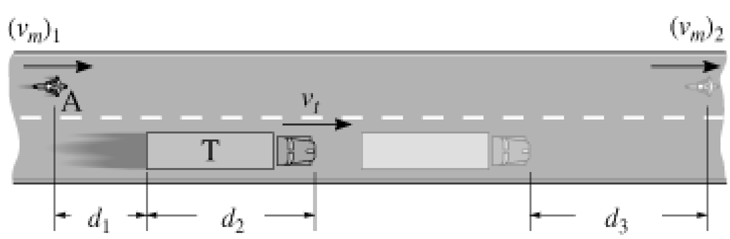
Given:

$$v_1 = 60\ \frac{ft}{s} \qquad d_1 = 40\ ft$$

$$v_2 = 60\ \frac{ft}{s} \qquad d_2 = 55\ ft$$

$$v_3 = 85\ \frac{ft}{s} \qquad d_3 = 100\ ft$$

$$a = 6\ \frac{ft}{s^2}$$



Solution: Let t_1 represent the time to full speed, t_2 the time to reache the required distance.

Guesses $t_1 = 10\ \text{s}$ $t_2 = 20\ \text{s}$

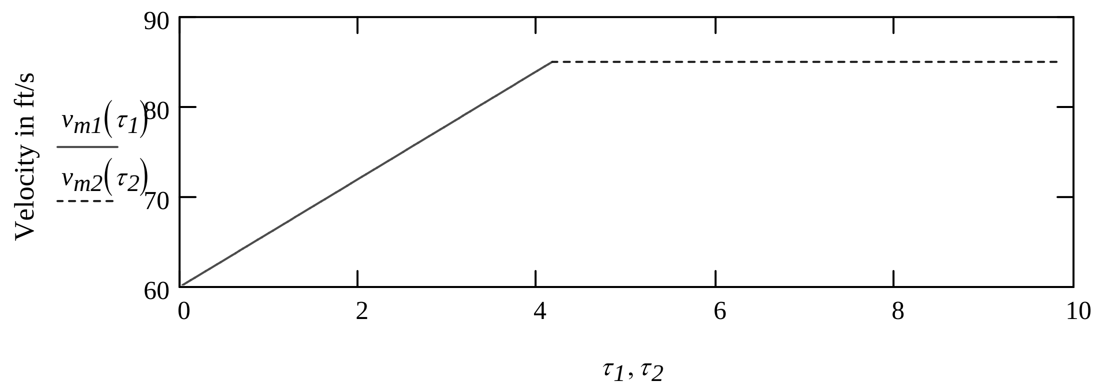
Given $v_3 = v_1 + a t_1$ $d_1 + d_2 + d_3 + v_2 t_2 = v_1 t_1 + \frac{1}{2} a t_1^2 + v_3(t_2 - t_1)$

$\begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \text{Find}(t_1, t_2)$ $t_1 = 4.167\ \text{s}$ $t_2 = 9.883\ \text{s}$

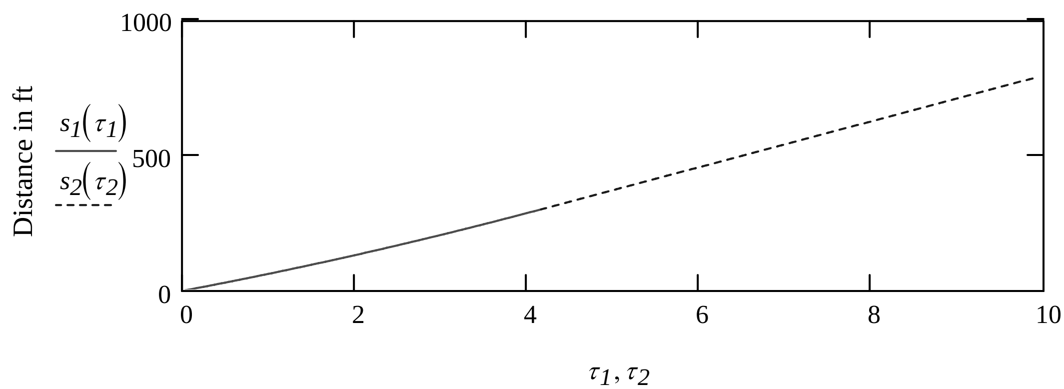
Now draw the graphs

$\tau_1 = 0, 0.01 t_1 .. t_1$ $s_1(\tau_1) = \left(v_1 \tau_1 + \frac{1}{2} a \tau_1^2\right)\frac{1}{\text{ft}}$ $v_{m1}(\tau_1) = (v_1 + a\tau_1)\frac{\text{s}}{\text{ft}}$

$\tau_2 = t_1, 1.01 t_1 .. t_2$ $s_2(\tau_2) = \left[v_1 t_1 + \frac{1}{2} a t_1^2 + v_3(\tau_2 - t_1)\right]\frac{1}{\text{ft}}$ $v_{m2}(\tau_2) = v_3 \frac{\text{s}}{\text{ft}}$



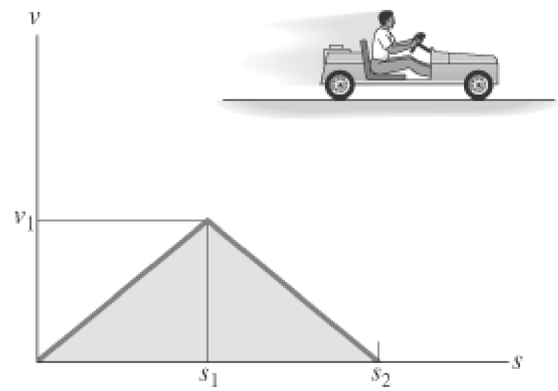
Distance in seconds



Time in seconds

Problem 12-59

The *v-s* graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at s_3 and s_4. Draw the *a-s* graph.



Given:

$v_1 = 8\ \frac{\text{m}}{\text{s}}$ $\quad s_3 = 50\ \text{m}$

$s_1 = 100\ \text{m}$ $\quad s_4 = 150\ \text{m}$

$s_2 = 200\ \text{m}$

Solution:

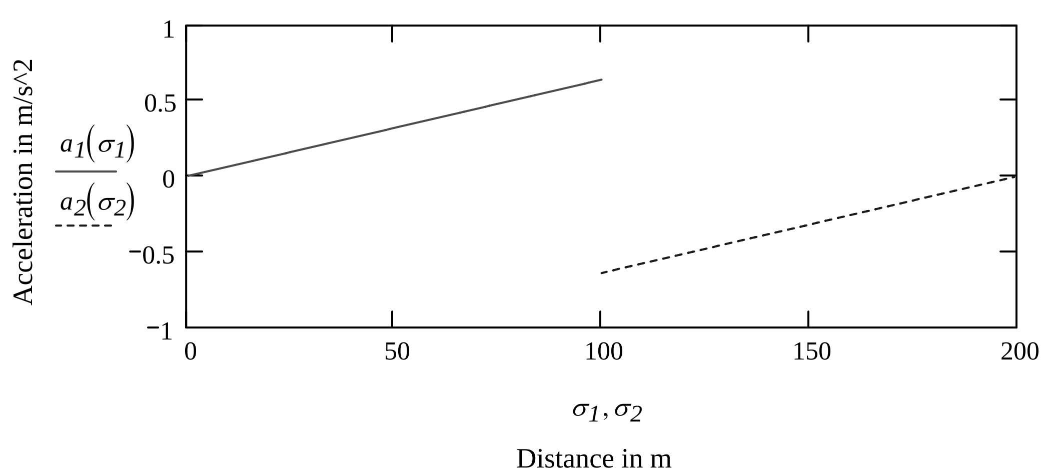
For $0 < s < s_1$ $\quad a = v\frac{dv}{ds} = v\frac{v_1}{s_1}$ $\quad a_3 = \frac{s_3}{s_1}v_1\frac{v_1}{s_1}$ $\quad \boxed{a_3 = 0.32\ \frac{\text{m}}{\text{s}^2}}$

For $s_1 < s < s_2$ $\quad a = v\frac{dv}{ds} = -v\frac{v_1}{s_2 - s_1}$

$$a_4 = -\frac{s_2 - s_4}{s_2 - s_1}v_1\frac{v_1}{s_2 - s_1} \qquad \boxed{a_4 = -0.32\ \frac{\text{m}}{\text{s}^2}}$$

$\sigma_1 = 0, 0.01s_1 .. s_1$ $\quad a_1(\sigma_1) = \frac{\sigma_1}{s_1}\frac{v_1^2}{s_1}\frac{\text{s}^2}{\text{m}}$

$\sigma_2 = s_1, 1.01s_1 .. s_2$ $\quad a_2(\sigma_2) = -\frac{s_2 - \sigma_2}{s_2 - s_1}\frac{v_1^2}{s_2 - s_1}\frac{\text{s}^2}{\text{m}}$



*__Problem 12–60__

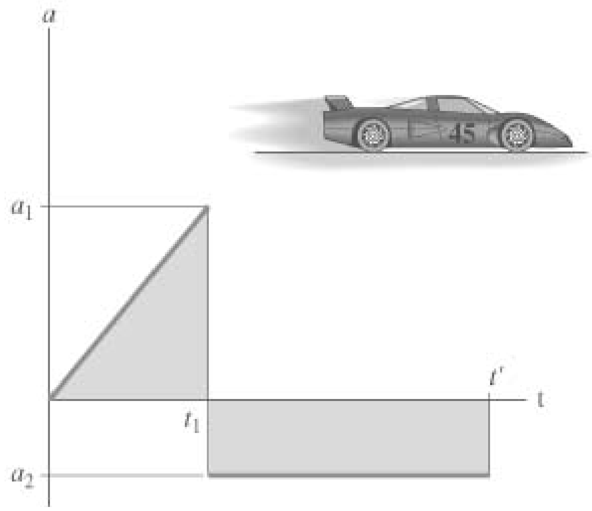
The *a–t* graph for a car is shown. Construct the *v–t* and *s–t* graphs if the car starts from rest at $t = 0$. At what time t' does the car stop?

Given:

$$a_1 = 5\ \frac{\text{m}}{\text{s}^2}$$

$$a_2 = -2\ \frac{\text{m}}{\text{s}^2}$$

$$t_1 = 10\ \text{s}$$



Solution:

$$k = \frac{a_1}{t_1}$$

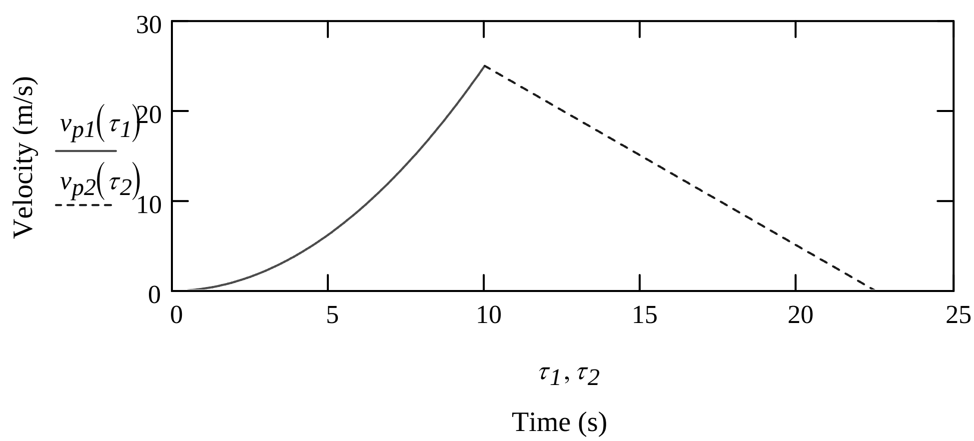
$$a_{p1}(t) = kt \qquad v_{p1}(t) = k\frac{t^2}{2} \qquad s_{p1}(t) = k\frac{t^3}{6}$$

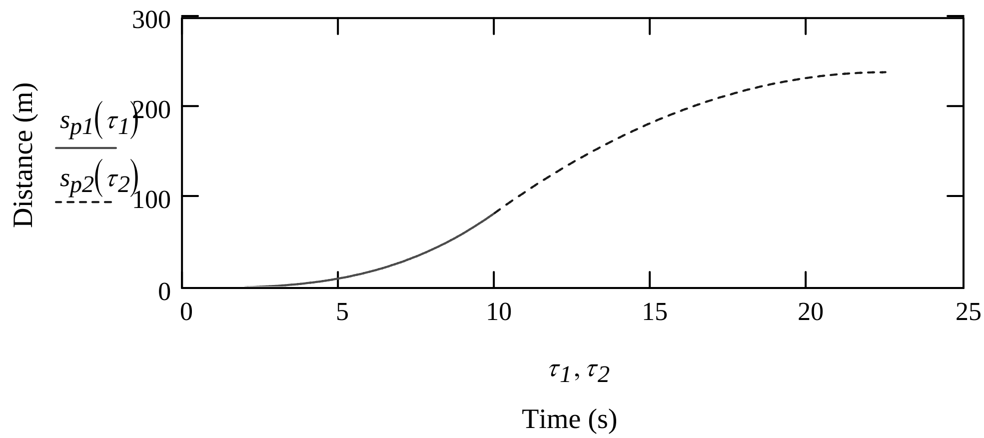
$$a_{p2}(t) = a_2 \qquad v_{p2}(t) = v_{p1}(t_1) + a_2(t - t_1)$$

$$s_{p2}(t) = s_{p1}(t_1) + v_{p1}(t_1)(t - t_1) + \frac{1}{2}a_2(t - t_1)^2$$

Guess $t' = 12\ \text{s}$ Given $v_{p2}(t') = 0$ $t' = \text{Find}(t')$ $t' = 22.5\ \text{s}$

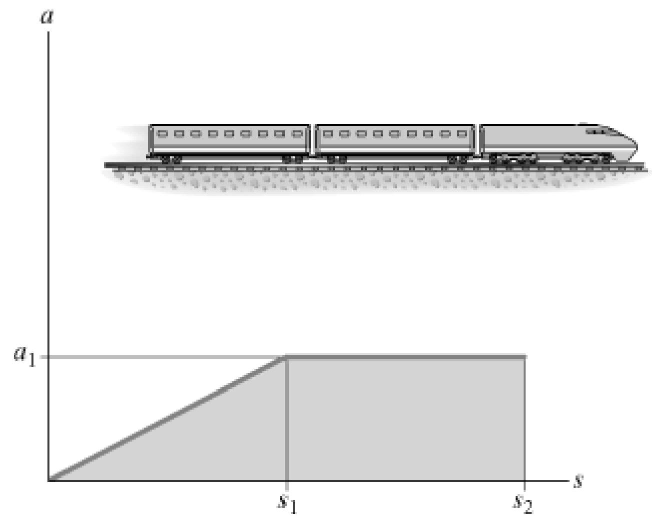
$$\tau_1 = 0, 0.01t_1 .. t_1 \qquad \tau_2 = t_1, 1.01t_1 .. t'$$





Problem 12-61

The *a*-*s* graph for a train traveling along a straight track is given for $0 \le s \le s_2$. Plot the *v*-*s* graph. $v = 0$ at $s = 0$.



Given:

$$s_1 = 200 \text{ m}$$

$$s_2 = 400 \text{ m}$$

$$a_1 = 2 \frac{\text{m}}{\text{s}^2}$$

Solution:

$$\sigma_1 = 0, 0.01s_1 .. s_1$$

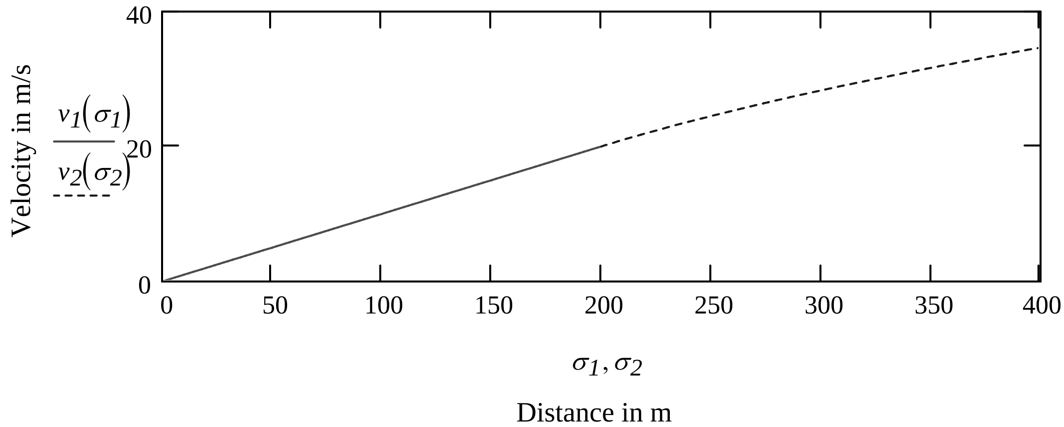
$$\sigma_2 = s_1, 1.01s_1 .. s_2$$

For $0 < s < s_1$ $\quad k = \dfrac{a_1}{s_1} \quad a_{c1} = ks$

$$a = ks = v\frac{dv}{ds} \qquad \int_0^v v\, dv = \int_0^s ks\, ds \qquad \frac{v^2}{2} = \frac{k}{2}s^2 \qquad v_1(\sigma_1) = \sqrt{k}\,\sigma_1 \frac{\text{s}}{\text{m}}$$

For $s_1 < s < s_2$ $\quad a_{c2} = a_1 \quad a = ks = v\dfrac{dv}{ds} \quad \displaystyle\int_{v_1}^v v\, dv = \int_{s_1}^s a_1\, ds$

$$\frac{v^2}{2} - \frac{v_1^2}{2} = a_1(s - s_1) \qquad v_2(\sigma_2) = \sqrt{2a_1(\sigma_2 - s_1) + ks_1^2}\,\frac{\text{s}}{\text{m}}$$



Problem 12-62

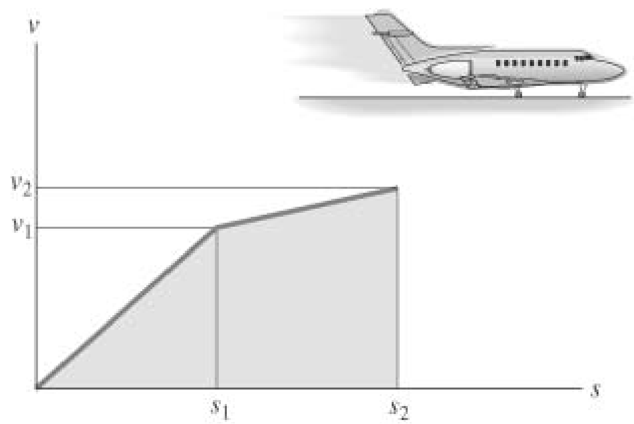
The *v*-*s* graph for an airplane traveling on a straight runway is shown. Determine the acceleration of the plane at $s = s_3$ and $s = s_4$. Draw the *a*-*s* graph.

Given:

$s_1 = 100 \text{ m}$ $\quad s_4 = 150 \text{ m}$

$s_2 = 200 \text{ m}$ $\quad v_1 = 40 \frac{\text{m}}{\text{s}}$

$s_3 = 50 \text{ m}$ $\quad v_2 = 50 \frac{\text{m}}{\text{s}}$



Solution:

$$a = v\frac{dv}{ds}$$

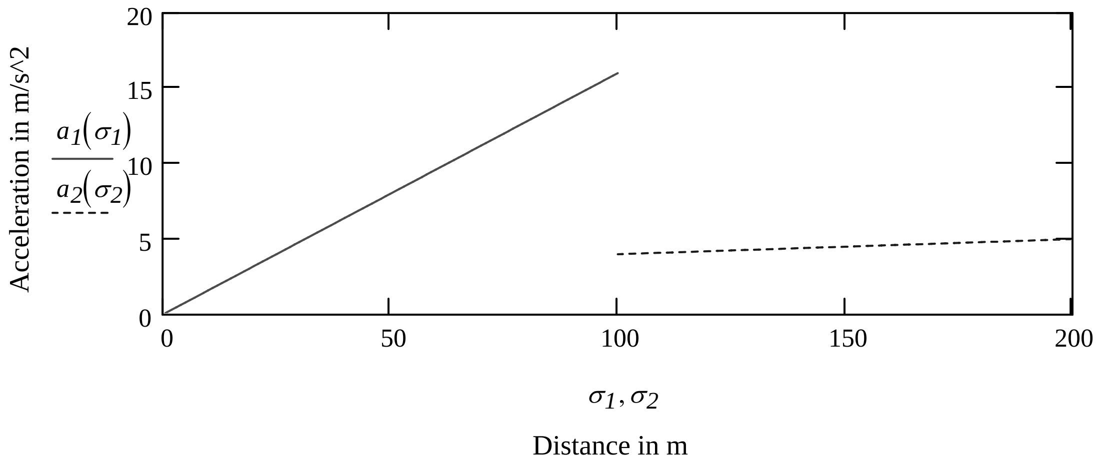
$0 < s_3 < s_1$ $\quad a_3 = \left(\frac{s_3}{s_1}\right) v_1 \left(\frac{v_1}{s_1}\right)$ $\quad a_3 = 8 \frac{\text{m}}{\text{s}^2}$

$s_1 < s_4 < s_2$ $\quad a_4 = \left[v_1 + \frac{s_4 - s_1}{s_2 - s_1}(v_2 - v_1)\right]\frac{v_2 - v_1}{s_2 - s_1}$ $\quad a_4 = 4.5 \frac{\text{m}}{\text{s}^2}$

The graph

$\sigma_1 = 0, 0.01s_1 .. s_1$ $\quad a_1(\sigma_1) = \frac{\sigma_1}{s_1}\frac{v_1^2}{s_1}\frac{\text{s}^2}{\text{m}}$

$\sigma_2 = s_1, 1.01s_1 .. s_2$ $\quad a_2(\sigma_2) = \left[v_1 + \frac{\sigma_2 - s_1}{s_2 - s_1}(v_2 - v_1)\right]\frac{v_2 - v_1}{s_2 - s_1}\frac{\text{s}^2}{\text{m}}$



Problem 12-63

Starting from rest at $s = 0$, a boat travels in a straight line with an acceleration as shown by the *a-s* graph. Determine the boat's speed when $s = s_4$, s_5, and s_6.

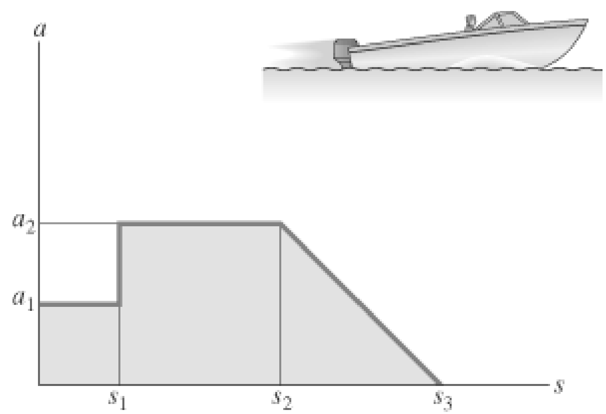
Given:

$s_1 = 50\ \text{ft}$ $\quad s_5 = 90\ \text{ft}$

$s_2 = 150\ \text{ft}$ $\quad s_6 = 200\ \text{ft}$

$s_3 = 250\ \text{ft}$ $\quad a_1 = 2\ \dfrac{\text{ft}}{\text{s}^2}$

$s_4 = 40\ \text{ft}$ $\quad a_2 = 4\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$0 < s_4 < s_1 \qquad a_4 = a_1 \qquad v_4 = \sqrt{2a_4 s_4}$ $\qquad v_4 = 12.649\ \dfrac{\text{ft}}{\text{s}}$

$$v_1 = \sqrt{2a_1 s_1}$$

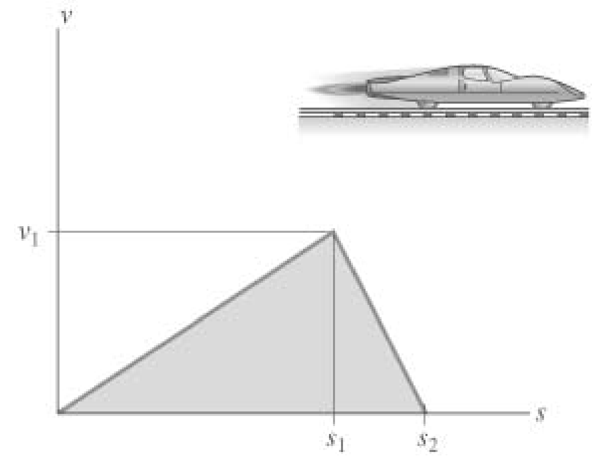
$s_1 < s_5 < s_2 \qquad a_5 = a_2 \qquad v_5 = \sqrt{2a_5(s_5 - s_1) + v_1^2}$ $\qquad v_5 = 22.804\ \dfrac{\text{ft}}{\text{s}}$

$$v_2 = \sqrt{2a_2(s_2 - s_1) + v_1^2}$$

$s_2 < s_6 < s_3 \qquad v_6 = \sqrt{v_2^2 + 2\int_{s_2}^{s_6} \dfrac{s_3 - s}{s_3 - s_2} a_2\, ds}$ $\qquad v_6 = 36.056\ \dfrac{\text{ft}}{\text{s}}$

***Problem 12–64**

The *v–s* graph for a test vehicle is shown. Determine its acceleration at $s = s_3$ and s_4.



Given:

$$v_1 = 50\ \frac{\text{m}}{\text{s}}$$

$$s_1 = 150\ \text{m} \quad s_3 = 100\ \text{m}$$

$$s_2 = 200\ \text{m} \quad s_4 = 175\ \text{m}$$

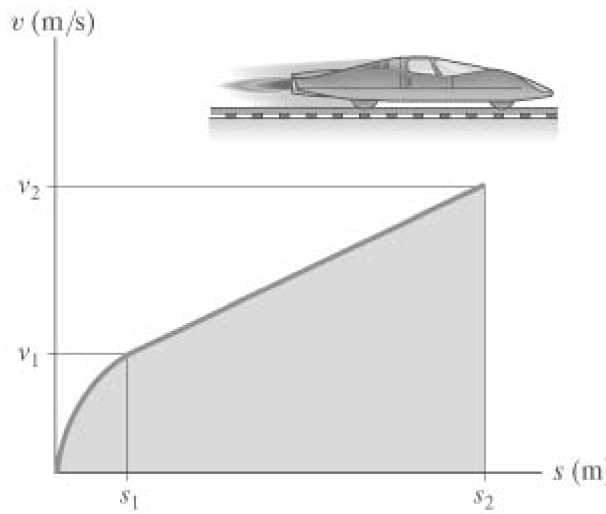
Solution:

$$a_3 = \left(\frac{s_3}{s_1}\right) v_1 \left(\frac{v_1}{s_1}\right) \qquad a_3 = 11.11\ \frac{\text{m}}{\text{s}^2}$$

$$a_4 = \left(\frac{s_2 - s_4}{s_2 - s_1}\right) v_1 \left(\frac{0 - v_1}{s_2 - s_1}\right) \qquad a_4 = -25\ \frac{\text{m}}{\text{s}^2}$$

Problem 12–65

The *v–s* graph was determined experimentally to describe the straight-line motion of a rocket sled. Determine the acceleration of the sled at $s = s_3$ and $s = s_4$.



Given:

$$v_1 = 20\ \frac{\text{m}}{\text{s}} \qquad s_1 = 50\ \text{m}$$

$$v_2 = 60\ \frac{\text{m}}{\text{s}} \qquad s_2 = 300\ \text{m}$$

$$s_3 = 100\ \text{m} \qquad s_4 = 200\ \text{m}$$

Solution:

$$a = v\frac{dv}{ds}$$

$$a_3 = \left[\frac{s_3 - s_1}{s_2 - s_1}(v_2 - v_1) + v_1\right]\frac{v_2 - v_1}{s_2 - s_1} \qquad a_3 = 4.48\ \frac{\text{m}}{\text{s}^2}$$

$$a_4 = \left[\frac{s_4 - s_1}{s_2 - s_1}(v_2 - v_1) + v_1\right]\frac{v_2 - v_1}{s_2 - s_1} \qquad a_4 = 7.04\ \frac{\text{m}}{\text{s}^2}$$

Problem 12-66

A particle, originally at rest and located at point (a, b, c), is subjected to an acceleration $\mathbf{a_c} = \{d\, t\, \mathbf{i} + e\, t^2\, \mathbf{k}\}$. Determine the particle's position (x, y, z) at time t_1.

Given: $a = 3\ \text{ft}$ $b = 2\ \text{ft}$ $c = 5\ \text{ft}$ $d = 6\ \dfrac{\text{ft}}{\text{s}^3}$ $e = 12\ \dfrac{\text{ft}}{\text{s}^4}$ $t_1 = 1\ \text{s}$

Solution:

$a_x = d\,t$ $v_x = \left(\dfrac{d}{2}\right)t^2$ $s_x = \left(\dfrac{d}{6}\right)t^3 + a$ $x = \left(\dfrac{d}{6}\right)t_1^{\ 3} + a$ $x = 4\,\text{ft}$

$a_y = 0$ $v_y = 0$ $s_y = b$ $y = b$ $y = 2\,\text{ft}$

$a_z = e\,t^2$ $v_z = \left(\dfrac{e}{3}\right)t^3$ $s_z = \left(\dfrac{e}{12}\right)t^4 + c$ $z = \left(\dfrac{e}{12}\right)t_1^{\ 4} + c$ $z = 6\,\text{ft}$

Problem 12-67

The velocity of a particle is given by $\mathbf{v} = [at^2\mathbf{i} + bt^3\mathbf{j} + (ct + d)\mathbf{k}]$. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = t_1$. Also, what is the x, y, z coordinate position of the particle at this instant?

Given: $a = 16\ \dfrac{\text{m}}{\text{s}^3}$ $b = 4\ \dfrac{\text{m}}{\text{s}^4}$ $c = 5\ \dfrac{\text{m}}{\text{s}^2}$ $d = 2\ \dfrac{\text{m}}{\text{s}}$ $t_1 = 2\ \text{s}$

Solution:

Acceleration

$a_x = 2a\,t_1$ $a_x = 64\ \dfrac{\text{m}}{\text{s}^2}$

$a_y = 3b\,t_1^{\ 2}$ $a_y = 48\ \dfrac{\text{m}}{\text{s}^2}$

$a_z = c$ $a_z = 5\ \dfrac{\text{m}}{\text{s}^2}$

$a_{mag} = \sqrt{a_x^{\ 2} + a_y^{\ 2} + a_z^{\ 2}}$ $a_{mag} = 80.2\ \dfrac{\text{m}}{\text{s}^2}$

Postition

$x = \dfrac{a}{3}t_1^{\ 3}$ $x = 42.667\ \text{m}$

$y = \dfrac{b}{4}t_1^{\ 4}$ $y = 16\ \text{m}$

$z = \frac{c}{2}t_1^2 + d\,t_1$ $z = 14\ \text{m}$

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*__Problem 12-68__

A particle is traveling with a velocity of $\mathbf{v} = \left(a\sqrt{t}\,e^{b\,t}\mathbf{i} + c\,e^{d\,t^2}\mathbf{j}\right)$. Determine the magnitude of the particle's displacement from t = 0 to t_1. Use Simpson's rule with n steps to evaluate the integrals. What is the magnitude of the particle's acceleration when $t = t_2$?

Given: $a = 3\ \frac{\text{m}}{\text{s}^{\frac{3}{2}}}$ $b = -0.2\ \frac{1}{\text{s}}$ $c = 4\ \frac{\text{m}}{\text{s}}$ $d = -0.8\ \frac{1}{\text{s}^2}$ $t_1 = 3\ \text{s}$ $t_2 = 2\ \text{s}$

$n = 100$

Displacement

$x_1 = \int_0^{t_1} a\sqrt{t}\,e^{b\,t}\,dt$ $x_1 = 7.34\ \text{m}$ $y_1 = \int_0^{t_1} c\,e^{d\,t^2}\,dt$ $y_1 = 3.96\ \text{m}$

$d_1 = \sqrt{x_1^2 + y_1^2}$ $d_1 = 8.34\ \text{m}$

Acceleration

$a_x = \frac{d}{dt}\left(a\sqrt{t}\,e^{b\,t}\right) = \frac{a}{2\sqrt{t}}e^{bt} + a\,b\sqrt{t}\,e^{b\,t}$ $a_{x2} = \frac{a}{\sqrt{t_2}}e^{b\,t_2}\left(\frac{1}{2} + b\,t_2\right)$

$a_y = \frac{d}{dt}\left(c\,e^{d\,t^2}\right) = 2c\,d\,t\,e^{d\,t^2}$ $a_{y2} = 2c\,d\,t_2\,e^{d\,t_2^2}$

$a_{x2} = 0.14\ \frac{\text{m}}{\text{s}^2}$ $a_{y2} = -0.52\ \frac{\text{m}}{\text{s}^2}$ $a_2 = \sqrt{a_{x2}^2 + a_{y2}^2}$ $a_2 = 0.541\ \frac{\text{m}}{\text{s}^2}$

Problem 12-69

The position of a particle is defined by $r = \{a\cos(bt)\,\mathbf{i} + c\sin(bt)\,\mathbf{j}\}$. Determine the magnitudes of the velocity and acceleration of the particle when $t = t_1$. Also, prove that the path of the particle is elliptical.

Given: $a = 5\ \text{m}$ $b = 2\ \frac{\text{rad}}{\text{s}}$ $c = 4\ \text{m}$ $t_1 = 1\ \text{s}$

Velocities

$v_{x1} = -a\,b\sin(b\,t_1)$ $v_{y1} = c\,b\cos(b\,t_1)$ $v_1 = \sqrt{v_{x1}^2 + v_{y1}^2}$

$v_{x1} = -9.093\ \dfrac{\text{m}}{\text{s}}$ $\qquad v_{y1} = -3.329\ \dfrac{\text{m}}{\text{s}}$ $\qquad v_1 = 9.683\ \dfrac{\text{m}}{\text{s}}$

Accelerations

$$a_{x1} = -a b^2 \cos(b t_1) \qquad a_{y1} = -c b^2 \sin(b t_1) \qquad a_1 = \sqrt{a_{x1}^2 + a_{y1}^2}$$

$$a_{x1} = 8.323\ \frac{\text{m}}{\text{s}^2} \qquad a_{y1} = -14.549\ \frac{\text{m}}{\text{s}^2} \qquad a_1 = 16.761\ \frac{\text{m}}{\text{s}^2}$$

Path

$$\frac{x}{a} = \cos(b t) \qquad \frac{y}{c} = \sin(b t) \qquad \text{Thus} \qquad \left(\frac{x}{a}\right)^2 + \left(\frac{y}{c}\right)^2 = 1 \qquad \text{QED}$$

Problem 12–70

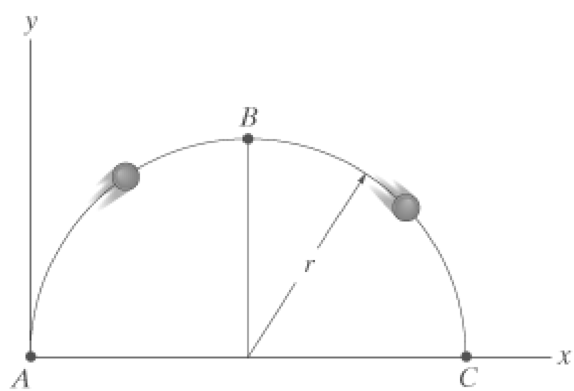
A particle travels along the curve from A to B in time t_1. If it takes time t_2 for it to go from A to C, determine its *average velocity* when it goes from B to C.

Given:

$t_1 = 1\ \text{s}$

$t_2 = 3\ \text{s}$

$r = 20\ \text{m}$



Solution:

$$\mathbf{r}_{\mathbf{AC}} = \begin{pmatrix} 2r \\ 0 \end{pmatrix}$$

$$\mathbf{r}_{\mathbf{AB}} = \begin{pmatrix} r \\ r \end{pmatrix}$$

$$\mathbf{v}_{\mathbf{ave}} = \frac{\mathbf{r}_{\mathbf{AC}} - \mathbf{r}_{\mathbf{AB}}}{t_2 - t_1} \qquad \mathbf{v}_{\mathbf{ave}} = \begin{pmatrix} 10 \\ -10 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

Problem 12-71

A particle travels along the curve from A to B in time t_1. It takes time t_2 for it to go from B to C and then time t_3 to go from C to D. Determine its average speed when it goes from A to D.

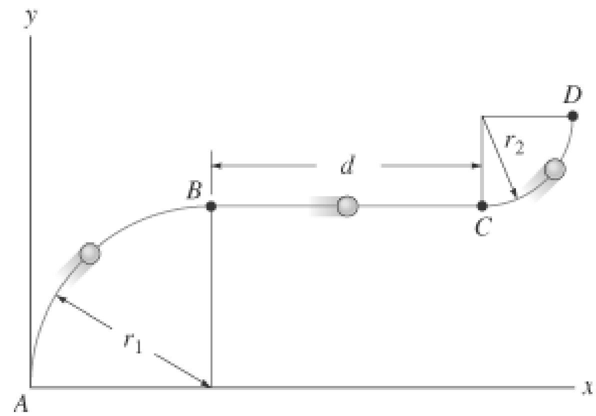
Given:

$t_1 = 2\ \text{s} \quad r_1 = 10\ \text{m}$

$t_2 = 4\ \text{s} \quad d = 15\ \text{m}$

$t_3 = 3\ \text{s} \quad r_2 = 5\ \text{m}$

Solution:



$$d = \left(\frac{\pi r_1}{2}\right) + d + \left(\frac{\pi r_2}{2}\right)$$

$$t = t_1 + t_2 + t_3 \qquad v_{ave} = \frac{d}{t}$$

$$v_{ave} = 4.285\ \frac{\text{m}}{\text{s}}$$

***Problem 12-72**

A car travels east a distance d_1 for time t_1, then north a distance d_2 for time t_2 and then west a distance d_3 for time t_3. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

Given: $d_1 = 2\ \text{km} \qquad d_2 = 3\ \text{km} \qquad d_3 = 4\ \text{km}$

$t_1 = 5\ \text{min} \qquad t_2 = 8\ \text{min} \qquad t_3 = 10\ \text{min}$

Solution:

Total Distance Traveled and Displacement: The total distance traveled is

$$s = d_1 + d_2 + d_3 \qquad s = 9\,\text{km}$$

and the magnitude of the displacement is

$$\Delta r = \sqrt{\left(d_1 - d_3\right)^2 + d_2{}^2} \qquad \Delta r = 3.606\,\text{km}$$

Average Velocity and Speed: The total time is $\Delta t = t_1 + t_2 + t_3 \qquad \Delta t = 1380\ \text{s}$

The magnitude of average velocity is

$$v_{avg} = \frac{\Delta r}{\Delta t} \qquad v_{avg} = 2.61\ \frac{\text{m}}{\text{s}}$$

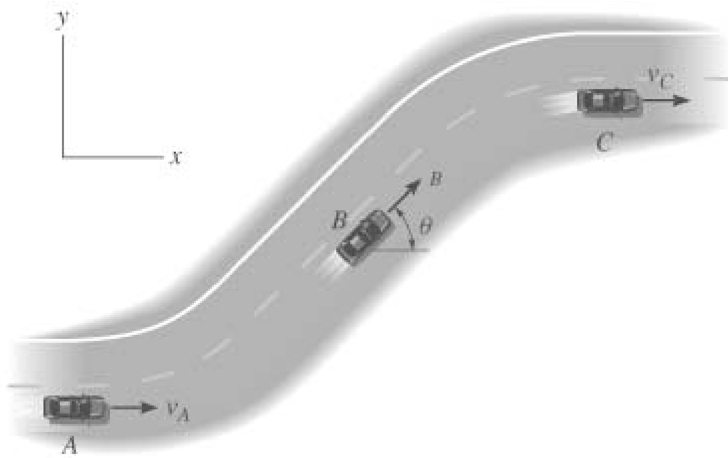
and the average speed is

$$v_{spavg} = \frac{s}{\Delta t} \qquad v_{spavg} = 6.522\ \frac{\text{m}}{\text{s}}$$

Problem 12-73

A car traveling along the straight portions of the road has the velocities indicated in the figure when it arrives at points *A*, *B*, and *C*. If it takes time t_{AB} to go from *A* to *B*, and then time t_{BC} to go from *B* to *C*, determine the average acceleration between points *A* and *B* and between points *A* and *C*.

Given:



$t_{AB} = 3 \text{ s}$

$t_{BC} = 5 \text{ s}$

$v_A = 20 \dfrac{\text{m}}{\text{s}}$

$v_B = 30 \dfrac{\text{m}}{\text{s}}$

$v_C = 40 \dfrac{\text{m}}{\text{s}}$

$\theta = 45 \text{ deg}$

Solution:

$$\mathbf{v_{Bv}} = v_B \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \qquad \mathbf{v_{Av}} = v_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \mathbf{v_{Cv}} = v_C \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{a_{ABave}} = \frac{\mathbf{v_{Bv}} - \mathbf{v_{Av}}}{t_{AB}} \qquad \mathbf{a_{ABave}} = \begin{pmatrix} 0.404 \\ 7.071 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

$$\mathbf{a_{ACave}} = \frac{\mathbf{v_{Cv}} - \mathbf{v_{Av}}}{t_{AB} + t_{BC}} \qquad \mathbf{a_{ACave}} = \begin{pmatrix} 2.5 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 12-74

A particle moves along the curve $y = ae^{bx}$ such that its velocity has a constant magnitude of $v = v_0$. Determine the *x* and *y* components of velocity when the particle is at $y = y_1$.

Given: $a = 1 \text{ ft}$ $\quad b = \dfrac{2}{\text{ft}}$ $\quad v_0 = 4 \dfrac{\text{ft}}{\text{s}}$ $\quad y_1 = 5 \text{ ft}$

In general we have

$$y = a e^{b x} \qquad v_y = a b e^{b x} v_x$$

$$v_x^2 + v_y^2 = v_x^2\left(1 + a^2 b^2 e^{2bx}\right) = v_0^2$$

$$v_x = \frac{v_0}{\sqrt{1 + a^2 b^2 e^{2bx}}} \qquad v_y = \frac{a b e^{bx} v_0}{\sqrt{1 + a^2 b^2 e^{2bx}}}$$

In specific case

$$x_1 = \frac{1}{b}\ln\left(\frac{y_1}{a}\right)$$

$$v_{x1} = \frac{v_0}{\sqrt{1 + a^2 b^2 e^{2bx_1}}} \qquad v_{x1} = 0.398\ \frac{\text{ft}}{\text{s}}$$

$$v_{y1} = \frac{a b e^{bx_1} v_0}{\sqrt{1 + a^2 b^2 e^{2bx_1}}} \qquad v_{y1} = 3.980\ \frac{\text{ft}}{\text{s}}$$

Problem 12-75

The path of a particle is defined by $y^2 = 4kx$, and the component of velocity along the y axis is $v_y = ct$, where both k and c are constants. Determine the x and y components of acceleration.

Solution:

$$y^2 = 4kx$$

$$2yv_y = 4kv_x$$

$$2v_y^2 + 2ya_y = 4ka_x$$

$$v_y = ct \qquad a_y = c$$

$$2(ct)^2 + 2yc = 4ka_x$$

$$a_x = \frac{c}{2k}\left(y + ct^2\right)$$

---

*__Problem 12-76__

A particle is moving along the curve $y = x - (x^2/a)$. If the velocity component in the x direction is $v_x = v_0$. and changes at the rate a_0, determine the magnitudes of the velocity and acceleration when $x = x_1$.

Given: $a = 400 \text{ ft}$ $\quad v_0 = 2 \frac{\text{ft}}{\text{s}}$ $\quad a_0 = 0 \frac{\text{ft}}{\text{s}^2}$ $\quad x_1 = 20 \text{ ft}$

Solution:

Velocity: Taking the first derivative of the path $y = x - \left(\frac{x^2}{a}\right)$ we have,

$$v_y = v_x\left(1 - \frac{2x}{a}\right) = v_0\left(1 - \frac{2x}{a}\right)$$

$$v_{x1} = v_0 \qquad v_{y1} = v_0\left(1 - \frac{2x_1}{a}\right) \qquad v_1 = \sqrt{v_{x1}^2 + v_{y1}^2}$$

$$v_{x1} = 2\frac{\text{ft}}{\text{s}} \qquad v_{y1} = 1.8\frac{\text{ft}}{\text{s}} \qquad v_1 = 2.691\frac{\text{ft}}{\text{s}}$$

Acceleration: Taking the second derivative:

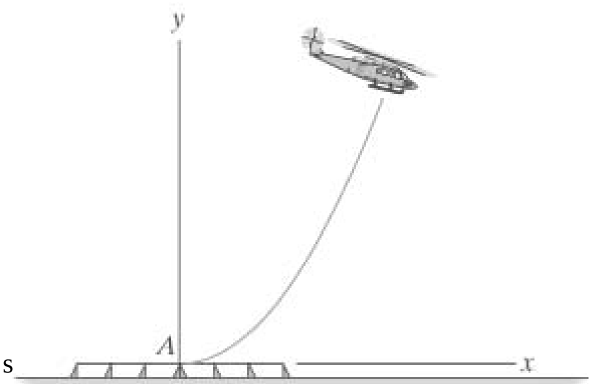
$$a_y = a_x\left(1 - \frac{2x}{a}\right) - 2\left(\frac{v_x^2}{a}\right) = a_0\left(1 - \frac{2x}{a}\right) - 2\left(\frac{v_0^2}{a}\right)$$

$$a_{x1} = a_0 \qquad a_{y1} = a_0\left(1 - \frac{2x_1}{a}\right) - 2\left(\frac{v_0^2}{a}\right) \qquad a_1 = \sqrt{a_{x1}^2 + a_{y1}^2}$$

$$a_{x1} = 0\frac{\text{ft}}{\text{s}^2} \qquad a_{y1} = -0.0200\frac{\text{ft}}{\text{s}^2} \qquad a_1 = 0.0200\frac{\text{ft}}{\text{s}^2}$$

Problem 12–77

The flight path of the helicopter as it takes off from *A* is defined by the parametric equations $x = bt^2$ and $y = ct^3$. Determine the distance the helicopter is from point *A* and the magnitudes of its velocity and acceleration when $t = t_1$.



Given:

$b = 2 \frac{\text{m}}{\text{s}^2}$ $\quad c = 0.04 \frac{\text{m}}{\text{s}^3}$ $\quad t_1 = 10 \text{ s}$

Solution:

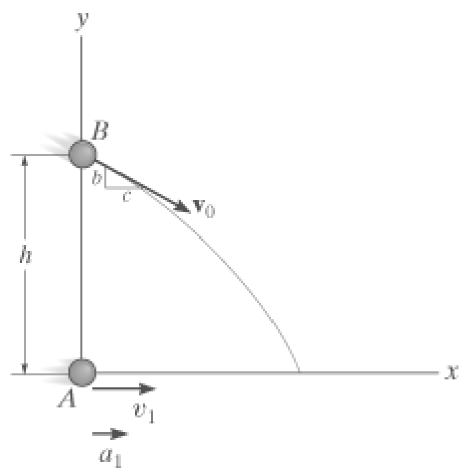
$$\mathbf{r_1} = \begin{pmatrix} b t_1^2 \\ c t_1^3 \end{pmatrix} \qquad \mathbf{v_1} = \begin{pmatrix} 2b t_1 \\ 3c t_1^2 \end{pmatrix} \qquad \mathbf{a_1} = \begin{pmatrix} 2b \\ 6c t_1 \end{pmatrix}$$

$$\mathbf{r_1} = \begin{pmatrix} 200 \\ 40 \end{pmatrix} \text{m} \qquad \mathbf{v_1} = \begin{pmatrix} 40 \\ 12 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_1} = \begin{pmatrix} 4 \\ 2.4 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

$$\left|\mathbf{r_1}\right| = 204 \text{ m} \qquad \left|\mathbf{v_1}\right| = 41.8 \frac{\text{m}}{\text{s}} \qquad \left|\mathbf{a_1}\right| = 4.66 \frac{\text{m}}{\text{s}^2}$$

Problem 12–78

At the instant shown particle A is traveling to the right at speed v_1 and has an acceleration a_1. Determine the initial speed v_0 of particle B so that when it is fired at the same instant from the angle shown it strikes A. Also, at what speed does it strike A?



Given:

$$v_1 = 10 \frac{\text{ft}}{\text{s}} \qquad a_1 = 2 \frac{\text{ft}}{\text{s}^2}$$

$$b = 3 \qquad c = 4$$

$$h = 100 \text{ ft} \qquad g = 32.2 \frac{\text{ft}}{\text{s}^2}$$

Solution:

Guesses $\quad v_0 = 1 \frac{\text{ft}}{\text{s}} \qquad t = 1 \text{ s}$

Given $\quad v_1 t + \frac{1}{2} a_1 t^2 = \left(\frac{c}{\sqrt{b^2 + c^2}}\right) v_0 t \qquad h - \frac{1}{2} g t^2 - \left(\frac{b}{\sqrt{b^2 + c^2}}\right) v_0 t = 0$

$$\begin{pmatrix} v_0 \\ t \end{pmatrix} = \text{Find}(v_0, t) \qquad t = 2.224 \text{ s} \qquad v_0 = 15.28 \frac{\text{ft}}{\text{s}}$$

$$\mathbf{v_B} = \begin{pmatrix} \dfrac{c}{\sqrt{b^2 + c^2}} v_0 \\ -g t - \dfrac{b}{\sqrt{b^2 + c^2}} v_0 \end{pmatrix} \qquad \mathbf{v_B} = \begin{pmatrix} 12.224 \\ -80.772 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \left|\mathbf{v_B}\right| = 81.691 \frac{\text{ft}}{\text{s}}$$

Problem 12-79

When a rocket reaches altitude h_1 it begins to travel along the parabolic path $(y - h_1)^2 = b\,x$. If the component of velocity in the vertical direction is constant at $v_y = v_0$, determine the magnitudes of the rocket's velocity and acceleration when it reaches altitude h_2.

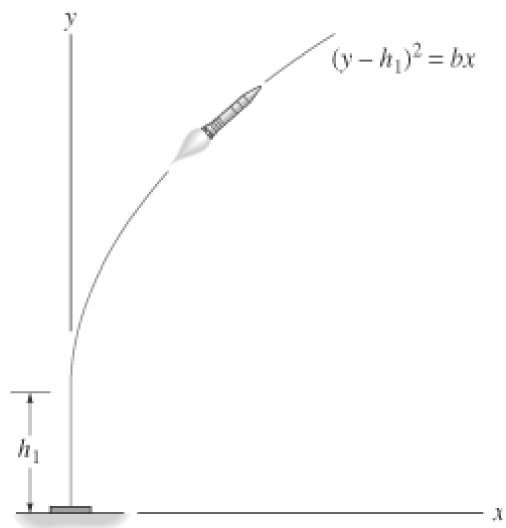
Given:

$h_1 = 40\ \text{m}$

$b = 160\ \text{m}$

$v_0 = 180\ \dfrac{\text{m}}{\text{s}}$

$h_2 = 80\ \text{m}$



Solution:

$$b\,x = (y - h_1)^2$$

$$b\,v_x = 2(y - h_1)v_y$$

$$b\,a_x = 2v_y^{\,2}$$

$v_{x2} = \dfrac{2}{b}(h_2 - h_1)v_0$ $\quad v_{y2} = v_0$ $\quad v_2 = \sqrt{v_{x2}^{\,2} + v_{y2}^{\,2}}$ $\quad v_2 = 201.246\ \dfrac{\text{m}}{\text{s}}$

$a_{x2} = \dfrac{2}{b}v_0^{\,2}$ $\quad a_{y2} = 0\ \dfrac{\text{m}}{\text{s}^2}$ $\quad a_2 = \sqrt{a_{x2}^{\,2} + a_{y2}^{\,2}}$ $\quad a_2 = 405\ \dfrac{\text{m}}{\text{s}^2}$

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*__Problem 12–80__

Determine the minimum speed of the stunt rider, so that when he leaves the ramp at A he passes through the center of the hoop at B. Also, how far h should the landing ramp be from the hoop so that he lands on it safely at C ? Neglect the size of the motorcycle and rider.

Given:

$a = 4\ \text{ft}$

$b = 24\ \text{ft}$

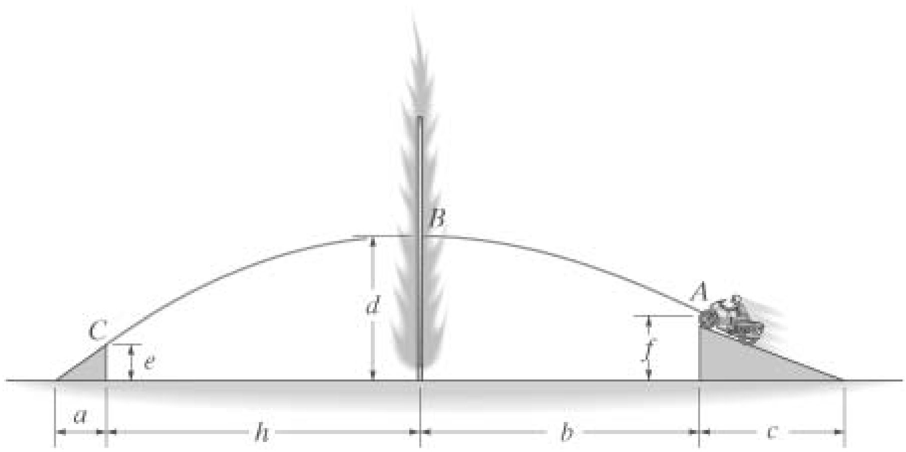
$c = 12\ \text{ft}$

$d = 12\ \text{ft}$

$e = 3\ \text{ft}$

$f = 5\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution: $\theta = \text{atan}\left(\dfrac{f}{c}\right)$

Guesses $v_A = 1\ \dfrac{\text{ft}}{\text{s}}$ $t_B = 1\ \text{s}$ $t_C = 1\ \text{s}$ $h = 1\ \text{ft}$

Given $b = v_A\cos(\theta)t_B$ $f + v_A\sin(\theta)t_B - \dfrac{1}{2}g\,t_B^{\,2} = d$

$b + h = v_A\cos(\theta)t_C$ $f + v_A\sin(\theta)t_C - \dfrac{1}{2}g\,t_C^{\,2} = e$

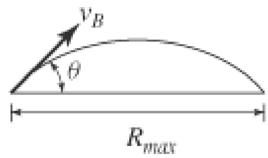
$$\begin{pmatrix} t_B \\ t_C \\ v_A \\ h \end{pmatrix} = \text{Find}(t_B, t_C, v_A, h) \qquad \begin{pmatrix} t_B \\ t_C \end{pmatrix} = \begin{pmatrix} 0.432 \\ 1.521 \end{pmatrix}\ \text{s} \qquad v_A = 60.2\ \frac{\text{ft}}{\text{s}}$$

$h = 60.5\ \text{ft}$

Problem 12–81

Show that if a projectile is fired at an angle θ from the horizontal with an initial velocity v_0, the *maximum* range the projectile can travel is given by $R_{max} = v_0^2/g$, where g is the acceleration of gravity. What is the angle θ for this condition?

Solution: After time t,



$$x = v_0 \cos(\theta) t \qquad t = \frac{x}{v_0 \cos(\theta)}$$

$$y = (v_0 \sin(\theta))t - \frac{1}{2} g t^2 \qquad y = x \tan(\theta) - \frac{g x^2}{2 v_0^2 \cos(\theta)^2}$$

Set $y = 0$ to determine the range, $x = R$:

$$R = \frac{2 v_0^2 \sin(\theta)\cos(\theta)}{g} = \frac{v_0^2 \sin(2\theta)}{g}$$

R_{max} occurs when $\sin(2\theta) = 1$ or, $\theta = 45\text{deg}$

This gives: $R_{max} = \dfrac{v_0^2}{g}$ Q.E.D

Problem 12-82

The balloon A is ascending at rate v_A and is being carried horizontally by the wind at v_w. If a ballast bag is dropped from the balloon when the balloon is at height h, determine the time needed for it to strike the ground. Assume that the bag was released from the balloon with the same velocity as the balloon. Also, with what speed does the bag strike the ground?

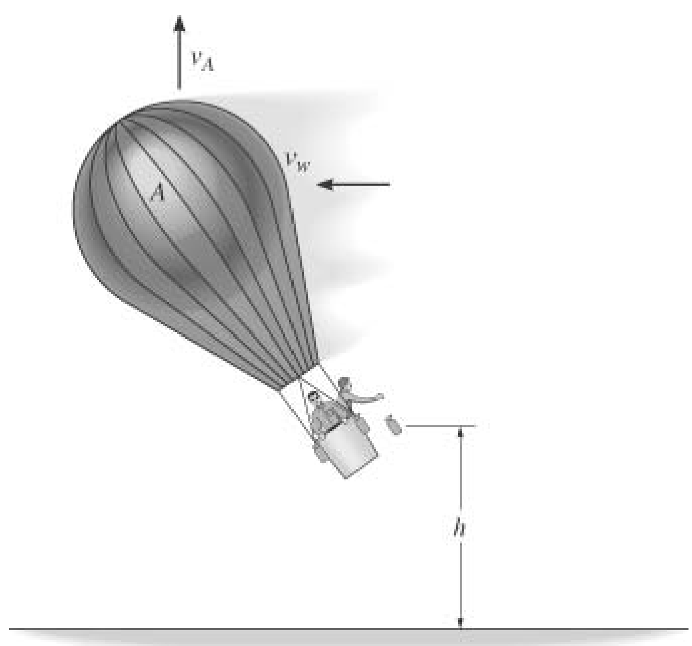
Given:

$$v_A = 12 \frac{\text{km}}{\text{hr}}$$

$$v_w = 20 \frac{\text{km}}{\text{hr}}$$

$$h = 50 \text{ m}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$



Solution:

$$a_x = 0 \qquad a_y = -g$$

$$v_x = v_w \qquad v_y = -g t + v_A$$

$s_x = v_w t \qquad s_y = \frac{-1}{2} g t^2 + v_A t + h$

Thus $\quad 0 = \frac{-1}{2} g t^2 + v_A t + h \qquad t = \frac{v_A + \sqrt{v_A^2 + 2gh}}{g} \qquad t = 3.551 \text{ s}$

$v_x = v_w \qquad v_y = -g t + v_A \qquad v = \sqrt{v_x^2 + v_y^2} \qquad v = 32.0 \frac{\text{m}}{\text{s}}$

Problem 12-83

Determine the height h on the wall to which the firefighter can project water from the hose, if the angle θ is as specified and the speed of the water at the nozzle is v_C.

Given:

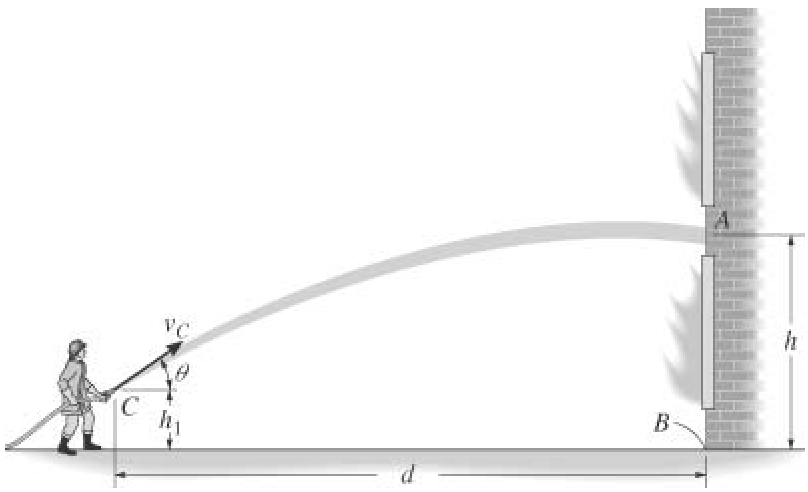
$v_C = 48 \frac{\text{ft}}{\text{s}}$

$h_1 = 3 \text{ ft}$

$d = 30 \text{ ft}$

$\theta = 40 \text{ deg}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

$a_x = 0 \qquad a_y = -g$

$v_x = v_C \cos(\theta) \qquad v_y = -g t + v_C \sin(\theta)$

$s_x = v_C \cos(\theta) t \qquad s_y = \left(\frac{-g}{2}\right) t^2 + v_C \sin(\theta) t + h_1$

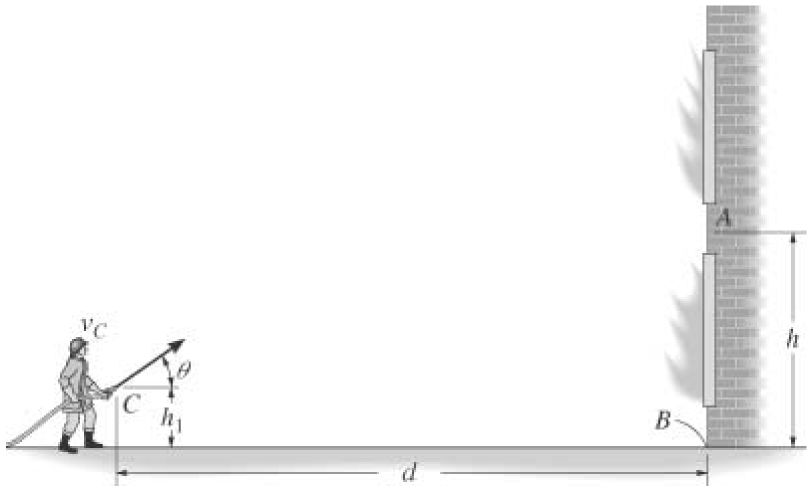
Guesses $\quad t = 1 \text{ s} \qquad h = 1 \text{ ft}$

Given $\quad d = v_C \cos(\theta) t \qquad h = \frac{-1}{2} g t^2 + v_C \sin(\theta) t + h_1$

$\begin{pmatrix} t \\ h \end{pmatrix} = \text{Find}(t, h) \qquad t = 0.816 \text{ s} \qquad h = 17.456 \text{ ft}$

***Problem 12-84**

Determine the smallest angle θ, measured above the horizontal, that the hose should be directed so that the water stream strikes the bottom of the wall at B. The speed of the water at the nozzle is v_C.



Given:

$$v_C = 48\ \frac{\text{ft}}{\text{s}}$$

$$h_1 = 3\ \text{ft}$$

$$d = 30\ \text{ft}$$

$$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

Solution:

$$a_x = 0 \qquad a_y = -g$$

$$v_x = v_C\cos(\theta) \qquad v_y = -g\,t + v_C\sin(\theta)$$

$$s_x = v_C\cos(\theta)t \qquad s_y = \frac{-g}{2}t^2 + v_C\sin(\theta)t + h_1$$

When it reaches the wall $\quad d = v_C\cos(\theta)t \qquad t = \dfrac{d}{v_C\cos(\theta)}$

$$0 = \frac{-g}{2}\left(\frac{d}{v_C\cos(\theta)}\right)^2 + v_C\sin(\theta)\frac{d}{v_C\cos(\theta)} + h_1 = \frac{d}{2\cos(\theta)^2}\left(\sin(2\theta) - \frac{d\,g}{v_C^{\,2}}\right) + h_1$$

Guess $\quad \theta = 10\ \text{deg}$

Given $\quad 0 = \dfrac{d}{2\cos(\theta)^2}\left(\sin(2\theta) - \dfrac{d\,g}{v_C^{\,2}}\right) + h_1 \qquad \theta = \text{Find}(\theta) \qquad \theta = 6.406\,\text{deg}$

Problem 12–85

The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes time t_1 to travel from A to B, determine the velocity v_A at which it was launched, the angle of release θ, and the height h.

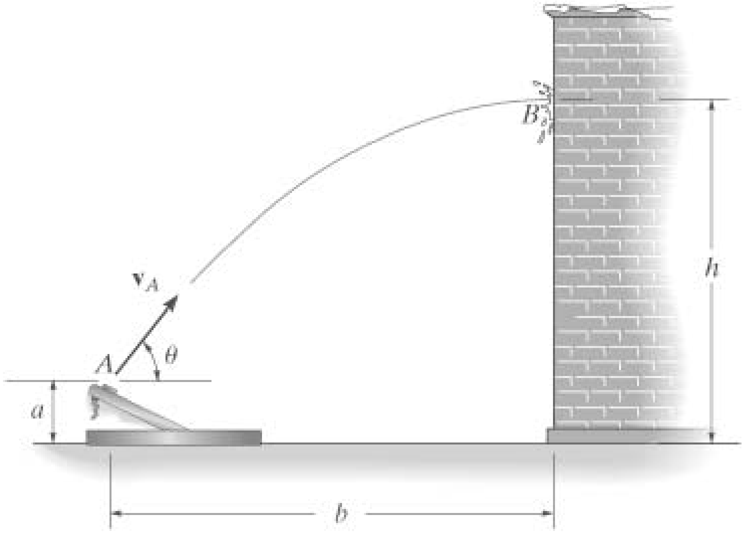
Given:

$a = 3.5 \text{ ft}$

$b = 18 \text{ ft}$

$t_1 = 1.5 \text{ s}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $v_A = 1 \frac{\text{ft}}{\text{s}}$ $\theta = 1 \text{ deg}$ $h = 1 \text{ ft}$

Given $v_A \cos(\theta) t_1 = b$ $v_A \sin(\theta) - g t_1 = 0$

$$a + v_A \sin(\theta) t_1 - \frac{1}{2} g t_1^2 = h$$

$$\begin{pmatrix} v_A \\ \theta \\ h \end{pmatrix} = \text{Find}(v_A, \theta, h) \qquad v_A = 49.8 \frac{\text{ft}}{\text{s}} \qquad \theta = 76 \text{ deg} \qquad h = 39.7 \text{ ft}$$

Problem 12–86

The buckets on the conveyor travel with a speed v. Each bucket contains a block which falls out of the bucket when $\theta = \theta_1$. Determine the distance d to where the block strikes the conveyor. Neglect the size of the block.

Given:

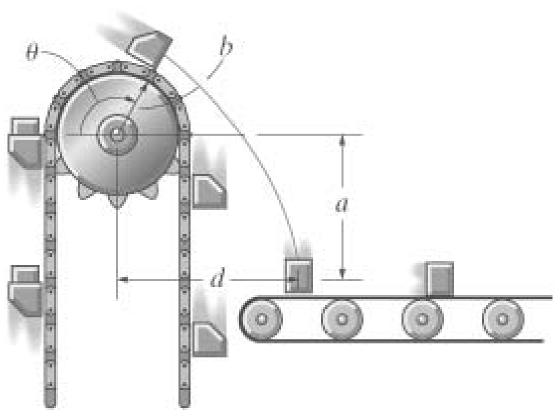
$a = 3 \text{ ft}$

$b = 1 \text{ ft}$

$\theta_1 = 120 \text{ deg}$

$v = 15 \frac{\text{ft}}{\text{s}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $d = 1\ \text{ft}$ $t = 1\ \text{s}$

Given $-b\cos(\theta_1) + v\sin(\theta_1)t = d$

$$a + b\sin(\theta_1) + v\cos(\theta_1)t - \frac{1}{2}g t^2 = 0$$

$\begin{pmatrix} d \\ t \end{pmatrix} = \text{Find}(d, t)$ $t = 0.31\ \text{s}$ $d = 4.52\,\text{ft}$

Problem 12-87

Measurements of a shot recorded on a videotape during a basketball game are shown. The ball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.

Given:

$a = 7\ \text{ft}$

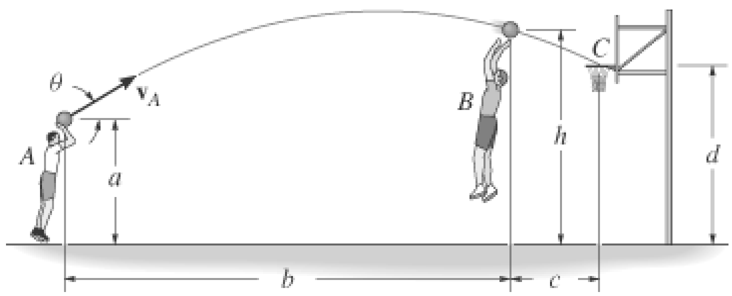
$b = 25\ \text{ft}$

$c = 5\ \text{ft}$

$d = 10\ \text{ft}$

$\theta = 30\ \text{deg}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $v_A = 10\ \dfrac{\text{ft}}{\text{s}}$ $t_B = 1\ \text{s}$ $t_C = 1\ \text{s}$ $h = 12\ \text{ft}$

Given $b + c = v_A\cos(\theta)t_C$ $b = v_A\cos(\theta)t_B$

$$d = \frac{-g}{2}t_C^{\,2} + v_A\sin(\theta)t_C + a \qquad h = \frac{-g}{2}t_B^{\,2} + v_A\sin(\theta)t_B + a$$

$\begin{pmatrix} v_A \\ t_B \\ t_C \\ h \end{pmatrix} = \text{Find}(v_A, t_B, t_C, h)$ $\begin{pmatrix} t_B \\ t_C \end{pmatrix} = \begin{pmatrix} 0.786 \\ 0.943 \end{pmatrix}\text{s}$ $v_A = 36.7\,\dfrac{\text{ft}}{\text{s}}$ $h = 11.489\,\text{ft}$

***Problem 12-88**

The snowmobile is traveling at speed v_0 when it leaves the embankment at A. Determine the time of flight from A to B and the range R of the trajectory.

Given:

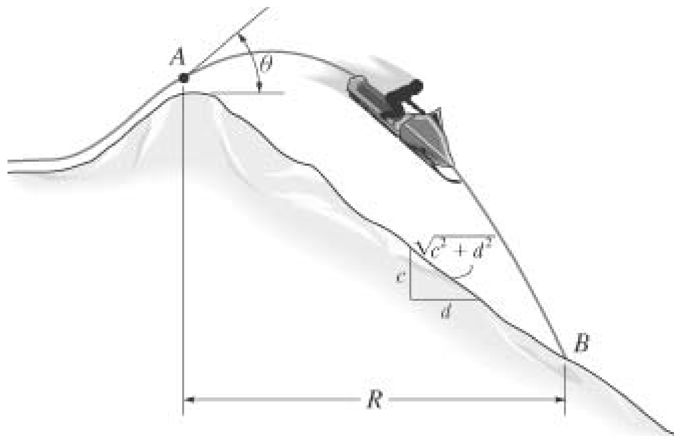
$v_0 = 10\ \frac{m}{s}$

$\theta = 40\ deg$

$c = 3$

$d = 4$

$g = 9.81\ \frac{m}{s^2}$



Solution:

Guesses $R = 1\ m$ $t = 1\ s$

Given $R = v_0 \cos(\theta) t$ $\left(\frac{-c}{d}\right) R = \left(\frac{-g}{2}\right) t^2 + v_0 \sin(\theta) t$

$\begin{pmatrix} R \\ t \end{pmatrix} = \mathrm{Find}(R, t)$ $t = 2.482\ s$ $R = 19.012\ m$

Problem 12–89

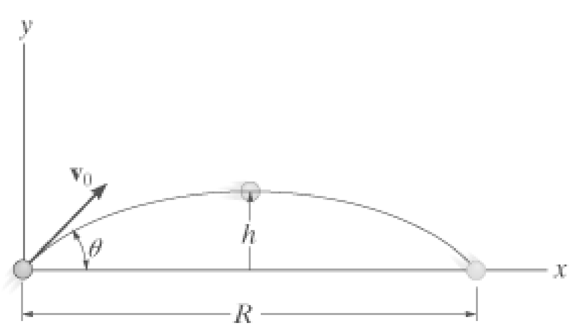
The projectile is launched with a velocity v_0. Determine the range R, the maximum height h attained, and the time of flight. Express the results in terms of the angle θ and v_0. The acceleration due to gravity is g.

Solution:

$a_x = 0$ $a_y = -g$

$v_x = v_0 \cos(\theta)$ $v_y = -g t + v_0 \sin(\theta)$

$s_x = v_0 \cos(\theta) t$ $s_y = \frac{-1}{2} g t^2 + v_0 \sin(\theta) t$



$0 = \frac{-1}{2} g t^2 + v_0 \sin(\theta) t$ $t = \frac{2 v_0 \sin(\theta)}{g}$

$R = v_0 \cos(\theta) t$ $\qquad$ $R = \frac{2v_0^2}{g} \sin(\theta)\cos(\theta)$

$h = \frac{-1}{2} g \left(\frac{t}{2}\right)^2 + v_0 \sin(\theta) \frac{t}{2}$ $\qquad$ $h = \frac{v_0^2 \sin(\theta)^2}{g}$

Problem 12–90

The fireman standing on the ladder directs the flow of water from his hose to the fire at B. Determine the velocity of the water at A if it is observed that the hose is held at angle θ.

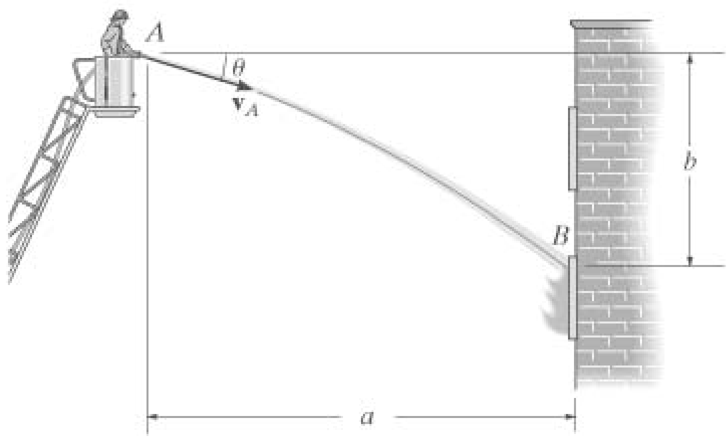
Given:

$\theta = 20 \text{ deg}$

$a = 60 \text{ ft}$

$b = 30 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $\quad v_A = 1 \frac{\text{ft}}{\text{s}}$

$t = 1 \text{ s}$

Given $\quad v_A \cos(\theta) t = a$ $\qquad$ $\frac{-1}{2} g t^2 - v_A \sin(\theta) t = -b$

$\begin{pmatrix} v_A \\ t \end{pmatrix} = \text{Find}(v_A, t)$ $\qquad$ $t = 0.712 \text{ s}$ $\qquad$ $v_A = 89.7 \frac{\text{ft}}{\text{s}}$

Problem 12–91

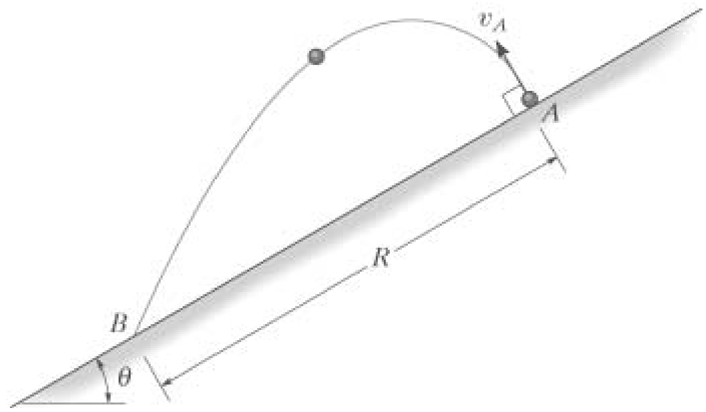
A ball bounces on the θ inclined plane such that it rebounds perpendicular to the incline with a velocity v_A. Determine the distance R to where it strikes the plane at B.

Given:

$\theta = 30 \text{ deg}$

$v_A = 40 \frac{\text{ft}}{\text{s}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $t = 10 \text{ s}$

$R = 1 \text{ ft}$

Given $v_A \sin(\theta) t = R \cos(\theta)$ $\frac{-1}{2} g t^2 + v_A \cos(\theta) t = -R \sin(\theta)$

$\begin{pmatrix} t \\ R \end{pmatrix} = \text{Find}(t, R)$ $t = 2.87 \text{ s}$ $R = 66.3 \text{ ft}$

***Problem 12-92**

The man stands a distance d from the wall and throws a ball at it with a speed v_0. Determine the angle θ at which he should release the ball so that it strikes the wall at the highest point possible. What is this height? The room has a ceiling height h_2.

Given:

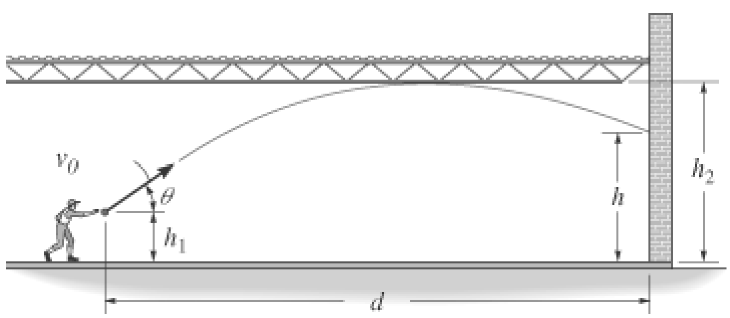
$d = 60 \text{ ft}$

$v_0 = 50 \frac{\text{ft}}{\text{s}}$

$h_1 = 5 \text{ ft}$

$h_2 = 20 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution: Guesses $t_1 = 1 \text{ s}$ $t_2 = 2 \text{ s}$ $\theta = 20 \text{ deg}$ $h = 10 \text{ ft}$

Given $d = v_0 \cos(\theta) t_2$ $h = \left(\frac{-g}{2}\right) t_2^2 + v_0 \sin(\theta) t_2 + h_1$

$$0 = -g\,t_1 + v_0 \sin(\theta) \qquad h_2 = \left(\frac{-g}{2}\right)t_1^{\,2} + v_0 \sin(\theta)t_1 + h_1$$

$$\begin{pmatrix} t_1 \\ t_2 \\ \theta \\ h \end{pmatrix} = \text{Find}(t_1, t_2, \theta, h) \qquad \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 0.965 \\ 1.532 \end{pmatrix} \text{s} \qquad \theta = 38.434\,\text{deg} \qquad h = 14.83\,\text{ft}$$

Problem 12–93

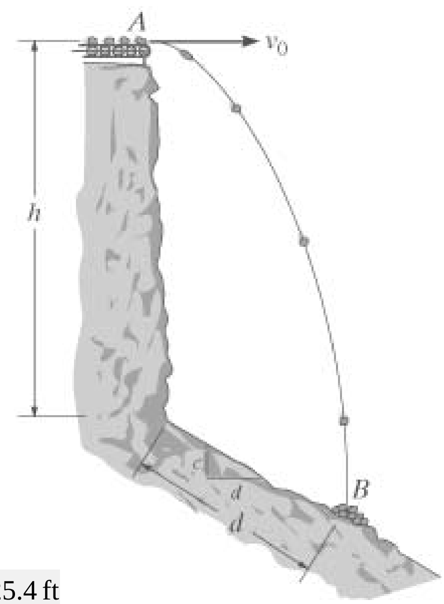
The stones are thrown off the conveyor with a horizontal velocity v_0 as shown. Determine the distance d down the slope to where the stones hit the ground at B.

Given:

$$v_0 = 10\,\frac{\text{ft}}{\text{s}} \qquad h = 100\text{ ft}$$

$$c = 1$$

$$g = 32.2\,\frac{\text{ft}}{\text{s}^2} \qquad d = 10$$



Solution:

$$\theta = \text{atan}\left(\frac{c}{d}\right)$$

Guesses $\quad t = 1\text{ s} \quad d = 1\text{ ft}$

Given $\quad v_0\,t = d\cos(\theta)$

$$\frac{-1}{2}g\,t^2 = -h - d\sin(\theta)$$

$$\begin{pmatrix} t \\ d \end{pmatrix} = \text{Find}(t, d) \qquad t = 2.523\text{ s} \qquad d = 25.4\,\text{ft}$$

Problem 12–94

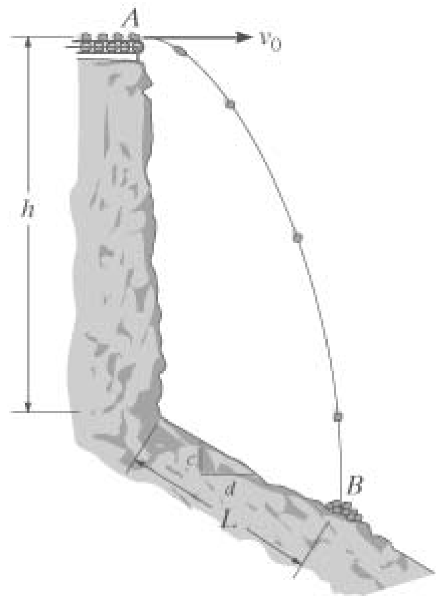
The stones are thrown off the conveyor with a horizontal velocity $v = v_0$ as shown. Determine the speed at which the stones hit the ground at B.

Given:

$$v_0 = 10\,\frac{\text{ft}}{\text{s}} \qquad h = 100\text{ ft}$$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad c = 1$

$d = 10$



Solution:

$\theta = \operatorname{atan}\left(\dfrac{c}{d}\right)$

Guesses $\quad t = 1\ \text{s} \quad L = 1\ \text{ft}$

Given $\quad v_0\, t = L\cos(\theta)$

$$\frac{-1}{2} g t^2 = -h - L\sin(\theta)$$

$\begin{pmatrix} t \\ L \end{pmatrix} = \operatorname{Find}(t, L) \qquad t = 2.523\ \text{s} \qquad L = 25.4\ \text{ft}$

$\mathbf{v_B} = \begin{pmatrix} v_0 \\ -g\,t \end{pmatrix} \qquad \mathbf{v_B} = \begin{pmatrix} 10 \\ -81.256 \end{pmatrix} \dfrac{\text{ft}}{\text{s}} \qquad \left|\mathbf{v_B}\right| = 81.9\ \dfrac{\text{ft}}{\text{s}}$

Problem 12–95

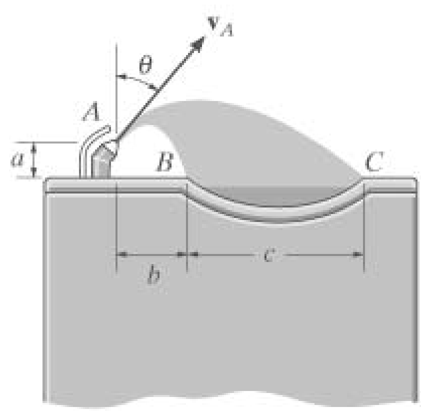
The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at B and C.

Given:

$\theta = 40\ \text{deg} \qquad a = 50\ \text{mm}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2} \qquad b = 100\ \text{mm}$

$c = 250\text{mm}$



Solution:

Guesses $\quad v_{min} = 1\ \dfrac{\text{m}}{\text{s}} \qquad t_{min} = 1\ \text{s}$

$v_{max} = 1\ \dfrac{\text{m}}{\text{s}} \qquad t_{max} = 1\ \text{s}$

Given $\quad b = v_{min}\sin(\theta)t_{min} \qquad a + v_{min}\cos(\theta)t_{min} - \dfrac{1}{2} g\, t_{min}^{\ 2} = 0$

$b + c = v_{max}\sin(\theta)t_{max} \qquad a + v_{max}\cos(\theta)t_{max} - \dfrac{1}{2} g\, t_{max}^{\ 2} = 0$

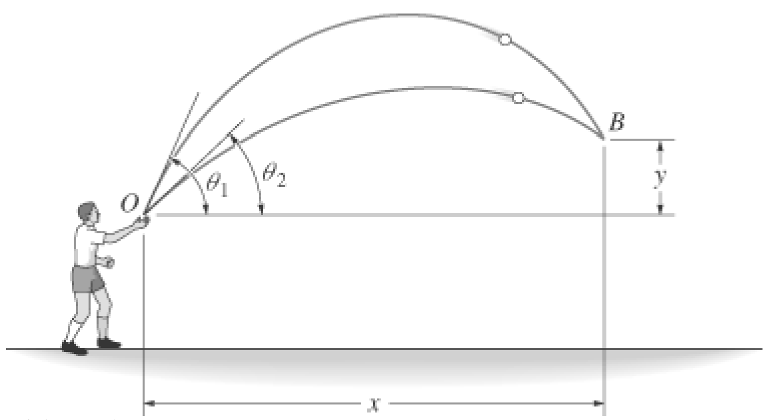
$$\begin{pmatrix} t_{min} \\ t_{max} \\ v_{min} \\ v_{max} \end{pmatrix} = \text{Find}(t_{min}, t_{max}, v_{min}, v_{max}) \qquad \begin{pmatrix} t_{min} \\ t_{max} \end{pmatrix} = \begin{pmatrix} 0.186 \\ 0.309 \end{pmatrix} \text{s}$$

$$\begin{pmatrix} v_{min} \\ v_{max} \end{pmatrix} = \begin{pmatrix} 0.838 \\ 1.764 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

---

*__Problem 12-96__

A boy at *O* throws a ball in the air with a speed v_0 at an angle θ_1. If he then throws another ball at the same speed v_0 at an angle $\theta_2 < \theta_1$, determine the time between the throws so the balls collide in mid air at *B*.



Solution:

$$x = v_0 \cos(\theta_1)t = v_0 \cos(\theta_2)(t - \Delta t)$$

$$y = \left(\frac{-g}{2}\right)t^2 + v_0 \sin(\theta_1)t = \left(\frac{-g}{2}\right)(t - \Delta t)^2 + v_0 \sin(\theta_2)(t - \Delta t)$$

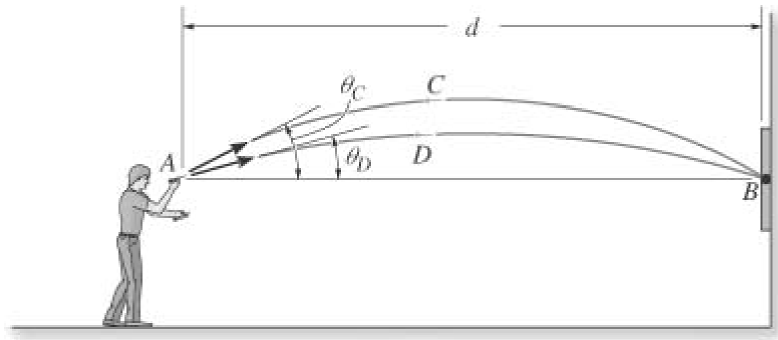
Eliminating time between these 2 equations we have

$$\Delta t = \frac{2v_0}{g}\left(\frac{\sin(\theta_1 - \theta_2)}{\cos(\theta_1) + \cos(\theta_2)}\right)$$

Problem 12-97

The man at *A* wishes to throw two darts at the target at *B* so that they arrive at the *same time.* If each dart is thrown with speed v_0, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at $\theta_C > \theta_D$ then the second dart is thrown at θ_D.

Given:



$v_0 = 10\ \dfrac{m}{s}$

$d = 5\ m$

$g = 9.81\ \dfrac{m}{s^2}$

Solution:

Guesses $\theta_C = 70\ deg$ $\theta_D = 15\ deg$ $\Delta t = 2\ s$ $t = 1\ s$

Given $d = v_0 \cos(\theta_C) t$ $0 = \dfrac{-g}{2} t^2 + v_0 \sin(\theta_C) t$

$d = v_0 \cos(\theta_D)(t - \Delta t)$ $0 = \dfrac{-g}{2}(t - \Delta t)^2 + v_0 \sin(\theta_D)(t - \Delta t)$

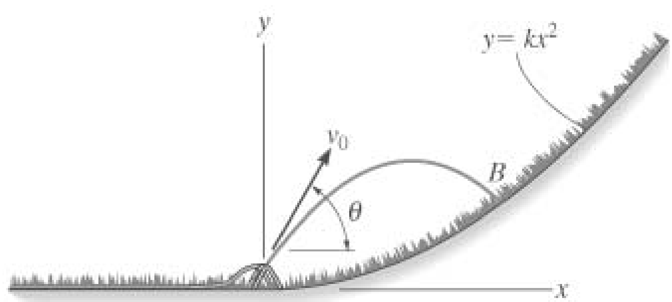
$\begin{pmatrix} \theta_C \\ \theta_D \\ t \\ \Delta t \end{pmatrix} = \mathrm{Find}(\theta_C, \theta_D, t, \Delta t)$ $t = 1.972\ s$ $\Delta t = 1.455\ s$ $\begin{pmatrix} \theta_C \\ \theta_D \end{pmatrix} = \begin{pmatrix} 75.313 \\ 14.687 \end{pmatrix} deg$

Problem 12–98

The water sprinkler, positioned at the base of a hill, releases a stream of water with a velocity v_0 as shown. Determine the point $B(x, y)$ where the water strikes the ground on the hill.

Assume that the hill is defined by the equation $y = kx^2$ and neglect the size of the sprinkler.

Given:



$v_0 = 15\ \dfrac{ft}{s}$ $k = \dfrac{0.05}{ft}$

$\theta = 60\ deg$

Solution:

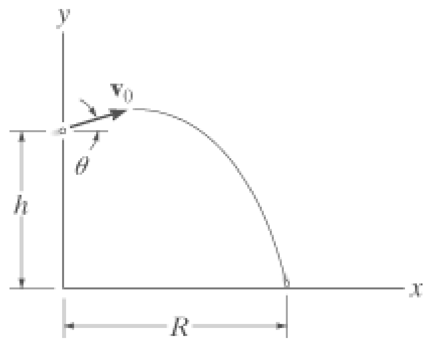
Guesses $x = 1\ ft$ $y = 1\ ft$ $t = 1\ s$

Given $x = v_0 \cos(\theta) t$ $y = v_0 \sin(\theta) t - \dfrac{1}{2} g t^2$ $y = k x^2$

$\begin{pmatrix} x \\ y \\ t \end{pmatrix} = \mathrm{Find}(x, y, t)$ $t = 0.687\ s$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5.154 \\ 1.328 \end{pmatrix} ft$

Problem 12–99

The projectile is launched from a height h with a velocity $\mathbf{v}_0$. Determine the range R.



Solution:

$a_x = 0 \qquad a_y = -g$

$v_x = v_0\cos(\theta) \qquad v_y = -g\,t + v_0\sin(\theta)$

$s_x = v_0\cos(\theta)t \qquad s_y = \frac{-1}{2}g\,t^2 + v_0\sin(\theta)t + h$

When it hits

$R = v_0\cos(\theta)t \qquad t = \frac{R}{v_0\cos(\theta)}$

$$0 = \frac{-1}{2}g\,t^2 + v_0\sin(\theta)t + h = \frac{-g}{2}\left(\frac{R}{v_0\cos(\theta)}\right)^2 + v_0\sin(\theta)\frac{R}{v_0\cos(\theta)} + h$$

Solving for R we find

$$R = \frac{v_0^2\cos(\theta)^2}{g}\left(\tan(\theta) + \sqrt{\tan(\theta)^2 + \frac{2g\,h}{v_0^2\cos(\theta)^2}}\right)$$

***Problem 12-100**

A car is traveling along a circular curve that has radius ρ. If its speed is v and the speed is increasing uniformly at rate a_t, determine the magnitude of its acceleration at this instant.

Given: $\rho = 50\text{ m} \qquad v = 16\ \frac{\text{m}}{\text{s}} \qquad a_t = 8\ \frac{\text{m}}{\text{s}^2}$

Solution:

$a_n = \frac{v^2}{\rho} \qquad a_n = 5.12\ \frac{\text{m}}{\text{s}^2} \qquad a = \sqrt{a_n^2 + a_t^2} \qquad a = 9.498\ \frac{\text{m}}{\text{s}^2}$

Problem 12-101

A car moves along a circular track of radius ρ such that its speed for a short period of time $0 \le t \le t_2$, is $v = b\,t + c\,t^2$. Determine the magnitude of its acceleration when $t = t_1$. How far has it traveled at time t_1?

Given: $\rho = 250\ \text{ft}$ $t_2 = 4\ \text{s}$ $b = 3\ \dfrac{\text{ft}}{\text{s}^2}$ $c = 3\ \dfrac{\text{ft}}{\text{s}^3}$ $t_1 = 3\ \text{s}$

Solution: $v = bt + ct^2$ $a_t = b + 2ct$

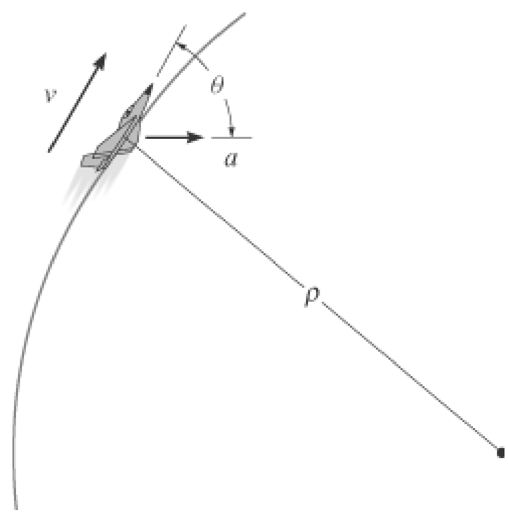
At t_1 $v_1 = bt_1 + ct_1^2$ $a_{t1} = b + 2ct_1$ $a_{n1} = \dfrac{v_1^2}{\rho}$

$a_1 = \sqrt{a_{t1}^2 + a_{n1}^2}$ $a_1 = 21.63\ \dfrac{\text{ft}}{\text{s}^2}$

Distance traveled $d_1 = \dfrac{b}{2}t_1^2 + \dfrac{c}{3}t_1^3$ $d_1 = 40.5\ \text{ft}$

Problem 12-102

At a given instant the jet plane has speed v and acceleration a acting in the directions shown. Determine the rate of increase in the plane's speed and the radius of curvature ρ of the path.



Given:

$v = 400\ \dfrac{\text{ft}}{\text{s}}$

$a = 70\ \dfrac{\text{ft}}{\text{s}^2}$

$\theta = 60\ \text{deg}$

Solution:

Rate of increase

$a_t = (a)\cos(\theta)$ $a_t = 35\ \dfrac{\text{ft}}{\text{s}^2}$

Radius of curvature

$a_n = (a)\sin(\theta) = \dfrac{v^2}{\rho}$ $\rho = \dfrac{v^2}{(a)\sin(\theta)}$ $\rho = 2639\ \text{ft}$

Problem 12–103

A particle is moving along a curved path at a constant speed v. The radii of curvature of the path at points P and P' are ρ and ρ', respectively. If it takes the particle time t to go from P to P', determine the acceleration of the particle at P and P'.

Given: $v = 60\ \dfrac{\text{ft}}{\text{s}}$ $\rho = 20\ \text{ft}$ $\rho' = 50\ \text{ft}$ $t = 20\ \text{s}$

Solution: $a = \frac{v^2}{\rho}$ $a = 180 \frac{\text{ft}}{\text{s}^2}$

$a' = \frac{v^2}{\rho'}$ $a' = 72 \frac{\text{ft}}{\text{s}^2}$

Note that the time doesn't matter here because the speed is constant.

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*__Problem 12-104__

A boat is traveling along a circular path having radius ρ. Determine the magnitude of the boat's acceleration when the speed is v and the rate of increase in the speed is a_t.

Given: $\rho = 20\ \text{m}$ $v = 5 \frac{\text{m}}{\text{s}}$ $a_t = 2 \frac{\text{m}}{\text{s}^2}$

Solution:

$a_n = \frac{v^2}{\rho}$ $a_n = 1.25 \frac{\text{m}}{\text{s}^2}$ $a = \sqrt{a_t^2 + a_n^2}$ $a = 2.358 \frac{\text{m}}{\text{s}^2}$

Problem 12-105

Starting from rest, a bicyclist travels around a horizontal circular path of radius ρ at a speed $v = b\,t^2 + c\,t$. Determine the magnitudes of his velocity and acceleration when he has traveled a distance s_1.

Given: $\rho = 10\ \text{m}$ $b = 0.09 \frac{\text{m}}{\text{s}^3}$ $c = 0.1 \frac{\text{m}}{\text{s}^2}$ $s_1 = 3\ \text{m}$

Solution: Guess $t_1 = 1\ \text{s}$

Given $s_1 = \left(\frac{b}{3}\right)t_1^3 + \left(\frac{c}{2}\right)t_1^2$ $t_1 = \text{Find}(t_1)$ $t_1 = 4.147\ \text{s}$

$v_1 = b\,t_1^2 + c\,t_1$ $v_1 = 1.963 \frac{\text{m}}{\text{s}}$

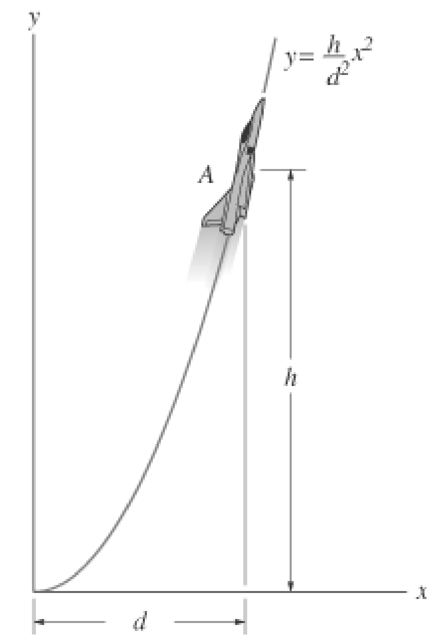
$a_{t1} = 2b\,t_1 + c$ $a_{t1} = 0.847 \frac{\text{m}}{\text{s}^2}$

$a_{n1} = \frac{v_1^2}{\rho}$ $a_{n1} = 0.385 \frac{\text{m}}{\text{s}^2}$

$a_1 = \sqrt{a_{t1}^2 + a_{n1}^2}$ $a_1 = 0.93 \frac{\text{m}}{\text{s}^2}$

Problem 12-106

The jet plane travels along the vertical parabolic path. When it is at point A it has speed v which is increasing at the rate a_t. Determine the magnitude of acceleration of the plane when it is at point A.



Given:

$$v = 200\ \frac{\text{m}}{\text{s}}$$

$$a_t = 0.8\ \frac{\text{m}}{\text{s}^2}$$

$$d = 5\ \text{km}$$

$$h = 10\ \text{km}$$

Solution:

$$y(x) = h\left(\frac{x}{d}\right)^2$$

$$y'(x) = \frac{d}{dx}y(x)$$

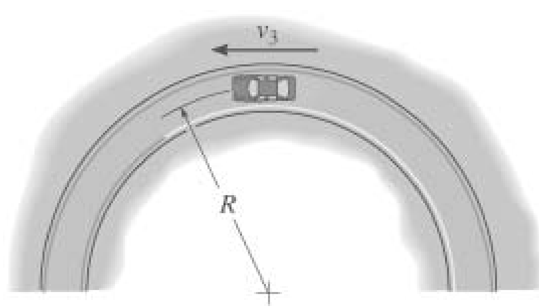
$$y''(x) = \frac{d}{dx}y'(x)$$

$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$a_n = \frac{v^2}{\rho(d)} \qquad a = \sqrt{a_t^2 + a_n^2} \qquad a = 0.921\ \frac{\text{m}}{\text{s}^2}$$

Problem 12–107

The car travels along the curve having a radius of R. If its speed is uniformly increased from v_1 to v_2 in time t, determine the magnitude of its acceleration at the instant its speed is v_3.



Given:

$$v_1 = 15\ \frac{\text{m}}{\text{s}} \qquad t = 3\ \text{s}$$

$v_2 = 27\ \frac{\text{m}}{\text{s}}$ $\qquad R = 300\ \text{m}$

$v_3 = 20\ \frac{\text{m}}{\text{s}}$

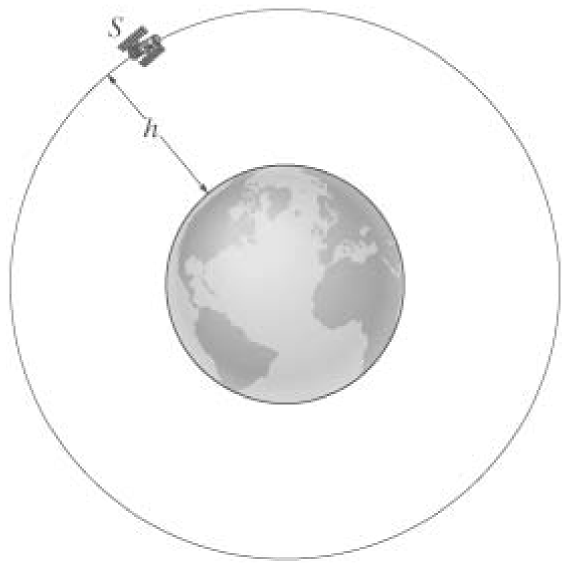
Solution:

$a_t = \frac{v_2 - v_1}{t} \qquad a_n = \frac{v_3^{\,2}}{R} \qquad a = \sqrt{a_t^{\,2} + a_n^{\,2}} \qquad a = 4.22\ \frac{\text{m}}{\text{s}^2}$

***Problem 12–108**

The satellite S travels around the earth in a circular path with a constant speed v_1. If the acceleration is a, determine the altitude h. Assume the earth's diameter to be d.

Units Used: $\quad \text{Mm} = 10^3\ \text{km}$



Given:

$v_1 = 20\ \frac{\text{Mm}}{\text{hr}}$

$a = 2.5\ \frac{\text{m}}{\text{s}^2}$

$d = 12713\ \text{km}$

Solution:

Guess $\quad h = 1\ \text{Mm}$

Given $\quad a = \frac{v_1^{\,2}}{h + \frac{d}{2}} \qquad h = \text{Find}(h) \qquad h = 5.99\,\text{Mm}$

Problem 12–109

A particle P moves along the curve $y = b\,x^2 + c$ with a constant speed v. Determine the point on the curve where the maximum magnitude of acceleration occurs and compute its value.

Given: $\quad b = 1\ \frac{1}{\text{m}} \qquad c = -4\ \text{m} \qquad v = 5\ \frac{\text{m}}{\text{s}}$

Solution: Maximum acceleration occurs where the radius of curvature is the smallest which occurs at $x = 0$.

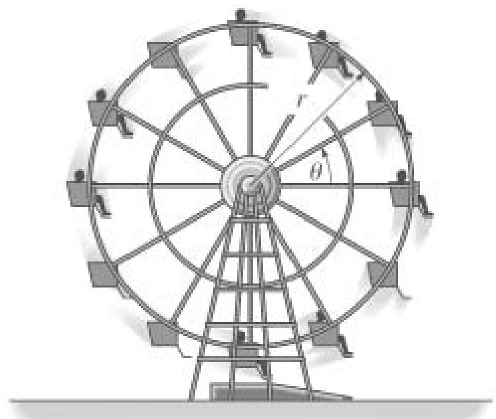
$$y(x) = bx^2 + c \qquad y'(x) = \frac{d}{dx}y(x) \qquad y''(x) = \frac{d}{dx}y'(x)$$

$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)} \qquad \rho_{min} = \rho(0\text{m}) \qquad \rho_{min} = 0.5 \text{ m}$$

$$a_{max} = \frac{v^2}{\rho_{min}} \qquad a_{max} = 50 \frac{\text{m}}{\text{s}^2}$$

Problem 12–110

The Ferris wheel turns such that the speed of the passengers is increased by $a_t = bt$. If the wheel starts from rest when $\theta = 0°$, determine the magnitudes of the velocity and acceleration of the passengers when the wheel turns $\theta = \theta_1$.



Given:

$$b = 4 \frac{\text{ft}}{\text{s}^3} \qquad \theta_1 = 30 \text{ deg} \qquad r = 40 \text{ ft}$$

Solution:

Guesses $\quad t_1 = 1 \text{ s} \qquad v_1 = 1 \frac{\text{ft}}{\text{s}}$

$$a_{t1} = 1 \frac{\text{ft}}{\text{s}^2}$$

Given $\quad a_{t1} = b\,t_1 \qquad v_1 = \left(\frac{b}{2}\right)t_1^2 \qquad r\theta_1 = \left(\frac{b}{6}\right)t_1^3$

$$\begin{pmatrix} a_{t1} \\ v_1 \\ t_1 \end{pmatrix} = \text{Find}(a_{t1}, v_1, t_1) \qquad t_1 = 3.16 \text{ s} \qquad v_1 = 19.91 \frac{\text{ft}}{\text{s}} \qquad a_{t1} = 12.62 \frac{\text{ft}}{\text{s}^2}$$

$$a_1 = \sqrt{a_{t1}^2 + \left(\frac{v_1^2}{r}\right)^2} \qquad v_1 = 19.91 \frac{\text{ft}}{\text{s}} \qquad a_1 = 16.05 \frac{\text{ft}}{\text{s}^2}$$

Problem 12-111

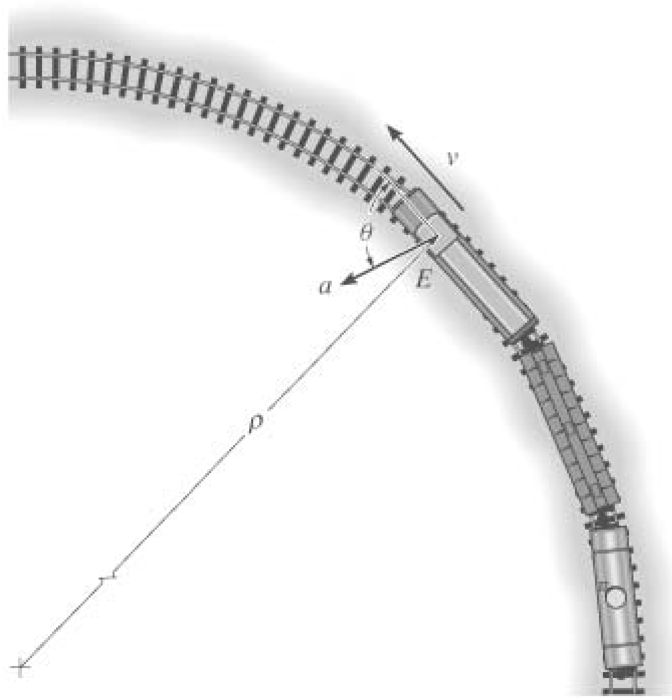
At a given instant the train engine at E has speed v and acceleration a acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.

Given:

$$v = 20\ \frac{m}{s}$$

$$a = 14\ \frac{m}{s^2}$$

$$\theta = 75\ deg$$



Solution:

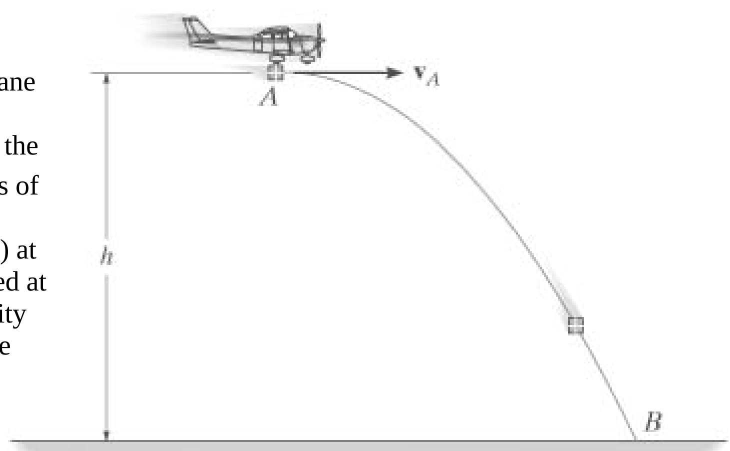
$$a_t = (a)\cos(\theta) \qquad a_t = 3.62\ \frac{m}{s^2}$$

$$a_n = (a)\sin(\theta) \qquad a_n = 13.523\ \frac{m}{s^2}$$

$$\rho = \frac{v^2}{a_n} \qquad \rho = 29.579\ m$$

***Problem 12–112**

A package is dropped from the plane which is flying with a constant horizontal velocity v_A. Determine the normal and tangential components of acceleration and the radius of curvature of the path of motion (a) at the moment the package is released at A, where it has a horizontal velocity v_A, and (b) *just before* it strikes the ground at B.



Given: $\quad v_A = 150\ \frac{ft}{s} \qquad h = 1500\ ft \qquad g = 32.2\ \frac{ft}{s^2}$

Solution:

At A:

$$a_{An} = g \qquad \rho_A = \frac{v_A^2}{a_{An}} \qquad \rho_A = 699\ ft$$

At B:

$$t = \sqrt{\frac{2h}{g}} \qquad v_x = v_A \qquad v_y = g\,t \qquad \theta = \operatorname{atan}\left(\frac{v_y}{v_x}\right)$$

$$v_B = \sqrt{v_x^{\,2} + v_y^{\,2}} \qquad a_{Bn} = g\cos(\theta) \qquad \rho_B = \frac{v_B^{\,2}}{a_{Bn}} \qquad \rho_B = 8510\,\text{ft}$$

Problem 12-113

The automobile is originally at rest at $s = 0$. If its speed is increased by $dv/dt = bt^2$, determine the magnitudes of its velocity and acceleration when $t = t_1$.

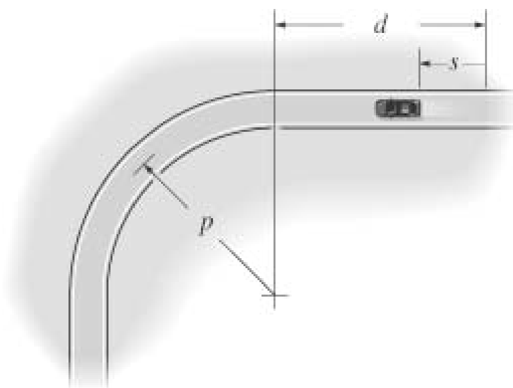
Given:

$$b = 0.05\ \frac{\text{ft}}{\text{s}^4}$$

$$t_1 = 18\ \text{s}$$

$$\rho = 240\ \text{ft}$$

$$d = 300\ \text{ft}$$



Solution:

$$a_{t1} = b\,t_1^{\,2} \qquad a_{t1} = 16.2\,\frac{\text{ft}}{\text{s}^2}$$

$$v_1 = \left(\frac{b}{3}\right)t_1^{\,3} \qquad v_1 = 97.2\,\frac{\text{ft}}{\text{s}}$$

$$s_1 = \left(\frac{b}{12}\right)t_1^{\,4} \qquad s_1 = 437.4\,\text{ft}$$

If $s_1 = 437.4\,\text{ft} > d = 300\,\text{ft}$ then we are on the curved part of the track.

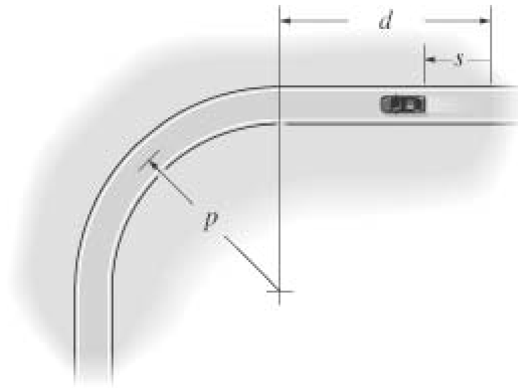
$$a_{n1} = \frac{v_1^{\,2}}{\rho} \qquad a_{n1} = 39.366\,\frac{\text{ft}}{\text{s}^2} \qquad a = \sqrt{a_{n1}^{\,2} + a_{t1}^{\,2}} \qquad a = 42.569\,\frac{\text{ft}}{\text{s}^2}$$

If $s_1 = 437.4\,\text{ft} < d = 300\,\text{ft}$ then we are on the straight part of the track.

$$a_{n1} = 0\,\frac{\text{ft}}{\text{s}^2} \qquad a_{n1} = 0\,\frac{\text{ft}}{\text{s}^2} \qquad a = \sqrt{a_{n1}^{\,2} + a_{t1}^{\,2}} \qquad a = 16.2\,\frac{\text{ft}}{\text{s}^2}$$

Problem 12-114

The automobile is originally at rest at $s = 0$. If it then starts to increase its speed at $dv/dt = bt^2$, determine the magnitudes of its velocity and acceleration at $s = s_1$.



Given:

$d = 300\ \text{ft}$

$\rho = 240\ \text{ft}$

$b = 0.05\ \frac{\text{ft}}{\text{s}^4}$

$s_1 = 550\ \text{ft}$

Solution:

$a_t = bt^2 \qquad v = \left(\frac{b}{3}\right)t^3 \qquad s = \left(\frac{b}{12}\right)t^4 \qquad t_1 = \left(\frac{12s_1}{b}\right)^{\frac{1}{4}} \qquad t_1 = 19.061\ \text{s}$

$v_1 = \left(\frac{b}{3}\right)t_1^3 \qquad v_1 = 115.4\ \frac{\text{ft}}{\text{s}}$

If $s_1 = 550\,\text{ft} > d = 300\,\text{ft}$ the car is on the curved path

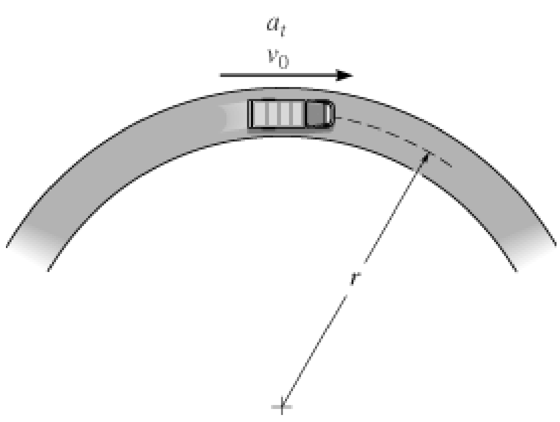
$a_t = bt_1^2 \qquad v = \left(\frac{b}{3}\right)t_1^3 \qquad a_n = \frac{v^2}{\rho} \qquad a = \sqrt{a_t^2 + a_n^2} \qquad a = 58.404\ \frac{\text{ft}}{\text{s}^2}$

If $s_1 = 550\,\text{ft} < d = 300\,\text{ft}$ the car is on the straight path

$a_t = bt_1^2 \qquad a_n = 0\ \frac{\text{ft}}{\text{s}^2} \qquad a = \sqrt{a_t^2 + a_n^2} \qquad a = 18.166\ \frac{\text{ft}}{\text{s}^2}$

Problem 12-115

The truck travels in a circular path having a radius ρ at a speed v_0. For a short distance from $s = 0$, its speed is increased by $a_t = bs$. Determine its speed and the magnitude of its acceleration when it has moved a distance $s = s_1$.



Given:

$\rho = 50\ \text{m} \qquad s_1 = 10\ \text{m}$

$v_0 = 4\ \frac{\text{m}}{\text{s}} \qquad b = 0.05\ \frac{1}{\text{s}^2}$

Solution:

$$a_t = b\,s \qquad \int_{v_0}^{v_1} v\,\mathrm{d}v = \int_0^{s_1} b\,s\,\mathrm{d}s \qquad \frac{v_1^2}{2} - \frac{v_0^2}{2} = \frac{b}{2}s_1^2$$

$$v_1 = \sqrt{v_0^2 + b\,s_1^2} \qquad v_1 = 4.583\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$a_{t1} = b\,s_1 \qquad a_{n1} = \frac{v_1^2}{\rho} \qquad a_1 = \sqrt{a_{t1}^2 + a_{n1}^2} \qquad a_1 = 0.653\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

***Problem 12–116**

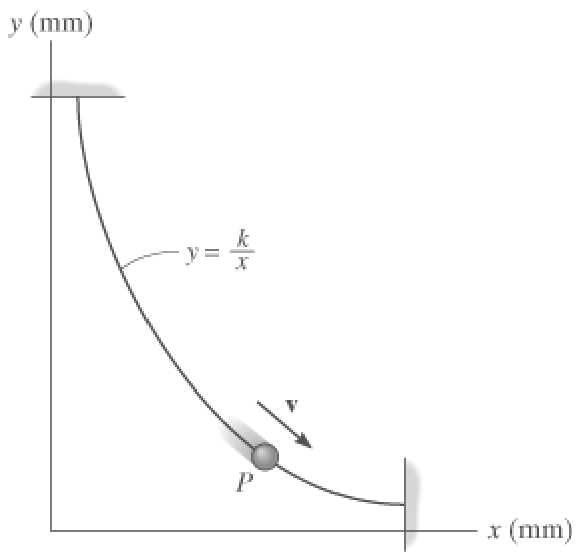
The particle travels with a constant speed v along the curve. Determine the particle’s acceleration when it is located at point $x = x_1$.

Given:

$$v = 300\ \frac{\mathrm{mm}}{\mathrm{s}}$$

$$k = 20 \times 10^3\ \mathrm{mm}^2$$

$$x_1 = 200\ \mathrm{mm}$$



Solution:

$$y(x) = \frac{k}{x}$$

$$y'(x) = \frac{\mathrm{d}}{\mathrm{d}x}y(x)$$

$$y''(x) = \frac{\mathrm{d}}{\mathrm{d}x}y'(x)$$

$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$\theta(x) = \operatorname{atan}(y'(x)) \qquad \theta_1 = \theta(x_1) \qquad \theta_1 = -26.6\,\mathrm{deg}$$

$$\mathbf{a} = \frac{v^2}{\rho(x_1)}\begin{pmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{pmatrix} \qquad \mathbf{a} = \begin{pmatrix} 144 \\ 288 \end{pmatrix}\frac{\mathrm{mm}}{\mathrm{s}^2} \qquad |\mathbf{a}| = 322\,\frac{\mathrm{mm}}{\mathrm{s}^2}$$

Problem 12–117

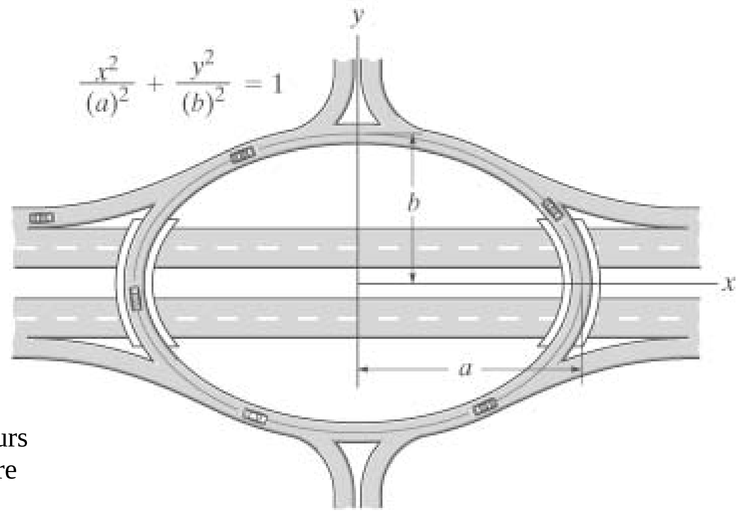
Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at *v*, determine the maximum acceleration experienced by the passengers.

Given:

$v = 60\ \frac{\text{km}}{\text{hr}}$

$a = 60\ \text{m}$

$b = 40\ \text{m}$



Solution:

Maximum acceleration occurs where the radius of curvature is the smallest. In this case that happens when $y = 0$.

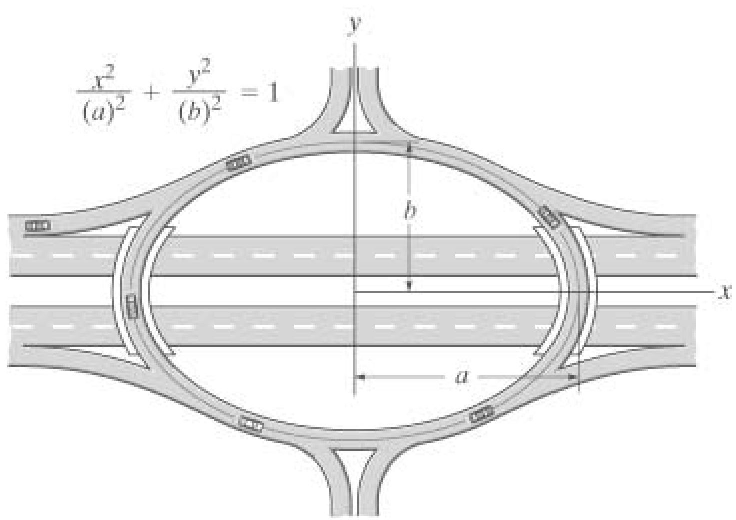
$x(y) = a\sqrt{1 - \left(\frac{y}{b}\right)^2}$ $\qquad x'(y) = \frac{d}{dy}x(y)$ $\qquad x''(y) = \frac{d}{dy}x'(y)$

$\rho(y) = \frac{-\sqrt{\left(1 + x'(y)^2\right)^3}}{x''(y)}$ $\qquad \rho_{min} = \rho(0\text{m})$ $\qquad \rho_{min} = 26.667\ \text{m}$

$a_{max} = \frac{v^2}{\rho_{min}}$ $\qquad \boxed{a_{max} = 10.42\ \frac{\text{m}}{\text{s}^2}}$

Problem 12–118

Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at *v*, determine the minimum acceleration experienced by the passengers.



Given:

$v = 60\ \frac{\text{km}}{\text{hr}}$

$a = 60\ \text{m}$

$b = 40\ \text{m}$

Solution:

Minimum acceleration occurs where the radius of curvature is the largest. In this case that happens when $x = 0$.

$$y(x) = b\sqrt{1-\left(\frac{x}{a}\right)^2} \qquad y'(x) = \frac{d}{dx}y(x) \qquad y''(x) = \frac{d}{dx}y'(x)$$

$$\rho(x) = \frac{-\sqrt{\left(1+y'(x)^2\right)^3}}{y''(x)} \qquad \rho_{max} = \rho(0\text{m}) \qquad \rho_{max} = 90 \text{ m}$$

$$a_{min} = \frac{v^2}{\rho_{max}} \qquad a_{min} = 3.09 \frac{\text{m}}{\text{s}^2}$$

Problem 12-119

The car B turns such that its speed is increased by $dv_B/dt = be^{ct}$. If the car starts from rest when $\theta = 0$, determine the magnitudes of its velocity and acceleration when the arm AB rotates to $\theta = \theta_1$. Neglect the size of the car.

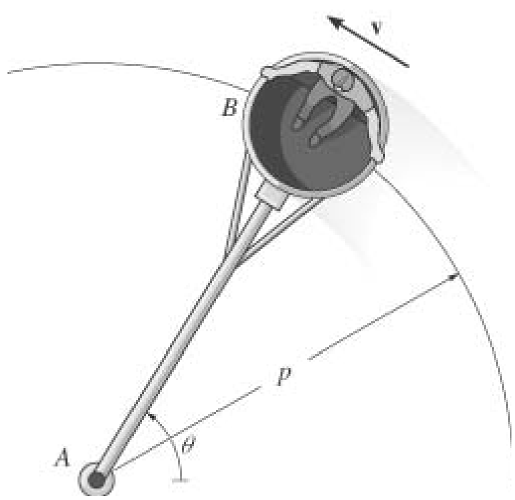
Given:

$$b = 0.5 \frac{\text{m}}{\text{s}^2}$$

$$c = 1 \text{ s}^{-1}$$

$$\theta_1 = 30 \text{ deg}$$

$$\rho = 5 \text{ m}$$



Solution:

$$a_{Bt} = b e^{ct}$$

$$v_B = \frac{b}{c}\left(e^{ct} - 1\right)$$

$$\rho\,\theta = \left(\frac{b}{c^2}\right)e^{ct} - \left(\frac{b}{c}\right)t - \frac{b}{c^2}$$

Guess $\quad t_1 = 1 \text{ s}$

Given $\rho\,\theta_1 = \left(\frac{b}{c^2}\right)e^{c\,t_1} - \left(\frac{b}{c}\right)t_1 - \frac{b}{c^2}$ $\quad t_1 = \text{Find}(t_1)$ $\quad t_1 = 2.123\ \text{s}$

$v_{B1} = \frac{b}{c}\left(e^{c\,t_1} - 1\right)$ $\quad v_{B1} = 3.68\ \frac{\text{m}}{\text{s}}$

$a_{Bt1} = b\,e^{c\,t_1}$ $\quad a_{Bn1} = \frac{v_{B1}^2}{\rho}$ $\quad a_{B1} = \sqrt{a_{Bt1}^2 + a_{Bn1}^2}$

$a_{Bt1} = 4.180\ \frac{\text{m}}{\text{s}^2}$ $\quad a_{Bn1} = 2.708\ \frac{\text{m}}{\text{s}^2}$ $\quad a_{B1} = 4.98\ \frac{\text{m}}{\text{s}^2}$

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*__Problem 12-120__

The car B turns such that its speed is increased by $dv_B/dt = b\,e^{ct}$. If the car starts from rest when $\theta = 0$, determine the magnitudes of its velocity and acceleration when $t = t_1$. Neglect the size of the car. Also, through what angle θ has it traveled?

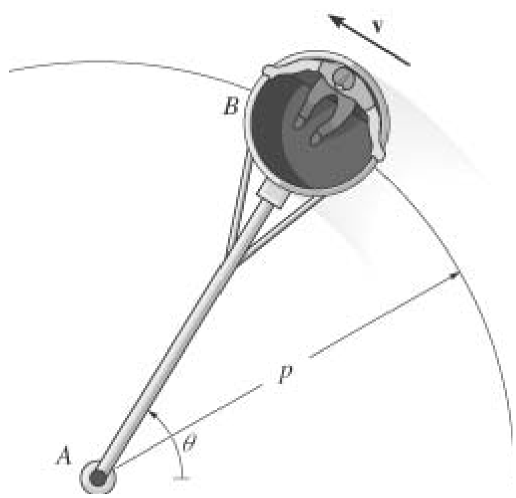
Given:

$b = 0.5\ \frac{\text{m}}{\text{s}^2}$

$c = 1\ \text{s}^{-1}$

$t_1 = 2\ \text{s}$

$\rho = 5\ \text{m}$



Solution:

$a_{Bt} = b\,e^{c\,t}$

$v_B = \frac{b}{c}\left(e^{c\,t} - 1\right)$

$\rho\,\theta = \left(\frac{b}{c^2}\right)e^{c\,t} - \left(\frac{b}{c}\right)t - \frac{b}{c^2}$

$v_{B1} = \frac{b}{c}\left(e^{c\,t_1} - 1\right)$ $\quad v_{B1} = 3.19\ \frac{\text{m}}{\text{s}}$

$a_{Bt1} = b\,e^{c\,t_1}$ $\quad a_{Bn1} = \frac{v_{B1}^2}{\rho}$ $\quad a_{B1} = \sqrt{a_{Bt1}^2 + a_{Bn1}^2}$

$a_{Bt1} = 3.695\ \frac{\text{m}}{\text{s}^2}$ $\quad a_{Bn1} = 2.041\ \frac{\text{m}}{\text{s}^2}$ $\quad a_{B1} = 4.22\ \frac{\text{m}}{\text{s}^2}$

$$\theta_1 = \frac{1}{\rho}\left[\left(\frac{b}{c^2}\right)e^{c\,t_1} - \left(\frac{b}{c}\right)t_1 - \frac{b}{c^2}\right] \qquad \theta_1 = 25.1\ \text{deg}$$

Problem 12–121

The motorcycle is traveling at v_0 when it is at A. If the speed is then increased at dv/dt = a_t, determine its speed and acceleration at the instant $t = t_1$.

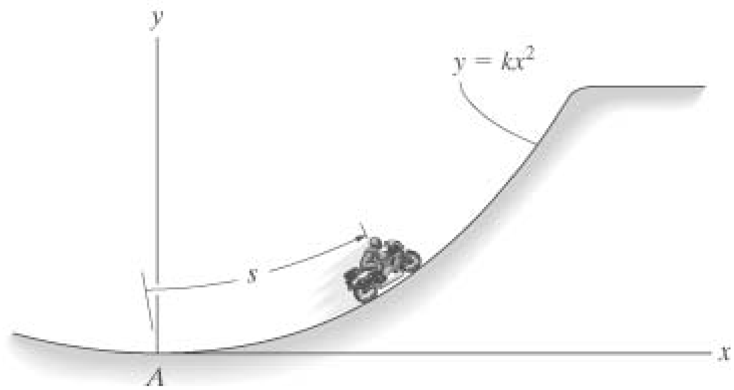
Given:

$k = 0.5\ \text{m}^{-1}$

$a_t = 0.1\ \frac{\text{m}}{\text{s}^2}$

$v_0 = 1\ \frac{\text{m}}{\text{s}}$

$t_1 = 5\ \text{s}$



Solution:

$$y(x) = kx^2 \qquad y'(x) = 2kx \qquad y''(x) = 2k \qquad \rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$v_1 = v_0 + a_t t_1 \qquad s_1 = v_0 t_1 + \frac{1}{2}a_t t_1^2 \qquad v_1 = 1.5\ \frac{\text{m}}{\text{s}}$$

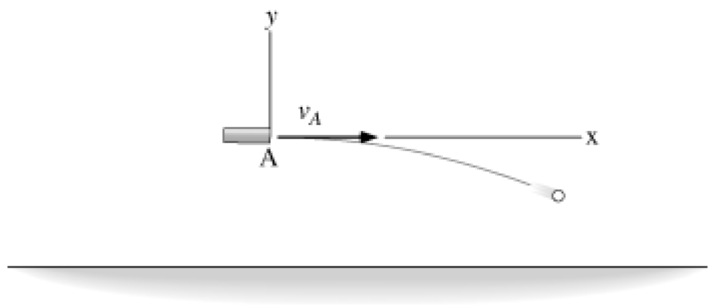
Guess $\quad x_1 = 1\ \text{m} \quad$ Given $\quad s_1 = \int_0^{x_1}\sqrt{1 + y'(x)^2}\,\text{d}x \qquad x_1 = \text{Find}(x_1)$

$$a_{1t} = a_t \qquad a_{1n} = \frac{v_1^2}{\rho(x_1)} \qquad a_1 = \sqrt{a_{1t}^2 + a_{1n}^2} \qquad a_1 = 0.117\ \frac{\text{m}}{\text{s}^2}$$

Problem 12-122

The ball is ejected horizontally from the tube with speed v_A. Find the equation of the path $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = t_1$.

Given:



$v_A = 8\ \frac{m}{s}$

$t_1 = 0.25\ s$

$g = 9.81\ \frac{m}{s^2}$

Solution:

$x = v_A t \qquad t = \frac{x}{v_A} \qquad y = \frac{-g}{2}t^2 \qquad y = \frac{-g}{2v_A^2}x^2 \qquad$ parabola

when $t = t_1$

$v_x = v_A \qquad v_y = -g t_1 \qquad \theta = \operatorname{atan}\left(\frac{-v_y}{v_x}\right) \qquad \theta = 17.044\ \text{deg}$

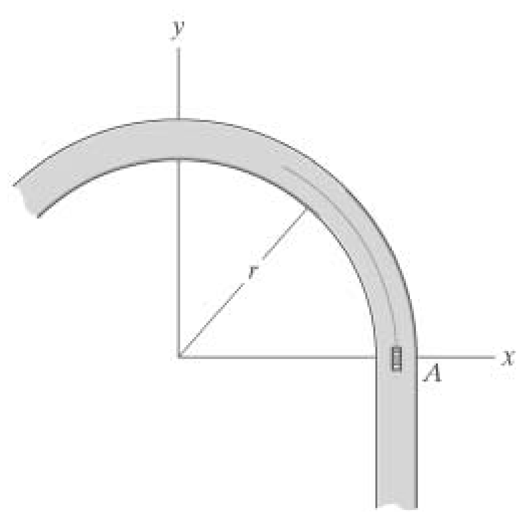
$a_n = g\cos(\theta) \qquad a_n = 9.379\ \frac{m}{s^2}$

$a_t = g\sin(\theta) \qquad a_t = 2.875\ \frac{m}{s^2}$

Problem 12–123

The car travels around the circular track having a radius r such that when it is at point A it has a velocity v_1 which is increasing at the rate dv/dt = kt. Determine the magnitudes of its velocity and acceleration when it has traveled one-third the way around the track.

Given:



$k = 0.06\ \frac{m}{s^3}$

$r = 300\ m$

$v_1 = 5\ \frac{m}{s}$

Solution:

$a_t(t) = kt$

$v(t) = v_1 + \frac{k}{2}t^2$

$s_p(t) = v_1 t + \frac{k}{6}t^3$

Guess $t_1 = 1\ \text{s}$ Given $s_p(t_1) = \dfrac{2\pi r}{3}$ $t_1 = \text{Find}(t_1)$ $t_1 = 35.58\ \text{s}$

$v_1 = v(t_1)$ $a_{t1} = a_t(t_1)$ $a_{n1} = \dfrac{v_1^2}{r}$ $a_1 = \sqrt{a_{t1}^2 + a_{n1}^2}$

$v_1 = 43.0\ \dfrac{\text{m}}{\text{s}}$ $a_1 = 6.52\ \dfrac{\text{m}}{\text{s}^2}$

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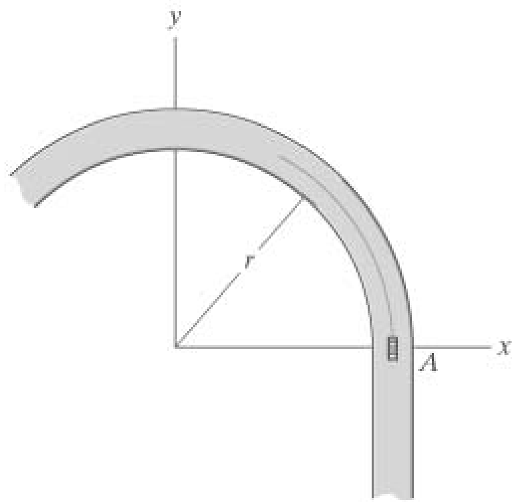
*__Problem 12–124__

The car travels around the portion of a circular track having a radius r such that when it is at point A it has a velocity v_1 which is increasing at the rate of $dv/dt = ks$. Determine the magnitudes of its velocity and acceleration when it has traveled three-fourths the way around the track.



Given:

$k = 0.002\ \text{s}^{-2}$

$r = 500\ \text{ft}$

$v_1 = 2\ \dfrac{\text{ft}}{\text{s}}$

Solution: $s_{p1} = \dfrac{3}{4}2\pi r$ $a_t = v\dfrac{d}{ds_p}v = k s_p$

Guess $v_1 = 1\ \dfrac{\text{ft}}{\text{s}}$ Given $\displaystyle\int_0^{v_1} v\, dv = \int_0^{s_{p1}} k s_p\, ds_p$ $v_1 = \text{Find}(v_1)$

$a_{t1} = k s_{p1}$ $a_{n1} = \dfrac{v_1^2}{r}$ $a_1 = \sqrt{a_{t1}^2 + a_{n1}^2}$ $v_1 = 105.4\ \dfrac{\text{ft}}{\text{s}}$

$a_1 = 22.7\ \dfrac{\text{ft}}{\text{s}^2}$

Problem 12-125

The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds v_A and v_B respectively. Determine at $t = t_1$, (a) the displacement along the path of each particle, (b) the position vector to each particle, and (c) the shortest distance between the particles.

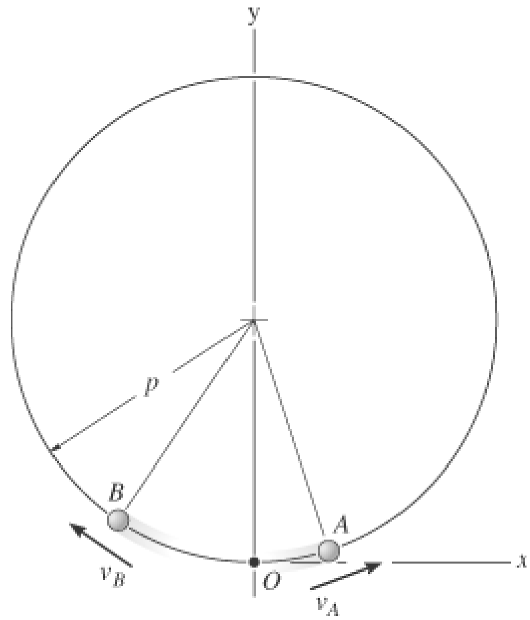
Given:

$$v_A = 0.7\ \frac{m}{s}$$

$$v_B = 1.5\ \frac{m}{s}$$

$$t_1 = 2\ s$$

$$\rho = 5\ m$$



Solution:

(a) The displacement along the path

$$s_A = v_A t_1 \qquad s_A = 1.4\ m$$

$$s_B = v_B t_1 \qquad s_B = 3\ m$$

(b) The position vector to each particle

$$\theta_A = \frac{s_A}{\rho} \qquad \mathbf{r_A} = \begin{pmatrix} \rho \sin(\theta_A) \\ \rho - \rho \cos(\theta_A) \end{pmatrix} \qquad \mathbf{r_A} = \begin{pmatrix} 1.382 \\ 0.195 \end{pmatrix} m$$

$$\theta_B = \frac{s_B}{\rho} \qquad \mathbf{r_B} = \begin{pmatrix} -\rho \sin(\theta_B) \\ \rho - \rho \cos(\theta_B) \end{pmatrix} \qquad \mathbf{r_B} = \begin{pmatrix} -2.823 \\ 0.873 \end{pmatrix} m$$

(c) The shortest distance between the particles

$$d = \left|\mathbf{r_B} - \mathbf{r_A}\right| \qquad d = 4.26\ m$$

Problem 12-126

The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds v_A and v_B respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens.

Given:

$$v_A = 0.7\ \frac{m}{s}$$

$$v_B = 1.5\ \frac{m}{s}$$

$$\rho = 5\ m$$

Solution:

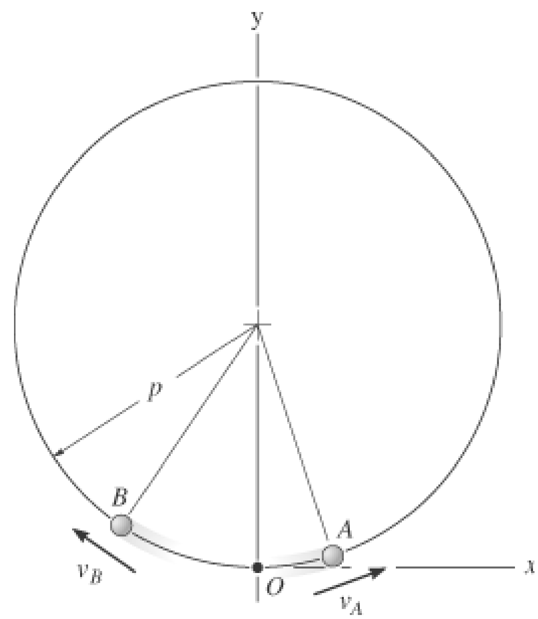
$$\left(v_A + v_B\right)t = 2\pi\rho$$

$$t = \frac{2\pi\rho}{v_A + v_B}$$

$$t = 14.28 \text{ s}$$

$$a_B = \frac{v_B^2}{\rho}$$

$$a_B = 0.45 \frac{\text{m}}{\text{s}^2}$$



Problem 12-127

The race car has an initial speed v_A at A. If it increases its speed along the circular track at the rate $a_t = bs$, determine the time needed for the car to travel distance s_1.

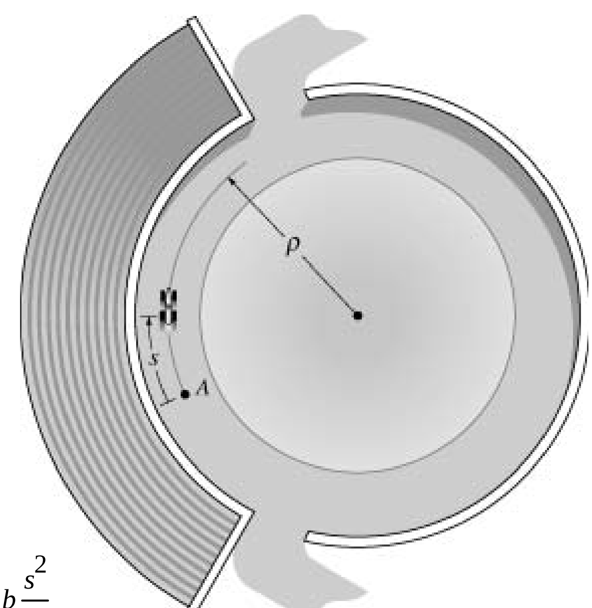
Given:

$$v_A = 15 \frac{\text{m}}{\text{s}}$$

$$b = 0.4 \text{ s}^{-2}$$

$$s_1 = 20 \text{ m}$$

$$\rho = 150 \text{ m}$$



Solution:

$$a_t = b\,s = v\frac{d}{ds}v$$

$$\int_{v_A}^{v} v\,dv = \int_0^s b\,s\,ds \qquad \frac{v^2}{2} - \frac{v_A^2}{2} = b\frac{s^2}{2}$$

$$v = \frac{d}{dt}s = \sqrt{v_A^2 + b\,s^2}$$

$$\int_0^s \frac{1}{\sqrt{v_A^2 + b s^2}}\, ds = \int_0^t 1\, dt \qquad t = \int_0^{s_1} \frac{1}{\sqrt{v_A^2 + b s^2}}\, ds \qquad \boxed{t = 1.211\ \text{s}}$$

***Problem 12-128**

A boy sits on a merry-go-round so that he is always located a distance r from the center of rotation. The merry-go-round is originally at rest, and then due to rotation the boy's speed is increased at the rate a_t. Determine the time needed for his acceleration to become a.

Given: $r = 8\ \text{ft}$ $\quad a_t = 2\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad a = 4\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$$a_n = \sqrt{a^2 - a_t^2} \qquad v = \sqrt{a_n r} \qquad t = \frac{v}{a_t} \qquad \boxed{t = 2.63\ \text{s}}$$

Problem 12–129

A particle moves along the curve $y = b\sin(cx)$ with a constant speed v. Determine the normal and tangential components of its velocity and acceleration at any instant.

Given: $v = 2\ \dfrac{\text{m}}{\text{s}}$ $\quad b = 1\ \text{m}$ $\quad c = \dfrac{1}{\text{m}}$

Solution:

$$y = b\sin(cx) \qquad y' = bc\cos(cx) \qquad y'' = -bc^2\sin(cx)$$

$$\rho = \frac{\sqrt{\left(1 + y'^2\right)^3}}{y''} = \frac{\left[1 + (bc\cos(cx))^2\right]^{\frac{3}{2}}}{-bc^2\sin(cx)}$$

$$\boxed{a_n = \frac{v^2 bc\sin(cx)}{\left[1 + (bc\cos(cx))^2\right]^{\frac{3}{2}}}} \qquad \boxed{a_t = 0} \qquad \boxed{v_t = 0} \qquad \boxed{v_n = 0}$$

Problem 12–130

The motion of a particle along a fixed path is defined by the parametric equations $r = b$, $\theta = ct$

and $z = dt^2$. Determine the unit vector that specifies the direction of the binormal axis to the osculating plane with respect to a set of fixed x, y, z coordinate axes when $t = t_1$. *Hint:* Formulate the particle's velocity v_p and acceleration a_p in terms of their **i**, **j**, **k** components. Note that $x = r\cos(\theta)$ and $y = r\sin(\theta)$. The binormal is parallel to $v_p \times a_p$. Why?

Given: $b = 8\ \text{ft}$ $\quad c = 4\ \dfrac{\text{rad}}{\text{s}}$ $\quad d = 6\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad t_1 = 2\ \text{s}$

Solution:

$$\mathbf{r_{p1}} = \begin{pmatrix} b\cos(c\,t_1) \\ b\sin(c\,t_1) \\ d\,t_1^{\,2} \end{pmatrix} \qquad \mathbf{v_{p1}} = \begin{pmatrix} -b\,c\sin(c\,t_1) \\ b\,c\cos(c\,t_1) \\ 2d\,t_1 \end{pmatrix} \qquad \mathbf{a_{p1}} = \begin{pmatrix} -b\,c^2\cos(c\,t_1) \\ -b\,c^2\sin(c\,t_1) \\ 2d \end{pmatrix}$$

Since v_p and a_p are in the normal plane and the binormal direction is perpendicular to this plane then we can use the cross product to define the binormal direction.

$$\mathbf{u} = \frac{\mathbf{v_{p1}} \times \mathbf{a_{p1}}}{|\mathbf{v_{p1}} \times \mathbf{a_{p1}}|} \qquad \mathbf{u} = \begin{pmatrix} 0.581 \\ 0.161 \\ 0.798 \end{pmatrix}$$

Problem 12-131

Particles A and B are traveling counter-clockwise around a circular track at constant speed v_0. If at the instant shown the speed of A is increased by $dv_A/dt = bs_A$, determine the distance measured counterclockwise along the track from B to A when $t = t_1$. What is the magnitude of the acceleration of each particle at this instant?

Given:

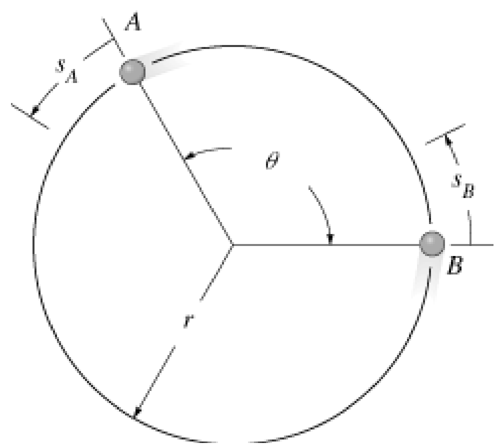
$v_0 = 8\ \dfrac{\text{m}}{\text{s}}$

$b = 4\ \text{s}^{-2}$

$t_1 = 1\ \text{s}$

$r = 5\ \text{m}$

$\theta = 120\ \text{deg}$



Solution: Distance

$$a_{At} = v_A\frac{dv_A}{ds_A} = b\,s_A \qquad \int_{v_0}^{v_A} v_A\,dv_A = \int_0^{s_A} b\,s_A\,ds_A$$

$$\frac{v_A^2}{2} - \frac{v_0^2}{2} = \frac{b}{2}s_A^2 \qquad v_A = \sqrt{v_0^2 + b\,s_A^2} = \frac{ds_A}{dt}$$

Guess $s_{A1} = 1\ \text{m}$ Given $\int_0^{t_1} 1\,dt = \int_0^{s_{A1}} \frac{1}{\sqrt{v_0^2 + b\,s_A^2}}\,ds_A$

$s_{A1} = \text{Find}(s_{A1})$ $s_{A1} = 14.507\ \text{m}$

$s_{B1} = v_0\,t_1$ $s_{B1} = 8\ \text{m}$ $s_{AB} = s_{A1} + r\theta - s_{B1}$ $s_{AB} = 16.979\ \text{m}$

$$a_A = \sqrt{(b\,s_{A1})^2 + \left(\frac{v_0^2 + b\,s_{A1}^2}{r}\right)^2} \qquad a_A = 190.24\ \frac{\text{m}}{\text{s}^2}$$

$$a_B = \frac{v_0^2}{r} \qquad a_B = 12.8\ \frac{\text{m}}{\text{s}^2}$$

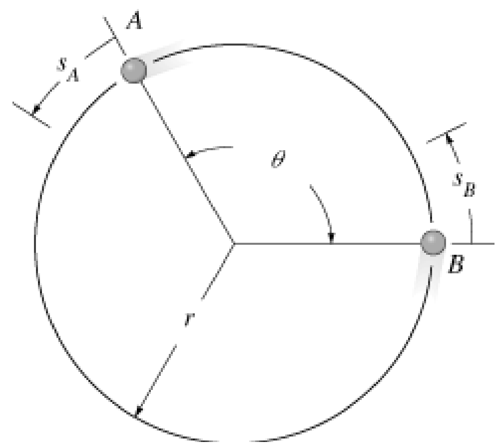
Problem 12-132

Particles A and B are traveling around a circular track at speed v_0 at the instant shown. If the speed of B is increased by $dv_B/dt = a_{Bt}$, and at the same instant A has an increase in speed $dv_A/dt = bt$, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?

Given: $v_0 = 8\ \frac{\text{m}}{\text{s}}$ $r = 5\ \text{m}$

$a_{Bt} = 4\ \frac{\text{m}}{\text{s}^2}$ $\theta = 120\ \text{deg}$

$b = 0.8\ \frac{\text{m}}{\text{s}^3}$



Solution:

$$v_B = a_{Bt}\,t + v_0$$

$$s_B = \frac{a_{Bt}}{2}t^2 + v_0\,t$$

$a_{At} = b\,t$ $v_A = \frac{b}{2}t^2 + v_0$ $s_A = \frac{b}{6}t^3 + v_0\,t$

Assume that B catches A Guess $t_1 = 1\ \text{s}$

Given $\frac{a_{Bt}}{2}t_1^{\,2} + v_0 t_1 = \frac{b}{6}t_1^{\,3} + v_0 t_1 + r\theta$ $t_1 = \text{Find}(t_1)$ $t_1 = 2.507 \text{ s}$

Assume that *A* catches *B* Guess $t_2 = 13 \text{ s}$

Given $\frac{a_{Bt}}{2}t_2^{\,2} + v_0 t_2 + r(2\pi - \theta) = \frac{b}{6}t_2^{\,3} + v_0 t_2$ $t_2 = \text{Find}(t_2)$ $t_2 = 15.642 \text{ s}$

Take the smaller time $t = \min(t_1, t_2)$ $t = 2.507 \text{ s}$

$$a_A = \sqrt{(b\,t)^2 + \left[\frac{\left(\frac{b}{2}t^2 + v_0\right)^2}{r}\right]^2} \qquad a_B = \sqrt{a_{Bt}^{\,2} + \left[\frac{(a_{Bt}\,t + v_0)^2}{r}\right]^2}$$

$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 22.2 \\ 65.14 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 12-133

The truck travels at speed v_0 along a circular road that has radius ρ. For a short distance from $s = 0$, its speed is then increased by dv/dt = bs. Determine its speed and the magnitude of its acceleration when it has moved a distance s_1.

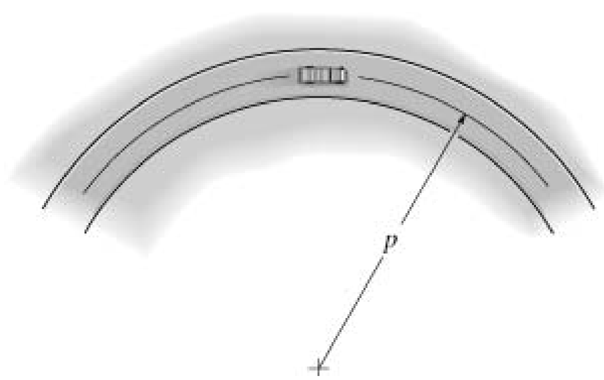
Given:

$v_0 = 4 \frac{\text{m}}{\text{s}}$

$\rho = 50 \text{ m}$

$b = \frac{0.05}{\text{s}^2}$

$s_1 = 10 \text{ m}$



Solution:

$$a_t = v\left(\frac{d}{ds}v\right) = b\,s \qquad \int_{v_0}^{v_1} v \, dv = \int_0^{s_1} b\,s \, ds \qquad \frac{v_1^{\,2}}{2} - \frac{v_0^{\,2}}{2} = \frac{b}{2}s_1^{\,2}$$

$$v_1 = \sqrt{v_0^{\,2} + b\,s_1^{\,2}} \qquad v_1 = 4.58 \frac{\text{m}}{\text{s}}$$

$a_t = b s_1$ $\qquad a_n = \dfrac{v_1^2}{\rho}$ $\qquad a = \sqrt{a_t^2 + a_n^2}$ $\qquad a = 0.653\ \dfrac{\text{m}}{\text{s}^2}$

Problem 12-134

A go-cart moves along a circular track of radius ρ such that its speed for a short period of time, $0 < t < t_1$, is $v = b\left(1 - e^{c\,t^2}\right)$. Determine the magnitude of its acceleration when $t = t_2$. How far has it traveled in $t = t_2$? Use Simpson's rule with n steps to evaluate the integral.

Given: $\rho = 100\ \text{ft}$ $\quad t_1 = 4\ \text{s}$ $\quad b = 60\ \dfrac{\text{ft}}{\text{s}}$ $\quad c = -1\ \text{s}^{-2}$ $\quad t_2 = 2\ \text{s}$ $\quad n = 50$

Solution: $t = t_2$ $\qquad v = b\left(1 - e^{c\,t^2}\right)$

$a_t = -2b\,c\,t\,e^{c\,t^2}$ $\qquad a_n = \dfrac{v^2}{\rho}$ $\qquad a = \sqrt{a_t^2 + a_n^2}$ $\qquad a = 35.0\ \dfrac{\text{ft}}{\text{s}^2}$

$$s_2 = \int_0^{t_2} b\left(1 - e^{c\,t^2}\right) \mathrm{d}t \qquad s_2 = 67.1\ \text{ft}$$

Problem 12-135

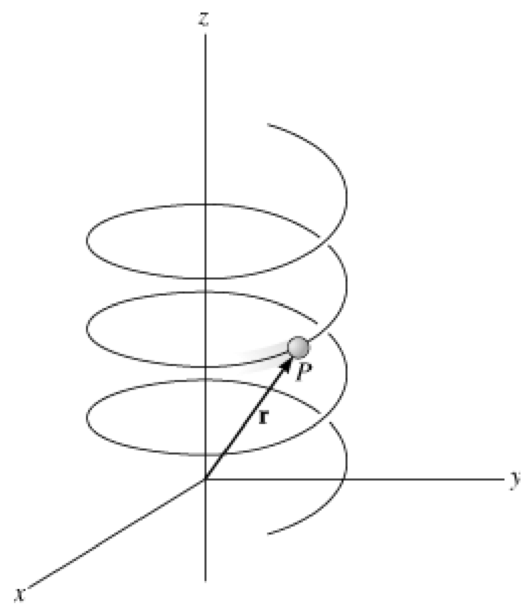
A particle P travels along an elliptical spiral path such that its position vector **r** is defined by **r** = (a cos bt **i** + c sin dt **j** + et **k**). When $t = t_1$, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint:* Solve for the velocity $\mathbf{v_p}$ and acceleration $\mathbf{a_p}$ of the particle in terms of their **i**, **j**, **k** components. The binormal is parallel to $\mathbf{v_p} \times \mathbf{a_p}$. Why?

Given:

$a = 2\ \text{m}$ $\quad d = 0.1\ \text{s}^{-1}$

$b = 0.1\ \text{s}^{-1}$ $\quad e = 2\ \frac{\text{m}}{\text{s}}$

$c = 1.5\ \text{m}$ $\quad t_1 = 8\ \text{s}$



Solution: $\quad t = t_1$

$$\mathbf{r_P} = \begin{bmatrix} (a)\cos(b\,t) \\ c\sin(d\,t) \\ e\,t \end{bmatrix}$$

$$\mathbf{v_P} = \begin{pmatrix} -a\,b\sin(b\,t) \\ c\,d\cos(d\,t) \\ e \end{pmatrix} \qquad \mathbf{a_P} = \begin{pmatrix} -a\,b^2\cos(b\,t) \\ -c\,d^2\sin(d\,t) \\ 0 \end{pmatrix}$$

$$\mathbf{u_b} = \frac{\mathbf{v_P}\times\mathbf{a_P}}{\left|\mathbf{v_P}\times\mathbf{a_P}\right|} \qquad \mathbf{u_b} = \begin{pmatrix} 0.609 \\ -0.789 \\ 0.085 \end{pmatrix} \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \operatorname{acos}(\mathbf{u_b}) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 52.5 \\ 142.1 \\ 85.1 \end{pmatrix}\text{deg}$$

***Problem 12-136**

The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, **a'**, in terms of its cylindrical components, using Eq. 12-32.

Solution:

$$\mathbf{a} = \left(r'' - r\theta'^2\right)\mathbf{u_r} + \left(r\theta'' + 2r'\theta'\right)\mathbf{u_\theta} + z''\mathbf{u_z}$$

$$\mathbf{a'} = \left(r''' - r'\theta'^2 - 2r\theta'\theta''\right)\mathbf{u_r} + \left(r'' - r\theta'^2\right)\mathbf{u'_r} \ldots$$
$$+ \left(r'\theta'' + r\theta''' + 2r''\theta' + 2r'\theta''\right)\mathbf{u_\theta} + \left(r\theta'' + 2r'\theta'\right)\mathbf{u'_\theta} + z'''\mathbf{u_z} + z''\mathbf{u'_z}$$

But $\quad \mathbf{u_r} = \theta'\mathbf{u_\theta} \qquad \mathbf{u'_\theta} = -\theta'\mathbf{u_r} \qquad \mathbf{u'_z} = 0$

Substituting and combining terms yields

$$\mathbf{a'} = \left(r''' - 3r'\theta'^2 - 3r\theta'\theta''\right)\mathbf{u_r} + \left(r\theta''' + 3r'\theta'' + 3r''\theta' - r\theta'^3\right)\mathbf{u_\theta} + (z''')\mathbf{u_z}$$

Problem 12-137

If a particle's position is described by the polar coordinates $r = a(1 + \sin bt)$ and $\theta = ce^{dt}$, determine the radial and tangential components of the particle's velocity and acceleration when $t = t_1$.

Given: $a = 4\ \text{m}$ $b = 1\ \text{s}^{-1}$ $c = 2\ \text{rad}$ $d = -1\ \text{s}^{-1}$ $t_1 = 2\ \text{s}$

Solution: When $t = t_1$

$r = a(1 + \sin(b\,t))$ $r' = a\,b\cos(b\,t)$ $r'' = -a\,b^2\sin(b\,t)$

$\theta = c\,e^{d\,t}$ $\theta' = c\,d\,e^{d\,t}$ $\theta'' = c\,d^2\,e^{d\,t}$

$v_r = r'$ $v_r = -1.66\ \frac{\text{m}}{\text{s}}$

$v_\theta = r\theta'$ $v_\theta = -2.07\ \frac{\text{m}}{\text{s}}$

$a_r = r'' - r\theta'^2$ $a_r = -4.20\ \frac{\text{m}}{\text{s}^2}$

$a_\theta = r\theta'' + 2r'\theta'$ $a_\theta = 2.97\ \frac{\text{m}}{\text{s}^2}$

Problem 12–138

The slotted fork is rotating about O at a constant rate θ'. Determine the radial and transverse components of the velocity and acceleration of the pin A at the instant $\theta = \theta_1$. The path is defined by the spiral groove $r = b + c\theta$, where θ is in radians.

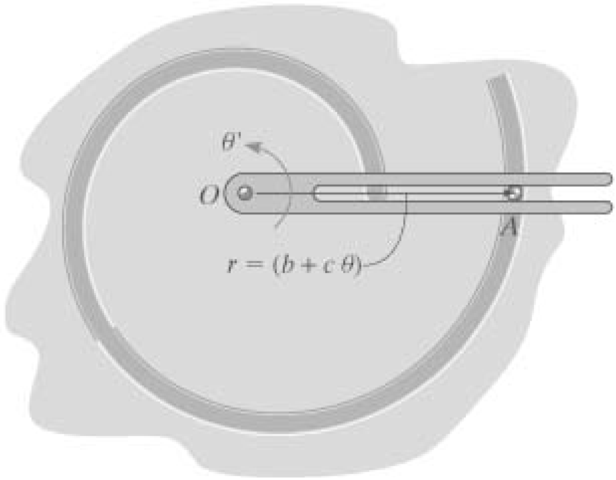
Given:

$\theta' = 3\ \frac{\text{rad}}{\text{s}}$

$b = 5\ \text{in}$

$c = \frac{1}{\pi}\ \text{in}$

$\theta_1 = 2\ \pi\ \text{rad}$



Solution: $\theta = \theta_1$

$r = b + c\theta$ $\quad r' = c\theta'$ $\quad r'' = 0\ \dfrac{\text{in}}{\text{s}^2}$ $\quad \theta'' = 0\ \dfrac{\text{rad}}{\text{s}^2}$

$v_r = r'$ $\quad v_\theta = r\theta'$ $\quad a_r = r'' - r\theta'^2$ $\quad a_\theta = r\theta'' + 2r'\theta'$

$v_r = 0.955\ \dfrac{\text{in}}{\text{s}}$ $\quad v_\theta = 21\ \dfrac{\text{in}}{\text{s}}$ $\quad a_r = -63\ \dfrac{\text{in}}{\text{s}^2}$ $\quad a_\theta = 5.73\ \dfrac{\text{in}}{\text{s}^2}$

Problem 12–139

The slotted fork is rotating about O at the rate θ' which is increasing at θ'' when $\theta = \theta_1$. Determine the radial and transverse components of the velocity and acceleration of the pin A at this instant. The path is defined by the spiral groove $r = (5 + \theta/\pi)$ in., where θ is in radians.

Given:

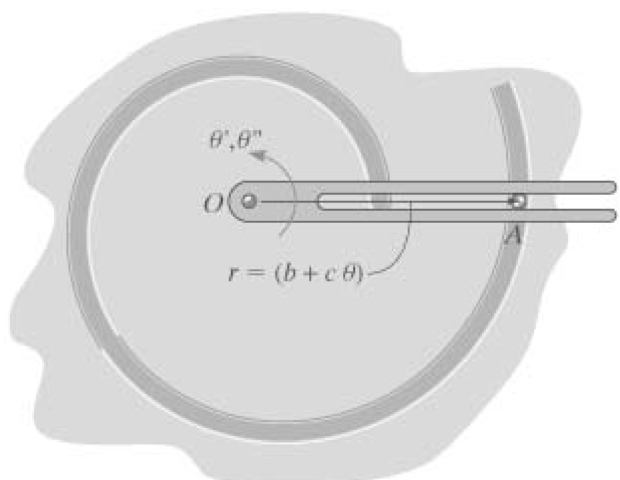
$\theta' = 3\ \dfrac{\text{rad}}{\text{s}}$

$\theta'' = 2\ \dfrac{\text{rad}}{\text{s}^2}$

$b = 5\ \text{in}$

$c = \dfrac{1}{\pi}\ \text{in}$

$\theta_1 = 2\ \pi\ \text{rad}$



Solution: $\quad \theta = \theta_1$

$r = b + c\theta$ $\quad r' = c\theta'$ $\quad r'' = c\theta''$

$v_r = r'$ $\quad v_\theta = r\theta'$

$a_r = r'' - r\theta'^2$ $\quad a_\theta = r\theta'' + 2r'\theta'$

$v_r = 0.955\ \dfrac{\text{in}}{\text{s}}$ $\quad v_\theta = 21\ \dfrac{\text{in}}{\text{s}}$ $\quad a_r = -62.363\ \dfrac{\text{in}}{\text{s}^2}$ $\quad a_\theta = 19.73\ \dfrac{\text{in}}{\text{s}^2}$

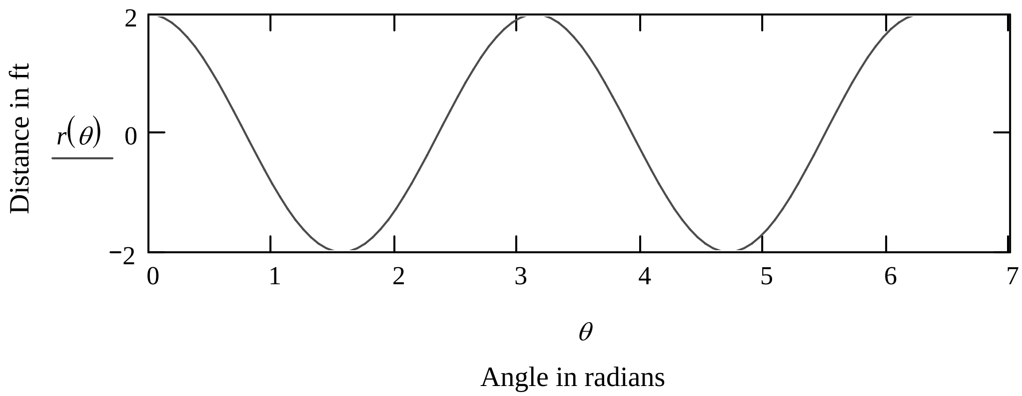
***Problem 12-140**

If a particle moves along a path such that $r = a\cos(bt)$ and $\theta = ct$, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

Given: $a = 2\ \text{ft}$ $b = 1\ \text{s}^{-1}$ $c = 0.5\ \frac{\text{rad}}{\text{s}}$

The plot $t = \frac{\theta}{c}$ $r = (a)\cos\left(b\frac{\theta}{c}\right)$

$\theta = 0, 0.01(2\pi) .. 2\pi$ $r(\theta) = (a)\cos\left(b\frac{\theta}{c}\right)\frac{1}{\text{ft}}$



$r = (a)\cos(b\,t)$ $r' = -a\,b\sin(b\,t)$ $r'' = -a\,b^2\cos(b\,t)$

$\theta = c\,t$ $\theta' = c$ $\theta'' = 0$

$v_r = r' = -a\,b\sin(b\,t)$ $a_r = r'' - r\theta'^2 = -a\left(b^2 + c^2\right)\cos(b\,t)$

$v_\theta = r\theta' = a\,c\cos(b\,t)$ $a_\theta = r\theta'' + 2r'\theta' = -2a\,b\,c\sin(b\,t)$

Problem 12-141

If a particle's position is described by the polar coordinates $r = a\sin b\theta$ and $\theta = ct$, determine the radial and tangential components of its velocity and acceleration when $t = t_1$.

Given: $a = 2\ \text{m}$ $b = 2\ \text{rad}$ $c = 4\ \frac{\text{rad}}{\text{s}}$ $t_1 = 1\ \text{s}$

Solution: $t = t_1$

$r = (a)\sin(b\,c\,t)$ $r' = a\,b\,c\cos(b\,c\,t)$ $r'' = -a\,b^2\,c^2\sin(b\,c\,t)$

$\theta = c\,t$ $\theta' = c$ $\theta'' = 0\ \frac{\text{rad}}{\text{s}^2}$

$v_r = r'$ $v_r = -2.328\ \frac{\text{m}}{\text{s}}$

$v_\theta = r\theta'$ $v_\theta = 7.915\ \frac{\text{m}}{\text{s}}$

$a_r = r'' - r\theta'^2$ $\qquad a_r = -158.3 \dfrac{\text{m}}{\text{s}^2}$

$a_\theta = r\theta'' + 2r'\theta'$ $\qquad a_\theta = -18.624 \dfrac{\text{m}}{\text{s}^2}$

Problem 12-142

A particle is moving along a circular path having a radius r. Its position as a function of time is given by $\theta = bt^2$. Determine the magnitude of the particle's acceleration when $\theta = \theta_1$. The particle starts from rest when $\theta = 0°$.

Given: $r = 400 \text{ mm}$ $\qquad b = 2 \dfrac{\text{rad}}{\text{s}^2}$ $\qquad \theta_1 = 30 \text{ deg}$

Solution: $t = \sqrt{\dfrac{\theta_1}{b}}$ $\qquad t = 0.512 \text{ s}$

$\theta = bt^2$ $\qquad \theta' = 2bt$ $\qquad \theta'' = 2b$

$a = \sqrt{\left(-r\theta'^2\right)^2 + \left(r\theta''\right)^2}$ $\qquad a = 2.317 \dfrac{\text{m}}{\text{s}^2}$

Problem 12-143

A particle moves in the x - y plane such that its position is defined by $\mathbf{r} = at\mathbf{i} + bt^2\mathbf{j}$. Determine the radial and tangential components of the particle's velocity and acceleration when $t = t_1$.

Given: $a = 2 \dfrac{\text{ft}}{\text{s}}$ $\qquad b = 4 \dfrac{\text{ft}}{\text{s}^2}$ $\qquad t_1 = 2 \text{ s}$

Solution: $t = t_1$

Rectangular

$x = at$ $\qquad v_x = a$ $\qquad a_x = 0 \dfrac{\text{ft}}{\text{s}^2}$

$y = bt^2$ $\qquad v_y = 2bt$ $\qquad a_y = 2b$

Polar

$\theta = \text{atan}\left(\dfrac{y}{x}\right)$ $\qquad \theta = 75.964 \text{ deg}$

$$v_r = v_x \cos(\theta) + v_y \sin(\theta) \qquad v_r = 16.007\ \frac{\text{ft}}{\text{s}}$$

$$v_\theta = -v_x \sin(\theta) + v_y \cos(\theta) \qquad v_\theta = 1.94\ \frac{\text{ft}}{\text{s}}$$

$$a_r = a_x \cos(\theta) + a_y \sin(\theta) \qquad a_r = 7.761\ \frac{\text{ft}}{\text{s}^2}$$

$$a_\theta = -a_x \sin(\theta) + a_y \cos(\theta) \qquad a_\theta = 1.94\ \frac{\text{ft}}{\text{s}^2}$$

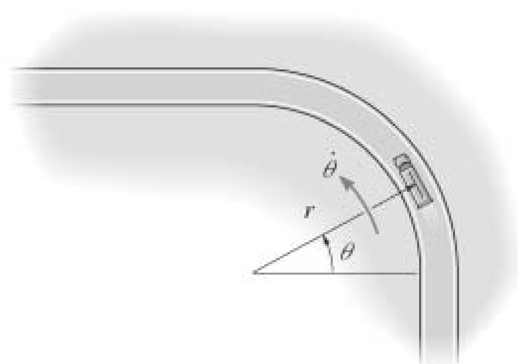
***Problem 12-144**

A truck is traveling along the horizontal circular curve of radius r with a constant speed v. Determine the angular rate of rotation θ' of the radial line r and the magnitude of the truck's acceleration.

Given:

$$r = 60\ \text{m}$$

$$v = 20\ \frac{\text{m}}{\text{s}}$$



Solution:

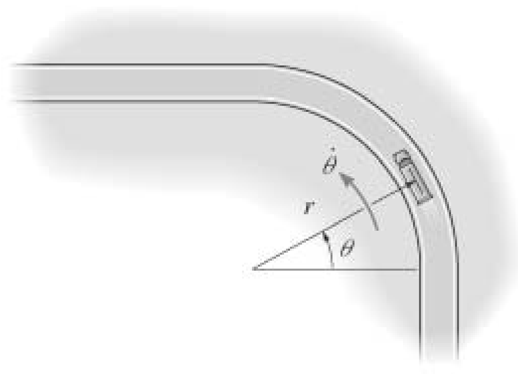
$$\theta' = \frac{v}{r} \qquad \theta' = 0.333\ \frac{\text{rad}}{\text{s}}$$

$$a = \left|-r\,\theta'^2\right| \qquad a = 6.667\ \frac{\text{m}}{\text{s}^2}$$

Problem 12-145

A truck is traveling along the horizontal circular curve of radius r with speed v which is increasing at the rate v'. Determine the truck's radial and transverse components of acceleration.

Given:



$r = 60\ \text{m}$

$v = 20\ \dfrac{\text{m}}{\text{s}}$

$v' = 3\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$a_r = \dfrac{-v^2}{r}$ $\qquad a_r = -6.667\ \dfrac{\text{m}}{\text{s}^2}$

$a_\theta = v'$ $\qquad a_\theta = 3\ \dfrac{\text{m}}{\text{s}^2}$

Problem 12-146

A particle is moving along a circular path having radius r such that its position as a function of time is given by $\theta = c \sin bt$. Determine the acceleration of the particle at $\theta = \theta_1$. The particle starts from rest at $\theta = 0°$.

Given: $r = 6\ \text{in}$ $\quad c = 1\ \text{rad}$ $\quad b = 3\ \text{s}^{-1}$ $\quad \theta_1 = 30\ \text{deg}$

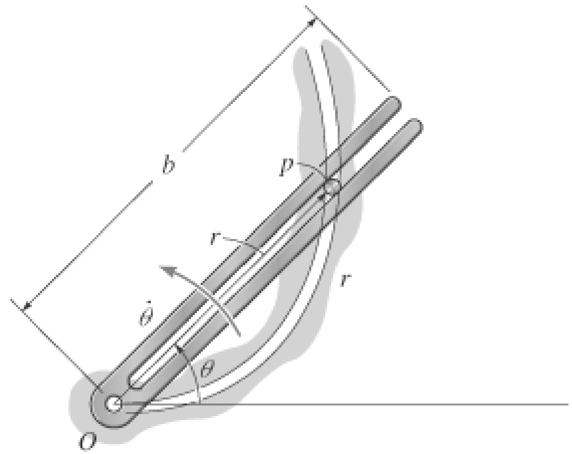
Solution: $t = \dfrac{1}{b}\operatorname{asin}\left(\dfrac{\theta_1}{c}\right)$ $\qquad t = 0.184\ \text{s}$

$\theta = c\sin(b\,t)$ $\qquad \theta' = c\,b\cos(b\,t)$ $\qquad \theta'' = c\,b^2\sin(b\,t)$

$a = \sqrt{\left(-r\,\theta'^2\right)^2 + \left(r\theta''\right)^2}$ $\qquad a = 48.329\ \dfrac{\text{in}}{\text{s}^2}$

Problem 12-147

The slotted link is pinned at O, and as a result of the constant angular velocity θ' it drives the peg P for a short distance along the spiral guide $r = a\theta$. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \theta_1$.



Given:

$\theta' = 3\,\dfrac{\text{rad}}{\text{s}}$ $\theta_1 = \dfrac{\pi}{3}\,\text{rad}$

$a = 0.4\ \text{m}$ $b = 0.5\ \text{m}$

Solution: $\theta = \theta_1$

$r = a\theta$ $r' = a\theta'$ $r'' = 0\,\dfrac{\text{m}}{\text{s}^2}$

$v_r = r'$ $v_r = 1.2\,\dfrac{\text{m}}{\text{s}}$

$v_\theta = r\theta'$ $v_\theta = 1.257\,\dfrac{\text{m}}{\text{s}}$

$a_r = r'' - r\theta'^2$ $a_r = -3.77\,\dfrac{\text{m}}{\text{s}^2}$

$a_\theta = 2r'\theta'$ $a_\theta = 7.2\,\dfrac{\text{m}}{\text{s}^2}$

***Problem 12-148**

The slotted link is pinned at O, and as a result of the angular velocity θ' and the angular acceleration θ'' it drives the peg P for a short distance along the spiral guide $r = a\theta$. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \theta_1$.

Given:

$\theta' = 3\,\dfrac{\text{rad}}{\text{s}}$ $\theta_1 = \dfrac{\pi}{3}\,\text{rad}$

$a = 0.4\ \text{m}$

$\theta'' = 8\,\dfrac{\text{rad}}{\text{s}^2}$

$b = 0.5\ \text{m}$

Solution: $\theta = \theta_1$

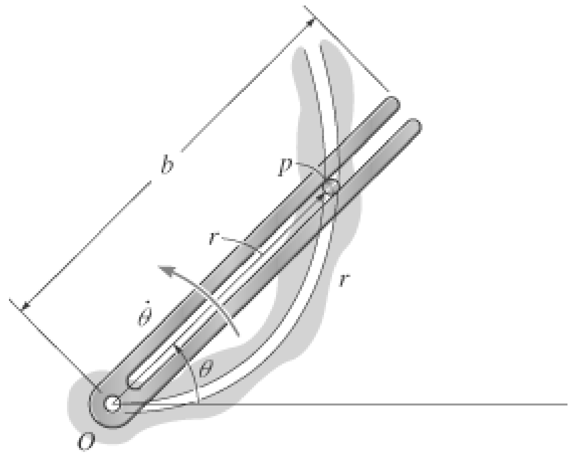
$r = a\theta \qquad r' = a\theta' \qquad r'' = a\theta''$

$v_r = r' \qquad \boxed{v_r = 1.2\ \frac{\text{m}}{\text{s}}}$

$v_\theta = r\theta' \qquad \boxed{v_\theta = 1.257\ \frac{\text{m}}{\text{s}}}$

$a_r = r'' - r\theta'^2 \qquad \boxed{a_r = -0.57\ \frac{\text{m}}{\text{s}^2}}$

$a_\theta = r\theta'' + 2r'\theta' \qquad \boxed{a_\theta = 10.551\ \frac{\text{m}}{\text{s}^2}}$



Problem 12-149

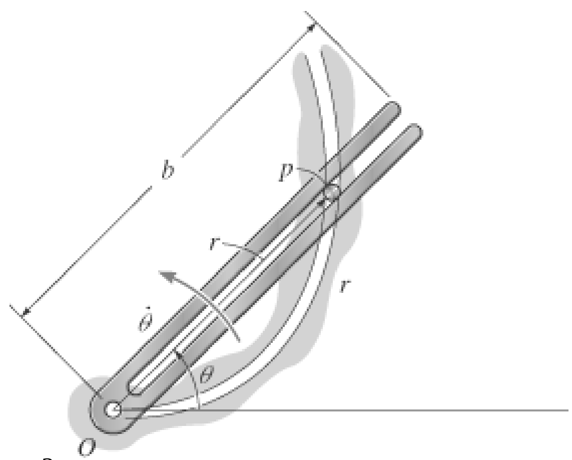
The slotted link is pinned at *O*, and as a result of the constant angular velocity θ' it drives the peg *P* for a short distance along the spiral guide $r = a\theta$ where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = b$.

Given:

$\theta' = 3\ \frac{\text{rad}}{\text{s}}$

$a = 0.4\ \text{m}$

$b = 0.5\ \text{m}$



Solution: $\theta = \frac{b}{a}$

$r = a\theta \qquad r' = a\theta' \qquad r'' = 0\ \frac{\text{m}}{\text{s}^2}$

$v_r = r' \qquad v_\theta = r\theta' \qquad a_r = r'' - r\theta'^2 \qquad a_\theta = 2r'\theta'$

$v = \sqrt{v_r^2 + v_\theta^2} \qquad a = \sqrt{a_r^2 + a_\theta^2} \qquad \boxed{v = 1.921\ \frac{\text{m}}{\text{s}}} \qquad \boxed{a = 8.491\ \frac{\text{m}}{\text{s}^2}}$

Problem 12–150

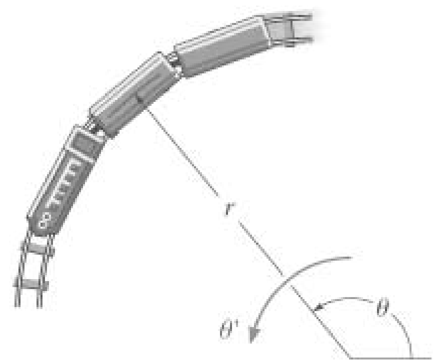
A train is traveling along the circular curve of radius *r*. At the instant shown, its angular rate of rotation is θ', which is decreasing at θ''. Determine the magnitudes of the train's velocity and acceleration at this instant.

Given:

$$r = 600 \text{ ft}$$

$$\theta' = 0.02 \frac{\text{rad}}{\text{s}}$$

$$\theta'' = -0.001 \frac{\text{rad}}{\text{s}^2}$$



Solution:

$$v = r\theta' \qquad v = 12 \frac{\text{ft}}{\text{s}}$$

$$a = \sqrt{\left(-r\,\theta'^2\right)^2 + \left(r\theta''\right)^2} \qquad a = 0.646 \frac{\text{ft}}{\text{s}^2}$$

Problem 12–151

A particle travels along a portion of the "four-leaf rose" defined by the equation $r = a\cos(b\theta)$. If the angular velocity of the radial coordinate line is $\theta' = ct^2$, determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = \theta_1$. When $t = 0$, $\theta = 0°$.

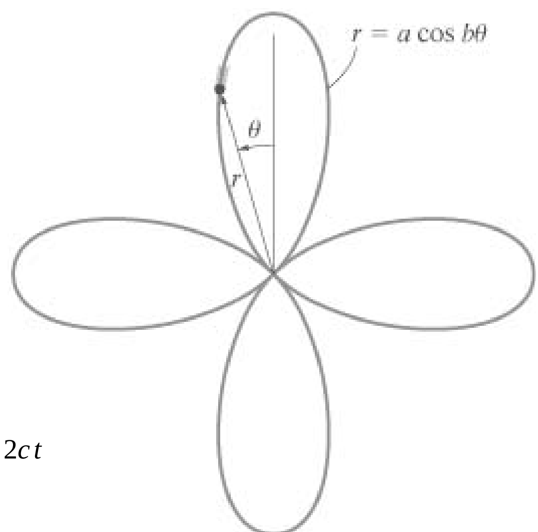
Given:

$$a = 5 \text{ m}$$

$$b = 2$$

$$c = 3 \frac{\text{rad}}{\text{s}^3}$$

$$\theta_1 = 30 \text{ deg}$$



Solution:

$$\theta(t) = \frac{c}{3}t^3 \qquad \theta'(t) = c\,t^2 \qquad \theta''(t) = 2c\,t$$

$$r(t) = (a)\cos\big(b\,\theta(t)\big) \qquad r'(t) = \frac{d}{dt}r(t) \qquad r''(t) = \frac{d}{dt}r'(t)$$

When $\theta = \theta_1$ $\qquad t_1 = \left(\frac{3\theta_1}{c}\right)^{\frac{1}{3}}$

$$v_r = r'(t_1) \qquad v_r = -16.88 \frac{\text{m}}{\text{s}}$$

$$v_\theta = r(t_1)\,\theta'(t_1) \qquad v_\theta = 4.87\ \frac{\text{m}}{\text{s}}$$

$$a_r = r''(t_1) - r(t_1)\,\theta'(t_1)^2 \qquad a_r = -89.4\ \frac{\text{m}}{\text{s}^2}$$

$$a_\theta = r(t_1)\,\theta''(t_1) + 2\,r'(t_1)\,\theta'(t_1) \qquad a_\theta = -53.7\ \frac{\text{m}}{\text{s}^2}$$

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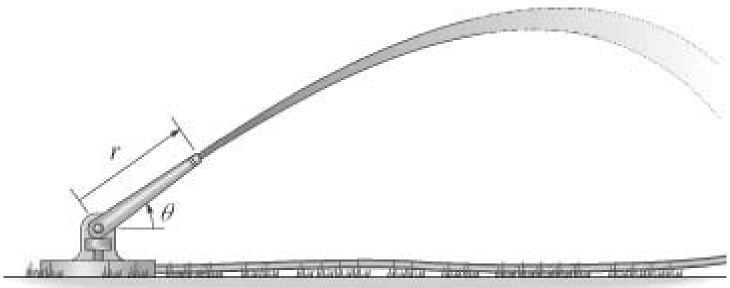
*__Problem 12-152__

At the instant shown, the watersprinkler is rotating with an angular speed θ' and an angular acceleration θ''. If the nozzle lies in the vertical plane and water is flowing through it at a constant rate r', determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, r.

Given:

$$\theta' = 2\ \frac{\text{rad}}{\text{s}} \qquad \theta'' = 3\ \frac{\text{rad}}{\text{s}^2}$$

$$r' = 3\ \frac{\text{m}}{\text{s}} \qquad r = 0.2\ \text{m}$$



Solution:

$$v = \sqrt{r'^2 + (r\theta')^2} \qquad v = 3.027\ \frac{\text{m}}{\text{s}}$$

$$a = \sqrt{\left(-r\,\theta'^2\right)^2 + \left(r\theta'' + 2r'\theta'\right)^2} \qquad a = 12.625\ \frac{\text{m}}{\text{s}^2}$$

Problem 12–153

The boy slides down the slide at a constant speed v. If the slide is in the form of a helix, defined by the equations r = constant and $z = -(h\theta)/(2\pi)$, determine the boy's angular velocity about the z axis, θ' and the magnitude of his acceleration.

Given:

$$v = 2\ \frac{\text{m}}{\text{s}}$$

$$r = 1.5\ \text{m}$$

$$h = 2\ \text{m}$$

Solution:

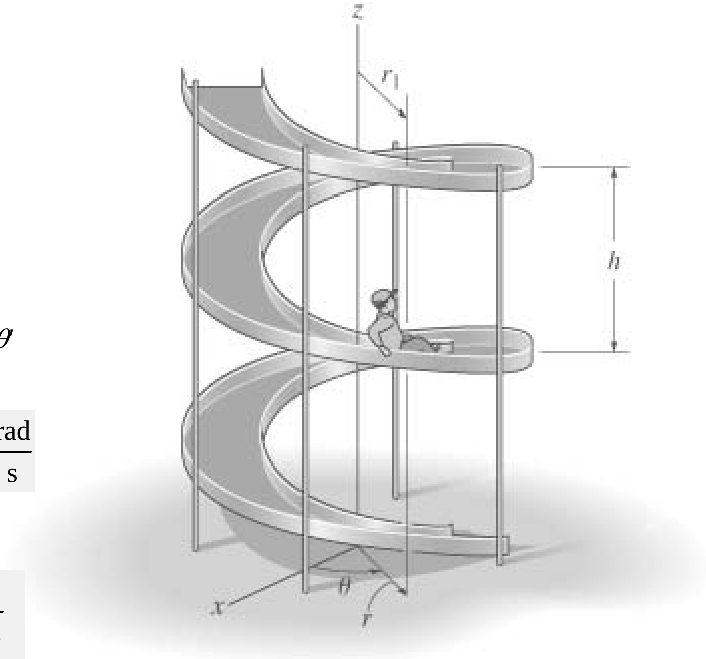
$$z = \frac{h}{2\pi}\theta$$

$$z' = \frac{h}{2\pi}\theta'$$

$$v = \sqrt{z'^2 + (r\theta')^2} = \sqrt{\left(\frac{h}{2\pi}\right)^2 + r^2}\,\theta'$$

$$\theta' = \frac{v}{\sqrt{\left(\frac{h}{2\pi}\right)^2 + r^2}} \qquad \theta' = 1.304\,\frac{\text{rad}}{\text{s}}$$

$$a = \left|-r\,\theta'^2\right| \qquad a = 2.55\,\frac{\text{m}}{\text{s}^2}$$

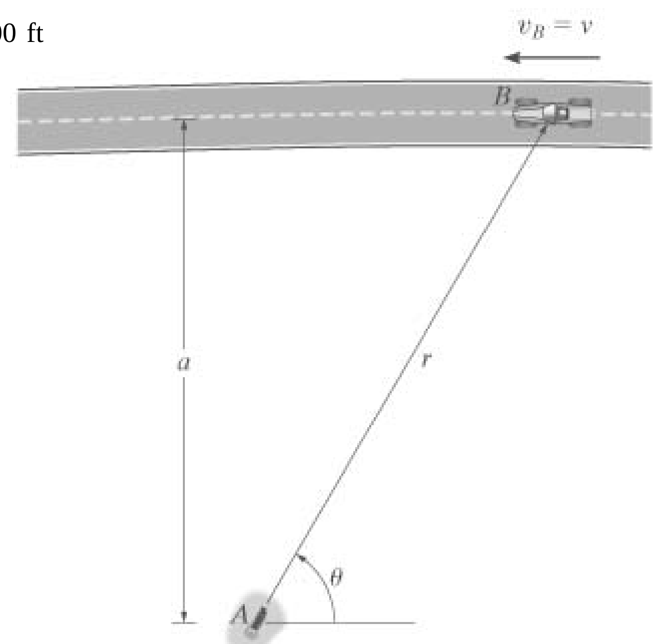


Problem 12–154

A cameraman standing at *A* is following the movement of a race car, *B*, which is traveling along a straight track at a constant speed *v*. Determine the angular rate at which he must turn in order to keep the camera directed on the car at the instant $\theta = \theta_1$.

Given:

$$v = 80\,\frac{\text{ft}}{\text{s}} \qquad \theta_1 = 60\text{ deg} \qquad a = 100\text{ ft}$$



Solution: $\qquad \theta = \theta_1$

$$a = r\sin(\theta)$$

$$0 = r'\sin(\theta) + r\theta'\cos(\theta)$$

$$x = r\cos(\theta)$$

$$v_x = -v = r'\cos(\theta) - r\theta'\sin(\theta)$$

Guess

$$r = 1\text{ ft} \qquad r' = 1\,\frac{\text{ft}}{\text{s}} \qquad \theta' = 1\,\frac{\text{rad}}{\text{s}}$$

Given

$$a = r\sin(\theta)$$

$$0 = r' \sin(\theta) + r\theta' \cos(\theta)$$

$$-v = r' \cos(\theta) - r\theta' \sin(\theta)$$

$$\begin{pmatrix} r \\ r' \\ \theta' \end{pmatrix} = \text{Find}(r, r', \theta')$$

$r = 115.47\ \text{ft}$ $\quad r' = -40\,\dfrac{\text{ft}}{\text{s}}$ $\quad \theta' = 0.6\,\dfrac{\text{rad}}{\text{s}}$

Problem 12-155

For a short distance the train travels along a track having the shape of a spiral, $r = a/\theta$. If it maintains a constant speed v, determine the radial and transverse components of its velocity when $\theta = \theta_1$.

Given: $\quad a = 1000\ \text{m} \quad v = 20\ \dfrac{\text{m}}{\text{s}} \quad \theta_1 = 9\ \dfrac{\pi}{4}\ \text{rad}$

Solution: $\quad \theta = \theta_1$

$$r = \frac{a}{\theta} \qquad r' = \frac{-a}{\theta^2}\theta' \qquad v^2 = r'^2 + r^2\,\theta'^2 = \left(\frac{a^2}{\theta^4} + \frac{a^2}{\theta^2}\right)\theta'^2$$

$$\theta' = \frac{v\theta^2}{a\sqrt{1+\theta^2}} \qquad r = \frac{a}{\theta} \qquad r' = \frac{-a}{\theta^2}\theta'$$

$v_r = r' \qquad v_r = -2.802\ \dfrac{\text{m}}{\text{s}}$

$v_\theta = r\theta' \qquad v_\theta = 19.803\ \dfrac{\text{m}}{\text{s}}$

***Problem 12-156**

For a short distance the train travels along a track having the shape of a spiral, $r = a / \theta$. If the angular rate θ' is constant, determine the radial and transverse components of its velocity and acceleration when $\theta = \theta_1$.

Given: $\quad a = 1000\ \text{m} \quad \theta' = 0.2\ \dfrac{\text{rad}}{\text{s}} \quad \theta_1 = 9\dfrac{\pi}{4}$

Solution: $\quad \theta = \theta_1$

$$r = \frac{a}{\theta} \qquad r' = \frac{-a}{\theta^2}\theta' \qquad r'' = \frac{2a}{\theta^3}\theta'^2$$

$v_r = r'$ $\qquad v_r = -4.003\ \frac{m}{s}$

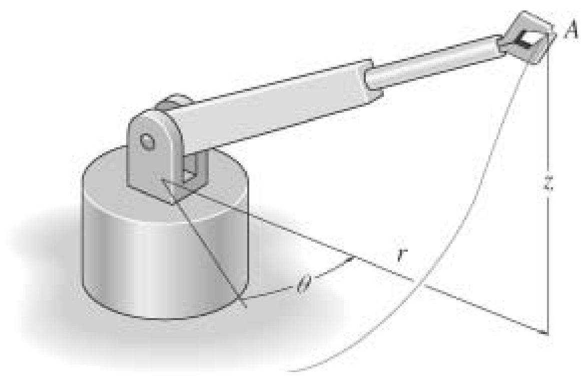
$v_\theta = r\theta'$ $\qquad v_\theta = 28.3\ \frac{m}{s}$

$a_r = r'' - r\theta'^2$ $\qquad a_r = -5.432\ \frac{m}{s^2}$

$a_\theta = 2r'\theta'$ $\qquad a_\theta = -1.601\ \frac{m}{s^2}$

Problem 12-157

The arm of the robot has a variable length so that r remains constant and its grip. A moves along the path $z = a \sin b\theta$. If $\theta = ct$, determine the magnitudes of the grip's velocity and acceleration when $t = t_1$.



Given:

$r = 3\text{ ft}$ $\quad c = 0.5\ \frac{\text{rad}}{\text{s}}$

$a = 3\text{ ft}$ $\quad t_1 = 3\text{ s}$

$b = 4$

Solution: $\quad t = t_1$

$\theta = ct \qquad r = r \qquad z = a\sin(bct)$

$\theta' = c \qquad r' = 0\frac{\text{ft}}{\text{s}} \qquad z' = abc\cos(bct)$

$\theta'' = 0\frac{\text{rad}}{\text{s}^2} \qquad r'' = 0\frac{\text{ft}}{\text{s}^2} \qquad z'' = -ab^2c^2\sin(bct)$

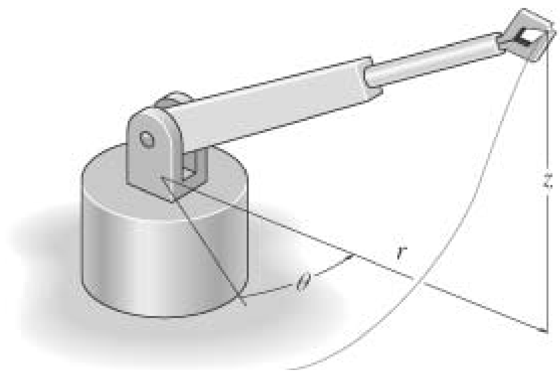
$v = \sqrt{r'^2 + (r\theta')^2 + z'^2}$ $\qquad v = 5.953\frac{\text{ft}}{\text{s}}$

$a = \sqrt{\left(r'' - r\theta'^2\right)^2 + (r\theta'' + 2r'\theta')^2 + z''^2}$ $\qquad a = 3.436\frac{\text{ft}}{\text{s}^2}$

Problem 12-158

For a short time the arm of the robot is extending so that r' remains constant, $z = bt^2$ and $\theta = ct$. Determine the magnitudes of the velocity and acceleration of the grip A when $t = t_1$ and $r = r_1$.

Given:



$r' = 1.5\ \frac{\text{ft}}{\text{s}}$

$b = 4\ \frac{\text{ft}}{\text{s}^2}$

$c = 0.5\ \frac{\text{rad}}{\text{s}}$

$t_1 = 3\ \text{s}$

$r_1 = 3\ \text{ft}$

Solution: $t = t_1$

$r = r_1 \qquad \theta = c\,t \qquad z = b\,t^2$

$\theta' = c \qquad z' = 2b\,t \qquad z'' = 2b$

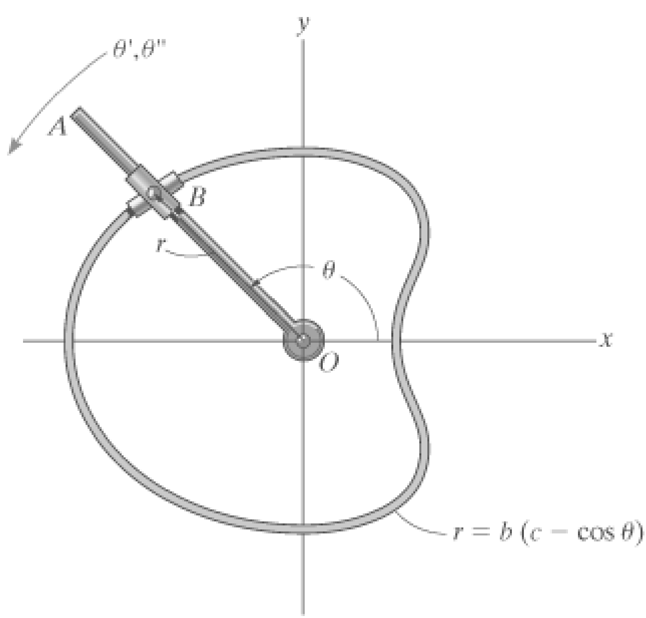
$v = \sqrt{r'^2 + (r\theta')^2 + z'^2}$ $\qquad v = 24.1\ \frac{\text{ft}}{\text{s}}$

$a = \sqrt{\left(-r\,\theta'^2\right)^2 + (2r'\theta')^2 + z''^2}$ $\qquad a = 8.174\ \frac{\text{ft}}{\text{s}^2}$

Problem 12–159

The rod OA rotates counterclockwise with a constant angular velocity of θ'. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = b(c - \cos(\theta))$. Determine the speed of the slider blocks at the instant $\theta = \theta_1$.

Given:



$\theta' = 5\ \frac{\text{rad}}{\text{s}}$

$b = 100\ \text{mm}$

$c = 2$

$\theta_1 = 120\ \text{deg}$

Solution:

$\theta = \theta_1$

$r = b(c - \cos(\theta))$

$r' = b\sin(\theta)\theta'$

$$v = \sqrt{r'^2 + (r\theta')^2} \qquad v = 1.323\ \frac{\text{m}}{\text{s}}$$

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*__Problem 12–160__

The rod OA rotates counterclockwise with a constant angular velocity of θ'. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = b(c - \cos(\theta))$. Determine the acceleration of the slider blocks at the instant $\theta = \theta_1$.

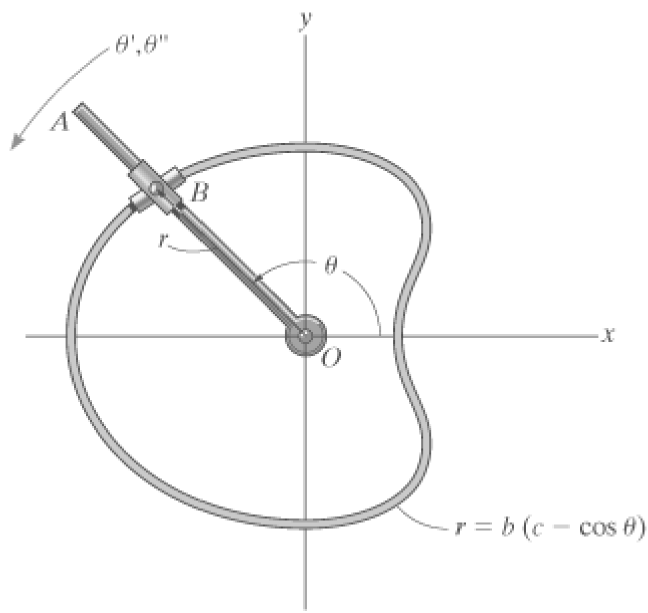
Given:

$$\theta' = 5\ \frac{\text{rad}}{\text{s}}$$

$$b = 100\ \text{mm}$$

$$c = 2$$

$$\theta_1 = 120\ \text{deg}$$



Solution:

$$\theta = \theta_1$$

$$r = b(c - \cos(\theta))$$

$$r' = b\sin(\theta)\theta'$$

$$r'' = b\cos(\theta)\,\theta'^2$$

$$a = \sqrt{\left(r'' - r\theta'^2\right)^2 + (2r'\theta')^2} \qquad a = 8.66\ \frac{\text{m}}{\text{s}^2}$$

Problem 12-161

The searchlight on the boat anchored a distance d from shore is turned on the automobile, which is traveling along the straight road at a constant speed v. Determine the angular rate of rotation of the light when the automobile is $r = r_1$ from the boat.

Given:

$$d = 2000\ \text{ft}$$

$$v = 80\ \frac{\text{ft}}{\text{s}}$$

$$r_1 = 3000\ \text{ft}$$

Solution:

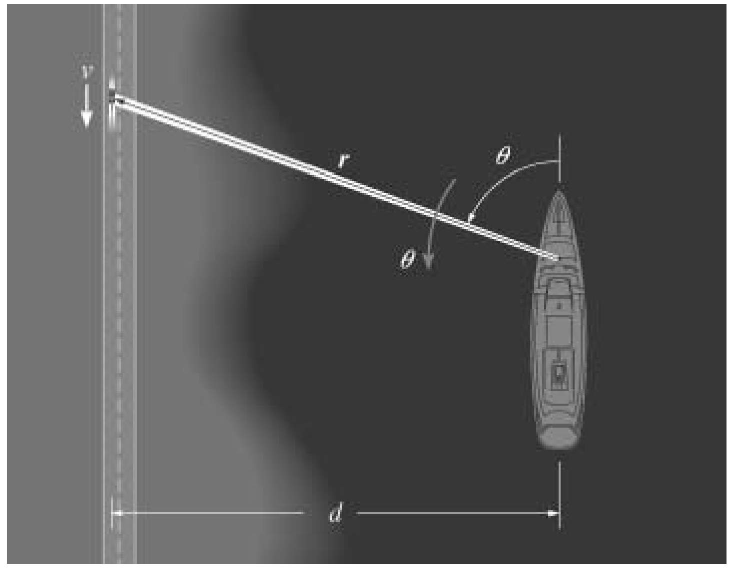
$r = r_1$

$\theta = \operatorname{asin}\left(\frac{d}{r}\right)$

$\theta = 41.81\ \text{deg}$

$\theta' = \frac{v \sin(\theta)}{r}$

$\theta' = 0.0178\ \frac{\text{rad}}{\text{s}}$



Problem 12-162

The searchlight on the boat anchored a distance d from shore is turned on the automobile, which is traveling along the straight road at speed v and acceleration a. Determine the required angular acceleration θ'' of the light when the automobile is $r = r_1$ from the boat.

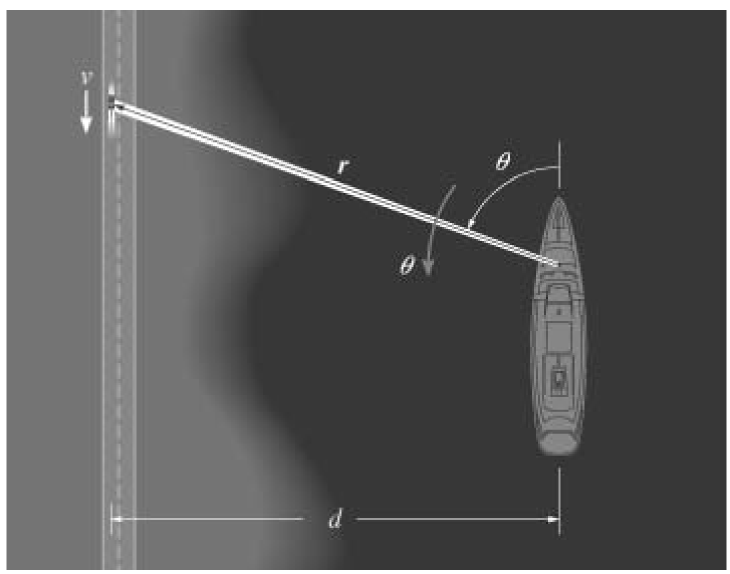
Given:

$d = 2000\ \text{ft}$

$v = 80\ \frac{\text{ft}}{\text{s}}$

$a = 15\ \frac{\text{ft}}{\text{s}^2}$

$r_1 = 3000\ \text{ft}$



Solution:

$r = r_1$

$\theta = \operatorname{asin}\left(\frac{d}{r}\right) \qquad \theta = 41.81\ \text{deg}$

$\theta' = \frac{v \sin(\theta)}{r} \qquad \theta' = 0.0178\ \frac{\text{rad}}{\text{s}}$

$r' = -v \cos(\theta) \qquad r' = -59.628\ \frac{\text{ft}}{\text{s}}$

$$\theta'' = \frac{a\sin(\theta) - 2r'\theta'}{r}$$

$$\theta'' = 0.00404\,\frac{\text{rad}}{\text{s}^2}$$

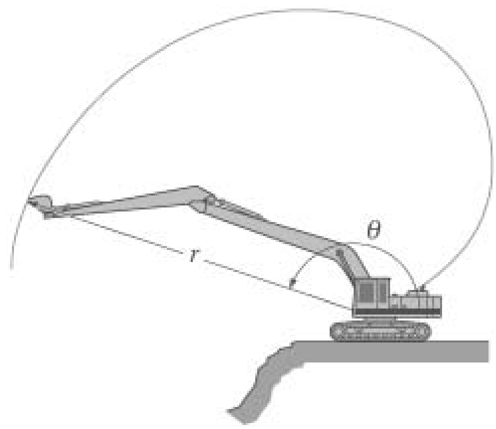
Problem 12–163

For a short time the bucket of the backhoe traces the path of the cardioid $r = a(1 - \cos\theta)$. Determine the magnitudes of the velocity and acceleration of the bucket at $\theta = \theta_1$ if the boom is rotating with an angular velocity θ' and an angular acceleration θ'' at the instant shown.

Given:

$a = 25\ \text{ft}$ $\qquad \theta' = 2\,\frac{\text{rad}}{\text{s}}$

$\theta_1 = 120\ \text{deg}$ $\qquad \theta'' = 0.2\,\frac{\text{rad}}{\text{s}^2}$



Solution:

$$\theta = \theta_1$$

$$r = a(1 - \cos(\theta)) \qquad r' = a\sin(\theta)\theta'$$

$$r'' = a\sin(\theta)\theta'' + a\cos(\theta)\,\theta'^2$$

$$v = \sqrt{r'^2 + (r\theta')^2} \qquad v = 86.6\,\frac{\text{ft}}{\text{s}}$$

$$a = \sqrt{\left(r'' - r\theta'^2\right)^2 + \left(r\theta'' + 2r'\theta'\right)^2} \qquad a = 266\,\frac{\text{ft}}{\text{s}^2}$$

***Problem 12-164**

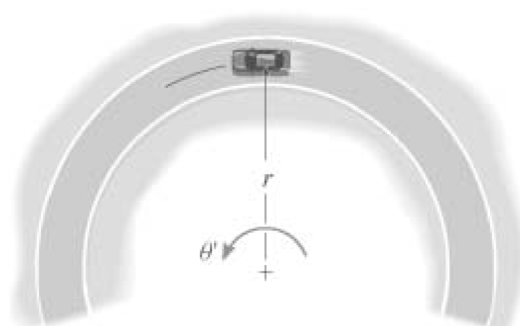
A car is traveling along the circular curve having a radius r. At the instance shown, its angular rate of rotation is θ', which is decreasing at the rate θ''. Determine the radial and transverse components of the car's velocity and acceleration at this instant.

Given:

$r = 400\ \text{ft}$

$\theta' = 0.025\,\frac{\text{rad}}{\text{s}}$

$\theta'' = -0.008\,\frac{\text{rad}}{\text{s}^2}$



Solution:

$v_r = r\theta'$ $\qquad v_r = 3.048\ \frac{\text{m}}{\text{s}}$

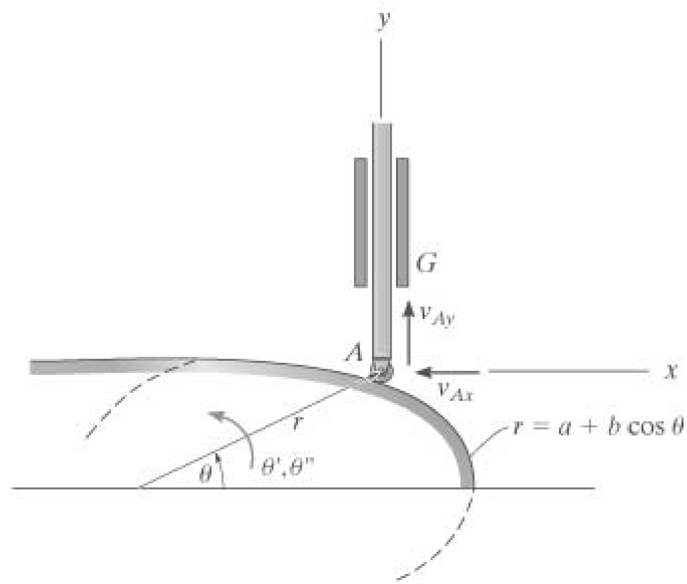
$v_\theta = 0$

$a_r = r\theta''$ $\qquad a_r = -0.975\ \frac{\text{m}}{\text{s}^2}$

$a_\theta = r\theta'^2$ $\qquad a_\theta = 0.076\ \frac{\text{m}}{\text{s}^2}$

Problem 12–165

The mechanism of a machine is constructed so that for a short time the roller at A follows the surface of the cam described by the equation $r = a + b\cos\theta$. If θ' and θ'' are given, determine the magnitudes of the roller's velocity and acceleration at the instant $\theta = \theta_1$. Neglect the size of the roller. Also determine the velocity components v_{Ax} and v_{Ay} of the roller at this instant. The rod to which the roller is attached remains vertical and can slide up or down along the guides while the guides move horizontally to the left.



Given:

$\theta' = 0.5\ \frac{\text{rad}}{\text{s}}$ $\qquad \theta_1 = 30\ \text{deg}$

$a = 0.3\ \text{m}$

$\theta'' = 0\ \frac{\text{rad}}{\text{s}^2}$ $\qquad b = 0.2\ \text{m}$

Solution:

$\theta = \theta_1$

$r = a + b\cos(\theta)$

$r' = -b\sin(\theta)\theta'$

$r'' = -b\sin(\theta)\theta'' - b\cos(\theta)\,\theta'^2$

$v = \sqrt{r'^2 + (r\theta')^2}$ $\qquad v = 0.242\ \frac{\text{m}}{\text{s}}$

$a = \sqrt{\left(r'' - r\theta'^2\right)^2 + \left(r\theta'' + 2r'\theta'\right)^2}$ $\qquad a = 0.169\ \frac{\text{m}}{\text{s}^2}$

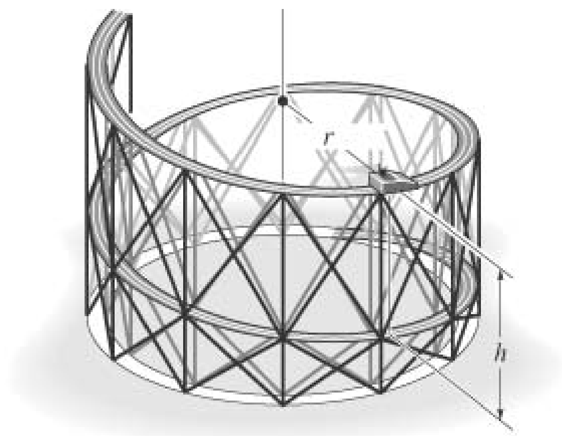
$$v_{Ax} = -r' \cos(\theta) + r\theta' \sin(\theta) \qquad v_{Ax} = 0.162\ \frac{\text{m}}{\text{s}}$$

$$v_{Ay} = r' \sin(\theta) + r\theta' \cos(\theta) \qquad v_{Ay} = 0.18\ \frac{\text{m}}{\text{s}}$$

Problem 12-166

The roller coaster is traveling down along the spiral ramp with a constant speed v. If the track descends a distance h for every full revolution, determine the magnitude of the roller coaster's acceleration as it moves along the track, r of radius. *Hint*: For part of the solution, note that the tangent to the ramp at any point is at an angle $\phi = \tan^{-1}(h/2\pi r)$ from the horizontal. Use this to determine the velocity components v_θ and v_z which in turn are used to determine θ and z.

Given:



$$v = 6\ \frac{\text{m}}{\text{s}} \qquad h = 10\ \text{m} \qquad r = 5\ \text{m}$$

Solution:

$$\phi = \operatorname{atan}\left(\frac{h}{2\pi r}\right) \qquad \phi = 17.657\ \text{deg}$$

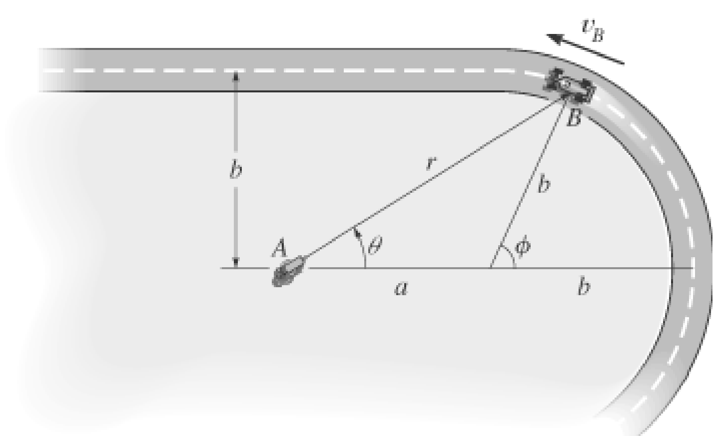
$$\theta' = \frac{v\cos(\phi)}{r} \qquad a = \left|-r\,\theta'^2\right|$$

$$a = 6.538\ \frac{\text{m}}{\text{s}^2}$$

Problem 12-167

A cameraman standing at A is following the movement of a race car, B, which is traveling around a curved track at constant speed v_B. Determine the angular rate at which the man must turn in order to keep the camera directed on the car at the instant $\theta = \theta_1$.

Given:



$$v_B = 30\ \frac{\text{m}}{\text{s}}$$

$$\theta_1 = 30\ \text{deg}$$

$$a = 20\ \text{m}$$

$$b = 20\ \text{m}$$

$$\theta = \theta_1$$

Solution:

Guess

$$r = 1\ \text{m} \qquad r' = 1\ \frac{\text{m}}{\text{s}} \qquad \theta' = 1\ \frac{\text{rad}}{\text{s}} \qquad \phi = 20\ \text{deg} \qquad \phi' = 2\ \frac{\text{rad}}{\text{s}}$$

Given $\quad r\sin(\theta) = b\sin(\phi)$

$$r'\sin(\theta) + r\cos(\theta)\theta' = b\cos(\phi)\phi'$$

$$r\cos(\theta) = a + b\cos(\phi)$$

$$r'\cos(\theta) - r\sin(\theta)\theta' = -b\sin(\phi)\phi'$$

$$v_B = b\phi'$$

$$\begin{pmatrix} r \\ r' \\ \theta' \\ \phi \\ \phi' \end{pmatrix} = \text{Find}(r, r', \theta', \phi, \phi') \qquad r = 34.641\ \text{m} \qquad r' = -15\ \frac{\text{m}}{\text{s}}$$

$$\phi = 60\,\text{deg} \qquad \phi' = 1.5\,\frac{\text{rad}}{\text{s}}$$

$$\theta' = 0.75\,\frac{\text{rad}}{\text{s}}$$

***Problem 12-168**

The pin follows the path described by the equation $r = a + b\cos\theta$. At the instant $\theta = \theta_1$. the angular velocity and angular acceleration are θ' and θ''. Determine the magnitudes of the pin's velocity and acceleration at this instant. Neglect the size of the pin.

Given:

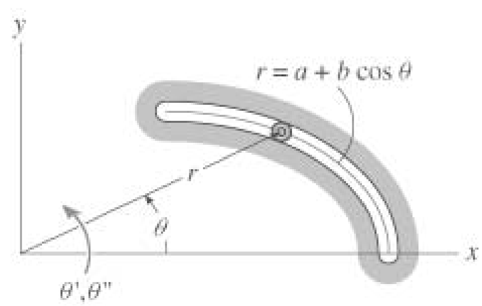
$$a = 0.2\ \text{m}$$

$$b = 0.15\ \text{m}$$

$$\theta_1 = 30\ \text{deg}$$

$$\theta' = 0.7\ \frac{\text{rad}}{\text{s}}$$

$$\theta'' = 0.5\,\frac{\text{rad}}{\text{s}^2}$$



Solution: $\qquad \theta = \theta_1$

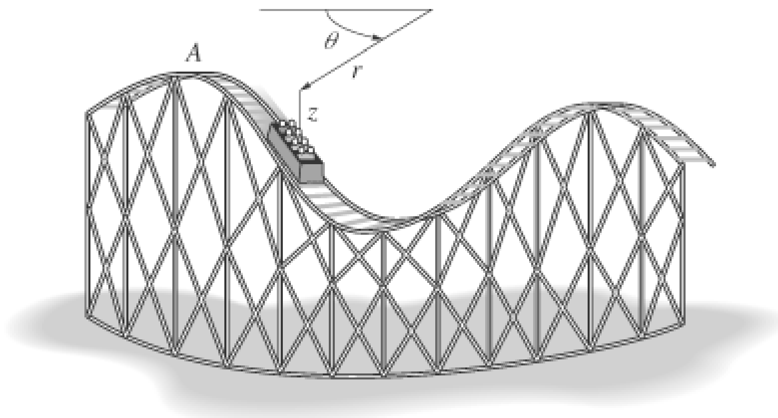
$$r = a + b\cos(\theta) \qquad r' = -b\sin(\theta)\theta' \qquad r'' = -b\cos(\theta)\,\theta'^2 - b\sin(\theta)\theta''$$

$$v = \sqrt{r'^2 + (r\theta')^2}$$ $v = 0.237 \frac{\text{m}}{\text{s}}$

$$a = \sqrt{\left(r'' - r\theta'^2\right)^2 + \left(r\theta'' + 2r'\theta'\right)^2}$$ $a = 0.278 \frac{\text{m}}{\text{s}^2}$

Problem 12-169

For a short time the position of the roller-coaster car along its path is defined by the equations $r = r_0$, $\theta = at$, and $z = b\cos\theta$. Determine the magnitude of the car's velocity and acceleration when $t = t_1$.



Given:

$r_0 = 25$ m

$a = 0.3 \frac{\text{rad}}{\text{s}}$

$b = -8$ m

$t_1 = 4$ s

Solution: $t = t_1$

$r = r_0$ $\theta = a\,t$ $z = b\cos(\theta)$

$\theta' = a$ $z' = -b\sin(\theta)\theta'$

$z'' = -b\cos(\theta)\,\theta'^2$

$$v = \sqrt{(r\theta')^2 + z'^2}$$ $v = 7.826 \frac{\text{m}}{\text{s}}$

$$a = \sqrt{\left(-r\,\theta'^2\right)^2 + z''^2}$$ $a = 2.265 \frac{\text{m}}{\text{s}^2}$

Problem 12-170

The small washer is sliding down the cord OA. When it is at the midpoint, its speed is v and its acceleration is a'. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.

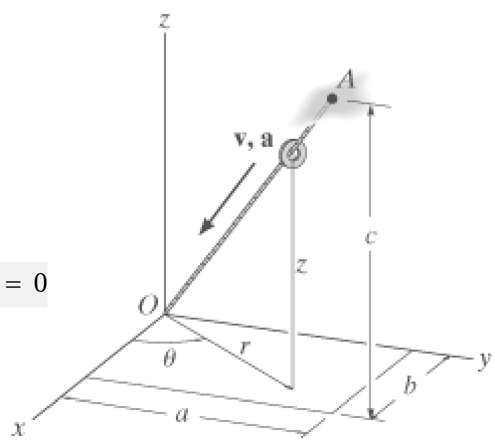
Given:

$v = 200 \frac{\text{mm}}{\text{s}}$ $a' = 10 \frac{\text{mm}}{\text{s}^2}$

$a = 400 \text{ mm}$

$b = 300 \text{ mm}$

$c = 700 \text{ mm}$



Solution:

$$v_r = \frac{-v\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2 + c^2}} \qquad v_r = -0.116 \frac{\text{m}}{\text{s}} \qquad v_\theta = 0$$

$$v_z = \frac{-v\,c}{\sqrt{a^2 + b^2 + c^2}} \qquad v_z = -0.163 \frac{\text{m}}{\text{s}}$$

$$a_r = -a\cos(\alpha)$$

$$a_r = \frac{-a'\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2 + c^2}} \qquad a_r = -5.812 \times 10^{-3} \frac{\text{m}}{\text{s}^2} \qquad a_\theta = 0$$

$$a_z = \frac{-v\,c}{\sqrt{a^2 + b^2 + c^2}} \qquad a_z = -0.163 \frac{\text{m}}{\text{s}}$$

Problem 12–171

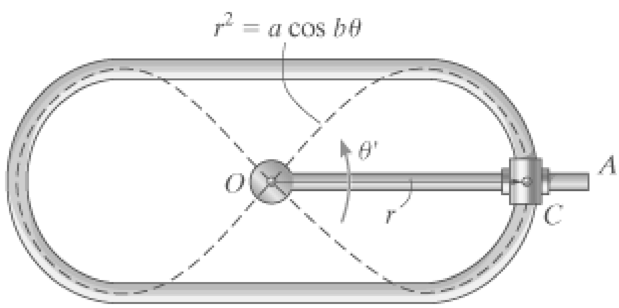
A double collar C is pin-connected together such that one collar slides over a fixed rod and the other slides over a rotating rod. If the geometry of the fixed rod for a short distance can be defined by a lemniscate, $r^2 = (a \cos b\theta)$, determine the collar's radial and transverse components of velocity and acceleration at the instant $\theta = 0°$ as shown. Rod OA is rotating at a constant rate of θ'.

Given:

$a = 4 \text{ ft}^2$

$b = 2$

$\theta' = 6 \frac{\text{rad}}{\text{s}}$



Solution:

$\theta = 0 \text{ deg} \qquad r = \sqrt{a\cos(b\theta)}$

$r^2 = a\cos(b\theta)$

$2rr' = -a\,b\sin(b\theta)\theta' \qquad r' = \dfrac{-a\,b\sin(b\theta)\theta'}{2r}$

$$2r\,r'' + 2r'^2 = -a\,b^2\cos(b\theta)\,\theta'^2 \qquad r'' = \frac{-a\,b^2\cos(b\theta)\,\theta'^2 - 2r'^2}{2r}$$

$$v_r = r' \qquad v_r = 0\ \frac{\text{m}}{\text{s}}$$

$$v_\theta = r\theta' \qquad v_\theta = 12\,\frac{\text{ft}}{\text{s}}$$

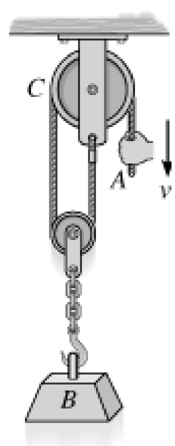
$$a_r = r'' - r\theta'^2 \qquad a_r = -216\,\frac{\text{ft}}{\text{s}^2}$$

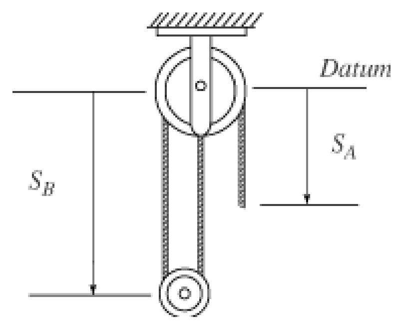
$$a_\theta = 2r'\theta' \qquad a_\theta = 0\,\frac{\text{ft}}{\text{s}^2}$$

***Problem 12-172**

If the end of the cable at *A* is pulled down with speed *v*, determine the speed at which block *B* rises.

Given: $v = 2\ \frac{\text{m}}{\text{s}}$





Solution:

$$v_A = v$$

$$L = 2s_B + s_A$$

$$0 = 2v_B + v_A$$

$$v_B = \frac{-v_A}{2}$$

$$v_B = -1\ \frac{\text{m}}{\text{s}}$$

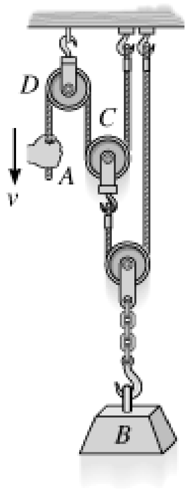
Problem 12-173

If the end of the cable at *A* is pulled down with speed *v*, determine the speed at which block *B* rises.

Given:

$$v = 2\ \frac{\text{m}}{\text{s}}$$

Solution:



$$v_A = v$$

$$L_1 = s_A + 2s_C$$

$$0 = v_A + 2v_C \qquad v_C = \frac{-v_A}{2}$$

$$L_2 = \left(s_B - s_C\right) + s_B \quad 0 = 2v_B - v_C$$

$$v_B = \frac{v_C}{2} \qquad \boxed{v_B = -0.5\ \frac{\text{m}}{\text{s}}}$$

Problem 12-174

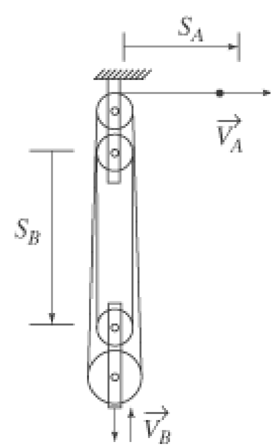
Determine the constant speed at which the cable at *A* must be drawn in by the motor in order to hoist the load at *B* a distance *d* in a time *t*.

Given:

$$d = 15\ \text{ft}$$

$$t = 5\ \text{s}$$

Solution:



$$L = 4s_B + s_A$$

$$0 = 4v_B + v_A$$

$$v_A = -4v_B$$

$$v_A = -4\left(\frac{-d}{t}\right)$$

$$\boxed{v_A = 12\frac{\text{ft}}{\text{s}}}$$

Problem 12-175

Determine the time needed for the load at *B* to attain speed *v*, starting from rest, if the cable is drawn into the motor with acceleration *a*.

Given:

$$v = -8\ \frac{\text{m}}{\text{s}}$$

$$a = 0.2\ \frac{\text{m}}{\text{s}^2}$$

Solution:

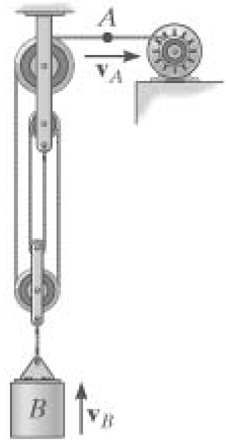
$$v_B = v$$

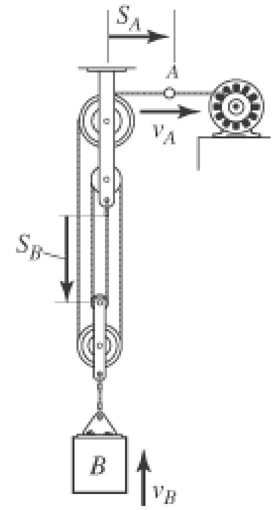
$$L = 4s_B + s_A$$

$$0 = 4v_B + v_A$$

$$v_B = \frac{-v_A}{4} = \frac{-1}{4}a\,t$$

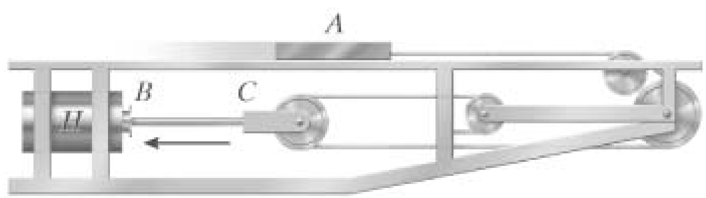
$$t = \frac{-4v_B}{a} \qquad t = 160\ \text{s}$$





***Problem 12–176**

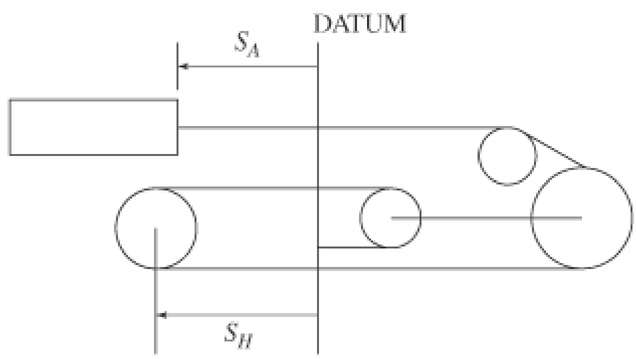
If the hydraulic cylinder at H draws rod BC in by a distance d, determine how far the slider at A moves.



Given:

$$d = 8\ \text{in}$$

Solution:



$$\Delta s_H = d$$

$$L = s_A + 2s_H \qquad 0 = \Delta s_A + 2\Delta s_H$$

$$\Delta s_A = -2\Delta s_H \qquad \Delta s_A = -16\,\text{in}$$

Problem 12-177

The crate is being lifted up the inclined plane using the motor M and the rope and pulley arrangement shown. Determine the speed at which the cable must be taken up by the motor in order to move the crate up the plane with constant speed v.

Given:

$$v = 4\ \frac{\text{ft}}{\text{s}}$$

Solution:

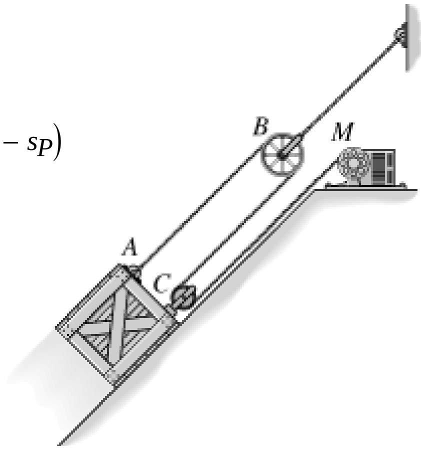
$v_A = v$

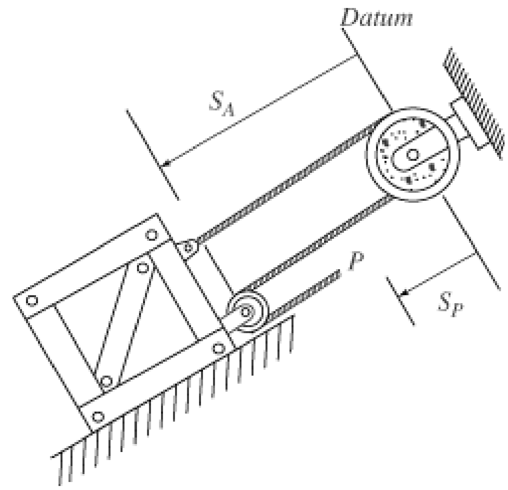
$L = 2s_A + (s_A - s_P)$

$0 = 3v_A - v_P$

$v_P = 3v_A$

$v_P = 12\,\frac{\text{ft}}{\text{s}}$





Problem 12-178

Determine the displacement of the block at B if A is pulled down a distance d.

Given:

$d = 4\text{ ft}$

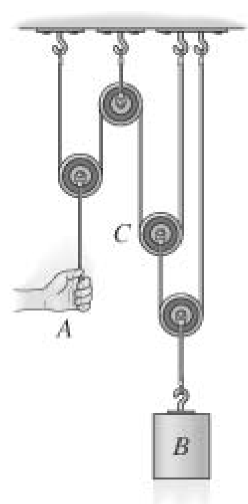
Solution:

$\Delta s_A = d$

$L_1 = 2s_A + 2s_C \qquad L_2 = (s_B - s_C) + s_B$

$0 = 2\Delta s_A + 2\Delta s_C \qquad 0 = 2\Delta s_B - \Delta s_C$

$\Delta s_C = -\Delta s_A \qquad \Delta s_B = \frac{\Delta s_C}{2} \qquad \Delta s_B = -2\text{ ft}$

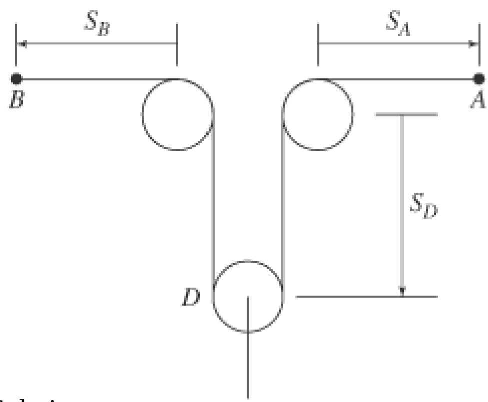


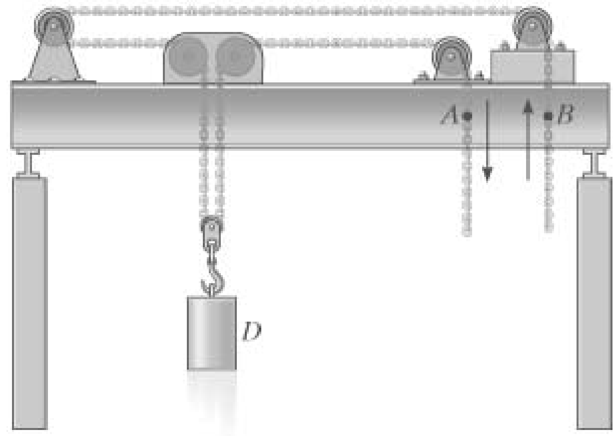
Problem 12–179

The hoist is used to lift the load at D. If the end A of the chain is travelling downward at v_A and the end B is travelling upward at v_B, determine the velocity of the load at D.

Given:

$v_A = 5\,\frac{\text{ft}}{\text{s}} \qquad v_B = 2\,\frac{\text{ft}}{\text{s}}$





Solution:

$L = s_B + s_A + 2s_D \qquad 0 = -v_B + v_A + 2v_D$

$v_D = \frac{v_B - v_A}{2} \qquad v_D = -1.5\,\frac{\text{ft}}{\text{s}}$ Positive means down, Negative means up

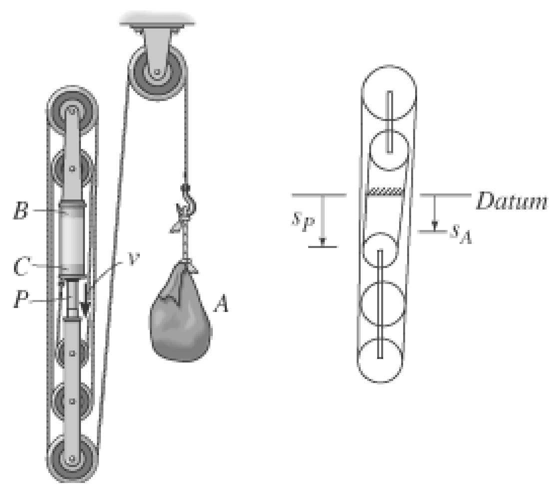
---

***Problem 12-180**

The pulley arrangement shown is designed for hoisting materials. If *BC remains fixed* while the plunger *P* is pushed downward with speed *v*, determine the speed of the load at *A*.

Given:

$v = 4\,\frac{\text{ft}}{\text{s}}$



Solution:

$v_P = v$

$L = 6s_P + s_A \qquad 0 = 6v_P + v_A$

$v_A = -6v_P \qquad v_A = -24\,\frac{\text{ft}}{\text{s}}$

Problem 12-181

If block *A* is moving downward with speed v_A while *C* is moving up at speed v_C, determine the speed of block *B*.

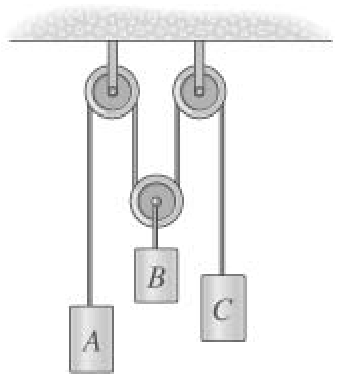
Given:

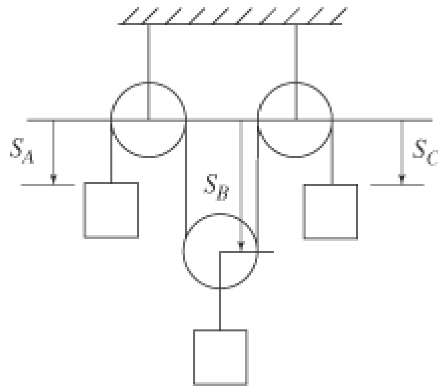
$v_A = 4\,\frac{\text{ft}}{\text{s}}$

$v_C = -2\ \frac{\text{ft}}{\text{s}}$

Solution:

$$S_A + 2S_B + S_C = L$$





Taking time derivative:

$$v_A + 2v_B + v_C = 0$$

$$v_B = \frac{-(v_C + v_A)}{2}$$

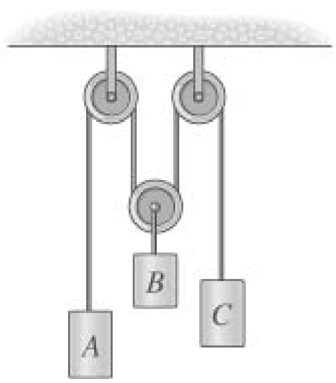
$$v_B = -1\ \frac{\text{ft}}{\text{s}}$$ Positive means down, negative means up.

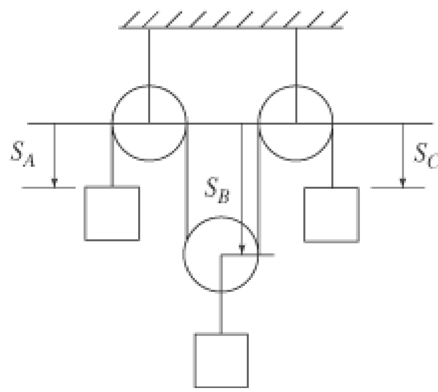
Problem 12-182

If block A is moving downward at speed v_A while block C is moving down at speed v_C, determine the relative velocity of block B with respect to C.

Given:

$v_A = 6\ \frac{\text{ft}}{\text{s}}$ $v_C = 18\ \frac{\text{ft}}{\text{s}}$





Solution:

$$S_A + 2S_B + S_C = L$$

Taking time derivative

$$v_A + 2v_B + v_C = 0$$

$$v_B = \frac{-(v_A + v_C)}{2} \qquad v_B = -12\ \frac{\text{ft}}{\text{s}}$$

$$v_{BC} = v_B - v_C \qquad v_{BC} = -30\ \frac{\text{ft}}{\text{s}}$$ Positive means down, negative means up

---

Problem 12–183

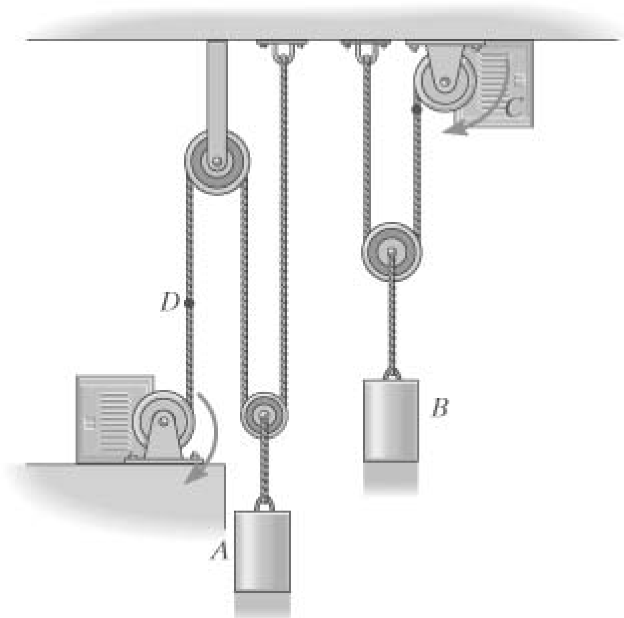
The motor draws in the cable at C with a constant velocity v_C. The motor draws in the cable at D with a constant acceleration of a_D. If $v_D = 0$ when $t = 0$, determine (a) the time needed for block A to rise a distance h, and (b) the relative velocity of block A with respect to block B when this occurs.

Given:

$$v_C = -4\ \frac{m}{s}$$

$$a_D = 8\ \frac{m}{s^2}$$

$$h = 3\ m$$



Solution:

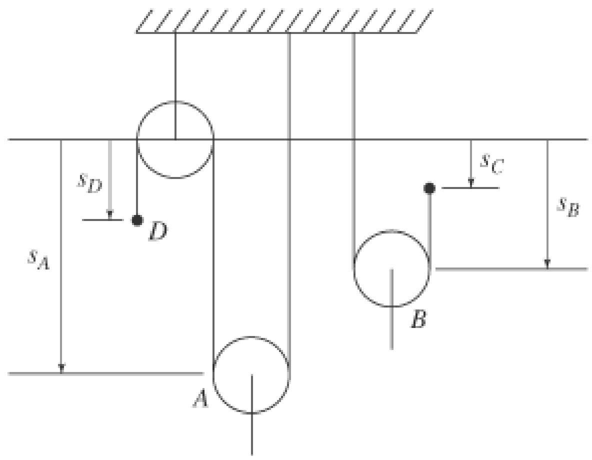
$$L_1 = s_D + 2s_A$$

$$0 = v_D + 2v_A$$

$$0 = a_D + 2a_A$$

$$L_2 = s_B + (s_B - s_C)$$

$$0 = 2v_B - v_C \qquad 0 = 2a_B - a_C$$



$$a_A = \frac{-a_D}{2}$$

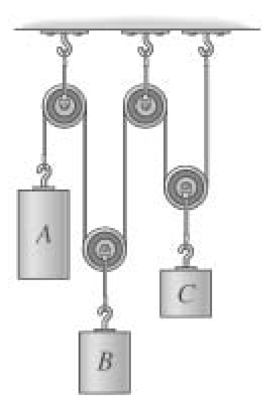
$$v_A = a_A t$$

$$s_A = -h = a_A\left(\frac{t^2}{2}\right)$$

$$t = \sqrt{\frac{-2h}{a_A}} \qquad t = 1.225\ s$$

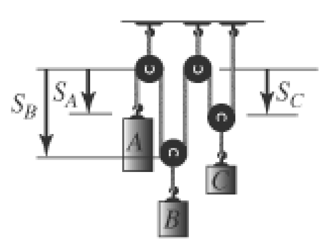
$$v_A = a_A t \qquad v_B = \frac{1}{2}v_C \qquad v_{AB} = v_A - v_B \qquad v_{AB} = -2.90\ \frac{m}{s}$$

***Problem 12-184**



If block A of the pulley system is moving downward with speed v_A while block C is moving up at v_C determine the speed of block B.

Given:



$$v_A = 4\ \frac{ft}{s}$$

$$v_C = -2\ \frac{ft}{s}$$

Solution:

$$S_A + 2S_B + 2S_C = L$$

$$v_A + 2v_B + 2v_C = 0 \qquad v_B = \frac{-2v_C - v_A}{2} \qquad v_B = 0\ \frac{\text{m}}{\text{s}}$$

Problem 12–185

If the point A on the cable is moving upwards at v_A, determine the speed of block B.

Given: $v_A = -14\ \frac{\text{m}}{\text{s}}$

Solution:

$$L_1 = (s_D - s_A) + (s_D - s_E)$$

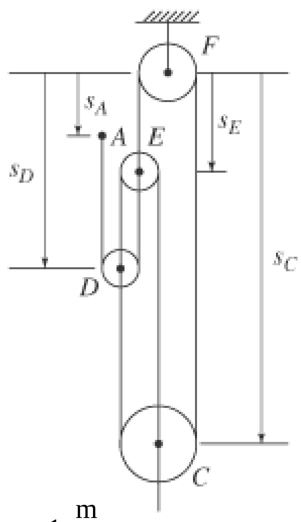
$$0 = 2v_D - v_A - v_E$$

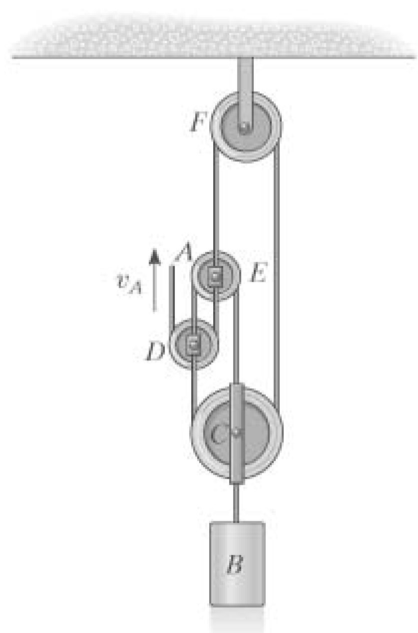
$$L_2 = (s_D - s_E) + (s_C - s_E)$$

$$0 = v_D + v_C - 2v_E$$

$$L_3 = (s_C - s_D) + s_C + s_E$$

$$0 = 2v_C - v_D + v_E$$





Guesses

$v_C = 1\ \frac{\text{m}}{\text{s}} \quad v_D = 1\ \frac{\text{m}}{\text{s}} \quad v_E = 1\ \frac{\text{m}}{\text{s}}$

Given $\quad 0 = 2v_D - v_A - v_E$

$0 = v_D + v_C - 2v_E$

$0 = 2v_C - v_D + v_E$

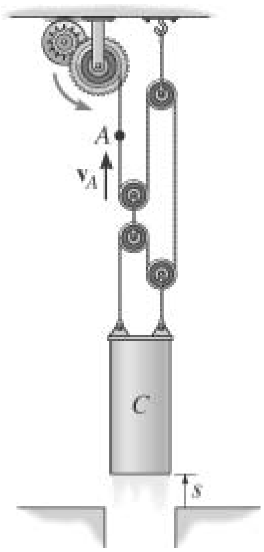
$$\begin{pmatrix} v_C \\ v_D \\ v_E \end{pmatrix} = \text{Find}(v_C, v_D, v_E) \qquad \begin{pmatrix} v_C \\ v_D \\ v_E \end{pmatrix} = \begin{pmatrix} -2 \\ -10 \\ -6 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

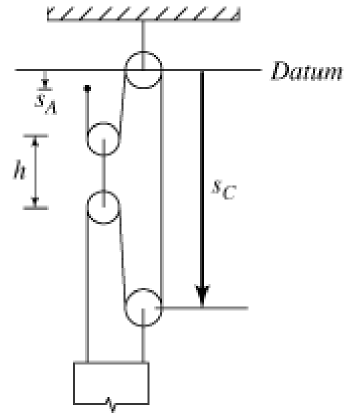
$v_B = v_C \qquad v_B = -2\ \frac{\text{m}}{\text{s}}$

Positive means down,
Negative means up

Problem 12-186

The cylinder C is being lifted using the cable and pulley system shown. If point A on the cable is being drawn toward the drum with speed of v_A, determine the speed of the cylinder.





Given:

$$v_A = -2\ \frac{\text{m}}{\text{s}}$$

Solution:

$$L = 2s_C + (s_C - s_A)$$

$$0 = 3v_C - v_A$$

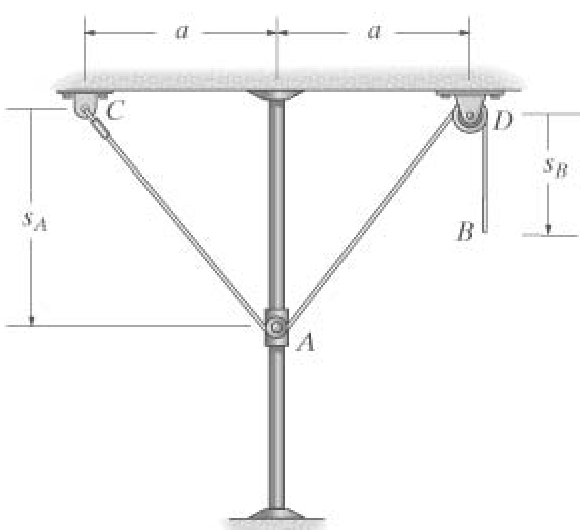
$$v_C = \frac{v_A}{3}$$

$$v_C = -0.667\ \frac{\text{m}}{\text{s}}$$

Positive means down, negative means up.

Problem 12–187

The cord is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to the smooth collar that travels along the vertical rod. Determine the velocity and acceleration of the end of the cord at B if at the instant $s_A = b$ the collar is moving upwards at speed v, which is decreasing at rate a.



Given:

$$a = 3\ \text{ft} \qquad v_A = -5\ \frac{\text{ft}}{\text{s}}$$

$$b = 4\ \text{ft} \qquad a_A = 2\ \frac{\text{ft}}{\text{s}^2}$$

Solution:

$$L = 2\sqrt{a^2 + s_A^2} + s_B \qquad s_A = b$$

Guesses $\quad v_B = 1\ \frac{\text{ft}}{\text{s}} \qquad a_B = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $\quad 0 = \frac{2s_A v_A}{\sqrt{a^2 + s_A^2}} + v_B$

$$0 = \frac{2s_A a_A + 2v_A^2}{\sqrt{a^2 + s_A^2}} - \frac{2s_A^2 v_A^2}{\sqrt{\left(a^2 + s_A^2\right)^3}} + a_B$$

$$\begin{pmatrix} v_B \\ a_B \end{pmatrix} = \text{Find}(v_B, a_B) \qquad v_B = 8\,\frac{\text{ft}}{\text{s}} \qquad a_B = -6.8\,\frac{\text{ft}}{\text{s}^2}$$

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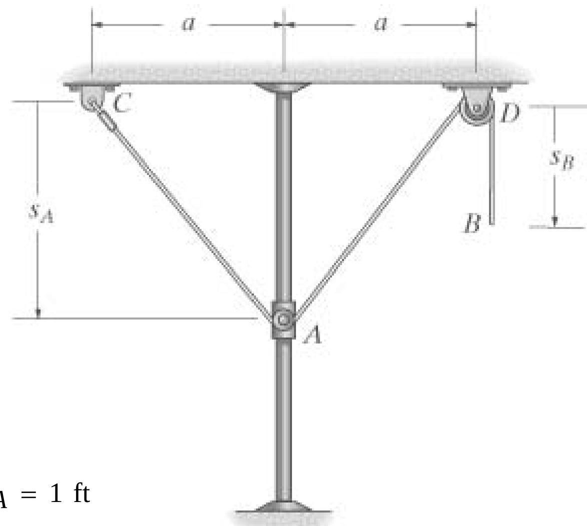
*__Problem 12–188__

The cord of length L is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to the smooth collar that travels along the vertical rod. When $s_B = b$, the end of the cord at B is pulled downwards with a velocity v_B and is given an acceleration a_B. Determine the velocity and acceleration of the collar A at this instant.

Given:



$L = 16$ ft

$a = 3$ ft $\qquad v_B = 4\,\frac{\text{ft}}{\text{s}}$

$b = 6$ ft $\qquad a_B = 3\,\frac{\text{ft}}{\text{s}^2}$

Solution: $\quad s_B = b$

Guesses

$v_A = 1\,\frac{\text{ft}}{\text{s}} \qquad a_A = 1\,\frac{\text{ft}}{\text{s}^2} \qquad s_A = 1$ ft

Given $\quad L = 2\sqrt{a^2 + s_A^2} + s_B$

$$0 = \frac{2s_A v_A}{\sqrt{a^2 + s_A^2}} + v_B$$

$$0 = \frac{2s_A a_A + 2v_A^2}{\sqrt{a^2 + s_A^2}} - \frac{2s_A^2 v_A^2}{\sqrt{\left(a^2 + s_A^2\right)^3}} + a_B$$

$$\begin{pmatrix} s_A \\ v_A \\ a_A \end{pmatrix} = \text{Find}(s_A, v_A, a_A) \qquad s_A = 4\,\text{ft} \qquad v_A = -2.50\,\frac{\text{ft}}{\text{s}} \qquad a_A = -2.44\,\frac{\text{ft}}{\text{s}^2}$$

Problem 12-189

The crate C is being lifted by moving the roller at A downward with constant speed v_A along the guide. Determine the velocity and acceleration of the crate at the instant $s = s_1$. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.

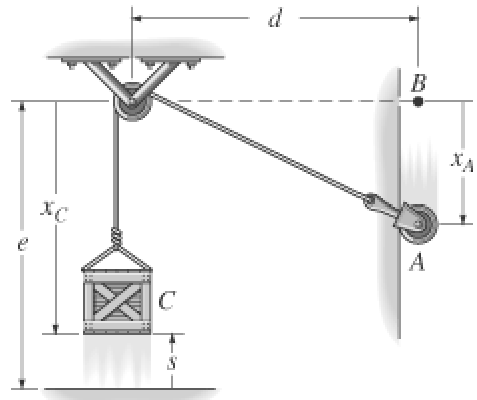
Given:

$v_A = 2\ \frac{\text{m}}{\text{s}}$

$s_1 = 1\ \text{m}$

$d = 4\ \text{m}$

$e = 4\ \text{m}$



Solution:

$x_C = e - s_1 \qquad L = d + e$

Guesses $\quad v_C = 1\ \frac{\text{m}}{\text{s}} \quad a_C = 1\ \frac{\text{m}}{\text{s}^2} \quad x_A = 1\ \text{m}$

Given $\quad L = x_C + \sqrt{x_A^2 + d^2} \qquad 0 = v_C + \frac{x_A v_A}{\sqrt{x_A^2 + d^2}}$

$$0 = a_C - \frac{x_A^2 v_A^2}{\sqrt{\left(x_A^2 + d^2\right)^3}} + \frac{v_A^2}{\sqrt{x_A^2 + d^2}}$$

$\begin{pmatrix} x_A \\ v_C \\ a_C \end{pmatrix} = \text{Find}(x_A, v_C, a_C) \qquad x_A = 3\ \text{m} \qquad v_C = -1.2\ \frac{\text{m}}{\text{s}} \qquad a_C = -0.512\ \frac{\text{m}}{\text{s}^2}$

Problem 12-190

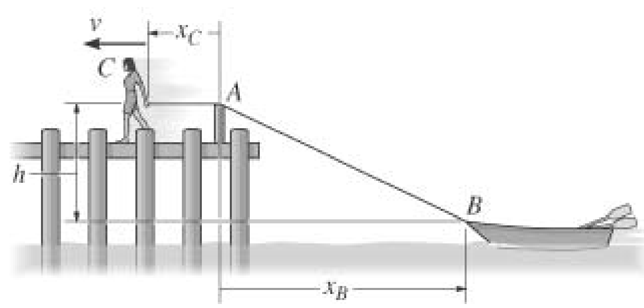
The girl at C stands near the edge of the pier and pulls in the rope *horizontally* at constant speed v_C. Determine how fast the boat approaches the pier at the instant the rope length AB is d.

Given:

$v_C = 6\ \frac{\text{ft}}{\text{s}}$

$h = 8\ \text{ft}$

$d = 50\ \text{ft}$



Solution: $x_B = \sqrt{d^2 - h^2}$

$L = x_C + \sqrt{h^2 + x_B^2}$ $\qquad 0 = v_C + \dfrac{x_B v_B}{\sqrt{h^2 + x_B^2}}$

$v_B = -v_C\left(\dfrac{\sqrt{h^2 + x_B^2}}{x_B}\right)$ $\qquad v_B = -6.078\ \dfrac{\text{ft}}{\text{s}}$ Positive means to the right, negative to the left.

Problem 12-191

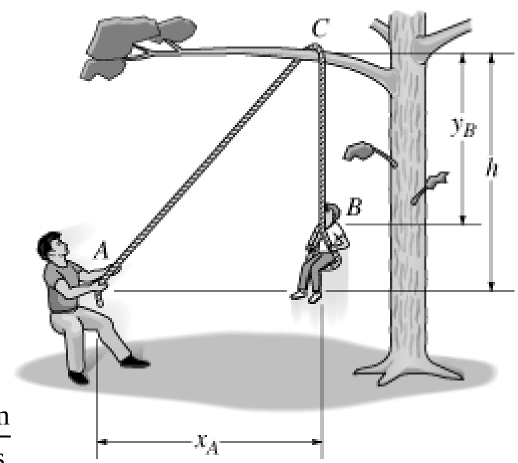
The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with constant acceleration a_A, determine the speed of the boy at the instant $y_B = y_{B1}$. Neglect the size of the limb. When $x_A = 0$, $y_B = h$ so that A and B are coincident, i.e., the rope is $2h$ long.

Given:

$a_A = 0.2\ \dfrac{\text{m}}{\text{s}^2}$

$y_{B1} = 4\ \text{m}$

$h = 8\ \text{m}$



Solution: $y_B = y_{B1}$

Guesses

$x_A = 1\ \text{m}$ $\quad v_A = 1\ \dfrac{\text{m}}{\text{s}}$ $\quad v_B = 1\ \dfrac{\text{m}}{\text{s}}$

Given $\quad 2h = \sqrt{x_A^2 + h^2} + y_B$ $\qquad 0 = \dfrac{x_A v_A}{\sqrt{x_A^2 + h^2}} + v_B$ $\qquad v_A^2 = 2a_A x_A$

$\begin{pmatrix} x_A \\ v_A \\ v_B \end{pmatrix} = \text{Find}(x_A, v_A, v_B)$ $\quad x_A = 8.944\ \text{m}$ $\quad v_A = 1.891\ \dfrac{\text{m}}{\text{s}}$ $\quad v_B = -1.41\ \dfrac{\text{m}}{\text{s}}$

Positive means down,
negative means up

***Problem 12-192**

Collars A and B are connected to the cord that passes over the small pulley at C. When A is located at D, B is a distance d_1 to the left of D. If A moves at a constant speed v_A, to the right, determine the speed of B when A is distance d_2 to the right of D.

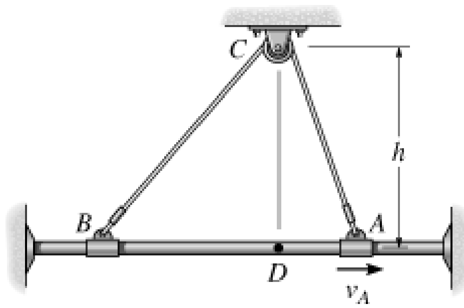
Given:

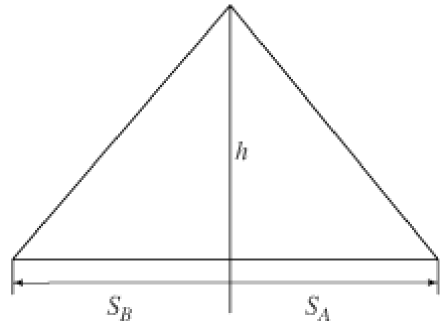
$h = 10\ \text{ft}$

$d_1 = 24\ \text{ft}$

$d_2 = 4\ \text{ft}$

$v_A = 2\ \frac{\text{ft}}{\text{s}}$





Solution:

$$L = \sqrt{h^2 + d_1^2} + h \qquad s_A = d_2$$

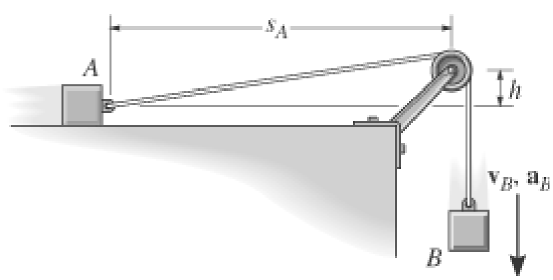
$$\sqrt{s_B^2 + h^2} = L - \sqrt{s_A^2 + h^2} \qquad s_B = \sqrt{\left(L - \sqrt{s_A^2 + h^2}\right)^2 - h^2} \qquad s_B = 23.163\ \text{ft}$$

$$\frac{s_B v_B}{\sqrt{s_B^2 + h^2}} = \frac{-s_A v_A}{\sqrt{s_A^2 + h^2}} \qquad v_B = \frac{-s_A v_A \sqrt{s_B^2 + h^2}}{s_B \sqrt{s_A^2 + h^2}} \qquad v_B = -0.809\ \frac{\text{ft}}{\text{s}}$$

Positive means to the left, negative to the right.

Problem 12-193

If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.

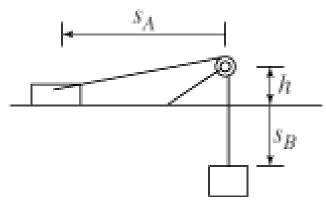


Solution:

$$L = s_B + \sqrt{s_A^2 + h^2}$$

$$0 = v_B + \frac{s_A v_A}{\sqrt{s_A^2 + h^2}}$$

$$v_A = \frac{-v_B \sqrt{s_A^2 + h^2}}{s_A}$$



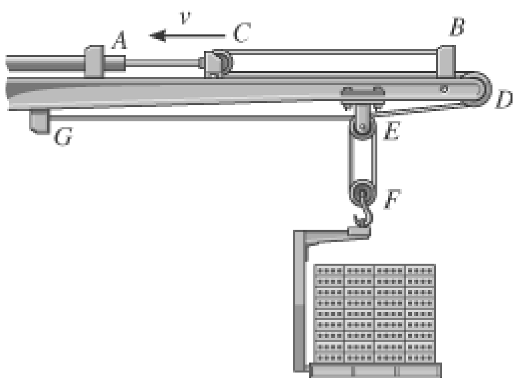
$$0 = a_B - \frac{s_A^2 v_A^2}{\left(s_A^2 + h^2\right)^{\frac{3}{2}}} + \frac{v_A^2 + s_A a_A}{\sqrt{s_A^2 + h^2}}$$

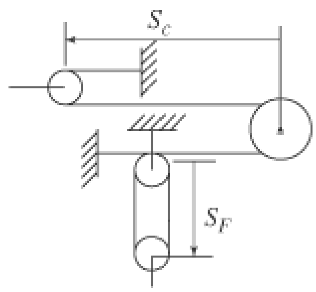
$$a_A = \frac{s_A v_A^2}{s_A^2 + h^2} - a_B \frac{\sqrt{s_A^2 + h^2}}{s_A} - \frac{v_A^2}{s_A} \qquad a_A = \frac{-a_B\sqrt{s_A^2 + h^2}}{s_A} - \frac{v_B^2 h^2}{s_A^3}$$

Problem 12-194

Vertical motion of the load is produced by movement of the piston at A on the boom. Determine the distance the piston or pulley at C must move to the left in order to lift the load a distance h. The cable is attached at B, passes over the pulley at C, then D, E, F, and again around E, and is attached at G.

Given:





$h = 2$ ft

Solution:

$\Delta s_F = -h$

$L = 2s_C + 2s_F$

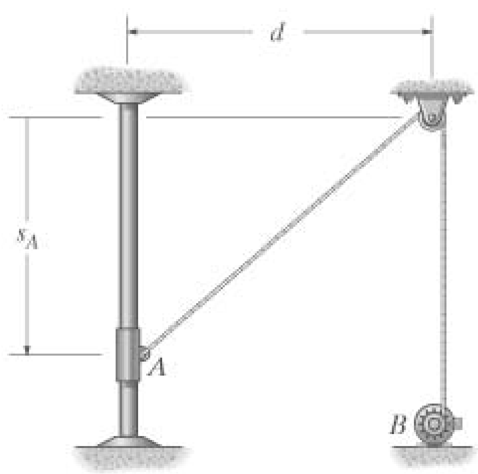
$2\Delta s_C = -2\Delta s_F$

$\Delta s_C = -\Delta s_F \qquad \Delta s_C = 2\,\text{ft}$

Problem 12-195

The motion of the collar at A is controlled by a motor at B such that when the collar is at s_A, it is moving upwards at v_A and slowing down at a_A. Determine the velocity and acceleration of the cable as it is drawn into the motor B at this instant.

Given:



$d = 4$ ft

$s_A = 3$ ft

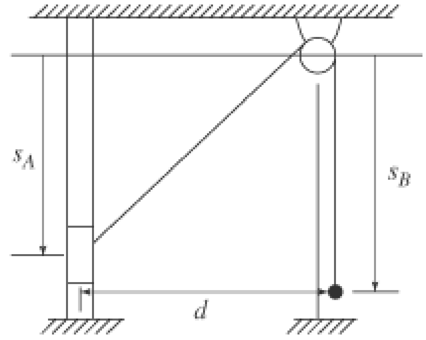
$v_A = -2\ \frac{\text{ft}}{\text{s}}$

$a_A = 1\ \frac{\text{ft}}{\text{s}^2}$

Solution: $L = \sqrt{s_A^2 + d^2} + s_B$

Guesses $v_B = 1\ \frac{\text{ft}}{\text{s}} \qquad a_B = 1\ \frac{\text{ft}}{\text{s}^2}$

$$v_B = -\frac{s_A v_A}{\sqrt{s_A^2 + d^2}}$$



$$a_B = -\frac{v_A^2 + s_A a_A}{\sqrt{s_A^2 + d^2}} + \frac{s_A^2 v_A^2}{\sqrt{\left(s_A^2 + d^2\right)^3}}$$

$v_B = 1.2\ \frac{\text{ft}}{\text{s}}$ $\quad a_B = -1.112\ \frac{\text{ft}}{\text{s}^2}$

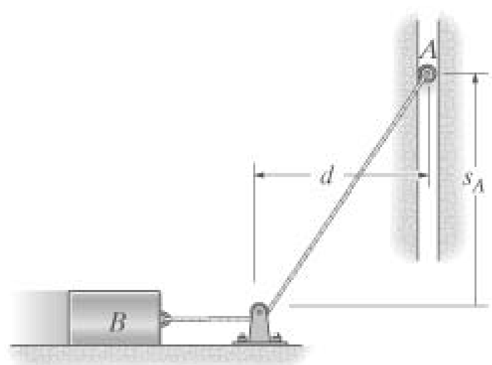
***Problem 12-196**

The roller at A is moving upward with a velocity v_A and has an acceleration a_A at s_A. Determine the velocity and acceleration of block B at this instant.

Given:

$s_A = 4\ \text{ft}$ $\quad a_A = 4\ \frac{\text{ft}}{\text{s}^2}$

$v_A = 3\ \frac{\text{ft}}{\text{s}}$ $\quad d = 3\ \text{ft}$



Solution:

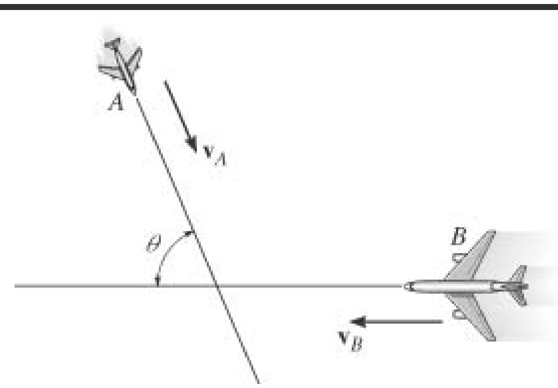
$l = s_B + \sqrt{s_A^2 + d^2}$ $\quad 0 = v_B + \frac{s_A v_A}{\sqrt{s_A^2 + d^2}}$

$v_B = \frac{-s_A v_A}{\sqrt{s_A^2 + d^2}}$ $\quad v_B = -2.4\ \frac{\text{ft}}{\text{s}}$

$a_B = \frac{-v_A^2 - s_A a_A}{\sqrt{s_A^2 + d^2}} + \frac{s_A^2 v_A^2}{\sqrt{\left(s_A^2 + d^2\right)^3}}$ $\quad a_B = -3.848\ \frac{\text{ft}}{\text{s}^2}$

Problem 12-197

Two planes, A and B, are flying at the same altitude. If their velocities are v_A and v_B such that the angle between their straight-line courses is θ, determine the velocity of plane B with respect to plane A.



Given:

$v_A = 600\ \frac{\text{km}}{\text{hr}}$

$v_B = 500\ \frac{\text{km}}{\text{hr}}$

$\theta = 75\ \text{deg}$

Solution:

$\mathbf{v_{Av}} = v_A\begin{pmatrix}\cos(\theta)\\ -\sin(\theta)\end{pmatrix}$ $\quad\quad$ $\mathbf{v_{Av}} = \begin{pmatrix}155.291\\ -579.555\end{pmatrix}\frac{\text{km}}{\text{hr}}$

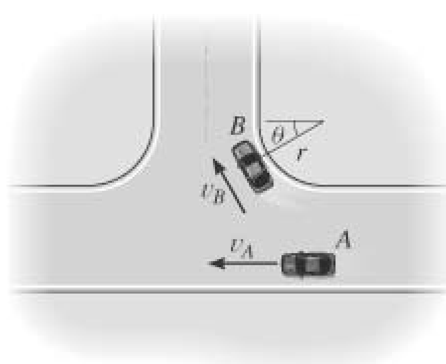
$\mathbf{v_{Bv}} = v_B\begin{pmatrix}-1\\ 0\end{pmatrix}$ $\quad\quad$ $\mathbf{v_{Bv}} = \begin{pmatrix}-500\\ 0\end{pmatrix}\frac{\text{km}}{\text{hr}}$

$\mathbf{v_{BA}} = \mathbf{v_{Bv}} - \mathbf{v_{Av}}$ $\quad\quad$ $\mathbf{v_{BA}} = \begin{pmatrix}-655\\ 580\end{pmatrix}\frac{\text{km}}{\text{hr}}$ $\quad\quad$ $|\mathbf{v_{BA}}| = 875\frac{\text{km}}{\text{hr}}$

Problem 12-198

At the instant shown, cars A and B are traveling at speeds v_A and v_B respectively. If B is increasing its speed at v'_A, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A.

Given:



$v_A = 30\ \frac{\text{mi}}{\text{hr}}$

$v_B = 20\ \frac{\text{mi}}{\text{hr}}$

$v'_A = 0\ \frac{\text{mi}}{\text{hr}^2}$

$v'_B = 1200\ \frac{\text{mi}}{\text{hr}^2}$

$\theta = 30\ \text{deg}$

$r = 0.3\ \text{mi}$

Solution:

$\mathbf{v_{Av}} = v_A\begin{pmatrix}-1\\ 0\end{pmatrix}$ $\quad\quad$ $\mathbf{v_{Av}} = \begin{pmatrix}-30\\ 0\end{pmatrix}\frac{\text{mi}}{\text{hr}}$

$$\mathbf{v_{Bv}} = v_B\begin{pmatrix}-\sin(\theta)\\ \cos(\theta)\end{pmatrix} \qquad \mathbf{v_{Bv}} = \begin{pmatrix}-10\\ 17.321\end{pmatrix}\frac{\text{mi}}{\text{hr}}$$

$$\mathbf{v_{BA}} = \mathbf{v_{Bv}} - \mathbf{v_{Av}} \qquad \mathbf{v_{BA}} = \begin{pmatrix}20\\ 17.321\end{pmatrix}\frac{\text{mi}}{\text{hr}} \qquad |\mathbf{v_{BA}}| = 26.5\,\frac{\text{mi}}{\text{hr}}$$

$$\mathbf{a_{Av}} = \begin{pmatrix}-v'_A\\ 0\end{pmatrix} \qquad \mathbf{a_{Av}} = \begin{pmatrix}0\\ 0\end{pmatrix}\frac{\text{mi}}{\text{hr}^2}$$

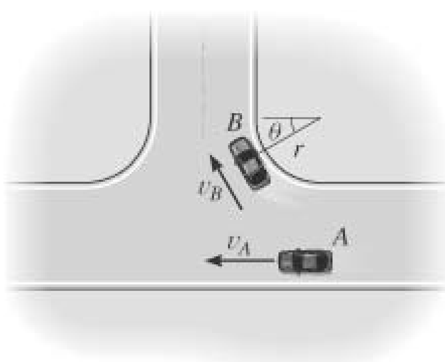
$$\mathbf{a_{Bv}} = v'_B\begin{pmatrix}-\sin(\theta)\\ \cos(\theta)\end{pmatrix} + \frac{v_B{}^2}{r}\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix} \qquad \mathbf{a_{Bv}} = \begin{pmatrix}554.701\\ 1.706\times 10^3\end{pmatrix}\frac{\text{mi}}{\text{hr}^2}$$

$$\mathbf{a_{BA}} = \mathbf{a_{Bv}} - \mathbf{a_{Av}} \qquad \mathbf{a_{BA}} = \begin{pmatrix}555\\ 1706\end{pmatrix}\frac{\text{mi}}{\text{hr}^2} \qquad |\mathbf{a_{BA}}| = 1794\,\frac{\text{mi}}{\text{hr}^2}$$

Problem 12-199

At the instant shown, cars *A* and *B* are traveling at speeds v_A and v_B respectively. If *A* is increasing its speed at v'_A whereas the speed of *B* is decreasing at v'_B, determine the velocity and acceleration of *B* with respect to *A*.

Given:



$$v_A = 30\,\frac{\text{mi}}{\text{hr}}$$

$$v_B = 20\,\frac{\text{mi}}{\text{hr}}$$

$$v'_A = 400\,\frac{\text{mi}}{\text{hr}^2}$$

$$v'_B = -800\,\frac{\text{mi}}{\text{hr}^2}$$

$$\theta = 30\text{ deg}$$

$$r = 0.3\text{ mi}$$

Solution:

$$\mathbf{v_{Av}} = v_A\begin{pmatrix}-1\\ 0\end{pmatrix} \qquad \mathbf{v_{Av}} = \begin{pmatrix}-30\\ 0\end{pmatrix}\frac{\text{mi}}{\text{hr}}$$

$$\mathbf{v_{Bv}} = v_B\begin{pmatrix}-\sin(\theta)\\ \cos(\theta)\end{pmatrix} \qquad \mathbf{v_{Bv}} = \begin{pmatrix}-10\\ 17.321\end{pmatrix}\frac{\text{mi}}{\text{hr}}$$

$$\mathbf{v_{BA}} = \mathbf{v_{Bv}} - \mathbf{v_{Av}} \qquad \mathbf{v_{BA}} = \begin{pmatrix} 20 \\ 17.321 \end{pmatrix} \frac{mi}{hr} \qquad |\mathbf{v_{BA}}| = 26.458 \frac{mi}{hr}$$

$$\mathbf{a_{Av}} = \begin{pmatrix} -v'_A \\ 0 \end{pmatrix} \qquad \mathbf{a_{Av}} = \begin{pmatrix} -400 \\ 0 \end{pmatrix} \frac{mi}{hr^2}$$

$$\mathbf{a_{Bv}} = v'_B \begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix} + \frac{v_B^2}{r} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \qquad \mathbf{a_{Bv}} = \begin{pmatrix} 1.555 \times 10^3 \\ -26.154 \end{pmatrix} \frac{mi}{hr^2}$$

$$\mathbf{a_{BA}} = \mathbf{a_{Bv}} - \mathbf{a_{Av}} \qquad \mathbf{a_{BA}} = \begin{pmatrix} 1955 \\ -26 \end{pmatrix} \frac{mi}{hr^2} \qquad |\mathbf{a_{BA}}| = 1955 \frac{mi}{hr^2}$$

---

*__Problem 12-200__

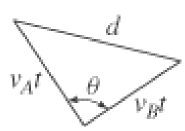
Two boats leave the shore at the same time and travel in the directions shown with the given speeds. Determine the speed of boat *A* with respect to boat *B*. How long after leaving the shore will the boats be at a distance *d* apart?

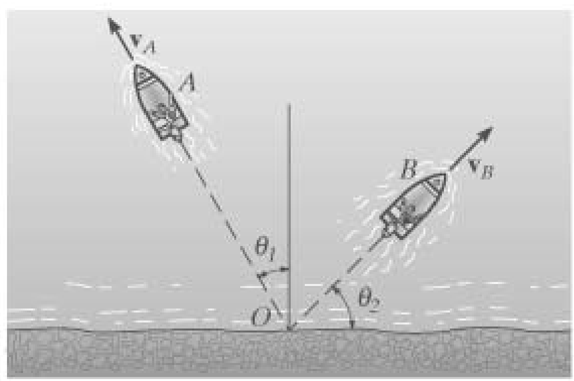
Given:

$v_A = 20 \frac{ft}{s}$ $\qquad \theta_1 = 30$ deg

$v_B = 15 \frac{ft}{s}$ $\qquad \theta_2 = 45$ deg

$d = 800$ ft





Solution:

$$\mathbf{v_{Av}} = v_A \begin{pmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{pmatrix} \qquad \mathbf{v_{Bv}} = v_B \begin{pmatrix} \cos(\theta_2) \\ \sin(\theta_2) \end{pmatrix}$$

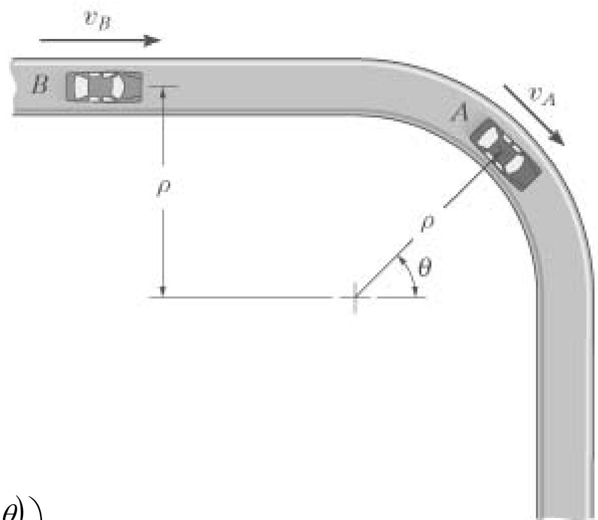
$$\mathbf{v_{AB}} = \mathbf{v_{Av}} - \mathbf{v_{Bv}} \qquad \mathbf{v_{AB}} = \begin{pmatrix} -20.607 \\ 6.714 \end{pmatrix} \frac{ft}{s} \qquad t = \frac{d}{|\mathbf{v_{AB}}|}$$

$$|\mathbf{v_{AB}}| = 21.673 \frac{ft}{s} \qquad t = 36.913 \text{ s}$$

Problem 12–201

At the instant shown, the car at A is traveling at v_A around the curve while increasing its speed at v'_A. The car at B is traveling at v_B along the straightaway and increasing its speed at v'_B. Determine the relative velocity and relative acceleration of A with respect to B at this instant.

Given:



$v_A = 10\,\frac{\text{m}}{\text{s}}$ $\quad v_B = 18.5\,\frac{\text{m}}{\text{s}}$

$v'_A = 5\,\frac{\text{m}}{\text{s}^2}$ $\quad v'_B = 2\,\frac{\text{m}}{\text{s}^2}$

$\theta = 45\text{ deg}$ $\quad \rho = 100\text{ m}$

Solution:

$$\mathbf{v_{Av}} = v_A\begin{pmatrix}\sin(\theta)\\ -\cos(\theta)\end{pmatrix}$$

$$\mathbf{a_{Av}} = v'_A\begin{pmatrix}\sin(\theta)\\ -\cos(\theta)\end{pmatrix} + \frac{v_A^{\,2}}{\rho}\begin{pmatrix}-\cos(\theta)\\ -\sin(\theta)\end{pmatrix}$$

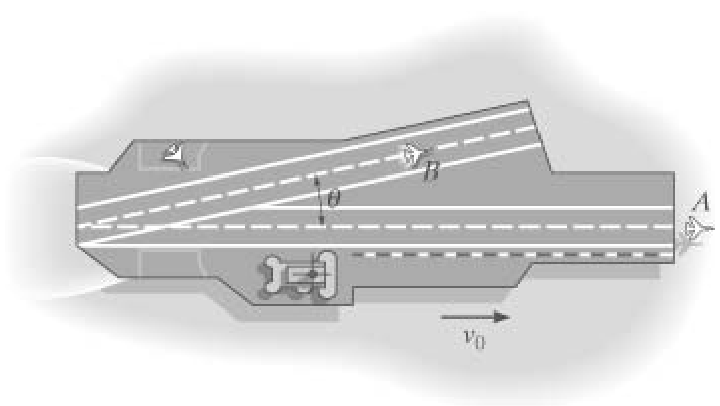
$$\mathbf{v_{Bv}} = \begin{pmatrix}v_B\\ 0\end{pmatrix} \qquad \mathbf{a_{Bv}} = \begin{pmatrix}v'_B\\ 0\end{pmatrix}$$

$$\mathbf{v_{AB}} = \mathbf{v_{Av}} - \mathbf{v_{Bv}} \qquad \mathbf{v_{AB}} = \begin{pmatrix}-11.43\\ -7.07\end{pmatrix}\frac{\text{m}}{\text{s}}$$

$$\mathbf{a_{AB}} = \mathbf{a_{Av}} - \mathbf{a_{Bv}} \qquad \mathbf{a_{AB}} = \begin{pmatrix}0.828\\ -4.243\end{pmatrix}\frac{\text{m}}{\text{s}^2}$$

Problem 12–202

An aircraft carrier is traveling forward with a velocity v_0. At the instant shown, the plane at A has just taken off and has attained a forward horizontal air speed v_A, measured from still water. If the plane at B is traveling along the runway of the carrier at v_B in the direction shown measured relative to the carrier, determine the velocity of A with respect to B.



Given:

$v_0 = 50\ \frac{km}{hr}$ $\qquad v_A = 200\ \frac{km}{hr}$

$\theta = 15\ deg$ $\qquad v_B = 175\ \frac{km}{hr}$

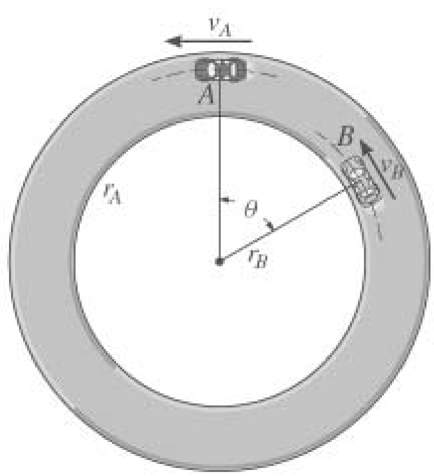
Solution:

$$\mathbf{v_A} = \begin{pmatrix} v_A \\ 0 \end{pmatrix} \qquad \mathbf{v_B} = \begin{pmatrix} v_0 \\ 0 \end{pmatrix} + v_B \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$$

$$\mathbf{v_{AB}} = \mathbf{v_A} - \mathbf{v_B} \qquad \mathbf{v_{AB}} = \begin{pmatrix} -19.04 \\ -45.29 \end{pmatrix} \frac{km}{hr} \qquad \left|\mathbf{v_{AB}}\right| = 49.1\ \frac{km}{hr}$$

Problem 12-203

Cars A and B are traveling around the circular race track. At the instant shown, A has speed v_A and is increasing its speed at the rate of v'_A, whereas B has speed v_B and is decreasing its speed at v'_B. Determine the relative velocity and relative acceleration of car A with respect to car B at this instant.



