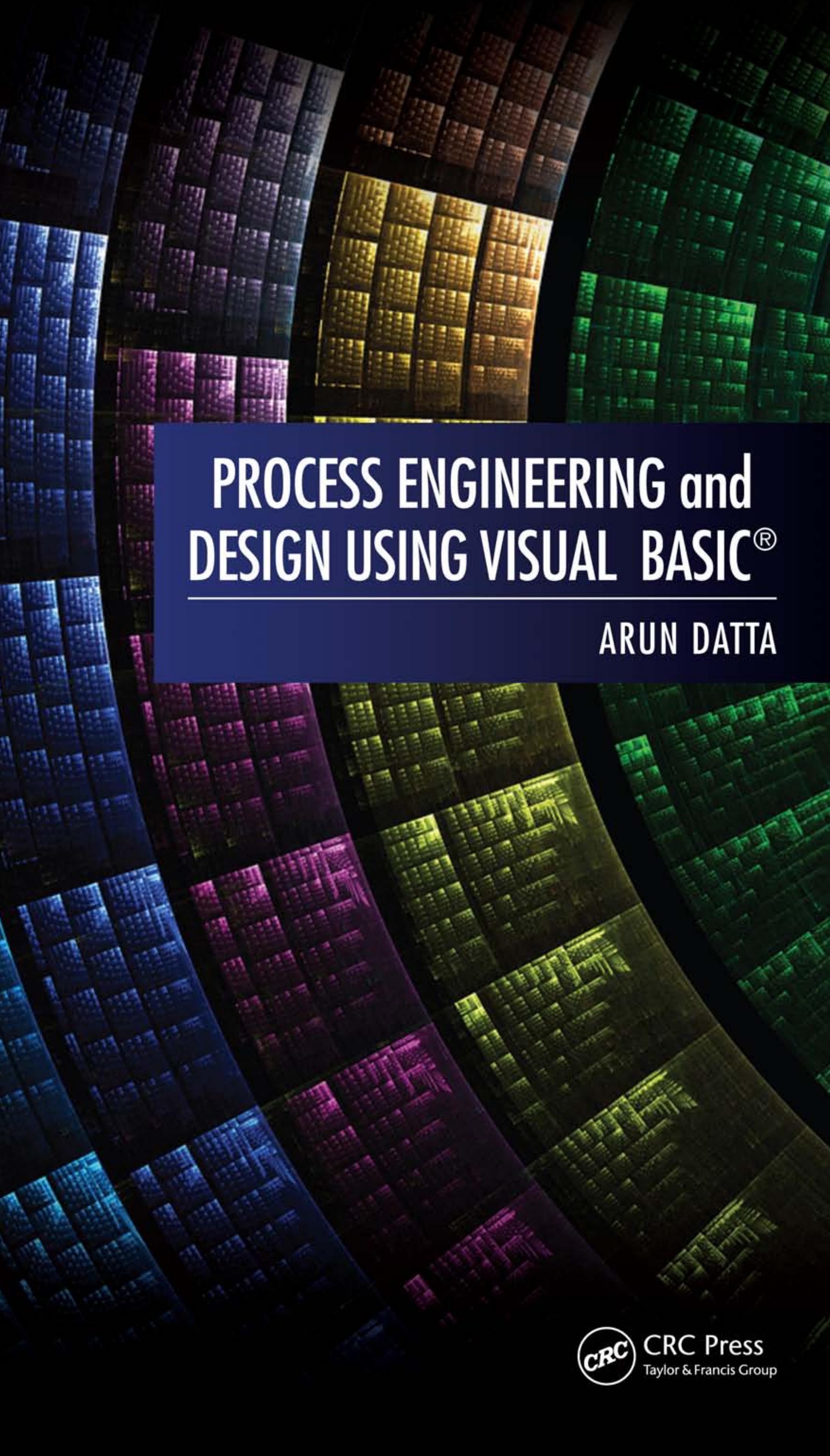


PROCESS ENGINEERING and DESIGN USING VISUAL BASIC®

ARUN DATTA

CRC Press
Taylor & Francis Group

PROCESS ENGINEERING and DESIGN USING VISUAL BASIC®

PROCESS ENGINEERING and DESIGN USING VISUAL BASIC®

ARUN DATTA

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

Visual Basic and Excel are registered trademarks of Microsoft.

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

© 2008 by Taylor & Francis Group, LLC
CRC Press is an imprint of Taylor & Francis Group, an Informa business

No claim to original U.S. Government works
Printed in the United States of America on acid-free paper
10 9 8 7 6 5 4 3 2 1

International Standard Book Number-13: 978-1-4200-4542-0 (Hardcover)

This book contains information obtained from authentic and highly regarded sources. Reprinted material is quoted with permission, and sources are indicated. A wide variety of references are listed. Reasonable efforts have been made to publish reliable data and information, but the author and the publisher cannot assume responsibility for the validity of all materials or for the consequences of their use.

No part of this book may be reprinted, reproduced, transmitted, or utilized in any form by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying, microfilming, and recording, or in any information storage or retrieval system, without written permission from the publishers.

For permission to photocopy or use material electronically from this work, please access www.copyright.com (http://www.copyright.com/) or contact the Copyright Clearance Center, Inc. (CCC) 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. CCC is a not-for-profit organization that provides licenses and registration for a variety of users. For organizations that have been granted a photocopy license by the CCC, a separate system of payment has been arranged.

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation without intent to infringe.

Library of Congress Cataloging-in-Publication Data

Datta, Arun.
Process engineering and design using Visual BASIC / Arun Datta.
p. cm.
Includes bibliographical references and index.
ISBN-13: 978-1-4200-4542-0 (alk. paper)
ISBN-10: 1-4200-4542-3 (alk. paper)
1. Chemical processes--Mathematical models. 2. Chemical processes--Computer simulation. 3. Microsoft Visual BASIC. I. Title.

TP155.7.D36 2006
660'.28--dc22 2006103307

Visit the Taylor & Francis Web site at
http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com

Dedication

To my mother
Smt. Narayani Datta

Contents

Preface

The availability of various design tools and software made process engineering and design simple but also, in some cases, paradoxically led to inadequate design. Unfortunately, not only young engineers but also reasonably experienced engineers are becoming more dependent on software tools without having a basic understanding of the design. I have seen people using HYSYS® to estimate water boiling temperature at atmospheric pressure. This reduced self-confidence is the key factor for inappropriate process design in many cases.

As everything in process design can't be covered by software tools like HYSYS®, HTRI®, etc., most design/consulting groups have developed their own design tools mostly using Excel® spreadsheets. Excel spreadsheets are extremely easy to develop and easy to corrupt through cut-and-paste modifications as well as with attempts to modify the macros of the spreadsheet. Sometimes we also failed to recognize the limitations of Excel spreadsheets and develop something that does not produce the intended output.

In spite of different procedures and design tools, we sometimes come across some typical design requirements that are not covered by any known procedure. Some understanding of basic mathematics and fundamental process engineering can solve a large number of problems without much difficulty. For example, a volume calculation of a horizontal vessel requires simple integration, volume calculation of an inclined vessel using Simpson's rule, etc.

The main purpose of this work is to identify small but important issues we normally come across during design and the best possible procedures to resolve them. This work gives a detailed analysis of the methodology used for a particular calculation and in some cases limitations of the procedure. This is done so that the user can check a calculation manually and develop an understanding of the basic design. This will improve the self-confidence of the user.

The programs have been developed in Visual Basic® to avoid limitations of other programming tools, e.g., Excel. All programs have been developed to use both the International System of Units (SI) and English units (the default choice is SI); however, the programs do not allow changing of individual units.

Though each program has been checked extensively for correctness, the possibility of program bugs can't be totally eliminated. Any feedback on program malfunctioning will be highly appreciated, as well as any other comments to improve future editions.

Arun K. Datta

Acknowledgements

I wish to express my gratitude to WorleyParsons, Brisbane, for their strong support while this work was being produced. Special thanks go to my wife, Dr. Nivedita Datta, and my daughter, Raka Datta, whose support and encouragements were the key factors in the completion of this work. I am also very grateful to Malcolm Rough of WorleyParsons for his full review of the manuscript and many useful suggestions. I also appreciate the encouragement I have received from my mentors over many years, particularly Tony Buckley and Ivan Broome of WorleyParsons. I have tried my best to thoroughly utilize my learning from them.

I also acknowledge the help received from the American Petroleum Institute, the Gas Processors Suppliers Association, and John M. Campbell and Company for allowing me to publish their relevant figures and tables. Special thanks are also due to the editorial and production staff of CRC Press, a Taylor & Francis Company, particularly Allison Shatkin, Marsha Pronin, and Ari Silver for publishing this work with outstanding quality.

I will thankfully acknowledge any suggestion to further improve future editions.

Arun K. Datta
Brisbane, Australia

About the author

Arun K. Datta, principal process engineer working with WorleyParsons, has twenty-seven years' experience in the field of process engineering and design. He holds a master's degree in chemical engineering from the Indian Institute of Technology, Delhi, and has worked with several process consultancy organizations in both India and Australia.

Arun has been a consultant for a large number of process engineering organizations including refineries, oil and gas industries, fine chemicals, and pharmaceuticals. His fields of expertise include heat and mass transfer, process simulations, exchanger design, pressure vessel design, design of safety systems, and design of control systems. Some of his clients include BP Refinery, Caltex Refinery, Santos, ExxonMobil, ONGC, Indian Oil, Incitec, and Oil Search.

Arun is a chartered professional engineer in Australia (Queensland chapter) and has membership with the Institute of Engineers, Australia. He is also a member of the interview committee for the Institute of Engineers.

chapter one

Basic mathematics

Introduction

Understanding mathematics is the most fundamental requirement in understanding engineering. Some fields in chemical engineering require the solution of complex mathematical equations. The purpose of this section is to get some idea in the field of mathematics commonly used for chemical engineering design and includes the following:

- Physical constants
- Mensuration
- Algebra
- Trigonometry
- Analytical geometry
- Calculus
- Differential equations
- Partial differential equations
- Numerical analysis
- Equation of states
- Unit conversions
- Programming

Physical constants

Commonly used physical constants are presented in Table 1.1.

SI prefixes

The International System of Units (SI) prefixes are presented in Table 1.2.

Table 1.1 Commonly Used Physical Constants

Name	Symbol	Unit	Value
Speed of light	c	cm/sec	2.99792458E10
Plank's constant	h	erg.sec	6.6260755E-27
	$hbar$	erg.sec	1.05457266E-27
Gravitational constant	G	$cm^3/(g.sec^2)$	6.67259E-8
	G	Nm^2/kg^2	6.67259E-11
Electron charge	e	ESU	4.8032068E-10
Mass of electron	m_e	g	9.1093897E-28
Mass of proton	m_p	g	1.6726231E-24
Mass of neutron	m_n	g	1.6749286E-24
Mass of hydrogen	m_H	g	1.6733E-24
Atomic mass unit	amu	g	1.6605402E-24
Avogadro's number	N_A		6.0221367E23
Boltzmann constant	k	erg/k	1.380658E-16
Electron volt	eV	erg	1.6021772E-12
Radiation density constant	a	$erg/(cm^3.K^4)$	7.5646E-15
Stefan–Boltzmann constant	$sigma$	$erg/(cm^2.K^4.sec)$	5.67051E-5
Rydberg constant		erg	2.1798741E-11
Gas constant	R	J/(mol.K)	8.31439
	R	cal/(mol.K)	1.98719
	R	l.atm/(mol.K)	0.0820567
Pi	π		3.1415926536
Napierian (natural) logarithm base	e		2.7182818285
Euler's constant	γ		0.5772156649
Logarithm conversion	$\log x$		0.4342944819 ln x
Logarithm conversion	$\ln x$		2.302585093 log x
Radian			57.2957795131°
Degree			0.0174532925 rad
Minute			0.0002908882 rad
Second			0.0000048481 rad

Mensuration

Triangles

$$\text{Area} = 1/2\ bh \tag{1.1}$$

where b = base and h = altitude.

Rectangles

$$\text{Area} = ab \tag{1.2}$$

where a and b are the length of the sides.

Table 1.2 SI Prefixes

Multiplication Factor			Prefix	Symbol
1 000 000 000 000 000 000	=	10^{18}	exa	E
1 000 000 000 000 000	=	10^{15}	peta	P
1 000 000 000 000	=	10^{12}	tera	T
1 000 000 000	=	10^{9}	giga	G
1 000 000	=	10^{6}	mega	M
1 000	=	10^{3}	kilo	k
100	=	10^{2}	hecto	h
10	=	10^{1}	deka	da
0.1	=	10^{-1}	deci	d
0.01	=	10^{-2}	centi	c
0.001	=	10^{-3}	milli	m
0.000 001	=	10^{-6}	micro	µ
0.000 000 001	=	10^{-9}	nano	n
0.000 000 000 001	=	10^{-12}	pico	p
0.000 000 000 000 001	=	10^{-15}	femto	f
0.000 000 000 000 000 001	=	10^{-18}	atto	a

Parallelogram (opposite sides parallel)

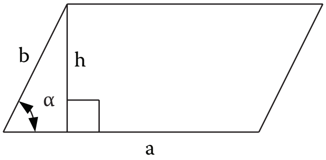


Figure 1.1 Parallelogram.

$$\text{Area} = ah = ab \sin\alpha \tag{1.3}$$

where a and b are the lengths of the sides, h the height, and α the angle between the sides. See Figure 1.1.

Rhombus (equilateral parallelogram)

$$\text{Area} = 1/2\, ab \tag{1.4}$$

where a and b are lengths of diagonals.

Trapezoid (four sides, two parallel)

$$\text{Area} = 1/2(a + b)h \tag{1.5}$$

where the lengths of parallel sides are a and b and h = height.

Quadrilateral (four sided)

$$\text{Area} = 1/2\, ab \sin\theta \tag{1.6}$$

where a and b are the lengths of the diagonals, and the acute angle between them is θ.

Regular polygon of n sides (refer to Figure 1.2)

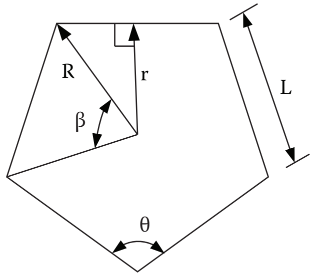


Figure 1.2 Regular polygon.

$$\text{Area} = 1/4\, nL^2 \cot(180°/n) \tag{1.7a}$$

$$R = L/2\, csc(180°/n) \tag{1.7b}$$

$$r = L/2 \cot(180°/n) \tag{1.7c}$$

$$\beta = 360°/n \tag{1.7d}$$

$$\theta = (n - 2)180°/n \tag{1.7e}$$

$$L = 2r \tan(\beta/2) = 2R \sin(\beta/2) \tag{1.7f}$$

Circle (refer to Figure 1.3)

Let

C = circumference
r = radius
D = diameter
A = area
S = arc length subtended by θ
L = chord length subtended by θ
H = maximum rise of arc above chord, $r - H = d$
θ = central angle (rad) subtended by arc S

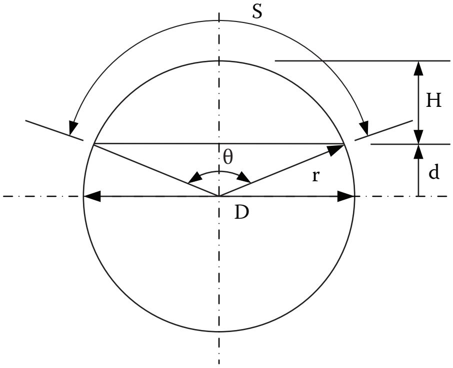


Figure 1.3 Circle.

Then,

$$C = 2\pi r = \pi D \tag{1.8a}$$

$$S = r\theta = 1/2\ D\theta \tag{1.8b}$$

$$L = 2\sqrt{r^2 - d^2} = 2r\sin(\theta/2) = 2d\tan(\theta/2) \tag{1.8c}$$

$$d = 1/2\sqrt{4r^2 - L^2} = 1/2L\cot(\theta/2) \tag{1.8d}$$

$$\theta = S/r = 2\cos^{-1}(d/r) = 2\sin^{-1}(L/D) \tag{1.8e}$$

$$A(\text{circle}) = \pi r^2 = 1/4\ \pi D^2 \tag{1.8f}$$

$$A(\text{sector}) = 1/2\ rS = 1/2\ r^2\theta \tag{1.8g}$$

$$A(\text{segment}) = A(\text{sector}) - A(\text{triangle}) = 1/2\ r^2\ (\theta - \sin\theta) \tag{1.8h}$$

$$= r^2\cos^{-1}\ (r - H)/r - (r - H)\ (2rH - H^2)^{0.5} \tag{1.8i}$$

$$\frac{A(\text{segment})}{A(\text{total})} = \frac{(\phi/360°)(\pi/4) - \sqrt{(h - h^2)}(0.5 - h)}{\pi/4} \tag{1.8j}$$

where

$h = H/D,\ h \le 0.5$

$\phi = 2\{90 - \sin^{-1}(1 - 2h)\}$

Ellipse (refer to Figure 1.4)

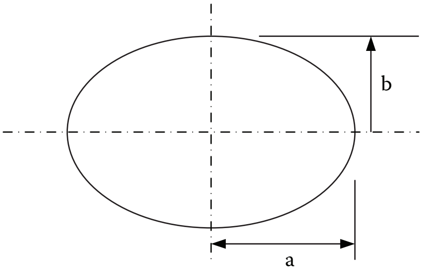


Figure 1.4 Ellipse.

$$\text{Area} = \pi ab \tag{1.9a}$$

$$\text{Circumference} = 2\pi\{(a^2 + b^2)/2\}^{0.5} \text{ (approximately)} \tag{1.9b}$$

Parabola (refer to Figure 1.5)

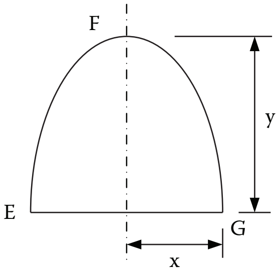


Figure 1.5 Parabola.

$$\text{Length of arc EFG} = \sqrt{\left(4x^2 + y^2\right)} + \frac{y^2}{2x}\ln\left\{\left(2x + \sqrt{4x^2 + y^2}\right)/y\right\} \tag{1.10a}$$

$$\text{Area of section EFG} = 4/3\ xy \tag{1.10b}$$

Prism

$$\text{Lateral surface area} = (\text{perimeter of right section}) * (\text{lateral edge}) \tag{1.11a}$$

$$\text{Volume} = (\text{area of base}) * (\text{altitude}) \tag{1.11b}$$

Pyramid

Lateral area of a regular pyramid = 1/2 (perimeter of base) * (slant height) (1.12a)

= 1/2 (number of sides)*(length of one side) * (slant height) (1.12b)

Volume = 1/3 (area of base) * (altitude) (1.12c)

Right circular cylinder

Lateral surface area = 2π (radius) * (altitude) (1.13a)

$$\text{Volume} = \pi\ (\text{radius})^2 * (\text{altitude}) \tag{1.13b}$$

Sphere (refer to Figure 1.6)

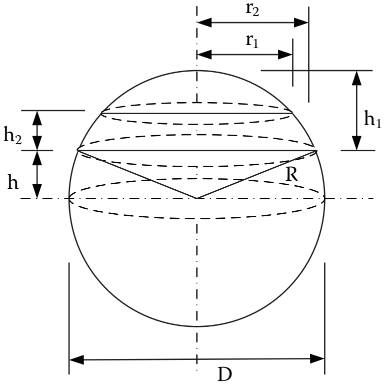


Figure 1.6 Sphere.

$$\text{Area (sphere)} = 4\pi R^2 = \pi D^2 \tag{1.14a}$$

$$\text{Area (zone)} = 2\pi Rh = \pi Dh \tag{1.14b}$$

$$\text{Volume (sphere)} = 4/3\pi R^3 = 1/6\pi D^3 \tag{1.14c}$$

$$\text{Volume (spherical sector)} = 2/3\pi R^2 h = 1/6\pi h_1(3r_2^2 + h_1^2) \tag{1.14d}$$

$$\text{Volume (spherical segment of one base)} = 1/6\pi h_1(3r_2^2 + h_1^2) \tag{1.14e}$$

$$\text{Volume (spherical segment of two bases)} = 1/6\pi h(3r_1^2 + 3r_2^2 + h_2^2) \tag{1.14f}$$

Right circular cone

$$\text{Curved surface area} = \pi r(r^2 + h^2)^{0.5} \tag{1.15a}$$

$$\text{Volume} = 1/3\ \pi r^2 h \tag{1.15b}$$

where

r = radius
h = height of the cone

Dished end

$$\text{Surface area (general)} = \pi a^2 + \pi/2(b^2/e)\ln\{(1 + e)/(1 - e)\} \tag{1.16a}$$

$$\text{Surface area (ellipsoidal, } b = a/2) = \pi a^2 + (\pi a^2/8e)\ln\{(1 + e)/(1 - e)\} = 4.336\ a^2 \tag{1.16b}$$

$$\text{Surface area (hemispherical, } b = a) = 2\pi a^2 \tag{1.16c}$$

$$\text{Volume (general)} = 2/3\ \pi a^2 b \tag{1.16d}$$

$$\text{Volume (ellipsoidal)} = 1/3\ \pi a^3 \tag{1.16e}$$

$$\text{Volume (hemispherical)} = 2/3\ \pi a^3 \tag{1.16f}$$

where

a = semimajor axis
b = semiminor axis
e = eccentricity = $(1 - b^2/a^2)^{0.5}$

Irregular shape

Let $y_o, y_1, y_2, \ldots, y_n$ be the lengths of a series of equally spaced parallel chords and h be the distance between them. The approximate area of the figure is given by using the trapezoidal rule or by Simpson's rule.

Trapezoidal rule

$$\text{Area} = h/2\ \{(y_o + y_n) + 2(y_1 + y_2 + y_3 + \ldots + y_{n-1})\} \tag{1.17}$$

Simpson's rule

$$\text{Area} = h/3\ \{(y_o + y_n) + 4(y_1 + y_3 + y_5 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\} \tag{1.18}$$

where n is even.

The greater the value of n, the greater the accuracy of approximation.

Irregular volume

To find irregular volume, replace y's by cross-sectional area A_j, and use the results in the preceding equation.

Algebra

Factoring

$$a^2 - b^2 = (a + b)(a - b) \tag{1.19a}$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2) \tag{1.19b}$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \tag{1.19c}$$

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1}) \tag{1.19d}$$

$$(a + b)^2 = a^2 + 2ab + b^2 \tag{1.19e}$$

$$(a - b)^2 = a^2 - 2ab + b^2 \tag{1.19f}$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc \tag{1.19g}$$

$$(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc \tag{1.19h}$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \tag{1.19i}$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \tag{1.19j}$$

$$\begin{gathered}(a + b)^n = a^n + na^{n-1}b + \{n(n - 1)/2!\}a^{n-2}b^2 + \\ \{n(n - 1)(n - 2)/3!\}a^{n-3}b^3 + \dots + b^n\end{gathered} \tag{1.19k}$$

Arithmetic progression

A series is said to be in arithmetic progression (AP), if the algebraic difference between any two successive terms be the same throughout the series. The following series is in AP.

$$a, a + b, a + 2b, a + 3b, \dots, n\text{th term}$$

where

a = first term
b = common difference

The nth term can be defined as

$$t_n = a + (n - 1)b \tag{1.20a}$$

Sum of the series

$$S = (n/2)\{2a + (n - 1)b\} \tag{1.20b}$$

Geometric progression

A series is said to be in geometric progression (GP) if the ratio of any term to the preceding one is the same throughout the series. The following series is in GP.

$$a, ar, ar^2, ar^3, \ldots, n\text{th term}$$

where

a = first term
r = common ratio

The nth term can be defined as

$$t_n = ar^{n-1} \tag{1.21a}$$

Sum of the series

$$S = a(1 - r^n)/(1 - r) \tag{1.21b}$$

Infinite series (in GP)

$$\text{When } r < 1, \text{ the sum of the series} = a/(1 - r) \tag{1.21c}$$

Example

Series $1 + 1/3 + 1/9 + 1/27 + \ldots + 1/\infty$.

The sum $= a/(1 - r) = 3/2$.

Best fit straight line (least squares method)

The straight line $y = a + bx$ should be fitted through given points (x_1,y_1), (x_2,y_2), …, (x_n,y_n) so that the sum of the squares of the distances of those points from the straight line is minimum, where the distances are measured in the vertical direction (y-direction).

The values of a and b are calculated from the following equations:

$$an + b\Sigma x_j = \Sigma y_j \tag{1.22a}$$

and

$$a\Sigma x_j + b\Sigma x_j^2 = \Sigma x_j y_j \tag{1.22b}$$

where n is the number of points.

Example 1.1

Let the four points be (–1,1), (–0.1,1.099), (0.2,0.808), and (1,1).

$$n = 4$$
$$\Sigma x_j = 0.1$$
$$\Sigma x_j^2 = 2.05$$
$$\Sigma y_j = 3.907$$
$$\Sigma x_j y_j = 0.0517$$
$$4a + 0.1b = 3.907$$
$$0.1a + 2.05b = 0.0517$$

The solution is

$$a = 0.9773$$
$$b = -0.0224$$

Therefore, the equation of the best fit straight line is $y = 0.9773 - 0.0224x$.

Binomial equation

The general Equation is:

$$ax^2 + bx + c = 0 \tag{1.23a}$$

General solution of Equation 1.23a

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{1.23b}$$

Polynomial equation

General solution of the following type of polynomial equation:

$$y = ax^2 + bx + c \tag{1.24a}$$

If three values (x_1,y_1), (x_2,y_2), and (x_3,y_3) are known, then

$$a = \frac{(y_1 - y_2)(x_2 - x_3) - (y_2 - y_3)(x_1 - x_2)}{({x_1}^2 - {x_2}^2)(x_2 - x_3) - ({x_2}^2 - {x_3}^2)(x_1 - x_2)} \tag{1.24b}$$

$$b = \frac{\left\{(y_1 - y_2) - a({x_1}^2 - {x_2}^2)\right\}}{(x_1 - x_2)} \tag{1.24c}$$

$$c = y_1 - a{x_1}^2 - bx_1 \tag{1.24d}$$

Example 1.2

Solve the equation $y = ax^2 + bx + c$, for three sets of values (1,10), (3,32), and (5,70).

Solution

$$a = \frac{(10 - 32)(3 - 5) - (32 - 70)(1 - 3)}{(1 - 9)(3 - 5) - (9 - 25)(1 - 3)} = 2$$

$$b = \frac{\left\{(10 - 32) - 2(1 - 9)\right\}}{-2} = 3$$

$$c = 10 - 2 - 3 = 5$$

Therefore, the general solution will be $y = 2x^2 + 3x + 5$.

Maxima/minima

Let a function be

$$y = f(x) \tag{1.25a}$$

If a solution for x at $y' = 0$ exists, then there will be either a maximum value or a minimum value for the function.

At a value of x, where $y' = 0$:

If y'' is negative, then there will be a maximum value for the function.
If y'' is positive, then there will be a minimum value for the function.

Example 1.3a

Let the equation be $y = ax + b$ (1.25b)

$$y' = a = 0$$

No solution exists at $y' = 0$; this indicates that there will be no maxima or minima for the function.

This is the equation of a straight line.

Example 1.3b

$$\text{Let the equation be } y = x^3 + x - 1 \quad (1.25c)$$

$$y' = 3x^2 + 1 = 0 \quad (1.25d)$$

$$\text{or, } x^2 = -1/3 \quad (1.25e)$$

No general solution exists for Equation 1.25e, meaning Equation 1.25c will have no general maxima or minima.

Example 1.3c

$$\text{Let the equation be } y = 4x^3 - 3x + 2 \quad (1.25f)$$

$$y' = 12x^2 - 3 = 0$$

$$x = \pm 1/2$$

At $x = 1/2$, $y'' = 12$ and at $x = -1/2$, $y'' = -12$.

Therefore at $x = 1/2$, the value of y will be minimum. Value of $y = 1$.

At $x = -1/2$ the value of y will be maximum. Value of $y = 3$.

The nature of this graph is such that for x values of less than –1/2, the value of y will decrease continuously, and for x values more than 1/2, the value of y will increase continuously.

Matrix

Addition and multiplication of matrices

- If two m by n matrices A and B are given, the sum $A + B$ can be defined as the m by n matrix adding corresponding elements, i.e.,

$$(A + B)\,[i,j] = A[i,j] + B[i,j] \quad (1.26a)$$

- If a matrix A and a number c are given, the multiplication of the matrix can be defined as

$$cA = cA[i,j] \quad (1.26b)$$

Addition of matrices

i. $$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} + \begin{bmatrix} u & v & w \\ x & y & z \end{bmatrix} = \begin{bmatrix} a+u & b+v & c+w \\ d+x & e+y & f+z \end{bmatrix}$$

ii. $$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} + \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \end{bmatrix} = 2\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

Multiplication of matrices

iii. $$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

iv. $$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax+by+cz \\ dx+ey+fz \end{bmatrix}$$

Multiplication of two matrices is well defined only if the number of columns of the first matrix is the same as the number of rows of the second matrix. If A is an m by n matrix (m rows, n columns) and B is an n by p matrix (n rows, p columns), then their product AB is the m by p matrix (m rows, p columns) given by $(AB)[i,j] = A[i,1] * B[1,j] + A[i,2] * B[2,j] + \ldots + A[i,n]*B[n,j]$ for each pair of i and j.

Multiplication has the following properties:

- $(AB)C = A(BC)$ for all k by m matrices A, m by n matrices B, and n by p matrices C (associativity).
- $(A + B)C = AC + BC$ for all m by n matrices A and B and k by m matrices C (distributivity).
- In general, commutativity does not hold, meaning generally $AB \neq BA$.

Matrix properties involving addition

Let A, B, and C be m by n matrices. Then,

a. $$A + B = B + A \tag{1.27a}$$

b. $$(A + B) + C = A + (B + C) \tag{1.27b}$$

c. $$A + 0 = A \tag{1.27c}$$

where 0 is the m by n zero matrix (all its entries are equal to 0).

d. $$A + B = 0 \text{ if and only if } B = -A \tag{1.27d}$$

Matrix properties involving multiplication

a. Let A, B, and C be three matrices. If the products AB, $(AB)C$, BC, and $A(BC)$ exist then

$$(AB)C = A(BC) \tag{1.28a}$$

b. If α and β are numbers and A is a matrix, then

$$\alpha(\beta A) = (\alpha\beta)A \tag{1.28b}$$

c. If α is a number, and A and B are two matrices such that the product AB is possible, then

$$\alpha(AB) = (\alpha A)B = A(\alpha B) \tag{1.28c}$$

d. If A is an n by m matrix and 0 is the m by k zero matrix, then

$$A0 = 0 \tag{1.28d}$$

Matrix properties involving addition and multiplication

a. Let A, B, and C be three matrices, then

$$(A + B)C = AC + BC \tag{1.29a}$$

$$\text{and } A(B + C) = AB + AC \tag{1.29b}$$

b. If α and β are numbers and A and B are matrices, then

$$\alpha(A + B) = \alpha A + \alpha B \tag{1.29c}$$

$$\text{and } (\alpha + \beta)A = \alpha A + \beta A \tag{1.29d}$$

Transpose

The transpose of a matrix is another matrix, produced by turning rows into columns and vice versa. The transpose of an n by m matrix A is the n by m matrix A^T defined by

$$A^T[i,j] = A[j,i] \tag{1.30}$$

For the matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix}$$

The transpose

$$A^T = \begin{bmatrix} a & d & g & j \\ b & e & h & k \\ c & f & i & l \end{bmatrix}$$

Symmetric matrix

A symmetric matrix is a matrix equal to its transpose. A symmetric matrix must be a square matrix, for example,

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix} \text{ and } \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$$

Diagonal matrix

A diagonal matrix is a symmetric matrix with all of its entries equal to zero, except possibly the values of the diagonal, for example,

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \text{ and } \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b \end{bmatrix}$$

Determinants

Determinants are important both in calculus and algebra.

The determinant of a 1 × 1 matrix is the element itself: $det[a] = a$.

The 2 × 2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

has determinant

$$det\ (A) = ad - bc \tag{1.31}$$

The 3 × 3 matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

has determinant

$$det(A) = a(ei - fh) - b(di - fg) + c(dh - eg) \tag{1.32}$$

Properties of determinants

1. Any matrix A and its transpose have the same determinant, meaning $det\ A = det\ A^T$.
2. If the elements of one row (or column) of a determinant are all zero, the value of the determinant is zero.
3. The determinant of a triangular matrix is the product of the entries on the diagonal, that is,

$$\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} = \begin{bmatrix} a & 0 \\ b & d \end{bmatrix} = ad$$

4. If one determinant is obtained from another by interchanging any two rows (or columns), the value of either is the negative of the value of the other.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = -\begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

5. If the elements of one row (or column) of a determinant are multiplied by the same constant factor, the value of the determinant is multiplied by this factor.

$$\begin{bmatrix} na & nb \\ c & d \end{bmatrix} = n\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ nc & nd \end{bmatrix}$$

6. If two rows (or columns) of a determinant are identical, the value of the determinant is zero.
7. If two determinants are identical except for one row (or column), the sum of their values is given by a single determinant obtained by adding corresponding elements of dissimilar rows (or columns) and leaving the remaining elements unchanged.

$$\begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 8 & 5 \end{bmatrix} = 19$$

8. The value of a determinant is not changed, if to the elements of any row (or column) are added a constant multiple of the corresponding elements of any other row (or column).

$$\begin{bmatrix} a+nc & b+nd \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c+na & d+nb \end{bmatrix}$$

9. If all elements but one in a row (or column) are zero, the value of the determinant is the product of that element and its cofactor.

Cofactor

Let A be an n × n matrix. The ij-th cofactor of A, denoted by A_{ij}, will be

$$A_{ij} = (-1)^{i+j} \left| M_{ij} \right| \tag{1.33}$$

Example 1.4

Find cofactors A_{32} and A_{24} of a 4 × 4 determinant A

$$A = \begin{bmatrix} 1 & -3 & 5 & 6 \\ 2 & 4 & 0 & 3 \\ 1 & 5 & 9 & -2 \\ 4 & 0 & 2 & 7 \end{bmatrix}$$

Solution

$$A_{32} = (-1)^{3+2} \left| M_{32} \right| = -\begin{bmatrix} 1 & 5 & 6 \\ 2 & 0 & 3 \\ 4 & 2 & 7 \end{bmatrix} = -8$$

$$A_{24} = (-1)^{2+4} \begin{bmatrix} 1 & -3 & 5 \\ 1 & 5 & 9 \\ 4 & 0 & 2 \end{bmatrix} = -192$$

Determinant and inverses

Theorem 1

If A is invertible, then $det\ A \neq 0$ and

$$det\ A^{-1} = 1/det\ A \tag{1.34}$$

Adjoint

Let a determinant be

$$B = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix} \tag{1.35}$$

Let A be an n × n matrix and let B, given in Equation 1.35, denote the matrix of its cofactors. Then the adjoint of A, written as $adjA$, is the transpose of the n × n matrix B.

$$adjA = B^T = \begin{bmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \vdots & \vdots & \vdots & \vdots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{bmatrix} \tag{1.36}$$

Theorem 2

Let A be an n × n matrix. Then,

$$(A)(adjA) = \begin{bmatrix} detA & 0 & 0 & \dots & 0 \\ 0 & detA & 0 & \dots & 0 \\ 0 & 0 & detA & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & detA \end{bmatrix} = (detA)I \tag{1.37}$$

Theorem 3

Let A be an n × n matrix. Then A is invertible if and only if $detA \neq 0$. If $detA \neq 0$, then

$$A^{-1} = \frac{1}{detA} adjA \tag{1.38}$$

Cramer's rule

Consider a system of n equations and n unknowns.

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \qquad (1.39a) \\
\vdots \qquad \vdots \qquad\quad \vdots \quad &\ \ \vdots \\
a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= b_n
\end{aligned}
$$

which can be written in the form

$$Ax = b \qquad (1.39b)$$

If $detA \neq 0$, then Equation 1.39b will have a unique solution given by

$$x = b/A \qquad (1.39c)$$

Let D be $detA$. The other matrix can be defined as

$$A_1 = \begin{bmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{bmatrix}, A_2 = \begin{bmatrix} a_{11} & b_1 & \cdots & a_{1n} \\ a_{21} & b_2 & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & b_n & \cdots & a_{nn} \end{bmatrix}, \text{etc.}$$

Let $D_1 = detA_1$, $D_2 = detA_2$ …, $D_n = detA_n$, then

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D} \qquad (1.40)$$

Example 1.5

Solve using Cramer's rule, the system

$$2x_1 + 4x_2 + 6x_3 = 18$$

$$4x_1 + 5x_2 + 6x_3 = 24$$

$$3x_1 + x_2 - 2x_3 = 4$$

Solution

The determinant is calculated first as

$$D = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 5 & 6 \\ 3 & 1 & -2 \end{bmatrix} = 2(-10-6) - 4(-8-18) + 6(4-15) = -32 + 104 - 66 = 6$$

Because the value of $D \neq 0$, the system will have a unique solution.

$$D_1 = \begin{bmatrix} 18 & 4 & 6 \\ 24 & 5 & 6 \\ 4 & 1 & -2 \end{bmatrix} = 18(-10-6) - 4\,(-48-24) + 6(24-20) = 24$$

$$D_2 = \begin{bmatrix} 2 & 18 & 6 \\ 4 & 24 & 6 \\ 3 & 4 & -2 \end{bmatrix} = 2(-48-24) - 18(-8-18) + 6(16-72) = -12$$

$$D_3 = \begin{bmatrix} 2 & 4 & 18 \\ 4 & 5 & 24 \\ 3 & 1 & 4 \end{bmatrix} = 2(20-24) - 4(16-72) + 18(4-15) = 18$$

Therefore,

$$x_1 = D_1/D = 24/6 = 4$$

$$x_2 = D_2/D = -12/6 = -2$$

$$x_3 = D_3/D = 18/6 = 3$$

Trigonometry

Functions of circular trigonometry

The trigonometric functions are the ratio of various sides of the reference angles presented in Figure 1.7.

The trigonometric relationships are:

Sine of α = $\sin \alpha = p/h$; Secant of α = $\sec \alpha = h/b$
Cosine of α = $\cos \alpha = b/h$; Cosecant of α = $\csc \alpha = h/p$

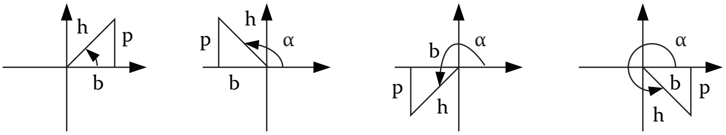


Figure 1.7 Triangles.

Table 1.3 Magnitude of Trigonometric Functions

Function	0–90°	90–180°	180–270°	270–360°
sin α	+0 to +1	+1 to +0	0 to –1	–1 to –0
csc α	+∞ to +1	+1 to +∞	–∞ to –1	–1 to –∞
cos α	+1 to 0	–0 to –1	–1 to –0	+0 to +1
sec α	+1 to +∞	–∞ to –1	–1 to –∞	+∞ to +1
tan α	+0 to +∞	–∞ to –0	+0 to +∞	–∞ to –0
cot α	+∞ to +0	–0 to –∞	+∞ to +0	–0 to –∞

Tangent of α = tan α = p/b; Cotangent of α = cot α = b/p

The magnitude and sign of trigonometric functions are presented in Table 1.3.

Periodic functions

The following periodic functions are commonly used in trigonometry:

$$\cos(\alpha + 2\pi) = \cos\alpha \tag{1.41a}$$

$$\sin(\alpha + 2\pi) = \sin\alpha \tag{1.41b}$$

$$\cos(\alpha + 2n\pi) = \cos\alpha, \quad n = 0, \pm 1, \pm 2, \ldots \tag{1.41c}$$

$$\sin(\alpha + 2n\pi) = \sin\alpha, \quad n = 0, \pm 1, \pm 2, \ldots \tag{1.41d}$$

The magic identity

The following identities are very basic to the analysis of trigonometric expressions:

$$\sin^2\alpha + \cos^2\alpha = 1 \tag{1.42a}$$

$$1 + \tan^2\alpha = \sec^2\alpha \tag{1.42b}$$

$$1 + \cot^2\alpha = \csc^2\alpha \tag{1.42c}$$

The addition formulas

The following addition formulas are very important for the analysis of trigonometric expressions:

$$\cos(a + b) = \cos a \cos b - \sin a \sin b \tag{1.43a}$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b \tag{1.43b}$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b \tag{1.43c}$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b \tag{1.43d}$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} \tag{1.43e}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b} \tag{1.43f}$$

Example 1.6

Find the exact value of cos 165°, given that $\cos 120° = -1/2$, $\sin 120° = \sqrt{3}/2$, and $\cos 45° = \sin 45° = \sqrt{2}/2$.

Solution

$$\cos 165° = \cos(120° + 45°)$$

$$= \cos 120° \cos 45° - \sin 120° \sin 45°$$

$$= \frac{-1}{2} * \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2}$$

$$= -\frac{\sqrt{6} + \sqrt{2}}{4}$$

Double angle and half angle

Double-angle and half-angle formulas are very useful in trigonometry. The following equations are widely used:

$$\cos(2a) = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a \tag{1.44a}$$

$$\sin(2a) = 2\sin a \cos a \tag{1.44b}$$

$$\tan(2a) = \frac{2\tan a}{1-\tan^2 a} \tag{1.44c}$$

$$\cos^2\left(\frac{a}{2}\right) = \frac{1}{2}\left(1+\cos a\right) \tag{1.44d}$$

$$\sin^2\left(\frac{a}{2}\right) = \frac{1}{2}\left(1-\cos a\right) \tag{1.44e}$$

$$\tan\left(\frac{a}{2}\right) = \frac{1-\cos a}{\sin a} = \frac{\sin a}{1+\cos a} \tag{1.44f}$$

Example 1.7

Use the half-angle formulas to find cos(π/8) and sin(π/8).

Solution

Let a = π/4, then π/8 = a/2.

Now, from the half-angle formula we have

$$\cos^2\left(\frac{\pi}{8}\right) = \frac{1}{2}\left(1+\cos\frac{\pi}{4}\right) = \frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)$$

Because 0 < π/8 < π/2, the value of cos (π/8) will be positive. Therefore,

$$\cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{2+\sqrt{2}}}{2}$$

Similarly,

$$\sin\left(\frac{\pi}{8}\right) = \frac{\sqrt{2-\sqrt{2}}}{2}$$

Product and sum formulas

The following product and sum formulas are commonly used in trigonometry:

$$\cos a \cos b = \frac{1}{2}\left\{\cos(a+b)+\cos(a-b)\right\} \tag{1.45a}$$

$$\sin a \sin b = \frac{1}{2}\left\{\cos(a-b)-\cos(a+b)\right\} \tag{1.45b}$$

$$\sin a \cos b = \frac{1}{2}\left\{\sin(a+b)+\sin(a-b)\right\} \tag{1.45c}$$

$$\cos a \sin b = \frac{1}{2}\left\{\sin(a+b)-\sin(a-b)\right\} \tag{1.45d}$$

$$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) \tag{1.45e}$$

$$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right) \tag{1.45f}$$

$$\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) \tag{1.45g}$$

$$\sin a - \sin b = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right) \tag{1.45h}$$

Relations between angles and sides of triangles

Refer to Figure 1.8.

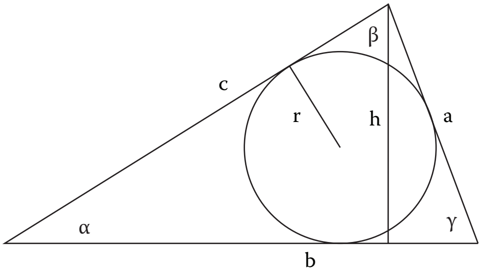


Figure 1.8 Triangle.

Let a, b, and c be the sides of a triangle.
Let α, β, and γ be the angles opposite the sides in the triangle.

$$2s = a + b + c$$

A = area
r = radius of the inscribed circle
R = radius of the circumscribed circle
h = altitude

Law of sines

$$\sin \alpha/a = \sin \beta/b = \sin \gamma/c \tag{1.46}$$

Law of tangents

$$\frac{a+b}{a-b} = \frac{\tan 1/2(\alpha+\beta)}{\tan 1/2(\alpha-\beta)} \tag{1.47a}$$

$$\frac{b+c}{b-c} = \frac{\tan 1/2(\beta+\gamma)}{\tan 1/2(\beta-\gamma)} \tag{1.47b}$$

$$\frac{a+c}{a-c} = \frac{\tan 1/2(\alpha+\gamma)}{\tan 1/2(\alpha-\gamma)} \tag{1.47c}$$

Law of cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \tag{1.48a}$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta \tag{1.48b}$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \tag{1.48c}$$

Other relations

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} \tag{1.49a}$$

$$a = b\cos\gamma + c\cos\beta \tag{1.49b}$$

$$\sin\alpha = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)} \tag{1.49c}$$

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}} \tag{1.49d}$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{s(s-a)}{bc}} \tag{1.49e}$$

$$A = \frac{1}{2}bh = \frac{1}{2}ab\sin\gamma = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha} = \sqrt{s(s-a)(s-b)(s-c)} = rs \tag{1.49f}$$

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}} \tag{1.49g}$$

$$R = \frac{a}{2\sin\alpha} = \frac{abc}{4A} \tag{1.49h}$$

$$h = c\sin\alpha = a\sin\gamma = \frac{2rs}{b} \tag{1.49i}$$

Inverse trigonometric functions

General inverse trigonometric functions with the following restrictions

$$-\pi/2 \le \sin^{-1}x \le \pi/2$$

$$0 \le \cos^{-1}x \le \pi$$

$$-\pi/2 \le \tan^{-1}x \le \pi/2$$

are:

$$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \sec^{-1}\frac{1}{\sqrt{1-x^2}} = \csc^{-1}\frac{1}{x} = \frac{\pi}{2} - \cos^{-1} x \tag{1.50a}$$

$$\cos^{-1} x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}} = \sec^{-1}\frac{1}{x} = \csc^{-1}\frac{1}{\sqrt{1-x^2}} = \frac{\pi}{2} - \sin^{-1} x \tag{1.50b}$$

$$\tan^{-1} x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \cot^{-1}\frac{1}{x} = \sec^{-1}\sqrt{1+x^2} = \csc^{-1}\frac{\sqrt{1+x^2}}{x} \tag{1.50c}$$

Hyperbolic functions

The hyperbolic functions are similar to the trigonometric functions and are presented as follows:

$$\cosh x = \frac{e^x + e^{-x}}{2} \tag{1.51a}$$

$$\sinh x = \frac{e^x - e^{-x}}{2} \tag{1.51b}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{1.51c}$$

$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}} \tag{1.51d}$$

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}} \tag{1.51e}$$

$$\text{csch}\,x = \frac{2}{e^{x} - e^{-x}} \tag{1.51f}$$

Other hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1 \tag{1.52a}$$

$$\text{sech}^2 x + \tanh^2 x = 1 \tag{1.52b}$$

$$\coth^2 x - \text{csch}^2 x = 1 \tag{1.52c}$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y \tag{1.52d}$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y \tag{1.52e}$$

$$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x} \tag{1.52f}$$

Inverse hyperbolic functions

The inverse of sinh x can be established as follows:

Let $y = \sinh x$

$$\text{or } 2y = e^{x} - e^{-x}$$
$$\text{or } 2ye^{x} = e^{2x} - 1$$
$$\text{or } (e^{x})^2 - 2y(e^{x}) - 1 = 0$$
$$\text{or } e^{x} = y \pm \sqrt{y^2 + 1}$$

Because $e^x > 0$ for all x, and $\sqrt{y^2 + 1} > y$ for all y, we can have the solution as

$$e^{x} = y + \sqrt{y^2 + 1}$$

Therefore,

$$x = \ln\left(y + \sqrt{y^2 + 1}\right)$$

and

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right) \tag{1.53a}$$

Similarly,

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right) \tag{1.53b}$$

$$\tanh^{-1} x = \frac{1}{2}\ln\frac{1 + x}{1 - x} \tag{1.53c}$$

$$\coth^{-1} x = \frac{1}{2}\ln\frac{x + 1}{x - 1} \tag{1.53d}$$

$$\operatorname{sech}^{-1} x = \ln\left(\frac{1 + \sqrt{1 - x^2}}{x}\right) \tag{1.53e}$$

$$\operatorname{csch}^{-1} x = \ln\left(\frac{1 + \sqrt{1 + x^2}}{x}\right) \tag{1.53f}$$

Analytical geometry

Straight line

The general equation of a straight line is

$$ax + by + c = 0 \tag{1.54}$$

If $b \neq 0$, the slope is $= -a/b$.
The intersection at y axis when $x = 0$ is $-c/b$.

Straight line through two points

It is possible to draw a straight line through two points. The general equation of a straight line through two points P_1 (a_1,b_1) and P_2 (a_2,b_2) can be represented as

$$\begin{bmatrix} x & y & 1 \\ a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \end{bmatrix} = 0 \tag{1.55a}$$

or

$$x(b_1 - b_2) - y(a_1 - a_2) + (a_1b_2 - a_2b_1) = 0 \qquad (1.55b)$$

Three points on one line

Although it is possible to draw a straight line through two points, any three points may not fall on a straight line. Three points P_1 (a_1,b_1), P_2 (a_2,b_2), and P_3 (a_3,b_3) can be on a straight line if and only if

$$\begin{bmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{bmatrix} = 0 \qquad (1.56)$$

The circle

The equation of a circle with coordinates of the center as (a,b) and radius r is

$$(x - a)^2 + (y - b)^2 = r^2 \qquad (1.57a)$$

Expanding Equation 1.57a, the general equation of a circle can be developed as

$$x^2 + y^2 + 2mx + 2ny + c = 0 \qquad (1.57b)$$

with

$$m^2 + n^2 - c > 0 \qquad (1.57c)$$

The coordinates of the center of the circle are $(-m, -n)$, and the radius is

$$r = \sqrt{m^2 + n^2 - c} \qquad (1.57d)$$

Tangent

The tangent at any point (h,k) of a circle is defined to be a straight line that meets the circle at the specified point, but when produced, does not cut it. This tangent is always perpendicular to the radius drawn from the center to the point of contact.

The equation of the tangent at a point (h,k) on a circle with center (a,b) and radius r is given by:

$$y - k = -\frac{a - h}{b - k}(x - h) \qquad (1.58)$$

The slope of the line is $-\dfrac{a-h}{b-k}$.

Normal

The normal at any point of a curve is the straight line that passes through the point and is perpendicular to the tangent at that point. The general equation of the normal with the coordinates of the center as (a,b) and point as (h,k) is

$$\begin{bmatrix} x & y & 1 \\ a & b & 1 \\ h & k & 1 \end{bmatrix} = 0 \tag{1.59}$$

Four points on a circle

Four points can be on a circle if the following condition is satisfied. Let the four points be (a_1,b_1), (a_2,b_2), (a_3,b_3), *and* (a_4,b_4). The four points can be on a circle if

$$\begin{bmatrix} a_1^2 + b_1^2 & a_1 & b_1 & 1 \\ a_2^2 + b_2^2 & a_2 & b_2 & 1 \\ a_3^2 + b_3^2 & a_3 & b_3 & 1 \\ a_4^2 + b_4^2 & a_4 & b_4 & 1 \end{bmatrix} = 0 \tag{1.60}$$

Example 1.8

Check if four points (5,0), (4.33,2.5), (2.5,–4.33), and (–3.536, –3.536) are on the same circle.

Solution

The condition of four points on the same circle is given in Equation 1.60. The value of the matrix will be

$$\begin{bmatrix} 25 & 5 & 0 & 1 \\ 25 & 4.33 & 2.5 & 1 \\ 25 & 2.5 & -4.33 & 1 \\ 25 & -3.536 & -3.536 & 1 \end{bmatrix} = 25\begin{bmatrix} 4.33 & 2.5 & 1 \\ 2.5 & -4.33 & 1 \\ -3.536 & -3.536 & 1 \end{bmatrix} - 5\begin{bmatrix} 25 & 2.5 & 1 \\ 25 & -4.33 & 1 \\ 25 & -3.536 & 1 \end{bmatrix}$$

$$-1\begin{bmatrix} 25 & 4.33 & 2.5 \\ 25 & 2.5 & -4.33 \\ 25 & -3.536 & -3.536 \end{bmatrix} = -1067 + 1067 = 0$$

Therefore, all four points will be on the same circle.

Circle through three points

The general equation of a circle through three points (a_1,b_1), (a_2,b_2), and (a_3,b_3) (provided three points do not fall on a straight line) can be given as

$$\begin{bmatrix} x^2+y^2 & x & y & 1 \\ a_1{}^2+b_1{}^2 & a_1 & b_1 & 1 \\ a_2{}^2+b_2{}^2 & a_2 & b_2 & 1 \\ a_3{}^2+b_3{}^2 & a_3 & b_3 & 1 \end{bmatrix} = 0 \tag{1.61}$$

Conic section

The locus of a point P that moves so that its distance from a fixed point is always in a constant ratio to its perpendicular distance from a fixed straight line is called a conic section.

Focus

The previously mentioned fixed point is called the focus and is usually denoted by S.

Eccentricity

The constant ratio is called eccentricity and is denoted by e.

Directrix

The fixed straight line is called the directrix.

An algebraic curve with an equation of the form

$$ax^2 + bxy + cy^2 + dxz + eyz + fz^2 = 0 \tag{1.62}$$

is called a conic section.

The Cartesian equation of the conic section is:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0 \tag{1.63}$$

Partial derivatives

The three partial derivatives of the equation of a conic section are:

$$Fx'\ (x,y,z) = 2ax + by + dz \tag{1.64a}$$

$$Fy'\ (x,y,z) = bx + 2cy + ez \tag{1.64b}$$

$$Fz'\ (x,y,z) = dx + ey + 2fz \tag{1.64c}$$

Table 1.4 Conic Section

	$b^2 - 4ac < 0$	$b^2 - 4ac = 0$	$b^2 - 4ac > 0$
$\Delta \neq 0$	$a\Delta < 0$ $a \neq c$, an ellipse $a = c$, a circle $a\Delta > 0$, no locus	Parabola	Hyperbola
$\Delta = 0$	Point	Two parallel lines if $Q = d^2 + e^2 - 4(a + c)f > 0$; one straight line if $Q = 0$; no locus if $Q < 0$	Two intersecting straight lines

The matrix formed by the coefficients of x, y, and z is called Δ.

$$\Delta = \begin{bmatrix} 2a & b & d \\ b & 2c & e \\ d & e & 2f \end{bmatrix} \tag{1.65}$$

The Table 1.4 characterizes the curve represented by the equation.

Example 1.9

Establish the type of curve for the equation $x^2 + 2xy + y^2 + 3x - 4y + 6 = 0$.

Solution

$$\Delta = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 2 & -4 \\ 3 & -4 & 12 \end{bmatrix} = -98 \neq 0$$

$$b^2 - 4ac = 0$$

Therefore, the curve will be a parabola.

Parabola

As in Figure 1.9, let AA′ be a straight line and e be a point (e is not on line AA′). The locus of all points B such that the distance of B from line AA′ is equal to the distance of B from point e will form a parabola.

The point e is the focus and the line AA′ is the directrix.

The general equation of a parabola is

$$y^2 = 4ax \tag{1.66}$$

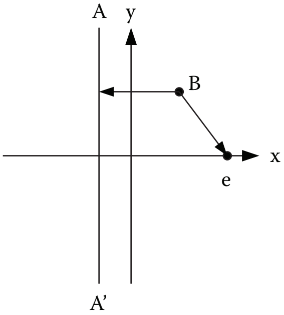


Figure 1.9 Parabola.

Parametric equations of the parabola. Let a and b be two lines in a plane with equations

$$x = 4at^2 \tag{1.67a}$$

$$y = 4at \tag{1.67b}$$

respectively.

The real number t is a parameter.

Now, to obtain an equation of the curve, we eliminate the parameter t from the two equations. Eliminating t from Equation 1.67a and Equation 1.67b we get

$$x = 4a\left(\frac{y}{4a}\right)^2$$

or

$$y^2 = 4ax$$

which is the equation of the parabola.

Equation of tangent of a parabola. The slope of the tangent line can be obtained as:

$$2yy' = 4a \text{ or slope } y' = 2a/y.$$

The slope of the tangent line at a point $(x_o, y_o) = 2a/y_o$.

The equation of the tangent line will be

$$y - y_o = \frac{2a}{y_o}(x - x_o) \tag{1.68a}$$

or

$$yy_o - 4ax_o = 2ax - 2ax_o \tag{1.68b}$$

or

$$yy_o = 2a\ (x + x_o) \tag{1.68c}$$

Equation 1.68c is the equation of the tangent of the parabola $y^2 = 4ax$ at point (x_o, y_o).

Tangent line with a given slope, m

For a parabola with equation $y^2 = 4ax$, the equation of the tangent line with a constant slope m can be presented by

$$y = mx + \frac{a}{m} \tag{1.69}$$

Ellipse (refer to Figure 1.10)

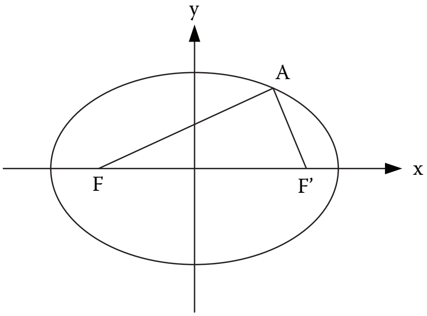


Figure 1.10 Ellipse.

Let there be two points F and F′ such that the distance F – F′ is always less than a fixed positive value 2*a*.

The locus of all points A such that distance AF + AF′ = 2*a* is an ellipse.

Two points F and F′ are called the foci of the ellipse.

The general equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{1.70}$$

where

a = semimajor axis and
b = semiminor axis

Parametric equations of the ellipse. Take in a plane two lines l and m with respective equations

$$x = a \cos t \tag{1.71a}$$

$$y = b \sin t \tag{1.71b}$$

The real number t is the parameter.

Now, to obtain an equation of the curve, we eliminate the parameter t from the two equations. Eliminating t from Equation 1.71a and Equation 1.71b, we get

$$x/a = \cos t \text{ and}$$

$$y/b = \sin t$$

or

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2 t + \sin^2 t = 1$$

is the general equation of the ellipse.

General properties of an ellipse.

Eccentricity. The eccentricity of an ellipse is defined as

$$e = \sqrt{1 - \frac{b^2}{a^2}}\ , e < 1 \tag{1.72}$$

Focus. Coordinates of the focus are (±ae, 0).

Tangent to a point P of an ellipse. The general equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The slope of the tangent line can be obtained by differentiating implicitly

$$\frac{2x}{a^2}+\frac{2yy'}{b^2}=0 \tag{1.73a}$$

or

$$y'=-\frac{b^2x}{a^2y} \tag{1.73b}$$

The equation of the tangent line at point (x_o,y_o) is

$$y-y_o=-\frac{b^2x_o}{a^2y_o}(x-x_o) \tag{1.73c}$$

or

$$\frac{x_ox}{a^2}+\frac{y_oy}{b^2}=1 \tag{1.73d}$$

Normal at a point P. The equation of a normal at a point P (x_o,y_o) can be given as

$$\frac{(x-x_o)a^2}{x_o}=\frac{(y-y_o)b^2}{y_o} \tag{1.74}$$

Hyperbola (refer to Figure 1.11)

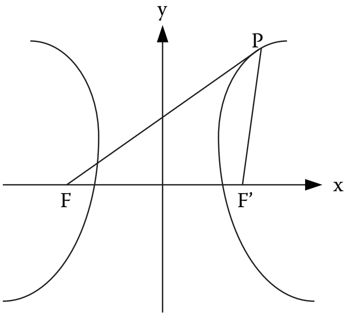


Figure 1.11 Hyperbola.

Let there be two points F and F′ and a strictly positive value 2*a*. The locus of all points P such that the absolute difference between PF′ and PF = 2*a* will be a hyperbola.

The general equation of a hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \tag{1.75}$$

Parametric equations of hyperbola. Take in a plane two lines l and m with respective equations

$$x = a \sec t \tag{1.76a}$$

$$y = b \tan t \tag{1.76b}$$

The real number t is the parameter.

Now, to obtain an equation of the curve, we eliminate the parameter t from the two equations. Eliminating t from Equation 1.76a and Equation 1.76b, we get

$$x/a = \sec t \text{ and}$$

$$y/b = \tan t$$

or

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2 t - \tan^2 t = 1$$

is the general equation of the hyperbola.

General properties of a hyperbola.

Eccentricity. The eccentricity of a hyperbola is defined as

$$e = \sqrt{1 + \frac{b^2}{a^2}}\ , e > 1 \tag{1.77}$$

Focus. The coordinates of the focus are $(\pm ae, 0)$.

Tangent to a point P of a hyperbola. The general equation of a hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The slope of the tangent line can be obtained by differentiating implicitly

$$\frac{2x}{a^2}-\frac{2yy'}{b^2}=0 \tag{1.78a}$$

or

$$y'=\frac{b^2x}{a^2y} \tag{1.78b}$$

The equation of the tangent line at point (x_o, y_o) is

$$y-y_o=\frac{b^2x_o}{a^2y_o}(x-x_o) \tag{1.78c}$$

or

$$\frac{x_ox}{a^2}-\frac{y_oy}{b^2}=1 \tag{1.78d}$$

Normal at a point P. The equation of a normal at a point P (x_o, y_o) can be given as

$$\frac{(x-x_o)a^2}{x_o}=-\frac{(y-y_o)b^2}{y_o} \tag{1.79}$$

Calculus

Differential calculus

A function f is said to be differentiable if the following limit exists:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \tag{1.80}$$

If this limit exists, it is called the derivative of f at x and denoted by $f'(x)$.

Example 1.10

Estimate the derivative of $1/x$.

Solution

The derivative can be established based on Equation 1.80 as

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\frac{-1}{x(x+h)}$$

Therefore,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{-1}{x(x+h)}=\frac{-1}{x^2}$$

provided $x \neq 0$.

Example 1.11

If $f(x) = 5 - x^3$, establish the value of $f'(2)$

Solution

$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$=\lim_{h\to 0}\frac{\left\{5-(2+h)^3\right\}-\left\{5-2^3\right\}}{h}$$

$$=\lim_{h\to 0}\frac{\left\{5-(8+12h+6h^2+h^3)\right\}+3}{h}$$

$$=\lim_{h\to 0}\left(-12-6h-h^2\right)$$

$$=-12$$

Understanding the derivatives

The purpose of derivatives can be explained by using the following example:

Example 1.12

Estimate the derivative of $f(x) = 2^x$ at $x = 0$.

Solution (See Figure 1.12)

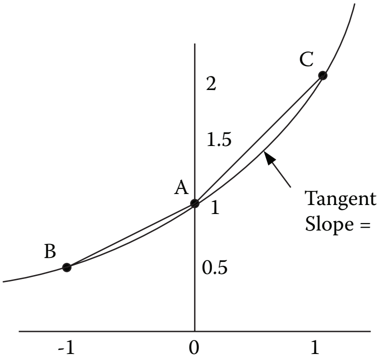


Figure 1.12 Graph $y = 2^x$.

Slope of line BA = $(2^0 - 2^{-1})/1 = 1/2$ and slope of line AC = $(2^1 - 2^0)/1 = 1$. Therefore, the derivative at $x = 0$ will be in between 1/2 and 1.

Now, for small values of h, Table 1.5 presents the value of 2^h with the values of the difference quotients.

Table 1.5 Solution of Example 1.12

h	2^h	Difference Quotient: $(2^h - 1)/h$
–0.003	0.99979	0.693075
–0.002	0.99986	0.693099
–0.001	0.99993	0.693123
0	1	
0.001	1.00007	0.693171
0.002	1.00014	0.693195
0.003	1.00021	0.693219

From Table 1.5, it is clear that difference quotients calculated for negative h values are less than calculated for positive h values, and the derivative is in between 0.693123 and 0.693171. If we use a value up to three decimal places, $f'(0) = 0.693$.

Therefore, the equation of the tangent at point A will be

$$y = 0.693x + 1 \text{ (because the intercept is at 1).}$$

Standard derivatives

$$d(x^n) = n\, x^{n-1}\, dx$$

$$d(e^x) = e^x\, dx$$

$$d(a^x) = a^x \ln a\, dx$$

$$d(\ln x) = (1/x)\,dx$$

$$d(\log x) = \{\log (e/x)\}\,dx$$

$$d(\sin x) = \cos x\,dx$$

$$d(\cos x) = -\sin x\,dx$$

$$d(\tan x) = \sec^2 x\,dx$$

$$d(\cot x) = -\csc^2 x\,dx$$

$$d(\sec x) = \tan x \sec x\,dx$$

$$d(\csc x) = -\csc x \cot x\,dx$$

$$d(\sin^{-1} x) = (1 - x^2)^{-1/2}\,dx$$

$$d(\cos^{-1} x) = -(1 - x^2)^{-1/2}\,dx$$

$$d(\tan^{-1} x) = (1 + x^2)^{-1}\,dx$$

$$d(\cot^{-1} x) = -(1 + x^2)^{-1}\,dx$$

$$d(\sec^{-1} x) = x^{-1}\,(x^2 - 1)^{-1/2}\,dx$$

$$d(\csc^{-1} x) = -x^{-1}\,(x^2 - 1)^{-1/2}\,dx$$

$$d(\sinh x) = \cosh x\,dx$$

$$d(\cosh x) = \sinh x\,dx$$

$$d(\tanh x) = \mathrm{sech}^2\,x\,dx$$

$$d(\coth x) = -\mathrm{csch}^2\,x\,dx$$

$$d(\mathrm{sech}\,x) = -\mathrm{sech}\,x \coth x\,dx$$

$$d(\sinh^{-1} x) = (x^2 + 1)^{-1/2}\,dx$$

$$d(\cosh^{-1} x) = (x^2 - 1)^{-1/2}\,dx$$

$$d(\tanh^{-1} x) = (1 - x^2)^{-1}\,dx$$

$$d(\coth^{-1} x) = -(x^2 - 1)^{-1}\,dx$$

$$d(\mathrm{sech}^{-1}\,x) = x(1 - x^2)^{-1/2}\,dx$$

$$d(\mathrm{csch}^{-1}\,x) = -x^{-1}\,(x^2 + 1)^{-1/2}\,dx$$

Integral calculus

Suppose f is continuous for $a \le t \le b$. The definite integral of f from a to b, written as

$$\int_a^b f(t)dt \tag{1.81}$$

is the limit of the left-hand or right-hand sums with n subdivisions of $[a, b]$, as n gets arbitrarily large. In other words,

$$\int_a^b f(t)dt = \lim_{n\to\infty}(Left\ \ handsum) = \lim_{n\to\infty}\left(\sum_{i=0}^{n-1} f(t_i)\Delta t\right) \tag{1.82a}$$

and

$$\int_a^b f(t)dt = \lim_{n\to\infty}(Right\ \ handsum) = \lim_{n\to\infty}\left(\sum_{i=1}^{n} f(t_i)\Delta t\right) \tag{1.82b}$$

Each of these sums is called a Riemann sum, f is called the integrand, and a and b are called the limits of integration.

Volume of horizontal dished end (refer to Figure 1.13)

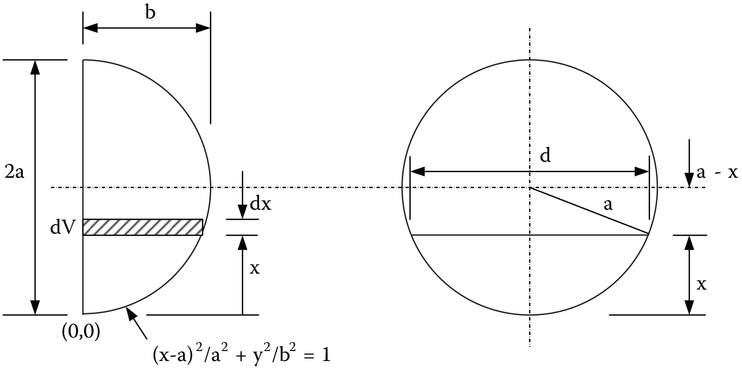


Figure 1.13 Volume of horizontal dished end.

The volume of the strip is

$$dV = \pi/2 * y * d/2 * dx \tag{1.83}$$

The volume up to a height of x is

$$V = \frac{\pi}{2}\int_{x=0}^{x=x} \frac{1}{2}\,dydx \tag{1.84}$$

$$(d/2)^2 = a^2 - (a - x)^2 = 2a\mathrm{x} - x^2 \tag{1.85a}$$

and

$$y^2/b^2 = 1 - (x - a)^2/a^2 = (2ax - x^2)/a^2 \tag{1.85b}$$

Therefore, Equation 1.83 will become

$$V = \frac{\pi b}{2a}\int_{x=0}^{x=x} (2ax - x^2)dx$$

$$= \frac{\pi b}{2a}\left(ax^2 - \frac{x^3}{3}\right) \tag{1.86}$$

Equation 1.86 is the general equation of the horizontal dished end volume.

Total volume (x = 2a). The total volume of the dished end

$$V_T = 2/3\ \pi b a^2 \tag{1.87}$$

Elliptical dished end (b = a/2). General equation of volume

$$V = \pi/4\ (ax^2 - x^3/3) \tag{1.88a}$$

Total volume ($x = 2a$)

$$V_T = \pi/3\ a^3 \tag{1.88b}$$

Hemispherical dished end (b = a). General equation of volume

$$V = \pi/2\ (ax^2 - x^3/3) \tag{1.89a}$$

Total volume ($x = 2a$)

$$V_T = 2/3\ \pi\ a^3 \tag{1.89b}$$

Volume of vertical dished end (refer to Figure 1.14)

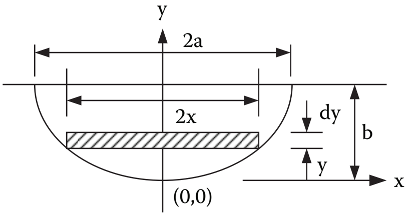


Figure 1.14 Volume of vertical dished end.

Equation of the curve with shifted coordinate is

$$\frac{x^2}{a^2} + \frac{(y-b)^2}{b^2} = 1 \tag{1.90}$$

Volume up to a distance y from the bottom will be

$$V = \int_{y=0}^{y=y} \frac{\pi}{4} 4x^2\,dy \tag{1.91}$$

Integrating Equation 1.91 and using the value of x^2 from Equation 1.90

$$V = \frac{\pi a^2}{b^2}\left\{by^2 - \frac{y^3}{3}\right\} \tag{1.92}$$

Equation 1.92 is the general equation of the volume of the vertical dished end up to a height of y from the bottom.

Total Volume ($y = b$). Total volume of the dished end

$$V_T = 2/3\ \pi b a^2 \tag{1.93}$$

Elliptical dished end ($b = a/2$)
General equation of volume

$$V = 4\pi\ (ay^2/2 - y^3/3) \tag{1.94a}$$

Total volume ($x = 2a$)

$$V_T = \pi/3\ a^3 \tag{1.94b}$$

Hemispherical dished end ($b = a$). General equation of volume

$$V = \pi\ (ay^2 - y^3/3) \tag{1.95a}$$

Total volume ($x = 2a$)

$$V_T = 2/3\ \pi\ a^3 \tag{1.95b}$$

Standard integrals

$$\int (du + dv + dw) = \int du + \int dv + \int dw$$

$$\int a\,dv = a \int dv$$

$$\int v^n dv = \frac{v^{n+1}}{n+1} + c (n \neq -1)$$

$$\int \frac{dv}{v} = \ln v + c$$

$$\int a^v dv = \frac{a^v}{\ln a} + c$$

$$\int e^v dv = e^v + c$$

$$\int \sin v\,dv = -\cos v + c$$

$$\int \cos v\,dv = \sin v + c$$

$$\int \sec^2 v\,dv = \tan v + c$$

$$\int \csc^2 v\,dv = -\cot v + c$$

$$\int \sec v \tan v\,dv = \sec v + c$$

$$\int \csc v \cot v\,dv = -\csc v + c$$

$$\int \frac{dv}{v^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{v}{a}\right) + c$$

$$\int \frac{dv}{\sqrt{a^2 - v^2}} = \sin^{-1}\left(\frac{v}{a}\right) + c$$

$$\int \frac{dv}{v^2 - a^2} = \frac{1}{2a}\ln\left\{\frac{v-a}{v+a}\right\} + c$$

$$\int \frac{dv}{\sqrt{v^2 \pm a^2}} = \ln\left\{v + \sqrt{v^2 \pm a^2}\right\} + c$$

$$\int \sec v\,dv = \ln(\sec v + \tan v) + c$$

$$\int \csc v\,dv = \ln(\csc v - \cot v) + c$$

Differential equations

First-order differential equations

An equation of the following type:

$$\frac{dy}{dx} = 50 - y \tag{1.96}$$

is called a first-order differential equation because it involves the first derivative, but no higher derivative.

Example 1.13

Solve the following first-order differential equation:

$$\frac{dy}{dx} = 2x$$

Solution

Integrating the preceding equation,

$$y = \int 2x dx = x^2 + C \quad \text{(C = constant of integration)}$$

The solution curves are parabolas.

Separation of variables

Let us use the following equation and solve the equation using the separating variables method.

$$\frac{dy}{dx} = ky$$

Separating variables,

$$\frac{1}{y} dy = k dx$$

Integrating,

$$\int \frac{1}{y} dy = \int k dx$$

The result is

$$\ln (y) = kx + C$$

or

$$y = e^{kx + C} = e^{kx}e^{C} = Ae^{kx}$$

Therefore, the general solution of the equation

$$\frac{dy}{dx} = ky$$

$$\text{is } y = Ae^{kx} \tag{1.97}$$

for any value of A.

Second-order differential equations

The concept of second-order differential equation can be expressed with the problem of free body movement under gravity. The general equation of a free-falling body under gravity is

$$\frac{d^2s}{dt^2} = -g \tag{1.98a}$$

where s = distance, t = time, and g = acceleration due to gravity. Equation 1.98a is a second-order differential equation.

Now we know the velocity can be expressed as $v = ds/dt$

Therefore, a solution of Equation 1.98a will be

$$\frac{ds}{dt} = -gt + v_0 \tag{1.98b}$$

where v_0 = the initial velocity. Integrating Equation 1.98b we get

$$s = -\frac{1}{2}gt^2 + v_0t + s_0 \tag{1.98c}$$

where s_0 = the initial height.

Example 1.14

Solve the following differential equation:

$$\frac{d^2s}{dt^2} + w^2s = 0 \tag{1.99a}$$

Solution

We need to find a solution whose second derivative is the negative of the original function. This can be achieved by using properties $s(t) = \cos wt$ and $s(t) = \sin wt$.

$$\frac{d^2}{dt^2}(\cos t) = \frac{d}{dt}(-\sin t) = -\cos t$$

and

$$\frac{d^2}{dt^2}(\sin t) = \frac{d}{dt}(\cos t) = -\sin t$$

Therefore, the general solution of Equation 1.99a is

$$s(t) = C_1 \cos wt + C_2 \sin wt \tag{1.99b}$$

where C_1 and C_2 are arbitrary constants, and the values of C_1 and C_2 depend on the boundary conditions.

Bessel function

Bessel's differential is one of the most important equations in applied mathematics, and the standard Bessel functions are used for the solution of the following types of equations:

$$x^2y'' + xy' + (x^2 - v^2)y = 0 \tag{1.100a}$$

or

$$y'' + \frac{1}{x}y' + \left(1 - \frac{v^2}{x^2}\right)y = 0 \tag{1.100b}$$

The general solution of Equation 1.100a or Equation 1.100b is denoted by $J_n(x)$ and is given by

$$J_n(x) = x^n \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+n}\, m!(n+m)!} \tag{1.100c}$$

The solution is called the Bessel function of the first kind of order n. This series converges for all x, as the ratio test shows, and in fact, converges very rapidly because of the factorials in the denominator.

Bessel functions $J_0(x)$ and $J_1(x)$.

$$J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m}(m!)^2} = 1 - \frac{x^2}{2^2(1!)^2} + \frac{x^4}{2^4(2!)^2} - \frac{x^6}{2^6(3!)^2} + \dots \quad (1.101a)$$

This looks similar to a cosine curve.

$$J_1(x) = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m+1}}{2^{2m+1} m!(m+1)!} = \frac{x}{2} - \frac{x^3}{2^3 1!2!} + \frac{x^5}{2^5 2!3!} - \frac{x^7}{2^7 3!4!} + \dots \quad (1.101b)$$

This looks similar to a sine curve.

Standard Bessel functions. Standard Bessel functions are very important in applied mathematics. These are presented in Table 1.6 [5].

Partial differential equations

An equation involving one or more partial derivatives of an (unknown) function of two or more independent variables is called a partial differential equation. The order of the highest derivative is called the order of the equation. A few important partial differential equations are presented in the following text:

One-dimensional wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (1.102a)$$

One-dimensional heat equation:

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (1.102b)$$

Two-dimensional Laplace equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (1.102c)$$

Table 1.6 Standard Bessel Functions

x	$J_0(x)$	$J_1(x)$	x	$J_0(x)$	$J_1(x)$	x	$J_0(x)$	$J_1(x)$
0.0	1.0000	0.0000	3.0	–0.2601	0.3991	6.0	0.1506	–0.2767
0.1	0.9975	0.0499	3.1	–0.2921	0.3009	6.1	0.1773	–0.2559
0.2	0.9900	0.0995	3.2	–0.3202	0.2613	6.2	0.2017	–0.2329
0.3	0.9776	0.1483	3.3	–0.3443	0.2207	6.3	0.2238	–0.2081
0.4	0.9604	0.1960	3.4	–0.3643	0.1792	6.4	0.2433	–0.1816
0.5	0.9385	0.2423	3.5	–0.3801	0.1374	6.5	0.2601	–0.1538
0.6	0.9120	0.2867	3.6	–0.3918	0.0955	6.6	0.2740	–0.1250
0.7	0.8812	0.3290	3.7	–0.3992	0.0538	6.7	0.2851	–0.0953
0.8	0.8463	0.3688	3.8	–0.4026	0.0128	6.8	0.2931	–0.0652
0.9	0.8075	0.4059	3.9	–0.4018	–0.0272	6.9	0.2981	–0.0349
1.0	0.7652	0.4401	4.0	–0.3971	–0.0660	7.0	0.3001	–0.0047
1.1	0.7196	0.4709	4.1	–0.3887	–0.1033	7.1	0.2991	0.0252
1.2	0.6711	0.4983	4.2	–0.3766	–0.1386	7.2	0.2951	0.0543
1.3	0.6201	0.5220	4.3	–0.3610	–0.1719	7.3	0.2882	0.0826
1.4	0.5669	0.5419	4.4	–0.3423	–0.2028	7.4	0.2786	0.1096
1.5	0.5118	0.5579	4.5	–0.3205	–0.2311	7.5	0.2663	0.1352
1.6	0.4554	0.5699	4.6	–0.2961	–0.2566	7.6	0.2516	0.1592
1.7	0.3980	0.5778	4.7	–0.2693	–0.2791	7.7	0.2346	0.1813
1.8	0.3400	0.5815	4.8	–0.2404	–0.2985	7.8	0.2154	0.2014
1.9	0.2818	0.5812	4.9	–0.2097	–0.3147	7.9	0.1944	0.2192
2.0	0.2239	0.5767	5.0	–0.1776	–0.3276	8.0	0.1717	0.2346
2.1	0.1666	0.5683	5.1	–0.1443	–0.3371	8.1	0.1475	0.2476
2.2	0.1104	0.5560	5.2	–0.1103	–0.3432	8.2	0.1222	0.2580
2.3	0.0555	0.5399	5.3	–0.0758	–0.3460	8.3	0.0960	0.2657
2.4	0.0025	0.5202	5.4	–0.0412	–0.3453	8.4	0.0692	0.2708
2.5	–0.0484	0.4971	5.5	–0.0068	–0.3414	8.5	0.0419	0.2731
2.6	–0.0968	0.4708	5.6	0.0270	–0.3343	8.6	0.0146	0.2728
2.7	–0.1424	0.4416	5.7	0.0599	–0.3241	8.7	–0.0125	0.2697
2.8	–0.1850	0.4097	5.8	0.0917	–0.3110	8.8	–0.0392	0.2641
2.9	–0.2243	0.3754	5.9	0.1220	–0.2951	8.9	–0.0653	0.2559

Two-dimensional Poisson equation:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f(x,y) \tag{1.102d}$$

Three-dimensional Laplace equation:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0 \tag{1.102e}$$

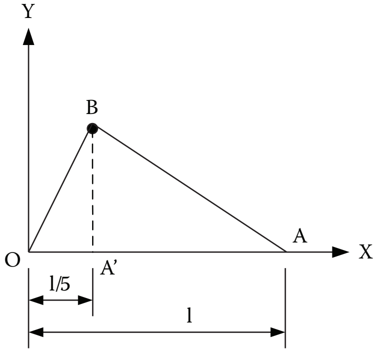


Figure 1.15 Example 1.15.

Example 1.15

Refer to Figure 1.15 above. A string of length l is stretched between two fixed points, and the motion is started by drawing aside, through a small distance b, a point of the string at distance 1/5 from one end. Establish the displacement y at distance x from the end at time t.

Solution

Let us take the fixed end O of the string as the origin and the equilibrium position OA along the x axis where OA = l. OY is taken in the plane OBA, and it is perpendicular to OX. The point A′ of the string, which is at a distance $l/5$ from the origin O, is drawn aside through a small distance b to the position B, and then released from rest. The motion starts from the position OB and BA (A is also a fixed point) of the string, and we measured time from this position of the string.

Boundary conditions

1. $y = 0$ when $x = 0$ and $x = l$, for all t
2. $y = 5bx/l$ for $0 \le x \le l/5$, when $t = 0$
3. $y = 5(l - x)b/(4l)$ for $l/5 \le x \le l$, when $t = 0$
4. $\frac{\partial y}{\partial t} = 0$ for $0 \le x \le l$, when $t = 0$

The differential equation for the vibrating string is the wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \tag{1.103a}$$

Because $y = 0$ when $x = 0$, we can assume

$$y = T \sin mx \tag{1.103b}$$

where T is a function of t only, and $m \neq 0$.

Substituting equation 1.103b in Equation 1.103a

$$T'' \sin mx = -T\, m^2c^2 \sin mx$$

For all values of x, $\sin mx \neq 0$.

Therefore,

$$T'' + m^2c^2\, T = 0 \tag{1.103c}$$

The general solution of Equation 1.103c is

$$T = A \cos(mct) + B \sin(mct)$$

$$\text{or, } y = [A \cos(mct) + B \sin(mct)] \sin(mx)$$

Because

$$\frac{\partial y}{\partial t} = 0\text{, when } t = 0$$

$0 = T' \sin mx$ when $t = 0$ or $T' = 0$ when $t = 0$ because $\sin mx \neq 0$. Now, $T' = mc\,[-A \sin(mct) + B \cos(mct)]$. Given $T' = 0$ when $t = 0$, or $B = 0$.

Therefore,

$$y = A \cos(mct) \sin mx;\ m \neq 0 \tag{1.103d}$$

Because $y = 0$ when $x = l$, from Equation 1.103b

$$0 = T \sin ml,\ T \neq 0$$

Therefore, $\sin ml = 0$ or $ml = n\pi$, when n is any integer positive or negative. Because positive and negative values of n will result in identical sets of solutions of Equation 1.103a, we can assume only positive values, i.e., $n = 1, 2, 3, 4 \ldots, \infty$.

Therefore, the general solution of Equation 1.103a will be

$$y = \sum_{n=1}^{\infty} b_n \cos(mct) \sin(mx)$$

$$= \sum_{n=1}^{\infty} b_n \cos\frac{n\pi ct}{l} \sin\frac{n\pi x}{l} \tag{1.103e}$$

Now $y = 5bx/l$, $0 \le x \le l/5$ for $t = 0$ and $y = 5b(l-x)/(4l)$, $l/5 \le x \le l$ for $t = 0$. Let us assume $y = V_0$ when $t = 0$.
Therefore,

$$V_0 = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \tag{1.103f}$$

The right-hand side of Equation 1.103f represents the Fourier half-range sine series for the function V_0 defined in (0,l); therefore,

$$b_n = \frac{2}{l}\left[\int_0^{l/5} \frac{5bx}{l} \sin\frac{n\pi x}{l}\,dx + \int_{l/5}^{l} \frac{5b(l-x)}{4l} \sin\frac{n\pi x}{l}\,dx\right]$$

$$= \frac{25b}{2n^2\pi^2} \sin\frac{n\pi}{5}$$

Therefore, the solution will be

$$y = \frac{25b}{2\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin\frac{n\pi}{5} \cos\frac{n\pi ct}{l} \sin\frac{n\pi x}{l} \tag{1.103g}$$

Example 1.16

Solve the heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{1.104a}$$

for the following boundary conditions:

$u = 0$ when $x = 0$ and $x = l$,
$u = x$ when $t = 0$.

Solution

Because $u = 0$ when $x = 0$, therefore we can assume $u = T \sin mx$, and T is the function of t only for a solution of the Equation 1.104a ($m \neq 0$).

Substituting in Equation 1.104a

$$T' \sin mx = -c^2 T m^2 \sin mx$$

or

$$T' = -Tc^2 m^2 \text{ (because } \sin mx \neq 0)$$

The solution is

$$T = Ae^{-c^2 m^2 t}$$

The Equation 1.104a will be

$$u = Ae^{-c^2 m^2 t} \sin mx$$

$$\text{Now, } u = 0 \text{ when } x = l$$

Therefore,

$$0 = Ae^{-c^2 m^2 t} \sin ml$$

Therefore, $\sin ml = 0$, or

$$m = \pi n / l \ (n = \pm 1, \pm 2, \pm 3, \pm 4, \ldots, \pm ^\circ)$$

Therefore,

$$u = Ae^{-c^2 (n\pi/l)^2 t} \sin(\frac{n\pi}{l})x \tag{1.104b}$$

Because positive and negative values of n will give the identical sets of solutions of Equation 1.104a, we can therefore assume either positive or negative values of n to have an independent set of solutions of Equation 1.104a.

The general solution will be

$$u = \sum_{n=1}^{\infty} b_n e^{-c^2(n\pi/x)^2 t} \sin\left(\frac{n\pi}{l}\right)x \ (m \neq n \neq 0)$$

Now, $u = x$ when $t = 0$. Therefore,

$$x = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right)x$$

The right-hand side of the above equation is the Fourier half-range sine series for the function $f(x)$ defined in (0,l).

Therefore,

$$b_n = \frac{2}{l}\int_0^l x \sin\left(\frac{n\pi}{l}\right)x dx$$

$$= \frac{2l}{n\pi}(-1)^{n+1}$$

Therefore, the solution will be

$$u = \frac{2l}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-c^2(n\pi/l)^2 t} \sin\left(\frac{n\pi x}{l}\right) \tag{1.104c}$$

Example 1.17

Solve the following Laplace equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{1.105a}$$

Subject to the conditions $u = 0$ when $x = 0$, $x = c$, and $y = b$ and $u = u_o$ when $y = 0$.

Solution

Because $u = 0$ when $x = 0$, therefore we assume $u = Y \sin mx$, where $m \neq 0$, for a solution of Equation 1.105a

Substituting in Equation 1.105a

$$-Y\,m^2 \sin mx + Y'' \sin mx = 0$$

or

$$Y'' - Y\,m^2 = 0 \tag{1.105b}$$

(sin mx cannot be zero for all values of x) whose solution is

$$Y = A \cosh my + B \sinh my$$

Therefore,

$$u = (A \cosh my + B \sinh my) \sin mx,\ m \neq 0 \tag{1.105c}$$

Because $u = 0$ when $x = c$, we have $0 = Y_{(y)} \sin mc$. $Y_{(y)}$ cannot be zero for all values of y. Therefore, sin $mc = 0$ or $mc = n\pi$, $n = 0 \pm 1 \pm 2 \pm 3 \pm 4 \ldots \pm \infty$. Because $u = 0$ when $y = b$.

$0 = (A \cosh mb + B \sinh mb) \sin mx$. Because x is arbitrary, therefore we have $0 = A \cosh mb + B \sinh mb$.

Therefore,

$$u = A[\cosh my - \frac{\cosh mb}{\sinh mb} \sinh my] \sin mx$$

$$= \frac{A}{\sinh mb}[\cosh my \sinh mb - \cosh mb \sinh my] \sin mx$$

$$= \frac{A}{\sinh mb} \sinh m(b - y) \sin mx$$

Because positive and negative values of m will result in identical sets of solutions of Equation 1.105a, we can use either positive or negative values of m to have an independent set of solutions of Equation 1.105a. In this case, we will use positive values of m.

Therefore, the general solution of Equation 1.105a will be

$$u = \sum_{n=1}^{\infty} A_n \frac{\sinh \frac{n\pi}{c}(b - y)}{\sinh \frac{n\pi b}{c}} \sin \frac{n\pi x}{c}$$

Because $u = u_0$ when $y = 0$, we have

$$u_0 = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{c} \tag{1.105d}$$

The right-hand side of Equation 1.105d represents the Fourier half-range sine series for all functions of u_o.

Therefore,

$$A_n = \frac{2}{c} \int_0^c u_0 \sin \frac{n\pi x}{c} dx$$

$$= \frac{2u_0}{n\pi}[1-(-1)^n]$$

When n is even the value will be zero, and when n is odd, we use $n = 2m - 1$ ($m = 1,2, \ldots, \infty$).

$$A_{2m-1} = \frac{4u_0}{(2m-1)\pi} \tag{1.105e}$$

Hence, the general solution of Equation 1.105a is

$$u = \sum_{m=1}^{\infty} \frac{4u_0}{(2m-1)\pi} \frac{\sinh \frac{2m-1}{c}\pi(b-y)}{\sinh \frac{2m-1}{c}\pi b} \sin \frac{2m-1}{c}\pi x \tag{1.105f}$$

Laplace transform

The Laplace transform of a function $x(t)$ is the function $\bar{x}(s)$ defined by

$$\bar{x}(s) = \int_0^{\infty} x(t)e^{-st} dt$$

where $s > 0$, and is written as $L[x(t)]$.

Therefore from definition,

$$L\left[x(t)\right]=\int_{0}^{\infty}x(t)e^{-st}dt \tag{1.106}$$

where $s > 0$, which is a function of $\bar{x}(s)$.

Example 1.18

Work out the Laplace transform of *t*.

Solution

$$x(t) = t$$

$$L[t]=\int_{0}^{\infty}te^{-st}dt$$

$$=\left[t\frac{e^{-st}}{-s}-\frac{e^{-st}}{s^2}\right]_0^{\infty}$$

$$=-\frac{1}{s}\lim_{t\to\infty}\frac{te^{-st}}{1}-\left[\frac{e^{-st}}{s^2}\right]_0^{\infty}$$

$$=-\frac{1}{s}\lim_{t\to\infty}\frac{t}{e^{st}}-\frac{1}{s^2}[-1]$$

$$=-\frac{1}{s}\lim_{t\to\infty}\frac{1}{se^{st}}+\frac{1}{s^2}=\frac{1}{s^2}\ \text{(L' Hospital's Rule)}$$

Therefore,

$$L[t] = 1/s^2 \tag{1.107}$$

Standard Laplace transforms

$$L[1]=\frac{1}{s}$$

$$L[t] = \frac{1}{s^2}$$

$$L[t^n] = \frac{n!}{s^{n+1}}$$

$$L[e^{at}] = \frac{1}{s-a}$$

$$L[e^{-at}] = \frac{1}{s+a}$$

$$L[\sin at] = \frac{a}{s^2+a^2}$$

$$L[\cos at] = \frac{s}{s^2+a^2}$$

$$L[\sinh at] = \frac{a}{s^2-a^2}$$

$$L[\cosh at] = \frac{s}{s^2-a^2}$$

$$L[t\cos at] = \frac{s^2-a^2}{\left(s^2+a^2\right)^2}$$

$$L[t\sin at] = \frac{2sa}{\left(s^2+a^2\right)^2}$$

Fourier half-range expansions [5]

In various applications, there is a practical need to use Fourier series in connection with function $f(x)$ that are given on some intervals only. We could extend $f(x)$ periodically with period L and then represent the extended function by a Fourier series, which in general would involve both cosine and sine terms.

Fourier half-range cosine series

The cosine half-range expansion is

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{L} x \tag{1.108a}$$

where

$$a_0 = \frac{1}{L} \int_0^L f(x) dx \tag{1.108b}$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx \tag{1.108c}$$

Fourier half-range sine series

The sine half-range expansion is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{L} x \tag{1.109a}$$

where

$$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx \tag{1.109b}$$

Example 1.19 [5]

Find the two half-range expansions of the following function:

$$f(x) = 2kx/L \quad \text{if } 0 < x < L/2$$

and

$$f(x) = 2k(L - x)/L \quad \text{if } L/2 < x < L$$

Solution

Even periodic extension:

$$a_0 = \frac{1}{L}\left[\frac{2k}{L}\int_0^{L/2} x\,dx + \frac{2k}{L}\int_{L/2}^{L}(L-x)\,dx\right] = \frac{k}{2}$$

$$a_n = \frac{2}{L}\left[\frac{2k}{L}\int_0^{L/2} x\cos\frac{n\pi}{L}x\,dx + \frac{2k}{L}\int_{L/2}^{L}(L-x)\cos\frac{n\pi}{L}x\,dx\right]$$

Integrating by parts,

$$\int_0^{L/2} x\cos\frac{n\pi}{L}x\,dx = \frac{L^2}{2n\pi}\sin\frac{n\pi}{2} + \frac{L^2}{n^2\pi^2}\left(\cos\frac{n\pi}{2} - 1\right)$$

and

$$\int_{L/2}^{L}(L-x)\cos\frac{n\pi}{L}x\,dx = -\frac{L^2}{2n\pi}\sin\frac{n\pi}{2} - \frac{L^2}{n^2\pi^2}\left(\cos n\pi - \cos\frac{n\pi}{2}\right)$$

Therefore,

$$a_n = \frac{4k}{n^2\pi^2}\left(2\cos\frac{n\pi}{2} - \cos n\pi - 1\right)$$

Thus,

$$a_2 = -\frac{16k}{2^2\pi^2}, a_4 = 0, a_6 = -\frac{16k}{6^2\pi^2}, a_8 = 0, \ldots$$

Therefore, the first half-range expansion of $f(x)$ is

$$f(x) = \frac{k}{2} - \frac{16k}{\pi^2}\left(\frac{1}{2^2}\cos\frac{2\pi}{L}x + \frac{1}{6^2}\cos\frac{6\pi}{L}x + \ldots\right)$$

Odd periodic extension:
Similarly, for odd periodic extension

$$b_n = \frac{8k}{n^2\pi^2}\sin\frac{n\pi}{2}$$

Hence, the second half-range expansion of $f(x)$ is

$$f(x)=\frac{8k}{\pi^2}\left(\frac{1}{1^2}\sin\frac{\pi}{L}x-\frac{1}{3^2}\sin\frac{3\pi}{L}x+\frac{1}{5^2}\sin\frac{5\pi}{L}x-\dots\right)$$

Numerical analysis

Solving linear equations (Newton's method)

Given a function $f(x)$ that is continuous and has a continuous derivative and also, given a starting value of x_o:

```
    For $n = 1,2,3, \ldots$ Until termination, do:
    Compute $f'(x_n)$
    If $f'(x_n) = 0$, signal and stop
    Else, compute
```

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \tag{1.110a}$$

```
    Test for termination
```

Example 1.20

Let the function $f(x) = x^3 + x - 1 = 0$.

$$x_{n+1}=x_n-\frac{x_n{}^3+x_n-1}{3x_n{}^2+1}=\frac{2x_n{}^3+1}{3x_n{}^2+1} \tag{1.110b}$$

Starting value $x_o = 1$.
Therefore, $x_1 = 3/4 = 0.75$.

$$x_2=\frac{2*0.75^3+1}{3*0.75^2+1}=0.686047$$

$$x_3 = 0.682340$$

$$x_4 = 0.682328$$

Value of x_4 is exact to the 6th decimal. A larger number of analyses may be required if the function does not converge rapidly.

Therefore, the solution of the function $x^3 + x - 1 = 0$ will be $x = 0.682328$.

Newton's method in two variables [1]

Let (x_o,y_o) be chosen. We need to generate the sequence of vectors (x_n,y_n) recursively as follows:

Let

$$D(x,y) = f_x(x,y)g_y(x,y) - f_y(x,y)g_x(x,y)$$

Then,

$$x_{n+1} = x_n - \frac{f(x_n,y_n)g_y(x_n,y_n) - f_y(x_n,y_n)g(x_n,y_n)}{D(x_n,y_n)} \tag{1.111a}$$

$$y_{n+1} = y_n - \frac{-f(x_n,y_n)g_x(x_n,y_n) + f_x(x_n,y_n)g(x_n,y_n)}{D(x_n,y_n)} \tag{1.111b}$$

Under suitable conditions, the sequence (x_n,y_n) will converge to a vector (u,v) that satisfies

$$f(u,v) = g(u,v) = 0 \tag{1.111c}$$

Newton's method is most likely to work when the initial vector (x_o,y_o) is close to a solution. It is not necessary that the system has only one solution.

Example 1.21

Use Newton's method to find a solution to the system

$$x^2 - 2x - y + 1/2 = 0$$

$$x^2 + 4y^2 - 4 = 0$$

starting at the initial vector (2,0.25).

Solution

$$f = x^2 - 2x - y + 1/2$$

$$f_x = 2x - 2 \text{ and } f_y = -1$$

$$g = x^2 + 4y^2 - 4$$

$$g_x = 2x \text{ and } g_y = 8y$$

$$D(x,y) = f_x g_y - f_y g_x = (2x - 2)(8y) - (-1)(2x)$$

$$= 2x - 16y + 16xy$$

$$fg_y - f_y g = (x^2 - 2x - y + 1/2)(8y) - (-1)(x^2 + 4y^2 - 4)$$

$$= -4 + 4y + x^2 - 4y^2 - 16xy + 8x^2y$$

$$-fg_x + f_x g = -(x^2 - 2x - y + 1/2)(2x) + (2x - 2)(x^2 + 4y^2 - 4)$$

$$= 8 - 9x + 2xy + 2x^2 - 8y^2 + 8xy^2$$

Thus,

$$x_{n+1} = x_n - \frac{-4 + 4y_n + x_n^2 - 4y_n^2 - 16x_n y_n + 8x_n^2 y_n}{2x_n - 16y_n + 16x_n y_n}$$

$$y_{n+1} = y_n - \frac{8 - 9x_n + 2x_n y_n + 2x_n^2 - 8y_n^2 + 8x_n y_n^2}{2x_n - 16y_n + 16x_n y_n}$$

Starting at $x_o = 2$ and $y_o = 0.25$,

$$x_1 = 2 - 0.75/8 = 1.90625$$

and

$$y_1 = 0.25 - (-)0.5/8 = 0.3125$$

The first four iterations are presented in Table 1.7.

Table 1.7 Solution of Example 1.21

n	x_n	y_n	x_{n+1}	y_{n+1}
0	2	0.25	1.90625	0.3125
1	1.90625	0.3125	1.900691	0.311213
2	1.900691	0.311213	1.900677	0.311219
3	1.900677	0.311219	1.900677	0.311219

The solution is correct up to six decimal places.

Values of above-mentioned two equations can be calculated using $x = 1.900677$ and $y = 0.311219$ from Table 1.7 as

$$x^2 - 2x - y + 1/2 = (1.900677)^2 - 2(1.900677) - 0.311219 + 0.5 = 0.000000058$$

$$x^2 + 4y^2 - 4 = (1.900677)^2 + 4(0.311219)^2 - 4 = 0.000002112$$

Numerical methods in linear algebra

Gauss elimination [6]

In this method unknowns are eliminated by combining equations such that the n equations and n unknowns are reduced to an equivalent upper triangular system, which is then solved by back substitution. Let us consider the following system:

$$\begin{array}{lll} a_{11}x_1 & a_{12}x_2 & a_{13}x_3 = b_1 \\ a_{21}x_1 & a_{22}x_2 & a_{23}x_3 = b_2 \\ a_{31}x_1 & a_{32}x_2 & x_{33}x_3 = b_3 \end{array} \tag{1.112}$$

For the first stage of elimination, multiply the first row of Equation 1.112 by a_{21}/a_{11} and a_{31}/a_{11}, respectively, and subtract from the second and third rows.

$$\begin{array}{ll} a_{22(2)}x_2 & a_{23(2)}x_3 = b_{2(2)} \\ a_{32(2)}x_2 & a_{33(2)}x_3 = b_{3(2)} \end{array} \tag{1.113}$$

where

$$a_{22(2)} = a_{22} - (a_{21}/a_{11})a_{12}$$

$$a_{23(2)} = a_{23} - (a_{21}/a_{11})a_{13}$$

$$a_{32(2)} = a_{32} - (a_{31}/a_{11})a_{12}$$

$$a_{33(2)} = a_{33} - (a_{31}/a_{11})a_{13}$$

$$b_{2(2)} = b_2 - (a_{21}/a_{11})b_1$$

$$b_{3(2)} = b_3 - (a_{31}/a_{11})b_1$$

In the next stage of elimination, multiply the first row of Equation 1.113 by $a_{32(2)}/a_{22(2)}$ and subtract from the second row in Equation 1.113.

$$a_{33(3)}\ x_3 = b_{3(3)}$$

where

$$a_{33(3)} = a_{33(2)} - (a_{32(2)}/a_{22(2)})a_{23(2)}$$

$$b_{3(3)} = b_{3(2)} - (a_{32(2)}/a_{22(2)})b_{2(2)}$$

The first equation from each stage results in

$$\begin{array}{rrr} a_{11}x_1 & a_{12}x_2 & a_{13}x_3 = b_1 \\ & a_{22(2)}x_2 & a_{23(2)}x_3 = b_{2(2)} \\ & & a_{33(3)}x_3 = b_{3(3)} \end{array} \tag{1.114}$$

The elements a_{11}, $a_{22(2)}$, and $a_{33(3)}$, which are assumed to be nonzero, are called pivot elements and this elimination method is called Gauss elimination.

One condition of analysis is that the pivots must be different from zero. To achieve this, it may be required to change the order of equations. This is called partial pivoting. In some cases, it may require not only an interchange of equations but also an interchange of the position of the variables. This is called complete pivoting.

Example 1.22

Solve the following equations using Gauss elimination:

$$x_1 + x_2 + x_3 = 6$$

$$3x_1 + 3x_2 + 4x_3 = 20$$

$$2x_1 + x_2 + 3x_3 = 13$$

Solution

Eliminating x_1 from the last two equations,

$$x_1 + x_2 + x_3 = 6$$

$$x_3 = 2$$

$$-x_2 + x_3 = 1$$

In this case, the pivot point in the second equation is zero, and we need to interchange the preceding second and third equations before the second step. Therefore,

$$x_1 + x_2 + x_3 = 6$$

$$-x_2 + x_3 = 1$$

$$+\, x_3 = 2$$

The solution is $x_3 = 2$, $x_2 = 1$, and $x_1 = 3$.

Cholesky method

This method is also known as the square root method. If the coefficient matrix A is symmetric and positive definite, then the matrix A can be decomposed as $A = LL^T$.

