The Effects of the Ionosphere and C/A Frequency on GPS Signal Shape: Considerations for GNSS-2

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BIOGRAPHIES

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ABSTRACT

By greatly increasing the chipping rate of the pseudo random noise (prn) code, it may be possible to *measure* the ionosphere (TEC) in near real time, by exploiting the dispersive nature of the ionosphere. The ionosphere is a major source of error for all stand alone GPS based navigation, and spatial decorrelation limits the accuracy of differential GPS users. The ionospheric distortion of the C/A code (1.023 Mbps), and P code (10.23 Mbps) may be too small to measure. However, the use of a *faster* (40-100 Mbps) chipping frequency for GNSS2, may permit measurement of the ionosphere's Total Electron Content (TEC) using only a single frequency receiver.

Currently, we can only 'measure' the ionosphere by using a dual frequency receiver (limited to military users), or with a cross correlating receiver (limited by poor signal to noise) or by observing 'code-carrier divergence' (limited by long observation times). The diurnal model of the atmosphere is only capable of removing 50-60% of the error.

This investigation is based on the standard model of the ionosphere, which assumes negligible attenuation in the L-band, but does produce a time *advance* proportional to k/f^2. Expanding in a Taylor Series about L1, the first two terms of this model lead to code-carrier divergence. The next term is *quadratic* in frequency and produces both amplitude and phase modulation of the received signal. The higher order terms produce relatively insignificant changes. Although these variations are present in the signal received from GPS, it is quite small and has previously been unmeasured.

A computer simulation was developed to analytically determine the shape of the GPS signal after passing through the ionosphere, and to determine the effect of faster chipping frequency on the shape of the received signal. The GPS signal is expressed in the frequency domain, then phase shifts, proportional to k/f, were applied. The signal was then transformed into the time domain, and this signal was compared with the original signal.

This paper will present analytical results from the simulation. The benefits of a faster chipping frequency for GNSS2 will be explored. Time domain plots of the modified signal will be presented, illustrating the changes in both amplitude and phase due to the ionosphere. The limits imposed by the presence of atmospheric noise and receiver noise will not be discussed. This paper will not discuss other aspects of GNSS2 design, such as constellation selection.

Although greater spreading requires greater bandwidth, the quadratic dependence of this distortion on frequency may justify the additional bandwidth if it allows us to directly measure, rather than estimate, the ionosphere with a single frequency receiver. GNSS2 should be designed with a much faster code frequency (40-100 Mbps) since it would improve code positioning accuracy, would reduce the carrier ambiguity space, and may permit real-time ionospheric measurements.

1. INTRODUCTION

The introduction of the Global Positioning System (GPS) by the Department of Defense (DoD) has led to hundreds of unanticipated applications, and the creation of an entirely new industry. Future growth of the satellite navigation industry is also very promising. Current applications range from navigational aids to standalone users, such as autos or hikers (~100 m), to the autonomous steering of tractors [10] and the autonomous piloting and landing of aircraft [3, 9] (~1 cm). The Standard Positioning Service (SPS) relies on the Clear Acquisition (C/A) Code which has 1023 bits and is transmitted at 1.023 Mbps on a carrier at L1 (1.57542 MHz). The addition of Selective Availability (SA) by the DoD reduces the SPS accuracy from approximately 50 meters CEP to 100 meters CEP.

GLONASS is a very similar system, controlled by the Russian military, but it uses frequency division, rather than code division to distinguish between satellite signals. The GLONASS acquisition code is the same for all satellites, and is transmitted at only 511 Kbps.

GNSS2 will be designed with the same four technical goals that shaped both GLONASS and GPS,

- Accuracy
- Availability
- Continuity
- Integrity

These goals are almost always conflicting with one another, and with the important cost constraint.

Although the both GPS and GLONASS are provided free of charge throughout the world, they are not without detractors.