Given: $\theta = 60\ deg$

$r_A = 300\ ft$ $\qquad r_B = 250\ ft$

$v_A = 90\ \frac{ft}{s}$ $\qquad v_B = 105\ \frac{ft}{s}$

$v'_A = 15\ \frac{ft}{s^2}$ $\qquad v'_B = -25\ \frac{ft}{s^2}$

Solution:

$$\mathbf{v_{Av}} = v_A \begin{pmatrix} -1 \\ 0 \end{pmatrix} \qquad \mathbf{v_{Av}} = \begin{pmatrix} -90 \\ 0 \end{pmatrix} \frac{ft}{s}$$

$$\mathbf{v_{Bv}} = v_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \end{pmatrix} \qquad \mathbf{v_{Bv}} = \begin{pmatrix} -52.5 \\ 90.933 \end{pmatrix} \frac{ft}{s}$$

$$\mathbf{v_{AB}} = \mathbf{v_{Av}} - \mathbf{v_{Bv}} \qquad \mathbf{v_{AB}} = \begin{pmatrix} -37.5 \\ -90.9 \end{pmatrix} \frac{ft}{s} \qquad \left|\mathbf{v_{AB}}\right| = 98.4\ \frac{ft}{s}$$

$$\mathbf{a_A} = v'_A \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \frac{{v_A}^2}{r_A} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \qquad \mathbf{a_A} = \begin{pmatrix} -15 \\ -27 \end{pmatrix} \frac{ft}{s^2}$$

$$\mathbf{a_B} = v'_B\begin{pmatrix}-\cos(\theta)\\ \sin(\theta)\end{pmatrix} + \frac{v_B^{\,2}}{r_B}\begin{pmatrix}-\sin(\theta)\\ -\cos(\theta)\end{pmatrix} \qquad \mathbf{a_B} = \begin{pmatrix}-25.692\\ -43.701\end{pmatrix}\frac{\text{ft}}{\text{s}^2}$$

$$\mathbf{a_{AB}} = \mathbf{a_A} - \mathbf{a_B} \qquad \mathbf{a_{AB}} = \begin{pmatrix}10.692\\ 16.701\end{pmatrix}\frac{\text{ft}}{\text{s}^2} \qquad \left|\mathbf{a_{AB}}\right| = 19.83\,\frac{\text{ft}}{\text{s}^2}$$

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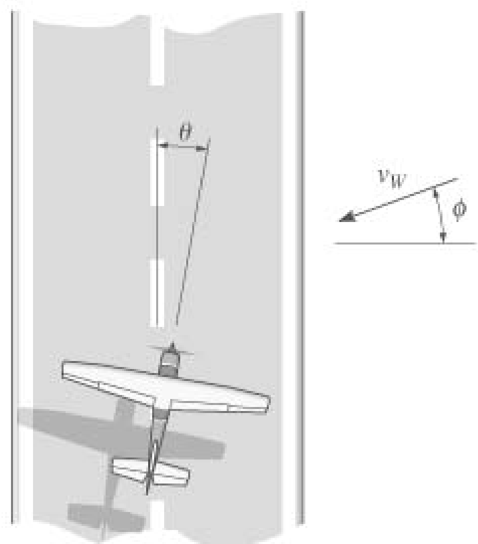
*__Problem 12–204__

The airplane has a speed relative to the wind of v_A. If the speed of the wind relative to the ground is v_W, determine the angle θ at which the plane must be directed in order to travel in the direction of the runway. Also, what is its speed relative to the runway?

Given:



$$v_A = 100\,\frac{\text{mi}}{\text{hr}}$$

$$v_W = 10\,\frac{\text{mi}}{\text{hr}}$$

$$\phi = 20\text{ deg}$$

Solution:

Guesses $\theta = 1\text{ deg}$

$$v_{Ag} = 1\,\frac{\text{mi}}{\text{hr}}$$

Given $$\begin{pmatrix}0\\ v_{Ag}\end{pmatrix} = v_A\begin{pmatrix}\sin(\theta)\\ \cos(\theta)\end{pmatrix} + v_W\begin{pmatrix}-\cos(\phi)\\ -\sin(\phi)\end{pmatrix}$$

$$\begin{pmatrix}\theta\\ v_{Ag}\end{pmatrix} = \text{Find}(\theta, v_{Ag}) \qquad \theta = 5.39\text{ deg} \qquad v_{Ag} = 96.1\,\frac{\text{mi}}{\text{hr}}$$

Problem 12–205

At the instant shown car A is traveling with a velocity v_A and has an acceleration a_A along the highway. At the same instant B is traveling on the trumpet interchange curve with a speed v_B which is decreasing at v'_B. Determine the relative velocity and relative acceleration of B with respect to A at this instant.

Given:

$v_A = 30\ \frac{m}{s}$

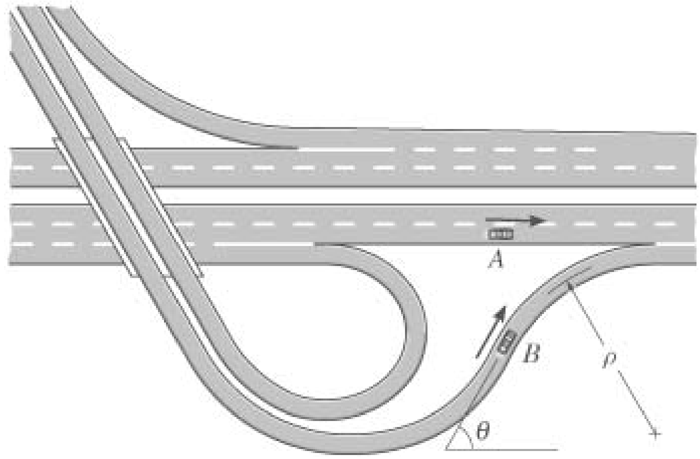
$v_B = 15\ \frac{m}{s}$

$a_A = 2\ \frac{m}{s^2}$

$v'_B = -0.8\ \frac{m}{s^2}$

$\rho = 250\ m$

$\theta = 60\ deg$



Solution:

$$\mathbf{v_{Av}} = \begin{pmatrix} v_A \\ 0 \end{pmatrix} \qquad \mathbf{a_{Av}} = \begin{pmatrix} a_A \\ 0 \end{pmatrix}$$

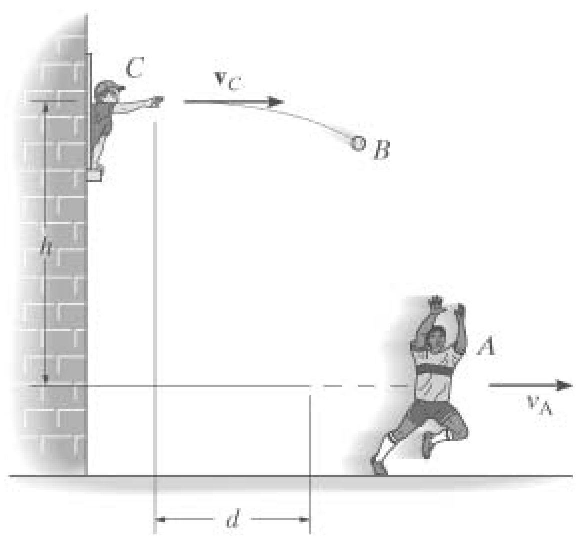
$$\mathbf{v_{Bv}} = v_B\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \qquad \mathbf{a_{Bv}} = v'_B\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} + \frac{v_B^{\,2}}{\rho}\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \end{pmatrix}$$

$$\mathbf{v_{BA}} = \mathbf{v_{Bv}} - \mathbf{v_{Av}} \qquad \mathbf{v_{BA}} = \begin{pmatrix} -22.5 \\ 12.99 \end{pmatrix} \frac{m}{s} \qquad \left|\mathbf{v_{BA}}\right| = 26.0\ \frac{m}{s}$$

$$\mathbf{a_{BA}} = \mathbf{a_{Bv}} - \mathbf{a_{Av}} \qquad \mathbf{a_{BA}} = \begin{pmatrix} -1.621 \\ -1.143 \end{pmatrix} \frac{m}{s^2} \qquad \left|\mathbf{a_{BA}}\right| = 1.983\ \frac{m}{s^2}$$

Problem 12–206

The boy A is moving in a straight line away from the building at a constant speed v_A. The boy C throws the ball B horizontally when A is at d. At what speed must C throw the ball so that A can catch it? Also determine the relative speed of the ball with respect to boy A at the instant the catch is made.



Given:

$v_A = 4\ \frac{ft}{s}$

$d = 10\ ft$

$h = 20\ ft$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $v_C = 1\ \frac{\text{ft}}{\text{s}}$

$t = 1\ \text{s}$

Given $h - \frac{1}{2} g t^2 = 0$

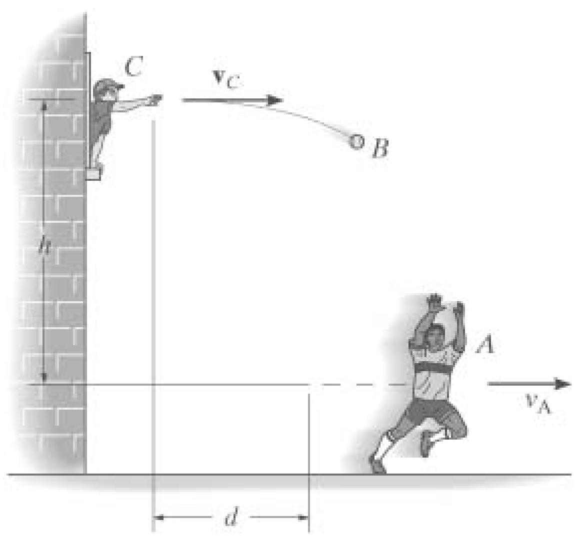
$v_C t = d + v_A t$

$\begin{pmatrix} t \\ v_C \end{pmatrix} = \text{Find}(t, v_C)$ $t = 1.115\ \text{s}$ $v_C = 12.97\ \frac{\text{ft}}{\text{s}}$

$\mathbf{v_{BA}} = \begin{pmatrix} v_C \\ -g t \end{pmatrix} - \begin{pmatrix} v_A \\ 0 \end{pmatrix}$ $\mathbf{v_{BA}} = \begin{pmatrix} 8.972 \\ -35.889 \end{pmatrix} \frac{\text{ft}}{\text{s}}$ $|\mathbf{v_{BA}}| = 37.0\ \frac{\text{ft}}{\text{s}}$

Problem 12–207

The boy A is moving in a straight line away from the building at a constant speed v_A. At what horizontal distance d must he be from C in order to make the catch if the ball is thrown with a horizontal velocity v_C? Also determine the relative speed of the ball with respect to the boy A at the instant the catch is made.



Given:

$v_A = 4\ \frac{\text{ft}}{\text{s}}$ $h = 20\ \text{ft}$

$v_C = 10\ \frac{\text{ft}}{\text{s}}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $d = 1\ \text{ft}$ $t = 1\ \text{s}$

Given $h - \frac{1}{2} g t^2 = 0$

$v_C t = d + v_A t$

$\begin{pmatrix} t \\ d \end{pmatrix} = \text{Find}(t, d)$ $t = 1.115\ \text{s}$ $d = 6.69\ \text{ft}$

$$\mathbf{v_{BA}} = \begin{pmatrix} v_C \\ -g\,t \end{pmatrix} - \begin{pmatrix} v_A \\ 0 \end{pmatrix} \qquad \mathbf{v_{BA}} = \begin{pmatrix} 6 \\ -35.889 \end{pmatrix}\frac{\text{ft}}{\text{s}} \qquad \left|\mathbf{v_{BA}}\right| = 36.4\,\frac{\text{ft}}{\text{s}}$$

---

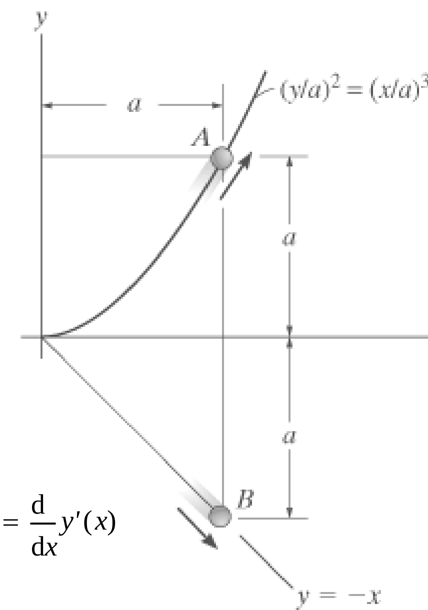
*__Problem 12–208__

At a given instant, two particles A and B are moving with a speed of v_0 along the paths shown. If B is decelerating at v'_B and the speed of A is increasing at v'_A, determine the acceleration of A with respect to B at this instant.

Given:

$$v_0 = 8\,\frac{\text{m}}{\text{s}} \qquad v'_A = 5\,\frac{\text{m}}{\text{s}^2}$$

$$a = 1\text{ m} \qquad v'_B = -6\,\frac{\text{m}}{\text{s}^2}$$



Solution:

$$y(x) = a\left(\frac{x}{a}\right)^{\frac{3}{2}} \qquad y'(x) = \frac{d}{dx}y(x) \qquad y''(x) = \frac{d}{dx}y'(x)$$

$$\rho = \frac{\sqrt{\left(1 + y'(a)^2\right)^3}}{y''(a)} \qquad \theta = \operatorname{atan}(y'(a)) \qquad \rho = 7.812\text{ m}$$

$$\mathbf{a_A} = v'_A\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} + \frac{v_0^2}{\rho}\begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix} \qquad \mathbf{a_B} = \frac{v'_B}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\mathbf{a_{AB}} = \mathbf{a_A} - \mathbf{a_B} \qquad \mathbf{a_{AB}} = \begin{pmatrix} 0.2 \\ 4.46 \end{pmatrix}\frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_{AB}}\right| = 4.47\,\frac{\text{m}}{\text{s}^2}$$

Problem 13-1

Determine the gravitational attraction between two spheres which are just touching each other. Each sphere has a mass M and radius r.

Given:

$r = 200 \text{ mm}$ $\quad M = 10 \text{ kg}$ $\quad G = 66.73 \times 10^{-12} \dfrac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$ $\quad \text{nN} = 1 \times 10^{-9} \text{ N}$

Solution:

$F = \dfrac{G M^2}{(2r)^2}$ $\quad F = 41.7 \text{ nN}$

Problem 13-2

By using an inclined plane to retard the motion of a falling object, and thus make the observations more accurate, Galileo was able to determine experimentally that the distance through which an object moves in free fall is proportional to the square of the time for travel. Show that this is the case, i.e., $s \propto t^2$ by determining the time t_B, t_C, and t_D needed for a block of mass m to slide from rest at A to points B, C, and D, respectively. Neglect the effects of friction.

Given:

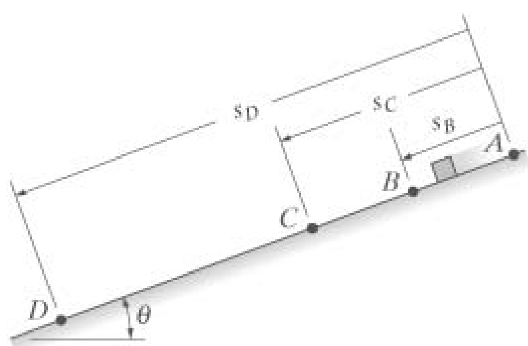
$s_B = 2 \text{ m}$

$s_C = 4 \text{ m}$

$s_D = 9 \text{ m}$

$\theta = 20 \text{ deg}$

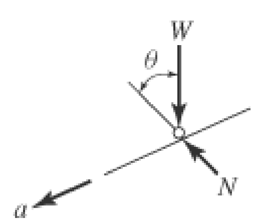
$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$



Solution:

$W \sin(\theta) = \left(\dfrac{W}{g}\right) a$

$a = g \sin(\theta)$ $\quad a = 3.355 \dfrac{\text{m}}{\text{s}^2}$



$s = \dfrac{1}{2} a t^2$

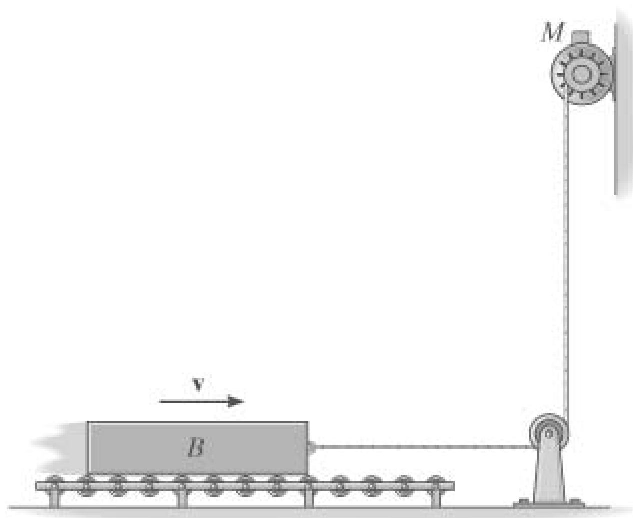
$t_B = \sqrt{\dfrac{2 s_B}{a}}$ $\quad t_B = 1.09 \text{ s}$

$$t_C = \sqrt{\frac{2s_C}{a}} \qquad t_C = 1.54\ \text{s}$$

$$t_D = \sqrt{\frac{2s_D}{a}} \qquad t_D = 2.32\ \text{s}$$

Problem 13-3

A bar B of mass M_1, originally at rest, is being towed over a series of small rollers. Determine the force in the cable at time t if the motor M is drawing in the cable for a short time at a rate $v = kt^2$. How far does the bar move in time t? Neglect the mass of the cable, pulley, and the rollers.



Given:

$$\text{kN} = 10^3\ \text{N}$$

$$M_1 = 300\ \text{kg}$$

$$t = 5\ \text{s}$$

$$k = 0.4\ \frac{\text{m}}{\text{s}^3}$$

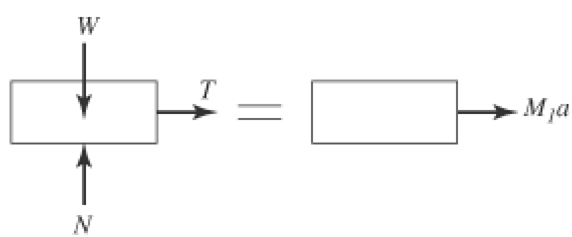
Solution:

$$v = kt^2 \qquad v = 10\ \frac{\text{m}}{\text{s}}$$

$$a = 2kt \qquad a = 4\ \frac{\text{m}}{\text{s}^2}$$

$$T = M_1 a \qquad T = 1.2\ \text{kN}$$

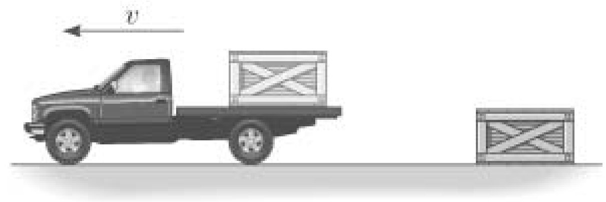
$$d = \int_0^t kt^2\, dt \qquad d = 16.7\ \text{m}$$



***Problem 13-4**

A crate having a mass M falls horizontally off the back of a truck which is traveling with speed v. Determine the coefficient of kinetic friction between the road and the crate if the crate slides a distance d on the ground with no tumbling along the road before coming to rest. Assume that the initial speed of the crate along the road is v.

Given:



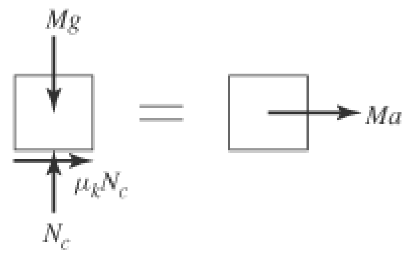
$M = 60\ \text{kg}$

$d = 45\ \text{m}$

$v = 80\ \dfrac{\text{km}}{\text{hr}}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:



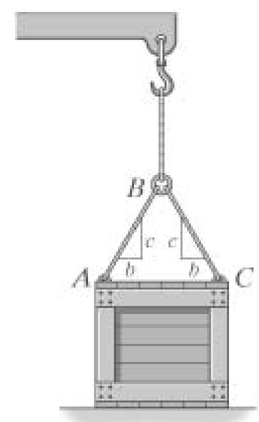
$N_C - Mg = 0 \qquad N_C = Mg$

$\mu_k N_C = Ma \qquad a = \mu_k g$

$$\frac{v^2}{2} = ad = \mu_k g d$$

$$\mu_k = \frac{v^2}{2gd} \qquad \mu_k = 0.559$$

Problem 13-5



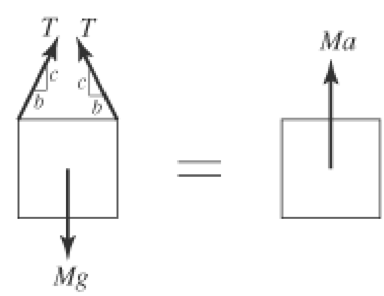
The crane lifts a bin of mass M with an initial acceleration a. Determine the force in each of the supporting cables due to this motion.

Given:

$M = 700\ \text{kg} \quad b = 3 \quad \text{kN} = 10^3\ \text{N}$

$a = 3\ \dfrac{\text{m}}{\text{s}^2} \quad c = 4$

Solution:



$$2T\left(\frac{c}{\sqrt{b^2 + c^2}}\right) - Mg = Ma$$

$$T = M(a+g)\left(\frac{\sqrt{b^2 + c^2}}{2c}\right) \qquad T = 5.60\,\text{kN}$$

Problem 13-6

The baggage truck A has mass m_t and is used to pull the two cars, each with mass m_c. The tractive force on the truck is F. Determine the initial acceleration of the truck. What is the acceleration of the truck if the coupling at C suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.

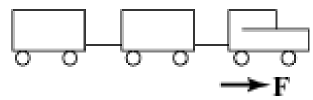
Given:

$m_t = 800\ \text{kg}$

$m_c = 300\ \text{kg}$

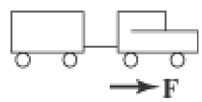
$F = 480\ \text{N}$

Solution:



$\overset{+}{\rightarrow}\ \Sigma F_x = ma_x;$ $F = (m_t + 2m_c)a$

$$a = \frac{F}{m_t + 2\,m_c} \qquad \boxed{a = 0.343\ \frac{\text{m}}{\text{s}^2}}$$



$\overset{+}{\rightarrow}\ \Sigma F_x = ma_x;$ $F = (m_t + m_c)a_{Fail}$

$$a_{Fail} = \frac{F}{m_t + m_c} \qquad \boxed{a_{Fail} = 0.436\ \frac{\text{m}}{\text{s}^2}}$$

Problem 13-7

The fuel assembly of mass M for a nuclear reactor is being lifted out from the core of the nuclear reactor using the pulley system shown. It is hoisted upward with a constant acceleration such that $s = 0$ and $v = 0$ when $t = 0$ and $s = s_1$ when $t = t_1$. Determine the tension in the cable at A during the motion.

Units Used:

$$kN = 10^3\ N$$

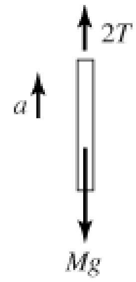
Given:

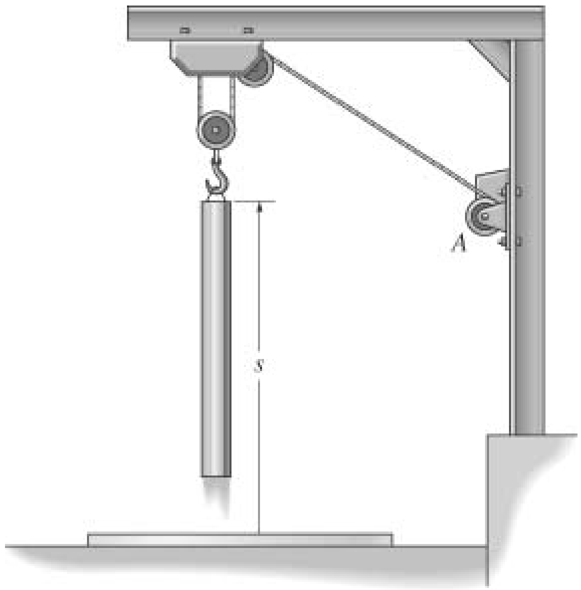
$$M = 500\ kg$$

$$s_1 = 2.5\ m$$

$$t_1 = 1.5\ s$$

$$g = 9.81\ \frac{m}{s^2}$$





Solution:

$$s = \left(\frac{a}{2}\right)t^2 \qquad a = \frac{2s_1}{t_1^2} \qquad a = 2.222\ \frac{m}{s^2}$$

$$2T - Mg = Ma \qquad T = \frac{M(a+g)}{2} \qquad T = 3.008\,kN$$

***Problem 13-8**

The crate of mass M is suspended from the cable of a crane. Determine the force in the cable at time t if the crate is moving upward with (a) a constant velocity v_1 and (b) a speed of $v = bt^2 + c$.

Units Used:

$$kN = 10^3\ N$$

Given:

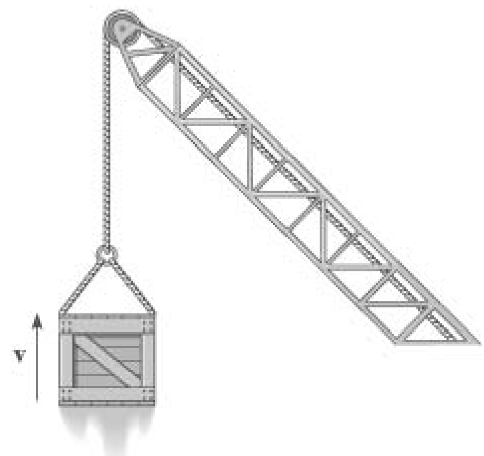
$$M = 200\ kg$$

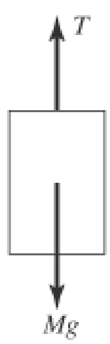
$$t = 2\ s$$

$$v_1 = 2\ \frac{m}{s}$$

$$b = 0.2\ \frac{m}{s^3}$$

$$c = 2\ \frac{m}{s}$$





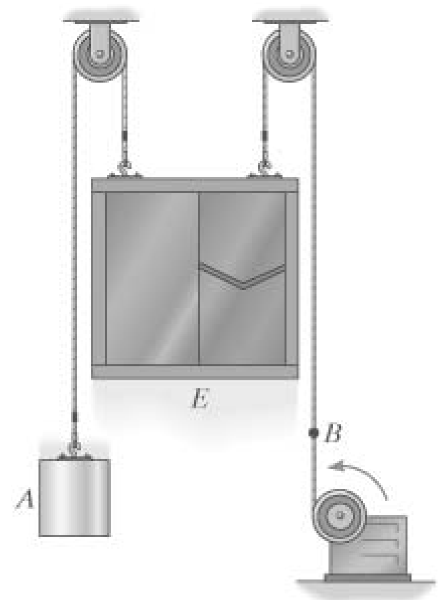
Solution:

$$\text{(a)} \qquad a = 0\ \frac{m}{s^2} \qquad T_a - Mg = Ma \qquad T_a = M(g+a) \qquad T_a = 1.962\,kN$$

(b) $v = bt^2 + c$ $a = 2bt$ $T_b = M(g+a)$ $T_b = 2.12\ \text{kN}$

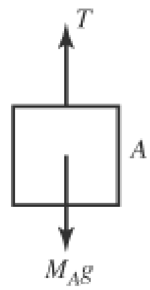
Problem 13-9

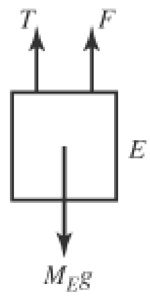
The elevator E has a mass M_E, and the counterweight at A has a mass M_A. If the motor supplies a constant force F on the cable at B, determine the speed of the elevator at time t starting from rest. Neglect the mass of the pulleys and cable.



Units Used:

$\text{kN} = 10^3\ \text{N}$





Given:

$M_E = 500\ \text{kg}$

$M_A = 150\ \text{kg}$

$F = 5\ \text{kN}$

$t = 3\ \text{s}$

Solution:

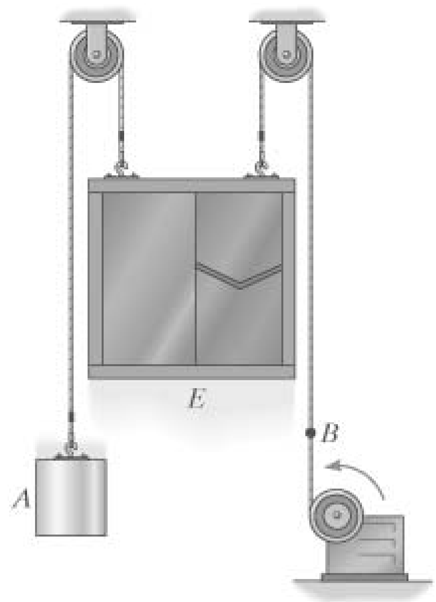
Guesses $T = 1\ \text{kN}$ $a = 1\ \dfrac{\text{m}}{\text{s}^2}$ $v = 1\ \dfrac{\text{m}}{\text{s}}$

Given $T - M_A g = -M_A a$ $F + T - M_E g = M_E a$ $v = at$

$$\begin{pmatrix} T \\ a \\ v \end{pmatrix} = \text{Find}(T, a, v) \qquad T = 1.11\ \text{kN} \qquad a = 2.41\ \frac{\text{m}}{\text{s}^2} \qquad v = 7.23\ \frac{\text{m}}{\text{s}}$$

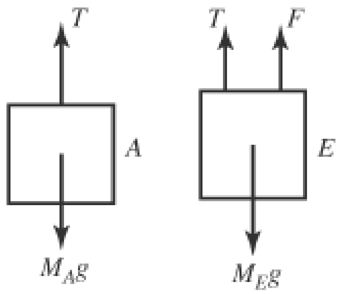
Problem 13-10

The elevator E has a mass M_E and the counterweight at A has a mass M_A. If the elevator attains a speed v after it rises a distance h, determine the constant force developed in the cable at B. Neglect the mass of the pulleys and cable.



Units Used:

$\mathrm{kN} = 10^3\ \mathrm{N}$



Given:

$M_E = 500\ \mathrm{kg}$

$M_A = 150\ \mathrm{kg}$

$v = 10\ \dfrac{\mathrm{m}}{\mathrm{s}}$

$h = 40\ \mathrm{m}$

Solution:

Guesses $\quad T = 1\ \mathrm{kN} \quad F = 1\ \mathrm{kN} \quad a = 1\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$

Given $\quad T - M_A g = -M_A a \qquad F + T - M_E g = M_E a \qquad v^2 = 2ah$

$$\begin{pmatrix} F \\ T \\ a \end{pmatrix} = \mathrm{Find}(F, T, a) \qquad a = 1.250\ \frac{\mathrm{m}}{\mathrm{s}^2} \qquad T = 1.28\ \mathrm{kN} \qquad F = 4.25\ \mathrm{kN}$$

Problem 13-11

The water-park ride consists of a sled of weight W which slides from rest down the incline and then into the pool. If the frictional resistance on the incline is F_{r1} and in the pool for a short distance is F_{r2}, determine how fast the sled is traveling when $s = s_2$.

Given:

$W = 800\ \text{lb}$

$F_{r1} = 30\ \text{lb}$

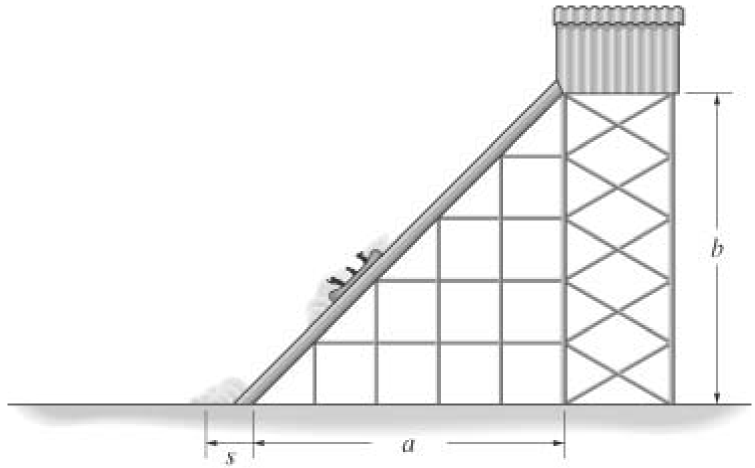
$F_{r2} = 80\ \text{lb}$

$s_2 = 5\ \text{ft}$

$a = 100\ \text{ft}$

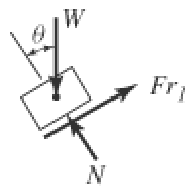
$b = 100\ \text{ft}$

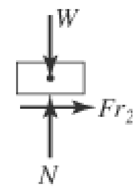
$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$\theta = \operatorname{atan}\left(\frac{b}{a}\right)$





On the incline

$W\sin(\theta) - F_{r1} = \left(\frac{W}{g}\right)a_1 \qquad a_1 = g\left(\frac{W\sin(\theta) - F_{r1}}{W}\right) \qquad a_1 = 21.561\ \frac{\text{ft}}{\text{s}^2}$

$v_1^{\,2} = 2a_1\sqrt{a^2 + b^2} \qquad v_1 = \sqrt{2a_1\sqrt{a^2 + b^2}} \qquad v_1 = 78.093\ \frac{\text{ft}}{\text{s}}$

In the water

$F_{r2} = \left(\frac{W}{g}\right)a_2 \qquad a_2 = \frac{g F_{r2}}{W} \qquad a_2 = 3.22\ \frac{\text{ft}}{\text{s}^2}$

$\frac{v_2^{\,2}}{2} - \frac{v_1^{\,2}}{2} = -a_2 s_2 \qquad v_2 = \sqrt{v_1^{\,2} - 2a_2 s_2} \qquad v_2 = 77.886\ \frac{\text{ft}}{\text{s}}$

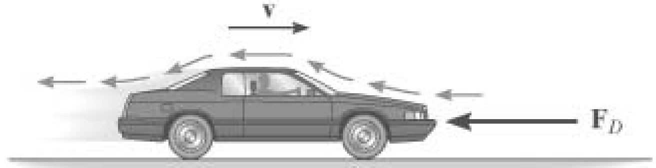
***Problem 13-12**

A car of mass m is traveling at a slow velocity v_0. If it is subjected to the drag resistance of the wind, which is proportional to its velocity, i.e., $F_D = kv$ determine the distance and the time the car will travel before its velocity becomes 0.5 v_0. Assume no other frictional forces act on the car.

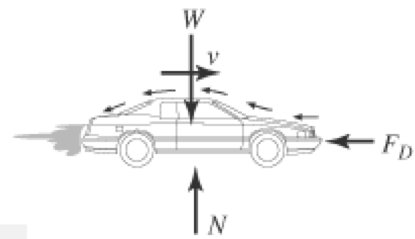
Solution:

$-F_D = m a$

$-k v = m a$



Find time $a = \frac{d}{dt}v = \frac{-k}{m}v$



$$\frac{-k}{m}\int_0^t 1\,dt = \int_{v_0}^{0.5v_0} \frac{1}{v}\,dv$$

$$t = \frac{m}{k}\ln\left(\frac{v_0}{0.5v_0}\right) \qquad t = \frac{m}{k}\ln(2) \qquad t = 0.693\frac{m}{k}$$

Find distance $a = v\frac{d}{dx}v = \frac{-k}{m}v$

$$-\int_0^x k\,dx = \int_{v_0}^{0.5v_0} m\,dv \qquad x = \frac{m}{k}(0.5v_0) \qquad x = 0.5\frac{m v_0}{k}$$

Problem 13-13

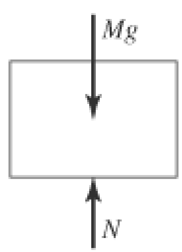
Determine the normal force the crate *A* of mass *M* exerts on the smooth cart if the cart is given an acceleration *a* down the plane. Also, what is the acceleration of the crate?

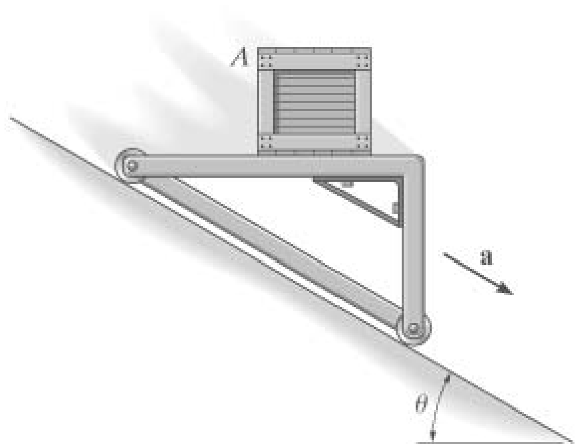
Given:

$M = 10$ kg

$a = 2\ \frac{m}{s^2}$

$\theta = 30$ deg





Solution:

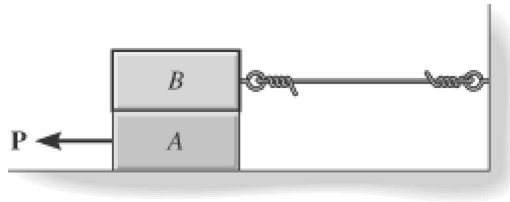
$$N - Mg = -M(a)\sin(\theta)$$

$$N = M[g - (a)\sin(\theta)] \qquad N = 88.1\ \text{N}$$

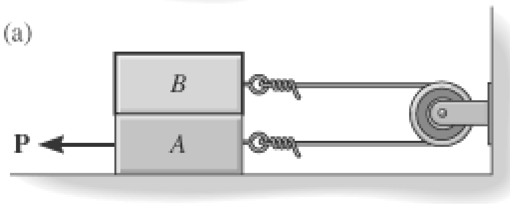
$$a_{crate} = (a)\sin(\theta) \qquad a_{crate} = 1\ \frac{m}{s^2}$$

Problem 13-14

Each of the two blocks has a mass *m*. The coefficient of kinetic friction at all surfaces of contact is μ. If a horizontal force **P** moves the bottom block, determine the acceleration of the bottom block in each case.



(a)



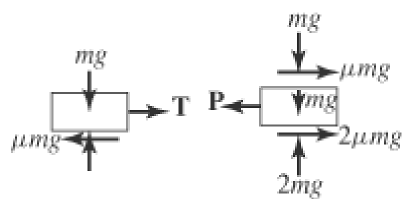
(b)

Solution:

(a) Block A:

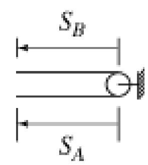
$\overset{+}{\leftarrow}$ $\Sigma F_x = ma_x;$ $P - 3\mu m g = m a_A$

$$\boxed{a_A = \frac{P}{m} - 3\mu g}$$



(b) $S_B + S_A = L$

$a_A = -a_B$



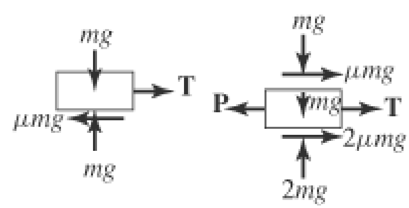
Block A:

$\overset{+}{\leftarrow}$ $\Sigma F_x = ma_x;$ $P - T - 3\mu m g = m a_A$

Block B:

$\overset{+}{\leftarrow}$ $\Sigma F_x = ma_x;$ $\mu m g - T = m a_B$

Solving simultaenously $$\boxed{a_A = \frac{P}{2m} - 2\mu g}$$



Problem 13-15

The driver attempts to tow the crate using a rope that has a tensile strength T_{max}. If the crate is originally at rest and has weight W, determine the greatest acceleration it can have if the coefficient of static friction between the crate and the road is μ_s and the coefficient of kinetic friction is μ_k.

Given:

$T_{max} = 200\ \text{lb}$

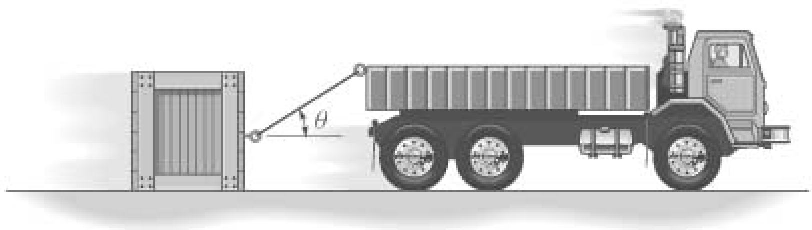
$W = 500\ \text{lb}$

$\mu_s = 0.4$

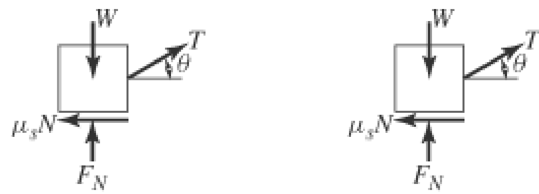
$\mu_k = 0.3$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$\theta = 30\ \text{deg}$



Solution:



Equilibrium : In order to slide the crate, the towing force must overcome static friction.

Initial guesses $F_N = 100\ \text{lb}$ $T = 50\ \text{lb}$

Given $T\cos(\theta) - \mu_s F_N = 0$ $F_N + T\sin(\theta) - W = 0$ $\begin{pmatrix} F_N \\ T \end{pmatrix} = \text{Find}(F_N, T)$

If $T = 187.613\,\text{lb} > T_{max} = 200\,\text{lb}$ then the truck will not be able to pull the create without breaking the rope.

If $T = 187.613\,\text{lb} < T_{max} = 200\,\text{lb}$ then the truck will be able to pull the create without breaking the rope and we will now calculate the acceleration for this case.

Initial guesses $F_N = 100\ \text{lb}$ $a = 1\ \dfrac{\text{ft}}{\text{s}^2}$ Require $T = T_{max}$

Given $T\cos(\theta) - \mu_k F_N = \dfrac{W}{g}a$ $F_N + T\sin(\theta) - W = 0$ $\begin{pmatrix} F_N \\ a \end{pmatrix} = \text{Find}(F_N, a)$

$a = 3.426\ \dfrac{\text{ft}}{\text{s}^2}$

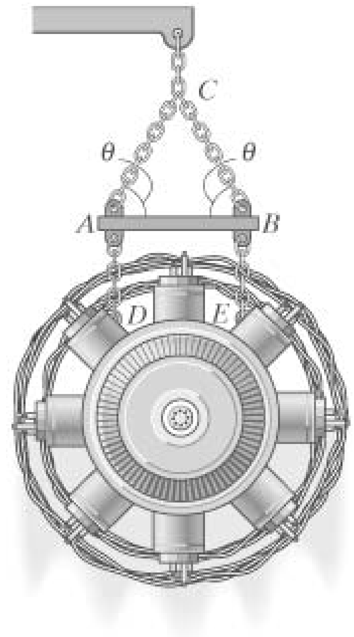
***Problem 13-16**

An engine of mass M_1 is suspended from a spreader beam of mass M_2 and hoisted by a crane which gives it an acceleration a when it has a velocity v. Determine the force in chains AC and AD during the lift.

Units Used:

$$Mg = 10^3\ kg \qquad kN = 10^3\ N$$

Given:



$$M_1 = 3.5\ Mg$$

$$M_2 = 500\ kg$$

$$a = 4\ \frac{m}{s^2}$$

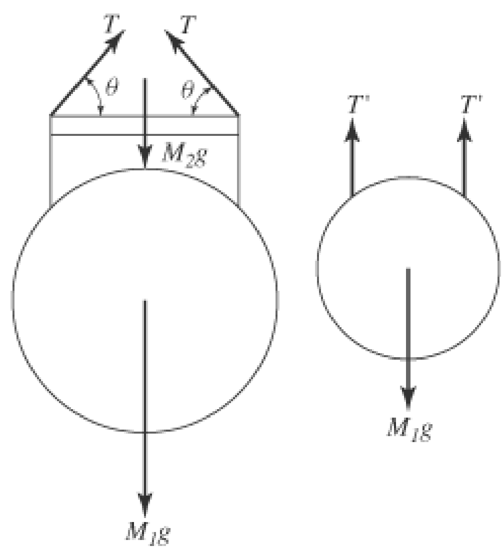
$$v = 2\ \frac{m}{s}$$

$$\theta = 60\ deg$$

Solution:

Guesses $T = 1\ N \quad T' = 1\ N$

Given



$$2T\sin(\theta) - (M_1 + M_2)g = (M_1 + M_2)a$$

$$2T' - M_1 g = M_1 a$$

$$\begin{pmatrix} T \\ T' \end{pmatrix} = \text{Find}(T, T') \qquad \begin{pmatrix} T_{AC} \\ T_{AD} \end{pmatrix} = \begin{pmatrix} T \\ T' \end{pmatrix}$$

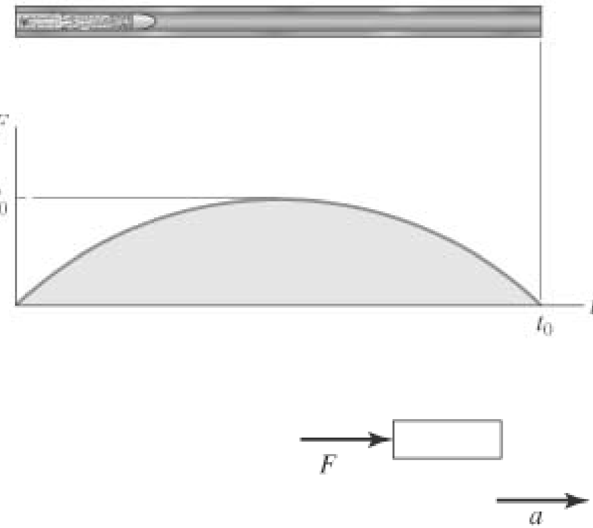
$$\begin{pmatrix} T_{AC} \\ T_{AD} \end{pmatrix} = \begin{pmatrix} 31.9 \\ 24.2 \end{pmatrix} kN$$

Problem 13-17

The bullet of mass *m* is given a velocity due to gas pressure caused by the burning of powder within the chamber of the gun. Assuming this pressure creates a force of $F = F_0 \sin(\pi t / t_0)$ on the bullet, determine the velocity of the bullet at any instant it is in the barrel. What is the bullet's maximum velocity? Also, determine the position of the bullet in the barrel as a function of time.

Solution:

$$F_0 \sin\left(\pi \frac{t}{t_0}\right) = m\,a \qquad a = \frac{dv}{dt} = \frac{F_0}{m}\sin\left(\frac{\pi t}{t_0}\right)$$



$$\int_0^v 1\,dv = \int_0^t \frac{F_0}{m}\sin\left(\frac{\pi t}{t_0}\right)dt$$

$$v = \frac{F_0 t_0}{\pi m}\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)$$

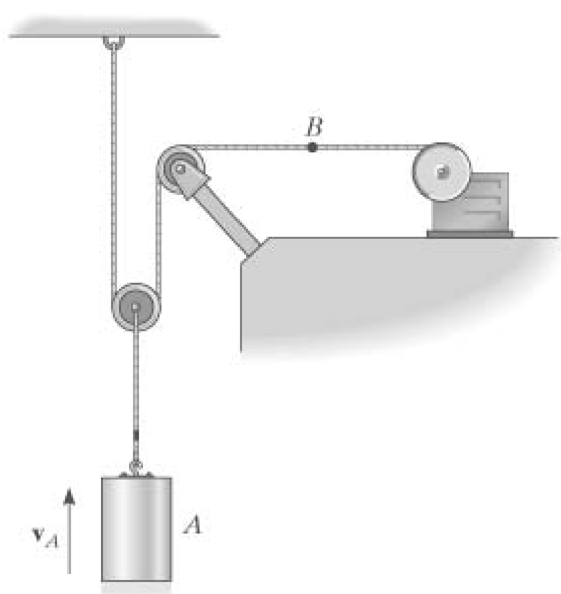
v_{max} occurs when $\cos\left(\frac{\pi t}{t_0}\right) = -1$, or $t = t_0$

$$v_{max} = \frac{2F_0 t_0}{\pi m}$$

$$\int_0^s 1\,ds = \int_0^t \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)dt \qquad s = \frac{F_0 t_0}{\pi m}\left(t - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right)$$

Problem 13-18

The cylinder of weight W at A is hoisted using the motor and the pulley system shown. If the speed of point B on the cable is increased at a constant rate from zero to v_B in time t, determine the tension in the cable at B to cause the motion.



Given:

$W = 400$ lb

$v_B = 10 \frac{ft}{s}$

$t = 5$ s

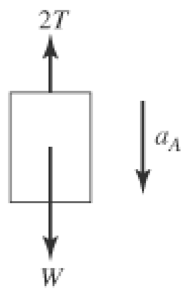
Solution:

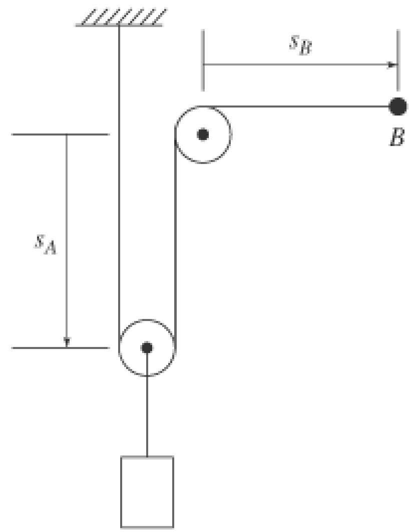
$$2s_A + s_B = l$$

$$a_B = \frac{v_B}{t}$$

$$a_A = \frac{-a_B}{2}$$

$$2T - W = -\frac{W}{g}a_A$$





$$T = \frac{W}{2}\left(1 - \frac{a_A}{g}\right) \qquad T = 206\,\text{lb}$$

Problem 13-19

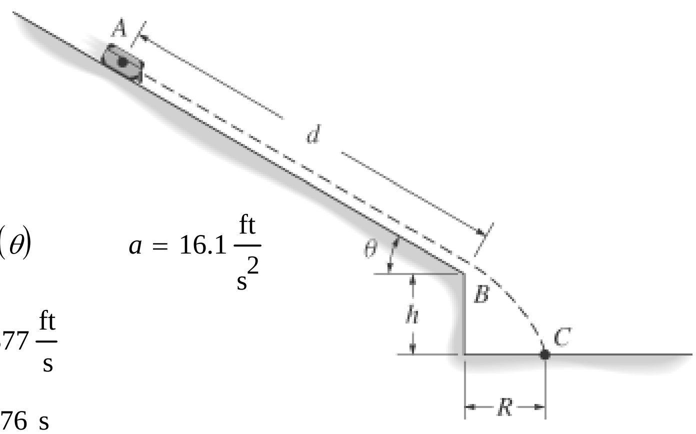
A suitcase of weight W slides from rest a distance d down the smooth ramp. Determine the point where it strikes the ground at C. How long does it take to go from A to C?

Given:

$W = 40\text{ lb}$ $\quad \theta = 30\text{ deg}$

$d = 20\text{ ft}$ $\quad g = 32.2\,\frac{\text{ft}}{\text{s}^2}$

$h = 4\text{ ft}$



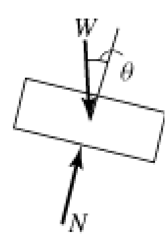
Solution:

$$W\sin(\theta) = \left(\frac{W}{g}\right)a \qquad a = g\sin(\theta) \qquad a = 16.1\,\frac{\text{ft}}{\text{s}^2}$$

$$v_B = \sqrt{2ad} \qquad v_B = 25.377\,\frac{\text{ft}}{\text{s}}$$

$$t_{AB} = \frac{v_B}{a} \qquad t_{AB} = 1.576\text{ s}$$

Guesses $\quad t_{BC} = 1\text{ s} \quad R = 1\text{ ft}$



Given $\quad \left(\frac{-g}{2}\right)t_{BC}^2 - v_B\sin(\theta)t_{BC} + h = 0 \qquad R = v_B\cos(\theta)t_{BC}$

$$\begin{pmatrix} t_{BC} \\ R \end{pmatrix} = \text{Find}(t_{BC}, R) \qquad t_{BC} = 0.241\text{ s}$$

$$R = 5.304\,\text{ft} \qquad t_{AB} + t_{BC} = 1.818\text{ s}$$

***Problem 13-20**

A suitcase of weight W slides from rest a distance d down the rough ramp. The coefficient of kinetic friction along ramp AB is μ_k. The suitcase has an initial velocity down the ramp v_0. Determine the point where it strikes the ground at C. How long does it take to go from A to C?

Given:

$W = 40\ \text{lb}$

$d = 20\ \text{ft}$

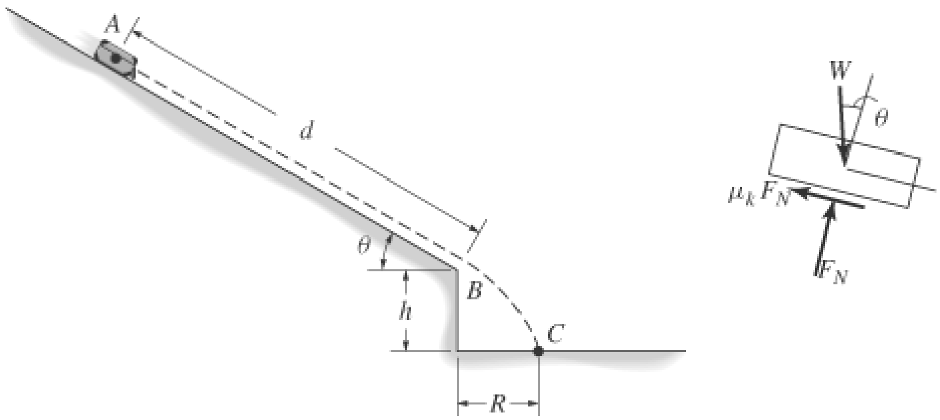
$h = 4\ \text{ft}$

$\mu_k = 0.2$

$\theta = 30\ \text{deg}$

$v_0 = 10\ \dfrac{\text{ft}}{\text{s}}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$F_N - W\cos(\theta) = 0 \qquad F_N = W\cos(\theta)$

$W\sin(\theta) - \mu_k W\cos(\theta) = \left(\dfrac{W}{g}\right)a$

$a = g\left(\sin(\theta) - \mu_k\cos(\theta)\right) \qquad a = 10.523\ \dfrac{\text{ft}}{\text{s}^2}$

$v_B = \sqrt{2ad + v_0^{\,2}} \qquad v_B = 22.823\ \dfrac{\text{ft}}{\text{s}}$

$t_{AB} = \dfrac{v_B - v_0}{a} \qquad t_{AB} = 1.219\ \text{s}$

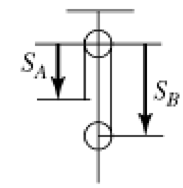
Guesses $\quad t_{BC} = 1\ \text{s} \qquad R = 1\ \text{ft}$

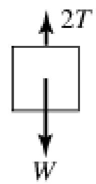
Given $\quad \left(\dfrac{-g}{2}\right)t_{BC}^{\,2} - v_B\sin(\theta)t_{BC} + h = 0 \qquad R = v_B\cos(\theta)t_{BC}$

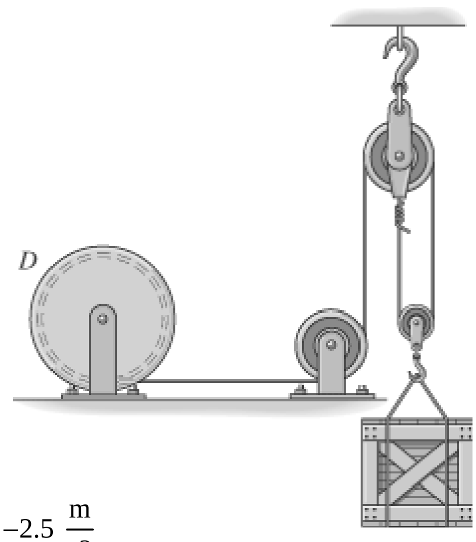
$\begin{pmatrix} t_{BC} \\ R \end{pmatrix} = \text{Find}(t_{BC}, R) \qquad t_{BC} = 0.257\ \text{s} \qquad R = 5.084\ \text{ft} \qquad t_{AB} + t_{BC} = 1.476\ \text{s}$

Problem 13-21

The winding drum D is drawing in the cable at an accelerated rate a. Determine the cable tension if the suspended crate has mass M.







Units Used:

$$kN = 1000\ N$$

Given:

$$a = 5\ \frac{m}{s^2}$$

$$M = 800\ kg$$

$$g = 9.81\ \frac{m}{s^2}$$

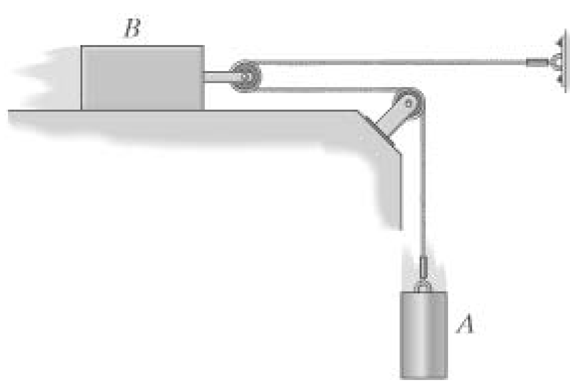
Solution:

$$L = s_A + 2s_B \qquad a_B = \frac{-a}{2} \qquad a_B = -2.5\ \frac{m}{s^2}$$

$$2T - Mg = -Ma_B \qquad T = \frac{M(g - a_B)}{2} \qquad T = 4.924\,\text{kN}$$

Problem 13-22

At a given instant block A of weight W_A is moving downward with a speed v_1. Determine its speed at the later time t. Block B has weight W_B, and the coefficient of kinetic friction between it and the horizontal plane is μ_k. Neglect the mass of the pulleys and cord.



Given:

$$W_A = 5\ lb \qquad v_1 = 4\ \frac{ft}{s}$$

$$W_B = 6\ lb \qquad t = 2\ s$$

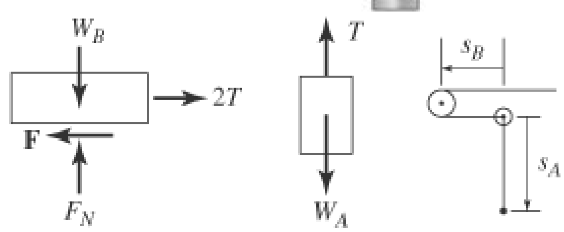
$$\mu_k = 0.3$$

Solution: $\quad 2s_B + s_A = L$

Guesses

$$a_A = 1\ \frac{ft}{s^2} \qquad a_B = 1\ \frac{ft}{s^2}$$

$$T = 1\ lb \qquad F_N = 1\ lb$$



Given $\quad F_N - W_B = 0 \qquad 2a_B + a_A = 0$

$$2T - \mu_k F_N = \left(\frac{-W_B}{g}\right) a_B \qquad T - W_A = \left(\frac{-W_A}{g}\right) a_A$$

$$\begin{pmatrix} F_N \\ T \\ a_A \\ a_B \end{pmatrix} = \text{Find}(F_N, T, a_A, a_B) \qquad \begin{pmatrix} F_N \\ T \end{pmatrix} = \begin{pmatrix} 6.000 \\ 1.846 \end{pmatrix} \text{lb} \qquad \begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 20.3 \\ -10.2 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

$$v_2 = v_1 + a_A t \qquad v_2 = 44.6 \frac{\text{ft}}{\text{s}}$$

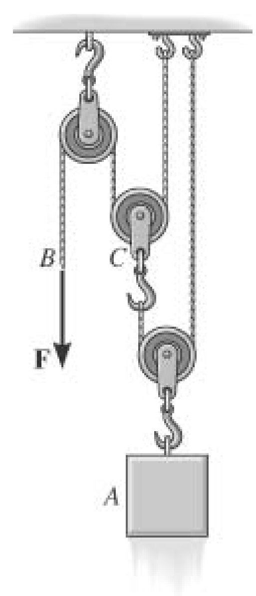
Problem 13-23

A force F is applied to the cord. Determine how high the block A of weight W rises in time t starting from rest. Neglect the weight of the pulleys and cord.

Given:

$$F = 15 \text{ lb} \qquad t = 2 \text{ s}$$

$$W = 30 \text{ lb} \qquad g = 32.2 \frac{\text{ft}}{\text{s}^2}$$



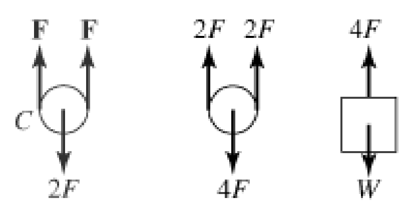
Solution:

$$4F - W = \left(\frac{W}{g}\right) a$$

$$a = \frac{g}{W}(4F - W)$$

$$a = 32.2 \frac{\text{ft}}{\text{s}^2}$$

$$d = \frac{1}{2} a t^2 \qquad d = 64.4 \text{ ft}$$

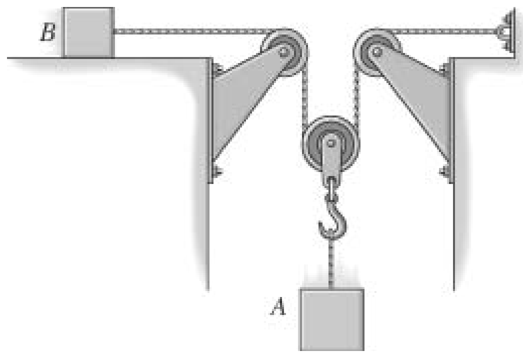


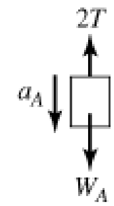
***Problem 13-24**

At a given instant block A of weight W_A is moving downward with speed v_{A0}. Determine its speed at a later time t. Block B has a weight W_B and the coefficient of kinetic friction between it and the horizontal plane is μ_k. Neglect the mass of the pulleys and cord.

Given:

$W_A = 10\ \text{lb}$

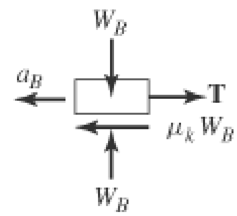




$v_{A0} = 6\ \dfrac{\text{ft}}{\text{s}}$

$t = 2\ \text{s}$

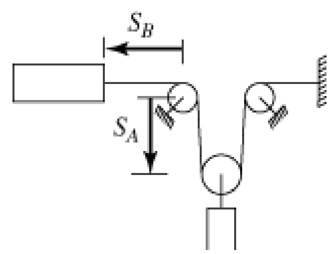
$W_B = 4\ \text{lb}$



$\mu_k = 0.2$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution: $L = s_B + 2s_A$



Guesses $a_A = 1\ \dfrac{\text{ft}}{\text{s}^2}$ $a_B = 1\ \dfrac{\text{ft}}{\text{s}^2}$ $T = 1\ \text{lb}$

Given $T - \mu_k W_B = \left(\dfrac{-W_B}{g}\right)a_B$

$$2T - W_A = \left(\dfrac{-W_A}{g}\right)a_A$$

$$0 = a_B + 2a_A$$

$$\begin{pmatrix} T \\ a_A \\ a_B \end{pmatrix} = \text{Find}(T, a_A, a_B) \qquad T = 3.385\,\text{lb} \qquad \begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 10.403 \\ -20.806 \end{pmatrix}\frac{\text{ft}}{\text{s}^2}$$

$$v_A = v_{A0} + a_A t \qquad v_A = 26.8\,\frac{\text{ft}}{\text{s}}$$

Problem 13-25

A freight elevator, including its load, has mass M_e. It is prevented from rotating due to the track and wheels mounted along its sides. If the motor M develops a constant tension T in its attached cable, determine the velocity of the elevator when it has moved upward at a distance d starting from rest. Neglect the mass of the pulleys and cables.

Units Used:

$kN = 10^3\ N$

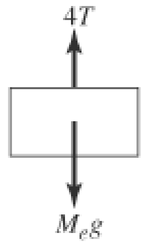
Given:

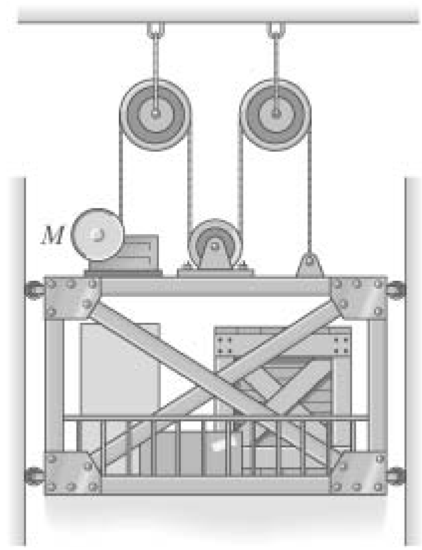
$M_e = 500\ kg$

$T = 1.50\ kN$

$d = 3\ m$

$g = 9.81\ \frac{m}{s^2}$





Solution:

$4T - M_e g = M_e a \qquad a = 4\left(\frac{T}{M_e}\right) - g$

$v = \sqrt{2ad} \qquad v = 3.62\ \frac{m}{s}$

Problem 13-26

At the instant shown the block A of weight W_A is moving down the plane at v_0 while being attached to the block B of weight W_B. If the coefficient of kinetic friction is μ_k, determine the acceleration of A and the distance A slides before it stops. Neglect the mass of the pulleys and cables.

Given:

$W_A = 100\ lb$

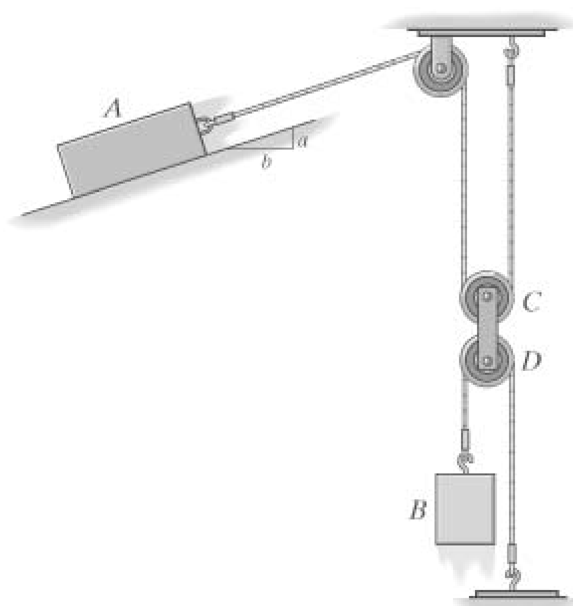
$W_B = 50\ lb$

$v_0 = 5\ \frac{ft}{s}$

$\mu_k = 0.2$

$a = 3$

$b = 4$



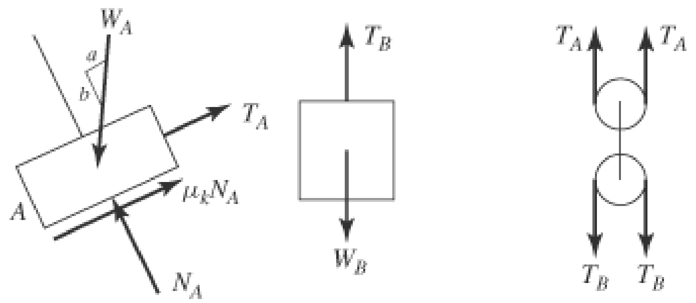
Solution: $\theta = \operatorname{atan}\left(\frac{a}{b}\right)$

Rope constraints

$s_A + 2s_C = L_1$

$s_D + (s_D - s_B) = L_2$

$s_C + s_D + d = d'$

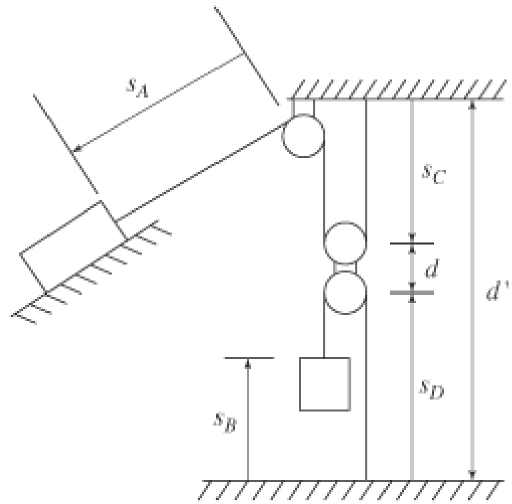


Guesses

$a_A = 1\ \frac{ft}{s^2}$ $a_B = 1\ \frac{ft}{s^2}$

$a_C = 1\ \frac{ft}{s^2}$ $a_D = 1\ \frac{ft}{s^2}$

$T_A = 1\ lb$ $T_B = 1\ lb$ $N_A = 1\ lb$



Given

$a_A + 2a_C = 0$ $2a_D - a_B = 0$

$a_C + a_D = 0$

$$T_B - W_B = \left(\frac{W_B}{g}\right)a_B$$

$$T_A - W_A\sin(\theta) + \mu_k N_A = \left(\frac{-W_A}{g}\right)a_A$$

$N_A - W_A\cos(\theta) = 0$ $2T_A - 2T_B = 0$

$$\begin{pmatrix} a_A \\ a_B \\ a_C \\ a_D \\ T_A \\ T_B \\ N_A \end{pmatrix} = \text{Find}(a_A, a_B, a_C, a_D, T_A, T_B, N_A) \quad \begin{pmatrix} T_A \\ T_B \\ N_A \end{pmatrix} = \begin{pmatrix} 48 \\ 48 \\ 80 \end{pmatrix} lb \quad \begin{pmatrix} a_A \\ a_B \\ a_C \\ a_D \end{pmatrix} = \begin{pmatrix} -1.287 \\ -1.287 \\ 0.644 \\ -0.644 \end{pmatrix} \frac{ft}{s^2}$$

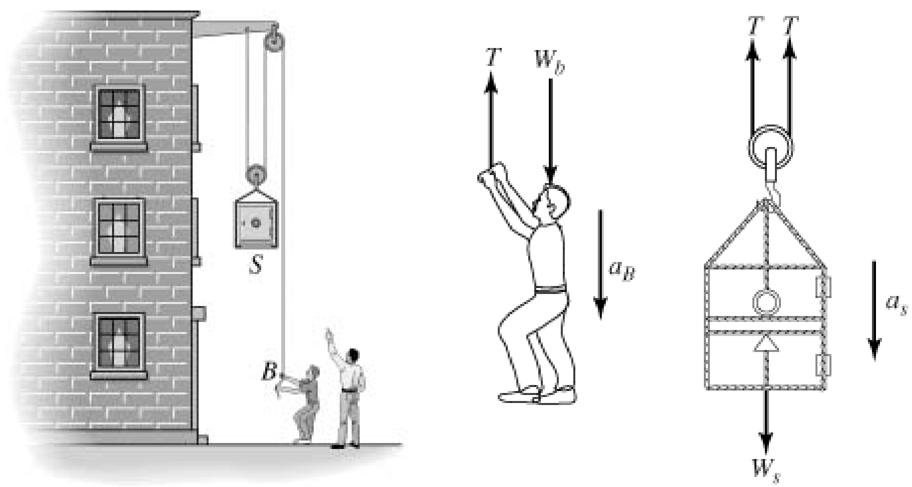
$d_A = \frac{-v_0^2}{2a_A}$ $a_A = -1.287\ \frac{ft}{s^2}$ $d_A = 9.71\ ft$

Problem 13-27

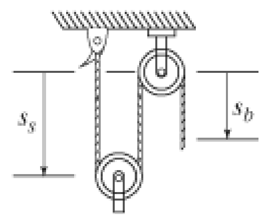
The safe *S* has weight W_s and is supported by the rope and pulley arrangement shown. If the end of the rope is given to a boy *B* of weight W_b, determine his acceleration if in the confusion he doesn't let go of the rope. Neglect the mass of the pulleys and rope.

Given:

$$W_s = 200\ \text{lb} \quad W_b = 90\ \text{lb} \quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$



Solution: $L = 2s_s + s_b$



Initial guesses: $a_b = 1\ \frac{\text{ft}}{\text{s}^2}$ $\quad a_s = 1\ \frac{\text{ft}}{\text{s}^2}$ $\quad T = 1\ \text{lb}$

Given $\quad 0 = 2a_s + a_b \quad 2T - W_s = \left(\frac{-W_s}{g}\right)a_s \quad T - W_b = \left(\frac{-W_b}{g}\right)a_b$

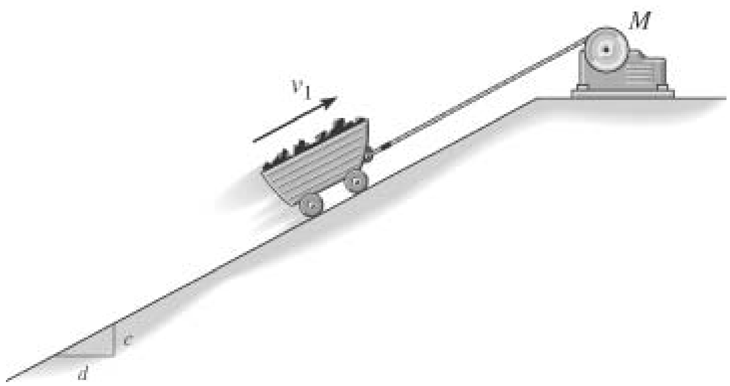
$$\begin{pmatrix} a_b \\ a_s \\ T \end{pmatrix} = \text{Find}(a_b, a_s, T) \quad T = 96.429\ \text{lb} \quad a_s = 1.15\ \frac{\text{ft}}{\text{s}^2} \quad a_b = -2.3\ \frac{\text{ft}}{\text{s}^2}$$

Negative means up

***Problem 13-28**

The mine car of mass m_{car} is hoisted up the incline using the cable and motor *M*. For a short time, the force in the cable is $F = bt^2$. If the car has an initial velocity v_0 when $t = 0$, determine its velocity when $t = t_1$.

Given:



$m_{car} = 400 \text{ kg}$

$b = 3200 \frac{\text{N}}{\text{s}^2}$

$v_0 = 2 \frac{\text{m}}{\text{s}}$

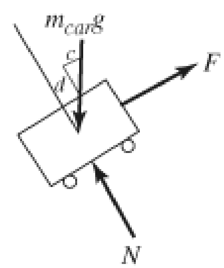
$t_1 = 2 \text{ s}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

$c = 8$

$d = 15$

Solution:



$$b t^2 - m_{car} g \left(\frac{c}{\sqrt{c^2 + d^2}} \right) = m_{car} a$$

$$a = \left(\frac{b}{m_{car}} \right) t^2 - \frac{g c}{\sqrt{c^2 + d^2}}$$

$$v_1 = \left(\frac{b}{m_{car}} \right) \frac{t_1^3}{3} - \frac{g c t_1}{\sqrt{c^2 + d^2}} + v_0 \qquad v_1 = 14.1 \frac{\text{m}}{\text{s}}$$

Problem 13-29

The mine car of mass m_{car} is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = bt^2$. If the car has an initial velocity v_0 when $t = 0$, determine the distance it moves up the plane when $t = t_1$.

Given:

$m_{car} = 400\ \text{kg}$

$b = 3200\ \dfrac{\text{N}}{\text{s}^2}$

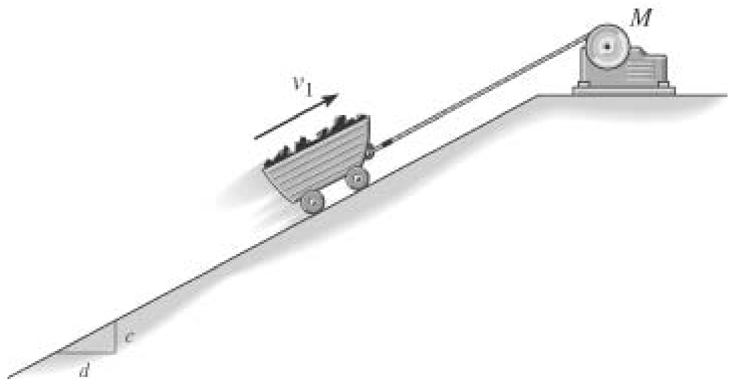
$v_0 = 2\ \dfrac{\text{m}}{\text{s}}$

$t_1 = 2\ \text{s}$

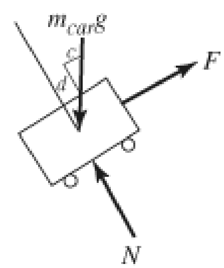
$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$c = 8$

$d = 15$



Solution:



$$b\,t^2 - m_{car}\,g\left(\frac{c}{\sqrt{c^2+d^2}}\right) = m_{car}\,a \qquad a = \left(\frac{b}{m_{car}}\right)t^2 - \frac{g\,c}{\sqrt{c^2+d^2}}$$

$$v = \left(\frac{b}{m_{car}}\right)\frac{t^3}{3} - \frac{g\,c\,t}{\sqrt{c^2+d^2}} + v_0$$

$$s_1 = \left(\frac{b}{m_{car}}\right)\frac{t_1^{\,4}}{12} - \left(\frac{g\,c}{\sqrt{c^2+d^2}}\right)\frac{t_1^{\,2}}{2} + v_0\,t_1 \qquad s_1 = 5.434\ \text{m}$$

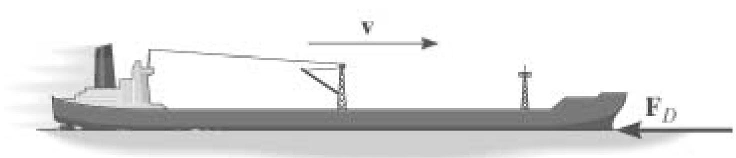
Problem 13-30

The tanker has a weight W and is traveling forward at speed v_0 in still water when the engines are shut off. If the drag resistance of the water is proportional to the speed of the tanker at any instant and can be approximated by $F_D = cv$, determine the time needed for the tanker's speed to become v_1. Given the initial velocity v_0 through what distance must the tanker travel before it stops?

Given:

$W = 800 \times 10^6\ \text{lb}$

$c = 400 \times 10^3\ \text{lb}\cdot\dfrac{\text{s}}{\text{ft}}$



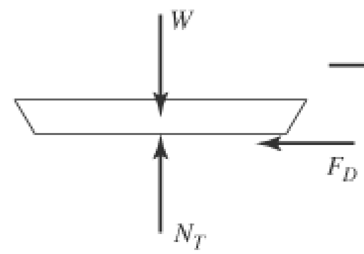
$v_0 = 3\ \dfrac{\text{ft}}{\text{s}} \qquad v_1 = 1.5\ \dfrac{\text{ft}}{\text{s}}$

Solution:

$$a(v) = \frac{-c\,g}{W}v$$

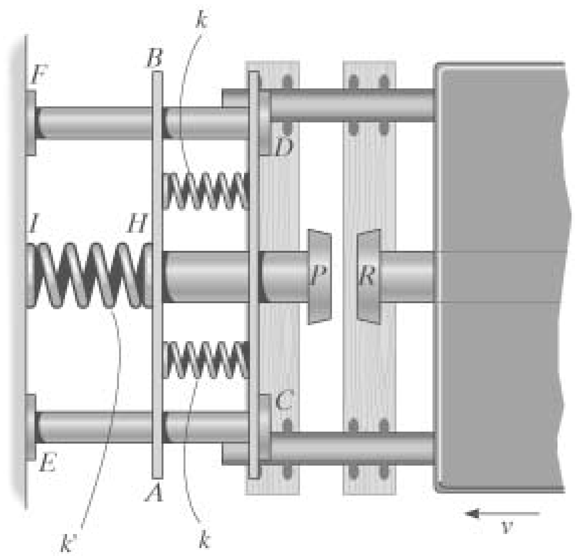
$$t = \int_{v_0}^{v_1} \frac{1}{a(v)}\, dv \qquad t = 43.1\ \text{s}$$

$$d = \int_{v_0}^{0} \frac{v}{a(v)}\, dv \qquad d = 186.4\ \text{ft}$$



Problem 13-31

The spring mechanism is used as a shock absorber for railroad cars. Determine the maximum compression of spring *HI* if the fixed bumper *R* of a railroad car of mass *M*, rolling freely at speed *v* strikes the plate *P*. Bar *AB* slides along the guide paths *CE* and *DF*. The ends of all springs are attached to their respective members and are originally unstretched.



Units Used:

$$\text{kN} = 10^3\ \text{N} \quad \text{Mg} = 10^3\ \text{kg}$$

Given:

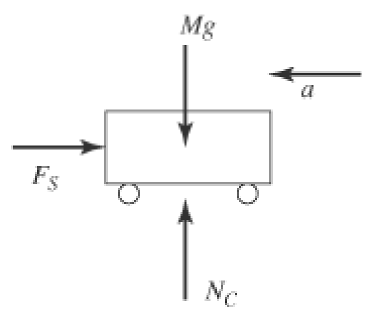
$$M = 5\ \text{Mg} \qquad k = 80\ \frac{\text{kN}}{\text{m}}$$

$$v = 2\ \frac{\text{m}}{\text{s}} \qquad k' = 160\ \frac{\text{kN}}{\text{m}}$$

Solution:

The springs stretch or compress an equal amount *x*. Thus,

$$(k' + 2k)x = -M a \qquad a = -\frac{k' + 2k}{M} x = v\frac{d}{dx}v$$



Guess $d = 1\ \text{m}$ Given $\int_{v}^{0} v\, dv = -\int_{0}^{d} \left(\frac{k' + 2k}{M}\right) x\, dx$ $d = \text{Find}(d)$

$$d = 0.250\ \text{m}$$

***Problem 13-32**

The collar *C* of mass m_c is free to slide along the smooth shaft *AB*. Determine the acceleration of collar *C* if (a) the shaft is fixed from moving, (b) collar *A*, which is fixed to shaft *AB*, moves downward at constant velocity along the vertical rod, and (c) collar A is subjected to downward acceleration a_A. In all cases, the collar moves in the plane.

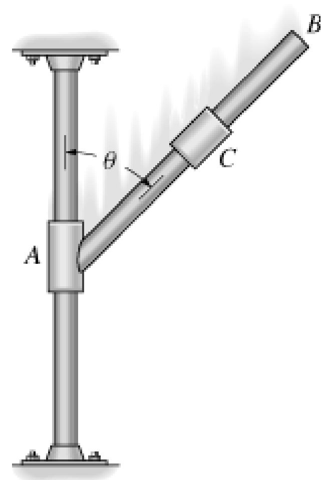
Given:

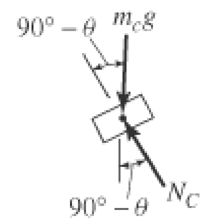
$m_C = 2\ \text{kg}$

$a_A = 2\ \frac{\text{m}}{\text{s}^2}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$\theta = 45\ \text{deg}$





Solution:

(a) $m_C\, g\cos(\theta) = m_C\, a_a$ $\qquad a_a = g\cos(\theta)$ $\qquad a_a = 6.937\ \frac{\text{m}}{\text{s}^2}$

(b) $m_C\, g\cos(\theta) = m_C\, a_b$ $\qquad a_b = g\cos(\theta)$ $\qquad a_b = 6.937\ \frac{\text{m}}{\text{s}^2}$

(c) $m_C(g - a_A)\cos(\theta) = m_C\, a_{crel}$ $\qquad a_{crel} = (g - a_A)\cos(\theta)$

$$\mathbf{a_C} = a_{crel}\begin{pmatrix} -\sin(\theta) \\ -\cos(\theta) \end{pmatrix} + a_A\begin{pmatrix} 0 \\ -1 \end{pmatrix} \qquad \mathbf{a_C} = \begin{pmatrix} -3.905 \\ -5.905 \end{pmatrix}\frac{\text{m}}{\text{s}^2} \qquad |\mathbf{a_C}| = 7.08\ \frac{\text{m}}{\text{s}^2}$$

Problem 13-33

The collar C of mass m_c is free to slide along the smooth shaft AB. Determine the acceleration of collar C if collar A is subjected to an upward acceleration a. The collar moves in the plane.

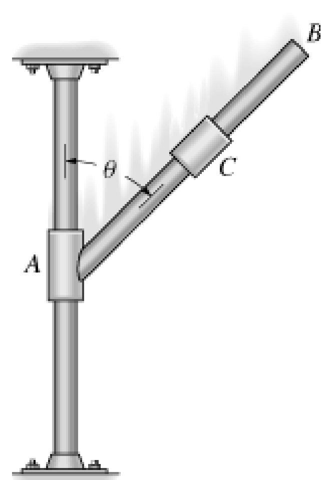
Given:

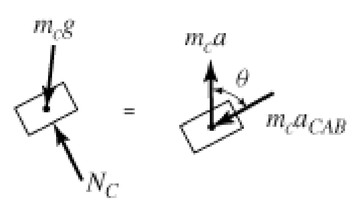
$m_C = 2\ \text{kg}$

$a = 4\ \frac{\text{m}}{\text{s}^2}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$\theta = 45\ \text{deg}$





Solution:

The collar accelerates along the rod and the rod accelerates upward.

$$m_C\, g \cos(\theta) = m_C\left[a_{CA} - (a)\cos(\theta)\right] \qquad a_{CA} = (g + a)\cos(\theta)$$

$$\mathbf{a_C} = \begin{pmatrix} -a_{CA}\sin(\theta) \\ -a_{CA}\cos(\theta) + a \end{pmatrix} \qquad \mathbf{a_C} = \begin{pmatrix} -6.905 \\ -2.905 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_C}\right| = 7.491\ \frac{\text{m}}{\text{s}^2}$$

Problem 13-34

The boy has weight W and hangs uniformly from the bar. Determine the force in each of his arms at time $t = t_1$ if the bar is moving upward with (a) a constant velocity v_0 and (b) a speed $v = bt^2$

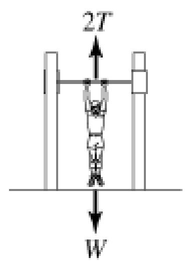
Given:

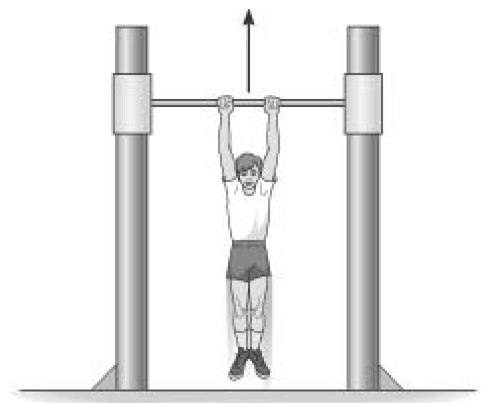
$W = 80\ \text{lb}$

$t_1 = 2\ \text{s}$

$v_0 = 3\ \frac{\text{ft}}{\text{s}}$

$b = 4\ \frac{\text{ft}}{\text{s}^3}$





Solution: (a) $2T_a - W = 0 \qquad T_a = \frac{W}{2} \qquad T_a = 40\,\text{lb}$

(b) $2T_b - W = \left(\frac{W}{g}\right)2b\,t \qquad T_b = \frac{1}{2}\left(W + \frac{W}{g}2b\,t_1\right) \qquad T_b = 59.885\,\text{lb}$

Problem 13-35

The block A of mass m_A rests on the plate B of mass m_B in the position shown. Neglecting the mass of the rope and pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block A to slide a distance s' *on the plate* when the system is released from rest.

Given:

$m_A = 10\ \text{kg}$

$m_B = 50\ \text{kg}$

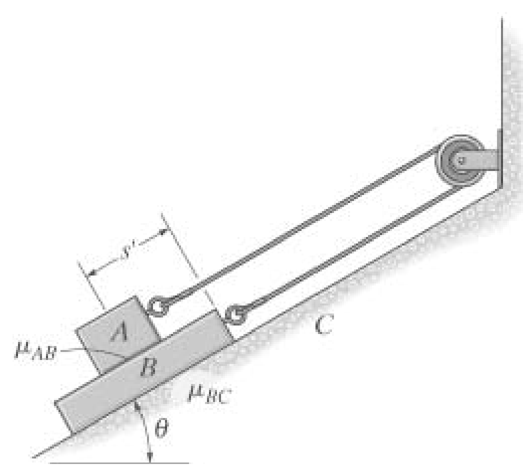
$s' = 0.5\ \text{m}$

$\mu_{AB} = 0.2$

$\mu_{BC} = 0.1$

$\theta = 30\ \text{deg}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

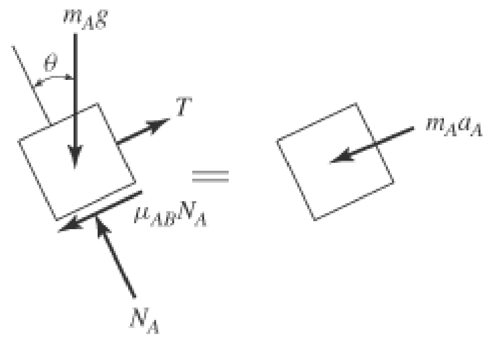
$$s_A + s_B = L$$

Guesses

$$a_A = 1 \frac{\text{m}}{\text{s}^2} \qquad a_B = 1 \frac{\text{m}}{\text{s}^2}$$

$$T = 1 \text{ N} \qquad N_A = 1 \text{ N} \qquad N_B = 1 \text{ N}$$

Given



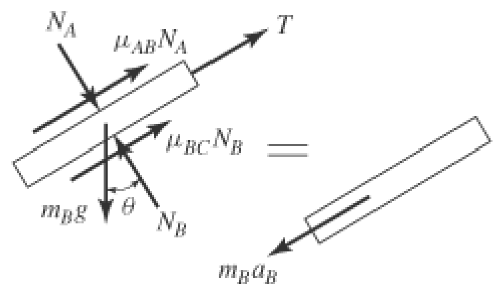
$$a_A + a_B = 0$$

$$N_A - m_A g \cos(\theta) = 0$$

$$N_B - N_A - m_B g \cos(\theta) = 0$$

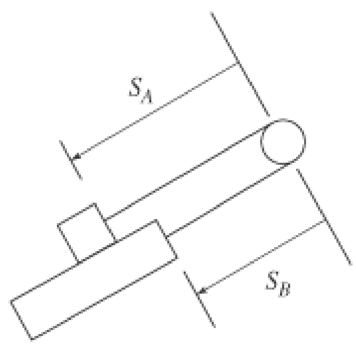
$$T - \mu_{AB} N_A - m_A g \sin(\theta) = -m_A a_A$$

$$T + \mu_{AB} N_A + \mu_{BC} N_B - m_B g \sin(\theta) = -m_B a_B$$



$$\begin{pmatrix} a_A \\ a_B \\ T \\ N_A \\ N_B \end{pmatrix} = \text{Find}(a_A, a_B, T, N_A, N_B) \qquad \begin{pmatrix} T \\ N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 84.58 \\ 84.96 \\ 509.74 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} -1.854 \\ 1.854 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$



$$a_{BA} = a_B - a_A \qquad a_{BA} = 3.708 \frac{\text{m}}{\text{s}^2} \qquad t = \sqrt{\frac{2s'}{a_{BA}}} \qquad t = 0.519 \text{ s}$$

***Problem 13-36**

Determine the acceleration of block *A* when the system is released from rest. The coefficient of kinetic friction and the weight of each block are indicated. Neglect the mass of the pulleys and cord.

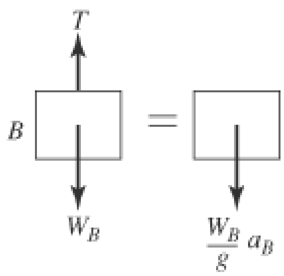
Given:

$$W_A = 80 \text{ lb}$$

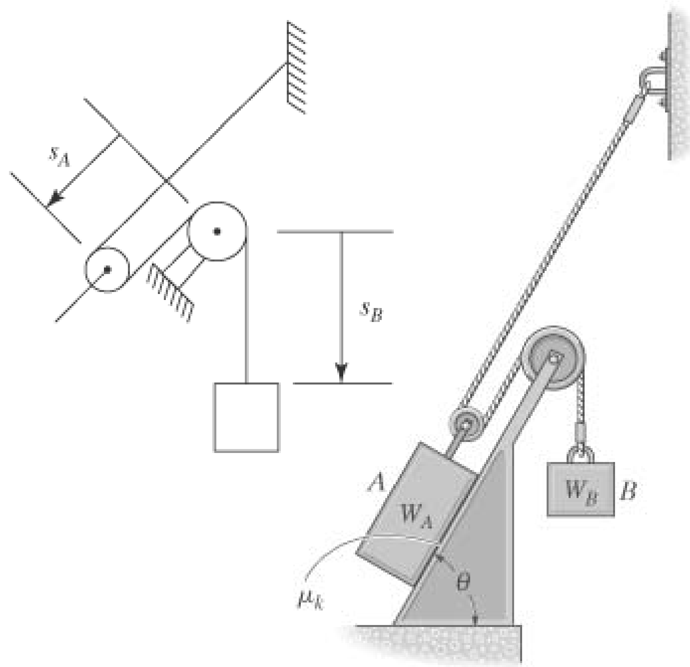
$$W_B = 20 \text{ lb}$$

$$\theta = 60 \text{ deg}$$

$$\mu_k = 0.2$$



$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: $2s_A + s_B = L$

Guesses

$a_A = 1\ \frac{\text{ft}}{\text{s}^2}$ $a_B = 1\ \frac{\text{ft}}{\text{s}^2}$

$T = 1\ \text{lb}$ $N_A = 1\ \text{lb}$

Given

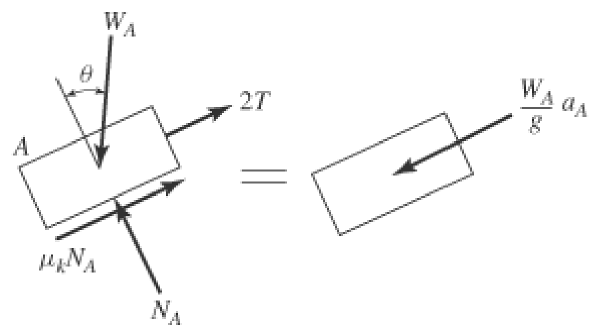
$$2T - W_A \sin(\theta) + \mu_k N_A = \left(\frac{-W_A}{g}\right) a_A$$

$$N_A - W_A \cos(\theta) = 0$$

$$T - W_B = \left(\frac{-W_B}{g}\right) a_B$$

$$2a_A + a_B = 0$$

$$\begin{pmatrix} a_A \\ a_B \\ T \\ N_A \end{pmatrix} = \text{Find}(a_A, a_B, T, N_A) \qquad a_A = 4.28\ \frac{\text{ft}}{\text{s}^2}$$



Problem 13-37

The conveyor belt is moving at speed *v*. If the coefficient of static friction between the conveyor and the package *B* of mass *M* is μ_s, determine the shortest time the belt can stop so that the package does not slide on the belt.

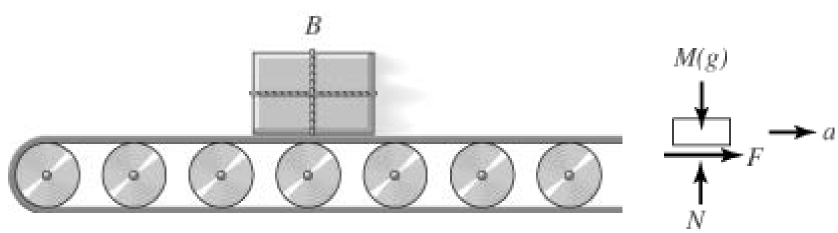
Given:

$v = 4\ \frac{\text{m}}{\text{s}}$

$M = 10\ \text{kg}$

$\mu_s = 0.2$

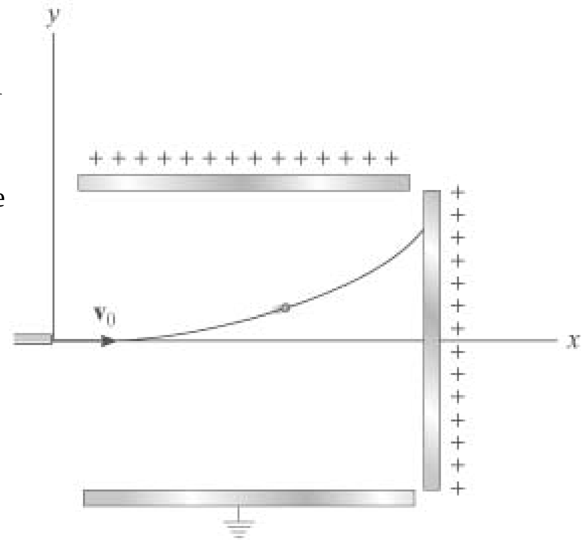
$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: $\mu_s M g = M a \qquad a = \mu_s g \qquad a = 1.962\ \frac{\text{m}}{\text{s}^2} \qquad t = \frac{v}{a} \qquad t = 2.039\ \text{s}$

Problem 13-38

An electron of mass *m* is discharged with an initial horizontal velocity of v_0. If it is subjected to two fields of force for which $F_x = F_0$ and $F_y = 0.3F_0$ where F_0 is constant, determine the equation of the path, and the speed of the electron at any time *t*.



Solution:

$F_0 = m a_x \qquad 0.3F_0 = m a_y$

$a_x = \dfrac{F_0}{m} \qquad a_y = 0.3\left(\dfrac{F_0}{m}\right)$

$v_x = \left(\dfrac{F_0}{m}\right)t + v_0 \qquad v_y = 0.3\left(\dfrac{F_0}{m}\right)t$

$s_x = \dfrac{F_0}{m}\left(\dfrac{t^2}{2}\right) + v_0 t \qquad s_y = 0.3\dfrac{F_0}{m}\left(\dfrac{t^2}{2}\right) \qquad t = \sqrt{\dfrac{20 s_y m}{3F_0}}$

Thus $\qquad s_x = \dfrac{10 s_y}{3} + v_0\sqrt{\dfrac{20 s_y m}{3F_0}}$

$v = \sqrt{v_x^2 + v_y^2} \qquad v = \sqrt{\left(\dfrac{F_0}{m}t + v_0\right)^2 + \left(\dfrac{0.3F_0}{m}t\right)^2}$

***Problem 13-39**

The conveyor belt delivers each crate of mass *M* to the ramp at *A* such that the crate's speed is v_A directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is μ_k, determine the speed at which each crate slides off the ramp at *B*. Assume that no tipping occurs.

Given:

$M = 12\ \text{kg}$

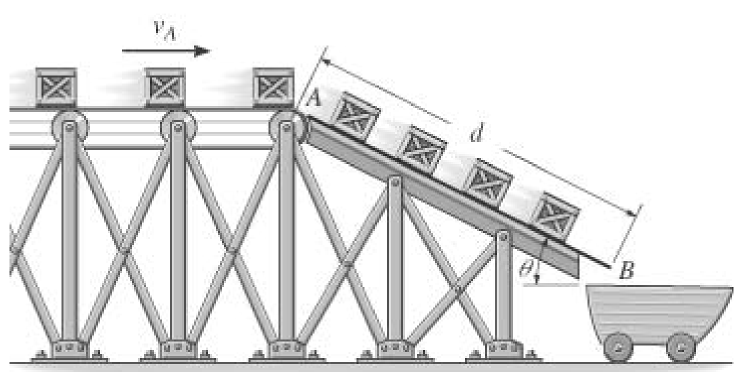
$v_A = 2.5\ \dfrac{\text{m}}{\text{s}}$

$d = 3\ \text{m}$

$\mu_k = 0.3$

$\theta = 30\ \text{deg}$

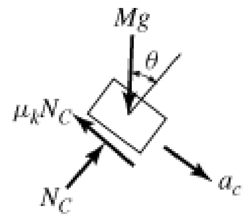
$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

$N_C - M g \cos(\theta) = 0 \qquad N_C = M g \cos(\theta)$

$M g \sin(\theta) - \mu_k N_C = M a \qquad a = g \sin(\theta) - \mu_k \left(\frac{N_C}{M}\right) \qquad a = 2.356 \ \frac{\text{m}}{\text{s}^2}$

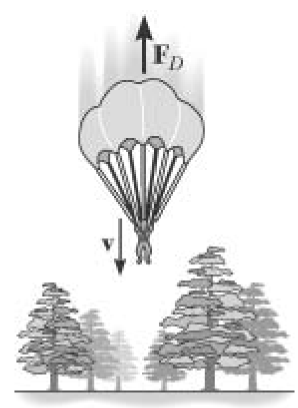


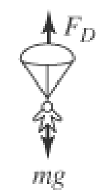
$v_B = \sqrt{v_A^2 + 2 a d} \qquad v_B = 4.515 \ \frac{\text{m}}{\text{s}}$

***Problem 13-40**

A parachutist having a mass *m* opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where *k* is a constant, determine his velocity when he has fallen for a time *t*. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \rightarrow \infty$.

Solution:





$$k v^2 - m g = -m a$$

$$a = g - \left(\frac{k}{m}\right) v^2 = \frac{dv}{dt}$$

$$t = \int_0^v \frac{1}{g - \left(\frac{k}{m}\right) v^2} \, dv$$

$$t = \left(\sqrt{\frac{m}{g k}}\right) \operatorname{atanh}\left(v \sqrt{\frac{k}{g m}}\right)$$

$$v = \left(\sqrt{\frac{mg}{k}}\right) \tanh\left(\sqrt{\frac{g k}{m}} \, t\right) \qquad \text{When } t \rightarrow \infty \qquad v = \sqrt{m \left(\frac{g}{k}\right)}$$

Problem 13-41

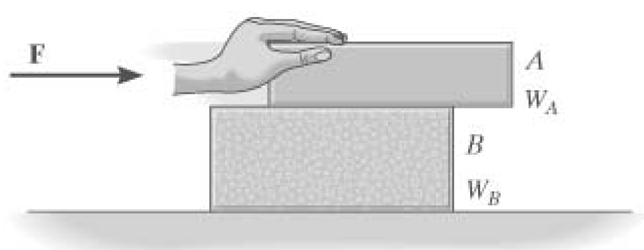
Block *B* rests on a smooth surface. If the coefficients of static and kinetic friction between *A* and *B* are μ_s and μ_k respectively, determine the acceleration of each block if someone pushes horizontally on block *A* with a force of (*a*) $F = F_a$ and (*b*) $F = F_b$.

Given:

$\mu_s = 0.4 \qquad F_a = 6 \text{ lb}$

$\mu_k = 0.3 \qquad F_b = 50 \text{ lb}$

$W_A = 20 \text{ lb} \qquad W_B = 30 \text{ lb}$

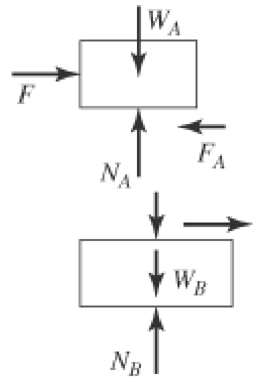


$$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

Solution:

Guesses $F_A = 1\ \text{lb}$ $F_{max} = 1\ \text{lb}$

$a_A = 1\ \frac{\text{ft}}{\text{s}^2}$ $a_B = 1\ \frac{\text{ft}}{\text{s}^2}$



(a) $F = F_a$ First assume no slip

Given $F - F_A = \left(\frac{W_A}{g}\right)a_A$ $F_A = \left(\frac{W_B}{g}\right)a_B$

$a_A = a_B$ $F_{max} = \mu_s W_A$

$$\begin{pmatrix} F_A \\ F_{max} \\ a_A \\ a_B \end{pmatrix} = \text{Find}(F_A, F_{max}, a_A, a_B)$$

If $F_A = 3.599\,\text{lb} < F_{max} = 8\,\text{lb}$ then our assumption is correct and $\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 3.86 \\ 3.86 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$

(b) $F = F_b$ First assume no slip

Given $F - F_A = \left(\frac{W_A}{g}\right)a_A$ $F_A = \left(\frac{W_B}{g}\right)a_B$

$a_A = a_B$ $F_{max} = \mu_s W_A$

$$\begin{pmatrix} F_A \\ F_{max} \\ a_A \\ a_B \end{pmatrix} = \text{Find}(F_A, F_{max}, a_A, a_B)$$

Since $F_A = 30\,\text{lb} > F_{max} = 8\,\text{lb}$ then our assumption is not correct.

Now we know that it slips

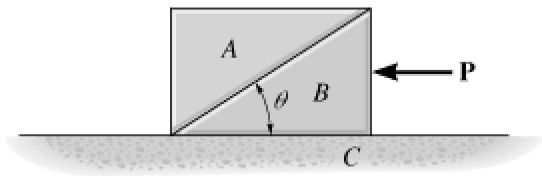
Given $F_A = \mu_k W_A$ $F - F_A = \left(\frac{W_A}{g}\right)a_A$ $F_A = \left(\frac{W_B}{g}\right)a_B$

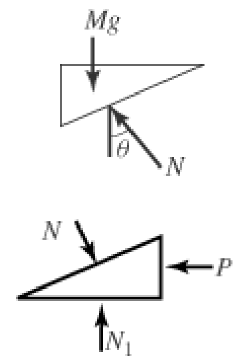
$$\begin{pmatrix} F_A \\ a_A \\ a_B \end{pmatrix} = \text{Find}(F_A, a_A, a_B)$$

$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 70.84 \\ 6.44 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 13-42

Blocks A and B each have a mass M. Determine the largest horizontal force P which can be applied to B so that A will not move relative to B. All surfaces are smooth.





Solution:

Require $a_A = a_B = a$

Block A:

$+\uparrow \ \Sigma F_y = 0;\qquad N\cos(\theta) - Mg = 0$

$\overset{+}{\leftarrow} \ \Sigma F_x = Ma_x;\qquad N\sin(\theta) = Ma$

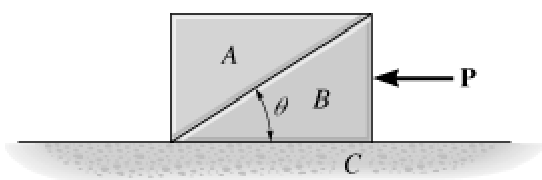
$a = g\tan(\theta)$

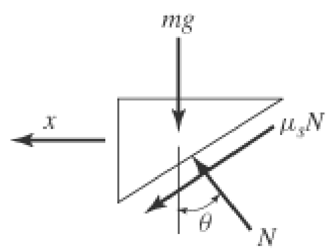
Block B:

$\overset{+}{\leftarrow} \ \Sigma F_x = Ma_x;\qquad P - N\sin(\theta) = Ma \qquad P = 2Mg\tan(\theta)$

Problem 13-43

Blocks A and B each have mass m. Determine the largest horizontal force P which can be applied to B so that A will not slip up B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.





Solution:

Require

$a_A = a_B = a$

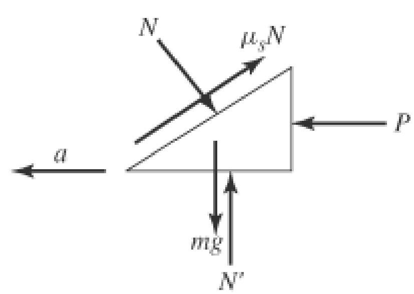
Block A:

$\Sigma F_y = 0;\qquad N\cos(\theta) - \mu_s N\sin(\theta) - mg = 0$

$\Sigma F_x = ma_x;\qquad N\sin(\theta) + \mu_s N\cos(\theta) = ma$

$$N = \frac{mg}{\cos(\theta) - \mu_s\sin(\theta)}$$

$$a = g\left(\frac{\sin(\theta) + \mu_s\cos(\theta)}{\cos(\theta) - \mu_s\sin(\theta)}\right)$$



Block B:

$\Sigma F_x = ma_x;\qquad P - \mu_s N\cos(\theta) - N\sin(\theta) = ma$

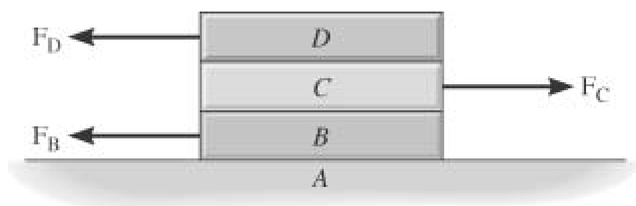
$$P - \frac{\mu_s m g \cos(\theta)}{\cos(\theta) - \mu_s \sin(\theta)} = m g \left(\frac{\sin(\theta) + \mu_s \cos(\theta)}{\cos(\theta) - \mu_s \sin(\theta)} \right)$$

$$p = 2 m g \left(\frac{\sin(\theta) + \mu_s \cos(\theta)}{\cos(\theta) - \mu_s \sin(\theta)} \right)$$

***Problem 13-44**

Each of the three plates has mass M. If the coefficients of static and kinetic friction at each surface of contact are μ_s and μ_k respectively, determine the acceleration of each plate when the three horizontal forces are applied.

Given:



$M = 10 \text{ kg}$

$\mu_s = 0.3$

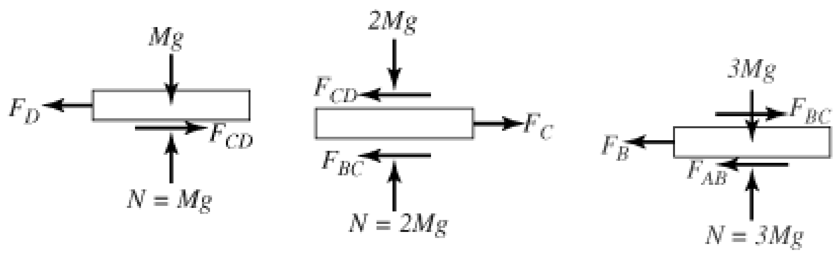
$\mu_k = 0.2$

$F_B = 15 \text{ N}$

$F_C = 100 \text{ N}$

$F_D = 18 \text{ N}$

$g = 9.81 \ \frac{\text{m}}{\text{s}^2}$



Solution:

Case 1: Assume that no slipping occurs anywhere.

$F_{ABmax} = \mu_s(3Mg)$ $\quad F_{BCmax} = \mu_s(2Mg)$ $\quad F_{CDmax} = \mu_s(Mg)$

Guesses $\quad F_{AB} = 1 \text{ N} \quad F_{BC} = 1 \text{ N} \quad F_{CD} = 1 \text{ N}$

Given $\quad -F_D + F_{CD} = 0 \quad F_C - F_{CD} - F_{BC} = 0 \quad -F_B - F_{AB} + F_{BC} = 0$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CD} \end{pmatrix} = \text{Find}(F_{AB}, F_{BC}, F_{CD}) \qquad \begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} 67 \\ 82 \\ 18 \end{pmatrix} \text{N} \qquad \begin{pmatrix} F_{ABmax} \\ F_{BCmax} \\ F_{CDmax} \end{pmatrix} = \begin{pmatrix} 88.29 \\ 58.86 \\ 29.43 \end{pmatrix} \text{N}$$

If $F_{AB} = 67\,\text{N} < F_{ABmax} = 88.29\,\text{N}$ and $F_{BC} = 82\,\text{N} > F_{BCmax} = 58.86\,\text{N}$ and $F_{CD} = 18\,\text{N} < F_{CDmax} = 29.43\,\text{N}$ then nothing moves and there is no acceleration.

Case 2: If $F_{AB} = 67\,\text{N} \;<\; F_{ABmax} = 88.29\,\text{N}$ and $F_{BC} = 82\,\text{N} \;>\; F_{BCmax} = 58.86\,\text{N}$ and $F_{CD} = 18\,\text{N} \;<\; F_{CDmax} = 29.43\,\text{N}$ then slipping occurs between *B* and *C*. We will assume that no slipping occurs at the other 2 surfaces.

Set $\quad F_{BC} = \mu_k(2Mg) \qquad a_B = 0 \qquad a_C = a_D = a$

Guesses $\quad F_{AB} = 1\ \text{N} \qquad F_{CD} = 1\ \text{N} \qquad a = 1\ \frac{\text{m}}{\text{s}^2}$

Given $\quad -F_D + F_{CD} = Ma \qquad F_C - F_{CD} - F_{BC} = Ma \qquad -F_B - F_{AB} + F_{BC} = 0$

$$\begin{pmatrix} F_{AB} \\ F_{CD} \\ a \end{pmatrix} = \text{Find}(F_{AB}, F_{CD}, a) \qquad \begin{pmatrix} F_{AB} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} 24.24 \\ 39.38 \end{pmatrix} \text{N} \qquad a = 2.138\ \frac{\text{m}}{\text{s}^2}$$

$$a_C = a \qquad a_D = a$$

If $F_{AB} = 24.24\,\text{N} \;<\; F_{ABmax} = 88.29\,\text{N}$ and $F_{CD} = 39.38\,\text{N} \;>\; F_{CDmax} = 29.43\,\text{N}$ then we have the correct answer and the accelerations are $a_B = 0$, $a_C = 2.138\,\frac{\text{m}}{\text{s}^2}$, $a_D = 2.138\,\frac{\text{m}}{\text{s}^2}$

Case 3: If $F_{AB} = 24.24\,\text{N} \;<\; F_{ABmax} = 88.29\,\text{N}$ and $F_{CD} = 39.38\,\text{N} \;>\; F_{CDmax} = 29.43\,\text{N}$ then slipping occurs between *C* and *D* as well as between *B* and *C*. We will assume that no slipping occurs at the other surface.

Set $\quad F_{BC} = \mu_k(2Mg) \qquad F_{CD} = \mu_k(Mg)$

Guesses $\quad F_{AB} = 1\ \text{N} \qquad a_C = 1\ \frac{\text{m}}{\text{s}^2} \qquad a_D = 1\ \frac{\text{m}}{\text{s}^2}$

Given $\quad -F_D + F_{CD} = Ma_D \qquad F_C - F_{CD} - F_{BC} = Ma_C \qquad -F_B - F_{AB} + F_{BC} = 0$

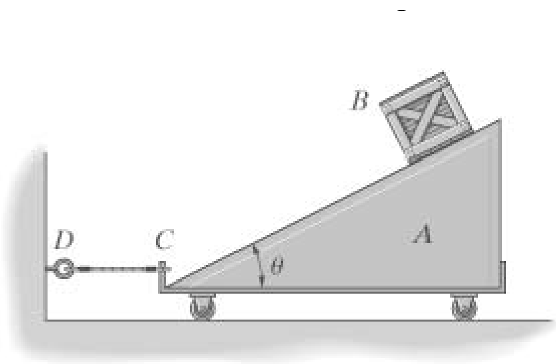
$$\begin{pmatrix} F_{AB} \\ a_C \\ a_D \end{pmatrix} = \text{Find}(F_{AB}, a_C, a_D) \qquad F_{AB} = 24.24\ \text{N} \qquad \begin{pmatrix} a_C \\ a_D \end{pmatrix} = \begin{pmatrix} 4.114 \\ 0.162 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

If $F_{AB} = 24.24\,\text{N} \;<\; F_{ABmax} = 88.29\,\text{N}$ then we have the correct answer and the accelerations are $a_B = 0$, $a_C = 4.114\,\frac{\text{m}}{\text{s}^2}$, $a_D = 0.162\,\frac{\text{m}}{\text{s}^2}$

There are other permutations of this problems depending on the numbers that one chooses.

Problem 13-45

Crate *B* has a mass *m* and is released from rest when it is on top of cart *A*, which has a mass 3*m*. Determine the tension in cord *CD* needed to hold the cart from moving while *B* is sliding down A. Neglect friction.



Solution:

Block B:

$$N_B - m\,g\cos(\theta) = 0$$

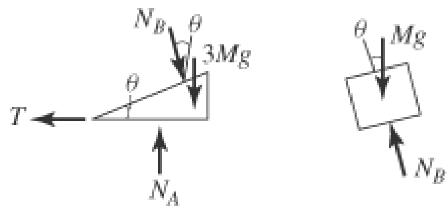
$$N_B = m\,g\cos(\theta)$$

Cart:

$$-T + N_B\sin(\theta) = 0$$

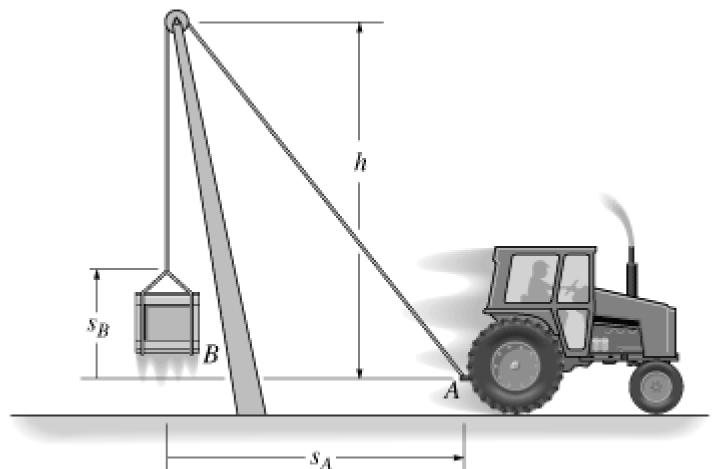
$$T = m\,g\sin(\theta)\cos(\theta)$$

$$T = \left(\frac{mg}{2}\right)\sin(2\theta)$$



Problem 13-46

The tractor is used to lift load B of mass M with the rope of length $2h$, and the boom, and pulley system. If the tractor is traveling to the right at constant speed v, determine the tension in the rope when $s_A = d$. When $s_A = 0$, $s_B = 0$



Units used:

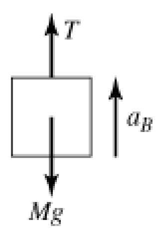
$$\mathrm{kN} = 10^3\ \mathrm{N}$$

Given:

$$M = 150\ \mathrm{kg}$$

$$v = 4\ \frac{\mathrm{m}}{\mathrm{s}} \qquad h = 12\ \mathrm{m}$$

$$d = 5\ \mathrm{m} \qquad g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$$



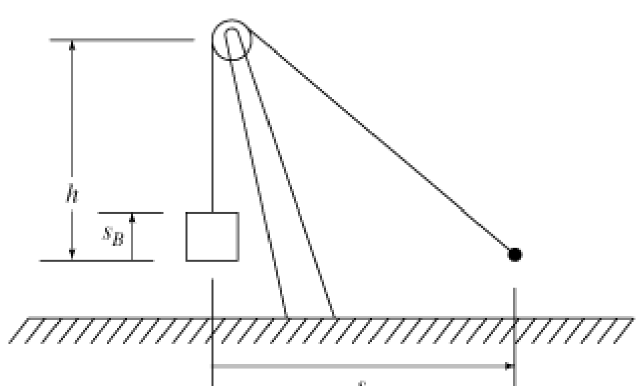
Solution: $\quad v_A = v \qquad s_A = d$

Guesses $\quad T = 1\ \mathrm{kN} \qquad s_B = 1\ \mathrm{m}$

$$a_B = 1\ \frac{\mathrm{m}}{\mathrm{s}^2} \qquad v_B = 1\ \frac{\mathrm{m}}{\mathrm{s}}$$

Given $\quad h - s_B + \sqrt{{s_A}^2 + h^2} = 2h$

$$-v_B + \frac{s_A v_A}{\sqrt{{s_A}^2 + h^2}} = 0$$



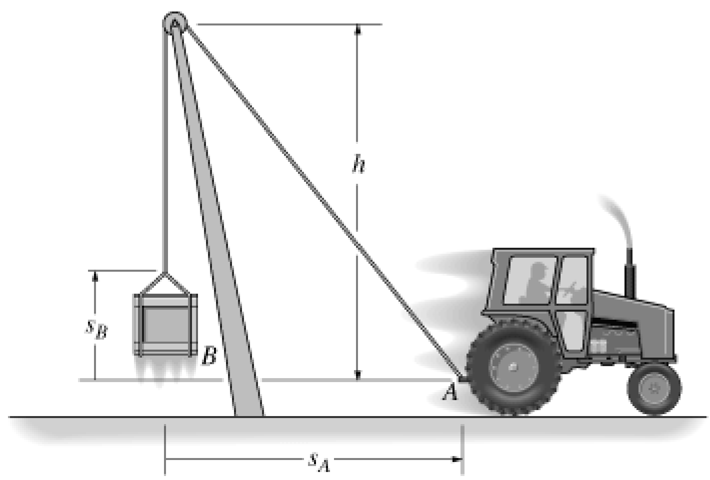
$$-a_B + \frac{v_A^2}{\sqrt{s_A^2 + h^2}} - \frac{s_A^2 v_A^2}{\left(s_A^2 + h^2\right)^{\frac{3}{2}}} = 0 \qquad T - Mg = M a_B$$

$$\begin{pmatrix} T \\ s_B \\ v_B \\ a_B \end{pmatrix} = \text{Find}\left(T, s_B, v_B, a_B\right) \qquad s_B = 1 \text{ m} \qquad a_B = 1.049 \frac{\text{m}}{\text{s}^2} \qquad T = 1.629\,\text{kN}$$

$$v_B = 1.538 \frac{\text{m}}{\text{s}}$$

Problem 13-47

The tractor is used to lift load *B* of mass *M* with the rope of length *2h*, and the boom, and pulley system. If the tractor is traveling to the right with an acceleration *a* and has speed *v* at the instant $s_A = d$, determine the tension in the rope. When $s_A = 0$, $s_B = 0$.



Units used: $\text{kN} = 10^3 \text{ N}$

Given:

$d = 5 \text{ m}$ $\qquad h = 12 \text{ m}$

$M = 150 \text{ kg}$ $\qquad g = 9.81 \frac{\text{m}}{\text{s}^2}$

$v = 4 \frac{\text{m}}{\text{s}}$

$a = 3 \frac{\text{m}}{\text{s}^2}$

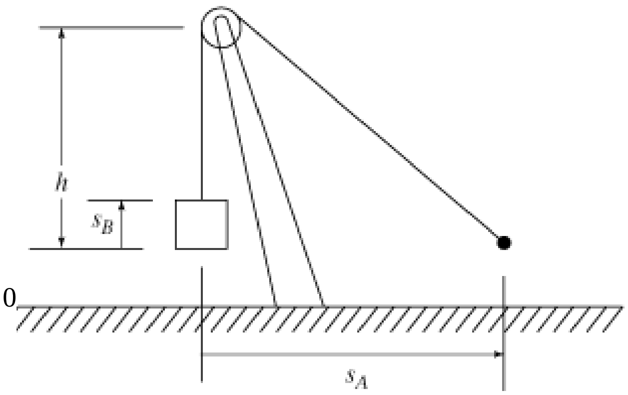
Solution: $\quad a_A = a \qquad v_A = v \qquad s_A = d$

Guesses $\quad T = 1 \text{ kN} \qquad s_B = 1 \text{ m} \qquad a_B = 1 \frac{\text{m}}{\text{s}^2} \qquad v_B = 1 \frac{\text{m}}{\text{s}}$

Given $h - s_B + \sqrt{s_A^2 + h^2} = 2h$

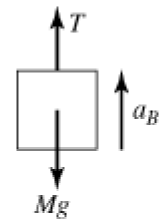
$$-v_B + \frac{s_A v_A}{\sqrt{s_A^2 + h^2}} = 0$$

$$-a_B + \frac{v_A^2 + s_A a_A}{\sqrt{s_A^2 + h^2}} - \frac{s_A^2 v_A^2}{\left(s_A^2 + h^2\right)^{\frac{3}{2}}} = 0$$



$$T - Mg = M a_B$$

$$\begin{pmatrix} T \\ s_B \\ v_B \\ a_B \end{pmatrix} = \text{Find}\left(T, s_B, v_B, a_B\right) \qquad s_B = 1\ \text{m}$$



$a_B = 2.203\ \frac{\text{m}}{\text{s}^2}$ $\qquad v_B = 1.538\ \frac{\text{m}}{\text{s}}$ $\qquad T = 1.802\ \text{kN}$

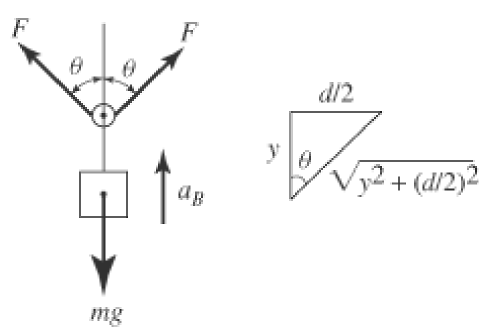
***Problem 13-48**

Block B has a mass m and is hoisted using the cord and pulley system shown. Determine the magnitude of force **F** as a function of the block's vertical position y so that when **F** is applied the block rises with a constant acceleration a_B. Neglect the mass of the cord and pulleys.

Solution:

$$2F\cos(\theta) - mg = m a_B$$

where $\cos(\theta) = \dfrac{y}{\sqrt{y^2 + \left(\dfrac{d}{2}\right)^2}}$



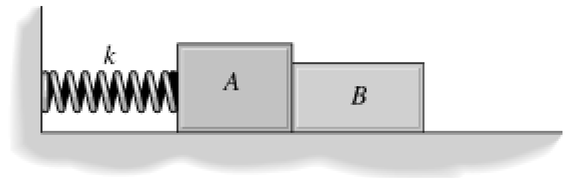
$$2F\left[\frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}\right] - mg = m a_B$$

$$F = m\frac{\left(a_B + g\right)\sqrt{4y^2 + d^2}}{4y}$$

Problem 13-49

Block A has mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having mass m_B is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?

Solution:



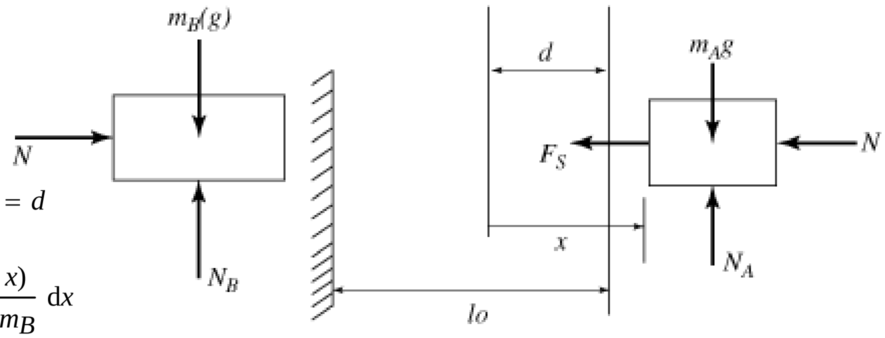
Block A: $\quad -k(x-d) - N = m_A a_A$

Block B: $\quad N = m_B a_B$

Since $a_A = a_B = a$,

$$a = \frac{k(d-x)}{m_A + m_B}$$

$$N = \frac{k m_B (d-x)}{m_A + m_B}$$



Separation occurs when

$N = 0 \quad$ or $\quad x = d$

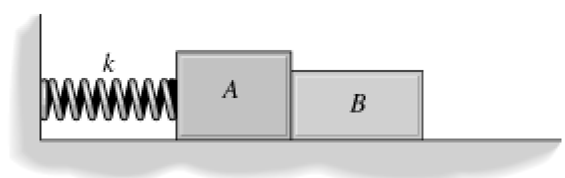
$$\int_0^v v\,dv = \int_0^d \frac{k(d-x)}{m_A + m_B}\,dx$$

$$\frac{v^2}{2} = \frac{k}{m_A + m_B}\left(d\,d - \frac{d^2}{2}\right) \qquad \boxed{v = \sqrt{\frac{kd^2}{m_A + m_B}}}$$

Problem 13-50

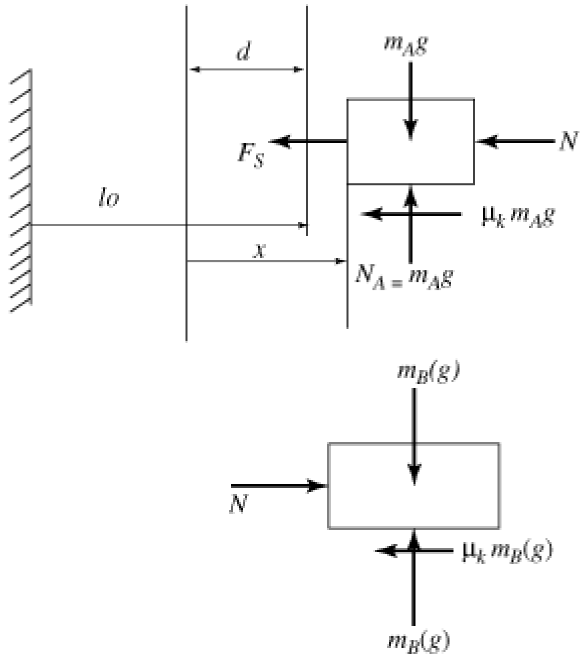
Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A+m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?

Solution: Block A:



$$-k(x-d) - N - \mu_k m_A g = m_A a_A$$

Block B: $\quad N - \mu_k m_B g = m_B a_B$



Since $a_A = a_B = a$

$$a = \frac{k(d-x)}{m_A + m_B} - \mu_k g$$

$$N = \frac{k m_B(d-x)}{m_A + m_B}$$

$N = 0$, then $x = d$ for separation.

At the moment of separation:

$$\int_0^v v\, dv = \int_0^d \left[\frac{k(d-x)}{m_A + m_B} dx - \mu_k g\right] dx$$

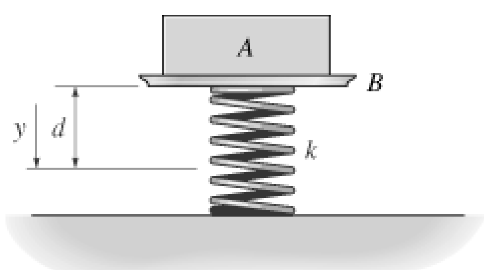
$$v = \sqrt{\frac{kd^2 - 2\mu_k g(m_A + m_B)d}{m_A + m_B}}$$

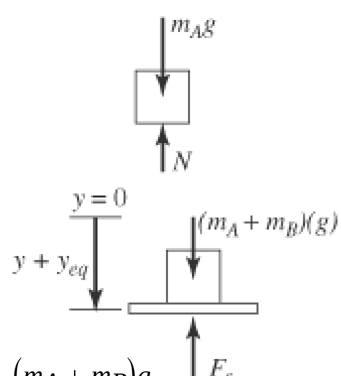
Require $v > 0$, so that

$$kd^2 - 2\mu_k g(m_A + m_B)d > 0 \qquad d > \frac{2\mu_k g}{k}(m_A + m_B) \qquad \text{Q.E.D}$$

Problem 13-51

The block A has mass m_A and rests on the pan B, which has mass m_B Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.





Solution:

For Equilibrium $\quad k\, y_{eq} - (m_A + m_B)g = 0 \qquad y_{eq} = \dfrac{(m_A + m_B)g}{k}$

Block: $\quad -m_A g + N = m_A a$

Block and Pan $\left(-m_A + m_B\right)g + k\left(y_{eq} + y\right) = \left(m_A + m_B\right)a$

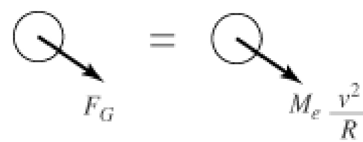
Thus, $-\left(m_A + m_B\right)g + k\left[\left(\frac{m_A + m_B}{k}\right)g + y\right] = \left(m_A + m_B\right)\left(\frac{-m_A g + N}{m_A}\right)$

Set $y = -d$, $N = 0$ Thus $d = y_{eq} = \frac{\left(m_A + m_B\right)g}{k}$

***Problem 13-52**

Determine the mass of the sun, knowing that the distance from the earth to the sun is R. *Hint:* Use Eq. 13-1 to represent the force of gravity acting on the earth.

Given: $R = 149.6 \times 10^6 \text{ km}$ $G = 6.673 \times 10^{-11} \text{ N}\cdot\frac{\text{m}^2}{\text{kg}^2}$

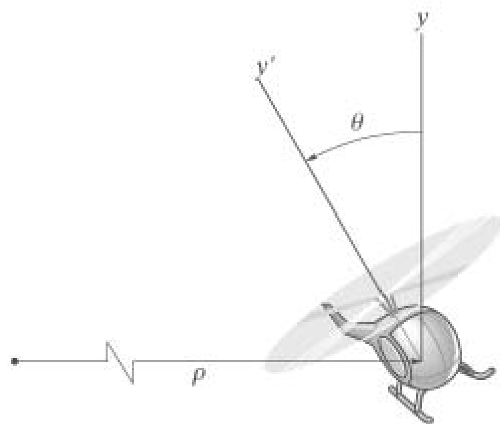


Solution: $v = \frac{s}{t}$ $v = \frac{2\pi R}{1 \text{ yr}}$ $v = 2.98 \times 10^4 \frac{\text{m}}{\text{s}}$

$\Sigma F_n = ma_n;$ $G\left(\frac{M_e M_s}{R^2}\right) = M_e\left(\frac{v^2}{R}\right)$ $M_s = v^2\left(\frac{R}{G}\right)$ $M_s = 1.99 \times 10^{30} \text{ kg}$

Problem 13-53

The helicopter of mass M is traveling at a constant speed v along the horizontal curved path while banking at angle θ. Determine the force acting normal to the blade, i.e., in the y' direction, and the radius of curvature of the path.



Units Used:

$$kN = 10^3\ N$$

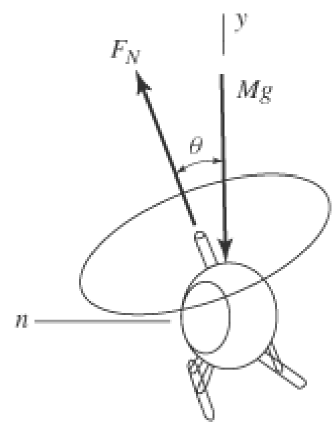
Given:

$$v = 40\ \frac{m}{s} \qquad M = 1.4 \times 10^3\ kg$$

$$\theta = 30\ deg \qquad g = 9.81\ \frac{m}{s^2}$$

Solution:

Guesses $\quad F_N = 1\ kN \qquad \rho = 1\ m$



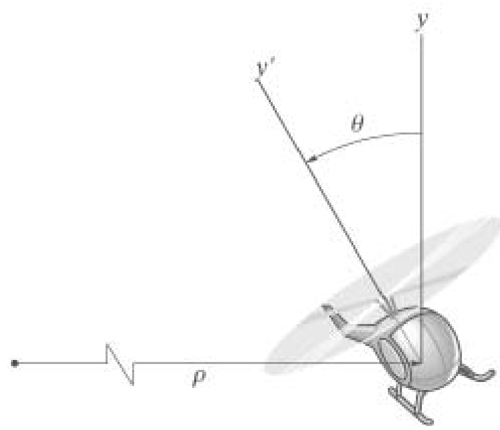
Given $\quad F_N \cos(\theta) - M g = 0$

$$F_N \sin(\theta) = M\left(\frac{v^2}{\rho}\right)$$

$$\begin{pmatrix} F_N \\ \rho \end{pmatrix} = \text{Find}(F_N, \rho) \qquad F_N = 15.86\,kN$$

$$\rho = 282\ m$$

Problem 13-54



The helicopter of mass *M* is traveling at a constant speed *v* along the horizontal curved path having a radius of curvature ρ. Determine the force the blade exerts on the frame and the bank angle θ.

Units Used:

$$kN = 10^3\ N$$

Given:

$$v = 33\ \frac{m}{s} \qquad M = 1.4 \times 10^3\ kg$$

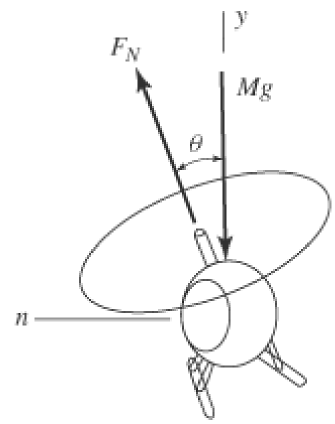
$$\rho = 300\ m \qquad g = 9.81\ \frac{m}{s^2}$$

Solution:

Guesses $F_N = 1\ \text{kN}$ $\theta = 1\ \text{deg}$

Given $F_N \cos(\theta) - Mg = 0$

$$F_N \sin(\theta) = M\left(\frac{v^2}{\rho}\right)$$



$$\begin{pmatrix} F_N \\ \theta \end{pmatrix} = \text{Find}(F_N, \theta) \qquad F_N = 14.64\,\text{kN}$$

$$\theta = 20\,\text{deg}$$

Problem 13-55

The plane is traveling at a constant speed v along the curve $y = bx^2 + c$. If the pilot has weight W, determine the normal and tangential components of the force the seat exerts on the pilot when the plane is at its lowest point.

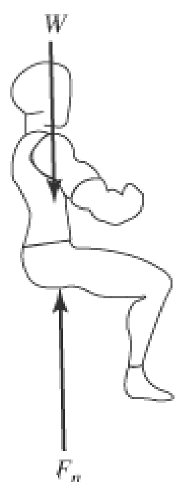
Given:

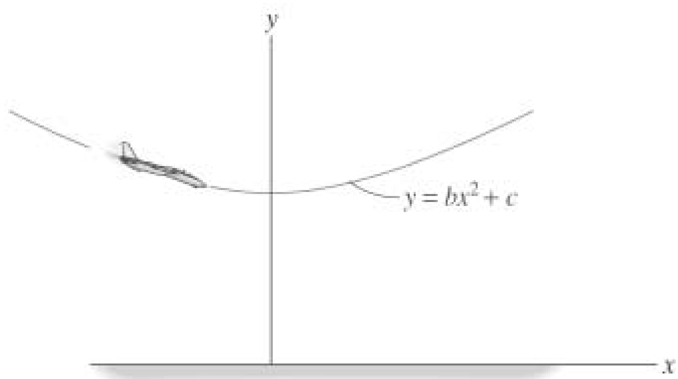
$$b = 20 \times 10^{-6}\ \frac{1}{\text{ft}}$$

$$c = 5000\ \text{ft}$$

$$W = 180\ \text{lb}$$

$$v = 800\ \frac{\text{ft}}{\text{s}}$$





Solution:

$x = 0\ \text{ft}$ $y = bx^2 + c$

$y' = 2bx$ $y'' = 2b$

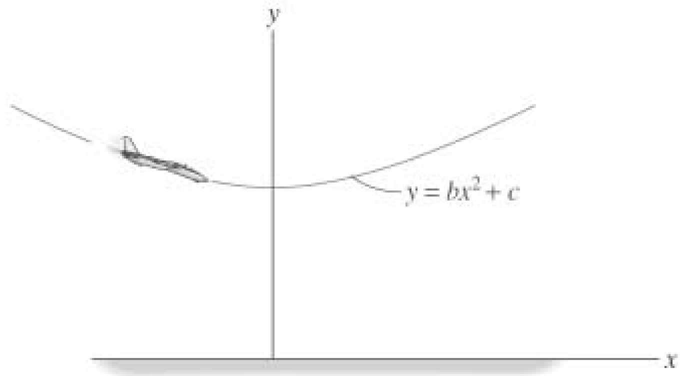
$$\rho = \frac{\sqrt{\left(1 + y'^2\right)^3}}{y''}$$

$$F_n - W = \frac{W}{g}\left(\frac{v^2}{\rho}\right) \qquad F_n = W + \frac{W}{g}\left(\frac{v^2}{\rho}\right) \qquad F_n = 323\,\text{lb}$$

$$a_t = 0 \qquad F_t = \left(\frac{W}{g}\right)a_t \qquad F_t = 0$$

***Problem 13-56**

The jet plane is traveling at a constant speed of v along the curve $y = bx^2 + c$. If the pilot has a weight W, determine the normal and tangential components of the force the seat exerts on the pilot when $y = y_1$.

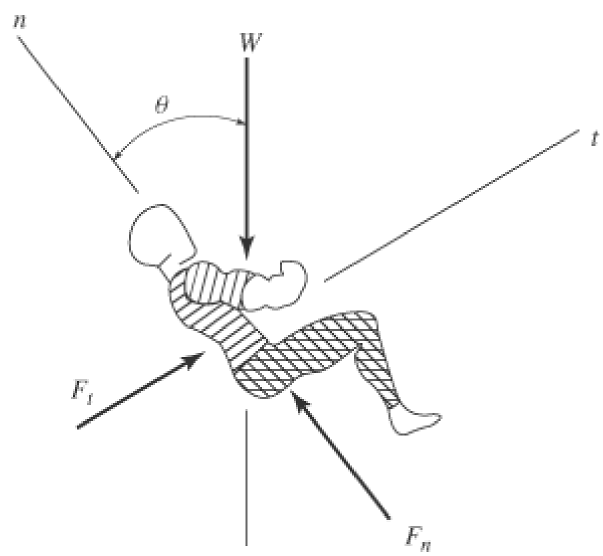


Given:

$b = 20 \times 10^{-6}\ \frac{1}{\text{ft}}$ $\quad W = 180\ \text{lb}$

$c = 5000\ \text{ft}$ $\quad v = 1000\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $\quad y_1 = 10000\ \text{ft}$



Solution:

$y(x) = bx^2 + c$

$y'(x) = 2bx$

$y''(x) = 2b$

$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

Guesses $\quad x_1 = 1\ \text{ft} \quad F_n = 1\ \text{lb}$

$\theta = 1\ \text{deg} \quad F_t = 1\ \text{lb}$

Given $\quad y_1 = y(x_1) \qquad \tan(\theta) = y'(x_1)$

$$F_n - W\cos(\theta) = \frac{W}{g}\left(\frac{v^2}{\rho(x_1)}\right) \qquad F_t - W\sin(\theta) = 0$$

$$\begin{pmatrix} x_1 \\ \theta \\ F_n \\ F_t \end{pmatrix} = \text{Find}(x_1, \theta, F_n, F_t) \qquad x_1 = 15811\,\text{ft} \qquad \theta = 32.3\,\text{deg} \qquad \begin{pmatrix} F_n \\ F_t \end{pmatrix} = \begin{pmatrix} 287.1 \\ 96.2 \end{pmatrix}\text{lb}$$

Problem 13-57

The wrecking ball of mass M is suspended from the crane by a cable having a negligible mass. If the ball has speed v at the instant it is at its lowest point θ, determine the tension in the cable at this instant. Also, determine the angle θ to which the ball swings before it stops.

Units Used:

$$\mathrm{kN} = 10^3\ \mathrm{N}$$

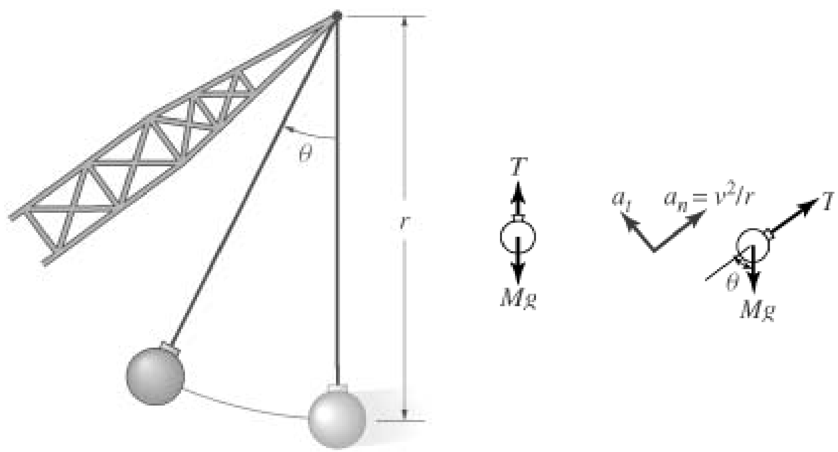
Given:

$$M = 600\ \mathrm{kg}$$

$$v = 8\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$r = 12\ \mathrm{m}$$

$$g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$$



Solution:

At the lowest point

$$T - Mg = M\left(\frac{v^2}{r}\right) \qquad T = Mg + M\left(\frac{v^2}{r}\right) \qquad T = 9.086\ \mathrm{kN}$$

At some arbitrary angle

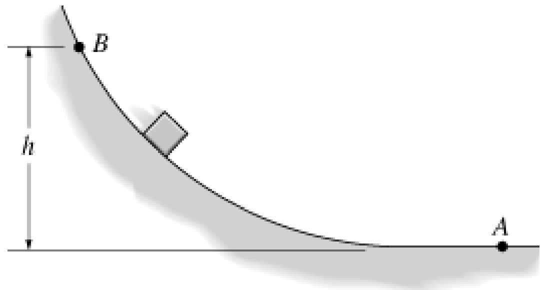
$$-Mg\sin(\theta) = M a_t \qquad a_t = -g\sin(\theta) = \frac{v}{r}\left(\frac{dv}{d\theta}\right)$$

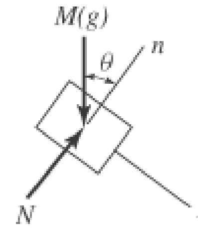
$$\int_v^0 v\ dv = -\int_0^\theta rg\sin(\theta)\ d\theta$$

$$\frac{-v^2}{2} = rg(\cos(\theta) - 1) \qquad \theta = \operatorname{acos}\left(1 - \frac{v^2}{2rg}\right) \qquad \theta = 43.3\ \mathrm{deg}$$

Problem 13-58

Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.





Solution:

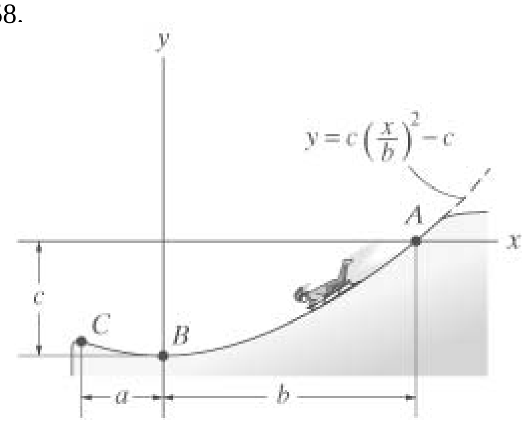
$\searrow^{+}\ \Sigma F_t = m a_t;\qquad (m\,g)\sin(\theta) = m\,a_t \qquad a_t = g\sin(\theta)$

$v\,\mathrm{d}v = a_t \mathrm{d}s = g\sin(\theta)\,\mathrm{d}s$ However $\mathrm{d}y = \mathrm{d}s\ \sin(\theta)$ $\quad \mathrm{d}y = \mathrm{d}s\ \sin(\theta)$

$$\int_0^v v\ \mathrm{d}v = \int_0^h g\ \mathrm{d}y \qquad \frac{v^2}{2} = gh \qquad v = \sqrt{2gh} \qquad \text{Q.E.D}$$

Problem 13-59

The sled and rider have a total mass M and start from rest at $A(b, 0)$. If the sled descends the smooth slope, which may be approximated by a parabola, determine the normal force that the ground exerts on the sled at the instant it arrives at point B. Neglect the size of the sled and rider. *Hint:* Use the result of Prob. 13–58.



Units Used:

$\mathrm{kN} = 10^3\ \mathrm{N}$

Given:

$a = 2\ \mathrm{m} \qquad b = 10\ \mathrm{m} \qquad c = 5\ \mathrm{m}$

$M = 80\ \mathrm{kg} \qquad g = 9.81\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$

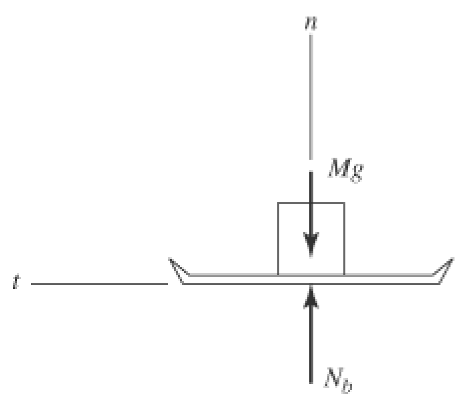
Solution:

$v = \sqrt{2g\,c}$

$$y(x) = c\left(\frac{x}{b}\right)^2 - c$$

$$y'(x) = \left(\frac{2c}{b^2}\right)x \qquad y''(x) = \frac{2c}{b^2}$$

$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$



$$N_b - M g = M\left(\frac{v^2}{\rho}\right)$$

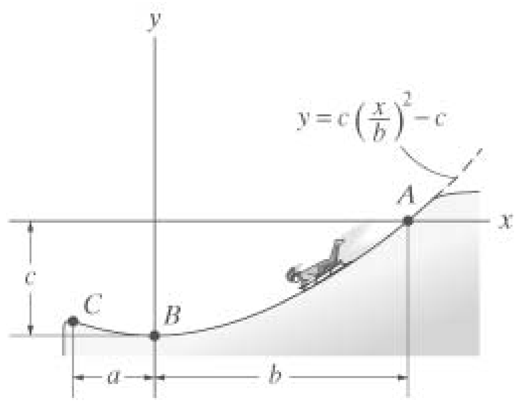
$$N_b = M g + M\left(\frac{v^2}{\rho(0\ \text{m})}\right) \qquad N_b = 1.57\,\text{kN}$$

***Problem 13-60**

The sled and rider have a total mass M and start from rest at $A(b, 0)$. If the sled descends the smooth slope which may be approximated by a parabola, determine the normal force that the ground exerts on the sled at the instant it arrives at point C. Neglect the size of the sled and rider. *Hint:* Use the result of Prob. 13–58.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$



Given:

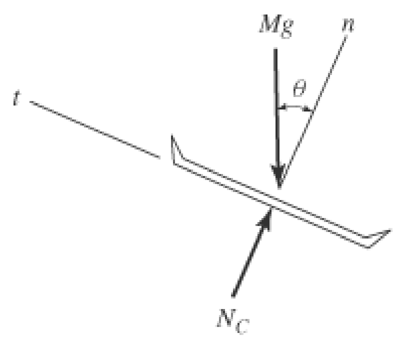
$a = 2\ \text{m} \qquad b = 10\ \text{m} \qquad c = 5\ \text{m}$

$M = 80\ \text{kg} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$$y(x) = c\left(\frac{x}{b}\right)^2 - c$$

$$y'(x) = \left(\frac{2c}{b^2}\right)x \qquad y''(x) = \frac{2c}{b^2}$$



$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$v = \sqrt{2g(-y(-a))}$$

$$\theta = \text{atan}(y'(-a))$$

$$N_C - M g\cos(\theta) = M\left(\frac{v^2}{\rho}\right) \qquad N_C = M\left(g\cos(\theta) + \frac{v^2}{\rho(-a)}\right) \qquad N_C = 1.48\,\text{kN}$$

Problem 13-61

At the instant $\theta = \theta_1$ the boy's center of mass G has a downward speed v_G. Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant.The boy has a weight W. Neglect his size and the mass of the seat and cords.

Given:

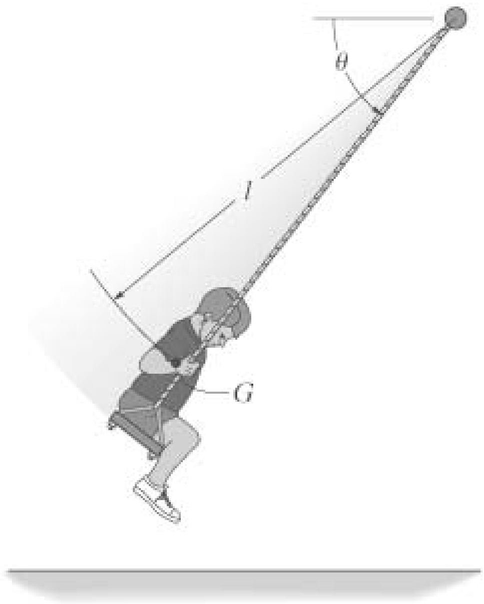
$W = 60\ \text{lb}$

$\theta_1 = 60\ \text{deg}$

$l = 10\ \text{ft}$

$v_G = 15\ \frac{\text{ft}}{\text{s}}$

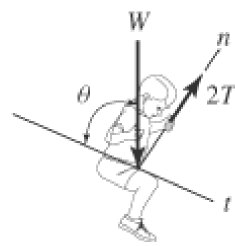
$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$W\cos(\theta_1) = \left(\frac{W}{g}\right)a_t$$

$$a_t = g\cos(\theta_1) \qquad a_t = 16.1\ \frac{\text{ft}}{\text{s}^2}$$

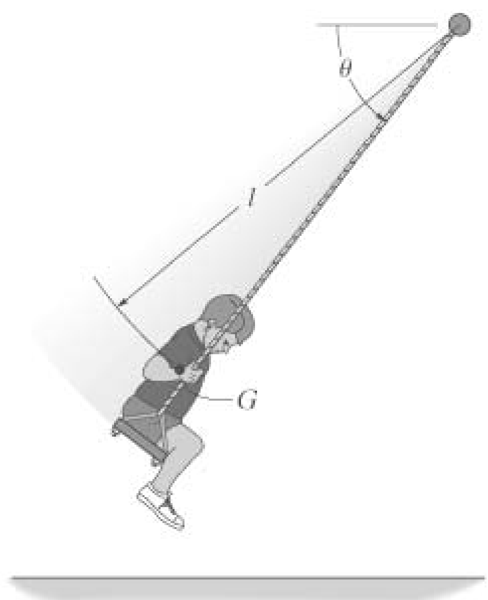


$$2T - W\sin(\theta_1) = \frac{W}{g}\left(\frac{v^2}{l}\right)$$

$$T = \frac{1}{2}\left[\frac{W}{g}\left(\frac{v_G^2}{l}\right) + W\sin(\theta_1)\right] \qquad T = 46.9\ \text{lb}$$

Problem 13-62

At the instant $\theta = \theta_1$ the boy's center of mass G is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = \theta_2$. The boy has a weight W. Neglect his size and the mass of the seat and cords.



Given:

$W = 60\ \text{lb} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$\theta_1 = 60\ \text{deg}$

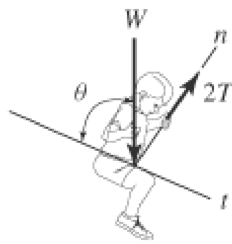
$\theta_2 = 90\ \text{deg}$

$l = 10\ \text{ft}$

Solution:

$$W\cos(\theta) = \left(\frac{W}{g}\right)a_t \qquad a_t = g\cos(\theta)$$

$$v_2 = \sqrt{2gl\int_{\theta_1}^{\theta_2}\cos(\theta)\,d\theta}$$



$$v_2 = 9.29\,\frac{\text{ft}}{\text{s}}$$

$$2T - W\sin(\theta_2) = \frac{W}{g}\left(\frac{v_2^{\,2}}{l}\right)$$

$$T = \frac{W}{2}\left(\sin(\theta_2) + \frac{v_2^{\,2}}{gl}\right) \qquad T = 38.0\,\text{lb}$$

Problem 13-63

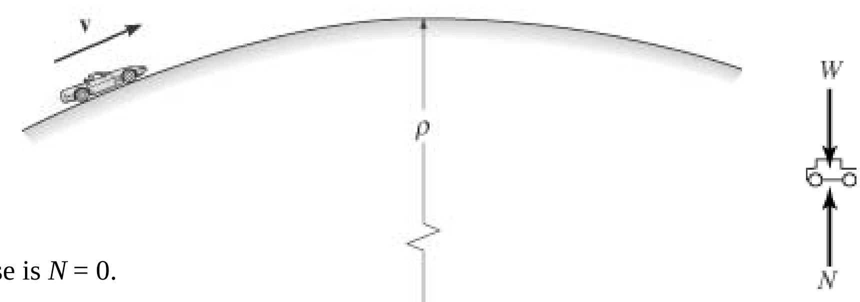
If the crest of the hill has a radius of curvature ρ, determine the maximum constant speed at which the car can travel over it without leaving the surface of the road. Neglect the size of the car in the calculation. The car has weight W.

Given:

$\rho = 200\ \text{ft}$

$W = 3500\ \text{lb}$

$g = 9.815\ \frac{\text{m}}{\text{s}^2}$



Solution: Limiting case is $N = 0$.

$\downarrow \Sigma F_n = ma_n;$ $\qquad W = \frac{W}{g}\left(\frac{v^2}{\rho}\right) \qquad v = \sqrt{g\rho} \qquad v = 80.25\,\frac{\text{ft}}{\text{s}}$

***Problem 13-64**

The airplane, traveling at constant speed v is executing a horizontal turn. If the plane is banked at angle θ when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has mass M?

Units Used:

$\text{kN} = 10^3\ \text{N}$

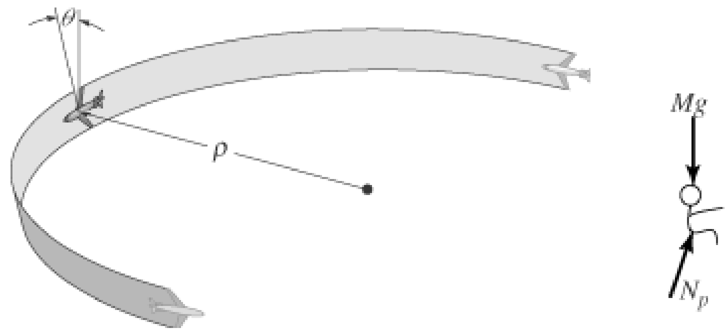
Given:

$v = 50\ \frac{m}{s}$

$\theta = 15\ \text{deg}$

$M = 70\ \text{kg}$

$g = 9.815\ \frac{m}{s^2}$



Solution:

$+\uparrow\ \Sigma F_b = ma_b;\qquad N_p \sin(\theta) - Mg = 0\qquad N_p = M\left(\frac{g}{\sin(\theta)}\right)\qquad N_p = 2.654\ \text{kN}$

$\xleftarrow{+}\ \Sigma F_n = ma_n;\qquad N_P \cos(\theta) = M\left(\frac{v^2}{\rho}\right)\qquad \rho = M\left(\frac{v^2}{N_p \cos(\theta)}\right)\qquad \rho = 68.3\ \text{m}$

Problem 13-65

The man has weight W and lies against the cushion for which the coefficient of static friction is μ_s. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the z axis, he has constant speed v. Neglect the size of the man.

Given:

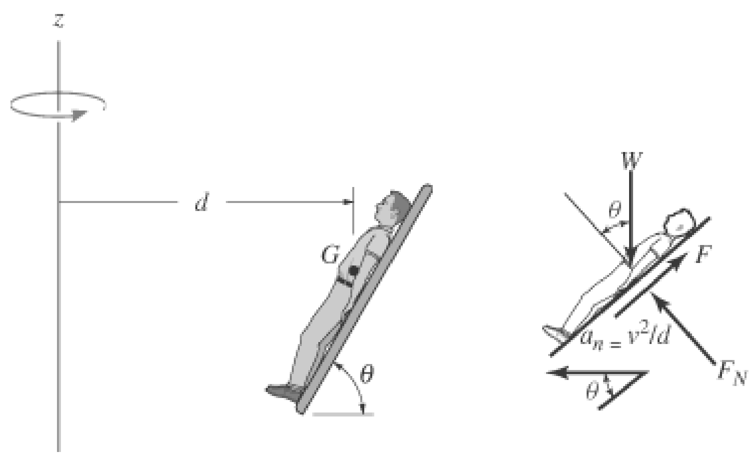
$W = 150\ \text{lb}$

$\mu_s = 0.5$

$v = 20\ \frac{ft}{s}$

$\theta = 60\ \text{deg}$

$d = 8\ \text{ft}$



Solution: Assume no slipping occurs Guesses $F_N = 1\ \text{lb}$ $F = 1\ \text{lb}$

Given $-F_N \sin(\theta) + F\cos(\theta) = \frac{-W}{g}\left(\frac{v^2}{d}\right)\qquad F_N \cos(\theta) - W + F\sin(\theta) = 0$

$\begin{pmatrix} F_N \\ F \end{pmatrix} = \text{Find}(F_N, F)\qquad \begin{pmatrix} F_N \\ F \end{pmatrix} = \begin{pmatrix} 276.714 \\ 13.444 \end{pmatrix}\text{lb}\qquad F_{max} = \mu_s F_N\qquad F_{max} = 138.357\ \text{lb}$

Since $F = 13.444\ \text{lb} < F_{max} = 138.357\ \text{lb}$ then our assumption is correct and there is no slipping.

Problem 13-66

The man has weight *W* and lies against the cushion for which the coefficient of static friction is μ_s. If he rotates about the *z* axis with a constant speed *v*, determine the smallest angle θ of the cushion at which he will begin to slip off.

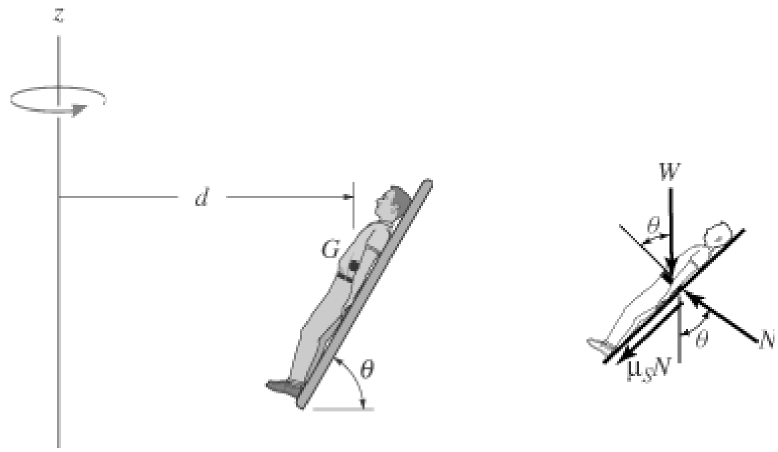
Given:

$W = 150 \text{ lb}$

$\mu_s = 0.5$

$v = 30 \frac{\text{ft}}{\text{s}}$

$d = 8 \text{ ft}$



Solution: Assume verge of slipping Guesses $F_N = 1 \text{ lb}$ $\theta = 20 \text{ deg}$

Given $-F_N \sin(\theta) - \mu_s F_N \cos(\theta) = \frac{-W}{g}\left(\frac{v^2}{d}\right)$ $F_N \cos(\theta) - W - \mu_s F_N \sin(\theta) = 0$

$\begin{pmatrix} F_N \\ \theta \end{pmatrix} = \text{Find}(F_N, \theta)$ $F_N = 487.563 \text{ lb}$ $\theta = 47.463 \text{ deg}$

Problem 13-67

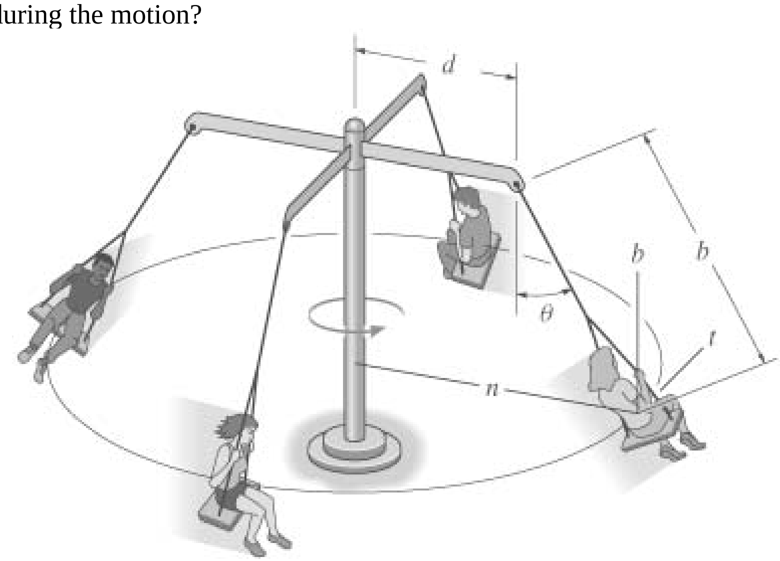
Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at angle *q* from the vertical. Each chair including its passenger has a mass m_c. Also, what are the components of force in the *n*, *t*, and *b* directions which the chair exerts on a passenger of mass m_p during the motion?

Given:

$\theta = 30 \text{ deg}$ $d = 4 \text{ m}$

$m_c = 80 \text{ kg}$ $b = 6 \text{ m}$

$m_p = 50\text{kg}$ $g = 9.81 \frac{\text{m}}{\text{s}^2}$



Solution:

The initial guesses:

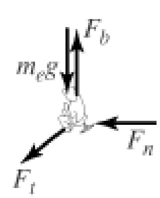
$T = 100 \text{ N}$ $v = 10 \frac{\text{m}}{\text{s}}$

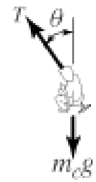
Given

$$T\sin(\theta) = m_c\left(\frac{v^2}{d + b\sin(\theta)}\right)$$

$$T\cos(\theta) - m_c g = 0$$

$$\begin{pmatrix} T \\ v \end{pmatrix} = \text{Find}(T, v) \qquad T = 906.209 \text{ N} \qquad v = 6.30\,\frac{\text{m}}{\text{s}}$$





$\Sigma F_n = ma_n;$ $\quad F_n = \dfrac{m_p v^2}{d + b\sin(\theta)}$ $\quad F_n = 283 \text{ N}$

$\Sigma F_t = ma_t;$ $\quad F_t = 0 \text{ N}$ $\quad F_t = 0$

$\Sigma F_b = ma_b;$ $\quad F_b - m_p g = 0 \quad F_b = m_p g$ $\quad F_b = 491 \text{ N}$

***Problem 13-68**

The snowmobile of mass M with passenger is traveling down the hill at a constant speed v. Determine the resultant normal force and the resultant frictional force exerted on the tracks at the instant it reaches point A. Neglect the size of the snowmobile.

Units Used:

$$\text{kN} = 10^3\text{N}$$

Given:

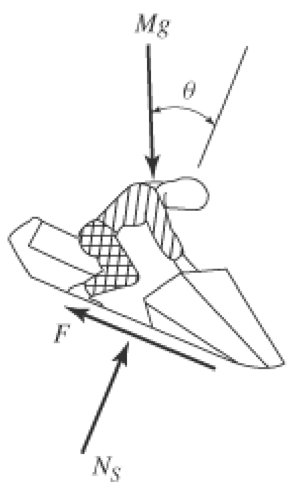
$$M = 200 \text{ kg}$$

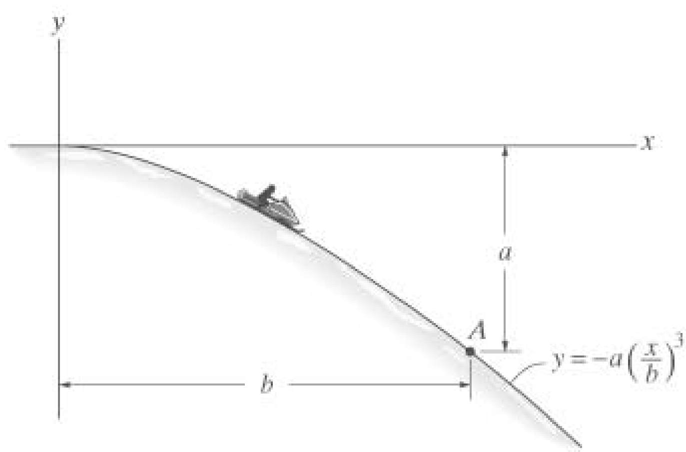
$$v = 6\,\frac{\text{m}}{\text{s}}$$

$$a = 5 \text{ m}$$

$$b = 10 \text{ m}$$

$$g = 9.81\,\frac{\text{m}}{\text{s}^2}$$





Solution:

$$y(x) = -a\left(\frac{x}{b}\right)^3 \qquad y'(x) = -3\left(\frac{a}{b^3}\right)x^2$$

$$y''(x) = -6\left(\frac{a}{b^3}\right)x \qquad \rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$\theta = \operatorname{atan}(y'(b))$

Guesses $N_S = 1\ \text{N}$ $F = 1\ \text{N}$

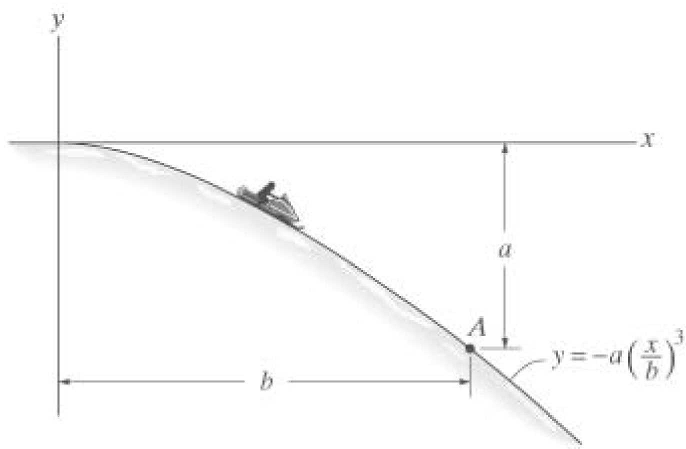
Given $N_S - M g \cos(\theta) = M\left(\dfrac{v^2}{\rho(b)}\right)$

$F - M g \sin(\theta) = 0$

$\begin{pmatrix} N_S \\ F \end{pmatrix} = \text{Find}(N_S, F)$ $\begin{pmatrix} N_S \\ F \end{pmatrix} = \begin{pmatrix} 0.72 \\ -1.632 \end{pmatrix} \text{kN}$

Problem 13-69

The snowmobile of mass M with passenger is traveling down the hill such that when it is at point A, it is traveling at speed v and increasing its speed at v'. Determine the resultant normal force and the resultant frictional force exerted on the tracks at this instant. Neglect the size of the snowmobile.



Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$M = 200\ \text{kg}$ $a = 5\ \text{m}$

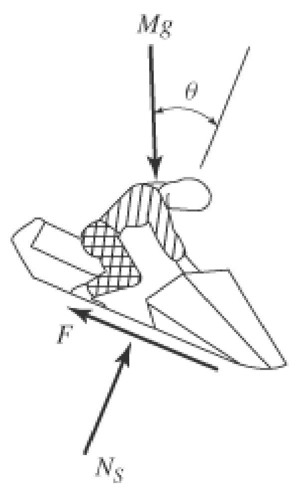
$v = 4\ \dfrac{\text{m}}{\text{s}}$ $b = 10\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$ $v' = 2\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$y(x) = -a\left(\dfrac{x}{b}\right)^3$ $y'(x) = -3\left(\dfrac{a}{b^3}\right)x^2$

$y''(x) = -6\left(\dfrac{a}{b^3}\right)x$ $\rho(x) = \dfrac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$



$\theta = \operatorname{atan}(y'(b))$

Guesses $N_S = 1\ \text{N}$ $F = 1\ \text{N}$

Given $N_S - M g \cos(\theta) = M\dfrac{v^2}{\rho(b)}$

$$F - M g \sin(\theta) = M v'$$

$$\begin{pmatrix} N_S \\ F \end{pmatrix} = \text{Find}(N_S, F) \qquad \begin{pmatrix} N_S \\ F \end{pmatrix} = \begin{pmatrix} 0.924 \\ -1.232 \end{pmatrix} \text{kN}$$

Problem 13-70

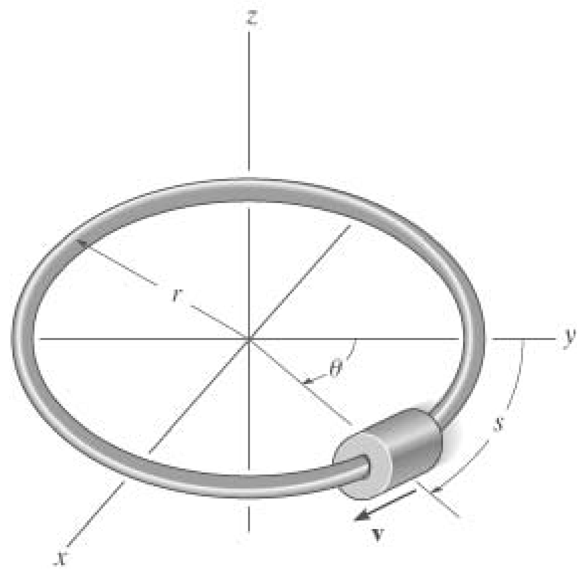
A collar having a mass M and negligible size slides over the surface of a horizontal circular rod for which the coefficient of kinetic friction is μ_k. If the collar is given a speed v_1 and then released at $\theta = 0$ deg, determine how far, d, it slides on the rod before coming to rest.

Given:

$M = 0.75 \text{ kg} \qquad r = 100 \text{ mm}$

$\mu_k = 0.3 \qquad g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

$v_1 = 4 \dfrac{\text{m}}{\text{s}}$



Solution:

$$N_{Cz} - M g = 0$$

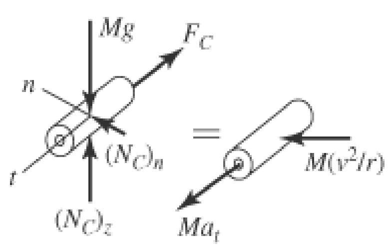
$$N_{Cn} = M\left(\frac{v^2}{r}\right)$$

$$N_C = \sqrt{N_{Cz}^2 + N_{Cn}^2}$$

$$F_C = \mu_k N_C = -M a_t$$

$$a_t(v) = -\mu_k \sqrt{g^2 + \frac{v^4}{r^2}}$$

$$d = \int_{v_1}^{0} \frac{v}{a_t(v)}\, dv \qquad d = 0.581 \text{ m}$$



Problem 13-71

The roller coaster car and passenger have a total weight W and starting from rest at A travel down the track that has the shape shown. Determine the normal force of the tracks on the car when the car is at point B, it has a velocity of v. Neglect friction and the size of the car and passenger.

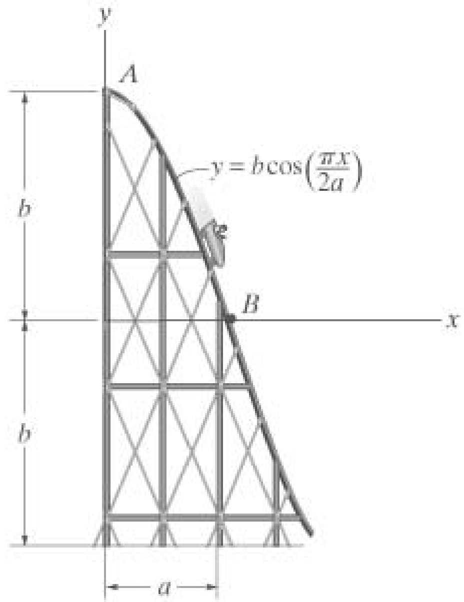
Given:

$W = 600 \text{ lb}$

$v = 15\ \frac{\text{ft}}{\text{s}}$

$a = 20\ \text{ft}$

$b = 40\ \text{ft}$



Solution:

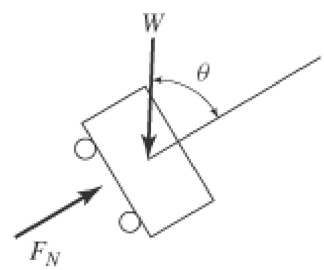
$$y(x) = b\cos\left(\frac{\pi x}{2a}\right) \qquad y'(x) = \frac{d}{dx}y(x)$$

$$y''(x) = \frac{d}{dx}y'(x) \qquad \rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

At B $\quad \theta = \operatorname{atan}(y'(a))$

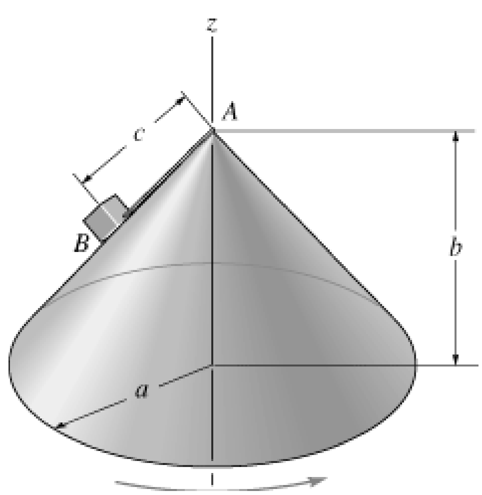
$$F_N - W\cos(\theta) = \frac{W}{g}\left(\frac{v^2}{\rho}\right)$$

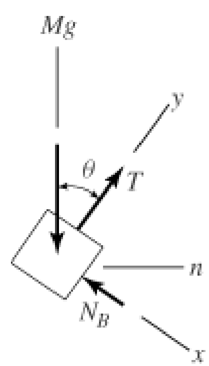
$$F_N = W\cos(\theta) + \frac{W}{g}\left(\frac{v^2}{\rho(a)}\right) \qquad F_N = 182.0\ \text{lb}$$

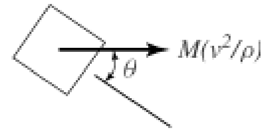


***Problem 13-72**

The smooth block *B*, having mass *M*, is attached to the vertex *A* of the right circular cone using a light cord. The cone is rotating at a constant angular rate about the *z* axis such that the block attains speed *v*. At this speed, determine the tension in the cord and the reaction which the cone exerts on the block. Neglect the size of the block.







Given: $M = 0.2 \text{ kg}$ $v = 0.5 \dfrac{\text{m}}{\text{s}}$ $a = 300 \text{ mm}$ $b = 400 \text{ mm}$

$c = 200 \text{ mm}$ $g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

Solution: Guesses $T = 1 \text{ N}$ $N_B = 1 \text{ N}$

Set $\theta = \operatorname{atan}\left(\dfrac{a}{b}\right)$ $\theta = 36.87 \deg$

$\rho = \left(\dfrac{c}{\sqrt{a^2 + b^2}}\right)a$ $\rho = 120 \text{mm}$

Given $T\sin(\theta) - N_B\cos(\theta) = M\left(\dfrac{v^2}{\rho}\right)$ $T\cos(\theta) + N_B\sin(\theta) - Mg = 0$

$\begin{pmatrix} T \\ N_B \end{pmatrix} = \text{Find}(T, N_B)$ $\begin{pmatrix} T \\ N_B \end{pmatrix} = \begin{pmatrix} 1.82 \\ 0.844 \end{pmatrix} \text{N}$

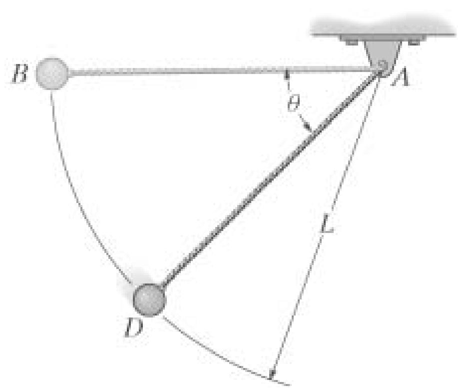
Problem 13-73

The pendulum bob B of mass M is released from rest when $\theta = 0°$. Determine the initial tension in the cord and also at the instant the bob reaches point D, $\theta = \theta_1$. Neglect the size of the bob.

Given:

$M = 5 \text{ kg}$ $\theta_1 = 45 \text{ deg}$

$L = 2 \text{ m}$ $g = 9.81 \dfrac{\text{m}}{\text{s}^2}$



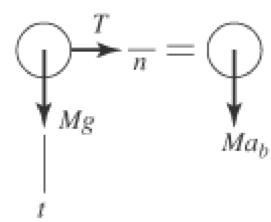
Solution:

Initially, $v = 0$ so $a_n = 0$ $T = 0$

At D we have

$Mg\cos(\theta_1) = Ma_t$

$a_t = g\cos(\theta_1)$ $a_t = 6.937 \dfrac{\text{m}}{\text{s}^2}$



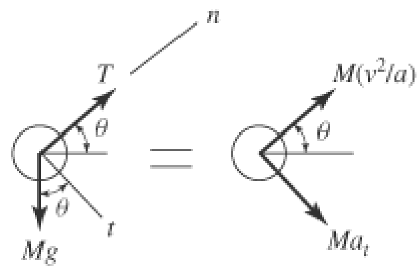
$T_D - Mg\sin(\theta_1) = \dfrac{Mv^2}{L}$

Now find the velocity v

Guess $v = 1 \dfrac{\text{m}}{\text{s}}$

Given $\displaystyle\int_0^v v\,\mathrm{d}v = \int_0^{\theta_1} g\cos(\theta)L\,\mathrm{d}\theta$

$v = \mathrm{Find}(v)$ $\quad v = 5.268\ \frac{\mathrm{m}}{\mathrm{s}}$



$T_D = M g\sin(\theta_1) + M\left(\frac{v^2}{L}\right)$ $\quad T_D = 104.1\ \mathrm{N}$

Problem 13-74

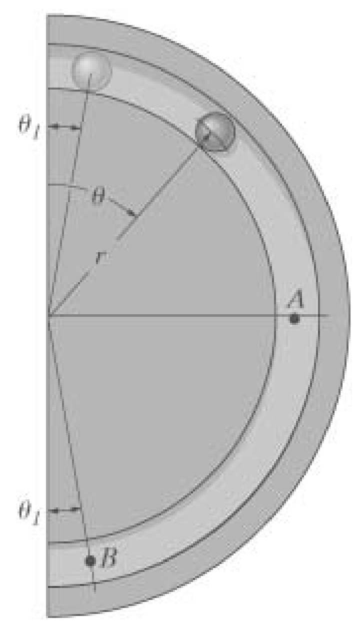
A ball having a mass M and negligible size moves within a smooth vertical circular slot. If it is released from rest at θ_1, determine the force of the slot on the ball when the ball arrives at points A and B.

Given:

$M = 2\ \mathrm{kg}$ $\quad \theta = 90\ \mathrm{deg}$ $\quad \theta_1 = 10\ \mathrm{deg}$ $\quad r = 0.8\ \mathrm{m}$ $\quad g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$

Solution:

$M g\sin(\theta) = M a_t$ $\quad a_t = g\sin(\theta)$



At A $\quad \theta_A = 90\ \mathrm{deg}$

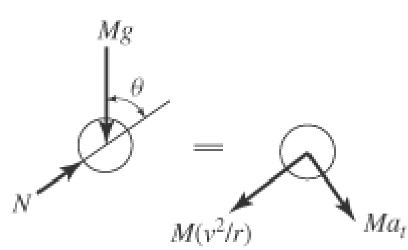
$$v_A = \sqrt{2g\left(\int_{\theta_1}^{\theta_A} \sin(\theta)\,r\,\mathrm{d}\theta\right)}$$

$$N_A - M g\cos(\theta_A) = -M\left(\frac{v_A^2}{r}\right)$$

$$N_A = M g\cos(\theta_A) - M\left(\frac{v_A^2}{r}\right) \qquad N_A = -38.6\ \mathrm{N}$$

At B $\quad \theta_B = 180\ \mathrm{deg} - \theta_1$

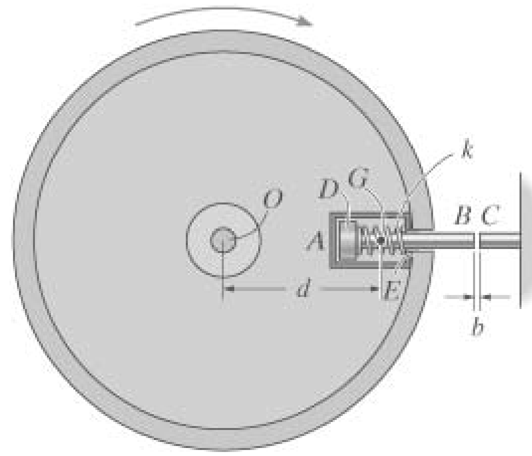
$$v_B = \sqrt{2g\left(\int_{\theta_1}^{\theta_B} \sin(\theta)\,r\,\mathrm{d}\theta\right)}$$



$$N_B - M g\cos(\theta_B) = -M\left(\frac{v_B^2}{r}\right)$$

$$N_B = M g\cos(\theta_B) - M\left(\frac{v_B^2}{r}\right) \qquad N_B = -96.6\ \mathrm{N}$$

Problem 13-75



The rotational speed of the disk is controlled by a smooth contact arm *AB* of mass *M* which is spring-mounted on the disk.When the disk is *at rest*, the center of mass *G* of the arm is located distance *d* from the center *O*, and the preset compression in the spring is *a*. If the initial gap between *B* and the contact at *C* is *b*, determine the (controlling) speed v_G of the arm's mass center, *G*, which will close the gap. The disk rotates in the horizontal plane. The spring has a stiffness *k* and its ends are attached to the contact arm at *D* and to the disk at *E*.

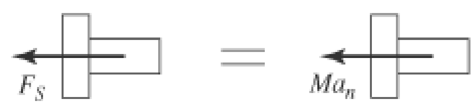
Given:

$M = 30 \text{ gm} \qquad a = 20 \text{ mm} \qquad b = 10 \text{ mm} \qquad d = 150 \text{ mm} \qquad k = 50 \frac{\text{N}}{\text{m}}$

Solution:

$F_S = k(a+b) \qquad F_S = 1.5 \text{ N}$

$a_n = \frac{v_G^2}{d+b} \qquad F_S = M\left(\frac{v_G^2}{d+b}\right)$



$v_G = \frac{1}{M}\sqrt{M\ k\ (a+b)(d+b)} \qquad v_G = 2.83 \frac{\text{m}}{\text{s}}$

***Problem 13-76**

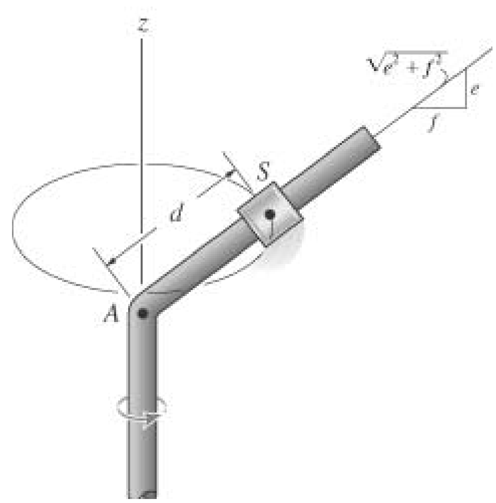
The spool *S* of mass *M* fits loosely on the inclined rod for which the coefficient of static friction is μ_s. If the spool is located a distance *d* from *A*, determine the maximum constant speed the spool can have so that it does not slip up the rod.

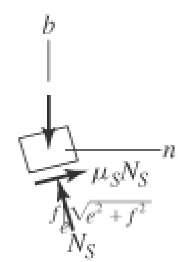
Given:

$M = 2 \text{ kg} \qquad e = 3$

$\mu_S = 0.2 \qquad f = 4$

$d = 0.25 \text{ m} \qquad g = 9.81 \frac{\text{m}}{\text{s}^2}$





Solution:

$\rho = d\left(\frac{f}{\sqrt{e^2+f^2}}\right)$

Guesses $\qquad N_S = 1 \text{ N} \qquad v = 1 \frac{\text{m}}{\text{s}}$

Given $\quad N_s\left(\frac{e}{\sqrt{e^2+f^2}}\right) - \mu_s N_s\left(\frac{f}{\sqrt{e^2+f^2}}\right) = M\left(\frac{v^2}{\rho}\right)$

$$N_s\left(\frac{f}{\sqrt{e^2+f^2}}\right) + \mu_s N_s\left(\frac{e}{\sqrt{e^2+f^2}}\right) - Mg = 0$$

$\begin{pmatrix} N_s \\ v \end{pmatrix} = \text{Find}(N_s, v) \qquad N_s = 21.326\ \text{N} \qquad v = 0.969\ \frac{\text{m}}{\text{s}}$

Problem 13-77

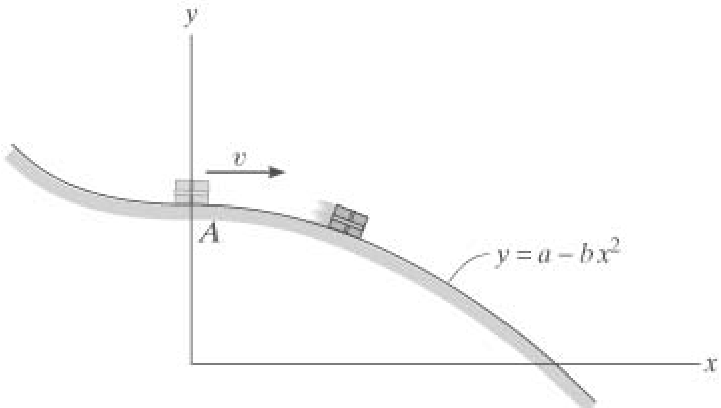
The box of mass M has a speed v_0 when it is at A on the smooth ramp. If the surface is in the shape of a parabola, determine the normal force on the box at the instant $x = x_1$. Also, what is the rate of increase in its speed at this instant?

Given:

$M = 35\ \text{kg} \qquad a = 4\ \text{m}$

$v_0 = 2\ \frac{\text{m}}{\text{s}} \qquad b = \frac{1}{9}\ \frac{1}{\text{m}}$

$x_1 = 3\ \text{m}$



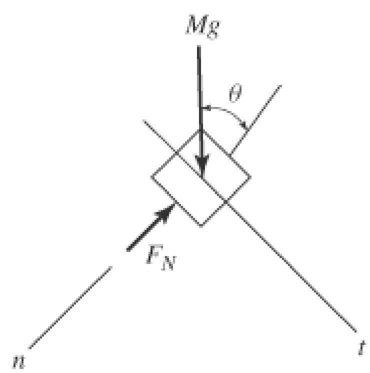
Solution:

$y(x) = a - bx^2$

$y'(x) = -2bx \qquad y''(x) = -2b$

$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$

$\theta(x) = \text{atan}(y'(x))$



Find the velocity

$v_1 = \sqrt{v_0^2 + 2g\left(y(0\ \text{m}) - y(x_1)\right)} \qquad v_1 = 4.86\ \frac{\text{m}}{\text{s}}$

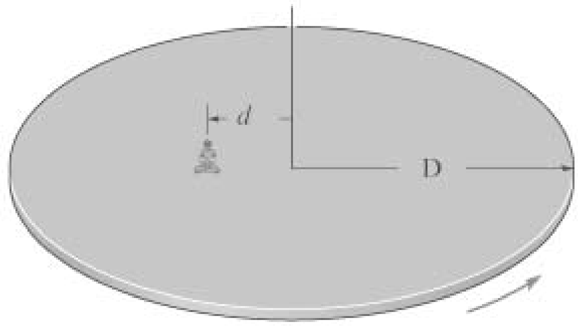
Guesses $\quad F_N = 1\ \text{N} \qquad v' = 1\ \frac{\text{m}}{\text{s}^2}$

Given $\quad F_N - Mg\cos\left(\theta(x_1)\right) = M\left(\frac{v_1^2}{\rho(x_1)}\right) \qquad -Mg\sin\left(\theta(x_1)\right) = Mv'$

$$\begin{pmatrix} F_N \\ v' \end{pmatrix} = \text{Find}(F_N, v') \qquad F_N = 179.9 \text{ N} \qquad v' = 5.442 \frac{\text{m}}{\text{s}^2}$$

Problem 13-78

The man has mass M and sits a distance d from the center of the rotating platform. Due to the rotation his speed is increased from rest by the rate v'. If the coefficient of static friction between his clothes and the platform is μ_s, determine the time required to cause him to slip.

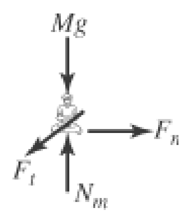


Given:

$M = 80 \text{ kg} \qquad \mu_s = 0.3$

$d = 3 \text{ m} \qquad D = 10 \text{ m}$

$v' = 0.4 \frac{\text{m}}{\text{s}^2} \qquad g = 9.81 \frac{\text{m}}{\text{s}^2}$



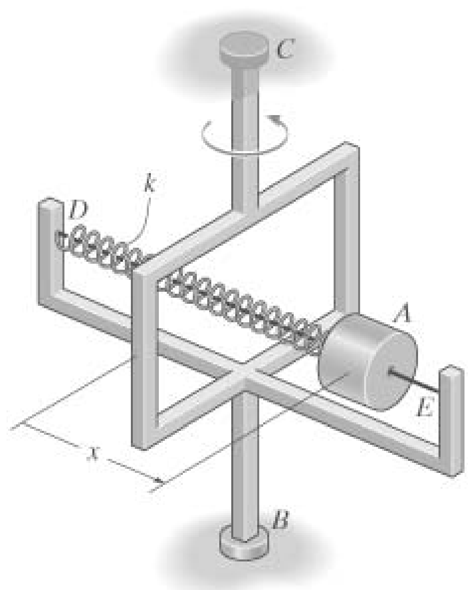
Solution: Guess $t = 1 \text{ s}$

Given $\mu_s M g = \sqrt{(M v')^2 + \left[M \frac{(v' t)^2}{d}\right]^2}$

$t = \text{Find}(t) \qquad t = 7.394 \text{ s}$

Problem 13-79

The collar A, having a mass M, is attached to a spring having a stiffness k. When rod BC rotates about the vertical axis, the collar slides outward along the smooth rod DE. If the spring is unstretched when $x = 0$, determine the constant speed of the collar in order that $x = x_1$. Also, what is the normal force of the rod on the collar? Neglect the size of the collar.



Given:

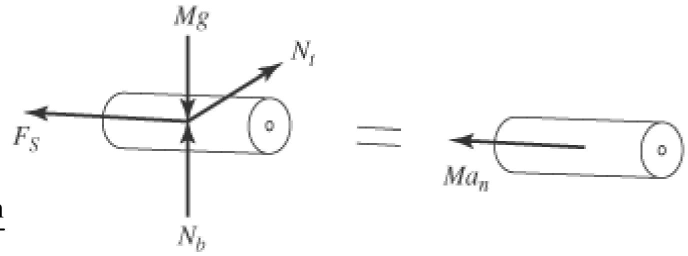
$M = 0.75 \text{ kg}$

$k = 200 \frac{\text{N}}{\text{m}}$

$x_1 = 100 \text{ mm}$

$g = 9.81\ \frac{m}{s^2}$

Solution:



Guesses

$N_b = 1\ N \quad N_t = 1\ N \quad v = 1\ \frac{m}{s}$

Given $\quad N_b - Mg = 0 \quad N_t = 0 \quad k x_1 = M\left(\frac{v^2}{x_1}\right)$

$\begin{pmatrix} N_b \\ N_t \\ v \end{pmatrix} = \text{Find}(N_b, N_t, v) \quad \begin{pmatrix} N_b \\ N_t \end{pmatrix} = \begin{pmatrix} 7.36 \\ 0 \end{pmatrix} N \quad \left|\begin{pmatrix} N_b \\ N_t \end{pmatrix}\right| = 7.36\ N \quad v = 1.633\ \frac{m}{s}$

***Problem 13-80**

The block has weight W and it is free to move along the smooth slot in the rotating disk. The spring has stiffness k and an unstretched length δ. Determine the force of the spring on the block and the tangential component of force which the slot exerts on the side of the block, when the block is at rest with respect to the disk and is traveling with constant speed v.

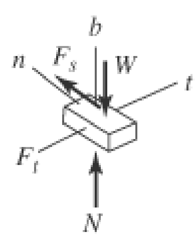
Given:

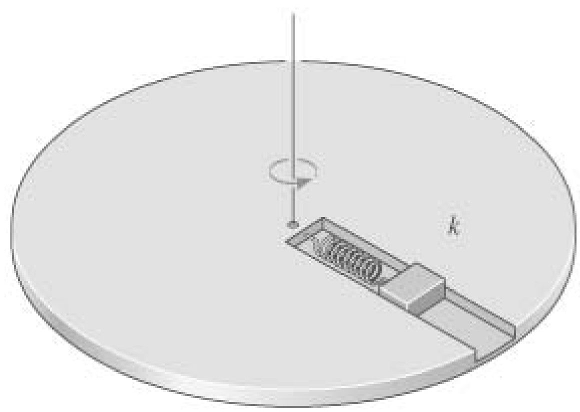
$W = 2\ lb$

$k = 2.5\ \frac{lb}{ft}$

$\delta = 1.25\ ft$

$v = 12\ \frac{ft}{s}$





Solution:

$\Sigma F_n = ma_n; \quad F_s = k(\rho - \delta) = \frac{W}{g}\left(\frac{v^2}{\rho}\right)$

Choosing the positive root,

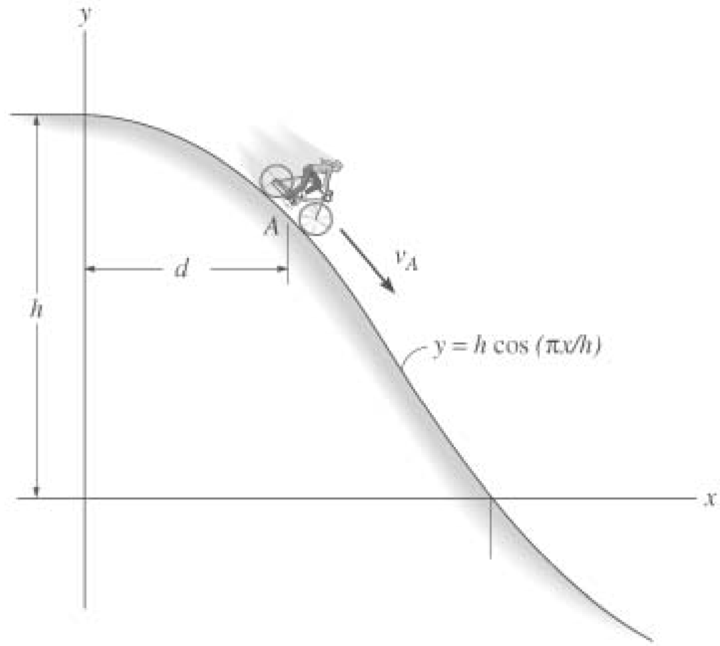
$\rho = \frac{1}{2kg}\left[kg\delta + \left(\sqrt{k^2 g^2 \delta^2 + 4kgWv^2}\right)\right] \quad \rho = 2.617\ ft$

$F_s = k(\rho - \delta) \quad F_s = 3.419\ lb$

$\Sigma F_t = ma_t; \quad \Sigma F_t = ma_t; \quad F_t = 0$

Problem 13-81

If the bicycle and rider have total weight W, determine the resultant normal force acting on the bicycle when it is at point A while it is freely coasting at speed v_A. Also, compute the increase in the bicyclist's speed at this point. Neglect the resistance due to the wind and the size of the bicycle and rider.



Given:

$W = 180\ \text{lb}$ $\quad d = 5\ \text{ft}$

$v_A = 6\ \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$h = 20\ \text{ft}$

Solution:

$$y(x) = h\cos\left(\pi\frac{x}{h}\right)$$

$$y'(x) = \frac{d}{dx}y(x) \qquad y''(x) = \frac{d}{dx}y'(x)$$

At A $\quad x = d \qquad \theta = \text{atan}(y'(x))$

$$\rho = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

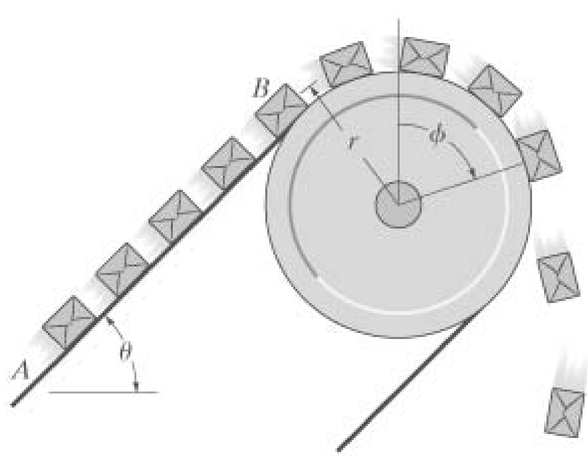
Guesses $\quad F_N = 1\ \text{lb} \qquad v' = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $\quad F_N - W\cos(\theta) = \frac{W}{g}\left(\frac{v_A^2}{\rho}\right) \qquad -W\sin(\theta) = \left(\frac{W}{g}\right)v'$

$$\begin{pmatrix} F_N \\ v' \end{pmatrix} = \text{Find}(F_N, v') \qquad F_N = 69.03\ \text{lb} \qquad v' = 29.362\ \frac{\text{ft}}{\text{s}^2}$$

Problem 13-82

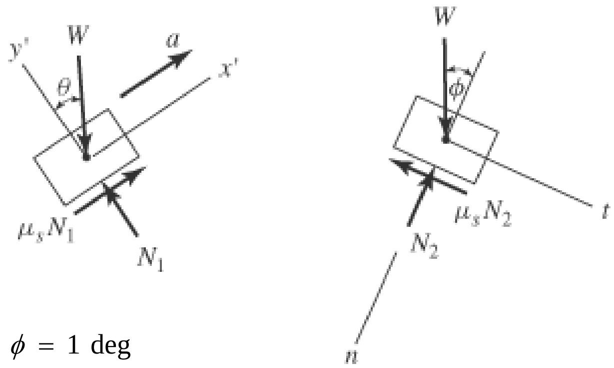
The packages of weight W ride on the surface of the conveyor belt. If the belt starts from rest and increases to a constant speed v_1 in time t_1, determine the maximum angle θ so that none of the packages slip on the inclined surface AB of the belt. The coefficient of static friction between the belt and a package is μ_s. At what angle ϕ do the packages first begin to slip off the surface of the belt after the belt is moving at its constant speed of v_1? Neglect the size of the packages.



Given:

$W = 5\ \text{lb}$ $\quad t_1 = 2\ \text{s}$ $\quad r = 6\ \text{in}$

$v_1 = 2\ \dfrac{\text{ft}}{\text{s}}$ $\quad \mu_s = 0.3$



Solution: $\quad a = \dfrac{v_1}{t_1}$

Guesses

$N_1 = 1\ \text{lb}$ $\quad N_2 = 1\ \text{lb}$ $\quad \theta = 1\ \text{deg}$ $\quad \phi = 1\ \text{deg}$

Given

$N_1 - W\cos(\theta) = 0$ $\qquad \mu_s N_1 - W\sin(\theta) = \left(\dfrac{W}{g}\right)a$

$N_2 - W\cos(\phi) = \dfrac{-W}{g}\left(\dfrac{v_1^{\,2}}{r}\right)$ $\qquad \mu_s N_2 - W\sin(\phi) = 0$

$$\begin{pmatrix} N_1 \\ N_2 \\ \theta \\ \phi \end{pmatrix} = \text{Find}(N_1, N_2, \theta, \phi) \qquad \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} 4.83 \\ 3.637 \end{pmatrix}\text{lb} \qquad \begin{pmatrix} \theta \\ \phi \end{pmatrix} = \begin{pmatrix} 14.99 \\ 12.61 \end{pmatrix}\text{deg}$$

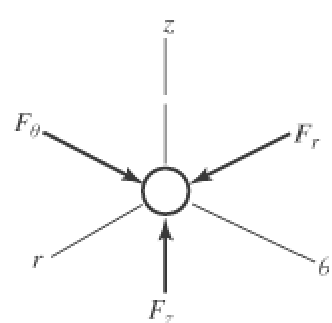
Problem 13-83

A particle having mass M moves along a path defined by the equations $r = a + bt$, $\theta = ct^2 + d$ and $z = e + ft^3$. Determine the r, θ, and z components of force which the path exerts on the particle when $t = t_1$.

Given: $\quad M = 1.5\ \text{kg}$ $\quad a = 4\ \text{m}$ $\quad b = 3\ \dfrac{\text{m}}{\text{s}}$

$c = 1\ \dfrac{\text{rad}}{\text{s}^2}$ $\quad d = 2\ \text{rad}$ $\quad e = 6\ \text{m}$

$f = -1\ \dfrac{\text{m}}{\text{s}^3}$ $\quad t_1 = 2\ \text{s}$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution: $\quad t = t_1$

$r = a + bt$ $\quad r' = b$ $\quad r'' = 0\ \dfrac{\text{m}}{\text{s}^2}$

$\theta = ct^2 + d$ $\quad \theta' = 2ct$ $\quad \theta'' = 2c$

$z = e + ft^3$ $\quad z' = 3ft^2$ $\quad z'' = 6ft$

$F_r = M\left(r'' - r\theta'^2\right)$ $\quad$ $F_r = -240\ \text{N}$

$F_\theta = M\left(r\theta'' + 2r'\theta'\right)$ $\quad$ $F_\theta = 66.0\ \text{N}$

$F_z = Mz'' + Mg$ $\quad$ $F_z = -3.285\ \text{N}$

***Problem 13-84**

The path of motion of a particle of weight W in the horizontal plane is described in terms of polar coordinates as $r = at + b$ and $\theta = ct^2 + dt$. Determine the magnitude of the unbalanced force acting on the particle when $t = t_1$.

Given: $W = 5\ \text{lb}$ $\quad a = 2\ \dfrac{\text{ft}}{\text{s}}$ $\quad b = 1\ \text{ft}$ $\quad c = 0.5\ \dfrac{\text{rad}}{\text{s}^2}$

$d = -1\ \dfrac{\text{rad}}{\text{s}}$ $\quad t_1 = 2\ \text{s}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution: $t = t_1$

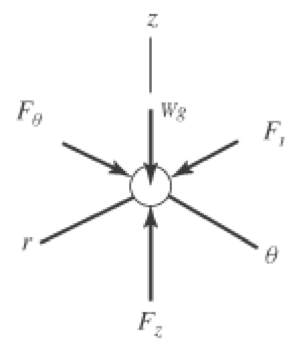
$r = at + b$ $\quad r' = a$ $\quad r'' = 0\ \dfrac{\text{ft}}{\text{s}^2}$

$\theta = ct^2 + dt$ $\quad \theta' = 2ct + d$ $\quad \theta'' = 2c$

$a_r = r'' - r\theta'^2$ $\quad a_r = -5\ \dfrac{\text{ft}}{\text{s}^2}$

$a_\theta = r\theta'' + 2r'\theta'$ $\quad a_\theta = 9\ \dfrac{\text{ft}}{\text{s}^2}$

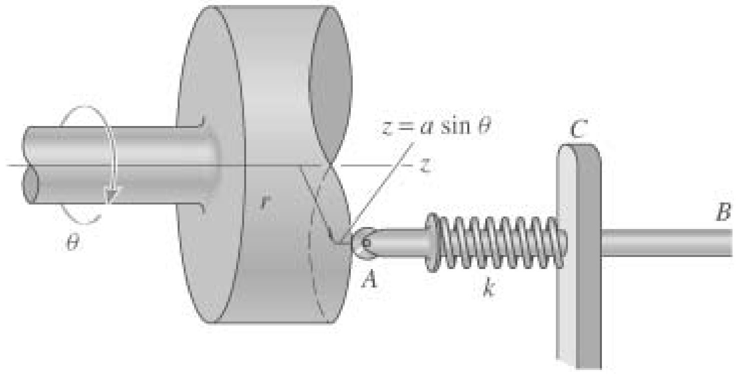
$F = \dfrac{W}{g}\sqrt{a_r^2 + a_\theta^2}$ $\quad F = 1.599\ \text{lb}$



Problem 13-85

The spring-held follower AB has weight W and moves back and forth as its end rolls on the contoured surface of the cam, where the radius is r and $z = a\sin(2\theta)$. If the cam is rotating at a constant rate θ', determine the force at the end A of the follower when $\theta = \theta_1$. In this position the spring is compressed δ_1. Neglect friction at the bearing C.

Given:



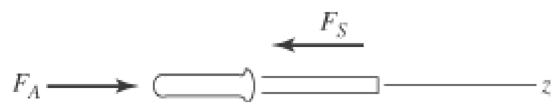
$W = 0.75\ \text{lb}$ $\delta_1 = 0.4\ \text{ft}$

$r = 0.2\ \text{ft}$ $\theta_1 = 45\ \text{deg}$

$a = 0.1\ \text{ft}$ $k = 12\ \frac{\text{lb}}{\text{ft}}$

$\theta' = 6\ \frac{\text{rad}}{\text{s}}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: $\theta = \theta_1$ $z = (a)\sin(2\theta)$



$z' = 2(a)\cos(2\theta)\theta'$ $z'' = -4(a)\sin(2\theta)\,\theta'^2$

$F_a - k\delta_1 = \left(\frac{W}{g}\right)z''$ $F_a = k\delta_1 + \left(\frac{W}{g}\right)z''$ $F_a = 4.464\,\text{lb}$

Problem 13-86

The spring-held follower *AB* has weight *W* and moves back and forth as its end rolls on the contoured surface of the cam, where the radius is *r* and $z = a\ \sin(2\theta)$. If the cam is rotating at a constant rate of θ', determine the maximum and minimum force the follower exerts on the cam if the spring is compressed δ_1 when $\theta = 45^\circ$.

Given:

$W = 0.75\ \text{lb}$

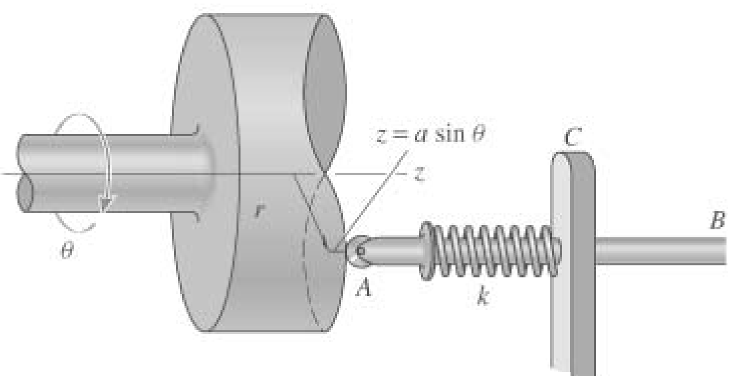
$r = 0.2\ \text{ft}$

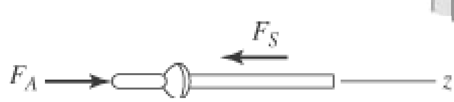
$a = 0.1\ \text{ft}$

$\theta' = 6\ \frac{\text{rad}}{\text{s}}$

$\delta_1 = 0.2\ \text{ft}$

$k = 12\ \frac{\text{lb}}{\text{ft}}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution: When $\theta = 45\ \text{deg}$ $z = (a)\cos(2\theta)$ $z = 0\ \text{m}$

So in other positions the spring is compresses a distance $\delta_1 + z$

$z = (a)\sin(2\theta)$ $z' = 2(a)\cos(2\theta)\theta'$ $z'' = -4(a)\sin(2\theta)\,\theta'^2$

$$F_a - k(\delta_1 + z) = \left(\frac{W}{g}\right)z'' \qquad F_a = k\left[\delta_1 + (a)\sin(2\theta)\right] - \left(\frac{W}{g}\right)4(a)\sin(2\theta)\,\theta'^2$$

The maximum values occurs when sin(2θ) = -1 and the minimum occurs when sin(2θ) = 1

$$F_{amin} = k(\delta_1 - a) + \left(\frac{W}{g}\right)4a\theta'^2 \qquad F_{amin} = 1.535\,\text{lb}$$

$$F_{amax} = k(\delta_1 + a) - \left(\frac{W}{g}\right)4a\theta'^2 \qquad F_{amax} = 3.265\,\text{lb}$$

Problem 13-87

The spool of mass M slides along the rotating rod. At the instant shown, the angular rate of rotation of the rod is θ', which is increasing at θ''. At this same instant, the spool is moving outward along the rod at r' which is increasing at r'' at r. Determine the radial frictional force and the normal force of the rod on the spool at this instant.

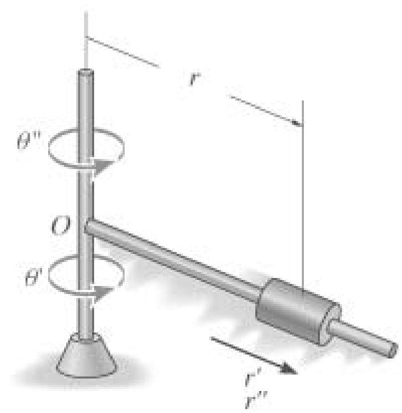
Given:

$M = 4\ \text{kg}$ $\qquad r = 0.5\ \text{m}$

$\theta' = 6\ \frac{\text{rad}}{\text{s}}$ $\qquad r' = 3\ \frac{\text{m}}{\text{s}}$

$\theta'' = 2\ \frac{\text{rad}}{\text{s}^2}$ $\qquad r'' = 1\ \frac{\text{m}}{\text{s}^2}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

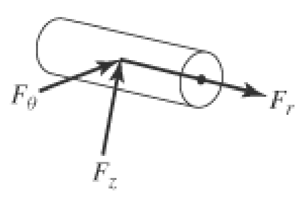


Solution:

$a_r = r'' - r\theta'^2$ $\qquad a_\theta = r\theta'' + 2r'\theta'$

$F_r = M a_r$ $\qquad F_\theta = M a_\theta$

$F_z = M g$



$F_r = -68.0\ \text{N}$ $\qquad \sqrt{F_\theta^{\,2} + F_z^{\,2}} = 153.1\ \text{N}$

***Problem 13-88**

The boy of mass M is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = r_0$, $\theta = bt$ and $z = ct$. Determine the components of force F_r, F_θ and F_z which the slide exerts on him at the instant $t = t_1$. Neglect the size of the boy.

Given:

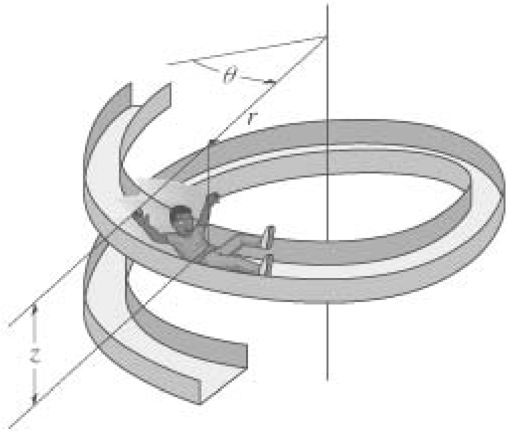
$M = 40\ \text{kg}$

$r_0 = 1.5\ \text{m}$

$b = 0.7\ \frac{\text{rad}}{\text{s}}$

$c = -0.5\ \frac{\text{m}}{\text{s}}$

$t_1 = 2\ \text{s}$ $\qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

$r = r_0 \qquad r' = 0\ \frac{\text{m}}{\text{s}} \qquad r'' = 0\ \frac{\text{m}}{\text{s}^2}$

$\theta = bt \qquad \theta' = b \qquad \theta'' = 0\ \frac{\text{rad}}{\text{s}^2}$

$z = ct \qquad z' = c \qquad z'' = 0\ \frac{\text{m}}{\text{s}^2}$

$F_r = M\left(r'' - r\theta'^2\right)$ $\qquad F_r = -29.4\ \text{N}$

$F_\theta = M\left(r\theta'' + 2r'\theta'\right)$ $\qquad F_\theta = 0$

$F_z - Mg = Mz'' \qquad F_z = M(g + z'')$ $\qquad F_z = 392\ \text{N}$

Problem 13-89

The girl has a mass M. She is seated on the horse of the merry-go-round which undergoes constant rotational motion θ'. If the path of the horse is defined by $r = r_0$, $z = b\sin(\theta)$, determine the maximum and minimum force F_z the horse exerts on her during the motion.

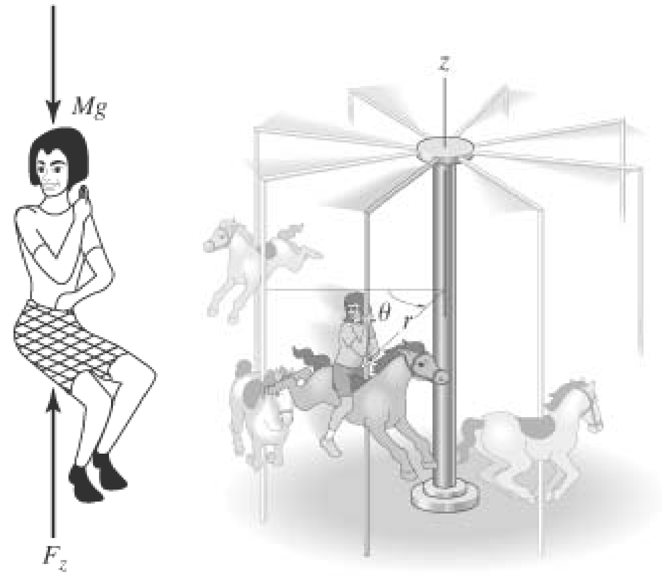
Given:

$M = 50\ \text{kg}$

$\theta' = 1.5\ \dfrac{\text{rad}}{\text{s}}$

$r_0 = 4\ \text{m}$

$b = 0.5\ \text{m}$



Solution:

$z = b\sin(\theta)$

$z' = b\cos(\theta)\theta'$

$z'' = -b\sin(\theta)\,\theta^2$

$F_z - Mg = Mz''$

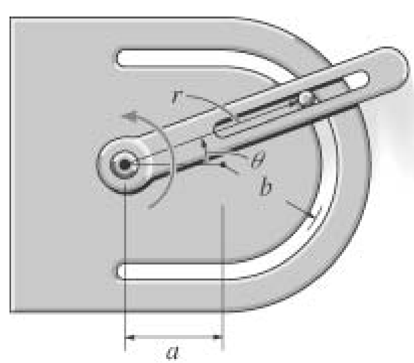
$F_z = M\left(g - b\sin(\theta)\,\theta^2\right)$

$F_{zmax} = M\left(g + b\theta'^2\right)$ $\qquad F_{zmax} = 547\ \text{N}$

$F_{zmin} = M\left(g - b\theta'^2\right)$ $\qquad F_{zmin} = 434\ \text{N}$

Problem 13-90

The particle of weight W is guided along the circular path using the slotted arm guide. If the arm has angular velocity θ' and angular acceleration θ'' at the instant $\theta = \theta_1$, determine the force of the guide on the particle. Motion occurs in the *horizontal plane*.

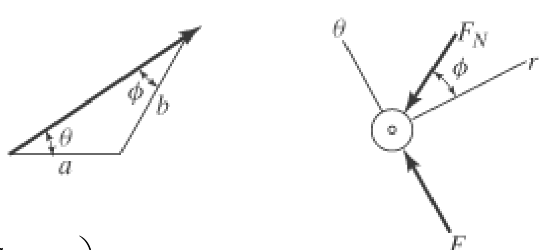


Given: $\qquad \theta_1 = 30\ \text{deg}$

$W = 0.5\ \text{lb}$ $\qquad a = 0.5\ \text{ft}$

$\theta' = 4\ \dfrac{\text{rad}}{\text{s}}$ $\qquad b = 0.5\ \text{ft}$

$\theta'' = 8\ \dfrac{\text{rad}}{\text{s}^2}$ $\qquad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution: $\qquad \theta = \theta_1$

$(a)\sin(\theta) = b\sin(\phi)$ $\qquad \phi = \operatorname{asin}\left(\dfrac{a}{b}\sin(\theta)\right)$ $\qquad \phi = 30\,\text{deg}$

$(a)\cos(\theta)\theta' = b\cos(\phi)\phi'$ $\phi' = \left[\frac{(a)\cos(\theta)}{b\cos(\phi)}\right]\theta'$ $\phi' = 4\,\frac{\text{rad}}{\text{s}}$

$$(a)\cos(\theta)\theta'' - (a)\sin(\theta)\,\theta'^2 = b\cos(\phi)\phi'' - b\sin(\phi)\,\phi'^2$$

$\phi'' = \dfrac{(a)\cos(\theta)\theta'' - (a)\sin(\theta)\,\theta'^2 + b\sin(\phi)\,\phi'^2}{b\cos(\phi)}$ $\phi'' = 8\,\frac{\text{rad}}{\text{s}^2}$

$r = (a)\cos(\theta) + b\cos(\phi)$ $r' = -(a)\sin(\theta)\theta' - b\sin(\phi)\phi'$

$$r'' = -(a)\sin(\theta)\theta'' - (a)\cos(\theta)\,\theta'^2 - b\sin(\phi)\phi'' - b\cos(\phi)\,\phi'^2$$

$-F_N\cos(\phi) = M\left(r'' - r\theta'^2\right)$ $F_N = \dfrac{-W\left(r'' - r\theta'^2\right)}{g\cos(\phi)}$ $F_N = 0.569\,\text{lb}$

$F - F_N\sin(\phi) = \left(\frac{W}{g}\right)\left(r\theta'' + 2r'\theta'\right)$ $F = F_N\sin(\phi) + \left(\frac{W}{g}\right)\left(r\theta'' + 2r'\theta'\right)$ $F = 0.143\,\text{lb}$

Problem 13-91

The particle has mass M and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = \theta_1$. The rod is rotating with a constant angular velocity θ'. Assume the particle contacts only one side of the slot at any instant.

Given:

$M = 0.5\ \text{kg}$

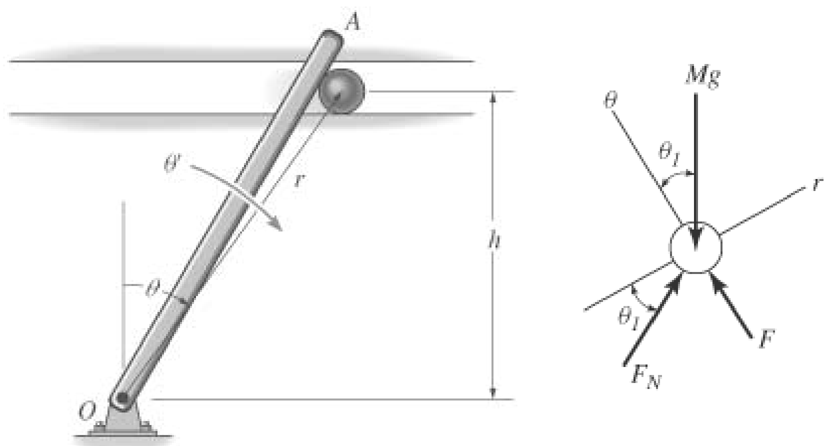
$\theta_1 = 30\ \text{deg}$

$\theta' = 2\ \frac{\text{rad}}{\text{s}}$

$\theta'' = 0\ \frac{\text{rad}}{\text{s}^2}$

$h = 0.5\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

$\theta = \theta_1$ $h = r\cos(\theta)$ $r = \dfrac{h}{\cos(\theta)}$ $r = 0.577\ \text{m}$

$$0 = r' \cos(\theta) - r \sin(\theta)\theta' \qquad r' = \left(\frac{r \sin(\theta)}{\cos(\theta)}\right)\theta' \qquad r' = 0.667 \ \frac{\text{m}}{\text{s}}$$

$$0 = r'' \cos(\theta) - 2r' \sin(\theta)\theta' - r\cos(\theta)\,\theta'^2 - r\sin(\theta)\theta''$$

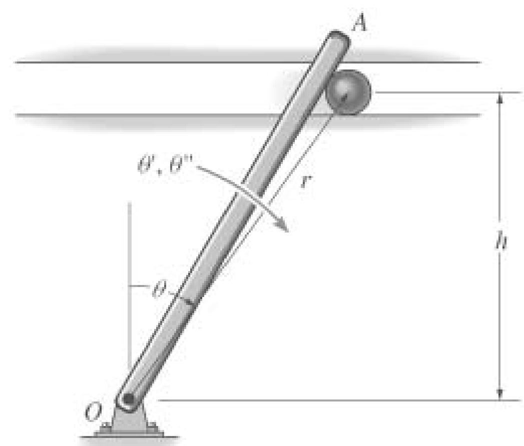
$$r'' = 2r'\theta' \tan(\theta) + r\theta'^2 + r\tan(\theta)\theta'' \qquad r'' = 3.849 \ \frac{\text{m}}{\text{s}^2}$$

$$(F_N - Mg)\cos(\theta) = M\left(r'' - r\theta'^2\right) \qquad F_N = Mg + M\left(\frac{r'' - r\theta'^2}{\cos(\theta)}\right) \qquad F_N = 5.794 \ \text{N}$$

$$-F + (F_N - Mg)\sin(\theta) = -M(r\theta'' + 2r'\theta')$$

$$F = (F_N - Mg)\sin(\theta) + M(r\theta'' + 2r'\theta') \qquad F = 1.778 \ \text{N}$$

***Problem 13-92**



The particle has mass M and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = \theta_1$. The rod is rotating with angular velocity θ' and angular acceleration θ''. Assume the particle contacts only one side of the slot at any instant.

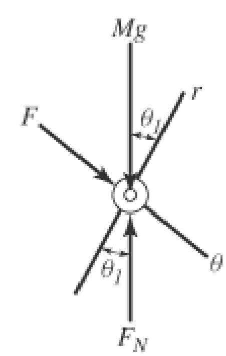
Given:

$$M = 0.5 \ \text{kg}$$

$$\theta_1 = 30 \ \text{deg}$$

$$\theta' = 2 \ \frac{\text{rad}}{\text{s}} \qquad h = 0.5 \ \text{m}$$

$$\theta'' = 3 \ \frac{\text{rad}}{\text{s}^2} \qquad g = 9.81 \ \frac{\text{m}}{\text{s}^2}$$



Solution:

$$\theta = \theta_1 \qquad h = r\cos(\theta) \qquad r = \frac{h}{\cos(\theta)} \qquad r = 0.577 \ \text{m}$$

$$0 = r' \cos(\theta) - r \sin(\theta)\theta' \qquad r' = \left(\frac{r \sin(\theta)}{\cos(\theta)}\right)\theta' \qquad r' = 0.667 \ \frac{\text{m}}{\text{s}}$$

$$0 = r'' \cos(\theta) - 2r' \sin(\theta)\theta' - r\cos(\theta)\,\theta'^2 - r\sin(\theta)\theta''$$

$r'' = 2r'\theta' \tan(\theta) + r\theta^2 + r\tan(\theta)\theta''$ $\quad r'' = 4.849\ \dfrac{\text{m}}{\text{s}^2}$

$(F_N - Mg)\cos(\theta) = M(r'' - r\theta^2)$ $\quad F_N = Mg + M\left(\dfrac{r'' - r\theta^2}{\cos(\theta)}\right)$ $\quad F_N = 6.371\ \text{N}$

$-F + (F_N - Mg)\sin(\theta) = -M(r\theta'' + 2r'\theta')$

$F = (F_N - Mg)\sin(\theta) + M(r\theta'' + 2r'\theta')$ $\quad F = 2.932\ \text{N}$

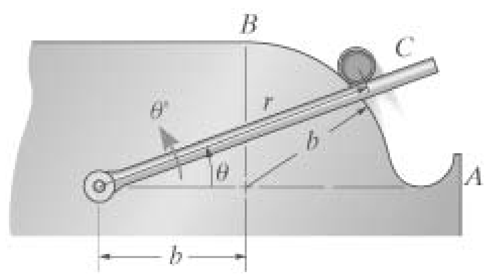
Problem 13-93

A smooth can *C*, having a mass *M*, is lifted from a feed at *A* to a ramp at *B* by a rotating rod. If the rod maintains a constant angular velocity of θ', determine the force which the rod exerts on the can at the instant $\theta = \theta_1$. Neglect the effects of friction in the calculation and the size of the can so that $r = 2b\cos\theta$. The ramp from *A* to *B* is circular, having a radius of *b*.

Given:

$M = 3\ \text{kg}$ $\quad \theta_1 = 30\ \text{deg}$

$\theta' = 0.5\ \dfrac{\text{rad}}{\text{s}}$ $\quad b = 600\ \text{mm}$



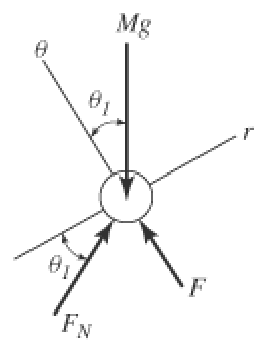
Solution:

$\theta = \theta_1$

$r = 2b\cos(\theta)$

$r' = -2b\sin(\theta)\theta'$

$r'' = -2b\cos(\theta)\,\theta'^2$



Guesses $\quad F_N = 1\ \text{N} \quad F = 1\ \text{N}$

Given

$F_N\cos(\theta) - Mg\sin(\theta) = M(r'' - r\theta'^2)$

$F + F_N\sin(\theta) - Mg\cos(\theta) = M(2r'\theta')$

$\begin{pmatrix} F_N \\ F \end{pmatrix} = \text{Find}(F_N, F)$ $\quad F_N = 15.191\ \text{N}$ $\quad F = 16.99\ \text{N}$

Problem 13-94

The collar of weight *W* slides along the smooth *horizontal* spiral rod $r = b\theta$, where θ is in radians. If its angular rate of rotation θ' is constant, determine the tangential force P needed to cause the motion and the normal force that the rod exerts on the spool at the instant $\theta = \theta_1$.

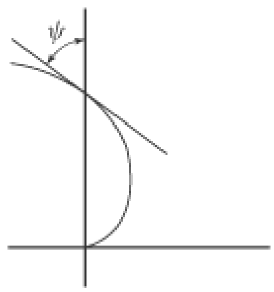
Given:

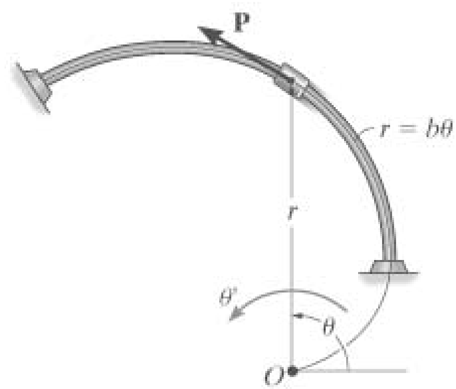
$W = 2 \text{ lb}$

$\theta_1 = 90 \text{ deg}$

$\theta' = 4 \dfrac{\text{rad}}{\text{s}}$

$b = 2 \text{ ft}$





Solution:

$\theta = \theta_1 \qquad r = b\theta \qquad r' = b\theta'$

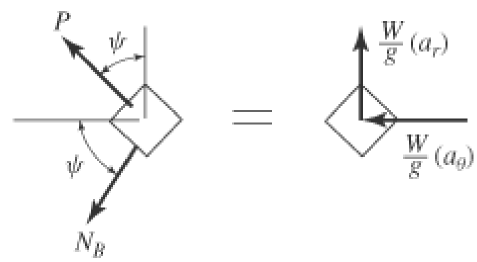
$$\psi = \operatorname{atan}\left(\frac{r\theta'}{r'}\right)$$

Guesses $\quad N_B = 1 \text{ lb} \qquad P = 1 \text{ lb}$

Given

$$-N_B \sin(\psi) + P\cos(\psi) = \left(\frac{W}{g}\right)\left(-r\,\theta'^2\right)$$

$$P\sin(\psi) + N_B\cos(\psi) = \left(\frac{W}{g}\right)\left(2r'\theta'\right)$$



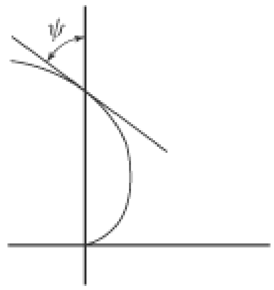
$$\begin{pmatrix} N_B \\ P \end{pmatrix} = \operatorname{Find}(N_B, P) \qquad \psi = 57.52 \text{ deg} \qquad N_B = 4.771 \text{ lb} \qquad P = 1.677 \text{ lb}$$

Problem 13-95

The collar of weight W slides along the smooth *vertical* spiral rod $r = b\theta$, where θ is in radians. If its angular rate of rotation θ' is constant, determine the tangential force P needed to cause the motion and the normal force that the rod exerts on the spool at the instant $\theta = \theta_1$.

Given:

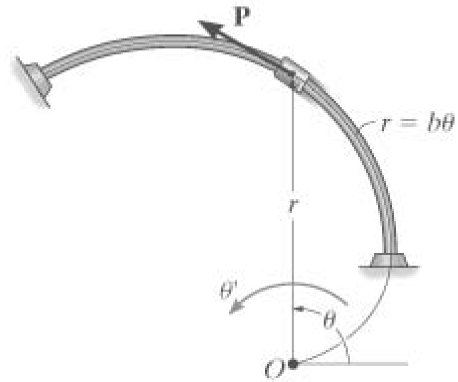
$W = 2\ \text{lb}$



$\theta_1 = 90\ \text{deg}$

$\theta' = 4\ \dfrac{\text{rad}}{\text{s}}$

$b = 2\ \text{ft}$

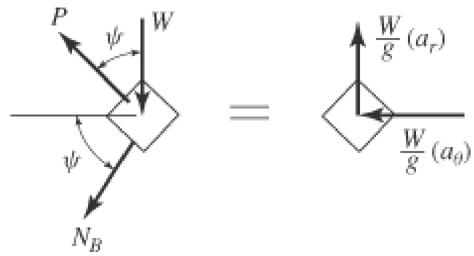


Solution:

$\theta = \theta_1 \qquad r = b\theta \qquad r' = b\theta'$

$$\psi = \operatorname{atan}\left(\frac{r\theta'}{r'}\right)$$

Guesses $\quad N_B = 1\ \text{lb} \qquad P = 1\ \text{lb}$



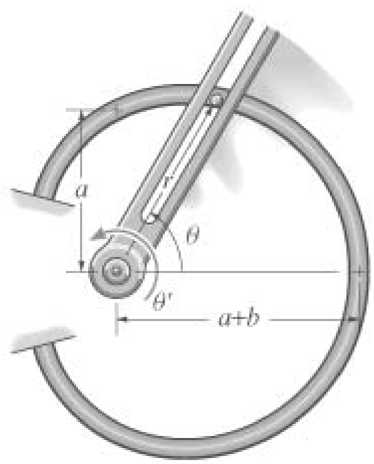
Given

$$-N_B\sin(\psi) + P\cos(\psi) - W = \left(\frac{W}{g}\right)\left(-r\,\theta'^2\right)$$

$$P\sin(\psi) + N_B\cos(\psi) = \left(\frac{W}{g}\right)\left(2r'\theta'\right)$$

$$\begin{pmatrix} N_B \\ P \end{pmatrix} = \operatorname{Find}(N_B, P) \qquad \psi = 57.52\ \text{deg} \qquad N_B = 3.084\ \text{lb} \qquad P = 2.751\ \text{lb}$$

***Problem 13-96**

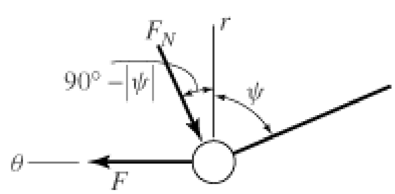


The forked rod is used to move the smooth particle of weight W around the horizontal path in the shape of a limacon $r = a + b\cos\theta$. If $\theta = ct^2$, determine the force which the rod exerts on the particle at the instant $t = t_1$. The fork and path contact the particle on only one side.

Given:

$W = 2\ \text{lb}$

$a = 2\ \text{ft}$

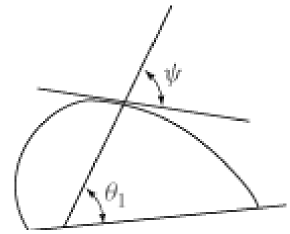


$b = 1\ \text{ft}$

$c = 0.5\ \dfrac{\text{rad}}{\text{s}^2}$

$t_1 = 1\ \text{s}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution: $t = t_1$ $\theta = c t^2$ $\theta' = 2ct$ $\theta'' = 2c$

Find the angel ψ using rectangular coordinates. The path is tangent to the velocity therefore.

$x = r\cos(\theta) = (a)\cos(\theta) + b\cos(\theta)^2$ $x' = [-(a)\sin(\theta) - 2b\cos(\theta)\sin(\theta)]\theta'$

$y = r\sin(\theta) = (a)\sin(\theta) + \dfrac{1}{2}b\sin(2\theta)$ $y' = [(a)\cos(\theta) + b\cos(2\theta)]\theta'$

$\psi = \theta - \operatorname{atan}\left(\dfrac{y'}{x'}\right)$ $\psi = 80.541\ \text{deg}$

Now do the dynamics using polar coordinates

$r = a + b\cos(\theta)$ $r' = -b\sin(\theta)\theta'$ $r'' = -b\cos(\theta)\theta'^2 - b\sin(\theta)\theta''$

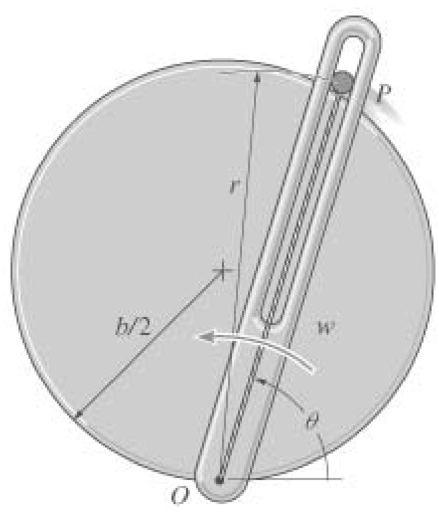
Guesses $F = 1\ \text{lb}$ $F_N = 1\ \text{lb}$

Given $F - F_N\cos(\psi) = \left(\dfrac{W}{g}\right)(r\theta'' + 2r'\theta')$ $-F_N\sin(\psi) = \left(\dfrac{W}{g}\right)(r'' - r\theta'^2)$

$\begin{pmatrix} F \\ F_N \end{pmatrix} = \text{Find}(F, F_N)$ $F_N = 0.267\ \text{lb}$ $F = 0.163\ \text{lb}$

Problem 13-97

The smooth particle has mass M. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = b\sin\theta$. If the cord has stiffness k and unstretched length δ determine the force of the guide on the particle when $\theta = \theta_1$. The guide has a constant angular velocity θ'.



Given:

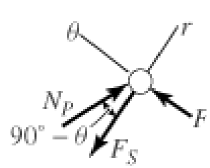
$M = 80\ \text{gm}$

$b = 0.8\ \text{m}$

$k = 30\ \dfrac{\text{N}}{\text{m}}$

$\delta = 0.25\ \text{m}$

$\theta_1 = 60\ \text{deg}$



$\theta' = 5\ \dfrac{\text{rad}}{\text{s}}$

$\theta'' = 0\ \dfrac{\text{rad}}{\text{s}^2}$

Solution: $\theta = \theta_1 \qquad r = b\sin(\theta) \qquad r' = b\cos(\theta)\theta' \qquad r'' = b\cos(\theta)\theta'' - b\sin(\theta)\,\theta'^2$

Guesses $N_P = 1\ \text{N} \qquad F = 1\ \text{N}$

Given $N_P\sin(\theta) - k(r - \delta) = M\left(r'' - r\theta'^2\right)$

$F - N_P\cos(\theta) = M(r\theta'' + 2r'\theta')$

$\begin{pmatrix} F \\ N_P \end{pmatrix} = \text{Find}(F, N_P) \qquad N_P = 12.14\ \text{N} \qquad F = 7.67\ \text{N}$

Problem 13-98

The smooth particle has mass M. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = b\sin\theta$. If the cord has stiffness k and unstretched length δ determine the force of the guide on the particle when $\theta = \theta_1$. The guide has angular velocity θ' and angular acceleration θ'' at this instant.

Given:

$M = 80\ \text{gm}$

$b = 0.8\ \text{m}$

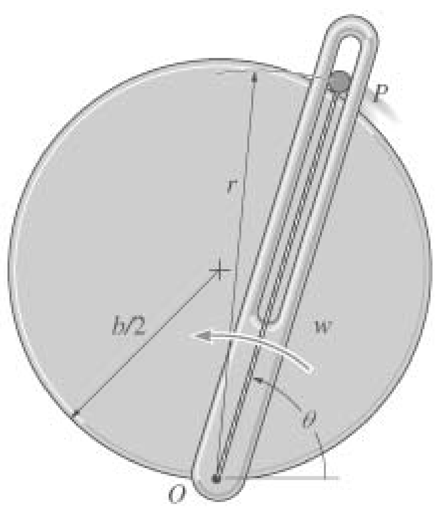
$k = 30\ \dfrac{\text{N}}{\text{m}}$

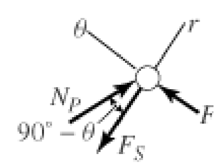
$\delta = 0.25\ \text{m}$

$\theta_1 = 60\ \text{deg}$

$\theta' = 5\ \dfrac{\text{rad}}{\text{s}}$

$\theta'' = 2\ \dfrac{\text{rad}}{\text{s}^2}$





Solution: $\theta = \theta_1 \qquad r = b\sin(\theta) \qquad r' = b\cos(\theta)\theta' \qquad r'' = b\cos(\theta)\theta'' - b\sin(\theta)\,\theta'^2$

Guesses $N_P = 1\ \text{N} \qquad F = 1\ \text{N}$

Given $\quad N_P \sin(\theta) - k(r - \delta) = M\left(r'' - r\theta'^2\right)$

$\quad F - N_P \cos(\theta) = M(r\theta'' + 2r'\theta')$

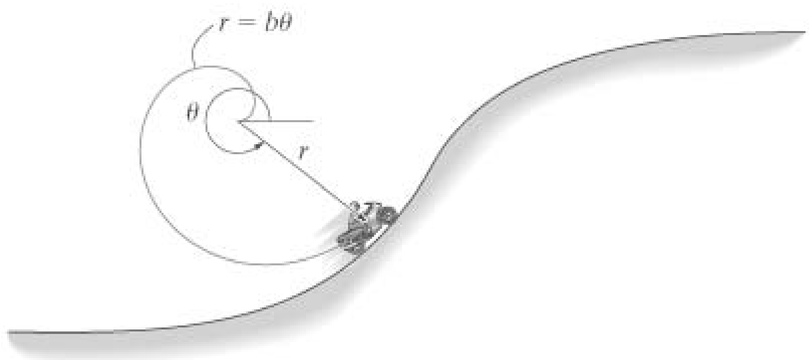
$$\begin{pmatrix} F \\ N_P \end{pmatrix} = \text{Find}(F, N_P) \qquad N_P = 12.214 \text{ N} \qquad F = 7.818 \text{ N}$$

Problem 13-99

Determine the normal and frictional driving forces that the partial spiral track exerts on the motorcycle of mass M at the instant θ, θ', and θ''. Neglect the size of the motorcycle.

Units Used:

$\quad \text{kN} = 10^3 \text{ N}$



Given:

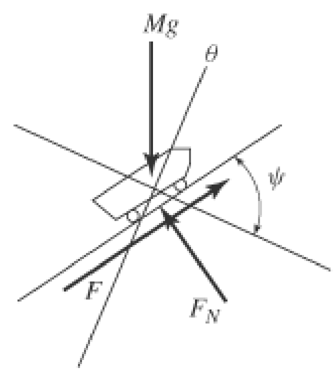
$\quad M = 200 \text{ kg}$

$\quad b = 5 \text{ m}$

$\quad \theta = \dfrac{5\pi}{3} \text{ rad}$

$\quad \theta' = 0.4 \dfrac{\text{rad}}{\text{s}}$

$\quad \theta'' = 0.8 \dfrac{\text{rad}}{\text{s}^2}$



Solution:

$\quad r = b\theta \qquad r' = b\theta' \qquad r'' = b\theta''$

$\quad \psi = \text{atan}\left(\dfrac{r\theta'}{r'}\right) \qquad \psi = 79.188 \deg$

Guesses $\quad F_N = 1 \text{ N} \qquad F = 1 \text{ N}$

Given $\quad -F_N \sin(\psi) + F\cos(\psi) - Mg\sin(\theta) = M\left(r'' - r\theta'^2\right)$

$\quad F_N \cos(\psi) + F\sin(\psi) - Mg\cos(\theta) = M(r\theta'' + 2r'\theta')$

$$\begin{pmatrix} F_N \\ F \end{pmatrix} = \text{Find}(F_N, F) \qquad \begin{pmatrix} F_N \\ F \end{pmatrix} = \begin{pmatrix} 2.74 \\ 5.07 \end{pmatrix} \text{kN}$$

***Problem 13-100**

Using a forked rod, a smooth cylinder C having a mass M is forced to move along the vertical slotted path $r = a\theta$. If the angular position of the arm is $\theta = bt^2$, determine the force of the rod on the cylinder and the normal force of the slot on the cylinder at the instant t. The cylinder is in contact with only one edge of the rod and slot at any instant.

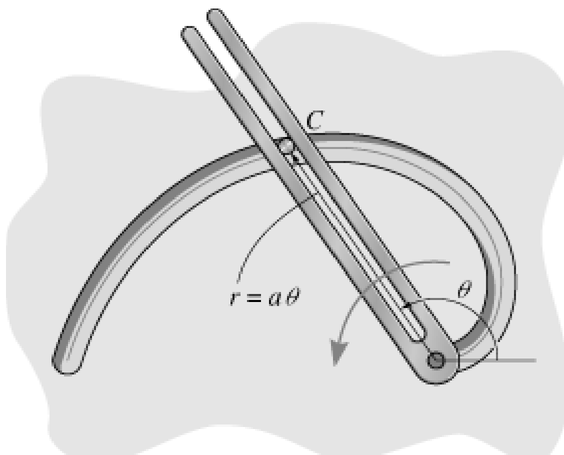
Given:

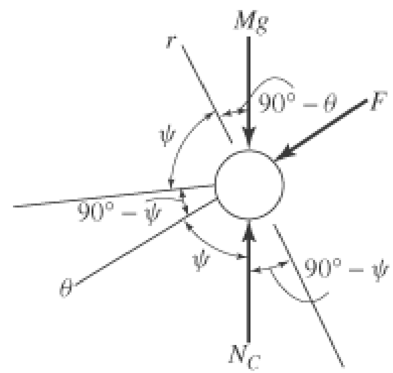
$M = 0.5\ \text{kg}$

$a = 0.5\ \text{m}$

$b = 0.5\ \dfrac{1}{\text{s}^2}$

$t_1 = 2\ \text{s}$





Solution:

$$t = t_1$$

Find the angle ψ using rectangular components. The velocity is parallel to the track therefore

$$x = r\cos(\theta) = \left(a\,b\,t^2\right)\cos\left(b\,t^2\right) \qquad x' = (2a\,b\,t)\cos\left(b\,t^2\right) - \left(2a\,b^2\,t^3\right)\sin\left(b\,t^2\right)$$

$$y = r\sin(\theta) = \left(a\,b\,t^2\right)\sin\left(b\,t^2\right) \qquad y' = (2a\,b\,t)\sin\left(b\,t^2\right) + \left(2a\,b^2\,t^3\right)\cos\left(b\,t^2\right)$$

$$\psi = \operatorname{atan}\left(\frac{y'}{x'}\right) - b\,t^2 + \pi \qquad \psi = 63.435\,\text{deg}$$

Now do the dynamics using polar coordinates

$$\theta = b\,t^2 \qquad \theta' = 2b\,t \qquad \theta'' = 2b \qquad r = a\theta \qquad r' = a\theta' \qquad r'' = a\theta''$$

Guesses $\quad F = 1\ \text{N} \quad N_C = 1\ \text{N}$

Given $\quad N_C\sin(\psi) - M\,g\sin(\theta) = M\left(r'' - r\theta'^2\right)$

$$F - N_C\cos(\psi) - M\,g\cos(\theta) = M(r\theta'' + 2r'\theta')$$

$$\begin{pmatrix} F \\ N_C \end{pmatrix} = \operatorname{Find}(F, N_C) \qquad \begin{pmatrix} F \\ N_C \end{pmatrix} = \begin{pmatrix} 1.814 \\ 3.032 \end{pmatrix}\ \text{N}$$

Problem 13-101

The ball has mass M and a negligible size. It is originally traveling around the horizontal circular path of radius r_0 such that the angular rate of rotation is θ'_0. If the attached cord ABC is drawn down through the hole at constant speed v, determine the tension the cord exerts on the ball at the instant $r = r_1$. Also, compute the angular velocity of the ball at this instant. Neglect the effects of friction between the ball and horizontal plane. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\theta'' + 2r'\theta' = (1/r)(d(r^2\theta')/dt) = 0$. When integrated, $r^2\theta' = c$ where the constant c is determined from the problem data.

Given:

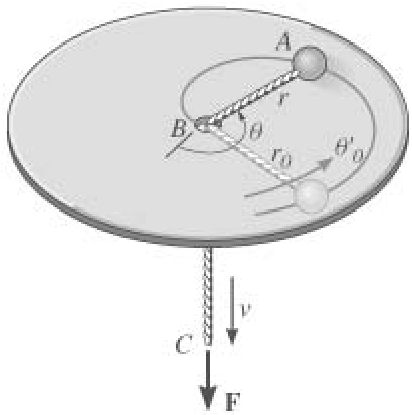
$M = 2\ \text{kg}$

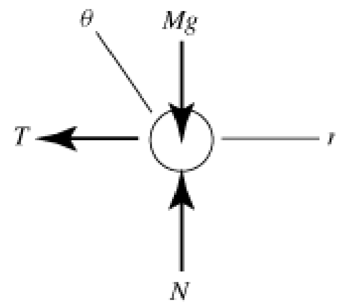
$r_0 = 0.5\ \text{m}$

$\theta'_0 = 1\ \frac{\text{rad}}{\text{s}}$

$v = 0.2\ \frac{\text{m}}{\text{s}}$

$r_1 = 0.25\ \text{m}$





Solution:

$$\Sigma F_\theta = Ma_\theta; \quad 0 = M(r\theta'' + 2r'\theta') = M\left[\frac{1}{r}\frac{d}{dt}\left(r^2\theta'\right)\right]$$

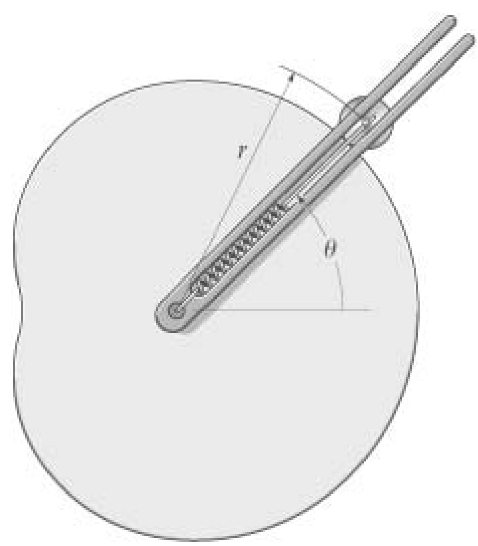
Thus $\quad c = r_0^{\,2}\theta'_0 = r_1^{\,2}\theta'_1 \qquad \theta'_1 = \left(\frac{r_0}{r_1}\right)^2\theta'_0 \qquad \theta'_1 = 4\ \frac{\text{rad}}{\text{s}}$

$$r = r_1 \qquad r' = -v \qquad r'' = 0\ \frac{\text{m}}{\text{s}^2} \qquad \theta' = \theta'_1$$

$$T = -M\left(r'' - r\theta'^2\right) \qquad T = 8\ \text{N}$$

Problem 13-102

The smooth surface of the vertical cam is defined in part by the curve $r = (a\cos\theta + b)$. If the forked rod is rotating with a constant angular velocity θ', determine the force the cam and the rod exert on the roller of mass M at angle θ. The attached spring has a stiffness k and an unstretched length l.



Given:

$a = 0.2\ \text{m} \qquad k = 30\ \frac{\text{N}}{\text{m}} \qquad \theta = 30\ \text{deg}$

$b = 0.3\ \text{m} \qquad l = 0.1\ \text{m} \qquad \theta' = 4\ \frac{\text{rad}}{\text{s}}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2} \qquad M = 2\ \text{kg} \qquad \theta'' = 0\ \frac{\text{rad}}{\text{s}^2}$

Solution:

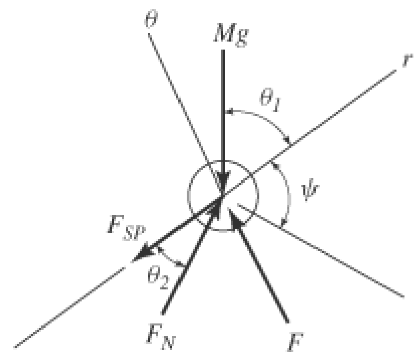
$$r = (a)\cos(\theta) + b$$

$$r' = -(a)\sin(\theta)\theta'$$

$$r'' = -(a)\cos(\theta)\theta'^2 - (a)\sin(\theta)\theta''$$

$$\psi = \operatorname{atan}\left(\frac{r\theta'}{r'}\right) + \pi$$

Guesses $F_N = 1\text{ N}$ $F = 1\text{ N}$



Given $F_N\sin(\psi) - M g\sin(\theta) - k(r-l) = M\left(r'' - r\theta'^2\right)$

$F - F_N\cos(\psi) - M g\cos(\theta) = M(r\theta'' + 2r'\theta')$

$$\begin{pmatrix} F \\ F_N \end{pmatrix} = \operatorname{Find}(F, F_N) \qquad \begin{pmatrix} F \\ F_N \end{pmatrix} = \begin{pmatrix} 10.524 \\ 0.328 \end{pmatrix}\text{ N}$$

Problem 13-103

The collar has mass M and travels along the smooth horizontal rod defined by the equiangular spiral $r = ae^{\theta}$. Determine the tangential force F and the normal force N_C acting on the collar when $\theta = \theta_1$ if the force F maintains a constant angular motion θ'.

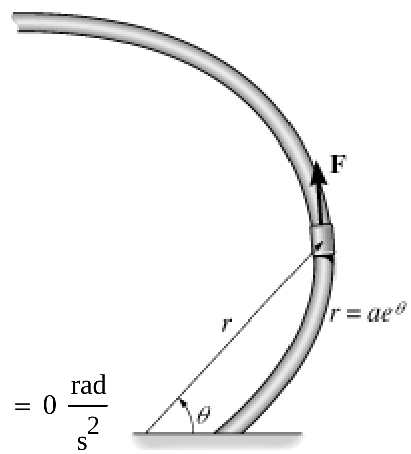
Given:

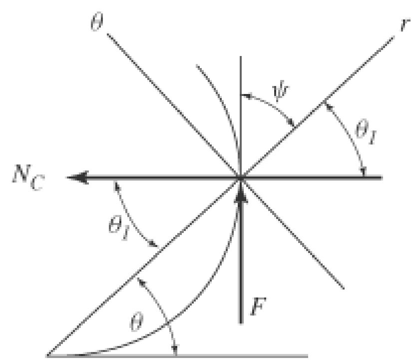
$M = 2\text{ kg}$

$a = 1\text{ m}$

$\theta_1 = 90\text{ deg}$

$\theta' = 2\ \dfrac{\text{rad}}{\text{s}}$





Solution:

$\theta = \theta_1 \quad \theta' = \theta' \quad \theta'' = 0\ \dfrac{\text{rad}}{\text{s}^2}$

$r = a e^{\theta} \qquad r' = a\theta' e^{\theta} \qquad r'' = a\left(\theta'' + \theta'^2\right)e^{\theta}$

Find the angle ψ using rectangular coordinates. The velocity is parallel to the path therefore

$x = r\cos(\theta) \qquad x' = r'\cos(\theta) - r\theta'\sin(\theta)$

$y = r\sin(\theta) \qquad y' = r'\sin(\theta) + r\theta'\cos(\theta)$

$\psi = \operatorname{atan}\left(\dfrac{y'}{x'}\right) - \theta + \pi \qquad \psi = 112.911\text{ deg}$

Now do the dynamics using polar coordinates Guesses $F = 1\text{ N}$ $N_C = 1\text{ N}$

Given $\quad F\cos(\psi) - N_C\cos(\psi) = M\left(r'' - r\theta'^2\right)$

$F\sin(\psi) + N_C\sin(\psi) = M(r\theta'' + 2r'\theta')$

$$\begin{pmatrix} F \\ N_C \end{pmatrix} = \text{Find}(F, N_C) \qquad \begin{pmatrix} F \\ N_C \end{pmatrix} = \begin{pmatrix} 10.2 \\ -13.7 \end{pmatrix} \text{ N}$$

***Problem 13-104**

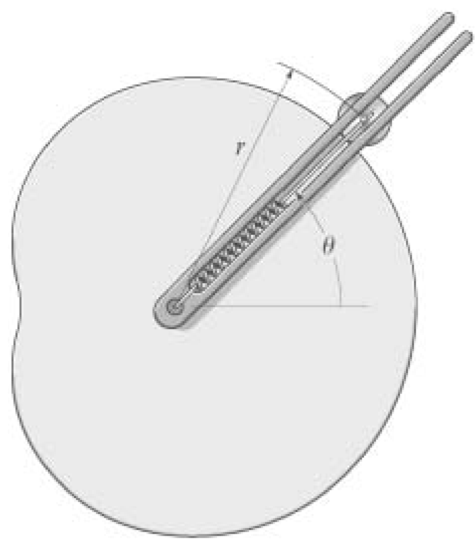
The smooth surface of the vertical cam is defined in part by the curve $r = (a\cos\theta + b)$. The forked rod is rotating with an angular acceleration θ'', and at angle θ the angular velocity is θ'. Determine the force the cam and the rod exert on the roller of mass M at this instant. The attached spring has a stiffness k and an unstretched length l.

Given:

$a = 0.2 \text{ m} \qquad k = 100 \frac{\text{N}}{\text{m}} \qquad \theta = 45 \text{ deg}$

$b = 0.3 \text{ m} \qquad l = 0.1 \text{ m} \qquad \theta' = 6 \frac{\text{rad}}{\text{s}}$

$g = 9.81 \frac{\text{m}}{\text{s}^2} \qquad M = 2 \text{ kg} \qquad \theta'' = 2 \frac{\text{rad}}{\text{s}^2}$



Solution:

$r = a\cos(\theta) + b \qquad r' = -(a)\sin(\theta)\theta'$

$r'' = -(a)\cos(\theta)\,\theta'^2 - (a)\sin(\theta)\theta''$

$\psi = \text{atan}\left(\frac{r\theta'}{r'}\right) + \pi$

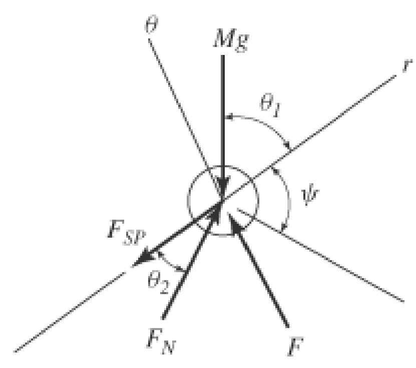
Guesses $\quad F_N = 1 \text{ N} \qquad F = 1 \text{ N}$

Given

$F_N\sin(\psi) - Mg\sin(\theta) - k(r - l) = M\left(r'' - r\theta'^2\right)$

$F - F_N\cos(\psi) - Mg\cos(\theta) = M(r\theta'' + 2r'\theta')$

$$\begin{pmatrix} F \\ F_N \end{pmatrix} = \text{Find}(F, F_N) \qquad \begin{pmatrix} F \\ F_N \end{pmatrix} = \begin{pmatrix} -6.483 \\ 5.76 \end{pmatrix} \text{ N}$$



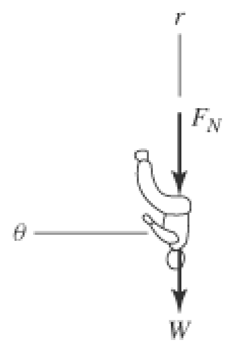
Problem 13-105

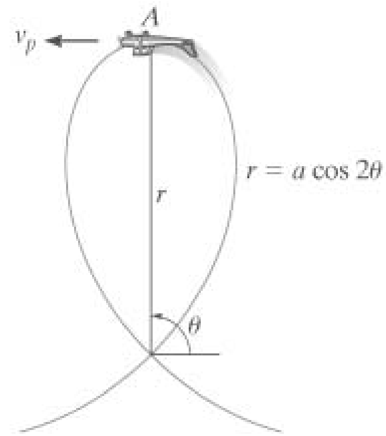
The pilot of an airplane executes a vertical loop which in part follows the path of a "four-leaved rose," $r = a\cos 2\theta$. If his speed at A is a constant v_p, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. His weight is W.

Given:

$a = -600 \text{ ft} \qquad W = 130 \text{ lb}$

$v_p = 80 \frac{\text{ft}}{\text{s}} \qquad g = 32.2 \frac{\text{ft}}{\text{s}^2}$





Solution:

$\theta = 90 \text{ deg}$

$r = (a)\cos(2\theta)$

Guesses

$r' = 1 \frac{\text{ft}}{\text{s}} \quad r'' = 1 \frac{\text{ft}}{\text{s}^2} \quad \theta' = 1 \frac{\text{rad}}{\text{s}} \quad \theta'' = 1 \frac{\text{rad}}{\text{s}^2}$

Given Note that v_p is constant so $dv_p/dt = 0$

$r' = -(a)\sin(2\theta)\,2\theta' \qquad r'' = -(a)\sin(2\theta)\,2\theta'' - (a)\cos(2\theta)\,4\theta'^2$

$v_p = \sqrt{r'^2 + (r\theta')^2} \qquad 0 = \dfrac{r'r'' + r\theta'(r\theta'' + r'\theta')}{\sqrt{r'^2 + (r\theta')^2}}$

$\begin{pmatrix} r' \\ r'' \\ \theta' \\ \theta'' \end{pmatrix} = \text{Find}(r', r'', \theta', \theta'') \qquad r' = 0.000 \frac{\text{ft}}{\text{s}} \qquad r'' = -42.7 \frac{\text{ft}}{\text{s}^2}$

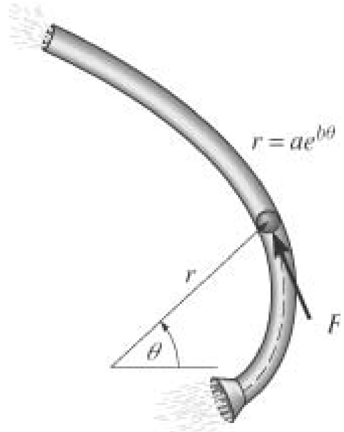
$\theta' = 0.133 \frac{\text{rad}}{\text{s}} \qquad \theta'' = 1.919 \times 10^{-14} \frac{\text{rad}}{\text{s}^2}$

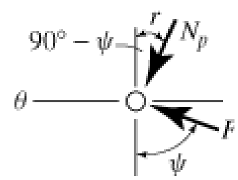
$-F_N - W = M(r'' - r\theta'^2) \qquad F_N = -W - \left(\frac{W}{g}\right)(r'' - r\theta'^2) \qquad F_N = 85.3 \text{ lb}$

Problem 13-106

Using air pressure, the ball of mass M is forced to move through the tube lying in the *horizontal plane* and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to the air is

F, determine the rate of increase in the ball's speed at the instant $\theta = \theta_1$ .What direction does it act in?





Given:

$M = 0.5$ kg $\quad a = 0.2$ m $\quad b = 0.1$

$\theta_1 = \dfrac{\pi}{2} \quad F = 6$ N

Solution:

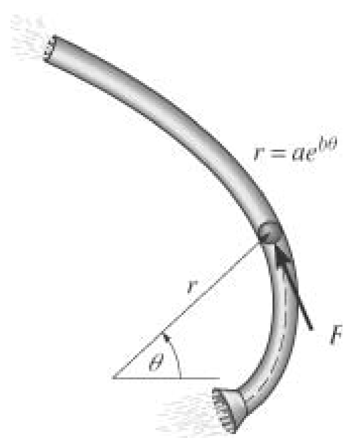
$$\tan(\psi) = \frac{r}{\frac{d}{d\theta}r} = \frac{a e^{b\theta}}{a b e^{b\theta}} = \frac{1}{b}$$

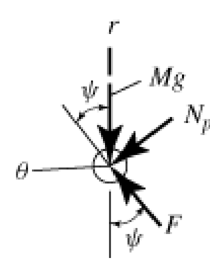
$$\psi = \operatorname{atan}\left(\frac{1}{b}\right) \qquad \psi = 84.289 \text{ deg}$$

$$F = M v' \qquad v' = \frac{F}{M} \qquad v' = 12 \frac{\text{m}}{\text{s}^2}$$

Problem 13-107

Using air pressure, the ball of mass M is forced to move through the tube lying in the *vertical plane* and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to the air is F, determine the rate of increase in the ball's speed at the instant $\theta = \theta_1$. What direction does it act in?





Given:

$M = 0.5$ kg $\quad a = 0.2$ m $\quad b = 0.1$

$F = 6$ N $\quad \theta_1 = \dfrac{\pi}{2}$

Solution:

$$\tan(\psi) = \frac{r}{\frac{d}{d\theta}r} = \frac{a e^{b\theta}}{a b e^{b\theta}} = \frac{1}{b}$$

$$\psi = \operatorname{atan}\left(\frac{1}{b}\right) \qquad \psi = 84.289 \text{ deg}$$

$$F - M g \cos(\psi) = M v' \qquad v' = \frac{F}{M} - g\cos(\psi) \qquad v' = 11.023 \frac{\text{m}}{\text{s}^2}$$

***Problem 13-108**

The arm is rotating at the rate θ' when the angular acceleration is θ'' and the angle is θ_0. Determine the normal force it must exert on the particle of mass M if the particle is confined to move along the slotted path defined by the *horizontal* hyperbolic spiral $r\theta = b$.

Given:

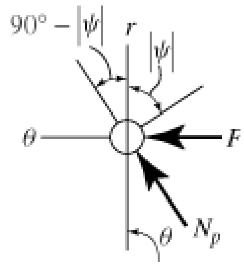
$$\theta' = 5\ \frac{\text{rad}}{\text{s}}$$

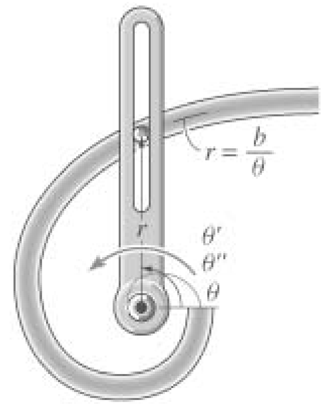
$$\theta'' = 2\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta_0 = 90\ \text{deg}$$

$$M = 0.5\ \text{kg}$$

$$b = 0.2\ \text{m}$$





Solution: $\theta = \theta_0 \qquad r = \dfrac{b}{\theta} \qquad r' = \left(\dfrac{-b}{\theta^2}\right)\theta' \qquad r'' = \left(\dfrac{-b}{\theta^2}\right)\theta'' + \left(\dfrac{2b}{\theta^3}\right)\theta'^2$

$$\tan(\psi) = \frac{r}{\dfrac{d}{d\theta}r} = \frac{\dfrac{b}{\theta}}{\dfrac{-b}{\theta^2}} = -\theta \qquad \psi = \operatorname{atan}(-\theta) \qquad \psi = -57.518\ \text{deg}$$

Guesses $\quad N_P = 1\ \text{N} \qquad F = 1\ \text{N}$

Given $\quad -N_P\sin(\psi) = M\left(r'' - r\theta'^2\right) \qquad F + N_P\cos(\psi) = M(r\theta'' + 2r'\theta')$

$$\begin{pmatrix} N_P \\ F \end{pmatrix} = \operatorname{Find}(N_P, F) \qquad \begin{pmatrix} N_P \\ F \end{pmatrix} = \begin{pmatrix} -0.453 \\ -1.656 \end{pmatrix}\ \text{N}$$

Problem 13-109

The collar, which has weight W, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = a/(1 - \cos\theta)$. If the collar's angular rate is θ', determine the tangential retarding force P needed to cause the motion and the normal force that the collar exerts on the rod at the instatnt $\theta = \theta_1$.

Given:

$$W = 3\ \text{lb}$$

$$a = 4\ \text{ft}$$

$$\theta' = 4\ \frac{\text{rad}}{\text{s}}$$

$$\theta'' = 0\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta_1 = 90\text{ deg} \qquad g = 32.2\,\frac{\text{ft}}{\text{s}^2} \qquad M = \frac{W}{g}$$

Solution: $\theta = \theta_1$

$$r = \frac{a}{1-\cos(\theta)} \qquad r' = \frac{-a\sin(\theta)}{(1-\cos(\theta))^2}\theta'$$

$$r'' = \frac{-a\sin(\theta)}{(1-\cos(\theta))^2}\theta'' + \frac{-a\cos(\theta)\,\theta'^2}{(1-\cos(\theta))^2} + \frac{2a\sin(\theta)^2\,\theta'^2}{(1-\cos(\theta))^3}$$

Find the angle ψ using rectangular coordinates. The velocity is parallel to the path

$$x = r\cos(\theta) \qquad x' = r'\cos(\theta) - r\theta'\sin(\theta) \qquad y = r\sin(\theta) \qquad y' = r'\sin(\theta) + r\theta'\cos(\theta)$$

$$x'' = r''\cos(\theta) - 2r'\theta'\sin(\theta) - r\theta''\sin(\theta) - r\theta'^2\cos(\theta)$$

$$y'' = r''\sin(\theta) + 2r'\theta'\cos(\theta) + r\theta''\sin(\theta) - r\theta'^2\sin(\theta)$$

$$\psi = \operatorname{atan}\left(\frac{y'}{x'}\right) \qquad \psi = 45\,\text{deg} \qquad \text{Guesses} \qquad P = 1\text{ lb} \qquad H = 1\text{ lb}$$

Given $\quad P\cos(\psi) + H\sin(\psi) = M\,x'' \qquad P\sin(\psi) - H\cos(\psi) = M\,y''$

$$\begin{pmatrix} P \\ H \end{pmatrix} = \operatorname{Find}(P, H) \qquad \begin{pmatrix} P \\ H \end{pmatrix} = \begin{pmatrix} 12.649 \\ 4.216 \end{pmatrix}\text{lb}$$

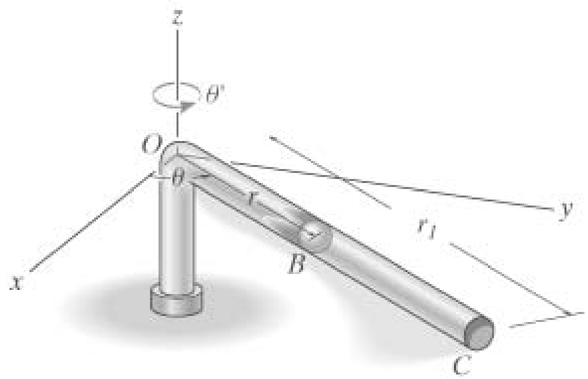
Problem 13-110

The tube rotates in the horizontal plane at a constant rate θ'. If a ball B of mass M starts at the origin O with an initial radial velocity r'_0 and moves outward through the tube, determine the radial and transverse components of the ball's velocity at the instant it leaves the outer end at C. *Hint:* Show that the equation of motion in the r direction is $r'' - r\theta'^2 = 0$. The solution is of the form $r = Ae^{-\theta' t} + Be^{\theta' t}$. Evaluate the integration constants A and B, and determine the time t at r_1. Proceed to obtain v_r and v_θ.

Given:

$\theta' = 4\ \frac{\text{rad}}{\text{s}}$ $\quad r'_0 = 1.5\ \frac{\text{m}}{\text{s}}$

$M = 0.2\ \text{kg}$ $\quad r_1 = 0.5\ \text{m}$



Solution:

$0 = M\left(r'' - r\theta'^2\right)$

$r(t) = A e^{\theta' t} + B e^{-\theta' t}$

$r'(t) = \theta'\left(A e^{\theta' t} - B e^{-\theta' t}\right)$

Guess $\quad A = 1\ \text{m} \quad B = 1\ \text{m}$

$t = 1\ \text{s}$

Given $\quad 0 = A + B \quad r'_0 = \theta'(A - B) \quad r_1 = A e^{\theta' t} + B e^{-\theta' t}$

$\begin{pmatrix} A \\ B \\ t_1 \end{pmatrix} = \text{Find}(A, B, t) \qquad \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0.188 \\ -0.188 \end{pmatrix}\ \text{m} \qquad t_1 = 0.275\ \text{s}$

$r(t) = A e^{\theta' t} + B e^{-\theta' t} \qquad r'(t) = \theta'\left(A e^{\theta' t} - B e^{-\theta' t}\right)$

$v_r = r'\left(t_1\right) \qquad v_\theta = r\left(t_1\right)\theta'$

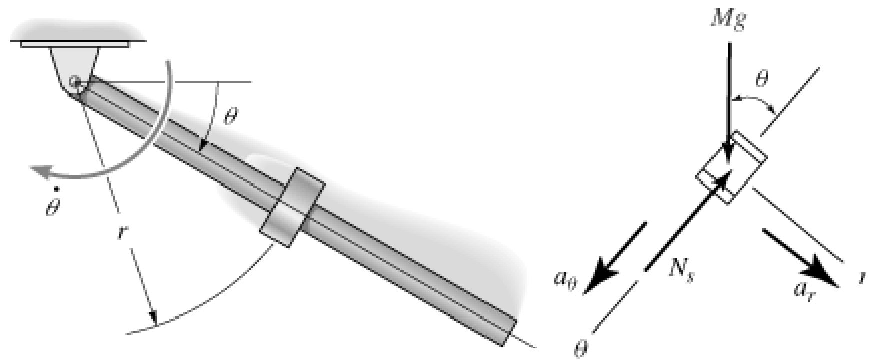
$\begin{pmatrix} v_r \\ v_\theta \end{pmatrix} = \begin{pmatrix} 2.5 \\ 2 \end{pmatrix} \frac{\text{m}}{\text{s}}$

Problem 13-111

A spool of mass M slides down along a smooth rod. If the rod has a constant angular rate of rotation θ' in the vertical plane, show that the equations of motion for the spool are $r'' - r\theta'^2 - g\sin\theta = 0$ and $2M\theta' r' + N_s - M g\cos\theta = 0$ where N_s is the magnitude of the normal force of the rod on the spool.

Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1 e^{-\theta' t} + C_2 e^{\theta' t} - (g/2\theta'^2)\sin(\theta' t)$. If r, r' and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 and determine r at the instant $\theta = \theta_1$.

Given:



$M = 0.2\ \text{kg}$

$\theta' = 2\ \dfrac{\text{rad}}{\text{s}}$

$\theta_1 = \dfrac{\pi}{4}$

$\theta'' = 0\ \dfrac{\text{rad}}{\text{s}^2}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$\Sigma F_r = Ma_r;$ $\quad M\,g\sin(\theta) = M\left(r'' - r\theta'^2\right)$ $\quad r'' - r\theta'^2 - g\sin(\theta) = 0$ [1]

$\Sigma F_\theta = Ma_\theta;$ $\quad M\,g\cos(\theta) - N_s = M(r\theta'' + 2r'\theta')$ $\quad 2M\theta'\,r' + N_s - M\,g\cos(\theta) = 0$

(Q.E.D)

The solution of the differential equation (Eq.[1] is given by

$$r = C_1 e^{-\theta' t} + C_2 e^{\theta' t} - \left(\frac{g}{2\theta'^2}\right)\sin(\theta' t)$$

$$r' = -\theta' C_1 e^{-\theta' t} + \theta' C_2 e^{\theta' t} - \left(\frac{g}{2\theta'}\right)\cos(\theta' t)$$

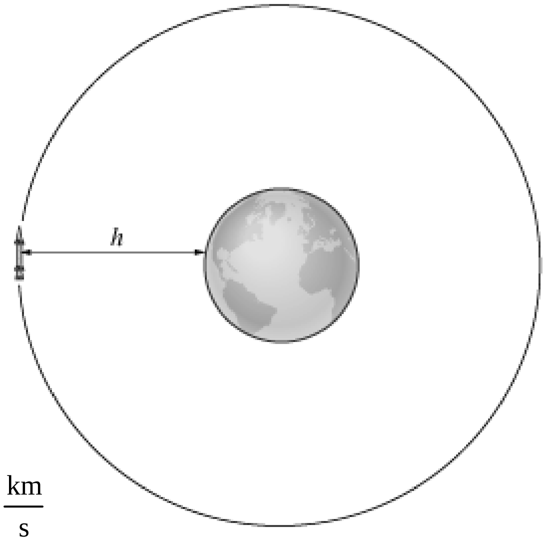
At $\quad t = 0 \quad r = 0 \quad 0 = C_1 + C_2 \quad r' = 0 \quad 0 = -\theta' C_1 + \theta' C_2 - \dfrac{g}{2\theta'}$

Thus $\quad C_1 = \dfrac{-g}{4\theta'^2} \quad C_2 = \dfrac{g}{4\theta'^2} \quad t = \dfrac{\theta_1}{\theta'} \quad t = 0.39\ \text{s}$

$r = C_1 e^{-\theta' t} + C_2 e^{\theta' t} - \left(\dfrac{g}{2\theta'^2}\right)\sin(\theta' t) \quad r = 0.198\ \text{m}$

***Problem 13-112**

The rocket is in circular orbit about the earth at altitude h. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



Given:

$$h = 4\,10^6 \text{ m}$$

$$G = 66.73 \times 10^{-12} \frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$$

$$M_e = 5.976 \times 10^{24} \text{ kg}$$

$$R_e = 6378 \text{ km}$$

Solution:

Circular orbit: $v_C = \sqrt{\dfrac{G M_e}{R_e + h}}$ $\qquad v_C = 6.199 \dfrac{\text{km}}{\text{s}}$

Parabolic orbit: $v_e = \sqrt{\dfrac{2G M_e}{R_e + h}}$ $\qquad v_e = 8.766 \dfrac{\text{km}}{\text{s}}$

$\Delta v = v_e - v_C$ $\qquad \Delta v = 2.57 \dfrac{\text{km}}{\text{s}}$

Problem 13-113

Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

Solution:

From Eq. 13-19, $\qquad \dfrac{1}{r} = C\cos(\theta) + \dfrac{GM_s}{h^2}$

For $\theta = 0\,\text{deg}$ and $\theta = 180\,\text{deg}$ $\qquad \dfrac{1}{r_\rho} = C + \dfrac{GM_s}{h^2} \qquad \dfrac{1}{r_a} = -C + \dfrac{GM_s}{h^2}$

Eliminating *C*, $\qquad$ From Eqs. 13-28 and 13-29, $\qquad \dfrac{2a}{b^2} = \dfrac{2GM_s}{h^2}$

From Eq. 13-31, $\qquad T = \dfrac{\pi}{h}(2a)(b)$

Thus, $\qquad b^2 = \dfrac{T^2 h^2}{4\pi^2 a^2} \qquad \dfrac{4\pi^2 a^2}{T^2 h^2} = \dfrac{GM_s}{h^2} \qquad T^2 = \left(\dfrac{4\pi^2}{GM_s}\right) a^2$

Problem 13-114

A satellite is to be placed into an elliptical orbit about the earth such that at the perigee of its orbit it has an *altitude* h_p, and at apogee its *altitude* is h_a. Determine its required launch velocity tangent to the earth's surface at perigee and the period of its orbit.

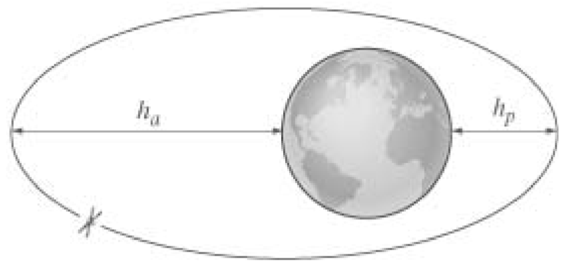
Given:

$h_p = 800 \text{ km}$ $\quad G = 66.73 \times 10^{-12} \dfrac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$

$h_a = 2400 \text{ km}$ $\quad M_e = 5.976 \times 10^{24} \text{ kg}$

$s_1 = 6378 \text{ km}$

Solution:



$r_p = h_p + s_1 \qquad r_p = 7178\,\text{km}$

$r_a = h_a + s_1 \qquad r_a = 8778\,\text{km}$

$$r_a = \frac{r_p}{\dfrac{2GM_e}{r_p v_0^2} - 1}$$

$$v_0 = \left(\frac{1}{r_a r_p + r_p^2}\right)\sqrt{2}\sqrt{r_p(r_a + r_p)r_a G M_e} \qquad v_0 = 7.82\,\frac{\text{km}}{\text{s}}$$

$$h = r_p v_0 \qquad h = 56.12 \times 10^9\,\frac{\text{m}^2}{\text{s}}$$

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a} \qquad T = 1.97\,\text{hr}$$

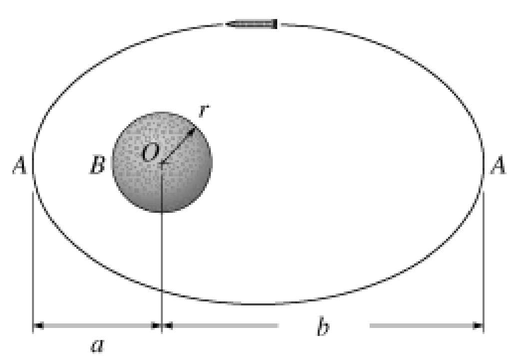
Problem 13-115

The rocket is traveling in free flight along an elliptical trajectory The planet has a mass k times that of the earth's. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.