The popular method of solving an equation

$$Ax = b \tag{1.115a}$$

is

$$A = LL^T \tag{1.115b}$$

or

$$LL^Tx = b \tag{1.115c}$$

This can be written as

$$L^Tx = z \tag{1.115d}$$

$$Lz = b \tag{1.115e}$$

Based on the preceding equations, a matrix can be factorized as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} m_{11} & 0 & 0 \\ m_{21} & m_{22} & 0 \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} m_{11} & m_{21} & m_{31} \\ 0 & m_{22} & m_{32} \\ 0 & 0 & m_{33} \end{bmatrix} \tag{1.115f}$$

where

$$m_{11} = \sqrt{a_{11}} \tag{1.115g}$$

$$m_{ii} = \sqrt{a_{ii} - \sum_{j=1}^{i-1} m^2{}_{ij}} \qquad i = 2, \ldots, n \tag{1.115h}$$

$$m_{i1} = \frac{a_{i1}}{m_{11}} \qquad i = 2, \ldots, n \tag{1.115i}$$

$$m_{ij} = \frac{1}{m_{jj}} \left(a_{ij} - \sum_{k=1}^{j-1} m_{jk} m_{ik} \right) \qquad i = j + 1, j + 2, \ldots, n, k \geq 2 \tag{1.115j}$$

Example 1.23

Using the Cholesky method, solve the following equations:

$$\begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The form of factorization

$$\begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} m_{11} & 0 & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ m_{31} & m_{32} & m_{33} & 0 \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} m_{11} & m_{21} & m_{31} & m_{41} \\ 0 & m_{22} & m_{32} & m_{42} \\ 0 & 0 & m_{33} & m_{43} \\ 0 & 0 & 0 & m_{44} \end{bmatrix}$$

Using the preceding equations,

$$m_{11} = 2$$

$$m_{21} = -1/2 \quad m_{22} = (15/4)^{0.5}$$

$$m_{31} = 0 \quad m_{32} = -(4/15)^{0.5} \quad m_{33} = (56/15)^{0.5}$$

$$m_{41} = 0 \quad m_{42} = 0 \quad m_{43} = -(15/56)^{0.5} \quad m_{44} = (209/56)^{0.5}$$

The solution for z will be

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ -1/2 & \sqrt{15/4} & 0 & 0 \\ 0 & -\sqrt{4/15} & \sqrt{56/15} & 0 \\ 0 & 0 & -\sqrt{15/56} & \sqrt{209/56} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

or

$$z_1 = \frac{1}{2}, \; z_2 = \frac{1}{\sqrt{60}}, \; z_3 = \frac{1}{\sqrt{840}}, \; z_4 = \frac{1}{\sqrt{11704}}$$

The final solution for x will be

$$\begin{bmatrix} 2 & -1/2 & 0 & 0 \\ 0 & \sqrt{15/4} & -\sqrt{4/15} & 0 \\ 0 & 0 & \sqrt{56/15} & -\sqrt{15/56} \\ 0 & 0 & 0 & \sqrt{209/56} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/\sqrt{60} \\ 1/\sqrt{840} \\ 1/\sqrt{11704} \end{bmatrix}$$

or

$$x_4 = \frac{1}{209},\ x_3 = \frac{4}{209},\ x_2 = \frac{15}{209},\ x_1 = \frac{56}{209}$$

Numerical integration

Two methods are extremely popular in numerical integration:

1. Trapezoidal rule
2. Simpson's rule

Trapezoidal rule

Under this rule, the interval (a,b) is divided into N subintervals of length $h = (b - a)/N$, and if the subintervals are denoted as (x_0,x_1), (x_1,x_2), (x_2,x_3), …, (x_{N-1},x_N), then the general integration can be written as

$$I = \int_a^b f(x)dx = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \ldots + \int_{x_{N-1}}^{x_N} f(x)dx \tag{1.116}$$

Evaluating each integral by the trapezoidal rule, the integration can be written as

$$I = \frac{h}{2}\left[f_0 + 2\left(f_1 + f_2 + \ldots + f_{N-1}\right) + f_N\right] \tag{1.117}$$

Simpson's rule

Under this rule, the interval of integration (a,b) is divided into an even number of equal subintervals, say into $N = 2M$ subintervals of length $h = (a - b)/2M$, with end points $x_0 = a$, x_1, x_2, …, x_{2M-1}, $x_{2M} = b$. The general form of the integration will be:

$$I = \int_a^b f(x)dx = \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \ldots + \int_{x_{2M-2}}^{x_{2M}} f(x)dx \tag{1.118}$$

Using Simpson's rule, the final integration will be

$$I = \frac{h}{3}\left[f_0 + 4\left(f_1 + f_3 + \ldots + f_{2M-1}\right) + 2\left(f_2 + f_4 + \ldots + f_{2M-2}\right) + f_{2M}\right] \quad (1.119)$$

Example 1.24

A slug catcher 60 m long, having a 0.5 m diameter, and at a 2° angle with the horizontal plane has a liquid-filled length of 5 m as shown in Figure 1.16. Calculate the filled volume of the slug catcher.

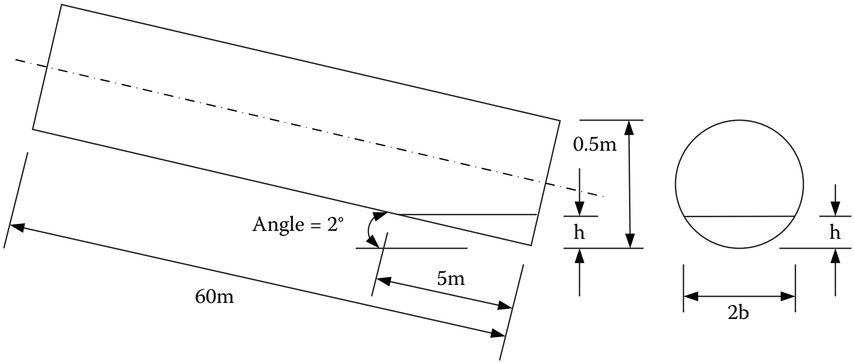


Figure 1.16 Slug catcher volume.

Solution

The volume can be calculated by integration using Simpson's rule.

The distance from the bottom corner of the slug catcher up to the liquid level (h) can be calculated as $h = 5\ \tan 2° = 0.175$ m. This is less than the radius of the pipe. For this condition, the top view of the surface will always look like a half-ellipse.

For a liquid length of l, if the major axis is $2x$ and minor axis is $2y$ (values of x and y depend on the value of l), the surface area (A) of the half-ellipse will be:

$$A = \pi xy/2$$

The volume of the liquid (V) can be estimated as

$$V = \int_{l=0}^{l=5} \frac{\pi xy}{2} \sin 2^0\, dl \quad (1.120a)$$

Now,

$$x = 1/\cos 2^\circ \tag{1.120b}$$

and

$$y = \left\{l \tan 2^\circ (2r - l \tan 2^\circ)\right\}^{0.5} \tag{1.120c}$$

where r = radius of the slug catcher = 0.25 m.

Therefore, the volume will be

$$V = \int_{l=0}^{l=5} \frac{\pi l}{2\cos 2^0} * \left\{l \tan 2^\circ (2r - l \tan 2^\circ)\right\}^{0.5} \sin 2^\circ dl$$

or

$$V = 0.054853 \int_{l=0}^{l=5} l * \left\{0.03492 l * (0.5 - 0.03492 l)\right\}^{0.5} dl \tag{1.120d}$$

Now, a = 0 and b = 5.

Number of divisions between limits (1 = 0 to 1 = 5), 2M = 50, and value of each division, h = 5/50 = 0.

The value of integrals will be

$$f_0 = 0, f_{50} = 0.054853 * 5 * \{0.1746 * (0.5 - 0.1746)\}^{0.5} = 0.06537$$

$$f_1 = 0.054853 * 0.1 * \left\{0.003492 * (0.5 - 0.003492)\right\}^{0.5} = 0.000228$$

$$f_2 = 0.054853 * 0.2 * \left\{0.006984 * (0.5 - 0.006984)\right\}^{0.5} = 0.000644$$

and so on.

The sum of the odd integrals will be

$$(f_1 + f_3 + f_5 + \ldots + f_{49}) = 0.701109$$

and the sum of the even integrals (2 to 48) will be

$$(f_2 + f_4 + f_6 + \ldots + f_{48}) = 0.668797$$

Therefore, the volume will be

$$V = \frac{0.1}{3}\left\{0 + 4 * 0.701109 + 2 * 0.668797 + 0.06537\right\} = 0.14 \quad \text{m}^3$$

Double integration using Simpson's rule [6]

A double integration in the following form:

$$I = \int_c^d \left(\int_a^b f(x,y)dx \right) dy \tag{1.121}$$

can be solved using Simpson's rule with $h = (b - a)/2$ and $k = (d - c)/2$. The value of the integral will be:

$$I = \frac{hk}{9}[f(a,c) + f(a,d) + f(b,c) + f(b,d) + 4\{f(a,c+k) + f(a+h,c) \\ + f(a+h,d) + f(b,c+k)\} + 16f(a+h,c+k)] \tag{1.122}$$

Example 1.25

Using Simpson's rule, evaluate the integral

$$I = \int_1^2 \int_1^2 \frac{dxdy}{(x+y)}$$

Solution

$h = 0.5$ and $k = 0.5$.

$$I = \frac{0.25}{9}[f(1,1) + f(1,2) + f(2,1) + f(2,2) + 4\{f(1,1.5) + f(1.5,1) + \\ f(1.5,2) + f(2,1.5)\} + 16f(1.5,1.5)]$$

or

$$I = \frac{0.25}{9}\left[0.5 + \frac{2}{3} + 0.25 + 4\left\{\frac{2}{2.5} + \frac{2}{3.5}\right\} + \frac{16}{3}\right] = 0.33988$$

A more accurate solution is 0.339789.

Numerical solution of first-order differential equations

Euler's method [1]

This method is very simple, but results are not very accurate. In this method, y_1 is obtained by assuming that $f(x,y)$ varies so little in the interval $x_0 \le x \le x_1$ that only a small error is made by replacing it by the constant value $f(x_0,y_0)$. Integrating

$$\frac{dy}{dx} = f(x,y)$$

from x_0 to x_1, we get

$$y(x_1) - y_0 = y(x_1) - y(x_0) = \int_{x_0}^{x_1} f(x,y)dx \approx f(x_0,y_0)(x_1 - x_0) \qquad (1.123)$$

Because $h = x_1 - x_0$,

$$y_1 = y_0 + hf(x_0,y_0)$$

The general equation is

$$y_{n+1} = y_n + hf(x_n,y_n) \qquad (1.124)$$

Improved Euler's method

The accuracy of Euler's method is improved by introducing the trapezoidal rule:

$$\int_{x_0}^{x_1} f(x,y)dx \approx \frac{h}{2}\{f(x_0,y_0) + f(x_1,y(x_1))\} \qquad (1.125)$$

The value of $y(x_1)$ is not known and is estimated by Euler's method, which is called z_1. This modification results in general equations such as:

$$z_{n+1} = y_n + hf(x_n,y_n) \qquad (1.126a)$$

and

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n,y_n) + f(x_{n+1},z_{n+1})] \qquad (1.126b)$$

Example 1.26

Using Euler's method, solve the following equation:

$$\frac{dy}{dx} = -2xy^2 \text{ , y(0) = 1}$$

at the value $x = 1$.

Solution

The value of $h = 0.2$, and the calculation is made by using values of x as 0.0, 0.2, 0.4, 0.6, 0.8, and 1.0.

$$z_1 = y_0 + hf(x_0, y_0) = 1$$

$$y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, z_1)\right] = 1 + 0.1 * [0 - 0.4] = 0.96$$

$$z_2 = y_1 + hf(x_1, y_1) = 0.96 + 0.2(-0.3686) = 0.8863$$

$$y_2 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, z_2)\right] = 0.96 + 0.1 * [-0.3686 - 0.6284] = 0.8603$$

and so on.

The results are presented in Table 1.8 (calculated volume = 0.5034).

Table 1.8 Solution of Example 1.26

x_n	y_n	f(x_n,y_n)	z_{n+1}	$f(x_{n+1},z_{n+1})$	y_{n+1}
0	1	0	1	–0.4	0.96
0.2	0.96	–0.3686	0.8863	–0.6284	0.8603
0.4	0.8603	–0.5921	0.7419	–0.6605	0.7350
0.6	0.7350	–0.6483	0.6053	–0.5862	0.6116
0.8	0.6116	–0.5985	0.4919	–0.4839	0.5034
1.0	0.5034				

The exact solution is 0.5.

Runge–Kutta method [5]

The Runge–Kutta method is widely used as a numerical method to solve differential equations. This method is more accurate than the improved Euler's method. This method computes the solution of the initial value problem.

$y' = f(x,y)$, $y(x_0) = y_0$ at equidistance points

$x_1 = x_0 + h$, $x_2 = x_0 + 2h$ … $x_N = x_0 + Nh$

The general method of solution is
For $n = 0, 1, 2, 3 \ldots, N - 1$
Evaluate

$$k_1 = hf(x_n, y_n) \tag{1.127a}$$

$$k_2 = hf(x_n + 0.5h, y_n + 0.5k_1) \tag{1.127b}$$

$$k_3 = hf(x_n + 0.5h, y_n + 0.5k_2) \tag{1.127c}$$

$$k_4 = hf(x_n + h, y_n + k_3) \tag{1.127d}$$

$$x_{n+1} = x_n + h \tag{1.127e}$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{1.127f}$$

This method is repeated till the solution is reached.

Example 1.27

Solve Example 1.26 using the Runge–Kutta method.

$$\frac{dy}{dx} = -2xy^2, \ \mathrm{y}(0) = 1$$

at the value $x = 1$.

Solution

For $n = 0$, $x_n = 0$, $y_n = 1$, and value of $h = 0.2$.
Using the preceding equations,

$$k_1 = 0.2f(0,1) = 0$$

$$k_2 = 0.2f(0.1,1) = -0.04$$

$$k_3 = 0.2f(0.1,0.98) = -0.0384$$

$$k_4 = 0.2f(0.2,0.9616) = -0.074$$

$$x_1 = 0.2$$

$$y_1 = 1 + \frac{1}{6}(0 - 0.08 - 0.0768 - 0.074) = 0.9615$$

Similarly, other values can be calculated and tabulated as in Table 1.9 (calculated volume is 0.500007).

Table 1.9 Solution of Example 1.27

n	x_n	y_n	k_1	k_2	k_3	k_4	x_{n+1}	y_{n+1}
0	0	1	0	–0.04	–0.038	–0.074	0.2	0.9615
1	0.2	0.9615	–0.074	–0.103	–0.099	–0.119	0.4	0.8621
2	0.4	0.8621	–0.119	–0.129	–0.127	–0.1296	0.6	0.7353
3	0.6	0.7353	–0.13	–0.126	–0.127	–0.119	0.8	0.6098
4	0.8	0.6098	–0.119	–0.109	–0.111	–0.0995	1.0	0.500007
5	1.0	0.500007						

The exact solution is 0.5.

Second-order differential equations

Runge–Kutta–Nystrom method [5]

This system computes the solution of the initial value problem for an equation $y'' = f(x, y, y')$, where initial values x_0, y_0, y_0', step size h, and number of steps N are known.

The general method of solution is as follows:

For $n = 0, 1, 2, \ldots, N-1$

Evaluate

$$k_1 = 0.5hf(x_n, y_n, y_n') \tag{1.128a}$$

$$K = 0.5h(y_n' + 0.5k_1) \tag{1.128b}$$

$$k_2 = 0.5hf(x_n + 0.5h, y_n + K, y_n' + k_1) \tag{1.128c}$$

$$k_3 = 0.5hf(x_n + 0.5h, y_n + K, y_n' + k_2) \tag{1.128d}$$

$$L = h(y'_n + k_3) \tag{1.128e}$$

$$k_4 = 0.5hf(x_n + h, y_n + L, y'_n + 2k_3) \tag{1.128f}$$

$$x_{n+1} = x_n + h \tag{1.128g}$$

$$y_{n+1} = y_n + h\left(y'_n + \frac{1}{3}(k_1 + k_2 + k_3)\right) \tag{1.128h}$$

$$y'_{n+1} = y'_n + \frac{1}{3}(k_1 + 2k_2 + 2k_3 + k_4) \tag{1.128i}$$

Example 1.28

Solve the following second-order differential equation:

$$\frac{d^2y}{dx^2} = 0.5(x + y + y' + 2) \text{ at the value x = 1}$$

Initial values: $x_0 = 0$, $y_0 = 0$, $y_0' = 0$, and $h = 0.2$.

Solution

For $n = 0$, $x_n = 0$, $y_n = 0$, $y_n' = 0$, and value of $h = 0.2$.
Using the preceding equations,

$$k_1 = 0.1$$

$$K = 0.005$$

$$k_2 = 0.11025$$

$$k_3 = 0.110763$$

$$\mathrm{L} = 0.022153$$

$$k_4 = 0.122184$$

$$x_{n+1} = 0.2$$

$$y_{n+1} = 0.021401$$

$$y_n{}'_{+1} = 0.221401$$

Similarly, other values can be calculated and tabulated as shown in Table 1.10.

Table 1.10 Solution of Example 1.28

Value n	x_n	y_n	y_n'	x_{n+1}	y_{n+1}	y_{n+1}'
0	0	0	0	0.2	0.021401	0.221401
1	0.2	0.021401	0.221401	0.4	0.091819	0.49182
2	0.4	0.091819	0.49182	0.6	0.222109	0.82211
3	0.6	0.222109	0.82211	0.8	0.425526	1.225526
4	0.8	0.425526	1.225526	1	0.718258	1.718259
5	1	0.718258	1.718259			

The solution up to four decimal places = 0.7183.

Partial differential equations

In chemical engineering, solutions of partial differential equations are often required, particularly for the problems associated with heat conduction. Different numerical methods are used for solving different types of partial differential equations.

Heat conduction problem [5]

Generally, the Crank–Nicolson method is used for numerical solution of heat conduction problems that are parabolic in nature. This method solves a mesh with mesh size h in x-direction and mesh size k in y-direction (time direction). It calculates the values of u at six points as shown in Figure 1.17.

The equation used to solve the problem is:

$$\frac{1}{k}(u_{i,j+1}-u_{ij})=\frac{1}{2h^2}(u_{i+1,j}-2u_{ij}+u_{i-1,j})+\frac{1}{2h^2}(u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}) \quad (1.129)$$

Using $r = k/h^2$, Equation 1.129 can be modified to

$$(2+2r)u_{i,j+1}-r(u_{i+1,j+1}+u_{i-1,j+1})=(2-2r)u_{ij}+r(u_{i+1,j}+u_{i-1,j}) \quad (1.130)$$

In Equation 1.130, the three values on the left side are unknown, whereas the three values on right side are known. Using the initial and boundary

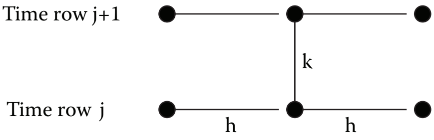


Figure 1.17 Mesh size.

conditions, most of the values can be made known except two values. Equation 1.130 is then solved for two consecutive values of m (m = 1,2, then m = 2,3, etc.). Under this situation, we will have two equations with two unknowns that can be solved easily.

Example 1.29

Solve the problem in Example 1.16 using theoretical and numerical methods.

The heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Initial condition: $u = x$ when $t = 0$.
Boundary condition: $u = 0$ when $x = 0$ and $x = l$.
This problem is further simplified by assuming $c = 1$ and $l = 1$.

Solution

Theoretical Solution

The general solution assuming $c = 1$ and $l = 1$ can be obtained from Equation 1.104c as follows:

$$u = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n} e^{-(n\pi)^2 t}\sin(n\pi x)$$

Value $i = 0$

Values of $u = 0$ for all grid points of n.

Value $i = 1$ ($x = 0.3333$)

Value of $j = 1$ ($t = 0.0278$)

For $n = 1$
sin ($n\pi x$) = 0.866
Exp ($-n^2\pi^2 t$) = 0.7602
Subvalue = 0.6584

For $n = 2$
sin ($n\pi x$) = 0.866
Exp ($-n^2\pi^2 t$) = 0.334
Subvalue = –0.1446

For $n = 3$
sin ($n\pi x$) = 0
Exp ($-n^2\pi^2 t$) = 0.0848
Subvalue = 0

For $n = 4$
sin ($n\pi x$) = –0.866

Exp $(-n^2\pi^2 t) = 0.0124$
Subvalue = 0.0027

It is clear from the preceding analysis that this infinite series is highly converging in nature, and it is adequate to calculate up to $n = 4$.

The sum = 0.3287.

Similarly, other values of j can be estimated as follows:

Value of $j = 2$ ($t = 0.0556$)
The sum = 0.2879

Value of $j = 3$ ($t = 0.0833$)
The sum = 0.2319

Value of $j = 4$ ($t = 0.1111$)
The sum = 0.1807

and so on.

Using the procedure given earlier, other values for $i = 2$, etc. can be estimated easily.

Numerical solution

The nodal points are represented in Figure 1.18 as follows:

For the solution, the values of h and k have been assumed as $h = 1/3$ and $k = 1/36$; therefore, $r = 1/4$.

Equation 1.130 will be changed to

$$-0.25u_{i-1,j+1} + 2.5u_{i,j+1} - 0.25u_{i+1,j+1} = 0.25u_{i-1,j} + 1.5u_{ij} + 0.25u_{i+1,j} \tag{1.131}$$

From initial and boundary conditions, we have

$$u_{0,0} = 0; u_{1,0} = 1/3; u_{2,0} = 2/3$$

$$u_{0,1} = u_{0,2} = u_{3,0} = u_{3,1} = u_{3,2} = 0$$

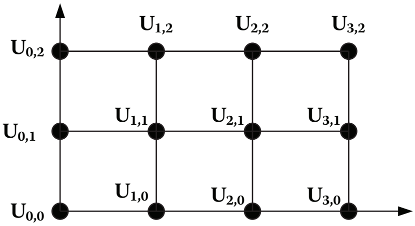


Figure 1.18 Representation of nodal points.

For *j* = 0

For *i* = 1

$$-0.25u_{0,1} + 2.5u_{1,1} - 0.25u_{2,1} = 0.25u_{0,0} + 1.5u_{1,0} + 0.25u_{2,0}$$

or

$$2.5u_{1,1} - 0.25u_{2,1} = 0.6667 \tag{1.132a}$$

For *i* = 2

$$-0.25u_{1,1} + 2.5u_{2,1} - 0.25u_{3,1} = 0.25u_{1,0} + 1.5u_{2,0} + 0.25u_{3,0}$$

or

$$-0.25u_{1,1} + 2.5u_{2,1} = 1.0833 \tag{1.132b}$$

Solving Equation 1.132a and Equation 1.132b, $u_{1,1} = 0.3131$ and $u_{2,1} = 0.4646$.

For j = 1

For *i* = 1

$$-0.25u_{0,2} + 2.5u_{1,2} - 0.25u_{2,2} = 0.25u_{0,1} + 1.5u_{1,1} + 0.25u_{2,1}$$

or

$$2.5u_{1,2} - 0.25u_{2,2} = 0.5858 \tag{1.133a}$$

For *i* = 2

$$-0.25u_{1,2} + 2.5u_{2,2} - 0.25u_{3,2} = 0.25u_{1,1} + 1.5u_{2,1} + 0.25u_{3,1}$$

or

$$-u_{1,2} + 2.5u_{2,2} = 0.7752 \tag{1.133b}$$

Solving Equation 1.133a and Equation 1.133b, $u_{2,2} = 0.3369$ and $u_{1,2} = 0.268$.

The theoretically calculated values and values obtained through numerical analysis are presented in Table 1.11.

Table 1.11 Example 1.29

Node Point	Theoretical	Numerical Analysis
$u_{1,1}$	0.3287	0.3131
$u_{2,1}$	0.5094	0.4646
$u_{1,2}$	0.2879	0.268
$u_{2,2}$	0.3494	0.3369

Alternating direction implicit (ADI) method [6]

These methods are two-step methods involving the solution of traditional systems of equations along lines parallel to the x- and y-axes, at the first and second step, respectively.

In the Peaceman–Rachford ADI method, in the first step, it is advanced from t_m to $t_{m+1/2}$, and in the second step, it is advanced from $t_{m+1/2}$ to t_{m+1}. The ADI method is written as

$$\frac{u_{i,j}^{m+1/2} - u_{i,j}^{m}}{k/2} = \frac{1}{h^2}\partial_x^2 u_{i,j}^{m+1/2} + \frac{1}{h^2}\partial_y^2 u_{i,j}^{m} \tag{1.134a}$$

and

$$\frac{u_{i,j}^{m+1} - u_{i,j}^{m+1/2}}{k/2} = \frac{1}{h^2}\partial_x^2 u_{i,j}^{m+1/2} + \frac{1}{h^2}\partial_y^2 u_{i,j}^{m+1} \tag{1.134b}$$

The equations can also be written as

$$\left(1 - \frac{r}{2}\partial_x^2\right)u_{i,j}^{m+1/2} = \left(1 + \frac{r}{2}\partial_y^2\right)u_{i,j}^{m} \tag{1.135a}$$

$$\left(1 - \frac{r}{2}\partial_y^2\right)u_{i,j}^{m+1} = \left(1 + \frac{r}{2}\partial_x^2\right)u_{i,j}^{m+1/2} \tag{1.135b}$$

The intermediate values can be obtained as

$$u_{i,j}^{m+1/2} = 0.5\left(1 + \frac{r}{2}\partial_y^2\right)u_{i,j}^{m} + 0.5\left(1 - \frac{r}{2}\partial_y^2\right)u_{i,j}^{m+1} \tag{1.136}$$

The boundary conditions to obtain the solution of Equation 1.135a can be obtained as

$$u_{i,j}^{m+1/2} = 0.5\left(1+\frac{r}{2}\partial_y^2\right)u_{i,j}^{m} + 0.5\left(1-\frac{r}{2}\partial_y^2\right)u_{i,j}^{m+1} \quad i = 0,M \tag{1.137}$$

Eliminating $u_{i,j}^{m+1/2}$ from Equation 1.135a and Equation 1.135b,

$$\left(1-\frac{r}{2}\partial_x^2\right)\left(1-\frac{r}{2}\partial_y^2\right)u_{i,j}^{m+1} = \left(1+\frac{r}{2}\partial_x^2\right)\left(1+\frac{r}{2}\partial_y^2\right)u_{i,j}^{m} \tag{1.138}$$

Equation 1.138 can be written in the D'yakonov split form as

$$\left(1-\frac{r}{2}\partial_x^2\right)u_{i,j}^{*m+1} = \left(1+\frac{r}{2}\partial_x^2\right)\left(1+\frac{r}{2}\partial_y^2\right)u_{i,j}^{m} \tag{1.139a}$$

$$\left(1-\frac{r}{2}\partial_y^2\right)u_{i,j}^{m+1} = u_{i,j}^{*m+1} \tag{1.139b}$$

Intermediate boundary conditions

$$u_{0,j}^{*m+1} = \left(1-\frac{r}{2}\partial_y^2\right)(g_1)_j^{m+1} \tag{1.140a}$$

and

$$u_{M,j}^{*m+1} = \left(1-\frac{r}{2}\partial_y^2\right)(g_2)_j^{m+1} \tag{1.140b}$$

Example 1.30

Solve the following two-dimensional heat conduction equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

Initial condition:

$$u(x,y,0) = \sin \pi x \sin \pi y,\ 0 \le x,\ y \le 1$$

Boundary conditions:

$$u = 0, \text{ on the boundary for } t \geq 0$$

Use the ADI method. Assume $h = 1/4$ and $k = 1/8$ and integrate for one time step.

Solution

The nodal points are

$$x_i = \frac{i}{4}, \quad 0 \leq i \leq 4$$

$$y_j = \frac{j}{4}, \quad 0 \leq j \leq 4$$

The initial and boundary conditions become

$$u_{i,j}^{0} = \sin\left(\frac{\pi i}{4}\right)\sin\left(\frac{\pi j}{4}\right)$$

$$u_{0,j}^{m+1/2} = u_{4,j}^{m+1/2} = 0, \quad 0 \leq j \leq 4$$

$$u_{i,0}^{m+1} = 0 = u_{i,4}^{m+1}, \quad 0 \leq i \leq 4$$

The ADI method is given by

$$\left(1 - \frac{1}{16}\partial_x^2\right)u_{i,j}^{m+1/2} = \left(1 + \frac{1}{16}\partial_y^2\right)u_{i,j}^{m}$$

$$\left(1 - \frac{1}{16}\partial_y^2\right)u_{i,j}^{m+1} = \left(1 + \frac{1}{16}\partial_x^2\right)u_{i,j}^{m+1/2}$$

For $m = 0$, the solution for $u_{i,j}^{1/2}$

$$-\frac{1}{16}u_{i-1,j}^{1/2} + \frac{9}{8}u_{i,j}^{1/2} - \frac{1}{16}u_{i+1,j}^{1/2} = \frac{1}{16}u_{i,j-1}^{0} + \frac{7}{8}u_{i,j}^{0} + \frac{1}{16}u_{i,j+1}^{0}$$

For $j = 1$

For $i = 1$

$$-\frac{1}{16}u_{0,1}^{1/2}+\frac{9}{8}u_{1,1}^{1/2}-\frac{1}{16}u_{2,1}^{1/2}=\frac{1}{16}u_{1,0}^{0}+\frac{7}{8}u_{1,1}^{0}+\frac{1}{16}u_{1,2}^{0}$$

For i = 2

$$-\frac{1}{16}u_{1,1}^{1/2}+\frac{9}{8}u_{2,1}^{1/2}-\frac{1}{16}u_{3,1}^{1/2}=\frac{1}{16}u_{2,0}^{0}+\frac{7}{8}u_{2,1}^{0}+\frac{1}{16}u_{2,2}^{0}$$

For i = 3

$$-\frac{1}{16}u_{2,1}^{1/2}+\frac{9}{8}u_{3,1}^{1/2}-\frac{1}{16}u_{4,1}^{1/2}=\frac{1}{16}u_{3,0}^{0}+\frac{7}{8}u_{3,1}^{0}+\frac{1}{16}u_{3,2}^{0}$$

Using boundary conditions,

$$\begin{bmatrix} 9/8 & -1/16 & 0 \\ -1/16 & 9/8 & -1/16 \\ 0 & -1/16 & 9/8 \end{bmatrix}\begin{bmatrix} u_{1,1}^{1/2} \\ u_{2,1}^{1/2} \\ u_{3,1}^{1/2} \end{bmatrix}=\begin{bmatrix} (7\sqrt{2}+1)/(16\sqrt{2}) \\ (14+\sqrt{2})/(16\sqrt{2}) \\ (7\sqrt{2}+1)/(16\sqrt{2}) \end{bmatrix}$$

The solution is:

$$u_{1,1}^{1/2}=u_{3,1}^{1/2}=0.46468$$

$$u_{2,1}^{1/2}=0.65716$$

For j = 2

For i = 1

$$-\frac{1}{16}u_{0,2}^{1/2}+\frac{9}{8}u_{1,2}^{1/2}-\frac{1}{16}u_{2,2}^{1/2}=\frac{1}{16}u_{1,1}^{0}+\frac{7}{8}u_{1,2}^{0}+\frac{1}{16}u_{1,3}^{0}$$

For i = 2

$$-\frac{1}{16}u_{1,2}^{1/2}+\frac{9}{8}u_{2,2}^{1/2}-\frac{1}{16}u_{3,2}^{1/2}=\frac{1}{16}u_{2,1}^{0}+\frac{7}{8}u_{2,2}^{0}+\frac{1}{16}u_{2,3}^{0}$$

For *i* = 3

$$-\frac{1}{16}u_{2,2}^{1/2}+\frac{9}{8}u_{3,2}^{1/2}-\frac{1}{16}u_{4,2}^{1/2}=\frac{1}{16}u_{3,1}^{0}+\frac{7}{8}u_{3,2}^{0}+\frac{1}{16}u_{3,3}^{0}$$

Using boundary conditions,

$$\begin{bmatrix} 9/8 & -1/16 & 0 \\ -1/16 & 9/8 & -1/16 \\ 0 & -1/16 & 9/8 \end{bmatrix}\begin{bmatrix} u_{1,2}^{1/2} \\ u_{2,2}^{1/2} \\ u_{3,2}^{1/2} \end{bmatrix}=\begin{bmatrix} (7\sqrt{2}+1)/16 \\ (14+\sqrt{2})/16 \\ (7\sqrt{2}+1)/16 \end{bmatrix}$$

The solution is:

$$u_{1,2}^{1/2}=u_{3,2}^{1/2}=0.65716$$

$$u_{2,2}^{1/2}=0.92936$$

For *j* = 3

For *i* = 1

$$-\frac{1}{16}u_{0,3}^{1/2}+\frac{9}{8}u_{1,3}^{1/2}-\frac{1}{16}u_{2,3}^{1/2}=\frac{1}{16}u_{1,2}^{0}+\frac{7}{8}u_{1,3}^{0}+\frac{1}{16}u_{1,4}^{0}$$

For *i* = 2

$$-\frac{1}{16}u_{1,3}^{1/2}+\frac{9}{8}u_{2,3}^{1/2}-\frac{1}{16}u_{3,3}^{1/2}=\frac{1}{16}u_{2,2}^{0}+\frac{7}{8}u_{2,3}^{0}+\frac{1}{16}u_{2,4}^{0}$$

For *i* = 3

$$-\frac{1}{16}u_{2,3}^{1/2}+\frac{9}{8}u_{3,3}^{1/2}-\frac{1}{16}u_{4,3}^{1/2}=\frac{1}{16}u_{3,2}^{0}+\frac{7}{8}u_{3,3}^{0}+\frac{1}{16}u_{3,4}^{0}$$

Using boundary conditions,

$$\begin{bmatrix} 9/8 & -1/16 & 0 \\ -1/16 & 9/8 & -1/16 \\ 0 & -1/16 & 9/8 \end{bmatrix}\begin{bmatrix} u_{1,3}^{1/2} \\ u_{2,3}^{1/2} \\ u_{3,3}^{1/2} \end{bmatrix}=\begin{bmatrix} (14+\sqrt{2})/32 \\ (7\sqrt{2}+1)/32 \\ (14+\sqrt{2})/32 \end{bmatrix}$$

The solution is:

$$u_{1,3}^{1/2} = u_{3,3}^{1/2} = 0.46468$$

$$u_{2,3}^{1/2} = 0.65716$$

For the solution of $u_{i,j}^{1}$,

$$-\frac{1}{16}u_{i,j-1}^{1} + \frac{9}{8}u_{i,j}^{1} - \frac{1}{16}u_{i,j+1}^{1} = \frac{1}{16}u_{i-1,j}^{1/2} + \frac{7}{8}u_{i,j}^{1/2} + \frac{1}{16}u_{i+1,j}^{1/2}$$

For $i = 1$

For $j = 1, 2$, and 3

$$\begin{bmatrix} 9/8 & -1/16 & 0 \\ -1/16 & 9/8 & -1/16 \\ 0 & -1/16 & 9/8 \end{bmatrix} \begin{bmatrix} u_{1,1}^{1} \\ u_{1,2}^{1} \\ u_{1,3}^{1} \end{bmatrix} = \begin{bmatrix} 0.44767 \\ 0.63310 \\ 0.44767 \end{bmatrix}$$

The solution is:

$$u_{1,1}^{1} = u_{1,3}^{1} = 0.43186$$

$$u_{1,2}^{1} = 0.61074$$

This method is repeated to find the solution at each mesh point.

Equation of state

Boyle's law and Charles' law

We can assume a cubical box of each side length l (as shown in Figure 1.19) and three axes are x, y, and z. The velocity v can be defined as

$$v^2 = x^2 + y^2 + z^2 \tag{1.141}$$

The momentum of a gas particle, mass m, in the x-direction will be mx, and if we assume the gas molecule rebounds in an elastic manner, the change in momentum will be $2mx$.

Now with the distance between two opposite walls equal to l, the number of collisions per unit time will be x/l. The change in momentum per unit

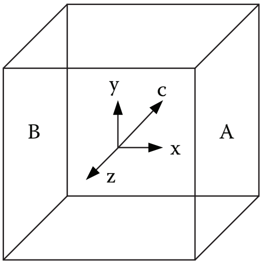


Figure 1.19 Pressure in a cube.

time will be $2mx^2/l$. Considering all sides, the total momentum imparted per unit time will be

$$\frac{2m(x^2+y^2+z^2)}{l}=\frac{2mc^2}{l}$$

Now considering N number of molecules, the total force exerted on all walls will be

$$F=\frac{2mNc^2}{l} \tag{1.142}$$

Again, pressure P can be defined as

$$P=\frac{F}{6l^2}=\frac{1}{3}\frac{mNc^2}{V} \tag{1.143a}$$

or

$$PV=\frac{1}{3}mNc^2 \tag{1.143b}$$

where

V = volume of the cube

Now,

$$PV=\frac{2}{3}N*\frac{1}{2}mc^2$$

For a given mass of gas, 2/3 N and 1/2 mc^2 (kinetic energy) are constant if the temperature is constant. Otherwise, PV is constant when the temperature is constant. This is Boyle's law.

Again, at constant pressure V/T is constant, which is Charles' law.

Combining Boyle's and Charles' laws,

$$PV = RT \tag{1.143c}$$

where

R = universal gas constant

Equation of state for real gas

Equation 1.143c is valid for a perfect gas. In actual practice, the equation is required to be modified to represent the actual world. There are large numbers of semiempirical correlations available in the literature and they are generally developed following identical logic. A gas molecule experiences uniform attraction in all directions when located at the center of the container. However, the gas molecule close to the wall will experience an unbalanced attraction (pulling inward). This will reduce the momentum of the gas molecule. If this decrease in pressure is denoted as p', then the ideal pressure will be $P + p'$, where P is the actual observed pressure.

Similarly, the gas molecule will occupy some space within the container and the actual volume will be reduced to some extent. If the reduction in volume is b, the actual volume will be $V - b$. With these modifications, the general equation is modified as

$$(P + a)\ (V - b) = RT \tag{1.144}$$

Various equations of state (EOSs) were developed to establish calculation procedures for a and b. Out of a large number of EOSs two are extensively used in process calculations and these are due to:

- Peng–Robinson [7]
- Soave–Redlich–Kwong (SRK) [8]

Comparison between Peng–Robinson and SRK EOSs

The comparison between Peng–Robinson (PR) and SRK EOSs is presented in Table 1.12, where

T = absolute temperature
P = absolute pressure
V = specific volume
Z = compressibility
T_c = critical temperature
P_c = critical pressure

Table 1.12 Comparison between PR and SRK EOSs

	Peng–Robinson	SRK	Equation Number
	$P=\frac{RT}{V-b}-\frac{a}{V(V+b)+b(V-b)}$	$P=\frac{RT}{V-b}-\frac{a}{V(V+b)}$	1.145a
	$Z^3-(1-B)Z^2+(A-2B-3B^2)Z-(AB-B^2-B^3)=0$	$Z^3-Z^2+(A-B-B^2)Z-AB=0$	1.145b
where			
$b =$	$\sum_{i=1}^{N} x_i b_i$	$\sum_{i=1}^{N} x_i b_i$	1.145c
$b_i =$	$0.077796\frac{RT_{ci}}{P_{ci}}$	$0.08664\frac{RT_{ci}}{P_{ci}}$	1.145d
$a =$	$\left(\sum_{i=1}^{N} x_i a_i^{0.5}\right)^2$	$\left(\sum_{i=1}^{N} x_i a_i^{0.5}\right)^2$	1.145e
$a_i =$	$a_{ci}\alpha_i$	$a_{ci}\alpha_i$	1.145f
$a_{ci} =$	$0.457235\frac{(RT_{ci})^2}{P_{ci}}$	$0.42747\frac{(RT_{ci})^2}{P_{ci}}$	1.145g
$\alpha_i^{0.5} =$	$1+m_i(1-T_{ri}^{0.5})$	$1+m_i(1-T_{ri}^{0.5})$	1.145h
$m_i =$	$0.37464+1.54226\omega_i-0.26992\omega_i^2$	$0.48+1.574\omega_i-0.176\omega_i^2$	1.145i
$A =$	$\frac{aP}{(RT)^2}$	$\frac{aP}{(RT)^2}$	1.145j
$B =$	$\frac{bP}{RT}$	$\frac{bP}{RT}$	1.145k

T_r = reduced temperature
ω = acentric factor
a, b = parameters used in the equations
$a(T)$ = parameter used in the equations
a_c = value of $a(T)$ at $T = T_c$
α = correction factor, a/a_c
m = slope of $\alpha^{0.5}$ against $T_r^{0.5}$
i = component i in the mixture
N = number of components

Acentric factor

One important parameter presented in the EOS is the acentric factor ω. The acentric factor is used to correlate physical and thermodynamic properties. Mathematically, the acentric factor is defined as [9]:

$$\omega = -\log p_r^* - 1.000 \tag{1.146}$$

where

p_r^* = reduced vapor pressure, p^*/p_c
p^* = vapor pressure at $T = 0.7\ T_c$, kPaA
p_c = critical pressure, kPaA
T = temperature, K
T_c = critical temperature, K

For hydrocarbon mixtures, the acentric factor is calculated as

$$\omega = \sum_{i=1}^{N} x_i \omega_i \tag{1.147}$$

where

N = number of components
ω_i = acentric factor of component i
x_i = mole fraction of component i

Other important factors used to establish physical and thermodynamic properties of hydrocarbon mixtures are the Watson characterization factor K and critical compressibility factor z_c. Table 1.13 presents values of the acentric factor, the Watson characterization factor, and the critical compressibility factor. A complete list is available in the API Technical Data Book [9].

Vapor pressure of pure components [7,8]

The fugacity coefficient of a pure component is estimated differently by SRK and PR EOSs.

SRK EOS

$$\ln \frac{f}{P} = Z - 1 - \ln(Z - B) - \frac{A}{B} \ln\left(\frac{Z + B}{Z}\right) \tag{1.148a}$$

Table 1.13 Acentric Factors, Watson Characterization Factors, and Critical Compressibility Factors of Pure Substances

Components	Acentric Factor (ω)	Watson Characterization Factor (K)	Critical Compressibility Factor (z_c)
Nonhydrocarbons			
Hydrogen	–0.2153		0.305
Nitrogen	0.0403		0.292
Oxygen	0.0218		0.288
Carbon monoxide	0.0663		0.295
Carbon dioxide	0.2276		0.274
Sulfur dioxide	0.2451		0.269
Hydrogen sulfide	0.0826		0.284
Water	0.3449		0.229
Ammonia	0.252		0.242
Paraffins			
Methane	0.0108	19.54	0.288
Ethane	0.0990	19.49	0.284
Propane	0.1517	14.69	0.280
n-Butane	0.1931	13.50	0.274
i-Butane	0.1770	13.78	0.282
n-Pentane	0.2486	13.03	0.269
i-Pentane	0.2275	13.01	0.270
2,2 Dimethylpropane	0.1964	13.36	0.269
n-Hexane	0.3047	12.78	0.264
i-Hexane	0.2781	12.82	0.267
3-Methylpentane	0.2773	12.65	0.273
2,2 Dimethylbutane	0.2339	12.77	0.272
2,3 Dimethylbutane	0.2476	12.62	0.269
n-Heptane	0.3494	12.68	0.263
i-Heptane	0.3282	12.72	0.261
3-Methylhexane	0.3216	12.55	0.255
3-Ethylpentane	0.3094	12.36	0.268
2,2 Dimethylpentane	0.2879	12.60	0.267
2,3 Dimethylpentane	0.2923	12.35	0.256
2,4 Dimethylpentane	0.3018	12.72	0.265
3,3 Dimethylpentane	0.2672	12.42	0.273
2,2,3 Trimethylbutane	0.2503	12.38	0.266
n-Octane	0.3962	12.68	0.259
i-Octane	0.3768	12.64	0.261
n-Nonane	0.4368	12.64	0.255
n-Decane	0.4842	12.64	0.249

Continued

Table 1.13 Acentric Factors, Watson Characterization Factors, and Critical Compressibility Factors of Pure Substances *(Continued)*

Components	Acentric Factor (ω)	Watson Characterization Factor (K)	Critical Compressibility Factor (z_c)
Napthenes			
Cyclopropane	0.1348	11.93	0.274
Cyclobutane	0.1866	11.45	0.274
Cyclopentane	0.1943	10.94	0.273
Methylcyclopentane	0.2302	11.32	0.272
Ethylcyclopentane	0.2715	11.39	0.269
Cyclohexane	0.2149	11.00	0.273
Methylcyclohexane	0.2350	11.31	0.269
Ethylcyclohexane	0.2455	11.36	0.270
Cyclooctane	0.2536	10.89	0.270
Olefins			
Ethylene	0.0852		0.277
Propylene	0.1424	14.21	0.275
1-Butene	0.1867	13.04	0.276
cis-2-Butene	0.2030	12.61	0.272
trans-2-Butene	0.2182	12.93	0.274
i-Butylene	0.1893	13.01	0.275
1-Pentene	0.2330	12.65	0.270
cis-2-Pentene	0.2406	12.48	0.279
trans-2-Pentene	0.2373	12.61	0.279
1-Hexene	0.2800	12.50	0.265
cis-2-Hexene	0.2722	12.30	0.266
trans-2-Hexene	0.2613	12.45	0.267
1-Heptene	0.3310	12.41	0.262
trans-2-Heptene	0.3389	12.39	0.256
1-Octene	0.3747	12.40	0.256
trans-2-Octene	0.3384	12.36	0.260
1-Nonene	0.4171	12.42	0.249
1-Decene	0.4645	12.46	0.247
Diolefins			
Propadiene	0.1594	12.58	0.271
1,2 Butadiene	0.2509	12.16	0.267
1,3 Butadiene	0.1932	12.51	0.270
1,2 Pentadiene	0.2235	11.90	0.256
1,3 Pentadiene	0.0837	12.21	0.285
2,3 Pentadiene	0.2194	11.91	0.253

Table 1.13 Acentric Factors, Watson Characterization Factors, and Critical Compressibility Factors of Pure Substances *(Continued)*

Components	Acentric Factor (ω)	Watson Characterization Factor (K)	Critical Compressibility Factor (z_c)
	Acetylenes		
Acetylene	0.1873	16.72	0.271
Methylacetylene	0.2161	12.34	0.276
Ethylacetylene	0.0500	12.14	0.270
	Aromatics		
Benzene	0.2108	9.74	0.271
Toluene	0.2641	10.11	0.264
Ethylbenzene	0.3036	10.33	0.263
o-Xylene	0.3127	10.27	0.263
m-Xylene	0.3260	10.41	0.259
p-Xylene	0.3259	10.44	0.260
n-Propylbenzene	0.3462	10.59	0.265
i-Propylbenzene	0.3377	10.54	0.262
n-Butylbenzene	0.3917	10.82	0.261
i-Butylbenzene	0.3811	10.84	0.256
Naphthalene	0.3019	9.32	0.269

Source: Technical Data Book — Petroleum Refining, 4th ed., American Petroleum Institute, Washington, D.C., 1982.

PR EOS

$$\ln\frac{f}{P} = Z - 1 - \ln(Z - B) - \frac{A}{2\sqrt{2}B}\ln\frac{Z + 2.414B}{Z - 0.414B} \qquad (1.148b)$$

Terms are as defined earlier.

The compressibility Z can be calculated using Equation 1.145b. Equation 1.145b can have three routes; the largest value is applicable for the vapor phase and the smallest one is applicable for the liquid phase. Knowing the value of compressibility, Equation 1.148 can be used to calculate fugacity for both liquid and vapor phases.

For a particular pressure, the fugacity coefficient for both liquid and vapor phases will be the same. This pressure is the vapor pressure of the component at that temperature. The pressure is established using a trial-and-error method.

Alternate method to estimate vapor pressure of pure components

There are other methods to estimate the vapor pressure of pure components by using the Antione equation. The Antione equation defines the vapor pressure of pure components as [10]

$$\ln(P) = A - \frac{B}{T + C} \tag{1.149}$$

where

P = vapor pressure, mmHg
T = temperature, K
A, B, C = constants in the Antione equation.

For commonly used hydrocarbon vapor, the values of constants A, B, and C are presented in Table 1.14. A more complete list is available in the literature [10].

Estimation of vapor pressure using the Antione equation is limited by the temperature limit. Estimation of vapor pressure for a wider range of temperature can be better achieved using the EOSs.

Table 1.14 Values of Antione Constants

	Constant in Antione Equation			Temperature Limit, °C	
Components	A	B	C	Minimum	Maximum
Methane	15.2243	597.84	–7.16	–180	–153
Ethane	15.6637	1511.42	–17.16	–143	–74
Propane	15.726	1872.46	–25.16	–109	–24
n-Butane	15.6782	2154.9	–34.42	–78	17
i-Butane	15.5381	2032.73	–33.15	–86	7
n-Pentane	15.8333	2477.07	–39.94	–53	57
Hexane	15.8366	2697.55	–48.78	–28	97
Heptane	15.8737	2911.32	–56.51	–3	127
Octane	15.9426	3120.29	–63.63	19	152
Nonane	15.9671	3291.45	–71.33	39	179
Decane	16.0114	2456.8	–78.67	57	203
Carbon monoxide	14.3686	530.22	–13.15	–210	–165
Carbon dioxide	22.5898	3103.39	–0.16	–119	–69
Hydrogen sulfide	16.1040	1768.69	–26.06	–83	–43
Nitrogen	14.9542	588.72	–6.6	–219	–183
Oxygen	15.4075	734.55	–6.45	–210	–173
Hydrogen	13.6333	164.9	3.19	–259	–248
Water	18.3036	3816.44	–46.13	11	168

Vapor pressure of pure water

Much better correlation can be developed for the estimation of pure water vapor pressure by using the following equation:

$$P = 10^{(7.97 - 1668.2/(T - 45.2))} \tag{1.150}$$

where

P = vapor pressure of pure water, mmHg
T = temperature, K

Equilibrium ratio (K)

For ideal conditions, the vapor and liquid compositions can be defined by Roult's law as

$$y_i = \frac{P_i^o x_i}{P} \tag{1.151}$$

where

y_i = mole fraction of component i in vapor phase
x_i = mole fraction of component i in liquid phase
P_i° = vapor pressure of pure component i
P = total pressure

The equilibrium ratio K of component i is defined as

$$K_i = y_i / x_i \tag{1.152}$$

For ideal mixtures (obeying Roult's law), the equilibrium ratio is defined as

$$K_i^o = P_i^o / P \tag{1.153}$$

and for real mixtures, the actual equilibrium is defined as

$$K = K_i^o \frac{\gamma_i}{\phi_i} = \frac{P_i^0 \gamma_i}{P \phi_i} \tag{1.154}$$

where

γ_i = liquid phase activity coefficient for component i
ϕ_i = gas phase fugacity coefficient for component i

Mixing rules [7,8]

Large numbers of mixing rules are available to modify the EOSs for mixed components. The equations used for SRK and PR EOSs are as follows:

SRK Equation of State:

$$a = \left(\sum x_i a_i^{0.5}\right)^2 \tag{1.155a}$$

$$b = \sum x_i b_i \tag{1.156a}$$

PR Equation of State:

$$a = \sum_i \sum_j x_i x_j a_{ij} \tag{1.155b}$$

$$b = \sum x_i b_i \tag{1.156b}$$

where

x_i = mole fraction of component i
x_j = mole fraction of component j
a_i = value of a of component i
b_i = value of b of component i

$$a_{ij} = (1 - \delta_{ij})(a_i a_j)^{0.5} \tag{1.157}$$

δ_{ij} = binary interaction coefficient

By applying mixing rules (Equation 1.155a and Equation 1.156a) to other equations, the following results can be achieved:

$$A = 0.42747 \frac{P}{T^2} \left(\sum x_i \frac{T_{ci} \alpha_i^{0.5}}{P_{ci}^{0.5}} \right)^2 \tag{1.158}$$

$$B = 0.08664 \frac{P}{T} \sum x_i \frac{T_{ci}}{P_{ci}} \tag{1.159}$$

Similar equations can be developed for PR EOS.
The fugacity coefficient of a component is estimated as
SRK Equation of State:

$$\ln \frac{f_i}{P x_i} = \frac{b_i}{b}(Z-1) - \ln(Z-B) - \frac{A}{B}\left(2\frac{a_i^{0.5}}{a^{0.5}} - \frac{b_i}{b} \right) \ln\left(1 + \frac{B}{Z} \right) \tag{1.160a}$$

PR Equation of State:

$$\ln\frac{f_k}{x_k P}=$$

$$\frac{b_k}{b}(Z-1)-\ln(Z-B)-\frac{A}{2\sqrt{2}B}\left(\frac{2\sum_i X_i a_{ik}}{a}-\frac{b_k}{b}\right)\ln\frac{Z+2.414B}{Z-0.414B} \tag{1.160b}$$

In Equation 1.160a, the ratios b_i/b and a_i/a are defined as

$$\frac{a_i^{0.5}}{a^{0.5}}=\frac{\alpha_i^{0.5}T_{ci}/P_{ci}^{0.5}}{\sum x_i\alpha_i^{0.5}T_{ci}/P_{ci}^{0.5}} \tag{1.161}$$

$$\frac{b_i}{b}=\frac{T_{ci}/P_{ci}}{\sum x_i T_{ci}/P_{ci}} \tag{1.162}$$

Example 1.31

Estimate the water content in saturated methane gas at 5000 kPa pressure and 30°C. Also, estimate the water dew point for dehydrated gas at 5000 kPa pressure with water content 65 mg/Sm^3.

Use the PR equation of state.

Solution

Vapor pressure of water at 30°C (Equation 1.150)
$= 10^{(7.97 - 1668.2/257.8)}$
= 31.56 mmHg = 4.24 kPa
For the ideal situation, vapor mol% will be
Water = 0.085 mol% (= 4.24*100/5000)
Methane = 99.915 mol%

Estimate fugacity coefficient of vapor

The PR equation of state is used for this. The calculation is presented in Table 1.15.

Equilibrium ratio (K) of water:
= vapor pressure of water at 30°C/(total pressure * fugacity of vapor)
= 0.000938
Assume the liquid phase activity coefficient = 1

Table 1.15 Example 1.31

Description	Unit	Methane	Water	Equation
Critical pressure	kPa	4599	22064	
Critical temperature	K	190.56	647.1	
Value of ω_i		0.0108	0.3449	Table 1.13
Value of m_i		0.3913	0.8744	1.145j
Reduced temperature		1.59	0.468	—
Value of $\alpha_i^{0.5}$		0.8979	1.2761	1.145h
Value of b_i		0.0268	0.019	1.145d
Value of b		0.0268	—	1.145c
Value of a_{ci}		249.6	599.8	1.145g
Value of a		201.6	—	1.145e
Value of A		0.15884	—	1.145j
Value of B		0.05318	—	1.145k
Value of Z		0.9049	—	1.145b
Value of f/P		0.9042	—	1.148b

Moisture content in the vapor phase = 16888.4 mg/mol
= 714.7 mg/Sm^3
(Neglecting the impact of interaction coefficient)

Influence of interaction coefficient

For a methane water system, the impact of the interaction coefficient can be established through regression analysis. The following multiplication factor (F) can be used as a correction factor for the interaction coefficient:

$$F = 0.973 \exp(0.00002 * P)$$

where

F = multiplication factor
P = pressure, kPa

Estimated multiplication factor at 5000 kPa = 1.0753.
Final estimated moisture content = 769 mg/Sm^3.
Value estimated through HYSYS® = 773 mg/Sm^3.

Estimation of dew point

Moisture content = 65 mg/Sm^3 methane.

Moisture content without the influence of interaction coefficient = 60.4 mg/Sm^3 methane.

Moisture content per mole of gas = 1427.3 mg/Sm^3.

Estimated K value = 0.000793.

Back calculation of temperature needs trial-and-error procedure.

Assume temperature = –10°C.

Vapor pressure of water at –10°C = 0.272 kPa.

Estimated $f/P = 0.8505$.

Estimated K value = 0.272/(5000 * 0.8505) = 0.000064.

This means the vapor pressure value needs to be increased slightly. This is achieved by increasing the temperature.

Assume temperature = –8°C.

Vapor pressure of water at –8°C = 0.32 kPa.

Estimated $f/P = 0.8505$.

Estimated K value = 0.000075 (close to previous estimate).

Therefore, estimated dew point = –8°C.

Dew point as per Figure 20.3 of Gas Processors Suppliers Association (GPSA) = –9°C.

Unit conversions

The basic unit conversions are presented in Table 1.16. An electronic conversion table is also included in this chapter.

Programming

General notes for all programs

- All programs have separate input and output frames;
- Most input data entry cells are white — means user inputs are required.
- Some input cells are yellow — means the values can be calculated using this program as well as can be modified externally;
- All output cells are red — means the values are calculated results and can't be modified externally;
- Both SI and English units can be used for all programs with SI unit being the default. These units are fixed units and can't be modified externally.

Vessel

The vessel.exe program has been developed to estimate the total and filled-liquid volumes and dry and wet surface areas for horizontal, vertical, and inclined (e.g., slug catcher) vessels. For horizontal and vertical vessels, the program also calculates the weight of the material and the hydro test weight.

Once the Vessel program executes, the following form (Figure 1.20) will appear. This form presents a general understanding of the program and has three menus:

- Vessel
- Project Details
- Help

Table 1.16 Unit Conversions

Converted from	Converted to	Multiplied by	Converted from	Converted to	Multiplied by
Acceleration					
cm/sec^2	free fall (g)	0.001097	cm/sec^2	m/sec^2	0.01
	ft/sec^2	0.032808		mm/sec^2	10
Angle					
degree	grad	1.11111	degree	revolution	0.0027778
	min	60		sec	3600
	rad	0.0174533			
Area					
acre	cm^2	40468564.2	acre	mi^2	0.0015625
	ft^2	43560		mm^2	4046856422.4
	ha	0.4046863		perch	160
	in.^{2}	6272640		square	435.6
	km^2	0.004046856		yd^2	4840
	m^2	4046.856			
Computer					
bit	byte	0.125	bit	nibble	0.25
	exabyte	1.0842E-19		petabyte	1.11022E-16
	Gb	1.16415E-10		terabyte	1.13687E-13
	Kb	0.00012207		yottabyte	1.03398E-25
	Mb	1.19209E-7		zettabyte	1.05879E-22
Concentration					
kmol/m^3	µM/cm^3	1000	kmol/m^3	mmol/m^3	1000000
	µM/l	1000000		mol/cm^3	0.001
	mmol/cm^3	1		mol/l	1
	mmol/l	1000		mol/m^3	1000
Density					
g/cm^3	g/m^3	1000000	g/cm^3	oz/gal (E)	160.35591
	kg/cm^3	0.001		oz/gal (U.S.)	133.52381
	kg/m^3	1000		lb/ft^3	62.42769
	mg/cm^3	1000		lb/in.^{3}	0.0361273
	mg/m^3	1000000000			
Distance					
angstrom	cm	1E-8	angstrom	µm	0.0001
	dm	1E-9		mil	0.00000393701
	furlong	4.97097E-13		mi	6.213712E-14
	ft	3.28084E-10		mm	0.0000001
	in.	3.93701E-9		nm	0.1
	km	1E-13		nmi	5.39957E-14
	lightyear	1.057023E-26		parsec	3.24078E-27
	m	1E-10		yd	1.09361E-10

Table 1.16 Unit Conversions *(Continued)*

Converted from	Converted to	Multiplied by	Converted from	Converted to	Multiplied by
Energy					
Btu	cal	251.99	Btu	kcal	0.25199
	erg	10550600000		kJ	1.05506
	GJ	0.00000105506		kWh	0.00029307
	gmf-cm	10758575.59		N-m	1055.06
	hp-h	0.000393		lbf-ft	778.1693
	J	1055.03		Wh	0.29307
Flow					
barrel/d	barrel/h	0.0416667	barrel/d	gal(U.S.)/min	0.0291667
	cm^3/h	6624.5833		gal(U.S.)/sec	0.000486111
	cm^3/sec	1.840162		l/h	6.6245833
	ft^3/h	0.2339458		l/sec	0.001840162
	ft^3/sec	0.000064984		m^3/d	0.15899
	gal(E)/d	34.973		m^3/h	0.00662458
	gal(E)/h	1.4572083		m^3/sec	0.00000184
	gal(E)/min	0.0242868		ml/h	6624.58333
	gal(E)/sec	0.00040478		ml/sec	1.840162
	gal(U.S.)/d	42		oz/h	224
	gal(U.S.)/h	1.75		oz/sec	0.0622222
Force					
dyn	g-force	0.00101972	dyn	N	0.00001
	kg-force	0.00000101972		oz-force	0.0000359694
	kN	1E-8		lb-force	0.000002248
	mN	0.01			
Heat Transfer Coefficient					
Btu/(h·ft^2·F)	Btu/(sec·ft^2·F)	0.00027778	Btu/(h·ft^2·F)	kcal/(h·m^2·C)	4.88251
	cal(h·cm^2·C)	0.488251		kJ/(h·m^2·C)	20.44175
	cal/(h·m^2·C)	4882.51		kW/(m^2·K)	0.005678263
	cal/(sec·cm^2·C)	0.000135625		W/(m^2·K)	5.678263
	cal/(sec·m^2·C)	1.35625			
Light					
flame	fc	4	flame	m-cd	43.055642
	lx	43.055642			
Mass					
carat	gr	3.08647	carat	oz	0.0070548
	g	0.2		lb	0.000440925
	kg	0.0002		stone	0.000031494
	μg	200000		t (metric)	0.0000002
	mg	200			
Power					
Btu/h	Btu/min	0.016667	Btu/h	kcal/min	0.00419993
	cal/min	4.199833		kW	0.00029281
	hp	0.000392665		lb-ft/min	12.958
	kcal/h	0.251996		W	0.2928104

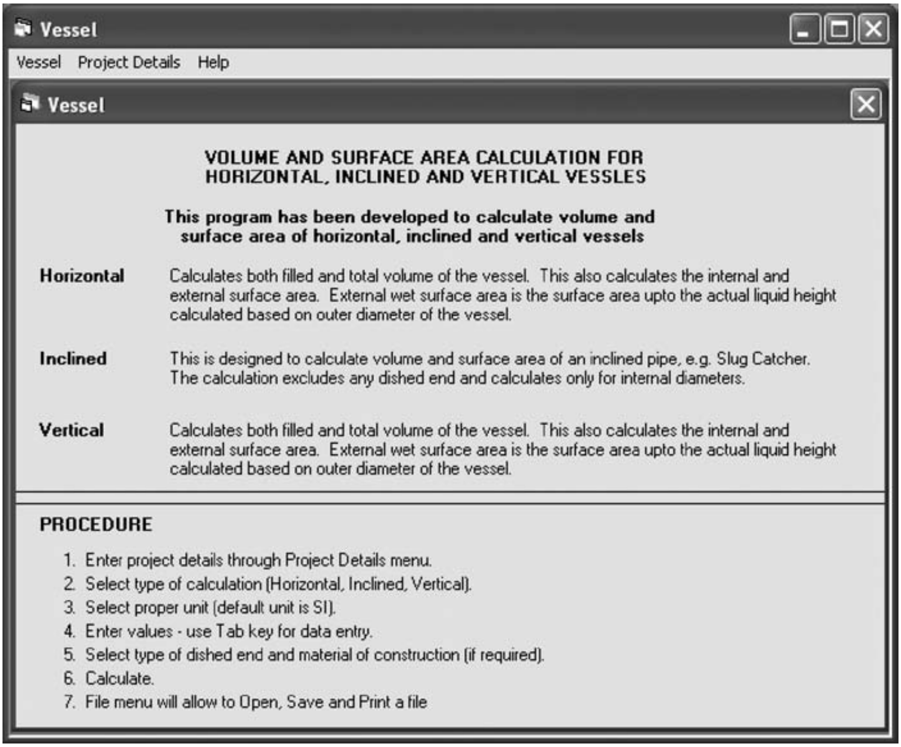
Continued

Table 1.16 Unit Conversions *(Continued)*

Converted from	Converted to	Multiplied by	Converted from	Converted to	Multiplied by
		Pressure			
Atm	bar	1.01325	atm	mbar	1013.25
	in. Hg	29.9213		mm Hg	760
	kg/cm^2	1.0332		lb/ft^2	2116.224
	kg/m^2	10332		lb/in.^{2}	14.696
	kPa	101.325		torr	760
	MPa	0.101325			
		Speed			
cm/h	cm/sec	0.000277778	cm/h	kn	5.399E-6
	ft/h	0.0328084		m/h	0.01
	ft/sec	0.000009113		m/sec	2.777778E-6
	km/h	0.00001		mi/h	6.21371E-6
	km/sec	2.777778E-9		mi/sec	1.72603E-9
		Surface Tension			
dyn/cm	mN/m	1	dyn/cm	pdl/ft	0.002205
	N/m	0.001			
		Temperature			
°C	°F	°C * 1.8 + 32	°C	°R	°C * 1.8 + 491.67
	°K	°C + 273.15			
		Thermal Conductivity			
Btu/(h·ft·F)	Btu/(s·ft·F)	0.00027778	Btu/(h·ft·F)	kcal/(h·m·C)	1.48813
	cal/(h·cm·C)	14.8813		kJ/(h·m·C)	6.229312
	cal/(h·m·C)	1488.13		kW/(m·K)	0.00173036
	cal/(s·cm·C)	0.00413369		W/(m·K)	1.73036
	cal/(s·m·C)	0.4133694			
		Time			
century	d	36525	century	msec	3.15576E12
	decade	10		min	52596000
	fortnight	2608.9286		month	1200
	h	876600		sec	3155760000
	leap year	25		week	5217.857
	millennium	0.1		year	100
		Torque			
dyn-cm	gmf-cm	0.001019716	dyn/cm	N-m	1.0E-7
	kgf-m	1.01972E-8		ouncef-ft	1.180097E-6
	kN-m	1.0E-10		poundf-ft	7.37561E-8
	MN-m	1.0E-13		poundf-in.	8.850728E-7
	mN-m	0.0001			

Table 1.16 Unit Conversions *(Continued)*

Converted from	Converted to	Multiplied by	Converted from	Converted to	Multiplied by
		Viscosity			
cP	gm/(cm·sec)	0.01	cP	milli Pas·sec	1
	kg/(m·h)	3.6		N·sec/m^2	0.001
	lb/(ft·h)	2.42		Pas·sec	0.001
	lb/(ft·sec)	0.00067222		P	0.01
	mN·sec/m^2	1			
		Volume			
barrel	cm^3	158990	barrel	l	158.99
	ft^3	5.6147		µl	158990000
	in^3	9702.202		ml	158990
	m^3	0.15899		oz (E)	5595.678
	yd^3	0.207952		oz (U.S.)	5376.006
	cup	672.0064		pt (E)	279.7834
	drop	2451625.8		pt (U.S.)	336.0053
	gal (E)	34.974		tablespoon	10752.012
	gal (U.S.)	42		teaspoon	32256.036



VOLUME AND SURFACE AREA CALCULATION FOR
HORIZONTAL, INCLINED AND VERTICAL VESSLES

This program has been developed to calculate volume and surface area of horizontal, inclined and vertical vessels

Horizontal Calculates both filled and total volume of the vessel. This also calculates the internal and external surface area. External wet surface area is the surface area upto the actual liquid height calculated based on outer diameter of the vessel.

Inclined This is designed to calculate volume and surface area of an inclined pipe, e.g. Slug Catcher. The calculation excludes any dished end and calculates only for internal diameters.

Vertical Calculates both filled and total volume of the vessel. This also calculates the internal and external surface area. External wet surface area is the surface area upto the actual liquid height calculated based on outer diameter of the vessel.

PROCEDURE

1. Enter project details through Project Details menu.
2. Select type of calculation (Horizontal, Inclined, Vertical).
3. Select proper unit (default unit is SI).
4. Enter values - use Tab key for data entry.
5. Select type of dished end and material of construction (if required).
6. Calculate.
7. File menu will allow to Open, Save and Print a file

Figure 1.20 Vessel.

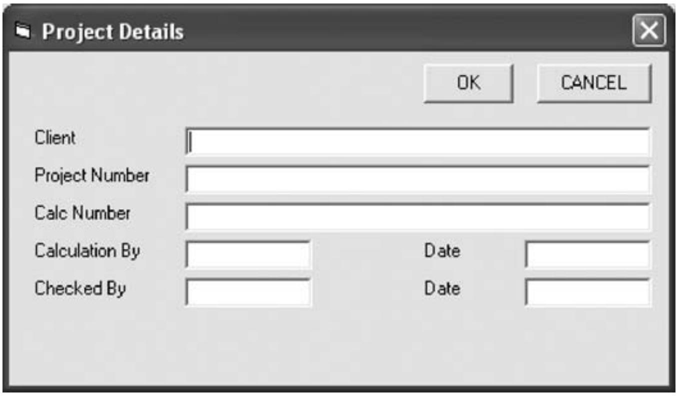


Figure 1.21 Project details.

The vessel menu gives the following calculation options:

- Horizontal
- Inclined
- Vertical

Program limitations

The following program limitations may be noted:

- The inclined vessel calculation is typically designed for a pipe without dished ends.
- If the filled Length of inclined vessel (Figure 1.23) is less than Total Length, the value of Height must be entered as 0 (default value).
- For inclined vessel any value of Height will ignore the value of Length.

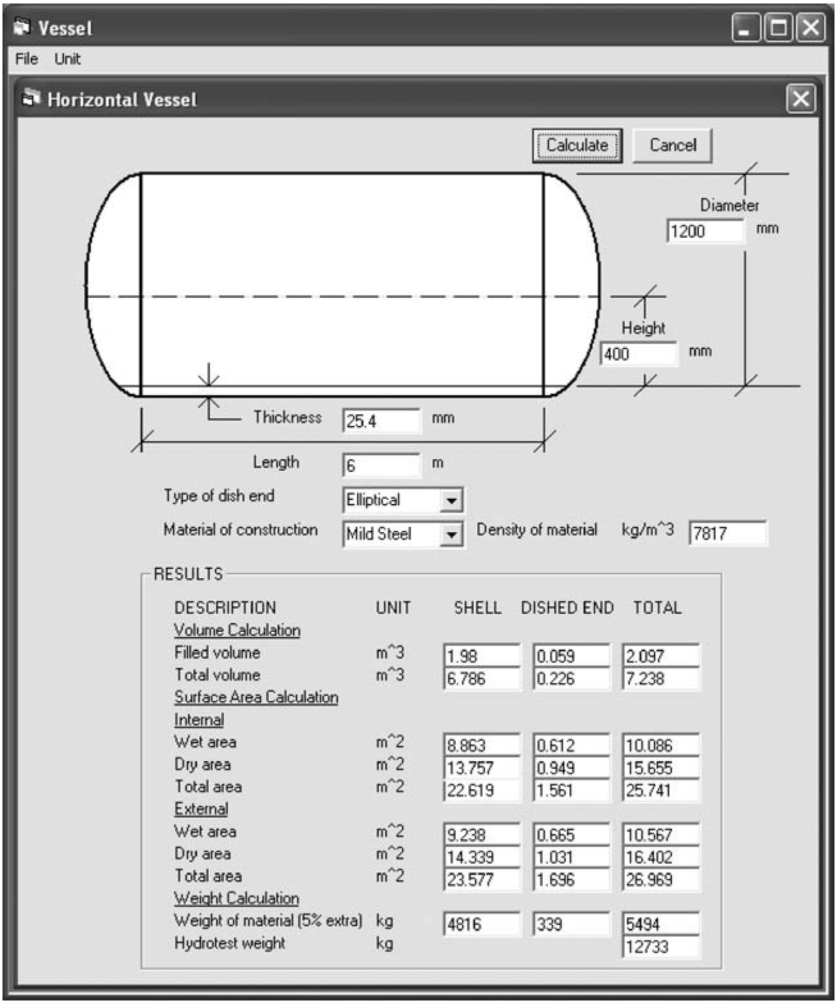
The Project Details menu will open a Project Details form as shown in Figure 1.21. This form will allow entry of the general details of the project and calculations. This also includes the Calculation Done by and Calculation Checked by options along with dates. Once the details are entered, pressing the OK button will transfer the information to the main Vessel form.

Horizontal

Once the menu Horizontal executes, the following form (Figure 1.22) will appear. This form is designed for horizontal vessel calculations.

This form has two menus:

- File
- Unit



Diameter 1200 mm

Height 400 mm

Thickness 25.4 mm

Length 6 m

Type of dish end Elliptical

Material of construction Mild Steel

Density of material kg/m^3 7817

DESCRIPTION	UNIT	SHELL	DISHED END	TOTAL
Volume Calculation				
Filled volume	m^3	1.98	0.059	2.097
Total volume	m^3	6.786	0.226	7.238
Surface Area Calculation				
Internal				
Wet area	m^2	8.863	0.612	10.086
Dry area	m^2	13.757	0.949	15.655
Total area	m^2	22.619	1.561	25.741
External				
Wet area	m^2	9.238	0.665	10.567
Dry area	m^2	14.339	1.031	16.402
Total area	m^2	23.577	1.696	26.969
Weight Calculation				
Weight of material (5% extra)	kg	4816	339	5494
Hydrotest weight	kg			12733

Figure 1.22 Horizontal vessel form.

The File menu is further divided to four submenus: (1) Open, (2) Save, (3) Print, and (4) Exit.

The Unit menu is used to select a proper unit for calculations. Both SI and Eng units can be used for calculation, the default unit being SI. It is not possible to change the unit of individual items.

Data entry

During data entry, care must be taken while entering the units. Because the basic units are fixed, the data must be entered for the unit shown in the form. The tab key is preferably used for data entry. The tab key will allow data entry in a sequential manner, and once the data entry is complete, the tab key will select the Calculate button for the calculation.

The data entry is simple; however, the program allows the following options:

Type of dish end:
  1. Elliptical
  2. Hemispherical

Selection of dish ends has significant impact on the calculation of volume, surface area, and weight.

Material of construction:
  1. Aluminum
  2. Copper
  3. Gold
  4. Iron
  5. Mild steel
  6. Monel
  7. Nichrome
  8. Platinum
  9. Stainless steel

Material of construction is only used to estimate the vessel weight and requires plate thickness for estimation. It must be noted that only the shell thickness is entered, and it is assumed that dish end thickness is the same as shell thickness. This may not be correct for all cases, and the designer should keep this in mind. However, for a preliminary estimate, the calculated weight will be adequate.

In the weight calculation, a 5% margin is added. This addition is mainly to account for the weight of nozzles, man way, etc.

For a particular material in the list, the density is automatically selected by the program. However, it is not possible to modify the density. In case the material is not present in the list, it is preferable to select one with closely matching density.

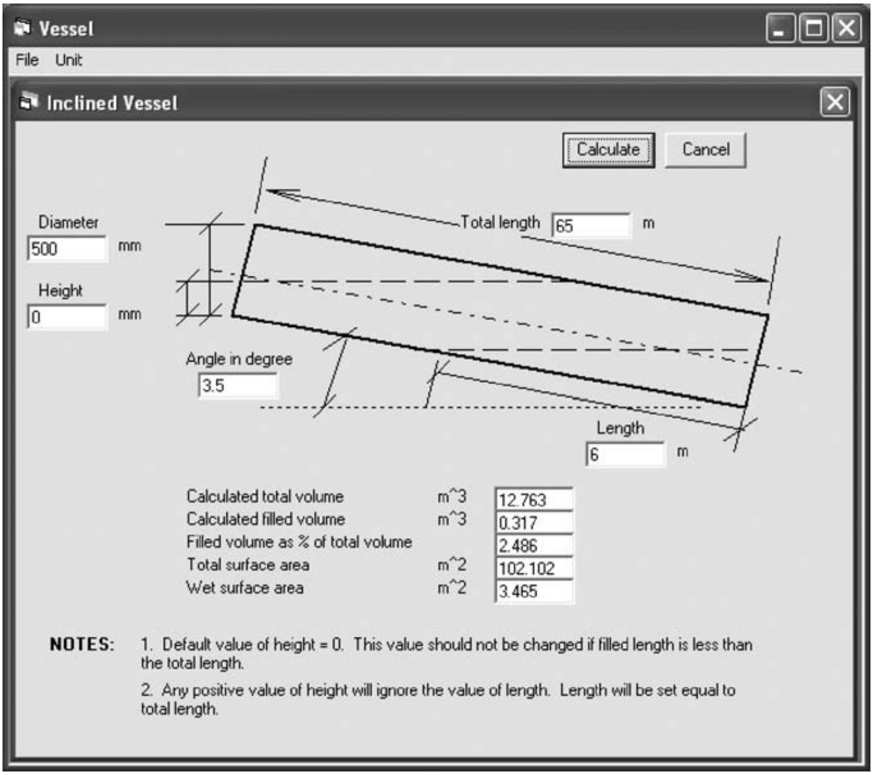
Buttons
  1. Calculate
  2. Cancel

The **Calculate** button is used to calculate the parameters after completion of data entry. If the tab key is used for data entry, this will automatically select the Calculate button once the data entry is complete. Pressing the Calculate button will calculate the parameters.

The **Cancel** button is used to cancel the execution. Once the Cancel button is pressed, the program will be transferred to the main menu for further calculation options.

Inclined

The general format of the Inclined vessel calculation is presented in Figure 1.23. General data entry and calculation principles are as discussed for the



Diameter 500 mm

Height 0 mm

Angle in degree 3.5

Total length 65 m

Length 6 m

Calculated total volume	m^3	12.763
Calculated filled volume	m^3	0.317
Filled volume as % of total volume		2.486
Total surface area	m^2	102.102
Wet surface area	m^2	3.465

NOTES: 1. Default value of height = 0. This value should not be changed if filled length is less than the total length.

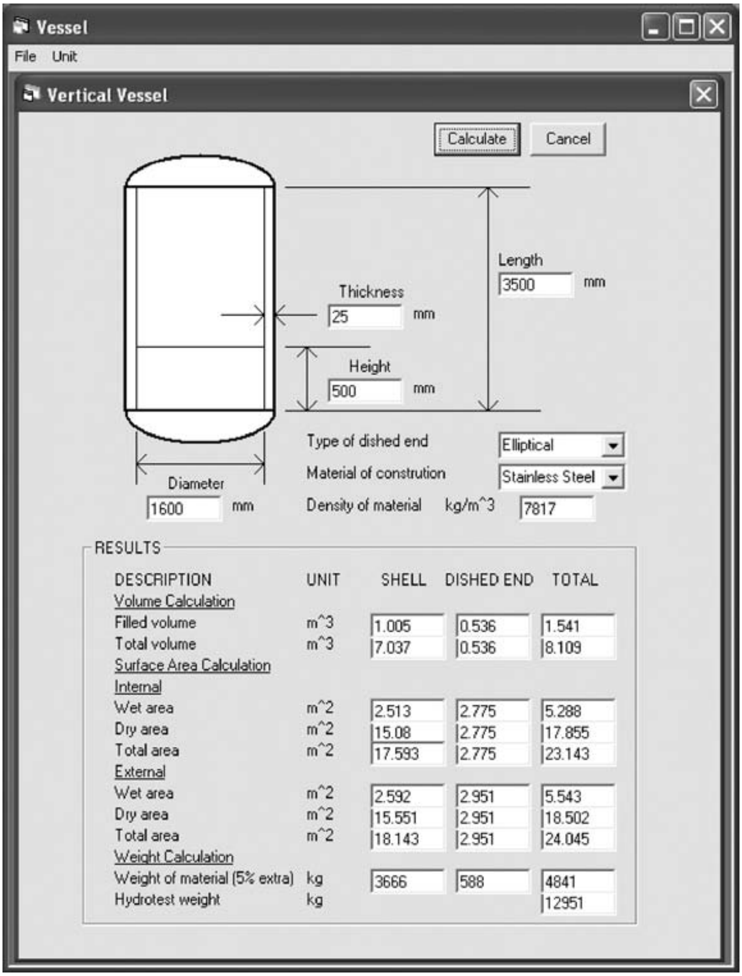
2. Any positive value of height will ignore the value of length. Length will be set equal to total length.

Figure 1.23 Inclined vessel form.

horizontal vessel calculation. However, the following points should be considered:

- **Height** indicated in the left side of the form is the height of liquid at the furthest left side of the inclined vessel. The default value of height is zero. In most slug-catcher type of calculations, the volume is calculated for partially filled vessels, and for such calculations, the height must be zero. The calculation is based on the Total Length and Length as indicated in the form. In general, the value of Length is smaller than the value of Total Length, and the value of Height must be zero. If any positive value is entered for Height, the program will ignore the value entered for Length and will calculate the volume and surface area based on the Total Length and Height.
- **Dished ends** have not been used in this calculation. In general, for the slug-catcher type of calculations, the impact of dished ends is negligible.

The inclined calculation does not include the calculation of weight, and material selection is not required for this. The calculation is only for the total and filled volumes and total and wet surface areas.



Length 3500 mm

Thickness 25 mm

Height 500 mm

Type of dished end Elliptical

Material of constrution Stainless Steel

Diameter 1600 mm

Density of material kg/m^3 7817

RESULTS

DESCRIPTION	UNIT	SHELL	DISHED END	TOTAL
Volume Calculation				
Filled volume	m^3	1.005	0.536	1.541
Total volume	m^3	7.037	0.536	8.109
Surface Area Calculation				
Internal				
Wet area	m^2	2.513	2.775	5.288
Dry area	m^2	15.08	2.775	17.855
Total area	m^2	17.593	2.775	23.143
External				
Wet area	m^2	2.592	2.951	5.543
Dry area	m^2	15.551	2.951	18.502
Total area	m^2	18.143	2.951	24.045
Weight Calculation				
Weight of material (5% extra)	kg	3666	588	4841
Hydrotest weight	kg			12951

Figure 1.24 Vertical vessel form.

Vertical

The general format of the Vertical vessel calculation is presented in Figure 1.24. General data entry and calculation principles are as discussed for the horizontal vessel calculation. However, the following points should be considered:

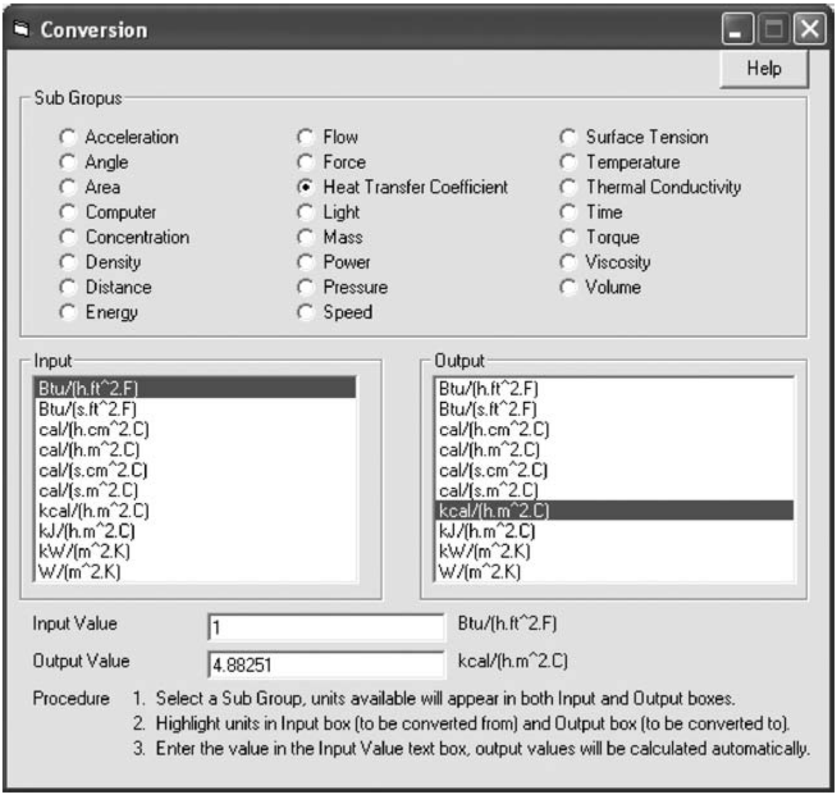
- **Height** indicated in the form is the height above the tan line of the vessel as almost all calculations are like this. The dished end volume added is based on the type of dished ends. In case a negative height is entered, the program will calculate the dished end volume and subtract the negative shell volume. The result will not represent the actual volume of the liquid in the dished end.

- For an elliptical dished end, the dished end volume is calculated for a dished end height 0.25 times the vessel diameter, and for a hemispherical dished end, the volume is calculated for a dished end height 0.5 times of the vessel diameter.

Conversion

Unit conversion has been divided into 23 subgroups, namely:

- Acceleration
- Angle
- Area
- Computer
- Concentration
- Density
- Distance
- Energy
- Flow



Input Value 1 Btu/(h.ft^2.F)

Output Value 4.88251 kcal/(h.m^2.C)

Procedure
1. Select a Sub Group, units available will appear in both Input and Output boxes.
2. Highlight units in Input box (to be converted from) and Output box (to be converted to).
3. Enter the value in the Input Value text box, output values will be calculated automatically.

Figure 1.25 Conversion.

- Force
- Heat transfer coefficient
- Light
- Mass
- Power
- Pressure
- Speed
- Surface tension
- Temperature
- Thermal conductivity
- Time
- Torque
- Viscosity
- Volume

Program limitations

The following program limitations may be noted:

- Program calculates output values automatically when the input values are changed.
- Program does not calculate the output values when the input/output sections are changed.
- If the section of input/output parameters is changed, the value in the Input Value cell must be reentered to calculate the Out Value.

Large numbers of commonly used units have been selected for each subgroup for conversion.

Procedure

The procedure for conversion is as follows:

- Select the required subgroup (from 23 radio buttons).
- Available units will appear in both Input and Output boxes.
- Select the input unit; this is the unit that is to be converted. The input value will indicate the selected unit.
- Select the output unit; this is the unit to be converted to. The output value will indicate the selected unit.
- Enter the value in the input value text box. Output values will be calculated automatically. The output value text box is read only and cannot be modified externally. The input value is to be modified every time a calculation is required. Changing unit selection will not automatically recalculate the output value.

References

1. Grossman, S.I., Multivariable Calculus, Liner Algebra and Differential Equations, 3rd ed., Sunders College Publishing, 1994.
2. Grossman, S.I., Elementary Algebra, Wadsworth Publishing Company, 1980.
3. Perry, R.H. and Green, D., Perry's Chemical Engineers Handbook, 6th ed., McGraw-Hill, Malaysia, 1984.
4. Hughes-Hallett, D., et al., Calculus Single Variable, 2nd ed., John Wiley and Sons, New York, 1998.
5. Kreyszig, E., Advanced Engineering Mathematics, 7th ed., John Wiley and Sons, New York, 1993.
6. Jain, M.K., Iyengar, S.R.K., and Jain, R.K., Numerical Methods for Scientific and Engineering Computation, Wiley Eastern Limited, New Delhi, 1985.
7. Peng, D.Y. and Robinson, D.B., A two constant equation of state, I.E.C Fundamentals, 15, 59–64, 1976.
8. Soave, G., Equilibrium constants from a modified Redlich-Kwong equation of state, *Chemical Engineering Science,* 27(6), 1197, 1972.
9. Technical Data Book — Petroleum Refining, 4th ed., American Petroleum Institute, Washington, D.C., 1982.
10. Coulson, J.M. and Richardson, J.F., Chemical Engineering Design, Vol. 6, Pergamon Press, Great Britain, 1986.

chapter two

Fluid mechanics

Introduction

Fluid mechanics is the most important unit operation in chemical industries. Nearly 30% of the total plant cost is from pipes and fittings. Proper knowledge is required for correct sizing of pipes, which in turn depends upon the proper estimation of pressure drop. For liquids and compressible fluids, system pressure drop can be calculated quite accurately; however, for two-phase flow, the calculation is extremely complex. The accuracy of a two-phase pressure drop calculation largely depends on the accuracy of liquid holdup calculations. Large numbers of correlations are available for liquid holdup and two-phase pressure drop calculations. It is often confusing for the designer to use the proper equation. Attempts will be made to establish general guideline for the use of two-phase correlations.

Flow measurements are also an important part of fluid mechanics. Flow in pipes is measured normally by using orifice or Venturi meters; however, notches are widely used for flow measurements in open channels. One aspect of oil and gas pipeline design is carbon dioxide corrosion. Several researches have been conducted to predict correct corrosion rate due to carbon dioxide; of these the Norsok Standard is probably the most reliable. For cross-country pipeline and for compressible flow, heat loss is an important design parameter. For a normal compressible fluid pressure drop calculation (within a plant battery limit), change in temperature is not significant. However, for cross-country pipelines, if the heat loss/gain is ignored (adiabatic flow), the temperature may drop significantly (due to a large pressure drop), resulting in an abnormally low temperature toward the outlet of the system. As density increases with decrease in temperature, the velocity will reduce significantly, resulting in a lower pressure drop than expected. Correct estimation of heat loss/gain is important for the design of cross-country pipelines.

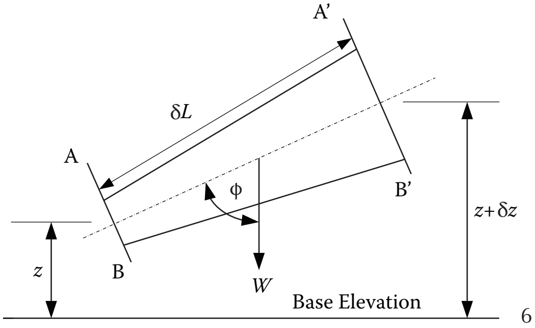


Figure 2.1 Bernoulli's theorem.

Table 2.1 Parameters in Section A–B and A′–B′

Descriptions	Section A–B	Section A′–B′
Area	a	$a + \delta a$
Velocity	v	$v + \delta v$
Pressure	p	$p + \delta p$

Bernoulli's theorem

The Bernoulli's theorem is based on the first law of thermodynamics: the total energy within a system is constant, provided no energy enters or leaves the system at any point.

Let a section with δL length lie between two sections A–B and A′–B′ (as shown in Figure 2.1). The parameters in both sections are presented in Table 2.1.

Between sections A–B and A′–B′ there is a change in momentum, and as per Newton's second law, the rate of change of momentum is proportional to the resultant force acting on the fluid.

Force at section A–B acting in the direction of motion = pa

Force at section A′–B′ acting in the opposite direction of motion = $(p + \delta p)(a + \delta a)$

Force due to pressure (at a point within δL) = $p + k\delta p$ (where k is a fraction)

Weight in the direction of motion = $-W \cos \phi = -W\, \delta z/\delta L$

Resultant force in the direction of motion

$pa - (p + \delta p)(a + \delta a) + (p + k\delta p)\delta a - W\delta z/\delta L$

Now the total weight (W) = density * volume of the element

$= \rho(a + 0.5\delta a)\delta L$

Weight in the direction of motion = $-\rho(a + 0.5\delta a)\delta z$

Eliminating the smaller quantities, the resultant force will be

$$-a\delta p - \rho a\delta z$$

The rate of change of momentum of a fluid can be expressed as a product of mass flow rate * change in velocity

$$\text{rate of change of momentum} = (\rho av/g)^{*}\delta v$$

Mathematically, rate of change of momentum = resultant force.

Therefore,

$$(\rho av/g)^{*}\delta v = -a\delta p - \rho a\delta z \tag{2.1}$$

or,

$$\delta z + \frac{v\delta v}{g} + \frac{\delta p}{\rho} = 0 \tag{2.2}$$

Integrating between two points (1 and 2) and balancing units,

$$z_1 + \frac{{v_1}^2}{2g} + \frac{1000p_1}{g\rho_1} = z_2 + \frac{{v_2}^2}{2g} + \frac{1000p_2}{g\rho_2} \tag{2.3}$$

Equation 2.3 is the general Bernoulli's equation. This equation ignores any energy loss/gain or any loss due to friction.

Velocity heads

The term $v^2/2g$ is normally called the *velocity head*, and this velocity head changes when the cross section of the pipe changes, typically for reducer and expander.

The Bernoulli's equation can be applied across a reducer as shown in Figure 2.2, and the velocity head change can be estimated with an assumption of no frictional head loss. Frictional loss across a reducer/expander will be discussed later in this chapter.

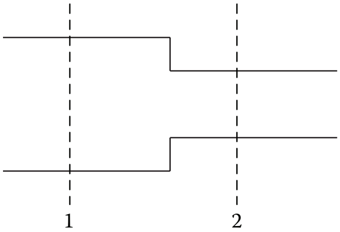


Figure 2.2 Velocity head loss across a reducer.

For the horizontal reducer, the elevation terms will cancel out; the incompressible fluid density at the inlet and outlet will be the same. With those changes the Bernoulli's equation can be written as:

$$\frac{1000p_2}{\rho} = \left(\frac{v_1^2}{2} - \frac{v_2^2}{2}\right) + \frac{1000p_1}{\rho} \tag{2.4}$$

For the reducer, because v_2 is more than v_1, the downstream pressure p_2 will reduce; however, for the expander, the downstream pressure p_2 will increase. This velocity head loss/gain is calculated with the consideration of no frictional loss.

Flow measurements

Flow measurement is an important process requirement. Generally flow measurement devices are designed to cause the flow to go through constrictions. Flow through pipes and open channels can be measured by using suitable devices. Most commonly used measuring devices for flow through pipes are orifice and Venturi meters, and the area meter (rotameter). Flow through open channels is measured using *V* or rectangular notches.

Orifice/Venturi meter [4,5,6]

The general equation of mass flow can be established from Bernoulli's equation

$$G = v_1 a_1 \rho_1 = v_2 a_2 \rho_2 \tag{2.5}$$

where 1 and 2 are upstream and downstream locations, respectively. Equation 2.5 cannot be solved directly because of two unknowns: mass flow rate and velocity. Now Bernoulli's equation can be used between points 1 and 2. For horizontal installation and incompressible fluid, $z_1 = z_2$ and $\rho_1 = \rho_2 = \rho_1$. Equation 2.3 can be rewritten as.

$$\frac{v_1^2}{2} + \frac{1000p_1}{\rho_1} = \frac{v_2^2}{2} + \frac{1000p_2}{\rho_1} \tag{2.6}$$

$$\text{and } v_1 = v_2 {}^* a_2/a_1 \tag{2.7}$$

From Equation 2.6 and Equation 2.7, the general equation for v_2 can be obtained as

$$v_2 = \sqrt{\frac{1000(p_1 - p_2)2}{\rho_1\left(1-\beta^4\right)}} \tag{2.8}$$

where β is the ratio of orifice and pipe diameter.

The mass flow rate from Equation 2.5 will be

$$G = a_2\rho_1\sqrt{\frac{1000(p_1 - p_2)2}{\rho_1\left(1-\beta^4\right)}} \tag{2.9}$$

In Equation 2.9, area a_2 is the area of flow at the point of pressure measurement (p_2). Generally, the area at the point of pressure measurement (at *vena contracta*) is less than the area of the orifice, and it is difficult to calculate the area at *vena contracta*. Mathematically, the area of *vena contracta* is calculated as:

$$a_o = a_2/C_D \tag{2.10}$$

where C_D is the coefficient of discharge, the value depends on the value of the Reynolds number and β ratio, and a_o is the orifice area.

The resultant equation of mass flow will be:

$$G = a_o C_D \rho_1\sqrt{\frac{1000(p_1 - p_2)2}{\rho_1\left(1-\beta^4\right)}} \tag{2.11}$$

The flow through the orifice/Venturi meter also depends on the thermal expansion of the flow meter and pipe (F_a), and the expansion factor (Y). With those corrections the final equation will be:

$$G = a_o C_D F_a Y \rho_1\sqrt{\frac{1000(p_1 - p_2)2}{\rho_1\left(1-\beta^4\right)}} \tag{2.12}$$

where

ρ_1 = upstream density,
Y = expansion factor, which allows for the change in gas density as it expands adiabatically from p_1 to p_2 [3],
F_a = thermal expansion factor.

Thermal expansion factor (F_a) [4]

The thermal expansion factor depends on the meter material, the pipe material, and the temperature of the process fluid. The following equation is used to calculate the thermal expansion factor:

$$F_a = 1 + \left(\frac{2}{1-\beta^4}\right)\left(\alpha_{PE} - \beta^4 \alpha_P\right)\left(t - t_R\right) \tag{2.13}$$

Coefficient of discharge C_D [4,5,6]

The value of C_D is normally established experimentally [1]. The coefficient of discharge is a function of Reynolds number, β ratio, and type of the meter.

Orifice meter

Corner taps. Corner taps are located such that the tap holes break through the wall flush with the faces of the meter plate. The discharge coefficient for corner taps is:

$$C_D = 0.5959 + 0.0312\beta^{2.1} - 0.1840\beta^8 + 91.71\beta^{2.5}N_{Re}^{-0.75} \tag{2.14}$$

The Reynolds number (N_{RE}) is based on the pipe diameter.

Flange taps. Flange pressure taps are located so that the inlet tap is 25 mm from the upstream face of the meter plate, and the outlet tap is 25 mm from the downstream face of the meter plate. The discharge coefficient depends on the diameter of the pipe (*d*).

Pipe diameter ≥ 58.4 mm.

$$C_D = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + 0.09(d*39.37)^{-1}\beta^4(1-\beta^4)^{-1} - 0.0337(d*39.37)^{-1}\beta^3 + 91.71\beta^{2.5}N_{Re}^{-0.75} \tag{2.15}$$

where *d* is the pipe diameter in meters and the Reynolds number (N_{Re}) is based on the pipe diameter.

Pipe diameter 50.8 mm < D < 58.4 mm.

$$C_D = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + 0.039(d*39.37)^{-1}\beta^4(1-\beta^4)^{-1} - 0.0337(d*39.37)^{-1}\beta^3 + 91.71\beta^{2.5}N_{Re}^{-0.75} \tag{2.16}$$

where *d* is the pipe diameter in meters and the Reynolds number (N_{Re}) is based on the pipe diameter.

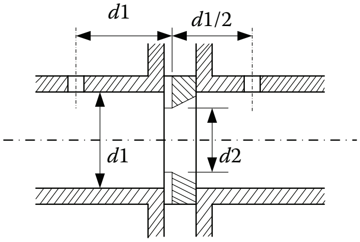


Figure **2.3** Square edge orifice meter.

Other pipe diameters.

$$C_D = K_{SB}(1 - \beta^4)^{1/2} \tag{2.17}$$

where

$$K_{SB} = 0.598 + 0.468(\beta^4 + 10\,\beta^{12}) + (0.00087 + 0.0081\beta^4)*1000/N_{Re}{}^{0.5} \tag{2.18}$$

1d – 1/2 d Taps. A typical 1*d* – 1/2 *d* tap is shown in Figure 2.3, where *d*1 is the inside diameter of the incoming pipe, and *d*2 is the diameter of the orifice. The upstream pressure tapping point is typically set at a distance *d*1, and the downstream tapping point is set at a distance *d*1/2. The discharge coefficient is defined as:

$$C_D = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + 0.039\beta^4(1 - \beta^4)^{-1}$$
$$- 0.0158\beta^3 + 91.71\beta^{2.5}N_{Re}{}^{-0.75} \tag{2.19}$$

The Reynolds number (N_{Re}) is based on the pipe diameter.

Restriction orifice

In the restriction orifice, no pressure measuring devices are installed, and the pressure drop is estimated as an actual pressure drop without the consideration of any pressure recovery. For this reason, the discharge coefficient for the restriction orifice is generally higher in comparison to other orifice flow measuring devices. The coefficient is also independent on Reynolds number. Values of discharge coefficient for restriction orifices are available in the literature [3]. For simplicity, this can be presented as:

$$C_D = 0.7858\beta^2 - 0.2354\beta + 0.6274 \tag{2.20}$$

The following points may be noted:

1. For β up to 0.2, the value of $C_D = 0.61$.

2. Equation 2.20 is applicable for $0.2 < C_D \leq 0.9$.
3. For critical flow, the value of $C_D = 0.73$.

Venturi meter

In the Venturi meter, the fluid is passed through converging/diverging sections, and because of gradual reduction of area, there is no *vena contracta*. The coefficient of discharge is in the range of 0.98 to 0.99 [2].

Expansion factor (Y)

When a fluid flows through a meter, there is a pressure drop as it passes through the constriction. When a compressible fluid flows through a meter, the resulting pressure drop causes a change in fluid density at the constriction. As a result, the fluid densities at the meter inlet and within the meter are different. The expansion factor corrects for density differences between pressure taps due to expansion to the lower pressure.

For liquids, the expansion factor = 1.

The expansion factor for a compressible fluid can be estimated using the following equations:

Orifices

$$Y = 1 - (0.41 + 0.35\beta^4)\left(\frac{\Delta p}{\gamma p_1}\right) \tag{2.21}$$

where

Δp = differential pressure, kPa
p_1 = upstream pressure, kPaA

Nozzles and Venturi

$$Y = \left\{\left(\frac{\gamma r^{2/\gamma}}{\gamma - 1}\right)\left(\frac{1-\beta^4}{1-\beta^4 r^{2/\gamma}}\right)\left(\frac{1-r^{(\gamma-1)/\gamma}}{1-r}\right)\right\}^{0.5} \tag{2.22}$$

Nonrecoverable pressure drop

The pressure drop calculated earlier does not represent the correct value. Downstream of the orifice/Venturi, the velocity gradually decreases, and some of the velocity head recovers. The actual pressure drop is the nonrecoverable pressure drop and is less than the pressure drop calculated earlier. The nonrecoverable pressure drop is calculated as:

Orifices

$$\Delta p_a = \left(\frac{(1-\beta^4)^{0.5} - C_D\beta^2}{(1-\beta^4)^{0.5} + C_D\beta^2} \right) \Delta p \tag{2.23}$$

Venturi with 15° divergent angle

$$\Delta p_a = (0.436 - 0.86\beta + 0.59\beta^2)\Delta p \tag{2.24}$$

Venturi with 7° divergent angle

$$\Delta p_a = (0.218 - 0.42\beta + 0.38\beta^2)\Delta p \tag{2.25}$$

Critical flow

From Equation 2.12, it is clear that for a given set of conditions, the flow through the orifice/Venturi will increase for a decrease in absolute pressure ratio p_2/p_1, until a linear velocity in the throat reaches the velocity of sound. The value of p_2/p_1 for which the acoustic flow is just attained is called the *critical pressure* ratio r_c, and flow at such a condition is called *critical* or *choked* flow. Under choked flow situations, the flow through the orifice depends only on the upstream pressure.

The theoretical equation for critical pressure ratio can be expressed as [3]:

$$r_c = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma+1}} \tag{2.26}$$

Critical flow through the orifice/Venturi can be expressed as [3]

$$G = a_o C_D F_a Y \sqrt{2000 p_1 \rho_1 \gamma \left(\frac{2}{\gamma + 1} \right)^{(\gamma+1)/(\gamma-1)}} \tag{2.27}$$

Area meter: rotameters [7]

In orifice and Venturi meters, the area of constriction is constant, and the pressure drop increases with the increase in flow rates. In area the meter, the pressure drop is constant, but the area of constriction increases with increase in flow rate. The rotameter consists of a gradually tapered glass tube mounted vertically in a frame with the large end up. The fluid flows upward and with increase in flow, the float moves upward.

For a given flow rate, the equilibrium position of the float depends upon three forces: (1) the weight of the float, (2) the bouncy force of the fluid on the float, and (3) the drag force on the float.

$$F_D = V_f \rho_f - V_f \rho \tag{2.28}$$

where

F_D = drag force, kg
V_f = volume of float, m^3
ρ_f = density of float, kg/m^3
ρ = density of fluid, kg/m^3

If the volume of the float is replaced by m_f/ρ_f, Equation 2.27 can be written as

$$F_D = m_f\,(1 - \rho/\rho_f) \tag{2.29}$$

The right-hand side of the equation means F_D is constant for any flow rate. A constant value of F_D is maintained by changing the flow area with volumetric flow rate (i.e., keeping constant velocity).

The flow rate through a rotameter can be obtained from [3]

$$G = Kd_f\sqrt{\frac{m_f(\rho_f - \rho)\rho}{\rho_f}} \tag{2.30}$$

Flow through an open channel

Flow through an open channel can be measured using notches. The V notch and rectangular notch are used for this purpose.

V notch

A typical V notch is shown in Figure 2.4.

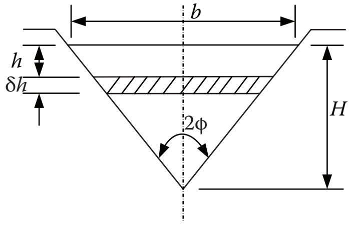


Figure 2.4 V notch.