Political shortcomings include

- Lack of International/Civilian control
  (military control has little foreign support)
- SA introduces needless and often unacceptable error.
  (GPS only - no such problem with GLONASS)

Technical shortcomings include

- Polar coverage of GPS constellation is weak.
- Carrier phase is very accurate but ambiguous
- Two frequencies are needed to measure ionosphere

With the advantage of hindsight, many authors [1,9,10] have already made suggestions about the configuration and operation of GNSS2. At least one paper [2] has suggested that multi-frequency systems are essential in GNSS2 in order to eliminate ionospheric errors. This paper is concerned only with the benefits of a significantly faster modulating code.

2.0 PREVIOUS IONOSPHERIC RESEARCH

Previous research has demonstrated that a single frequency L Band signal that traverses the ionosphere will arrive slightly earlier than it would have if it were traveling through a vacuum. This does not violate any relativistic principles since no *information* is conveyed by just a single frequency. The time *advance* is a function of frequency, and to first order, is equal to k/f^2. It turns out that the information conveyed on a modulated L band carrier, arrives late (group delay) by exactly the same amount of time that the carrier arrives early.

At L1 = 1575.42 MHz, the group delay can occasionally be much as 50 meters (167 nanoseconds) for a signal passing vertically through the ionosphere. Satellites near the horizon experience nearly three times the group delay due to the obliquity.

Many of the previous measurements of the ionospheric delay utilized Faraday Rotation of linearly polarized signals. Unfortunately this same technique can not be used with the Circularly Polarized GPS signal. From this past research, we know that the advance for a signal frequency signal can be given by.

$$1)\quad \Delta\tau_{phase}(f) = \frac{k_2}{f^2} + \frac{k_3}{f^3} + \frac{k_4}{f^4} \le 0$$

This equation can be simplified by focusing only on the first term, which is directly proportional to the number of free electrons (TEC) along the ray's path.

$$2)\quad \Delta\tau_{phase}(f) = \frac{-40.3TEC}{f^2} \le 0$$

The time delay can be expressed as a change in phase by multiplying by the frequency in radians/sec.

$$3)\quad \Delta\phi = 2\pi f\tau = 2\pi\frac{\kappa}{f} = 4\pi^2\frac{\kappa}{\omega}$$

We must express the GPS signal in the frequency domain to make use of this model of the ionosphere.

2.1 GROUP DELAY AND PHASE ADVANCE

Group velocity and phase velocity are clearly explained in [5] From the following definitions of group velocity and phase velocity

$$4)\quad v_{GROUP} \overset{d}{=} \frac{\delta\omega}{\delta\beta} = \frac{1}{\delta\beta/\delta\omega} \le c$$

$$5)\quad v_{PHASE} \overset{d}{=} \frac{\omega}{\beta} = \frac{1}{\beta/\omega}$$

It is possible to determine the difference in arrival time for the carrier and the code independently.

$$6)\quad \Delta t_{group} = t_{group} - t_0 = \frac{\delta\left(\phi_0 + \Delta\phi\right)}{\delta\omega} - t_0 = \frac{\delta\Delta\phi}{\delta\omega}$$

$$7)\quad \Delta t_{phase} = t_{phase} - t_0 = \frac{\phi_0 + \Delta\phi}{\omega} - t_0 = \frac{\Delta\phi}{\omega}$$

Substituting in the expression for the phase change.

$$8)\quad \boxed{\Delta t_{group} = \frac{\delta\Delta\phi}{\delta\omega} = -4\pi^2\frac{\kappa}{\omega^2} = -\frac{\kappa}{f^2} = +\left|\Delta\tau\right|}$$

$$9)\quad \boxed{\Delta t_{phase} = \frac{\Delta\phi}{\omega} = +4\pi^2\frac{\kappa}{\omega^2} = +\frac{\kappa}{f^2} = -\left|\Delta\tau\right|}$$

Equations 8 and 9 show that the group delay and phase advance are exactly equal and opposite, but this is true only for the first term in the ionospheric model (k/f^2). This would not be true if higher order terms were included. Furthermore, Equations 8 and 9 reveal why the standard range equations for code and carrier have opposite signs for the ionospheric correction term [6].