Units Used:

$\text{Mm} = 10^3 \text{ km}$

Given:



$k = 0.60$

$a = 6.40 \text{ Mm}$

$b = 16 \text{ Mm}$

$r = 3.20 \text{ Mm}$

$$G = 6.673 \times 10^{-11}\ \text{N}\cdot\frac{\text{m}^2}{\text{kg}^2}$$

$$M_e = 5.976 \times 10^{24}\ \text{kg}$$

Solution: *Central - Force Motion*: Substitute Eq 13-27

$$r_a = \frac{r_0}{\frac{2GM}{r_0 v_0^2} - 1} \qquad \text{with} \quad r_0 = r_p = a \quad \text{and} \quad M = kM_e$$

$$b = \frac{a}{\frac{2GM}{a v_0^2} - 1} \qquad \frac{a}{b} = \left(\frac{2GM}{a v_0^2} - 1\right) \qquad \left(1 + \frac{a}{b}\right) = \frac{2GkM_e}{a v_p^2}$$

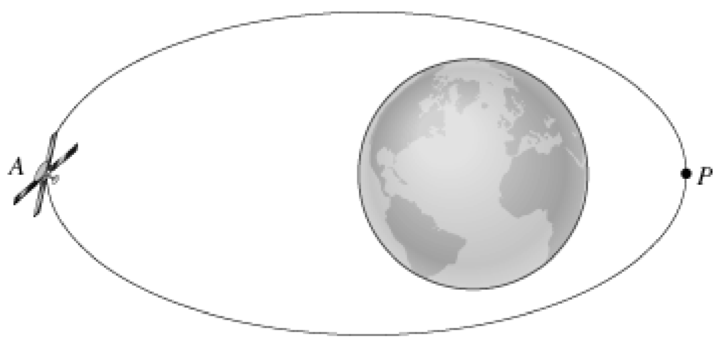
$$v_p = \sqrt{\frac{2GkM_e b}{(a+b)a}} \qquad v_p = 7.308\ \frac{\text{km}}{\text{s}}$$

***Problem 13-116**

An elliptical path of a satellite has an eccentricity e. If it has speed v_p when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?

Units Used:

$$\text{Mm} = 10^3\ \text{km}$$



Given:

$$e = 0.130$$

$$v_p = 15\ \frac{\text{Mm}}{\text{hr}}$$

$$G = 6.673 \times 10^{-11}\ \text{N}\cdot\frac{\text{m}^2}{\text{kg}^2}$$

$$M_e = 5.976 \times 10^{24}\ \text{kg}$$

$$R_e = 6.378 \times 10^6\ \text{m}$$

Solution: $v_0 = v_p \qquad e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right) \qquad r_0 = \frac{(e+1)GM_e}{v_0^2} \qquad r_0 = 25.956\ \text{Mm}$

$$r_A = \frac{r_0(e+1)}{1-e} \qquad r_A = 33.7\,\text{Mm} \qquad v_A = \frac{v_0 r_0}{r_A} \qquad v_A = 11.5\,\frac{\text{Mm}}{\text{hr}}$$

$$d = r_A - R_e \qquad d = 27.3\,\text{Mm}$$

Problem 13-117

A communications satellite is to be placed into an equatorial circular orbit around the earth so that it always remains directly over a point on the earth's surface. If this requires the period T (approximately), determine the radius of the orbit and the satellite's velocity.

Units Used: $\text{Mm} = 10^3\ \text{km}$

Given: $T = 24\ \text{hr}$ $\quad G = 66.73 \times 10^{-12}\ \frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$ $\quad M_e = 5.976 \times 10^{24}\ \text{kg}$

Solution:

$$\frac{G M_e M_s}{r^2} = \frac{M_s v^2}{r} \qquad \frac{G M_e}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$r = \frac{1}{2\pi}\, 2^{\frac{1}{3}} \left(G\ M_e\ T^2\ \pi\right)^{\frac{1}{3}} \qquad r = 42.2\,\text{Mm}$$

$$v = \frac{2\pi r}{T} \qquad v = 3.07\,\frac{\text{km}}{\text{s}}$$

Problem 13-118

The rocket is traveling in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is c times that of the earth's. If the rocket has the apogee and perigee shown, determine the rocket's velocity when it is at point A.

Given:

$a = 4000\ \text{mi}$

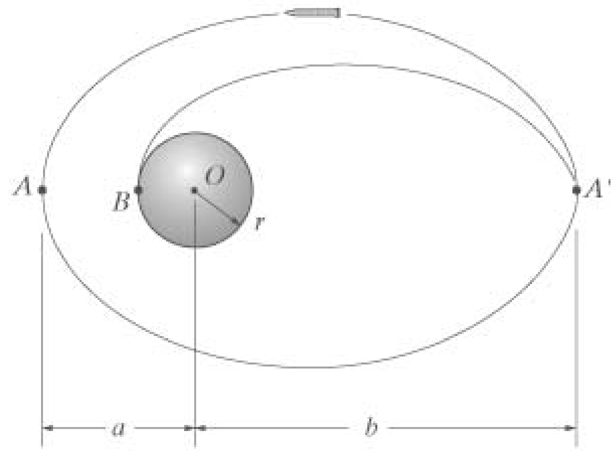
$b = 10000\ \text{mi}$

$c = 0.6$

$r = 2000\ \text{mi}$

$G = 34.4 \times 10^{-9}\ \dfrac{\text{lbf}\cdot\text{ft}^2}{\text{slug}^2}$

$M_e = 409 \times 10^{21}\ \text{slug}$



Solution:

$r_0 = a \qquad OA' = b \qquad M_p = M_e c$

$$OA' = \frac{r_0}{2\left(\dfrac{G M_p}{r_0 v_0^2}\right) - 1} \qquad v_0 = \sqrt{\frac{2 G M_p}{r_0\left(\dfrac{r_0}{OA'} + 1\right)}} \qquad v_0 = 23.9 \times 10^3\ \frac{\text{ft}}{\text{s}}$$

Problem 13-119

The rocket is traveling in free flight along an elliptical trajectory $A'A$. If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that the landing occurs at B. How long does it take for the rocket to land, in going from A' to B? The planet has no atmosphere, and its mass is 0.6 times that of the earth's.

Units Used:

$\text{Mm} = 10^3\ \text{km}$

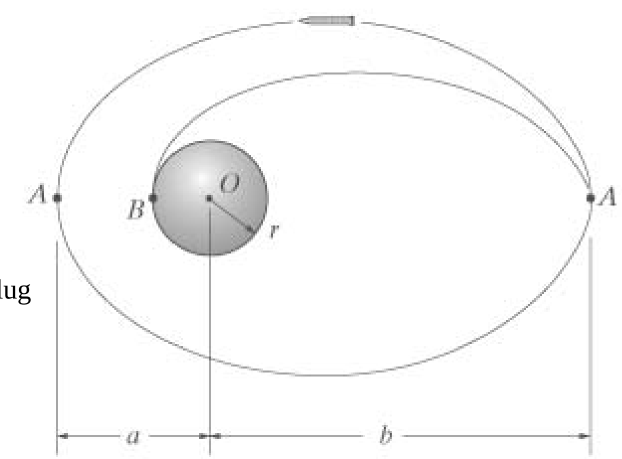
Given:

$a = 4000\ \text{mi} \qquad r = 2000\ \text{mi}$

$b = 10000\ \text{mi} \qquad M_e = 409 \times 10^{21}\ \text{slug}$

$c = 0.6$

$G = 34.4 \times 10^{-9}\ \dfrac{\text{lbf}\cdot\text{ft}^2}{\text{slug}^2}$



Solution:

$M_p = M_e c \qquad OA' = b \qquad OB = r \qquad OA' = \dfrac{OB}{2\left(\dfrac{G M_p}{OB\, v_0^2}\right) - 1}$

$$v_0 = \frac{\sqrt{2}}{OA'\,OB + OB^2}\sqrt{OB\,(OA' + OB)\,OA'\,G\,M_p} \qquad v_0 = 36.5 \times 10^3\ \frac{\text{ft}}{\text{s}} \qquad \text{(speed at } B\text{)}$$

$$v_{A'} = \frac{OB\,v_0}{OA'} \qquad v_{A'} = 7.3 \times 10^3\ \frac{\text{ft}}{\text{s}} \qquad h = OB\,v_0 \qquad h = 385.5 \times 10^9\ \frac{\text{ft}^2}{\text{s}}$$

Thus,

$$T = \frac{\pi(OB + OA')}{h}\sqrt{OB\,OA'} \qquad T = 12.19 \times 10^3\ \text{s} \qquad \frac{T}{2} = 1.69\,\text{hr}$$

***Problem 13-120**

The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13-25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit a distance d from the earth's surface.

Given: $d = 800\ \text{km}$ $\qquad G = 6.673 \times 10^{-11}\ \text{N}\cdot\frac{\text{m}^2}{\text{kg}^2}$ $\qquad M_e = 5.976 \times 10^{24}\ \text{kg}$

$r_e = 6378\ \text{km}$

Solution:

$$v = \sqrt{\frac{G\,M_e}{d + r_e}} \qquad v = 7.454\ \frac{\text{km}}{\text{s}}$$

Problem 13-121

The rocket is traveling in free flight along an elliptical trajectory $A'A$ .The planet has no atmosphere, and its mass is k times that of the earth's. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.

Units used:

$\text{Mm} = 10^3\ \text{km}$

Given:

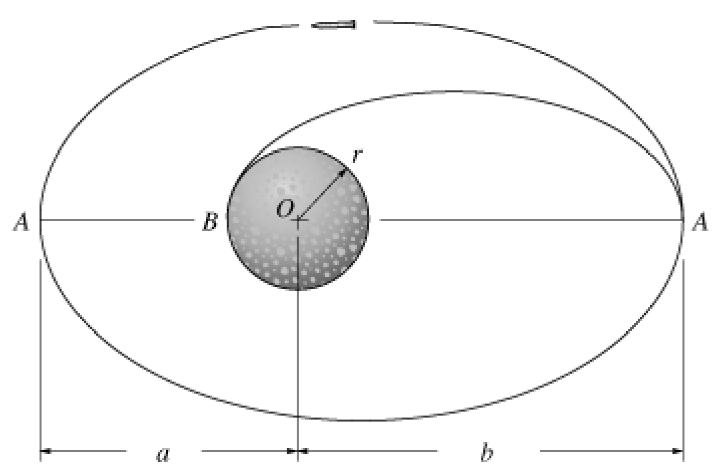
$k = 0.70$

$a = 6\ \text{Mm}$

$b = 9\ \text{Mm}$

$r = 3\ \text{Mm}$

$M_e = 5.976 \times 10^{24}\ \text{kg}$



$$G = 6.673 \times 10^{-11}\ \text{N}\cdot\frac{\text{m}^2}{\text{kg}^2}$$

Solution:

Central - Force motion:

$$r_a = \frac{r_0}{\frac{2GM}{r_0 v_0^2} - 1} \qquad b = \frac{a}{\frac{2G(kM_e)}{a v_p^2} - 1} \qquad v_p = \sqrt{\frac{2GkM_e b}{a(a+b)}} \qquad v_p = 7.472\ \frac{\text{km}}{\text{s}}$$

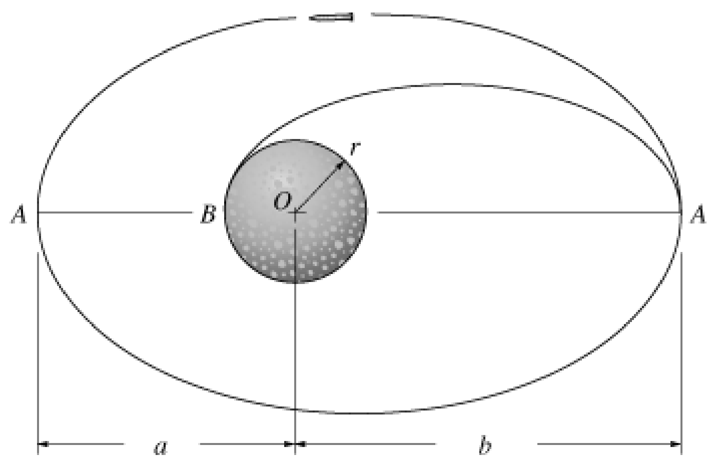
Problem 13-122

The rocket is traveling in free flight along an elliptical trajectory $A'A$ .The planet has no atmosphere, and its mass is k times that of the earth's. The rocket has an apoapsis and periapsis as shown in the figure. If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that it strikes the planet at B. How long does it take for the rocket to land, going from A' to B along an elliptical path?

Units used:

$$\text{Mm} = 10^3\ \text{km}$$

Given:



$k = 0.70$

$a = 6\ \text{Mm}$

$b = 9\ \text{Mm}$

$r = 3\ \text{Mm}$

$M_e = 5.976 \times 10^{24}\ \text{kg}$

$G = 6.673 \times 10^{-11}\ \text{N}\cdot\frac{\text{m}^2}{\text{kg}^2}$

Solution:

Central Force motion:

$$r_a = \frac{r_0}{\frac{2GM}{r_0 v_0^2} - 1} \qquad b = \frac{r}{\frac{2G(kM_e)}{r v_p^2} - 1} \qquad v_p = \sqrt{\frac{2GkM_e b}{r(r+b)}} \qquad v_p = 11.814\ \frac{\text{km}}{\text{s}}$$

$$r_a v_a = r_p v_p \qquad v_a = \left(\frac{r}{b}\right) v_p \qquad v_a = 3.938\ \frac{\text{km}}{\text{s}}$$

Eq.13-20 gives $h = v_p r$ $h = 35.44 \times 10^9 \frac{m^2}{s}$

Thus, applying Eq.13-31 we have $T = \frac{\pi}{h}(r+b)\sqrt{rb}$ $T = 5.527 \times 10^3$ s

The time required for the rocket to go from A' to B (half the orbit) is given by

$t = \frac{T}{2}$ $t = 46.1$ min

Problem 13-123

A satellite S travels in a circular orbit around the earth. A rocket is located at the apogee of its elliptical orbit for which the eccentricity is e. Determine the sudden change in speed that must occur at A so that the rocket can enter the satellite's orbit while in free flight along the blue elliptical trajectory. When it arrives at B, determine the sudden adjustment in speed that must be given to the rocket in order to maintain the circular orbit.

Units used:

$\text{Mm} = 10^3 \text{ km}$

Given:

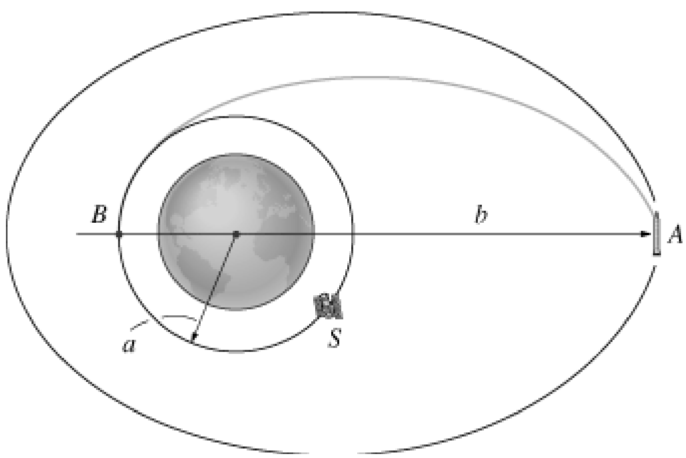
$e = 0.58$

$a = 10 \text{ Mm}$

$b = 120 \text{ Mm}$

$M_e = 5.976 \times 10^{24} \text{ kg}$

$G = 6.673 \times 10^{-11} \text{ N} \cdot \frac{m^2}{kg^2}$



Solution:

Central - Force motion: $C = \frac{1}{r_0}\left(1 - \frac{G M_e}{r_0 v_0^2}\right)$ $h = r_0 v_0$ $e = \frac{C h^2}{G M_e} = \frac{r_0 v_0^2}{G M_e} - 1$

$$v_0 = \sqrt{\frac{(1+e) G M_e}{r_0}} \qquad r_a = \frac{r_0}{\left(\frac{2 G M_e}{r v_0^2}\right) - 1} = \frac{r_0}{2\left(\frac{1}{1+e}\right) - 1}$$

$$r_0 = r_a\left(\frac{1-e}{1+e}\right) \qquad r_0 = b\left(\frac{1-e}{1+e}\right) \qquad r_0 = 31.90 \times 10^6 \text{ m}$$

Substitute $r_{p1} = r_0$ $v_{p1} = \sqrt{\frac{(1+e)(G)(M_e)}{r_{p1}}}$ $v_{p1} = 4.444 \times 10^3 \frac{m}{s}$

$$v_{a1} = \left(\frac{r_{p1}}{b}\right)v_{p1} \qquad v_{a1} = 1.181 \times 10^3\ \frac{\text{m}}{\text{s}}$$

When the rocket travels along the second elliptical orbit , from Eq.[4] , we have

$$a = \left(\frac{1-e'}{1+e'}\right)b \qquad e' = \frac{-a+b}{b+a} \qquad e' = 0.8462$$

Substitute $r_0 = r_{p2} = a$ $\quad r_{p2} = a \quad v_{p2} = \sqrt{\frac{(1+e')(G)(M_e)}{r_{p2}}} \quad v_{p2} = 8.58 \times 10^3\ \frac{\text{m}}{\text{s}}$

Applying Eq. 13-20, we have $\quad v_{a2} = \frac{r_{p2}}{b}v_{p2} \qquad v_{a2} = 715.021\ \frac{\text{m}}{\text{s}}$

For the rocket to enter into orbit two from orbit one at A, its speed must be decreased by

$$\Delta v = v_{a1} - v_{a2} \qquad \Delta v = 466\ \frac{\text{m}}{\text{s}}$$

If the rocket travels in a circular free - flight trajectory , its speed is given by Eq. 13-25

$$v_c = \sqrt{\frac{GM_e}{a}} \qquad v_c = 6.315 \times 10^3\ \frac{\text{m}}{\text{s}}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = v_{p2} - v_c \qquad \Delta v = 2.27\ \frac{\text{km}}{\text{s}}$$

***Problem 13-124**

An asteroid is in an elliptical orbit about the sun such that its perihelion distance is d. If the eccentricity of the orbit is e, determine the aphelion distance of the orbit.

Given: $\quad d = 9.30 \times 10^9$ km $\qquad e = 0.073$

Solution: $\quad r_p = d \qquad r_0 = d$

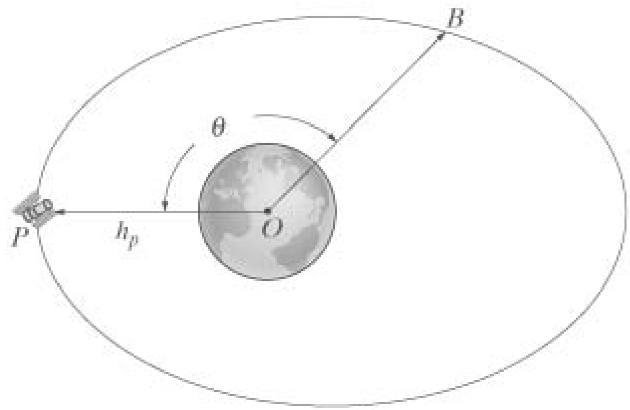
$$e = \frac{Ch^2}{GM_s} = \frac{1}{r_0}\left(1 - \frac{GM_s}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_s}\right) \qquad e = \left(\frac{r_0 v_0^2}{GM_s} - 1\right)$$

$$\frac{GM_s}{r_0 v_0^2} = \frac{1}{e+1} \qquad r_A = \frac{r_0}{\frac{2}{e+1} - 1} \qquad r_A = \frac{r_0(e+1)}{1-e} \qquad r_A = 10.76 \times 10^9\ \text{km}$$

Problem 13-125

A satellite is in an elliptical orbit around the earth with eccentricity e. If its perigee is h_p, determine its velocity at this point and also the distance OB when it is at point B, located at angle θ from perigee as shown.

Units Used: $\mathrm{Mm} = 10^3\ \mathrm{km}$



Given:

$e = 0.156$

$\theta = 135\ \mathrm{deg}$

$h_p = 5\ \mathrm{Mm}$

$G = 66.73 \times 10^{-12}\ \dfrac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}$

$M_e = 5.976 \times 10^{24}\ \mathrm{kg}$

Solution:

$$e = \frac{Ch^2}{GM_e} = \frac{1}{h_p}\left(1 - \frac{GM_e}{h_p v_0^{\,2}}\right)\left(\frac{h_p^{\,2} v_0^{\,2}}{GM_e}\right) \qquad \frac{h_p v^2}{GM_e} = e + 1$$

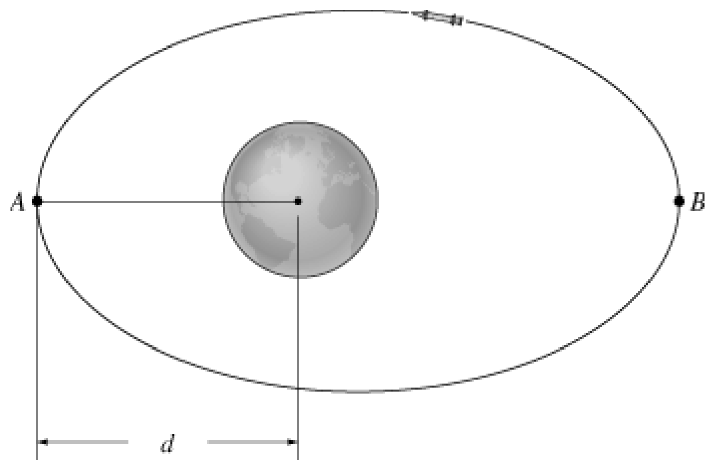
$$v_0 = \frac{1}{h_p}\sqrt{h_p GM_e(e+1)} \qquad v_0 = 9.6\,\frac{\mathrm{km}}{\mathrm{s}}$$

$$\frac{1}{r} = \frac{1}{h_p}\left(1 - \frac{GM_e}{h_p v_0^{\,2}}\right)\cos(\theta) + \frac{GM_e}{h_p^{\,2} v_0^{\,2}}$$

$$\frac{1}{r} = \frac{1}{h_p}\left(1 - \frac{1}{e+1}\right)\cos(\theta) + \frac{1}{h_p}\left(\frac{1}{e+1}\right)$$

$$r = h_p\left(\frac{e+1}{e\cdot\cos(\theta) + 1}\right) \qquad r = 6.5\,\mathrm{Mm}$$

Problem 13-126



The rocket is traveling in a free-flight elliptical orbit about the earth such that the eccentricity is *e* and its perigee is a distanced *d* as shown. Determine its speed when it is at point *B*. Also determine the sudden decrease in speed the rocket must experience at *A* in order to travel in a circular orbit about the earth.

Given:

$e = 0.76$

$d = 9 \times 10^{6}\ \text{m}$

$G = 6.673 \times 10^{-11}\ \text{N}\cdot\frac{\text{m}^2}{\text{kg}^2}$

$M_e = 5.976 \times 10^{24}\ \text{kg}$

Solution:

Central - Force motion:

$$C = \frac{1}{r_0}\left(1 - \frac{G M_e}{r_0 v_0^{\,2}}\right) \qquad h = r_0 v_0$$

$$e = \frac{c h^2}{G M_e} = \frac{r_0 v_0^{\,2}}{G M_e} - 1 \qquad \frac{1}{1+e} = \frac{G M_e}{r_0 v_0^{\,2}} \qquad v_0 = \sqrt{\frac{(1+e) G M_e}{r_0}}$$

$$r_a = \left(\frac{1+e}{1-e}\right)d \qquad r_a = 66 \times 10^{6}\ \text{m} \qquad r_p = d$$

$$v_p = \sqrt{\frac{(1+e) G M_e}{d}} \qquad v_p = 8.831\ \frac{\text{km}}{\text{s}} \qquad v_a = \left(\frac{d}{r_a}\right) v_p \qquad v_a = 1.2\ \frac{\text{km}}{\text{s}}$$

If the rockets in a cicular free - fright trajectory, its speed is given by eq.13-25

$$v_c = \sqrt{\frac{G M_e}{d}} \qquad v_c = 6656.48\ \frac{\text{m}}{\text{s}}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = v_p - v_c \qquad \Delta v = 2.17\ \frac{\text{km}}{\text{s}}$$

Problem 13-127

A rocket is in free-flight elliptical orbit around the planet Venus. Knowing that the periapsis and apoapsis of the orbit are r_p and a_p, respectively, determine (a) the speed of the rocket at point A', (b) the required speed it must attain at A just after braking so that it undergoes a free-flight circular orbit around Venus, and (c) the periods of both the circular and elliptical orbits. The mass of Venus is a times the mass of the earth.

Units Used:

$$Mm = 10^3\ km$$

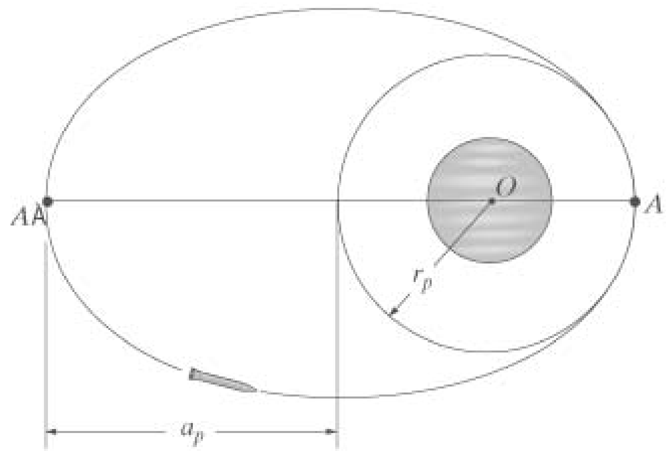
Given:

$$a = 0.816 \qquad a_p = 26\ Mm$$

$$f = 8\ Mm \qquad r_p = 8\ Mm$$

$$G = 66.73 \times 10^{-12}\ \frac{m^3}{kg \cdot s^2}$$

$$M_e = 5.976 \times 10^{24}\ kg$$



Solution:

$$M_v = a M_e \qquad M_v = 4.876 \times 10^{24}\ kg$$

$$OA' = \frac{OA}{2\left(\frac{G M_p}{OA\, v_0{}^2}\right) - 1} \qquad a_p = \frac{r_p}{\frac{2 G M_v}{r_p v_A{}^2} - 1}$$

$$v_A = \left(\frac{1}{a_p r_p + r_p{}^2}\right)\sqrt{2}\sqrt{r_p\left(a_p + r_p\right) a_p G M_v} \qquad v_A = 7.89\ \frac{km}{s}$$

$$v'_A = \frac{r_p v_A}{a_p} \qquad v'_A = 2.43\ \frac{km}{s}$$

$$v''_A = \sqrt{\frac{G M_v}{r_p}} \qquad v''_A = 6.38\ \frac{km}{s}$$

Circular Orbit: $T_c = \dfrac{2\pi r_p}{v''_A}$ $\qquad T_c = 2.19\ hr$

Elliptic Orbit: $T_e = \dfrac{\pi}{r_p v_A}\left(r_p + a_p\right)\sqrt{r_p a_p}$ $\qquad T_e = 6.78\ hr$

Problem 14-1

A woman having a mass M stands in an elevator which has a downward acceleration a starting from rest. Determine the work done by her weight and the work of the normal force which the floor exerts on her when the elevator descends a distance s. Explain why the work of these forces is different.

Units Used: $\mathrm{kJ} = 10^3\,\mathrm{J}$

Given: $M = 70\ \mathrm{kg}$ $\quad g = 9.81\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$ $\quad a = 4\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$ $\quad s = 6\ \mathrm{m}$

Solution:

$$M g - N_p = M a \qquad N_p = M g - M a \qquad N_p = 406.7\ \mathrm{N}$$

$$U_W = M g s \qquad U_W = 4.12\,\mathrm{kJ}$$

$$U_{NP} = -s N_p \qquad U_{NP} = -2.44\,\mathrm{kJ}$$

The difference accounts for a change in kinetic energy.

Problem 14-2

The crate of weight W has a velocity v_A when it is at A. Determine its velocity after it slides down the plane to $s = s'$. The coefficient of kinetic friction between the crate and the plane is μ_k.

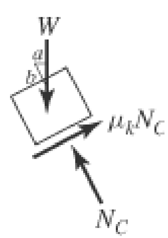
Given:

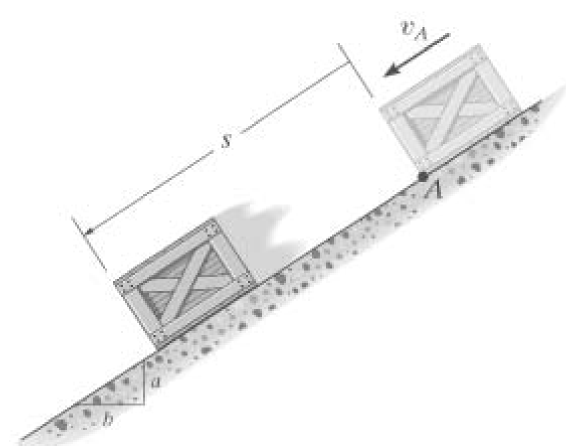
$W = 20\ \mathrm{lb}$ $\qquad a = 3$

$v_A = 12\ \dfrac{\mathrm{ft}}{\mathrm{s}}$ $\qquad b = 4$

$s' = 6\ \mathrm{ft}$

$\mu_k = 0.2$





Solution:

$$\theta = \mathrm{atan}\left(\frac{a}{b}\right) \qquad N_C = W\cos(\theta) \qquad F = \mu_k N_C$$

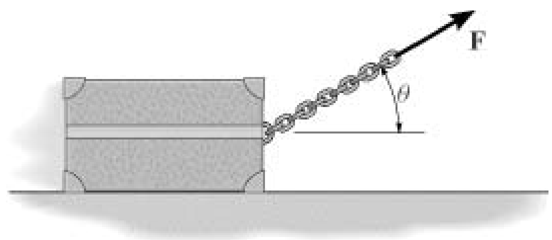
Guess $\quad v' = 1\,\dfrac{\mathrm{m}}{\mathrm{s}}$

Given $\quad \dfrac{1}{2}\left(\dfrac{W}{g}\right) v_A^{\,2} + W\sin(\theta) s' - F s' = \dfrac{1}{2}\left(\dfrac{W}{g}\right) v'^2 \qquad v' = \mathrm{Find}(v') \quad v' = 17.72\,\dfrac{\mathrm{ft}}{\mathrm{s}}$

Problem 14-3

The crate of mass M is subjected to a force having a constant direction and a magnitude F, where s is measured in meters. When $s = s_1$, the crate is moving to the right with a speed v_1. Determine its speed when $s = s_2$. The coefficient of kinetic friction between the crate and the ground is μ_k.

Given:



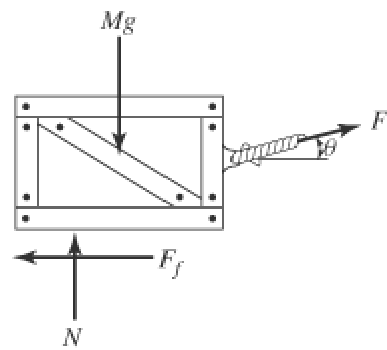
$M = 20\ \text{kg}$ $\quad F = 100\ \text{N}$

$s_1 = 4\ \text{m}$ $\quad \theta = 30\ \text{deg}$

$v_1 = 8\ \dfrac{\text{m}}{\text{s}}$ $\quad a = 1$

$s_2 = 25\ \text{m}$ $\quad b = 1\ \text{m}^{-1}$

$\mu_k = 0.25$



Solution:

Equation of motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N$

$$N + F\sin(\theta) - Mg = 0 \qquad N = Mg - F\sin(\theta)$$

Principle of work and Energy: The horizontal component of force **F** which acts in the direction of displacement does positive work, whereas the friction force $F_f = \mu_k(Mg - F\sin(\theta))$ does negative work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force **F** and the weight of the crate do not displace hence do no work.

$$F\cos(\theta) - \mu_k N = Ma$$

$$F\cos(\theta) - \mu_k(Mg - F\sin(\theta)) = Ma$$

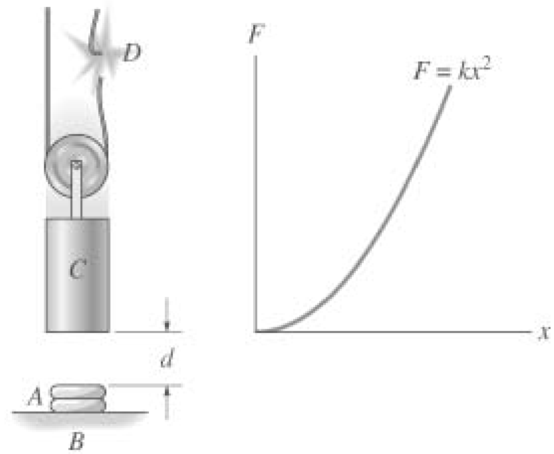
$$a = \frac{F\cos(\theta) - \mu_k(Mg - F\sin(\theta))}{M} \qquad a = 2.503\ \frac{\text{m}}{\text{s}^2}$$

$$v\frac{dv}{ds} = a \qquad \frac{v^2}{2} = \frac{{v_1}^2}{2} + a(s_2 - s_1)$$

$$v = \sqrt{2\left[\frac{{v_1}^2}{2} + a(s_2 - s_1)\right]} \qquad v = 13.004\ \frac{\text{m}}{\text{s}}$$

***Problem 14-4**

The "air spring" A is used to protect the support structure B and prevent damage to the conveyor-belt tensioning weight C in the event of a belt failure D. The force developed by the spring as a function of its deflection is shown by the graph. If the weight is W and it is suspended a height d above the top of the spring, determine the maximum deformation of the spring in the event the conveyor belt fails. Neglect the mass of the pulley and belt.



Given:

$$W = 50\ \text{lb} \qquad k = 8000\ \frac{\text{lb}}{\text{ft}^2}$$

$$d = 1.5\ \text{ft}$$

Solution:

$$T_1 + U = T_2$$

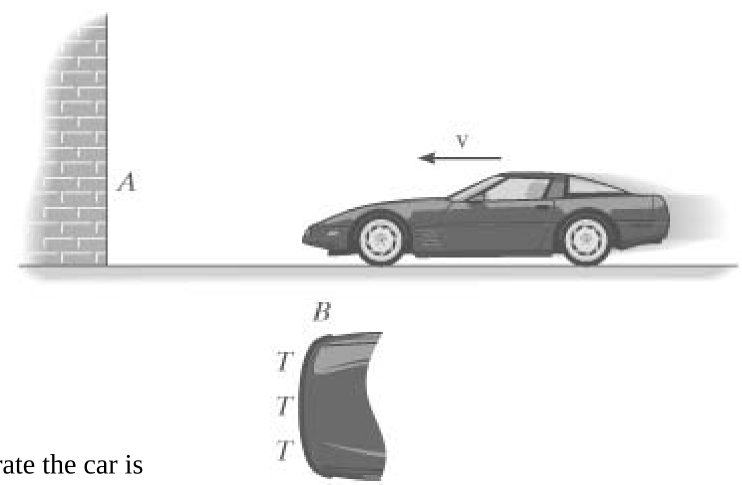
$$0 + W(d + \delta) - \int_0^{\delta} k x^2\ dx = 0$$

Guess $\delta = 1\ \text{in}$

Given $W(d+\delta) - k\left(\frac{\delta^3}{3}\right) = 0$ $\delta = \text{Find}(\delta)$ $\delta = 3.896\ \text{in}$

Problem 14-5

A car is equipped with a bumper B designed to absorb collisions. The bumper is mounted to the car using pieces of flexible tubing T. Upon collision with a rigid barrier at A, a constant horizontal force **F** is developed which causes a car deceleration kg (the highest safe deceleration for a passenger without a seatbelt). If the car and passenger have a total mass M and the car is initially coasting with a speed v, determine the magnitude of **F** needed to stop the car and the deformation x of the bumper tubing.



Units Used:

$$\text{Mm} = 10^3\ \text{kg}$$

$$\text{kN} = 10^3\ \text{N}$$

Given:

$$M = 1.5\,10^3\ \text{kg}$$

$$v = 1.5\ \frac{\text{m}}{\text{s}} \qquad k = 3$$

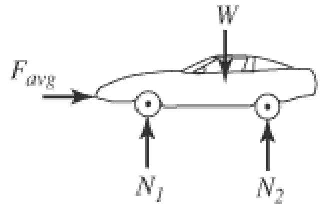
Solution:

The average force needed to decelerate the car is

$$F_{avg} = M k g \qquad F_{avg} = 44.1\ \text{kN}$$

The deformation is

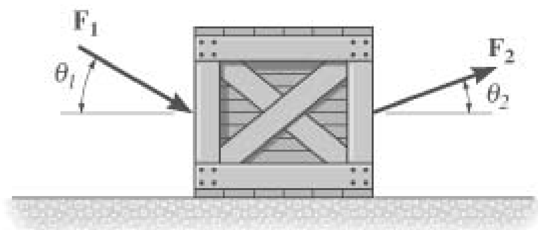
$$T_1 + U_{12} = T_2 \qquad \frac{1}{2}Mv^2 - F_{avg}x = 0$$



$$x = \frac{1}{2}M\left(\frac{v^2}{F_{avg}}\right) \qquad x = 38.2\ \text{mm}$$

Problem 14-6

The crate of mass M is subjected to forces $\mathbf{F_1}$ and $\mathbf{F_2}$, as shown. If it is originally at rest, determine the distance it slides in order to attain a speed v. The coefficient of kinetic friction between the crate and the surface is μ_k.



Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

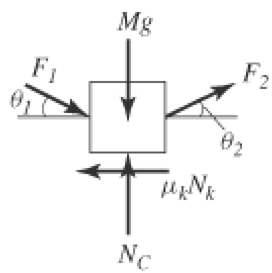
$$M = 100\ \text{kg} \qquad v = 6\ \frac{\text{m}}{\text{s}}$$

$$F_1 = 800\ \text{N} \qquad \mu_k = 0.2$$

$$F_2 = 1.5\ \text{kN} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

$$\theta_1 = 30\ \text{deg}$$

$$\theta_2 = 20\ \text{deg}$$



Solution:

$$N_C - F_1\sin(\theta_1) - Mg + F_2\sin(\theta_2) = 0$$

$$N_C = F_1\sin(\theta_1) + Mg - F_2\sin(\theta_2) \qquad N_C = 867.97\ \text{N}$$

$$T_1 + U_{12} = T_2$$

$$F_1\cos(\theta_1)s - \mu_k N_c s + F_2\cos(\theta_2)s = \frac{1}{2}Mv^2$$

$$s = \frac{Mv^2}{2(F_1\cos(\theta_1) - \mu_k N_C + F_2\cos(\theta_2))} \qquad s = 0.933\ \text{m}$$

Problem 14-7

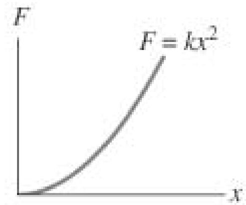
Design considerations for the bumper B on the train car of mass M require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to a distance d when the car, traveling at speed v, strikes the rigid stop. Neglect the mass of the car wheels.

Units Used:

$\mathrm{Mg} = 10^3\ \mathrm{kg}$

$\mathrm{kN} = 10^3\ \mathrm{N}$

$\mathrm{MN} = 10^3\ \mathrm{kN}$

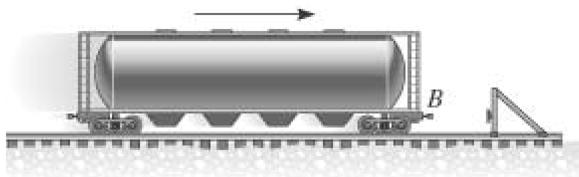


Given:

$M = 5\ \mathrm{Mg}$

$d = 0.2\ \mathrm{m}$

$v = 4\ \dfrac{\mathrm{m}}{\mathrm{s}}$



Solution:

$$\frac{1}{2}Mv^2 - \int_0^d kx^2\,dx = 0 \qquad \frac{1}{2}Mv^2 - \frac{kd^3}{3} = 0 \qquad k = \frac{3Mv^2}{2d^3} \qquad k = 15\,\frac{\mathrm{MN}}{\mathrm{m}^2}$$

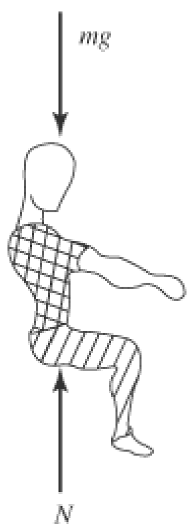
***Problem 14-8**

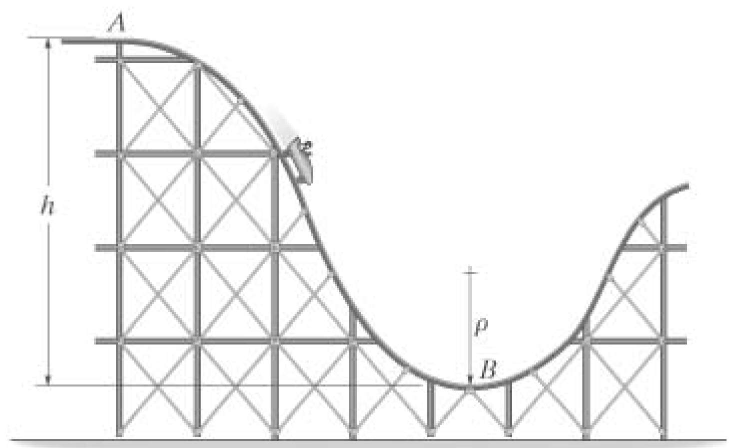
Determine the required height h of the roller coaster so that when it is essentially at rest at the crest of the hill it will reach a speed v when it comes to the bottom. Also, what should be the minimum radius of curvature ρ for the track at B so that the passengers do not experience a normal force greater than kmg? Neglect the size of the car and passengers.

Given:

$v = 100\ \dfrac{\mathrm{km}}{\mathrm{hr}}$

$k = 4$





Solution:

$T_1 + U_{12} = T_2$

$mgh = \dfrac{1}{2}mv^2$

$h = \dfrac{1}{2}\dfrac{v^2}{g}$

$h = 39.3\ \mathrm{m}$

$$kmg - mg = \frac{mv^2}{\rho} \qquad \rho = \frac{v^2}{g(k-1)} \qquad \rho = 26.2\ \mathrm{m}$$

Problem 14-9

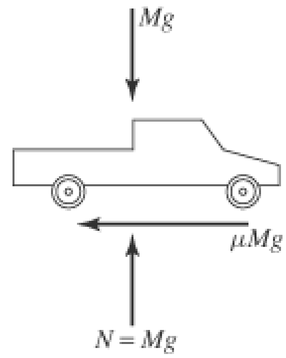
When the driver applies the brakes of a light truck traveling at speed v_1 it skids a distance d_1 before stopping. How far will the truck skid if it is traveling at speed v_2 when the brakes are applied?

Given:

$v_1 = 40\ \frac{\text{km}}{\text{hr}}$

$d_1 = 3\ \text{m}$

$v_2 = 80\ \frac{\text{km}}{\text{hr}}$



Solution:

$\frac{1}{2}Mv_1^2 - \mu_k M g d_1 = 0$ $\mu_k = \frac{v_1^2}{2gd_1}$ $\mu_k = 2.097$

$\frac{1}{2}Mv_2^2 - \mu_k M g d_2 = 0$ $d_2 = \frac{v_2^2}{2\mu_k g}$ $d_2 = 12\ \text{m}$

Problem 14-10

The ball of mass M of negligible size is fired up the vertical circular track using the spring plunger. The plunger keeps the spring compressed a distance δ when $x = 0$. Determine how far x it must be pulled back and released so that the ball will begin to leave the track when $\theta = \theta_1$.

Given:

$M = 0.5\ \text{kg}$

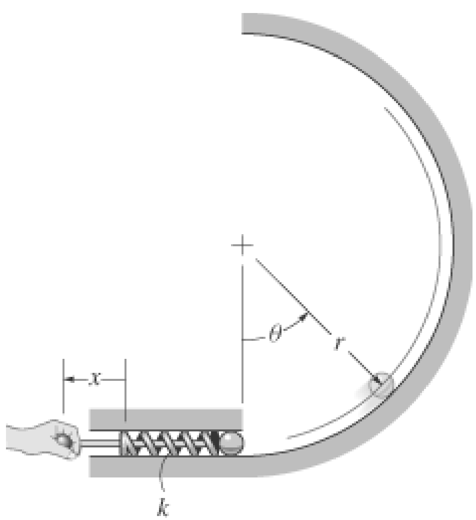
$\delta = 0.08\ \text{m}$

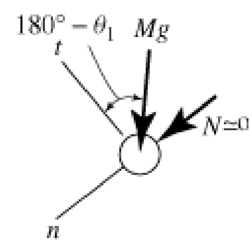
$\theta_1 = 135\ \text{deg}$

$r = 1.5\ \text{m}$

$k = 500\ \frac{\text{N}}{\text{m}}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$





Solution:

$N = 0$ $\theta = \theta_1$

$\Sigma F_n = m a_n$ $N - Mg\cos(\theta) = M\left(\frac{v^2}{r}\right)$ $v = \sqrt{-g r \cos(\theta)}$ $v = 3.226\ \frac{\text{m}}{\text{s}}$

Guess $x = 10\ \text{mm}$

Given $\displaystyle\int_{x+\delta}^{\delta} -k x\, dx - M g r(1-\cos(\theta)) = \frac{1}{2} M v^2$ $\quad x = \text{Find}(x) \quad x = 178.9\,\text{mm}$

Problem 14-11

The force **F**, acting in a constant direction on the block of mass M, has a magnitude which varies with the position x of the block. Determine how far the block slides before its velocity becomes v_1. When $x = 0$, the block is moving to the right at speed v_0 . The coefficient of kinetic friction between the block and surface is μ_k.

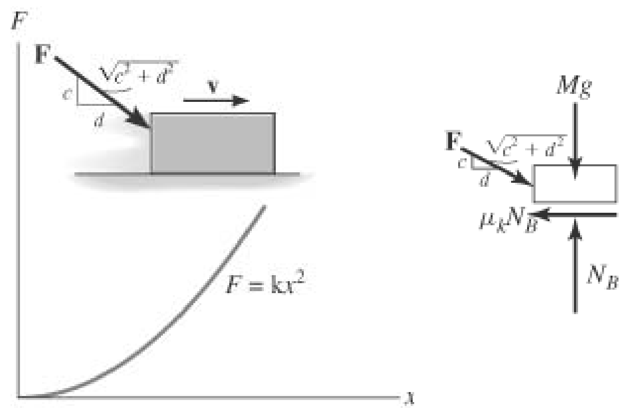
Given:

$M = 20\ \text{kg}$ $\quad c = 3$

$v_1 = 5\ \dfrac{\text{m}}{\text{s}}$ $\quad d = 4$

$v_0 = 2\ \dfrac{\text{m}}{\text{s}}$ $\quad k = 50\ \dfrac{\text{N}}{\text{m}^2}$

$\mu_k = 0.3$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

$$N_B - M g - \left(\frac{c}{\sqrt{c^2+d^2}}\right) k x^2 = 0 \qquad N_B = M g + \left(\frac{c}{\sqrt{c^2+d^2}}\right) k x^2$$

Guess $\delta = 2\ \text{m}$

Given

$$\frac{1}{2} M v_0^2 + \left(\frac{d}{\sqrt{c^2+d^2}}\right)\int_0^{\delta} k x^2\, dx - \mu_k M g \delta - \mu_k \int_0^{\delta} \left(\frac{c}{\sqrt{c^2+d^2}}\right) k x^2\, dx = \frac{1}{2} M v_1^2$$

$\delta = \text{Find}(\delta) \quad \delta = 3.413\ \text{m}$

***Problem 14-12**

The force **F**, acting in a constant direction on the block of mass M, has a magnitude which varies with position x of the block. Determine the speed of the block after it slides a distance d_1 . When $x = 0$, the block is moving to the right at v_0 . The coefficient of kinetic friction between the block and surface is μ_k.

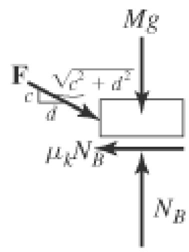
Given:

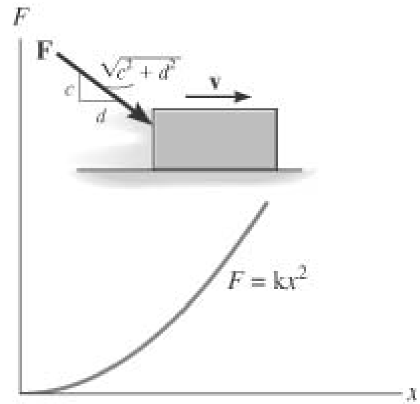
$M = 20\ \text{kg}$ $\quad c = 3$

$d_1 = 3\ \text{m}$ $\quad d = 4$

$v_0 = 2\ \dfrac{\text{m}}{\text{s}}$ $\quad k = 50\ \dfrac{\text{N}}{\text{m}^2}$

$\mu_k = 0.3$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$





Solution:

$$N_B - Mg - \left(\frac{c}{\sqrt{c^2 + d^2}}\right)kx^2 = 0 \quad N_B = Mg + \left(\frac{c}{\sqrt{c^2 + d^2}}\right)kx^2$$

Guess $\quad v_1 = 2\dfrac{\text{m}}{\text{s}}$

Given

$$\frac{1}{2}Mv_0^2 + \left(\frac{d}{\sqrt{c^2 + d^2}}\right)\int_0^{d_1} kx^2\,dx - \mu_k M g d_1 - \mu_k \int_0^{d_1} \left(\frac{c}{\sqrt{c^2 + d^2}}\right)kx^2\,dx = \frac{1}{2}Mv_1^2$$

$v_1 = \text{Find}(v_1)$ $\quad v_1 = 3.774\ \dfrac{\text{m}}{\text{s}}$

Problem 14-13

As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so by considering the block of mass M which rests on the smooth surface and is subjected to horizontal force **F**. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of v_0 and travels a distance d, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of v_B relative to A. *Hint:* The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.

Given:

$M = 10\ \text{kg}$

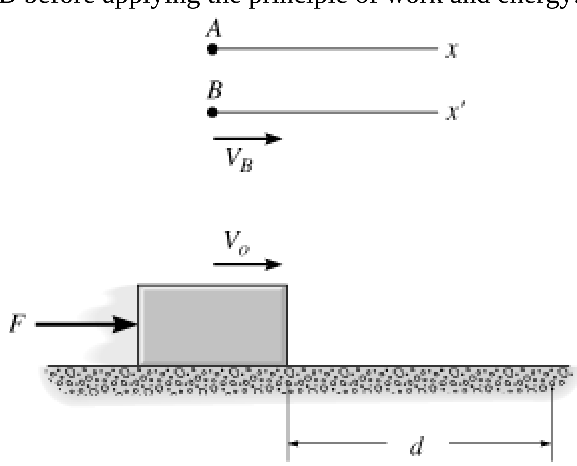
$F = 6\ \text{N}$

$v_0 = 5\ \dfrac{\text{m}}{\text{s}}$

$d = 10\ \text{m}$

$v_B = 2\ \dfrac{\text{m}}{\text{s}}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

Observer A:

$$\frac{1}{2}M v_0^2 + F d = \frac{1}{2}M v_2^2 \qquad v_2 = \sqrt{v_0^2 + \frac{2F d}{M}} \qquad v_2 = 6.083\ \frac{\text{m}}{\text{s}}$$

$$F = M a \qquad a = \frac{F}{M} \qquad a = 0.6\ \frac{\text{m}}{\text{s}^2} \qquad \text{Guess} \quad t = 1\text{s}$$

Given $\quad d = 0 + v_0 t + \frac{1}{2}a t^2 \qquad t = \text{Find}(t) \qquad t = 1.805\ \text{s}$

Observer B:

$$d' = (v_0 - v_B)t + \frac{1}{2}a t^2 \qquad d' = 6.391\ \text{m}$$

The distance that the block moves as seen by observer B.

$$\frac{1}{2}M(v_0 - v_B)^2 + F d' = \frac{1}{2}M v'_2{}^2 \qquad v'_2 = \sqrt{(v_0 - v_B)^2 + \frac{2F d'}{M}} \qquad v'_2 = 4.083\ \frac{\text{m}}{\text{s}}$$

Notice that $\quad v_2 = v'_2 + v_B$

Problem 14-14

Determine the velocity of the block A of weight W_A if the two blocks are released from rest and the block B of weight W_B moves a distance d up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is μ_k.

Given:

$W_A = 60\ \text{lb}$

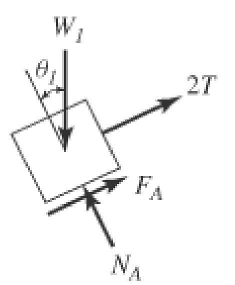
$W_B = 40\ \text{lb}$

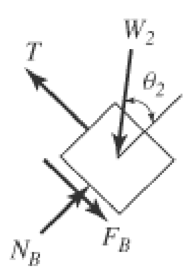
$\theta_1 = 60\ \text{deg}$

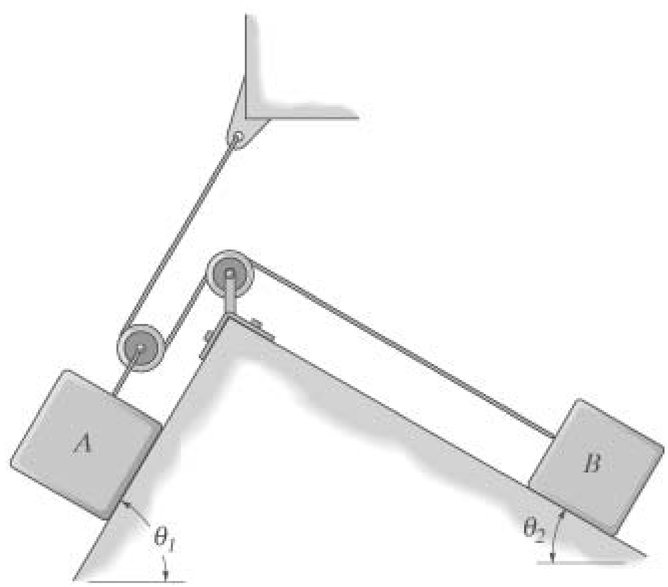
$\theta_2 = 30\ \text{deg}$

$d = 2\ \text{ft}$

$\mu_k = 0.10$







Solution:

$L = 2s_A + s_B$

$0 = 2v_A + v_B$

Guesses

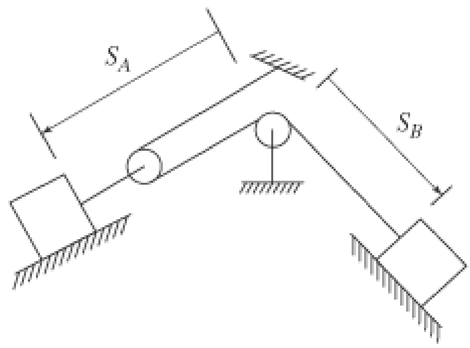
$v_A = 1\frac{\text{ft}}{\text{s}} \qquad v_B = -1\frac{\text{ft}}{\text{s}}$

Given

$$0 = 2v_A + v_B$$

$$W_A\left(\frac{d}{2}\right)\sin(\theta_1) - W_B\,d\sin(\theta_2) - \mu_k W_A\cos(\theta_1)\frac{d}{2}\ ... = \frac{1}{2g}\left(W_A v_A{}^2 + W_B v_B{}^2\right)$$
$$+ -\mu_k W_B\cos(\theta_2)d$$

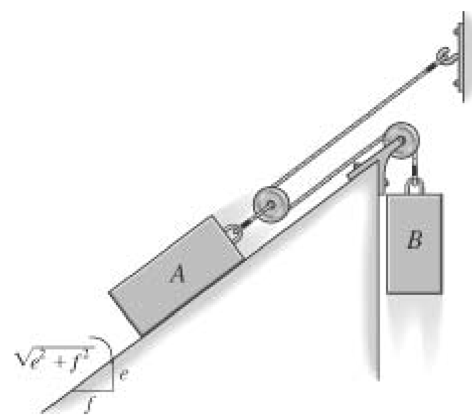
$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \mathrm{Find}(v_A, v_B)$$



$$v_B = -1.543\ \frac{\mathrm{ft}}{\mathrm{s}} \qquad v_A = 0.771\ \frac{\mathrm{ft}}{\mathrm{s}}$$

Problem 14-15

Block A has weight W_A and block B has weight W_B. Determine the speed of block A after it moves a distance d down the plane, starting from rest. Neglect friction and the mass of the cord and pulleys.



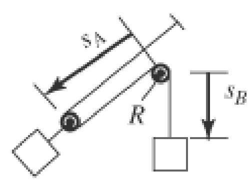
Given:

$$W_A = 60\ \mathrm{lb} \qquad e = 3$$

$$W_B = 10\ \mathrm{lb} \qquad f = 4$$

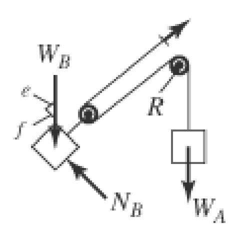
$$d = 5\ \mathrm{ft} \qquad g = 32.2\ \frac{\mathrm{ft}}{\mathrm{s}^2}$$

Solution:



$$L = 2s_A + s_B \qquad 0 = 2\Delta s_A + \Delta s_B \qquad 0 = 2v_A + v_B$$

$$0 + W_A\left(\frac{e}{\sqrt{e^2+f^2}}\right)d - W_B 2d = \frac{1}{2}\left(\frac{W_A}{g}\right)v_A{}^2 + \frac{1}{2}\left(\frac{W_B}{g}\right)(2v_A)^2$$



$$v_A = \sqrt{\frac{2g\,d}{W_A + 4W_B}\left(W_A\frac{e}{\sqrt{e^2+f^2}} - 2W_B\right)} \qquad v_A = 7.178\ \frac{\mathrm{ft}}{\mathrm{s}}$$

***Problem 14-16**

The block A of weight W_A rests on a surface for which the coefficient of kinetic friction is μ_k. Determine the distance the cylinder B of weight W_B must descend so that A has a speed v_A starting from rest.

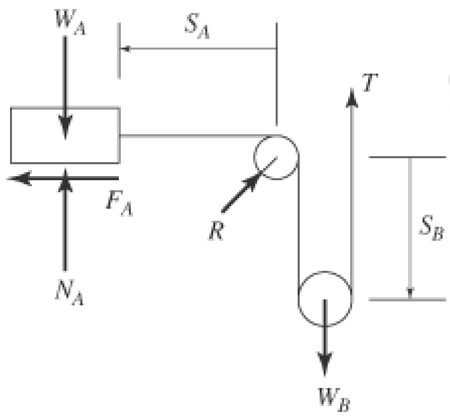
Given:

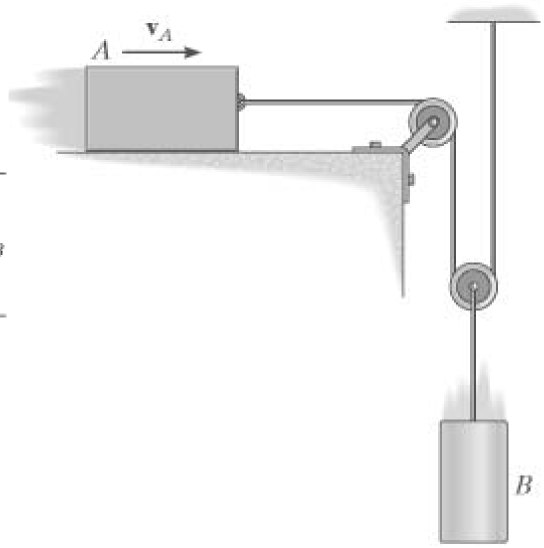
$W_A = 3\ \text{lb}$

$W_B = 8\ \text{lb}$

$\mu_k = 0.3$

$v_A = 5\ \frac{\text{ft}}{\text{s}}$





Solution:

$L = s_A + 2s_B$

Guesses $\quad d = 1\ \text{ft}$

Given

$$W_B d - \mu_k W_A 2d = \frac{1}{2g}\left[W_A v_A^{\,2} + W_B\left(\frac{v_A}{2}\right)^2\right]$$

$d = \text{Find}(d) \qquad d = 0.313\,\text{ft}$

Problem 14-17

The block of weight W slides down the inclined plane for which the coefficient of kinetic friction is μ_k. If it is moving at speed v when it reaches point A, determine the maximum deformation of the spring needed to momentarily arrest the motion.

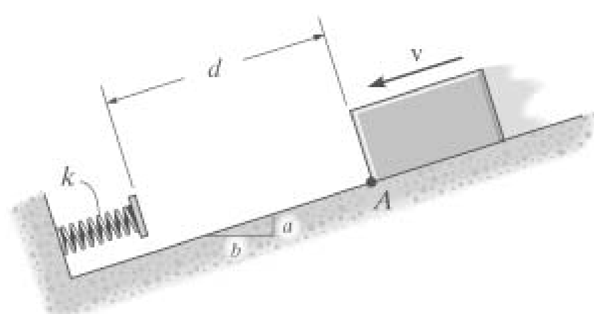
Given:

$W = 100\ \text{lb} \qquad a = 3\ \text{m}$

$v = 10\ \frac{\text{ft}}{\text{s}} \qquad b = 4\ \text{m}$

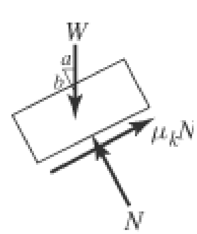
$d = 10\ \text{ft}$

$k = 200\ \frac{\text{lb}}{\text{ft}} \qquad \mu_k = 0.25$



Solution:

$$N = \left(\frac{b}{\sqrt{a^2 + b^2}}\right)W \qquad N = 80\,\text{lb}$$



Initial Guess

$d_{max} = 5\ \text{m}$

Given $$\frac{1}{2}\left(\frac{W}{g}\right)v^2 - \mu_k N\left(d + d_{max}\right) - \frac{1}{2}k\,d_{max}^{\;2} + W\left(d + d_{max}\right)\left(\frac{a}{\sqrt{a^2 + b^2}}\right) = 0$$

$$d_{max} = \text{Find}\left(d_{max}\right) \qquad d_{max} = 2.56\,\text{ft}$$

Problem 14-18

The collar has mass M and rests on the smooth rod. Two springs are attached to it and the ends of the rod as shown. Each spring has an uncompressed length l. If the collar is displaced a distance $s = s'$ and released from rest, determine its velocity at the instant it returns to the point $s = 0$.

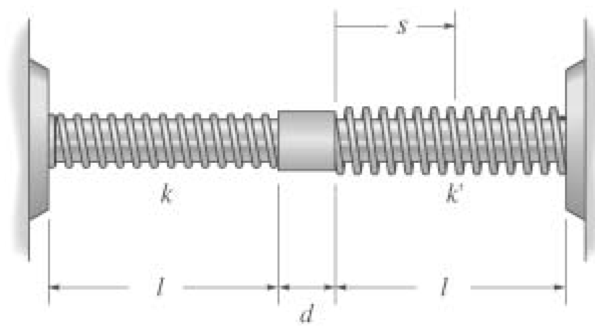
Given:

$M = 20\ \text{kg}$ $\quad k = 50\ \frac{\text{N}}{\text{m}}$

$s' = 0.5\ \text{m}$

$l = 1\ \text{m}$ $\quad k' = 100\ \frac{\text{N}}{\text{m}}$

$d = 0.25\ \text{m}$

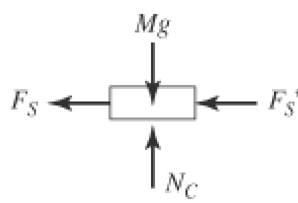


Solution:

$$\frac{1}{2}k s'^2 + \frac{1}{2}k' s'^2 = \frac{1}{2}M v_C^{\,2}$$

$$v_C = \sqrt{\frac{k + k'}{M}}\cdot s'$$

$$v_C = 1.37\ \frac{\text{m}}{\text{s}}$$



Problem 14-19

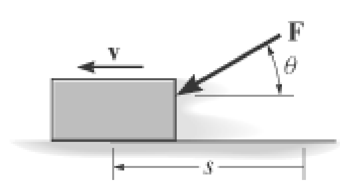
The block of mass M is subjected to a force having a constant direction and a magnitude $F = k/(a+bx)$. When $x = x_1$, the block is moving to the left with a speed v_1. Determine its speed when $x = x_2$. The coefficient of kinetic friction between the block and the ground is μ_k.

Given:

$M = 2\ \text{kg}$ $\quad b = 1\ \text{m}^{-1}$ $\quad x_2 = 12\ \text{m}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

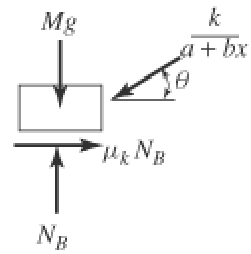
$k = 300\ \text{N}$ $\quad x_1 = 4\ \text{m}$ $\quad \theta = 30\ \text{deg}$

$a = 1$ $\quad v_1 = 8\ \frac{\text{m}}{\text{s}}$ $\quad \mu_k = 0.25$



Solution:

$$N_B - Mg - \left(\frac{k}{a+bx}\right)\sin(\theta) = 0 \qquad N_B = Mg + \frac{k\sin(\theta)}{a+bx}$$



$$U = \int_{x_1}^{x_2} \frac{k\cos(\theta)}{a+bx}\,dx - \mu_k \int_{x_1}^{x_2} Mg + \frac{k\sin(\theta)}{a+bx}\,dx \qquad U = 173.177\ \text{N}\cdot\text{m}$$

$$\frac{1}{2}Mv_1^2 + U = \frac{1}{2}Mv_2^2 \qquad v_2 = \sqrt{v_1^2 + \frac{2U}{M}} \qquad v_2 = 15.401\ \frac{\text{m}}{\text{s}}$$

***Problem 14-20**

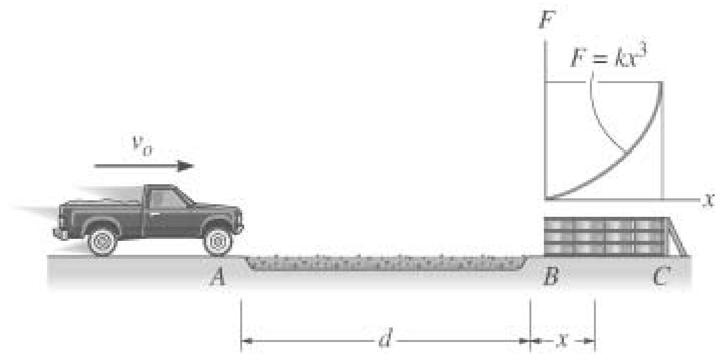
The motion of a truck is arrested using a bed of loose stones AB and a set of crash barrels BC. If experiments show that the stones provide a rolling resistance F_t per wheel and the crash barrels provide a resistance as shown in the graph, determine the distance x the truck of weight W penetrates the barrels if the truck is coasting at speed v_0 when it approaches A. Neglect the size of the truck.

Given:

$F_t = 160$ lb $\quad d = 50$ ft

$W = 4500$ lb $\quad k = 1000\ \frac{\text{lb}}{\text{ft}^3}$

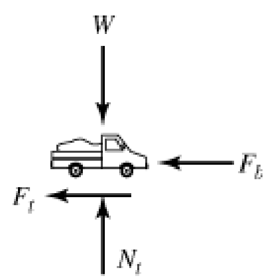
$v_0 = 60\ \frac{\text{ft}}{\text{s}} \quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$\frac{1}{2}\left(\frac{W}{g}\right)v_0^2 - 4F_t d - k\frac{x^4}{4} = 0$$

$$x = \left(\frac{2Wv_0^2}{kg} - \frac{16F_t d}{k}\right)^{\frac{1}{4}} \qquad x = 5.444\ \text{ft}$$



Problem 14-21

The crash cushion for a highway barrier consists of a nest of barrels filled with an impact-absorbing material. The barrier stopping force is measured versus the vehicle penetration into the barrier. Determine the distance a car having weight W will penetrate the barrier if it is originally traveling at speed v_0 when it strikes the first barrel.

Units Used:

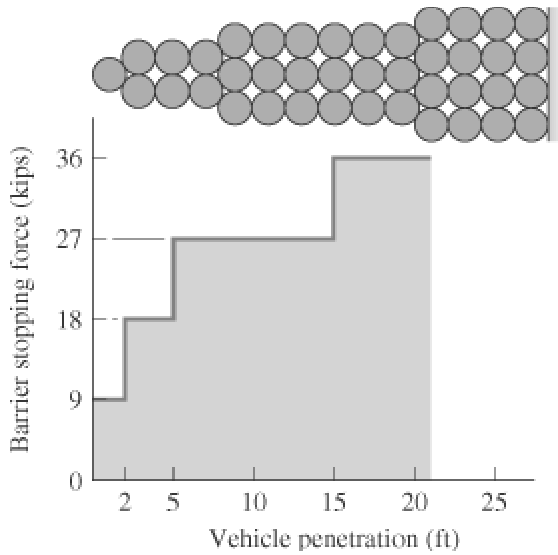
$\text{kip} = 10^3 \text{ lb}$

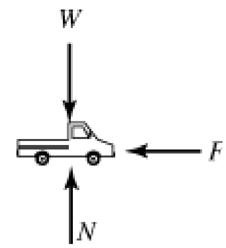
Given:

$W = 4000 \text{ lb}$

$v_0 = 55 \frac{\text{ft}}{\text{s}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$





Solution:

$$\frac{1}{2}\left(\frac{W}{g}\right)v_0^2 - Area = 0$$

$Area = \frac{1}{2}\left(\frac{W}{g}\right)v_0^2$ $\quad Area = 187.888 \text{ kip}\cdot\text{ft}$ We must produce this much work with the barrels.

Assume that $5 \text{ ft} < x < 15 \text{ ft}$

$$Area = (2 \text{ ft})(9 \text{ kip}) + (3 \text{ ft})(18 \text{ kip}) + (x - 5 \text{ ft})(27 \text{ kip})$$

$x = \frac{Area - 72 \text{ kip}\cdot\text{ft}}{27 \text{ kip}} + 5 \text{ ft}$ $\quad x = 9.292 \text{ ft}$ Check that the assumption is corrrect!

Problem 14-22

The collar has a mass M and is supported on the rod having a coefficient of kinetic friction μ_k. The attached spring has an unstretched length l and a stiffness k. Determine the speed of the collar after the applied force F causes it to be displaced a distance $s = s_1$ from point A. When s = 0 the collar is held at rest.

Given:

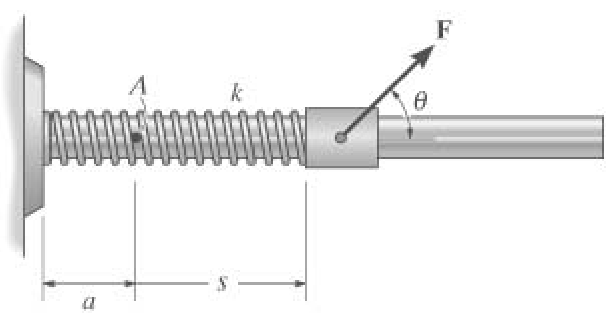
$M = 30 \text{ kg}$ $\quad \mu_k = 0.4$

$a = 0.5 \text{ m}$ $\quad \theta = 45 \text{ deg}$

$F = 200 \text{ N}$ $\quad s_1 = 1.5 \text{ m}$

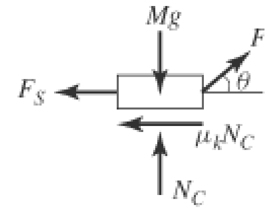
$l = 0.2 \text{ m}$ $\quad g = 9.81 \frac{\text{m}}{\text{s}^2}$

$k = 50 \frac{\text{N}}{\text{m}}$



Solution:

Guesses $N_C = 1\ \text{N}$ $v = 1\ \dfrac{\text{m}}{\text{s}}$



Given

$$N_C - M g + F \sin(\theta) = 0$$

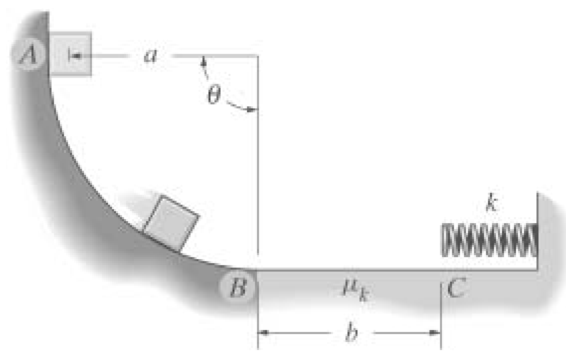
$$F \cos(\theta) s_1 - \mu_k N_C s_1 + \frac{1}{2} k (a - l)^2 - \frac{1}{2} k (s_1 + a - l)^2 = \frac{1}{2} M v^2$$

$$\begin{pmatrix} N_C \\ v \end{pmatrix} = \text{Find}(N_C, v) \qquad N_C = 152.9\ \text{N} \qquad v = 1.666\ \frac{\text{m}}{\text{s}}$$

Problem 14-23

The block of weight W is released from rest at A and slides down the smooth circular surface AB. It then continues to slide along the horizontal rough surface until it strikes the spring. Determine how far it compresses the spring before stopping.

Given:



$W = 5\ \text{lb}$ $\mu_k = 0.2$

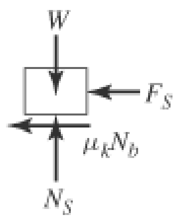
$a = 3\ \text{ft}$ $\theta = 90\ \text{deg}$

$b = 2\ \text{ft}$ $k = 40\ \dfrac{\text{lb}}{\text{ft}}$

Solution:

Guess $d = 1\ \text{ft}$

Given



$$W a - \mu_k W (b + d) - \frac{1}{2} k d^2 = 0$$

$$d = \text{Find}(d) \qquad d = 0.782\ \text{ft}$$

***Problem 14-24**

The block has a mass M and moves within the smooth vertical slot. If it starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force **F** which must be applied to the cord so that the block attains a speed v_B when it reaches s_B.

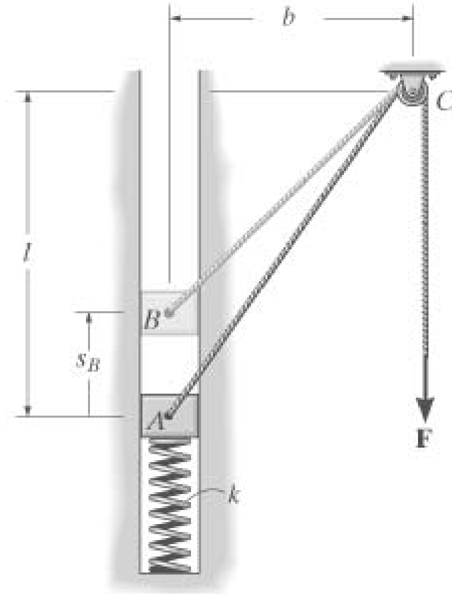
Neglect the size and mass of the pulley. *Hint:* The work of **F** can be determined by finding the difference Δl in cord lengths AC and BC and using $U_F = F\ \Delta l$.

Given:

$M = 0.8\ \text{kg}$ $l = 0.4\ \text{m}$

$v_B = 2.5\ \dfrac{\text{m}}{\text{s}}$ $b = 0.3\ \text{m}$

$s_B = 0.15\ \text{m} \qquad k = 100\ \frac{\text{N}}{\text{m}}$



Solution:

$$\Delta l = \sqrt{l^2 + b^2} - \sqrt{(l - s_B)^2 + b^2}$$

Guess $\quad F = 1\ \text{N}$

Given

$$F\Delta l - M g s_B - \frac{1}{2}k s_B^{\ 2} = \frac{1}{2}M v_B^{\ 2}$$

$F = \text{Find}(F) \qquad F = 43.9\ \text{N}$

Problem 14-25

The collar has a mass M and is moving at speed v_1 when $x = 0$ and a force of **F** is applied to it. The direction θ of this force varies such that $\theta = ax$, where θ is clockwise, measured in degrees. Determine the speed of the collar when $x = x_1$. The coefficient of kinetic friction between the collar and the rod is μ_k.

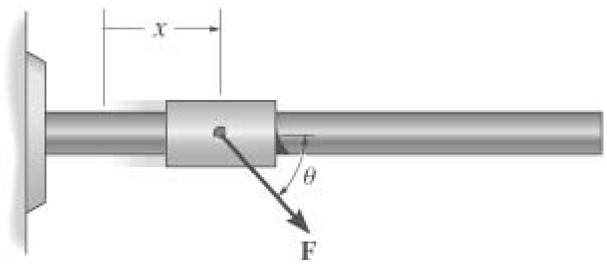
Given:

$M = 5\ \text{kg} \qquad v_1 = 8\ \frac{\text{m}}{\text{s}}$

$F = 60\ \text{N} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

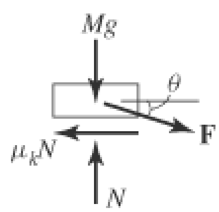
$\mu_k = 0.3$

$x_1 = 3\ \text{m} \qquad a = 10\ \frac{\text{deg}}{\text{m}}$



Solution:

$$N = F\sin(\theta) + M g$$



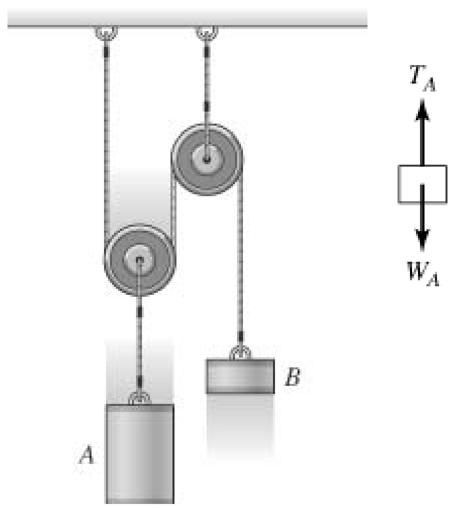
Guess $\quad v = 5\ \frac{\text{m}}{\text{s}}$

Given $\quad \frac{1}{2}M v_1^{\ 2} + \int_0^{x_1} F\cos(a x)\ dx - \mu_k \int_0^{x_1} F\sin(a x) + M g\ dx = \frac{1}{2}M v^2$

$v = \text{Find}(v) \qquad v = 10.47\ \frac{\text{m}}{\text{s}}$

Problem 14-26

Cylinder A has weight W_A and block B has weight W_B. Determine the distance A must descend from rest before it obtains speed v_A. Also, what is the tension in the cord supporting block A? Neglect the mass of the cord and pulleys.



Given:

$W_A = 60\ \text{lb} \qquad v_A = 8\ \frac{\text{ft}}{\text{s}}$

$W_B = 10\ \text{lb} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$L = 2s_A + s_B \qquad 0 = 2v_A + v_B$

System

$$0 + W_A d - W_B 2d = \frac{1}{2}\left(\frac{W_A}{g}\right)v_A^2 + \frac{1}{2}\left(\frac{W_B}{g}\right)(2v_A)^2$$

$$d = \frac{\left(\frac{W_A + 4W_B}{2g}\right)v_A^2}{W_A - 2W_B} \qquad d = 2.484\,\text{ft}$$

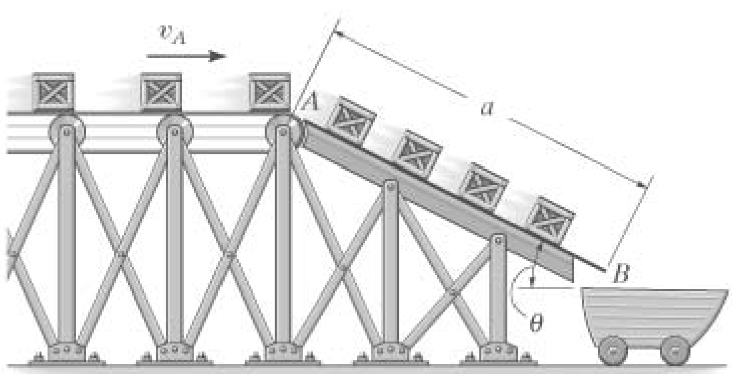
Block A alone

$$0 + W_A d - T d = \frac{1}{2}\left(\frac{W_A}{g}\right)v_A^2 \qquad T = W_A - \frac{W_A v_A^2}{2g\,d} \qquad T = 36\,\text{lb}$$

Problem 14-27

The conveyor belt delivers crate each of mass M to the ramp at A such that the crate's velocity is v_A, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is μ_k, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs.

Given:



$M = 12\ \text{kg}$

$v_A = 2.5\ \frac{\text{m}}{\text{s}}$

$\mu_k = 0.3$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$\theta = 30\ \text{deg}$

$a = 3\ \text{m}$

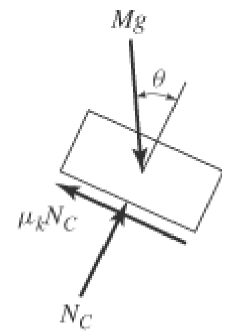
Solution:

$$N_C = M g \cos(\theta)$$

$$\frac{1}{2} M v_A^2 + (M g a)\sin(\theta) - \mu_k N_C a = \frac{1}{2} M v_B^2$$

$$v_B = \sqrt{v_A^2 + (2 g a)\sin(\theta) - (2\mu_k g)\cos(\theta) a}$$

$$v_B = 4.52 \ \frac{\text{m}}{\text{s}}$$



***Problem 14-28**

When the skier of weight W is at point A he has a speed v_A. Determine his speed when he reaches point B on the smooth slope. For this distance the slope follows the cosine curve shown. Also, what is the normal force on his skis at B and his rate of increase in speed? Neglect friction and air resistance.

Given:

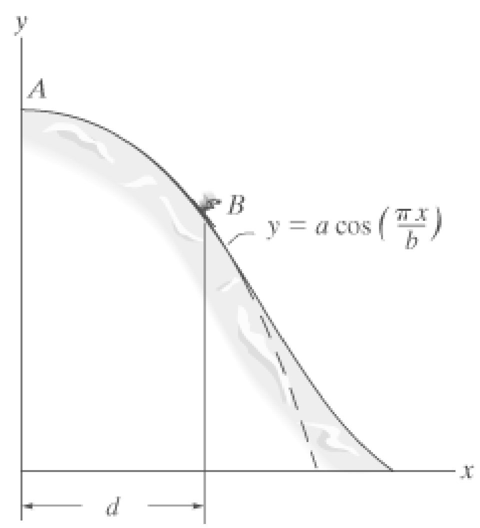
$W = 150 \text{ lb}$

$v_A = 5 \ \frac{\text{ft}}{\text{s}}$

$a = 50 \text{ ft}$

$b = 100 \text{ ft}$

$d = 35 \text{ ft}$



Solution:

$$y(x) = (a)\cos\left(\pi \frac{x}{b}\right) \qquad y'(x) = \frac{d}{dx} y(x)$$

$$y''(x) = \frac{d}{dx} y'(x) \qquad \rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$\theta_B = \operatorname{atan}(y'(d)) \qquad \rho_B = \rho(d)$$

Guesses $F_N = 1 \text{ lb}$ $\quad v' = 1 \ \frac{\text{ft}}{\text{s}^2}$ $\quad v_B = 1 \ \frac{\text{ft}}{\text{s}}$

Given $\quad \frac{1}{2}\left(\frac{W}{g}\right) v_A^2 + W(y(0 \text{ ft}) - y(d)) = \frac{1}{2}\left(\frac{W}{g}\right) v_B^2$

$$F_N - W\cos(\theta_B) = \left(\frac{W}{g}\right)\frac{v_B^2}{\rho_B} \qquad -W\sin(\theta_B) = \left(\frac{W}{g}\right) v'$$

$$\begin{pmatrix} v_B \\ F_N \\ v' \end{pmatrix} = \text{Find}(v_B, F_N, v') \qquad v_B = 42.2\,\frac{\text{ft}}{\text{s}} \qquad F_N = 50.6\,\text{lb} \qquad v' = 26.2\,\frac{\text{ft}}{\text{s}^2}$$

Problem 14-29

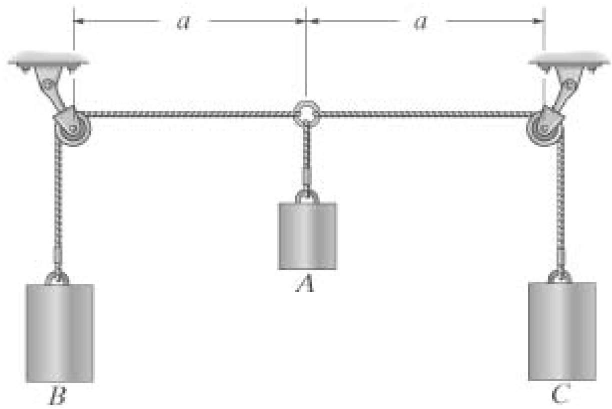
When the block A of weight W_1 is released from rest it lifts the two weights B and C each of weight W_2. Determine the maximum distance A will fall before its motion is momentarily stopped. Neglect the weight of the cord and the size of the pulleys.

Given:

$W_1 = 12$ lb

$W_2 = 15$ lb

$a = 4$ ft



Solution:

Guess $y = 10$ ft

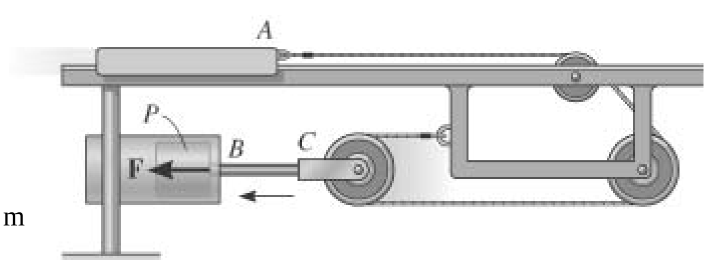
Given $W_1 y - 2W_2\left(\sqrt{a^2 + y^2} - a\right) = 0 \qquad y = \text{Find}(y) \qquad y = 3.81\,\text{ft}$

Problem 14-30

The catapulting mechanism is used to propel slider A of mass M to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies constant force $\mathbf{F}$ to rod BC such that it moves it a distance d, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.

Units Used:

$\text{kN} = 10^3\ \text{N}$



Given:

$M = 10$ kg $\quad F = 20$ kN $\quad d = 0.2$ m

Solution:

$$0 + F d = \frac{1}{2} M v^2 \qquad v = \sqrt{\frac{2F d}{M}} \qquad v = 28.284\ \frac{\text{m}}{\text{s}}$$

Problem 14-31

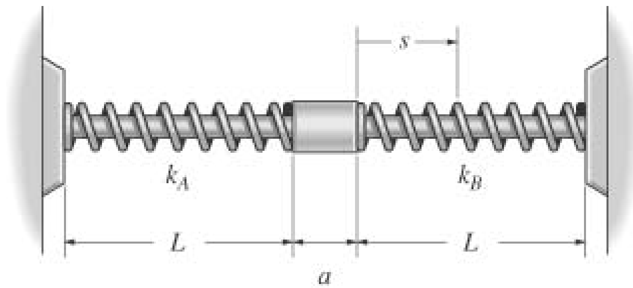
The collar has mass M and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length L and the collar has speed v_0 when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.

Given:

$M = 20\ \text{kg}$ $\quad a = 0.25\ \text{m}$

$L = 1\ \text{m}$ $\quad k_A = 50\ \dfrac{\text{N}}{\text{m}}$

$v_0 = 2\ \dfrac{\text{m}}{\text{s}}$ $\quad k_B = 100\ \dfrac{\text{N}}{\text{m}}$



Solution:

$$\frac{1}{2}M v_0^2 - \frac{1}{2}(k_A + k_B) d^2 = 0 \qquad d = \sqrt{\frac{M}{k_A + k_B}}\, v_0 \qquad d = 0.73\ \text{m}$$

---

*__Problem 14-32__

The cyclist travels to point A, pedaling until he reaches speed v_A. He then coasts freely up the curved surface. Determine the normal force he exerts on the surface when he reaches point B. The total mass of the bike and man is M. Neglect friction, the mass of the wheels, and the size of the bicycle.

Units Used:

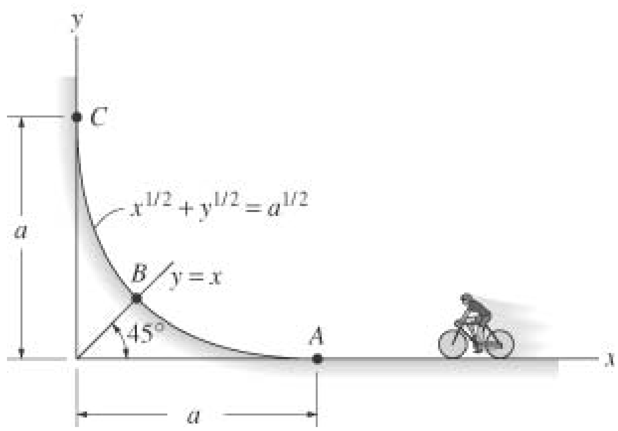
$\text{kN} = 10^3\ \text{N}$

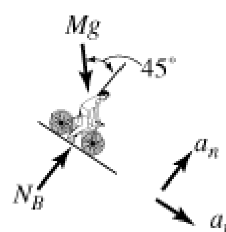
Given:

$v_A = 8\ \dfrac{\text{m}}{\text{s}}$

$M = 75\ \text{kg}$

$a = 4\ \text{m}$





Solution:

When $\quad y = x \qquad 2\sqrt{y} = \sqrt{a} \qquad y = \dfrac{a}{4} \qquad y = 1\ \text{m}$

$$\frac{1}{2}M v_A^2 - M g y = \frac{1}{2} M v_B^2 \qquad v_B = \sqrt{v_A^2 - 2 g y} \qquad v_B = 6.662\ \frac{\text{m}}{\text{s}}$$

Now find the radius of curvature $\quad \sqrt{x} + \sqrt{y} = \sqrt{a} \qquad \dfrac{1}{2\sqrt{x}}\text{d}x + \dfrac{1}{2\sqrt{y}}\text{d}y = 0$

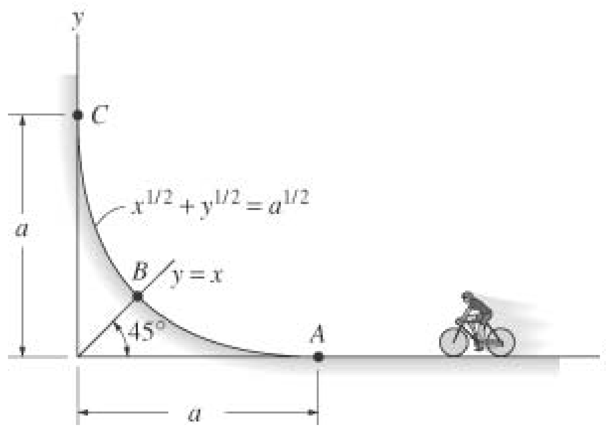
$$y' = -\sqrt{\frac{y}{x}} \qquad y'' = \frac{y - x\frac{d}{dx}y}{2x^2}\sqrt{\frac{x}{y}} \qquad \text{When} \qquad y = x \qquad y' = -1 \qquad y'' = \frac{1}{y}$$

Thus $\rho = \dfrac{\sqrt{\left(1 + y'^2\right)^3}}{y''}$ $\qquad \rho = \sqrt{8}\,y \qquad \rho = 2.828\ \text{m}$

$$N_B - M g \cos(45\ \text{deg}) = M\left(\frac{v_B^2}{\rho}\right) \qquad N_B = M g \cos(45\ \text{deg}) + M\left(\frac{v_B^2}{\rho}\right) \qquad N_B = 1.697\ \text{kN}$$

Problem 14-33

The cyclist travels to point A, pedaling until he reaches speed v_A. He then coasts freely up the curved surface. Determine how high he reaches up the surface before he comes to a stop. Also, what are the resultant normal force on the surface at this point and his acceleration? The total mass of the bike and man is M. Neglect friction, the mass of the wheels, and the size of the bicycle.



Given:

$$v_A = 4\ \frac{\text{m}}{\text{s}} \qquad M = 75\ \text{kg} \qquad a = 4\ \text{m}$$

Solution:

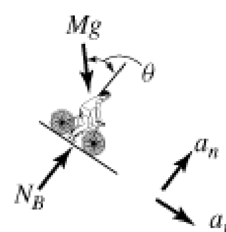
$$\frac{1}{2}M v_A^2 - M g y = 0 \qquad y = \frac{v_A^2}{2g} \qquad y = 0.815\ \text{m}$$

$$x = \left(\sqrt{a} - \sqrt{y}\right)^2 \qquad x = 1.203\ \text{m}$$

$$y' = -\sqrt{\frac{y}{x}} \qquad \theta = \operatorname{atan}\left(\left|y'\right|\right) \qquad \theta = 39.462\ \text{deg}$$

$$N_B - M g \cos(\theta) = 0 \qquad N_B = M g \cos(\theta) \qquad N_B = 568.03\ \text{N}$$

$$M g \sin(\theta) = M a_t \qquad a_t = g \sin(\theta) \qquad a_t = 6.235\ \frac{\text{m}}{\text{s}^2}$$



Problem 14-34

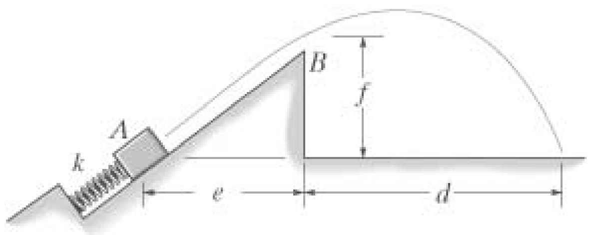
The block of weight W is pressed against the spring so as to compress it a distance δ when it is at A. If the plane is smooth, determine the distance d, measured from the wall, to where the block strikes the ground. Neglect the size of the block.

Given:

$W = 10 \text{ lb}$ $\quad e = 4 \text{ ft}$

$\delta = 2 \text{ ft}$ $\quad f = 3 \text{ ft}$

$k = 100 \frac{\text{lb}}{\text{ft}}$ $\quad g = 9.81 \frac{\text{m}}{\text{s}^2}$



Solution: $\theta = \text{atan}\left(\frac{f}{e}\right)$

Guesses $v_B = 1 \frac{\text{ft}}{\text{s}}$ $\quad t = 1 \text{ s}$ $\quad d = 1 \text{ ft}$

Given

$$\frac{1}{2}k\delta^2 - W f = \frac{1}{2}\left(\frac{W}{g}\right)v_B^2 \qquad d = v_B\cos(\theta)t \qquad 0 = f + v_B\sin(\theta)t - \left(\frac{g}{2}\right)t^2$$

$$\begin{pmatrix} v_B \\ t \\ d \end{pmatrix} = \text{Find}(v_B, t, d) \qquad v_B = 33.08 \frac{\text{ft}}{\text{s}} \qquad t = 1.369 \text{ s} \qquad d = 36.2 \text{ ft}$$

Problem 14-35

The man at the window A wishes to throw a sack of mass M onto the ground. To do this he allows it to swing from rest at B to point C, when he releases the cord at $\theta = \theta_1$. Determine the speed at which it strikes the ground and the distance R.

Given:

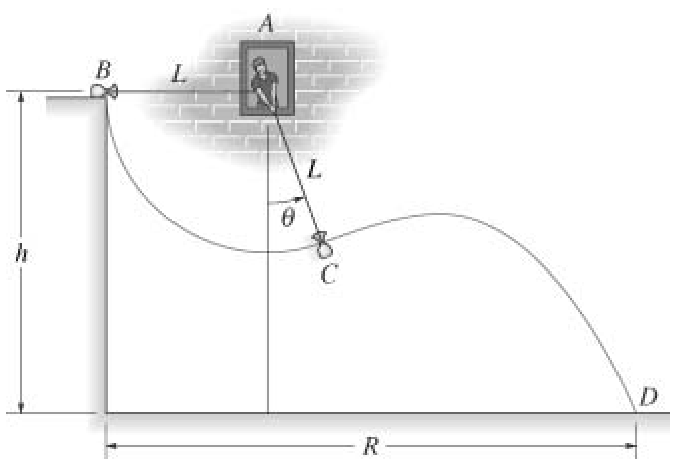
$\theta_1 = 30 \text{ deg}$

$h = 16 \text{ m}$

$L = 8 \text{ m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

$M = 30 \text{ kg}$



Solution:

$$0 + M g L\cos(\theta_1) = \frac{1}{2}M v_C^2 \qquad v_C = \sqrt{2 g L\cos(\theta_1)} \qquad v_C = 11.659 \frac{\text{m}}{\text{s}}$$

$0 + Mgh = \frac{1}{2}Mv_D^2 \qquad v_D = \sqrt{2gh} \qquad v_D = 17.718\ \frac{\text{m}}{\text{s}}$

Free Flight Guess $t = 2\ \text{s}$ $R = 1\ \text{m}$

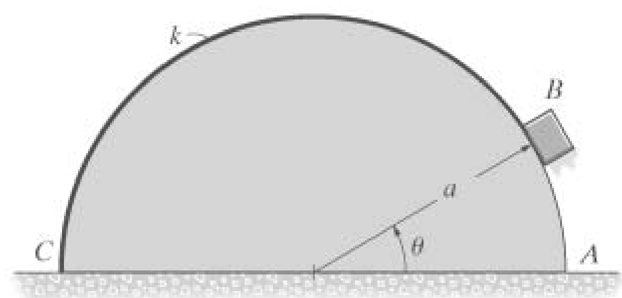
Given $0 = \left(\frac{-g}{2}\right)t^2 + v_C\sin(\theta_1)t + h - L\cos(\theta_1) \qquad R = v_C\cos(\theta_1)t + L(1 + \sin(\theta_1))$

$\begin{pmatrix} t \\ R \end{pmatrix} = \text{Find}(t, R) \qquad t = 2.078\ \text{s} \qquad R = 33.0\ \text{m}$

***Problem 14-36**

A block of weight W rests on the smooth semicylindrical surface. An elastic cord having a stiffness k is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at $A(\theta = 0°)$, determine the unstretched length of the cord so the block begins to leave the semicylinder at the instant $\theta = \theta_1$. Neglect the size of the block.

Given:



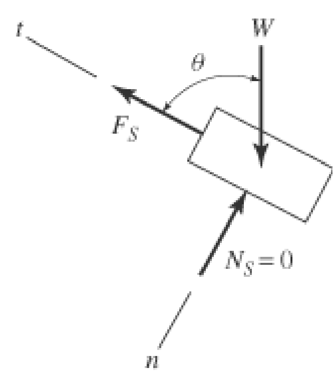
$W = 2\ \text{lb}$

$k = 2\ \frac{\text{lb}}{\text{ft}}$

$\theta_1 = 45\ \text{deg}$

$a = 1.5\ \text{ft}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guess $\delta = 1\ \text{ft}$ $v_1 = 1\ \frac{\text{ft}}{\text{s}}$

Given

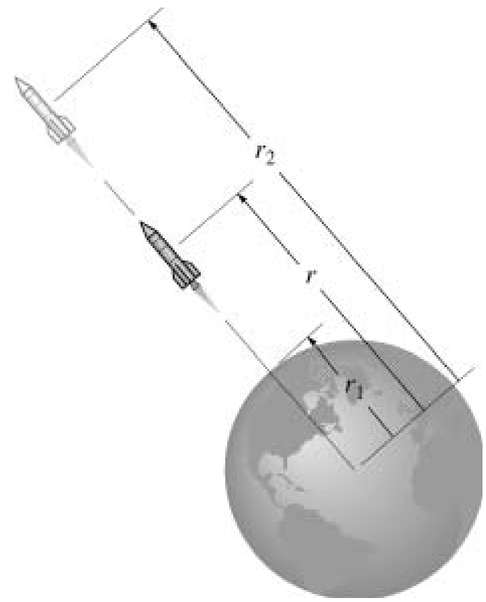
$$W\sin(\theta_1) = \left(\frac{W}{g}\right)\frac{v_1^2}{a}$$

$$\frac{1}{2}k(\pi a - \delta)^2 - \frac{1}{2}k[(\pi - \theta_1)a - \delta]^2 - Wa\sin(\theta_1) = \left(\frac{W}{g}\right)\frac{v_1^2}{2}$$

$\begin{pmatrix} v_1 \\ \delta \end{pmatrix} = \text{Find}(v_1, \delta) \qquad v_1 = 5.843\ \frac{\text{ft}}{\text{s}} \qquad \delta = 2.77\ \text{ft}$

Problem 14-37

A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming that no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13-1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.



Solution:

$$F = G\left(\frac{M_e m}{r^2}\right)$$

$$U_{12} = \int F\,dr = -G M_e m \int_{r_1}^{r_2} \frac{1}{r^2}\,dr$$

$$U_{12} = G M_e m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Problem 14-38

The spring has a stiffness k and an *unstretched length* l_0. As shown, it is confined by the plate and wall using cables so that its length is l. A block of weight W is given a speed v_A when it is at A, and it slides down the incline having a coefficient of kinetic friction μ_k. If it strikes the plate and pushes it forward a distance l_1 before stopping, determine its speed at A. Neglect the mass of the plate and spring.

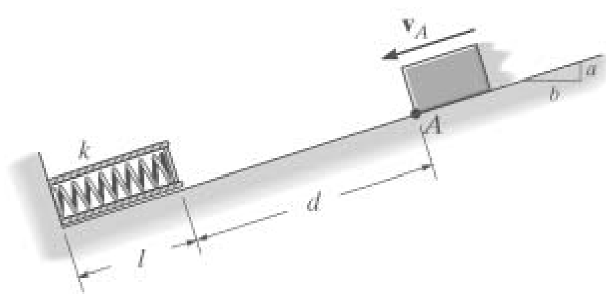
Given:

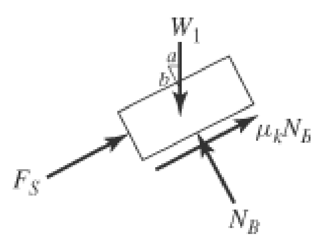
$W = 4$ lb $\quad d = 3$ ft

$l_0 = 2$ ft $\quad k = 50\ \frac{\text{lb}}{\text{ft}}$

$l = 1.5$ ft $\quad \mu_k = 0.2$

$l_1 = 0.25$ ft $\quad a = 3 \quad b = 4$





Solution: $\quad \theta = \operatorname{atan}\left(\frac{a}{b}\right)$

Guess $\quad v_A = 1\ \frac{\text{ft}}{\text{s}}$

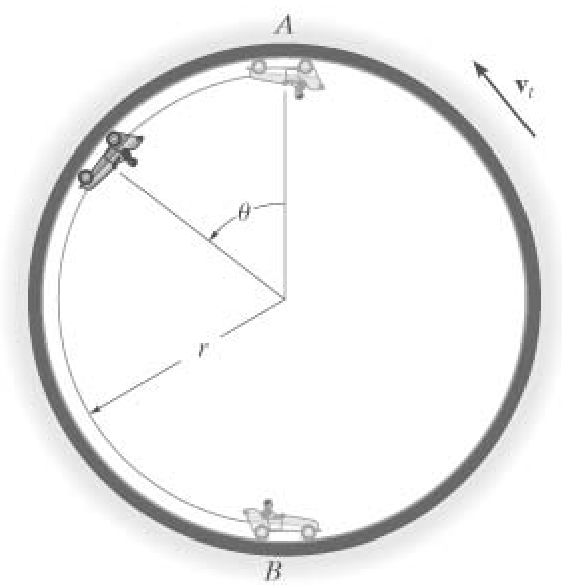
Given

$$\frac{1}{2}\left(\frac{W}{g}\right)v_A^{\,2} + W\left(\sin(\theta) - \mu_k\cos(\theta)\right)\left(d + l_1\right) - \frac{1}{2}k\left[\left(l_0 - l + l_1\right)^2 - \left(l_0 - l\right)^2\right] = 0$$

$v_A = \text{Find}(v_A)$ $\qquad$ $v_A = 5.80\,\frac{\text{ft}}{\text{s}}$

Problem 14-39

The “flying car” is a ride at an amusement park which consists of a car having wheels that roll along a track mounted inside a rotating drum. By design the car cannot fall off the track, however motion of the car is developed by applying the car’s brake, thereby gripping the car to the track and allowing it to move with a constant speed of the track, v_t. If the rider applies the brake when going from B to A and then releases it at the top of the drum, A, so that the car coasts freely down along the track to B ($\theta = \pi$ rad), determine the speed of the car at B and the normal reaction which the drum exerts on the car at B. Neglect friction during the motion from A to B. The rider and car have a total mass M and the center of mass of the car and rider moves along a circular path having a radius r.



Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$M = 250\ \text{kg}$

$r = 8\ \text{m}$

$v_t = 3\ \frac{\text{m}}{\text{s}}$

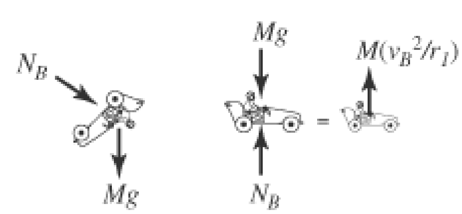
Solution:

$$\frac{1}{2}M v_t^{\,2} + M g 2r = \frac{1}{2}M v_B^{\,2}$$

$v_B = \sqrt{v_t^{\,2} + 4gr}$ $\qquad$ $v_B = 18.0\ \frac{\text{m}}{\text{s}}$

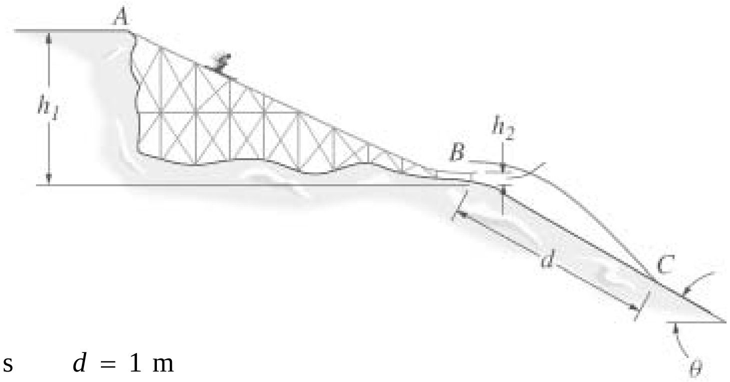
$$N_B - Mg = M\left(\frac{v_B^{\,2}}{r}\right)$$

$N_B = M\left(g + \frac{v_B^{\,2}}{r}\right)$ $\qquad$ $N_B = 12.5\,\text{kN}$



***Problem 14-40**

The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance d to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass M.



Given:

$M = 70\ \text{kg}$ $\quad h_1 = 50\ \text{m}$

$\theta = 30\ \text{deg}$ $\quad h_2 = 4\ \text{m}$

Solution:

Guesses $\quad v_B = 1\ \dfrac{\text{m}}{\text{s}} \quad t = 1\ \text{s} \quad d = 1\ \text{m}$

Given $\quad M g(h_1 - h_2) = \dfrac{1}{2} M v_B^2 \quad v_B t = d\cos(\theta) \quad -h_2 - d\sin(\theta) = \dfrac{-1}{2} g t^2$

$$\begin{pmatrix} v_B \\ t \\ d \end{pmatrix} = \text{Find}(v_B, t, d) \qquad t = 3.753\ \text{s} \qquad v_B = 30.0\ \frac{\text{m}}{\text{s}} \qquad d = 130.2\ \text{m}$$

Problem 14-41

A spring having a stiffness k is compressed a distance δ. The stored energy in the spring is used to drive a machine which requires power P. Determine how long the spring can supply energy at the required rate.

Units Used: $\quad \text{kN} = 10^3\ \text{N}$

Given: $\quad k = 5\ \dfrac{\text{kN}}{\text{m}} \quad \delta = 400\ \text{mm} \quad P = 90\ \text{W}$

Solution: $\quad U_{12} = \dfrac{1}{2} k\delta^2 = P t \quad t = \dfrac{1}{2} k\left(\dfrac{\delta^2}{P}\right) \quad t = 4.44\ \text{s}$

Problem 14-42

Determine the power input for a motor necessary to lift a weight W at a constant rate v. The efficiency of the motor is ε.

Given: $\quad W = 300\ \text{lbf} \quad v = 5\ \dfrac{\text{ft}}{\text{s}} \quad \varepsilon = 0.65$

Solution: $P = \frac{W v}{\varepsilon}$ $P = 4.20\,\text{hp}$

Problem 14-43

An electrically powered train car draws a power P. If the car has weight W and starts from rest, determine the maximum speed it attains in time t. The mechanical efficiency is ε.

Given: $P = 30\ \text{kW}$ $W = 40000\ \text{lbf}$ $t = 30\ \text{s}$ $\varepsilon = 0.8$

Solution: $\varepsilon P = F v = \frac{W}{g}\left(\frac{d}{dt}v\right)v$

$$\int_0^v v\,dv = \int_0^t \frac{\varepsilon P g}{W}\,dt \qquad v = \sqrt{\frac{2\varepsilon P g t}{W}} \qquad v = 29.2\ \frac{\text{ft}}{\text{s}}$$

***Problem 14-44**

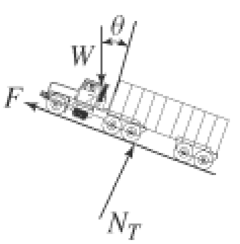
A truck has a weight W and an engine which transmits a power P to *all* the wheels. Assuming that the wheels do not slip on the ground, determine the angle θ of the largest incline the truck can climb at a constant speed v.

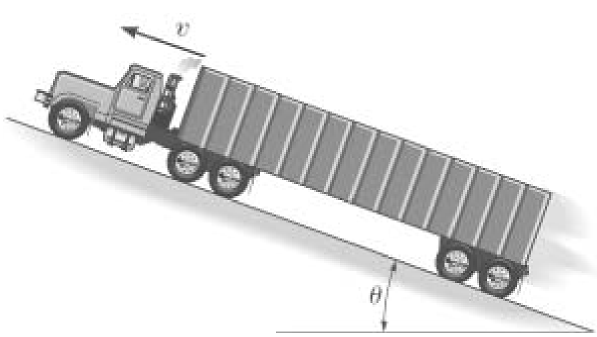
Given:

$W = 25000\ \text{lbf}$

$v = 50\ \frac{\text{ft}}{\text{s}}$

$P = 350\ \text{hp}$





Solution:

$F = W\sin(\theta)$ $P = W\sin(\theta)v$

$\theta = \operatorname{asin}\left(\frac{P}{W v}\right)$ $\theta = 8.86\,\text{deg}$

Problem 14-45

An automobile having mass M travels up a slope at constant speed v. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has efficiency ε.

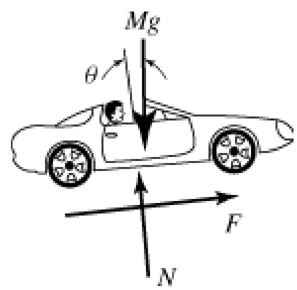
Units Used:

$\text{Mg} = 10^3\ \text{kg}$

Given:

$M = 2\ \text{Mg}$ $\quad v = 100\ \dfrac{\text{km}}{\text{hr}}$

$\theta = 7\ \text{deg}$ $\quad \varepsilon = 0.65$



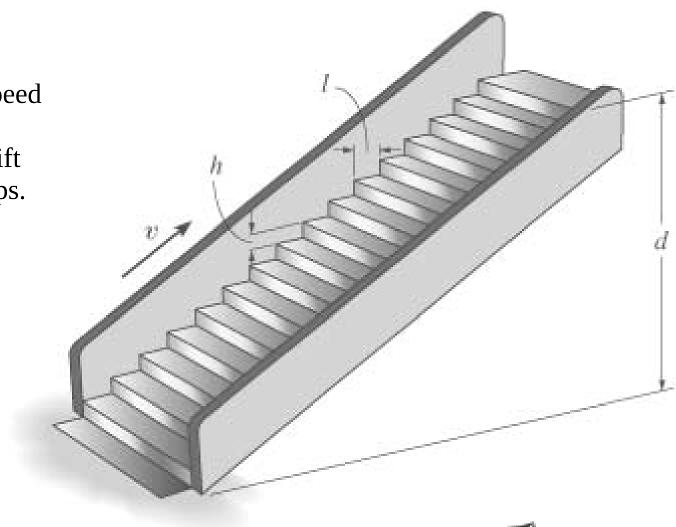
Solution:

$P = M g \sin(\theta) v$ $\qquad P = 66.419\ \text{kW}$

$P_{eng} = \dfrac{P}{\varepsilon}$ $\qquad P_{eng} = 102.2\ \text{kW}$

Problem 14-46

The escalator steps move with a constant speed *v*. If the steps are of height *h* and length *l*, determine the power of a motor needed to lift an average mass *M* per step.There are *n* steps.



Given:

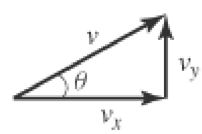
$M = 150\ \text{kg}$ $\qquad h = 125\ \text{mm}$

$n = 32$ $\qquad l = 250\ \text{mm}$

$v = 0.6\ \dfrac{\text{m}}{\text{s}}$ $\qquad d = n h$

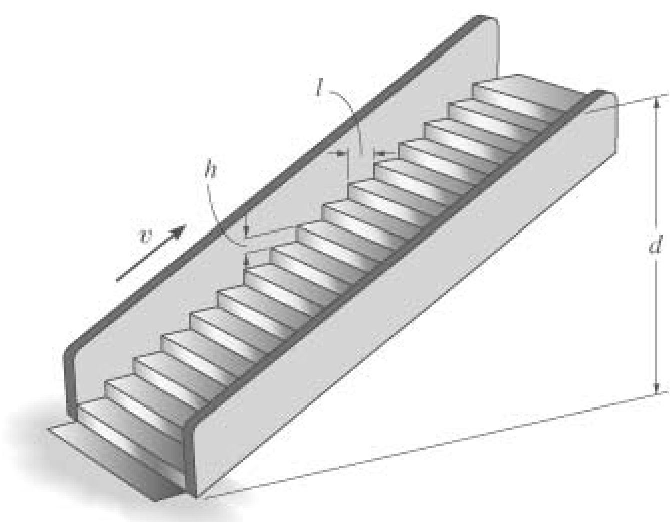
Solution:

$\theta = \text{atan}\left(\dfrac{h}{l}\right)$ $\qquad P = n M g v \sin(\theta)$ $\qquad P = 12.63\ \text{kW}$



Problem 14-47

If the escalator in Prob. 14–46 is *not moving*, determine the constant speed at which a man having a mass *M* must walk up the steps to generate power *P*—the same amount that is needed to power a standard light bulb.



Given:

$M = 80\ \text{kg}$ $\qquad h = 125\ \text{mm}$

$n = 32$ $\qquad l = 250\ \text{mm}$

$v = 0.6\ \dfrac{\text{m}}{\text{s}}$ $\qquad P = 100\ \text{W}$

Solution:

$$\theta = \operatorname{atan}\left(\frac{h}{l}\right) \qquad P = F v \sin(\theta) \qquad v = \frac{P}{M g \sin(\theta)} \qquad v = 0.285\ \frac{\text{m}}{\text{s}}$$

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*__Problem 14-48__

An electric streetcar has a weight W and accelerates along a horizontal straight road from rest such that the power is always P. Determine how far it must travel to reach a speed of v.

Given: $W = 15000\ \text{lbf}$ $\qquad v = 40\ \frac{\text{ft}}{\text{s}}$ $\qquad P = 100\ \text{hp}$

Solution:

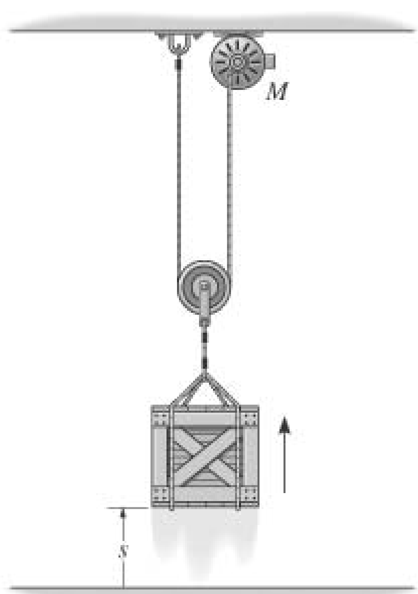
$$P = F v = \left(\frac{W}{g}\right) a v = \left(\frac{W}{g}\right) v^2 \left(\frac{\text{d}}{\text{d}s_c} v\right)$$

Guess $\qquad d = 1\ \text{ft}$

Given $\qquad \int_0^d P\, \text{d}s_c = \int_0^v \left(\frac{W}{g}\right) v^2\, \text{d}v \qquad d = \text{Find}(d) \qquad d = 180.8\ \text{ft}$

Problem 14-49

The crate of weight W is given speed v in time t_1 starting from rest. If the acceleration is constant, determine the power that must be supplied to the motor when $t = t_2$. The motor has an efficiency ε. Neglect the mass of the pulley and cable.



Given:

$W = 50\ \text{lbf}$ $\qquad t_2 = 2\ \text{s}$

$v = 10\ \frac{\text{ft}}{\text{s}}$ $\qquad \varepsilon = 0.76$

$t_1 = 4\ \text{s}$ $\qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$a = \frac{v}{t_1} \qquad a = 2.5\ \frac{\text{ft}}{\text{s}^2}$$

$$v_2 = a t_2 \qquad v_2 = 5\ \frac{\text{ft}}{\text{s}}$$

$$F - W = \left(\frac{W}{g}\right)a \qquad F = W + \left(\frac{W}{g}\right)a \qquad F = 53.882\,\text{lbf}$$

$$P = F v_2 \qquad P = 0.49\,\text{hp} \qquad P_{motor} = \frac{P}{\varepsilon} \qquad P_{motor} = 0.645\,\text{hp}$$

Problem 14-50

A car has a mass M and accelerates along a horizontal straight road from rest such that the power is always a constant amount P. Determine how far it must travel to reach a speed of v.

Solution:

Power: Since the power output is constant, then the traction force F varies with v. Applying Eq. 14-10, we have

$$P = F v \qquad F = \frac{P}{v}$$

Equation of Motion: $\dfrac{P}{v} = M a \qquad a = \dfrac{P}{M v}$

Kinematics: Applying equation $\mathrm{d}s = \dfrac{v\,\mathrm{d}v}{a}$, we have

$$\int_0^s 1\,\mathrm{d}s = \int_0^v \frac{M v^2}{P}\,\mathrm{d}v \qquad s = \frac{M v^3}{3P}$$

Problem 14-51

To dramatize the loss of energy in an automobile, consider a car having a weight W_{car} that is traveling at velocity v. If the car is brought to a stop, determine how long a light bulb with power P_{bulb} must burn to expend the same amount of energy.

Given: $W_{car} = 5000\ \text{lbf} \qquad P_{bulb} = 100\ \text{W}$

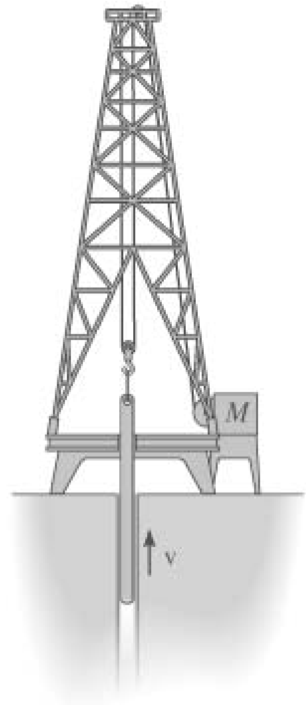
$v = 35\ \dfrac{\text{mi}}{\text{hr}} \qquad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$$\frac{1}{2}\left(\frac{W_{car}}{g}\right)v^2 = P_{bulb}\,t \qquad t = \frac{W_{car} v^2}{2g P_{bulb}} \qquad t = 46.2\,\text{min}$$

***Problem 14-52**

Determine the power output of the draw-works motor M necessary to lift the drill pipe of weight W upward with a constant speed v. The cable is tied to the top of the oil rig, wraps around the lower pulley, then around the top pulley, and then to the motor.



Given:

$$W = 600 \text{ lbf}$$

$$v = 4 \frac{\text{ft}}{\text{s}}$$

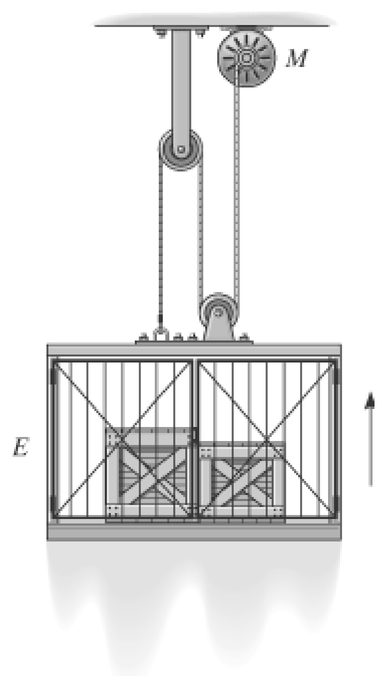
Solution:

$$P = W v$$

$$P = 4.36 \text{ hp}$$

Problem 14-53

The elevator of mass m_{el} starts from rest and travels upward with a constant acceleration a_c. Determine the power output of the motor M when $t = t_1$. Neglect the mass of the pulleys and cable.



Given:

$$m_{el} = 500 \text{ kg}$$

$$a_c = 2 \frac{\text{m}}{\text{s}^2}$$

$$t_1 = 3 \text{ s}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$

Solution:

$$F - m_{el} g = m_{el} a_c \qquad F = m_{el}(g + a_c)$$

$$F = 5.905 \times 10^3 \text{ N}$$

$v_1 = a_C t_1$ $\qquad v_1 = 6\ \frac{\text{m}}{\text{s}}$

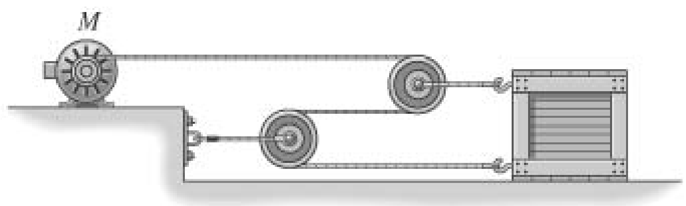
$P = F v_1$ $\qquad P = 35.4\,\text{kW}$

Problem 14-54

The crate has mass m_c and rests on a surface for which the coefficients of static and kinetic friction are μ_s and μ_k respectively. If the motor M supplies a cable force of $F = at^2 + b$, determine the power output developed by the motor when $t = t_1$.

Given:

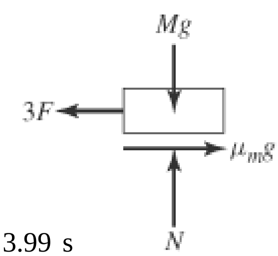
$m_C = 150\ \text{kg}$ $\qquad a = 8\ \frac{\text{N}}{\text{s}^2}$



$\mu_S = 0.3$ $\qquad b = 20\ \text{N}$

$\mu_k = 0.2$ $\qquad t_1 = 5\ \text{s}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Time to start motion

$3\left(a t^2 + b\right) = \mu_S m_C g$ $\qquad t = \sqrt{\frac{1}{a}\left(\frac{\mu_S m_C g}{3} - b\right)}$ $\qquad t = 3.99\ \text{s}$

Speed at t_1 $\qquad 3\left(a t^2 + b\right) - \mu_k m_C g = m_C a = m_C \frac{d}{dt} v$

$v = \int_t^{t_1} \frac{3}{m_C}\left(a t^2 + b\right) - \mu_k g\ dt$ $\qquad v = 1.70\ \frac{\text{m}}{\text{s}}$

$P = 3\left(a t_1^{\,2} + b\right) v$ $\qquad P = 1.12\,\text{kW}$

Problem 14-55

The elevator E and its freight have total mass m_E. Hoisting is provided by the motor M and the block C of mass m_C .If the motor has an efficiency ε, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed v_E.

Given:

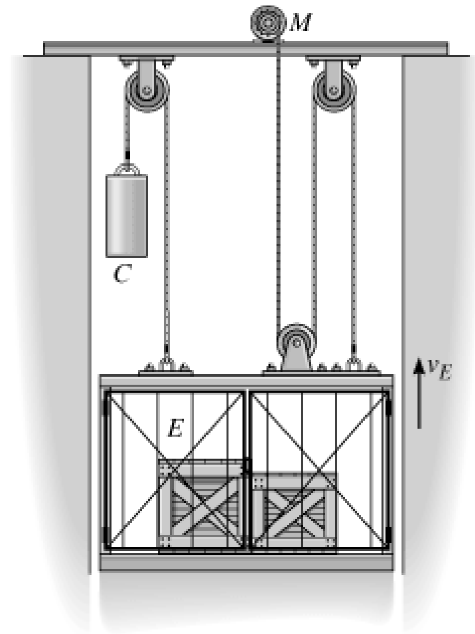
$m_C = 60\ \text{kg}$

$m_E = 400\ \text{kg}$

$\varepsilon = 0.6$

$v_E = 4\ \dfrac{\text{m}}{\text{s}}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



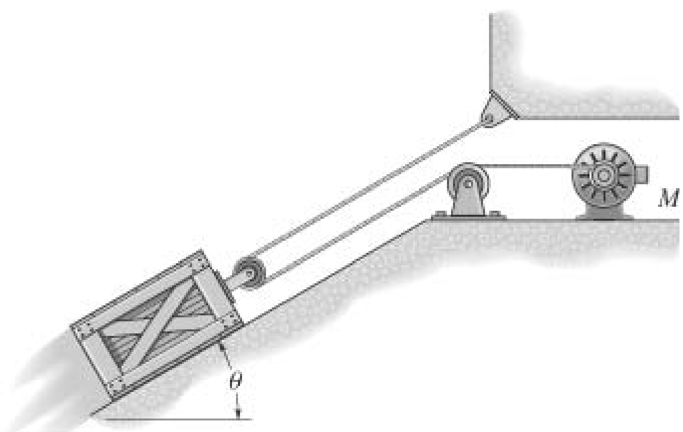
Solution:

$F = (m_E - m_C)g$

$P = \dfrac{F v_E}{\varepsilon}$ $\qquad P = 22.236\,\text{kW}$

***Problem 14-56**

The crate of mass m_c is hoisted up the incline of angle θ by the pulley system and motor M. If the crate starts from rest and by constant acceleration attains speed v after traveling a distance d along the plane, determine the power that must be supplied to the motor at this instant. Neglect friction along the plane. The motor has efficiency ε.



Given:

$m_C = 50\ \text{kg}$ $\qquad \theta = 30\ \text{deg}$

$d = 8\ \text{m}$ $\qquad \varepsilon = 0.74$

$v = 4\ \dfrac{\text{m}}{\text{s}}$ $\qquad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$a_C = \dfrac{v^2}{2d}$ $\qquad a_C = 1\ \dfrac{\text{m}}{\text{s}^2}$

$F - (m_C g)\sin(\theta) = m a_C$ $\qquad F = m_C(g\sin(\theta) + a_C)$ $\qquad F = 295.25\ \text{N}$

$P = \dfrac{F v}{\varepsilon}$ $\qquad P = 1.596\,\text{kW}$

Problem 14-57

The block has mass M and rests on a surface for which the coefficients of static and kinetic friction are μ_s and μ_k respectively. If a force $F = kt^2$ is applied to the cable, determine the power developed by the force at $t = t_2$. *Hint:* First determine the time needed for the force to cause motion.

Given:

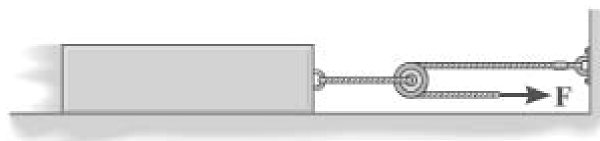
$M = 150 \text{ kg} \quad k = 60 \frac{\text{N}}{\text{s}^2}$

$\mu_s = 0.5$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

$\mu_k = 0.4$

$t_2 = 5 \text{ s}$



Solution:

$2F = 2kt_1^2 = \mu_s M g \qquad t_1 = \sqrt{\frac{\mu_s M g}{2k}} \qquad t_1 = 2.476 \text{ s}$

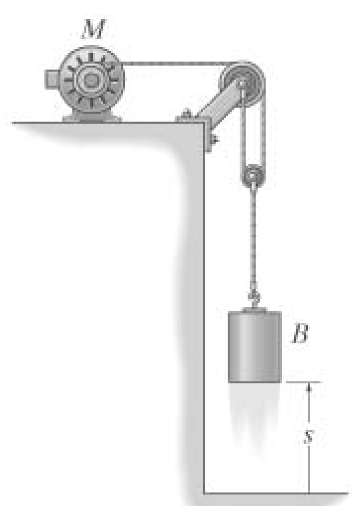
$2kt^2 - \mu_k M g = M a = M\left(\frac{d}{dt}v\right)$

$v_2 = \int_{t_1}^{t_2} \left(\frac{2kt^2}{M} - \mu_k g\right) dt \qquad v_2 = 19.381 \frac{\text{m}}{\text{s}}$

$P = 2kt_2^2 v_2 \qquad P = 58.144 \text{ kW}$

Problem 14-58

The load of weight W is hoisted by the pulley system and motor M. If the crate starts from rest and by constant acceleration attains a speed v after rising a distance $s = s_1$, determine the power that must be supplied to the motor at this instant. The motor has an efficiency ε. Neglect the mass of the pulleys and cable.



Given:

$W = 50 \text{ lbf} \qquad \varepsilon = 0.76$

$v = 15 \frac{\text{ft}}{\text{s}} \qquad g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$s_1 = 6 \text{ ft}$

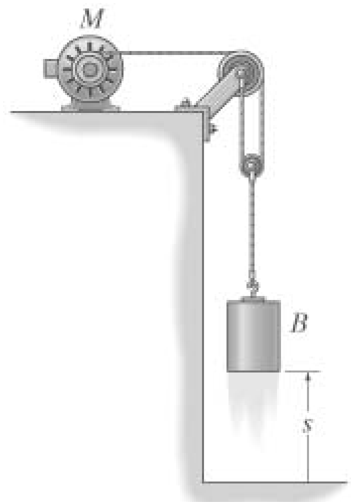
Solution:

$$a = \frac{v^2}{2s_1} \qquad F = W + \left(\frac{W}{g}\right)a$$

$$P = \frac{F\,v}{\varepsilon} \qquad P = 2.84\,\text{hp}$$

Problem 14-59

The load of weight W is hoisted by the pulley system and motor M. If the motor exerts a constant force **F** on the cable, determine the power that must be supplied to the motor if the load has been hoisted at $s = s'$ starting from rest.The motor has an efficiency ε.



Given:

$W = 50\text{ lbf} \quad \varepsilon = 0.76$

$F = 30\text{ lbf} \quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$s' = 10\text{ ft}$

Solution:

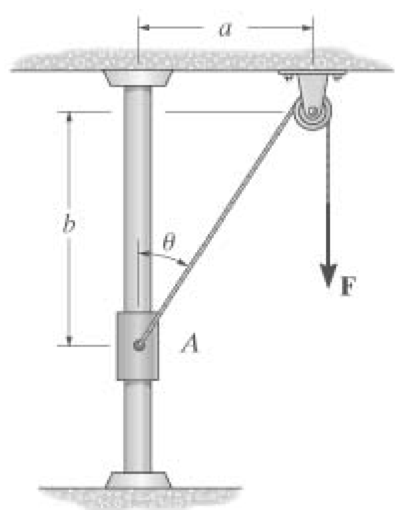
$$2F - W = \frac{W}{g}a$$

$$a = \left(\frac{2F}{W} - 1\right)g \qquad a = 6.44\,\frac{\text{ft}}{\text{s}^2}$$

$$v = \sqrt{2a\,s'} \qquad v = 11.349\,\frac{\text{ft}}{\text{s}}$$

$$P = \frac{2F\,v}{\varepsilon} \qquad P = 1.629\,\text{hp}$$

***Problem 14-60**



The collar of weight W starts from rest at A and is lifted by applying a constant vertical force **F** to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = \theta_2$.

Given:

$W = 10\text{ lbf} \qquad a = 3\text{ ft}$

$F = 25\text{ lbf} \qquad b = 4\text{ ft}$

$\theta_2 = 60\text{ deg}$

Solution:

$$h = b - (a)\cot(\theta_2)$$

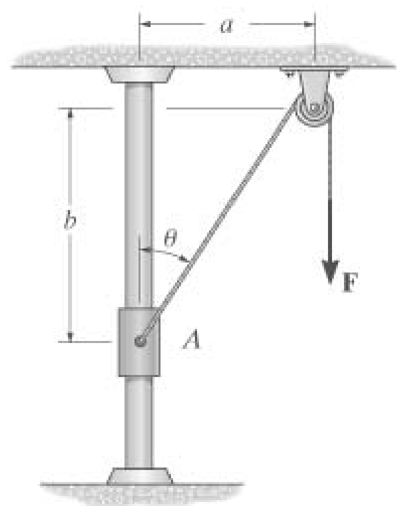
$L_1 = \sqrt{a^2 + b^2}$ $\qquad$ $L_2 = \sqrt{a^2 + (b - h)^2}$

$F(L_1 - L_2) - W h = \frac{1}{2}\left(\frac{W}{g}\right) v_2^2$ $\qquad$ $v_2 = \sqrt{2\left(\frac{F}{W}\right)(L_1 - L_2)g - 2g h}$

$P = F v_2 \cos(\theta_2)$ $\qquad$ $P = 0.229\ \text{hp}$

Problem 14-61

The collar of weight *W* starts from rest at *A* and is lifted with a constant speed *v* along the smooth rod. Determine the power developed by the force **F** at the instant shown.



Given:

$W = 10\ \text{lbf}$

$v = 2\ \frac{\text{ft}}{\text{s}}$

$a = 3\ \text{ft}$

$b = 4\ \text{ft}$

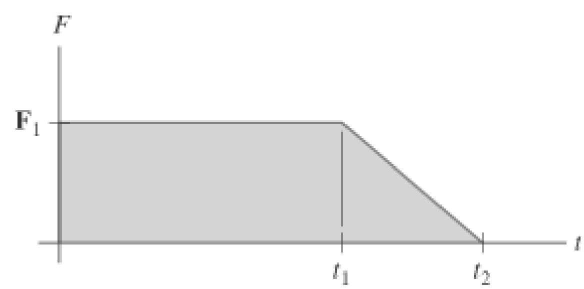
Solution:

$\theta = \text{atan}\left(\frac{a}{b}\right)$ $\qquad$ $F\cos(\theta) - W = 0$ $\qquad$ $F = \frac{W}{\cos(\theta)}$

$P = F v \cos(\theta)$ $\qquad$ $P = 0.0364\ \text{hp}$

Problem 14-62

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in time $t = t_2$.

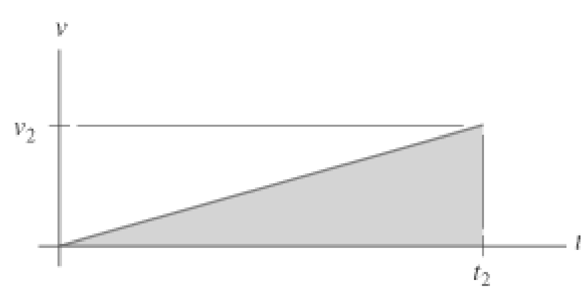


Units Used: $\qquad$ $\text{kJ} = 10^3\ \text{J}$

Given:

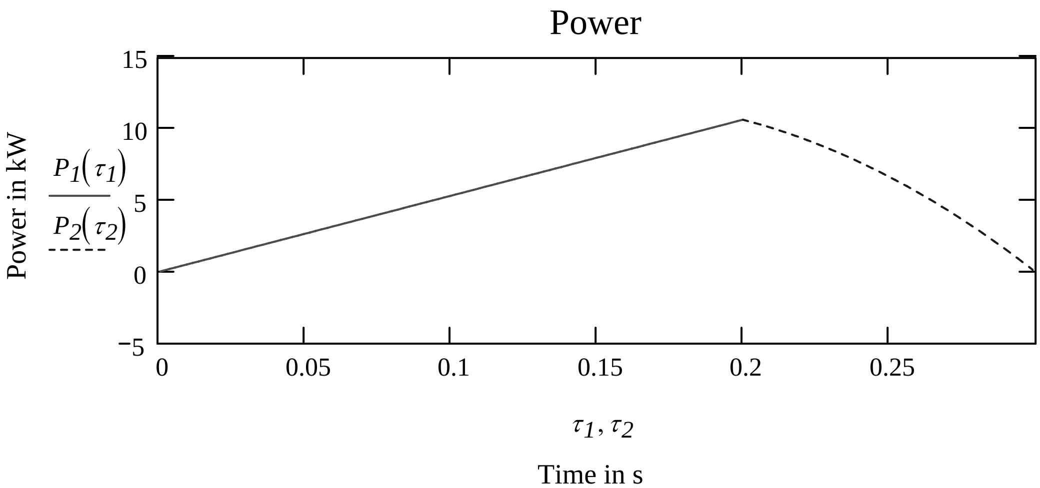
$F_1 = 800\ \text{N}$ $\qquad$ $t_1 = 0.2\ \text{s}$

$v_2 = 20\ \frac{\text{m}}{\text{s}}$ $\qquad$ $t_2 = 0.3\ \text{s}$



Solution: $\tau_1 = 0, 0.01t_1 .. t_1$ $\qquad P_1(\tau_1) = F_1 \frac{v_2}{t_2} \tau_1 \frac{1}{\text{kW}}$

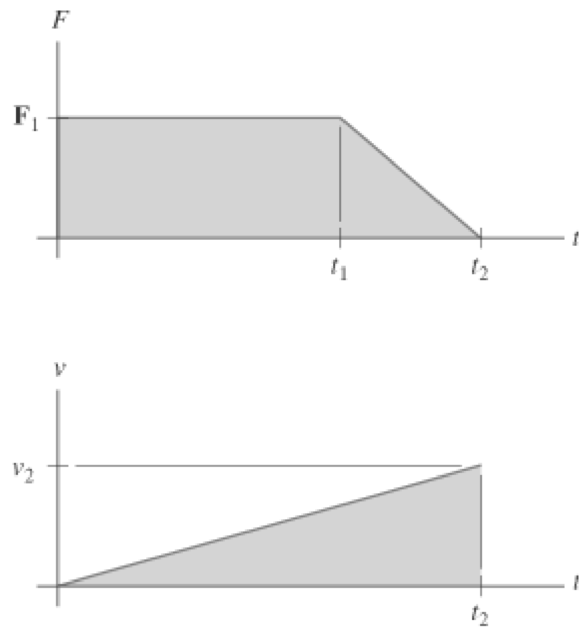
$\tau_2 = t_1, 1.01t_1 .. t_2$ $\qquad P_2(\tau_2) = F_1 \left(\frac{\tau_2 - t_2}{t_1 - t_2} \right) \frac{v_2}{t_2} \tau_2 \frac{1}{\text{kW}}$



$$U = \left(\int_0^{t_1} P_1(\tau)\, d\tau + \int_{t_1}^{t_2} P_2(\tau)\, d\tau \right) \text{kW} \qquad U = 1.689\ \text{kJ}$$

Problem 14-63

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the time period $0 < t < t_2$.



Given:

$F_1 = 800\ \text{N}$

$v_2 = 20\ \frac{\text{m}}{\text{s}}$

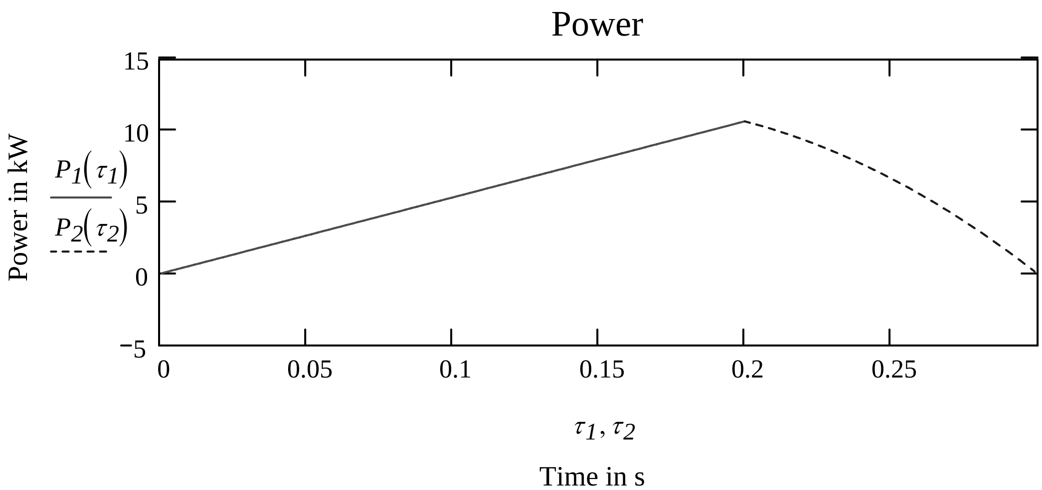
$t_1 = 0.2\ \text{s}$

$t_2 = 0.3\ \text{s}$

Solution:

$$\tau_1 = 0, 0.01t_1 .. t_1 \qquad P_1(\tau_1) = F_1\left(\frac{v_2}{t_2}\right)\tau_1 \frac{1}{\text{kW}}$$

$$\tau_2 = t_1, 1.01t_1 .. t_2 \qquad P_2(\tau_2) = F_1\left(\frac{\tau_2 - t_2}{t_1 - t_2}\right)\left(\frac{v_2}{t_2}\right)\tau_2 \frac{1}{\text{kW}}$$



$$P_{max} = P_1(t_1)\,\text{kW} \qquad P_{max} = 10.667\,\text{kW}$$

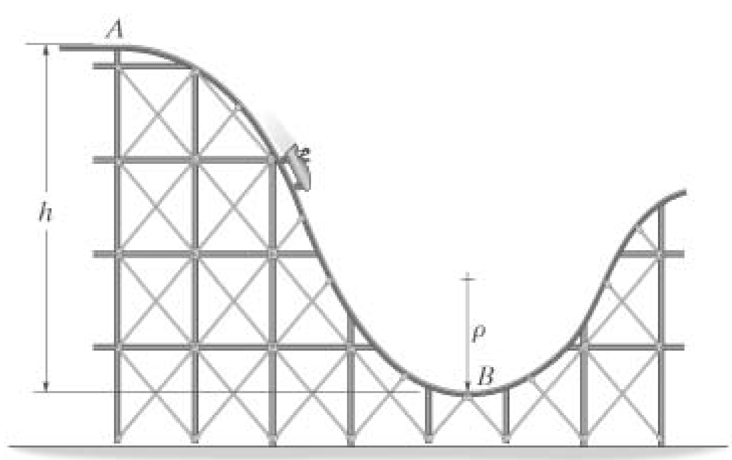
***Problem 14-64**

Determine the required height h of the roller coaster so that when it is essentially at rest at the crest of the hill it will reach a speed v when it comes to the bottom. Also, what should be the minimum radius of curvature ρ for the track at B so that the passengers do not experience a normal force greater than kmg? Neglect the size of the car and passengers.

Given:

$$v = 100\,\frac{\text{km}}{\text{hr}}$$

$$k = 4$$



Solution:

$$T_1 = 0 \qquad V_1 = mgh$$

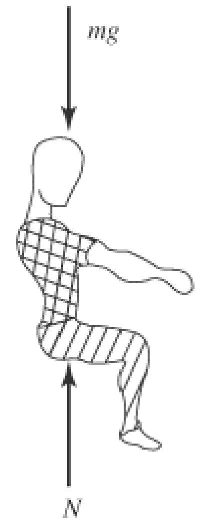
$$T_2 = \frac{1}{2}mv^2 \qquad V_2 = 0$$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$h = \frac{1}{2}\left(\frac{v^2}{g}\right) \qquad h = 39.3\ \text{m}$$

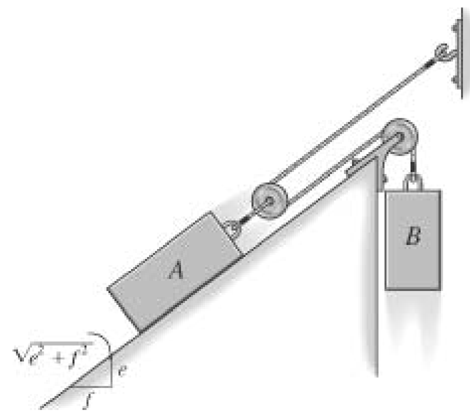
$$kmg - mg = \frac{mv^2}{\rho}$$

$$\rho = \frac{v^2}{g(k-1)} \qquad \rho = 26.2\ \text{m}$$



Problem 14-65

Block A has weight W_A and block B has weight W_B. Determine the speed of block A after it moves a distance d down the plane, starting from rest. Neglect friction and the mass of the cord and pulleys.



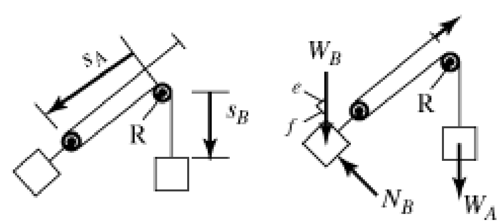
Given:

$W_A = 60\ \text{lb} \qquad e = 3$

$W_B = 10\ \text{lb} \qquad f = 4$

$d = 5\ \text{ft} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:



$L = 2s_A + s_B \qquad 0 = 2\Delta s_A + \Delta s_B \qquad 0 = 2v_A + v_B$

$T_1 = 0 \qquad V_1 = 0$

$$T_2 = \frac{1}{2}\left(\frac{W_A}{g}\right)v_A^2 + \frac{1}{2}\left(\frac{W_B}{g}\right)v_B^2 \qquad V_2 = -W_A\left(\frac{e}{\sqrt{e^2+f^2}}\right)d + W_B 2d$$

$$0 + 0 = \frac{1}{2}\left(\frac{W_A}{g}\right)v_A^2 + \frac{1}{2}\left(\frac{W_B}{g}\right) - W_A\left(\frac{e}{\sqrt{e^2+f^2}}\right)d + W_B 2d$$

$$v_A = \sqrt{\frac{2gd}{W_A + 4W_B}\left[W_A\left(\frac{e}{\sqrt{e^2+f^2}}\right) - 2W_B\right]} \qquad v_A = 7.178\ \frac{\text{ft}}{\text{s}}$$

Problem 14-66

The collar has mass M and rests on the smooth rod. Two springs are attached to it and the ends of the rod as shown. Each spring has an uncompressed length l. If the collar is displaced a distance $s = s'$ and released from rest, determine its velocity at the instant it returns to the point $s = 0$.

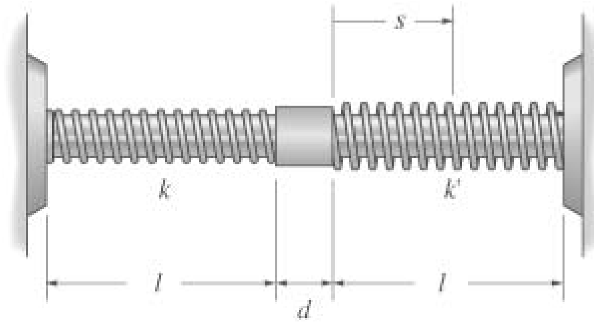
Given:

$M = 20\ \text{kg}$ $\quad k = 50\ \dfrac{\text{N}}{\text{m}}$

$s' = 0.5\ \text{m}$

$l = 1\ \text{m}$ $\quad k' = 100\ \dfrac{\text{N}}{\text{m}}$

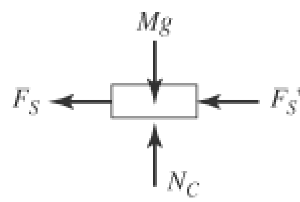
$d = 0.25\ \text{m}$



Solution:

$T_1 = 0 \qquad V_1 = \dfrac{1}{2}(k + k')\,s'^2$

$T_2 = \dfrac{1}{2}M v^2 \qquad V_2 = 0$



$$0 + \frac{1}{2}(k + k')\,s'^2 = \frac{1}{2}M v_C^{\,2} + 0$$

$$v_C = \sqrt{\frac{k + k'}{M}}\,s' \qquad \boxed{v_C = 1.37\ \frac{\text{m}}{\text{s}}}$$

Problem 14-67

The collar has mass M and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length L and the collar has speed v_0 when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.

Given:

$M = 20\ \text{kg}$

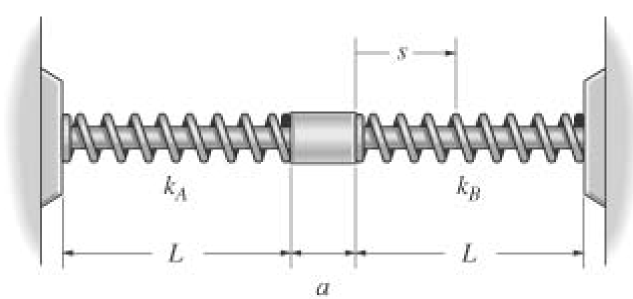
$L = 1\ \text{m}$

$a = 0.25\ \text{m}$

$v_0 = 2\ \dfrac{\text{m}}{\text{s}}$

$k_A = 50\ \dfrac{\text{N}}{\text{m}}$

$k_B = 100\ \dfrac{\text{N}}{\text{m}}$



Solution:

$T_1 = \frac{1}{2}M v_0^2 \qquad V_1 = 0 \qquad T_2 = 0 \qquad V_2 = \frac{1}{2}(k_A + k_B)d^2$

$\frac{1}{2}M v_0^2 + 0 = 0 + \frac{1}{2}(k_A + k_B)d^2 \qquad d = \sqrt{\frac{M}{k_A + k_B}}v_0 \qquad d = 0.73\ \text{m}$

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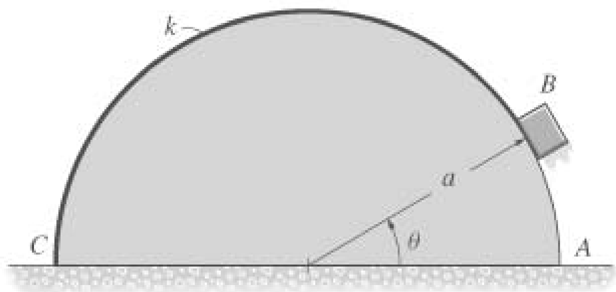
*__Problem 14-68__

A block of weight W rests on the smooth semicylindrical surface. An elastic cord having a stiffness k is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at $A(\theta = 0°)$, determine the unstretched length of the cord so the block begins to leave the semicylinder at the instant $\theta = \theta_2$. Neglect the size of the block.

Given:

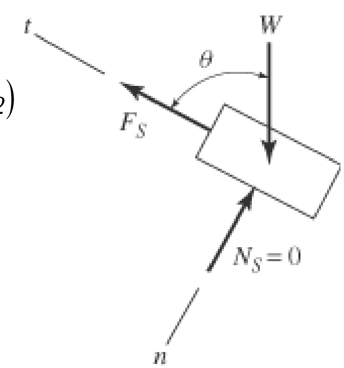


$W = 2\ \text{lb} \qquad a = 1.5\ \text{ft}$

$k = 2\ \frac{\text{lb}}{\text{ft}} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$\theta_2 = 45\ \text{deg}$

Solution:



$T_1 = 0 \qquad V_1 = \frac{1}{2}k(\pi a - \delta)^2$

$T_2 = \frac{1}{2}\left(\frac{W}{g}\right)v_2^2 \qquad V_2 = \frac{1}{2}k\left[(\pi - \theta_2)a - \delta\right]^2 + (W a)\sin(\theta_2)$

Guess $\qquad \delta = 1\ \text{ft} \qquad v_2 = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$W\sin(\theta_2) = \frac{W}{g}\left(\frac{v_2^2}{a}\right)$$

$$0 + \frac{1}{2}k(\pi a - \delta)^2 = \frac{1}{2}\left(\frac{W}{g}\right)v_2^2 + \left[\frac{1}{2}k\left[(\pi - \theta_2)a - \delta\right]^2 + (W a)\sin(\theta_2)\right]$$

$$\begin{pmatrix} v_2 \\ \delta \end{pmatrix} = \text{Find}(v_2, \delta) \qquad v_2 = 5.843\ \frac{\text{ft}}{\text{s}} \qquad \delta = 2.77\ \text{ft}$$

Problem 14-69

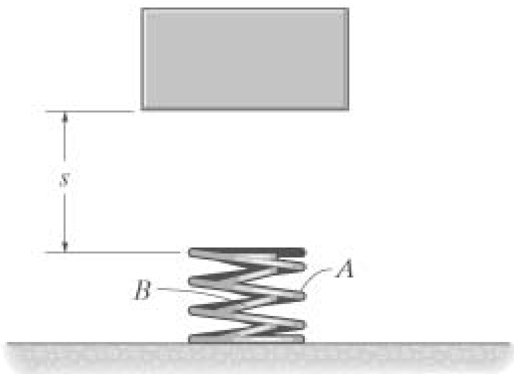
Two equal-length springs are "nested" together in order to form a shock absorber. If it is designed to arrest the motion of mass M that is dropped from a height s_1 above the top of the springs from an at-rest position, and the maximum compression of the springs is to be δ, determine the required stiffness of the inner spring, k_B, if the outer spring has a stiffness k_A.

Given:

$M = 2\ \text{kg}$ $\qquad \delta = 0.2\ \text{m}$

$k_A = 400\ \dfrac{\text{N}}{\text{m}}$ $\qquad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$s_1 = 0.5\ \text{m}$



Solution:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + Mg(s_1 + \delta) = 0 + \frac{1}{2}(k_A + k_B)\,\delta^2$$

$$k_B = \frac{2Mg(s_1 + \delta)}{\delta^2} - k_A \qquad k_B = 287\ \frac{\text{N}}{\text{m}}$$

Problem 14-70

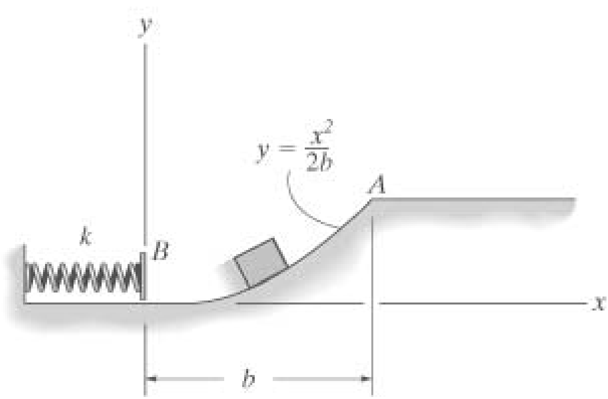
Determine the smallest amount the spring at B must be compressed against the block of weight W so that when it is released from B it slides along the smooth surface and reaches point A.

Given:

$W = 0.5\ \text{lb}$

$b = 1\ \text{ft}$

$k = 5\ \dfrac{\text{lb}}{\text{in}}$



Solution:

$$y(x) = \frac{x^2}{2b}$$

$T_B = 0 \qquad V_B = \dfrac{1}{2}k\delta^2 \qquad T_A = 0 \qquad V_A = W\,y(b)$

$0 + \dfrac{1}{2}k\delta^2 = 0 + W\,y(b) \qquad \delta = \sqrt{\dfrac{2W\,y(b)}{k}} \qquad \delta = 1.095\ \text{in}$

Problem 14-71

If the spring is compressed a distance δ against the block of weight W and it is released from rest, determine the normal force of the smooth surface on the block when it reaches the point x_1.

Given:

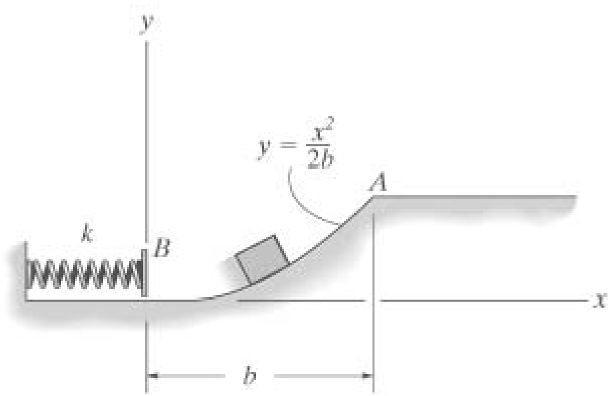
$W = 0.5 \text{ lb}$

$b = 1 \text{ ft}$

$k = 5 \dfrac{\text{lb}}{\text{in}}$

$\delta = 3 \text{ in}$

$x_1 = 0.5 \text{ ft}$



Solution:

$$y(x) = \frac{x^2}{2b} \qquad y'(x) = \frac{x}{b} \qquad y''(x) = \frac{1}{b} \qquad \rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$\theta(x) = \operatorname{atan}(y'(x))$$

$$T_1 = 0 \qquad V_1 = \frac{1}{2}k\delta^2 \qquad T_1 = \frac{1}{2}\left(\frac{W}{g}\right)v_1^2$$

$$0 + \frac{1}{2}k\delta^2 = \frac{1}{2}\left(\frac{W}{g}\right)v_1^2 + W\,y(x_1) \qquad v_1 = \sqrt{\left(k\delta^2 - 2\,V_2 = W\,y(x_1)\right)}$$

$$F_N - W\cos(\theta(x_1)) = \frac{W}{g}\left(\frac{v_1^2}{\rho(x_1)}\right)$$

$$F_N = W\cos(\theta(x_1)) + \frac{W}{g}\left(\frac{v_1^2}{\rho(x_1)}\right) \qquad F_N = 3.041 \text{ lb}$$

***Problem 14-72**

The girl has mass M and center of mass at G. If she is swinging to a maximum height defined by $\theta = \theta_1$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.

Given:

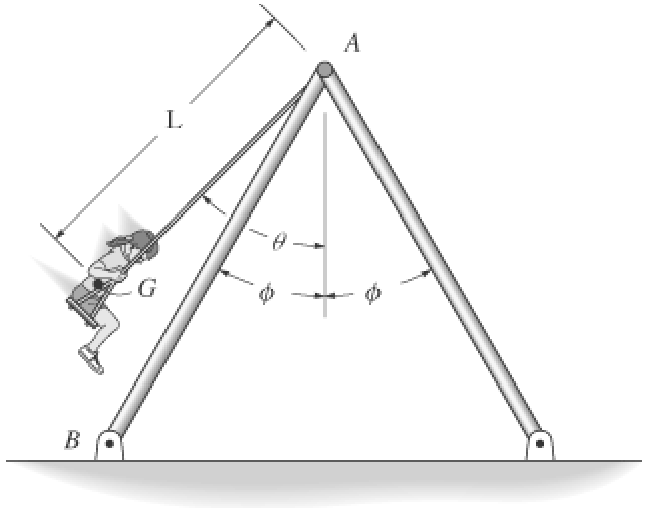
$M = 40 \text{ kg}$

$\theta_1 = 60 \text{ deg}$

$\phi = 30 \text{ deg}$

$L = 2 \text{ m}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$



Solution:

$$T_1 + V_1 = T_2 + V_2$$

$$0 - MgL\cos(\theta_1) = \frac{1}{2}Mv^2 - MgL$$

$v = \sqrt{2gL(1 - \cos(\theta_1))}$ $\quad v = 4.429 \dfrac{\text{m}}{\text{s}}$

$T - Mg = M\left(\dfrac{v^2}{L}\right)$ $\quad T = Mg + M\left(\dfrac{v^2}{L}\right)$ $\quad T = 784.8 \text{ N}$

$4F_{AB}\cos(\phi) - T = 0$ $\quad F_{AB} = \dfrac{T}{4\cos(\phi)}$ $\quad F_{AB} = 226.552 \text{ N}$

Problem 14-73

Each of the two elastic rubber bands of the slingshot has an unstretched length l. If they are pulled back to the position shown and released from rest, determine the speed of the pellet of mass M just after the rubber bands become unstretched. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness k.

Given:

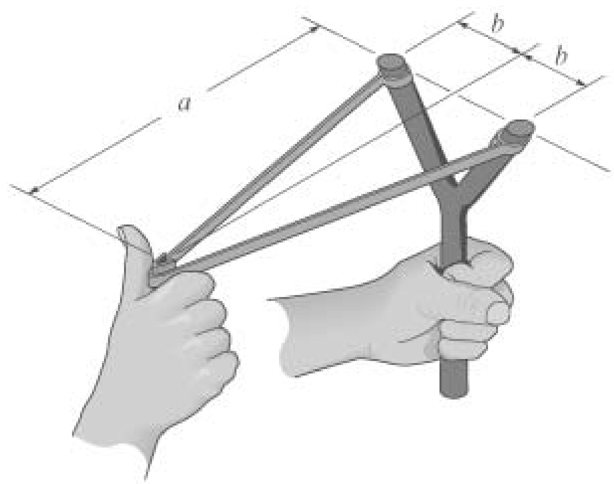
$l = 200 \text{ mm}$

$M = 25 \text{ gm}$

$a = 240 \text{ mm}$

$b = 50 \text{ mm}$

$k = 50 \dfrac{\text{N}}{\text{m}}$



Solution:

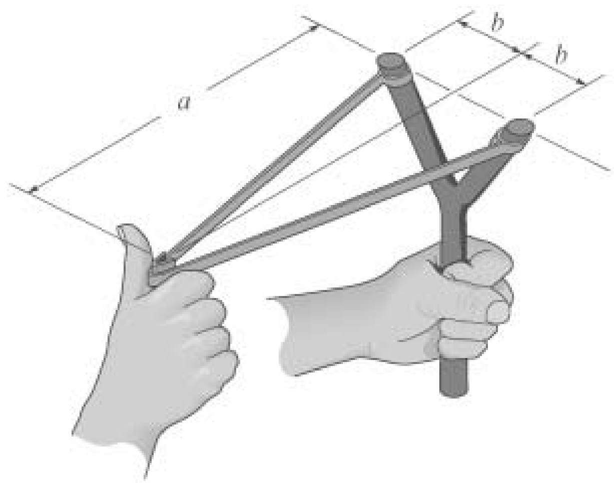
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}k\left(\sqrt{b^2 + a^2} - l\right)^2\right] = \frac{1}{2}Mv^2$$

$$v = \sqrt{\frac{2k}{M}}\left(\sqrt{b^2 + a^2} - l\right) \qquad v = 2.86\ \frac{\text{m}}{\text{s}}$$

Problem 14-74

Each of the two elastic rubber bands of the slingshot has an unstretched length l. If they are pulled back to the position shown and released from rest, determine the maximum height the pellet of mass M will reach if it is fired vertically upward. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness k.



Given:

$l = 200\ \text{mm}$

$M = 25\ \text{gm}$

$a = 240\ \text{mm}$

$b = 50\ \text{mm}$

$k = 50\ \frac{\text{N}}{\text{m}}$

Solution:

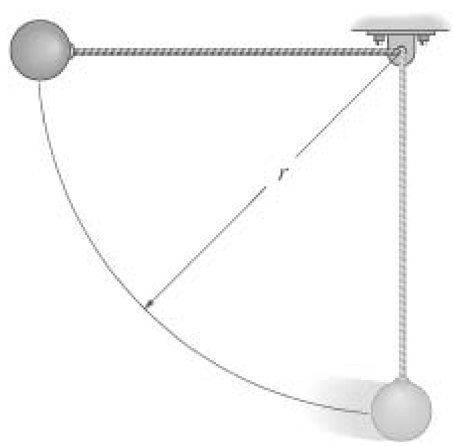
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}k\left(\sqrt{b^2 + a^2} - l\right)^2\right] = Mgh$$

$$h = \frac{k}{Mg}\left(\sqrt{b^2 + a^2} - l\right)^2 \qquad h = 416\,\text{mm}$$

Problem 14-75

The bob of the pendulum has a mass M and is released from rest when it is in the horizontal position shown. Determine its speed and the tension in the cord at the instant the bob passes through its lowest position.



Given:

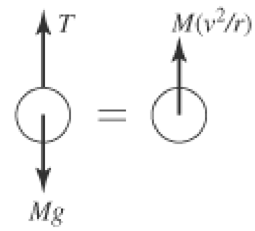
$M = 0.2\ \text{kg}$

$r = 0.75\ \text{m} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

Datum at initial position:

$$T_1 + V_1 = T_2 + V_2$$



$$0 + 0 = \frac{1}{2}M v_2^2 - M g r$$

$$v_2 = \sqrt{2g r} \qquad v_2 = 3.84\ \frac{\text{m}}{\text{s}}$$

$$\Sigma F_n = Ma_n$$

$$T - M g = M\left(\frac{v_2^2}{r}\right)$$

$$T = M\left(g + \frac{v_2^2}{r}\right) \qquad T = 5.89\ \text{N}$$

***Problem 14-76**

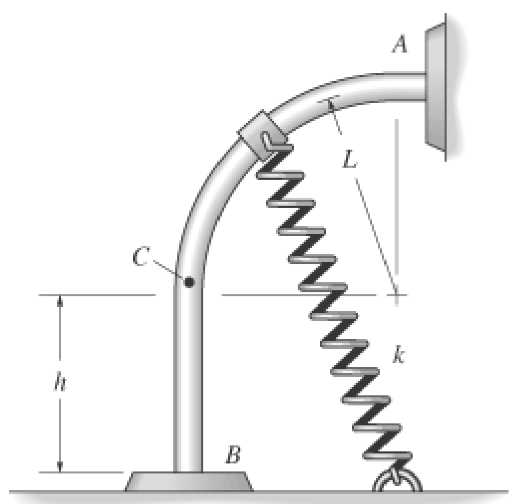
The collar of weight W is released from rest at A and travels along the smooth guide. Determine the speed of the collar just before it strikes the stop at B. The spring has an unstretched length L.

Given:

$W = 5\ \text{lb} \quad k = 2\ \frac{\text{lb}}{\text{in}}$

$L = 12\ \text{in} \quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$h = 10\ \text{in}$



Solution:

$$T_A + V_A = T_B + V_B$$

$$0 + W(L + h) + \frac{1}{2}kh^2 = \frac{1}{2}\left(\frac{W}{g}\right)v_B^2$$

$$v_B = \sqrt{\left(\frac{kg}{W}\right)h^2 + 2g(L + h)} \qquad v_B = 15.013\ \frac{\text{ft}}{\text{s}}$$

Problem 14-77

The collar of weight W is released from rest at A and travels along the smooth guide. Determine its speed when its center reaches point C and the normal force it exerts on the rod at this point. The spring has an unstretched length L, and point C is located just before the end of the curved portion of the rod.

Given:

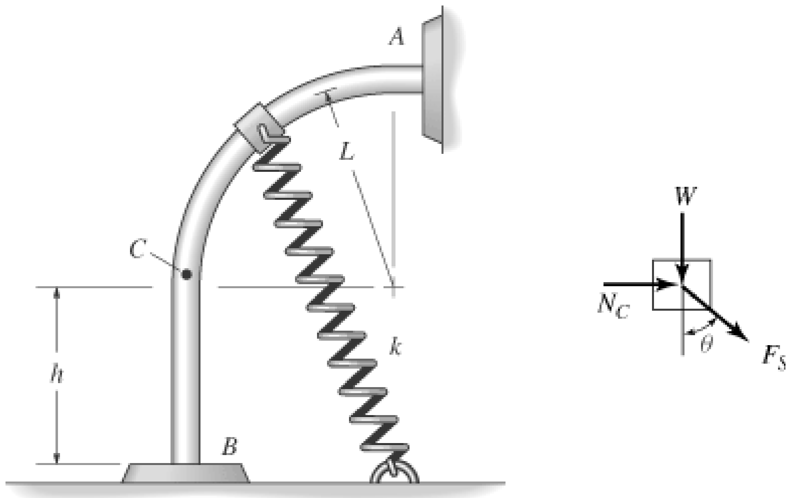
$W = 5 \text{ lb}$

$L = 12 \text{ in}$

$h = 10 \text{ in}$

$k = 2 \frac{\text{lb}}{\text{in}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

$T_A + V_A = T_C + V_C \qquad 0 + WL + \frac{1}{2}kh^2 = \frac{1}{2}\left(\frac{W}{g}\right)v_C^2 + \frac{1}{2}k\left(\sqrt{L^2+h^2} - L\right)^2$

$v_C = \sqrt{2gL + \left(\frac{kg}{W}\right)h^2 - \left(\frac{kg}{W}\right)\left(\sqrt{L^2+h^2} - L\right)^2} \qquad v_C = 12.556 \frac{\text{ft}}{\text{s}}$

$N_C + k\left(\sqrt{L^2+h^2} - L\right)\left(\frac{L}{\sqrt{L^2+h^2}}\right) = \frac{W}{g}\left(\frac{v_C^2}{L}\right)$

$N_C = \frac{W}{g}\left(\frac{v_C^2}{L}\right) - \left(\frac{kL}{\sqrt{L^2+h^2}}\right)\left(\sqrt{L^2+h^2} - L\right) \qquad N_C = 18.919 \text{ lb}$

Problem 14-78

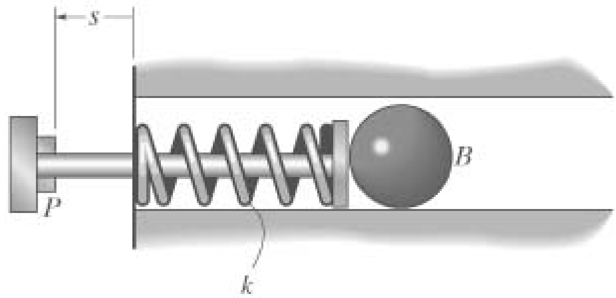
The firing mechanism of a pinball machine consists of a plunger P having a mass M_p and a spring stiffness k. When $s = 0$, the spring is compressed a distance δ. If the arm is pulled back such that $s = s_1$ and released, determine the speed of the pinball B of mass M_b *just before* the plunger strikes the stop, i.e., assume all surfaces of contact to be smooth. The ball moves in the horizontal plane. Neglect friction, the mass of the spring, and the rolling motion of the ball.

Given:

$M_p = 0.25 \text{ kg} \qquad s_1 = 100 \text{ mm}$

$M_b = 0.3\ \text{kg}$ $\quad k = 300\ \dfrac{\text{N}}{\text{m}}$

$\delta = 50\ \text{mm}$



Solution:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}k\left(s_1 + \delta\right)^2 = \frac{1}{2}\left(M_p + M_b\right)v_2^{\,2} + \frac{1}{2}k\delta^2$$

$$v_2 = \sqrt{\frac{k}{M_p + M_b}\left[\left(s_1 + \delta\right)^2 - \delta^2\right]} \qquad v_2 = 3.30\ \frac{\text{m}}{\text{s}}$$

Problem 14-79

The roller-coaster car has mass M, including its passenger, and starts from the top of the hill A with a speed v_A. Determine the minimum height h of the hill crest so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C?

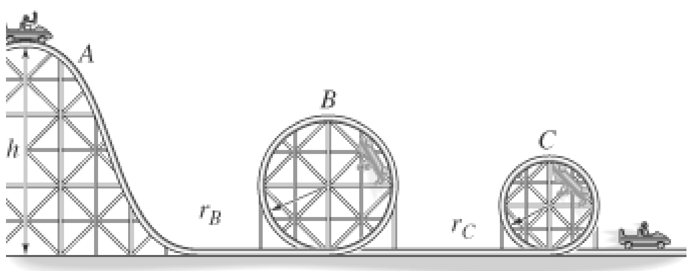
Units Used: $\quad \text{kN} = 10^3\ \text{N}$

Given:

$M = 800\ \text{kg}$ $\quad v_A = 3\ \dfrac{\text{m}}{\text{s}}$

$r_B = 10\ \text{m}$

$r_C = 7\ \text{m}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

Check the loop at B first $\quad$ We require that $\quad N_B = 0$

$$-N_B - Mg = -M\left(\frac{v_B^{\,2}}{r_B}\right) \qquad v_B = \sqrt{g r_B} \qquad v_B = 9.907\ \frac{\text{m}}{\text{s}}$$

$$T_A + V_A = T_B + V_B \qquad \frac{1}{2}Mv_A^{\,2} + Mgh = \frac{1}{2}Mv_B^{\,2} + Mg2r_B$$

$$h = \frac{v_B^{\,2} - v_A^{\,2}}{2g} + 2r_B \qquad h = 24.541\ \text{m}$$

Now check the loop at C

$$T_A + V_A = T_C + V_C \qquad \frac{1}{2}M v_A^2 + M g h = \frac{1}{2}M v_C^2 + M g 2 r_C$$

$$v_C = \sqrt{v_A^2 + 2g(h - 2r_C)} \qquad v_C = 14.694\ \frac{\text{m}}{\text{s}}$$

$$-N_C - M g = -M\left(\frac{v_C^2}{r_C}\right) \qquad N_C = M\left(\frac{v_C^2}{r_C}\right) - M g \qquad N_C = 16.825\ \text{kN}$$

Since $N_C > 0$ then the coaster successfully passes through loop C.

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*__Problem 14-80__

The roller-coaster car has mass M, including its passenger, and starts from the top of the hill A with a speed v_A. Determine the minimum height h of the hill crest so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C?

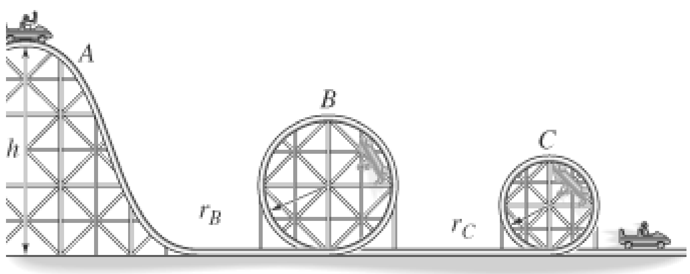
Units Used: $\text{kN} = 10^3\ \text{N}$

Given:

$M = 800\ \text{kg}$ $\quad v_A = 0\ \frac{\text{m}}{\text{s}}$

$r_B = 10\ \text{m}$

$r_C = 7\ \text{m}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: Check the loop at B first We require that $N_B = 0$

$$-N_B - M g = -M\left(\frac{v_B^2}{r_B}\right) \qquad v_B = \sqrt{g r_B} \qquad v_B = 9.907\ \frac{\text{m}}{\text{s}}$$

$$T_A + V_A = T_B + V_B \qquad \frac{1}{2}M v_A^2 + M g h = \frac{1}{2}M v_B^2 + M g 2 r_B$$

$$h = \frac{v_B^2 - v_A^2}{2g} + 2r_B \qquad h = 25\ \text{m}$$

Now check the loop at C

$$T_A + V_A = T_C + V_C \qquad \frac{1}{2}M v_A^2 + M g h = \frac{1}{2}M v_C^2 + M g 2 r_C$$

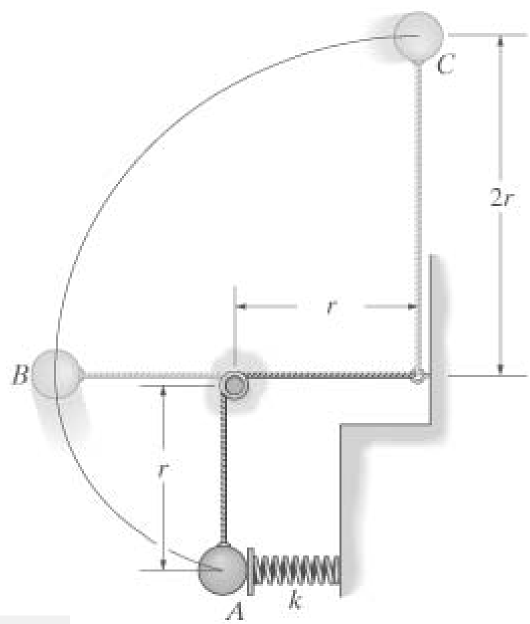
$$v_C = \sqrt{v_A^2 + 2g(h - 2r_C)} \qquad v_C = 14.694\ \frac{\text{m}}{\text{s}}$$

$$-N_C - Mg = -M\left(\frac{v_C^{\,2}}{r_C}\right) \qquad N_C = M\left(\frac{v_C^{\,2}}{r_C}\right) - Mg \qquad N_C = 16.825\,\text{kN}$$

Since $N_C > 0$ then the coaster successfully passes through loop C.

Problem 14-81

The bob of mass M of a pendulum is fired from rest at position A by a spring which has a stiffness k and is compressed a distance δ. Determine the speed of the bob and the tension in the cord when the bob is at positions B and C. Point B is located on the path where the radius of curvature is still r, i.e., just before the cord becomes horizontal.



Units Used: $\text{kN} = 10^3\ \text{N}$

Given:

$M = 0.75\ \text{kg}$

$k = 6\ \frac{\text{kN}}{\text{m}}$

$\delta = 125\ \text{mm}$

$r = 0.6\ \text{m}$

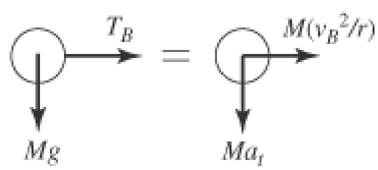
Solution:

At B:

$$0 + \frac{1}{2}k\delta^2 = \frac{1}{2}Mv_B^{\,2} + Mgr$$

$$v_B = \sqrt{\left(\frac{k}{M}\right)\delta^2 - 2gr} \qquad v_B = 10.6\ \frac{\text{m}}{\text{s}}$$

$$T_B = M\left(\frac{v_B^{\,2}}{r}\right) \qquad T_B = 142\ \text{N}$$

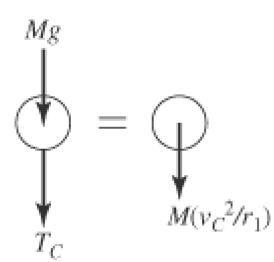


At C:

$$0 + \frac{1}{2}k\delta^2 = \frac{1}{2}Mv_C^{\,2} + Mg3r$$

$$v_C = \sqrt{\left(\frac{k}{M}\right)\delta^2 - 6gr} \qquad v_C = 9.47\ \frac{\text{m}}{\text{s}}$$

$$T_C + Mg = M\left(\frac{v_C^{\,2}}{2r}\right)$$



$$T_C = M\left(\frac{v_C^2}{2r} - g\right) \qquad T_C = 48.7\ \text{N}$$

Problem 14-82

The spring has stiffness k and unstretched length L. If it is attached to the smooth collar of weight W and the collar is released from rest at A, determine the speed of the collar just before it strikes the end of the rod at B. Neglect the size of the collar.

Given:

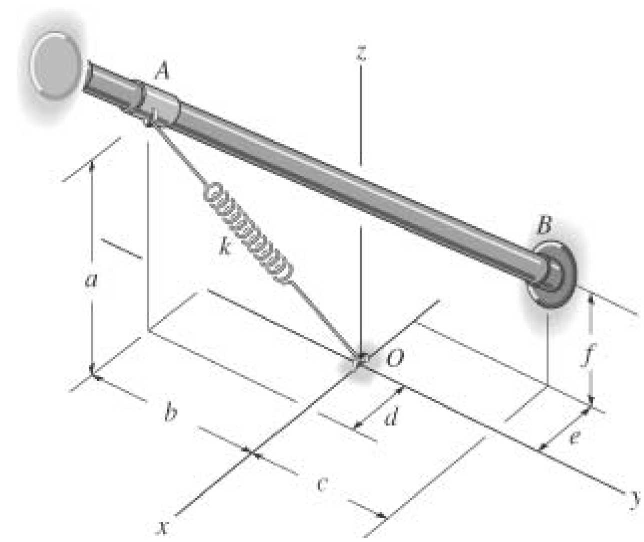
$k = 3\ \frac{\text{lb}}{\text{ft}}$ $\quad c = 3\ \text{ft}$

$L = 2\ \text{ft}$ $\quad d = 1\ \text{ft}$

$W = 5\ \text{lb}$ $\quad e = 1\ \text{ft}$

$a = 6\ \text{ft}$ $\quad f = 2\ \text{ft}$

$b = 4\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

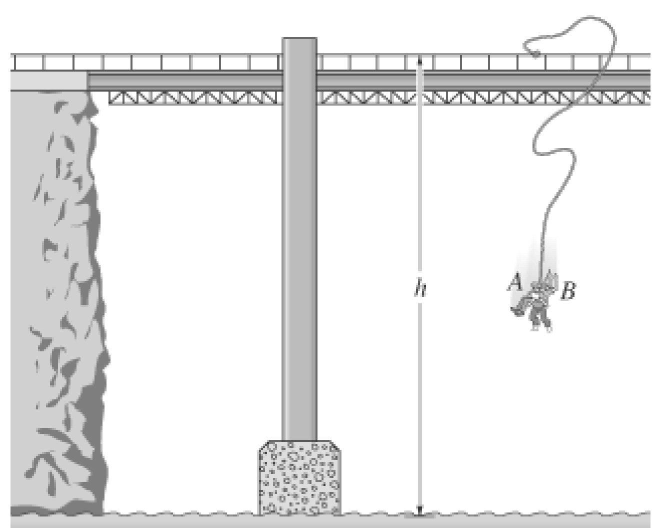
$$T_A + V_A = T_B + V_B$$

$$0 + W(a - f) + \frac{1}{2}k\left(\sqrt{a^2 + b^2 + d^2} - L\right)^2 = \frac{1}{2}\left(\frac{W}{g}\right)v_B^2 + \frac{1}{2}k\left(\sqrt{c^2 + e^2 + f^2} - L\right)^2$$

$$v_B = \sqrt{2g(a - f) + \frac{kg}{W}\left[\left(\sqrt{a^2 + b^2 + d^2} - L\right)^2 - \left(\sqrt{c^2 + e^2 + f^2} - L\right)^2\right]} \qquad v_B = 27.2\ \frac{\text{ft}}{\text{s}}$$

Problem 14-83

Just for fun, two engineering students each of weight W, A and B, intend to jump off the bridge from rest using an elastic cord (bungee cord) having stiffness k. They wish to just reach the surface of the river, when A, attached to the cord, lets go of B at the instant they touch the water. Determine the proper unstretched length of the cord to do the stunt, and calculate the maximum acceleration of student A and the maximum height he reaches above the water after the rebound. From your results, comment on the feasibility of doing this stunt.



Given:

$W = 150\ \text{lb}$ $k = 80\ \dfrac{\text{lb}}{\text{ft}}$ $h = 120\ \text{ft}$

Solution:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 - 2Wh + \frac{1}{2}k(h - L)^2$$

$$L = h - \sqrt{\frac{4Wh}{k}} \qquad L = 90\,\text{ft}$$

At the bottom, after A lets go of B

$$k(h - L) - W = \left(\frac{W}{g}\right)a \qquad a = \frac{kg}{W}(h - L) - g \qquad a = 483\,\frac{\text{ft}}{\text{s}^2} \qquad \frac{a}{g} = 15$$

Maximum height

$T_2 + V_2 = T_3 + V_3$ Guess $H = 2h$ Given

$$0 + \frac{1}{2}k(h - L)^2 = WH + \frac{1}{2}k(H - h - L)^2 \qquad H = \text{Find}(H) \qquad H = 218.896\,\text{ft}$$

This stunt should not be attempted since $\dfrac{a}{g} = 15$ (excessive) and the rebound height is above the bridge!!

Problem 14-84

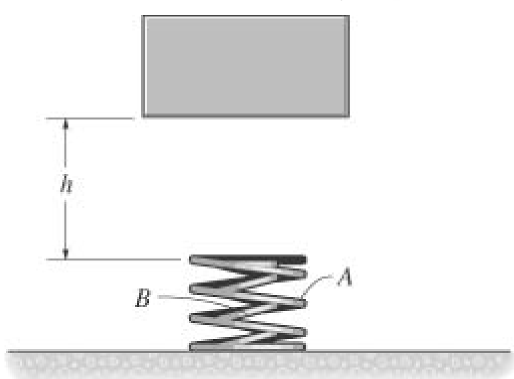
Two equal-length springs having stiffnesses k_A and k_B are “nested” together in order to form a shock absorber. If a block of mass M is dropped from an at-rest position a distance h above the top of the springs, determine their deformation when the block momentarily stops.

Given:

$k_A = 300\ \dfrac{\text{N}}{\text{m}}$ $M = 2\ \text{kg}$

$h = 0.6\ \text{m}$

$k_B = 200\ \dfrac{\text{N}}{\text{m}}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

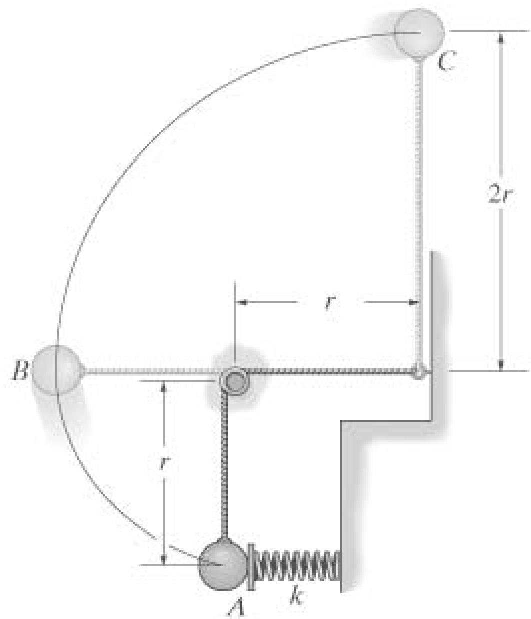
$$T_1 + V_1 = T_2 + V_2$$

Guess $\delta = 0.1\ \text{m}$

Given $0 + Mgh = \dfrac{1}{2}(k_A + k_B)\,\delta^2 - Mg\delta$ $\delta = \text{Find}(\delta)$ $\delta = 0.260\ \text{m}$

Problem 14-85

The bob of mass M of a pendulum is fired from rest at position A. If the spring is compressed to a distance δ and released, determine (a) its stiffness k so that the speed of the bob is zero when it reaches point B, where the radius of curvature is still r, and (b) the stiffness k so that when the bob reaches point C the tension in the cord is zero.



Units Used: $\mathrm{kN} = 10^3\ \mathrm{N}$

Given:

$M = 0.75\ \mathrm{kg}$ $g = 9.81\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$

$\delta = 50\ \mathrm{mm}$ $r = 0.6\ \mathrm{m}$

Solution:

At B:

$$\frac{1}{2}k\delta^2 = Mgr$$

$$k = \frac{2Mgr}{\delta^2} \qquad k = 3.53\ \frac{\mathrm{kN}}{\mathrm{m}}$$

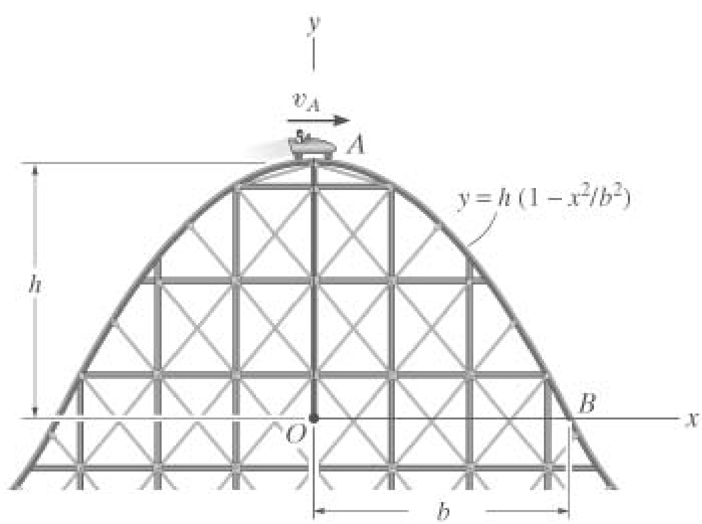
At C:

$$-Mg = -M\left(\frac{v_C^2}{2r}\right) \qquad v_C = \sqrt{2gr}$$

$$\frac{1}{2}k\delta^2 = Mg3r + \frac{1}{2}Mv_C^2 \qquad k = \frac{M}{\delta^2}\left(6gr + v_C^2\right) \qquad k = 14.13\ \frac{\mathrm{kN}}{\mathrm{m}}$$

Problem 14-86

The roller-coaster car has a speed v_A when it is at the crest of a vertical parabolic track. Determine the car's velocity and the normal force it exerts on the track when it reaches point B. Neglect friction and the mass of the wheels. The total weight of the car and the passengers is W.



Given:

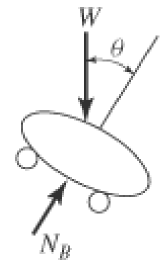
$W = 350\ \mathrm{lb}$ $b = 200\ \mathrm{ft}$

$v_A = 15\ \dfrac{\mathrm{ft}}{\mathrm{s}}$ $h = 200\ \mathrm{ft}$

Solution:

$$y(x) = h\left(1 - \frac{x^2}{b^2}\right) \qquad y'(x) = -2\left(\frac{hx}{b^2}\right) \qquad y''(x) = -2\left(\frac{h}{b^2}\right)$$

$$\theta_B = \operatorname{atan}(y'(b)) \qquad \rho_B = \frac{\sqrt{\left(1 + y'(b)^2\right)^3}}{y''(b)}$$



$$\frac{1}{2}\left(\frac{W}{g}\right)v_A^{\,2} + Wh = \frac{1}{2}\left(\frac{W}{g}\right)v_B^{\,2}$$

$$v_B = \sqrt{v_A^{\,2} + 2gh} \qquad v_B = 114.5\,\frac{\text{ft}}{\text{s}}$$

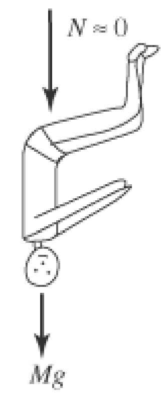
$$N_B - W\cos(\theta_B) = \frac{W}{g}\left(\frac{v_B^{\,2}}{\rho_B}\right) \qquad N_B = W\cos(\theta_B) + \frac{W}{g}\left(\frac{v_B^{\,2}}{\rho_B}\right) \qquad N_B = 29.1\ \text{lb}$$

Problem 14-87

The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. Determine the minimum speed v_0 at which the cars should coast down from the top of the hill, so that passengers can just make the loop without leaving contact with their seats. Neglect friction, the size of the car and passenger, and assume each passenger and car has a mass m.

Solution:

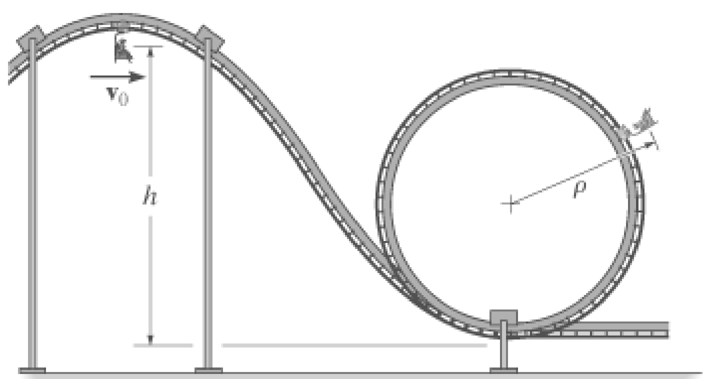
Datum at ground:



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_0^{\,2} + mgh = \frac{1}{2}mv_1^{\,2} + mg2\rho$$

$$v_1 = \sqrt{v_0^{\,2} + 2g(h - 2\rho)}$$



$$mg = m\left(\frac{v_1^{\,2}}{\rho}\right)$$

$$v_1 = \sqrt{g\rho}$$

Thus,

$$g\rho = v_0^{\,2} + 2gh - 4g\rho$$

$$v_0 = \sqrt{g(5\rho - 2h)}$$

***Problem 14-88**

The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. If the cars travel at v_0 when they are at the top of the hill, determine their speed when they are at the top of the loop and the reaction of the passenger of mass M_p on his seat at this instant.The car has a mass M_c. Neglect friction and the size of the car and passenger.

Given:

$M_p = 70\ \text{kg}$

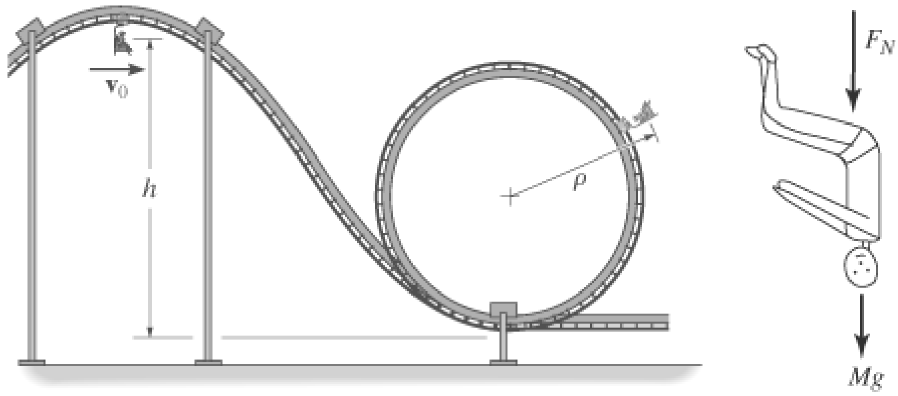
$M_c = 50\ \text{kg}$

$v_0 = 4\ \frac{\text{m}}{\text{s}}$

$h = 12\ \text{m}$

$\rho = 5\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



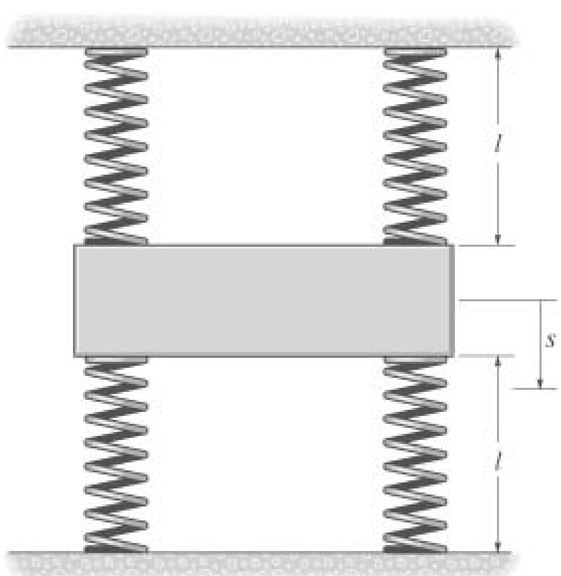
Solution:

$$\frac{1}{2}M v_0^2 + M g h = \frac{1}{2}M v_1^2 + M g 2\rho \qquad v_1 = \sqrt{v_0^2 + 2gh - 4g\rho} \qquad v_1 = 7.432\ \frac{\text{m}}{\text{s}}$$

$$M_p g + N = M_p\left(\frac{v_1^2}{\rho}\right) \qquad F_N = M_p\left(\frac{v_1^2}{\rho} - g\right) \qquad F_N = 86.7\ \text{N}$$

Problem 14-89

A block having a mass M is attached to four springs. If each spring has a stiffness k and an unstretched length δ, determine the *maximum* downward vertical displacement s_{max} of the block if it is released from rest at $s = 0$.



Units Used: $\text{kN} = 10^3\ \text{N}$

Given:

$M = 20\ \text{kg}$

$k = 2\ \frac{\text{kN}}{\text{m}}$

$l = 100\ \text{mm}$

$\delta = 150\ \text{mm}$

Solution:

Guess $s_{max} = 100\ \text{mm}$

Given $4\frac{1}{2}k(l-\delta)^2 = -M\,g\,s_{max} + 2\left[\frac{1}{2}k\left(l-\delta+s_{max}\right)^2\right] + 2\left[\frac{1}{2}k\left(l-\delta-s_{max}\right)^2\right]$

$s_{max} = \text{Find}(s_{max})$ $\qquad s_{max} = 49.0\,\text{mm}$

Problem 14-90

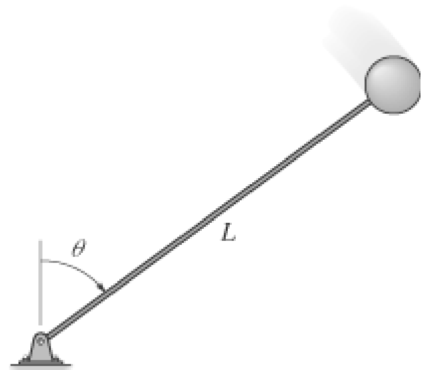
The ball has weight W and is fixed to a rod having a negligible mass. If it is released from rest when $\theta = 0°$, determine the angle θ at which the compressive force in the rod becomes zero.

Given:

$W = 15\ \text{lb}$

$L = 3\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

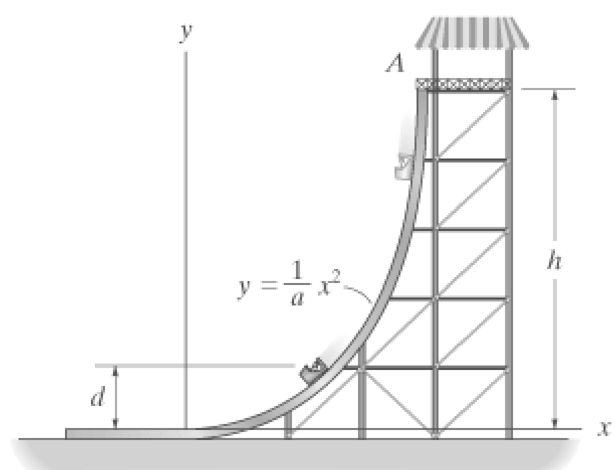
Guesses $v = 1\ \frac{\text{m}}{\text{s}}$ $\qquad \theta = 10\ \text{deg}$

Given $W\,L = \frac{1}{2}\left(\frac{W}{g}\right)v^2 + W\,L\cos(\theta)$ $\qquad -W\cos(\theta) = \frac{-W}{g}\left(\frac{v^2}{L}\right)$

$\begin{pmatrix} v \\ \theta \end{pmatrix} = \text{Find}(v, \theta)$ $\qquad v = 8.025\,\frac{\text{ft}}{\text{s}}$ $\qquad \theta = 48.2\,\text{deg}$

Problem 14-91

The ride at an amusement park consists of a gondola which is lifted to a height h at A. If it is released from rest and falls along the parabolic track, determine the speed at the instant $y = d$. Also determine the normal reaction of the tracks on the gondola at this instant. The gondola and passenger have a total weight W. Neglect the effects of friction.



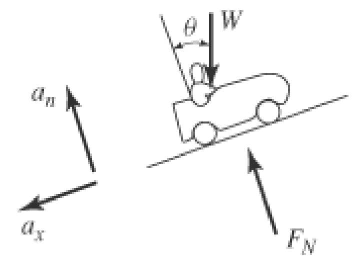
Given:

$W = 500\ \text{lb}$ $\qquad d = 20\ \text{ft}$

$h = 120\ \text{ft}$ $\qquad a = 260\ \text{ft}$

Solution:

$$y(x) = \frac{x^2}{a} \qquad y'(x) = 2\frac{x}{a} \qquad y''(x) = \frac{2}{a}$$



$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$\theta(x) = \operatorname{atan}(y'(x))$$

Guesses

$$x_2 = 1\ \text{ft} \qquad v_2 = 10\ \frac{\text{ft}}{\text{s}} \qquad F_N = 1\ \text{lb}$$

Given

$$W h = \frac{1}{2}\left(\frac{W}{g}\right) v_2^2 + W d \qquad d = y(x_2) \qquad F_N - W\cos(\theta(x_2)) = \frac{W}{g}\left(\frac{v_2^2}{\rho(x_2)}\right)$$

$$\begin{pmatrix} x_2 \\ v_2 \\ F_N \end{pmatrix} = \text{Find}(x_2, v_2, F_N) \qquad x_2 = 72.1\ \text{ft} \qquad v_2 = -80.2\ \frac{\text{ft}}{\text{s}} \qquad F_N = 952\ \text{lb}$$

---

*__Problem 14-92__

The collar of weight W has a speed v at A. The attached spring has an unstretched length δ and a stiffness k. If the collar moves over the smooth rod, determine its speed when it reaches point B, the normal force of the rod on the collar, and the rate of decrease in its speed.

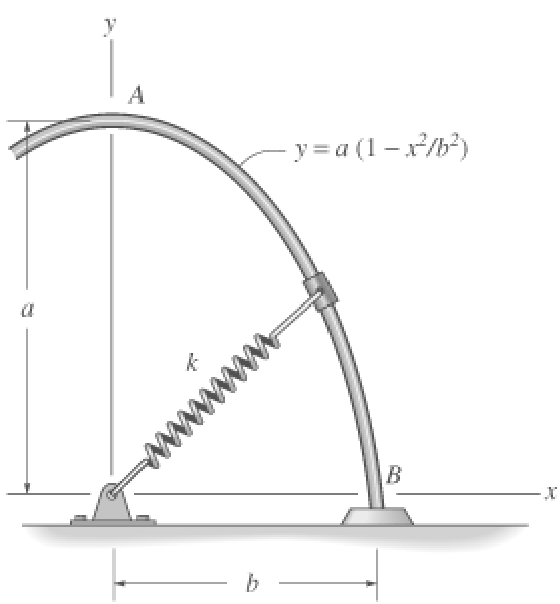
Given:

$$W = 2\ \text{lb} \qquad \delta = 2\ \text{ft}$$

$$a = 4.5\ \text{ft} \qquad k = 10\ \frac{\text{lb}}{\text{ft}}$$

$$b = 3\ \text{ft} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

$$v = 5\ \frac{\text{ft}}{\text{s}}$$



Solution:

$$y(x) = a\left(1 - \frac{x^2}{b^2}\right)$$

$$y'(x) = -2\left(\frac{a\,x}{b^2}\right) \qquad y''(x) = -2\left(\frac{a}{b^2}\right) \qquad \rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$\theta = \mathrm{atan}(y'(b)) \qquad \rho_B = \rho(b)$$

Guesses $\quad v_B = 1\ \frac{\text{ft}}{\text{s}} \qquad F_N = 1\ \text{lb} \qquad v'_B = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $\quad \frac{1}{2}\left(\frac{W}{g}\right)v^2 + \frac{1}{2}k(a-\delta)^2 + W\,a = \frac{1}{2}\left(\frac{W}{g}\right)v_B{}^2 + \frac{1}{2}k(b-\delta)^2$

$$F_N + k(b-\delta)\sin(\theta) - W\cos(\theta) = \frac{W}{g}\left(\frac{v_B{}^2}{\rho_B}\right)$$

$$-k(b-\delta)\cos(\theta) - W\sin(\theta) = \left(\frac{W}{g}\right)v'_B$$

$$\begin{pmatrix} v_B \\ F_N \\ v'_B \end{pmatrix} = \mathrm{Find}(v_B, F_N, v'_B) \qquad v_B = 34.1\ \frac{\text{ft}}{\text{s}} \qquad F_N = 7.84\ \text{lb} \qquad v'_B = -20.4\ \frac{\text{ft}}{\text{s}^2}$$

Problem 14-93

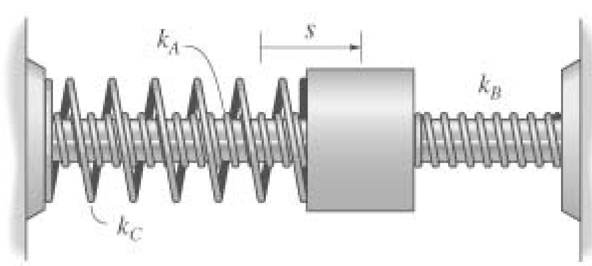
The collar of weight W is constrained to move on the smooth rod. It is attached to the three springs which are unstretched at $s = 0$. If the collar is displaced a distance $s = s_1$ and released from rest, determine its speed when $s = 0$.

Given:

$$W = 20\ \text{lb} \qquad k_A = 10\ \frac{\text{lb}}{\text{ft}}$$

$$s_1 = 0.5\ \text{ft} \qquad k_B = 10\ \frac{\text{lb}}{\text{ft}}$$

$$g = 32.2\ \frac{\text{ft}}{\text{s}^2} \qquad k_C = 30\ \frac{\text{lb}}{\text{ft}}$$



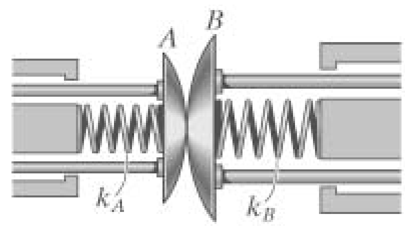
Solution:

$$\frac{1}{2}\left(k_A + k_B + k_C\right)s_1{}^2 = \frac{1}{2}\left(\frac{W}{g}\right)v^2$$

$$v = \sqrt{\frac{g}{W}\left(k_A + k_B + k_C\right)}\,s_1 \qquad v = 4.49\ \frac{\text{ft}}{\text{s}}$$

Problem 14-94

A tank car is stopped by two spring bumpers A and B, having stiffness k_A and k_B respectively. Bumper A is attached to the car, whereas bumper B is attached to the wall. If the car has a weight W and is freely coasting at speed v_c determine the maximum deflection of each spring at the instant the bumpers stop the car.

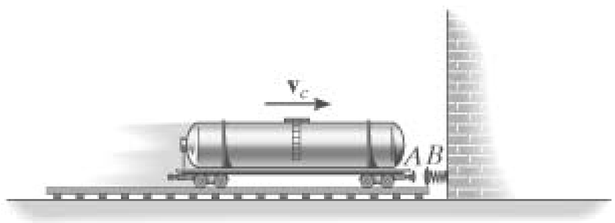


Given:

$k_A = 15 \times 10^3 \ \frac{\text{lb}}{\text{ft}}$ $\quad k_B = 20 \times 10^3 \ \frac{\text{lb}}{\text{ft}}$

$W = 25 \times 10^3 \ \text{lb}$ $\quad v_c = 3 \ \frac{\text{ft}}{\text{s}}$

Solution:



Guesses $\quad s_A = 1 \ \text{ft} \qquad s_B = 1 \ \text{ft}$

Given $\quad \frac{1}{2}\left(\frac{W}{g}\right) v_c^2 = \frac{1}{2} k_A s_A^2 + \frac{1}{2} k_B s_B^2$

$k_A s_A = k_B s_B$

$\begin{pmatrix} s_A \\ s_B \end{pmatrix} = \text{Find}(s_A, s_B) \qquad \begin{pmatrix} s_A \\ s_B \end{pmatrix} = \begin{pmatrix} 0.516 \\ 0.387 \end{pmatrix} \text{ft}$

Problem 14-95

If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_e m/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_e m/r^2)$, Eq. 13−1. For the calculation, locate the datum at $r \to \infty$. Also, prove that F is a conservative force.

Solution:

$$V = -\int_{\infty}^{r} \frac{-G M_e m}{r^2} \, dr = \frac{-G M_e m}{r} \qquad \text{QED}$$

$$F = -Grad\, V = -\frac{d}{dr} V = -\frac{d}{dr} \frac{-G M_e m}{r} = \frac{-G M_e m}{r^2} \qquad \text{QED}$$

---

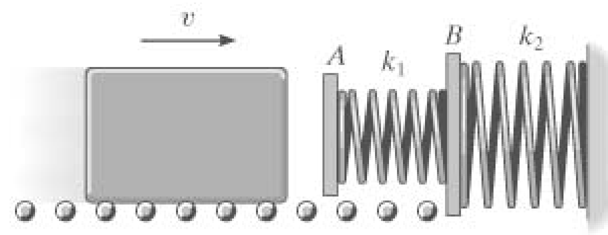
*__Problem 14-96__

The double-spring bumper is used to stop the steel billet of weight W in the rolling mill. Determine the maximum deflection of the plate A caused by the billet if it strikes the plate with a speed v. Neglect the mass of the springs, rollers and the plates A and B.

Given:

$W = 1500\ \text{lb}$ $\quad k_1 = 3000\ \frac{\text{lb}}{\text{ft}}$

$v = 8\ \frac{\text{ft}}{\text{s}}$ $\quad k_2 = 4500\ \frac{\text{lb}}{\text{ft}}$



Solution:

$$k_1 x_1 = k_2 x_2$$

$$\frac{1}{2}\left(\frac{W}{g}\right)v^2 = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2$$

$$\frac{1}{2}\left(\frac{W}{g}\right)v^2 = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2\left(\frac{k_1 x_1}{k_2}\right)^2$$

$$\left(\frac{W}{g}\right)v^2 = \left(k_1 + \frac{k_1^2 x_1^2}{k_2}\right)x_1^2 \qquad x_1 = \sqrt{\frac{W v^2}{g\left(k_1 + \frac{k_1^2}{k_2}\right)}} \qquad x_1 = 0.235\ \text{m}$$

Problem 15-1

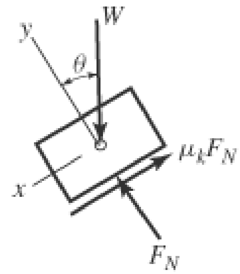
A block of weight W slides down an inclined plane of angle θ with initial velocity v_0. Determine the velocity of the block at time t_1 if the coefficient of kinetic friction between the block and the plane is μ_k.

Given:

$W = 20 \text{ lb}$ $\quad t_1 = 3 \text{ s}$

$\theta = 30 \text{ deg}$ $\quad \mu_k = 0.25$

$v_0 = 2 \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

$$(\nwarrow +) \quad m v_{y1} + \Sigma \int_{t_1}^{t_2} F_{y'}\, dt = m v_{y2}$$

$$0 + F_N t_1 - W \cos(\theta) t_1 = 0 \qquad F_N = W\cos(\theta) \qquad F_N = 17.32 \text{ lb}$$

$$(\swarrow +) \quad m(v_{x'1}) + \Sigma \int_{t_1}^{t_2} F_{x'}\, dt = m(v_{x'2})$$

$$\left(\frac{W}{g}\right) v_0 + W\sin(\theta) t_1 - \mu_k F_N t_1 = \left(\frac{W}{g}\right) v$$

$$v = \frac{W v_0 + W\sin(\theta)\, t_1\, g - \mu_k F_N t_1\, g}{W} \qquad v = 29.4 \frac{\text{ft}}{\text{s}}$$

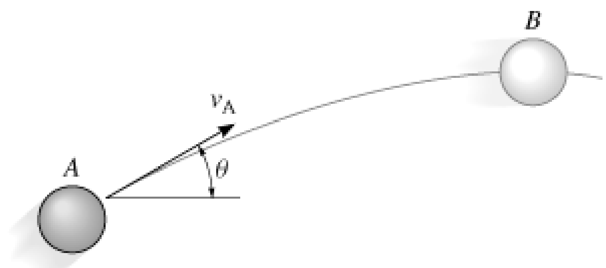
Problem 15-2

A ball of weight W is thrown in the direction shown with an initial speed v_A. Determine the time needed for it to reach its highest point B and the speed at which it is traveling at B. Use the principle of impulse and momentum for the solution.

Given:

$W = 2 \text{ lb}$ $\quad \theta = 30 \text{ deg}$

$v_A = 18 \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

$$\left(\frac{W}{g}\right)v_A\sin(\theta) - W\,t = \left(\frac{W}{g}\right)0 \qquad t = \frac{v_A\sin(\theta)}{g} \qquad t = 0.280\ \text{s}$$

$$\left(\frac{W}{g}\right)v_A\cos(\theta) + 0 = \left(\frac{W}{g}\right)v_x \qquad v_x = v_A\cos(\theta) \qquad v_x = 15.59\ \frac{\text{ft}}{\text{s}}$$

Problem 15-3

A block of weight W is given an initial velocity v_0 up a smooth slope of angle θ. Determine the time it will take to travel up the slope before it stops.

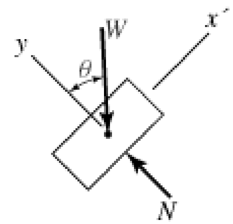
Given:

$W = 5\ \text{lb}$

$v_0 = 10\ \frac{\text{ft}}{\text{s}}$

$\theta = 45\ \text{deg}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$\left(\frac{W}{g}\right)v_0 - W\sin(\theta)t = 0 \qquad t = \frac{v_0}{g\sin(\theta)} \qquad t = 0.439\ \text{s}$$

*__Problem 15-4__

The baseball has a horizontal speed v_1 when it is struck by the bat B. If it then travels away at an angle θ from the horizontal and reaches a maximum height h, measured from the height of the bat, determine the magnitude of the net impulse of the bat on the ball.The ball has a mass M. Neglect the weight of the ball during the time the bat strikes the ball.

Given:

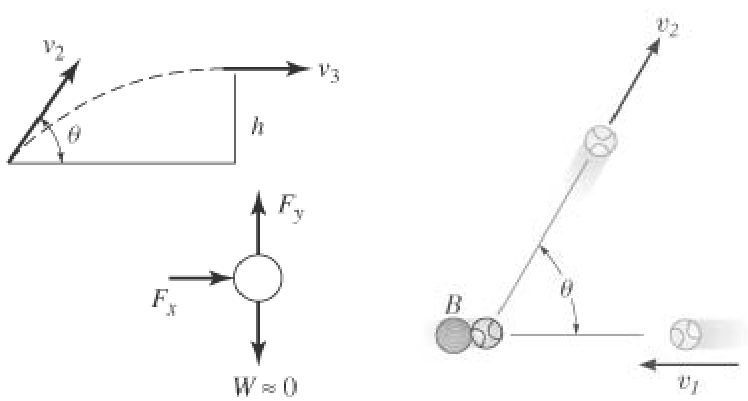
$M = 0.4\ \text{kg}$

$v_1 = 35\ \frac{\text{m}}{\text{s}}$

$h = 50\ \text{m}$

$\theta = 60\ \text{deg}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guesses

$$v_2 = 20\ \frac{\text{m}}{\text{s}} \qquad Imp_x = 1\ \text{N}\cdot\text{s} \qquad Imp_y = 10\ \text{N}\cdot\text{s}$$

Given

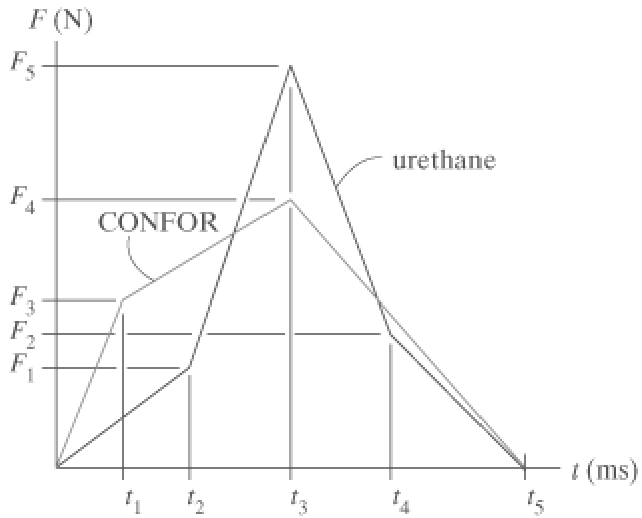
$$\frac{1}{2}M\left(v_2\sin(\theta)\right)^2 = M\,g\,h \qquad -M\,v_1 + Imp_x = M\,v_2\cos(\theta) \qquad 0 + Imp_y = M\,v_2\sin(\theta)$$

$$\begin{pmatrix} v_2 \\ Imp_x \\ Imp_y \end{pmatrix} = \text{Find}\left(v_2, Imp_x, Imp_y\right) \qquad v_2 = 36.2\ \frac{\text{m}}{\text{s}} \qquad \begin{pmatrix} Imp_x \\ Imp_y \end{pmatrix} = \begin{pmatrix} 21.2 \\ 12.5 \end{pmatrix}\text{N}\cdot\text{s}$$

$$\left|\begin{pmatrix} Imp_x \\ Imp_y \end{pmatrix}\right| = 24.7\,\text{N}\cdot\text{s}$$

Problem 15-5

The choice of a seating material for moving vehicles depends upon its ability to resist shock and vibration. From the data shown in the graphs, determine the impulses created by a falling weight onto a sample of urethane foam and CONFOR foam.



Units Used:

$$\text{ms} = 10^{-3}\ \text{s}$$

Given:

$F_1 = 0.3\ \text{N} \quad t_1 = 2\ \text{ms}$

$F_2 = 0.4\ \text{N} \quad t_2 = 4\ \text{ms}$

$F_3 = 0.5\ \text{N} \quad t_3 = 7\ \text{ms}$

$F_4 = 0.8\ \text{N} \quad t_4 = 10\ \text{ms}$

$F_5 = 1.2\ \text{N} \quad t_5 = 14\ \text{ms}$

Solution:

CONFOR foam:

$$I_c = \frac{1}{2}t_1 F_3 + \frac{1}{2}\left(F_3 + F_4\right)\left(t_3 - t_1\right) + \frac{1}{2}F_4\left(t_5 - t_3\right)$$

$$I_c = 6.55\,\text{N}\cdot\text{ms}$$

Urethane foam:

$$I_U = \frac{1}{2}t_2 F_1 + \frac{1}{2}(F_5 + F_1)(t_3 - t_2) + \frac{1}{2}(F_5 + F_2)(t_4 - t_3) + \frac{1}{2}(t_5 - t_4)F_2$$

$$I_U = 6.05\ \text{N}\cdot\text{ms}$$

Problem 15-6

A man hits the golf ball of mass M such that it leaves the tee at angle θ with the horizontal and strikes the ground at the same elevation a distance d away. Determine the impulse of the club C on the ball. Neglect the impulse caused by the ball's weight while the club is striking the ball.

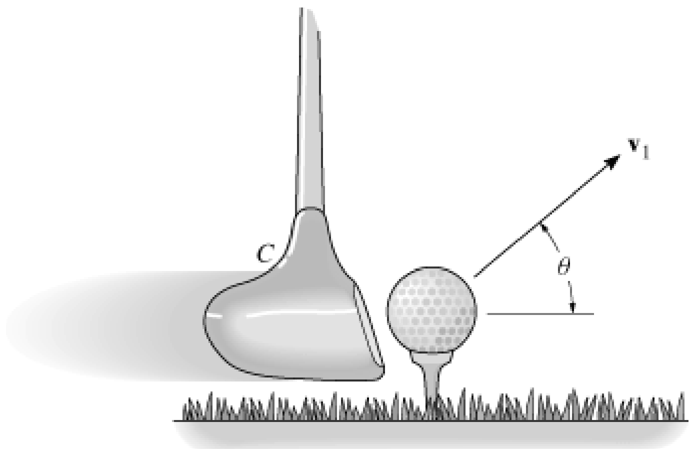
Given:

$M = 50\ \text{gm}$

$\theta = 40\ \text{deg}$

$d = 20\ \text{m}$

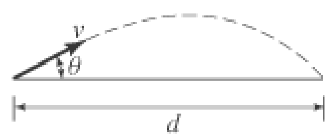
$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: First find the velocity v_1

Guesses $v_1 = 1\ \frac{\text{m}}{\text{s}}$ $t = 1\ \text{s}$

Given $0 = \left(\frac{-g}{2}\right)t^2 + v_1 \sin(\theta)t$ $d = v_1 \cos(\theta)t$



$\begin{pmatrix} t \\ v_1 \end{pmatrix} = \text{Find}(t, v_1)$ $t = 1.85\ \text{s}$ $v_1 = 14.11\ \frac{\text{m}}{\text{s}}$

Impulse - Momentum

$0 + Imp = M v_1$ $Imp = M v_1$ $Imp = 0.706\ \text{N}\cdot\text{s}$

Problem 15-7

A solid-fueled rocket can be made using a fuel grain with either a hole (a), or starred cavity (b), in the cross section. From experiment the engine thrust-time curves (T vs. t) for the same amount of propellant using these geometries are shown. Determine the total impulse in both cases.

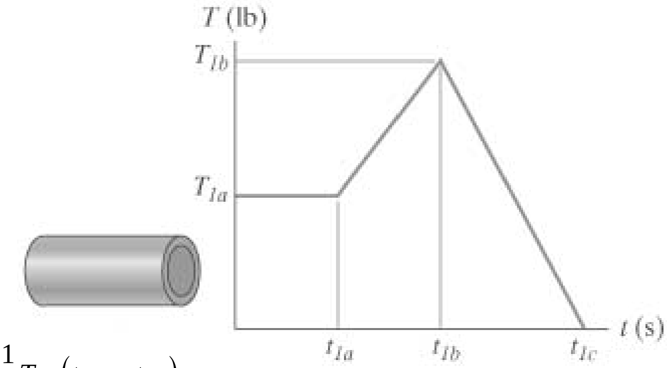
Given:

$T_{1a} = 4\ \text{lb}$ $t_{1a} = 3\ \text{s}$

$T_{1b} = 8\ \text{lb}$ $t_{1b} = 6\ \text{s}$

$T_{2a} = 6 \text{ lb} \quad t_{1c} = 10 \text{ s}$

$t_{2a} = 8 \text{ s} \quad t_{2b} = 10 \text{ s}$

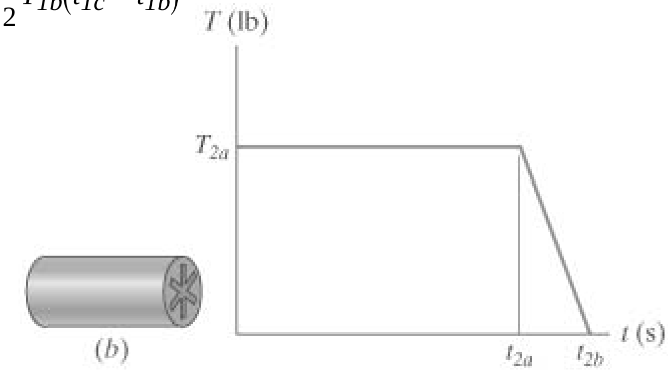


Solution:

Impulse is area under curve for hole cavity.

$$I_a = T_{1a}t_{1a} + \frac{1}{2}\left(T_{1a} + T_{1b}\right)\left(t_{1b} - t_{1a}\right) + \frac{1}{2}T_{1b}\left(t_{1c} - t_{1b}\right)$$

$$I_a = 46.00 \text{ lb}\cdot\text{s}$$



For starred cavity:

$$I_b = T_{2a}t_{2a} + \frac{1}{2}T_{2a}\left(t_{2b} - t_{2a}\right)$$

$$I_b = 54.00 \text{ lb}\cdot\text{s}$$

***Problem 15-8**

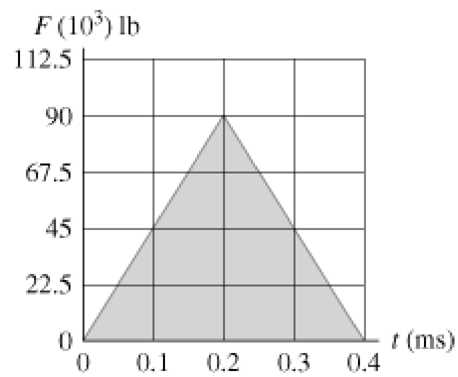
During operation the breaker hammer develops on the concrete surface a force which is indicated in the graph. To achieve this the spike S of weight W is fired from rest into the surface at speed v. Determine the speed of the spike just after rebounding.

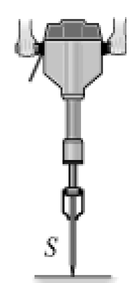
Given:

$W = 2 \text{ lb}$

$v = 200 \dfrac{\text{ft}}{\text{s}}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$





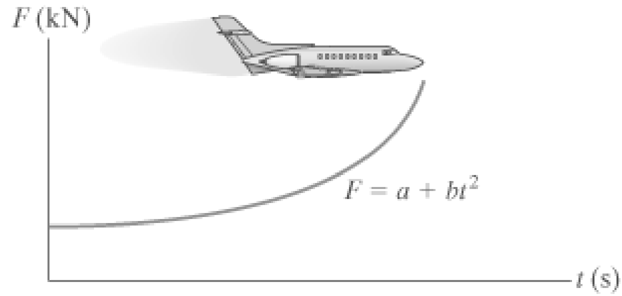
Solution:

$$I = \left(\frac{1}{2} \times 90 \times 10^3 \text{ lb}\right)\left(0.4\ 10^{-3}\text{s}\right) \qquad I = 18.00 \text{ lb}\cdot\text{s} \qquad \Delta t = 0.4 \times 10^{-3} \text{ s}$$

$$\left(\frac{-W}{g}\right)v + I - W\Delta t = \left(\frac{W}{g}\right)v' \qquad v' = -v + \left(\frac{Ig}{W}\right) - g\Delta t \qquad v' = 89.8 \frac{\text{ft}}{\text{s}}$$

Problem 15-9

The jet plane has a mass M and a horizontal velocity v_0 when $t = 0$. If *both* engines provide a horizontal thrust which varies as shown in the graph, determine the plane's velocity at time t_1. Neglect air resistance and the loss of fuel during the motion.



Units Used:

$\text{Mg} = 10^3\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$

Given:

$M = 250\ \text{Mg}$

$v_0 = 100\ \dfrac{\text{m}}{\text{s}}$

$t_1 = 15\ \text{s}$

$a = 200\ \text{kN}$

$b = 2\ \dfrac{\text{kN}}{\text{s}^2}$

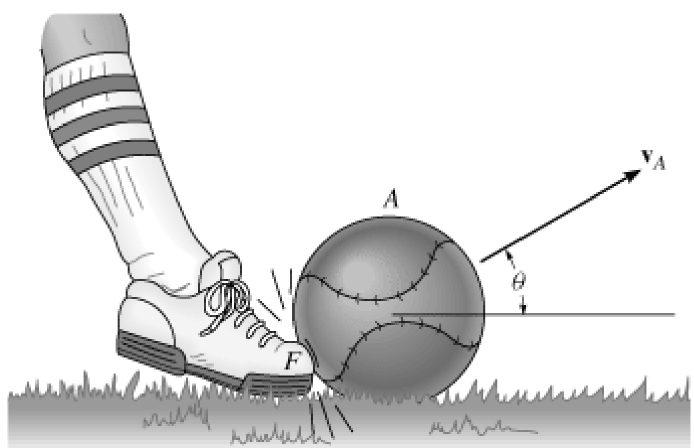
Solution:

$$M v_0 + \int_0^{t_1} a + b t^2\ dt = M v_1$$

$$v_1 = v_0 + \frac{1}{M}\int_0^{t_1} a + b t^2\ dt \qquad v_1 = 121.00\ \frac{\text{m}}{\text{s}}$$

Problem 15-10

A man kicks the ball of mass M such that it leaves the ground at angle θ with the horizontal and strikes the ground at the same elevation a distance d away. Determine the impulse of his foot F on the ball. Neglect the impulse caused by the ball's weight while it's being kicked.



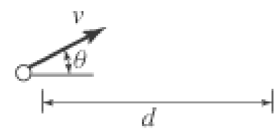
Given:

$M = 200\ \text{gm} \quad d = 15\ \text{m}$

$\theta = 30\ \text{deg} \quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution: First find the velocity v_A

Guesses $v_A = 1\ \frac{\text{m}}{\text{s}}$ $t = 1\ \text{s}$



Given $0 = \left(\frac{-g}{2}\right)t^2 + v_A \sin(\theta)t$ $d = v_A \cos(\theta)t$

$\begin{pmatrix} t \\ v_A \end{pmatrix} = \text{Find}(t, v_A)$ $t = 1.33\ \text{s}$ $v_A = 13.04\ \frac{\text{m}}{\text{s}}$

Impulse - Momentum

$0 + I = M v_A$ $I = M v_A$ $I = 2.61\ \text{N}\cdot\text{s}$

Problem 15-11

The particle P is acted upon by its weight W and forces $\mathbf{F}_1 = (a\mathbf{i} + bt\mathbf{j} + ct\mathbf{k})$ and $\mathbf{F}_2 = dt^2\mathbf{i}$. If the particle originally has a velocity of $\mathbf{v}_1 = (v_{1x}\mathbf{i} + v_{1y}\mathbf{j} + v_{1z}\mathbf{k})$, determine its speed after time t_1.

Given:

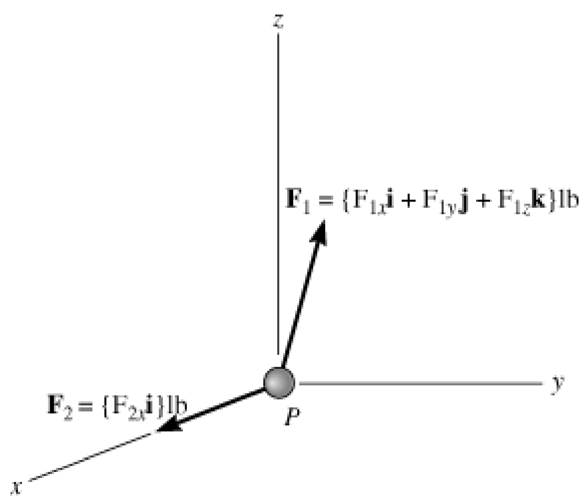
$W = 3\ \text{lb}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$v_{1x} = 3\ \frac{\text{ft}}{\text{s}}$ $a = 5\ \text{lb}$

$v_{1y} = 1\ \frac{\text{ft}}{\text{s}}$ $b = 2\ \frac{\text{lb}}{\text{s}}$

$v_{1z} = 6\ \frac{\text{ft}}{\text{s}}$ $c = 1\ \frac{\text{lb}}{\text{s}}$

$t_1 = 2\ \text{s}$ $d = 1\ \frac{\text{lb}}{\text{s}^2}$



Solution:

$$m v_1 + \int_0^{t_1} (F_1 + F_2 - W k)\ \text{d}t = m v_2 \qquad v_2 = v_1 + \frac{1}{m}\int_0^{t_1} (F_1 + F_2 - W k)\ \text{d}t$$

$$v_{2x} = v_{1x} + \frac{g}{W}\int_0^{t_1} a + d t^2\ \text{d}t \qquad v_{2x} = 138.96\ \frac{\text{ft}}{\text{s}}$$

$$v_{2y} = v_{1y} + \frac{g}{W}\int_0^{t_1} b\, t\ \text{d}t \qquad v_{2y} = 43.93\ \frac{\text{ft}}{\text{s}}$$

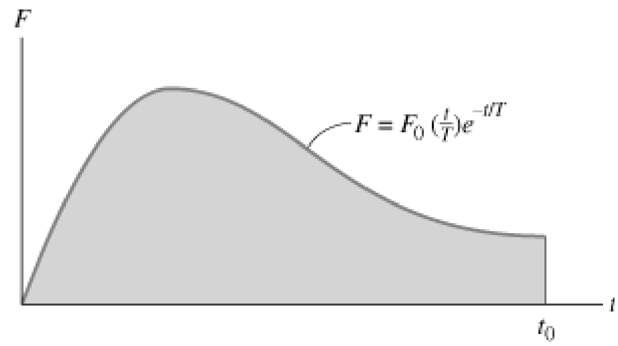
$$v_{2z} = v_{1z} + \frac{g}{W}\int_0^{t_1} c\,t - W\,\mathrm{d}t \qquad v_{2z} = -36.93\,\frac{\text{ft}}{\text{s}}$$

$$v_2 = \sqrt{v_{2x}^2 + v_{2y}^2 + v_{2z}^2} \qquad v_2 = 150.34\,\frac{\text{ft}}{\text{s}}$$

---

*__Problem 15-12__

The twitch in a muscle of the arm develops a force which can be measured as a function of time as shown in the graph. If the effective contraction of the muscle lasts for a time t_0, determine the impulse developed by the muscle.



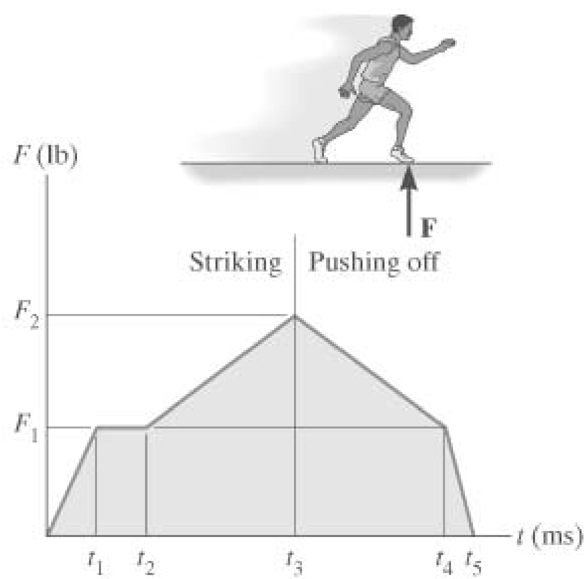
Solution:

$$I = \int_0^{t_0} F_0\left(\frac{t}{T}\right) e^{\frac{-t}{T}}\,\mathrm{d}t = F_0\left(-t_0 - T\right) e^{\frac{-t_0}{T}} + T\,F_0$$

$$I = F_0\,T\left[1 - \left(1 + \frac{t_0}{T}\right) e^{\frac{-t_0}{T}}\right]$$

Problem 15-13

From experiments, the time variation of the vertical force on a runner’s foot as he strikes and pushes off the ground is shown in the graph.These results are reported for a 1-lb *static* load, i.e., in terms of unit weight. If a runner has weight W, determine the approximate vertical impulse he exerts on the ground if the impulse occurs in time t_5.



Units Used:

$$\text{ms} = 10^{-3}\ \text{s}$$

Given:

$$W = 175\ \text{lb}$$

$$t_1 = 25\ \text{ms} \qquad t = 210\ \text{ms}$$

$t_2 = 50\ \text{ms}$ $t_3 = 125\ \text{ms}$

$t_4 = 200\ \text{ms}$ $t_5 = 210\ \text{ms}$

$F_2 = 3.0\ \text{lb}$ $F_1 = 1.5\ \text{lb}$

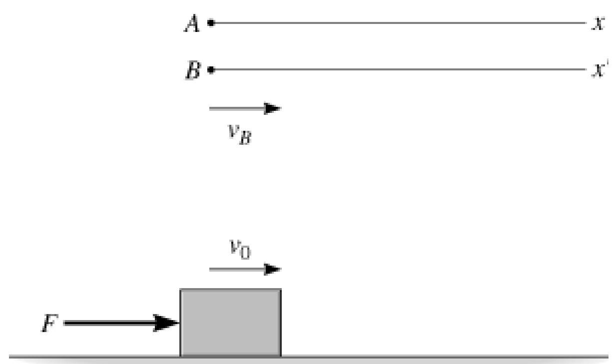
Solution:

$$Area = \frac{1}{2}t_1 F_1 + F_1(t_2 - t_1) + F_1(t_4 - t_2) + \frac{1}{2}(t_5 - t_4)F_1 + \frac{1}{2}(F_2 - F_1)(t_4 - t_2)$$

$$Imp = Area\frac{W}{\text{lb}} \qquad Imp = 70.2\ \text{lb}\cdot\text{s}$$

Problem 15-14

As indicated by the derivation, the principle of impulse and momentum is valid for observers in *any* inertial reference frame. Show that this is so, by considering the block of mass M which rests on the smooth surface and is subjected to horizontal force **F**. If observer A is in a *fixed* frame x, determine the final speed of the block at time t_1 if it has an initial speed v_0 measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis that moves at constant velocity v_B relative to A.



Given:

$M = 10\ \text{kg}$ $v_0 = 5\ \frac{\text{m}}{\text{s}}$

$F = 6\ \text{N}$

$t_1 = 4\ \text{s}$ $v_B = 2\ \frac{\text{m}}{\text{s}}$

Solution:

Observer A:

$$M v_0 + F t_1 = M v_{1A} \qquad v_{1A} = v_0 + \left(\frac{F}{M}\right)t_1 \qquad v_{1A} = 7.40\ \frac{\text{m}}{\text{s}}$$

Observer B:

$$M(v_0 - v_B) + F t_1 = M v_{1B} \qquad v_{1B} = v_0 - v_B + \left(\frac{F}{M}\right)t_1 \qquad v_{1B} = 5.40\ \frac{\text{m}}{\text{s}}$$

Note that $v_{1A} = v_{1B} + v_B$

Problem 15-15

The cabinet of weight W is subjected to the force **F** = $a(bt+c)$. If the cabinet is initially moving up the plane with velocity v_0, determine how long it will take before the cabinet comes to a stop. **F** always acts parallel to the plane. Neglect the size of the rollers.

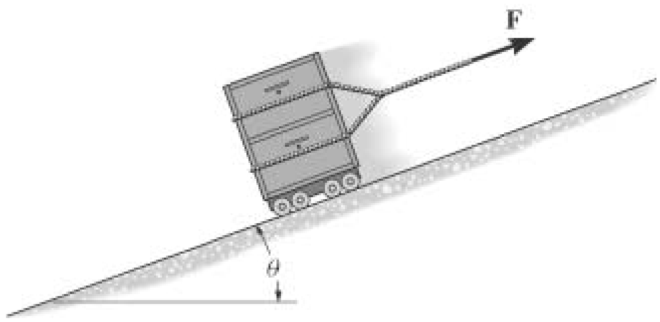
Given:

$W = 4\ \text{lb}$ $v_0 = 10\ \frac{\text{ft}}{\text{s}}$

$a = 20\ \text{lb}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$b = \frac{1}{\text{s}}$ $\theta = 20\ \text{deg}$

$c = 1$



Solution: Guess $t = 10\ \text{s}$ Given

$$\left(\frac{W}{g}\right)v_0 + \int_0^t a(b\tau + c)\,d\tau - W\sin(\theta)t = 0 \qquad t = \text{Find}(t) \qquad t = -0.069256619\ \text{s}$$

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*__Problem 15-16__

If it takes time t_1 for the tugboat of mass m_t to increase its speed uniformly to v_1 starting from rest, determine the force of the rope on the tugboat. The propeller provides the propulsion force **F** which gives the tugboat forward motion, whereas the barge moves freely. Also, determine the force F acting on the tugboat. The barge has mass of m_b.

Units Used:

$\text{Mg} = 1000\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$

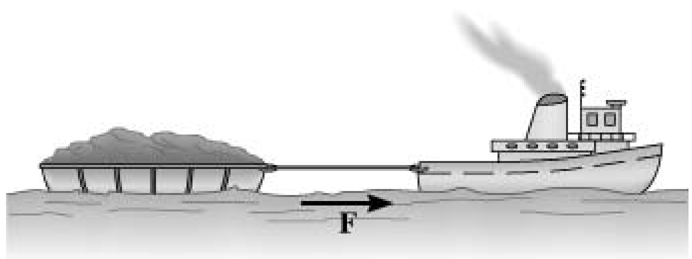
Given:

$t_1 = 35\ \text{s}$

$m_t = 50\ \text{Mg}$

$v_1 = 25\ \frac{\text{km}}{\text{hr}}$

$m_b = 75\ \text{Mg}$



Solution:

The barge alone

$$0 + T t_1 = m_b v_1 \qquad T = \frac{m_b v_1}{t_1} \qquad T = 14.88\,\text{kN}$$

The barge and the tug

$$0 + F t_1 = (m_t + m_b)v_1 \qquad F = \frac{(m_t + m_b)v_1}{t_1} \qquad F = 24.80\,\text{kN}$$

Problem 15-17

When the ball of weight W is fired, it leaves the ground at an angle θ from the horizontal and strikes the ground at the same elevation a distance d away. Determine the impulse given to the ball.

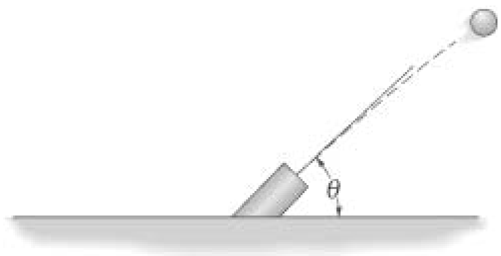
Given:

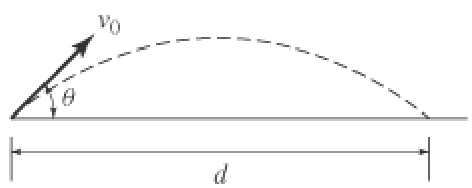
$W = 0.4\ \text{lb}$

$d = 130\ \text{ft}$

$\theta = 40\ \text{deg}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

Guesses $\quad v_0 = 1\ \frac{\text{ft}}{\text{s}} \quad t = 1\ \text{s} \quad Imp = 1\ \text{lb}\cdot\text{s}$

Given $\quad v_0 \cos(\theta) t = d \quad \frac{-1}{2} g t^2 + v_0 \sin(\theta) t = 0 \quad Imp = \left(\frac{W}{g}\right) v_0$

$$\begin{pmatrix} v_0 \\ t \\ Imp \end{pmatrix} = \text{Find}(v_0, t, Imp) \qquad v_0 = 65.2\ \frac{\text{ft}}{\text{s}} \qquad t = 2.6\ \text{s} \qquad Imp = 0.810\ \text{lb}\cdot\text{s}$$

Problem 15-18

The uniform beam has weight W. Determine the average tension in each of the two cables AB and AC if the beam is given an upward speed v in time t starting from rest. Neglect the mass of the cables.

Units Used:

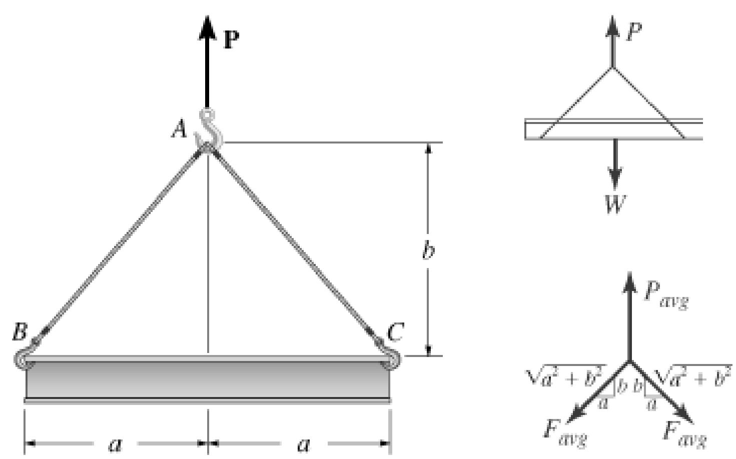
$\text{kip} = 10^3\ \text{lb}$

Given:

$W = 5000\ \text{lb} \quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$v = 8\ \frac{\text{ft}}{\text{s}} \quad a = 3\ \text{ft}$

$t = 1.5\ \text{s} \quad b = 4\ \text{ft}$



Solution:

$$0 - Wt + 2\left(\frac{b}{\sqrt{a^2 + b^2}}\right)F_{AB}t = \left(\frac{W}{g}\right)v$$

$$F_{AB} = \left(\frac{W}{g}v + Wt\right)\left(\frac{\sqrt{a^2 + b^2}}{2bt}\right) \qquad F_{AB} = 3.64\,\text{kip}$$

Problem 15-19

The block of mass M is moving downward at speed v_1 when it is a distance h from the sandy surface. Determine the impulse of the sand on the block necessary to stop its motion. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.

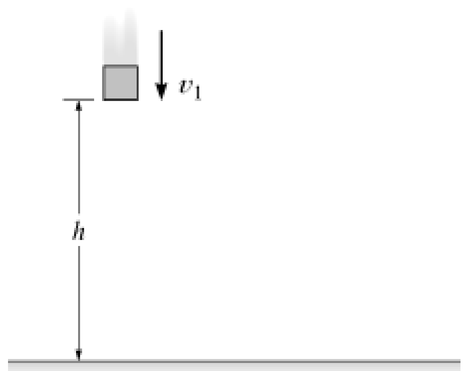
Given:

$M = 5\ \text{kg}$

$v_1 = 2\ \frac{\text{m}}{\text{s}}$

$h = 8\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



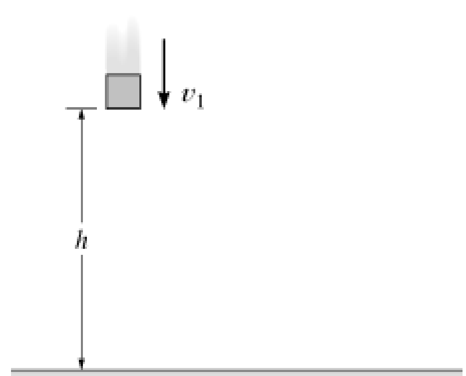
Solution:

Just before impact $\qquad v_2 = \sqrt{v_1^{\,2} + 2gh} \qquad v_2 = 12.69\ \frac{\text{m}}{\text{s}}$

Collision $\qquad Mv_2 - I = 0 \qquad I = Mv_2 \qquad I = 63.4\,\text{N}\cdot\text{s}$

***Problem 15-20**

The block of mass M is falling downward at speed v_1 when it is a distance h from the sandy surface. Determine the average impulsive force acting on the block by the sand if the motion of the block is stopped in time Δt once the block strikes the sand. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.



Given:

$M = 5\ \text{kg}$

$v_1 = 2\ \dfrac{\text{m}}{\text{s}}$ $\quad h = 8\ \text{m}$

$\Delta t = 0.9\ \text{s}$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

Just before impact $\quad v_2 = \sqrt{v_1^2 + 2gh}$ $\quad v_2 = 12.69\ \dfrac{\text{m}}{\text{s}}$

Collision $\quad M v_2 - F\Delta t = 0$ $\quad F = \dfrac{M v_2}{\Delta t}$ $\quad F = 70.5\ \text{N}$

Problem 15-21

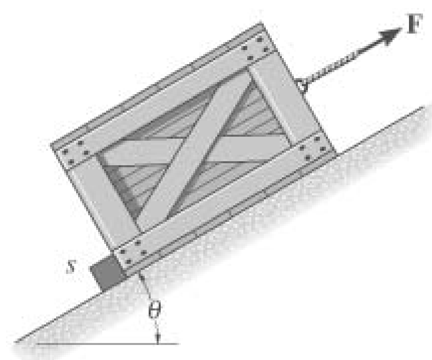
A crate of mass M rests against a stop block s, which prevents the crate from moving down the plane. If the coefficients of static and kinetic friction between the plane and the crate are μ_s and μ_k respectively, determine the time needed for the force **F** to give the crate a speed v up the plane. The force always acts parallel to the plane and has a magnitude of $F = at$. *Hint:* First determine the time needed to overcome static friction and start the crate moving.

Given:

$M = 50\ \text{kg}$ $\quad \theta = 30\ \text{deg}$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$v = 2\ \dfrac{\text{m}}{\text{s}}$ $\quad \mu_s = 0.3$

$a = 300\ \dfrac{\text{N}}{\text{s}}$ $\quad \mu_k = 0.2$



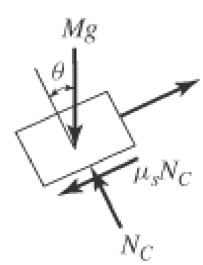
Solution:

Guesses $\quad t_1 = 1\ \text{s}$ $\quad N_C = 1\ \text{N}$ $\quad t_2 = 1\ \text{s}$

Given $\quad N_C - Mg\cos(\theta) = 0$

$a t_1 - \mu_s N_C - Mg\sin(\theta) = 0$

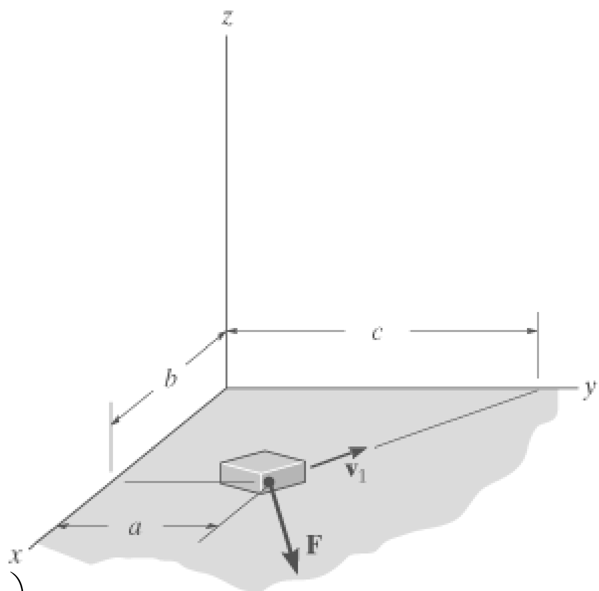
$$\int_{t_1}^{t_2} \left(a t - Mg\sin(\theta) - \mu_k N_C\right) dt = M v$$



$\begin{pmatrix} t_1 \\ t_2 \\ N_C \end{pmatrix} = \text{Find}(t_1, t_2, N_C)$ $\quad t_1 = 1.24\ \text{s}$ $\quad t_2 = 1.93\ \text{s}$

Problem 15-22

The block of weight W has an initial velocity v_1 in the direction shown. If a force $\mathbf{F} = \{f_1\mathbf{i} + f_2\mathbf{j}\}$ acts on the block for time t, determine the final speed of the block. Neglect friction.



Given:

$W = 2\ \text{lb}$ $\quad a = 2\ \text{ft}$ $\quad f_1 = 0.5\ \text{lb}$

$v_1 = 10\ \frac{\text{ft}}{\text{s}}$ $\quad b = 2\ \text{ft}$ $\quad f_2 = 0.2\ \text{lb}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $\quad c = 5\ \text{ft}$ $\quad t = 5\ \text{s}$

Solution:

$$\theta = \operatorname{atan}\left(\frac{b}{c-a}\right)$$

Guesses $\quad v_{2x} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_{2y} = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$\left(\frac{W}{g}\right)v_1\begin{pmatrix}-\sin(\theta)\\ \cos(\theta)\end{pmatrix} + \begin{pmatrix}f_1\\ f_2\end{pmatrix}t = \left(\frac{W}{g}\right)\begin{pmatrix}v_{2x}\\ v_{2y}\end{pmatrix}$$

$$\begin{pmatrix}v_{2x}\\ v_{2y}\end{pmatrix} = \text{Find}(v_{2x}, v_{2y}) \qquad \begin{pmatrix}v_{2x}\\ v_{2y}\end{pmatrix} = \begin{pmatrix}34.7\\ 24.4\end{pmatrix}\frac{\text{ft}}{\text{s}} \qquad \left|\begin{pmatrix}v_{2x}\\ v_{2y}\end{pmatrix}\right| = 42.4\ \frac{\text{ft}}{\text{s}}$$

Problem 15-23

The tennis ball has a horizontal speed v_1 when it is struck by the racket. If it then travels away at angle θ from the horizontal and reaches maximum altitude h, measured from the height of the racket, determine the magnitude of the net impulse of the racket on the ball. The ball has mass M. Neglect the weight of the ball during the time the racket strikes the ball.

Given:

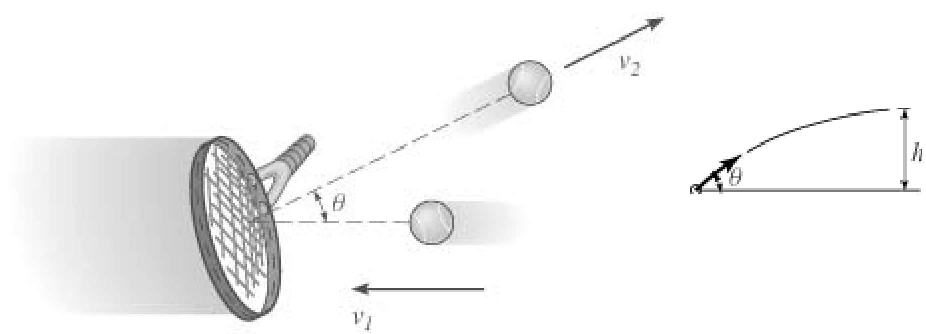
$v_1 = 15\ \frac{\text{m}}{\text{s}}$

$\theta = 25\ \text{deg}$

$h = 10\ \text{m}$

$M = 180\ \text{gm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: Free flight $v_2 \sin(\theta) = \sqrt{2gh}$ $v_2 = \frac{\sqrt{2gh}}{\sin(\theta)}$ $v_2 = 33.14\ \frac{m}{s}$

Impulse - momentum

$-M v_1 + I_x = M v_2 \cos(\theta)$ $I_x = M(v_2 \cos(\theta) + v_1)$ $I_x = 8.11\ N\cdot s$

$0 + I_y = M v_2 \sin(\theta)$ $I_y = M v_2 \sin(\theta)$ $I_y = 2.52\ N\cdot s$

$I = \sqrt{I_x^2 + I_y^2}$ $I = 8.49\ N\cdot s$

---

*__Problem 15-24__

The slider block of mass M is moving to the right with speed v when it is acted upon by the forces $\mathbf{F_1}$ and $\mathbf{F_2}$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = t_3$. Neglect friction and the mass of the pulleys and cords.

Given:

$M = 40\ kg$

$v = 1.5\ \frac{m}{s}$

$t_3 = 6\ s$

$t_2 = 4\ s$

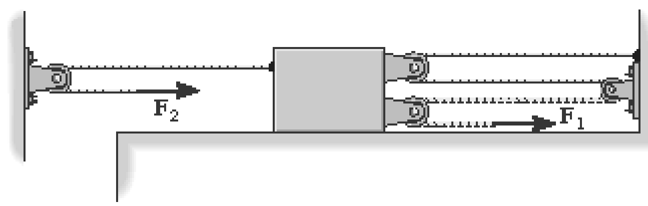
$t_1 = 2\ s$

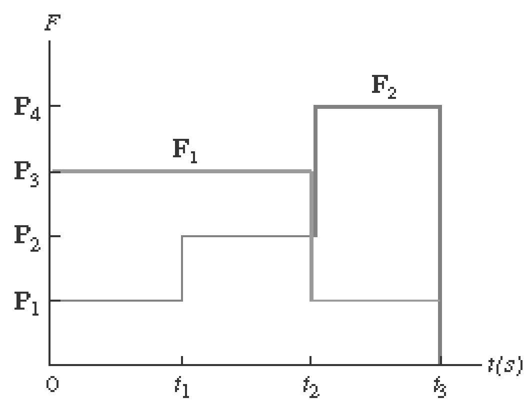
$P_1 = 10\ N$

$P_2 = 20\ N$

$P_3 = 30\ N$

$P_4 = 40\ N$





Solution: The impulses acting on the block are found from the areas under the graph.

$I = 4\left[P_3 t_2 + P_1(t_3 - t_2)\right] - \left[P_1 t_1 + P_2(t_2 - t_1) + P_4(t_3 - t_2)\right]$

$M v + I = M v_3$ $v_3 = v + \frac{I}{M}$ $v_3 = 12.00\ \frac{m}{s}$

Problem 15-25

Determine the velocities of blocks A and B at time t after they are released from rest. Neglect the mass of the pulleys and cables.

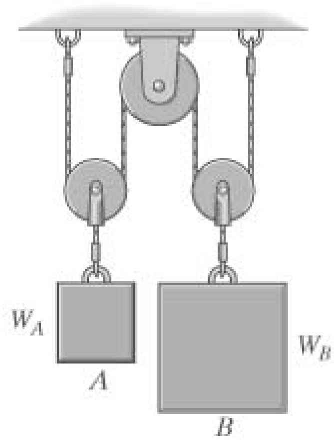
Given:

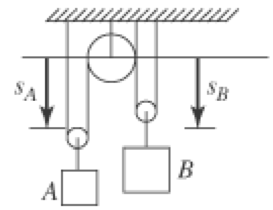
$W_A = 2\ \text{lb}$

$W_B = 4\ \text{lb}$

$t = 2\ \text{s}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution:

$2s_A + 2s_B = L$

$v_A = -v_B$

Block A $\quad 0 + (2T - W_A)t = \dfrac{W_A}{g} v_A$

Block B $\quad 0 + (2T - W_B)t = \dfrac{W_B}{g}(-v_A)$

Combining $\quad (W_B - W_A)t = \left(\dfrac{W_B + W_A}{g}\right) v_A$

$$v_A = \left(\frac{W_B - W_A}{W_B + W_A}\right) g\,t \qquad v_B = -v_A \qquad \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 21.47 \\ -21.47 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

Problem 15-26

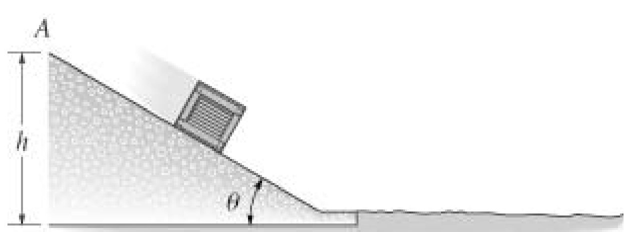
The package of mass M is released from rest at A. It slides down the smooth plane which is inclined at angle θ onto the rough surface having a coefficient of kinetic friction of μ_k. Determine the total time of travel before the package stops sliding. Neglect the size of the package.

Given:

$M = 5\ \text{kg} \qquad h = 3\ \text{m}$

$\theta = 30\ \text{deg} \qquad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$\mu_k = 0.2$



Solution:

On the slope $v_1 = \sqrt{2gh}$ $v_1 = 7.67\ \frac{\text{m}}{\text{s}}$ $t_1 = \frac{v_1}{g\sin(\theta)}$ $t_1 = 1.56\ \text{s}$

On the flat $M v_1 - \mu_k M g t_2 = 0$ $t_2 = \frac{v_1}{\mu_k g}$ $t_2 = 3.91\ \text{s}$

$t = t_1 + t_2$ $t = 5.47\ \text{s}$

Problem 15-27

Block A has weight W_A and block B has weight W_B. If B is moving downward with a velocity v_{B0} at $t = 0$, determine the velocity of A when $t = t_1$. Assume that block A slides smoothly.

Given:

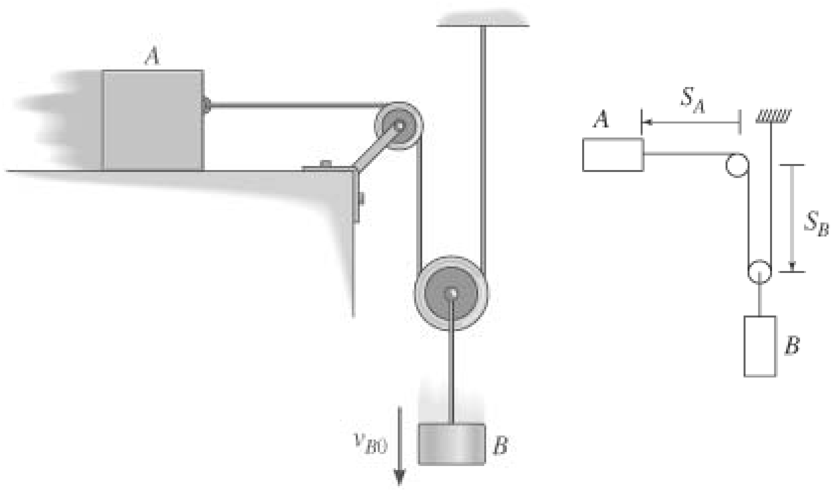
$W_A = 10\ \text{lb}$

$W_B = 3\ \text{lb}$

$v_{B0} = 3\ \frac{\text{ft}}{\text{s}}$

$t_1 = 1\ \text{s}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: $s_A + 2s_B = L$ $v_A = -2v_B$ Guess $v_{A1} = 1\ \frac{\text{ft}}{\text{s}}$ $T = 1\ \text{lb}$

Given

Block A $\left(\frac{W_A}{g}\right)2v_{B0} + T t_1 = \left(\frac{W_A}{g}\right)v_{A1}$

Block B $\left(\frac{-W_B}{g}\right)v_{B0} + 2T t_1 - W_B t_1 = \left(\frac{-W_B}{g}\right)\left(\frac{v_{A1}}{2}\right)$

$\begin{pmatrix} v_{A1} \\ T \end{pmatrix} = \text{Find}(v_{A1}, T)$ $T = 1.40\ \text{lb}$ $v_{A1} = 10.49\ \frac{\text{ft}}{\text{s}}$

***Problem 15-28**

Block A has weight W_A and block B has weight W_B. If B is moving downward with a velocity v_{B1} at $t = 0$, determine the velocity of A when $t = t_1$. The coefficient of kinetic friction between the horizontal plane and block A is μ_k.

Given:

$W_A = 10 \text{ lb}$

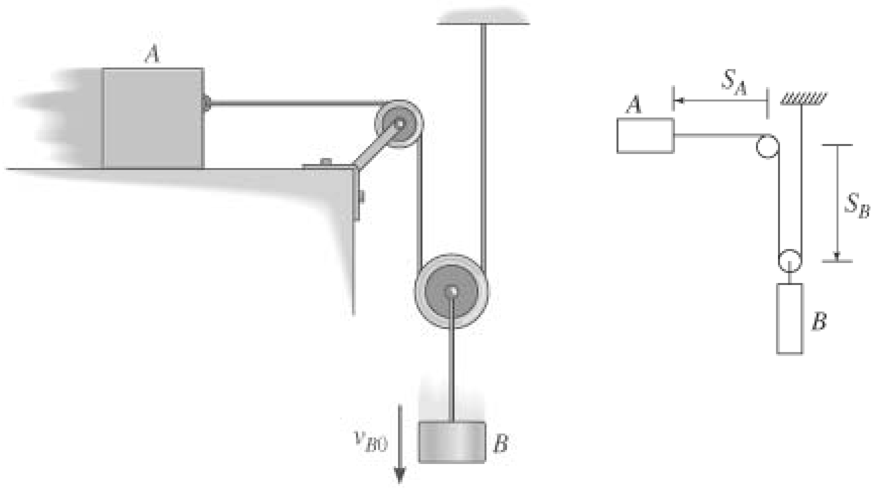
$W_B = 3 \text{ lb}$

$v_{B1} = 3 \frac{\text{ft}}{\text{s}}$

$\mu_k = 0.15$

$t_1 = 1 \text{ s}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution: $s_A + 2s_B = L$ $\quad v_A = -2v_B$ $\quad$ Guess $\quad v_{A2} = 1 \frac{\text{ft}}{\text{s}}$ $\quad T = 1 \text{ lb}$

Given

Block A $\quad \left(\frac{W_A}{g}\right)2v_{B1} + T t_1 - \mu_k W_A t_1 = \left(\frac{W_A}{g}\right)v_{A2}$

Block B $\quad \left(\frac{-W_B}{g}\right)v_{B1} + 2T t_1 - W_B t_1 = \frac{-W_B}{g}\left(\frac{v_{A2}}{2}\right)$

$\begin{pmatrix} v_{A2} \\ T \end{pmatrix} = \text{Find}(v_{A2}, T) \qquad T = 1.50 \text{ lb} \qquad v_{A2} = 6.00 \frac{\text{ft}}{\text{s}}$

Problem 15-29

A jet plane having a mass M takes off from an aircraft carrier such that the engine thrust varies as shown by the graph. If the carrier is traveling forward with a speed v, determine the plane's airspeed after time t.

Units Used:

$\text{Mg} = 10^3 \text{ kg}$

$\text{kN} = 10^3 \text{ N}$

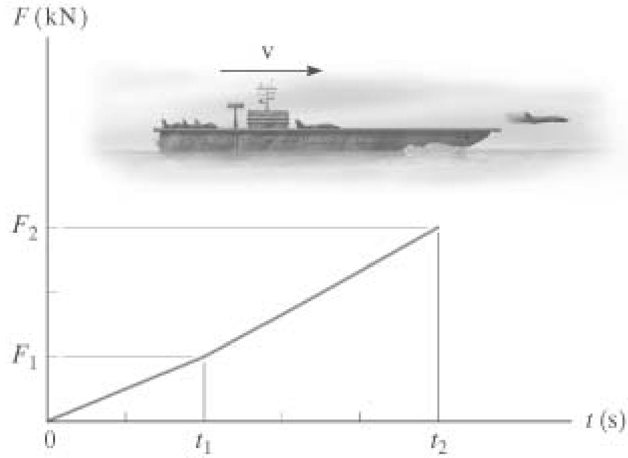
Given:

$M = 7 \text{ Mg}$ $\quad t_1 = 2 \text{ s}$

$v = 40 \dfrac{\text{km}}{\text{hr}}$ $\quad t_2 = 5 \text{ s}$

$F_1 = 5 \text{ kN}$ $\quad t = 5 \text{ s}$

$F_2 = 15 \text{ kN}$



Solution:

The impulse exerted on the plane is equal to the area under the graph.

$$M v + \frac{1}{2} F_1 t_1 + \frac{1}{2}(F_1 + F_2)(t_2 - t_1) = M v_1$$

$$v_1 = v + \frac{1}{2M}\left[F_1 t_1 + (F_1 + F_2)(t_2 - t_1)\right] \qquad v_1 = 16.11 \frac{\text{m}}{\text{s}}$$

Problem 15-30

The motor pulls on the cable at A with a force $\mathbf{F} = a + bt^2$. If the crate of weight W is originally at rest at $t = 0$, determine its speed at time $t = t_2$. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.

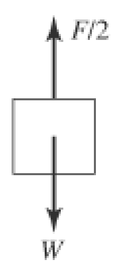
Given:

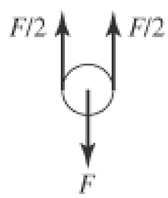
$W = 17 \text{ lb}$

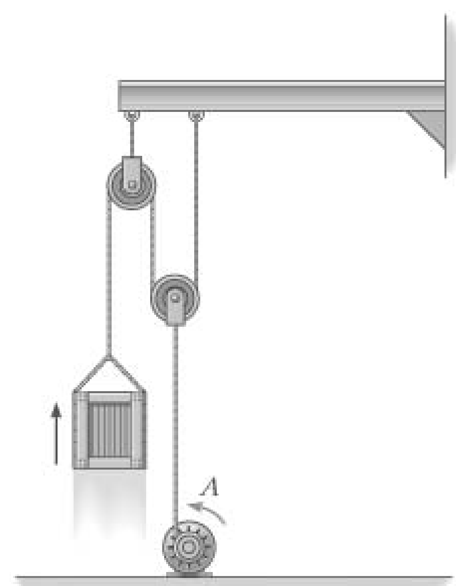
$a = 30 \text{ lb}$

$b = 1 \dfrac{\text{lb}}{\text{s}^2}$

$t_2 = 4 \text{ s}$







Solution:

$$\frac{1}{2}\left(a + b t_1^2\right) - W = 0$$

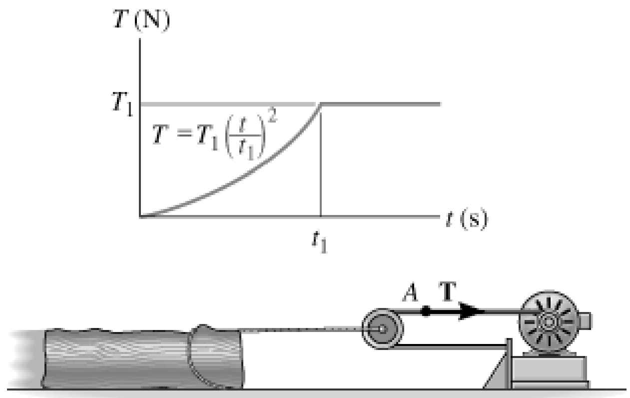
$$t_1 = \sqrt{\frac{2W - a}{b}} \qquad t_1 = 2.00 \text{ s}$$

$$\frac{1}{2}\int_{t_1}^{t_2} a + b t^2 \, dt - W(t_2 - t_1) = \left(\frac{W}{g}\right) v_2 \qquad v_2 = \frac{g}{2W}\int_{t_1}^{t_2} a + b t^2 \, dt - g(t_2 - t_1) \qquad v_2 = 10.10 \frac{\text{ft}}{\text{s}}$$

Problem 15-31

The log has mass M and rests on the ground for which the coefficients of static and kinetic friction are μ_s and μ_k respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the log when $t = t_2$. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the log.

Given:



$M = 500 \text{ kg}$ $t_1 = 3 \text{ s}$

$\mu_s = 0.5$ $T_1 = 1800 \text{ N}$

$\mu_k = 0.4$ $g = 9.81 \frac{\text{m}}{\text{s}^2}$

$t_2 = 5 \text{ s}$

Solution:

To begin motion we need $2T_1\left(\frac{t_0^2}{t_1^2}\right) = \mu_s M g$ $t_0 = \sqrt{\frac{\mu_s M g}{2T_1}}\, t_1$ $t_0 = 2.48 \text{ s}$

Impulse - Momentum

$$0 + \int_{t_0}^{t_1} 2T_1\left(\frac{t}{t_1}\right)^2 dt + 2T_1(t_2 - t_1) - \mu_k M g(t_2 - t_0) = M v_2$$

$$v_2 = \frac{1}{M}\left[\int_{t_0}^{t_1} 2T_1\left(\frac{t}{t_1}\right)^2 dt + 2T_1(t_2 - t_1) - \mu_k M g(t_2 - t_0)\right]$$

$v_2 = 7.65 \frac{\text{m}}{\text{s}}$

***Problem 15-32**

A railroad car having mass m_1 is coasting with speed v_1 on a horizontal track. At the same time another car having mass m_2 is coasting with speed v_2 in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

Units used: $\text{Mg} = 10^3 \text{ kg}$ $\text{kJ} = 10^3 \text{ J}$

Given: $m_1 = 15 \text{ Mg}$ $m_2 = 12 \text{ Mg}$

$v_1 = 1.5\ \frac{\text{m}}{\text{s}}$ $\quad v_2 = 0.75\ \frac{\text{m}}{\text{s}}$

Solution:

$m_1 v_1 - m_2 v_2 = (m_1 + m_2)v$ $\quad v = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2}$ $\quad v = 0.50\ \frac{\text{m}}{\text{s}}$

$T_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$ $\quad T_1 = 20.25\,\text{kJ}$

$T_2 = \frac{1}{2}(m_1 + m_2) v^2$ $\quad T_2 = 3.38\,\text{kJ}$

$\Delta T = T_2 - T_1$ $\quad \Delta T = -16.88\,\text{kJ}$

$\frac{-\Delta T}{T_1} 100 = 83.33$ % loss

The energy is dissipated as noise, shock, and heat during the coupling.

Problem 15-33

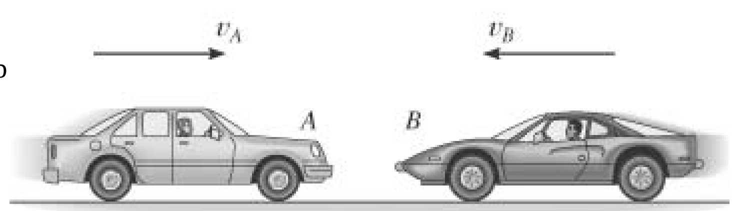
Car A has weight W_A and is traveling to the right at speed v_A Meanwhile car B of weight W_B is traveling at speed v_B to the left. If the cars crash head-on and become entangled, determine their common velocity just after the collision. Assume that the brakes are not applied during collision.

Given:

$W_A = 4500\ \text{lb}$ $\quad W_B = 3000\ \text{lb}$

$v_A = 3\ \frac{\text{ft}}{\text{s}}$ $\quad v_B = 6\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: $\left(\frac{W_A}{g}\right) v_A - \left(\frac{W_B}{g}\right) v_B = \left(\frac{W_A + W_B}{g}\right) v$ $\quad v = \frac{W_A v_A - W_B v_B}{W_A + W_B}$ $\quad v = -0.60\ \frac{\text{ft}}{\text{s}}$

Problem 15-34

The bus B has weight W_B and is traveling to the right at speed v_B. Meanwhile car A of weight W_A is traveling at speed v_A to the left. If the vehicles crash head-on and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.

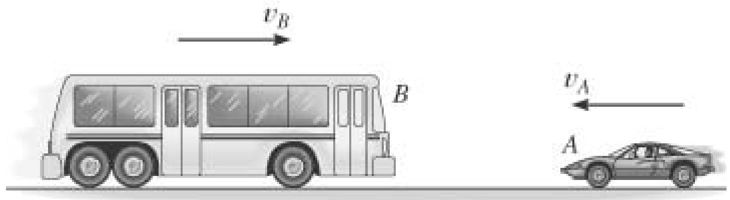
Given:

$W_B = 15000\ \text{lb}$ $\quad v_B = 5\ \frac{\text{ft}}{\text{s}}$

$W_A = 3000\ \text{lb}$

$v_A = 4\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$\left(\frac{W_B}{g}\right)v_B - \left(\frac{W_A}{g}\right)v_A = \left(\frac{W_B + W_A}{g}\right)v \qquad v = \frac{W_B v_B - W_A v_A}{W_B + W_A} \qquad v = 3.50\ \frac{\text{ft}}{\text{s}}$$

Positive means to the right,
negative means to the left.

Problem 15-35

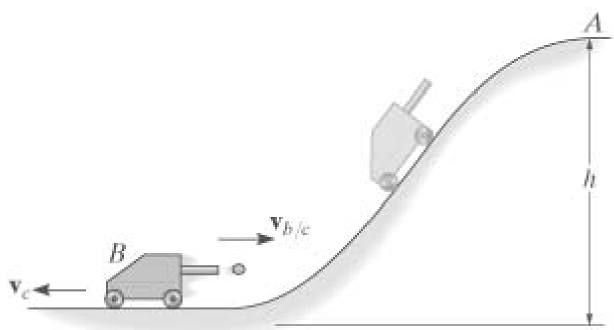
The cart has mass M and rolls freely down the slope. When it reaches the bottom, a spring loaded gun fires a ball of mass M_1 out the back with a horizontal velocity v_{bc} measured relative to the cart. Determine the final velocity of the cart.

Given:

$M = 3\ \text{kg}$ $\quad h = 1.25\ \text{m}$

$M_1 = 0.5\ \text{kg}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$v_{bc} = 0.6\ \frac{\text{m}}{\text{s}}$



Solution:

$$v_1 = \sqrt{2gh}$$

$$(M + M_1)v_1 = M v_c + M_1(v_c - v_{bc})$$

$$v_c = v_1 + \left(\frac{M_1}{M + M_1}\right)v_{bc}$$

$$v_c = 5.04\ \frac{\text{m}}{\text{s}}$$

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*__Problem 15-36__

Two men A and B, each having weight W_m, stand on the cart of weight W_c. Each runs with speed v measured relative to the cart. Determine the final speed of the cart if (a) A runs and jumps off, then B runs and jumps off the same end, and (b) both run at the same time and jump off at the same time. Neglect the mass of the wheels and assume the jumps are made horizontally.

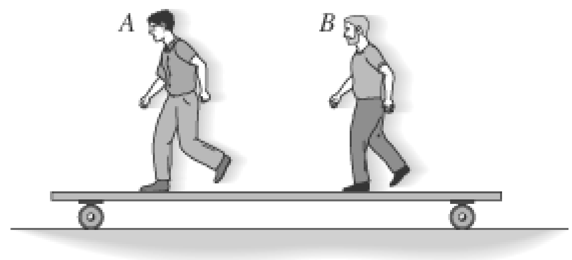
Given:

$W_m = 160 \text{ lb}$

$W_c = 200 \text{ lb}$

$v = 3 \frac{\text{ft}}{\text{s}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution: $m_m = \frac{W_m}{g}$ $\quad m_c = \frac{W_c}{g}$

(a) A jumps first

$0 = -m_m(v - v_c) + (m_m + m_c)v_{c1}$ $\quad v_{c1} = \frac{m_m v}{m_c + 2m_m}$ $\quad v_{c1} = 0.923 \frac{\text{ft}}{\text{s}}$

And then B jumps

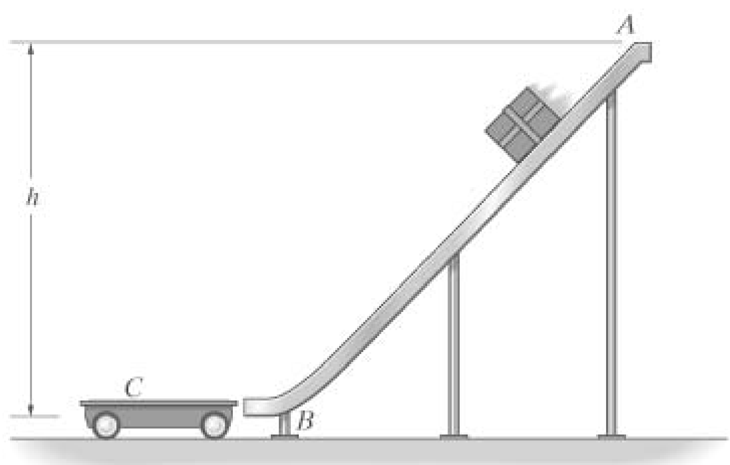
$(m_m + m_c)v_{c1} = -m_m(v - v_{c2}) + m_c v_{c2}$ $\quad v_{c2} = \frac{m_m v + (m_m + m_c)v_{c1}}{m_m + m_c}$ $\quad v_{c2} = 2.26 \frac{\text{ft}}{\text{s}}$

(b) Both men jump at the same time

$0 = -2m_m(v - v_{c3}) + m_c v_{c3}$ $\quad v_{c3} = \frac{2m_m v}{2m_m + m_c}$ $\quad v_{c3} = 1.85 \frac{\text{ft}}{\text{s}}$

Problem 15-37

A box of weight W_1 slides from rest down the smooth ramp onto the surface of a cart of weight W_2. Determine the speed of the box at the instant it stops sliding on the cart. If someone ties the cart to the ramp at B, determine the horizontal impulse the box will exert at C in order to stop its motion. Neglect friction on the ramp and neglect the size of the box.



Given:

$W_1 = 40\ \text{lb} \qquad W_2 = 20\ \text{lb} \qquad h = 15\ \text{ft} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

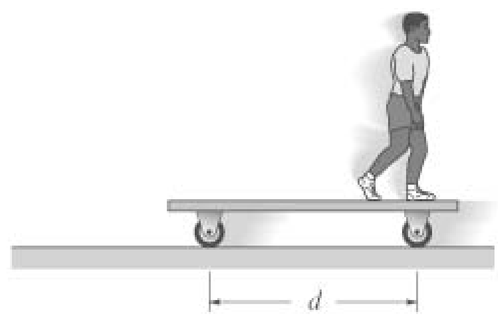
$$v_1 = \sqrt{2gh}$$

$$\frac{W_1}{g}v_1 = \left(\frac{W_1 + W_2}{g}\right)v_2 \qquad v_2 = \left(\frac{W_1}{W_1 + W_2}\right)v_1 \qquad v_2 = 20.7\ \frac{\text{ft}}{\text{s}}$$

$$\left(\frac{W_1}{g}\right)v_1 - Imp = 0 \qquad Imp = \left(\frac{W_1}{g}\right)v_1 \qquad Imp = 38.6\ \text{lb}\cdot\text{s}$$

Problem 15-38

A boy of weight W_1 walks forward over the surface of the cart of weight W_2 with a constant speed v relative to the cart. Determine the cart's speed and its displacement at the moment he is about to step off. Neglect the mass of the wheels and assume the cart and boy are originally at rest.