Consider a strip of thickness δh at a depth h below the surface. If the velocity of approach is small:

Head producing flow = h
Area of the strip = $b\delta h$
Velocity through the strip, $v = \sqrt{2gh}$
Volume flow through the strip, $\delta Q = vb\delta h$

Now, width b depends on the value of h, and from trigonometric consideration, $b = 2(H - h)\tan\phi$.

Therefore,

$$\delta Q = \sqrt{2gh} * 2(H - h)\tan\phi\delta h$$

Integrating between the limits $h = 0$ to $h = H$

$$Q = \frac{8}{15}\sqrt{2g}\tan\phi H^{5/2} \tag{2.31}$$

Equation 2.31 is the general equation of flow through a V notch.

Rectangular notch

Similarly, the volumetric flow through a rectangular notch can be estimated as

$$Q = \frac{2}{3}b\sqrt{2g}H^{3/2} \tag{2.32}$$

where

b = width of the notch
H = height of the notch

Frictional pressure drop

Darcy equation

Assume a fluid is flowing through a pipe of uniform cross section area (diameter d), upstream pressure of p_1, and downstream pressure of p_2, for a pipe length of L.

The force due to pressure difference

$$= (p_1 - p_2)\,A$$

$$= (p_1 - p_2) \times \pi/4 \times d^2$$

Let q be frictional resistance per unit surface area at unit velocity and the frictional resistance vary as v^2.

Frictional resistance/unit area at velocity $v = qv^2$.

Force due to friction on surface of pipe = $\pi d \times L \times qv^2$.

Now, force due to pressure difference = force due to friction.

$$(p_1 - p_2) \times d/4 = L \times qv^2 \tag{2.33}$$

If h_f = head loss in friction for the length L, then

$$h_f = (p_1 - p_2)/\rho$$

$$h_f = \frac{2gq}{\rho} v^2 \frac{4L}{2gd} \tag{2.34}$$

The constant $2gq/\rho$ is called the resistance coefficient and is denoted by f (called friction factor). Equation 2.34 can be modified as

$$h_f = \frac{4fL}{d} \frac{v^2}{2g} \tag{2.35}$$

Equation 2.35 is the Darcy formula for the loss of head in a pipeline.

The friction factor f is called the Fanning friction factor. Another friction factor commonly used is called the Moody friction factor and is equal to $4f$.

Flow in open channel [2]

Flow through an open channel of uniform cross section and slope (angle θ) can be calculated mathematically, when the depth of liquid is constant throughout the length of the channel. For a length of L, the accelerating force acting on the liquid

$$= L * a * \rho * g * \sin\theta$$

The force resisting the motion = $R_m * w * L$

where

w = wetted perimeter

a = flow area, m^2

The hydraulic mean diameter $d_m = 4a/w$.

Rearranging the preceding equations, and dividing both sides by ρv^2,

$$v^2 = \frac{d_m}{4(R_m / \rho v^2)} g * \sin\theta \tag{2.36}$$

The value of $R_m/\rho v^2$ is almost independent of velocity but is a function of surface roughness (ε) and is normally defined as

$$\frac{R_m}{\rho v^2} = \frac{1}{16} * \left(\frac{\varepsilon}{d_m}\right)^{1/3} \tag{2.37}$$

Combining Equation 2.36 and Equation 2.37, the volumetric flow rate can be calculated as

$$Q = 2 * a * (d_m)^{2/3} * (\varepsilon)^{-1/6} * (g * \sin\theta)^{0.5} \tag{2.38}$$

Estimation of friction factor

A correct value of friction factor is required for the estimation of pressure drop. The value of the friction factor depends upon the flow characteristics. For laminar flow (N_{Re} less than 2100), the friction factor varies inversely with the Reynolds number, whereas for turbulent flow, the friction factor has a complex relationship with the pipe diameter, roughness of the pipe, and the Reynolds number.

Friction factor — laminar flow

The Fanning friction factor for laminar flow (Reynolds number less than 2100) is defined as [7]:

$$f = 16/N_{Re} \tag{2.39}$$

Friction factor — turbulent flow

For turbulent flow, the friction factor is estimated by using the well-known Moody diagram. This can also be calculated by using the Colebrook equation, which is the basis of the Moody diagram [3]:

$$\frac{1}{\sqrt{f}} = -4\log\left\{\frac{\varepsilon}{3.7d} + \frac{1.256}{N_{\mathrm{Re}}\sqrt{f}}\right\} \tag{2.40}$$

where

d = pipe ID, m
ε = roughness of pipe wall, m

Trial and error are required to solve the Colebrook equation; however, the trial-and-error method does converge very rapidly.

Friction factors calculated using Equation 2.39 and Equation 2.40 are valid for Equation 2.35.

Two-K method [6,8,9]

The pressure drop established in Equation 2.35 can be presented in two unrelated heads. The first term $4fL/d$ depends on the friction factor and pipe geometry — normally expressed as "K-factor" — whereas the second term v^2/g only depends on the velocity of the fluid, the velocity head. Using the two factors, the general equation of pressure drop can be modified as

$$h_f = K\,H_d \tag{2.41}$$

K is a dimensionless factor defined as the excess head loss in pipe fitting, expressed in velocity heads. In general, it does not depend on the roughness of the fitting (or the attached pipe) or the size of the system, but is a function of the Reynolds number and of the exact geometry of the fitting. The two-K method takes these dependencies into account in the following equation:

$$K = \frac{K_1}{N_{\text{Re}}} + K_\infty\left(1 + \frac{0.0254}{d}\right) \tag{2.42}$$

where

d = pipe ID, m

The values of K_1 and K_∞ for different fittings are presented in Table 2.2.

K for reducer/expander (refer to Figure 2.5)

K values for reducer and expander depend on the angle θ and the ratio of the smaller to larger diameter ($\beta = d_1/d_2$).

Reducer

If $\theta \le 45°$

$$K = \frac{0.8\sin(\theta/2)(1-\beta^2)}{\beta^4} \tag{2.43}$$

If $45° < \theta \le 180°$

Table 2.2 Constants for Two-K Method

Fitting Types		Description	K_1	K_∞
Elbows	90°	Standard (R/D = 1), screwed	800	0.4
		Standard (R/D = 1), flanged/welded	800	0.25
		Long radius (R/D = 1.5), all types	800	0.20
		Mitered, 1 weld, 90° angle	1000	1.15
		Mitered, 2 welds, 45° angles	800	0.35
		Mitered, 3 welds, 30° angles	800	0.30
		Mitered, 4 welds, 22.5° angles	800	0.27
		Mitered, 5 welds, 18° angles	800	0.25
	45°	Standard (R/D = 1), all types	500	0.20
		Long radius (R/D = 1.5), all types	500	0.15
		Mitered, 1 weld, 45° angle	500	0.25
		Mitered, 2 welds, 22.5° angles	500	0.15
	180°	Standard (R/D = 1), screwed	1000	0.60
		Standard (R/D = 1), flanged/welded	1000	0.35
		Standard (R/D = 1.5), all types	1000	0.30
Tees	Used as elbow	Standard, screwed	500	0.70
		Long radius, screwed	800	0.40
		Standard, flanged, or welded	800	0.80
		Stub-in type branch	1000	1.00
	Run through tee	Screwed	200	0.10
		Flanged or welded	150	0.50
		Stub-in type branch	100	0.00
Valves	Gate, ball, plug	Full line size, β= 1.0	300	0.10
		Reduced trim, β= 0.9	500	0.15
		Reduced trim, β= 0.8	1000	0.25
	Globe, standard		1500	4.00
	Globe, angle, or Y-type		1000	2.00
	Diaphragm, dam type		1000	2.00
	Butterfly		800	0.25
	Check	Lift	2000	10.00
		Swing	1500	1.5
		Tilting-disk	1000	0.5

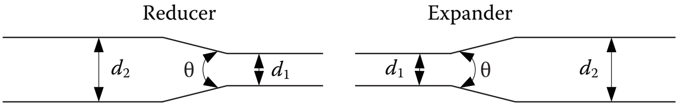


Figure 2.5 Sudden and gradual contraction/enlargement.

$$K = \frac{0.5(1-\beta^2)\sqrt{\sin(\theta/2)}}{\beta^4} \tag{2.44}$$

The overall pressure drop through a reducer (including change in velocity head):

$$\Delta p = \frac{\rho_2 * v_2^{\,2}}{2000}(K-1) + \frac{\rho_1 * v_1^{\,2}}{2000} \tag{2.45}$$

In both cases, K is calculated for a larger pipe diameter.

Expander

If $\theta \le 45°$

$$K = \frac{2.6\sin(\theta/2)(1-\beta^2)^2}{\beta^4} \tag{2.46}$$

If $45° < \theta \le 180°$

$$K = \frac{(1-\beta^2)^2}{\beta^4} \tag{2.47}$$

The overall pressure drop through an expander (including change in velocity head):

$$\Delta p = \frac{\rho_2 * v_2^{\,2}}{2000}(K-1) + \frac{\rho_1 * v_1^{\,2}}{2000} \tag{2.48}$$

In both cases, K is calculated for a larger pipe diameter.

Pipe entrance

For normal entrance, $K_1 = 160$ and $K_\infty = 0.5$.

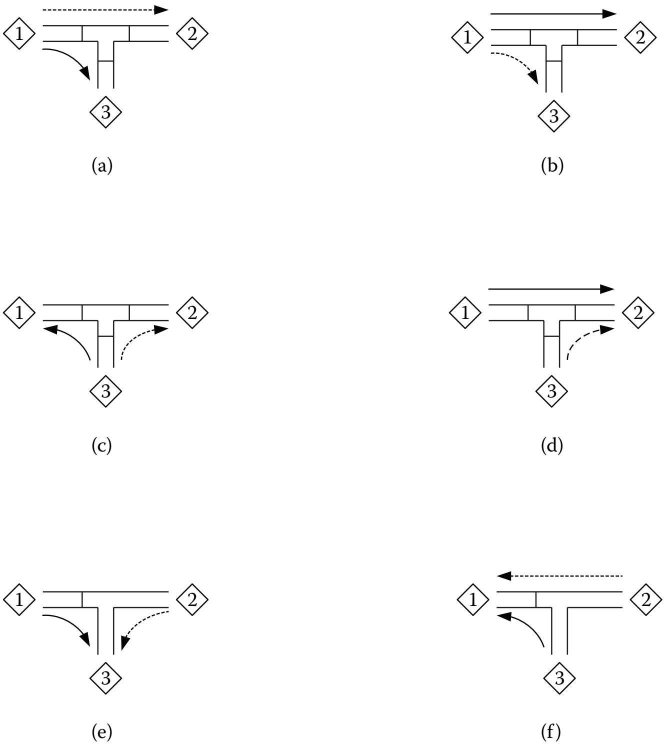
For "Borda" entrance, $K = 1.0$.

Pipe exit

For pipe exit, $K_1 = 0$ and $K_\infty = 1.0$.

Split flow

Table 2.2 is used to estimate the dimensionless K values for pipes and fittings. However, using values presented in Table 2.2, it is difficult to establish K values for various split flows. The method developed by Zenz [32] is normally used to establish split-flow pressure drop. Six different types of splits,



(a) (b) (c) (d) (e) (f)

Figure 2.6 (a) Split 1,3; (b) Split 1,2; (c) Split 3,1; (d) Split 1,2,3; (e) Split 1,3,2; and (f) Split 3,1,2.

as shown in Figures 2.6a to f, are commonly used in hydraulic calculations. This calculation is mainly derived for liquid flow but can be used for compressible flow if the split pressure drop is low in comparison to the inlet pressure. In such a case, the density of the outlet stream will be equal to the density of the inlet stream.

Split 1,3

$$\Delta p = \frac{\rho_1}{2000}\left(1.8v_3^2 - 0.368v_1v_3\right) \tag{2.49}$$

Split 1,2

$$\Delta p = \frac{\rho_1}{2000}\left(1.36{v_2}^2 - 0.64{v_1}^2 - 0.72v_1v_2\right) \tag{2.50}$$

Split 3,1

$$\Delta p = \frac{\rho_3}{2000}\left(1.8v_1^2 - 0.368v_1v_3\right) \tag{2.51}$$

Split 1,2,3

$$\Delta p = \frac{\rho_1}{2000}\left\{2v_2^2 - 0.05v_1^2 - 2v_2\left(0.205v_3\frac{Q_3}{Q_2} + v_1\frac{Q_1}{Q_2}\right)\right\} \tag{2.52}$$

Split 1,3,2

$$\Delta p = \frac{\rho_1}{2000}\left\{2v_3{}^2 - 0.4v_1{}^2 - 0.41v_3\left(v_1\frac{Q_1}{Q_3} + v_2\frac{Q_2}{Q_3}\right)\right\} \tag{2.53}$$

Split 3,1,2

$$\Delta p = \frac{\rho_3}{2000}\left\{2v_1{}^2 - 0.4v_3{}^2 - 2v_1\left(0.205v_3\frac{Q_3}{Q_1} + v_2\frac{Q_2}{Q_1}\right)\right\} \tag{2.54}$$

Split pressure drops are extremely useful for accurate pressure drop calculations, typically for the pressure release valve (PRV) inlet line frictional pressure drop.

Hydraulics — general guidelines

For incompressible fluids, the fluid density does not change with pressure drop. It is also assumed that the temperature drop is not significant to change the fluid density appreciably. The following input parameters are required to calculate the pressure drop:

- Mass flow rate
- Density of fluid
- Viscosity of fluid
- Inlet pressure
- Pipe length

Table 2.3 Roughness of Pipe Wall

Piping Material	Roughness (m)
Riveted steel	0.0009–0.009
Concrete	0.0003–0.003
Wood stave	0.00018–0.0009
Cast iron	0.00025
Galvanized iron	0.00015
Asphalted cast iron	0.00012
Commercial steel	0.000046
Wrought iron	0.000046
Drawn tubing	0.0000015
PE pipes	0.000003
Glass, plastic pipe	0 (for all practical purposes)

- Pipe inlet diameter
- Change in elevation
- Roughness factor
- Fitting details, as explained earlier
- *CV* of control valve (if any)
- Additional *K* or pressure drop

Roughness of pipe wall

Roughness of the pipe wall is an important design parameter for pressure drop calculation. The roughness depends on the type of pipe being used. Most commonly used pipe materials have roughnesses as given in Table 2.3.

Control valve CV

It is sometimes required to calculate the pressure drop through a control valve; this is best achieved by using the coefficient of valve (CV) of the control valve. For incompressible fluids, the pressure drop through the control valve is calculated using the following equation:

$$\Delta p = 0.1337 * \rho * (Q/CV)^2 \tag{2.55}$$

The value of the *CV* can be taken from Table 2.4 [10].

Care must be taken to use the control valve *CV* to calculate the pressure drop. For fixed *CV*, with an increase in flow rate, the pressure drop increases (pressure drop increases with the square of the volumetric flow rate). In most cases, the control valve is used for a fixed pressure drop (in fact, with increase in flow rate, pressure drop through the control valve decreases). The designer must consider this fact to estimate the pressure drop through the control

Table 2.4 Control Valve *CV*

Size, NB (mm)	Valve opening (%) 10	20	30	40	50	60	70	80	90	100
25	0.783	1.54	2.2	2.89	4.21	5.76	7.83	10.9	14.1	17.2
40	1.52	2.63	3.87	5.41	7.45	11.2	17.4	24.5	30.8	35.8
50	1.66	2.93	4.66	6.98	10.8	16.5	25.4	37.3	50.7	59.7
65	3.43	7.13	10.8	15.1	22.4	33.7	49.2	71.1	89.5	99.4
80	4.32	7.53	10.9	17.1	27.2	43.5	66	97	120	136
100	5.85	11.6	18.3	30.2	49.7	79.7	125	171	205	224
150	12.9	25.8	43.3	67.4	104	162	239	316	368	394
200	27	58.1	105	188	307	478	605	695	761	818

valve. It is often preferable to use a fixed pressure drop through the control valve, rather than specifying a fixed *CV*.

Line sizing criteria for liquid lines

Table 2.5 presents the general guidelines used for sizing liquid lines. Though these guidelines are extensively used, the final sizing is based on the hydraulic calculation of the section.

Line sizing for gravity flow lines

Equation 2.38 is used to estimate the flow rate for gravity flow. Normally, for gravity flow, a slope of 1:100 is maintained (0.57°). For gravity flow, pipe with a diameter 200 mm or less is designed for 50% liquid capacity, and for a diameter more than 200 mm, 75% capacity.

Downpipe sizing

Downpipe sizing is an interesting sizing requirement particularly used to design distillation column internals. For distillation columns the downpipes are designed such that the pipes are self-venting, i.e., while the liquid flows downward in the pipe, it disengages the vapor that flows upward. This type of downpipe is designed for a Froude number < 0.3. The Froude number is defined as

$$N_f = \frac{v}{\left(g * d * \frac{(\rho_l - \rho_g)}{\rho_l} \right)^{0.5}} \tag{2.56}$$

If N_f is less than 0.3, the liquid flow will be self-venting. For an N_f of 0.3 to 2.0, the pipe will be full of liquid, with some possibility of vapor ingress at

Table 2.5 Sizing Criteria for Carbon Steel Liquid Lines

Description	Velocity (m/sec)			ΔP (kPa/100 m)
Nominal pipe size (mm)	≤50	80–250	≥300	
Hydrocarbon, organic liquids				
Pump suction	0.3–0.9	0.9–1.8	1.2–2.4	5–20
Nonboiling	0.3–0.6	0.6–1.2	0.9–1.8	5–10
Boiling				
Pump discharge				
Short length	1.2–2.7	1.5–3.0	2.4–3.7	24–60
Long length	0.6–0.9	0.9–1.5	1.2–2.1	24–35
		For all sizes		
Liquid from condenser		0.9–1.8		5–10
Refrigeration lines		0.6–1.2		5–9
Gravity flow lines		0.9–2.5		9, Maximum
Reboiler inlet		0.3–1.2		2–6
Liquid feed to tower		1.2–1.8		6–10
Water				
General service	0.6–1.5	1.5–3.7	3.0–4.9	10–45
Pump suction		0.6–2.1		5–11
Pump discharge		0.6–4.9		10–45
Boiler feed		2.5–4.5		
Cooling water		3.6–4.8		10–45
Caustic solution		1.2–1.6		
Viscous Oil				
Pump suction	0.2 Maximum	0.2–0.3	0.2–0.3	NPSH
Pump discharge	0.3–0.9	0.9–1.5		

the top of the pipe. For $N_f > 2.0$, liquid will only flow through the pipe, and vapor disengagement should occur before any liquid enters the downpipe.

Line sizing criteria for vapor lines

Table 2.6 presents the general guidelines for sizing vapor lines. Single-phase gas lines are normally designed based on velocity criteria as specified in API 14E [11]. The Maximum recommended design velocity is 18 m/sec (60 ft/sec) though higher velocities are also acceptable. Some operating companies prefer to use ρv^2 as a design criterion. For natural gas, the value of ρv^2, presented in Table 2.6, represents a maximum velocity of 18 m/sec.

Relief valve inlet line sizing

The inlet line to the relief valve is designed for a maximum pressure drop of 3% of the relief valve set pressure. The pressure drop should be in between the equipment being protected and the inlet to the relief valve. However, the line size should not be smaller than the inlet flange size of the relief valve.

Table 2.6 Sizing Criteria for Carbon Steel Vapor Lines

Description	Maximum ρv^2 (kg/[m·sec^2])	Maximum Velocity (m/sec)	Maximum *DP* (kPa/100 m)
Continuous Operation			
p < 2,000 kPaG	6,000		4–8
2,000 < p < 5,000	7,500		8–11
5,000 < p < 8,000	10,000		11–20
p > 8,000 kPaG	15,000		20–27
Intermittent Operation			
p < 2,000 kPaG	7,500		
2,000 < p < 5,000	9,500		
5,000 < p < 8,000	12,500		
p > 8,000 kPaG	20,000		
Compressor suction lines		20	3–7
Compressor Discharge Lines			
p < 500 kPaG		30	4
500 < p < 1,000		30	7
1,000 < p < 5,000		30	10
p > 5,000		30	12

Relief valve outlet line sizing

The outlet line from the relief valve is designed based on the *Mach* number. Generally, the *Mach* number in between 0.5 to 0.75 is maintained. However, the outlet line size shall not be smaller than the outlet flange of the relief valve.

Line sizing criteria for two-phase flow

Maximum allowable velocity is calculated, based on an empirical relationship of erosional velocity [11].

$$V_e = \frac{c}{\rho_m} \tag{2.57}$$

The mixed density is defined as

$$\rho_m = \rho_g * (1 - \lambda) + \lambda * \rho_l \tag{2.58}$$

The value of the empirical constant *c* is tabulated as follows:

Type of Services	Value of *c*
Continuous service	122
Intermittent service	152

For solid-free fluids where corrosion is not anticipated, or when corrosion is controlled by inhibition, or by employing corrosion-resistant alloys, values of *c* from 183 to 244 may be used for continuous flow, and values up to 305 can be used for intermittent service.

Hydraulics — compressible fluids [12]

In case a compressible fluid is flowing from point 1 to point 2, the general equation of flow under steady state can be given by:

$$\left(\frac{p_2}{\rho_2}+\frac{v_2{}^2}{2}+gz_2\right)-\left(\frac{p_1}{\rho_1}+\frac{v_1{}^2}{2}+gz_1\right)+e_2-e_1=0 \tag{2.59}$$

Because $e + p/\rho$ may be written as specific enthalpy h, and as points 1 and 2 were arbitrarily chosen, Equation 2.59 can be expressed as

$$h + 1/2\ v^2 + gz = \text{constant along a streamline} \tag{2.60}$$

Adiabatic flow in a pipe

For gases, the term gz is negligible, and the Equation 2.60 can be rewritten as

$$h + 1/2\ v^2 = \text{constant} \tag{2.61}$$

In Equation 2.61, it is clear that with increase in velocity (low-pressure downstream side) the specific enthalpy will reduce, and with decrease in velocity, the specific enthalpy will increase. The specific enthalpy will be a maximum when the velocity is zero, and this maximum value is termed as stagnation enthalpy, h_o. The Equation 2.61 can further be modified as

$$C_p * T + 1/2\ v^2 = C_p * T_o \tag{2.62}$$

Differentiating Equation 2.62,

$$C_p * dT + v\ dv = 0 \tag{2.63}$$

Now, sonic velocity (S) is defined as

$$S = 91.1833\sqrt{\frac{Z * \gamma * T}{MW}} \tag{2.64}$$

and the *Mach* number is

$$M = \frac{v}{S} \tag{2.65}$$

Integrating Equation 2.63 and replacing the velocity with the *Mach* number,

$$C_p * T_1 + 1/2\, M_1^2 \gamma R T_1 = C_p * T_2 + 1/2\, M_2^2 \gamma R T_2 \tag{2.66}$$

where

$$\frac{T_1}{T_2} = \frac{1 + \frac{1}{2}(\gamma - 1)M_2^{\,2}}{1 + \frac{1}{2}(\gamma - 1)M_1^{\,2}} \tag{2.67}$$

$$\frac{v_1}{v_2} = \frac{M_1}{M_2}\sqrt{\frac{T_1}{T_2}} = \frac{M_1}{M_2}\left\{\frac{1 + \frac{1}{2}(\gamma - 1)M_2^{\,2}}{1 + \frac{1}{2}(\gamma - 1)M_1^{\,2}}\right\}^{0.5} \tag{2.68}$$

$$\frac{p_1}{p_2} = \frac{\rho_1 T_1}{\rho_2 T_2} = \frac{v_2 T_1}{v_1 T_2} = \frac{M_2}{M_1}\left\{\frac{1 + \frac{1}{2}(\gamma - 1)M_2^{\,2}}{1 + \frac{1}{2}(\gamma - 1)M_1^{\,2}}\right\}^{0.5} \tag{2.69}$$

The limiting conditions under which an initially subsonic flow is choked are given by putting $M_2 = 1$. Then,

$$\frac{T_1}{T_c} = \frac{\frac{1}{2}(\gamma + 1)}{1 + \frac{1}{2}(\gamma - 1)M_1^{\,2}} \tag{2.70a}$$

$$\frac{v_1}{v_c} = M_1\left\{\frac{\frac{1}{2}(\gamma + 1)}{1 + \frac{1}{2}(\gamma - 1)M_1^{\,2}}\right\}^{0.5} \tag{2.70b}$$

$$\frac{p_1}{p_c} = \frac{1}{M_1}\left\{\frac{\frac{1}{2}(\gamma+1)}{1+\frac{1}{2}(\gamma-1)M_1^2}\right\}^{0.5} \tag{2.70c}$$

The general equation for a compressible fluid in a circular pipe is

$$\frac{dp}{\rho} + vdv + \frac{fdl}{a/P}\frac{v^2}{2} = 0 \tag{2.71}$$

also,

$$\frac{dv}{v} = \frac{\frac{dM}{M}}{\frac{1}{2}(\gamma-1)M^2+1} \tag{2.72}$$

and

$$\frac{dp}{\rho} = \frac{d\rho}{\rho} + \frac{dT}{T} = -\left\{\frac{(\gamma-1)M^2+1}{\frac{1}{2}(\gamma-1)M^2+1}\right\}\frac{dM}{M} \tag{2.73}$$

From Equation 2.71, Equation 2.72, and Equation 2.73,

$$\left\{\frac{M^2-1}{\frac{1}{2}(\gamma-1)M^2+1}\right\}\frac{dM}{\gamma M^3} + \frac{fdl}{2a/P} = 0 \tag{2.74}$$

Integrating Equation 2.74 between points 1 and 2 at a distance *l* apart:

$$\frac{1}{2\gamma}\left(\frac{1}{M_2^2} - \frac{1}{M_1^2}\right) + \frac{\gamma+1}{4\gamma}\ln\left\{\left(\frac{M_2}{M_1}\right)^2 \frac{(\gamma-1)M_1^2+2}{(\gamma-1)M_2^2+2}\right\} + \frac{fl}{2a/P} = 0 \tag{2.75}$$

From Equation 2.75, the limiting length (i.e., the length necessary for the *Mach* number to change from M_1 to unity) is given by

$$\frac{f\gamma l_{\max}}{a/P} = \frac{1}{M_1^2} - 1 - \frac{\gamma+1}{2}\ln\left\{\frac{\gamma-1}{\gamma+1} + \frac{2}{M_1^2(\gamma+1)}\right\} \tag{2.76}$$

Isothermal flow in a pipe

In isothermal flow, the flowing temperature is constant. Constant temperature means the viscosity is constant. Viscosity generally depends on temperature; pressure has negligible influence on the value of viscosity (unless the pressure drop is extremely high to influence the value of viscosity). The value of the Reynolds number is expressed as $dv\rho/\mu$. For isothermal flow and with comparatively low pressure drop condition, the Reynolds' number as well as the friction factor will remain unchanged.

For a perfect gas and for isothermal flow, from the continuity equation, ρv = constant, and the velocity term can be defined as

$$\frac{dv}{v} = -\frac{d\rho}{\rho} = -\frac{dp}{p} \tag{2.77}$$

Dividing Equation 2.71 by v^2, and substituting for dv/v from Equation 2.77 we get

$$\frac{fdl}{2a/P} = -\frac{dp}{\rho v^2} + \frac{dp}{p} \tag{2.78}$$

The velocity $v = m/\rho a$, where m is mass flow rate (kg/sec) and density $\rho = p/RT$. Equation 2.78 can be rearranged as

$$\frac{fdl}{2a/P} = -\frac{a^2}{m^2}\frac{p}{RT}dp + \frac{dp}{p} \tag{2.79}$$

Integrating Equation 2.79 between points 1 and 2 at a distance l apart and considering a circular pipe:

$$\frac{2fl}{d} = \frac{a^2}{2m^2RT}(p_1^2 - p_2^2) - \ln\frac{p_1}{p_2} \tag{2.80}$$

Using the *Mach* number for inlet and outlet conditions, Equation 2.80 can be rearranged as

$$\frac{2fl}{d} = \frac{1}{2\gamma}\left(\frac{1}{M_1^2} - \frac{1}{M_2^2}\right) - \ln\frac{M_2}{M_1} \tag{2.81}$$

Equation 2.80 or Equation 2.81 can be solved by trial and error.

Heat loss

Heat loss is one important factor in calculating pressure drop for compressible and multiphase fluids. For compressible fluids, heat loss reduces the temperature of the fluid and thus increases the fluid density, which reduces the pressure drop of the fluid. On the other hand, heat gain increases the fluid temperature as well as pressure drop. However, for compressible fluids, the impact of heat loss/gain for plant piping (within battery limit) is not severe in pressure drop calculation, and it is recommended to ignore heat loss or gain. For long pipelines (cross-country pipelines), the heat loss/gain has a substantial impact in pressure drop calculation. Most cross-country pipelines are either undersea or underground, and proper considerations are required to calculate heat loss or gain. With heat loss/gain, the physical properties of fluid, such as viscosity and compressibility, change, but the influence is not very significant.

For two-phase flow, heat gain/loss is also an important consideration, because this dictates the vapor/liquid fraction of the flow. Apart from changing the overall physical properties, this also changes the flow regime and will have a substantial impact in pressure drop calculation. The heat loss/gain can be used for compressible fluids for better results; however, it is of less importance for multiphase flow, unless the thermodynamic correlations are used to estimate the vapor/liquid fraction of the fluid.

Types of cross-country buried pipelines

Two types of underground piping are commonly used in industry: (1) two-layer extruded polyethylene coatings (yellow jacket) and (2) fusion-bonded epoxy coating (FBE).

Yellow jacket [13]

This is basically an external coating of high-density polyethylene and suitable where the pipe temperature during operation is in the range of –20°C to 55°C. The coating is intended for use in applications where it is not subject to weather or ultraviolet light exposure.

Coating thickness

The sheath is in accordance with requirements specified in Table 2.7. The maximum polyethylene layer thickness does not exceed twice the minimum polyethylene layer thickness. Where additional mechanical protection is

Table 2.7 Thickness of Polyethylene Coating

Outside Diameter of Pipe (mm)	Polyethylene Thickness Minimum (mm)
Over 60.3	1.0
48.3–60.3	0.75
Under 48.3	0.60

required, a thicker layer of polyethylene may be specified, or a double coating may be specified.

Fusion-bonded epoxy coating [14]

Generally, FBE is designed to operate up to 105°C when located in a dry environment, and up to 90°C when wet. Standard thickness of the single coating is 400 μm (0.4 mm) and for double coatings is 800 μm (0.8 mm).

Rate of heat transfer

The rate of heat transfer depends on the overall heat transfer coefficient, the surface area, and the temperature difference between the fluid and surrounding. Mathematically, the average rate of heat transfer is defined as

$$Q_a = U_o\, A_o\, \Delta T \tag{2.82}$$

The heat transfer to/from the fluid to the surroundings depends on the total thermal resistances or overall heat transfer coefficient. The thermal resistances depend on a large number of factors including the physical properties of the fluid and the environmental conditions. In general, the total thermal resistance is the summation of the following resistances:

- Gas film resistance (R_{film})
- Pipe resistance (R_{pipe})
- Coating resistances ($R_{coating}$)
- Environment resistance (R_{env})

Mathematically, the overall heat transfer coefficient is defined as

$$\frac{1000}{U_o D_{ot}} = R_{film} + R_{pipe} + R_{coating} + R_{env} \tag{2.83}$$

Typical overall heat transfer coefficients are presented in Table 2.8.

Film resistance (R_{film}) [15]

The resistance due to the film can be calculated using the following equation:

Table 2.8 Overall Heat Transfer Coefficient (W/m²·K)

		Pipe NB (mm)		
Location	Coatings	250	500	750
Buried in dry sand	1.5-mm PVC	2.4	1.4	1.1
Buried in soil	1.5-mm PVC	3.8	2.3	1.8
Buried in soil (insulated)	1.5-mm PVC 50-mm PUF 10-mm polystyrene	0.74	0.6	0.53
Buried under seabed	1.5-mm PVC 75-mm concrete	4.9	3.1	2.4
Exposed in water (on seabed)	1.5-mm PVC 75-mm concrete	16	14	13
Exposed in water (on seabed — insulated)	1.5-mm PVC 50-mm PUF 10-mm sleeve pipe	0.91	0.80	0.76
Exposed in water (riser)	3-mm polyethylene	100	86	73
Exposed in air	None	26	19	16
Exposed in air (insulated)	1.5-mm PVC 50-mm epoxy foam 5-mm polypropylene	1.7	1.44	1.36

$$R_{film} = \frac{1000}{h_i D_i} \tag{2.84}$$

The value of the inside film coefficient is a function of the Reynolds number and Prandtl number.

Reynolds number > 10,000. For fully turbulent flow (Reynolds number > 10,000), the inside film coefficient is defined as

$$h_i = 0.027 \frac{1000 k_f}{D_i} \left(N_{\mathrm{Re}}\right)^{0.8} \left(P_r\right)^{1/3} \tag{2.85}$$

where the Prandtl number is defined as

$$P_r = \left(\frac{C_p \mu}{1000 k_f} \right) \tag{2.86}$$

Reynolds number < 2100. In case of laminar flow (Reynolds number less than 2100), the inside film coefficient is defined as [16]

$$h_i = 3.66\frac{1000k_f}{D_i} \tag{2.87}$$

Intermediate Reynolds number. The calculation of the inside film coefficient for the intermediate Reynolds number is relatively complex. The following conservative approach can be used to calculate the inside film coefficient:

- For a Reynolds number from 2100 to 3000 set the value as 3000 and use Equation 2.85.
- For a Reynolds number above 3000, use Equation 2.85.

Equation 2.85 and Equation 2.86 are developed for tubes, but can be used for pipes. No experimental data are available for pipes. Pipes are rougher than tubes and produce more turbulence than tubes for the same Reynolds number. Coefficients calculated from tube–data correlations are actually lower and safer than corresponding calculations based on pipe data.

Resistance of pipe (R_{pipe})

The resistance of heat transfer due to the pipe wall is defined as

$$R_{pipe} = \frac{\ln(D_o / D_i)}{2k_{pipe}} \tag{2.88}$$

Thermal conductivities of some commonly used piping materials are presented in Table 2.9.

Table 2.9 Thermal Conductivities of Commonly Used Pipe Materials

Material	Thermal Conductivity (W/[m.°C])
Carbon steel	45–52
Ductile iron	52
Aluminum	199
Copper	380
Polyethylene (high density)	0.36
Polyvinylchloride (PVC)	0.17
Concrete	0.8–1.4

Resistance of coatings ($R_{coating}$)

Thermal resistance of coatings is the total resistance of all coatings. This is simply the sum of all coatings calculated separately. The general equation to calculate a single layer resistance is

Table 2.10 Thermal Conductivities of Commonly Used Coating Materials

Material	Thermal Conductivity (W/[m.°C])
Asphalt	0.29
Concrete	0.8–1.4
Concrete (weight)	0.04
Glass fiber	0.25
Neoprene	0.19
PVC	0.17
Polyurethane foam	0.035

$$R_{coating} = \frac{1}{2k_c}\ln\left(1+\frac{2t_c}{D_{ci}}\right) \tag{2.89}$$

Thermal conductivities of some commonly used coating materials is presented in Table 2.10.

Resistance of environment (R_{env})

There are two main methods of pipe laying: (1) buried pipe and (2) aboveground pipe. The heat transfer coefficient largely depends on the type of environment.

Buried pipe [17]. The equation for buried pipe is derived by Carslaw and Jaeger and gives the thermal resistance of the ground for a buried pipe (Figure 2.7):

$$R_{env} = \frac{\cosh^{-1}(2Z_b / D_{ot})}{2k_s} \tag{2.90}$$

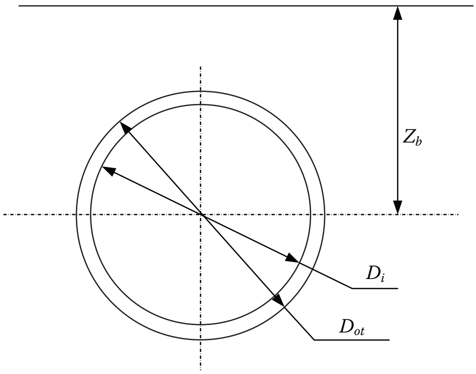


Figure 2.7 Buried pipe.

Table 2.11 Thermal Conductivities of Surroundings

Material	Thermal Conductivity (W/[m.°C])
Peat (dry)	0.17
Peat (wet)	0.54
Peat (icy)	1.89
Sandy soil (dry)	0.43–0.69
Sandy soil (moist)	0.87–1.04
Sandy soil (soaked)	1.90–2.42
Clay soil (dry)	0.35–0.52
Clay soil (moist)	0.69–0.87
Clay soil (wet)	1.04–1.56
Clay soil (frozen)	2.51
Gravel	0.95–1.25
Gravel (sandy)	2.51
Limestone	1.30
Sandstone	1.63–2.08
Ice (0°C)	2.2
Ice (–40°C)	2.66
Snow (loose)	0.09–0.23
Snow (hard-packed)	0.69–1.21

The inverse hyperbolic cosine can be expressed as

$$\cosh^{-1}(x) = \ln\left\{x + \sqrt{x^2 - 1}\right\} \tag{2.91}$$

The Equation 2.90 can be rewritten as

$$R_{env} = \frac{1}{2k_s}\ln\left\{\frac{2Z_b + \sqrt{4Z_b{}^2 - D_{ot}{}^2}}{D_{ot}}\right\} \tag{2.92}$$

The thermal conductivities of typical surroundings are presented in Table 2.11.

Submerged pipe [18]. Submerged pipe is mostly applicable for seawater, and Parker et al. recommended that the following empirical equation should be used:

$$R_{env} = \frac{1}{k_s C_2 N_{\mathrm{Re}}{}^n P_r{}^{1/3}} \tag{2.93}$$

Table 2.12 Constants Used for Equation 2.93 and Equation 2.94

N_{Re}	C_1	C_2	n
1–4	0.891	0.989	0.330
4–40	0.821	0.911	0.385
40–4,000	0.615	0.683	0.466
4,000–40,000	0.174	0.193	0.618
40,000–250,000	0.0239	0.026	0.805

where the Reynolds number (N_{Re}) and Prandtl number (P_r) are based on surrounding fluid and overall pipe outside diameter.

The values of n and C_2 are presented in Table 2.12.

Above-ground pipe [19]. The resistance due to air (above-ground consideration) can be calculated based on the following empirical equation. In this equation, a constant Prandtl number has been used for air:

$$R_{env} = \frac{1}{k_s C_1 N_{\mathrm{Re}}{}^{n}} \tag{2.94}$$

The Reynolds number is based on the surrounding fluid and overall pipe outside diameter. The values of constants n, C_1, and C_2 are presented in Table 2.12.

Calculation of environmental resistance requires the estimation of thermal conductivity, viscosity, and density of the surrounding air and water.

Viscosity of water

The viscosity of water varies with temperature and can be expressed using the following equation:

$$\mu = \exp\left\{-3.827 + \frac{609.246}{138.89 + t}\right\} \tag{2.95}$$

where viscosity μ is in cP and temperature t is in °C. Equation 2.95 is valid for a temperature range of 0 to 150°C.

Thermal conductivity of water

From 0 to 100°C, the thermal conductivity of water shows a linear relationship with temperature and can be expressed as

$$k_s = 0.0009t + 0.5937 \tag{2.96}$$

where thermal conductivity k_s is in W/(m.°C) and t is in °C.

Viscosity of air

The viscosity of air increases with temperature and can be expressed as

$$\mu = \exp\left\{-4.364 + \frac{49.375}{190.225 - t}\right\} \tag{2.97}$$

where viscosity μ is in cP and temperature t is in °C. Equation 2.97 is valid for a temperature range of 0 to 100°C.

Thermal conductivity of air

At a low temperature range of 0 to 60°C, the thermal conductivity of air varies linearly with temperature and can be expressed as

$$k_s = 0.00007t + 0.0241 \tag{2.98}$$

where thermal conductivity k_s is in W/(m.°C) and temperature t is in °C.

Choked flow

Compressible fluid, when stored in a vessel at high pressure, can escape at very high velocity to a relatively low downstream pressure. If the pressure difference between the vessel and downstream equipment is high, the fluid will escape at sonic velocity, and the situation is called *choked flow.*

There are several instances where choked flow rates are required to be estimated. Typical examples are overpressuring drain lines due to gas breakthrough, overpressuring flare lines during blowdown, etc. The general equation of compressible fluid flow [6] is

$$Q = 53.64 * Y * d^2 \sqrt{\frac{\Delta p * p_1}{K * T * s_g}} \tag{2.99}$$

where

Q = volumetric flow, Sm^3/s
Y = expansion factor, dimensionless
K = flow resistance factor of pipes and fittings
d = diameter, m
Δp = differential pressure, kPa
p_1 = vessel pressure, kPaA
T = temperature, K
s_g = specific gravity (air = 1)

Limiting differential pressure

The difference between the high and low pressures is Δp; however, in a choked flow situation, the value of Δp will depend on the sonic velocity limiting factor. The sonic velocity limiting factor depends on the flow resistance (K factor) and the ratio of specific heats. For oil and gas applications, the ratio of specific heats is approximately 1.3, and for this value the limiting value of $\Delta p/p_1$ can be defined as [6]:

$$\frac{\Delta p}{p_1} = \left\{0.9953 + \frac{0.9054}{K^{0.5}} + \frac{0.1173}{K} - \frac{0.0195}{K^{1.5}}\right\}^{-1.0} \tag{2.100}$$

Limiting expansion factor Y

The limiting value of the expansion factor also depends upon the flow resistance (K factor) and ratio of specific heats. For a specific heats ratio of 1.3, the limiting value of the expansion factor can be defined as [6]

$$Y = 0.0415\ \ln(K) + 0.6097 \tag{2.101}$$

The maximum value of the expansion factor is 0.718.

In a choked flow situation, the flow rate can be calculated using Equation 2.99 where the limiting Δp is established from Equation 2.100, and limiting Y is established from Equation 2.101 (subject to a maximum Y value as 0.718).

In case of normal flow (not choked), the actual value of expansion coefficient is required to estimate the flow rate. The value of the expansion coefficient (nonchoked condition) is defined as

$$Y = 1 - m * \frac{\Delta p}{p_1} \tag{2.102}$$

where the slope m is defined as

$$m = \frac{1 - chokedflow Y}{chokedflow \Delta p\ /\ p_1} \tag{2.103}$$

Example 2.1

A gas at 60°C having a specific gravity of 0.42 is stored in a cylinder at 800 kPaG and is released to the atmosphere through a 6-m 80 NB (sch 80) pipe. Estimate the flow rate in Sm^3/h. Assume the ratio of specific heats is 1.3, and the gas in the cylinder is not limiting.

Solution

Pipe ID = 0.0779 m
Friction factor (for fully turbulent flow) = 0.0175
K for pipe (0.0175 * 6/0.0779) = 1.35
K (entrance) = 0.5
K (exit) =1.0
Total K = 2.85
$\Delta p/p_1$ =(901.3 – 101.3)/901.3 = 0.888

Choked flow situation

$\Delta p/p_1$ (Equation 2.100) = 0.638

Because the limiting $\Delta p/p_1$ is less than the estimated $\Delta p/p_1$, the flow will be sonic.

$$\Delta p = 0.638 * 901.3 = 575 \text{ kPa}$$

Limiting value of Y (Equation 2.101) = 0.653

$$Q = 53.64 * 0.653 * (0.0779)^2 \sqrt{\frac{575 * 901.3}{2.85 * 333 * 0.42}}$$

$$= 7.664 \text{ Sm}^3/\text{sec}$$

$$= 27592 \text{ Sm}^3/\text{h}$$

Example 2.2

Solve the problem in Example 2.1 assuming the cylinder pressure is 150 kPaG.

Solution

Total K = 2.85
$\Delta p/p_1$ = 0.597

The flow will be subsonic.

Limiting value of $\Delta p/p_1$ (Equation 2.100) = 0.638
Limiting value of Y (Equation 2.101) = 0.653
Value of slope m (Equation 2.103) = 0.544
Value of Y (Equation 2.102) (1 – 0.544 * 0.597) = 0.675
Flow rate Q = 7692 Sm^3/h

Hydraulics — two-phase flow

Two-phase flow is probably the most complex unit operation in chemical engineering. To date no single method has been developed to cover all ranges of two-phase flow. Large numbers of correlations are available in the literature, and in most cases those correlations are limited for a very narrow range of process parameters. From among the large numbers of correlations available in the literature, the following are popular correlations used extensively in the industry:

- Beggs and Brill — original [20]
- Beggs and Brill — revised
- Mukherjee and Brill [21,22]
- Gregory et al. [23]
- Bendiksen et al. [26]

The first version of OLGA was developed through a joint research program between the Institute for Energy Technology and SINTEF, supported by Conoco Norway, Esso Norge, Mobil Exploration Norway, Norsk Hydro A/S, Petro Canada, Saga Petroleum, Statoil, and Texaco Exploration Norway.

The physical model of OLGA was originally based on small-diameter data for low-pressure air/water flow. The 1983 data from the SINTEF Two-Phase Flow Laboratory showed that, although the bubble/slug flow regime was described adequately, the stratified/annular regime was not. In vertical annular flow, the predicted pressure drops were up to 50% too high. In horizontal flow, the predicted holdups were too high by a factor of two in extreme cases.

OLGA 2000 (two phase) from Scandpower is a revision of the OLGA model. OLGA 2000 includes improved correlations for wall shear stress, gas entrainment in the liquid layer, and droplet deposition on the pipe wall in stratified flow.

A detailed study of the accuracy of other different correlations was conducted by Baker et al. [24,25] with 107-km 500 NB Marlin pipeline and another 64-km 500 NB pipeline data. The study conclusions were extremely useful to understand the complexity and accuracy of the correlations. Generally, the impact of pressure is not very well established, whereas Baker et al. indicated that pipeline operating pressure is an important parameter in pipeline performance. The high-pressure correlations (high pressure is defined as pressure above 7000 kPaG) are different from low-pressure correlations. The program developed later in this chapter is for general use within plant battery limits and can be considered as low-pressure pipeline. The general comparison of commonly used correlations is presented in Table 2.13 through Table 2.15.

Beggs and Brill correlations showed a consistent tendency seriously to overpredict liquid holdup on all data sets except the low-pressure pipeline.

Table 2.13 High-Pressure Pipeline Performance

Gas Flow MMcum/d	Beggs and Brill (Original)	Beggs and Brill (Revised)	Mukherjee and Brill	No-Slip
		Liquid Holdup (Percentage Error)		
1.4	32	18	22	–78
2.8	62	45	50	–69
4.1	91	74	82	–60
5.5	99	85	92	–54
6.8	109	94	107	–48
8.2	120	107	127	–39
Mean	86	70	80	–58
Standard deviation	33	33	38	14

Table 2.14 Low-Pressure Holdup

Gas flow MMcum/d	Beggs and Brill (Original)	Beggs and Brill (Revised)	Mukherjee and Brill	No-Slip
		Liquid Holdup (Percentage Error)		
1.2	–24	–34	–61	–94
1.2	–40	–47	–71	–96
1.1	–21	–32	–72	–96
1.1	–14	–26	–63	–94
1.1	–9	–21	–59	–93
Mean	–22	–32	–65	–95
Standard deviation	12	10	6	1

Table 2.15 Low-Pressure Pressure Drop

Gas flow MMcum/d	Beggs and Brill (Original)	Beggs and Brill (Revised)	Mukherjee and Brill	No-Slip
		Pressure Loss (Percentage Error)		
1.2	–48	–45	–27	–65
1.2	–55	–55	–41	–70
1.1	–55	–49	–43	–66
1.1	–58	–54	–43	–70
1.1	–50	–41	–23	–63
Mean	–53	–49	–35	–67
Standard deviation	4	6	10	3
Overall mean	–34	5	–12	–24
Overall standard deviation	17	49	21	39

The Beggs and Brill correlation significantly underpredicts pressure loss (34%) due to the assumption of smooth pipe friction factors. The smooth pipe friction factors in the original correlations were substituted with rough pipe friction factors to produce revised Beggs and Brill correlations.

Mukherjee and Brill pressure-loss correlation performed very well at low liquid loading but performed less well on the low-pressure pipeline where liquid loadings were significantly greater.

Beggs and Brill correlations

The following steps are used to estimate the pressure drop using Beggs and Brill correlations.

Step 1: Estimation of flow regime

Calculate the Froude number.

$$N_f = v_m^2/(g * d) \tag{2.104}$$

Calculate factor $X = \ln(\lambda)$

Calculate factors $L1$ and $L2$ as

$$L1 = \exp(4.62 - 3.757\,X - 0.481\,X^2 - 0.0207\,X^3) \tag{2.105}$$

$$L2 = \exp(1.061 - 4.602\,X - 1.609\,X^2 - 0.179\,X^3 + 0.000635\,X^5) \tag{2.106}$$

If

1. $N_f < L1$, the flow pattern is segregated.
2. $N_f > L1$ and $> L2$, the flow pattern is distributed.
3. $L1 < N_f < L2$, the flow pattern is intermittent.

Step 2: Estimation of horizontal holdup

Holdup for horizontal pipe depends on the flow regime and is estimated using the following equations:

Segregated flow:

$$H_L(0) = \frac{0.98\lambda^{0.4846}}{N_f^{\;0.0668}} \tag{2.107}$$

Intermittent flow:

$$H_L(0) = \frac{0.845\lambda^{0.5351}}{N_f^{\;0.0173}} \tag{2.108}$$

Distributed flow:

$$H_L(0) = \frac{1.065\lambda^{0.5824}}{N_f^{\ 0.0609}} \tag{2.109}$$

Step 3: Estimation of uphill holdup

Liquid velocity number

$$N_{Lv} = v_l * \left\{ \frac{\rho_l}{g^2 * 0.00010197\sigma} \right\}^{0.25} \tag{2.110}$$

Calculate the C_+ parameter for a different flow regime.

Segregated flow:

$$C_+ = (1-\lambda)\ln\left\{ \frac{0.011 N_{Lv}^{\ 3.539}}{\lambda^{3.768} N_f^{\ 1.614}} \right\} \tag{2.111}$$

Intermittent flow:

$$C_+ = (1-\lambda)\ln\left\{ \frac{2.96\lambda^{0.305} N_f^{\ 0.0978}}{N_{Lv}^{\ 0.4473}} \right\} \tag{2.112}$$

Distributed flow:

$$C_+ = 0 \tag{2.113}$$

The inclined holdup is calculated as

$$H_L(\phi) = H_L(0) * \left\{ 1 + C_+ \left(\sin(1.8\phi) - 1/3 \sin^3(1.8\phi) \right) \right\} \tag{2.114}$$

Step 4: Estimation of downhill holdup

Calculate the C-parameter (for all flow regimes) as

$$C_- = (1-\lambda)\ln\left\{ \frac{4.7 N_{Lv}^{\ 0.1244}}{\lambda^{0.3692} N_f^{\ 0.5056}} \right\} \tag{2.115}$$

The inclined holdup is calculated as

$$H_L(\phi) = H_L(0) * \left\{1 + C_{-}\left(\sin(1.8\phi) - 1/3\sin^3(1.8\phi)\right)\right\} \tag{2.116}$$

Step 5: Estimation of friction factor

The Friction factor is a function of the Reynolds number. For a two-phase flow, the Reynolds number is defined as

$$N_{\mathrm{Re}} = \frac{\left(\rho_l * \lambda + \rho_g * (1-\lambda)\right) * v_m * d}{\mu_m} \tag{2.117}$$

The normal friction factor is calculated using either Equation 2.39 or Equation 2.40.

Factor y is calculated as

$$y = \frac{\lambda}{\left[H_L(\phi)\right]^2} \tag{2.118}$$

If $1 < y < 1.2$, the factor s is calculated as

$$s = \ln(2.2y - 1.2) \tag{2.119}$$

else,

$$s = \frac{\ln(y)}{\left\{-0.0523 + 3.182\ln(y) - 0.8725\left[\ln(y)\right]^2 + 0.01853\left[\ln(y)\right]^4\right\}} \tag{2.120}$$

The two-phase friction factor is calculated as

$$f_{tp} = f * e^s \tag{2.121}$$

Step 6: Estimation of pressure drop

Mixture flow rate is defined as

$$G_m = \frac{G_l}{a} + \frac{G_v}{a} \tag{2.122}$$

where a = cross section area of the pipe, m^2.

Frictional pressure gradient

$$\left(\frac{dp}{dL}\right)_f = \frac{4f_{tp} * G_m * v_m}{2 * g * (d / 1000)} \tag{2.123}$$

where d = diameter of pipe, mm.

Two-phase density

$$\rho_{tp} = \rho_l * H_L + \rho_g * (1 - H_L) \tag{2.124}$$

Total pressure drop

$$-\frac{dp}{dL} = \frac{\sin\phi * \rho_{tp} + \left(\frac{dp}{dL}\right)_f}{1 - \frac{\rho_{tp} * v_m * v_g}{g * p}} \tag{2.125}$$

Mukherjee and Brill correlations

The following steps are used to calculate the pressure drop using Mukherjee and Brill correlations.

Step 1: Estimation of flow regime [27]

The following procedure is used to establish the flow regime:

Calculate dimensionless numbers.

N_{Lv} as calculated in Equation 2.110

$$N_{gv} = v_g \left\{\frac{\rho_l}{g^2 * 0.00010197\sigma}\right\}^{0.25} \tag{2.126}$$

$$N_L = 0.001 * \mu_l \left\{\frac{1}{g^2 * \rho_l * (0.00010197\sigma)^3}\right\}^{0.25} \tag{2.127}$$

The uphill bubble-slug transition equation is

$$N_{LvBS} = 10^{\left\{\log(N_{gv}) + 0.94 + 0.074\sin\phi - 0.855\sin^2\phi + 3.695N_L\right\}} \tag{2.128}$$

The uphill slug–annular mist transition equation is

$$N_{GvSM} = 10^{\left\{1.401-2.694N_L+0.521{N_{Lv}}^{0.329}\right\}} \tag{2.129}$$

The downhill and horizontal bubble-slug transition equation is

$$N_{GvBS} = 10^{\left\{0.431+1.132\sin\phi-3.003N_L-1.138\log N_{Lv}\sin\phi-0.429(\log N_{Lv})^2\sin\phi\right\}} \tag{2.130}$$

The downhill and horizontal stratified flow boundary is given by

$$N_{LvST} = 10^{\left\{0.321-0.017N_{gv}-4.267\sin\phi-2.972N_L-0.033(\log N_{gv})^2-3.925\sin^2\phi\right\}} \tag{2.131}$$

The following flowchart is used to establish the flow regime (refer to Figure 2.8):

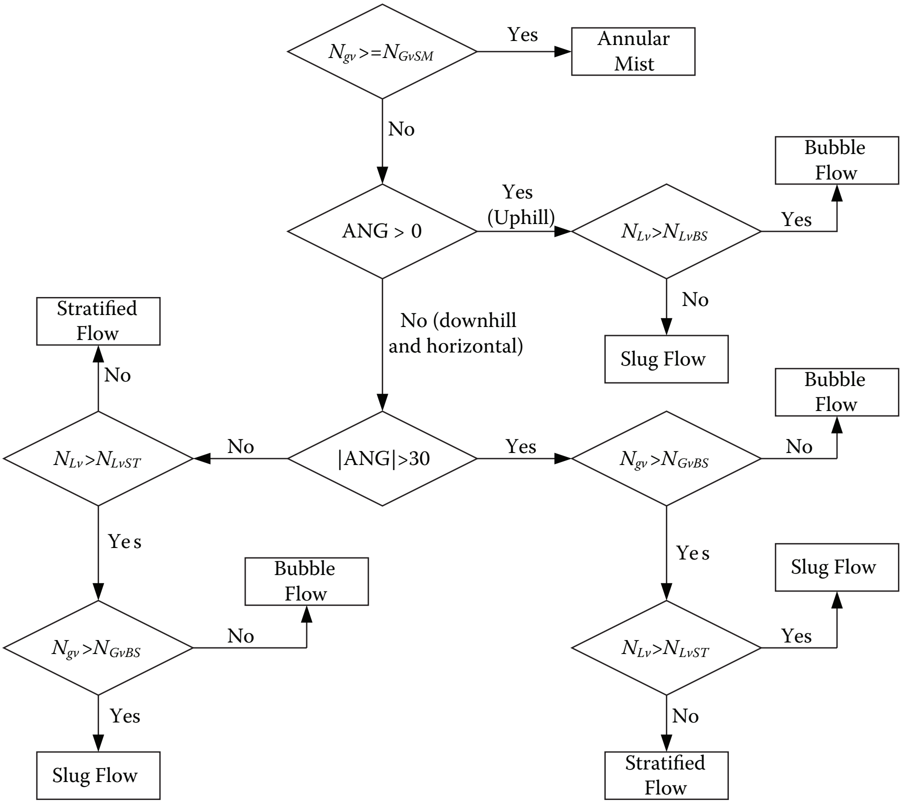


Figure 2.8 Flowchart for determining flow regime.

Table 2.16 Coefficients of Liquid Holdup Equation

Flow Direction	Flow Pattern	*c*1	*c*2	*c*3	*c*4	*c*5	*c*6
Uphill and horizontal flow	All	–0.380113	0.129875	–0.119788	2.343227	0.475686	0.288657
Downhill flow	Stratified	–1.330282	4.808139	4.171584	56.262268	0.079951	0.504887
	Other	–0.516644	0.789805	0.551627	15.519214	0.371771	0.393952

Step 2: Estimation of holdup

The general equation used to estimate liquid holdup is

$$H_L = \exp\left\{\left(c1 + c2\sin\phi + c3\sin^2\phi + c4{N_L}^2\right) * \frac{{N_{gv}}^{c5}}{{N_{Lv}}^{c6}}\right\} \qquad (2.132)$$

Values of coefficients used in Equation 2.132 are presented in Table 2.16.

The liquid viscosity number N_L is calculated using Equation 2.127.

The liquid velocity number is calculated using Equation 2.110, and the gas velocity number is calculated using Equation 2.126.

Step 3: Estimation of hydrostatic head

For stratified flow,

$$\Delta p_h = \gamma_g * L\sin\phi * 0.00981 \qquad (2.133)$$

For other flow,

$$\Delta p_h = \gamma_m * L\sin\phi * 0.00981 \qquad (2.134)$$

where the specific weight of the mixed phase is defined as

$$\gamma_m = H_L * \gamma_l + (1 - H_L) * \gamma_g \qquad (2.135)$$

Step 4: Estimation of acceleration head

For stratified flow,

$$\Delta p_{acl} = 0 \qquad (2.136)$$

For all other flows,

$$\Delta p_{acl} = \frac{\gamma_m * v_m * v_g}{g} * \frac{dp}{p_{av}} * 0.00981 \tag{2.137}$$

where dp/p_{av} is the ratio of differential pressure and average actual pressure. This term is calculated after estimating the frictional pressure drop.

Step 5: Estimation of friction factor

A two-phase Reynolds number is calculated by using Equation 2.117. The friction factor is calculated by using either Equation 2.39 or Equation 2.40 depending upon the Reynolds number.

Step 6: Estimation of frictional pressure drop

Bubble and slug flow. The frictional pressure drop is calculated as follows:

$$\Delta p_f = \frac{4 f_{tp} * L * {v_m}^2 * \gamma_m}{2 * g * d} * 0.00981 \tag{2.138}$$

Stratified flow. Geometric parameters are defined as (refer to Figure 2.9)

$$\delta = 2\cos^{-1}\left(1 - \frac{2h_L}{d}\right) \tag{2.139}$$

$$W_i = 2d\sqrt{\frac{h_L}{d} - \left(\frac{h_L}{d}\right)^2} \tag{2.140}$$

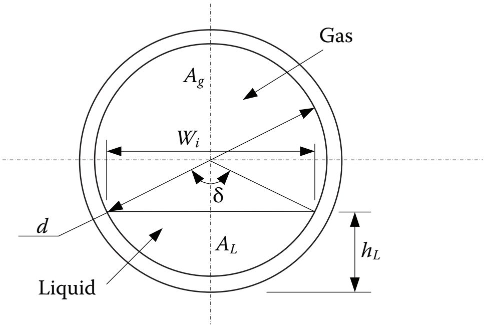


Figure 2.9 Stratified flow model.

$$H_L = \frac{1}{2\pi}\left(\delta - \sin\delta\right) \tag{2.141}$$

The equivalent diameter of the cross section occupied by liquid and gas phase can be calculated as follows:

$$D_{EL} = d\,\frac{\left(\delta - \sin\delta\right)}{\delta + 2\sin(\delta/2)} \tag{2.142}$$

$$D_{Eg} = d\,\frac{2\pi - \left(\delta - \sin\delta\right)}{2\pi - \delta + 2\sin(\delta/2)} \tag{2.143}$$

The gas phase perimeter is calculated as follows:

$$P_g = (1 - \frac{\delta}{2\pi}) * P \tag{2.144}$$

and the total perimeter is

$$P = P_g + P_L \tag{2.145}$$

The wall shear stress for liquid and gas phase can be calculated using the following equations:

$$T_{wL} = \frac{4 f_l \gamma_l v_l^{\,2}}{2g} \tag{2.146}$$

$$T_{wg} = \frac{4 f_g \gamma_g v_g^{\,2}}{2g} \tag{2.147}$$

where f_l and f_g are based on liquid-phase and vapor-phase Reynolds number, respectively.

The frictional pressure drop can be calculated as follows:

$$\Delta p_f = \frac{L}{a} * 0.00981 * \left\{\left(T_{wL}(\pi d - P_g) + T_{wg} P_g\right) + \left(\gamma_l A_L + \gamma_g A_g\right)\sin\phi\right\} \tag{2.148}$$

Procedure.

1. Calculate H_L based on Equation 2.132.
2. With this value of H_L, Equation 2.141 is used to calculate the value of δ.
3. With the value of δ known, h_L/d can be calculated using Equation 2.139, and D_{EL} and D_{Eg} are evaluated from Equation 2.142 and Equation 2.143, respectively.
4. With the value of d known, P, P_g, and P_L are calculated using Equation 2.144 and 2.145.
5. The total frictional pressure loss is calculated by using Equation 2.148.

Annular flow. The ratio of liquid fraction to holdup fraction is defined as follows:

$$H_R = \frac{\lambda}{H_L} \tag{2.149}$$

With the holdup ratio (H_R) known, the value of the friction factor ratio (f_R) can be estimated as follows:

$$\text{If } H_R \le 0.01, \text{ then } f_R = 1.0 \tag{2.150a}$$

$$\text{If } 0.01 < H_R \le 0.2, \text{ then } f_R = -0.1053 {*} H_R + 1.0011 \tag{2.150b}$$

$$\text{If } 0.2 < H_R \le 1, \text{ then } f_R = 2.1863 * H_R^3 - 5.7901 * H_R^2 + 4.2488 * H_R + 0.3561 \tag{2.150c}$$

$$\text{If } H_R > 1, \text{ then } f_R = 1.0 \tag{2.150d}$$

The design friction factor is estimated as follows:

$$f = f_R {*} f_{ns} \tag{2.151}$$

and the frictional pressure drop is

$$\Delta p_f = \frac{4f * \gamma_m * v_m^2 * L}{2 * g * d} * 0.00981 \tag{2.152}$$

Procedure.

1. Calculate H_L using Equation 2.132.

2. Calculate H_R using Equation 2.149.
3. Calculate f_R using Equation 2.150a through Equation 2.150d.
4. Calculate f using Equation 2.151.
5. Calculate the friction pressure drop using Equation 2.152.

CO_2 corrosion [28–31]

CO_2 corrosion is an important parameter, particularly in the design of cross-country pipelines. Although dry CO_2 is noncorrosive, CO_2 in the presence of water forms carbonic acid, which is corrosive to the carbon steel material. The degree of corrosion largely depends on the partial pressure of carbon dioxide, the pH of the flowing fluid, and the temperature of the fluid.

CO_2 corrosion mechanism [28]

In the presence of water, CO_2 forms carbonic acid:

$$H_2O + CO_2 \rightarrow \underset{\text{Carbonic acid}}{H_2CO_3^-} \tag{2.153a}$$

Carbonic acid is a weak acid and dissociates as

$$H_2CO_3 \rightarrow H^+ + HCO_3^- \tag{2.153b}$$

Because the dissociation is weak, the H^+ ion is not a major corrosive species. Although H_2CO_3 is a weak acid, its corrosiveness is due to the cathodic part of the corrosion reaction, which does not need H^+ ions. The direct reduction of undissociated acid is

$$H_2CO_3 + e^- \rightarrow H + HCO_3^- \tag{2.153c}$$

The anodic reaction is

$$Fe \rightarrow Fe^{++} + 2e^- \tag{2.153d}$$

and the overall corrosion reaction is

$$Fe + H_2CO_3 \rightarrow FeCO_3 + H_2 \tag{2.153e}$$

The iron carbonate can form a protective film and thus can reduce the corrosion rate. However, it is extremely difficult to quantify the protective film and estimate the actual protection this film can provide.

NACE requirements [31]

One important check in pipeline design is the checking of NACE (National Association of Corrosion Engineers) requirements. According to Section 1.3.1.1, NACE applies when the partial pressure of H_2S in a wet (water as a liquid) gas phase of a gas, gas condensate, or crude oil system is equal to or exceeds 0.0003 MPa abs (0.05 psia).

Rate of corrosion [29]

The general rate of corrosion depends on temperature. It is estimated as follows:

At 5°C,

$$CR = K_t * f_{CO2}{}^{0.36} * f(pH) \tag{2.154a}$$

At 15°C,

$$CR = K_t * f_{CO2}{}^{0.36} * \left(S / 19\right)^{0.146+0.0324\log(f_{CO2})} * f(pH) \tag{2.154b}$$

At a temperature above 20°C,

$$CR = K_t * f_{CO2}{}^{0.62} * (S / 19)^{0.146+0.0324\log(f_{CO_2})} * f(pH) \tag{2.154c}$$

Constant K_t

The value of K_t is temperature dependent and applicable for a temperature range of 5 to 150°C. If below 5°C, the temperature is assumed as 5°C for the purpose of calculation. Similarly, if the temperature is above 150°C, it is assumed as 150°C.

For the operating temperature range, K_t is defined as follows:

For $5 \leq t < 15$,

$$K_t = 0.117 * t - 0.165 \tag{2.155a}$$

For $15 \leq t < 20$,

$$K_t = 0.6344 * t - 7.926 \tag{2.155b}$$

For $20 \leq t < 40$,

$$K_t = 0.20825 * t + 0.597 \tag{2.155c}$$

For $40 \le t < 60$,

$$K_t = 0.0884 * t + 5.391 \tag{2.155d}$$

For $60 \le t < 80$,

$$K_t = -0.0373 * t + 12.933 \tag{2.155e}$$

For $80 \le t < 90$,

$$K_t = -0.3699 * t + 39.541 \tag{2.155f}$$

For $90 \le t < 120$,

$$K_t = 0.0507 * t + 1.69 \tag{2.155g}$$

For $120 \le t <= 150$,

$$K_t = -0.0856 * t + 18.045 \tag{2.155h}$$

Fugacity of CO_2 [29]

The fugacity of CO_2 is defined as

$$f_{CO2} = a * p_{CO2} \tag{2.156}$$

where

f_{CO2} = fugacity of CO_2, bar
a = fugacity constant, dimensionless
p_{CO2} = partial pressure of CO_2, bar

The fugacity constant depends on pressure and temperature and is defined as follows:

For $p \le 250$ bar,

$$a = 10^{p(0.0031-1.4/T)} \tag{2.157a}$$

For $p > 250$ bar,

$$a = 10^{250(0.0031-1.4/T)} \tag{2.157b}$$

where

p = total pressure, bar
T = temperature, K

Calculation of pH factor f(pH) [29]

Mathematically, pH is defined as a negative logarithm of the concentration of hydrogen ions. The concentration of hydrogen ion is calculated as follows:

$$C_{H^+}{}^3 + C_{O,Bicarb}C_{H^+}{}^2 - (K_H K_0 K_1 p_{CO2} + K_W)C_{H^+} - 2K_H K_0 K_1 K_2 p_{CO2} = 0 \tag{2.158}$$

where

C_{H+} = concentration of hydrogen ion, molar
$C_{O,Bicarb}$ = initial amount of sodium bicarbonate, molar
= $C_{Na^+} - C_{Cl^-}$
K_0 = 0.00258, dimensionless

Other parameters are calculated as follows:

$$K_H = 55.5084 * 10^{-(0.000179P+0.107I)} * e^{-\left(4.8+3934.4/T_K-941290.2/T_K{}^2\right)} \tag{2.159a}$$

(For a temperature range of 0 to 80°C)

$$K_H = 55.5084 * 10^{-\left(0.000179P+0.107I\right)} * e^{-\left(\frac{1713.53*(1-T_K/647)^{0.333}}{T_K}+3.875+3680.09/T_K-1198506.1/T_K{}^2\right)} \tag{2.159b}$$

(For a temperature range of 80 to 200°C)

$$K_1 = 10^{-\left(\begin{array}{l}356.3094+0.06091964T_K-\frac{21834.37}{T_K}-126.8339*\log(T_K)+\frac{168491.5}{T_K{}^2}-2.564*10^{-5}P-0.491I^{0.5}+\\ 0.379I-0.06506I^{1.5}-1.458*10^{-3}I*T_f\end{array}\right)} \tag{2.160a}$$

However, the value of K_1 estimated using the equation given in the previous version of the standard results in a more stable value of *pH*. The equation is as follows:

$$K_1 = 387.6 * 10^{-(6.41-1.594*10^{-3}T_f+8.52*10^{-6}T_f{}^2-3.07*10^{-5}p-0.4772I^{0.5}+0.1180I)} \tag{2.160b}$$

$$K_2 = 10^{-\left(\begin{array}{l}107.8871+0.03252849T_K-\frac{5151.79}{T_K}-38.92561\log(T_K)+\frac{563713.9}{T_K^{\,2}}-2.118*10^{-5}P-1.255I^{0.5}+\\ 0.867I-0.174I^{1.5}-1.588*10^{-3}T_f*I\end{array}\right)} \tag{2.161}$$

$$K_W = 10^{-(29.3868-0.0737549T+7.47881*10^{-5}T^2)} \tag{2.162}$$

where

K_H = equilibrium constant, molar/bar
K_1 = equilibrium constant, molar
K_2 = equilibrium constant, molar
K_W = equilibrium constant, molar^2
T_f = temperature, °F
p = pressure, psia
I = ionic strength, molar
T = temperature, K

The *pH* function *f*(*pH*) is calculated using the equations presented in Table 2.17. These equations are valid for a *pH* range of 3.5 to 6.5 and a temperature range of 5 to 150°C.

Calculation of shear stress [29]

The shear stress is calculated as follows:

$$SS = 0.5 * \rho_m * f * u_m^{\,2} \tag{2.163}$$

where

SS = shear stress, Pa
ρ_m = mixed density, kg/m^3
u_m = mixed velocity, m/sec
f = friction factor, dimensionless

The friction factor is defined as follows:

$$f = 0.001375\left[1+\left\{20000\frac{\varepsilon}{d}+10^6\frac{\mu_m}{\rho_m u_m d}\right\}^{0.33}\right] \tag{2.164}$$

where

ε = pipe roughness, mm
d = pipe diameter, mm
μ_m = mixed viscosity, cP

Table 2.17 pH Function

pH Range	Temperature (°C)	$f(pH)$
$3.5 \le pH < 4.6$	5	$f(pH) = 2.0676 - (0.2309 * pH)$
$4.6 \le pH \le 6.5$		$f(pH) = 4.342 - (1.051 * pH) + (0.0708 * pH^2)$
$3.5 \le pH < 4.6$	15	$f(pH) = 2.0676 - (0.2309 * pH)$
$4.6 \le pH \le 6.5$		$f(pH) = 4.986 - (1.191 * pH) + (0.0708 * pH^2)$
$3.5 \le pH < 4.6$	20	$f(pH) = 2.0676 - (0.2309 * pH)$
$4.6 \le pH \le 6.5$		$f(pH) = 5.1885 - (1.2353 * pH) + (0.0708 * pH^2)$
$3.5 \le pH < 4.6$	40	$f(pH) = 2.0676 - (0.2309 * pH)$
$4.6 \le pH \le 6.5$		$f(pH) = 5.1885 - (1.2353 * pH) + (0.0708 * pH^2)$
$3.5 \le pH < 4.6$	60	$f(pH) = 1.836 - (0.1818 * pH)$
$4.6 \le pH \le 6.5$		$f(pH) = 15.444 - (6.1291 * pH) + (0.8204 * pH^2) - (0.0371 * pH^3)$
$3.5 \le pH < 4.6$	80	$f(pH) = 2.6727 - (0.3636 * pH)$
$4.6 \le pH \le 6.5$		$f(pH) = 331.68 * e^{(-1.2618 * pH)}$
$3.5 \le pH < 4.57$	90	$f(pH) = 3.1355 - (0.4673 * pH)$
$4.57 \le pH < 5.62$		$f(pH) = 21254 * e^{(-2.1811 * pH)}$
$5.62 \le pH \le 6.5$		$f(pH) = 0.4014 - (0.0538 * pH)$
$3.5 \le pH < 4.3$	120	$f(pH) = 1.5375 - (0.125 * pH)$
$4.3 \le pH < 5.0$		$f(pH) = 5.9757 - (1.157 * pH)$
$5.0 \le pH \le 6.5$		$f(pH) = 0.546125 - (0.071225 * pH)$
$3.5 \le pH < 3.8$	150	$f(pH) = 1$
$3.8 \le pH < 5.0$		$f(pH) = 17.634 - (7.0945 * pH) + (0.715 * pH^2)$
$5.0 \le pH \le 6.5$		$f(pH) = 0.037$

Effect of temperature [30]

It is thought that at high temperature the precipitation of $FeCO_3$ (or Fe_3O_4) results in the formation of some protective layer. At a low temperature, this protective layer is easily removed by the flowing liquid. However, at a high temperature, the layer is more stable and less easily washed away. It is assumed that the protection will only be available if the flowing temperature exceeds a certain value called the *scaling temperature*. The scaling temperature is calculated as follows:

$$T_{scale} = 2400/(6.7 + 0.6 \log(f_{CO2})) \tag{2.165}$$

where

T_{scale} = scaling temperature, K
f_{CO2} = fugacity of CO_2, bar

The scale factor is calculated as follows:

$$\log (F_{scale}) = 2400 * (1/T - 1/T_{scale}) \tag{2.166}$$

where

F_{scale} = scale factor, dimensionless
T = flowing temperature, K
T_{scale} = scaling temperature, K

If $T \leq T_{scale}$, $F_{scale} = 1$.

Effect of glycol [30]

Glycol is often added to prevent the formation of hydrate. The presence of glycol also acts on CO_2 corrosion in two ways:

1. By reducing the corrosivity of the water phase it is mixed with
2. By absorbing water from the gas phase

The glycol factor is expressed as follows:

$$\log F_{glyc} = A \log (W) - 2A \tag{2.167}$$

where

F_{glyc} = glycol factor, dimensionless
$A = 1.6$
W = water content in water–glycol mixture, % (w/w)

For $W < 5$, the glycol reduction factor is set to 0.008.

Effect of corrosion inhibitor

The effect of corrosion inhibitor is used directly in the calculation and defined as inhibitor efficiency. If the inhibitor efficiency is 90%, the corrosion rate will be reduced 10-fold.

Condensation factor [30]

Normally, CO_2 corrosion is possible if water is present in the system. It is expected that the nonwet surface of the pipeline (not wet with water) will have no CO_2 corrosion; however, this is not the case. In a cross-country pipeline, water condenses as a result of heat loss, and free water is formed. This free water behaves as a mist and wets the dry surface of the pipe. CO_2 corrosion of the dry area depends on the rate of water condensation and is defined as a condensation factor.

$$F_{cond} = 0.4 * CR, \text{ where } CR < 2.5 \tag{2.168a}$$

$$F_{cond} = 1, \text{ where } CR \geq 2.5 \qquad (2.168\text{ b})$$

where

F_{cond} = condensation factor

CR = water condensation rate, g/(m^2·sec)

The minimum condensation rate is assumed as 0.1.

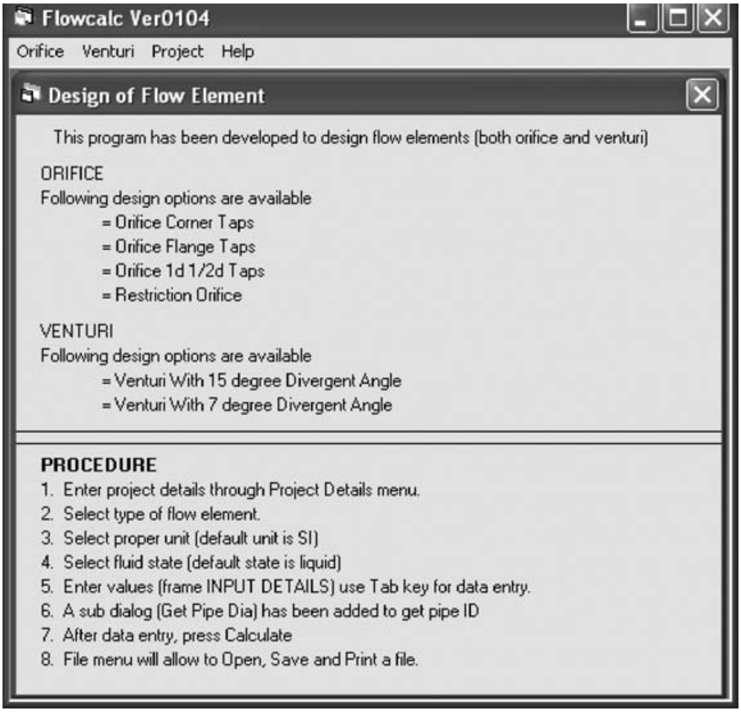
Programming

Program for flow elements

A program to design flow elements, flowcalc.exe, has been developed to design both orifice and Venturi type flow elements. The main form is divided into two frames, "Input Parameters" and "Results." Flow elements can be designed using SI or English units (SI unit by default). The units are fixed, and the program does not allow for change of individual units.

General overview

Double-click flowcalc.exe, and the first form, as shown in Figure 2.10, will be displayed. This form has the following options:



This program has been developed to design flow elements (both orifice and venturi)

ORIFICE

Following design options are available

= Orifice Corner Taps

= Orifice Flange Taps

= Orifice 1d 1/2d Taps

= Restriction Orifice

VENTURI

Following design options are available

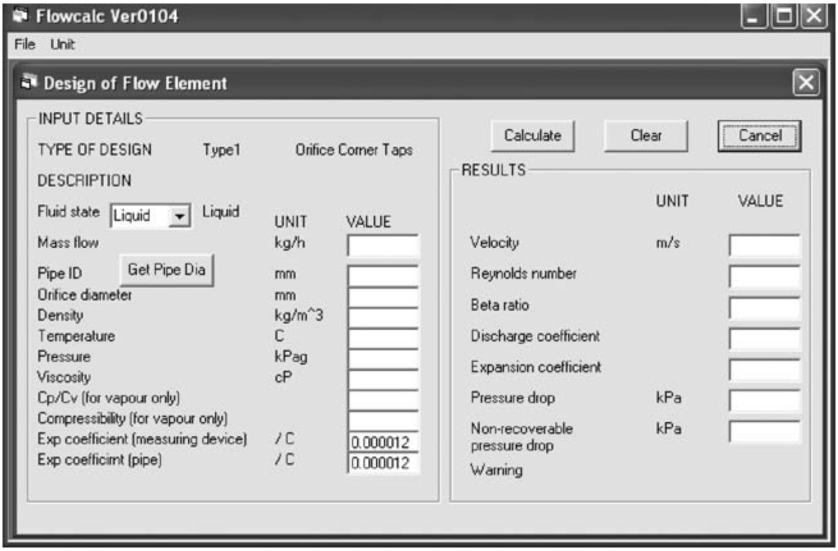
= Venturi With 15 degree Divergent Angle

= Venturi With 7 degree Divergent Angle

PROCEDURE

1. Enter project details through Project Details menu.
2. Select type of flow element.
3. Select proper unit (default unit is SI)
4. Select fluid state (default state is liquid)
5. Enter values (frame INPUT DETAILS) use Tab key for data entry.
6. A sub dialog (Get Pipe Dia) has been added to get pipe ID
7. After data entry, press Calculate
8. File menu will allow to Open, Save and Print a file.

Figure 2.10 Flow element design — main form.



INPUT DETAILS

TYPE OF DESIGN Type1 Orifice Corner Taps

DESCRIPTION

Fluid state Liquid Liquid

	UNIT	VALUE
Mass flow	kg/h	
Pipe ID	mm	
Orifice diameter	mm	
Density	kg/m^3	
Temperature	C	
Pressure	kPag	
Viscosity	cP	
Cp/Cv (for vapour only)		
Compressibility (for vapour only)		
Exp coefficient (measuring device)	/ C	0.000012
Exp coefficimt (pipe)	/ C	0.000012

RESULTS

	UNIT	VALUE
Velocity	m/s	
Reynolds number		
Beta ratio		
Discharge coefficient		
Expansion coefficient		
Pressure drop	kPa	
Non-recoverable pressure drop	kPa	
Warning		

Figure 2.11 Flow element design — liquid.

Orifice: Orifice corner taps
    Orifice flange taps
    Orifice 1*D*–1/2 *D* taps
    Restriction orifice
Venturi: Venturi with 15° divergent angle
    Venturi with 7° divergent angle
Project: Project details

Project details

The form Project Details is used to get project information. Once the details are filled in, press the "OK" button to transfer the information to the main form.

Calculation form

The calculation form is as shown in Figure 2.11. In this form, the available menus are (1) File and (2) Unit. The following options are available through the File menu:

Open: This will open a previously saved file. Only file extension *.flw is acceptable for the File menu.
Save: Once a calculation is done, the Save menu will save the input parameters, including the project details.
Print: Once a calculation is complete, the Print menu will print the output in a standard format.
Exit: Exit the program.

In the Unit menu, the following options are available:

SI Unit: This will set all units in the SI format (this is also the unit by default).
Eng Unit: This will set all units in the English format.

Program limitations and notes

- The program only calculates pressure drop for a certain orifice diameter.
- A few trials are required to achieve a correct orifice diameter.
- For a compressible fluid, different equations are used for critical and noncritical flows. This may result in some mismatch in the calculated flow.
- For a critical flow, the program automatically calculates maximum possible flow for a certain orifice diameter.

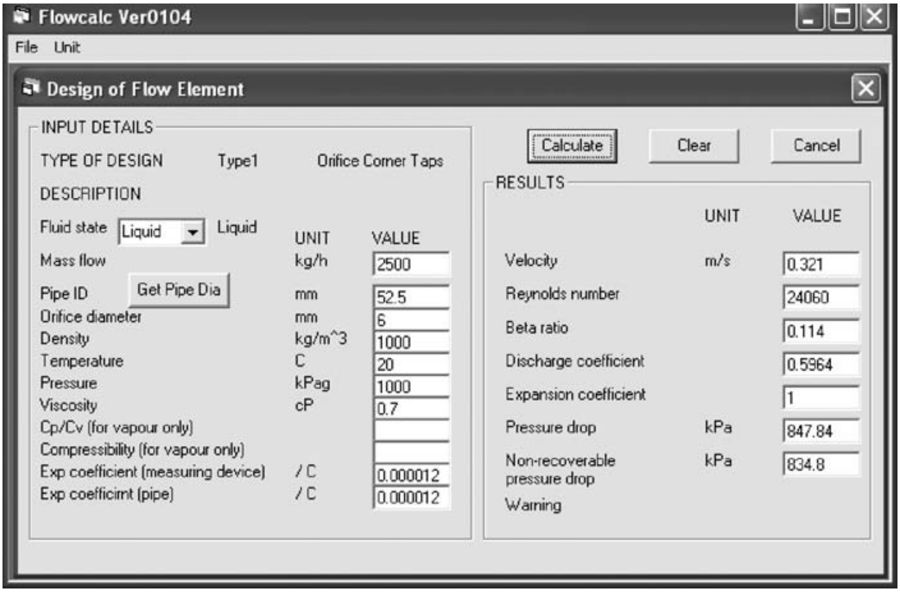
Example 2.3

A liquid is flowing through a 6-mm-diameter orifice with flange taps in a 50 NB sch. 40 pipe. The flow parameters are presented as follows. Estimate the maximum nonrecoverable pressure drop.

Flow rate = 2500 kg/h
Pipe ID = 52.48 mm
Liquid density = 1000 kg/m^3
Liquid viscosity = 0.7 cP
Operating pressure = 1100 kPaA
Operating temperature = 20°C

Solution

Cross section area of pipe = 0.00216 m^2
Volumetric flow = 0.000694 m^3/sec
Velocity = 0.32 m/sec
Reynolds number = 24000
Beta ratio = 0.1143
Pipe diameter = 0.05248 m
C_D (from Equation 2.16) = 0.5964
F_a = Not to be calculated because temperature is 20°C
Expansion factor, $Y = 1$ (liquid)
Orifice area, $a_o = 0.00002827$ m^2
Flow rate, $G = 0.69444$ kg/sec
Differential pressure = 848.5 kPa (from Equation 2.12)
Nonrecoverable pressure drop = 835.4 kPa (from Equation 2.20)



INPUT DETAILS

TYPE OF DESIGN Type1 Orifice Corner Taps

DESCRIPTION

Fluid state Liquid Liquid

	UNIT	VALUE
Mass flow	kg/h	2500
Pipe ID	mm	52.5
Orifice diameter	mm	6
Density	kg/m^3	1000
Temperature	C	20
Pressure	kPag	1000
Viscosity	cP	0.7
Cp/Cv (for vapour only)		
Compressibility (for vapour only)		
Exp coefficient (measuring device)	/ C	0.000012
Exp coefficirnt (pipe)	/ C	0.000012

RESULTS

	UNIT	VALUE
Velocity	m/s	0.321
Reynolds number		24060
Beta ratio		0.114
Discharge coefficient		0.5964
Expansion coefficient		1
Pressure drop	kPa	847.84
Non-recoverable pressure drop	kPa	834.8
Warning		

Figure 2.12 Standard calculation format — Example 2.3.

The calculation is also performed using the standard calculation format as shown in Figure 2.12.

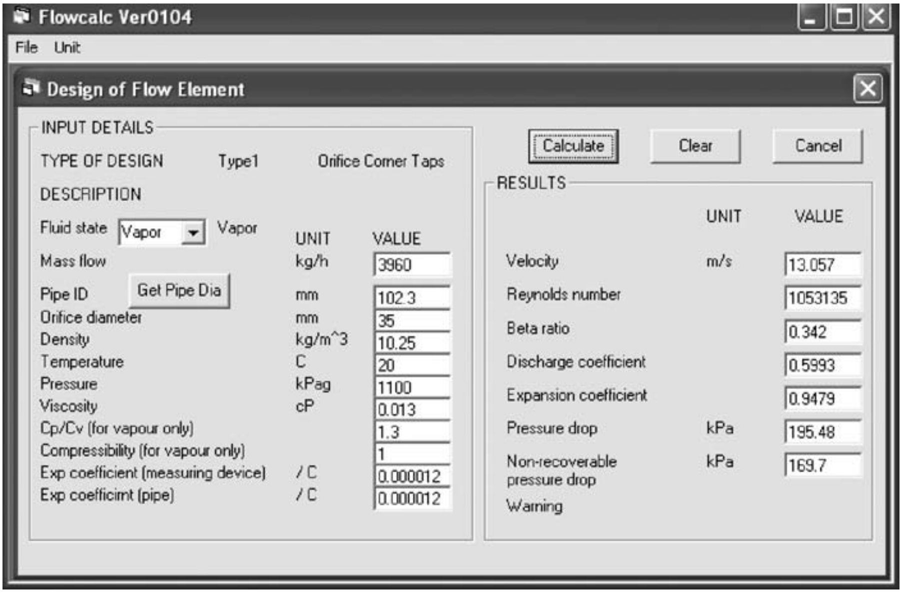
Example 2.4

A compressible gas is flowing through a 100 NB, sch. 40 pipe. A 35-mm restriction orifice is installed to reduce the pressure of the flowing gas. Estimate the pressure drop for the following parameters:

Flow rate = 3960 kg/h
Pipe *ID* = 102.26 mm
Flowing temperature = 20°C
Gas density = 10.25 kg/m^3
Gas viscosity = 0.013 cP
Specific heat ratio = 1.3
Compressibility = 1
Operating pressure = 1200 kPaA

Solution

Expansion coefficient = 0.9548 (calculated through trial and error)
Beta ratio (β) = 0.34226
C_D (from Equation 2.18) = 0.6388
Differential pressure = 169.9 kPa (from Equation 2.21)
Orifice area, a_o = 0.000962 m^2



INPUT DETAILS

TYPE OF DESIGN Type1 Orifice Corner Taps

DESCRIPTION

Fluid state Vapor Vapor

	UNIT	VALUE
Mass flow	kg/h	3960
Pipe ID	mm	102.3
Orifice diameter	mm	35
Density	kg/m^3	10.25
Temperature	C	20
Pressure	kPag	1100
Viscosity	cP	0.013
Cp/Cv (for vapour only)		1.3
Compressibility (for vapour only)		1
Exp coefficient (measuring device)	/C	0.000012
Exp coefficimt (pipe)	/C	0.000012

RESULTS

	UNIT	VALUE
Velocity	m/s	13.057
Reynolds number		1053135
Beta ratio		0.342
Discharge coefficient		0.5993
Expansion coefficient		0.9479
Pressure drop	kPa	195.48
Non-recoverable pressure drop	kPa	169.7
Warning		

Figure 2.13 Standard calculation format — Example 2.4.

Value of Fa = 1 (temperature is 20°C)
Recalculated Y = 0.9526 (recalculated using Equation 2.12)

The difference in the Y value is less than 1%, which is acceptable.

The calculation is also performed using the standard calculation format as presented in Figure 2.13. Both calculations match extremely well.

Example 2.5

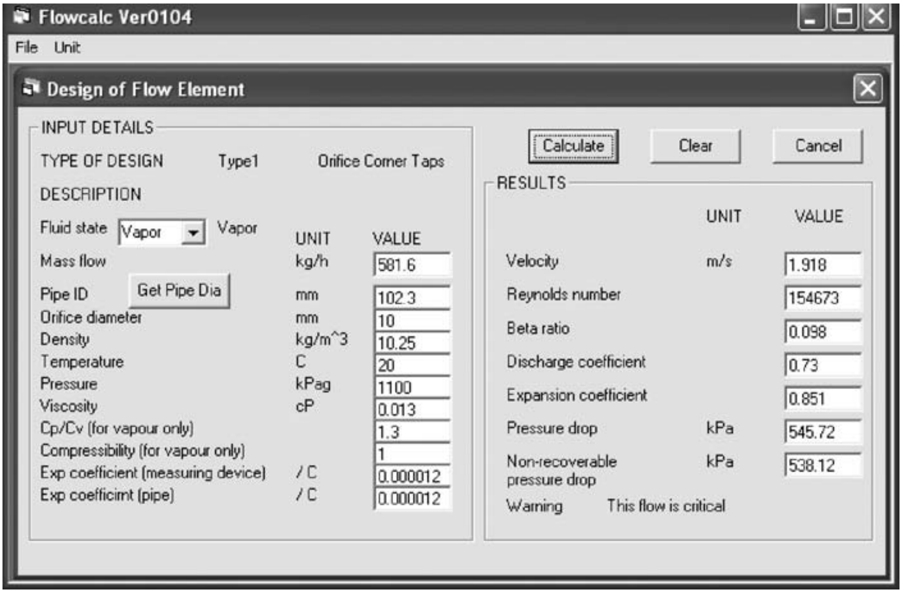
This is the same as Example 2.4 but with the diameter of the restriction orifice 10 mm.

Solution

It is normally assumed, if both Equation 2.12 and Equation 2.21 do not solve till a Y value of 0.67, the flow is critical. In the first trial, the value of Y can be assumed as 0.67.

First Trial

Value of Y = 0.67
Beta ratio (β) = 0.0978
Value of C_D = 0.61 (because β is less than 0.2)
Orifice area, a_o = 0.0000785 m^2
Differential pressure = 57433 kPa



INPUT DETAILS

TYPE OF DESIGN Type1 Orifice Corner Taps

DESCRIPTION

Fluid state Vapor Vapor

	UNIT	VALUE
Mass flow	kg/h	581.6
Pipe ID	mm	102.3
Orifice diameter	mm	10
Density	kg/m^3	10.25
Temperature	C	20
Pressure	kPag	1100
Viscosity	cP	0.013
Cp/Cv (for vapour only)		1.3
Compressibility (for vapour only)		1
Exp coefficient (measuring device)	/C	0.000012
Exp coefficimt (pipe)	/C	0.000012

RESULTS

	UNIT	VALUE
Velocity	m/s	1.918
Reynolds number		154673
Beta ratio		0.098
Discharge coefficient		0.73
Expansion coefficient		0.851
Pressure drop	kPa	545.72
Non-recoverable pressure drop	kPa	538.12

Warning This flow is critical

Figure 2.14 Standard calculation format — Example 2.5.

It is clear the flow is critical and the calculation should be done accordingly.

Second Trial

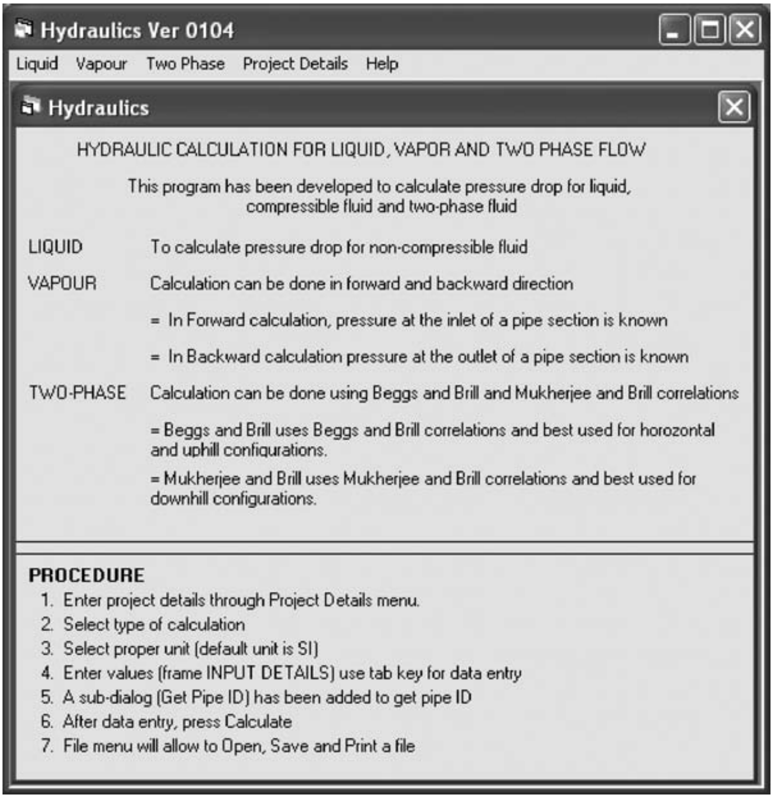
The flow is critical.

Critical pressure ratio, $r_c = 0.5457$ (Equation 2.26)
Pressure, $p_2 = 654.8$ kPa
Pressure drop = 545.2 kPa
$C_D = 0.73$
Expansion factor, $Y = 0.851$
Flow rate = 0.1142 kg/sec (from Equation 2.27)

The calculation is also performed using the standard calculation format as shown in Figure 2.14. Both calculations match extremely well.

Program for hydraulic calculations

A program for hydraulic calculations, hydraulics.exe, has been developed to estimate pressure drop for noncompressible fluids, compressible fluids, and two-phase fluids. The main form has been divided into a number of frames for input and results. Calculations can be performed using either SI or English units (SI unit by default). The units are fixed ones, and the program does not allow for change of individual units.



HYDRAULIC CALCULATION FOR LIQUID, VAPOR AND TWO PHASE FLOW

This program has been developed to calculate pressure drop for liquid, compressible fluid and two-phase fluid

LIQUID To calculate pressure drop for non-compressible fluid

VAPOUR Calculation can be done in forward and backward direction

= In Forward calculation, pressure at the inlet of a pipe section is known

= In Backward calculation pressure at the outlet of a pipe section is known

TWO-PHASE Calculation can be done using Beggs and Brill and Mukherjee and Brill correlations

= Beggs and Brill uses Beggs and Brill correlations and best used for horozontal and uphill configurations.

= Mukherjee and Brill uses Mukherjee and Brill correlations and best used for downhill configurations.

PROCEDURE

1. Enter project details through Project Details menu.
2. Select type of calculation
3. Select proper unit (default unit is SI)
4. Enter values (frame INPUT DETAILS) use tab key for data entry
5. A sub-dialog (Get Pipe ID) has been added to get pipe ID
6. After data entry, press Calculate
7. File menu will allow to Open, Save and Print a file

Figure 2.15 Hydraulic calculation — main form.

General overview

Double-click hydraulics.exe, and the first form, as shown in Figure 2.15, will be displayed. This form has the following options:

Liquid: Hydraulic calculation for noncompressible fluid
Vapor: Forward — where inlet pressure of a section is known
  Backward — where outlet pressure of a section is known
Two-Phase: Using Beggs and Brill correlations. This is preferably used for horizontal and uphill configuration.
  Using Mukherjee and Brill correlations. This is preferably used for downhill configuration.

Project details

The form Project Details is used to get project information. Once the details are filled in, the information is transferred to the main form by pressing the OK button.

Table 2.18 File Extension Used in Hydraulics.exe Program

Type of Calculation	File Extension
Liquid	*.h01
Vapor	*.h02
Two phase	*.h03

The available menus are (1) File and (2) Unit. Only input data are saved or opened. Because each type of hydraulic calculation has a different format, different extensions are used to open or save a file. This is indicated in Table 2.18.

File Save: Once a calculation is done, the "File Save" menu will save the input parameters.

File Print: Once the calculation is complete, the "File Print" menu will print the output.

Exit: Exit the program.

Program limitations and notes

- For compressible flows, the pressure drop can be calculated for both forward and backward directions. However, the forward pressure drop calculation is more reliable than the backward calculation.
- Heat transfer calculation is possible only in forward calculations.
- There are limitations in overall heat transfer calculations for underwater and above-ground pipelines.
- Two-phase pressure drop calculations are not applicable for all flow regimes.

Form incompressible fluid

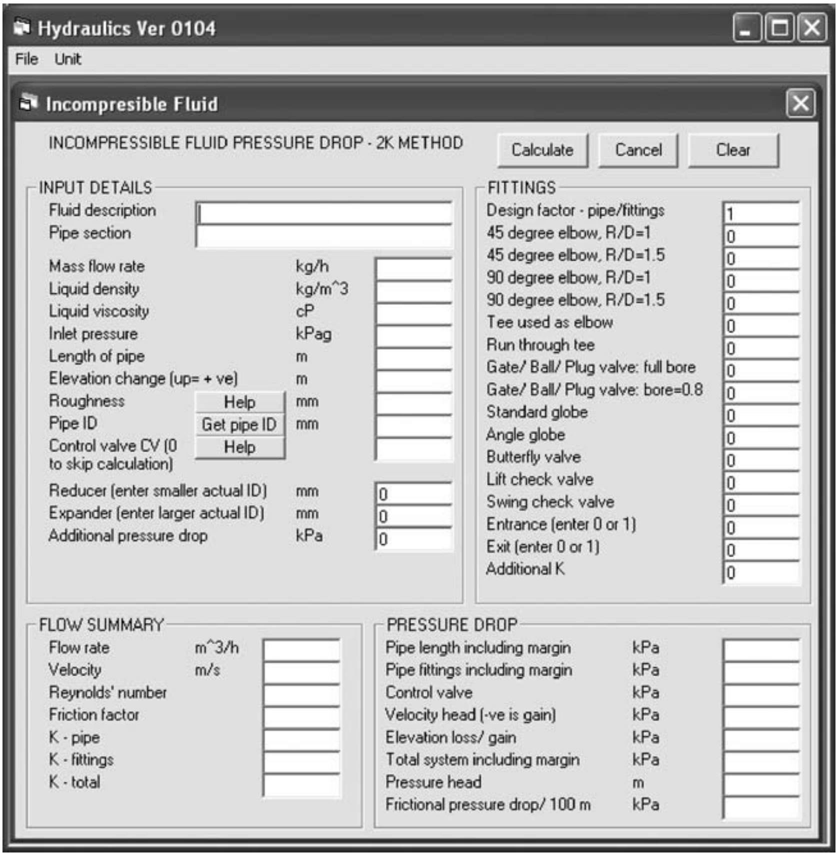
The form is presented in Figure 2.16. This form has four frames. The frames "Input Details" and "Fittings" are used to enter design parameters, whereas the frames "Flow Summary" and "Pressure Drop" are design outputs. The main form has six buttons:

Calculate: This is the calculation button and must be pressed once the data entry is complete.

Cancel: This button is used to cancel the calculation and go back to the main form.

Clear: By pressing this button, all input data are cleared. This will not clear the calculated values.

Roughness: This help button will open a separate form for recommended pipe roughness values. These are suggested values and normally used in calculations.



INCOMPRESSIBLE FLUID PRESSURE DROP - 2K METHOD

INPUT DETAILS

Fluid description		
Pipe section		
Mass flow rate	kg/h	
Liquid density	kg/m^3	
Liquid viscosity	cP	
Inlet pressure	kPag	
Length of pipe	m	
Elevation change (up= + ve)	m	
Roughness	mm	
Pipe ID	mm	
Control valve CV (0 to skip calculation)		
Reducer (enter smaller actual ID)	mm	0
Expander (enter larger actual ID)	mm	0
Additional pressure drop	kPa	0

FITTINGS

Design factor - pipe/fittings	1
45 degree elbow, R/D=1	0
45 degree elbow, R/D=1.5	0
90 degree elbow, R/D=1	0
90 degree elbow, R/D=1.5	0
Tee used as elbow	0
Run through tee	0
Gate/ Ball/ Plug valve: full bore	0
Gate/ Ball/ Plug valve: bore=0.8	0
Standard globe	0
Angle globe	0
Butterfly valve	0
Lift check valve	0
Swing check valve	0
Entrance (enter 0 or 1)	0
Exit (enter 0 or 1)	0
Additional K	0

FLOW SUMMARY

Flow rate	m^3/h	
Velocity	m/s	
Reynolds' number		
Friction factor		
K - pipe		
K - fittings		
K - total		

PRESSURE DROP

Pipe length including margin	kPa	
Pipe fittings including margin	kPa	
Control valve	kPa	
Velocity head (-ve is gain)	kPa	
Elevation loss/ gain	kPa	
Total system including margin	kPa	
Pressure head	m	
Frictional pressure drop/ 100 m	kPa	

Figure 2.16 Incompressible fluid form.

Pipe *ID*: Get Pipe *ID* button will open a separate form for calculation of pipe *ID* and *OD*. After the calculation of pipe diameter, the values are transferred to the main calculation form by pressing OK.