Equations 8 and 9 contain terms that appear to come from a Taylor Series, in particular, the derivative of phase change with respect to frequency. This derivation makes it clear that the first two terms in such a Taylor Series Expansion produce the well known 'code-carrier divergence'. Although it is well known, 'code-carrier divergence' has very low observability, and requires several hours of observation in order to measure the ionosphere [4]. It is the higher order terms that are more interesting and were the focus of the simulation.

3.0 COMPUTER SIMULAION

The computer simulation of the ionosphere that was performed for this paper decomposes the model in Equation (1) into Taylor Series, as shown in Figure 3.0. By using Taylor Series, it was possible to isolate the contribution of each term individually.

MODEL → $\Delta\tau(f) = \frac{k_2}{f^2} + \frac{k_3}{f^3} + \frac{k_4}{f^4}$

TAYLOR	0th Term Nominal	$\phi\big\rvert_{f=L_1}$	Carrier Advance ~ - 10 m	Carrier Advance ~ - 1 cm	Carrier Advance ~ - 1 ε
SERIES	1st Term Slope	$\frac{d\phi}{df}\Big\rvert_{f=L_1}$	Code Delay ~ + 10 m	Code Delay ~ + 2 cm	Code Delay ~ + 3 ε
TERMS	2nd Term Curvature	$\frac{d^2\phi}{df^2}\Big\rvert_{f=L_1}$	AM ? PM ?	AM ? PM ?	AM ? PM ?
	3rd Term	$\frac{d^3\phi}{df^3}\Big\rvert_{f=L_1}$	AM ? PM ?	AM ? PM ?	AM ? PM ?

Figure 3.0 Contributions of model terms to distortion

In particular, the first two terms have very simple explanations, as mentioned previously. The 'zeroeth' term produces carrier advance. The 'first' term, or the first derivative of delta phase with respect to frequency, leads to code delay only. Note that for the first term in the model (k/f^2), the code-carrier divergence is exactly equal and opposite. However, for the next highest term of the model (k/f^3), the code-carrier divergence is in the ratio of +1:-2.

The block diagram in Figure 3.1 reveals the basic steps undertaken in this simulation. First the simulation parameters (prn #, chipping frequency, center frequency, TEC) are selected. The GPS signal is then expressed in the frequency domain, and the appropriate phase shift is introduced for a given TEC. Note that the obliquity is not considered in this simulation, since results for any given value of TEC are applicable for any particular elevation.

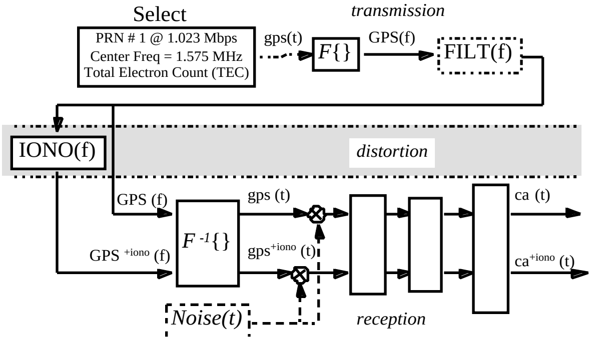

Figure 3.1 Schematic of computer simulation

Both the original and the modified signals are then transformed back to the time domain. Noise can then be added in the time domain. However, for the sake of clarity, all of the following results are from noise free simulations. Time domain noise will be used in future simulations since time domain averaging is likely in any future receiver.

3.1 DERIVATION OF GPS(f)

A computer simulation was developed to examine the theoretical shape of the GPS signal after passing through the ionosphere, and to determine what effect faster C/A codes would have on the shape of the received signal. This computer simulation involves several relatively simple steps. Various difficulties, such as the length of the C/A (prn) code (1023 ≠ 2^N), the narrow signal bandwidth and the high carrier frequency prevent the direct use of a Fast Fourier Transform. Instead the Frequency Domain representation of the GPS signal is derived analytically, combining the known spectra of a rectangular pulse, the appropriate C/A code, and the modulation at L1. Phase shifts, equal to -80.6πTEC/f, are applied in the Frequency Domain, and an Inverse Fourier Transformation is calculated analytically. The new time domain signals can then be compared to the original signal to show how the ionosphere effect is different from a pure time delay.