Given:

$W_1 = 100\ \text{lb} \qquad W_2 = 60\ \text{lb} \qquad v = 3\ \frac{\text{ft}}{\text{s}} \qquad d = 6\ \text{ft}$

Solution:

$$0 = \left(\frac{W_1}{g}\right)(v_c + v) + \left(\frac{W_2}{g}\right)v_c \qquad v_c = -\frac{W_1}{W_1 + W_2}v \qquad v_c = -1.88\ \frac{\text{ft}}{\text{s}}$$

Assuming that the boy walks the distance d

$$t = \frac{d}{v} \qquad s_c = v_c t \qquad s_c = -3.75\ \text{ft}$$

Problem 15-39

The barge B has weight W_B and supports an automobile weighing W_a. If the barge is not tied to the pier P and someone drives the automobile to the other side of the barge for unloading, determine how far the barge moves away from the pier. Neglect the resistance of the water.

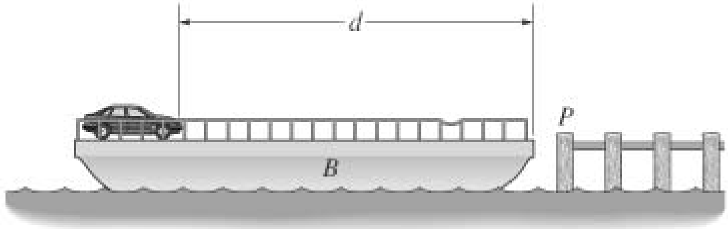
Given:

$W_B = 30000\ \text{lb}$

$W_a = 3000\ \text{lb}$

$d = 200\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$m_B = \frac{W_B}{g} \qquad m_a = \frac{W_a}{g}$

v is the velocity of the car relative to the barge. The answer is independent of the acceleration so we will do the problem for a constant speed.

$m_B v_B + m_a(v + v_B) = 0 \qquad v_B = \frac{-m_a v}{m_B + m_a}$

$t = \frac{d}{v} \qquad s_B = -v_B t \qquad s_B = \frac{m_a d}{m_a + m_B} \qquad s_B = 18.18\ \text{ft}$

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*__Problem 15-40__

A bullet of weight W_1 traveling at speed v_1 strikes the wooden block of weight W_2 and exits the other side at speed v_2 as shown. Determine the speed of the block just after the bullet exits the block, and also determine how far the block slides before it stops. The coefficient of kinetic friction between the block and the surface is μ_k.

Given:

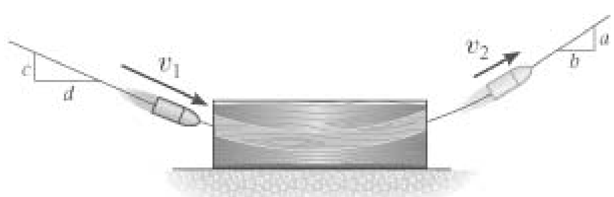
$W_1 = 0.03\ \text{lb} \qquad a = 3\ \text{ft}$

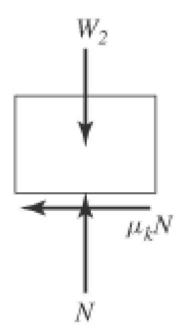
$W_2 = 10\ \text{lb} \qquad b = 4\ \text{ft}$

$v_1 = 1300\ \frac{\text{ft}}{\text{s}} \qquad c = 5\ \text{ft}$

$d = 12\ \text{ft}$

$v_2 = 50\ \frac{\text{ft}}{\text{s}} \qquad \mu_k = 0.5$





Solution:

$$\left(\frac{W_1}{g}\right) v_1 \left(\frac{d}{\sqrt{c^2 + d^2}}\right) = \left(\frac{W_2}{g}\right) v_B + \left(\frac{W_1}{g}\right) v_2 \left(\frac{b}{\sqrt{a^2 + b^2}}\right)$$

$$v_B = \frac{W_1}{W_2}\left(\frac{v_1 d}{\sqrt{c^2 + d^2}} - \frac{v_2 b}{\sqrt{a^2 + b^2}}\right) \qquad v_B = 3.48\,\frac{\text{ft}}{\text{s}}$$

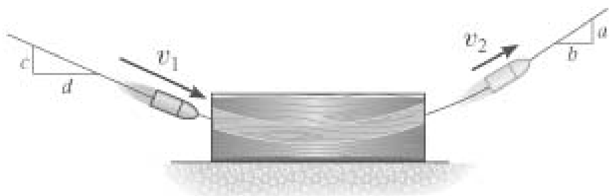
$$\frac{1}{2}\left(\frac{W_2}{g}\right)v_B^2 - \mu_k W_2 d = 0 \qquad d = \frac{v_B^2}{2g\mu_k} \qquad d = 0.38\,\text{ft}$$

Problem 15-41

A bullet of weight W_1 traveling at v_1 strikes the wooden block of weight W_2 and exits the other side at v_2 as shown. Determine the speed of the block just after the bullet exits the block. Also, determine the average normal force on the block if the bullet passes through it in time Δt, and the time the block slides before it stops. The coefficient of kinetic friction between the block and the surface is μ_k.

Units Used: $\text{ms} = 10^{-3}\,\text{s}$

Given:



$W_1 = 0.03\,\text{lb}$ $\quad a = 3\,\text{ft}$

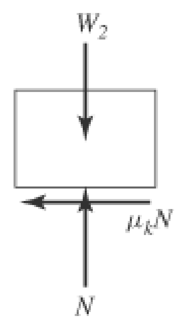
$W_2 = 10\,\text{lb}$ $\quad b = 4\,\text{ft}$

$\mu_k = 0.5$ $\quad c = 5\,\text{ft}$

$\Delta t = 1\,\text{ms}$ $\quad d = 12\,\text{ft}$

$v_1 = 1300\,\frac{\text{ft}}{\text{s}}$ $\quad v_2 = 50\,\frac{\text{ft}}{\text{s}}$

Solution:



$$\frac{W_1}{g}v_1\left(\frac{d}{\sqrt{c^2 + d^2}}\right) = \frac{W_2}{g}v_B + \frac{W_1}{g}v_2\left(\frac{b}{\sqrt{a^2 + b^2}}\right)$$

$$v_B = \frac{W_1}{W_2}\left(\frac{v_1 d}{\sqrt{c^2 + d^2}} - \frac{v_2 b}{\sqrt{a^2 + b^2}}\right) \qquad v_B = 3.48\,\frac{\text{ft}}{\text{s}}$$

$$\frac{-W_1}{g}v_1\left(\frac{c}{\sqrt{c^2 + d^2}}\right) + (N - W_2)\Delta t = \frac{W_1}{g}v_2\left(\frac{a}{\sqrt{a^2 + b^2}}\right)$$

$$N = \frac{W_1}{g\Delta t}\left(\frac{v_2 a}{\sqrt{a^2 + b^2}} + \frac{v_1 c}{\sqrt{c^2 + d^2}}\right) + W_2 \qquad N = 503.79\,\text{lb}$$

$$\left(\frac{W_2}{g}\right)v_B - \mu_k W_2 t = 0 \qquad t = \frac{v_B}{g\mu_k} \qquad t = 0.22\,\text{s}$$

Problem 15-42

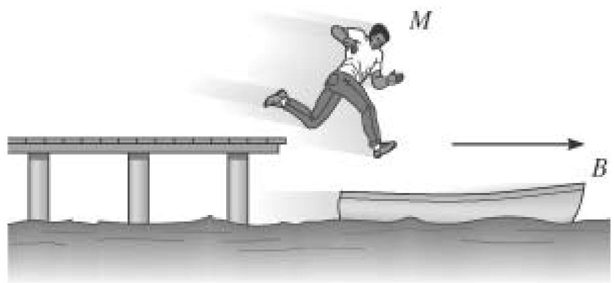
The man *M* has weight W_M and jumps onto the boat *B* which has weight W_B. If he has a horizontal component of velocity *v relative to the boat*, just before he enters the boat, and the boat is traveling at speed v_B away from the pier when he makes the jump, determine the resulting velocity of the man and boat.

Given:

$W_M = 150\ \text{lb}$ $\quad v_B = 2\ \dfrac{\text{ft}}{\text{s}}$

$W_B = 200\ \text{lb}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$v = 3\ \dfrac{\text{ft}}{\text{s}}$



Solution:

$$\frac{W_M}{g}(v + v_B) + \frac{W_B}{g} v_B = \left(\frac{W_M + W_B}{g}\right) v'$$

$$v' = \frac{W_M v + (W_M + W_B) v_B}{W_M + W_B} \qquad v' = 3.29\ \frac{\text{ft}}{\text{s}}$$

Problem 15-43

The man *M* has weight W_M and jumps onto the boat *B* which is originally at rest. If he has a horizontal component of velocity *v* just before he enters the boat, determine the weight of the boat if it has velocity *v'* once the man enters it.

Given:

$W_M = 150\ \text{lb}$

$v = 3\ \dfrac{\text{ft}}{\text{s}}$

$v' = 2\ \dfrac{\text{ft}}{\text{s}}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$$\left(\frac{W_M}{g}\right) v = \left(\frac{W_M + W_B}{g}\right) v' \qquad W_B = \left(\frac{v - v'}{v'}\right) W_M \qquad W_B = 75.00\ \text{lb}$$

***Problem 15-44**

A boy A having weight W_A and a girl B having weight W_B stand motionless at the ends of the toboggan, which has weight W_t. If A walks to B and stops, and both walk back together to the original position of A (both positions measured on the toboggan), determine the final position of the toboggan just after the motion stops. Neglect friction.

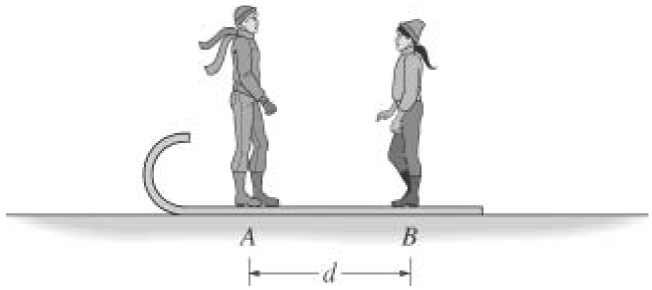
Given:

$W_A = 80\ \text{lb}$

$W_B = 65\ \text{lb}$

$W_t = 20\ \text{lb}$

$d = 4\ \text{ft}$



Solution: The center of mass doesn't move during the motion since there is no friction and therefore no net horizontal force

$$W_B d = (W_A + W_B + W_t)d' \qquad d' = \frac{W_B d}{W_A + W_B + W_t} \qquad d' = 1.58\ \text{ft}$$

Problem 15-45

The projectile of weight W is fired from ground level with initial velocity v_A in the direction shown. When it reaches its highest point B it explodes into two fragments of weight $W/2$. If one fragment travels vertically upward at speed v_1, determine the distance between the fragments after they strike the ground. Neglect the size of the gun.

Given:

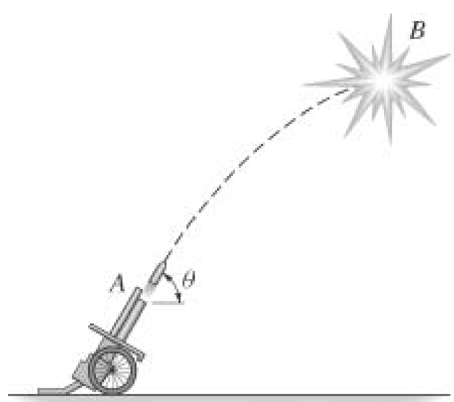
$W = 10\ \text{lb}$

$v_A = 80\ \dfrac{\text{ft}}{\text{s}}$

$v_1 = 12\ \dfrac{\text{ft}}{\text{s}}$

$\theta = 60\ \text{deg}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution: At the top $v = v_A \cos(\theta)$

Explosion $\left(\frac{W}{g}\right)v = 0 + \left(\frac{W}{2g}\right)v_{2x}$ $v_{2x} = 2v$ $v_{2x} = 80.00\ \frac{\text{ft}}{\text{s}}$

$0 = \left(\frac{W}{2g}\right)v_1 - \left(\frac{W}{2g}\right)v_{2y}$ $v_{2y} = v_1$ $v_{2y} = 12.00\ \frac{\text{ft}}{\text{s}}$

Kinematics $h = \frac{(v_A \sin(\theta))^2}{2g}$ $h = 74.53\text{ ft}$ Guess $t = 1\text{ s}$

Given $0 = \left(\frac{-g}{2}\right)t^2 - v_{2y}t + h$ $t = \text{Find}(t)$ $t = 1.81\text{ s}$

$d = v_{2x}t$ $d = 144.9\text{ ft}$

Problem 15-46

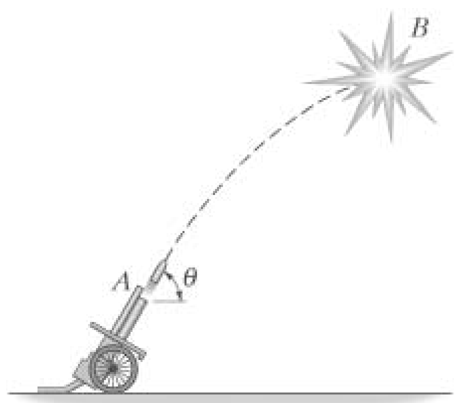
The projectile of weight W is fired from ground level with an initial velocity v_A in the direction shown. When it reaches its highest point B it explodes into two fragments of weight $W/2$. If one fragment is seen to travel vertically upward, and after they fall they are a distance d apart, determine the speed of each fragment just after the explosion. Neglect the size of the gun.

Given:

$W = 10\text{ lb}$ $\theta = 60\text{ deg}$

$v_A = 80\ \frac{\text{ft}}{\text{s}}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$d = 150\text{ ft}$



Solution:

$$h = \frac{(v_A \sin(\theta))^2}{2g}$$

Guesses

$v_1 = 1\ \frac{\text{ft}}{\text{s}}$ $v_{2x} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{2y} = 1\ \frac{\text{ft}}{\text{s}}$ $t = 1\text{ s}$

Given $\left(\frac{W}{g}\right)v_A\cos(\theta) = \left(\frac{W}{2g}\right)v_{2x}$ $0 = \left(\frac{W}{2g}\right)v_1 + \left(\frac{W}{2g}\right)v_{2y}$

$d = v_{2x}t$ $0 = h - \frac{1}{2}gt^2 + v_{2y}t$

$$\begin{pmatrix} v_1 \\ v_{2x} \\ v_{2y} \\ t \end{pmatrix} = \text{Find}(v_1, v_{2x}, v_{2y}, t) \qquad t = 1.87 \text{ s} \qquad \begin{pmatrix} v_1 \\ v_{2x} \\ v_{2y} \end{pmatrix} = \begin{pmatrix} 9.56 \\ 80.00 \\ -9.56 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

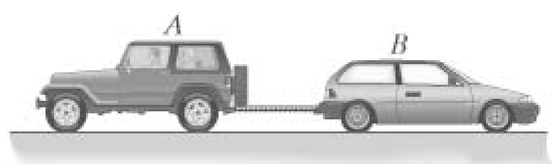
$$v_1 = 9.56 \frac{\text{ft}}{\text{s}} \qquad \left| \begin{pmatrix} v_{2x} \\ v_{2y} \end{pmatrix} \right| = 80.57 \frac{\text{ft}}{\text{s}}$$

Problem 15-47

The winch on the back of the jeep A is turned on and pulls in the tow rope at speed v_{rel}. If both the car B of mass M_B and the jeep A of mass M_A are free to roll, determine their velocities at the instant they meet. If the rope is of length L, how long will this take?

Units Used:

$$\text{Mg} = 10^3 \text{ kg}$$



Given:

$$M_A = 2.5 \text{ Mg} \qquad v_{rel} = 2 \frac{\text{m}}{\text{s}}$$

$$M_B = 1.25 \text{ Mg} \qquad L = 5 \text{ m}$$

Solution:

Guess $\quad v_A = 1 \frac{\text{m}}{\text{s}} \qquad v_B = 1 \frac{\text{m}}{\text{s}}$

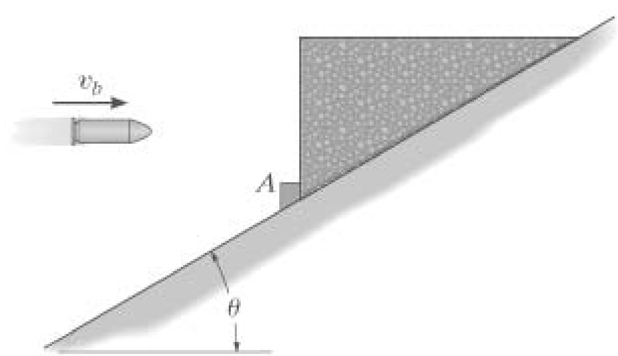
Given $\quad 0 = M_A v_A + M_B v_B \qquad v_A - v_B = v_{rel} \qquad \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \text{Find}(v_A, v_B)$

$$t = \frac{L}{v_{rel}} \qquad t = 2.50 \text{ s} \qquad \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 0.67 \\ -1.33 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

---

*__Problem 15-48__

The block of mass M_a is held at rest on the smooth inclined plane by the stop block at A. If the bullet of mass M_b is traveling at speed v when it becomes embedded in the block of mass M_c, determine the distance the block will slide up along the plane before momentarily stopping.



Given:

$M_a = 10\text{ kg}$ $\quad v = 300\ \frac{\text{m}}{\text{s}}$

$M_b = 10\text{ gm}$ $\quad \theta = 30\text{ deg}$

$M_c = 10\text{ kg}$

Solution:

Conservation of Linear Momentum: If we consider the block and the bullet as a system, then from the *FBD*, the *impulsive* force **F** caused by the impact is *internal* to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are *nonimpulsive forces*. As the result, linear momentum is conserved along the *x* axis

$$M_b v_{bx} = (M_b + M_a)v_x$$

$$M_b v\cos(\theta) = (M_b + M_a)v_x$$

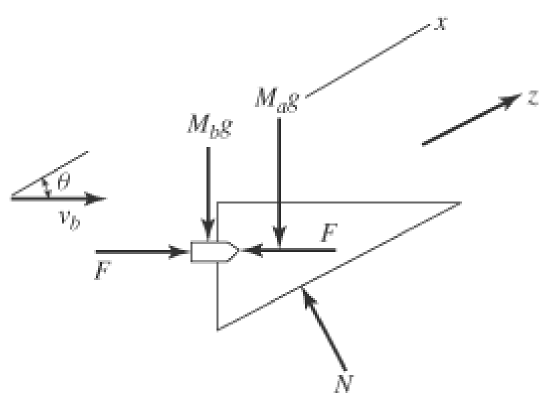
$$v_x = M_b\, v\left(\frac{\cos(\theta)}{M_b + M_a}\right) \qquad v_x = 0.2595\ \frac{\text{m}}{\text{s}}$$

Conservation of Energy: The datum is set at the block's initial position. When the block and the embedded bullet are at their highest point they are a distance *h above* the datum. Their gravitational potential energy is $(M_a + M_b)gh$. Applying Eq. 14-21, we have

$$0 + \frac{1}{2}(M_a + M_b)v_x^{\,2} = 0 + (M_a + M_b)g\,h$$

$$h = \frac{1}{2}\left(\frac{v_x^{\,2}}{g}\right) \qquad h = 3.43\,\text{mm}$$

$$d = \frac{h}{\sin(\theta)} \qquad d = 6.86\,\text{mm}$$



Problem 15-49

A tugboat *T* having mass m_T is tied to a barge *B* having mass m_B. If the rope is "elastic" such that it has stiffness *k*, determine the maximum stretch in the rope during the initial towing. Originally both the tugboat and barge are moving in the same direction with speeds v_{T1} and v_{B1} respectively. Neglect the resistance of the water.

Units Used:

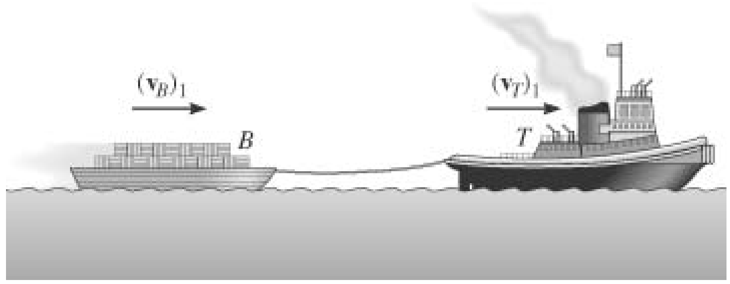
$\text{Mg} = 10^3\text{ kg} \qquad \text{kN} = 10^3\text{ N}$

Given:

$m_T = 19\ \text{Mg}$ $\quad v_{B1} = 10\ \dfrac{\text{km}}{\text{hr}}$

$m_B = 75\ \text{Mg}$ $\quad v_{T1} = 15\ \dfrac{\text{km}}{\text{hr}}$

$k = 600\ \dfrac{\text{kN}}{\text{m}}$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

At maximum stretch the velocities are the same.

Guesses $\quad v_2 = 1\ \dfrac{\text{km}}{\text{hr}} \quad \delta = 1\ \text{m}$

Given

momentum $\quad m_T v_{T1} + m_B v_{B1} = (m_T + m_B)v_2$

energy $\quad \dfrac{1}{2}m_T v_{T1}^2 + \dfrac{1}{2}m_B v_{B1}^2 = \dfrac{1}{2}(m_T + m_B)v_2^2 + \dfrac{1}{2}k\delta^2$

$\begin{pmatrix} v_2 \\ \delta \end{pmatrix} = \text{Find}(v_2, \delta) \qquad v_2 = 11.01\ \dfrac{\text{km}}{\text{hr}} \qquad \delta = 0.221\ \text{m}$

Problem 15-50

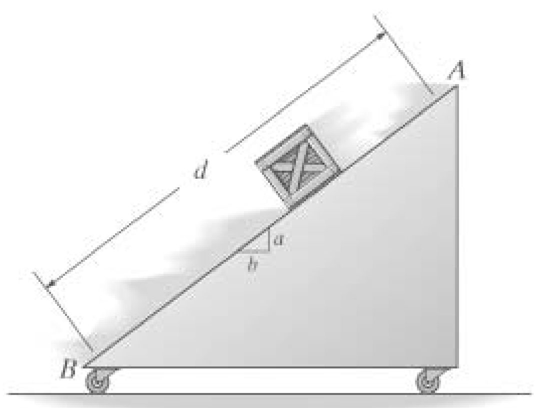
The free-rolling ramp has a weight W_r. The crate, whose weight is W_c, slides a distance d from rest at A, down the ramp to B. Determine the ramp's speed when the crate reaches B. Assume that the ramp is smooth, and neglect the mass of the wheels.

Given:

$W_r = 120\ \text{lb}$ $\quad a = 3$

$W_c = 80\ \text{lb}$ $\quad b = 4$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad d = 15\ \text{ft}$



Solution:

$\theta = \text{atan}\left(\dfrac{a}{b}\right)$

Guesses $\quad v_r = 1\ \dfrac{\text{ft}}{\text{s}} \quad v_{cr} = 1\ \dfrac{\text{ft}}{\text{s}}$

Given

$$W_c\, d\sin(\theta) = \frac{1}{2}\left(\frac{W_r}{g}\right)v_r^{\,2} + \frac{1}{2}\left(\frac{W_c}{g}\right)\left[\left(v_r - v_{cr}\cos(\theta)\right)^2 + \left(v_{cr}\sin(\theta)\right)^2\right]$$

$$0 = \left(\frac{W_r}{g}\right)v_r + \left(\frac{W_c}{g}\right)\left(v_r - v_{cr}\cos(\theta)\right)$$

$$\begin{pmatrix} v_r \\ v_{cr} \end{pmatrix} = \text{Find}(v_r, v_{cr}) \qquad v_{cr} = 27.9\,\frac{\text{ft}}{\text{s}} \qquad v_r = 8.93\,\frac{\text{ft}}{\text{s}}$$

Problem 15-51

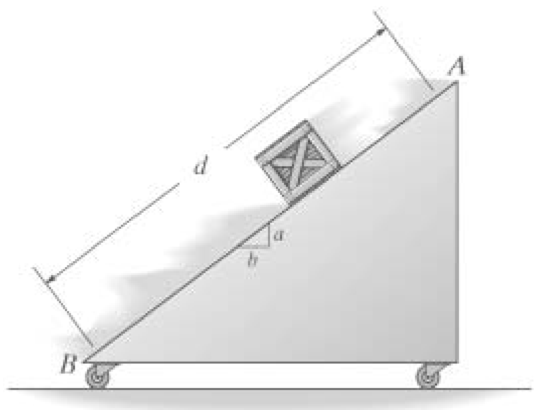
The free-rolling ramp has a weight W_r. If the crate, whose weight is W_c, is released from rest at A, determine the distance the ramp moves when the crate slides a distance d down the ramp and reaches the bottom B.

Given:

$W_r = 120\ \text{lb}$ $\quad a = 3$

$W_c = 80\ \text{lb}$ $\quad b = 4$

$g = 32.2\,\frac{\text{ft}}{\text{s}^2}$ $\quad d = 15\ \text{ft}$



Solution:

$$\theta = \text{atan}\left(\frac{a}{b}\right)$$

Momentum

$$0 = \left(\frac{W_r}{g}\right)v_r + \left(\frac{W_c}{g}\right)\left(v_r - v_{cr}\cos(\theta)\right) \qquad v_r = \left(\frac{W_c}{W_c + W_r}\right)\cos(\theta)v_{cr}$$

Integrate

$$s_r = \left(\frac{W_c}{W_c + W_r}\right)\cos(\theta)\, d \qquad s_r = 4.80\ \text{ft}$$

---

*__Problem 15-52__

The boy B jumps off the canoe at A with a velocity v_{BA} relative to the canoe as shown. If he lands in the second canoe C, determine the final speed of both canoes after the motion. Each canoe has a mass M_c. The boy's mass is M_B, and the girl D has a mass M_D. Both canoes are originally at rest.

Given:

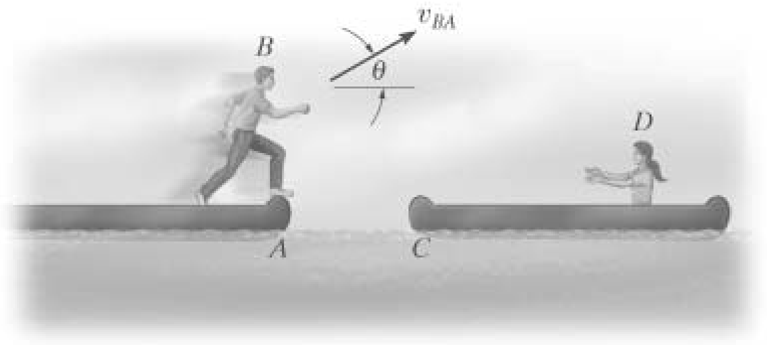
$M_c = 40$ kg

$M_B = 30$ kg

$M_D = 25$ kg

$v_{BA} = 5\ \frac{\text{m}}{\text{s}}$

$\theta = 30$ deg



Solution:

Guesses $v_A = 1\ \frac{\text{m}}{\text{s}}$ $v_C = 1\ \frac{\text{m}}{\text{s}}$

Given $0 = M_c v_A + M_B\left(v_A + v_{BA}\cos(\theta)\right)$

$$M_B\left(v_A + v_{BA}\cos(\theta)\right) = \left(M_c + M_B + M_D\right)v_C$$

$$\begin{pmatrix} v_A \\ v_C \end{pmatrix} = \text{Find}\left(v_A, v_C\right) \qquad \begin{pmatrix} v_A \\ v_C \end{pmatrix} = \begin{pmatrix} -1.86 \\ 0.78 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

Problem 15-53

The free-rolling ramp has a mass M_r. A crate of mass M_c is released from rest at A and slides down d to point B. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches B. Also, what is the velocity of the crate?

Given:

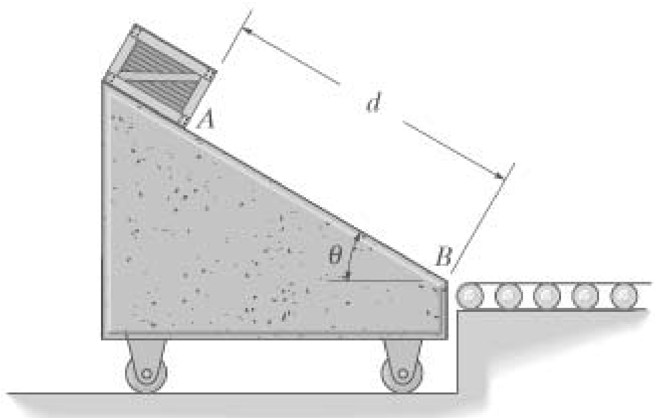
$M_r = 40$ kg

$M_c = 10$ kg

$d = 3.5$ m

$\theta = 30$ deg

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guesses $v_c = 1\ \frac{\text{m}}{\text{s}}$ $v_r = 1\ \frac{\text{m}}{\text{s}}$ $v_{cr} = 1\ \frac{\text{m}}{\text{s}}$

Given $0 + M_c g d \sin(\theta) = \frac{1}{2} M_c v_c^2 + \frac{1}{2} M_r v_r^2$

$$\left(v_r + v_{cr}\cos(\theta)\right)^2 + \left(v_{cr}\sin(\theta)\right)^2 = v_c^{\,2}$$

$$0 = M_r v_r + M_c\left(v_r + v_{cr}\cos(\theta)\right)$$

$$\begin{pmatrix} v_c \\ v_r \\ v_{cr} \end{pmatrix} = \text{Find}\left(v_c, v_r, v_{cr}\right) \qquad v_{cr} = 6.36\ \frac{\text{m}}{\text{s}} \qquad \begin{pmatrix} v_r \\ v_c \end{pmatrix} = \begin{pmatrix} -1.101 \\ 5.430 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

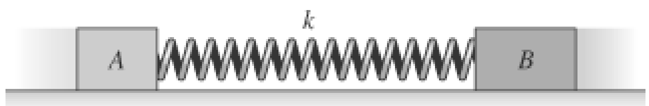
Problem 15-54

Blocks *A* and *B* have masses m_A and m_B respectively. They are placed on a smooth surface and the spring connected between them is stretched a distance *d*. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.

Given:

$$m_A = 40\ \text{kg} \qquad d = 2\ \text{m}$$

$$m_B = 60\ \text{kg} \qquad k = 180\ \frac{\text{N}}{\text{m}}$$



Solution: Guesses $v_A = 1\ \frac{\text{m}}{\text{s}}$ $\quad v_B = -1\ \frac{\text{m}}{\text{s}}$ Given

momentum $\quad 0 = m_A v_A + m_B v_B$

energy $\quad \frac{1}{2}k d^2 = \frac{1}{2}m_A v_A^{\,2} + \frac{1}{2}m_B v_B^{\,2}$

$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \text{Find}\left(v_A, v_B\right) \qquad \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 3.29 \\ -2.19 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

Problem 15-55

Block *A* has a mass M_A and is sliding on a rough horizontal surface with a velocity v_{A1} when it makes a direct collision with block *B*, which has a mass M_B and is originally at rest. If the collision is perfectly elastic, determine the velocity of each block just after collision and the distance between the blocks when they stop sliding. The coefficient of kinetic friction between the blocks and the plane is μ_k.

Given:

$$M_A = 3\ \text{kg} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

$$M_B = 2\ \text{kg} \qquad e = 1$$

$$v_{A1} = 2\ \frac{\text{m}}{\text{s}} \qquad \mu_k = 0.3$$

Solution:

Guesess

$v_{A2} = 3\ \frac{m}{s}$ $\quad v_{B2} = 5\ \frac{m}{s}$ $\quad d_2 = 1\ m$

Given

$M_A v_{A1} = M_A v_{A2} + M_B v_{B2}$ $\quad e\, v_{A1} = v_{B2} - v_{A2}$ $\quad d_2 = \frac{v_{B2}^2 - v_{A2}^2}{2g\mu_k}$

$\begin{pmatrix} v_{A2} \\ v_{B2} \\ d_2 \end{pmatrix} = \mathrm{Find}(v_{A2}, v_{B2}, d_2)$ $\quad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} 0.40 \\ 2.40 \end{pmatrix} \frac{m}{s}$ $\quad d_2 = 0.951\ m$

---

*__Problem 15-56__

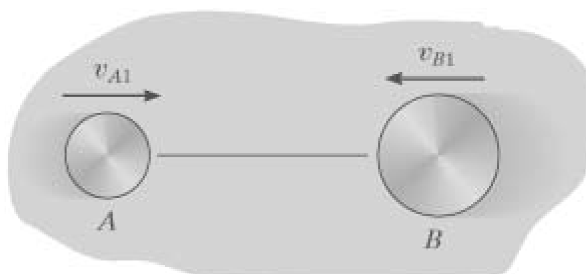
Disks A and B have masses M_A and M_B respectively. If they have the velocities shown, determine their velocities just after direct central impact.

Given:

$M_A = 2\ kg$ $\quad v_{A1} = 2\ \frac{m}{s}$

$M_B = 4\ kg$ $\quad v_{B1} = 5\ \frac{m}{s}$

$e = 0.4$



Solution: Guesses $v_{A2} = 1\ \frac{m}{s}$ $\quad v_{B2} = 1\ \frac{m}{s}$

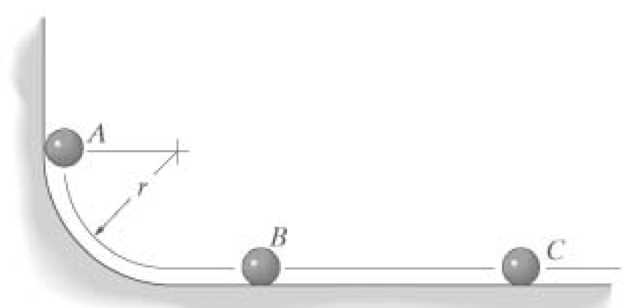
Given $M_A v_{A1} - M_B v_{B1} = M_A v_{A2} + M_B v_{B2}$

$e(v_{A1} + v_{B1}) = v_{B2} - v_{A2}$

$\begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \mathrm{Find}(v_{A2}, v_{B2})$ $\quad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -4.53 \\ -1.73 \end{pmatrix} \frac{m}{s}$

Problem 15-57

The three balls each have weight W and have a coefficient of restitution e. If ball A is released from rest and strikes ball B and then ball B strikes ball C, determine the velocity of each ball after the second collision has occurred. The balls slide without friction.



Given:

$W = 0.5\ lb$ $\quad r = 3\ ft$

$e = 0.85 \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$v_A = \sqrt{2gr}$

Guesses

$v_{A'} = 1\ \frac{\text{ft}}{\text{s}} \qquad v_{B'} = 1\ \frac{\text{ft}}{\text{s}} \qquad v_{B''} = 1\ \frac{\text{ft}}{\text{s}} \qquad v_{C''} = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$\left(\frac{W}{g}\right)v_A = \left(\frac{W}{g}\right)v_{A'} + \left(\frac{W}{g}\right)v_{B'} \qquad e\,v_A = v_{B'} - v_{A'}$$

$$\left(\frac{W}{g}\right)v_{B'} = \left(\frac{W}{g}\right)v_{B''} + \left(\frac{W}{g}\right)v_{C''} \qquad e\,v_{B'} = v_{C''} - v_{B''}$$

$$\begin{pmatrix} v_{A'} \\ v_{B'} \\ v_{B''} \\ v_{C''} \end{pmatrix} = \text{Find}(v_{A'}, v_{B'}, v_{B''}, v_{C''}) \qquad \begin{pmatrix} v_{A'} \\ v_{B''} \\ v_{C''} \end{pmatrix} = \begin{pmatrix} 1.04 \\ 0.96 \\ 11.89 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

Problem 15-58

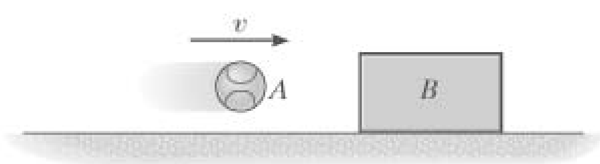
The ball A of weight W_A is thrown so that when it strikes the block B of weight W_B it is traveling horizontally at speed v. If the coefficient of restitution between A and B is e, and the coefficient of kinetic friction between the plane and the block is μ_k, determine the time before block B stops sliding.

Given:

$W_A = 1\ \text{lb} \qquad \mu_k = 0.4$

$W_B = 10\ \text{lb} \qquad v = 20\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2} \qquad e = 0.6$



Solution:

Guesses $\quad v_{A2} = 1\ \frac{\text{ft}}{\text{s}} \qquad v_{B2} = 1\ \frac{\text{ft}}{\text{s}} \qquad t = 1\ \text{s}$

Given $\quad \left(\frac{W_A}{g}\right)v = \left(\frac{W_A}{g}\right)v_{A2} + \left(\frac{W_B}{g}\right)v_{B2} \qquad e\,v = v_{B2} - v_{A2}$

$$\left(\frac{W_B}{g}\right)v_{B2} - \mu_k W_B t = 0$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ t \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, t) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -9.09 \\ 2.91 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad t = 0.23\ \text{s}$$

Problem 15-59

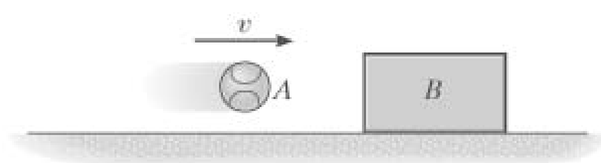
The ball A of weight W_A is thrown so that when it strikes the block B of weight W_B it is traveling horizontally at speed v. If the coefficient of restitution between A and B is e, and the coefficient of kinetic friction between the plane and the block is μ_k, determine the distance block B slides before stopping.

Given:

$W_A = 1\ \text{lb}$ $\quad \mu_k = 0.4$

$W_B = 10\ \text{lb}$ $\quad v = 20\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $\quad e = 0.6$



Solution:

Guesses $\quad v_{A2} = 1\ \frac{\text{ft}}{\text{s}} \quad v_{B2} = 1\ \frac{\text{ft}}{\text{s}} \quad d = 1\ \text{ft}$

Given $\quad \left(\frac{W_A}{g}\right)v = \left(\frac{W_A}{g}\right)v_{A2} + \left(\frac{W_B}{g}\right)v_{B2} \qquad e\,v = v_{B2} - v_{A2}$

$$\frac{1}{2}\left(\frac{W_B}{g}\right)v_{B2}^{\ 2} - \mu_k W_B d = 0$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ d \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, d) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -9.09 \\ 2.91 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad d = 0.33\ \text{ft}$$

Problem 15-60

The ball A of weight W_A is thrown so that when it strikes the block B of weight W_B it is traveling horizontally at speed v. Determine the average normal force exerted between A and B if the impact occurs in time Δt. The coefficient of restitution between A and B is e.

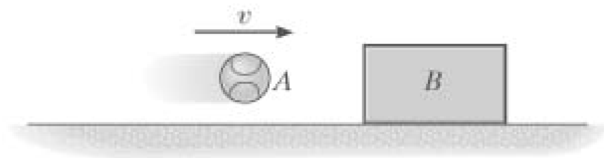
Given:

$W_A = 1\ \text{lb}$ $\quad \mu_k = 0.4$

$W_B = 10 \text{ lb} \quad v = 20 \frac{\text{ft}}{\text{s}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2} \quad e = 0.6$

$\Delta t = 0.02 \text{ s}$



Solution:

Guesses $\quad v_{A2} = 1 \frac{\text{ft}}{\text{s}} \quad v_{B2} = 1 \frac{\text{ft}}{\text{s}} \quad F_N = 1 \text{ lb}$

Given $\quad \left(\frac{W_A}{g}\right)v = \left(\frac{W_A}{g}\right)v_{A2} + \left(\frac{W_B}{g}\right)v_{B2} \qquad e\,v = v_{B2} - v_{A2}$

$$\left(\frac{W_A}{g}\right)v - F_N \Delta t = \left(\frac{W_A}{g}\right)v_{A2}$$

$\begin{pmatrix} v_{A2} \\ v_{B2} \\ F_N \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, F_N) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -9.09 \\ 2.91 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad F_N = 45.2 \text{ lb}$

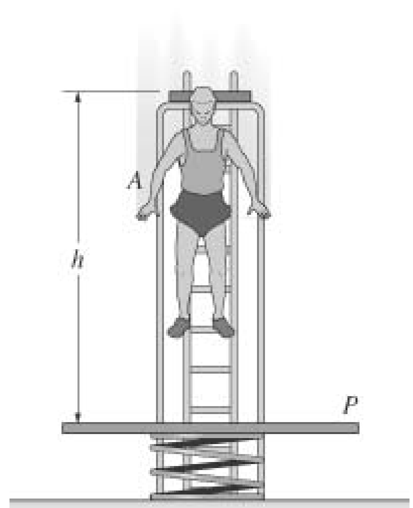
Problem 15-61

The man A has weight W_A and jumps from rest from a height h onto a platform P that has weight W_P. The platform is mounted on a spring, which has stiffness k. Determine (a) the velocities of A and P just after impact and (b) the maximum compression imparted to the spring by the impact. Assume the coefficient of restitution between the man and the platform is e, and the man holds himself rigid during the motion.

Given:

$W_A = 175 \text{ lb} \quad W_P = 60 \text{ lb} \quad k = 200 \frac{\text{lb}}{\text{ft}}$

$h = 8 \text{ ft} \quad e = 0.6 \quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

$m_A = \frac{W_A}{g} \qquad m_P = \frac{W_P}{g} \qquad \delta_{st} = \frac{W_P}{k}$

Guesses $\quad v_{A1} = 1 \frac{\text{ft}}{\text{s}} \quad v_{A2} = 1 \frac{\text{ft}}{\text{s}}$

$v_{P2} = -1 \frac{\text{ft}}{\text{s}} \quad \delta = 21 \text{ ft}$

Given

energy $W_A h = \frac{1}{2} m_A v_{A1}^2$

momentum $-m_A v_{A1} = m_A v_{A2} + m_P v_{P2}$

restitution $e v_{A1} = v_{A2} - v_{P2}$

energy $\frac{1}{2} m_P v_{P2}^2 + \frac{1}{2} k \delta_{st}^2 = \frac{1}{2} k \left(\delta + \delta_{st}\right)^2 - W_P \delta$

$$\begin{pmatrix} v_{A1} \\ v_{A2} \\ v_{P2} \\ \delta \end{pmatrix} = \text{Find}\left(v_{A1}, v_{A2}, v_{P2}, \delta\right) \qquad \begin{pmatrix} v_{A2} \\ v_{P2} \end{pmatrix} = \begin{pmatrix} -13.43 \\ -27.04 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \delta = 2.61\ \text{ft}$$

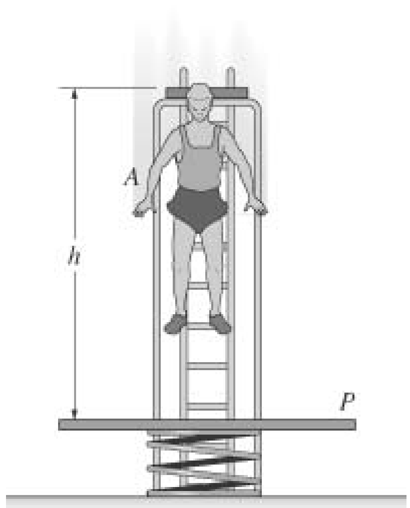
Problem 15-62

The man A has weight W_A and jumps from rest onto a platform P that has weight W_P. The platform is mounted on a spring, which has stiffness k. If the coefficient of restitution between the man and the platform is e, and the man holds himself rigid during the motion, determine the required height h of the jump if the maximum compression of the spring becomes δ.

Given:

$W_A = 100\ \text{lb}$ $\quad W_P = 60\ \text{lb}$ $\quad \delta = 2\ \text{ft}$

$k = 200\ \frac{\text{lb}}{\text{ft}}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $\quad e = 0.6$



Solution:

$m_A = \frac{W_A}{g}$ $\quad m_P = \frac{W_P}{g}$ $\quad \delta_{st} = \frac{W_P}{k}$

Guesses $\quad v_{A1} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_{A2} = 1\ \frac{\text{ft}}{\text{s}}$

$v_{P2} = -1\ \frac{\text{ft}}{\text{s}}$ $\quad h = 21\ \text{ft}$

Given

energy $W_A h = \frac{1}{2} m_A v_{A1}^2$

momentum $-m_A v_{A1} = m_A v_{A2} + m_P v_{P2}$

restitution $e v_{A1} = v_{A2} - v_{P2}$

energy $\frac{1}{2}m_P v_{P2}^2 + \frac{1}{2}k\delta_{st}^2 = \frac{1}{2}k\delta^2 - W_P(\delta - \delta_{st})$

$$\begin{pmatrix} v_{A1} \\ v_{A2} \\ v_{P2} \\ h \end{pmatrix} = \text{Find}(v_{A1}, v_{A2}, v_{P2}, h) \qquad \begin{pmatrix} v_{A2} \\ v_{P2} \end{pmatrix} = \begin{pmatrix} -7.04 \\ -17.61 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad h = 4.82\ \text{ft}$$

Problem 15-63

The collar B of weight W_B is at rest, and when it is in the position shown the spring is unstretched. If another collar A of weight W_A strikes it so that B slides a distance b on the smooth rod before momentarily stopping, determine the velocity of A just after impact, and the average force exerted between A and B during the impact if the impact occurs in time Δt. The coefficient of restitution between A and B is e.

Units Used: $\text{kip} = 10^3\ \text{lb}$

Given:

$W_B = 10\ \text{lb}$

$W_A = 1\ \text{lb}$

$k = 20\ \frac{\text{lb}}{\text{ft}}$

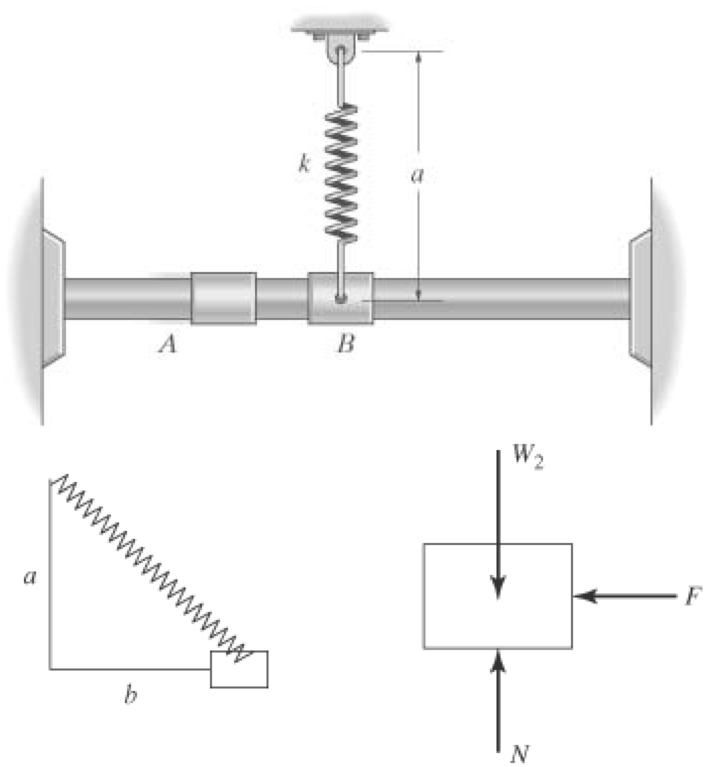
$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$a = 3\ \text{ft}$

$b = 4\ \text{ft}$

$\Delta t = 0.002\ \text{s}$

$e = 0.5$



Solution:

Guesses $v_{A1} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_{A2} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_{B2} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad F = 1\ \text{lb}$

Given $\left(\frac{W_A}{g}\right)v_{A1} = \left(\frac{W_A}{g}\right)v_{A2} + \left(\frac{W_B}{g}\right)v_{B2} \qquad e\,v_{A1} = v_{B2} - v_{A2}$

$$\left(\frac{W_A}{g}\right)v_{A1} - F\Delta t = \left(\frac{W_A}{g}\right)v_{A2} \qquad \frac{1}{2}\left(\frac{W_B}{g}\right)v_{B2}^2 = \frac{1}{2}k\left(\sqrt{a^2+b^2} - a\right)^2$$

$$\begin{pmatrix} v_{A1} \\ v_{A2} \\ v_{B2} \\ F \end{pmatrix} = \text{Find}(v_{A1}, v_{A2}, v_{B2}, F) \qquad v_{A2} = -42.80\,\frac{\text{ft}}{\text{s}} \qquad F = 2.49\,\text{kip}$$

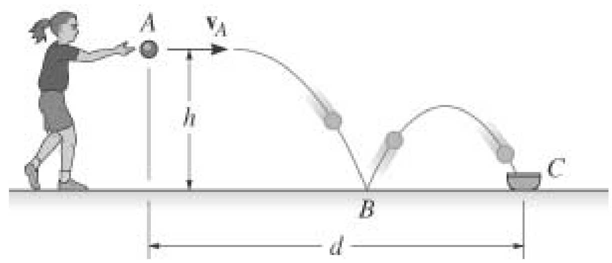
***Problem 15-64**

If the girl throws the ball with horizontal velocity v_A, determine the distance d so that the ball bounces once on the smooth surface and then lands in the cup at C.

Given:

$v_A = 8\,\frac{\text{ft}}{\text{s}}$ $\quad g = 32.2\,\frac{\text{ft}}{\text{s}^2}$

$e = 0.8$ $\quad h = 3\text{ ft}$



Solution:

$t_B = \sqrt{2\frac{h}{g}}$ $\qquad t_B = 0.43\text{ s}$

$v_{By1} = g\,t_B$ $\qquad v_{By1} = 13.90\,\frac{\text{ft}}{\text{s}}$

$v_{By2} = e\,v_{By1}$ $\qquad v_{By2} = 11.12\,\frac{\text{ft}}{\text{s}}$

$t_C = \frac{2v_{By2}}{g}$ $\qquad t_C = 0.69\text{ s}$

$d = v_A(t_B + t_C)$ $\qquad d = 8.98\,\text{ft}$

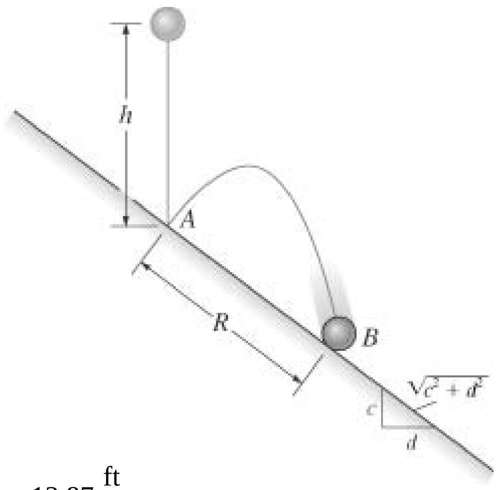
Problem 15-65

The ball is dropped from rest and falls a distance h before striking the smooth plane at A. If the coefficient of restitution is e, determine the distance R to where it again strikes the plane at B.

Given:

$h = 4\text{ ft}$ $\quad c = 3$

$e = 0.8 \qquad d = 4 \qquad g = 32.2\ \frac{ft}{s^2}$



Solution:

$\theta = \operatorname{atan}\left(\frac{c}{d}\right) \qquad \theta = 36.87\ deg$

$v_{A1} = \sqrt{2gh} \qquad v_{A1} = 16.05\ \frac{ft}{s}$

$v_{A1n} = v_{A1}\cos(\theta) \qquad v_{A1t} = v_{A1}\sin(\theta)$

$v_{A2n} = e\,v_{A1n} \qquad v_{A2t} = v_{A1t}$

$v_{A2x} = v_{A2n}\sin(\theta) + v_{A2t}\cos(\theta) \qquad v_{A2x} = 13.87\ \frac{ft}{s}$

$v_{A2y} = v_{A2n}\cos(\theta) - v_{A2t}\sin(\theta) \qquad v_{A2y} = 2.44\ \frac{ft}{s}$

Guesses $\quad t = 1\ s \qquad R = 10\ ft$

Given $\quad R\cos(\theta) = v_{A2x}t \qquad -R\sin(\theta) = \left(\frac{-g}{2}\right)t^2 + v_{A2y}t$

$\begin{pmatrix} R \\ t \end{pmatrix} = \operatorname{Find}(R, t) \qquad t = 0.80\ s \qquad R = 13.82\ ft$

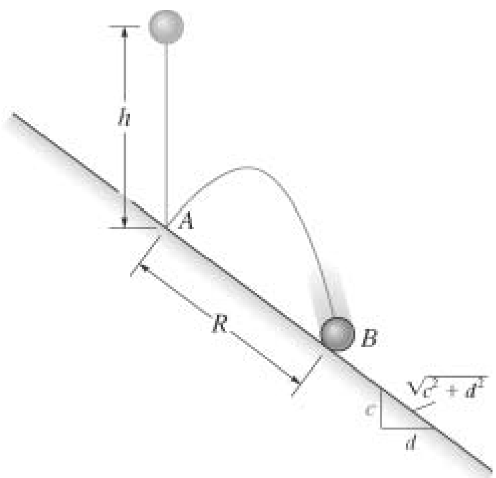
Problem 15-66

The ball is dropped from rest and falls a distance h before striking the smooth plane at A. If it rebounds and in time t again strikes the plane at B, determine the coefficient of restitution e between the ball and the plane. Also, what is the distance R?

Given:

$h = 4\ ft \qquad c = 3 \qquad g = 32.2\ \frac{ft}{s^2}$

$t = 0.5\ s \qquad d = 4$



Solution:

$\theta = \operatorname{atan}\left(\frac{c}{d}\right) \qquad \theta = 36.87\ deg$

$v_{A1} = \sqrt{2gh} \qquad v_{A1} = 16.05\ \frac{ft}{s}$

$v_{A1n} = v_{A1}\cos(\theta) \qquad v_{A1t} = v_{A1}\sin(\theta)$

$v_{A2t} = v_{A1t}$

Guesses $e = 0.8$ $R = 10\ \text{ft}$ $v_{A2n} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{A2x} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{A2y} = 1\ \frac{\text{ft}}{\text{s}}$

Given $v_{A2n} = e\,v_{A1n}$

$v_{A2x} = v_{A2n}\sin(\theta) + v_{A2t}\cos(\theta)$ $v_{A2y} = v_{A2n}\cos(\theta) - v_{A2t}\sin(\theta)$

$R\cos(\theta) = v_{A2x}t$ $-R\sin(\theta) = \frac{-g}{2}t^2 + v_{A2y}t$

$$\begin{pmatrix} e \\ R \\ v_{A2n} \\ v_{A2x} \\ v_{A2y} \end{pmatrix} = \text{Find}(e, R, v_{A2n}, v_{A2x}, v_{A2y})$$ $R = 7.23\,\text{ft}$ $e = 0.502$

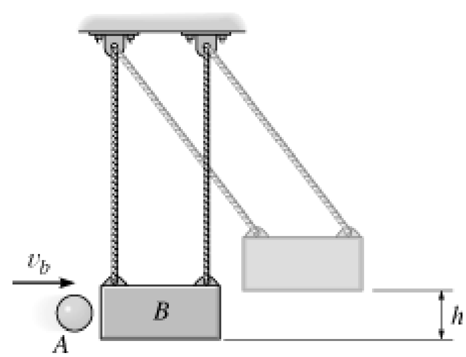
Problem 15-67

The ball of mass m_b is thrown at the suspended block of mass m_B with velocity v_b. If the coefficient of restitution between the ball and the block is e, determine the maximum height h to which the block will swing before it momentarily stops.

Given:

$m_b = 2\ \text{kg}$ $m_B = 20\ \text{kg}$ $e = 0.8$ $v_b = 4\ \frac{\text{m}}{\text{s}}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:



Guesses $v_A = 1\ \frac{\text{m}}{\text{s}}$ $v_B = 1\ \frac{\text{m}}{\text{s}}$ $h = 1\ \text{m}$

Given

momentum $m_b v_b = m_b v_A + m_B v_B$

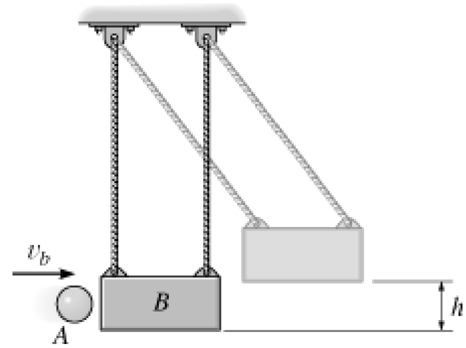
restitution $e\,v_b = v_B - v_A$

energy $\frac{1}{2}m_B v_B^2 = m_B g h$

$$\begin{pmatrix} v_A \\ v_B \\ h \end{pmatrix} = \text{Find}(v_A, v_B, h)$$ $$\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} -2.55 \\ 0.65 \end{pmatrix}\frac{\text{m}}{\text{s}}$$ $h = 21.84\,\text{mm}$

*__Problem 15-68__

The ball of mass m_b is thrown at the suspended block of mass m_B with a velocity of v_b. If the time of impact between the ball and the block is Δt, determine the average normal force exerted on the block during this time.



Given: $\text{kN} = 10^3\ \text{N}$

$m_b = 2\ \text{kg}$ $\quad v_b = 4\ \frac{\text{m}}{\text{s}}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$m_B = 20\ \text{kg}$ $\quad e = 0.8$ $\quad \Delta t = 0.005\ \text{s}$

Solution:

Guesses $\quad v_A = 1\ \frac{\text{m}}{\text{s}} \quad v_B = 1\ \frac{\text{m}}{\text{s}} \quad F = 1\ \text{N}$

Given

momentum $\quad m_b v_b = m_b v_A + m_B v_B$

restitution $\quad e v_b = v_B - v_A$

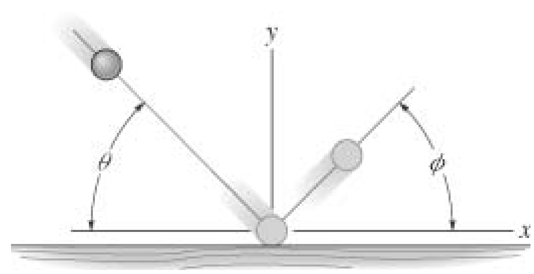
momentum B $\quad 0 + F\Delta t = m_B v_B$

$$\begin{pmatrix} v_A \\ v_B \\ F \end{pmatrix} = \text{Find}(v_A, v_B, F) \qquad \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} -2.55 \\ 0.65 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad F = 2.62\ \text{kN}$$

Problem 15-69

A ball is thrown onto a rough floor at an angle θ. If it rebounds at an angle ϕ and the coefficient of kinetic friction is μ, determine the coefficient of restitution e. Neglect the size of the ball. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.

Solution:



$$e v_1 \sin(\theta) = v_2 \sin(\phi)$$

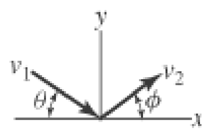
$$\frac{v_2}{v_1} = e\left(\frac{\sin(\theta)}{\sin(\phi)}\right) \qquad [1]$$

$$(\xrightarrow{+}) \qquad m v_1 \cos(\theta) - F_x \Delta t = m v_2 \cos(\phi)$$

$$F_x = \frac{m v_1 \cos(\theta) - m v_2 \cos(\phi)}{\Delta t} \qquad [2]$$

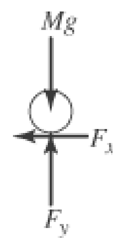
$$(+\downarrow) \qquad m v_1 \sin(\theta) - F_y \Delta t = -m v_2 \sin(\phi)$$

$$F_y = \frac{m v_1 \sin(\theta) + m v_2 \sin(\phi)}{\Delta t} \qquad [3]$$



Since $F_x = \mu F_y$, from Eqs [2] and [3]

$$\frac{m v_1 \cos(\theta) - m v_2 \cos(\phi)}{\Delta t} = \frac{\mu(m v_1 \sin(\theta) + m v_2 \sin(\phi))}{\Delta t}$$



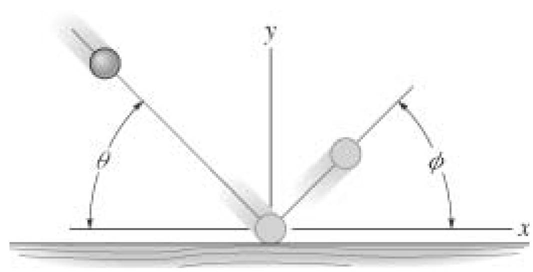
$$\frac{v_2}{v_1} = \frac{\cos(\theta) - \mu \sin(\theta)}{\mu \sin(\phi) + \cos(\phi)} \qquad [4]$$

Substituting Eq. [4] into [1] yields:

$$e = \frac{\sin(\phi)}{\sin(\theta)}\left(\frac{\cos(\theta) - \mu \sin(\theta)}{\mu \sin(\phi) + \cos(\phi)}\right)$$

Problem 15-70

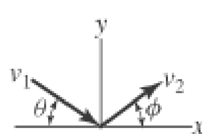
A ball is thrown onto a rough floor at an angle of θ. If it rebounds at the same angle ϕ, determine the coefficient of kinetic friction between the floor and the ball. The coefficient of restitution is e. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.



Solution:

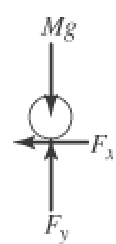
$$e v_1 \sin(\theta) = v_2 \sin(\phi)$$

$$\frac{v_2}{v_1} = e\left(\frac{\sin(\theta)}{\sin(\phi)}\right) \qquad [1]$$



$(\xrightarrow{+})$ $\quad m v_1 \cos(\theta) - F_x \Delta t = m v_2 \cos(\phi)$

$$F_x = \frac{m v_1 \cos(\theta) - m v_2 \cos(\phi)}{\Delta t} \qquad [2]$$



$(+\downarrow)$ $\quad m v_1 \sin(\theta) - F_y \Delta t = -m v_2 \sin(\phi)$

$$F_y = \frac{m v_1 \sin(\theta) + m v_2 \sin(\phi)}{\Delta t} \qquad [3]$$

Since $F_x = \mu F_y$, from Eqs [2] and [3]

$$\frac{m v_1 \cos(\theta) - m v_2 \cos(\phi)}{\Delta t} = \frac{\mu(m v_1 \sin(\theta) + m v_2 \sin(\phi))}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos(\theta) - \mu \sin(\theta)}{\mu \sin(\phi) + \cos(\phi)} \qquad [4]$$

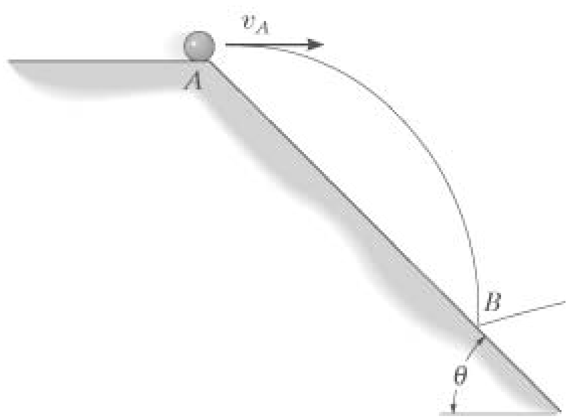
Substituting Eq. [4] into [1] yields: $e = \frac{\sin(\phi)}{\sin(\theta)}\left(\frac{\cos(\theta) - \mu \sin(\theta)}{\mu \sin(\phi) + \cos(\phi)}\right)$

Given $\theta = 45$ deg $\phi = 45$ deg $e = 0.6$ Guess $\mu = 0.2$

Given $e = \frac{\sin(\phi)}{\sin(\theta)}\left(\frac{\cos(\theta) - \mu \sin(\theta)}{\mu \sin(\phi) + \cos(\phi)}\right)$ $\mu = \text{Find}(\mu)$ $\mu = 0.25$

Problem 15-71

The ball bearing of weight W travels over the edge A with velocity v_A. Determine the speed at which it rebounds from the smooth inclined plane at B. Take $e = 0.8$.



Given:

$W = 0.2$ lb $\theta = 45$ deg

$v_A = 3\ \frac{\text{ft}}{\text{s}}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $e = 0.8$

Solution:

Guesses $v_{B1x} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{B1y} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{B2n} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{B2t} = 1\ \frac{\text{ft}}{\text{s}}$

$t = 1$ s $R = 1$ ft

Given $v_{B1x} = v_A$ $v_A t = R\cos(\theta)$

$\frac{-1}{2} g t^2 = -R\sin(\theta)$ $v_{B1y} = -g t$

$v_{B1x}\cos(\theta) - v_{B1y}\sin(\theta) = v_{B2t}$

$$e\left(-v_{B1y}\cos(\theta) - v_{B1x}\sin(\theta)\right) = v_{B2n}$$

$$\begin{pmatrix} v_{B1x} \\ v_{B1y} \\ v_{B2n} \\ v_{B2t} \\ t \\ R \end{pmatrix} = \text{Find}\left(v_{B1x}, v_{B1y}, v_{B2n}, v_{B2t}, t, R\right) \qquad \begin{pmatrix} v_{B1x} \\ v_{B1y} \end{pmatrix} = \begin{pmatrix} 3.00 \\ -6.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$t = 0.19\ \text{s}$

$R = 0.79\,\text{ft}$

$$\begin{pmatrix} v_{B2n} \\ v_{B2t} \end{pmatrix} = \begin{pmatrix} 1.70 \\ 6.36 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \left|\begin{pmatrix} v_{B2n} \\ v_{B2t} \end{pmatrix}\right| = 6.59\,\frac{\text{ft}}{\text{s}}$$

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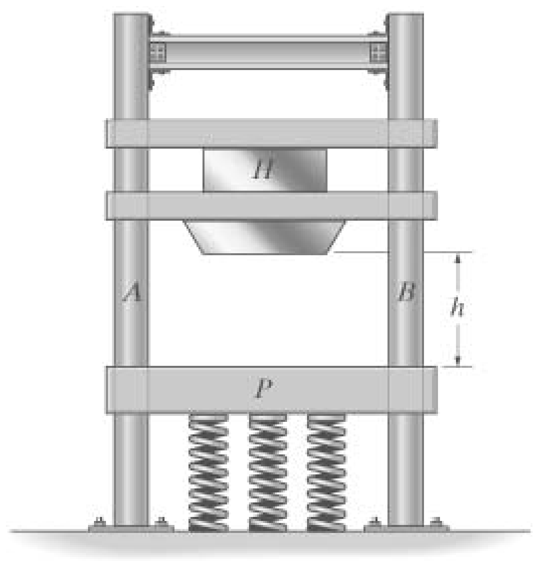
*__Problem 15-72__

The drop hammer H has a weight W_H and falls from rest h onto a forged anvil plate P that has a weight W_P. The plate is mounted on a set of springs that have a combined stiffness k_T. Determine (a) the velocity of P and H just after collision and (b) the maximum compression in the springs caused by the impact. The coefficient of restitution between the hammer and the plate is e. Neglect friction along the vertical guideposts A and B.



Given:

$W_H = 900\ \text{lb} \qquad k_T = 500\ \frac{\text{lb}}{\text{ft}}$

$W_P = 500\ \text{lb} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$h = 3\ \text{ft} \qquad e = 0.6$

Solution:

$\delta_{st} = \frac{W_P}{k_T} \qquad v_{H1} = \sqrt{2gh}$

Guesses

$v_{H2} = 1\ \frac{\text{ft}}{\text{s}} \qquad v_{P2} = 1\ \frac{\text{ft}}{\text{s}} \qquad \delta = 2\ \text{ft}$

Given $\quad \left(\frac{W_H}{g}\right)v_{H1} = \left(\frac{W_H}{g}\right)v_{H2} + \left(\frac{W_P}{g}\right)v_{P2}$

$e\,v_{H1} = v_{P2} - v_{H2}$

$$\frac{1}{2}k_T\delta_{st}^{\;2} + \frac{1}{2}\left(\frac{W_P}{g}\right)v_{P2}^{\;2} = \frac{1}{2}k_T\delta^2 - W_P(\delta - \delta_{st})$$

$$\begin{pmatrix} v_{H2} \\ v_{P2} \\ \delta \end{pmatrix} = \text{Find}(v_{H2}, v_{P2}, \delta) \qquad \begin{pmatrix} v_{H2} \\ v_{P2} \end{pmatrix} = \begin{pmatrix} 5.96 \\ 14.30 \end{pmatrix}\frac{\text{ft}}{\text{s}} \qquad \delta = 3.52\,\text{ft}$$

Problem 15-73

It was observed that a tennis ball when served horizontally a distance h above the ground strikes the smooth ground at B a distance d away. Determine the initial velocity v_A of the ball and the velocity v_B (and θ) of the ball just after it strikes the court at B. The coefficient of restitution is e.

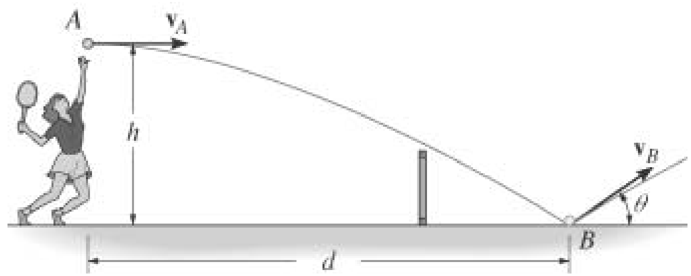
Given:

$h = 7.5\ \text{ft}$

$d = 20\ \text{ft}$

$e = 0.7$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $\quad v_A = 1\ \dfrac{\text{ft}}{\text{s}} \qquad v_{B2} = 1\ \dfrac{\text{ft}}{\text{s}}$

$v_{By1} = 1\ \dfrac{\text{ft}}{\text{s}} \qquad \theta = 10\ \text{deg} \qquad t = 1\ \text{s}$

Given $\quad h = \dfrac{1}{2}g\,t^2 \qquad d = v_A\,t$

$e\,v_{By1} = v_{B2}\sin(\theta) \qquad v_{By1} = g\,t$

$v_A = v_{B2}\cos(\theta)$

$$\begin{pmatrix} v_A \\ t \\ v_{By1} \\ v_{B2} \\ \theta \end{pmatrix} = \text{Find}(v_A, t, v_{By1}, v_{B2}, \theta) \qquad v_A = 29.30\,\frac{\text{ft}}{\text{s}} \qquad v_{B2} = 33.10\,\frac{\text{ft}}{\text{s}} \qquad \theta = 27.70\,\text{deg}$$

Problem 15-74

The tennis ball is struck with a horizontal velocity v_A, strikes the smooth ground at B, and bounces upward at $\theta = \theta_1$. Determine the initial velocity v_A, the final velocity v_B, and the coefficient of restitution between the ball and the ground.

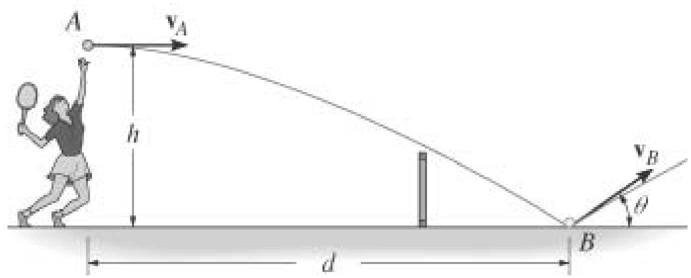
Given:

$h = 7.5\ \text{ft}$

$d = 20\ \text{ft}$

$\theta_1 = 30\ \text{deg}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution: $\theta = \theta_1$

Guesses $v_A = 1\ \dfrac{\text{ft}}{\text{s}}$ $t = 1\ \text{s}$ $v_{By1} = 1\ \dfrac{\text{ft}}{\text{s}}$ $v_{B2} = 1\ \dfrac{\text{ft}}{\text{s}}$ $e = 0.5$

Given $h = \dfrac{1}{2} g t^2$ $d = v_A t$ $v_{By1} = g t$

$e\, v_{By1} = v_{B2} \sin(\theta)$ $v_A = v_{B2} \cos(\theta)$

$$\begin{pmatrix} v_A \\ t \\ v_{By1} \\ v_{B2} \\ e \end{pmatrix} = \text{Find}(v_A, t, v_{By1}, v_{B2}, e) \qquad v_A = 29.30\,\frac{\text{ft}}{\text{s}} \qquad v_{B2} = 33.84\,\frac{\text{ft}}{\text{s}} \qquad e = 0.77$$

Problem 15-75

The ping-pong ball has mass M. If it is struck with the velocity shown, determine how high h it rises above the end of the smooth table after the rebound. The coefficient of restitution is e.

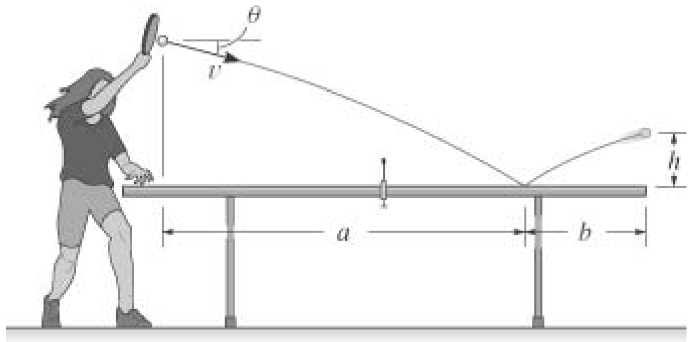
Given:

$M = 2\ \text{gm}$ $a = 2.25\ \text{m}$

$e = 0.8$ $b = 0.75\ \text{m}$

$\theta = 30\ \text{deg}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$v = 18\ \frac{\text{m}}{\text{s}}$



Solution: Guesses $v_{1x} = 1\ \frac{\text{m}}{\text{s}}$ $v_{1y} = 1\ \frac{\text{m}}{\text{s}}$ $v_{2x} = 1\ \frac{\text{m}}{\text{s}}$ $v_{2y} = 1\ \frac{\text{m}}{\text{s}}$

$t_1 = 1\ \text{s}$ $t_2 = 2\ \text{s}$ $h = 1\ \text{m}$

Given $v_{1x} = v\cos(\theta)$ $a = v\cos(\theta)t_1$ $v_{1y} = g\,t_1 + v\sin(\theta)$

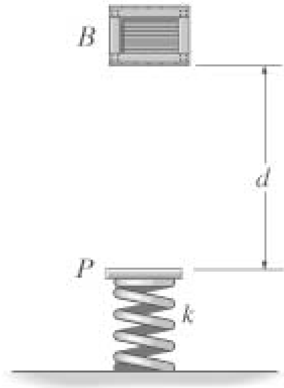
$v_{2x} = v_{1x}$ $e\,v_{1y} = v_{2y}$ $b = v_{2x}t_2$ $h = v_{2y}t_2 - \left(\frac{g}{2}\right)t_2^{\,2}$

$$\begin{pmatrix} v_{1x} \\ v_{1y} \\ v_{2x} \\ v_{2y} \\ t_1 \\ t_2 \\ h \end{pmatrix} = \text{Find}(v_{1x}, v_{1y}, v_{2x}, v_{2y}, t_1, t_2, h) \qquad \begin{pmatrix} v_{1x} \\ v_{1y} \\ v_{2x} \\ v_{2y} \end{pmatrix} = \begin{pmatrix} 15.59 \\ 10.42 \\ 15.59 \\ 8.33 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 0.14 \\ 0.05 \end{pmatrix} \text{s}$$

$h = 390\,\text{mm}$

***Problem 15-76**

The box B of weight W_B is dropped from rest a distance d from the top of the plate P of weight W_P, which is supported by the spring having a stiffness k. Determine the maximum compression imparted to the spring. Neglect the mass of the spring.



Given: $W_B = 5\ \text{lb}$ $W_P = 10\ \text{lb}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$k = 30\ \frac{\text{lb}}{\text{ft}}$ $d = 5\ \text{ft}$ $e = 0.6$

Solution:

$\delta_{st} = \frac{W_P}{k}$ $v_{B1} = \sqrt{2 g d}$

Guesses $v_{B2} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{P2} = 1\ \frac{\text{ft}}{\text{s}}$ $\delta = 2\ \text{ft}$

Given $\left(\frac{W_B}{g}\right) v_{B1} = \left(\frac{W_B}{g}\right) v_{B2} + \left(\frac{W_P}{g}\right) v_{P2}$ $e\, v_{B1} = v_{P2} - v_{B2}$

$$\frac{1}{2} k \delta_{st}^{\ 2} + \frac{1}{2}\left(\frac{W_P}{g}\right) v_{P2}^{\ 2} = \frac{1}{2} k \delta^2 - W_P(\delta - \delta_{st})$$

$$\begin{pmatrix} v_{B2} \\ v_{P2} \\ \delta \end{pmatrix} = \text{Find}(v_{B2}, v_{P2}, \delta) \qquad \begin{pmatrix} v_{B2} \\ v_{P2} \end{pmatrix} = \begin{pmatrix} -1.20 \\ 9.57 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \delta = 1.31\ \text{ft}$$

Problem 15-77

A pitching machine throws the ball of weight M towards the wall with an initial velocity v_A as shown. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall and (c) the distance d from the wall to where it strikes the ground at C.

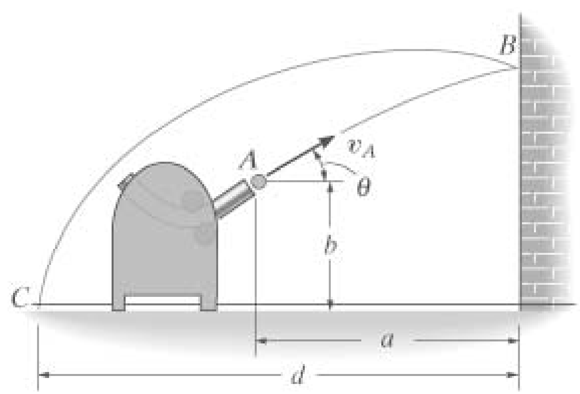
Given:

$M = 0.5\ \text{kg}$ $a = 3\ \text{m}$

$v_A = 10\ \frac{\text{m}}{\text{s}}$ $b = 1.5\ \text{m}$

$\theta = 30\ \text{deg}$ $e = 0.5$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: Guesses

$v_{Bx1} = 1\ \frac{\text{m}}{\text{s}}$ $v_{Bx2} = 1\ \frac{\text{m}}{\text{s}}$

$v_{By1} = 1\ \frac{\text{m}}{\text{s}}$ $v_{By2} = 1\ \frac{\text{m}}{\text{s}}$

$h = 1\ \text{m}$ $d = 1\ \text{m}$

$t_1 = 1\ \text{s}$ $t_2 = 1\ \text{s}$

Given

$v_A \cos(\theta) t_1 = a$ $b + v_A \sin(\theta) t_1 - \frac{1}{2} g t_1^{\,2} = h$

$v_{By2} = v_{By1}$ $v_A \sin(\theta) - g t_1 = v_{By1}$

$d = v_{Bx2} t_2$ $h + v_{By2} t_2 - \frac{1}{2} g t_2^{\,2} = 0$

$v_A \cos(\theta) = v_{Bx1}$ $e v_{Bx1} = v_{Bx2}$

$$\begin{pmatrix} v_{Bx1} \\ v_{By1} \\ v_{Bx2} \\ v_{By2} \\ h \\ t_1 \\ t_2 \\ d \end{pmatrix} = \text{Find}(v_{Bx1}, v_{By1}, v_{Bx2}, v_{By2}, h, t_1, t_2, d)$$

$$\left|\begin{pmatrix} v_{Bx1} \\ v_{By1} \end{pmatrix}\right| = 8.81\ \frac{\text{m}}{\text{s}}$$

$$\left|\begin{pmatrix} v_{Bx2} \\ v_{By2} \end{pmatrix}\right| = 4.62\ \frac{\text{m}}{\text{s}}$$

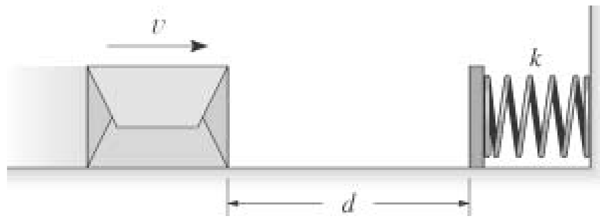
$d = 3.96\ \text{m}$

Problem 15-78

The box of weight W_b slides on the surface for which the coefficient of friction is μ_k. The box has velocity v when it is a distance d from the plate. If it strikes the plate, which has weight W_p and is held in position by an unstretched spring of stiffness k, determine the maximum compression imparted to the spring. The coefficient of restitution between the box and the plate is e. Assume that the plate slides smoothly.

Given:

$W_b = 20\ \text{lb}$ $\quad W_p = 10\ \text{lb}$



$\mu_k = 0.3$ $\quad k = 400\ \dfrac{\text{lb}}{\text{ft}}$

$v = 15\ \dfrac{\text{ft}}{\text{s}}$ $\quad e = 0.8$

$d = 2\ \text{ft}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $\quad v_{b1} = 1\ \dfrac{\text{ft}}{\text{s}}$ $\quad v_{b2} = 1\ \dfrac{\text{ft}}{\text{s}}$ $\quad v_{p2} = 1\ \dfrac{\text{ft}}{\text{s}}$ $\quad \delta = 1\ \text{ft}$

Given $\quad \dfrac{1}{2}\left(\dfrac{W_b}{g}\right)v^2 - \mu_k W_b d = \dfrac{1}{2}\left(\dfrac{W_b}{g}\right)v_{b1}^2$ $\quad \left(\dfrac{W_b}{g}\right)v_{b1} = \left(\dfrac{W_b}{g}\right)v_{b2} + \left(\dfrac{W_p}{g}\right)v_{p2}$

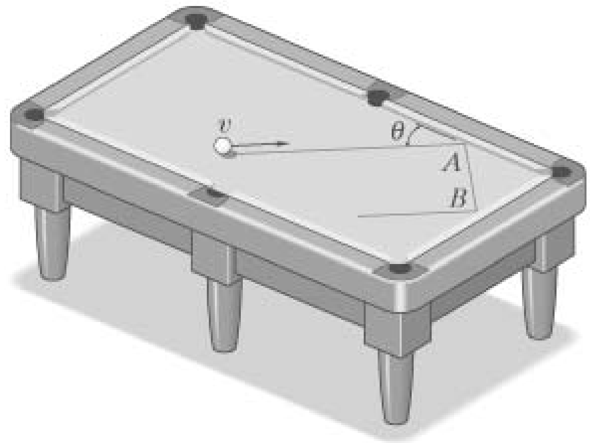
$e\,v_{b1} = v_{p2} - v_{b2}$ $\quad \dfrac{1}{2}\left(\dfrac{W_p}{g}\right)v_{p2}^2 = \dfrac{1}{2}k\delta^2$

$$\begin{pmatrix} v_{b1} \\ v_{b2} \\ v_{p2} \\ \delta \end{pmatrix} = \text{Find}(v_{b1}, v_{b2}, v_{p2}, \delta) \qquad \begin{pmatrix} v_{b1} \\ v_{b2} \\ v_{p2} \end{pmatrix} = \begin{pmatrix} 13.65 \\ 5.46 \\ 16.38 \end{pmatrix}\frac{\text{ft}}{\text{s}} \qquad \delta = 0.456\,\text{ft}$$

Problem 15-79

The billiard ball of mass M is moving with a speed v when it strikes the side of the pool table at A. If the coefficient of restitution between the ball and the side of the table is e, determine the speed of the ball just after striking the table twice, i.e., at A, then at B. Neglect the size of the ball.

Given:



$M = 200\ \text{gm}$

$v = 2.5\ \frac{\text{m}}{\text{s}}$

$\theta = 45\ \text{deg}$

$e = 0.6$

Solution:

Guesses

$v_2 = 1\ \frac{\text{m}}{\text{s}}$ $\quad \theta_2 = 1\ \text{deg}$ $\quad v_3 = 1\ \frac{\text{m}}{\text{s}}$ $\quad \theta_3 = 1\ \text{deg}$

Given $\quad e\,v\sin(\theta) = v_2\sin(\theta_2)$ $\qquad v\cos(\theta) = v_2\cos(\theta_2)$

$e\,v_2\cos(\theta_2) = v_3\sin(\theta_3)$ $\qquad v_2\sin(\theta_2) = v_3\cos(\theta_3)$

$$\begin{pmatrix} v_2 \\ v_3 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \text{Find}(v_2, v_3, \theta_2, \theta_3) \qquad \begin{pmatrix} v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 2.06 \\ 1.50 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \begin{pmatrix} \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} 31.0 \\ 45.0 \end{pmatrix} \text{deg}$$

$$v_3 = 1.500\ \frac{\text{m}}{\text{s}}$$

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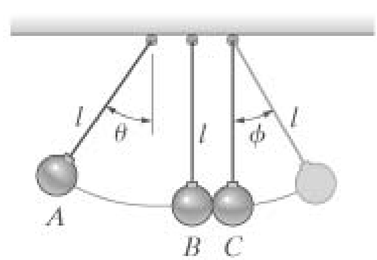
*__Problem 15-80__

The three balls each have the same mass m. If A is released from rest at θ, determine the angle ϕ to which C rises after collision. The coefficient of restitution between each ball is e.



Solution:

Energy

$$0 + l(1 - \cos(\theta))mg = \frac{1}{2} m v_A^{\,2}$$

$$v_A = \sqrt{2(1 - \cos(\theta))g\,l}$$

Collision of ball A with B:

$m v_A + 0 = m v'_A + m v'_B$ $\qquad e v_A = v'_B - v'_A$ $\qquad v'_B = \frac{1}{2}(1 + e)v'_B$

Collision of ball B with C:

$m v'_B + 0 = m v''_B + m v''_C \qquad e v'_B = v''_C - v''_B \qquad v''_C = \frac{1}{4}(1+e)^2 v_A$

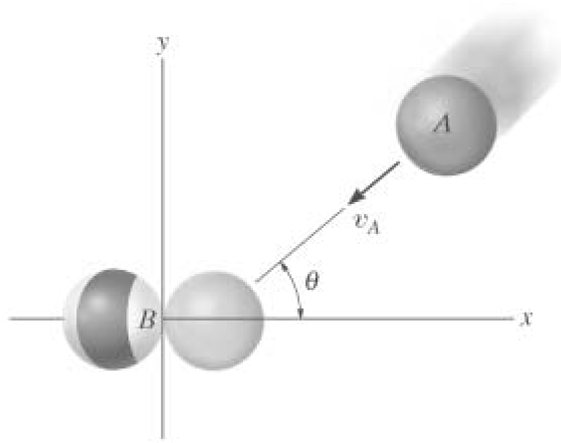
Energy

$\frac{1}{2} m v''_c{}^2 + 0 = 0 + l(1 - cos(\phi)) m g \qquad \frac{1}{2}\left(\frac{1}{16}\right)(1+e)^4(2)(1-\cos(\theta)) = (1-\cos(\phi))$

$\left(\frac{1+e}{2}\right)^4 (1-\cos(\theta)) = 1-\cos(\phi) \qquad \phi = \text{acos}\left[1 - \left(\frac{1+e}{2}\right)^4 (1-\cos(\theta))\right]$

Problem 15-81

Two smooth billiard balls *A* and *B* each have mass *M*. If *A* strikes *B* with a velocity v_A as shown, determine their final velocities just after collision. Ball *B* is originally at rest and the coefficient of restitution is *e*. Neglect the size of each ball.



Given:

$M = 0.2\ \text{kg}$

$\theta = 40\ \text{deg}$

$v_A = 1.5\ \frac{\text{m}}{\text{s}}$

$e = 0.85$

Solution: Guesses $v_{A2} = 1\ \frac{\text{m}}{\text{s}} \qquad v_{B2} = 1\ \frac{\text{m}}{\text{s}} \qquad \theta_2 = 20\ \text{deg}$

Given $-M v_A \cos(\theta) = M v_{B2} + M v_{A2} \cos(\theta_2)$

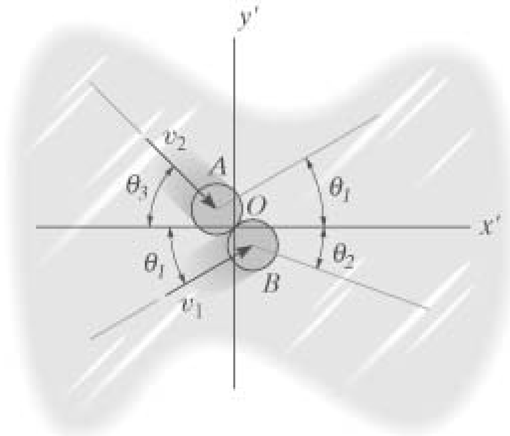
$e v_A \cos(\theta) = v_{A2} \cos(\theta_2) - v_{B2}$

$v_A \sin(\theta) = v_{A2} \sin(\theta_2)$

$\begin{pmatrix} v_{A2} \\ v_{B2} \\ \theta_2 \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, \theta_2) \qquad \theta_2 = 95.1\ \text{deg} \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} 0.968 \\ -1.063 \end{pmatrix} \frac{\text{m}}{\text{s}}$

Problem 15-82

The two hockey pucks *A* and *B* each have a mass *M*. If they collide at *O* and are deflected along the colored paths, determine their speeds just after impact. Assume that the icy surface over which they slide is smooth. *Hint:* Since the *y'* axis is *not* along the line of impact, apply the conservation of momentum along the *x'* and *y'* axes.



Given:

$M = 250\ \text{g}$ $\theta_1 = 30\ \text{deg}$

$v_1 = 40\ \frac{\text{m}}{\text{s}}$ $\theta_2 = 20\ \text{deg}$

$v_2 = 60\ \frac{\text{m}}{\text{s}}$ $\theta_3 = 45\ \text{deg}$

Solution:

Initial Guess:

$v_{A2} = 5\ \frac{\text{m}}{\text{s}}$ $v_{B2} = 4\ \frac{\text{m}}{\text{s}}$

Given

$$M v_2 \cos(\theta_3) + M v_1 \cos(\theta_1) = M v_{A2} \cos(\theta_1) + M v_{B2} \cos(\theta_2)$$

$$-M v_2 \sin(\theta_3) + M v_1 \sin(\theta_1) = M v_{A2} \sin(\theta_1) - M v_{B2} \sin(\theta_2)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} 6.90 \\ 75.66 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

Problem 15-83

Two smooth coins *A* and *B*, each having the same mass, slide on a smooth surface with the motion shown. Determine the speed of each coin after collision if they move off along the dashed paths. *Hint:* Since the line of impact has not been defined, apply the conservation of momentum along the *x* and *y* axes, respectively.

Given:

$v_{A1} = 0.5 \frac{\text{ft}}{\text{s}}$

$v_{B1} = 0.8 \frac{\text{ft}}{\text{s}}$

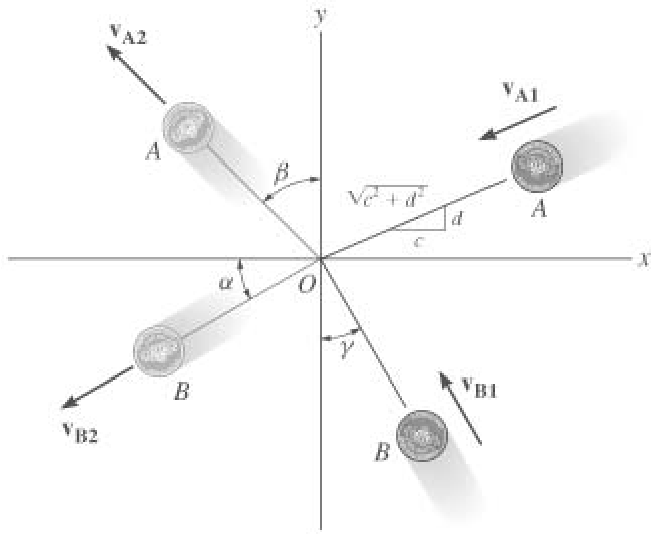
$\alpha = 30 \text{ deg}$

$\beta = 45 \text{ deg}$

$\gamma = 30 \text{ deg}$

$c = 4$

$d = 3$



Solution:

Guesses $v_{B2} = 0.25 \frac{\text{ft}}{\text{s}}$ $v_{A2} = 0.5 \frac{\text{ft}}{\text{s}}$

Given

$$-v_{A1}\left(\frac{c}{\sqrt{c^2+d^2}}\right) - v_{B1}\sin(\gamma) = -v_{A2}\sin(\beta) - v_{B2}\cos(\alpha)$$

$$-v_{A1}\left(\frac{d}{\sqrt{c^2+d^2}}\right) + v_{B1}\cos(\gamma) = v_{A2}\cos(\beta) - v_{B2}\sin(\alpha)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} 0.766 \\ 0.298 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

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*__Problem 15-84__

The two disks A and B have a mass M_A and M_B, respectively. If they collide with the initial velocities shown, determine their velocities just after impact. The coefficient of restitution is e.

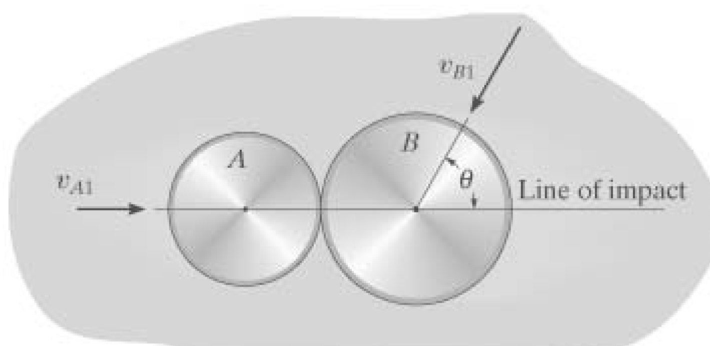
Given:

$M_A = 3 \text{ kg}$

$M_B = 5 \text{ kg}$

$\theta = 60 \text{ deg}$

$v_{B1} = 7 \frac{\text{m}}{\text{s}}$



$$v_{A1} = 6\ \frac{m}{s}$$

$$e = 0.65$$

Solution: Guesses $v_{A2} = 1\ \frac{m}{s}$ $v_{B2} = 1\ \frac{m}{s}$ $\theta_2 = 20\ \text{deg}$

Given

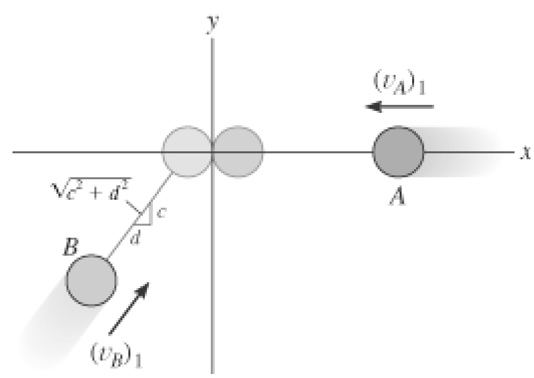
$$M_A v_{A1} - M_B v_{B1} \cos(\theta) = M_A v_{A2} + M_B v_{B2} \cos(\theta_2)$$

$$e(v_{A1} + v_{B1} \cos(\theta)) = v_{B2} \cos(\theta_2) - v_{A2} \qquad v_{B1} \sin(\theta) = v_{B2} \sin(\theta_2)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ \theta_2 \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, \theta_2) \qquad \theta_2 = 68.6\,\text{deg} \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -3.80 \\ 6.51 \end{pmatrix} \frac{m}{s}$$

Problem 15-85

Two smooth disks *A* and *B* each have mass *M*. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is *e*.



Given:

$M = 0.5\ \text{kg}$ $c = 4$ $v_{A1} = 6\ \frac{m}{s}$

$e = 0.75$ $d = 3$ $v_{B1} = 4\ \frac{m}{s}$

Solution:

Guesses

$$v_{A2} = 1\ \frac{m}{s} \qquad v_{B2} = 1\ \frac{m}{s} \qquad \theta_A = 10\ \text{deg} \qquad \theta_B = 10\ \text{deg}$$

Given $\quad v_{A1}(0) = v_{A2} \sin(\theta_A) \qquad v_{B1}\left(\frac{c}{\sqrt{c^2 + d^2}}\right) = v_{B2} \sin(\theta_B)$

$$M v_{B1}\left(\frac{d}{\sqrt{c^2 + d^2}}\right) - M v_{A1} = M v_{A2} \cos(\theta_A) - M v_{B2} \cos(\theta_B)$$

$$e\left[v_{A1} + v_{B1}\left(\frac{d}{\sqrt{c^2 + d^2}}\right)\right] = v_{A2} \cos(\theta_A) + v_{B2} \cos(\theta_B)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ \theta_A \\ \theta_B \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, \theta_A, \theta_B) \qquad \begin{pmatrix} \theta_A \\ \theta_B \end{pmatrix} = \begin{pmatrix} 0.00 \\ 32.88 \end{pmatrix} \text{deg} \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} 1.35 \\ 5.89 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

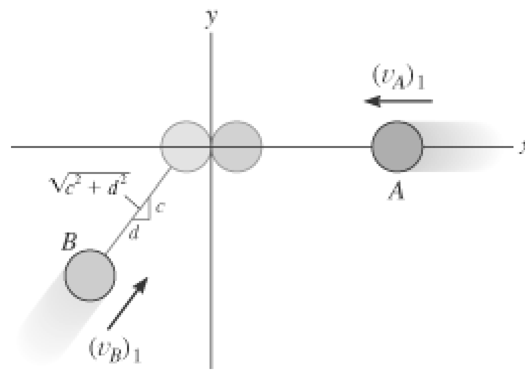
Problem 15-86

Two smooth disks A and B each have mass M. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision B travels along a line angle θ counterclockwise from the y axis.

Given:

$M = 0.5\ \text{kg} \qquad c = 4 \qquad v_{A1} = 6\ \frac{\text{m}}{\text{s}}$

$\theta_B = 30\ \text{deg} \qquad d = 3 \qquad v_{B1} = 4\ \frac{\text{m}}{\text{s}}$



Solution:

Guesses

$v_{A2} = 2\ \frac{\text{m}}{\text{s}} \qquad v_{B2} = 1\ \frac{\text{m}}{\text{s}} \qquad \theta_A = 10\ \text{deg} \qquad e = 0.5$

Given $\quad v_{A1} 0 = v_{A2} \sin(\theta_A) \qquad v_{B1}\left(\frac{c}{\sqrt{c^2 + d^2}}\right) = v_{B2} \cos(\theta_B)$

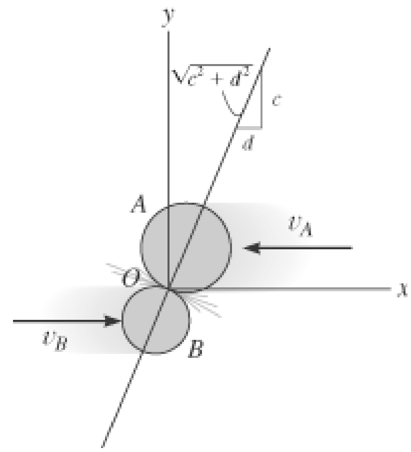
$$M v_{B1}\left(\frac{d}{\sqrt{c^2 + d^2}}\right) - M v_{A1} = M v_{A2} \cos(\theta_A) - M v_{B2} \sin(\theta_B)$$

$$e\left[v_{A1} + v_{B1}\left(\frac{d}{\sqrt{c^2 + d^2}}\right)\right] = v_{A2} \cos(\theta_A) + v_{B2} \sin(\theta_B)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ \theta_A \\ e \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, \theta_A, e) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -1.75 \\ 3.70 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad e = 0.0113$$

Problem 15-87

Two smooth disks A and B have the initial velocities shown just before they collide at O. If they have masses m_A and m_B, determine their speeds just after impact. The coefficient of restitution is e.



Given:

$v_A = 7\ \frac{m}{s}$ $\quad m_A = 8\ kg$ $\quad c = 12$ $\quad e = 0.5$

$v_B = 3\ \frac{m}{s}$ $\quad m_B = 6\ kg$ $\quad d = 5$

Solution: $\theta = \operatorname{atan}\left(\frac{d}{c}\right)$ $\quad \theta = 22.62\,deg$

Guesses $v_{A2t} = 1\ \frac{m}{s}$ $\quad v_{A2n} = 1\ \frac{m}{s}$

$v_{B2t} = 1\ \frac{m}{s}$ $\quad v_{B2n} = 1\ \frac{m}{s}$

Given $v_B\cos(\theta) = v_{B2t}$ $\qquad -v_A\cos(\theta) = v_{A2t}$

$$m_B v_B\sin(\theta) - m_A v_A\sin(\theta) = m_B v_{B2n} + m_A v_{A2n}$$

$$e(v_B + v_A)\sin(\theta) = v_{A2n} - v_{B2n}$$

$$\begin{pmatrix} v_{A2t} \\ v_{A2n} \\ v_{B2t} \\ v_{B2n} \end{pmatrix} = \operatorname{Find}(v_{A2t}, v_{A2n}, v_{B2t}, v_{B2n}) \qquad \begin{pmatrix} v_{A2t} \\ v_{A2n} \\ v_{B2t} \\ v_{B2n} \end{pmatrix} = \begin{pmatrix} -6.46 \\ -0.22 \\ 2.77 \\ -2.14 \end{pmatrix} \frac{m}{s}$$

$v_{A2} = \sqrt{v_{A2t}^2 + v_{A2n}^2}$ $\qquad v_{A2} = 6.47\ \frac{m}{s}$

$v_{B2} = \sqrt{v_{B2t}^2 + v_{B2n}^2}$ $\qquad v_{B2} = 3.50\ \frac{m}{s}$

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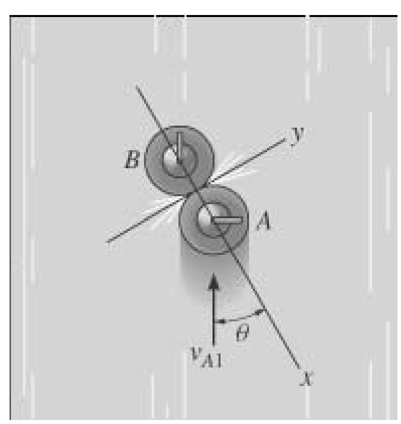
*__Problem 15-88__

The “stone” A used in the sport of curling slides over the ice track and strikes another “stone” B as shown. If each “stone” is smooth and has weight W, and the coefficient of restitution between the “stones” is e, determine their speeds just after collision. Initially A has velocity v_{A1} and B is at rest. Neglect friction.



Given: $W = 47\ lb$ $\quad v_{A1} = 8\ \frac{ft}{s}$

$e = 0.8$ $\quad \theta = 30\ deg$

Solution:

Guesses $v_{A2t} = 1\ \frac{ft}{s}$ $v_{A2n} = 1\ \frac{ft}{s}$

$v_{B2t} = 1\ \frac{ft}{s}$ $v_{B2n} = 1\ \frac{ft}{s}$

Given $v_{A1}\sin(\theta) = v_{A2t}$ $0 = v_{B2t}$

$v_{A1}\cos(\theta) = v_{A2n} + v_{B2n}$

$e\,v_{A1}\cos(\theta) = v_{B2n} - v_{A2n}$

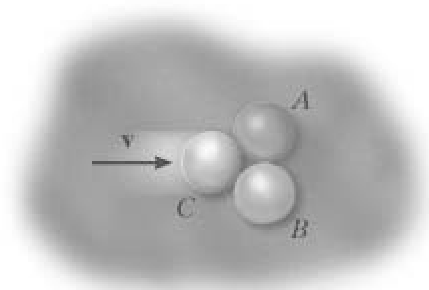
$$\begin{pmatrix} v_{A2t} \\ v_{A2n} \\ v_{B2t} \\ v_{B2n} \end{pmatrix} = \text{Find}(v_{A2t}, v_{A2n}, v_{B2t}, v_{B2n}) \qquad \begin{pmatrix} v_{A2t} \\ v_{A2n} \\ v_{B2t} \\ v_{B2n} \end{pmatrix} = \begin{pmatrix} 4.00 \\ 0.69 \\ 0.00 \\ 6.24 \end{pmatrix} \frac{ft}{s}$$

$v_{A2} = \sqrt{{v_{A2t}}^2 + {v_{A2n}}^2}$ $v_{A2} = 4.06\ \frac{ft}{s}$

$v_{B2} = \sqrt{{v_{B2t}}^2 + {v_{B2n}}^2}$ $v_{B2} = 6.24\ \frac{ft}{s}$

Problem 15-89

The two billiard balls *A* and *B* are originally in contact with one another when a third ball *C* strikes each of them at the same time as shown. If ball *C* remains at rest after the collision, determine the coefficient of restitution. All the balls have the same mass. Neglect the size of each ball.

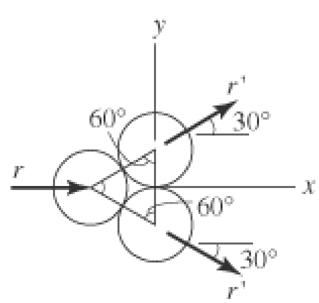


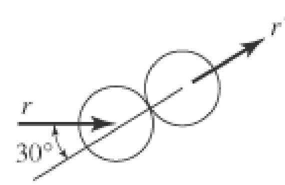
Solution:

Conservation of "*x*" momentum:

$$m\,v = 2m\,v'\cos(30\ \text{deg})$$

$$v = 2v'\cos(30\ \text{deg}) \qquad (1)$$





Coefficient of restitution:

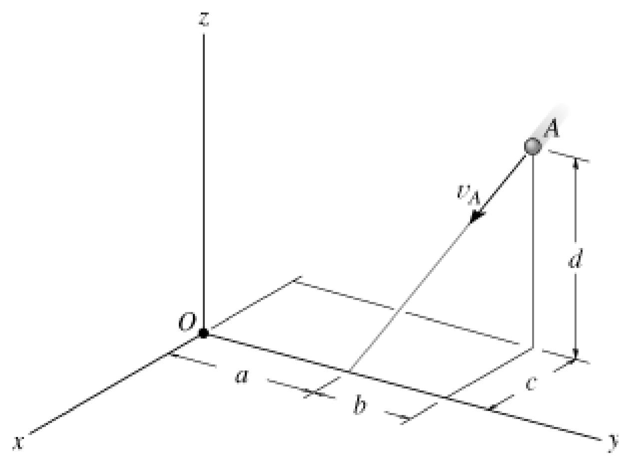
$$e = \frac{v'}{v\cos(30\ \text{deg})} \qquad (2)$$

Substituiting Eq. (1) into Eq. (2) yields:

$$e = \frac{v'}{2v'\cos(30\ \text{deg})^2} \qquad e = \frac{2}{3}$$

Problem 15-90

Determine the angular momentum of particle A of weight W about point O. Use a Cartesian vector solution.



Given:

$W = 2\ \text{lb}$ $\qquad a = 3\ \text{ft}$

$v_A = 12\ \frac{\text{ft}}{\text{s}}$ $\qquad b = 2\ \text{ft}$

$c = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $\qquad d = 4\ \text{ft}$

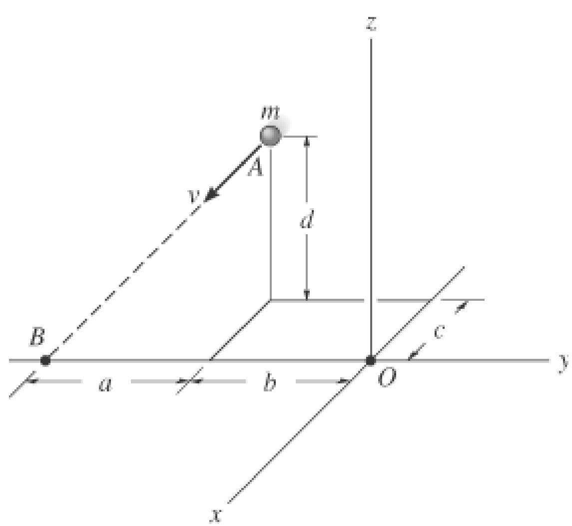
Solution:

$$\mathbf{r_{OA}} = \begin{pmatrix} -c \\ a+b \\ d \end{pmatrix} \qquad \mathbf{r_v} = \begin{pmatrix} c \\ -b \\ -d \end{pmatrix} \qquad \mathbf{v_{Av}} = v_A \frac{\mathbf{r_v}}{|\mathbf{r_v}|}$$

$$\mathbf{H_O} = \mathbf{r_{OA}} \times (W\mathbf{v_{Av}}) \qquad \mathbf{H_O} = \begin{pmatrix} -1.827 \\ 0.000 \\ -0.914 \end{pmatrix} \text{slug}\cdot\frac{\text{ft}^2}{\text{s}}$$

Problem 15-91

Determine the angular momentum $\mathbf{H_O}$ of the particle about point O.



Given:

$M = 1.5\ \text{kg}$

$v = 6\ \frac{\text{m}}{\text{s}}$

$a = 4\ \text{m}$

$b = 3\ \text{m}$

$c = 2\ \text{m}$

$d = 4\ \text{m}$

Solution:

$$\mathbf{r_{OA}} = \begin{pmatrix} -c \\ -b \\ d \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} c \\ -a \\ -d \end{pmatrix} \qquad \mathbf{v_A} = v\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|}$$

$$\mathbf{H_O} = \mathbf{r_{OA}} \times (M\mathbf{v_A}) \qquad \mathbf{H_O} = \begin{pmatrix} 42.0 \\ 0.0 \\ 21.0 \end{pmatrix} \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

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*__Problem 15-92__

Determine the angular momentum $\mathbf{H_O}$ of each of the particles about point O.

Given: $\theta = 30 \text{ deg}$ $\quad \phi = 60 \text{ deg}$

$m_A = 6 \text{ kg}$ $\quad c = 2 \text{ m}$

$m_B = 4 \text{ kg}$ $\quad d = 5 \text{ m}$

$m_C = 2 \text{ kg}$ $\quad e = 2 \text{ m}$

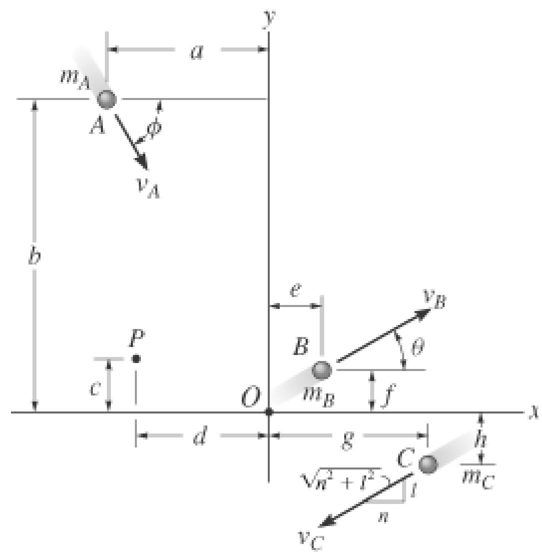
$v_A = 4 \frac{\text{m}}{\text{s}}$ $\quad f = 1.5 \text{ m}$

$v_B = 6 \frac{\text{m}}{\text{s}}$ $\quad g = 6 \text{ m}$

$v_C = 2.6 \frac{\text{m}}{\text{s}}$ $\quad h = 2 \text{ m}$

$a = 8 \text{ m}$ $\quad l = 5$

$b = 12 \text{ m}$ $\quad n = 12$



Solution:

$$\mathbf{H_{AO}} = a\,m_A v_A \sin(\phi) - b\,m_A v_A \cos(\phi) \qquad \mathbf{H_{AO}} = 22.3 \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\mathbf{H_{BO}} = -f\,m_B v_B \cos(\theta) + e\,m_B v_B \sin(\theta) \qquad \mathbf{H_{BO}} = -7.18 \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\mathbf{H_{CO}} = -h\,m_C\left(\frac{n}{\sqrt{l^2+n^2}}\right)v_C - g\,m_C\left(\frac{l}{\sqrt{l^2+n^2}}\right)v_C \qquad \mathbf{H_{CO}} = -21.60 \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

Problem 15-93

Determine the angular momentum $\mathbf{H_P}$ of each of the particles about point P.

Given: $\theta = 30\ \text{deg}$ $\phi = 60\ \text{deg}$ $a = 8\ \text{m}$ $f = 1.5\ \text{m}$

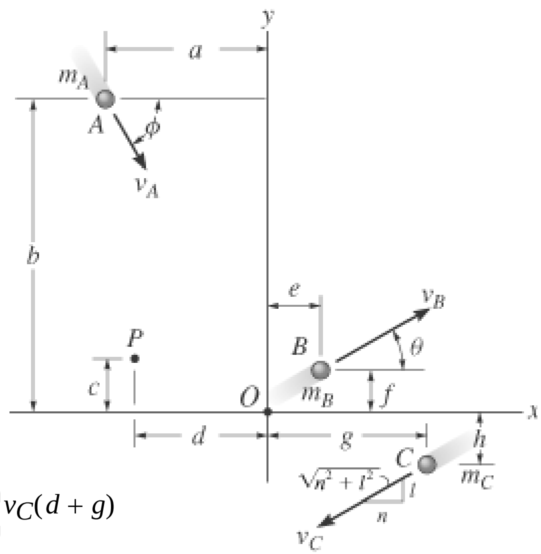
$m_A = 6\ \text{kg}$ $v_A = 4\ \frac{\text{m}}{\text{s}}$ $b = 12\ \text{m}$ $g = 6\ \text{m}$

$c = 2\ \text{m}$ $h = 2\ \text{m}$

$m_B = 4\ \text{kg}$ $v_B = 6\ \frac{\text{m}}{\text{s}}$ $d = 5\ \text{m}$ $l = 5$

$m_C = 2\ \text{kg}$ $v_C = 2.6\ \frac{\text{m}}{\text{s}}$ $e = 2\ \text{m}$ $n = 12$

Solution:



$$\mathbf{H_{AP}} = m_A v_A \sin(\phi)(a-d) - m_A v_A \cos(\phi)(b-c)$$

$$\mathbf{H_{AP}} = -57.6\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\mathbf{H_{BP}} = m_B v_B \cos(\theta)(c-f) + m_B v_B \sin(\theta)(d+e)$$

$$\mathbf{H_{BP}} = 94.4\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\mathbf{H_{CP}} = -m_C\left(\frac{n}{\sqrt{l^2+n^2}}\right)v_C(c+h) - m_C\left(\frac{l}{\sqrt{l^2+n^2}}\right)v_C(d+g)$$

$$\mathbf{H_{CP}} = -41.2\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

Problem 15-94

Determine the angular momentum $\mathbf{H_O}$ of the particle about point O.

Given:

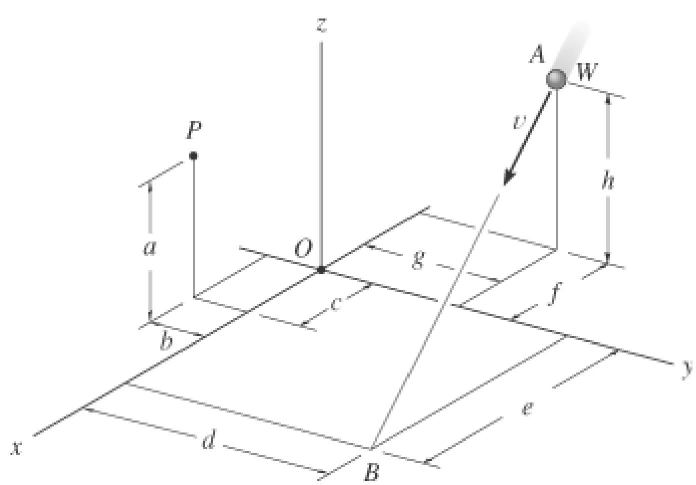
$W = 10\ \text{lb}$ $d = 9\ \text{ft}$

$v = 14\ \frac{\text{ft}}{\text{s}}$ $e = 8\ \text{ft}$

$a = 5\ \text{ft}$ $f = 4\ \text{ft}$

$b = 2\ \text{ft}$ $g = 5\ \text{ft}$

$c = 3\ \text{ft}$ $h = 6\ \text{ft}$



Solution:

$$\mathbf{r_{OA}} = \begin{pmatrix} -f \\ g \\ h \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} f+e \\ d-g \\ -h \end{pmatrix}$$

$$\mathbf{v_A} = v\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{H_O} = \mathbf{r_{OA}} \times (W\mathbf{v_A}) \qquad \mathbf{H_O} = \begin{pmatrix} -16.78 \\ 14.92 \\ -23.62 \end{pmatrix} \text{slug}\cdot\frac{\text{ft}^2}{\text{s}}$$

Problem 15-95

Determine the angular momentum $\mathbf{H_P}$ of the particle about point P.

Given:

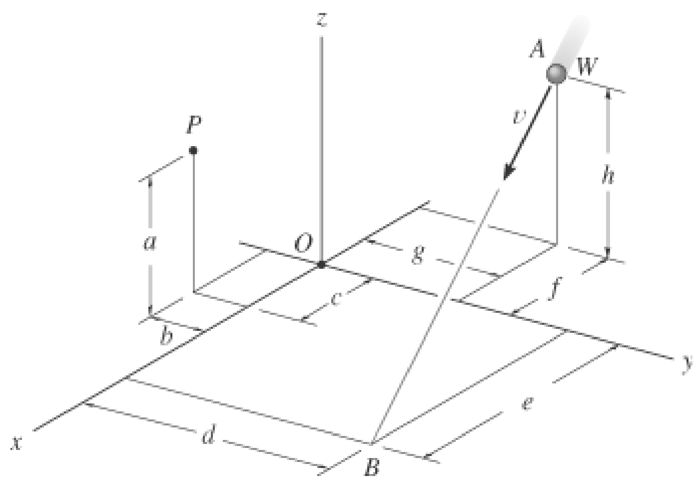
$W = 10$ lb $\qquad d = 9$ ft

$v = 14\ \frac{\text{ft}}{\text{s}} \qquad e = 8$ ft

$a = 5$ ft $\qquad f = 4$ ft

$b = 2$ ft $\qquad g = 5$ ft

$c = 3$ ft $\qquad h = 6$ ft



Solution:

$$\mathbf{r_{PA}} = \begin{pmatrix} -f-c \\ b+g \\ h-a \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} f+e \\ d-g \\ -h \end{pmatrix}$$

$$\mathbf{v_A} = v\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{H_P} = \mathbf{r_{PA}} \times (W\mathbf{v_A}) \qquad \mathbf{H_P} = \begin{pmatrix} -14.30 \\ -9.32 \\ -34.81 \end{pmatrix} \text{slug}\cdot\frac{\text{ft}^2}{\text{s}}$$

*__Problem 15-96__

Determine the total angular momentum $\mathbf{H_O}$ for the system of three particles about point O. All the particles are moving in the x-y plane.

Given:

$m_A = 1.5$ kg $\qquad a = 900$ mm

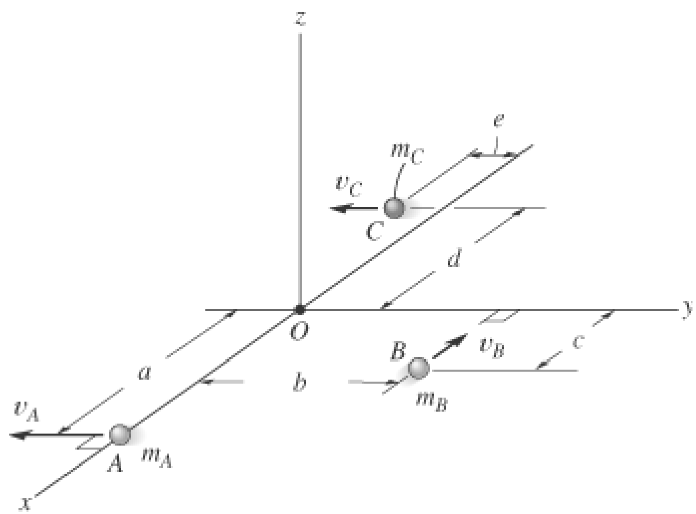
$v_A = 4\ \frac{\text{m}}{\text{s}}$ $\quad b = 700\ \text{mm}$

$m_B = 2.5\ \text{kg}$ $\quad c = 600\ \text{mm}$

$v_B = 2\ \frac{\text{m}}{\text{s}}$ $\quad d = 800\ \text{mm}$

$m_C = 3\ \text{kg}$ $\quad e = 200\ \text{mm}$

$v_C = 6\ \frac{\text{m}}{\text{s}}$



Solution:

$$\mathbf{H_O} = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} \times \left[m_A \begin{pmatrix} 0 \\ -v_A \\ 0 \end{pmatrix} \right] + \begin{pmatrix} c \\ b \\ 0 \end{pmatrix} \times \left[m_B \begin{pmatrix} -v_B \\ 0 \\ 0 \end{pmatrix} \right] + \begin{pmatrix} -d \\ -e \\ 0 \end{pmatrix} \times \left[m_C \begin{pmatrix} 0 \\ -v_C \\ 0 \end{pmatrix} \right]$$

$$\mathbf{H_O} = \begin{pmatrix} 0.00 \\ 0.00 \\ 12.50 \end{pmatrix} \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

Problem 15-97

Determine the angular momentum $\mathbf{H_O}$ of each of the two particles about point O. Use a scalar solution.

Given:

$m_A = 2\ \text{kg}$ $\quad c = 1.5\ \text{m}$

$m_B = 1.5\ \text{kg}$ $\quad d = 2\ \text{m}$

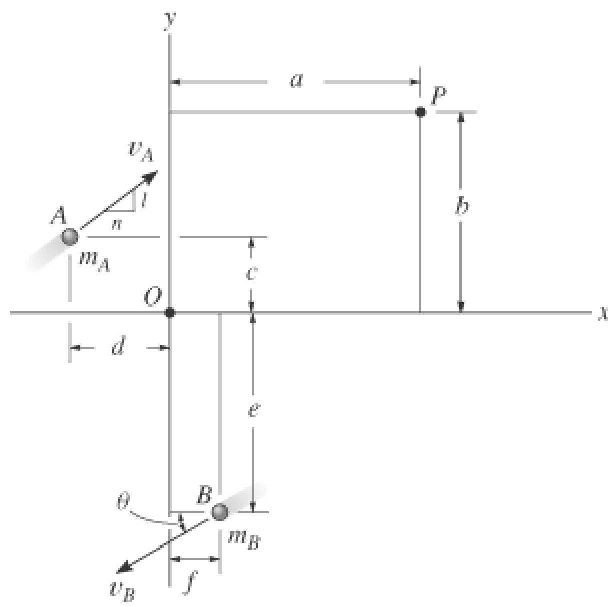
$v_A = 15\ \frac{\text{m}}{\text{s}}$ $\quad e = 4\ \text{m}$

$f = 1\ \text{m}$

$v_B = 10\ \frac{\text{m}}{\text{s}}$ $\quad \theta = 30\ \text{deg}$

$a = 5\ \text{m}$ $\quad l = 3$

$b = 4\ \text{m}$ $\quad n = 4$



Solution:

$$H_{OA} = -m_A\left(\frac{n}{\sqrt{n^2+l^2}}\right)v_A c - m_A\left(\frac{l}{\sqrt{n^2+l^2}}\right)v_A d \qquad H_{OA} = -72.0\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$H_{OB} = -m_B v_B \cos(\theta)e - m_B v_B \sin(\theta) f \qquad H_{OB} = -59.5\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

Problem 15-98

Determine the angular momentum $\mathbf{H_P}$ of each of the two particles about point P. Use a scalar solution.

Given:

$m_A = 2\ \text{kg}$ $\quad c = 1.5\ \text{m}$

$m_B = 1.5\ \text{kg}$ $\quad d = 2\ \text{m}$

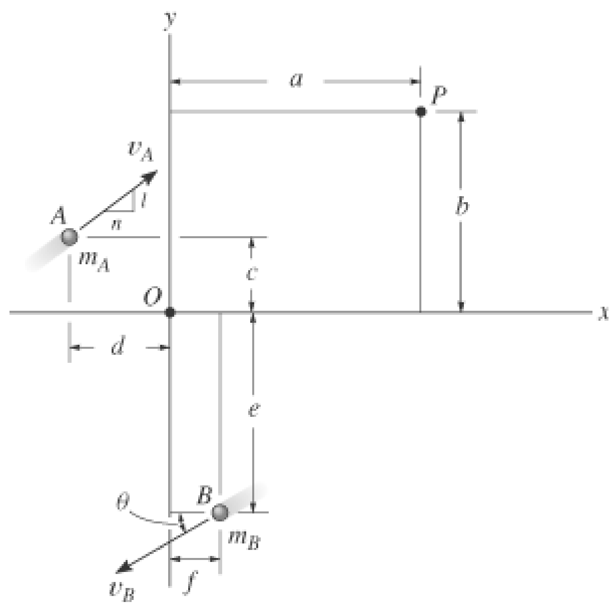
$v_A = 15\ \frac{\text{m}}{\text{s}}$ $\quad e = 4\ \text{m}$

$v_B = 10\ \frac{\text{m}}{\text{s}}$ $\quad f = 1\ \text{m}$

$a = 5\ \text{m}$ $\quad \theta = 30\ \text{deg}$

$b = 4\ \text{m}$ $\quad l = 3$

$n = 4$



Solution:

$$H_{PA} = m_A\frac{n}{\sqrt{n^2+l^2}}v_A(b-c) - m_A\frac{l}{\sqrt{n^2+l^2}}v_A(a+d) \qquad H_{PA} = -66.0\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$H_{PB} = -m_B v_B \cos(\theta)(b+e) + m_B v_B \sin(\theta)(a-f) \qquad H_{PB} = -73.9\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

Problem 15-99

The ball B has mass M and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = at^2 + bt + c$, determine the speed of the ball when $t = t_1$. The ball has a speed $v = v_0$ when $t = 0$.

Given:

$M = 10 \text{ kg}$

$a = 3 \dfrac{\text{N}\cdot\text{m}}{\text{s}^2}$

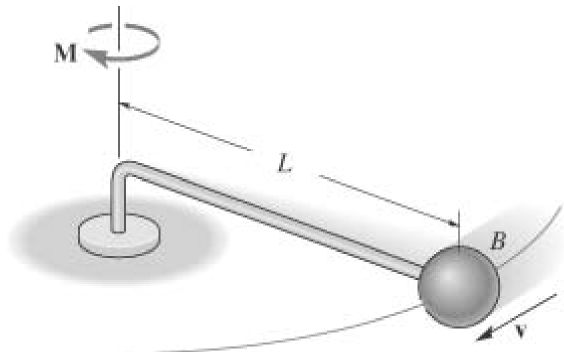
$b = 5 \dfrac{\text{N}\cdot\text{m}}{\text{s}}$

$c = 2 \text{ N}\cdot\text{m}$

$t_1 = 2 \text{ s}$

$v_0 = 2 \dfrac{\text{m}}{\text{s}}$

$L = 1.5 \text{ m}$



Solution: Principle of angular impulse momentum

$$M v_0 L + \int_0^{t_1} a t^2 + b t + c \, dt = M v_1 L$$

$$v_1 = v_0 + \frac{1}{M L} \int_0^{t_1} a t^2 + b t + c \, dt \qquad v_1 = 3.47 \frac{\text{m}}{\text{s}}$$

***Problem 15-100**

The two blocks A and B each have a mass M_0. The blocks are fixed to the horizontal rods, and their initial velocity is v' in the direction shown. If a couple moment of M is applied about shaft CD of the frame, determine the speed of the blocks at time t. The mass of the frame is negligible, and it is free to rotate about CD. Neglect the size of the blocks.

Given:

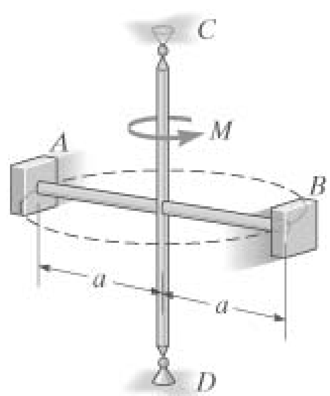
$M_0 = 0.4 \text{ kg}$

$a = 0.3 \text{ m}$

$v' = 2 \dfrac{\text{m}}{\text{s}}$

$M = 0.6 \text{ N}\cdot\text{m}$

$t = 3 \text{ s}$



Solution:

$$2a M_0 v' + M t = 2a M_0 v$$

$$v = v' + \frac{M t}{2a M_0} \qquad v = 9.50 \ \frac{\text{m}}{\text{s}}$$

Problem 15-101

The small cylinder C has mass m_C and is attached to the end of a rod whose mass may be neglected. If the frame is subjected to a couple $M = at^2 + b$, and the cylinder is subjected to force F, which is always directed as shown, determine the speed of the cylinder when $t = t_1$. The cylinder has a speed v_0 when $t = 0$.

Given:

$m_C = 10 \text{ kg}$ $\qquad t_1 = 2 \text{ s}$

$a = 8 \text{ N} \dfrac{\text{m}}{\text{s}^2}$ $\qquad v_0 = 2 \dfrac{\text{m}}{\text{s}}$

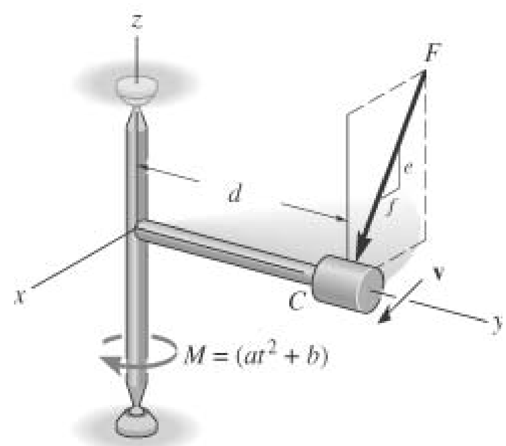
$d = 0.75 \text{ m}$

$b = 5 \text{ N·m}$

$e = 4$

$F = 60 \text{ N}$

$f = 3$



Solution:

$$m_C v_0 d + \int_0^{t_1} a t^2 + b \, dt + \left(\frac{f}{\sqrt{e^2 + f^2}}\right) F d t_1 = m_C v_1 d$$

$$v_1 = v_0 + \frac{1}{m_C d}\left[\int_0^{t_1} a t^2 + b \, dt + \left(\frac{f}{\sqrt{e^2 + f^2}}\right) F d t_1\right] \qquad v_1 = 13.38 \frac{\text{m}}{\text{s}}$$

Problem 15-102

A box having a weight W is moving around in a circle of radius r_A with a speed v_{A1} while connected to the end of a rope. If the rope is pulled inward with a constant speed v_r, determine the speed of the box at the instant $r = r_B$. How much work is done after pulling in the rope from A to B? Neglect friction and the size of the box.

Given:

$W = 8 \text{ lb}$

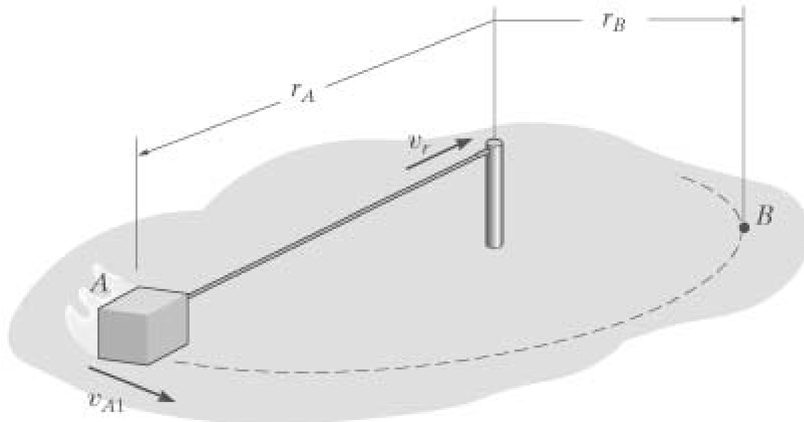
$r_A = 2 \text{ ft}$

$v_{A1} = 5\ \frac{\text{ft}}{\text{s}}$

$v_r = 4\ \frac{\text{ft}}{\text{s}}$

$r_B = 1\ \text{ft}$

$g = 32.21\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$\left(\frac{W}{g}\right) r_A v_{A1} = \left(\frac{W}{g}\right) r_B v_{Btangent}$$

$$v_{Btangent} = r_A \left(\frac{v_{A1}}{r_B}\right) \qquad v_{Btangent} = 10.00\ \frac{\text{ft}}{\text{s}}$$

$$v_B = \sqrt{v_{Btangent}^2 + v_r^2} \qquad v_B = 10.8\ \frac{\text{ft}}{\text{s}}$$

$$U_{AB} = \frac{1}{2}\left(\frac{W}{g}\right) v_B^2 - \frac{1}{2}\left(\frac{W}{g}\right) v_{A1}^2 \qquad U_{AB} = 11.3\ \text{ft}\cdot\text{lb}$$

Problem 15-103

An earth satellite of mass M is launched into a free-flight trajectory about the earth with initial speed v_A when the distance from the center of the earth is r_A. If the launch angle at this position is ϕ_A determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass M_e. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, **F**, Eq. 13-1. For part of the solution, use the conservation of energy.

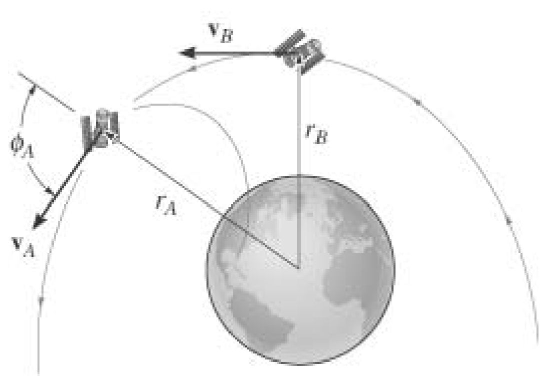
Units used: $\text{Mm} = 10^3\ \text{km}$

Given:

$M = 700\ \text{kg}$ $\quad \phi_A = 70\ \text{deg}$

$v_A = 10\ \frac{\text{km}}{\text{s}}$ $\quad G = 6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

$r_A = 15\ \text{Mm}$ $\quad M_e = 5.976 \times 10^{24}\ \text{kg}$



Solution: Guesses $v_B = 10\ \frac{\text{km}}{\text{s}}$ $r_B = 10\ \text{Mm}$

Given $M v_A \sin(\phi_A) r_A = M v_B r_B$

$$\frac{1}{2} M v_A^2 - \frac{G M_e M}{r_A} = \frac{1}{2} M v_B^2 - \frac{G M_e M}{r_B}$$

$$\begin{pmatrix} v_B \\ r_B \end{pmatrix} = \text{Find}(v_B, r_B) \qquad v_B = 10.2\ \frac{\text{km}}{\text{s}} \qquad r_B = 13.8\ \text{Mm}$$

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*__Problem 15-104__

The ball *B* has weight *W* and is originally rotating in a circle. As shown, the cord *AB* has a length of *L* and passes through the hole *A*, which is a distance *h* above the plane of motion. If *L*/2 of the cord is pulled through the hole, determine the speed of the ball when it moves in a circular path at *C*.

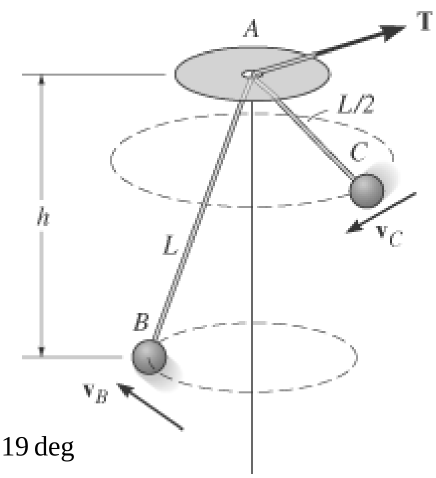
Given:

$W = 5\ \text{lb}$

$L = 3\ \text{ft}$

$h = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: $\theta_B = \text{acos}\left(\frac{h}{L}\right)$ $\theta_B = 48.19\ \text{deg}$

Guesses $T_B = 1\ \text{lb}$ $T_C = 1\ \text{lb}$ $v_B = 1\ \frac{\text{ft}}{\text{s}}$ $v_C = 1\ \frac{\text{ft}}{\text{s}}$ $\theta_C = 10\ \text{deg}$

Given $T_B \cos(\theta_B) - W = 0$ $\qquad T_B \sin(\theta_B) = \frac{W}{g}\left(\frac{v_B^2}{L \sin(\theta_B)}\right)$

$$T_C \cos(\theta_C) - W = 0 \qquad T_C \sin(\theta_C) = \frac{W}{g}\left(\frac{v_C^2}{\frac{L}{2}\sin(\theta_C)}\right)$$

$$\left(\frac{W}{g}\right) v_B L \sin(\theta_B) = \left(\frac{W}{g}\right) v_C \left(\frac{L}{2}\right) \sin(\theta_C)$$

$$\begin{pmatrix} T_B \\ T_C \\ v_B \\ v_C \\ \theta_C \end{pmatrix} = \text{Find}(T_B, T_C, v_B, v_C, \theta_C) \qquad \begin{pmatrix} T_B \\ T_C \end{pmatrix} = \begin{pmatrix} 7.50 \\ 20.85 \end{pmatrix} \text{lb} \qquad \theta_C = 76.12 \text{ deg}$$

$$v_B = 8.97 \frac{\text{ft}}{\text{s}} \qquad v_C = 13.78 \frac{\text{ft}}{\text{s}}$$

Problem 15-105

The block of weight W rests on a surface for which the kinetic coefficient of friction is μ_k. It is acted upon by a radial force $\mathbf{F_R}$ and a horizontal force $\mathbf{F_H}$, always directed at angle θ from the tangent to the path as shown. If the block is initially moving in a circular path with a speed v_1 at the instant the forces are applied, determine the time required before the tension in cord AB becomes T. Neglect the size of the block for the calculation.

Given:

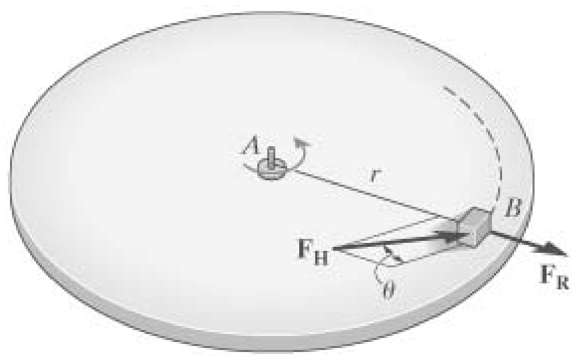
$W = 10 \text{ lb}$ $\quad \mu_k = 0.5$

$F_R = 2 \text{ lb}$ $\quad T = 20 \text{ lb}$

$F_H = 7 \text{ lb}$ $\quad r = 4 \text{ ft}$

$v_1 = 2 \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$\theta = 30 \text{ deg}$



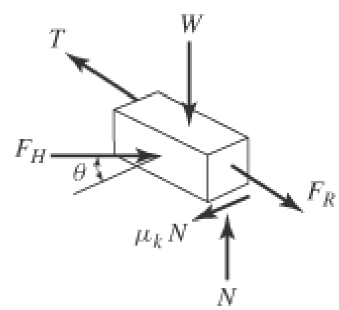
Solution:

Guesses $\quad t = 1 \text{ s} \quad v_2 = 1 \frac{\text{ft}}{\text{s}}$

Given

$$\left(\frac{W}{g}\right)v_1 r + F_H\cos(\theta)rt - \mu_k W r t = \left(\frac{W}{g}\right)v_2 r$$

$$F_R + F_H\sin(\theta) - T = -\frac{W}{g}\left(\frac{v_2^2}{r}\right)$$



$$\begin{pmatrix} t \\ v_2 \end{pmatrix} = \text{Find}(t, v_2) \qquad v_2 = 13.67 \frac{\text{ft}}{\text{s}} \qquad t = 3.41 \text{ s}$$

Problem 15-106

The block of weight W is originally at rest on the smooth surface. It is acted upon by a radial force $\mathbf{F_R}$ and a horizontal force $\mathbf{F_H}$, always directed at θ from the tangent to the path as shown. Determine the time required to break the cord, which requires a tension T. What is the speed of the block when this occurs? Neglect the size of the block for the calculation.

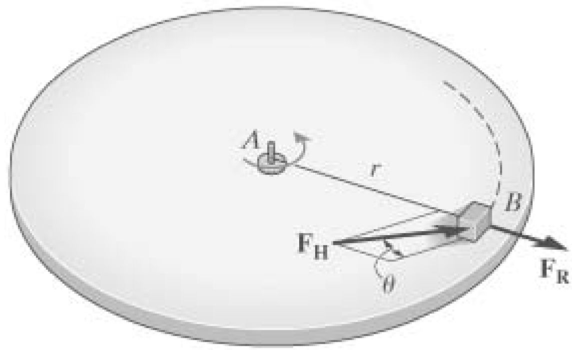
Given:

$W = 10\ \text{lb}$ $\quad\theta = 30\ \text{deg}$

$F_R = 2\ \text{lb}$ $\quad T = 30\ \text{lb}$

$F_H = 7\ \text{lb}$ $\quad r = 4\ \text{ft}$

$v_1 = 0\ \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

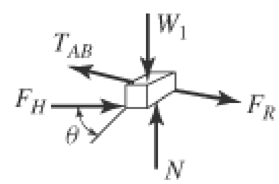


Solution:

Guesses $\quad t = 1\ \text{s} \quad v_2 = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$\left(\frac{W}{g}\right)v_1 r + F_H\cos(\theta) r t = \left(\frac{W}{g}\right)v_2 r$$

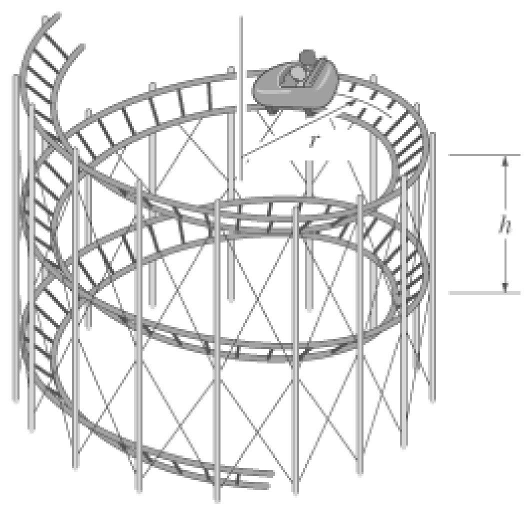


$$F_R + F_H\sin(\theta) - T = -\frac{W}{g}\left(\frac{v_2^{\,2}}{r}\right)$$

$$\begin{pmatrix} t \\ v_2 \end{pmatrix} = \text{Find}(t, v_2) \qquad v_2 = 17.76\ \frac{\text{ft}}{\text{s}} \qquad t = 0.91\ \text{s}$$

Problem 15-107

The roller-coaster car of weight W starts from rest on the track having the shape of a cylindrical helix. If the helix descends a distance h for every one revolution, determine the time required for the car to attain a speed v. Neglect friction and the size of the car.



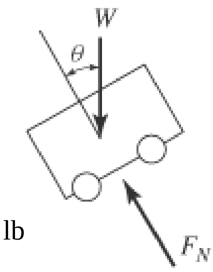
Given:

$W = 800\ \text{lb}$

$h = 8\ \text{ft}$

$v = 60\ \frac{\text{ft}}{\text{s}}$

$r = 8 \text{ ft}$



Solution:

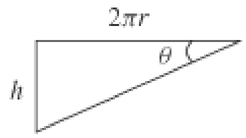
$\theta = \operatorname{atan}\left(\frac{h}{2\pi r}\right)$ $\theta = 9.04 \,\text{deg}$

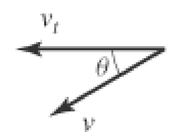
$F_N - W\cos(\theta) = 0$ $F_N = W\cos(\theta)$ $F_N = 790.06 \,\text{lb}$

$v_t = v\cos(\theta)$ $v_t = 59.25 \,\frac{\text{ft}}{\text{s}}$

$H_A + \int M \, dt = H_2$ $\int_0^t F_N \sin(\theta) r \, dt = \left(\frac{W}{g}\right) h v_t$ $F_N \sin(\theta) r t = \left(\frac{W}{g}\right) h v_t$

$t = W\left(\frac{v_t h}{F_N \sin(\theta) g r}\right)$ $t = 11.9 \text{ s}$





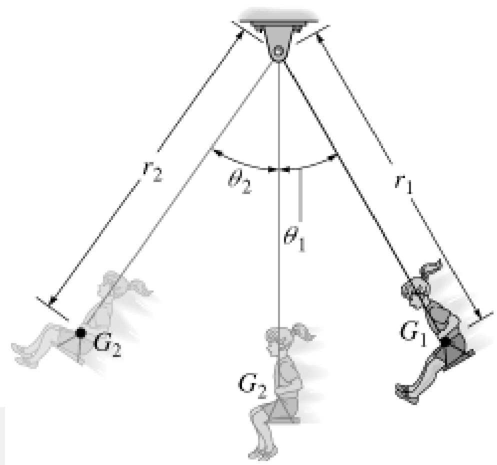
***Problem 15-108**

A child having mass M holds her legs up as shown as she swings downward from rest at θ_1. Her center of mass is located at point G_1. When she is at the bottom position $\theta = 0°$, she *suddenly* lets her legs come down, shifting her center of mass to position G_2. Determine her speed in the upswing due to this sudden movement and the angle θ_2 to which she swings before momentarily coming to rest. Treat the child's body as a particle.

Given:

$M = 50 \text{ kg}$ $r_1 = 2.80 \text{ m}$ $g = 9.81 \,\frac{\text{m}}{\text{s}^2}$

$\theta_1 = 30 \text{ deg}$ $r_2 = 3 \text{ m}$



Solution:

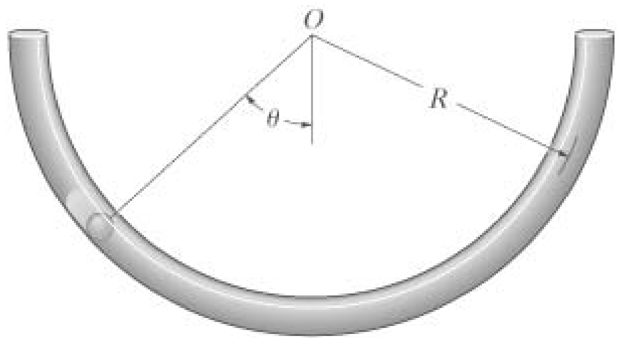
$v_{2b} = \sqrt{2 g r_1 (1 - \cos(\theta_1))}$ $v_{2b} = 2.71 \,\frac{\text{m}}{\text{s}}$

$r_1 v_{2b} = r_2 v_{2a}$ $v_{2a} = \frac{r_1}{r_2} v_{2b}$ $v_{2a} = 2.53 \,\frac{\text{m}}{\text{s}}$

$\theta_2 = \operatorname{acos}\left(1 - \frac{v_{2a}^2}{2 g r_2}\right)$ $\theta_2 = 27.0 \,\text{deg}$

Problem 15-109

A small particle having a mass m is placed inside the semicircular tube. The particle is placed at the position shown and released. Apply the principle of angular momentum about point O ($\Sigma M_0 = H_0$), and show that the motion of the particle is governed by the differential equation $\theta'' + (g / R) \sin \theta = 0$.



Solution:

$$\Sigma M_0 = \frac{d}{dt} H_0$$

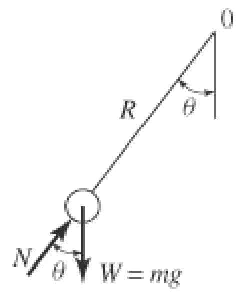
$$-R\, m\, g \sin(\theta) = \frac{d}{dt}(m\, v\, R)$$

$$g \sin(\theta) = -\frac{d}{dt} v = -\frac{d^2}{dt^2} s$$

But, $s = R\theta$

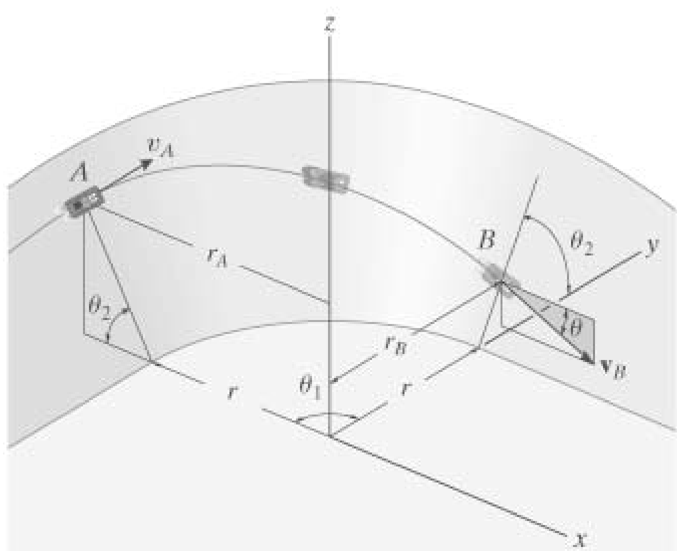
Thus, $g \sin(\theta) = -R\, \theta''$

or, $\boxed{\theta'' + \left(\frac{g}{R}\right) \sin(\theta) = 0}$



Problem 15-110

A toboggan and rider, having a total mass M, enter horizontally tangent to a circular curve (θ_1) with a velocity v_A. If the track is flat and banked at angle θ_2, determine the speed v_B and the angle θ of "descent", measured from the horizontal in a vertical x–z plane, at which the toboggan exists at B. Neglect friction in the calculation.



Given:

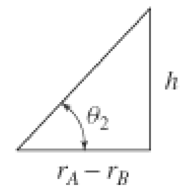
$M = 150$ kg $\quad \theta_1 = 90$ deg $\quad v_A = 70\ \frac{\text{km}}{\text{hr}} \quad \theta_2 = 60$ deg

$r_A = 60$ m $\quad r_B = 57$ m $\quad r = 55$ m

Solution:

$$h = (r_A - r_B)\tan(\theta_2)$$

Guesses $v_B = 10\ \frac{m}{s}$ $\theta = 1\ \text{deg}$



Given $\frac{1}{2}M v_A^2 + M g h = \frac{1}{2}M v_B^2$ $M v_A r_A = M v_B \cos(\theta) r_B$

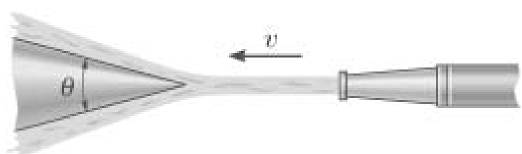
$$\begin{pmatrix} v_B \\ \theta \end{pmatrix} = \text{Find}(v_B, \theta) \qquad v_B = 21.9\ \frac{m}{s} \qquad \theta = -1.1 \times 10^3\ \text{deg}$$

Problem 15-111

Water is discharged at speed *v* against the fixed cone diffuser. If the opening diameter of the nozzle is *d*, determine the horizontal force exerted by the water on the diffuser.

Units Used:

$$\text{Mg} = 10^3\ \text{kg}$$

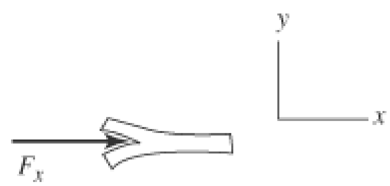


Given:

$$v = 16\ \frac{m}{s} \qquad \theta = 30\ \text{deg}$$

$$d = 40\ \text{mm} \qquad \rho_w = 1\ \frac{\text{Mg}}{m^3}$$

Solution:



$$Q = \frac{\pi}{4}d^2 v \qquad m' = \rho_w Q$$

$$F_x = m'\left(-v\cos\left(\frac{\theta}{2}\right) + v\right)$$

$$F_x = 11.0\ \text{N}$$

***Problem 15-112**

A jet of water having cross-sectional area *A* strikes the fixed blade with speed *v*. Determine the horizontal and vertical components of force which the blade exerts on the water.

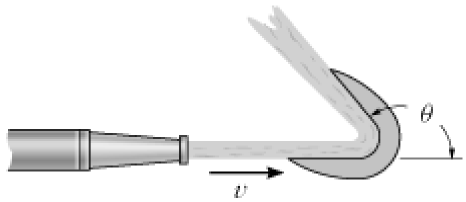
Given:

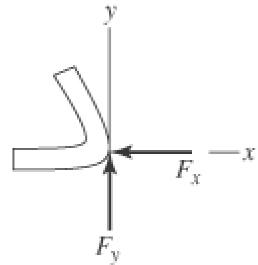
$$A = 4\ \text{in}^2$$

$v = 25\ \frac{\text{ft}}{\text{s}}$

$\theta = 130\ \text{deg}$

$\gamma_w = 62.4\ \frac{\text{lb}}{\text{ft}^3}$





Solution: $Q = A\,v \qquad Q = 0.69\ \frac{\text{ft}^3}{\text{s}}$

$\frac{d}{dt}m = m' = \rho Q \qquad m' = \gamma_w Q \qquad m' = 1.3468\ \frac{\text{slug}}{\text{s}}$

$v_{Ax} = v \qquad v_{Ay} = 0\ \frac{\text{ft}}{\text{s}} \qquad v_{Bx} = v\cos(\theta) \qquad v_{By} = v\sin(\theta)$

$F_x = \frac{-m'}{g}(v_{Bx} - v_{Ax}) \qquad F_x = 55.3\ \text{lb}$

$F_y = \frac{m'}{g}(v_{By} - v_{Ay}) \qquad F_y = 25.8\ \text{lb}$

Problem 15-113

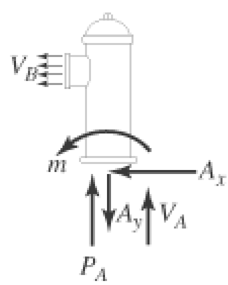
Water is flowing from the fire hydrant opening of diameter d_B with velocity v_B. Determine the horizontal and vertical components of force and the moment developed at the base joint A, if the static (gauge) pressure at A is P_A. The diameter of the fire hydrant at A is d_A.

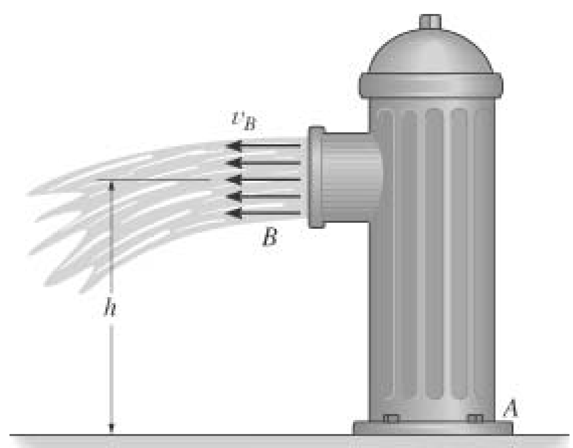
Units Used:

$\text{kPa} = 10^3\ \text{Pa}$

$\text{Mg} = 10^3\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$





Given:

$d_B = 150\ \text{mm} \qquad h = 500\ \text{mm}$

$v_B = 15\ \frac{\text{m}}{\text{s}} \qquad d_A = 200\ \text{mm}$

$P_A = 50\ \text{kPa} \qquad \rho_w = 1\ \frac{\text{Mg}}{\text{m}^3}$

Solution:

$A_B = \pi\left(\frac{d_B}{2}\right)^2 \qquad A_A = \pi\left(\frac{d_A}{2}\right)^2 \qquad m' = \rho_w v_B \pi\left(\frac{d_B}{2}\right)^2 \qquad v_A = \frac{m'}{\rho_w A_A}$

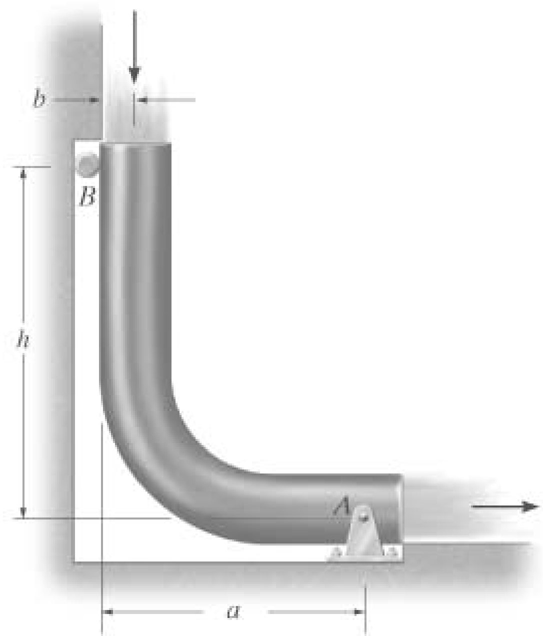
$$A_x = m' v_B \qquad \boxed{A_x = 3.98\,\text{kN}}$$

$$-A_y + 50\pi\left(\frac{d_A}{2}\right)^2 = m'(0 - v_A) \qquad A_y = m' v_A + P_A\pi\left(\frac{d_A}{2}\right)^2 \qquad \boxed{A_y = 3.81\,\text{kN}}$$

$$M = m' h v_B \qquad \boxed{M = 1.99\,\text{kN}\cdot\text{m}}$$

Problem 15-114

The chute is used to divert the flow of water Q. If the water has a cross-sectional area A, determine the force components at the pin A and roller B necessary for equilibrium. Neglect both the weight of the chute and the weight of the water on the chute.



Units Used:

$$\text{Mg} = 10^3\ \text{kg} \qquad \text{kN} = 10^3\ \text{N}$$

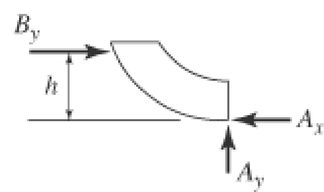
Given:

$$Q = 0.6\ \frac{\text{m}^3}{\text{s}} \qquad \rho_w = 1\ \frac{\text{Mg}}{\text{m}^3}$$

$$A = 0.05\ \text{m}^2 \qquad h = 2\ \text{m}$$

$$a = 1.5\ \text{m} \qquad b = 0.12\ \text{m}$$

Solution:



$$\frac{d}{dt}m = m' \qquad m' = \rho_w Q$$

$$v_A = \frac{Q}{A} \qquad v_B = v_A$$

$$\Sigma F_x = m'(v_{Ax} - v_{Bx}) \qquad B_x - A_x = m'(v_{Ax} - v_{Bx})$$

$$\Sigma F_y = m'(v_{Ay} - v_{By}) \qquad A_y = m'[0 - (-v_B)] \qquad \boxed{A_y = 7.20\,\text{kN}}$$

$$\Sigma M_A = m'(d_{0A} v_A - d_{0B} v_B) \qquad B_x = \frac{1}{h}m'[b v_A + (a-b)v_A] \qquad \boxed{B_x = 5.40\,\text{kN}}$$

$$A_x = B_x - m' v_A \qquad \boxed{A_x = -1.80\,\text{kN}}$$

Problem 15-115

The fan draws air through a vent with speed v. If the cross-sectional area of the vent is A, determine the horizontal thrust on the blade. The specific weight of the air is γ_a.

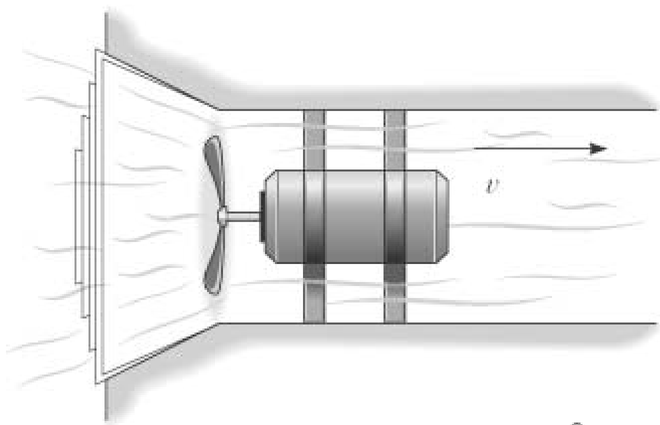
Given:

$$v = 12\ \frac{\text{ft}}{\text{s}}$$

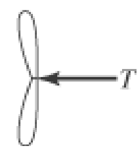
$$A = 2\ \text{ft}^2$$

$$\gamma_a = 0.076\ \frac{\text{lb}}{\text{ft}^3}$$

$$g = 32.20\ \frac{\text{ft}}{\text{s}^2}$$



Solution:

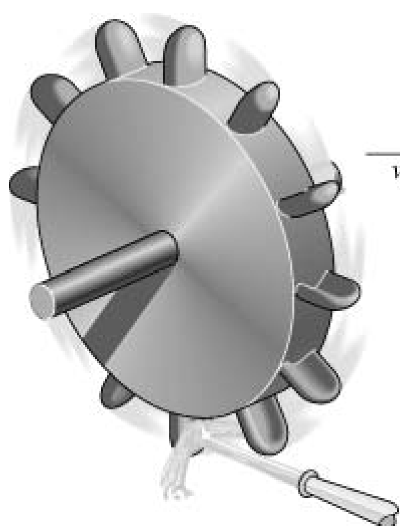


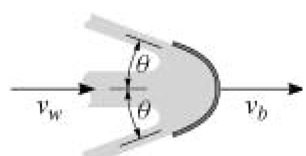
$$m' = \frac{d}{dt}m \qquad m' = \gamma_a v A \qquad m' = 0.05669\ \frac{\text{slug}}{\text{s}}$$

$$T = \frac{m'(v-0)}{g} \qquad \boxed{T = 0.68\ \text{lb}}$$

*****Problem 15-116**

The buckets on the *Pelton wheel* are subjected to a jet of water of diameter d, which has velocity v_w. If each bucket is traveling at speed v_b when the water strikes it, determine the power developed by the wheel. The density of water is γ_w.





Given:

$$d = 2\ \text{in} \qquad \theta = 20\ \text{deg}$$

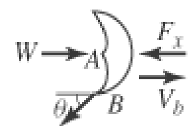
$$v_w = 150\ \frac{\text{ft}}{\text{s}} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

$$v_b = 95\ \frac{\text{ft}}{\text{s}}$$

$$\gamma_w = 62.4\ \frac{\text{lbf}}{\text{ft}^3}$$

Solution: $v_A = v_w - v_b \qquad v_A = 55\,\frac{\text{ft}}{\text{s}}$

$v_{Bx} = -v_A\cos(\theta) + v_b \qquad v_{Bx} = 43.317\,\frac{\text{ft}}{\text{s}}$



$\overset{+}{\longleftarrow} \quad \Sigma F_x = m'(v_{Bx} - v_{Ax})$

$F_x = \left(\frac{\gamma_w}{g}\right)\pi\left(\frac{d^2}{4}\right)v_A\left[-v_{Bx} - (-v_A)\right] \qquad F_x = 266.41\,\frac{\text{m}}{\text{s}^2}\cdot\text{lb}$

$P = F_x v_b \qquad P = 4.69\,\text{hp}$

Problem 15-117

The boat of mass M is powered by a fan F which develops a slipstream having a diameter d. If the fan ejects air with a speed v, measured relative to the boat, determine the initial acceleration of the boat if it is initially at rest. Assume that air has a constant density ρ_a and that the entering air is essentially at rest. Neglect the drag resistance of the water.

Given:

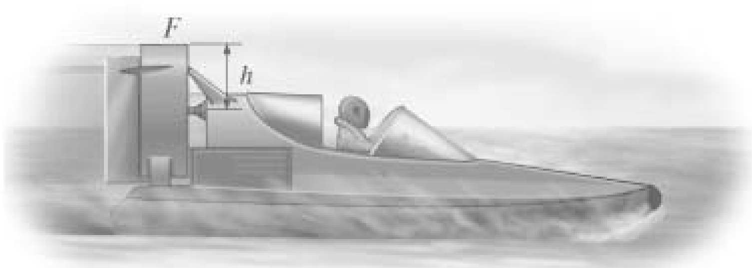
$M = 200\ \text{kg}$

$h = 0.375\ \text{m}$

$d = 0.75\ \text{m}$

$v = 14\ \frac{\text{m}}{\text{s}}$

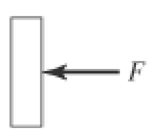
$\rho_a = 1.22\ \frac{\text{kg}}{\text{m}^3}$



Solution:

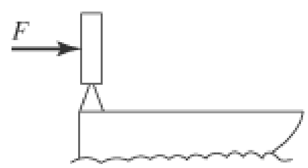
$Q = A v \qquad Q = \frac{\pi}{4}d^2 v \qquad Q = 6.1850\ \frac{\text{m}^3}{\text{s}}$

$\frac{\text{d}}{\text{d}t}m = m' \qquad m' = \rho_a Q \qquad m' = 7.5457\ \frac{\text{kg}}{\text{s}}$



$\Sigma F_x = m'(v_{Bx} - v_{Ax})$

$F = \rho_a Q v \qquad F = 105.64\ \text{N}$



$\Sigma F_x = M a_x \qquad F = M a$

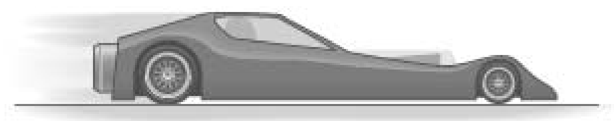
$a = \dfrac{F}{M}$ $\quad$ $a = 0.53\ \dfrac{\text{m}}{\text{s}^2}$

Problem 15-118

The rocket car has a mass M_C (empty) and carries fuel of mass M_F. If the fuel is consumed at a constant rate c and ejected from the car with a relative velocity v_{DR}, determine the maximum speed attained by the car starting from rest. The drag resistance due to the atmosphere is $F_D = kv^2$ and the speed is measured in m/s.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$



Given:

$M_C = 3\ \text{Mg}$ $\quad$ $M_F = 150\ \text{kg}$

$v_{DR} = 250\ \dfrac{\text{m}}{\text{s}}$ $\quad$ $c = 4\ \dfrac{\text{kg}}{\text{s}}$

$k = 60\ \text{N}\cdot\dfrac{\text{s}^2}{\text{m}^2}$

Solution:

$m_0 = M_C + M_F$ $\quad$ At time t the mass of the car is $m_0 - ct$

Set $F = kv^2$, then $\quad -kv^2 = (m_0 - ct)\dfrac{d}{dt}v - v_{DR}c$

Maximum speed occurs at the instant the fuel runs out. $\quad t = \dfrac{M_F}{c}$ $\quad t = 37.50\ \text{s}$

Thus, Initial Guess: $\quad v = 4\ \dfrac{\text{m}}{\text{s}}$

Given $\quad \displaystyle\int_0^v \frac{1}{cv_{DR} - kv^2}\,dv = \int_0^t \frac{1}{m_0 - ct}\,dt$

$v = \text{Find}(v)$ $\quad$ $v = 4.06\ \dfrac{\text{m}}{\text{s}}$

Problem 15-119

A power lawn mower hovers very close over the ground. This is done by drawing air in at speed v_A through an intake unit A, which has cross-sectional area A_A and then discharging it at the ground, B, where the cross-sectional area is A_B. If air at A is subjected only to atmospheric pressure, determine the air pressure which the lawn mower exerts on the ground when the weight of the mower is freely supported and no load is placed on the handle. The mower has mass M with center of mass at G. Assume that air has a constant density of ρ_a.

Given:

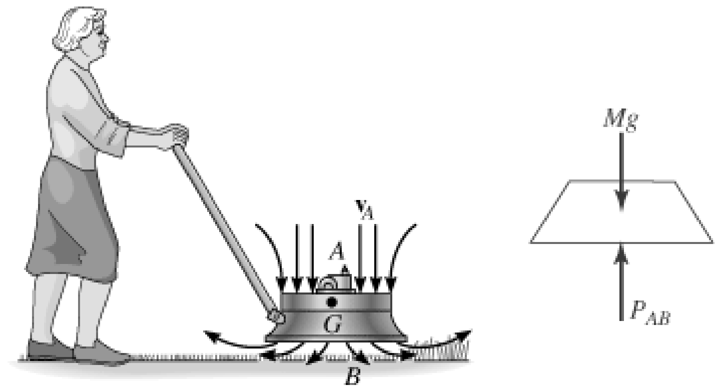
$v_A = 6\ \frac{\text{m}}{\text{s}}$

$A_A = 0.25\ \text{m}^2$

$A_B = 0.35\ \text{m}^2$

$M = 15\ \text{kg}$

$\rho_a = 1.22\ \frac{\text{kg}}{\text{m}^3}$



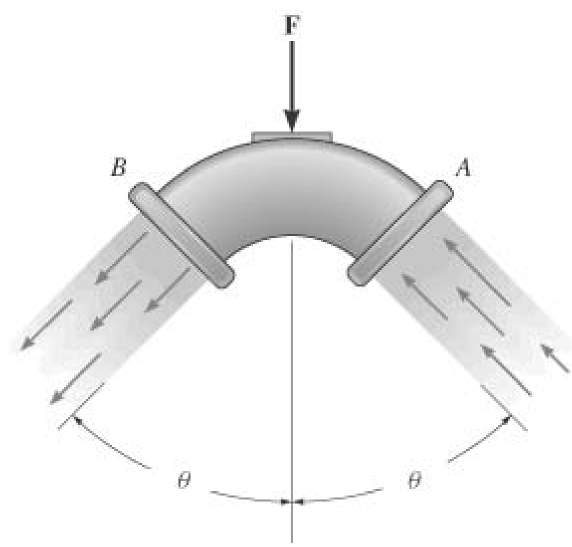
Solution: $m' = \rho_a A_A v_A \qquad m' = 1.83\ \frac{\text{kg}}{\text{s}}$

$+\uparrow\ \Sigma F_y = m'(v_{By} - v_{Ay}) \qquad P\,A_B - M\,g = m'[0 - (-v_A)]$

$P = \frac{1}{A_B}(m' v_A + M g) \qquad P = 452\ \text{Pa}$

***Problem 15-120**

The elbow for a buried pipe of diameter d is subjected to static pressure P. The speed of the water passing through it is v. Assuming the pipe connection at A and B do not offer any vertical force resistance on the elbow, determine the resultant vertical force **F** that the soil must then exert on the elbow in order to hold it in equilibrium. Neglect the weight of the elbow and the water within it. The density of water is γ_w.

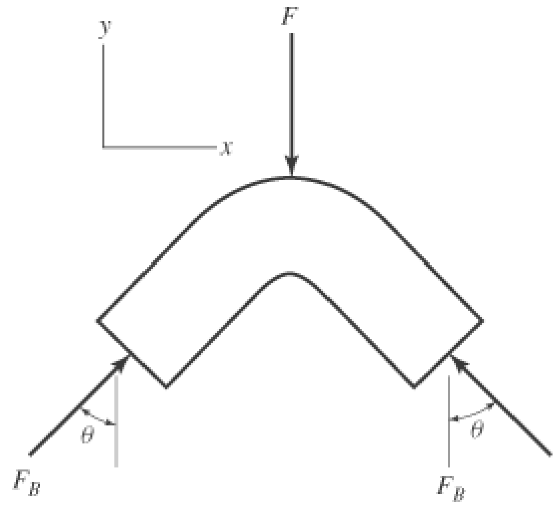


Given:

$d = 5\ \text{in} \qquad \theta = 45\ \text{deg}$

$P = 10\ \frac{\text{lb}}{\text{in}^2}$ $\gamma_w = 62.4\ \frac{\text{lb}}{\text{ft}^3}$

$v = 8\ \frac{\text{ft}}{\text{s}}$



Solution:

$$Q = v\left(\frac{\pi}{4}d^2\right)$$

$$m' = \frac{\gamma_w}{g}Q$$

Also, the force induced by the water pressure at A is

$$A = \frac{\pi}{4}d^2$$

$$F = P\,A \qquad F = 196.35\ \text{lb}$$

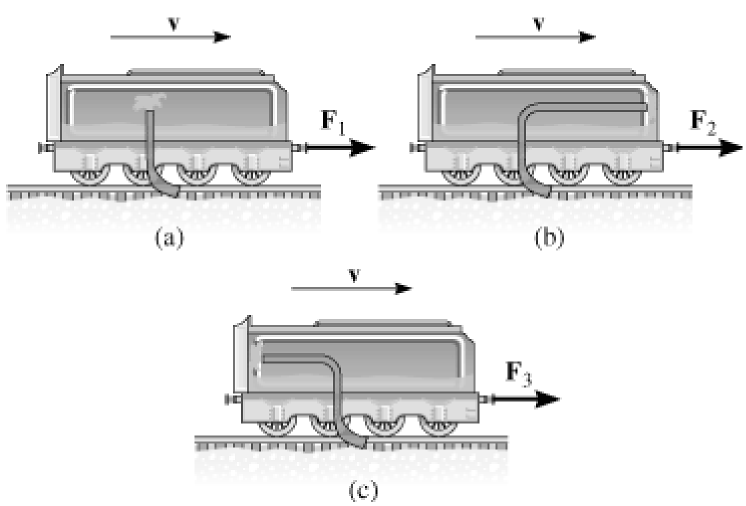
$$2F\cos(\theta) - F_1 = m'(-v\cos(\theta) - v\cos(\theta))$$

$$F_1 = 2(F\cos(\theta) + m'\,v\cos(\theta))$$

$$F_1 = 302\ \text{lb}$$

Problem 15-121

The car is used to scoop up water that is lying in a trough at the tracks. Determine the force needed to pull the car forward at constant velocity **v** for each of the three cases. The scoop has a cross-sectional area A and the density of water is ρ_w.



Solution:

The system consists of the car and the scoop. In all cases

$$\Sigma F_s = m\frac{d}{dt}v - V_{De}\frac{d}{dt}m_e$$

$$F = 0 - V\rho A\, V \qquad F = V^2\rho A$$

Problem 15-122

A rocket has an empty weight W_1 and carries fuel of weight W_2. If the fuel is burned at the rate c and ejected with a relative velocity v_{DR}, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.

Given: $W_1 = 500\ \text{lb}$ $\quad W_2 = 300\ \text{lb}$ $\quad c = 15\ \frac{\text{lb}}{\text{s}}$ $\quad v_{DR} = 4400\ \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution: $\quad m_0 = \dfrac{W_1 + W_2}{g}$

The maximum speed occurs when all the fuel is consumed, that is, where $t = \dfrac{W_2}{c}$ $\qquad t = 20.00\ \text{s}$

$$\Sigma F_x = m\frac{d}{dt}v - v_{DR}\frac{d}{dt}m_e$$

At a time t, $M = m_0 - \frac{c}{g}t$, where $\frac{c}{g} = \frac{d}{dt}m_e$. In space the weight of the rocket is zero.

$$0 = \left(m_0 - c\,t\right)\frac{d}{dt}v - v_{DR}c$$

Guess $\quad v_{max} = 1\ \frac{\text{ft}}{\text{s}}$

Given $\quad \displaystyle\int_0^{v_{max}} 1\, dv = \int_0^t \frac{\frac{c}{g}v_{DR}}{m_0 - \frac{c}{g}t}\, dt$

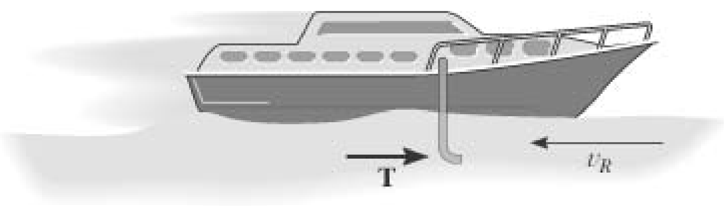
$$v_{max} = \text{Find}\left(v_{max}\right) \qquad v_{max} = 2068\ \frac{\text{ft}}{\text{s}}$$

Problem 15-123

The boat has mass M and is traveling forward on a river with constant velocity v_b, measured relative to the river. The river is flowing in the opposite direction at speed v_R. If a tube is placed in the water, as shown, and it collects water of mass M_w in the boat in time t, determine the horizontal thrust T on the tube that is required to overcome the resistance to the water collection.

Units Used:

$\text{Mg} = 10^3 \text{ kg}$



Given:

$M = 180 \text{ kg}$ $\quad M_w = 40 \text{ kg}$

$v_b = 70 \dfrac{\text{km}}{\text{hr}}$ $\quad t = 80 \text{ s}$

$v_R = 5 \dfrac{\text{km}}{\text{hr}}$ $\quad \rho w = 1 \dfrac{\text{Mg}}{\text{m}^3}$

Solution: $\quad m' = \dfrac{M_w}{t}$ $\quad m' = 0.50 \dfrac{\text{kg}}{\text{s}}$

$v_{di} = v_b$ $\quad v_{di} = 19.44 \dfrac{\text{m}}{\text{s}}$

$$\Sigma F_i = m\frac{d}{dt}v + v_{di}m'$$

$T = v_{di}m'$ $\quad T = 9.72 \text{ N}$

***Problem 15-124**

The second stage of a two-stage rocket has weight W_2 and is launched from the first stage with velocity v. The fuel in the second stage has weight W_f. If it is consumed at rate r and ejected with relative velocity v_r, determine the acceleration of the second stage just after the engine is fired. What is the rocket's acceleration just before all the fuel is consumed? Neglect the effect of gravitation.

Given:

$W_2 = 2000 \text{ lb}$ $\quad W_f = 1000 \text{ lb}$ $\quad r = 50 \dfrac{\text{lb}}{\text{s}}$

$v = 3000 \dfrac{\text{mi}}{\text{hr}}$ $\quad v_r = 8000 \dfrac{\text{ft}}{\text{s}}$ $\quad g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

Solution:

Initially,

$$\Sigma F_s = m\frac{d}{dt}v - v_{di}\left(\frac{d}{dt}m_e\right)$$

$$0 = \left(\frac{W_2 + W_f}{g}\right)a - v_r\frac{r}{g} \qquad a = v_r\left(\frac{r}{W_2 + W_f}\right) \qquad a = 133\frac{\text{ft}}{\text{s}^2}$$

Finally,

$$0 = \left(\frac{W_2}{g}\right)a_1 - v_r\left(\frac{r}{g}\right) \qquad a_1 = v_r\left(\frac{r}{W_2}\right) \qquad a_1 = 200\frac{\text{ft}}{\text{s}^2}$$

Problem 15-125

The earthmover initially carries volume V of sand having a density ρ. The sand is unloaded horizontally through A dumping port P at a rate m' measured relative to the port. If the earthmover maintains a constant resultant tractive force **F** at its front wheels to provide forward motion, determine its acceleration when half the sand is dumped. When empty, the earthmover has a mass M. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.

Units Used:

$$\text{Mg} = 10^3\ \text{kg}$$

$$\text{kN} = 10^3\ \text{N}$$

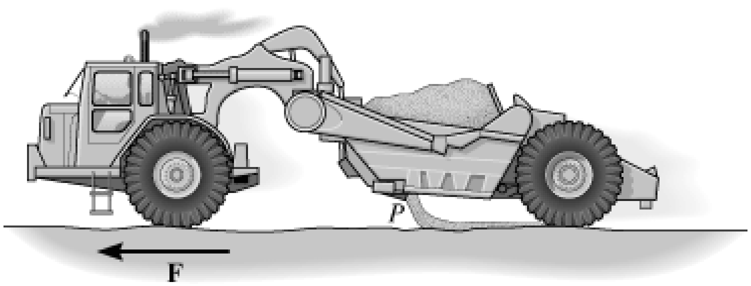
Given:

$$A = 2.5\ \text{m}^2 \qquad \rho = 1520\ \frac{\text{kg}}{\text{m}^3}$$

$$m' = 900\ \frac{\text{kg}}{\text{s}} \qquad V = 10\ \text{m}^3$$

$$F = 4\ \text{kN}$$

$$M = 30\ \text{Mg}$$



Solution:

When half the sand remains,

$$M_1 = M + \frac{1}{2}V\rho \qquad M_1 = 37600\ \text{kg}$$

$$\frac{d}{dt}m = m' = \rho v A \qquad v = \frac{m'}{\rho A} \quad v = 0.24\ \frac{m}{s}$$

$$\Sigma\ F = m\frac{d}{dt}v - \frac{d}{dt}m\,v_{DR} \qquad F = M_1\,a - m'\,v$$

$$a = \frac{F + m'\,v}{M_1} \qquad a = 0.11\ \frac{m}{s^2}$$

$$\boxed{a = 112\,\frac{mm}{s^2}}$$

Problem 15-126

The earthmover initially carries sand of volume V having density ρ. The sand is unloaded horizontally through a dumping port P of area A at rate of r measured relative to the port. Determine the resultant tractive force **F** at its front wheels if the acceleration of the earthmover is a when half the sand is dumped. When empty, the earthmover has mass M. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.

Units Used:

$kN = 10^3\ N$

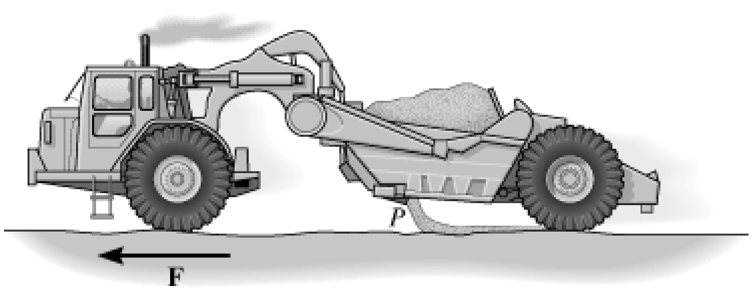
$Mg = 1000\ kg$

Given:

$V = 10\ m^3 \qquad r = 900\ \frac{kg}{s}$

$\rho = 1520\ \frac{kg}{m^3} \qquad a = 0.1\ \frac{m}{s^2}$

$A = 2.5\ m^2 \qquad M = 30\ Mg$



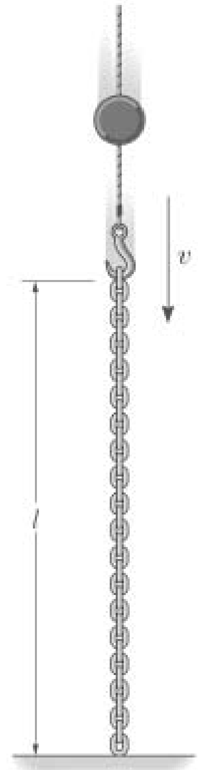
Solution:

When half the sand remains, $\quad M_1 = M + \frac{1}{2}V\rho \qquad M_1 = 37600\ kg$

$$\frac{d}{dt}m = r \qquad r = \rho v A \qquad v = \frac{r}{\rho A} \qquad v = 0.237\ \frac{m}{s}$$

$$F = m\frac{d}{dt}v - \frac{d}{dt}m\,v \qquad F = M_1\,a - r\,v \qquad \boxed{F = 3.55\,kN}$$

Problem 15-127



If the chain is lowered at a constant speed v, determine the normal reaction exerted on the floor as a function of time. The chain has a weight W and a total length l.

Given:

$$W = 5\ \frac{\text{lb}}{\text{ft}}$$

$$l = 20\ \text{ft}$$

$$v = 4\ \frac{\text{ft}}{\text{s}}$$

Solution:

At time t, the weight of the chain on the floor is $W = M\,g(v\,t)$

$$\frac{d}{dt}v = 0 \qquad M_t = M(v\,t)$$

$$\frac{d}{dt}M_t = M\,v$$

$$\Sigma\ F_s = M\frac{d}{dt}v + v_{Dt}\frac{d}{dt}M_t$$

$$R - M\,g(v\,t) = 0 + v(M\,v)$$

$$R = M\left(g\,v\,t + v^2\right) \qquad R = \frac{W}{g}\left(g\,v\,t + v^2\right)$$

***Problem 15-128**

The rocket has mass M including the fuel. Determine the constant rate at which the fuel must be burned so that its thrust gives the rocket a speed v in time t starting from rest. The fuel is expelled from the rocket at relative speed v_r. Neglect the effects of air resistance and assume that g is constant.

Given:

$$M = 65000\ \text{lb} \qquad v_r = 3000\ \frac{\text{ft}}{\text{s}}$$

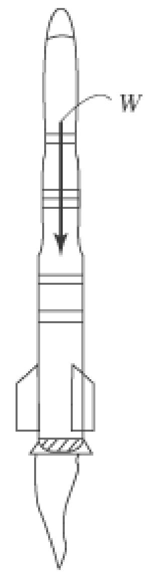
$$v = 200\ \frac{\text{ft}}{\text{s}}$$

$$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

$$t = 10\ \text{s}$$

Solution:

A System That Losses Mass: Here,



$$W = \left(m_0 - \frac{d}{dt}m_e\, t\right)g$$

Applying Eq. 15-29, we have

$$+\uparrow \Sigma\ F_z = m\frac{d}{dt}v - v_{DE}\frac{d}{dt}m_e$$ integrating we find

$$v = v_{DE}\ln\left(\frac{m_o}{m_0 - \frac{d}{dt}m_e\, t}\right) - g\, t$$

with $m_0 = M$ $v_{DE} = v_r$

$$v = v_r\ln\left(\frac{m_0}{m_0 - \frac{d}{dt}m_e\, t}\right) - g(t)$$

$$\frac{d}{dt}m_e = A = \left(\frac{-m_0}{e^{\frac{v+g\,t}{v_r}}} + m_0\right)\frac{1}{t} \qquad A = \left(\frac{-m_0}{e^{\frac{v+g\,t}{v_r}}} + m_0\right)\frac{1}{t} \qquad A = 43.3\,\frac{\text{slug}}{\text{s}}$$

Problem 15-129

The rocket has an initial mass m_0, including the fuel. For practical reasons desired for the crew, it is required that it maintain a constant upward acceleration a_0. If the fuel is expelled from the rocket at a relative speed v_{er}, determine the rate at which the fuel should be consumed to maintain the motion. Neglect air resistance, and assume that the gravitational acceleration is constant.

Solution:

$$a_0 = \frac{d}{dt}v$$

$$+\uparrow\ \Sigma F_s = m\frac{d}{dt}v - v_{er}\frac{d}{dt}m_e$$

$$-mg = ma_o - v_{er}\frac{d}{dt}m$$

$$v_{er}\frac{dm}{m} = (a_0 + g)dt$$

Since v_{er} is constant, integrating, with $t = 0$ when $m = m_0$ yields

$$v_{er}\ln\left(\frac{m}{m_0}\right) = \left(a_0 + g\right)t \qquad \frac{m}{m_0} = e^{\left(\frac{a_0+g}{v_{er}}\right)t}$$

The time rate fuel consumption is determined from Eq.[1]

$$\frac{d}{dt}m = m\frac{a_0 + g}{v_{er}} \qquad \boxed{\frac{d}{dt}m = m_0\left(\frac{a_0 + g}{v_{er}}\right)e^{\left(\frac{a_0+g}{v_{er}}\right)t}}$$

Note : v_{er} must be considered a negative quantity.

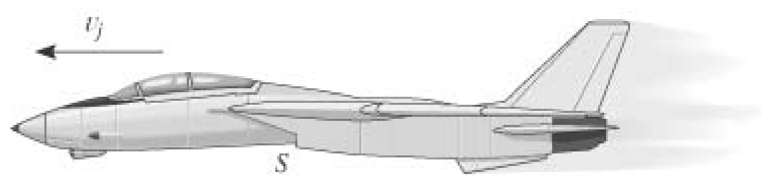
Problem 15-130

The jet airplane of mass M has constant speed v_j when it is flying along a horizontal straight line. Air enters the intake scoops S at rate r_1. If the engine burns fuel at the rate r_2 and the gas (air and fuel) is exhausted relative to the plane with speed v_e, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density ρ. *Hint*: Since mass both enters and exits the plane, Eqs. 15-29 and 15-30 must be combined.

Units Used:

$\text{Mg} = 1000\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$

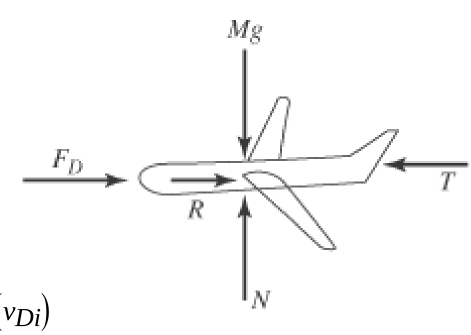


Given:

$M = 12\ \text{Mg}$ $\qquad r_2 = 0.4\ \frac{\text{kg}}{\text{s}}$

$v_j = 950\ \frac{\text{km}}{\text{hr}}$ $\qquad v_e = 450\ \frac{\text{m}}{\text{s}}$

$r_1 = 50\ \frac{\text{m}^3}{\text{s}}$ $\qquad \rho = 1.22\ \frac{\text{kg}}{\text{m}^3}$



Solution:

$$\Sigma F_s = m\frac{d}{dt}v - \frac{d}{dt}m_e\left(v_{DE}\right) + \frac{d}{dt}m_i\left(v_{Di}\right)$$

$$\frac{d}{dt}v = 0 \qquad v_{DE} = V_e \qquad v_{Di} = v_j \qquad \frac{d}{dt}m_i = r_1\rho$$

$$A = r_1 \rho \qquad \frac{d}{dt}m_e = r_2 + A \qquad B = r_2 + A$$

Forces T and F_D are incorporated as the last two terms in the equation,

$$F_D = v_e B - v_j A \qquad F_D = 11.5\ \text{kN}$$

Problem 15-131

The jet is traveling at speed v, angle θ with the horizontal. If the fuel is being spent at rate r_1 and the engine takes in air at r_2 whereas the exhaust gas (air and fuel) has relative speed v_e, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = kv^2$ The jet has weight W. *Hint:* See Prob. 15-130.

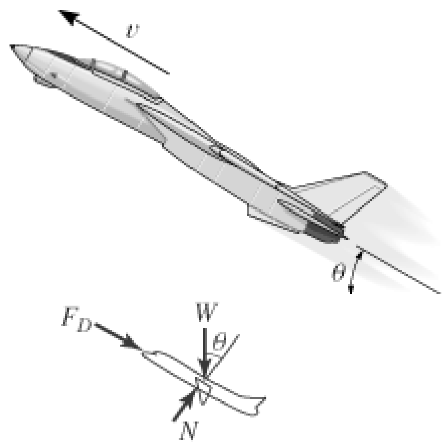
Given:

$$v = 500\ \frac{\text{mi}}{\text{hr}} \qquad v_e = 32800\ \frac{\text{ft}}{\text{s}}$$

$$\theta = 30\ \text{deg} \qquad k_1 = 0.7\ \text{lb}\ \frac{\text{s}^2}{\text{ft}^2}$$

$$r_1 = 3\ \frac{\text{lb}}{\text{s}} \qquad W = 15000\ \text{lb}$$

$$r_2 = 400\ \frac{\text{lb}}{\text{s}} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$



Solution:

$$\frac{d}{dt}m_i = \frac{r_2}{g_1} \qquad A_1 = r_2 \qquad \frac{d}{dt}m_e = \frac{r_1 + r_2}{g_1} \qquad B = r_1 + r_2 \qquad v_1 = v$$

$$+\ \Sigma F_s = m\frac{d}{dt}v - v_{De}\frac{d}{dt}m_e + v_{Di}\frac{d}{dt}m_i$$

$$-W\sin(\theta) - k_1 v_1^{\,2} = W a - v_e B + v_1 A_1$$

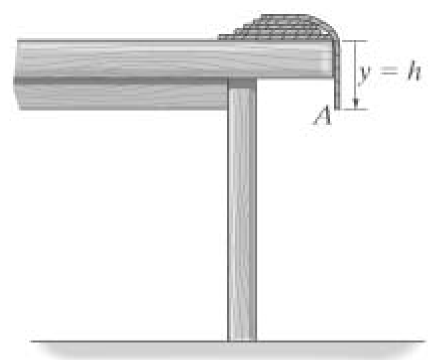
$$a = \frac{\left(-W\sin(\theta) - k_1 v_1^{\,2} + v_e\frac{B}{g} - v_1\frac{A_1}{g}\right)g}{W} \qquad a = 37.5\ \frac{\text{ft}}{\text{s}^2}$$

***Problem 15-132**

The rope has a mass m' per unit length. If the end length $y = h$ is draped off the edge of the table, and released, determine the velocity of its end A for any position y, as the rope uncoils and begins to fall.

Solution:

$$F_s = m\frac{d}{dt}v + v_{Di}\frac{d}{dt}m_i$$ At a time t, $m = m'y$ and $\frac{d}{dt}m_i = m'\frac{d}{dt}y = m'v$.



Here, $v_{Di} = v$, $\frac{d}{dt}v = g$.

$$m'gy = m'y\frac{d}{dt}v + v(m'v)$$

$$gy = y\frac{d}{dt}v + v^2$$ Since $v = \frac{d}{dt}y$, then $dt = \frac{dy}{v}$

$$gy = vy\frac{d}{dy}v + v^2$$

Multiply both sides by $2y\,dy$

$$2gy^2\,dy = 2vy^2\,dv + 2yv^2\,dy$$

$$\int 2gy^2\,dy = \int 1\,dv^2y^2 \qquad \frac{2}{3}gy^3 + C = v^2y^2$$

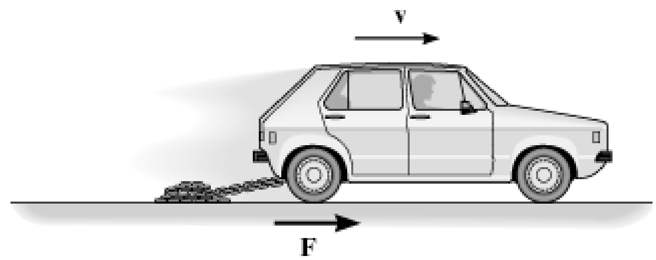
$$v = 0 \quad \text{at} \quad y = h \qquad \frac{2}{3}gh^3 + C = 0 \qquad C = \frac{-2}{3}gh^3$$

$$\frac{2}{3}gy^3 - \frac{2}{3}gh^3 = v^2y^2 \qquad v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)}$$

Problem 15-133

The car has a mass m_0 and is used to tow the smooth chain having a total length l and a mass per unit of length m'. If the chain is originally piled up, determine the tractive force **F** that must be supplied by the rear wheels of the car, necessary to maintain a constant speed v while the chain is being drawn out.

Solution:



$\xrightarrow{+}$ $\Sigma F_s = m\frac{d}{dt}v + v_{Di}\frac{d}{dt}m_i$

At a time t, $m = m_0 + c\,t$

Where, $c = \frac{d}{dt}m_i = m'\frac{d}{dt}x = m'v$

Here, $v_{Di} = v$ $\frac{d}{dt}v = 0$

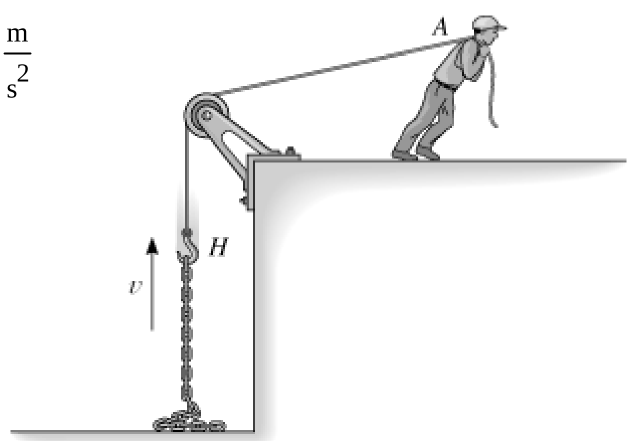
$F = (m_0 - m'vt)(0) + v(m'v) = m'v^2$ $F = m'v^2$

Problem 15-134

Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at A to raise the hook H with a constant speed v. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass density ρ.

Given:

$v = 0.4\,\frac{m}{s}$ $\rho = 2\,\frac{kg}{m}$ $g = 9.81\,\frac{m}{s^2}$



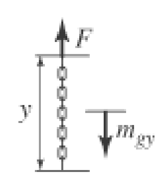
Solution:

$\frac{d}{dt}v = 0$ $y = v\,t$

$m_i = m\,y = m\,v\,t$

$\frac{d}{dt}m_i = m\,v$

$+\uparrow$ $\Sigma F_s = m\frac{d}{dt}v + v_{Di}\left(\frac{d}{dt}m_i\right)$



$F - m\,g\,v\,t = 0 + v\,m\,v$ $F = m\,g\,v\,t + v\,m\,v$

$F = \rho g\,v\,t + v^2$

$f_1 = \rho g\,v$ $f_1 = 7.85\,\frac{N}{s}$ $f_2 = \rho v^2$ $f_2 = 0.320\,N$

$F = f_1\,t + f_2$

Problem 16-1

A wheel has an initial clockwise angular velocity ω and a constant angular acceleration α. Determine the number of revolutions it must undergo to acquire a clockwise angular velocity ω_f. What time is required?

Units Used: $\text{rev} = 2\pi\ \text{rad}$

Given: $\omega = 10\ \dfrac{\text{rad}}{\text{s}}$ $\quad \alpha = 3\ \dfrac{\text{rad}}{\text{s}^2}$ $\quad \omega_f = 15\ \dfrac{\text{rad}}{\text{s}}$

Solution: $\omega_f^2 = \omega^2 + 2\alpha\,\theta$ $\quad \theta = \dfrac{\omega_f^2 - \omega^2}{2\alpha}$ $\quad \theta = 3.32\ \text{rev}$

$\omega_f = \omega + \alpha t$ $\quad t = \dfrac{\omega_f - \omega}{\alpha}$ $\quad t = 1.67\ \text{s}$

Problem 16-2

A flywheel has its angular speed increased uniformly from ω_1 to ω_2 in time t. If the diameter of the wheel is D, determine the magnitudes of the normal and tangential components of acceleration of a point on the rim of the wheel at time t, and the total distance the point travels during the time period.

Given: $\omega_1 = 15\ \dfrac{\text{rad}}{\text{s}}$ $\quad \omega_2 = 60\ \dfrac{\text{rad}}{\text{s}}$ $\quad t = 80\ \text{s}$ $\quad D = 2\ \text{ft}$

Solution: $r = \dfrac{D}{2}$

$\omega_2 = \omega_1 + \alpha t$ $\quad \alpha = \dfrac{\omega_2 - \omega_1}{t}$ $\quad \alpha = 0.56\ \dfrac{\text{rad}}{\text{s}^2}$

$a_t = \alpha r$ $\quad a_t = 0.563\ \dfrac{\text{ft}}{\text{s}^2}$

$a_n = \omega_2^2 r$ $\quad a_n = 3600\ \dfrac{\text{ft}}{\text{s}^2}$

$\theta = \dfrac{\omega_2^2 - \omega_1^2}{2\alpha}$ $\quad \theta = 3000\ \text{rad}$

$d = \theta r$ $\quad d = 3000\ \text{ft}$

Problem 16-3

The angular velocity of the disk is defined by $\omega = at^2 + b$. Determine the magnitudes of the velocity and acceleration of point A on the disk when $t = t_1$.

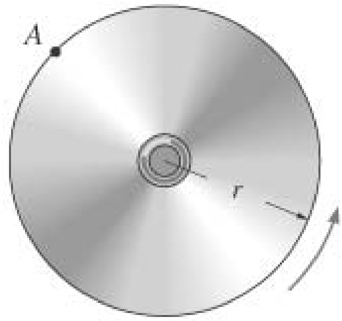
Given:

$$a = 5\ \frac{\text{rad}}{\text{s}^3}$$

$$b = 2\ \frac{\text{rad}}{\text{s}}$$

$$r = 0.8\ \text{m}$$

$$t_1 = 0.5\ \text{s}$$



Solution: $t = t_1$

$$\omega = a t^2 + b \qquad \omega = 3.25\ \frac{\text{rad}}{\text{s}}$$

$$\alpha = 2a t \qquad \alpha = 5.00\ \frac{\text{rad}}{\text{s}^2}$$

$$v = \omega r \qquad v = 2.60\ \frac{\text{m}}{\text{s}}$$

$$a = \sqrt{(\alpha r)^2 + \left(\omega^2 r\right)^2} \qquad a = 9.35\ \frac{\text{m}}{\text{s}^2}$$

***Problem 16-4**

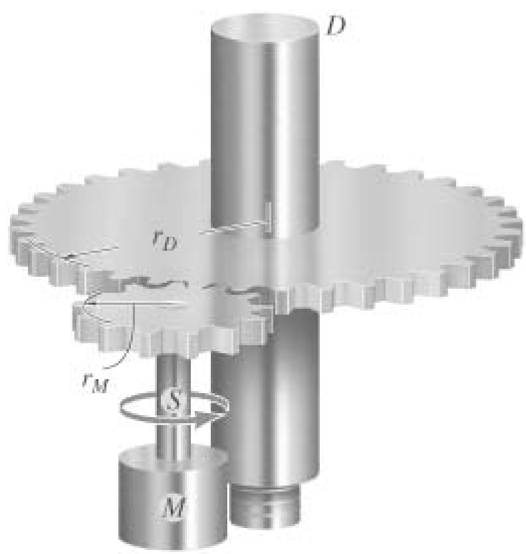
The figure shows the internal gearing of a "spinner" used for drilling wells. With constant angular acceleration, the motor M rotates the shaft S to angular velocity ω_M in time t starting from rest. Determine the angular acceleration of the drill-pipe connection D and the number of revolutions it makes during the start up at t.

Units Used: $\text{rev} = 2\pi$

Given:

$$\omega_M = 100\ \frac{\text{rev}}{\text{min}} \qquad r_M = 60\ \text{mm}$$

$$r_D = 150\ \text{mm} \qquad t = 2\ \text{s}$$



Solution:

$$\omega_M = \alpha_M t$$

$$\alpha_M = \frac{\omega_M}{t} \qquad \alpha_M = 5.24\ \frac{\text{rad}}{\text{s}^2}$$

$$\alpha_M r_M = \alpha_D r_D$$

$$\alpha_D = \alpha_M \left(\frac{r_M}{r_D}\right) \qquad \alpha_D = 2.09 \frac{\text{rad}}{\text{s}^2}$$

$$\theta = \frac{1}{2}\alpha_D t^2 \qquad \theta = 0.67 \text{ rev}$$

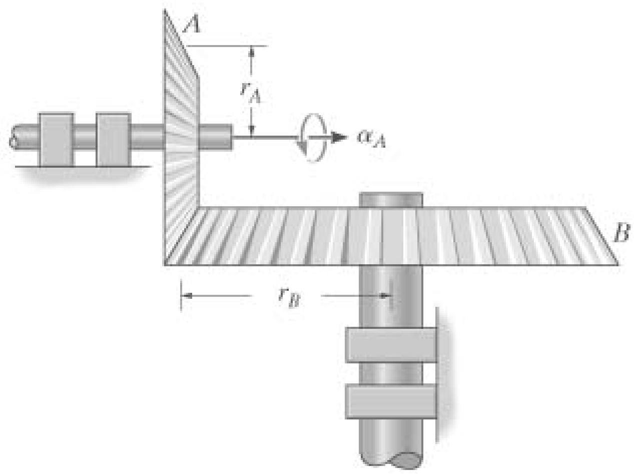
Problem 16-5

If gear A starts from rest and has a constant angular acceleration α_A, determine the time needed for gear B to attain an angular velocity ω_B.

Given:

$$\alpha_A = 2 \frac{\text{rad}}{\text{s}^2} \qquad r_B = 0.5 \text{ ft}$$

$$\omega_B = 50 \frac{\text{rad}}{\text{s}} \qquad r_A = 0.2 \text{ ft}$$



Solution:

The point in contact with both gears has a speed of

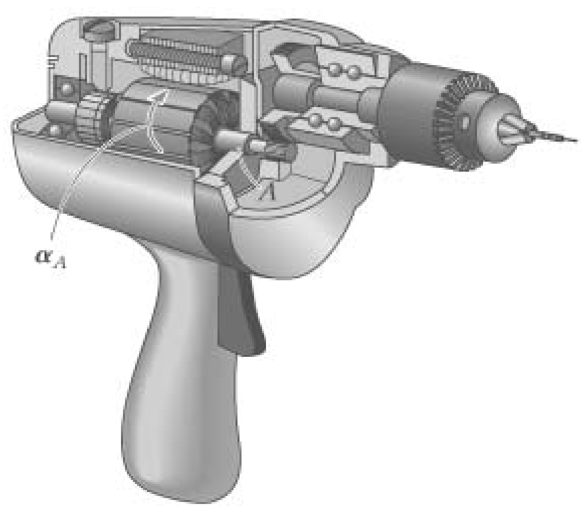
$$v_p = \omega_B r_B \qquad v_p = 25.00 \frac{\text{ft}}{\text{s}}$$

Thus,

$$\omega_A = \frac{v_p}{r_A} \qquad \omega_A = 125.00 \frac{\text{rad}}{\text{s}}$$

So that $\quad \omega = \alpha_c t \quad t = \dfrac{\omega_A}{\alpha_A} \quad t = 62.50 \text{ s}$

Problem 16-6



If the armature A of the electric motor in the drill has a constant angular acceleration α_A, determine its angular velocity and angular displacement at time t. The motor starts from rest.

Given:

$$\alpha_A = 20 \frac{\text{rad}}{\text{s}^2} \qquad t = 3 \text{ s}$$

Solution:

$$\omega = \alpha_c t \qquad \omega = \alpha_A t \qquad \omega = 60.00 \frac{\text{rad}}{\text{s}}$$

$\theta = \frac{1}{2}\alpha_A t^2 \qquad \theta = 90.00\,\text{rad}$

Problem 16-7

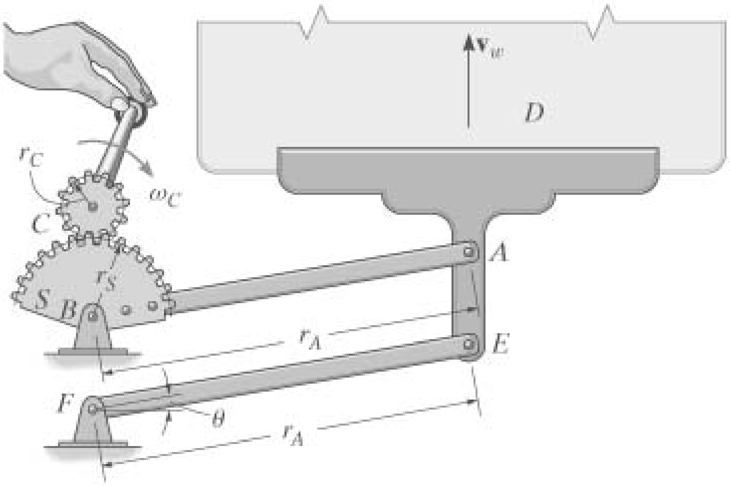
The mechanism for a car window winder is shown in the figure. Here the handle turns the small cog *C*, which rotates the spur gear *S*, thereby rotating the fixed-connected lever *AB* which raises track *D* in which the window rests. The window is free to slide on the track. If the handle is wound with angular velocity ω_c, determine the speed of points *A* and *E* and the speed v_w of the window at the instant θ.

Given:

$\omega_C = 0.5\,\frac{\text{rad}}{\text{s}} \qquad r_C = 20\text{ mm}$

$\theta = 30\text{ deg} \qquad r_S = 50\text{ mm}$

$r_A = 200\text{ mm}$



Solution:

$v_C = \omega_C r_C$

$v_C = 0.01\,\frac{\text{m}}{\text{s}}$

$\omega_S = \frac{v_C}{r_S} \qquad \omega_S = 0.20\,\frac{\text{rad}}{\text{s}}$

$v_A = v_E = \omega_S r_A$

$v_A = \omega_S r_A \qquad v_A = v_E = 40.00\,\frac{\text{mm}}{\text{s}}$

Points *A* and *E* move along circular paths. The vertical component closes the window.

$v_w = v_A\cos(\theta) \qquad v_w = 34.6\,\frac{\text{mm}}{\text{s}}$

***Problem 16-8**

The pinion gear *A* on the motor shaft is given a constant angular acceleration α. If the gears *A* and *B* have the dimensions shown, determine the angular velocity and angular displacement of the output shaft *C*, when $t = t_1$ starting from rest. The shaft is fixed to *B* and turns with it.

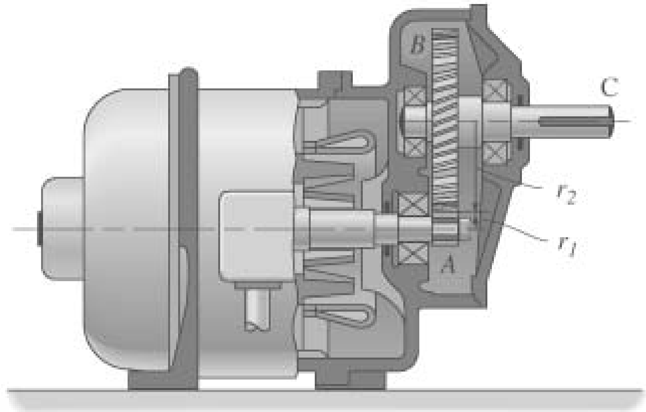
Given:

$\alpha = 3\,\frac{\text{rad}}{\text{s}^2}$

$t_1 = 2\text{ s}$

$r_1 = 35\text{ mm}$

$r_2 = 125\text{ mm}$



Solution:

$\alpha_A = \alpha$

$r_1\alpha_A = r_2\alpha_C \qquad \alpha_C = \left(\frac{r_1}{r_2}\right)\alpha_A$

$\omega_C = \alpha_C t_1 \qquad \omega_C = 1.68\,\frac{\text{rad}}{\text{s}}$

$\theta_C = \frac{1}{2}\alpha_C t_1^{\,2} \qquad \theta_C = 1.68\text{ rad}$

Problem 16-9

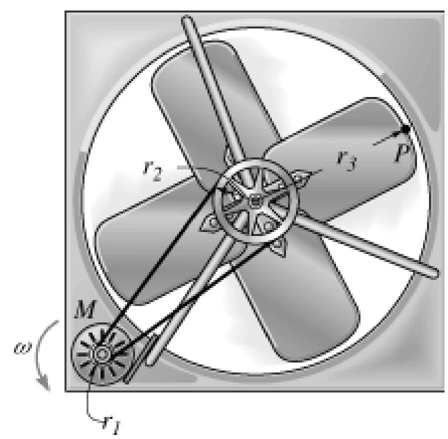
The motor M begins rotating at an angular rate $\omega = a(1 - e^{bt})$. If the pulleys and fan have the radii shown, determine the magnitudes of the velocity and acceleration of point P on the fan blade when $t = t_1$. Also, what is the maximum speed of this point?

Given:

$a = 4\,\frac{\text{rad}}{\text{s}} \qquad r_1 = 1\text{ in}$

$b = -1\,\frac{1}{\text{s}} \qquad r_2 = 4\text{ in}$

$t_1 = 0.5\text{ s} \qquad r_3 = 16\text{ in}$



Solution:

$t = t_1 \qquad r_1\omega_1 = r_2\omega_2$

$\omega_1 = a\left(1 - e^{b\,t}\right) \qquad \omega_2 = \left(\frac{r_1}{r_2}\right)\omega_1$

$v_P = r_3\omega_2 \qquad v_P = 6.30\,\frac{\text{in}}{\text{s}}$

$$\alpha_1 = -a b e^{b t} \qquad \alpha_2 = \left(\frac{r_1}{r_2}\right)\alpha_1$$

$$a_P = \sqrt{\left(\alpha_2 r_3\right)^2 + \left(\omega_2{}^2 r_3\right)^2} \qquad a_P = 10.02\ \frac{\text{in}}{\text{s}^2}$$

As t approaches ∞

$$\omega_1 = a \qquad \omega_f = \frac{r_1}{r_2}\omega_1 \qquad v_f = r_3\omega_f \qquad v_f = 16.00\ \frac{\text{in}}{\text{s}}$$

Problem 16-10

The disk is originally rotating at angular velocity ω_0. If it is subjected to a constant angular acceleration α, determine the magnitudes of the velocity and the n and t components of acceleration of point A at the instant t.

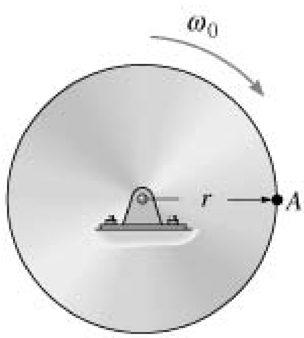
Given:

$$\omega_0 = 8\ \frac{\text{rad}}{\text{s}}$$

$$\alpha = 6\ \frac{\text{rad}}{\text{s}^2}$$

$$t = 0.5\ \text{s}$$

$$r = 2\ \text{ft}$$



Solution:

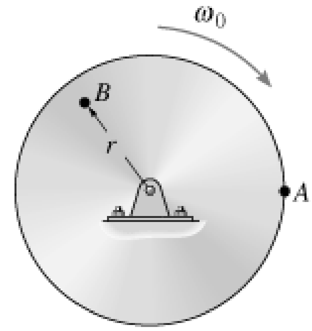
$$\omega = \omega_0 + \alpha t \qquad v_A = r\omega \qquad v_A = 22.00\ \frac{\text{ft}}{\text{s}}$$

$$a_t = r\alpha \qquad a_t = 12.00\ \frac{\text{ft}}{\text{s}^2}$$

$$a_n = r\omega^2 \qquad a_n = 242.00\ \frac{\text{ft}}{\text{s}^2}$$

Problem 16-11

The disk is originally rotating at angular velocity ω_0. If it is subjected to a constant angular acceleration α, determine the magnitudes of the velocity and the n and t components of acceleration of point B just after the wheel undergoes a rotation θ.



Given:

$$\text{rev} = 2\pi\ \text{rad} \qquad \alpha = 6\ \frac{\text{rad}}{\text{s}^2} \qquad r = 1.5\ \text{ft}$$

$$\omega_0 = 8\ \frac{\text{rad}}{\text{s}} \qquad \theta = 2\ \text{rev}$$

Solution:

$$\omega = \sqrt{\omega_0^{\ 2} + 2\,\alpha\,\theta} \qquad \omega = 14.66\ \frac{\text{rad}}{\text{s}}$$

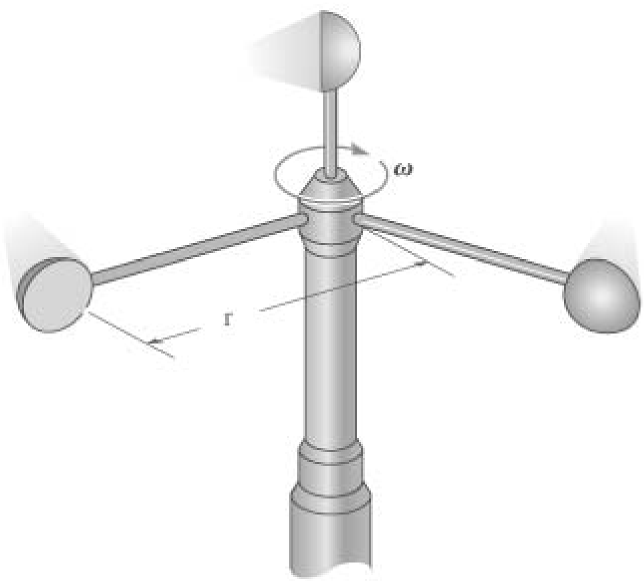
$$v_B = r\omega \qquad v_B = 22\ \frac{\text{ft}}{\text{s}}$$

$$a_{Bt} = r\alpha \qquad a_{Bt} = 9\ \frac{\text{ft}}{\text{s}^2}$$

$$a_{Bn} = r\omega^2 \qquad a_{Bn} = 322\ \frac{\text{ft}}{\text{s}^2}$$

***Problem 16-12**

The anemometer measures the speed of the wind due to the rotation of the three cups. If during a time period t_1 a wind gust causes the cups to have an angular velocity $\omega = (At^2 + B)$, determine (a) the speed of the cups when $t = t_2$, (b) the total distance traveled by each cup during the time period t_1, and (c) the angular acceleration of the cups when $t = t_2$. Neglect the size of the cups for the calculation.



Given:

$$t_1 = 3\ \text{s} \qquad t_2 = 2\ \text{s} \qquad r = 1.5\ \text{ft}$$

$$A = 2\ \frac{1}{\text{s}^3} \qquad B = 3\ \frac{1}{\text{s}}$$

Solution:

$\omega_2 = A t_2^{\,2} + B \qquad v_2 = r\omega_2 \qquad v_2 = 16.50\,\frac{\text{ft}}{\text{s}}$

$d = r\int_0^{t_1} A t^2 + B\,\mathrm{d}t \qquad d = 40.50\,\text{ft}$

$\alpha = \frac{\mathrm{d}\omega_2}{\mathrm{d}t} \qquad \alpha = 2A t_2 \qquad \alpha = 8.00\,\frac{\text{rad}}{\text{s}^2}$

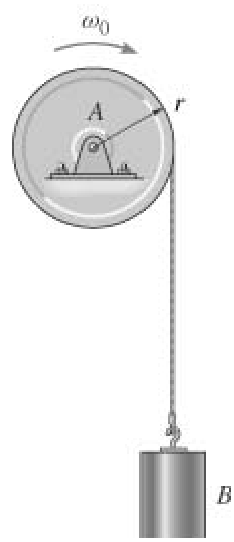
Problem 16-13

A motor gives disk A a clockwise angular acceleration $\alpha_A = at^2 + b$. If the initial angular velocity of the disk is ω_0, determine the magnitudes of the velocity and acceleration of block B when $t = t_1$.

Given:

$a = 0.6\,\frac{\text{rad}}{\text{s}^4} \qquad \omega_0 = 6\,\frac{\text{rad}}{\text{s}} \qquad r = 0.15\ \text{m}$

$b = 0.75\,\frac{\text{rad}}{\text{s}^2} \qquad t_1 = 2\ \text{s}$



Solution:

$\alpha_A = a t_1^{\,2} + b$

$\omega_A = \frac{a}{3} t_1^{\,3} + b t_1 + \omega_0$

$v_B = \omega_A r \qquad v_B = 1.365\,\frac{\text{m}}{\text{s}}$

$a_B = \alpha_A r \qquad a_B = 0.472\,\frac{\text{m}}{\text{s}^2}$

Problem 16-14

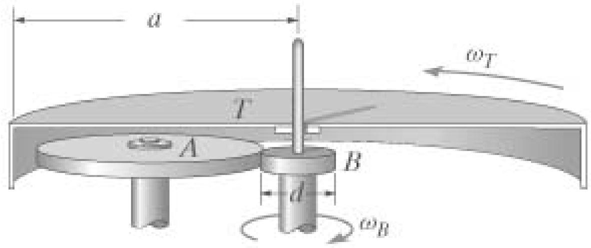
The turntable T is driven by the frictional idler wheel A, which simultaneously bears against the inner rim of the turntable and the motor-shaft spindle B. Determine the required diameter d of the spindle if the motor turns it with angular velocity ω_B and it is required that the turntable rotate with angular velocity ω_T.

Given:

$\omega_B = 25 \frac{\text{rad}}{\text{s}}$

$\omega_T = 2 \frac{\text{rad}}{\text{s}}$

$a = 9 \text{ in}$



Solution:

$$\omega_B \frac{d}{2} = \omega_A \left(\frac{a - \frac{d}{2}}{2} \right) \qquad \omega_A = \frac{\omega_B d}{a - \frac{d}{2}}$$

$$\omega_A \left(\frac{a - \frac{d}{2}}{2} \right) = \omega_T a \qquad \frac{\omega_B d}{2} = \omega_T a \qquad d = \frac{2\omega_T a}{\omega_B} \qquad d = 1.44 \text{ in}$$

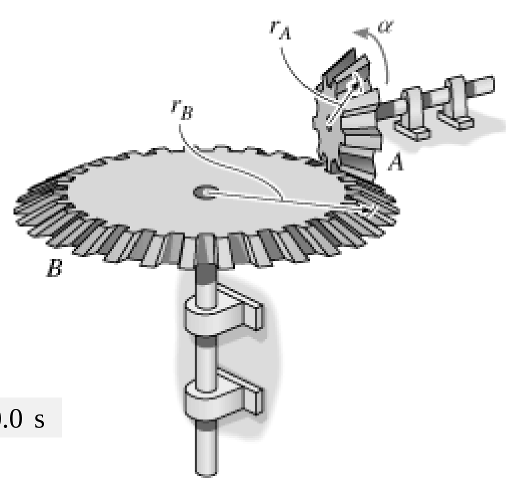
Problem 16-15

Gear A is in mesh with gear B as shown. If A starts from rest and has constant angular acceleration α_A, determine the time needed for B to attain an angular velocity ω_B.

Given:

$\alpha_A = 2 \frac{\text{rad}}{\text{s}^2} \qquad r_A = 25 \text{ mm}$

$\omega_B = 50 \frac{\text{rad}}{\text{s}} \qquad r_B = 100 \text{ mm}$



Solution:

$$\alpha_A r_A = \alpha_B r_B \qquad \alpha_B = \left(\frac{r_A}{r_B} \right) \alpha_A$$

$$\omega_B = \alpha_B t \qquad t = \frac{\omega_B}{\alpha_B} \qquad t = 100.0 \text{ s}$$

***Problem 16-16**

The blade on the horizontal-axis windmill is turning with an angular velocity ω_0. Determine the distance point *P* on the tip of the blade has traveled if the blade attains an angular velocity ω in time *t*. The angular acceleration is constant. Also, what is the magnitude of the acceleration of this point at time *t*?

Given:

$\omega_0 = 2\ \frac{\text{rad}}{\text{s}}$ $\omega = 5\ \frac{\text{rad}}{\text{s}}$

$t = 3\ \text{s}$ $r_p = 15\ \text{ft}$

Solution:

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$d_p = r_p \int_0^t \omega_0 + \alpha t\ dt \qquad d_p = 157.50\ \text{ft}$$

$$a_n = r_p \omega^2 \qquad a_t = r_p \alpha$$

$$a_p = \left| \begin{pmatrix} a_n \\ a_t \end{pmatrix} \right| \qquad a_p = 375.30\ \frac{\text{ft}}{\text{s}^2}$$

Problem 16-17

The blade on the horizontal-axis windmill is turning with an angular velocity ω_0. If it is given an angular acceleration α, determine the angular velocity and the magnitude of acceleration of point *P* on the tip of the blade at time *t*.

Given:

$\omega_0 = 2\ \frac{\text{rad}}{\text{s}}$ $\alpha = 0.6\ \frac{\text{rad}}{\text{s}^2}$ $r = 15\ \text{ft}$ $t = 3\ \text{s}$

Solution:

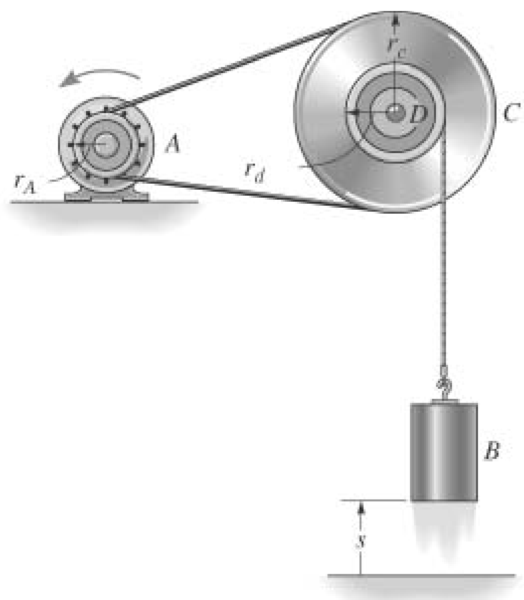
$\omega = \omega_0 + \alpha t$ $\omega = 3.80\ \frac{\text{rad}}{\text{s}}$

$a_{pt} = \alpha r$ $a_{pt} = 9.00\ \frac{\text{ft}}{\text{s}^2}$

$a_{pn} = \omega^2 r$ $a_{pn} = 216.60\ \frac{\text{ft}}{\text{s}^2}$

$a_p = \sqrt{a_{pt}^2 + a_{pn}^2}$ $a_p = 217\ \frac{\text{ft}}{\text{s}^2}$

Problem 16-18



Starting from rest when $s = 0$, pulley A is given an angular acceleration $\alpha_A = k\theta$. Determine the speed of block B when it has risen to $s = s_1$. The pulley has an inner hub D which is fixed to C and turns with it.

Given:

$k = 6\ \text{s}^{-2}$ $r_C = 150\ \text{mm}$

$s_1 = 6\ \text{m}$ $r_D = 75\ \text{mm}$

$r_A = 50\ \text{mm}$

Solution:

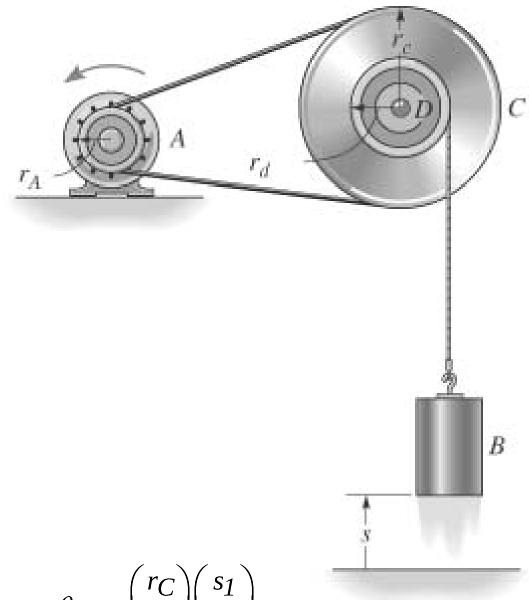
$\theta_A r_A = \theta_C r_C$ $\theta_C r_D = s_1$ $\theta_A = \left(\frac{r_C}{r_A}\right)\frac{s_1}{r_D}$

$\alpha_A = k\theta$ $\frac{\omega_A^2}{2} = k\left(\frac{\theta_A^2}{2}\right)$ $\omega_A = \sqrt{k}\,\theta_A$

$\omega_A r_A = \omega_C r_C$ $\omega_C = \left(\frac{r_A}{r_C}\right)\omega_A$ $v_B = \omega_C r_D$ $v_B = 14.70\ \frac{\text{m}}{\text{s}}$

Problem 16-19

Starting from rest when $s = 0$, pulley A is given a constant angular acceleration α_A. Determine the speed of block B when it has risen to $s = s_1$. The pulley has an inner hub D which is fixed to C and turns with it.



Given:

$\alpha_A = 6\ \frac{\text{rad}}{\text{s}^2}$

$s_1 = 6\ \text{m}$

$r_A = 50\ \text{mm}$

$r_C = 150\ \text{mm}$

$r_D = 75\ \text{mm}$

Solution:

$\theta_A r_A = \theta_C r_C$ $\qquad \theta_C r_D = s_1$ $\qquad \theta_A = \left(\frac{r_C}{r_A}\right)\left(\frac{s_1}{r_D}\right)$

$\frac{\omega_A^2}{2} = \alpha_A \theta_A$ $\qquad \omega_A = \sqrt{2\alpha_A\theta_A}$

$\omega_A r_A = \omega_C r_C$ $\qquad \omega_C = \frac{r_A}{r_C}\omega_A$ $\qquad v_B = \omega_C r_D$ $\qquad v_B = 1.34\ \frac{\text{m}}{\text{s}}$

***Problem 16-20**

Initially the motor on the circular saw turns its drive shaft at $\omega = kt^{2/3}$. If the radii of gears A and B are r_A and r_B respectively, determine the magnitudes of the velocity and acceleration of a tooth C on the saw blade after the drive shaft rotates through angle $\theta = \theta_1$ starting from rest.

Given:

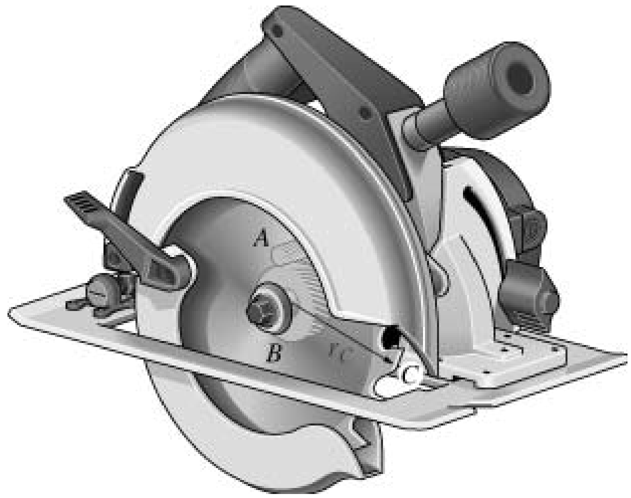
$r_A = 0.25\ \text{in}$

$r_B = 1\ \text{in}$

$r_C = 2.5\ \text{in}$

$\theta_1 = 5\ \text{rad}$

$k = 20\ \dfrac{\text{rad}}{\text{s}^{\frac{5}{3}}}$



Solution:

$$\omega_A = k t^{\frac{2}{3}} \qquad \theta_A = \frac{3}{5} k t^{\frac{5}{3}}$$

$$t_1 = \left(\frac{5\theta_1}{3k}\right)^{\frac{3}{5}} \qquad t_1 = 0.59\ \text{s}$$

$$\omega_A = k t_1^{\frac{2}{3}} \qquad \omega_A = 14.09\ \frac{\text{rad}}{\text{s}}$$

$$\omega_B = \frac{r_A}{r_B}\omega_A \qquad \omega_B = 3.52\ \frac{\text{rad}}{\text{s}}$$

$$\alpha_A = \frac{2}{3} k t_1^{\frac{-1}{3}} \qquad \alpha_A = 15.88\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_B = \frac{r_A}{r_B}\alpha_A \qquad \alpha_B = 3.97\ \frac{\text{rad}}{\text{s}^2}$$

$$v_C = r_C\omega_B \qquad v_C = 8.81\ \frac{\text{in}}{\text{s}}$$

$$a_C = \sqrt{\left(r_C\alpha_B\right)^2 + \left(r_C\omega_B^{\,2}\right)^2} \qquad a_C = 32.6\ \frac{\text{in}}{\text{s}^2}$$

Problem 16-21

Due to the screw at E, the actuator provides linear motion to the arm at F when the motor turns the gear at A. If the gears have the radii listed, and the screw at E has pitch p, determine the speed at F when the motor turns A with angular velocity ω_A. *Hint*: The screw pitch indicates the amount of advance of the screw for each full revolution.

Given:

$\text{rev} = 2\pi \text{ rad}$

$p = 2 \frac{\text{mm}}{\text{rev}}$

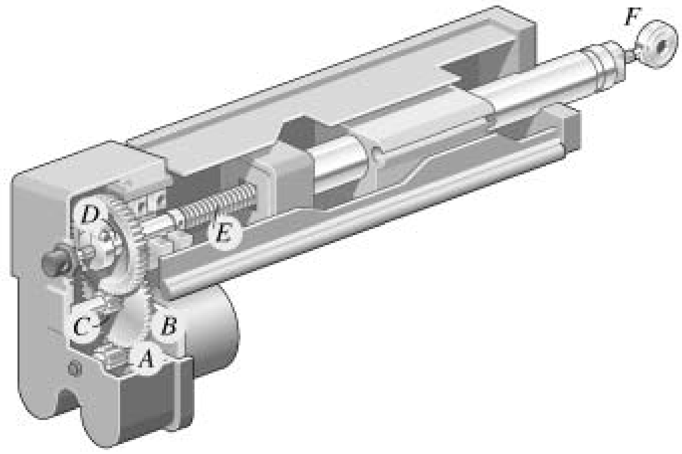
$\omega_A = 20 \frac{\text{rad}}{\text{s}}$

$r_A = 10 \text{ mm}$

$r_B = 50 \text{ mm}$

$r_C = 15 \text{ mm}$

$r_D = 60 \text{ mm}$



Solution:

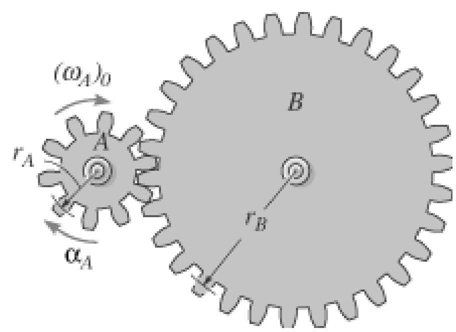
$\omega_A r_A = \omega_B r_B \qquad \omega_B r_C = \omega_D r_D$

$\omega_D = \left(\frac{r_A}{r_B}\right)\left(\frac{r_C}{r_D}\right)\omega_A \qquad \omega_D = 1 \frac{\text{rad}}{\text{s}}$

$v_F = \omega_D p \qquad v_F = 0.318 \frac{\text{mm}}{\text{s}}$

Problem 16-22

A motor gives gear A angular acceleration $\alpha_A = a\theta^3 + b$. If this gear is initially turning with angular velocity ω_{A0}, determine the angular velocity of gear B after A undergoes an angular displacement θ_1.



Given:

$\text{rev} = 2\pi \text{ rad}$

$a = 0.25 \frac{\text{rad}}{\text{s}^2}$

$b = 0.5 \frac{\text{rad}}{\text{s}^2}$

$\omega_{A0} = 20 \frac{\text{rad}}{\text{s}}$

$r_A = 0.05\ \text{m}$

$r_B = 0.15\ \text{m}$

$\theta_1 = 10\ \text{rev}$

Solution:

$\alpha_A = a\theta^3 + b$ $\qquad \omega_A{}^2 = \omega_{A0}{}^2 + 2\int_0^{\theta_1} \left(a\theta^3 + b\right) d\theta$

$\omega_A = \sqrt{\omega_{A0}{}^2 + 2\int_0^{\theta_1} a\theta^3 + b\ d\theta}$ $\qquad \omega_A = 1395.94\ \frac{\text{rad}}{\text{s}}$

$\omega_B = \frac{r_A}{r_B}\omega_A$ $\qquad \omega_B = 465\ \frac{\text{rad}}{\text{s}}$

Problem 16-23

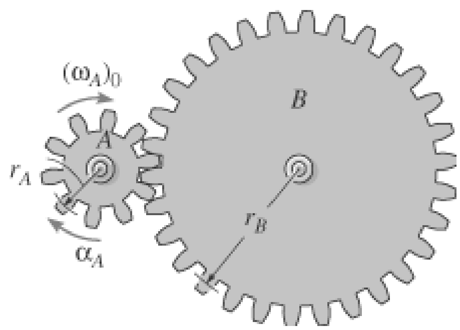
A motor gives gear A angular acceleration $\alpha_A = kt^3$. If this gear is initially turning with angular velocity ω_{A0}, determine the angular velocity of gear B when $t = t_1$.

Given:

$k = 4\ \frac{\text{rad}}{\text{s}^5}$ $\qquad t_1 = 2\ \text{s}$

$r_A = 0.05\ \text{m}$

$\omega_{A0} = 20\ \frac{\text{rad}}{\text{s}}$ $\qquad r_B = 0.15\ \text{m}$



Solution: $\qquad t = t_1$

$\alpha_A = kt^3$ $\qquad \omega_A = \left(\frac{k}{4}\right)t^4 + \omega_{A0}$ $\qquad \omega_A = 36.00\ \frac{\text{rad}}{\text{s}}$

$\omega_B = \frac{r_A}{r_B}\omega_A$ $\qquad \omega_B = 12.00\ \frac{\text{rad}}{\text{s}}$

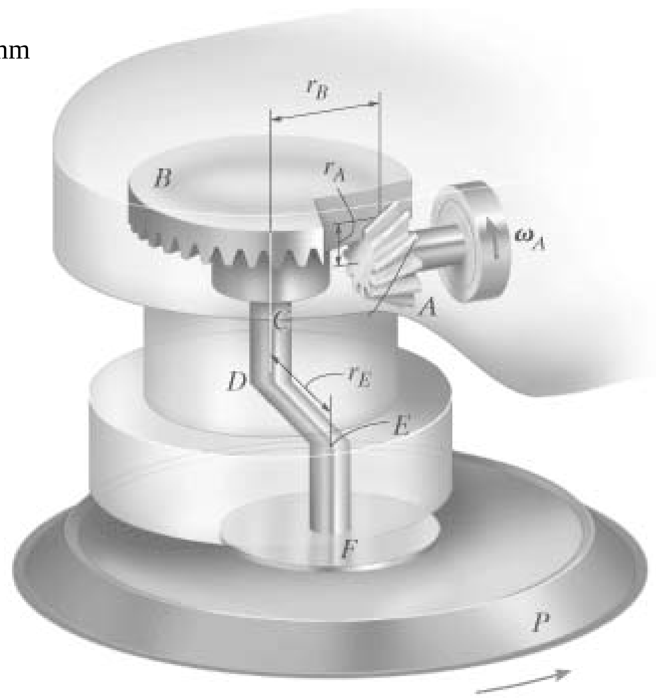
***Problem 16-24**

For a short time a motor of the random-orbit sander drives the gear A with an angular velocity $\omega_A = A(t^3 + Bt)$. This gear is connected to gear B, which is fixed connected to the shaft CD. The end of this shaft is connected to the eccentric spindle EF and pad P, which causes the pad to orbit around shaft CD at a radius r_E. Determine the magnitudes of the velocity and the tangential and normal components of acceleration of the spindle EF at time t after starting from rest.

Given:

$r_A = 10$ mm $\quad r_B = 40$ mm $\quad r_E = 15$ mm

$A = 40 \dfrac{\text{rad}}{\text{s}^4} \quad B = 6\ \text{s}^2 \quad t = 2$ s



Solution:

$\omega_A = A\left(t^3 + Bt\right) \qquad \omega_B = \dfrac{r_A}{r_B}\omega_A$

$\alpha_A = A\left(3t^2 + B\right) \qquad \alpha_B = \dfrac{r_A}{r_B}\alpha_A$

$v = \omega_B r_E \qquad \boxed{v = 3.00 \dfrac{\text{m}}{\text{s}}}$

$a_t = \alpha_B r_E \qquad \boxed{a_t = 2.70 \dfrac{\text{m}}{\text{s}^2}}$

$a_n = \omega_B^{\ 2} r_E \qquad \boxed{a_n = 600.00 \dfrac{\text{m}}{\text{s}^2}}$

Problem 16-25

For a short time the motor of the random-orbit sander drives the gear A with an angular velocity $\omega_A = k\theta^2$. This gear is connected to gear B, which is fixed connected to the shaft CD. The end of this shaft is connected to the eccentric spindle EF and pad P, which causes the pad to orbit around shaft CD at a radius r_E. Determine the magnitudes of the velocity and the tangential and normal components of acceleration of the spindle EF when $\theta = \theta_1$ starting from rest.

Units Used:

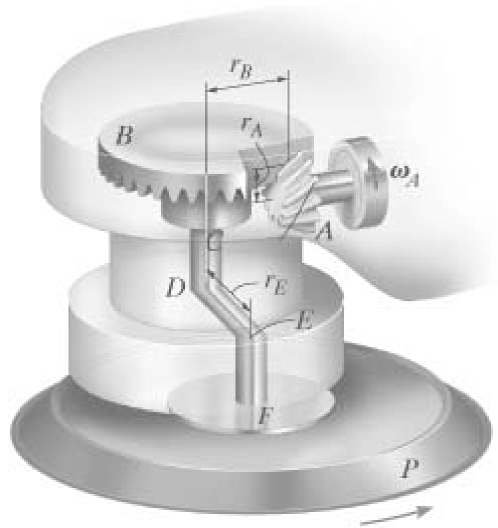
$\text{rev} = 2\pi\ \text{rad}$

Given:

$k = 5 \frac{\text{rad}}{\text{s}}$ $\quad r_A = 10\text{ mm}$

$r_B = 40\text{ mm}$

$\theta_1 = 0.5\text{ rev}$ $\quad r_E = 15\text{ mm}$



Solution:

$$\omega_A = k\theta_1^{\,2}$$

$$\alpha_A = \left(k\theta_1^{\,2}\right)\left(2k\theta_1\right)$$

$$\omega_B = \frac{r_A}{r_B}\omega_A \qquad \alpha_B = \frac{r_A}{r_B}\alpha_A$$

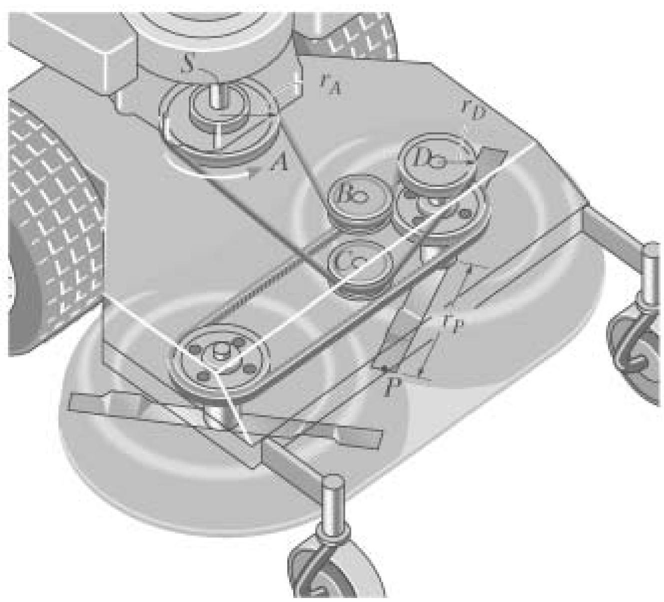
$$v = \omega_B r_E \qquad v = 0.19\ \frac{\text{m}}{\text{s}}$$

$$a_t = \alpha_B r_E \qquad a_t = 5.81\ \frac{\text{m}}{\text{s}^2}$$

$$a_n = \omega_B^{\,2} r_E \qquad a_n = 2.28\ \frac{\text{m}}{\text{s}^2}$$

Problem 16-26

The engine shaft S on the lawnmower rotates at a constant angular rate ω_A. Determine the magnitudes of the velocity and acceleration of point P on the blade and the distance P travels in time t. The shaft S is connected to the driver pulley A, and the motion is transmitted to the belt that passes over the idler pulleys at B and C and to the pulley at D. This pulley is connected to the blade and to another belt that drives the other blade.



Given:

$\omega_A = 40 \frac{\text{rad}}{\text{s}}$ $\quad r_P = 200\text{ mm}$

$r_A = 75\text{ mm}$ $\quad \alpha_A = 0$

$r_D = 50\text{ mm}$ $\quad t = 3\text{ s}$

Solution:

$$\omega_D = \frac{r_A}{r_D}\omega_A$$

$$v_P = \omega_D r_P \qquad v_P = 12.00\ \frac{\text{m}}{\text{s}}$$

$$a_P = \omega_D^{\,2} r_P \qquad a_P = 720.00\ \frac{\text{m}}{\text{s}^2}$$

$$s_P = r_P\left(\frac{\omega_A t\, r_A}{r_D}\right) \qquad s_P = 36.00\ \text{m}$$

Problem 16-27

The operation of "reverse" for a three-speed automotive transmission is illustrated schematically in the figure. If the crank shaft *G* is turning with angular speed ω_G, determine the angular speed of the drive shaft *H*. Each of the gears rotates about a fixed axis. Note that gears *A* and *B*, *C* and *D*, and *E* and *F* are in mesh. The radii of each of these gears are listed.

Given:

$$\omega_G = 60\ \frac{\text{rad}}{\text{s}}$$

$r_A = 90\ \text{mm}$

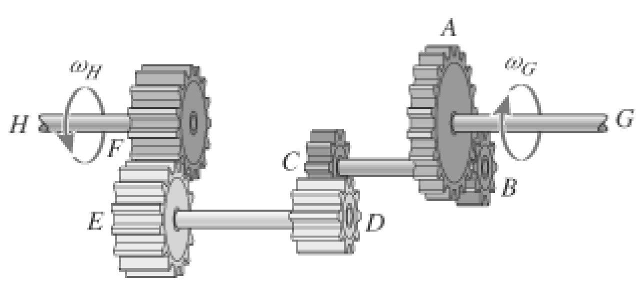
$r_B = 30\ \text{mm}$

$r_C = 30\ \text{mm}$

$r_D = 50\ \text{mm}$

$r_E = 70\ \text{mm}$

$r_F = 60\ \text{mm}$



Solution:

$$\omega_B = \frac{r_A}{r_B}\omega_G \qquad \omega_B = 180.00\ \frac{\text{rad}}{\text{s}}$$

$$\omega_D = \frac{r_C}{r_D}\omega_B \qquad \omega_D = 108.00\ \frac{\text{rad}}{\text{s}}$$

$$\omega_H = \frac{r_E}{r_F}\omega_D \qquad \omega_H = 126.00\ \frac{\text{rad}}{\text{s}}$$

***Problem 16-28**

Morse Industrial manufactures the speed reducer shown. If a motor drives the gear shaft S with an angular acceleration $\alpha = ke^{bt}$, determine the angular velocity of shaft E at time t after starting from rest. The radius of each gear is listed. Note that gears B and C are fixed connected to the same shaft.

Given:

$r_A = 20\ \text{mm}$

$r_B = 80\ \text{mm}$

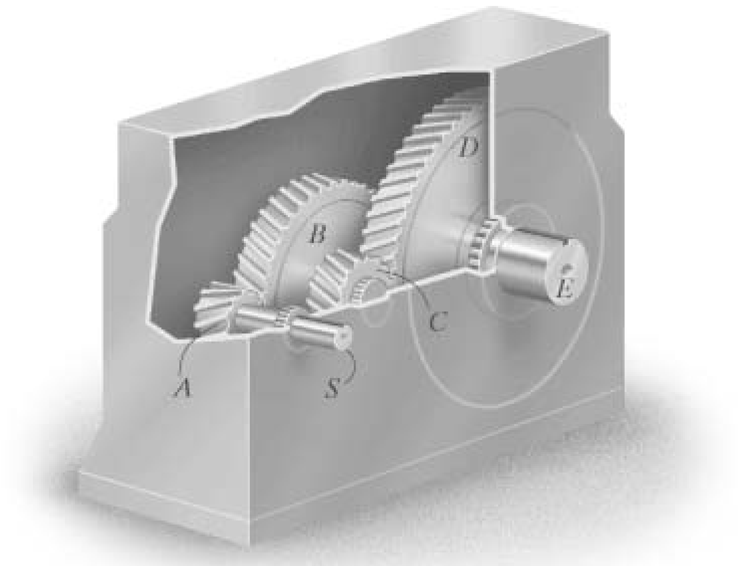
$r_C = 30\ \text{mm}$

$r_D = 120\ \text{mm}$

$k = 0.4\ \dfrac{\text{rad}}{\text{s}^2}$

$b = 1\ \text{s}^{-1}$

$t = 2\ \text{s}$



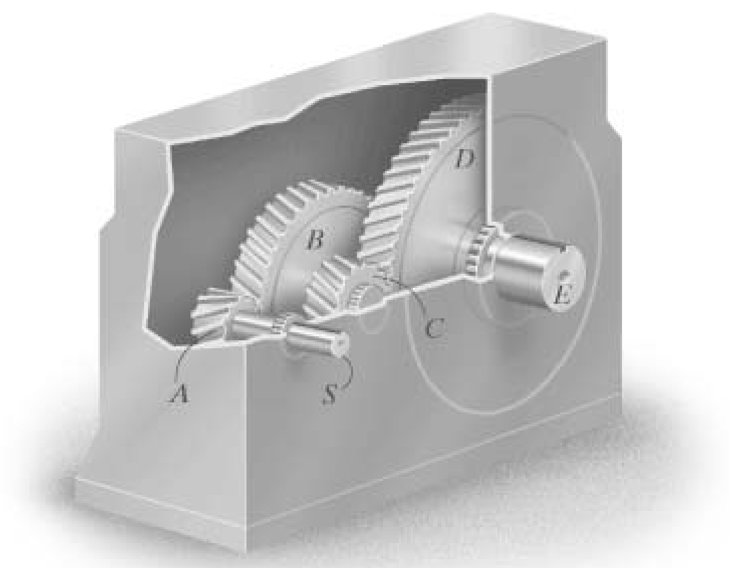
Solution:

$$\omega = \int_0^t k e^{b\,t}\,dt \qquad \omega = 2.56\ \frac{\text{rad}}{\text{s}}$$

$$\omega_E = \left(\frac{r_A}{r_B}\right)\left(\frac{r_C}{r_D}\right)\omega \qquad \omega_E = 0.160\ \frac{\text{rad}}{\text{s}}$$

Problem 16-29

Morse Industrial manufactures the speed reducer shown. If a motor drives the gear shaft S with an angular acceleration $\alpha = k\omega^3$, determine the angular velocity of shaft E at time t_1 after gear S starts from an angular velocity ω_0 when $t = 0$. The radius of each gear is listed. Note that gears B and C are fixed connected to the same shaft.



Given:

$r_A = 20\ \text{mm}$

$r_B = 80\ \text{mm}$

$r_C = 30\ \text{mm}$

$r_D = 120\ \text{mm}$

$\omega_0 = 1\ \frac{\text{rad}}{\text{s}}$

$k = 4\ \frac{\text{rad}}{\text{s}^5}$

$t_1 = 2\ \text{s}$

Solution:

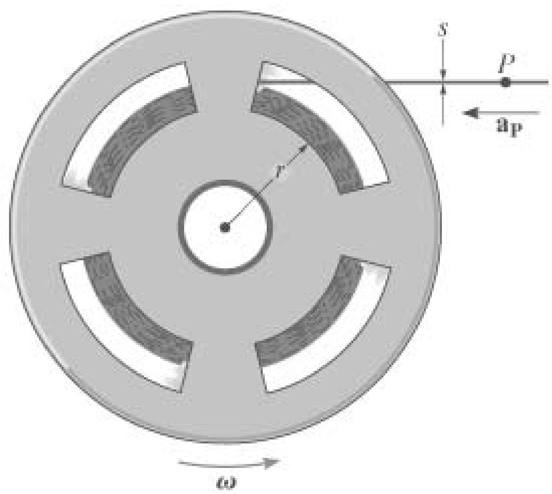
Guess $\omega_1 = 1\ \frac{\text{rad}}{\text{s}}$

Given $\int_0^{t_1} k\,\text{d}t = \int_{\omega_0}^{\omega_1} \omega^3\,\text{d}\omega \qquad \omega_1 = \text{Find}(\omega_1)$

$\omega_1 = 2.40\,\frac{\text{rad}}{\text{s}} \qquad \omega_E = \left(\frac{r_A}{r_B}\right)\left(\frac{r_C}{r_D}\right)\omega_1 \qquad \omega_E = 0.150\,\frac{\text{rad}}{\text{s}}$

Problem 16-30

A tape having a thickness s wraps around the wheel which is turning at a constant rate ω. Assuming the unwrapped portion of tape remains horizontal, determine the acceleration of point P of the unwrapped tape when the radius of the wrapped tape is r. *Hint:* Since $v_P = \omega r$, take the time derivative and note that $\text{d}r/\text{d}t = \omega\,(s/2\pi)$.



Solution:

$v_P = \omega r$

$a_p = \frac{\text{d}v_p}{\text{d}t} = \frac{\text{d}\omega}{\text{d}t}r + \omega\frac{\text{d}r}{\text{d}t}$

since $\frac{\text{d}\omega}{\text{d}t} = 0, \qquad a_p = \omega\left(\frac{\text{d}r}{\text{d}t}\right)$

In one revolution r is increased by s, so that

$\frac{2\pi}{\Delta\theta} = \frac{s}{\Delta r}$

Hence,

$$\Delta r = \frac{s}{2\pi}\Delta\theta \qquad \frac{dr}{dt} = \frac{s}{2\pi}\omega$$

$$\boxed{a_p = \frac{s}{2\pi}\omega^2}$$

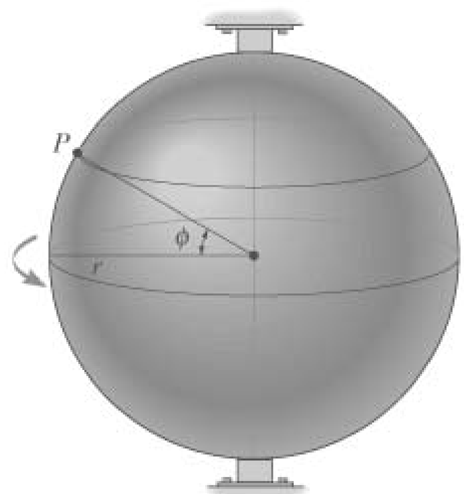
Problem 16-31

The sphere starts from rest at $\theta = 0°$ and rotates with an angular acceleration $\alpha = k\theta$. Determine the magnitudes of the velocity and acceleration of point P on the sphere at the instant $\theta = \theta_1$.

Given:

$\theta_1 = 6\ \text{rad}$ $\qquad r = 8\ \text{in}$

$\phi = 30\ \text{deg}$ $\qquad k = 4\ \frac{\text{rad}}{\text{s}^2}$



Solution:

$$\alpha = k\theta_1$$

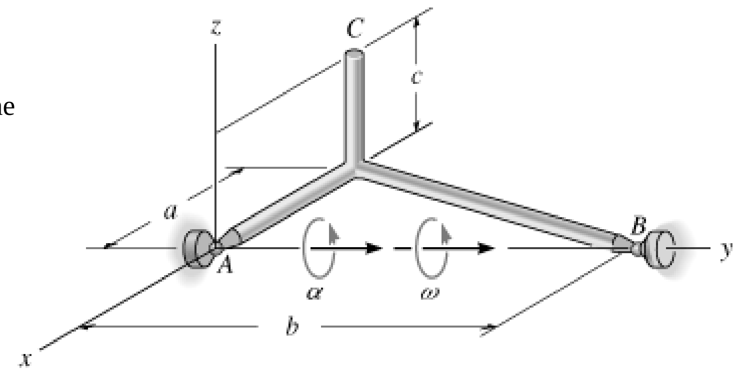
$$\frac{\omega^2}{2} = k\left(\frac{\theta_1^2}{2}\right) \qquad \omega = \sqrt{k}\,\theta_1$$

$$v_P = \omega r\cos(\phi) \qquad \boxed{v_P = 6.93\ \frac{\text{ft}}{\text{s}}}$$

$$a_P = \sqrt{(\alpha r\cos(\phi))^2 + \left(\omega^2 r\cos(\phi)\right)^2} \qquad \boxed{a_P = 84.3\ \frac{\text{ft}}{\text{s}^2}}$$

***Problem 16-32**

The rod assembly is supported by ball-and-socket joints at A and B. At the instant shown it is rotating about the y axis with angular velocity ω and has angular acceleration α. Determine the magnitudes of the velocity and acceleration of point C at this instant. Solve the problem using Cartesian vectors and Eqs. 16-9 and 16-13.



Given:

$\omega = 5\ \frac{\text{rad}}{\text{s}}$ $a = 0.4\ \text{m}$

$b = 0.4\ \text{m}$

$\alpha = 8\ \frac{\text{rad}}{\text{s}^2}$ $c = 0.3\ \text{m}$

Solution:

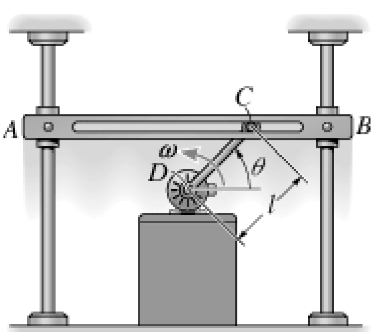
$$\mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \mathbf{r_{AC}} = \begin{pmatrix} -a \\ 0 \\ c \end{pmatrix}$$

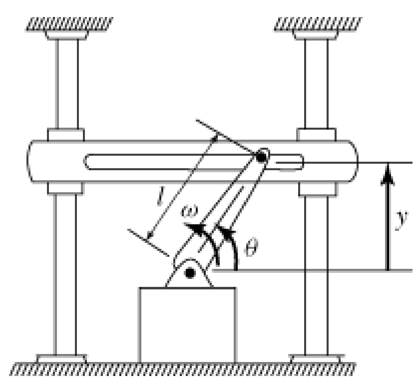
$$\mathbf{v_C} = (\omega\mathbf{j}) \times \mathbf{r_{AC}} \qquad \mathbf{v_C} = \begin{pmatrix} 1.50 \\ 0.00 \\ 2.00 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad |\mathbf{v_C}| = 2.50\ \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_C} = (\alpha\mathbf{j}) \times \mathbf{r_{AC}} + (\omega\mathbf{j}) \times \left[(\omega\mathbf{j}) \times \mathbf{r_{AC}}\right] \qquad \mathbf{a_C} = \begin{pmatrix} 12.40 \\ 0.00 \\ -4.30 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad |\mathbf{a_C}| = 13.12\ \frac{\text{m}}{\text{s}^2}$$

Problem 16-33

The bar *DC* rotates uniformly about the shaft at *D* with a constant angular velocity ω. Determine the velocity and acceleration of the bar *AB*, which is confined by the guides to move vertically.





Solution: $\theta' = \omega$ $\theta'' = \alpha = 0$

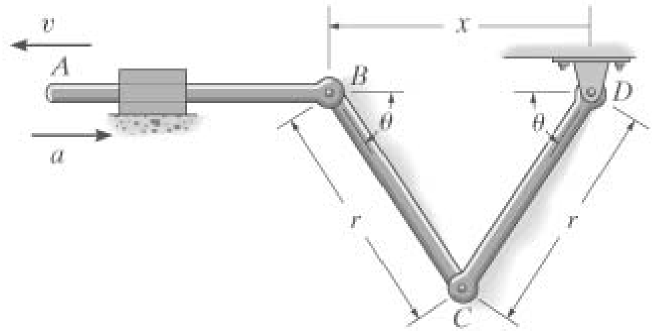
$$y = l\sin(\theta)$$

$$y' = v_y = l\cos(\theta)\theta' \qquad v_{AB} = \omega l\cos(\theta)$$

$$y'' = a_y = l\left(\cos(\theta)\,\theta'' - \sin(\theta)\,\theta'^2\right) \qquad a_{AB} = -\omega^2 l\sin(\theta)$$

Problem 16-34

At the instant shown, θ is given, and rod *AB* is subjected to a deceleration *a* when the velocity is *v*. Determine the angular velocity and angular acceleration of link *CD* at this instant.



Given:

$v = 10\ \frac{m}{s}$ $\quad a = 16\ \frac{m}{s^2}$

$\theta = 60\ deg$ $\quad r = 300\ mm$

Solution:

$x = 2r\cos(\theta)$ $\quad x = 0.30\ m$

$x' = -2r\sin(\theta)\theta'$

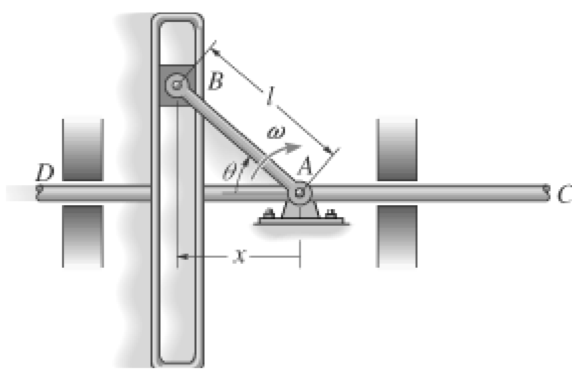
$$\omega = \frac{-v}{2r\sin(\theta)} \qquad \boxed{\omega = -19.2\ \frac{rad}{s}}$$

$x'' = -2r\cos(\theta)\,\theta'^2 - 2r\sin(\theta)\theta''$

$$\alpha = \frac{a - 2r\cos(\theta)\ \omega^2}{2r\sin(\theta)} \qquad \boxed{\alpha = -183\ \frac{rad}{s^2}}$$

Problem 16-35

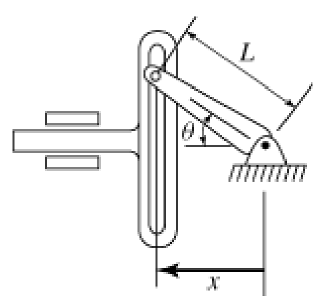
The mechanism is used to convert the constant circular motion ω of rod *AB* into translating motion of rod *CD*. Determine the velocity and acceleration of *CD* for any angle θ of *AB*.



Solution:

$x = l\cos(\theta)$ $\quad x' = v_x = -l\sin(\theta)\theta'$

$x'' = a_x = -l\left(\sin(\theta)\,\theta'' + \cos(\theta)\,\theta'^2\right)$



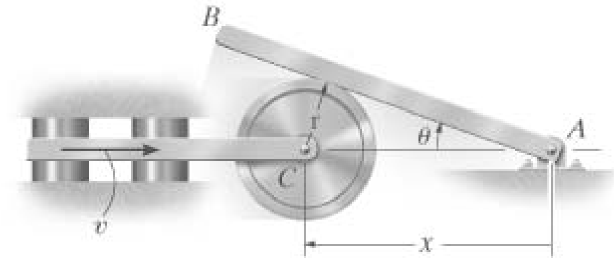
$v_x = v_{CD}$ $\quad a_x = a_{CD}$ $\quad$ and $\quad \theta' = \omega$ $\quad \theta'' = \alpha = 0$

$\boxed{v_{CD} = -\omega l\sin(\theta)}$ $\quad \boxed{a_{CD} = -\omega^2 l\cos(\theta)}$

***Problem 16-36**

Determine the angular velocity of rod *AB* for the given θ. The shaft and the center of the roller *C* move forward at a constant rate *v*.

Given:



$$v = 5\ \frac{\text{m}}{\text{s}}$$

$$\theta = 30\ \text{deg}$$

$$r = 100\ \text{mm}$$

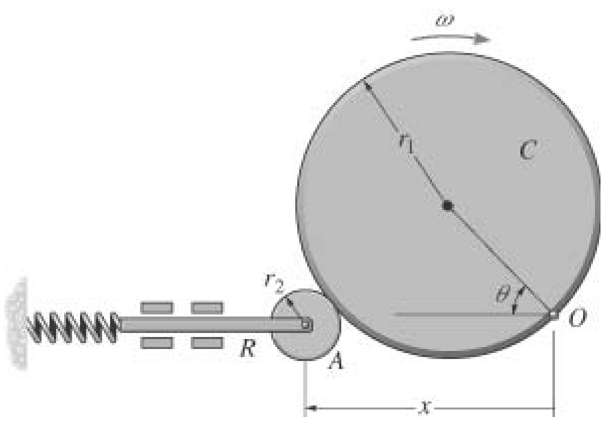
Solution:

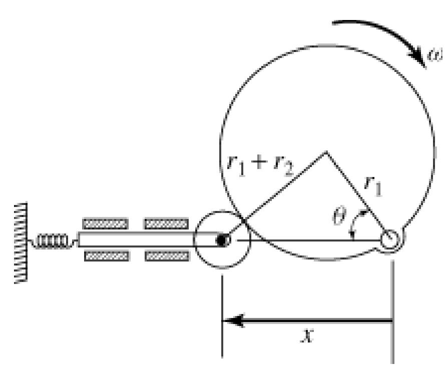
$$r = x\sin(\theta) \quad 0 = x'\sin(\theta) + x\cos(\theta)\theta' = -v\sin(\theta) + x\cos(\theta)\omega$$

$$x = \frac{r}{\sin(\theta)} \qquad \omega = \left(\frac{v}{x}\right)\tan(\theta) \qquad \omega = 14.43\ \frac{\text{rad}}{\text{s}}$$

Problem 16-37

Determine the velocity of rod *R* for any angle θ of the cam *C* if the cam rotates with a constant angular velocity ω. The pin connection at *O* does not cause an interference with the motion of *A* on *C*.





Solution:

Position Coordinate Equation: Using law of cosines.

$$(r_1 + r_2)^2 = x^2 + r_1^2 - 2r_1 x\cos(\theta)$$

$$x = r_1\cos(\theta) + \sqrt{r_1^2\cos(\theta)^2 + 2r_1 r_2 + r_2^2}$$

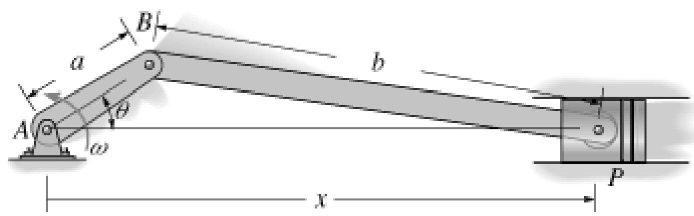
$$0 = 2xx' - 2r_1 x'\cos(\theta) + 2r_1 x\sin(\theta)\theta'$$

$$x' = \frac{-r_1 x \sin(\theta)\theta'}{x - r_1 \cos(\theta)} \qquad v = -r_1 \sin(\theta)\,\omega\left(1 + \frac{r_1 \cos(\theta)}{\sqrt{r_1{}^2 \cos(\theta)^2 + 2r_1 r_2 + r_2{}^2}}\right)$$

Problem 16-38

The crankshaft AB is rotating at constant angular velocity ω. Determine the velocity of the piston P for the given θ.

Given:



$$\omega = 150 \frac{\text{rad}}{\text{s}}$$

$$\theta = 30 \text{ deg}$$

$$a = 0.2 \text{ ft}$$

$$b = 0.75 \text{ ft}$$

Solution:

$$x = (a)\cos(\theta) + \sqrt{b^2 - a^2 \sin^2(\theta)}$$

$$x' = -(a)\sin(\theta)\theta' - \frac{a^2 \cos(\theta)\sin(\theta)\theta'}{\sqrt{b^2 - a^2 \sin(\theta)^2}}$$

$$v = -(a)\sin(\theta)\omega - \frac{a^2 \cos(\theta)\sin(\theta)\omega}{\sqrt{b^2 - a^2 \sin(\theta)^2}} \qquad v = -18.50 \frac{\text{ft}}{\text{s}}$$

Problem 16-39

At the instant $\theta = \theta_1$ the slotted guide is moving upward with acceleration a and velocity v. Determine the angular acceleration and angular velocity of link AB at this instant. *Note*: The upward motion of the guide is in the negative y direction.

Given:

$$\theta_1 = 50 \text{ deg} \quad v = 2 \frac{\text{m}}{\text{s}}$$

$$a = 3 \frac{\text{m}}{\text{s}^2} \qquad L = 300 \text{ mm}$$

Solution:

$$y = L\cos(\theta)$$

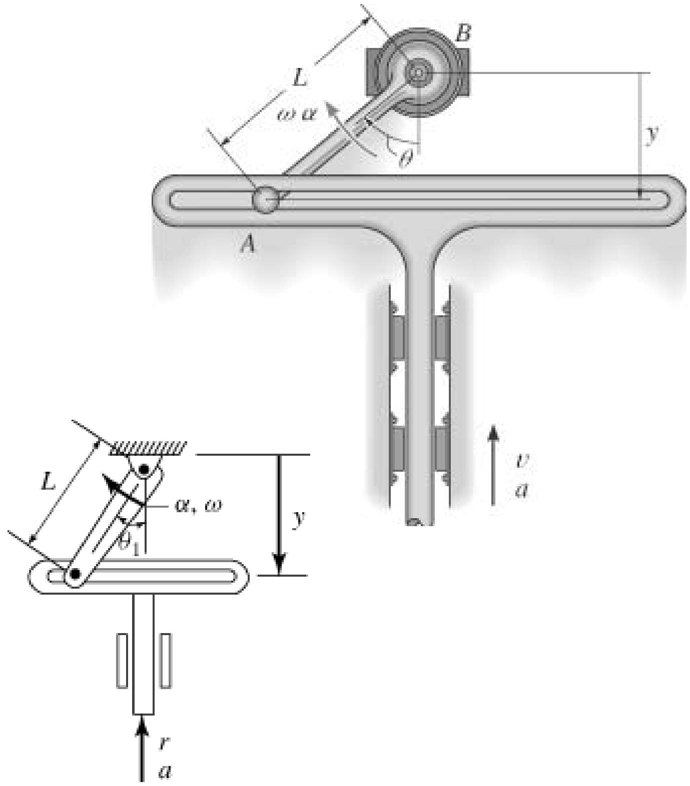
$$y' = -L\sin(\theta)\theta'$$

$$y'' = -L\sin(\theta)\theta'' - L\cos(\theta)\,\theta'^2$$

$$\omega = \frac{v}{L\sin(\theta_1)} \qquad \omega = 8.70\,\frac{\text{rad}}{\text{s}}$$

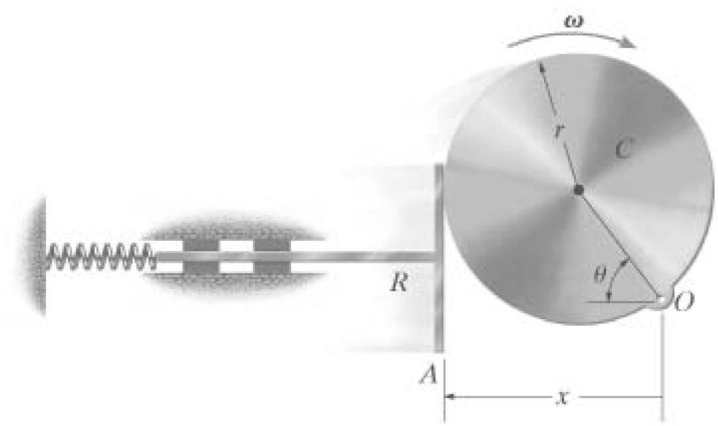
$$\alpha = \frac{a - L\cos(\theta_1)\,\omega^2}{L\sin(\theta_1)}$$

$$\alpha = -50.50\,\frac{\text{rad}}{\text{s}^2}$$



***Problem 16-40**

Determine the velocity of the rod R for any angle θ of cam C as the cam rotates with a constant angular velocity ω. The pin connection at O does not cause an interference with the motion of plate A on C.



Solution:

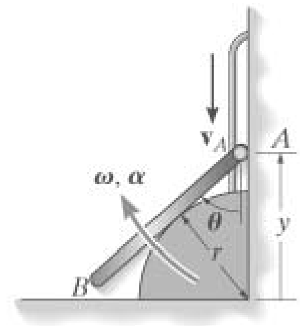
$$x = r + r\cos(\theta)$$

$$x' = -r\sin(\theta)\theta'$$

$$v = -r\,\omega\sin(\theta)$$

Problem 16-41

The end A of the bar is moving downward along the slotted guide with a constant velocity v_A. Determine the angular velocity ω and angular acceleration a of the bar as a function of its position y.



Solution: $y' = -v_A \qquad y'' = 0$

$$r = y\sin(\theta)$$

$$0 = y'\sin(\theta) + y\cos(\theta)\theta'$$

$$0 = y''\sin(\theta) + 2y'\cos(\theta)\theta' - y\sin(\theta)\,\theta'^2 + y\cos(\theta)\theta''$$

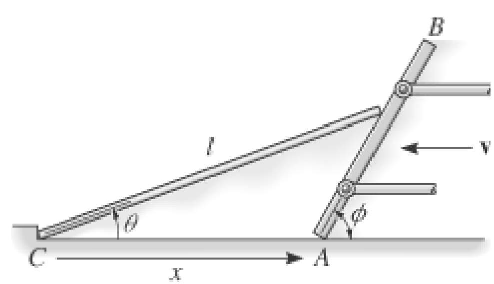
$$\omega = \left(\frac{v_A}{y}\right)\tan(\theta) \qquad \boxed{\omega = \frac{v_A}{y}\left(\frac{r}{\sqrt{y^2 - r^2}}\right)}$$

$$\alpha = 2\left(\frac{v_A}{y}\right)\omega + \tan(\theta)\,\omega^2$$

$$\alpha = 2\frac{v_A^2}{y^2}\left(\frac{r}{\sqrt{y^2 - r^2}}\right) + \frac{v_A^2 r^3}{y^2\sqrt{(y^2 - r^2)^3}} \qquad \boxed{\alpha = \frac{r v_A^2(2y^2 - r^2)}{y^2\sqrt{(y^2 - r^2)^3}}}$$

Problem 16-42

The inclined plate moves to the left with a constant velocity **v**. Determine the angular velocity and angular acceleration of the slender rod of length *l*. The rod pivots about the step at *C* as it slides on the plate.



Solution: $x' = -v$

$$\frac{x}{\sin(\phi - \theta)} = \frac{l}{\sin(180\text{ deg} - \phi)} = \frac{l}{\sin(\phi)}$$

$$x\sin(\phi) = l\sin(\phi - \theta)$$

$$x'\sin(\phi) = -l\cos(\phi - \theta)\theta'$$

Thus, $\boxed{\omega = \dfrac{-v\sin(\phi)}{l\cos(\phi - \theta)}}$

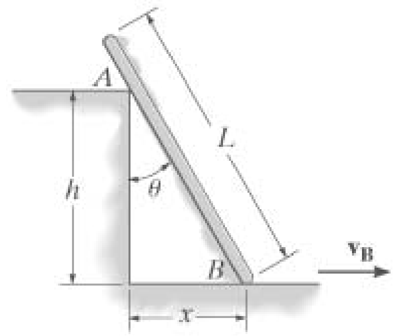
$$x''\sin(\phi) = -l\cos(\phi - \theta)\theta'' - l\sin(\phi - \theta)\,\theta'^2$$

$$0 = -\cos(\phi - \theta)\,\alpha - \sin(\phi - \theta)\,\omega^2$$

$$\alpha = \frac{-\sin(\phi - \theta)}{\cos(\phi - \theta)}\left[\frac{v^2\sin\phi^2}{l^2\cos(\phi - \theta)^2}\right] \qquad \boxed{\alpha = \frac{-v^2\sin^2(\phi)\sin(\phi - \theta)}{l^2\cos(\phi - \theta)^3}}$$

Problem 16-43

The bar remains in contact with the floor and with point A. If point B moves to the right with a constant velocity v_B, determine the angular velocity and angular acceleration of the bar as a function of x.



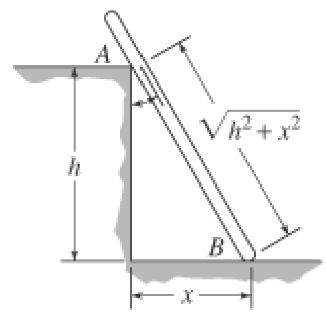
Solution: $x' = v_B \qquad x'' = 0$

$$x = h\tan(\theta)$$

$$x' = h\sec(\theta)^2\theta'$$

$$x'' = h\sec(\theta)^2\theta'' + 2h\sec(\theta)^2\tan(\theta)\,\theta'^2$$

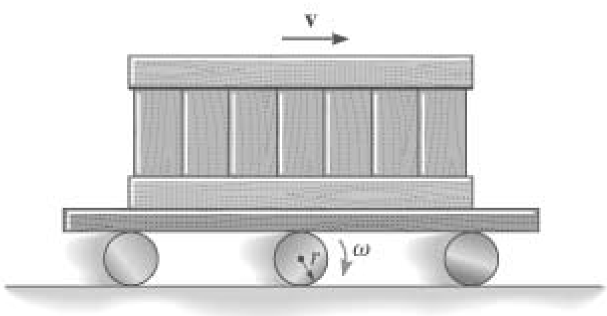
$$\sec(\theta) = \frac{\sqrt{h^2+x^2}}{h} \qquad \tan(\theta) = \frac{x}{h}$$

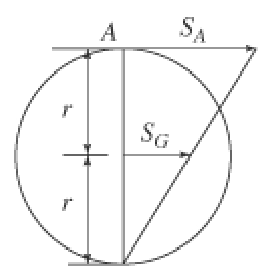


$$\omega = \frac{h v_B}{h^2+x^2} \qquad \alpha = \frac{-2hxv_B^{\,2}}{\left(h^2+x^2\right)^2}$$

***Problem 16-44**

The crate is transported on a platform which rests on rollers, each having a radius r. If the rollers do not slip, determine their angular velocity if the platform moves forward with a velocity **v**.





Solution:

Position coordinate equation: $s_G = r\theta$. Using similar triangles $s_A = 2s_G = 2r\theta$

$s'_A = v = 2r\theta'$ where $\theta' = \omega$

$$\omega = \frac{v}{2r}$$

Problem 16-45

Bar *AB* rotates uniformly about the fixed pin *A* with a constant angular velocity ω. Determine the velocity and acceleration of block *C* when $\theta = \theta_1$.

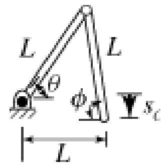
Given:

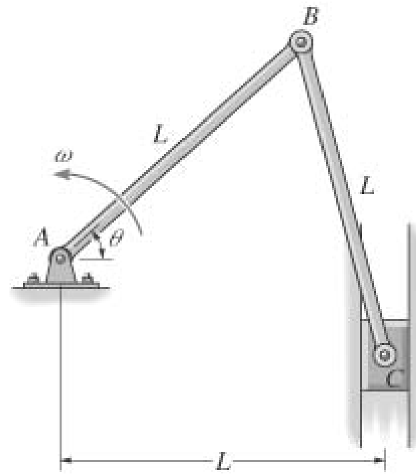
$L = 1\ \text{m}$

$\theta_1 = 60\ \text{deg}$

$\omega = 2\ \dfrac{\text{rad}}{\text{s}}$

$\alpha = 0\ \dfrac{\text{rad}}{\text{s}^2}$





Solution:

$\theta = \theta_1 \qquad \theta' = \omega \qquad \theta'' = \alpha$

Guesses $\quad \phi = 60\ \text{deg} \qquad \phi' = 1\ \dfrac{\text{rad}}{\text{s}} \qquad \phi'' = 1\ \dfrac{\text{rad}}{\text{s}^2}$

$s_C = 1\ \text{m} \qquad v_C = -1\ \dfrac{\text{m}}{\text{s}} \qquad a_C = -2\ \dfrac{\text{m}}{\text{s}^2}$

Given

$$L\cos(\theta) + L\cos(\phi) = L$$

$$\sin(\theta)\,\theta' + \sin(\phi)\,\phi' = 0$$

$$\cos(\theta)\,\theta'^2 + \sin(\theta)\,\theta'' + \sin(\phi)\,\phi'' + \cos(\phi)\,\phi'^2 = 0$$

$$s_C = L\sin(\phi) - L\sin(\theta)$$

$$v_C = L\cos(\phi)\phi' - L\cos(\theta)\theta'$$

$$a_C = -L\sin(\phi)\,\phi'^2 + L\cos(\phi)\phi'' + L\sin(\theta)\,\theta'^2 - L\cos(\theta)\theta''$$

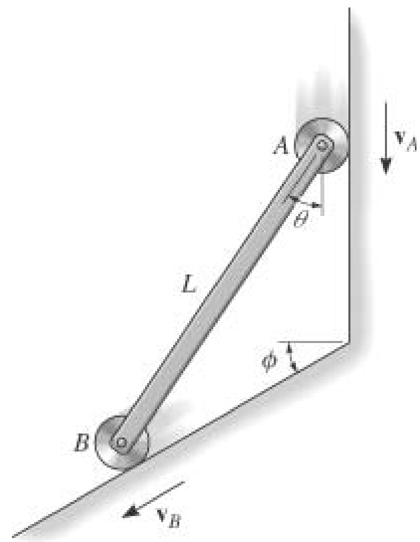
$$\begin{pmatrix} \phi \\ \phi' \\ \phi'' \\ s_C \\ v_C \\ a_C \end{pmatrix} = \text{Find}(\phi, \phi', \phi'', s_C, v_C, a_C)$$

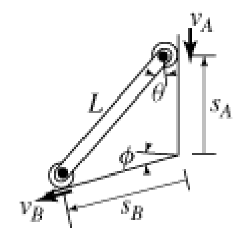
$\phi = 60.00\ \text{deg} \qquad \phi' = -2.00\ \dfrac{\text{rad}}{\text{s}} \qquad \phi'' = -4.62\ \dfrac{\text{rad}}{\text{s}^2}$

$s_C = 0.00\ \text{m} \qquad v_C = -2.00\ \dfrac{\text{m}}{\text{s}} \qquad a_C = -2.31\ \dfrac{\text{m}}{\text{s}^2}$

Problem 16-46

The bar is confined to move along the vertical and inclined planes. If the velocity of the roller at A is v_A downward when $\theta = \theta_1$. determine the bar's angular velocity and the velocity of roller B at this instant.