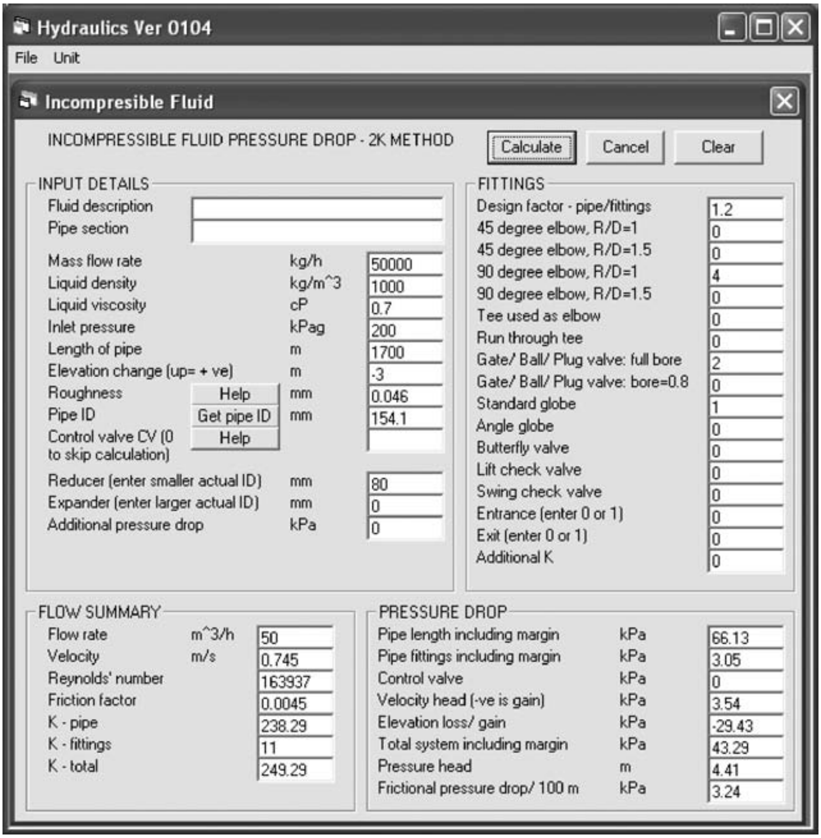
Control valve *CV*: This from is used for suggested values of control valve *CV*. If control valve *CV* is entered, the system will calculate the pressure drop through the control valve for a particular *CV*. This is generally not recommended because the opening of control valve depends on the control signal and the valve *CV* varies with flow rate. It is often a better idea to use a fixed pressure drop through the control valve for variable flow rates. In actual practice, pressure drop through the control valve decreases with the increase in flow rate.

Example 2.6

Calculate the pressure drop through a 1,700-m-long pipe with water flowing at a rate of 50,000 kg/h at an inlet pressure of 200 kPaG. Other parameters are as follows:

Design margin	20%
Pipe nominal bore	150 NB (sch. 40)
Change in elevation	–3 m
Welded elbow, 90° (R/D = 1)	4
Gate valve	2
Globe valve	1
Reducer	80 mm
Results	see Figure 2.17

Flow Summary shows the values of *K*-pipe and *K*-fittings. These are the indications of pressure drop contribution by the pipe and fittings. In the Pressure Drop frame, there are a few pressure drop calculations, which include the contribution of static pressure drop (elevation loss or gain), velocity head (change in velocity due to reducer or expander), and control valve. The result also shows the frictional pressure drop per 100 m. This is sometimes very useful, because most operating companies establish some basic value of pressure drop per 100 m.



INCOMPRESSIBLE FLUID PRESSURE DROP - 2K METHOD

INPUT DETAILS

Fluid description		
Pipe section		
Mass flow rate	kg/h	50000
Liquid density	kg/m^3	1000
Liquid viscosity	cP	0.7
Inlet pressure	kPag	200
Length of pipe	m	1700
Elevation change (up= + ve)	m	-3
Roughness	mm	0.046
Pipe ID	mm	154.1
Control valve CV (0 to skip calculation)		
Reducer (enter smaller actual ID)	mm	80
Expander (enter larger actual ID)	mm	0
Additional pressure drop	kPa	0

FITTINGS

Design factor - pipe/fittings	1.2
45 degree elbow, R/D=1	0
45 degree elbow, R/D=1.5	0
90 degree elbow, R/D=1	4
90 degree elbow, R/D=1.5	0
Tee used as elbow	0
Run through tee	0
Gate/ Ball/ Plug valve: full bore	2
Gate/ Ball/ Plug valve: bore=0.8	0
Standard globe	1
Angle globe	0
Butterfly valve	0
Lift check valve	0
Swing check valve	0
Entrance (enter 0 or 1)	0
Exit (enter 0 or 1)	0
Additional K	0

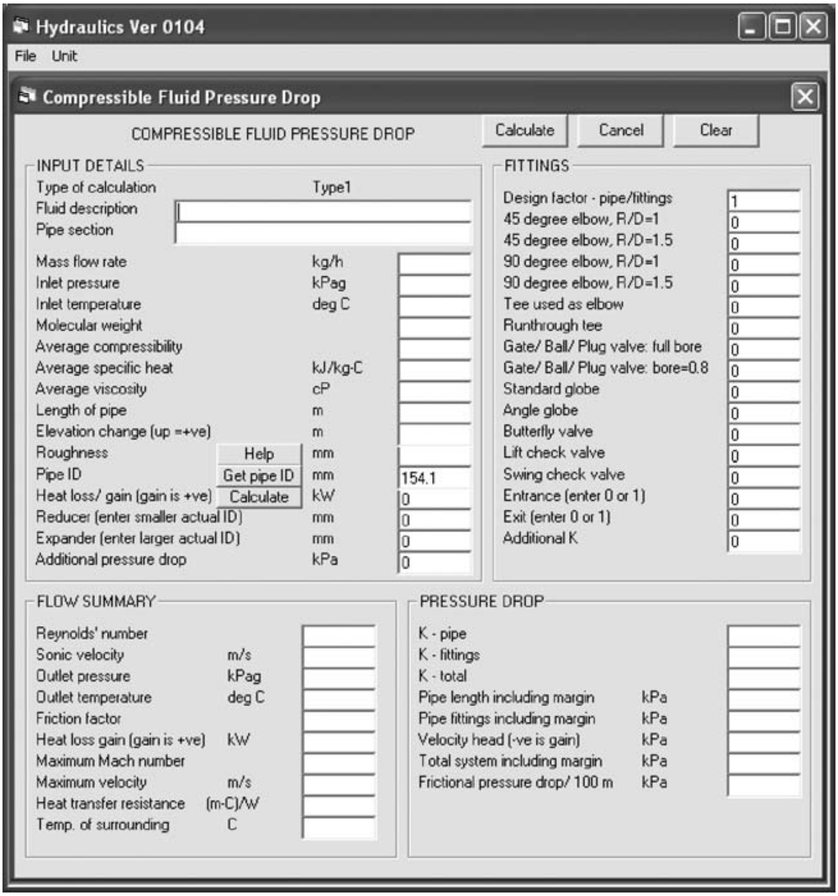
FLOW SUMMARY

Flow rate	m^3/h	50
Velocity	m/s	0.745
Reynolds' number		163937
Friction factor		0.0045
K - pipe		238.29
K - fittings		11
K - total		249.29

PRESSURE DROP

Pipe length including margin	kPa	66.13
Pipe fittings including margin	kPa	3.05
Control valve	kPa	0
Velocity head (-ve is gain)	kPa	3.54
Elevation loss/ gain	kPa	-29.43
Total system including margin	kPa	43.29
Pressure head	m	4.41
Frictional pressure drop/ 100 m	kPa	3.24

Figure 2.17 Incompressible fluid calculation.



Compressible Fluid Pressure Drop

COMPRESSIBLE FLUID PRESSURE DROP

INPUT DETAILS

Type of calculation	Type1	
Fluid description		
Pipe section		
Mass flow rate	kg/h	
Inlet pressure	kPag	
Inlet temperature	deg C	
Molecular weight		
Average compressibility		
Average specific heat	kJ/kg-C	
Average viscosity	cP	
Length of pipe	m	
Elevation change (up =+ve)	m	
Roughness	mm	
Pipe ID	mm	154.1
Heat loss/ gain (gain is +ve)	kW	0
Reducer (enter smaller actual ID)	mm	0
Expander (enter larger actual ID)	mm	0
Additional pressure drop	kPa	0

FITTINGS

Design factor - pipe/fittings	1
45 degree elbow, R/D=1	0
45 degree elbow, R/D=1.5	0
90 degree elbow, R/D=1	0
90 degree elbow, R/D=1.5	0
Tee used as elbow	0
Runthrough tee	0
Gate/ Ball/ Plug valve: full bore	0
Gate/ Ball/ Plug valve: bore=0.8	0
Standard globe	0
Angle globe	0
Butterfly valve	0
Lift check valve	0
Swing check valve	0
Entrance (enter 0 or 1)	0
Exit (enter 0 or 1)	0
Additional K	0

FLOW SUMMARY

Reynolds' number		
Sonic velocity	m/s	
Outlet pressure	kPag	
Outlet temperature	deg C	
Friction factor		
Heat loss gain (gain is +ve)	kW	
Maximum Mach number		
Maximum velocity	m/s	
Heat transfer resistance	(m-C)/W	
Temp. of surrounding	C	

PRESSURE DROP

K - pipe		
K - fittings		
K - total		
Pipe length including margin	kPa	
Pipe fittings including margin	kPa	
Velocity head (-ve is gain)	kPa	
Total system including margin	kPa	
Frictional pressure drop/ 100 m	kPa	

Figure 2.18 Compressible fluid form.

Form compressible fluid

The structure of the form, data entry, and other facilities are as discussed before (Figure 2.18). However, this form is unique for its ability to calculate the heat loss/gain. In the calculation of compressible fluids, the temperature has a significant impact on pressure drop. With a decrease in temperature, the gas density increases, resulting in a lower flow velocity. The pressure drop varies as a square of velocity and thus a large change in pressure drop is possible for some change in temperature. The impact of heat loss or gain may not be significant for battery limit piping design (within the plant); however, for a long-distance pipeline, the impact of heat loss or gain cannot be ignored.

A separate form has been introduced to calculate the heat transfer resistance. The heat transfer resistance calculation form is presented in Figure 2.19. In the "Input Details" frame of the main form, it is possible to enter the heat loss/gain (gain is +ve) value. The design is structured as follows:

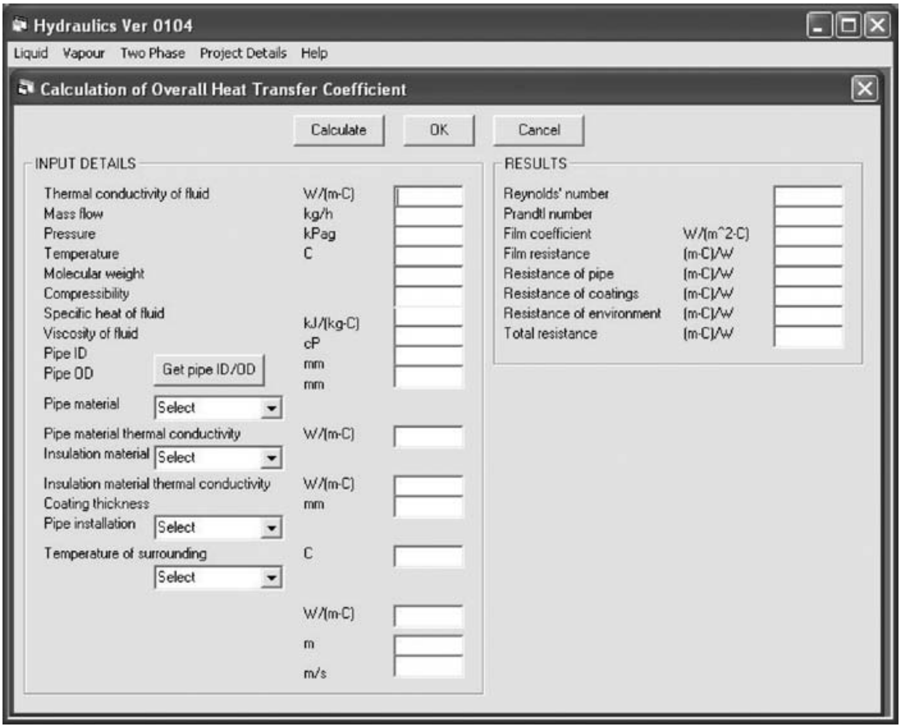


Figure 2.19 Calculation of heat transfer resistance.

- Enter 0 in the heat loss/gain cell in Figure 2.18 (0 is entered by default). In this situation, the heat loss/gain button can be pressed to calculate the heat transfer resistance; alternatively, just do nothing to let the program know that heat loss/gain calculation is not required.
- Add the heat loss/gain value. If this is nonzero, the program will assume the value as calculated externally and use it to estimate the temperature changes in the pipeline.
- Input of elevation change (up = +ve) needs to be established carefully, particularly when the calculation is performed in a backward direction. The change of elevation is always to be considered in the direction of flow.

A few points must be noted to fully understand the heat gain/loss calculation. These are explained as follows:

- Normally, the calculation output is outlet temperature in the frame Flow Summary. If this calculation is not performed, the outlet temperature will be indicated as 0, which is not a calculation output.
- Sometimes the heat loss/gain calculation can diverge to an abnormal value, which will be indicated by some abnormal value of outlet

temperature. If this happens, the designer should calculate the maximum possible heat loss/gain value and enter the data manually in frame Input Details.

- It should be noted that normally the outlet temperature should not cross the surrounding temperature limit, and the maximum heat gain/loss can be calculated easily by multiplying the gas flow rate with the specific heat of the gas and the difference between the fluid inlet temperature and the surrounding temperature.
- The heat gain/loss calculation has been disabled for backward calculation of pressure drop.

Information ranging from mass flow to pipe *OD* is available from the main form, and it is recommended that the values are not changed. Other values, including the thermal conductivity of the fluid, are to be entered manually by the designer. A large number of options are available for pipe material, insulation material, pipe installation, and type of soils, and the program will select a default value once a particular option is selected. If required, the default values can be changed by the designer. The structure of the Input Details frame changes depending on the selection of pipe installation as buried, submerged, or above ground. Once the data entry is complete, "calculate" is pressed to obtain the value of total resistance. It should be noted that the purpose of this form is to calculate the value of total resistance, not the actual heat loss/gain. After the calculation, the OK button is pressed to transfer the value to the main form. The cancel button will cancel the calculation and return to the main form.

Example 2.7

Design a gas lift line for the following parameters. The fluid is a high-pressure sales gas without any free water or formation liquid.

- Length of pipe = 13930 m (200 pipe segments due to hilly terrain according to pipe survey data)
- Inlet pressure = 15040 kPaA
- Inlet temperature = 50°C
- Line size = 100 NB, sch. 80
- Flow rate = 11770 kg/h (12 MMscfd)
- Average viscosity = 0.0184
- Average specific heat = 3.3465
- Molecular weight = 19.7
- Average compressibility = 0.7359
- Overall change in elevation = 737 m
- Heat loss from the system (calculated through HYSYS®) = 238.5 kW

The designer decided to solve the problem using the most advanced method available. He employed PIPESYS available within HYSYS®, and the method selected was OLGA. He had to spend long hours to enter the pipe profile (200 pipe segments). Because of the large number of segments, the computation time was also high. Moreover, with the very new token philosophy of HYSYS®, access to PIPESYS was a problem. However, the designer was able to work out the total pressure drop.

Although there was no direct method to actually calculate the total cost of the exercise, it can be expected that total man-hour and computation costs to perform that type of calculation would be quite high. This is a typical case of a calculation done without an understanding of the background and overall philosophy. The designer could have done this calculation with much less computing and man-hour time by modifying the calculation philosophy, for example, as follows:

- OLGA correlations are typically used for two-phase calculations. For single-phase calculation, selection of OLGA correlations (which require high tokens) adds no extra value to the calculation. Most designers ignore this fact and end up spending more expensive computation time.
- Only overall elevation difference is required for single-phase pressure drop calculations. Instead of entering 200 pipe segments, the designer should enter only total pipe length and overall elevation difference.

This problem has been solved in the compressible fluid program here and is presented in Figure 2.20.

Pressure drop comparison

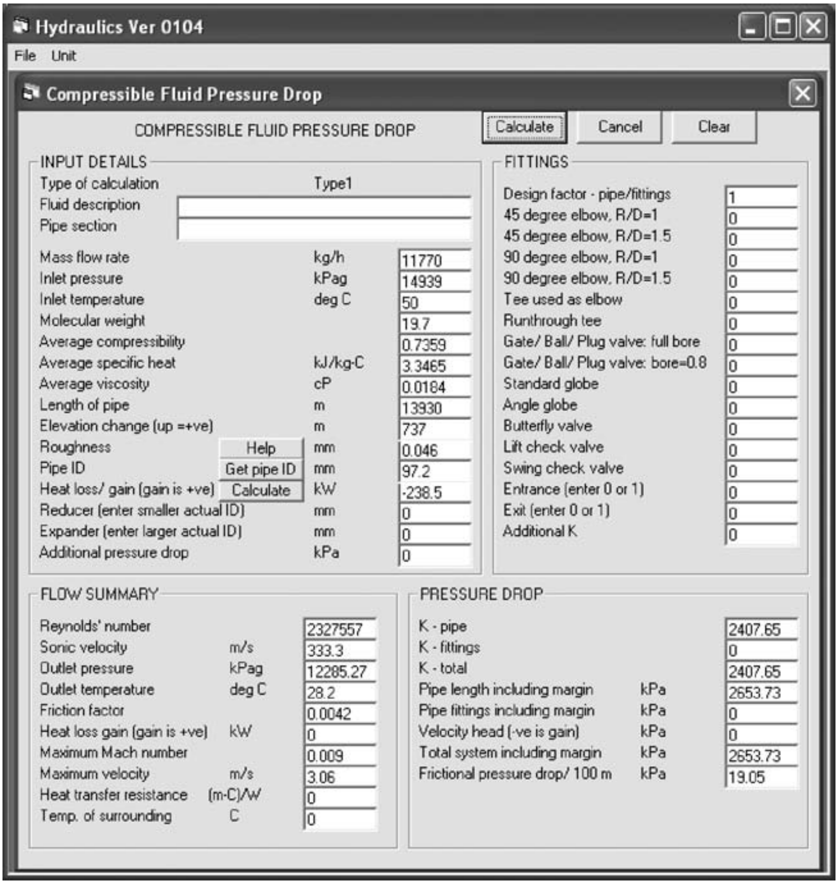
Pressure drop calculated by HYSYS® = 2650 kPa
Pressure drop calculated with this program = 2654 kPa

Compressible fluid program performance has also been checked using the HYSYS® simulation package and the results are presented Table 2.19.

Form for two-phase flow

The structure of the form is as shown in Figure 2.21. The calculation can be done for two correlations: (1) Beggs and Brill and (2) Mukherjee and Brill. The existing calculation method uses only the vapor density as entered by the designer. The program does not calculate vapor density from the user input, as is the case for compressible fluid calculations. Therefore, the data entered in the cells inlet temperature, and vapor molecular weight will not be used for the calculation, and the designer may omit these cells.

Unlike previous calculations, the two-phase calculation is extremely complex, and no single correlation can cover the entire range of calculations. The major problem with two-phase calculation is in regard to the correct prediction of liquid holdup; in some cases, the calculated liquid holdup is



COMPRESSIBLE FLUID PRESSURE DROP

INPUT DETAILS

Type of calculation		Type1
Fluid description		
Pipe section		
Mass flow rate	kg/h	11770
Inlet pressure	kPag	14939
Inlet temperature	deg C	50
Molecular weight		19.7
Average compressibility		0.7359
Average specific heat	kJ/kg-C	3.3465
Average viscosity	cP	0.0184
Length of pipe	m	13930
Elevation change (up =+ve)	m	737
Roughness	mm	0.046
Pipe ID	mm	97.2
Heat loss/ gain (gain is +ve)	kW	-238.5
Reducer (enter smaller actual ID)	mm	0
Expander (enter larger actual ID)	mm	0
Additional pressure drop	kPa	0

FITTINGS

Design factor - pipe/fittings	1
45 degree elbow, R/D=1	0
45 degree elbow, R/D=1.5	0
90 degree elbow, R/D=1	0
90 degree elbow, R/D=1.5	0
Tee used as elbow	0
Runthrough tee	0
Gate/ Ball/ Plug valve: full bore	0
Gate/ Ball/ Plug valve: bore=0.8	0
Standard globe	0
Angle globe	0
Butterfly valve	0
Lift check valve	0
Swing check valve	0
Entrance (enter 0 or 1)	0
Exit (enter 0 or 1)	0
Additional K	0

FLOW SUMMARY

Reynolds' number		2327557
Sonic velocity	m/s	333.3
Outlet pressure	kPag	12285.27
Outlet temperature	deg C	28.2
Friction factor		0.0042
Heat loss gain (gain is +ve)	kW	0
Maximum Mach number		0.009
Maximum velocity	m/s	3.06
Heat transfer resistance	(m-C)/W	0
Temp. of surrounding	C	0

PRESSURE DROP

K - pipe		2407.65
K - fittings		0
K - total		2407.65
Pipe length including margin	kPa	2653.73
Pipe fittings including margin	kPa	0
Velocity head (-ve is gain)	kPa	0
Total system including margin	kPa	2653.73
Frictional pressure drop/ 100 m	kPa	19.05

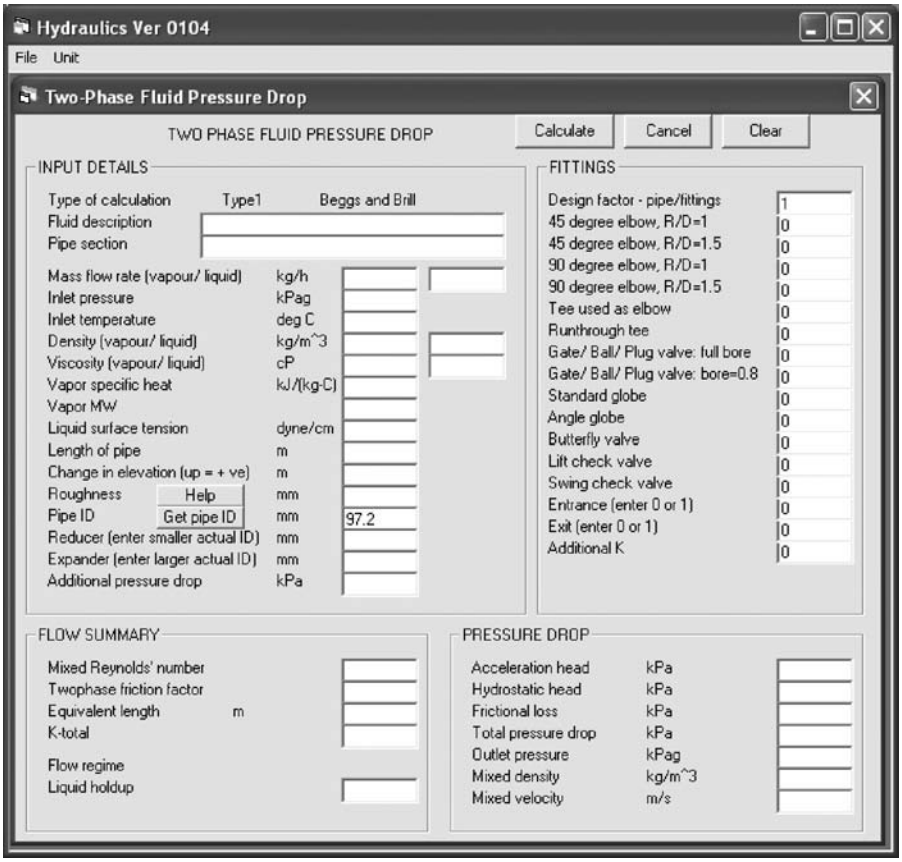
Figure 2.20 Example 2.7.

to some extent not realistic. In today's simulation environment, the most authentic two-phase correlations have been developed by Scandpower, and the simulation package HYSYS® has a license to use the correlation in its OLGA correlation package. The adequacy of the program developed here will be checked with the OLGA results for (1) horizontal pipes, (2) uphill pipes, and (3) downhill pipes.

It should be noted that this two-phase correlation has not been developed for cross-country pipelines. This program is to be used to design plant battery limit piping. This calculation does not attempt to establish the impact of temperature (heat loss/gain) simply because the impact can only be established once the full fluid dynamics are available. For example, if there is heat loss, some liquid will condense, and vapor and liquid physical properties will change. It is not possible to calculate these without full fluid properties.

Table 2.19 Program Performance

		Program Input					Program Output	
Case Number	Method	Flow (kg/h)	Length (m)	Temperature (°C)	Pipe ID (mm)	Heat (kW)	Pressure Drop (kPa)	Temperature (°C)
1	HYSYS®	30,000	100	50	254.5	0	9.74	50
	Program	30,000	100	50	254.5	0	9.83	50
2	HYSYS®	30,000	1,000	50	254.5	0	101.6	50
	Program	30,000	1,000	50	254.5	0	102.7	50
3	HYSYS®	16,000	10,000	50	254.5	0	331.4	50
	Program	16,000	10,000	50	254.4	0	327.2	50
4	HYSYS®	16,000	100,000	50	477.8	0	122.3	50
	Program	16,000	100,000	50	477.8	0	122.8	50
5	HYSYS®	10,000	500,000	50	477.8	0	269.2	50
	Program	10,000	500,000	50	477.8	0	265.1	50
6	HYSYS®	10,000	1,000	50	202.7	200	35.2	14.8
	Program	10,000	1,000	50	202.7	200	34.2	15.5
7	HYSYS®	10,000	1,000	70	202.7	–100	77.6	86.7
	Program	10,000	1,000	70	202.7	–100	77.6	87.9
8	HYSYS®	10,000	2,000	70	202.7	–200	174.2	103.1
	Program	10,000	2,000	70	202.7	–200	174.3	105.8



TWO PHASE FLUID PRESSURE DROP

INPUT DETAILS

Type of calculation Type1 Beggs and Brill

Fluid description

Pipe section

Mass flow rate (vapour/ liquid)	kg/h		
Inlet pressure	kPag		
Inlet temperature	deg C		
Density (vapour/ liquid)	kg/m^3		
Viscosity (vapour/ liquid)	cP		
Vapor specific heat	kJ/(kg-C)		
Vapor MW			
Liquid surface tension	dyne/cm		
Length of pipe	m		
Change in elevation (up = + ve)	m		
Roughness	mm		
Pipe ID	mm	97.2	
Reducer (enter smaller actual ID)	mm		
Expander (enter larger actual ID)	mm		
Additional pressure drop	kPa		

FITTINGS

Design factor - pipe/fittings	1
45 degree elbow, R/D=1	0
45 degree elbow, R/D=1.5	0
90 degree elbow, R/D=1	0
90 degree elbow, R/D=1.5	0
Tee used as elbow	0
Runthrough tee	0
Gate/ Ball/ Plug valve: full bore	0
Gate/ Ball/ Plug valve: bore=0.8	0
Standard globe	0
Angle globe	0
Butterfly valve	0
Lift check valve	0
Swing check valve	0
Entrance (enter 0 or 1)	0
Exit (enter 0 or 1)	0
Additional K	0

FLOW SUMMARY

Mixed Reynolds' number		
Twophase friction factor		
Equivalent length	m	
K-total		
Flow regime		
Liquid holdup		

PRESSURE DROP

Acceleration head	kPa	
Hydrostatic head	kPa	
Frictional loss	kPa	
Total pressure drop	kPa	
Outlet pressure	kPag	
Mixed density	kg/m^3	
Mixed velocity	m/s	

Figure 2.21 Two-phase fluid form.

Horizontal pipe section

Table 2.20 compares the outcomes of pressure drop calculations using OLGA, Beggs and Brill, and Mukherjee and Brill correlations. This comparison has been made for the following parameters:

Length of pipe = 4,000 m
Change in elevation = 0 m
Pipe *ID* = 477.8 mm
Total flow range = 10,000 to 200,000 kg/h
Liquid mole fraction = 0.0034 to 0.8

It is clear from Table 2.20 that, in comparison to the OLGA correlation, the pressure drop as calculated by Beggs and Brill is conservative, whereas Mukherjee and Brill underestimate the pressure drop. The Beggs and Brill correlation can be used without major concern to calculate pressure drop for horizontal pipe segments.

Table 2.20 Horizontal Pipe Pressure Drop

Case Number	Correlations	Pressure Drop, kPa Frictional	Acceleration	Total	Regime
1	OLGA	70.9	0	70.9	Stratified
	Beggs and Brill	94.1	0	94.1	Segregated
	Mukherjee and Brill	63.5	0	63.5	Annular
2	OLGA	235.6	1	236.6	Stratified
	Beggs and Brill	256.5	0	256.5	Segregated
	Mukherjee and Brill	170.9	0	170.9	Annular
3	OLGA	30.4	0	30.4	Stratified
	Beggs and Brill	34.4	0	34.4	Segregated
	Mukherjee and Brill	28.2	0	28.2	Annular
4	OLGA	2.0	0	2.0	Stratified
	Beggs and Brill	2.7	0	2.7	Segregated
	Mukherjee and Brill	7.8	0	7.8	Stratified
5	OLGA	2.4	0	2.4	Stratified
	Beggs and Brill	8.0	0	8.0	Segregated
	Mukherjee and Brill	3.5	0	3.5	Stratified
6	OLGA	15.0	0	15.0	Stratified
	Beggs and Brill	45.7	0	45.7	Segregated
	Mukherjee and Brill	34.7	0	34.7	Slug
7	OLGA	84.3	0	84.3	Slug
	Beggs and Brill	113.5	0	113.5	Segregated
	Mukherjee and Brill	78.0	0	78.0	Slug
8	OLGA	173.9	0	173.9	Slug
	Beggs and Brill	202.6	0	202.6	Segregated
	Mukherjee and Brill	138.7	0	138.7	Slug

Uphill pipe section

Uphill analysis is always very complex because of the influence of liquid holdup. Table 2.21 presents the comparison among different methods for both inclined and vertical pipe sections. The following parameters have been used for the comparison:

Description	Uphill Inclined	Uphill Vertical
Length of pipe, m	4,000	15
Change in elevation, m	–100	–15
Pipe diameter, mm	477.8	102.3
Flow range, kg/h	10,000 to 100,000	10,000 to 30,000
Liquid mole fraction	0.0035 to 0.76	0.0035 to 0.8

Where the liquid loading is very low, OLGA predicts high pressure drop in comparison to other methods (cases 1 and 2). This high pressure drop is mainly due to the static pressure drop. An interesting calculation in the OLGA method is the liquid drop time. This dropout time indicates the time required to achieve such a situation. For example, case 1 has a static pressure

Table 2.21 Uphill Pressure Drop

Case Number	Correlations	Liquid Drop Time (h)	Pressure Drop (kPa)		
			Frictional	Static	Total
	Uphill Inclined				
1	OLGA	6376	–65.0	332.2	267.2
	Beggs and Brill		2.6	14.5	17.1
	Mukherjee and Brill		2.0	6.9	8.9
2	OLGA	1789	13.7	189.9	203.7
	Beggs and Brill		10	13.8	23.8
	Mukherjee and Brill		7.5	6.9	14.4
3	OLGA	1	71.4	7.2	78.7
	Beggs and Brill		85.8	13	98.8
	Mukherjee and Brill		63.5	6.9	70.4
4	OLGA	–1	236.9	6.4	244.4
	Beggs and Brill		233.3	12.7	246
	Mukherjee and Brill		170.7	6.9	177.8
5	OLGA	23	–88.2	486.5	398.3
	Beggs and Brill		2.5	214	216.5
	Mukherjee and Brill		2	207.8	209.8
6	OLGA	8	–68.6	352.5	284.0
	Beggs and Brill		9.5	197.2	206.7
	Mukherjee and Brill		7.1	168.7	175.8
7	OLGA	4	–13.1	265.5	252.5
	Beggs and Brill		20.8	188.1	208.9
	Mukherjee and Brill		15.5	147.5	163
8	OLGA	2	80.6	174.6	255.2
	Beggs and Brill		56.7	177.6	234.3
	Mukherjee and Brill		41.9	164.7	206.6
	Uphill Vertical				
9	OLGA		1.2	0.4	26.3
	Beggs and Brill		44.3	1.3	45.6
	Mukherjee and Brill		19.4	1.0	20.5
10	OLGA		256.8	1.0	318.1
	Beggs and Brill		505.0	1.4	506.4
	Mukherjee and Brill		174.6	1.0	182.6
11	OLGA		1.0	33.3	34.3
	Beggs and Brill		6.0	19.3	25.3
	Mukherjee and Brill		3.9	17.5	21.4
12	OLGA		41.6	8.7	51.5
	Beggs and Brill		62.0	15.1	77.1
	Mukherjee and Brill		34.6	11.1	46.0

drop of 332 kPa, which is equivalent to a mixed holdup density of 338.6 kg/m^3 (for a 100-m change in elevation). The system needs to accumulate nearly 242.3 m^3 of liquid to achieve such high mixed density, and for this particular case, the liquid inlet rate is 0.038 m^3/h. This indicates that the liquid accumulation time will be 6376 h. In short, this calculation indicates that the system will achieve a steady state after 6376 h, and static pressure drop after 6376 h will be 332 kPa.

Low liquid rates are typical when water is condensed because of a drop in fluid temperature. The condensed water is generally drained using some automatic or manual drain system; sometimes, pigging is used to drain liquid from the system. In short, no design will allow much time for liquid to accumulate within the system. If the liquid is drained at a regular interval of 24 h, the amount of liquid accumulated and the static pressure drop will be substantially reduced. Thus, the OLGA correlation may be quite accurate but impractical for very low liquid rates.

In conclusion, the OLGA method should be used with caution for very low liquid loading, and the designer must check the liquid dropout time (which cannot be calculated directly) and make sure that it is acceptable.

On the other hand, the Beggs and Brill method is consistent and gives much more convincing results. The Mukherjee and Brill method also provides consistent results but predicts a little less pressure drop in comparison to the Beggs and Brill method.

Downhill pipe section

Similar to uphill analysis, downhill analysis is also complex mainly because the correct liquid holdup is required to be calculated. Table 2.22 presents the comparison among different methods for both inclined and vertical pipe sections. The following parameters have been used for the comparison:

Description	Downhill Inclined	Downhill Vertical
Length of pipe, m	4,000	15
Change in elevation, m	–100	–15
Pipe diameter, mm	477.8	102.3
Flow range, kg/h	10,000 to 100,000	10,000 to 30,000
Liquid mole fraction	0.0035 to 0.76	0.0035 to 0.8

From the results presented in Table 2.22, it is difficult to predict which of the three correlations gives consistent results. In some cases, the Beggs and Brill method and, in others, the Mukherjee and Brill method match well with OLGA.

One disadvantage of Beggs and Brill correlations is that it calculates rather high liquid holdup for downhill piping. Liquid holdup calculated by the Mukherjee and Brill method is reasonable when compared with the OLGA method. From this point of view, the Mukherjee and Brill method gives a more conservative pressure drop.

Table 2.22 Downhill Pressure Drop

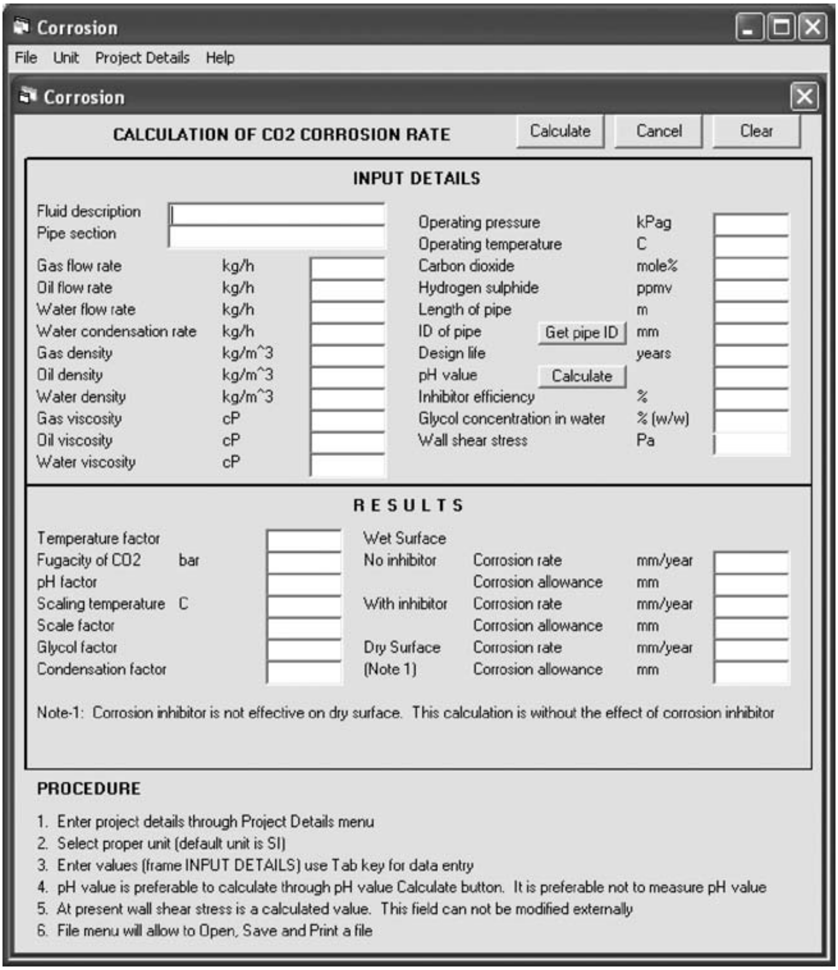
Case Number	Correlations	Pressure Drop (kPa) Frictional	Static	Total
	Downhill Inclined			
1	OLGA	2.3	–7.3	–5.0
	Beggs and Brill	2.7	–13.8	–11.2
	Mukherjee and Brill	14.7	–6.9	7.8
2	OLGA	7.8	–7.4	0.4
	Beggs and Brill	10.0	–13.1	–3.1
	Mukherjee and Brill	36.7	–6.9	29.8
3	OLGA	71.2	–7.2	64.0
	Beggs and Brill	79.5	–12.0	67.5
	Mukherjee and Brill	63.5	–6.9	56.5
4	OLGA	239.3	–6.5	232.8
	Beggs and Brill	227.4	–11.7	215.7
	Mukherjee and Brill	170.7	–6.9	163.8
5	OLGA	19.6	–26.1	–6.5
	Beggs and Brill	2.5	–200.7	–198.2
	Mukherjee and Brill	20.9	–6.9	14.0
6	OLGA	32.5	–37.9	–5.4
	Beggs and Brill	9.5	–179.0	–169.5
	Mukherjee and Brill	49.7	–6.9	42.8
7	OLGA	44.7	–48.1	–3.4
	Beggs and Brill	20.8	–167.8	–146.9
	Mukherjee and Brill	80.8	–6.9	73.9
8	OLGA	69.1	–65.8	3.4
	Beggs and Brill	48.3	–154.8	–106.5
	Mukherjee and Brill	41.9	–117.0	–75.2
	Uphill Vertical			
9	OLGA	25.3	–1.2	24.5
	Beggs and Brill	38.9	–1.3	37.6
	Mukherjee and Brill	19.4	–1.0	18.5
10	OLGA	256.7	–1.0	315.2
	Beggs and Brill	510.2	–1.5	508.7
	Mukherjee and Brill	174.6	–1.0	180.6
11	OLGA	17.4	–13.4	4.0
	Beggs and Brill	5.9	–21.5	–15.6
	Mukherjee and Brill	3.9	–10.3	–6.4
12	OLGA	28.4	–9.4	19.0
	Beggs and Brill	63.5	–13.7	49.8
	Mukherjee and Brill	34.6	–10.9	24.0

General conclusion

- Use the Beggs and Brill method for horizontal and uphill piping configuration.
- Use the Mukherjee and Brill method for downhill piping configuration.

Program for corrosion calculations

The program corrosion.exe has been developed to calculate CO_2 corrosion in pipelines. The structure of the main form is similar to the structures discussed before (refer to Figure 2.22). Although the input structure is straightforward, a few points are worth noting:



Note-1: Corrosion inhibitor is not effective on dry surface. This calculation is without the effect of corrosion inhibitor

1. Enter project details through Project Details menu
2. Select proper unit (default unit is SI)
3. Enter values (frame INPUT DETAILS) use Tab key for data entry
4. pH value is preferable to calculate through pH value Calculate button. It is preferable not to measure pH value
5. At present wall shear stress is a calculated value. This field can not be modified externally
6. File menu will allow to Open, Save and Print a file

Figure 2.22 Corrosion calculation.

1. Water condensation rate relates to the water actually condensing in the pipeline from heat loss. This is used to obtain the condensation factor for the calculation of the dry surface corrosion rate. If the water condensation rate is 0 (zero), a default condensation factor of 0.04 is used in the calculation.
2. The length of pipe is also used to calculate the water condensation rate per unit area.
3. The concentration of hydrogen sulfide is used to establish the NACE requirement. This information is not used to calculate the actual corrosion rate.
4. The *pH* value can either be calculated or entered directly. This is estimated based on the ionic strength and may be substantially different from the actual measurement. It is always preferable to estimate the *pH* rather than enter the actual *pH*.
5. Wall shear stress is calculated and cannot be entered directly.

Program limitations and notes

- The *pH* is calculated based on water analysis. It is preferable to estimate the *pH* based on this program. Any field estimation may result in an inadequate *pH* value. The program is limited to a *pH* range of 3.5 to 6.5. If the estimated *pH* is less than 3.5, a value of 3.5 is used by the program. Similarly, a *pH* value of 6.5 is used if the estimated value is more than 6.5.
- The program is valid for a temperature range of 5 to 150°C. Any value below 5°C or above 150°C is readjusted to 5 and 150°C, respectively.

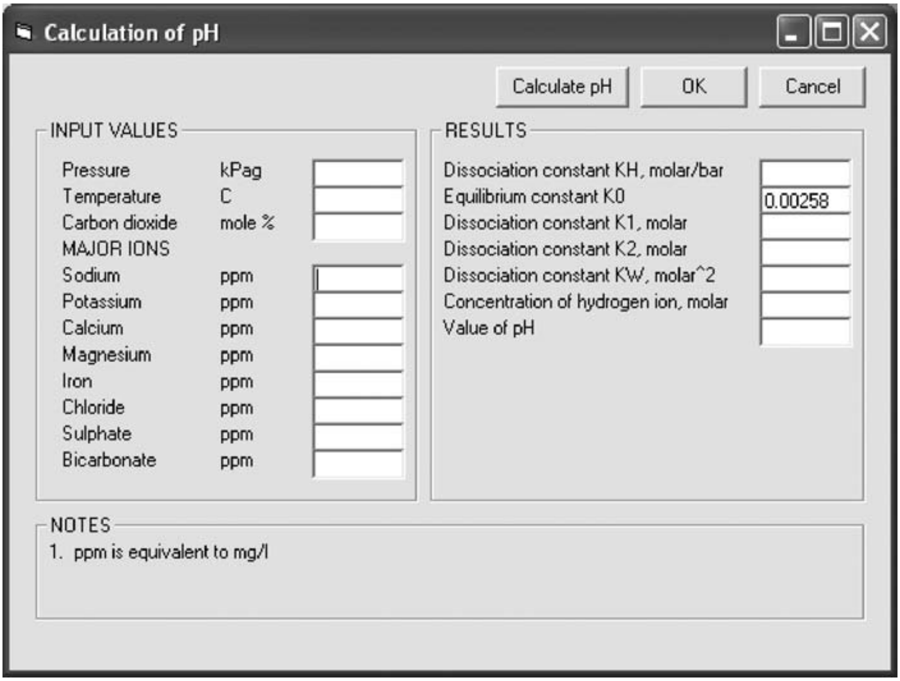
Calculation of pH

The format used to calculate *pH* is shown in Figure 2.23.

As explained before, the *pH* is calculated based on ionic strength of the water phase. The input values of pressure, temperature, and carbon dioxide are taken directly from the main form and cannot be modified in this form. Alhough commonly available ions have been listed in the form, there might be ions outside this list. In such a case, the ionic strength should be converted to that of an available ion.

In the case of a new cation, this is to be converted to an existing cation, except sodium; in case of an anion, it is to be converted to an existing anion, except chlorine. Sodium and chlorine concentrations are used to calculate initial bicarbonate strength and should not be modified through additional ions.

For example, for 500 ppm cupric ion (Cu^{++}), the equivalent potassium ion concentration can be calculated as follows:



Equilibrium constant K0 0.00258

1. ppm is equivalent to mg/l

Figure 2.23 Calculation of *pH*.

Mole of cupric ion = 500/1000/63.5 = 0.00787 mol
Equivalent potassium ions = 0.00787 * 4
= 0.0315 mol
(Multiplication factor 4 is due to conversion of Cu^{++} to K^{+})
Equivalent potassium ion concentration = 0.0315 * 1000 * 39.1
= 1232 ppm

That means the concentration of potassium ion is to be increased by 1232 ppm.

Iron is used as ferrous (Fe^{++}) ions. If there are ferric ions (Fe^{+++}), then the ionic strength is to be converted to that of ferric ion. For example, 50 ppm of ferric (Fe^{+++}) ion is equivalent to 112.5 (= 50 * 9/4) ppm of ferrous (Fe^{++}) ion.

Example 2.8

A 15,000-m-long cross-country pipeline is transporting a fluid with the following properties:

Gas flow rate = 20,000 kg/h
Oil flow rate = 50,000 kg/h
Water flow rate = 5,000 kg/h
Water condensation rate = 0 kg/h (heat loss is not significant)
Gas density = 50 kg/m^3

Oil density = 750 kg/m^3
Water density = 1,000 kg/m^3
Gas viscosity = 0.013 cP
Oil viscosity = 5.6 cP
Water viscosity = 0.7 cP
Operating pressure = 5,000 kPaG
Operating temperature = 35°C
CO_2 concentration = 2.5 mol%
H_2S concentration = 50 ppm
Pipe diameter = 200 NB, sch. 40
Design life of the pipeline = 15 years
Glycol concentration = 0
Maximum inhibitor efficiency = 90%

Water analysis:
Sodium ion = 7,000 ppm
Potassium ion = 50 ppm
Ferric ion = 20 ppm
Chloride ion = 8,000 ppm
Sulfate ion = 50 ppm
Bicarbonate ion = 2,000

Solution

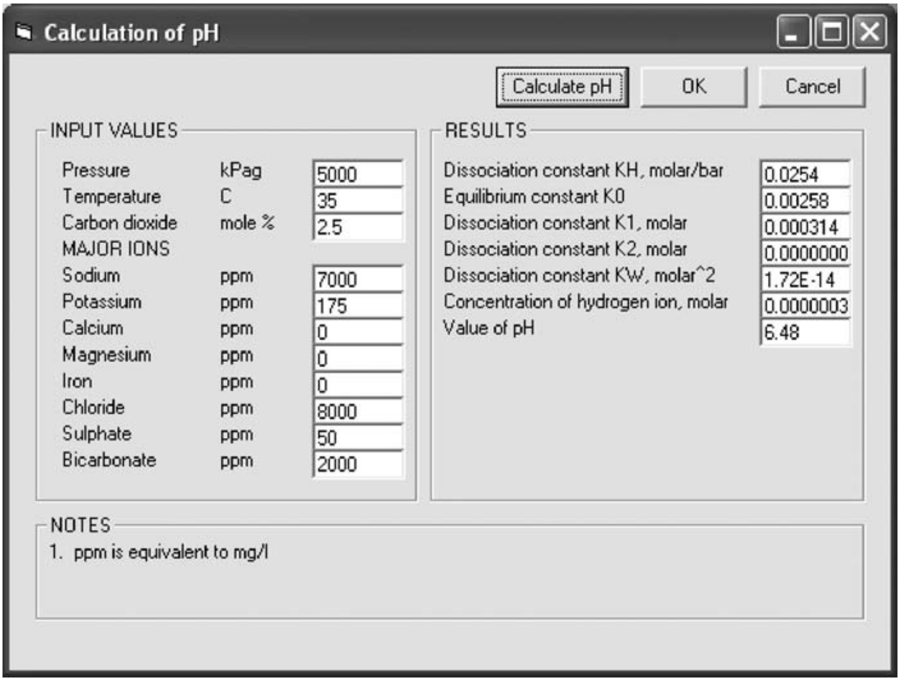
Because ferric ion is not a part of the standard format, we need to convert the ion to any other ion (except sodium), say, potassium ion.

Mole of ferric ion = 20/55.85/1000
= 0.00036 mol
Equivalent potassium ion = 0.00036 * 9
= 0.0032 mol
Potassium ion concentration = 125 ppm
Total concentration of potassium ion = 175 ppm

The *pH* calculation is shown in Figure 2.24. The OK button of the *pH* calculation form will transfer the information to the main form for further calculation. The corrosion allowance calculation is presented in Figure 2.25.

From the main form (Figure 2.25) it is clear that the required corrosion allowance without corrosion inhibitor is 16.3 mm, which means that corrosion inhibitor is a must. The calculated corrosion allowance with 90% corrosion inhibitor efficiency is 1.6 mm, and the calculated corrosion allowance for a dry surface is 0.65 mm. Hence, the design conclusion will be as follows:

Required corrosion inhibitor efficiency = 90%
Required corrosion allowance = 1.63 mm
Whether material selection should follow NACE = No



INPUT VALUES

Pressure	kPag	5000
Temperature	C	35
Carbon dioxide	mole %	2.5
MAJOR IONS		
Sodium	ppm	7000
Potassium	ppm	175
Calcium	ppm	0
Magnesium	ppm	0
Iron	ppm	0
Chloride	ppm	8000
Sulphate	ppm	50
Bicarbonate	ppm	2000

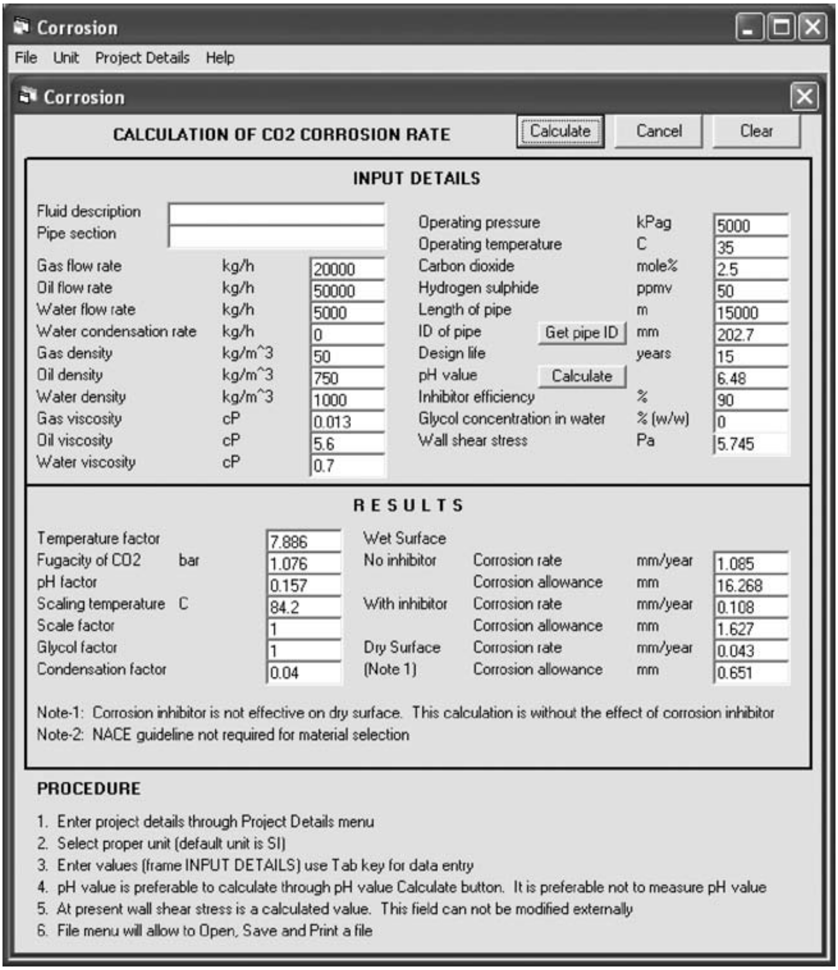
RESULTS

Dissociation constant KH, molar/bar	0.0254
Equilibrium constant K0	0.00258
Dissociation constant K1, molar	0.000314
Dissociation constant K2, molar	0.0000000
Dissociation constant KW, molar^2	1.72E-14
Concentration of hydrogen ion, molar	0.0000003
Value of pH	6.48

NOTES

1. ppm is equivalent to mg/l

Figure 2.24 Calculation of *pH* Example 2.8.



CALCULATION OF CO2 CORROSION RATE

INPUT DETAILS

Fluid description		
Pipe section		
Gas flow rate	kg/h	20000
Oil flow rate	kg/h	50000
Water flow rate	kg/h	5000
Water condensation rate	kg/h	0
Gas density	kg/m^3	50
Oil density	kg/m^3	750
Water density	kg/m^3	1000
Gas viscosity	cP	0.013
Oil viscosity	cP	5.6
Water viscosity	cP	0.7

Operating pressure	kPag	5000
Operating temperature	C	35
Carbon dioxide	mole%	2.5
Hydrogen sulphide	ppmv	50
Length of pipe	m	15000
ID of pipe	mm	202.7
Design life	years	15
pH value		6.48
Inhibitor efficiency	%	90
Glycol concentration in water	% (w/w)	0
Wall shear stress	Pa	5.745

RESULTS

Temperature factor		7.886
Fugacity of CO2	bar	1.076
pH factor		0.157
Scaling temperature	C	84.2
Scale factor		1
Glycol factor		1
Condensation factor		0.04

Wet Surface No inhibitor	Corrosion rate	mm/year	1.085
	Corrosion allowance	mm	16.268
With inhibitor	Corrosion rate	mm/year	0.108
	Corrosion allowance	mm	1.627
Dry Surface	Corrosion rate	mm/year	0.043
(Note 1)	Corrosion allowance	mm	0.651

Note-1: Corrosion inhibitor is not effective on dry surface. This calculation is without the effect of corrosion inhibitor
Note-2: NACE guideline not required for material selection

PROCEDURE

1. Enter project details through Project Details menu
2. Select proper unit (default unit is SI)
3. Enter values (frame INPUT DETAILS) use Tab key for data entry
4. pH value is preferable to calculate through pH value Calculate button. It is preferable not to measure pH value
5. At present wall shear stress is a calculated value. This field can not be modified externally
6. File menu will allow to Open, Save and Print a file

Figure 2.25 Corrosion Allowance Example 2.8.

Nomenclature

a	Area, m^2
A_o	Heat transfer area, m^2
A_g	Area of gas phase, m^2
A_L	Area of liquid phase, m^2
b	Width, m
c	Empirical constant
C_D	Coefficient of discharge, dimensionless
Cp	Specific heat at constant pressure, kJ/(kg)(°C)
CR	Rate of corrosion, mm/year
CV	CV of control valve

d	Diameter, m
d_f	Float diameter, m
d_m	Hydraulic mean diameter, m
D_{ci}	Inside diameter of a particular layer, mm
D_{Eg}	Equivalent diameter of gas phase, m
D_{EL}	Equivalent diameter of liquid phase, m
D_i	Inside diameter of pipe, mm
D_o	Outside diameter of pipe, mm
D_{ot}	Overall outside diameter of pipe, including all layers, mm
e	Energy, m^2/sec^2
f	Friction factor, dimensionless
f_{CO2}	Fugacity of CO_2, bar
f_g	Gas phase friction factor, dimensionless
f_l	Liquid phase friction factor, dimensionless
f_{ns}	No slip friction factor, dimensionless
$f(pH)$	The *pH* factor, dimensionless
f_R	Friction factor ratio, dimensionless
f_{tp}	Two-phase friction factor, dimensionless
F_a	Thermal expansion correction factor, dimensionless
F_D	Drag force, kg
g	Acceleration due to gravity, m/sec^2
G	Mass flow rate, kg/sec
G_l	Liquid flux rate, $kg/(m^2 \cdot sec)$
G_m	Mixture flux rate, $kg/(m^2 \cdot sec)$
G_v	Vapor flux rate, $kg/(m^2 \cdot sec)$
h	Specific enthalpy, m^2/sec^2
h,H	Height, m
h_f	Head loss due to friction, m
h_i	Inside film coefficient, $kW/(m^2)(°C)$
h_L	Height of liquid phase, m
H_d	Velocity head, m
$H_L(0)$	Horizontal holdup fraction, dimensionless
$H_L(\theta)$	Inclined holdup fraction, dimensionless
H_R	Holdup ratio, dimensionless
k	Fraction, dimensionless
k_c	Thermal conductivity of coating material, kW/(m-°C)
k_f	Thermal conductivity of fluid, kW/(m-°C)
k_{pipe}	Thermal conductivity of pipe material, kW/(m-°C)
k_s	Thermal conductivity of surrounding, kW/(m-°C)
K	Flow parameter, $m^{1/2}/sec$
K	"K factor," dimensionless
K_1	Term used to calculate "K factor"
K_t	Constant for temperature t, dimensionless
K_∞	Term used to calculate "K factor"
L	Length, m
m	Mass, kg

m_f	Mass of float, kg
M	Mach number, dimensionless
MW	Molecular weight
N_f	Froude number, dimensionless
N_{gv}	Gas velocity number, dimensionless
N_L	Liquid viscosity number, dimensionless
N_{Lv}	Liquid velocity number, dimensionless
N_{Re}	Reynolds number
p	Pressure, kPaA
p_2	Pressure at orifice, kPaA
p_1	Pressure at upstream pipe section, kPaA
p_{CO2}	Partial pressure of CO_2, bar
P	Perimeter, m
P_g	Perimeter of gas phase, m
P_L	Perimeter of liquid phase, m
P_r	Prandtl number, dimensionless
q	Frictional resistance per unit area
Q	Volumetric flow, m^3/sec
Q_a	Average rate of heat transfer, kW
r	Pressure ratio (p_2/p_1), dimensionless
r_c	Critical pressure ratio, dimensionless
$R_{coating}$	Thermal resistance for coating, (m)(°C)/kW
R_{env}	Thermal resistance due to environment, (m)(°C)/kW
R_{film}	Thermal resistance due to film, (m)(°C)/kW
R_m	Mean value of shear stress at surface, kg/(m)(s^2)
R_{pipe}	Thermal resistance due to pipe material, (m)(°C)/kW
s_g	Specific gravity, dimensionless
S	Sonic velocity, m/sec
SS	Shear stress, Pa
t	Temperature, °C
t_c	Thickness of coating, mm
t_R	Reference temperature, °C (default value 20°C)
T	Temperature, K
T_{wg}	Gas phase wall shear stress, kg/m^2
T_{wL}	Liquid phase wall shear stress, kg/m^2
Uo	Overall heat transfer coefficient, kW/(m^2-°C)
v	Velocity, m/sec
v_g	Superficial gas velocity, m/sec
v_l	Superficial liquid velocity, m/sec
v_m	Mixture velocity, m/sec
V	Volume, m^3
V_e	Erosional velocity, m/sec
w	Wetted perimeter, m
W	Weight, kg
W_i	Width of interface, m
Y	Expansion factor based on upstream pressure, dimensionless

z	Elevation, m
Z	Compressibility, dimensionless
Z_b	Depth of buried pipe, m

Greek characters

α_{P}	Thermal expansion of pipe/°C (default value 12e-6/°C)
α_{PE}	Thermal expansion of flow meter/°C (default value 12e-6/°C)
β	Beta ratio, ratio between orifice to pipe diameter, dimensionless
δ	Angle
ΔT	Differential temperature, °C
ε	Roughness of pipe wall, m
γ	Ratio of specific heats, dimensionless
γ_{g}	Specific weight of gas phase, kg/m^3
γ_{l}	Specific weight of liquid phase, kg/m^3
γ_{m}	Specific weight of mixed phase, kg/m^3
Δp	Differential pressure, kPa
Δp_a	Nonrecoverable (actual) pressure drop, kPa
Δp_{acl}	Acceleration head, kPa
Δp_f	Frictional pressure drop, kPa
Δp_h	Hydrostatic head, kPa
Φ	Angle
λ	Liquid volume fraction, dimensionless
ρ	Density, kg/m^3
ρ_{f}	Density of float, kg/m^3
ρ_{g}	Density of gas, kg/m^3
ρ_{l}	Density of liquid, kg/m^3
ρ_{m}	Mixed density, kg/m^3
ρ_{tp}	Two-phase density, kg/m^3
μ	Viscosity, cP
μ_{l}	Viscosity of liquid, cP
μ_{m}	Mixed viscosity, cP
σ	Surface tension, dyn/cm

References

1. Tuve, G.L. and Sprenkle, R.E., Instruments, 6, 201, 1933.
2. Coulson, J.M. and Richardson, J.F., Chemical Engineering, 4th ed., Vol. 1, Pergamon Press, Great Britain, 1990.
3. Perry, R.H. and Green, D., Perry's Chemical Engineers' Handbook, 6th ed., McGraw Hill, Malasia, 1984.
4. American Society of Mechanical Engineers Standard, Measurement of Fluid Flow in Pipes Using Orifice, Nozzle, and Venturi.
5. Fluid Meters Their Theory and Application, 6th ed., Report of the American Society of Mechanical Engineers Research Committee on Fluid Meters, 1971.

6. Crane Technical Paper No. 410, Flow of Fluids through Valves, Fittings, and Pipe, Metric edition — SI Unit, 1995.
7. McCabe, W.L. and Smith, J.C., Unit Operations of Chemical Engineering, 3rd ed., McGraw Hill, Tokyo, 1976.
8. Hooper, W.B., Calculated head loss caused by change in pipe size, Chemical Engineering, November 7, 89–92, 1988.
9. Hooper, W.B., The two-K method predict head loss in pipe fittings, Chemical Engineering, August 24, 96–100, 1981.
10. Fisher ED full port equal percentage case, Fisher Catalog-10, pp. 1–73.
11. API RP 14E, Recommended Practice for Design and Installation of Offshore Production Platform Piping Systems, 5th ed., American Petroleum Institute, 1991.
12. Massey, B.S., Mechanics of Fluids, 6th ed., Van Nostrand Reinhold, chap. 12, London, 1989.
13. External Extruded High-density Polyethylene Coating System for Pipes, Australian/New Zealand Standard AS/NZS 1518:2002.
14. External Fusion-bonded Epoxy Coating for Still Pipes, Australian/New Zealand Standard AS/NZS 3862:2002.
15. Kern, D.Q., Process Heat Transfer, International edition, McGraw Hill, Singapore, 1988.
16. Hausen, H., Darstellung des Warmeuberganges in Rohren durch verallgemeinerte Potenzbeziehungen, Zeitsche, V.D.I. Beihefte Verfahrenstechnik, No. 4, 1943.
17. Carslaw, H.S. and Jaeger, J.C., Conduction of Heat in Solids, Oxford University Press, Great Britain, 1959.
18. Parker, J.D., Boggs, J.H., and Blick, E.F., Introduction to Fluid Mechanics and Heat Transfer, Addison-Wesley, Reading, MA, 1969.
19. Hilpert, R., Warmeabgabe von geheizen Drahten und Rohren, Forsch Gebiete Iingenieurw, Vol. 4, 1933.
20. Beggs, H.D. and Brill, J.P., A study of two-phase flow in inclined pipe, Journal of Petroleum Technology, May, 607–617, 1973.
21. Mukherjee, H. and Brill, J.P., Liquid holdup correlations for inclined two-phase flow, Journal of Petroleum Technology, May, 1003–1008, 1983.
22. Mukherjee, H. and Brill, J.P., Pressure drop correlations for inclined two-phase flow, Journal of Energy Resources Technology, 107, December, 549–554, 1985.
23. Gregory, G.A., Mandhane, J., and Aziz, K., Some design consideration for two-phase flow in pipes, Journal of Canadian Petroleum Technology, January–March, 65–71, 1975.
24. Baker, A., Nielsen, K., and Gabb, A., Pressure loss, liquid-holdup calculations developed, Oil and Gas Journal, March 14, 55–59, 1988.
25. Baker, A., Nielson, K., and Gabb, A., Holdup, pressure-loss calculation confirmed, Oil and Gas Journal, March 28, 44–49, 1988.
26. Bendiksen, K.H., Malnes, D., Moe, R., and Nuland, S., The dynamic two-fluid model OLGA: theory and application, SPE Production Engineering, May, 171–180, 1991.
27. Mukherjee, H. and Brill, J.P., Empirical equations to predict flow patterns in two-phase inclined flow, International Journal of Multiphase Flow, 11(3), 299–315, 1985.
28. Waard, C. and Lotz, U., Corrosion 93, The NACE Annual Conference and Corrosion Show, Paper No. 69.

29. CO_2 Corrosion Rate Calculation Model, Norsok Standard, M-506, Rev2, June 2005.
30. Waard, C., Lotz, U., and Millams, D.E., Corrosion 91, The NACE Annual Conference and Corrosion Show, Paper No. 577.
31. NACE Standard MR0175-2001, Item No. 21304.
32. Zenz, F.A., Minimize manifold pressure drop, Hydrocarbon Processing and Petroleum Refiner, 41(12), 125–130, 1962.

chapter three

Separators

Introduction

Separators are the most important static equipment used in chemical industries. These are principally used to separate vapor from liquid phase; the liquid phase can be a single continuous phase or a mixture of two immiscible phases. Both horizontal and vertical separators are extensively used in industries, and some typical applications include the following:

- Compressor knockout (KO) drum
- Inlet separator in a gas processing plant
- Distillation column overhead separator
- Flare KO drum
- Desalter

Separators are normally designed as steady-state equipment, i.e., the fluid physical properties do not change with time, though a dynamic state in a separator is not uncommon.

General principles of separation

Droplet in a vertical vessel

In a vertical vessel, the gas flow is always in the upward direction, and the liquid droplet will have two forces acting on it. The gravitational force will act downward, trying to settle the droplet, and the drag force will act upward, trying to carry the droplet with the gas phase (Figure 3.1). The gravitational force depends on the density of the liquid droplet, and the drag force depends on the Reynolds number and the size and shape of the particle.

If the drag force exceeds the gravitational force, the liquid droplet will not settle out unless it coalesces into larger droplets.

If a high-density liquid droplet of diameter D_p and density ρ_{heavy} moves a distance d at a velocity V in a light continuous phase of density ρ_{light}, then the kinetic energy required to displace this volume of light phase is given by

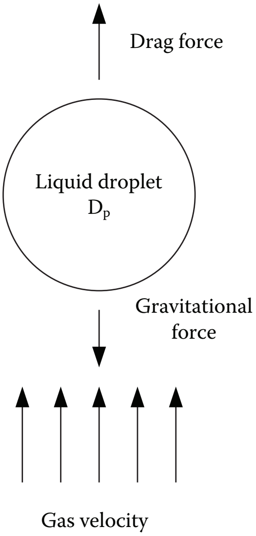


Figure 3.1 Liquid droplet in gas field.

$$E = \frac{1}{2} mV^2 \tag{3.1a}$$

and the mass of light phase displaced by

$$m = \frac{\pi}{4} D_p{}^2 \rho_{light} d \tag{3.1b}$$

Combining Equation 3.1a and Equation 3.1b,

$$E = \frac{\pi}{8} D_p{}^2 \rho_{light} d V^2 \tag{3.1c}$$

As energy is force * displacement, the drag force will be

$$F_D = \frac{\pi}{8} D_p{}^2 \rho_{light} V^2 \tag{3.1d}$$

Equation 3.1d is the theoretical equation for drag force on a spherical droplet; however, this equation does not consider the impact of viscosity or the dynamics of the flow around the sphere. The impact of viscosity and flow dynamics around the sphere is introduced through another coefficient, popularly known as drag coefficient (C_D), and Equation 3.1d becomes

$$F_D = C_D \frac{\pi}{8} {D_p}^2 \rho_{light} V^2 \tag{3.1e}$$

The drag coefficient depends on the Reynolds number of the falling droplet, and for very low Reynolds numbers ($N_{RE} < 2$), the drag coefficient is defined as

$$C_D = 24/N_{RE} \tag{3.1f}$$

The Reynolds number is defined as

$$N_{RE} = \frac{1000 D_p V \rho_{light}}{\mu} \tag{3.1g}$$

Combining Equation 3.1e, Equation 3.1f, and Equation 3.1g,

$$F_D = \frac{3}{1000} \mu \pi D_p V \tag{3.1h}$$

When the heavy droplet reaches the terminal velocity, this drag force is balanced by the immersed weight of the droplet. The immersed weight is defined by

$$I = W - U \tag{3.1i}$$

The weight of a heavy droplet (volume * density) is

$$W = \frac{4}{3} \pi \frac{{D_p}^3}{8} g \rho_{heavy} \tag{3.1j}$$

The upthrust is equal to the weight of the light phase that the heavy droplet displaces:

$$U = \frac{4\pi}{3} \frac{{D_p}^3}{8} \rho_{light} g \tag{3.1k}$$

Combining Equation 3.1i, Equation 3.1j, and Equation 3.1k, the immersed weight of the droplet is

$$I = \frac{4\pi}{3} \frac{{D_p}^3}{8} g \left(\rho_{heavy} - \rho_{light} \right) \tag{3.1l}$$

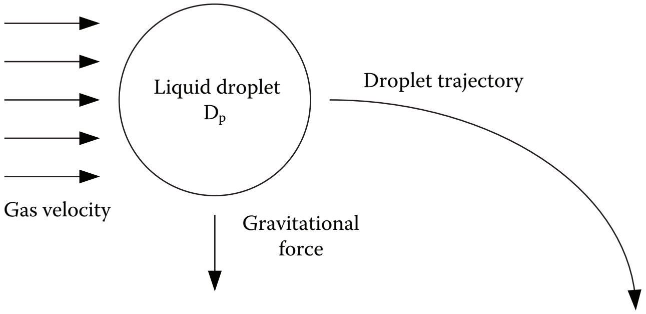


Figure 3.2 Liquid droplet trajectory.

At very low Reynolds numbers ($N_{RE} < 2$), the drag force equals the immersed weight of the droplet, and from Equation 3.1h and Equation 3.1l,

$$3\mu D_p V = \frac{4000\pi}{3}\frac{D_p^{\;3}}{8} g\left(\rho_{heavy} - \rho_{light}\right) \tag{3.1m}$$

Rearranging for velocity,

$$V = \frac{1000 D_p^{\;2} g\left(\rho_{heavy} - \rho_{light}\right)}{18\mu} \tag{3.1n}$$

Equation 3.1n gives the droplet fall velocity under Stokes' law.

Droplet in a horizontal vessel

Settling in a horizontal vessel is more complex in nature than settling in a vertical vessel. In a horizontal vessel, vapor flows horizontally (parallel to the vessel's center line), and the gravitational force acts vertically in the downward direction. As a result, the liquid droplet travels in an elliptical path (Figure 3.2). Settling of a liquid droplet is possible if the following are satisfied:

- Actual gas velocity is less than the settling velocity
- Liquid droplet fall time is less than the gas residence time

Gravity settling: limiting conditions

When two immiscible fluids, which may be gas–liquid or liquid–liquid, are kept for a sufficiently long time, the denser droplets settle down, resulting in two separate phases. For a gas–liquid system, the terminal or free settling velocity can be mathematically defined as [1,2]

$$V_t = \sqrt{\frac{4gD_p(\rho_L - \rho_G)}{3\rho_G C'}} \tag{3.2}$$

The drag coefficient C' is a function of the Reynolds number, which is defined as

$$N_{RE} = \frac{1000 D_p V_t \rho_G}{\mu} \tag{3.3}$$

Depending on the Reynolds number, the terminal velocity can be defined further.

Newton's law

Newton's law is applicable when the Reynolds number is above 500; the terminal velocity would then be defined as [1]

$$V_t = 1.74\sqrt{\frac{gD_p(\rho_L - \rho_G)}{\rho_G}} \tag{3.4}$$

Stokes' law

Stokes' law is applicable when the Reynolds number is very low (less than 2); in such cases, the terminal velocity would be defined as (Equation 3.1n)

$$V_t = \frac{1000 g D_p^2(\rho_L - \rho_G)}{18\mu} \tag{3.5}$$

Intermediate law

This law is applicable for intermediate Reynolds numbers (from 2 to 500); the terminal velocity for intermediate situations is

$$V_t = \frac{3.54 g^{0.71} D_p^{1.14}(\rho_L - \rho_G)^{0.71}}{\rho_G^{0.29}\mu^{0.43}} \tag{3.6}$$

Critical particle diameter

The application of the aforementioned settling laws are normally restricted by the particle diameter. Newton's law, being applicable for very high Reynolds numbers, can handle larger particles without a major upset of settling characteristics, whereas the Stokes' law can handle very small particle diameters.

Normally, the limit of droplet size applicable to individual settling equations dictates the use of proper settling equations. The maximum allowable droplet diameter is defined as

$$D_p = K_{CR}\left\{\frac{\mu^2}{g\rho_G(\rho_L - \rho_G)}\right\}^{0.33} \tag{3.7}$$

The value of proportionality constant K_{CR} depends on the Reynolds number and the following values are used for design [1]:

Newton's law	18.13
Intermediate law	0.334
Stokes' law	0.025

In Equation 3.7, the viscosity used is for continuous phase, which means the following:

- Viscosity of gas phase, when oil or water droplets are settling through the gas phase
- Viscosity of oil phase, when water droplets are settling through light oil phase
- Viscosity of water phase, when oil droplets are rising through heavy water phase

Vertical vs. horizontal separators

Proper selection of the separator type is important. For three-phase separation, a horizontal separator is more effective than a vertical separator. Also, it is easier to design the control system for a three-phase horizontal separator.

For a two-phase vapor–liquid separator, both vertical and horizontal separators are used, and the selection should be made on a case-by-case basis. In a horizontal separator, with an increase in liquid level, the area of the vapor space is reduced and the possibility of liquid entrainment increases. On the other hand, the vapor-flow area remains constant in the vertical separator, and liquid entrainment is not an issue at high liquid level. Vertical separators also have the advantage of lower space requirement and easy-to-install control systems.

For a relief KO drum, the horizontal separator is popular simply because of the use of split flow. In this design, one inlet nozzle is used at the vessel center with two outlets on either side. This split-flow advantage is available only in horizontal separators.

Advantages of the horizontal separator

- It has high separation efficiency in comparison with a vertical separator.
- It is the only choice for a single inlet and two vapor outlets.
- It is easy to design for three-phase separation.
- It is more suitable for handling large liquid volumes.

Disadvantages of the horizontal separator

- It requires a larger footprint area in comparison with a vertical separator.
- At high liquid levels, the liquid entrainment rate progressively increases with the increase in liquid level.

Advantages of the vertical separator

- The liquid surface area does not change with liquid height, hence liquid entrainment is reasonably constant.
- It requires a smaller footprint area.
- It is easier to install level instruments, alarms, and shutdown systems.
- It is usually more efficient than a horizontal separator for high vapor–liquid ratios.

Disadvantages of the vertical separator

- It is not suitable for three-phase separation.
- It is less suitable for high liquid–vapor ratios.

Design of a gas–liquid separator

Critical settling velocity

Once a liquid droplet is allowed to settle through a continuous vapor phase, the settling velocity depends on the particle size, resistance to settling (defined as drag), and densities on vapor and liquid phases. The critical settling velocity is defined as

$$V_c = K_D * \left\{ \frac{(\rho_L - \rho_G)}{\rho_G} \right\}^{0.5} \tag{3.8}$$

Table 3.1 Typical K Factors for Sizing Woven Wire Demisters

Separator Type	K Factor (m/sec)	Typical Value (m/sec)
Horizontal	0.12 to 0.15	0.135
Vertical (height ≤ 1.5 m)	0.036 to 0.073	0.05
Vertical (height > 1.5 m)	0.05 to 0.11	0.08
Spherical	0.05 to 0.11	0.08
Wet steam	0.076	0.076
Most vapors under vacuum	0.061	0.061
Salt and caustic evaporators	0.046	0.046

- For glycol and amine solutions, multiply K by 0.6 to 0.8.
- Typically, use one-half of the above K values for approximate sizing of vertical separators without wire mesh demisters.
- For compressor suction scrubbers and expander-inlet separators, multiply K by 0.7 to 0.8.

Design constant K_D

The sizing of a separator depends on the value of empirical constant K_D. The value of K_D is largely influenced by the presence of internals. Normally, internal manufacturers specify the particular value of K_D, based on their test results. It is always advisable to get the value of K_D from the internal manufacturers before finalizing a separator design. In the absence of manufacturer data, the data presented in Table 3.1 are often used [1,3].

The value of K also depends upon the operating pressure, and the effect of the pressure on the allowable K factor can be defined as given in Reference 3. Table 3.2 gives the effect of pressure on allowable K factor.

For horizontal separators, the separation efficiency depends on the total vapor travel length within the vessel. A longer vessel makes it easier to

Table 3.2 Effect of Pressure on Allowable K Factor

Allowable Pressure, kPa	K Factor, % of Design Value
5	100
20	100
50	100
100	100
500	94
1000	90
2000	85
4000	80
8000	75

remove liquid droplets. The value of K for a horizontal vessel, shown in Table 3.1, is typically used for a vessel length of 3.05 m. A smaller K value is used when the vessel length is less and a larger K value is used when the vessel length is more. A typical design K value for horizontal separators is defined as [4]

$$K_D = K\left(\frac{L}{3.05}\right)^{0.56} \tag{3.9}$$

Sometimes, for compressor suction, intermediate and discharge scrubbers with the following design K values are used [5]:

- K_D = 0.107 m/sec: When designed for design conditions only, with no capacity of liquid slugs from inlet piping or condensation mist produced by inlet valve throttling and no capacity for gas-flow increases due to start-up surges, pressure changes, or compressor clearance adjustments. This is typically used for compressor discharge scrubbers.
- K_D = 0.076 m/sec: Scrubbers for design conditions with moderate additional scrubber capacity; typically used for interstage scrubbers without side streams.
- K_D = 0.055 m/sec: Scrubbers for design conditions with substantial additional capacity; generally used for suction scrubbers.

Liquid phase separation is dictated by the settling velocity and droplet falling time. Settling velocity is calculated by using Equation 3.4, Equation 3.5, or Equation 3.6. It is also possible that the calculated settling velocity is too high to achieve undisturbed settling. Generally, the maximum settling velocity for liquid phase separation is limited to 4 mm/sec.

API 521 method [9]

The method presented in API 521 is mostly used in the sizing of KO drums. This method is based on the droplet settling velocity and does not calculate the K factor velocity.

The droplet velocity is calculated as

$$U_c = 1.15\sqrt{\frac{g * D_p * (\rho_L - \rho_G)}{\rho_G * C_D}} \tag{3.10}$$

The value of the dimensionless drag coefficient C_D depends on the Reynolds number ($C(Re)^2$), which is calculated as

$$C(\mathrm{Re})^2 = \frac{0.13 * 10^8 * \rho_G * D_p{}^3 * (\rho_L - \rho_G)}{\mu_G} \quad (3.11)$$

The value of the drag coefficient is calculated as

$$C_D = -34.8312 + \frac{352.3078}{X^{0.5}} - \frac{1195.63}{X} + \frac{1385.236}{X^{1.5}} \quad (3.12)$$

where

$X = \ln(C(Re)^2)$

The vapor-flow area is calculated as the area available above the high liquid-level shutdown. Once the vapor-flow area is known, the vapor residence time can easily be calculated, using the actual vapor velocity. The required length of the KO drum is calculated by multiplying the droplet velocity and vapor residence time. This length is then checked with the available length.

Design of liquid–liquid separators

Separation of two immiscible liquids is possible due to the difference in their densities. This general principle of liquid–liquid separation is applicable for both horizontal and vertical separators. Liquid–liquid separation is better achieved in a horizontal separator due to the larger interfacial area.

Though difference in densities is the main force for liquid–liquid separation, there are two factors that may prevent separation in spite of large difference in densities.

1. If the droplet particles are very small (less than 0.1 μm), they may be suspended by the Brownian movement and will not be separated.
2. The droplets may carry electric charges due to dissolved ions. These charged droplets may repel each other and prevent separation.

For liquid–liquid separation, the capacity of the separator is determined by using the following equations [1]:

Vertical vessels:

$$W_{cl} = C^* \left(\frac{\rho_{heavy} - \rho_{light}}{1000\mu} \right) * 0.785 * ID^2 \quad (3.13)$$

Table 3.3 Value of C*

Emulsion Characteristics	Droplet Diameter (μm)	Constant, C*
Free liquid	200	1880
Loose emulsion	150	1060
Moderate emulsion	100	470
Tight emulsion	60	170

Horizontal vessels:

$$W_{cl} = C^* \left(\frac{\rho_{heavy} - \rho_{light}}{1000\mu} \right) * L_1 * H_1 \tag{3.14}$$

The value of C* is obtained from Table 3.3.

The total retention time is calculated using the following equation [1]:

$$VS = \frac{F * T}{1440} \tag{3.15}$$

Retention time T depends on the type of separation and can be estimated from Table 3.4.

For ethylene glycol/hydrocarbon and amine/hydrocarbon systems, the calculation should be checked for interface flow. The allowable interface flow for these two systems is 100 $m^3/(m^2\text{-day})$.

Table 3.4 Typical Retention Time for Liquid–Liquid Separation

Type of Separation	Retention Time (Min)
Hydrocarbon water separators	
Below 0.85 relative density hydrocarbon	3 to 5
Above 0.85 relative density hydrocarbon	
38°C and above	5 to 10
27°C	10 to 20
15°C	20 to 30
Ethylene glycol/hydrocarbon separator	20 to 60
Amine/hydrocarbon separator	20 to 30
Coalescers, hydrocarbon/water separator	
38°C	5 to 10
27°C	10 to 20
15°C	20 to 30
Caustic/propane	30 to 45
Caustic/heavy gasoline	30 to 90

Table 3.5 Features of Mist Eliminators

Description	Wire Mesh	Vane Type
Cost	Lowest	2 to 3 times of wire mesh unit
Efficiency	100% (for droplets larger than 3 to 10 μm)	100% (for mists > 10 to 40 μm)
Gas capacity	Good	Up to twice that of a wire mesh unit
Liquid capacity	Good	Best
Solids	Good	Best
Typical installation	Vertical separator	Horizontal separator

Mist eliminator

The mist eliminator is the most important internal component of a separator. Though conventionally it is used in most separators, its use can be best judged by the quality of the liquid separated. In oil and gas service, a tight and gummy emulsion is not uncommon, and the mist eliminator may create a large pressure drop, resulting in inadequate vapor–liquid separation. Although for clean liquids, the mist eliminator is used in almost all cases, better judgment is required for gummy liquids. Generally, three types of mist eliminators are used: (1) wire mesh, (2) vane type, and (3) fiber bed. Of these, the first two are most widely used in chemical industries. The main differences between wire mesh and vane types are presented in Table 3.5 [3].

Wire mesh mist eliminator

The most common type of mist eliminator is the wire mesh, and it is made with knitted wire. This type of mist eliminator has high surface area and void volume. A mesh is made with wire having a diameter in the range of 0.1 to 0.28 mm and a typical void fraction of 0.95 to 0.99. Due to very low wire thickness, the material of construction requires careful consideration. Stainless steel wire is the most popular mesh material.

Efficiency of the mist eliminator

The efficiency of a wire mesh mist eliminator is defined as [3]

$$\eta = 1 - \exp\{-(2/3)\,\pi\, a\, H\, \eta_t\} \tag{3.16}$$

where

a = surface area of the wire per unit volume of the mesh pad, m^{-1}
H = thickness of the mesh pad, m
η_t = target collection efficiency

If the specific mesh surface area is not available, this can be approximated as

$$a = 4^*(1 - \varepsilon)/d_w \tag{3.16a}$$

where

ε = mesh void fraction
d_w = wire diameter, m

Target collection efficiency [3]

The target collection efficiency depends on a dimensionless number, called the separation number (N_s). This separation number further depends on the separation mechanism, and the following methods are used to calculate this number.

Inertial impaction

This is applicable for larger droplets. Larger droplets have sufficient momentum to break through the gas streamlines and move in a straight line, until they impinge on the target. The separation number is estimated by using the following equation:

$$N\text{s} = 1000 * K_m * \rho_L * D_p^2 * V/(18 * \mu_g * D_b) \tag{3.17}$$

where

K_m = Stokes–Cunningham correction factor; it can be set to 1.0 for droplet sizes significantly bigger than 15 μm.
ρ_L = droplet density, kg/m^3
D_p = droplet diameter, m
V = superficial gas velocity, m/sec
μ_g = gas viscosity, cP
D_b = target diameter, m

Direct interception

For smaller particles, the momentum is not sufficient to break through the gas streamlines; instead, they are carried around the object by the gas stream. In such cases, the separation number is expressed as

$$N_s = D_p/D_b \tag{3.18}$$

All terms have been explained earlier.

Diffusion

Very small particles (usually below 1 μm) exhibit random Brownian motion, caused by collision with the gas molecule. In such cases, the separation number is estimated as

$$N_s = D_v/(V^*D_b) \tag{3.19}$$

where

D_v = diffusivity of droplets in gas, m^2/sec
V = superficial face velocity, m/sec
D_b = target diameter, m

Target collection efficiency

This is calculated based on the value of the separation number. The following method is used to calculate the target separation efficiency:

$$\text{If } N_s < 0.15; \quad \eta_t = 0 \tag{3.20a}$$

$$\text{For } 0.15 \le N_s<2; \quad \eta_t = 0.2626\ln(N_s) + 0.5578 \tag{3.20b}$$

$$\text{For } 2 \le N_s<10; \quad \eta_t = 0.1293\ln(N_s) + 0.6432 \tag{3.20c}$$

$$\text{For } 10 \le N_s<30; \quad \eta_t = -0.0001N_s^2 + 0.006N_s + 0.88 \tag{3.20d}$$

$$\text{For } 30 \le N_s<50; \quad \eta_t = 0.98 \tag{3.20e}$$

$$\text{For } N_s \ge 50; \quad \eta_t = 0.99 \tag{3.20f}$$

Pressure drop of mist eliminator

Pressure drop of the mist eliminator is defined by the following equation [3]:

$$P_{dry} = 0.001\, fHa\rho_G V^2/\varepsilon^3 \tag{3.21}$$

where

ΔP_{dry} = dry pressure drop, kPa
f = friction factor
ε = void fraction

Other terms have been defined earlier.

Friction factor f depends on the Reynolds number, and Table 3.6 can be used to estimate the friction factor [3].

Table 3.6 Variation of Friction Factor with Reynolds Number

Friction Factor f	Reynolds Number
2	10
0.4	100
0.2	1,000
0.15	10,000

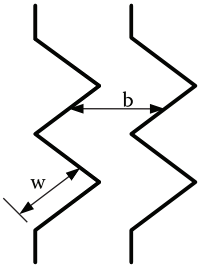


Figure 3.3 Vane type mist eliminator.

Vane type mist eliminator

The vane type mist eliminator consists of a series of baffles or plates, and the gas flows between them, as shown in Figure 3.3. The droplet that is impinged on the surface has a free fall downward. The space between the baffles varies from as low as 5 mm to 75 mm, and the depth in the direction of the flow varies from between 150 to 300 mm.

In case of a horizontal separator, significantly higher gas flow can be achieved in comparison to vertical gas flow, simply because, in the horizontal mode the liquid can easily drain downwards due to gravity. This is the reason the vane type mist eliminator is very popular in horizontal separators.

Efficiency of vane pack [3]

The efficiency of the vane type mist eliminator is defined as [3]

$$\eta = 1 - \exp\,(-V_t n w \theta / (57.3\ Vb \tan\theta) \qquad (3.22)$$

where

η = fractional primary collection efficiency
V_t = droplet terminal centrifugal velocity in normal direction, m/sec
n = number of rows of baffles or bends
w = width of baffle, m
θ = angle of inclination of the baffle to the flow path, degree
V = superficial gas velocity, m/sec
b = spacing between baffles

Terminal centrifugal velocity [2]

For low Reynolds numbers, where Stokes' law applies, the drop terminal centrifugal velocity can be defined as

$$V_t = \frac{1000 * D_p^{\,2} * \rho_p * 2 * V^2 \sin\theta}{18 * \mu_g * w * \cos^3\theta} \tag{3.23}$$

where

D_p = particle diameter, m
ρ_p = particle density, kg/m^3
μ_g = gas viscosity, cP

Other terms are as defined earlier.

Pressure drop through the vane pack

The pressure drop for a vane type mist eliminator can be calculated as the summation of the number of rows of baffles present [2]:

$$\Delta P = \sum_{i=1}^{n} 0.00102 * f_D * \rho_g \frac{VA_p}{2A_t * \cos\theta} \tag{3.24}$$

where

ΔP = pressure drop, cm of water
ρ_g = gas density, gm/cm^3
V = superficial gas velocity, cm/sec
A_p = total projected area of an entire row of baffles in the direction of the inlet gas flow, cm^2
A_t = duct cross-sectional area, cm^2
n = number of rows
θ = angle of incidence (it must be noted that the angle of incidence θ for the second and successive rows of baffles is twice the angle of incidence for the first row)
f_D = drag coefficient; it depends on the angle of incidence, as follows:

Angle of Incidence	Drag Coefficient, f_D
20	0.45
30	0.6
40	0.75
50	0.9
60	1.0
70	1.05
80	1.08
90	1.1

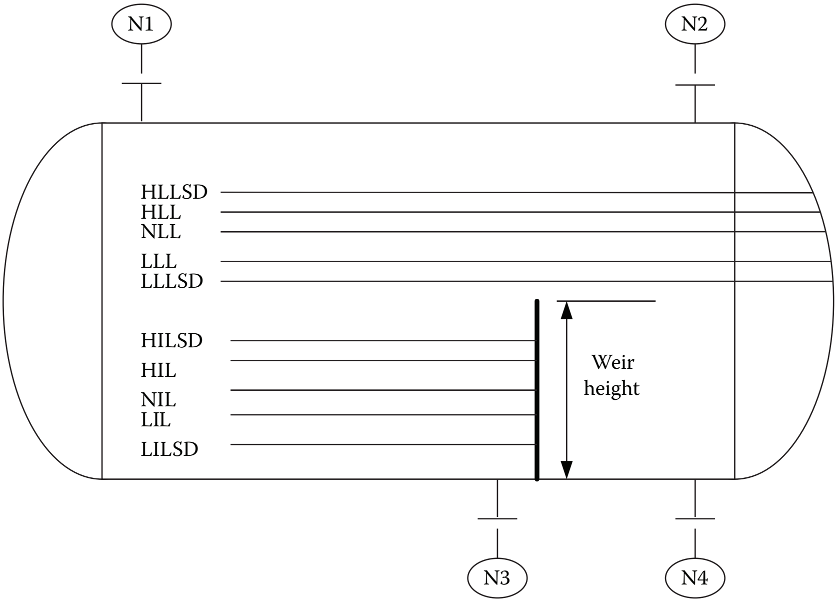


Figure 3.4 A typical horizontal separator.

General dimensions and setting of levels

The horizontal separator

A typical horizontal three-phase separator with flooded weir is shown in Figure 3.4. The inlet nozzle (N1) and the gas outlet nozzles (N2) are placed as close as possible to the vessel tangent line. The distance of the nozzle center line and the tan line is dictated by the mechanical limitations of the vessel; however, distances in Table 3.7 can be used for initial design.

Table 3.7 Minimum Distance of Nozzles from the Tan Line

Nozzle Size, NB (mm)	Minimum Distances (Tan Line to Nozzle Center Line; mm)
40	150
50	150
80	225
100	260
150	330
200	400
250	450
300	500
400	675
500	850
600	925

The flow path distance is normally defined as

$$d = L - d_i - d_o - 0.3 \tag{3.25}$$

where

d = flow path distance, m
L = tan-tan distance, m
d_i = inlet nozzle diameter, m
d_o = outlet nozzle diameter, m

The heavy liquid outlet nozzle (N3) is placed as close as possible to the weir location, and the light liquid outlet nozzle (N4) is placed close to the tangent line.

Elevation of low interface level shutdown (LILSD) is normally set at 150 mm from the bottom of the vessel; however, it is always advisable to check the liquid residence time for LILSD. A too short liquid residence time will increase the chances of a gas breakthrough from the separator. Generally, 1 to 3 min residence time is adequate for the shutdown level. The same principle can be used to set the low liquid level shutdown (LLLSD). This setting also depends on the type of separator being used. For the flooded weir separator, this level can be set either above or below the weir level, depending on the residence time requirement.

A low interface level (LIL) is usually used to raise an alarm for the operator's intervention. The required distance and liquid residence time between LIL and LILSD depends on the fluctuation of the incoming liquid and the possible liquid withdrawal rate. For a relatively steady flow, a 150 mm gap between LIL and LILSD is often used, with a liquid residence time ranging from 1 to 3 min. A low liquid level (LLL) can also be set for a 1 to 3 min residence time. Like LLLSD, LLL can also be set either above or below the weir level, depending on the application.

A normal interface level (NIL) is usually set to achieve the required residence time. However, a distance of 300 mm between LIL and NIL is preferable for proper working of the control system. The recommended liquid residence time (normal level) is presented in Table 3.8. Although the setting of NIL is easy, the setting of normal liquid level (NLL) is a bit of an involved procedure. In case a slug is not a major design problem, NLL is taken as the liquid level to calculate gas velocity above the liquid surface. In case slug is a problem and the design needs to consider instantaneous flow of slug, the high liquid level (HLL) should be taken to calculate the gas velocity. In such a case, the volume between NLL and HLL will be equal to the slug volume. For a three-phase application, and for this design, if water is the major source of a slug, the capacity between NIL and HIL should be equal to the slug volume.

Setting of a high liquid level shutdown (HLLSD) depends on the other internals present in the separator. Generally, the highest possible liquid level

Table 3.8 Recommended Liquid Residence Time

Service	Residence Time (Min)
Normal oil and gas inlet separator	
Heavy phase	10
Light phase	10
Glycol separation	30
Reflux to column	5
Distillate to storage	2
Compressor interstage KO drums	5
Refrigeration systems	
Intermediate stage drums	5
Main accumulator	15
Hot oil surge drum	10
Very low liquid rate (manual draining)	8 to 24 hr
Other general purpose	3 to 10

Note: Residence time is based on normal level; for combined reflux and distillate, the larger volume is used as liquid holdup.

Table 3.9 Guidelines for Residence Time Provision

Time between Control Points (Min)

	LLSD	LLL	NLL	HLL	HLLSD
LLSD		6	8	—	—
LLL	5		2[a]	—	—
NLL	6.5	1.5[a]		2[a]	8
HLL	—	—	1.5[a]		6
HLLSD	—	—	6.5	5	

Note: Values to the right of the shaded boxes are for surging flow or feed to a fired heater. Values to the left are for normal operation.

[a] Time and volume between LLL and HLL may be set by surge volume to be accepted by vessel.

Source: Gas Conditioning and Processing, 7th ed., Vol. 2, Campbell Petroleum Series, chap. 11.

within the separator is kept at least 150 mm below the lowest point of any internals present.

Table 3.9 may be used to get some idea about the liquid residence time. This table should be used only as a guide, and the designer should make his or her choice depending on the requirements.

Boot

The boot is generally used when the water flow rate is relatively small, i.e., when it is less than 15 to 20% (w/w) of the total liquid. The boot diameter

Table 3.10 Boot Diameter

Separator Diameter (D), mm	Minimum Boot Diameter (mm)	Maximum Boot Diameter
Less than 750	No boot recommended	
750 to 1500	300	0.4 D
1500 to 2400	450	0.33 D
Above 2400	500	0.33 D

usually depends on the diameter of the separator. Table 3.10 is normally used for preliminary estimation of the boot diameter.

The boot height is usually calculated by the required holdup volume of the heavy phase. A minimum boot height of 800 mm is normally recommended to accommodate the control system. However, it is advisable to limit the boot height to 1500 mm.

Vertical separator

A typical configuration of a vertical separator is shown as Figure 3.5.

As explained earlier, the height of NLL is dictated by the residence time and height requirements for the control system to work properly. Setting of HLL depends on the slug characteristics of the system. In case the design is required to handle slug, the volume between NLL and HLL should be equal to or greater than the slug volume. If slug is not a design criteria, a 150 mm gap is generally recommended.

HLLSD is normally used for system shutdown and is most commonly used in the compressor suction scrubber. It is required to avoid liquid droplets entering the compressor. A gap of 150 mm is normally kept between HLL and HLLSD.

The inlet nozzle is generally fitted with an inlet diverter. The diameter of the inlet diverter ($h2$ in Figure 3.5) is normally twice that of the inlet nozzle, as shown in Equation 3.26a:

$$h2 = 2 * di \tag{3.26a}$$

The distance between the bottom of the inlet diverter and HLLSD is kept at 0.3 m ($h1$ in Figure 3.5).

The space between the inlet nozzle and the mist eliminator is provided for the droplets to fall back to the bottom of the separator. Too small a distance may cause the droplets to be carried away with the gas stream. The following criteria is often used for estimating the distance $h3$:

$$h3 = \text{maximum of } (0.6 \text{ m or } 0.7ID) \tag{3.26b}$$

The thickness of the mist eliminator is normally in the range of 100 to 150 mm, depending on the service. The distance between the top of the mist eliminator and the top of the dished end ($h4$ in Figure 3.5) is defined as

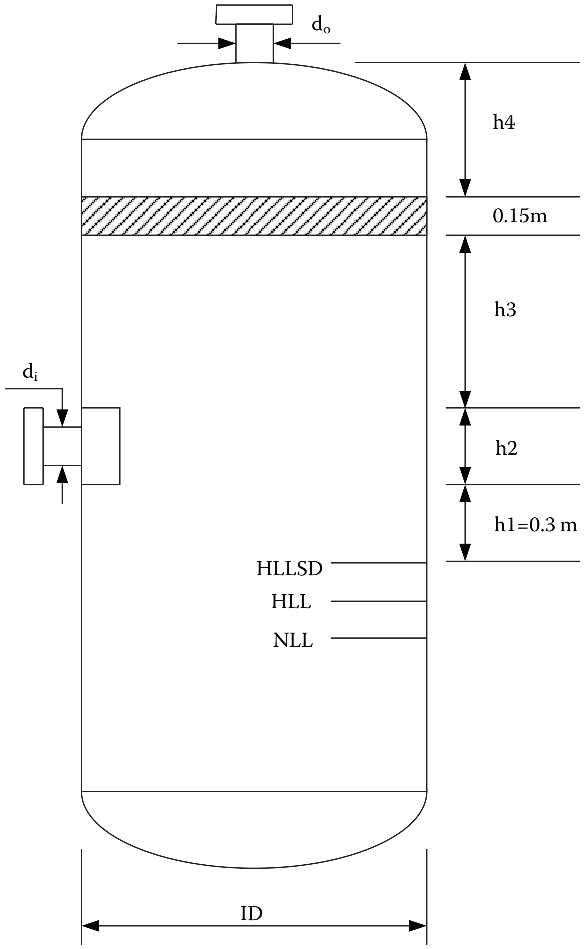


Figure 3.5 A typical vertical separator.

$$h4 = 0.5*(\text{mist eliminator OD} - do) \tag{3.26c}$$

Separator internals

The inlet nozzle

Generally, a top entry is used for the inlet nozzle. However, in some cases, side entry is also used. Except for high performance separators, internal arrangements for the inlet nozzle are quite simple. The commonly used internal arrangements for top- and side-entry separators are shown in Figure 3.6a and Figure 3.6b. For top entry, the half-pipe arrangement is quite popular, as shown in Figure 3.6a. The length of the half pipe is normally equal to the inlet pipe diameter.

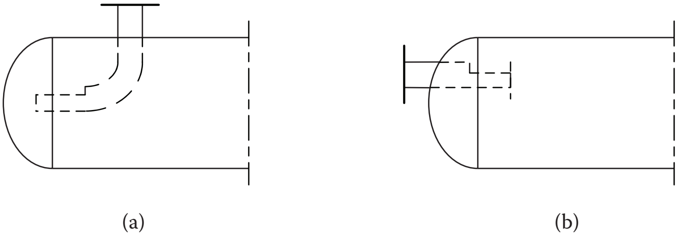

(a) (b)

Figure 3.6 (a) Inlet nozzle — top entry and (b) inlet nozzle — side entry.

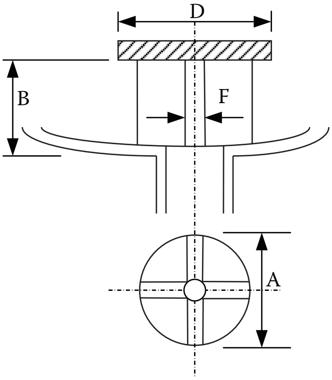


Figure 3.7 Vortex breaker.

For the side-entry separator also, the half-pipe arrangement can be used, as shown in Figure 3.6b.

The vortex breaker

The vortex breaker is used in almost all liquid-draw nozzles. There are several types of vortex breakers used; the most common one is shown as Figure 3.7.

In this design, vertical baffles are used to break the vortex. The number of baffles used depends on the nozzle diameter. Table 3.11 is often used as a preliminary guideline to design a vortex breaker. The position of the draw nozzle is also important; a vessel-flushed nozzle may not be always recommended, particularly if the incoming fluid contains inert or gummy material. The internal nozzle projection (50 to 100 mm) is often used to avoid draining of solids along with the liquid stream.

Separator control

Normally, separator controls can be broadly classified into four major headings:

Table 3.11 Vortex Breaker Dimensions

Nominal Size (mm)	Number of Baffles	A (mm)	B (mm)	D (mm)	F (mm)
40	4	75	50	85	—
50		85		100	
80		115		130	
100		140		155	
150		215	75	230	
200		290	105	305	
250	8	365	130	380	50
300		445	150	460	
350		515	180	530	
400		590	200	605	
450		665	230	680	
500	12	735	255	750	75
600		890	305	890	

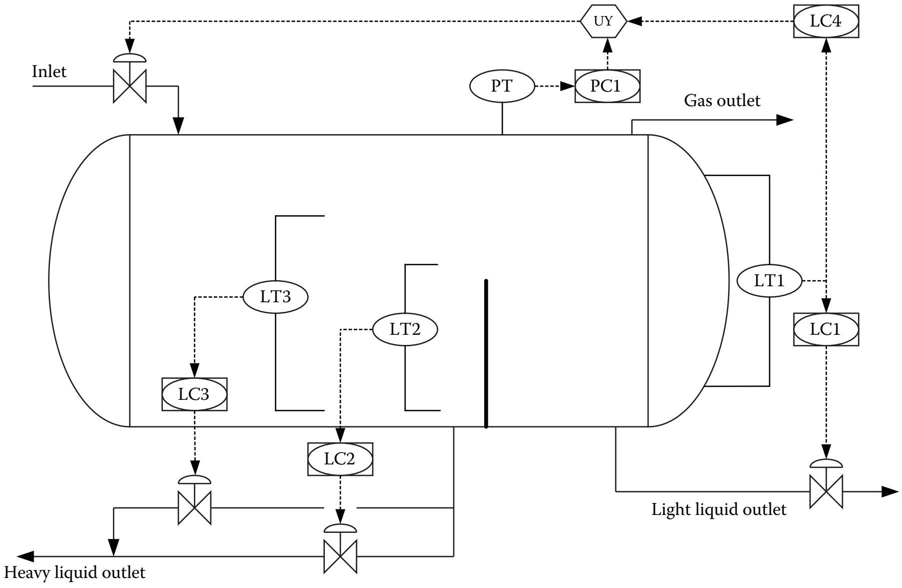


Figure 3.8 Separator control scheme.

- Pressure and flow control
- Light liquid level control
- Heavy liquid level control
- Slug control

The general control scheme is shown in Figure 3.8.

Pressure and flow control

The separator pressure and flow rate are interrelated — a change in flow rate will influence the separator pressure. Normally, an inlet pressure control valve is used to control the separator pressure (as shown in Figure 3.8). Another frequently used pressure control system is to install a control valve on the vapor outlet line; this control system is rather popular when the separator downstream pressure is quite low compared to the separator pressure.

Light liquid level control

A light liquid level signal (LT1) is used to control the level of the light liquid. This control valve is designed to enable maximum possible light liquid flow. However, care should be taken not to overdesign this control valve, particularly if the condensate collection system operates at substantially lower pressure in comparison to the separator pressure. A larger control valve increases the gas breakthrough quantity and may cause overpressure on the condensate recovery system.

Under a large slug condition, the condensate level can go very high. This situation causes severe liquid entrainment into the gas phase. It is a practice to reduce the inlet flow rate when the light liquid level increases substantially; at high liquid level, the level control overrides the pressure control (through a low select UY) and starts reducing the inlet flow.