A sample of the GPS Signal spectrum is shown below for regular C/A Code with a signal bandwidth of 20.46 MHz.

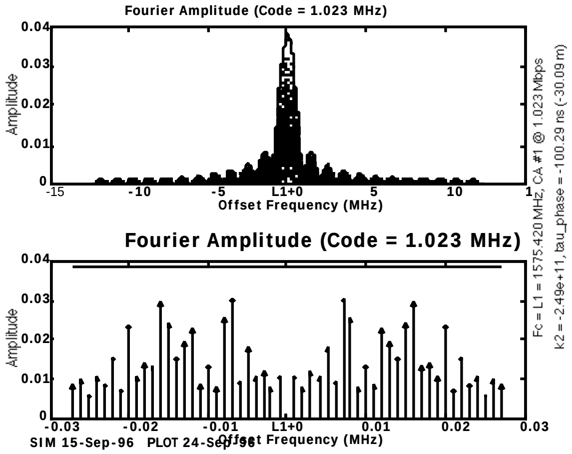

Figure 3.1 GPS Signal in Frequency Domain

The first two terms in the Taylor Series can be shown to produce 'code-carrier divergence', which is illustrated below in Figure 3.2. This effect is well known and is not relevant to this simulation. Therefore the corresponding terms were neglected, and the effects of only the 2nd derivative and beyond were considered.

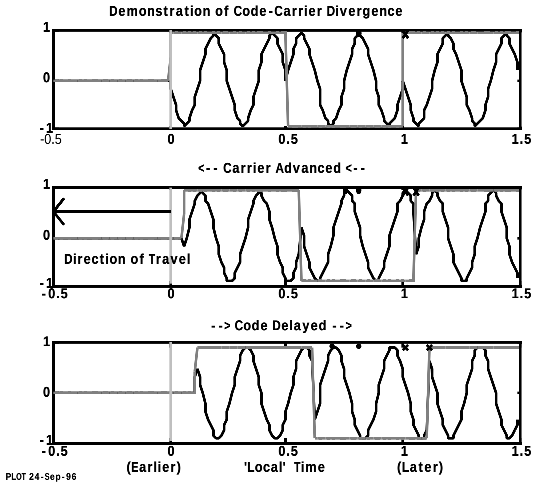

Figure 3.2 Simple Dispersion, without Higher Order Terms

4.0 RESULTS

The changes produced by the ionosphere, above and beyond 'code-carrier divergence', are typically rather small, particularly at the current C/A frequency of 1.023 Mbps. For that reason, all of these plots show both the original signal and the post-ionospheric signal on the same (upper) plot, with the *difference* plotted beneath them. It is the difference plots that are most revealing. In all plots the Ionospheric Delay was held constant at 30.09 meters = 100.29 nanoseconds = 158.0 cycles of L1. The group delay was fixed to an integer number of cycles in an attempt to limit discontinuities when the bit flips, particularly for the phase modulation plots. The chipping frequency increases from 1.023 Mbps (C/A) to 10.23 Mbps (P/Y) to 112.530 Mbps (GNSS2).

Note that this simulation assumes a bandpass transmit filter which causes the bit transitions to be somewhat rounded. The filter also introduces a delay of approximately 22 ns, which was subtracted before applying the ionospheric phase corrections, so that the beginning of the prn code is always aligned with cycle 'zero'.

The discontinuities as the signal passes through zero amplitude seem to be a numerical artifact. Even if these discontinuities did exist, they are of extremely short duration, and would probably be undetectable.

4.1 AMPLITUDE MODULATION

The following three graphs show that a small amount of amplitude modulation is introduced due to the GPS signal transmission through the ionosphere. The amplitude modulation *difference* seems to be most pronounced right near the bit transitions. Note that the difference plots in Figures 4.0 (1.023 Mbps) and 4.1 (10.23 Mbps) are very similar. This is probably due to the fact that both simulations used a signal bandwidth of 20.46 MHz, and that may be more important. The vertical hash marks in Figures 4.2 and 4.3 indicate the bit transitions. The vertical axis scale is the same for all four of the upper plots, but varies in the lower subplots, as the magnitude of the effect increases.