Given:

$$v_A = 6\ \frac{\text{ft}}{\text{s}}$$

$$\theta_1 = 45\ \text{deg}$$

$$\phi = 30\ \text{deg}$$

$$L = 5\ \text{ft}$$

Solution: $\theta = \theta_1$

Guesses

$$s_A = 1\ \text{ft} \qquad s_B = 1\ \text{ft} \qquad \omega = 1\ \frac{\text{rad}}{\text{s}} \qquad v_B = 1\ \frac{\text{ft}}{\text{s}}$$

Given

$$L\sin(\theta) = s_B\cos(\phi) \qquad L\cos(\theta)\omega = v_B\cos(\phi)$$

$$L\cos(\theta) = s_A + s_B\sin(\phi) \qquad -L\sin(\theta)\omega = -v_A + v_B\sin(\phi)$$

$$\begin{pmatrix} s_A \\ s_B \\ \omega \\ v_B \end{pmatrix} = \text{Find}(s_A, s_B, \omega, v_B) \qquad \begin{pmatrix} s_A \\ s_B \end{pmatrix} = \begin{pmatrix} 1.49 \\ 4.08 \end{pmatrix}\text{ft} \qquad \omega = 1.08\ \frac{\text{rad}}{\text{s}} \qquad v_B = 4.39\ \frac{\text{ft}}{\text{s}}$$

Problem 16-47

When the bar is at the angle θ the rod is rotating clockwise at ω and has an angular acceleration α. Determine the velocity and acceleration of the weight A at this instant. The cord is of length L.

Given:

$$L = 20\ \text{ft}$$

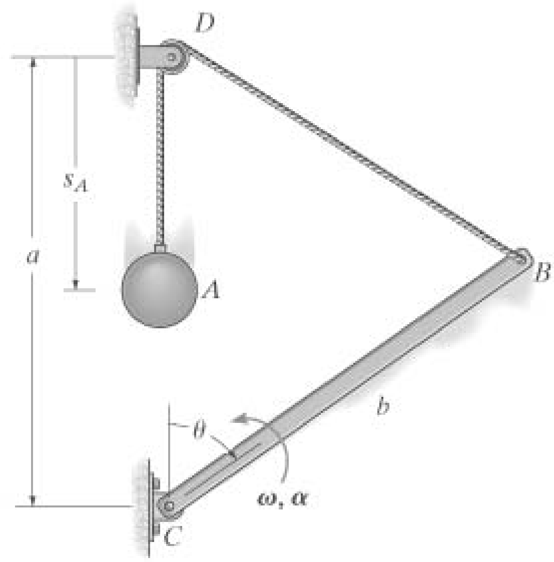
$$a = 10\ \text{ft}$$

$b = 10\ \text{ft}$

$\theta = 30\ \text{deg}$

$\omega = 3\ \dfrac{\text{rad}}{\text{s}}$

$\alpha = 5\ \dfrac{\text{rad}}{\text{s}^2}$



Solution: $\theta' = -\omega \qquad \theta'' = -\alpha$

$$s_A = L - \sqrt{a^2 + b^2 - 2a\,b\cos(\theta)}$$

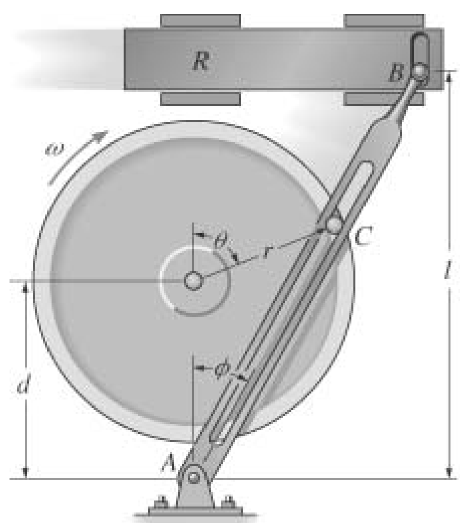
$$v_A = \frac{-a\,b\sin(\theta)\theta'}{\sqrt{a^2 + b^2 - 2a\,b\cos(\theta)}}$$

$$a_A = \frac{-a\,b\sin(\theta)\theta'' - a\,b\cos(\theta)\,\theta'^2}{\sqrt{a^2 + b^2 - 2a\,b\cos(\theta)}} + \frac{\left(a\,b\sin(\theta)\theta'\right)^2}{\sqrt{\left(a^2 + b^2 - 2a\,b\cos(\theta)\right)^3}}$$

$s_A = 14.82\ \text{ft} \qquad v_A = 29.0\ \dfrac{\text{ft}}{\text{s}} \qquad a_A = 59.9\ \dfrac{\text{ft}}{\text{s}^2}$

***Problem 16-48**

The slotted yoke is pinned at A while end B is used to move the ram R horizontally. If the disk rotates with a constant angular velocity ω, determine the velocity and acceleration of the ram. The crank pin C is fixed to the disk and turns with it. The length of AB is L.

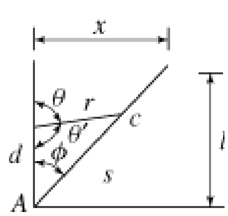


Solution:

$$x = L\sin(\phi)$$

$$s = \sqrt{d^2 + r^2 + 2r\,d\cos(\theta)}$$

$$s\sin(\phi) = r\sin(\theta)$$



Thus $$x = \frac{L\,r\sin(\theta)}{\sqrt{d^2 + r^2 + 2r\,d\cos(\theta)}}$$

$$v = \frac{L r \cos(\theta)\omega}{\sqrt{d^2 + r^2 + 2 r d \cos(\theta)}} + \frac{d L r^2 \sin(\theta)\omega}{\sqrt{\left(d^2 + r^2 + 2 r d \cos(\theta)\right)^3}}$$

$$a = \frac{-L r \sin(\theta)\,\omega^2}{\sqrt{d^2 + r^2 + 2 r d \cos(\theta)}} + \frac{3 d L r^2 \sin(\theta)\cos(\theta)\,\omega^2}{\sqrt{\left(d^2 + r^2 + 2 r d \cos(\theta)\right)^3}} + \frac{3 d^2 L r^3 \sin(\theta)\,\omega^2}{\sqrt{\left(d^2 + r^2 + 2 r d \cos(\theta)\right)^5}}$$

Problem 16-49

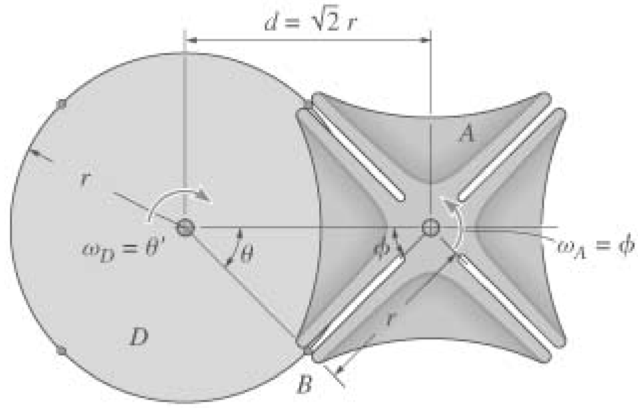
The Geneva wheel A provides intermittent rotary motion ω_A for continuous motion ω_D of disk D. By choosing $d = \sqrt{2}\,r$, the wheel has zero angular velocity at the instant pin B enters or leaves one of the four slots. Determine the magnitude of the angular velocity ω_A of the Geneva wheel when $\theta = \theta_1$ so that pin B is in contact with the slot.

Given:

$\omega_D = 2\ \frac{\text{rad}}{\text{s}}$

$r = 100\ \text{mm}$

$\theta_1 = 30\ \text{deg}$



Solution:

$\theta = \theta_1$

Guesses $\phi = 10\ \text{deg}$ $\omega_A = 1\ \frac{\text{rad}}{\text{s}}$

$s_{BA} = 10\ \text{mm}$ $s'_{BA} = 10\ \frac{\text{mm}}{\text{s}}$

Given $r\cos(\theta) + s_{BA}\cos(\phi) = \sqrt{2}\,r$

$-r\sin(\theta)\omega_D + s'_{BA}\cos(\phi) - s_{BA}\sin(\phi)\omega_A = 0$

$r\sin(\theta) = s_{BA}\sin(\phi)$

$r\cos(\theta)\omega_D = s'_{BA}\sin(\phi) + s_{BA}\cos(\phi)\omega_A$

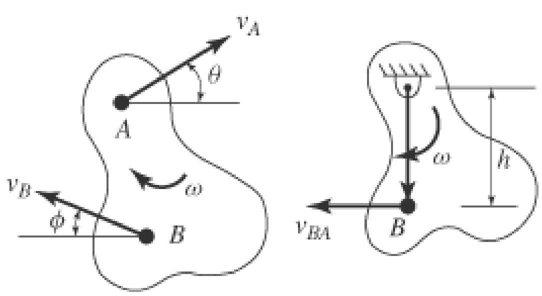
$$\begin{pmatrix} \phi \\ \omega_A \\ s_{BA} \\ s'_{BA} \end{pmatrix} = \text{Find}(\phi, \omega_A, s_{BA}, s'_{BA}) \qquad \phi = 42.37\,\text{deg} \qquad \boxed{\omega_A = 0.816\,\frac{\text{rad}}{\text{s}}}$$

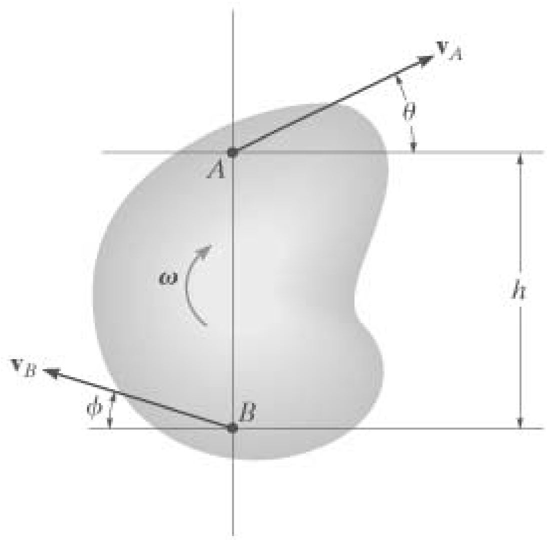
$$s_{BA} = 74.20\,\text{mm} \qquad s'_{BA} = 190.60\,\frac{\text{mm}}{\text{s}}$$

The general solution is $\boxed{\omega_A = \omega_D\left(\frac{\sqrt{2}\cos(\theta) - 1}{3 - 2\sqrt{2}\cos(\theta)}\right)}$

Problem 16-50

If h and θ are known, and the speed of A and B is $v_A = v_B = v$, determine the angular velocity ω of the body and the direction ϕ of v_B.





Solution:

$$v_B = v_A + \omega \times r_{BA}$$

$$-v\cos(\phi)\mathbf{i} + v\sin(\phi)\mathbf{j} = v\cos(\theta)\mathbf{i} + v\sin(\theta)\mathbf{j} + (-\omega\,\mathbf{k}) \times (-h\,\mathbf{j})$$

$$-v\cos(\phi) = v\cos(\theta) - \omega h \qquad (1)$$

$$v\sin(\phi) = v\sin(\theta) \qquad (2)$$

From Eq. (2), $\boxed{\phi = \theta}$

From Eq. (1), $\boxed{\omega = \frac{2v}{h}\cos(\theta)}$

Problem 16-51

The wheel is rotating with an angular velocity ω. Determine the velocity of the collar A for the given values of θ and ϕ.

Given:

$\theta = 30\ \text{deg}$

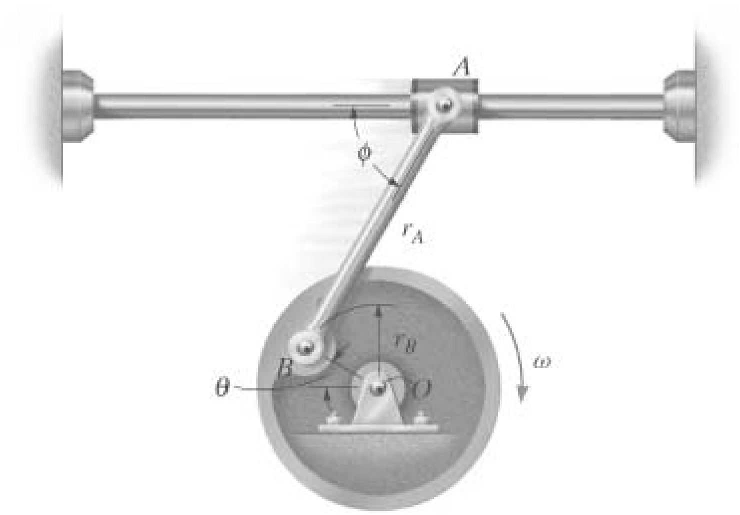
$\phi = 60\ \text{deg}$

$\omega = 8\ \dfrac{\text{rad}}{\text{s}}$

$r_A = 500\ \text{mm}$

$r_B = 150\ \text{mm}$

$v_B = 1.2\ \dfrac{\text{m}}{\text{s}}$



Solution:

Guesses $\quad \omega_{AB} = 1\ \dfrac{\text{rad}}{\text{s}} \qquad v_A = 1\ \dfrac{\text{m}}{\text{s}}$

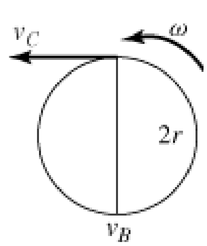
Given
$$\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -r_B\cos(\theta) \\ r_B\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} r_A\cos(\phi) \\ r_A\sin(\phi) \\ 0 \end{pmatrix} = \begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix}$$

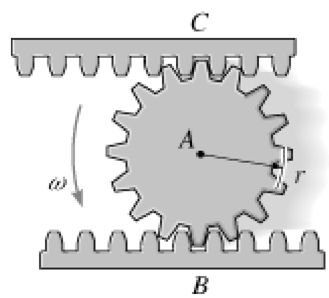
$$\begin{pmatrix} \omega_{AB} \\ v_A \end{pmatrix} = \text{Find}(\omega_{AB}, v_A) \qquad \omega_{AB} = -4.16\ \frac{\text{rad}}{\text{s}} \qquad v_A = 2.40\ \frac{\text{m}}{\text{s}}$$

***Problem 16-52**

The pinion gear A rolls on the fixed gear rack B with angular velocity ω. Determine the velocity of the gear rack C.

Given: $\quad \omega = 4\ \dfrac{\text{rad}}{\text{s}} \qquad r = 0.3\ \text{ft}$





Solution: $\quad v_C = v_B + v_{CB}$

$v_C = \omega(2r) \qquad v_C = -6.56\ \dfrac{\text{ft}}{\text{s}}$

Problem 16-53

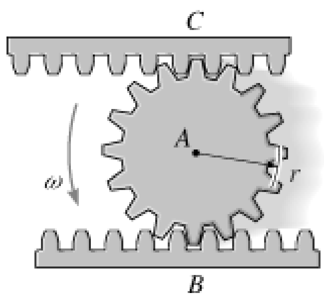
The pinion gear rolls on the gear racks. If B is moving to the right at speed v_B and C is moving to the left at speed v_C determine the angular velocity of the pinion gear and the velocity of its center A.

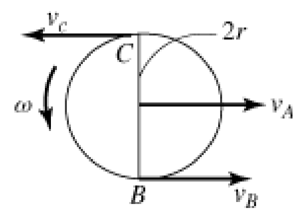
Given:

$v_B = 8 \ \frac{\text{ft}}{\text{s}}$

$v_C = 4 \ \frac{\text{ft}}{\text{s}}$

$r = 0.3 \ \text{ft}$





Solution:

$$v_C = v_B + v_{CB}$$

$$-v_C = v_B - (2r)\omega$$

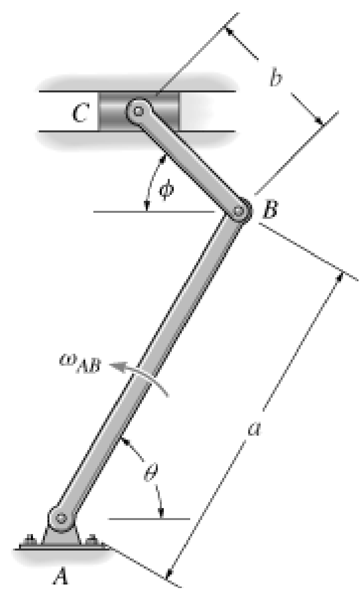
$$\omega = \frac{v_C + v_B}{2r} \qquad \omega = 20.00 \ \frac{\text{rad}}{\text{s}}$$

$$v_A = v_B + v_{AB}$$

$$v_A = v_B + (-\omega)r \qquad v_A = 2 \ \frac{\text{ft}}{\text{s}}$$

Problem 16-54

The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. Determine the velocity of the slider block C at the instant shown, if link AB is rotating at angular velocity ω_{AB}.



Given:

$\theta = 60 \ \text{deg}$

$\phi = 45 \ \text{deg}$

$\omega_{AB} = 4 \ \frac{\text{rad}}{\text{s}}$

$a = 300 \ \text{mm}$

$b = 125 \ \text{mm}$

Solution:

Guesses $\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad v_C = 1\ \frac{\text{m}}{\text{s}}$

Given $$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[a \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[b \begin{pmatrix} -\cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} \right] = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C) \qquad \omega_{BC} = 6.79\ \frac{\text{rad}}{\text{s}} \qquad v_C = -1.64\ \frac{\text{m}}{\text{s}}$$

Problem 16-55

The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at *C*. Determine the velocity of the slider block *C* at the instant shown, if link *AB* is rotating at angular velocity ω_{AB}.

Given:

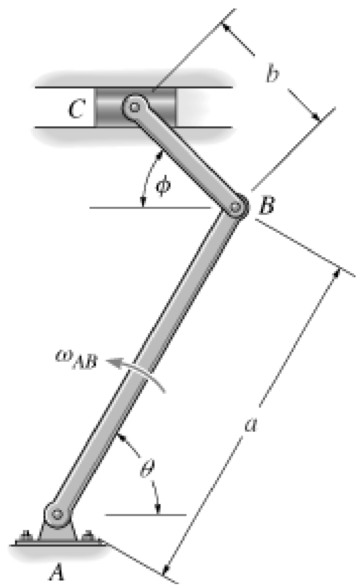
$\theta = 45\ \text{deg}$

$\phi = 45\ \text{deg}$

$\omega_{AB} = 4\ \frac{\text{rad}}{\text{s}}$

$a = 300\ \text{mm}$

$b = 125\ \text{mm}$



Solution:

Guesses $\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad v_C = 1\ \frac{\text{m}}{\text{s}}$

Given $$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[a \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[b \begin{pmatrix} -\cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} \right] = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$$

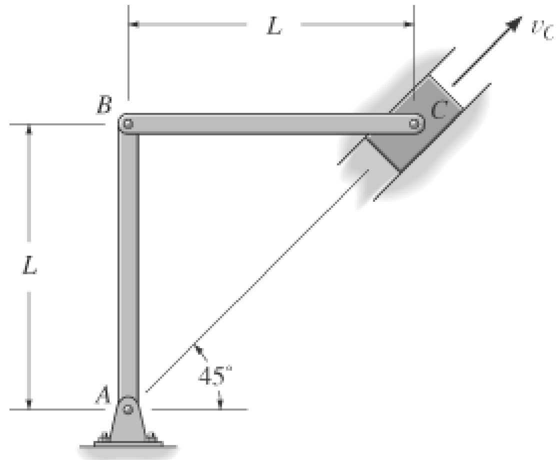
$$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C) \qquad \omega_{BC} = 9.60\ \frac{\text{rad}}{\text{s}} \qquad v_C = -1.70\ \frac{\text{m}}{\text{s}}$$

Problem 16-56

The velocity of the slider block C is v_C up the inclined groove. Determine the angular velocity of links AB and BC and the velocity of point B at the instant shown.

Given:

$v_C = 4\ \frac{\text{ft}}{\text{s}}$ $\quad L = 1\ \text{ft}$



Guesses

$v_{Bx} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_{By} = 1\ \frac{\text{ft}}{\text{s}}$

$\omega_{AB} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$

Given $$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} 0 \\ L \\ 0 \end{pmatrix} = \begin{pmatrix} v_{Bx} \\ v_{By} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_{Bx} \\ v_{By} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} v_C \cos(45\ \text{deg}) \\ v_C \sin(45\ \text{deg}) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_{Bx} \\ v_{By} \\ \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \text{Find}(v_{Bx}, v_{By}, \omega_{AB}, \omega_{BC})$$

$$\begin{pmatrix} \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \begin{pmatrix} -2.83 \\ 2.83 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\begin{pmatrix} v_{Bx} \\ v_{By} \end{pmatrix} = \begin{pmatrix} 2.83 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \left|\begin{pmatrix} v_{Bx} \\ v_{By} \end{pmatrix}\right| = 2.83\ \frac{\text{ft}}{\text{s}}$$

Problem 16-57

Rod AB is rotating with an angular velocity ω_{AB}. Determine the velocity of the collar C for the given angles θ and ϕ.

Given:

$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}}$

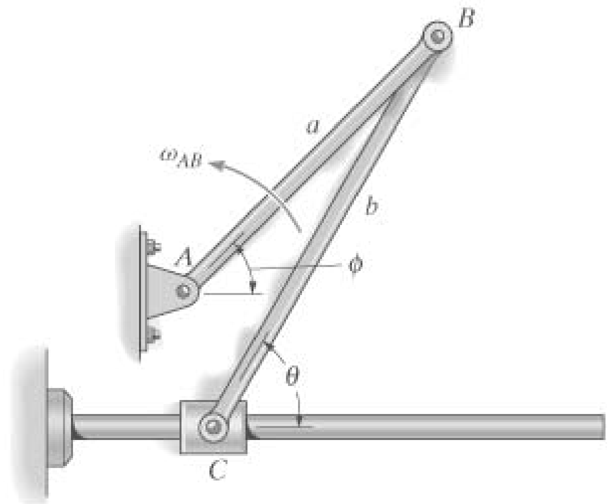
$v_B = 10\ \frac{\text{ft}}{\text{s}}$

$a = 2\ \text{ft}$

$b = 2.5\ \text{ft}$

$\theta = 60\ \text{deg}$

$\phi = 45\ \text{deg}$



Solution:

Guesses

$v_C = 4\ \dfrac{\text{ft}}{\text{s}} \qquad \omega_{CB} = 7\ \dfrac{\text{rad}}{\text{s}}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{bmatrix} (a)\cos(\phi) \\ (a)\sin(\phi) \\ 0 \end{bmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{CB} \end{pmatrix} \times \begin{pmatrix} -b\cos(\theta) \\ -b\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_C \\ \omega_{CB} \end{pmatrix} = \text{Find}(v_C, \omega_{CB}) \qquad \omega_{CB} = 5.66\ \frac{\text{rad}}{\text{s}} \qquad v_C = 5.18\ \frac{\text{ft}}{\text{s}}$$

Problem 16-58

If rod *CD* is rotating with an angular velocity ω_{DC}, determine the angular velocities of rods *AB* and *BC* at the instant shown.

Given:

$\omega_{DC} = 8\ \dfrac{\text{rad}}{\text{s}}$

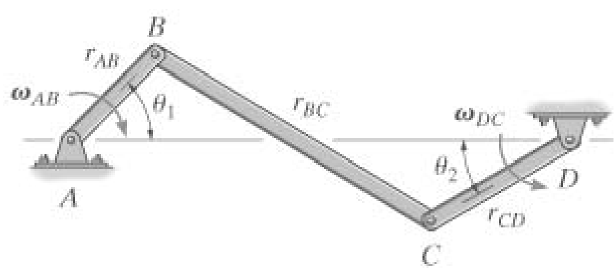
$\theta_1 = 45\ \text{deg}$

$\theta_2 = 30\ \text{deg}$

$r_{AB} = 150\ \text{mm}$

$r_{BC} = 400\ \text{mm}$

$r_{CD} = 200\ \text{mm}$



Solution:

Guesses $\theta_3 = 20\ \text{deg} \quad \omega_{AB} = 1\ \dfrac{\text{rad}}{\text{s}} \qquad \omega_{BC} = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

$$r_{AB}\sin(\theta_1) - r_{BC}\sin(\theta_3) + r_{CD}\sin(\theta_2) = 0$$

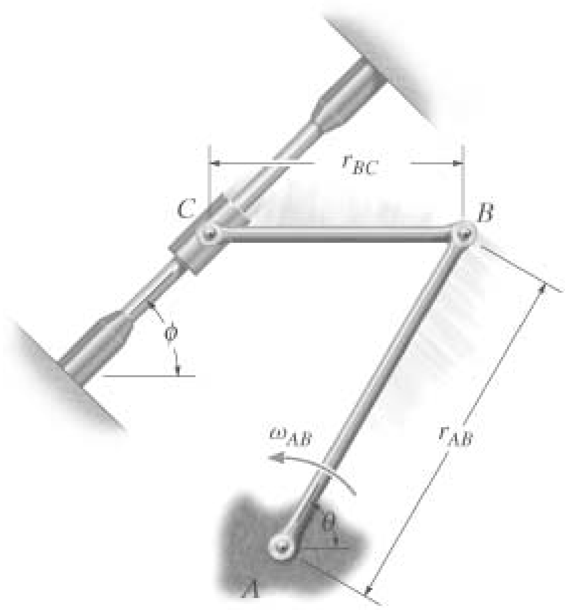
$$\begin{pmatrix} 0 \\ 0 \\ \omega_{DC} \end{pmatrix} \times \begin{pmatrix} -r_{CD}\cos(\theta_2) \\ -r_{CD}\sin(\theta_2) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} -r_{BC}\cos(\theta_3) \\ r_{BC}\sin(\theta_3) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -r_{AB}\cos(\theta_1) \\ -r_{AB}\sin(\theta_1) \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} \theta_3 \\ \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \text{Find}(\theta_3, \omega_{AB}, \omega_{BC}) \qquad \theta_3 = 31.01\text{ deg} \qquad \begin{pmatrix} \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \begin{pmatrix} -9.615 \\ -1.067 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

Positive means *CCW*
Negative means *CW*

Problem 16-59

The angular velocity of link *AB* is ω_{AB}. Determine the velocity of the collar at *C* and the angular velocity of link *CB* for the given angles θ and ϕ. Link *CB* is horizontal at this instant.



Given:

$$\omega_{AB} = 4\ \frac{\text{rad}}{\text{s}} \qquad \phi = 45\text{ deg}$$

$$r_{AB} = 500\text{ mm} \qquad \theta = 60\text{ deg}$$

$$r_{BC} = 350\text{ mm} \qquad \theta_1 = 30\text{ deg}$$

Solution:

Guesses $\quad v_C = 1\ \dfrac{\text{m}}{\text{s}} \qquad \omega_{CB} = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

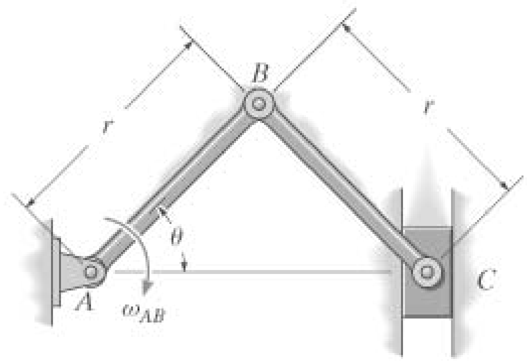
$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} r_{AB}\cos(\theta) \\ r_{AB}\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{CB} \end{pmatrix} \times \begin{pmatrix} -r_{BC} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -v_C\cos(\phi) \\ -v_C\sin(\phi) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_C \\ \omega_{CB} \end{pmatrix} = \text{Find}(v_C, \omega_{CB}) \qquad \omega_{CB} = 7.81\ \frac{\text{rad}}{\text{s}} \qquad v_C = 2.45\ \frac{\text{m}}{\text{s}}$$

***Problem 16-60**

The link AB has an angular velocity ω_{AB}. Determine the velocity of block C at the instant shown when θ = 45 deg.

Given:



$$\omega_{AB} = 2\ \frac{\text{rad}}{\text{s}} \qquad r = 15\ \text{in}$$

$$\theta = 45\ \text{deg} \qquad r = 15\ \text{in}$$

Solution:

Guesses

$$\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}} \qquad v_C = 1\ \frac{\text{in}}{\text{s}}$$

Given

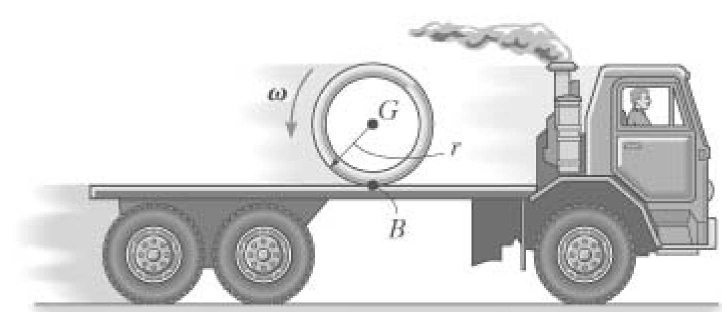
$$\begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \begin{pmatrix} r\cos(\theta) \\ r\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega_{BC} \end{pmatrix} \times \begin{pmatrix} r\cos(\theta) \\ -r\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_C \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C) \qquad \omega_{BC} = 2.00\ \frac{\text{rad}}{\text{s}} \qquad v_C = 3.54\ \frac{\text{ft}}{\text{s}}$$

Problem 16-61

At the instant shown, the truck is traveling to the right at speed v, while the pipe is rolling counterclockwise at angular velocity ω without slipping at B. Determine the velocity of the pipe's center G.

Given:



$$v = 3\ \frac{\text{m}}{\text{s}}$$

$$\omega = 8\ \frac{\text{rad}}{\text{s}}$$

$$r = 1.5\ \text{m}$$

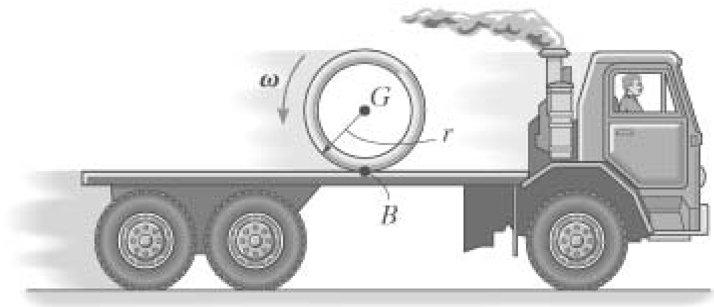
Solution:

$$v_G = v + v_{GB}$$

$$v_G = v - \omega r \qquad v_G = -9.00\ \frac{\text{m}}{\text{s}}$$

Problem 16-62

At the instant shown, the truck is traveling to the right at speed v. If the spool does not slip at B, determine its angular velocity so that its mass center G appears to an observer on the ground to remain stationary.



Given:

$v = 8 \frac{\text{m}}{\text{s}}$

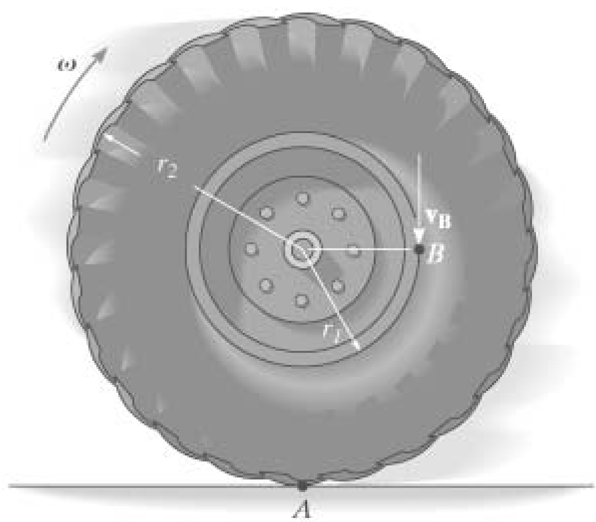
$r = 1.5 \text{ m}$

Solution:

$v_G = v + v_{GB}$ $\qquad 0 = v - \omega r$ $\qquad \omega = \frac{v}{r}$ $\qquad \omega = 5.33 \frac{\text{rad}}{\text{s}}$

Problem 16-63

If, at a given instant, point B has a downward velocity of v_B, determine the velocity of point A at this instant. Notice that for this motion to occur, the wheel must slip at A.



Given:

$v_B = 3 \frac{\text{m}}{\text{s}}$

$r_1 = 0.15 \text{ m}$

$r_2 = 0.4 \text{ m}$

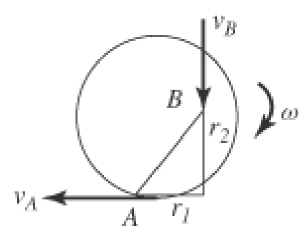
Solution:

Guesses

$v_A = 1 \frac{\text{m}}{\text{s}}$ $\qquad \omega = 1 \frac{\text{rad}}{\text{s}}$

Given

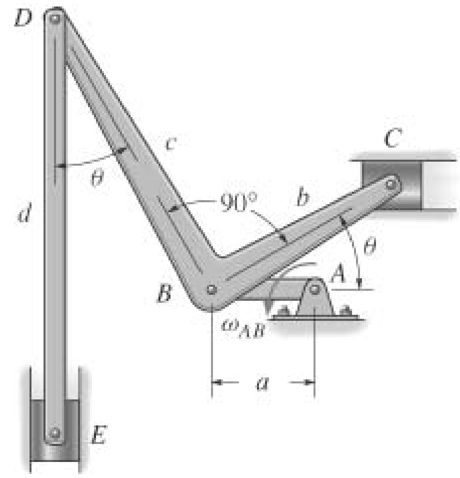
$$\begin{pmatrix} -v_A \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_B \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -r_1 \\ -r_2 \\ 0 \end{pmatrix}$$



$$\begin{pmatrix} v_A \\ \omega \end{pmatrix} = \text{Find}(v_A, \omega) \qquad \omega = 20.00 \frac{\text{rad}}{\text{s}} \qquad v_A = 8.00 \frac{\text{m}}{\text{s}}$$

Problem 16-64

If the link AB is rotating about the pin at A with angular velocity ω_{AB}, determine the velocities of blocks C and E at the instant shown.



Given:

$$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}}$$

$$a = 1\ \text{ft}$$

$$b = 2\ \text{ft}$$

$$\theta = 30\ \text{deg}$$

$$c = 3\ \text{ft}$$

$$d = 4\ \text{ft}$$

Solution:

Guesses $\omega_{BCD} = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_{DE} = 1\ \frac{\text{rad}}{\text{s}}$ $v_C = 1\ \frac{\text{ft}}{\text{s}}$ $v_E = 1\ \frac{\text{ft}}{\text{s}}$

Given

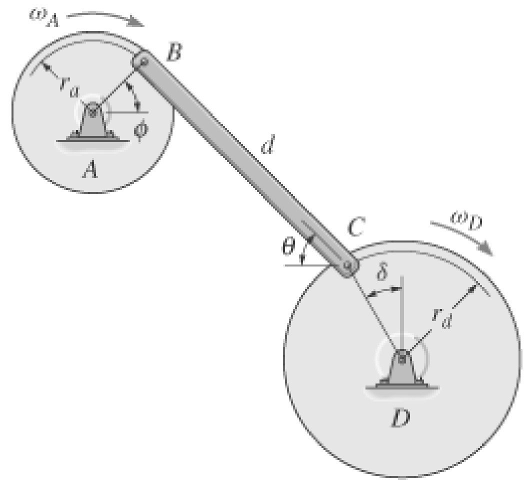
$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BCD} \end{pmatrix} \times \begin{pmatrix} b\cos(\theta) \\ b\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} -v_C \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BCD} \end{pmatrix} \times \begin{pmatrix} -c\sin(\theta) \\ c\cos(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{DE} \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_E \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{BCD} \\ \omega_{DE} \\ v_C \\ v_E \end{pmatrix} = \text{Find}(\omega_{BCD}, \omega_{DE}, v_C, v_E) \qquad \begin{pmatrix} \omega_{BCD} \\ \omega_{DE} \end{pmatrix} = \begin{pmatrix} 2.89 \\ 1.88 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} v_E \\ v_C \end{pmatrix} = \begin{pmatrix} 9.33 \\ 2.89 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

Problem 16-65

If disk D has constant angular velocity ω_D, determine the angular velocity of disk A at the instant shown.



Given:

$\omega_D = 2\ \frac{\text{rad}}{\text{s}}$ $\quad r_a = 0.5\ \text{ft}$

$\theta = 60\ \text{deg}$ $\quad r_d = 0.75\ \text{ft}$

$\phi = 45\ \text{deg}$ $\quad d = 2\ \text{ft}$

$\delta = 30\ \text{deg}$

Solution:

Guesses $\quad \omega_A = 1\ \frac{\text{rad}}{\text{s}} \quad \omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$

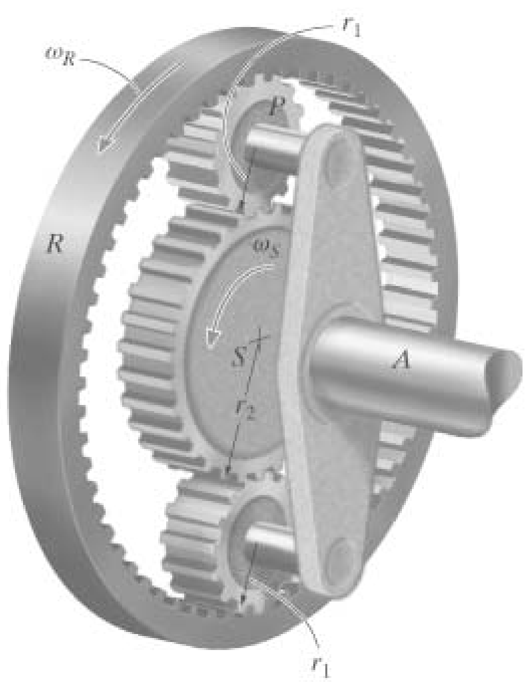
Given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_D \end{pmatrix} \times \begin{pmatrix} -r_d \sin(\delta) \\ r_d \cos(\delta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} -d \cos(\theta) \\ d \sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_A \end{pmatrix} \times \begin{pmatrix} -r_a \cos(\phi) \\ -r_a \sin(\phi) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} \omega_A \\ \omega_{BC} \end{pmatrix} = \text{Find}(\omega_A, \omega_{BC}) \qquad \omega_{BC} = -0.75\ \frac{\text{rad}}{\text{s}} \qquad \omega_A = 0.00\ \frac{\text{rad}}{\text{s}}$$

Problem 16-66

The planetary gear system is used in an automatic transmission for an automobile. By locking or releasing certain gears, it has the advantage of operating the car at different speeds. Consider the case where the ring gear *R* is rotating with angular velocity ω_R, and the sun gear *S* is held fixed, $\omega_S = 0$. Determine the angular velocity of each of the planet gears *P* and shaft *A*.



Given:

$r_1 = 40\ \text{mm}$ $\quad \omega_R = 3\ \frac{\text{rad}}{\text{s}}$

$r_2 = 80\ \text{mm}$ $\quad v_B = 0$

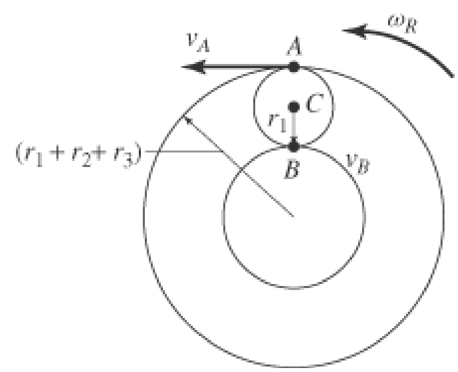
Solution:

$$v_A = \omega_R (r_2 + 2r_1)$$

$$\omega_P = \frac{v_A}{2r_1} \qquad \omega_P = 6.00\,\frac{\text{rad}}{\text{s}}$$

$$v_C = \omega_P\, r_1$$

$$\omega_A = \frac{v_C}{r_2 + r_1} \qquad \omega_A = 2.00\,\frac{\text{rad}}{\text{s}}$$



Problem 16-67

If bar AB has an angular velocity ω_{AB}, determine the velocity of the slider block C at the instant shown.

Given:

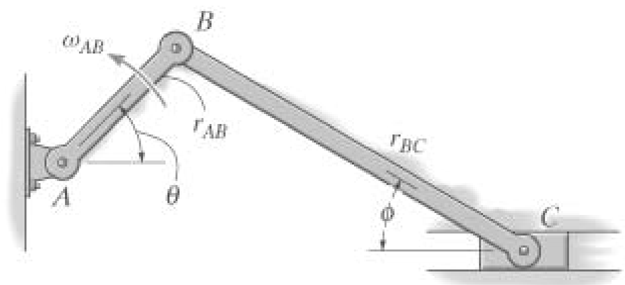
$$\omega_{AB} = 6\,\frac{\text{rad}}{\text{s}}$$

$$\theta = 45 \text{ deg}$$

$$\phi = 30 \text{ deg}$$

$$r_{AB} = 200 \text{ mm}$$

$$r_{BC} = 500 \text{ mm}$$



Solution:

Guesses $\quad \omega_{BC} = 2\,\frac{\text{rad}}{\text{s}}$

$$v_C = 4\,\frac{\text{m}}{\text{s}}$$

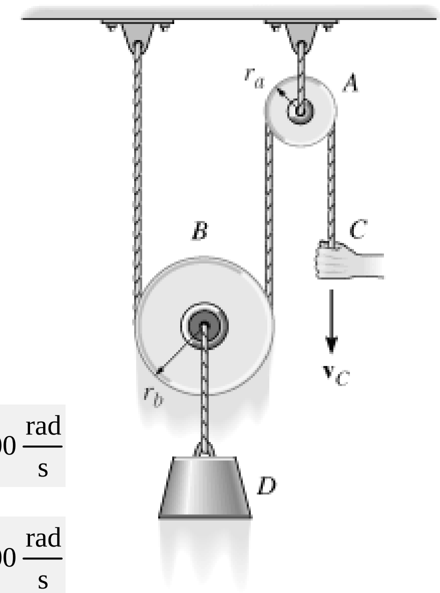
Given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} r_{AB}\cos(\theta) \\ r_{AB}\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} r_{BC}\cos(\phi) \\ -r_{BC}\sin(\phi) \\ 0 \end{pmatrix} = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C) \qquad \omega_{BC} = -1.96\,\frac{\text{rad}}{\text{s}} \qquad v_C = -1.34\,\frac{\text{m}}{\text{s}}$$

***Problem 16-68**

If the end of the cord is pulled downward with speed v_C, determine the angular velocities of pulleys A and B and the speed of block D. Assume that the cord does not slip on the pulleys.



Given:

$$v_C = 120\ \frac{\text{mm}}{\text{s}}$$

$$r_a = 30\ \text{mm}$$

$$r_b = 60\ \text{mm}$$

Solution:

$$v_C = \omega_A r_a \qquad \omega_A = \frac{v_C}{r_a} \qquad \omega_A = 4.00\ \frac{\text{rad}}{\text{s}}$$

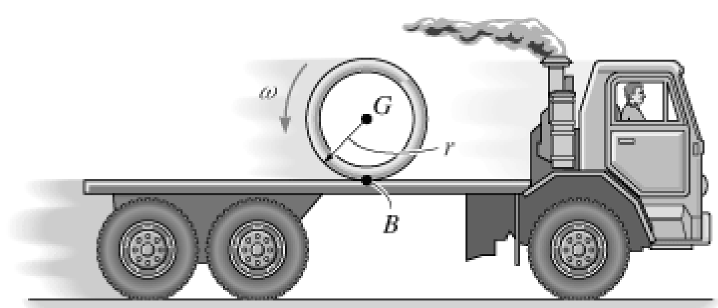
$$v_C = \omega_B 2 r_b \qquad \omega_B = \frac{v_C}{2 r_b} \qquad \omega_B = 1.00\ \frac{\text{rad}}{\text{s}}$$

$$v_D = \omega_B r_b \qquad v_D = 60.00\ \frac{\text{mm}}{\text{s}}$$

Problem 16-69

At the instant shown, the truck is traveling to the right at speed $v = at$, while the pipe is rolling counterclockwise at angular velocity $\omega = bt$, without slipping at B. Determine the velocity of the pipe's center G at time t.

Given:



$$a = 8\ \frac{\text{m}}{\text{s}^2}$$

$$b = 2\ \frac{\text{rad}}{\text{s}^2}$$

$$r = 1.5\ \text{m}$$

$$t = 3\ \text{s}$$

Solution:

$$v = a\,t \qquad \omega = b\,t$$

$v_G = v - \omega r$ $\quad$ $v_G = (a - b\,r)t$

where $\quad a - b\,r = 5.00\ \dfrac{\text{m}}{\text{s}^2}$

Problem 16-70

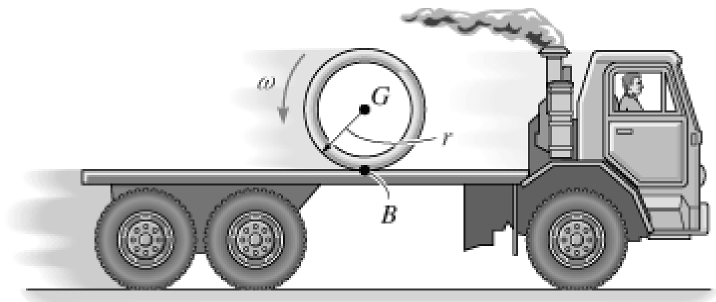
At the instant shown, the truck is traveling to the right at speed v_t. If the spool does not slip at B, determine its angular velocity if its mass center appears to an observer on the ground to be moving to the right at speed v_G.

Given:

$v_t = 12\ \dfrac{\text{m}}{\text{s}}$

$v_G = 3\ \dfrac{\text{m}}{\text{s}}$

$r = 1.5\ \text{m}$



Solution:

$v_G = v_t - \omega r$

$\omega = \dfrac{v_t - v_G}{r}$ $\quad$ $\omega = 6.00\ \dfrac{\text{rad}}{\text{s}}$

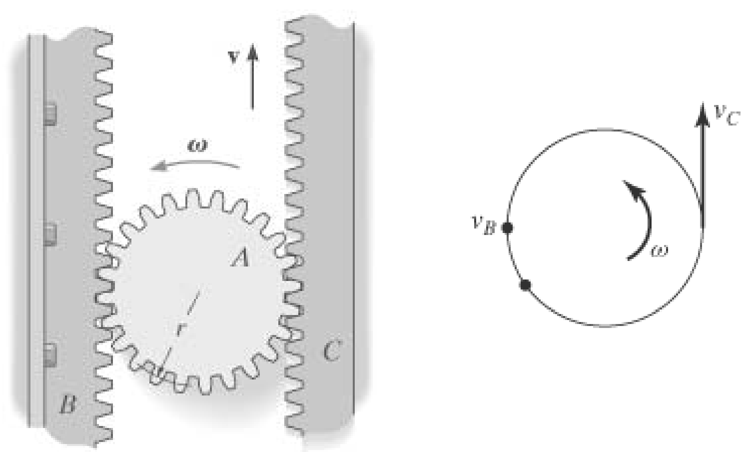
Problem 16-71

The pinion gear A rolls on the fixed gear rack B with an angular velocity ω. Determine the velocity of the gear rack C.

Given:

$\omega = 4\ \dfrac{\text{rad}}{\text{s}}$

$r = 0.3\ \text{ft}$



Solution:

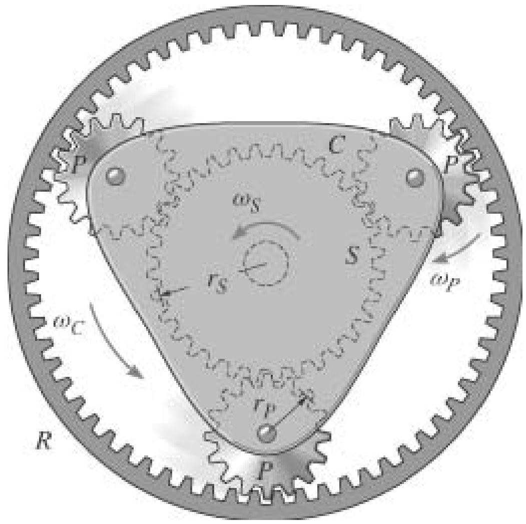
$v_C = v_B + v_{CB}$

$v_C = 2\omega r$

$v_C = 2.40\ \dfrac{\text{ft}}{\text{s}}$

***Problem 16-72**

Part of an automatic transmission consists of a *fixed* ring gear *R*, three equal planet gears *P*, the sun gear *S*, and the planet carrier *C*, which is shaded. If the sun gear is rotating with angular velocity ω_s determine the angular velocity ω_c of the *planet carrier*. Note that *C* is pin-connected to the center of each of the planet gears.



Given:

$$\omega_S = 6\,\frac{\text{rad}}{\text{s}}$$

$$r_S = 4\text{ in}$$

$$r_P = 2\text{ in}$$

Solution:

$$\omega_S r_S = \omega_P 2 r_P$$

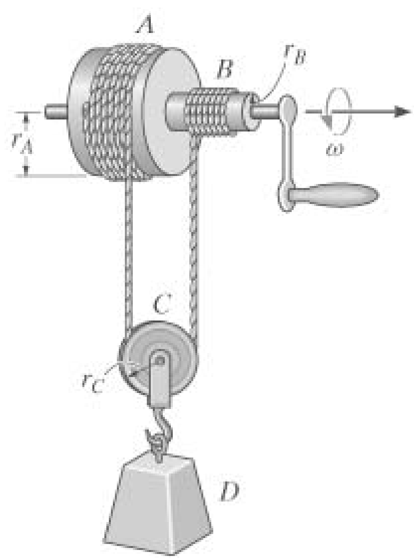
$$\omega_P = \omega_S\left(\frac{r_S}{2r_P}\right) \qquad \omega_P = 6.00\,\frac{\text{rad}}{\text{s}}$$

$$\omega_P r_P = \omega_C\left(r_S + r_P\right)$$

$$\omega_C = \omega_P\left(\frac{r_P}{r_S + r_P}\right) \qquad \omega_C = 2.00\,\frac{\text{rad}}{\text{s}}$$

Problem 16-73

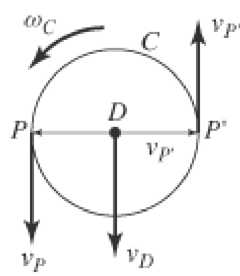
When the crank on the Chinese windlass is turning, the rope on shaft *A* unwinds while that on shaft *B* winds up. Determine the speed of block *D* if the crank is turning with an angular velocity ω. What is the angular velocity of the pulley at *C*? The rope segments on each side of the pulley are both parallel and vertical, and the rope does not slip on the pulley.



Given:

$$\omega = 4\,\frac{\text{rad}}{\text{s}} \qquad r_A = 75\text{ mm}$$

$$r_C = 50\text{ mm} \qquad r_B = 25\text{ mm}$$



Solution:

$$v_P = \omega r_A \qquad v_{P'} = \omega r_B$$

$$\omega_C = \frac{v_P + v_{P'}}{2r_C} \qquad \omega_C = 4.00\,\frac{\text{rad}}{\text{s}}$$

$$v_D = -v_{P'} + \omega_C r_C \qquad v_D = 100.00\,\frac{\text{mm}}{\text{s}}$$

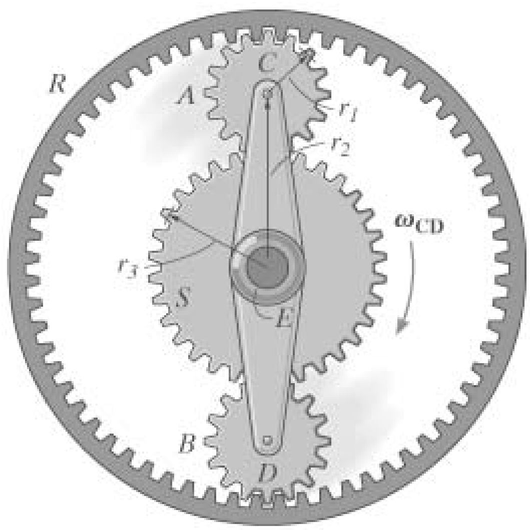
Problem 16-74

In an automobile transmission the planet pinions *A* and *B* rotate on shafts that are mounted on the planet pinion carrier *CD*. As shown, *CD* is attached to a shaft at *E* which is aligned with the center of the *fixed* sun-gear *S*. This shaft is not attached to the sun gear. If *CD* is rotating with angular velocity ω_{CD}, determine the angular velocity of the ring gear *R*.

Given:

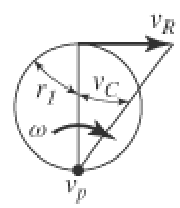
$$\omega_{CD} = 8\,\frac{\text{rad}}{\text{s}} \qquad r_1 = 50\text{ mm}$$

$$r_2 = 125\text{ mm} \qquad r_3 = 75\text{ mm}$$



Solution:

$$v_C = \omega_{CD} r_2 \qquad \omega_A = \frac{v_C}{r_1}$$



$$v_R = \omega_A 2r_1$$

$$\omega_R = \frac{v_R}{r_2 + r_1} \qquad \omega_R = 11.4\,\frac{\text{rad}}{\text{s}}$$

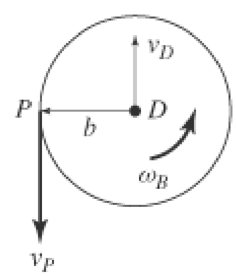
Problem 16-75

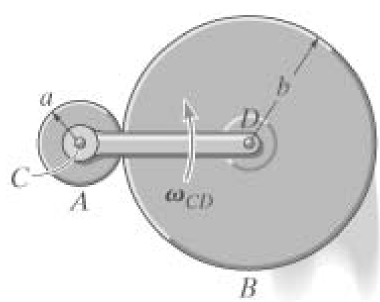
The cylinder *B* rolls on the *fixed cylinder A* without slipping. If the connected bar *CD* is rotating with an angular velocity ω_{CD}. Determine the angular velocity of cylinder *B*.

Given:

$$\omega_{CD} = 5\,\frac{\text{rad}}{\text{s}} \qquad a = 0.1\text{ m}$$

$$b = 0.3\text{ m}$$





Solution:

$$v_D = \omega_{CD}(a + b)$$

$$\omega_B = \frac{v_D}{b} \qquad \omega_B = 6.67\,\frac{\text{rad}}{\text{s}}$$

***Problem 16-76**

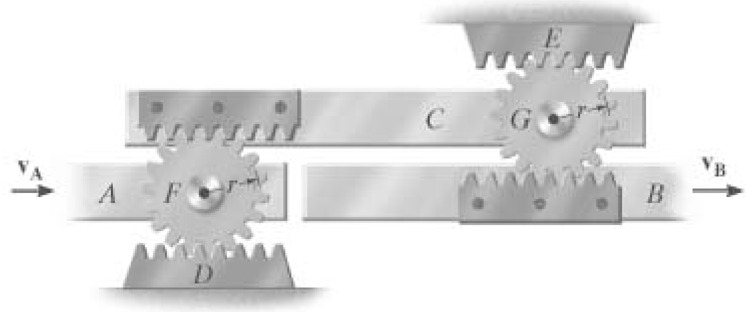
The slider mechanism is used to increase the stroke of travel of one slider with respect to that of another. As shown, when the slider *A* is moving forward, the attached pinion *F* rolls on the fixed rack *D*, forcing slider *C* to move forward. This in turn causes the attached pinion *G* to roll on the fixed rack *E*, thereby moving slider *B*. If *A* has a velocity $\mathbf{v_A}$ at the instant shown, determine the velocity of *B*.

Given:

$v_A = 4\ \frac{\text{ft}}{\text{s}}$

$r = 0.2\ \text{ft}$

$v_c = 8\ \frac{\text{ft}}{\text{s}}$



Solution:

$\omega_F = \frac{v_A}{r}$ $\quad v_C = \omega_F(2r)$

$\omega_G = \frac{v_C}{r}$ $\quad v_B = \omega_G(2r)$ $\quad v_B = 16.00\ \frac{\text{ft}}{\text{s}}$

Problem 16-77

The gauge is used to indicate the safe load acting at the end of the boom, *B*, when it is in any angular position. It consists of a fixed dial plate *D* and an indicator arm *ACE* which is pinned to the plate at *C* and to a short link *EF*. If the boom is pin-connected to the trunk frame at *G* and is rotating downward with angular velocity ω_B, determine the velocity of the dial pointer *A* at the instant shown, i.e., when *EF* and *AC* are in the vertical position.

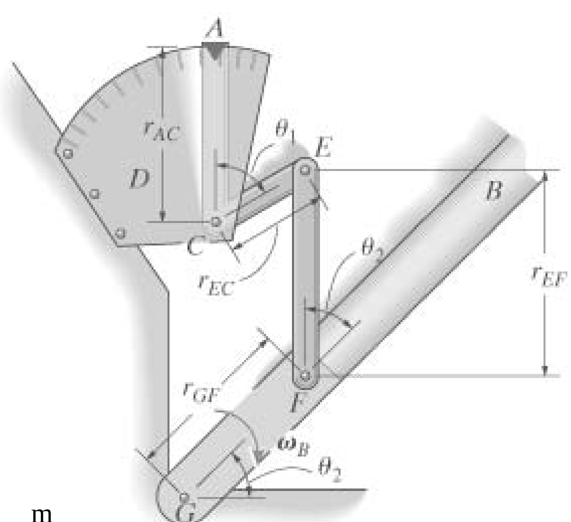
Given:

$r_{AC} = 250\ \text{mm}$ $\quad \omega_B = 4\ \frac{\text{rad}}{\text{s}}$

$r_{EC} = 150\ \text{mm}$ $\quad \theta_1 = 60\ \text{deg}$

$r_{GF} = 250\ \text{mm}$ $\quad \theta_2 = 45\ \text{deg}$

$r_{EF} = 300\ \text{mm}$



Solution:

Guesses

$\omega_{EF} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \omega_{ACE} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad v_A = 1\ \frac{\text{m}}{\text{s}}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ -\omega_B \end{pmatrix} \times \begin{pmatrix} r_{GF}\cos(\theta_2) \\ r_{GF}\sin(\theta_2) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{EF} \end{pmatrix} \times \begin{pmatrix} 0 \\ r_{EF} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{ACE} \end{pmatrix} \times \begin{pmatrix} -r_{EC}\sin(\theta_1) \\ -r_{EC}\cos(\theta_1) \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{ACE} \end{pmatrix} \times \begin{pmatrix} 0 \\ r_{AC} \\ 0 \end{pmatrix} = \begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{EF} \\ \omega_{ACE} \\ v_A \end{pmatrix} = \text{Find}(\omega_{EF}, \omega_{ACE}, v_A) \qquad \begin{pmatrix} \omega_{EF} \\ \omega_{ACE} \end{pmatrix} = \begin{pmatrix} 1.00 \\ -5.44 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_A = 1.00\ \frac{\text{m}}{\text{s}}$$

Problem 16-78

The wheel is rotating with an angular velocity ω. Determine the velocity of the collar A at the instant θ and ϕ using the method of instantaneous center of zero velocity.

Given:

$r_A = 500\ \text{mm}$

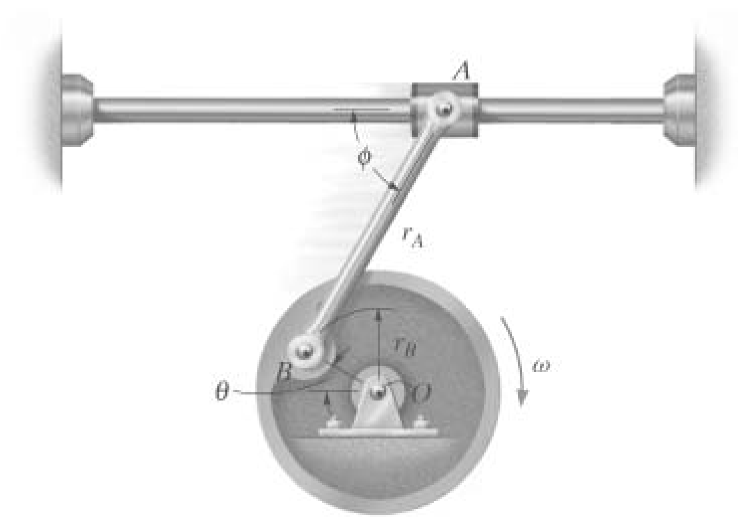
$r_B = 150\ \text{mm}$

$\theta = 30\ \text{deg}$

$\theta_1 = 90\ \text{deg}$

$\phi = 60\ \text{deg}$

$\omega = 8\ \frac{\text{rad}}{\text{s}}$



Solution:

$v_B = \omega r_B$

$v_B = 1.20\ \frac{\text{m}}{\text{s}}$

$r_{BC} = r_A \tan(\theta)$

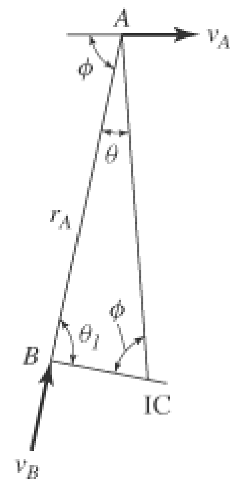
$r_{BC} = 0.289\ \text{m}$

$$\omega_{AB} = \frac{v_B}{r_{BC}}$$

$$\omega_{AB} = 4.16\ \frac{\text{rad}}{\text{s}}$$

$$r_{AC} = \frac{r_A}{\sin(\phi)} \qquad r_{AC} = 0.577\ \text{m}$$

$$v_A = r_{AC}\omega_{AB} \qquad v_A = 2.40\ \frac{\text{m}}{\text{s}}$$



Problem 16-79

The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at *C*. Determine the velocity of the slider block *C* at the instant shown, if link *AB* is rotating with angular velocity ω_{AB}. Solve using the method of instantaneous center of zero velocity.

Given:

$$r_{AB} = 300\ \text{mm}$$

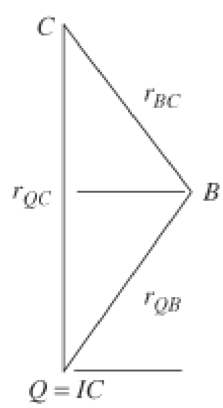
$$r_{BC} = 125\ \text{mm}$$

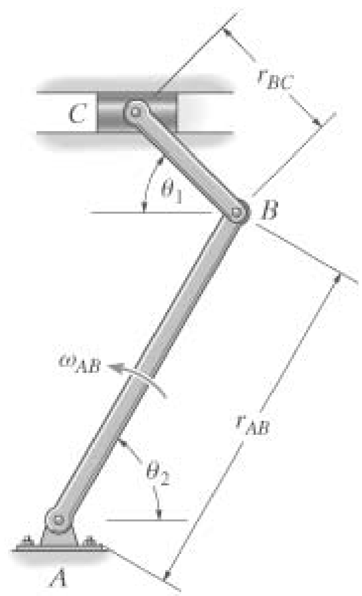
$$\omega_{AB} = 4\ \frac{\text{rad}}{\text{s}}$$

$$\theta = 90\ \text{deg}$$

$$\theta_1 = 45\ \text{deg}$$

$$\theta_2 = 60\ \text{deg}$$





Solution:

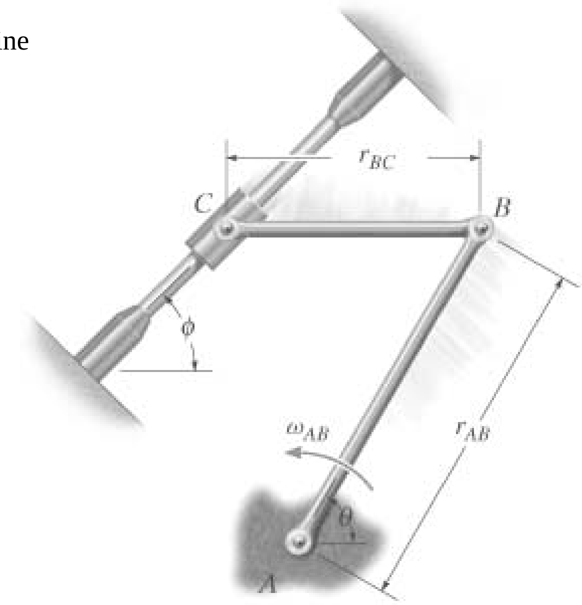
$$r_{QB} = r_{BC}\frac{\cos(\theta_1)}{\cos(\theta_2)}$$

$$r_{QC} = r_{BC}\sin(\theta_1) + r_{QB}\sin(\theta_2)$$

$$v_B = \omega_{AB} r_{AB} \qquad \omega_{BC} = \frac{v_B}{r_{QB}} \qquad v_C = \omega_{BC} r_{QC} \qquad v_C = 1.64\ \frac{\text{m}}{\text{s}}$$

***Problem 16-80**

The angular velocity of link AB is ω_{AB}. Determine the velocity of the collar at C and the angular velocity of link CB in the position shown using the method of instantaneous center of zero velocity. Link CB is horizontal at this instant.



Given:

$$\omega_{AB} = 4\ \frac{\text{rad}}{\text{s}}$$

$$r_{AB} = 500\ \text{mm}$$

$$r_{BC} = 350\ \text{mm}$$

$$\phi = 45\ \text{deg}$$

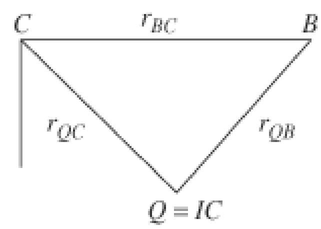
$$\theta = 60\ \text{deg}$$

Solution:

Guesses $\quad r_{QB} = 1\ \text{mm} \qquad r_{QC} = 1\ \text{mm}$

Given $\quad r_{BC} = r_{QB}\cos(\theta) + r_{QC}\sin(\phi)$

$$r_{QB}\sin(\theta) = r_{QC}\cos(\phi)$$



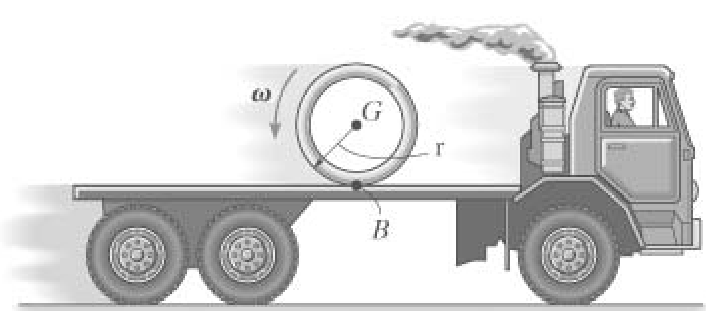
$$\begin{pmatrix} r_{QC} \\ r_{QB} \end{pmatrix} = \text{Find}(r_{QC}, r_{QB}) \qquad \begin{pmatrix} r_{QC} \\ r_{QB} \end{pmatrix} = \begin{pmatrix} 314 \\ 256 \end{pmatrix} \text{mm}$$

$$v_B = \omega_{AB} r_{AB} \qquad \omega_{CB} = \frac{v_B}{r_{QB}} \qquad v_C = \omega_{CB} r_{QC}$$

$$\omega_{CB} = 7.81\ \frac{\text{rad}}{\text{s}} \qquad v_C = 2.45\ \frac{\text{m}}{\text{s}}$$

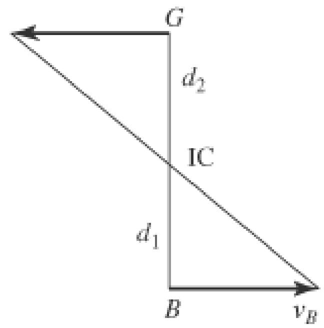
Problem 16-81

At the instant shown, the truck is traveling to the right with speed v_B, while the pipe is rolling counterclockwise with angular velocity ω without slipping at B. Determine the velocity of the pipe's center G using the method of instantaneous center of zero velocity.



Given:

$\omega = 8\ \frac{\text{rad}}{\text{s}}$ $\quad v_B = 3\ \frac{\text{m}}{\text{s}}$ $\quad r = 1.5\ \text{m}$



Solution:

$d_1 = \frac{v_B}{\omega}$ $\qquad d_1 = 0.38\ \text{m}$

$d_2 = r - d_1$ $\qquad d_2 = 1.13\ \text{m}$

$v_G = \omega d_2$ $\qquad v_G = 9.00\ \frac{\text{m}}{\text{s}}$

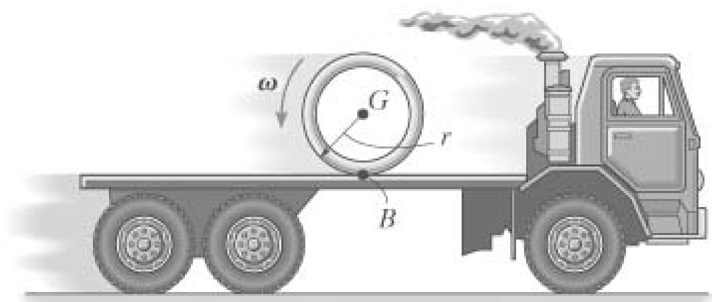
Problem 16-82

At the instant shown, the truck is traveling to the right with speed v_B. If the spool does not slip at B, determine its angular velocity so that its mass center G appears to an observer on the ground to remain stationary. Use the method of instantaneous center of zero velocity.

Given:

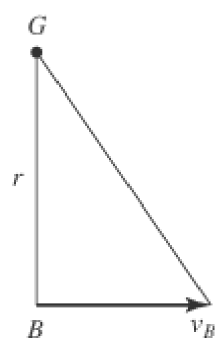
$r = 1.5\ \text{m}$

$v_B = 8\ \frac{\text{m}}{\text{s}}$



Solution:

Mass center G is the instantaneous center.



$r\omega = v_B$

$\omega = \frac{v_B}{r}$

$\omega = 5.33\ \frac{\text{rad}}{\text{s}}$

Problem 16-83

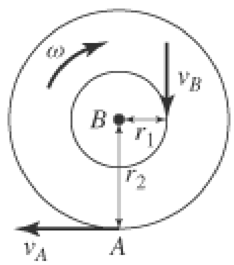
If, at a given instant, point B has a downward velocity v_B determine the velocity of point A at this instant using the method of instantaneous center of zero velocity. Notice that for this motion to occur, the wheel must slip at A.

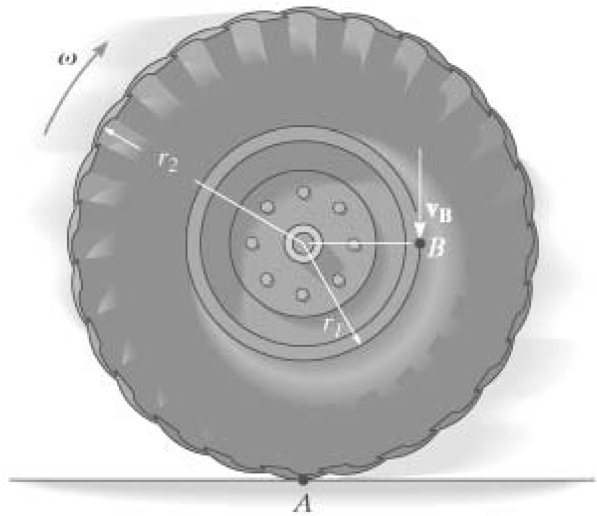
Given:

$$v_B = 3\ \frac{\text{m}}{\text{s}}$$

$$r_1 = 0.15\ \text{m}$$

$$r_2 = 0.4\ \text{m}$$





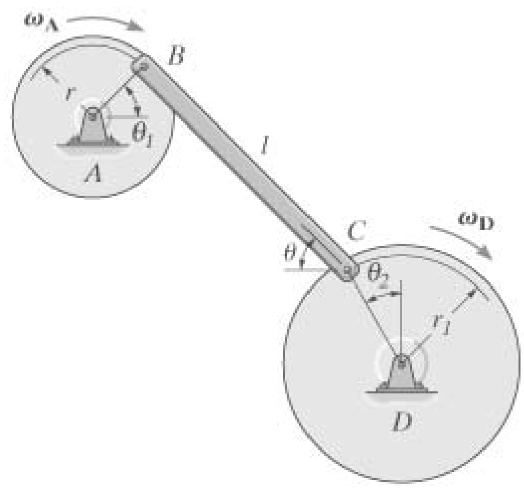
Solution:

$$\omega = \frac{v_B}{r_1} \qquad \omega = 20.00\ \frac{\text{rad}}{\text{s}}$$

$$v_A = r_2\omega \qquad v_A = 8.00\ \frac{\text{m}}{\text{s}}$$

***Problem 16-84**

If disk D has a constant angular velocity ω_D, determine the angular velocity of disk A at the instant θ, using the method of instantaneous center of zero velocity.



Given:

$$r = 0.5\ \text{ft} \qquad \theta = 60\ \text{deg}$$

$$r_1 = 0.75\ \text{ft} \qquad \theta_1 = 45\text{deg}$$

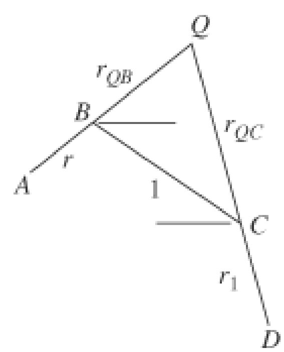
$$l = 2\ \text{ft} \qquad \theta_2 = 30\ \text{deg} \qquad \omega_D = 2\ \frac{\text{rad}}{\text{s}}$$

Solution:

$$\alpha = \theta_1 + \theta \qquad \beta = \frac{\pi}{2} - \theta - \theta_2 \qquad \gamma = \pi - \alpha - \beta$$

$$r_{QB} = l\left(\frac{\sin(\beta)}{\sin(\gamma)}\right) \qquad r_{QB} = 0\ \text{m}$$

$$r_{QC} = l\left(\frac{\sin(\alpha)}{\sin(\gamma)}\right) \qquad r_{QC} = 0.61\ \text{m}$$

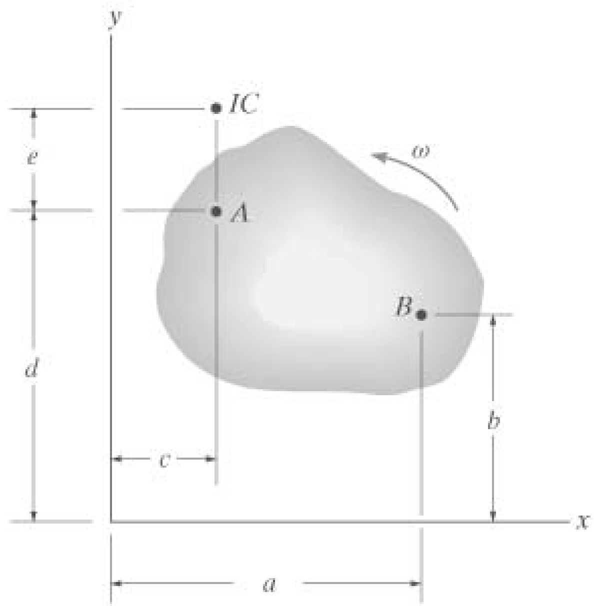


$$v_C = \omega_D r_1 \qquad \omega_{BC} = \frac{v_C}{r_{QC}} \qquad v_B = \omega_{BC} r_{QB}$$

$$\omega_A = \frac{v_B}{r} \qquad \omega_A = 0\ \frac{1}{\text{s}}$$

Problem 16-85

The instantaneous center of zero velocity for the body is located at point *IC*. If the body has an angular velocity ω, as shown, determine the velocity of *B* with respect to *A*.



Given:

$$\omega = 4\ \frac{\text{rad}}{\text{s}}$$

$$a = 1.5\ \text{m}$$

$$b = 1\ \text{m}$$

$$c = 0.5\ \text{m}$$

$$d = 1.5\ \text{m}$$

$$e = 0.5\ \text{m}$$

Solution:

$$\phi = \text{atan}\left(\frac{c + d - b}{a - c}\right)$$

$$\mathbf{v_A} = \begin{pmatrix} \omega e \\ 0 \\ 0 \end{pmatrix}$$

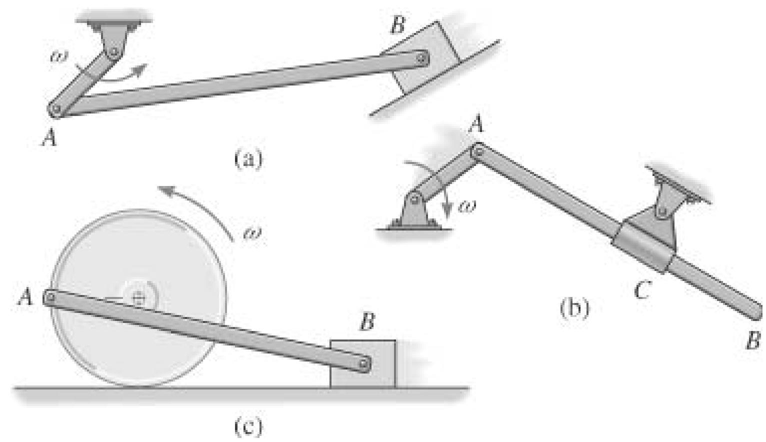
$$\mathbf{v_B} = \omega\sqrt{(a-c)^2 + (e+d-b)^2}\begin{pmatrix} \sin(\phi) \\ \cos(\phi) \\ 0 \end{pmatrix}$$

$$\mathbf{v_{BA}} = \mathbf{v_B} - \mathbf{v_A} \qquad \mathbf{v_{BA}} = \begin{pmatrix} 2.00 \\ 4.00 \\ 0.00 \end{pmatrix}\frac{\text{m}}{\text{s}} \qquad \left|\mathbf{v_{BA}}\right| = 4.47\ \frac{\text{m}}{\text{s}}$$

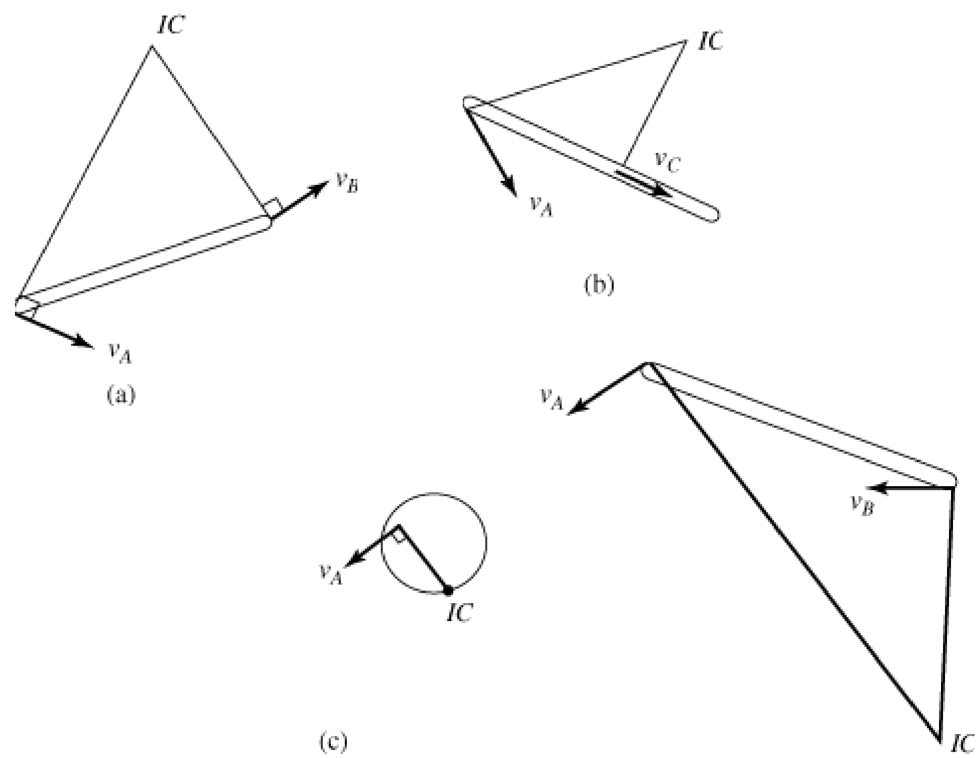
$$\theta = \text{atan}\left(\frac{\mathbf{v_{BA}}_1}{\mathbf{v_{BA}}_0}\right) \qquad \theta = 63.4\ \text{deg}$$

Problem 16-86

In each case show graphically how to locate the instantaneous center of zero velocity of link *AB*. Assume the geometry is known.

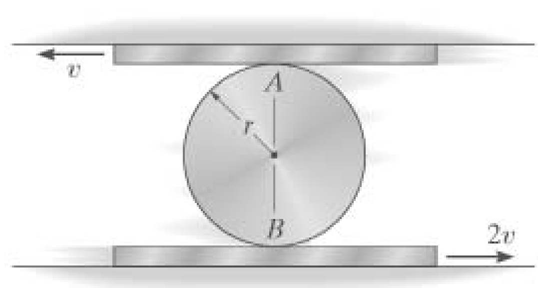


Solution:



Problem 16-87

The disk of radius *r* is confined to roll without slipping at *A* and *B*. If the plates have the velocities shown, determine the angular velocity of the disk.



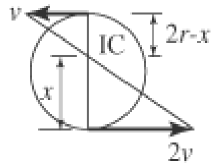
Solution:

$$\frac{v}{2r - x} = \frac{2v}{x}$$

$x = 4r - 2x$

$3x = 4\mathbf{r} \qquad x = \frac{4}{3}r$

$\omega = \frac{2v}{\frac{4r}{3}} \qquad \omega = \frac{3v}{2r}$



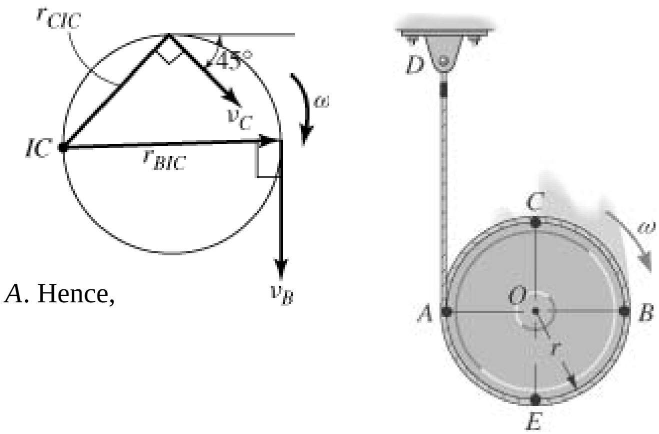
***Problem 16-88**

At the instant shown, the disk is rotating with angular velocity ω. Determine the velocities of points A, B, and C.

Given:

$\omega = 4 \frac{\text{rad}}{\text{s}}$

$r = 0.15 \text{ m}$



Solution:

The instantaneous center is located at point A. Hence,

$v_A = 0$

$v_C = \sqrt{2}r\omega \qquad v_C = 0.849 \frac{\text{m}}{\text{s}}$

$v_B = 2r\omega \qquad v_B = 1.20 \frac{\text{m}}{\text{s}}$

Problem 16-89

The slider block C is moving with speed v_C up the incline. Determine the angular velocities of links AB and BC and the velocity of point B at the instant shown.

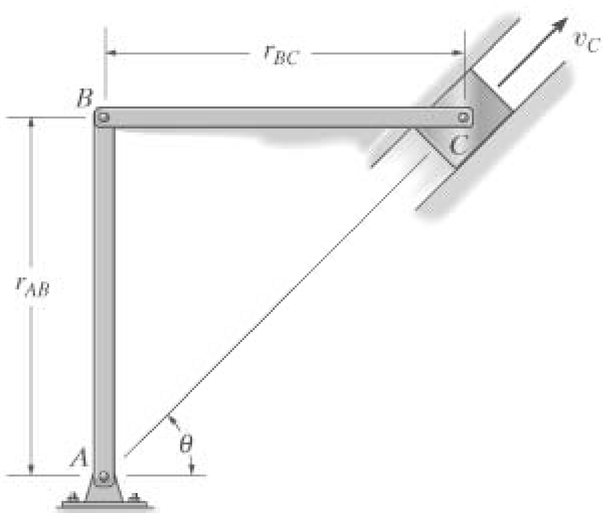
Given:

$v_C = 4 \frac{\text{ft}}{\text{s}}$

$r_{AB} = 1 \text{ ft}$

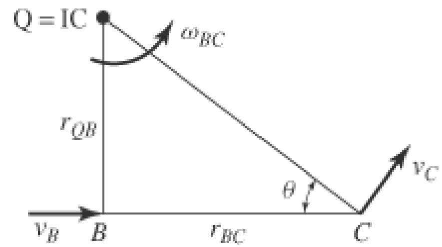
$r_{BC} = 1 \text{ ft}$

$\theta = 45 \text{ deg}$



Solution:

$r_{QB} = r_{BC} \tan(\theta)$



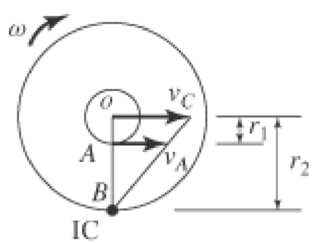
$\omega_{BC} = \frac{v_C}{r_{QB}}$ $\boxed{\omega_{BC} = 4.00 \frac{\text{rad}}{\text{s}}}$

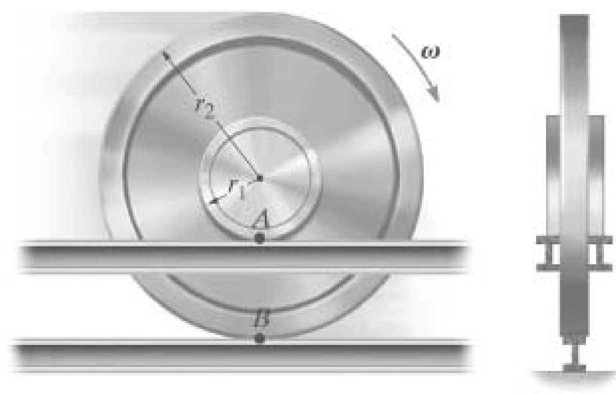
$v_B = \omega_{BC} r_{QB}$ $\boxed{v_B = 4.00 \frac{\text{ft}}{\text{s}}}$

$\omega_{AB} = \frac{v_B}{r_{AB}}$ $\boxed{\omega_{AB} = 4.00 \frac{\text{rad}}{\text{s}}}$

Problem 16-90

Show that if the rim of the wheel and its hub maintain contact with the three tracks as the wheel rolls, it is necessary that slipping occurs at the hub *A* if no slipping occurs at *B*. Under these conditions, what is the speed at *A* if the wheel has an angular velocity ω?





Solution:

IC is at *B*. $\boxed{v_A = \omega(r_2 - r_1)}$

Problem 16-91

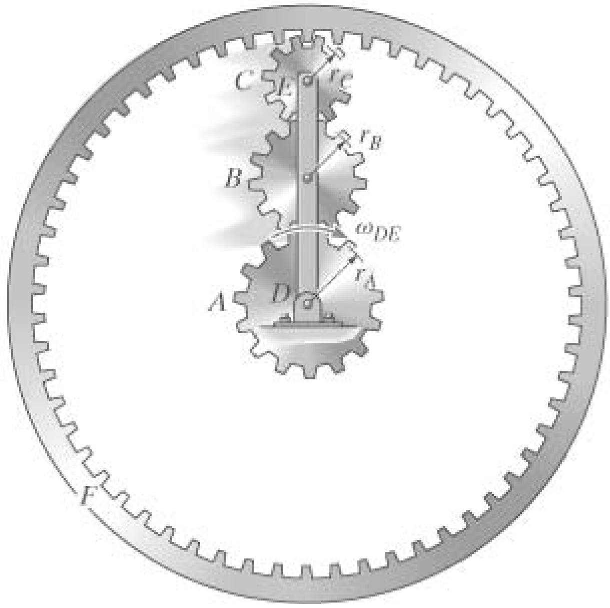
The epicyclic gear train is driven by the rotating link *DE*, which has an angular velocity ω_{DE}. If the ring gear *F* is fixed, determine the angular velocities of gears *A*, *B*, and *C*.

Given:

$r_A = 50$ mm $r_C = 30$ mm

$r_B = 40$ mm $\omega_{DE} = 5 \frac{\text{rad}}{\text{s}}$

Solution:



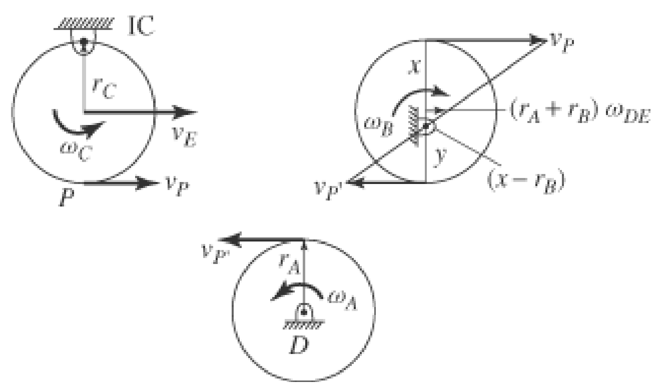
$$v_E = (r_A + 2r_B + r_C)\omega_{DE}$$

$$\omega_C = \frac{v_E}{r_C} \qquad \omega_C = 26.7\,\frac{\text{rad}}{\text{s}}$$

$$v_P = 2r_C\omega_C$$

$$\frac{v_P}{x} = \frac{(r_A + r_B)\omega_{DE}}{x - r_B}$$

$$x = \frac{r_B v_P}{v_P - (r_A + r_B)\omega_{DE}}$$



$$\omega_B = \frac{v_P}{x} \qquad \omega_B = 28.75\,\frac{\text{rad}}{\text{s}}$$

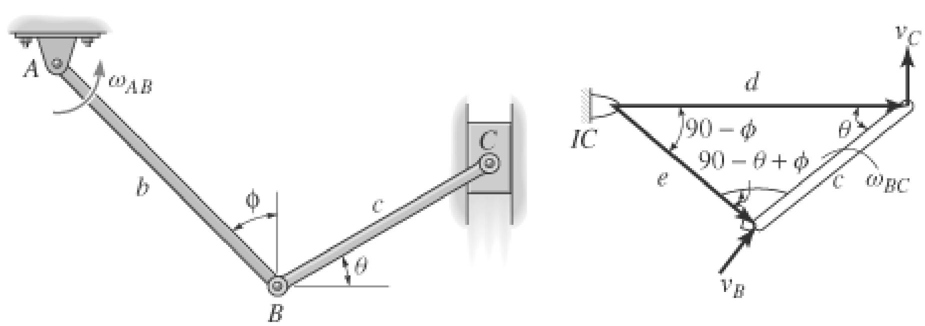
$$v_{P'} = \omega_B(2r_B - x)$$

$$\omega_A = \frac{v_{P'}}{r_A} \qquad \omega_A = 14.0\,\frac{\text{rad}}{\text{s}}$$

***Problem 16-92**

Determine the angular velocity of link *AB* at the instant shown if block *C* is moving upward at speed v_C.

Given:



$$v_C = 12\,\frac{\text{in}}{\text{s}}$$

$$c = 4 \text{ in}$$

$$b = 5 \text{ in}$$

$$\theta = 30 \text{ deg}$$

$$\phi = 45 \text{ deg}$$

Solution:

$$d = c\left(\frac{\sin(90\ \text{deg} - \theta + \phi)}{\sin(90\ \text{deg} - \phi)}\right) \qquad d = 5.46\ \text{in}$$

$$e = c\left(\frac{\sin(\theta)}{\sin(90\ \text{deg} - \phi)}\right) \qquad e = 2.83\ \text{in}$$

$$\omega_{BC} = \frac{v_C}{d} \qquad \omega_{BC} = 2.20\ \frac{\text{rad}}{\text{s}}$$

$$v_B = \omega_{BC}\, e \qquad v_B = 6.21\ \frac{\text{in}}{\text{s}}$$

$$\omega_{AB} = \frac{v_B}{b} \qquad \omega_{AB} = 1.24\ \frac{\text{rad}}{\text{s}}$$

Problem 16-93

In an automobile transmission the planet pinions *A* and *B* rotate on shafts that are mounted on the planet pinion carrier *CD*. As shown, *CD* is attached to a shaft at *E* which is aligned with the center of the *fixed* sun gear *S*. This shaft is not attached to the sun gear. If *CD* is rotating with angular velocity ω_{CD}, determine the angular velocity of the ring gear *R*.

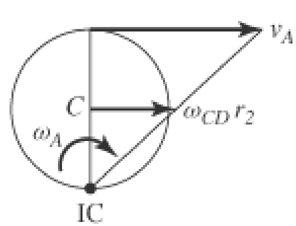
Given:

$r_1 = 50\ \text{mm} \qquad r_2 = r_1 + r_3$

$r_3 = 75\ \text{mm} \qquad \omega_{CD} = 8\ \frac{\text{rad}}{\text{s}}$

Solution:

Pinion *A*:



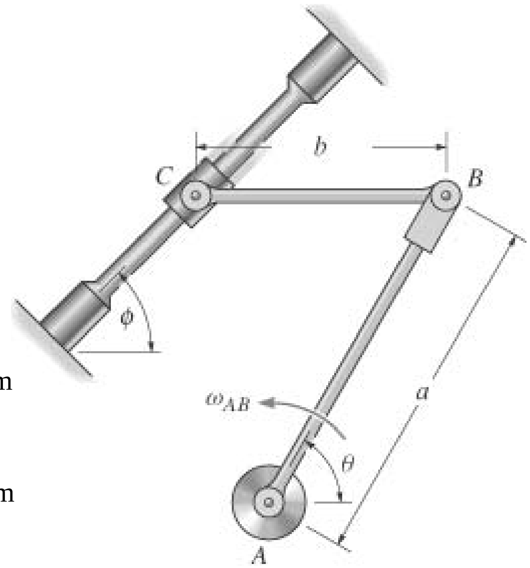
$$\omega_A = \frac{r_2\omega_{CD}}{r_1} \qquad \omega_A = 20.00\ \frac{\text{rad}}{\text{s}}$$

$$v_R = \omega_A(2r_1) \qquad v_R = 2.00\ \frac{\text{m}}{\text{s}}$$

$$\omega_R = \frac{v_R}{r_2 + r_1} \qquad \omega_R = 11.4\ \frac{\text{rad}}{\text{s}}$$

Problem 16-94

Knowing that the angular velocity of link *AB* is ω_{AB}, determine the velocity of the collar at *C* and the angular velocity of link *CB* at the instant shown. Link *CB* is horizontal at this instant.



Given:

$\omega_{AB} = 4\ \frac{\text{rad}}{\text{s}}$ $\theta = 60\ \text{deg}$

$a = 500\ \text{mm}$ $\phi = 45\ \text{deg}$

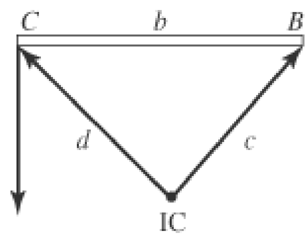
$b = 350\ \text{mm}$

Solution:

$$c = b\left(\frac{\sin(90\ \text{deg} - \phi)}{\sin(90\ \text{deg} - \theta + \phi)}\right) \qquad c = 256.22\ \text{mm}$$

$$d = b\left(\frac{\sin(\theta)}{\sin(90\ \text{deg} - \theta + \phi)}\right) \qquad d = 313.80\ \text{mm}$$

$$v_B = \omega_{AB}\, a \qquad v_B = 2.00\ \frac{\text{m}}{\text{s}}$$

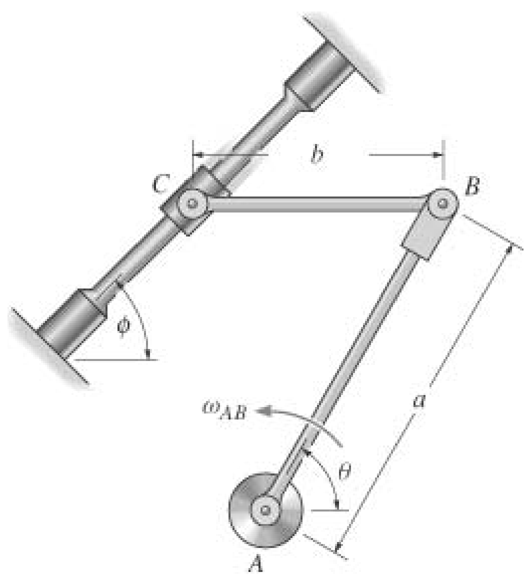


$$\omega_{BC} = \frac{v_B}{c} \qquad \omega_{BC} = 7.81\ \frac{\text{rad}}{\text{s}}$$

$$v_C = \omega_{BC}\, d \qquad v_C = 2.45\ \frac{\text{m}}{\text{s}}$$

Problem 16-95

If the collar at *C* is moving downward to the left with speed v_C, determine the angular velocity of link *AB* at the instant shown.



Given:

$v_C = 8\ \frac{\text{m}}{\text{s}}$

$a = 500\ \text{mm}$

$b = 350\ \text{mm}$

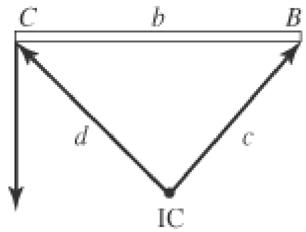
$\theta = 60\ \text{deg}$

$\phi = 45\ \text{deg}$

Solution:

$$c = b\left(\frac{\sin(90\ \deg - \phi)}{\sin(90\ \deg - \theta + \phi)}\right) \qquad c = 256.22\,\text{mm}$$

$$d = b\left(\frac{\sin(\theta)}{\sin(90\ \deg - \theta + \phi)}\right) \qquad d = 313.80\,\text{mm}$$



$$\omega_{BC} = \frac{v_C}{d} \qquad \omega_{BC} = 25.49\,\frac{\text{rad}}{\text{s}}$$

$$v_B = \omega_{BC}\,c \qquad v_B = 6.53\,\frac{\text{m}}{\text{s}}$$

$$\omega_{AB} = \frac{v_B}{a} \qquad \omega_{AB} = 13.1\,\frac{\text{rad}}{\text{s}}$$

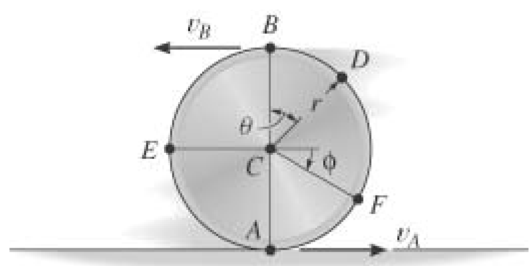
***Problem 16-96**

Due to slipping, points A and B on the rim of the disk have the velocities v_A and v_B. Determine the velocities of the center point C and point D at this instant.

Given:

$$v_A = 5\,\frac{\text{ft}}{\text{s}} \qquad \theta = 45\ \deg \qquad r = 0.8\ \text{ft}$$

$$v_B = 10\,\frac{\text{ft}}{\text{s}} \qquad \phi = 30\ \deg$$

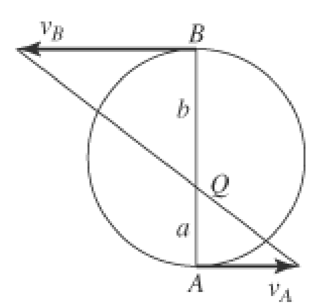


Solution:

Guesses $\quad a = 1\ \text{ft} \qquad b = 1\ \text{ft}$

Given $\quad \dfrac{a}{v_A} = \dfrac{b}{v_B} \qquad a + b = 2r$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \text{Find}(a, b) \qquad \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0.53 \\ 1.07 \end{pmatrix}\text{ft}$$



$$\omega = \frac{v_A}{a} \qquad \omega = 9.38\,\frac{\text{rad}}{\text{s}}$$

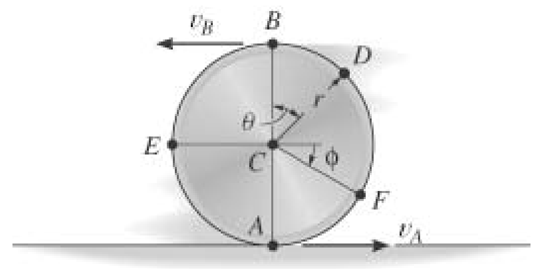
$$v_C = \omega(r - a) \qquad v_C = 2.50\,\frac{\text{ft}}{\text{s}}$$

$$v_D = \omega\sqrt{(r - a + r\cos(\theta))^2 + (r\sin(\theta))^2} \qquad v_D = 9.43\,\frac{\text{ft}}{\text{s}}$$

Problem 16-97

Due to slipping, points A and B on the rim of the disk have the velocities v_A and v_B. Determine the velocities of the center point C and point E at this instant.

Given:



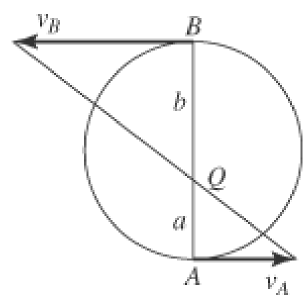
$v_A = 5\ \frac{\text{ft}}{\text{s}}$ $\theta = 45\ \text{deg}$ $r = 0.8\ \text{ft}$

$v_B = 10\ \frac{\text{ft}}{\text{s}}$ $\phi = 30\ \text{deg}$

Solution:

Guesses $a = 1\ \text{ft}$ $b = 1\ \text{ft}$

Given $\frac{a}{v_A} = \frac{b}{v_B}$ $a + b = 2r$



$\begin{pmatrix} a \\ b \end{pmatrix} = \text{Find}(a, b)$ $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0.53 \\ 1.07 \end{pmatrix} \text{ft}$

$\omega = \frac{v_A}{a}$ $\omega = 9.38\ \frac{\text{rad}}{\text{s}}$

$v_C = \omega(r - a)$ $v_C = 2.50\ \frac{\text{ft}}{\text{s}}$

$v_E = \omega\sqrt{(r-a)^2 + r^2}$ $v_E = 7.91\ \frac{\text{ft}}{\text{s}}$

Problem 16-98

The mechanism used in a marine engine consists of a single crank AB and two connecting rods BC and BD. Determine the velocity of the piston at C the instant the crank is in the position shown and has an angular velocity ω_{AB}.

Given:

$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}}$

$a = 0.2\ \text{m}$

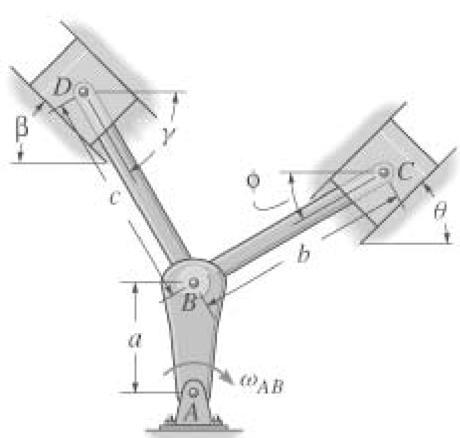
$b = 0.4\ \text{m}$

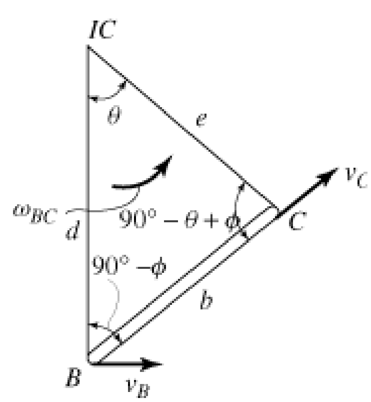
$c = 0.4\ \text{m}$

$\theta = 45\ \text{deg}$

$\phi = 30\ \text{deg}$

$\beta = 45\ \text{deg}$





Solution:

$$d = b\left(\frac{\sin(90\ \text{deg} - \phi)}{\sin(\theta)}\right) \qquad d = 0.49\ \text{m}$$

$$e = b\left(\frac{\sin(90\ \text{deg} + \phi - \theta)}{\sin(\theta)}\right) \qquad e = 0.55\ \text{m}$$

$$v_B = \omega_{AB}\, a \qquad v_B = 1.00\ \frac{\text{m}}{\text{s}}$$

$$\omega_{BC} = \frac{v_B}{e} \qquad \omega_{BC} = 1.83\ \frac{\text{rad}}{\text{s}}$$

$$v_C = \omega_{BC}\, d \qquad v_C = 0.897\ \frac{\text{m}}{\text{s}}$$

Problem 16-99

The mechanism used in a marine engine consists of a single crank *AB* and two connecting rods *BC* and *BD*. Determine the velocity of the piston at *D* the instant the crank is in the position shown and has an angular velocity ω_{AB}.

Given:

$$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}}$$

$a = 0.2\ \text{m}$

$b = 0.4\ \text{m}$

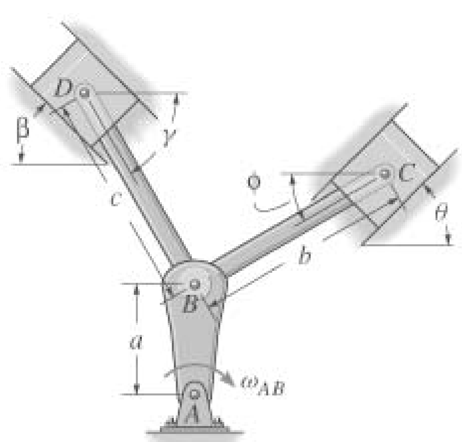
$c = 0.4\ \text{m}$

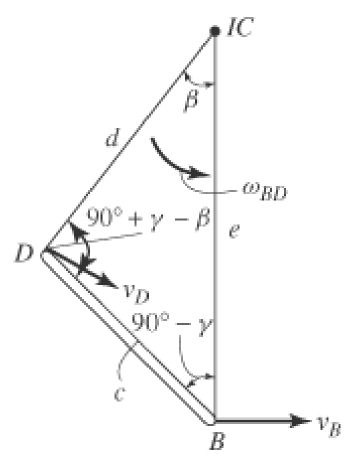
$\theta = 45\ \text{deg}$

$\phi = 30\ \text{deg}$

$\gamma = 60\ \text{deg}$

$\beta = 45\ \text{deg}$





Solution:

$$d = c\left(\frac{\sin(90\ \text{deg} - \gamma)}{\sin(\beta)}\right) \qquad d = 0.28\ \text{m}$$

$$e = c\left(\frac{\sin(90\ \text{deg} + \gamma - \beta)}{\sin(\beta)}\right) \qquad e = 0.55\ \text{m}$$

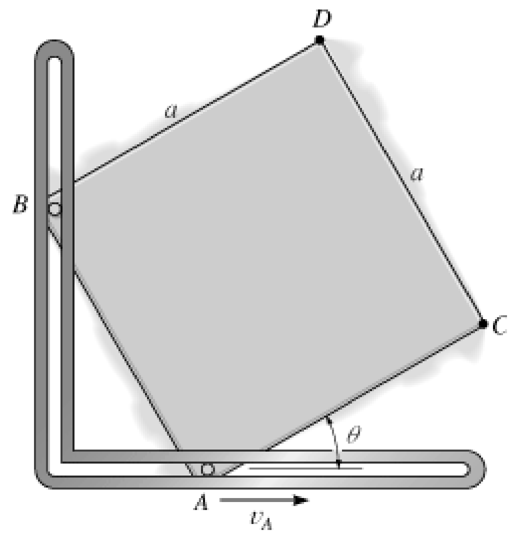
$$v_B = \omega_{AB}\, a \qquad v_B = 1.00\ \frac{\text{m}}{\text{s}}$$

$$\omega_{BC} = \frac{v_B}{e} \qquad \omega_{BC} = 1.83\ \frac{\text{rad}}{\text{s}}$$

$$v_D = \omega_{BC}\, d \qquad v_D = 0.518\ \frac{\text{m}}{\text{s}}$$

***Problem 16-100**

The square plate is confined within the slots at A and B. In the position shown, point A is moving to the right with speed v_A. Determine the velocity of point C at this instant.

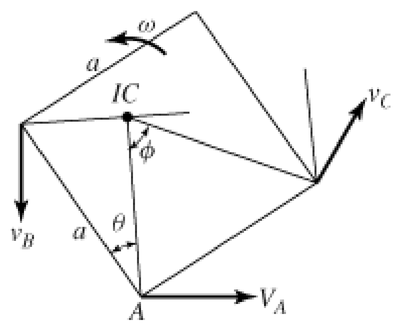


Given:

$\theta = 30\ \text{deg}$

$v_A = 8\ \frac{\text{m}}{\text{s}}$

$a = 0.3\ \text{m}$



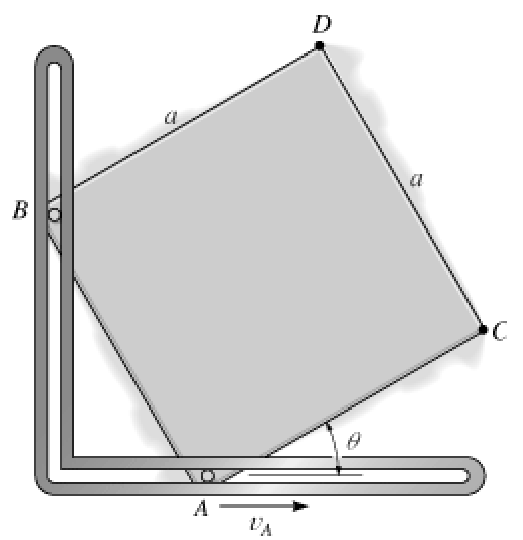
Solution:

$$\omega = \frac{v_A}{a\cos(\theta)} \qquad \omega = 30.79\ \frac{\text{rad}}{\text{s}}$$

$$v_C = \omega\sqrt{(a\cos(\theta))^2 + (a\cos(\theta) - a\sin(\theta))^2} \qquad v_C = 8.69\ \frac{\text{m}}{\text{s}}$$

Problem 16-101

The square plate is confined within the slots at A and B. In the position shown, point A is moving to the right at speed v_A. Determine the velocity of point D at this instant.

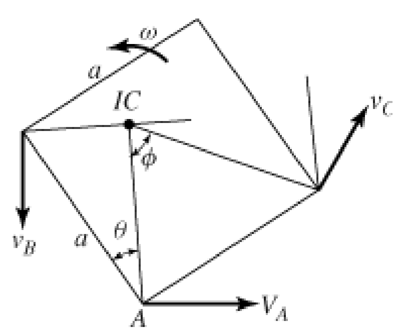


Given:

$\theta = 30\ \text{deg}$

$v_A = 8\ \frac{\text{m}}{\text{s}}$

$a = 0.3\ \text{m}$



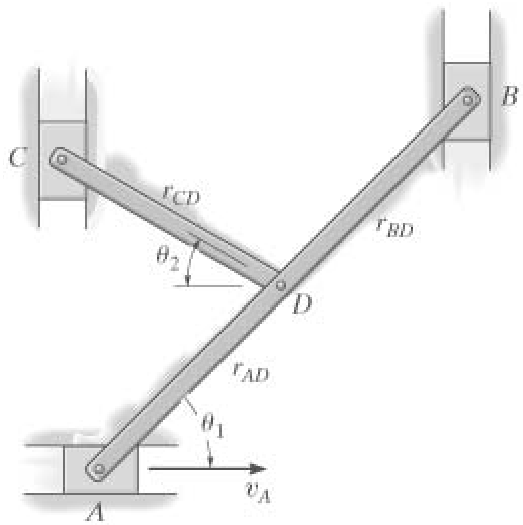
Solution:

$$\omega = \frac{v_A}{a\cos(\theta)} \qquad \omega = 30.79\,\frac{\text{rad}}{\text{s}}$$

$$v_D = \omega\sqrt{(-a\sin(\theta) + a\cos(\theta))^2 + (a\sin(\theta))^2} \qquad v_D = 5.72\,\frac{\text{m}}{\text{s}}$$

Problem 16-102

If the slider block A is moving to the right with speed v_A, determine the velocities of blocks B and C at the instant shown.



Given:

$$v_A = 8\,\frac{\text{ft}}{\text{s}} \qquad r_{AD} = 2\text{ ft}$$

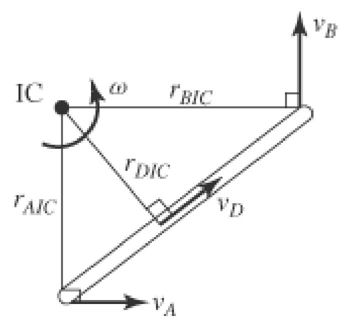
$$\theta_1 = 45\text{ deg} \qquad r_{BD} = 2\text{ ft}$$

$$\theta_2 = 30\text{ deg} \qquad r_{CD} = 2\text{ ft}$$

Solution:

$$r_{AIC} = (r_{AD} + r_{BD})\sin(\theta_1)$$

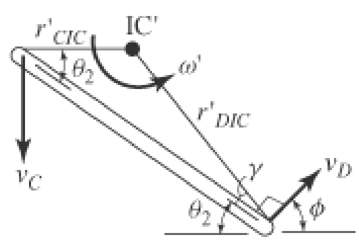
$$r_{BIC} = (r_{AD} + r_{BD})\cos(\theta_1)$$



$$r_{CID} = \sqrt{r_{AIC}^2 + r_{AD}^2 - 2r_{AIC}r_{AD}\sin(\theta_1)}$$

$$\phi = \operatorname{asin}\left(\frac{r_{AD}}{r_{CID}}\cos(\theta_1)\right)$$

$$\gamma = 90\text{ deg} - \phi - \theta_2$$



$$r'_{CIC} = r_{CD}\left(\frac{\sin(\gamma)}{\sin(90\text{ deg} + \phi)}\right)$$

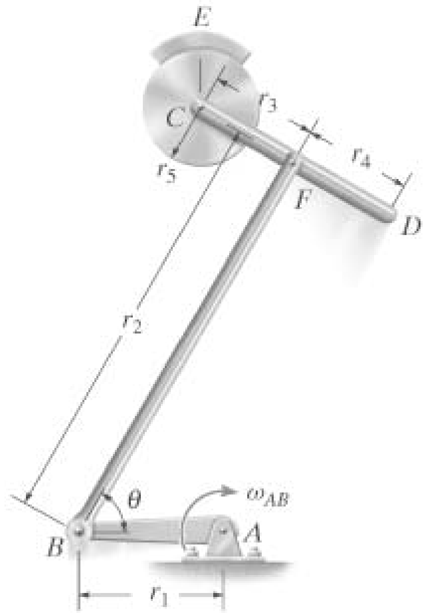
$$r'_{DIC} = r_{CD}\left(\frac{\sin(\theta_2)}{\sin(90\text{ deg} + \phi)}\right)$$

$$\omega_{AB} = \frac{v_A}{r_{AIC}} \qquad v_B = \omega_{AB}r_{BIC} \qquad v_B = 8.00\,\frac{\text{ft}}{\text{s}}$$

$v_D = \omega_{AB} r_{CID}$ $\omega_{CD} = \frac{v_D}{r'_{DIC}}$ $v_C = \omega_{CD} r'_{CIC}$ $v_C = 2.93\,\frac{\text{ft}}{\text{s}}$

Problem 16-103

The crankshaft *AB* rotates with angular velocity ω_{AB} about the fixed axis through point *A*, and the disk at *C* is held fixed in its support at *E*. Determine the angular velocity of rod *CD* at the instant shown where *CD* is perpendicular to *BF*.



Given:

$r_1 = 100\ \text{mm}$ $\theta = 60\ \text{deg}$

$r_2 = 300\ \text{mm}$ $\omega_{AB} = 50\ \frac{\text{rad}}{\text{s}}$

$r_3 = 75\ \text{mm}$

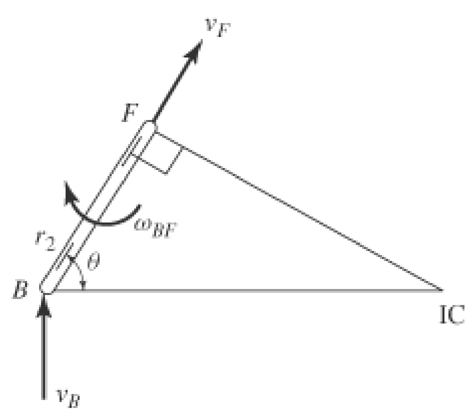
$r_4 = 75\ \text{mm}$

$r_5 = 40\ \text{mm}$

Solution:

$r_{BIC} = \frac{r_2}{\cos(\theta)}$ $r_{BIC} = 0.60\ \text{m}$

$r_{FIC} = r_2 \tan(\theta)$ $r_{FIC} = 0.5196\ \text{m}$



$v_B = \omega_{AB} r_1$ $v_B = 5.00\ \frac{\text{m}}{\text{s}}$

$\omega_{BF} = \frac{v_B}{r_{BIC}}$ $\omega_{BF} = 8.33\ \frac{\text{rad}}{\text{s}}$

$v_F = \omega_{BF} r_{FIC}$ $v_F = 4.330\ \frac{\text{m}}{\text{s}}$

Thus,

$\omega_{CD} = \frac{v_F}{r_3}$ $\omega_{CD} = 57.7\ \frac{\text{rad}}{\text{s}}$

*Problem 16-104

The mechanism shown is used in a riveting machine. It consists of a driving piston *A*, three members, and a riveter which is attached to the slider block *D*. Determine the velocity of *D* at the instant shown, when the piston at *A* is traveling at v_A.

Given:

$r_{AC} = 300\ \text{mm}$

$r_{BC} = 200\ \text{mm}$

$r_{CD} = 150\ \text{mm}$

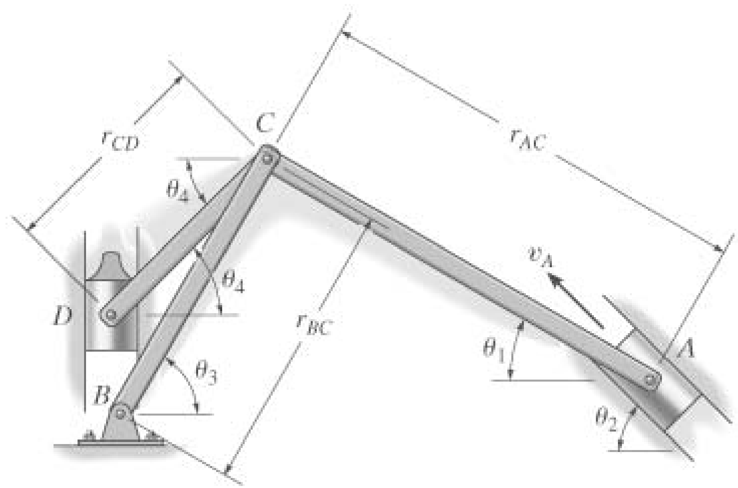
$v_A = 30\ \frac{\text{m}}{\text{s}}$

$\theta_1 = 30\ \text{deg}$

$\theta_2 = 45\ \text{deg}$

$\theta_3 = 60\ \text{deg}$

$\theta_4 = 45\ \text{deg}$



Solution:

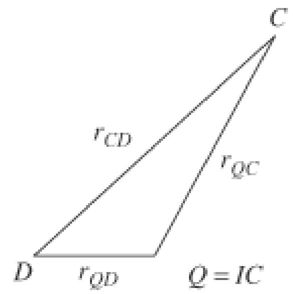
$\alpha = \theta_3 - \theta_2$

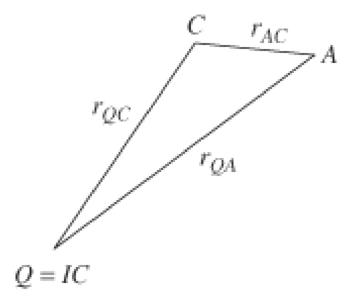
$\beta = 90\ \text{deg} - \theta_2 + \theta_1$

$\gamma = 180\ \text{deg} - \alpha - \beta$

$\delta = \theta_3 - \theta_4$

$\varepsilon = 180\ \text{deg} - \theta_4 - \delta$





$$r_{QC} = r_{AC}\left(\frac{\sin(\beta)}{\sin(\alpha)}\right) \qquad r_{QA} = r_{AC}\left(\frac{\sin(\gamma)}{\sin(\alpha)}\right)$$

$$r_{Q'D} = r_{CD}\left(\frac{\sin(\delta)}{\sin(\varepsilon)}\right) \qquad r_{Q'C} = r_{CD}\left(\frac{\sin(\theta_4)}{\sin(\varepsilon)}\right)$$

$$\omega_{AC} = \frac{v_A}{r_{QA}} \qquad v_C = \omega_{AC} r_{QC} \qquad \omega_{CD} = \frac{v_C}{r_{Q'C}} \qquad v_D = \omega_{CD} r_{Q'D}$$

$$v_D = 10.61\ \frac{\text{m}}{\text{s}}$$

Problem 16-105

At a given instant the bottom A of the ladder has acceleration a_A and velocity v_A, both acting to the left. Determine the acceleration of the top of the ladder, B, and the ladder's angular acceleration at this same instant.

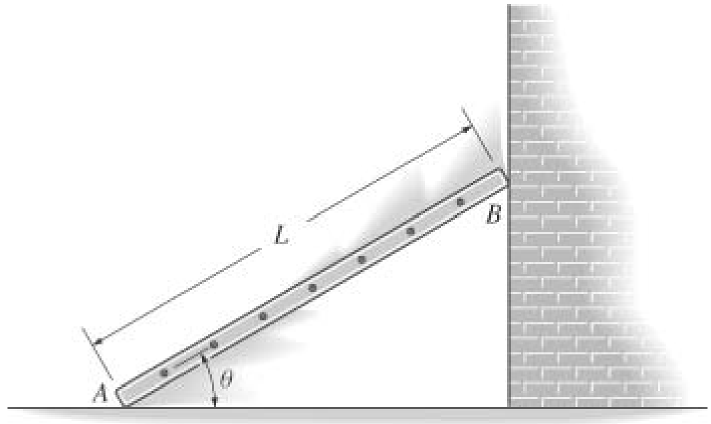
Given:

$$a_A = 4\ \frac{\text{ft}}{\text{s}^2}$$

$$v_A = 6\ \frac{\text{ft}}{\text{s}}$$

$$L = 16\ \text{ft}$$

$$\theta = 30\ \text{deg}$$



Solution: Guesses $\omega = 1\ \frac{\text{rad}}{\text{s}}$ $v_B = 1\ \frac{\text{ft}}{\text{s}}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $a_B = 1\ \frac{\text{ft}}{\text{s}^2}$

Given
$$\begin{pmatrix} -v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix}\right] = \begin{pmatrix} 0 \\ a_B \\ 0 \end{pmatrix}$$

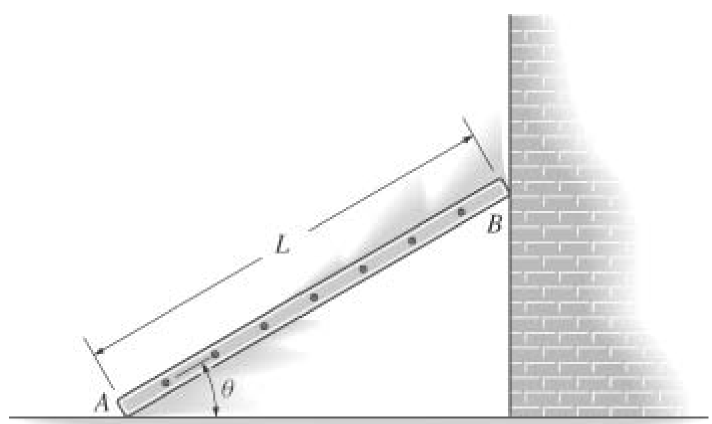
$$\begin{pmatrix} \omega \\ \alpha \\ v_B \\ a_B \end{pmatrix} = \text{Find}(\omega, \alpha, v_B, a_B)$$

$\omega = -0.75\ \frac{\text{rad}}{\text{s}}$ $v_B = -10.39\ \frac{\text{ft}}{\text{s}}$

$\alpha = -1.47\ \frac{\text{rad}}{\text{s}^2}$ $a_B = -24.93\ \frac{\text{ft}}{\text{s}^2}$

Problem 16-106

At a given instant the top B of the ladder has acceleration a_B and velocity v_B both acting downward. Determine the acceleration of the bottom A of the ladder, and the ladder's angular acceleration at this instant.



Given:

$a_B = 2\ \frac{\text{ft}}{\text{s}^2}$ $v_B = 4\ \frac{\text{ft}}{\text{s}}$

$L = 16\ \text{ft} \qquad \theta = 30\ \text{deg}$

Solution: Guesses $\omega = 1\ \frac{\text{rad}}{\text{s}} \qquad v_A = 1\ \frac{\text{ft}}{\text{s}} \qquad \alpha = 1\ \frac{\text{rad}}{\text{s}^2} \qquad a_A = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $\begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_B \\ 0 \end{pmatrix}$

$$\begin{pmatrix} a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix}\right] = \begin{pmatrix} 0 \\ -a_B \\ 0 \end{pmatrix}$$

$\begin{pmatrix} \omega \\ \alpha \\ v_A \\ a_A \end{pmatrix} = \text{Find}(\omega, \alpha, v_A, a_A) \qquad \omega = -0.289\ \frac{\text{rad}}{\text{s}} \qquad v_A = -2.31\ \frac{\text{ft}}{\text{s}}$

$\alpha = -0.0962\ \frac{\text{rad}}{\text{s}^2} \qquad a_A = 0.385\ \frac{\text{ft}}{\text{s}^2}$

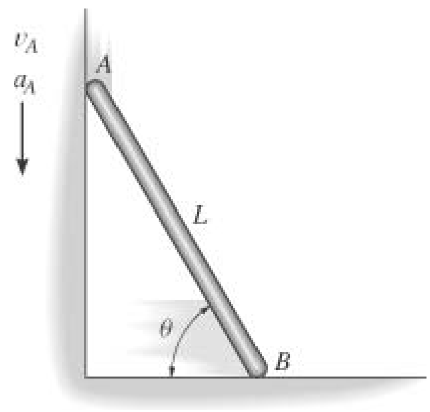
Problem 16-107

At a given instant the top end *A* of the bar has the velocity and acceleration shown. Determine the acceleration of the bottom *B* and the bar's angular acceleration at this instant.

Given:

$v_A = 5\ \frac{\text{ft}}{\text{s}} \qquad L = 10\ \text{ft}$

$a_A = 7\ \frac{\text{ft}}{\text{s}^2} \qquad \theta = 60\ \text{deg}$



Solution:

Guesses $\omega = 1\ \frac{\text{rad}}{\text{s}} \qquad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

$v_B = 1\ \frac{\text{ft}}{\text{s}} \qquad a_B = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $\begin{pmatrix} 0 \\ -v_A \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ -L\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} v_B \\ 0 \\ 0 \end{pmatrix}$

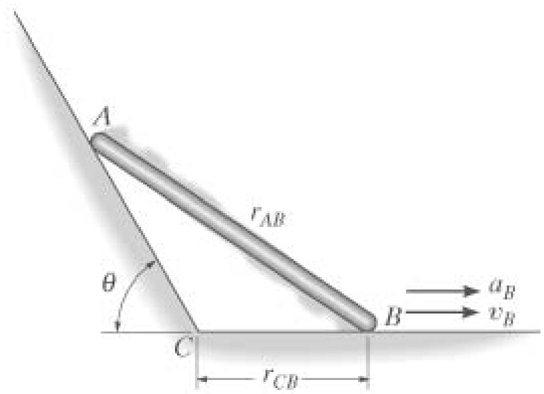
$$\begin{pmatrix} 0 \\ -a_A \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ -L\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ -L\sin(\theta) \\ 0 \end{pmatrix} \right] = \begin{pmatrix} a_B \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega \\ v_B \\ \alpha \\ a_B \end{pmatrix} = \text{Find}(\omega, v_B, \alpha, a_B) \qquad \omega = 1.00\,\frac{\text{rad}}{\text{s}} \qquad v_B = 8.66\,\frac{\text{ft}}{\text{s}}$$

$$\alpha = -0.332\,\frac{\text{rad}}{\text{s}^2} \qquad a_B = -7.88\,\frac{\text{ft}}{\text{s}^2}$$

***Problem 16-108**

The rod of length r_{AB} slides down the inclined plane, such that when it is at B it has the motion shown. Determine the velocity and acceleration of A at this instant.



Given:

$$r_{AB} = 10\ \text{ft} \qquad v_B = 2\ \frac{\text{ft}}{\text{s}}$$

$$r_{CB} = 4\ \text{ft} \qquad \theta = 60\ \text{deg}$$

$$a_B = 1\ \frac{\text{ft}}{\text{s}^2}$$

Solution:

Guesses $\quad \omega = 1\ \frac{\text{rad}}{\text{s}} \quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2} \quad v_A = 1\ \frac{\text{ft}}{\text{s}} \quad a_A = 1\ \frac{\text{ft}}{\text{s}^2} \quad \phi = 1\ \text{deg}$

Given $\quad r_{AB}\sin(\theta - \phi) = r_{CB}\sin(\theta)$

$$\begin{pmatrix} v_B \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} -r_{AB}\cos(\phi) \\ r_{AB}\sin(\phi) \\ 0 \end{pmatrix} = \begin{pmatrix} v_A\cos(\theta) \\ -v_A\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_B \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} -r_{AB}\cos(\phi) \\ r_{AB}\sin(\phi) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} -r_{AB}\cos(\phi) \\ r_{AB}\sin(\phi) \\ 0 \end{pmatrix} \right] = \begin{pmatrix} a_A\cos(\theta) \\ -a_A\sin(\theta) \\ 0 \end{pmatrix}$$

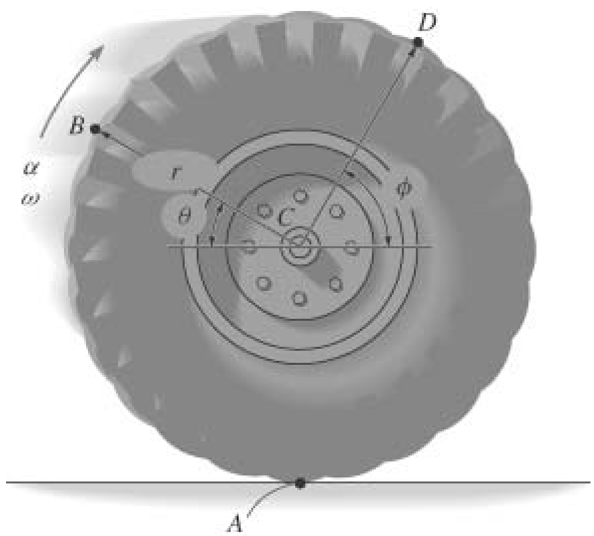
$$\begin{pmatrix} \phi \\ \omega \\ \alpha \\ v_A \\ a_A \end{pmatrix} = \text{Find}(\phi, \omega, \alpha, v_A, a_A) \qquad \phi = 39.73\,\text{deg}$$

$$\omega = 0.18\,\frac{\text{rad}}{\text{s}} \qquad \alpha = 0.1049\,\frac{\text{rad}}{\text{s}^2} \qquad v_A = 1.640\,\frac{\text{ft}}{\text{s}} \qquad a_A = 1.18\,\frac{\text{ft}}{\text{s}^2}$$

Problem 16-109

The wheel is moving to the right such that it has angular velocity ω and angular acceleration α at the instant shown. If it does not slip at *A*, determine the acceleration of point *B*.

Given:



$$\omega = 2\,\frac{\text{rad}}{\text{s}}$$

$$\alpha = 4\,\frac{\text{rad}}{\text{s}^2}$$

$$r = 1.45\ \text{ft}$$

$$\theta = 30\ \text{deg}$$

$$\phi = 60\ \text{deg}$$

Solution:

$$\mathbf{a_B} = \begin{pmatrix} \alpha r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\alpha \end{pmatrix} \times \begin{pmatrix} -r\cos(\theta) \\ r\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -r\cos(\theta) \\ r\sin(\theta) \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_B} = \begin{pmatrix} 13.72 \\ 2.12 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_B}| = 13.89\,\frac{\text{ft}}{\text{s}^2}$$

Problem 16-110

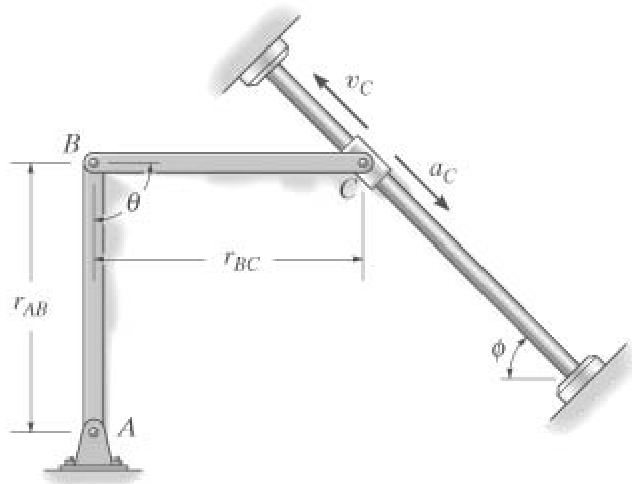
Determine the angular acceleration of link *AB* at the instant shown if the collar *C* has velocity v_c and deceleration a_c as shown.

Given:

$v_C = 4\ \frac{\text{ft}}{\text{s}}$ $\qquad r_{AB} = 0.5\ \text{ft}$

$a_C = 3\ \frac{\text{ft}}{\text{s}^2}$ $\qquad r_{BC} = 0.5\ \text{ft}$

$\theta = 90\ \text{deg}$ $\qquad \phi = 45\ \text{deg}$



Solution:

Guesses $\omega_{AB} = 1\ \frac{\text{rad}}{\text{s}}$ $\qquad \omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$

$\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2}$ $\qquad \alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix}0\\0\\\omega_{AB}\end{pmatrix}\times\begin{pmatrix}0\\r_{AB}\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{BC}\end{pmatrix}\times\begin{pmatrix}r_{BC}\sin(\theta)\\-r_{BC}\cos(\theta)\\0\end{pmatrix}=\begin{pmatrix}-v_C\cos(\phi)\\v_C\sin(\phi)\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\0\\\alpha_{AB}\end{pmatrix}\times\begin{pmatrix}0\\r_{AB}\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{AB}\end{pmatrix}\times\left[\begin{pmatrix}0\\0\\\omega_{AB}\end{pmatrix}\times\begin{pmatrix}0\\r_{AB}\\0\end{pmatrix}\right]\dots$$
$$+\begin{pmatrix}0\\0\\\alpha_{BC}\end{pmatrix}\times\begin{pmatrix}r_{BC}\sin(\theta)\\-r_{BC}\cos(\theta)\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{BC}\end{pmatrix}\times\left[\begin{pmatrix}0\\0\\\omega_{BC}\end{pmatrix}\times\begin{pmatrix}r_{BC}\sin(\theta)\\-r_{BC}\cos(\theta)\\0\end{pmatrix}\right]=\begin{pmatrix}a_C\cos(\phi)\\-a_C\sin(\phi)\\0\end{pmatrix}$$

$$\begin{pmatrix}\omega_{AB}\\\omega_{BC}\\\alpha_{AB}\\\alpha_{BC}\end{pmatrix}=\text{Find}(\omega_{AB},\omega_{BC},\alpha_{AB},\alpha_{BC})$$

$$\begin{pmatrix}\omega_{AB}\\\omega_{BC}\end{pmatrix}=\begin{pmatrix}5.66\\5.66\end{pmatrix}\frac{\text{rad}}{\text{s}}$$

$\alpha_{BC} = 27.8\ \frac{\text{rad}}{\text{s}^2}$ $\qquad \alpha_{AB} = -36.2\ \frac{\text{rad}}{\text{s}^2}$

Problem 16-111

The flywheel rotates with angular velocity ω and angular acceleration α. Determine the angular acceleration of links *AB* and *BC* at the instant shown.

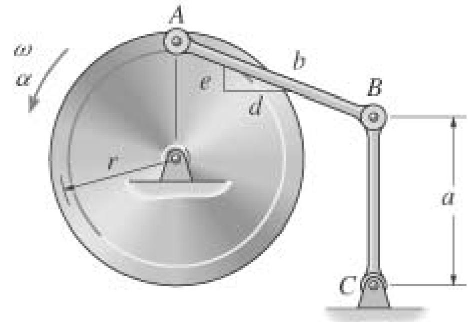
Given:

$\omega = 2\ \frac{\text{rad}}{\text{s}}$ $\quad a = 0.4\ \text{m}$

$\alpha = 6\ \frac{\text{rad}}{\text{s}^2}$ $\quad b = 0.5\ \text{m}$

$r = 0.3\ \text{m}$ $\quad e = 3$

$d = 4$



Solution:

$$\mathbf{r_1} = \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} \qquad \mathbf{r_2} = \frac{b}{\sqrt{e^2 + d^2}}\begin{pmatrix} d \\ -e \\ 0 \end{pmatrix} \qquad \mathbf{r_3} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \omega_{AB} = 1\ \frac{\text{rad}}{\text{s}} \quad \omega_{BC} = 1\ \frac{\text{rad}}{\text{s}} \quad \alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2} \quad \alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\omega\mathbf{k} \times \mathbf{r_1} + \omega_{AB}\mathbf{k} \times \mathbf{r_2} + \omega_{BC}\mathbf{k} \times \mathbf{r_3} = 0$$

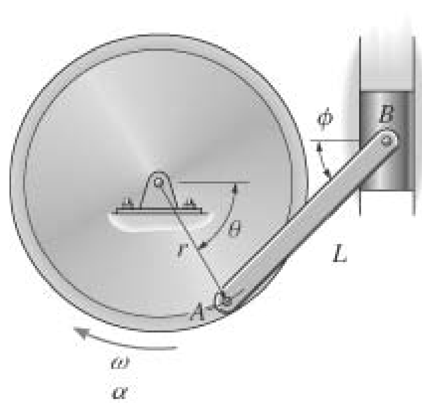
$$\alpha\mathbf{k} \times \mathbf{r_1} + \omega\mathbf{k} \times (\omega\mathbf{k} \times \mathbf{r_1}) + \alpha_{AB}\mathbf{k} \times \mathbf{r_2} + \omega_{AB}\mathbf{k} \times (\omega_{AB}\mathbf{k} \times \mathbf{r_2}) \ ... = 0$$
$$+ \alpha_{BC}\mathbf{k} \times \mathbf{r_3} + \omega_{BC}\mathbf{k} \times (\omega_{BC}\mathbf{k} \times \mathbf{r_3})$$

$$\begin{pmatrix} \omega_{AB} \\ \omega_{BC} \\ \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \text{Find}(\omega_{AB}, \omega_{BC}, \alpha_{AB}, \alpha_{BC}) \qquad \begin{pmatrix} \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \begin{pmatrix} 0.00 \\ 1.50 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\begin{pmatrix} \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \begin{pmatrix} 0.75 \\ 3.94 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

***Problem 16-112**

At a given instant the wheel is rotating with the angular velocity and angular acceleration shown. Determine the acceleration of block *B* at this instant.



Given:

$\omega = 2\ \frac{\text{rad}}{\text{s}}$ $\quad \theta = 60\ \text{deg}$

$\alpha = 6\ \frac{\text{rad}}{\text{s}^2}$ $\quad L = 0.5\ \text{m}$

$r = 0.3\ \text{m}$ $\quad \phi = 45\ \text{deg}$

Solution:

$$\mathbf{r_1} = r\begin{pmatrix} \cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{r_2} = L\begin{pmatrix} \cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \omega_{AB} = 1\ \frac{\text{rad}}{\text{s}} \quad \alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2} \quad v_B = 1\ \frac{\text{m}}{\text{s}} \quad a_B = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$-\omega\mathbf{k}\times\mathbf{r_1} + \omega_{AB}\mathbf{k}\times\mathbf{r_2} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix}$$

$$-\alpha\,\mathbf{k}\times\mathbf{r_1} - \omega\mathbf{k}\times\left(-\omega\mathbf{k}\times\mathbf{r_1}\right) + \alpha_{AB}\mathbf{k}\times\mathbf{r_2} + \omega_{AB}\mathbf{k}\times\left(\omega_{AB}\mathbf{k}\times\mathbf{r_2}\right) = \begin{pmatrix} 0 \\ a_B \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{AB} \\ \alpha_{AB} \\ v_B \\ a_B \end{pmatrix} = \text{Find}\left(\omega_{AB}, \alpha_{AB}, v_B, a_B\right) \qquad \omega_{AB} = -1.47\ \frac{\text{rad}}{\text{s}} \qquad v_B = -0.82\ \frac{\text{m}}{\text{s}}$$

$$\alpha_{AB} = -8.27\ \frac{\text{rad}}{\text{s}^2} \qquad a_B = -3.55\ \frac{\text{m}}{\text{s}^2}$$

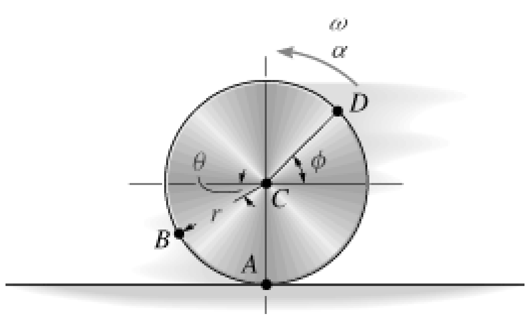
Problem 16-113

The disk is moving to the left such that it has angular acceleration α and angular velocity ω at the instant shown. If it does not slip at A, determine the acceleration of point B.

Given:

$$\alpha = 8\ \frac{\text{rad}}{\text{s}^2} \qquad r = 0.5\ \text{m} \qquad \phi = 45\ \text{deg}$$

$$\omega = 3\ \frac{\text{rad}}{\text{s}} \qquad \theta = 30\ \text{deg}$$



Solution:

$$\mathbf{a_B} = \begin{pmatrix} -\alpha\,r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix}\times\begin{pmatrix} -r\cos(\theta) \\ -r\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}\times\left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}\times\begin{pmatrix} -r\cos(\theta) \\ -r\sin(\theta) \\ 0 \end{pmatrix}\right]$$

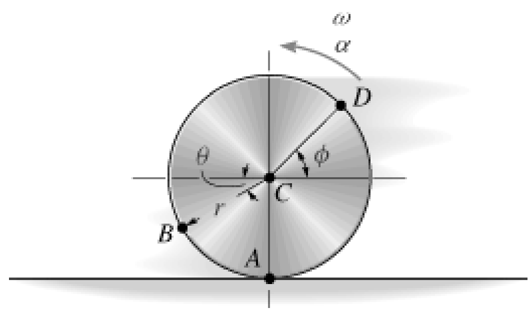
$$\mathbf{a_B} = \begin{pmatrix} 1.90 \\ -1.21 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad |\mathbf{a_B}| = 2.25 \frac{\text{m}}{\text{s}^2}$$

$$\theta = \operatorname{atan}\left(\frac{-\alpha r\cos(\theta) + \omega^2 r\sin(\theta)}{-\alpha r + \alpha r\sin(\theta) + \omega^2 r\cos(\theta)}\right) \qquad \theta = -32.6\,\text{deg} \qquad |\theta| = 32.62\,\text{deg}$$

Problem 16-114

The disk is moving to the left such that it has angular acceleration α and angular velocity ω at the instant shown. If it does not slip at A, determine the acceleration of point D.

Given:



$$\alpha = 8 \frac{\text{rad}}{\text{s}^2} \qquad r = 0.5\ \text{m} \qquad \phi = 45\ \text{deg}$$

$$\omega = 3 \frac{\text{rad}}{\text{s}} \qquad \theta = 30\ \text{deg}$$

Solution:

$$\mathbf{a_D} = \begin{pmatrix} -\alpha r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} r\cos(\phi) \\ r\sin(\phi) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r\cos(\phi) \\ r\sin(\phi) \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_D} = \begin{pmatrix} -10.01 \\ -0.35 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad |\mathbf{a_D}| = 10.02 \frac{\text{m}}{\text{s}^2}$$

$$\theta = \operatorname{atan}\left(\frac{\alpha r\cos(\phi) - \omega^2 r\sin(\phi)}{-\alpha r - \alpha r\sin(\phi) - \omega^2 r\cos(\phi)}\right) \qquad \theta = 2.02\,\text{deg}$$

Problem 16-115

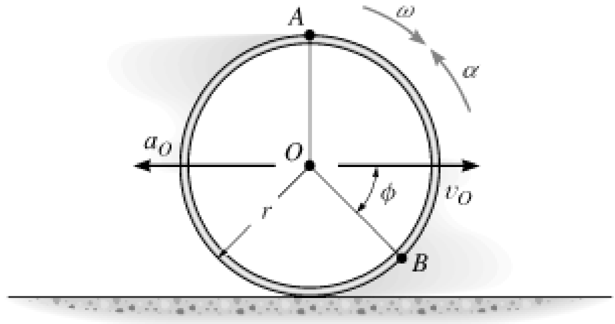
The hoop is cast on the rough surface such that it has angular velocity ω and angular acceleration α.. Also, its center has a velocity v_0 and a deceleration a_0. Determine the acceleration of point A at this instant.

Given:

$$\omega = 4 \frac{\text{rad}}{\text{s}} \qquad a_0 = 2 \frac{\text{m}}{\text{s}^2}$$

$\alpha = 5\ \frac{\text{rad}}{\text{s}^2}$ $\quad r = 0.3\ \text{m}$

$v_0 = 5\ \frac{\text{m}}{\text{s}}$ $\quad \phi = 45\ \text{deg}$



Solution:

$$\mathbf{a_A} = \begin{pmatrix} -a_0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_A} = \begin{pmatrix} -3.50 \\ -4.80 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_A}\right| = 5.94\ \frac{\text{m}}{\text{s}^2}$$

$$\theta = \operatorname{atan}\left(\frac{-\omega^2 r}{-a_0 - \alpha r}\right) \qquad \theta = 53.9\ \text{deg}$$

***Problem 16-116**

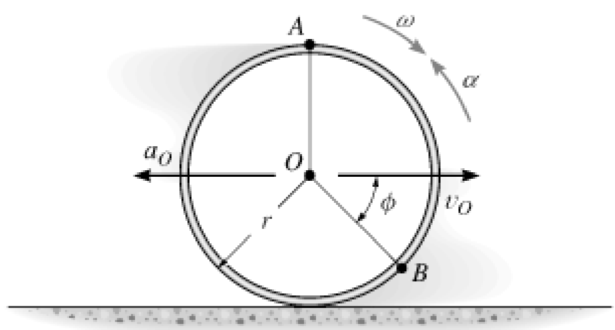
The hoop is cast on the rough surface such that it has angular velocity ω and angular acceleration α. Also, its center has a velocity v_0 and a deceleration a_0. Determine the acceleration of point B at this instant.

Given:

$\omega = 4\ \frac{\text{rad}}{\text{s}}$ $\quad a_0 = 2\ \frac{\text{m}}{\text{s}^2}$

$\alpha = 5\ \frac{\text{rad}}{\text{s}^2}$ $\quad r = 0.3\ \text{m}$

$v_0 = 5\ \frac{\text{m}}{\text{s}}$ $\quad \phi = 45\ \text{deg}$



Solution:

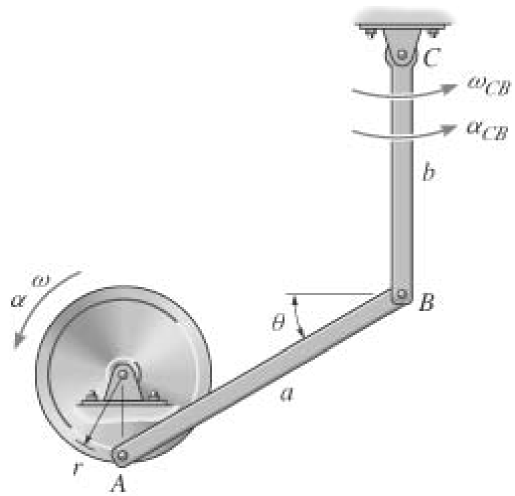
$$\mathbf{a_B} = \begin{pmatrix} -a_0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} r\cos(\phi) \\ -r\sin(\phi) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} r\cos(\phi) \\ -r\sin(\phi) \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_B} = \begin{pmatrix} -4.33 \\ 4.45 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad |\mathbf{a_B}| = 6.21 \frac{\text{m}}{\text{s}^2}$$

$$\theta = \text{atan}\left(\frac{\alpha r\cos(\phi) + \omega^2 r\sin(\phi)}{-a_0 + \alpha r\sin(\phi) - \omega^2 r\cos(\phi)}\right) \qquad \theta = -45.8\,\text{deg} \qquad |\theta| = 45.8\,\text{deg}$$

Problem 16-117

The disk rotates with angular velocity ω and angular acceleration α. Determine the angular acceleration of link *CB* at this instant.



Given:

$\omega = 5 \frac{\text{rad}}{\text{s}}$ $\quad a = 2\ \text{ft}$

$\alpha = 6 \frac{\text{rad}}{\text{s}^2}$ $\quad b = 1.5\ \text{ft}$

$r = 0.5\ \text{ft}$ $\quad \theta = 30\ \text{deg}$

Solution:

$$\mathbf{r_1} = \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \qquad \mathbf{r_2} = \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix}$$

$$\mathbf{r_3} = \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \omega_{AB} = 1 \frac{\text{rad}}{\text{s}} \quad \omega_{BC} = 1 \frac{\text{rad}}{\text{s}} \quad \alpha_{AB} = 1 \frac{\text{rad}}{\text{s}^2} \quad \alpha_{BC} = 1 \frac{\text{rad}}{\text{s}^2}$

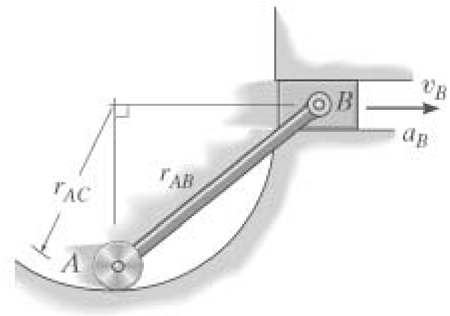
Given $\quad \omega\mathbf{k} \times \mathbf{r_1} + \omega_{AB}\mathbf{k} \times \mathbf{r_2} + \omega_{BC}\mathbf{k} \times \mathbf{r_3} = 0$

$$\alpha\mathbf{k} \times \mathbf{r_1} + \omega\mathbf{k} \times (\omega\mathbf{k} \times \mathbf{r_1}) + \alpha_{AB}\mathbf{k} \times \mathbf{r_2} + \omega_{AB}\mathbf{k} \times (\omega_{AB}\mathbf{k} \times \mathbf{r_2}) \ldots = 0$$
$$+ \alpha_{BC}\mathbf{k} \times \mathbf{r_3} + \omega_{BC}\mathbf{k} \times (\omega_{BC}\mathbf{k} \times \mathbf{r_3})$$

$$\begin{pmatrix} \omega_{AB} \\ \omega_{BC} \\ \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \text{Find}(\omega_{AB}, \omega_{BC}, \alpha_{AB}, \alpha_{BC}) \qquad \begin{pmatrix} \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \begin{pmatrix} 0.00 \\ 1.67 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\alpha_{AB} = -4.81 \frac{\text{rad}}{\text{s}^2} \qquad \alpha_{BC} = 5.21 \frac{\text{rad}}{\text{s}^2}$$

Problem 16-118



At a given instant the slider block *B* is moving to the right with the motion shown. Determine the angular acceleration of link *AB* and the acceleration of point *A* at this instant.

Given:

$a_B = 2\,\frac{\text{ft}}{\text{s}^2}$ $\quad r_{AB} = 5\text{ ft}$

$v_B = 6\,\frac{\text{ft}}{\text{s}}$ $\quad r_{AC} = 3\text{ ft}$

Solution: $\quad d = \sqrt{r_{AB}^{\;2} - r_{AC}^{\;2}}$

From an instantaneous center analysis we find that $\quad \omega_{AB} = 0$

Guesses $\quad \alpha_{AB} = 1\,\frac{\text{rad}}{\text{s}^2} \quad a_{Ax} = 1\,\frac{\text{ft}}{\text{s}^2}$

Given

$$\begin{pmatrix} a_B \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{Ax} \\ \dfrac{v_B^{\;2}}{r_{AC}} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} d \\ r_{AC} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{AB} \\ a_{Ax} \end{pmatrix} = \text{Find}(\alpha_{AB}, a_{Ax}) \qquad \mathbf{a_A} = \begin{pmatrix} a_{Ax} \\ \dfrac{v_B^{\;2}}{r_{AC}} \end{pmatrix} \qquad \mathbf{a_A} = \begin{pmatrix} -7.00 \\ 12.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

$$\alpha_{AB} = -3.00\,\frac{\text{rad}}{\text{s}^2}$$

$$\theta = \text{atan}\left(\frac{\alpha_{AB}\, d}{a_{Ax}}\right) \qquad \theta = 59.7\,\text{deg}$$

Problem 16-119

The closure is manufactured by the LCN Company and is used to control the restricted motion of a heavy door. If the door to which is it connected has an angular acceleration α, determine the angular accelerations of links *BC* and *CD*. Originally the door is not rotating but is hinged at *A*.

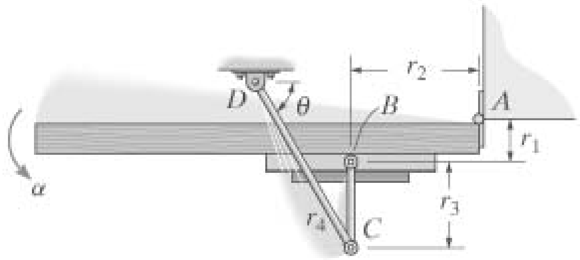
Given:

$r_1 = 2.5\ \text{in} \qquad \alpha = 3\ \dfrac{\text{rad}}{\text{s}^2}$

$r_2 = 6\ \text{in}$

$r_3 = 4\ \text{in} \qquad \theta = 60\ \text{deg}$

$r_4 = 12\ \text{in}$



Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} -r_2 \\ -r_1 \\ 0 \end{pmatrix} \qquad \mathbf{r_{BC}} = \begin{pmatrix} 0 \\ -r_3 \\ 0 \end{pmatrix} \qquad \mathbf{r_{CD}} = \begin{pmatrix} -r_4 \cos(\theta) \\ r_4 \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\qquad \alpha_{BC} = 1\ \dfrac{\text{rad}}{\text{s}^2} \qquad \alpha_{CD} = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given $\qquad \alpha \mathbf{k} \times \mathbf{r_{AB}} + \alpha_{BC}\mathbf{k} \times \mathbf{r_{BC}} + \alpha_{CD}\mathbf{k} \times \mathbf{r_{CD}} = 0$

$$\begin{pmatrix} \alpha_{BC} \\ \alpha_{CD} \end{pmatrix} = \text{Find}(\alpha_{BC}, \alpha_{CD}) \qquad \begin{pmatrix} \alpha_{BC} \\ \alpha_{CD} \end{pmatrix} = \begin{pmatrix} -9.67 \\ -3.00 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

***Problem 16-120**

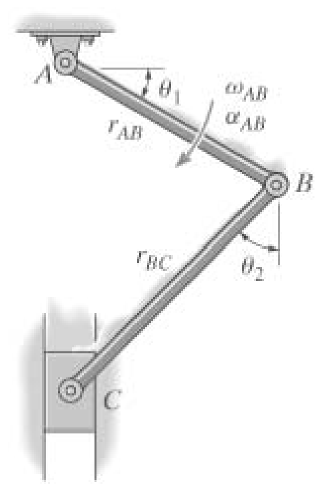
Rod AB has the angular motion shown. Determine the acceleration of the collar C at this instant.

Given:

$\omega_{AB} = 3\ \dfrac{\text{rad}}{\text{s}} \qquad \alpha_{AB} = 5\ \dfrac{\text{rad}}{\text{s}^2}$

$r_{AB} = 0.5\ \text{m} \qquad r_{BC} = 0.6\ \text{m}$

$\theta_1 = 30\ \text{deg} \qquad \theta_2 = 45\ \text{deg}$



Solution:

$$\mathbf{r_1} = \begin{pmatrix} r_{AB} \cos(\theta_1) \\ -r_{AB} \sin(\theta_1) \\ 0 \end{pmatrix} \qquad \mathbf{r_2} = \begin{pmatrix} -r_{BC} \sin(\theta_2) \\ -r_{BC} \cos(\theta_2) \\ 0 \end{pmatrix}$$

Guesses $\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$ $v_C = 1\ \frac{\text{m}}{\text{s}}$ $a_C = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \mathbf{r_1} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \mathbf{r_2} = \begin{pmatrix} 0 \\ -v_C \\ 0 \end{pmatrix}$$

$$\left[\begin{pmatrix} 0 \\ 0 \\ -\alpha_{AB} \end{pmatrix} \times \mathbf{r_1} + \begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \mathbf{r_1} \right] \cdots + \begin{pmatrix} 0 \\ 0 \\ \alpha_{BC} \end{pmatrix} \times \mathbf{r_2} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \mathbf{r_2} \right] = \begin{pmatrix} 0 \\ -a_C \\ 0 \end{pmatrix} \right]$$

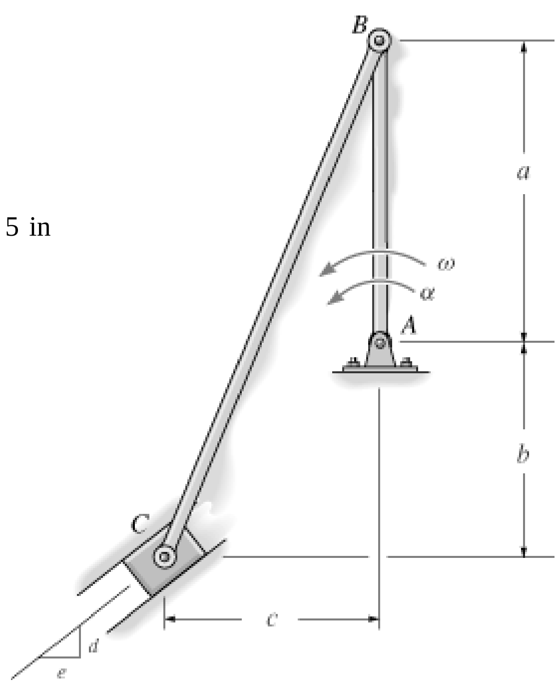
$$\begin{pmatrix} \omega_{BC} \\ \alpha_{BC} \\ v_C \\ a_C \end{pmatrix} = \text{Find}(\omega_{BC}, \alpha_{BC}, v_C, a_C)$$

$\omega_{BC} = 1.77\ \frac{\text{rad}}{\text{s}}$ $\alpha_{BC} = 9.01\ \frac{\text{rad}}{\text{s}^2}$

$v_C = 2.05\ \frac{\text{m}}{\text{s}}$ $a_C = 2.41\ \frac{\text{m}}{\text{s}^2}$

Problem 16-121

At the given instant member AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.



Given:

$\omega = 3\ \frac{\text{rad}}{\text{s}}$ $a = 7\text{ in}$ $d = 3$ $c = 5\text{ in}$

$\alpha = 2\ \frac{\text{rad}}{\text{s}^2}$ $b = 5\text{ in}$ $e = 4$

Solution:

Guesses

$\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$

$v_C = 1\ \frac{\text{in}}{\text{s}}$ $a_C = 1\ \frac{\text{in}}{\text{s}^2}$

Given $\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} -c \\ -a-b \\ 0 \end{pmatrix} = \frac{v_C}{\sqrt{e^2+d^2}} \begin{pmatrix} e \\ d \\ 0 \end{pmatrix}$

$$\left[\begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix}\right]\right] \ldots = \frac{a_C}{\sqrt{e^2+d^2}} \begin{pmatrix} e \\ d \\ 0 \end{pmatrix}$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{BC} \end{pmatrix} \times \begin{pmatrix} -c \\ -a-b \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} -c \\ -a-b \\ 0 \end{pmatrix}\right]$$

$$\begin{pmatrix} \omega_{BC} \\ \alpha_{BC} \\ v_C \\ a_C \end{pmatrix} = \text{Find}(\omega_{BC}, \alpha_{BC}, v_C, a_C) \qquad \omega_{BC} = 1.13\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{BC} = -3.00\ \frac{\text{rad}}{\text{s}^2}$$

$$v_C = -9.38\ \frac{\text{in}}{\text{s}} \qquad a_C = -54.7\ \frac{\text{in}}{\text{s}^2}$$

Problem 16-122

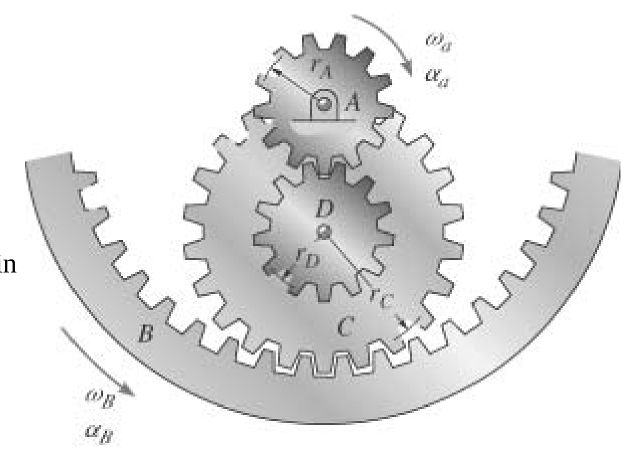
At a given instant gears *A* and *B* have the angular motions shown. Determine the angular acceleration of gear *C* and the acceleration of its center point *D* at this instant. Note that the inner hub of gear *C* is in mesh with gear *A* and its outer rim is in mesh with gear *B*.

Given:

$\omega_B = 1\ \frac{\text{rad}}{\text{s}}$ $\qquad \omega_A = 4\ \frac{\text{rad}}{\text{s}}$

$\alpha_B = 6\ \frac{\text{rad}}{\text{s}^2}$ $\qquad \alpha_A = 8\ \frac{\text{rad}}{\text{s}^2}$

$r_A = 5\ \text{in}$ $\qquad r_C = 10\ \text{in}$ $\qquad r_D = 5\ \text{in}$



Solution:

Guesses $\omega_C = 1\ \frac{\text{rad}}{\text{s}}$ $\qquad \alpha_C = 1\ \frac{\text{rad}}{\text{s}^2}$

Given $-\omega_A r_A = \omega_B(r_A + r_D + r_C) - \omega_C(r_C + r_D)$

$-\alpha_A r_A = \alpha_B(r_A + r_D + r_C) - \alpha_C(r_C + r_D)$

$$\begin{pmatrix} \omega_C \\ \alpha_C \end{pmatrix} = \text{Find}(\omega_C, \alpha_C) \qquad \omega_C = 2.67\,\frac{\text{rad}}{\text{s}} \qquad \alpha_C = 10.67\,\frac{\text{rad}}{\text{s}^2}$$

$$v_D = -\omega_A r_A + \omega_C r_D \qquad v_D = -6.67\,\frac{\text{in}}{\text{s}}$$

$$a_{Dt} = -\alpha_A r_A + \alpha_C r_D \qquad a_{Dt} = 13.33\,\frac{\text{in}}{\text{s}^2}$$

$$a_{Dn} = \frac{v_D^{\,2}}{r_A + r_D} \qquad a_{Dn} = 4.44\,\frac{\text{in}}{\text{s}^2}$$

$$\mathbf{a_D} = \begin{pmatrix} a_{Dt} \\ a_{Dn} \end{pmatrix} \qquad \mathbf{a_D} = \begin{pmatrix} 13.33 \\ 4.44 \end{pmatrix}\frac{\text{in}}{\text{s}^2} \qquad |\mathbf{a_D}| = 14.05\,\frac{\text{in}}{\text{s}^2}$$

Problem 16-123

The tied crank and gear mechanism gives rocking motion to crank *AC*, necessary for the operation of a printing press. If link *DE* has the angular motion shown, determine the respective angular velocities of gear *F* and crank *AC* at this instant, and the angular acceleration of crank *AC*.

Given:

$$\omega_{DE} = 4\,\frac{\text{rad}}{\text{s}}$$

$$\alpha_{DE} = 20\,\frac{\text{rad}}{\text{s}^2}$$

$a = 100$ mm

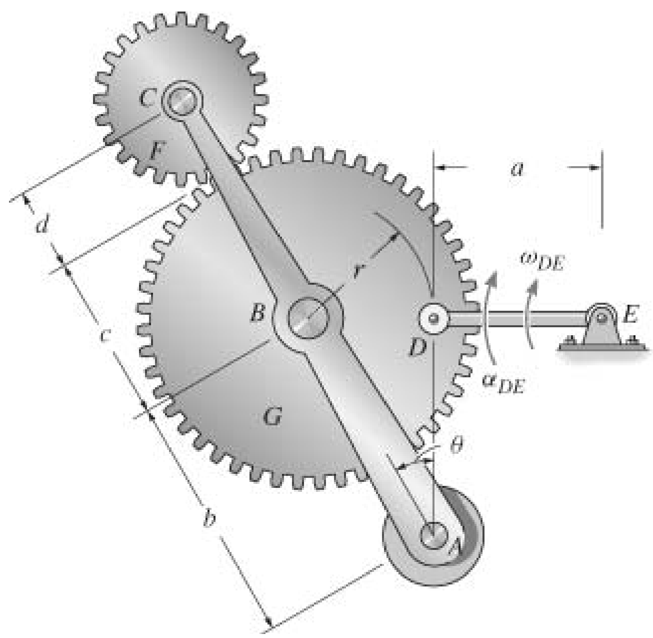
$b = 150$ mm

$c = 100$ mm

$d = 50$ mm

$r = 75$ mm

$\theta = 30$ deg



Solution: Guesses

$$\omega_G = 1\,\frac{\text{rad}}{\text{s}} \qquad \omega_{AC} = 1\,\frac{\text{rad}}{\text{s}}$$

$$\alpha_G = 1\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_{AC} = 1\ \frac{\text{rad}}{\text{s}^2}$$

Given $$\begin{pmatrix} 0 \\ 0 \\ -\omega_{DE} \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega_G \end{pmatrix} \times \begin{pmatrix} -r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} b\sin(\theta) \\ -b\cos(\theta) \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ 0 \\ -\alpha_{DE} \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega_{DE} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega_{DE} \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \alpha_G \end{pmatrix} \times \begin{pmatrix} -r \\ 0 \\ 0 \end{pmatrix} \dots = 0$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \omega_G \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_G \end{pmatrix} \times \begin{pmatrix} -r \\ 0 \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \alpha_{AC} \end{pmatrix} \times \begin{pmatrix} b\sin(\theta) \\ -b\cos(\theta) \\ 0 \end{pmatrix} \dots$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} b\sin(\theta) \\ -b\cos(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_G \\ \omega_{AC} \\ \alpha_G \\ \alpha_{AC} \end{pmatrix} = \text{Find}(\omega_G, \omega_{AC}, \alpha_G, \alpha_{AC})$$

$$\omega_G = -5.33\ \frac{\text{rad}}{\text{s}} \qquad \omega_{AC} = 0.00\ \frac{\text{rad}}{\text{s}}$$

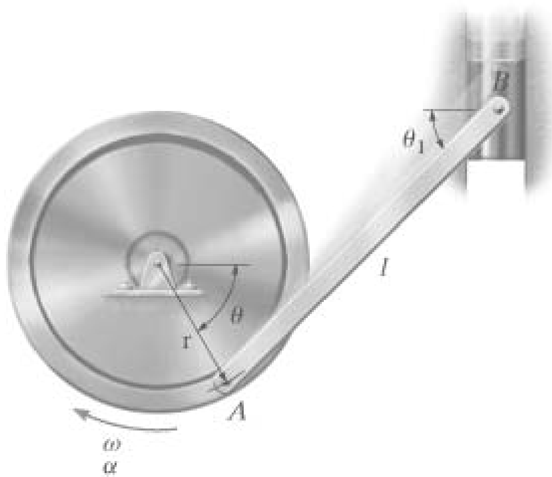
$$\alpha_G = -2.07\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_{AC} = -28.7\ \frac{\text{rad}}{\text{s}^2}$$

Now find the motion of gear F.

$$\omega_{AC}\,b + \omega_G\,c = \omega_{AC}(b + c + d) - \omega_F\,d \qquad \omega_F = \frac{\omega_{AC}(c+d) - \omega_G\,c}{d} \qquad \omega_F = 10.67\ \frac{\text{rad}}{\text{s}}$$

***Problem 16-124**

At a given instant the wheel is rotating with the angular velocity and angular acceleration shown. Determine the acceleration of block *B* at this instant.



Given:

$\omega = 2\ \frac{\text{rad}}{\text{s}}$ $\theta = 60\ \text{deg}$

$\alpha = 6\ \frac{\text{rad}}{\text{s}^2}$ $\phi = 45\ \text{deg}$

$l = 1.5\ \text{m}$ $r = 0.3\ \text{m}$

Solution:

$$\mathbf{r_1} = r\begin{pmatrix}\cos(\theta)\\ -\sin(\theta)\\ 0\end{pmatrix} \qquad \mathbf{r_2} = l\begin{pmatrix}\cos(\phi)\\ \sin(\phi)\\ 0\end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$$

Guesses $\omega_{AB} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2}$ $v_B = 1\ \frac{\text{m}}{\text{s}}$ $a_B = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$-\omega\mathbf{k}\times\mathbf{r_1} + \omega_{AB}\mathbf{k}\times\mathbf{r_2} = \begin{pmatrix}0\\ v_B\\ 0\end{pmatrix}$$

$$-\alpha\,\mathbf{k}\times\mathbf{r_1} - \omega\mathbf{k}\times\left(-\omega\mathbf{k}\times\mathbf{r_1}\right) + \alpha_{AB}\mathbf{k}\times\mathbf{r_2} + \omega_{AB}\mathbf{k}\times\left(\omega_{AB}\mathbf{k}\times\mathbf{r_2}\right) = \begin{pmatrix}0\\ a_B\\ 0\end{pmatrix}$$

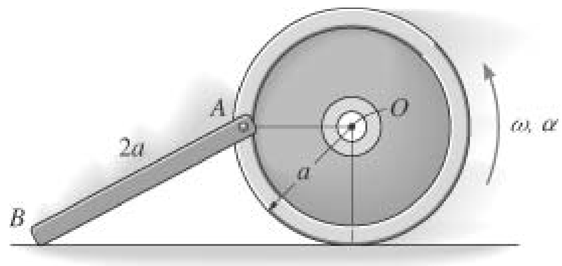
$$\begin{pmatrix}\omega_{AB}\\ \alpha_{AB}\\ v_B\\ a_B\end{pmatrix} = \text{Find}\left(\omega_{AB}, \alpha_{AB}, v_B, a_B\right)$$

$\omega_{AB} = -0.49\ \frac{\text{rad}}{\text{s}}$ $v_B = -0.82\ \frac{\text{m}}{\text{s}}$

$\alpha_{AB} = -2.28\ \frac{\text{rad}}{\text{s}^2}$ $a_B = -2.53\ \frac{\text{m}}{\text{s}^2}$

Problem 16-125

The wheel rolls without slipping such that at the instant shown it has an angular velocity ω and angular acceleration α. Determine the velocity and acceleration of point B on the rod at this instant.



Solution:

Velocity

$$v_B = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} -a \\ a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -\sqrt{3}\,a \\ -a \\ 0 \end{pmatrix} = \begin{bmatrix} (\omega_{AB} - \omega)a \\ -(\sqrt{3}\,\omega_{AB} + \omega)\,a \\ 0 \end{bmatrix}$$

Since B stays in contact with the ground we have $\omega_{AB} = \dfrac{-\omega}{\sqrt{3}}$ $\quad v_B = -\dfrac{1+\sqrt{3}}{\sqrt{3}}\,\omega a$

Acceleration

$$a_B = \begin{pmatrix} -\alpha\, a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} -\sqrt{3}\,a \\ -a \\ 0 \end{pmatrix} \dots$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -\sqrt{3}\,a \\ -a \\ 0 \end{pmatrix}\right]$$

$$a_B = a\begin{pmatrix} -\alpha + \omega^2 + \alpha_{AB} + \sqrt{3}\,\omega_{AB}^2 \\ -\alpha - \sqrt{3}\,\alpha_{AB} + \omega_{AB}^2 \\ 0 \end{pmatrix}$$

Since B stays in contact with the ground we find

$$\alpha_{AB} = \frac{\omega_{AB}^2 - \alpha}{\sqrt{3}} = \frac{\omega^2}{3\sqrt{3}} - \frac{\alpha}{\sqrt{3}}$$

$$a_B = \left(\frac{4+3\sqrt{3}}{3\sqrt{3}}\omega^2 - \frac{1+\sqrt{3}}{\sqrt{3}}\alpha\right)a$$

Problem 16-126

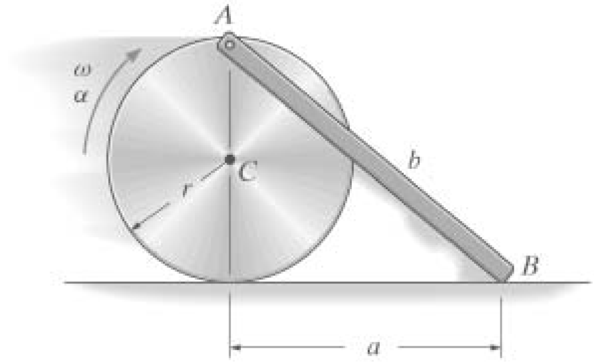
The disk rolls without slipping such that it has angular acceleration α and angular velocity ω at the instant shown. Determine the accelerations of points A and B on the link and the link's angular acceleration at this instant. Assume point A lies on the periphery of the disk, a distance r from C.

Given:

$\alpha = 4 \dfrac{\text{rad}}{\text{s}^2}$ $a = 400\ \text{mm}$

$b = 500\ \text{mm}$

$\omega = 2 \dfrac{\text{rad}}{\text{s}}$ $r = 150\ \text{mm}$



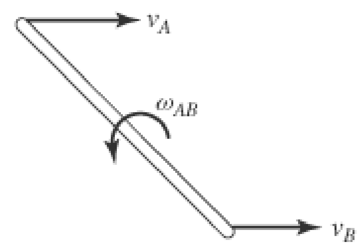
Solution:

The IC is at ∞, so $\omega_{AB} = 0$

$a_C = \alpha r$

$\mathbf{a_A} = a_C + \alpha \times r_{AC} - \omega^2 r_{AC}$

$$\mathbf{a_A} = \begin{pmatrix} a_C \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\alpha \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} - \omega^2 \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix}$$



$\mathbf{a_A} = \begin{pmatrix} 1.20 \\ -0.60 \\ 0.00 \end{pmatrix} \dfrac{\text{m}}{\text{s}^2}$ $|\mathbf{a_A}| = 1.342 \dfrac{\text{m}}{\text{s}^2}$

$\theta = \operatorname{atan}\left(\dfrac{-\omega^2 r}{a_C + \alpha r}\right)$ $\theta = -26.6\ \text{deg}$ $|\theta| = 26.6\ \text{deg}$

Guesses $a_B = 1 \dfrac{\text{m}}{\text{s}^2}$ $\alpha_{AB} = 1 \dfrac{\text{rad}}{\text{s}^2}$

Given $\mathbf{a_A} + \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} a \\ -2r \\ 0 \end{pmatrix} = \begin{pmatrix} a_B \\ 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} a_B \\ \alpha_{AB} \end{pmatrix} = \text{Find}(a_B, \alpha_{AB})$

$\alpha_{AB} = 1.500 \dfrac{\text{rad}}{\text{s}^2}$ $a_B = 1.650 \dfrac{\text{m}}{\text{s}^2}$

Problem 16-127

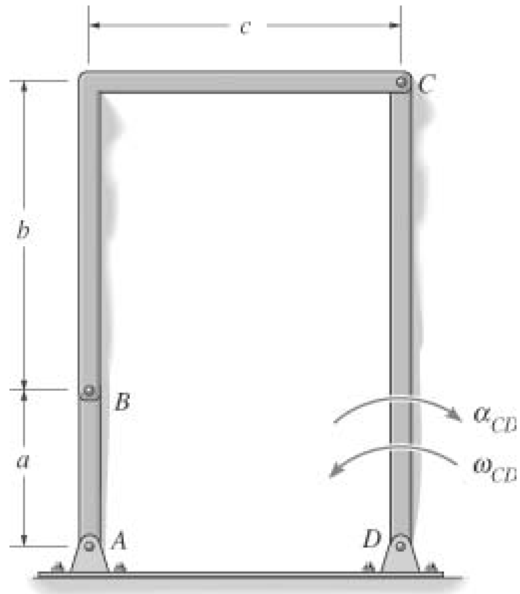
Determine the angular acceleration of link *AB* if link *CD* has the angular velocity and angular deceleration shown.

Given:

$\alpha_{CD} = 4\ \frac{\text{rad}}{\text{s}^2}$ $a = 0.3\ \text{m}$

$b = 0.6\ \text{m}$

$\omega_{CD} = 2\ \frac{\text{rad}}{\text{s}}$ $c = 0.6\ \text{m}$



Solution:

$\omega_{BC} = 0 \qquad \omega_{AB} = \omega_{CD}\frac{a+b}{a}$

Guesses

$\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$

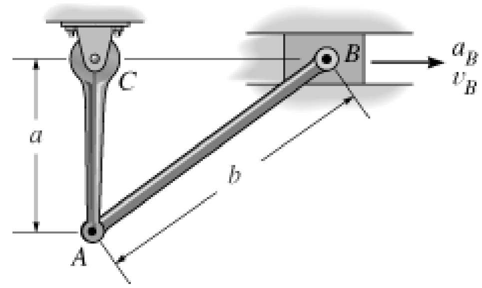
Given

$$\begin{pmatrix}0\\0\\-\alpha_{CD}\end{pmatrix} \times \begin{pmatrix}0\\a+b\\0\end{pmatrix} + \begin{pmatrix}0\\0\\\omega_{CD}\end{pmatrix} \times \left[\begin{pmatrix}0\\0\\\omega_{CD}\end{pmatrix} \times \begin{pmatrix}0\\a+b\\0\end{pmatrix}\right] + \begin{pmatrix}0\\0\\\alpha_{BC}\end{pmatrix} \times \begin{pmatrix}-c\\-b\\0\end{pmatrix} \ldots = \begin{pmatrix}0\\0\\0\end{pmatrix}\frac{\text{m}}{\text{s}^2}$$
$$+ \begin{pmatrix}0\\0\\\alpha_{AB}\end{pmatrix} \times \begin{pmatrix}0\\-a\\0\end{pmatrix} + \begin{pmatrix}0\\0\\\omega_{AB}\end{pmatrix} \times \left[\begin{pmatrix}0\\0\\\omega_{AB}\end{pmatrix} \times \begin{pmatrix}0\\-a\\0\end{pmatrix}\right]$$

$$\begin{pmatrix}\alpha_{AB}\\\alpha_{BC}\end{pmatrix} = \text{Find}(\alpha_{AB}, \alpha_{BC}) \qquad \alpha_{BC} = 12.00\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_{AB} = -36.00\ \frac{\text{rad}}{\text{s}^2}$$

***Problem 16-128**

The slider block B is moving to the right with acceleration a_B. At the instant shown, its velocity is v_B. Determine the angular acceleration of link AB and the acceleration of point A at this instant.



Given: $a_B = 2\ \frac{\text{ft}}{\text{s}^2}$ $v_B = 6\ \frac{\text{ft}}{\text{s}}$

$a = 3\ \text{ft}$ $b = 5\ \text{ft}$

Solution: $\omega_{AB} = 0$ $v_A = v_B$

$\omega_{AC} = \frac{v_A}{a}$ $\omega_{AC} = 2\ \frac{\text{rad}}{\text{s}}$

Guesses $\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_{AC} = 1\ \frac{\text{rad}}{\text{s}^2}$ Given

$$\begin{pmatrix} 0 \\ 0 \\ \alpha_{AC} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} \sqrt{b^2 - a^2} \\ a \\ 0 \end{pmatrix} = \begin{pmatrix} a_B \\ 0 \\ 0 \end{pmatrix}$$

$\begin{pmatrix} \alpha_{AB} \\ \alpha_{AC} \end{pmatrix} = \text{Find}(\alpha_{AB}, \alpha_{AC})$ $\alpha_{AC} = -2.33\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_{AB} = -3.00\ \frac{\text{rad}}{\text{s}^2}$

$$\mathbf{a_A} = \begin{pmatrix} 0 \\ 0 \\ \alpha_{AC} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix}\right] \qquad \mathbf{a_A} = \begin{pmatrix} -7.00 \\ 12.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

$|\mathbf{a_A}| = 13.89\ \frac{\text{ft}}{\text{s}^2}$

$\theta = \text{atan}\left(\frac{\omega_{AC}^2 a}{-\alpha_{AC} a}\right)$ $\theta = 59.74\ \text{deg}$

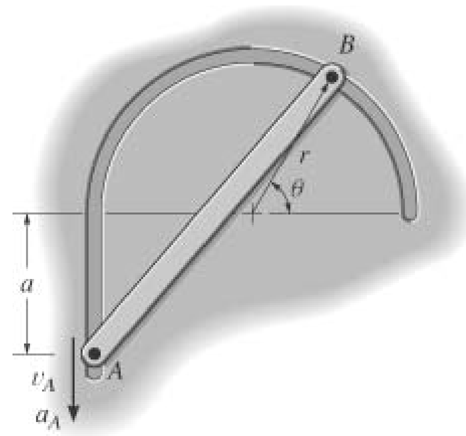
Problem 16-129

The ends of the bar *AB* are confined to move along the paths shown. At a given instant, *A* has velocity v_A and acceleration a_A. Determine the angular velocity and angular acceleration of *AB* at this instant.

Given:

$v_A = 4\ \frac{\text{ft}}{\text{s}}$ $a = 2\ \text{ft}$

$$a_A = 7\ \frac{\text{ft}}{\text{s}^2} \qquad r = 2\ \text{ft} \qquad \theta = 60\ \text{deg}$$



Solution: Guessses

$$\omega = 1\ \frac{\text{rad}}{\text{s}} \qquad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$$

$$v_B = 1\ \frac{\text{ft}}{\text{s}} \qquad a_{Bt} = 1\ \frac{\text{ft}}{\text{s}^2}$$

Given

$$\begin{pmatrix} -v_B\sin(\theta) \\ v_B\cos(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_A \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r + r\cos(\theta) \\ a + r\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -a_{Bt}\sin(\theta) \\ a_{Bt}\cos(\theta) \\ 0 \end{pmatrix} + \frac{v_B{}^2}{r}\begin{pmatrix} -\cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -a_A \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} r + r\cos(\theta) \\ a + r\sin(\theta) \\ 0 \end{pmatrix} \cdots$$

$$+ \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r + r\cos(\theta) \\ a + r\sin(\theta) \\ 0 \end{pmatrix}\right]$$

$$\begin{pmatrix} \omega \\ \alpha \\ v_B \\ a_{Bt} \end{pmatrix} = \text{Find}(\omega, \alpha, v_B, a_{Bt}) \qquad v_B = 20.39\ \frac{\text{ft}}{\text{s}} \qquad a_{Bt} = -607.01\ \frac{\text{ft}}{\text{s}^2}$$

$$\omega = 4.73\ \frac{\text{rad}}{\text{s}} \qquad \alpha = -131.00\ \frac{\text{rad}}{\text{s}^2}$$

Problem 16-130

The mechanism produces intermittent motion of link *AB*. If the sprocket *S* is turning with an angular acceleration α_s and has an angular velocity ω_s at the instant shown, determine the angular velocity and angular acceleration of link *AB* at this instant. The sprocket *S* is mounted on a shaft which is *separate* from a collinear shaft attached to *AB* at *A*. The pin at *C* is attached to one of the chain links such that it moves vertically downward.

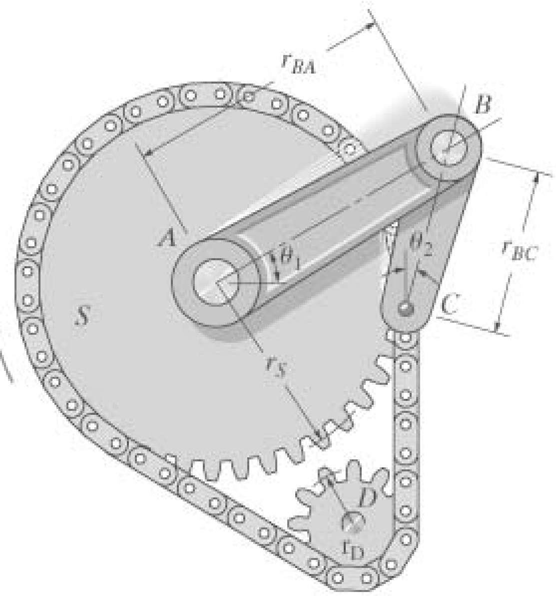
Given:

$$\omega_s = 6\ \frac{\text{rad}}{\text{s}} \qquad r_{BA} = 200\ \text{mm}$$

$\alpha_S = 2 \frac{\text{rad}}{\text{s}^2}$ $\quad r_{BC} = 150\ \text{mm}$

$\theta_1 = 30\ \text{deg}$ $\quad r_S = 175\ \text{mm}$

$\theta_2 = 15\ \text{deg}$ $\quad r_D = 50\ \text{mm}$



Solution:

$$\mathbf{r_1} = r_{BA}\begin{pmatrix}\cos(\theta_1)\\ \sin(\theta_1)\\ 0\end{pmatrix} \qquad \mathbf{r_2} = r_{BC}\begin{pmatrix}-\sin(\theta_2)\\ -\cos(\theta_2)\\ 0\end{pmatrix}$$

$$\mathbf{k} = \begin{pmatrix}0\\0\\1\end{pmatrix} \qquad \mathbf{j} = \begin{pmatrix}0\\1\\0\end{pmatrix}$$

Guesses $\quad \omega_{AB} = 1 \frac{\text{rad}}{\text{s}} \quad \omega_{BC} = 1 \frac{\text{rad}}{\text{s}} \quad \alpha_{AB} = 1 \frac{\text{rad}}{\text{s}^2} \quad \alpha_{BC} = 1 \frac{\text{rad}}{\text{s}^2}$

Given $\quad \omega_{AB}\mathbf{k}\times\mathbf{r_1} + \omega_{BC}\mathbf{k}\times\mathbf{r_2} = -\omega_S r_S \mathbf{j}$

$$\alpha_{AB}\mathbf{k}\times\mathbf{r_1} - \omega_{AB}^2\mathbf{r_1} + \alpha_{BC}\mathbf{k}\times\mathbf{r_2} - \omega_{BC}^2\mathbf{r_2} = -\alpha_S r_S \mathbf{j}$$

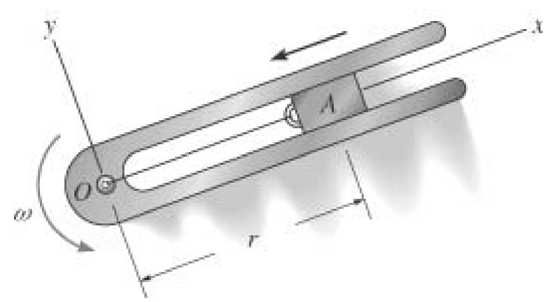
$$\begin{pmatrix}\omega_{AB}\\ \omega_{BC}\\ \alpha_{AB}\\ \alpha_{BC}\end{pmatrix} = \text{Find}(\omega_{AB}, \omega_{BC}, \alpha_{AB}, \alpha_{BC})$$

$\omega_{BC} = -4.95 \frac{\text{rad}}{\text{s}}$ $\quad \alpha_{BC} = 70.8 \frac{\text{rad}}{\text{s}^2}$

$\omega_{AB} = -7.17 \frac{\text{rad}}{\text{s}}$ $\quad \alpha_{AB} = 23.1 \frac{\text{rad}}{\text{s}^2}$

Problem 16-131

Block *A*, which is attached to a cord, moves along the slot of a horizontal forked rod. At the instant shown, the cord is pulled down through the hole at *O* with acceleration *a* and velocity *v*. Determine the acceleration of the block at this instant. The rod rotates about *O* with constant angular velocity.



Given:

$a = 4 \frac{\text{m}}{\text{s}^2} \qquad \omega = 4 \frac{\text{rad}}{\text{s}}$

$$v = 2\ \frac{\text{m}}{\text{s}} \qquad r = 100\ \text{mm}$$

Solution:

$$\mathbf{a_A} = \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} -v \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{a_A} = \begin{pmatrix} -5.60 \\ -16.00 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_A}\right| = 16.95\ \frac{\text{m}}{\text{s}^2}$$

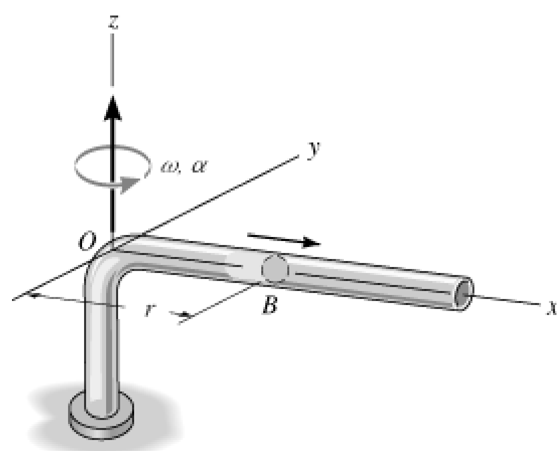
Problem 16-132

The ball B of negligible size rolls through the tube such that at the instant shown it has velocity v and acceleration a, measured relative to the tube. If the tube has angular velocity ω and angular acceleration α at this same instant, determine the velocity and acceleration of the ball.

Given:

$$v = 5\ \frac{\text{ft}}{\text{s}} \qquad \omega = 3\ \frac{\text{rad}}{\text{s}} \qquad r = 2\ \text{ft}$$

$$a = 3\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 5\ \frac{\text{rad}}{\text{s}^2}$$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

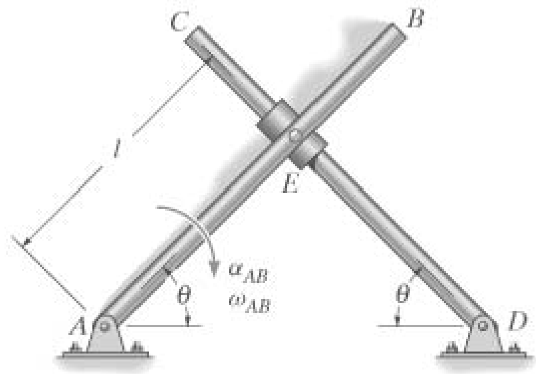
$$\mathbf{v_B} = \begin{pmatrix} 5.00 \\ 6.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \left|\mathbf{v_B}\right| = 7.81\ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_B} = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{a_B} = \begin{pmatrix} -15.00 \\ 40.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad \left|\mathbf{a_B}\right| = 42.72\ \frac{\text{ft}}{\text{s}^2}$$

Problem 16-133

The collar *E* is attached to, and pivots about, rod *AB* while it slides on rod *CD*. If rod *AB* has an angular velocity of ω_{AB} and an angular acceleration of α_{AB} both acting clockwise, determine the angular velocity and the angular acceleration of rod *CD* at the instant shown.



Given:

$$\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2} \qquad \omega_{AB} = 6\ \frac{\text{rad}}{\text{s}}$$

$$l = 4\ \text{ft} \qquad \theta = 45\ \text{deg}$$

Solution:

$$\mathbf{u_1} = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{u_2} = \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{r_1} = l\mathbf{u_1} \qquad \mathbf{r_2} = l\mathbf{u_2} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \omega_{CD} = 1\ \frac{\text{rad}}{\text{s}} \qquad v_{rel} = 1\ \frac{\text{ft}}{\text{s}} \qquad \alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2} \qquad a_{rel} = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$-\omega_{AB}\mathbf{k} \times \mathbf{r_1} = \omega_{CD}\mathbf{k} \times \mathbf{r_2} + v_{rel}\mathbf{u_2}$$

$$-\alpha_{AB}\mathbf{k} \times \mathbf{r_1} - \omega_{AB}^2\mathbf{r_1} = \alpha_{CD}\mathbf{k} \times \mathbf{r_2} - \omega_{CD}^2\mathbf{r_2} + a_{rel}\mathbf{u_2} + 2\omega_{CD}\mathbf{k} \times \left(v_{rel}\mathbf{u_2}\right)$$

$$\begin{pmatrix} \omega_{CD} \\ \alpha_{CD} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \text{Find}\left(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel}\right) \qquad v_{rel} = -24.00\ \frac{\text{ft}}{\text{s}} \qquad a_{rel} = -4.00\ \frac{\text{ft}}{\text{s}^2}$$

$$\omega_{CD} = -0.00\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 36\ \frac{\text{rad}}{\text{s}^2}$$

Problem 16-134

Block *B* moves along the slot in the platform with constant speed *v*, measured relative to the platform in the direction shown. If the platform is rotating at constant rate ω,determine the velocity and acceleration of the block at the instant shown.

Given:

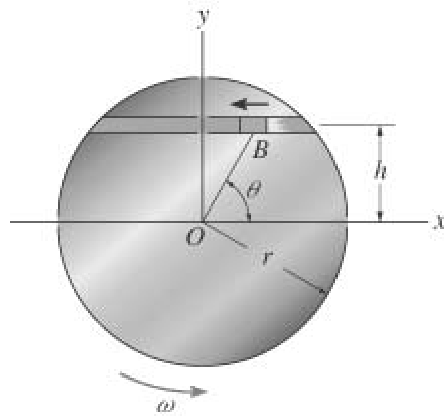
$$v = 2\ \frac{\text{ft}}{\text{s}}$$

$$\theta = 60\ \text{deg}$$

$\omega = 5 \frac{\text{rad}}{\text{s}}$

$r = 3 \text{ ft}$

$h = 2 \text{ ft}$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} -v \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} h\cot(\theta) \\ h \\ 0 \end{pmatrix}$$

$$\mathbf{v_B} = \begin{pmatrix} -12.00 \\ 5.77 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_B}| = 13.32 \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_B} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} h\cot(\theta) \\ h \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} -v \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{a_B} = \begin{pmatrix} -28.9 \\ -70.0 \\ 0.0 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_B}| = 75.7 \frac{\text{ft}}{\text{s}^2}$$

Problem 16-135

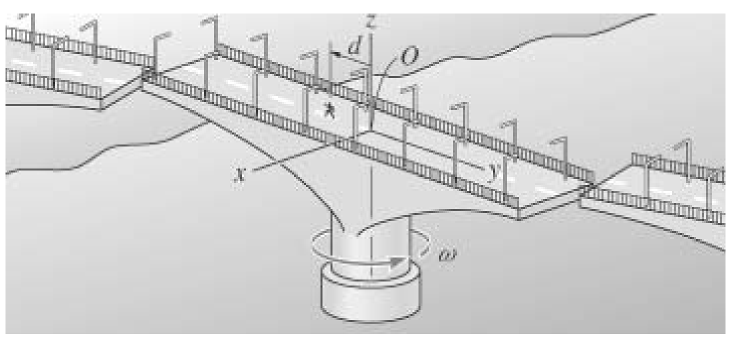
While the swing bridge is closing with constant rotation ω, a man runs along the roadway at constant speed v relative to the roadway. Determine his velocity and acceleration at the instant shown.

Given:

$\omega = 0.5 \frac{\text{rad}}{\text{s}}$

$v = 5 \frac{\text{ft}}{\text{s}}$

$d = 15 \text{ ft}$



Solution:

$$\mathbf{v_{man}} = \begin{pmatrix} 0 \\ -v \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} \qquad \mathbf{v_{man}} = \begin{pmatrix} 7.50 \\ -5.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_{man}}| = 9.01 \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_{man}} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -v \\ 0 \end{pmatrix}$$

$$\mathbf{a_{man}} = \begin{pmatrix} 5.00 \\ 3.75 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad \left|\mathbf{a_{man}}\right| = 6.25\,\frac{\text{ft}}{\text{s}^2}$$

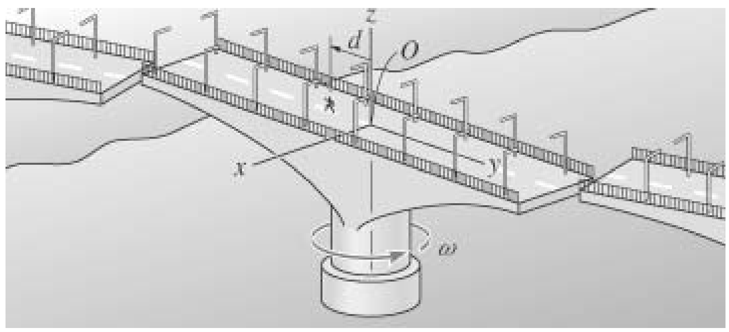
***Problem 16-136**

While the swing bridge is closing with constant rotation ω, a man runs along the roadway such that he is running outward from the center at speed v with acceleration a, both measured relative to the roadway. Determine his velocity and acceleration at this instant.

Given:

$\omega = 0.5\,\frac{\text{rad}}{\text{s}}$ $\qquad a = 2\,\frac{\text{ft}}{\text{s}^2}$

$v = 5\,\frac{\text{ft}}{\text{s}}$ $\qquad d = 10\,\text{ft}$



Solution:

$$\mathbf{v_{man}} = \begin{pmatrix} 0 \\ -v \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} \qquad \mathbf{v_{man}} = \begin{pmatrix} 5.00 \\ -5.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \left|\mathbf{v_{man}}\right| = 7.07\,\frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_{man}} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -v \\ 0 \end{pmatrix}$$

$$\mathbf{a_{man}} = \begin{pmatrix} 5.00 \\ 0.50 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad \left|\mathbf{a_{man}}\right| = 5.02\,\frac{\text{ft}}{\text{s}^2}$$

Problem 16-137

A girl stands at *A* on a platform which is rotating with constant angular velocity ω. If she walks at constant speed v measured relative to the platform, determine her acceleration (a) when she reaches point *D* in going along the path *ADC*, and (b) when she reaches point *B* if she follows the path *ABC*.

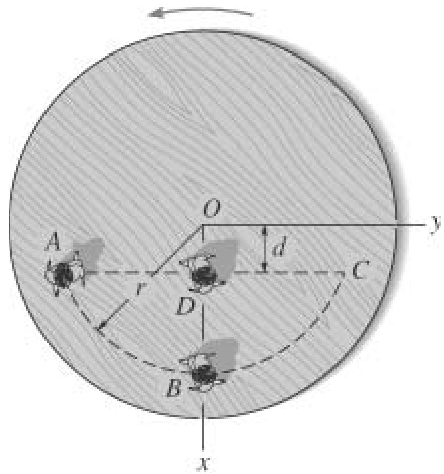
Given:

$$\omega = 0.5\ \frac{\text{rad}}{\text{s}}$$

$$v = 0.75\ \frac{\text{m}}{\text{s}}$$

$$d = 1\ \text{m}$$

$$r = 3\ \text{m}$$



Solution:

(a)

$$\mathbf{a_{girl}} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \qquad \mathbf{a_{girl}} = \begin{pmatrix} -1.00 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

(b)

$$\mathbf{a_{girl}} = \begin{pmatrix} \frac{-v^2}{r} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \qquad \mathbf{a_{girl}} = \begin{pmatrix} -1.69 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 16-138

A girl stands at *A* on a platform which is rotating with angular acceleration α and at the instant shown has angular velocity ω.If she walks at constant speed *v* measured relative to the platform, determine her acceleration (a) when she reaches point *D* in going along the path *ADC*, and (b) when she reaches point *B* if she follows the path *ABC*.

Given:

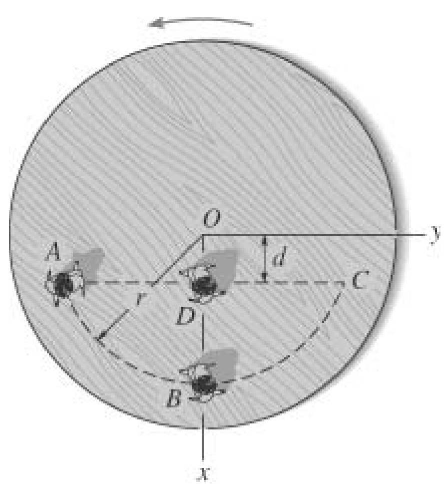
$$\alpha = 0.2\ \frac{\text{rad}}{\text{s}^2}$$

$$\omega = 0.5\ \frac{\text{rad}}{\text{s}}$$

$$v = 0.75\ \frac{\text{m}}{\text{s}}$$

$$d = 1\ \text{m}$$

$$r = 3\ \text{m}$$



Solution:

(a)

$$\mathbf{a_{girl}} = \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \qquad \mathbf{a_{girl}} = \begin{pmatrix} -1.00 \\ 0.20 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

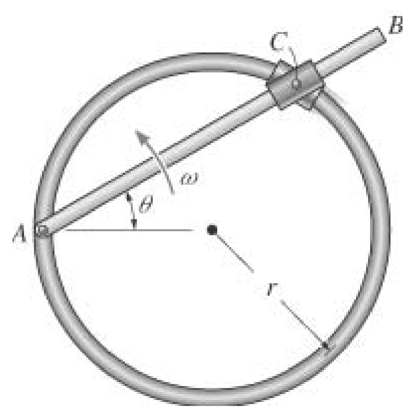
(b)

$$\mathbf{a_{girl}} = \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{-v^2}{r} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}$$

$$\mathbf{a_{girl}} = \begin{pmatrix} -1.69 \\ 0.60 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 16-139

Rod *AB* rotates counterclockwise with constant angular velocity ω. Determine the velocity and acceleration of point *C* located on the double collar when at the position shown. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod *AB*.



Given: $\omega = 3\ \frac{\text{rad}}{\text{s}}$ $\theta = 45\ \text{deg}$ $r = 0.4\ \text{m}$

Solution: Guesses

$v_{rel} = 1\ \frac{\text{m}}{\text{s}}$ $v_C = 1\ \frac{\text{m}}{\text{s}}$ $a_{rel} = 1\ \frac{\text{m}}{\text{s}^2}$ $a_{Ct} = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$v_C \begin{pmatrix} -\sin(2\theta) \\ \cos(2\theta) \\ 0 \end{pmatrix} = v_{rel} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r + r\cos(2\theta) \\ r\sin(2\theta) \\ 0 \end{pmatrix}$$

$$a_{Ct} \begin{pmatrix} -\sin(2\theta) \\ \cos(2\theta) \\ 0 \end{pmatrix} + \frac{v_C^2}{r} \begin{pmatrix} -\cos(2\theta) \\ -\sin(2\theta) \\ 0 \end{pmatrix} = a_{rel} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r + r\cos(2\theta) \\ r\sin(2\theta) \\ 0 \end{pmatrix}\right] \dots$$
$$+ 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[v_{rel} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}\right]$$

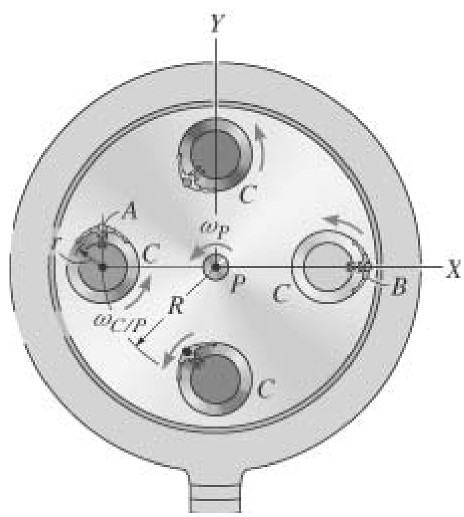
$$\begin{pmatrix} v_{rel} \\ a_{rel} \\ v_C \\ a_{Ct} \end{pmatrix} = \text{Find}(v_{rel}, a_{rel}, v_C, a_{Ct}) \qquad \begin{pmatrix} v_{rel} \\ v_C \end{pmatrix} = \begin{pmatrix} -1.70 \\ 2.40 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \begin{pmatrix} a_{rel} \\ a_{Ct} \end{pmatrix} = \begin{pmatrix} -5.09 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

$$\mathbf{v_{Cv}} = v_C \begin{pmatrix} -\sin(2\theta) \\ \cos(2\theta) \\ 0 \end{pmatrix} \qquad \mathbf{a_{Cv}} = a_{Ct} \begin{pmatrix} -\sin(2\theta) \\ \cos(2\theta) \\ 0 \end{pmatrix} + \frac{v_C^{\,2}}{r} \begin{pmatrix} -\cos(2\theta) \\ -\sin(2\theta) \\ 0 \end{pmatrix}$$

$$\mathbf{v_{Cv}} = \begin{pmatrix} -2.40 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_{Cv}} = \begin{pmatrix} -0.00 \\ -14.40 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 16-140

A ride in an amusement park consists of a rotating platform P, having constant angular velocity ω_P and four cars, C, mounted on the platform, which have constant angular velocities ω_{CP} measured relative to the platform. Determine the velocity and acceleration of the passenger at B at the instant shown.



Given: $\omega_P = 1.5 \frac{\text{rad}}{\text{s}}$ $\qquad r = 0.75 \text{ m}$

$\omega_{CP} = 2 \frac{\text{rad}}{\text{s}}$ $\qquad R = 3 \text{ m}$

Solution:

$$\mathbf{v_B} = \begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \begin{pmatrix} R \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

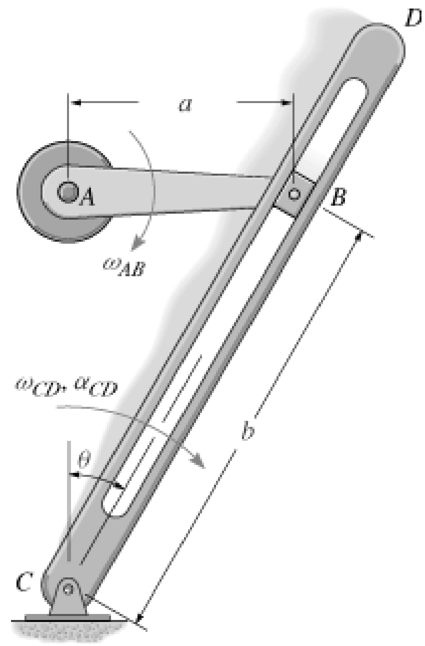
$$\mathbf{v_B} = \begin{pmatrix} 0.00 \\ 7.13 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad |\mathbf{v_B}| = 7.13 \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_B} = \begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \begin{pmatrix} R \\ 0 \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_B} = \begin{pmatrix} -15.94 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad |\mathbf{a_B}| = 15.94 \frac{\text{m}}{\text{s}^2}$$

Problem 16-141

Block *B* of the mechanism is confined to move within the slot member *CD*. If *AB* is rotating at constant rate ω_{AB}, determine the angular velocity and angular acceleration of member *CD* at the instant shown.



Given: $\omega_{AB} = 3\ \frac{\text{rad}}{\text{s}}$ $\quad a = 100\ \text{mm}$

$\theta = 30\ \text{deg}$ $\quad b = 200\ \text{mm}$

Solution: Guesses

$\omega_{CD} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2}$

$v_{rel} = 1\ \frac{\text{m}}{\text{s}}$ $\quad a_{rel} = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} = v_{rel}\begin{pmatrix} \sin(\theta) \\ \cos(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega_{CD} \end{pmatrix} \times \begin{pmatrix} b\sin(\theta) \\ b\cos(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}\right] = a_{rel}\begin{pmatrix} \sin(\theta) \\ \cos(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\alpha_{CD} \end{pmatrix} \times \begin{pmatrix} b\sin(\theta) \\ b\cos(\theta) \\ 0 \end{pmatrix} \ldots$$

$$+ \begin{pmatrix} 0 \\ 0 \\ -\omega_{CD} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega_{CD} \end{pmatrix} \times \begin{pmatrix} b\sin(\theta) \\ b\cos(\theta) \\ 0 \end{pmatrix}\right] \ldots$$

$$+ 2\begin{pmatrix} 0 \\ 0 \\ -\omega_{CD} \end{pmatrix} \times \left[v_{rel}\begin{pmatrix} \sin(\theta) \\ \cos(\theta) \\ 0 \end{pmatrix}\right]$$

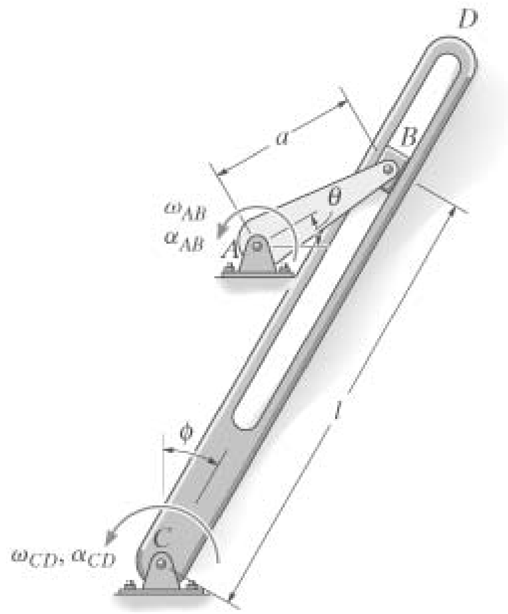
$$\begin{pmatrix} \omega_{CD} \\ \alpha_{CD} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \text{Find}(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel})$$

$v_{rel} = -0.26\ \frac{\text{m}}{\text{s}}$ $\quad a_{rel} = -0.34\ \frac{\text{m}}{\text{s}^2}$

$\omega_{CD} = 0.75\ \frac{\text{rad}}{\text{s}}$ $\quad \alpha_{CD} = -1.95\ \frac{\text{rad}}{\text{s}^2}$

Problem 16-142

The "quick-return" mechanism consists of a crank *AB*, slider block *B*, and slotted link *CD*. If the crank has the angular motion shown, determine the angular motion of the slotted link at this instant.



Given:

$\omega_{AB} = 3\ \frac{\text{rad}}{\text{s}}$ $\quad a = 100\ \text{mm}$

$l = 300\ \text{mm}$

$\alpha_{AB} = 9\ \frac{\text{rad}}{\text{s}^2}$ $\quad \theta = 30\ \text{deg}$

$\phi = 30\ \text{deg}$

Solution:

$$\mathbf{u_1} = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{u_2} = \begin{pmatrix} \sin(\phi) \\ \cos(\phi) \\ 0 \end{pmatrix} \qquad \mathbf{r_1} = a\mathbf{u_1} \qquad \mathbf{r_2} = l\mathbf{u_2} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \omega_{CD} = 1\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2} \qquad v_{rel} = 1\ \frac{\text{m}}{\text{s}} \qquad a_{rel} = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$\omega_{AB}\mathbf{k} \times \mathbf{r_1} = \omega_{CD}\mathbf{k} \times \mathbf{r_2} + v_{rel}\mathbf{u_2}$$

$$\alpha_{AB}\mathbf{k} \times \mathbf{r_1} - \omega_{AB}^{\,2}\mathbf{r_1} = \alpha_{CD}\mathbf{k} \times \mathbf{r_2} - \omega_{CD}^{\,2}\mathbf{r_2} + a_{rel}\mathbf{u_2} + 2\omega_{CD}\mathbf{k} \times \left(v_{rel}\mathbf{u_2}\right)$$

$$\begin{pmatrix} \omega_{CD} \\ \alpha_{CD} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \text{Find}\left(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel}\right) \qquad v_{rel} = 0.15\ \frac{\text{m}}{\text{s}} \qquad a_{rel} = -0.10\ \frac{\text{m}}{\text{s}^2}$$

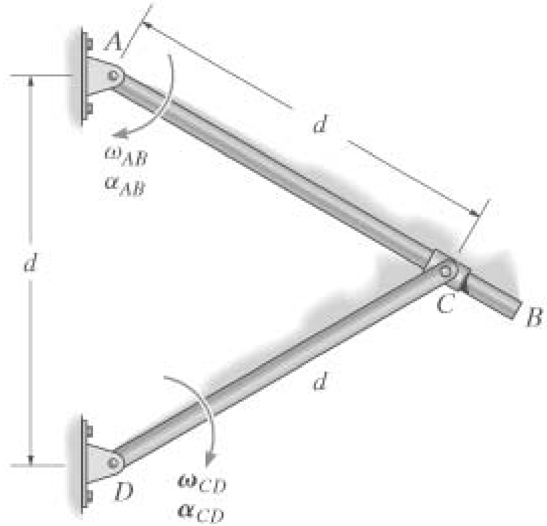
$$\omega_{CD} = 0.87\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 3.23\ \frac{\text{rad}}{\text{s}^2}$$

Problem 16-143

At a given instant, rod *AB* has the angular motions shown. Determine the angular velocity and angular acceleration of rod *CD* at this instant.There is a collar at *C*.

Given:

$$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}} \quad \alpha_{AB} = 12\ \frac{\text{rad}}{\text{s}^2} \quad d = 2\ \text{ft}$$



Solution:

$$\mathbf{u_1} = \begin{pmatrix} \sin(60\ \text{deg}) \\ -\cos(60\ \text{deg}) \\ 0 \end{pmatrix} \quad \mathbf{u_2} = \begin{pmatrix} \sin(60\ \text{deg}) \\ \cos(60\ \text{deg}) \\ 0 \end{pmatrix}$$

$$\mathbf{r_1} = d\mathbf{u_1} \quad \mathbf{r_2} = d\mathbf{u_2} \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\omega_{CD} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2}$

$v_{rel} = 1\ \frac{\text{ft}}{\text{s}}$ $a_{rel} = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$-\omega_{CD}\mathbf{k} \times \mathbf{r_2} = -\omega_{AB}\mathbf{k} \times \mathbf{r_1} + v_{rel}\mathbf{u_1}$$

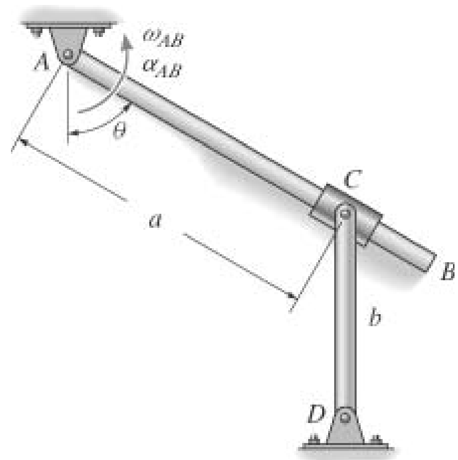
$$-\alpha_{CD}\mathbf{k} \times \mathbf{r_2} - \omega_{CD}^2\mathbf{r_2} = -\alpha_{AB}\mathbf{k} \times \mathbf{r_1} - \omega_{AB}^2\mathbf{r_1} + a_{rel}\mathbf{u_1} - 2\omega_{AB}\mathbf{k} \times \left(v_{rel}\mathbf{u_1}\right)$$

$$\begin{pmatrix} \omega_{CD} \\ \alpha_{CD} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \text{Find}\left(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel}\right) \qquad v_{rel} = 17.32\ \frac{\text{ft}}{\text{s}} \qquad a_{rel} = -8.43\ \frac{\text{ft}}{\text{s}^2}$$

$$\omega_{CD} = 10.00\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 24.00\ \frac{\text{rad}}{\text{s}^2}$$

Problem 16-144

At the instant shown, rod *AB* has angular velocity ω_{AB} and angular acceleration $\alpha_{AB.}$ Determine the angular velocity and angular acceleration of rod *CD* at this instant. The collar at *C* is pin-connected to *CD* and slides over *AB*.



Given: $\theta = 60\ \text{deg}$ $\quad a = 0.75\ \text{m}$

$\omega_{AB} = 3\ \frac{\text{rad}}{\text{s}}$ $\quad \alpha_{AB} = 5\ \frac{\text{rad}}{\text{s}^2}$ $\quad b = 0.5\ \text{m}$

Solution: Guesses

$\omega_{CD} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2}$ $\quad v_{rel} = 1\ \frac{\text{m}}{\text{s}}$

$a_{rel} = 1\ \frac{\text{m}}{\text{s}^2}$ $\quad a_{Cx} = 1\ \frac{\text{m}}{\text{s}^2}$ $\quad a_{Cy} = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{CD} \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\sin(\theta) \\ -a\cos(\theta) \\ 0 \end{pmatrix} + v_{rel}\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{Cx} \\ a_{Cy} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \alpha_{CD} \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{CD} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{CD} \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}\right]$$

$$\begin{pmatrix} a_{Cx} \\ a_{Cy} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} a\sin(\theta) \\ -a\cos(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\sin(\theta) \\ -a\cos(\theta) \\ 0 \end{pmatrix}\right] \ldots$$
$$+ a_{rel}\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[v_{rel}\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix}\right]$$

$$\begin{pmatrix} \omega_{CD} \\ \alpha_{CD} \\ v_{rel} \\ a_{rel} \\ a_{Cx} \\ a_{Cy} \end{pmatrix} = \text{Find}(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel}, a_{Cx}, a_{Cy})$$

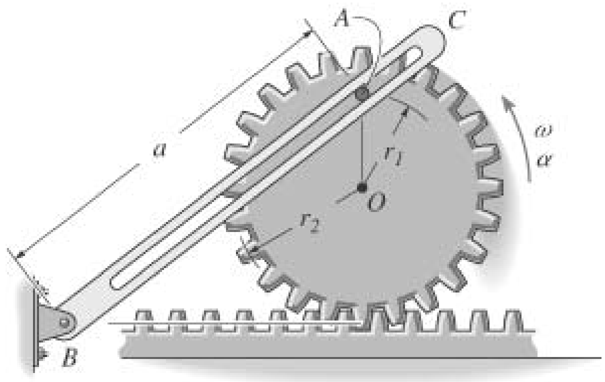
$$\begin{pmatrix} a_{Cx} \\ a_{Cy} \end{pmatrix} = \begin{pmatrix} 124.41 \\ -40.50 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

$v_{rel} = 3.90\ \frac{\text{m}}{\text{s}}$ $\quad \omega_{CD} = -9.00\ \frac{\text{rad}}{\text{s}}$

$a_{rel} = 134.75\ \frac{\text{m}}{\text{s}^2}$ $\quad \alpha_{CD} = -249\ \frac{\text{rad}}{\text{s}^2}$

Problem 16-145

The gear has the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link *BC* at this instant. The peg at *A* is fixed to the gear.



Given:

$\omega = 2\ \frac{\text{rad}}{\text{s}}$ $\quad r_1 = 0.5\ \text{ft}$

$r_2 = 0.7\ \text{ft}$

$\alpha = 4\ \frac{\text{rad}}{\text{s}^2}$ $\quad a = 2\ \text{ft}$

Solution: $b = \sqrt{a^2 - (r_1 + r_2)^2}$

$$\theta = \operatorname{atan}\left(\frac{r_1 + r_2}{b}\right)$$

Guesses $\quad \omega_{BC} = 1\ \frac{\text{rad}}{\text{s}} \quad \alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2} \quad v_{rel} = 1\ \frac{\text{ft}}{\text{s}} \quad a_{rel} = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$\begin{bmatrix} -\omega(r_1 + r_2) \\ 0 \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} b \\ r_1 + r_2 \\ 0 \end{pmatrix} + v_{rel}\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}$$

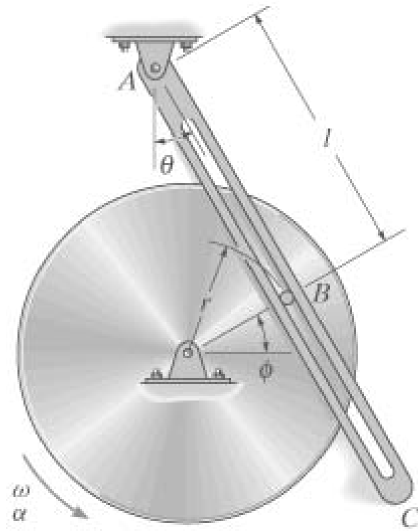
$$\begin{bmatrix} -\alpha(r_1 + r_2) \\ -r_1\,\omega^2 \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ \alpha_{BC} \end{pmatrix} \times \begin{pmatrix} b \\ r_1 + r_2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{bmatrix} b \\ (r_1 + r_2) \\ 0 \end{bmatrix}\right] \ldots$$
$$+ a_{rel}\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[v_{rel}\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}\right]$$

$$\begin{pmatrix} \omega_{BC} \\ \alpha_{BC} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \operatorname{Find}(\omega_{BC}, \alpha_{BC}, v_{rel}, a_{rel})$$

$v_{rel} = -1.92\ \frac{\text{ft}}{\text{s}} \quad a_{rel} = -4.00\ \frac{\text{ft}}{\text{s}^2}$

$\omega_{BC} = 0.72\ \frac{\text{rad}}{\text{s}} \quad \alpha_{BC} = 2.02\ \frac{\text{rad}}{\text{s}^2}$

Problem 16-146



The disk rotates with the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link *AC* at this instant. The peg at *B* is fixed to the disk.

Given:

$\omega = 6\ \frac{\text{rad}}{\text{s}}$ $\quad \alpha = 10\ \frac{\text{rad}}{\text{s}^2}$ $\quad l = 0.75\ \text{m}$

$\theta = 30\ \text{deg}$ $\quad \phi = 30\ \text{deg}$ $\quad r = 0.3\ \text{m}$

Solution:

$$\mathbf{u_1} = \begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \quad \mathbf{u_2} = \begin{pmatrix} \cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \mathbf{r_1} = l\mathbf{u_1} \quad \mathbf{r_2} = r\mathbf{u_2}$$

Guesses $\quad \omega_{AC} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \alpha_{AC} = 1\ \frac{\text{rad}}{\text{s}^2}$ $\quad v_{rel} = 1\ \frac{\text{m}}{\text{s}}$ $\quad a_{rel} = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$\omega\mathbf{k} \times \mathbf{r_2} = \omega_{AC}\mathbf{k} \times \mathbf{r_1} + v_{rel}\mathbf{u_1}$$

$$\alpha\mathbf{k} \times \mathbf{r_2} - \omega^2\mathbf{r_2} = \alpha_{AC}\mathbf{k} \times \mathbf{r_1} - \omega_{AC}{}^2\mathbf{r_1} + a_{rel}\mathbf{u_1} + 2\omega_{AC}\mathbf{k} \times \left(v_{rel}\mathbf{u_1}\right)$$

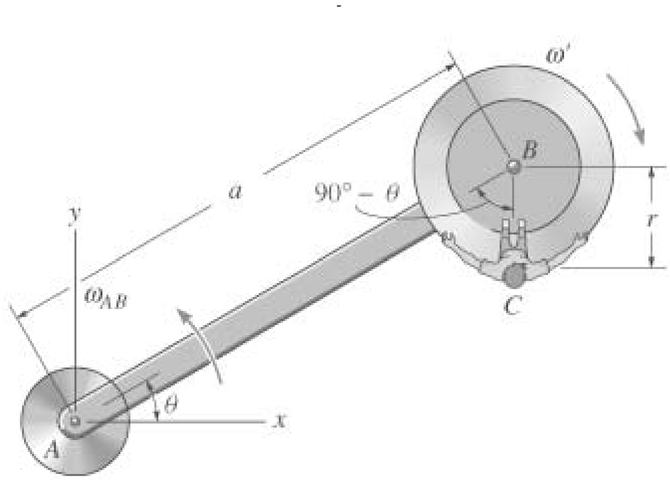
$$\begin{pmatrix} \omega_{AC} \\ \alpha_{AC} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \text{Find}\left(\omega_{AC}, \alpha_{AC}, v_{rel}, a_{rel}\right)$$

$v_{rel} = -1.80\ \frac{\text{m}}{\text{s}}$ $\quad a_{rel} = -3.00\ \frac{\text{m}}{\text{s}^2}$

$\omega_{AC} = 0.00\ \frac{\text{rad}}{\text{s}}$ $\quad \alpha_{AC} = -14.40\ \frac{\text{rad}}{\text{s}^2}$

Problem 16-147

A ride in an amusement park consists of a rotating arm *AB* having constant angular velocity ω_{AB} about point *A* and a car mounted at the end of the arm which has constant angular velocity $-\omega'\,\mathbf{k}$ measured relative to the arm. At the instant shown, determine the velocity and acceleration of the passenger at *C*.



Given:

$\omega_{AB} = 2\ \frac{\text{rad}}{\text{s}}$ $\omega' = 0.5\ \frac{\text{rad}}{\text{s}}$

$a = 10\ \text{ft}$ $r = 2\ \text{ft}$

$\theta = 30\ \text{deg}$

Solution:

$$\mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -7.00 \\ 17.32 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_C} = \begin{pmatrix} -34.64 \\ -15.50 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 16-148

A ride in an amusement park consists of a rotating arm *AB* that has angular acceleration α_{AB} when the angular velocity is ω_{AB} at the instant shown. Also at this instant the car mounted at the end of the arm has a relative angular acceleration $-\alpha'\,\mathbf{k}$ of when the angular velocity is $-\omega'\,\mathbf{k}$. Determine the velocity and acceleration of the passenger C at this instant.

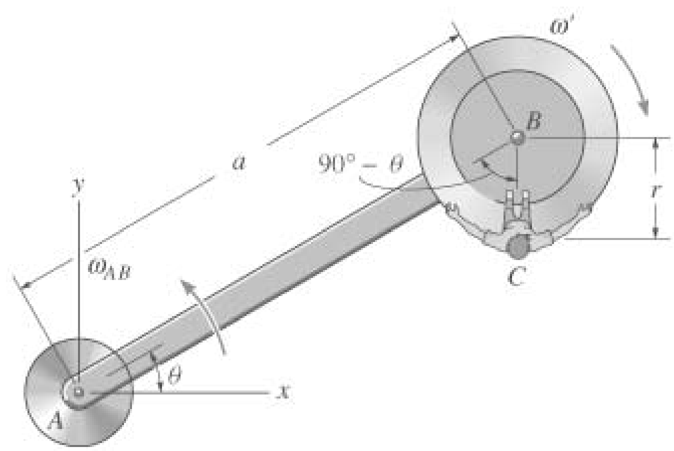
Given:

$\omega_{AB} = 2\ \frac{\text{rad}}{\text{s}}$ $\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2}$

$\omega' = 0.5\ \frac{\text{rad}}{\text{s}}$ $\alpha' = 0.6\ \frac{\text{rad}}{\text{s}^2}$

$a = 10\ \text{ft}$ $r = 2\ \text{ft}$

$\theta = 30\ \text{deg}$



Solution:

$$\mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -7.00 \\ 17.32 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_C} = \left[\begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix}\right]\right] \ldots$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} - \alpha' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_C} = \begin{pmatrix} -38.84 \\ -6.84 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 16-149

The cars on the amusement-park ride rotate around the axle at A with constant angular velocity ω_{Af} measured relative to the frame AB. At the same time the frame rotates around the main axle support at B with constant angular velocity ω_f .Determine the velocity and acceleration of the passenger at C at the instant shown.

Given:

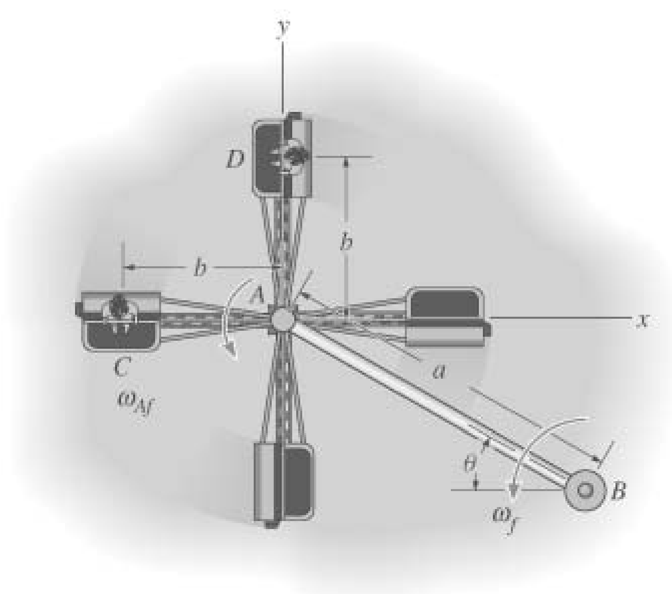
$$\omega_{Af} = 2\ \frac{\text{rad}}{\text{s}}$$

$$\omega_f = 1\ \frac{\text{rad}}{\text{s}}$$

$$a = 15\ \text{ft}$$

$$b = 8\ \text{ft}$$

$$\theta = 30\ \text{deg}$$



Solution:

$$\mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_f \end{pmatrix} \times \begin{pmatrix} -a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_f + \omega_{Af} \end{pmatrix} \times \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -7.50 \\ -36.99 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_f \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_f \end{pmatrix} \times \begin{pmatrix} -a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_f + \omega_{Af} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_f + \omega_{Af} \end{pmatrix} \times \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_C} = \begin{pmatrix} 84.99 \\ -7.50 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 16-150

The block *B* of the "quick-return" mechanism is confined to move within the slot in member *CD*. If *AB* is rotating at a constant rate of ω_{AB}, determine the angular velocity and angular acceleration of member *CD* at the instant shown.

Given:

$$\omega_{AB} = 3\,\frac{\text{rad}}{\text{s}}$$

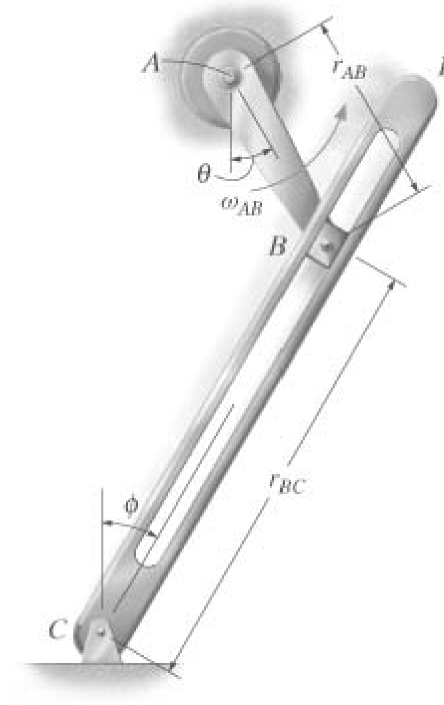
$$r_{AB} = 50\ \text{mm}$$

$$r_{BC} = 200\ \text{mm}$$

$$\theta = 30\ \text{deg}$$

$$\phi = 30\ \text{deg}$$

Solution:



$$\mathbf{u_1} = \begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \quad \mathbf{u_2} = \begin{pmatrix} \sin(\phi) \\ \cos(\phi) \\ 0 \end{pmatrix} \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbf{r_1} = r_{AB}\mathbf{u_1} \qquad \mathbf{r_2} = r_{BC}\mathbf{u_2}$$

Guesses

$$\omega_{CD} = 1\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2}$$

$$v_{rel} = 1\ \frac{\text{m}}{\text{s}} \qquad a_{rel} = 1\ \frac{\text{m}}{\text{s}^2}$$

Given

$$\omega_{AB}\mathbf{k} \times \mathbf{r_1} = \omega_{CD}\mathbf{k} \times \mathbf{r_2} + v_{rel}\mathbf{u_2}$$

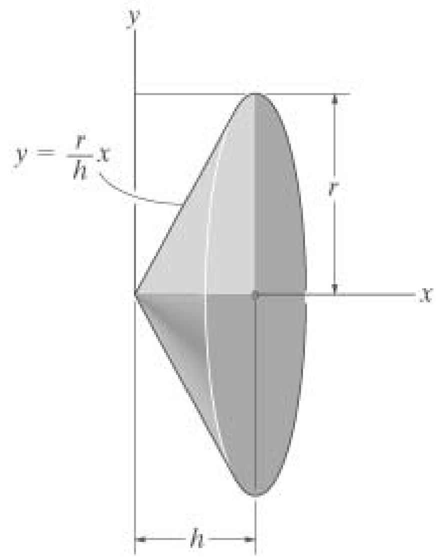
$$-\omega_{AB}^2\,\mathbf{r_1} = \alpha_{CD}\mathbf{k} \times \mathbf{r_2} - \omega_{CD}^2\,\mathbf{r_2} + a_{rel}\mathbf{u_2} + 2\omega_{CD}\mathbf{k} \times (v_{rel}\mathbf{u_2})$$

$$\begin{pmatrix} \omega_{CD} \\ \alpha_{CD} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \text{Find}(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel}) \qquad v_{rel} = 0.13\ \frac{\text{m}}{\text{s}} \qquad a_{rel} = 0.25\ \frac{\text{m}}{\text{s}^2}$$

$$\omega_{CD} = -0.38\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 2.44\ \frac{\text{rad}}{\text{s}^2}$$

Problem 17-1

The right circular cone is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.



Solution:

$$m = \int_0^h \rho\,\pi\left(\frac{r\,x}{h}\right)^2 \mathrm{d}x = \frac{1}{3}\,h\,\rho\,\pi r^2$$

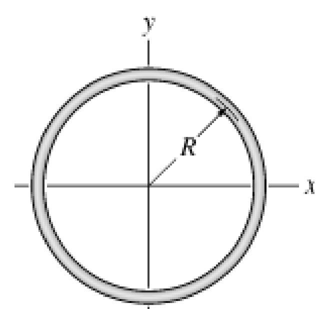
$$\rho = \frac{3m}{h\pi r^2}$$

$$I_x = \frac{3m}{h\pi r^2}\int_0^h \frac{1}{2}\pi\left(\frac{r\,x}{h}\right)^2\left(\frac{r\,x}{h}\right)^2 \mathrm{d}x \qquad I_x = \frac{3}{10}m\,r^2$$

Problem 17-2

Determine the moment of inertia of the thin ring about the z axis. The ring has a mass m.

Solution:



$$m = \int_0^{2\pi} \rho R\,\mathrm{d}\theta = 2\,\pi\,\rho R \qquad \rho = \frac{m}{2\pi R}$$

$$I_z = \frac{m}{2\pi R}\int_0^{2\pi} R\,R^2\,\mathrm{d}\theta = m\,R^2 \qquad I_z = m\,R^2$$

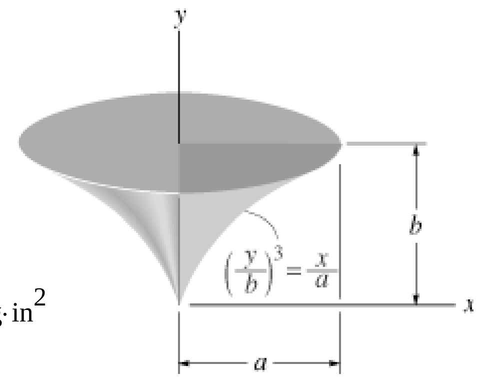
Problem 17-3

The solid is formed by revolving the shaded area around the y axis. Determine the radius of gyration k_y. The specific weight of the material is γ.

Given:

$$\gamma = 380\,\frac{\text{lb}}{\text{ft}^3} \qquad a = 3\text{ in} \qquad b = 3\text{ in}$$

Solution:



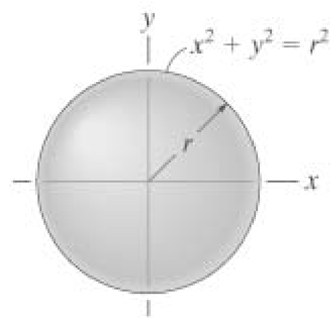
$$m = \int_0^b \gamma \pi \left(a \frac{y^3}{b^3}\right)^2 \mathrm{d}y \qquad m = 0.083\,\text{slug}$$

$$I_y = \int_0^b \gamma \frac{1}{2} \pi \left(a \frac{y^3}{b^3}\right)^2 \left(a \frac{y^3}{b^3}\right)^2 \mathrm{d}y \qquad I_y = 0.201\,\text{slug}\cdot\text{in}^2$$

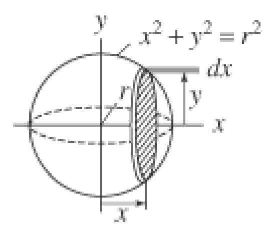
$$k = \sqrt{\frac{I_y}{m}} \qquad k = 1.56\,\text{in}$$

***Problem 17-4**

Determine the moment of inertia I_x of the sphere and express the result in terms of the total mass m of the sphere.The sphere has a constant density ρ.



Solution:

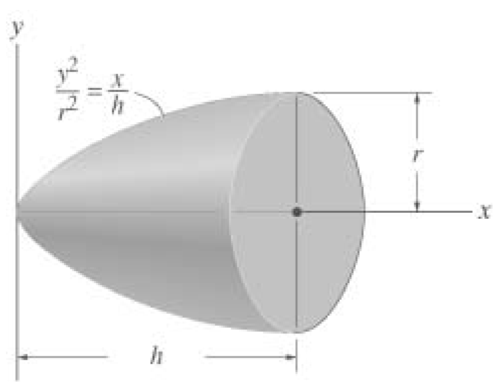


$$m = \int_{-r}^{r} \rho \pi \left(r^2 - x^2\right) \mathrm{d}x = \frac{4}{3} r^3 \rho \pi \qquad \rho = \frac{3m}{4\pi r^3}$$

$$I_x = \frac{3m}{4\pi r^3} \int_{-r}^{r} \frac{\pi}{2} \left(r^2 - x^2\right)^2 \mathrm{d}x \qquad I_x = \frac{2}{5} m r^2$$

Problem 17-5

Determine the radius of gyration k_x of the paraboloid. The density of the material is ρ.



Units Used: $\mathrm{Mg} = 10^6\ \mathrm{gm}$

Given:

$h = 200\ \mathrm{mm}$

$r = 100\ \mathrm{mm}$

$\rho = 5\ \frac{\mathrm{Mg}}{\mathrm{m}^3}$

Solution:

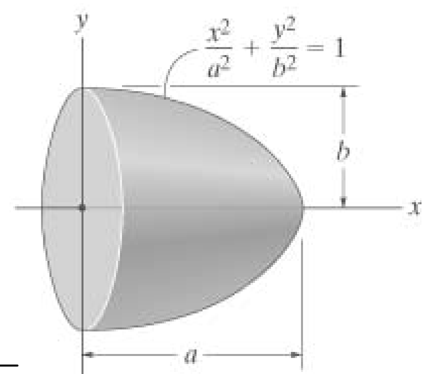
$$M = \int_0^h \rho\,\pi\left(\frac{x r^2}{h}\right) dx \qquad M = 15.708\ \text{kg}$$

$$I_x = \int_0^h \frac{1}{2}\rho\,\pi\left(\frac{x r^2}{h}\right)^2 dx \qquad I_x = 0.052\ \text{kg}\cdot\text{m}^2$$

$$k_x = \sqrt{\frac{I_x}{M}} \qquad k_x = 57.7\,\text{mm}$$

Problem 17-6

Determine the moment of inertia of the semiellipsoid with respect to the x axis and express the result in terms of the mass m of the semiellipsoid. The material has a constant density ρ.



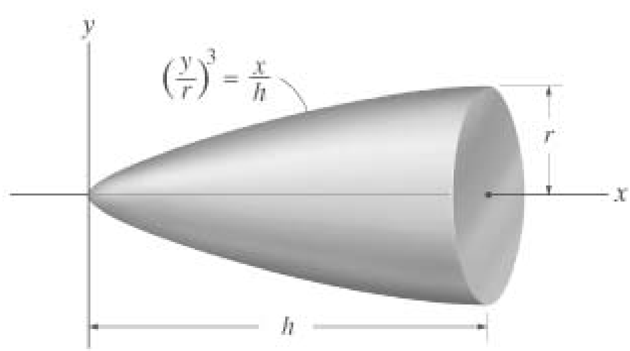
Solution:

$$m = \int_0^a \rho\,\pi b^2\left(1 - \frac{x^2}{a^2}\right) dx = \frac{2}{3}a\,\rho\,\pi b^2 \qquad \rho = \frac{3m}{2a\pi b^2}$$

$$I_x = \frac{3m}{2a\pi b^2}\int_0^a \frac{1}{2}\pi\left[b^2\left(1 - \frac{x^2}{a^2}\right)\right]^2 dx \qquad I_x = \frac{2}{5}m\,b^2$$

Problem 17-7

Determine the radius of gyration k_x of the body. The specific weight of the material is γ.



Given:

$$\gamma = 380\ \frac{\text{lb}}{\text{ft}^3}$$

$$h = 8\ \text{in}$$

$$r = 2\ \text{in}$$

Solution:

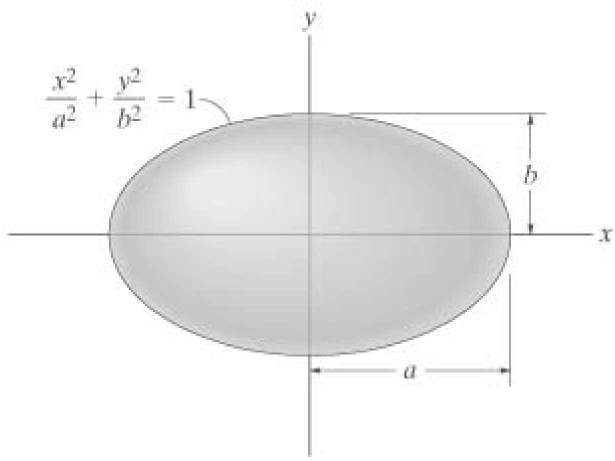
$$M = \int_0^h \gamma\pi\left[r\left(\frac{x}{h}\right)^{\frac{1}{3}}\right]^2 dx \qquad M = 0.412\,\text{slug}$$

$$I_x = \int_0^h \frac{1}{2}\gamma\pi\left[r\left(\frac{x}{h}\right)^{\frac{1}{3}}\right]^4 dx \qquad I_x = 0.589\,\text{slug}\cdot\text{in}^2$$

$$k_x = \sqrt{\frac{I_x}{M}} \qquad k_x = 1.20\,\text{in}$$

***Problem 17-8**

Determine the moment of inertia of the ellipsoid with respect to the x axis and express the result in terms of the mass m of the ellipsoid. The material has a constant density ρ.



Solution:

$$m = \int_{-a}^{a} \rho\pi b^2\left(1 - \frac{x^2}{a^2}\right) dx = \frac{4}{3}a\rho\pi b^2$$

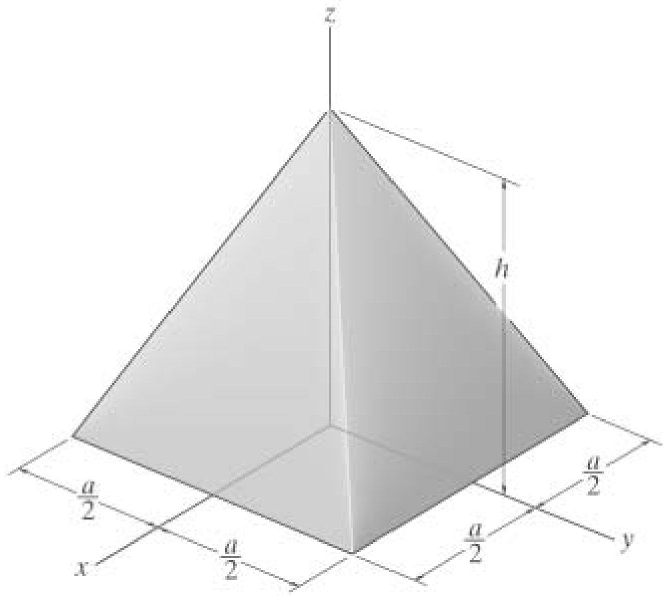
$$\rho = \frac{3m}{4a\pi b^2}$$

$$I_x = \frac{3m}{4a\pi b^2}\int_{-a}^{a} \frac{1}{2}\pi\left[b^2\left(1 - \frac{x^2}{a^2}\right)\right]^2 dx \qquad I_x = \frac{2}{5}mb^2$$

Problem 17-9

Determine the moment of inertia of the homogeneous pyramid of mass m with respect to the z axis.The density of the material is ρ. *Suggestion*: Use a rectangular plate element having a volume of $dV = (2x)(2y)\,dz$.

Solution:



$$x = \frac{a(h-z)}{2h}$$

$$m = \int_0^h \rho\left[\frac{a(h-z)}{h}\right]^2 \, dz = \frac{1}{3} h\rho a^2$$

$$\rho = \frac{3m}{h a^2}$$

$$I_z = \frac{3m}{h a^2}\int_0^h \frac{1}{6}\left[\frac{a(h-z)}{h}\right]^4 \, dz$$

$$I_z = \frac{1}{10} m a^2$$

Problem 17-10

The concrete shape is formed by rotating the shaded area about the y axis. Determine the moment of inertia I_y. The specific weight of concrete is γ.

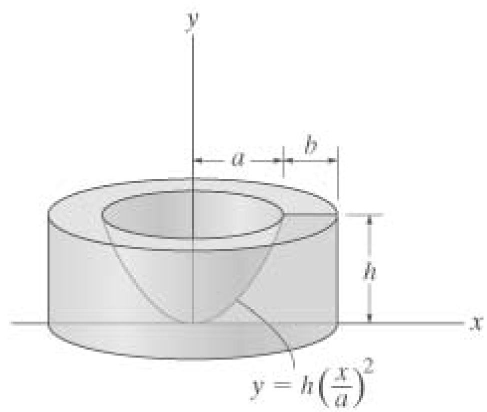
Given:

$a = 6$ in

$b = 4$ in

$h = 8$ in

$\gamma = 150 \frac{\text{lb}}{\text{ft}^3}$



Solution:

$$I_y = \int_0^h \gamma\left[\frac{1}{2}\pi(a+b)^4 - \frac{1}{2}\pi\left(a^2\frac{y}{h}\right)^2\right] dy \qquad I_y = 2.25 \, \text{slug}\cdot\text{ft}^2$$

Problem 17-11

Determine the moment of inertia of the thin plate about an axis perpendicular to the page and passing through the pin at O. The plate has a hole in its center. Its thickness is t, and the material has a density of ρ.

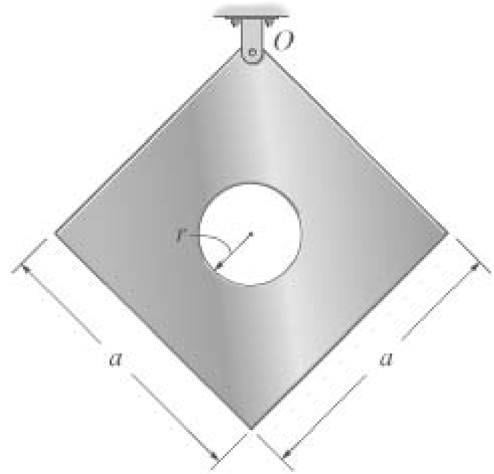
Given:

$a = 1.40\ \text{m}$

$r = 150\ \text{mm}$

$t = 50\ \text{mm}$

$\rho = 50\ \frac{\text{kg}}{\text{m}^3}$



Solution:

$$I_O = \rho a^2 t\left(\frac{2a^2}{3}\right) - \left[\rho\,\pi r^2 t\left(\frac{r^2}{2}\right) + \rho\,\pi r^2 t\left(\frac{\sqrt{2}\,a}{2}\right)^2\right]$$

$I_O = 6.227\ \text{kg}\cdot\text{m}^2$

***Problem 17-12**

Determine the moment of inertia I_z of the frustum of the cone which has a conical depression. The material has a density ρ.

Given:

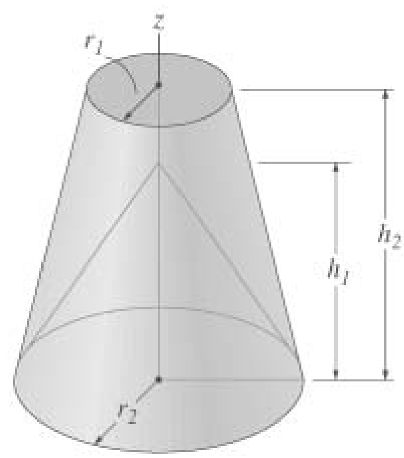
$r_1 = 0.2\ \text{m}$

$r_2 = 0.4\ \text{m}$

$h_1 = 0.6\ \text{m}$

$h_2 = 0.8\ \text{m}$

$\rho = 200\ \frac{\text{kg}}{\text{m}^3}$



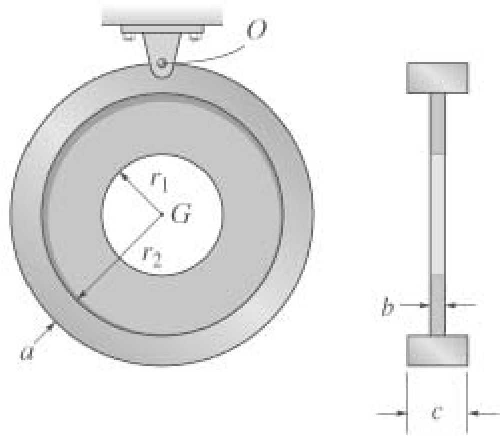
Solution:

$$h_3 = \frac{r_2 h_2}{r_2 - r_1} \qquad h_4 = h_3 - h_2$$

$$I_z = \rho\left(\frac{\pi r_2^2 h_3}{3}\right)\left(\frac{3}{10}\right) r_2^2 - \left(\frac{\rho\,\pi r_1^2 h_4}{3}\right)\left(\frac{3}{10}\right) r_1^2 - \left(\frac{\rho\,\pi r_2^2 h_1}{3}\right)\left(\frac{3}{10}\right) r_2^2$$

$I_z = 1.53\ \text{kg}\cdot\text{m}^2$

Problem 17-13



Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through the center of mass *G*. The material has a specific weight γ.

Given:

$a = 0.5\ \text{ft}$ $\quad r_1 = 1\ \text{ft}$

$b = 0.25\ \text{ft}$ $\quad r_2 = 2\ \text{ft}$

$c = 1\ \text{ft}$ $\quad \gamma = 90\ \dfrac{\text{lb}}{\text{ft}^3}$

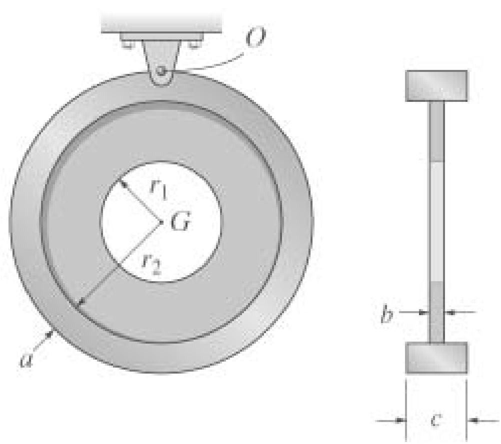
Solution:

$$I_G = \frac{1}{2}\gamma\pi c\left(r_2 + a\right)^4 - \frac{1}{2}\gamma\pi(c-b)\,r_2^{\,4} - \frac{1}{2}\gamma\pi b\,r_1^{\,4} \qquad I_G = 118\,\text{slug}\cdot\text{ft}^2$$

Problem 17-14

Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through point *O*. The material has a specific weight γ.

Given:



$a = 0.5\ \text{ft}$

$b = 0.25\ \text{ft}$

$c = 1\ \text{ft}$

$r_1 = 1\ \text{ft}$

$r_2 = 2\ \text{ft}$

$\gamma = 90\ \dfrac{\text{lb}}{\text{ft}^3}$

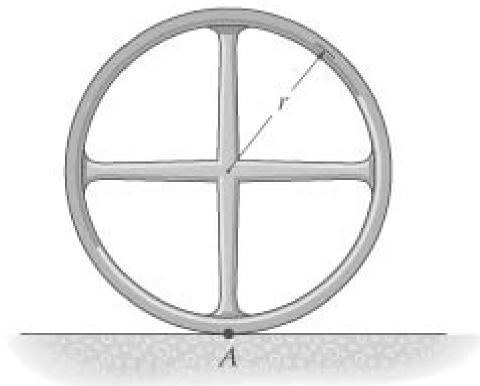
Solution:

$$I_O = \frac{3}{2}\gamma\pi c\left(r_2 + a\right)^4 - \left[\frac{1}{2}\gamma\pi(c-b)\,r_2^{\,4} + \gamma\pi(c-b)\,r_2^{\,2}\left(r_2 + a\right)^2\right] \dots$$
$$+ -\left[\frac{1}{2}\gamma\pi b\,r_1^{\,4} + \gamma\pi b\,r_1^{\,2}\left(r_2 + a\right)^2\right]$$

$I_O = 283\,\text{slug}\cdot\text{ft}^2$

Problem 17-15

The wheel consists of a thin ring having a mass M_1 and four spokes made from slender rods, each having a mass M_2. Determine the wheel's moment of inertia about an axis perpendicular to the page and passing through point A.



Given:

$M_1 = 10\ \text{kg}$

$M_2 = 2\ \text{kg}$

$r = 500\ \text{mm}$

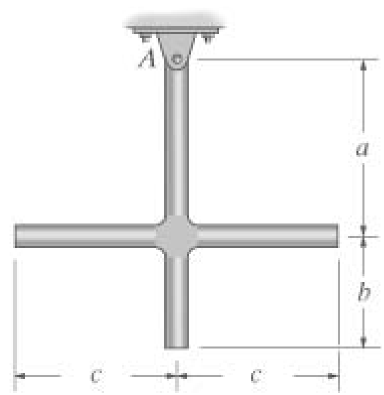
Solution:

$$I_G = M_1 r^2 + 4M_2\left(\frac{r^2}{3}\right)$$

$$I_A = I_G + (M_1 + 4M_2)r^2 \qquad I_A = 7.67\ \text{kg}\cdot\text{m}^2$$

Problem 17-16

The slender rods have a weight density γ. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through point A.



Given:

$\gamma = 3\ \frac{\text{lb}}{\text{ft}}$

$a = 2\ \text{ft}$

$b = 1\ \text{ft}$

$c = 1.5\ \text{ft}$

Solution:

$$I_A = \gamma(a+b)\left[\frac{(a+b)^2}{3}\right] + \gamma(2c)\frac{(2c)^2}{12} + \gamma(2c)\,a^2 \qquad I_A = 2.17\ \text{slug}\cdot\text{ft}^2$$

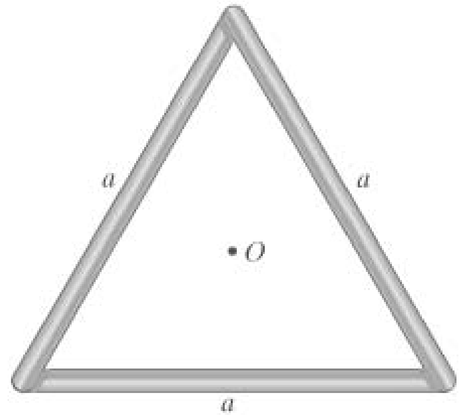
Problem 17-17

Each of the three rods has a mass m. Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through the center point O.

Solution:

$$I_O = 3\left[\frac{1}{12}m a^2 + m\left[\frac{(a)\sin(60\ \text{deg})}{3}\right]^2\right]$$

$$I_O = \frac{1}{2}m a^2$$



Problem 17-18

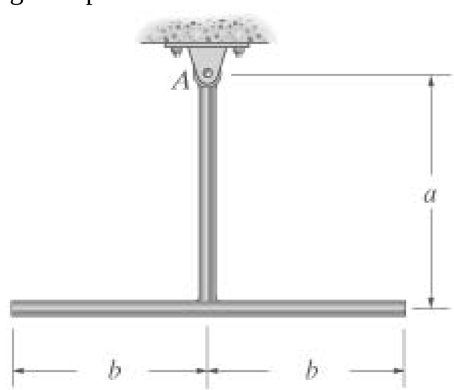
The slender rods have weight density γ. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through the pin at A.

Given:

$$\gamma = 3\ \frac{\text{lb}}{\text{ft}}$$

$$a = 2\ \text{ft}$$

$$b = 1.5\ \text{ft}$$



Solution:

$$I_A = \frac{1}{3}\gamma a^3 + \frac{1}{12}\gamma(2b)^3 + \gamma(2b)\,a^2 \qquad I_A = 1.58\ \text{slug}\cdot\text{ft}^2$$

Problem 17-19

The pendulum consists of a plate having weight W_p and a slender rod having weight W_r. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.

Given:

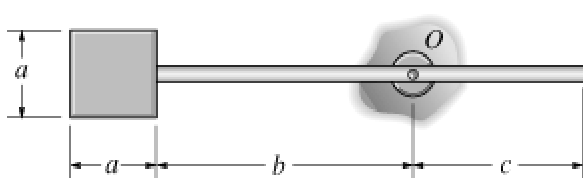
$$W_p = 12\ \text{lb}$$

$$W_r = 4\ \text{lb}$$

$$a = 1\ \text{ft}$$

$$b = 3\ \text{ft}$$

$$c = 2\ \text{ft}$$



Solution:

$$I_O = \frac{1}{3}\left(\frac{b}{b+c}\right)W_r b^2 + \frac{1}{3}\left(\frac{c}{b+c}\right)W_r c^2 + \frac{1}{6}W_p a^2 + W_p\left(b + \frac{a}{2}\right)^2$$

$$I_O = 4.921\,\text{slug}\cdot\text{ft}^2$$

$$k_O = \sqrt{\frac{I_O}{W_r + W_p}} \qquad k_O = 3.146\,\text{ft}$$

***Problem 17-20**

Determine the moment of inertia of the overhung crank about the x axis. The material is steel having a density ρ.

Units Used:

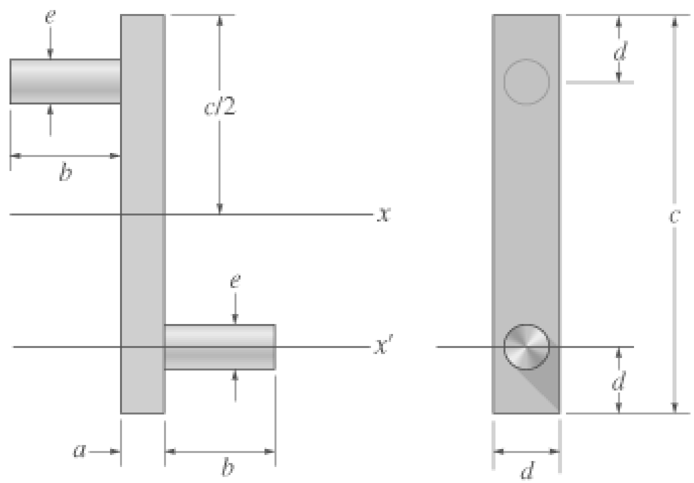
$$\text{Mg} = 10^3\ \text{kg}$$

Given:

$$\rho = 7.85\ \frac{\text{Mg}}{\text{m}^3} \qquad c = 180\ \text{mm}$$

$$a = 20\ \text{mm} \qquad d = 30\ \text{mm}$$

$$b = 50\ \text{mm} \qquad e = 20\ \text{mm}$$



Solution:

$$I_x = 2\left[\frac{\rho\pi}{2}\left(\frac{e}{2}\right)^2 b\left(\frac{e}{2}\right)^2 + \rho\pi\left(\frac{e}{2}\right)^2 b\left(\frac{c-2d}{2}\right)^2\right] + \frac{\rho a d c}{12}\left(d^2 + c^2\right)$$

$$I_x = 3.25 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

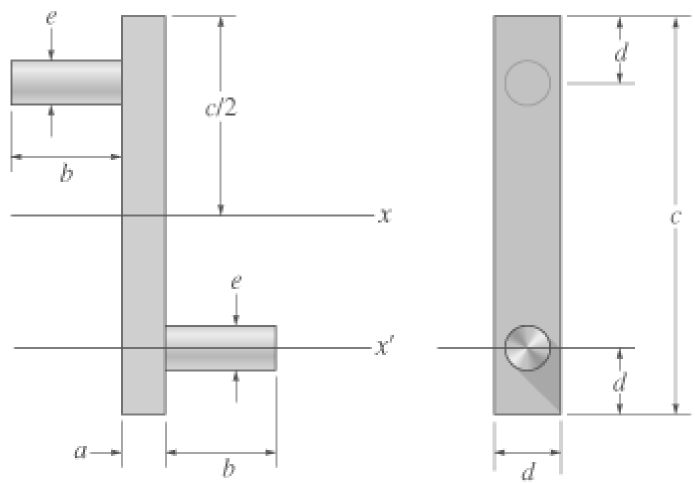
Problem 17-21

Determine the moment of inertia of the overhung crank about the x' axis. The material is steel having a density ρ.

Units Used:

$$\text{Mg} = 10^3\ \text{kg}$$

Given:



$$\rho = 7.85 \frac{\text{Mg}}{\text{m}^3}$$

$$a = 20 \text{ mm}$$

$$b = 50 \text{ mm}$$

$$c = 180 \text{ mm}$$

$$d = 30 \text{ mm}$$

$$e = 20 \text{ mm}$$

Solution:

$$I_x = 2\left[\frac{\rho \pi}{2}\left(\frac{e}{2}\right)^2 b\left(\frac{e}{2}\right)^2 + \rho \pi\left(\frac{e}{2}\right)^2 b\left(\frac{c-2d}{2}\right)^2\right] + \frac{\rho a d c}{12}\left(d^2 + c^2\right)$$

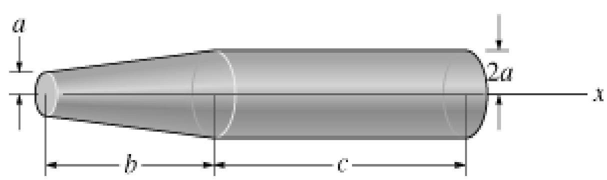
$$I_{x'} = I_x + \left[2\rho \pi\left(\frac{e}{2}\right)^2 b + \rho a d c\right]\left(\frac{c-2d}{2}\right)^2$$

$$I_{x'} = 7.19 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Problem 17-22

Determine the moment of inertia of the solid steel assembly about the x axis. Steel has specific weight γ_{st}.

Given:



$$a = 0.25 \text{ ft}$$

$$b = 2 \text{ ft}$$

$$c = 3 \text{ ft}$$

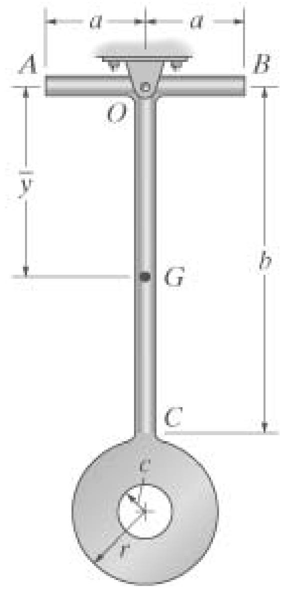
$$\gamma_{st} = 490 \frac{\text{lb}}{\text{ft}^3}$$

Solution:

$$I_x = \left[\frac{1}{2}\pi(2a)^2 c(2a)^2 + \frac{3}{10}\frac{1}{3}\pi(2a)^2(2b)(2a)^2 - \frac{3}{10}\frac{1}{3}\pi a^2 b a^2\right]\gamma_{st}$$

$$I_x = 5.644 \text{ slug} \cdot \text{ft}^2$$

Problem 17-23



The pendulum consists of two slender rods *AB* and *OC* which have a mass density ρ_1. The thin plate has a mass density ρ_2. Determine the location y' of the center of mass *G* of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through *G*.

Given:

$\rho_1 = 3\ \frac{\text{kg}}{\text{m}}$ $a = 0.4\ \text{m}$ $c = 0.1\ \text{m}$

$\rho_2 = 12\ \frac{\text{kg}}{\text{m}^2}$ $b = 1.5\ \text{m}$ $r = 0.3\ \text{m}$

Solution:

$$y' = \frac{\rho_1 b\left(\frac{b}{2}\right) + \rho_2\pi\left(r^2 - c^2\right)(b + r)}{\rho_1(b + 2a) + \rho_2\pi\left(r^2 - c^2\right)} \qquad y' = 0.888\ \text{m}$$

$$I_O = \frac{1}{12}\rho_1(2a)^3 + \frac{1}{3}\rho_1 b^3 + \left(\frac{\rho_2}{2}\right)\pi r^4 + \rho_2\pi r^2(r + b)^2 - \left[\left(\frac{\rho_2}{2}\right)\pi c^4 + \rho_2\pi c^2(r + b)^2\right]$$

$$I_G = I_O - \left[\rho_1(2a + b) + \rho_2\pi\left(r^2 - c^2\right)\right]y'^2$$

$$I_G = 5.61\ \text{kg}\cdot\text{m}^2$$

***Problem 17-24**

Determine the greatest possible acceleration of the race car of mass *M* so that its front tires do not leave the ground or the tires slip on the track. The coefficients of static and kinetic friction are μ_s and μ_k respectively. Neglect the mass of the tires. The car has rear-wheel drive and the front tires are free to roll.

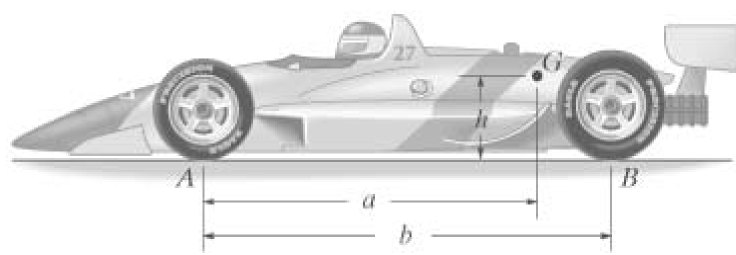
Given:

$M = 975\ \text{kg}$ $\mu_s = 0.8$

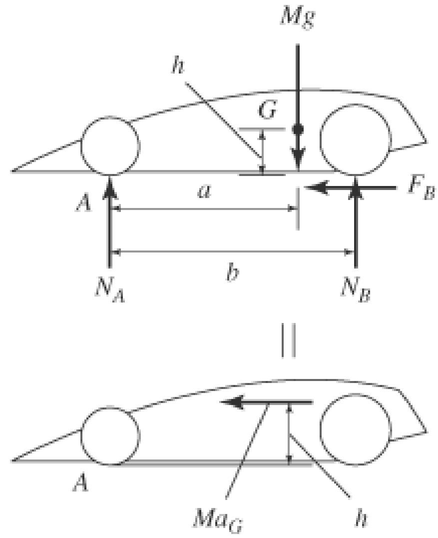
$a = 1.82\ \text{m}$ $\mu_k = 0.6$

$b = 2.20\ \text{m}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$h = 0.55\ \text{m}$



Solution:



First assume that the rear wheels are on the verge of slipping $F_B = \mu_s N_B$

Guesses

$N_A = 1\ \text{N} \qquad N_B = 1\ \text{N} \qquad a_G = 1\ \dfrac{\text{m}}{\text{s}^2}$

Given

$\mu_s N_B = M a_G \qquad N_A + N_B - M g = 0$

$-N_A a + N_B(b - a) - \mu_s N_B h = 0$

$\begin{pmatrix} N_A \\ N_B \\ a_{G1} \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} -326 \\ 9891 \end{pmatrix} \text{N} \qquad a_{G1} = 8.12\ \dfrac{\text{m}}{\text{s}^2}$

Next assume that the front wheels lose contact with the ground $N_A = 0$

Guesses $N_B = 1\ \text{N} \qquad F_B = 1\ \text{N} \qquad a_G = 1\ \dfrac{\text{m}}{\text{s}^2}$

Given $F_B = M a_G \qquad N_B - M g = 0 \qquad N_B(b - a) - F_B h = 0$

$\begin{pmatrix} N_B \\ F_B \\ a_{G2} \end{pmatrix} = \text{Find}(N_B, F_B, a_G) \qquad \begin{pmatrix} N_B \\ F_B \end{pmatrix} = \begin{pmatrix} 9565 \\ 6608 \end{pmatrix} \text{N} \qquad a_{G2} = 6.78\ \dfrac{\text{m}}{\text{s}^2}$

Choose the critical case $a_G = \min(a_{G1}, a_{G2}) \qquad a_G = 6.78\ \dfrac{\text{m}}{\text{s}^2}$

Problem 17-25

Determine the greatest possible acceleration of the race car of mass M so that its front tires do not leave the ground nor the tires slip on the track. The coefficients of static and kinetic friction are μ_s and μ_k respectively. Neglect the mass of the tires. The car has four-wheel drive.

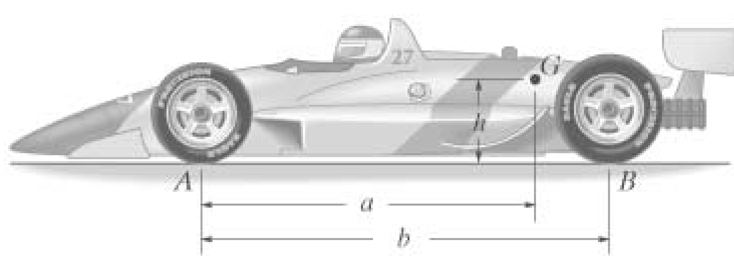
Given:

$M = 975\ \text{kg} \quad \mu_s = 0.8$

$a = 1.82\ \text{m} \quad \mu_k = 0.6$

$b = 2.20\ \text{m} \quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

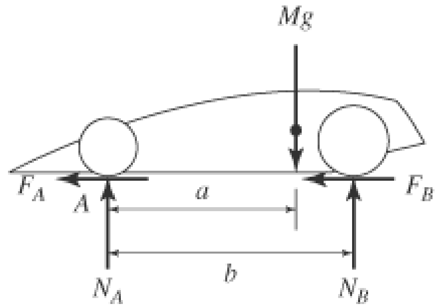
$h = 0.55\ \text{m}$



Solution:

First assume that all wheels are on the verge of slipping $F_A = \mu_s N_A$

$F_B = \mu_s N_B$



Guesses

$N_A = 1\ \text{N}$ $\quad N_B = 1\ \text{N}$ $\quad a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$\mu_s N_B + \mu_s N_A = M a_G$ $\quad N_A + N_B - M g = 0$

$-N_A a + N_B(b - a) - \mu_s N_B h - \mu_s N_A h = 0$

$\begin{pmatrix} N_A \\ N_B \\ a_{G1} \end{pmatrix} = \text{Find}(N_A, N_B, a_G)$ $\quad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} -261 \\ 9826 \end{pmatrix} \text{N}$ $\quad a_{G1} = 7.85\ \frac{\text{m}}{\text{s}^2}$

Next assume that the front wheels lose contact with the ground $N_A = 0$

Guesses $N_B = 1\ \text{N}$ $\quad F_B = 1\ \text{N}$ $\quad a_G = 1\ \frac{\text{m}}{\text{s}^2}$

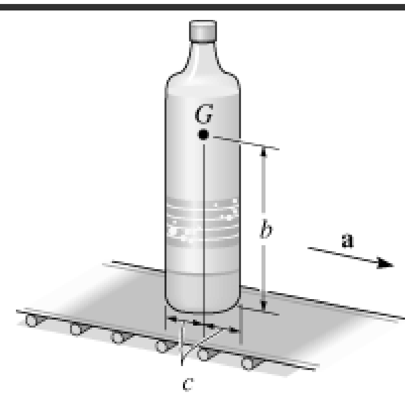
Given $F_B = M a_G$ $\quad N_B - M g = 0$ $\quad N_B(b - a) - F_B h = 0$

$\begin{pmatrix} N_B \\ F_B \\ a_{G2} \end{pmatrix} = \text{Find}(N_B, F_B, a_G)$ $\quad \begin{pmatrix} N_B \\ F_B \end{pmatrix} = \begin{pmatrix} 9565 \\ 6608 \end{pmatrix} \text{N}$ $\quad a_{G2} = 6.78\ \frac{\text{m}}{\text{s}^2}$

Choose the critical case $a_G = \min(a_{G1}, a_{G2})$ $\quad a_G = 6.78\ \frac{\text{m}}{\text{s}^2}$

Problem 17-26

The bottle of weight W rests on the check-out conveyor at a grocery store. If the coefficient of static friction is μ_s, determine the largest acceleration the conveyor can have without causing the bottle to slip or tip. The center of gravity is at G.



Given:

$W = 2\ \text{lb}$

$\mu_s = 0.2$

$b = 8 \text{ in}$

$c = 1.5 \text{ in}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

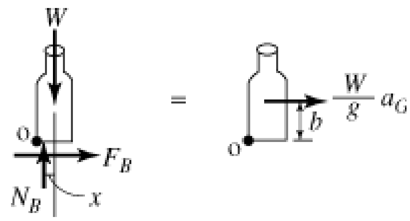
Solution: Assume that bottle tips before slipping

$x = c$

Guesses $a_G = 1 \dfrac{\text{ft}}{\text{s}^2}$ $F_B = 1 \text{ lb}$ $N_B = 1 \text{ lb}$ $F_{max} = 1 \text{ lb}$

Given $F_B = \left(\dfrac{W}{g}\right)a_G$ $N_B - W = 0$

$F_B b - N_B x = 0$ $F_{max} = \mu_s N_B$



$$\begin{pmatrix} a_{Gt} \\ F_B \\ N_B \\ F_{max} \end{pmatrix} = \text{Find}(a_G, F_B, N_B, F_{max}) \qquad \begin{pmatrix} F_B \\ N_B \\ F_{max} \end{pmatrix} = \begin{pmatrix} 0.375 \\ 2 \\ 0.4 \end{pmatrix} \text{lb} \qquad a_{Gt} = 6.037 \frac{\text{ft}}{\text{s}^2}$$

If $F_B = 0.375 \text{ lb} < F_{max} = 0.4 \text{ lb}$ then we have the correct answer.

If $F_B = 0.375 \text{ lb} > F_{max} = 0.4 \text{ lb}$ then we know that slipping occurs first. If this is the case,

$F_B = \mu_s N_B$

Given $F_B = \left(\dfrac{W}{g}\right)a_G$ $N_B - W = 0$ $F_B b - N_B x = 0$

$$\begin{pmatrix} a_{Gs} \\ N_B \\ x \end{pmatrix} = \text{Find}(a_G, N_B, x) \qquad N_B = 2 \text{ lb} \qquad x = 1.6 \text{ in} \qquad a_{Gs} = 6.44 \frac{\text{ft}}{\text{s}^2}$$

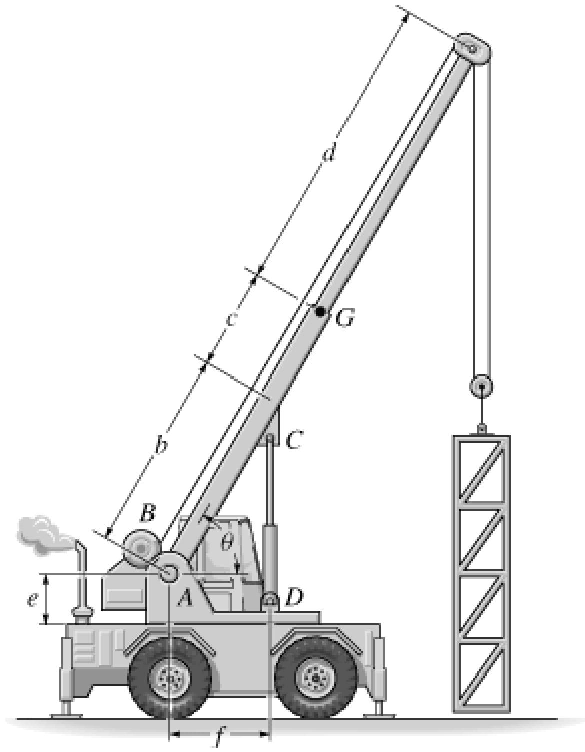
As a check, we should have $x = 1.6 \text{ in} < c = 1.5 \text{ in}$ if slipping occurs first

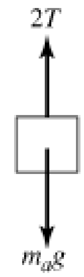
In either case, the answer is $a_G = \min(a_{Gs}, a_{Gt})$ $a_G = 6.037 \dfrac{\text{ft}}{\text{s}^2}$

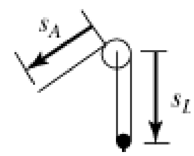
Problem 17-27

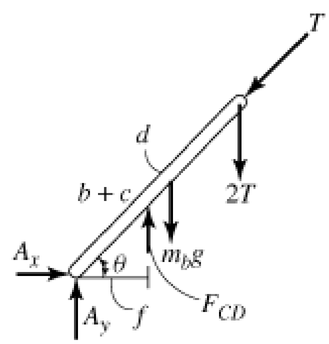
The assembly has mass m_a and is hoisted using the boom and pulley system. If the winch at B

draws in the cable with acceleration a, determine the compressive force in the hydraulic cylinder needed to support the boom. The boom has mass m_b and mass center at G.









Units Used:

$$\text{Mg} = 10^3 \text{ kg}$$

$$\text{kN} = 10^3 \text{ N}$$

Given:

$$m_a = 8 \text{ Mg}$$

$$m_b = 2 \text{ Mg}$$

$$a = 2 \frac{\text{m}}{\text{s}^2}$$

$$b = 4 \text{ m}$$

$$c = 2 \text{ m}$$

$$d = 6 \text{ m}$$

$$e = 1 \text{ m}$$

$$f = 2 \text{ m}$$

$$\theta = 60 \text{ deg}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$

Solution:

$$2T - m_a g = m_a \frac{a}{2} \qquad T = \frac{1}{2}\left(m_a g + m_a \frac{a}{2}\right) \qquad T = 43.24 \text{ kN}$$

$$\left[-2T(b+c+d) - m_b g(b+c) + F_{CD} b\right]\cos(\theta) = 0$$

$$F_{CD} = \frac{2T(b+c+d) + m_b g(b+c)}{b} \qquad F_{CD} = 289 \text{ kN}$$

*Problem 17-28

The jet aircraft has total mass M and a center of mass at G. Initially at take-off the engines provide thrusts $2T$ and T'. Determine the acceleration of the plane and the normal reactions on the nose wheel and each of the *two* wing wheels located at B. Neglect the mass of the wheels and, due to low velocity, neglect any lift caused by the wings.

Units Used:

$$\text{Mg} = 10^3 \text{ kg}$$

$\text{kN} = 10^3\ \text{N}$

Given:

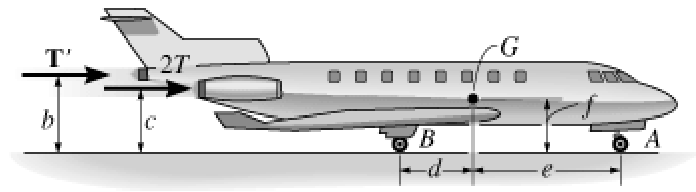
$b = 2.5\ \text{m}$ $M = 22\ \text{Mg}$

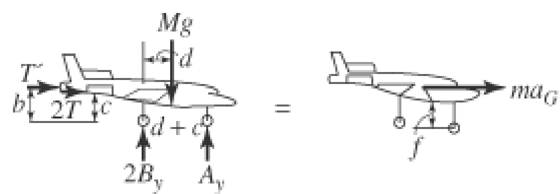
$c = 2.3\ \text{m}$ $T = 2\ \text{kN}$

$d = 3\ \text{m}$ $T' = 1.5\ \text{kN}$

$e = 6\ \text{m}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$f = 1.2\ \text{m}$





Solution:

Guesses $a_G = 1\ \dfrac{\text{m}}{\text{s}^2}$ $B_y = 1\ \text{kN}$ $A_y = 1\ \text{kN}$

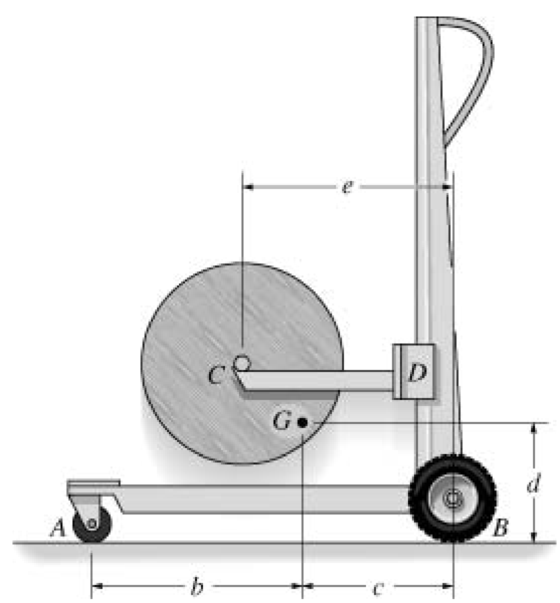
Given $T' + 2T = M a_G$ $2B_y + A_y - M g = 0$

$$-T' b - 2T c - M g d + A_y(d + e) = -M a_G f$$

$$\begin{pmatrix} a_G \\ B_y \\ A_y \end{pmatrix} = \text{Find}(a_G, B_y, A_y) \qquad a_G = 0.250\ \frac{\text{m}}{\text{s}^2} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 72.6 \\ 71.6 \end{pmatrix} \text{kN}$$

Problem 17-29

The lift truck has mass m_t and mass center at G. If it lifts the spool of mass m_s with acceleration a, determine the reactions of each of the four wheels on the ground. The loading is symmetric. Neglect the mass of the movable arm CD.