Heavy liquid level and slug control

Heavy liquid level control is actually the control of the interface level. For three-phase separators with weir, the interface level must be below the weir height; this is to avoid water passing into the condensate phase. In case of severe slug problems, two control valve systems are often used. A control through LC2 is the normal control system to maintain the interface level. In case there is a sudden flow of slug, the interface level increases in a very short time, the control system LC3 gets activated, and the corresponding control valve opens. Normally, LC3 operates as an on/off control to remove excess water. When the interface level falls below a certain set point, the LC3 signal closes the corresponding control valve.

High performance separator

The high performance separator technology has been developed to reduce the size of the separator. A reduced size not only reduces the weight of the separator, but the space requirement is also reduced. In offshore platforms, the high performance separator is always preferred due to its weight and space advantage. In general, high performance internals are used to increase the separation efficiency and the design K value, which reduces the overall dimensions of the separator.

One high performance separator is the Gas-Liquid Cylindrical Cyclone (GLCC) separator. It is a simple, compact, lightweight, and inexpensive bulk gas or liquid separator, suited for many different applications. The GLCC, patented by Chevron, is a device that is available to the oil and gas industry through members of Tulsa University Separation Technology Project (TUSIP). TUSIP is a joint industry program involving thirteen companies from petroleum and petroleum equipment industries. Among others, NATCO, as a member of TUSIP, has a license and the necessary expertise to design and manufacture the GLCC and related separation systems. A typical control scheme of GLCC is shown in Figure 3.9

The process fluid enters a sloped tangential inlet nozzle, sized to deliver a preconditioned flow stream into the body of the separator. The momentum of the process fluid, combined with the tangential inlet, generates a liquid

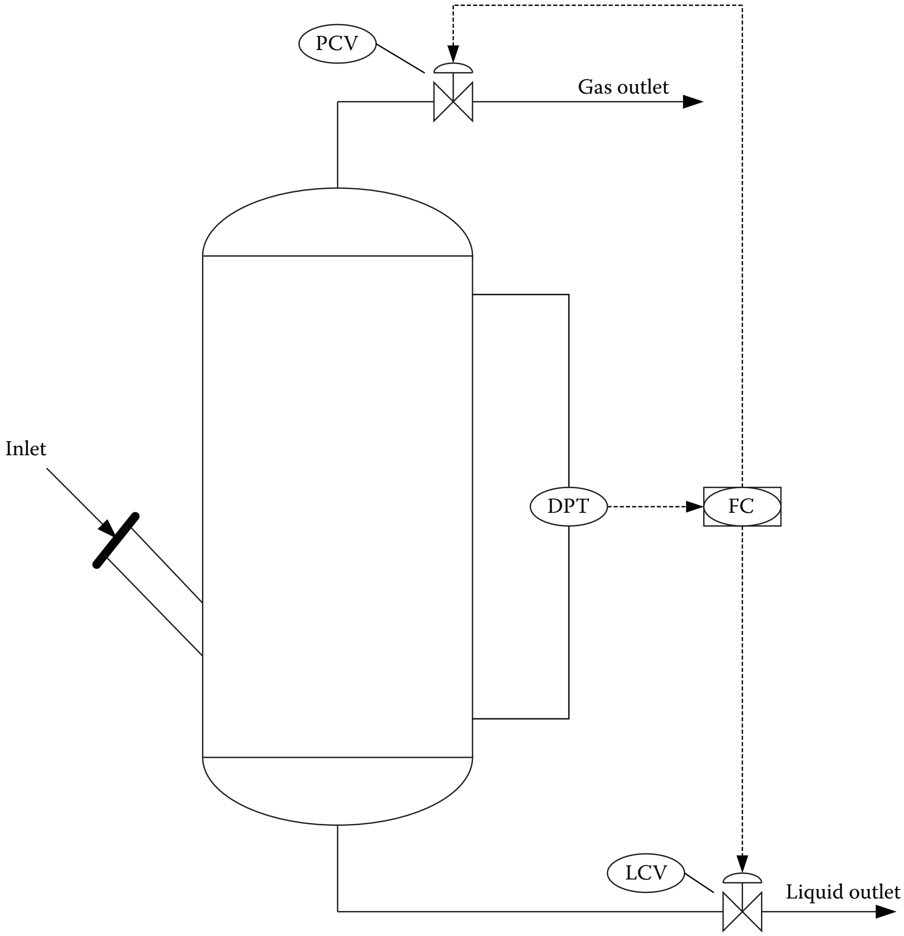


Figure 3.9 Control scheme of GLCC.

vortex with sufficient "*g*" force for the bulk gas and liquid separation to rapidly occur. The liquid is held in the base of the unit, below the level of the inlet, and is bled down with some form of control system. Gas exits through the top of the GLCC.

Salient features of GLCC

- The GLCC is used primarily for test separators, though other uses are not uncommon.
- The separation is largely influenced by the high "*g*" value; the typical value varies from 15 to 100+, with lower "*g*" for bigger vessels.
- The typical recommended performance level is 0.27 m^3 liquid removal per $MMsm^3$ of gas.
- GLCC is not very effective for slug handling.

Design parameters

Physical properties of fluids are normally available through simulation packages. For a good design, it is often required to modify these available physical parameters. Also, it is important to get a reasonable idea about the design margin required for upset conditions.

Flow rates

Flow rates for gases, light liquids, and heavy liquids that are required are normally available from other sources. Variation of gas flows needs to be carefully estimated, and for a reasonably steady situation, a margin of 10% is often adequate.

Estimation of liquid flow variation is more complex due to our inadequate understanding of slug behavior. Slug flow is very common in the oil and gas industry, particularly in long pipelines. In gas processing plants, sometimes the design of inlet separators fails due to poor estimation of slug flow. Dynamic analysis is required for a correct estimation of slug flow; however, it is not uncommon to get a highly inaccurate slug flow level through dynamic simulation. Dynamic simulation must be done with a clear understanding of the pipe profile, the piping geometry, and most importantly, the dynamic simulation itself. Normally, the slug arrival period is very short, but for design purposes, the flow can be assumed to be instantaneous. A provision for slug holding is required between the normal liquid level and the highest acceptable operating level.

Slug length [8]

A large number of studies have been conducted to predict slug length, but none of these studies have covered the full range of operations. One study was conducted in the Prudhoe Bay field in Alaska [8]. During this study, more than 19 million data points were collected on 300, 400, 500, and 600 nominal bore (NB) pipes. This study was conducted for horizontal pipes, and as per the study, the slug length is a function of pipe diameter, superficial gas velocity, and length of pipe. The slug length can be calculated by using the correlations developed through this study; however, it is recommended to use a minimum slug length of 32*di*, where *di* is the pipe diameter in meters. The reference slug length is defined as

$$\ln\left(\frac{L_{sr}}{0.3048}\right) = -25.4144 + 28.4948 * \left\{\ln\left(39.37 d_i\right)\right\}^{0.1} \tag{3.27}$$

and

$$L_s = L_{sr} G_D \tag{3.28}$$

The dimensionless slug growth (G_D) is defined as

$$G_D = (C + G_D\,|_{x_t})\,\frac{\ln(x/0.3048)}{\ln(x_t/0.3048)} - C \tag{3.29}$$

where $C = 5.29$ when $x \le x_t$ and $C = 2.35$ when $x > x_t$.

The value of x_t depends on the pipe diameter and superficial gas velocity; it is expressed by the following equation:

$$x_t = 880\ d\ v_{sg} \tag{3.30}$$

The dimensionless slug growth at the point of transition x_t can be expressed as

$$G_D\,|_{x_t} = 0.309 e^{0.1329 v_{sg}} \tag{3.31}$$

Example 3.1

Estimate the slug volume for a 400 NB (ID = 406 mm), 3000 m long horizontal pipeline. The superficial gas velocity is 10 m/sec.

Solution

$$x = 3000 \text{ m}$$

$$x_t \text{ (from Equation 3.30)} = 880 * 10 * 0.406$$

$$= 3572.8 \text{ m}$$

As x < x_t, the value of C = 5.29

$$G_D\,|_{x_t} \text{ (from Equation 3.31)} = 0.309e^{0.1329*10}$$

$$= 1.1672$$

$$G_D \text{ (from Equation 3.29)} = (5.29+1.1672) * 0.9814 - 5.29$$

$$= 1.047$$

From Equation 3.27,

$$\ln\frac{L_{sr}}{0.3048} = -25.4144 + 28.4948\left\{\ln\left(39.37 * 0.406\right)\right\}^{0.1}$$

or, L_{sr} = 141.2 m.

Finally, from Equation 3.28, the slug length

$$L_s = 141.2 * 1.047$$

$$= 147.8 \text{ m}$$

The calculated slug volume is 19.1 m^3.

Density

Density is normally obtained from simulation, and it is used directly. Sometimes a two-liquid phase is separated as a single phase, and the density of the mixed liquid is calculated as

$$\rho_m = \frac{m_l + m_h}{m_l / \rho_{light} + m_h / \rho_{heavy}} \tag{3.32}$$

Though the design of a separator is done assuming a single-liquid phase, in actual practice, there will be phase separation within the separator. In case a two-liquid phase is separated as a single phase, the impact of the phase separation within the vessel needs to be established carefully.

Viscosity

Generally, viscosity is obtained from the simulation. Viscosity is the most important parameter that influences the design of a separator. As the simulation package normally establishes physical parameters based on pure components, the impact of gummy substances and other impurities is ignored. An actual viscosity that is 20 to 50 times more than the simulated viscosity is not uncommon. Sometimes it is required to establish the viscosity in the laboratory.

The general equation of viscosity to temperature can be defined as

$$\ln \mu = A + \frac{B}{C + t} \tag{3.33}$$

This equation can be used to calculate viscosity at any temperature, if the value is known for three different temperatures.

Oil–water emulsion is very common in the oil and gas industry, and the viscosity of the emulsion is quite different from that of the individual phases. Emulsion viscosity also depends on the inversion point. If the water cut is below the inversion point, emulsion viscosity is defined as [6]

$$\mu_L = \mu_O \left\{ 1 + \frac{\frac{\phi}{K_O}}{1.187 - \frac{\phi}{K_O}} \right\}^{2.5} \tag{3.34}$$

The factor K_O is calculated as

$$K_O = \frac{\phi_C}{1.187 \left\{ 1 - \left(\frac{1}{\mu_{rel\max}} \right)^{0.4} \right\}} \tag{3.35}$$

If the value of the maximum relative viscosity (μ_{relmax}) is not known, a value of 7.06 is often used. If the water cut is above the inversion point, the emulsion viscosity is defined as [6]

$$\mu_L = \mu_W \left\{ 1 + \frac{\frac{1 - \phi}{K_W}}{1.187 - \frac{1 - \phi}{K_W}} \right\}^{2.5} \tag{3.36}$$

The factor K_W is calculated as

$$K_W = \frac{1 - \phi_C}{1.187\left\{1 - \left(\dfrac{R}{\mu_{rel\max}}\right)^{0.4}\right\}} \tag{3.37}$$

where R is defined as the ratio of water and oil viscosity at a given temperature. For medium-viscosity oil at 60°C, the value of *R* can be approximated to 0.42.

Water viscosity is normally calculated using the following equation. This calculation is generally valid for the temperature range of 0 to 150°C.

$$\eta_W = \exp\left(-3.82684 + \frac{609.2462}{138.8927 + t}\right) \tag{3.38}$$

Oil in gas droplet size

The separation efficiency of oil in gas depends upon the separation requirements of the oil droplets. Under gravity separation, it is not easy to separate small droplets from the gas phase. Many vendors claim that they can remove all the particles down to 10 μm. This is not commercially viable using an entrainment-type separator. In the oil and gas industry, most separators are sized to remove liquid droplets larger than 150 μm. Smaller droplets normally coalesce to form larger droplets (in the range of 150 μm) before they can be separated.

Oil in water droplet size

Oil in water droplets indicate that water is in continuous phase and oil is in dispersible phase. This is in fact the rising of oil droplets through the bottom water phase. Some oil droplets may settle out along with the water droplets, and these oil droplets should be separated from the water phase before the water phase exits from the separator. The oil droplet separation time must be less than water phase residence time. The separation efficiency depends on the size of the oil droplets, and the size depends on the droplet distribution in the incoming fluid. Droplets of 150 μm and below will normally pass through the gas phase; larger droplets have to be separated from the water phase. For design purposes, an average droplet size in the range of 200 to 500 μm is generally considered. For better results, a droplet distribution analysis is required to obtain the correct droplet size.

Water in oil droplet size

In this case, oil is in a continuous phase and water is in a dispersible phase. Water droplets settle down through the oil phase, and as explained earlier, the settling time must be less than the oil residence time. Water droplets are normally large in size, and for design purposes, a range of 300 to 600 μm can be used.

Inlet nozzle velocity

It is a common practice to design the inlet nozzle based on the momentum (ρv^2) of the entering fluid. The nozzle size depends on the presence of internals in the inlet nozzle. The following guidelines are often used to size the inlet nozzle:

$$(\rho v^2)_{\text{max}} \text{ inlet nozzle: no inlet device} \leq 1000 \text{ kg/ms}^2$$

$$\text{half-open pipe inlet} \leq 1500 \text{ kg/ms}^2$$

Gas outlet nozzle velocity

As the gas outlet nozzle is not required to be designed for a two-phase flow, a higher momentum can be used for the design. Normally, the following value is used:

$$(\rho v^2)_{\text{max}} \text{ gas outlet nozzle: } \leq 3600 \text{ kg/ms}^2$$

Liquid outlet velocity

Liquid outlet nozzles are generally designed based on the liquid superficial velocity of 1 m/sec.

Separator program

A separator program, separator.exe, has been developed to design both horizontal and vertical separators. In fact, this program has been developed to rate a separator for certain configurations. Each form is subdivided into several frames, and the frames, Physical Properties and Separator Dimensions, are the input frames. Other frames are program output frames. The Check Results frame normally captures the design problems, and if the design is acceptable, all check parameters will be marked as OK. Separators can be designed using SI or English units. The units are fixed and the program does not allow changing individual units. Five horizontal and two vertical types of separator can be designed using this program.

Program limitations/notes

- This program has been developed to rate a separator rather than to design a separator.
- For horizontal separators, the separator performance is checked for normal liquid level. If the performance is to be checked for high liquid level, the value of the high liquid level should be entered as normal liquid level.
- For a two-phase liquid–liquid separator, the program assumes that the interface level is at the center of the separator.
- The separator design using the API 521 method has been developed only for the horizontal separator.
- The separator design using the API 521 method cannot check for *K* factor velocity; it is always better to check the separator adequacy for *K* velocity.
- The separator design using API 521 method uses a high-level shutdown (instead of normal liquid level, as in other cases) to calculate the separator length.

Horizontal separators

Three-phase flooded weir

A three-phase flooded weir separator is normally used where gas, water, and condensate flow rates are very high. In this design, NLL must be above the weir height. In the flooded-weir design, water and condensate holdups are high. This type of separator is very common where slug is a design consideration.

Three-phase nonflooded-weir separator

A three-phase nonflooded-weir design is commonly used for low condensate loading. In this design, NLL must be below the weir height. Though this design can give a high water holdup capacity, the condensate holdup is normally low. This design is often used for steady flow rate.

Three-phase with boot separator

A three-phase with boot design is used where water load is low (typically, reflux drums). The boot section is used to hold the water, and the cylindrical section is used to hold the condensate. In this design, the interface level is kept within the boot, though a design with the interface in the cylindrical section is also possible. The separator.exe design only considers interface level within the boot.

Two-phase vapor–liquid separator

The two-phase separator is basically a gas–liquid separator. In case a two-liquid phase is separated as one, the mixed liquid properties are to be considered for the design. Using two immiscible liquid phases as one sometimes creates other design problems. In fact, there will be phase separation within the separator and, as the draw nozzle is at the bottom of the vessel, the light liquid will gradually build up within the separator; at one stage, only the light liquid will be discharged from the separator.

Two-phase liquid–liquid separator

A two-phase liquid–liquid horizontal separator is designed based on the residence time and interface flow. For horizontal configuration, the separator diameter depends on the location of the interface, and the diameter will be minimum if the interface is at the center of the vessel. It is better to design a liquid–liquid separator with the interface level at the center of the vessel. The separator.exe program assumes the interface level is at the center of the separator.

Vertical separators

Two-phase vapor–liquid separator

The design principle is the same as discussed earlier.

Two-phase liquid–liquid separator

The design principle is the same as discussed earlier, except mainly that the interface area is constant and is not dependent on the location of the interface.

General overview of the separator.exe program

Double-click separator.exe, and the first form will be as shown in Figure 3.10. This form gives the option to select the type of separator, as explained earlier.

In case a three-phase flooded-weir separator is selected, the form will appear as shown in Figure 3.11.

The available menus are (1) File and (2) Unit. The following operations are possible through the File menu:

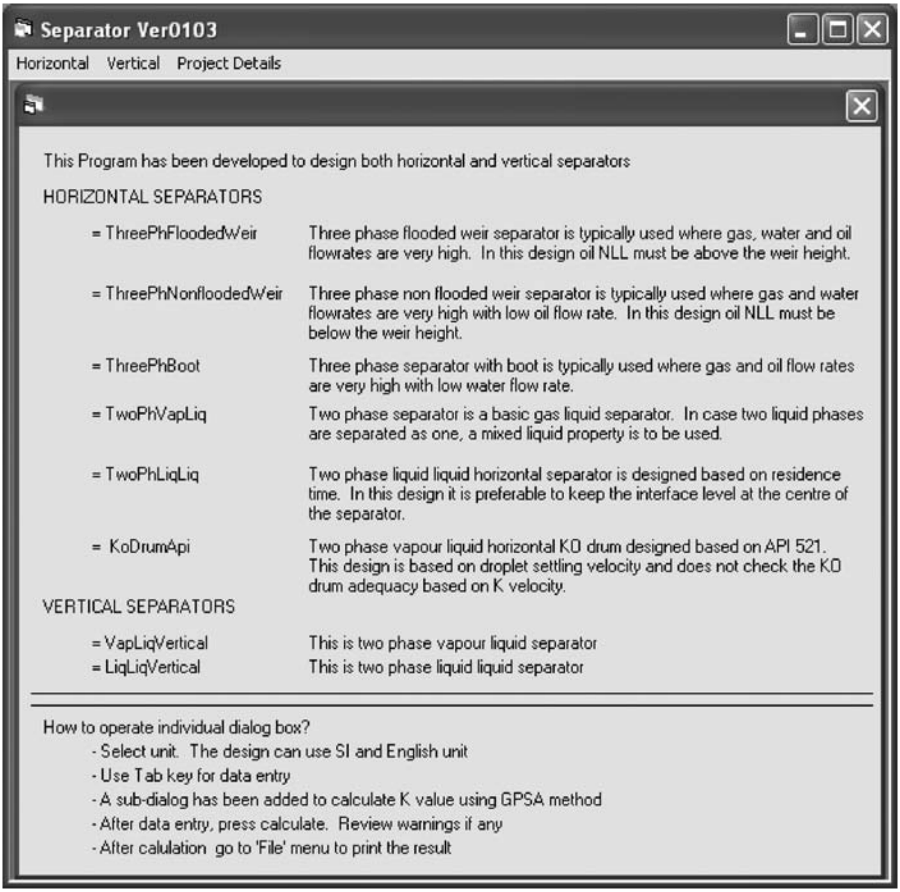
File Open: This menu will open a previously saved file. Only the input data are saved and opened. As each type of separator has a different format, different file extensions are used to open or save a file. This is indicated in Table 3.12

File Save: Once a calculation has been done this menu will save the input parameters.

File Print: Once a calculation has been completed this menu will print the output.

Exit: This menu is to exit from the program.

Under Unit menu, the following operations are available:



This Program has been developed to design both horizontal and vertical separators

HORIZONTAL SEPARATORS

= ThreePhFloodedWeir	Three phase flooded weir separator is typically used where gas, water and oil flowrates are very high. In this design oil NLL must be above the weir height.
= ThreePhNonfloodedWeir	Three phase non flooded weir separator is typically used where gas and water flowrates are very high with low oil flow rate. In this design oil NLL must be below the weir height.
= ThreePhBoot	Three phase separator with boot is typically used where gas and oil flow rates are very high with low water flow rate.
= TwoPhVapLiq	Two phase separator is a basic gas liquid separator. In case two liquid phases are separated as one, a mixed liquid property is to be used.
= TwoPhLiqLiq	Two phase liquid liquid horizontal separator is designed based on residence time. In this design it is preferable to keep the interface level at the centre of the separator.
= KoDrumApi	Two phase vapour liquid horizontal KO drum designed based on API 521. This design is based on droplet settling velocity and does not check the KO drum adequacy based on K velocity.

VERTICAL SEPARATORS

= VapLiqVertical	This is two phase vapour liquid separator
= LiqLiqVertical	This is two phase liquid liquid separator

How to operate individual dialog box?

- Select unit. The design can use SI and English unit
- Use Tab key for data entry
- A sub-dialog has been added to calculate K value using GPSA method
- After data entry, press calculate. Review warnings if any
- After calulation go to 'File' menu to print the result

Figure 3.10 Separator design — main form.

SI Unit: This will set all the units in SI format.
Eng Unit: This will set all the units in English format.

Each separator design form has been subdivided into a number of frames. The frames Physical Properties and Separator Dimensions are used as input frames. Other frames are the Results and Check Frames.

Each form contains three main buttons: Calculate, Clear, and Cancel.

Calculate: This button is to carry out the design once the input information is complete. The unit must be selected before performing the calculation, otherwise a message box will appear saying, "Select proper unit before calculation."
Clear: This button is to clear all data (except units); a fresh calculation can then be carried out.
Cancel: This button will return the program to the main form.

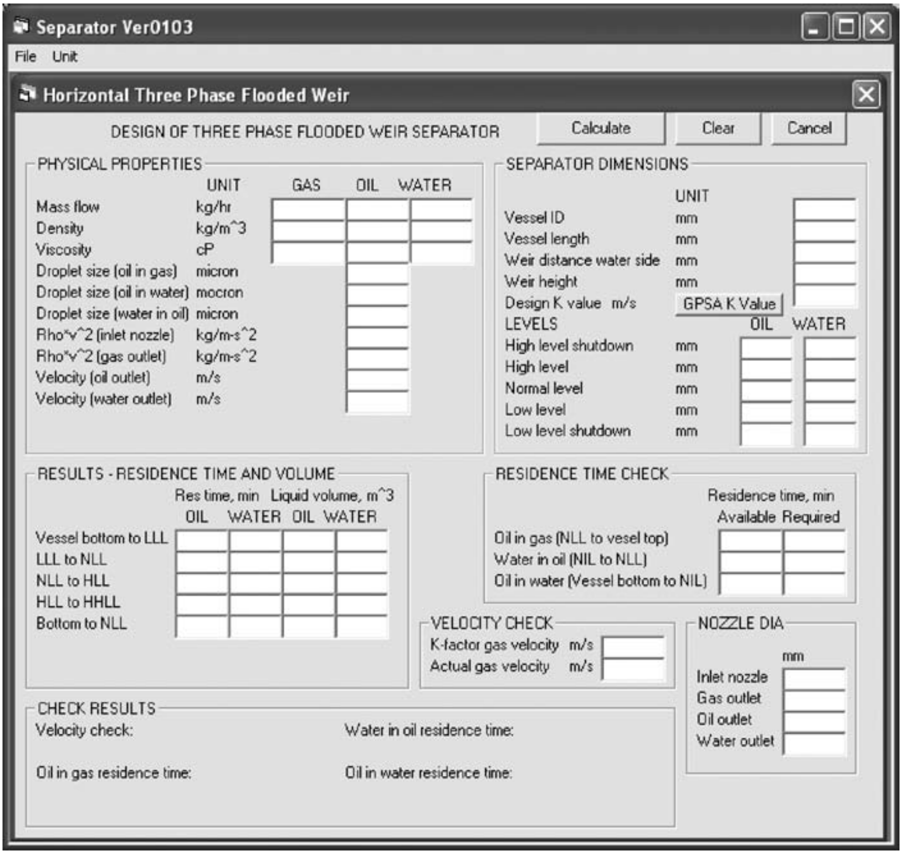
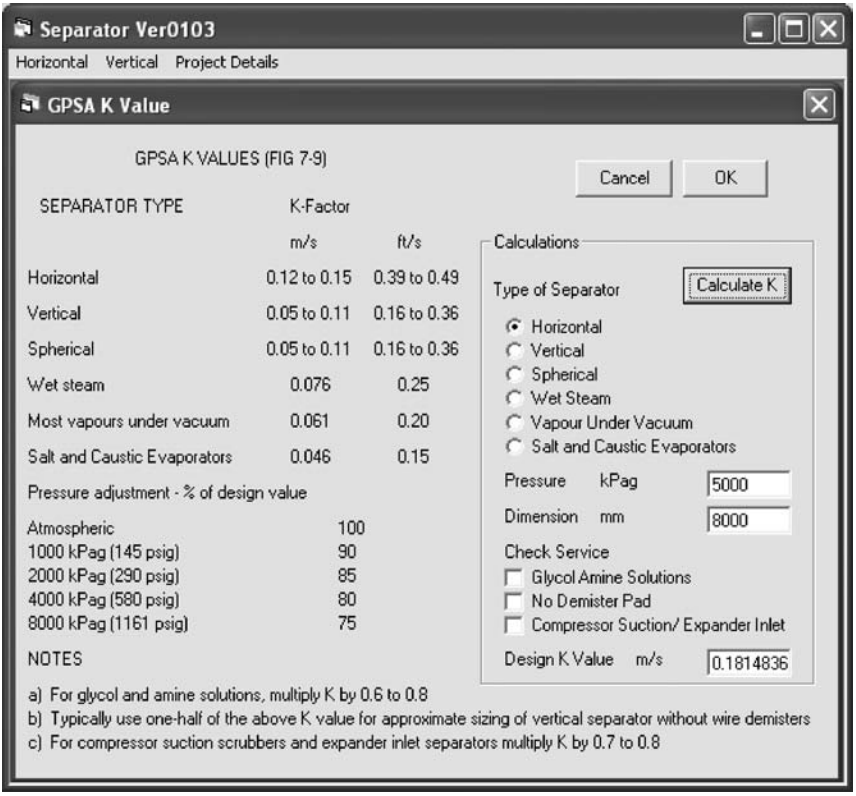


Figure 3.11 Three-phase flooded-weir separator design.

Table 3.12 File Extension Used in Separator.exe Program

Type of Separator	File Extension
Three-phase flooded-weir	*.s01
Three-phase nonflooded-weir	*.s02
Three-phase boot	*.s03
Two-phase vapor–liquid	*.s04
Two-phase liquid–liquid	*.s05
Vapor–liquid vertical	*.s06
Liquid–liquid vertical	*.s07
KO drum as per API 521	*.s08



GPSA K VALUES (FIG 7-9)

SEPARATOR TYPE	K-Factor m/s	ft/s
Horizontal	0.12 to 0.15	0.39 to 0.49
Vertical	0.05 to 0.11	0.16 to 0.36
Spherical	0.05 to 0.11	0.16 to 0.36
Wet steam	0.076	0.25
Most vapours under vacuum	0.061	0.20
Salt and Caustic Evaporators	0.046	0.15

Pressure adjustment - % of design value

Atmospheric	100
1000 kPag (145 psig)	90
2000 kPag (290 psig)	85
4000 kPag (580 psig)	80
8000 kPag (1161 psig)	75

NOTES

a) For glycol and amine solutions, multiply K by 0.6 to 0.8
b) Typically use one-half of the above K value for approximate sizing of vertical separator without wire demisters
c) For compressor suction scrubbers and expander inlet separators multiply K by 0.7 to 0.8

Calculations

Type of Separator
● Horizontal
○ Vertical
○ Spherical
○ Wet Steam
○ Vapour Under Vacuum
○ Salt and Caustic Evaporators

Pressure kPag: 5000
Dimension mm: 8000

Check Service
☐ Glycol Amine Solutions
☐ No Demister Pad
☐ Compressor Suction/ Expander Inlet

Design K Value m/s: 0.1814836

Figure 3.12 Calculation of *K* value.

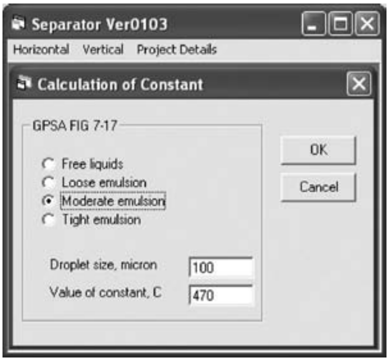
For a gas–liquid separator design, the button GPSA *K* value is used to calculate the *K* value as per GPSA. It will open a form, as shown in Figure 3.12. The unit will automatically be selected based on the original unit. The dimension will also be taken from the main calculation sheet. The user has to complete the following parameters:

- Type of separator
- Operating pressure (at default unit)
- Check service

After completing the data, press Calculate to calculate the *K* value. The OK button is used to transfer the *K* value to the main calculation form.

The Cancel button is used to cancel the calculation and to go back to the main calculation form.

For a liquid–liquid two-phase separation, the value of constant *C*, the retention time, and the interface flow can be calculated by clicking the respective Suggestion buttons. The typical formats are shown as Figure 3.13 and Figure 3.14, respectively.

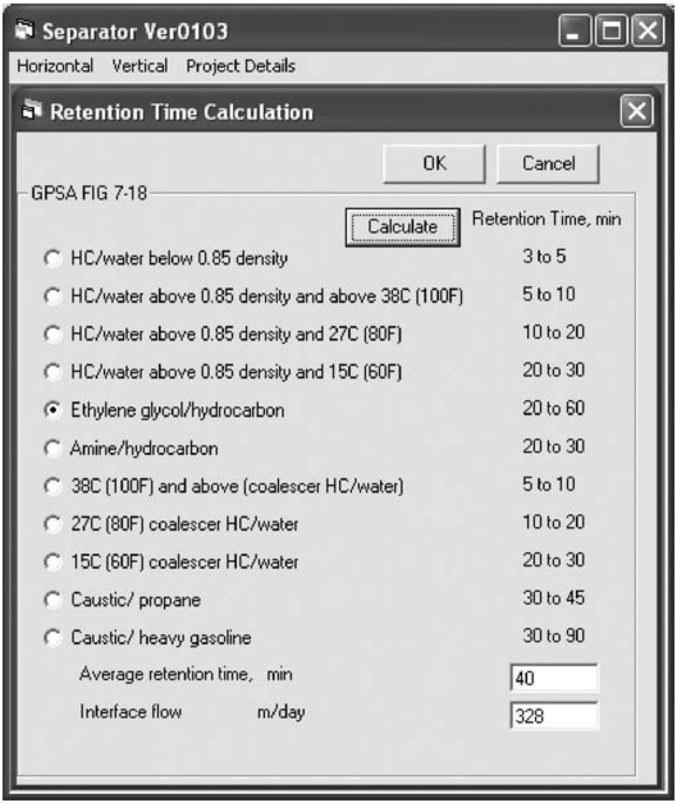


GPSA FIG 7-17

○ Free liquids
○ Loose emulsion
● Moderate emulsion
○ Tight emulsion

Droplet size, micron 100
Value of constant, C 470

Figure 3.13 Calculation of constant.



GPSA FIG 7-18

	Retention Time, min
○ HC/water below 0.85 density	3 to 5
○ HC/water above 0.85 density and above 38C (100F)	5 to 10
○ HC/water above 0.85 density and 27C (80F)	10 to 20
○ HC/water above 0.85 density and 15C (60F)	20 to 30
● Ethylene glycol/hydrocarbon	20 to 60
○ Amine/hydrocarbon	20 to 30
○ 38C (100F) and above (coalescer HC/water)	5 to 10
○ 27C (80F) coalescer HC/water	10 to 20
○ 15C (60F) coalescer HC/water	20 to 30
○ Caustic/ propane	30 to 45
○ Caustic/ heavy gasoline	30 to 90

Average retention time, min 40
Interface flow m/day 328

Figure 3.14 Residence time.

Example 3.2

Design a three-phase separator with the following parameters:

Gas	Flow	100000 kg/h
	Density	55.2 kg/m^3
	Viscosity	0.013 cP
Condensate	Flow	60000 kg/h
	Density	720 kg/m^3
	Viscosity	3.0 cP
Water	Flow	50000 kg/h
	Density	1000 kg/m^3
	Viscosity	0.7 cP
Other parameters	Oil in gas droplet size	150 μm
	Oil in water droplet size	350 μm
	Water in oil droplet size	500 μm
	Slug volume (water slug)	7 m^3
	Inlet nozzle momentum	1500 kg/(m)(s^2)
	Momentum (gas outlet)	3600 kg/(m)(s^2)
	Liquid outlet velocity	1 m/sec
	Operating pressure	5000 kPag

Design

As the flow rates are quite high and the design needs to accommodate 7 m^3 of slug, a horizontal three-phase flooded-weir separator has been selected.

Slug volume

It is assumed that slug arrival will be instantaneous. In the water phase, the slug capacity will be equal to the volume difference between NIL and HIL. In this case it will be 7 m^3. With the addition of slug, the normal condensate level will also increase, and the volume difference between NLL and HLL should also be to 7 m^3.

The separator.exe program calculates the vapor space area above NLL. However, due to the presence of slug, the calculation should be performed for the vapor space above HLL. This can be done by checking the calculation, taking NLL as HLL, i.e., NLL level = 1800 mm.

This is a rating program, and the designer should have a fairly good idea of the separator design. A trial-and-error approach is required for working out the final sizing. The following dimensions are used for the separator design:

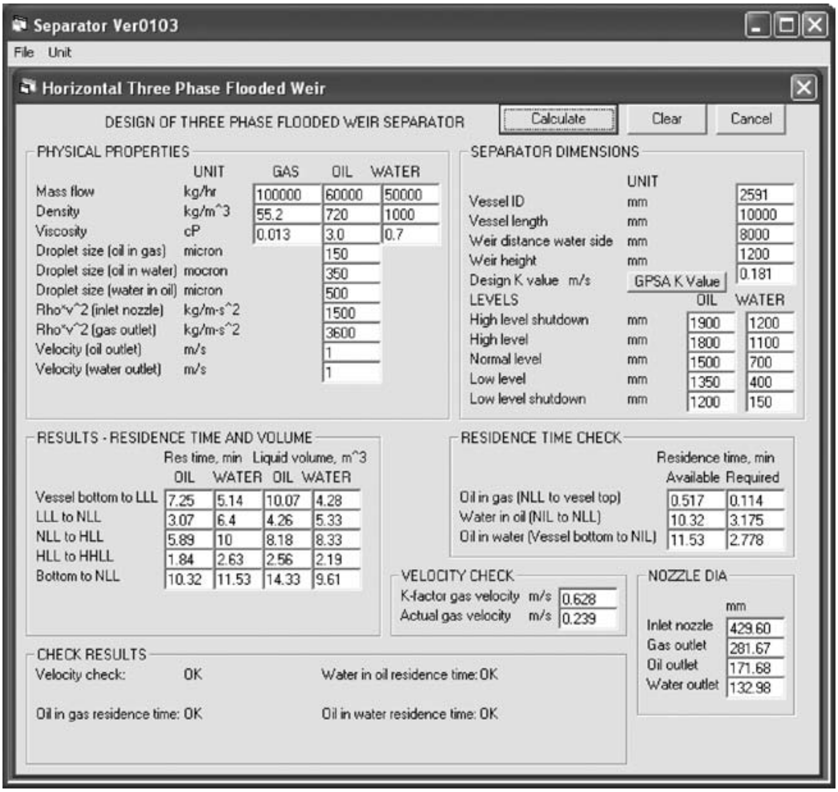
Inner diameter	2,591 mm
Total length	10,000 mm

Weir distance (water side)	8,000 mm
Weir height	1,200 mm

Level Setting, mm

	Condensate	Water	Remarks
High level shutdown	1900	1200	Water shutdown level should not be set above the weir height.
High level	1800	1100	
Normal level	1500	700	
Low level	1350	400	
Low level shutdown	1200	150	Condensate low level shutdown should not be set below weir height.

The program output is shown as Figure 3.15. The result can also be printed in a standard format.



DESIGN OF THREE PHASE FLOODED WEIR SEPARATOR

PHYSICAL PROPERTIES

	UNIT	GAS	OIL	WATER
Mass flow	kg/hr	100000	60000	50000
Density	kg/m^3	55.2	720	1000
Viscosity	cP	0.013	3.0	0.7
Droplet size (oil in gas)	micron		150	
Droplet size (oil in water)	mocron		350	
Droplet size (water in oil)	micron		500	
Rho*v^2 (inlet nozzle)	kg/m-s^2		1500	
Rho*v^2 (gas outlet)	kg/m-s^2		3600	
Velocity (oil outlet)	m/s		1	
Velocity (water outlet)	m/s		1	

SEPARATOR DIMENSIONS

	UNIT	
Vessel ID	mm	2591
Vessel length	mm	10000
Weir distance water side	mm	8000
Weir height	mm	1200
Design K value m/s		0.181

LEVELS		OIL	WATER
High level shutdown	mm	1900	1200
High level	mm	1800	1100
Normal level	mm	1500	700
Low level	mm	1350	400
Low level shutdown	mm	1200	150

RESULTS - RESIDENCE TIME AND VOLUME

	Res time, min		Liquid volume, m^3	
	OIL	WATER	OIL	WATER
Vessel bottom to LLL	7.25	5.14	10.07	4.28
LLL to NLL	3.07	6.4	4.26	5.33
NLL to HLL	5.89	10	8.18	8.33
HLL to HHLL	1.84	2.63	2.56	2.19
Bottom to NLL	10.32	11.53	14.33	9.61

RESIDENCE TIME CHECK

	Residence time, min	
	Available	Required
Oil in gas (NLL to vesel top)	0.517	0.114
Water in oil (NIL to NLL)	10.32	3.175
Oil in water (Vessel bottom to NIL)	11.53	2.778

VELOCITY CHECK

K-factor gas velocity	m/s	0.628
Actual gas velocity	m/s	0.239

NOZZLE DIA

	mm
Inlet nozzle	429.60
Gas outlet	281.67
Oil outlet	171.68
Water outlet	132.98

CHECK RESULTS

Velocity check: OK

Water in oil residence time: OK

Oil in gas residence time: OK

Oil in water residence time: OK

Figure 3.15 Example 3.2.

Further checking and analysis

- Under slugging, the condensate NLL will increase to HLL (for a 7 m^3 slug). The velocity check should be performed taking NLL as 1800 mm. The values obtained from the program are:
  K-factor gas velocity = 0.628 m/sec
  Actual gas velocity = 0.369 m/sec

  The above figures indicate that some margin is available in the design, i.e., the diameter can be reduced further. However, reduction of the diameter will have the following problems:
  1. The space between HLL (1900 mm) and the vessel top (2591 mm) will be reduced and installation of the demister pad will be difficult.
  2. The slug holdup volume will be reduced.
- The residence time check indicates that available residence time is much more than required. This does not necessarily mean that the separator is overdesigned; it shows that smaller droplets can be separated.
- In the residence time and volume frame, the volume between NLL and HLL for water is 8.33 m^3 and the condensate is 8.18 m^3. These are adequate for a slug volume of 7 m^3.
- The residence time from bottom to NLL is 10.32 min for the condensate and 11.13 min for water. The normal practice is to have a 10 min residence time.
- Check results show no design issues.

The design is adequate.

Example 3.3

Design a reflux drum with the following conditions:

Gas	Flow	60000 kg/h
	Density	3.5 kg/m^3
	Viscosity	0.012 cP
Condensate	Flow	20000 kg/h
	Density	600 kg/m^3
	Viscosity	1.0 cP
Water	Flow	200 kg/h
	Density	1000 kg/m^3
	Viscosity	0.7 cP
Other parameters	Oil-in-gas droplet size	200 µm
	Oil-in-water droplet size	300 µm
	Water-in-oil droplet size	400 µm

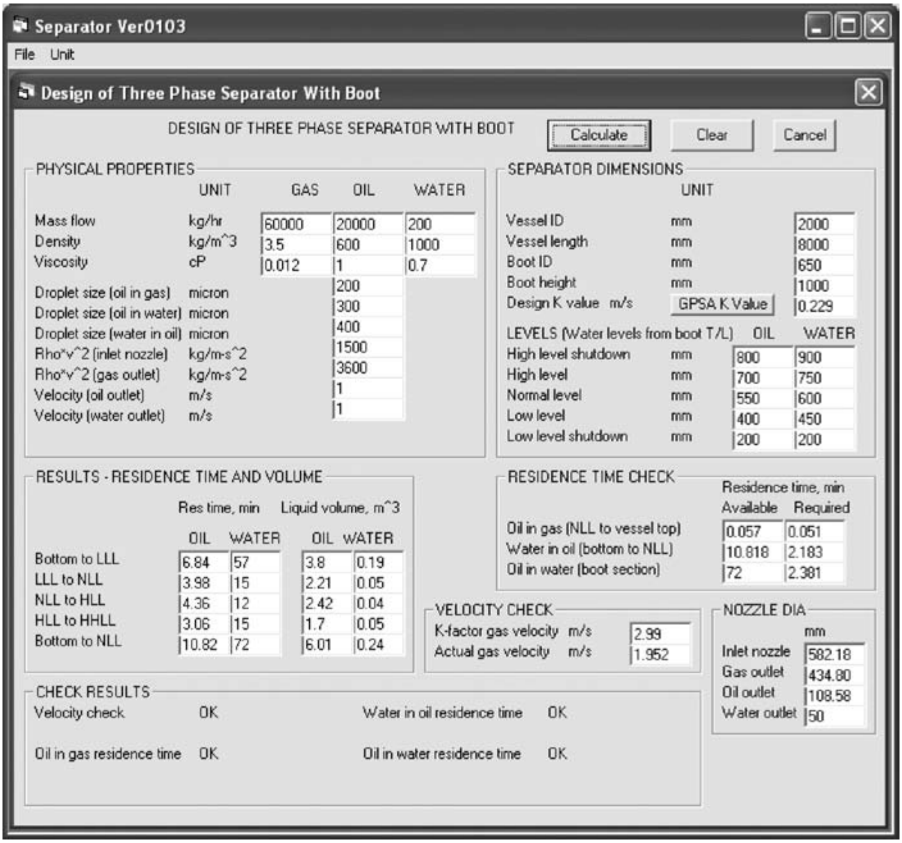
Inlet nozzle momentum	1500 kg/(m)(s^2)
Momentum (gas outlet)	3600 kg/(m)(s^2)
Liquid outlet velocity	1 m/sec
Operating pressure	350 kPag

Design

As the water rate is very low, a horizontal separator with boot is selected. The design output is shown in Figure 3.16.

The following dimensions are used:

Vessel diameter	2000 mm
Vessel length	8000 mm
Boot diameter	650 mm
Boot height	1000 mm



DESIGN OF THREE PHASE SEPARATOR WITH BOOT

PHYSICAL PROPERTIES

	UNIT	GAS	OIL	WATER
Mass flow	kg/hr	60000	20000	200
Density	kg/m^3	3.5	600	1000
Viscosity	cP	0.012	1	0.7
Droplet size (oil in gas)	micron		200	
Droplet size (oil in water)	micron		300	
Droplet size (water in oil)	micron		400	
Rho*v^2 (inlet nozzle)	kg/m-s^2		1500	
Rho*v^2 (gas outlet)	kg/m-s^2		3600	
Velocity (oil outlet)	m/s		1	
Velocity (water outlet)	m/s		1	

SEPARATOR DIMENSIONS

	UNIT	
Vessel ID	mm	2000
Vessel length	mm	8000
Boot ID	mm	650
Boot height	mm	1000
Design K value	m/s	0.229

LEVELS (Water levels from boot T/L)		OIL	WATER
High level shutdown	mm	800	900
High level	mm	700	750
Normal level	mm	550	600
Low level	mm	400	450
Low level shutdown	mm	200	200

RESULTS - RESIDENCE TIME AND VOLUME

	Res time, min		Liquid volume, m^3	
	OIL	WATER	OIL	WATER
Bottom to LLL	6.84	57	3.8	0.19
LLL to NLL	3.98	15	2.21	0.05
NLL to HLL	4.36	12	2.42	0.04
HLL to HHLL	3.06	15	1.7	0.05
Bottom to NLL	10.82	72	6.01	0.24

RESIDENCE TIME CHECK

	Residence time, min	
	Available	Required
Oil in gas (NLL to vessel top)	0.057	0.051
Water in oil (bottom to NLL)	10.818	2.183
Oil in water (boot section)	72	2.381

VELOCITY CHECK

K-factor gas velocity	m/s	2.99
Actual gas velocity	m/s	1.952

NOZZLE DIA

	mm
Inlet nozzle	582.18
Gas outlet	434.80
Oil outlet	108.58
Water outlet	50

CHECK RESULTS

Velocity check	OK	Water in oil residence time	OK
Oil in gas residence time	OK	Oil in water residence time	OK

Figure 3.16 Example 3.3.

Analysis

A velocity check shows some margin between the actual gas velocity and the K factor gas velocity. This means that the vessel diameter can be reduced further. However, the oil-in-gas residence time shows that available and required residence times are nearly the same, which means that the separator is not oversized.

One noteworthy point is the selection of the droplet size. The oil-in-gas droplet was assumed as 200 μm. If this is reduced to 100 μm, the following warning will appear:

"Required gas residence time exceeds actual residence time.
Following suggestions recommended:

1. Increase vessel diameter.
2. Reduce NLL level."

In the Check Results frame, the oil-in-gas residence time will show the error, "Required residence time exceeds actual."

The other residence times, oil-in-water and water-in-oil, show that the required residence time is much less than the available residence time; this indicates that oil-in-water and water-in-oil droplet sizes can be reduced further.

Nomenclature

a	Surface area/volume, m^{-1}
A,B,C	Constant used in Equation 3.30
A_p	Projected area of baffles, m^2
A_t	Cross sectional area, m^2
b	Depth as in Figure 3.3
C	Constant used in Equation 3.26
C^*	Empirical constant for liquid–liquid separator
C_D	Drag coefficient, dimensionless
d	Distance, m
d_i	Inlet diameter, m
d_o	Outlet diameter, m
d_w	Diameter of wire, m
D_p	Droplet diameter, m
E	Kinetic energy, $kg*m^2/sec^2$
f	Friction factor, dimensionless
F	Total liquid flow rate, m^3/day
F_D	Drag force, $kg*m/sec^2$
g	Acceleration due to gravity, m/sec^2
G_D	Dimensionless slug growth
$h1,h2,h3,h4$	Distance, m

H	Thickness, m
H_1	Width of liquid interface area, m
I	Immersed weight, kg
ID	Inlet diameter of vessel, m
K	Empirical constant for separator sizing, m/sec
K_{CR}	Proportionality constant, dimensionless
K_D	Empirical design constant for separator sizing, m/sec
L	Length, m
L_1	Length of liquid interface area, m
L_s	Average slug length, m
L_{sr}	Reference slug length, m
m	Mass, kg
m_h	Mass flow of heavy phase, kg/h
m_1	Mass flow of light phase, kg/h
n	Number of bends
N_{RE}	Reynolds number, dimensionless
N_s	Separation number, dimensionless
t	Temperature, °C
T	Retention time, min
U	Fluid up-thrust, kg
U_c	Droplet velocity, m/sec
v_{sg}	Superficial gas velocity, m/s
V	Velocity, m/s
V_a	Actual gas velocity (= V/cos)
V_c	Critical settling velocity, m/sec
V_t	Terminal velocity, m/sec
VS	Volume of settling section, m^3
w	Width, m
W	Weight of particle, kg
W_{cl}	Flow rate of light condensate liquid, m^3/day
x	Distance from initiation of slug flow, m
x_t	Point of transition from developing to long-term slug flow, m

Greek characters

ΔP	Pressure drop, kPa
ΔP_{dry}	Dry pressure drop, kPa
ε	Void fraction
ϕ	Water cut
ϕ_c	Water cut at inversion point
	Density, kg/m^3
ρ_G	Gas phase density, kg/m^3
ρ_{heavy}	Heavy phase density, kg/m^3
ρ_L	Liquid phase density, kg/m^3
ρ_{light}	Light phase density, kg/m^3

ρ_m	Mixed liquid density, kg/m^3
μ	Viscosity of continuous phase, cP
μ_G	Viscosity of gas, cP
μ_L	Viscosity of liquid, cP
μ_O	Viscosity of oil, cP
μ_{relmax}	Maximum relative viscosity (relative to oil), dimensionless
μ_W	Viscosity of water, cP
η	Efficiency
η_t	Target collection efficiency
θ	Vane angle, degree

References

1. Gas Processors Suppliers Association, GPSA Engineering Databook, 11th ed. — SI, Oklahoma, 1998.
2. Perry, R.H., Ed., Chemical Engineers' Handbook, 5th ed., McGraw-Hill Book, chap. 5, Tokyo, 1973.
3. Fabian, P., Cusack, R., Hennessey, P., and Neuman, M., Demystifying the selection of mist eliminators, Part I, Chemical Engineering, November, 148–156, 1993.
4. API Specification 12J, Specification for Oil and Gas Separators, 7th ed., American Petroleum Institute, 1989.
5. API Specification 11P, Specification for Packaged Reciprocating Compressors for Oil and Gas Production Services, American Petroleum Institute, 2nd ed., 1989.
6. CO_2 Corrosion Rate Calculation Model, NORSOK M-506, Revision 1, June 1998.
7. Campbell, J.M., Gas Conditioning and Processing, John M. Campbell and Company, 8th ed., Vol. 2, Oklahoma, 2004.
8. Scott, S.L., Shoham, O., and Brill, J.P., Prediction of slug length in horizontal, large-diameter pipes, SPE Production Engineering, August, 335–340, 1989.
9. API RP 521, Guide for Pressure-Relieving and Depressuring Systems, American Petroleum Institute, 4th ed., 1997.

chapter four

Overpressure protection

Introduction

Overpressure protection is the most fundamental safety requirement for plant design. A large number of situations are possible for a particular pressure relief valve (PRV), and careful consideration is required to quantify all scenarios. The design of a particular pressure protection system depends upon the contingency calculated for a particular situation, and often more than one contingency situation controls the design. The design aspects of a pressure relief system are dictated by the mechanical design code; however, two API-recommended practices, API RP 520 [1,2] and API RP 521 [3], are widely used. Although these API codes are only recommendatory, the impact of violation requires careful consideration. Contingencies for the design of a pressure protection system can be divided into two major groups:

- Impact on plant design
- Impact on individual design

Impact on plant design

There are contingencies that need not necessarily be considered for the design of a particular PRV but will have an impact in the design of the plant relieving system (flare or cold vent). Relieving contingencies under all possible scenarios need to be established for the following situations:

- Overall plant power failure
- Failure of a particular motor control center (MCC)
- Cooling water failure
- Instrument air failure
- Steam failure
- Fire (sometimes fire in a particular fire circle controls the total contingency)

Impact on individual design

There are a large number of contingencies that influence the design of a particular PRV, but may have no impact on the design of the overall plant. The controlling contingencies need to be established and reported for the following situations:

- Check valve failure
- Blocked discharge
- Control valve failure
- Thermal expansion of liquid
- Heat exchanger tube rupture
- Reflux failure and overhead system
- Loss of reboiler heat
- Venting of storage tank
- Failure of individual motor
- Accidental closure of valve

Most design houses develop a typical table, normally called contingency quantification, which tabulates all possible contingencies for a particular PRV. Although tabulation of all contingencies under the situation "impact on plant design" is essential, only the single controlling contingency (if applicable) under "impact on individual design" requires tabulation. Some operating houses have the practice of tabulating all contingencies under impact on individual design, which are preferably avoided. These tabulations do not serve any fruitful design purpose but sometimes confuse the designer.

Definition

Terms commonly used in the design of PRV systems and their definitions are presented in the following subsections.

Accumulation

Accumulation is the pressure increase over the maximum allowable working pressure of a vessel during discharge through PRV, expressed as either a pressure unit or percentage. Maximum allowable accumulations are established by applicable codes for operating and fire contingencies. ASME (American Society of Mechanical Engineers), Section VIII [4], allows maximum accumulation as follows:

- Accumulation for pressure vessels protected by multiple relief valves should not exceed 16% of the maximum allowable working pressure (MAWP), except in case of fire.
- External fire accumulation for pressure vessel shall not exceed 21% of MAWP.

Atmospheric discharge

This is the release of vapors and gases from a PRV and a depressuring system to the atmosphere. The following points must be considered before deciding on any discharge to the atmosphere:

- Discharge of flammable or toxic materials to the atmosphere must be minimized as far as possible.
- The discharge point of steam or air to the atmosphere should be located at least 3 m above the nearest operating platform.
- The tailpipe for hydrocarbon or other flammable vapor release should be designed for a maximum velocity of 0.5 *Mach*.
- The release of hydrocarbons or other flammable vapors to the atmosphere should comply with API RP 521, Section 4.3.
- The tailpipe must have a 20-NB drain hole at the lowest point.
- The tailpipe must have a suitable cover flap over its end.
- The tailpipe should have a 25-NB snuffing steam connection located 300 mm below the pipe outlet.

Built-up back pressure

This is the increase in pressure at the outlet of the PRV as a result of flow after the pressure relief device or devices open.

General back pressure

This is the pressure at the outlet of the PRV and is the sum of built-up and superimposed back pressures. The impact of back pressure to the selection of the type of PRV can be summarized as under:

- In the case of a conventional relief valve, general back pressure shall not exceed 10% of the relief valve set pressure.
- In the case of a balanced-bellows relief valve, the impact of general back pressure depends on the percentage of overpressure and is known as the back-pressure correction factor. For 10% overpressure, the back-pressure correction factor is 1 up to 30% general back pressure. For 16% overpressure, the correction factor does not change (value = 1) up to 37% back pressure, and for 21% overpressure, the correction factor does not change (value = 1) up to 50% back pressure.
- For pilot-operated PRVs, the variation in superimposed back pressure does not affect the opening pressure.
- The rupture disk is designed based on the differential pressure and is greatly influenced by the variation of back pressure.

Superimposed back pressure

This is the pressure at the outlet of the PRV when the valve is in a closed position. The superimposed back pressure arises from other sources in the discharge system and it may be either constant or variable.

Balanced-bellows PRV

This is a PRV designed to minimize the effect of back pressure on its performance characteristics.

Blowdown

Blowdown is the difference between the set pressure and the closing pressure of the PRV, expressed as a percentage of the set pressure.

- In design, blowdown is normally set at 7% and can vary from 7 to 10%.
- For a pilot-operated PRV, the blowdown can be reduced to 3% provided the pilot has a remote pressure sensing line from an independent and separate vessel nozzle.

Closed discharge system

This is a discharge system that releases to a closed system, such as blowdown drum and flare header.

Cold differential test pressure

This is the pressure at which the PRV is adjusted to open the test stand. The cold differential test pressure includes corrections for the service conditions of superimposed back pressure, temperature, or both.

Conventional PRV

This is a closed-bonnet spring-loaded PRV whose performance characteristics are directly affected by the changes in back pressure on the valve.

Design capacity

This is the maximum estimated contingency and is used to determine the area of the PRV.

Design pressure

This is the pressure in the equipment or piping under the most severe condition of coincidence temperature, liquid level, and pressure drop within the vessel that is expected during operation. It may be used in place of MAWP in all cases in which it has not been established. The design pressure is the pressure used in the design of a vessel to determine the minimum permissible thickness or other physical characteristics of the different parts of the vessel.

Maximum allowable accumulated pressure

This is the sum of MAWP and maximum accumulation. The following guidelines are used to set the maximum allowable accumulated pressure:

- Such pressure for single-valve installation (except for a fire case) shall not exceed 110%.
- It must not exceed 116% for multiple-valve installation (except for a fire case).
- It shall not exceed 121% for a fire case.

Maximum allowable working pressure

This is the maximum gauge pressure permissible at the top of a completed vessel in its operating position for a designated temperature. The pressure is based on calculations for each element in a vessel using normal thicknesses, exclusive of additional metal thicknesses allowed for corrosion and loadings other than pressure. MAWP is generally equal to, or slightly more than, the design pressure. MAWP is the basis for the pressure setting of the pressure relief devices that protect the vessel.

Operating pressure

This is the pressure to which the vessel is usually subjected in service. A pressure vessel is normally designed for a MAWP that will provide a suitable margin above the operating pressure to prevent any undesirable operation of the relieving device.

Overpressure

This is the pressure increase of the vessel over the set pressure of the relieving device, expressed either as a pressure unit or as a percentage. It is the same as accumulation when the relieving device is set at the MAWP of the vessel, assuming no inlet pipe loss to the relieving device.

Pilot-operated PRV

This is a PRV in which the main valve is combined with and controlled by an auxiliary PRV.

Pressure relief valve (PRV)

This is a general term used for pressure relief devices that reclose when the system pressure returns to normal. The following terms are often used for PRVs:

- PRV — pressure relief valve
- PSV — pressure safety valve
- RV — relief valve
- SRV — safety relief valve
- SV — safety valve
- TRV — temperature relief valve
- TSV — temperature safety valve

Pressure safety valve

This is an alternative term for a PRV.

Rated relieving capacity

This is the measured relieving capacity permitted by the applicable code or regulation to be used as a basis for the application of a pressure relief device.

Relief valve

This is a spring-loaded PRV used for liquid service. The valve opens normally in proportion to the pressure increase over the opening pressure.

Relieving conditions

Used to indicate the inlet pressure and temperature of a pressure relief device at a specific overpressure. The relieving pressure is equal to the valve set pressure (or rupture disk burst pressure) plus the overpressure. The temperature of the flowing fluid at relieving conditions may be higher or lower than the operating temperature.

Rupture disk

This is a nonclosing differential pressure relief device designed to function by bursting the pressure-containing rupture disk. A rupture disk device includes a rupture disk and a rupture disk holder.

Safety relief valve

A spring-loaded PRV that may be used as either a safety or a relief valve, depending upon the application.

Safety valve

This is a spring-loaded pressure relief valve actuated by the static pressure upstream of the valve and characterized by rapid opening or pop action. A safety valve is normally used with compressible fluids.

Set pressure

This is the inlet gauge pressure at which the PRV is set to open under service conditions. The PRV set pressure shall be based on the following guidelines:

- Normally, set pressure is equal to 10% of, or 150 kPa above, operating pressure, whichever is higher.
- For high operating pressure (above 7000 kPaG), the set pressure can be reduced to 7% above operating pressure.
- The PRV set pressure on storage tanks (e.g., LPG bullets) shall not be less than the fluid vapor pressure at maximum anticipated temperature or 46°C, whichever is higher.
- Set pressure of a single PRV shall not exceed the MAWP of the equipment. If the MAWP is not known, the set pressure shall not exceed the design pressure of the equipment.
- For multiple PRVs, only one operating valve needs to be set at the MAWP of the vessel. Other valves can be set at higher pressure with a maximum limit of 105% of MAWP. For example, if a vessel is protected by three PRVs, the set pressures can be adjusted at 100, 103, and 105% of MAWP.
- In case one PRV protects multiple equipment with different operating pressures, the minimum set pressure shall be 105% of the settled-out pressure.
- PRVs may have a set pressure less than MAWP or design pressure, depending on the requirement.
- Set pressure for piping shall not exceed 90% of the short-term pressure rating of piping. The piping short-term pressure rating must be according to ANSI B31.3.

Vapor depressuring system

This is a protective arrangement of valves and piping intended to provide for rapid reduction of pressure in equipment by releasing vapors. The actuation of the system may be automatic or manual.

Vent stack

This is an elevated vertical termination of a disposal system that discharges vapors into the atmosphere without combustion or conversion of the relieved fluid.

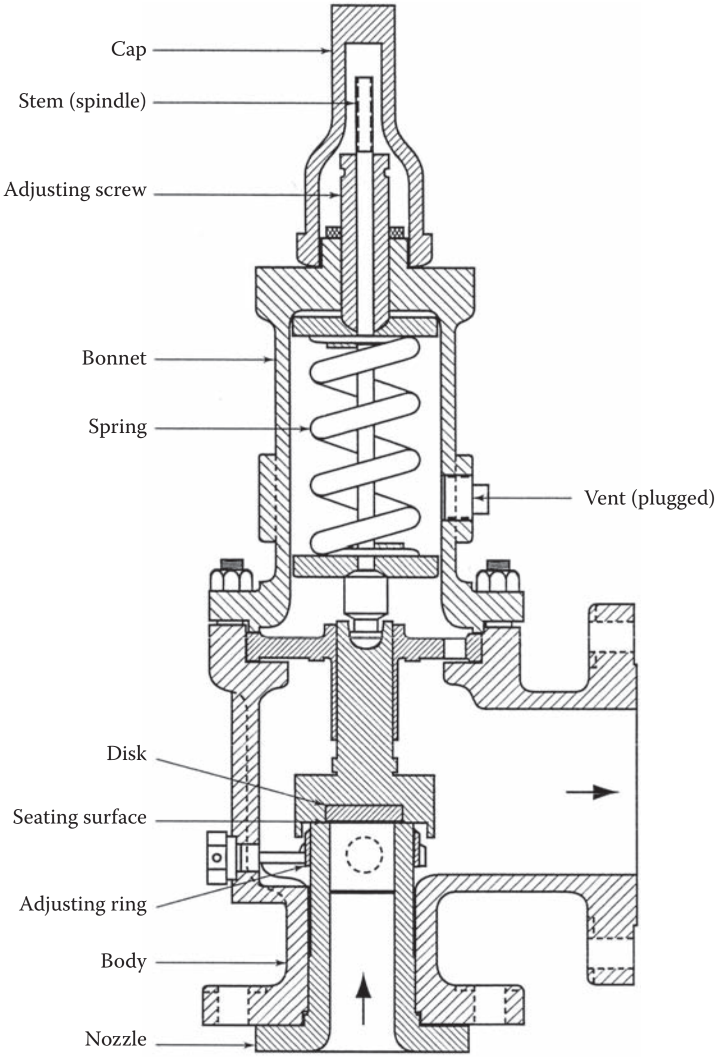


Figure 4.1 Conventional pressure relief valve with a single adjusting ring for blowdown control. (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

Types of pressure relief valves

Conventional pressure relief valve (vapor service)

Conventional PRVs are shown in Figure 4.1 and Figure 4.2. This is a self-actuated spring-loaded valve designed to open at a certain set pressure. The basic elements of the valve are:

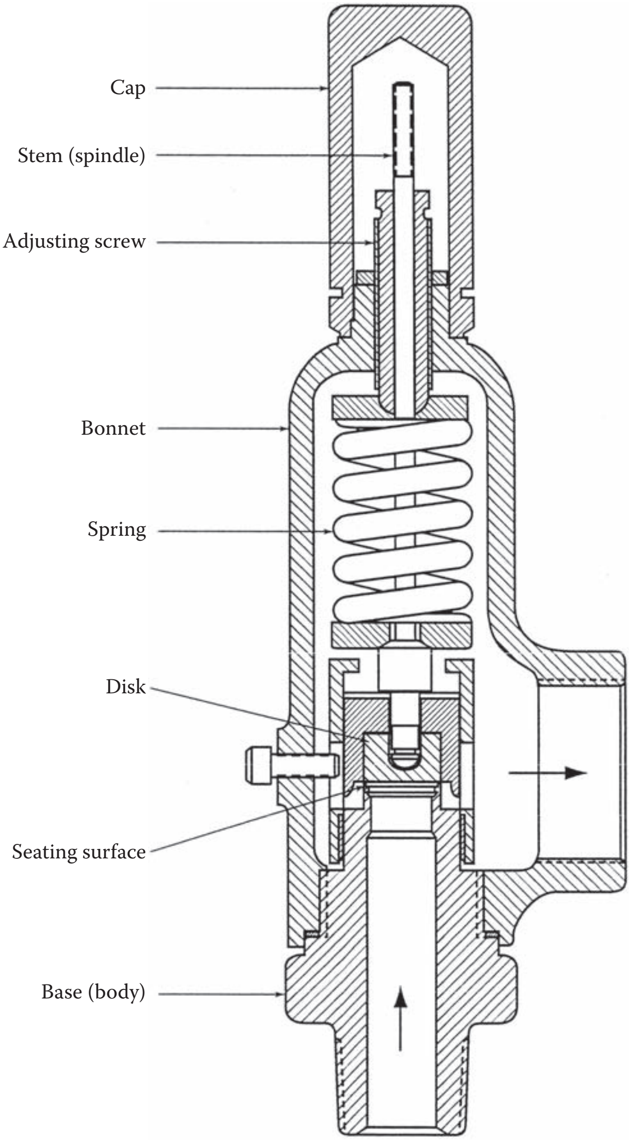


Figure 4.2 Conventional pressure relief valve with threaded connections. (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

- The inlet nozzle connected to the vessel through a suitably sized piping
- The outlet nozzle connected to the discharge system, which can open to the atmosphere or to a closed system
- The disk, a movable element that controls the flow through the valve
- The spring that controls the position of the disk

The spring and the disk control the flow through the valve. When the operating pressure in the vessel or system is less than the set pressure, the spring force allows the disk to sit on the nozzle, and no fluid passes through the inlet nozzle. This continues till the vessel or system pressure is equal to the set pressure of the valve. When the pressure of the vessel or system exceeds the set pressure, its force moves the valve upward, thus opening the valve to release fluid through the outlet nozzle. Once the pressure falls below the set pressure, the disk falls back on the nozzle and the flow stops.

The design of the valve is such that when the pressure approaches the set pressure the fluid moves past to another chamber called the *huddling chamber*. Because of the larger area available, the pressure force exerted over the disk exceeds the spring force, and the valve pops open.

Now the valve is open, and the pressure force is acting over a larger area. This means that the valve will not close when the pressure falls to the set pressure (because the pressure is acting on a larger area). The disk will fall back when the vessel or system pressure falls a little below the set pressure. The difference between the set pressure and closing pressure is called *blowdown*, which is an important design criterion.

Conventional pressure relief valve (liquid service)

There are fundamental differences in popping action between vapor and liquid service. This is due to the fact that the expansive force produced by vapor is not present in liquid flow. At the initial opening, the liquid escapes through the slits and expands radially between two seating surfaces. At the first 2 to 4% overpressure, the liquid creates a reactive force and moves the disk upward.

With the increase in liquid flow rate through the opening slit, the velocity head also increases, causing a larger opening, and the valve suddenly surges to 50 to 100% lift. This typically happens at 2 to 6% overpressure. The valves are required to reach 100% capacity at 10% or less overpressure.

During the reverse cycle, the overpressure and momentum decrease, and the spring force moves the disk back into contact with the seat.

Balanced-bellows pressure relief valve

Balanced-bellows PRVs are spring-loaded valves with a bellows or some other means of balancing the valve disk to minimize the effect of back pressure. Typical valve configurations are shown in Figure 4.3 and Figure 4.4.

The main operating principle of balanced-bellows PRVs can be explained through Figure 4.5. In this configuration, the nozzle seat area (A_N) and effective bellows area (A_B) are the same, and the spring force always keeps the disk shut when the operating pressure is below the set pressure. There is a possibility that noncondensables will accumulate inside the bellows, creating extra pressure over the disk. This problem can be avoided by venting the bellows to the atmosphere. If the bellows cannot be vented to the atmo-

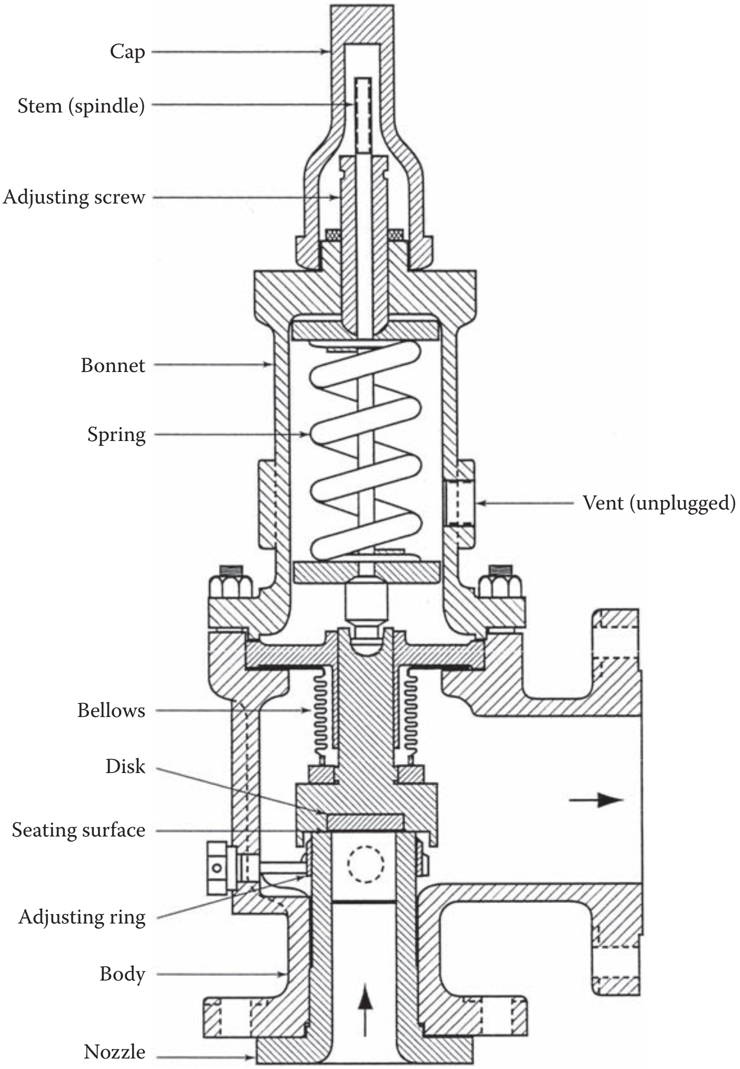


Figure 4.3 Balanced-bellows pressure relief valve. (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

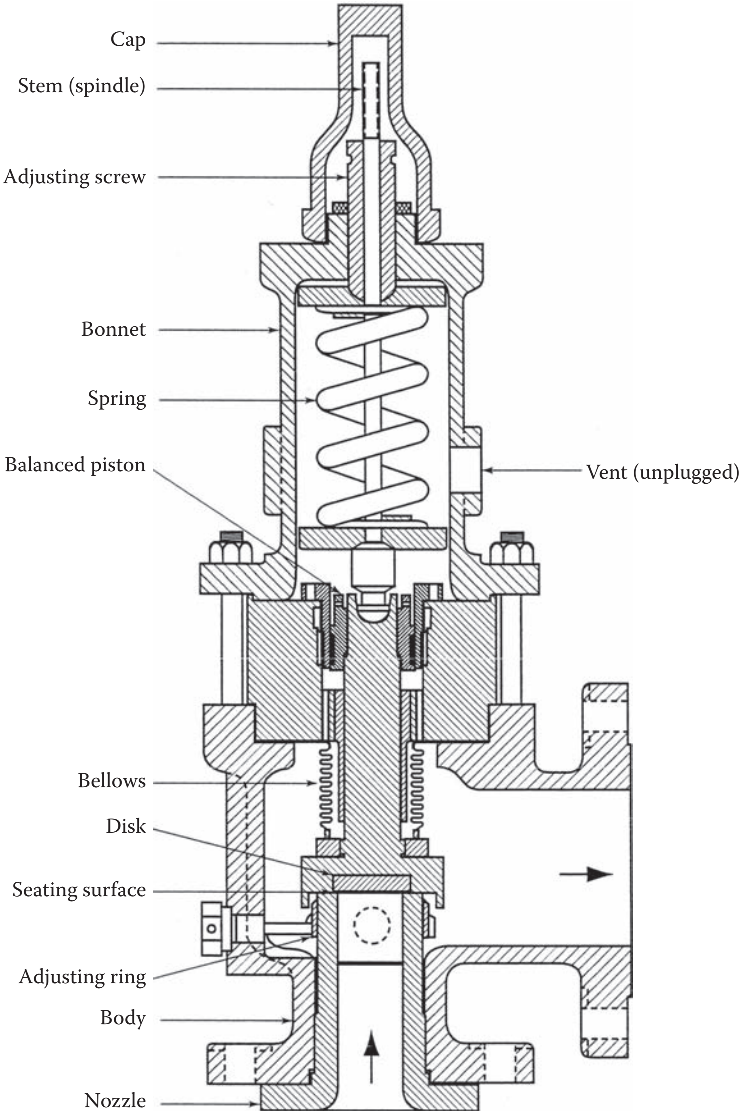
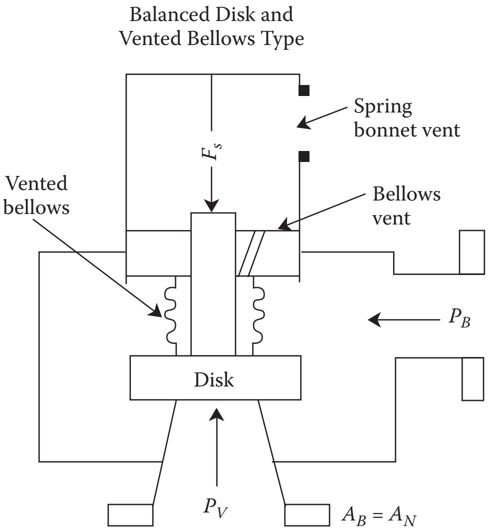


Figure 4.4 Balanced-bellows pressure relief valve with an auxiliary balanced piston. (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)



A_B = effective bellows area,
A_D = disk area,
A_N = nozzle seat area,
A_P = piston area (top),
F_S = spring force,
P_V = vessel gauge pressure,
P_B = superimposed back pressure in pounds per square inch gauge,
P_S = set pressure, in pounds per square inch gauge.

Note: In this figure, $P_V = P_S$; $(P_V)(A_N) = F_S$ (typical); and $P_S = F_S/A_N$.

Figure 4.5 Typical effects of back pressure on the set pressure of balanced pressure relief valves. (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

sphere, it will result in malfunctioning of the relief valve. In case there are difficulties in venting the bellows locally, the vent must be connected to a tailpipe to vent in a safe location. The pressure drop in the venting pipe should be negligible.

The bellows isolate an area of the disk, approximately equal to the disk seat area, from the back pressure. Thus, the installation of bellows will allow the set pressure to remain constant in spite of variation in back pressure.

Pilot-operated pressure relief valve

Typical pilot-operated PRVs are shown in Figure 4.6 and Figure 4.7. A pilot-operated PRV consists of a piston and an external pilot assembly. The piston has a larger area at the top in comparison to the bottom. Before the pressure reaches the set pressure, the balancing line maintains the same

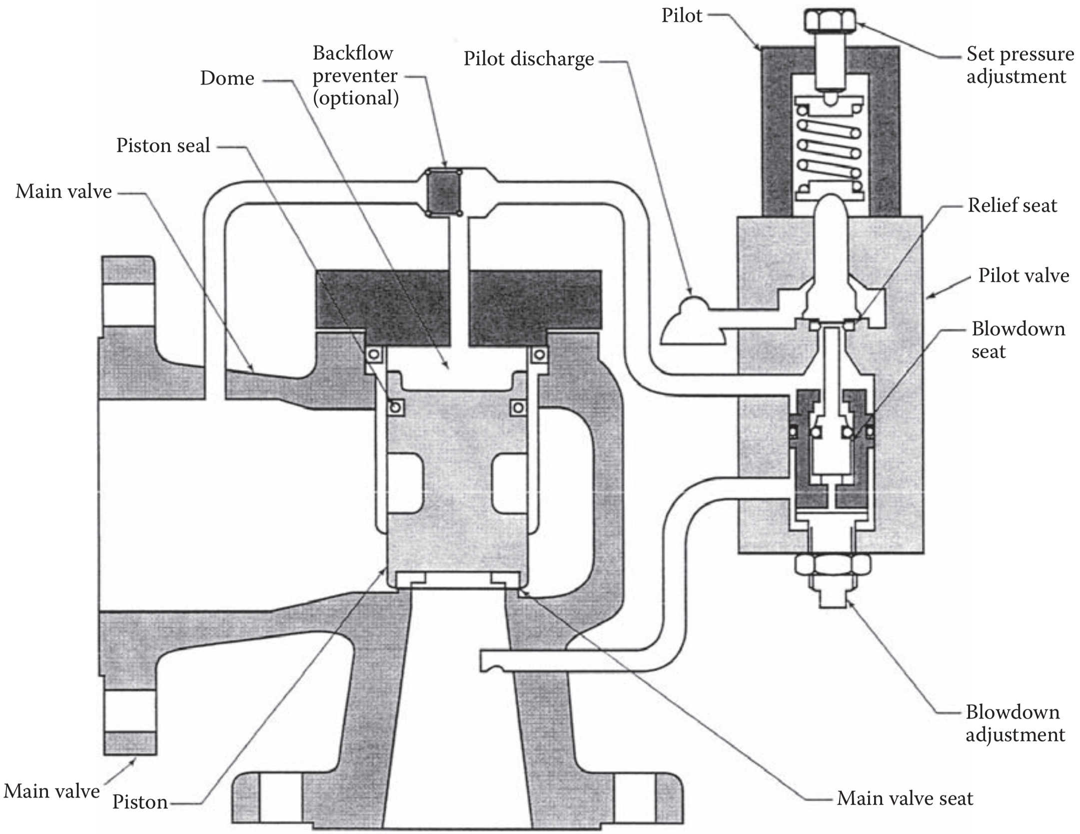


Figure 4.6 Pop-action pilot-operated valve (nonflowing type). (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

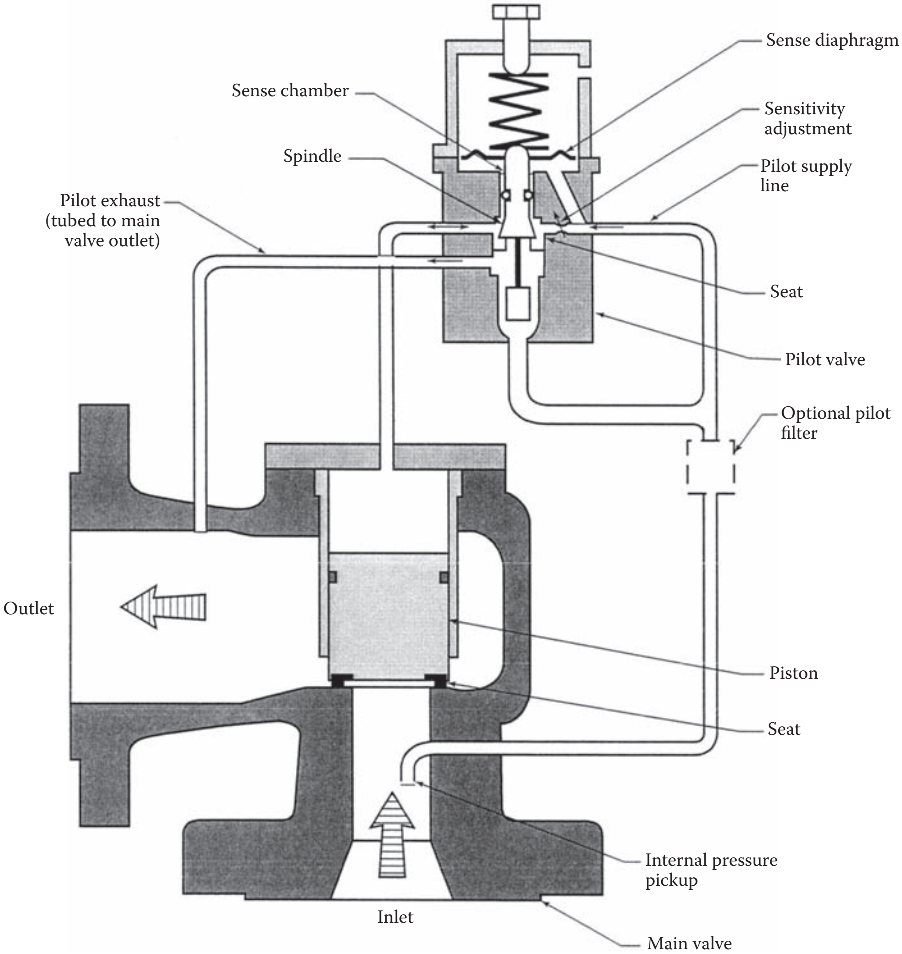


Figure 4.7 Modulating pilot-operated valve (flowing type). (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

pressure at both ends of the piston. Because the top section of the piston has a larger area, the net force acts downward, keeping the valve tight shut. This feature is a unique advantage of pilot-operated PRVs. When the pressure reaches the set pressure, the pilot vents the pressure from the top of the piston, causing the net force to act upward. The upward force moves the piston, and the flow is established through the valve. Once the pressure falls, the pilot will close the vent, and the net downward force will cause the piston to reset.

One unique advantage of the pilot-operated PRV is that the lift of the piston does not depend on the back pressure of the system and the performance of the valve is not affected by the built-up back pressure.

The pilot can vent to the atmosphere or to the valve outlet if atmosphere venting is not allowed. In case the pilot is vented to the atmosphere, the performance of the valve will be unaffected by the back pressure of the valve. In case the vent is connected to the valve outlet with variable pressure, a balanced-type pilot will be required for proper performance of the valve.

The performance of pilot-operated PRVs depends on the pressure balancing line. If this line is blocked, pressure balancing between the top and bottom of the piston will not be possible, and the valve will lift at comparatively low operating pressure. This is the major disadvantage of pilot-operated valves. This type of valve can be recommended only if the operating fluid is clean. The pilot-operated valve is not very popular for hydrocarbon service.

Rupture disk

A typical rupture disk and its assembly are shown in Figure 4.8. These devices are the nonclosing type and once opened require replacement. A rupture disk can be installed to protect vessels, piping systems, and other pressure-containing components.

One advantage of the rupture disk that it has no moving parts and reacts quickly enough to relieve some of the pressure spikes. The quick-acting characteristics of the rupture disk have made it popular for applications in which pressure buildup is instantaneous, e.g., runaway reaction and exchanger tube rupture.

The rupture disk operates through differential pressure and may not work satisfactorily if there are large variations in back pressure. This disk is also temperature sensitive. Burst pressure can vary substantially with temperature. With increase in temperature, the burst pressure decreases, simply because with increase in temperature the mechanical strength of the disk is reduced. Rupture disks are generally designed for a particular differential pressure and temperature and are not recommended if the design differential pressure and temperature vary substantially.

Selection of pressure relief valves

The following subsections describe the guidelines often used in selecting the type of pressure relief device.

Conventional pressure relief valve

Conventional PRVs are the most widely used relief devices in chemical industries, particularly in a system in which the superimposed back pressure is relatively low. Conventional PRVs are not very suitable for high back pressure.

Correct Installation

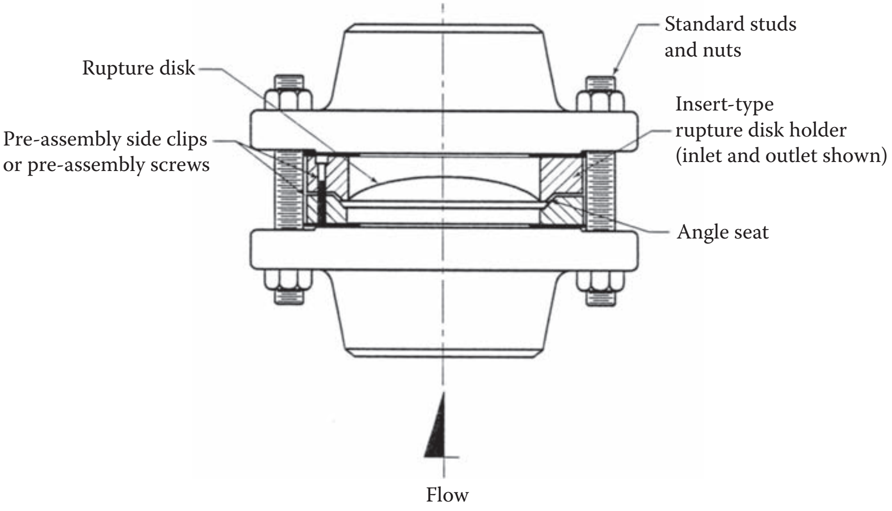


Figure 4.8 Forward-acting solid metal rupture disk. (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

Advantages of conventional valves:

- Simple in design.
- Lowest in cost.
- The conventional PRV is very common when the discharge is open to the atmosphere.
- It is the preferred PRV.

Disadvantages of conventional valves:

- A conventional PRV should not be used if the back pressure exceeds 10% of the set pressure.
- It is not recommended for flashing service, simply because a little change in fluid composition may increase the flushing vapor rate substantially. High vapor flow may increase the back pressure beyond 10% of the set pressure.
- Conventional PRVs are not preferable for slurry service.
- They are not preferable for corrosive service.

Balanced-bellows pressure relief valve

Balanced-bellows PRVs are not normally used if a conventional PRV is acceptable. However, some operating companies prefer to install balanced-bellows PRVs for all scenarios simply because of possible change in future service conditions.

Advantages of balanced-bellows PRVs:

- Balanced-bellows PRVs can be used for higher back pressure up to 50% back pressure without reducing the back-pressure correction factor (for a 21% overpressure situation).
- Can be used for slurry and corrosive service.
- Balanced-bellows PRVs are normally used when the back pressure exceeds 10%.
- Balanced-bellows PRVs are normally used if the PRV is connected to the high-pressure relief header.
- Balanced-bellows PRVs are widely used for flashing service.

Disadvantages of balanced-bellows PRVs:

- If there is a choice between conventional and balanced-bellows PRVs, the conventional PRVs are a better option.

Pilot-operated pressure relief valve

The pilot-operated PRV is best avoided and may be considered only if other types of valves are not suitable.

Advantages of pilot-operated valves:

- Suitable for very low set pressure
- Often used when back pressure is too high

- Suitable if inlet line pressure drop exceeds the 3% limit of set pressure
- Tight up to the set pressure and often used if high tightness is required
- Suitable for liquid service where good modulation is required

Disadvantages of pilot-operated valves:

- Not suitable for slurry and corrosive service
- Not suitable for steam service

Rupture disk

Unlike other pressure relief devices, rupture disks do not fall back to the original position when the pressure is reduced. The rupture disk is of the nonclosing type and needs to be replaced manually.

Advantages of rupture disks:

- A rupture disk is used where the rate of pressure buildup is very high.
- It is used in reactors with a possibility of runaway reaction.
- Sometimes rupture disks are used in series with PRVs if valve leakage is a problem. If a rupture disk is installed on the inlet of a PRV, the pressure between the disk and the valve can be monitored to detect any pinhole leakage of the rupture disk.

Disadvantages of rupture disks:

- Rupture disks do not reclose as PRVs do. This results in loss due to increased production time.
- Vessel/system depressurization is required once the rupture disk is opened.
- The rupture disk operates on differential pressure and is not suitable in case of high fluctuation in such pressure.
- It is difficult to get a rupture disk for a specific differential burst pressure, the range of which is normally 10% or more than specified.
- Rupture disks can be inspected only by destructive testing.

PRV installation and line sizing

The following guidelines are useful in deciding the location of a PRV.

Compressors and pumps

- For the positive displacement type of compressors or pumps, a PRV should be installed at the discharge of each stage. For a multistage positive displacement compressor, a PRV is required at the suction of the compressor if the settled-out pressure is more than the design pressure of the suction side. In case the settled-out pressure exceeds the design pressure, the relief valve will lift every time the compressor is shut down; this is a situation the designer should prevent. This can be avoided by installing an automatic blowdown system that will release an adequate amount of gas to reduce the settled-out pressure below the design pressure of the suction side.

- A PRV must be installed at the discharge of a centrifugal pump if the maximum possible discharge pressure exceeds the design pressure of downstream piping and equipment that are not protected by another PRV. In general, the pump itself is designed for a much higher pressure rating. The maximum discharge pressure should be estimated, using the following procedure:
    a. Estimate the maximum suction pressure. This will be the set pressure of the PRV protecting the suction side of the pump plus the maximum liquid head. Maximum liquid density is to be used in estimating the liquid head.
    b. From the pump curve, estimate the no-flow head of the pump at the maximum possible speed of the pump (in case of variable-speed pump). If the pump curve is not available, a preliminary no-flow head can be estimated at 25% higher than the normal head of the pump. This needs to be rechecked once the pump curve is available.
    c. Maximum discharge pressure should be estimated based on the maximum possible suction pressure and maximum differential head as calculated previously.
- A PRV should be installed at the discharge of the compressor if the maximum possible discharge pressure exceeds the design pressure of the downstream piping and equipment. The maximum discharge pressure is estimated as follows:
    a. Estimate the maximum suction pressure; this is generally the PRV set pressure at the suction side of the compressor.
    b. Calculate the maximum compressor head at no-flow and maximum speed conditions. This is to be obtained from the manufacturer curve. In case the value is not available, a 25% margin over the design head can be used as a preliminary calculation. This needs to be rechecked once the manufacturer curve is available.

Fired heaters

- Normally, a PRV is located on the downstream equipment of the fired heater, e.g., the distillation column at the discharge of a fired heater. This is acceptable provided:
    a. There is no isolation valve on the transfer line.
    b. The isolation valve on the transfer line is locked open.

Heat exchangers

- A PRV is required for a blocked-in condition if the blocked-in pressure exceeds the design pressure.
- A PRV is required for blocked-in thermal expansion as well as for external fire.
- A PRV can be avoided in a heat exchanger train by installing a locked-open valve.
- For long heat exchanger trains (e.g., a heat exchanger train for a crude column), there are significant pressure drops in the train. The train is often divided into two design pressure classes, and each class is protected by a suitably designed PRV and locked-open isolation valves.
- A PRV is required for a tube rupture situation to protect the low design pressure side. Pressure relief for tube rupture is not required where the low-pressure exchanger side is designed at or above two thirds of the design pressure of the high-pressure side.

Piping

- Pressure protection for the piping system should be designed based on ASME B31.3 [5].
- Thermal expansion is one major concern for the safety of the piping system. For a liquid-filled situation, thermal expansion should be considered for the protection of the piping system.
- The piping system must also be protected against external fire.

Pressure vessels

- All pressure vessels should have PRVs.
- Normally, PRVs are located on the vessel. However, location of a PRV on the interconnecting piping is also common.
- For vessels with a mist eliminator, it is preferable to locate the PRV upstream of the mist eliminator. This is to avoid any pressure problem with the blockage of the mist eliminator.
- It is possible that a single PRV protects multiple interconnected vessels, provided there is no isolation (or isolation valves are locked open) between the vessels.
- In a fractionating system, two PRVs are normally installed, one on the fractionating column or on the interconnecting piping and the other on the reflux drum. The following guidelines should be followed in designing the protection system for a fractionating column:
  a. The PRV for the column should be designed based on the general column contingency, including fire.

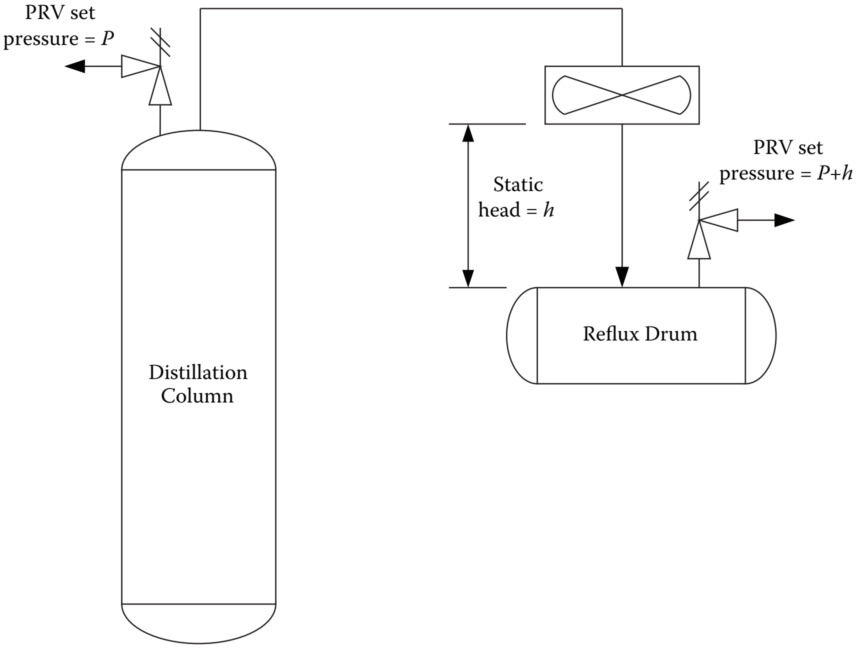


Figure 4.9 PRV set pressure in a distillation column.

b. The PRV should be located either on the column or on the interconnecting pipe between the column and the overhead cooler.
c. The PRV should be set at column design pressure or MAWP of the column.
d. Fire contingency for the cooler should be added to the column contingency.
e. The reflux drum design pressure (and the PRV set pressure) must be equal to the set pressure of the column PRV plus the static head up to the flooded condenser (the reflux drum PRV should never be set below the column PRV set pressure).
f. The PRV on the reflux drum must be designed based on the fire contingency of the reflux drum.
g. The general PRV arrangement is shown in Figure 4.9.

PRV isolation valves

Isolation valves are quite common at the inlet and outlet of PRVs. However, certain aspects need to be considered in designing the isolation system of a pressure relief valve.

- All isolation valves should be the full-bore type.
- Isolation valves must have the same pressure rating as the line.

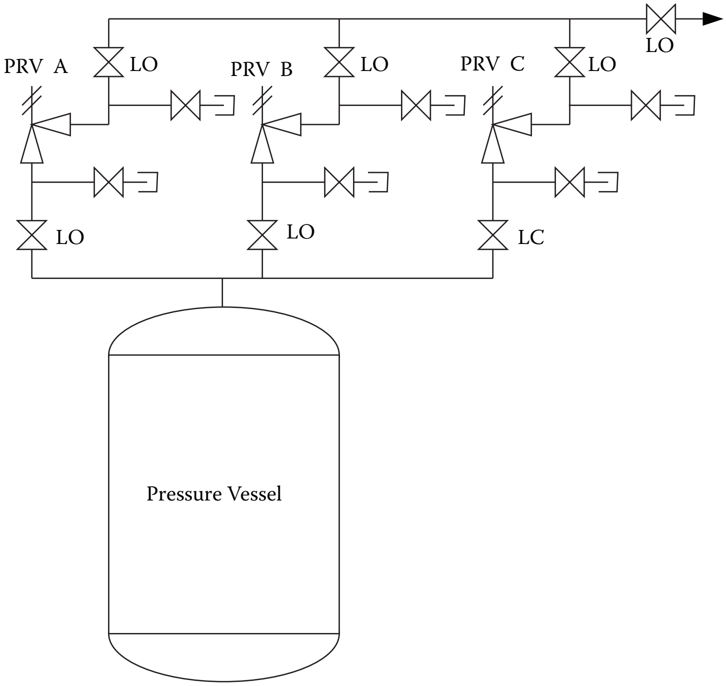


Figure 4.10 Typical isolation valve arrangements.

- Isolation valves should have the capability of being locked open and locked closed.
- The isolation valve at the inlet of the PRV should have a low pressure drop. This is to avoid an inlet piping pressure drop exceeding 3% of the PRV set pressure.
- Gate valves should preferably be installed on the vertical line. If installation on the horizontal line cannot be avoided, the valve stem must be positioned downward. This is to avoid the valve's falling off and blocking the flow.
- In case of a single PRV, it is preferable to avoid an isolation valve. For maintenance requirements, a header isolation valve can be used (refer to Figure 4.10).
- An isolation valve downstream of the pressure relief device may be installed at the battery limits of the process unit. This is to allow the process units to be removed from service for maintenance while other units are in operation.
- Installation of a standby PRV is shown in Figure 4.10. The following points are to be considered for the installation.
  1. A mechanical interlock or interlocking procedure, which manages proper closing and opening sequences of the isolation valves,

should be provided to ensure that overpressure protection of the vessel is not compromised.

2. The inlet isolation valve for the PRV in service must be locked open.
3. The inlet isolation valve of the spare PRV must be locked closed.
4. Discharge isolation valves for the PRV in service should be locked open.
5. The discharge isolation valve for the spare PRV is preferably locked open. This is to avoid pressure buildup between the PRV and discharge isolation valve for a leaky relief valve. In case the valve is locked closed, pressure monitoring and draining facilities shall be installed.
6. Bleed valves are to be installed at the inlet and outlet of the PRV as shown in Figure 4.10.

Inlet piping to PRVs

A PRV inlet line is required to be sized properly to keep the pressure drop within the limit. For vapor flow, a large pressure drop through the inlet line may reduce the PRV inlet pressure below its blowdown pressure and result in the closing of the valve. Closing of the valve against a high-velocity stream will result in rapid rise in pressure, reopening the valve. This cycle of closing and opening of the valve will repeat at high frequency, which is known as chattering of the valve. Chattering may damage the valve or may reduce valve efficiency.

Normally, the blowdown of the valve is set at 7 to 10% of the set pressure (except in the pilot-operated valve, in which it can be set as low as 3%), and based on the normal setting of blowdown, the mechanical design code restricts the inlet line nonrecoverable pressure drop to a maximum of 3%. The estimated pressure drop is only the frictional pressure drop and excludes any drop or gain due to change in velocity head. This means the velocity head change due to the reducer installed at the PRV inlet should be ignored in estimating the pressure drop.

Because of differences in operating characteristics, liquid relief valves are not subject to chattering. However, the pressure drop is still to be kept within the limit as a large pressure drop will significantly reduce the capacity of the valve.

The following criteria are used for preliminary inlet line sizing:

- If piping layout is not available, the following piping configuration may be assumed:
  a. Single relief valve:
    i. One T
    ii. Two long radius elbows
    iii. Straight pipe length — 6 m
  b. Duplicate relief valve (common line):
    i. One T
    ii. Two long radius elbows
    iii. Straight pipe length — 5 m

  c. Duplicate relief valve (individual line):
    i. One T
    ii. One gate valve
    iii. Straight pipe length — 2 m
- The flow contingency shall be the maximum contingency flow through the valve. This is the maximum capacity of the relief valve.
- For conventional and balanced-bellows PRVs, the nonrecoverable pressure drop shall not exceed 3% of the PRV set pressure.
- There are instances when the PRV protects interconnected vessels and equipment (e.g., number of exchangers in a heat exchanger train). For such an installation, the 3% limit will apply for the closest equipment protected by the relief valve. The pressure on any equipment must not exceed the MAWP, and no piping should exceed the pressure limit according to ASME B31.3 [5].
- In case of a major problem in meeting the 3% line loss criterion, a pilot-operated relief valve can be installed with a pressure sensing tapping taken from the protected equipment. With such an installation, it is still required to reduce the inlet pressure drop as far as possible.
- For a revamp situation, it is sometimes difficult to keep the frictional pressure drop within 3%. In such a situation, some operating companies increase the frictional pressure drop to 5% provided the blowdown is increased to a minimum of 9% of the set pressure. Increasing the pressure drop to 5% is not a good practice. Moreover, the following points are to be considered before accepting the higher pressure drop:
  a. Higher pressure drops must preferably be avoided.
  b. For PRVs with adjustable blowdown, AS 1271 [6] recommends a maximum blowdown of 7% if the PRV set pressure is more than 300 kPaG.
  c. For the nonadjustable type, the blowdown can be increased up to 15%.
- The inlet line size shall not be less than the inlet nozzle size of the PRV.
- When two or more relief valves are placed on one connection, the inlet internal cross-sectional area of this connection must either be sized to avoid restricting flow to the PRVs or made at least equal to the combined inlet areas of the relief valves connected to it.

Discharge piping from PRVs

Whereas sizing of the inlet line is straightforward, that of the outlet line is more complex, and sometimes, more involved procedures are required. The sizing of the outlet line depends on a large number of factors, including type of relief valve, final location of the discharge, and so on. Velocity and pressure drop are two criteria normally used in estimating the outlet line size. The general guidelines can be explained as follows:

- For relief valves venting to the atmosphere, the outlet line is sized for a maximum velocity of 0.75 *Mach* at atmospheric pressure.
- For conventional relief valves venting to a closed system, the line is sized to keep the back pressure within 10% of the relief valve set pressure. This is subject to a maximum velocity of 0.75 *Mach*. API 521 recommends a velocity up to 1 *Mach*. However, proper analysis of piping integrity is required for a velocity close to *Mach* 1; such high velocity should not be used for the design of a new system.
- Outlet line sizing for balanced-bellows and pilot-operated relief valves is based on a maximum velocity of 0.75 *Mach*.
- In revamp situations, it is sometimes difficult to maintain the velocity within 0.75 *Mach*. In such cases, the velocity can be increased up to 1 *Mach*.
- Velocity checking requires a full network analysis of the total system. This can only be done once the piping layout and all contingencies are finalized.
- Outlet line size must not be less than the outlet nozzle size of the PRV.

Contingency quantification

General

Generally, the design of a relief system is governed by the mechanical design code. Recommended practices such as API RP 520 [1] and API RP 521 [3] are extensively used in the design of relief systems. In general, the following approaches are used for quantifying relief contingencies.

All conditions that can result in a relief shall be considered. The possibility of two unrelated emergency conditions, i.e., "double jeopardy," must not be evaluated.

No credit should be taken for the presence of emergency instrumentation and the response of plant operators in estimating the relief contingency. However, instrumentation credit is permitted where high-integrity protection systems (HIPSs) are provided. For example, if the flow to an inlet separator is stopped through a properly designed HIPS, the blocked outlet contingency for the separator relief valve can be eliminated.

Flow rates through equipment and other conditions during a particular contingency shall be assumed to be at the maximum operating flow except for a fire case. In a fire situation, the instrumentation system may fail to operate, and the vessel on fire may have a blocked-in condition. For this reason, no flow is assumed from the equipment on fire.

Credit may be taken for reduced flow due to increase in system pressure from normal operating pressure. For example, flow through a centrifugal compressor may be reduced if differential pressure across the compressor is increased at relief condition.

Where loss of liquid level in an upstream vessel may result in gas breakthrough, the upstream vessel must be considered at normal operating pressure and downstream equipment at relieving condition. In some cases, the liquid from the vessel is drained to a pond or drain tank and is connected through piping with a lower rating (150#). In such circumstances, if the upstream vessel is operating at higher pressure (rating 300# or more), the following points should be considered:

- The isolation in 150# piping must be of lock open.
- Maximum gas breakthrough should not overpressure the 150# piping beyond its design pressure limit.
- If pressure increase in the 150# piping is too high, a suitably sized restriction orifice should be installed.

In the event of heating media failure on a column reboiler, the effect on downstream equipment must be considered. With the loss of reboiler feed, it can be assumed that the flow through the column bottom is identical to the feed (same flow rate and composition). In case the bottom is fed to another column, the impact of this additional feed must be considered.

Loss of reflux on the column should be considered as a total loss of cooling. It is assumed that with loss of reflux the liquid accumulates and floods the reflux drum, and then the cooler. The cooler loses the cooling duty and the column pressure will increase to its relieving condition. No credit should be taken for the normal outflow of uncondensed vapor. The static head of the flooded system should be allowed for in design pressure consideration as explained previously.

For fan failure on air coolers, credit may be taken for normal convection according to API 521. The credit shall be 25% of the normal design duty. In case precise calculation is required, a check shall be performed for natural convection using a heat exchanger rating program. In case of a water-cooled cooler, credit is not allowed if the cooling water flow stops.

For vapor relief from columns (other than fire relief), the rate of vapor relief and the fluid properties must be determined by simulation at relieving pressure. The relieving will be 110% or 116%, depending on the requirement. For a fire case, the relief rate depends upon the latent heat and wetted surface area. The latent heat shall be estimated at 121% relieving pressure and for the average liquid composition within the column.

When equipment in a fire area may contribute a significant relief load in the event of a fire, which the relief valves discharge into another vessel outside the fire area, then the relief load from the equipment shall be considered as a contingency for the vessel to which the relief valves discharge.