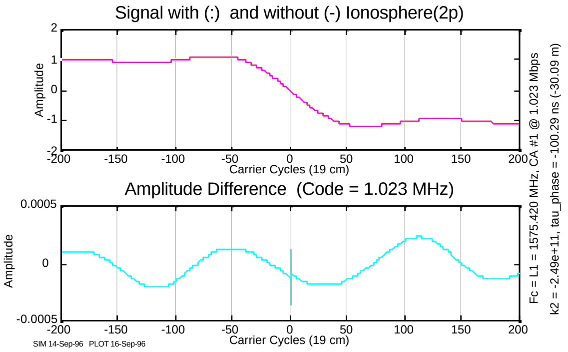

Figure 4.0 AM Variation at 1.023 Mbps

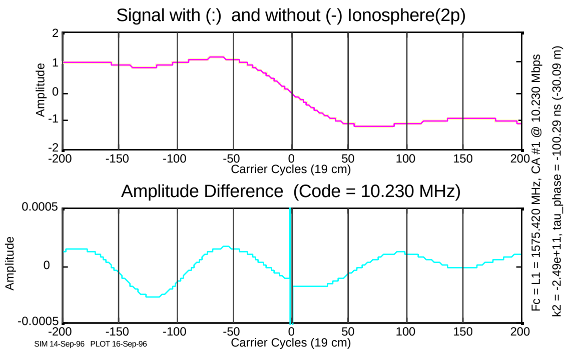

Figure 4.1 AM Variation at 10.23 Mbps

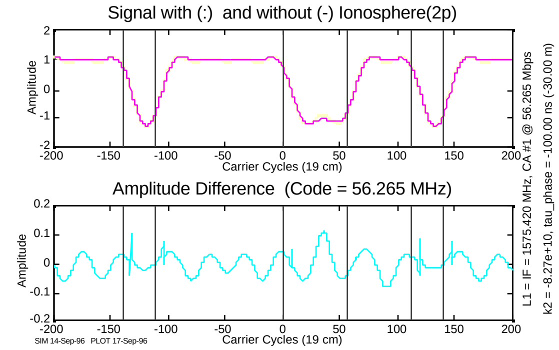

Figure 4.2 AM Variation at 56.265 Mbps

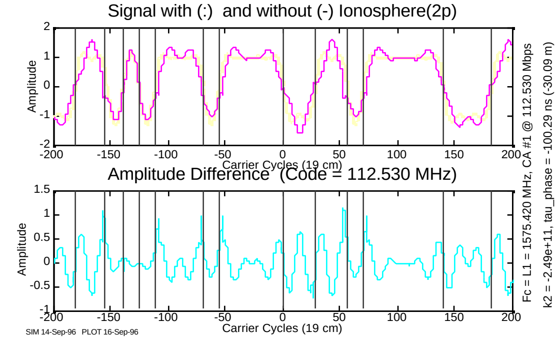

Figure 4.3 AM Variation at 112.530 Mbps

4.2 PHASE MODULATION

The following graphs show that a small amount of phase modulation is introduced due to the GPS signal transmission through the ionosphere. The phase variation *difference* seems to be most pronounced right near the bit transitions, as was the case in the preceding section. Theoretically the phase is discontinuous at the bit transitions, and the phase switches by 180 degrees. Thus, a phase discontinuity as the signal passes through zero amplitude is to be expected.

The quantity plotted in the upper subplot represents the phase difference from the *nominal* phase of $\varphi = \omega t$. The actual cycles are too numerous to show individually. The vertical hash marks in Figures 4.6 and 4.7 indicate the bit transitions. The vertical axis scale is the same for all four of the upper plots, but varies in the lower subplots, as the magnitude of the effect increases.