Given: $a = 3\ \dfrac{\text{m}}{\text{s}^2}$ $d = 0.4\ \text{m}$

$b = 0.75\ \text{m}$ $e = 0.7\ \text{m}$

$c = 0.5\ \text{m}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$m_t = 70\ \text{kg}$ $m_s = 120\ \text{kg}$

Solution:

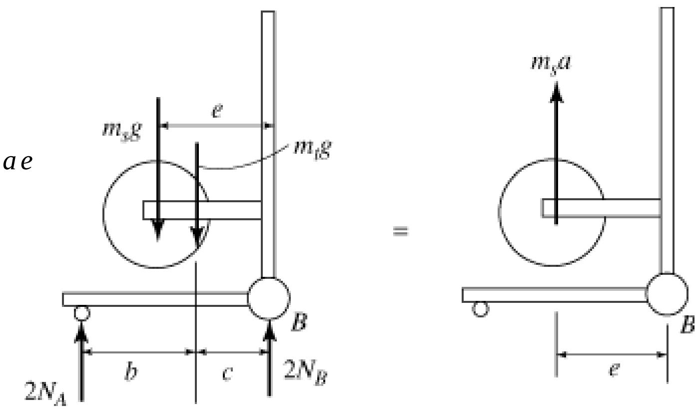
Guesses $N_A = 1\ \text{N}$ $N_B = 1\ \text{N}$

Given

$$2(N_A + N_B) - (m_t + m_s)g = m_s a$$

$$-2N_A(b + c) + m_s g e + m_t g c = -m_s a e$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \text{Find}(N_A, N_B)$$



$$\boxed{\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 568 \\ 544 \end{pmatrix} \text{N}}$$

Problem 17-30

The lift truck has mass m_t and mass center at G. Determine the largest upward acceleration of the spool of mass m_s so that no reaction of the wheels on the ground exceeds F_{max}.

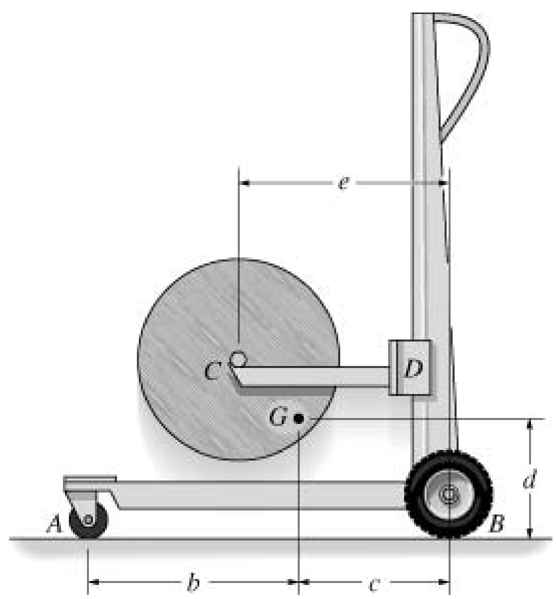
Given:

$m_t = 70\ \text{kg}$ $b = 0.75\ \text{m}$

$m_s = 120\ \text{kg}$ $c = 0.5\ \text{m}$

$F_{max} = 600\ \text{N}$ $d = 0.4\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$ $e = 0.7\ \text{m}$



Solution: Assume $N_A = F_{max}$

Guesses $a = 1\ \dfrac{\text{m}}{\text{s}^2}$ $N_B = 1\ \text{N}$

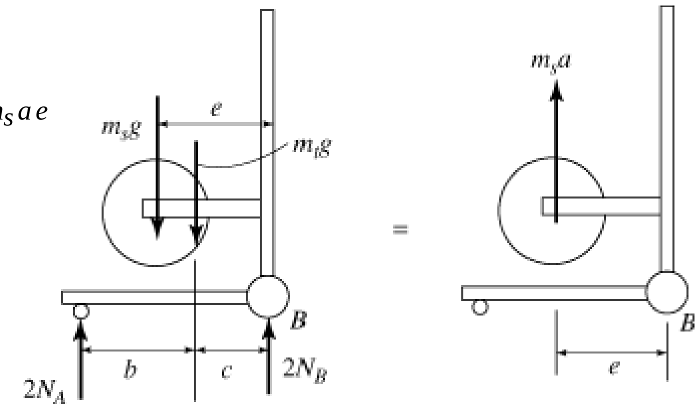
Given

$$2(N_A + N_B) - (m_t + m_s)g = m_s a$$

$$-2N_A(b + c) + m_s g e + m_t g c = -m_s a e$$

$$\begin{pmatrix} a \\ N_B \end{pmatrix} = \text{Find}(a, N_B)$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 600 \\ 569.529 \end{pmatrix} \text{N}$$



$$a = 3.96 \ \frac{\text{m}}{\text{s}^2}$$

Check: Since $N_B = 570 \text{ N}$ < $F_{max} = 600 \text{ N}$ then our assumption is good.

Problem 17-31

The door has weight W and center of gravity at G. Determine how far the door moves in time t starting from rest, if a man pushes on it at C with a horizontal force F. Also, find the vertical reactions at the rollers A and B.

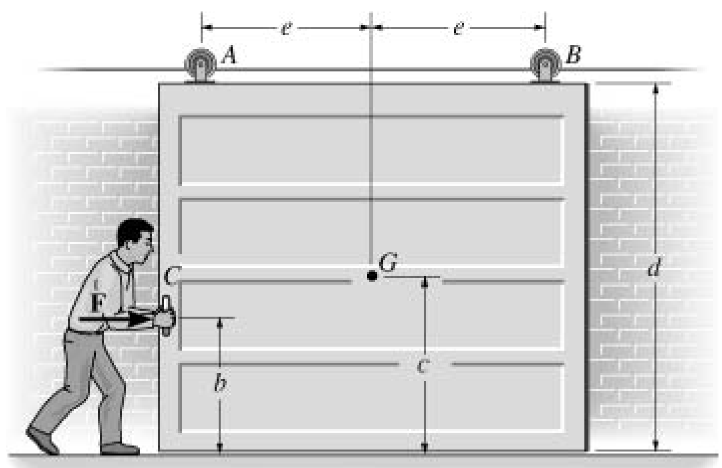
Given:

$W = 200 \text{ lb}$ $\quad c = 5 \text{ ft}$

$t = 2 \text{ s}$ $\quad d = 12 \text{ ft}$

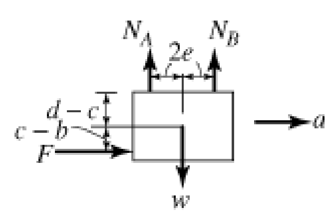
$F = 30 \text{ lb}$ $\quad e = 6 \text{ ft}$

$b = 3 \text{ ft}$ $\quad g = 32.2 \ \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $\quad a = 1 \ \frac{\text{ft}}{\text{s}^2} \quad N_A = 1 \text{ lb} \quad N_B = 1 \text{ lb}$



Given $\quad F = \left(\frac{W}{g}\right)a \qquad N_A + N_B - W = 0$

$$F(c - b) + N_B e - N_A e = 0$$

$$\begin{pmatrix} a \\ N_A \\ N_B \end{pmatrix} = \text{Find}(a, N_A, N_B) \qquad a = 4.83\,\frac{\text{ft}}{\text{s}^2} \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 105.0 \\ 95.0 \end{pmatrix}\text{lb}$$

$$d = \frac{1}{2}a t^2 \qquad d = 9.66\text{ ft}$$

***Problem 17-32**

The door has weight W and center of gravity at G. Determine the constant force F that must be applied to the door to push it open a distance d to the right in time t, starting from rest. Also, find the vertical reactions at the rollers A and B.

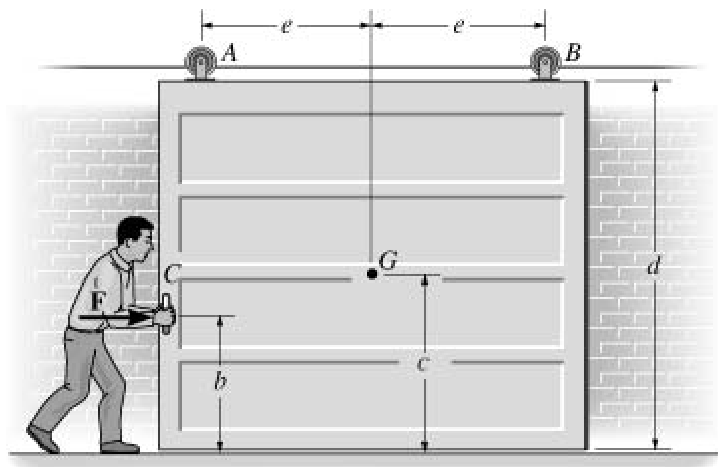
Given:

$W = 200\text{ lb}$ $\quad c = 5\text{ ft}$

$t = 5\text{ s}$ $\quad d = 12\text{ ft}$

$d = 12\text{ ft}$ $\quad e = 6\text{ ft}$

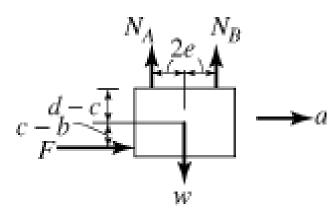
$b = 3\text{ ft}$ $\quad g = 32.2\,\frac{\text{ft}}{\text{s}^2}$



Solution:

$$a = 2\left(\frac{d}{t^2}\right) \qquad a = 0.96\,\frac{\text{ft}}{\text{s}^2}$$

Guesses $\quad F = 1\text{ lb} \quad N_A = 1\text{ lb} \quad N_B = 1\text{ lb}$



Given $\quad F = \left(\frac{W}{g}\right)a \qquad N_A + N_B - W = 0$

$$F(c - b) + N_B e - N_A e = 0$$

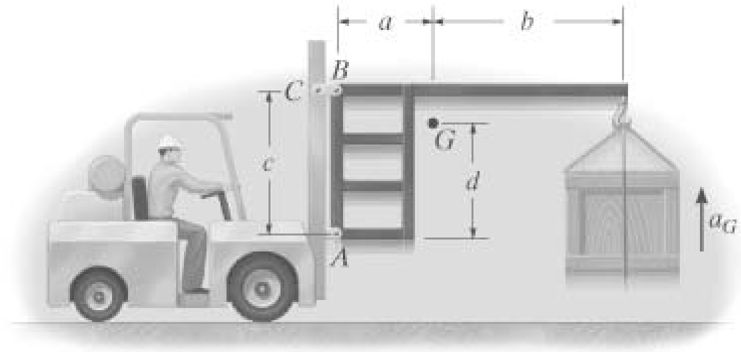
$$\begin{pmatrix} F \\ N_A \\ N_B \end{pmatrix} = \text{Find}(F, N_A, N_B) \qquad \begin{pmatrix} F \\ N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 5.96 \\ 100.99 \\ 99.01 \end{pmatrix}\text{lb}$$

Problem 17-33

The fork lift has a boom with mass M_1 and a mass center at G. If the vertical acceleration of the boom is a_G, determine the horizontal and vertical reactions at the pin A and on the short link BC when the load M_2 is lifted.

Units Used:

$\mathrm{Mg} = 10^3\ \mathrm{kg}$ $\quad$ $\mathrm{kN} = 10^3\ \mathrm{N}$



Given:

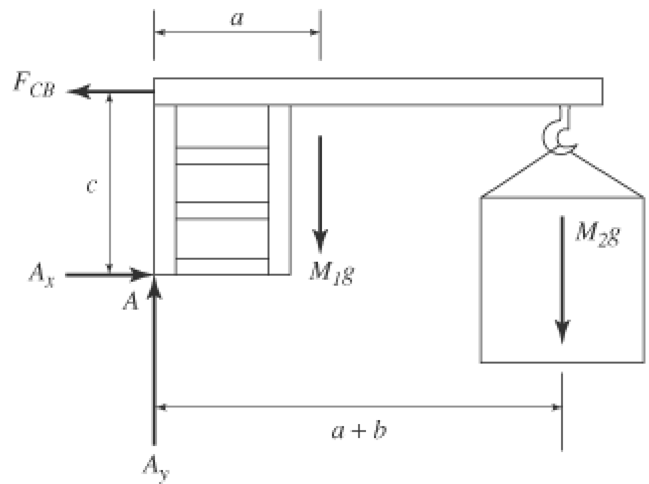
$M_1 = 800\ \mathrm{kg}$ $\quad$ $a = 1\ \mathrm{m}$

$M_2 = 1.25\ \mathrm{Mg}$ $\quad$ $b = 2\ \mathrm{m}$

$c = 1.5\ \mathrm{m}$

$a_G = 4\ \frac{\mathrm{m}}{\mathrm{s}^2}$ $\quad$ $d = 1.25\ \mathrm{m}$

Solution:



Guesses

$A_x = 1\ \mathrm{N}$ $\quad$ $F_{CB} = 1\ \mathrm{N}$

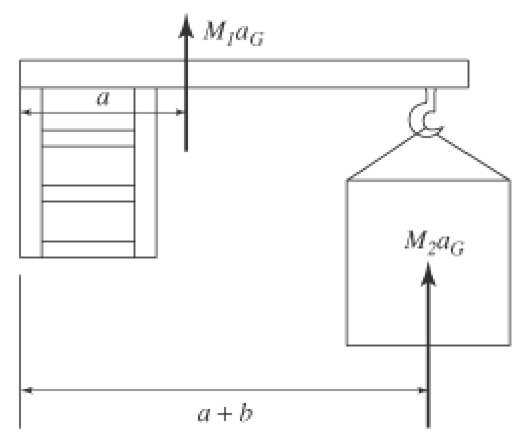
$A_y = 1\ \mathrm{N}$

Given

$$-F_{CB} + A_x = 0$$

$$A_y - (M_1 + M_2)g = (M_1 + M_2)a_G$$

$$F_{CB}\,c - M_1\,g\,a - M_2\,g(a+b) = M_1\,a_G\,a + M_2\,a_G(a+b)$$



$$\begin{pmatrix} A_x \\ A_y \\ F_{CB} \end{pmatrix} = \mathrm{Find}(A_x, A_y, F_{CB})$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{CB} \end{pmatrix} = \begin{pmatrix} 41.9 \\ 28.3 \\ 41.9 \end{pmatrix} \mathrm{kN}$$

Problem 17-34

The pipe has mass M and is being towed behind the truck. If the acceleration of the truck is a_t, determine the angle θ and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is μ_k.

Units Used:

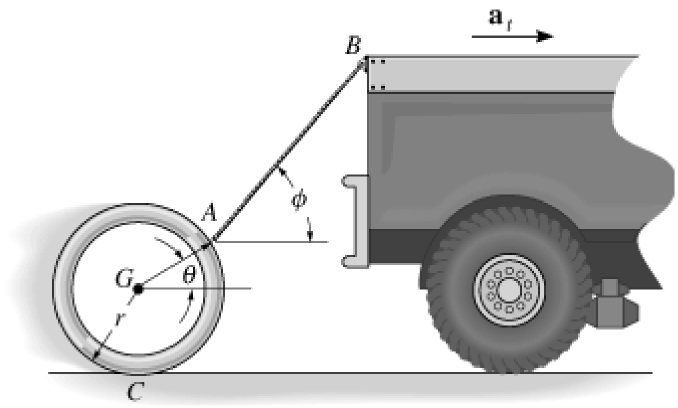
$$kN = 10^3\ N$$

Given:

$M = 800\ kg$ $\quad r = 0.4\ m$

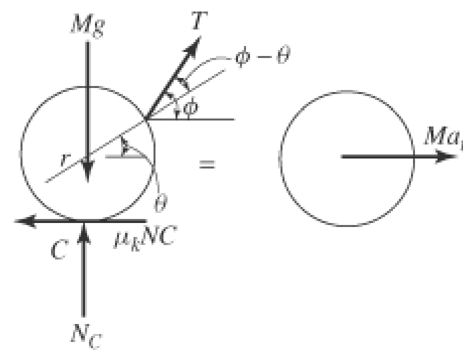
$a_t = 0.5\ \frac{m}{s^2}$ $\quad \phi = 45\ deg$

$\mu_k = 0.1$ $\quad g = 9.81\ \frac{m}{s^2}$



Solution:

Guesses $\quad \theta = 10\ deg \quad N_C = 1\ N \quad T = 1\ N$



Given

$$T\cos(\phi) - \mu_k N_C = M a_t$$

$$T\sin(\phi) - M g + N_C = 0$$

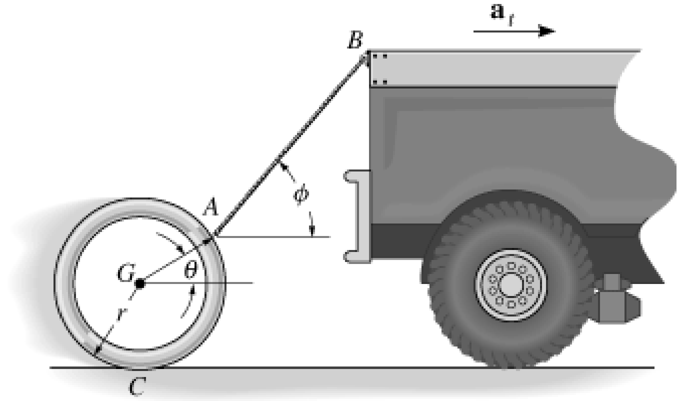
$$T\sin(\phi - \theta) r - \mu_k N_C r = 0$$

$$\begin{pmatrix} \theta \\ N_C \\ T \end{pmatrix} = \text{Find}(\theta, N_C, T) \qquad N_C = 6.771\,\text{kN} \qquad T = 1.523\,\text{kN} \qquad \theta = 18.608\,\text{deg}$$

Problem 17-35

The pipe has mass M and is being towed behind a truck. Determine the acceleration of the truck and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is μ_k.

Units Used: $\text{kN} = 10^3\ \text{N}$



Given:

$M = 800\ \text{kg}$ $\quad r = 0.4\ \text{m}$

$\theta = 30\ \text{deg}$ $\quad \phi = 45\ \text{deg}$

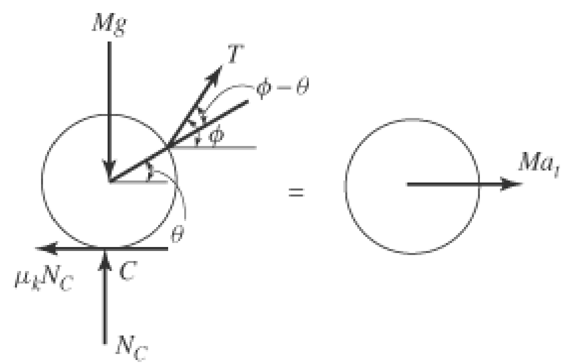
$\mu_k = 0.1$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

Guesses $\quad a_t = 1\ \dfrac{\text{m}}{\text{s}^2}$

$N_C = 1\ \text{N}$

$T = 1\ \text{N}$



Given

$$T\cos(\phi) - \mu_k N_C = M a_t$$

$$T\sin(\phi) - M g + N_C = 0$$

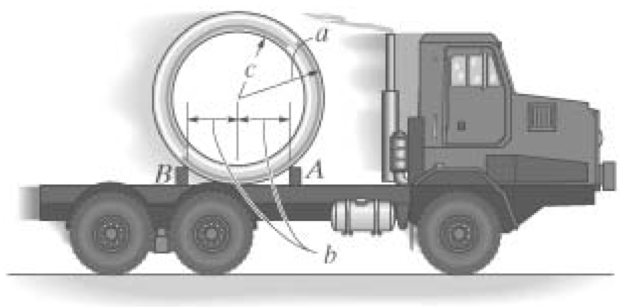
$$T\sin(\phi - \theta)r - \mu_k N_C r = 0$$

$$\begin{pmatrix} a_t \\ N_C \\ T \end{pmatrix} = \text{Find}(a_t, N_C, T) \qquad N_C = 6.164\,\text{kN} \qquad T = 2.382\,\text{kN} \qquad a_t = 1.335\,\frac{\text{m}}{\text{s}^2}$$

***Problem 17-36**

The pipe has a mass M and is held in place on the truck bed using the two boards A and B. Determine the acceleration of the truck so that the pipe begins to lose contact at A and the bed of the truck and starts to pivot about B. Assume board B will not slip on the bed of the truck, and the pipe is smooth. Also, what force does board B exert on the pipe during the acceleration?

Units Used: $\mathrm{kN} = 10^3\ \mathrm{N}$



Given:

$M = 460\ \mathrm{kg}$

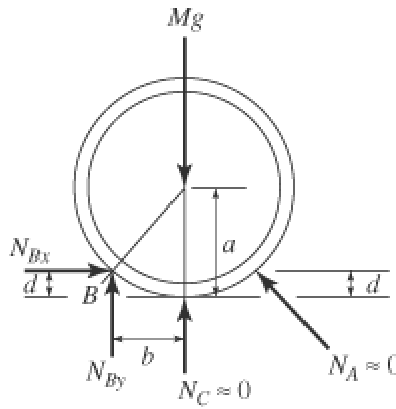
$a = 0.5\ \mathrm{m}$

$b = 0.3\ \mathrm{m}$

$c = 0.4\ \mathrm{m}$

$d = 0.1\ \mathrm{m}$

$g = 9.81\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$



Solution: $\theta = \operatorname{asin}\left(\dfrac{b}{a}\right)$

Guesses

$N_{Bx} = 1\ \mathrm{N}$ $\quad N_{By} = 1\ \mathrm{N}$ $\quad a_t = 1\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$

Given

$N_{Bx} = M a_t$ $\quad N_{By} - M g = 0$ $\quad N_{Bx}(a)\cos(\theta) - N_{By} b = 0$

$$\begin{pmatrix} N_{Bx} \\ N_{By} \\ a_t \end{pmatrix} = \operatorname{Find}(N_{Bx}, N_{By}, a_t) \qquad \begin{pmatrix} N_{Bx} \\ N_{By} \end{pmatrix} = \begin{pmatrix} 3.384 \\ 4.513 \end{pmatrix} \mathrm{kN} \qquad a_t = 7.36\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

$$\left|\begin{pmatrix} N_{Bx} \\ N_{By} \end{pmatrix}\right| = 5.64\ \mathrm{kN}$$

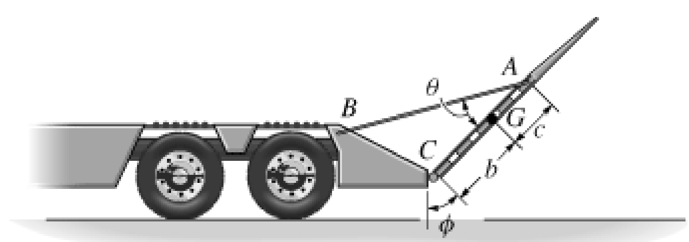
Problem 17-37

The drop gate at the end of the trailer has mass *M* and mass center at *G*. If it is supported by the cable *AB* and hinge at *C*, determine the tension in the cable when the truck begins to accelerate at rate *a*. Also, what are the horizontal and vertical components of reaction at the hinge *C*?

Given: $\mathrm{kN} = 10^3\ \mathrm{N}$

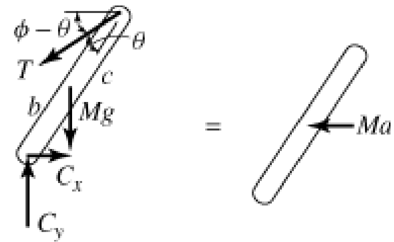
$M = 1.25 \times 10^3\ \mathrm{kg}$

$a = 5\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$ $\quad \theta = 30\ \mathrm{deg}$



$b = 1.5\ \text{m}$ $\phi = 45\ \text{deg}$

$c = 1\ \text{m}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

Guesses $T = 1\ \text{N}$ $C_x = 1\ \text{N}$ $C_y = 1\ \text{N}$

Given $-T\cos(\phi-\theta) + C_x = -M a$

$$-T\sin(\phi-\theta) - M g + C_y = 0$$

$$T\sin(\theta)(b+c) - M g b\cos(\phi) = M a b\sin(\phi)$$

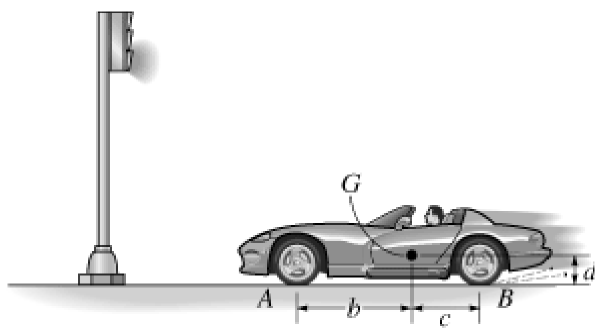
$$\begin{pmatrix} T \\ C_x \\ C_y \end{pmatrix} = \text{Find}(T, C_x, C_y) \qquad \begin{pmatrix} T \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} 15.708 \\ 8.923 \\ 16.328 \end{pmatrix} \text{kN}$$

Problem 17-38

The sports car has mass M and a center of mass at G. Determine the shortest time it takes for it to reach speed v, starting from rest, if the engine only drives the rear wheels, whereas the front wheels are free rolling. The coefficient of static friction between the wheels and the road is μ_s. Neglect the mass of the wheels for the calculation. If driving power could be supplied to all four wheels, what would be the shortest time for the car to reach a speed of v?

Given:

$M = 1.5 \times 10^3\ \text{kg}$



$v = 80\ \dfrac{\text{km}}{\text{hr}}$

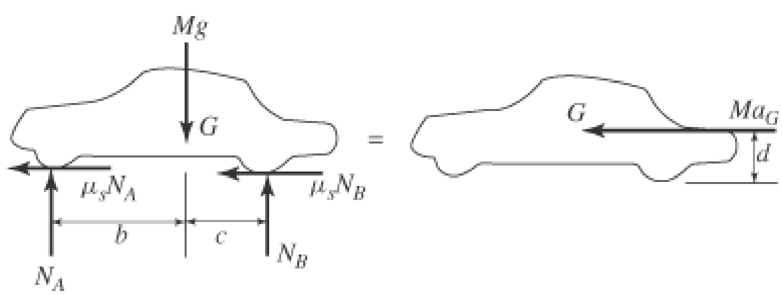
$\mu_s = 0.2$

$b = 1.25\ \text{m}$

$c = 0.75\ \text{m}$

$d = 0.35\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

(a) Rear wheel drive only Guesses $N_A = 1\ \text{N}$ $N_B = 1\ \text{N}$ $a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given $N_A + N_B - Mg = 0$ $\mu_s N_B = M a_G$

$Mgc - N_A(b+c) = M a_G d$

$$\begin{pmatrix} N_A \\ N_B \\ a_G \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 5.185 \times 10^3 \\ 9.53 \times 10^3 \end{pmatrix} \text{N} \qquad a_G = 1.271\ \frac{\text{m}}{\text{s}^2}$$

$$t_{rw} = \frac{v}{a_G} \qquad t_{rw} = 17.488\ \text{s}$$

(b) Four wheel drive Guesses $N_A = 1\ \text{N}$ $N_B = 1\ \text{N}$ $a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given $N_A + N_B - Mg = 0$ $\mu_s N_B + \mu_s N_A = M a_G$

$Mgc - N_A(b+c) = M a_G d$

$$\begin{pmatrix} N_A \\ N_B \\ a_G \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 5.003 \times 10^3 \\ 9.712 \times 10^3 \end{pmatrix} \text{N} \qquad a_G = 1.962\ \frac{\text{m}}{\text{s}^2}$$

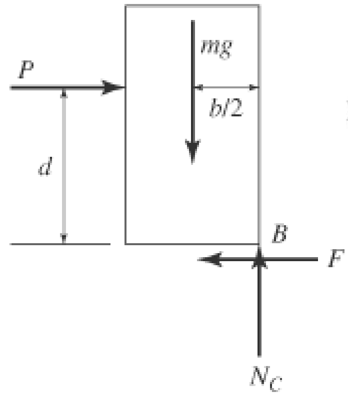
$$t_{rw} = \frac{v}{a_G} \qquad t_{rw} = 11.326\ \text{s}$$

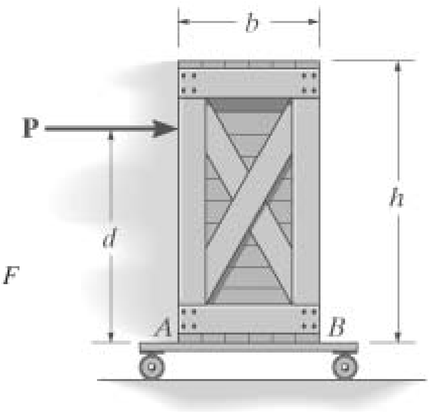
Problem 17-39

The crate of mass m is supported on a cart of negligible mass. Determine the maximum force P that can be applied a distance d from the cart bottom without causing the crate to tip on the cart.

Solution:

Require N_c to act at corner B for tipping.





$$P\,d - m\,g\left(\frac{b}{2}\right) = m\,a_G\left(\frac{h}{2}\right)$$

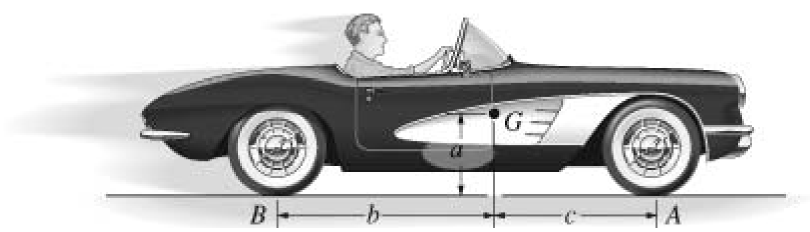
$$P = m\,a_G$$

$$P\,d - m\mathbf{g}\left(\frac{b}{2}\right) = P\left(\frac{h}{2}\right)$$

$$\boxed{P_{max} = \frac{m\,g\,b}{2\left(d - \frac{h}{2}\right)}}$$

***Problem 17-40**

The car accelerates uniformly from rest to speed v in time t. If it has weight W and a center of gravity at G, determine the normal reaction of *each wheel* on the pavement during the motion. Power is developed at the front wheels, whereas the rear wheels are free to roll. Neglect the mass of the wheels and take the coefficients of static and kinetic friction to be μ_s and μ_k respectively.



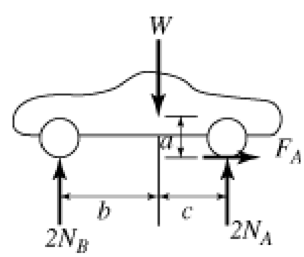
Given: $a = 2.5\ \text{ft}$

$v = 88\ \frac{\text{ft}}{\text{s}}$ $\quad \mu_k = 0.2$

$t = 15\ \text{s}$ $\quad b = 4\ \text{ft}$

$W = 3800\ \text{lb}$ $\quad c = 3\ \text{ft}$

$\mu_s = 0.4$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: Assume no slipping $\quad a_G = \frac{v}{t} \quad a_G = 5.867\ \frac{\text{ft}}{\text{s}^2}$

Guesses $\quad N_B = 1\ \text{lb} \quad N_A = 1\ \text{lb} \quad F_A = 1\ \text{lb}$

Given $\quad 2N_B + 2N_A - W = 0$

$$2F_A = \left(\frac{W}{g}\right)a_G$$

$$-2N_B(b + c) + W\,c = \left(\frac{-W}{g}\right)a_G\,a$$

$$\begin{pmatrix} N_A \\ N_B \\ F_A \end{pmatrix} = \text{Find}(N_A, N_B, F_A) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 962 \\ 938 \end{pmatrix} \text{lb} \qquad F_A = 346\,\text{lb}$$

$$F_{max} = \mu_s N_A \qquad F_{max} = 385\,\text{lb}$$

Check: Our no-slip assumption is true if $F_A = 346\,\text{lb} < F_{max} = 385\,\text{lb}$

Problem 17-41

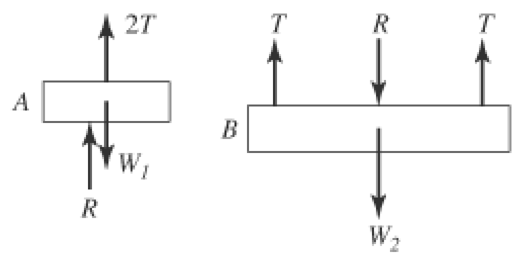
Block A has weight W_1 and the platform has weight W_2. Determine the normal force exerted by block A on B. Neglect the weight of the pulleys and bars of the triangular frame.

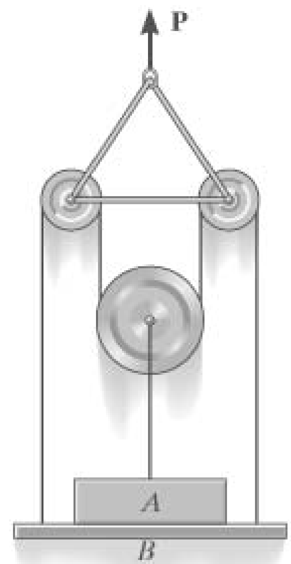
Given:

$W_1 = 50$ lb

$W_2 = 10$ lb

$P = 100$ lb

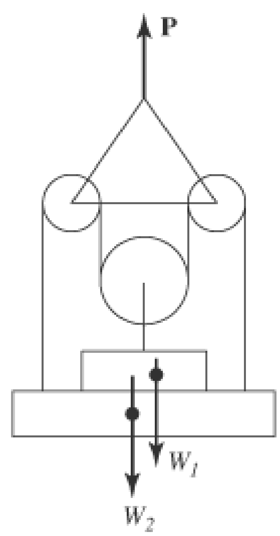




Solution:

$$P - (W_1 + W_2) = \left(\frac{W_1 + W_2}{g}\right) a_G$$

$$a_G = g\left(\frac{P - 1\ W_1 - 1\ W_2}{W_1 + W_2}\right) \qquad a_G = 21.47\,\frac{\text{ft}}{\text{s}^2}$$



Guesses $\quad R = 25$ lb $\quad T = 22$ lb

Given

$$2T + R - W_1 = \left(\frac{W_1}{g}\right) a_G$$

$$2T - R - W_2 = \left(\frac{W_2}{g}\right) a_G$$

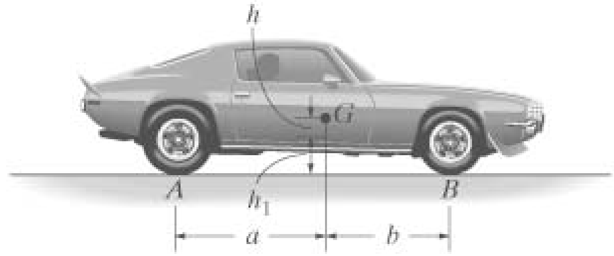
$$\begin{pmatrix} R \\ T \end{pmatrix} = \text{Find}(R, T) \qquad R = 33.3\,\text{lb} \qquad T = 25\,\text{lb}$$

Problem 17-42

The car of mass M shown has been "raked" by increasing the height of its center of mass to h This was done by raising the springs on the rear axle. If the coefficient of kinetic friction between the rear wheels and the ground is μ_k, show that the car can accelerate slightly faster than its counterpart for which $h = 0$. Neglect the mass of the wheels and driver and assume the front wheels at B are free to roll while the rear wheels slip.

Units Used:

$$\text{Mg} = 10^3 \text{ kg} \quad \text{kN} = 10^3 \text{ N}$$



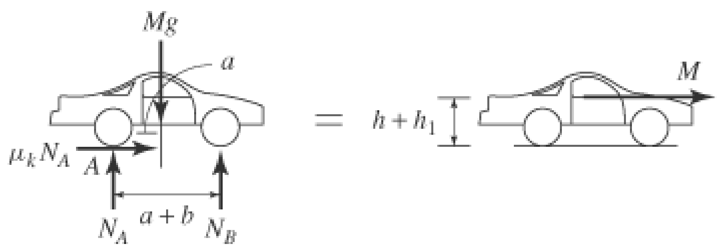
Given:

$$M = 1.6 \text{ Mg} \quad a = 1.6 \text{ m}$$

$$\mu_k = 0.3 \quad b = 1.3 \text{ m}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2} \quad h = 0.2 \text{ m}$$

$$h_1 = 0.4 \text{ m}$$



Solution:

In the raised position

Guesses $\quad a_G = 1 \frac{\text{m}}{\text{s}^2} \quad N_A = 4 \text{ N} \quad N_B = 5 \text{ N}$

Given $\quad \mu_k N_A = M a_G$

$$N_A + N_B - M g = 0$$

$$-M g a + N_B(a + b) = -M a_G(h + h_1)$$

$$\begin{pmatrix} a_{Gr} \\ N_A \\ N_B \end{pmatrix} = \text{Find}(a_G, N_A, N_B) \qquad a_{Gr} = 1.41 \frac{\text{m}}{\text{s}^2}$$

In the lower (regular) position

Given

$$\mu_k N_A = M a_G \qquad N_A + N_B - M g = 0 \qquad -M g a + N_B(a + b) = -M a_G h_1$$

$$\begin{pmatrix} a_{Gl} \\ N_A \\ N_B \end{pmatrix} = \text{Find}(a_G, N_A, N_B) \qquad a_{Gl} = 1.38 \frac{\text{m}}{\text{s}^2}$$

Thus the advantage in the raised position is $\quad a_{Gr} - a_{Gl} = 0.03 \frac{\text{m}}{\text{s}^2}$

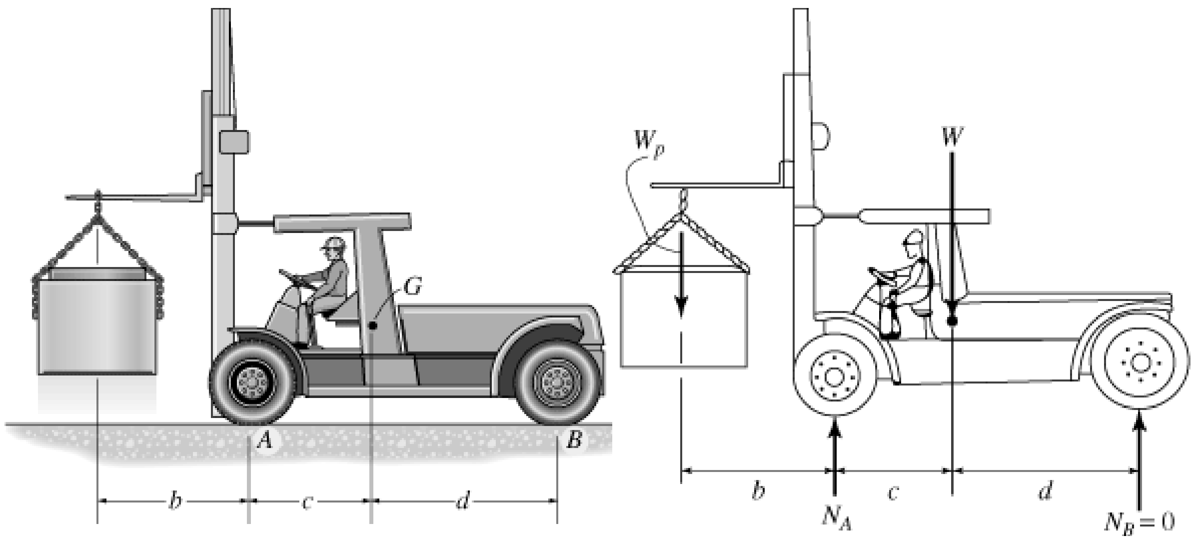
Problem 17-43

The forklift and operator have combined weight W and center of mass at G. If the forklift is used to lift the concrete pipe of weight W_p determine the maximum vertical acceleration it can give to the pipe so that it does not tip forward on its front wheels.

Given:

$W = 10000\ \text{lb}$ $\quad b = 5\ \text{ft}$ $\quad d = 6\ \text{ft}$

$W_p = 2000\ \text{lb}$ $\quad c = 4\ \text{ft}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



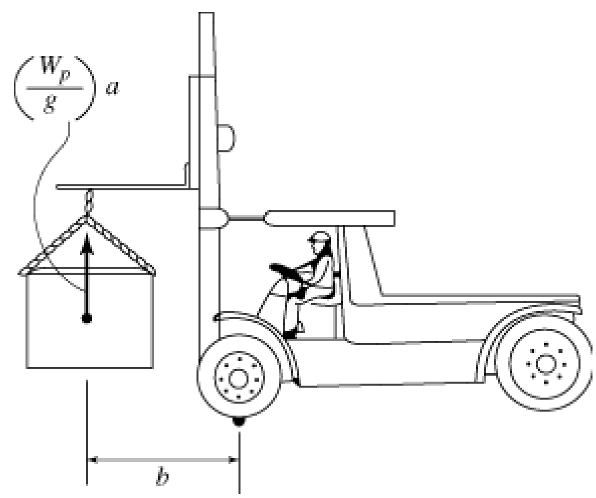
Solution:

It is required that $N_B = 0$

$$W_p b - W c = -\frac{W_p}{g} a b$$

$$a = \left(\frac{W c - W_p b}{W_p}\right)\frac{g}{b}$$

$$a = 96.6\ \frac{\text{ft}}{\text{s}^2}$$



***Problem 17-44**

The forklift and operator have combined weight W and center of mass at G. If the forklift is used to lift the concrete pipe of weight W_p determine the normal reactions on each of its four wheels if the pipe is given upward acceleration a.

Units Used:

$$\text{kip} = 10^3\ \text{lb}$$

Given:

$$W_p = 2000\ \text{lb}$$

$$W = 10000\ \text{lb}$$

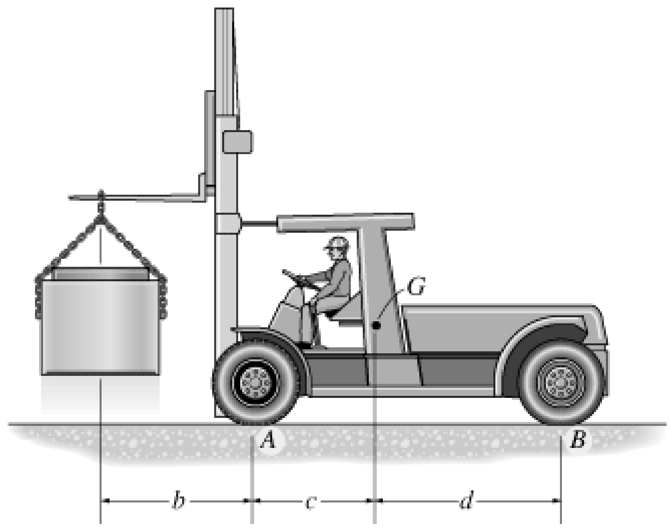
$$a = 4\ \frac{\text{ft}}{\text{s}^2}$$

$$b = 5\ \text{ft}$$

$$c = 4\ \text{ft}$$

$$d = 6\ \text{ft}$$

$$g = 9.81\ \frac{\text{m}}{\text{s}^2}$$



Solution:

Guesses

$$N_A = 1\ \text{lb}$$

$$N_B = 1\ \text{lb}$$

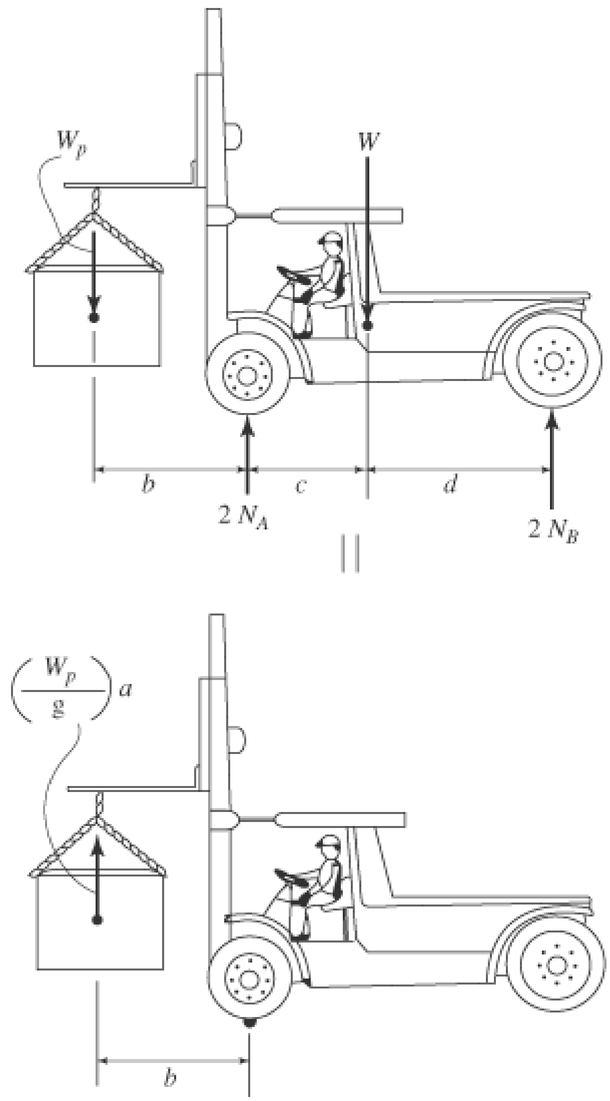
Given

$$2N_A + 2N_B - W - W_p = \left(\frac{W_p}{g}\right)a$$

$$2N_A b - W(b+c) + 2N_B(b+c+d) = 0$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \text{Find}(N_A, N_B)$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 4.686 \\ 1.438 \end{pmatrix}\text{kip}$$



Problem 17-45

The van has weight W_v and center of gravity at G_v. It carries fixed load W_l which has center of gravity at G_l. If the van is traveling at speed v, determine the distance it skids before stopping. The brakes cause *all* the wheels to lock or skid. The coefficient of kinetic friction between the wheels and the pavement is μ_k. Compare this distance with that of the van being empty. Neglect the mass of the wheels.

Given:

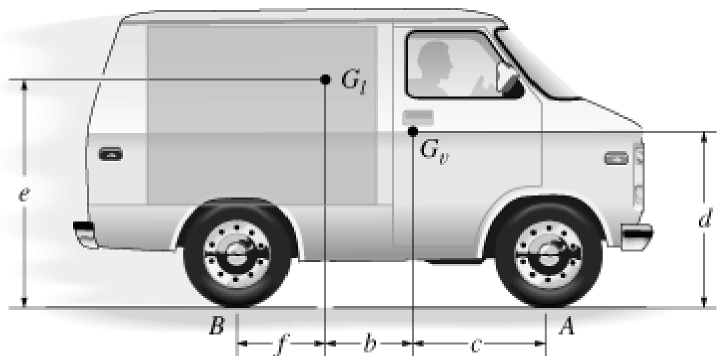
$W_v = 4500\ \text{lb}$ $\quad b = 2\ \text{ft}$

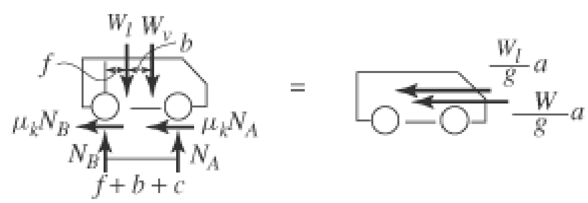
$W_l = 800\ \text{lb}$ $\quad c = 3\ \text{ft}$

$v = 40\ \dfrac{\text{ft}}{\text{s}}$ $\quad d = 4\ \text{ft}$

$\mu_k = 0.3$ $\quad e = 6\ \text{ft}$

$f = 2\ \text{ft}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution: Loaded

Guesses $\quad N_A = 1\ \text{lb} \quad N_B = 1\ \text{lb} \quad a = 1\ \dfrac{\text{ft}}{\text{s}^2}$

Given

$$N_A + N_B - W_v - W_l = 0$$

$$\mu_k(N_A + N_B) = \left(\frac{W_v + W_l}{g}\right)a$$

$$-N_B(f + b + c) + W_l(b + c) + W_v c = \left(\frac{W_l}{g}\right)a\,e + \left(\frac{W_v}{g}\right)a\,d$$

$$\begin{pmatrix} N_A \\ N_B \\ a \end{pmatrix} = \text{Find}(N_A, N_B, a) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 3777 \\ 1523 \end{pmatrix}\text{lb}$$

$$a = 9.66\,\frac{\text{ft}}{\text{s}^2} \qquad d_l = \frac{v^2}{2a} \qquad d_l = 82.816\,\text{ft}$$

Unloaded $\quad W_l = 0\ \text{lb}$

Guesses $\quad N_A = 1\ \text{lb} \quad N_B = 1\ \text{lb} \quad a = 1\ \dfrac{\text{ft}}{\text{s}^2}$

Given $\quad N_A + N_B - W_v - W_l = 0$

$$\mu_k(N_A + N_B) = \left(\frac{W_v + W_l}{g}\right)a$$

$$-N_B(f + b + c) + W_l(b + c) + W_v c = \left(\frac{W_l}{g}\right)a\,e + \left(\frac{W_v}{g}\right)a\,d$$

$$\begin{pmatrix} N_A \\ N_B \\ a \end{pmatrix} = \text{Find}(N_A, N_B, a) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 3343 \\ 1157 \end{pmatrix} \text{lb}$$

$$a = 9.66 \frac{\text{ft}}{\text{s}^2} \qquad d_{ul} = \frac{v^2}{2a}$$

$$d_{ul} = 82.816 \text{ ft}$$

The distance is the same in both cases although the forces on the tires are different.

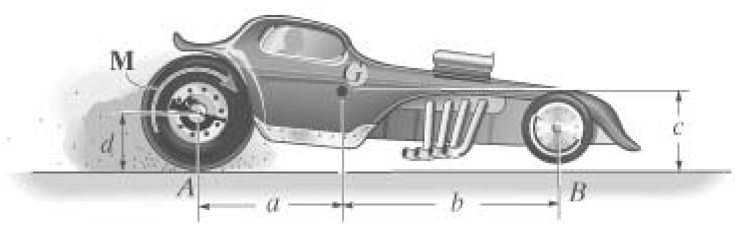
Problem 17-46

The "muscle car" is designed to do a "wheeley", i.e., to be able to lift its front wheels off the ground in the manner shown when it accelerates. If the car of mass M_1 has a center of mass at G, determine the minimum torque that must be developed at both rear wheels in order to do this. Also, what is the smallest necessary coefficient of static friction assuming the thick-walled rear wheels do not slip on the pavement? Neglect the mass of the wheels.

Units Used:

$\text{Mg} = 10^3 \text{ kg}$

$\text{kN} = 10^3 \text{ N}$



Given:

$M_1 = 1.35 \text{ Mg}$

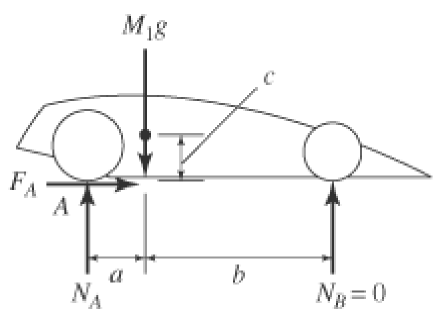
$a = 1.10 \text{ m}$

$b = 1.76 \text{ m}$

$c = 0.67 \text{ m}$

$d = 0.31 \text{ m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$



Solution:

Guesses $\quad a_G = 1 \frac{\text{m}}{\text{s}^2} \qquad F_A = 1 \text{ N} \qquad N_A = 1 \text{ N} \qquad M = 1 \text{ N m} \qquad \mu_s = 0.1$

Given

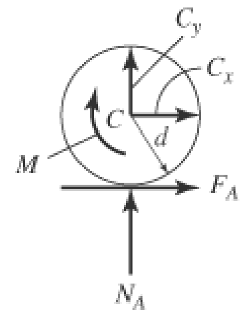
$F_A = M_1 a_G$

$N_A - M_1 g = 0$

$M_1 g a = M_1 a_G c$

$-M + F_A d = 0$

$F_A = \mu_s N_A$



$$\begin{pmatrix} a_G \\ F_A \\ N_A \\ M \\ \mu_s \end{pmatrix} = \text{Find}(a_G, F_A, N_A, M, \mu_s)$$

$$\begin{pmatrix} F_A \\ N_A \end{pmatrix} = \begin{pmatrix} 21.7 \\ 13.2 \end{pmatrix} \text{kN} \qquad M = 6.74 \text{ kN·m}$$

$$a_G = 16.11 \frac{\text{m}}{\text{s}^2} \qquad \mu_s = 1.642$$

Problem 17-47

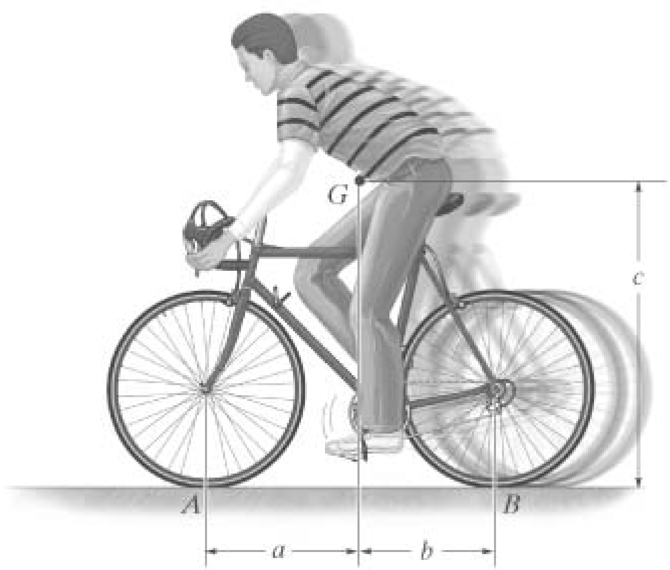
The bicycle and rider have a mass M with center of mass located at G. If the coefficient of kinetic friction at the rear tire is μ_B, determine the normal reactions at the tires A and B, and the deceleration of the rider, when the rear wheel locks for braking. What is the normal reaction at the rear wheel when the bicycle is traveling at constant velocity and the brakes are not applied? Neglect the mass of the wheels.

Given:

$M = 80$ kg $\quad \mu_B = 0.8$

$a = 0.55$ m $\quad b = 0.4$ m

$c = 1.2$ m



Solution: Deceleration:

Guesses $\quad a_G = 1 \dfrac{\text{m}}{\text{s}^2}$

$N_B = 1$ N $\quad N_A = 1$ N

Given $\quad \mu_B N_B = M a_G \qquad N_A + N_B - M g = 0$

$-N_B(a + b) + M g a = M a_G c$

$$\begin{pmatrix} a_G \\ N_B \\ N_A \end{pmatrix} = \text{Find}(a_G, N_B, N_A) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 559 \\ 226 \end{pmatrix} \text{N} \qquad a_G = 2.26 \ \frac{\text{m}}{\text{s}^2}$$

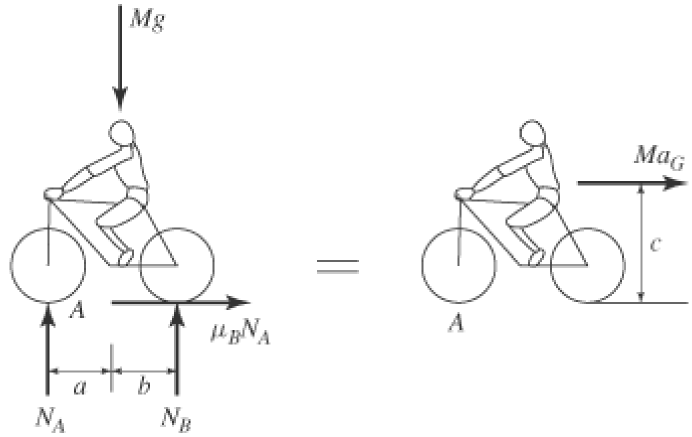
Equilibrium

Given $N_A + N_B - M g = 0$

$$-N_B(a + b) + M g a = 0$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \text{Find}(N_A, N_B)$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 330 \\ 454 \end{pmatrix} \text{N}$$



***Problem 17-48**

The bicycle and rider have a mass M with center of mass located at G. Determine the minimum coefficient of kinetic friction between the road and the wheels so that the rear wheel B starts to lift off the ground when the rider applies the brakes to the front wheel. Neglect the mass of the wheels.

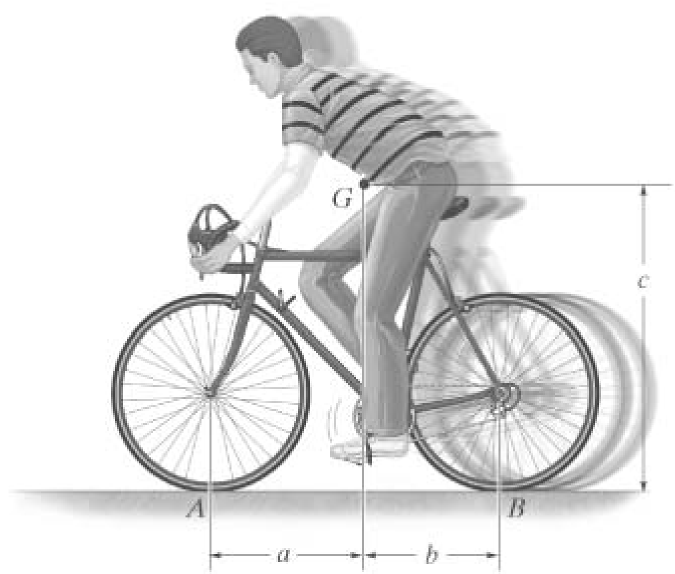
Given:

$M = 80$ kg

$a = 0.55$ m

$b = 0.4$ m

$c = 1.2$ m



Solution: $N_B = 0$

Guesses $a_G = 1 \ \frac{\text{m}}{\text{s}^2}$

$\mu_k = 0.1$

$N_A = 1$ N

Given $\mu_k N_A = M a_G$

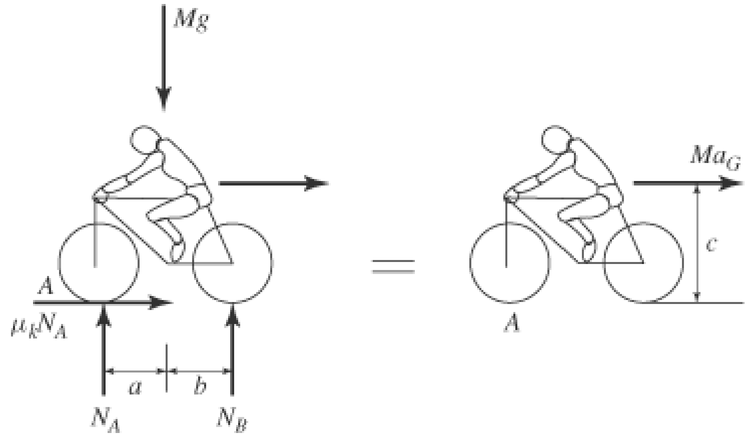
$$N_A - M g = 0$$

$$M g a = M a_G c$$

$$\begin{pmatrix} a_G \\ \mu_k \\ N_A \end{pmatrix} = \text{Find}(a_G, \mu_k, N_A)$$

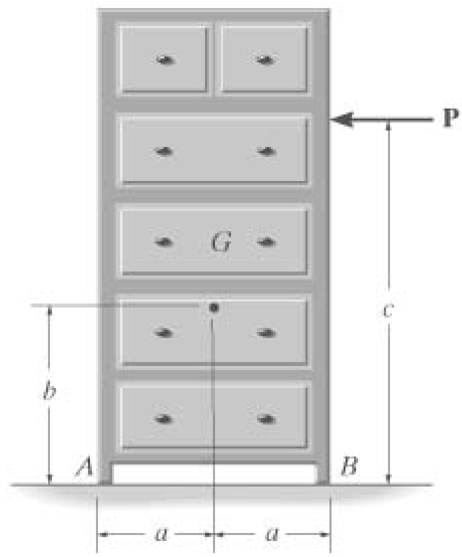
$N_A = 785 \text{ N}$

$a_G = 4.50 \dfrac{\text{m}}{\text{s}^2}$ $\quad$ $\mu_k = 0.458$



Problem 17-49

The dresser has a weight W and is pushed along the floor. If the coefficient of static friction at A and B is μ_s and the coefficient of kinetic friction is μ_k, determine the smallest horizontal force P needed to cause motion. If this force is increased slightly, determine the acceleration of the dresser. Also, what are the normal reactions at A and B when it begins to move?



Given:

$W = 80 \text{ lb}$

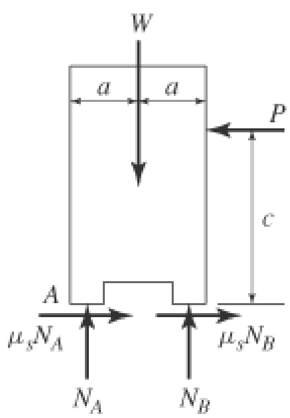
$\mu_s = 0.3$

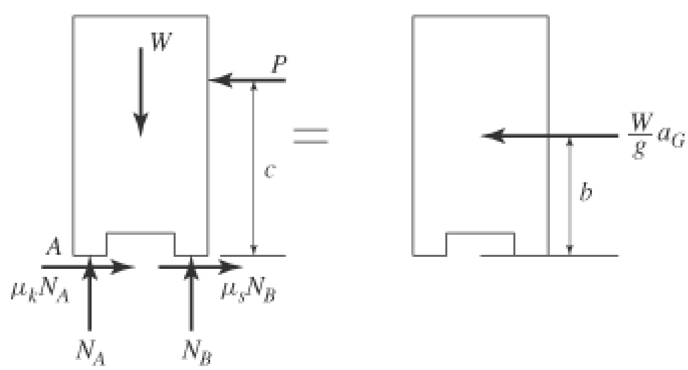
$\mu_k = 0.2$

$a = 1.5 \text{ ft}$

$b = 2.5 \text{ ft}$

$c = 4\text{ft}$





Solution: Impending Motion

Guesses $\quad P = 1 \text{ lb}$

$N_A = 1 \text{ lb} \quad N_B = 1 \text{ lb}$

Given $\quad N_A + N_B - W = 0$

$\mu_s N_A + \mu_s N_B - P = 0$

$P c + W a - N_A(2a) = 0$

$$\begin{pmatrix} P \\ N_A \\ N_B \end{pmatrix} = \text{Find}(P, N_A, N_B) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 72 \\ 8 \end{pmatrix} \text{lb} \qquad P = 24\,\text{lb}$$

Motion Guesses $a_G = 1\ \frac{\text{ft}}{\text{s}^2}$

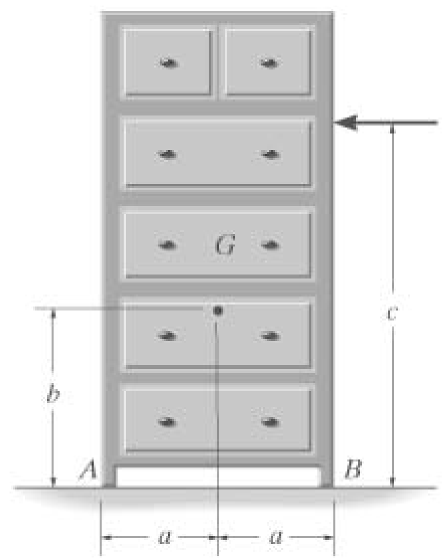
Given $N_A + N_B - W = 0 \qquad \mu_k N_A + \mu_k N_B - P = \left(\frac{-W}{g}\right) a_G$

$$P(c - b) + \mu_k (N_A + N_B) b + N_B a - N_A a = 0$$

$$\begin{pmatrix} N_A \\ N_B \\ a_G \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 65.3 \\ 14.7 \end{pmatrix} \text{lb} \qquad a_G = 3.22\ \frac{\text{ft}}{\text{s}^2}$$

Problem 17-50

The dresser has a weight W and is pushed along the floor. If the coefficient of static friction at A and B is μ_s and the coefficient of kinetic friction is μ_k, determine the maximum horizontal force P that can be applied without causing the dresser to tip over.



Given:

$W = 80\ \text{lb}$

$\mu_s = 0.3$

$\mu_k = 0.2$

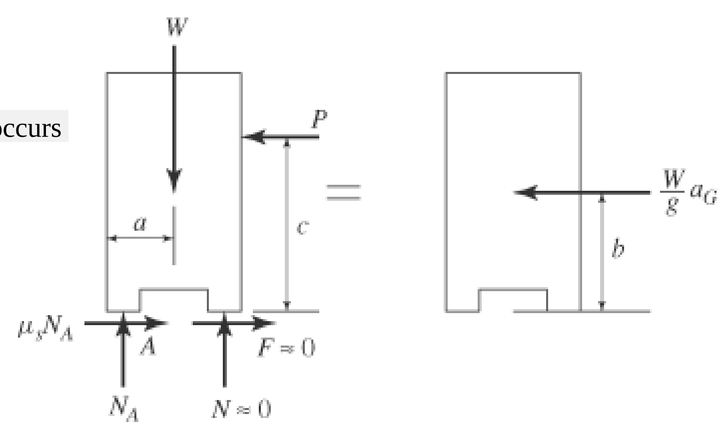
$a = 1.5\ \text{ft}$

$b = 2.5\ \text{ft}$

$c = 4\ \text{ft}$

Solution:

Dresser slides before tipping occurs



Guesses

$N_A = 1\ \text{lb}$

$P = 1\ \text{lb}$

$a_G = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$N_A - W = 0 \qquad \mu_k N_A - P = \left(\frac{-W}{g}\right)a_G \qquad P(c-b) - N_A a + \mu_k N_A b = 0$$

$$\begin{pmatrix} N_A \\ P \\ a_G \end{pmatrix} = \text{Find}(N_A, P, a_G) \qquad a_G = 15.02\,\frac{\text{ft}}{\text{s}^2} \qquad N_A = 80\,\text{lb} \qquad P = 53.3\,\text{lb}$$

Problem 17-51

The crate C has weight W and rests on the truck elevator for which the coefficient of static friction is μ_s. Determine the largest initial angular acceleration α starting from rest, which the parallel links AB and DE can have without causing the crate to slip. No tipping occurs.

Given:

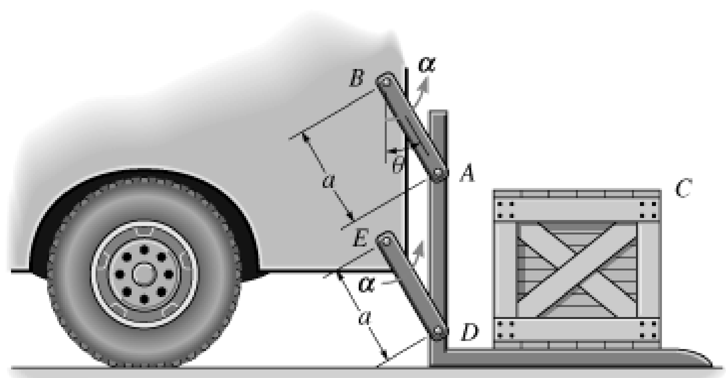
$W = 150$ lb

$\mu_s = 0.4$

$a = 2$ ft

$\theta = 30$ deg

$g = 32.2\,\frac{\text{ft}}{\text{s}^2}$



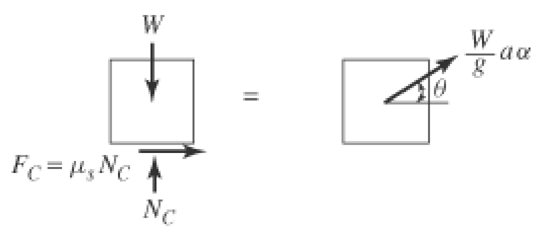
Solution:

Initial Guesses: $N_C = 1$ lb $\qquad \alpha = 1\,\frac{\text{rad}}{\text{s}^2}$

Given

$$\mu_s N_C = \left(\frac{W}{g}\right)\alpha a \cos(\theta)$$

$$N_C - W = \left(\frac{W}{g}\right)\alpha(a)\sin(\theta)$$



$$\begin{pmatrix} N_C \\ \alpha \end{pmatrix} = \text{Find}(N_C, \alpha) \qquad N_C = 195.043\,\text{lb} \qquad \alpha = 9.669\,\frac{\text{rad}}{\text{s}^2}$$

***Problem 17-52**

The two rods EF and HI each of weight W are fixed (welded) to the link AC at E. Determine the normal force N_E, shear force V_E, and moment M_E, which the bar AC exerts on FE at E if at the instant θ link AB has an angular velocity ω and an angular acceleration α as shown.

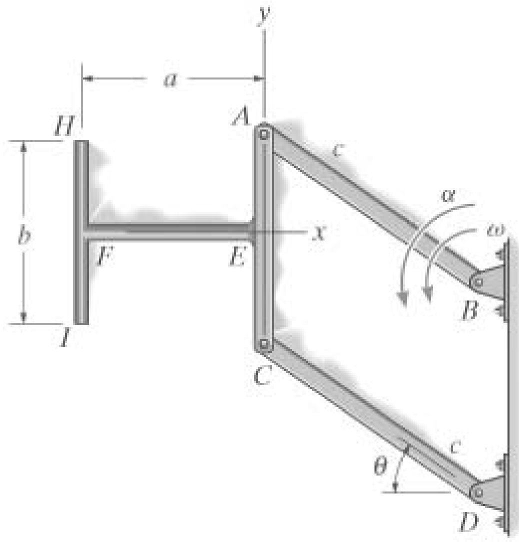
Given:

$W = 3\ \text{lb}$ $\quad a = 2\ \text{ft}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$\theta = 30\ \text{deg}$ $\quad b = 2\ \text{ft}$

$\omega = 5\ \dfrac{\text{rad}}{\text{s}}$ $\quad c = 3\ \text{ft}$

$\alpha = 8\ \dfrac{\text{rad}}{\text{s}^2}$ $\quad d = 3\ \text{ft}$

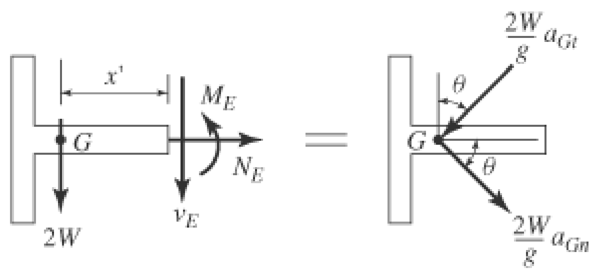


Solution:

$$x' = \frac{W a + W\left(\frac{a}{2}\right)}{2W}$$

$$a_{Gn} = c\omega^2$$

$$a_{Gt} = c\alpha$$



Guesses

$V_E = 1\ \text{lb}$

$N_E = 1\ \text{lb}$

$M_E = 1\ \text{lb}\cdot\text{ft}$

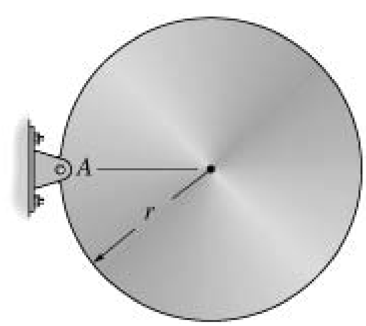
Given $\quad -2W - V_E = -2\left(\dfrac{W}{g}\right)a_{Gt}\cos(\theta) - 2\left(\dfrac{W}{g}\right)a_{Gn}\sin(\theta)$

$$N_E = 2\left(\frac{W}{g}\right)a_{Gn}\cos(\theta) - 2\left(\frac{W}{g}\right)a_{Gt}\sin(\theta) \qquad M_E - V_E x' = 0$$

$$\begin{pmatrix} V_E \\ N_E \\ M_E \end{pmatrix} = \text{Find}(V_E, N_E, M_E) \qquad \begin{pmatrix} N_E \\ V_E \end{pmatrix} = \begin{pmatrix} 9.87 \\ 4.86 \end{pmatrix} \text{lb} \qquad M_E = 7.29\ \text{lb}\cdot\text{ft}$$

Problem 17-53

The disk of mass M is supported by a pin at A. If it is released from rest from the position shown, determine the initial horizontal and vertical components of reaction at the pin.

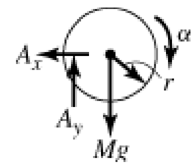


Given:

$M = 80\ \text{kg}$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$r = 1.5\ \text{m}$

Solution: Guesses $A_x = 1\ \text{N}$ $A_y = 1\ \text{N}$ $\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$



Given $M g r = \dfrac{3}{2} M r^2 \alpha$ $-A_x = 0$ $A_y - M g = -M r \alpha$

$$\begin{pmatrix} A_x \\ A_y \\ \alpha \end{pmatrix} = \text{Find}(A_x, A_y, \alpha) \qquad \alpha = 4.36\ \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 \\ 262 \end{pmatrix} \text{N}$$

Problem 17-54

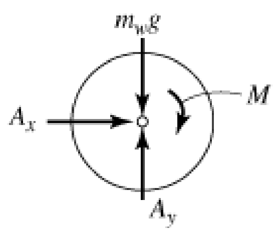
The wheel of mass m_w has a radius of gyration k_A. If the wheel is subjected to a moment $M = bt$, determine its angular velocity at time t starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.

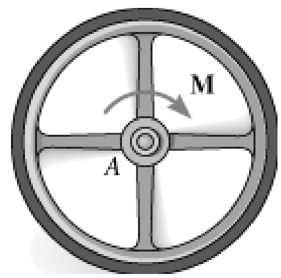
Given:

$m_w = 10\ \text{kg}$ $t = 3\ \text{s}$

$k_A = 200\ \text{mm}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$b = 5\ \text{N}\ \dfrac{\text{m}}{\text{s}}$





Solution:

$b t = m_w k_A^2 \alpha$ $\alpha = \dfrac{b t}{m_w k_A^2}$

$\omega = \dfrac{b t^2}{2 m_w k_A^2}$ $\omega = 56.2\ \dfrac{\text{rad}}{\text{s}}$

$A_x = 0\ \text{N}$ $A_y - m_w g = 0$

$A_y = m_w g$ $\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 \\ 98.1 \end{pmatrix} \text{N}$

Problem 17-55

The fan blade has mass m_b and a moment of inertia I_0 about an axis passing through its center O. If it is subjected to moment $M = A(1 - e^{bt})$ determine its angular velocity when $t = t_1$ starting from rest.

Given:

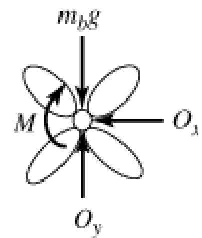
$m_b = 2\ \text{kg}$

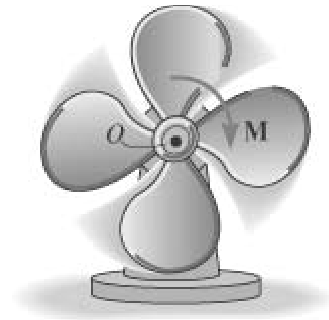
$I_O = 0.18\ \text{kg}\cdot\text{m}^2$

$A = 3\ \text{N}\cdot\text{m}$

$b = -0.2\ \text{s}^{-1}$

$t_1 = 4\ \text{s}$





Solution:

$$A\left(1 - e^{b\,t}\right) = I_O\alpha \qquad \alpha = \frac{A}{I_O}\left(1 - e^{b\,t_1}\right)$$

$$\omega = \frac{A}{I_O}\left(t_1 + \frac{1}{b} - \frac{1}{b}e^{b\,t_1}\right) \qquad \omega = 20.8\ \frac{\text{rad}}{\text{s}}$$

***Problem 17-56**

The rod of weight W is pin-connected to its support at A and has an angular velocity ω when it is in the horizontal position shown. Determine its angular acceleration and the horizontal and vertical components of reaction which the pin exerts on the rod at this instant.

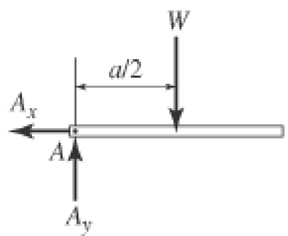
Given:

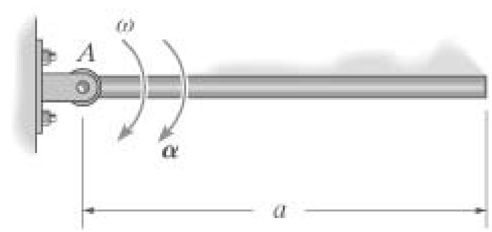
$\omega = 4\ \frac{\text{rad}}{\text{s}}$

$W = 10\ \text{lb}$

$a = 6\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

$$A_x = \left(\frac{W}{g}\right)\omega^2\left(\frac{a}{2}\right) \qquad A_x = 14.9\,\text{lb}$$

$$W\frac{a}{2} = \frac{1}{3}\left(\frac{W}{g}\right)a^2\alpha \qquad \alpha = \frac{3}{2a}g \qquad \alpha = 8.05\,\frac{\text{rad}}{\text{s}^2}$$

$$W - A_y = \left(\frac{W}{g}\right)\alpha\left(\frac{a}{2}\right)$$

$$A_y = W - \left(\frac{W}{g}\right)\alpha\left(\frac{a}{2}\right) \qquad A_y = 2.50\,\text{lb}$$

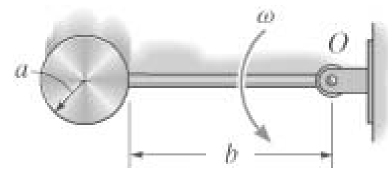
Problem 17-57

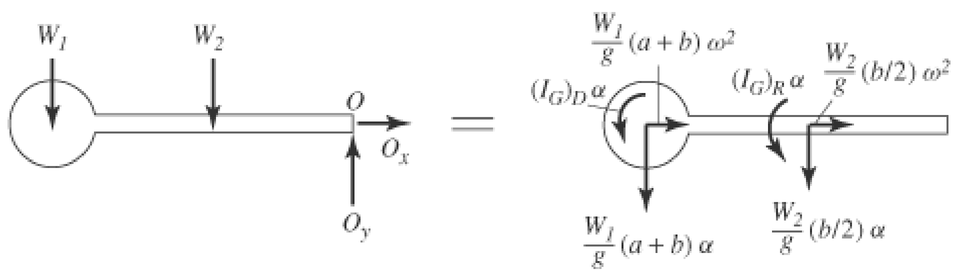
The pendulum consists of a disk of weight W_1 and a slender rod of weight W_2. Determine the horizontal and vertical components of reaction that the pin O exerts on the rod just as it passes the horizontal position, at which time its angular velocity is ω.

Given:

$W_1 = 15\ \text{lb}$ $\quad a = 0.75\ \text{ft}$ $\quad \omega = 8\ \dfrac{\text{rad}}{\text{s}}$

$W_2 = 10\ \text{lb}$ $\quad b = 3\ \text{ft}$





Solution: $$I_O = \frac{1}{2}\left(\frac{W_1}{g}\right)a^2 + \left(\frac{W_1}{g}\right)(a+b)^2 + \frac{1}{3}\left(\frac{W_2}{g}\right)b^2$$

Guesses $\quad \alpha = 10\ \dfrac{\text{rad}}{\text{s}^2} \quad O_x = 100\ \text{lb} \quad O_y = 5\ \text{lb}$

Given

$$O_y - W_1 - W_2 = -\frac{W_1}{g}(a+b)\alpha - \left(\frac{W_2}{g}\right)\frac{b}{2}\alpha$$

$$O_x = \left(\frac{W_1}{g}\right)(a+b)\,\omega^2 + \left(\frac{W_2}{g}\right)\left(\frac{b}{2}\right)\omega^2$$

$$W_1(a+b) + W_2\left(\frac{b}{2}\right) = I_O\alpha$$

$$\begin{pmatrix}\alpha\\ O_x\\ O_y\end{pmatrix} = \text{Find}(\alpha, O_x, O_y) \qquad \alpha = 9.36\ \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix}O_x\\ O_y\end{pmatrix} = \begin{pmatrix}141.61\\ 4.29\end{pmatrix}\text{lb}$$

Problem 17-58

The pendulum consists of a uniform plate of mass M_1 and a slender rod of mass M_2. Determine the horizontal and vertical components of reaction that the pin O exerts on the rod at the instant shown at which time its angular velocity is ω.

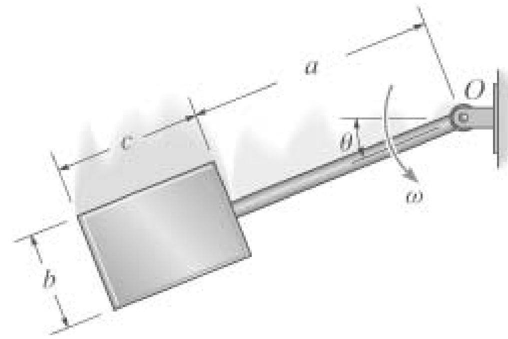
Given:

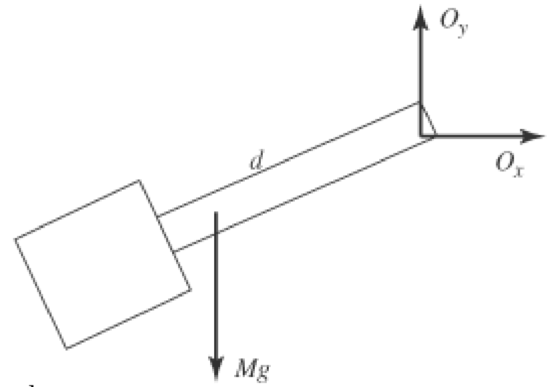
$M_1 = 5\ \text{kg}$ $\quad a = 0.5\ \text{m}$

$M_2 = 2\ \text{kg}$ $\quad b = 0.2\ \text{m}$

$\omega = 3\ \frac{\text{rad}}{\text{s}}$ $\quad c = 0.3\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$ $\quad \theta = 30\ \text{deg}$





Solution: $\quad M = M_1 + M_2$

$$I_O = \frac{1}{12}M_1\left(b^2 + c^2\right) + M_1\left(a + \frac{c}{2}\right)^2 + \frac{1}{3}M_2 a^2$$

$$d = \frac{M_1\left(a + \frac{c}{2}\right) + M_2\left(\frac{a}{2}\right)}{M}$$

Guesses $\quad O_x = 1\ \text{N} \quad O_y = 1\ \text{N} \quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

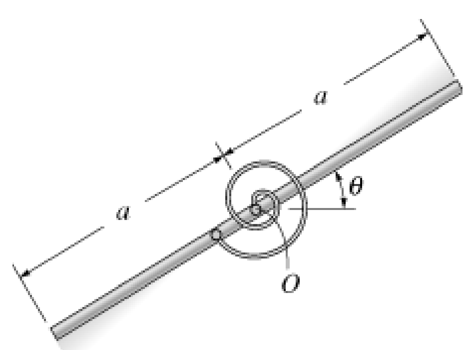
Given $\quad O_x = M d\alpha \sin(\theta) + M d\omega^2 \cos(\theta) \qquad M g d \cos(\theta) = I_O\alpha$

$$O_y - Mg = -M d\alpha \cos(\theta) + M d\omega^2 \sin(\theta)$$

$$\begin{pmatrix} O_x \\ O_y \\ \alpha \end{pmatrix} = \text{Find}\left(O_x, O_y, \alpha\right) \qquad \alpha = 13.65\ \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix} O_x \\ O_y \end{pmatrix} = \begin{pmatrix} 54.8 \\ 41.2 \end{pmatrix}\ \text{N}$$

Problem 17-59

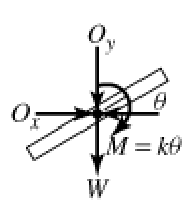
The bar of weight W is pinned at its center O and connected to a torsional spring. The spring has a stiffness k, so that the torque developed is $M = k\theta$. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$.



Given:

$W = 10\ \text{lb}$

$k = 5\ \frac{\text{lb}\cdot\text{ft}}{\text{rad}}$



$a = 1 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution:

$-k\theta = \frac{1}{12}\left(\frac{W}{g}\right)(2a)^2\alpha \qquad \alpha = \frac{-3kg}{Wa^2}\theta \qquad \frac{\omega^2}{2} - \frac{\omega_0^2}{2} = \frac{-3kg}{Wa^2}\left(\frac{\theta^2}{2} - \frac{\theta_0^2}{2}\right)$

$\omega = \sqrt{\frac{3kg}{Wa^2}\left(\frac{\pi}{2}\right)^2} \qquad \omega = 10.917 \frac{\text{rad}}{\text{s}}$

***Problem 17-60**

The bar of weight *w* is pinned at its center *O* and connected to a torsional spring. The spring has a stiffness *k*, so that the torque developed is $M = k\theta$. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = \theta_1$.

Given:

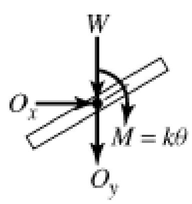
$W = 10 \text{ lb}$

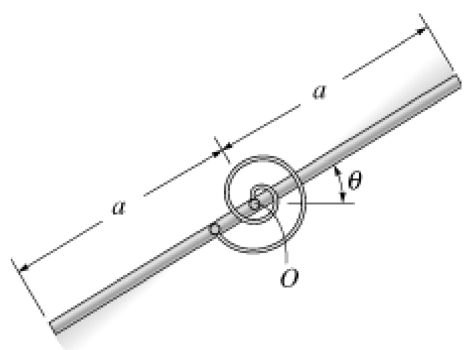
$k = 5 \frac{\text{lb}\cdot\text{ft}}{\text{rad}}$

$a = 1 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$\theta_1 = 45 \text{ deg}$





Solution:

$-k\theta = \frac{1}{12}\left(\frac{W}{g}\right)(2a)^2\alpha \qquad \alpha = \frac{-3kg}{Wa^2}\theta$

$\frac{\omega^2}{2} - \frac{\omega_0^2}{2} = \frac{-3kg}{Wa^2}\left(\frac{\theta^2}{2} - \frac{\theta_0^2}{2}\right)$

$\omega = \sqrt{\frac{3kg}{Wa^2}\left[(90 \text{ deg})^2 - \theta_1^2\right]} \qquad \omega = 9.454 \frac{\text{rad}}{\text{s}}$

Problem 17-61

The roll of paper of mass M has radius of gyration k_A about an axis passing through point A. It is pin-supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is μ_k and a vertical force F is applied to the end of the paper, determine the angular acceleration of the roll as the paper unrolls.

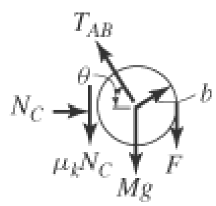
Given:

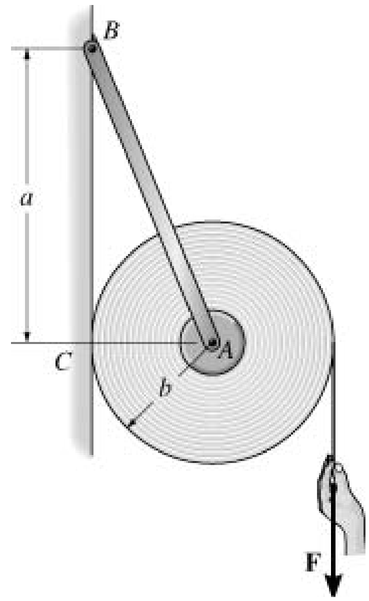
$M = 20\text{ kg}$ $a = 300\text{ mm}$

$k_A = 90\text{ mm}$ $b = 125\text{ mm}$

$\mu_k = 0.2$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$F = 30\text{ N}$





Solution: $\theta = \operatorname{atan}\left(\frac{a}{b}\right)$

Guesses $N_C = 1\text{ N}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $T_{AB} = 1\text{ N}$

Given $N_C - T_{AB}\cos(\theta) = 0$

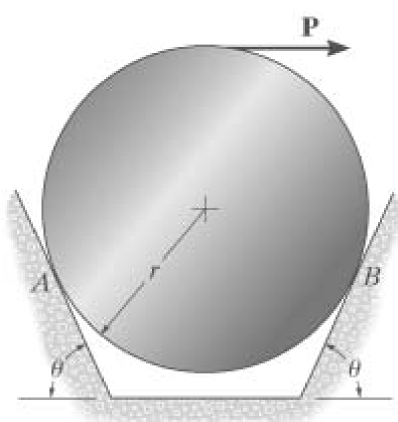
$T_{AB}\sin(\theta) - \mu_k N_C - M g - F = 0$

$F b - \mu_k N_C b = M k_A{}^2 \alpha$

$$\begin{pmatrix} N_C \\ \alpha \\ T_{AB} \end{pmatrix} = \text{Find}(N_C, \alpha, T_{AB}) \qquad \begin{pmatrix} N_C \\ T_{AB} \end{pmatrix} = \begin{pmatrix} 102.818 \\ 267.327 \end{pmatrix}\text{ N} \qquad \alpha = 7.281\ \frac{\text{rad}}{\text{s}^2}$$

Problem 17-62

The cylinder has a radius r and mass m and rests in the trough for which the coefficient of kinetic friction at A and B is μ_k. If a horizontal force **P** is applied to the cylinder, determine the cylinder's angular acceleration when it begins to spin.

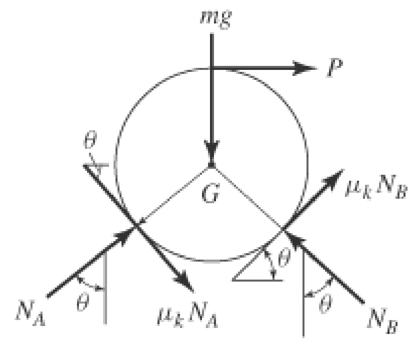


Solution:

$$P - (N_B - N_A)\sin(\theta) + \mu_k(N_A + N_B)\cos(\theta) = 0$$

$$(N_A + N_B)\cos(\theta) + \mu_k(N_B - N_A)\sin(\theta) - m g = 0$$

$$\left[\mu_k\left(N_A + N_B\right) - P\right]r = \frac{-1}{2}m r^2 \alpha$$



Solving

$$N_A + N_B = \frac{m g - \mu_k P}{\cos(\theta)\left(1 + \mu_k{}^2\right)}$$

$$N_B - N_A = \frac{\mu_k m g + P}{\sin(\theta)\left(1 + \mu_k{}^2\right)}$$

$$\alpha = \frac{-2\mu_k}{m r}\left[\frac{m g - \mu_k P}{\cos(\theta)\left(1 + \mu_k{}^2\right)}\right] + \frac{2P}{m r}$$

Problem 17-63

The uniform slender rod has a mass M. If the cord at A is cut, determine the reaction at the pin O, (a) when the rod is still in the horizontal position, and (b) when the rod swings to the vertical position.

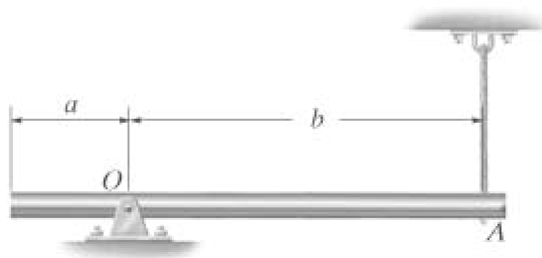
Given:

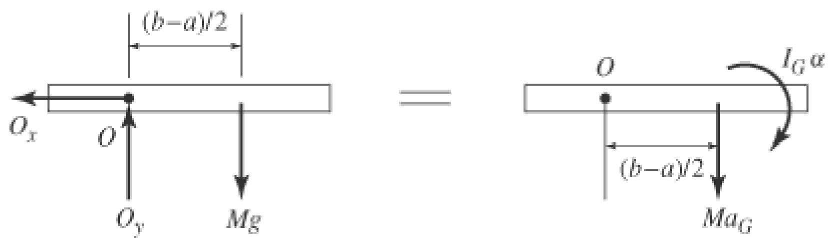
$M = 5\ \text{kg}$

$a = 200\ \text{mm}$

$b = 600\ \text{mm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$





Solution: $L = a + b$ $\quad d = \frac{b - a}{2}$ $\quad I_O = \frac{1}{12}M L^2 + M d^2$

(a) In the horizontal position

Guesses $a_G = 1\ \frac{\text{m}}{\text{s}^2}$ $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $\quad O_x = 1\ \text{N}$ $\quad O_y = 1\ \text{N}$

Given $a_G = \alpha d$ $\quad M g d = I_O\alpha$ $\quad -O_x = 0$ $\quad O_y - M g = -M a_G$

$$\begin{pmatrix} O_x \\ O_y \\ a_G \\ \alpha \end{pmatrix} = \text{Find}(O_x, O_y, a_G, \alpha) \quad a_G = 4.20\ \frac{\text{m}}{\text{s}^2} \quad \alpha = 21.0\ \frac{\text{rad}}{\text{s}^2} \quad \begin{pmatrix} O_x \\ O_y \end{pmatrix} = \begin{pmatrix} 0.0 \\ 28.0 \end{pmatrix} \text{N}$$

$$\left| \begin{pmatrix} O_x \\ O_y \end{pmatrix} \right| = 28.0\ \text{N}$$

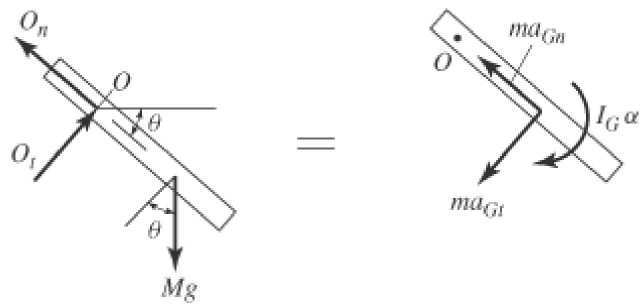
Next examine a general position

$$M g d \cos(\theta) = I_O \alpha$$

$$\alpha = \frac{M g d}{I_O} \cos(\theta)$$

$$\frac{\omega^2}{2} = \frac{M g d}{I_O} \sin(\theta)$$

$$\omega = \sqrt{\frac{2 M g d}{I_O}} \sin(\theta)$$



(b) In the vertical position (θ = 90 deg) $\quad \omega = \sqrt{\frac{2 M g d}{I_O}} \sin(90\ \text{deg})$

Guesses $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2} \quad O_x = 1\ \text{N} \quad O_y = 1\ \text{N}$

Given $\quad 0 = I_O \alpha \quad -O_x = -M \alpha d \quad O_y - M g = M d \omega^2$

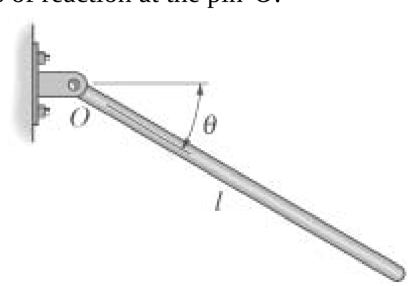
$$\begin{pmatrix} \alpha \\ O_x \\ O_y \end{pmatrix} = \text{Find}(\alpha, O_x, O_y) \quad \alpha = 0.0\ \frac{\text{rad}}{\text{s}^2} \quad \begin{pmatrix} O_x \\ O_y \end{pmatrix} = \begin{pmatrix} 0.0 \\ 91.1 \end{pmatrix} \text{N} \quad \left| \begin{pmatrix} O_x \\ O_y \end{pmatrix} \right| = 91.1\ \text{N}$$

***Problem 17-64**

The bar has a mass *m* and length *l*. If it is released from rest from the position shown, determine its angular acceleration and the horizontal and vertical components of reaction at the pin *O*.

Given:

$\theta = 30\ \text{deg}$

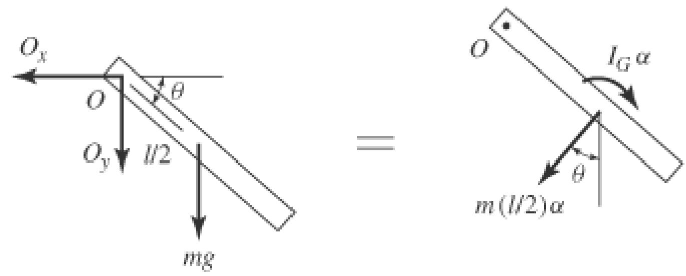


Solution:

$$m g \frac{l}{2} \cos(\theta) = \frac{1}{3} m l^2 \alpha$$

$$O_x = m\frac{l}{2}\alpha\sin(\theta)$$

$$O_y - mg = -m\frac{l}{2}\alpha\cos(\theta)$$



Solving

$$\alpha = \frac{3g}{2l}\cos(\theta) \qquad O_x = \frac{3mg}{8}\sin(2\theta) \qquad O_y = mg\left(1 - \frac{3}{4}\cos(\theta)^2\right)$$

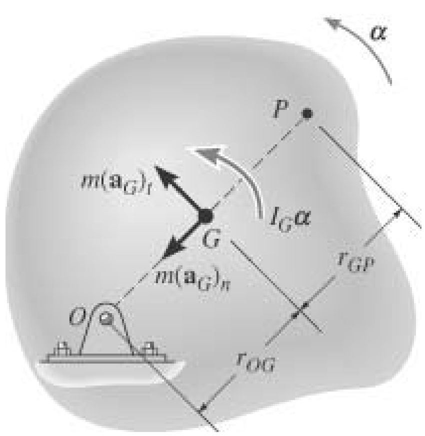
$$k_1 = \frac{3}{2}\cos(\theta) \qquad k_2 = \frac{3}{8}\sin(2\theta) \qquad k_3 = 1 - \frac{3}{4}\cos(\theta)^2$$

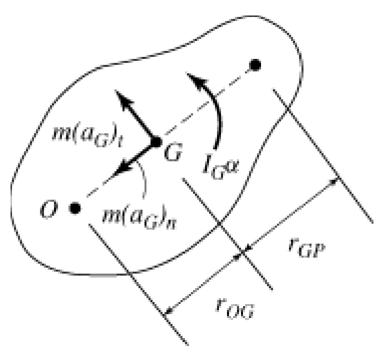
$$\alpha = k_1\frac{g}{l} \qquad O_x = k_2 mg \qquad O_y = k_3 mg$$

$$k_1 = 1.30 \qquad k_2 = 0.325 \qquad k_3 = 0.437$$

Problem 17-65

The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis at O is shown in the figure. Show that $I_G\alpha$ may be eliminated by moving the vectors $m(a_{Gt})$ and $m(a_{Gn})$ to point P, located a distance $r_{GP} = k_G^2/r_{OG}$ from the center of mass G of the body. Here k_G represents the radius of gyration of the body about G. The point P is called the *center of percussion* of the body.





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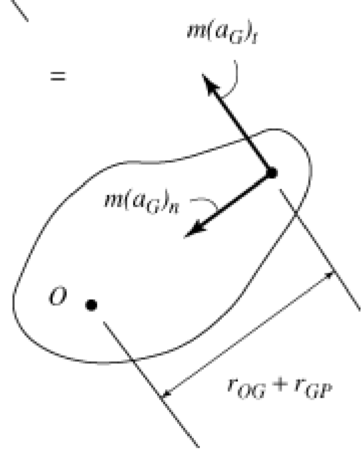


Solution:

$$I_G = mk_G^2 = mr_{GP}r_{OG} \qquad \alpha = \frac{a_{Gt}}{r_{OG}}$$

$$m\,a_{Gt}r_{OG} + I_G\alpha = m\,a_{Gt}r_{OG} + \left(m\,r_{OG}r_{GP}\right)\left(\frac{a_{Gt}}{r_{OG}}\right)$$

$$m\,a_{Gt}r_{OG} + I_G\alpha = m\,a_{Gt}\left(r_{OG} + r_{GP}\right)$$ Q.E.D.

Problem 17-66

Determine the position of the center of percussion P of the slender bar of weight W. (See Prob. 17-65.) What is the horizontal force at the pin when the bar is struck at P with force F?

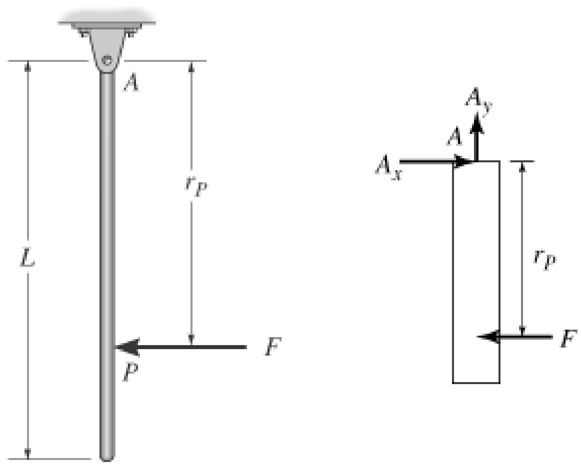
Given:

$W = 10\ \text{lb}$

$F = 20\ \text{lb}$

$L = 4\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution: $k_G = \dfrac{L}{\sqrt{12}}$

From Prob 17-65

$$r_P = \frac{L}{2} + \frac{k_G^{\ 2}}{\frac{L}{2}} \qquad r_P = 2.667\ \text{ft}$$

Guesses $A_x = 1\ \text{lb}$ $\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given $A_x - F = \left(\dfrac{-W}{g}\right)\alpha\left(\dfrac{L}{2}\right)$ $-F\,r_P = \dfrac{-1}{3}\left(\dfrac{W}{g}\right)L^2\alpha$

$$\begin{pmatrix} A_x \\ \alpha \end{pmatrix} = \text{Find}\left(A_x, \alpha\right) \qquad \alpha = 32.2\ \frac{\text{rad}}{\text{s}^2} \qquad A_x = 2.35 \times 10^{-14}\ \text{lb}$$

A zero horizontal force is the condition used to define the center of percussion.

Problem 17-67

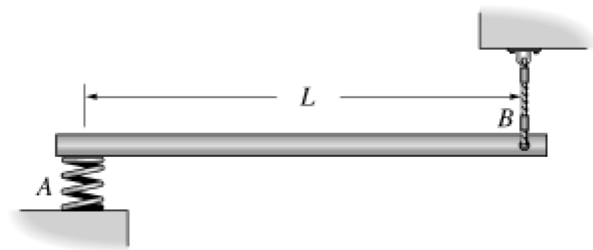
The slender rod of mass M is supported horizontally by a spring at A and a cord at B. Determine the angular acceleration of the rod and the acceleration of the rod's mass center at the instant the cord at B is cut. *Hint*: The stiffness of the spring is not needed for the calculation.

Given:

$M = 4\ \text{kg}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

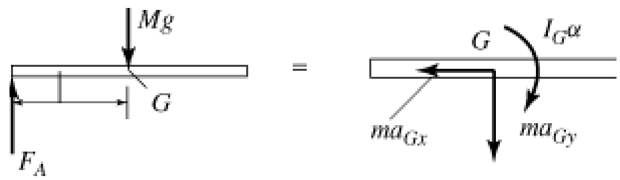
$L = 2\ \text{m}$



Solution:

Since the deflection of the spring is unchanged, we have

$F_A = \dfrac{Mg}{2}$



$F_A\dfrac{L}{2} = \dfrac{1}{12}ML^2\alpha$

$\alpha = \dfrac{6F_A}{ML}$ $\qquad \alpha = 14.7\ \dfrac{\text{rad}}{\text{s}^2}$

$F_A - Mg = -Ma_{Gy}$ $\qquad a_{Gy} = g - \dfrac{F_A}{M}$ $\qquad a_{Gy} = 4.91\ \dfrac{\text{m}}{\text{s}^2}$ $\qquad a_{Gx} = 0\ \dfrac{\text{m}}{\text{s}^2}$

***Problem 17-68**

In order to experimentally determine the moment of inertia I_G of a connecting rod of mass M, the rod is suspended horizontally at A by a cord and at B by a bearing and piezoelectric sensor, an instrument used for measuring force. Under these equilibrium conditions, the force at B is measured as F_1. If, at the instant the cord is released, the reaction at B is measured as F_2, determine the value of I_G. The support at B does not move when the measurement is taken. For the calculation, the horizontal location of G must be determined.

Given:

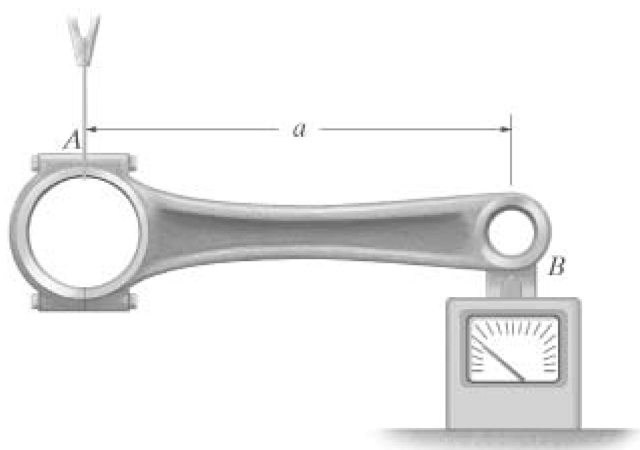
$M = 4\ \text{kg}$

$F_1 = 14.6\ \text{N}$

$F_2 = 9.3\ \text{N}$

$a = 350\ \text{mm}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

Guesses

$x = 1\ \text{mm}$ $\quad I_G = 1\ \text{kg}\cdot\text{m}^2$ $\quad A_y = 1\ \text{N}$ $\quad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

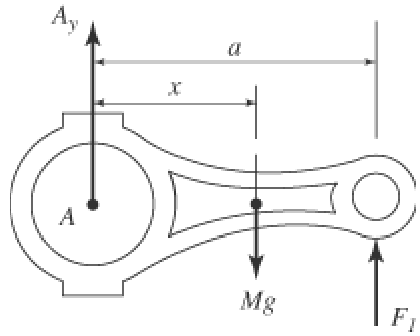
Given

$$A_y - M g + F_1 = 0$$

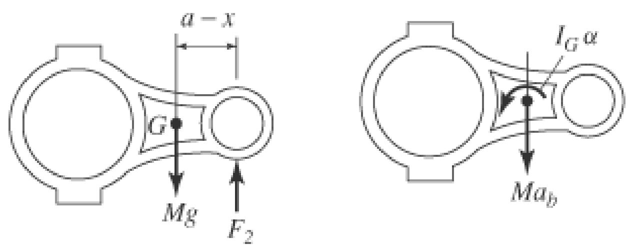
$$-M g x + F_1 a = 0$$

$$F_2 - M g = -M \alpha (a - x)$$

$$M g(a - x) = \left[I_G + M (a - x)^2\right]\alpha$$



$$\begin{pmatrix} x \\ I_G \\ A_y \\ \alpha \end{pmatrix} = \text{Find}\left(x, I_G, A_y, \alpha\right)$$



$x = 130\,\text{mm}$ $\qquad A_y = 24.6\ \text{N}$ $\qquad \alpha = 34.1\ \dfrac{\text{rad}}{\text{s}^2}$ $\qquad I_G = 0.0600\ \text{kg}\cdot\text{m}^2$

Problem 17-69

Disk *D* of weight *W* is subjected to counterclockwise moment *M* = *bt*. Determine the angular velocity of the disk at time *t* after the moment is applied. Due to the spring the plate *P* exerts constant force *P* on the disk. The coefficients of static and kinetic friction between the disk and the plate are μ_s and μ_k respectively. *Hint:* First find the time needed to start the disk rotating.

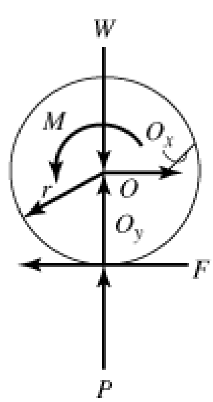
Given:

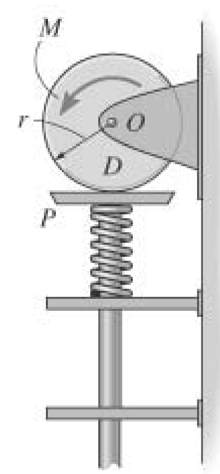
$W = 10\ \text{lb}$ $\qquad \mu_s = 0.3$

$b = 10\ \dfrac{\text{lb}\cdot\text{ft}}{\text{s}}$ $\qquad \mu_k = 0.2$

$t = 2\ \text{s}$ $\qquad r = 0.5\ \text{ft}$

$P = 100\ \text{lb}$ $\qquad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution: When motion begins

$$b t_1 = \mu_s P r \qquad t_1 = \frac{\mu_s P r}{b} \qquad t_1 = 1.5\ \text{s}$$

At a later time we have

$$b t - \mu_k P r = \frac{1}{2}\left(\frac{W}{g}\right) r^2 \alpha \qquad\qquad \alpha = \frac{2g}{W r^2}\left(b t - \mu_k P r\right)$$

$$\omega = \frac{2g}{W r^2}\left[\frac{b}{2}\left(t^2 - t_1^{\,2}\right) - \mu_k P r\left(t - t_1\right)\right] \qquad \omega = 96.6\,\frac{\text{rad}}{\text{s}}$$

Problem 17-70

The furnace cover has a mass M and a radius of gyration k_G about its mass center G. If an operator applies a force F to the handle in order to open the cover, determine the cover's initial angular acceleration and the horizontal and vertical components of reaction which the pin at A exerts on the cover at the instant the cover begins to open. Neglect the mass of the handle BAC in the calculation.

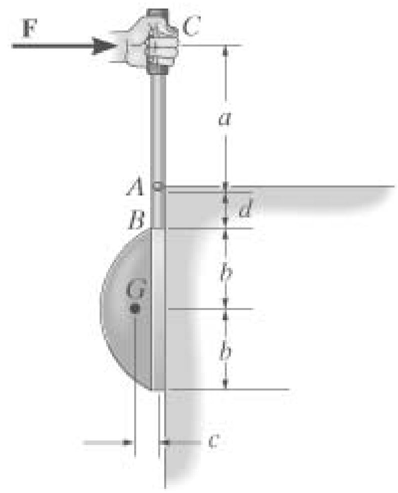
Given:

$M = 20\ \text{kg}$ $\quad a = 0.7\ \text{m}$

$k_G = 0.25\ \text{m}$ $\quad b = 0.4\ \text{m}$

$F = 120\ \text{N}$ $\quad c = 0.25\ \text{m}$

$d = 0.2\ \text{m}$



Solution: $\quad \theta = \operatorname{atan}\left(\frac{c}{b+d}\right)$

Guesses $\quad \alpha = 5\,\frac{\text{rad}}{\text{s}^2}$

$A_x = 50\ \text{N}$

$A_y = 20\ \text{N}$

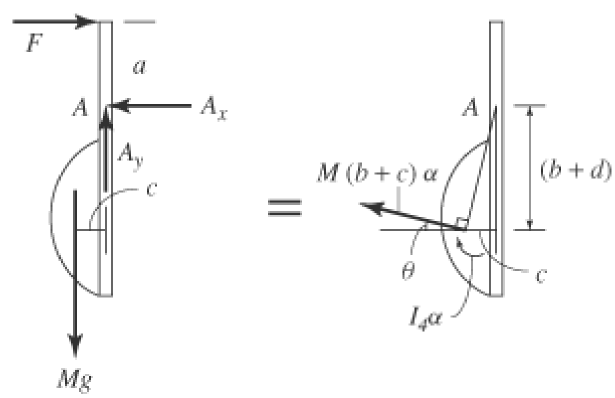
Given

$$A_x - F = M(c+b)\,\alpha\cos(\theta)$$

$$A_y - Mg = M(c+b)\,\alpha\sin(\theta)$$

$$F a - M g c = M c^2 \alpha + M (c+b)^2 \alpha$$

$$\begin{pmatrix}\alpha \\ A_x \\ A_y\end{pmatrix} = \text{Find}\left(\alpha, A_x, A_y\right)$$



$\alpha = 3.60\,\frac{\text{rad}}{\text{s}^2}$ $\qquad \begin{pmatrix}A_x \\ A_y\end{pmatrix} = \begin{pmatrix}163 \\ 214\end{pmatrix}\ \text{N}$

Problem 17-71

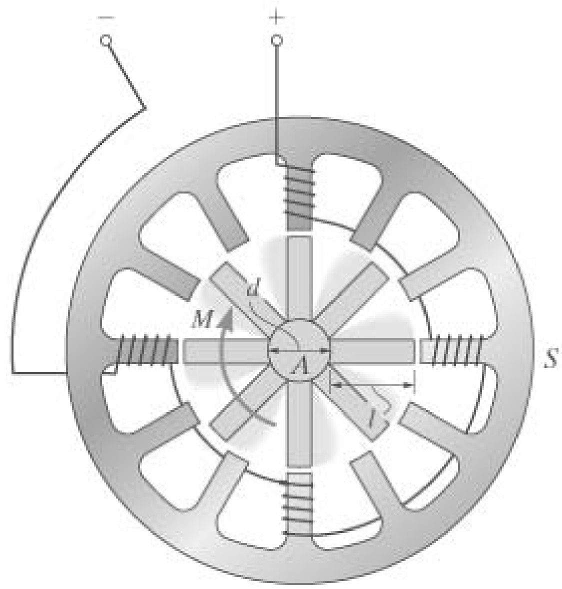
The variable-resistance motor is often used for appliances, pumps, and blowers. By applying a current through the stator S, an electromagnetic field is created that "pulls in" the nearest rotor poles. The result of this is to create a torque M about the bearing at A. If the rotor is made from iron and has a cylindrical core of mass M_1, diameter d and eight extended slender rods, each having a mass M_2 and length l, determine its angular velocity at time t starting from rest.

Given:

$M_1 = 3\ \text{kg}$ $\quad l = 100\text{mm}$

$M_2 = 1\ \text{kg}$ $\quad d = 50\ \text{mm}$

$M = 4\ \text{N}\cdot\text{m}$ $\quad t = 5\ \text{s}$



Solution:

$$I_A = \frac{1}{2}M_1\left(\frac{d}{2}\right)^2 + 8\left[\frac{1}{12}M_2 l^2 + M_2\left(\frac{d}{2}+\frac{l}{2}\right)^2\right]$$

$$M = I_A\alpha \qquad \alpha = \frac{M}{I_A}$$

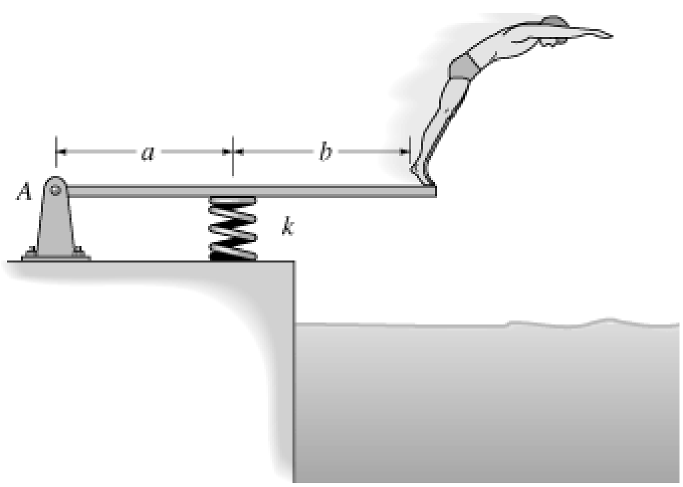
$$\omega = \alpha t \qquad \omega = 380\ \frac{\text{rad}}{\text{s}}$$

***Problem 17-72**

Determine the angular acceleration of the diving board of mass M and the horizontal and vertical components of reaction at the pin A the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount δ and the board is horizontal.

Units Used:

$\text{kN} = 10^3\ \text{N}$



Given:

$M = 25\ \text{kg}$

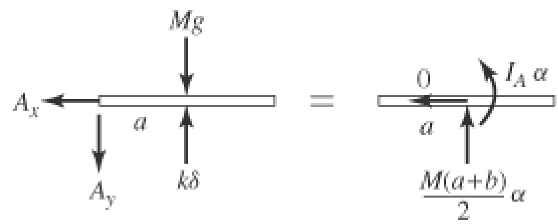
$\delta = 200\ \text{mm}$

$k = 7\ \frac{\text{kN}}{\text{m}}$

$a = 1.5\ \text{m}$

$b = 1.5\ \text{m}$

$g = 9.815\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

Guesses $A_x = 1\ \text{N}$ $A_y = 1\ \text{N}$ $\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given $-M\,g\,a + k\delta a = \dfrac{1}{3}M\,(a+b)^2\alpha$

$A_x = 0$ $-A_y - M\,g + k\delta = M\alpha\left(\dfrac{a+b}{2}\right)$

$\begin{pmatrix} A_x \\ A_y \\ \alpha \end{pmatrix} = \text{Find}(A_x, A_y, \alpha)$ $\alpha = 23.1\ \dfrac{\text{rad}}{\text{s}^2}$ $\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 \\ 289 \end{pmatrix}\ \text{N}$

Problem 17-73

The disk has mass M and is originally spinning at the end of the strut with angular velocity ω. If it is then placed against the wall, for which the coefficient of kinetic friction is μ_k, determine the time required for the motion to stop. What is the force in strut BC during this time?

Given:

$M = 20\ \text{kg}$

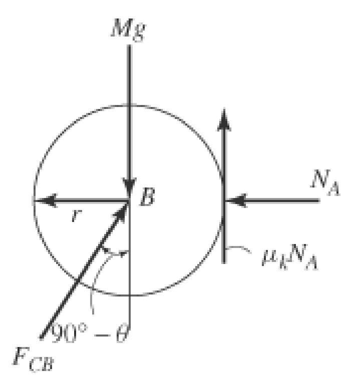
$\omega = 60\ \dfrac{\text{rad}}{\text{s}}$

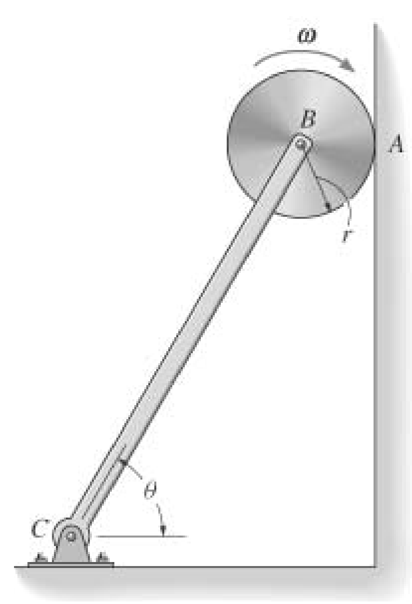
$\mu_k = 0.3$

$\theta = 60\ \text{deg}$

$r = 150\ \text{mm}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$





Solution:

Initial Guess:

$F_{CB} = 1\ \text{N}$ $\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$ $N_A = 1\ \text{N}$

Given $F_{CB}\cos(\theta) - N_A = 0$

$F_{CB}\sin(\theta) - M g + \mu_k N_A = 0$

$\mu_k N_A r = \frac{1}{2} M r^2 \alpha$

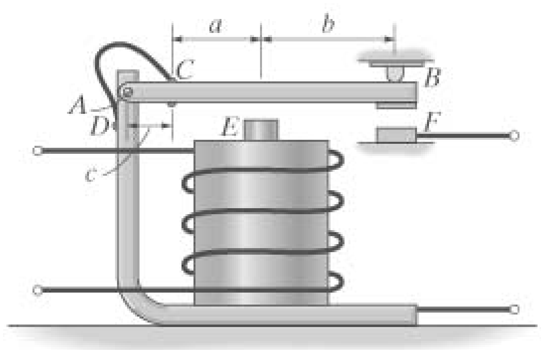
$\begin{pmatrix} F_{CB} \\ N_A \\ \alpha \end{pmatrix} = \text{Find}(F_{CB}, N_A, \alpha)$ $\alpha = 19.311 \frac{\text{rad}}{\text{s}^2}$ $N_A = 96.6 \text{ N}$ $F_{CB} = 193 \text{ N}$

$t = \frac{\omega}{\alpha}$ $t = 3.107 \text{ s}$

Problem 17-74

The relay switch consists of an electromagnet *E* and an armature *AB* (slender bar) of mass *M* which is pinned at *A* and lies in the vertical plane.When the current is turned off, the armature is held open against the smooth stop at *B* by the spring *CD*, which exerts an upward vertical force F_s on the armature at *C*. When the current is turned on, the electromagnet attracts the armature at *E* with a vertical force *F*. Determine the initial angular acceleration of the armature when the contact *BF* begins to close.

Given:



$M = 20 \text{ gm}$

$F = 0.8 \text{ N}$

$F_s = 0.85 \text{ N}$

$a = 20 \text{ mm}$

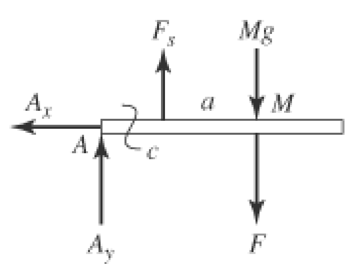
$b = 30 \text{ mm}$

$c = 10 \text{ mm}$

Solution:

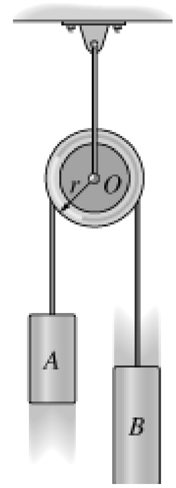
$$F_s c - M g\left(\frac{a+b+c}{2}\right) - F(a+c) = -\frac{1}{3} M (a+b+c)^2 \alpha$$

$$\alpha = 3\left[\frac{M g\left(\frac{a+b+c}{2}\right) + F(a+c) - F_s c}{M(a+b+c)^2}\right]$$



$\alpha = 891 \frac{\text{rad}}{\text{s}^2}$

Problem 17-75



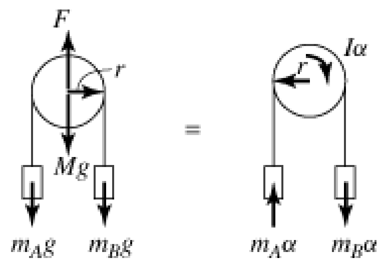
The two blocks A and B have a mass m_A and m_B, respectively, where $m_B > m_A$. If the pulley can be treated as a disk of mass M, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.

Solution:

$$a = \alpha r$$

$$m_B g r - m_A g r = \frac{1}{2} M r^2 \alpha + m_B r^2 \alpha + m_A r^2 \alpha$$

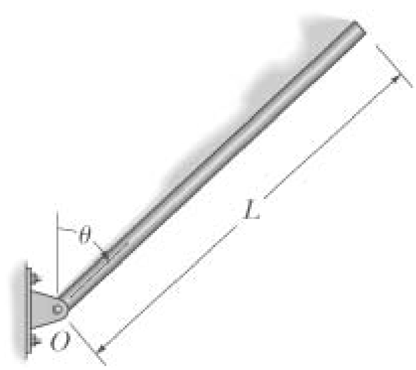
$$\alpha = \frac{g(m_B - m_A)}{r\left(\frac{1}{2}M + m_B + m_A\right)}$$



$$\boxed{a = \frac{g(m_B - m_A)}{\frac{1}{2}M + m_B + m_A}}$$

***Problem 17-76**

The rod has a length L and mass m. If it is released from rest when $\theta = 0°$, determine its angular velocity as a function of θ. Also, express the horizontal and vertical components of reaction at the pin O as a function of θ.



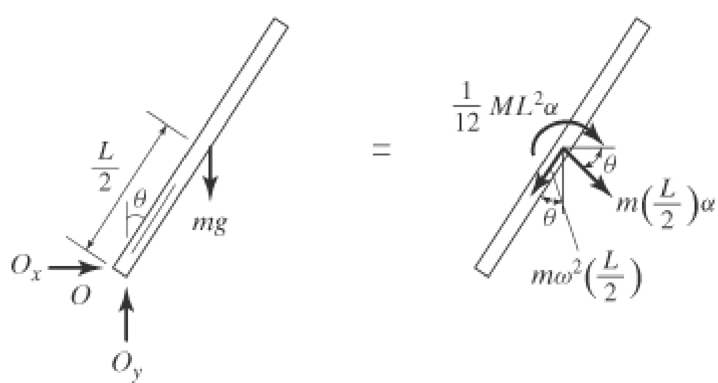
Solution:

$$m g \frac{L}{2} \sin(\theta) = \frac{1}{3} m L^2 \alpha$$

$$\alpha = \frac{3g}{2L} \sin(\theta)$$

$$\frac{\omega^2}{2} = \frac{3g}{2L}(1 - \cos(\theta))$$

$$\boxed{\omega = \sqrt{\frac{3g}{L}(1 - \cos(\theta))}}$$



$$O_x = m \frac{L}{2} \alpha \cos(\theta) - m \frac{L}{2} \omega^2 \sin(\theta)$$

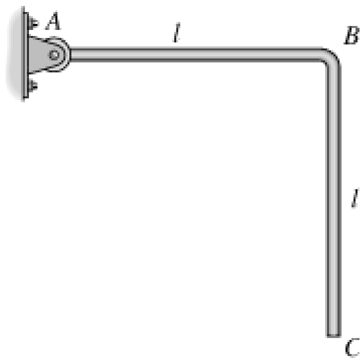
$$\boxed{O_x = m g \sin(\theta)\left(\frac{9}{4}\cos(\theta) - \frac{3}{2}\right)}$$

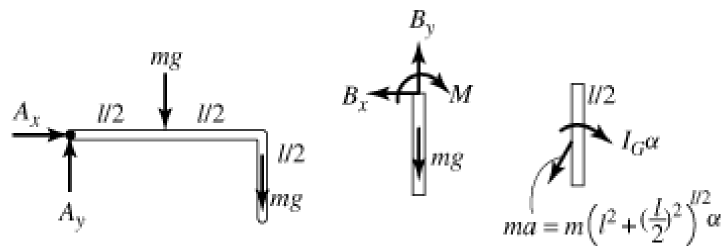
$$O_y - mg = -m\left(\frac{L}{2}\right)\alpha\sin(\theta) - m\left(\frac{L}{2}\right)\omega^2\cos(\theta)$$

$$O_y = mg\left(1 - \frac{3}{2}\cos(\theta) + \frac{3}{2}\cos(\theta)^2 - \frac{3}{4}\sin(\theta)^2\right)$$

Problem 17-77

The two-bar assembly is released from rest in the position shown. Determine the initial bending moment at the fixed joint *B*. Each bar has mass *m* and length *l*.





Solution:

$$I_A = \frac{1}{3}ml^2 + \frac{1}{12}ml^2 + m\left[l^2 + \left(\frac{l}{2}\right)^2\right] = \frac{5}{3}ml^2$$

$$mg\frac{l}{2} + mgl = I_A\alpha \qquad \alpha = \frac{9}{10}\frac{g}{l}$$

$$M = \frac{1}{12}ml^2\alpha + m\left(\frac{l}{2}\right)\alpha\left(\frac{l}{2}\right) = \frac{1}{3}ml^2\alpha \qquad M_A = \frac{3}{10}mgl$$

Problem 17-78

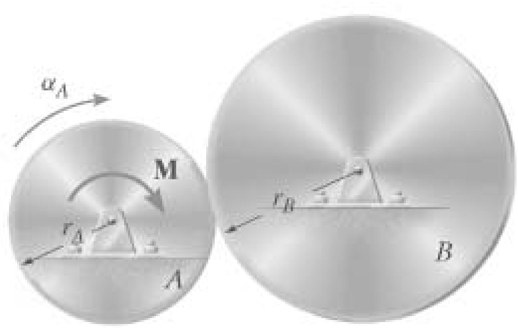
Disk *A* has weight W_A and disk *B* has weight W_B. If no slipping occurs between them, determine the couple moment *M* which must be applied to disk *A* to give it an angular acceleration α_A.

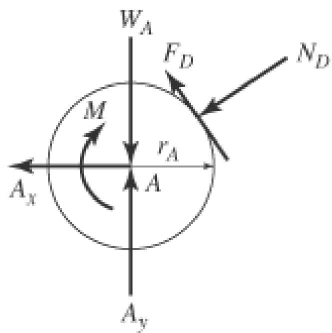
Given:

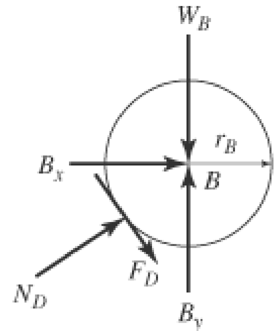
$W_A = 5$ lb $\quad r_A = 0.5$ ft

$W_B = 10$ lb $\quad r_B = 0.75$ ft

$\alpha_A = 4\ \frac{\text{rad}}{\text{s}^2}$







Solution:

Guesses $\alpha_B = 1\ \frac{\text{rad}}{\text{s}^2}$ $M = 1\ \text{lb}\cdot\text{ft}$ $F_D = 1\ \text{lb}$

Given

$$M - F_D r_A = \frac{1}{2}\left(\frac{W_A}{g}\right) r_A{}^2 \alpha_A \qquad F_D r_B = \frac{1}{2}\left(\frac{W_B}{g}\right) r_B{}^2 \alpha_B \qquad r_A\alpha_A = r_B\alpha_B$$

$$\begin{pmatrix} M \\ \alpha_B \\ F_D \end{pmatrix} = \text{Find}(M, \alpha_B, F_D) \qquad \alpha_B = 2.67\ \frac{\text{rad}}{\text{s}^2} \qquad F_D = 0.311\ \text{lb} \qquad M = 0.233\ \text{lb}\cdot\text{ft}$$

Problem 17-79

The wheel has mass M and radius of gyration k_B. It is originally spinning with angular velocity ω_1. If it is placed on the ground, for which the coefficient of kinetic friction is μ_c, determine the time required for the motion to stop. What are the horizontal and vertical components of reaction which the pin at A exerts on AB during this time? Neglect the mass of AB.

Given:

$M = 25\ \text{kg}$

$k_B = 0.15\ \text{m}$

$\omega_1 = 40\ \frac{\text{rad}}{\text{s}}$

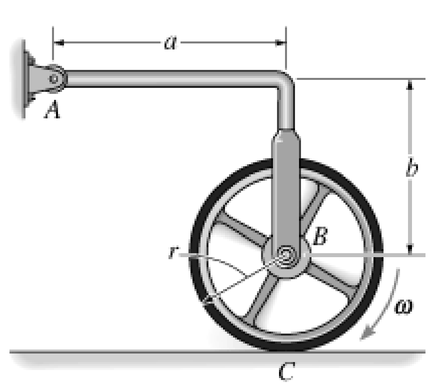
$\mu_C = 0.5$

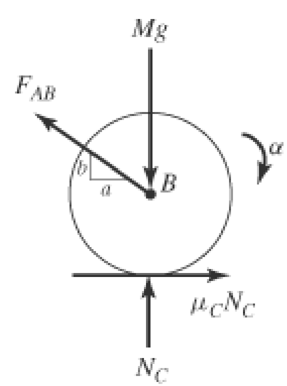
$a = 0.4\ \text{m}$

$b = 0.3\ \text{m}$

$r = 0.2\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$





Solution: Guesses $F_{AB} = 1\ \text{N}$ $\quad N_C = 1\ \text{N}$ $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given
$$\mu_C N_C - \left(\frac{a}{\sqrt{a^2+b^2}}\right) F_{AB} = 0$$

$$N_C - M g + \left(\frac{b}{\sqrt{a^2+b^2}}\right) F_{AB} = 0$$

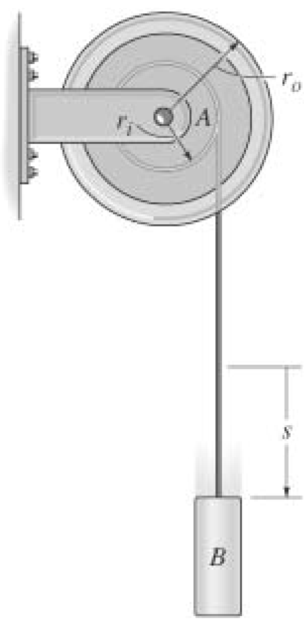
$$\mu_C N_C r = -M k_B^2 \alpha$$

$$\begin{pmatrix} F_{AB} \\ N_C \\ \alpha \end{pmatrix} = \text{Find}(F_{AB}, N_C, \alpha) \qquad \begin{pmatrix} F_{AB} \\ N_C \end{pmatrix} = \begin{pmatrix} 111.477 \\ 178.364 \end{pmatrix} \text{N} \qquad \alpha = -31.709\ \frac{\text{rad}}{\text{s}^2}$$

$$t = \frac{\omega_1}{-\alpha} \qquad t = 1.261\ \text{s}$$

$$\mathbf{F_A} = \frac{F_{AB}}{\sqrt{a^2+b^2}}\begin{pmatrix} a \\ b \end{pmatrix} \qquad \mathbf{F_A} = \begin{pmatrix} 89.2 \\ 66.9 \end{pmatrix} \text{N}$$

Problem 17-80



The cord is wrapped around the inner core of the spool. If block B of weight W_B is suspended from the cord and released from rest, determine the spool's angular velocity when $t = t_1$. Neglect the mass of the cord. The spool has weight W_S and the radius of gyration about the axle A is k_A. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.

Given:

$W_B = 5\ \text{lb}$

$t_1 = 3\ \text{s}$

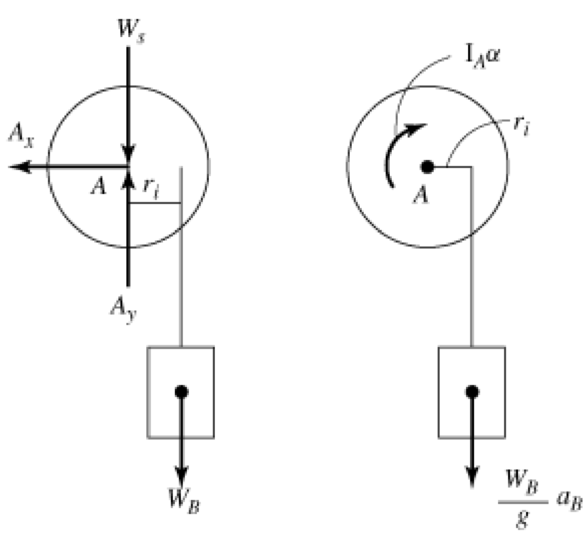
$W_S = 180\ \text{lb}$

$k_A = 1.25\ \text{ft}$

$r_i = 1.5\ \text{ft}$

$r_o = 2.75\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

(a) System as a whole

$$W_B r_i = \left(\frac{W_S}{g}\right) k_A{}^2 \alpha + \left(\frac{W_B}{g}\right)(r_i \alpha r_i) \qquad \alpha = \frac{W_B r_i g}{W_B r_i{}^2 + W_S k_A{}^2} \qquad \alpha = 0.826\ \frac{\text{rad}}{\text{s}^2}$$

$$\omega = \alpha t_1 \qquad \omega = 2.477\ \frac{\text{rad}}{\text{s}}$$

(b) Parts separately Guesses $T = 1\ \text{lb}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given $T r_i = \left(\frac{W_S}{g}\right) k_A{}^2 \alpha$ $T - W_B = \left(\frac{-W_B}{g}\right) \alpha r_i$ $\begin{pmatrix} T \\ \alpha \end{pmatrix} = \text{Find}(T, \alpha)$

$T = 4.808\ \text{lb}$ $\alpha = 0.826\ \frac{\text{rad}}{\text{s}^2}$ $\omega = \alpha t_1$ $\omega = 2.477\ \frac{\text{rad}}{\text{s}}$

Problem 17-81

A boy of mass m_b sits on top of the large wheel which has mass m_w and a radius of gyration k_G. If the boy essentially starts from rest at $\theta = 0°$, and the wheel begins to rotate freely, determine the angle at which the boy begins to slip. The coefficient of static friction between the wheel and the boy is μ_s. Neglect the size of the boy in the calculation.

Given:

$m_b = 40\ \text{kg}$

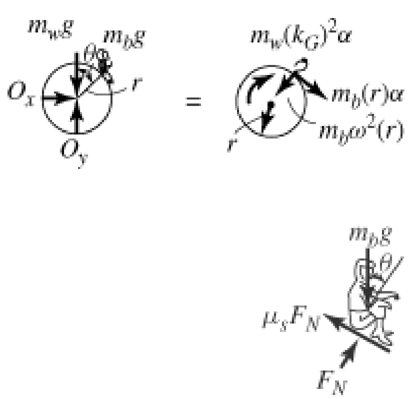
$m_w = 400\ \text{kg}$

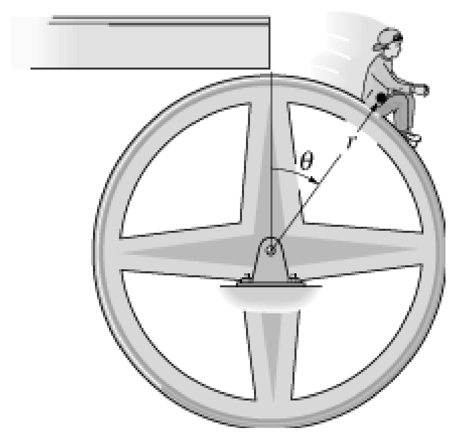
$k_G = 5.5\ \text{m}$

$\mu_s = 0.5$

$r = 8\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$





Solution: Assume slipping occurs before contact is lost

$$m_b g r \sin(\theta) = \left(m_b r^2 + m_w k_G{}^2\right)\alpha \qquad \alpha = \frac{m_b g r}{m_b r^2 + m_w k_G{}^2} \sin(\theta)$$

Guesses $\theta = 10\ \text{deg}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $\omega = 1\ \frac{\text{rad}}{\text{s}}$ $F_N = 1\ \text{N}$

Given $F_N - m_b\, g \cos(\theta) = -m_b\, r\omega^2$ $\mu_s F_N - m_b\, g \sin(\theta) = -m_b\, r\alpha$

$$\alpha = \frac{m_b\, g\, r}{m_b\, r^2 + m_w\, k_G^2} \sin(\theta) \qquad \frac{\omega^2}{2} = \frac{m_b\, g\, r}{m_b\, r^2 + m_w\, k_G^2}(1 - \cos(\theta))$$

$$\begin{pmatrix} \theta \\ \alpha \\ \omega \\ F_N \end{pmatrix} = \mathrm{Find}(\theta, \alpha, \omega, F_N)$$

Since $F_N = 322\ \text{N}$ > 0 our assumption is correct.

$\alpha = 0.107 \frac{\text{rad}}{\text{s}^2}$ $\omega = 0.238 \frac{\text{rad}}{\text{s}}$ $\theta = 29.8\ \text{deg}$

Problem 17-82

The “Catherine wheel” is a firework that consists of a coiled tube of powder which is pinned at its center. If the powder burns at a constant rate m' such that the exhaust gases always exert a force having a constant magnitude of F, directed tangent to the wheel, determine the angular velocity of the wheel when k of the mass is burned off. Initially, the wheel is at rest and has mass m_0 and radius r_0. For the calculation, consider the wheel to always be a thin disk.

Given:

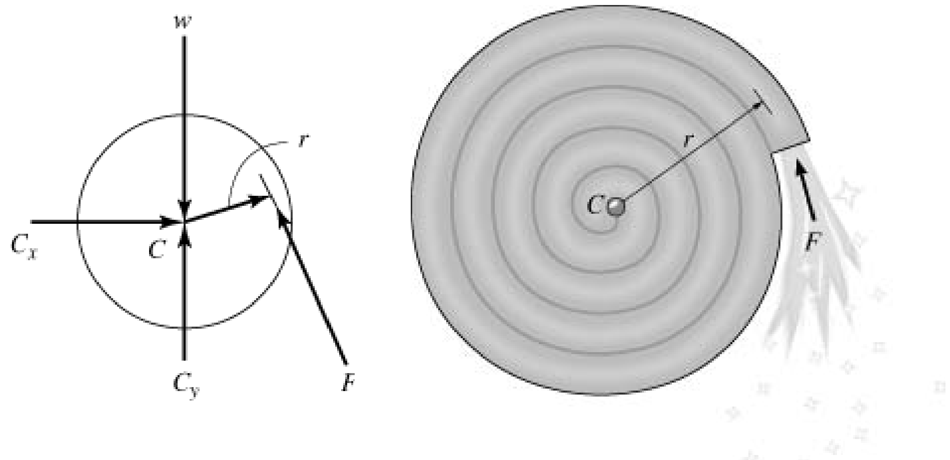
$m' = 20 \frac{\text{gm}}{\text{s}}$

$F = 0.3\ \text{N}$

$m_0 = 100\ \text{gm}$

$r_0 = 75\ \text{mm}$

$k = 0.75$



Solution:

The density is $\rho = \frac{m_0}{\pi r_0^2}$

The mass is $m = m_0 - m' t = (1 - k) m_0$

$t_1 = \frac{k m_0}{m'}$ $t_1 = 3.75\ \text{s}$

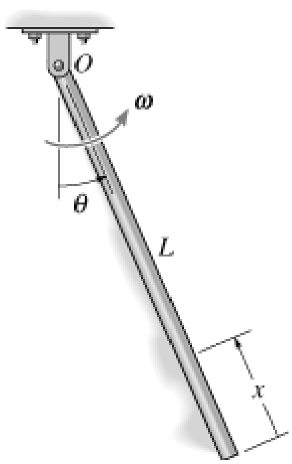
Find the radius $m_0 - m' t = \rho\, \pi r^2$ $r = \sqrt{\frac{m_0 - m' t}{\rho\, \pi}}$

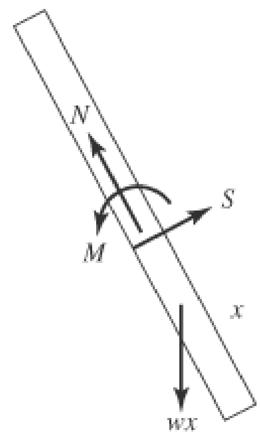
Dynamics $\quad F\,r = \dfrac{1}{2} m\,r^2 \alpha \qquad \alpha = \dfrac{2F\sqrt{\rho\,\pi}}{\sqrt{\left(m_0 - m'\,t\right)^3}}$

$$\omega = \int_0^{t_1} \frac{2F\sqrt{\rho\,\pi}}{\sqrt{\left(m_0 - m'\,t\right)^3}}\,dt \qquad \omega = 800\,\frac{\text{rad}}{\text{s}}$$

Problem 17-83

The bar has a weight per length of w. If it is rotating in the vertical plane at a constant rate ω about point O, determine the internal normal force, shear force, and moment as a function of x and θ.





Solution:

$$N - w\,x\cos(\theta) = \left(\frac{w\,x}{g}\right)\omega^2\left(L - \frac{x}{2}\right) \qquad N = w\,x\left[\cos(\theta) + \left(\frac{\omega^2}{g}\right)\left(L - \frac{x}{2}\right)\right]$$

$$S - w\,x\sin(\theta) = 0 \qquad S = w\,x\,\text{win}(\theta)$$

$$M - S\frac{x}{2} = 0 \qquad M = w\frac{x^2}{2}\sin(\theta)$$

Problem 17-84

A force F is applied perpendicular to the axis of the rod of weight W and moves from O to A at a constant rate v. If the rod is at rest when $\theta = 0°$ and F is at O when $t = 0$, determine the rod's angular velocity at the instant the force is at A. Through what angle has the rod rotated when this occurs? The rod rotates in the *horizontal plane*.

Given:

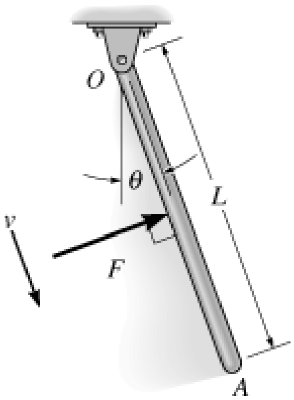
$F = 2$ lb

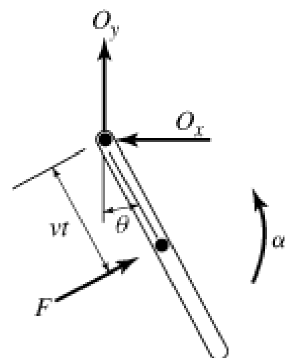
$W = 5\ \text{lb}$

$v = 4\dfrac{\text{ft}}{\text{s}}$

$L = 4\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution:

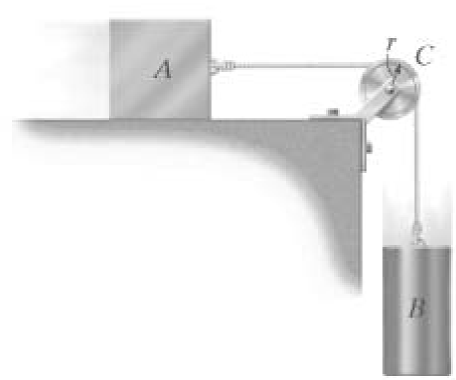
$F\,v\,t = \dfrac{1}{3}\dfrac{W}{g}L^2\alpha \qquad t = \dfrac{L}{v} \qquad t = 1\ \text{s}$

$\alpha = \left(\dfrac{3F\,v\,g}{WL^2}\right)t \qquad \omega = \left(\dfrac{3F\,v\,g}{2WL^2}\right)t^2 \qquad \omega = 4.83\dfrac{\text{rad}}{\text{s}}$

$\theta = \left(\dfrac{F\,v\,g}{2WL^2}\right)t^3 \qquad \theta = 92.2\,\text{deg}$

Problem 17-85

Block A has a mass m and rests on a surface having a coefficient of kinetic friction μ_k. The cord attached to A passes over a pulley at C and is attached to a block B having a mass $2m$. If B is released, determine the acceleration of A. Assume that the cord does not slip over the pulley. The pulley can be approximated as a thin disk of radius r and mass $m/4$. Neglect the mass of the cord.



Solution:

Given

$T_1 - \mu_k m\,g = m\,a$

$T_2 - 2m\,g = -2m\,a$

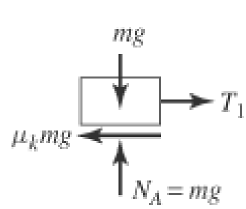
$T_1 r - T_2 r = -\dfrac{1}{2}\left(\dfrac{m}{4}\right)r^2\alpha$

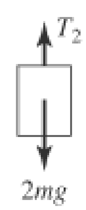
$a = \alpha r$

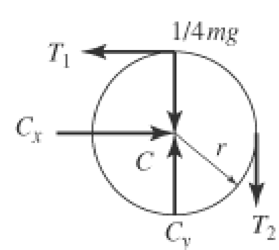
Solving

$T_1 = \dfrac{m\,g}{25}\left(16 + 17\mu_k\right)$

$T_2 = \dfrac{2m\,g}{25}\left(9 + 8\mu_k\right)$







$$\alpha = \frac{8g}{25r}(2 - \mu_k)$$

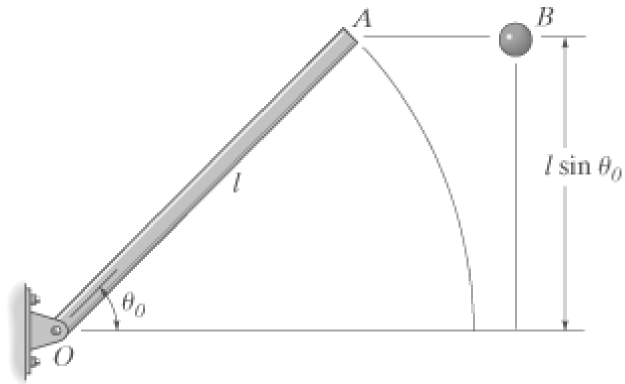
$$a = \frac{8g}{25}(2 - \mu_k)$$

Problem 17-86

The slender rod of mass m is released from rest when $\theta = \theta_0$. At the same instant ball B having the same mass m is released.Will B or the end A of the rod have the greatest speed when they pass the horizontal? What is the difference in their speeds?

Given:

$$\theta_0 = 45 \text{ deg}$$



Solution: At horizontal $\theta_f = 0 \text{ deg}$

Rod

$$m g \frac{1}{2} \cos(\theta) = \frac{1}{3} m l^2 \alpha$$

$$\alpha = \frac{3g}{2l} \cos(\theta)$$

$$\frac{\omega^2}{2} = \frac{3g}{2l}(\sin(\theta_0) - \sin(\theta_f))$$

$$\omega = \sqrt{\frac{3g}{l}(\sin(\theta_0) - \sin(\theta_f))} \qquad v_A = \omega l = \sqrt{3g\,l(\sin(\theta_0) - \sin(\theta_f))}$$

Ball

$$m g = m a \qquad a = g$$

$$\frac{v^2}{2} = g\,l(\sin(\theta_0) - \sin(\theta_f))$$

$$v_B = \sqrt{2g\,l(\sin(\theta_0) - \sin(\theta_f))}$$

Define the constant $k = (\sqrt{3} - \sqrt{2})\sqrt{\sin(\theta_0) - \sin(\theta_f)}$

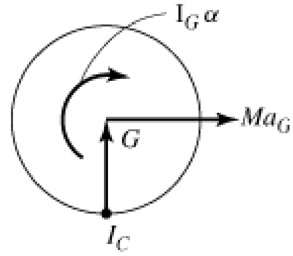
A has the greater speed and the difference is given by $\Delta v = k\sqrt{g\,l}$

$$k = 0.267$$

Problem 17-87

If a disk *rolls without slipping*, show that when moments are summed about the instantaneous center of zero velocity, *IC*, it is possible to use the moment equation $\Sigma M_{IC} = I_{IC}\alpha$, where I_{IC} represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.

Solution:



$\Sigma M_{IC} = \Sigma(M_k)_{IC}$; $\quad \Sigma M_{IC} = I_G\alpha + ma_G r$

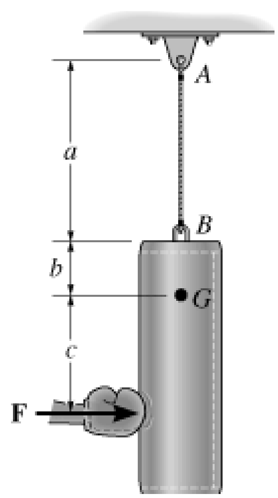
Since there is no slipping, $a_G = \alpha r$

Thus, $\quad \Sigma M_{IC} = \left(I_G + m r^2\right)\alpha$

By the parallel - axis theorem, the term in parenthesis represents I_{IC}.

$\Sigma M_{IC} = I_{IC}\alpha \qquad$ Q.E.D

***Problem 17-88**



The punching bag of mass M has a radius of gyration about its center of mass G of k_G. If it is subjected to a horizontal force F, determine the initial angular acceleration of the bag and the tension in the supporting cable AB.

Given:

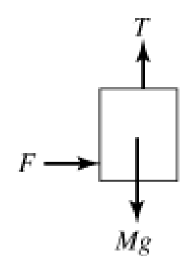
$M = 20\ \text{kg} \qquad b = 0.3\ \text{m}$

$k_G = 0.4\ \text{m} \qquad c = 0.6\ \text{m}$

$F = 30\ \text{N} \qquad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$a = 1\ \text{m}$

Solution:

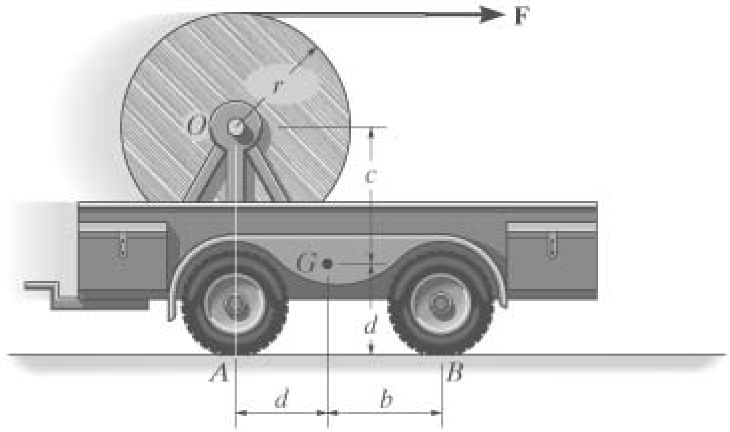


$T - Mg = 0 \qquad T = Mg \qquad T = 196.2\ \text{N}$

$F c = M k_G^2 \alpha \qquad \alpha = \dfrac{F c}{M k_G^2} \qquad \alpha = 5.625\ \dfrac{\text{rad}}{\text{s}^2}$

Problem 17-89

The trailer has mass M_1 and a mass center at G, whereas the spool has mass M_2, mass center at O, and radius of gyration about an axis passing through O k_O. If a force F is applied to the cable, determine the angular acceleration of the spool and the acceleration of the trailer. The wheels have negligible mass and are free to roll.



Given:

$M_1 = 580\ \text{kg}$ $\quad b = 0.5\ \text{m}$

$M_2 = 200\ \text{kg}$ $\quad c = 0.6\ \text{m}$

$k_O = 0.45\ \text{m}$ $\quad d = 0.4\ \text{m}$

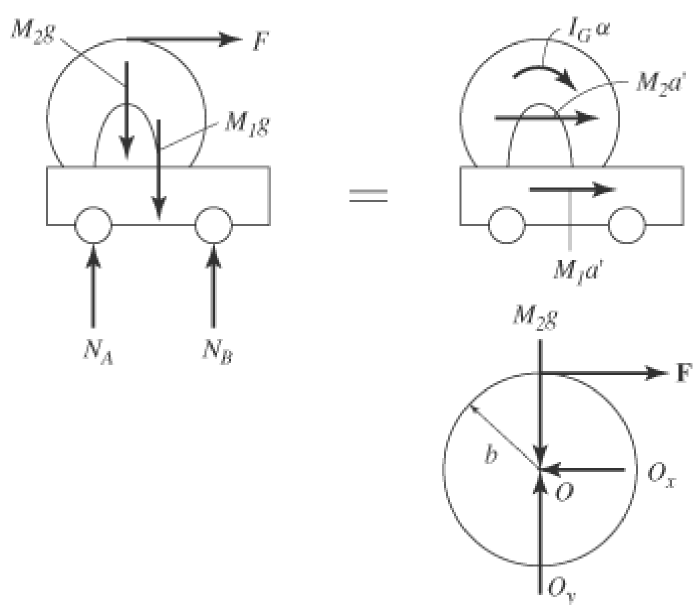
$F = 60\ \text{N}$ $\quad r = 0.5\ \text{m}$

Solution:

$$F = (M_1 + M_2)a$$

$$a = \frac{F}{M_1 + M_2}$$

$$a = 0.0769\ \frac{\text{m}}{\text{s}^2}$$



$$F r = M_2 k_O^2 \alpha$$

$$\alpha = F\left(\frac{r}{M_2 k_O^2}\right)$$

$$\alpha = 0.741\ \frac{\text{rad}}{\text{s}^2}$$

Problem 17-90

The rocket has weight W, mass center at G, and radius of gyration about the mass center k_G when it is fired. Each of its two engines provides a thrust T. At a given instant, engine A suddenly fails to operate. Determine the angular acceleration of the rocket and the acceleration of its nose B.

Given:

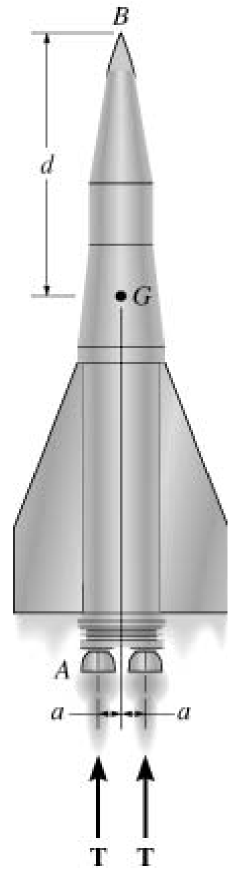
$W = 20000\ \text{lb}$ $\quad T = 50000\ \text{lb}$

$k_G = 21\ \text{ft}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$d = 30\ \text{ft}$

$a = 1.5$ ft

Solution:



$$T a = \left(\frac{W}{g}\right) k_G^{\ 2} \alpha$$

$$\alpha = \frac{T a g}{W k_G^{\ 2}} \qquad \alpha = 0.274\ \frac{\text{rad}}{\text{s}^2}$$

$$T - W = \left(\frac{W}{g}\right) a_{Gy}$$

$$a_{Gy} = \frac{(T - W)g}{W} \qquad a_{Gy} = 14.715\ \frac{\text{m}}{\text{s}^2}$$

$$\mathbf{a_B} = \begin{pmatrix} 0 \\ a_{Gy} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} 0 \\ d \\ 0 \end{pmatrix} \qquad \mathbf{a_B} = \begin{pmatrix} -8.2 \\ 48.3 \\ 0.0 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

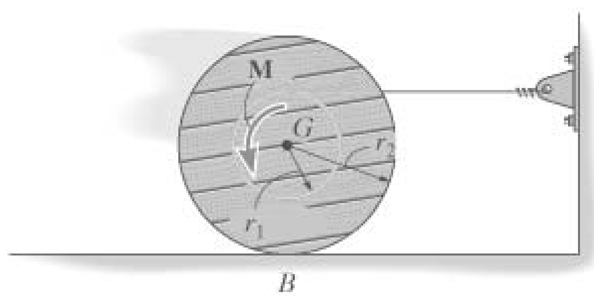
$$\left|\mathbf{a_B}\right| = 49.0\ \frac{\text{ft}}{\text{s}^2}$$

$$\theta = \text{atan}\left(\frac{a_{Gy}}{\alpha d}\right) \qquad \theta = 80.3\ \text{deg}$$

Problem 17-91

The spool and wire wrapped around its core have a mass m_s and a centroidal radius of gyration k_G. If the coefficient of kinetic friction at the ground is μ_k, determine the angular acceleration of the spool when the couple M is applied.

Given:



$m_s = 20$ kg $\qquad M = 30$ N m

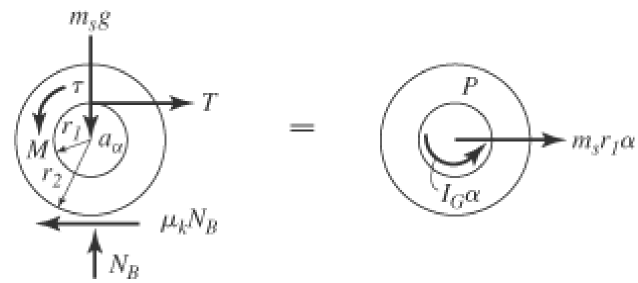
$k_G = 250$ mm $\qquad r_1 = 200$ mm

$\mu_k = 0.1 \qquad r_2 = 400$ mm

Solution:

Guesses $T = 1\ \text{N}$ $N_B = 1\ \text{N}$

$\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$



Given

$$T - \mu_k N_B = m_s r_1 \alpha$$

$$N_B - m_s g = 0$$

$$M - \mu_k N_B r_2 - T r_1 = m_s k_G^2 \alpha$$

$$\begin{pmatrix} T \\ N_B \\ \alpha \end{pmatrix} = \text{Find}(T, N_B, \alpha) \qquad \begin{pmatrix} T \\ N_B \end{pmatrix} = \begin{pmatrix} 55.2 \\ 196.2 \end{pmatrix} \text{N} \qquad \alpha = 8.89\ \frac{\text{rad}}{\text{s}^2}$$

***Problem 17-92**

The uniform board of weight W is suspended from cords at C and D. If these cords are subjected to constant forces F_A and F_B respectively, determine the acceleration of the board's center and the board's angular acceleration. Assume the board is a thin plate. Neglect the mass of the pulleys at E and F.

Given:

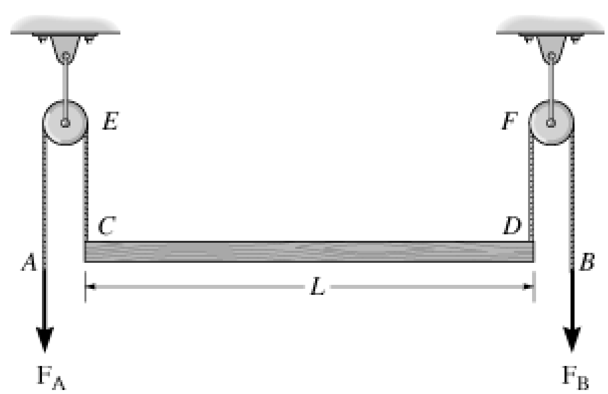
$W = 50\ \text{lb}$

$F_A = 30\ \text{lb}$

$F_B = 45\ \text{lb}$

$L = 10\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

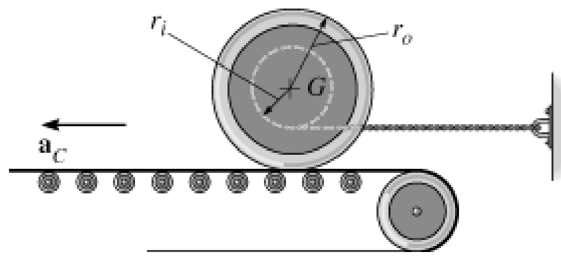
$$F_A + F_B - W = \left(\frac{W}{g}\right) a_{Gy} \qquad a_{Gy} = \left(\frac{F_A + F_B - W}{W}\right) g \qquad a_{Gy} = 16.1\ \frac{\text{ft}}{\text{s}^2}$$

$$F_B\left(\frac{L}{2}\right) - F_A\left(\frac{L}{2}\right) = \frac{1}{12}\left(\frac{W}{g}\right)L^2\alpha \qquad \alpha = \frac{6(F_B - F_A)g}{WL} \qquad \alpha = 5.796\ \frac{\text{rad}}{\text{s}^2}$$

Problem 17-93

The spool has mass M and radius of gyration k_G. It rests on the surface of a conveyor belt for which the coefficient of static friction is μ_s and the coefficient of kinetic friction is μ_k. If the conveyor accelerates at rate a_C, determine the initial tension in the wire and the angular acceleration of the spool. The spool is originally at rest.

Units Used: $\text{kN} = 10^3\ \text{N}$



Given:

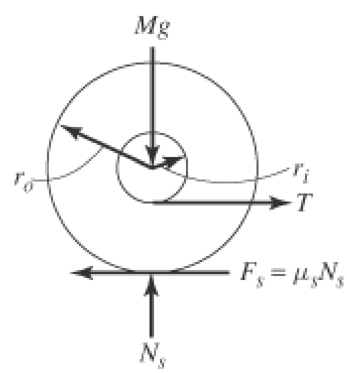
$M = 500\ \text{kg} \qquad a_C = 1\ \frac{\text{m}}{\text{s}^2}$

$k_G = 1.30\ \text{m} \qquad r_i = 0.8\ \text{m}$

$\mu_s = 0.5 \qquad r_o = 1.6\ \text{m}$

$\mu_k = 0.4$

Solution: Assume no slip



Guesses $\alpha = 1\ \frac{\text{rad}}{\text{s}^2} \qquad a_x = 1\ \frac{\text{m}}{\text{s}^2} \qquad T = 1\text{N}$

$F_{max} = 1\text{N} \qquad N_s = 1\text{N} \qquad F_s = 1\text{N}$

Given $T - F_s = M a_x \qquad N_s - Mg = 0$

$T r_i - F_s r_o = -M k_G^2 \alpha \qquad F_{max} = \mu_s N_s$

$a_x = r_i \alpha \qquad a_C = (r_o - r_i)\alpha$

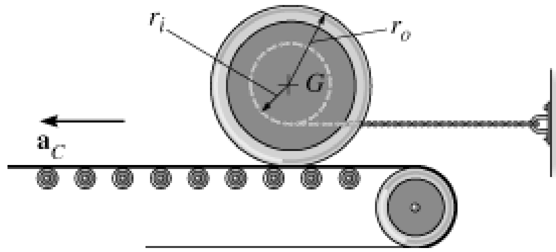
$$\begin{pmatrix} \alpha \\ a_x \\ F_{max} \\ N_s \\ F_s \\ T \end{pmatrix} = \text{Find}(\alpha, a_x, F_{max}, N_s, F_s, T) \qquad \begin{pmatrix} N_s \\ F_s \\ F_{max} \end{pmatrix} = \begin{pmatrix} 4.907 \\ 1.82 \\ 2.454 \end{pmatrix}\text{kN} \qquad \alpha = 1.25\ \frac{\text{rad}}{\text{s}^2}$$

$$T = 2.32\ \text{kN}$$

Since $F_s = 1.82\ \text{kN} < F_{max} = 2.454\ \text{kN}$ then our no-slip assumption is correct.

Problem 17-94

The spool has mass M and radius of gyration k_G. It rests on the surface of a conveyor belt for which the coefficient of static friction is μ_s. Determine the greatest acceleration of the conveyor so that the spool will not slip. Also, what are the initial tension in the wire and the angular acceleration of the spool? The spool is originally at rest.



Units Used: $\quad kN = 10^3\ N$

Given: $\quad M = 500\ kg$

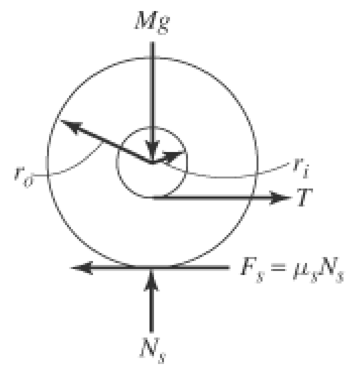
$k_G = 1.30\ m \qquad r_i = 0.8\ m$

$\mu_s = 0.5 \qquad r_o = 1.6\ m$

Solution:

Guesses $\quad \alpha = 1\ \frac{rad}{s^2} \qquad a_x = 1\ \frac{m}{s^2} \qquad a_C = 1\ \frac{m}{s^2}$

$N_s = 1\ N \qquad F_s = 1\ N \qquad T = 1\ N$



Given $\quad T - F_s = M a_x \qquad N_s - M g = 0$

$T r_i - F_s r_o = -M k_G^2 \alpha \qquad F_s = \mu_s N_s$

$a_x = r_i \alpha \qquad a_C = (r_o - r_i)\alpha$

$$\begin{pmatrix} \alpha \\ a_x \\ a_C \\ N_s \\ F_s \\ T \end{pmatrix} = \mathrm{Find}(\alpha, a_x, a_C, N_s, F_s, T) \qquad \begin{pmatrix} N_s \\ F_s \end{pmatrix} = \begin{pmatrix} 4.907 \\ 2.454 \end{pmatrix} kN \qquad a_C = 1.348\ \frac{m}{s^2}$$

$$\alpha = 1.685\ \frac{rad}{s^2} \qquad T = 3.128\ kN$$

Problem 17-95

The wheel has weight W and radius of gyration k_G. If the coefficients of static and kinetic friction between the wheel and the plane are μ_s and μ_k, determine the wheel's angular acceleration as it rolls down the incline.

Given:

$W = 30\ \text{lb}$

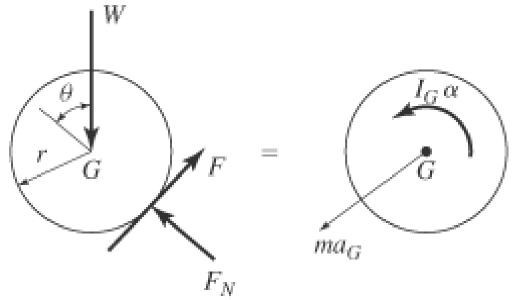
$k_G = 0.6\ \text{ft}$

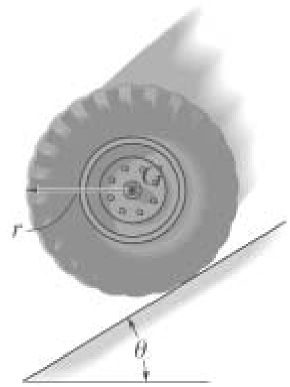
$\mu_s = 0.2$

$\mu_k = 0.15$

$r = 1.25\ \text{ft}$

$\theta = 12\ \text{deg}$





Solution: Assume no slipping

Guesses $F_N = 1\ \text{lb}$ $F = 1\ \text{lb}$ $a_G = 1\ \frac{\text{ft}}{\text{s}^2}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $F_{max} = 1\ \text{lb}$

Given $F - W\sin(\theta) = \frac{-W}{g}a_G$ $F_N - W\cos(\theta) = 0$ $F_{max} = \mu_s F_N$

$F\,r = \frac{W}{g}k_G^{\,2}\alpha$ $a_G = r\alpha$

$$\begin{pmatrix} F \\ F_N \\ F_{max} \\ a_G \\ \alpha \end{pmatrix} = \text{Find}(F, F_N, F_{max}, a_G, \alpha)$$

$$\begin{pmatrix} F \\ F_N \\ F_{max} \end{pmatrix} = \begin{pmatrix} 1.17 \\ 29.34 \\ 5.87 \end{pmatrix}\text{lb} \qquad a_G = 5.44\ \frac{\text{ft}}{\text{s}^2}$$

$$\alpha = 4.35\ \frac{\text{rad}}{\text{s}^2}$$

Since $F = 1.17\ \text{lb} < F_{max} = 5.87\ \text{lb}$ then our no-slip assumption is correct.

***Problem 17-96**

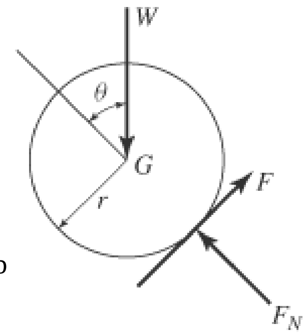
The wheel has a weight W and a radius of gyration k_G. If the coefficients of static and kinetic friction between the wheel and the plane are μ_s and μ_k, determine the maximum angle θ of the inclined plane so that the wheel rolls without slipping.

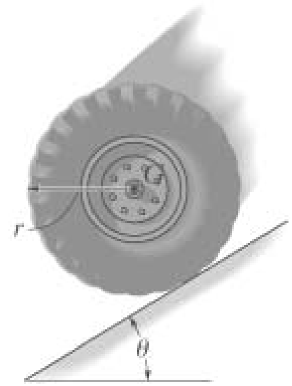
Given:

$W = 30\ \text{lb}$ $\quad r = 1.25\ \text{ft}$

$k_G = 0.6\ \text{ft}$ $\quad \theta = 12\ \text{deg}$

$\mu_s = 0.2$ $\quad \mu_k = 0.15$





Solution:

Guesses

$\theta = 1\ \text{deg}$ $\quad F_N = 1\ \text{lb}$ $\quad F = 1\ \text{lb}$

$a_G = 1\ \frac{\text{ft}}{\text{s}^2}$ $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$F - W\sin(\theta) = \left(\frac{-W}{g}\right)a_G \qquad F\,r = \left(\frac{W}{g}\right)k_G^{\,2}\alpha$$

$$F_N - W\cos(\theta) = 0 \qquad F = \mu_s F_N$$

$$a_G = r\alpha$$

$$\begin{pmatrix} \theta \\ F_N \\ F \\ a_G \\ \alpha \end{pmatrix} = \text{Find}(\theta, F_N, F, a_G, \alpha) \qquad \begin{pmatrix} F \\ F_N \end{pmatrix} = \begin{pmatrix} 4.10 \\ 20.50 \end{pmatrix}\text{lb} \qquad a_G = 19.1\,\frac{\text{ft}}{\text{s}^2} \qquad \alpha = 15.3\,\frac{\text{rad}}{\text{s}^2}$$

$$\theta = 46.9\,\text{deg}$$

Problem 17-97

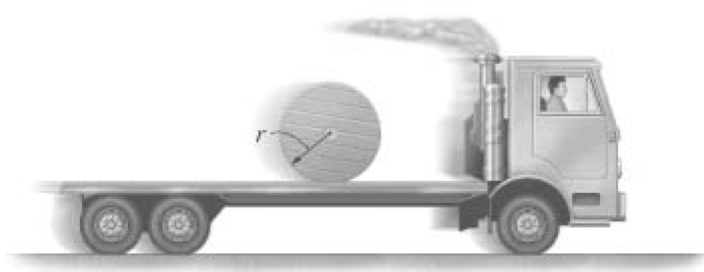
The truck carries the spool which has weight W and radius of gyration k_G. Determine the angular acceleration of the spool if it is not tied down on the truck and the truck begins to accelerate at the rate a_{At}. Assume the spool does not slip on the bed of the truck.

Given:

$W = 500\ \text{lb}$ $\quad r = 3\ \text{ft}$

$k_G = 2\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

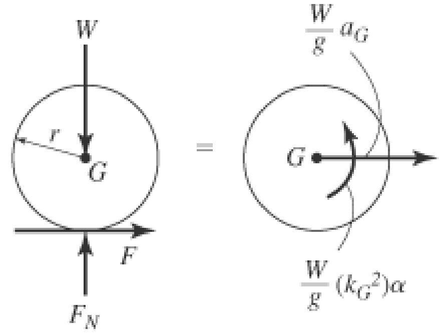
$a_{At} = 3\ \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $a_G = 1\ \frac{ft}{s^2}$ $\alpha = 1\ \frac{rad}{s^2}$

Given



$$a_G = a_{At} - \alpha r$$

$$0 = \left(\frac{-W}{g}\right) a_G r + \left(\frac{W}{g}\right) k_G^2 \alpha$$

$$\begin{pmatrix} a_G \\ \alpha \end{pmatrix} = \text{Find}(a_G, \alpha) \qquad a_G = 0.923\ \frac{ft}{s^2} \qquad \alpha = 0.692\ \frac{rad}{s^2}$$

Problem 17-98

The truck carries the spool which has weight W and radius of gyration k_G. Determine the angular acceleration of the spool if it is not tied down on the truck and the truck begins to accelerate at the rate a_{At}. The coefficients of static and kinetic friction between the spool and the truck bed are μ_s and μ_k, respectively.

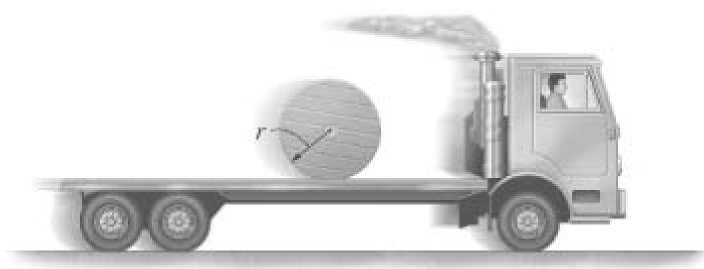
Given:

$W = 200\ \text{lb}$ $g = 32.2\ \frac{ft}{s^2}$

$k_G = 2\ \text{ft}$ $\mu_s = 0.15$

$a_{At} = 5\ \frac{ft}{s^2}$ $\mu_k = 0.1$

$r = 3\ \text{ft}$

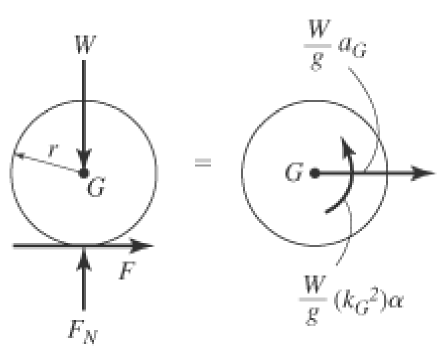


Solution: Assume no slip

Guesses

$F = 1\ \text{lb}$ $F_N = 1\ \text{lb}$ $F_{max} = 1\ \text{lb}$

$a_G = 1\ \frac{ft}{s^2}$ $\alpha = 1\ \frac{rad}{s^2}$



Given $F = \frac{W}{g} a_G$ $F r = \frac{W}{g} k_G^2 \alpha$ $F_N - W = 0$ $F_{max} = \mu_s F_N$

$$a_G = a_{At} - \alpha r$$

$$\begin{pmatrix} F \\ F_N \\ F_{max} \\ a_G \\ \alpha \end{pmatrix} = \text{Find}(F, F_N, F_{max}, a_G, \alpha) \qquad \begin{pmatrix} F \\ F_{max} \\ F_N \end{pmatrix} = \begin{pmatrix} 9.56 \\ 30.00 \\ 200.00 \end{pmatrix} \text{lb} \qquad a_G = 1.538 \frac{\text{ft}}{\text{s}^2}$$

$$\alpha = 1.154 \frac{\text{rad}}{\text{s}^2}$$

Since $F = 9.56\,\text{lb} < F_{max} = 30\,\text{lb}$ then our no-slip assumption is correct.

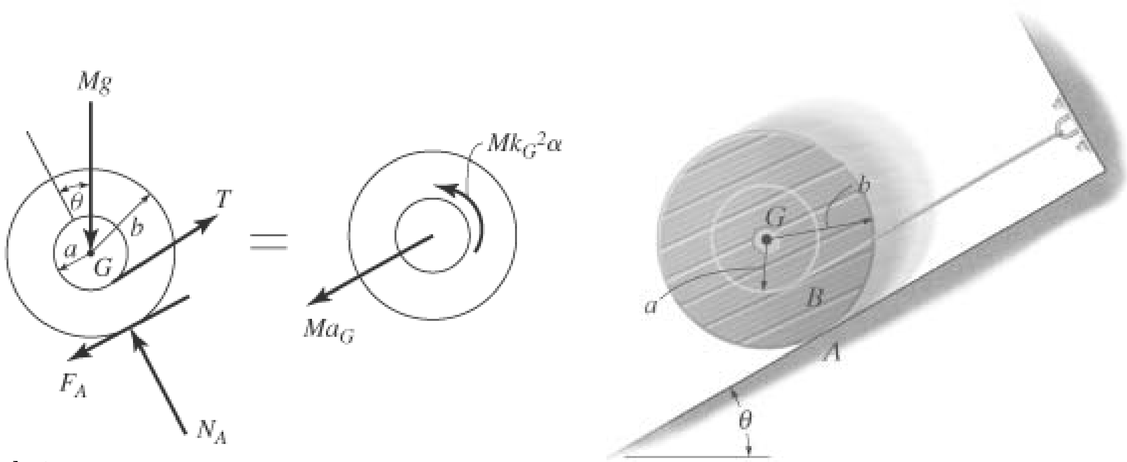
Problem 17-99

The spool has mass M and radius of gyration k_G. It rests on the inclined surface for which the coefficient of kinetic friction is μ_k. If the spool is released from rest and slips at A, determine the initial tension in the cord and the angular acceleration of the spool.

Given:

$M = 75\ \text{kg}$ $\quad k_G = 0.380\ \text{m}$ $\quad a = 0.3\ \text{m}$

$\mu_k = 0.15$ $\quad \theta = 30\ \text{deg}$ $\quad b = 0.6\ \text{m}$



Solution:

Guesses $\quad T = 1\ \text{N}$ $\quad N_A = 1\ \text{N}$ $\quad a_G = 1\ \frac{\text{m}}{\text{s}^2}$ $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$T - M g \sin(\theta) - \mu_k N_A = -M a_G \qquad N_A - M g \cos(\theta) = 0$$

$$T a - \mu_k N_A b = M k_G^{\,2} \alpha \qquad a_G = \alpha a$$

$$\begin{pmatrix} T \\ N_A \\ \alpha \\ a_G \end{pmatrix} = \text{Find}(T, N_A, \alpha, a_G) \qquad a_G = 1.395\ \frac{\text{m}}{\text{s}^2} \qquad \alpha = 4.65\ \frac{\text{rad}}{\text{s}^2} \qquad T = 359\ \text{N}$$

***Problem 17-100**

A uniform rod having weight *W* is pin-supported at *A* from a roller which rides on horizontal track. If the rod is originally at rest, and horizontal force **F** is applied to the roller, determine the acceleration of the roller. Neglect the mass of the roller and its size *d* in the computations.

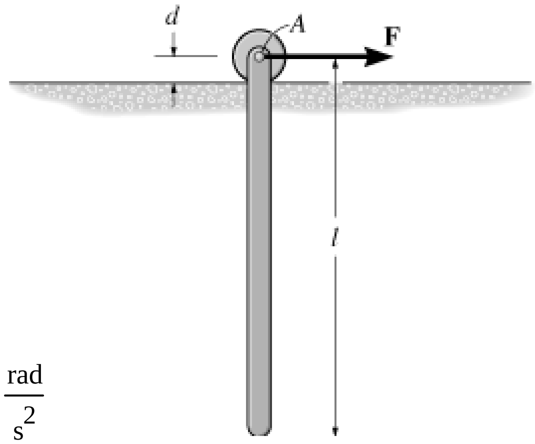
Given:

$W = 10\ \text{lb}$

$F = 15\ \text{lb}$

$l = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

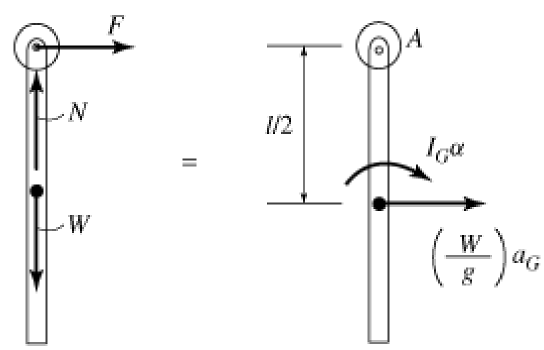
Guesses $\quad a_G = 1\ \frac{\text{ft}}{\text{s}^2} \quad a_A = 1\ \frac{\text{ft}}{\text{s}^2} \quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$F = \left(\frac{W}{g}\right)a_G \qquad F\left(\frac{l}{2}\right) = \frac{1}{12}\left(\frac{W}{g}\right)l^2\alpha$$

$$a_A = a_G + \alpha\frac{l}{2}$$

$$\begin{pmatrix} a_G \\ a_A \\ \alpha \end{pmatrix} = \text{Find}(a_G, a_A, \alpha)$$



$$a_G = 48.3\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 145\ \frac{\text{rad}}{\text{s}^2} \qquad a_A = 193\ \frac{\text{ft}}{\text{s}^2}$$

Problem 17-101

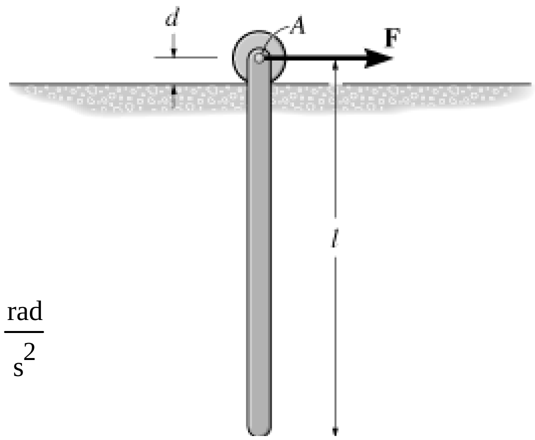
A uniform rod having weight *W* is pin-supported at *A* from a roller which rides on horizontal track. Assume that the roller at *A* is replaced by a slider block having a negligible mass. If the rod is initially at rest, and a horizontal force **F** is applied to the slider, determine the slider's acceleration. The coefficient of kinetic friction between the block and the track is μ_k. Neglect the dimension *d* and the size of the block in the computations.

Given:

$W = 10\ \text{lb}$ $\mu_k = 0.2$

$F = 15\ \text{lb}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$l = 2\ \text{ft}$



Solution:

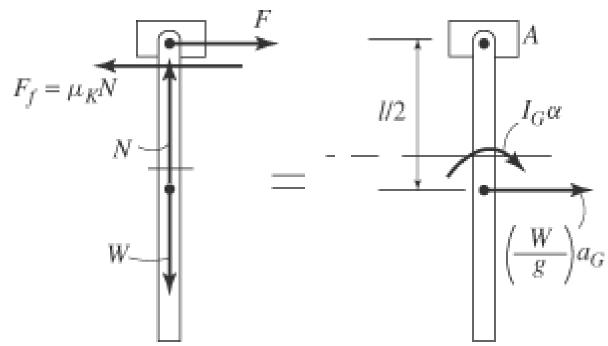
Guesses $a_G = 1\ \frac{\text{ft}}{\text{s}^2}$ $a_A = 1\ \frac{\text{ft}}{\text{s}^2}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$F - \mu_k W = \left(\frac{W}{g}\right)a_G$$

$$\left(F - \mu_k W\right)\frac{l}{2} = \frac{1}{12}\left(\frac{W}{g}\right)l^2\alpha$$

$$a_A = a_G + \alpha\left(\frac{l}{2}\right)$$

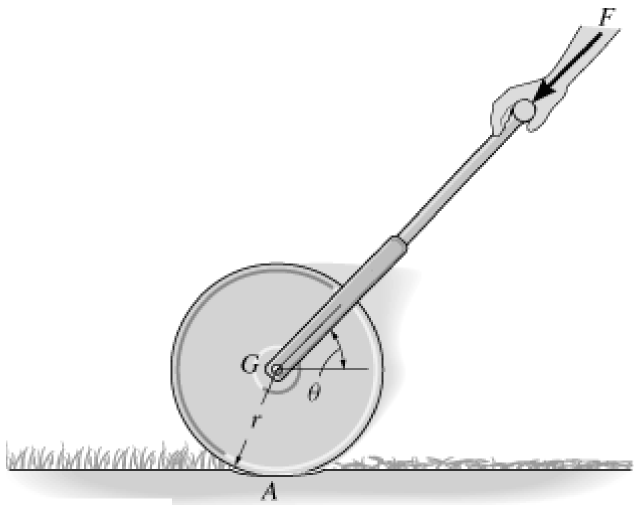


$$\begin{pmatrix} a_G \\ a_A \\ \alpha \end{pmatrix} = \text{Find}\left(a_G, a_A, \alpha\right)$$

$a_G = 41.86\ \frac{\text{ft}}{\text{s}^2}$ $\alpha = 126\ \frac{\text{rad}}{\text{s}^2}$ $a_A = 167\ \frac{\text{ft}}{\text{s}^2}$

Problem 17-102

The lawn roller has mass M and radius of gyration k_G. If it is pushed forward with a force **F** when the handle is in the position shown, determine its angular acceleration. The coefficients of static and kinetic friction between the ground and the roller are μ_s and μ_k, respectively.



Given:

$M = 80 \text{ kg}$ $\mu_s = 0.12$

$k_G = 0.175 \text{ m}$ $\mu_k = 0.1$

$F = 200 \text{ N}$ $r = 200 \text{ mm}$

$\theta = 45 \text{ deg}$ $g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution: Assume no slipping,

Guesses

$F_f = 1 \text{ N}$ $F_N = 1 \text{ N}$ $a_x = 1 \frac{\text{m}}{\text{s}^2}$ $\alpha = 1 \frac{\text{rad}}{\text{s}^2}$ $F_{max} = 1 \text{ N}$

Given $-F\cos(\theta) + F_f = -M a_x$ $F_N - M g - F\sin(\theta) = 0$

$F_f r = M k_G^2 \alpha$ $a_x = \alpha r$ $F_{max} = \mu_s F_N$

$$\begin{pmatrix} F_f \\ F_N \\ F_{max} \\ a_x \\ \alpha \end{pmatrix} = \text{Find}(F_f, F_N, F_{max}, a_x, \alpha) \qquad \begin{pmatrix} F_N \\ F_f \\ F_{max} \end{pmatrix} = \begin{pmatrix} 926.221 \\ 61.324 \\ 111.147 \end{pmatrix} \text{N} \qquad a_x = 1.001 \frac{\text{m}}{\text{s}^2}$$

$$\alpha = 5.01 \frac{\text{rad}}{\text{s}^2}$$

Since $F_f = 61.3 \text{ N} < F_{max} = 111.1 \text{ N}$ then our no-slip assumption is true.

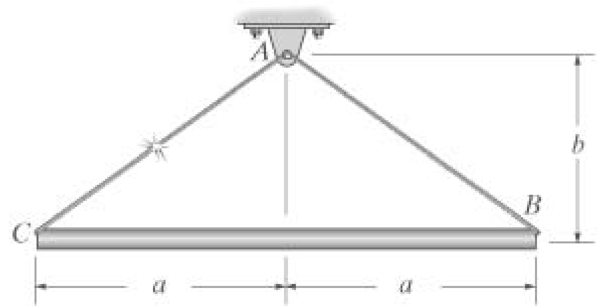
Problem 17-103

The slender bar of weight W is supported by two cords AB and AC. If cord AC suddenly breaks, determine the initial angular acceleration of the bar and the tension in cord AB.

Given:

$W = 150\ \text{lb}$ $\quad a = 4\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $\quad b = 3\ \text{ft}$

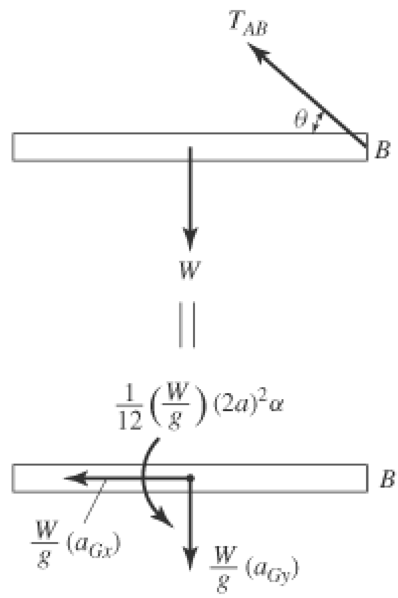


Solution: $\theta = \operatorname{atan}\left(\frac{b}{a}\right)$

Guesses

$T_{AB} = 1\ \text{lb}$ $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

$a_{Gx} = 1\ \frac{\text{ft}}{\text{s}^2}$ $\quad a_{Gy} = 1\ \frac{\text{ft}}{\text{s}^2}$ $\quad a_B = 1\ \frac{\text{ft}}{\text{s}^2}$



Given

$$-T_{AB}\cos(\theta) = \left(\frac{-W}{g}\right)a_{Gx}$$

$$T_{AB}\sin(\theta) - W = \left(\frac{-W}{g}\right)a_{Gy} \qquad -a_{Gx} = -a_B\sin(\theta)$$

$$T_{AB}\sin(\theta)a = \frac{1}{12}\left(\frac{W}{g}\right)(2a)^2\alpha \qquad -a_{Gy} + \alpha a = -a_B\cos(\theta)$$

$$\begin{pmatrix} T_{AB} \\ \alpha \\ a_{Gx} \\ a_{Gy} \\ a_B \end{pmatrix} = \operatorname{Find}(T_{AB}, \alpha, a_{Gx}, a_{Gy}, a_B) \qquad \begin{pmatrix} a_{Gx} \\ a_{Gy} \\ a_B \end{pmatrix} = \begin{pmatrix} 7.43 \\ 26.63 \\ 12.38 \end{pmatrix}\frac{\text{ft}}{\text{s}^2} \qquad \alpha = 4.18\ \frac{\text{rad}}{\text{s}^2}$$

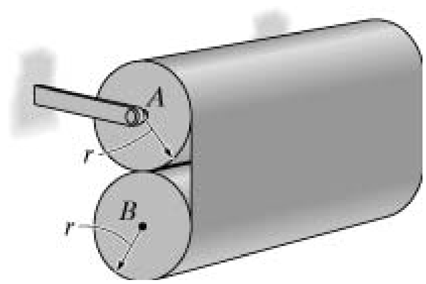
$$T_{AB} = 43.3\ \text{lb}$$

***Problem 17-104**

A long strip of paper is wrapped into two rolls, each having mass M. Roll A is pin-supported about its center whereas roll B is not centrally supported. If B is brought into contact with A and released from rest, determine the initial tension in the paper between the rolls and the angular acceleration of each roll. For the calculation, assume the rolls to be approximated by cylinders.

Given:

$M = 8\ \text{kg}$ $\quad r = 90\ \text{mm}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$



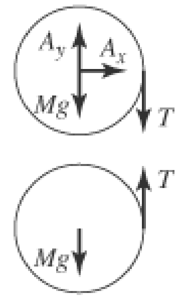
Solution: Guesses

$T = 1\ \text{N}$ $\quad a_{By} = 1\ \frac{\text{m}}{\text{s}^2}$ $\quad \alpha_A = 1\ \frac{\text{rad}}{\text{s}^2}$ $\quad \alpha_B = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$T r = \frac{1}{2} M r^2 \alpha_A \qquad T r = \frac{1}{2} M r^2 \alpha_B$$

$$T - M g = M a_{By} \qquad -\alpha_A r = a_{By} + \alpha_B r$$



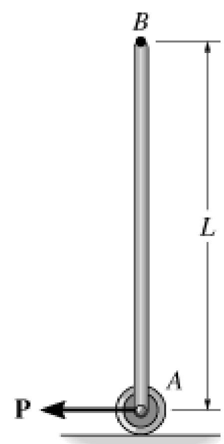
$$\begin{pmatrix} T \\ a_{By} \\ \alpha_A \\ \alpha_B \end{pmatrix} = \text{Find}(T, a_{By}, \alpha_A, \alpha_B) \qquad T = 15.7 \text{ N}$$

$$\begin{pmatrix} \alpha_A \\ \alpha_B \end{pmatrix} = \begin{pmatrix} 43.6 \\ 43.6 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

$$a_{By} = -7.848 \ \frac{\text{m}}{\text{s}^2}$$

Problem 17-105

The uniform bar of mass *m* and length *L* is balanced in the vertical position when the horizontal force **P** is applied to the roller at *A*. Determine the bar's initial angular acceleration and the acceleration of its top point *B*.



Solution:

$$-P = m a_x \qquad a_x = \frac{-P}{m}$$

$$-P\left(\frac{L}{2}\right) = \frac{1}{12} m L^2 \alpha \qquad \alpha = \frac{-6P}{mL}$$

$$a_B = a_x - \alpha\left(\frac{L}{2}\right) \qquad a_B = \frac{2P}{m}$$

positive means to the right

Problem 17-106

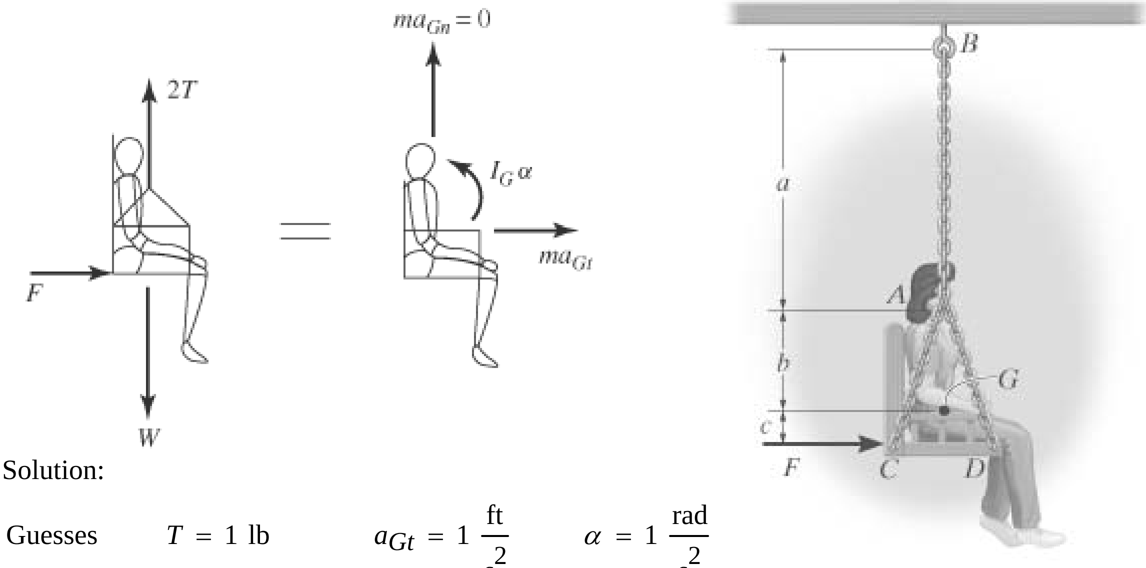
A woman sits in a rigid position in the middle of the swing. The combined weight of the woman and swing is *W* and the radius of gyration about the center of mass *G* is k_G. If a man pushes on the swing with a horizontal force **F** as shown, determine the initial angular acceleration and the tension in each of the two supporting chains *AB*. During the motion, assume that the chain segment *CAD* remains rigid. The swing is originally at rest.

Given:

$$W = 180 \text{ lb} \qquad a = 4 \text{ ft} \qquad g = 32.2 \ \frac{\text{ft}}{\text{s}^2}$$

$$k_G = 2.5 \text{ ft} \qquad b = 1.5 \text{ ft}$$

$$F = 20 \text{ lb} \qquad c = 0.4 \text{ ft}$$



Solution:

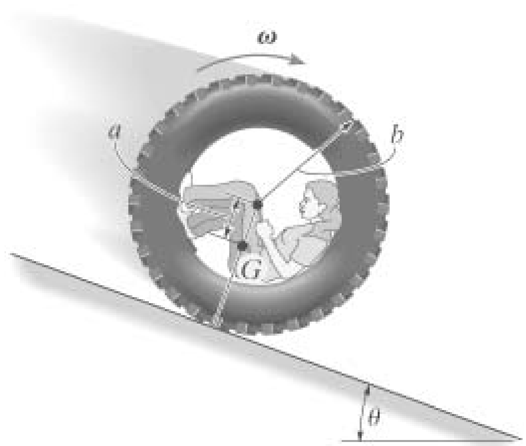
Guesses $\quad T = 1\ \text{lb} \qquad a_{Gt} = 1\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given $\quad F = \left(\frac{W}{g}\right)a_{Gt} \qquad 2T - W = 0 \qquad F\,c = \left(\frac{W}{g}\right)k_G{}^2\,\alpha$

$$\begin{pmatrix} T \\ a_{Gt} \\ \alpha \end{pmatrix} = \text{Find}(T, a_{Gt}, \alpha) \qquad a_{Gt} = 3.58\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 0.229\ \frac{\text{rad}}{\text{s}^2} \qquad T = 90.0\ \text{lb}$$

Problem 17-107

A girl sits snugly inside a large tire such that together the girl and tire have a total weight W, a center of mass at G, and a radius of gyration k_G about G. If the tire rolls freely down the incline, determine the normal and frictional forces it exerts on the ground when it is in the position shown and has an angular velocity ω. Assume that the tire does not slip as it rolls.



Given:

$W = 185\ \text{lb} \qquad b = 2\ \text{ft}$

$k_G = 1.65\ \text{ft} \qquad a = 0.75\ \text{ft}$

$\omega = 6\ \frac{\text{rad}}{\text{s}} \qquad \theta = 20\ \text{deg}$

Solution:

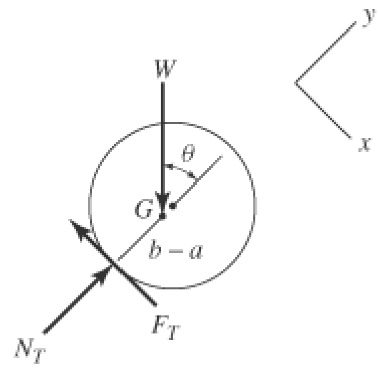
Guesses $\quad N_T = 1\ \text{lb} \qquad F_T = 1\ \text{lb}$

$\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$N_T - W\cos(\theta) = \frac{W}{g} a\omega^2 \qquad F_T(b-a) = \frac{W}{g} k_G^2 \alpha$$

$$F_T - W\sin(\theta) = \frac{-W}{g}(b-a)\alpha$$



$$\begin{pmatrix} N_T \\ F_T \\ \alpha \end{pmatrix} = \text{Find}(N_T, F_T, \alpha) \qquad \alpha = 3.2 \frac{\text{rad}}{\text{s}^2}$$

$$\begin{pmatrix} N_T \\ F_T \end{pmatrix} = \begin{pmatrix} 329.0 \\ 40.2 \end{pmatrix} \text{lb}$$

***Problem 17-108**

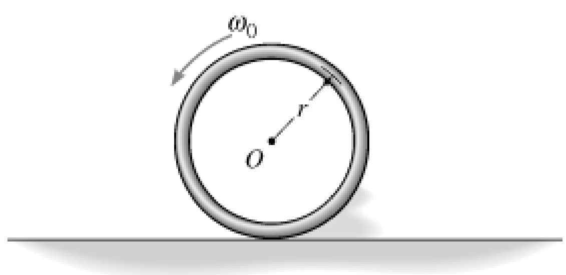
The hoop or thin ring of weight W is given an initial angular velocity ω_0 when it is placed on the surface. If the coefficient of kinetic friction between the hoop and the surface is μ_k, determine the distance the hoop moves before it stops slipping.

Given:

$$W = 10 \text{ lb} \qquad \mu_k = 0.3$$

$$\omega_0 = 6 \frac{\text{rad}}{\text{s}} \qquad r = 6 \text{ in}$$

$$g = 32.2 \frac{\text{ft}}{\text{s}^2}$$

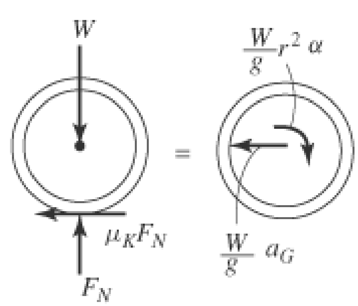


Solution:

$$F_N - W = 0 \qquad F_N = W \qquad F_N = 10 \text{lb}$$

$$\mu_k F_N = \left(\frac{W}{g}\right) a_G \qquad a_G = \mu_k g \qquad a_G = 9.66 \frac{\text{ft}}{\text{s}^2}$$

$$\mu_k F_N r = \left(\frac{W}{g}\right) r^2 \alpha \qquad \alpha = \frac{\mu_k g}{r} \qquad \alpha = 19.32 \frac{\text{rad}}{\text{s}^2}$$



When it stops slipping

$$v_G = \omega r$$

$$a_G t = (\omega_0 - \alpha t) r \qquad t = \frac{\omega_0 r}{a_G + \alpha r} \qquad t = 0.155 \text{ s}$$

$d = \frac{1}{2} a_G t^2$ $d = 1.398\ \text{in}$

Problem 17-109

The circular plate of weight W is suspended from a pin at A. If the pin is connected to a track which is given acceleration a_A, determine the horizontal and vertical components of reaction at A and the acceleration of the plate's mass center G. The plate is originally at rest.

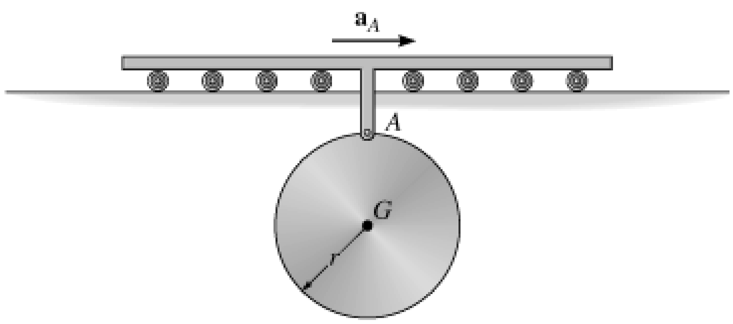
Given:

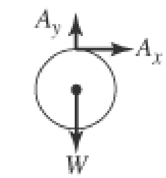
$W = 15\ \text{lb}$

$a_A = 3\ \frac{\text{ft}}{\text{s}^2}$

$r = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

Guesses $a_{Gx} = 1\ \frac{\text{ft}}{\text{s}^2}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $A_x = 1\ \text{lb}$ $A_y = 1\ \text{lb}$

Given $A_x = \frac{W}{g} a_{Gx}$ $A_y - W = 0$ $-A_x r = \frac{-1}{2} \frac{W}{g} r^2 \alpha$ $a_{Gx} + \alpha r = a_A$

$$\begin{pmatrix} a_{Gx} \\ \alpha \\ A_x \\ A_y \end{pmatrix} = \text{Find}(a_{Gx}, \alpha, A_x, A_y) \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0.466 \\ 15.000 \end{pmatrix} \text{lb} \qquad a_{Gx} = 1\ \frac{\text{ft}}{\text{s}^2}$$

$$\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$$

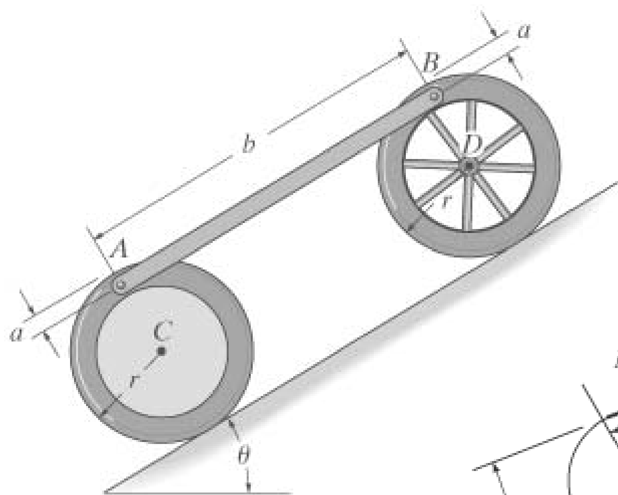
Problem 17-110

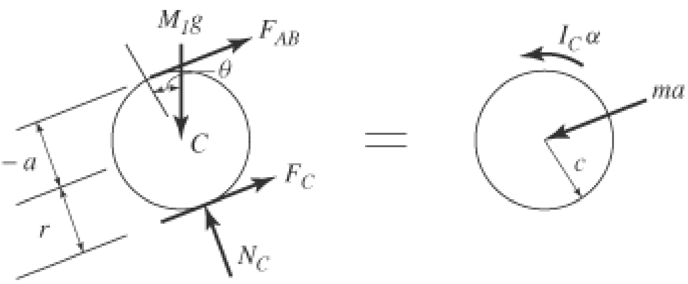
Wheel C has a mass M_1 and a radius of gyration k_C, whereas wheel D has a mass M_2 and a radius of gyration k_D. Determine the angular acceleration of each wheel at the instant shown. Neglect the mass of the link and assume that the assembly does not slip on the plane.

Given:

$M_1 = 60\ \text{kg}$ $k_C = 0.4\ \text{m}$ $r = 0.5\ \text{m}$ $b = 2\ \text{m}$

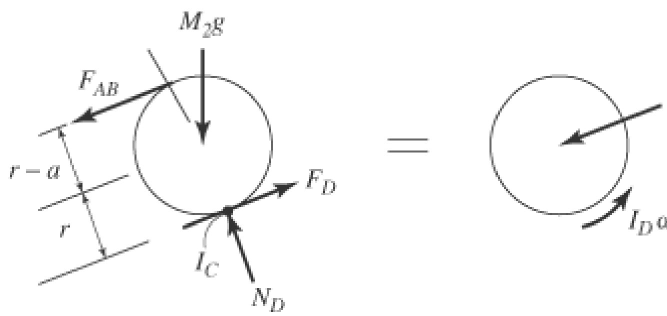
$M_2 = 40\ \text{kg}$ $k_D = 0.35\ \text{m}$ $a = 0.1\ \text{m}$ $\theta = 30\ \text{deg}$





Solution:

Both wheels have the same angular acceleration.



Guesses

$F_{AB} = 1 \text{ N}$

$\alpha = 1 \dfrac{\text{rad}}{\text{s}^2}$

Given

$$-F_{AB}(2r - a) + M_1 g \sin(\theta) r = M_1 k_C{}^2 \alpha + M_1 (r\alpha) r$$

$$F_{AB}(2r - a) + M_2 g \sin(\theta) r = M_2 k_D{}^2 \alpha + M_2 (r\alpha) r$$

$\begin{pmatrix} F_{AB} \\ \alpha \end{pmatrix} = \text{Find}(F_{AB}, \alpha)$ $\quad F_{AB} = -6.21 \text{ N}$ $\quad \alpha = 6.21 \dfrac{\text{rad}}{\text{s}^2}$

Problem 17-111

The assembly consists of a disk of mass m_D and a bar of mass m_b which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are μ_s and μ_k respectively. Neglect friction at B.

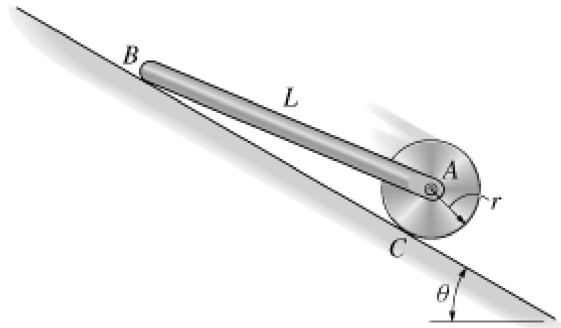
Given:

$m_D = 8\ \text{kg}$ $\quad L = 1\ \text{m}$

$m_b = 10\ \text{kg}$ $\quad r = 0.3\ \text{m}$

$\mu_s = 0.6$ $\quad \theta = 30\ \text{deg}$

$\mu_k = 0.4$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution: $\quad \phi = \operatorname{asin}\left(\dfrac{r}{L}\right)$

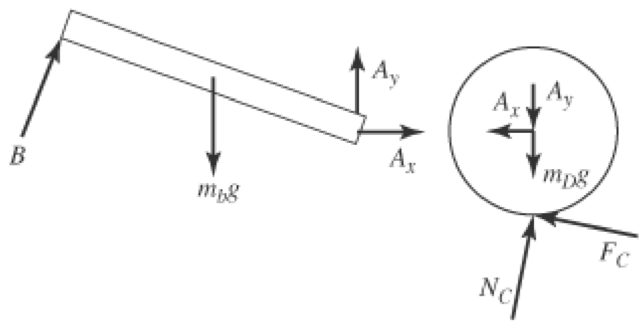
Assume no slip

Guesses

$N_C = 1\ \text{N}$ $\quad F_C = 1\ \text{N}$

$\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$ $\quad a_A = 1\ \dfrac{\text{m}}{\text{s}^2}$

$F_{max} = 1\ \text{N}$



Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta - \phi) - m_b g\frac{L}{2}\cos(\theta - \phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A\frac{r}{2}$$

$$-F_C + (m_D + m_b)g\sin(\theta) = (m_D + m_b)a_A$$

$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad a_A = r\alpha \qquad F_{max} = \mu_s N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \text{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 109.042 \\ 16.053 \\ 65.425 \end{pmatrix}\text{N} \qquad \alpha = 13.377\ \frac{\text{rad}}{\text{s}^2}$$

Since $F_C = 16.053\ \text{N}$ $< F_{max} = 65.425\ \text{N}$ then our no-slip assumption is correct.

Problem 17-112

The assembly consists of a disk of mass m_D and a bar of mass m_b which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are μ_s and μ_k respectively. Neglect friction at B. Solve if the bar is removed.

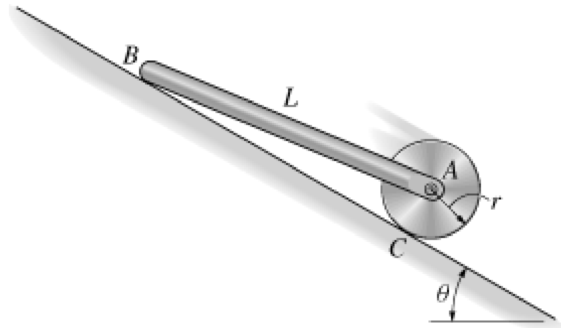
Given:

$m_D = 8 \text{ kg}$ $\quad L = 1 \text{ m}$

$m_b = 0 \text{ kg}$ $\quad r = 0.3 \text{ m}$

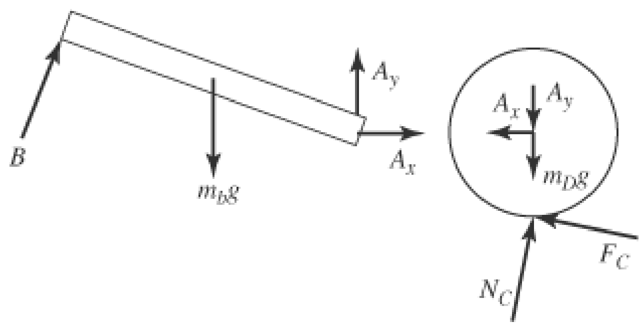
$\mu_s = 0.15$ $\quad \theta = 30 \text{ deg}$

$\mu_k = 0.1$ $\quad g = 9.81 \dfrac{\text{m}}{\text{s}^2}$



Solution: $\quad \phi = \operatorname{asin}\left(\dfrac{r}{L}\right)$

Assume no slip



Guesses

$N_C = 1 \text{ N}$ $\quad F_C = 1 \text{ N}$

$\alpha = 1 \dfrac{\text{rad}}{\text{s}^2}$ $\quad a_A = 1 \dfrac{\text{m}}{\text{s}^2}$

$F_{max} = 1 \text{ N}$

Given

$$N_C L \cos(\phi) - m_D g L \cos(\theta - \phi) - m_b g \frac{L}{2} \cos(\theta - \phi) = \frac{-1}{2} m_D r^2 \alpha - m_D a_A r - m_b a_A \frac{r}{2}$$

$$-F_C + (m_D + m_b) g \sin(\theta) = (m_D + m_b) a_A$$

$$F_C r = \frac{1}{2} m_D r^2 \alpha \qquad a_A = r\alpha \qquad F_{max} = \mu_s N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \operatorname{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 67.966 \\ 13.08 \\ 10.195 \end{pmatrix} \text{N} \qquad \alpha = 10.9 \frac{\text{rad}}{\text{s}^2}$$

Since $F_C = 13.08 \text{ N}$ $> F_{max} = 10.195 \text{ N}$ then our no-slip assumption is wrong and we know that slipping does occur.

Guesses

$N_C = 1 \text{ N}$ $\quad F_C = 1 \text{ N}$ $\quad \alpha = 1 \dfrac{\text{rad}}{\text{s}^2}$ $\quad a_A = 1 \dfrac{\text{m}}{\text{s}^2}$ $\quad F_{max} = 1 \text{ N}$

Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta - \phi) - m_b g \frac{L}{2}\cos(\theta - \phi) = \frac{-1}{2} m_D r^2 \alpha - m_D a_A r - m_b a_A \frac{r}{2}$$

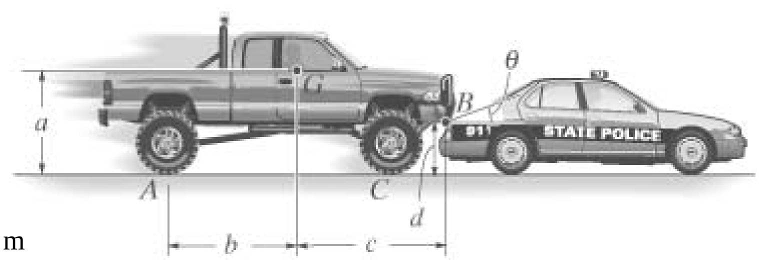
$$-F_C + (m_D + m_b) g \sin(\theta) = (m_D + m_b) a_A$$

$$F_C r = \frac{1}{2} m_D r^2 \alpha \qquad F_{max} = \mu_s N_C \qquad F_C = \mu_k N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \text{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 67.966 \\ 6.797 \\ 10.195 \end{pmatrix} \text{N} \qquad \alpha = 5.664 \frac{\text{rad}}{\text{s}^2}$$

Problem 17-113

A “lifted” truck can become a road hazard since the bumper is high enough to ride up a standard car in the event the car is rear-ended. As a model of this case consider the truck to have a mass M, a mass center G, and a radius of gyration k_G about G. Determine the horizontal and vertical components of acceleration of the mass center G, and the angular acceleration of the truck, at the moment its front wheels at C have just left the ground and its smooth front bumper begins to ride up the back of the stopped car so that point B has a velocity of v_B at angle θ from the horizontal. Assume the wheels are free to roll, and neglect the size of the wheels and the deformation of the material.



Units Used:

$\text{Mg} = 10^3 \text{ kg}$

$\text{kN} = 10^3 \text{ N}$

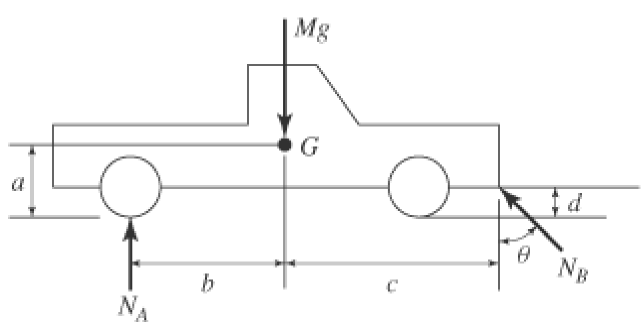
Given:

$M = 2.70 \text{ Mg}$ $\quad a = 1.3 \text{ m}$

$\theta = 20 \text{ deg}$ $\quad b = 1.6 \text{ m}$

$k_G = 1.45 \text{ m}$ $\quad c = 1.2 \text{ m}$

$v_B = 8 \frac{\text{m}}{\text{s}}$ $\quad d = 0.4 \text{ m}$

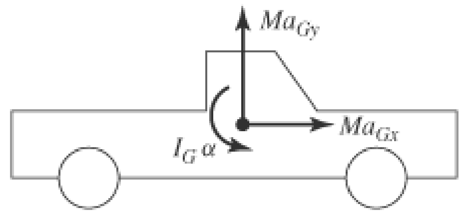


Solution:

Guesses $\quad v_A = 1 \frac{\text{m}}{\text{s}} \qquad \omega = 1 \frac{\text{rad}}{\text{s}}$

$a_B = 1 \frac{\text{m}}{\text{s}^2} \qquad \alpha = 1 \frac{\text{rad}}{\text{s}^2}$

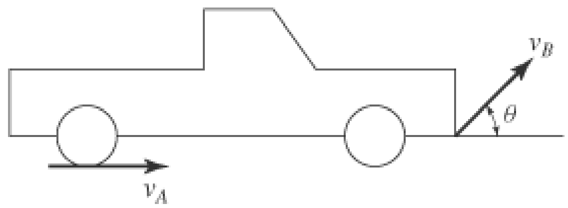
$a_{Gx} = 1\ \frac{m}{s^2} \quad a_{Gy} = 1\ \frac{m}{s^2}$



$N_A = 1\ N \quad N_B = 1\ N$

$a_A = 1\ \frac{m}{s^2}$

Given



$$N_A + N_B\cos(\theta) - Mg = Ma_{Gy}$$

$$-N_B\sin(\theta) = Ma_{Gx}$$

$$N_B\cos(\theta)c - N_B\sin(\theta)(a-d) - N_A b = Mk_G^2\alpha$$

$$\begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} = \begin{pmatrix} v_B\cos(\theta) \\ v_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} - \omega^2\begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} = \begin{pmatrix} a_B\cos(\theta) \\ a_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{Gx} \\ a_{Gy} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} c \\ -a+d \\ 0 \end{pmatrix} - \omega^2\begin{pmatrix} c \\ -a+d \\ 0 \end{pmatrix} = \begin{pmatrix} a_B\cos(\theta) \\ a_B\sin(\theta) \\ 0 \end{pmatrix}$$

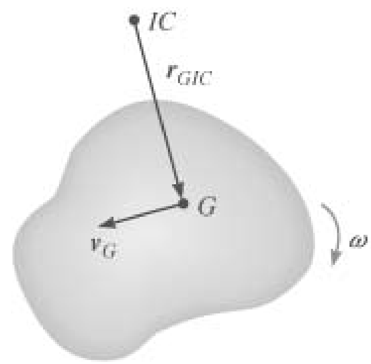
$$\begin{pmatrix} v_A \\ \omega \\ a_A \\ a_B \\ a_{Gx} \\ a_{Gy} \\ \alpha \\ N_A \\ N_B \end{pmatrix} = \text{Find}(v_A, \omega, a_A, a_B, a_{Gx}, a_{Gy}, \alpha, N_A, N_B)$$

$$\begin{pmatrix} a_{Gx} \\ a_{Gy} \end{pmatrix} = \begin{pmatrix} -1.82 \\ -1.69 \end{pmatrix} \frac{m}{s^2}$$

$$\omega = 0.977\ \frac{rad}{s} \quad \alpha = -0.283\ \frac{rad}{s^2}$$

$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} -0.664 \\ -3.431 \end{pmatrix} \frac{m}{s^2} \quad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 8.38 \\ 14.40 \end{pmatrix} kN$$

Problem 18-1



At a given instant the body of mass m has an angular velocity ω and its mass center has a velocity $\mathbf{v}_G$. Show that its kinetic energy can be represented as $T = 1/2\ I_{IC}\omega^2$, where I_{IC} is the moment of inertia of the body computed about the instantaneous axis of zero velocity, located a distance r_{GIC} from the mass center as shown.

Solution:

$$T = \left(\frac{1}{2}\right) m v_G^2 + \left(\frac{1}{2}\right)(I_G)\,\omega^2 \qquad \text{where } v_G = \omega\, r_{GIC}$$

$$T = \left(\frac{1}{2}\right) m \left(\omega r_{GIC}\right)^2 + \frac{1}{2} I_G \omega^2$$

$$T = \left(\frac{1}{2}\right)\left(m\, r_{GIC}^2 + I_G\right)\omega^2 \qquad \text{However } m(r_{GIC})^2 + I_G = I_{IC}$$

$$T = \left(\frac{1}{2}\right) I_{IC}\omega^2$$

Problem 18-2

The wheel is made from a thin ring of mass m_{ring} and two slender rods each of mass m_{rod}. If the torsional spring attached to the wheel's center has stiffness k, so that the torque on the center of the wheel is $M = -k\theta$, determine the maximum angular velocity of the wheel if it is rotated two revolutions and then released from rest.

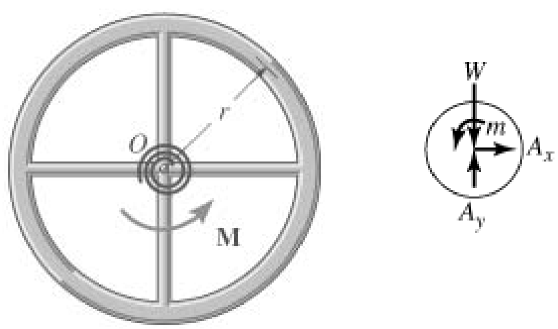
Given:

$$m_{ring} = 5\ \text{kg}$$

$$m_{rod} = 2\ \text{kg}$$

$$k = 2\ \frac{\text{N}\cdot\text{m}}{\text{rad}}$$

$$r = 0.5\ \text{m}$$



Solution:

$$I_O = 2\left[\frac{1}{12} m_{rod}(2r)^2\right] + m_{ring} r^2$$

$$I_O = 1.583\ \text{kg}\cdot\text{m}^2$$

$$T_1 + \Sigma U_{12} = T_2$$

$$0 + \int_{4\pi}^{0} -k\,\theta\,d\theta = \frac{1}{2} I_O \omega^2 \qquad \omega = \sqrt{\frac{k}{I_O}}\,4\pi \qquad \omega = 14.1\,\frac{\text{rad}}{\text{s}}$$

Problem 18-3

At the instant shown, the disk of weight W has counterclockwise angular velocity ω when its center has velocity v. Determine the kinetic energy of the disk at this instant.

Given:

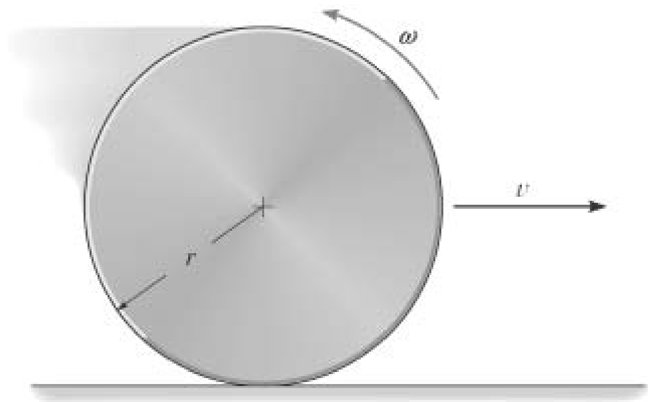
$W = 30\ \text{lb}$

$\omega = 5\ \frac{\text{rad}}{\text{s}}$

$v = 20\ \frac{\text{ft}}{\text{s}}$

$r = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

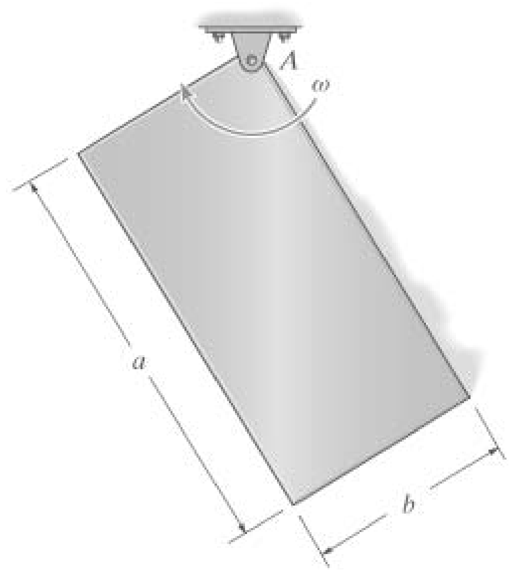


Solution:

$$T = \frac{1}{2}\left(\frac{1}{2}\frac{W}{g}r^2\right)\omega^2 + \frac{1}{2}\left(\frac{W}{g}\right)v^2 \qquad T = 210\,\text{ft}\cdot\text{lb}$$

***Problem 18-4**

The uniform rectangular plate has weight W. If the plate is pinned at A and has an angular velocity ω, determine the kinetic energy of the plate.



Given:

$W = 30\ \text{lb}$

$\omega = 3\ \frac{\text{rad}}{\text{s}}$

$a = 2\ \text{ft}$

$b = 1\ \text{ft}$

Solution:

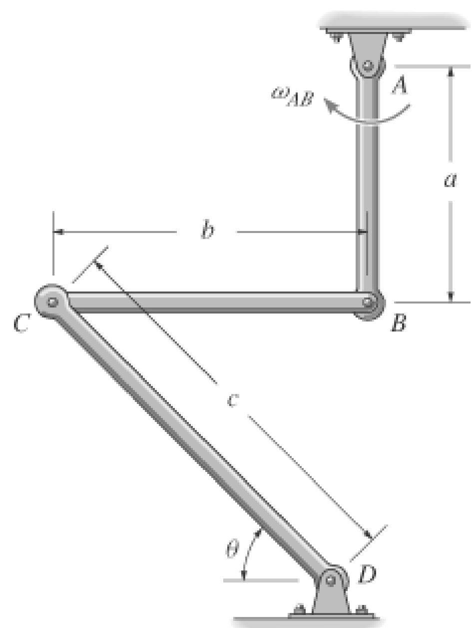
$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

$$T = \frac{1}{2}\left(\frac{W}{g}\right)\left(\omega\frac{\sqrt{b^2+a^2}}{2}\right)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{W}{g}\right)\left(b^2+a^2\right)\right]\omega^2$$

$T = 6.99\ \mathrm{ft \cdot lb}$

Problem 18-5

At the instant shown, link *AB* has angular velocity ω_{AB}. If each link is considered as a uniform slender bar with weight density γ, determine the total kinetic energy of the system.



Given: $\omega_{AB} = 2\ \frac{\mathrm{rad}}{\mathrm{s}}$ $a = 3\ \mathrm{in}$

$\gamma = 0.5\ \frac{\mathrm{lb}}{\mathrm{in}}$ $b = 4\ \mathrm{in}$

$\theta = 45\ \mathrm{deg}$ $c = 5\ \mathrm{in}$

Solution: $\rho = \frac{\gamma}{g}$

Guesses

$\omega_{BC} = 1\ \frac{\mathrm{rad}}{\mathrm{s}}$ $\omega_{CD} = 1\ \frac{\mathrm{rad}}{\mathrm{s}}$

$v_{Gx} = 1\ \frac{\mathrm{in}}{\mathrm{s}}$ $v_{Gy} = 1\ \frac{\mathrm{in}}{\mathrm{s}}$ $T = 1\ \mathrm{lb \cdot ft}$

Given

$$\begin{pmatrix}0\\0\\-\omega_{AB}\end{pmatrix}\times\begin{pmatrix}0\\-a\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{BC}\end{pmatrix}\times\begin{pmatrix}-b\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{CD}\end{pmatrix}\times\begin{pmatrix}c\cos(\theta)\\-c\sin(\theta)\\0\end{pmatrix}=0$$

$$\begin{pmatrix}0\\0\\-\omega_{AB}\end{pmatrix}\times\begin{pmatrix}0\\-a\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{BC}\end{pmatrix}\times\begin{pmatrix}\frac{-b}{2}\\0\\0\end{pmatrix}=\begin{pmatrix}v_{Gx}\\v_{Gy}\\0\end{pmatrix}$$

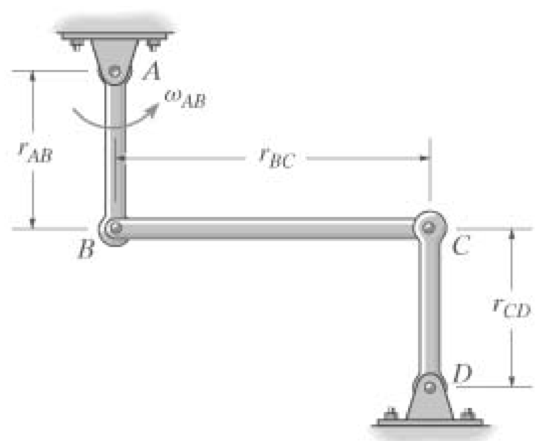
$$T = \frac{1}{2}\left(\frac{\rho a^3}{3}\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{\rho b^3}{12}\right)\omega_{BC}^2 + \frac{1}{2}\rho b\left(v_{Gx}^2+v_{Gy}^2\right) + \frac{1}{2}\left(\frac{\rho c^3}{3}\right)\omega_{CD}^2$$

$$\begin{pmatrix} \omega_{BC} \\ \omega_{CD} \\ v_{Gx} \\ v_{Gy} \\ T \end{pmatrix} = \text{Find}(\omega_{BC}, \omega_{CD}, v_{Gx}, v_{Gy}, T) \qquad \begin{pmatrix} \omega_{BC} \\ \omega_{CD} \end{pmatrix} = \begin{pmatrix} 1.5 \\ 1.697 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} v_{Gx} \\ v_{Gy} \end{pmatrix} = \begin{pmatrix} -0.5 \\ -0.25 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$T = 0.0188\ \text{ft}\cdot\text{lb}$

Problem 18-6

Determine the kinetic energy of the system of three links. Links *AB* and *CD* each have weight W_1, and link *BC* has weight W_2.



Given:

$W_1 = 10\ \text{lb}$

$W_2 = 20\ \text{lb}$

$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}}$

$r_{AB} = 1\ \text{ft}$

$r_{BC} = 2\ \text{ft}$

$r_{CD} = 1\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$\omega_{BC} = 0\ \frac{\text{rad}}{\text{s}} \qquad \omega_{CD} = \omega_{AB}\left(\frac{r_{AB}}{r_{CD}}\right)$

$$T = \frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{r_{AB}^2}{3}\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)(\omega_{AB} r_{AB})^2 + \frac{1}{2}\left(\frac{W_1}{g}\right)\frac{r_{CD}^2}{3}\omega_{CD}^2$$

$T = 10.4\ \text{ft}\cdot\text{lb}$

Problem 18-7

The mechanism consists of two rods, *AB* and *BC*, which have weights W_1 and W_2, respectively, and a block at *C* of weight W_3. Determine the kinetic energy of the system at the instant shown, when the block is moving at speed v_C.

Given:

$W_1 = 10\ \text{lb}$

$W_2 = 20\ \text{lb}$

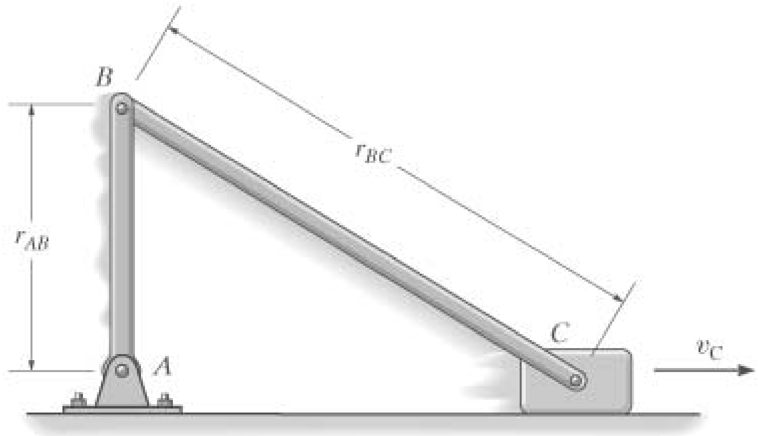
$W_3 = 4\ \text{lb}$

$r_{AB} = 2\ \text{ft}$

$r_{BC} = 4\ \text{ft}$

$v_C = 3\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$\omega_{BC} = 0\ \frac{\text{rad}}{\text{s}} \qquad \omega_{AB} = \frac{v_C}{r_{AB}}$$

$$T = \frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{r_{AB}^2}{3}\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)v_C^2 + \frac{1}{2}\left(\frac{W_3}{g}\right)v_C^2 \qquad T = 3.82\ \text{lb}\cdot\text{ft}$$

***Problem 18-8**

The bar of weight W is pinned at its center O and connected to a torsional spring. The spring has a stiffness k, so that the torque developed is $M = k\theta$. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$. Use the prinicple of work and energy.

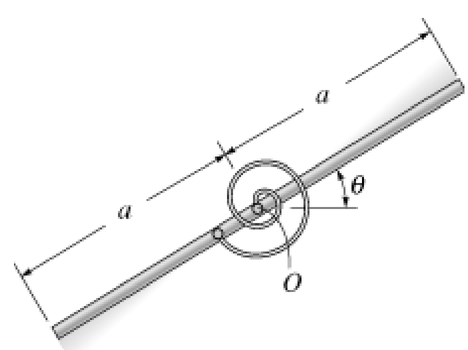
Given:

$W = 10\ \text{lb}$

$k = 5\ \frac{\text{lb}\cdot\text{ft}}{\text{rad}}$

$a = 1\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$\theta_0 = 90\ \text{deg} \qquad \theta_f = 0\ \text{deg}$

Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $\frac{1}{2}k\theta_0^{\ 2} = \frac{1}{2}k\theta_f^{\ 2} + \frac{1}{2}\left(\frac{W}{g}\right)\frac{(2a)^2}{12}\ \omega^2$

$\omega = \text{Find}(\omega)$ $\omega = 10.9\ \frac{\text{rad}}{\text{s}}$

Problem 18-9

A force P is applied to the cable which causes the reel of mass M to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. The radius of gyration of the reel about its center axis is k_G.

Given:

$P = 20\ \text{N}$

$M = 175\ \text{kg}$

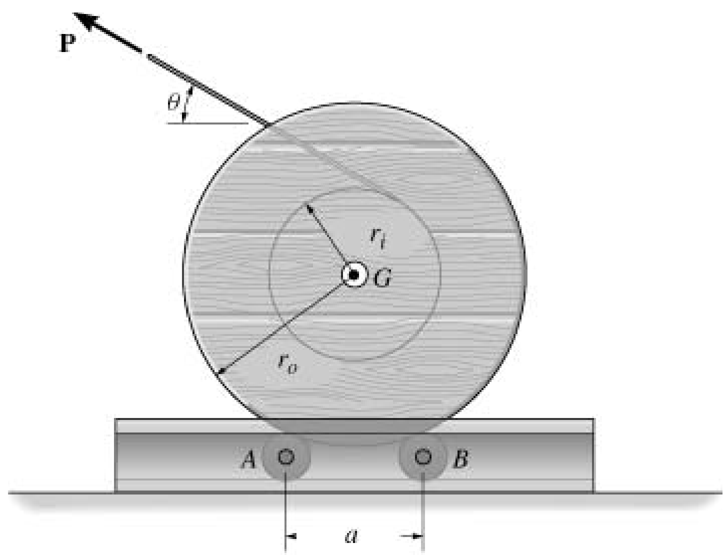
$k_G = 0.42\ \text{m}$

$\theta = 30\ \text{deg}$

$r_i = 250\ \text{mm}$

$r_0 = 500\ \text{mm}$

$a = 400\ \text{mm}$



Solution:

$$0 + P2(2\pi r_i) = \frac{1}{2}M k_G^{\ 2}\ \omega^2$$

$$\omega = \sqrt{\frac{8\pi P\, r_i}{M\, k_G^{\ 2}}} \qquad \omega = 2.02\ \frac{\text{rad}}{\text{s}}$$

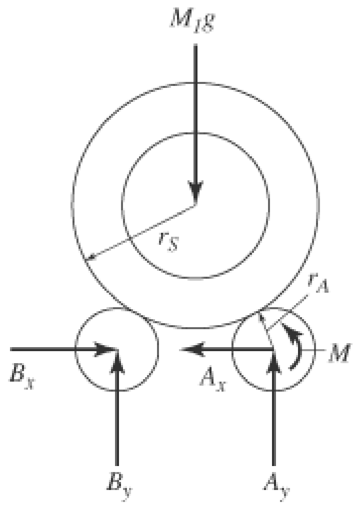
Problem 18-10

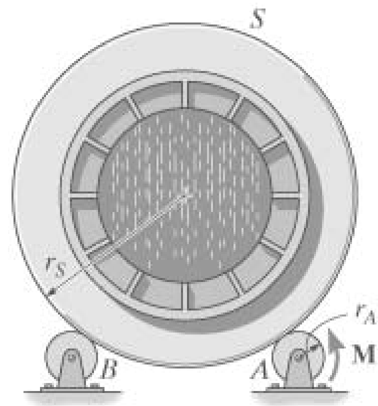
The rotary screen S is used to wash limestone. When empty it has a mass M_1 and a radius of gyration k_G. Rotation is achieved by applying a torque M about the drive wheel A. If no slipping occurs at A and the supporting wheel at B is free to roll, determine the angular velocity of the screen after it has rotated n revolutions. Neglect the mass of A and B.

Unit Used:

$\text{rev} = 2\pi \text{ rad}$

Given:





$M_1 = 800 \text{ kg}$

$k_G = 1.75 \text{ m}$

$M = 280 \text{ N·m}$

$r_S = 2 \text{ m}$

$r_A = 0.3 \text{ m}$

$n = 5 \text{ rev}$

Solution:

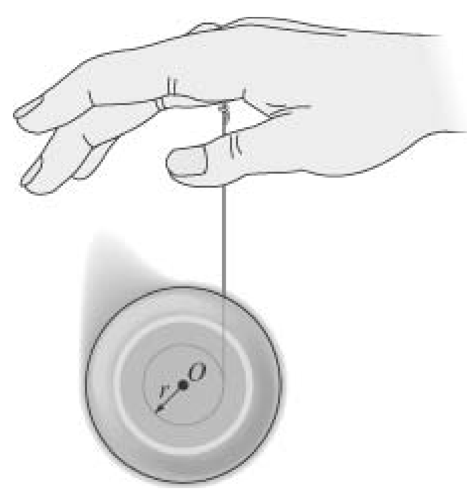
$$M\left(\frac{r_S}{r_A}n\right) = \frac{1}{2}M_1 k_G^2 \omega^2$$

$$\omega = \sqrt{\frac{2M r_S n}{r_A M_1 k_G^2}} \qquad \omega = 6.92 \frac{\text{rad}}{\text{s}}$$

Problem 18-11

A yo-yo has weight W and radius of gyration k_O. If it is released from rest, determine how far it must descend in order to attain angular velocity ω. Neglect the mass of the string and assume that the string is wound around the central peg such that the mean radius at which it unravels is r.

Given:



$W = 0.3 \text{ lb}$

$k_O = 0.06 \text{ ft}$

$\omega = 70 \frac{\text{rad}}{\text{s}}$

$r = 0.02 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution:

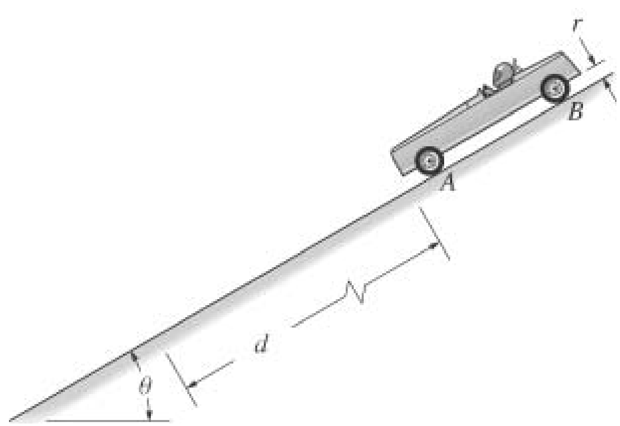
$$0 + W h = \frac{1}{2}\left(\frac{W}{g}\right)(r\omega)^2 + \frac{1}{2}\left(\frac{W}{g}k_O^{\ 2}\right)\omega^2$$

$$h = \frac{r^2 + k_O^{\ 2}}{2g}\omega^2 \qquad h = 0.304\,\text{ft}$$

***Problem 18-12**

The soap-box car has weight W_c including the passenger but *excluding* its four wheels. Each wheel has weight W_w, radius r, and radius of gyration k, computed about an axis passing through the wheel's axle. Determine the car's speed after it has traveled a distance d starting from rest. The wheels roll without slipping. Neglect air resistance.

Given:



$W_c = 110\ \text{lb}$

$W_w = 5\ \text{lb}$

$r = 0.5\ \text{ft}$

$k = 0.3\ \text{ft}$

$d = 100\ \text{ft}$

$\theta = 30\ \text{deg}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

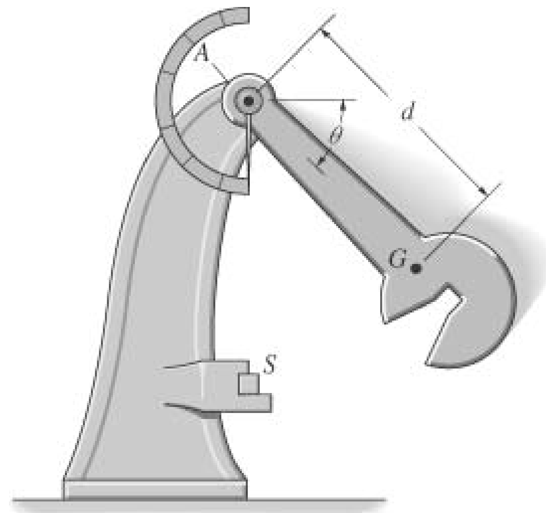
$$\left(W_c + 4W_w\right)d\sin(\theta) = \frac{1}{2}\left(\frac{W_c + 4W_w}{g}\right)v^2 + \frac{1}{2}4\left(\frac{W_w}{g}k^2\right)\left(\frac{v}{r}\right)^2$$

$$v = \sqrt{\frac{2\left(W_c + 4W_w\right)d\sin(\theta)g}{W_c + 4W_w + 4W_w\frac{k^2}{r^2}}} \qquad v = 55.2\,\frac{\text{ft}}{\text{s}}$$

Problem 18-13

The pendulum of the Charpy impact machine has mass M and radius of gyration k_A. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen S, $\theta = 90°$.

Given:



$M = 50$ kg

$k_A = 1.75$ m

$d = 1.25$ m

$g = 9.81 \frac{m}{s^2}$

Solution:

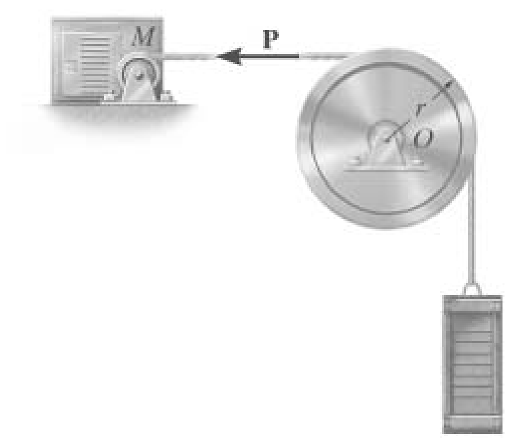
$$0 + M g d = \frac{1}{2} M k_A^2 \omega_2^2$$

$$\omega_2 = \sqrt{\frac{2 g d}{k_A^2}} \qquad \omega_2 = 2.83 \frac{rad}{s}$$

Problem 18-14

The pulley of mass M_p has a radius of gyration about O of k_O. If a motor M supplies a force to the cable of $P = a\,(b - ce^{-dx})$, where x is the amount of cable wound up, determine the speed of the crate of mass M_c when it has been hoisted a distance h starting from rest. Neglect the mass of the cable and assume the cable does not slip on the pulley.

Given:



$M_p = 10$ kg $\qquad a = 800$ N

$M_c = 50$ kg $\qquad b = 3$

$k_O = 0.21$ m $\qquad c = 2$

$r = 0.3$ m $\qquad d = \frac{1}{m}$

$h = 2$ m

Solution:

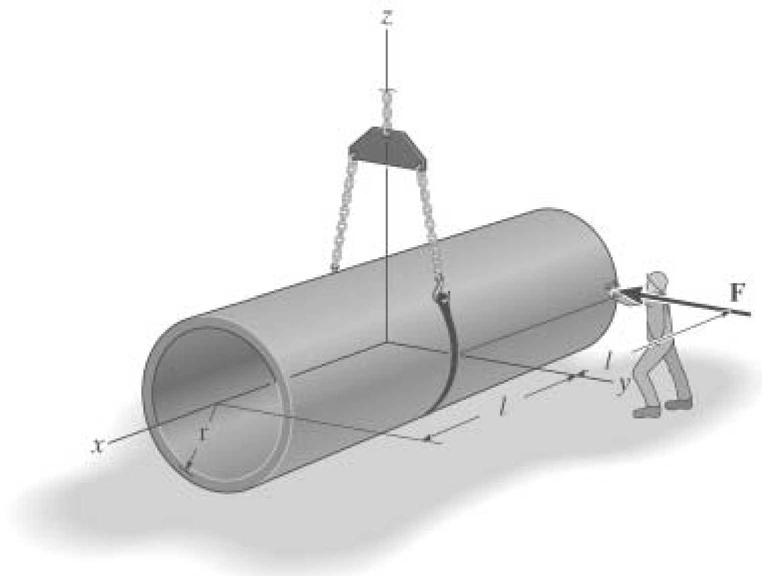
Guesses $\qquad v_c = 1 \frac{m}{s}$

Given $\displaystyle\int_0^h a\left(b - c\,e^{-d\,x}\right)\,dx = \frac{1}{2}M_p\,k_O^{\,2}\left(\frac{v_c}{r}\right)^2 + \frac{1}{2}M_c\,v_c^{\,2} + M_c\,g\,h$

$v_c = \mathrm{Find}(v_c)$ $\qquad v_c = 9.419\ \dfrac{\mathrm{m}}{\mathrm{s}}$

Problem 18-15

The uniform pipe has a mass M and radius of gyration about the z axis of k_G. If the worker pushes on it with a horizontal force F, applied perpendicular to the pipe, determine the pipe's angular velocity when it has rotated through angle θ about the z axis, starting from rest. Assume the pipe does not swing.



Units Used:

$\mathrm{Mg} = 10^3\ \mathrm{kg}$

Given:

$M = 16\ \mathrm{Mg}$ $\quad \theta = 90\ \mathrm{deg}$

$k_G = 2.7\ \mathrm{m}$ $\quad r = 0.75\ \mathrm{m}$

$F = 50\ \mathrm{N}$ $\quad l = 3\ \mathrm{m}$

Solution:

$$0 + F\,l\theta = \frac{1}{2}M\,k_G^{\,2}\,\omega^2$$

$\omega = \dfrac{1}{M}\dfrac{\sqrt{M\,F\,l\pi}}{k_G}$ $\qquad \omega = 0.0636\ \dfrac{\mathrm{rad}}{\mathrm{s}}$

***Problem 18-16**

The slender rod of mass m_{rod} is subjected to the force and couple moment. When it is in the position shown it has angular velocity ω_1. Determine its angular velocity at the instant it has rotated downward 90°. The force is always applied perpendicular to the axis of the rod. Motion occurs in the vertical plane.

Given:

$m_{rod} = 4\ \text{kg}$

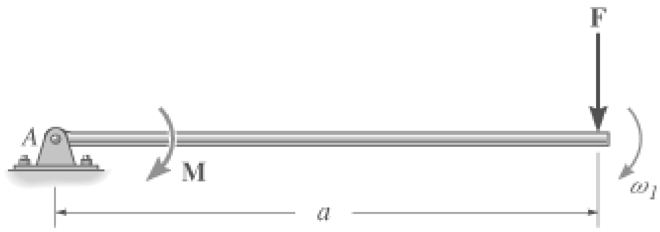
$\omega_1 = 6\ \frac{\text{rad}}{\text{s}}$

$F = 15\ \text{N}$

$M = 40\ \text{N}\cdot\text{m}$

$a = 3\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: Guess $\omega_2 = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_1^{\ 2} + F\,a\left(\frac{\pi}{2}\right) + m_{rod}g\left(\frac{a}{2}\right) + M\frac{\pi}{2} = \frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_2^{\ 2}$$

$\omega_2 = \text{Find}(\omega_2)$ $\omega_2 = 8.25\ \frac{\text{rad}}{\text{s}}$

Problem 18-17

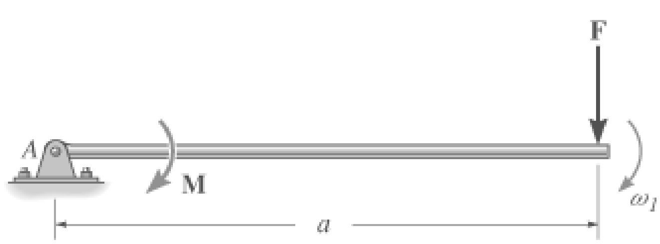
The slender rod of mass M is subjected to the force and couple moment. When the rod is in the position shown it has angular velocity ω_1. Determine its angular velocity at the instant it has rotated 360°. The force is always applied perpendicular to the axis of the rod and motion occurs in the vertical plane.

Given:

$m_{rod} = 4\ \text{kg}$ $M = 40\ \text{N}\cdot\text{m}$

$\omega_1 = 6\ \frac{\text{rad}}{\text{s}}$ $a = 3\ \text{m}$

$F = 15\ \text{N}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guess $\omega_2 = 1\ \frac{\text{rad}}{\text{s}}$

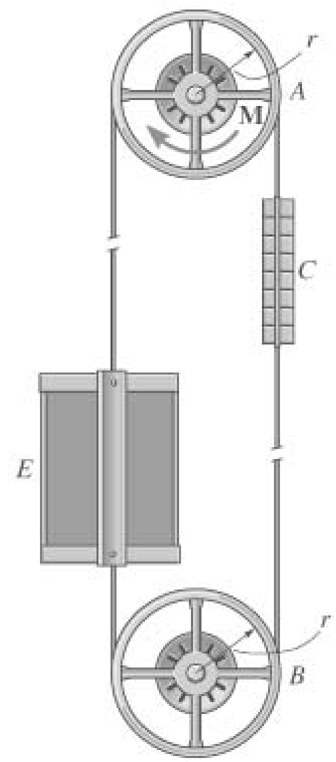
Given

$$\frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_1^2 + F\,a2\pi + M2\pi = \frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_2^2$$

$\omega_2 = \text{Find}(\omega_2)$ $\qquad \omega_2 = 11.2\ \frac{\text{rad}}{\text{s}}$

Problem 18-18

The elevator car E has mass m_E and the counterweight C has mass m_C. If a motor turns the driving sheave A with constant torque M, determine the speed of the elevator when it has ascended a distance d starting from rest. Each sheave A and B has mass m_S and radius of gyration k about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.



Units Used: $\quad \text{Mg} = 1000\ \text{kg}$

Given:

$m_E = 1.80\ \text{Mg} \qquad d = 10\ \text{m}$

$m_C = 2.30\ \text{Mg} \qquad r = 0.35\ \text{m}$

$m_S = 150\ \text{kg} \qquad k = 0.2\ \text{m}$

$M = 100\ \text{N}\cdot\text{m} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$$M\frac{d}{r} - (m_E - m_C)g\,d = \frac{1}{2}\left(m_E + m_C + 2m_S\frac{k^2}{r^2}\right)v^2$$

$$v = \sqrt{\frac{2\left[M\frac{d}{r} - (m_E - m_C)g\,d\right]}{m_E + m_C + 2m_S\frac{k^2}{r^2}}} \qquad v = 4.973\ \frac{\text{m}}{\text{s}}$$

Problem 18-19

The elevator car E has mass m_E and the counterweight C has mass m_C. If a motor turns the driving sheave A with torque $a\theta^2 + b$, determine the speed of the elevator when it has ascended a distance d starting from rest. Each sheave A and B has mass m_S and radius of gyration k about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.

Units Used:

$Mg = 1000\ kg$

Given:

$m_E = 1.80\ Mg$

$m_C = 2.30\ Mg$

$m_S = 150\ kg$

$a = 0.06\ N{\cdot}m$

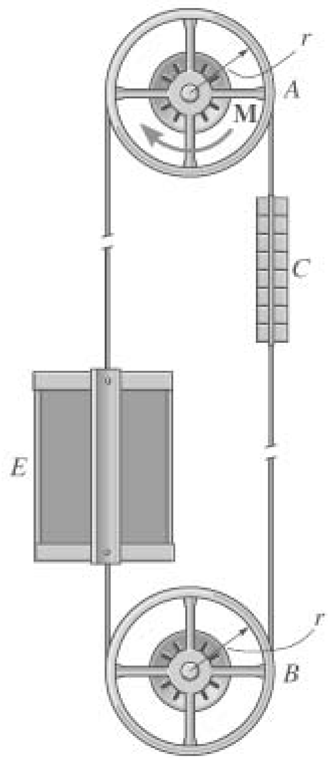
$b = 7.5\ N{\cdot}m$

$d = 12\ m$

$r = 0.35\ m$

$k = 0.2\ m$

$g = 9.81\ \frac{m}{s^2}$



Solution:

Guess $v = 1\ \frac{m}{s}$

Given $\displaystyle\int_0^{\frac{d}{r}} a\theta^2 + b\ d\theta - (m_E - m_C)g\,d = \frac{1}{2}\left(m_E + m_C + 2m_S\frac{k^2}{r^2}\right)v^2$

$v = \text{Find}(v)$ $v = 5.343\ \frac{m}{s}$

***Problem 18-20**

The wheel has a mass M_1 and a radius of gyration k_O. A motor supplies a torque **M** = $(a\theta + b)$, about the drive shaft at O. Determine the speed of the loading car, which has a mass M_2, after it travels a distance $s = d$. Initially the car is at rest when $s = 0$ and $\theta = 0°$. Neglect the mass of the attached cable and the mass of the car's wheels.

Given:

$M_1 = 100\ \text{kg}$

$M_2 = 300\ \text{kg}$

$d = 4\ \text{m}$

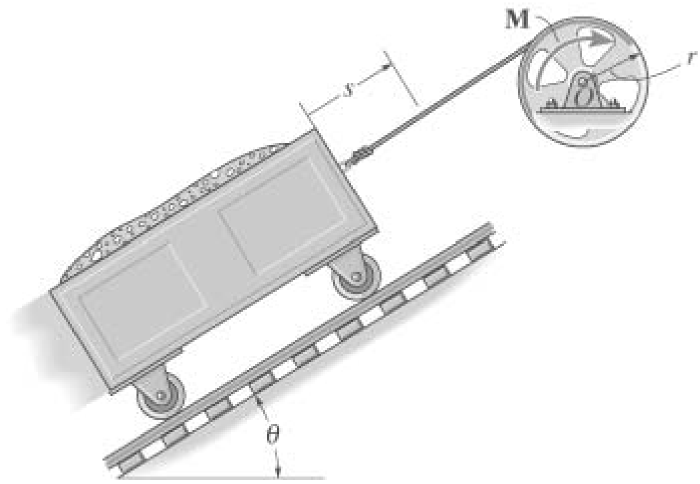
$a = 40\ \text{N}\cdot\text{m}$

$b = 900\ \text{N}\cdot\text{m}$

$r = 0.3\ \text{m}$

$k_O = 0.2\ \text{m}$

$\theta = 30\ \text{deg}$



Solution:

Guess $v = 1\ \dfrac{\text{m}}{\text{s}}$

Given $$\int_0^{\frac{d}{r}} (a\theta + b)\, d\theta = \frac{1}{2}M_2 v^2 + \frac{1}{2}M_1 k_O^2\left(\frac{v}{r}\right)^2 + M_2\, g\, d \sin(\theta)$$

$v = \text{Find}(v)$ $\quad v = 7.49\ \dfrac{\text{m}}{\text{s}}$

Problem 18-21

The gear has a weight W and a radius of gyration k_G. If the spring is unstretched when the torque M is applied, determine the gear's angular velocity after its mass center G has moved to the left a distance d.

Given:

$W = 15\ \text{lb}$

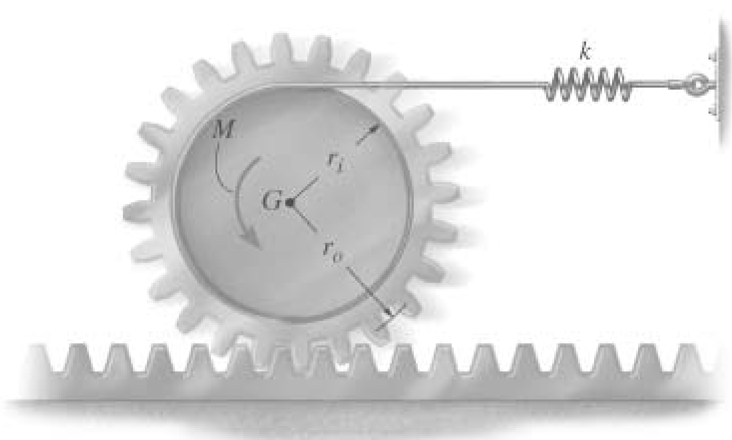
$M = 6\ \text{lb}\cdot\text{ft}$

$r_o = 0.5\ \text{ft}$

$r_i = 0.4\ \text{ft}$

$d = 2\ \text{ft}$

$k = 3\ \dfrac{\text{lb}}{\text{ft}}$



$k_G = 0.375$ ft

Solution:

Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $M\left(\frac{d}{r_o}\right) = \frac{1}{2}\left(\frac{W}{g}\right)(\omega r_o)^2 + \frac{1}{2}\left(\frac{W}{g}\right)k_G^{\ 2}\omega^2 + \frac{1}{2}k\left(\frac{r_i + r_o}{r_o}d\right)^2$

$\omega = \text{Find}(\omega)$ $\omega = 7.08\ \frac{\text{rad}}{\text{s}}$

Problem 18-22

The disk of mass m_d is originally at rest, and the spring holds it in equilibrium. A couple moment M is then applied to the disk as shown. Determine its angular velocity at the instant its mass center G has moved distance d down along the inclined plane. The disk rolls without slipping.

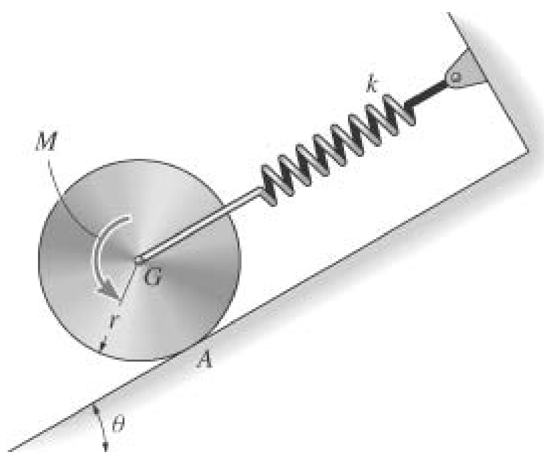
Given:

$m_d = 20$ kg $\theta = 30$ deg

$M = 30$ N·m $r = 0.2$ m

$d = 0.8$ m $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$k = 150\ \frac{\text{N}}{\text{m}}$



Solution: Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Initial stretch in the spring $k d_0 = m_d g \sin(\theta)$

$d_0 = \frac{m_d g \sin(\theta)}{k}$ $d_0 = 0.654$ m

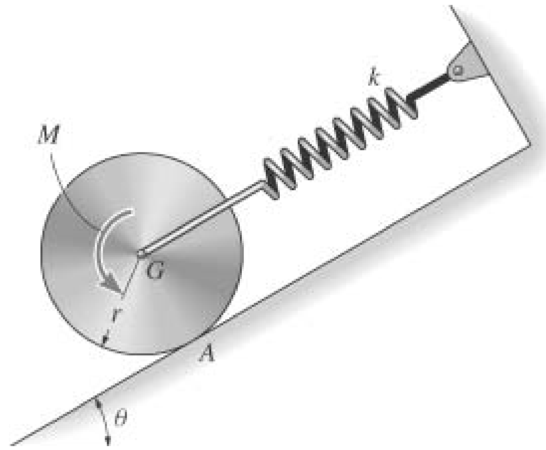
Given

$$M\frac{d}{r} + m_d g d \sin(\theta) - \frac{k}{2}\left[(d + d_0)^2 - d_0^{\ 2}\right] = \frac{1}{2}m_d(\omega r)^2 + \frac{1}{2}\left(\frac{1}{2}m_d r^2\right)\omega^2$$

$\omega = \text{Find}(\omega)$ $\quad\quad$ $\omega = 11.0\,\frac{\text{rad}}{\text{s}}$

Problem 18-23

The disk of mass m_d is originally at rest, and the spring holds it in equilibrium. A couple moment M is then applied to the disk as shown. Determine how far the center of mass of the disk travels down along the incline, measured from the equilibrium position, before it stops. The disk rolls without slipping.



Given:

$m_d = 20\text{ kg}$

$M = 30\text{ N}\cdot\text{m}$

$k = 150\,\frac{\text{N}}{\text{m}}$

$\theta = 30\text{ deg}$

$r = 0.2\text{ m}$

$g = 9.81\,\frac{\text{m}}{\text{s}^2}$

Solution: Guess $d = 3\text{ m}$

Initial stretch in the spring $\quad k\,d_0 = m_d\,g\sin(\theta)$

$d_0 = \dfrac{m_d\,g\sin(\theta)}{k}$ $\quad\quad$ $d_0 = 0.654\text{ m}$

Given $\quad M\frac{d}{r} + m_d\,g\,d\sin(\theta) - \frac{k}{2}\left[(d+d_0)^2 - d_0^{\,2}\right] = 0$

$d = \text{Find}(d)$ $\quad\quad$ $d = 2\text{ m}$

***Problem 18-24**

The linkage consists of two rods AB and CD each of weight W_1 and bar AD of weight W_2. When $\theta = 0$, rod AB is rotating with angular velocity ω_O. If rod CD is subjected to a couple moment M

and bar AD is subjected to a horizontal force P as shown, determine ω_{AB} at the instant $\theta = \theta_1$.

Given:

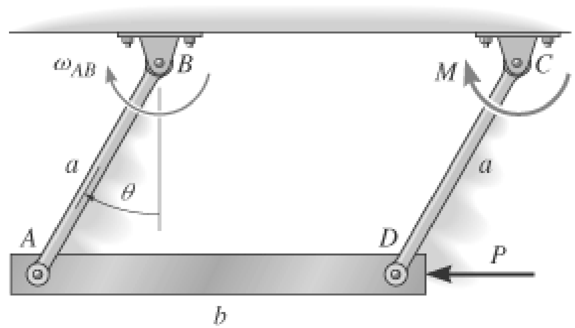
$W_1 = 8\ \text{lb}$ $\quad a = 2\ \text{ft}$

$W_2 = 10\ \text{lb}$ $\quad b = 3\ \text{ft}$

$\omega_0 = 2\ \frac{\text{rad}}{\text{s}}$ $\quad \theta_1 = 90\ \text{deg}$

$P = 20\ \text{lb}$ $\quad M = 15\ \text{lb}\cdot\text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$U = P\,a\sin(\theta_1) + M\theta_1 - 2W_1\frac{a}{2}(1-\cos(\theta_1)) - W_2\,a(1-\cos(\theta_1))$$

Guess $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\frac{1}{2}2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega_0^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)(a\omega_0)^2 + U = \frac{1}{2}2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)(a\omega)^2$$

$\omega = \text{Find}(\omega)$ $\quad \omega = 5.739\ \frac{\text{rad}}{\text{s}}$

Problem 18-25

The linkage consists of two rods AB and CD each of weight W_1 and bar AD of weight W_2. When $\theta = 0$, rod AB is rotating with angular velocity ω_O. If rod CD is subjected to a couple moment M and bar AD is subjected to a horizontal force P as shown, determine ω_{AB} at the instant $\theta = \theta_1$.

Given:

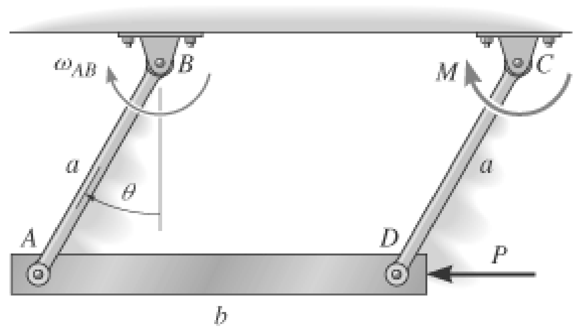
$W_1 = 8 \text{ lb}$ $a = 2 \text{ ft}$

$W_2 = 10 \text{ lb}$ $b = 3 \text{ ft}$

$\omega_0 = 2 \frac{\text{rad}}{\text{s}}$ $\theta_1 = 45 \text{ deg}$

$M = 15 \text{ lb}\cdot\text{ft}$ $P = 20 \text{ lb}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

$$U = P a \sin(\theta_1) + M\theta_1 - 2W_1 \frac{a}{2}(1 - \cos(\theta_1)) - W_2 a(1 - \cos(\theta_1))$$

Guess $\omega = 1 \frac{\text{rad}}{\text{s}}$

Given $$\frac{1}{2} 2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega_0^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)(a\omega_0)^2 + U = \frac{1}{2} 2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)(a\omega)^2$$

$\omega = \text{Find}(\omega)$ $\omega = 5.916 \frac{\text{rad}}{\text{s}}$

Problem 18-26

The spool has weight *W* and radius of gyration k_G. A horizontal force *P* is applied to a cable wrapped around its inner core. If the spool is originally at rest, determine its angular velocity after the mass center *G* has moved distance *d* to the left. The spool rolls without slipping. Neglect the mass of the cable.

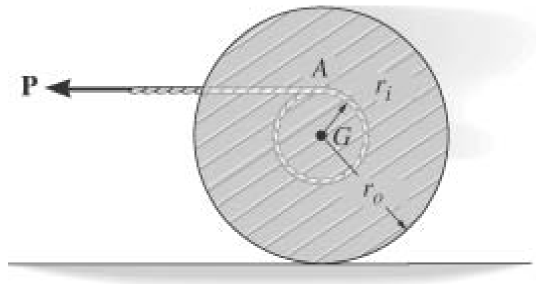
Given:

$W = 500 \text{ lb}$ $d = 6 \text{ ft}$

$k_G = 1.75\ \text{ft}$ $\quad r_i = 0.8\ \text{ft}$

$P = 15\ \text{lb}$ $\quad r_o = 2.4\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

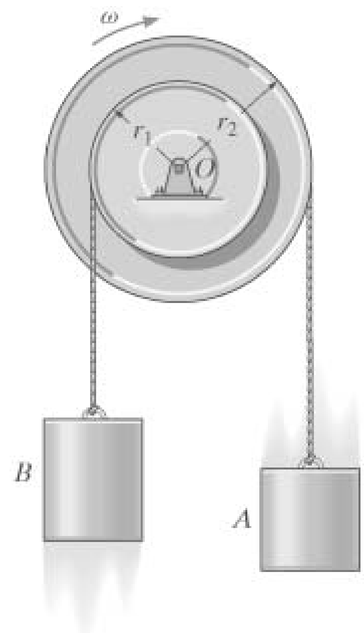


Solution: Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $P\left(\dfrac{r_o + r_i}{r_o}\right)d = \dfrac{1}{2}\left(\dfrac{W}{g}\right)k_G{}^2\,\omega^2 + \dfrac{1}{2}\left(\dfrac{W}{g}\right)(r_o\omega)^2$ $\quad \omega = \text{Find}(\omega)$ $\quad \omega = 1.324\ \dfrac{\text{rad}}{\text{s}}$

Problem 18-27

The double pulley consists of two parts that are attached to one another. It has a weight W_p and a centroidal radius of gyration k_O and is turning with an angular velocity ω clockwise. Determine the kinetic energy of the system. Assume that neither cable slips on the pulley.



Given:

$W_P = 50\ \text{lb}$ $\quad r_1 = 0.5\ \text{ft}$

$W_A = 20\ \text{lb}$ $\quad r_2 = 1\ \text{ft}$

$W_B = 30\ \text{lb}$ $\quad k_O = 0.6\ \text{ft}$

$\omega = 20\ \dfrac{\text{rad}}{\text{s}}$

Solution:

$$K_E = \frac{1}{2}I\omega^2 + \frac{1}{2}W_A v_A{}^2 + \frac{1}{2}W_B v_B{}^2$$

$$K_E = \frac{1}{2}\left(\frac{W_P}{g}\right)k_O{}^2\,\omega^2 + \frac{1}{2}\left(\frac{W_A}{g}\right)(r_2\omega)^2 + \frac{1}{2}\left(\frac{W_B}{g}\right)(r_1\omega)^2$$

$K_E = 283\,\text{ft}\cdot\text{lb}$

***Problem 18-28**

The system consists of disk A of weight W_A, slender rod BC of weight W_{BC}, and smooth collar C of weight W_C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e. $\theta = 0°$. The system is released from rest when $\theta = \theta_0$.

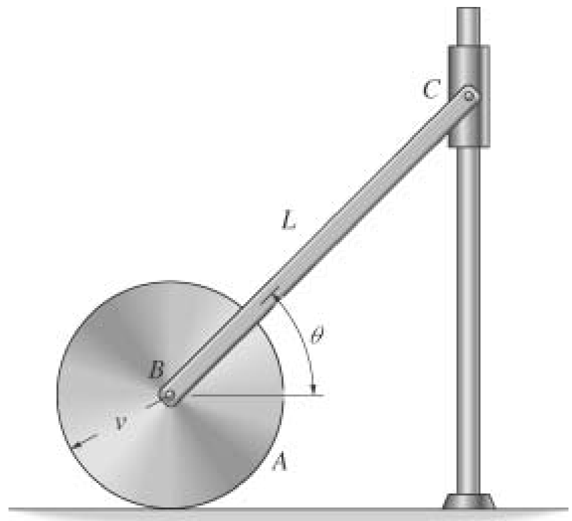
Given:

$W_A = 20\ \text{lb}$ $\quad L = 3\ \text{ft}$

$W_{BC} = 4\ \text{lb}$ $\quad r = 0.8\ \text{ft}$

$W_C = 1\ \text{lb}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$\theta_0 = 45\ \text{deg}$



Solution:

Guess $\quad v_C = 1\ \dfrac{\text{ft}}{\text{s}}$

Given

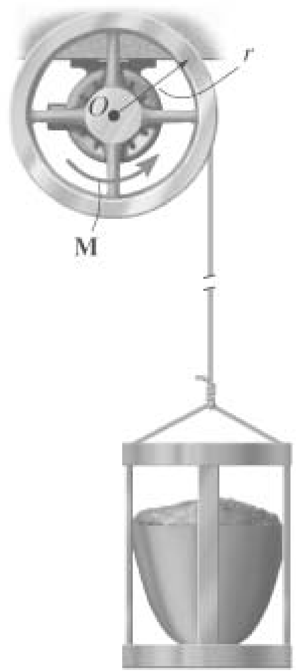
$$W_{BC}\frac{L}{2}\cos(\theta_0) + W_C L\cos(\theta_0) = \frac{1}{2}\left(\frac{W_C}{g}\right)v_C^{\,2} + \frac{1}{2}\left(\frac{W_{BC}}{g}\frac{L^2}{3}\right)\left(\frac{v_C}{L}\right)^2$$

$v_C = \text{Find}(v_C) \qquad v_C = 13.3\ \dfrac{\text{ft}}{\text{s}}$

Problem 18-29

The cement bucket of weight W_1 is hoisted using a motor that supplies a torque **M** to the axle of the wheel. If the wheel has a weight W_2 and a radius of gyration about O of k_O, determine the speed of the bucket when it has been hoisted a distance h starting from rest.

Given:



$W_1 = 1500\ \text{lb}$

$W_2 = 115\ \text{lb}$

$M = 2000\ \text{lb}\cdot\text{ft}$

$k_O = 0.95\ \text{ft}$

$h = 10\ \text{ft}$

$r = 1.25\ \text{ft}$

Solution:

Guess $\quad v = 1\ \dfrac{\text{ft}}{\text{s}}$

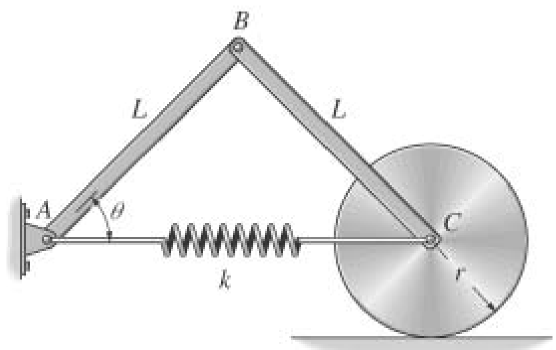
Given $\quad M\dfrac{h}{r} = \dfrac{1}{2}\left(\dfrac{W_1}{g}\right)v^2 + \dfrac{1}{2}\left(\dfrac{W_2}{g}\right)k_O{}^2\left(\dfrac{v}{r}\right)^2 + W_1 h$

$v = \text{Find}(v) \qquad v = 6.41\ \dfrac{\text{ft}}{\text{s}}$

Problem 18-30

The assembly consists of two slender rods each of weight W_r and a disk of weight W_d. If the spring is unstretched when $\theta = \theta_1$ and the assembly is released from rest at this position, determine the angular velocity of rod AB at the instant $\theta = 0$. The disk rolls without slipping.

Given:



$W_r = 15\ \text{lb}$

$W_d = 20\ \text{lb}$

$\theta_1 = 45\ \text{deg}$

$k = 4\ \dfrac{\text{lb}}{\text{ft}}$

$L = 3\ \text{ft}$

$r = 1\ \text{ft}$

Solution:

Guess $\quad \omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $$2W_r\left(\frac{L}{2}\right)\sin(\theta_1) - \frac{1}{2}k\left(2L - 2L\cos(\theta_1)\right)^2 = 2\frac{1}{2}\left(\frac{1}{3}\frac{W_r}{g}L^2\right)\omega^2$$

$\omega = \text{Find}(\omega)$ $\omega = 4.284\,\frac{\text{rad}}{\text{s}}$

Problem 18-31

The uniform door has mass M and can be treated as a thin plate having the dimensions shown. If it is connected to a torsional spring at A, which has stiffness k, determine the required initial twist of the spring in radians so that the door has an angular velocity ω when it closes at $\theta = 0°$ after being opened at $\theta = 90°$ and released from rest. *Hint:* For a torsional spring $M = k\theta$, where k is the stiffness and θ is the angle of twist.

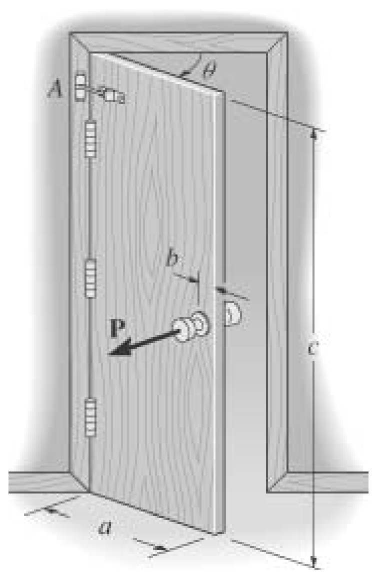
Given:

$M = 20\ \text{kg}$ $a = 0.8\ \text{m}$

$k = 80\ \frac{\text{N}\cdot\text{m}}{\text{rad}}$ $b = 0.1\ \text{m}$

$\omega = 12\ \frac{\text{rad}}{\text{s}}$ $c = 2\ \text{m}$

$P = 0\ \text{N}$



Solution:

Guess $\theta_0 = 1\ \text{rad}$

Given

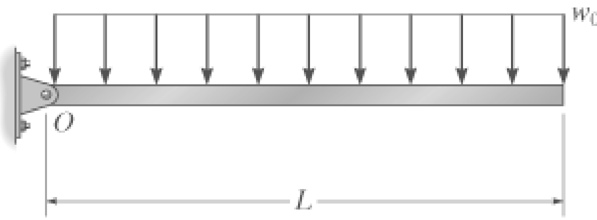
$$\int_{\theta_0+90\,\text{deg}}^{\theta_0} -k\,\theta\,d\theta = \frac{1}{2}\frac{1}{3}M a^2\omega^2$$

$\theta_0 = \text{Find}(\theta_0)$ $\theta_0 = 1.659\,\text{rad}$

***Problem 18-32**

The uniform slender bar has a mass m and a length L. It is subjected to a uniform distributed load w_0 which is always directed perpendicular to the axis of the bar. If it is released from the position shown, determine its angular velocity at the instant it has rotated 90°. Solve the problem for rotation in (a) the horizontal plane, and (b) the vertical plane.

Solution:



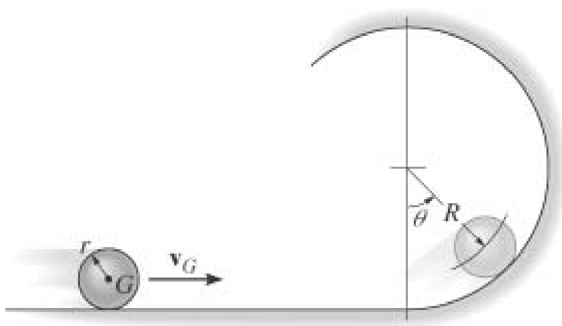
$$\int_0^{\frac{\pi}{2}} \int_0^L x w_0 \, dx \, d\theta = \frac{1}{4}\pi L^2 w_0$$

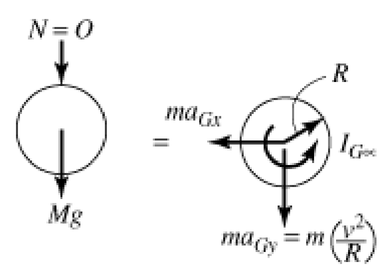
(a) $\frac{1}{4}\pi L^2 w_0 = \frac{1}{2}\left(\frac{1}{3}m L^2\right)\omega^2$ $\omega = \sqrt{\frac{3\pi w_0}{2m}}$

(b) $\frac{1}{4}\pi L^2 w_0 = \frac{1}{2}\left(\frac{1}{3}m L^2\right)\omega^2 - m g\frac{L}{2}$ $\omega = \sqrt{\frac{3\pi w_0}{2m} + \frac{3g}{L}}$

Problem 18-33

A ball of mass *m* and radius *r* is cast onto the horizontal surface such that it rolls without slipping. Determine the minimum speed v_G of its mass center *G* so that it rolls completely around the loop of radius *R* + *r* without leaving the track.