Power failure

The following possible scenarios shall be considered in analyzing a power failure situation:

- Local power failure, such as a motor for a pump or compressor, or fan for an air cooler
- Failure of a distribution center, such as a particular MCC in the production area
- Total power failure, where all electrically operated equipment will be affected

Local power failure

This essentially means the failure of an individual drive of a pump, compressor, fan, and so on. Failure of a single motor does not always mean it will affect only one relief valve. Depending on the drive that fails, the relief can be substantial. The following situations should be considered carefully in evaluating the impact of local power failure:

- In case of failure of a drive affecting a single equipment, the contingency can easily be quantified. For example, the failure of a reflux pump will affect cooling of the overhead vapor, and the column will relieve under a reflux failure situation.
- Failure of a cooling water distribution pump may create a complex situation. If a single pump is used for the water distribution (which is not the case in general), the situation will change to total failure of cooling water. This situation will result in substantial relief from the plant. In most designs, multiple pumps operate in cooling water distribution circuits, and failure of one operating pump will result in low water flow in the cooling water circuit. The operator may start a standby pump after receiving the low water flow alarm; the time required to restore the full cooling water flow needs to be considered in estimating any possible relief. It is also possible that the system will operate at low water flow for a considerable period, and in such a situation the impact of low water flow needs to be considered for the entire plant.
- Failure of one instrument air compressor may result in the failure of instrument air supply. Once the instrument air supply stops, the system will still operate with the instrument air available in the receiver, which will give the operator time to restart the standby air compressor. The sizing of the instrument air receiver is important in estimating any relief possibility.

Failure of a distribution center

This may create more serious problems than individual local power failure. The impact of a particular bus bar design and the motors connected to a particular bus bar are very important in estimating contingency. It is often preferred that the electrical engineer and process engineer work together to establish the design of the bus bar.

Some designs allow for power from a different source in the event of failure of the main power source. Because this type of system requires operator intervention to prevent overpressure in the equipment, no credit shall be allowed for the availability of an alternative power supply.

Another possibility exists in the design of a bus bar in the same substation where half of the motors are connected through a different bus bar. In such a situation, availability of 50% power can be assumed in estimating the relief contingency.

Total power failure

The impact of total power failure is very complex and needs to be established carefully. Total power failure does not always result in maximum relief contingency.

In some cases, the operation of a plant depends on the pumping of feed. In such a system, total power failure will result in loss of feed and, in turn, may stop any substantial relief from the plant. However, if the feed pump is partly turbine driven, feed will be available at a reduced rate, and the relief can be significantly higher.

Total loss of power may not necessarily mean the total loss of steam. With loss of power, the boiler can still operate for some time at reduced capacity, and all the steam turbines can operate at reduce capacity. This impact needs to be considered carefully.

Failure of the cooling water pump is an important consideration, and the number of cooling water circulation pumps needs to be established carefully. The number of cooling water circulation pumps can be equal to the number of steam turbine-driven pumps that normally operate.

Regarding total power failure, additional study is required to analyze and evaluate the combined effects of multiple equipment failures. Special consideration should be given to the effect of simultaneous opening of relief valves in several services, particularly if the relief valves discharge into a closed header system.

Cooling water failure

There are several reasons of cooling water failure, such as system leak or rupture, pump failure, loss of cooling water in a particular supply branch due to accidental closure of a valve, etc. In some cases, partial failure of cooling water may cause more relief contingency than total cooling water failure. For example, in case of a compressor feeding a distillation column, total cooling water failure will result in compressor trip, whereas a failure of the column condenser while the compressor cooler is operating will result in column relief.

Whereas total loss in cooling water supply is easy to analyze, partial losses are not. Proper identification of partial losses is essential before quantifying the impact of partial loss. There are many possibilities of partial losses, such as:

- Accidental closure of a valve. This may stop supply to one exchanger or to a group of exchangers connected to that particular branch.
- In most cases, cooling water is supplied through multiple operating pumps. Failure of one pump will result in reduced cooling water flow, the impact largely depending on the capacity of the pump.
- In case some cooling water transfer pumps are operated through turbine drives, the impact of steam failure and electric power failure needs to be analyzed carefully.

Adequate backup features are required to minimize the impact of cooling water failure. The following are minimum requirements that should be considered in the system design:

- Multiple pumps are to be used for a normal supply of cooling water. In case of an operating pump failure, the automatic introduction of a spare pump is highly beneficial.
- The cooling tower sump should have adequate volume with a low-level alarm. In case the sump level gets low, operator intervention is essential to increase the makeup water rate, so water is held up to a reasonable level.
- Failure of the cooling tower fan will not reduce the cooling water flow, but the cooling duty will be reduced. It is often beneficial to have a secondary power supply to the cooling tower fans to restore the cooling tower duty in the shortest possible time.
- A multiple cooling tower system is better than having a single large system. In a multiple cooling tower system, the probability of failure of all cooling towers is low, and the overall impact of cooling water failure can be reduced further by cross-connecting the individual loops.
- Adequate instrumentation and alarm systems are required to reduce the impact of cooling water failure. For example, a low-pressure alarm on the makeup cooling water header is helpful to identify low makeup water rate.

Instrument air failure

Loss of instrument air to the control valve means the valves will assume either the closed or open position, depending on the original design. Generally, the control valve failure position is determined in such a way that overpressure is minimized. The overall plant contingency due to instrument air failure depends on the actual number of PSVs that are likely to relieve. In practice, few PRVs are likely to relieve owing to instrument air failure, but this does not generate very high contingency for the plant.

There are however other situations that may produce very high contingency due to instrument air failure. In some plants, blowdown contingency

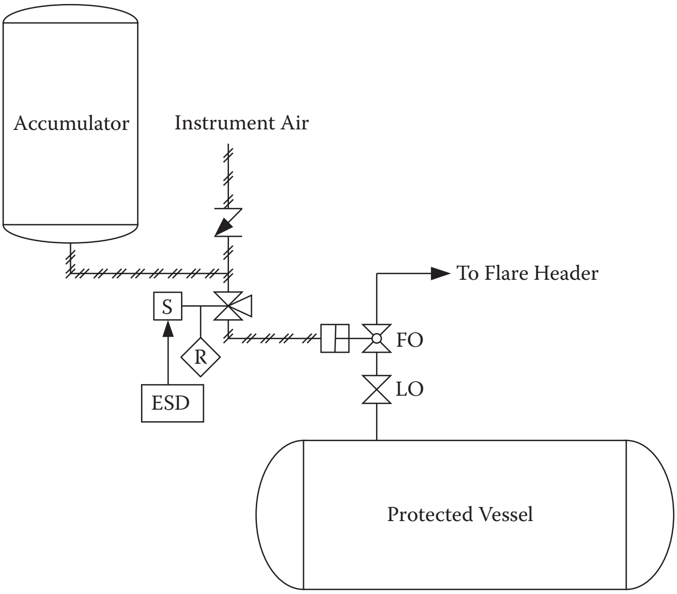


Figure 4.11 Instrument air accumulator to delay blowdown.

is very high, and in most cases the blowdown is done in stages (through delay action) to minimize the total contingency. For example, if the whole plant is divided into several fire areas, then fire in one area will blowdown that area instantaneously. Other areas are not required to blowdown instantaneously, and a delay action is introduced. The delay can be a range of time for different fire areas, and the blowdown contingency can be reduced substantially through delay blowdown action.

Instrument air failure may have a substantial impact on blowdown. The blowdown valves normally fail open (i.e., the valve will open with instrument air failure). In fact, the blowdown valve opens when the instrument air pressure drops below a particular design pressure (approximately 400 kPaG). In case of instrument air failure, if the instrument air pressure drops below the minimum pressure required to keep the blowdown valves shut, all blowdown valves will open at the same time. This may produce a large contingency and, in some cases, more than the capacity of the plant. This situation can be improved by introducing an instrument air accumulator as shown in Figure 4.11. The instrument air accumulator will allow the blowdown valve to close for a substantial period of time to enable any delay action. With this modification, the total blowdown contingency can be reduced substantially.

Steam failure

Steam failure directly affects the operation of the following equipment:

- Turbine-driven pumps, compressors, blowers, and generators
- Reboilers using steam as the energy source
- Steam ejectors
- Equipment using direct steam injection

Steam failure may be complex in nature. This can be total steam failure, partial steam failure, or failure of a particular type of steam (e.g., high-pressure steam or medium-pressure steam).

Total steam failure

For a large plant, this is not very common. Steam is used as energy to the system, and loss of steam in some equipment (e.g., reboiler) may not result in relief, whereas in some other equipment (e.g., turbine-driven cooling water pump) it may cause a large contingency. In general, total failure of steam will not lead to a controlling plant contingency; however, it is essential to establish an overall plant contingency for total steam failure.

Loss of steam to specific equipment

Loss of steam to the turbine for cooling water pumps may cause substantial plant relief. If all cooling water pumps are steam turbine driven, the contingency will be maximum, but this is seldom the case. In most cases, cooling water pumps are a combination of electric motor drive and steam turbine drive. The impact of other drives needs to be analyzed carefully.

For a cogeneration system, the total plant power may be available from generators powered by steam. In such a case, the total loss of steam will be equal to the total loss of power.

Partial steam failure

This may cause greater contingency in a particular relief scenario. For example, in a distillation system, the cooling water may stop from loss of high-pressure steam (loss of condenser cooling), whereas the reboiler can still use low-pressure steam as a source of energy. This will result in maximum contingency in the event of partial steam failure.

Check valve failure

In general, check valves are not very reliable and often fail; they can cause significant overpressure problems. In practice, a single check valve will be considered as fully open to calculate the back flow and pressure increase due to back flow. In situations in which the impact of pressure increase due to back flow is substantial, two or more check valves are installed in series. Multiple check valves are normally used in the following instances:

- The differential pressure is 7000 kPa or more.
- Full back flow will result in significant relief.

If two or more check valves are used in series, it is always assumed that one check valve will be fully open and the other will leak. The leakage rate is approximated as follows:

Liquid flow: The leakage rate through a single check valve will be the higher of the following:
  a. 10% of forward flow
  b. $W = 0.00034 * \rho * d * \Delta p^{0.5}$ (4.1)

Vapor flow: The leakage rate through a single check valve will be the higher of the following:
  a. 10% of forward flow
  b. $W = 0.00026 * d*P_1 * MW^{0.5}$ (4.2)

Blocked outlet

This contingency depends on the nature of the blocked outlet. A large number of scenarios are possible, and it is difficult to cover all these here. However, the contingency for major blocked-outlet scenarios can be estimated by the following procedure.

Pump or compressor discharge

Blocked-outlet pressure protection may be required because of overpressure by pumps or compressors. The following general guidelines should be used to establish the protection requirement and contingency:

- For centrifugal pumps or compressors, blocked-discharge overpressure protection is not required if the sum of no-flow differential head and relieving pressure of the suction PRV is less than the design pressure of the equipment on the discharge side.
- If maximum possible discharge pressure from the centrifugal pump or compressor exceeds the design pressure of the downstream equipment, pressure protection will be required for a blocked-discharge scenario. The contingency should be estimated by the following procedure:
  1. Assume normal suction pressure.
  2. Estimate the pump or compressor discharge pressure as the sum of PRV relieving pressure and the pressure drop between the pump or compressor discharge point to the location of PRV.
  3. Estimate the differential head, which will be higher than the normal.
  4. From the pump or compressor performance curve, establish the flow rate, which will be the required contingency.
- For positive displacement pumps or compressors, PRV for a blocked-discharge situation will always be required. For pumps, the contingency will be the normal flow rate. For compressors, the contingency will depend on the change in the suction condition in a

blocked-discharge situation. The blocked-discharge contingency shall be the normal flow rate if the suction pressure does not change during blocked-discharge relief. In case the suction pressure increases, the contingency will be equal to the compressor flow rate at a higher suction pressure.

Multiple outlet

Blocked-outlet relief is possible even in a multioutlet case. The following general guidelines will apply:

- In a multioutlet situation, each outlet will be considered individually, and the maximum contingency established.
- Simultaneous blockage of two or more valves (double jeopardy) should not be considered unless the reason of blockage (e.g., instrument failure) is the same.
- Blockage due to instrument air failure is established in an instrument air failure scenario and need not be considered in a blocked-discharge scenario.
- Multiple outlet blockage should be considered if the blocked outlet is caused by operator intervention and the same logic causes multiple outlet blockage.
- It is normally assumed that locked-open valves are not blocked in a normal operating scenario.
- In a single or multiple blocked-outlet situation, the inlet valve is considered at its normal operating position unless the blocked situation itself alters the position of the inlet valve.

Block valve downstream of control valve

In this situation, the following methodology is used to establish the blocked-outlet contingency:

- A blocked-discharge situation may increase the downstream pressure of the control valve, and this will reduce the differential pressure across the control valve.
- In most cases, the control valve goes fully open and the contingency shall be estimated for the reduced differential pressure across the control valve and maximum control valve CV. For more explanations, see the following subsection.

Control valve failure

Control valve failure is an important scenario in contingency calculation. The following general guidelines are normally used to estimate control valve failure contingency.

- Both fail-close and fail-open control valves can fail to cause an overpressure relief.
- It is assumed that the control valve will be fully open (CV = 100%) at a failure situation.
- Control valve contingency is calculated with the upstream pressure assumed as maximum operating pressure and downstream pressure as PRV relieving pressure.
- The control valve bypass valve will be considered fully open when it is required to be open, e.g., startup or shutdown situation.
- The bypass can be assumed fully closed if the control valve capacity is adequate and bypass is not required to be opened for maximum possible flow condition.
- Sometimes the operator requires to partially open the bypass isolation valve to meet the demand of high flow. In such cases, the bypass valve will be considered partially open and equal to 50% of the fully open CV of the control valve. This means the contingency shall be estimated for 150% of fully open control valve CV. A bypass valve partially open need not be considered if the valve is locked closed.
- Sometimes a control valve is installed with a mechanical travel limit to reduce the maximum possible flow through the valve. In such circumstances, the relief contingency will be estimated for the actual CV (not the reduced CV) of the control valve.
- For a liquid-level control valve when the liquid is discharged to a lower pressure, the contingency shall be estimated for maximum possible liquid flow or maximum gas breakthrough rate.

Vapor breakthrough

Vapor breakthrough is an important criterion in estimating the control valve failure contingency. In case a liquid-level control valve that dumps liquid to a lower pressure fails, the liquid will be drained out from the vessel. Once all liquid is drained, the high-pressure vapor will start flowing through the control valve, which is commonly known as vapor breakthrough. If the ratio of upstream to downstream pressure exceeds the critical pressure ratio, the flow will be choked. The choked-flow situation can overpressure the downstream piping even after the installation of a pressure reducing valve. In such cases, a restriction orifice is to be installed downstream of the control valve to limit the flow and to avoid a choked-flow situation.

Example 4.1

The suction scrubber of a gas compressor operates at 4000 kPaG. The liquid collected in the scrubber is discharged to a separator tank, which is open to the atmosphere, through a level-control valve installed on the liquid outlet line. In a particular situation of control valve failure (control valve goes fully open), there will be vapor breakthrough to the separator. Analyze the vapor breakthrough situation based on the following information:

Liquid drain line size = 50 NB
Design pressure of the drain line = 1960 kPaG (150#)
Equivalent length of the line = 50 m
Ratio of specific heat = 1.3
Molecular weight = 20
Operating temperature = 25°C

No isolation valve in between the control valve and the separator.

Solution

Pipe ID = 52 mm
f for fully turbulent flow = 0.019
K for fully open valve = 0.2
K for 50-m pipe (0.019 * 50/0.052) = 18.3
K for entrance = 0.5
K for exit = 1
Total *K* = 20
$\Delta p/P_1$ = (4101.3 – 101.3)/4101.3 = 0.975

Limiting value of $\Delta p/P_1$ (from Equation 2.100):

$$\frac{\Delta p}{P_1} = \left\{0.9953 + \frac{0.9054}{K^{0.5}} + \frac{0.1173}{K} - \frac{0.0195}{K^{1.5}}\right\}^{-1.0}$$

$\Delta p/P_1$ (limiting value) = 0.831

Because the limiting value is less than the calculated value, the flow will be choked.

Δp = 0.831 * 4101.3 = 3408 kPaA

The differential pressure is too high, and the pressure in downstream piping will exceed its design pressure of 1960 kPaG. The installation of PRV may not be practical in this particular situation because of very high flow rate and extremely high frictional pressure drop. The most practical solution is to install a restriction orifice.

Sizing of the restriction orifice depends on the maximum allowable flow rate (to keep the downstream pressure within the design pressure limit) and minimum orifice size (so that the orifice does not get blocked from impurities). It is in fact a trial-and-error analysis.

For a reasonably clean service, a 10-mm orifice can be used without much problem, and the pressure/flow situation can be analyzed for a 10-mm orifice.

Maximum flow

Because the pressure in the scrubber is 4000 kPaG, the flow across the restriction orifice will be choked; the flow rate can be estimated using Equation 2.27:

$$G = a_o C_D F_a Y \sqrt{2000 P_1 \rho_1 \gamma \left(\frac{2}{\gamma + 1} \right)^{(\gamma+1)/(\gamma-1)}}$$

where

a_o = area of the restriction orifice, m^2 = 0.0000785 m^2
ρ_1 = upstream density, kg/m^3 = 33.1 kg/m^3
C_D (from Equation 2.20) = 0.61
F_a = 1
Y (from Equation 2.21) = 0.85 (approximately)
G = 0.4475 kg/sec
= 1611 kg/h
Density at atmospheric pressure = 0.818 kg/m^3
Volumetric flow = 1969 m^3/h
Velocity = 257 m/sec (conservative analysis)
Total K = 19.8
Pressure drop = 534 kPa (conservative analysis)

In actual practice, the pressure drop will be lower than estimated previously (247 kPa), using the program developed in Chapter 2, because of higher average density. However, the calculation indicates that the 10-mm restriction orifice is adequate to restrict the pressure of the downstream piping within the 150# rating.

Thermal relief

It is quite common that a liquid-carrying line can be isolated between two isolation valves. The isolated liquid is under thermal expansion due to heat gain from high ambient temperature. Small thermal expansion can be adequate to increase the pressure beyond the design pressure limit of the pipe. The exact calculation of thermal expansion and contingency is complex and is often not required. The volume released is small and a nominal 20 × 25 NB relief valve is installed to protect the system. However, the thermal expansion can be established mathematically as follows:

Pipe expansion due to pressure can be defined as

$$f_c = \frac{\Delta PD}{2h} \tag{4.3a}$$

$$f_l = \frac{\Delta PD}{4h} \tag{4.3b}$$

Change in pipe diameter:

$$\Delta D = \frac{f_c D}{E} - \frac{\sigma f_l D}{E}$$

$$= \frac{f_c D}{E}(1 - 0.5\sigma) \tag{4.4}$$

because $f_l/f_c = 0.5$.

Pipe cross-sectional area:

$$A = \frac{\pi}{4} D^2 \tag{4.5}$$

Change in pipe cross-sectional area:

$$\Delta A = \frac{\pi}{4}(D + \Delta D)^2 - \frac{\pi}{4} D^2$$

$$= \frac{\pi}{2} D \Delta D \tag{4.6}$$

The value of ΔD^2 is too small and therefore neglected.

Change in length:

$$\Delta L = \frac{f_l L}{E} - \sigma \frac{f_c L}{E}$$

$$= \frac{f_c L}{E}(0.5 - \sigma) \tag{4.7}$$

Now, the volume of the pipe is defined as

$$V = A * L$$

Therefore, the change in pipe volume will be

$$\Delta V = A\Delta L + L\Delta A$$

$$= \frac{Af_cL}{E}(0.5 - \sigma) + \frac{L\pi}{2}D\frac{f_cD}{E}(1 - 0.5\sigma)$$

$$= \frac{Vf_c}{E}(0.5 - \sigma) + \frac{2Vf_c}{E}(1 - 0.5\sigma) \tag{4.8}$$

$$\frac{\Delta V}{V} = \frac{f_c}{E}(2.5 - 2\sigma) \tag{4.9}$$

Using the value of f_c from Equation 4.3a,

$$\frac{\Delta V}{V} = \frac{\Delta PD}{2hE}(2.5 - 2\sigma) \tag{4.10}$$

Now, the thermal expansion of the pipe is

$$\frac{\Delta V}{V} = 3\alpha\Delta T \tag{4.11}$$

Therefore, the total expansion will be

$$\frac{\Delta V}{V} = \frac{\Delta PD}{2hE}(2.5 - 2\sigma) + 3\alpha\Delta T \tag{4.12}$$

Again, the thermal expansion of the liquid is

$$\frac{\Delta V}{V} = \beta\Delta T \tag{4.13}$$

Liquid compression due to pressure is

$$\frac{\Delta V}{V} = -\Delta PZ \tag{4.14}$$

Now, it is assumed that the liquid is blocked in by at least one isolation valve. However, the isolation valve can never be fully liquid-tight and will pass some quantity of liquid. This liquid volume leaked out of the system should be considered in calculating the total liquid expansion due to change in temperature. Therefore, effective liquid expansion will be

$$\frac{\Delta V}{V} = \beta \Delta T - \Delta PZ - \frac{qt}{V} \tag{4.15}$$

where qt/V is the volume of liquid leakage per unit volume of fluid. From Equation 4.12 and Equation 4.15,

$$\frac{\Delta PD}{2hE}(2.5 - 2\sigma) + 3\alpha\Delta T = \beta\Delta T - \Delta PZ - \frac{qt}{V}$$

or

$$\Delta P = \frac{\Delta T(\beta - 3\alpha) - qt/V}{Z + \dfrac{D}{2hE}(2.5 - 2\sigma)} \tag{4.16}$$

Equation 4.16 is the general equation to estimate the pressure increase due to thermal expansion.

The following information is required to solve Equation 4.16:

- Modulus of elasticity of the pipe materials (E)
- Coefficient of linear thermal expansion (α)
- Valve leakage rate (q)
- Compressibility of liquid (Z)
- Coefficient of cubic expansion of liquid (β)

Modulus of elasticity of pipe material (E)

The modulus of elasticity of metals varies slightly with temperature. Values of the modulus of elasticity for commonly used metals at 21°C and 149°C are presented in Table 4.1 [5].

Coefficient of linear thermal expansion (α)

The coefficient of linear thermal expansion of metals also varies slightly with temperature. Coefficients for commonly used metals at 21°C and 149°C are presented in Table 4.2 [5].

Table 4.1 Modulus of Elasticity of Commonly Used Metals

Metals	Temperature 21°C Million (psi)	 Million (kPa)	Temperature 149°C Million (psi)	 Million (kPa)
Gray cast iron	13.4	92.4	12.9	88.9
Carbon steels, C ≤ 0.3%	29.5	203.3	28.3	195.1
Carbon steels, C > 0.3%	29.3	202.0	28.1	193.7
Carbon-moly steels	29.2	201.3	28.0	193.0
Cr-Mo steels, Cr 1/2–2%	29.7	204.7	28.5	196.5
Cr-Mo steels, Cr 5–9%	30.9	213.0	29.7	204.7
Austenitic stainless steels (TP 304,310,316,321,347)	28.3	195.1	27.0	186.1

Table 4.2 Coefficients of Linear Thermal Expansion for Commonly Used Metals

Metals	Temperature 21°C 1/°F	 1/°C	Temperature 149°C 1/°F	 1/°C
Carbon steels	$6.07 * 10^{-6}$	$10.93 * 10^{-6}$	$6.6 * 10^{-6}$	$11.88 * 10^{-6}$
Low chrome alloys	$6.07 * 10^{-6}$	$10.93 * 10^{-6}$	$6.6 * 10^{-6}$	$11.88 * 10^{-6}$
5Cr-Mo through 9Cr-Mo	$5.73 * 10^{-6}$	$10.31 * 10^{-6}$	$6.19 * 10^{-6}$	$11.14 * 10^{-6}$
Austenitic stainless steels	$9.11 * 10^{-6}$	$16.4 * 10^{-6}$	$9.47 * 10^{-6}$	$17.05 * 10^{-6}$

Valve leakage rate (q)

Valve leakage rates are functions of the valve size and type of valve seat. Recommended valves and their maximum liquid leakage rate are presented in Table 4.3 [11].

The values presented in Table 4.3 are maximum allowable valve leakage rates according to API Std 598 [11] but may not be the actual valve leakage rates. However, the impact of valve leakage rate is not significant in overall calculations, and values presented in Table 4.3 can be used.

Table 4.3 Maximum Valve Leakage Rates (for Liquid)

Valve Size (mm)	Leakage Rate (m^3/sec)
≤50	0
65–150	$1.25 * 10^{-8}$
200–300	$2.08 * 10^{-8}$
≥350[a]	—

Note: (1) The leakage rates given relate to all metal-seated valves except check valves. (2) The leakage rate of metal-seated check valves is $5.0 * 10^{-8}$ m^3/sec per inch of nominal pipe size. (3) The leakage rate of soft-seated valves is zero for all sizes.

[a] The valve leakage rate is $2.08 * 10^{-9}$ m^3/sec per inch of pipe size.

Table 4.4 Compressibility of Commonly Used Liquids

Liquid	Compressibility, 1/psi	Compressibility, 1/kPa
Propane	$1.89 * 10^{-5}$	$2.74 * 10^{-6}$
Butane	$1.23 * 10^{-5}$	$1.78 * 10^{-6}$
Gasoline	$0.52 * 10^{-5}$	$0.75 * 10^{-6}$
Diesel	$0.44 * 10^{-5}$	$0.64 * 10^{-6}$
Water	$0.34 * 10^{-5}$	$0.49 * 10^{-6}$

Compressibility of liquid (Z)

The compressibility of liquid is a function of pressure. The compressibility of commonly used liquids is presented in Table 4.4.

Coefficient of cubic expansion of liquids (β)

The coefficient of thermal expansion of liquid is a function of temperature and can be expressed as [12]

$$\beta = a\left(1 - \frac{T}{T_c}\right)^m \tag{4.17}$$

where a and m are constants, depending on the fluid. Values of constants for commonly used components and coefficients of expansion at 25°C are presented in Table 4.5.

For refinery operations as well as oil and gas applications, the cubical expansion coefficient of hydrocarbon liquids and water at 15°C is presented in Table 4.6 below [2].

Installation of thermal relief valve

The following guidelines are normally followed in installing a thermal relief valve:

- Though Equation 4.16 can be used, it is often difficult to establish the relief contingency. However, because the contingency is small, a relief valve of 20 × 25 is used to protect a pipeline against thermal overpressure.
- The thermal relief valve is set at the maximum possible pressure. In most cases, the relief valve is used only to protect the pipeline, and if the pipeline is designed according to ASME B31.3 [5], the set pressure can be 1.2 times the design pressure of the pipe.
- In some cases, the thermal relief valve discharges into a closed system; in such a situation, the back pressure should be considered in the design of the relief valve.
- The relief valve should be installed in the most appropriate location on the pipeline. A careful consideration is required for the discharge route of the relief valve.

Table 4.5 Coefficient of Cubic Expansion of Liquids

Liquid	*a*	T_c (K)	*m*	Expansion at 25°C (1/°C)
Propane	0.000995	369.82	–0.713	0.003206
Isopropanol	0.0006353	508.31	–0.757	0.00124
n-Propanol	0.000605	536.71	–0.7506	0.001112
Glycerol	0.0002963	723.0	–0.8459	0.0004646
n-Propylamine	0.0007093	496.95	–0.7539	0.001415
Isopropylamine	0.0007955	471.85	–0.7028	0.001606
Trimethylamine	0.0008426	433.25	–0.7313	0.001976
Cyclobutene	0.0008195	446.33	–0.7143	0.001801
1,2-Butadiene	0.0008498	444.0	–0.7143	0.001882
1,3-Butadiene	0.0008892	425.37	–0.7093	0.002093
Acetic anhydride	0.0006752	569.15	–0.7301	0.001161
1-Butene	0.0008997	419.59	–0.7147	0.002182
Cis-2-butene	0.0008575	435.58	–0.7143	0.001955
Trans-2-butene	0.0008674	428.63	–0.7143	0.002029
Cyclobutane	0.0006297	459.93	–0.7619	0.001396
Isobutene	0.0008846	417.9	–0.7204	0.002177
n-Butane	0.0008757	425.18	–0.7137	0.002074
Isobutane	0.0008686	408.14	–0.7270	0.002253
n-Butanol	0.0005768	562.93	–0.7543	0.001019
Isobutanol	0.0005570	547.73	–0.7657	0.001017

Table 4.6 Coefficient of Cubic Expansion of Hydrocarbon Liquids and Water

Gravity of Liquid (°API)	Expansion at 15°C (1/°C)
3–34.9	0.00072
35–50.9	0.0009
51–63.9	0.00108
64–78.9	0.00126
79–88.9	0.00144
89–93.9	0.00153
94–100 and lighter	0.00162
Water	0.00018

- A thermal relief valve that releases toxic chemicals must discharge to a closed system.

Fire exposure

The general guidelines outlined in API RP 521 [2] should be followed for the design of protection against fire. Both liquid- and gas-filled vessels are to be analyzed for protection against fire. For liquid wet vessels, the required relief is the generated vapor due to fire heat input, and for gas-filled vessels the required relief is calculated for the thermal expansion of the gas.

General guidelines

The following general guidelines are used to design pressure protection from fire:

- The contingency due to fire is calculated with an assumption that both inlet and outlet valves are closed. No vapor outlet is allowed except through the PRV.
- No additional heat inputs or heat rejections are considered except the fire heat input.
- During a fire, it is assumed all pumps, compressors, fans, etc., will be stopped.
- Relieving pressure during a fire is 121% of the PRV set pressure.
- A minimum of 116 kJ/kg is to be used if the estimated latent heat is less than this value.
- If the liquid content of the vessel is above the critical point, a minimum latent heat of 116 kJ/kg is to be used to estimate the fire contingency. Liquid thermal expansion can also be used to estimate the contingency if reliable data are available. Normally, physical properties estimated through simulation packages are not very reliable when the liquid is above the critical point.

Estimation of wetted surface area

The heat input into a liquid wet vessel depends upon the wetted area of the vessel. The following guidelines are used to estimate the wetted surface area:

- A maximum liquid height of 7.62 m above the grade is assumed. The general height guideline can be tabulated based on API RP 521 [3]. See Table 4.7.
- The grade can be ground or any floor or platform that can hold a liquid pool.
- Liquid height in the vessel with automatic level control is the normal operating level plus any additional liquid that could be in the vessel, up to a maximum of 7.62 m.
- Liquid height in the vessel without automatic level control is based on the maximum filling height up to a maximum of 7.62 m.
- For the tray column, liquid height is estimated as follows:

$$H_{liq} = H_{NLL} + H_W * N \tag{4.18}$$

where

H_{liq} = liquid height subject to a maximum of 7.62 m

H_{NLL} = height of normal liquid level

Table 4.7 Effect of Fire on the Wetted Surface of a Vessel

Vessel Type	Liquid Inventory	Remarks
Liquid full, such as treater	All liquids up to height of 7.62 m	
Surge drums, knockout drums, process vessels	Normal operating level up to the height of 7.62 m	
Fractionating column	Normal liquid level at bottom plus liquid holdup from all trays — up to the height of 7.62 m	Level of the reboiler must be included if the reboiler is an integrated part of the column
Working storage	Average inventory level up to the height of 7.62 m	
Spheres and spheroids	Up to the maximum horizontal diameter or up to the height of 7.62 m, whichever is greater	

H_W = weir height of the tray (if not available, assume 50 mm)
N = number of actual trays

- A margin of 10% is added to the calculated surface area to account for the associated piping.
- For the shell side of a shell and tube heat exchanger, the wetted surface area is estimated as follows:

$$A = \pi * D * L \tag{4.19}$$

where
A = surface area
D = outer diameter of shell
L = tube length

- For the tube side of a shell and tube heat exchanger, the wetted surface area is calculated as follows:

$$A = \pi * D * L_C + A_H \tag{4.20}$$

where
A = surface area
D = outer diameter of shell
L_C = length of the channel (consider both sides if required)
A_H = area of ends

- General guidelines for estimating the tube area for finned-tube air coolers are as follows:

a. For finned tube air cooler, the bare tube area is used instead of the finned area. This is done because in most cases fins are destroyed in the first few minutes of a fire.
b. For condensing but not subcooling air coolers, the wetted surface area is 0.3 times the bare tube area.
c. For condensing and subcooling air coolers, the condensing section is 0.3 times the bare tube area of the section, and the subcooling section is the bare tube area of the section.
d. For gas cooling, the area is equal to the bare tube area.
e. For liquid cooling exchangers, the area is equal to the bare tube area.

Fire circle

It is important to define a fire circle and to establish the number of pieces of equipment affected by the fire. The following guidelines are used for design:

- According to API RP 521 [3], fire circle area is 230 to 460 m^2 (diameter 17.1 to 24.2 m). A larger fire circle area is used for a plant processing flammable liquid, whereas a lower fire circle area is used for a gas processing plant with little flammable liquid.
- If two or more vessels are in the same fire circle and protected by the same PRV, contingencies from both or all vessels must be considered for the PRV design.
- Fire circles are to be developed so as to allow maximum possible equipment protected by the same PRV in the same fire circle. Some overlap in fire circles may happen to achieve this.

Estimation of latent heat and physical properties

Latent heat and other physical properties are important in estimating the fire contingency and are established at relieving pressure (121% of PRV set pressure). At high ranges of molecular weight, the latent heat can vary up to 4-fold, and a careful estimation is required. In a fire situation, it is assumed that a particular vessel is blocked in and the liquid inventory within the vessel will not increase with time. Initially, the wetted surface area is maximum, with maximum latent heat and minimum vapor molecular weight. With time, the wetted surface area is reduced with reduction in latent heat and increase in vapor molecular weight. The maximum vapor contingency and maximum PRV size can occur at any point in between, and it is difficult to assess the point of maximum PRV size. The following guidelines are used in estimating the latent heat and other physical properties.

For narrow boiling range hydrocarbons:

- The wetted surface area is the initial surface area.
- Latent heat is established at 50 mol% vaporization point.
- Vapor properties are established at 50 mol% vaporization point.

For high-boiling range hydrocarbons:

- The wetted surface area is the initial surface area.
- Latent heat and vapor physical properties are estimated at 5 mol% vaporization point.

Latent heat can be estimated either by using a simulation package or as shown in Figure 4.12. In case of a simulation package, the following procedure can be used to establish the latent heat:

- Increase the liquid stream pressure to the relieving pressure (121% of PRV set pressure).
- Heat the stream to its bubble point.
- Heat the stream further to achieve 5 or 50 mol% vapor, and record the heat input, say, Q_1 kJ/h.
- Estimate the vapor generated at the outlet of the second exchanger, say, W_1 kg/h.
- Estimate the increase in sensible heat of the liquid, say, S_1 kJ/h.
- Latent heat = $(Q_1 - S_1)/W_1$ kJ/kg.
- Physical properties are to be estimated from the vapor properties at the outlet of the second exchanger.
- In no case should the latent heat be less than 116 kJ/kg.

Liquid wet vessel

If N numbers of vessels are in the same fire circle and are protected by the same PRV, the heat input in the absence of proper drainage and where fire-fighting equipment does not exist is [3]:

$$Q = 255600 * F * \sum_{i=1}^{i=N} A_i^{0.82} \tag{4.21}$$

where

Q = total heat input, kJ/h
F = environment factor
A_i = wetted surface of individual vessel, m^2

If proper drainage and fire-fighting systems are available, the heat absorption is defined as [3]

$$Q = 155600 * F * \sum_{i=1}^{i=N} A_i^{0.82} \tag{4.22}$$

For air-cooled exchangers, the area term A would be raised to an exponent of 1.0 instead of 0.82 as is done for vessels. The environment factor, *F*, would be 1.0.

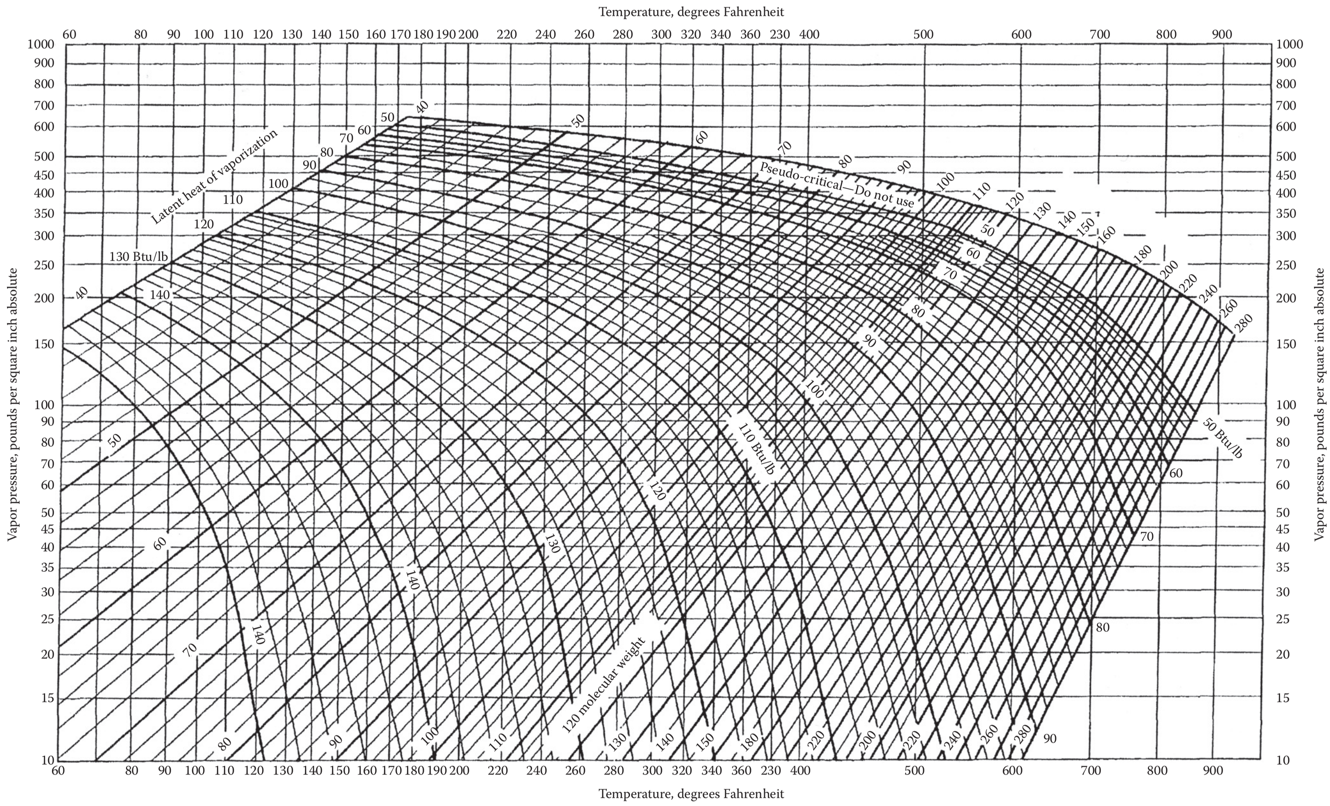


Figure 4.12 Vapor pressure and heat of vaporization of pure single-component paraffin hydrocarbon liquids. (Courtesy American Petroleum Institute, 1220L Street, NW, Washington, D.C.)

Table 4.8 Environment Factor

Type of Vessel	Thermal Conductivity (kJ/h·m²·°C)	Factor (F)
Bare	N/A	1.0
Insulated	81.767	0.3
	40.884	0.15
	20.442	0.075
	13.696	0.05
	10.221	0.0376
	8.177	0.03
	6.746	0.026
Below ground storage	N/A	0.00

Environment factor (*F*): The environment factor can be estimated using Table 4.8 [3].

The Following points are to be considered before taking advantage of insulation [3]:

- The insulating materials should be effective up to a temperature of 940°C during a fire.
- There should not be any dislodgement of insulating material (from fire and water jets) for 2 h of fire exposure.
- Insulation should be externally covered by a steel jacket up to a height of 7.62 m from grade.

Vessels with only gas

Heat input to a vessel containing only gas is different from a liquid wet vessel, and the following procedure is normally used to design pressure protection of a gas-filled vessel:

- The total surface area of the vessel is considered for design. A margin of 10% is added to allow for the associated piping.
- The effective PRV discharge area is estimated as [3]

$$A = \frac{F'A'}{\sqrt{P_1}} \tag{4.23}$$

where

A = effective PRV area, in.^{2}

A' = exposed area of the vessel, ft^2

P_1 = relieving pressure (121% of the set pressure), psia

- The value of F' in Equation 4.23 is estimated as [3]

$$F' = \frac{0.1406}{CK_d}\left\{\frac{(T_w - T_1)^{1.25}}{T_1^{\;0.6506}}\right\} \tag{4.24}$$

where

$$C = 520\sqrt{k\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}} \tag{4.25}$$

K_d = coefficient of discharge to be obtained from the valve manufacturers. A preliminary value of 0.975 can be assumed.
T_w = vessel wall temperature, °R. Recommended maximum vessel wall temperature for carbon steel plate is 1100°F.
T_1 = gas temperature at relieving condition, °R, defined as

$$T_1 = (P_1/P_o) * T_o \tag{4.26}$$

where
P_o = normal operating pressure, psia
T_o = normal operating temperature, °R

The recommended minimum value of F' is 0.01, and when the minimum value of F' is not known, F' = 0.045 can be used.

If the relieving temperature exceeds 1100°F, then it can be assumed that the vessel, if carbon steel, will fail before the PRV could relieve. In such a situation, PRV protection may not be very helpful. The use of a rupture disk or an alternative for PRV can be considered. Additional protective measures such as water spray, depressuring, or fireproofing can also be considered.

The relief contingency can also be calculated as [3]

$$W = 0.1406\sqrt{MWP_1}\left\{\frac{A'(T_w - T_1)^{1.25}}{T_1^{\;1.1506}}\right\} \tag{4.27}$$

where
W = relieving rate, lb/h
MW = molecular weight of vapor

Two liquid phases

Whereas it is easy to estimate the contingency for a single liquid phase, it is rather difficult to estimate it for two liquid phases, e.g., liquid hydrocarbon phase and water phase. All three-phase separators have two liquid phases, and a different approach is required to estimate the fire contingency:

- The heat input is calculated by the total wetted surface area.
- The hydrocarbon (HC) liquid phase is heated to achieve 5 or 50 mol% vapor (as the case may be).
- The water phase is heated to achieve 100% vapor (water being a single component).
- The temperature for both cases must be the same and is decided such that the total vapor pressure of HC and water streams equals the PRV relieving pressure (121% of PRV set pressure).
- The average vapor physical properties are estimated based on the weight fraction of the HC and water vapor phase.

Example 4.2

A three-phase separator, operating at 4000 kPaG and 25°C, has a total length (T/T) of 8000 mm and an inner diameter of 2134 mm. The normal liquid level (NLL) of the condensate is 900 mm from the bottom of the vessel. The vessel PRV is to be designed for a fire contingency based on the following information:

PRV set pressure = 5000 kPaG
Percentage overpressure = 21

Composition of inlet stream:

Components	Mol%
C1	72.25
C2	6.05
C3	4.04
iC4	3.03
nC4	3.03
iC5	2.02
nC5	2.02
C6	1.52
C7	1.0
C8	0.5
Water	4.54

The HYSYS® simulation package is used for general flash calculation and to estimate the physical properties.

Solution

Wetted surface area = 28.52 m^2
PRV relieving pressure = 6050 kPaG

The HC liquid has a high boiling range, and the physical properties are to be obtained at 5 mol% vaporization. The water stream is vaporized up to 100%.

The temperature of the hot stream is adjusted such that the total pressure equals the PRV relieving pressure. The adjusted temperature is 119°C and will be used as the relieving temperature.

HC vapor pressure at 119°C = 5947 kPaG (at 5 mol% vaporization)
Water vapor pressure at 119°C = 91 kPaG (at 100 mol% vaporization)
Total pressure = 6038 kPaG (compared to 6050 kPaG relieving pressure)

Wt% of vapor

Type	Mol%	*MW*	Wt%
HC vapor (=5947 * 100/6038)	98.5	35	99.2
Water (=100 – 98.5)	1.5	18	0.8

Average *MW* = 34.86
Average Cp/Cv = 1.39
Average compressibility = 0.747
Average latent heat = 133 kJ/kg (as calculated below)

For a HC stream with a *MW* of 35 and a temperature of 119°C, the latent heat, from Figure 4.12, is less than 116 kJ/kg. The minimum latent heat is assumed as 116 kJ/kg.

Latent heat of the water stream is 2239 kJ/kg.

Average latent heat = 133 kJ/kg
Heat input = 3,995,140 kJ/h
Contingency = 30,040 kg/h
PRV orifice = J

Heat exchanger tube rupture

API RP 521 [3], Section 3.18.2, states that for relatively low-pressure equipment, complete tube failure is not a viable contingency when the design pressure of the low-pressure side is equal to or greater than two thirds of the design pressure of the high-pressure side. Minor leakage can seldom result in overpressure of the low-pressure side during operation. Conversely, complete tube failure is considered a viable contingency when the design pressure of the low-pressure side is less than two thirds of the design pressure of the high-pressure side. However, if the high-pressure side of the exchanger operates at 6900 kPaG (1000 psig) or more and contains a vapor or a liquid that can flash or result in vaporization of liquid on the low-pressure side, complete tube failure should be considered, regardless of the pressure differential.

API RP 521, Section 3.18.5, indicates that, where the low-pressure side is in the vapor phase, full credit can be taken for the vapor handling capacity of the outlet and inlet lines, provided that the inlet lines do not contain check valves or other equipment that could prevent back flow. The same approach would apply in cases in which the low-pressure side is liquid full, provided that the released material also remains in the liquid state. However, when the low-pressure side contains liquid and vapor is released or generated through a rupture tube, the effective relieving capacity with which the piping system can be credited should be based solely on an equivalent vapor flow.

The following points need to be considered in quantifying the tube rupture contingency:

- Both transient and steady-state situations need to be considered. In some cases, the transient situation may result in a controlling contingency; e.g., gas in the high-pressure side and liquid in the low-pressure side; in other cases, the steady-state contingency may do so. The transient state is the state just after the tube rupture, and any steady state is a stable flow after the transient phase is over.
- It is required to provide a suitable pressure relief device. If the transient analysis indicates an instantaneous increase in pressure, up to the PSV relief pressure, a rupture disk is needed to protect the system (because the PSV response time is much slower). However, the disadvantages of a rupture disk should also be considered before finalizing the type of protection required.
- Transient analysis is a complex analysis and should be avoided if possible. It is preferable to improve the mechanical design of the exchanger to avoid transient analysis.
- The flow through the ruptured tube is calculated based on normal pressure of the high-pressure side and relieving pressure of the low-pressure side.
- Pressure protection is not required when an open flow path can be guaranteed to a destination, which can accommodate the burst tube flow without overpressuring the low-pressure side. For such situations, no credit is allowed for the control valve action located in the flow path, and either the control valve is to be relocated outside the flow path or a mechanical minimum stop is to be installed on the control valve.
- Heat transfer within the rupture tube may be ignored because of very short residence time. This makes the calculation simple for a single-phase fluid on both sides at considerably different temperatures.

Contingency calculation

The following procedure can be followed in calculating the steady-state contingency of tube rupture. The calculation procedure is different for compressible and noncompressible flow.

Noncompressible flow. The normal Darcy equation is used in calculating the flow through a ruptured tube. The pressure drop calculated using the Darcy equation is

$$\Delta p = K\frac{v^2}{2} * \frac{\rho}{1000} \tag{4.28}$$

The equation can be rearranged to calculate the mass flow rate for a certain pressure drop as follows:

$$W = 0.1265d^2\sqrt{\frac{\Delta p\rho}{K}} \tag{4.29}$$

Equation 4.29 is normally used to calculate the flow rate for tube rupture under a steady condition.

The number velocity K is calculated for both the short and long ends based on entry loss, exit loss, and frictional loss in the tube head.

Short end:

$$K = 1.5 \tag{4.30a}$$

Long end:

$$K = 1.5 + fL/D \tag{4.30b}$$

In most cases, a tube rupture situation causes a choked flow or a very high flow due to a large difference in shell and tube pressure. The large flow causes a fully turbulent flow with a very high Reynolds number. Friction factors for different tube diameters are presented in Table 4.9 [8].

Table 4.9 Friction Factor for Tube Rupture Situation

Tube Diameter (mm)	Friction Factor (f)
19	0.024
25.4	0.0225
32	0.0215
40	0.0205
50	0.019
80	0.0175

Compressible flow. For compressible fluids, the flow through a ruptured tube is calculated based on the Lapple equation [9,10].

$$Mach^2 = (r^2 - 1)/(K + \ln r^2) \tag{4.31}$$

The general equation of sonic velocity is

$$S = 91.1833\sqrt{\frac{kTZ}{MW}} \tag{4.32}$$

Mach = v/S. The velocity at downstream pressure P_2 can be calculated using the following equation:

$$v = 2938.74 * \frac{W * T * Z}{MW * P_2 * d^2} \tag{4.33}$$

$$\frac{v^2}{S^2} = 1038.7 * \frac{W^2 * T * Z}{MW * P_2{}^2 * d^4 * k} \tag{4.34}$$

Combining Equation 4.33 and Equation 4.34, the general equation of flow can be obtained as follows:

$$W = 0.031 * d^2 * P_2\sqrt{\frac{MW * k}{T * Z}} * \sqrt{\frac{r^2 - 1}{K + \ln r^2}} \tag{4.35}$$

To calculate the flow rate for tube rupture contingency, it is first required to establish the critical downstream pressure by using the following equations:

The critical pressure ratio is calculated as follows:

$$\frac{r_c{}^2 - 1}{K + \ln r_c{}^2} = 1 \tag{4.36}$$

where the critical pressure ratio is

$$r_c = P_1/P_c \tag{4.37}$$

The flow will be critical if critical pressure (P_c) is more than the downstream pressure (P_2), and in such cases the flow is calculated using the following equation:

$$W = 0.031 * d^2 * P_c \sqrt{\frac{MW * k}{T * Z}} \tag{4.38}$$

If the flow is subcritical, the flow is calculated using Equation 4.35.

Reflux failure and overhead system

There are several reasons for loss of reflux, including misoperation of isolation valves, reflux pump failure, failure of the control valve, etc. Reflux failure will result in accumulation of more liquid in the reflux drum, eventually flooding the drum and overhead condenser. This will result in loss of cooling. The column pressure will increase and the PRV will lift.

This situation is dynamic; however, in most cases, the contingency is estimated based on static operation of the column. The analysis of the columns and reflux drums are done separately, and it is assumed that the column PRV will be designed for all contingencies generated by the column, including column fire contingency. This left only the fire contingency to design the PRV on the reflux drum. The PRV on the reflux drum is set at column PRV set pressure plus the liquid head between the column and the reflux drum.

A conservative approach to design the column PRV is to assume the normal overhead vapor rate as reflux/condenser cooling failure contingency. However, the actual contingency will be less than the normal overhead vapor flow rate and can only be established using a simulation package. The simulation needs to be modified to achieve this. The following approach is used to establish the contingency for reflux/condenser cooling failure:

- Loss of duty of the overhead condenser or any pump around cooling is a possibility, with each case to be analyzed separately.
- The column is to be simulated at relieving pressure — 110% if protected by a single PRV or 116% if protected by multiple PRVs.
- In case of pumped feed, the feed rate may decrease because of increased column pressure. If possible and if the pump curve is available, the reduced feed rate can be estimated. In most cases, this involves complex calculation, which can be avoided with a little sacrifice of accuracy.
- Because of high column pressure, the column bottom temperature increases, which will have some impact on the reboiler heat input. In case steam or hot oil is used as the heat source, the LMTD (log mean temperature difference) of the reboiler is reestimated. Because of the increase in bottom temperature, the LMTD and reboiler heat duty would reduce. In case of a fired heater, reboiler heat duty is assumed to be the same.
- In general, the temperature profile in the column increases — this will result in higher pump-around duty (due to higher LMTD).

However, the impact of this is generally ignored for a conservative analysis.

- Logically, simulation is to be done at zero reflux rate, but the column will not converge at zero reflux rate. The reflux rate is reduced to the minimum possible value to achieve a converged solution.
- The contingency will be the vapor flow rate from the top of the column.
- The physical properties will be the properties of the vapor from the top of the column.

Loss of reboiler heat

Loss of reboiler heat means loss of heat input to the column, and in most cases, this will not result in any relief. However, during fractionation, failure of reboiler duty will increase the light ends fed into the bottom product, and if the bottom product is further processed to a second fractionation column, the feed to the second column will be lighter. The impact of lighter feed to the second column needs to be analyzed properly.

Venting of storage tank [13,14]

API Std 2000 [13] is widely used in designing the venting requirement for atmospheric- and low-pressure storage tanks. The total venting capacity is to be at least the sum of the venting requirements for liquid movement and thermal effect. Venting requirements are estimated for three different scenarios:

1. Pumping liquid into the tank or from the tank. During liquid filling, the tank needs to outbreathe the air at a sufficient rate so that the tank pressure does not exceed the design pressure. Similarly, during tank emptying, the air inbreath rate should be adequate to prevent tank failure due to vacuum.
2. Thermal venting is required to inbreathe or outbreathe the air in the event of a sudden change in tank temperature, e.g., due to rain. The general guideline allows for a mean temperature change of 37.8°C/h. This is typically based on conditions in the U.S. and may vary from country to country.
3. Venting due to external fire is also required to be considered. In some cases a weak roof-to-shell attachment is provided, which fails during a fire; if this is not provided, a proper estimation of the venting rate is required. The following measures are also considered for a high-emergency venting rate:
    - Larger or additional open vents.
    - Larger or additional pressure-vacuum valves.
    - A gauge hatch that permits the cover to lift under abnormal pressure.
    - A manhole cover that lifts at a certain pressure.

Table 4.10 Venting Due to Liquid Movements

Flash Point/Boiling Point	Inbreathing (Nm^3/h per m^3/h Liquid Flow)	Outbreathing (Nm^3/h per m^3/h Liquid Flow)
Flash point ≥ 37.8°C	0.94	1.01
Boiling point ≥ 148.9°C	0.94	1.01
Flash point < 37.8°C	0.94	2.02
Boiling point < 149°C	0.94	2.02

Source: From API Std. 2000, Venting Atmospheric and Low-Pressure Storage Tanks, American Petroleum Institute, 5th ed., 1998.

- Weak shell-to-roof connection.
- A rupture disk.

Venting due to liquid movements

This is the venting requirement due to pumping to and from the tank. The venting requirement is tabulated in Table 4.10.

Thermal venting

As explained previously, thermal venting has been established for a temperature variation of 37.8°C/h. The rate of thermal venting is also flash-point and boiling-point dependent (see Table 4.11).

Fire exposure

Venting requirements due to fire depend upon the wetted surface area of the tank. The following methods are used to estimate the wetted surface area of the tank:

- Sphere and spheroids — The wetted area is equal to 55% of the total surface area or the surface area up to a height of 9.14 m above grade, whichever is greater.
- Horizontal tanks — The wetted surface area is equal to 75% of the total surface area or the surface area up to a height of 9.14 m above grade, whichever is greater.
- Vertical tank — The wetted area is equal to the total surface area of the vertical shell to a height of 9.14 m above the grade.

Venting requirements are presented in Table 4.12.

Minimum flow area

The minimum flow area is estimated as follows:

$$F = 1250300 P_1 A \sqrt{\frac{k}{MWTZ(k-1)} \left\{ \left(\frac{P_2}{P_1} \right)^{2/k} - \left(\frac{P_2}{P_1} \right)^{(k+1)/k} \right\}} \tag{4.39}$$

Table 4.11 Thermal Venting

Tank Capacity (m^3)	Inbreathing, Vacuum (Nm^3/h)	Outbreathing (Nm^3/h) Flash Point ≥ 37.8°C or Normal Boiling Point ≥ 148.9°C	Outbreathing (Nm^3/h) Flash Point < 37.8°C or Normal Boiling Point < 148.9°C
10	1.69	1.01	1.69
20	3.37	2.02	3.37
100	16.9	10.1	16.9
200	33.7	20.2	33.7
300	50.6	30.3	50.6
500	84.3	50.6	84.3
700	118	70.8	118
1,000	169	101	169
1,500	253	152	253
2,000	337	202	337
3,000	506	303	506
3,180	536	388	536
4,000	647	472	647
5,000	787	537	787
6,000	896	602	896
7,000	1,003	646	1,003
8,000	1,077	682	1,077
9,000	1,136	726	1,136
10,000	1,210	807	1,210
12,000	1,345	888	1,345
14,000	1,480	969	1,480
16,000	1,615	1,047	1,615
18,000	1,745	1,126	1,745
20,000	1,877	1,370	1,877
25,000	2,179	1,378	2,179
30,000	2,495	1,497	2,495

Source: From API Std. 2000, Venting Atmospheric and Low-Pressure Storage Tanks, American Petroleum Institute, 5th ed., 1998.

Table 4.12 Fire Venting Requirements

Wetted Area (m^2)	Venting (Nm^3/h)	Wetted Area (m^2)	Venting (Nm^3/h)
2	608	35	8,086
3	913	40	8,721
4	1,217	45	9,322
5	1,521	50	9,895
6	1,825	60	10,971
7	2,130	70	11,971
8	2,434	80	12,911
9	2,738	90	13,801
11	3,347	110	15,461
13	3,955	130	15,751
15	4,563	150	16,532
17	5,172	175	17,416
19	5,780	200	18,220
22	6,217	230	19,102
25	6,684	260	19,910
30	7,411	>260[a,b]	—

[a] If the design pressure is ≤ 7 kPaG, venting rate = 19,910 Nm^3/h.

[b] If the design pressure is 7 to 103.4 kPaG, venting rate = $208.2FA^{0.82}$ Nm^3/h (F = environmental factor, A = wetted surface area, m^2).

Example 4.3

A hydrocarbon liquid is stored at 25°C in a 30,000-m^3 storage tank designed according to API standard 620 for the pressure of a 400-mm water column and vacuum of a 200-mm water column. The tank is maintained with a nitrogen atmosphere. The maximum pumped-in rate is 50 m^3/h and the pump-out rate is 40 m^3/h. What will be the size of the vent? Assume the flash point is 20°C, fire venting is not required, and the atmospheric pressure is 100 kPaA.

Solution

Pressure case:

P_1 = 104 kPaA
P_2 = 100 kPaA
Pump-in rate is 50m^3/h, outbreathing rate = 2.02 * 50 = 101 Nm^3/h
Thermal outbreathing (for 30,000-m^3 tank) = 2495 Nm^3/h
Total outbreathing = 2596 Nm^3/h
Ratio of specific heat = 1.4
Molecular weight = 28
Temperature = 298K
Compressibility = 1

Calculated area (Equation 4.39) = 0.0095 m^2
Diameter = 110 mm

Vacuum case:

P_1 = 100 kPaA
P_2 = 98 kPaA
Calculated area (Equation 4.39) = 0.0136 m^2
Diameter = 132 mm
Required size = 150-mm NB

Sizing procedure

Sizing for vapor and liquid relief largely follows the API RP 520 [1] procedures. API RP 520 also outlines the procedures to design a relief valve for two-phase flow. The Design Institute for Emergency Relief, a consortium of 29 companies under AIChE (American Institute of Chemical Engineers), spent 10 years to investigate emergency relief systems and published a large number of papers [15–19]. Some methods described in API RP 520 are from the cited works.

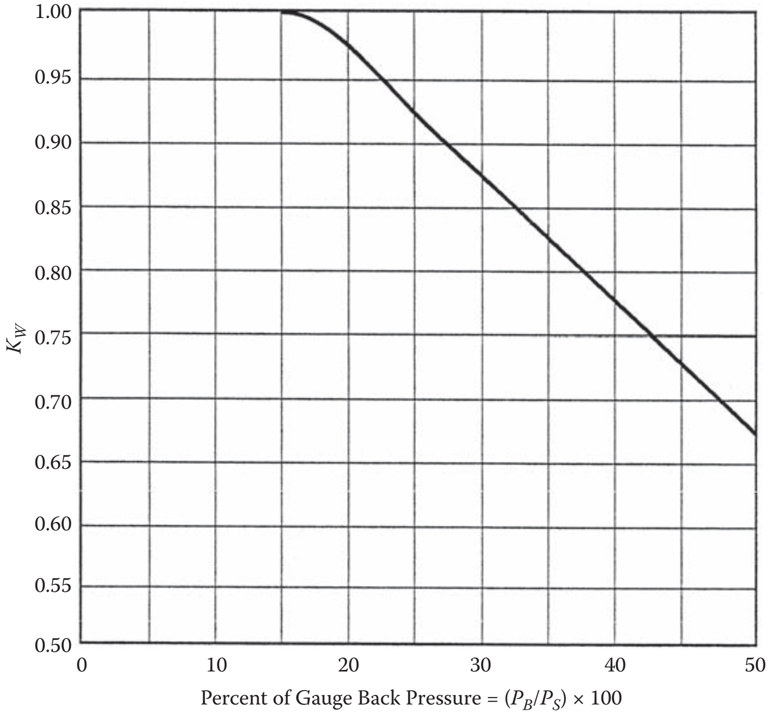
Sizing of liquid relief

Pressure relief valves in liquid service that require ASME certification can be sized using the following equation:

$$A = \frac{11.78Q}{K_d K_w K_c K_v}\sqrt{\frac{G}{P_1 - P_2}} \tag{4.40}$$

where

A = required discharge area, mm^2
Q = flow rate, l/min
G = specific gravity of the liquid at flowing temperature referred to water at standard condition
P_1 = upstream relieving pressure, kPaA
P_2 = back pressure, kPaA
K_d = coefficient of discharge, for preliminary sizing
= 0.65 when PRV is installed with or without rupture disk
= 0.62 for rupture disk design
K_w = for PRVs relieving to atmosphere, K_w = 1.0. For balanced-bellows PRVs, the value of K_w is obtained from Figure 4.13. For PRVs with back pressure less than 15% of the set pressure, K_w = 1.0
K_c = combination factor
= 1 when rupture disk is not installed



K_W = correction factor due to back pressure.
P_B = back pressure, in kPaG.
P_S = set pressure, in kPaG.

Note: The curve above represents values recommended by various manufacturers. This curve may be used when the manufacturer is not known. Otherwise, the manufacturer should be consulted for the applicable correction factor.

Figure 4.13 Capacity correction factor, K_w, due to back pressure on balanced-bellows pressure relief valves in liquid service. (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

= 0.9 when rupture disk is installed
K_v = viscosity correction factor, determined using the following equation

$$K_v = \left\{0.9935 + \frac{2.878}{R^{0.5}} + \frac{342.75}{R^{1.5}}\right\}^{-1.0} \tag{4.41}$$

where R = Reynolds number, calculated as follows:

$$R = \frac{18800QG}{\mu\sqrt{A}} \tag{4.42}$$

where

μ = viscosity at flowing temperature, cP
A = actual area of the orifice, mm^2

The general calculation requires a trial-and-error method. As a first step, the PRV size is calculated by using Equation 4.40, with $K_v = 1$. Once the area is known, the next orifice size is established, and then the value of A in Equation 4.42 is ascertained. From Equation 4.42, the Reynolds number is established, and the viscosity correction factor is calculated using Equation 4.41. Once the value of K_v is known, the area is recalculated using Equation 4.40 and compared with the previous value.

Sizing of vapor relief

The procedure of vapor relief valve sizing depends on the type of vapor flow. Sizing for critical flow is different from that for subcritical flow. It is therefore required to establish the flow behavior first.

$$\frac{P_c}{P_1} = \left\{\frac{2}{k+1}\right\}^{k/(k-1)} \tag{4.43}$$

where

P_c = critical pressure, kPaA
P_1 = relieving pressure, kPaA
k = ratio of specific heat

If the critical pressure is equal to or more than the downstream (back) pressure, the flow will be critical; otherwise, the flow will be subcritical.

Critical flow

The general equation to estimate the orifice area for critical flow is

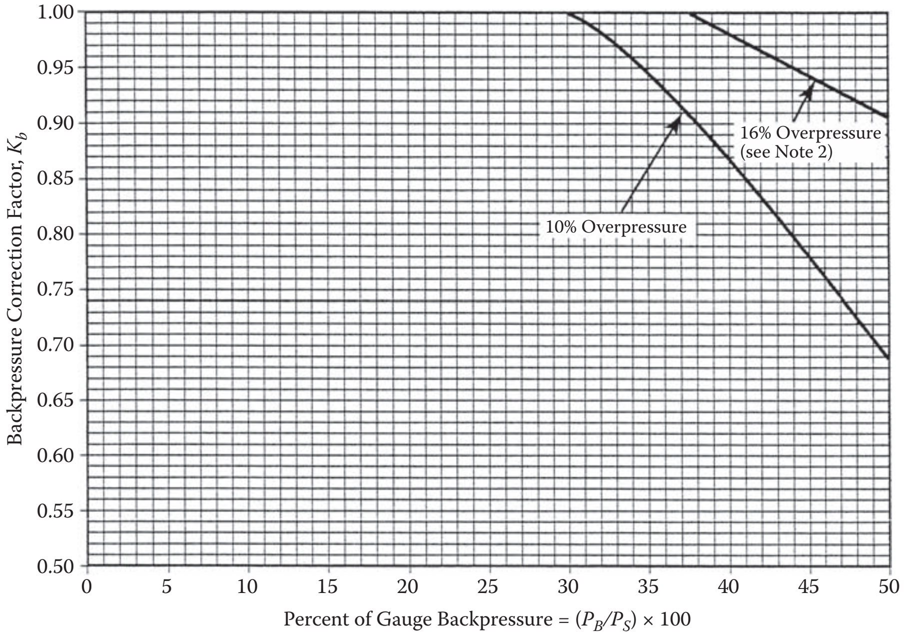
$$A = \frac{13160W}{CK_dP_1K_bK_c}\sqrt{\frac{TZ}{MW}} \tag{4.44}$$

where

A = required discharge area, mm^2
W = design flow, kg/h
T = relieving temperature, K
Z = compressibility factor, dimensionless
MW = vapor molecular weight
C = coefficient of discharge, established as follows:

$$C = 520\sqrt{k\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}} \tag{4.45}$$

K_d = coefficient of discharge, for preliminary analysis
= 0.975 when PRV is installed with or without rupture disk
= 0.62 for rupture disk only
K_b = back-pressure correction factor, obtained from Figure 4.14
= 1 for conventional and pilot-operated PRV
= 1 if back pressure is less than 30% of the set pressure



P_B = back pressure, in kPag.
P_S = set pressure, in kPag.

Notes:

1. The curves above represent a compromise of the values recommended by a number of relief valve manufacturers and may be used when the make of the valve or the critical flow pressure point for the vapor or gas is unknown. When the make of the valve is known, the manufacturer should be consulted for the correction factor. These curves are for set pressures of 50 psig and above. they are limited to back pressure below critical flow pressure for a given set pressure. For set pressures below 50 psig or for subcritical flow, the manufacturer must be dconsulted for values of K_b
2. See paragraph 3.3.3.
3. For 21% overpressure, K_b equals 1.0 up to P_B/P_S = 50%.

Figure 4.14 Back-pressure correction factor, K_b, for balanced-bellows pressure relief valves (for vapors and gases). (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

Subcritical flow

The general equation to estimate the orifice area for subcritical flow depends on the type of PRV.

Conventional and pilot-operated PRV

The general equation of the orifice area is

$$A = \frac{17.9W}{F_2 K_d K_c} \sqrt{\frac{ZT}{MWP_1(P_1 - P_2)}} \tag{4.46}$$

where

A = required discharge area, mm^2
F_2 = coefficient of subcritical flow, estimated as follows:

$$F_2 = \sqrt{\left(\frac{k}{k+1}\right) r^{2/k} \left\{\frac{1 - r^{(k-1)/k}}{1 - r}\right\}} \tag{4.47}$$

where

r = ratio of back pressure to relieving pressure, P_2/P_1

Other terms are as defined earlier.

Balanced-bellows PRV

Balanced-bellows PRVs for subcritical flow are designed using Equation 4.44. The back-pressure correction factor is obtained from the manufacturer.

Sizing for steam relief

The general equation to estimate the orifice area is

$$A = \frac{190.4W}{P_1 K_d K_b K_c K_N K_{SH}} \tag{4.48}$$

where

K_N = correction factor for the Napier equation
= 1 when $P_1 \le 10{,}339$ kPaA

$$= \frac{0.02764 P_1 - 1000}{0.03324 P_1 - 1061} \tag{4.49}$$

when $P_1 > 10{,}339$ kPaA and ≤ 22057 kPaA

Table 4.13 Steam Superheat Correction Factor (K_{SH})

Set Pressure	Temperature (°C)									
(kPaG)	150	200	260	320	370	430	480	540	590	650
103	1.00	0.98	0.93	0.88	0.84	0.80	0.77	0.74	0.72	0.70
140	1.00	0.98	0.93	0.88	0.84	0.80	0.77	0.74	0.72	0.70
275	1.00	0.99	0.93	0.88	0.84	0.81	0.77	0.74	0.72	0.70
415	1.00	0.99	0.93	0.88	0.84	0.81	0.77	0.75	0.72	0.70
550	1.00	0.99	0.93	0.88	0.84	0.81	0.77	0.75	0.72	0.70
690	1.00	0.99	0.94	0.89	0.84	0.81	0.77	0.75	0.72	0.70
825	1.00	0.99	0.94	0.89	0.84	0.81	0.78	0.75	0.72	0.70
965	1.00	0.99	0.94	0.89	0.85	0.81	0.78	0.75	0.72	0.70
1,100	1.00	0.99	0.94	0.89	0.85	0.81	0.78	0.75	0.72	0.70
1,240	1.00	0.99	0.94	0.89	0.85	0.81	0.78	0.75	0.72	0.70
1,380	1.00	0.99	0.95	0.89	0.85	0.81	0.78	0.75	0.72	0.70
1,515	1.00	0.99	0.95	0.89	0.85	0.81	0.78	0.75	0.72	0.70
1,655	—	1.00	0.95	0.90	0.85	0.81	0.78	0.75	0.72	0.70
1,790	—	1.00	0.95	0.90	0.85	0.81	0.78	0.75	0.72	0.70
1,930	—	1.00	0.96	0.90	0.85	0.81	0.78	0.75	0.72	0.70
2,070	—	1.00	0.96	0.90	0.85	0.81	0.78	0.75	0.72	0.70
2,410	—	1.00	0.96	0.90	0.86	0.82	0.78	0.75	0.72	0.70
2,760	—	1.00	0.96	0.91	0.86	0.82	0.78	0.75	0.72	0.70
3,450	—	1.00	0.96	0.92	0.86	0.82	0.78	0.75	0.73	0.70
4,135	—	1.00	0.97	0.92	0.87	0.82	0.79	0.75	0.73	0.70
5,515	—	—	1.00	0.95	0.88	0.83	0.79	0.76	0.73	0.70
6,900	—	—	1.00	0.96	0.89	0.84	0.78	0.76	0.73	0.71
8,600	—	—	1.00	0.97	0.91	0.85	0.80	0.77	0.74	0.71
10,350	—	—	—	1.00	0.93	0.86	0.81	0.77	0.74	0.71
12,050	—	—	—	1.00	0.94	0.86	0.81	0.77	0.73	0.70
13,800	—	—	—	1.00	0.95	0.86	0.80	0.76	0.72	0.69
17,200	—	—	—	1.00	0.95	0.85	0.78	0.73	0.69	0.66
20,700	—	—	—	—	1.00	0.82	0.74	0.69	0.65	0.62

K_{SH} = superheat steam correction factor, obtained from Table 4.13. Table 4.13 has been developed based on Table 9 of API RP 520 in SI unit. Some differences will be observed because of rounding off. The value of K_{SH} for saturated steam is 1.

Other terms are as defined earlier.

Example 4.4

A PRV is to be sized to relieve superheated steam. The PRV set pressure is 1100 kPaG, and the steam temperature is 250°C. The required relieving rate is 20,000 kg/h.

Solution

Relieving pressure, $P_1 = 1100 * 1.1 + 101 = 1311$ kPaA
Correction factor for Napier equation, $K_N = 1$

Superheated steam correction factor, K_{SH} (Table 4.13) = 0.948
Correction factor for rupture disk, $K_c = 1$
Coefficient of discharge, $K_d = 0.975$
Back-pressure correction factor, $K_b = 1$
The required orifice area, A (Equation 4.48) = 3142 mm^2
Required orifice = P (area 4116 mm^2)

Sizing for two-phase fluids [1]

Sizing for two-phase fluids is done based on the procedure outlined in API RP 520. In this design, the physical parameters are to be established carefully, because a little change in the physical parameters may change the result substantially. In this design procedure, the following definitions are used:

Noncondensable gas: This is a gas that is not easily condensed under normal pressure and temperature conditions. Common noncondensable gases are air, nitrogen, oxygen, hydrogen, carbon dioxide, hydrogen sulfide, and carbon monoxide.
Highly subcooled liquid: It is a liquid that does not flash after passing through the PRV.
Nominal boiling range: Nominal boiling range is the difference in the atmospheric boiling points of the lightest and heaviest components in the system.
Low subcooling region: This region is defined when the flashing occurs upstream of the throat.
High subcooling region: This region is defined when the flashing occurs at the throat.

The following four different types are possible for designing a two-phase PRV:

Type 1: Saturated liquid and vapor enter the PRV and flash. No noncondensable gas is present.
Type 2: Highly subcooled liquid and noncondensable gas, condensable vapor, or both enter the PRV and the liquid does not flash.
Type 3: Subcooled liquid enters the PRV and flashes. No condensable vapor or noncondensable gas is present.
Type 4: Noncondensable gas or both condensable vapor and noncondensable gas and either subcooled or saturated liquid enter the PRV and flash. Noncondensable gas is present.

Type 1

The calculation depends on the range of nominal boiling point. If the range is less than 83°C, the omega parameter (ω) is estimated by using the controlling of Equation 4.50, Equation 4.51, or Equation 4.52. If the nominal boiling range is more than 83°C or a single-component system near its

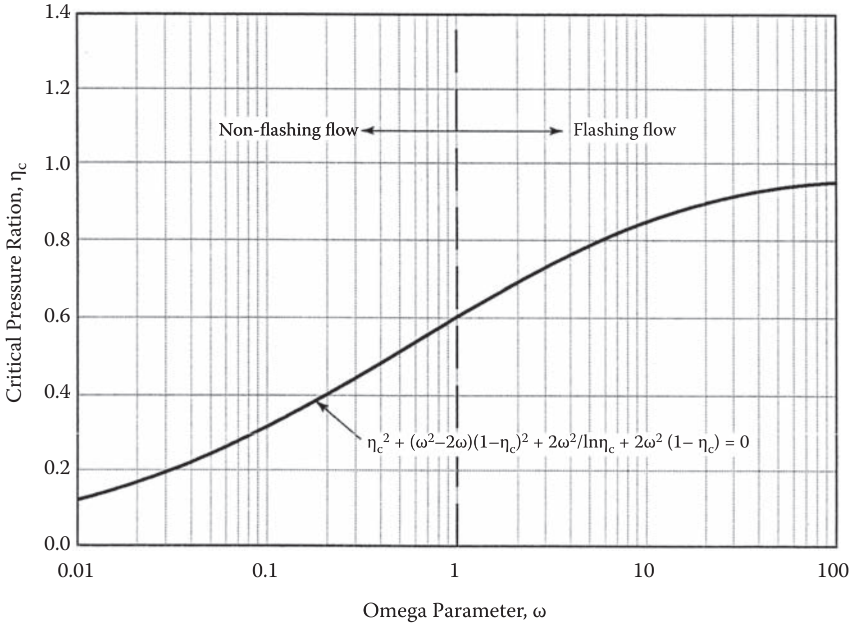


Figure 4.15 Correlation for nozzle critical flow of flashing and nonflashing systems. (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

thermodynamic critical point, Equation 4.52 is used to estimate the ω parameter.

$$\omega = \frac{x_o v_{vo}}{v_o}\left(1 - \frac{2P_o v_{vlo}}{h_{vlo}}\right) + \frac{C_P T_o P_o}{v_o}\left(\frac{v_{vlo}}{h_{vlo}}\right)^2 \tag{4.50}$$

$$\omega = \frac{x_o v_{vo}}{v_o k} + \frac{C_P T_o P_o}{v_o}\left(\frac{v_{vlo}}{h_{vlo}}\right)^2 \tag{4.51}$$

$$\omega = 9\left(\frac{v_9}{v_o} - 1\right) \tag{4.52}$$

The critical pressure ratio and critical pressure are estimated using the following equation or obtained from Figure 4.15:

$$\eta_c^2 + (\omega^2 - 2\omega)(1-\eta_c)^2 + 2\omega^2 \ln\eta_c + 2\omega^2(1-\eta_c) = 0 \tag{4.53}$$

$$P_c = \eta_c P_o \tag{4.54}$$

If $P_c \geq P_a$, the flow is critical; if $P_c < P_a$ the flow is subcritical.

Equation 4.55 and Equation 4.56 are used to estimate the mass flux for critical and subcritical flows, respectively:

Critical flow

$$G = 31.637\eta_c\sqrt{\frac{P_o}{v_o\omega}} \tag{4.55}$$

Subcritical flow

$$G = \frac{31.637\left\{-2\left[\omega \ln \eta_a + (\omega - 1)(1 - \eta_a)\right]\right\}^{0.5}}{\omega\left(\frac{1}{\eta_a} - 1\right) + 1}\sqrt{\frac{P_o}{v_o}} \tag{4.56}$$

The required area of the PRV is estimated as follows:

$$A = \frac{277.777W}{K_d K_b K_c G} \tag{4.57}$$

where

x_o = vapor mass fraction at the PRV inlet. This is estimated at the PRV relieving conditions.

v_{vo} = specific volume of the vapor at the PRV inlet, estimated at PRV relieving conditions, m^3/kg.

v_o = specific volume of the two-phase system at the PRV inlet, estimated at PRV relieving conditions, m^3/kg.

P_o = PRV relieving pressure, kPaA.

v_{vlo} = difference between the vapor and liquid specific volumes at PRV relieving conditions, m^3/kg.

h_{vlo} = latent heat of vaporization estimated at the PRV relieving conditions (kJ/kg). For multicomponent systems, the difference between the vapor and liquid specific enthalpies can be used as latent heat. For narrow-boiling systems, Figure 4.12 can be used to calculate the latent heat.

C_P = liquid specific heat at constant pressure at PRV relieving conditions, kJ/(kg·K).

T_o = PRV relieving temperature, K.

k = ratio of specific heat of the vapor.

v_9 = specific volume of the two-phase system, estimated at 90% of the PRV relieving pressure, m^3/kg.

P_c = critical pressure, kPaA.
η_c = critical pressure ratio.
P_a = downstream back pressure, kPaA.
G = mass flux, kg/sec·m^2.
η_a = back-pressure ratio, estimated at relieving pressure, P_a/P_o.
A = required effective discharge area, mm^2.
W = mass flow rate, kg/h.
K_d = discharge coefficient to be obtained from valve manufacturer. For a preliminary estimation $K_d = 0.85$ can be used.
K_b = back-pressure correction factor applied for balanced-bellows PRV. For preliminary sizing, the K_b value can be obtained from Figure 4.14.
K_c = combination factor.
= 1.0 when rupture disk is not installed.
= 0.9 when rupture disk is installed in combination with PRV.

Estimation of v_o, v_9, and v_{vlo}. The specific volume of a two-phase system at relieving pressure and 90% of relieving pressure can be estimated using the following procedure:

- Flash the inlet stream at relieving pressure (P_o) and relieving temperature (T_o), and estimate the liquid and vapor specific volumes and mass fraction.
- Flash the inlet stream isentropically from P_o,T_o to 90% of P_o and estimate the liquid and vapor specific volumes and mass fraction.
- The value of v_o will be

$$v_o = (1 - x_o)\, v_{lo} + x_o\, v_{vo} \tag{4.58}$$

- The value of v_9 will be

$$v_9 = (1 - x_9) v_{l9} + x_9\, v_{v9} \tag{4.59}$$

and

$$v_{vlo} = v_{vo} - v_{lo} \tag{4.60}$$

where

v_{lo} = liquid specific volume at relieving pressure, m^3/kg
x_9 = vapor mass fraction at 90% of relieving pressure
v_{l9} = liquid specific volume at 90% of relieving pressure, m^3/kg
v_{v9} = vapor specific volume at 90% of relieving pressure, m^3/kg

Other terms are as defined before.

Notes:

1. Equation 4.50 to Equation 4.52 are used when the fluid is far from its thermodynamic critical point, reduced temperature ≤ 0.9, and reduced pressure ≤ 0.5.
2. For multicomponent systems with nominal boiling range less than 83°C, the design ω parameter should be the controlling ω, calculated using Equation 4.50 to Equation 4.52.
3. For a multicomponent system with nominal boiling range more than 83°C, the ω parameter is to be estimated using Equation 4.52.
4. The estimated value of v_9 should always be more than v_o.

Type 2

For this nonflashing system, the method described in type 1 is used with the exception of the estimation of ω parameter. The ω parameter is to be estimated using the following equation:

$$\omega = \frac{x_o v_{vgo}}{v_o k} \tag{4.61}$$

where

x_o = combined vapor and gas mass fraction at PRV relieving conditions
v_{vgo} = specific volume of combined vapor and gas at PRV relieving conditions, m^3/kg
v_o = specific volume of two-phase system at PRV relieving conditions, m^3/kg
k = ratio of specific heat

Example 4.5

Design a pressure relieving valve for a two-phase fluid with the following properties:

Flow rate = 20000 kg/h
PRV set pressure = 300 kPaG
PRV relieving pressure = 330 kPaG (10% overpressure)
PRV back pressure = 20 kPaG
PRV relieving temperature = 60°C
Fluid composition: nC4 = 25 mol%
    iC4 = 25 mol%
    nC5 = 25 mol%
    iC5 = 25 mol%

Solution

The flash calculation is performed using the HYSYS® simulation package and the Peng–Robinson equation of state. The condition corresponds to type 1.

Nominal boiling point of the lightest component, iC4, atm pressure = –11.7°C
Nominal boiling point of the heaviest component, nC5, atm pressure = 27.8°C
Difference in boiling point = 39.5°C

$$\text{Pseudocritical pressure} = \sum_{i=1}^{N} n_i P_{ci} = 3437 \text{ kPaG}$$

$$\text{Pseudocritical temperature} = \sum_{i=1}^{N} n_i T_{ci} = 170.7°\text{C}$$

where

n_i = mole fraction of component i
P_{ci} = critical pressure of component i
T_{ci} = critical temperature of component i

The fluid is far from its thermodynamic critical point.
At relieving pressure and temperature:

Liquid specific volume, $v_{lo} = 0.00179 \text{ m}^3/\text{kg}$
Vapor specific volume, $v_{vo} = 0.0908 \text{ m}^3/\text{kg}$
Vapor mass fraction, $x_o = 0.409$

The following are estimated by flash calculations performed from P_o, T_o to 90% of P_o (297 kPaG):

Liquid specific volume, $v_{l9} = 0.00178 \text{ m}^3/\text{kg}$
Vapor specific volume, $v_{v9} = 0.098 \text{ m}^3/\text{kg}$
Vapor mass fraction, $x_9 = 0.428$
$v_o = (1 - 0.409) * 0.00179 + 0.409 * 0.0908 = 0.0382 \text{ m}^3/\text{kg}$
$v_9 = (1 - 0.428) * 0.00178 + 0.428 * 0.098 = 0.043 \text{ m}^3/\text{kg}$
$v_{vlo} = v_{vo} - v_{lo} = 0.089 \text{ m}^3/\text{kg}$
$h_{vlo} = 2412 - 2135 = 277 \text{ kJ/kg}$
$C_P = 2.555 \text{ kJ/kg·K}$
$k = 1.113$
$K_d = 0.85$
$K_c = 1$
$K_b = 1$

The following are now calculated:

ω (from Equation 4.50) = 1.695
ω (from Equation 4.51) = 1.865
ω (from Equation 4.52) = 1.131

The design ω is the maximum of the preceding values.

Critical pressure ratio, calculated from Equation 4.53, = 0.6843
Critical pressure (Equation 4.54) = 295.1 kPaA

The flow is critical (critical pressure is more than the back pressure). Mass flux (from Equation 4.55):

$$G = 1684.4\ \text{kg/sec·m}^2$$

Required area (from Equation 4.57):

$$A = 3880.3\ \text{mm}^2$$

The required orifice is P.

Type 3

Similar to type 1, the calculation for subcooled liquid that flashes at the PRV also depends on the nominal boiling range. If the nominal boiling range is less than 83°C, the saturation point ω parameter is estimated by using the controlling of Equation 4.62 and Equation 4.63. In case Equation 4.62 is used, the fluid must be far from its thermodynamic critical point (reduced temperature ≤ 0.9 and reduced pressure ≤ 0.5). If the nominal boiling range is more than 83°C or the fluid is near its thermodynamic critical point, Equation 4.63 is used to estimate the ω parameter at saturation point.

$$\omega_s = \rho_{lo} C_P T_o P_s \left(\frac{v_{vls}}{h_{vls}} \right)^2 \tag{4.62}$$

$$\omega_s = 9\left(\frac{\rho_{lo}}{\rho_9} - 1 \right) \tag{4.63}$$

where

ρ_{lo} = liquid density at PRV relieving condition, kg/m^3
C_P = liquid specific heat at PRV relieving condition, kJ/(kg.°C)
T_o = temperature at PRV relieving condition, K
P_s = saturation pressure corresponding to temperature T_o, kPaA
v_{vls} = difference between vapor and liquid specific volumes at P_s, m^3/kg
h_{vls} = latent heat of vaporization, kJ/kg
ρ_9 = density evaluated at 90% of the saturation vapor pressure P_s corresponding to PRV inlet temperature T_o, kg/m^3
P_o = relieving pressure of the PRV, kPaA

Estimation of P_s, v_{vls}, h_{vls}, and ρ_9. These values are normally estimated using a simulation package. The following procedure is used:

- Consider the inlet liquid at relieving temperature, T_o.
- Adjust the pressure till the bubble point is reached. In simulation packages, the pressure is estimated using vapor fraction as 0.0. The estimated pressure is saturation pressure P_s. Because this calculation is based on the subcooled liquid at the PRV inlet, the saturation pressure (P_s) will always be less than the relieving pressure (P_o).
- Perform a flash calculation using a small vapor fraction, say, 0.01, keeping pressure as P_s; the temperature will change slightly.
- From above flash (vapor = 0.01 mole fraction), v_{vls} is estimated as the difference between vapor and liquid specific volumes. Similarly, the latent heat of vaporization is established as the difference between the vapor and liquid specific enthalpies.
- Perform an isentropic flash calculation from P_s, T_o to 90% of P_s.
- The density is estimated from the above flash calculation as follows:

$$\rho_9 = (1 - x_9) * \rho_{l9} + x_9 * \rho_{v9} \tag{4.64}$$

where

x_9 = vapor mass fraction at 90% of bubble point pressure (P_s)
ρ_{l9} = liquid density at 90% of bubble point pressure, kg/m^3
ρ_{v9} = vapor density at 90% of bubble point pressure, kg/m^3

Subcooling region. The subcooling region is defined based on the point of liquid flash. In case of a low subcooling region, the flashing occurs upstream of the throat, and in case of a high subcooling region, the flashing occurs at the throat.

Low subcooling region:

$$P_s > \eta_{st} * P_o \tag{4.65a}$$

High subcooling region:

$$P_s < \eta_{st} * P_o \tag{4.65b}$$

where η_{st} is the transition saturation pressure ratio, defined as:

$$\eta_{st} = \frac{2\omega_s}{1+2\omega_s} \tag{4.66}$$

Critical flow. The critical flow condition depends on the subcooling region.

Low subcooling region.

Critical flow if $P_c > P_a$
Subcritical flow if $P_c < P_a$

High subcooling region.

Critical flow if $P_s > P_a$
Subcritical flow if $P_s < P_a$

where

P_c = critical pressure, kPaA
P_a = downstream back pressure, kPaA

Critical pressure estimation. The following procedure is used to estimate the critical pressure:

- Estimate the saturation pressure ratio η_s as follows:

$$\eta_s = P_s/P_o \tag{4.67}$$

- From Figure 4.16, establish the value of η_c.

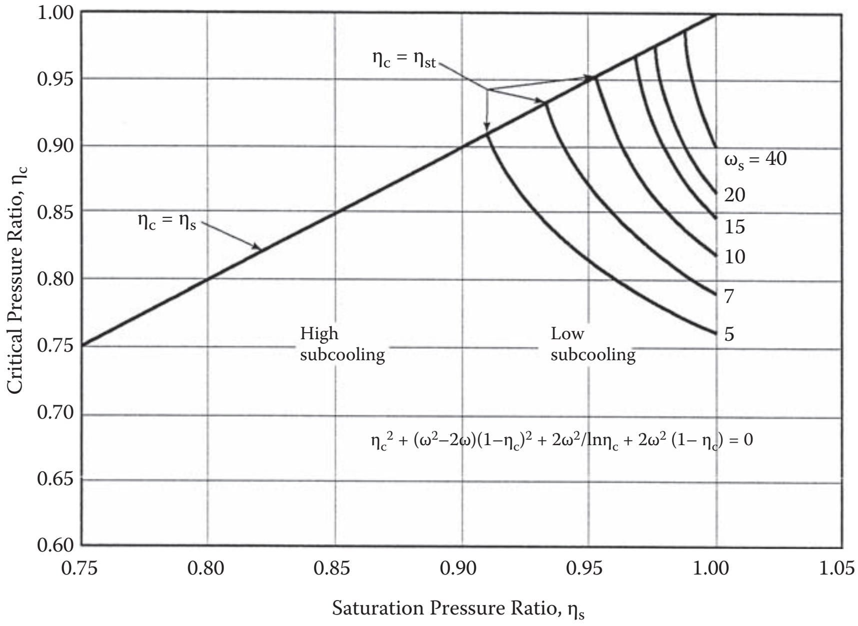


Figure 4.16 Correlation for nozzle critical flow of inlet subcooled liquids. (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

- The critical pressure is given by the following equation:

$$P_c = \eta_c P_o \tag{4.68}$$

Mass flux for low subcooling region. The mass flux depends on the type of flow.

Critical flow.

$$G = \frac{31.637\left\{2(1-\eta_s)+2\left[\omega_s\eta_s\ln\left(\dfrac{\eta_s}{\eta_c}\right)-(\omega_s-1)(\eta_s-\eta_c)\right]\right\}^{0.5}}{\omega_s\left(\dfrac{\eta_s}{\eta_c}-1\right)+1}\sqrt{P_o\rho_{lo}} \tag{4.69}$$

Subcritical flow.

$$G = \frac{31.637\left\{2(1-\eta_s)+2\left[\omega_s\eta_s\ln\left(\dfrac{\eta_s}{\eta_a}\right)-(\omega_s-1)(\eta_s-\eta_a)\right]\right\}^{0.5}}{\omega_s\left(\dfrac{\eta_s}{\eta_a}-1\right)+1}\sqrt{P_o\rho_{lo}} \tag{4.70}$$

Mass flux for high subcooling region. For this region also, the mass flux depends on the type of flow.

Critical flow.

$$G = 44.745\left[\rho_{lo}\left(P_o - P_s\right)\right]^{0.5} \tag{4.71}$$

Subcritical flow.

$$G = 44.745\left[\rho_{lo}\left(P_o - P_a\right)\right]^{0.5} \tag{4.72}$$

where

η_s = saturation pressure ratio, P_s/P_o
η_a = back-pressure ratio, P_a/P_o
G = mass flux, kg/sec·m^2

The required area of the PRV is estimated as follows:

$$A = 277.777 \frac{Q\rho_{lo}}{K_d K_b K_c G} \tag{4.73}$$

where

A = required effective discharge area, mm^2.
Q = volumetric flow rate, m^3/h.
K_d = discharge coefficient, to be obtained from valve manufacturer. For a preliminary estimation, $K_d = 0.85$ can be used for low subcooling regions and $K_d = 0.65$ can be used for high subcooling regions.
K_b = back-pressure correction factor. For preliminary sizing, K_b can be obtained from Figure 4.14.
K_c = combination factor
= 1.0 when rupture disk is not installed.
= 0.9 when rupture disk is installed in combination with PRV.

Example 4.6

This example is the same as Example 4.5 but with a higher PRV set pressure of 500 kPaG, so that the fluid at relieving condition is all liquid.

Solution

The flash calculation is performed using the HYSYS® simulation package and Peng–Robinson equation of state.
Properties at relieving condition:

ρ_{lo} = density of liquid at relieving condition = 552.3 kg/m^3
C_P = specific heat of the liquid at relieving condition = 2.576 kJ/kg·K
T_o = 333 K
P_o = 651 kPaA

Bubble point pressure at 60°C inlet temperature, P_s = 485.8 kPaA. Flash the liquid from 60°C and 485.8 kPaA to 0.01 mole fraction of vapor.

v_{vls} = difference between vapor and liquid specific volumes = 0.0796 m^3/kg
h_{vls} = latent heat of vaporization = 274 kJ/kg

Flash the liquid from P_s, T_o to 90% of P_s.

x_9 = vapor mass fraction = 0.0246
ρ_{l9} = liquid density = 556 kg/m^3
ρ_{v9} = vapor density = 11.33 kg/m^3
ρ_9 = 542.6 kg/m^3

The required values can now be calculated:

ω_s (from Equation 4.62) = 19.42
ω_s (from Equation 4.63) = 0.16
Design ω_s = 19.42
Transition saturation pressure ratio, η_{st} = 0.975
(This is a high subcooling region and the flow is critical.)

Mass flux (from Equation 4.71) = 13528 kg/(m^2·sec)
K_d (high subcooling region) = 0.65
Required discharge area (Equation 4.73) = 631.8 mm^2
Required orifice = J

Type 4

The inlet void fraction, α_o, is estimated as follows:

$$\alpha_o = \frac{x_o v_{vgo}}{v_o} \tag{4.74}$$

where

α_o = inlet void fraction
x_o = combined gas and vapor mass fraction at PRV relieving conditions
v_{vgo} = specific volume of combined gas and vapor at PRV relieving conditions, m^3/kg
v_o = specific volume of two-phase system at the PRV relieving conditions, m^3/kg

Calculation of the ω parameter and mass flux depend on two different sets of conditions as explained in the following subsections.

Scenario 1. This scenario is applicable if the system satisfies all of the following conditions:

1. Contains less than 0.1% (w/w) hydrogen
2. Nominal boiling range less than 83°C
3. Either P_{vo}/P_o is less than 0.9 or P_{go}/P_o is greater than 0.1
4. Far from its thermodynamic critical point ($T_r \leq 0.9$ or $P_r \leq 0.5$)

The ω parameter is estimated as follows:

$$\omega = \frac{\alpha_o}{k} + (1 - \alpha_o)\rho_{lo} C_P T_o P_{vo} \left(\frac{v_{vlo}}{h_{vlo}} \right)^2 \tag{4.75}$$

where

P_{vo} = saturation or bubble point pressure corresponding to T_o, kPaA. This is estimated without considering any noncondensable gas in the system.
P_o = relieving pressure of the PRV, kPaA.
P_{go} = noncondensable gas partial pressure at PRV relieving conditions, kPaA.
k = ratio of specific heats of the combined gas and vapor.
ρ_{lo} = liquid density at PRV relieving conditions, kg/m^3.
C_P = liquid specific heat at PRV relieving conditions, kJ/kg·K.
T_o = PRV relieving temperature, K.
v_{vlo} = difference between the vapor (excluding any noncondensable gas present) and liquid specific volumes at PRV relieving conditions, m^3/kg.
h_{vlo} = latent heat of vaporization at PRV relieving conditions, kJ/kg. For multicomponent systems, h_{vlo} is the difference between vapor (excluding noncondensables) and liquid specific enthalpies.

The nonflashing critical pressure ratio, η_{gc}, is calculated as follows:

$$\eta_{gc}{}^2 + \left\{\left(\frac{\alpha_o}{k}\right)^2 - \frac{2\alpha_o}{k}\right\}\left(1-\eta_{gc}\right)^2 + 2\left(\frac{\alpha_o}{k}\right)^2 \ln \eta_{gc} + 2\left(\frac{\alpha_o}{k}\right)^2 \left(1-\eta_{gc}\right) = 0 \quad (4.76)$$

Similarly, the flashing critical pressure ratio, η_{vc}, is calculated as follows:

$$\eta_{vc}{}^2 + \left(\omega^2 - 2\omega\right)(1-\eta_{vc})^2 + 2\omega^2 \ln \eta_{vc} + 2\omega^2(1-\eta_{vc}) = 0 \quad (4.77)$$

The mole fraction of noncondensables at the PRV relieving conditions, y_{go}, is calculated as follows:

$$y_{go} = \frac{P_{go}}{P_o} \quad (4.78)$$

The critical pressure, P_c , will be

$$P_c = \left\{y_{go}\eta_{gc} + (1-y_{go})\eta_{vc}\right\}P_o \quad (4.79)$$

Critical flow if $P_c \geq P_a$
Subcritical flow if $P_c < P_a$

where

P_a = downstream back pressure, kPaA
P_c = critical pressure, kPaA

Critical flow mass flux. Critical flow mass flux is estimated using Equation 4.80.

$$G = 31.637\left[\frac{P_o}{v_o}\left\{\frac{y_{go}\eta_{gc}{}^2k}{\alpha_o} + \frac{(1-y_{go})\eta_{vc}{}^2}{\omega}\right\}\right]^{0.5} \tag{4.80}$$

where

G = mass flux, kg/sec·m^2

Subcritical mass flux. The following procedure is used to estimate the subcritical mass flux.

The nonflashing partial pressure ratio and flashing partial pressure ratio are estimated by solving Equation 4.81 and Equation 4.82.

$$\eta_a = y_{go}\eta_g + (1-y_{go})\eta_v \tag{4.81}$$

$$\frac{\alpha_o}{k}\left(\frac{1}{\eta_g}-1\right) = \omega\left(\frac{1}{\eta_v}-1\right) \tag{4.82}$$

where

η_g = nonflashing partial pressure ratio
η_v = flashing partial pressure ratio
η_a = back-pressure ratio = P_a/P_o
P_a = downstream back pressure, kPaA

The nonflashing mass flux is estimated as follows:

$$G_g = \frac{31.637\left\{-2\left[\frac{\alpha_o}{k}\ln\eta_g + \left(\frac{\alpha_o}{k}-1\right)(1-\eta_g)\right]\right\}^{0.5}}{\frac{\alpha_o}{k}\left(\frac{1}{\eta_g}-1\right)+1}\sqrt{\frac{P_o}{v_o}} \tag{4.83}$$

The flashing mass flux is estimated as follows:

$$G_v = \frac{31.637\left\{-2\left[\omega \ln \eta_v + (\omega - 1)(1 - \eta_v)\right]\right\}^{0.5}}{\omega\left(\dfrac{1}{\eta_v} - 1\right) + 1}\sqrt{\frac{P_0}{v_o}} \tag{4.84}$$

The design mass flux is estimated as follows:

$$G = \left\{y_{go}G_g^{\,2} + (1 - y_{go})G_v^{\,2}\right\}^{0.5} \tag{4.85}$$

where

G_g = nonflashing mass flux, kg/sec·m^2
G_v = flashing mass flux, kg/sec·m^2
G = design mass flux, kg/sec·m^2

Scenario 2. This scenario is applicable if the system satisfies one of the following conditions:

1. Contains more than 0.1% (w/w) hydrogen
2. Normal boiling range is more than 83°C
3. Either P_{vo}/P_o is greater than 0.9 or P_{go}/P_o is less than 0.1
4. Near its thermodynamic critical point ($T_r \geq 0.9$ or $P_r \geq 0.5$)

The ω parameter is estimated as follows:

$$\omega = 9\left(\frac{v_9}{v_o} - 1\right) \tag{4.86}$$

where

v_9 = specific volume evaluated at 90% of the PRV relieving pressure (m^3/kg), evaluated through an isentropic flash calculation

The critical pressure ratio, η_c, is calculated by solving the following equation:

$$\eta_c^{\,2} + (\omega^2 - 2\omega)(1 - \eta_c)^2 + 2\omega^2 \ln \eta_c + 2\omega^2(1 - \eta_c) = 0 \tag{4.87}$$

The critical pressure P_c is defined as

$$P_c = \eta_c P_o \tag{4.88}$$

Critical flow if $P_c \geq P_a$
Subcritical flow if $P_c < P_a$

where

P_a = downstream back pressure, kPaA
P_c = critical pressure, kPaA

Critical flow mass flux. This is estimated using Equation 4.89.

$$G = 31.637\eta_c \sqrt{\frac{P_o}{v_o \omega}} \tag{4.89}$$

Subcritical mass flux. This is estimated using Equation 4.90.

$$G = \frac{31.637\left\{-2\left[\omega \ln \eta_a + (\omega - 1)(1 - \eta_a)\right]\right\}^{0.5}}{\omega\left(\frac{1}{\eta_a} - 1\right) + 1} \sqrt{\frac{P_o}{v_o}} \tag{4.90}$$

where

G = mass flux, kg/sec·m^2
η_a = back-pressure ratio, P_a/P_o

Effective discharge area. For both scenario 1 and scenario 2, the effective discharge area is estimated by using Equation 4.91.

$$A = \frac{277.777W}{K_d K_b K_c G} \tag{4.91}$$

where

A = required effective discharge area, mm^2
W = mass flow rate, kg/h
K_d, K_b, K_c = as defined before

Example 4.7

Design a pressure relieving valve with a two-phase fluid and a noncondensable gas according to the following requirements:

Flow rate = 20,000 kg/h
PRV set pressure = 500 kPaG
PRV relieving pressure = 651.3 kPaA
PRV back pressure = 310 kPaG
PRV relieving temperature = 60°C

Fluid compositions: nC4 = 20 mol%
iC4 = 20 mol%
nC5 = 20 mol%
iC5 = 20 mol%
Nitrogen = 20 mol%

Solution

This is a type 4 system and will be designed accordingly.

The flash calculation is performed using the HYSYS® simulation package and the Peng–Robinson equation of state.

Check whether the design corresponds to scenario 1 or scenario 2.

P_{vo} = saturation bubble point pressure without nitrogen at 60°C = 485.8 kPaA
$P_{vo}/P_o = 0.75$

The design meets all four conditions of scenario 1; therefore, the procedure for that situation will be used.

Combined gas and vapor mass fraction, $x_o = 0.551$
Specific volume of combined gas and vapor, $v_{vgo} = 0.0752$ m^3/kg
Specific volume of two-phase system, $v_o = 0.0422$ m^3/kg
Inlet void fraction, α_o (Equation 4.74) = 0.982

Estimation of latent heat: Because the fluid at the PRV inlet, without nitrogen, is all liquid it is difficult to estimate the specific enthalpy of vapor at PRV relieving conditions. The latent heat is estimated from Figure 4.12 for a liquid molecular weight of 65.1 and a temperature of 60°C.

Latent heat, h_{vlo} = 332 kJ/kg

Estimation of vapor specific volume:
Vapor composition:
Condensable = 0.6758 mole fraction
Noncondensable (gas) = 0.3242 mole fraction
Molecular weight of vapor and gas = 52 (from simulation)
Molecular weight on noncondensable gas (nitrogen) = 28
Calculated molecular weight of condensable vapor = 63.5
Calculated partial pressure of condensable vapor = 440.1 kPaA
Specific volume of condensable vapor ($R * T/MW/P$) = 0.099 m^3/kg
P_{vo} = saturation bubble point pressure = 485.8 kPaA
P_o = relieving pressure = 651.3 kPaA
P_{go} = noncondensable gas partial pressure
= mole fraction of nitrogen in vapor phase * P_o = 211.2 kPaA
k = ratio of specific heat of combined gas and vapor = 1.141

ρ_{lo} = liquid density at PRV relieving conditions = 559.9 kg/m^3
C_P = liquid specific heat = 2.548 kJ/kg·K
T_o = 333 K
v_{vlo} = 0.099 – 0.0018 = 0.0972 m^3/kg
h_{vlo} = latent heat = 332 kJ/kg
ω parameter (Equation 4.75) = 1.217
Nonflashing critical pressure ratio, η_{gc} (Equation 4.76) = 0.587
Flashing critical pressure ratio, η_{vc} (Equation 4.77) = 0.632
Mole fraction of noncondensable gas, y_{go} (Equation 4.78) = 0.324
Critical pressure, P_c (Equation 4.79) = 402.1 kPaA

Because back pressure is more than the critical pressure, the flow will be subcritical. Nonflashing and flashing partial pressure ratios are estimated using Equation 4.81 and Equation 4.82.