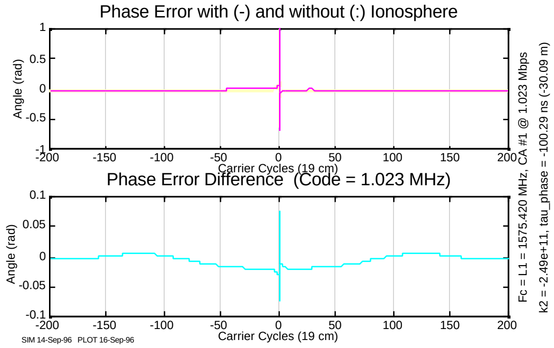

Figure 4.4 PM Variation at 1.023 Mbps

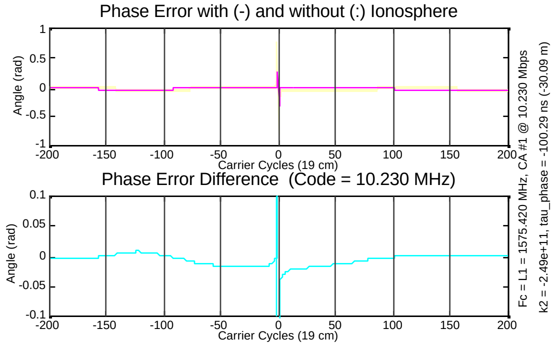

Figure 4.5 PM Variation at 10.23 Mbps

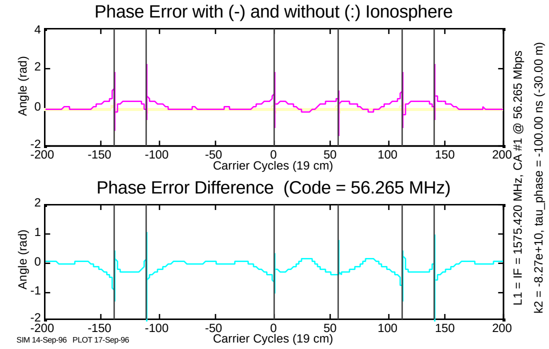

Figure 4.6 PM Variation at 55.625 Mbps

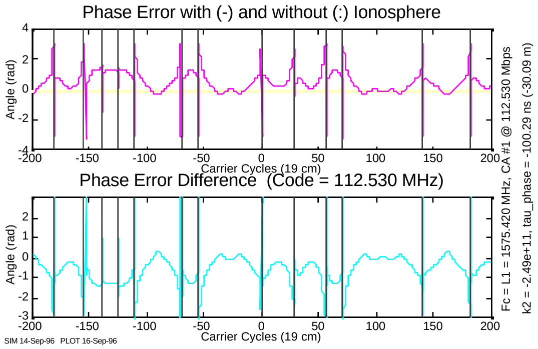

Figure 4.7 PM Variation at 112.530 Mbps

4.3 CORRELATION PROFILES

Correlation profiles were also calculated for the sample GPS signal both with and without the ionosphere. The difference of these two signals was surprisingly small and does not seem to offer a viable pathway for estimating the Total Electron Count. The reason for the small difference is believed to be the fact that correlation uses the entire duration of the signal for comparison, and the interesting phenomenon is confined to the bit transitions. Thus the 'interesting' variation introduced by the ionosphere seems to be getting 'diluted' by the correlation process.

Two graphs are shown below, depicting the correlation profile for the highest chipping frequency examined (112.530 Mbps). As in the previous result plots, the upper plots show the signal with and without the ionosphere, and the lower plot shows the difference in magnitude. The second pair of plots focuses on the central peak +/- 10 bits.

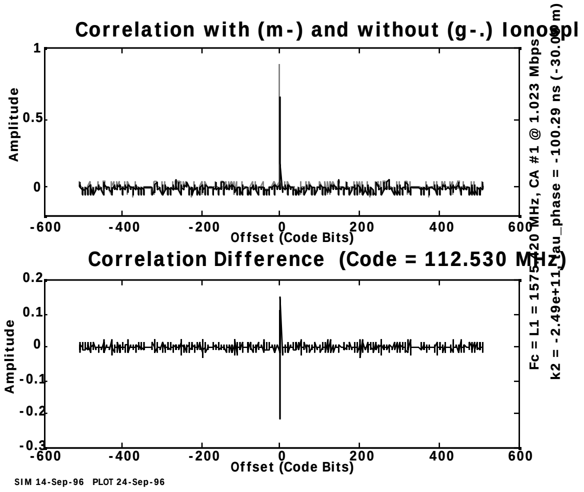

Figure 4.8 Correlation Peak (112.530 Mbps)