Solution:

$$m g = m\left(\frac{v^2}{R}\right) \qquad v^2 = g R$$

$$\frac{1}{2}\left(\frac{2}{5}m r^2\right)\left(\frac{v_G}{r}\right)^2 + \frac{1}{2}m v_G^2 - m g 2R = \frac{1}{2}\left(\frac{2}{5}m r^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}m v^2$$

$$\frac{1}{5}v_G^2 + \frac{1}{2}v_G^2 = 2g R + \frac{1}{5}g R + \frac{1}{2}g R \qquad v_G = 3\sqrt{\frac{3}{7}g R}$$

Problem 18-34

The beam has weight W and is being raised to a vertical position by pulling very slowly on its bottom end A. If the cord fails when $\theta = \theta_1$ and the beam is essentially at rest, determine the speed of A at the instant cord BC becomes vertical. Neglect friction and the mass of the cords, and treat the beam as a slender rod.

Given:

$W = 1500 \text{ lb}$

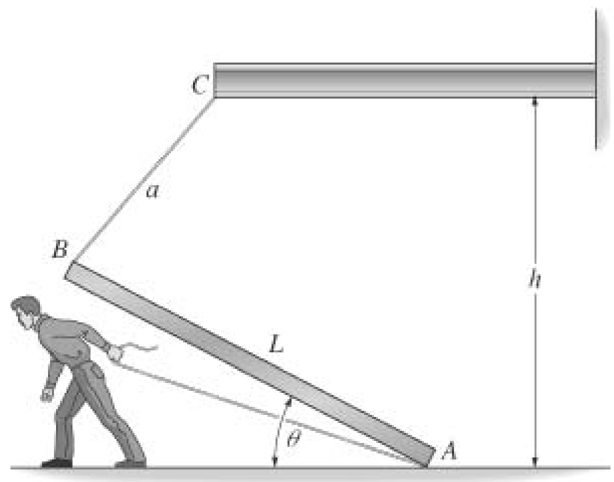
$\theta_1 = 60 \text{ deg}$

$L = 13 \text{ ft}$

$h = 12 \text{ ft}$

$a = 7 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



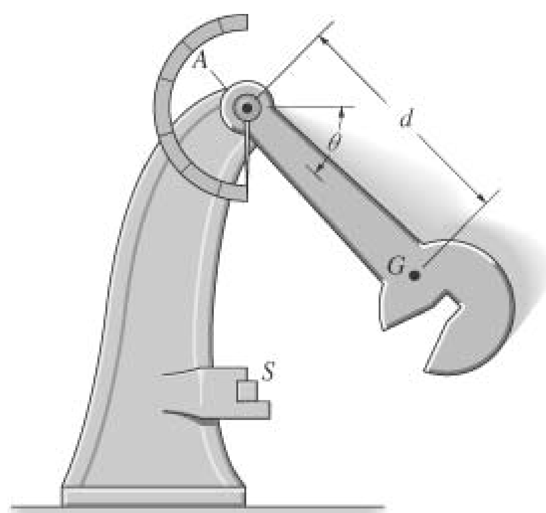
Solution:

$$W\left[\frac{L}{2}\sin(\theta_1) - \left(\frac{h-a}{2}\right)\right] = \frac{1}{2}\left(\frac{W}{g}\right)v_A^2$$

$$v_A = \sqrt{2g\left[\frac{L}{2}\sin(\theta_1) - \left(\frac{h-a}{2}\right)\right]} \qquad v_A = 14.2 \frac{\text{ft}}{\text{s}}$$

Problem 18-35

The pendulum of the Charpy impact machine has mass M and radius of gyration k_A. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen S, $\theta = 90°$, using the conservation of energy equation.



Given:

$M = 50 \text{ kg}$

$k_A = 1.75 \text{ m}$

$d = 1.25 \text{ m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

$$0 + M g d = 0 + \frac{1}{2} M k_A^2 \omega_2^2 \qquad \omega_2 = \sqrt{\frac{2 g d}{k_A^2}}$$

$$\omega_2 = 2.83 \frac{\text{rad}}{\text{s}}$$

***Problem 18-36**

The soap-box car has weight W_c including the passenger but *excluding* its four wheels. Each wheel has weight W_w radius r, and radius of gyration k, computed about an axis passing through the wheel's axle. Determine the car's speed after it has traveled distance d starting from rest. The wheels roll without slipping. Neglect air resistance. Solve using conservation of energy.

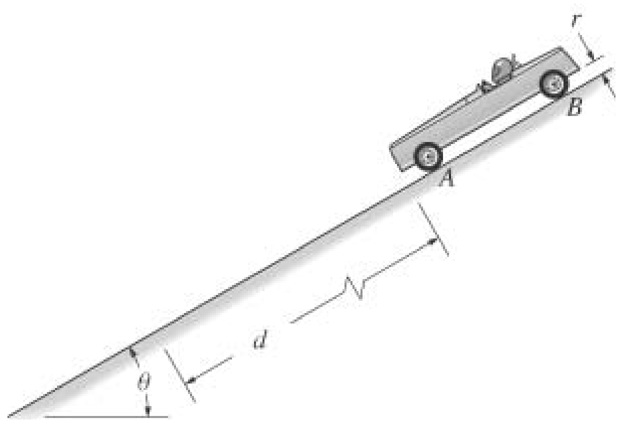
Given:

$W_c = 110$ lb $\quad d = 100$ ft

$W_w = 5$ lb $\quad \theta = 30$ deg

$r = 0.5$ ft $\quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$k = 0.3$ ft



Solution:

$$0 + (W_c + 4W_w) d \sin(\theta) = 0 + \frac{1}{2}\left(\frac{W_c + 4W_w}{g}\right) v^2 + \frac{1}{2} 4\left(\frac{W_w}{g} k^2\right)\left(\frac{v}{r}\right)^2$$

$$v = \sqrt{\frac{2(W_c + 4W_w) d \sin(\theta) g}{W_c + 4W_w + 4W_w\left(\frac{k^2}{r^2}\right)}} \qquad v = 55.2 \frac{\text{ft}}{\text{s}}$$

Problem 18-37

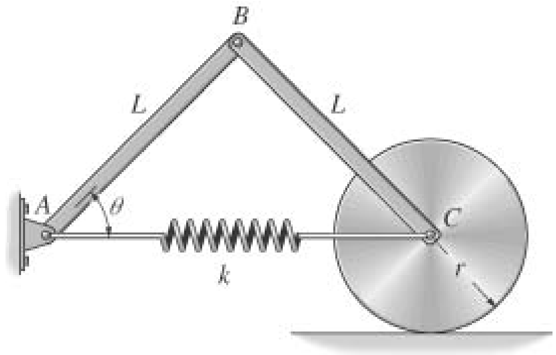
The assembly consists of two slender rods each of weight W_r and a disk of weight W_d. If the spring is unstretched when $\theta = \theta_1$ and the assembly is released from rest at this position, determine the angular velocity of rod AB at the instant $\theta = 0$. The disk rolls without slipping. Solve using the conservation of energy.

Given:

$W_r = 15\ \text{lb}$ $\quad k = 4\ \dfrac{\text{lb}}{\text{ft}}$

$W_d = 20\ \text{lb}$ $\quad L = 3\ \text{ft}$

$\theta_1 = 45\ \text{deg}$ $\quad r = 1\ \text{ft}$



Solution: Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

$$0 + 2W_r\left(\frac{L}{2}\right)\sin(\theta_1) = 2\frac{1}{2}\left[\frac{1}{3}\left(\frac{W_r}{g}\right)L^2\right]\omega^2 + \frac{1}{2}k\left(2L - 2L\cos(\theta_1)\right)^2$$

$\omega = \text{Find}(\omega)$ $\qquad \omega = 4.284\ \dfrac{\text{rad}}{\text{s}}$

Problem 18-38

A yo-yo has weight W and radius of gyration k_O. If it is released from rest, determine how far it must descend in order to attain angular velocity ω. Neglect the mass of the string and assume that the string is wound around the central peg such that the mean radius at which it unravels is r. Solve using the conservation of energy.

Given:

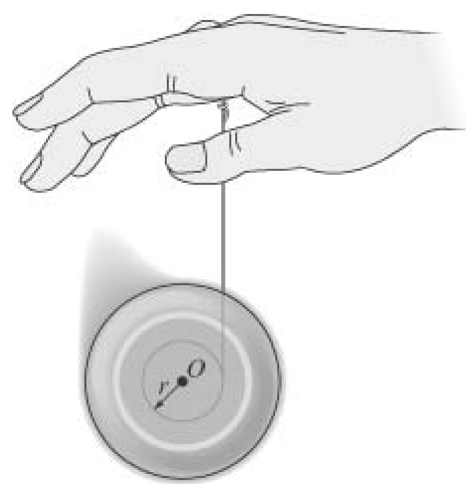
$W = 0.3\ \text{lb}$

$k_O = 0.06\ \text{ft}$

$\omega = 70\ \dfrac{\text{rad}}{\text{s}}$

$r = 0.02\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

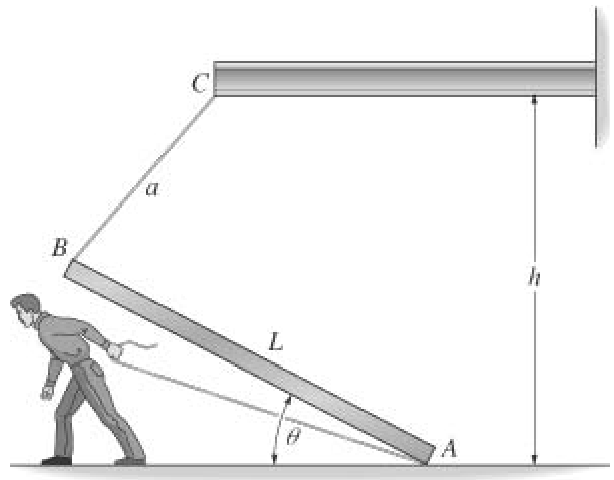
$$0 + Ws = \frac{1}{2}\left(\frac{W}{g}\right)(r\omega)^2 + \frac{1}{2}\left(\frac{W}{g}k_O^2\right)\omega^2 + 0$$

$s = \left(\dfrac{r^2 + k_O^2}{2g}\right)\omega^2$ $\qquad s = 0.304\ \text{ft}$

Problem 18-39

The beam has weight W and is being raised to a vertical position by pulling very slowly on its bottom end *A*. If the cord fails when $\theta = \theta_1$ and the beam is essentially at rest, determine the speed of *A* at the instant cord *BC* becomes vertical. Neglect friction and the mass of the cords, and treat the beam as a slender rod. Solve using the conservation of energy.

Given:



$W = 1500\ \text{lb}$

$\theta_1 = 60\ \text{deg}$

$L = 13\ \text{ft}$

$h = 12\ \text{ft}$

$a = 7\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

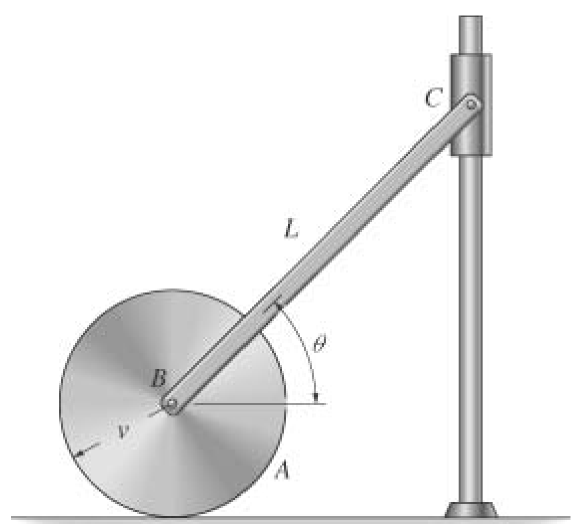
Solution:

$$0 + W\left(\frac{L}{2}\right)\sin(\theta_1) = \frac{1}{2}\left(\frac{W}{g}\right)v_A^2 + W\left(\frac{h-a}{2}\right)$$

$$v_A = \sqrt{2g\left[\frac{L}{2}\sin(\theta_1) - \left(\frac{h-a}{2}\right)\right]} \qquad v_A = 14.2\ \frac{\text{ft}}{\text{s}}$$

***Problem 18-40**

The system consists of disk *A* of weight W_A, slender rod *BC* of weight W_{BC}, and smooth collar *C* of weight W_C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e. $\theta = 0°$. The system is released from rest when $\theta = \theta_0$. Solve using the conservation of energy.



Given:

$W_A = 20\ \text{lb}$ $\quad L = 3\ \text{ft}$

$W_{BC} = 4\ \text{lb}$ $\quad r = 0.8\ \text{ft}$

$W_C = 1\ \text{lb} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$\theta_0 = 45\ \text{deg}$

Solution:

Guess $\quad v_C = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$0 + W_{BC}\left(\frac{L}{2}\right)\cos(\theta_0) + W_C L \cos(\theta_0) = \frac{1}{2}\left(\frac{W_C}{g}\right)v_C^2 + \frac{1}{2}\left(\frac{W_{BC}}{g}\frac{L^2}{3}\right)\left(\frac{v_C}{L}\right)^2 + 0$$

$v_C = \text{Find}(v_C) \qquad v_C = 13.3\ \frac{\text{ft}}{\text{s}}$

Problem 18-41

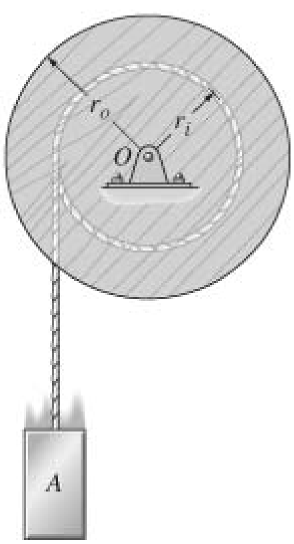
The spool has mass m_S and radius of gyration k_O. If block A of mass m_A is released from rest, determine the distance the block must fall in order for the spool to have angular velocity ω. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.

Given:

$m_S = 50\ \text{kg} \qquad r_i = 0.2\ \text{m} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$m_A = 20\ \text{kg} \qquad r_o = 0.3\ \text{m}$

$\omega = 5\ \frac{\text{rad}}{\text{s}} \qquad k_O = 0.280\ \text{m}$



Solution:

Guesses $\quad d = 1\ \text{m} \qquad T = 1\ \text{N}$

Given

$$0 + 0 = \frac{1}{2}m_S k_O^2\,\omega^2 + \frac{1}{2}m_A(r_i\omega)^2 - m_A g\,d$$

$$0 + 0 - T\,d = \frac{1}{2}m_A(r_i\omega)^2 - m_A g\,d$$

$\begin{pmatrix} d \\ T \end{pmatrix} = \text{Find}(d, T) \qquad d = 0.301\ \text{m} \qquad T = 163\ \text{N}$

Problem 18-42

When slender bar *AB* of mass *M* is horizontal it is at rest and the spring is unstretched. Determine the stiffness *k* of the spring so that the motion of the bar is momentarily stopped when it has rotated downward 90°.

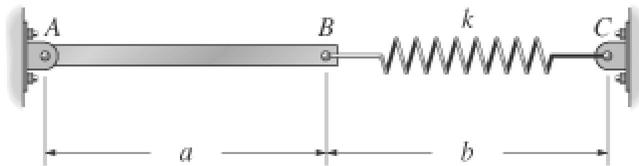
Given:

$M = 10\ \text{kg}$

$a = 1.5\ \text{m}$

$b = 1.5\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

$$0 + 0 = 0 + \frac{1}{2}k\left[\sqrt{(a+b)^2 + a^2} - b\right]^2 - Mg\frac{a}{2}$$

$$k = \frac{Mga}{\left[\sqrt{(a+b)^2 + a^2} - b\right]^2} \qquad k = 42.8\ \frac{\text{N}}{\text{m}}$$

Problem 18-43

The disk of weight *W* is rotating about pin *A* in the vertical plane with an angular velocity ω_1 when $\theta = 0°$. Determine its angular velocity at the instant shown, $\theta = 90$ deg. Also, compute the horizontal and vertical components of reaction at *A* at this instant.

Given:

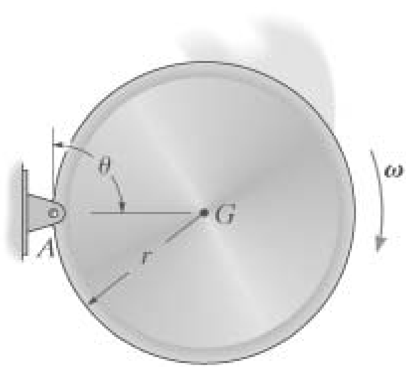
$W = 15\ \text{lb}$

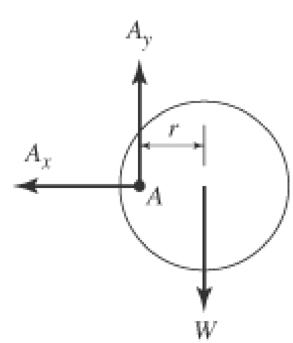
$\omega_1 = 2\ \dfrac{\text{rad}}{\text{s}}$

$\theta = 90\ \text{deg}$

$r = 0.5\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution:

Guesses $\quad A_x = 1\ \text{lb} \quad A_y = 1\ \text{lb} \quad \omega_2 = 1\ \dfrac{\text{rad}}{\text{s}} \quad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given $$\frac{1}{2}\left(\frac{3}{2}\frac{W}{g}r^2\right)\omega_1^{\,2} + W\,r = \frac{1}{2}\left(\frac{3}{2}\frac{W}{g}r^2\right)\omega_2^{\,2}$$

$$-A_x = \left(\frac{-W}{g}\right)r\omega_2^{\,2} \qquad A_y - W = \left(\frac{-W}{g}\right)\alpha r \qquad -W\,r = \frac{-3}{2}\left(\frac{W}{g}\right)r^2\alpha$$

$$\begin{pmatrix} A_x \\ A_y \\ \omega_2 \\ \alpha \end{pmatrix} = \text{Find}(A_x, A_y, \omega_2, \alpha) \qquad \alpha = 42.9\,\frac{\text{rad}}{\text{s}^2} \qquad \omega_2 = 9.48\,\frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 20.9 \\ 5.0 \end{pmatrix}\text{lb}$$

***Problem 18-44**

The door is made from one piece, whose ends move along the horizontal and vertical tracks. If the door is in the open position $\theta = 0°$, and then released, determine the speed at which its end A strikes the stop at C. Assume the door is a thin plate of weight W having width c.

Given:

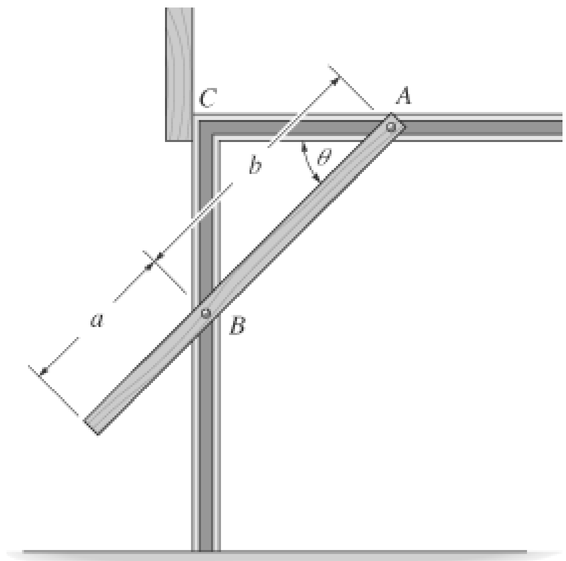
$W = 180$ lb

$a = 3$ ft

$b = 5$ ft

$c = 10$ ft

$g = 32.2\,\frac{\text{ft}}{\text{s}^2}$



Solution:

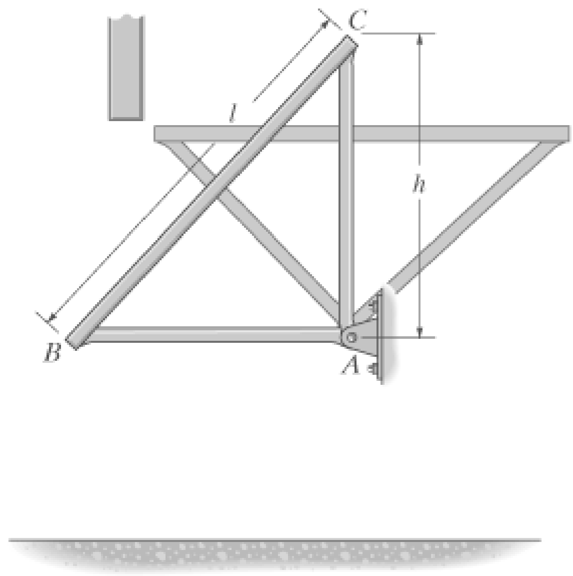
Guesses $v_A = 1\,\frac{\text{ft}}{\text{s}}$ $\qquad \omega = 1\,\frac{\text{rad}}{\text{s}}$

Given

$$0 + 0 = \frac{1}{2}\frac{W}{g}\left[\frac{(a+b)^2}{12} + \left(b - \frac{a+b}{2}\right)^2\right]\omega^2 - W\left(\frac{a+b}{2}\right) \qquad v_A = \omega b$$

$$\begin{pmatrix} \omega \\ v_A \end{pmatrix} = \text{Find}(\omega, v_A) \qquad \omega = 6.378\,\frac{\text{rad}}{\text{s}} \qquad v_A = 31.9\,\frac{\text{ft}}{\text{s}}$$

Problem 18-45



The overhead door *BC* is pushed slightly from its open position and then rotates downward about the pin at *A*. Determine its angular velocity just before its end *B* strikes the floor. Assume the door is a thin plate having a mass *M* and length *l*. Neglect the mass of the supporting frame *AB* and *AC*.

Given:

$M = 180\ \text{kg}$

$l = 6\ \text{m}$

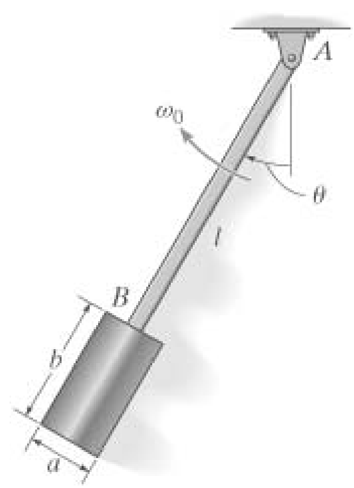
$h = 5\ \text{m}$

Solution: $d = \sqrt{h^2 - \left(\frac{l}{2}\right)^2}$

Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $M g d = \frac{1}{2}\left(\frac{M l^2}{12} + M d^2\right)\omega^2$ $\omega = \text{Find}(\omega)$ $\omega = 2.03\ \frac{\text{rad}}{\text{s}}$

Problem 18-46



The cylinder of weight W_1 is attached to the slender rod of weight W_2 which is pinned at point *A*. At the instant $\theta = \theta_0$ the rod has an angular velocity ω_0 as shown. Determine the angle θ_f to which the rod swings before it momentarily stops.

Given:

$W_1 = 80\ \text{lb}$ $a = 1\ \text{ft}$

$W_2 = 10\ \text{lb}$ $b = 2\ \text{ft}$

$\omega_0 = 1\ \frac{\text{rad}}{\text{s}}$ $l = 5\ \text{ft}$

$\theta_0 = 30\ \text{deg}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$I_A = \frac{W_1}{g}\left[\frac{1}{4}\left(\frac{a}{2}\right)^2 + \frac{b^2}{12}\right] + \frac{W_1}{g}\left(l + \frac{b}{2}\right)^2 + \left(\frac{W_2}{g}\right)\frac{l^2}{3}$$

$$d = \frac{W_1\left(l + \frac{b}{2}\right) + W_2\left(\frac{l}{2}\right)}{W_1 + W_2}$$

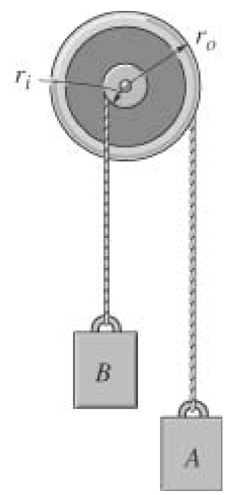
Guess $\theta_f = 1 \text{ deg}$

Given $\frac{1}{2}I_A\omega_0^{\;2} - (W_1 + W_2)d\cos(\theta_0) = -(W_1 + W_2)\,d\cos(\theta_f)$

$\theta_f = \text{Find}(\theta_f)$ $\theta_f = 39.3\,\text{deg}$

Problem 18-47

The compound disk pulley consists of a hub and attached outer rim. If it has mass m_P and radius of gyration k_G, determine the speed of block A after A descends distance d from rest. Blocks A and B each have a mass m_b. Neglect the mass of the cords.



Given:

$m_p = 3 \text{ kg}$ $r_i = 30 \text{ mm}$ $m_b = 2 \text{ kg}$

$k_G = 45 \text{ mm}$ $r_o = 100 \text{ mm}$

$d = 0.2 \text{ m}$ $g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

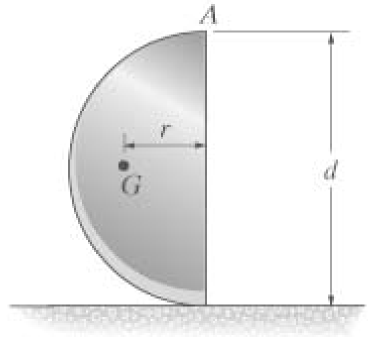
Guess $v_A = 1 \frac{\text{m}}{\text{s}}$

Given

$$0 + 0 = \frac{1}{2}m_b v_A^{\;2} + \frac{1}{2}m_b\left(\frac{r_i}{r_o}v_A\right)^2 + \frac{1}{2}\left(m_p k_G^{\;2}\right)\left(\frac{v_A}{r_o}\right)^2 - m_b\,g\,d + m_b\,g\left(\frac{r_i}{r_o}\right)d$$

$v_A = \text{Find}(v_A)$ $v_A = 1.404\,\frac{\text{m}}{\text{s}}$

***Problem 18-48**



The semicircular segment of mass M is released from rest in the position shown. Determine the velocity of point A when it has rotated counterclockwise 90°. Assume that the segment rolls without slipping on the surface. The moment of inertia about its mass center is I_G.

Given:

$M = 15\ \text{kg}$ $\quad r = 0.15\ \text{m}$

$I_G = 0.25\ \text{kg}\cdot\text{m}^2$ $\quad d = 0.4\ \text{m}$

Solution:

Guesses $\quad \omega = 1\ \frac{\text{rad}}{\text{s}} \quad v_G = 1\ \frac{\text{m}}{\text{s}}$

Given $\quad M g d = \frac{1}{2} M v_G^2 + \frac{1}{2} I_G \omega^2 + M g(d - r) \qquad v_G = \omega\left(\frac{d}{2} - r\right)$

$$\begin{pmatrix} \omega \\ v_G \end{pmatrix} = \text{Find}(\omega, v_G) \qquad \omega = 12.4\ \frac{\text{rad}}{\text{s}} \qquad v_G = 0.62\ \frac{\text{m}}{\text{s}}$$

$$\mathbf{v_A} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} \frac{-d}{2} \\ \frac{d}{2} \\ 0 \end{pmatrix} \qquad \mathbf{v_A} = \begin{pmatrix} -2.48 \\ -2.48 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \left|\mathbf{v_A}\right| = 3.50\ \frac{\text{m}}{\text{s}}$$

Problem 18-49

The uniform stone (rectangular block) of weight W is being turned over on its side by pulling the vertical cable *slowly* upward until the stone begins to tip. If it then falls freely (**T** = 0) from an essentially balanced at-rest position, determine the speed at which the corner A strikes the pad at B. The stone does not slip at its corner C as it falls.

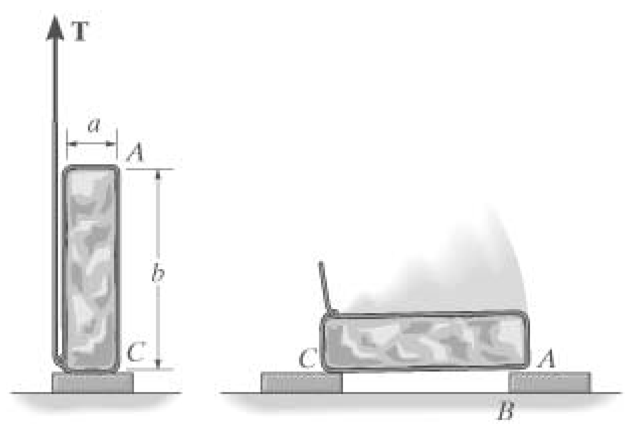
Given:

$W = 150\ \text{lb}$

$a = 0.5\ \text{ft}$

$b = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

Guess $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

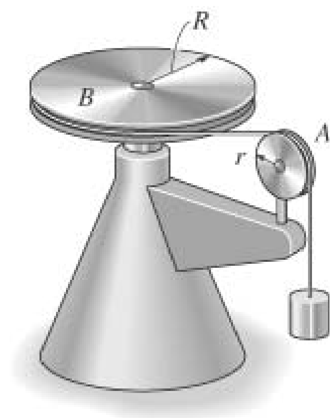
Given $$W\left(\frac{\sqrt{a^2+b^2}}{2}\right) = \frac{1}{2}\frac{W}{g}\left(\frac{a^2+b^2}{3}\right)\omega^2 + W\frac{a}{2} \qquad \omega = \text{Find}(\omega)$$

$$v_A = \omega b \qquad v_A = 11.9\ \frac{\text{ft}}{\text{s}}$$

Problem 18-50

The assembly consists of pulley A of mass m_A and pulley B of mass m_B. If a block of mass m_b is suspended from the cord, determine the block's speed after it descends a distance d starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.

Given:



$m_A = 3\ \text{kg}$

$m_B = 10\ \text{kg}$

$m_b = 2\ \text{kg}$

$d = 0.5\ \text{m}$

$r = 30\ \text{mm}$

$R = 100\ \text{mm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Guess $v_b = 1\ \frac{\text{m}}{\text{s}}$

Given

$$0 + 0 = \frac{1}{2}\left(\frac{m_A r^2}{2}\right)\left(\frac{v_b}{r}\right)^2 + \frac{1}{2}\left(\frac{m_B R^2}{2}\right)\left(\frac{v_b}{R}\right)^2 + \frac{1}{2}m_b v_b^2 - m_b g d$$

$$v_b = \text{Find}(v_b) \qquad v_b = 1.519\ \frac{\text{m}}{\text{s}}$$

Problem 18-51

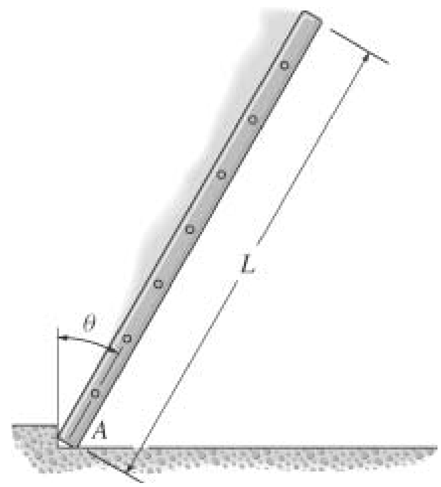
A uniform ladder having weight W is released from rest when it is in the vertical position. If it is allowed to fall freely, determine the angle at which the bottom end A starts to lift off the ground. For the calculation, assume the ladder to be a slender rod and neglect friction at A.

Given:

$W = 30\ \text{lb}$

$L = 10\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

(a) The rod will rotate around point A until it loses contact with the horizontal constraint ($A_x = 0$). We will find this point first

Guesses

$\theta_1 = 30\ \text{deg} \quad \omega_1 = 1\ \dfrac{\text{rad}}{\text{s}} \quad \alpha_1 = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given

$$0 + W\left(\frac{L}{2}\right) = \frac{1}{2}\left[\frac{1}{3}\left(\frac{W}{g}\right)L^2\right]\omega_1{}^2 + W\left(\frac{L}{2}\right)\cos(\theta_1)$$

$$W\left(\frac{L}{2}\right)\sin(\theta_1) = \left[\frac{1}{3}\left(\frac{W}{g}\right)L^2\right]\alpha_1$$

$$\alpha_1\left(\frac{L}{2}\right)\cos(\theta_1) - \omega_1{}^2\left(\frac{L}{2}\right)\sin(\theta_1) = 0$$

$$\begin{pmatrix}\omega_1\\ \alpha_1\\ \theta_1\end{pmatrix} = \text{Find}(\omega_1, \alpha_1, \theta_1) \qquad \theta_1 = 48.19\ \text{deg} \qquad \omega_1 = 1.794\ \frac{\text{rad}}{\text{s}} \qquad \alpha_1 = 3.6\ \frac{\text{rad}}{\text{s}^2}$$

(b) Now the rod moves without any horizontal constraint. If we look for the point at which it loses contact with the floor ($A_y = 0$) we will find that this condition never occurs.

***Problem 18-52**

The slender rod *AB* of weight *W* is attached to a spring *BC* which, has unstretched length *L*. If the rod is released from rest when $\theta = \theta_1$,determine its angular velocity at the instant $\theta = \theta_2$.

Given:

$W = 25\ \text{lb}$

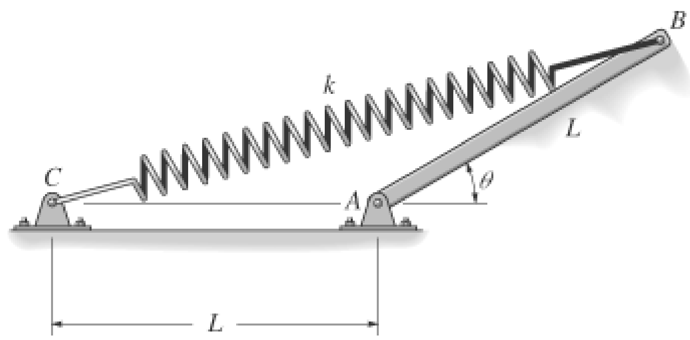
$L = 4\ \text{ft}$

$k = 5\ \dfrac{\text{lb}}{\text{ft}}$

$\theta_1 = 30\ \text{deg}$

$\theta_2 = 90\ \text{deg}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

$$0 + W\left(\frac{L}{2}\right)\sin(\theta_1) + \frac{1}{2}kL^2\left[\sqrt{2(1+\cos(\theta_1))} - 1\right]^2 = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{L^2}{3}\right)\omega^2 + W\left(\frac{L}{2}\right)\sin(\theta_2) \ldots$$
$$+ \frac{1}{2}kL^2\left[\sqrt{2(1+\cos(\theta_2))} - 1\right]^2$$

$\omega = \text{Find}(\omega)$ $\quad \omega = 1.178\ \dfrac{\text{rad}}{\text{s}}$

Problem 18-53

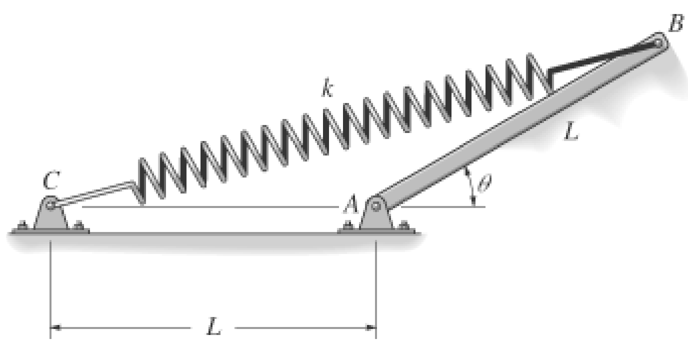
The slender rod *AB* of weight *w* is attached to a spring *BC* which has an unstretched length *L*. If the rod is released from rest when $\theta = \theta_1$, determine the angular velocity of the rod the instant the spring becomes unstretched.

Given:

$W = 25\ \text{lb}$ $\quad \theta_1 = 30\ \text{deg}$

$L = 4\ \text{ft}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$k = 5\ \dfrac{\text{lb}}{\text{ft}}$



Solution:

When the spring is unstretched $\quad \theta_2 = 120\ \text{deg}$

Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

$$0 + W\left(\frac{L}{2}\right)\sin\left(\theta_1\right) + \frac{1}{2}kL^2\left[\sqrt{2\left(1+\cos\left(\theta_1\right)\right)} - 1\right]^2 = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{L^2}{3}\right)\omega^2 + W\left(\frac{L}{2}\right)\sin\left(\theta_2\right) \ ... \\ + \frac{1}{2}kL^2\left[\sqrt{2\left(1+\cos\left(\theta_2\right)\right)} - 1\right]^2$$

$\omega = \mathrm{Find}(\omega)$ $\qquad \omega = 2.817\ \frac{\mathrm{rad}}{\mathrm{s}}$

Problem 18-54

A chain that has a negligible mass is draped over the sprocket which has mass m_s and radius of gyration k_O. If block A of mass m_A is released from rest in the position $s = s_1$, determine the angular velocity of the sprocket at the instant $s = s_2$.

Given:

$m_S = 2\ \mathrm{kg}$

$k_O = 50\ \mathrm{mm}$

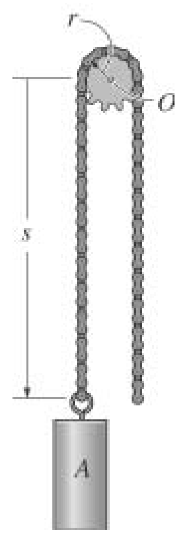
$m_A = 4\ \mathrm{kg}$

$s_1 = 1\ \mathrm{m}$

$s_2 = 2\ \mathrm{m}$

$r = 0.1\ \mathrm{m}$

$g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$



Solution:

Guess $\qquad \omega = 1\ \frac{\mathrm{rad}}{\mathrm{s}}$

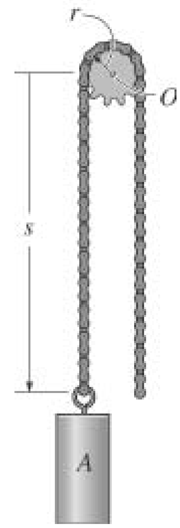
Given

$$0 - m_A g s_1 = \frac{1}{2}m_A(r\omega)^2 + \frac{1}{2}m_S k_O^2\,\omega^2 - m_A g s_2$$

$\omega = \mathrm{Find}(\omega)$ $\qquad \omega = 41.8\ \frac{\mathrm{rad}}{\mathrm{s}}$

Problem 18-55

A chain that has a mass density ρ is draped over the sprocket which has mass m_s and radius of gyration k_O. If block A of mass m_A is released from rest in the position $s = s_1$, determine the angular velocity of the sprocket at the instant $s = s_2$. When released there is an equal amount of chain on each side. Neglect the portion of the chain that wraps over the sprocket.



Given:

$m_s = 2 \text{ kg}$ $\quad s_1 = 1 \text{ m}$

$k_O = 50 \text{ mm}$ $\quad s_2 = 2 \text{ m}$

$m_A = 4 \text{ kg}$ $\quad r = 0.1 \text{ m}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$ $\quad \rho = 0.8 \dfrac{\text{kg}}{\text{m}}$

Solution:

Guess $\quad T_1 = 1 \text{ N m} \quad T_2 = 1 \text{ N m} \quad \omega = 10 \dfrac{\text{rad}}{\text{s}}$

$V_1 = 1 \text{ N m} \quad V_2 = 1 \text{ N m}$

Given

$$T_1 = 0$$

$$V_1 = -m_A g s_1 - 2\rho s_1 g\left(\frac{s_1}{2}\right)$$

$$T_2 = \frac{1}{2}m_A(r\omega)^2 + \frac{1}{2}m_s k_O^{\,2}\,\omega^2 + \frac{1}{2}\rho(2s_1)(r\omega)^2$$

$$V_2 = -m_A g s_2 - \rho s_2 g\left(\frac{s_2}{2}\right) - \rho(2s_1 - s_2)g\left(\frac{2s_1 - s_2}{2}\right)$$

$$T_1 + V_1 = T_2 + V_2$$

$$\begin{pmatrix} T_1 \\ V_1 \\ T_2 \\ V_2 \\ \omega \end{pmatrix} = \text{Find}(T_1, V_1, T_2, V_2, \omega) \qquad \omega = 39.3 \frac{\text{rad}}{\text{s}}$$

***Problem 18-56**

Pulley A has weight W_A and centroidal radius of gyration k_B. Determine the speed of the crate C of weight W_C at the instant $s = s_2$. Initially, the crate is released from rest when $s = s_1$. The pulley at P "rolls" downward on the cord without slipping. For the calculation, neglect the mass of this pulley and the cord as it unwinds from the inner and outer hubs of pulley A.

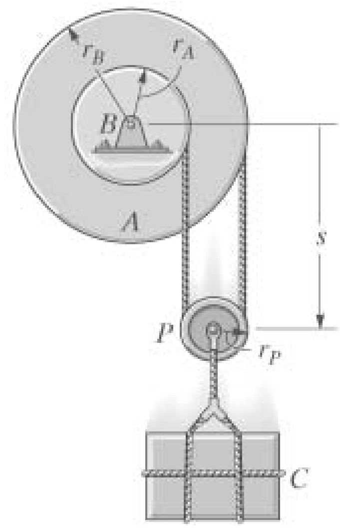
Given:

$W_A = 30\text{ lb}$ $\quad r_A = 0.4\text{ ft}$

$W_C = 20\text{ lb}$ $\quad r_B = 0.8\text{ ft}$

$k_B = 0.6\text{ ft}$ $\quad r_P = \dfrac{r_B - r_A}{2}$

$s_1 = 5\text{ ft}$ $\quad s_2 = 10\text{ ft}$



Solution:

Guess $\quad \omega = 1\ \dfrac{\text{rad}}{\text{s}} \quad v_C = 1\ \dfrac{\text{ft}}{\text{s}}$

Given

$$-W_C s_1 = \frac{1}{2}\left(\frac{W_A}{g}\right)k_B^{\,2}\,\omega^2 + \frac{1}{2}\left(\frac{W_C}{g}\right)v_C^{\,2} - W_C s_2 \qquad v_C = \omega\left(\frac{r_A + r_B}{2}\right)$$

$$\begin{pmatrix}\omega \\ v_C\end{pmatrix} = \text{Find}(\omega, v_C) \qquad \omega = 18.9\ \frac{\text{rad}}{\text{s}} \qquad v_C = 11.3\ \frac{\text{ft}}{\text{s}}$$

Problem 18-57

The assembly consists of two bars of weight W_1 which are pinconnected to the two disks of weight W_2. If the bars are released from rest at $\theta = \theta_0$, determine their angular velocities at the instant $\theta = 0°$. Assume the disks roll without slipping.

Given:

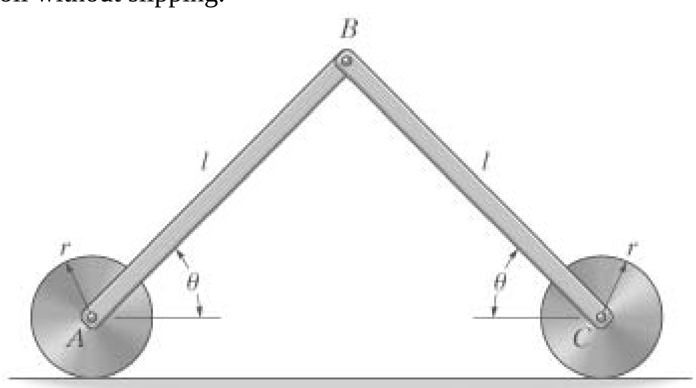
$W_1 = 8\text{ lb}$

$W_2 = 10\text{ lb}$

$r = 0.5\text{ ft}$

$l = 3\text{ ft}$

$\theta_0 = 60\text{ deg}$



Solution:

Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $2W_1\left(\frac{l}{2}\right)\sin(\theta_0) = 2\frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{l^2}{3}\right)\omega^2$

$\omega = \text{Find}(\omega)$ $\omega = 5.28\ \frac{\text{rad}}{\text{s}}$

Problem 18-58

The assembly consists of two bars of weight W_1 which are pin-connected to the two disks of weight W_2. If the bars are released from rest at $\theta = \theta_1$, determine their angular velocities at the instant $\theta = \theta_2$. Assume the disks roll without slipping.

Given:

$W_1 = 8\ \text{lb}$

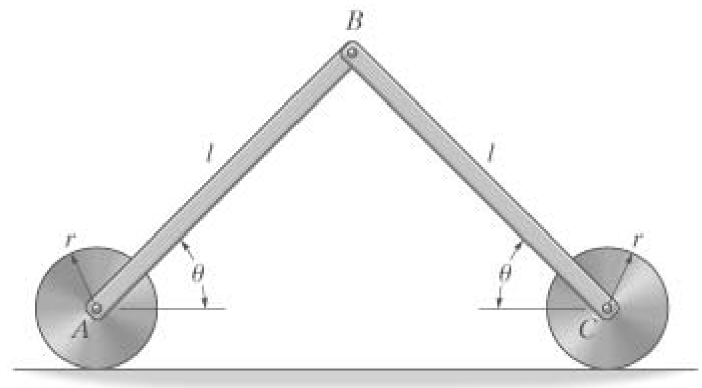
$W_2 = 10\ \text{lb}$

$\theta_1 = 60\ \text{deg}$

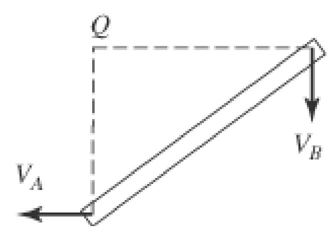
$\theta_2 = 30\ \text{deg}$

$r = 0.5\ \text{ft}$

$l = 3\ \text{ft}$



Solution:



Guesses

$\omega = 1\ \frac{\text{rad}}{\text{s}}$ $v_A = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$2W_1\left(\frac{l}{2}\right)\sin(\theta_1) = 2W_1\left(\frac{l}{2}\right)\sin(\theta_2) + 2\frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{l^2}{3}\right)\omega^2 + 2\frac{1}{2}\left[\frac{3}{2}\left(\frac{W_2}{g}\right)r^2\right]\left(\frac{v_A}{r}\right)^2$$

$$v_A = \omega l \sin(\theta_2)$$

$$\begin{pmatrix} \omega \\ v_A \end{pmatrix} = \text{Find}(\omega, v_A) \qquad v_A = 3.32\,\frac{\text{ft}}{\text{s}} \qquad \omega = 2.21\,\frac{\text{rad}}{\text{s}}$$

Problem 18-59

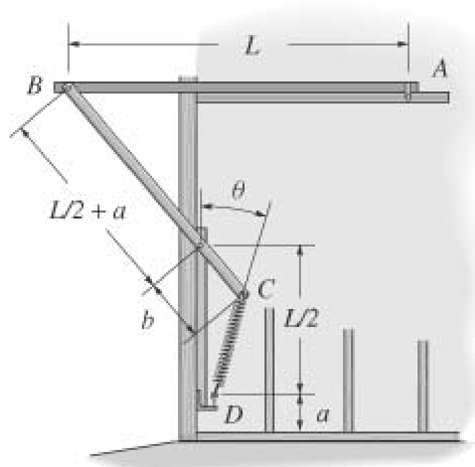
The end *A* of the garage door *AB* travels along the horizontal track, and the end of member *BC* is attached to a spring at *C*. If the spring is originally unstretched, determine the stiffness *k* so that when the door falls downward from rest in the position shown, it will have zero angular velocity the moment it closes, i.e., when it and *BC* become vertical. Neglect the mass of member *BC* and assume the door is a thin plate having weight *W* and a width and height of length *L*. There is a similar connection and spring on the other side of the door.

Given:

$W = 200\ \text{lb}$ $\qquad b = 2\ \text{ft}$

$L = 12\ \text{ft}$ $\qquad \theta = 15\ \text{deg}$

$a = 1\ \text{ft}$



Solution:

Guess $\quad k = 1\,\frac{\text{lb}}{\text{ft}} \qquad d = 1\ \text{ft}$

Given

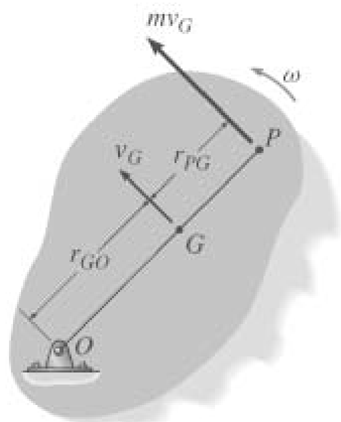
$$b^2 = \left(\frac{L}{2}\right)^2 + d^2 - 2d\left(\frac{L}{2}\right)\cos(\theta)$$

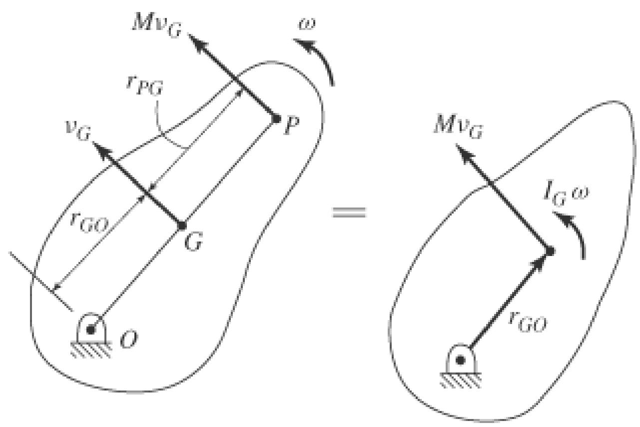
$$0 = -W\left(\frac{L}{2}\right) + 2\,\frac{1}{2}k\left(\frac{L}{2} + b - d\right)^2$$

$$\begin{pmatrix} k \\ d \end{pmatrix} = \text{Find}(k, d) \qquad d = 4.535\,\text{ft} \qquad k = 100.0\,\frac{\text{lb}}{\text{ft}}$$

Problem 19-1

The rigid body (slab) has a mass m and is rotating with an angular velocity ω about an axis passing through the fixed point O. Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude mv_G and acting through point P, called the center of *percussion*, which lies at a distance $r_{PG} = k^2_G / r_{GO}$ from the mass center G. Here k_G is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through G.





Solution:

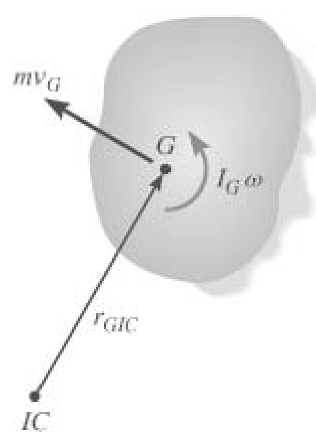
$$H_o = (r_{GO} + r_{PG})m\,v_G = r_{GO}\,m\,v_G + I_G\omega \qquad \text{Where} \qquad I_G = m\,k_G^{\,2}$$

$$r_{GO}\,m\,v_G + r_{PG}\,m\,v_G = r_{GO}\,m\,v_G + m\,k_G^{\,2}\,\omega$$

$$r_{PG} = \frac{k_G^{\,2}\,\omega}{v_G} = \frac{k_G^{\,2}}{v_G}\left(\frac{v_G}{r_{GO}}\right) = \frac{k_G^{\,2}}{r_{GO}} \qquad \text{Q.E.D}$$

Problem 19-2

At a given instant, the body has a linear momentum $L = mv_G$ and an angular momentum $H_G = I_G\omega$ computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity IC can be expressed as $H_{IC} = I_{IC}\omega$ where I_{IC} represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the IC is located at a distance r_{GIC} away from the mass center G.



Solution:

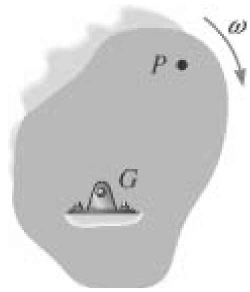
$$H_{IC} = r_{GIC}\,m\,v_G + I_G\omega \qquad \text{Where} \qquad v_G = \omega r_{GIC}$$

$$H_{IC} = r_{GIC} m \omega r_{GIC} + I_G \omega$$

$$H_{IC} = \left(I_G + m r_{GIC}^2\right)\omega$$

$$H_{IC} = I_{IC}\omega \qquad \text{Q.E.D.}$$

Problem 19-3



Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center G, the angular momentum is the same when computed about any other point P on the slab.

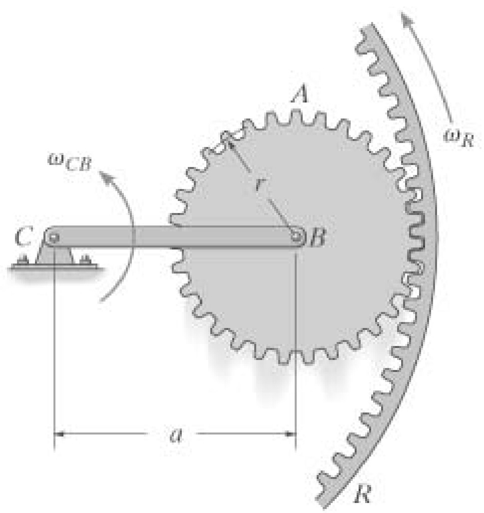
Solution:

Since $v_G = 0$, the linear momentum $L = mv_G = 0$. Hence the angular momentum about any point P is

$$H_P = I_G\omega$$

Since ω is a free vector , so is H_P. Q.E.D.

***Problem 19-4**



Gear A rotates along the inside of the circular gear rack R. If A has weight W and radius of gyration k_B, determine its angular momentum about point C when (a) $\omega_R = 0$, (b) $\omega_R = \omega$.

Given:

$W = 4 \text{ lbf}$ $\qquad r = 0.75 \text{ ft}$

$\omega_{CB} = 30 \dfrac{\text{rad}}{\text{s}}$ $\qquad a = 1.5 \text{ ft}$

$\omega = 20 \dfrac{\text{rad}}{\text{s}}$ $\qquad k_B = 0.5 \text{ ft}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

Solution:

(a) $\omega_R = 0 \dfrac{\text{rad}}{\text{s}}$

$v_B = a\omega_{CB}$ $\qquad \omega_A = \dfrac{\omega_R(a+r) - \omega_{CB}a}{r}$

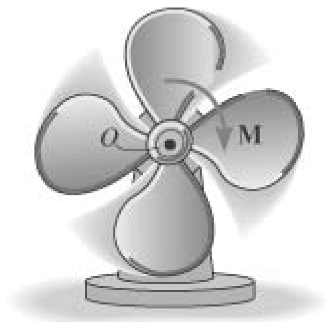
$H_c = \left(\dfrac{W}{g}\right)v_B a + \left(\dfrac{W}{g}\right)k_B^2\omega_A$ $\qquad H_c = 6.52 \text{ slug}\cdot\dfrac{\text{ft}^2}{\text{s}}$

(b) $\omega_R = \omega$

$v_B = a\omega_{CB}$ $\qquad$ $\omega_A = \dfrac{\omega_R(a+r) - \omega_{CB}\,a}{r}$

$H_c = \left(\dfrac{W}{g}\right) v_B\,a + \left(\dfrac{W}{g}\right) k_B^{\,2}\,\omega_A$ $\qquad$ $H_c = 8.39\,\mathrm{slug}\cdot\dfrac{\mathrm{ft}^2}{\mathrm{s}}$

Problem 19-5



The fan blade has mass m_b and a moment of inertia I_0 about an axis passing through its center O. If it is subjected to moment $M = A(1 - e^{bt})$ determine its angular velocity when $t = t_1$ starting from rest.

Given:

$m_b = 2\ \mathrm{kg}$ $\qquad$ $A = 3\ \mathrm{N\cdot m}$ $\qquad$ $t_1 = 4\ \mathrm{s}$

$I_O = 0.18\ \mathrm{kg\cdot m^2}$ $\qquad$ $b = -0.2\ \mathrm{s}^{-1}$

Solution:

$0 + \displaystyle\int_0^{t_1} A\left(1 - e^{b\,t}\right)\,dt = I_O\omega_1$ $\qquad$ $\omega_1 = \dfrac{1}{I_O}\displaystyle\int_0^{t_1} A\left(1 - e^{b\,t}\right)\,dt$ $\qquad$ $\omega_1 = 20.8\,\dfrac{\mathrm{rad}}{\mathrm{s}}$

Problem 19-6

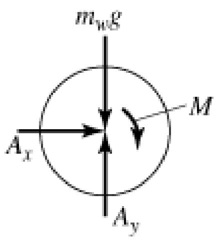
The wheel of mass m_w has a radius of gyration k_A. If the wheel is subjected to a moment $M = bt$, determine its angular velocity at time t_1 starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.

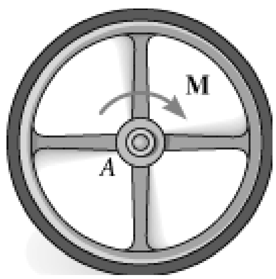
Given:

$m_w = 10\ \mathrm{kg}$ $\qquad$ $t_1 = 3\ \mathrm{s}$

$k_A = 200\ \mathrm{mm}$ $\qquad$ $g = 9.81\,\dfrac{\mathrm{m}}{\mathrm{s}^2}$

$b = 5\ \mathrm{N}\cdot\dfrac{\mathrm{m}}{\mathrm{s}}$





Solution:

$0 + \displaystyle\int_0^{t_1} b\,t\,dt = m_w k_A^{\,2}\omega_1$ $\qquad$ $\omega_1 = \dfrac{1}{m_w k_A^{\,2}}\displaystyle\int_0^{t_1} b\,t\,dt$ $\qquad$ $\omega_1 = 56.25\,\dfrac{\mathrm{rad}}{\mathrm{s}}$

$0 + Ax\,t_1 = 0$ $\qquad$ $A_x = 0$ $\qquad$ $A_x = 0.00$

$0 + A_y t_1 - m_W g t_1 = 0$ $\qquad A_y = m_W g$ $\qquad A_y = 98.10\ \text{N}$

Problem 19-7

Disk D of weight W is subjected to counterclockwise moment $M = bt$. Determine the angular velocity of the disk at time t_2 after the moment is applied. Due to the spring the plate P exerts constant force P on the disk. The coefficients of static and kinetic friction between the disk and the plate are μ_s and μ_k respectively. *Hint*: First find the time needed to start the disk rotating.

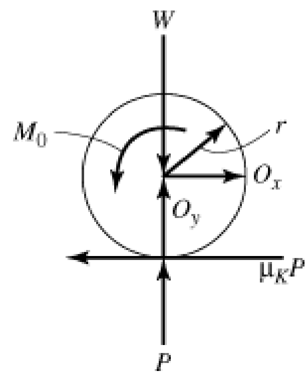
Given:

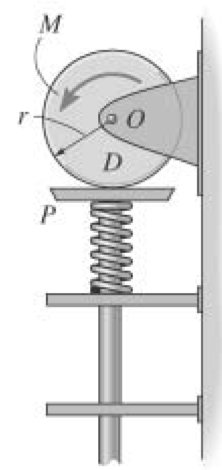
$W = 10\ \text{lb}$ $\qquad \mu_s = 0.3$

$b = 10\ \text{lb}\cdot\frac{\text{ft}}{\text{s}}$ $\qquad \mu_k = 0.2$

$t_2 = 2\ \text{s}$ $\qquad r = 0.5\ \text{ft}$

$P = 100\text{lb}$ $\qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution: When motion begins

$b t_1 = \mu_s P r$ $\qquad t_1 = \frac{\mu_s P r}{b}$ $\qquad t_1 = 1.50\ \text{s}$

At a later time we have

$$0 + \int_{t_1}^{t_2} (b t - \mu_k P r)\, dt = \left(\frac{W}{g}\right)\frac{r^2}{2}\omega_2$$

$$\omega_2 = \frac{2g}{W r^2}\int_{t_1}^{t_2} (b t - \mu_k P r)\, dt \qquad \omega_2 = 96.6\ \frac{\text{rad}}{\text{s}}$$

***Problem 19-8**

The cord is wrapped around the inner core of the spool. If block B of weight W_B is suspended from the cord and released from rest, determine the spool's angular velocity when $t = t_1$. Neglect the mass of the cord. The spool has weight W_S and the radius of gyration about the axle A is k_A. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.

Given:

$W_B = 5\ \text{lb}$

$t_1 = 3\ \text{s}$

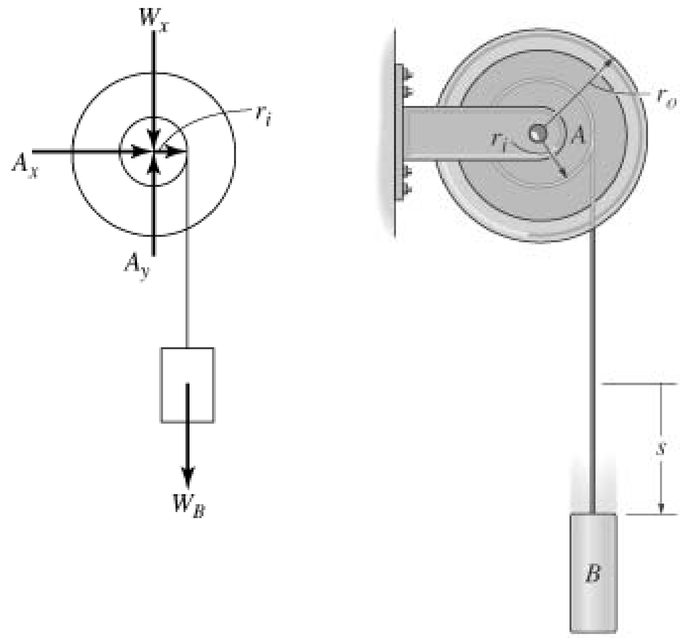
$W_S = 180\ \text{lb}$

$k_A = 1.25\ \text{ft}$

$r_i = 1.5\text{ft}$

$r_o = 2.75\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

(a) System as a whole

$$0 + W_B r_i t_1 = \left(\frac{W_S}{g}\right) k_A{}^2 \omega + \left(\frac{W_B}{g}\right)(r_i \omega) r_i \qquad \omega = \frac{W_B r_i t_1 g}{W_S k_A{}^2 + W_B r_i{}^2} \qquad \omega = 2.48\ \frac{\text{rad}}{\text{s}}$$

(b) Parts separately Guesses $T = 1\ \text{lb}$ $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $0 + T t_1 r_i = \left(\dfrac{W_S}{g}\right) k_A{}^2 \omega \qquad T t_1 - W_B t_1 = \left(\dfrac{-W_B}{g}\right)(r_i \omega)$

$$\begin{pmatrix} T \\ \omega \end{pmatrix} = \text{Find}(T, \omega) \qquad T = 4.81\ \text{lb} \qquad \omega = 2.48\ \frac{\text{rad}}{\text{s}}$$

Problem 19-9

The disk has mass M and is originally spinning at the end of the strut with angular velocity ω. If it is then placed against the wall, for which the coefficient of kinetic friction is μ_k determine the time required for the motion to stop. What is the force in strut BC during this time?

Given:

$M = 20\ \text{kg}$ $\theta = 60\ \text{deg}$

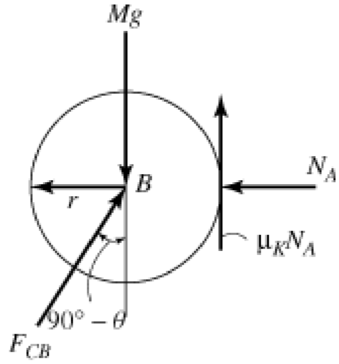
$\omega = 60\ \dfrac{\text{rad}}{\text{s}}$ $r = 150\ \text{mm}$

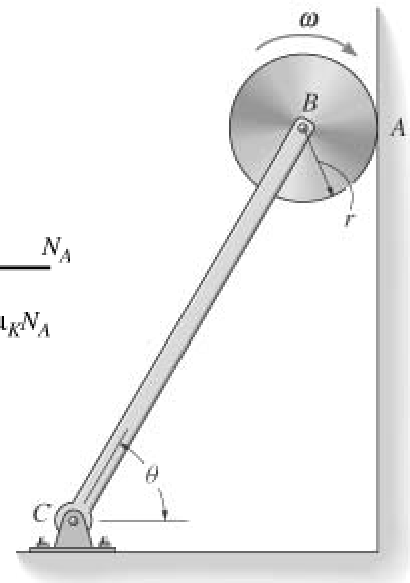
$\mu_k = 0.3$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

Initial Guess:

$F_{CB} = 1\ \text{N} \qquad t = 1\ \text{s} \qquad N_A = 1\ \text{N}$





Given

$$-M\left(\frac{r^2}{2}\right)\omega + \mu_k N_A r t = 0$$

$$F_{CB}\cos(\theta)t - N_A t = 0$$

$$F_{CB}\sin(\theta)t - M g t + \mu_k N_A t = 0$$

$$\begin{pmatrix} F_{CB} \\ N_A \\ t \end{pmatrix} = \text{Find}(F_{CB}, N_A, t) \qquad N_A = 96.55\ \text{N} \qquad F_{CB} = 193\ \text{N} \qquad t = 3.11\ \text{s}$$

Problem 19-10

A flywheel has a mass M and radius of gyration k_G about an axis of rotation passing through its mass center. If a motor supplies a clockwise torque having a magnitude $M = kt$, determine the flywheel's angular veliocity at time t_1. Initially the flywheel is rotating clockwise at angular velocity ω_0.

Given:

$$M = 60\ \text{kg} \qquad k_G = 150\ \text{mm} \qquad k = 5\ \frac{\text{N}\cdot\text{m}}{\text{s}} \qquad t_1 = 3\ \text{s} \qquad \omega_0 = 2\ \frac{\text{rad}}{\text{s}}$$

Solution:

$$M k_G^2 \omega_0 + \int_0^{t_1} k t\ \text{d}t = M k_G^2 \omega_1$$

$$\omega_1 = \omega_0 + \frac{1}{M k_G^2}\int_0^{t_1} k t\ \text{d}t \qquad \omega_1 = 18.7\ \frac{\text{rad}}{\text{s}}$$

Problem 19-11

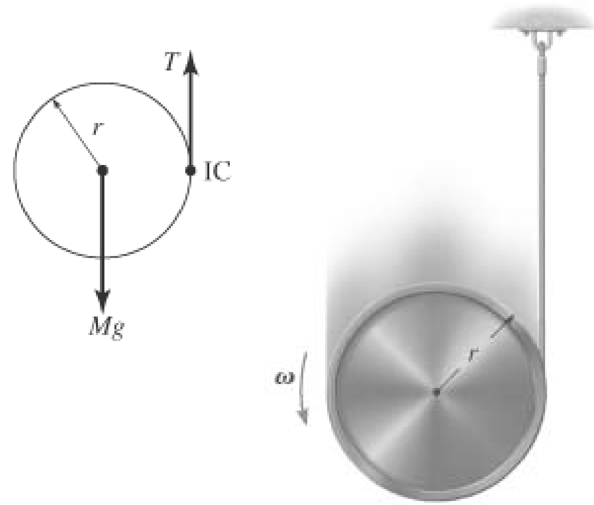
A wire of negligible mass is wrapped around the outer surface of the disk of mass M. If the disk is released from rest, determine its angular velocity at time t.

Given:

$M = 2$ kg

$t = 3$ s

$r = 80$ mm



Solution:

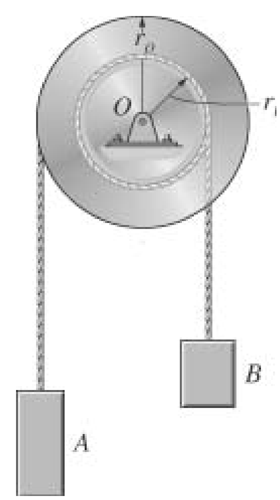
$$0 + M g r t = \frac{3}{2} M r^2 \omega$$

$$\omega = \frac{2}{3}\left(\frac{g}{r}\right) t \qquad \boxed{\omega = 245 \frac{\text{rad}}{\text{s}}}$$

***Problem 19-12**

The spool has mass m_S and radius of gyration k_O. Block A has mass m_A, and block B has mass m_B. If they are released from rest, determine the time required for block A to attain speed v_A. Neglect the mass of the ropes.

Given:



$m_S = 30$ kg $\quad m_B = 10$ kg $\quad r_o = 0.3$ m

$k_O = 0.25$ m $\quad v_A = 2 \frac{\text{m}}{\text{s}} \quad g = 9.81 \frac{\text{m}}{\text{s}^2}$

$m_A = 25$ kg $\quad r_i = 0.18$ m

Solution:

Guesses $\quad t = 1$ s $\quad v_B = 1 \frac{\text{m}}{\text{s}} \quad \omega = 1 \frac{\text{rad}}{\text{s}}$

Given $\quad v_A = \omega r_o \quad v_B = \omega r_i$

$$0 + m_A g t r_o - m_B g t r_i = m_A v_A r_o + m_B v_B r_i + m_S k_O^2 \omega$$

$$\begin{pmatrix} t \\ v_B \\ \omega \end{pmatrix} = \text{Find}(t, v_B, \omega) \qquad v_B = 1.20 \frac{\text{m}}{\text{s}} \qquad \omega = 6.67 \frac{\text{rad}}{\text{s}} \qquad \boxed{t = 0.530 \text{ s}}$$

Problem 19-13

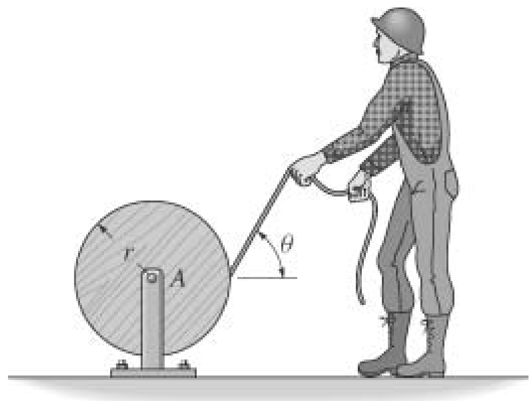
The man pulls the rope off the reel with a constant force P in the direction shown. If the reel has weight W and radius of gyration k_G about the trunnion (pin) at A, determine the angular velocity of the reel at time t starting from rest. Neglect friction and the weight of rope that is removed.

Given:

$P = 8\ \text{lb}$ $\quad t = 3\ \text{s}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$W = 250\ \text{lb}$ $\quad \theta = 60\ \text{deg}$

$k_G = 0.8\ \text{ft}$ $\quad r = 1.25\ \text{ft}$



Solution:

$$0 + P r t = \left(\frac{W}{g}\right) k_G{}^2 \omega$$

$$\omega = \frac{P r t g}{W k_G{}^2} \qquad \boxed{\omega = 6.04\ \frac{\text{rad}}{\text{s}}}$$

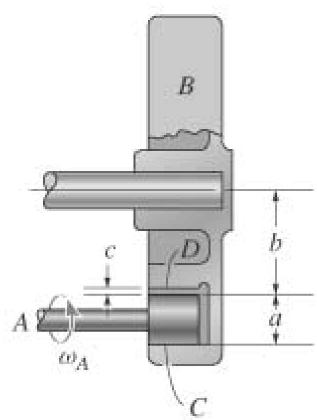
Problem 19-14

Angular motion is transmitted from a driver wheel A to the driven wheel B by friction between the wheels at C. If A always rotates at constant rate ω_A and the coefficient of kinetic friction between the wheels is μ_k, determine the time required for B to reach a constant angular velocity once the wheels make contact with a normal force F_N. What is the final angular velocity of wheel B? Wheel B has mass m_B and radius of gyration about its axis of rotation k_G.

Given:

$\omega_A = 16\ \dfrac{\text{rad}}{\text{s}}$ $\quad m_B = 90\ \text{kg}$ $\quad a = 40\ \text{mm}$ $\quad c = 4\ \text{mm}$

$\mu_k = 0.2$ $\quad k_G = 120\ \text{mm}$ $\quad b = 50\ \text{mm}$ $\quad F_N = 50\ \text{N}$



Solution: Guesses $t = 1\ \text{s}$ $\quad \omega_B = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $\quad \mu_k F_N (a + b) t = m_B k_G{}^2 \omega_B$

$$\omega_B (a + b) = \omega_A \left(\frac{a}{2}\right)$$

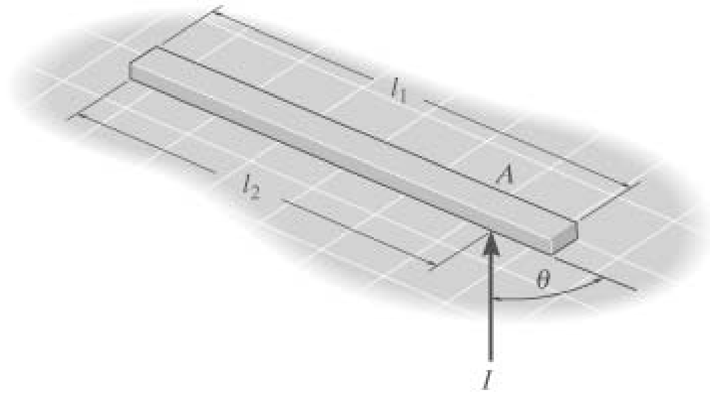
$$\begin{pmatrix} t \\ \omega_B \end{pmatrix} = \text{Find}(t, \omega_B) \qquad \boxed{\omega_B = 3.56\ \frac{\text{rad}}{\text{s}}} \qquad \boxed{t = 5.12\ \text{s}}$$

Problem 19-15

The slender rod of mass M rests on a smooth floor. If it is kicked so as to receive a horizontal impulse I at point A as shown, determine its angular velocity and the speed of its mass center.

Given:

$M = 4\ \text{kg}$



$l_1 = 2\ \text{m}$

$l_2 = 1.75\ \text{m}$

$I = 8\ \text{N s}$

$\theta = 60\ \text{deg}$

Solution:

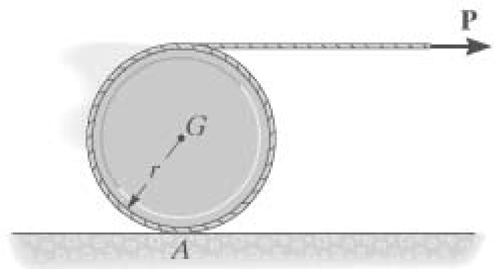
Guesses $\quad v = 1\ \frac{\text{m}}{\text{s}} \qquad \omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $\quad I\sin(\theta)\left(l_2 - \frac{l_1}{2}\right) = \frac{1}{12}M\,l_1^{\,2}\,\omega \qquad I = M\,v$

$\begin{pmatrix}\omega \\ v\end{pmatrix} = \text{Find}(\omega, v) \qquad \omega = 3.90\ \frac{\text{rad}}{\text{s}} \qquad v = 2.00\ \frac{\text{m}}{\text{s}}$

***Problem 19-16**

A cord of negligible mass is wrapped around the outer surface of the cylinder of weight W and its end is subjected to a constant horizontal force **P**. If the cylinder rolls without slipping at A, determine its angular velocity in time t starting from rest. Neglect the thickness of the cord.



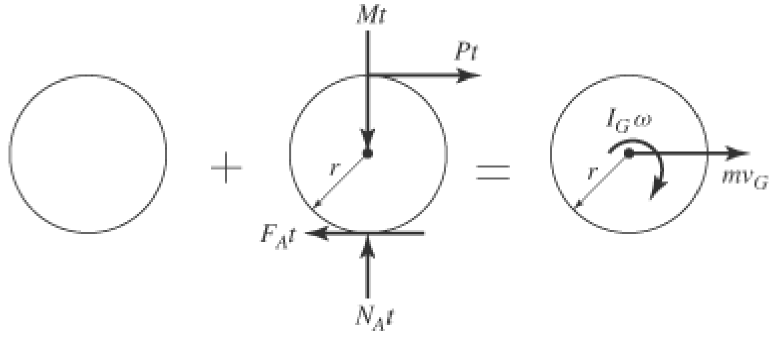
Given:

$W = 50\ \text{lb}$

$P = 2\ \text{lb}$

$t = 4\ \text{s}$

$r = 0.6\ \text{ft}$



Solution:

$$0 + P\,t(2r) = \left[\frac{1}{2}\left(\frac{W}{g}\right)r^2\right]\omega + \left(\frac{W}{g}\right)(r\omega)r$$

$$\omega = \frac{4P\,t\,g}{3r\,W} \qquad \omega = 11.4\ \frac{\text{rad}}{\text{s}}$$

Problem 19-17

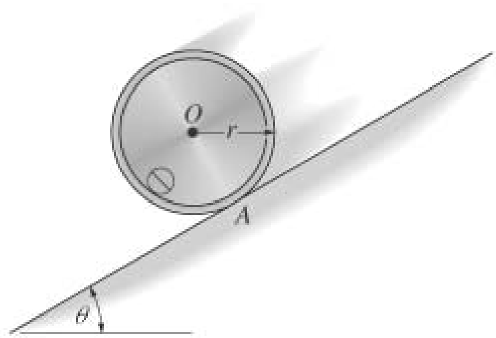
The drum has mass M, radius r, and radius of gyration k_O. If the coefficients of static and kinetic friction at A are μ_s and μ_k respectively, determine the drum's angular velocity at time t after it is released from rest.

Given:

$M = 70\ \text{kg}$ $\quad \mu_s = 0.4$ $\quad \theta = 30\ \text{deg}$

$r = 300\text{mm}$ $\quad \mu_k = 0.3$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$k_O = 125\ \text{mm}$ $\quad t = 2\ \text{s}$



Solution: Assume no slip

Guesses $\quad F_f = 1\ \text{N} \quad F_N = 1\ \text{N}$

$\omega = 1\ \frac{\text{rad}}{\text{s}} \quad v = 1\ \frac{\text{m}}{\text{s}} \quad F_{max} = 1\ \text{N}$

Given $\quad 0 + F_f r\,t = M k_O^2\,\omega \qquad v = \omega r \qquad F_{max} = \mu_s F_N$

$M\,g\sin(\theta)t - F_f t = M\,v \qquad F_N\,t - M\,g\cos(\theta)t = 0$

$$\begin{pmatrix} F_f \\ F_{max} \\ F_N \\ \omega \\ v \end{pmatrix} = \text{Find}(F_f, F_{max}, F_N, \omega, v) \qquad \begin{pmatrix} F_f \\ F_{max} \\ F_N \end{pmatrix} = \begin{pmatrix} 51 \\ 238 \\ 595 \end{pmatrix} \text{N} \qquad v = 8.36\ \frac{\text{m}}{\text{s}}$$

$$\omega = 27.9\ \frac{\text{rad}}{\text{s}}$$

Since $F_f = 51\ \text{N} < F_{max} = 238\ \text{N}$ then our no-slip assumption is good.

Problem 19-18

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has mass m_p and radius of gyration k_O. If the block at A has mass m_A, determine the speed of the block at time t after a constant force **F** is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest. Neglect the mass of the rope.

Units Used: $kN = 10^3\ N$

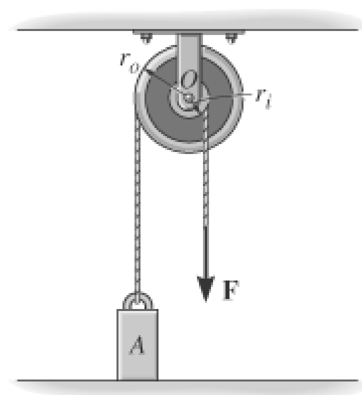
Given:

$m_p = 15\ kg$ $F = 2\ kN$

$k_O = 110\ mm$ $r_i = 75\ mm$

$m_A = 40\ kg$ $r_o = 200\ mm$

$t = 3\ s$



Solution: Guess $v_A = 1\ \frac{m}{s}$ $\omega = 1\ \frac{rad}{s}$

Given $-F\, r_i\, t + m_A\, g\, r_o\, t = -m_p\, k_O^2\, \omega - m_A\, v_A\, r_o$

$v_A = \omega r_o$

$\begin{pmatrix} v_A \\ \omega \end{pmatrix} = \text{Find}(v_A, \omega)$ $\omega = 120.44\ \frac{rad}{s}$ $v_A = 24.1\ \frac{m}{s}$

Problem 19-19

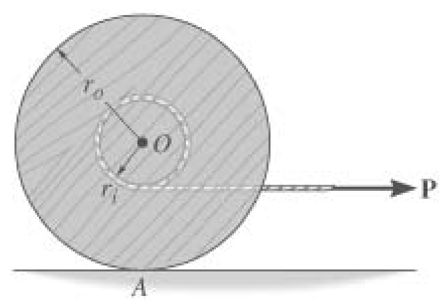
The spool has weight W and radius of gyration k_O. A cord is wrapped around its inner hub and the end subjected to a horizontal force **P**. Determine the spool's angular velocity at time t starting from rest. Assume the spool rolls without slipping.

Given:

$W = 30\ lb$ $t = 4\ s$

$k_O = 0.45\ ft$ $r_i = 0.3\ ft$

$P = 5\ lb$ $r_o = 0.9\ ft$



Solution: Guesses $\omega = 1\ \frac{rad}{s}$ $v_O = 1\ \frac{ft}{s}$

Given $-P(r_o - r_i)t = \left(\frac{-W}{g}\right) v_O\, r_o - \left(\frac{W}{g}\right) k_O^2\, \omega$ $v_O = \omega r_o$

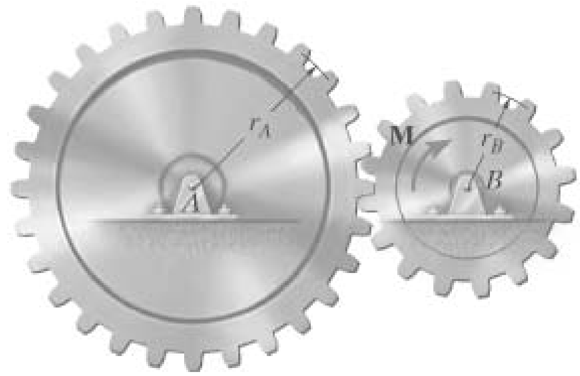
$\begin{pmatrix} v_O \\ \omega \end{pmatrix} = \text{Find}(v_O, \omega)$ $v_O = 3.49\ \frac{m}{s}$ $\omega = 12.7\ \frac{rad}{s}$

***Problem 19-20**

The two gears A and B have weights W_A, W_B and radii of gyration k_A and k_B respectively. If a motor transmits a couple moment to gear B of $M = M_0\,(1 - e^{-bt})$, determine the angular velocity of gear A

at time t, starting from rest.

Given:



$W_A = 15\ \text{lb} \qquad W_B = 10\ \text{lb}$

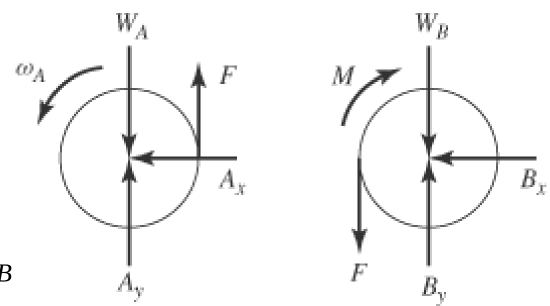
$r_A = 0.8\ \text{ft} \qquad r_B = 0.5\ \text{ft}$

$k_A = 0.5\ \text{ft} \qquad k_B = 0.35\ \text{ft}$

$M_0 = 2\ \text{lb}\cdot\text{ft} \qquad b = 0.5\ \text{s}^{-1}$

$t = 5\ \text{s}$

Solution:



Guesses

$\omega_A = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_B = 1\ \frac{\text{rad}}{\text{s}} \qquad ImpF = 1\ \text{lb}\cdot\text{s}$

Given $\displaystyle\int_0^t M_0\left(1 - e^{-b\,t}\right) dt - ImpF\, r_B = \left(\frac{W_B}{g}\right) k_B^{\,2}\, \omega_B$

$$ImpF\, r_A = \left(\frac{W_A}{g}\right) k_A^{\,2}\, \omega_A \qquad \omega_A r_A = \omega_B r_B$$

$$\begin{pmatrix} \omega_A \\ \omega_B \\ ImpF \end{pmatrix} = \text{Find}\left(\omega_A, \omega_B, ImpF\right) \qquad ImpF = 6.89\,\text{lb}\,\text{s} \qquad \omega_B = 75.7\,\frac{\text{rad}}{\text{s}}$$

$$\omega_A = 47.3\,\frac{\text{rad}}{\text{s}}$$

Problem 19-21

Spool B is at rest and spool A is rotating at ω when the slack in the cord connecting them is taken up. Determine the angular velocity of each spool immediately after the cord is jerked tight by the spinning of spool A. The weights and radii of gyration of A and B are W_A, k_A, and W_B, k_A, respectively.

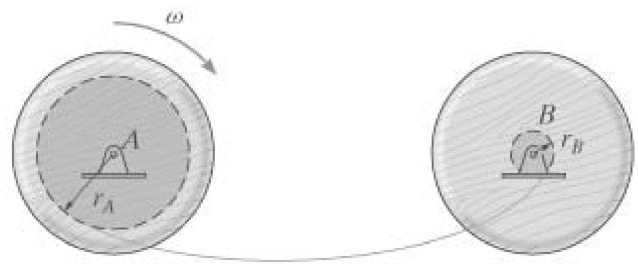
Given:

$W_A = 30\ \text{lb}$ $W_B = 15\ \text{lb}$

$k_A = 0.8\ \text{ft}$ $k_B = 0.6\ \text{ft}$

$r_A = 1.2\ \text{ft}$ $r_B = 0.4\ \text{ft}$

$\omega = 6\ \frac{\text{rad}}{\text{s}}$

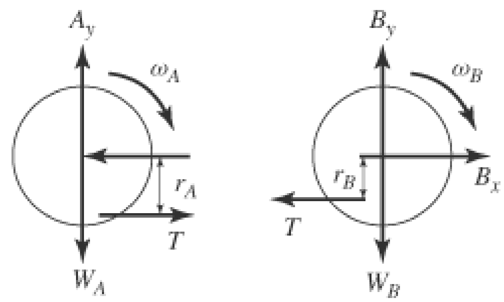


Solution:

Guesses

$\omega_A = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_B = 1\ \frac{\text{rad}}{\text{s}}$

$Imp = 1\ \text{lb}\cdot\text{s}$



Given

$$\left(\frac{W_A}{g}\right)k_A{}^2\omega - Imp\, r_A = \left(\frac{W_A}{g}\right)k_A{}^2\omega_A \qquad Imp\, r_B = \left(\frac{W_B}{g}\right)k_B{}^2\omega_B \qquad \omega_A r_A = \omega_B r_B$$

$$\begin{pmatrix}\omega_A\\ \omega_B\\ Imp\end{pmatrix} = \text{Find}(\omega_A, \omega_B, Imp) \qquad Imp = 2.14\ \text{lb}\cdot\text{s} \qquad \begin{pmatrix}\omega_A\\ \omega_B\end{pmatrix} = \begin{pmatrix}1.70\\ 5.10\end{pmatrix}\frac{\text{rad}}{\text{s}}$$

Problem 19-22

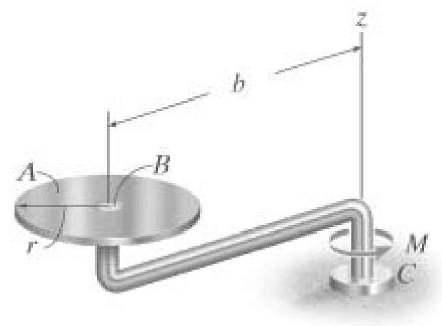
Disk A of mass m_A is mounted on arm BC, which has a negligible mass. If a torque of $M = M_0 e^{at}$ is applied to the arm at C, determine the angular velocity of BC at time t starting from rest. Solve the problem assuming that (a) the disk is set in a smooth bearing at B so that it rotates with curvilinear translation, (b) the disk is fixed to the shaft BC, and (c) the disk is given an initial freely spinning angular velocity $\omega_D\mathbf{k}$ prior to application of the torque.

Given:

$m_A = 4\ \text{kg}$ $\quad M_0 = 5\ \text{N}\cdot\text{m}$ $\quad \omega_D = -80\ \frac{\text{rad}}{\text{s}}$

$r = 60\ \text{mm}$ $\quad a = 0.5\ \text{s}^{-1}$

$b = 250\ \text{mm}$ $\quad t = 2\ \text{s}$



Solution: Guess $\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$

(a) Given $\displaystyle\int_0^t M_0 e^{a\,t}\,dt = m_A \omega_{BC} b^2$

$\omega_{BC} = \text{Find}(\omega_{BC})$ $\quad \omega_{BC} = 68.7\ \frac{\text{rad}}{\text{s}}$

(b) Given $\displaystyle\int_0^t M_0 e^{a\,t}\,dt = m_A \omega_{BC} b^2 + m_A\left(\frac{r^2}{2}\right)\omega_{BC}$

$\omega_{BC} = \text{Find}(\omega_{BC})$ $\quad \omega_{BC} = 66.8\ \frac{\text{rad}}{\text{s}}$

(c) Given $\displaystyle -m_A\left(\frac{r^2}{2}\right)\omega_D + \int_0^t M_0 e^{a\,t}\,dt = m_A \omega_{BC} b^2 - m_A\left(\frac{r^2}{2}\right)\omega_D$

$\omega_{BC} = \text{Find}(\omega_{BC})$ $\quad \omega_{BC} = 68.7\ \frac{\text{rad}}{\text{s}}$

Problem 19-23

The inner hub of the wheel rests on the horizontal track. If it does not slip at A, determine the speed of the block of weight W_b at time t after the block is released from rest. The wheel has weight W_w and radius of gyration k_G. Neglect the mass of the pulley and cord.

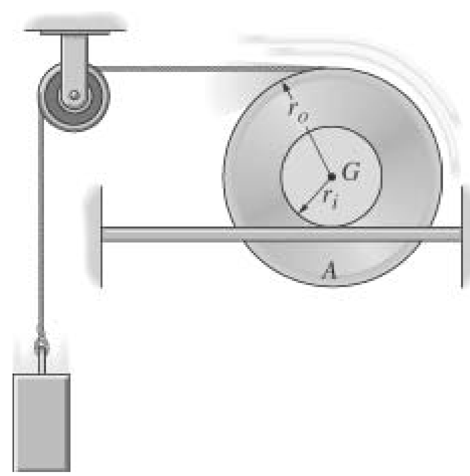
Given:

$W_b = 10\ \text{lb}$ $\quad r_i = 1\ \text{ft}$

$t = 2\ \text{s}$ $\quad r_o = 2\ \text{ft}$

$W_w = 30\ \text{lb}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$k_G = 1.30\ \text{ft}$



Solution: Guesses $v_G = 1\ \frac{\text{ft}}{\text{s}}$

$v_B = 1\ \frac{\text{ft}}{\text{s}}$ $\quad T = 1\ \text{lb}$ $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

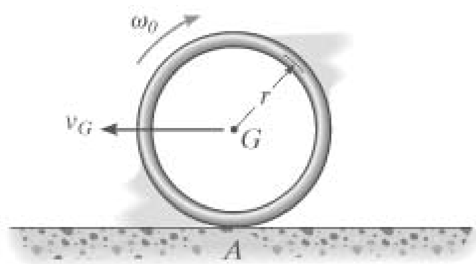
Given $\quad T\,t - W_b\,t = \left(\frac{-W_b}{g}\right)v_B \qquad T(r_o + r_i)t = \left(\frac{W_w}{g}\right)v_G r_i + \left(\frac{W_w}{g}\right)k_G{}^2\omega$

$v_G = \omega r_i \qquad v_B = \omega(r_i + r_o)$

$$\begin{pmatrix} v_G \\ v_B \\ \omega \\ T \end{pmatrix} = \text{Find}(v_G, v_B, \omega, T) \qquad T = 4.73\,\text{lb} \quad \omega = 11.3\,\frac{\text{rad}}{\text{s}} \qquad v_G = 11.3\,\frac{\text{ft}}{\text{s}} \qquad v_B = 34.0\,\frac{\text{ft}}{\text{s}}$$

***Problem 19-24**

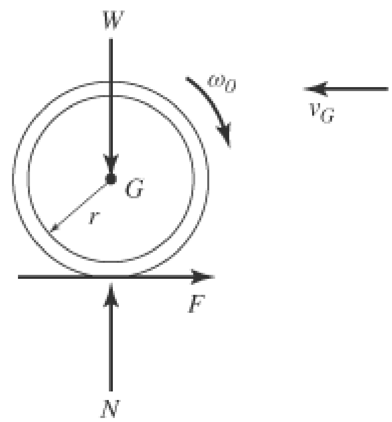
If the hoop has a weight W and radius r and is thrown onto a *rough surface* with a velocity v_G parallel to the surface, determine the amount of backspin, ω_0, it must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of kinetic friction at A for the calculation.



Solution:

$$\left(\frac{W}{g}\right)v_G\,r - \left(\frac{W}{g}\right)r^2\omega_0 = 0$$

$$\omega_0 = \frac{v_G}{r}$$



Problem 19-25

The rectangular plate of weight W is at rest on a smooth *horizontal* floor. If it is given the horizontal impulses shown, determine its angular velocity and the velocity of the mass center.

Given:

$W = 10\ \text{lb}$ $\quad d = 0.5\ \text{ft}$

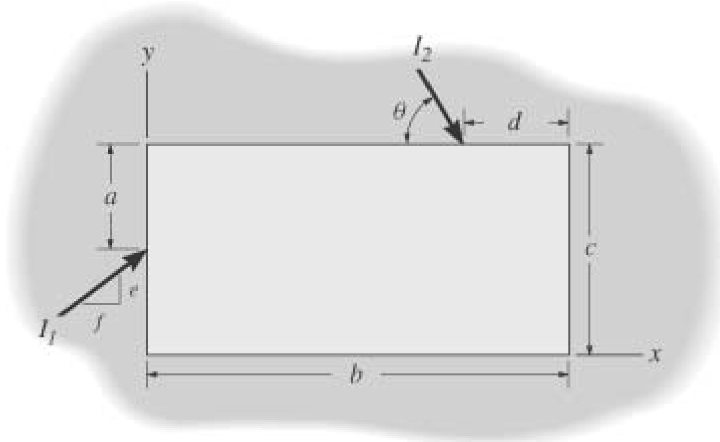
$I_1 = 20\ \text{lb}\cdot\text{s}$ $\quad \theta = 60\ \text{deg}$

$I_2 = 5\ \text{lb}\cdot\text{s}$ $\quad e = 3$

$a = 0.5\ \text{ft}$ $\quad f = 4$

$b = 2\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$c = 1\ \text{ft}$



Solution: Guesses $v_x = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_y = 1\ \frac{\text{ft}}{\text{s}}$ $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\left(\frac{f}{\sqrt{e^2+f^2}}\right)I_1 + I_2\cos(\theta) = \left(\frac{W}{g}\right)v_x$$

$$\left(\frac{e}{\sqrt{e^2+f^2}}\right)I_1 - I_2\sin(\theta) = \left(\frac{W}{g}\right)v_y$$

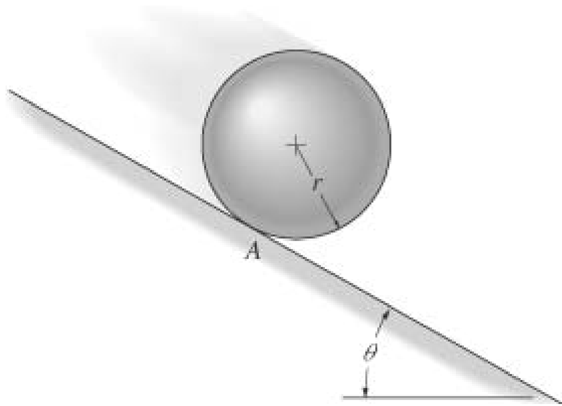
$$\left(\frac{f}{\sqrt{e^2+f^2}}\right)I_1\,a - I_2\sin(\theta)(b-d) = \frac{1}{12}\left(\frac{W}{g}\right)\left(b^2+c^2\right)\omega + \left(\frac{W}{g}\right)v_x\left(\frac{c}{2}\right) + \left(\frac{W}{g}\right)v_y\left(\frac{b}{2}\right)$$

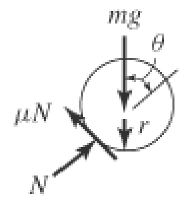
$$\begin{pmatrix} v_x \\ v_y \\ \omega \end{pmatrix} = \text{Find}(v_x, v_y, \omega) \qquad \mathbf{v_G} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} \qquad \omega = -119\ \frac{\text{rad}}{\text{s}} \qquad \mathbf{v_G} = \begin{pmatrix} 59.6 \\ 24.7 \end{pmatrix}\frac{\text{ft}}{\text{s}}$$

$$\left|\mathbf{v_G}\right| = 64.5\ \frac{\text{ft}}{\text{s}}$$

Problem 19-26

The ball of mass m and radius r rolls along an inclined plane for which the coefficient of static friction is μ. If the ball is released from rest, determine the maximum angle θ for the incline so that it rolls without slipping at A.





Solution:

$$N t - m g \cos(\theta) t = 0 \qquad N = m g \cos(\theta)$$

$$\mu N r t = \frac{2}{5} m r^2 \omega \qquad t = \frac{2\omega r}{5\mu g \cos(\theta)}$$

$$m g \sin(\theta) t - \mu N t = m r \omega \qquad t = \frac{r\omega}{g(\sin(\theta) - \mu \cos(\theta))}$$

Thus

$$\frac{2\omega r}{5\mu g \cos(\theta)} = \frac{r\omega}{g(\sin(\theta) - \mu \cos(\theta))} \qquad \theta = \operatorname{atan}\left(\frac{7\mu}{2}\right)$$

Problem 19-27

The spool has weight W_s and radius of gyration $k_{O.}$ If the block B has weight W_b and a force **P** is applied to the cord, determine the speed of the block at time t starting from rest. Neglect the mass of the cord.

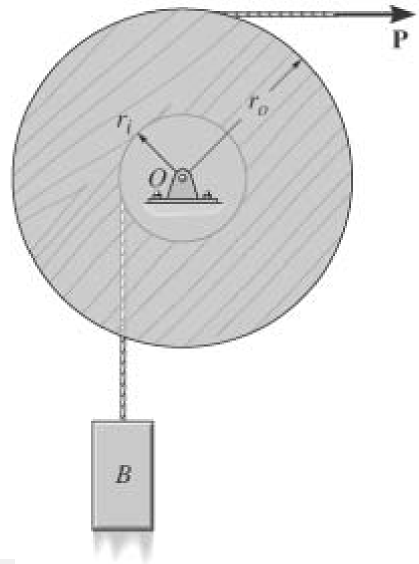
Given:

$W_S = 75\text{lb} \qquad t = 5\text{s}$

$k_O = 1.2\text{ft} \qquad r_o = 2\text{ft}$

$W_b = 60\text{lb} \qquad r_i = 0.75\text{ft}$

$P = 25\text{lb}$



Solution:

Guess $\quad v_B = 1\dfrac{\text{ft}}{\text{s}} \qquad \omega = 1\dfrac{\text{rad}}{\text{s}}$

Given

$$-P\,r_o\,t + W_b\,r_i\,t = \left(\frac{-W_b}{g}\right)v_B\,r_i - \left(\frac{W_s}{g}\right)k_O^{\,2}\,\omega$$

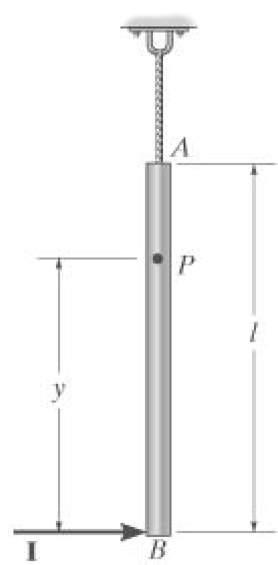
$$v_B = \omega r_i$$

$$\begin{pmatrix} v_B \\ \omega \end{pmatrix} = \text{Find}(v_B, \omega) \qquad \omega = 5.68\,\frac{\text{rad}}{\text{s}} \qquad v_B = 4.26\,\frac{\text{ft}}{\text{s}}$$

***Problem 19-28**

The slender rod has a mass *m* and is suspended at its end *A* by a cord. If the rod receives a horizontal blow giving it an impulse **I** at its bottom *B*, determine the location *y* of the point *P* about which the rod appears to rotate during the impact.

Solution:



$$F\Delta t = m\,v_{cm}$$

$$m\Delta t = I\omega$$

$$I\,y = I'\,\omega$$

$$I\,y = m\left[\frac{l^2}{12} + \left(y - \frac{l}{2}\right)^2\right]\omega \qquad (1)$$

$$I = m\,v_{cm}$$

$$I = m\frac{l}{2}\omega \qquad (2)$$

Divide Eqs (1) by (2)

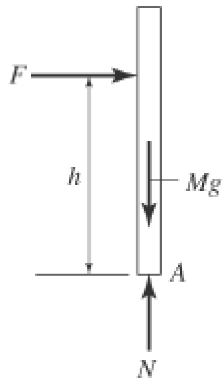
$$\frac{y\,l}{2} = \left[\frac{l^2}{12} + \left(y - \frac{l}{2}\right)^2\right] \qquad y = \left(\frac{3}{4} + \frac{\sqrt{33}}{12}\right)l$$

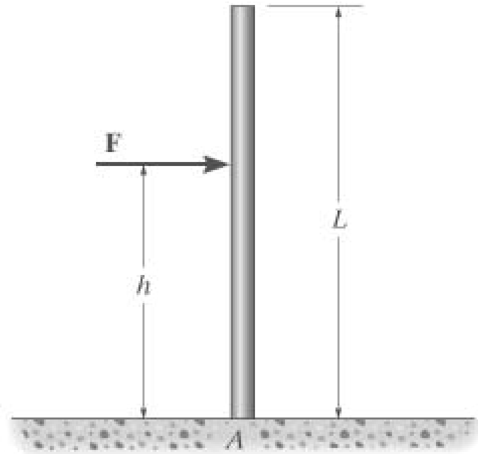
Problem 19-29

A thin rod having mass M is balanced vertically as shown. Determine the height h at which it can be struck with a horizontal force **F** and not slip on the floor. This requires that the frictional force at A be essentially zero.

Given:

$M = 4\ \text{kg} \qquad L = 0.8\ \text{m}$





Solution:

$$F\,t = M\omega\frac{L}{2}$$

$$F\,t\,h = M\frac{L^2}{3}\omega$$

Thus

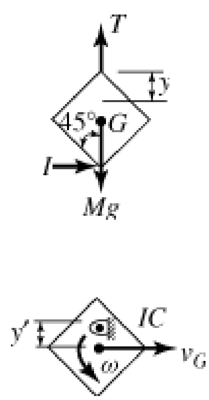
$$M\omega\frac{L}{2}h = M\frac{L^2}{3}\omega$$

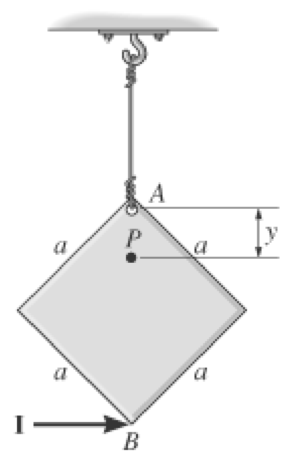
$$h = \frac{2}{3}L \qquad h = 0.53\ \text{m}$$

Problem 19-30

The square plate has a mass M and is suspended at its corner A by a cord. If it receives a horizontal impulse **I** at corner B, determine the location y' of the point P about which the plate appears to rotate during the impact.

Solution:





$$I\frac{a}{\sqrt{2}} = \frac{1}{6}Ma^2\omega \qquad \omega = \frac{6}{\sqrt{2}}\frac{I}{Ma}$$

$$I = Mv_G \qquad v_G = \frac{I}{M}$$

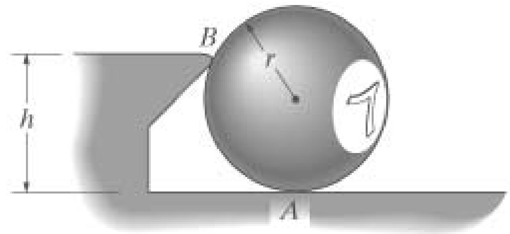
$$y' = \frac{v_G}{\omega} \qquad y' = \frac{\sqrt{2}}{6}a$$

$$y = \frac{a}{\sqrt{2}} - y' \qquad y = \frac{\sqrt{2}}{3}a$$

Problem 19-31

Determine the height h of the bumper of the pool table, so that when the pool ball of mass m strikes it, no frictional force will be developed between the ball and the table at A. Assume the bumper exerts only a horizontal force on the ball.

Solution:



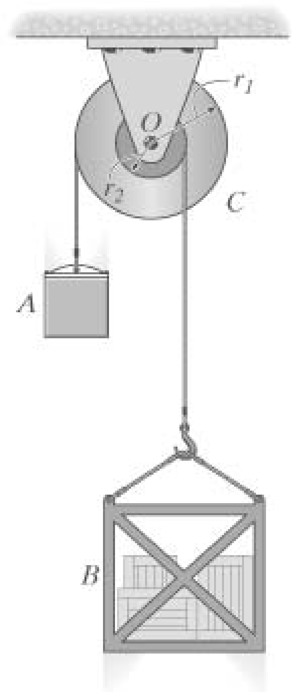
$$F\Delta t = M\Delta v \qquad F\Delta t\,h = \frac{7}{5}M r^2 \Delta\omega \qquad \Delta v = r\Delta\omega$$

Thus

$$M r\Delta\omega h = \frac{7}{5}M r^2 \Delta\omega \qquad h = \frac{7}{5}r$$

***Problem 19-32**

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass M_C and a radius of gyration k_O. If the block at A has a mass M_A and the container at B has a mass M_B, including its contents, determine the speed of the container at time t after it is released from rest.



Given:

$M_C = 15\ \text{kg} \quad k_O = 110\ \text{mm}$

$M_A = 40\ \text{kg} \quad r_1 = 200\ \text{mm}$

$M_B = 85\ \text{kg} \quad r_2 = 75\ \text{mm}$

$t = 3\ \text{s}$

Solution:

Guess $\quad v_A = 1\frac{\text{m}}{\text{s}} \qquad v_B = 1\frac{\text{m}}{\text{s}} \qquad \omega = 1\frac{\text{rad}}{\text{s}}$

Given $\quad M_A g t r_1 - M_B g t r_2 = M_A v_A r_1 + M_B v_B r_2 + M_C k_O^2 \omega$

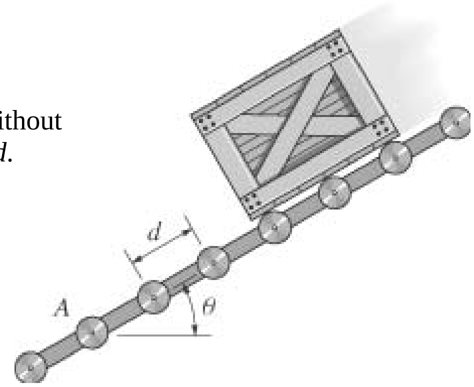
$v_A = \omega r_1 \qquad v_B = \omega r_2$

$$\begin{pmatrix} v_A \\ v_B \\ \omega \end{pmatrix} = \text{Find}(v_A, v_B, \omega) \qquad \omega = 21.2\frac{\text{rad}}{\text{s}} \qquad v_A = 4.23\ \frac{\text{m}}{\text{s}} \qquad v_B = 1.59\ \frac{\text{m}}{\text{s}}$$

Problem 19-33

The crate has a mass M_c. Determine the constant speed v_0 it acquires as it moves down the conveyor. The rollers each have radius r, mass M, and are spaced distance d apart. Note that friction causes each roller to rotate when the crate comes in contact with it.

Solution:



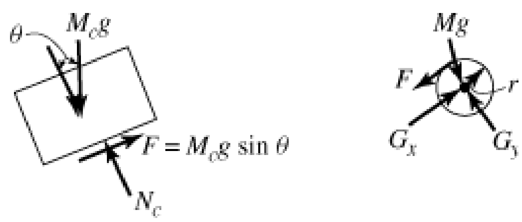
Assume each roller is brought to the condition of roll without slipping. In time t, the number of rollers affected is $v_0 t/d$.

$$M_C\, g \sin(\theta) t - F\, t = 0$$

$$F = M_C\, g \sin(\theta)$$

$$F\, t\, r = \left(\frac{1}{2} M\, r^2\right) \frac{v_0}{r} \left(\frac{v_0}{d} t\right)$$

$$v_0 = \sqrt{2g \sin(\theta) d \frac{M_C}{M}}$$



Problem 19-34

Two wheels A and B have masses m_A and m_B and radii of gyration about their central vertical axes of k_A and k_B respectively. If they are freely rotating in the same direction at ω_A and ω_B about the same vertical axis, determine their common angular velocity after they are brought into contact and slipping between them stops.

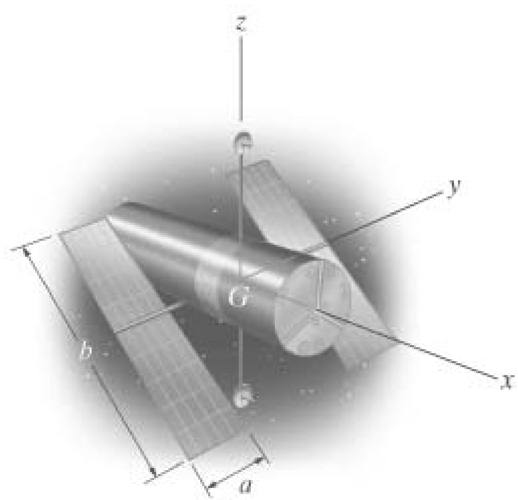
Solution:

$$m_A k_A^2 \omega_A + m_B k_B^2 \omega_B = \left(m_A k_A^2 + m_B k_B^2\right)\omega$$

$$\omega = \frac{m_A k_A^2 \omega_A + m_B k_B^2 \omega_B}{m_A k_A^2 + m_B k_B^2}$$

Problem 19-35

The Hubble Space Telescope is powered by two solar panels as shown. The body of the telescope has a mass M_1 and radii of gyration k_x and k_y, whereas the solar panels can be considered as thin plates, each having a mass M_2. Due to an internal drive, the panels are given an angular velocity of $\omega_0\mathbf{j}$, measured relative to the telescope. Determine the angular velocity of the telescope due to the rotation of the panels. Prior to rotating the panels, the telescope was originally traveling at $\mathbf{v_G} = (v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k})$. Neglect its orbital rotation.



Units Used: $\mathrm{Mg} = 10^3 \mathrm{kg}$

Given:

$M_1 = 11\ \mathrm{Mg}$ $\quad \omega_0 = 0.6\ \frac{\mathrm{rad}}{\mathrm{s}}$ $\quad v_x = -400\ \frac{\mathrm{m}}{\mathrm{s}}$

$M_2 = 54\ \mathrm{kg}$ $\quad a = 1.5\ \mathrm{m}$ $\quad v_y = 250\ \frac{\mathrm{m}}{\mathrm{s}}$

$k_x = 1.64\ \mathrm{m}$

$k_y = 3.85\ \mathrm{m}$ $\quad b = 6\ \mathrm{m}$ $\quad v_z = 175\ \frac{\mathrm{m}}{\mathrm{s}}$

Solution: Angular momentum is conserved.

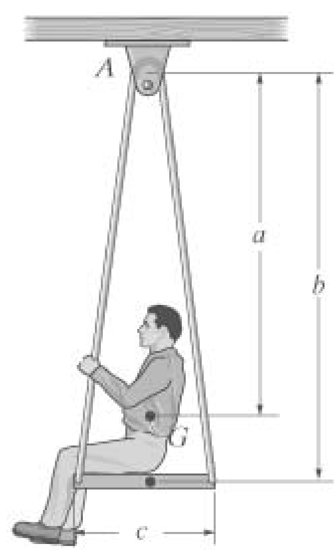
Guess $\omega_T = 1\frac{\mathrm{rad}}{\mathrm{s}}$

Given $0 = 2\left(\frac{1}{12}M_2 b^2\right)(\omega_0 - \omega_T) - \left(M_1 k_y^2\right)\omega_T$ $\quad \omega_T = \mathrm{Find}(\omega_T)$

$\omega_T = 0.00\,\frac{\mathrm{rad}}{\mathrm{s}}$

***Problem 19-36**

The platform swing consists of a flat plate of weight W_p suspended by four rods of negligible weight. When the swing is at rest, the man of weight W_m jumps off the platform when his center of gravity G is at distance a from the pin at A. This is done with a horizontal velocity v, measured relative to the swing at the level of G. Determine the angular velocity he imparts to the swing just after jumping off.



Given:

$W_p = 200\mathrm{lb}$ $\quad a = 10\mathrm{ft}$

$W_m = 150\mathrm{lb}$ $\quad b = 11\mathrm{ft}$

$v = 5\frac{\mathrm{ft}}{\mathrm{s}}$ $\quad c = 4\mathrm{ft}$

Solution:

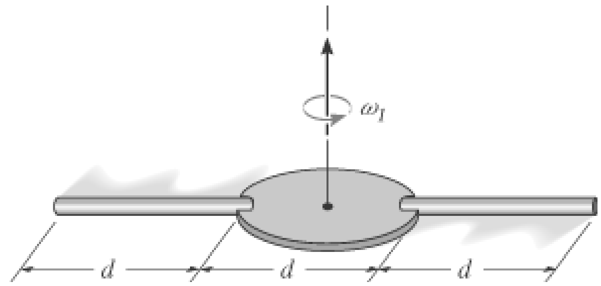
Guess $\omega = 1\frac{\mathrm{rad}}{\mathrm{s}}$

Given $0 = \frac{-W_m}{g}(v - \omega a)a + \frac{W_p}{g}\left(\frac{c^2}{12} + b^2\right)\omega$

$\omega = \mathrm{Find}(\omega)$ $\quad \omega = 0.190\,\frac{\mathrm{rad}}{\mathrm{s}}$

Problem 19-37

Each of the two slender rods and the disk have the same mass *m*. Also, the length of each rod is equal to the diameter *d* of the disk. If the assembly is rotating with an angular velocity ω_1 when the rods are directed outward, determine the angular velocity of the assembly if by internal means the rods are brought to an upright vertical position.



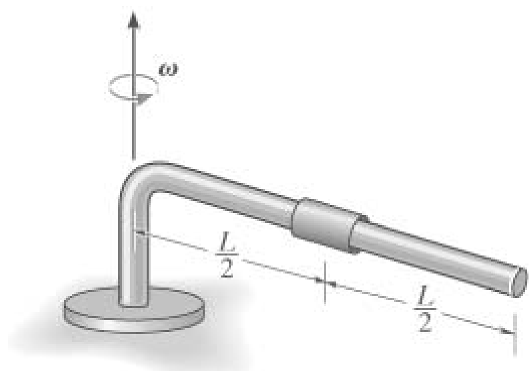
Solution:

$$H_1 = H_2$$

$$\left[\frac{1}{2}m\left(\frac{d}{2}\right)^2 + 2\frac{1}{12}m d^2 + 2m d^2\right]\omega_1 = \left[\frac{1}{2}m\left(\frac{d}{2}\right)^2 + 2m\left(\frac{d}{2}\right)^2\right]\omega_2 \qquad \omega_2 = \frac{11}{3}\omega_1$$

Problem 19-38

The rod has a length *L* and mass *m*. A smooth collar having a negligible size and one-fourth the mass of the rod is placed on the rod at its midpoint. If the rod is freely rotating with angular velocity ω about its end and the collar is released, determine the rod's angular velocity just before the collar flies off the rod. Also, what is the speed of the collar as it leaves the rod?



Solution:

$$H_1 = H_2$$

$$\frac{1}{3}mL^2\omega + \left(\frac{m}{4}\right)\left(\frac{L}{2}\right)\omega\left(\frac{L}{2}\right) = \frac{1}{3}mL^2\omega' + \left(\frac{m}{4}\right)L\omega' L \qquad \omega' = \frac{19}{28}\omega$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 + \frac{1}{2}\left(\frac{m}{4}\right)\left(\frac{L}{2}\omega\right)^2 = \frac{1}{2}\left(\frac{m}{4}\right)v'^2 + \frac{1}{2}\left(\frac{m}{4}\right)(L\omega')^2 + \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega'^2$$

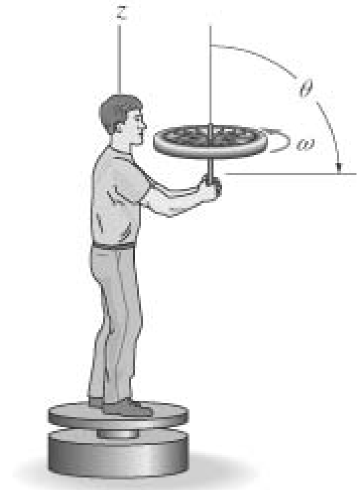
$$v'^2 = \frac{57}{112}L^2\omega^2$$

$$v'' = \sqrt{\frac{57}{112}L^2\omega^2 + \left[L\left(\frac{19}{28}\omega\right)\right]^2} \qquad v'' = \sqrt{\frac{95}{98}}\,\omega L \qquad v'' = 0.985\,\omega L$$

Problem 19-39

A man has a moment of inertia I_z about the z axis. He is originally at rest and standing on a small platform which can turn freely. If he is handed a wheel which is rotating at angular velocity ω and has a moment of inertia I about its spinning axis, determine his angular velocity if (a) he holds the wheel upright as shown, (b) turns the wheel out $\theta = 90°$, and (c) turns the wheel downward, $\theta = 180°$.

Solution:



(a) $0 + I\omega = I_z\omega_M + I\omega$ $\qquad \omega_M = 0$

(b) $0 + I\omega = I_z\omega_M + 0$ $\qquad \omega_M = \dfrac{I}{I_z}\omega$

(c) $0 + I\omega = I_z\omega_M - I\omega$ $\qquad \omega_M = \dfrac{2I}{I_z}\omega$

*** Problem 19-40**

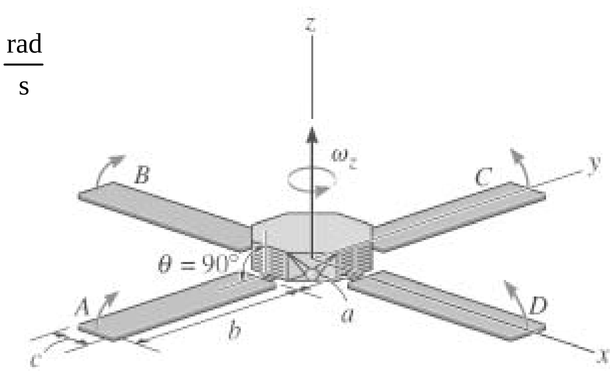
The space satellite has mass m_{ss} and moment of inertia I_z, excluding the four solar panels A,B,C, and D. Each solar panel has mass m_p and can be approximated as a thin plate.If the satellite is originally spinning about the z axis at aconstant rate ω_z, when $\theta = 90°$, determine the rate of spin if all the panels are raised and reach the upward position, $\theta = 0°$, at the same instsnt.

Given:

$m_{SS} = 125\ \text{kg} \qquad a = 0.2\ \text{m} \qquad \omega_z = 0.5\ \dfrac{\text{rad}}{\text{s}}$

$I_z = 0.940\ \text{kg}\cdot\text{m}^2 \qquad b = 0.75\ \text{m}$

$m_{sp} = 20\ \text{kg} \qquad c = 0.2\ \text{m}$



Solution: Guess $\omega_{z2} = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

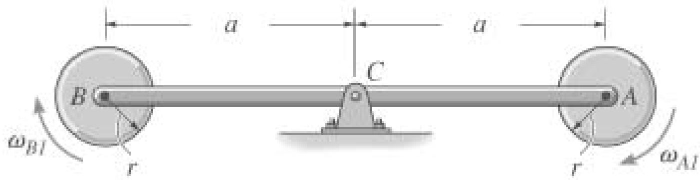
$$\left[I_z + 4\left[\frac{m_{sp}}{12}\left(b^2 + c^2\right) + m_{sp}\left(a + \frac{b}{2}\right)^2\right]\right]\omega_z = \left[I_z + 4\left(\frac{m_{sp}}{12}c^2 + m_{sp}a^2\right)\right]\omega_{z2}$$

$\omega_{z2} = \text{Find}(\omega_{z2}) \qquad \omega_{z2} = 3.56\ \dfrac{\text{rad}}{\text{s}}$

Problem 19-41

Rod *ACB* of mass m_r supports the two disks each of mass m_d at its ends. If both disks are given a clockwise angular velocity $\omega_{A1} = \omega_{B1} = \omega_0$ while the rod is held stationary and then released, determine the angular velocity of the rod after both disks have stopped spinning relative to the rod due to frictional resistance at the pins *A* and *B*. Motion is in the *horizontal plane.* Neglect friction at pin *C*.

Given:



$m_r = 2\ \text{kg}$

$m_d = 4\ \text{kg}$

$\omega_0 = 5\ \frac{\text{rad}}{\text{s}}$

$a = 0.75\ \text{m}$

$r = 0.15\ \text{m}$

Solution:

$$2\left(\frac{1}{2}m_d\right)r^2\omega_0 = \left[2\left(\frac{1}{2}m_d\right)r^2 + 2m_d a^2 + \frac{m_r}{12}(2a)^2\right]\omega_2$$

$$\omega_2 = \frac{m_d r^2}{m_d r^2 + 2m_d a^2 + \left(\frac{m_r}{3}\right)a^2}\omega_0 \qquad \omega_2 = 0.0906\ \frac{\text{rad}}{\text{s}}$$

Problem 19-42

Disk *A* has a weight W_A. An inextensible cable is attached to the weight *W* and wrapped around the disk.The weight is dropped distance *h* before the slack is taken up. If the impact is perfectly elastic, i.e., $e = 1$, determine the angular velocity of the disk just after impact.

Given:

$W_A = 20\ \text{lb} \qquad h = 2\ \text{ft}$

$W = 10\ \text{lb} \qquad r = 0.5\ \text{ft}$

Solution:

$v_1 = \sqrt{2gh}$

Guess $\qquad \omega_2 = 1\ \frac{\text{rad}}{\text{s}} \qquad v_2 = 1\ \frac{\text{ft}}{\text{s}}$

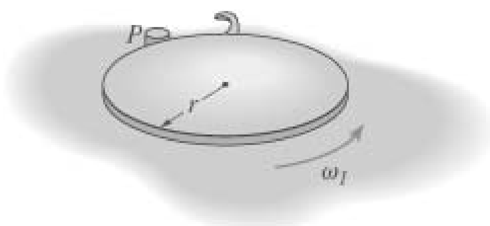
Given $\left(\frac{W}{g}\right) v_1 r = \left(\frac{W}{g}\right) v_2 r + \left(\frac{W_A}{g}\right) \frac{r^2}{2} \omega_2 \qquad v_2 = \omega_2 r$

$\begin{pmatrix} v_2 \\ \omega_2 \end{pmatrix} = \mathrm{Find}(v_2, \omega_2) \qquad v_2 = 5.67 \frac{\mathrm{ft}}{\mathrm{s}} \qquad \omega_2 = 11.3 \frac{\mathrm{rad}}{\mathrm{s}}$

Problem 19-43

A thin disk of mass m has an angular velocity ω_1 while rotating on a smooth surface. Determine its new angular velocity just after the hook at its edge strikes the peg P and the disk starts to rotate about P without rebounding.

Solution:

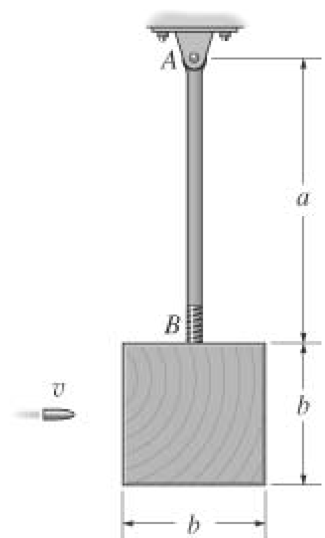


$H_1 = H_2$

$\left(\frac{1}{2} m r^2\right) \omega_1 = \left(\frac{1}{2} m r^2 + m r^2\right) \omega_2$

$\omega_2 = \frac{1}{3} \omega_1$

***Problem 19-44**



The pendulum consists of a slender rod AB of weight W_r and a wooden block of weight W_b. A projectile of weight W_p is fired into the center of the block with velocity v. If the pendulum is initially at rest, and the projectile embeds itself into the block, determine the angular velocity of the pendulum just after the impact.

Given:

$W_r = 5 \text{ lb} \quad W_p = 0.2 \text{ lb} \quad v = 1000 \frac{\mathrm{ft}}{\mathrm{s}} \quad g = 32.2 \frac{\mathrm{ft}}{\mathrm{s}^2}$

$W_b = 10 \text{ lb} \quad a = 2 \text{ ft} \quad b = 1 \text{ ft}$

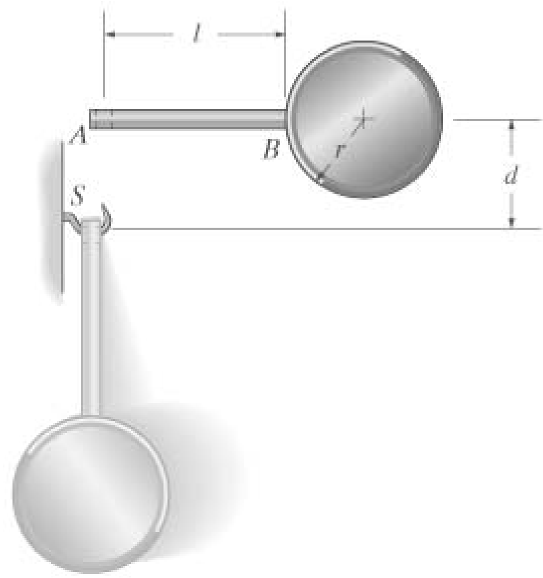
Solution:

$$\left(\frac{W_p}{g}\right) v \left(a + \frac{b}{2}\right) = \left[\left(\frac{W_r}{g}\right)\left(\frac{a^2}{3}\right) + \left(\frac{W_b}{g}\right)\left(\frac{b^2}{6}\right) + \left(\frac{W_b}{g}\right)\left(a + \frac{b}{2}\right)^2 + \left(\frac{W_p}{g}\right)\left(a + \frac{b}{2}\right)^2\right] \omega_2$$

$$\omega_2 = \frac{W_p v \left(a + \frac{b}{2}\right)}{W_r \frac{a^2}{3} + W_b \frac{b^2}{6} + W_b \left(a + \frac{b}{2}\right)^2 + W_p \left(a + \frac{b}{2}\right)^2} \qquad \omega_2 = 6.94 \frac{\mathrm{rad}}{\mathrm{s}}$$

Problem 19-45

The pendulum consists of a slender rod *AB* of mass M_1 and a disk of mass M_2. It is released from rest without rotating. When it falls a distance *d*, the end *A* strikes the hook *S*, which provides a permanent connection. Determine the angular velocity of the pendulum after it has rotated 90°. Treat the pendulum's weight during impact as a nonimpulsive force.



Given:

$M_1 = 2\text{kg}$ $\quad r = 0.2\text{m}$

$M_2 = 5\text{kg}$ $\quad l = 0.5\text{m}$

$d = 0.3\text{m}$

Solution:

$$v_1 = \sqrt{2g\,d}$$

$$I_A = M_1\frac{l^2}{3} + M_2\frac{r^2}{2} + M_2(l+r)^2$$

Guesses

$$\omega_2 = 1\frac{\text{rad}}{\text{s}} \qquad \omega_3 = 1\frac{\text{rad}}{\text{s}}$$

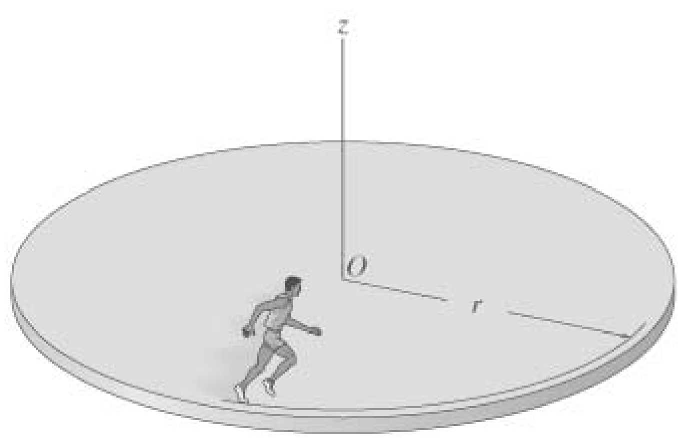
Given

$$M_1 v_1\frac{l}{2} + M_2 v_1(l+r) = I_A\omega_2 \qquad \frac{1}{2}I_A\omega_2^{\,2} = \frac{1}{2}I_A\omega_3^{\,2} - M_1 g\frac{l}{2} - M_2 g(l+r)$$

$$\begin{pmatrix}\omega_2\\ \omega_3\end{pmatrix} = \text{Find}(\omega_2, \omega_3) \qquad \omega_2 = 3.57\frac{\text{rad}}{\text{s}} \qquad \omega_3 = 6.46\frac{\text{rad}}{\text{s}}$$

Problem 19-46

A horizontal circular platform has a weight W_1 and a radius of gyration k_z about the *z* axis passing through its center *O*. The platform is free to rotate about the *z* axis and is initially at rest. A man having a weight W_2 begins to run along the edge in a circular path of radius *r*. If he has a speed *v* and maintains this speed relative to the platform, determine the angular velocity of the platform. Neglect friction.



Given:

$W_1 = 300\ \text{lb}$ $v = 4\ \frac{\text{ft}}{\text{s}}$

$W_2 = 150\ \text{lb}$

$r = 10\ \text{ft}$ $k_z = 8\ \text{ft}$

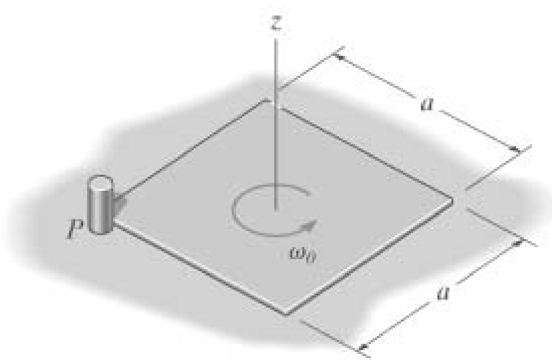
Solution:

$$Mvr = I\omega$$

$$\frac{W_2}{g} v r = \frac{W_1}{g} {k_z}^2 \omega \qquad \omega = W_2 v \frac{r}{W_1 {k_z}^2} \qquad \omega = 0.312\ \frac{\text{rad}}{\text{s}}$$

Problem 19-47

The square plate has a weight W and is rotating on the smooth surface with a constant angular velocity ω_0. Determine the new angular velocity of the plate just after its corner strikes the peg P and the plate starts to rotate about P without rebounding.

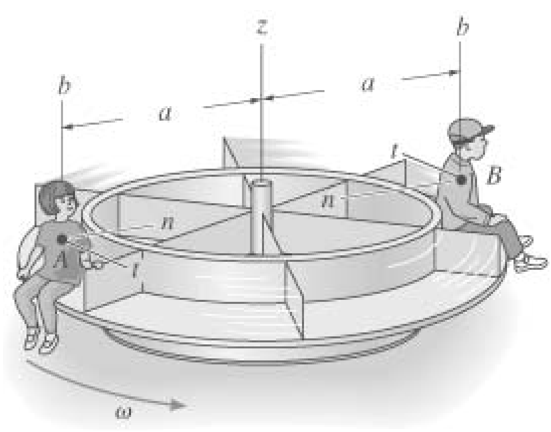


Solution:

$$\left(\frac{W}{g}\right)\left(\frac{a^2}{6}\right)\omega_0 = \left(\frac{W}{g}\right)\left(\frac{2a^2}{3}\right)\omega \qquad \omega = \frac{1}{4}\omega_0$$

***Problem 19-48**

Two children A and B, each having a mass M_1, sit at the edge of the merry-go-round which is rotating with angular velocity ω. Excluding the children, the merry-go-round has a mass M_2 and a radius of gyration k_z. Determine the angular velocity of the merry-go-round if A jumps off horizontally in the $-n$ direction with a speed v, measured with respect to the merry-go-round. What is the merry-go-round's angular velocity if B then jumps off horizontally in the $+t$ direction with a speed v, measured with respect to the merry-go-round? Neglect friction and the size of each child.



Given:

$M_1 = 30\ \text{kg}$ $k_z = 0.6\ \text{m}$

$M_2 = 180\ \text{kg}$ $a = 0.75\ \text{m}$

$\omega = 2\ \frac{\text{rad}}{\text{s}}$ $v = 2\ \frac{\text{m}}{\text{s}}$

Solution:

(a) Guess $\omega_2 = 1\frac{\text{rad}}{\text{s}}$

Given $\left(M_2 k_z^2 + 2M_1 a^2\right)\omega = \left(M_2 k_z^2 + M_1 a^2\right)\omega_2$

$\omega_2 = \text{Find}(\omega_2)$ $\omega_2 = 2.41\frac{\text{rad}}{\text{s}}$

(b) Guess $\omega_3 = 1\frac{\text{rad}}{\text{s}}$

Given $\left(M_2 k_z^2 + M_1 a^2\right)\omega_2 = M_2 k_z^2 \omega_3 + M_1(v + \omega_3 a)a$

$\omega_3 = \text{Find}(\omega_3)$ $\omega_3 = 1.86\frac{\text{rad}}{\text{s}}$

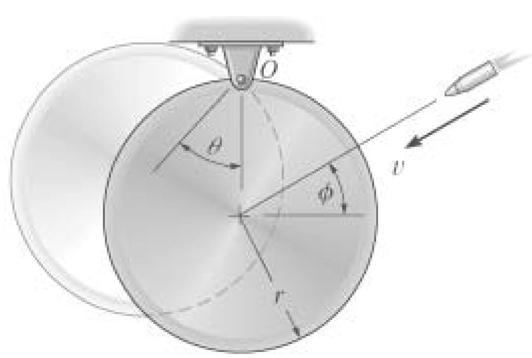
Problem 19-49

A bullet of mass m_b having velocity v is fired into the edge of the disk of mass m_d as shown. Determine the angular velocity of the disk of mass m_d just after the bullet becomes embedded in it. Also, calculate how far θ the disk will swing until it stops. The disk is originally at rest.

Given:

$m_b = 7\ \text{gm}$ $m_d = 5\ \text{kg}$ $\phi = 30\ \text{deg}$

$v = 800\ \frac{\text{m}}{\text{s}}$ $r = 0.2\ \text{m}$



Solution:

Guesses $\omega = 1\ \frac{\text{rad}}{\text{s}}$ $\theta = 10\ \text{deg}$

Given $m_b v\cos(\phi)r = \frac{3}{2}m_d r^2 \omega$ $-m_d g r + \frac{1}{2}\left(\frac{3}{2}m_d r^2\right)\omega^2 = -m_d g r\cos(\theta)$

$\begin{pmatrix}\omega \\ \theta\end{pmatrix} = \text{Find}(\omega, \theta)$ $\omega = 3.23\frac{\text{rad}}{\text{s}}$ $\theta = 32.8\,\text{deg}$

Problem 19-50

The two disks each have weight W. If they are released from rest when $\theta = \theta_1$, determine the maximum angle θ_2 after they collide and rebound from each other. The coefficient of restitution is e. When $\theta = 0°$ the disks hang so that they just touch one another.

Given:

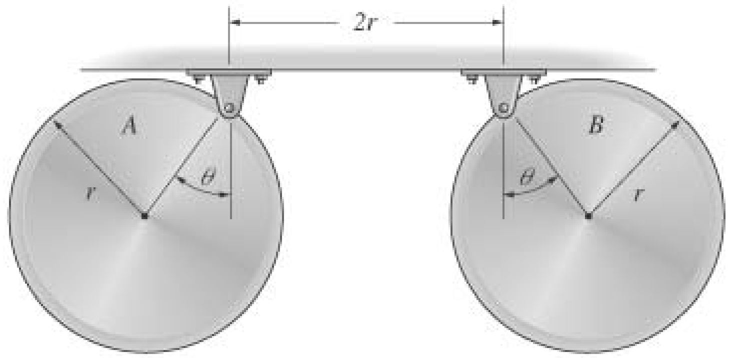
$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$W = 10\ \text{lb}$

$\theta_1 = 30\ \text{deg}$

$e = 0.75$

$r = 1\ \text{ft}$



Solution:

Guesses $\omega_1 = 1\dfrac{\text{rad}}{\text{s}}$ $\omega_2 = 1\dfrac{\text{rad}}{\text{s}}$ $\theta_2 = 10\text{deg}$

Given $-W r\cos(\theta_1) = \dfrac{1}{2}\left(\dfrac{3}{2}\dfrac{W}{g}r^2\right)\omega_1^{\,2} - W r$

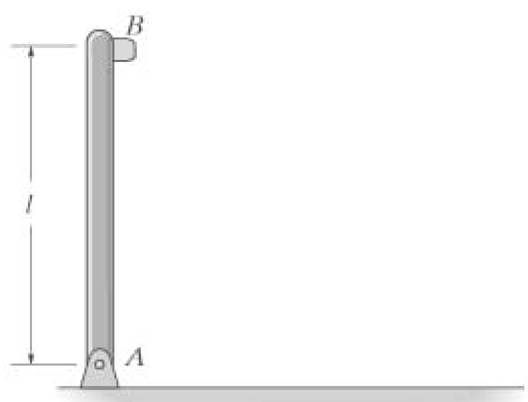
$e r\omega_1 = r\omega_2$

$-W r + \dfrac{1}{2}\left(\dfrac{3}{2}\dfrac{W}{g}r^2\right)\omega_2^{\,2} = -W r\cos(\theta_2)$

$\begin{pmatrix}\omega_1\\ \omega_2\\ \theta_2\end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta_2)$ $\begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix} = \begin{pmatrix}2.40\\ 1.80\end{pmatrix}\dfrac{\text{rad}}{\text{s}}$ $\theta_2 = 22.4\,\text{deg}$

Problem 19-51

The rod AB of weight W is released from rest in the vertical position. If the coefficient of restitution between the floor and the cushion at B is e, determine how high the end of the rod rebounds after impact with the floor.



Given:

$W = 15\ \text{lb}$

$l = 2\ \text{ft}$

$e = 0.7$

Solution:

Guesses $\omega_1 = 1\frac{\text{rad}}{\text{s}}$ $\omega_2 = 1\frac{\text{rad}}{\text{s}}$ $\theta = 1\text{deg}$

Given

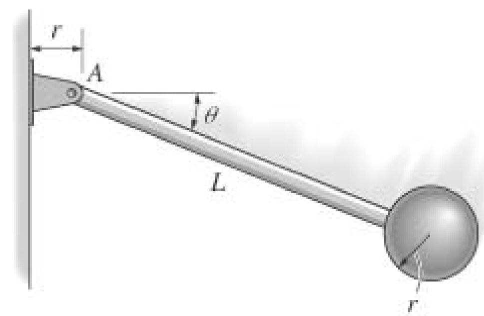
$$W\left(\frac{l}{2}\right) = \frac{1}{2}\left(\frac{W}{g}\right)\frac{l^2}{3}\omega_1^2 \qquad e\omega_1 l = \omega_2 l \qquad W\left(\frac{l}{2}\right)\sin(\theta) = \frac{1}{2}\left(\frac{W}{g}\right)\frac{l^2}{3}\omega_2^2$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \theta\end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta) \qquad \begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix} = \begin{pmatrix}6.95\\ 4.86\end{pmatrix}\frac{\text{rad}}{\text{s}} \qquad \theta = 29.34\,\text{deg}$$

$$h = l\sin(\theta) \qquad h = 0.980\,\text{ft}$$

***Problem 19-52**

The pendulum consists of a solid ball of weight W_b and a rod of weight W_r. If it is released from rest when $\theta_1 = 0°$, determine the angle θ_2 after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest.



Given:

$W_b = 10\text{lb}$ $e = 0.6$ $L = 2\text{ft}$

$W_r = 4\text{lb}$ $r = 0.3\text{ft}$ $g = 32.2\frac{\text{ft}}{\text{s}^2}$

Solution:

$$I_A = \left(\frac{W_r}{g}\right)\left(\frac{L^2}{3}\right) + \frac{2}{5}\left(\frac{W_b}{g}\right)r^2 + \frac{W_b}{g}(L+r)^2$$

Guesses $\omega_1 = 1\frac{\text{rad}}{\text{s}}$ $\omega_2 = 1\frac{\text{rad}}{\text{s}}$ $\theta_2 = 10\text{deg}$

Given $0 = -W_b(L+r) - W_r\left(\frac{L}{2}\right) + \frac{1}{2}I_A\omega_1^2$

$$e(L+r)\omega_1 = (L+r)\omega_2$$

$$-W_b(L+r) - W_r\left(\frac{L}{2}\right) + \frac{1}{2}I_A\omega_2^2 = -\left[W_b(L+r) + W_r\left(\frac{L}{2}\right)\right]\sin(\theta_2)$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \theta_2\end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta_2) \qquad \begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix} = \begin{pmatrix}5.45\\ 3.27\end{pmatrix}\frac{\text{rad}}{\text{s}} \qquad \theta_2 = 39.8\,\text{deg}$$

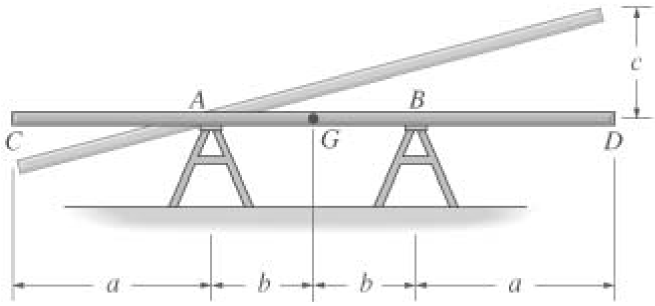
Problem 19-53

The plank has a weight W, center of gravity at G, and it rests on the two sawhorses at A and B. If the end D is raised a distance c above the top of the sawhorses and is released from rest, determine how high end C will rise from the top of the sawhorses after the plank falls so that it rotates clockwise about A, strikes and pivots on the sawhorses at B, and rotates clockwise off the sawhorse at A.

Given:

$W = 30\text{lb}$ $b = 1.5\text{ft}$

$a = 3\text{ft}$ $c = 2\text{ft}$



Solution:

$$I_G = \frac{1}{12}\left(\frac{W}{g}\right)4(a+b)^2 \qquad I_A = I_G + \left(\frac{W}{g}\right)b^2$$

Guesses $\omega_1 = 1\frac{\text{rad}}{\text{s}}$ $\omega_2 = 1\frac{\text{rad}}{\text{s}}$ $\theta = 1\text{deg}$ $h = 1\text{ft}$

Given $$W\left(\frac{b}{2b+a}\right)c = \frac{1}{2}I_A\omega_1^2 \qquad -I_G\,\omega_1 + \left(\frac{W}{g}\right)\omega_1\,b\,b = -I_A\,\omega_2$$

$$W\,b\sin(\theta) = \frac{1}{2}I_A\omega_2^2 \qquad h = (a+2b)\sin(\theta)$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \theta\\ h\end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta, h) \qquad \begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix} = \begin{pmatrix}1.89\\ 0.95\end{pmatrix}\frac{\text{rad}}{\text{s}} \qquad \theta = 4.78\,\text{deg} \qquad h = 0.500\,\text{ft}$$

Problem 19-54

Tests of impact on the fixed crash dummy are conducted using the ram of weight W that is released from rest at $\theta = \theta_1$ and allowed to fall and strike the dummy at $\theta = \theta_2$. If the coefficient of restitution between the dummy and the ram is e, determine the angle θ_3 to which the ram will rebound before momentarily coming to rest.

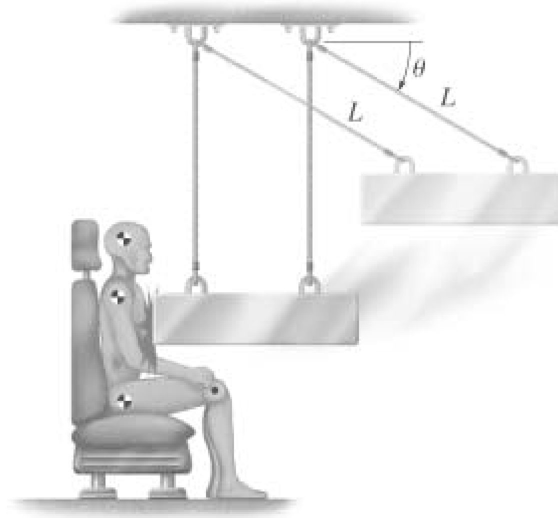
Given:

$W = 300\text{ lb}$ $e = 0.4$

$\theta_1 = 30\text{ deg}$ $L = 10\text{ ft}$

$\theta_2 = 90\text{ deg}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:



Guesses $\quad v_1 = 1\,\frac{\text{ft}}{\text{s}} \qquad v_2 = 1\,\frac{\text{ft}}{\text{s}}$

$\theta_3 = 1\ \text{deg}$

Given

$$-W L \sin(\theta_1) = \frac{1}{2}\left(\frac{W}{g}\right) v_1^{\,2} - W L \sin(\theta_2)$$

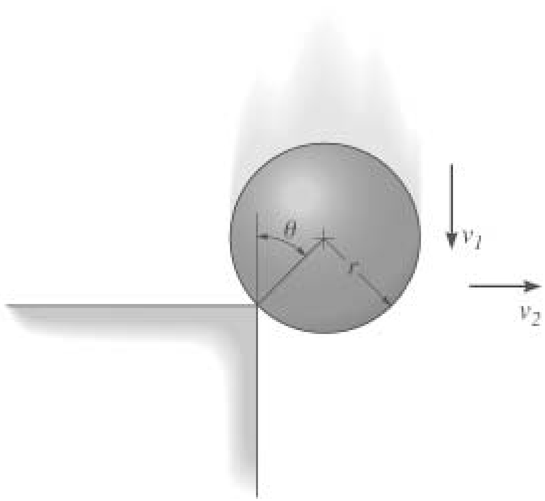
$$-W L \sin(\theta_2) + \frac{1}{2}\left(\frac{W}{g}\right) v_2^{\,2} = -W L \sin(\theta_3)$$

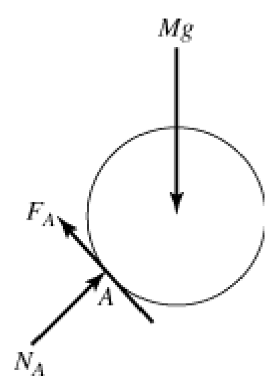
$$e\,v_1 = v_2$$

$$\begin{pmatrix} v_1 \\ v_2 \\ \theta_3 \end{pmatrix} = \text{Find}(v_1, v_2, \theta_3) \qquad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 17.94 \\ 7.18 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \theta_3 = 66.9\,\text{deg}$$

Problem 19-55

The solid ball of mass *m* is dropped with a velocity v_1 onto the edge of the rough step. If it rebounds horizontally off the step with a velocity v_2, determine the angle θ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is *e*.





Solution:

No slip $\qquad \omega_2\, r = v_2 \cos(\theta)$

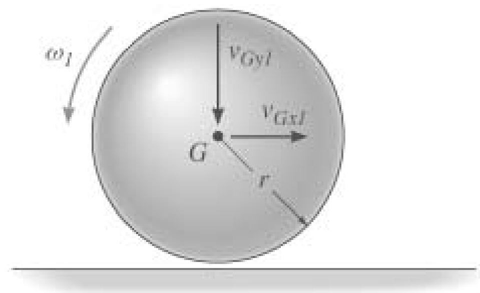
Angular Momentum about A $\quad m v_1 r \sin(\theta) = m v_2 r \cos(\theta) + \frac{2}{5} m r^2 \omega_2$

Restitution $\quad e v_1 \cos(\theta) = v_2 \sin(\theta)$

Combining we find $\quad \boxed{\theta = \operatorname{atan}\left(\sqrt{\frac{7e}{5}}\right)}$

***Problem 19-56**

A solid ball with a mass m is thrown on the ground such that at the instant of contact it has an angular velocity ω_1 and velocity components v_{Gx1} and v_{Gy1} as shown. If the ground is rough so no slipping occurs, determine the components of the velocity of its mass center just after impact. The coefficient of restitution is e.



Solution:

Restitution $\quad e\, v_{Gy1} = v_{Gy2}$

Angular Momentum $\quad \frac{2}{5} m r^2 \omega_1{}^2 - m v_{Gx1} r = \frac{2}{5} m r^2 \omega_2 + m v_{Gx2} r$

No slip $\quad v_{Gx2} = \omega_2 r$

Combining $\quad \boxed{v_{G2} = \begin{pmatrix} \frac{5}{7} v_{Gx1} - \frac{2}{7} r\omega_1 \\ e\, v_{Gy1} \end{pmatrix}}$

Problem 20-1

The ladder of the fire truck rotates around the z axis with angular velocity ω_1 which is increasing at rate α_1. At the same instant it is rotating upwards at the constant rate ω_2. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.

Given:

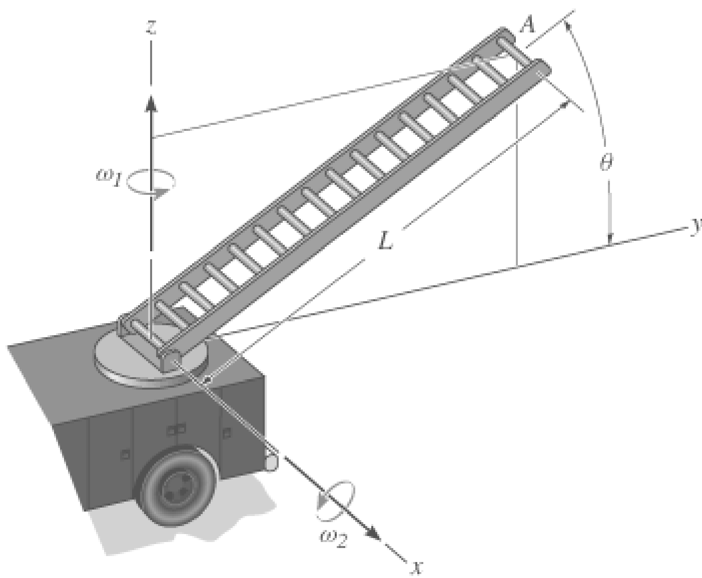
$$\omega_1 = 0.15 \frac{\text{rad}}{\text{s}}$$

$$\alpha_1 = 0.8 \frac{\text{rad}}{\text{s}^2}$$

$$\omega_2 = 0.6 \frac{\text{rad}}{\text{s}}$$

$$\theta = 30 \text{ deg}$$

$$L = 40 \text{ ft}$$



Solution:

$$\mathbf{r} = \begin{pmatrix} 0 \\ L\cos(\theta) \\ L\sin(\theta) \end{pmatrix}$$

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \omega_1 \omega_2 \\ \alpha_1 \end{pmatrix}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_A} = \begin{pmatrix} -5.20 \\ -12.00 \\ 20.78 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_A}| = 24.6 \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_A} = \begin{pmatrix} -24.11 \\ -13.25 \\ -7.20 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_A}| = 28.4 \frac{\text{ft}}{\text{s}^2}$$

Problem 20-2

The ladder of the fire truck rotates around the z axis with angular velocity ω_1 which is increasing at rate α_1. At the same instant it is rotating upwards at rate ω_2 while increasing at rate α_2. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.

Given:

$$\omega_1 = 0.15 \frac{\text{rad}}{\text{s}}$$

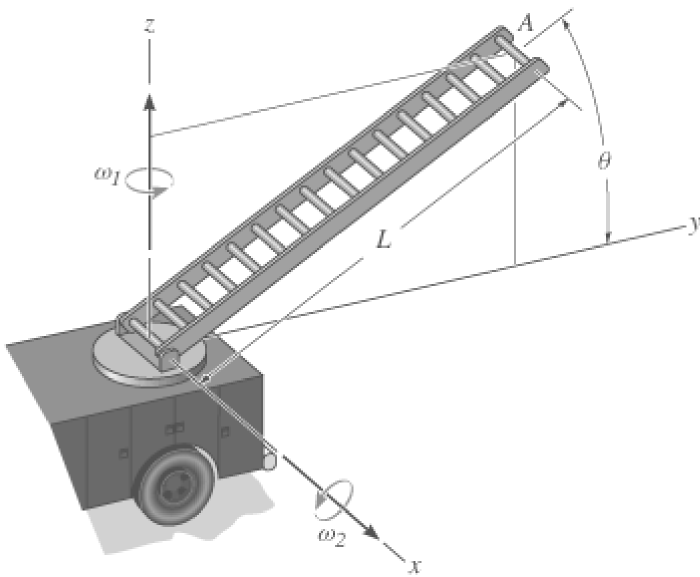
$$\alpha_1 = 0.2 \frac{\text{rad}}{\text{s}^2}$$

$$\omega_2 = 0.6 \frac{\text{rad}}{\text{s}}$$

$$\alpha_2 = 0.4 \frac{\text{rad}}{\text{s}^2}$$

$$\theta = 30 \text{ deg}$$

$$L = 40 \text{ ft}$$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_2 \\ \omega_1 \omega_2 \\ \alpha_1 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 0 \\ L\cos(\theta) \\ L\sin(\theta) \end{pmatrix}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_A} = \begin{pmatrix} -5.20 \\ -12.00 \\ 20.78 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_A}| = 24.6 \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_A} = \begin{pmatrix} -3.33 \\ -21.25 \\ 6.66 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_A}| = 22.5 \frac{\text{ft}}{\text{s}^2}$$

Problem 20-3

The antenna is following the motion of a jet plane. At the instant shown, the constant angular rates of change are θ' and ϕ'. Determine the velocity and acceleration of the signal horn A at this instant. The distance OA is d.

Given:

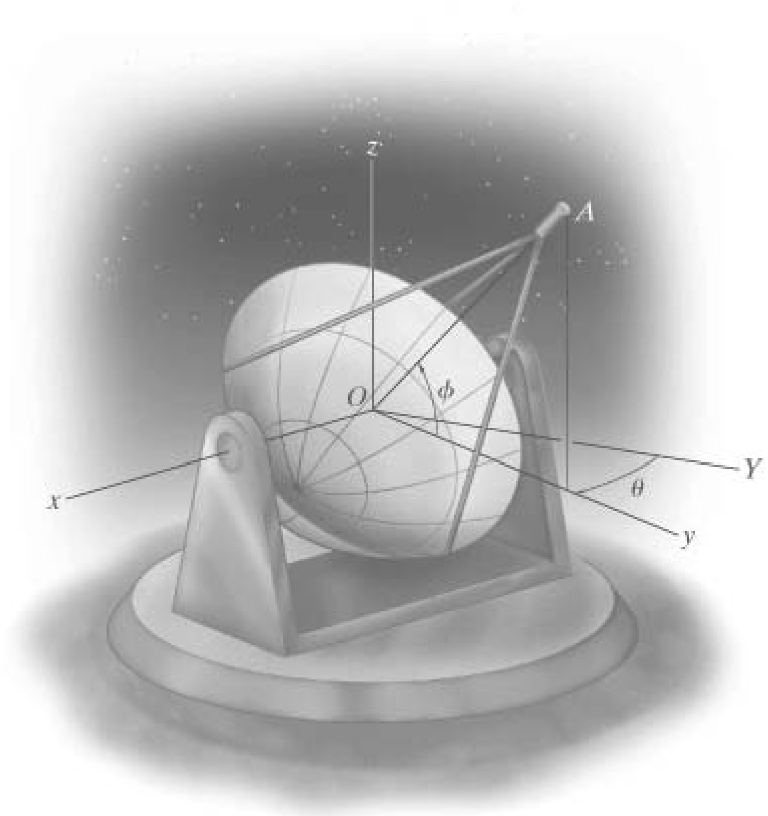
$\theta = 25\ \text{deg}$

$\theta' = 0.4\ \frac{\text{rad}}{\text{s}}$

$\phi = 75\ \text{deg}$

$\phi' = 0.6\ \frac{\text{rad}}{\text{s}}$

$d = 0.8\ \text{m}$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} \phi' \\ 0 \\ -\theta' \end{pmatrix}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 \\ -\theta'\phi' \\ 0 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} 0 \\ d\cos(\phi) \\ d\sin(\phi) \end{pmatrix}$$

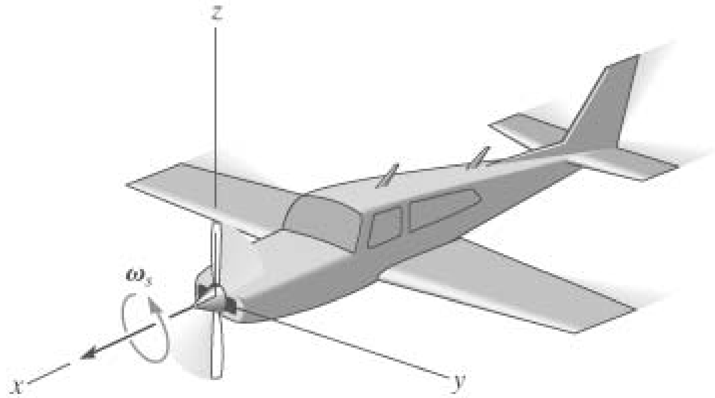
$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_A} = \begin{pmatrix} 0.083 \\ -0.464 \\ 0.124 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_A} = \begin{pmatrix} -0.371 \\ -0.108 \\ -0.278 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

***Problem 20-4**

The propeller of an airplane is rotating at a constant speed $\omega_s \mathbf{i}$, while the plane is undergoing a turn at a constant rate ω_t. Determine the angular acceleration of the propeller if (a) the turn is horizontal, i.e., $\omega_t \mathbf{k}$, and (b) the turn is vertical, downward, i.e., $\omega_t \mathbf{j}$.

Solution:



(a) $$\boldsymbol{\alpha} = \left(\omega_t \vec{\mathbf{k}}\right) \times \left(\omega_s \vec{\mathbf{i}}\right)$$

$$\boldsymbol{\alpha} = \omega_s \omega_t \vec{\mathbf{j}}$$

(b) $$\boldsymbol{\alpha} = \left(\omega_t \vec{\mathbf{j}}\right) \times \left(\omega_s \vec{\mathbf{i}}\right)$$

$$\boldsymbol{\alpha} = -\omega_s \omega_t \vec{\mathbf{k}}$$

Problem 20-5

Gear A is fixed while gear B is free to rotate on the shaft S. If the shaft is turning about the z axis with angular velocity ω_z, while increasing at rate α_z, determine the velocity and acceleration of point C at the instant shown. The face of gear B lies in a vertical plane.

Given:

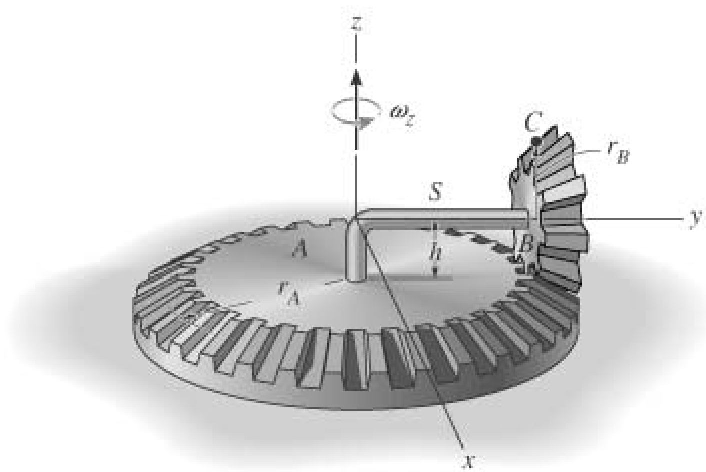
$$\omega_z = 5 \frac{\text{rad}}{\text{s}}$$

$$\alpha_z = 2 \frac{\text{rad}}{\text{s}^2}$$

$$r_A = 160 \text{ mm}$$

$$r_B = 80 \text{ mm}$$

$$h = 80 \text{ mm}$$



Solution:

$$\omega_z r_A = \omega_B r_B \qquad \omega_B = \omega_z \frac{r_A}{r_B} \qquad \omega_B = 10 \frac{\text{rad}}{\text{s}}$$

$$\alpha_z r_A = \alpha_B r_B \qquad \alpha_B = \alpha_z \frac{r_A}{r_B} \qquad \alpha_B = 4 \frac{\text{rad}}{\text{s}^2}$$

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ -\omega_B \\ \omega_z \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ -\alpha_B \\ \alpha_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \boldsymbol{\omega} \qquad \mathbf{r} = \begin{pmatrix} 0 \\ r_A \\ r_B \end{pmatrix}$$

$$\mathbf{v_C} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_C} = \begin{pmatrix} -1.6 \\ 0 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \left|\mathbf{v_C}\right| = 1.6\ \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_C} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_C} = \begin{pmatrix} -0.64 \\ -12 \\ -8 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_C}\right| = 14.436\ \frac{\text{m}}{\text{s}^2}$$

Problem 20-6

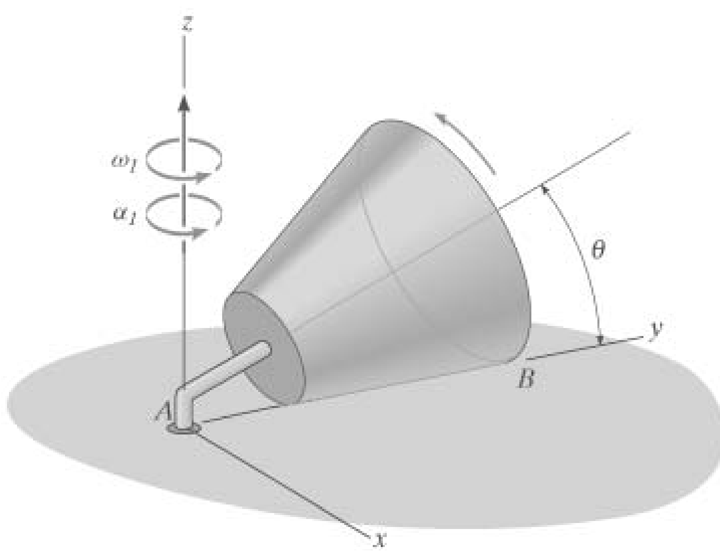
The conical spool rolls on the plane without slipping. If the axle has an angular velocity ω_1 and an angular acceleration α_1 at the instant shown, determine the angular velocity and angular acceleration of the spool at this instant. Neglect the small vertical part of the rod at A.

Given:

$$\omega_1 = 3\ \frac{\text{rad}}{\text{s}}$$

$$\alpha_1 = 2\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta = 20\ \text{deg}$$



Solution:

$$L = 1\ \text{m}$$

$$R = L\tan(\theta)$$

$$\mathbf{r} = \begin{pmatrix} 0 \\ L\cos(\theta) + R\sin(\theta) \\ L\sin(\theta) - R\cos(\theta) \end{pmatrix}$$

Guesses $\omega_2 = 1\,\frac{\text{rad}}{\text{s}}$ $\alpha_2 = 1\,\frac{\text{rad}}{\text{s}^2}$ $a_y = 1\,\frac{\text{m}}{\text{s}^2}$ $a_z = 1\,\frac{\text{m}}{\text{s}^2}$

Given

Enforce the no-slip constraint

$$\begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_1 + \omega_2\sin(\theta) \end{pmatrix} \times \mathbf{r} = 0$$

$$\left[\begin{pmatrix} 0 \\ \alpha_2\cos(\theta) \\ \alpha_1 + \alpha_2\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_1 + \omega_2\sin(\theta) \end{pmatrix}\right] \times \mathbf{r} = \begin{pmatrix} 0 \\ a_y \\ a_z \end{pmatrix}$$

$$\begin{pmatrix} \omega_2 \\ \alpha_2 \\ a_y \\ a_z \end{pmatrix} = \text{Find}(\omega_2, \alpha_2, a_y, a_z) \qquad \begin{pmatrix} a_y \\ a_z \end{pmatrix} = \begin{pmatrix} 0 \\ 26.3 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \omega_2 = -8.77\,\frac{\text{rad}}{\text{s}}$$

$$\alpha_2 = -5.85\,\frac{\text{rad}}{\text{s}^2}$$

Now construct the angular velocity and angular acceleration.

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_1 + \omega_2\sin(\theta) \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0.00 \\ -8.24 \\ 0.00 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \alpha_2\cos(\theta) \\ \alpha_1 + \alpha_2\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_1 + \omega_2\sin(\theta) \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 24.73 \\ -5.49 \\ 0.00 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

Problem 20-7

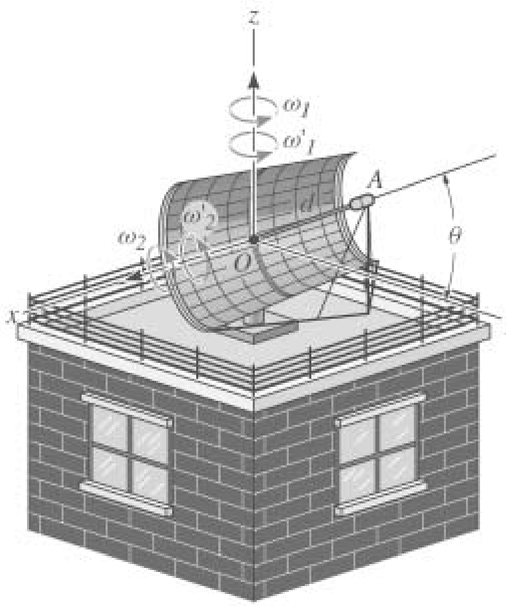
At a given instant, the antenna has an angular motion ω_1 and ω'_1 about the z axis. At the same instant $\theta = \theta_1$, the angular motion about the x axis is ω_2 and ω'_2. Determine the velocity and acceleration of the signal horn A at this instant. The distance from O to A is d.

Given:

$\omega_1 = 3\ \frac{\text{rad}}{\text{s}}$ $\omega_2 = 1.5\ \frac{\text{rad}}{\text{s}}$

$\omega'_1 = 2\ \frac{\text{rad}}{\text{s}^2}$ $\omega'_2 = 4\ \frac{\text{rad}}{\text{s}^2}$

$\theta_1 = 30\ \text{deg}$ $d = 3\ \text{ft}$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} \omega'_2 \\ 0 \\ \omega'_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \boldsymbol{\omega}$$

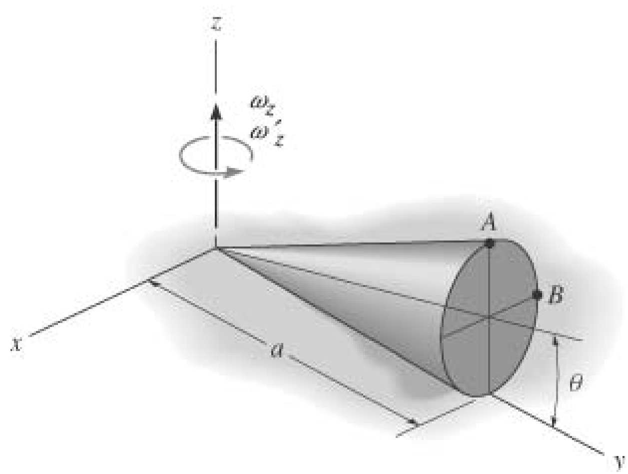
$$\mathbf{r_A} = \begin{pmatrix} 0 \\ d\cos(\theta_1) \\ d\sin(\theta_1) \end{pmatrix}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r_A} \qquad \mathbf{v_A} = \begin{pmatrix} -7.79 \\ -2.25 \\ 3.90 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_A}| = 9\ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r_A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_A}) \qquad \mathbf{a_A} = \begin{pmatrix} 8.30 \\ -35.23 \\ 7.02 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_A}| = 36.868\ \frac{\text{ft}}{\text{s}^2}$$

***Problem 20-8**

The cone rolls without slipping such that at the instant shown ω_z and ω'_z are as given. Determine the velocity and acceleration of point *A* at this instant.



Given:

$\omega_z = 4\ \frac{\text{rad}}{\text{s}}$

$\omega'_z = 3\ \frac{\text{rad}}{\text{s}^2}$

$\theta = 20\ \text{deg}$

$a = 2\ \text{ft}$

Solution:

$b = a\sin(\theta)$

$\omega_z - \omega_2 \sin(\theta) = 0 \qquad \omega_2 = \dfrac{\omega_z}{\sin(\theta)} \qquad \omega_2 = 11.695\ \dfrac{\text{rad}}{\text{s}}$

$\omega'_z - \omega'_2 \sin(\theta) = 0 \qquad \omega'_2 = \dfrac{\omega'_z}{\sin(\theta)} \qquad \omega'_2 = 8.771\ \dfrac{\text{rad}}{\text{s}^2}$

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ -\omega_2\cos(\theta) \\ -\omega_2\sin(\theta) + \omega_z \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ -\omega'_2\cos(\theta) \\ -\omega'_2\sin(\theta) + \omega'_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \boldsymbol{\omega}$$

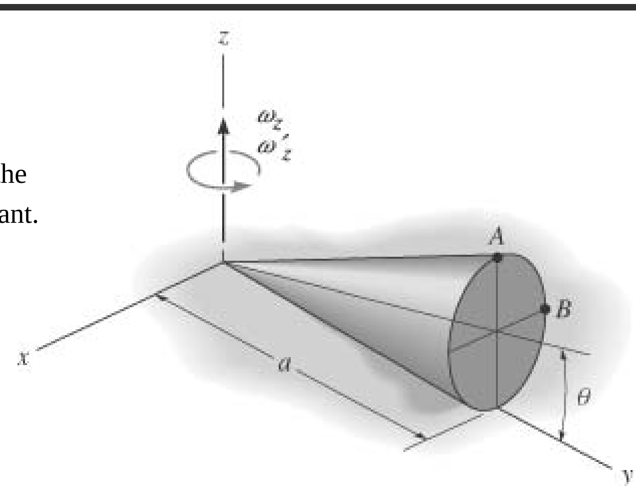
$$\mathbf{r_A} = \begin{pmatrix} 0 \\ a - 2b\sin(\theta) \\ 2b\cos(\theta) \end{pmatrix} \qquad \mathbf{r_A} = \begin{pmatrix} 0 \\ 1.532 \\ 1.286 \end{pmatrix} \text{ft}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r_A} \qquad \mathbf{v_A} = \begin{pmatrix} -14.1 \\ 0.0 \\ 0.0 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \left|\mathbf{v_A}\right| = 14.128\ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r_A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_A}) \qquad \mathbf{a_A} = \begin{pmatrix} -10.6 \\ -56.5 \\ -87.9 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad \left|\mathbf{a_A}\right| = 105.052\ \frac{\text{ft}}{\text{s}^2}$$

Problem 20-9

The cone rolls without slipping such that at the instant shown ω_z and ω'_z are given. Determine the velocity and acceleration of point B at this instant.



Given:

$\omega_z = 4\ \dfrac{\text{rad}}{\text{s}}$

$\omega'_z = 3\ \dfrac{\text{rad}}{\text{s}^2}$

$$\theta = 20\ \text{deg}$$

$$a = 2\ \text{ft}$$

Solution:

$$b = a\sin(\theta)$$

$$\omega_z - \omega_2 \sin(\theta) = 0 \qquad \omega_2 = \frac{\omega_z}{\sin(\theta)} \qquad \omega_2 = 11.695\ \frac{\text{rad}}{\text{s}}$$

$$\omega'_z - \omega'_2 \sin(\theta) = 0 \qquad \omega'_2 = \frac{\omega'_z}{\sin(\theta)} \qquad \omega'_2 = 8.771\ \frac{\text{rad}}{\text{s}^2}$$

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ -\omega_2 \cos(\theta) \\ -\omega_2 \sin(\theta) + \omega_z \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ -\omega'_2 \cos(\theta) \\ -\omega'_2 \sin(\theta) + \omega'_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \boldsymbol{\omega}$$

$$\mathbf{r_B} = \begin{pmatrix} -b \\ a - b\sin(\theta) \\ b\cos(\theta) \end{pmatrix} \qquad \mathbf{r_B} = \begin{pmatrix} -0.684 \\ 1.766 \\ 0.643 \end{pmatrix} \text{ft}$$

$$\mathbf{v_B} = \boldsymbol{\omega} \times \mathbf{r_B} \qquad \mathbf{v_B} = \begin{pmatrix} -7.064 \\ 0 \\ -7.518 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \left|\mathbf{v_B}\right| = 10.316\ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_B} = \boldsymbol{\alpha} \times \mathbf{r_B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_B}) \qquad \mathbf{a_B} = \begin{pmatrix} 77.319 \\ -28.257 \\ -5.638 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad \left|\mathbf{a_B}\right| = 82.513\ \frac{\text{ft}}{\text{s}^2}$$

Problem 20-10

If the plate gears *A* and *B* are rotating with the angular velocities shown, determine the angular velocity of gear *C* about the shaft *DE*. What is the angular velocity of *DE* about the *y* axis?

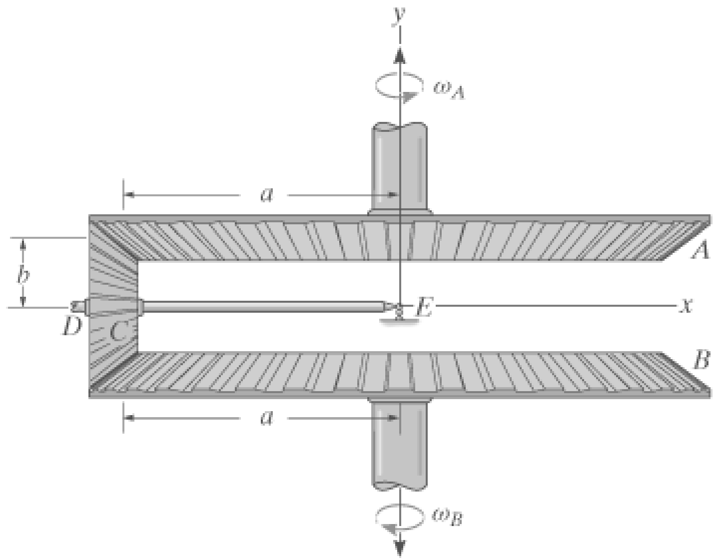
Given:

$$\omega_A = 5\ \frac{\text{rad}}{\text{s}}$$

$$\omega_B = 15\ \frac{\text{rad}}{\text{s}}$$

$a = 100\ \text{mm}$

$b = 25\ \text{mm}$



Solution:

Guesses

$\omega_{DE} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \omega_x = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_y = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \omega_z = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_A a \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\omega_B a \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} 0 \\ 2\ b \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ -\omega_B a \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \omega_{DE} \\ 0 \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \omega_{DE} \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, \omega_{DE}) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 40 \\ 0 \\ 0 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \omega_{DE} = -5\,\frac{\text{rad}}{\text{s}}$$

Problem 20-11

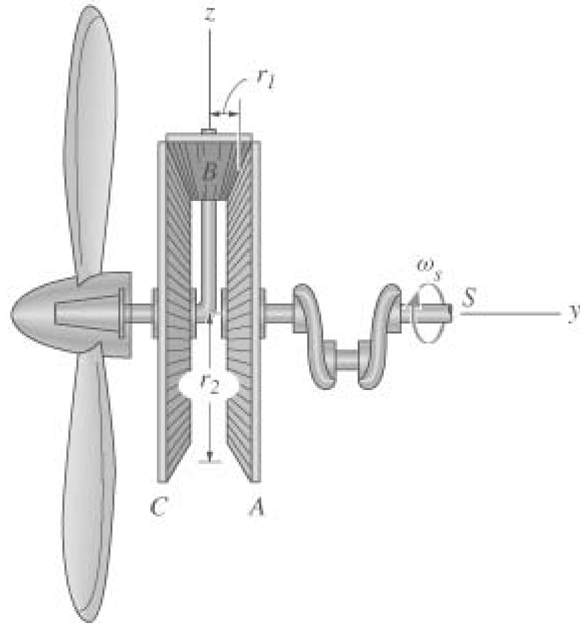
Gear A is fixed to the crankshaft S, while gear C is fixed and gear B and the propeller are free to rotate. The crankshaft is turning with angular velocity ω_s about its axis. Determine the magnitudes of the angular velocity of the propeller and the angular acceleration of gear B.

Given:

$\omega_s = 80\ \frac{\text{rad}}{\text{s}}$

$r_2 = 0.4$ ft

$r_1 = 0.1$ ft



Solution:

$v_P = \omega_s r_2 = \omega_B(2r_1)$

$\omega_B = \dfrac{\omega_s r_2}{2r_1}$ $\qquad \omega_B = 160 \dfrac{\text{rad}}{\text{s}}$

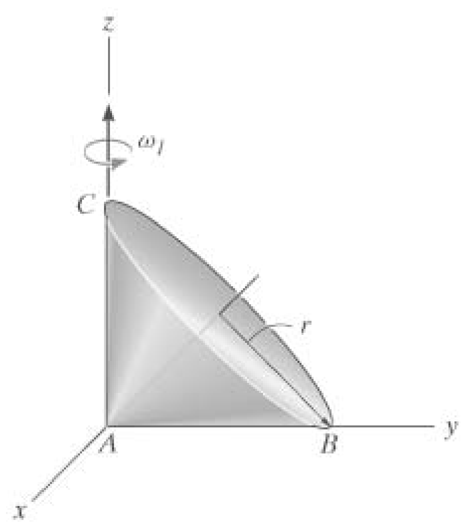
$v_B = \omega_B r_1$ $\qquad v_B = 16 \dfrac{\text{ft}}{\text{s}}$

$\omega_{prop} = \dfrac{v_B}{r_2}$ $\qquad \boldsymbol{\omega} = \begin{pmatrix} 0 \\ -\omega_{prop} \\ 0 \end{pmatrix}$ $\qquad \boldsymbol{\omega} = \begin{pmatrix} 0.0 \\ -40.0 \\ 0.0 \end{pmatrix} \dfrac{\text{rad}}{\text{s}}$

$\boldsymbol{\alpha_B} = \begin{pmatrix} 0 \\ -\omega_{prop} \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \omega_B \end{pmatrix}$ $\qquad \boldsymbol{\alpha_B} = \begin{pmatrix} -6400 \\ 0 \\ 0 \end{pmatrix} \dfrac{\text{rad}}{\text{s}^2}$

***Problem 20-12**

The right circular cone rotates about the *z* axis at a constant rate ω_1 without slipping on the horizontal plane. Determine the magnitudes of the velocity and acceleration of points *B* and *C*.



Given:

$\omega_1 = 4 \dfrac{\text{rad}}{\text{s}}$ $\qquad r = 50$ mm $\qquad \theta = 45$ deg

Solution:

Enforce no-slip condition $\qquad$ Guess $\qquad \omega_2 = 1 \dfrac{\text{rad}}{\text{s}}$

Given $\begin{pmatrix} 0 \\ \omega_2 \cos(\theta) \\ \omega_2 \sin(\theta) + \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \sqrt{2}\,r \\ 0 \end{pmatrix} = 0$ $\quad \omega_2 = \text{Find}(\omega_2) \quad \omega_2 = -5.66\,\frac{\text{rad}}{\text{s}}$

Define terms

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_2 \cos(\theta) \\ \omega_2 \sin(\theta) + \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2 \cos(\theta) \\ \omega_2 \sin(\theta) + \omega_1 \end{pmatrix}$$

$$\mathbf{r_B} = \begin{pmatrix} 0 \\ \sqrt{2}\,r \\ 0 \end{pmatrix} \qquad \mathbf{r_C} = \begin{pmatrix} 0 \\ 0 \\ \sqrt{2}\,r \end{pmatrix}$$

Find velocities and accelerations

$$\mathbf{v_B} = \boldsymbol{\omega} \times \mathbf{r_B} \qquad \mathbf{v_C} = \boldsymbol{\omega} \times \mathbf{r_C}$$

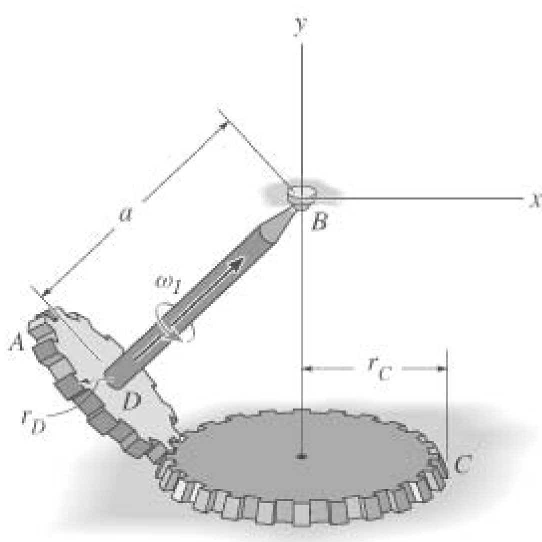
$$\mathbf{a_B} = \boldsymbol{\alpha} \times \mathbf{r_B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_B}) \qquad \mathbf{a_C} = \boldsymbol{\alpha} \times \mathbf{r_C} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_C})$$

$$\mathbf{v_B} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_B} = \begin{pmatrix} 0 \\ 0 \\ 1.131 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \mathbf{v_C} = \begin{pmatrix} -0.283 \\ 0 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_C} = \begin{pmatrix} 0 \\ -1.131 \\ -1.131 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

$$|\mathbf{v_B}| = 0\,\frac{\text{m}}{\text{s}} \qquad |\mathbf{a_B}| = 1.131\,\frac{\text{m}}{\text{s}^2} \qquad |\mathbf{v_C}| = 0.283\,\frac{\text{m}}{\text{s}} \qquad |\mathbf{a_C}| = 1.6\,\frac{\text{m}}{\text{s}^2}$$

Problem 20-13

Shaft *BD* is connected to a ball-and-socket joint at *B*, and a beveled gear *A* is attached to its other end. The gear is in mesh with a fixed gear *C*. If the shaft and gear *A* are *spinning* with a constant angular velocity ω_1, determine the angular velocity and angular acceleration of gear *A*.



Given:

$$\omega_1 = 8\,\frac{\text{rad}}{\text{s}}$$

$$a = 300\text{ mm}$$

$r_D = 75 \text{ mm}$

$r_C = 100 \text{ mm}$

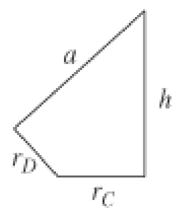
Solution:

Guesses $\quad \theta = 10 \text{ deg} \quad h = 10 \text{ mm}$

Given

$$a\cos(\theta) + r_D\sin(\theta) = h \qquad a\sin(\theta) = r_C + r_D\cos(\theta)$$

$$\begin{pmatrix} h \\ \theta \end{pmatrix} = \text{Find}(h, \theta) \qquad h = 0.293 \text{ m} \qquad \theta = 32.904 \text{ deg}$$



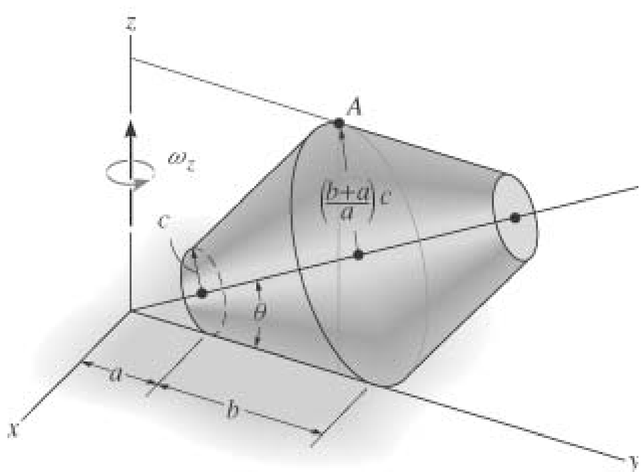
$$\omega_y = \frac{\omega_1 r_D}{a\sin(\theta) - r_D\cos(\theta)} \qquad \omega_y = 6\,\frac{\text{rad}}{\text{s}}$$

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_1\sin(\theta) \\ \omega_1\cos(\theta) + \omega_y \\ 0 \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 4.346 \\ 12.717 \\ 0 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \omega_y \\ 0 \end{pmatrix} \times \boldsymbol{\omega} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0.0 \\ 0.0 \\ -26.1 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

Problem 20-14

The truncated cone rotates about the z axis at a constant rate ω_z without slipping on the horizontal plane. Determine the velocity and acceleration of point A on the cone.



Given:

$\omega_z = 0.4\,\frac{\text{rad}}{\text{s}}$

$a = 1 \text{ ft}$

$b = 2 \text{ ft}$

$c = 0.5 \text{ ft}$

Solution: $\theta = \operatorname{asin}\left(\frac{c}{a}\right)$

$\omega_z + \omega_S \sin(\theta) = 0 \qquad \omega_S = \frac{-\omega_z}{\sin(\theta)}$

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_S \cos(\theta) \\ \omega_z + \omega_S \sin(\theta) \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0 \\ -0.693 \\ 0 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \boldsymbol{\omega} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0.277 \\ 0 \\ 0 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

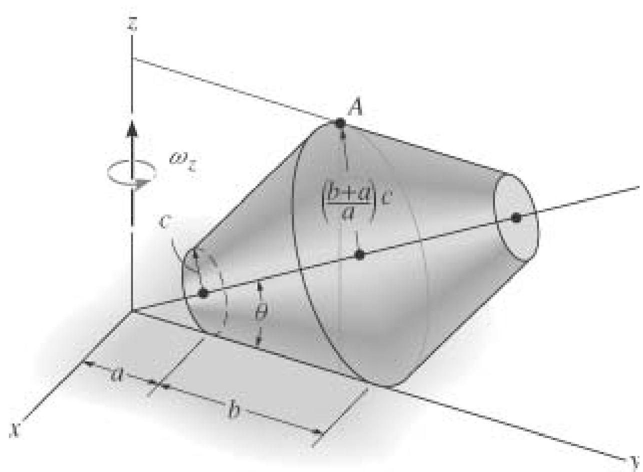
$$\mathbf{r_A} = \begin{bmatrix} 0 \\ a + b - 2\left(\frac{b+a}{a}\right) c \sin(\theta) \\ 2\left(\frac{b+a}{a}\right) c \cos(\theta) \end{bmatrix} \qquad \mathbf{r_A} = \begin{pmatrix} 0 \\ 1.5 \\ 2.598 \end{pmatrix} \text{ft}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r_A} \qquad \mathbf{v_A} = \begin{pmatrix} -1.8 \\ 0 \\ 0 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r_A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_A}) \qquad \mathbf{a_A} = \begin{pmatrix} 0.000 \\ -0.720 \\ -0.831 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 20-15

The truncated cone rotates about the z axis at ω_z without slipping on the horizontal plane. If at this same instant ω_z is increasing at ω'_z, determine the velocity and acceleration of point A on the cone.



Given:

$\omega_z = 0.4 \frac{\text{rad}}{\text{s}} \qquad a = 1 \text{ ft}$

$\omega'_z = 0.5 \frac{\text{rad}}{\text{s}^2} \qquad b = 2 \text{ ft}$

$\theta = 30\ \text{deg} \qquad c = 0.5\ \text{ft}$

Solution: $r = \left(\dfrac{b+a}{a}\right)c \qquad \mathbf{r_A} = \begin{pmatrix} 0 \\ a + b - 2r\sin(\theta) \\ 2r\cos(\theta) \end{pmatrix}$

Guesses $\omega_2 = 1\ \dfrac{\text{rad}}{\text{s}} \qquad \alpha_2 = 1\ \dfrac{\text{rad}}{\text{s}^2} \qquad a_y = 1\ \dfrac{\text{ft}}{\text{s}^2} \qquad a_z = 1\ \dfrac{\text{ft}}{\text{s}^2}$

Given Enforce the no-slip constraints

$$\begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_z + \omega_2\sin(\theta) \end{pmatrix} \times \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} = 0$$

$$\left[\begin{pmatrix} 0 \\ \alpha_2\cos(\theta) \\ \omega'_z + \alpha_2\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_z + \omega_2\sin(\theta) \end{pmatrix}\right] \times \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ a_y \\ a_z \end{pmatrix}$$

$$\begin{pmatrix} \omega_2 \\ \alpha_2 \\ a_y \\ a_z \end{pmatrix} = \text{Find}(\omega_2, \alpha_2, a_y, a_z) \qquad \omega_2 = -0.8\ \frac{\text{rad}}{\text{s}} \qquad \alpha_2 = -1\ \frac{\text{rad}}{\text{s}^2}$$

Define terms

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_z + \omega_2\sin(\theta) \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \alpha_2\cos(\theta) \\ \omega'_z + \alpha_2\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_z + \omega_2\sin(\theta) \end{pmatrix}$$

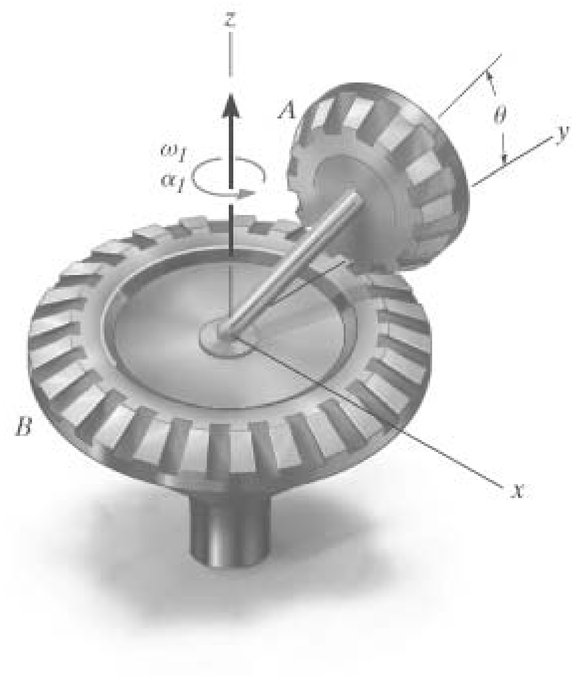
Calculate velocity and acceleration.

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r_A} \qquad \mathbf{v_A} = \begin{pmatrix} -1.80 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r_A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_A}) \qquad \mathbf{a_A} = \begin{pmatrix} -2.25 \\ -0.72 \\ -0.831 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

***Problem 20-16**

The bevel gear *A* rolls on the fixed gear *B*. If at the instant shown the shaft to which *A* is attached is rotating with angular velocity ω_1 and has angular acceleration α_1, determine the angular velocity and angular acceleration of gear *A*.



Given:

$$\omega_1 = 2\ \frac{\text{rad}}{\text{s}}$$

$$\alpha_1 = 4\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta = 30\ \text{deg}$$

Solution:

$$L = 1\ \text{m}$$

$$R = L\tan(\theta) \qquad b = L\sec(\theta)$$

Guesses

$$\omega_2 = 1\ \frac{\text{rad}}{\text{s}} \qquad \alpha_2 = 1\ \frac{\text{rad}}{\text{s}^2}$$

$$a_y = 1\ \frac{\text{m}}{\text{s}^2} \qquad a_z = 1\ \frac{\text{m}}{\text{s}^2}$$

Given Enforce the no-slip constraints.

$$\begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_2\sin(\theta)+\omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} = 0$$

$$\left[\begin{pmatrix} 0 \\ \alpha_2\cos(\theta) \\ \alpha_2\sin(\theta)+\alpha_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_2\sin(\theta)+\omega_1 \end{pmatrix}\right] \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ a_y \\ a_z \end{pmatrix}$$

$$\begin{pmatrix} \omega_2 \\ \alpha_2 \\ a_y \\ a_z \end{pmatrix} = \text{Find}(\omega_2, \alpha_2, a_y, a_z) \qquad \begin{pmatrix} a_y \\ a_z \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \omega_2 = -4\,\frac{\text{rad}}{\text{s}} \qquad \alpha_2 = -8\,\frac{\text{rad}}{\text{s}^2}$$

Build the angular velocity and angular acceleration.

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_2 \cos(\theta) \\ \omega_2 \sin(\theta) + \omega_1 \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0.00 \\ -3.46 \\ 0.00 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \alpha_2 \cos(\theta) \\ \alpha_2 \sin(\theta) + \alpha_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2 \cos(\theta) \\ \omega_2 \sin(\theta) + \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 6.93 \\ -6.93 \\ 0.00 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

Problem 20-17

The differential of an automobile allows the two rear wheels to rotate at different speeds when the automobile travels along a curve. For operation, the rear axles are attached to the wheels at one end and have beveled gears *A* and *B* on their other ends. The differential case *D* is placed over the left axle but can rotate about *C* independent of the axle. The case supports a pinion gear *E* on a shaft, which meshes with gears *A* and *B*. Finally, a ring gear *G* is *fixed* to the differential case so that the case rotates with the ring gear when the latter is driven by the drive pinion H. This gear, like the differential case, is free to rotate about the left wheel axle. If the drive pinion is turning with angular velocity ω_H and the pinion gear *E* is spinning about its shaft with angular velocity ω_E, determine the angular velocity ω_A and ω_B of each axle.

Given:

$\omega_H = 100\ \frac{\text{rad}}{\text{s}}$

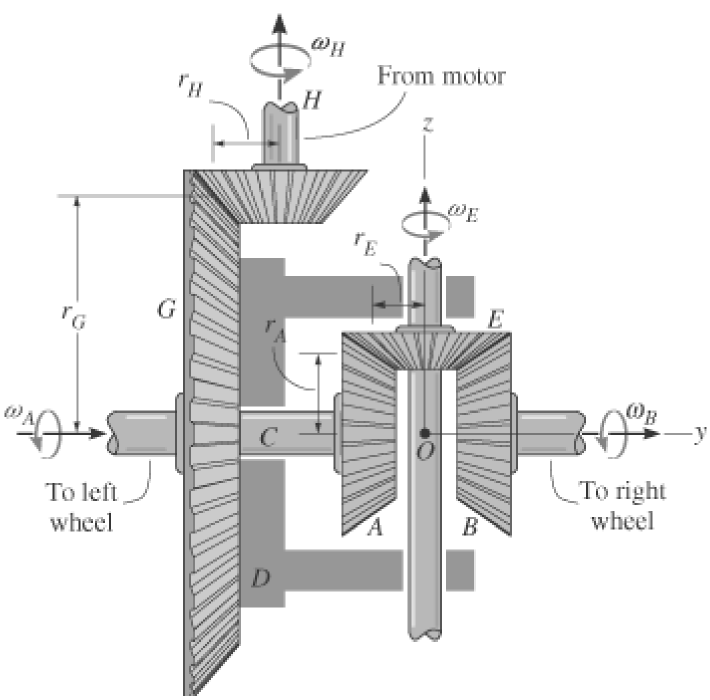
$\omega_E = 30\ \frac{\text{rad}}{\text{s}}$

$r_G = 180\ \text{mm}$

$r_H = 50\ \text{mm}$

$r_E = 40\ \text{mm}$

$r_A = 60\ \text{mm}$



Solution:

$\omega_H r_H = \omega_G r_G$

$$\omega_G = \omega_H \frac{r_H}{r_G}$$

$$\omega_G = 27.778\ \frac{\text{rad}}{\text{s}}$$

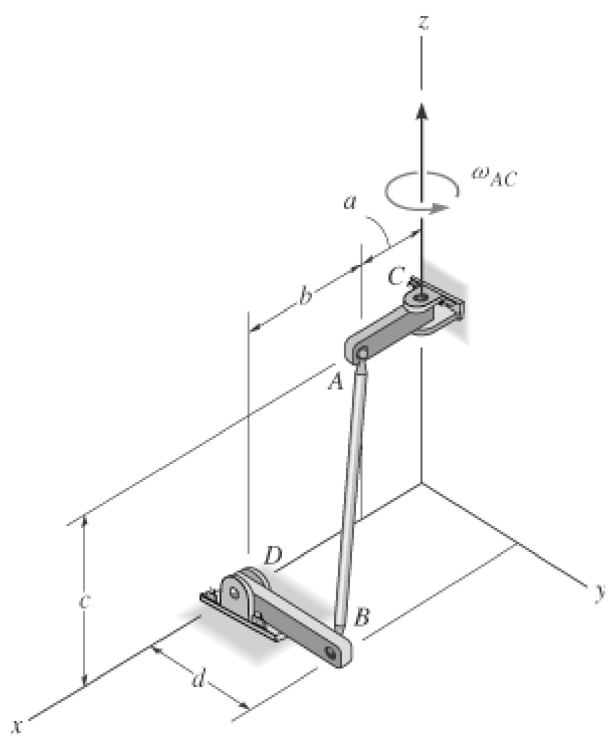
$$v_E = \omega_G r_A \qquad v_E = 1.667\ \frac{\text{m}}{\text{s}}$$

$$v_E - \omega_E r_E = \omega_B r_A \qquad \omega_B = \frac{v_E - \omega_E r_E}{r_A} \qquad \omega_B = 7.778\ \frac{\text{rad}}{\text{s}}$$

$$v_E + \omega_E r_E = \omega_A r_A \qquad \omega_A = \frac{v_E + \omega_E r_E}{r_A} \qquad \omega_A = 47.8\ \frac{\text{rad}}{\text{s}}$$

Problem 20-18

Rod *AB* is attached to the rotating arm using ball-and-socket joints. If *AC* is rotating with constant angular velocity ω_{AC} about the pin at *C*, determine the angular velocity of link *BD* at the instant shown.



Given:

$a = 1.5\ \text{ft} \qquad d = 2\ \text{ft}$

$b = 3\ \text{ft} \qquad \omega_{AC} = 8\ \frac{\text{rad}}{\text{s}}$

$c = 6\ \text{ft}$

Solution:

Guesses

$\omega_{BD} = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_{ABx} = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_{ABy} = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_{ABz} = 1\ \frac{\text{rad}}{\text{s}}$

Given

Note that $\boldsymbol{\omega}_{\mathbf{AB}}$ is perpendicular to $\mathbf{r}_{\mathbf{AB}}$.

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} \times \begin{pmatrix} b \\ d \\ -c \end{pmatrix} + \begin{pmatrix} \omega_{BD} \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} \begin{pmatrix} b \\ d \\ -c \end{pmatrix} = 0\frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} \omega_{BD} \\ \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} = \text{Find}(\omega_{BD}, \omega_{ABx}, \omega_{ABy}, \omega_{ABz}) \qquad \begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} = \begin{pmatrix} -1.633 \\ 0.245 \\ -0.735 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\omega_{BD} = -2\,\frac{\text{rad}}{\text{s}}$$

Problem 20-19

Rod AB is attached to the rotating arm using ball-and-socket joints. If AC is rotating about the pin at C with angular velocity ω_{AC} and angular acceleration α_{AC}, determine the angular velocity and angular acceleration of link BD at the instant shown.

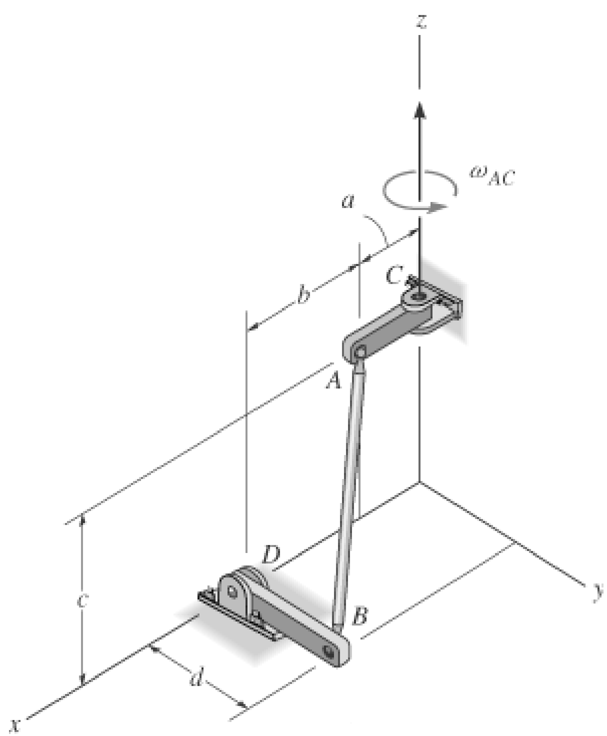
Given:

$a = 1.5\text{ ft}$

$\omega_{AC} = 8\,\frac{\text{rad}}{\text{s}}$ $\quad b = 3\text{ ft}$

$\alpha_{AC} = 6\,\frac{\text{rad}}{\text{s}^2}$ $\quad c = 6\text{ ft}$

$d = 2\text{ ft}$



Solution:

Guesses

$\omega_{BD} = 1\,\frac{\text{rad}}{\text{s}}$ $\quad \alpha_{BD} = 1\,\frac{\text{rad}}{\text{s}^2}$

$\omega_{ABx} = 1\,\frac{\text{rad}}{\text{s}}$ $\quad \alpha_{ABx} = 1\,\frac{\text{rad}}{\text{s}^2}$

$\omega_{ABy} = 1\,\frac{\text{rad}}{\text{s}}$ $\quad \alpha_{ABy} = 1\,\frac{\text{rad}}{\text{s}^2}$

$\omega_{ABz} = 1\,\frac{\text{rad}}{\text{s}}$ $\quad \alpha_{ABz} = 1\,\frac{\text{rad}}{\text{s}^2}$

Given Note that ω_{AB} and α_{AB} are perpendicular to r_{AB}.

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} \times \begin{pmatrix} b \\ d \\ -c \end{pmatrix} + \begin{pmatrix} \omega_{BD} \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} -a\,\omega_{AC}^{2} \\ a\alpha_{AC} \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha_{ABx} \\ \alpha_{ABy} \\ \alpha_{ABz} \end{pmatrix} \times \begin{pmatrix} b \\ d \\ -c \end{pmatrix} + \begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABy} \end{pmatrix} \times \left[\begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} \times \begin{pmatrix} b \\ d \\ -c \end{pmatrix}\right] = \begin{pmatrix} 0 \\ -d\,\omega_{BD}^{2} \\ d\alpha_{BD} \end{pmatrix}$$

$$\begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} \begin{pmatrix} b \\ d \\ -c \end{pmatrix} = 0 \qquad \begin{pmatrix} \alpha_{ABx} \\ \alpha_{ABy} \\ \alpha_{ABz} \end{pmatrix} \begin{pmatrix} b \\ d \\ -c \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_{BD} \\ \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \\ \alpha_{BD} \\ \alpha_{ABx} \\ \alpha_{ABy} \\ \alpha_{ABz} \end{pmatrix} = \text{Find}\left(\omega_{BD}, \omega_{ABx}, \omega_{ABy}, \omega_{ABz}, \alpha_{BD}, \alpha_{ABx}, \alpha_{ABy}, \alpha_{ABz}\right)$$

$$\begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} = \begin{pmatrix} -1.633 \\ 0.245 \\ -0.735 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} \alpha_{ABx} \\ \alpha_{ABy} \\ \alpha_{ABz} \end{pmatrix} = \begin{pmatrix} 0.495 \\ -14.18 \\ -4.479 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

$$\omega_{BD} = -2\,\frac{\text{rad}}{\text{s}}$$

$$\alpha_{BD} = 31.6\,\frac{\text{rad}}{\text{s}^2}$$

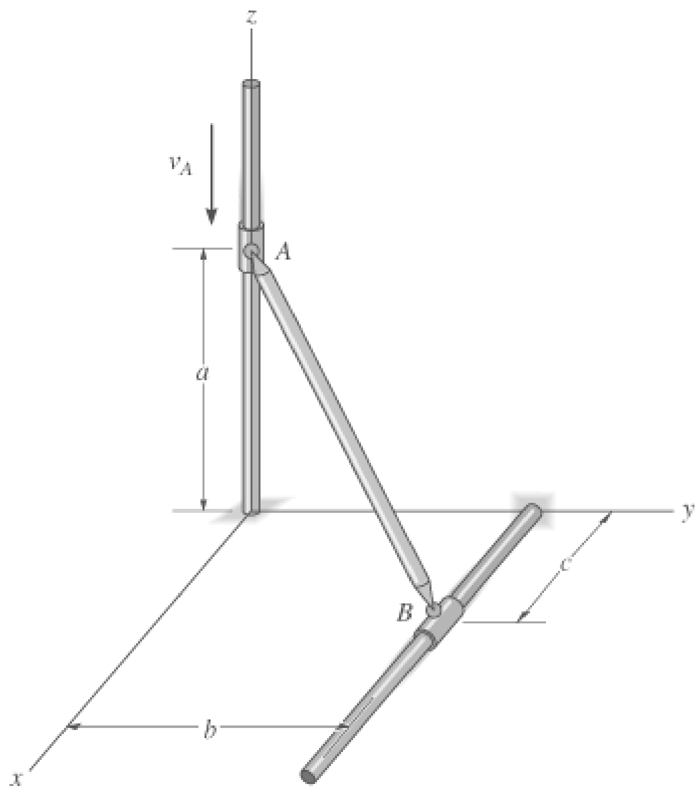
***Problem 20-20**

If the rod is attached with ball-and-socket joints to smooth collars A and B at its end points, determine the speed of B at the instant shown if A is moving downward at constant speed v_A. Also, determine the angular velocity of the rod if it is directed perpendicular to the axis of the rod.

Given:

$$v_A = 8\ \frac{\text{ft}}{\text{s}} \qquad b = 6\ \text{ft}$$

$$a = 3\ \text{ft} \qquad c = 2\ \text{ft}$$



Solution:

Guesses

$$v_B = 1\ \frac{\text{ft}}{\text{s}}$$

$$\omega_x = 1\ \frac{\text{rad}}{\text{s}}$$

$$\omega_y = 1\ \frac{\text{rad}}{\text{s}}$$

$$\omega_z = 1\ \frac{\text{rad}}{\text{s}}$$

Given

$$\begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = \begin{pmatrix} v_B \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0\ \frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} v_B \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \text{Find}(v_B, \omega_x, \omega_y, \omega_z) \qquad \boldsymbol{\omega} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0.980 \\ -1.061 \\ -1.469 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = 12\ \frac{\text{ft}}{\text{s}}$$

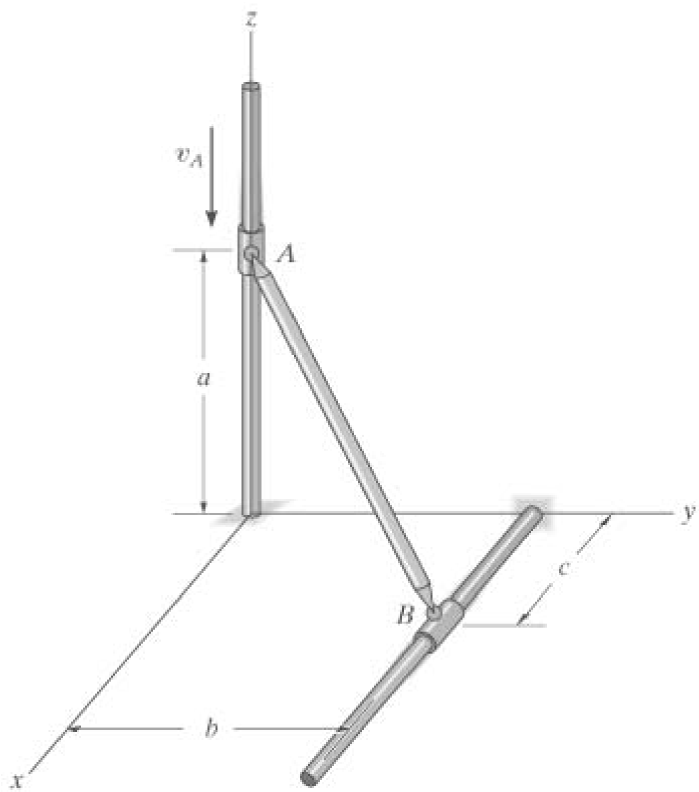
Problem 20-21

If the collar at A is moving downward with an acceleration a_A, at the instant its speed is v_A, determine the acceleration of the collar at B at this instant.

Given:

$$v_A = 8\ \frac{\text{ft}}{\text{s}} \qquad a = 3\ \text{ft} \qquad c = 2\ \text{ft}$$

$a_A = 5\ \frac{\text{ft}}{\text{s}^2} \qquad b = 6\ \text{ft}$



Solution:

Guesses

$v_B = 1\ \frac{\text{ft}}{\text{s}} \qquad \omega_x = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_y = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_z = 1\ \frac{\text{rad}}{\text{s}}$

$a_B = 1\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha_x = 1\ \frac{\text{rad}}{\text{s}^2}$

$\alpha_y = 1\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_z = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = \begin{pmatrix} v_B \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ 0 \\ -a_A \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} \right] = \begin{pmatrix} a_B \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \\ v_B \\ a_B \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, \alpha_x, \alpha_y, \alpha_z, v_B, a_B)$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.98 \\ -1.06 \\ -1.47 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} 0.61 \\ 5.70 \\ 11.82 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

$$v_B = 12\,\frac{\text{ft}}{\text{s}}$$

$$a_B = -96.5\,\frac{\text{ft}}{\text{s}^2}$$

Problem 20-22

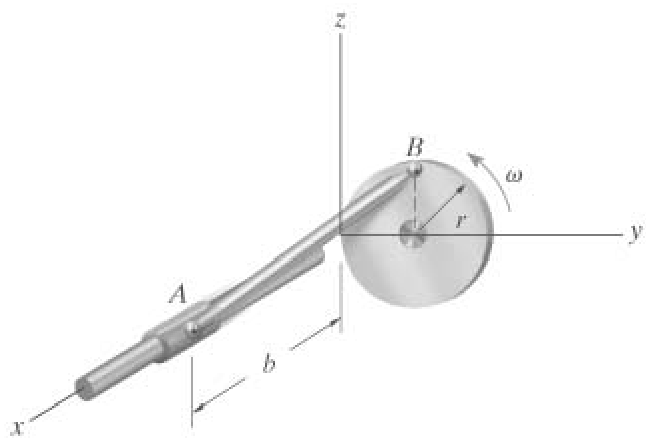
Rod AB is attached to a disk and a collar by ball-and-socket joints. If the disk is rotating at a constant angular velocity ω, determine the velocity and acceleration of the collar at A at the instant shown. Assume the angular velocity is directed perpendicular to the rod.

Given:

$$\omega = 2\,\frac{\text{rad}}{\text{s}}$$

$$r = 1\text{ ft}$$

$$b = 3\text{ ft}$$



Solution:

Guesses

$$\omega_x = 1\,\frac{\text{rad}}{\text{s}} \qquad \alpha_x = 1\,\frac{\text{rad}}{\text{s}^2}$$

$$\omega_y = 1\,\frac{\text{rad}}{\text{s}} \qquad \alpha_y = 1\,\frac{\text{rad}}{\text{s}^2}$$

$$\omega_z = 1\,\frac{\text{rad}}{\text{s}} \qquad \alpha_z = 1\,\frac{\text{rad}}{\text{s}^2}$$

$$v_A = 1\,\frac{\text{ft}}{\text{s}} \qquad a_A = 1\,\frac{\text{ft}}{\text{s}^2}$$

Given

$$\begin{pmatrix} 0 \\ -\omega r \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = \begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}\begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = 0 \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix}\begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ 0 \\ -\omega^2 r \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} \right] = \begin{pmatrix} a_A \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \\ v_A \\ a_A \end{pmatrix} = \mathrm{Find}(\omega_x, \omega_y, \omega_z, \alpha_x, \alpha_y, \alpha_z, v_A, a_A) \qquad v_A = 0.667\ \frac{\mathrm{ft}}{\mathrm{s}} \qquad a_A = -0.148\ \frac{\mathrm{ft}}{\mathrm{s}^2}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.182 \\ -0.061 \\ 0.606 \end{pmatrix} \frac{\mathrm{rad}}{\mathrm{s}} \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} -0.364 \\ -1.077 \\ -0.013 \end{pmatrix} \frac{\mathrm{rad}}{\mathrm{s}^2}$$

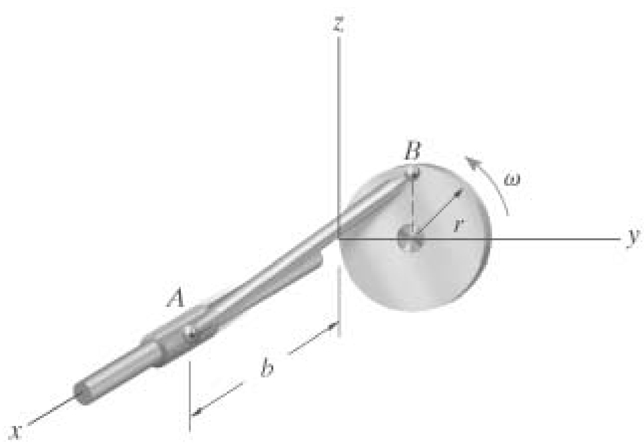
Problem 20-23

Rod *AB* is attached to a disk and a collar by ball and-socket joints. If the disk is rotating with an angular acceleration α, and at the instant shown has an angular velocity ω, determine the velocity and acceleration of the collar at *A* at the instant shown.

Given:

$\omega = 2\ \frac{\mathrm{rad}}{\mathrm{s}}$ $\qquad r = 1\ \mathrm{ft}$

$\alpha = 4\ \frac{\mathrm{rad}}{\mathrm{s}^2}$ $\qquad b = 3\ \mathrm{ft}$



Solution:

Guesses $\omega_x = 1\ \frac{\mathrm{rad}}{\mathrm{s}}$ $\qquad \omega_y = 1\ \frac{\mathrm{rad}}{\mathrm{s}}$

$\omega_z = 1\ \frac{\mathrm{rad}}{\mathrm{s}}$ $\qquad v_A = 1\ \frac{\mathrm{ft}}{\mathrm{s}}$

$$\alpha_x = 1\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_y = 1\ \frac{\text{rad}}{\text{s}^2}$$

$$\alpha_z = 1\ \frac{\text{rad}}{\text{s}^2} \qquad a_A = 1\ \frac{\text{ft}}{\text{s}^2}$$

Given

$$\begin{pmatrix} 0 \\ -\omega r \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = \begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = 0 \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ -\alpha r \\ -\omega^2 r \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} \right] = \begin{pmatrix} a_A \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \\ v_A \\ a_A \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, \alpha_x, \alpha_y, \alpha_z, v_A, a_A) \qquad v_A = 0.667\ \frac{\text{ft}}{\text{s}} \qquad a_A = 1.185\ \frac{\text{ft}}{\text{s}^2}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.182 \\ -0.061 \\ 0.606 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} -3.636 \times 10^{-7} \\ -1.199 \\ 1.199 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

***Problem 20-24**

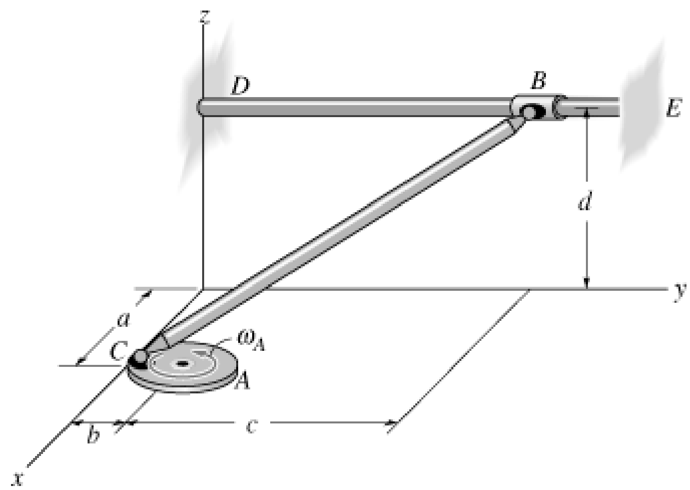
The rod BC is attached to collars at its ends by ball-and-socket joints. If disk A has angular velocity ω_A, determine the angular velocity of the rod and the velocity of collar B at the instant shown. Assume the angular velocity of the rod is directed perpendicular to the rod.

Given:

$$a = 200\ \text{mm}$$

$b = 100 \text{ mm} \qquad \omega_A = 10 \dfrac{\text{rad}}{\text{s}}$

$c = 500 \text{ mm} \qquad d = 300 \text{ mm}$



Solution:

Guesses

$v_B = 1 \dfrac{\text{m}}{\text{s}} \qquad \omega_x = 1 \dfrac{\text{rad}}{\text{s}}$

$\omega_y = 1 \dfrac{\text{rad}}{\text{s}} \qquad \omega_z = 1 \dfrac{\text{rad}}{\text{s}}$

Given

$$\begin{pmatrix} \omega_A b \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} -a \\ b+c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} -a \\ b+c \\ d \end{pmatrix} = 0 \frac{\text{m}}{\text{s}}$$

$$\begin{pmatrix} v_B \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \text{Find}(v_B, \omega_x, \omega_y, \omega_z) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.204 \\ -0.612 \\ 1.361 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = -0.333 \frac{\text{m}}{\text{s}}$$

Problem 20-25

The rod *BC* is attached to collars at its ends. There is a ball-and-socket at *C*. The connection at *B* now consists of a pin as shown in the bottom figure. If disk *A* has angular velocity ω_A, determine the angular velocity of the rod and the velocity of collar *B* at the instant shown. *Hint*: The constraint allows rotation of the rod both along the bar *DE* (**j** direction) and along the axis of the pin (**n** direction). Since there is no rotational component in the **u** direction, i.e., perpendicular to **n** and **j** where $\mathbf{u} = \mathbf{j} \times \mathbf{n}$, an additional equation for solution can be obtained from $\boldsymbol{\omega} \cdot \mathbf{u} = 0$. The vector **n** is in the same direction as $\mathbf{r_{BC}} \times \mathbf{r_{DC}}$.

Given:

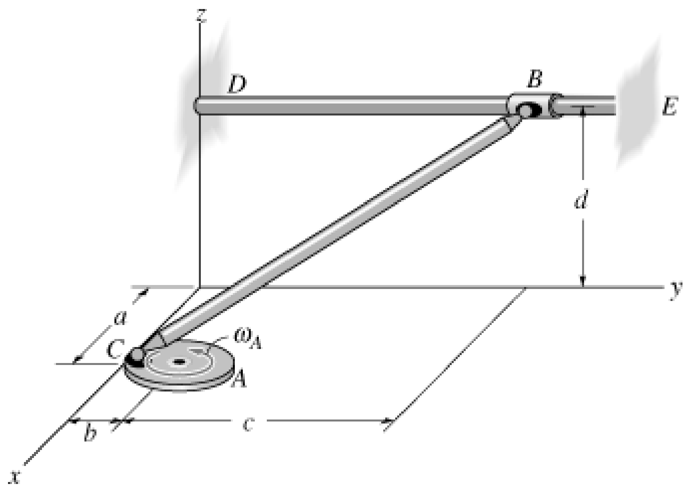
$\omega_A = 10 \dfrac{\text{rad}}{\text{s}}$

$a = 200 \text{ mm}$

$b = 100 \text{ mm}$

$c = 500\ \text{mm}$

$d = 300\ \text{mm}$

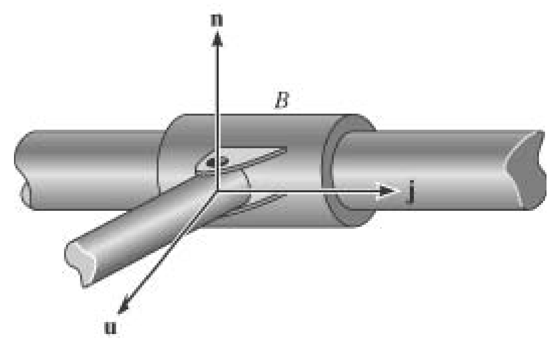


Solution:

$$\mathbf{r_{BC}} = \begin{pmatrix} a \\ -b-c \\ d \end{pmatrix}$$

$$\mathbf{r_{DC}} = \begin{pmatrix} a \\ 0 \\ -d \end{pmatrix}$$

$$\mathbf{n} = \frac{\mathbf{r_{BC}} \times \mathbf{r_{DC}}}{\left|\mathbf{r_{BC}} \times \mathbf{r_{DC}}\right|} \qquad \mathbf{n} = \begin{pmatrix} 0.728 \\ 0.485 \\ 0.485 \end{pmatrix}$$



$$\mathbf{u} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \mathbf{n} \qquad \mathbf{u} = \begin{pmatrix} 0.485 \\ 0 \\ -0.728 \end{pmatrix}$$

Guesses $\quad v_B = 1\ \frac{\text{m}}{\text{s}} \quad \omega_x = 1\ \frac{\text{rad}}{\text{s}} \quad \omega_y = 1\ \frac{\text{rad}}{\text{s}} \quad \omega_z = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\begin{pmatrix} \omega_A b \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} -a \\ b+c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \mathbf{u} = 0\ \frac{\text{rad}}{\text{s}}$$

$$\begin{pmatrix} v_B \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \text{Find}\left(v_B, \omega_x, \omega_y, \omega_z\right) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.769 \\ -2.308 \\ 0.513 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = -0.333\ \frac{\text{m}}{\text{s}}$$

Problem 20-26

The rod *AB* is attached to collars at its ends by ball-and-socket joints. If collar *A* has a speed v_A, determine the speed of collar *B* at the instant shown.

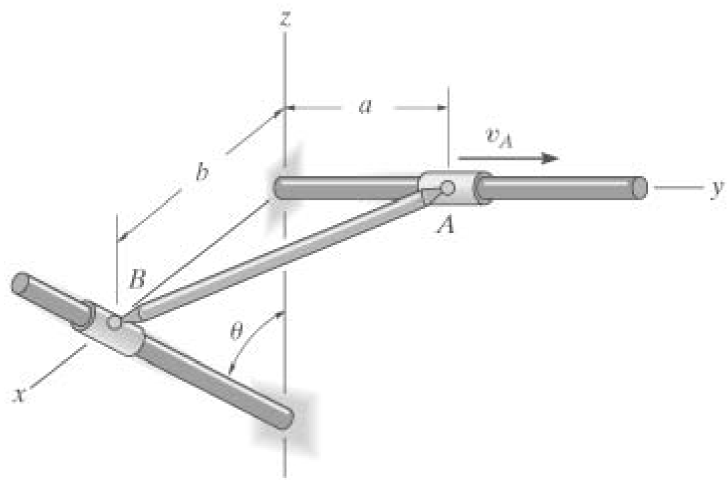
Given:

$v_A = 20\ \frac{ft}{s}$

$a = 2\ ft$

$b = 6\ ft$

$\theta = 45\ deg$



Solution:

Guesses $\omega_x = 1\ \frac{rad}{s}$ $\omega_y = 1\ \frac{rad}{s}$ $\omega_z = 1\ \frac{rad}{s}$ $v_B = 1\ \frac{ft}{s}$

Given

$$\begin{pmatrix} 0 \\ v_A \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix} = v_B \begin{pmatrix} -\sin(\theta) \\ 0 \\ -\cos(\theta) \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_B \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, v_B) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.333 \\ 1 \\ -3.333 \end{pmatrix} \frac{rad}{s} \qquad v_B = 9.43\ \frac{ft}{s}$$

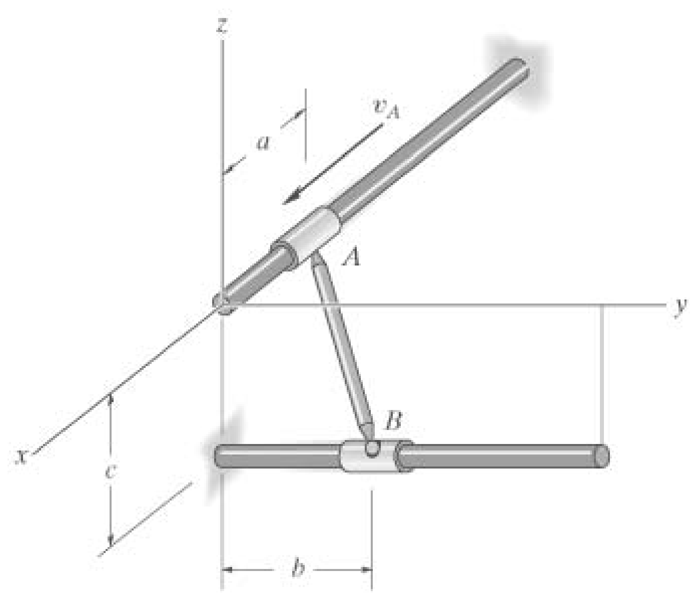
Problem 20-27

The rod is attached to smooth collars A and B at its ends using ball-and-socket joints. Determine the speed of B at the instant shown if A is moving with speed v_A. Also, determine the angular velocity of the rod if it is directed perpendicular to the axis of the rod.

Given:

$v_A = 6\ \frac{m}{s}$ $b = 1\ m$

$a = 0.5\ m$ $c = 1\ m$



Solution:

Guesses

$$\omega_x = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_y = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_z = 1\ \frac{\text{rad}}{\text{s}} \qquad v_B = 1\ \frac{\text{m}}{\text{s}}$$

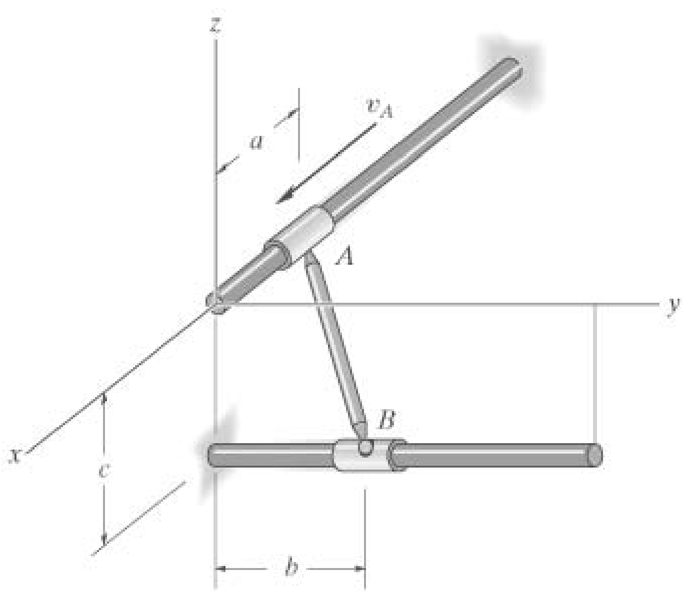
Given

$$\begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} a \\ b \\ -c \end{pmatrix} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} a \\ b \\ -c \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_B \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, v_B) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 1.33 \\ 2.67 \\ 3.33 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = 3.00\ \frac{\text{m}}{\text{s}}$$

***Problem 20-28**

The rod is attached to smooth collars A and B at its ends using ball-and-socket joints. At the instant shown, A is moving with speed v_A and is decelerating at the rate a_A. Determine the acceleration of collar B at this instant.



Given:

$$v_A = 6\ \frac{\text{m}}{\text{s}} \qquad a = 0.5\ \text{m}$$

$$a_A = 5\ \frac{\text{m}}{\text{s}^2} \qquad b = 1\ \text{m}$$

$$c = 1\ \text{m}$$

Solution:

Guesses

$$\omega_x = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_y = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_z = 1\ \frac{\text{rad}}{\text{s}} \qquad v_B = 1\ \frac{\text{m}}{\text{s}}$$

$$\alpha_x = 1\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_y = 1\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_z = 1\ \frac{\text{rad}}{\text{s}^2} \qquad a_B = 1\ \frac{\text{m}}{\text{s}^2}$$

Given

$$\begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} a \\ b \\ -c \end{pmatrix} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} a \\ b \\ -c \end{pmatrix} = 0 \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \begin{pmatrix} a \\ b \\ -c \end{pmatrix} = 0$$

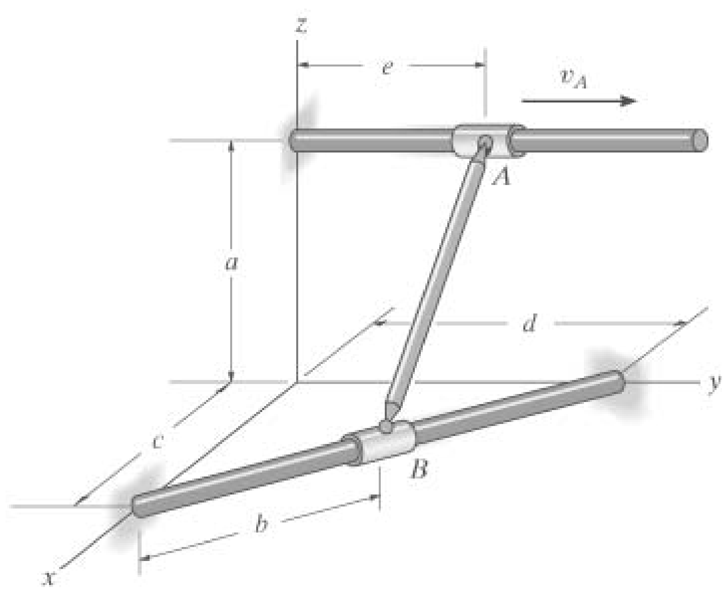
$$\begin{pmatrix} -a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \times \begin{pmatrix} a \\ b \\ -c \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} a \\ b \\ -c \end{pmatrix} \right] = \begin{pmatrix} 0 \\ a_B \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \\ v_B \\ a_B \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, \alpha_x, \alpha_y, \alpha_z, v_B, a_B)$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 1.33 \\ 2.67 \\ 3.33 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} -21.11 \\ -2.22 \\ -12.78 \end{pmatrix} \frac{\text{rad}}{\text{s}^2} \qquad v_B = 3.00\ \frac{\text{m}}{\text{s}} \qquad a_B = -47.5\ \frac{\text{m}}{\text{s}^2}$$

Problem 20-29

Rod *AB* is attached to collars at its ends by using ball-and-socket joints. If collar *A* moves along the fixed rod with speed v_A, determine the angular velocity of the rod and the velocity of collar *B* at the instant shown. Assume that the rod's angular velocity is directed perpendicular to the axis of the rod.



Given:

$v_A = 8\ \frac{\text{ft}}{\text{s}} \qquad c = 6\ \text{ft}$

$a = 8\ \text{ft} \qquad d = 8\ \text{ft}$

$b = 5\ \text{ft} \qquad e = 6\ \text{ft}$

Solution:

$$\theta = \text{atan}\left(\frac{d}{c}\right)$$

Guesses $\omega_x = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_y = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_z = 1\ \frac{\text{rad}}{\text{s}} \qquad v_B = 1\ \frac{\text{ft}}{\text{s}}$

Given

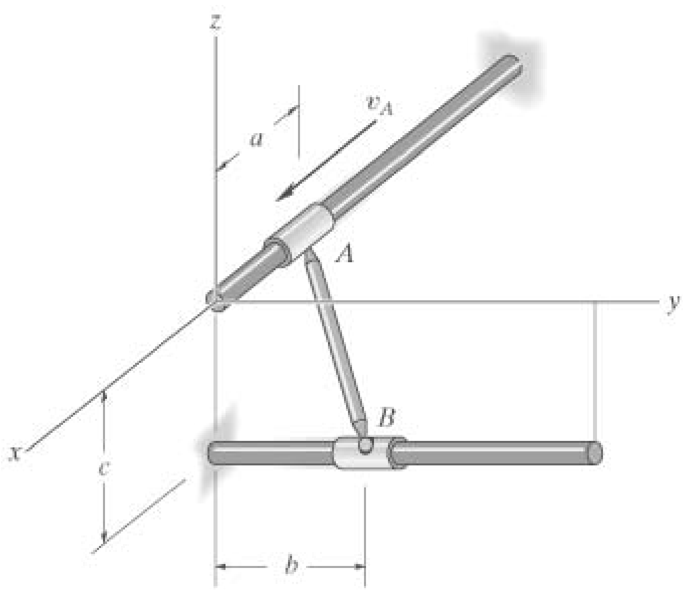
$$\begin{pmatrix} 0 \\ v_A \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} = v_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_B \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, v_B) \qquad \mathbf{v_{Bv}} = v_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{v_{Bv}} = \begin{pmatrix} -2.82 \\ 3.76 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} -0.440 \\ 0.293 \\ -0.238 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

Problem 20-30

Rod AB is attached to collars at its ends by using ball-and-socket joints. If collar A moves along the fixed rod with a velocity v_A and has an acceleration a_A at the instant shown, determine the angular acceleration of the rod and the acceleration of collar B at this instant. Assume that the rod' s angular velocity and angular acceleration are directed perpendicular to the axis of the rod.



Given:

$v_A = 8\ \frac{\text{ft}}{\text{s}} \qquad a_A = 4\ \frac{\text{ft}}{\text{s}^2}$

$a = 8\ \text{ft} \qquad b = 5\ \text{ft} \qquad c = 6\ \text{ft} \qquad d = 8\ \text{ft} \qquad e = 6\ \text{ft}$

Solution:

$$\theta = \operatorname{atan}\left(\frac{d}{c}\right)$$

Guesses $\omega_x = 1\,\frac{\text{rad}}{\text{s}}$ $\omega_y = 1\,\frac{\text{rad}}{\text{s}}$ $\omega_z = 1\,\frac{\text{rad}}{\text{s}}$ $v_B = 1\,\frac{\text{ft}}{\text{s}}$

$\alpha_x = 1\,\frac{\text{rad}}{\text{s}^2}$ $\alpha_y = 1\,\frac{\text{rad}}{\text{s}^2}$ $\alpha_z = 1\,\frac{\text{rad}}{\text{s}^2}$ $a_B = 1\,\frac{\text{ft}}{\text{s}^2}$

Given

$$\begin{pmatrix} 0 \\ v_A \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} = v_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ a_A \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \times \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix}\right] = a_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} = 0 \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \\ v_B \\ a_B \end{pmatrix} = \operatorname{Find}(\omega_x, \omega_y, \omega_z, \alpha_x, \alpha_y, \alpha_z, v_B, a_B)$$

$$\mathbf{v_{Bv}} = v_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{a_{Bv}} = a_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{a_{Bv}} = \begin{pmatrix} -5.98 \\ 7.98 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} -0.440 \\ 0.293 \\ -0.238 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} 0.413 \\ 0.622 \\ -0.000 \end{pmatrix} \frac{\text{rad}}{\text{s}^2} \qquad \mathbf{v_{Bv}} = \begin{pmatrix} -2.824 \\ 3.765 \\ 0 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

Problem 20-31

Consider again Example 20.5. The pendulum consists of two rods: *AB* is pin supported at *A* and swings only in the *y*-*z* plane, whereas a bearing at *B* allows the attached rod *BD* to spin about rod *AB*. At a given instant, the rods have the angular motions shown. Also, a collar *C* has velocity v_C and acceleration a_C along the rod. Determine the velocity and acceleration of the collar at this instant. Solve such that the *x*, *y*, *z* axes move with curvilinear translation, $\boldsymbol{\Omega} = 0$, in which case the collar appears to have both an angular velocity $\boldsymbol{\Omega_{xyz}} = \boldsymbol{\omega_1} + \boldsymbol{\omega_2}$ and radial motion.

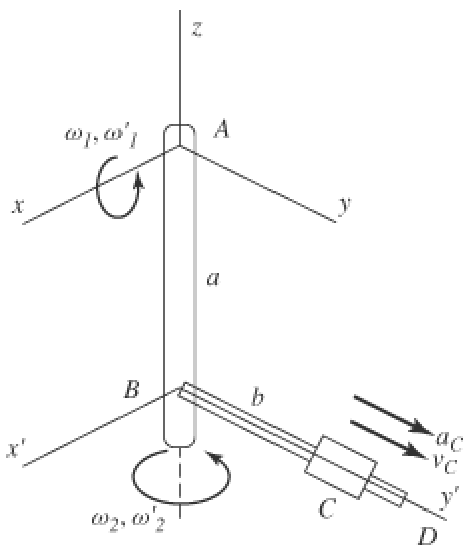
Given:

$$\omega_1 = 4\ \frac{\text{rad}}{\text{s}} \qquad v_{CB} = 3\ \frac{\text{m}}{\text{s}}$$

$$\omega_2 = 5\ \frac{\text{rad}}{\text{s}} \qquad a_{CB} = 2\ \frac{\text{m}}{\text{s}^2}$$

$$\omega'_1 = 1.5\ \frac{\text{rad}}{\text{s}^2} \qquad a = 0.5\ \text{m}$$

$$\omega'_2 = -6\ \frac{\text{rad}}{\text{s}^2} \qquad b = 0.2\ \text{m}$$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \qquad \mathbf{v_B} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_B} = \begin{pmatrix} \omega'_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \left[\begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \right] \qquad \mathbf{a_B} = \begin{pmatrix} 0 \\ 0.75 \\ 8 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

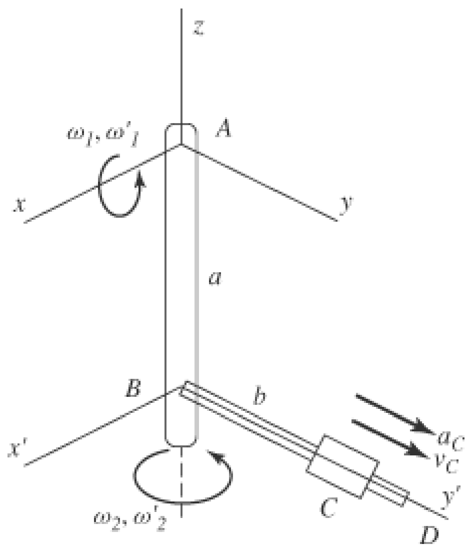
$$\mathbf{v_C} = \mathbf{v_B} + \begin{pmatrix} 0 \\ v_{CB} \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -1.00 \\ 5.00 \\ 0.80 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_C} = \mathbf{a_B} + \begin{pmatrix} 0 \\ a_{CB} \\ 0 \end{pmatrix} + \left[\begin{pmatrix} \omega'_1 \\ 0 \\ \omega'_2 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix}\right] \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \left[\begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}\right] \dots$$
$$+ 2\begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ v_{CB} \\ 0 \end{pmatrix}$$

$$\mathbf{a_C} = \begin{pmatrix} -28.8 \\ -5.45 \\ 32.3 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-32

Consider again Example 20.5. The pendulum consists of two rods: *AB* is pin supported at *A* and swings only in the *y-z* plane, whereas a bearing at *B* allows the attached rod *BD* to spin about rod *AB*. At a given instant, the rods have the angular motions shown. Also, a collar *C* has velocity v_C and acceleration a_C along the rod. Determine the velocity and acceleration of the collar at this instant. Solve by fixing the *x, y, z* axes to rod *BD* in which case the collar appears only to have radial motion.



Given:

$\omega_1 = 4 \frac{\text{rad}}{\text{s}}$ $\omega'_1 = 1.5 \frac{\text{rad}}{\text{s}^2}$

$\omega_2 = 5 \frac{\text{rad}}{\text{s}}$ $\omega'_2 = -6 \frac{\text{rad}}{\text{s}^2}$

$a = 0.5 \text{ m}$ $b = 0.2 \text{ m}$

$v_{CB} = 3 \frac{\text{m}}{\text{s}}$ $a_{CB} = 2 \frac{\text{m}}{\text{s}^2}$

Solution:

$$\mathbf{v_B} = \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix}$$

$$\mathbf{a_B} = \begin{pmatrix} \omega'_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \left[\begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix}\right]$$

$$\mathbf{v_C} = \mathbf{v_B} + \begin{pmatrix} 0 \\ v_{CB} \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -1.00 \\ 5.00 \\ 0.80 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_C} = \mathbf{a_B} + \begin{pmatrix} 0 \\ a_{CB} \\ 0 \end{pmatrix} + \left[\begin{pmatrix} \omega'_1 \\ 0 \\ \omega'_2 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix}\right] \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \left[\begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}\right] \ldots$$
$$+ 2\begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ v_{CB} \\ 0 \end{pmatrix}$$

$$\mathbf{a_C} = \begin{pmatrix} -28.8 \\ -5.45 \\ 32.3 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-33

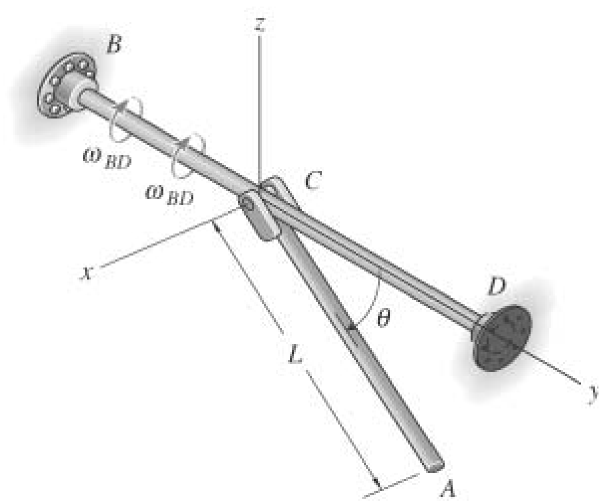
At a given instant, rod *BD* is rotating about the *y* axis with angular velocity ω_{BD} and angular acceleration ω'_{BD}. Also, when $\theta = \theta_1$, link *AC* is rotating downward such that $\theta' = \omega_2$ and $\theta'' = \alpha_2$. Determine the velocity and acceleration of point *A* on the link at this instant.

Given:

$\omega_{BD} = 2\ \frac{\text{rad}}{\text{s}}$ $\qquad \theta_1 = 60\ \text{deg}$

$\omega'_{BD} = 5\ \frac{\text{rad}}{\text{s}^2}$ $\qquad \omega_2 = 2\ \frac{\text{rad}}{\text{s}}$

$L = 3\ \text{ft}$ $\qquad \alpha_2 = 8\ \frac{\text{rad}}{\text{s}^2}$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} -\omega_2 \\ -\omega_{BD} \\ 0 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} -\alpha_2 \\ -\omega'_{BD} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\omega_{BD} \\ 0 \end{pmatrix} \times \boldsymbol{\omega} \qquad \mathbf{r_A} = \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ -L\sin(\theta_1) \end{pmatrix}$$

$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r_A}$ $\mathbf{v_A} = \begin{pmatrix} 5.196 \\ -5.196 \\ -3 \end{pmatrix} \frac{\text{ft}}{\text{s}}$

$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r_A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_A})$ $\mathbf{a_A} = \begin{pmatrix} 24.99 \\ -26.785 \\ 8.785 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$

Problem 20-34

During the instant shown the frame of the X-ray camera is rotating about the vertical axis at ω_z and ω'_z. Relative to the frame the arm is rotating at ω_{rel} and ω'_{rel}. Determine the velocity and acceleration of the center of the camera C at this instant.

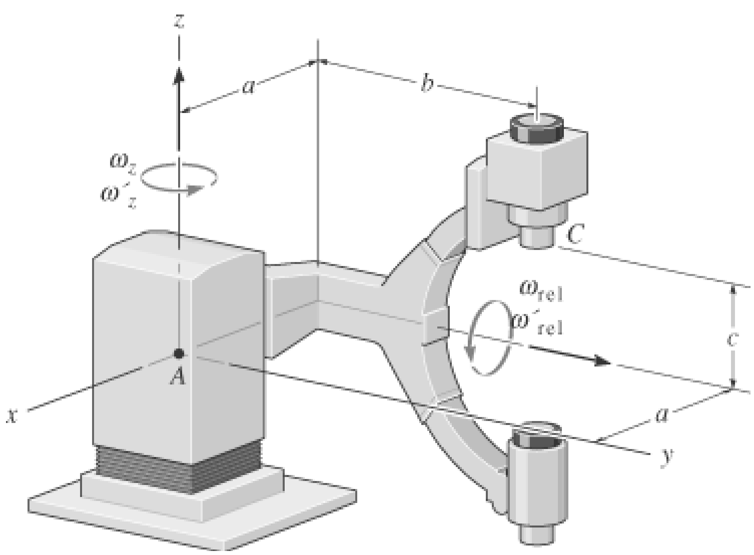
Given:

$\omega_z = 5\ \frac{\text{rad}}{\text{s}}$ $a = 1.25\ \text{m}$

$\omega'_z = 2\ \frac{\text{rad}}{\text{s}^2}$ $b = 1.75\ \text{m}$

$\omega_{rel} = 2\ \frac{\text{rad}}{\text{s}}$ $c = 1\ \text{m}$

$\omega'_{rel} = 1\ \frac{\text{rad}}{\text{s}^2}$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_{rel} \\ \omega_z \end{pmatrix}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \omega'_{rel} \\ \omega'_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \boldsymbol{\omega}$$

$$\mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \boldsymbol{\omega} \times \begin{pmatrix} 0 \\ b \\ c \end{pmatrix}$$ $\mathbf{v_C} = \begin{pmatrix} -6.75 \\ -6.25 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}}$

$$\mathbf{a_C} = \begin{pmatrix} a\omega_z^{\,2} \\ -a\,\omega'_z \\ 0 \end{pmatrix} + \boldsymbol{\alpha} \times \begin{pmatrix} 0 \\ b \\ c \end{pmatrix} + \boldsymbol{\omega} \times \left[\boldsymbol{\omega} \times \begin{pmatrix} 0 \\ b \\ c \end{pmatrix} \right] \qquad \mathbf{a_C} = \begin{pmatrix} 28.75 \\ -26.25 \\ -4 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-35

At the instant shown, the frame of the brush cutter is traveling forward in the x direction with a constant velocity v, and the cab is rotating about the vertical axis with a constant angular velocity ω_1. At the same instant the boom AB has a constant angular velocity θ', in the direction shown. Determine the velocity and acceleration of point B at the connection to the mower at this instant.

Given:

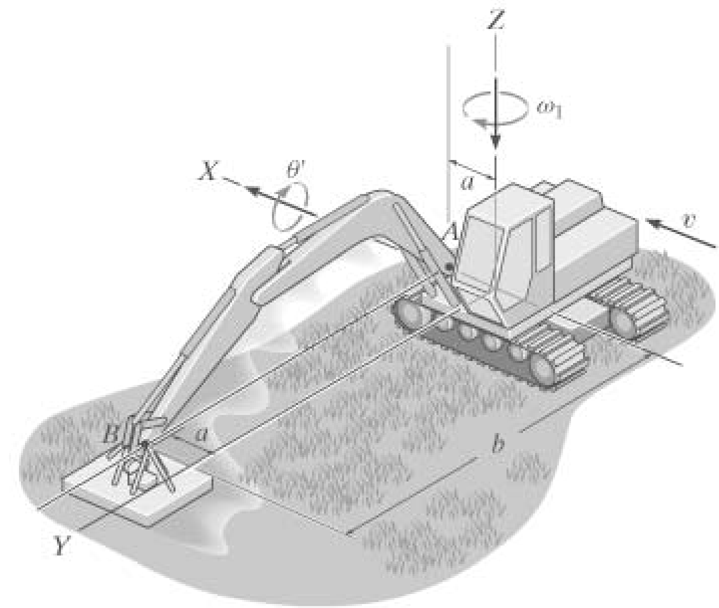
$$\omega_1 = 0.5\ \frac{\text{rad}}{\text{s}}$$

$$\theta' = 0.8\ \frac{\text{rad}}{\text{s}}$$

$$v = 1\ \frac{\text{m}}{\text{s}}$$

$$a = 1\ \text{m}$$

$$b = 8\ \text{m}$$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} \qquad \mathbf{v_B} = \begin{pmatrix} 5 \\ -0.5 \\ 6.4 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

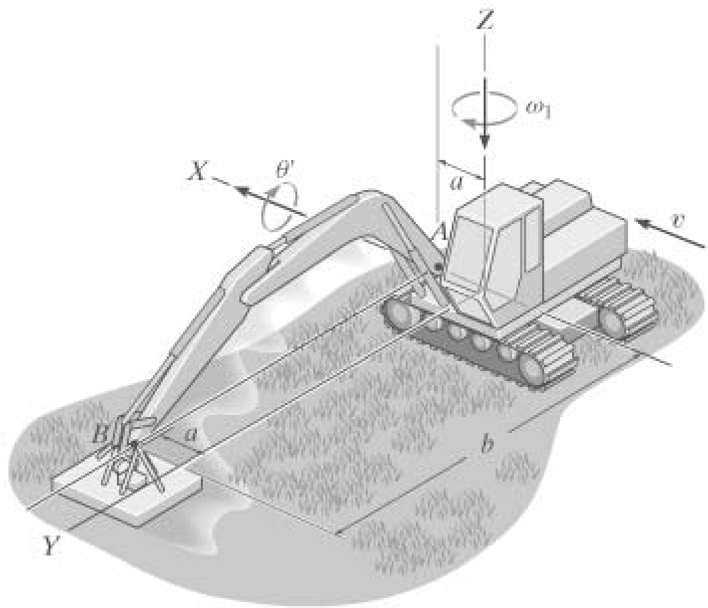
$$\mathbf{a_B} = \begin{pmatrix} 0 \\ -\omega_1\,\theta' \\ 0 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_B} = \begin{pmatrix} -0.25 \\ -7.12 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

***Problem 20-36**

At the instant shown, the frame of the brush cutter is traveling forward in the *x* direction with a constant velocity *v*, and the cab is rotating about the vertical axis with an angular velocity ω_1, which is increasing at ω'_1. At the same instant the boom *AB* has an angular velocity θ', which is increasing at θ''. Determine the velocity and acceleration of point *B* at the connection to the mower at this instant.

Given:



$$\omega_1 = 0.5\ \frac{\text{rad}}{\text{s}}$$

$$\omega'_1 = 0.4\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta' = 0.8\ \frac{\text{rad}}{\text{s}}$$

$$\theta'' = 0.9\ \frac{\text{rad}}{\text{s}^2}$$

$$v = 1\ \frac{\text{m}}{\text{s}}$$

$$a = 1\ \text{m}$$

$$b = 8\ \text{m}$$

Solution:

$$\mathbf{v_B} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} \qquad \mathbf{v_B} = \begin{pmatrix} 5 \\ -0.5 \\ 6.4 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_B} = \begin{pmatrix} \theta'' \\ -\omega_1\,\theta' \\ -\omega'_1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix}\right] \qquad \mathbf{a_B} = \begin{pmatrix} 2.95 \\ -7.52 \\ 7.20 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-37

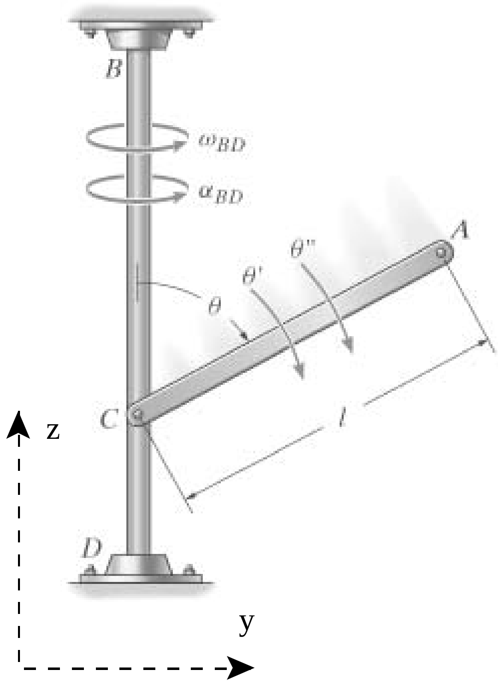
At the instant shown, rod *BD* is rotating about the vertical axis with an angular velocity ω_{BD} and an angular acceleration α_{BD}. Link *AC* is rotating downward. Determine the velocity and acceleration of point *A* on the link at this instant.

Given:

$\omega_{BD} = 7\ \frac{\text{rad}}{\text{s}}$ $\theta = 60\ \text{deg}$

$\alpha_{BD} = 4\ \frac{\text{rad}}{\text{s}^2}$ $\theta' = 2\ \frac{\text{rad}}{\text{s}}$

$l = 0.8\ \text{m}$ $\theta'' = 3\ \frac{\text{rad}}{\text{s}^2}$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} -\theta' \\ 0 \\ \omega_{BD} \end{pmatrix} \qquad \mathbf{r} = l \begin{pmatrix} 0 \\ \sin(\theta) \\ \cos(\theta) \end{pmatrix}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} -\theta'' \\ 0 \\ \alpha_{BD} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BD} \end{pmatrix} \times \boldsymbol{\omega}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_A} = \begin{pmatrix} -4.85 \\ 0.80 \\ -1.39 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_A} = \begin{pmatrix} -13.97 \\ -35.52 \\ -3.68 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-38

The boom *AB* of the locomotive crane is rotating about the *Z* axis with angular velocity ω_1 which is increasing at ω'_1. At this same instant, $\theta = \theta_1$ and the boom is rotating upward at a constant rate of $\theta' = \omega_2$. Determine the velocity and acceleration of the tip *B* of the boom at this instant.

Given:

$\omega_1 = 0.5\ \frac{\text{rad}}{\text{s}}$

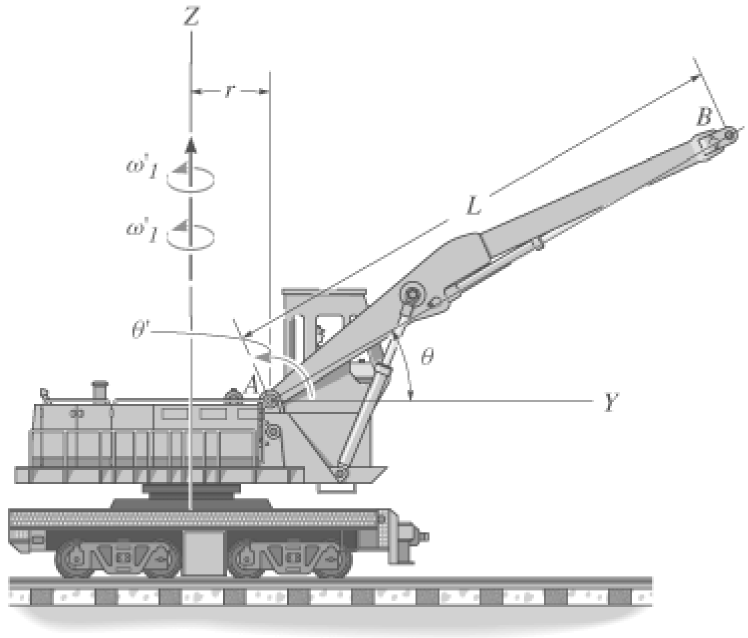
$\omega'_1 = 3\ \frac{\text{rad}}{\text{s}^2}$

$\theta_1 = 30\ \text{deg}$

$\omega_2 = 3\ \frac{\text{rad}}{\text{s}}$

$L = 20\ \text{m}$

$r = 3\ \text{m}$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix}$$

$$\mathbf{a_B} = \begin{pmatrix} -\omega'_1 r \\ -\omega_1^{\ 2} r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_1\omega_2 \\ \omega'_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix}\right]$$

$$\mathbf{v_B} = \begin{pmatrix} -10.2 \\ -30.0 \\ 52.0 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_B} = \begin{pmatrix} -31.0 \\ -161.0 \\ -90.0 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-39

The locomotive crane is traveling to the right with speed v and acceleration a. The boom AB is rotating about the Z axis with angular velocity ω_1 which is increasing at ω'_1. At this same instant, $\theta = \theta_1$ and the boom is rotating upward at a constant rate of $\theta' = \omega_2$. Determine the velocity and acceleration of the tip B of the boom at this instant.

Given:

$v = 2\ \frac{\text{m}}{\text{s}}$

$a = 1.5\ \frac{\text{m}}{\text{s}^2}$

$\omega_1 = 0.5\ \frac{\text{rad}}{\text{s}}$

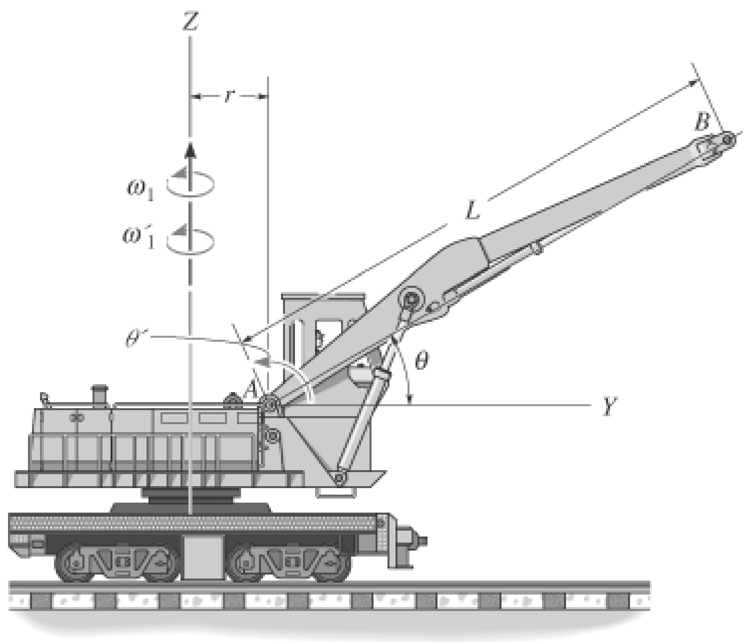
$\omega'_1 = 3\ \frac{\text{rad}}{\text{s}^2}$

$\theta_1 = 30\ \text{deg}$

$\omega_2 = 3\ \frac{\text{rad}}{\text{s}}$

$L = 20\ \text{m}$

$r = 3\ \text{m}$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix}$$

$$\mathbf{a_B} = \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} + \begin{pmatrix} -\omega'_1 r \\ -\omega_1^2 r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_1\omega_2 \\ \omega'_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix}\right]$$

$$\mathbf{v_B} = \begin{pmatrix} -10.2 \\ -28.0 \\ 52.0 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_B} = \begin{pmatrix} -31.0 \\ -159.5 \\ -90.0 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

***Problem 20-40**

At a given instant, the rod has the angular motions shown, while the collar C is moving down *relative* to the rod with a velocity v and an acceleration a. Determine the collar's velocity and acceleration at this instant.

Given:

$v = 6\ \frac{\text{ft}}{\text{s}}$

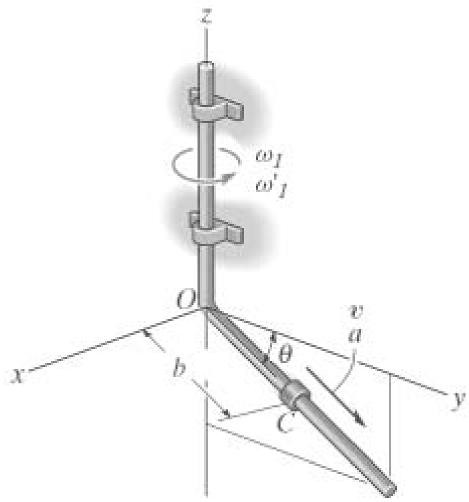
$\omega_1 = 8\ \frac{\text{rad}}{\text{s}}$

$\omega'_1 = 12\ \frac{\text{rad}}{\text{s}^2}$

$\theta = 30\ \text{deg}$

$a = 2\ \frac{\text{ft}}{\text{s}^2}$

$b = 0.8\ \text{ft}$



Solution:

$$\mathbf{v_C} = v\begin{pmatrix} 0 \\ \cos(\theta) \\ -\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ b\cos(\theta) \\ -b\sin(\theta) \end{pmatrix}$$

$$\mathbf{a_C} = a\begin{pmatrix} 0 \\ \cos(\theta) \\ -\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega'_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ b\cos(\theta) \\ -b\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ b\cos(\theta) \\ -b\sin(\theta) \end{pmatrix}\right] \dots$$
$$+ 2\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ v\cos(\theta) \\ -v\sin(\theta) \end{pmatrix}$$

$$\mathbf{v_C} = \begin{pmatrix} -5.54 \\ 5.20 \\ -3.00 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \mathbf{a_C} = \begin{pmatrix} -91.45 \\ -42.61 \\ -1.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 20-41

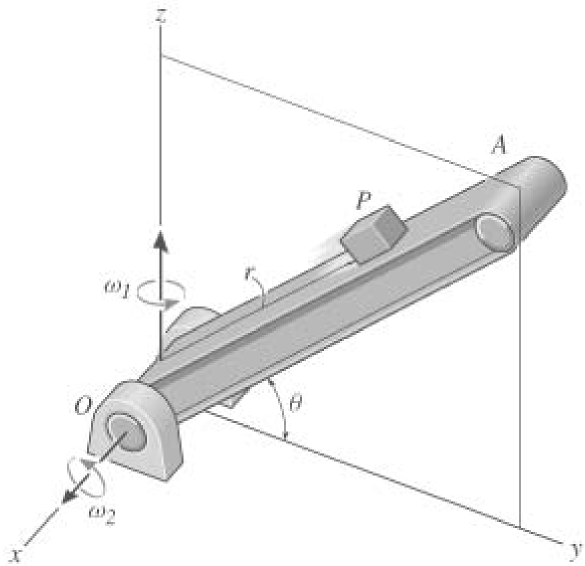
At the instant shown, the arm *OA* of the conveyor belt is rotating about the z axis with a constant angular velocity ω_1, while at the same instant the arm is rotating upward at a constant rate ω_2. If the conveyor is running at a constant rate $r' = v$, determine the velocity and acceleration of the package *P* at the instant shown. Neglect the size of the package.

Given:

$\omega_1 = 6\ \frac{\text{rad}}{\text{s}} \qquad v = 5\ \frac{\text{ft}}{\text{s}}$

$\omega_2 = 4\ \frac{\text{rad}}{\text{s}}$ $\quad r = 6\ \text{ft}$

$\theta = 45\ \text{deg}$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \boldsymbol{\omega}$$

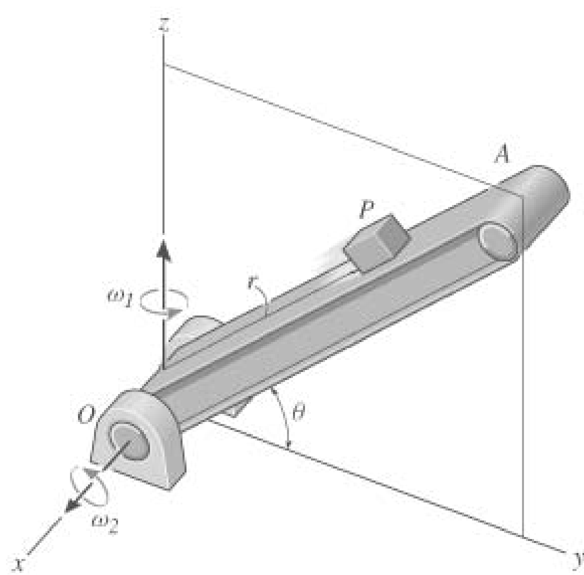
$$\mathbf{r_P} = \begin{pmatrix} 0 \\ r\cos(\theta) \\ r\sin(\theta) \end{pmatrix} \qquad \mathbf{v_{rel}} = \begin{pmatrix} 0 \\ v\cos(\theta) \\ v\sin(\theta) \end{pmatrix}$$

$$\mathbf{v_P} = \mathbf{v_{rel}} + \boldsymbol{\omega} \times \mathbf{r_P} \qquad \mathbf{a_P} = \boldsymbol{\alpha} \times \mathbf{r_P} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_P}) + 2\boldsymbol{\omega} \times \mathbf{v_{rel}}$$

$$\mathbf{v_P} = \begin{pmatrix} -25.5 \\ -13.4 \\ 20.5 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \mathbf{a_P} = \begin{pmatrix} 161.2 \\ -248.9 \\ -39.6 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 20-42

At the instant shown, the arm *OA* of the conveyor belt is rotating about the z axis with a constant angular velocity ω_1, while at the same instant the arm is rotating upward at a constant rate ω_2. If the conveyor is running at the rate $r' = v$ which is increasing at the rate $r'' = a$, determine the velocity and acceleration of the package *P* at the instant shown. Neglect the size of the package.



Given:

$\omega_1 = 6\ \frac{\text{rad}}{\text{s}}$ $\quad \omega_2 = 4\ \frac{\text{rad}}{\text{s}}$

$v = 5\ \frac{\text{ft}}{\text{s}}$ $\quad a = 8\ \frac{\text{ft}}{\text{s}^2}$

$r = 6\ \text{ft}$ $\quad \theta = 45\ \text{deg}$

Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \boldsymbol{\omega} \qquad \mathbf{r_P} = \begin{pmatrix} 0 \\ r\cos(\theta) \\ r\sin(\theta) \end{pmatrix}$$

$$\mathbf{v_{rel}} = \begin{pmatrix} 0 \\ v\cos(\theta) \\ v\sin(\theta) \end{pmatrix} \qquad \mathbf{a_{rel}} = \begin{pmatrix} 0 \\ a\cos(\theta) \\ a\sin(\theta) \end{pmatrix}$$

$$\mathbf{v_P} = \mathbf{v_{rel}} + \boldsymbol{\omega} \times \mathbf{r_P} \qquad \mathbf{a_P} = \mathbf{a_{rel}} + \boldsymbol{\alpha} \times \mathbf{r_P} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_P}) + 2\boldsymbol{\omega} \times \mathbf{v_{rel}}$$

$$\mathbf{v_P} = \begin{pmatrix} -25.5 \\ -13.4 \\ 20.5 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \mathbf{a_P} = \begin{pmatrix} 161.2 \\ -243.2 \\ -33.9 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 20-43

At the given instant, the rod is spinning about the z axis with an angular velocity ω_1 and angular acceleration ω'_1. At this same instant, the disk is spinning, with ω_2 and ω'_2 both measured *relative* to the rod. Determine the velocity and acceleration of point P on the disk at this instant.

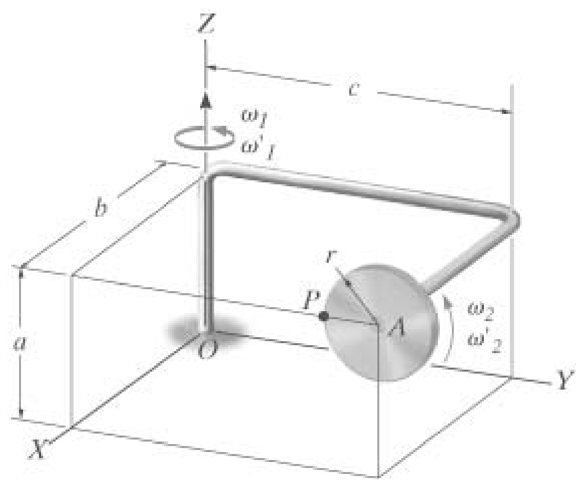
Given:

$\omega_1 = 3\ \frac{\text{rad}}{\text{s}}$ $\qquad a = 2\ \text{ft}$

$\omega'_1 = 4\ \frac{\text{rad}}{\text{s}^2}$ $\qquad b = 3\ \text{ft}$

$\omega_2 = 2\ \frac{\text{rad}}{\text{s}}$ $\qquad c = 4\ \text{ft}$

$\omega'_2 = 1\ \frac{\text{rad}}{\text{s}^2}$ $\qquad r = 0.5\ \text{ft}$



Solution:

$$\mathbf{v_P} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} b \\ c \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix}$$

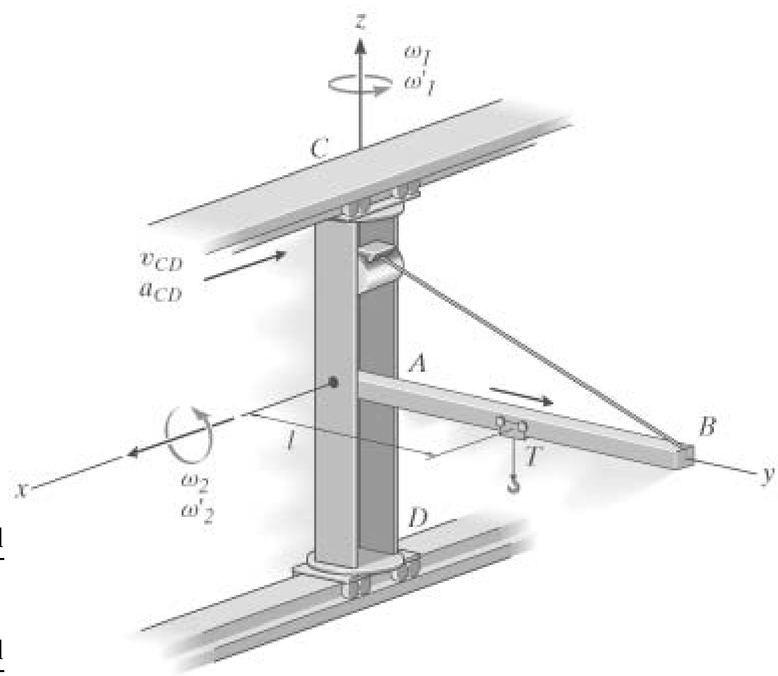
$$\mathbf{v_P} = \begin{pmatrix} -10.50 \\ 9.00 \\ -1.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\mathbf{ap} = \begin{pmatrix} 0 \\ 0 \\ \omega'_1 \end{pmatrix} \times \begin{pmatrix} b \\ c \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} b \\ c \\ 0 \end{pmatrix}\right] \dots$$
$$+ \begin{pmatrix} \omega'_2 \\ \omega_1 \omega_2 \\ \omega'_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix}\right]$$

$$\mathbf{ap} = \begin{pmatrix} -41.00 \\ -17.50 \\ -0.50 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

***Problem 20-44**

At a given instant, the crane is moving along the track with a velocity v_{CD} and acceleration a_{CD}. Simultaneously, it has the angular motions shown. If the trolley T is moving outwards along the boom AB with a relative speed v_r and relative acceleration a_r, determine the velocity and acceleration of the trolley.



Given:

$\omega_1 = 0.5 \frac{\text{rad}}{\text{s}}$ $\quad \omega'_1 = 0.8 \frac{\text{rad}}{\text{s}^2}$

$\omega_2 = 0.4 \frac{\text{rad}}{\text{s}}$ $\quad \omega'_2 = 0.6 \frac{\text{rad}}{\text{s}^2}$

$v_{CD} = 8 \frac{\text{m}}{\text{s}}$ $\quad a_{CD} = 9 \frac{\text{m}}{\text{s}^2}$

$v_r = 3 \frac{\text{m}}{\text{s}}$ $\quad a_r = 5 \frac{\text{m}}{\text{s}^2}$ $\quad l = 3 \text{ m}$

Solution:

$$\mathbf{v_A} = \begin{pmatrix} -v_{CD} \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{a_A} = \begin{pmatrix} -a_{CD} \\ 0 \\ 0 \end{pmatrix} \quad \boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \quad \boldsymbol{\alpha} = \begin{pmatrix} \omega'_2 \\ \omega_1 \omega_2 \\ \omega'_1 \end{pmatrix}$$

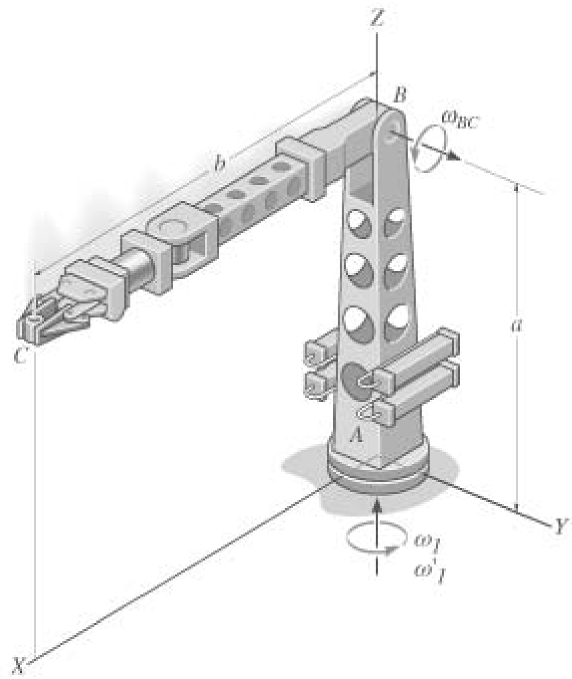
$$\mathbf{r} = \begin{pmatrix} 0 \\ l \\ 0 \end{pmatrix} \quad \mathbf{v_{rel}} = \begin{pmatrix} 0 \\ v_r \\ 0 \end{pmatrix} \quad \mathbf{a_{rel}} = \begin{pmatrix} 0 \\ a_r \\ 0 \end{pmatrix}$$

$$\mathbf{v_T} = \mathbf{v_A} + \mathbf{v_{rel}} + \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_T} = \begin{pmatrix} -9.50 \\ 3.00 \\ 1.20 \end{pmatrix} \frac{m}{s}$$

$$\mathbf{a_T} = \mathbf{a_A} + \mathbf{a_{rel}} + \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v_{rel}} \qquad \mathbf{a_T} = \begin{pmatrix} -14.40 \\ 3.77 \\ 4.20 \end{pmatrix} \frac{m}{s^2}$$

Problem 20-45

At the instant shown, the base of the robotic arm is turning about the z axis with angular velocity ω_1, which is increasing at ω'_1. Also, the boom segment BC is rotating at constant rate ω_{BC}. Determine the velocity and acceleration of the part C held in its grip at this instant.



Given:

$$\omega_1 = 4 \frac{rad}{s} \qquad a = 0.5\ m$$

$$\omega'_1 = 3 \frac{rad}{s^2} \qquad b = 0.7\ m$$

$$\omega_{BC} = 8 \frac{rad}{s}$$

Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_{BC} \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} -\omega_1\,\omega_{BC} \\ 0 \\ \omega'_1 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix}$$

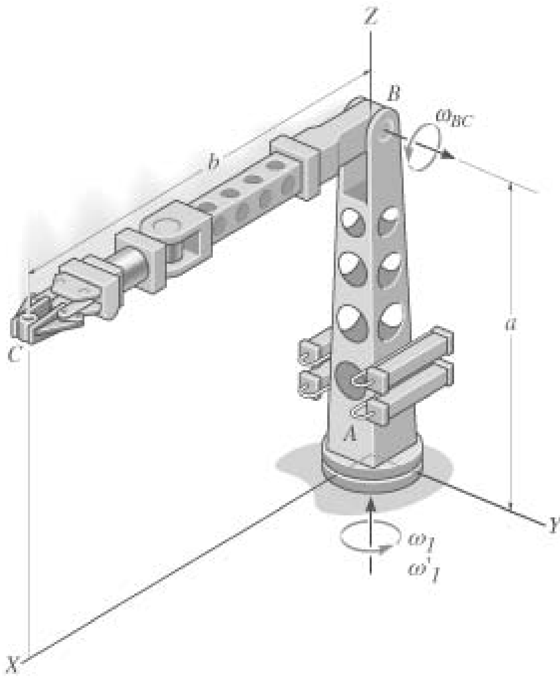
$$\mathbf{v_C} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_C} = \begin{pmatrix} 0 \\ 2.8 \\ -5.6 \end{pmatrix} \frac{m}{s}$$

$$\mathbf{a_C} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_C} = \begin{pmatrix} -56 \\ 2.1 \\ 0 \end{pmatrix} \frac{m}{s^2}$$

Problem 20-46

At the instant shown, the base of the robotic arm is turning about the z axis with angular velocity ω_1, which is increasing at ω'_1. Also, the boom segment BC is rotating with angular velociy ω_{BC} which is incrasing at ω'_{BC}. Determine the velocity and acceleration of the part C held in its grip at this instant.

Given:



$$\omega_1 = 4\ \frac{\text{rad}}{\text{s}} \qquad \omega'_1 = 3\ \frac{\text{rad}}{\text{s}^2}$$

$$\omega_{BC} = 8\ \frac{\text{rad}}{\text{s}} \qquad \omega'_{BC} = 2\ \frac{\text{rad}}{\text{s}^2}$$

$$a = 0.5\ \text{m} \qquad b = 0.7\ \text{m}$$

Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_{BC} \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} -\omega_1\,\omega_{BC} \\ \omega'_{BC} \\ \omega'_1 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{v_C} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_C} = \begin{pmatrix} 0 \\ 2.8 \\ -5.6 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_C} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

$$\mathbf{a_C} = \begin{pmatrix} -56 \\ 2.1 \\ -1.4 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-47

The load is being lifted upward at a constant rate v relative to the crane boom AB. At the instant shown, the boom is rotating about the vertical axis at a constant rate ω_1, and the trolley T is moving outward along the boom at a constant rate v_t. Furthermore, at this same instant the rectractable arm supporting the load is vertical and is swinging in the y-z plane at an angular rate ω_2, with an increase in the rate of swing α_2. Determine the velocity and acceleration of the center G of the load at this instant.

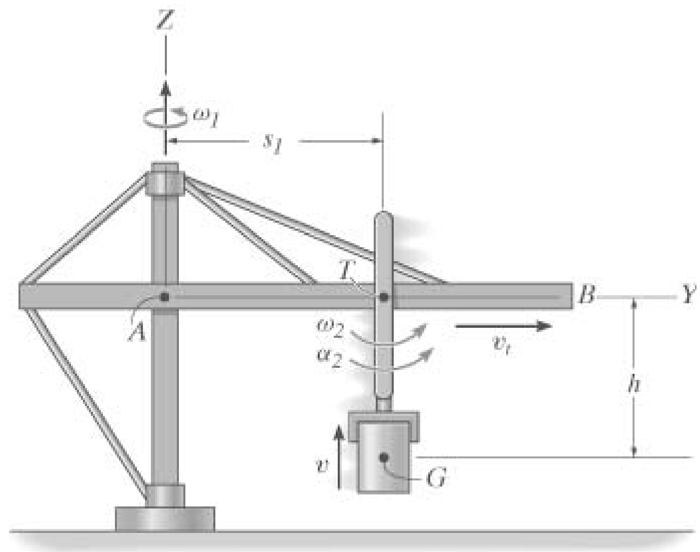
Given:

$\omega_1 = 4 \frac{\text{rad}}{\text{s}}$ $\alpha_2 = 7 \frac{\text{rad}}{\text{s}^2}$

$\omega_2 = 5 \frac{\text{rad}}{\text{s}}$ $h = 3 \text{ m}$

$v_t = 2 \frac{\text{m}}{\text{s}}$ $s_1 = 4 \text{ m}$

$v = 9 \frac{\text{m}}{\text{s}}$



Solution:

$$\mathbf{v_T} = \begin{pmatrix} 0 \\ v_t \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ s_1 \\ 0 \end{pmatrix} \qquad \mathbf{a_T} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ s_1 \\ 0 \end{pmatrix} \right] + 2 \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ v_t \\ 0 \end{pmatrix}$$

$$\mathbf{v_G} = \mathbf{v_T} + \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}$$

$$\mathbf{a_G} = \mathbf{a_T} + \begin{pmatrix} \alpha_2 \\ \omega_1 \omega_2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix} \right] + 2 \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

$$\mathbf{v_G} = \begin{pmatrix} -16.0 \\ 17.0 \\ 9.0 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_G} = \begin{pmatrix} -136.0 \\ -133.0 \\ 75.0 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 21-1

Show that the sum of the moments of inertia of a body, $I_{xx}+I_{yy}+I_{zz}$, is independent of the orientation of the x, y, z axes and thus depends only on the location of the origin.

Solution:

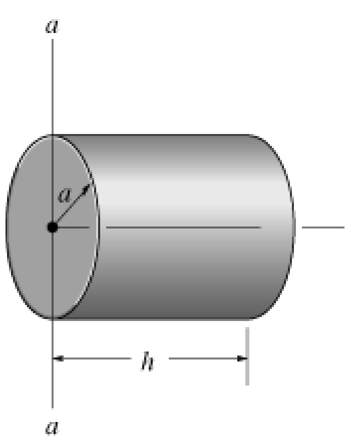
$$I_{xx} + I_{yy} + I_{zz} = \int_m y^2 + z^2 \, \mathrm{d}m + \int_m x^2 + z^2 \, \mathrm{d}m + \int_m x^2 + y^2 \, \mathrm{d}m$$

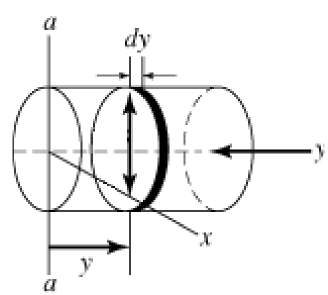
$$I_{xx} + I_{yy} + I_{zz} = 2\int_m x^2 + y^2 + z^2 \, \mathrm{d}m$$

However, $x^2 + y^2 + z^2 = r^2$, where r is the distance from the origin O to dm. The magnitude $| r |$ does not depend on the orientation of the x, y, z axes. Consequently, $I_{xx}+I_{yy}+I_{zz}$ is also independent of the x, y, z axes.