Nonflashing partial pressure ratio, η_g = 0.576
Flashing partial pressure ratio, η_v = 0.658
Nonflashing mass flux, G_g (Equation 4.83) = 2486.1 kg/sec·m^2
Flashing mass flux, G_v (Equation 4.84) = 2246.2 kg/sec·m^2
Design mass flux, G (Equation 4.85) = 2326.6 kg/sec·m^2

Required PRV area:

W = mass flow rate = 20,000 kg/h
K_d = discharge coefficient = 0.85 (assumed)
K_c = combination factor = 1.0
K_b = back-pressure correction factor = 0.69 (assumed; cannot be calculated using Figure 4.14)
A = required effective discharge area (Equation 4.91) = 4,071.3 mm^2
Required orifice = P

Design of flare stack [3]

The flare stack is designed based on allowable radiation intensity. For the purpose of design, the recommended total radiation is the sum of flare radiation and solar radiation. Solar radiation depends on the geographical location and varies from 0.79 to 1.04 kW/m^2. The recommended total radiation is presented in Table 4.14.

Minimum distance

The minimum distance from the center of the flare to the point of exposure is estimated as follows:

$$D = \sqrt{\frac{\tau F Q}{4\pi K}} \tag{4.92}$$

Table 4.14 Recommended Total Radiations

Radiation[a] (kW/m^2)	Condition
15.77	Heat intensity on structures and in areas where operators are not likely to be performing duties and where shelter from radiant heat is available (e.g., behind equipment)
9.46	Value of *K* at design flare release at any location to which people have access (e.g., at grade below the flare or a service platform of a nearby tower); exposure should be limited to a few seconds, sufficient for escape only
6.31	Heat intensity in areas where emergency actions lasting up to 1 min may be required by personnel without shielding but with appropriate clothing
4.73	Heat intensity in areas where emergency actions lasting several minutes may be required by personnel without shielding but with appropriate clothing
1.58	Value of *K* at any location where personnel with appropriate clothing may be continuously exposed

[a] Includes solar radiation from 0.79 to 1.04 kW/m^2.

where

D = minimum distance from flame center, m
τ = fraction of heat intensity transmitted
F = fraction of heat radiated
Q = heat release, kW
K = allowable radiation, kW/m^2

Fraction of heat intensity transmitted, τ

This can be estimated by the procedure outlined by Brzustowski and Sommer [20]. However, for a conservative analysis, the value of τ is assumed as 1.

Fraction of heat radiated, F

This depends on the composition of gas and the burner diameter. An approximate value of *F* can be applied based on Table 4.15. The values presented in Table 4.15 are applicable to radiation from a gas. If liquid droplets of the

Table 4.15 Radiation from Gaseous Diffusion Flames

Gas Type	Value of *F*
Hydrogen	0.15
Butane	0.30
Methane	0.15
Natural gas	0.25

Table 4.16 Heating Value of Commonly Used Gases

Gases	Heating Value (MJ/kg)
Methane	55.53
Ethane	51.91
Propane	50.38
i-Butane	49.44
n-Butane	49.55
i-Pentane	48.96
n-Pentane	48.77
n-Hexane	48.70
n-Heptane	48.07
n-Octane	47.88
Hydrogen	142.10
Carbon monoxide	10.11
Carbon dioxide	0
Nitrogen	0

hydrocarbon larger than 150 μm in size are present in the flame, the values should be increased.

Heat release, Q

For gases with known compositions, the heat release is estimated as follows:

$$Q = \frac{W}{3.6}\sum_i w_i q_i \tag{4.93}$$

where

Q = heat release, kW
W = gas flow rate, kg/h
w_i = mass fraction of component i
q_i = heating value of component i, MJ/kg

If the gas composition is not known, the heating value of the gas can be assumed as 50 MJ/kg.

Heating values of commonly used gases are presented in Table 4.16.

Sizing of a flare stack: simple approach

Calculation of stack diameter

Flare stack diameter depends on the *Mach* number and is estimated by using the following equation:

$$Mach = (3.23 * 10^{-5})\left(\frac{W}{Pd^2}\right)\left(\frac{zT}{kMW}\right)^{0.5} \tag{4.94}$$

where

$Mach$ = design Mach number
W = flow rate, kg/h
P = pressure at flare tip, kPaA
d = flare stack diameter, m
z = compressibility of the flowing gas
T = temperature of the flowing gas, K
k = ratio of specific heat
MW = molecular weight of the flowing gas

Calculation of flame length

The flame length is calculated by using the following equation:

$$L = \exp\left\{0.4562\ln(Q) - 5.3603\right\} \tag{4.95}$$

where

L = flame length, m
Q = heat release, watt

(Note: Heat release calculated by Equation 4.93 is expressed in kW.)

Flame distortion caused by wind velocity

This depends on the actual flow rate of the gas and the wind velocity. The actual flow rate is calculated using Equation 4.96.

$$F = \frac{22.4 * W * T}{3600 * 273 * MW} \tag{4.96}$$

where

F = actual volumetric flow, m^3/sec
W = mass flow rate, kg/h
MW = molecular weight of the flowing gas
T = temperature of the flowing gas, K

The flare tip exit velocity is calculated as follows:

$$U_j = \frac{4F}{\pi d^2} \tag{4.97}$$

where

U_j = flare tip exit velocity, m/sec

Flame distortion caused by wind velocity is calculated as follows:

$$U = \frac{U_\infty}{U_j} \tag{4.98}$$

where

U = velocity factor

U_∞ = wind velocity, m/sec

Flame vertical length, Δy, is estimated by using the following equation:

$$\Delta y = L\left\{-0.0392 + \frac{0.1267}{U^{0.5}} + \frac{0.0178}{U} - \frac{0.003}{U^{1.5}}\right\} \tag{4.99}$$

Equation 4.99 is valid for $1.2 \geq U > 0.022$. Similarly, flame horizontal length, Δx, is estimated by using the following equation:

$$\Delta x = L\left\{0.9402 + \frac{0.1067}{U^{0.5}} - \frac{0.0165}{U} + \frac{0.0038}{U^{1.5}}\right\}^{-1.0} \tag{4.100}$$

Equation 4.100 is valid for $1.2 \geq U > 0.005$. Both Δy and Δx can also be calculated using Figure 4.17.

The center of the flame from the top of the flare stack can be calculated as follows:

$$y_c = \frac{1}{2}\Delta y \tag{4.101a}$$

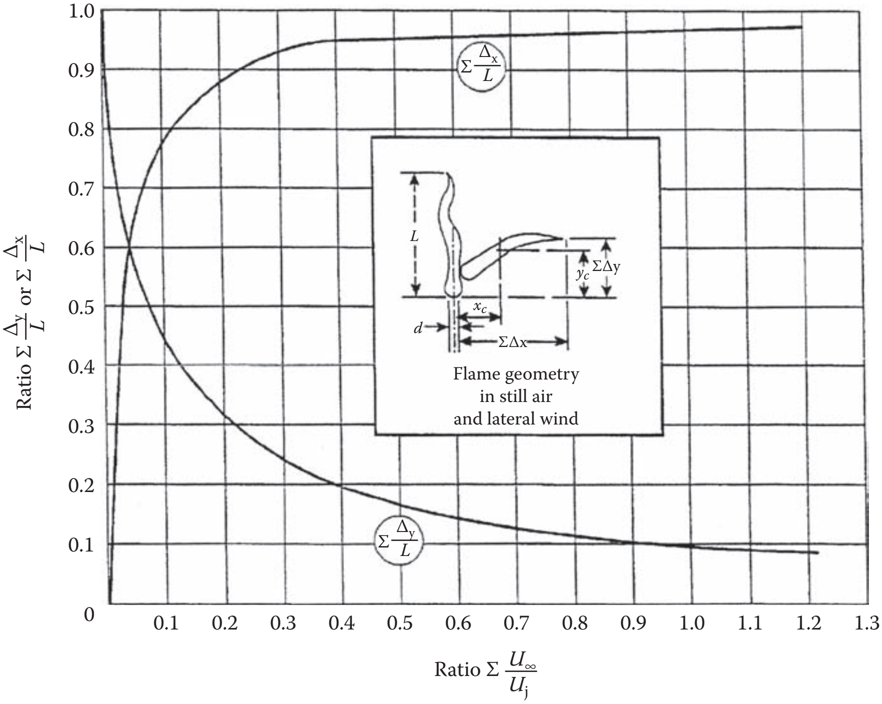
$$x_c = \frac{1}{2}\Delta x \tag{4.101b}$$

where

y_c = vertical distance of flame center from the top of flare stack, m

x_c = horizontal distance of flame center from the top of flare stack, m

From the geometry of the flare stack, Figure 4.18, the flare stack height can be calculated easily.



U_x = Lateral wind velocity.
U_j = Exit gas velocity from stack.

Figure 4.17 Approximate flame distortion due to lateral wind on jet velocity from flare stack. (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

Sizing of flare stack: Brzustowski and Sommer approach [20]

Calculation of flare stack diameter

Flare stack diameter is calculated using Equation 4.94 as explained before.

Location of flame center x_c, y_c

A different approach is used to estimate the vertical and horizontal distances of the flame center from the top of the flare stack.

Lower explosive limit of mixtures

This limit is estimated as follows:

$$C_L = \left\{ \sum_i \frac{y_i}{C_{Li}} \right\}^{-1.0} \tag{4.102}$$

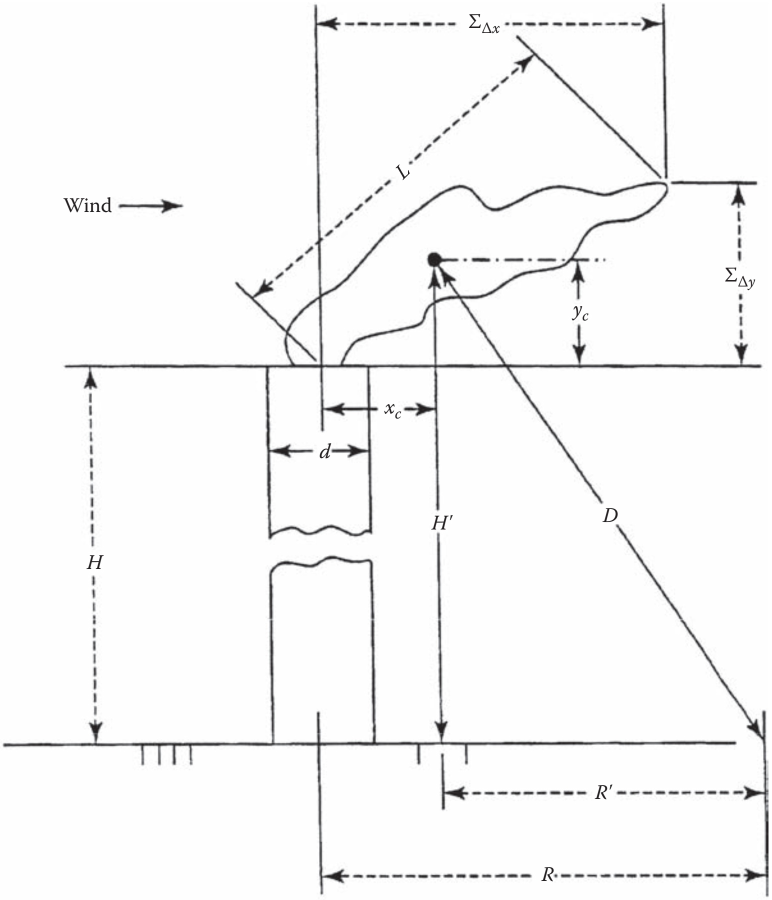


Figure 4.18 Dimensional references for sizing a flare stack. (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

where

C_L = lower explosive concentration of the gas in air
C_{Li} = lower explosive concentration of component i in the air
y_i = mole fraction of component i in the mixture

The lower explosive concentrations of commonly used gases are presented in Table 4.17 [21].

The lower explosive limit concentration parameter is calculated as follows:

$$C_L' = C_L \left(\frac{U_j}{U_\infty} \right) \left(\frac{M_j}{M_\infty} \right) \tag{4.103}$$

Table 4.17 Data for Flammable Materials

Component	Melting Point (°C)	Boiling Point (°C)	Flash Point (°C)	Explosive Limit (mol%) Lower	Explosive Limit (mol%) Higher
Acetylene	–81	–84	—	1.5	100
Benzene	–6	80	–11	1.2	8
Butane	–138	–1	–60	1.5	8.5
Carbon monoxide	–205	–191	—	12.5	74.2
Cyclobutane	–91	13	—	1.8	—
Cyclopentane	—	119	<21	—	—
Cyclohexane	7	81	–18	1.2	7.8
Cyclopropane	–127	–33	—	2.4	10.4
Ethane	–183	–87	—	3.0	15.5
Ethanol	–144	78	12	3.3	19.0
Ethylene	–169	–104	—	2.7	34.0
Heptane	–91	98	4	1.1	6.7
Methane	–182	–161	—	5.0	15.0
Naphtha	—	35	6	0.9	6.0
Nonane	–54	151	30	0.8	5.6
Octane	–56	126	13	1.0	3.2
Pentane	–130	36	≤20	1.4	8.0
Propane	–188	–42	—	2.0	9.5
Propene	–185	–48	—	2.0	11.7

Source: From BS 5345, Selection, Installation and Maintenance of Electrical Apparatus for Use in Potentially Explosive Atmospheres (Other than Mining Applications or Explosives Processing and Manufacture) Part 1: General Recommendations, British Standards, 1989.

where

U_j = exit velocity, calculated using Equation 4.97, m/sec
U_∞ = wind velocity, m/sec
M_j = molecular weight of the flowing gas
M_∞ = molecular weight of air (29)

The jet and wind thrust parameter is estimated as follows:

$$d_j R = d_j \left(\frac{U_j}{U_\infty}\right)\left(\frac{T_\infty M_j}{T_j}\right)^{0.5} \tag{4.104}$$

where

d_j = flare stack diameter, m
T_∞ = ambient temperature, K
T_j = temperature of the flowing gas, K

Other terms are as defined before. From the values of C_L and $d_j R$, y_c and x_c can be calculated.

Vertical distance y_c

This distance is estimated using Figure 4.19; it can also be calculated by the following procedure. The value estimated by this procedure is not valid for the entire range of the graph, and the calculated value should be checked with Figure 4.19. The value of y_c is estimated as follows:

For d_jR value $1200 \geq d_jR \geq 400$:

$$\ln(y_c) = A1 * \ln(C_L') + B1 \tag{4.105}$$

where

$$A1 = -2 * 10^{-7} * (d_jR)^2 + 6 * 10^{-5} * (d_jR) - 0.4304 \tag{4.106a}$$

$$B1 = 3 * 10^{-7} * (d_jR)^2 + 0.0013 * (d_jR) + 4.2069 \tag{4.106b}$$

For d_jR value $400 > d_jR \geq 10$:

$$\ln(y_c) = A2 * \{\ln(C_L')\}^2 - 0.42 * \ln(C_L') + C2 \tag{4.107}$$

where

$$A2 = 6 * 10^{-8} * (d_jR)^2 - 5 * 10^{-5} * (d_jR) - 0.0119 \tag{4.108a}$$

$$C2 = 0.9731 * \ln(d_jR) - 1.1158 \tag{4.108b}$$

For d_jR value $10 > d_jR \geq 3$:

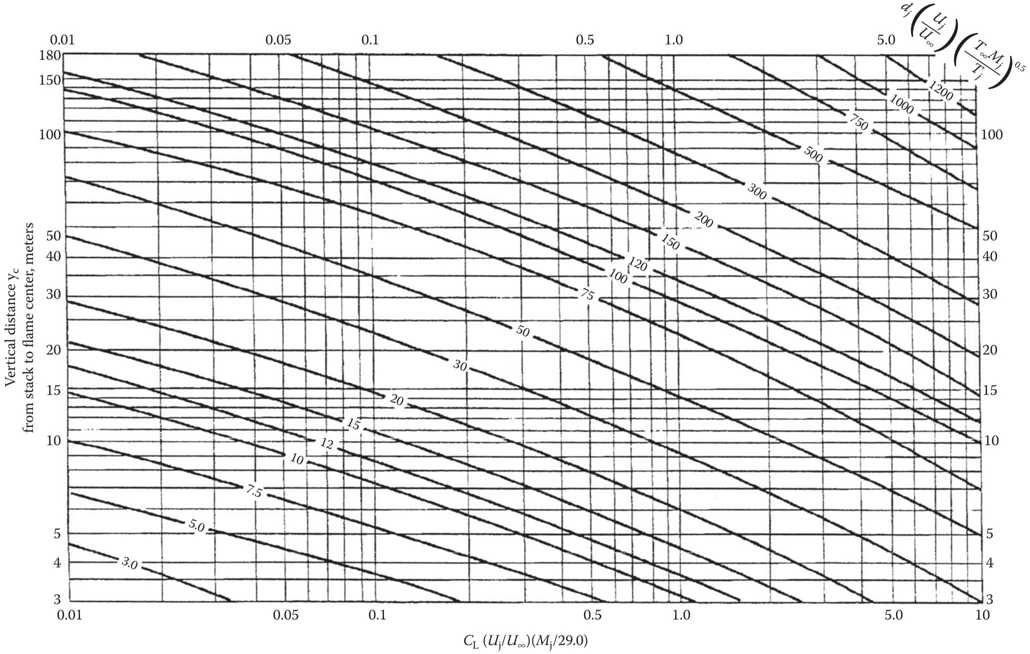
$$\ln(y_c) = -0.30 * \ln(C_L') + B3 \tag{4.109}$$

where

$$B3 = 0.9504 * \ln(d_jR) - 0.9679 \tag{4.110}$$

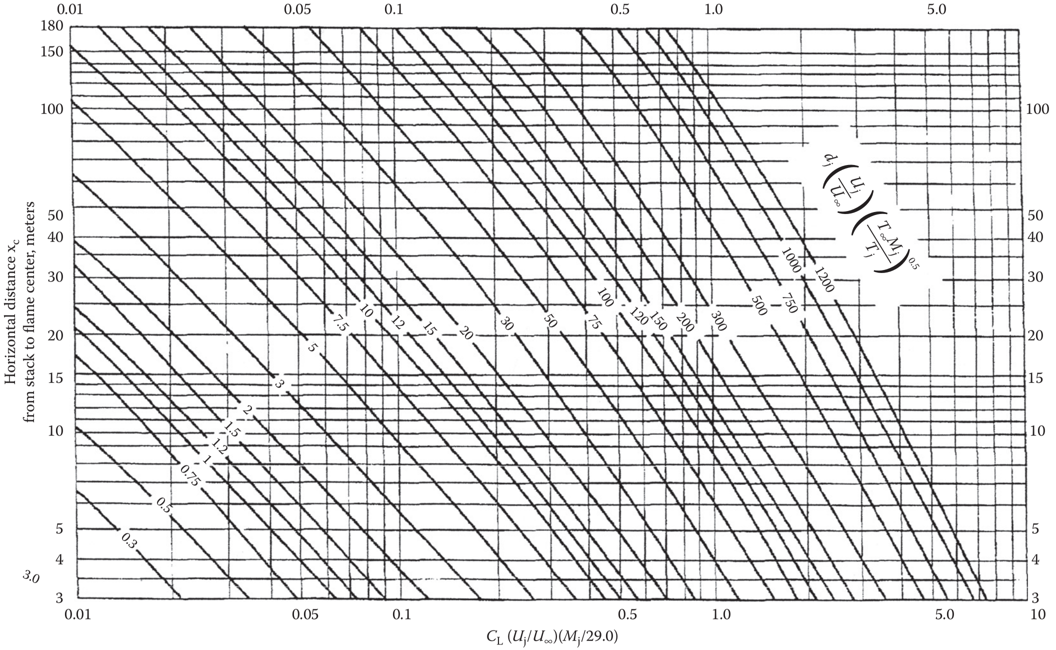
Horizontal distance x_c

The horizontal distance x_c is estimated using Figure 4.20; it can also be calculated by the following procedure. The value estimated by this procedure



C_L = lower explosive limit, volume fraction.
d_j = inside diameter of flare tip, in meters.
M_j = molecular weight of the gas.
T_j = temperature of the gas, in degress Kelvin.
T_x = temperature of the air, in degrees Kelvin.
U_j = gas exit velocity, in meters per second.
U_x = verage wind velocity, in meters per second.

Figure 4.19 Flame center for flares and ignited vents: vertical distance y_c (SI units). (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)



C_L = lower explosive limit, volume fraction.
d_j = inside diameter of flare tip, in meters.
M_j = molecular weight of the gas.
T_j = temperature of the gas, in degress Kelvin.
T_x = temperature of the air, in degrees Kelvin.
U_j = gas exit velocity, in meters per second.
U_x = average wind velocity, in meters per second.

Figure 4.20 Flame center for flares and ignited vents: horizontal distance x_c (SI units). (Courtesy American Petroleum Institute, 1220 L Street, NW, Washington, D.C.)

is not valid for the entire range of the graph, and the calculated value should be checked with Figure 4.20. The value of x_c is estimated as follows:

For d_jR value $1200 \geq d_jR \geq 10$:

$$\ln(x_c) = A4 * \{\ln(C_L')\}^2 + B4 * \ln(C_L') + C4 \tag{4.111}$$

where

$$A4 = -0.0382 * \ln(d_jR) + 0.0368 \tag{4.112a}$$

$$B4 = 0.0522 * \left\{\ln(d_jR)\right\}^2 - 0.4859 * \ln(d_jR) - 0.5487 \tag{4.112b}$$

$$C4 = \ln(d_jR) - 2.4125 \tag{4.112c}$$

For d_jR value $10 > d_jR \geq 0.5$:

$$\ln(x_c) = A5 * \left\{\ln(C_L')\right\}^2 + B5 * \ln(C_L') + C5 \tag{4.113}$$

where

$$A5 = 0.0133 * \ln(d_jR) - 0.0887 \tag{4.114a}$$

$$B5 = 0.1106 * \ln(d_jR) - 1.6518 \tag{4.114b}$$

$$C5 = 1.2209 * \ln(d_jR) - 2.7643 \tag{4.114c}$$

Example 4.8

A flare stack is to be designed for hydrocarbon vapors of the following conditions:

Flow rate = 100,000 kg/h
Flowing temperature = 100°C
Compressibility factor = 1
Ratio of specific heat = 1.2
Flare tip pressure = 101.3 kPaA
Maximum allowable radiation = 6.31 kW/m^2 at a distance of 50 m from the stack

Design wind velocity = 10 m/sec
Design Mach number = 0.5
Fluid composition:
  Methane = 80 mol%
  Ethane = 10 mol%
  Propane = 5 mol%
  i-Butane = 3 mol%
  n-Butane = 2 mol%

Solution

Average molecular weight = 20.95
Heating value of the gas (Table 4.16) = 53.63 MJ/kg

Because the gas is predominantly methane, the radiation factor, F, is assumed as 0.15.

Total heat released (Equation 4.93) = $1.49 * 10^6$ kW
Minimum distance from flame center (Equation 4.92) = 53 m
Flare stack diameter (Equation 4.94) = 0.496 m

Simple Approach

Flame length (Equation 4.95) = 71.9 m
Actual volumetric flow (Equation 4.96) = 40.58 m^3/sec
Flare tip exit velocity (Equation 4.97) = 210 m/sec
Velocity factor (Equation 4.98) = 0.048
Flame vertical length, Δy (Equation 4.99) = 71.9 * 0.625 = 44.9 m
Flame horizontal length, Δx (Equation 4.100) = 71.9 * 0.692 = 49.8 m
y_c (Equation 4.101a) = 22.45 m
x_c (Equation 4.101b) = 24.9 m

From the geometry (Figure 4.18),

$R' = R - x_c = 25.1$ m
$H' = (D^2 - R'^2)^{0.5} = 46.7$ m
$H = H' - y_c = 24.25$ m (stack height)

Brzustowski and Sommer Approach

Flare stack diameter (as calculated before) = 0.496 m
Tip exit velocity (as calculated before) = 210 m/sec
Lower explosive limit of gas (Equation 4.102, Table 4.17) = 0.0397 mol fraction
Concentration parameter, C_L', (Equation 4.103) = 0.6

Parameter for jet and wind thrust, d_jR (Equation 104) = 42.6
x_c (Equation 4.111) = 8.6 m
y_c (Equation 4.107) = 15.6 m

From the geometry (Figure 4.18),

$$R' = R - x_c = 41.4 \text{ m}$$
$$H' = (D^2 - R'^2)^{0.5} = 33.1 \text{ m}$$
$$H = H' - y_c = 17.5 \text{ m (stack height)}$$

SIL analysis

The design of safety systems of a process industry is the most important part of the overall design. International standards IEC 61508 [22], IEC 61511 [23], and ANSI/ISA-S84.01 [24] outline the detailed design procedure. Detailed analysis of the safety system design is not a part of this section. However, an overview will be presented in the following section for the purpose of general understanding.

The relationship between IEC 61511 and IEC 61508 is presented in Figure 4.21.

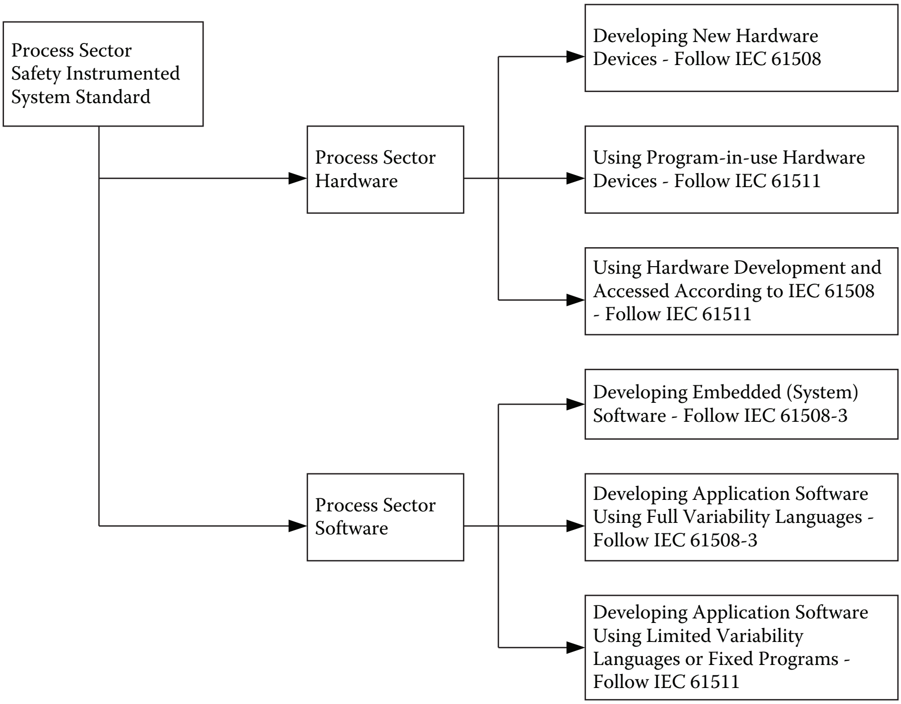


Figure 4.21 Relationship between IEC 61511 and IEC 61508.

Definitions

Definitions of frequently used terms in *SIL* analysis are presented in the following text. For detailed definitions, refer to IEC 61511 (Part 1) [23].

Diagnostic coverage (DC)

This is the ratio of the detected failure rate to the total failure rate of the component or subsystem as detected by diagnostic tests. Diagnostic coverage does not include any faults detected by proof tests.

Final element

This is a part of a safety-instrumented system, which implements the physical action necessary to achieve a safe state. Examples are valves, switch gear, motors, solenoid valves, actuators, etc.

MooN

This is a safety-instrumented system, or part thereof, made up of "**N**" independent channels, which are so connected that "**M**" channels are sufficient to perform the safety-instrumented function.

Programmable electronics (PE)

This is an electronic component or device forming part of a PE system, based on computer technology. The term encompasses both hardware and software and input and output units.

Programmable electronic system (PES)

This is a system for control, protection, or monitoring based on one or more programmable electronic devices, including all elements of the system, such as power supplies, sensors, and other input devices, data highways and other communication paths, and actuators and other input devices.

Protection layer

This is an independent mechanism that reduces risk by control, prevention, or mitigation. It could be a process engineering mechanism such as the size of vessels containing hazardous chemicals, a mechanical engineering mechanism such as a relief valve, or an administrative procedure such as an emergency plan against an imminent hazard.

Safety-instrumented function (SIF)

This is a safety function with a specified integrity level which is necessary to achieve functional safety and which can be either a safety-instrumented protection function or a safety-instrumented control function.

Table 4.18 SIL: Probability of Failure on Demand

Demand Mode of Operation		
SIL	Target Average Probability of Failure on Demand	Target Risk Reduction
1	$\geq 10^{-2}$ to $< 10^{-1}$	> 10 to ≤ 100
2	$\geq 10^{-3}$ to $< 10^{-2}$	> 100 to $\leq 1{,}000$
3	$\geq 10^{-4}$ to $< 10^{-3}$	$> 1{,}000$ to $\leq 10{,}000$
4	$\geq 10^{-5}$ to $< 10^{-4}$	$> 10{,}000$ to $\leq 100{,}000$

Note: SIL = Safety integrity level.

Table 4.19 SIL: Frequency of Dangerous Failures of the *SIF*

Continuous Mode of Operation	
SIL	Target Frequency of Dangerous Failures to Perform the Safety-Instrumented Function (per Hour)
1	$\geq 10^{-6}$ to $< 10^{-5}$
2	$\geq 10^{-7}$ to $< 10^{-6}$
3	$\geq 10^{-8}$ to $< 10^{-7}$
4	$\geq 10^{-9}$ to $< 10^{-8}$

Safety-instrumented systems (SIS)

These are systems composed of sensors, logic solvers, and final control elements for the purpose of taking the process to a safe state when predetermined conditions are violated.

Safety integrity

This is the average probability of a safety-instrumented system satisfactorily performing the required safety-instrumented functions under all stated conditions within a stated period of time.

Safety integrity level (SIL)

This is a discrete level (one of four) for specifying the safety integrity requirements of the safety-instrumented functions to be allocated to the safety-instrumented systems. *SIL* 4 has the highest level of safety integrity, and *SIL* 1 has the lowest.

The target failure measures for the *SIL* are specified in Table 4.18 and Table 4.19.

Safety life cycle

This is the sequence of activities involved in the implementation of the safety-instrumented systems from conception through decommissioning.

Matrix for SIL determination

A simplified matrix is often used to determine the *SIL* for a particular scenario. The SIL is determined based on the frequency of a particular occurrence and the consequence severity. Consequence severity can further be subclassified as follows:

- Financial consequence
- Health and safety consequence
- Environment and asset consequence

The consequence severity is generally industry specific, and most industrial houses have their own guidelines for consequence severity. The *SIL* matrix is presented in Table 4.20. The frequency of occurrence can be established based on Table 4.21. The consequence of a particular severity can be established based on Table 4.22.

Table 4.20 *SIL* Matrix

Consequence Severity	Frequency of Occurrence					
	A	B	C	D	E	F
1	Above 4	4	3	2	2	1
2	4	3	2	1	1	—
3	3	2	1	1	—	—
4	2	1	1	—	—	—
5	1	—	—	—	—	—

Note: Empty cells indicate that *SIL* analysis is not required. Normally, a process industry is not designed for *SIL* 4 and above.

Table 4.21 Frequency of Occurrence

A	1 to 0.1
B	0.1 to 0.01
C	0.01 to 0.001
D	0.001 to 0.0001
E	0.0001 to 0.00001
F	Less than 0.00001

Table 4.22 Consequence Severity

Category	Financial	Health and Safety	Environment and Asset
1	Extensive financial loss	Multiple fatalities	Extensive damage to the environment and facility
2	Major financial loss	Single fatality	Major damage to the environment and facility
3	Moderate financial loss	Serious injury	Localized damage to the environment and facility
4	Minor financial loss	Medical treatment	Minor damage to the environment and facility
5	Negligible	Minor impact	No significant impact

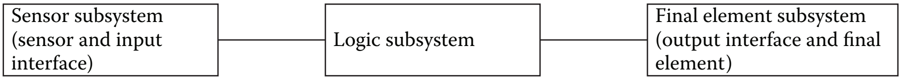


Figure 4.22 Subsystem structure.

Probability of failure on demand

The average probability of failure on demand can be expressed as follows:

$$PFD_{SYS} = PFD_S + PFD_L + PFD_{FE}$$

where

PFD_{SYS} = average probability of failure on demand of a safety function for the E/E/PE safety-related system
PFD_S = average probability of failure on demand for the sensor subsystem
PFD_L = average probability of failure on demand for the logic subsystem
PFD_{FE} = average probability of failure on demand for the final element subsystem

The structure can be explained as in Figure 4.22.

Sensor subsystem: Sensor subsystem components may be sensors (e.g., pressure transmitter, temperature transmitter, etc.), barriers, input conditioning circuits, etc.
Logic subsystem: This may be a *PES*, processor, scanning device, etc.
Final element subsystem: This may be an actuator (e.g., shutdown valve, blowdown valve, etc.), barrier, etc.

Each subsystem can be designed as 1oo1, 1oo2, 2oo2, 1oo2D, or 2oo3 voted groups.

The following definitions and terms are useful in understanding the calculation procedure [22]. The values used for different situations are only for understanding the calculation procedure. Establishment of the actual values is beyond the scope of this section. Detailed procedures are available in IEC 61508 [22].

MTTR: Mean time to resolution, the value normally used being 8 h.
Failure rate (λ): Failure rate (per hour) of a channel in a subsystem. The failure rate is generally estimated for a particular case.
Definition of β_D: The fraction of those failures detected by diagnostic tests that have a common cause (expressed as a fraction in equations and as a percentage elsewhere). Commonly used values of β_D are

$$\beta_D = 1\%$$

$$\beta_D = 5\%$$

$$\beta_D = 10\%$$

Definition of β: The fraction of undetected failures that have a common cause (expressed as a fraction in equations and as a percentage elsewhere). Commonly used values of β are ($\beta = 2 * \beta_D$)

$$\beta = 2\%$$

$$\beta = 10\%$$

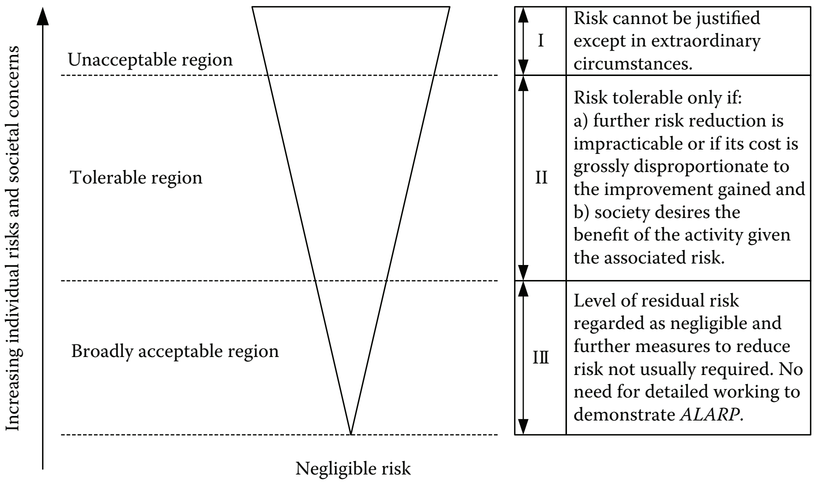
$$\beta = 20\%$$

ALARP model

The as low as reasonably practicable (*ALARP*) model is a concept that is used in the design of safety and integrity level. The design of a safety system is a trade-off between expenditure and improvement in safety. In most cases, a very safe design involves the spending of a large amount of money. The ALARP concept defines a design practice so as to have adequate safety at a reasonable cost. The concept can be indicated as in Figure 4.23.

Determination of SIL

The following examples explain the overall method of *SIL* determination. These examples provide an idea of the method; the actual analysis is a more involved procedure.



I	Risk cannot be justified except in extraordinary circumstances.
II	Risk tolerable only if: a) further risk reduction is impracticable or if its cost is grossly disproportionate to the improvement gained and b) society desires the benefit of the activity given the associated risk.
III	Level of residual risk regarded as negligible and further measures to reduce risk not usually required. No need for detailed working to demonstrate *ALARP*.

Figure 4.23 Tolerable risk and *ALARP*.

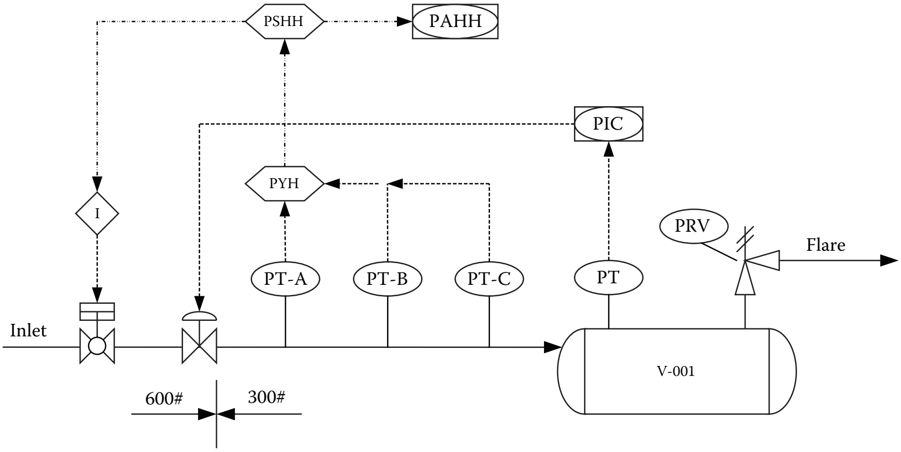


Figure 4.24 High-integrity protection.

Example 4.9

Refer to Figure 4.24. Vessel V-001 is an inlet separator for a gas and liquid processing plant. A two-phase fluid is available from a well and processed in the central processing plant. The inlet separator is designed at 4000 kPaG (PRV set point = 4000 kPaG), and the fluid inlet pressure at the plant battery limit is 6000 kPaG. The pressure is reduced to its operating pressure of 3500 kPaG before the fluid enters the inlet separator. The existing flare system has a limited capacity and cannot handle the full flow from the wellhead. The design needs to avoid a block discharge scenario for the separator. The PRV has been designed only for fire relief.

To avoid block discharge overpressure, a high-integrity system has been adopted into the design. This includes the installation of three pressure transmitters (PT-A, B, and C); if two of three pressure signals indicate a high pressure, the PSHH (pressure switch high high) will be activated and the inlet shutdown valve will be closed. The closing of the shutdown valve will avoid a high-pressure situation in the inlet separator. There is also the high-pressure alarm to inform the operator for an appropriate action. It is essential to keep the pressure signals healthy, which is done by installing a deviation alarm (not shown in Figure 4.24). The following additional information is used for the analysis:

- The production consists mainly of 50,000 barrels of oil per day (for cost purpose).
- The impact of gas flow is neglected for cost analysis.
- There is a bund around the separator.
- Gas and liquid production lines from the separator are not shown.

Solution

Vessel V-001 is protected by a PRV. To reduce the flare load, the PRV has been designed only for a fire contingency, and it is expected that block discharge will not be a design contingency for the inlet separator.

The purpose of this exercise is to establish at what *SIL* this design needs to be maintained. As explained before, the *SIL* analysis will be conducted for all three different scenarios: (1) financial, (2) health and safety, and (3) environment and asset.

Financial

In this case, the consequence will be established based on the financial loss of the plant. In case the design fails, the vessel will be subjected to high pressure, and because the PRV is not designed for full blocked discharge flow, the high pressure will cause vessel failure and, ultimately, loss of production. The analysis will be done keeping the following in mind:

- The V-001 is still protected by a PRV — under a high-pressure scenario, the PRV will have some mitigation effect.
- The pressure drop across the control valve is not very high.

Consequence severity: With the preceding facts in mind, it is expected that extensive vessel damage will not occur. The extent of damage will be failure of a gasket or nozzle. However, if the nozzle fails, repair of the nozzle and postweld heat treatment will be time consuming. This indicates that, although the damage is not significant, the loss of production can be significantly high. It is assumed that a loss of 4 weeks' production will result.
Production loss (4 weeks) = 1,400,000 barrels
Cost (50 US$/barrel) = 70 million US$
Actual loss (assumed 20% of above cost) = 14 million US$
Financial loss = major
Consequence severity (Table 4.22) = 2

Frequency: The protection is developed using the 2003 voting system. This type of design will have a failure rate in the range of 0.1 to 0.01. From Table 4.21, the frequency will be B.

Protection layer: There is a DCS alarm available to inform the operator of pressure rise, and operator intervention will have some mitigation effect. This is equivalent to one protection layer. The PRV is not designed for a blocked-discharge scenario, and no protection layer has been considered for the PRV. Considering one protection layer, the frequency will be modified to C.

From Table 4.20, for consequence severity of 2 and frequency of C, the estimated *SIL* will be 2.

Health and safety

The situation may cause serious damage to the personnel at a site to the extent of a single fatality.

The consequence severity category = 2
Overall frequency (as estimated before) = C
The estimated *SIL* (from Table 4.20) = 2

Environment and asset

Under an abnormal situation, all vessel inlet and outlet shutdown valves will be closed through operator intervention. This will stop the spillage of a large quantity of oil to the land. The vessel also has a dyke to prevent the spillage of oil into the land. Considering these facts, the consequence severity will be equivalent to localized damage and the category will be 3. The protection layer will be the same as indicated before, and therefore, the overall frequency will be C. From Table 4.20, the estimated *SIL* = 1. Therefore, the controlling *SIL* will be 2.

Example 4.10

In Example 4.9, the safety function requires a *SIL* 2 system. It is required to establish if the available architecture can achieve *SIL* 2.

For this analysis, a 1-year proof-test interval and an 8-h MTTR have been assumed.

Solution

The configuration is presented in Figure 4.25.

For the sensor subsystem,

$$PFD_S \text{ (from IEC 61508)} = 2.3 * 10^{-4}$$

For the logic subsystem,

$$PFD_L \text{ (from IEC 61508)} = 8.8 * 10^{-3}$$

For the final element subsystem,

$$PFD_{FE} \text{ (from IEC 61508)} = 8.8 * 10^{-3}$$

Therefore, for the safety function,

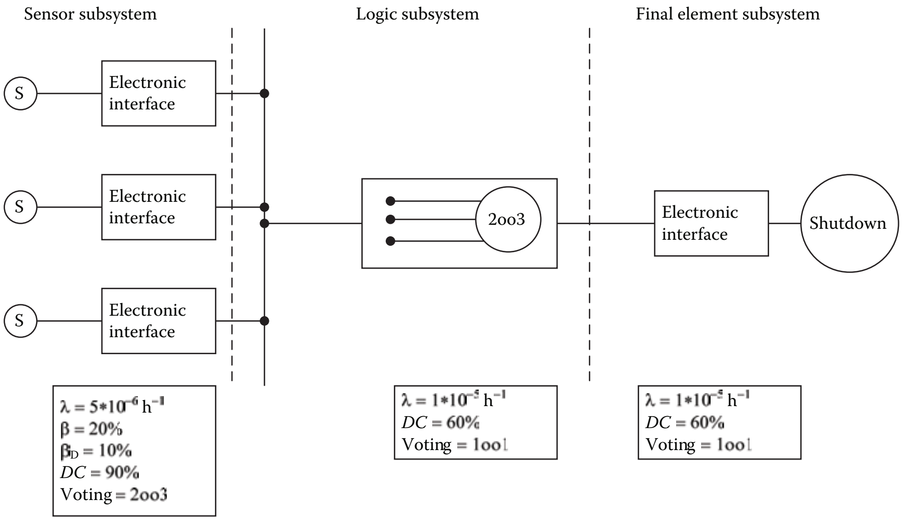


Figure 4.25 Architecture of Example 4.10.

$$PFD_{SYS} = PFD_S + PFD_L + PFD_{FE}$$

$$= 0.018$$

$$= SIL\ 1$$

This indicates that the existing hardware is not adequate to maintain *SIL* 2.

Modification 1

In this modification, it is assumed that the proof-test interval will be reduced from 1 year to 6 months. The estimated values will be

$$PFD_S = 1.1 * 10^{-4}$$

$$PFD_L = 4.4 * 10^{-3}$$

$$PFD_{FE} = 4.4 * 10^{-3}$$

$$PFD_{SYS} = 0.009$$

$$= SIL\ 2$$

Modification 2

In this modification it is assumed that the proof-test interval will be 1 year, but changing the 1oo1 shutdown valve to 1oo2, which means the installation of one more shutdown valve and closing another, will make the system safe. Also assume $\beta = 10\%$ and $\beta_D = 5\%$. The estimated values will be

$$PFD_S = 2.3 * 10^{-4}$$

$$PFD_L = 8.8 * 10^{-3}$$

$$PFD_{FE} = 9.7 * 10^{-4}$$

$$PFD_{SYS} = 0.01$$

$$= SIL\ 2$$

The preceding calculation indicates that the existing architecture cannot satisfy the *SIL* 2 requirements with a proof-test interval of 1 year. *SIL* 2 can be satisfied if one of the following two options is adopted:

- Reduce proof-test interval to 6 months.
- Add one more shutdown valve.

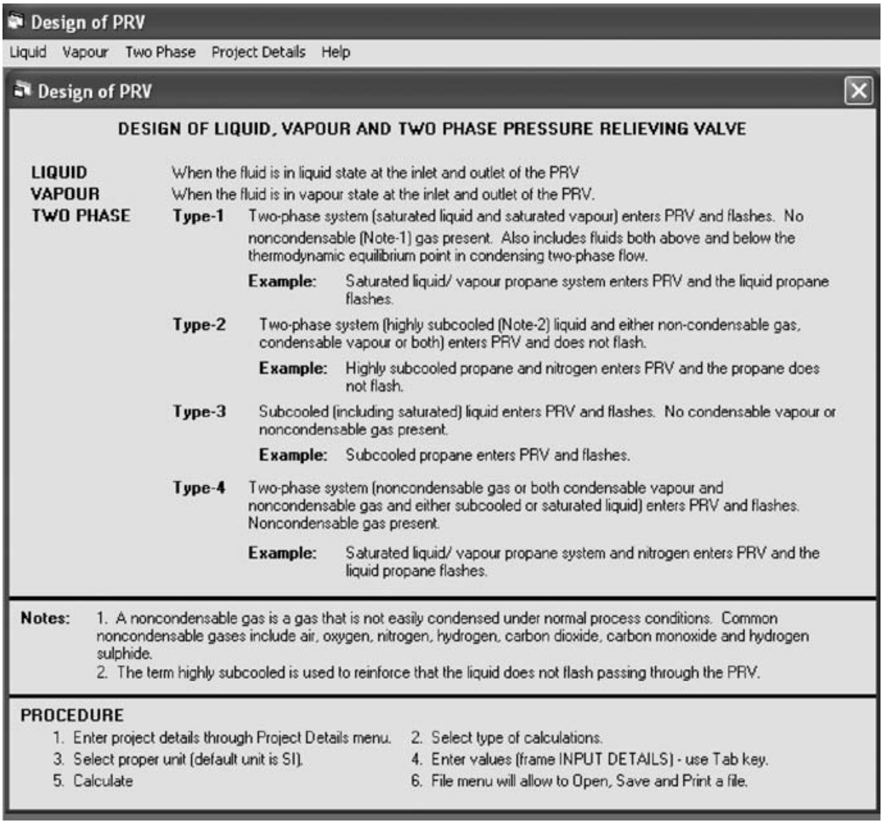
Programming

Program for pressure relief valve

A program to design a pressure relief valve, prvdesign, has been developed for liquid flow, vapor flow, and two-phase flow. The main form has been divided into a number of frames for input and results. The calculation can be performed using either SI or English units (the unit by default is SI). The units are fixed, and the program does not allow for change of individual units.

Program limitations and notes

- Whereas the single-phase PRV design is simple, the design of a two-phase PRV needs careful attention.
- In two-phase PRV design, a minor change in some specific parameters can result in a large difference in the orifice area.
- If the back pressure is more than 10% of the set pressure, the conventional PRV is not suitable. The program generates a message box in such cases.
- For balanced bellows, if the back pressure is more than 50% of the set pressure, the program asks for manual input of the back-pressure correction factor (which by default is 0.69).
- In some calculations, the discharge coefficient is a program output and cannot be entered manually.



DESIGN OF LIQUID, VAPOUR AND TWO PHASE PRESSURE RELIEVING VALVE

LIQUID When the fluid is in liquid state at the inlet and outlet of the PRV

VAPOUR When the fluid is in vapour state at the inlet and outlet of the PRV.

TWO PHASE

Type-1 Two-phase system (saturated liquid and saturated vapour) enters PRV and flashes. No noncondensable (Note-1) gas present. Also includes fluids both above and below the thermodynamic equilibrium point in condensing two-phase flow.

Example: Saturated liquid/ vapour propane system enters PRV and the liquid propane flashes.

Type-2 Two-phase system (highly subcooled (Note-2) liquid and either non-condensable gas, condensable vapour or both) enters PRV and does not flash.

Example: Highly subcooled propane and nitrogen enters PRV and the propane does not flash.

Type-3 Subcooled (including saturated) liquid enters PRV and flashes. No condensable vapour or noncondensable gas present.

Example: Subcooled propane enters PRV and flashes.

Type-4 Two-phase system (noncondensable gas or both condensable vapour and noncondensable gas and either subcooled or saturated liquid) enters PRV and flashes. Noncondensable gas present.

Example: Saturated liquid/ vapour propane system and nitrogen enters PRV and the liquid propane flashes.

Notes:
1. A noncondensable gas is a gas that is not easily condensed under normal process conditions. Common noncondensable gases include air, oxygen, nitrogen, hydrogen, carbon dioxide, carbon monoxide and hydrogen sulphide.
2. The term highly subcooled is used to reinforce that the liquid does not flash passing through the PRV.

PROCEDURE
1. Enter project details through Project Details menu.
2. Select type of calculations.
3. Select proper unit (default unit is SI).
4. Enter values (frame INPUT DETAILS) - use Tab key.
5. Calculate
6. File menu will allow to Open, Save and Print a file.

Figure 4.26 Pressure relief valve calculation — main form.

- The program calculates up to a single "T" orifice. The flow rate needs to be reduced if the calculated orifice size exceeds T.

General overview

Double click prvdesign.exe and the first form will be as shown in Figure 4.26. This form has the following options:

Liquid: The sizing of PRVs for liquid at the inlet and outlet of the relief valve.

Vapor: The sizing of PRVs for vapor at the inlet and outlet of the relief valve.

Two phase: The sizing of two-phase PRVs has further been divided into four different types:

Type 1: Saturated liquid and vapor enter the PRV and flash. No noncondensable gas is present.

Type 2: Highly subcooled liquid and either noncondensable gas or condensable vapor or both enter the PRV and the liquid does not flash.

Table 4.23 File Extension Used in Prvdesign.exe Program

Type of Calculation	File Extension
Two phase — type 1	*.p01
Two phase — type 2	*.p01
Two phase — type 3	*.p02
Two phase — type 4	*.p03
Liquid	*.p04
Vapor	*.p05

Type 3: Subcooled liquid enters the PRV and flashes. No condensable vapor or noncondensable gas is present.

Type 4: Noncondensable gas or both condensable vapor and noncondensable gas and either subcooled or saturated liquid enter the PRV and flash.

Project details

The form Project Details helps the user obtain project information. Once the details are filled in, press the “OK” button to transfer the information to the main form.

Available menus are (1) File and (2) Unit. Only input data are saved or opened. Because each type of PRV calculation has a different format, different extensions are used to open or save a file. The extensions are indicated in Table 4.23.

File save

Once the calculation is complete, the File Save menu will save the input parameters using the proper file extension.

File open

File Open is used to open a previously saved file.

File print

Once the calculation is complete, the File Print menu will print the output to a particular format.

Exit

Exit the program.

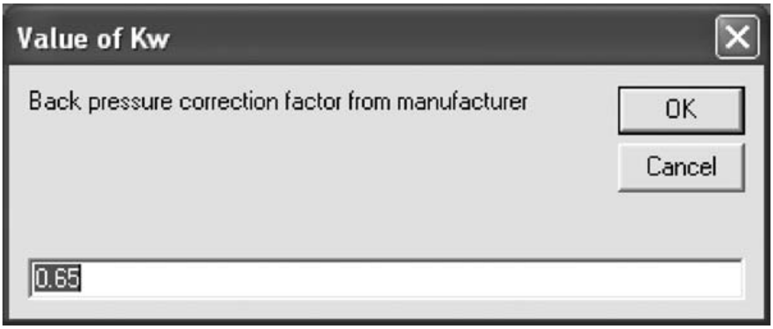
Specific message or warning: back pressure

If the PRV is of the conventional type and back pressure is more than 10%, a warning message appears (Figure 4.27), and the calculation is terminated. To proceed with the calculation, the PRV has to be changed to the balanced-bellows type.

Prvdesign

Conventional PRV is not suitable
Recalculate using Balanced Bellow PRV

Figure 4.27 Warning for high back pressure.



Value of Kw

Back pressure correction factor from manufacturer

0.65

Figure 4.28 Back-pressure correction factor.

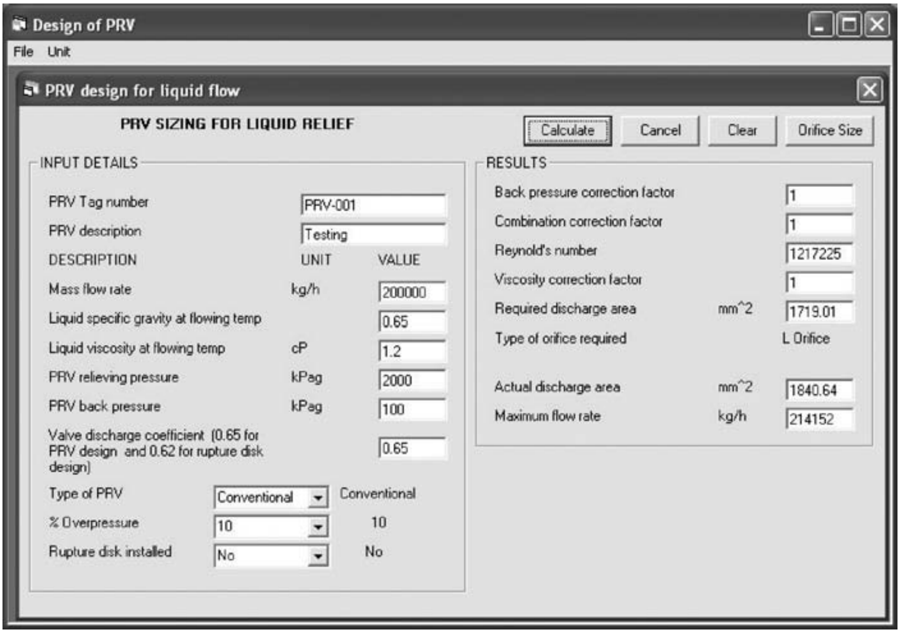
Back-pressure correction factor

The program cannot calculate the back-pressure correction factor, if the back pressure is more than 50% of the PRV set pressure. An input form (Figure 4.28) appears for the back-pressure correction factor (normally available from the manufacturer). A default value of 0.69 is used, which is modified by the vendor-supplied value.

Pilot-operated PRV

This program has been developed for conventional and balanced-bellows PRVs. However, this program can be used for pilot-operated PRVs, too. For pilot-operated PRVs, use:

- Conventional procedure if the back pressure is less than 10% of the set pressure.
- Balanced-bellows procedure if the back pressure is more than 10% of the set pressure.



PRV SIZING FOR LIQUID RELIEF

INPUT DETAILS

PRV Tag number: PRV-001
PRV description: Testing

DESCRIPTION	UNIT	VALUE
Mass flow rate	kg/h	200000
Liquid specific gravity at flowing temp		0.65
Liquid viscosity at flowing temp	cP	1.2
PRV relieving pressure	kPag	2000
PRV back pressure	kPag	100
Valve discharge coefficient (0.65 for PRV design and 0.62 for rupture disk design)		0.65

Type of PRV: Conventional — Conventional
% Overpressure: 10 — 10
Rupture disk installed: No — No

RESULTS

Back pressure correction factor		1
Combination correction factor		1
Reynold's number		1217225
Viscosity correction factor		1
Required discharge area	mm^2	1719.01
Type of orifice required		L Orifice
Actual discharge area	mm^2	1840.64
Maximum flow rate	kg/h	214152

Figure 4.29 PRV design for liquid flow.

Liquid

The liquid form is shown in Figure 4.29. This form has four buttons.

Calculate: This is to calculate the PRV area, type of orifice, actual discharge area, and maximum flow rate. In case the PRV area exceeds the "T" orifice, the program terminates with the following note:
"Orifice exceeds T.
Please reduce the contingency and try again."
To proceed with the recalculations, the contingency has to be reduced.

Cancel: This is to cancel a particular calculation and go back to the main form.

Clear: This is to clear the input parameters to enter a new set of data.

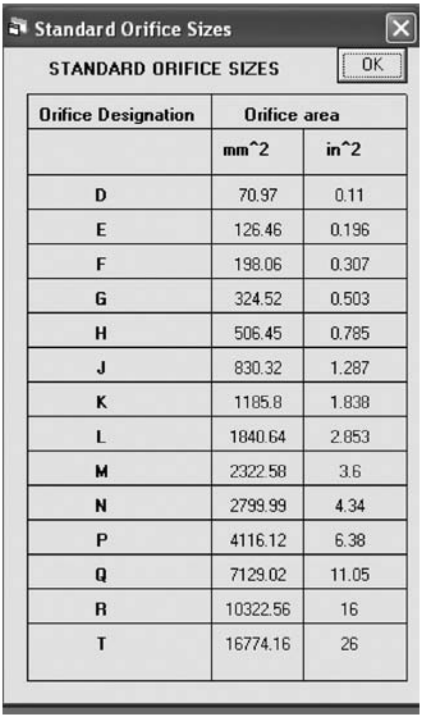
Orifice size: This presents standard orifice sizes as shown in Figure 4.30.

Vapor

Calculation for vapor flow is identical to the calculation of liquid flow.

Two-phase type 1 and type 2 calculations

The same form has been used for type 1 and type 2 calculations. There are some basic differences in the inputs of type 1 and type 2 calculations. These differences are presented in Table 4.24.



STANDARD ORIFICE SIZES

Orifice Designation	Orifice area	
	mm^2	in^2
D	70.97	0.11
E	126.46	0.196
F	198.06	0.307
G	324.52	0.503
H	506.45	0.785
J	830.32	1.287
K	1185.8	1.838
L	1840.64	2.853
M	2322.58	3.6
N	2799.99	4.34
P	4116.12	6.38
Q	7129.02	11.05
R	10322.56	16
T	16774.16	26

Figure 4.30 Standard orifice sizes.

Example 4.11

Use the program to solve Example 4.5.

Parameters:
Mass flow = 20000 kg/h
Vapor mass fraction = 0.409
Specific volume of vapor = 0.0908 m^3/kg
Specific volume of two-phase system = 0.0382 m^3/kg
PRV relieving pressure = 330 kPaG
Difference between vapor and liquid specific volume = 0.089 m^3/kg
Latent heat of vaporization = 277 kJ/kg
Liquid specific heat at constant pressure = 2.555 kJ/(kg.°C)
Temperature at PRV inlet = 60°C
Ratio of specific heat = 1.113
Specific volume at 90% of PRV relieving pressure = 0.043 m^3/kg
PRV back pressure = 20 kPaG
Valve discharge coefficient = 0.85
Type of PRV = conventional
Percentage overpressure = 10
Rupture disk installed = no

Table 4.24 Difference in Inputs for Type 1 and Type 2 Calculation

Description	Type 1 Boiling Range > 83°C	Type 1 Boiling Range < 83°C	Type 2
Boiling point range	To be defined		Not to be defined
Vapor/vapor plus mass fraction at PRV inlet	Not required	Required	Required
Specific volume of vapor/vapor plus gas at PRV inlet	Not required	Required	Required
Difference between vapor and liquid specific volume at PRV inlet	Not required	Required	Not required
Latent heat of vaporization at PRV inlet	Not required	Required	Not required
Liquid specific heat at constant pressure at PRV inlet	Not required	Required	Not required
Ratio of specific heat of vapors	Not required	Required	Required
Specific volume (two-phase system) at 90% of PRV relieving pressure	Required	Required	Not required

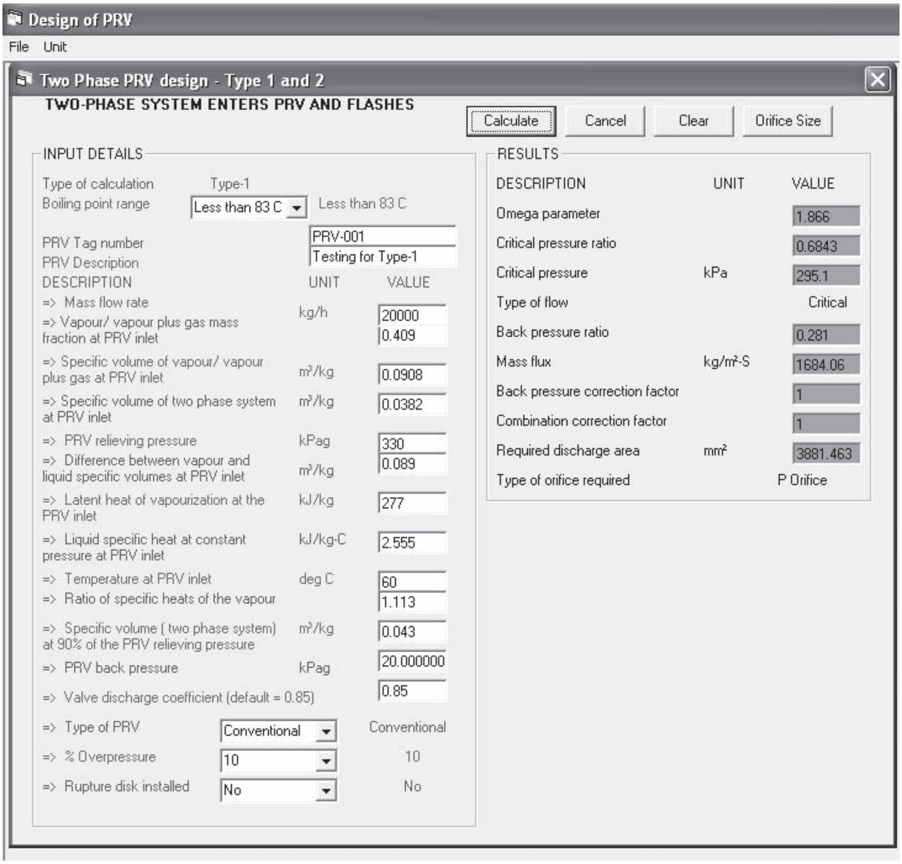
Solution

The solution is presented in Figure 4.31. The difference between manual calculation and program calculation is presented in Table 4.25.

Example 4.12

Use the program to solve Example 4.6.

Parameters:
Boiling point range = less than 83°C
Mass flow = 20000 kg/h
Liquid density at PRV inlet = 552.3 kg/m^3
Liquid specific heat = 2.576 kJ/(kg.°C)
PRV relieving pressure = 550 kpaG
Bubble point pressure = 384.5 kpaG
Difference between vapor and liquid specific volume = 0.0796 m^3/kg
Latent heat of vaporization = 274 kJ/kg
Temperature at PRV inlet = 60°C
PRV back pressure = 20 kpaG
Valve discharge coefficient = 0.65 (see the following note)
Type of PRV = conventional
Percentage overpressure = 10
Rupture disk installed = no



TWO-PHASE SYSTEM ENTERS PRV AND FLASHES

INPUT DETAILS

Type of calculation	Type-1	
Boiling point range	Less than 83 C	Less than 83 C
PRV Tag number	PRV-001	
PRV Description	Testing for Type-1	

DESCRIPTION	UNIT	VALUE
=> Mass flow rate	kg/h	20000
=> Vapour/ vapour plus gas mass fraction at PRV inlet		0.409
=> Specific volume of vapour/ vapour plus gas at PRV inlet	m³/kg	0.0908
=> Specific volume of two phase system at PRV inlet	m³/kg	0.0382
=> PRV relieving pressure	kPag	330
=> Difference between vapour and liquid specific volumes at PRV inlet	m³/kg	0.089
=> Latent heat of vapourization at the PRV inlet	kJ/kg	277
=> Liquid specific heat at constant pressure at PRV inlet	kJ/kg-C	2.555
=> Temperature at PRV inlet	deg C	60
=> Ratio of specific heats of the vapour		1.113
=> Specific volume (two phase system) at 90% of the PRV relieving pressure	m³/kg	0.043
=> PRV back pressure	kPag	20.000000
=> Valve discharge coefficient (default = 0.85)		0.85

=> Type of PRV	Conventional	Conventional
=> % Overpressure	10	10
=> Rupture disk installed	No	No

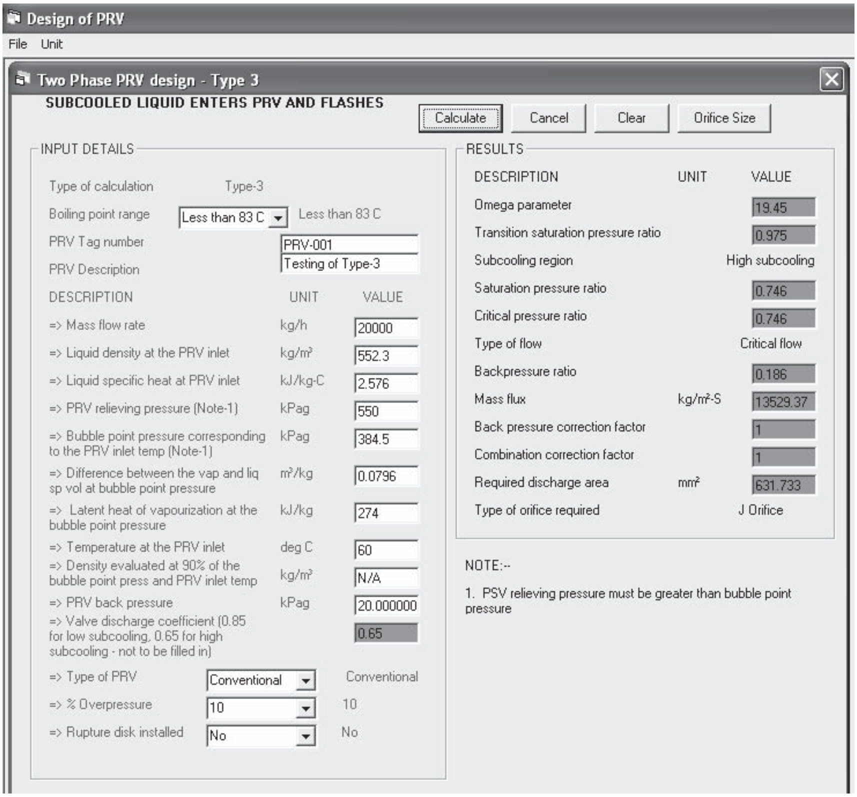
RESULTS

DESCRIPTION	UNIT	VALUE
Omega parameter		1.866
Critical pressure ratio		0.6843
Critical pressure	kPa	295.1
Type of flow		Critical
Back pressure ratio		0.281
Mass flux	kg/m²-S	1684.06
Back pressure correction factor		1
Combination correction factor		1
Required discharge area	mm²	3881.463
Type of orifice required		P Orifice

Figure 4.31 Solution of Example 4.11.

Table 4.25 Difference between Manual and Program Calculation (Example 4.11)

Description	Unit	Manual Calculation	Program Calculation
Omega parameter		1.865	1.866
Critical pressure ratio		0.6843	0.6843
Critical pressure	kPaA	295.1	295.1
Back-pressure ratio		0.281	0.281
Mass flux	kg/sec·m²	1684.4	1684.06
Required area	mm²	3880.3	3881.46



SUBCOOLED LIQUID ENTERS PRV AND FLASHES

INPUT DETAILS

Type of calculation Type-3
Boiling point range: Less than 83 C — Less than 83 C
PRV Tag number: PRV-001
PRV Description: Testing of Type-3

DESCRIPTION	UNIT	VALUE
=> Mass flow rate	kg/h	20000
=> Liquid density at the PRV inlet	kg/m³	552.3
=> Liquid specific heat at PRV inlet	kJ/kg-C	2.576
=> PRV relieving pressure (Note-1)	kPag	550
=> Bubble point pressure corresponding to the PRV inlet temp (Note-1)	kPag	384.5
=> Difference between the vap and liq sp vol at bubble point pressure	m³/kg	0.0796
=> Latent heat of vapourization at the bubble point pressure	kJ/kg	274
=> Temperature at the PRV inlet	deg C	60
=> Density evaluated at 90% of the bubble point press and PRV inlet temp	kg/m³	N/A
=> PRV back pressure	kPag	20.000000
=> Valve discharge coefficient (0.85 for low subcooling, 0.65 for high subcooling - not to be filled in)		0.65

=> Type of PRV: Conventional — Conventional
=> % Overpressure: 10 — 10
=> Rupture disk installed: No — No

RESULTS

DESCRIPTION	UNIT	VALUE
Omega parameter		19.45
Transition saturation pressure ratio		0.975
Subcooling region		High subcooling
Saturation pressure ratio		0.746
Critical pressure ratio		0.746
Type of flow		Critical flow
Backpressure ratio		0.186
Mass flux	kg/m²-S	13529.37
Back pressure correction factor		1
Combination correction factor		1
Required discharge area	mm²	631.733
Type of orifice required		J Orifice

NOTE:--

1. PSV relieving pressure must be greater than bubble point pressure

Figure 4.32 Solution of Example 4.12.

> ***Note:*** The valve discharge coefficient depends on the subcooling range, and the program calculates the coefficient. To make this consistent with other forms, this parameter has been retained in the input frame.

Solution

The solution is presented in Figure 4.32.

The difference between manual calculation and program calculation is presented in Table 4.26.

Example 4.13

Use the program to solve Example 4.7 (type 4).

Parameters:
- Boiling point range = less than 83°C
- Flow rate = 20,000 kg/h
- Combined vapor and gas mass fraction = 0.551
- Specific volume of combined vapor and gas = 0.0752 m^3/kg

Table 4.26 Difference between Manual and Program Calculation (Example 4.12)

Description	Unit	Manual Calculation	Program Calculation
Omega parameter		19.42	19.45
Transient saturation pressure ratio		0.975	0.975
Subcooling region		High	High
Saturation pressure ratio			0.746
Critical pressure ratio			0.746
Type of flow		Critical	Critical
Back-pressure ratio		0.186	0.186
Mass flux	kg/sec·m^2	13528	13529.37
Required discharge area	mm^2	631.8	631.73

Specific volume of two-phase system = 0.0422 m^3/kg
Saturation pressure corresponding to inlet temperature = 384.5 kPaG
PRV relieving pressure = 550 kPaG
Noncondensable gas partial pressure = 110 kPaG
Ratio of specific heats (vapor and gas) = 1.141
Liquid density at PRV inlet = 559.9 kg/m^3
Liquid specific heat = 2.548 kJ/(kg.°C)
Temperature at PRV inlet = 60°C
Difference between vapor and liquid specific volumes = 0.0972 m^3/kg
Latent heat of vaporization = 332 kJ/kg
PRV back pressure = 310 kPaG
Valve discharge coefficient = 0.85
Type of PRV = balanced bellows
Percentage overpressure = 10
Rupture disk installed = no

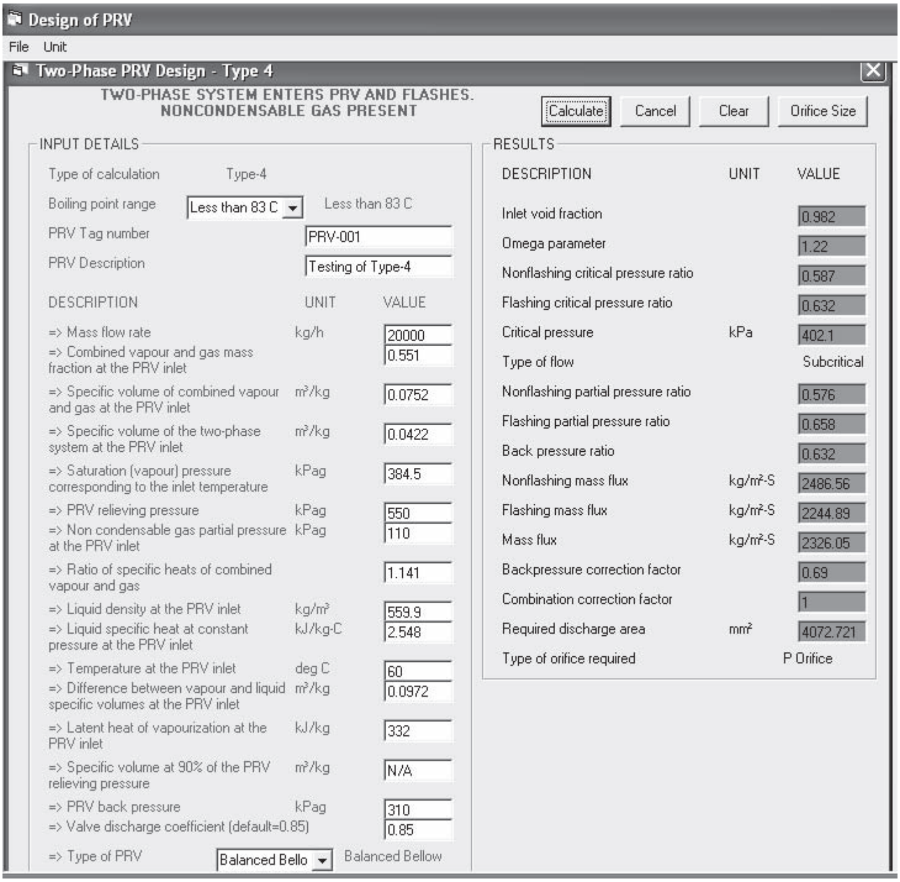
Solution

When the calculate button is pressed, an input form appears as shown in Figure 4.28. This is because the back-pressure ratio exceeds 50%, and the program will require the value of the back-pressure correction factor. A value of 0.69 is entered.

The solution is presented in Figure 4.33. The difference between manual calculation and program calculation is presented in Table 4.27.

Program for flare stack estimation

A program to estimate the height of flare stack, Flarestack.exe, has been developed for simple and Brzustowski and Sommer approaches. One sketch has been added to the main form to understand the dimensions and parameters. The calculation can be performed by using either SI or English units



TWO-PHASE SYSTEM ENTERS PRV AND FLASHES.
NONCONDENSABLE GAS PRESENT

INPUT DETAILS

Type of calculation Type-4
Boiling point range: Less than 83 C
PRV Tag number: PRV-001
PRV Description: Testing of Type-4

DESCRIPTION	UNIT	VALUE
=> Mass flow rate	kg/h	20000
=> Combined vapour and gas mass fraction at the PRV inlet		0.551
=> Specific volume of combined vapour and gas at the PRV inlet	m³/kg	0.0752
=> Specific volume of the two-phase system at the PRV inlet	m³/kg	0.0422
=> Saturation (vapour) pressure corresponding to the inlet temperature	kPag	384.5
=> PRV relieving pressure	kPag	550
=> Non condensable gas partial pressure at the PRV inlet	kPag	110
=> Ratio of specific heats of combined vapour and gas		1.141
=> Liquid density at the PRV inlet	kg/m³	559.9
=> Liquid specific heat at constant pressure at the PRV inlet	kJ/kg-C	2.548
=> Temperature at the PRV inlet	deg C	60
=> Difference between vapour and liquid specific volumes at the PRV inlet	m³/kg	0.0972
=> Latent heat of vapourization at the PRV inlet	kJ/kg	332
=> Specific volume at 90% of the PRV relieving pressure	m³/kg	N/A
=> PRV back pressure	kPag	310
=> Valve discharge coefficient (default=0.85)		0.85

=> Type of PRV: Balanced Bellow

RESULTS

DESCRIPTION	UNIT	VALUE
Inlet void fraction		0.982
Omega parameter		1.22
Nonflashing critical pressure ratio		0.587
Flashing critical pressure ratio		0.632
Critical pressure	kPa	402.1
Type of flow		Subcritical
Nonflashing partial pressure ratio		0.576
Flashing partial pressure ratio		0.658
Back pressure ratio		0.632
Nonflashing mass flux	kg/m²-S	2486.56
Flashing mass flux	kg/m²-S	2244.89
Mass flux	kg/m²-S	2326.05
Backpressure correction factor		0.69
Combination correction factor		1
Required discharge area	mm²	4072.721
Type of orifice required		P Orifice

Figure 4.33 Solution of Example 4.13.

(by default the unit is SI). The units are fixed, and the program does not allow for change of individual units.

There is only one file extension used for this program, which is *.flr.

Other operations and file systems (e.g., Project Details, File Save, File Open, File Print, and Exit) are as explained before.

Program limitations and notes

- In general, the simple approach results in greater stack height in comparison to Brzustowski and Sommer approach.
- The values of y_c and x_c are to be entered as 0 (default). The program calculates the value of y_c and x_c.
- If required, the calculated values of y_c and x_c can be checked from standard charts.

Table 4.27 Difference between Manual and Program Calculation (Example 4.13)

Description	Unit	Manual Calculation	Program Calculation
Inlet void fraction		0.982	0.982
Omega parameter		1.217	1.22
Nonflashing critical pressure ratio		0.587	0.587
Flashing critical pressure ratio		0.632	0.632
Critical pressure	kPaA	402.1	402.1
Type of flow		Subcritical	Subcritical
Nonflashing partial pressure ratio		0.576	0.576
Flashing partial pressure ratio		0.658	0.658
Back-pressure ratio		0.632	0.632
Nonflashing mass flux	$kg/m^2 \cdot sec$	2486.1	2486.56
Flashing mass flux	$kg/m^2 \cdot sec$	2246.2	2244.89
Mass flux	$kg/m^2 \cdot sec$	2326.6	2326.05
Back pressure correction factor		0.69	0.69
Required discharge area	mm^2	4071.3	4072.72

- In this calculation, the value of distance (R) is an input, whereas the program calculates the stack height.

Specific message/warning

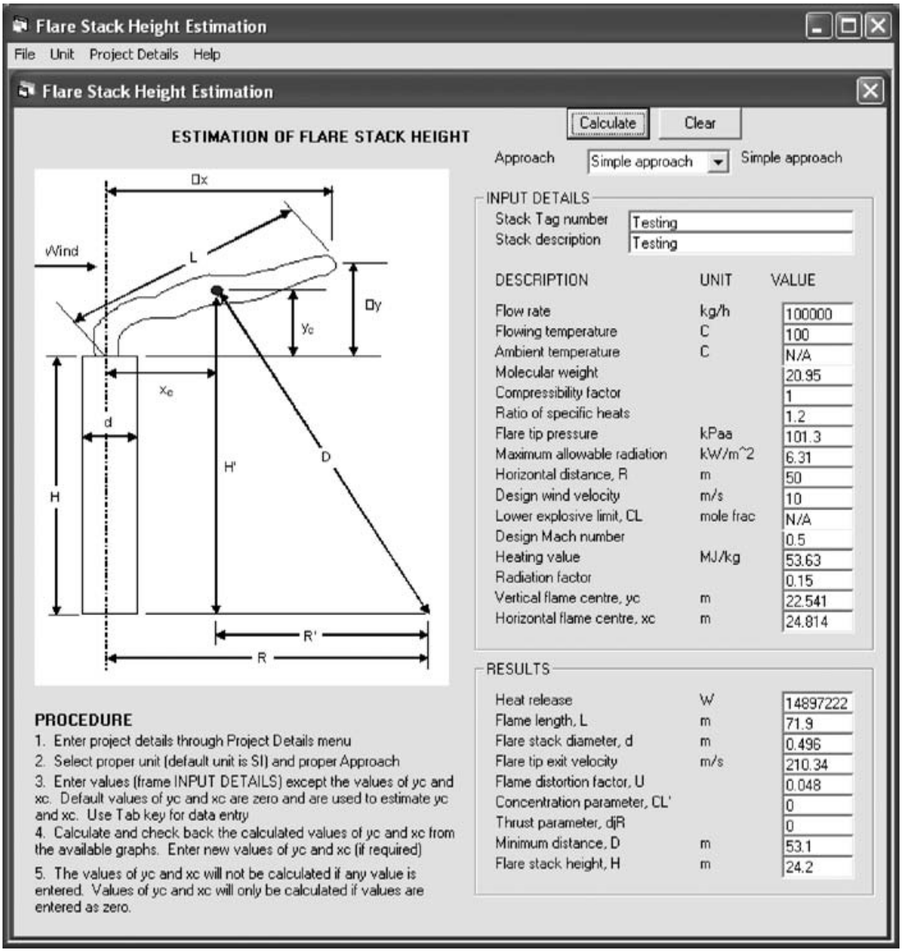
Estimation of stack height is a function of the horizontal distance defined for the heat radiation. Simple trigonometry indicates that too large a horizontal distance will result in negative stack height. In this situation, a warning will indicate that the horizontal distance is to be reduced for a reasonable solution of the problem.

Example 4.14

Solve Example 4.8 for the simple approach.

Solution

The solution is shown in Figure 4.34. The result matches well with the manual calculation.



ESTIMATION OF FLARE STACK HEIGHT

Approach: Simple approach — Simple approach

INPUT DETAILS

Stack Tag number: Testing
Stack description: Testing

DESCRIPTION	UNIT	VALUE
Flow rate	kg/h	100000
Flowing temperature	C	100
Ambient temperature	C	N/A
Molecular weight		20.95
Compressibility factor		1
Ratio of specific heats		1.2
Flare tip pressure	kPaa	101.3
Maximum allowable radiation	kW/m^2	6.31
Horizontal distance, R	m	50
Design wind velocity	m/s	10
Lower explosive limit, CL	mole frac	N/A
Design Mach number		0.5
Heating value	MJ/kg	53.63
Radiation factor		0.15
Vertical flame centre, yc	m	22.541
Horizontal flame centre, xc	m	24.814

RESULTS

Heat release	W	14897222
Flame length, L	m	71.9
Flare stack diameter, d	m	0.496
Flare tip exit velocity	m/s	210.34
Flame distortion factor, U		0.048
Concentration parameter, CL'		0
Thrust parameter, djR		0
Minimum distance, D	m	53.1
Flare stack height, H	m	24.2

PROCEDURE

1. Enter project details through Project Details menu
2. Select proper unit (default unit is SI) and proper Approach
3. Enter values (frame INPUT DETAILS) except the values of yc and xc. Default values of yc and xc are zero and are used to estimate yc and xc. Use Tab key for data entry
4. Calculate and check back the calculated values of yc and xc from the available graphs. Enter new values of yc and xc (if required)
5. The values of yc and xc will not be calculated if any value is entered. Values of yc and xc will only be calculated if values are entered as zero.

Figure 4.34 Solution of Example 4.14.

Nomenclature

A	Area, m^2
d	Diameter, mm
C_D	Coefficient of discharge, dimensionless
D	Diameter, m
E	Modulus of elasticity for the metal, kPa
f	Moody friction factor
f_c	Circumferential stress, kPa
f_l	Longitudinal stress, kPa
F	Flow, Nm^3/h
F_a	Thermal expansion correction factor, dimensionless
G	Mass flow, kg/sec

h	Thickness, m
k	Ratio of specific heats
K	Number of velocity head
K_d	Coefficient of discharge, dimensionless
L	Length, m
$Mach$	Ratio of velocity to the sonic velocity
MW	Molecular weight
N_{Re}	Reynolds number
P_1	Upstream pressure, kpaA
P_2	Downstream pressure, kpaA
P_c	Critical pressure, kpaA
q	Leakage rate, m^3/sec
Q	Heat energy, kJ/h
r	Ratio of upstream and downstream pressure, P_1/P_2
r_c	Critical pressure ratio
S	Sonic velocity, m/sec
t	Time, sec
T	Temperature, K
T_c	Critical temperature, K
v	Velocity, m/sec
V	Volume, m^3
W	Mass flow, kg/h
Y	Expansion factor based on upstream pressure, dimensionless
Z	Compressibility

Greek characters

α	Coefficient of linear expansion of metal, 1/°C
β	Coefficient of cubic expansion of liquid, 1/°C
γ	Ratio of specific heats, dimensionless
ΔA	Change in area, m^2
ΔL	Change in length, m
Δp	Differential pressure, kPa
ΔP	Increase in pressure, kPa
ΔT	Increase in temperature, °C
ε	Roughness of pipe wall
ρ	Fluid density, kg/m^3
σ	Poisson's ratio, usually 0.3

References

1. API RP 520, Sizing, Selection, and Installation of Pressure-Relieving Devices in Refineries, Part-1 — Sizing and Selection, American Petroleum Institute, 7th ed., 2000.

2. API RP 520, Sizing, Selection, and Installation of Pressure-Relieving Devices in Refineries, Part-II — Installation, American Petroleum Institute, 4th ed., 1994.
3. API RP 521, *Guide for Pressure-Relieving and Depressuring Systems*, American Petroleum Institute, 4th ed., 1997.
4. ASME Section VIII, Rules for Construction of Pressure Vessels, Division 1, American Society of Mechanical Engineers, 2003.
5. ASME B31.3, Process Piping, American Society of Mechanical Engineers, 2002.
6. AS 1271, Safety Valves, Other Valves, Liquid Level Gauges and Other Fittings for Boiler and Unfired Pressure Vessels, Australian Standard, 1997.
7. McCabe, W.L. and Smith, J.C., *Unit Operations of Chemical Engineering*, 3rd ed., McGraw Hill, Tokyo, 1976.
8. Perry, R.H. and Green, D., *Perry's Chemical Engineers' Handbook*, 6th ed., McGraw Hill, Malaysia, 1984.
9. Lapple, C.E., Isothermal and adiabatic flow of compressible fluids, *Transactions of AIChE*, Vol. 39, 1943.
10. Mak, H.Y., New method speeds pressure relief manifold design, *Oil and Gas Journal*, November 20, 1978.
11. API Std 598, Valve Inspection and Testing, American Petroleum Institute, 8th ed., 2004.
12. Yaws, C.L., Calculate thermal-expansion coefficients, *Chemical Engineering*, August 1995.
13. API Std. 2000, Venting Atmospheric and Low-Pressure Storage Tanks, American Petroleum Institute, 5th ed., 1998.
14. AS 1940, The Storage and Handling of Flammable and Combustible Liquids, Australian Standard, 1993.
15. Fisher, H.G., DIERS research program on emergency relief systems, *CEP*, August, 33–36, 1985.
16. Fauske, H.K., Emergency relief system (ERS) design, *CEP*, August, 53–56, 1985.
17. Leung, J.C. and Grolmes, M.A., A generalized correlation for flashing choked flow of initially subcooled liquid, *AIChE*, April, 688–691, 1988.
18. Grolmes, M.A., Leung, J.C. and Fauske, H.K., Large-scale experiments of emergency relief systems, *CEP*, 57–62, 1985.
19. Fauske, H.K. and Leung, J.C., New experimental technique for characterizing runaway chemical reactions, *CEP*, August, 39–46, 1985.
20. Brzustowski, T.A. and Sommer, E.C., Jr., Predicting radiant heating from flares, *Proceeding — Division of Refining*, Vol. 53, 865–898, 1973.
21. BS 5345, Selection, Installation and Maintenance of Electrical Apparatus for Use in Potentially Explosive Atmospheres (Other than Mining Applications or Explosives Processing and Manufacture) Part 1: General Recommendations, 1989.
22. IEC 61508, Functional Safety of Electrical/Electronics/Programmable Electronic Safety-related Systems (Part 1 to Part 7).
23. IEC 61511, Functional Safety — Safety Instrumented Systems for the Process Industry Sector (Part 1 to Part 3).
24. ANSI/ISA — S84.01, Application of Safety Instrumented Systems for the Process Industries, 1996.

chapter five

Glycol dehydration

Introduction

The glycol dehydration unit is extensively used in the oil and gas industry to produce dehydrated gas. The saturated gas, if left without treatment, can cause major problems, including:

- Hydrate formation
- Water dropouts that can cause an excessively corrosive environment if CO_2 and H_2S are present
- Ice formation
- Excess water vapor that will reduce the heating value of the gas
- Two-phase flow (due to water dropouts) that can lead to pressure-drop problems

Normally, the gas is dehydrated to a moisture level of 65 mg/Sm^3 (sometimes, a higher value of 127 mg/Sm^3 is used). Two commonly used methods to dehydrate the gas are:

- Using solid desiccant: commonly used desiccants are silica gel, activated alumina, molecular sieve, etc.
- Using liquid desiccant: by far, the most widely used liquid desiccant is glycol.

A glycol dehydration unit is most commonly used in the industry because of the following advantages over solid desiccant:

- The glycol dehydration unit is comparatively cheaper to install.
- Easy to operate.
- Generates less greenhouse gas.
- Can dehydrate the gas to a very low water dew point.
- Requires less regeneration heat per kilogram of water removed from the saturated gas.

Basic scheme

The basic scheme is presented in Figure 5.1. The glycol dehydration unit can be installed either up- or downstream of the compressor. The following advantages/disadvantages need to be considered before finalizing the location.

Advantages

- High pressure requires a smaller unit (though design pressure can be considerably high). In general, a high-pressure unit is cheaper in comparison with a low-pressure unit. Sometimes, an intermediate pressure can also be considered.
- It is easy to dehydrate high-pressure saturated gas (high-pressure gas contains less amount of water in vapor phase).
- It is easy to operate and maintain a high-pressure dehydration unit in comparison with a low-pressure unit.

Disadvantages

- High-pressure operation has some typical problems such as more chances of gas leakage. More space is required to comply with the hazardous area requirement.
- In case an energy exchange pump is used, there are limits up to which a reliable energy exchange pump is available.

Pre-TEG coalescer

In general, the glycol dehydration unit is installed downstream of a compressor, and there will be a substantial quantity of lubricating oil mixed with the wet gas. It is preferable to remove the lubricating oil before the wet gas is sent to the contactor. This is done by installing a pre-TEG (triethylene glycol) coalescer. The bottom chamber of the coalescer generally collects the water and large oil droplets, and the top chamber collects coalesced oil droplets. The quantity of water or oil collection depends on factors such as availability of a discharge scrubber at the outlet of the compressor, the temperature to which the compressed gas is cooled down, the type of compressors, etc.

In case the quantity of water and oil coming to the coalescer is small, the control systems are designed for a gap control. Gap control means the control valve will open at some high liquid level and will close at some low level in the chamber.

Contactor

The contactor is designed as a reflux absorber where the wet gas is flowed from bottom to top and the reflux (lean glycol) flows from top to bottom.

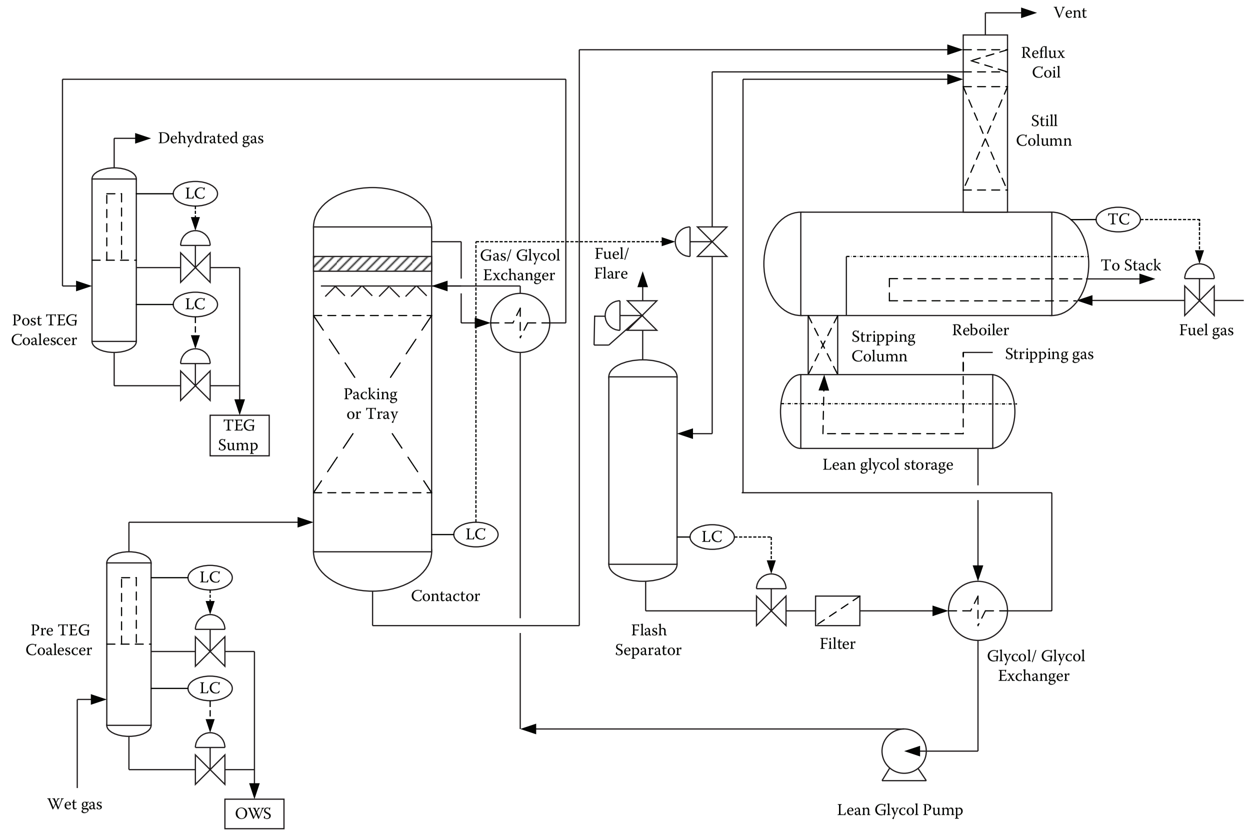


Figure 5.1 Basic glycol dehydration scheme.

There is no specific reboiler or condenser attached with the column. Lean glycol absorbs the moisture from the wet gas stream making the gas stream dry, and the glycol leaving the column containing water becomes rich glycol.

Bubble cap trays are commonly used as column internals with a trend to change the internals to structured packing. Use of structured packing reduces the column diameter substantially and, hence, is more economical.

Dehydrated gas leaving the contactor passes through a gas/glycol exchanger to reduce the reflux (lean-glycol) temperature. In general, the reflux temperature is maintained at 6 to 20°C above the dehydrated gas outlet temperature.

Flash separator

Generally, the contactor is operated at a substantially high pressure, whereas the regeneration system works at atmospheric pressure. A flash separator is used to reduce the pressure of rich glycol to approximately 250 to 500 kPaG. The flash separator needs to be maintained at a pressure required to transfer the liquid from the separator vessel to the regenerator column. Sometimes, the rich glycol is passed through a condensing coil in the regenerator column. The condensing coil condenses part of the hot gas and provides reflux to the regenerator column.

Hot, rich glycol at reduced pressure generates flash gas that can be used as a fuel gas to the regenerator reboiler or as a stripping gas or for both. Excess flash gas is normally vented to the flare system. The flash separator pressure is maintained through a back-pressure control valve on the gas outlet line from the separator. The liquid flow is maintained through a level-control valve.

Filters

Generally, one glycol filter and one activated carbon filter are installed in series on the rich-glycol line from the flash separator. These filters are required to remove inert particles from the system as well as some oil mist that escapes the pre-TEG coalescer.

Pumping

Lean glycol that is used as reflux is available at atmospheric pressure, and the pressure has to be increased to the column pressure. The following methods are commonly used to increase the pressure:

- Use of electric motor driven pump
- Use of energy exchange pump

An electric motor driven pump is simple in design and can be used only in places where power is available. Sometimes, the dehydration units are remotely placed and with practically no power. In such cases, the use of an electric motor driven pump may be difficult, though not impossible.

Energy exchange pumps are frequently used for this type of dehydration unit. One side of the pump reduces the high contactor pressure to the separator pressure, and the other side of the pump uses the energy to boost the lean-glycol pressure to the contactor pressure. A slip gas stream is required from the contactor to operate the energy exchange pump. These pumps are quite small in size and require less space. However, at very high contactor pressures (above 12,000 kPaG), these pumps are not very reliable.

Glycol/glycol exchanger

A glycol/glycol exchanger is used to reduce overall energy requirements. In this exchanger, the temperature of rich glycol is increased by exchanging heat with the high-temperature lean glycol from the regenerator reboiler. Generally, a shell and tube or plate type or pipe in pipe exchanger is used for this purpose. Sometimes, a coil in the lean-glycol storage tank is used to heat the rich-glycol stream. The heating coil is generally less efficient and requires careful design consideration.

Generally, the glycol/glycol exchanger is placed upstream of the lean-glycol pump. This reduces the glycol temperature at the pump inlet and thus increases the available net positive suction head (NPSH) of the pump.

Gas/glycol exchanger

This exchanger is used to control the reflux temperature (lean glycol) of the contactor. Generally, lean glycol entering the contactor is heated up to 6 to 20°C above the contactor temperature [1]. This is required to maintain a proper equilibrium between the glycol and water vapor. Generally, a shell and tube or pipe in pipe exchanger is used for this purpose. This heat exchanger exchanges heat between the dry gas from the contactor and the hot lean glycol; as a result, the temperature of the dehydrated gas increases. Because the flow of dehydrated gas is much more than the flow of the lean glycol, the temperature increase of the dehydrated gas is not substantial.

Regenerator

A typical regenerator consists of the following parts:

- Regenerator column (still column)
- Reboiler
- Lean-glycol storage
- Reflux coil
- Stripping coil (if required)

The main purpose of the regenerator is to drive the absorbed water from the rich glycol and increase the glycol concentration. The required lean-glycol concentration depends on the process parameters, and sometimes, stripping gas and a stripping column are required to achieve high lean-glycol concentration. Concentration of lean glycol depends on the reboiler temperature and the design of the stripping column. Most commonly, the reboiler temperature is maintained at 204°C, and the achievable lean-glycol concentration is approximately 98.5 wt%. Sometimes, to maintain higher dehydration specifications a higher lean-glycol concentration is required, and that is achieved by using stripping gas and a stripping column.

The still column is generally packed with random packing (structured packing can also be used for larger units) with one or two equivalent theoretical stages. One reflux coil is attached at the top section of the column, and the rich glycol from the contactor is used to condense some vapor in the column. This is done to generate some reflux for the proper performance of the column.

In case a high lean-glycol concentration is required (higher than 98.5 wt%), stripping gas and a stripping column are used. Any inert gas (e.g., nitrogen) or fuel gas can be used as a stripping gas. If fuel gas is used as a stripping gas, proper hazardous area requirements are to be met. The stripping column is a packed column with one or two theoretical stages, depending on the concentration required.

To achieve required reboiler duty, the reboiler is either designed using a fire tube design or hot recirculation fluid is used. In case of fire tube design, care should be taken to maintain proper fire tube surface temperature to avoid glycol degradation as well as to achieve proper heat flux. Sometimes, the lean-glycol storage tank is built as a part of the reboiler (separate compartment), and sometimes as a separate vessel. Both designs have their own advantages and disadvantages that need to be considered carefully.

Physical properties

Selection of type of glycol

Physical properties of different glycol components are important in selecting the type of glycol used for a particular design. Although triethylene glycol is typically widely used for glycol dehydration design, other types of glycol can also be used, depending on the requirement. The following Table 5.1 gives a general guideline of the use of a particular glycol component [2].

The reboiler can be operated at a temperature close to the decomposition temperature, without significant decomposition. Depending on the requirement, any type of glycol can be selected based on Table 5.1.

Common properties of glycol

The common properties of glycol are presented in Table 5.2.

Table 5.1 General Design Parameters

Glycol	Decomposition Temperature (°C)	Lean Glycol Concentration (wt%)[a]	Equilibrium Water Dew Point at 38°C (°C)[b]
Ethylene glycol	165	96	3
Diethylene glycol	164	97.1	3
Triethylene glycol	206	98.7	–8
Tetraethylene glycol	238	99	–18

[a] Equilibrium concentration at decomposition temperature and 1 atmospheric pressure.

[b] At concentration in column at left.

Table 5.2 Common Properties of Glycol

Description	Ethylene Glycol	Diethylene Glycol	Triethylene Glycol	Tetraethylene Glycol
Formula	$C_2H_6O_2$	$C_4H_{10}O_3$	$C_6H_{14}O_4$	$C_8H_{18}O_5$
Molecular weight	62.07	106.12	150.17	194.32
Freezing point, °C	–13.4	–8.7	–7.2	–9.4
Boiling point @ 760 mmHg, °C	197.4	245.5	287.8	325.6
Density at 25°C, kg/m^3	1110	1111	1120	1123
Density at 60°C, kg/m^3	1085	1087	1093	1096
Pour point, °C	≤59	–54	–58	–41
Flash point (open cup)	115.6	143.3	165.6	176.7
Viscosity at 25°C, cP	16.9	25.3	39.4	43.0
Viscosity at 60°C	5.2	7.3	10.3	11.4
Surface tension at 25°C, dyn/cm	48	44	45	—
Refractive index at 25°C	1.430	1.446	1.454	1.457
Solubility in water	Complete	Complete	Complete	Complete
Specific heat at 25°C, cal/gm/°C	0.58	0.55	0.52	0.52
Coefficient of expansion ×10^4 (0–60°C)	6.5	6.6	7.2	7.3
Thermal conductivity at 25°C, kJ/(h·m °C)	0.928	0.732	0.706	0.689
Heat of formation, kcal/g-mol	–93	–131	–171	–212
Vapor pressure at 20°C, mmHg	<0.1	<0.01	<0.01	<0.01

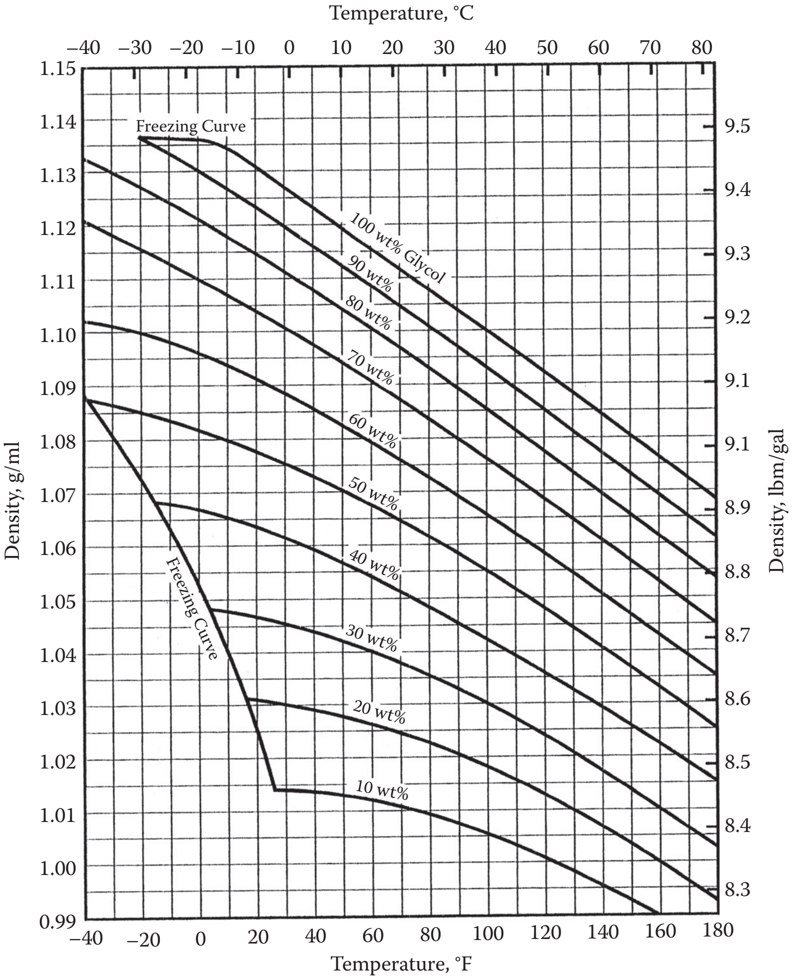


Figure 5.2A Densities of aqueous ethylene glycol solutions. (Courtesy of John M. Campbell.)