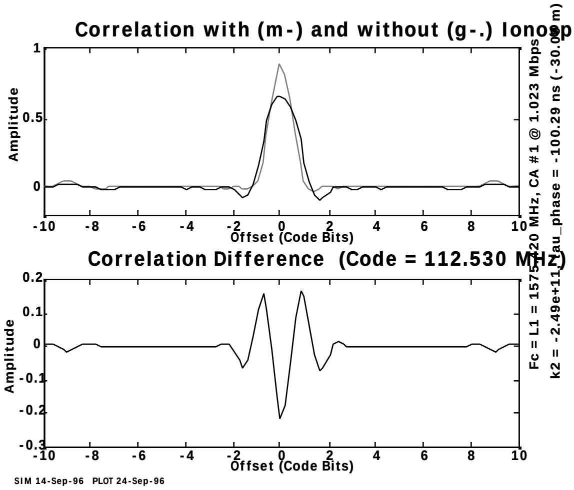

Figure 4.9 Correlation Peak Zoomed (112.530 Mbps)

5.0 GNSS-2 IMPLEMENTATION ISSUES

The introduction of GNSS2 would permit an opportunity to utilize lessons learned from both GPS and GLONASS. There are several advantages to a substantially faster code frequency, roughly 40-100 Mbps. In particular, a faster code would increase code positioning accuracy and reduce the ambiguity space for carrier tracking receivers. Most importantly, the previous results suggest that a faster code frequency may permit real-time single frequency ionospheric measurements. Despite the inclusion of 8 model coefficients for ionospheric correction in the GPS data messages, only 50-60% of the error can be eliminated. Dual frequency measurements also exploit the dispersive nature of the ionosphere, but these are used principally in military receivers.

The obvious drawback to this technique for ionospheric measurement is the necessity of protecting an even larger portion of the L band than the current 20 MHz band already protected for GPS use. Of course by utilizing a much greater spreading, the signal will be better able to withstand narrow band interference.

If we are going to look for amplitude modulation of the received GPS signal, the characteristics of the transmit filter are important. Since the source of the '3rd Order Dispersion' is proportional to δf^2, it is important to have as wide a spectrum transmitted as possible. Modeling errors of the transmit filter may lead to errors in the estimated ionosphere.

Since the original specifications were set forth for the Global Positioning System, digital electronics have undergone tremendous reductions in size and price. Technologically it would be quite straightforward to build a receiver capable of tracking such a fast code.

5.1 FREQUENCY ALLOCATION

One of the biggest issues that might hinder the development and deployment of GNSS2 with very fast code (40-100 Mbps) is the limited available Electromagnetic Spectrum. Spectrum allocations are governed internationally by the ITU, and domestically by the Federal Communications Commission (FCC).

Ideally, the carrier frequency would still be quite near L1 (and GLONASS) to permit some hardwareheritage in newer receivers. Also by working in the same part of the spectrum, it would be easier to design hybrid receivers for the transition market between GPS/GLONASS and GNSS2

While 40-100 Mbps is a substantial portion of the spectrum, satellite TV uplinks and downlinks currently employ bandwidths up to 40 MHz. The advantages of Code Division Multiple Access require only a single such band to be allocated for GNSS2.

6 CONCLUSIONS

The previous graphs show that the transmission of the GPS signal through the ionosphere introduces some changes in both amplitude modulation and phase modulation. While these changes are quite small for both C/A Code and P Code, increasing the chipping frequency does produce significant variations in both amplitude and phase of the received signal. Adoption of a substantially higher (40-100 Mbps) code chipping frequency might permit real-time single frequency measurements of the ionosphere. A single frequency receiver for measuring the ionospheric group delay, would be simpler than a dual frequency receiver, and would not require calibration of the interfrequency bias (Tgd). Faster codes in conjunction with longer codes would reduce code measurement noise and reduce carrier cycle ambiguity.

7 ACKNOWLEDGEMENTS

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