Problem 21-2

Determine the moment of inertia of the cylinder with respect to the a-a axis of the cylinder. The cylinder has a mass m.





Solution:

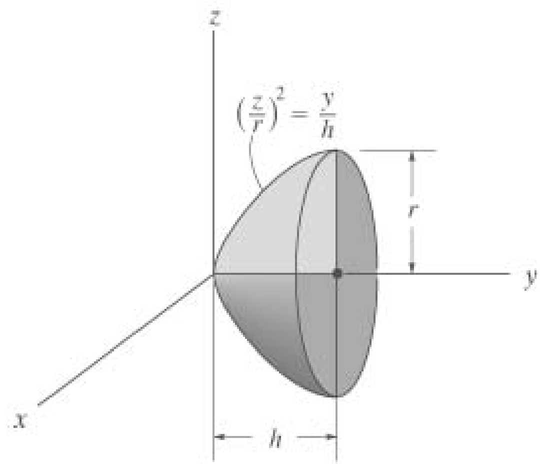
$$m = \int_0^h \rho\,\pi a^2 \,\mathrm{d}y = h\,\rho\,\pi a^2 \qquad\qquad \rho = \frac{m}{h\pi a^2}$$

$$I_{aa} = \frac{m}{h\pi a^2}\left[\int_0^h \left(\frac{a^2}{4} + y^2\right)\pi a^2 \,\mathrm{d}y\right] = \frac{m}{h\,\pi a^2}\left(\frac{1}{4}\,h\,\pi a^4 + \frac{1}{3}\,\pi\,a^2\,h^3\right)$$

$$I_{aa} = m\left(\frac{a^2}{4} + \frac{h^2}{3}\right)$$

Problem 21-3

Determine the moments of inerta I_x and I_y of the paraboloid of revolution. The mass of the paraboloid is M.



Given:

$M = 20\ \text{slug}$

$r = 2\ \text{ft}$

$h = 2\ \text{ft}$

Solution:

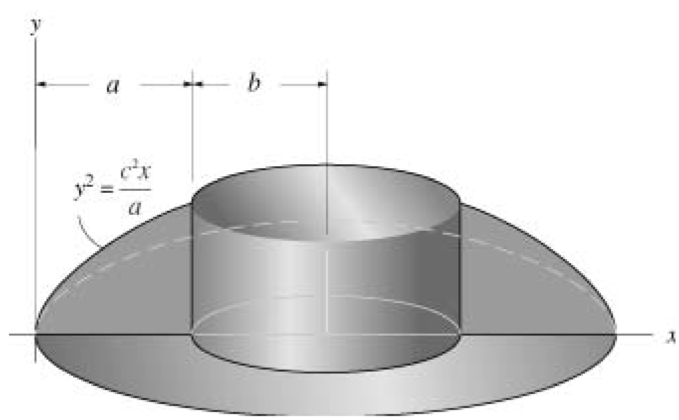
$$V = \int_0^h \pi r^2 \frac{y}{h}\, dy \qquad \rho = \frac{M}{V}$$

$$I_x = \int_0^h \rho \frac{1}{2}\left(\pi r^2 \frac{y}{h}\right)\left(r^2 \frac{y}{h}\right) dy \qquad I_x = 26.7\ \text{slug}\cdot\text{ft}^2$$

$$I_y = \int_0^h \rho\left(\pi r^2 \frac{y}{h}\right)\left[\frac{1}{4}\left(r^2 \frac{y}{h}\right) + y^2\right] dy \qquad I_y = 53.3\ \text{slug}\cdot\text{ft}^2$$

***Problem 21-4**

Determine the product of inertia I_{xy} of the body formed by revolving the shaded area about the line $x = a + b$. Express your answer in terms of the density ρ.



Given:

$a = 3\ \text{ft}$

$b = 2\ \text{ft}$

$c = 3\ \text{ft}$

Solution:

$$k = \int_0^a \left[2\pi c\sqrt{\frac{x}{a}}(a+b-x)\right](a+b)\frac{c}{2}\sqrt{\frac{x}{a}}\,dx$$

$I_{xy} = k\rho$ $\quad k = 636\,\text{ft}^5$

Problem 21-5

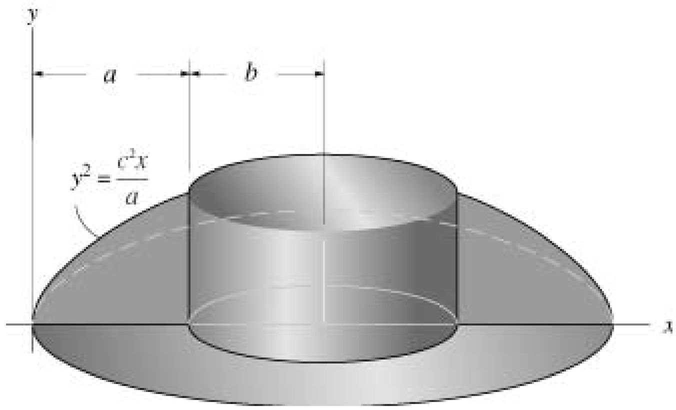
Determine the moment of inertia I_y of the body formed by revolving the shaded area about the line $x = a + b$. Express your answer in terms of the density ρ.

Given:

$a = 3$ ft

$b = 2$ ft

$c = 3$ ft



Solution:

$$k = \int_0^a \left[2\pi c\sqrt{\frac{x}{a}}(a+b-x)\right]\left[(a+b-x)^2 + (a+b)^2\right]dx$$

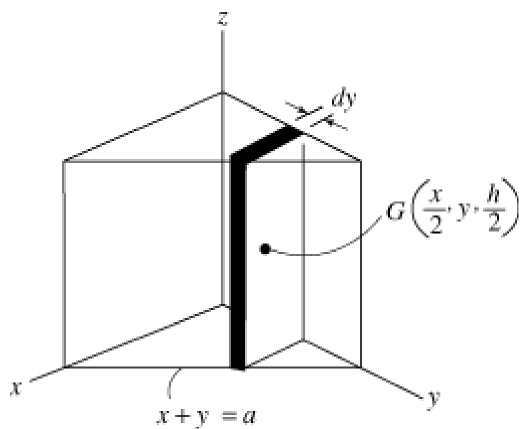
$I_y = k\rho$ $\quad k = 4481\,\text{ft}^5$

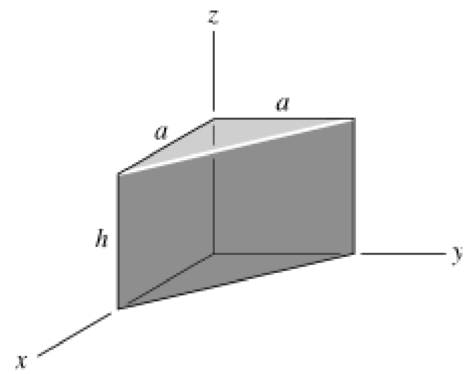
Problem 21-6

Determine by direct integration the product of inertia I_{yz} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the mass m of the prism.

Solution:

$$\rho = \frac{2m}{a^2 h}$$



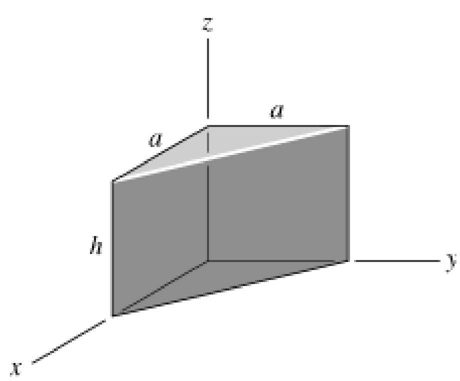


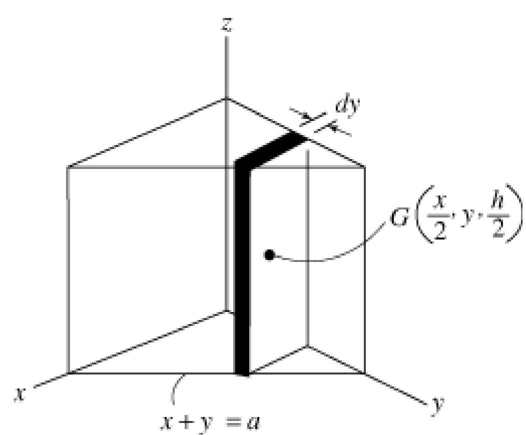
$$I_{yz} = \int_0^a \left(\frac{2m}{a^2 h}\right) h(a-y)\,\frac{h}{2}\left(\frac{a-y}{2}\right) \mathrm{d}y = \frac{1}{6} m\,a\,h$$

$$I_{yz} = \frac{1}{6} m\,a\,h$$

Problem 21-7

Determine by direct integration the product of inertia I_{xy} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the mass m of the prism.





Solution:

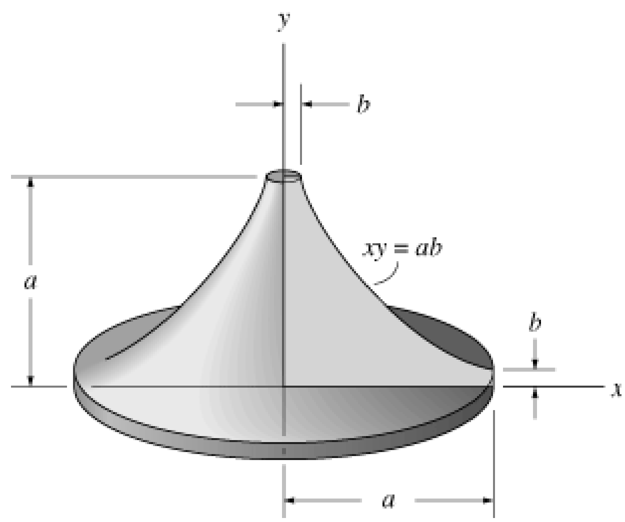
$$\rho = \frac{2m}{a^2 h}$$

$$I_{xy} = \int_0^a \left(\frac{2m}{a^2 h}\right) h(a-y)\, y\left(\frac{a-y}{2}\right) dy = \frac{1}{12} a^4 \left(\frac{m}{a^2}\right)$$

$$I_{xy} = \frac{1}{12} a^2 m$$

***Problem 21-8**

Determine the radii of gyration k_x and k_y for the solid formed by revolving the shaded area about the y axis. The density of the material is ρ.



Given:

$a = 4\ \text{ft}$

$b = 0.25\ \text{ft}$

$\rho = 12\ \frac{\text{slug}}{\text{ft}^3}$

Solution:

$$M = \int_0^b \rho \pi a^2\, dy + \int_b^a \rho \pi \frac{a^2 b^2}{y^2}\, dy \qquad M = 292.17\ \text{slug}$$

$$I_x = \int_0^b \rho\left(\frac{a^2}{4} + y^2\right)\pi a^2\, dy + \int_b^a \rho\left(\frac{a^2 b^2}{4y^2} + y^2\right)\pi \frac{a^2 b^2}{y^2}\, dy \qquad I_x = 948.71\ \text{slug}\cdot\text{ft}^2$$

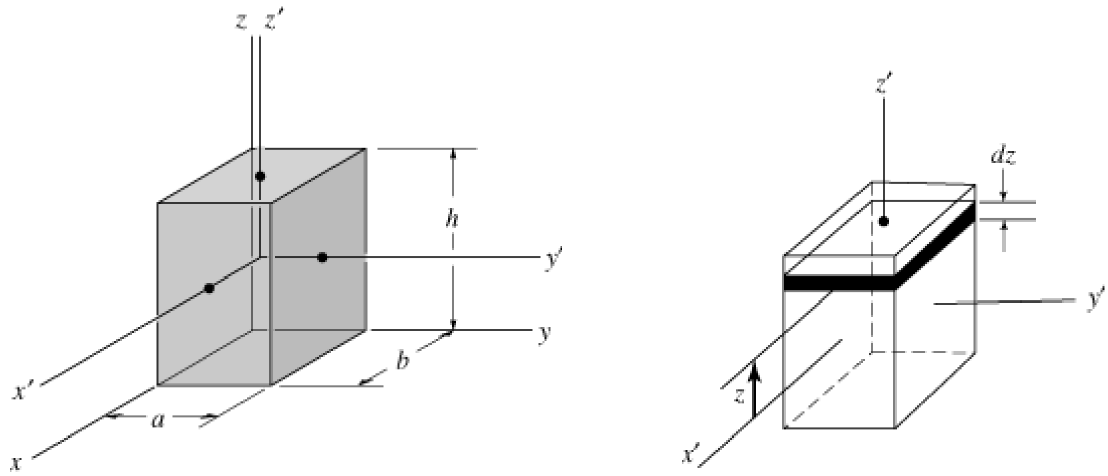
$$I_y = \int_0^b \rho\left(\frac{a^2}{2}\right)\pi a^2\, dy + \int_b^a \rho\left(\frac{a^2 b^2}{2y^2}\right)\pi \frac{a^2 b^2}{y^2}\, dy \qquad I_y = 1608.40\ \text{slug}\cdot\text{ft}^2$$

$$k_x = \sqrt{\frac{I_x}{M}} \qquad k_x = 1.80\ \text{ft}$$

$$k_y = \sqrt{\frac{I_y}{M}} \qquad k_y = 2.35\ \text{ft}$$

Problem 21-9

Determine the mass moment of inertia of the homogeneous block with respect to its centroidal x' axis. The mass of the block is m.



Solution:

$$m = \rho a b h \qquad \rho = \frac{m}{a b h}$$

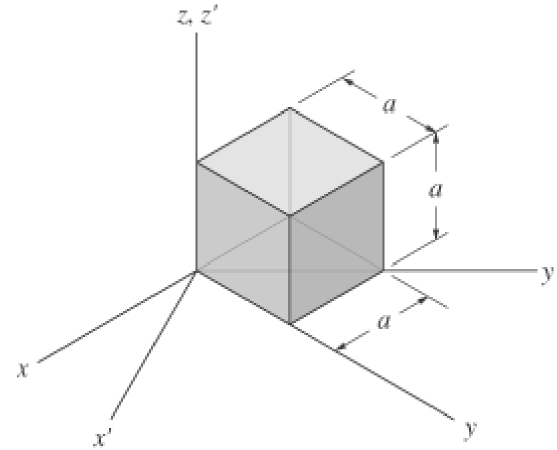
$$I_{x'} = \frac{m}{a b h}\int_{\frac{-h}{2}}^{\frac{h}{2}} \left(\frac{1}{12}a^2 + z^2\right) a b\ \text{dz} = \frac{m}{a b h}\left(\frac{1}{12}a^3 b h + \frac{1}{12}a b h^3\right)$$

$$I_{x'} = \frac{m}{12}\left(a^2 + h^2\right)$$

Problem 21-10

Determine the elements of the inertia tensor for the cube with respect to the x, y, z coordinate system. The mass of the cube is m.

Solution:



$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3} m a^2$$

$$I_{xy} = I_{xz} = -m\left(\frac{a}{2}\right)^2 = -m\frac{a^2}{4}$$

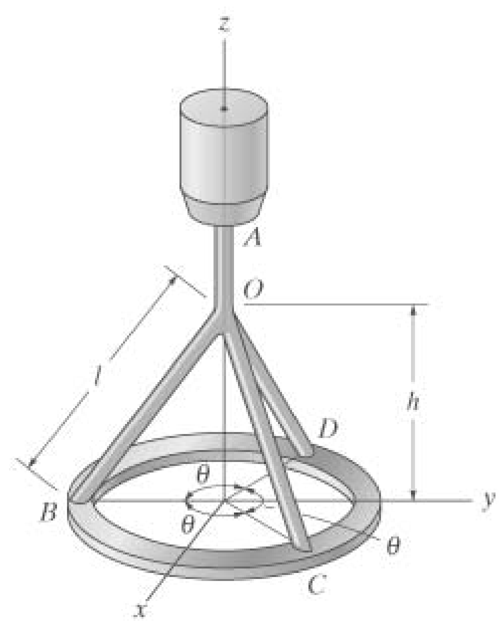
$$I_{yz} = m\left(\frac{a}{2}\right)^2 = m\frac{a^2}{4}$$

$$\mathbf{I} = \frac{m a^2}{12}\begin{pmatrix} 8 & 3 & 3 \\ 3 & 8 & -3 \\ 3 & -3 & 8 \end{pmatrix}$$

Remember to change the signs of the products of inertia to put them in the inertia tensor

Problem 21-11

Compute the moment of inertia of the rod-and-thin-ring assembly about the *z* axis. The rods and ring have a mass density ρ.



Given:

$$\rho = 2\ \frac{\text{kg}}{\text{m}}$$

$$l = 500\ \text{mm}$$

$$h = 400\ \text{mm}$$

$$\theta = 120\ \text{deg}$$

Solution:

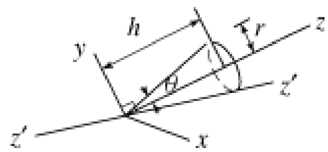
$$r = \sqrt{l^2 - h^2}$$

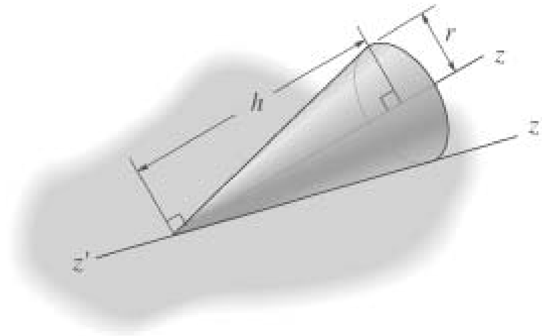
$$\phi = \operatorname{acos}\left(\frac{h}{l}\right)$$

$$I_z = 3\left(\rho l \frac{l^2}{3}\sin(\phi)^2\right) + \rho(2\pi r) r^2 \qquad I_z = 0.43\ \text{kg}\cdot\text{m}^2$$

***Problem 21-12**

Determine the moment of inertia of the cone about the z' axis. The weight of the cone is W, the *height* is h, and the radius is r.





Given:

$W = 15\ \text{lb}$

$h = 1.5\ \text{ft}$

$r = 0.5\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$$\theta = \text{atan}\left(\frac{r}{h}\right)$$

$$I_x = \frac{3}{80}W\left(4r^2 + h^2\right) + W\left(\frac{3h}{4}\right)^2$$

$$I_y = I_x \qquad I_z = \frac{3}{10}W r^2$$

$$I_{z'} = I_x \sin(\theta)^2 + I_z \cos(\theta)^2 \qquad I_{z'} = 0.0962\ \text{slug}\cdot\text{ft}^2$$

Problem 21-13

The bent rod has weight density γ. Locate the center of gravity $G(x', y')$ and determine the principal moments of inertia $I_{x'}$, $I_{y'}$, and $I_{z'}$ of the rod with respect to the x', y', z' axes.

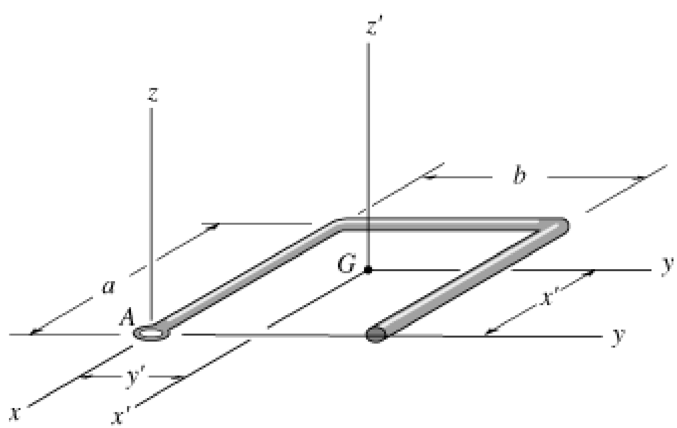
Given:

$\gamma = 1.5\ \dfrac{\text{lb}}{\text{ft}}$

$a = 1\ \text{ft}$

$b = 1\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$$x' = \frac{2a\frac{a}{2} + b\,a}{2a + b} \qquad x' = 0.667\ \text{ft}$$

$$y' = \frac{a\,b + b\frac{b}{2}}{2a + b} \qquad y' = 0.50\ \text{ft}$$

$$I_{x'} = \gamma a\,y'^2 + \gamma a\,(b - y')^2 + \frac{1}{12}\gamma b\,b^2 + \gamma b\left(\frac{b}{2} - y'\right)^2 \qquad I_{x'} = 0.0272\ \text{slug}\cdot\text{ft}^2$$

$$I_{y'} = \frac{2}{12}\gamma a\,a^2 + 2\gamma a\left(\frac{a}{2} - x'\right)^2 + \gamma b\,(a - x')^2 \qquad I_{y'} = 0.0155\ \text{slug}\cdot\text{ft}^2$$

$$I_{z'} = I_{x'} + I_{y'} \qquad I_{z'} = 0.0427\ \text{slug}\cdot\text{ft}^2$$

Problem 21-14

The assembly consists of two square plates *A* and *B* which have a mass M_A each and a rectangular plate *C* which has a mass M_C. Determine the moments of inertia I_x, I_y and I_z.

Given:

$M_A = 3\ \text{kg}$

$M_C = 4.5\ \text{kg}$

$\theta = 60\ \text{deg}$

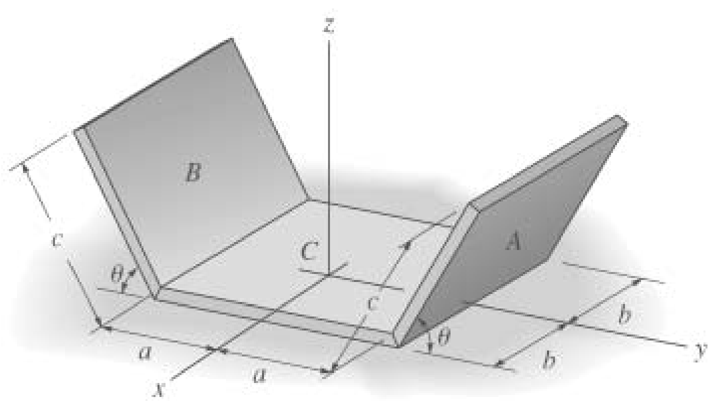
$\theta_1 = 90\ \text{deg}$

$\theta_2 = 30\ \text{deg}$

$a = 0.3\ \text{m}$

$b = 0.2\ \text{m}$

$c = 0.4\ \text{m}$



Solution:

$$\rho_A = \frac{M_A}{c(2b)}$$

$$I_x = \frac{1}{12}M_C(2a)^2 + 2\int_0^c \rho_A(2b)\left[(a + \xi\cos(\theta))^2 + (\xi\sin(\theta))^2\right] d\xi$$

$$I_y = \frac{1}{12}M_C(2b)^2 + 2\int_{-b}^{b}\int_0^c \rho_A\left(x^2 + \xi^2\sin(\theta)^2\right) d\xi\, dx$$

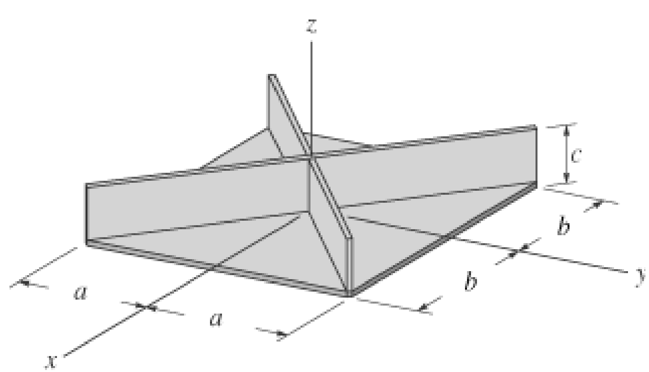
$$I_z = \frac{1}{12}M_C\left[(2b)^2 + (2a)^2\right] + 2\int_{-b}^{b}\int_0^c \rho_A\left[x^2 + (a + \xi\cos(\theta))^2\right] d\xi\, dx$$

$I_x = 1.36\ \text{kg}\cdot\text{m}^2$ $I_y = 0.380\ \text{kg}\cdot\text{m}^2$ $I_z = 1.25\ \text{kg}\cdot\text{m}^2$

Problem 21-15

Determine the moment of inertia I_x of the composite plate assembly. The plates have a specific weight γ.

Given:



$$\gamma = 6\ \frac{\text{lb}}{\text{ft}^2}$$

$$a = 0.5\ \text{ft}$$

$$b = 0.5\ \text{ft}$$

$$c = 0.25\ \text{ft}$$

$$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

Solution:

$$\theta = \text{atan}\left(\frac{a}{b}\right)$$

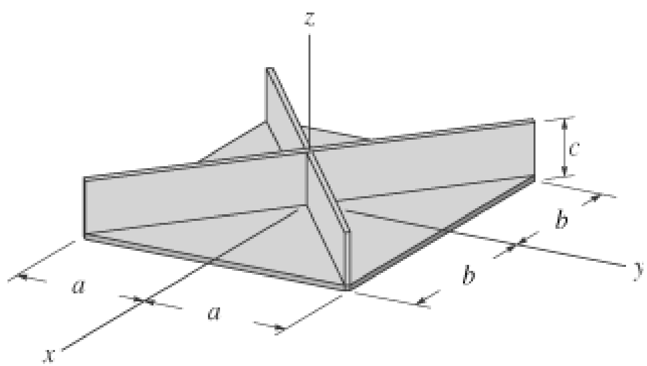
$$I_1 = \gamma c 2\sqrt{a^2 + b^2}\left[\frac{c^2}{3} + \frac{(2a)^2 + (2b)^2}{12}\right]$$

$$I_2 = \gamma c 2\sqrt{a^2 + b^2}\,\frac{c^2}{3}$$

$$I_x = 2\left(I_1 \sin(\theta)^2 + I_2 \cos(\theta)^2\right) + \gamma(2a)(2b)\,\frac{(2a)^2}{12} \qquad I_x = 0.0293\ \text{slug}\cdot\text{ft}^2$$

***Problem 21-16**

Determine the product of inertia I_{yz} of the composite plate assembly. The plates have a specific weight γ.



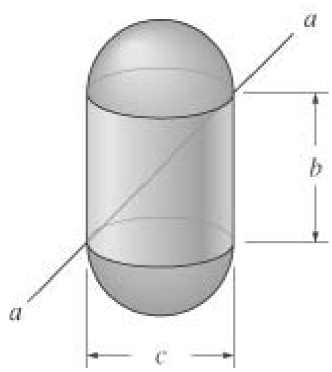
Solution:

Due to symmetry,

$I_{yz} = 0$

Problem 21-17

Determine the moment of inertia of the composite body about the *aa* axis. The cylinder has weight W_c and each hemisphere has weight W_h.



Given:

$W_c = 20\ \text{lb}$

$W_h = 10\ \text{lb}$

$b = 2\ \text{ft}$

$c = 2\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$$\theta = \operatorname{atan}\left(\frac{c}{b}\right)$$

$$I_z = 2\,\frac{2}{5} W_h \left(\frac{c}{2}\right)^2 + \frac{1}{2} W_c \left(\frac{c}{2}\right)^2 \qquad I_z = 0.56\ \text{slug}\cdot\text{ft}^2$$

$$I_y = 2\frac{83}{320}W_h\left(\frac{c}{2}\right)^2 + 2W_h\left(\frac{b}{2} + \frac{3}{8}\frac{c}{2}\right)^2 + W_c\left[\frac{b^2}{12} + \left(\frac{c}{2}\right)^2\frac{1}{4}\right]$$

$$I_y = 1.70\,\text{slug}\cdot\text{ft}^2$$

$$I_{aa} = I_z\cos(\theta)^2 + I_y\sin(\theta)^2 \qquad I_{aa} = 1.13\,\text{slug}\cdot\text{ft}^2$$

Problem 21-18

Determine the moment of inertia about the *z* axis of the assembly which consists of the rod *CD* of mass M_R and disk of mass M_D.

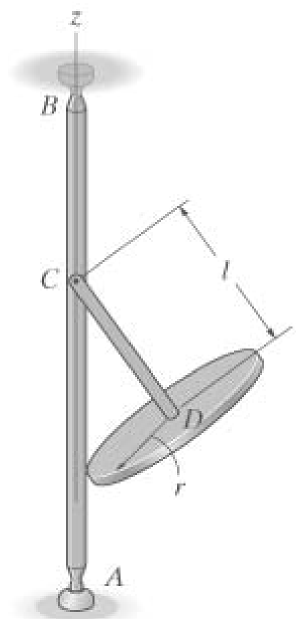
Given:

$M_R = 1.5$ kg

$M_D = 7$ kg

$r = 100$ mm

$l = 200$ mm



Solution:

$$\theta = \text{atan}\left(\frac{r}{l}\right)$$

$$I_1 = \frac{1}{3}M_R l^2 + \frac{1}{4}M_D r^2 + M_D l^2$$

$$I_2 = I_1$$

$$I_3 = \frac{1}{2}M_D r^2$$

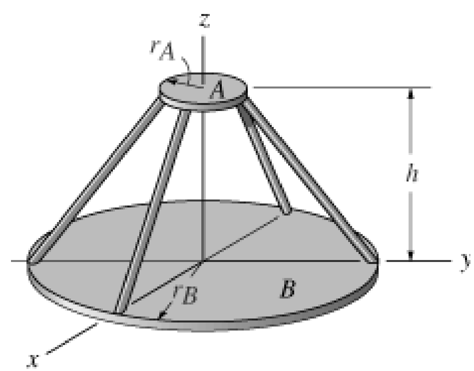
$$\mathbf{I_{mat}} = \left[\begin{pmatrix}1 & 0 & 0\\ 0 & \cos(\theta) & \sin(\theta)\\ 0 & -\sin(\theta) & \cos(\theta)\end{pmatrix}\begin{pmatrix}I_1 & 0 & 0\\ 0 & I_2 & 0\\ 0 & 0 & I_3\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \cos(\theta) & -\sin(\theta)\\ 0 & \sin(\theta) & \cos(\theta)\end{pmatrix}\right]$$

$$I_z = \mathbf{I_{mat}}_{2,2} \qquad I_z = 0.0915\,\text{kg}\cdot\text{m}^2$$

Problem 21-19

The assembly consists of a plate A of weight W_A, plate B of weight W_B, and four rods each of weight W_r. Determine the moments of inertia of the assembly with respect to the principal x, y, z axes.

Given:



$W_A = 15\text{ lb}$

$W_B = 40\text{ lb}$

$W_r = 7\text{ lb}$

$r_A = 1\text{ ft}$

$r_B = 4\text{ ft}$

$h = 4\text{ ft}$

Solution:

$$L = \sqrt{(r_B - r_A)^2 + h^2} \qquad L = 5.00\text{ ft}$$

$$\theta = \text{asin}\left(\frac{h}{L}\right) \qquad \theta = 53.13\text{ deg}$$

$$I_x = 2W_r\left(\frac{L^2}{3}\right)\sin(\theta)^2 + 2\left[W_r\left(\frac{L^2}{12}\right) + W_r\left[\left(\frac{h}{2}\right)^2 + \left(\frac{r_A + r_B}{2}\right)^2\right]\right] + W_B\frac{r_B^2}{4}\ \ldots$$
$$+ W_A\left(\frac{r_A^2}{4}\right) + W_A h^2$$

$I_y = I_x$ by symmetry

$$I_z = 4\left[W_r\frac{L^2}{12}\cos(\theta)^2 + W_r\left(\frac{r_B + r_A}{2}\right)^2\right] + W_A\left(\frac{r_A^2}{2}\right) + W_B\left(\frac{r_B^2}{2}\right)$$

$$\begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} = \begin{pmatrix} 20.2 \\ 20.2 \\ 16.3 \end{pmatrix}\text{slug}\cdot\text{ft}^2$$

***Problem 21-20**

The thin plate has a weight W_p and each of the four rods has weight W_r. Determine the moment of inertia of the assembly about the z axis.

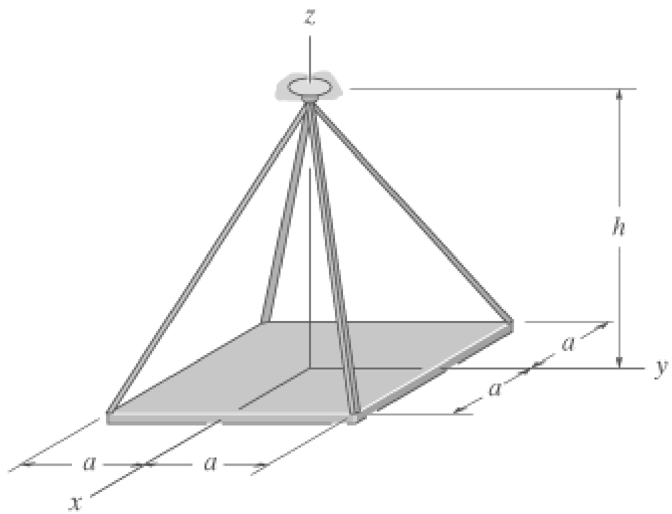
Given:

$W_p = 5\ \text{lb}$

$W_r = 3\ \text{lb}$

$h = 1.5\ \text{ft}$

$a = 0.5\ \text{ft}$



Solution:

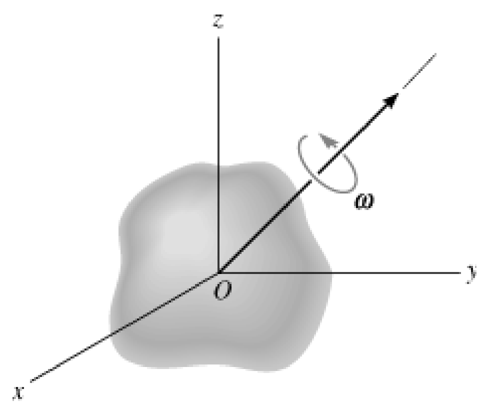
$$L = \sqrt{h^2 + a^2 + a^2}$$

$$\theta = \operatorname{acos}\left(\frac{h}{L}\right)$$

$$I_z = 4\frac{1}{3}W_r L^2 \sin(\theta)^2 + \frac{1}{12}W_p\left[(2a)^2 + (2a)^2\right] \qquad I_z = 0.0881\ \text{slug}\cdot\text{ft}^2$$

Problem 21-21

If a body contains *no planes of symmetry*, the principal moments of inertia can be determined mathematically. To show how this is done, consider the rigid body which is spinning with an angular velocity ω, directed along one of its principal axes of inertia. If the principal moment of inertia about this axis is I, the angular momentum can be expressed as $\mathbf{H} = I\boldsymbol{\omega} = I\omega_x\mathbf{i} + I\omega_y\mathbf{j} + I\omega_z\mathbf{k}$. The components of $\mathbf{H}$ may also be expressed by Eqs. 21-10, where the inertia tensor is assumed to be known. Equate the **i**, **j**, and **k** components of both expressions for $\mathbf{H}$ and consider ω_x, ω_y, and ω_z to be unknown. The solution of these three equations is obtained provided the determinant of the coefficients is zero. Show that this determinant, when expanded, yields the cubic equation $I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + (I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I^2_{xy} - I^2_{yz} - I^2_{zx})I - (I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I^2_{yz} - I_{yy}I^2_{zx} - I_{zz}I^2_{xy}) = 0$. The three positive roots of I, obtained from the solution of this equation, represent the principal moments of inertia I_x, I_y, and I_z.



Solution:

$$\mathbf{H} = I\boldsymbol{\omega} = I\omega_x\mathbf{i} + I\omega_y\mathbf{j} + I\omega_z\mathbf{k}$$

Equating the **i**, **j**, and **k** components to the scalar (Eq. 21 - 10) yields

$$(I_{xx} - I)\omega_x - I_{xy}\omega_y - I_{xz}\omega_z = 0$$

$$-I_{yx}\,\omega_x + (I_{yy} - I)\omega_y - I_{yz}\omega_z = 0$$

$$-I_{zx}\,\omega_x - I_{zy}\omega_y + \left(I_{zz} - I\right)\omega_z = 0$$

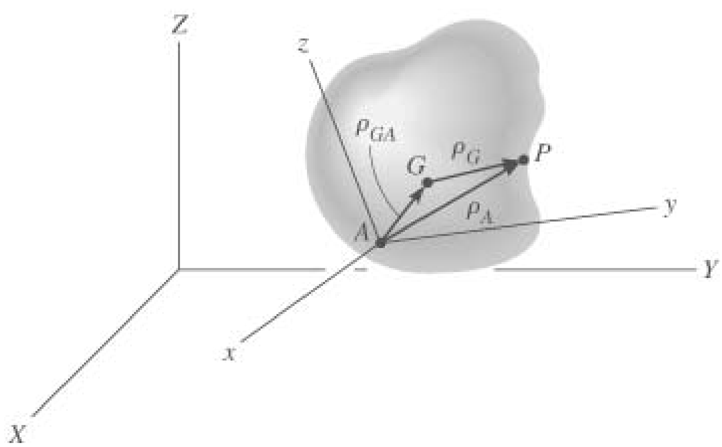
Solution for nontrivial ω_x, ω_y, and ω_z requires

$$\begin{bmatrix} \left(I_{xx} - I\right) & -I_{xy} & -I_{xz} \\ -Iyx & \left(I_{yy} - I\right) & -I_{yz} \\ -I_{zx} & -I_{zy} & \left(I_{zz} - I\right) \end{bmatrix} = 0$$

Expanding the determinant produces the required equation QED

Problem 21-22

Show that if the angular momentum of a body is determined with respect to an arbitrary point A, then $\mathbf{H_A}$ can be expressed by Eq. 21-9. This requires substituting $\boldsymbol{\rho}_\mathbf{A} = \boldsymbol{\rho}_\mathbf{G} + \boldsymbol{\rho}_\mathbf{GA}$ into Eq. 21-6 and expanding, noting that $\int \boldsymbol{\rho}_\mathbf{G} dm = 0$ by definition of the mass center and $\mathbf{v_G} = \mathbf{v_A} + \boldsymbol{\omega} \times \boldsymbol{\rho}_\mathbf{GA}$.



Solution:

$$\mathbf{H_A} = \int \boldsymbol{\rho}_\mathbf{A}\, dm \times \mathbf{v_A} + \int \boldsymbol{\rho}_\mathbf{A} \times \left(\boldsymbol{\omega} \times \boldsymbol{\rho}_\mathbf{A}\right) dm$$

$$\mathbf{H_A} = \int \boldsymbol{\rho}_\mathbf{G} + \boldsymbol{\rho}_\mathbf{GA}\, dm \times \mathbf{v_A} + \int \left(\boldsymbol{\rho}_\mathbf{G} + \boldsymbol{\rho}_\mathbf{GA}\right) \times \left[\boldsymbol{\omega} \times \left(\boldsymbol{\rho}_\mathbf{G} + \boldsymbol{\rho}_\mathbf{GA}\right)\right] dm$$

$$\mathbf{H_A} = \int \boldsymbol{\rho}_\mathbf{G}\, dm \times \mathbf{v_A} + \boldsymbol{\rho}_\mathbf{GA} \times m\mathbf{v_A} + \int \boldsymbol{\rho}_\mathbf{G} \times \left(\boldsymbol{\omega} \times \boldsymbol{\rho}_\mathbf{G}\right) dm + \int \boldsymbol{\rho}_\mathbf{G}\, dm \times \left(\boldsymbol{\omega} \times \boldsymbol{\rho}_\mathbf{GA}\right) \dots$$

$$+ \boldsymbol{\rho}_\mathbf{GA} \times \left(\boldsymbol{\omega} \times \int \boldsymbol{\rho}_\mathbf{G}\, dm\right) + \boldsymbol{\rho}_\mathbf{GA} \times \left(\boldsymbol{\omega} \times \boldsymbol{\rho}_\mathbf{GA}\right) m$$

Since $\int \boldsymbol{\rho}_{\mathbf{G}}\,dm = 0$ and $\mathbf{H_G} = \int \boldsymbol{\rho}_{\mathbf{G}} \times (\boldsymbol{\omega} \times \boldsymbol{\rho}_{\mathbf{G}})\,dm$

$$\mathbf{H_A} = \boldsymbol{\rho}_{\mathbf{GA}} \times m\mathbf{v_A} + \mathbf{H_G} + \boldsymbol{\rho}_{\mathbf{GA}} \times \boldsymbol{\omega} \times (\boldsymbol{\rho}_{\mathbf{GA}})\,m = \boldsymbol{\rho}_{\mathbf{GA}} \times m(\mathbf{v_A} + \boldsymbol{\omega} \times \boldsymbol{\rho}_{\mathbf{GA}}) + \mathbf{H_G}$$

$$\mathbf{H_A} = \boldsymbol{\rho}_{\mathbf{G}} \times m\mathbf{v_G} + \mathbf{H_G} \qquad \text{Q.E.D}$$

Problem 21-23

The thin plate of mass M is suspended at O using a ball-and-socket joint. It is rotating with a constant angular velocity $\omega = \omega_1\mathbf{k}$ when the corner A strikes the hook at S, which provides a permanent connection. Determine the angular velocity of the plate immediately after impact.

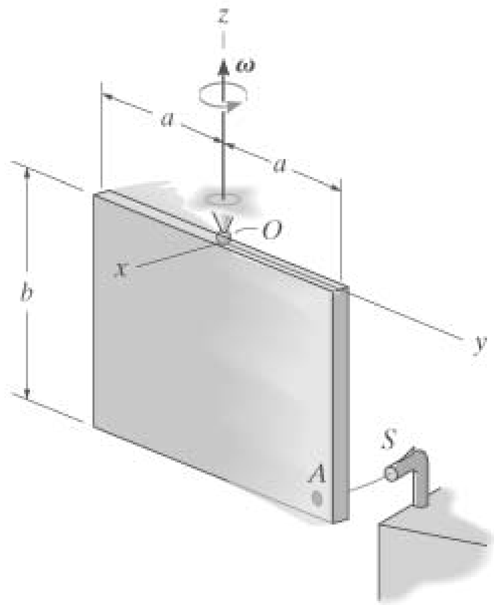
Given:

$M = 5\ \text{kg}$

$\omega_1 = 2\ \dfrac{\text{rad}}{\text{s}}$

$a = 300\ \text{mm}$

$b = 400\ \text{mm}$



Solution:

Angular Momentum is conserved about the line OA.

$$\mathbf{OA} = \begin{pmatrix} 0 \\ a \\ -b \end{pmatrix} \qquad \mathbf{oa} = \frac{\mathbf{OA}}{|\mathbf{OA}|}$$

$$I_2 = \frac{1}{3}Mb^2 \qquad I_3 = \frac{1}{12}M(2a)^2 \qquad I_1 = I_2 + I_3$$

$$\mathbf{I_{mat}} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix}$$

$$I_{oa} = \mathbf{oa}^T\mathbf{I_{mat}}\,\mathbf{oa}$$

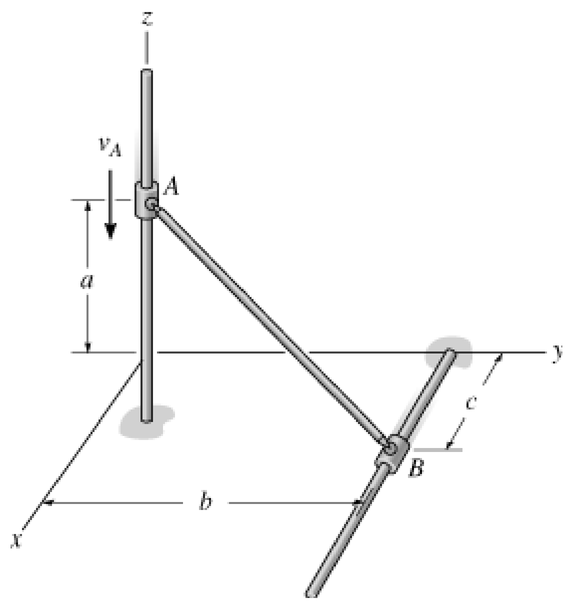
Guess $\omega_2 = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $\mathbf{I_{mat}} \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \mathbf{oa} = I_{oa}\omega_2$

$\omega_2 = \text{Find}(\omega_2)$ $\omega_2\mathbf{oa} = \begin{pmatrix} 0.00 \\ -0.75 \\ 1.00 \end{pmatrix} \frac{\text{rad}}{\text{s}}$

***Problem 21-24**

Rod AB has weight W and is attached to two smooth collars at its end points by ball-and-socket joints. If collar A is moving downward at speed v_A, determine the kinetic energy of the rod at the instant shown. Assume that at this instant the angular velocity of the rod is directed perpendicular to the rod's axis.



Given:

$W = 6 \text{ lb}$

$v_A = 8 \frac{\text{ft}}{\text{s}}$

$a = 3 \text{ ft}$

$b = 6 \text{ ft}$

$c = 2 \text{ ft}$

Solution: $L = \sqrt{a^2 + b^2 + c^2}$

Guesses

$v_B = 1 \frac{\text{ft}}{\text{s}}$ $\omega_x = 1 \frac{\text{rad}}{\text{s}}$

$\omega_y = 1 \frac{\text{rad}}{\text{s}}$ $\omega_z = 1 \frac{\text{rad}}{\text{s}}$

Given $\begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = \begin{pmatrix} v_B \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0 \frac{\text{ft}}{\text{s}}$

$$\begin{pmatrix} v_B \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \text{Find}(v_B, \omega_x, \omega_y, \omega_z) \qquad \boldsymbol{\omega} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0.98 \\ -1.06 \\ -1.47 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = 12.00 \, \frac{\text{ft}}{\text{s}}$$

$$\mathbf{v_G} = \begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \boldsymbol{\omega} \times \begin{pmatrix} \frac{c}{2} \\ \frac{b}{2} \\ \frac{-a}{2} \end{pmatrix} \qquad T = \frac{1}{2}\left(\frac{W}{g}\right)(\mathbf{v_G}\,\mathbf{v_G}) + \frac{1}{2}\left(\frac{W}{g}\right)\frac{L^2}{12}(\boldsymbol{\omega}\,\boldsymbol{\omega}) \qquad T = 6.46 \, \text{lb} \cdot \text{ft}$$

Problem 21-25

At the instant shown the collar at *A* on rod *AB* of weight *W* has velocity v_A. Determine the kinetic energy of the rod after the collar has descended a distance *d*. Neglect friction and the thickness of the rod. Neglect the mass of the collar and the collar is attached to the rod using ball-and-socket joints.

Given:

$W = 6 \text{ lb}$

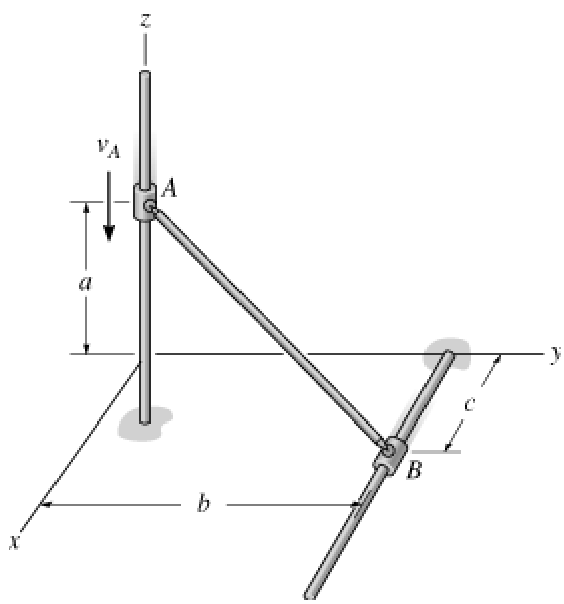
$v_A = 8 \, \frac{\text{ft}}{\text{s}}$

$a = 3 \text{ ft}$

$b = 6 \text{ ft}$

$c = 2 \text{ ft}$

$d = 3 \text{ ft}$



Solution:

$L = \sqrt{a^2 + b^2 + c^2}$

Guesses

$v_B = 1 \, \frac{\text{ft}}{\text{s}} \qquad \omega_x = 1 \, \frac{\text{rad}}{\text{s}}$

$\omega_y = 1 \, \frac{\text{rad}}{\text{s}} \qquad \omega_z = 1 \, \frac{\text{rad}}{\text{s}}$

Given $\begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = \begin{pmatrix} v_B \\ 0 \\ 0 \end{pmatrix}$ $\qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0 \ \frac{\text{ft}}{\text{s}}$

$$\begin{pmatrix} v_B \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \text{Find}(v_B, \omega_x, \omega_y, \omega_z)$$

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0.98 \\ -1.06 \\ -1.47 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = 12.00 \ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{v_G} = \begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \boldsymbol{\omega} \times \begin{pmatrix} \frac{c}{2} \\ \frac{b}{2} \\ \frac{-a}{2} \end{pmatrix}$$

$$T_1 = \frac{1}{2}\left(\frac{W}{g}\right)(\mathbf{v_G}\cdot\mathbf{v_G}) + \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{L^2}{12}\right)(\boldsymbol{\omega}\cdot\boldsymbol{\omega}) \qquad T_1 = 6.46 \ \text{lb}\cdot\text{ft}$$

In position 2 the center of mass has fallen a distance $d/2$

$$T_1 + 0 = T_2 - W\left(\frac{d}{2}\right)$$

$$T_2 = T_1 + W\left(\frac{d}{2}\right) \qquad T_2 = 15.5 \ \text{lb}\cdot\text{ft}$$

Problem 21-26

The rod AB of mass M_{AB} is attached to the collar of mass M_A at A and a link BC of mass M_{BC} using ball-and-socket joints. If the rod is released from rest in the position shown, determine the angular velocity of the link after it has rotated 180°.

Given:

$M_{AB} = 4 \text{ kg}$

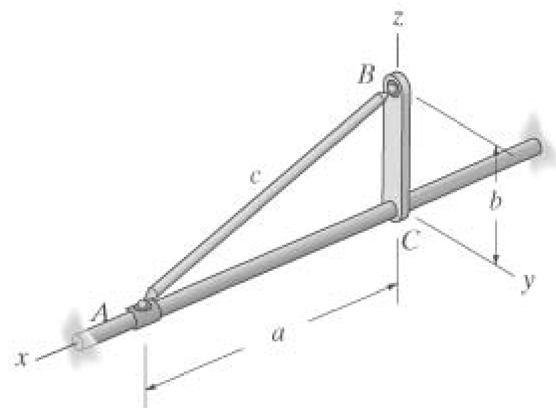
$M_A = 1 \text{ kg}$

$M_{BC} = 2 \text{ kg}$

$a = 1.2 \text{ m}$

$b = 0.5 \text{ m}$

$c = \sqrt{a^2 + b^2}$



Solution:

$\theta = \text{atan}\left(\frac{b}{a}\right)$

$I = \frac{1}{3} M_{AB} c^2 \sin(\theta)^2 + \frac{1}{3} M_{BC} b^2$

Guess $\omega = 1 \frac{\text{rad}}{\text{s}}$

Given $(M_{AB} + M_{BC}) g b = \frac{1}{2} I \omega^2$

$\omega = \text{Find}(\omega)$ $\omega = 10.85 \frac{\text{rad}}{\text{s}}$

Problem 21-27

The rod has weight density γ and is suspended from parallel cords at A and B. If the rod has angular velocity ω about the z axis at the instant shown, determine how high the center of the rod rises at the instant the rod momentarily stops swinging.

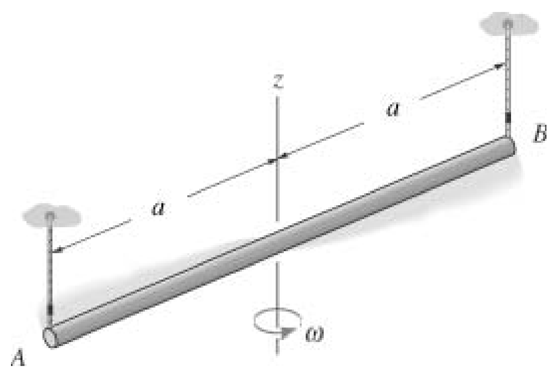
Given:

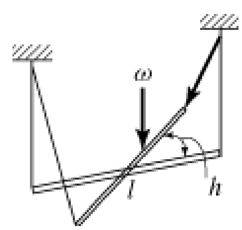
$\gamma = 3 \frac{\text{lb}}{\text{ft}}$

$\omega = 2 \frac{\text{rad}}{\text{s}}$

$a = 3 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$





Solution:

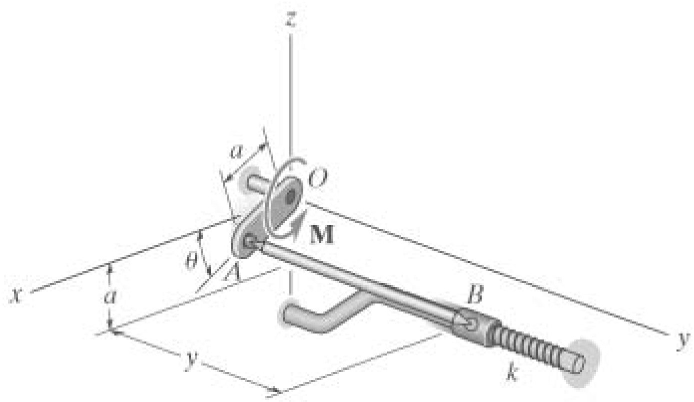
$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left[\frac{1}{12}\frac{\gamma(2a)}{g}\right](2a)^2\,\omega^2 = \gamma 2a\,h$$

$$h = \frac{1}{6}a^2\left(\frac{\omega^2}{g}\right) \qquad h = 2.24\,\text{in}$$

***Problem 21-28**

The assembly consists of a rod AB of mass m_{AB} which is connected to link OA and the collar at B by ball-and-socket joints. When $\theta = 0$ and $y = y_1$, the system is at rest, the spring is unstretched, and a couple moment M, is applied to the link at O. Determine the angular velocity of the link at the instant $\theta = 90°$. Neglect the mass of the link.



Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

$$m_{AB} = 4\ \text{kg}$$

$$M = 7\ \text{N m}$$

$$a = 200\ \text{mm}$$

$$y_1 = 600\ \text{mm}$$

$$k = 2\ \frac{\text{kN}}{\text{m}}$$

Solution:

$$L = \sqrt{a^2 + a^2 + y_1^{\,2}}$$

$$I = \frac{1}{3}m_{AB}L^2$$

$$\delta = L - y_1$$

Guess $\qquad \omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $\qquad m_{AB}\,g\,\frac{a}{2} + M(90\ \text{deg}) = \frac{1}{2}I\omega^2 + \frac{1}{2}k\delta^2$

$\omega = \text{Find}(\omega) \qquad \omega = 6.10\,\frac{\text{rad}}{\text{s}}$

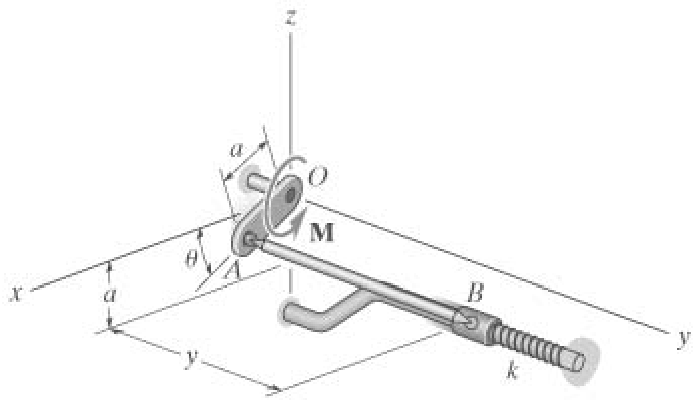
$\omega_{OA} = \omega\frac{L}{a} \qquad \omega_{OA} = 20.2\,\frac{\text{rad}}{\text{s}}$

Problem 21-29

The assembly consists of a rod AB of mass m_{AB} which is connected to link OA and the collar at B by ball-and-socket joints. When $\theta = 0$ and $y = y_1$, the system is at rest, the spring is unstretched, and a couple moment $M = M_0(b\theta + c)$, is applied to the link at O. Determine the angular velocity of the link at the instant $\theta = 90°$. Neglect the mass of the link.

Units Used: $\text{kN} = 10^3\ \text{N}$

Given:



$m_{AB} = 4\ \text{kg}$

$M_0 = 1\ \text{N m}$

$y_1 = 600\ \text{mm}$

$a = 200\ \text{mm}$

$b = 4$

$c = 2$

$k = 2\,\frac{\text{kN}}{\text{m}}$

Solution:

$L = \sqrt{a^2 + a^2 + y_1{}^2}$

$I = \frac{1}{3}m_{AB}L^2$

$\delta = L - y_1$

Guess $\omega = 1\,\frac{\text{rad}}{\text{s}}$

Given $m_{AB}\,g\,\frac{a}{2} + \int_0^{90\,\text{deg}} M_0(b\theta + c)\,d\theta = \frac{1}{2}I\omega^2 + \frac{1}{2}k\delta^2$

$\omega = \text{Find}(\omega)$

$$\omega = 5.22\ \frac{\text{rad}}{\text{s}} \qquad \omega_{OA} = \omega\frac{L}{a} \qquad \omega_{OA} = 17.3\ \frac{\text{rad}}{\text{s}}$$

Problem 21-30

The circular plate has weight W and diameter d. If it is released from rest and falls horizontally a distance h onto the hook at S, which provides a permanent connection, determine the velocity of the mass center of the plate just after the connection with the hook is made.

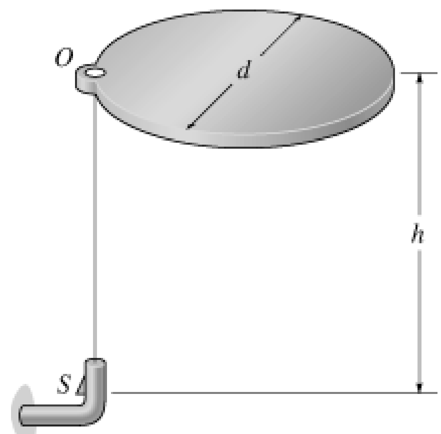
Given:

$W = 19\ \text{lb}$

$d = 1.5\ \text{ft}$

$h = 2.5\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$v_{G1} = \sqrt{2gh} \qquad v_{G1} = 12.69\ \frac{\text{ft}}{\text{s}}$$

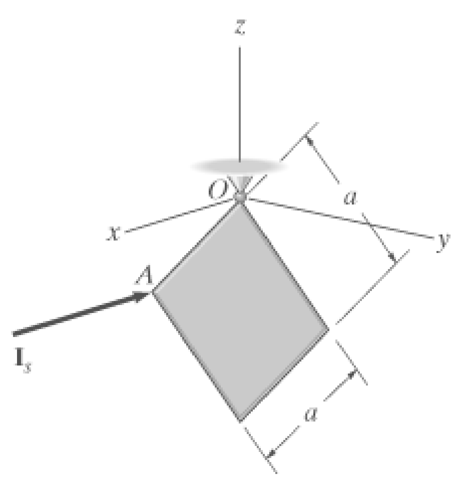
$$\left(\frac{W}{g}\right)v_{G1}\left(\frac{d}{2}\right) = \frac{5}{4}\left(\frac{W}{g}\right)\left(\frac{d}{2}\right)^2\omega_2$$

$$\omega_2 = \frac{8v_{G1}}{5d} \qquad \omega_2 = 13.53\ \frac{\text{rad}}{\text{s}}$$

$$v_{G2} = \omega_2\frac{d}{2} \qquad v_{G2} = 10.2\ \frac{\text{ft}}{\text{s}}$$

Problem 21-31

A thin plate, having mass M, is suspended from one of its corners by a ball-and-socket joint O. If a stone strikes the plate perpendicular to its surface at an adjacent corner A with an impulse $\mathbf{I_s}$, determine the instantaneous axis of rotation for the plate and the impulse created at O.



Given:

$M = 4\ \text{kg}$

$a = 200\ \text{mm}$

$$\theta = -45 \text{ deg}$$

$$\mathbf{I_s} = \begin{pmatrix} -60 \\ 0 \\ 0 \end{pmatrix} \text{N}\cdot\text{s}$$

Solution:

$$I_1 = \frac{2}{3} M a^2 \qquad I_2 = \frac{1}{3} M a^2$$

$$I_3 = I_2 \qquad I_{23} = M \frac{a^2}{4}$$

$$\mathbf{C_{mat}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

$$\mathbf{I_{mat}} = \mathbf{C_{mat}} \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & -I_{23} \\ 0 & -I_{23} & I_3 \end{pmatrix} \mathbf{C_{mat}}^T$$

Guesses $\begin{pmatrix} I_{Ox} \\ I_{Oy} \\ I_{Oz} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{N s} \qquad \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \frac{\text{rad}}{\text{s}}$

Given

$$\mathbf{I_s} + \begin{pmatrix} I_{Ox} \\ I_{Oy} \\ I_{Oz} \end{pmatrix} = M \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

$$\frac{a}{\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \times \mathbf{I_s} = \mathbf{I_{mat}} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \frac{-a}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

$$\begin{pmatrix} I_{Ox} \\ I_{Oy} \\ I_{Oz} \\ \omega_x \\ \omega_y \\ \omega_z \\ v_x \\ v_y \\ v_z \end{pmatrix} = \text{Find}\left(I_{Ox}, I_{Oy}, I_{Oz}, \omega_x, \omega_y, \omega_z, v_x, v_y, v_z\right)$$

$$\textbf{Impulse} = \begin{pmatrix} I_{Ox} \\ I_{Oy} \\ I_{Oz} \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \qquad \textbf{axis} = \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|} \qquad \textbf{Impulse} = \begin{pmatrix} 8.57 \\ 0.00 \\ 0.00 \end{pmatrix} \text{N}\cdot\text{s}$$

$$\textbf{axis} = \begin{pmatrix} 0.00 \\ 0.14 \\ -0.99 \end{pmatrix}$$

***Problem 21-32**

Rod *AB* has weight *W* and is attached to two smooth collars at its ends by ball-and-socket joints. If collar *A* is moving downward with speed v_A when $z = a$, determine the speed of *A* at the instant $z = 0$. The spring has unstretched length *c*. Neglect the mass of the collars. Assume the angular velocity of rod *AB* is perpendicular to its axis.

Given:

$W = 6\ \text{lb}$

$v_A = 8\ \dfrac{\text{ft}}{\text{s}}$

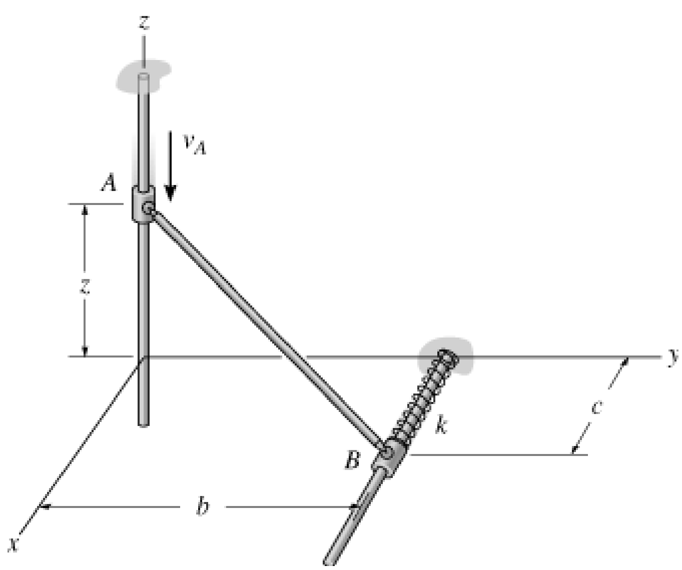
$a = 3\ \text{ft}$

$b = 6\ \text{ft}$

$c = 2\ \text{ft}$

$k = 4\ \dfrac{\text{lb}}{\text{ft}}$

$\delta = 2\ \text{ft}$



$$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

Solution: $L = \sqrt{a^2 + b^2 + c^2}$

First Position

Guesses

$$v_{B1} = 1\ \frac{\text{ft}}{\text{s}} \qquad \omega_{x1} = 1\ \frac{\text{rad}}{\text{s}}$$

$$\omega_{y1} = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_{z1} = 1\ \frac{\text{rad}}{\text{s}}$$

Given $$\begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \begin{pmatrix} \omega_{x1} \\ \omega_{y1} \\ \omega_{z1} \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = \begin{pmatrix} v_{B1} \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{x1} \\ \omega_{y1} \\ \omega_{z1} \end{pmatrix} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0\ \frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} \omega_{x1} \\ \omega_{y1} \\ \omega_{z1} \\ v_{B1} \end{pmatrix} = \text{Find}\left(\omega_{x1}, \omega_{y1}, \omega_{z1}, v_{B1}\right) \qquad \boldsymbol{\omega_1} = \begin{pmatrix} \omega_{x1} \\ \omega_{y1} \\ \omega_{z1} \end{pmatrix} \qquad \boldsymbol{\omega_1} = \begin{pmatrix} 0.98 \\ -1.06 \\ -1.47 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\mathbf{v_{G1}} = \begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \boldsymbol{\omega_1} \times \left[\frac{1}{2}\begin{pmatrix} c \\ b \\ -a \end{pmatrix}\right]$$

$$\mathbf{v_{G1}} = \begin{pmatrix} 6.00 \\ 0.00 \\ -4.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$T_1 = \frac{1}{2}\frac{W}{g}\left(\mathbf{v_{G1}}\cdot\mathbf{v_{G1}}\right) + \frac{1}{2}\frac{W}{g}\frac{L^2}{12}\left(\boldsymbol{\omega_1}\cdot\boldsymbol{\omega_1}\right) \qquad T_1 = 6.46\ \text{lb}\cdot\text{ft}$$

Work - Energy $T_1 + V_1 = T_2 + V_2$

$$T_2 = T_1 + W\frac{a}{2} - \frac{1}{2}k\left(\sqrt{L^2 - b^2} - c\right)^2 \qquad T_2 = 10.30\ \text{lb}\cdot\text{ft}$$

Second Position Note that B becomes the instantaneous center

Guesses $v_{A2} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{B2} = 1\ \frac{\text{ft}}{\text{s}}$

$\omega_{x2} = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_{y2} = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_{z2} = 1\ \frac{\text{rad}}{\text{s}}$

Given $$\begin{pmatrix} 0 \\ 0 \\ -v_{A2} \end{pmatrix} + \begin{pmatrix} \omega_{x2} \\ \omega_{y2} \\ \omega_{z2} \end{pmatrix} \times \begin{pmatrix} \sqrt{L^2 - b^2} \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} v_{B2} \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{x2} \\ \omega_{y2} \\ \omega_{z2} \end{pmatrix}\begin{pmatrix} \sqrt{L^2 - b^2} \\ b \\ 0 \end{pmatrix} = 0\ \frac{\text{ft}}{\text{s}}$$

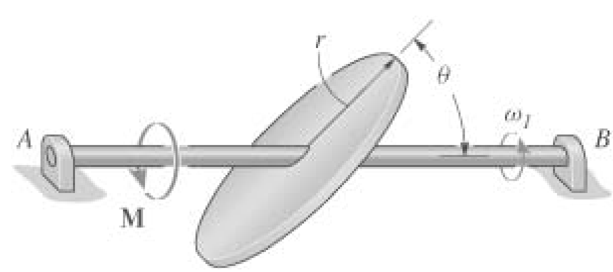
$$T_2 = \frac{1}{2}\frac{W}{g}\frac{L^2}{3}\left(\omega_{x2}^2 + \omega_{y2}^2 + \omega_{z2}^2\right)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ \omega_{x2} \\ \omega_{y2} \\ \omega_{z2} \end{pmatrix} = \text{Find}\left(v_{A2}, v_{B2}, \omega_{x2}, \omega_{y2}, \omega_{z2}\right) \qquad \begin{pmatrix} \omega_{x2} \\ \omega_{y2} \\ \omega_{z2} \end{pmatrix} = \begin{pmatrix} 2.23 \\ -1.34 \\ 0.00 \end{pmatrix}\frac{\text{rad}}{\text{s}} \qquad v_{B2} = 0.00\ \frac{\text{ft}}{\text{s}}$$

$$v_{A2} = 18.2\ \frac{\text{ft}}{\text{s}}$$

Problem 21-33

The circular disk has weight W and is mounted on the shaft AB at angle θ with the horizontal. Determine the angular velocity of the shaft when $t = t_1$ if a constant torque **M** is applied to the shaft. The shaft is originally spinning with angular velocity ω_1 when the torque is applied.



Given:

$W = 15\ \text{lb}$

$\theta = 45\ \text{deg}$

$t_1 = 3\ \text{s}$

$M = 2\ \text{lb}\cdot\text{ft}$

$\omega_1 = 8\ \frac{\text{rad}}{\text{s}}$

$r = 0.8\text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$I_{AB} = \left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right)\cos(\theta)^2 + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\sin(\theta)^2 \qquad I_{AB} = 0.11\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\alpha = \frac{M}{I_{AB}} \qquad \omega_2 = \omega_1 + \alpha t_1 \qquad \omega_2 = 61.7\ \frac{\text{rad}}{\text{s}}$$

Problem 21-34

The circular disk has weight W and is mounted on the shaft AB at angle of θ with the horizontal. Determine the angular velocity of the shaft when $t = t_1$ if a torque $\mathbf{M} = \mathbf{M_0}\, e^{bt}$ applied to the shaft. The shaft is originally spinning at ω_1 when the torque is applied.

Given:

$W = 15\ \text{lb}$

$\theta = 45\ \text{deg}$

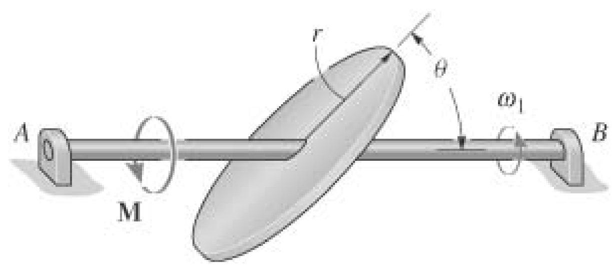
$t_1 = 2\ \text{s}$

$M_0 = 4\ \text{lb}\cdot\text{ft}$

$\omega_1 = 8\ \frac{\text{rad}}{\text{s}}$

$r = 0.8\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



$b = 0.1\ \mathrm{s}^{-1}$

Solution:

$$I_{AB} = \left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right)\cos(\theta)^2 + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\sin(\theta)^2 \qquad I_{AB} = 0.11\ \mathrm{lb\cdot ft\cdot s^2}$$

$$\omega_2 = \omega_1 + \frac{1}{I_{AB}}\left(\int_0^{t_1} M_0 e^{b\,t}\,dt\right) \qquad \omega_2 = 87.2\ \frac{\mathrm{rad}}{\mathrm{s}}$$

Problem 21-35

The rectangular plate of mass m_p is free to rotate about the y axis because of the bearing supports at A and B. When the plate is balanced in the vertical plane, a bullet of mass m_b is fired into it, perpendicular to its surface, with a velocity v. Compute the angular velocity of the plate at the instant it has rotated 180°. If the bullet strikes corner D with the same velocity v, instead of at C, does the angular velocity remain the same? Why or why not?

Given:

$m_p = 15\ \mathrm{kg}$

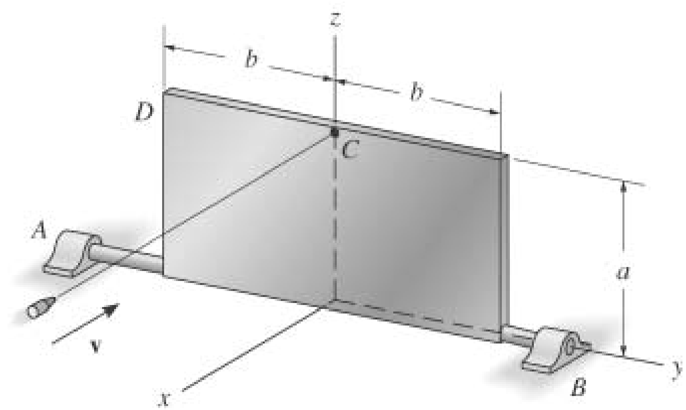
$m_b = 0.003\ \mathrm{kg}$

$v = 2000\ \frac{\mathrm{m}}{\mathrm{s}}$

$a = 150\ \mathrm{mm}$

$b = 150\ \mathrm{mm}$

$g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$



Solution: Guesses $\omega_2 = 1\ \frac{\mathrm{rad}}{\mathrm{s}}$ $\omega_3 = 1\ \frac{\mathrm{rad}}{\mathrm{s}}$

Given $m_b\,v\,a = \frac{1}{3} m_p\,a^2\,\omega_2$

$$\frac{1}{2} m_p\,\frac{a^2}{3}\,\omega_2^{\,2} + m_p\,g\,\frac{a}{2} = \frac{1}{2} m_p\,\frac{a^2}{3}\,\omega_3^{\,2} - m_p\,g\,\frac{a}{2}$$

$$\begin{pmatrix} \omega_2 \\ \omega_3 \end{pmatrix} = \text{Find}(\omega_2, \omega_3) \qquad \omega_2 = 8.00\,\frac{\text{rad}}{\text{s}} \qquad \omega_3 = 21.4\,\frac{\text{rad}}{\text{s}}$$

If the bullet strikes at *D*, the result will be the same.

***Problem 21-36**

The rod assembly has a mass density ρ and is rotating with a constant angular velocity $\omega = \omega_1\mathbf{k}$ when the loop end at *C* encounters a hook at *S*, which provides a permanent connection. Determine the angular velocity of the assembly immediately after impact.

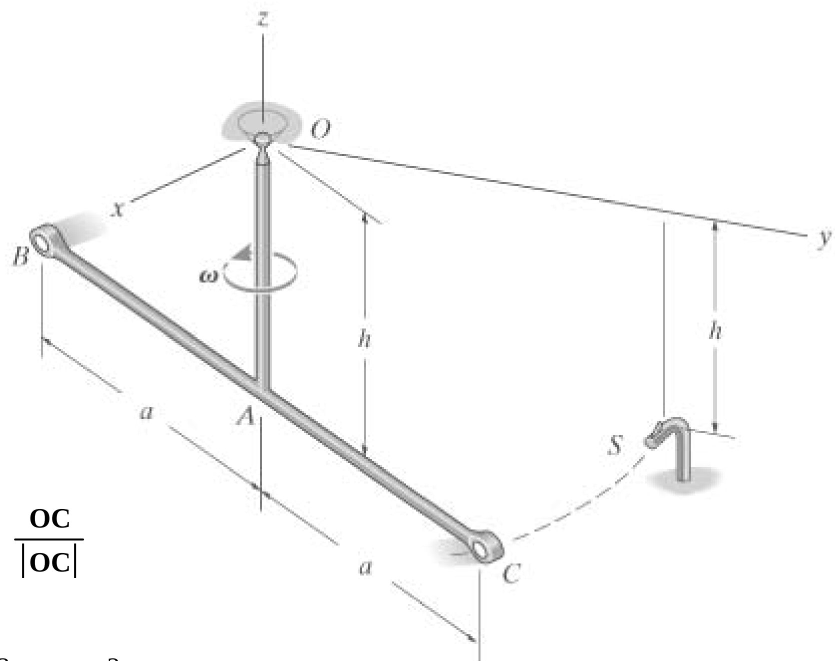
Given:

$$\rho = 2.5\,\frac{\text{kg}}{\text{m}}$$

$$a = 0.5\text{ m}$$

$$\omega_1 = 2\,\frac{\text{rad}}{\text{s}}$$

$$h = 0.5\text{ m}$$



Solution:

$$\mathbf{OC} = \begin{pmatrix} 0 \\ a \\ -h \end{pmatrix} \qquad \mathbf{oc} = \frac{\mathbf{OC}}{|\mathbf{OC}|}$$

$$I_1 = \frac{1}{3}\rho h^3 + \frac{1}{12}\rho(2a)^3 + \rho 2a\,h^2$$

$$I_2 = \frac{1}{3}\rho h^3 + \rho 2a\,h^2 \qquad I_3 = I_1 - I_2$$

$$\mathbf{I_{mat}} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix} \qquad \mathbf{I_{OC}} = \mathbf{oc}^T\,\mathbf{I_{mat}}\,\mathbf{oc}$$

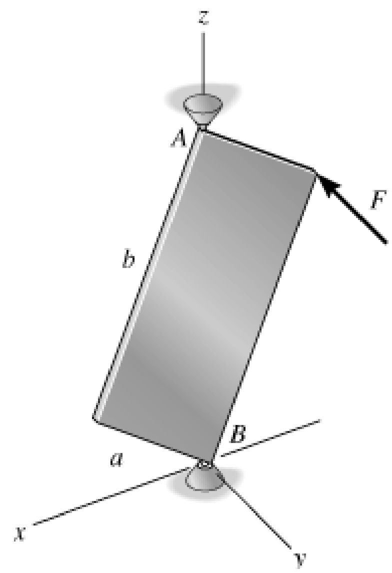
Guess $\qquad \omega_2 = 1\,\frac{\text{rad}}{\text{s}}$

Given $\quad \mathbf{I_{mat}}\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix}\mathbf{oc} = \mathbf{I_{OC}}\,\omega_2 \qquad \omega_2 = \text{Find}(\omega_2) \qquad \omega_2 = -0.63\,\dfrac{\text{rad}}{\text{s}}$

$$\omega_2\mathbf{oc} = \begin{pmatrix} 0.00 \\ -0.44 \\ 0.44 \end{pmatrix}\frac{\text{rad}}{\text{s}}$$

Problem 21-37

The plate of weight W is subjected to force F which is always directed perpendicular to the face of the plate. If the plate is originally at rest, determine its angular velocity after it has rotated one revolution (360°). The plate is supported by ball-and-socket joints at A and B.



Given:

$W = 15\ \text{lb}$

$F = 8\ \text{lb}$

$a = 0.4\ \text{ft}$

$b = 1.2\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$\theta = \text{atan}\left(\dfrac{a}{b}\right) \qquad \theta = 18.43\,\text{deg}$

$$I_{AB} = \left(\frac{W}{g}\right)\left(\frac{a^2}{12}\right)\cos(\theta)^2 + \left(\frac{W}{g}\right)\left(\frac{b^2}{12}\right)\sin(\theta)^2$$

$I_{AB} = 0.0112\,\text{lb}\cdot\text{ft}\cdot\text{s}^2$

Guess $\quad \omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $\quad F\,a\cos(\theta)(2\pi) = \dfrac{1}{2}I_{AB}\omega^2$

$\omega = \text{Find}(\omega) \qquad \omega = 58.4\ \dfrac{\text{rad}}{\text{s}}$

Problem 21-38

The space capsule has mass m_c and the radii of gyration are $k_x = k_z$ and k_y. If it is traveling with a velocity v_G, compute its angular velocity just after it is struck by a meteoroid having mass m_m and a velocity $\mathbf{v_m} = (v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k})$. Assume that the meteoroid embeds itself into the capsule at point A and that the capsule initially has no angular velocity.

Units Used:

$$\text{Mg} = 1000\ \text{kg}$$

Given:

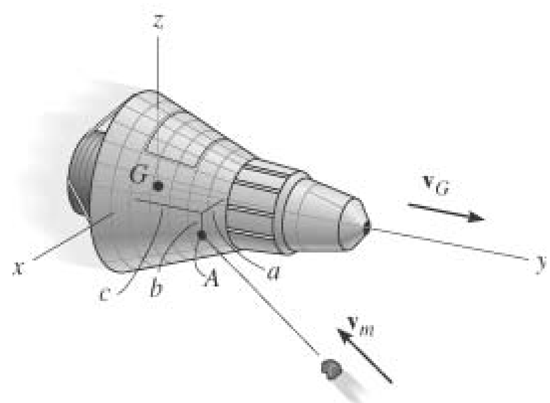
$$m_c = 3.5\ \text{Mg} \qquad m_m = 0.60\ \text{kg}$$

$$k_x = 0.8\ \text{m} \qquad v_x = -200\ \frac{\text{m}}{\text{s}}$$

$$k_y = 0.5\ \text{m} \qquad v_y = -400\ \frac{\text{m}}{\text{s}}$$

$$v_G = 600\ \frac{\text{m}}{\text{s}} \qquad v_z = 200\ \frac{\text{m}}{\text{s}}$$

$$a = 1\ \text{m} \quad b = 1\ \text{m} \quad c = 3\ \text{m}$$



Solution:

Guesses

$$\omega_x = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_y = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_z = 1\ \frac{\text{rad}}{\text{s}}$$

Given

$$\begin{pmatrix} a \\ c \\ -b \end{pmatrix} \times \left[m_m \begin{pmatrix} v_x \\ v_y - v_G \\ v_z \end{pmatrix} \right] = m_c \begin{pmatrix} k_x^2 & 0 & 0 \\ 0 & k_y^2 & 0 \\ 0 & 0 & k_x^2 \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} -0.107 \\ 0.000 \\ -0.107 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

Problem 21-39

Derive the scalar form of the rotational equation of motion along the x axis when $\mathbf{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *not constant* with respect to time.

Solution:

In general

$$\mathbf{M} = \frac{d}{dt}\left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

$$\mathbf{M} = \left(H'_x\mathbf{i} + H'_y\mathbf{j} + H'_z\mathbf{k}\right) + \mathbf{\Omega} \times \left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

Substitute $\mathbf{\Omega} = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(H'_x - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left(H'_y - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} \ \ldots$$
$$+ \left(H'_z - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Substitute H_x, H_y, and H_z using Eq. 21 - 10. For the i component

$$\Sigma M_x = \frac{d}{dt}\begin{bmatrix}\left(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right) - \Omega_z\left(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x\right) \ \ldots \\ + \Omega_y\left(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y\right)\end{bmatrix}$$

One can obtain y and z components in a similar manner.

***Problem 21-40**

Derive the scalar form of the rotational equation of motion along the x axis when $\mathbf{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *constant* with respect to time.

Solution:

In general

$$\mathbf{M} = \frac{d}{dt}\left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

$$\mathbf{M} = \left(H'_x\mathbf{i} + H'_y\mathbf{j} + H'_z\mathbf{k}\right) + \mathbf{\Omega} \times \left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

Substitute $\mathbf{\Omega} = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(H'_x - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left(H'_y - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} \ \ldots$$
$$+ \left(H'_z - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Substitute H_x, H_y, and H_z using Eq. 21 - 10. For the i component

$$\Sigma M_x = \frac{d}{dt}\begin{bmatrix}\left(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right) - \Omega_z\left(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x\right) \ \ldots \\ + \Omega_y\left(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y\right)\end{bmatrix}$$

For constant inertia, expanding the time derivative of the above equation yields

$$\Sigma M_x = \left(I_x\omega'_x - I_{xy}\omega'_y - I_{xz}\omega'_z\right) - \Omega_z\left(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x\right) \ldots$$
$$+ \Omega_y\left(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y\right)$$

One can obtain y and z components in a similar manner.

Problem 21-41

Derive the Euler equations of motion for $\mathbf{\Omega} \neq \boldsymbol{\omega}$ i.e., Eqs. 21-26.

Solution:

In general

$$\mathbf{M} = \frac{d}{dt}\left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

$$\mathbf{M} = \left(H'_x\mathbf{i} + H'_y\mathbf{j} + H'_z\mathbf{k}\right) + \mathbf{\Omega} \times \left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

Substitute $\mathbf{\Omega} = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(H'_x - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left(H'_y - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} \ldots$$
$$+ \left(H'_z - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Substitute H_x, H_y, and H_z using Eq. 21 - 10. For the i component

$$\Sigma M_x = \frac{d}{dt}\begin{bmatrix}\left(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right) - \Omega_z\left(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x\right) \ldots \\ + \Omega_y\left(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y\right)\end{bmatrix}$$

Set $I_{xy} = I_{yz} = I_{zx} = 0$ and require I_x, I_y, I_z to be constant. This yields

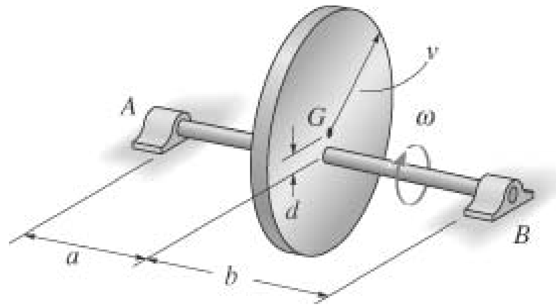
$$\Sigma M_x = I_x\omega'_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z$$

One can obtain y and z components in a similar manner.

Problem 21-42

The flywheel (disk of mass M) is mounted a distance d off its true center at G. If the shaft is rotating at constant speed ω, determine the maximum reactions exerted on the journal bearings at A and B.

Given:

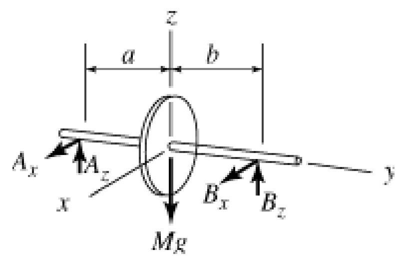


$M = 40\ \text{kg}$ $a = 0.75\ \text{m}$

$d = 20\ \text{mm}$ $b = 1.25\ \text{m}$

$\omega = 8\ \frac{\text{rad}}{\text{s}}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Check both up and down positions



Guesses $A_{up} = 1\ \text{N}$ $B_{up} = 1\ \text{N}$

Given $A_{up} + B_{up} - M g = -M d \omega^2$

$-A_{up} a + B_{up} b = 0$

$\begin{pmatrix} A_{up} \\ B_{up} \end{pmatrix} = \text{Find}(A_{up}, B_{up})$ $\begin{pmatrix} A_{up} \\ B_{up} \end{pmatrix} = \begin{pmatrix} 213.25 \\ 127.95 \end{pmatrix} \text{N}$

Guesses $A_{down} = 1\ \text{N}$ $B_{down} = 1\ \text{N}$

Given $A_{down} + B_{down} - M g = M d \omega^2$

$-A_{down} a + B_{down} b = 0$

$\begin{pmatrix} A_{down} \\ B_{down} \end{pmatrix} = \text{Find}(A_{down}, B_{down})$ $\begin{pmatrix} A_{down} \\ B_{down} \end{pmatrix} = \begin{pmatrix} 277.25 \\ 166.35 \end{pmatrix} \text{N}$

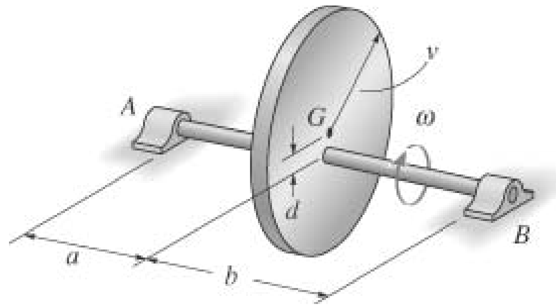
Thus $A_{max} = \max(A_{up}, A_{down})$ $A_{max} = 277\ \text{N}$

$B_{max} = \max(B_{up}, B_{down})$ $B_{max} = 166\ \text{N}$

Problem 21-43

The flywheel (disk of mass M) is mounted a distance d off its true center at G. If the shaft is rotating at constant speed ω, determine the minimum reactions exerted on the journal bearings at A and B during the motion.

Given:

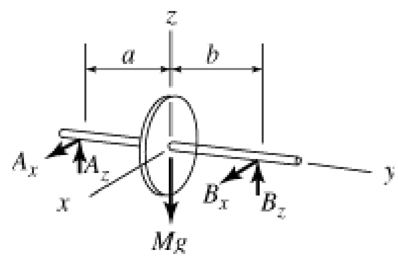


$M = 40\ \text{kg}$ $\quad a = 0.75\ \text{m}$

$d = 20\ \text{mm}$ $\quad b = 1.25\ \text{m}$

$\omega = 8\ \frac{\text{rad}}{\text{s}}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Check both up and down positions



Guesses $A_{up} = 1\ \text{N}$ $\quad B_{up} = 1\ \text{N}$

Given $A_{up} + B_{up} - Mg = -Md\omega^2$

$-A_{up}a + B_{up}b = 0$

$\begin{pmatrix} A_{up} \\ B_{up} \end{pmatrix} = \text{Find}(A_{up}, B_{up})$ $\qquad \begin{pmatrix} A_{up} \\ B_{up} \end{pmatrix} = \begin{pmatrix} 213.25 \\ 127.95 \end{pmatrix}\ \text{N}$

Guesses $A_{down} = 1\ \text{N}$ $\quad B_{down} = 1\ \text{N}$

Given $A_{down} + B_{down} - Mg = Md\omega^2$

$-A_{down}a + B_{down}b = 0$

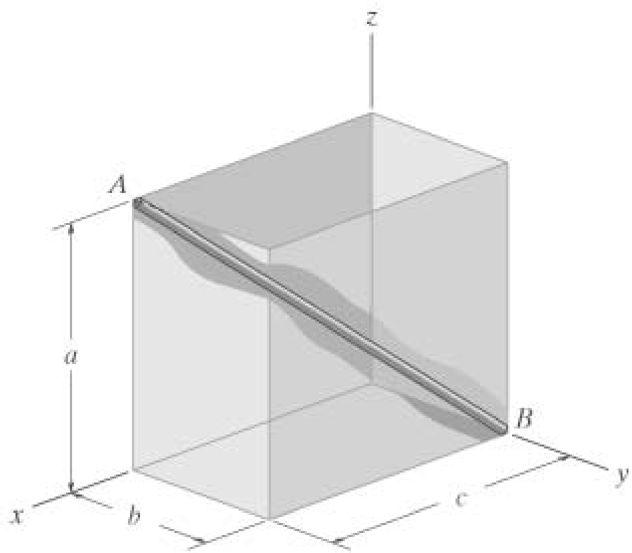
$\begin{pmatrix} A_{down} \\ B_{down} \end{pmatrix} = \text{Find}(A_{down}, B_{down})$ $\qquad \begin{pmatrix} A_{down} \\ B_{down} \end{pmatrix} = \begin{pmatrix} 277.25 \\ 166.35 \end{pmatrix}\ \text{N}$

Thus $A_{min} = \min(A_{up}, A_{down})$ $\qquad A_{min} = 213\ \text{N}$

$B_{min} = \min(B_{up}, B_{down})$ $\qquad B_{min} = 128\ \text{N}$

***Problem 21-44**

The bar of weight W rests along the smooth corners of an open box. At the instant shown, the box has a velocity $v = v_1\mathbf{k}$ and an acceleration $a = a_1\mathbf{k}$. Determine the x, y, z components of force which the corners exert on the bar.



Given:

$W = 4\text{ lb}$ $a = 2\text{ ft}$

$v_1 = 5\dfrac{\text{ft}}{\text{s}}$ $b = 1\text{ ft}$

$a_1 = 2\dfrac{\text{ft}}{\text{s}^2}$ $c = 2\text{ ft}$

Solution:

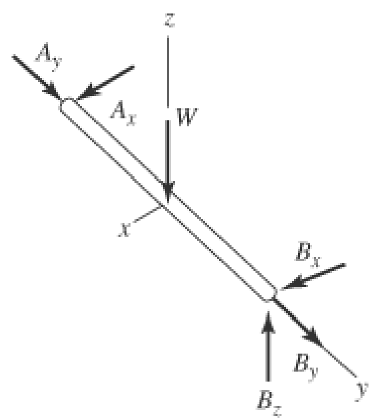
Guesses

$A_x = 1\text{ lb}$ $B_x = 1\text{ lb}$

$A_y = 1\text{ lb}$ $B_y = 1\text{ lb}$

$B_z = 1\text{ lb}$

Given



$$A_x + B_x = 0$$

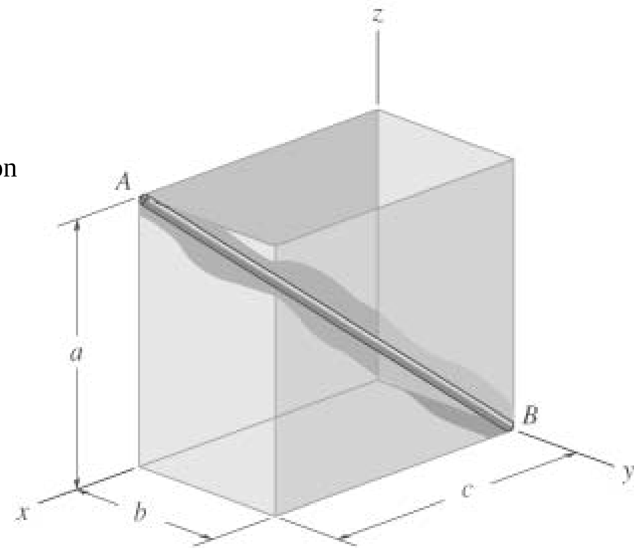
$$A_y + B_y = 0$$

$$B_z - W = \frac{W}{g}a_1$$

$$\frac{1}{2}\begin{pmatrix}-c\\ b\\ -a\end{pmatrix} \times \begin{pmatrix}B_x\\ B_y\\ B_z\end{pmatrix} + \frac{1}{2}\begin{pmatrix}c\\ -b\\ a\end{pmatrix} \times \begin{pmatrix}A_x\\ A_y\\ 0\end{pmatrix} = 0$$

$$\begin{pmatrix}A_x\\ A_y\\ B_x\\ B_y\\ B_z\end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y, B_z) \qquad \begin{pmatrix}A_x\\ A_y\end{pmatrix} = \begin{pmatrix}-2.12\\ 1.06\end{pmatrix}\text{lb} \qquad \begin{pmatrix}B_x\\ B_y\\ B_z\end{pmatrix} = \begin{pmatrix}2.12\\ -1.06\\ 4.25\end{pmatrix}\text{lb}$$

Problem 21-45



The bar of weight W rests along the smooth corners of an open box. At the instant shown, the box has a velocity $v = v_1\mathbf{j}$ and an acceleration $a = a_1\mathbf{j}$. Determine the x, y, z components of force which the corners exert on the bar.

Given:

$W = 4\text{ lb}$ $\quad a = 2\text{ ft}$

$v_1 = 3\,\dfrac{\text{ft}}{\text{s}}$ $\quad b = 1\text{ ft}$

$a_1 = -6\,\dfrac{\text{ft}}{\text{s}^2}$ $\quad c = 2\text{ ft}$

Solution:

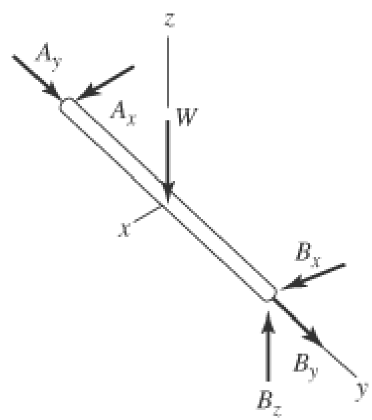
Guesses

$A_x = 1\text{ lb}$ $\quad B_x = 1\text{ lb}$

$A_y = 1\text{ lb}$ $\quad B_y = 1\text{ lb}$

$B_z = 1\text{ lb}$

Given



$$A_x + B_x = 0$$

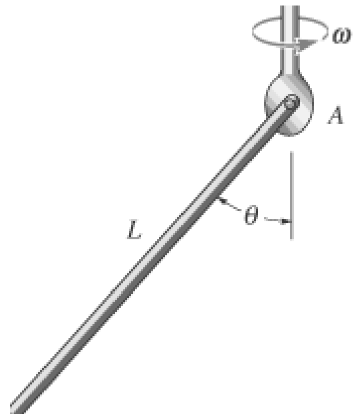
$$A_y + B_y = \frac{W}{g}a_1$$

$$B_z - W = 0$$

$$\frac{1}{2}\begin{pmatrix} -c \\ b \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + \frac{1}{2}\begin{pmatrix} c \\ -b \\ a \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \\ B_z \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y, B_z) \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} -2.00 \\ 0.63 \end{pmatrix}\text{lb} \qquad \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 2.00 \\ -1.37 \\ 4.00 \end{pmatrix}\text{lb}$$

Problem 21-46



The conical pendulum consists of a bar of mass *m* and length *L* that is supported by the pin at its end *A*. If the pin is subjected to a rotation ω, determine the angle θ that the bar makes with the vertical as it rotates.

Solution:

$$I_x = I_z = \frac{1}{3}mL^2$$

$$I_y = 0 \qquad \omega_x = 0$$

$$\omega_y = -\omega cos(\theta)$$

$$\omega_z = \omega sin(\theta)$$

$$\omega'_x = 0 \qquad \omega'_y = 0 \qquad \omega'_z = 0$$

$$\Sigma M_x = I_x\omega'_x - (I_y - I_z)\omega_y\omega_x$$

$$-mg\left(\frac{L}{2}\right)sin(\theta) = 0 - \left(0 - \frac{1}{3}mL^2\right)(-\omega cos(\theta))(\omega sin(\theta))$$

$$\frac{g}{2} = \frac{1}{3}L\omega^2 cos(\theta) \qquad \theta = acos\left(\frac{3g}{2L\omega^2}\right)$$

Problem 21-47

The plate of weight *W* is mounted on the shaft *AB* so that the plane of the plate makes an angle θ with the vertical. If the shaft is turning in the direction shown with angular velocity ω, determine the vertical reactions at the bearing supports *A* and *B* when the plate is in the position shown.

Given:

$W = 20 \text{ lb}$

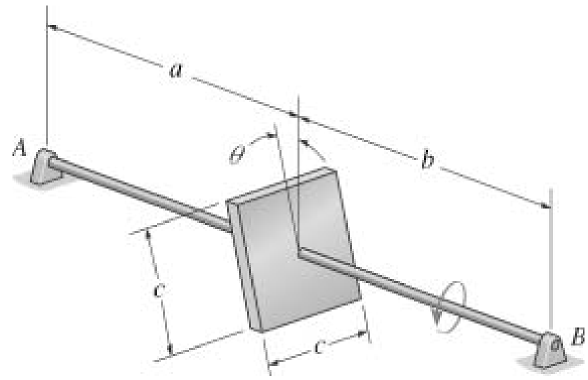
$\theta = 30 \text{ deg}$

$\omega = 25 \frac{\text{rad}}{\text{s}}$

$a = 18 \text{ in}$

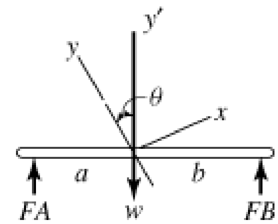
$b = 18 \text{ in}$

$c = 6 \text{ in}$



Solution:

$I_x = \left(\frac{W}{g}\right)\left(\frac{c^2}{6}\right)$



$I_z = \frac{I_x}{2} \qquad I_y = I_z$

$\omega_x = \omega \sin(\theta) \qquad \omega_y = -\omega \cos(\theta)$

$\omega_z = 0 \frac{\text{rad}}{\text{s}}$

Guesses $\quad F_A = 1 \text{ lb} \qquad F_B = 1 \text{ lb}$

Given $\quad F_A + F_B - W = 0$

$$\begin{pmatrix} 0 \\ 0 \\ F_B b - F_A a \end{pmatrix} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix}\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}\right]$$

$\begin{pmatrix} F_A \\ F_B \end{pmatrix} = \text{Find}(F_A, F_B) \qquad \begin{pmatrix} F_A \\ F_B \end{pmatrix} = \begin{pmatrix} 8.83 \\ 11.17 \end{pmatrix} \text{lb}$

***Problem 21-48**

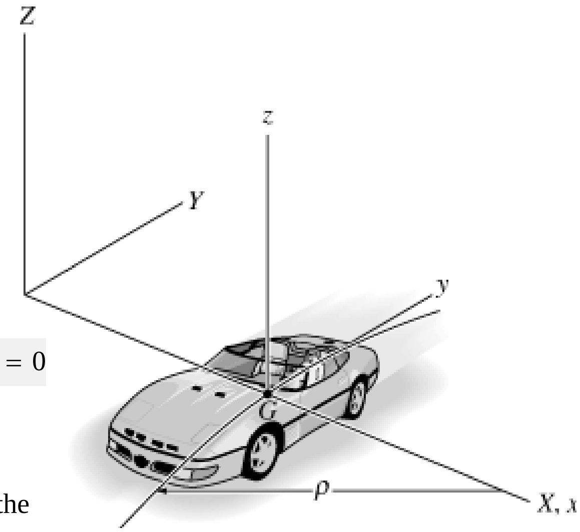
The car is traveling around the curved road of radius ρ such that its mass center has a constant speed v_G. Write the equations of rotational motion with respect to the x, y, z axes. Assume that the car's six moments and products of inertia with respect to these axes are known.

Solution:

Applying Eq. 21-24 with $\omega_x = 0 \quad \omega_y = 0$

$\omega_z = \dfrac{v_G}{\rho} \qquad \omega'_x = \omega'_y = \omega'_z = 0$

$\Sigma M_x = I_{yz}\left(\dfrac{v_G}{\rho}\right)^2 \qquad \Sigma M_y = I_{zx}\left(\dfrac{v_G}{\rho}\right)^2 \qquad \Sigma M_z = 0$



Note: This result indicates the normal reactions of the tires on the ground are not all necessarily equal. Instead, they depend upon the speed of the car, radius of curvature, and the products of inertia, I_{yz} and I_{zx}. (See Example 13-6.)

Problem 21-49

The rod assembly is supported by journal bearings at *A* and *B*, which develops only *x* and *z* force reactions on the shaft. If the shaft *AB* is rotating in the direction shown with angular velocity ω, determine the reactions at the bearings when the assembly is in the position shown. Also, what is the shaft's angular acceleration? The mass density of each rod is ρ.

Given:

$\omega = -5 \dfrac{\text{rad}}{\text{s}}$

$\rho = 1.5 \dfrac{\text{kg}}{\text{m}}$

$a = 500 \text{ mm}$

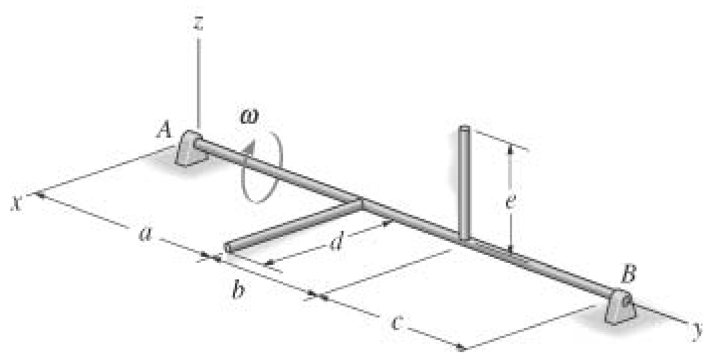
$b = 300 \text{ mm}$

$c = 500 \text{ mm}$

$d = 400 \text{ mm}$

$e = 300 \text{ mm}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$



Solution:

$$I_{xx} = \rho(a+b+c)\frac{(a+b+c)^2}{3} + \rho d a^2 + \rho e \frac{e^2}{12} + \rho e\left[(a+b)^2 + \left(\frac{e}{2}\right)^2\right]$$

$$I_{zz} = \rho(a+b+c)\frac{(a+b+c)^2}{3} + \rho d \frac{d^2}{12} + \rho d\left[a^2 + \left(\frac{d}{2}\right)^2\right] + \rho e(a+b)^2$$

$$I_{yy} = \rho d \frac{d^2}{3} + \rho e \frac{e^2}{3} \qquad I_{xy} = \rho d a \frac{d}{2} \qquad I_{yz} = \rho e(a+b)\frac{e}{2}$$

$$\mathbf{I_{mat}} = \begin{pmatrix} I_{xx} & -I_{xy} & 0 \\ -I_{xy} & I_{yy} & -I_{yz} \\ 0 & -I_{yz} & I_{zz} \end{pmatrix} \qquad \mathbf{I_{mat}} = \begin{pmatrix} 1.5500 & -0.0600 & 0.0000 \\ -0.0600 & 0.0455 & -0.0540 \\ 0.0000 & -0.0540 & 1.5685 \end{pmatrix} \text{kg}\cdot\text{m}^2$$

Guesses $A_x = 1\text{ N}$ $A_z = 1\text{ N}$ $B_x = 1\text{ N}$ $B_z = 1\text{ N}$ $\omega'_y = 1\frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} - \begin{bmatrix} 0 \\ 0 \\ \rho(a+b+c+d+e)g \end{bmatrix} = \begin{pmatrix} -\rho d \frac{d}{2}\omega^2 + \rho e \frac{e}{2}\omega'_y \\ 0 \\ -\rho d \frac{d}{2}\omega'_y - \rho e \frac{e}{2}\omega^2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ \frac{a+b+c}{2} \\ 0 \end{pmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -\rho(a+b+c)g \end{bmatrix} + \begin{pmatrix} \frac{d}{2} \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho d g \end{pmatrix} \ldots = \mathbf{I_{mat}}\begin{pmatrix} 0 \\ \omega'_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \times \left[\mathbf{I_{mat}}\begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix}\right]$$

$$+ \begin{pmatrix} 0 \\ a+b \\ \frac{e}{2} \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho e g \end{pmatrix} + \begin{pmatrix} 0 \\ a+b+c \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix}$$

$$\begin{pmatrix} A_x \\ A_z \\ B_x \\ B_z \\ \omega'_y \end{pmatrix} = \text{Find}(A_x, A_z, B_x, B_z, \omega'_y) \qquad \begin{pmatrix} A_x \\ A_z \end{pmatrix} = \begin{pmatrix} -1.17 \\ 12.33 \end{pmatrix}\text{N} \qquad \begin{pmatrix} B_x \\ B_z \end{pmatrix} = \begin{pmatrix} -0.0791 \\ 12.3126 \end{pmatrix}\text{N}$$

$$\omega'_y = 25.9\frac{\text{rad}}{\text{s}^2}$$

Problem 21-50

The rod assembly is supported by journal bearings at *A* and *B*, which develops only *x* and *z* force reactions on the shaft. If the shaft *AB* is subjected to a couple moment M_0 **j** and at the instant shown the shaft has an angular velocity ω **j**, determine the reactions at the bearings when the assembly is in the position shown. Also, what is the shaft's angular acceleration? The mass density of each rod is ρ.

Given:

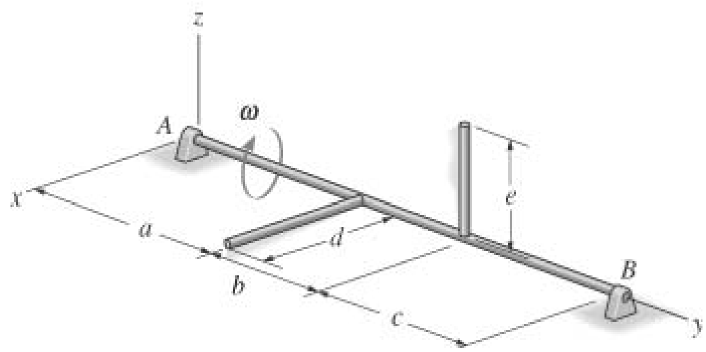
$\omega = -5 \frac{\text{rad}}{\text{s}}$ $c = 500 \text{ mm}$

$\rho = 1.5 \frac{\text{kg}}{\text{m}}$ $d = 400 \text{ mm}$

$a = 500 \text{ mm}$ $e = 300 \text{ mm}$

$b = 300 \text{ mm}$ $g = 9.81 \frac{\text{m}}{\text{s}^2}$

$M_0 = 8 \text{ N}\cdot\text{m}$



Solution:

$$I_{xx} = \rho(a+b+c)\frac{(a+b+c)^2}{3} + \rho d a^2 + \rho e\frac{e^2}{12} + \rho e\left[(a+b)^2 + \left(\frac{e}{2}\right)^2\right]$$

$$I_{zz} = \rho(a+b+c)\frac{(a+b+c)^2}{3} + \rho d\frac{d^2}{12} + \rho d\left[a^2 + \left(\frac{d}{2}\right)^2\right] + \rho e(a+b)^2$$

$$I_{yy} = \rho d\frac{d^2}{3} + \rho e\frac{e^2}{3} \qquad I_{xy} = \rho d a\frac{d}{2} \qquad I_{yz} = \rho e(a+b)\frac{e}{2}$$

$$\mathbf{I_{mat}} = \begin{pmatrix} I_{xx} & -I_{xy} & 0 \\ -I_{xy} & I_{yy} & -I_{yz} \\ 0 & -I_{yz} & I_{zz} \end{pmatrix} \qquad \mathbf{I_{mat}} = \begin{pmatrix} 1.5500 & -0.0600 & 0.0000 \\ -0.0600 & 0.0455 & -0.0540 \\ 0.0000 & -0.0540 & 1.5685 \end{pmatrix} \text{kg}\cdot\text{m}^2$$

Guesses $A_x = 1 \text{ N}$ $A_z = 1 \text{ N}$ $B_x = 1 \text{ N}$ $B_z = 1 \text{ N}$ $\omega'_y = 1 \frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} - \begin{bmatrix} 0 \\ 0 \\ \rho(a+b+c+d+e)g \end{bmatrix} = \begin{pmatrix} -\rho\, d \frac{d}{2}\omega^2 + \rho e \frac{e}{2}\omega'_y \\ 0 \\ -\rho\, d \frac{d}{2}\omega'_y - \rho e \frac{e}{2}\omega^2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ \dfrac{a+b+c}{2} \\ 0 \end{pmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -\rho(a+b+c)g \end{bmatrix} + \begin{pmatrix} \frac{d}{2} \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho\, d\, g \end{pmatrix} \ldots = \mathbf{I_{mat}} \begin{pmatrix} 0 \\ \omega'_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \times \left[\mathbf{I_{mat}} \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \right]$$

$$+ \begin{pmatrix} 0 \\ a+b \\ \frac{e}{2} \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho\, e\, g \end{pmatrix} + \begin{pmatrix} 0 \\ a+b+c \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} A_x \\ A_z \\ B_x \\ B_z \\ \omega'_y \end{pmatrix} = \text{Find}(A_x, A_z, B_x, B_z, \omega'_y) \qquad \begin{pmatrix} A_x \\ A_z \end{pmatrix} = \begin{pmatrix} 3.39 \\ -0.66 \end{pmatrix} \text{N} \qquad \begin{pmatrix} B_x \\ B_z \end{pmatrix} = \begin{pmatrix} 7.2243 \\ 4.1977 \end{pmatrix} \text{N}$$

$$\omega'_y = 201.7 \frac{\text{rad}}{\text{s}^2}$$

Problem 21-51

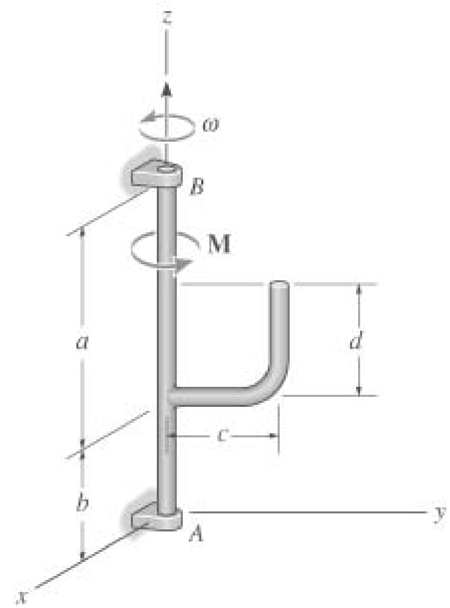
The rod assembly has a weight density *r*. It is supported at *B* by a smooth journal bearing, which develops *x* and *y* force reactions, and at *A* by a smooth thrust bearing, which develops *x*, *y*, and *z* force reactions. If torque **M** is applied along rod *AB*, determine the components of reaction at the bearings when the assembly has angular velocity ω at the instant shown.

Given:

$\gamma = 5\ \frac{\text{lb}}{\text{ft}}$ $\quad a = 4\ \text{ft}$ $\quad d = 2\ \text{ft}$

$M = 50\ \text{lb}\cdot\text{ft}$ $\quad b = 2\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$\omega = 10\ \frac{\text{rad}}{\text{s}}$ $\quad c = 2\ \text{ft}$



Solution:

$$\rho = \frac{\gamma}{g}$$

$$I_{yz} = \rho c b \frac{c}{2} + \rho d c\left(b + \frac{d}{2}\right)$$

$$I_{zz} = \rho c \frac{c^2}{3} + \rho d c^2$$

$$I_{xx} = \rho(a+b)\frac{(a+b)^2}{3} + \rho c \frac{c^2}{12} + \rho c\left[b^2 + \left(\frac{c}{2}\right)^2\right] + \rho d \frac{d^2}{12} + \rho d\left[c^2 + \left(b + \frac{d}{2}\right)^2\right]$$

$$I_{yy} = \rho(a+b)\frac{(a+b)^2}{3} + \rho c b^2 + \rho d \frac{d^2}{12} + \rho d\left(b + \frac{d}{2}\right)^2$$

$$\mathbf{I_{mat}} = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & -I_{yz} \\ 0 & -I_{yz} & I_{zz} \end{pmatrix} \qquad \mathbf{I_{mat}} = \begin{pmatrix} 16.98 & 0.00 & 0.00 \\ 0.00 & 15.32 & -2.48 \\ 0.00 & -2.48 & 1.66 \end{pmatrix} \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Guesses $\quad A_x = 1\ \text{lb}$ $\quad A_y = 1\ \text{lb}$ $\quad A_z = 1\ \text{lb}$ $\quad B_x = 1\ \text{lb}$ $\quad B_y = 1\ \text{lb}$ $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} - \begin{bmatrix} 0 \\ 0 \\ \rho(a+b+c+d)g \end{bmatrix} = \begin{pmatrix} -\rho c \frac{c}{2}\alpha - \rho d c \alpha \\ -\rho c \frac{c}{2}\omega^2 - \rho d c \omega^2 \\ 0 \end{pmatrix}$$

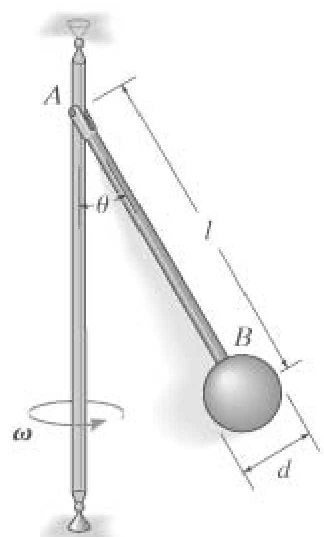
$$\begin{pmatrix} 0 \\ 0 \\ a+b \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{c}{2} \\ b \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho c g \end{pmatrix} \ldots = \mathbf{I_{mat}} \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\mathbf{I_{mat}} \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \right]$$
$$+ \begin{pmatrix} 0 \\ c \\ b + \frac{d}{2} \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho d g \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ M \end{pmatrix}$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ B_x \\ B_y \\ \alpha \end{pmatrix} = \mathrm{Find}(A_x, A_y, A_z, B_x, B_y, \alpha) \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} -15.6 \\ -46.8 \\ 50.0 \end{pmatrix} \mathrm{lb} \qquad \begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} -12.5 \\ -46.4 \end{pmatrix} \mathrm{lb}$$

$$\alpha = 30.19 \frac{\mathrm{rad}}{\mathrm{s}^2}$$

***Problem 21-52**

The rod *AB* supports the sphere of weight *W*. If the rod is pinned at *A* to the vertical shaft which is rotating at a constant rate $\omega\,\mathbf{k}$, determine the angle θ of the rod during the motion. Neglect the mass of the rod in the calculation.



Given:

$$W = 10 \text{ lb}$$
$$\omega = 7 \frac{\mathrm{rad}}{\mathrm{s}}$$
$$d = 0.5 \text{ ft}$$
$$l = 2 \text{ ft}$$
$$g = 32.2 \frac{\mathrm{ft}}{\mathrm{s}^2}$$

Solution:

$$I_3 = \frac{2}{5}\left(\frac{W}{g}\right)\left(\frac{d}{2}\right)^2$$

$$I_1 = I_3 + \left(\frac{W}{g}\right) l^2$$

Guess $\theta = 50\ \text{deg}$

Given $W l \sin(\theta) = -(I_3 - I_1)\,\omega\cos(\theta)\omega\sin(\theta)$

$\theta = \text{Find}(\theta)$ $\theta = 70.8\,\text{deg}$

Problem 21-53

The rod *AB* supports the sphere of weight *W*. If the rod is pinned at *A* to the vertical shaft which is rotating with angular acceleration $\alpha\,\mathbf{k}$, and at the instant shown the shaft has an angular velocity $\omega\mathbf{k}$, determine the angle θ of the rod during the motion. Neglect the mass of the rod in the calculation.

Given:

$W = 10\ \text{lb}$

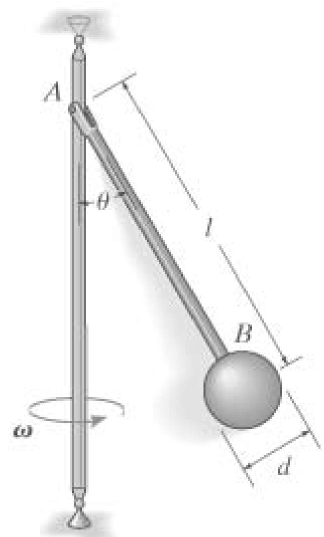
$\alpha = 2\ \dfrac{\text{rad}}{\text{s}^2}$

$\omega = 7\ \dfrac{\text{rad}}{\text{s}}$

$d = 0.5\ \text{ft}$

$l = 2\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$$I_3 = \frac{2}{5}\left(\frac{W}{g}\right)\left(\frac{d}{2}\right)^2$$

$$I_1 = I_3 + \left(\frac{W}{g}\right) l^2$$

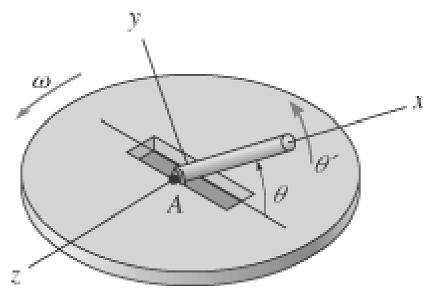
Guess $\theta = 50\ \text{deg}$

Given $W l \sin(\theta) = -(I_3 - I_1)\,\omega\cos(\theta)\omega\sin(\theta)$

$\theta = \text{Find}(\theta)$ $\theta = 70.8\,\text{deg}$

Problem 21-54

The *thin* rod has mass m_{rod} and total length L. Only half of the rod is visible in the figure. It is rotating about its midpoint at a constant rate θ', while the table to which its axle A is fastened is rotating at angular velocity ω. Determine the x, y, z moment components which the axle exerts on the rod when the rod is in position θ.



Given:

$$m_{rod} = 0.8\ \text{kg}$$

$$L = 150\ \text{mm}$$

$$\theta' = 6\ \frac{\text{rad}}{\text{s}}$$

$$\omega = 2\ \frac{\text{rad}}{\text{s}}$$

Solution:

$$I_A = m_{rod}\frac{L^2}{12}$$

$$\boldsymbol{\omega}_\mathbf{v} = \begin{pmatrix} \omega\sin(\theta) \\ \omega\cos(\theta) \\ \theta' \end{pmatrix}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} \omega\sin(\theta) \\ \omega\cos(\theta) \\ 0 \end{pmatrix} \times \boldsymbol{\omega}_\mathbf{v} = \begin{pmatrix} \omega\,\theta'\cos(\theta) \\ -\omega\,\theta'\sin(\theta) \\ 0 \end{pmatrix}$$

$$\mathbf{M} = \mathbf{I_{mat}}\boldsymbol{\alpha} + \boldsymbol{\omega}_\mathbf{v} \times \left(\mathbf{I_{mat}}\boldsymbol{\omega}_\mathbf{v}\right)$$

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & I_A & 0 \\ 0 & 0 & I_A \end{pmatrix} \times \begin{pmatrix} \omega\,\theta'\cos(\theta) \\ -\omega\,\theta'\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} \omega\sin(\theta) \\ \omega\cos(\theta) \\ \theta' \end{pmatrix} \times \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & I_A & 0 \\ 0 & 0 & I_A \end{pmatrix}\begin{pmatrix} \omega\sin(\theta) \\ \omega\cos(\theta) \\ \theta' \end{pmatrix}\right]$$

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0 \\ -2I_A\theta'\omega\sin(\theta) \\ \frac{1}{2}I_A\omega^2\sin(2\theta) \end{pmatrix} \qquad \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix} = \begin{pmatrix} 0 \\ -2I_A\theta'\omega \\ \frac{1}{2}I_A\omega^2 \end{pmatrix}$$

$M_x = 0$

$M_y = k_y \sin(\theta)$ $\qquad k_y = -0.036\ \text{N}\cdot\text{m}$

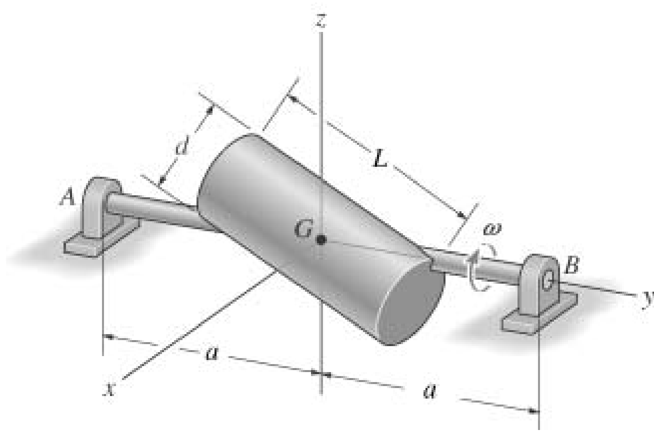
$M_z = k_z \sin(2\ \theta)$ $\qquad k_z = 0.0030\ \text{N}\cdot\text{m}$

Problem 21-55

The cylinder has mass m_c and is mounted on an axle that is supported by bearings at A and B. If the axle is turning at $\omega\,\mathbf{j}$, determine the vertical components of force acting at the bearings at this instant.

Units Used:

$\text{kN} = 10^3\ \text{N}$



Given:

$m_C = 30\ \text{kg}$

$a = 1\ \text{m}$

$\omega = -40\ \dfrac{\text{rad}}{\text{s}}$

$d = 0.5\ \text{m}$

$L = 1.5\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$$\theta = \text{atan}\left(\frac{d}{L}\right)$$

$$I_{x'} = m_C \frac{L^2}{12} + \frac{m_C}{4}\left(\frac{d}{2}\right)^2 \qquad I_{z'} = I_{x'} \qquad I_{y'} = \frac{m_C}{2}\left(\frac{d}{2}\right)^2$$

$$\mathbf{I_G} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} I_{x'} & 0 & 0 \\ 0 & I_{y'} & 0 \\ 0 & 0 & I_{z'} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

Guesses $A_x = 1\ \text{N}$ $A_z = 1\ \text{N}$ $B_x = 1\ \text{N}$ $B_z = 1\ \text{N}$

Given

$$\begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ m_c g \end{pmatrix} = 0 \qquad \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \times \begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} = \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \times \left[\mathbf{I_G} \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_z \\ B_x \\ B_z \end{pmatrix} = \text{Find}(A_x, A_z, B_x, B_z) \qquad \begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix} \text{N} \qquad \begin{pmatrix} A_z \\ B_z \end{pmatrix} = \begin{pmatrix} 1.38 \\ -1.09 \end{pmatrix} \text{kN}$$

***Problem 21-56**

The cylinder has mass m_c and is mounted on an axle that is supported by bearings at A and B. If the axle is subjected to a couple moment $M\,\mathbf{j}$ and at the instant shown has an angular velocity $\omega\,\mathbf{j}$, determine the vertical components of force acting at the bearings at this instant.

Units Used: $\text{kN} = 10^3\ \text{N}$

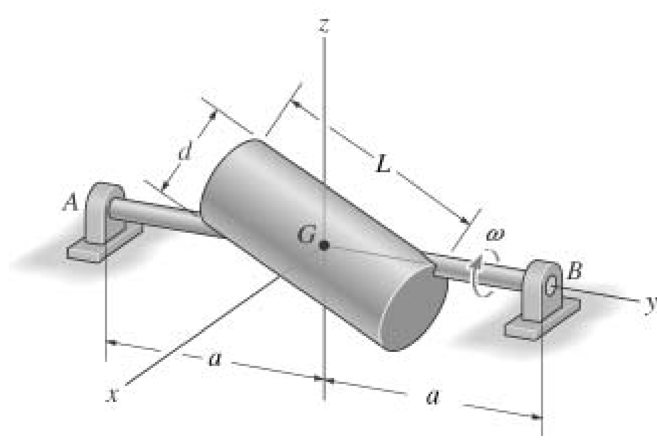
Given:

$m_c = 30\ \text{kg}$ $d = 0.5\ \text{m}$

$a = 1\ \text{m}$ $L = 1.5\ \text{m}$

$\omega = -40\ \dfrac{\text{rad}}{\text{s}}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$M = -30\ \text{N}\cdot\text{m}$



Solution:

$$\theta = \text{atan}\left(\frac{d}{L}\right)$$

$$I_{x'} = m_c \frac{L^2}{12} + \frac{m_c}{4}\left(\frac{d}{2}\right)^2$$

$$I_{z'} = I_{x'} \qquad I_{y'} = \frac{m_c}{2}\left(\frac{d}{2}\right)^2$$

$$\mathbf{I_G} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} I_{x'} & 0 & 0 \\ 0 & I_{y'} & 0 \\ 0 & 0 & I_{z'} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

Guesses $A_x = 1\ \text{N}$ $A_z = 1\ \text{N}$ $B_x = 1\ \text{N}$ $B_z = 1\ \text{N}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ m_c g \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ M \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \times \begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} = \mathbf{I_G} \begin{pmatrix} 0 \\ \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \times \left[\mathbf{I_G} \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_z \\ B_x \\ B_z \\ \alpha \end{pmatrix} = \text{Find}(A_x, A_z, B_x, B_z, \alpha)$$

$$\begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} 15.97 \\ -15.97 \end{pmatrix} \text{N} \qquad \begin{pmatrix} A_z \\ B_z \end{pmatrix} = \begin{pmatrix} 1.38 \\ -1.09 \end{pmatrix} \text{kN}$$

$$\alpha = -20.65\ \frac{\text{rad}}{\text{s}^2}$$

Problem 21-57

The uniform hatch door, having mass M and mass center G, is supported in the horizontal plane by bearings at A and B. If a vertical force **F** is applied to the door as shown, determine the components of reaction at the bearings and the angular acceleration of the door. The bearing at A will resist a component of force in the y direction, whereas the bearing at B will not. For the calculation, assume the door to be a thin plate and neglect the size of each bearing. The door is originally at rest.

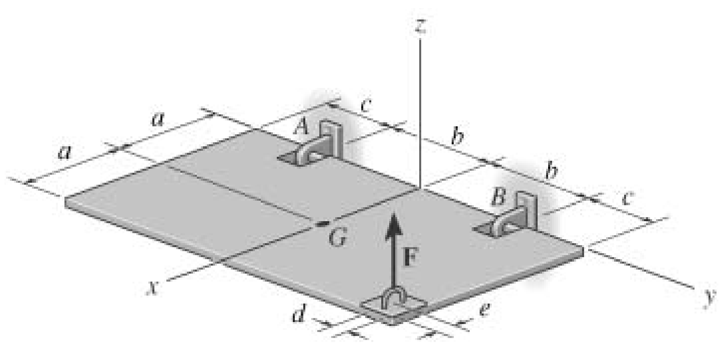
Given:

$M = 15\ \text{kg}$ $c = 100\ \text{mm}$

$F = 300\ \text{N}$ $d = 30\ \text{mm}$

$a = 200\ \text{mm}$ $e = 30\ \text{mm}$

$b = 150\ \text{mm}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: Guesses

$A_x = 1\text{ N} \quad A_y = 1\text{ N} \quad A_z = 1\text{ N} \quad B_x = 1\text{ N} \quad B_z = 1\text{ N} \quad \omega'_y = 1\ \frac{\text{rad}}{\text{s}^2}$

Given
$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ F - Mg \end{pmatrix} = M\begin{pmatrix} 0 \\ 0 \\ -\omega'_y a \end{pmatrix}$$

$$\begin{pmatrix} -a \\ b \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} -a \\ -b \\ 0 \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} a - e \\ b + c - d \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ F \end{pmatrix} = \begin{bmatrix} 0 \\ \frac{M(2a)^2}{12}\omega'_y \\ 0 \end{bmatrix}$$

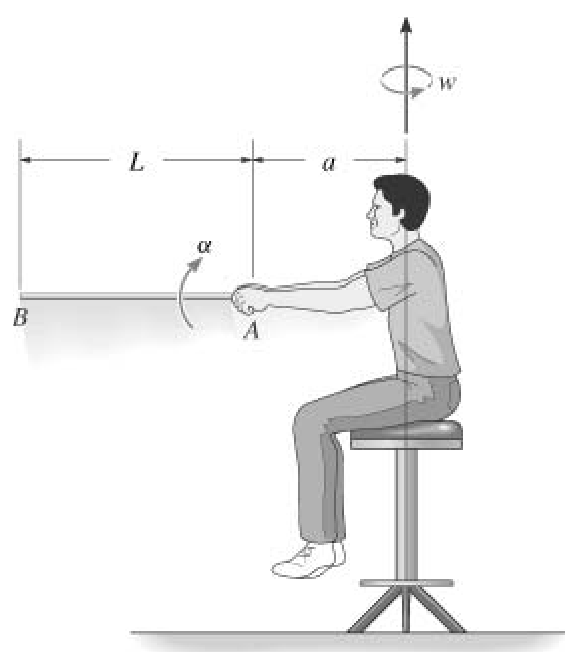
$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ B_x \\ B_z \\ \omega'_y \end{pmatrix} = \text{Find}(A_x, A_y, A_z, B_x, B_z, \omega'_y)$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 297 \end{pmatrix}\text{ N} \qquad \begin{pmatrix} B_x \\ B_z \end{pmatrix} = \begin{pmatrix} 0 \\ -143 \end{pmatrix}\text{ N}$$

$$\omega'_y = -102\ \frac{\text{rad}}{\text{s}^2}$$

Problem 21-58

The man sits on a swivel chair which is rotating with constant angular velocity ω. He holds the uniform rod AB of weight W horizontal. He suddenly gives it an angular acceleration α measured relative to him, as shown. Determine the required force and moment components at the grip, A, necessary to do this. Establish axes at the rod's center of mass G, with $+z$ upward, and $+y$ directed along the axis of the rod towards A.



Given:

$\omega = 3\ \frac{\text{rad}}{\text{s}}$

$W = 5\text{ lb}$

$L = 3\text{ ft}$

$a = 2\ \text{ft}$

$\alpha = 2\ \frac{\text{rad}}{\text{s}^2}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution: $I_G = \frac{W}{g}\frac{L^2}{12}$

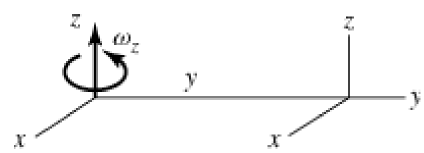
Guesses

$A_x = 1\ \text{lb}$ $A_y = 1\ \text{lb}$ $A_z = 1\ \text{lb}$

$M_x = 1\ \text{lb}\cdot\text{ft}$ $M_y = 1\ \text{lb}\cdot\text{ft}$ $M_z = 1\ \text{lb}\cdot\text{ft}$

Given

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -W \end{pmatrix} = \frac{W}{g}\begin{bmatrix} 0 \\ \left(a + \frac{L}{2}\right)\omega^2 \\ \frac{L}{2}\alpha \end{bmatrix}$$



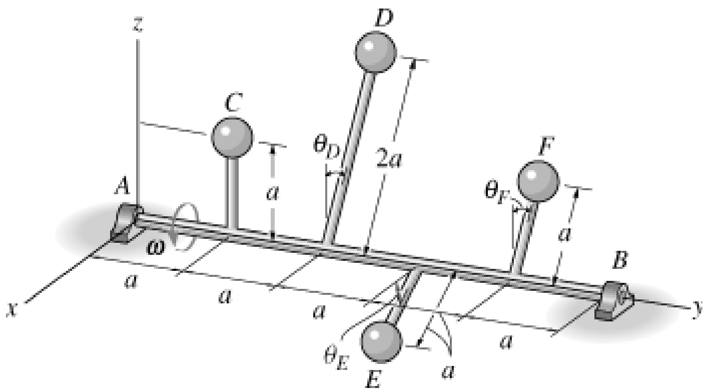
$$\begin{pmatrix} 0 \\ \frac{L}{2} \\ 0 \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} I_G & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_G \end{pmatrix}\begin{pmatrix} -\alpha \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} I_G & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_G \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}\right]$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ M_x \\ M_y \\ M_z \end{pmatrix} = \text{Find}(A_x, A_y, A_z, M_x, M_y, M_z) \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 0.00 \\ 4.89 \\ 5.47 \end{pmatrix} \text{lb} \qquad \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} -8.43 \\ 0.00 \\ 0.00 \end{pmatrix} \text{lb}\cdot\text{ft}$$

Problem 21-59

Four spheres are connected to shaft *AB*. If you know m_C and m_E, determine the mass of *D* and *F* and the angles of the rods, θ_D and θ_F so that the shaft is dynamically balanced, that is, so that the bearings at *A* and *B* exert only vertical reactions on the shaft as it rotates. Neglect the mass of the rods.

Given:



$m_C = 1\ \text{kg}$

$m_E = 2\ \text{kg}$

$\theta_E = 30\ \text{deg}$

$a = 0.1\ \text{m}$

Solution: We need to put the center of mass along *AB* and to make the product of inetia go to zero.

Guesses $m_D = 1\ \text{kg}$ $m_F = 1\ \text{kg}$ $\theta_D = 40\ \text{deg}$ $\theta_F = 10\ \text{deg}$

Given

$$m_E\, a \cos(\theta_E) - m_D(2a)\sin(\theta_D) - m_F\, a \sin(\theta_F) = 0$$

$$m_C\, a + m_D(2a)\cos(\theta_D) - m_E\, a \sin(\theta_E) + m_F\, a\cos(\theta_F) = 0$$

$$m_C\, a\, a + m_D(2a)(2a)\cos(\theta_D) - m_E(3a)a\sin(\theta_E) + m_F(4a)a\cos(\theta_F) = 0$$

$$-m_D(2a)(2a)\sin(\theta_D) + m_E(3a)a\cos(\theta_E) - m_F(4a)a\sin(\theta_F) = 0$$

$$\begin{pmatrix} m_D \\ m_F \\ \theta_D \\ \theta_F \end{pmatrix} = \text{Find}(m_D, m_F, \theta_D, \theta_F) \qquad \begin{pmatrix} m_D \\ m_F \end{pmatrix} = \begin{pmatrix} 0.661 \\ 1.323 \end{pmatrix} \text{kg} \qquad \begin{pmatrix} \theta_D \\ \theta_F \end{pmatrix} = \begin{pmatrix} 139.1 \\ 40.9 \end{pmatrix} \text{deg}$$

***Problem 21-60**

The bent uniform rod *ACD* has a weight density γ, and is supported at *A* by a pin and at *B* by a cord. If the vertical shaft rotates with a constant angular velocity ω, determine the *x*, *y*, *z* components of force and moment developed at *A* and the tension of the cord.

Given:

$\gamma = 5\ \dfrac{\text{lb}}{\text{ft}}$

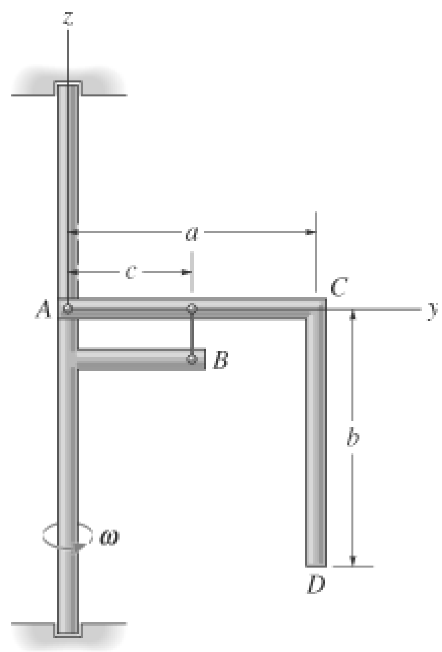
$a = 1\ \text{ft}$

$b = 1\ \text{ft}$

$c = 0.5\ \text{ft}$

$\omega = 20\ \frac{\text{rad}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: $\rho = \frac{\gamma}{g}$

$$I_{xx} = \rho a\left(\frac{a^2}{3}\right) + \rho b\left(\frac{b^2}{12}\right) + \rho b\left[a^2 + \left(\frac{b}{2}\right)^2\right]$$

$$I_{yy} = \rho b\left(\frac{b^2}{3}\right)$$

$$I_{zz} = \rho a\left(\frac{a^2}{3}\right) + \rho b\, a^2$$

$$I_{yz} = -\rho\, b\, a\frac{b}{2}$$

$$\mathbf{I_A} = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & -I_{yz} \\ 0 & -I_{yz} & I_{zz} \end{pmatrix}$$

Guesses $M_y = 1\ \text{lb}\cdot\text{ft}$ $A_x = 1\ \text{lb}$ $A_z = 1\ \text{lb}$

$M_z = 1\ \text{lb}\cdot\text{ft}$ $A_y = 1\ \text{lb}$ $T = 1\ \text{lb}$

Given

$$\begin{pmatrix} A_x \\ A_y \\ A_z - T \end{pmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\gamma(a+b) \end{bmatrix} = \begin{pmatrix} 0 \\ -\rho\, a\frac{a}{2}\,\omega^2 - \rho b\, a\omega^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -\gamma a\frac{a}{2} - \gamma b\, a - T c \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\mathbf{I_A}\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}\right]$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ T \\ M_y \\ M_z \end{pmatrix} = \text{Find}(A_x, A_y, A_z, T, M_y, M_z) \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \\ T \end{pmatrix} = \begin{pmatrix} 0.0 \\ -93.2 \\ 57.1 \\ 47.1 \end{pmatrix} \text{lb} \qquad \begin{pmatrix} M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix} \text{lb}\cdot\text{ft}$$

Problem 21-61

Show that the angular velocity of a body, in terms of Euler angles ϕ, θ and ψ may be expressed as $\boldsymbol{\omega} = (\phi' \sin\theta \sin\psi + \theta' \cos\psi)\,\mathbf{i} + (\phi' \sin\theta \cos\psi - \theta' \sin\psi)\,\mathbf{j} + (\phi' \cos\theta + \psi')\,\mathbf{k}$, where **i**, **j**, and **k** are directed along the *x*, *y*, *z* axes as shown in Fig. 21-15*d*.

Solution:

From Fig. 21 - 15*b*, due to rotation ϕ, the *x*, *y*, *z* components of ϕ' are simply ϕ' along *z* axis

From Fig. 21 - 15*c*, due to rotation θ, the *x*, *y*, *z* components of ϕ' and θ' are $\phi' \sin\theta$ in the *y* direction, $\phi' \cos\theta$ in the *z* direction, and θ' in the *x* direction.

Lastly, rotation ψ, Fig 21 - 15*d*, produces the final components which yields

$$\boldsymbol{\omega} = (\phi' \sin(\theta)\sin(\psi) + \theta' \cos(\psi))\mathbf{i} + (\phi' \sin(\theta)\cos(\psi) - \theta' \sin(\psi))\mathbf{j} + (\phi' \cos(\theta) + \psi')\mathbf{k}$$

Q.E.D

Problem 21-62

A thin rod is initially coincident with the *Z* axis when it is given three rotations defined by the Euler angles ϕ, θ, and ψ. If these rotations are given in the order stated, determine the coordinate direction angles α, β, γ of the axis of the rod with respect to the *X*, *Y*, and *Z* axes. Are these directions the same for any order of the rotations? Why?

Given:

$\phi = 30\ \text{deg}$

$\theta = 45\ \text{deg}$

$\psi = 60\ \text{deg}$

Solution:

$$\mathbf{u} = \begin{pmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

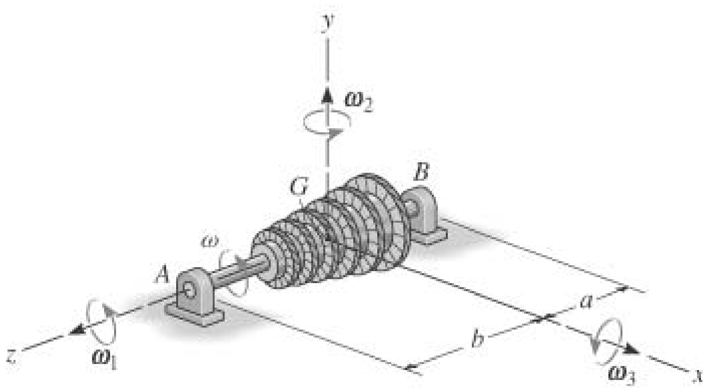
$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \operatorname{acos}(\mathbf{u}) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 69.3 \\ 127.8 \\ 45.0 \end{pmatrix} \text{deg}$$

The last rotation (ψ) does not affect the result because the rod just spins around its own axis.

The order of application of the rotations does affect the final result since rotational position is not a vector quantity.

Problem 21-63

The turbine on a ship has mass M and is mounted on bearings A and B as shown. Its center of mass is at G, its radius of gyration is k_z, and $k_x = k_y$. If it is spinning at angular velocity ω, determine the vertical reactions at the bearings when the ship undergoes each of the following motions: (a) rolling ω_1, (b) turning ω_2, (c) pitching ω_3.



Units Used:

$\text{kN} = 1000\ \text{N}$

Given:

$M = 400\ \text{kg}$ $\qquad k_x = 0.5\ \text{m}$

$\omega = 200\ \dfrac{\text{rad}}{\text{s}}$ $\qquad k_z = 0.3\ \text{m}$

$\omega_1 = 0.2\ \dfrac{\text{rad}}{\text{s}}$ $\qquad a = 0.8\ \text{m}$

$\omega_2 = 0.8\ \dfrac{\text{rad}}{\text{s}}$ $\qquad b = 1.3\ \text{m}$

$\omega_3 = 1.4\ \dfrac{\text{rad}}{\text{s}}$

Solution:

$$\mathbf{I_G} = M\begin{pmatrix} k_x^{\,2} & 0 & 0 \\ 0 & k_x^{\,2} & 0 \\ 0 & 0 & k_z^{\,2} \end{pmatrix}$$

Guesses $\qquad A_x = 1\ \text{N} \qquad A_y = 1\ \text{N}$

$\qquad B_x = 1\ \text{N} \qquad B_y = 1\ \text{N}$

(a) Rolling Given

$$\begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -Mg \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \mathbf{I_G}\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{rad}}{\text{s}^2} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\mathbf{I_G}\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}\right]$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y) \qquad \begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 1.50 \\ 2.43 \end{pmatrix} \text{kN}$$

(b) Turning Given

$$\begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -Mg \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \mathbf{I_G}\begin{pmatrix} \omega\,\omega_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_2 \\ \omega \end{pmatrix} \times \left[\mathbf{I_G}\begin{pmatrix} 0 \\ \omega_2 \\ \omega \end{pmatrix}\right]$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y) \qquad \begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} -1.25 \\ 5.17 \end{pmatrix} \text{kN}$$

(c) Pitching Given

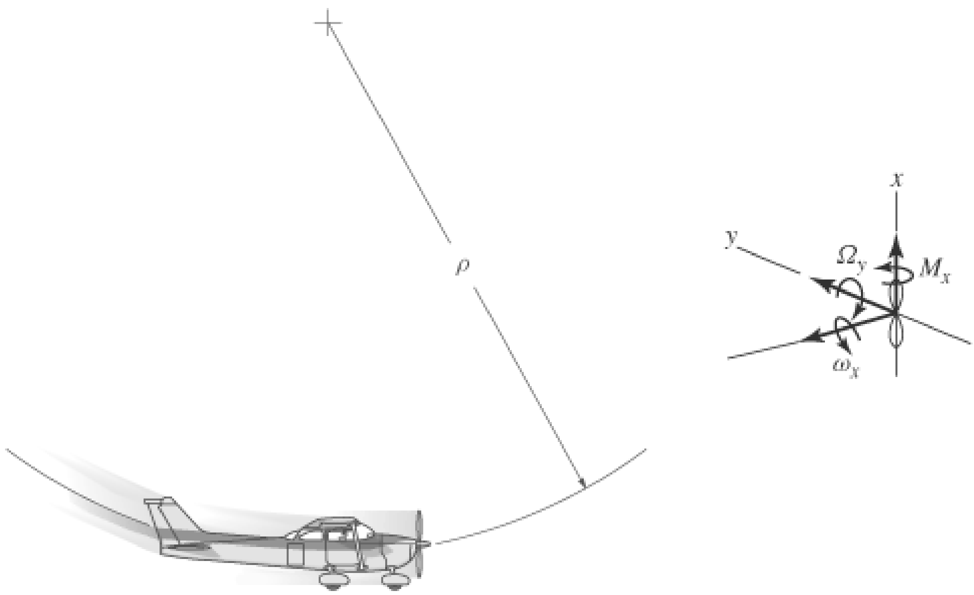
$$\begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -Mg \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \mathbf{I_G} \begin{pmatrix} 0 \\ -\omega\,\omega_3 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_3 \\ 0 \\ \omega \end{pmatrix} \times \left[\mathbf{I_G} \begin{pmatrix} \omega_3 \\ 0 \\ \omega \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y) \qquad \begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} -4.80 \\ 4.80 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 1.50 \\ 2.43 \end{pmatrix} \text{kN}$$

***Problem 21-64**

An airplane descends at a steep angle and then levels off horizontally to land. If the propeller is turning clockwise when observed from the rear of the plane, determine the direction in which the plane tends to turn as caused by the gyroscopic effect as it levels off.



Solution:

As noted on the diagram M_x represents the effect of the plane on the propeller. The opposite effect occurs on the plane. Hence, the plane tends to **turn to the right when viewed from above.**

Problem 21-65

The propeller on a single-engine airplane has a mass M and a centroidal radius of gyration k_G computed about the axis of spin. When viewed from the front of the airplane, the propeller is turning clockwise at ω_s about the spin axis. If the airplane enters a vertical curve having a radius ρ and is traveling at speed v, determine the gyroscopic bending moment which the propeller exerts on the bearings of the engine when the airplane is in its lowest position.

Given:

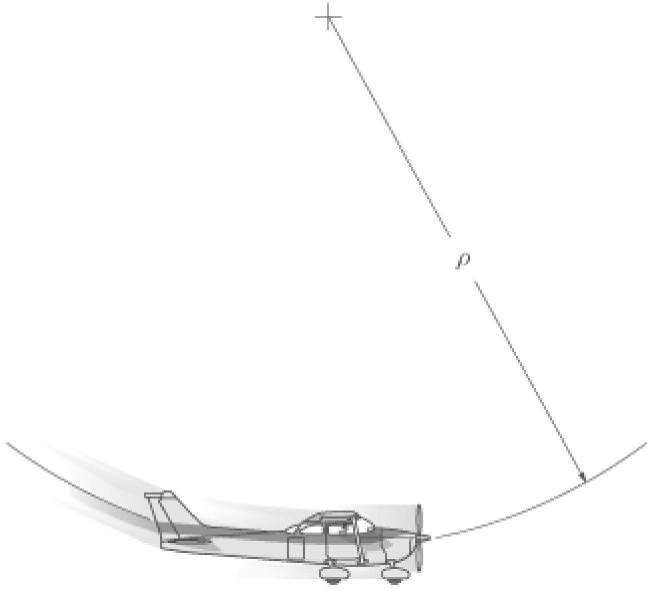
$M = 15 \text{ kg}$

$k_G = 0.3 \text{ m}$

$v = 200 \dfrac{\text{km}}{\text{hr}}$

$\omega_s = 350 \dfrac{\text{rad}}{\text{s}}$

$\rho = 80 \text{ m}$



Solution:

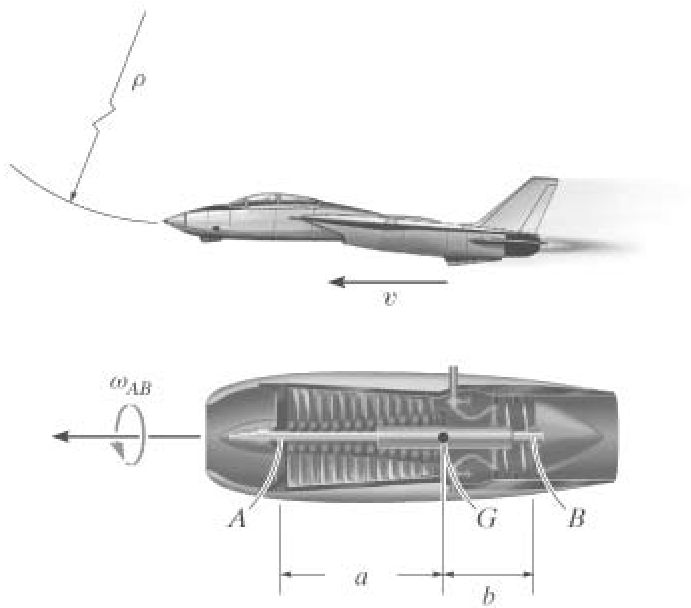
$\Omega_y = \dfrac{v}{\rho}$

$M_z = \left(M k_G^2\right)\Omega_y \omega_s$

$M_z = 328 \text{ N}\cdot\text{m}$

Problem 21-66

The rotor assembly on the engine of a jet airplane consists of the turbine, drive shaft, and compressor. The total mass is m_r, the radius of gyration about the shaft axis is k_{AB}, and the mass center is at G. If the rotor has an angular velocity ω_{AB}, and the plane is pulling out of a vertical curve while traveling at speed v, determine the components of reaction at the bearings A and B due to the gyroscopic effect.



Units Used:

$\text{kN} = 10^3 \text{ N}$

Given:

$m_r = 700 \text{ kg}$

$k_{AB} = 0.35 \text{ m}$

$\omega_{AB} = 1000 \dfrac{\text{rad}}{\text{s}}$

$\rho = 1.30\ \text{km}$

$a = 0.8\ \text{m}$

$b = 0.4\ \text{m}$

$v = 250\ \dfrac{\text{m}}{\text{s}}$

Solution: $M = m_r k_{AB}{}^2 \omega_{AB} \dfrac{v}{\rho}$

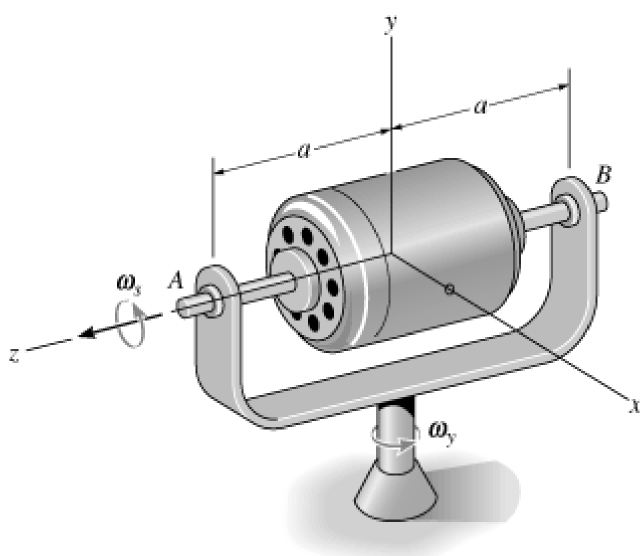
Guesses $A = 1\ \text{N}$ $B = 1\ \text{N}$

Given $A a - B b = M$ $A + B = 0$ $\begin{pmatrix} A \\ B \end{pmatrix} = \text{Find}(A, B)$ $\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 13.7 \\ -13.7 \end{pmatrix} \text{kN}$

Problem 21-67

A motor has weight W and has radius of gyration k_z about the z axis. The shaft of the motor is supported by bearings at A and B, and is turning at a constant rate $\omega_s = \omega_z\mathbf{k}$, while the frame has an angular velocity of $\omega_y = \omega_y\mathbf{j}$. Determine the moment which the bearing forces at A and B exert on the shaft due to this motion.

Given:



$W = 50\ \text{lb}$

$k_z = 0.2\ \text{ft}$

$\omega_z = 100\ \dfrac{\text{rad}}{\text{s}}$

$\omega_y = 2\ \dfrac{\text{rad}}{\text{s}}$

$a = 0.5\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

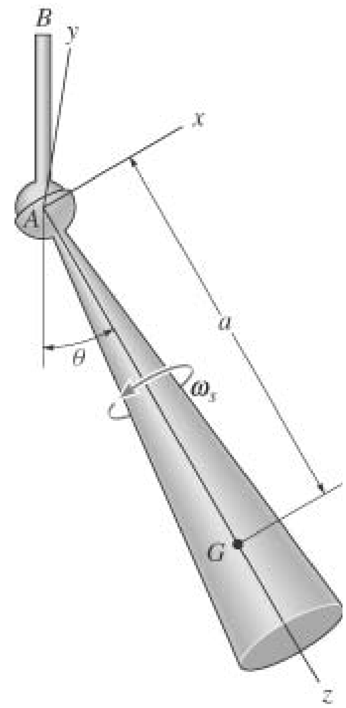
Solution:

$$\mathbf{M} = \begin{pmatrix} 0 \\ \omega_y \\ 0 \end{pmatrix} \times \begin{bmatrix} 0 \\ 0 \\ \left(\dfrac{W}{g}\right) k_z{}^2 \omega_z \end{bmatrix} \qquad \mathbf{M} = \begin{pmatrix} 12.4 \\ 0.0 \\ 0.0 \end{pmatrix} \text{lb}\cdot\text{ft}$$

***Problem 21-68**

The conical top has mass M, and the moments of inertia are $I_x = I_y$ and I_z. If it spins freely in the ball-and-socket joint at A with angular velocity ω_s compute the precession of the top about the axis of the shaft AB.

Given:



$M = 0.8\ \text{kg}$ $\qquad a = 100\ \text{mm}$

$I_x = 3.5\ 10^{-3}\ \text{kg}\cdot\text{m}^2$ $\qquad \theta = 30\ \text{deg}$

$I_z = 0.8 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ $\qquad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$\omega_s = 750\ \dfrac{\text{rad}}{\text{s}}$

Solution: Using Eq. 21-30.

$$\Sigma M_x = -I_x \phi'^2 \sin(\theta)\cos(\theta) + I_z \phi' \sin(\theta)(\phi' \cos(\theta) + \psi')$$

Guess $\quad \phi' = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $\quad M g \sin(\theta) a = -I_x \phi'^2 \sin(\theta)\cos(\theta) + I_z \phi' \sin(\theta)(\phi' \cos(\theta) + \omega_s)$

$\phi' = \text{Find}(\phi')$ $\qquad \phi' = 1.31\ \dfrac{\text{rad}}{\text{s}}$ $\qquad$ low precession

Guess $\quad \phi' = 200\ \dfrac{\text{rad}}{\text{s}}$

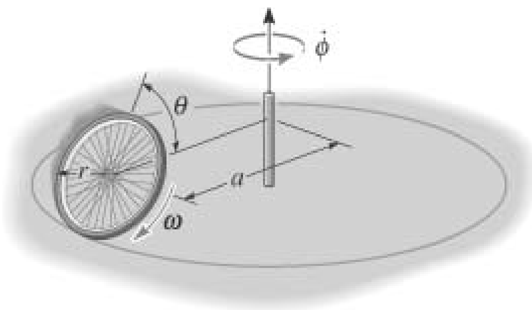
Given $\quad M g \sin(\theta) a = -I_x \phi'^2 \sin(\theta)\cos(\theta) + I_z \phi' \sin(\theta)(\phi' \cos(\theta) + \omega_s)$

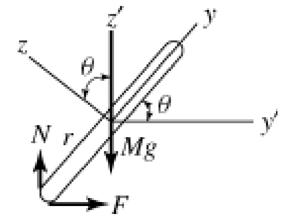
$\phi' = \text{Find}(\phi')$ $\qquad \phi' = 255\ \dfrac{\text{rad}}{\text{s}}$ $\qquad$ high precession

---

Problem 21-69

A wheel of mass m and radius r rolls with constant spin ω about a circular path having a radius a. If the angle of inclination is θ, determine the rate of precession. Treat the wheel as a thin ring. No slipping occurs.





Solution:

Since no sipping occurs, $r\psi' = a + r\cos(\theta)\phi'$ $\psi' = \left(\frac{a + r\cos(\theta)}{r}\right)\phi'$

Also, $\omega = \phi' + \psi'$ $F = m\left(a\phi'^2\right)$ $N - mg = 0$

$$I_x = I_y = \frac{mr^2}{2} \qquad I_z = mr^2$$

$$\omega = \phi' \sin(\theta)\mathbf{j} + (-\psi' + \phi' \cos(\theta)\mathbf{k}$$

Thus, $\omega_x = 0$ $\omega_y = \phi' \sin(\theta)$ $\omega_z = -\psi' + \phi' \cos(\theta)$

$$\omega' = \phi' \times \psi' = -\phi' \psi' \sin(\theta)$$

$$\omega'_x = -\phi' \psi' \sin(\theta) \qquad \omega'_y = \omega'_z = 0$$

Applying

$$\Sigma M_x = I_x\omega'_x + (I_z - I_y)\omega_z\omega_y$$

$$F r\sin(\theta) - N r\cos(\theta) = \frac{mr^2}{2}(-\phi' \psi' \sin(\theta)) + \left(mr^2 - \frac{mr^2}{2}\right)(-\psi' + \phi' \cos(\theta))(\phi' \sin(\theta))$$

Solving we find

$$m a\phi'^2 r\sin(\theta) - m g r\cos(\theta) = \left(\frac{-mr^2}{2}\right)\phi'^2 \sin(\theta)\left(\frac{a + r\cos(\theta)}{r}\right) - \left(\frac{mr^2}{2}\right)\left(\frac{a}{r}\right)\phi'^2 \sin(\theta)$$

$$2g\cos(\theta) = a\phi'^2 \sin(\theta) + r\phi'^2 \sin(\theta)\cos(\theta) \qquad \phi' = \sqrt{\frac{2g\cot(\theta)}{a + r\cos(\theta)}}$$

Problem 21-70

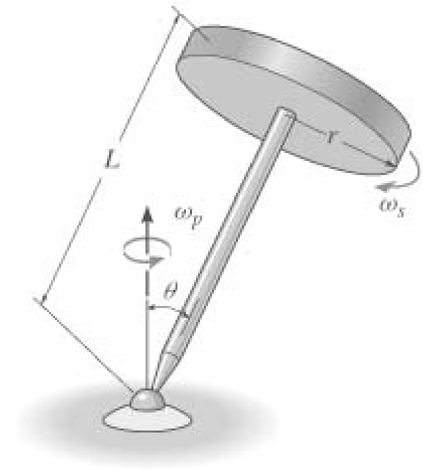
The top consists of a thin disk that has weight W and radius r. The rod has a negligible mass and length L. If the top is spinning with an angular velocity ω_s, determine the steady-state precessional angular velocity ω_p.

Given:

$W = 8\ \text{lb}$ $\theta = 40\ \text{deg}$

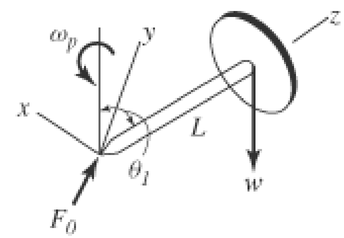
$r = 0.3\ \text{ft}$ $g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$L = 0.5\ \text{ft}$ $\omega_s = 300\ \dfrac{\text{rad}}{\text{s}}$



Solution:

$$\Sigma M_x = -I\,\phi'^2 \sin(\theta)\cos(\theta) + I_z \phi' \sin(\theta)(\phi' \cos(\theta) + \psi')$$



Guess $\omega_p = 1\ \dfrac{\text{rad}}{\text{s}}$ Given

$$W L \sin(\theta) = -\left[\left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right) + \left(\frac{W}{g}\right)L^2\right]\omega_p^{\,2}\sin(\theta)\cos(\theta) + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\omega_p \sin(\theta)(\omega_p\cos(\theta) + \omega_s)$$

$\omega_p = \text{Find}(\omega_p)$ $\omega_p = 1.21\ \dfrac{\text{rad}}{\text{s}}$ low precession

Guess $\omega_p = 70\ \dfrac{\text{rad}}{\text{s}}$ Given

$$W L \sin(\theta) = -\left[\left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right) + \left(\frac{W}{g}\right)L^2\right]\omega_p^{\,2}\sin(\theta)\cos(\theta) + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\omega_p \sin(\theta)(\omega_p\cos(\theta) + \omega_s)$$

$\omega_p = \text{Find}(\omega_p)$ $\omega_p = 76.3\ \dfrac{\text{rad}}{\text{s}}$ high precession

---

Problem 21-71

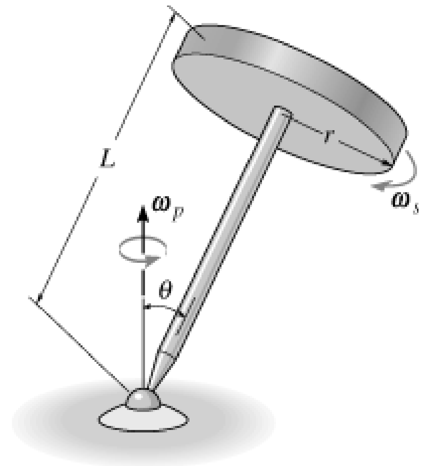
The top consists of a thin disk that has weight W and radius r. The rod has a negligible mass and length L. If the top is spinning with an angular velocity ω_s, determine the steady-state precessional angular velocity ω_p.

Given:

$W = 8\ \text{lb}$ $\theta = 90\ \text{deg}$

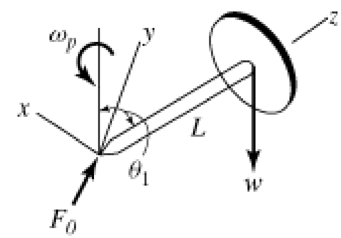
$r = 0.3\ \text{ft}$ $g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$L = 0.5\ \text{ft}$ $\omega_s = 300\ \dfrac{\text{rad}}{\text{s}}$



Solution:

$$\Sigma M_x = -I\,\phi'^2 \sin(\theta)\cos(\theta) + I_z\phi' \sin(\theta)(\phi' \cos(\theta) + \psi')$$



Guess $\omega_p = 1\ \dfrac{\text{rad}}{\text{s}}$ Given

$$W L \sin(\theta) = -\left[\left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right) + \left(\frac{W}{g}\right)L^2\right]\omega_p^{\ 2}\sin(\theta)\cos(\theta) + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\omega_p \sin(\theta)(\omega_p \cos(\theta) + \omega_s)$$

$\omega_p = \text{Find}(\omega_p)$ $\omega_p = 1.19\ \dfrac{\text{rad}}{\text{s}}$

***Problem 21-72**

The top has weight *W* and can be considered as a solid cone. If it is observed to precess about the vertical axis at a constant rate of ω_y, determine its spin ω_s.

Given:

$W = 3\ \text{lb}$

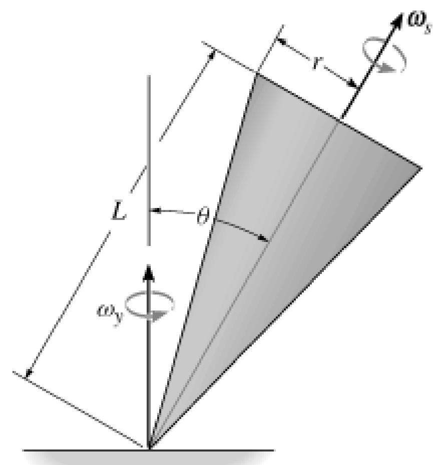
$\omega_y = 5\ \dfrac{\text{rad}}{\text{s}}$

$\theta = 30\ \text{deg}$

$L = 6\ \text{in}$

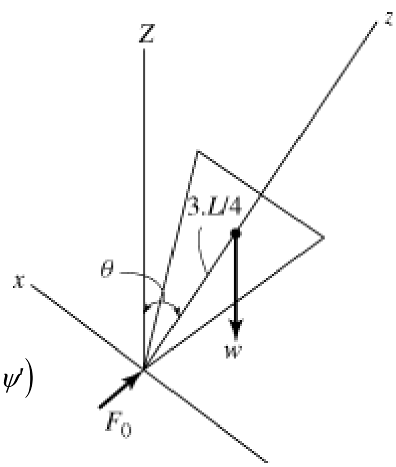
$r = 1.5\ \text{in}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$$I = \frac{3}{80}\left(\frac{W}{g}\right)\left(4r^2 + L^2\right) + \left(\frac{W}{g}\right)\left(\frac{3L}{4}\right)^2$$



$$I_z = \frac{3}{10}\left(\frac{W}{g}\right)r^2$$

$$\Sigma M_x = -I\,\phi'^2 \sin(\theta)\cos(\theta) + I_z\phi' \sin(\theta)\left(\phi' \cos(\theta) + \psi'\right)$$

$$W\frac{3L}{4}\sin(\theta) = -I\,\omega_y^{\;2} \sin(\theta)\cos(\theta) + I_z\omega_y \sin(\theta)\left(\omega_y \cos(\theta) + \psi'\right)$$

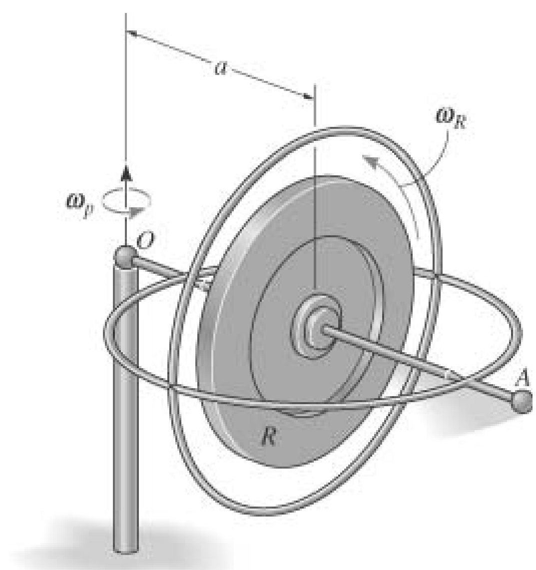
$$\psi' = \frac{1}{4}\left(\frac{3\,WL + 4I\,\omega_y^{\;2}\cos(\theta) - 4I_z\,\omega_y^{\;2}\cos(\theta)}{I_z\omega_y}\right)$$

$$\psi' = 652\,\frac{\text{rad}}{\text{s}}$$

Problem 21-73

The toy gyroscope consists of a rotor R which is attached to the frame of negligible mass. If it is observed that the frame is precessing about the pivot point O at rate ω_p determine the angular velocity ω_R of the rotor. The stem OA moves in the horizontal plane. The rotor has mass M and a radius of gyration k_{OA} about OA.

Given:



$$\omega_p = 2\,\frac{\text{rad}}{\text{s}}$$

$$M = 200\text{ gm}$$

$$k_{OA} = 20\text{ mm}$$

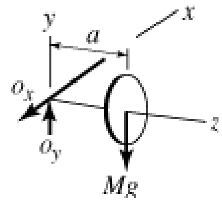
$$a = 30\text{ mm}$$

$$g = 9.81\,\frac{\text{m}}{\text{s}^2}$$

Solution:

$$\Sigma M_x = I_z\Omega_y\omega_z$$

$M g a = M k_{OA}^2 \omega_p \omega_R$



$\omega_R = \dfrac{g a}{k_{OA}^2 \omega_p}$

$\omega_R = 368 \dfrac{\text{rad}}{\text{s}}$

Problem 21-74

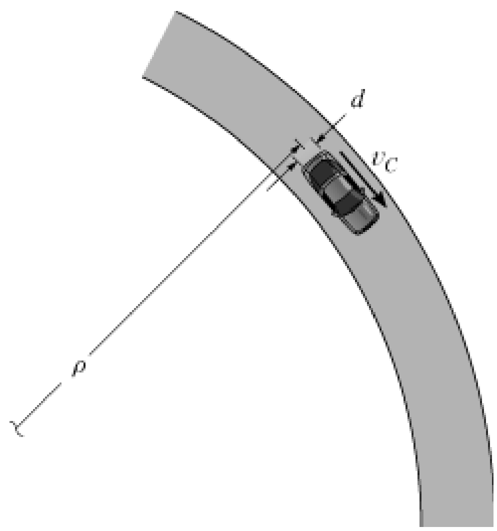
The car is traveling at velocity v_c around the horizontal curve having radius ρ. If each wheel has mass M, radius of gyration k_G about its spinning axis, and radius r, determine the difference between the normal forces of the rear wheels, caused by the gyroscopic effect. The distance between the wheels is d.

Given:

$v_c = 100 \dfrac{\text{km}}{\text{hr}}$ $\quad k_G = 300 \text{ mm}$

$\rho = 80 \text{ m}$ $\quad r = 400 \text{ mm}$

$M = 16 \text{ kg}$ $\quad d = 1.3 \text{ m}$



Solution:

$I = 2M k_G^2$ $\quad I = 2.88 \text{ kg} \cdot \text{m}^2$

$\omega_s = \dfrac{v_c}{r}$ $\quad \omega_s = 69.44 \dfrac{\text{rad}}{\text{s}}$

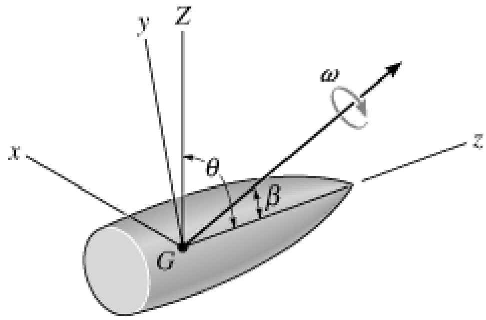
$\omega_p = \dfrac{v_c}{\rho}$ $\quad \omega_p = 0.35 \dfrac{\text{rad}}{\text{s}}$

$M = I \omega_s \omega_p$

$\Delta F\, d = I \omega_s \omega_p$ $\quad \Delta F = I\, \omega_s \dfrac{\omega_p}{d}$ $\quad \Delta F = 53.4 \text{ N}$

Problem 21-75

The projectile shown is subjected to torque-free motion. The transverse and axial moments of inertia are I and I_z respectively. If θ represents the angle between the precessional axis Z and the axis of symmetry z, and β is the angle between the angular velocity ω and the z axis, show that β and θ are related by the equation tan $\theta = (I/I_z)$tan β.



Solution:

From Eq. 21-34 $\omega_y = \dfrac{H_G \sin(\theta)}{I}$ and $\omega_z = \dfrac{H_G \cos(\theta)}{I_z}$

Hence $\dfrac{\omega_y}{\omega_z} = \dfrac{I_z}{I} \tan(\theta)$

However, $\omega_y = \omega \sin(\beta)$ and $\omega_z = \omega \cos(\beta)$

$$\frac{\omega_y}{\omega_z} = \tan(\beta) = \frac{I_z}{I} \tan(\theta)$$

$$\tan(\theta) = \frac{I}{I_z} \tan(\beta)$$ Q.E.D

***Problem 21-76**

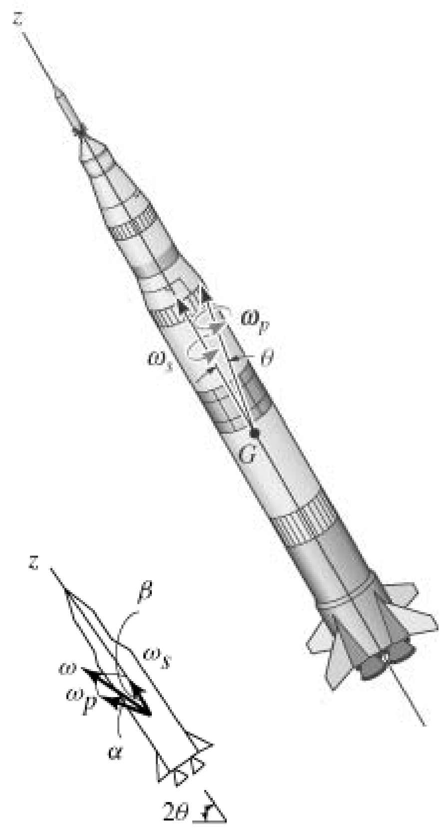
While the rocket is in free flight, it has a spin ω_s and precesses about an axis measured angle θ from the axis of spin. If the ratio of the axial to transverse moments of inertia of the rocket is r, computed about axes which pass through the mass center G, determine the angle which the resultant angular velocity makes with the spin axis. Construct the body and space cones used to describe the motion. Is the precession regular or retrograde?

Given:

$\omega_s = 3 \dfrac{\text{rad}}{\text{s}}$

$\theta = 10 \text{ deg}$

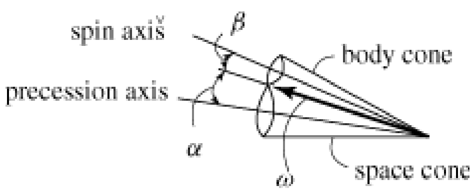
$$r = \frac{1}{15}$$



Solution:

Determine the angle β from the result of prob.21-75

$$\tan(\theta) = \frac{\tan(\beta)}{r}$$



$$\beta = \operatorname{atan}(r\tan(\theta)) \qquad \beta = 0.673\,\text{deg}$$

Thus,

$$\alpha = \theta - \beta \qquad \alpha = 9.33\,\text{deg}$$

Regular Precession Since $I_z < I$

Problem 21-77

The projectile has a mass M and axial and transverse radii of gyration k_z and k_t, respectively. If it is spinning at ω_s when it leaves the barrel of a gun, determine its angular momentum. Precession occurs about the Z axis.

Given:

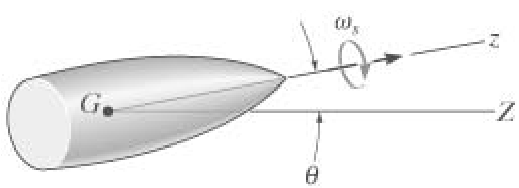
$M = 0.9$ kg

$k_z = 20$ mm

$k_t = 25$ mm

$\omega_s = 6\ \frac{\text{rad}}{\text{s}}$

$\theta = 10$ deg



Solution:

$$I = M k_t^{\,2} \qquad I = 5.625 \times 10^{-4}\,\text{kg}\cdot\text{m}^2$$

$$I_z = M k_z^{\,2} \qquad I_z = 3.600 \times 10^{-4}\,\text{kg}\cdot\text{m}^2$$

$$\psi = \omega_s$$

$$\psi = \left(\frac{I - I_z}{I I_z}\right) H_G \cos(\theta)$$

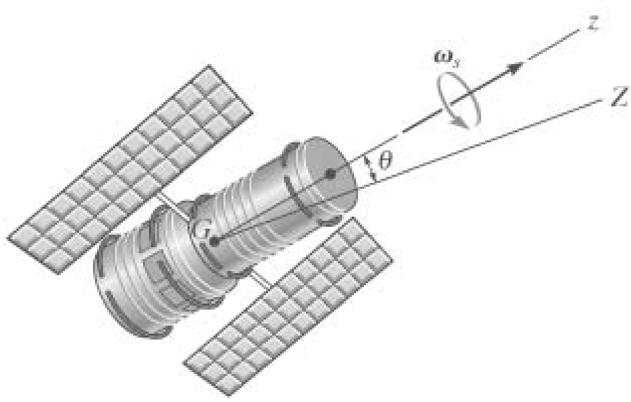
$$H_G = \psi I \left[\frac{I_z}{\cos(\theta)(I - I_z)}\right] \qquad H_G = 6.09 \times 10^{-3} \frac{\text{kg} \cdot \text{m}^2}{\text{s}}$$

Problem 21-78

The satellite has mass M, and about axes passing through the mass center G the axial and transverse radii of gyration are k_z and k_t, respectively. If it is spinning at ω_s when it is launched, determine its angular momentum. Precession occurs about the Z axis.

Units Used:

$\text{Mg} = 10^3 \text{ kg}$



Given:

$M = 1.8 \text{ Mg}$ $\qquad \omega_s = 6 \frac{\text{rad}}{\text{s}}$

$k_z = 0.8 \text{ m}$

$k_t = 1.2 \text{ m}$ $\qquad \theta = 5 \text{ deg}$

Solution:

$I = M k_t^2 \qquad I = 2592 \text{ kg} \cdot \text{m}^2$

$I_z = M k_z^2 \qquad I_z = 1152 \text{ kg} \cdot \text{m}^2$

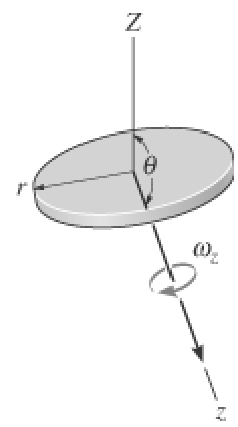
$\psi' = \omega_s$

$$\psi' = \left(\frac{I - I_z}{I I_z}\right) H_G \cos(\theta)$$

$$H_G = \psi' I \left[\frac{I_z}{\cos(\theta)(I - I_z)}\right] \qquad H_G = 12.5 \text{ Mg} \cdot \frac{\text{m}^2}{\text{s}}$$

Problem 21-79

The disk of mass M is thrown with a spin ω_z. The angle θ is measured as shown. Determine the precession about the Z axis.



Given: $M = 4\ \text{kg}$

$\theta = 160\ \text{deg}$

$r = 125\ \text{mm}$

$\omega_z = 6\ \dfrac{\text{rad}}{\text{s}}$

Solution:

$$I = \frac{1}{4} M r^2 \qquad I_z = \frac{1}{2} M r^2$$

Applying Eq.21 - 36

$$\psi' = \omega_z = \frac{I - I_z}{I I_z} H_G \cos(\theta)$$

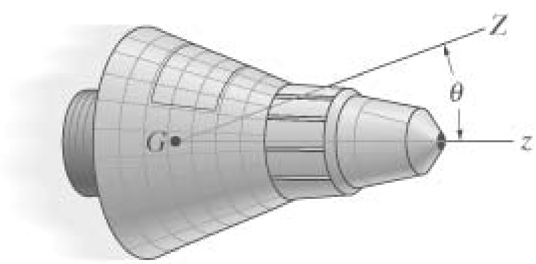
$$H_G = \omega_z \frac{I I_z}{\cos(\theta)(I - I_z)} \qquad H_G = 0.1995\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\phi' = \frac{H_G}{I} \qquad \phi' = 12.8\ \frac{\text{rad}}{\text{s}}$$

Note that this is a case of retrograde precession since $I_z > I$

***Problem 21-80**

The radius of gyration about an axis passing through the axis of symmetry of the space capsule of mass M is k_z, and about any transverse axis passing through the center of mass G, is k_t. If the capsule has a known steady-state precession of two revolutions per hour about the Z axis, determine the rate of spin about the z axis.



Units Used:

$\text{Mg} = 10^3\ \text{kg}$

Given:

$M = 1.6\ \text{Mg}$

$k_z = 1.2\ \text{m}$

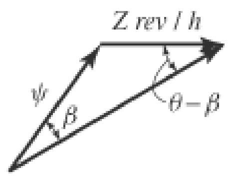
$k_t = 1.8 \text{ m}$

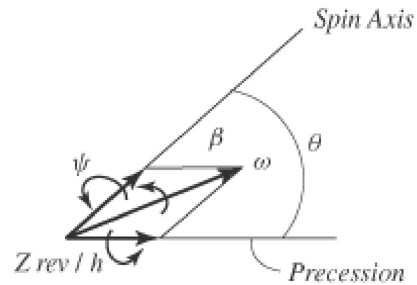
$\theta = 20 \text{ deg}$

Solution:

$I = M k_t^2$

$I_z = M k_z^2$





Using the Eqn.

$$\tan(\theta) = \left(\frac{I}{I_z}\right)\tan(\beta)$$

$$\beta = \operatorname{atan}\left(\tan(\theta)\frac{I_z}{I}\right)$$

$$\beta = 9.19 \text{ deg}$$

Problem 22-1

When a load of weight W_1 is suspended from a spring, the spring is stretched a distance d. Determine the natural frequency and the period of vibration for a load of weight W_2 attached to the same spring.

Given: $W_1 = 20\text{ lb}$ $W_2 = 10\text{ lb}$ $d = 4\text{ in}$

Solution:

$$k = \frac{W_1}{d} \qquad k = 60.00\,\frac{\text{lb}}{\text{ft}}$$

$$\omega_n = \sqrt{\frac{k}{\frac{W_2}{g}}} \qquad \omega_n = 13.89\,\frac{\text{rad}}{\text{s}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 0.45\text{ s}$$

$$f = \frac{\omega_n}{2\pi} \qquad f = 2.21\,\frac{1}{\text{s}}$$

Problem 22-2

A spring has stiffness k. If a block of mass M is attached to the spring, pushed a distance d above its equilibrium position, and released from rest, determine the equation which describes the block's motion. Assume that positive displacement is measured downward.

Given: $k = 600\,\frac{\text{N}}{\text{m}}$ $M = 4\text{ kg}$ $d = 50\text{ mm}$

Solution:

$$\omega_n = \sqrt{\frac{k}{M}} \qquad \omega_n = 12.2\,\frac{\text{rad}}{\text{s}}$$

$$v = 0 \quad x = -d \quad \text{at} \quad t = 0$$

$$x = A\sin(\omega_n t) + B\cos(\omega_n t) \qquad A = 0 \qquad B = -d$$

Thus,

$$x = B\cos(\omega_n t) \qquad B = -0.05\text{ m} \qquad \omega_n = 12.2\,\frac{\text{rad}}{\text{s}}$$

Problem 22-3

When a block of mass m_1 is suspended from a spring, the spring is stretched a distance δ. Determine the natural frequency and the period of vibration for a block of mass m_2 attached to the same spring.

Given: $m_1 = 3\ \text{kg}$ $m_2 = 0.2\ \text{kg}$ $\delta = 60\ \text{mm}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$$k = \frac{m_1 g}{\delta} \qquad k = 490.50\ \frac{\text{N}}{\text{m}}$$

$$\omega_n = \sqrt{\frac{k}{m_2}} \qquad \omega_n = 49.52\ \frac{\text{rad}}{\text{s}}$$

$$f = \frac{\omega_n}{2\pi} \qquad f = 7.88\ \text{Hz}$$

$$r = \frac{1}{f} \qquad r = 0.127\ \text{s}$$

***Problem 22-4**

A block of mass M is suspended from a spring having a stiffness k. If the block is given an upward velocity v when it is distance d above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that the positive displacement is measured downward.

Given: $M = 8\ \text{kg}$ $k = 80\ \dfrac{\text{N}}{\text{m}}$ $v = 0.4\ \dfrac{\text{m}}{\text{s}}$ $d = 90\ \text{mm}$

Solution:

$$\omega_n = \sqrt{\frac{k}{M}} \qquad \omega_n = 3.16\ \frac{\text{rad}}{\text{s}}$$

$$x = A\sin(\omega_n)t + B\cos(\omega_n)t$$

$$B = -d \qquad A = \frac{-v}{\omega_n}$$

$$x = A\sin(\omega_n)t + B\cos(\omega_n)t \qquad A = -0.13\ \text{m} \qquad B = -0.09\ \text{m} \qquad \omega_n = 3.16\ \frac{\text{rad}}{\text{s}}$$

$x_{max} = \sqrt{A^2 + B^2}$ $\qquad x_{max} = 0.16\ \text{m}$

Problem 22-5

A weight W is suspended from a spring having a stiffness k. If the weight is pushed distance d upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What is the amplitude and the natural frequency of the vibration?

Given: $\quad W = 2\ \text{lb} \quad k = 2\ \frac{\text{lb}}{\text{in}} \quad d = 1\ \text{in} \quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$\omega_n = \sqrt{\frac{k}{\frac{W}{g}}} \qquad \omega_n = 19.7\ \frac{\text{rad}}{\text{s}}$$

$$y = A\cos(\omega_n t) + B\sin(\omega_n t)$$

$$A = -d \qquad B = 0\ \text{in}$$

$$y = A\cos(\omega_n t) \qquad A = -0.08\ \text{ft} \qquad \omega_n = 19.7\ \frac{\text{rad}}{\text{s}}$$

$$f = \frac{\omega_n}{2\pi} \qquad f = 3.13\ \text{Hz}$$

$$C = \sqrt{A^2 + B^2} \qquad C = 1.00\ \text{in}$$

Problem 22-6

A weight W is suspended from a spring having a stiffness k. If the weight is given an upward velocity of v when it is distance d above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the weight, measured from the equilibrium position. Assume positive displacement is downward.

Given:

$W = 6\ \text{lb}$

$k = 3\ \frac{\text{lb}}{\text{in}}$

$v = 20\ \frac{\text{ft}}{\text{s}}$

$d = 2\ \text{in}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$\omega_n = \sqrt{\frac{k g}{W}}$ $\qquad \omega_n = 13.90\ \frac{\text{rad}}{\text{s}}$ $\qquad y = A\cos(\omega_n t) + B\sin(\omega_n t)$

$A = -d \qquad B = \frac{-v}{\omega_n}$

$y = A\cos(\omega_n t) + B\sin(\omega_n t)$

$A = -0.17\ \text{ft} \qquad B = -1.44\ \text{ft}$

$\omega_n = 13.90\ \frac{\text{rad}}{\text{s}}$

$C = \sqrt{A^2 + B^2} \qquad C = 1.45\ \text{ft}$

Problem 22-7

A spring is stretched a distance d by a block of mass M. If the block is displaced a distance b downward from its equilibrium position and given a downward velocity v, determine the differential equation which describes the motion. Assume that positive displacement is measured downward. Use the Runge–Kutta method to determine the position of the block, measured from its unstretched position, at time t_1 (See Appendix B.) Use a time increment Δt.

Given:

$M = 8\ \text{kg}$

$d = 175\ \text{mm}$

$b = 100\ \text{mm}$

$v = 1.50\ \frac{\text{m}}{\text{s}}$

$t_1 = 0.22\ \text{s}$

$\Delta t = 0.02\ \text{s}$

$$g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

Solution:

$$k = \frac{M g}{d} \qquad \omega_n = \sqrt{\frac{k}{M}}$$

$$y'' + \omega_n^{\,2} y = 0 \qquad \omega_n^{\,2} = 56.1\ \frac{\text{rad}}{\text{s}^2}$$

To numerically integrate in Mathcad we have to switch to nondimensional variables

$$\Omega_n = \omega_n \frac{\text{s}}{\text{rad}} \qquad B = \frac{b}{\text{mm}} \qquad V = v\frac{\text{s}}{\text{mm}} \qquad T_1 = \frac{t_1}{\text{s}}$$

Given $\quad y''(t) + \Omega_n^{\,2}\, y(t) = 0 \qquad y(0) = B \qquad y'(0) = V$

$$y = \text{Odesolve}\left(t, T_1\right) \qquad y\left(T_1\right) = 192\ \text{mm}$$

***Problem 22-8**

A spring is stretched a distance *d* by a block of mass *M*. If the block is displaced a distance *b* downward from its equilibrium position and given an upward velocity *v*, determine the differential equation which describes the motion. Assume that positive displacement is measured downward. What is the amplitude of the motion?

Given:

$$M = 8\ \text{kg}$$

$$d = 175\ \text{mm}$$

$$b = 60\ \text{mm}$$

$$v = 4\ \frac{\text{m}}{\text{s}}$$

$$t_1 = 0.22\ \text{s}$$

$$\Delta t = 0.02\ \text{s} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

Solution:

$$k = \frac{M g}{d} \qquad \omega_n = \sqrt{\frac{k}{M}} \qquad y'' + \omega_n^{\,2} y = 0$$

$$A = b \qquad B = \frac{-v}{\omega_n}$$

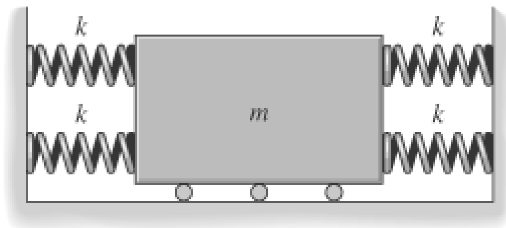
$$y(t) = A\cos(\omega_n t) + B\sin(\omega_n t) \qquad A = 0.06\ \text{m} \qquad B = -0.53\ \text{m} \qquad \omega_n = 7.49\ \frac{\text{rad}}{\text{s}}$$

$$C = \sqrt{A^2 + B^2} \qquad C = 0.54\ \text{m}$$

Problem 22-9

Determine the frequency of vibration for the block. The springs are originally compressed Δ.

Solution:



$$m x'' + 4k x = 0$$

$$x'' + \frac{4k}{m}x = 0$$

$$f = \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$$

Problem 22-10

A pendulum has a cord of length L and is given a tangential velocity v toward the vertical from a position θ_0. Determine the equation which describes the angular motion.

Given:

$$L = 0.4\ \text{m} \qquad v = 0.2\ \frac{\text{m}}{\text{s}}$$

$$\theta_0 = 0.3\ \text{rad} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

Solution:

Since the motion remains small $\quad \omega_n = \sqrt{\frac{g}{L}}$

$$\theta = A\sin(\omega_n t) + B\cos(\omega_n t)$$

$$A = \frac{-v}{\omega_n L} \qquad B = \theta_0$$

$$\theta = A\sin(\omega_n t) + B\cos(\omega_n t)$$

$$A = -0.101\,\text{rad} \qquad B = 0.30\,\text{rad} \qquad \omega_n = 4.95\,\frac{\text{rad}}{\text{s}}$$

Problem 22-11

A platform, having an unknown mass, is supported by *four* springs, each having the same stiffness k. When nothing is on the platform, the period of vertical vibration is measured as t_1; whereas if a block of mass M_2 is supported on the platform, the period of vertical vibration is t_2. Determine the mass of a block placed on the (empty) platform which causes the platform to vibrate vertically with a period t_3. What is the stiffness k of each of the springs?

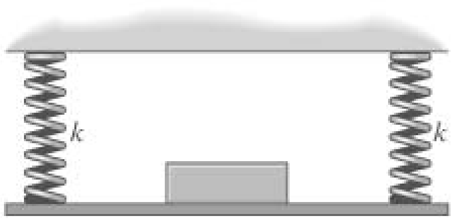
Given:

$M_2 = 3\ \text{kg}$

$t_1 = 2.35\ \text{s}$

$t_2 = 5.23\ \text{s}$

$t_3 = 5.62\ \text{s}$



Solution:

Guesses $M_1 = 1\ \text{kg}$

$M_3 = 1\ \text{kg}$

$k = 1\,\frac{\text{N}}{\text{m}}$

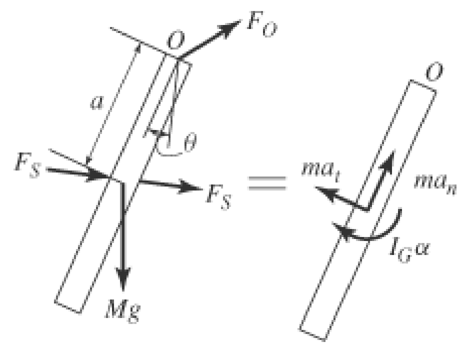
Given $t_1 = 2\pi\sqrt{\frac{M_1}{4k}} \qquad t_2 = 2\pi\sqrt{\frac{M_1 + M_2}{4k}} \qquad t_3 = 2\pi\sqrt{\frac{M_1 + M_3}{4k}}$

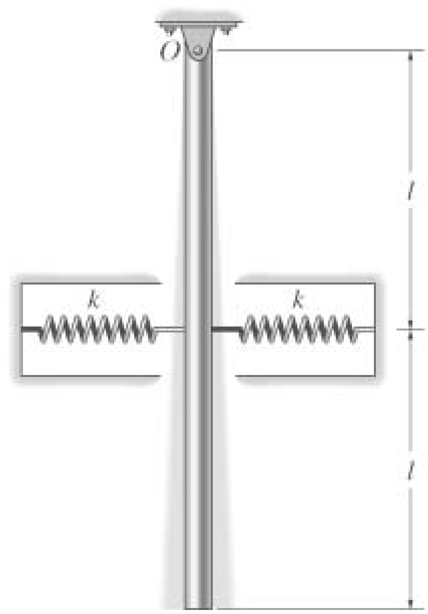
$$\begin{pmatrix} M_1 \\ M_3 \\ k \end{pmatrix} = \text{Find}(M_1, M_3, k) \qquad M_1 = 0.759\ \text{kg} \qquad M_3 = 3.58\ \text{kg}$$

$$k = 1.36\,\frac{\text{N}}{\text{m}}$$

***Problem 22-12**

If the lower end of the slender rod of mass M is displaced a small amount and released from rest, determine the natural frequency of vibration. Each spring has a stiffness k and is unstretched when the rod is hanging vertically.





Given:

$M = 30\ \text{kg} \qquad l = 1\ \text{m}$

$k = 500\ \dfrac{\text{N}}{\text{m}}$

Solution:

$$M g l \sin(\theta) + 2k l \sin(\theta) l \cos(\theta) = -M \frac{(2l)^2}{3} \theta'$$

For small angles

$$\frac{4Ml^2}{3}\theta' + \left(M g l + 2kl^2\right)\theta = 0$$

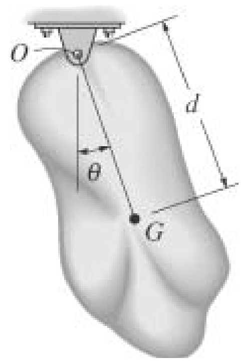
$$\theta' + \left(\frac{3g}{4l} + \frac{3k}{2M}\right)\theta = 0$$

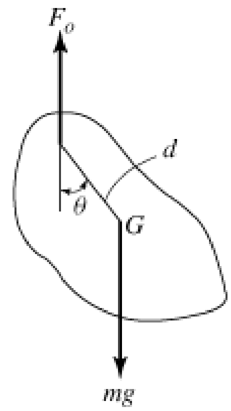
$$\omega_n = \sqrt{\frac{3g}{4l} + \frac{3k}{2M}} \qquad \omega_n = 5.69\ \frac{\text{rad}}{\text{s}}$$

$$f = \frac{\omega_n}{2\pi} \qquad f = 0.91\ \frac{1}{\text{s}}$$

Problem 22-13

The body of arbitrary shape has a mass *m*, mass center at *G*, and a radius of gyration about *G* of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration.





Solution:

$$\Sigma M_O = I_O\alpha \qquad -m\,g\,d\sin(\theta) = \left(m\,k_G^{\,2} + m\,d^2\right)\theta''$$

$$\theta'' + \left(\frac{g\,d}{k_G^{\,2} + d^2}\right)\sin(\theta) = 0$$

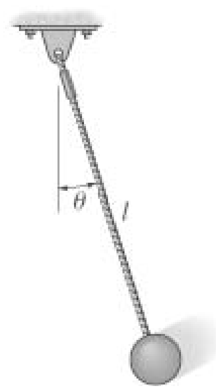
However, for small rotation $\sin(\theta) = \theta$. Hence $\qquad \theta'' + \left(\frac{g\,d}{k_G^{\,2} + d^2}\right)\theta = 0$

From the above differential equation,

$$\omega_n = \sqrt{\frac{g\,d}{k_G^{\,2} + d^2}} \qquad \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{k_G^{\,2} + d^2}{g\,d}}$$

Problem 22-14

Determine to the nearest degree the maximum angular displacement of the bob if it is initially displaced θ_0 from the vertical and given a tangential velocity *v* away from the vertical.



Given:

$\theta_0 = 0.2 \text{ rad} \qquad v = 0.4 \ \frac{\text{m}}{\text{s}}$

$l = 0.4 \text{ m}$

Solution:

$$\omega_n = \sqrt{\frac{g}{l}} \qquad A = \theta_0 \qquad B = \frac{v}{l\omega_n}$$

$$\theta(t) = A\cos(\omega_n t) + B\sin(\omega_n t)$$

$$C = \sqrt{A^2 + B^2} \qquad C = 16\,\text{deg}$$

Problem 22-15

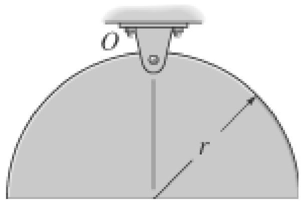
The semicircular disk has weight W. Determine the natural period of vibration if it is displaced a small amount and released.

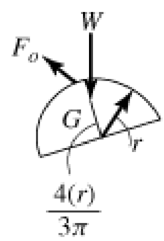
Given:

$W = 20$ lb

$r = 1$ ft

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

$$I_O = \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right) - \left(\frac{W}{g}\right)\left(\frac{4r}{3\pi}\right)^2 + \left(\frac{W}{g}\right)\left(r - \frac{4r}{3\pi}\right)^2$$

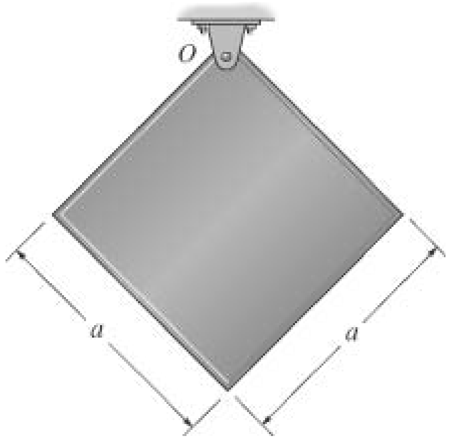
$$-W\left(r - \frac{4r}{3\pi}\right)\sin(\theta) = I_O\theta'' \qquad \theta'' + \left(r - \frac{4r}{3\pi}\right)\left(\frac{W}{I_O}\right)\theta = 0$$

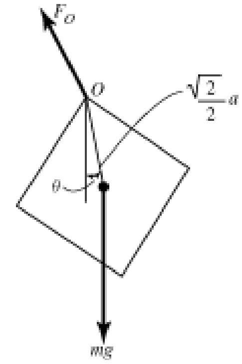
$$\omega_n = \sqrt{\left(r - \frac{4r}{3\pi}\right)\left(\frac{W}{I_O}\right)} \qquad \omega_n = 5.34\ \frac{\text{rad}}{\text{s}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 1.18\ \text{s}$$

***Problem 22-16**

The square plate has a mass m and is suspended at its corner by the pin O. Determine the natural period of vibration if it is displaced a small amount and released.





Solution:

$$I_O = \frac{2}{3}m\,a^2 \qquad -m\,g\left(\frac{\sqrt{2}}{2}a\right)\theta = \left(\frac{2}{3}m\,a^2\right)\theta'$$

$$\theta' + \left(\frac{3\sqrt{2}g}{4a}\right)\theta = 0 \qquad \omega_n = \sqrt{\frac{3\sqrt{2}g}{4a}} \qquad \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{4}{3\sqrt{2}}}\sqrt{\frac{a}{g}}$$

$$b = 2\pi\sqrt{\frac{4}{3\sqrt{2}}} \qquad \tau = b\sqrt{\frac{a}{g}} \qquad b = 6.10$$

Problem 22-17

The disk has weight W and rolls without slipping on the horizontal surface as it oscillates about its equilibrium position. If the disk is displaced by rolling it counterclockwise through angle θ_0, determine the equation which describes its oscillatory motion when it is released.

Given:

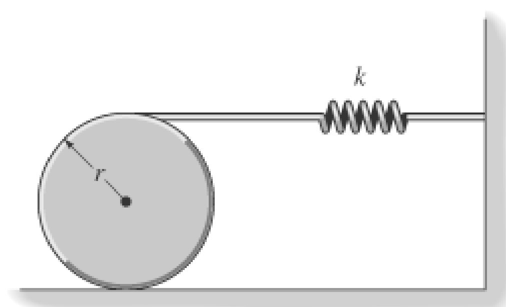
$W = 10$ lb

$\theta_0 = 0.4$ rad

$r = 1$ ft

$k = 100 \frac{\text{lb}}{\text{ft}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

$$-k(2r\theta)\,2r = \left(\frac{W}{g}\frac{r^2}{2} + \frac{W}{g}r^2\right)\theta'' \qquad \theta'' + \frac{8kg}{3W}\theta = 0 \qquad \omega_n = \sqrt{\frac{8kg}{3W}}$$

$$\theta = \theta_0 \cos(\omega_n t) \qquad \theta_0 = 0.40\,\text{rad} \qquad \omega_n = 29.3\,\frac{\text{rad}}{\text{s}}$$

Problem 22-18

The pointer on a metronome supports slider A of weight W, which is positioned at a fixed distance a from the pivot O of the pointer. When the pointer is displaced, a torsional spring at O exerts a restoring torque on the pointer having a magnitude $M = k\theta$ where θ represents the angle of displacement from the vertical. Determine the natural period of vibration when the pointer is displaced a small amount and released. Neglect the mass of the pointer.

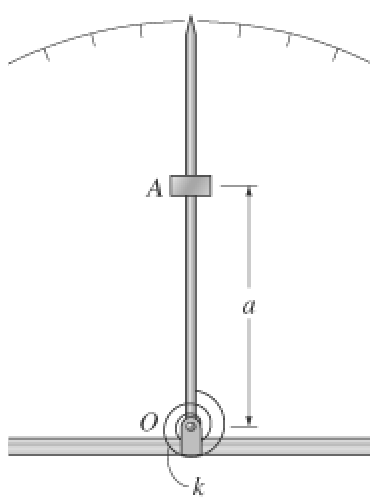
Given:

$W = 0.4\ \text{lb}$

$k = 1.2\ \text{lb}\ \frac{\text{ft}}{\text{rad}}$

$a = 0.25\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$-Wa\theta + k\theta = \left(\frac{-W}{g}\right)a^2\theta'' \qquad \theta'' + \frac{g}{a}\left(\frac{k}{aW} - 1\right)\theta = 0$$

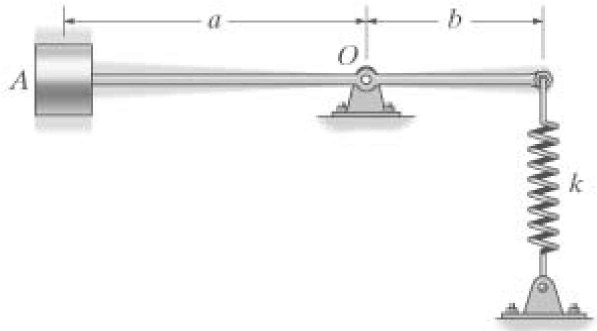
$$\omega_n = \sqrt{\frac{g}{a}\left(\frac{k}{aW} - 1\right)} \qquad \omega_n = 37.64\,\frac{\text{rad}}{\text{s}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 0.167\ \text{s}$$

Problem 22-19

The block has a mass *m* and is supported by a rigid bar of negligible mass. If the spring has a stiffness *k*, determine the natural period of vibration for the block.

Solution:



$$-kb\theta b = ma\theta' a$$

$$ma^2\theta' + kb^2\theta = 0$$

$$\theta' + \left(\frac{kb^2}{ma^2}\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{k}{m}}\left(\frac{b}{a}\right)$$

$$\tau = 2\pi\left(\frac{a}{b}\right)\sqrt{\frac{m}{k}}$$

***Problem 22-20**

The disk, having weight *W*, is pinned at its center *O* and supports the block *A* that has weight W_A. If the belt which passes over the disk is not allowed to slip at its contacting surface, determine the natural period of vibration of the system.

Given:

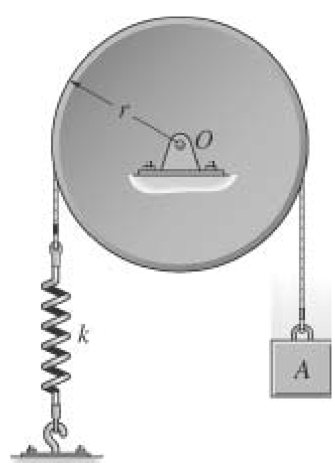
$W = 15$ lb

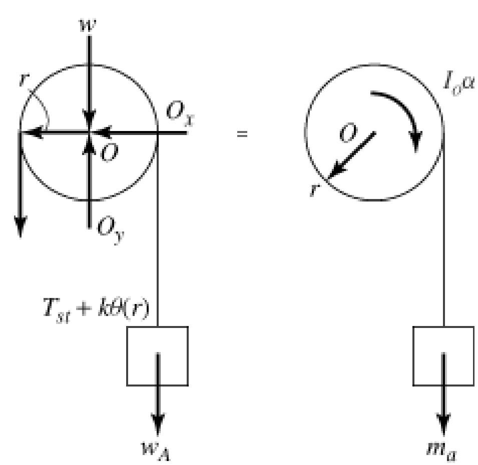
$W_A = 3$ lb

$k = 80\ \frac{\text{lb}}{\text{ft}}$

$r = 0.75$ ft

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

$$k\,r\theta r = \left(\frac{-W}{g}\right)\left(\frac{r^2}{2}\right)\theta'' - \left(\frac{W_A}{g}\right)r\theta'' r$$

$$\frac{r^2}{g}\left(W_A + \frac{W}{2}\right)\theta'' + k\,r^2\theta = 0$$

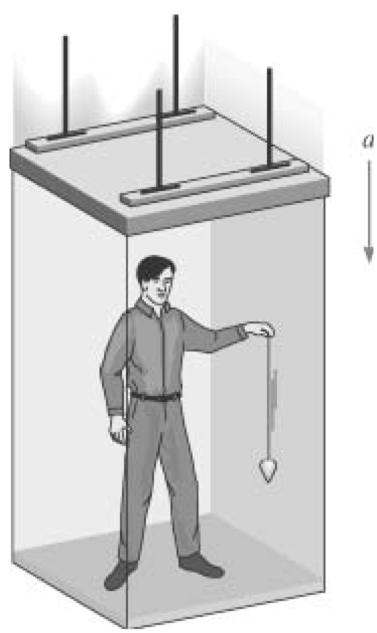
$$\theta'' + \frac{k\,g}{W_A + \frac{W}{2}}\theta = 0$$

$$\omega_n = \sqrt{\frac{k\,g}{W_A + \frac{W}{2}}} \qquad \omega_n = 15.66\,\frac{\text{rad}}{\text{s}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 0.401\ \text{s}$$

Problem 22-21

While standing in an elevator, the man holds a pendulum which consists of cord of length L and a bob of weight W. If the elevator is descending with an acceleration a, determine the natural period of vibration for small amplitudes of swing.



Given:

$$L = 18\ \text{in}$$

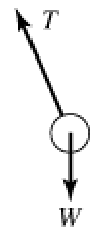
$$W = 0.5\ \text{lb}$$

$$a = 4\,\frac{\text{ft}}{\text{s}^2}$$

$$g = 32.2\,\frac{\text{ft}}{\text{s}^2}$$

Solution:

Since the acceleration of the pendulum is



$$a' = (g - a) \qquad a' = 28.2\ \frac{\text{ft}}{\text{s}^2}$$

Using the result of Example 22-1, we have

$$\omega_n = \sqrt{\frac{a'}{L}} \qquad \omega_n = 4.34\ \frac{\text{rad}}{\text{s}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 1.45\ \text{s}$$

Problem 22-22

The spool of weight W is attached to two springs. If the spool is displaced a small amount and released, determine the natural period of vibration. The radius of gyration of the spool is k_G. The spool rolls without slipping.

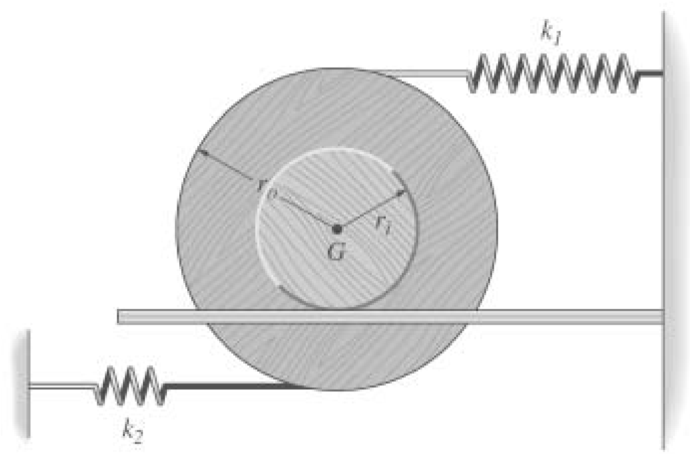
Given:

$W = 50\ \text{lb} \qquad r_i = 1\ \text{ft}$

$k_G = 1.5\ \text{ft} \qquad r_o = 2\ \text{ft}$

$k_1 = 3\ \frac{\text{lb}}{\text{ft}} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$k_2 = 1\ \frac{\text{lb}}{\text{ft}}$



Solution:

$$-k_1(r_o + r_i)\theta(r_o + r_i) - k_2(r_o - r_i)\theta(r_o - r_i) = \left(\frac{W}{g}k_G^2 + \frac{W}{g}r_i^2\right)\theta''$$

$$\left(\frac{W}{g}k_G^2 + \frac{W}{g}r_i^2\right)\theta'' + \left[k_1(r_o + r_i)^2 + k_2(r_o - r_i)^2\right]\theta = 0$$

$$\omega_n = \sqrt{\frac{g}{W\left(k_G^2 + r_i^2\right)}\left[k_1(r_o + r_i)^2 + k_2(r_o - r_i)^2\right]} \qquad \omega_n = 2.36\ \frac{\text{rad}}{\text{s}}$$

$\tau = \dfrac{2\pi}{\omega_n}$ $\boxed{\tau = 2.67 \text{ s}}$

Problem 22-23

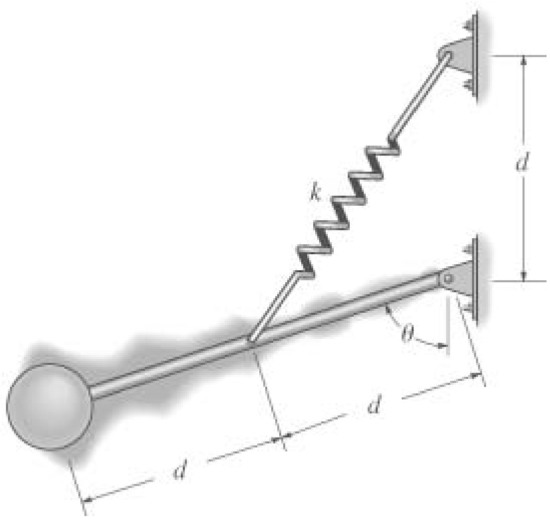
Determine the natural frequency for small oscillations of the sphere of weight W when the rod is displaced a slight distance and released. Neglect the size of the sphere and the mass of the rod. The spring has an unstretched length d.

Given:

$W = 10 \text{ lb}$

$k = 5 \dfrac{\text{lb}}{\text{ft}}$

$d = 1 \text{ ft}$



Solution:

Geometry $L = \sqrt{d^2 + d^2 + 2d\,d\cos(\theta)} = d\sqrt{2(1+\cos(\theta))} = 2d\cos\left(\dfrac{\theta}{2}\right)$

Dynamics $W(2d)\sin(\theta) - k\left(2d\cos\left(\dfrac{\theta}{2}\right) - d\right)\sin\left(\dfrac{\theta}{2}\right)d = \left(\dfrac{-W}{g}\right)(2d)^2\theta''$

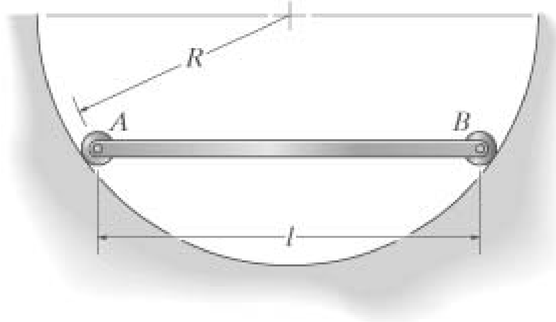
Linearize around $\theta = 0$.

$$\left(\frac{4Wd^2}{g}\right)\theta'' + \left[W(2d) - \frac{kd^2}{2}\right]\theta = 0$$

$\theta'' + \left(\dfrac{g}{2d} - \dfrac{kg}{8W}\right)\theta = 0$ $\omega_n = \sqrt{\dfrac{g}{2d} - \dfrac{kg}{8W}}$ $\omega_n = 3.75 \dfrac{\text{rad}}{\text{s}}$

$f = \dfrac{\omega_n}{2\pi}$ $\boxed{f = 0.60 \dfrac{1}{\text{s}}}$

***Problem 22-24**



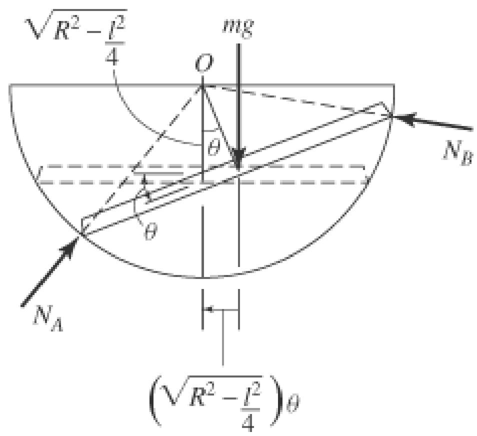
The bar has length l and mass m. It is supported at its ends by rollers of negligible mass. If it is given a small displacement and released, determine the natural frequency of vibration.

Solution:

Moment of inertia about point O:

$$I_O = \frac{1}{12}ml^2 + m\left(\sqrt{R^2 - \frac{l^2}{4}}\right)^2 = m\left(R^2 - \frac{1}{6}l^2\right)$$

$$mg\left(\sqrt{R^2 - \frac{l^2}{4}}\right)\theta = -m\left(R^2 - \frac{1}{6}l^2\right)\theta''$$



$$\theta'' + \frac{3g\sqrt{4R^2 - l^2}}{6R^2 - l^2}\theta = 0$$

From the above differential equation,

$$p = \sqrt{\frac{3g\sqrt{4R^2 - l^2}}{6R^2 - l^2}}$$

$$f = \frac{p}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3g\sqrt{4R^2 - l^2}}{6R^2 - l^2}}$$

Problem 22-25

The weight W is fixed to the end of the rod assembly. If both springs are unstretched when the assembly is in the position shown, determine the natural period of vibration for the weight when it is displaced slightly and released. Neglect the size of the block and the mass of the rods.

Given:

$$W = 25 \text{ lb}$$

$$k = 2 \frac{\text{lb}}{\text{in}}$$

$$l = 12 \text{ in}$$

$$d = 6 \text{ in}$$

Solution:

$$-Wl\theta - 2kd\theta d = \left(\frac{W}{g}\right)l^2\theta''$$

$$\left(\frac{W}{g}\right)l^2\theta'' + \left(Wl + 2kd^2\right)\theta = 0$$

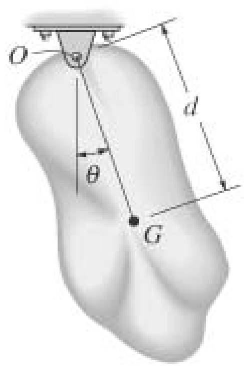
$$\theta'' + \left(\frac{g}{l} + \frac{2kgd^2}{Wl^2}\right)\theta = 0$$

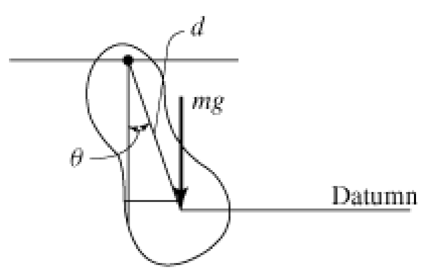
$$\omega_n = \sqrt{\frac{g}{l} + \frac{2kgd^2}{Wl^2}} \qquad T = \frac{2\pi}{\omega_n}$$

$$T = 0.91 \text{ s}$$

Problem 22-26

The body of arbitrary shape has a mass *m*, mass center at *G*, and a radius of gyration about *G* of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration. Solve using energy methods





Solution:

$$T + V = \frac{1}{2}\left(mk_G^{\,2} + md^2\right)\theta'^2 + mgd(1 - \cos(\theta))$$

$$m\left(k_G^{\,2} + d^2\right)\theta'\theta'' + mgd(\sin(\theta))\theta' = 0$$

$$\sin(\theta) \approx \theta$$

$$\theta' + \left(\frac{g\,d}{k_G{}^2 + d^2}\right)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n}$$

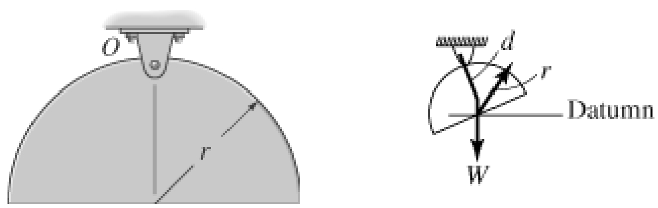
$$\tau = 2\pi\sqrt{\frac{k_G{}^2 + d^2}{g\,d}}$$

Problem 22-27

The semicircular disk has weight W. Determine the natural period of vibration if it is displaced a small amount and released. Solve using energy methods.

Given:

$W = 20 \text{ lb}$



$r = 1 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution:

$$I_O = \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right) - \left(\frac{W}{g}\right)\left(\frac{4r}{3\pi}\right)^2 + \left(\frac{W}{g}\right)\left(r - \frac{4r}{3\pi}\right)^2$$

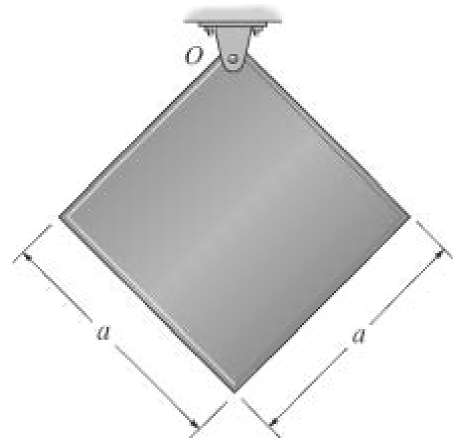
$$T + V = \frac{1}{2}I_O\theta'^2 - W\left(r - \frac{4r}{3\pi}\right)\cos(\theta)$$

$$I_O\theta' + W\left(r - \frac{4r}{3\pi}\right)\theta = 0 \qquad \theta' + \frac{W}{I_O}\left(r - \frac{4r}{3\pi}\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{W}{I_O}\left(r - \frac{4r}{3\pi}\right)} \qquad \omega_n = 5.34\frac{\text{rad}}{\text{s}} \qquad \tau = \frac{2\pi}{\omega_n} \qquad \tau = 1.18 \text{ s}$$

***Problem 22-28**

The square plate has a mass *m* and is suspended at its corner by the pin *O*. Determine the natural period of vibration if it is displaced a small amount and released. Solve using energy methods.



Solution:

$$T + V = \frac{1}{2}\left[\frac{1}{12}m\left(a^2 + a^2\right) + m\left(\frac{a}{\sqrt{2}}\right)^2\right]\theta'^2 + mg\left(\frac{a}{\sqrt{2}}\right)(1 - \cos(\theta))$$

$$\frac{2}{3}ma^2\theta'\theta'' + mg\left(\frac{a}{\sqrt{2}}\right)(\sin(\theta))\theta' = 0$$

$$\theta'' + \left(\frac{3\sqrt{2}g}{4a}\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{3\sqrt{2}g}{4a}} \qquad \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{4}{3\sqrt{2}}}\sqrt{\frac{a}{g}}$$

$$b = 2\pi\sqrt{\frac{4}{3\sqrt{2}}} \qquad \tau = b\sqrt{\frac{a}{g}}$$

$$b = 6.10$$

Problem 22-29

The disk, having weight *W*, is pinned at its center *O* and supports the block *A* that has weight W_A. If the belt which passes over the disk is not allowed to slip at its contacting surface, determine the natural period of vibration of the system.

Given:

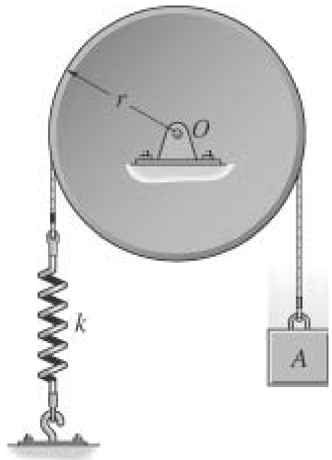
$W = 15$ lb

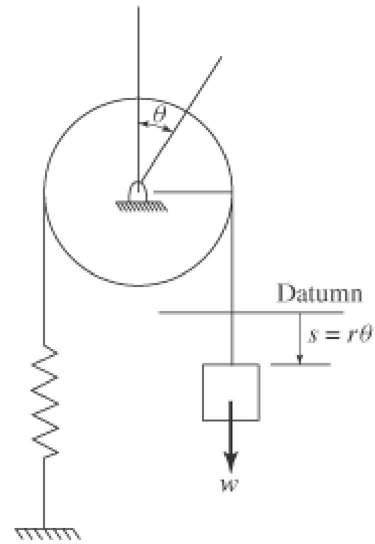
$W_A = 3$ lb

$k = 80\ \frac{\text{lb}}{\text{ft}}$

$r = 0.75\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

$$T + V = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\theta^2 + \frac{1}{2}\left(\frac{W_A}{g}\right)r^2\,\theta^2 + \frac{1}{2}k(r\theta)^2$$

$$\left(\frac{W}{g}\frac{r^2}{2} + \frac{W_A}{g}r^2\right)\theta' + kr^2\theta = 0$$

$$\theta' + \frac{kg}{W_A + \frac{W}{2}}\theta = 0 \qquad \omega_n = \sqrt{\frac{kg}{W_A + \frac{W}{2}}} \qquad \omega_n = 15.66\ \frac{\text{rad}}{\text{s}}$$

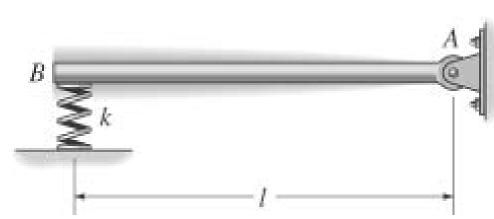
$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 0.401\ \text{s}$$

Problem 22-30

The uniform rod of mass m is supported by a pin at A and a spring at B. If the end B is given a small downward displacement and released, determine the natural period of vibration.

Solution:

$$T + V = \frac{1}{2}m\left(\frac{l^2}{3}\right)\theta^2 + \frac{1}{2}k(l\theta)^2$$



$$m\frac{l^2}{3}\theta' + kl^2\theta = 0 \qquad \theta' + \frac{3k}{m}\theta = 0$$

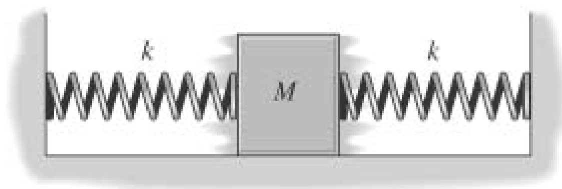
$$\omega_n = \sqrt{\frac{3k}{m}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 2\pi\sqrt{\frac{m}{3k}}$$

Problem 22-31

Determine the differential equation of motion of the block of mass M when it is displaced slightly and released. The surface is smooth and the springs are originally unstretched.

Given:



$$M = 3 \text{ kg}$$

$$k = 500 \frac{\text{N}}{\text{m}}$$

Solution:

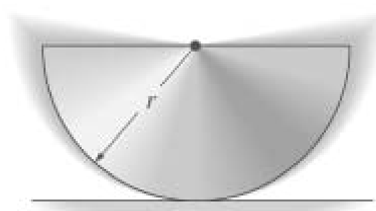
$$T + V = \frac{1}{2}Mx'^2 + 2\frac{1}{2}kx^2$$

$$Mx'' + 2kx = 0 \qquad x'' + \left(\frac{2k}{M}\right)x = 0$$

$$b = \frac{2k}{M} \qquad x'' + bx = 0 \qquad b = 333 \frac{\text{rad}}{\text{s}^2}$$

***Problem 22-32**

Determine the natural period of vibration of the semicircular disk of weight W.



Given:

$$W = 10 \text{ lb} \qquad r = 0.5 \text{ ft}$$

Solution:

$$T + V = \frac{1}{2}\left[\left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right) - \left(\frac{W}{g}\right)\left(\frac{4r}{3\pi}\right)^2 + \left(\frac{W}{g}\right)\left(r - \frac{4r}{3\pi}\right)^2\right]\theta'^2 - W\left(\frac{4r}{3\pi}\right)(1 - \cos(\theta))$$

$$\frac{1}{2}\left(\frac{W}{g}\right)r^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\theta'^2 - W\left(\frac{4r}{3\pi}\right)(1 - \cos(\theta)) = 0$$

$$\left(\frac{W}{g}\right)r^2\left(\frac{3}{2}-\frac{8}{3\pi}\right)\theta'' + W\left(\frac{4r}{3\pi}\right)\theta = 0$$

$$\theta'' + \frac{4g}{3r\pi\left(\frac{3}{2}-\frac{8}{3\pi}\right)}\theta = 0 \qquad \omega_n = \sqrt{\frac{4g}{3r\pi\left(\frac{3}{2}-\frac{8}{3\pi}\right)}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 0.970\ \text{s}$$

Problem 22-33

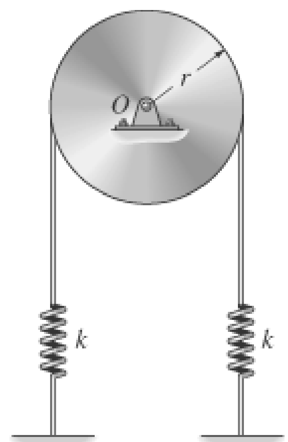
The disk of mass M is pin-connected at its midpoint. Determine the natural period of vibration of the disk if the springs have sufficient tension in them to prevent the cord from slipping on the disk as it oscillates. *Hint:* Assume that the initial stretch in each spring is δ_0. This term will cancel out after taking the time derivative of the energy equation.

Given:

$M = 7\ \text{kg}$

$k = 600\ \frac{\text{N}}{\text{m}}$

$r = 100\ \text{mm}$



Solution:

$$T + V = \frac{1}{2}M\left(\frac{r^2}{2}\right)\theta'^2 + \frac{1}{2}k\left(r\theta + \delta_0\right)^2 + \frac{1}{2}k\left(r\theta - \delta_0\right)^2$$

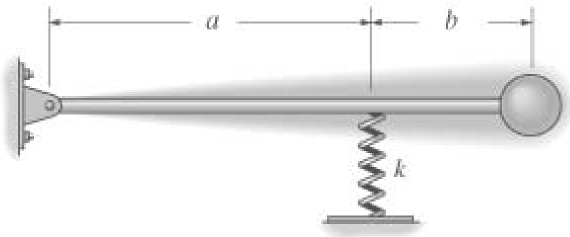
$$M\left(\frac{r^2}{2}\right)\theta'' + 2kr^2\theta = 0 \qquad \theta'' + \left(\frac{4k}{M}\right)\theta = 0$$

$$\tau = 2\pi\sqrt{\frac{M}{4k}} \qquad \tau = 0.339\ \text{s}$$

Problem 22-34

The sphere of weight W is attached to a rod of negligible mass and rests in the horizontal position. Determine the natural frequency of vibration. Neglect the size of the sphere.

Given:



$W = 5\ \text{lb}$ $\quad a = 1\ \text{ft}$

$k = 10\ \dfrac{\text{lb}}{\text{ft}}$ $\quad b = 0.5\ \text{ft}$

Solution:

$$T + V = \frac{1}{2}\left(\frac{W}{g}\right)(a+b)^2\,\theta'^2 + \frac{1}{2}k(a\theta)^2$$

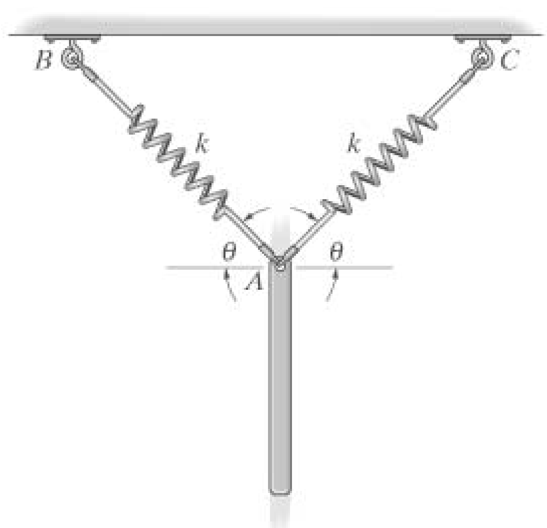
$$\left(\frac{W}{g}\right)(a+b)^2\theta'' + ka^2\theta = 0$$

$$\theta'' + \left[\frac{ka^2 g}{W(a+b)^2}\right]\theta = 0 \qquad \omega_n = \sqrt{\frac{ka^2 g}{W(a+b)^2}}$$

$$f = \frac{\omega_n}{2\pi} \qquad f = 0.85\ \frac{1}{\text{s}}$$

Problem 22-35

The bar has a mass M and is suspended from two springs such that when it is in equilibrium, the springs make an angle θ with the horizontal as shown. Determine the natural period of vibration if the bar is pulled down a short distance and released. Each spring has a stiffness k.



Given:

$M = 8\ \text{kg}$

$\theta = 45\ \text{deg}$

$k = 40\ \dfrac{\text{N}}{\text{m}}$

Solution:

Let $2b$ be the distance between B and C.

$$T + V = \frac{1}{2}My'^2 + \frac{1}{2}(2k)\,\delta^2$$

where

$$\delta = \sqrt{(b\tan(\theta) + y)^2 + b^2} - \sqrt{(b\tan(\theta))^2 + b^2} = \sin(\theta)\,y \qquad \text{for small } y$$

thus

$$\frac{1}{2}M y'^2 + k\sin(\theta)^2 y^2 = 0 \qquad M y'' + 2k\sin(\theta)^2 y = 0$$

$$y'' + \left(\frac{2k\sin(\theta)^2}{M}\right)y = 0$$

$$\omega_n = \sqrt{\frac{2k\sin(\theta)^2}{M}} \qquad \tau = \frac{2\pi}{\omega_n} \qquad \tau = 2.81\ \text{s}$$

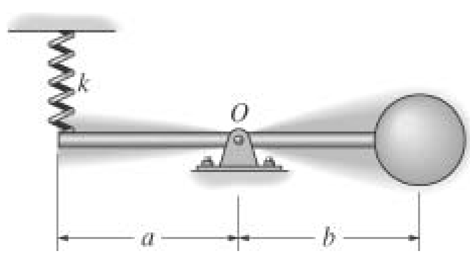
***Problem 22-36**

Determine the natural period of vibration of the sphere of mass M. Neglect the mass of the rod and the size of the sphere.

Given:

$M = 3\ \text{kg}$ $\quad a = 300\ \text{mm}$

$k = 500\ \dfrac{\text{N}}{\text{m}}$ $\quad b = 300\ \text{mm}$



Solution:

$$T + V = \frac{1}{2}M(b\theta')^2 + \frac{1}{2}k(a\theta)^2$$

$$M b^2 \theta'' + k a^2 \theta = 0 \qquad \theta'' + \frac{k a^2}{M b^2}\theta = 0$$

$$\tau = 2\pi\sqrt{\frac{M b^2}{k a^2}} \qquad \tau = 0.487\ \text{s}$$

Problem 22-37

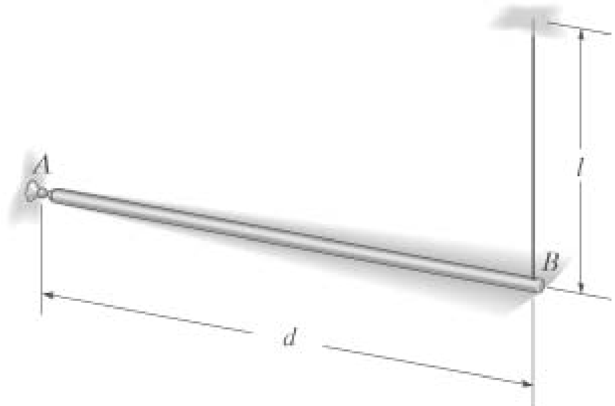
The slender rod has a weight W. If it is supported in the horizontal plane by a ball-and-socket joint at A and a cable at B, determine the natural frequency of vibration when the end B is given a small horizontal displacement and then released.

Given:

$$W = 4\ \frac{\text{lb}}{\text{ft}}$$

$$d = 1.5\ \text{ft}$$

$$l = 0.75\ \text{ft}$$



Solution:

$$d\theta = l\phi \qquad \phi = \frac{d\theta}{l}$$

$$T + V = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{d^2}{3}\right)\theta'^2 + \frac{W}{2}l\left(1 - \cos\left(\frac{d\theta}{l}\right)\right)$$

$$\left(\frac{Wd^2}{3g}\right)\theta'' + \left(\frac{W}{2}\right)l\sin\left(\frac{d\theta}{l}\right)\frac{d}{l} = 0$$

$$\left(\frac{Wd^2}{3g}\right)\theta'' + \left(\frac{Wd^2}{2l}\right)\theta = 0$$

$$\theta'' + \frac{3g}{2l}\theta = 0$$

$$f = \frac{1}{2\pi}\sqrt{\frac{3g}{2l}} \qquad f = 1.28\ \frac{1}{\text{s}}$$

Problem 22-38

Determine the natural frequency of vibration of the disk of weight W. Assume the disk does not slip on the inclined surface.

Given:

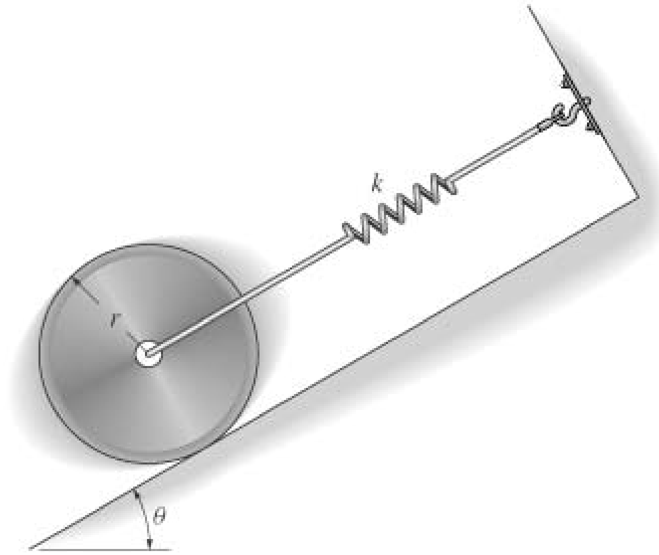
$$W = 20\ \text{lb}$$

$k = 10\ \frac{\text{lb}}{\text{in}}$

$\theta = 30\ \text{deg}$

$r = 1\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$T + V = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{3r^2}{2}\right)\theta'^2 + \frac{1}{2}k(r\theta)^2$$

$$\left(\frac{W}{g}\right)\left(\frac{3r^2}{2}\right)\theta'' + kr^2\theta = 0$$

$$\theta'' + \frac{2kg}{3W}\theta = 0 \qquad f = \frac{1}{2\pi}\sqrt{\frac{2kg}{3W}} \qquad f = 1.81\ \frac{1}{\text{s}}$$

Problem 22-39

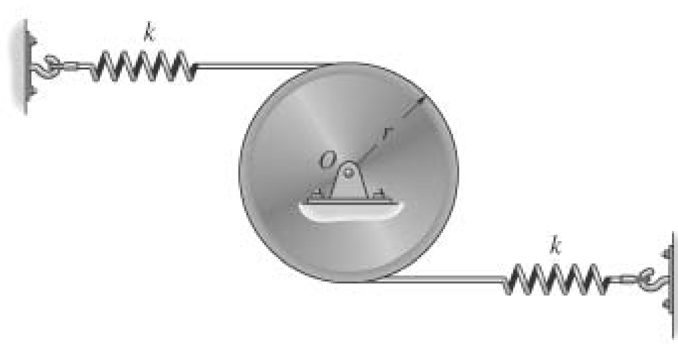
If the disk has mass *M*, determine the natural frequency of vibration. The springs are originally unstretched.

Given:

$M = 8\ \text{kg}$

$k = 400\ \frac{\text{N}}{\text{m}}$

$r = 100\ \text{mm}$



Solution:

$$T + V = \frac{1}{2}\left(\frac{Mr^2}{2}\right)\theta'^2 + 2\,\frac{1}{2}k(r\theta)^2$$

$$M\left(\frac{r^2}{2}\right)\theta'' + 2kr^2\theta = 0$$

$$\theta'' + \frac{4k}{M}\theta = 0 \qquad f = \frac{1}{2\pi}\sqrt{\frac{4k}{M}} \qquad f = 2.25\ \frac{1}{s}$$

***Problem 22-40**

Determine the differential equation of motion of the spool of mass M. Assume that it does not slip at the surface of contact as it oscillates. The radius of gyration of the spool about its center of mass is k_G.

Given:

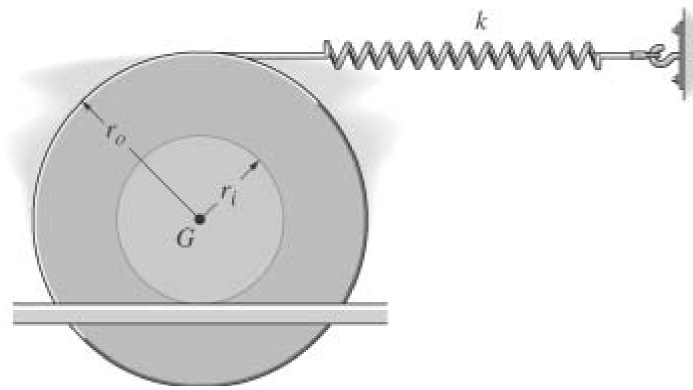
$M = 3\ \text{kg}$

$k_G = 125\ \text{mm}$

$r_i = 100\ \text{mm}$

$r_o = 200\ \text{mm}$

$k = 400\ \frac{\text{N}}{\text{m}}$



Solution:

$$T + V = \frac{1}{2}M\left(k_G^2 + r_i^2\right)\theta'^2 + \frac{1}{2}k\left[\left(r_o + r_i\right)\theta\right]^2$$

$$M\left(k_G^2 + r_i^2\right)\theta'' + k\left(r_o + r_i\right)^2\theta = 0$$

$$\omega_n = \sqrt{\frac{k\left(r_o + r_i\right)^2}{M\left(k_G^2 + r_i^2\right)}}$$

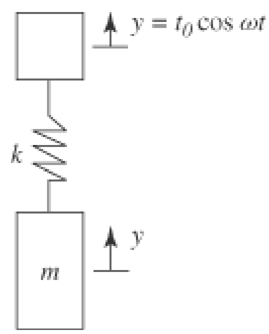
$$\theta'' + \omega_n^2\theta = 0$$

where $\omega_n^2 = 468\ \frac{\text{rad}}{\text{s}^2}$

Problem 22-41

Use a block-and-spring model like that shown in Fig. 22-14*a* but suspended from a vertical position and subjected to a periodic support displacement of $\delta = \delta_0 \cos\omega t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.

Solution:



For Static Equilibrium $\quad mg = k\delta_{st}$

Equation of Motion is then

$$k(\delta + \delta_{st} - y) - mg = my''$$

$$my'' + ky = k\delta_0 \cos(\omega t)$$

$$y'' + \frac{k}{m}y = \frac{k}{m}\delta_0 \cos(\omega t)$$

The solution consists of a homogeneous part and a particular part

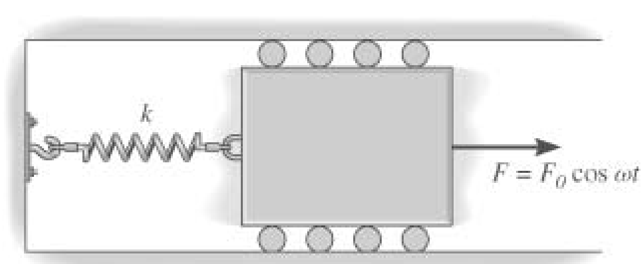
$$y(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t\right) + B\sin\left(\sqrt{\frac{k}{m}}\,t\right) + \frac{\delta_0}{1 - \frac{m\omega^2}{k}}\cos(\omega t)$$

The constants A and B are determined from the initial conditions.

Problem 22-42

The block of weight W is attached to a spring having stiffness k. A force $F = F_0 \cos\omega t$ is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.

Given:



$W = 20\ \text{lb}$

$k = 20\ \frac{\text{lb}}{\text{ft}}$

$F_0 = 6\ \text{lb}$

$$\omega = 2\ \frac{\text{rad}}{\text{s}}$$

$$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

Solution:

$$\omega_n = \sqrt{\frac{k\,g}{W}} \qquad C = \frac{\frac{F_0\,g}{W}}{\frac{k\,g}{W} - \omega^2}$$

$$x = C\cos(\omega t) \qquad v = -C\,\omega\sin(\omega t)$$

$$v_{max} = C\omega \qquad \boxed{v_{max} = 0.685\ \frac{\text{ft}}{\text{s}}}$$

Problem 22-43

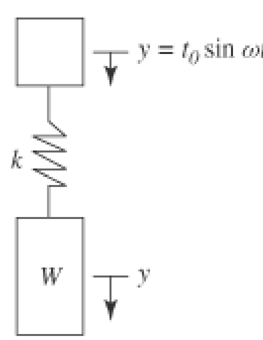
A weight W is attached to a spring having a stiffness k. The weight is drawn downward a distance d and released from rest. If the support moves with a vertical displacement $\delta = \delta_0 \sin\omega t$, determine the equation which describes the position of the weight as a function of time.

Given:

$$W = 4\ \text{lb} \qquad \delta_0 = 0.5\ \text{in}$$

$$k = 10\ \frac{\text{lb}}{\text{ft}} \qquad \omega = 4\ \frac{\text{rad}}{\text{s}}$$

$$d = 4\ \text{in}$$



Solution:

For Static Equilibrium $\quad W = k\delta_{st}$

Equation of Motion is then

$$k(y + \delta_{st} - \delta) - W = \left(\frac{-W}{g}\right)y''$$

$$\left(\frac{W}{g}\right)y'' + k\,y = k\delta_0 \sin(\omega t)$$

$$y'' + \left(\frac{kg}{W}\right)y = \left(\frac{kg}{W}\right)\delta_0 \sin(\omega t)$$

The solution consists of a homogeneous part and a particular part

$$y(t) = A\cos\left(\sqrt{\frac{kg}{W}}\,t\right) + B\sin\left(\sqrt{\frac{kg}{W}}\,t\right) + \frac{\delta_0}{1 - \frac{W\omega^2}{kg}}\sin(\omega t)$$

The constants A and B are determined from the initial conditions.

$$A = d \qquad B = \frac{-\delta_0\,\omega}{\left(1 - \frac{W\omega^2}{kg}\right)\sqrt{\frac{kg}{W}}} \qquad C = \frac{\delta_0}{1 - \frac{W\omega^2}{kg}} \qquad p = \sqrt{\frac{kg}{W}}$$

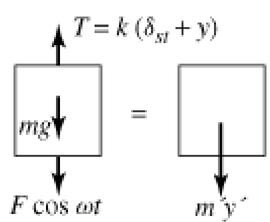
$$y = A\cos(p\,t) + B\sin(p\,t) + C\sin(\omega t)$$

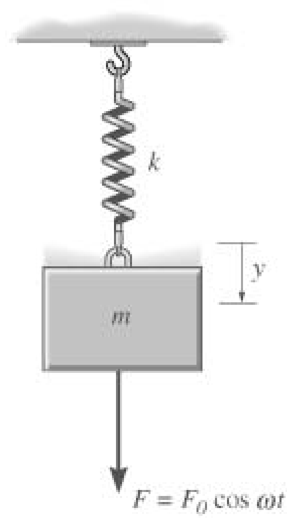
where $A = 0.33\ \text{ft}$ $B = -0.0232\ \text{ft}$

$C = 0.05\ \text{ft}$ $p = 8.97\ \frac{\text{rad}}{\text{s}}$ $\omega = 4.00\ \frac{\text{rad}}{\text{s}}$

***Problem 22-44**

If the block is subjected to the impressed force $F = F_0\cos(\omega\, t)$, show that the differential equation of motion is $y'' + (k/m)y = (F_0/m)\cos(\omega\, t)$, where y is measured from the equilibrium position of the block. What is the general solution of this equation ?





Solution:

$$W = k\delta_{st}$$

$$F_0 \cos(\omega t) + mg - k(\delta_{st} + y) = m y''$$

$$y'' + \left(\frac{k}{m}\right)y = \frac{F_0}{m}\cos(\omega t)$$ Q.E.D.

$$y = A\sin\left(\sqrt{\frac{k}{m}}\,t\right) + B\cos\left(\sqrt{\frac{k}{m}}\,t\right) + \left(\frac{F_o}{k - m\omega^2}\right)\cos(\omega t)$$

Problem 22-45

The light elastic rod supports the sphere of mass M. When a vertical force P is applied to the sphere, the rod deflects a distance d. If the wall oscillates with harmonic frequency f and has amplitude A, determine the amplitude of vibration for the sphere.

Given:

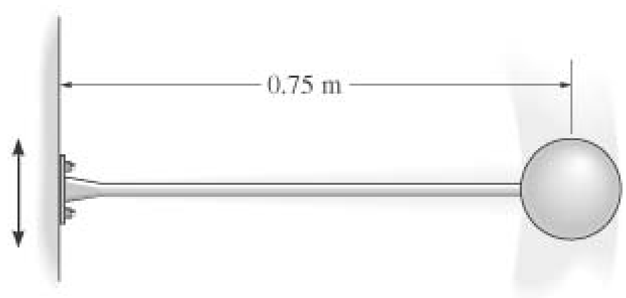
$M = 4\ \text{kg}$

$P = 18\ \text{N}$

$\delta = 14\ \text{mm}$

$f = 2\ \text{Hz}$

$A = 15\ \text{mm}$



Solution:

$$k = \frac{P}{\delta}$$

$$\omega = 2\pi f \qquad \omega_n = \sqrt{\frac{k}{M}}$$

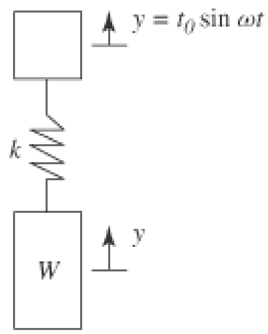
$$x_{pmax} = \left|\frac{A}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right| \qquad x_{pmax} = 29.5\ \text{mm}$$

Problem 22-46

Use a block-and-spring model like that shown in Fig. 22-14*a* but suspended from a vertical position and subjected to a periodic support displacement of $\delta = \delta_0 \sin\omega t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.

Solution:

For Static Equilibrium $\quad m g = k\delta_{st}$



Equation of Motion is then

$$k\left(\delta + \delta_{st} - y\right) - m g = m y''$$

$$m y'' + k y = k\delta_0 \sin(\omega t)$$

$$y'' + \frac{k}{m} y = \frac{k}{m}\delta_0 \sin(\omega t)$$

The solution consists of a homogeneous part and a particular part

$$y(t) = A\sin\left(\sqrt{\frac{k}{m}}\, t\right) + B\cos\left(\sqrt{\frac{k}{m}}\, t\right) + \frac{\delta_0}{1 - \dfrac{m\omega^2}{k}} \sin(\omega t)$$

The constants *A* and *B* are determined from the initial conditions.

Problem 22-47

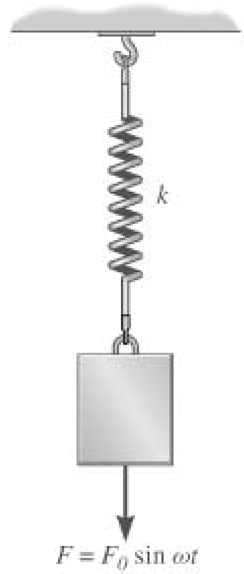
A block of mass *M* is suspended from a spring having a stiffness *k*. If the block is acted upon by a vertical force $F = F_0 \sin\omega t$, determine the equation which describes the motion of the block when it is pulled down a distance *d* from the equilibrium position and released from rest at $t = 0$. Assume that positive displacement is downward.

Given:

$M = 5\ \text{kg}$

$k = 300\ \frac{\text{N}}{\text{m}}$

$F_0 = 7\ \text{N}$



$\omega = 8 \frac{\text{rad}}{\text{s}}$

$d = 100 \text{ mm}$

Solution:

$\omega_n = \sqrt{\frac{k}{M}}$ $\quad C = \frac{F_0}{M(\omega_n^2 - \omega^2)}$

$y = A\sin(\omega_n t) + B\cos(\omega_n t) + C\sin(\omega t)$

$y' = A\omega_n \cos(\omega_n t) - B\omega_n \sin(\omega_n t) + C\omega\cos(\omega t)$

$y = d$ when $t = 0$ $\quad B = d$

$y = y' = 0$ when $t = 0$ $\quad A = -C\frac{\omega}{\omega_n}$

$y = A\sin(\omega_n t) + B\cos(\omega_n t) + C\sin(\omega t)$

$\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 361 \\ 100 \\ -350 \end{pmatrix} \text{mm}$ $\quad \begin{pmatrix} \omega_n \\ \omega \end{pmatrix} = \begin{pmatrix} 7.75 \\ 8.00 \end{pmatrix} \frac{\text{rad}}{\text{s}}$

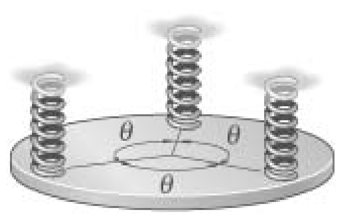
***Problem 22-48**

The circular disk of mass M is attached to three springs, each spring having a stiffness k. If the disk is immersed in a fluid and given a downward velocity v at the equilibrium position, determine the equation which describes the motion. Assume that positive displacement is measured downward, and that fluid resistance acting on the disk furnishes a damping force having a magnitude $F = cv$.

Given:

$M = 4 \text{ kg}$ $\quad c = 60 \frac{\text{kg}}{\text{s}}$

$k = 180 \frac{\text{N}}{\text{m}}$ $\quad \theta = 120 \text{ deg}$



$v = 0.3 \frac{\text{m}}{\text{s}}$

Solution:

$$M y'' + c y' + 3k y = 0 \qquad y'' + \frac{c}{M} y' + \frac{3k}{M} y = 0$$

$$\omega_n = \sqrt{\frac{3k}{M}} \qquad \zeta = \frac{c}{2M\omega_n}$$ Since $\zeta = 0.65 < 1$ the system is underdamped

$$b = \zeta\,\omega_n \qquad \omega_d = \sqrt{1-\zeta^2}\,\omega_n$$

$$y(t) = e^{-b\,t}\left(A\cos\left(\omega_d\,t\right) + B\sin\left(\omega_d\,t\right)\right)$$

Now find A and B from initial conditions. Guesses $A = 1\text{ m}$ $B = 1\text{ m}$

Given $0 = A$ $v = -A\,b + B\omega_d$ $\begin{pmatrix} A \\ B \end{pmatrix} = \text{Find}(A, B)$

$$y(t) = e^{-b\,t}\left(A\cos\left(\omega_d\,t\right) + B\sin\left(\omega_d\,t\right)\right)$$

where $b = 7.50\,\frac{\text{rad}}{\text{s}}$ $\omega_d = 8.87\,\frac{\text{rad}}{\text{s}}$ $A = 0.0\,\text{mm}$ $B = 33.8\,\text{mm}$

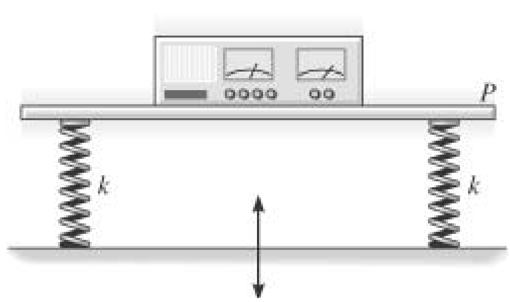
Problem 22-49

The instrument is centered uniformly on a platform P, which in turn is supported by *four* springs, each spring having stiffness k. If the floor is subjected to a vibration f, having a vertical displacement amplitude δ_0, determine the vertical displacement amplitude of the platform and instrument. The instrument and the platform have a total weight W.

Given:

$k = 130\,\frac{\text{lb}}{\text{ft}}$ $\delta_0 = 0.17\text{ ft}$

$f = 7\text{ Hz}$ $W = 18\text{ lb}$



Solution:

$$\omega_n = \sqrt{\frac{4kg}{W}} \qquad \omega_n = 30.50\,\frac{\text{rad}}{\text{s}}$$

$\omega = 2\pi f \qquad \omega = 43.98 \frac{\text{rad}}{\text{s}}$

Using Eq. 22-22, the amplitude is $\quad x_{pmax} = \left| \dfrac{\delta_0}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2} \right| \qquad x_{pmax} = 1.89 \text{ in}$

Problem 22-50

A trailer of mass M is pulled with a constant speed over the surface of a bumpy road, which may be approximated by a cosine curve having an amplitude a and wave length $2d$. If the two springs s which support the trailer each have a stiffness k, determine the speed v which will cause the greatest vibration (resonance) of the trailer. Neglect the weight of the wheels.

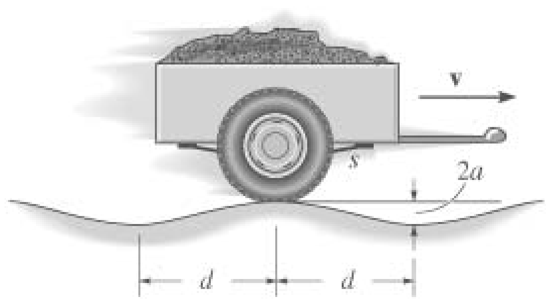
Given:

$M = 450 \text{ kg}$

$k = 800 \frac{\text{N}}{\text{m}}$

$d = 2 \text{ m}$

$a = 50 \text{ mm}$



Solution:

$p = \sqrt{\dfrac{2k}{M}} \qquad \tau = \dfrac{2\pi}{p} \qquad \tau = 3.33 \text{ s}$

For maximum vibration of the trailer, resonance must occur, $\quad \omega = p$

Thus the trailer must travel so that $\quad v_R = \dfrac{2d}{\tau} \qquad v_R = 1.20 \frac{\text{m}}{\text{s}}$

Problem 22-51

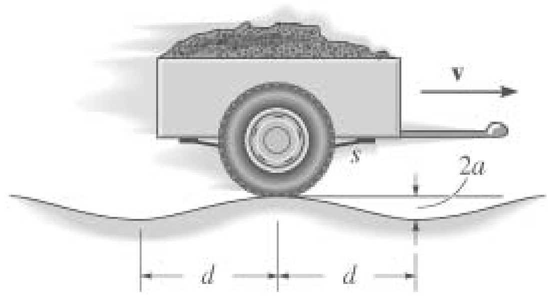
The trailer of mass M is pulled with a constant speed over the surface of a bumpy road, which may be approximated by a cosine curve having an amplitude a and wave length $2d$. If the two springs s which support the trailer each have a stiffness k, determine the amplitude of vibration of the trailer if the speed is v.

Given:

$M = 450\ \text{kg}$ $d = 2\ \text{m}$

$k = 800\ \dfrac{\text{N}}{\text{m}}$ $v = 15\ \dfrac{\text{km}}{\text{hr}}$

$a = 50\ \text{mm}$



Solution:

$$p = \sqrt{\frac{2k}{M}} \qquad \tau = \frac{2d}{v} \qquad \omega = \frac{2\pi}{\tau}$$

$$x_{max} = \left|\frac{a}{1 - \left(\frac{\omega}{p}\right)^2}\right| \qquad x_{max} = 4.53\ \text{mm}$$

***Problem 22-52**

The electric motor turns an eccentric flywheel which is equivalent to an unbalanced weight W_b located distance d from the axis of rotation. If the static deflection of the beam is δ because of the weight of the motor, determine the angular velocity of the flywheel at which resonance will occur. The motor has weight W_m. Neglect the mass of the beam.

Given:

$W_b = 0.25\ \text{lb}$ $W_m = 150\ \text{lb}$

$d = 10\ \text{in}$ $g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$\delta = 1\ \text{in}$

Solution:

$$k = \frac{W_m}{\delta} \qquad k = 1800\ \frac{\text{lb}}{\text{ft}}$$

$$\omega_n = \sqrt{\frac{kg}{W_m}} \qquad \omega_n = 19.66\ \frac{\text{rad}}{\text{s}}$$

Resonance occurs when $\omega = \omega_n$ $\qquad \omega = 19.7\ \dfrac{\text{rad}}{\text{s}}$

Problem 22-53

The electric motor turns an eccentric flywheel which is equivalent to an unbalanced weight W_b located distance d from the axis of rotation. The static deflection of the beam is δ because of the weight of the motor. The motor has weight W_m. Neglect the mass of the beam. What will be the amplitude of steady-state vibration of the motor if the angular velocity of the flywheel is ω?

Given:

$W_b = 0.25\ \text{lb}$ $\quad W_m = 150\ \text{lb}$

$d = 10\ \text{in}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$\delta = 1\ \text{in}$ $\quad \omega = 20\ \dfrac{\text{rad}}{\text{s}}$

Solution:

$$k = \frac{W_m}{\delta} \qquad k = 1800\,\frac{\text{lb}}{\text{ft}}$$

$$\omega_n = \sqrt{\frac{kg}{W_m}} \qquad \omega_n = 19.66\,\frac{\text{rad}}{\text{s}}$$

$$F_0 = \frac{W_b}{g}d\omega^2 \qquad F_0 = 2.59\,\text{lb}$$

From Eq. 22-21, the amplitude of the steady state motion is

$$C = \left[\frac{\dfrac{F_0}{k}}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}\right] \qquad |C| = 0.490\,\text{in}$$

Problem 22-54

The electric motor turns an eccentric flywheel which is equivalent to an unbalanced weight W_b located distance d from the axis of rotation. The static deflection of the beam is δ because of the weight of the motor. The motor has weight W_m. Neglect the mass of the beam. Determine the angular velocity of the flywheel which will produce an amplitude of vibration C.

Given:

$W_b = 0.25\ \text{lb}$

$W_m = 150\ \text{lb}$

$d = 10\ \text{in}$

$\delta = 1\ \text{in}$

$C = 0.25\ \text{in}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$k = \frac{W_m}{\delta}$ $\quad k = 1800\frac{\text{lb}}{\text{ft}}$ $\quad \omega_n = \sqrt{\frac{k}{\frac{W_m}{g}}}$ $\quad \omega_n = 19.657\frac{\text{rad}}{\text{s}}$

There are 2 correct answers to this problem. We can find these 2 answers by starting with different inital guesses.

$\omega = 25\frac{\text{rad}}{\text{s}}$ Given $C = \frac{\frac{W_b d\omega^2}{g k}}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|}$ $\omega = \text{Find}(\omega)$ $\omega = 20.3\ \frac{1}{\text{s}}$

$\omega = 18\frac{\text{rad}}{\text{s}}$ Given $C = \frac{\frac{W_b d\omega^2}{g k}}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|}$ $\omega = \text{Find}(\omega)$ $\omega = 19.0\ \frac{1}{\text{s}}$

Problem 22-55

The engine is mounted on a foundation block which is spring-supported. Describe the steady-state vibration of the system if the block and engine have total weight W and the engine, when running, creates an impressed force $F = F_0 \sin\omega t$. Assume that the system vibrates only in the vertical direction, with the positive displacement measured downward, and that the total stiffness of the springs can be represented as k.

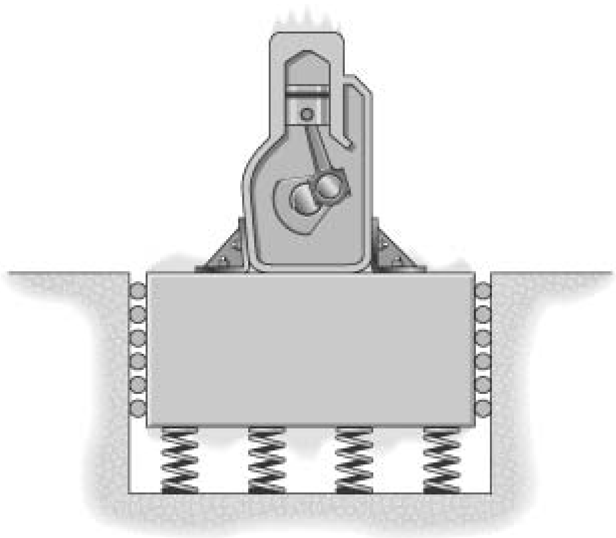
Given:

$W = 1500\ \text{lb}$

$F_0 = 50 \text{ lb}$

$\omega = 2 \frac{\text{rad}}{\text{s}}$

$k = 2000 \frac{\text{lb}}{\text{ft}}$



Solution:

$\omega_n = \sqrt{\frac{k g}{W}}$ $\quad \omega_n = 6.55 \frac{\text{rad}}{\text{s}}$

$$C = \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$x_p = C \sin(\omega t)$ $\quad \omega = 2.00 \frac{\text{rad}}{\text{s}}$ $\quad C = 0.0276 \text{ ft}$

***Problem 22-56**

The engine is mounted on a foundation block which is spring-supported. The block and engine have total weight W and the engine, when running, creates an impressed force $F = F_0 \sin\omega t$. Assume that the system vibrates only in the vertical direction, with the positive displacement measured downward, and that the total stiffness of the springs can be represented as k. What rotational speed ω will cause resonance?

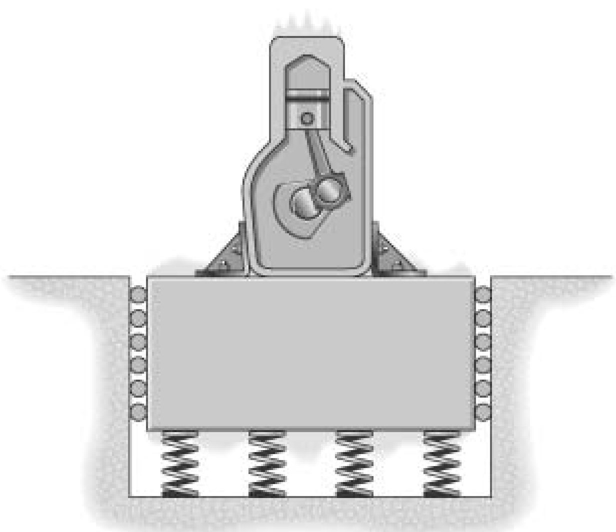
Given:

$W = 1500 \text{ lb}$

$F_0 = 50 \text{ lb}$

$\omega = 2 \frac{\text{rad}}{\text{s}}$

$k = 2000 \frac{\text{lb}}{\text{ft}}$

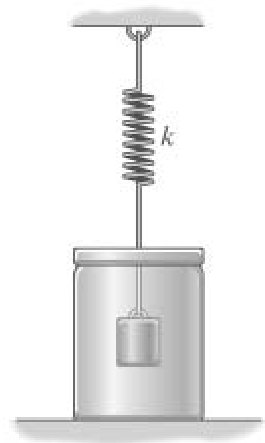


Solution:

$$\omega_n = \sqrt{\frac{k g}{W}} \qquad \omega_n = 6.55 \frac{\text{rad}}{\text{s}} \qquad \omega = \omega_n \qquad \omega = 6.55 \frac{\text{rad}}{\text{s}}$$

Problem 22-57

The block, having weight W, is immersed in a liquid such that the damping force acting on the block has a magnitude of $F = c|v|$. If the block is pulled down at a distance d and released from rest, determine the position of the block as a function of time. The spring has a stiffness k. Assume that positive displacement is downward.



Given:

$$W = 12 \text{ lb} \qquad d = 0.62 \text{ ft}$$

$$c = 0.7 \frac{\text{lb s}}{\text{ft}} \qquad g = 32.2 \frac{\text{ft}}{\text{s}^2}$$

$$k = 53 \frac{\text{lb}}{\text{ft}}$$

Solution:

$$\omega_n = \sqrt{\frac{k g}{W}} \qquad \zeta = \frac{c g}{2 W \omega_n}$$

Since $\zeta = 0.08 \quad < 1$ the system is underdamped

$$b = \zeta \omega_n \qquad \omega_d = \sqrt{1 - \zeta^2}\, \omega_n$$

We can write the solution as $\quad y(t) = B e^{-b t} \sin(\omega_d t + \phi)$

To solve for the constants B and ϕ

Guesses $\quad B = 1 \text{ ft} \qquad \phi = 1 \text{ rad}$

Given $\quad B \sin(\phi) = d \qquad B \omega_d \cos(\phi) - b B \sin(\phi) = 0 \qquad \begin{pmatrix} B \\ \phi \end{pmatrix} = \text{Find}(B, \phi)$

Thus

$$y(t) = B e^{-b t} \sin(\omega_d t + \phi)$$

where $B = 0.62\ \text{ft}$ $b = 0.94\ \frac{\text{rad}}{\text{s}}$

$\omega_d = 11.9\ \frac{\text{rad}}{\text{s}}$ $\phi = 1.49\ \text{rad}$

Problem 22-58

A block of weight W is suspended from a spring having stiffness k. The support to which the spring is attached is given simple harmonic motion which may be expressed as $\delta = \delta_0 \sin\omega t$. If the damping factor is C_{ratio}, determine the phase angle ϕ of the forced vibration.

Given:

$W = 7\ \text{lb}$ $\omega = 2\ \frac{\text{rad}}{\text{s}}$

$k = 75\ \frac{\text{lb}}{\text{ft}}$ $C_{ratio} = 0.8$

$\delta_0 = 0.15\ \text{ft}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$\omega_n = \sqrt{\frac{kg}{W}}$ $\omega_n = 18.57\ \frac{\text{rad}}{\text{s}}$

$$\phi = \text{atan}\left[\frac{2C_{ratio}\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right] \qquad \phi = 9.89\ \text{deg}$$

Problem 22-59

A block of weight W is suspended from a spring having stiffness k. The support to which the spring is attached is given simple harmonic motion which may be expressed as $\delta = \delta_0 \sin\omega t$. If the damping factor is C_{ratio}, determine the magnification factor of the forced vibration.

Given:

$W = 7\ \text{lb}$ $\omega = 2\ \frac{\text{rad}}{\text{s}}$

$k = 75\ \frac{\text{lb}}{\text{ft}}$ $\quad C_{ratio} = 0.8$

$\delta_0 = 0.15\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$\omega_n = \sqrt{\frac{kg}{W}}$ $\quad \omega_n = 18.57\ \frac{\text{rad}}{\text{s}}$

$$MF = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2(C_{ratio})\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$MF = 0.997$

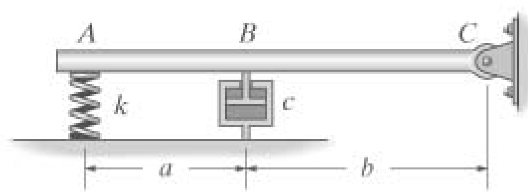
***Problem 22-60**

The bar has a weight W. If the stiffness of the spring is k and the dashpot has a damping coefficient c, determine the differential equation which describes the motion in terms of the angle θ of the bar's rotation. Also, what should be the damping coefficient of the dashpot if the bar is to be critically damped?

Given:

$W = 6\ \text{lb}$ $\quad b = 3\ \text{ft}$ $\quad c = 60\ \frac{\text{lb}\cdot\text{s}}{\text{ft}}$

$k = 8\ \frac{\text{lb}}{\text{ft}}$ $\quad a = 2\ \text{ft}$



Solution:

$$\left(\frac{W}{g}\right)\frac{(a+b)^2}{3}\theta'' + cb^2\theta' + k(a+b)^2\theta = 0$$

$M = \left(\frac{W}{g}\right)\frac{(a+b)^2}{3}$ $\quad C = cb^2$ $\quad K = k(a+b)^2$

$M\theta'' + C\theta' + K\theta = 0$

where $M = 1.55\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

$C = 540.00\ \text{lb}\cdot\text{ft}\cdot\text{s}$

$K = 200.00\ \text{lb}\cdot\text{ft}$

To find critical damping

$$\omega_n = \sqrt{\frac{K}{M}} \qquad C = 2M\omega_n$$

$$c = \frac{C}{b^2} \qquad c = 3.92\,\frac{\text{lb}\cdot\text{s}}{\text{ft}}$$

Problem 22-61

A block having mass M is suspended from a spring that has stiffness k. If the block is given an upward velocity v from its equilibrium position at $t = 0$, determine its position as a function of time. Assume that positive displacement of the block is downward and that motion takes place in a medium which furnishes a damping force $F = C\,|v|$.

Given: $M = 7\ \text{kg}$ $\quad k = 600\ \frac{\text{N}}{\text{m}}$ $\quad C = 50\ \frac{\text{N}\cdot\text{s}}{\text{m}}$ $\quad v = 0.6\,\frac{\text{m}}{\text{s}}$

Solution:

$$\omega_n = \sqrt{\frac{k}{M}} \qquad \omega_n = 9.258\,\frac{\text{rad}}{\text{s}}$$

$$C_c = 2M\omega_n \qquad C_c = 129.6\,\frac{\text{N}\cdot\text{s}}{\text{m}}$$

If $C = 50.00\,\text{N}\cdot\frac{\text{s}}{\text{m}} < C_c = 129.61\,\text{N}\cdot\frac{\text{s}}{\text{m}}$ the system is underdamped

$$b = \frac{-C}{C_c}\omega_n \qquad \omega_d = \omega_n\sqrt{1-\left(\frac{C}{C_c}\right)^2}$$

$$y = 0\ \text{m} \quad y' = -v \qquad A = 1\ \text{m} \qquad \phi = 1\ \text{rad} \qquad t = 0\ \text{s}$$

Given $\quad y = A\,e^{b\,t}\sin\left(\omega_d\,t + \phi\right)$

$$y' = A\,b\,e^{b\,t}\sin\left(\omega_d\,t+\phi\right) + A\,\omega_d\,e^{b\,t}\cos\left(\omega_d\,t+\phi\right)$$

$$\begin{pmatrix} A \\ \phi \end{pmatrix} = \text{Find}(A, \phi)$$

$$y = A e^{b\,t} \sin(\omega_d t + \phi)$$

$A = -0.0702\ \text{m}$ $\quad b = -3.57\ \dfrac{\text{rad}}{\text{s}}$ $\quad \omega_d = 8.54\ \dfrac{\text{rad}}{\text{s}}$ $\quad \phi = 0.00\ \text{rad}$

Problem 22-62

The damping factor C/C_c, may be determined experimentally by measuring the successive amplitudes of vibrating motion of a system. If two of these maximum displacements can be approximated by x_1 and x_2, as shown in Fig. 22-17, show that the ratio $\ln(x_1/x_2) = 2\pi(C/C_c)/(1-(C/C_e)^2)^{1/2}$. The quantity $\ln(x_1/x_2)$ is called the logarithmic decrement.

Solution:

$$x = D\left(e^{-\frac{C}{2m}t}\sin(\omega_d t + \phi)\right)$$

$$x_{max} = D e^{-\frac{C}{2m}t} \qquad x_1 = D e^{-\frac{C}{2m}t_1} \qquad x_2 = D e^{-\frac{C}{2m}t_2}$$

$$\frac{x_1}{x_2} = \frac{D e^{-\frac{C}{2m}t_1}}{D e^{-\frac{C}{2m}t_2}} = e^{\frac{C}{2m}(t_2 - t_1)}$$

Since $\omega_d t_2 - \omega_d t_1 = 2\pi$ then $t_2 - t_1 = \dfrac{2\pi}{\omega_d}$

so that $\ln\left(\dfrac{x_1}{x_2}\right) = \dfrac{C\pi}{m\omega_d}$

$$C_c = 2m\omega_n \qquad \omega_d = \omega_n\sqrt{1 - \left(\frac{C}{C_c}\right)^2} = \frac{C_c}{2m}\sqrt{1 - \left(\frac{C}{C_c}\right)^2}$$

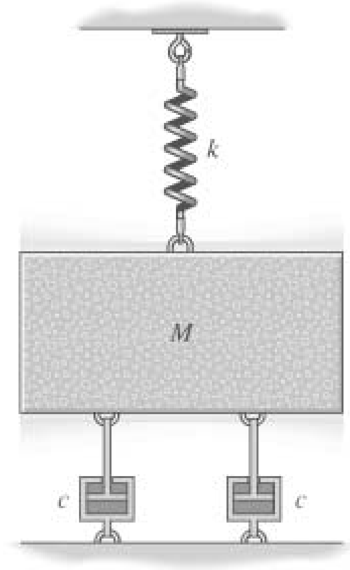
So that,

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{C}{C_c}\right)}{\sqrt{1 - \left(\frac{C}{C_c}\right)^2}}$$

Q.E.D.

Problem 22-63

Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs?



Given:

$M = 25\ \text{kg} \qquad k = 100\ \frac{\text{N}}{\text{m}} \qquad c = 200\ \frac{\text{N}\cdot\text{s}}{\text{m}}$

Solution:

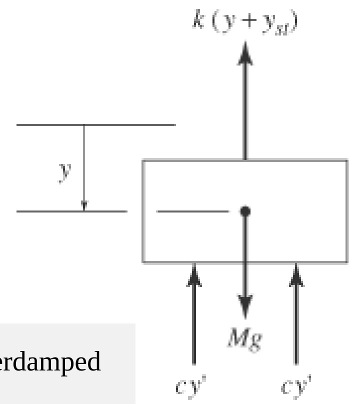
$$M g - k(y + y_{st}) - 2c y' = M y''$$

$$M y'' + k y + 2c y' + k y_{st} - M g = 0$$

Equilibrium $\quad k y_{st} - M g = 0$

$$M y'' + 2c y' + k y = 0 \qquad (1)$$

$$y'' + \frac{2c}{M} y' + \frac{k}{M} y = 0$$



By comparing Eq.(1) to Eq. 22-27

$p = \sqrt{\frac{k}{M}} \qquad p = 2.00\ \frac{\text{rad}}{\text{s}}$

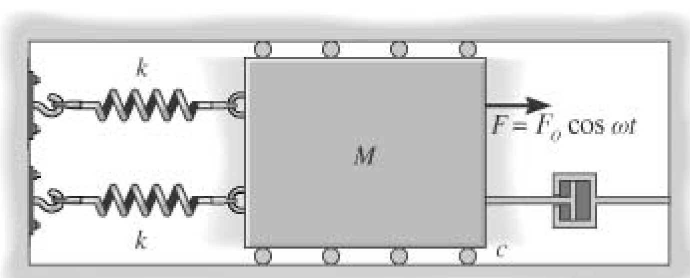
$c_c = 2M p \qquad c_c = 100.00\ \text{N}\cdot\frac{\text{s}}{\text{m}}$

Since $c = 200.00\ \frac{\text{N}\cdot\text{s}}{\text{m}} > c_c = 100.00\ \frac{\text{N}\cdot\text{s}}{\text{m}}$, the system is overdamped and will not oscillate. The motion is an exponential decay.

***Problem 22-64**

The block of mass M is subjected to the action of the harmonic force $F = F_0 \cos\omega t$. Write the equation which describes the steady-state motion.

Given:



$M = 20\ \text{kg} \qquad k = 400\ \frac{\text{N}}{\text{m}}$

$F_0 = 90\ \text{N} \qquad C = 125\ \frac{\text{N}\cdot\text{s}}{\text{m}}$

$\omega = 6\ \frac{\text{rad}}{\text{s}}$

Solution:

$$\omega_n = \sqrt{\frac{2k}{M}} \qquad \omega_n = 6.32\,\frac{\text{rad}}{\text{s}}$$

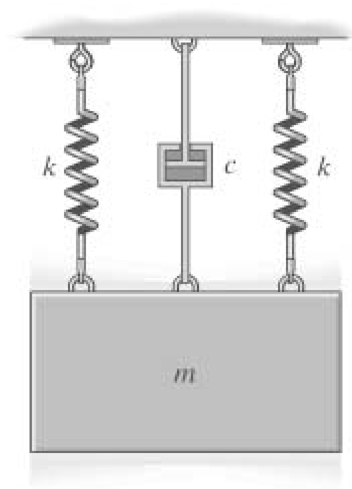
$$C_c = 2M\omega_n \qquad C_c = 253.0\,\text{N}\cdot\frac{\text{s}}{\text{m}}$$

$$A = \frac{\dfrac{F_0}{2k}}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\left(\dfrac{C}{C_c}\right)\left(\dfrac{\omega}{\omega_n}\right)\right]^2}} \qquad \phi = \text{atan}\left[\frac{\dfrac{C\omega}{2k}}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}\right]$$

$$x = A\cos(\omega t - \phi) \qquad A = 0.119\ \text{m} \qquad \omega = 6.00\,\frac{\text{rad}}{\text{s}} \qquad \phi = 83.9\,\text{deg}$$

Problem 22-65

Draw the electrical circuit that is equivalent to the mechanical system shown. What is the differential equation which describes the charge q in the circuit?



Solution:

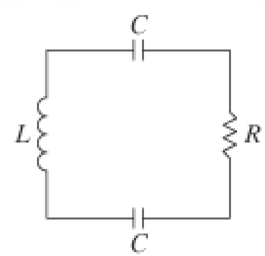
For the block,

$$mx'' + cx' + 2kx = 0$$

Let

$$m = L \qquad c = R \qquad x = q \qquad k = \frac{1}{C}$$

$$Lq'' + Rq' + \left(\frac{2}{C}\right)q = 0$$



Problem 22-66

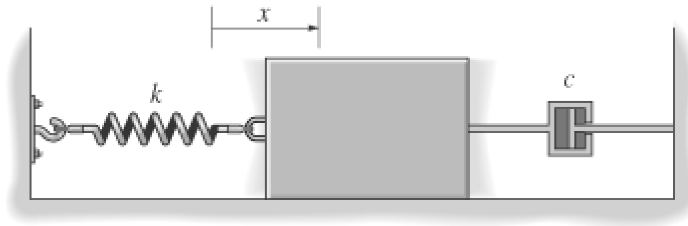
The block of mass M is continually damped. If the block is displaced $x = x_1$ and released from rest, determine the time required for it to return to the position $x = x_2$.

Given:

$M = 10\ \text{kg} \qquad k = 60\ \frac{\text{N}}{\text{m}}$

$x_1 = 50\ \text{mm} \qquad C = 80\ \text{N}\cdot\frac{\text{s}}{\text{m}}$

$x_2 = 2\ \text{mm}$



Solution:

$\omega_n = \sqrt{\frac{k}{M}} \qquad \omega_n = 2.45\ \frac{\text{rad}}{\text{s}} \qquad C_c = 2M\omega_n \qquad C_c = 48.99\ \text{N}\cdot\frac{\text{s}}{\text{m}}$

Since $C = 80.00\ \text{N}\cdot\frac{\text{s}}{\text{m}} > C_c = 48.99\ \text{N}\cdot\frac{\text{s}}{\text{m}}$ the system is overdamped

$$\lambda_1 = \frac{-C}{2M} + \sqrt{\left(\frac{C}{2M}\right)^2 - \frac{k}{M}} \qquad \lambda_2 = \frac{-C}{2M} - \sqrt{\left(\frac{C}{2M}\right)^2 - \frac{k}{M}}$$

$t = 0\ \text{s} \qquad x = x_1 \qquad x' = 0\ \frac{\text{m}}{\text{s}} \qquad A_1 = 1\ \text{m} \qquad A_2 = 1\ \text{m}$

Given $\qquad x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \qquad x' = A_1\lambda_1 e^{\lambda_1 t} + A_2\lambda_2 e^{\lambda_2 t}$

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \text{Find}(A_1, A_2) \qquad \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} -0.84 \\ -7.16 \end{pmatrix}\frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0.06 \\ -0.01 \end{pmatrix}\text{m}$$

Given $\qquad x_2 = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \qquad t = \text{Find}(t) \qquad t = 3.99\ \text{s}$