Densities of aqueous glycol solutions

Densities of aqueous glycol solutions are presented in Figure 5.2A–D [2].

Solubility of various compounds

Solubility of various compounds in glycol is presented in Table 5.3. Values presented are wt% in glycol at 25°C.

Fire hazard information

Fire hazard information, e.g., flammable limit, autoignition temperature, etc., is presented in Table 5.4.

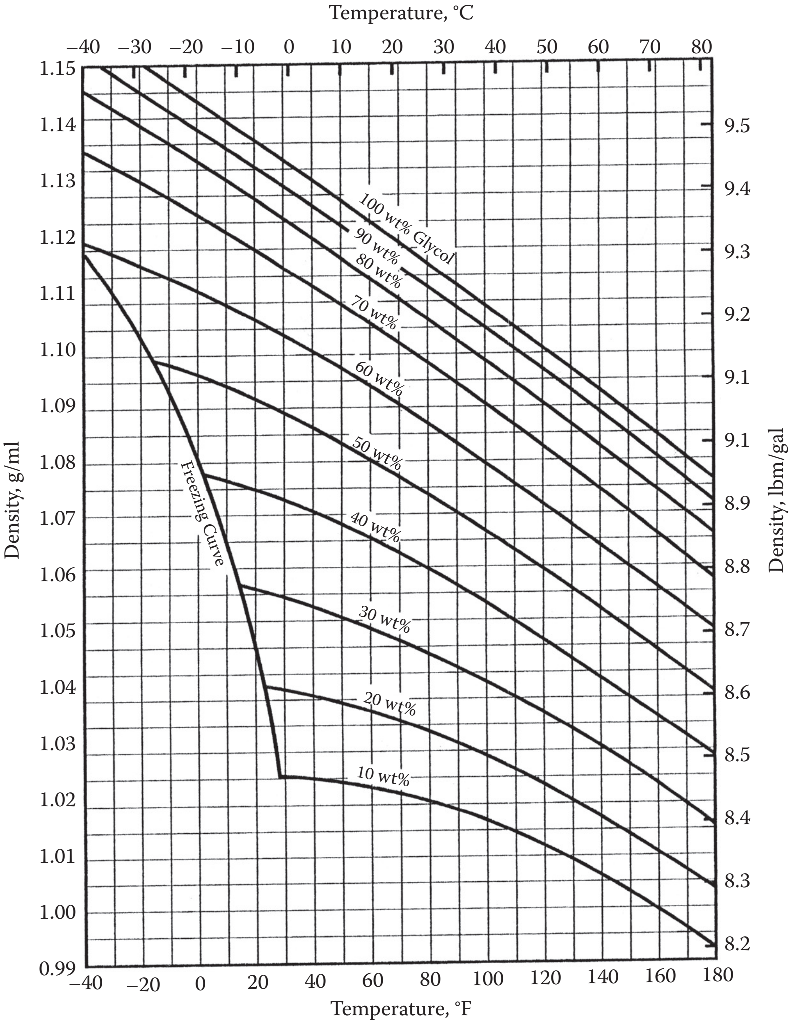


Figure 5.2B Densities of aqueous diethylene glycol solutions. (Courtesy of John M. Campbell.)

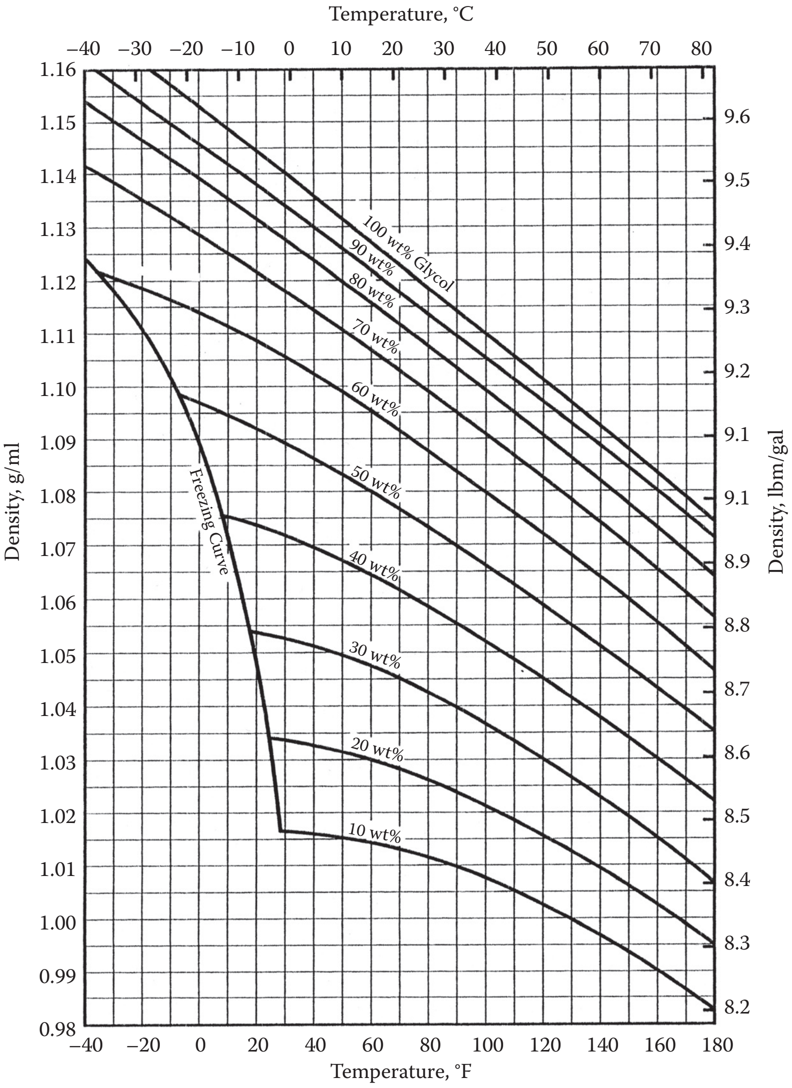


Figure 5.2C Densities of aqueous triethylene glycol solutions. (Courtesy of John M. Campbell.)

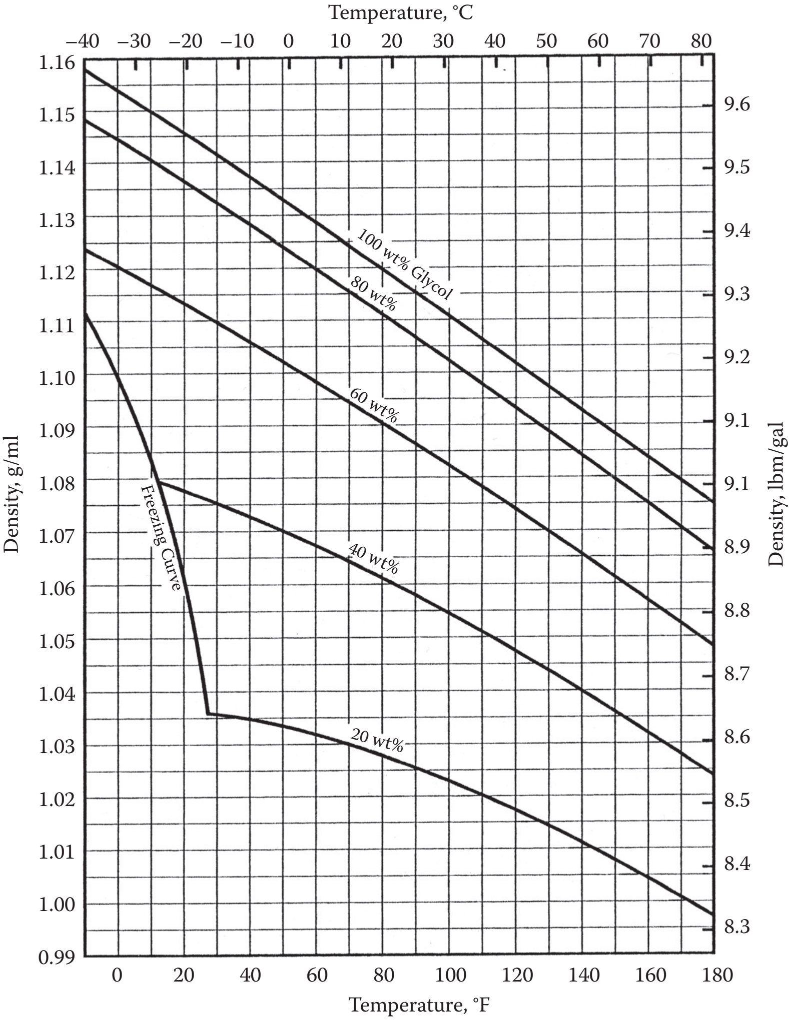


Figure 5.2D Densities of aqueous tetraethylene glycol solutions. (Courtesy of John M. Campbell.)

Table 5.3 Solubility of Various Compounds

Component	Ethylene Glycol	Diethylene Glycol	Triethylene Glycol	Tetraethylene Glycol
Benzene	5.7	31.3	S	S
Carbon tetrachloride	6.2	26.2	33.6	62
Dibutyl phthalate	0.5	10.6	16.5	S
Ethyl alcohol	S	S	S	S
Ethyl ether	8.2	16.3	16.9	20
Methyl alcohol	S	S	S	S
Methyl isobutyl carbinol	S	S	S	S
Methyl isobutyl ketone	12	S	S	S
Monochlorobenzene	5.7	S	S	S
Monoethanolamine	S	S	S	S
Ortho-dichlorobenzene	4.5	48.4	S	S
Perchloroethylene	0.7	10.7	15	19
Phenol	S	S	S	S
Styrene	3.4	36	S	S
Toluene	2.9	17.2	24.8	89
Urea	48	30	37	28
Castor oil	1	<0.5	<0.5	<1
Coconut oil	1	1	1	<1
Cottonseed oil	1	1	1	<1
Lard oil	1	1	1	<1
Linseed oil	1	1	1	<1
Olive oil	1	1	1	<1
Pine oil	S	S	S	S
Soya bean oil	1	1	1	<1
Paraffin oil	1	1	1	<1

Note: S = completely soluble.

Table 5.4 Fire Hazard Information

Glycol	Flammable Limits (vol%)		Autoignition Temperature (°C)[a]	NFPA[b] Hazard Identification			Flash Point (°C)
	Lower	Upper		Health	Flammability	Reactivity	
Ethylene glycol	3.2	—	397.8	1	1	0	119
Diethylene glycol	—	—	223.9	1	1	0	138
Triethylene glycol	0.9	9.2	371.1	1	1	0	163
Tetraethylene glycol	—	—	—	1	1	0	204

[a] In presence of air.

[b] NFPA — National Fire Protection Association.

Viscosities of aqueous glycol solutions

Viscosities of aqueous glycol solutions are presented in Figure 5.3A–D [2].

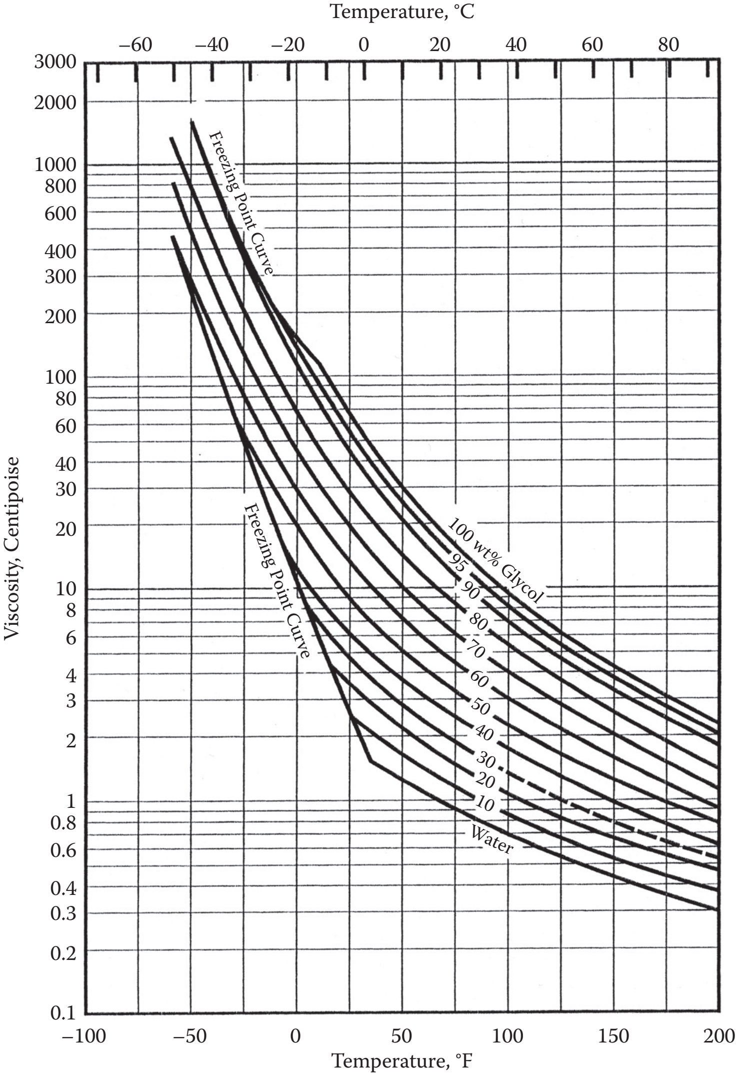


Figure 5.3A Viscosities of aqueous ethylene glycol solutions. (Courtesy of John M. Campbell.)

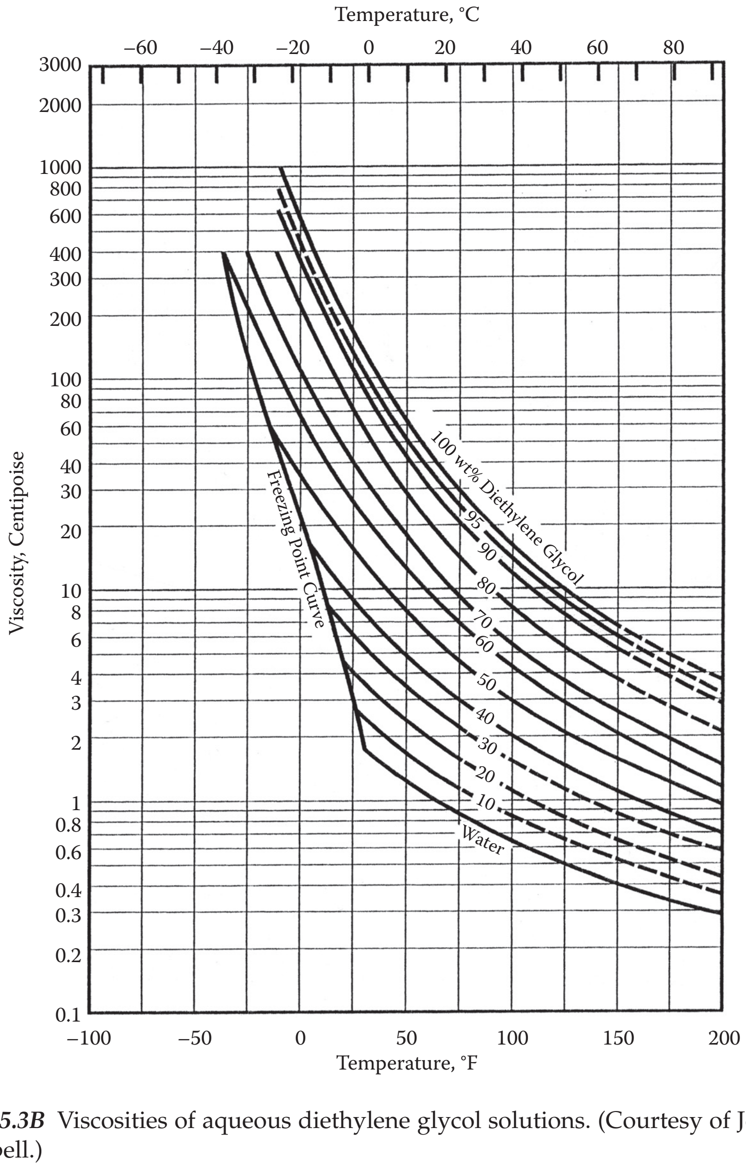


Figure 5.3B Viscosities of aqueous diethylene glycol solutions. (Courtesy of John M. Campbell.)

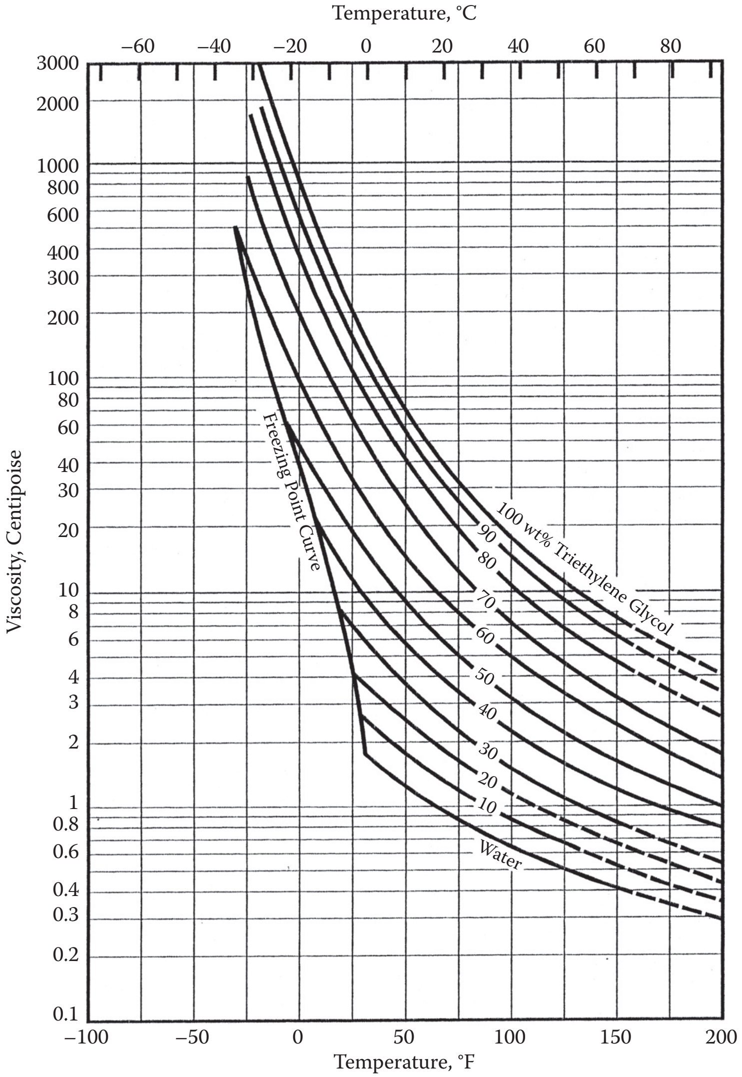


Figure 5.3C Viscosities of aqueous triethylene glycol solutions. (Courtesy of John M. Campbell.)

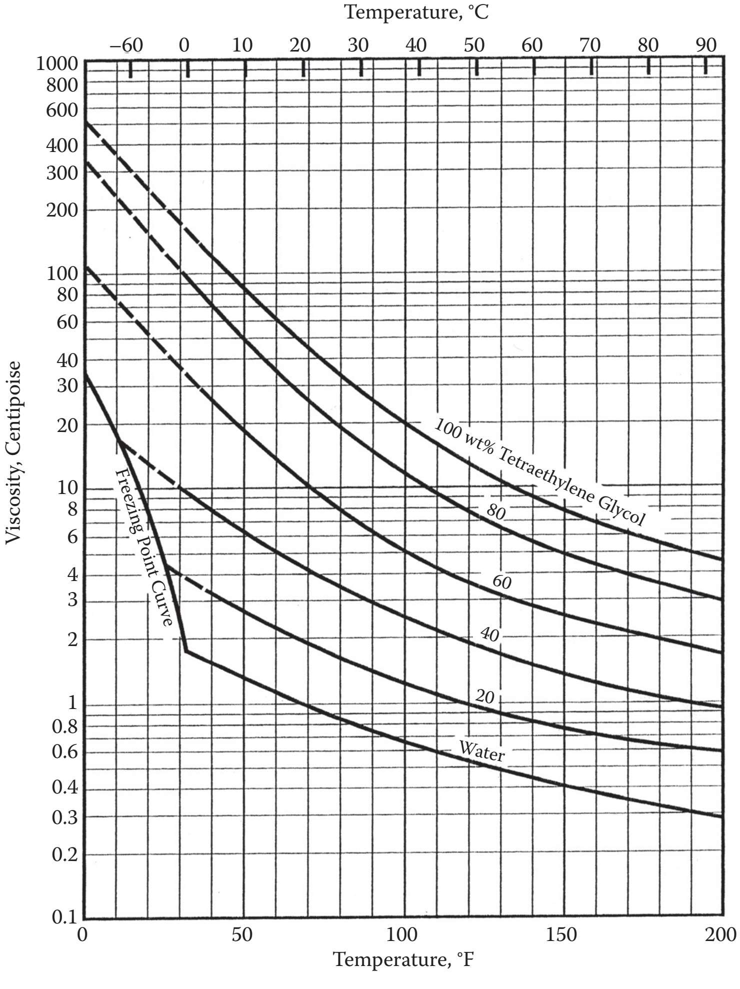


Figure 5.3D Viscosities of aqueous tetraethylene glycol solutions. (Courtesy of John M. Campbell.)

Specific heats of aqueous glycol solutions

Specific heats of aqueous glycol solutions are presented in Figure 5.4A–C [3].

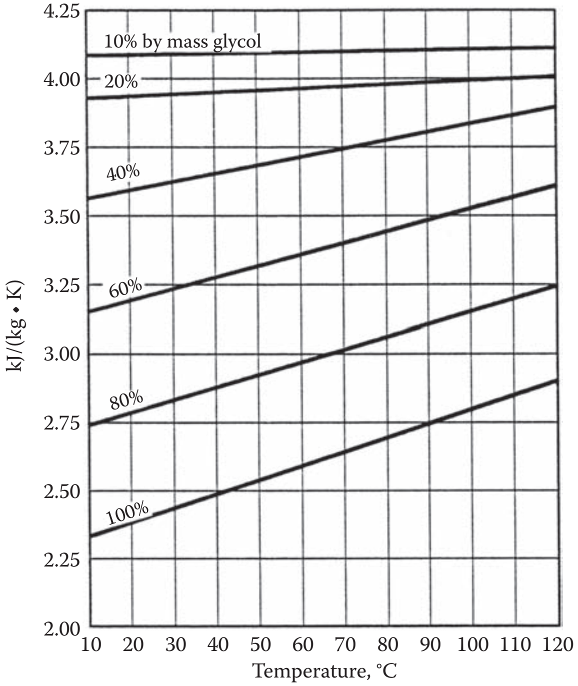


Figure 5.4A Specific heats of aqueous ethylene glycol solutions. (Courtesy Gas Processing Supplier Association.)

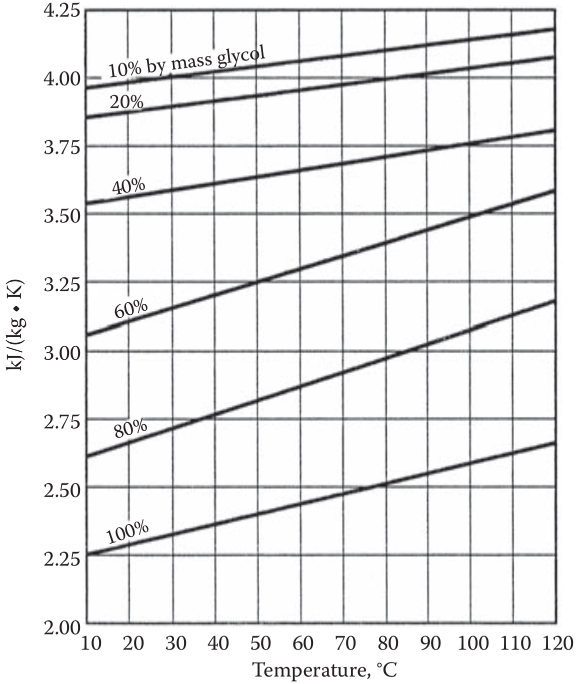


Figure 5.4B Specific heats of aqueous diethylene glycol solutions. (Courtesy Gas Processing Supplier Association.)

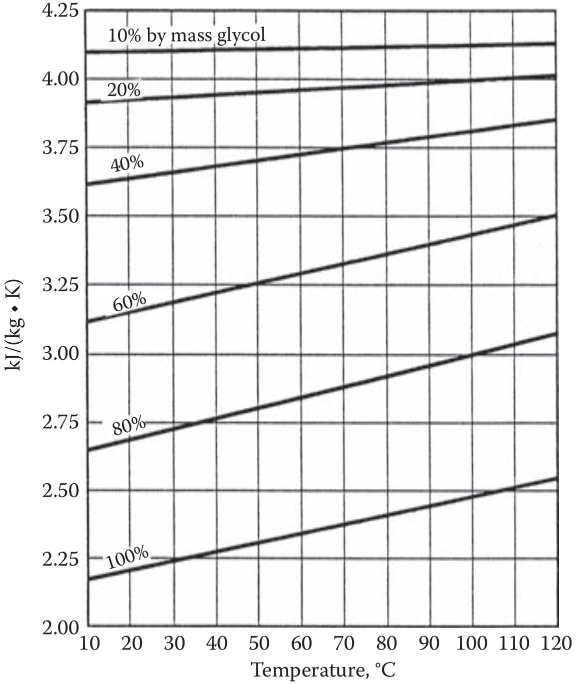


Figure 5.4C Specific heats of aqueous triethylene glycol solutions. (Courtesy Gas Processing Supplier Association.)

Thermal conductivities of aqueous glycol solutions

Thermal conductivities of aqueous glycol solutions are presented in Figure 5.5A–C [3].

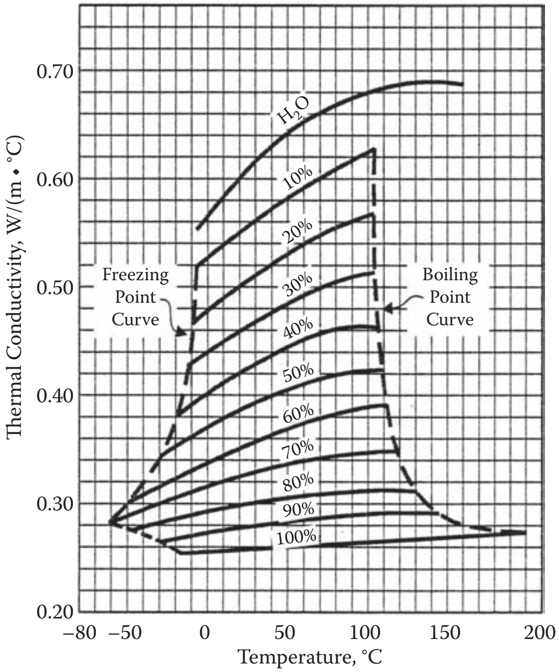


Figure 5.5A Thermal conductivities of aqueous ethylene glycol solutions. (Courtesy Gas Processing Supplier Association.)

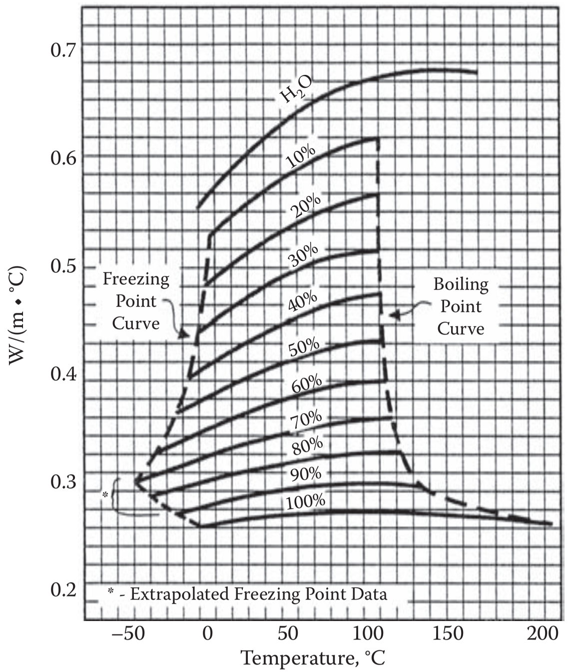


Figure 5.5B Thermal conductivities of aqueous diethylene glycol solutions. (Courtesy Gas Processing Supplier Association.)

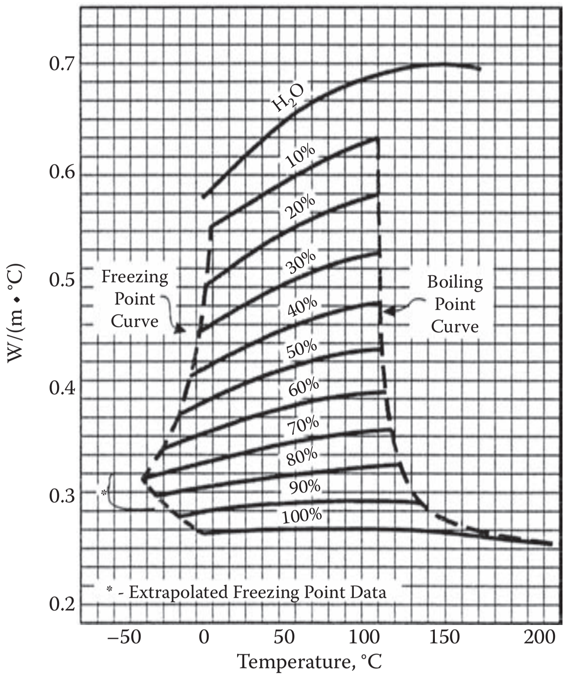


Figure 5.5C Thermal conductivities of aqueous triethylene glycol solutions. (Courtesy Gas Processing Supplier Association.)

Design aspects

Though all types of glycols can be used in the design of dehydration units, the most commonly used glycol, for the dehydration of natural gas, is ethylene glycol. The discussions of design aspects will be based on the use of ethylene glycol.

Water content in hydrocarbon gas

It is essential to estimate correctly the saturated water content in the incoming natural gas (generally expressed in mg/Sm^3). This can be estimated by using simulation software or it can be calculated using equation of states. The most commonly used method to estimate the saturated water content is to use a saturated water content chart developed by McKetta and Wehe as presented in Figure 5.6 [3].

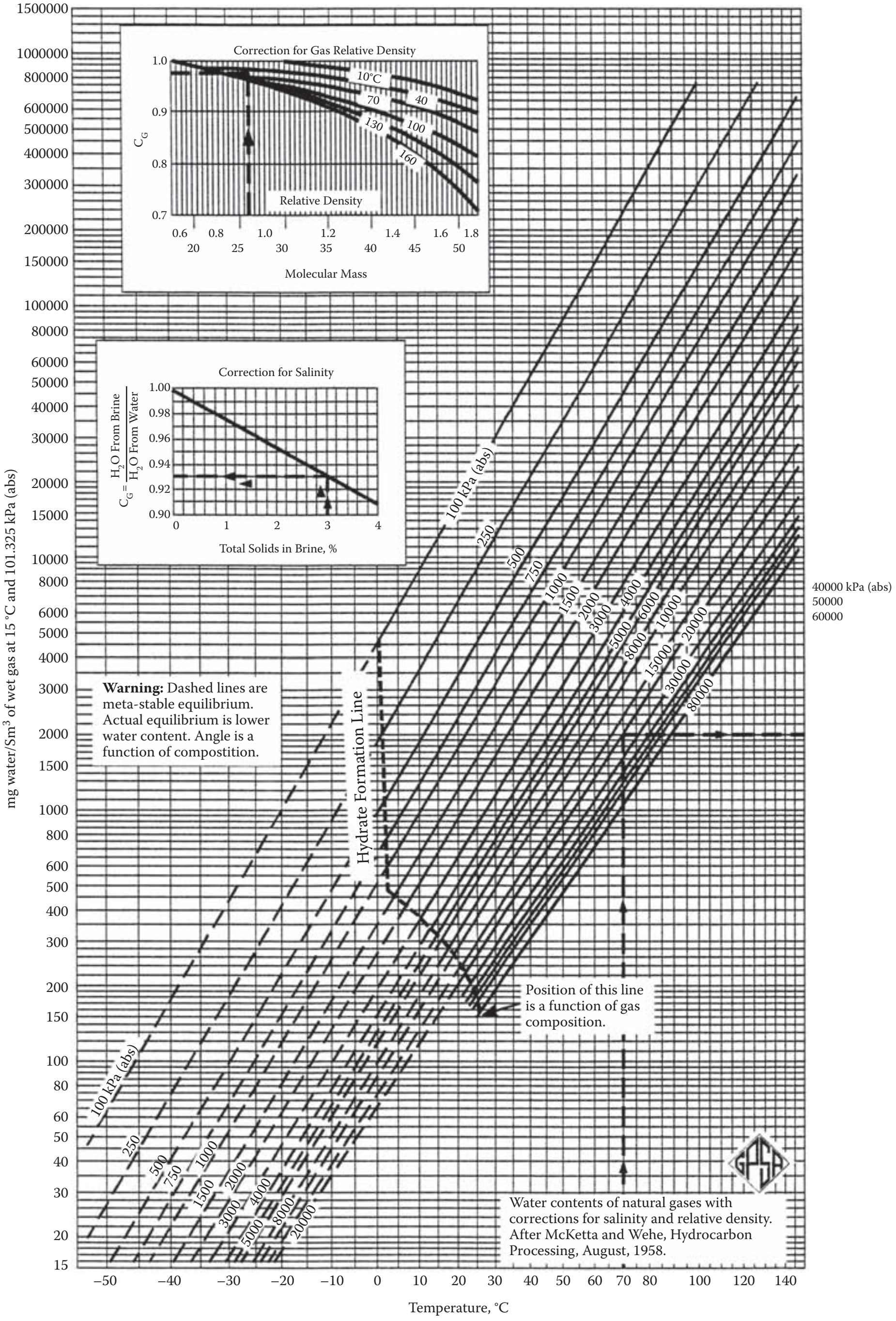
Equilibrium dew point

Equilibrium dew point of the dehydrated gas is an important design parameter for the design of the dehydration system. In general, the moisture content of the dehydrated gas varies between 65 and 127 mg/Sm^3. The equilibrium dew point is estimated as per the methods discussed earlier or by using Figure 5.6.

Equilibrium dew point calculated as in the preceding text may not be a good design parameter. In general, it is easy to achieve equilibrium dew point in a test cell, but in the actual absorber, the gas–liquid contact time may not be high enough to achieve the equilibrium dew point. Due to this reason, an approach temperature of 5 to 10°C is maintained. This approach temperature reduces the design dew point temperature and results in a more effective design. The lean-glycol concentration is a function of equilibrium dew point and contactor temperature. Sometimes, the equilibrium dew point is not low enough to achieve the required lean-glycol concentration. The problem of lean-glycol concentration can be eliminated easily by adding an approach to the calculated equilibrium dew point.

Minimum lean-TEG concentration

Lean TEG is used as a reflux to the contactor, and the water dew point of the dehydrated gas depends on the concentration of the lean TEG. With the increase in lean-TEG purity, the equilibrium dew point of the dehydrated gas is reduced. The minimum TEG concentration required for a specified dehydration depends on the contactor temperature and equilibrium dew point. Achieving the required lean-TEG concentration depends on the design of the regenerator. Normally, a 98.5 wt% lean-TEG concentration can be achieved if the regenerator reboiler is operated at 204°C. Generally, the following Figure 5.7 is used to establish the minimum lean-TEG concentration [3].



Warning: Dashed lines are meta-stable equilibrium. Actual equilibrium is lower water content. Angle is a function of compostition.

Position of this line is a function of gas composition.

Water contents of natural gases with corrections for salinity and relative density. After McKetta and Wehe, Hydrocarbon Processing, August, 1958.

Figure 5.6 Water content of hydrocarbon gas. (Courtesy Gas Processing Supplier Association.)

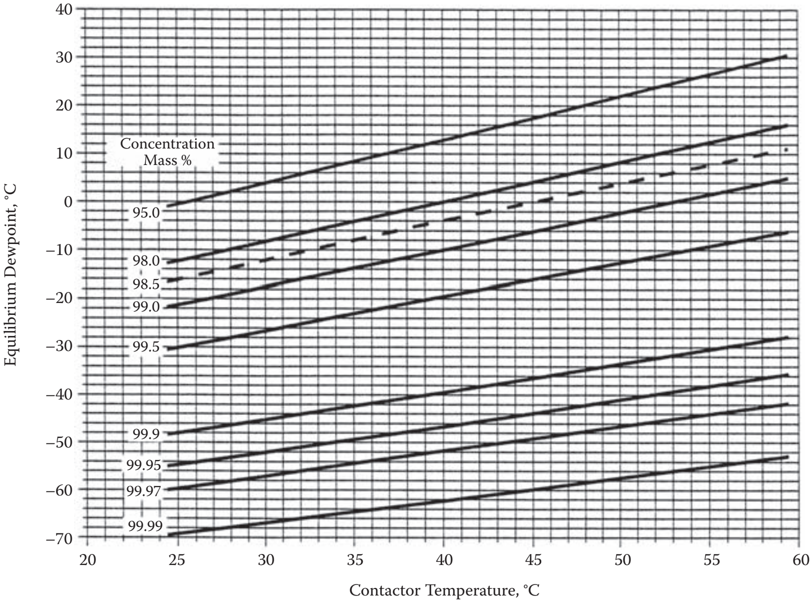


Figure 5.7 Minimum lean TEG concentration. (Courtesy Gas Processing Supplier Association.)

Number of theoretical stages of the contactor

The number of theoretical stages required depends on the reflux rate, lean-TEG concentration, and the water removal requirements. Water removal is defined as

$$W_r = (W_{in} - W_{out})/W_{in} \tag{5.1}$$

where

W_r = water removal
W_{in} = water content of the saturated gas at contactor inlet in mg/Sm^3
W_{out} = water content of the dehydrated gas at the contactor outlet in mg/Sm^3

Reflux rate or lean-glycol recirculation rate is expressed as m^3 TEG per kg of water removed. Generally, a lean-glycol recirculation rate of 0.025 to 0.04 m^3 TEG per kg of water removed is a reasonable value for design. Design of a contactor is a balance of the following design parameters:

- Water removal
- TEG circulation rate
- Number of theoretical stages
- Concentration of lean glycol

Figure 5.8A–E [3] is used to optimize a contactor design.

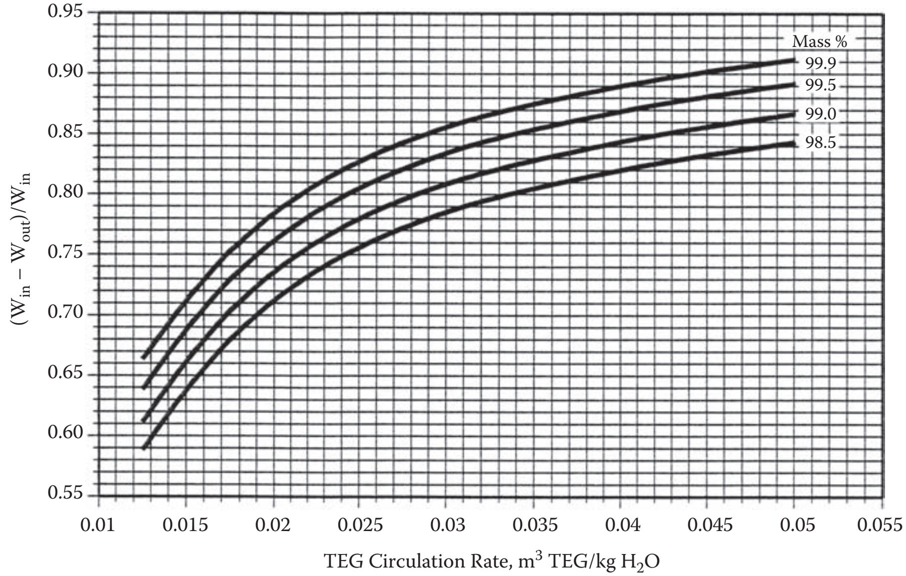


Figure 5.8A Water removal vs. TEG circulation rate (theoretical stage = 1). (Courtesy Gas Processing Supplier Association.)

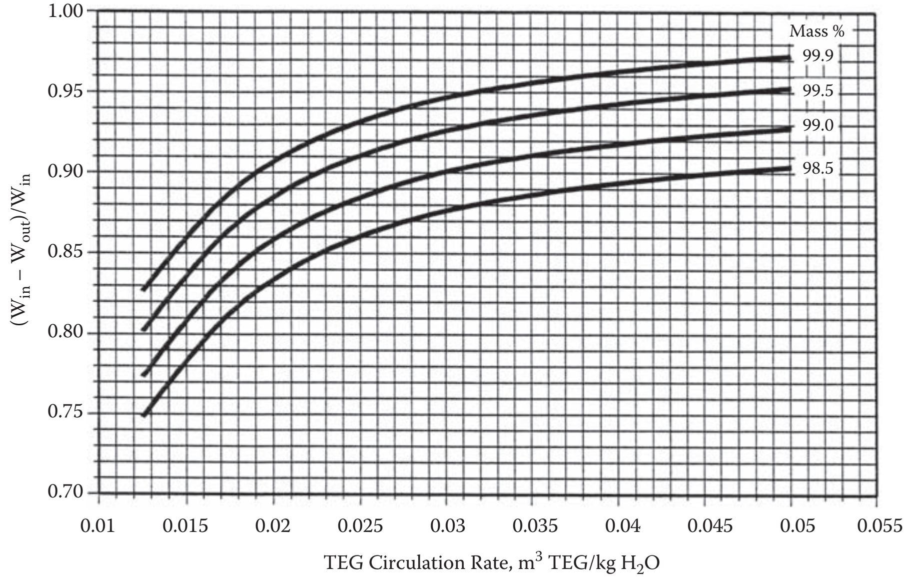


Figure 5.8B Water removal vs. TEG circulation rate (theoretical stage = 1.5). (Courtesy Gas Processing Supplier Association.)

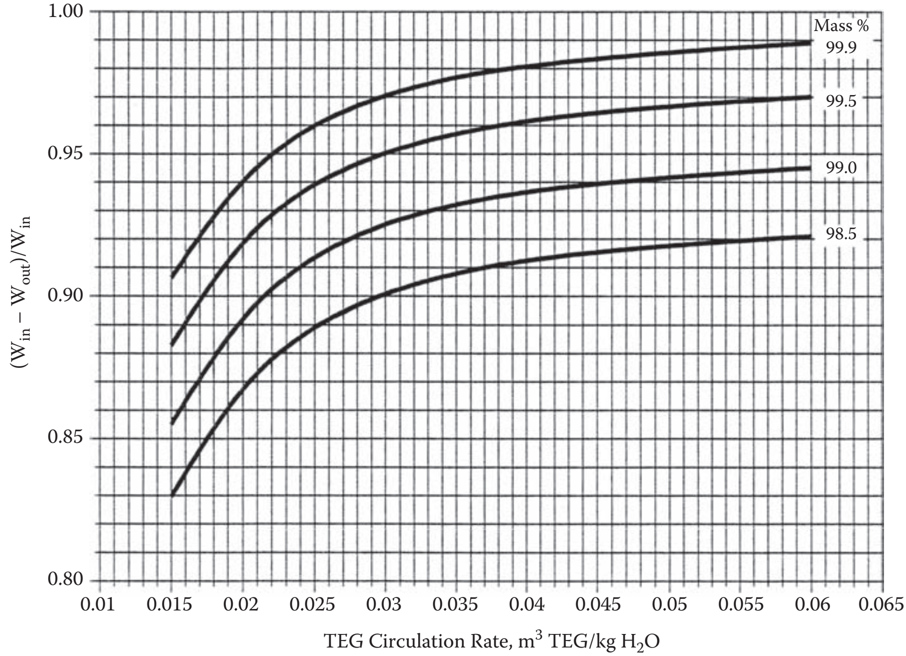


Figure 5.8C Water removal vs. TEG circulation rate (theoretical stage = 2). (Courtesy Gas Processing Supplier Association.)

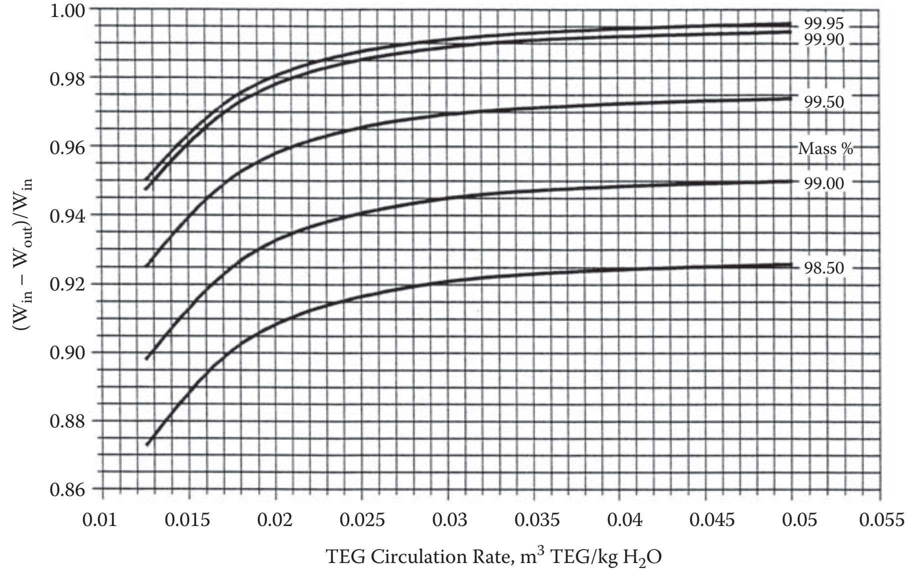


Figure 5.8D Water removal vs. TEG circulation rate (theoretical stage = 2.5). (Courtesy Gas Processing Supplier Association.)

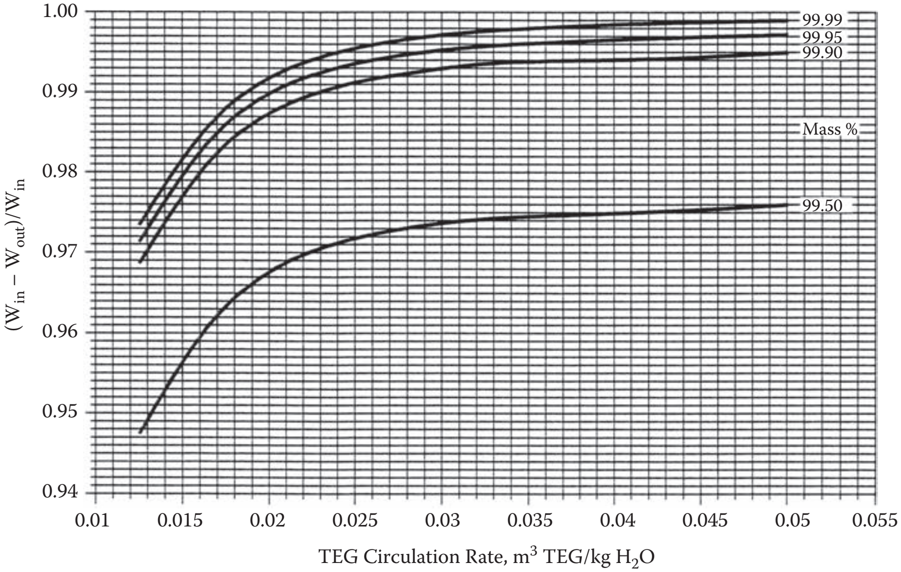


Figure 5.8E Water removal vs. TEG circulation rate (theoretical stage = 3). (Courtesy Gas Processing Supplier Association.)

Design of contactor

In general, design of the contactor is largely involved in the design and selection of the following items:

- Type of internals
- Liquid distributor

Type of internals

The diameter and height of the contactor depend on the type of internals. Generally, bubble cap tray or structured packing is used as contactor internals. The following methods are used in the estimation of contactor diameter [3]:

The gas loading rate is estimated as

$$G = C\{\rho_v(\rho_L - \rho_v)\}^{0.5} \tag{5.2}$$

where

G = gas loading, kg/(m^2·h)
ρ_v = actual density of the inlet vapor, kg/m^3
ρ_L = density of lean TEG (reflux), kg/m^3
C = design factor and is defined as follows:

Type of Internals	C Factor (m/h)
Bubble cap trays	
500-mm spacing	154
600-mm spacing	176
750-mm spacing	187
Packing	
Structured	329–439
25-mm Pall rings	143–198
50-mm Pall rings	208–285

The preceding C value for structured packing depends on the type of packing. A value of 329 is used for a structured packing of 125Y or equivalent. A higher value of C can be used for denser structured packing.

The cross-section area of the contactor is estimated as

$$A = m/G \tag{5.3}$$

where

A = cross-section area of the contactor
m = mass flow of inlet gas, kg/h

The diameter of the contactor is

$$d = \left(\frac{4A}{\pi}\right)^{0.5} \tag{5.4}$$

where

d = diameter of contactor, m

The actual number of trays is generally estimated by using a stage efficiency between 25 to 30% for bubble cup trays. For most engineering calculations, a tray efficiency of 25% is used [2].

Height equivalent to theoretical plates (HETP) is usually a function of the type of packing, and surprisingly less dependent on the fluid flow within the column. For structured packing, a HETP of 1.5 m is commonly used.

Once the packing height or tray spacing is known, the total height of the contactor can be calculated easily. A spacing of 2.5 m is used to estimate tan to tan height of the contactor.

Example 5.1

Estimate the diameter of a contactor for the following design parameters:

- Flow rate = 50,000 Sm^3/h
- Inlet pressure = 8,000 kPaG
- Inlet temperature = 50°C
- Molecular weight = 16.8
- Compressibility = 0.9
- Density of lean TEG = 1,108 kg/m^3
- Type of packing = structured 125 Y

Solution

Density of inlet gas = 16.8/22.4 * (8101.3/101.3) * (273/323)/0.9
= 56.33 kg/m^3
Standard density of gas = 16.8/22.4 * 273/288
= 0.711 kg/Sm^3
Gas mass flow = 50,000 * 0.711
= 35,550 kg/h
Design C value = 329
Gas loading rate = 329 * $\{56.33 * (1108 - 56.33)\}^{0.5}$
= 80,077 $kg/(m^2 \cdot h)$
Contactor cross-section area = 35,550/80,077
= 0.444 m^2
Contactor diameter = 0.752 m

Liquid distributor. Liquid distribution for packing is very important, and improper liquid distribution is probably the main cause of contactor underperformance. The liquid (reflux) needs to be distributed evenly throughout the packing surface. Both spray-type and gravity-type distributors can be used. Generally, the spray-type distributor is used for heat transfer duty, and the gravity-type distributor is used for mass transfer duty. The main reason to use the gravity-type distributor for mass transfer is that the gravity type distributor results in a more uniform distribution and thus achieves better efficiency.

Spray-type distributor. It can be designed using the following criteria:

- Pressure drop through the spray nozzle — 80 to 150 kPa
- Spray angle (typical) — 90°
- Coverage ratio (coverage ratio is the ratio of the spray area to the cross-section area of the contactor) — 2
- Distributor height (from top of packing) — 0.2 to 0.4 m

Gravity-type distributor. This is generally designed using the following criteria [2]:

- The distributor must be level.
- Weir openings or drip tubes should not be plugged.
- Drip density should be 80 to 100/m^2.

- Minimum glycol circulation rate should be about 1 $m^3/m^2/h$.
- Area available for gas flow should be a minimum 40 to 50% of cross-sectional area.
- Drip points should be less than 10 mm from the top of the packing.
- The distance between the top of the distributor and the mist eliminator should be a minimum of 0.6 m.

One typical problem with gravity-type distributors is low turndown ratio. However, in the operation of the dehydration unit the liquid and vapor traffic are independent to each other. The liquid flow can be maintained close to the design flow rate even if the vapor flow is reduced substantially.

Flash separator

Depending on the fluid compositions, either a two- or three-phase separator can be used. Normally, the flash separator should not have any hydrocarbon liquid; however, due to poor upstream separation, the separator may have a three-phase fluid. A two-phase separator is adequate in most cases, though, depending on the requirements, a three-phase separator is also used. In case of a three-phase separator, the glycol–water is fed to the regenerator column, and the hydrocarbon phase is drained to the oily water system.

Liquid retention is another important parameter in the design of the flash separator. Generally, a 20-min retention time is used for the separation of glycol–water from the hydrocarbon liquid or from the gas phase. In most cases, the gas outflow from the separator is not significant, and the sizing of the separator is dictated by the liquid retention time. The procedure outlined in Chapter 3 can be used for the design of the separator using the program separator.exe.

The following points should be considered for the design of the flash separator:

- Generally, a two-phase separator is adequate, though a three-phase separator is not uncommon.
- When an energy exchange pump is used, the separator is maintained at approximately 15% of the contactor pressure. A higher separator pressure may have some problem in the flow of lean glycol (reflux) to the contactor.
- If CO_2 and H_2S are present in the system, this will result in a corrosive environment and corrode the regenerator. Degassing the flash separator helps mitigate the corrosion effect in the regenerator.
- The flash separator can be designed using the program separator.exe.

Filters

Filters are normally installed downstream of the flash separator. Generally, particulate and carbon filters are used in series.

Particulate filter

The particulate filter is used to remove solid impurities from the system. In glycol dehydration systems solid impurities cause the fouling of the pump, contactor trays (packing), regenerator packing, etc. Properly designed particulate filters can reduce fouling problems substantially. Generally, the solid impurity concentration is kept below 0.01 wt%. A maximum glycol flow of 11.5 l/min per 900-mm-long element is used as a basis for design.

Carbon filter

This is used to remove dissolved impurities, particularly high-boiling hydrocarbons, surfactants, compressor lubricants, etc. These components cause major foaming problems in the contactor. Generally, the carbon filter is placed downstream of the particulate filter to avoid filter blockage by particulates. Four kilograms of activated carbon per liter of glycol per minute is used as a basis for design.

Glycol/glycol exchanger

Lean glycol is available from the reboiler at around 204°C and needs to be cooled down before pumping back to the contactor. The glycol/glycol exchanger is designed to cool the lean glycol and heat the rich glycol (from the contactor). Generally, through the glycol/glycol exchanger, the temperature of the rich glycol can be increased to approximately 150°C. Depending on the exchanger duty, the following types of exchangers can be used:

- Coil into the hot lean-glycol storage vessel: This type of coil has a very poor heat transfer coefficient, though the design and fabrication are simple.
- Double pipe exchanger: This type of design is more efficient than the coil design and is more expensive than the previous design.
- Shell and tube or plate and frame design: This type of design is the most efficient one and can be effected with very little temperature approach. With this design, the energy recovery can be maximized, and the reflux pump can be operated at highest possible NPSH.
- The recommended overall fouling factor for the glycol/glycol exchanger is 0.0015 m^2 K/W.

Gas/glycol exchanger

The gas/glycol exchanger can be of several types, namely, double pipe exchanger, shell and tube exchanger, plate type exchanger, etc. The main purpose of the gas/glycol exchanger is to have a controlled reflux temperature. As discussed earlier, for proper design, a reflux temperature typically 6 to 20°C higher than the contactor gas inlet temperature is required for efficient operation of the contactor. In general, glycol/glycol and gas/glycol exchangers are designed with consideration of total energy savings. The recommended overall fouling factor for gas/glycol exchangers is 0.0015 m^2 K/W. Energy savings can be maximized with a larger glycol/glycol exchanger. However, larger exchangers are sometimes financially uneconomical, and a cost-benefit analysis is required. Also, with high energy recovery, the temperature of lean glycol (reflux) can be reduced substantially and will result in inefficient contactor operation. The following procedure can be used for the design of glycol/glycol and gas/glycol exchangers:

- Rich-glycol temperature from the contactor is generally assumed to be equal to the contactor temperature.
- Assume rich-glycol exit temperature, from the glycol/glycol exchanger, as 150°C.
- Lean-glycol temperature, from the reboiler, is generally known, and a typical value is 204°C.
- From heat balance, the lean-glycol exit temperature from the glycol/glycol exchanger can be estimated.
- If the lean-glycol exit temperature does not exceed the contactor temperature by 6 to 20°C (as required by the design), the temperature of the rich-glycol exit temperature (from the glycol/glycol exchanger) is to be reduced suitably.

Another problem associated with the design of the gas/glycol exchanger is that the high-duty exchanger increases the gas outlet temperature from the contactor. Generally, the dry gas from the contactor is transported through an underground (typically, yellow jacket) pipe. The yellow jacket type of pipe is generally designed for a maximum of 60°C temperature. In tropical countries, the ambient air temperature can go as high as 45°C, resulting in an air cooler gas outlet temperature (from the compressor and inlet to the contactor) of 55°C. High contactor outlet temperature restricts the design of the gas/glycol exchanger, because the maximum dry gas temperature generally should not exceed 60°C.

For proper estimation of exchanger heat duty, it is essential to calculate the density and specific heat of glycol. For lean TEG, the density is approximately estimated as

$$\rho = -0.783 * T + 1140 \tag{5.5}$$

where

ρ = density of lean TEG, kg/m^3
T = temperature, °C

Specific heat of TEG is estimated as follows:

Lean TEG:

$$C_P = 4.186 * (0.00122 * T + 0.471) \tag{5.6}$$

Rich TEG (valid for 90 to 100 wt% TEG):

$$C_P = m * T + c \tag{5.7}$$

where

$$m = 0.000073 * TEG_c - 0.00219 \tag{5.8a}$$

$$c = -0.03534 * TEG_c + 5.5056 \tag{5.8b}$$

TEG_c = concentration of TEG (wt%)
C_P = specific heat at constant pressure, kJ/(kg·°C)
T = temperature, °C

Regenerator

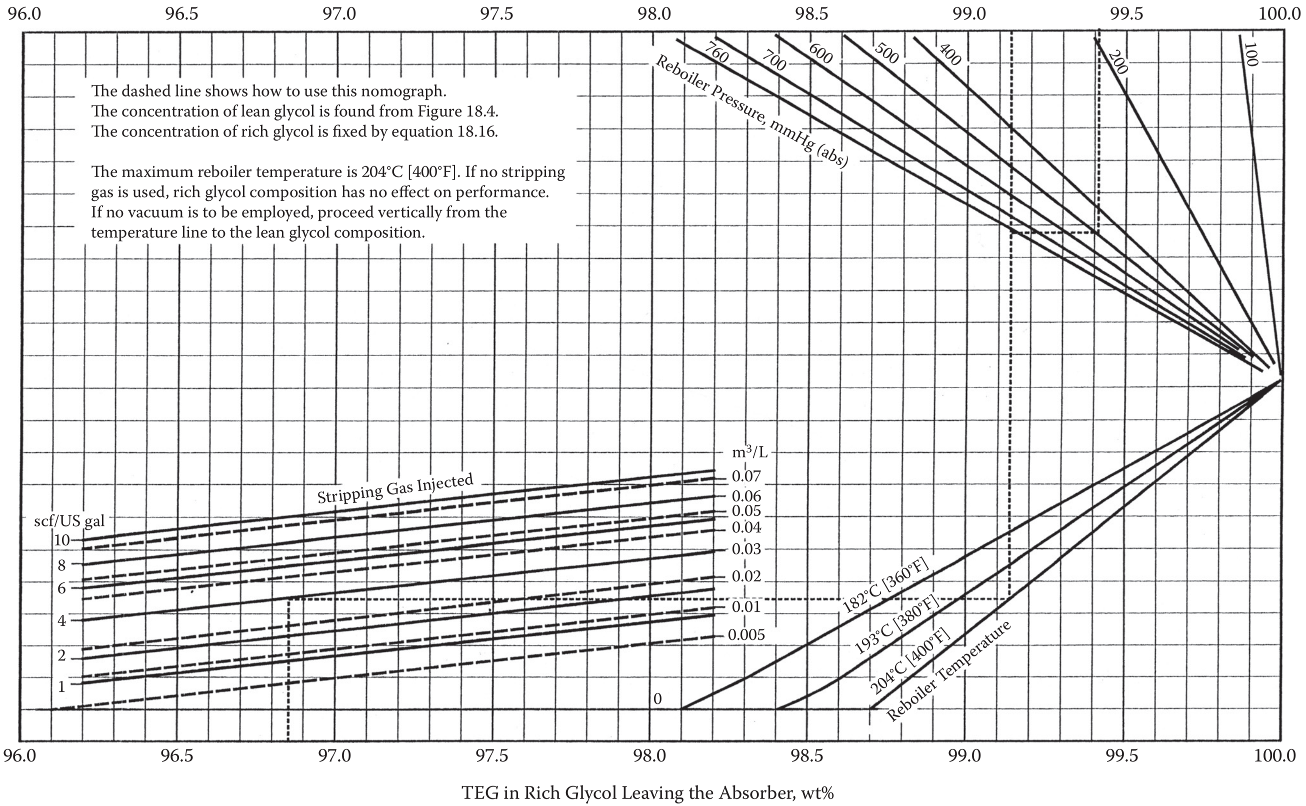
Regeneration is required to remove the absorbed water from rich glycol. Generally, the water-absorbed glycol (rich glycol) is heated to a sufficiently high temperature to drive the water out and produce nearly pure glycol (lean glycol). This operation is generally performed at atmospheric pressure. For material balance and proper design, it is essential to know the concentrations of both rich and lean glycol. The following equation can be used to estimate the concentration of rich TEG.

$$\text{Rich TEG wt\%} = \frac{M_{leanTEG} * 100}{M_{leanTEG} + M_W + M_{WleanTEG}} \tag{5.9}$$

where
$M_{leanTEG}$ = mass flow of lean TEG, kg/h
M_W = mass of water absorbed
= mass of water entering the contactor, kg/h
$M_{WleanTEG}$ = water in lean TEG, kg/h

The required concentration of lean TEG is a function of reboiler temperature and stripping gas flow. Generally, the reboiler for the TEG regeneration system is maintained at 204°C, and at this reboiler temperature, 98.5 wt% TEG is achievable without using any stripping gas. The reboiler temperature is generally set to a maximum value of 204°C, though a lower reboiler temperature is possible.

The lean-TEG concentration is first estimated using Figure 5.7, and then Figure 5.9 is used to optimize the reboiler temperature and stripping gas flow [2].



The dashed line shows how to use this nomograph.
The concentration of lean glycol is found from Figure 18.4.
The concentration of rich glycol is fixed by equation 18.16.

The maximum reboiler temperature is 204°C [400°F]. If no stripping gas is used, rich glycol composition has no effect on performance. If no vacuum is to be employed, proceed vertically from the temperature line to the lean glycol composition.

Figure 5.9 Regenerator performance as a function of pressure, reboiler temperature, and stripping gas. (Courtesy of John M. Campbell.)

Still column

The still column is mounted above the reboiler and mostly the water vapors enter the still-column bottom. The bottom section of the still column is packed with random packing — the 25-mm pall ring is most widely used. The still column operates as a standard packing column, and for proper operation the still column needs a reflux stream. The reflux stream is generated by introducing a reflux coil just above the packing. Cold rich glycol is passed through the reflux coil to condense most of the glycol vapors and some water vapors, to generate the reflux. Generally, 10 to 20% of the total overhead vapor is condensed as a reflux.

Small-diameter still columns (diameter less than 500 mm) are generally packed with 25-mm SS 316 pall ring or 25-mm ceramic intalox saddles. Larger columns are normally designed with bubble cap trays. Generally, one theoretical stage (packing height approximately 2 m) is adequate for the still column. However, if the lean-glycol concentration needs to be increased by using stripping gas, the still column is preferably designed for two theoretical stages. This will reduce the consumption of stripping gas. The requirement of stripping gas can be estimated by using the following equation:

$$S = 0.4335 - \frac{5.4482}{x^{0.5}} + \frac{8.237}{x} - \frac{0.8758}{x^{1.5}} \tag{5.10}$$

where

S = stripping gas requirement, Sm^3/m^3 TEG
x = 100 – lean TEG wt%

Equation 5.10 is valid for the following conditions:

- Two theoretical stages in stripping column.
- Reboiler temperature is 204°C.
- Lean TEG concentration in the range of 98.5 to 99.97 wt%.

For a 25-mm Pall ring, the diameter of the still column can be calculated using the following equation [2]:

$$d = 210 * (q_{TEG})^{0.5} \tag{5.11}$$

where

d = diameter of the still column, mm
q_{TEG} = TEG circulation rate, m^3/h

A reflux coil is used to maintain the outgoing vapor temperature a little above 100°C and to condense 10 to 20% of the vapor as a reflux. For a small-capacity plant, the cool rich glycol is circulated through the reflux coil to maintain the necessary reflux. For larger plants, a water-cooled condenser with temperature control is used. The overall heat transfer coefficient of the reflux coil is assumed in the range of 110 to 230 W/(m^2·K) [2].

Reboiler

The reboiler provides the required heat for the regenerator. Though different heat sources can be used for the reboiler, a fire tube reboiler is most commonly used. For fire tube design, it is important to design the correct fire tube diameter and length. At the inlet of the fire tube, the temperature of the flue gas is in the range of 1450 to 1500°C, and at the stack inlet point, the flue gas temperature drops to approximately 500°C. It is important to keep the fire tube metal temperature low enough to prevent excessive glycol degradation. This is done by controlling the heat density and heat flux of the fire tube. The reboiler is operated at high temperature and has a corrosive environment due to the presence of water, carbon dioxide, and so on. Generally, the bottom section of the reboiler corrodes at a higher rate than the top section, and the reboiler is designed with some corrosion allowance.

Fire tube heat density

Fire tube heat density depends on the type of draft used for design. Most TEG dehydration units are designed for natural draft, and for such conditions, the maximum fire tube heat density used is 24 kJ/(mm^2·h) [1]. The general equation of heat density is

$$H_D = \frac{Q}{A * \eta} \tag{5.12}$$

where

H_D = fire tube heat density, kJ/(mm^2·h)
Q = total reboiler duty, kJ/h
A = cross-section area of the fire tube, mm^2
η = efficiency (generally 0.7)

Fire tube heat flux

Fire tube heat flux generally should not exceed 114,000 kJ/(m^2·h) [1]. Higher heat flux increases the fire tube metal temperature and causes glycol degradation. The general equation of heat flux is:

$$H_F = \frac{Q}{A} \tag{5.13}$$

where

H_F = fire tube heat flux, kJ/(m^2·h)
Q = total reboiler duty, kJ/h
A = fire tube surface area, m^2

Lean glycol storage

Lean glycol storage can be designed as a part of the reboiler (with the addition of a partition) or can be a separate vessel. Because this vessel only stores lean glycol, without any corrosive environment, no corrosion allowance is required for this vessel. The following points should be considered in the design of the lean-glycol storage vessel [2]:

- Design residence time between the normal and low level as 20 min. The residence time is estimated based on the rich-glycol circulation rate.
- The capacity of the storage vessel should be large enough to hold the liquid from the reboiler. This is required to transfer the glycol from the reboiler for reboiler maintenance.
- Provision is required to add fresh glycol, using an air-driven or hand pump. Also, provision for chemical addition and stripping gas injection is required.

Energy exchange pump [4,5]

Energy exchange pumps are extremely popular in the design of glycol dehydration systems. These pumps are low-cost items and require no electrical power. In most remote locations, availability of electrical power is a problem, and the general emphasis is to reduce power consumption. For such plant locations, energy exchange pumps are widely used.

Though energy exchange pumps are very popular, the operation is not trouble-free, particularly if the operating pressure is very high. For that reason, it is common to install a 100% standby pumping arrangement. Electric motor driven reflux pumps are less problematic, though for electric motor driven pumps, the 100% standby concept is quite common. The pumping arrangement is the most maintenance-intensive section, and the standby design concept offers more trouble-free operation.

Figure 5.10 explains the operation of energy exchange pumps. Liquid is drawn straight from the contactor through a high-pressure filter to the motor end of the pump. The high-pressure filter is used to avoid any dust or foreign particles from entering into the motor end of the pump. The high-pressure end is called the motor end because the energy available by dropping the pressure in the high-pressure end is used to drive the pump end. The liquid draw rate to the pump is a function of liquid availability and is controlled automatically. Initially, the needle valve is opened gradually to introduce the liquid and to prime the pump. Once the pump is primed and the system operates at full capacity, the needle valve is kept fully open. This system does not need a level-control arrangement, because the level is controlled automatically by the pump itself. In this arrangement it is important to maintain a specific piping layout, and the following recommendations are generally useful:

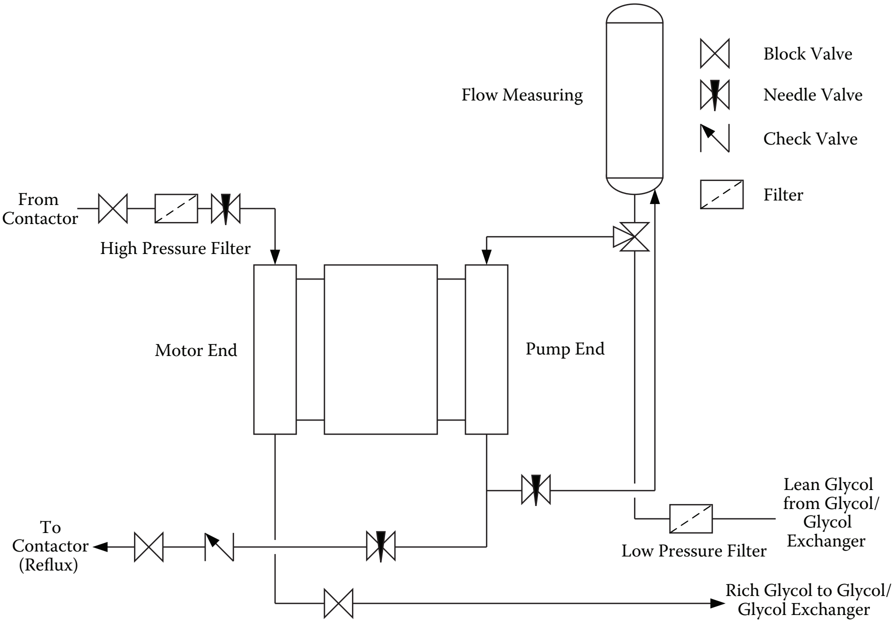


Figure 5.10 Energy exchange pump.

- Piping between the contactor and the motor should be as short as possible.
- Piping between the contactor and the motor should be straight. Bends and pockets affect the pump performance.
- The piping arrangement should be such that a small quantity of gas, from the contactor, is always available.

For proper system operation, a small quantity of contactor gas is always required. The gas required depends on the contactor pressure and is presented in Table 5.5 [5].

The pressure in the motor end is reduced to the flash separator pressure with some margins for frictional and static pressure drops. In some designs, rich glycol is used to generate reflux in the regenerator column, and for such

Table 5.5 Gas Consumption to Operate Energy Exchange Pump

Operating Pressure (kPaG)	Gas Consumption (Sm^3/m^3 TEG)	Operating Pressure (kPaG)	Gas Consumption (Sm^3/m^3 TEG)
2,750	17.2	6,900	41.9
3,450	20.9	7,600	45.6
4,150	25.4	8,300	50.1
4,800	29.2	8,950	53.9
5,500	33.7	9,650	29.1
6,200	37.4	10,350	62.1

designs, the outlet pressure from the motor end should be higher. In fact, the pressure is controlled through the back-pressure control valve at the outlet of the flash separator, and the pressure profile is maintained automatically.

The low-pressure filter is installed on the lean-glycol circuit and is extremely useful to filter all degraded products from the glycol reboiler.

The lean glycol after the low-pressure filter flows to the pump end through a three-way valve. The three-way valve is very useful to calibrate the pump when required. General practice is to precalibrate the pump to the desired flow, and a particular setting is maintained. It is also required to check the flow rate periodically (using the three-way valve and flow measuring vessel). The needle valve on the line connecting pump discharge to the flow measuring vessel is essential, because this valve is used to bleed the air out of the dry pump.

The needle valve at the pump discharge side is used to control the flow rate through the pump. This flow control is important and can be done by using either a needle valve or a globe valve. The check valve downstream of the needle valve is an absolute requirement. Without this check valve, the fluid will flow backward from the contactor to the pump and will cause the pump to rotate in the reverse direction. Without this check valve, it is almost impossible to start the pump operation.

The system needs some block valves as indicated in Figure 5.10. A ball valve is widely used as a block valve because of its better sealing facility.

Burner management

The burner management system is one of the key factors in good design of a dehydration system. A typical burner management system (for medium plants) is presented in Figure 5.11.

The flow of main gas and pilot gas needs to be controlled in a particular manner and is mostly laid down in codes and standards. The following

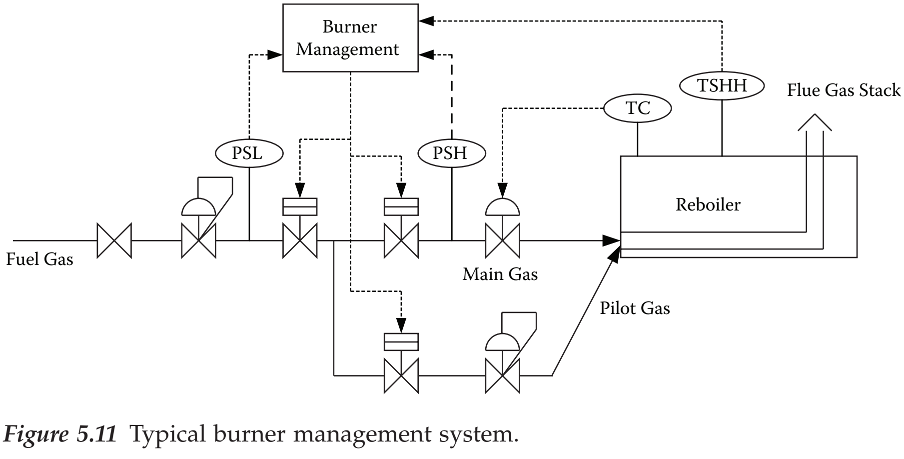


Figure 5.11 Typical burner management system.

discussions on the burner management system will be in accordance to Australian Standard (AS) 3814 [6].

The first requirement of a burner management system is to maintain the required reboiler temperature. This is done through a temperature control system as indicated in Figure 5.11. The temperature control valve maintains a certain flow of fuel gas to the burner. In most cases, the fuel gas is available at a reasonably high pressure, and a pressure regulating valve is required at the inlet. This pressure regulating valve maintains a constant downstream pressure.

Development of low and high pressure is a possibility and this may cause malfunction of burner operation. In general, a low-pressure switch (PSL) is installed upstream of the first shutdown valve, and a high-pressure switch (PSH) is installed downstream of the second shutdown valve. In case of low and high pressures, the signal will close all shutdown valves on the main gas line, as well as the shutdown valve on the pilot gas line. In most cases, a combination of shutdown and venting is installed together. This operates as a double block and bleed manner, and under shutdown situations, the vent valve opens and releases the gas trapped within the double-block shutdown valves.

The pilot gas can be a separate connection or can be connected downstream of the main gas line, depending on the burner capacity. Generally, a single shutdown valve on the pilot gas line is adequate. A pressure regulating valve is also required on the pilot gas line and is generally installed downstream of the shutdown valve. This maintains certain downstream pressure, and no attempts are made to control reboiler temperature while the system is on pilot operation.

In practice, all shutdown valves on the main as well as pilot gas lines are fitted with manual resets. In the event of a shutdown, the manual reset is required to put the system back in operation.

When a double block and bleed is installed, the following minimum vent valve sizes as shown in Table 5.6 are used [6].

Table 5.6 Minimum Size of Vent Valve

	Minimum Size of Vent Valve (mm)	
Size of Largest Safety Valve	Double Block and Bleed Systems	Position Proofing Systems
Less than 20	Same as SDV	Same as SDV
20–40	20	15
50	25	15
65	32	20
80	40	20
100	50	25
150	65	32
200	80	50

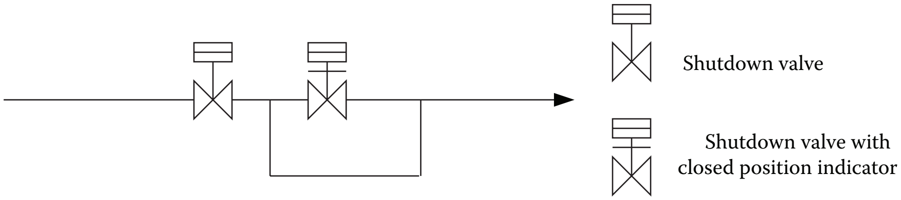


Figure 5.12A Burner capacity up to 500 MJ/h.

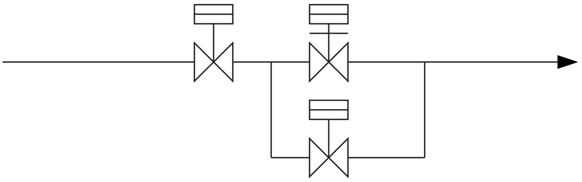


Figure 5.12B Burner capacity in between 500 MJ/h and 5 GJ/h.

The start gas connection is taken from the main gas line and is taken from between two main gas shutdown valves. Depending on the burner capacity, different arrangements are possible, and these are discussed in the following text:

Burner capacity not exceeding 500 MJ/h: The configuration is shown in Figure 5.12A. In this arrangement, no isolation valve is provided on the start gas connection.

Burner capacity exceeds 500 MJ/h but is less than 5 GJ/h: The configuration is shown in Figure 5.12B. This configuration is similar to the previous one with a shutdown valve on the start gas line.

Burner capacity exceeds 5 GJ/h but is less than 20 GJ/h: The configuration is shown in Figure 5.12C. For this capacity, one shutdown valve, as shown in Figure 5.12C, needs to be installed with a proof-of-closure switch.

Burner capacity exceeds 20 GJ/h: The configuration is shown in Figure 5.12D. For this configuration, all shutdown valves are to be installed with proof-of-closure switch.

The start gas rate is required to be established for proper design of the burner management system. The start gas rate generally should not exceed the value calculated by using the following methods [6]:

1. The gas concentration in relation to the proven airflow rate is not to exceed the percentage of the lower explosive limit (% LEL) as indicated in Table 5.7.
2. The energy released during the start flame establishment period does not exceed 35 kJ/m^3 of the combustion chamber volume.

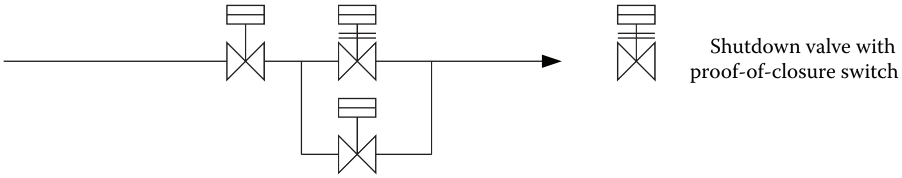


Figure 5.12C Burner capacity in between 5 GJ/h and 20 GJ/h.

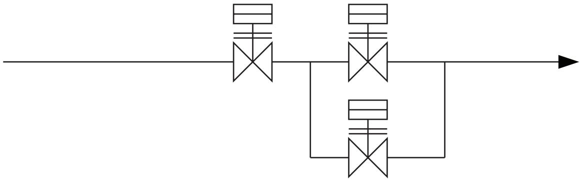


Figure 5.12D Burner capacity exceeds 20 GJ/h.

Example 5.2

Design a glycol dehydration system for the following conditions:

- Gas flow rate = 50,000 Sm^3/h
- Inlet pressure = 8,000 kPaG
- Inlet temperature = 50°C
- Molecular weight = 17
- Moisture of dehydrated gas = 65 mg/Sm^3
- Flash separator pressure = 500 kPaG
- Separator retention time = 20 min
- Approach to equilibrium temperature = 4°C
- TEG recirculation rate = 0.025 m^3/kg water removed
- Contactor internals = bubble cup with 610-mm spacing
- Rich-glycol outlet temperature = 150°C (from glycol/glycol exchanger)
- Heat flux for fire tube design = 90,000 kJ/(m^2·h)
- Heat density for fire tube design = 12 kJ/(mm^2·h)

Solution

Step 1: Estimation of Moisture Content

Using the procedure outlined in Chapter 1, the moisture content of saturated natural gas at 8,000 kPaG and 50°C is estimated using the PR equation of state.

Moisture content at inlet condition = 1540 mg/Sm^3
Similarly adjusted water dew point of dehydrated gas = –9.5°C
Compressibility factor = 0.8905

Table 5.7 Limits for Start Gas Estimation

Type of Gas	LEL Percentage Gas in Mix	Ratio[a]	Percentage of LEL	Percentage of Ratio	Percentage Excess Air	Percentage O_2 in Flue Gas	Percentage CO_2 in Flue Gas
Natural	5.0	9.55:1	50	24	310	16.2	2.7
Town	6.0	4.63:1	50	14	600	18.2	1.6
Propane	2.2	22.80:1	25	13	690	18.5	1.6
Butane	1.8	30.97:1	25	14	610	18.3	1.8
TLP (26% LP gas)	8.46	5.19:1	25	11	790	18.5	1.6
SNG (55% LP gas)	4	12.09:1	25	12	720	18.5	1.6

[a] The ratio indicates the stoichiometric ratio.

Step 2: Estimation of Lean-TEG Concentration

Lean-TEG concentration for 50°C contactor temperature and – 9.5°C equilibrium dew point (from Figure 5.7) = 99.4 wt%

Step 3: Estimation of Theoretical Stages

Water removal efficiency = (1,540 – 65)/1,540 = 0.958
TEG circulation rate = 0.025 m^3 TEG/kg water
Minimum lean-TEG concentration = 99.4 wt%
Number of theoretical stages (from Figure 5.8A–Figure 5.8E) = 3
Number of actual stages = 12

Step 4: Contactor Design

Density of inlet gas = (17/22.4) * (8101.3/101.3) * (273/323)/0.8905 = 57.61 kg/m^3
Standard density of gas = 0.719 kg/Sm^3
Gas mass flow rate = 35,970 kg/h
Design C value (for 610-mm spacing) = 176 m/h
Density of lean TEG at 50°C (Equation 5.5) = 1,101 kg/m^3
Gas loading rate (Equation 5.2) = 43,150 kg/(m^2·h)
Contactor cross-section area = 35,970/43,150 = 0.834 m^2
Contactor diameter = 1.03 m
Approximate contactor height = 0.61 * 12 + 2.5 = 9.82 m

Step 5: Design of Flash Separator

Water removal rate = 50,000 * (1540 – 65)/1,000/1,000 = 73.75 kg/h
TEG circulation rate = 1.844 m^3/h
TEG mass flow rate = 2,030 kg/h
Flow of pure TEG = 2,018 kg/h
Flow of rich TEG = 2,091.8 kg/h
Volumetric flow of rich TEG = 1.9 m^3/h
TEG volume in the flash separator (20-min residence) = 0.633 m^3
Flash separator volume (approximately 3 * TEG volume) = 1.9 m^3
Flash separator diameter (approximate) = 1.06 m
Flash separator height (approximate) = 2.131 m

Step 6: Estimation of Exchanger Duty

Rich-glycol outlet temperature (assumed) = 150°C
Rich-glycol average temperature = 177°C
Glycol specific heat at 177°C (Equation 5.6) = 2.876 kJ/(kg·°C)
TEG mass flow = 2,030 kg/h
Sensible heat = 315.27 MJ/h

Latent heat of water = 166.68 MJ/h
Condenser duty (25% of latent heat) = 41.67 MJ/h
Reboiler duty (considering 10% extra) = 575.98 MJ/h
Rich-TEG flow = 2,091.8 kg/h
Rich-TEG concentration = 96.5 wt%
c value (Equation 5.8b) = 2.095
m value (Equation 5.8a) = 0.00485
Rich-TEG average temperature = 100°C
Rich-TEG specific heat (Equation 5.7) = 2.58 kJ/(kg·°C)
Duty of glycol/glycol exchanger = 498 MJ/h
Reflux temperature = 58°C
Lean-glycol outlet temperature (glycol/glycol exchanger) = 115°C
Average temperature (for gas/glycol exchanger) = 86.5°C
Average specific heat (Equation 5.6) = 2.413 kJ/(kg·°C)
Duty of gas/glycol exchanger = 279.2 MJ/h

Step 7: Regenerator Design

Reboiler temperature = 204°C
Diameter of still column (Equation 5.11) = 285.1 mm
(for 25-mm pall ring packing)
Estimated LMTD (for reflux condenser) = 91.2°C
Reflux coil area = 6,346 cm^2
Fire tube cross-section area (Equation 5.12) = 68,569 mm^2
Fire tube OD (10-mm thickness assumed) = 0.315 m
Fire tube surface area (Equation 5.13) = 7.04 m^2
Fire tube length = 7.1 m
Stripping gas required (Equation 5.10) = 4.61 Sm^3/m^3 TEG
Stripping column theoretical stages (used for Equation 5.10) = 2

Specifications

General specifications of a typical TEG dehydration unit are discussed Table 5.8. These specifications have been developed to get some understanding of the different parameters used in the design. The values furnished in the following table are typical and will vary from case to case.

Table 5.8 General Specification of a TEG Dehydration Unit

S. No	Description	Unit	Value	Notes
1	Basic process information			
	• Gas flow rate	Sm^3/h	50,000	
	• Inlet pressure	kPaG	8,000	
	• Inlet temperature	°C	50	
	• Molecular weight		17	
	• Approach to equilibrium temperature	°C	4	1
	• TEG circulation rate	m^3/kg water	0.025	
	• Compressibility of gas		0.891	
2	Contactor design			
	• Moisture of dehydrated gas	mg/Sm^3	65	
	• Contactor design pressure	kPaG	12,000	
	• Contactor design temperature	°C	100	
	• Reflux temperature (approximate)	°C	58	
	• Material of construction		CS	
2.1	Contactor internals			
	• Type		Structured packing	2
	• Packing material		SS 316	
	• Packing type	m^2/m^3	125Y	3
2.2	Liquid distributor			
	• Type		Gravity	4
	• Drip density	$Numbers/m^2$	80 to 100	
	• Minimum glycol circulation rate	$m^3/m^2/h$	1	
	• Minimum area available for gas flow	%	40 to 50	
	• Minimum distance between distributor and mist eliminator	mm	600	
3	Flash separator design			
	• Operating pressure	kPaG	400	
	• Liquid residence time	min	20	5
	• Design *K* value	m/sec	0.06	6
	• Design pressure	kPaG	1,000	
	• Design temperature	°C	100	
	• Material of construction	CS		
4	Heat exchanger design			
	• Type		Shell and tube	7
	• Over design required	%	15	
	• Fouling factor	$m^2 \cdot K/W$	0.0015	
	• Material of construction		CS	
5	Regenerator			
5.1	Reboiler and lean-glycol storage			
	• Reboiler temperature	°C	204	8
	• Residence time of lean-glycol storage	min	20	9
	• Fire tube heat flux	$kJ/(m^2 \cdot h)$	90,000	
	• Fire tube heat density	$kJ/(mm^2 \cdot h)$	12	
	• Design pressure	kPaG	Liquid head	
	• Design temperature	°C	220	
	• Material of construction		CS	

Continued

Table 5.8 General Specification of a TEG Dehydration Unit *(Continued)*

S. No	Description	Unit	Value	Notes
5.2	Still column			
	• Type of packing		25-mm Pall ring	
	• Number of theoretical stages		1	
	• Reflux coil duty (condensing percentage of vapor)		20	
	• Reflux coil heat transfer coefficient	W/(m^2·°K)	200	
	• Design pressure	kPaG	Atm	
	• Design temperature	°C	220	
	• Material of construction		CS	
5.3	Stripping column			10
	• Type of packing		25-mm Pall ring	
	• Number of theoretical stages		2	11
	• Type of stripping gas		Dehydrated gas	
6	Other design information			
	• Type of reflux pump		Electric motor-driven	
	• Particulate filter		Required	
	• Solid impurity at the outlet of particulate filter	wt%	0.01	
	• Maximum glycol flow per 900-mm-long element	l/min	11.5	
	• Carbon filter		Required	
	• Activated carbon per liter glycol per minute	kg	4	
	• Burner management			12
7	Battery limit conditions			
7.1	Inlet to the contactor			
	• Location		Note 13	13
	• Pressure	kPaG	8,000	
	• Temperature	°C	50	
7.2	Outlet from the contactor			
	• Location		Note 14	14
	• Pressure	kPaG	7,990	
	• Temperature (maximum)	°C	60	
7.3	Flash separator gas			
	• Location		Note 15	15
	• Pressure (maximum)	kPaG	400	
	• Temperature	°C	50	
7.4	Stripping gas			
	• Location		Note 16	16
	• Pressure	kPaG	400	
	• Temperature	°C	50	
7.5	Fuel gas			
	• Location		Note 17	17
	• Pressure	kPaG	400	
	• Temperature	°C	50	

Continued

Table 5.8 General Specification of a TEG Dehydration Unit *(Continued)*

S. No	Description	Unit	Value	Notes
7.6	Instrument air			
	• Location		Single point	
	• Pressure	kPaG	700	
	• Temperature	°C	40	
	• Dew point	°C	–40	18

Notes:

1. Approach to equilibrium temperature may need to be changed to suit lean-glycol concentration.
2. Structured packing and bubble cup trays are commonly used in the design of contactor.
3. 125Y is the commonly used packing. More dense packing (e.g., 250Y) can also be used to reduce the HETP of packing. Normally, the type of packing is decided by the vendor; however, for fouling services, it is recommended that the packing density not exceed beyond 125Y.
4. Spray-type distributor can also be used for some cases.
5. The residence time can be substantially higher for tight emulsions.
6. *K* value does not normally control the size of the flash separator.
7. The designer may define the type of exchanger. Depending on the exchanger duty, a double pipe, shell and tube, or plate-type exchanger can be used.
8. Without any stripping gas, 204°C reboiler temperature produces 98.5 wt% lean glycol.
9. The residence time between the normal and low level in the storage tank.
10. Requirement of stripping column and stripping gas should be carefully established.
11. Depending on the layout restriction, one theoretical stage can be used.
12. Burner management system should be designed to meet international standards.
13. Inlet flange to the contactor inlet shutdown valve. It is preferable to include both inlet and outlet shutdown valve in vendor's scope; this will help to design a proper shutdown system by the vendor.
14. Outlet flange to the shutdown valve at the outlet of gas/glycol exchanger.
15. At the outlet of the back-pressure control valve.
16. Stripping gas should be available at the inlet flange of the lean-glycol storage tank.
17. Fuel gas should be available at the inlet of the main isolation valve.
18. Instrument air dew point is measured at atmospheric pressure.
19. Inlet gas composition has not been entered in the table. It is recommended to furnish the gas composition at the inlet. This will be helpful in deciding the material of construction or corrosion allowance.

Programming

The program to design TEG dehydration unit, dehydration.exe, has been developed for sizing major equipment and estimate the heat exchanger loads. Designing individual exchangers has not been considered in this program.

Program limitations

The following limitations can be noted:

- The unit performance is estimated for a 204°C reboiler temperature. It is not possible to establish impact of variable reboiler temperature.
- Stripping gas requirement is calculated for two theoretical stages in the stripping column.
- Though not specifically indicated, preferred structured packing is 125Y.
- Estimated fire tube length is the total length. Normally one "U" is used as the fire tube.

- Fire tube OD can be finalized based on the nearest higher standard size. The length can be adjusted accordingly.
- Glycol temperature to the regenerator column is the temperature at the lean-glycol pump discharge.

General overview

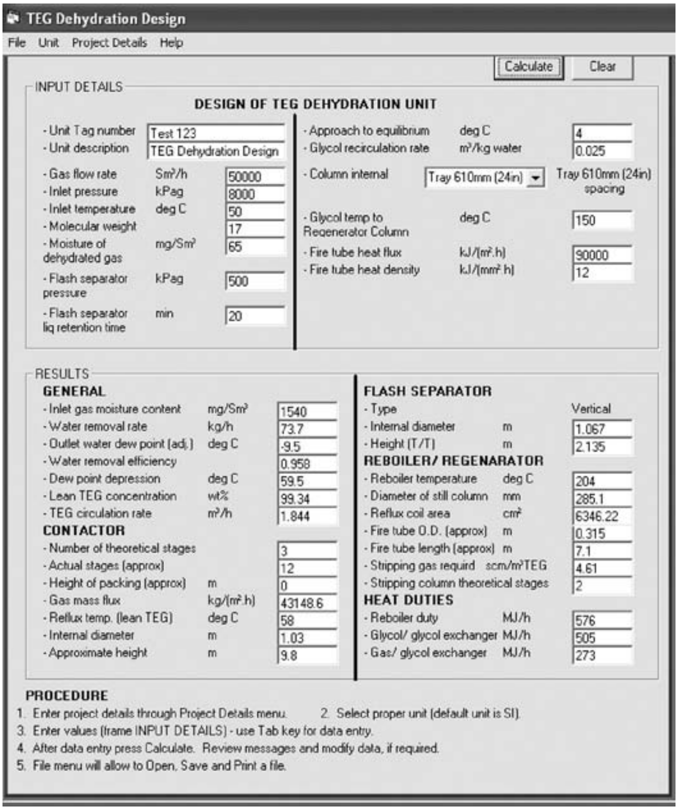
Double-click dehydration.exe, and a form similar to Figure 5.13 will open. The form will be a blank one, without any default data.

File menu

The following options are available under File Menu:

Open: This will open a previously saved file. Only file extension *.teg is acceptable for file menu.

Save: Once a calculation is complete, the "Save" submenu will save the input parameters including the project details.



INPUT DETAILS

DESIGN OF TEG DEHYDRATION UNIT

- Unit Tag number		Test 123
- Unit description		TEG Dehydration Design
- Gas flow rate	Sm^3/h	50000
- Inlet pressure	kPag	8000
- Inlet temperature	deg C	50
- Molecular weight		17
- Moisture of dehydrated gas	mg/Sm^3	65
- Flash separator pressure	kPag	500
- Flash separator liq retention time	min	20
- Approach to equilibrium	deg C	4
- Glycol recirculation rate	m^3/kg water	0.025
- Column internal	Tray 610mm (24in)	Tray 610mm (24in) spacing
- Glycol temp to Regenerator Column	deg C	150
- Fire tube heat flux	kJ/(m^2.h)	90000
- Fire tube heat density	kJ/(mm^2 h)	12

RESULTS

GENERAL		
- Inlet gas moisture content	mg/Sm^3	1540
- Water removal rate	kg/h	73.7
- Outlet water dew point (adj.)	deg C	-9.5
- Water removal efficiency		0.958
- Dew point depression	deg C	59.5
- Lean TEG concentration	wt%	99.34
- TEG circulation rate	m^3/h	1.844
CONTACTOR		
- Number of theoretical stages		3
- Actual stages (approx)		12
- Height of packing (approx)	m	0
- Gas mass flux	kg/(m^2.h)	43148.6
- Reflux temp. (lean TEG)	deg C	58
- Internal diameter	m	1.03
- Approximate height	m	9.8

FLASH SEPARATOR		
- Type		Vertical
- Internal diameter	m	1.067
- Height (T/T)	m	2.135
REBOILER/ REGENARATOR		
- Reboiler temperature	deg C	204
- Diameter of still column	mm	285.1
- Reflux coil area	cm^2	6346.22
- Fire tube O.D. (approx)	m	0.315
- Fire tube length (approx)	m	7.1
- Stripping gas requird	scm/m^3TEG	4.61
- Stripping column theoretical stages		2
HEAT DUTIES		
- Reboiler duty	MJ/h	576
- Glycol/ glycol exchanger	MJ/h	505
- Gas/ glycol exchanger	MJ/h	273

PROCEDURE

1. Enter project details through Project Details menu. 2. Select proper unit (default unit is SI).
3. Enter values (frame INPUT DETAILS) - use Tab key for data entry.
4. After data entry press Calculate. Review messages and modify data, if required.
5. File menu will allow to Open, Save and Print a file.

Figure 5.13 Design of dehydration unit.

Print: Once a calculation is complete, the "Print" submenu will print the input as well as output in a standard format.

Exit: Exit the program.

Unit menu

Under Unit Menu, the following options are available. Changing of units for individual items is not possible for this program, only a particular type of unit can be used.

SI Unit: This will set all units in SI format. This is also the default unit.

Eng Unit: This will set all units in English format.

Project details

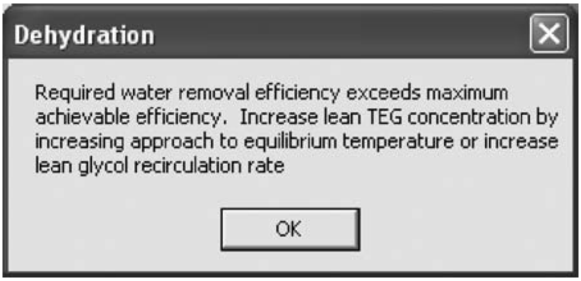
This menu is to enter the details about the project. Information, such as project name, calculation number, calculation performed and checked by, etc., can be entered under this menu.

Data entry

Once the proper unit is selected, and the project information entered, the form is ready for data entry. Information is entered only in the INPUT DETAILS box. The Tab key is generally used to enter data; the Enter key does not allow to movement from field to field.

There are a few problems anticipated in the design. The design is developed based on certain recommended practices and within certain limits of theoretical stages. Generally the contactor design is limited to 3 maximum theoretical stages (which is equivalent to 12 actual stages). It may happen that lean glycol concentration calculated using Figure 5.7 is lower than calculated by using Figure 5.8A–Figure 5.8E. Under this situation, a message box will appear as shown in Figure 5.14. This message box indicates that the lean-glycol concentration, estimated is not adequate to achieve required specification using a maximum of three theoretical stages. In other words, either the water dew point is to be reduced or the lean-glycol circulation rate (mostly equilibrium dew point) is to be increased. The water dew point is reduced by increasing the approach to equilibrium temperature. There are other reasons to use a certain approach to equilibrium temperature as discussed in the main section.

In the design of the contactor, a reflux temperature higher than the feed temperature is required for proper operation of the unit. In this program, reflux temperature is assumed as 8°C higher than the feed temperature. Reflux temperature is a function of the glycol temperature to the regenerator column (this is used as a design input). With increase in glycol to regenerator column temperature, the duty of the glycol/glycol exchanger will increase and the gas/glycol exchanger will reduce. If the glycol temperature to the regenerator column is too high, the lean-glycol temperature at the outlet of the glycol/glycol exchanger will be below the required reflux temperature.



Required water removal efficiency exceeds maximum achievable efficiency. Increase lean TEG concentration by increasing approach to equilibrium temperature or increase lean glycol recirculation rate

Figure 5.14 Message box.

Expected reflux twmperature is too low
Reduce glycol temp to Regenerator Column and recalculate

Figure 5.15 Message box.

Under this situation, the message box in Figure 5.15 will appear. In this situation the glycol temperature to the regenerator column is to be reduced and recalculated.

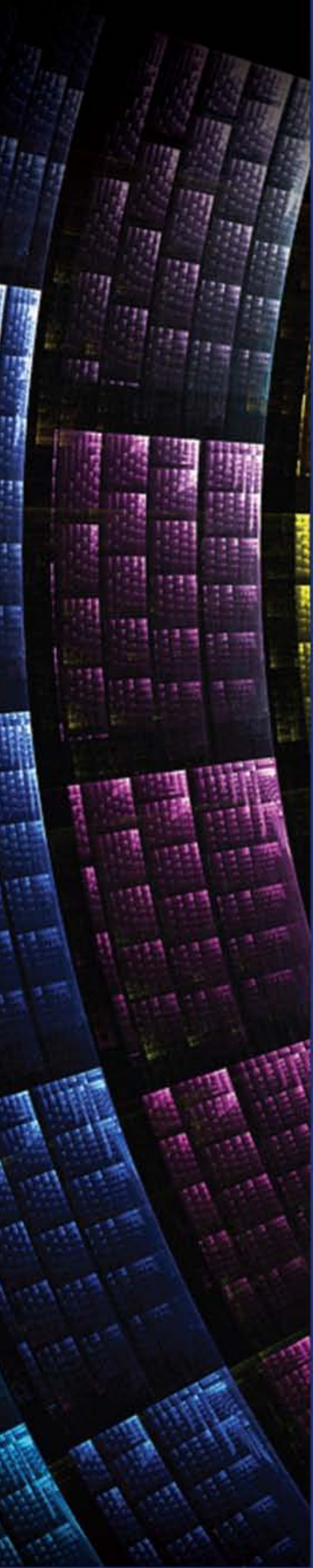
The calculation using this program can be misleading and should be checked by a competent process engineer. The glycol to regenerator column dictates the duty of glycol/glycol and gas/glycol exchangers. This program does not check the possibility of temperature cross or any problem, to achieve such a temperature. However, the total duty of two exchangers will not change. It is advisable to check the heat exchanger adequacy using better exchanger programs, e.g., HTRI®, and recalculate.

References

1. API Specification 12GDU, Specification for Glycol-Type Gas Dehydration Units, 1st ed., December 15, 1990.
2. Campbell, J.M., Gas Conditioning and Processing, John M. Campbell and Company, 8th ed., Vol. 2, Oklahoma, 2004.
3. Gas Processors Suppliers Association, GPSA Engineering Data Book — SI, Oklahoma, 1998.
4. Glycol Pumps for the Next Generation, General Product Information, Rotor-Tech.
5. Glycol Pumps and Accessories, Kimray Inc.
6. AS 3814 — 2005, Australian Standard, Industrial and Commercial Gas-fired Applications.

Index

Note: Page numbers in **bold** are for figures and tables. A note is indicated by an "n" after the page number.



Chemical Engineering

PROCESS ENGINEERING and DESIGN USING VISUAL BASIC®

ARUN DATTA

Software tools are a great aid to process engineers, but too much dependence on those tools can often lead to suboptimal, even inappropriate, designs. Reliance on specific software can also be a hindrance to the design process, if one lacks a firm understanding of the principles underlying its operation.

In ***Process Engineering and Design Using Visual Basic***®, Arun K. Datta provides you with a unique and versatile suite of programs along with the underlying concepts, principles, and mathematics needed to make the best use of them. Each chapter details the theory and techniques that provide the basis for specific design and engineering software and then showcases its development and functionality. This all-inclusive guide moves systematically from basic mathematics to fluid mechanics, separators, overpressure protection, and glycol dehydration, providing the necessary design guidelines based on international codes. Worked examples demonstrate the utility of each program, while the author also explains problems and limitations associated with the simulations.

Providing nine ready-to-use programs in Visual Basic® and the principles on which they are built, this guide—

- Builds a practical and intuitive understanding of design guidelines and how to apply them to the use of software tools
- Explains the fundamentals of process design and development underlying the use of software tools
- Enables you to verify and evaluate the accuracy of software results to make sure your design is on track
- Identifies the important issues in process design and the best procedures to resolve them

After reading this book you will be able to immediately put these programs into action and have total confidence in the result, regardless of your level of experience. All nine programs are available on the companion CD-ROM, which also includes a useful unit conversion tool.

45423
ISBN 1-4200-4542-3
90000
9 781420 045420
www.crcpress.com

CRC Press
Taylor & Francis Group
an **informa** business
www.taylorandfrancisgroup.com

6000 Broken Sound Parkway, NW
Suite 300, Boca Raton, FL 33487
270 Madison Avenue
New York, NY 10016
2 Park Square, Milton Park
Abingdon, Oxon OX14 4RN, UK